SIGN RISK-SHARING CONTRACT WITH CONTRACT MANUFACTURING ORGANIZATIONS IN PHARMACEUTICAL INDUSTRY

A Thesis in
Industrial Engineering and Operations Research
by
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ABSTRACT

Outsourcing risks problem for the new ethical drugs manufacturing and commercialization is one of the most challenging problems for the pharmaceutical firms due to the big uncertainty of the FDA testing result, fluctuating market performance, changing government and finance environment. Motivated by the need of the ethical drugs risks sharing in the pharmaceutical industry, we are introducing a finite period analysis based on three different types of contracts which distinguished by the level of risk sharing including price discount, quantity flexibility and forecasting methods. These contracts are short term contract, long term time flexible contract and long term time inflexible contract. In order to analyze the performances of risk sharing of those contracts, we use mathematical functions to express the price discount, quantity flexibility and the demand risk and put them into the model of total extra outsourcing cost. By successfully use the concept of the Leibnitz’s Rule and newsvendor model, we successfully simplified the problem and classified the level of risk sharing for each contract, and also realized the complexity for those contracts. At the end of this thesis, we use a numerical analysis to give an introduction of how our model could be used in the contract selection. A quantitative solution is followed after that introduction and will select the best contract strategy under certain circumstance. The purpose of this thesis is to generate cost functions for the contracts, and help the firm to select the best contract strategy under different circumstances.
# TABLE OF CONTENTS

LIST OF FIGURES ...................................................................................................................... vi
LIST OF TABLES .......................................................................................................................... vii
ACKNOWLEDGEMENT ................................................................................................................. viii

CHAPTER 1 INTRODUCTION ....................................................................................................... 1

CHAPTER 2 LITERATURE REVIEW ............................................................................................ 7

  2.1 Risk management in outsourcing ........................................................................................ 7
  2.2 Risk sharing contract ........................................................................................................... 8
  2.3 Piecewise Price mechanism ................................................................................................ 9
  2.4 Newsvendor Model ............................................................................................................ 10

CHAPTER 3 METHODOLOGY ...................................................................................................... 11

  3.1 Introduction ......................................................................................................................... 11
      3.1.1 Price Sharing ................................................................................................................ 11
      3.1.2 Quantity Flexibility ..................................................................................................... 12
      3.1.3 Demand Probability .................................................................................................... 13
      3.1.4 Probability of the risk ................................................................................................. 15
  3.2 The simple case-Short-term contract .................................................................................. 15
  3.3 Long-term time-flexible contract ....................................................................................... 17
      3.3.1 Forecasted demand ...................................................................................................... 18
      3.3.2 Cost function ............................................................................................................... 19
  3.4 Long-term time-inflexible contract ...................................................................................... 20
      3.4.1 Forecasted demand ...................................................................................................... 21
      3.4.2 Quantity flexibility ..................................................................................................... 21
      3.4.3 Cost function ............................................................................................................... 21
  3.5 Special case ........................................................................................................................ 22
      3.5.1 Phase III clinic trial .................................................................................................... 22
      3.5.2 FDA review: .............................................................................................................. 23
  3.6 Main Algorithm ................................................................................................................... 24

CHAPTER 4 ANALYSIS ................................................................................................................ 27

  4.1 Optimal policy for the short term contract ......................................................................... 27
  4.2 Optimal policy for the long term time flexible contract .................................................... 28
  4.3 Optimal policy for the long term time inflexible contract .................................................. 31
CHAPTER 5 NUMERICAL STUDIES .......................................................... 34
  5.1 Parameter setting ........................................................................ 35
  5.2 Analytical calculation ................................................................. 36
    5.2.1 Short term contact ............................................................... 36
    5.2.2 Long term time flexible contract ....................................... 38
    5.2.3 Long term time inflexible contract .................................... 39
    5.2.4 Analytical calculation comparison .................................... 40
  5.3 Monte Carlo simulation ............................................................. 41
  5.4 Numerical study conclusion ...................................................... 44

CHAPTER 6 CONCLUSION ................................................................... 46
References ..................................................................................... 48

APPENDIX A ................................................................................. 50
APPENDIX B ................................................................................. 58
LIST OF FIGURES

Figure 1 Changing trend of drug manufacturing outsourcing [19] ................................................. 2
Figure 2 CMO market worth from 2008 to 2018 ($ billion) .......................................................... 3
Figure 3 CMO market worth in different markets from 2011 to 2017 ($ billion) ............................. 4
Figure 4 Process chart of the time inflexible contract ...................................................................... 25
Figure 5 Process chart of the time flexible contract ....................................................................... 26
Figure 6 Forecasted order quantity of the short term contract ...................................................... 37
Figure 7 the expected cost of the short term contract under analytical calculation ...................... 38
Figure 8 the optimal order quantities of the time inflexible contract .............................................. 39
Figure 9 the expected cost of time inflexible contract under analytical calculation ....................... 40
Figure 10 the expected costs of different contracts under analytical calculation ............................ 41
Figure 11 the expected cost of time short term contract under simulation ...................................... 42
Figure 12 the expected cost of time flexible contract under simulation ......................................... 42
Figure 13 the expected cost of time inflexible contract under simulation ......................................... 43
Figure 14 comparison between mean and SD of the expected cost under simulation .................... 44
LIST OF TABLES

Table 1 Table of Parameters for short term contract .................................................. 16
Table 2 Table of Parameters for long term time flexible contract ................................. 17
Table 3 Table of Parameters for time inflexible contract ................................................ 20
Table 4 Parameter Values in Numerical Analysis .......................................................... 36
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CHAPTER 1  INTRODUCTION

The globalization has changed the appearance of the world economic structure. Firms, especially for pharmaceutical firms, are facing unprecedented competition inside or even outside their industry. Everyone is now trying to reduce its cost, maximize efficiency and at the same time, develop a new product as quickly as possible. [1] This situation has pushed the pharmaceutical firms to outsource their noncore business functions and product to a third-party service or manufacturing organization. Therefore, firms could concentrate on the new drug development and let the rest of the work done by their contract development organization (CRO) and contract manufacturing organization (CMO).

Organization which receives the outsourcing contract is called the Contract Manufacturing Organization (CMO). It usually serves the pharmaceutical firms and provides its client with comprehensive services including manufacturing, sometimes drug development. CMO is used in the whole life cycle of drugs from the FDA clinical trial phases to the commercial production period.
As we know, in recent years, pharmaceutical firms are increasingly outsourcing their internal functions such as management, QA or purchasing in order to free up resources that could be put into effective use for other purposes such as finding new biological target, research and development. Since manufacturing requires the firms to run their own capacities, which will distract the firm from developing and other core business, more and more pharmaceutical firm choose to outsource their manufacturing functions. Compare with traditional Pharmaceutical firms, which are usually fully integrated, they run the in-house research, development, manufacturing, clinical and everything else by themselves. A recent survey indicated that

Figure 1 Changing trend of drug manufacturing outsourcing [19]
pharmaceutical firms are increasingly outsourcing their manufacturing functions to CMOs, both in the commercial period and clinic trial phrases, during most recently 5 years.

![CMO market worth from 2008 to 2018 ($ billion)](image)

**Figure 2 CMO market worth from 2008 to 2018 ($ billion)**

The market size of CMO is also increasing. Based on the historical data and the prediction, the CMO market worth was $21.2 billion in 2008 and will rise up to more than $60 billion in the year 2018. The biggest emerging markets are India and China. According to the most recent research, China and India will have an annual increase rate 17.4% and 16.9% respectively from year 2011 to 2017, increasing their market share to 8% and 7% compared to their recent market share 6% in 2011. Meanwhile, since the contracted manufacturing is a growing market, although the market share for both USA and Western Europe will decrease, their market value will still increasing with an annual rate 10.9% and 8.9% [18].
However, although outsourcing is a good way to reduce unnecessary cost, some extra costs will be generated due to the risks of outsourcing. These risks are the quality risk due to bad performance of the CMOs, also risks in economic, new competitors, exchange rate and hyperinflation, demand and capacity uncertainty risk. These risks can lead to extra cost in many aspects and finally put firms into crisis. What’s more, in the real world, not only the pharmaceutical firms suffer these risks, but also the CMOs. Hence, finding a way to share the risk has become one of the most important considerations when pharmaceutical firms and CMOs, are making strategic decisions. The purpose of this thesis is to introduce several single-source outsourcing strategies and meanwhile help the firm to find the best strategy under different circumstances.

Figure 3 CMO market worth in different markets from 2011 to 2017 ($ billion)
Our study is focusing on the periods from the date that the drug starts the phase III clinical trial to the end of its patent protection years. Before the phase II clinical trial, the drug is under testing in small groups, which include people from 20-80 in phase I to 100-300 in phase II. Because the quantity is not large, so there is no reason to outsource with CMOs. And also, the approval rate for the clinical trial in phase I and phase II are not high, with 60%-70% for phase I [2], and 20%-35% for phase II [3]. And after the patent is expired during the patent protection years, lower-price generics would quickly siphon off almost 90% of its sales. Although it will benefit the most of the customers, for the original pharmaceutical firm, the better decision would be to stop manufacturing and selling the drug and send new drug into the market as soon as possible. As a result, since the patent protection years only last for 20 years, we take almost 7 years of the clinic trial into consideration; we will take a planning horizon with the length of 14 periods, which include the years before the drug is finally approval from the phase III clinic trial.

Since the purpose of our study is to share the risk between the pharmaceutical firm and its CMO to minimize their probability of losing money and maximize their profits together, we needed to identify what risks will occur during the contracted period and what effects they would create. The first risk would be the public relations (PR) crisis risk. One of the most famous examples is Johnson & Johnson’s Tylenol crisis happened in 1982; 7 people died as a result. This PR crisis have caused $1 billion loss of market value for the firm, and Tylenol’s market share fell to 7% from originally 35% in the US market [4]. The second one would be a risk for the government policy. In 2008, acetonitrile was suffered a worldwide shortage, because its main supplier China was busily involved in hosting the Olympics Games and its government want to minimize air pollution, the factories producing acetonitrile were asked to close temporarily [5]. Also, other risks as we have already mentioned before such as quality risk, demand and capacity risk will
cause great effect on the beneficial result for both pharmaceutical firm and its CMOs. It is obviously that, in the supply chain level, we cannot control the probability for the occurrence of the risk; however, we can build a contract to minimize the loss when the risks happen in a supply chain level.

Chapter 2 of this thesis presents a detailed review of current researches in risk-sharing strategies, by first examining the key factors that are needed to be considered in a risk sharing contract, and afterwards discussing the long term contract and short term contract. Cost models will be demonstrated in Chapter 3 using the risk sharing functions such as unit price sharing, quantity sharing, etc. We then generate the cost model for different types of contracts separately. In Chapter 4, several methods, for example, a bisection method, will be used to simplify the models we have generated in Chapter 3. Once the models are simplified, we can easily solve the optimal cost for each contract under different circumstances. Chapter 5 is a numerical study that shows how the solving and contract selecting process works. And the last Chapter will include the conclusion and suggestions that consider the real world constraints.
CHAPTER 2    LITERATURE REVIEW

2.1. Risk management in outsourcing

Risk management is the identification, assessment, and prioritization of risk followed by coordinated and economical application of resources to minimize, monitor, and control the probability and impact of unfortunate events and to maximize the realization of opportunities [15]. It has always been heat topics in supply chain management, especially in the outsourcing industry, which have reached the value of $6 trillion worldwide, depend on research in 2004. [6]

According to research done by C.K.M. Lee, Yu Ching Yeung and Zhen Hong, the main three important outsourcing risks are transportation risk, inventory risk and demand risk, based on qualitative risk assessment method named SCR-failure mode and effect analysis. They calculated the Risk Priority Number (RPN) and Risk Score, and then use risk map to represent the risk level for each factor to make their conclusion. Also, they have generated a quantitative risk assessment method by using Monte Carlo simulation to examine the total estimated customer lead time and total estimated customer supply chain cost.

Timothy G.Fowler and Eirik Sorgard have concentrated in the transportation risk[9], so did some other researchers have concentrated their attentions in the inventory risk and demand risk area. Those methods could make contributes in the optimization of the risk and control it. However, only consider in one area will not help the whole outsourcing system to be optimized. In 2006, Emanuele Padovani and David Young demonstrated three perspectives that could be used to identify the risk in a potentially outsourcing activity, which are citizen sensitivity, the supplier market and the costs of switching.

Later in year 2009, Liu Qinghua and Xu Zhongwei studied outsourcing risk management from the perspective of the value network, where value network is a kind of structure of value creation, assignment, shift and utilization, which is formed by stakeholders influencing mutually.
[7] Ravi Aron, Erick Clemons and Sashi Reddi have discussed the just right outsourcing in 2005. In their perspective, firm should not outsource their product as much as possible but to achieve the very best long-term risk-adjusted rate of return [16]. However, it cares much in the facilitate outsourcing by redesign the process of outsourcing.

2.2 Risk sharing contract.

Konstantinos Serfes had discovered in 2005 that, risk can be important in contract design although some research before him had concluded that the market data unveil a positive relationship between risk and incentives.[8] There are different kinds of contracts style that firms could choose when they are starting a negotiation with CMOs.

Li and Kouvelis have mentioned two different kinds of supply contracts, time-inflexible contract and time-flexible contract [10]. A time-inflexible contract requires firm to specify how many units it intends to purchase and the exact time the units will be purchased in the future, while a time-flexible contract requires firm to specify at time 0, but it does not require the firm to specify the exact purchase time. Moreover, in risk sharing contract, quantity flexibility is required to reduce the risk of demand decrease. A supply contract with the quantity flexibility property can be defined as follows: the firm signs a contract with a supplier which the order quantity is Q, and the contract has quantity flexibility index α, where $0 \leq \alpha \leq 1$. Then it is possible for the firm to purchase a total quantity x less than or more than Q, where $(1 - \alpha)Q \leq x \leq (1 + \alpha)Q$. If $\alpha = 0$, we call the contract a quantity inflexible contract.
2.3 Piecewise Price mechanism

Piecewise price mechanism is one of the key parts in the risk-sharing structure. Since in the real world the drug demand depended on many factors, part of them can be predicted but the rests are hard to be forecasted. By setting a good piecewise price mechanism, can allow both supplier and pharmaceutical firm bear little price risks and maximize the profit for both of them.

Xi Jia, Qing Xia, Qixin Chen had proposed a piecewise price mechanism for the Smart Grid construction. They divided the system load into several intervals, and defined the piecewise points as the interval demarcation parameters. The values of the interval demarcation parameters are depend on the reshaping target of the system load curve and the predicted consumers’ price demand. They have compared different modes of price-based DR, TOU, CPP and RTP, and finally conclude that the best price strategy is to share the price risks between generators, transmission operators and consumers. [11]

Li and Kouvelis have proposed one of the significant risk-sharing price models in 1999. Supplier charge for $g(p)>0$ dollars per unit of product from the firm, where $g(p)$ is a nondecreasing function. They defined the function as:

$$g(P) = \begin{cases} 
P - \lambda (P - \overline{P}) & \text{if } P > \overline{P}, \\
P & \text{if } \overline{P} \leq P \leq \overline{P}, \\
P + \lambda (\overline{P} - P) & \text{if } P < \overline{P},
\end{cases}$$

$P$ actually is the unit price followed by the market discipline. $\lambda \in [0, 1]$ represents the level of price risk the supplier is going to share with the firm. And $\overline{P}, \overline{P}$ are the upper and lower bound of the nonrisk sharing price. This method is very popular in handling exchange rate risk. In our
thesis, we will use this formula in a demand from that to share the price risk caused by the demand uncertainty. [10]

2.4 Newsvendor Model

In our model, we have assumed that the inventory does not hold from one period to the next. Therefore, although we have defined holding cost here in our thesis, it is more precisely a cost per unit left over after demand is realized. It means that the holding cost is the holding and disposal cost of the drug. In this case, the multi-period lot size model will no longer available to be used in this single period. Thus, we are introducing the newsvendor model in this situation. The newsvendor problem, also known as t single-period problem (SPP), is always used by firms to find out order quantity which maximized the expected profit in a single period probabilistic demand framework [12]. Moutaz introduced a classical single-period problem, which evaluate the maximize expected value of the profit. He also mentioned that both approaches that minimize the expected cost or maximize the profit could yield the same results [14]. Jucker considered three types of quantity discounts, one of them is all unit quantity discount while using newsvendor model to decide the maximize profit [13].
CHAPTER 3 METHODOLOGY

3.1 Introduction

3.1.1 Price Sharing:

Whatever kinds of risk, it finally shows in the form of demand variance or price variance. Therefore, in our long-term contract risk-sharing model, we define the firm pay the supplier $g(D)$ for each unit of drug. For a non-risk sharing contract, the $g(D)$ may always equal to a constant value, say $P$ for all $P>0$. However, in more complicated pricing and contracting strategy, the price could be shown as follow:

$$g(D) = \begin{cases} 
P - \lambda d(D - \bar{D}) & \text{if } D > \bar{D}, \\
P & \text{if } \bar{D} \leq D \leq \bar{D}, \\
P + \lambda d(\bar{D} - D) & \text{if } D < \bar{D},
\end{cases}$$

We define the $d(\Delta D)$ as the influence function during the demand change to the price change. $\lambda \in [0, 1]$ represents how much risk the supplier is going to share with the firm. The term price sharing also refers to quantity discount in many related studies. The most popular three of the quantity discount methods are all unit quantity discounts, incremental-quantity discounts and Carload-Lot discounts, the difference between these three methods is whether the discounts could adapt to the total quantity or part of it[17]. While the all unit quantity discounts method reduce the price for all the quantities, the other two methods discount on the price for the part of the quantity when quantity reaches the discount point. Here in our study, we only concern all unit quantity discount situation.

For the short-term contract, the fluctuation of price and demand will not be huge within a single year. Risk sharing can be included in the period of contract negotiation, thus, the price could vary from year to year, but should be stable in every single year.
3.1.2 Quantity Flexibility

Quantity flexibility is a contract will either reduce the CMO’s extra cost risk caused by the uncertainty of demand or reduce the firm’s penalty cost as much as possible. In the case when demand reduced unexpectedly, the reason for that may be a serious Public Relation crisis or trade barriers incident, set a lower bound for the order quantity ahead of time when contract was signed will make sure that the CMO will as least sell a certain quantity of product to the firm. On the contrary, while demand increase more than expected, a risk-sharing contract will offer extra quantity to the firm with a planned quantity upper bound.

For the risks we have mentioned before, the long-term contract may better reduce the risk of the Exchange rate and Hyperinflation. We assume there are two kinds of long term contract in the thesis. One is the time-inflexible contract; this contract requires the firm to specify how many units it intends to purchase and when those units will be purchased in the future. The other kind is time-flexible contract, firm specify the units it intends to purchase in the future, but it does not require the firm to specify the actual purchase time. We define a Quantity interval $[-\alpha, \alpha]$, in which actual order quantity $x$ units could have a slight difference with the contracted Quantity $Q$, $(1 - \alpha)Q \leq x \leq (1 + \alpha)Q$. The actual quantity flexibility strategy in different types of contracts will be discussed in the following sections.

The quantities ordered more than the demand incurs a unit holding and disposal cost $h$. Since in the drug model, we suppose that the drug can only last for one period, and expired in the next period, so the holding cost $h$ can also be the disposal cost. Because of the same reason, we will use newsvendor model in our contract model. We also assume lost sale cost for unsatisfied demand $p$, when the demand is more than the order quantities. A relatively high service level is also required to minimize the cost of a lost sale. Our final objective is to minimize the cost.
3.1.3 Demand Probability

Empirical evidences show that the demand of newly introduced drug is stochastically increasing. And during the first decade the growth is nearly linear. Patent expiration will lead to the slowing down and decreasing of the demand. Moreover, the ethical drugs market is a high viscous market; prescribers are not willing to change from one drug to another on the same indication. Only when the patient asks for or some accidents make the drug in a very bad reputation, prescribers will then start to give alternated drug to the patient. In the meanwhile, the demand for the original drug will decline due to the action of the prescribers. In conclusion, prescribers of a drug are more willing to be a long-term user unless there is a certain event changed the patients’ mind. Therefore, we define the process as a discrete-time Markov process, \{D_t, 3 \leq t \leq N\}.

Initially, we denote the demand in period 3 as \(D_3\), with its mean \(\mu_3 > 0\) and standard deviation equal to \(\sigma_D\). Then we denote \(X_n^D \sim N(\mu_D, \sigma_D)\) in this thesis where \(X_n^D\) is identical and independent distributed variable representing the net change of sales in the future periods, \(\mu_D, \sigma_D\) can be the growth rate and volatility, respectively. Therefore, \(D_{3+t} = D_3 + \sum_{n=1}^{t} X_n^D\)

For:

\[D_3 \sim N(\mu_3, \sigma_D)\]

\[X_n^D \sim N(\mu_D, \sigma_D)\]

Hence:

\[D_{3+t} \sim N(\mu_3 + \mu_D, \sqrt{t + 1}\sigma_D) = N(\mu_{3+t-1} + \mu_D, \sqrt{t + 1}\sigma_D)\]

Normally, the demand is distributed like the form we mentioned above. However, as we mentioned before, the public relationship have a huge effect on the demand of the product of the
firm. Sometimes the effect will act only on a single drug, for other situation, the whole firm will be affected.

Assume the information require a period to spread. In that case, once the public relationship problem happens at the period $2+t$, demand in the next period $3+t$ will be:

$$D_{3+t} \sim N(P_{2+t}(\mu_3 + t\mu_D), \sqrt{P_{2+t}(t + 1)}\sigma_D)$$

Suppose $P_{2+t}$ is the decline rate of the demand, which mean that once the public relationship problem happens, the demand will decline with the rate $P$ in the next period. Therefore, the expected drug demand for the next period will be $P_{2+t}(\mu_3 + t\mu_D)$, and the variance of demand will change either.

Due to the property of Markov chain, the demand for the next period is only affected by the demand in the current period. Therefore, after the period of demand decline, the demand will increase again with the rate $\mu_D$, which would be

$$D_{3+t+1} \sim N(P_{2+t+1} \left[ P_{2+t}(\mu_3 + t\mu_D) + \mu_D \right], \sqrt{P_{2+t+1} \left( P_{2+t}(t + 1) + 1 \right) \sigma_D})$$

Therefore, in general

$$D_{t+1} \sim N(P_t \mu_{t+1}, \sqrt{P_t \sigma_{t+1}})$$

Where

$$\mu_{t+1} = P_{t-1}\mu_t + \mu_D$$

$$\sigma_{t+1}^2 = P_{t-1}V_t + V_D$$
We can treat the demand process as a discrete-time Brownian motion (DTBM) with a positive drift rate. DTBM is a great estimator in the approximation of the drug demand, which have a linear growth rate. The only difference between our model and the DTBM motion is we have added a PR-crisis risk probability in the model. Every time the PR-crisis happened in the demand process with a certain probability, the demand in the next period will reduce by a certain rate.

3.1.4 Probability of the risk

As it has been mentioned in the previous section, $P_t$ is the decline rate of the demand when the public relationship problem happens. Thus, there should have a probability distribution for the incidence of the public relation problems. Once the problem happened, a certain rate $\rho$ will be given to $P_t$ and let the mean of the demand reduce with it; if the problem is not happening, $P_t$ will be 1, and the demand will not be changed.

The probability for the incidence of the problem (notated as $R_t$) will get from the statistics historical data of the familiar product. We set that is $b$ in our model, and $b$ follow a Bernoulli distribution, therefore, $R_t \sim Bernoulli(b)$ and $R_t \in \{0, 1\}$.

$$P_t = \rho^{R_t}$$

If $R_t = 0$, $P_t = 1$, it means the public relationship crisis will not happen, otherwise $R_t = 1$, based on the equation, $P_t = \rho$, which means the demand in the next period will reduce with the rate $\rho$.

3.2 The simple case-Short-term contract

The traditional way to handle risks is to sign a short-term contract. The short-term contract we defined here is one-year contract with one-year lead-time. Once the contract decision for the short-term contract is made, firm will wait one year for the orders to be fulfilled. And at the
beginning of the next year, firm should make forecast the demand for the year after. Therefore, the firm will be able to better deal with the demand uncertainty risk since the outsourcing decision is made every year. Also, the economic risk for the CMOs will be reduced. However, since the contract is signed every year and may not be the same CMOs, the possibility of leaking information and losing proprietary product will increase. What’s more, once the exchange rate increase, a new contract will cost more due to this change. The variables and parameters we are going to use here are listed below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>Demand quantity, a random variable in period t</td>
</tr>
<tr>
<td>$f(x_t)$</td>
<td>The probability density function of $x_t$</td>
</tr>
<tr>
<td>$F(x_t)$</td>
<td>The cumulative distribution function of $x_t$</td>
</tr>
<tr>
<td>$p$</td>
<td>Understock unit cost, or penalty cost</td>
</tr>
<tr>
<td>$h$</td>
<td>Holding cost, or disposal and overstocking cost</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>Order quantity, a decision variable at period t</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Demand reduce rate in period t, equal to P or 1</td>
</tr>
</tbody>
</table>

Since the contract is making every year, so that $Q_t$ only depends on the forecasted demand. Also, the price of the drug will be constant here because both CMOs and Firm will not willing to sign a risk-sharing price for a single-year contract. The cost per period is:

$$C(Q_t) = \begin{cases} p(x_t - Q_t) + c Q_t, & \text{if } x_t > Q_t \\ h(Q_t - x_t) + c Q_t, & \text{if } Q_t > x_t \end{cases}$$

We can simplify and take the expected value of $C(Q_t)$:

$$E[C(Q_t)] = \int_{Q_t}^{\infty} [p(x_t - Q_t) + c Q_t] f(x_t) dx_t + \int_{0}^{Q_t} [h(Q_t - x_t) + c Q_t] f(x_t) dx_t$$  \hspace{1cm} (1)
Hence, for the total 14 periods:

\[ \mathbb{E}[C] = \sum_{t=1}^{14} \mathbb{E}[C(Q_t)] \]  

(2)

With one period lead-time, the order decision is made every year. Thus, the firm’s decision at period \( t \) is the quantity of drug \( Q_t \). To optimize the model and get the minimized cost, we need to use the newsvendor model and treat \( Q_t \) as the decision variable, and find out the optimal value of the cost.

3.3 Long-term time-flexible contract

By analyzing the short-term contract, we find that although by sign the contract every year could perfectly handle the risk of demand uncertainty, it is not cost efficiency in a more volatility situation. With time-flexible contract, the firm is required to specify the demands at time 0, but it does not require the firm to specify the exact purchase time. After that, the drug was sent to FDA to take the phase III clinic trial test and FDA review. If the drug has passed both of them, the firm will start to commercialize the drug and put it into the market. The first year of commercialization would be either surplus or shortage under a normal, predictable demand. But after that year, a PR-crisis will occur with a rate \( b \) and would reduce the demand with a rate \( \rho \) once it happens. However, due to the exact purchase time is not specified at beginning, the exact order quantity could be as much as possible. The variables and parameters we are going to use here are listed below:

<table>
<thead>
<tr>
<th>( x_t )</th>
<th>Quantity demanded, a random variable in period ( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X )</td>
<td>( X = \sum_{t=1}^{14} x_t )</td>
</tr>
</tbody>
</table>
Forecasted demand in period $t$  

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>Forecasted demand in period $t$</td>
</tr>
<tr>
<td>$S$</td>
<td>A decision variable that should be optimized in this problem</td>
</tr>
<tr>
<td>$f(x_t)$</td>
<td>The probability density function of $x_t$</td>
</tr>
<tr>
<td>$F(x_t)$</td>
<td>The cumulative distribution function of $x_t$</td>
</tr>
<tr>
<td>$p$</td>
<td>Understock unit cost, or penalty cost</td>
</tr>
<tr>
<td>$h$</td>
<td>Holding cost, or disposal and overstocking cost</td>
</tr>
<tr>
<td>$g(X)$</td>
<td>Function of unit price</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Demand reduce rate in period $t$, equal to $\rho$ or 1</td>
</tr>
<tr>
<td>$R_t$</td>
<td>Binary variable following the Bernoulli that determine the PR crisis</td>
</tr>
<tr>
<td>$Q(t)$</td>
<td>Actual order quantity at time $t$</td>
</tr>
</tbody>
</table>

3.3.1 Forecasted demand

As we have mentioned before, the order quantity should be optimized in this problem. Let the matrix $F = \{y_1, y_2, \ldots, y_t, \ldots, y_n\}$. $S$ is the total quantities in the contract, which $S = \sum_{t=1}^{n} y_t$ was determined at the time the contract was signed.

As the time-flexible property has been demonstrated, the total order during $n$ years could not be more than $(1 + \alpha)S$ or less than $(1 - \alpha)S$. The advantage of this contract is that the pharmaceutical firm could easily handle the variance of the demand and minimize the loss from their CMO while the backlog cost is high and the variance of the demand is very large.

Due to the constraint of the quantity sharing, the price share function need to be modified by the upper and lower bound of the quantity, therefore, the unit price function would be. In order to make the price sharing function work, we set the transaction completion data at the end of the periods.
As we already assumed above, the demand in every period is followed by the normal distribution with a DTBM characteristic, hence we can also assume that \( X \) follow the following distribution.

\[
X \sim (12\mu_t + 66\mu_d, \sqrt{12\nu_t + 66\nu_d})
\]

### 3.3.2 Cost function

The state of the system in the period \( t \) is then defined as \((x_t, Q(t), g(X))\), the purchasing price has been defined follow the fluctuation of \( X \), so that the purchasing price will be related to the value of \( X \) and \( S \). The only decision that should be made for the time-flexible situation the total quantity \( S \) and this \( S \) needed to be decided at the time the contract was signed.

Since we take the whole 14 processes into consideration in this situation, the cost function below will evaluate the cost of the 14 periods as a whole instead consider it separately in each period.

\[
C(S) = \begin{cases} (1 + \alpha)Sg(X) + p[\sum_{t=1}^{14} x_t - (1 + \alpha)S], & X \geq (1 + \alpha)S \\ Xg(X), & (1 - \alpha)S \leq X \leq (1 + \alpha)S \\ (1 - \alpha)Sg(X) + h[(1 - \alpha)S - \sum_{t=1}^{14} x_t], & X \leq (1 - \alpha)S \end{cases}
\]

(3)

Assume \( \sum_{t=1}^{14} x_t = X \) Thus:

\[
E[C(S)] = \int_{(1+\alpha)S}^{\infty} ((1 + \alpha)Sg(X) + p[X - (1 + \alpha)S])f(X)dX + \int_{(1-\alpha)S}^{(1+\alpha)S} Xg(X)f(X)dX + \int_{0}^{(1-\alpha)S} \{(1 - \alpha)Sg(X) + h[(1 - \alpha)S - X]}f(X)dX
\]

(4)
The decision is made at the time when contract was signed, thus the firm need to decide how many drugs it want to order for the coming 14 periods. The optimal solution would minimize the total cost for the contract.

3.4 Long-term time-inflexible contract

With the property of time-inflexible, the decision will be made at the beginning of the every period when the contract had been signed. Hence the firm’s decision would be $F = \{Q_1, Q_2, \ldots, Q_t, \ldots, Q_n\} = \{y_1, y_2, \ldots, y_t, \ldots, y_n\}$. After that, the drug was sent to FDA to take phase III clinic trial test and FDA review. If the drug has passed both of them, the firm will start to commercialize the drug and put it into the market. The first year of commercialization would be either surplus or shortage under a normal predictable demand. But after that year, a PR-crisis will occur with a rate $b$ and would reduce the demand with a rate $\rho$ once it happens. The variables and parameters we are going to use here are listed below:

Table 3 Table of Parameters for time inflexible contract

<table>
<thead>
<tr>
<th>$x_t$</th>
<th>Quantity demanded, a random variable in period $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_t = y_t$</td>
<td>Forecasted demand in period $t$, the decision variable here</td>
</tr>
<tr>
<td>$f(x_t)$</td>
<td>The probability density function of $x_t$</td>
</tr>
<tr>
<td>$F(x_t)$</td>
<td>The cumulative distribution function of $x_t$</td>
</tr>
<tr>
<td>$p$</td>
<td>Understock unit cost, or penalty cost</td>
</tr>
<tr>
<td>$h$</td>
<td>Holding cost, or disposal and overstocking cost</td>
</tr>
<tr>
<td>$g(X)$</td>
<td>Unit price</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Demand reduce rate in period $t$, equal to $P$ or $1$</td>
</tr>
<tr>
<td>$R_t$</td>
<td>Binary variable following the Bernoulli that determine the PR crisis</td>
</tr>
<tr>
<td>$Q(t)$</td>
<td>Actual order quantity at time $t$</td>
</tr>
</tbody>
</table>
3.4.1 Forecasted demand

We have mentioned that the time-inflexible contract requires firm to specify how many units it intends to purchase and the exact time the units will be purchased in the future. Thus, firm need to specify the order quantity before each period. Matrix of forecasted demand for n years is shown below; assume that the annual demands are forecasted by certain forecasting strategy. So that $F = \{Q_1Q_2, ..., Q_t, ..., Q_n\} = \{y_1y_2, ..., y_t, ..., y_n\}$. Here, $y_t$ is determined at the time the contract signed.

3.4.2 Quantity flexibility

In the time-inflexible contract, the forecasted demands have been determined before each period. Thus actual capacity $Q_t$ is depended on the actual demand, forecasted demand and the quantity index $\alpha$:

$$Q(t) = \begin{cases} 
  x_t & \text{if } (1 - \alpha)Q_t \leq x_t \leq (1 + \alpha)Q_t, \\
  (1 + \alpha)Q_t & \text{if } x_t \geq (1 + \alpha)Q_t, \\
  (1 - \alpha)Q_t & \text{if } x_t \leq (1 - \alpha)Q_t,
\end{cases}$$

Same with the time flexible contract, since the quantity is flexible, there should have an upper and lower bound for the discount price strategy. The unit price model should be:

$$g(x_t) = \begin{cases} 
  P - \lambda((1 + \alpha)Q_t - \bar{D}) & \text{if } x_t > (1 + \alpha)Q_t, \\
  P - \lambda(x_t - \bar{D}) & \text{if } \bar{D} \leq x_t \leq (1 + \alpha)Q_t, \\
  P & \text{if } x_t \leq \bar{D}, \\
  P + \lambda(\bar{D} - x_t) & \text{if } (1 - \alpha)Q_t \leq x_t \leq \bar{D}, \\
  P + \lambda(\bar{D} - (1 - \alpha)Q_t) & \text{if } x_t < (1 - \alpha)Q_t.
\end{cases}$$

3.4.3 Cost function
Set the backlog cost is \( p \), inventory holding cost is \( h \). Here, since the product has timeliness property, we define the overproduction could not hold until next year. Hence, the holding cost here is representing the holding and disposal cost. Demand function for each year: \( D_t \) was given in section 3.1. Therefore, the minimum expected total cost from period \( t \) to the end of the planning horizon should be \( C(Q_t) \), which is:

\[
C(Q_t) = \begin{cases} 
  p[x_t - (1 + \alpha)Q_t] + g(x_t)(1 + \alpha)Q_t, & \text{if } x_t \geq (1 + \alpha)Q_t \\
  g(x_t)x_t, & \text{if } (1 - \alpha)Q_t \leq x_t \leq (1 + \alpha)Q_t \\
  h[(1 - \alpha)Q_t - x_t] + g(x_t)(1 - \alpha)Q_t, & \text{if } x_t \leq (1 - \alpha)Q_t
\end{cases}
\]

Therefore, the expected cost will be:

\[
E[C(Q_t)] = \\
\int_{(1+\alpha)Q_t}^{\infty} [p[x_t - (1 + \alpha)Q_t] + g(x_t)(1 + \alpha)Q_t] f(x_t) dx_t + \int_{(1-\alpha)Q_t}^{(1+\alpha)Q_t} g(x_t)x_t f(x_t) dx_t + \\
\int_0^{(1-\alpha)Q_t} [h[(1 - \alpha)Q_t - x_t] + g(x_t)(1 - \alpha)Q_t] f(x_t) dx_t
\]

3.5 Special case

Sections 3.2, 3.3 and 3.4 have described the general situations when the drug has successfully passed the Phase III clinical trial and the FDA review. But in fact, the outsourcing decision usually is making at the period of Phase III clinical trial. We here assume the phase III clinical trial lasts one period and the probability of success is \( \gamma_1 \). If alive, the drug will next undergo the FDA review in period 2 and the success probability is \( \gamma_2 \). If the period 2 also alive, demand will starts in period 3 and the drug will be produced in the next \( N-2 \) periods until its patent expires at the end of period \( N \).

3.5.1 Phase III clinic trial
Demand $D_1 = 0$, Order quantity $Q_1 = 0$, the only cost is the contract sign up cost. We denote it as $A$ here.

Therefore, for the long-term contract, the expected cost from the first period is a set up cost $A$ plus the expected cost from the rest of the periods. Since the probability for the drug to pass the phase III is $\gamma_1$, the expected cost from the next period to the end of the patent expires should be $\gamma_1 E(\sum_{n=2}^{14} C(Q_n))$. So that if the drug successfully past the phase III clinical trial test, $C(Q_1, Q_2, Q_3, ..., Q_{14}) = A + E(\sum_{n=2}^{14} C(Q_n))$ , otherwise if the drug failed $C(Q_1, Q_2, Q_3, ..., Q_{14}) = A + \infty$, $\infty$ is the infinity value that will make this decision will not be chosen when the drug failed.

However, we have noticed the lead time for outsourcing is 1 period; therefore, for the strategy of short-term outsourcing, there is no setup cost at the first period.

3.5.2 FDA review:

Long-term time-inflexible contract: Compared to period 1, period 2 is relatively more complex. For the long-term time-inflexible contract, after the contract was signed in the 1$^{st}$ period, the contract manufacturing organization will start to produce a product in the 2$^{nd}$ period. As we have mentioned before, FDA will review the drug and finally decide whether to allow the drug to pass or not. The drug is not commercialized until period 3 after the FDA tests, thus the demand in period 2 will be 0. If the drug has failed in the FDA review with a rate $\gamma_2$, then the pharmaceutical firm will receive a penalty charging from the CMO, which is equal to the value of the forecasted the first month demand with a none risk sharing price. So $C(Q_2, Q_3, ..., Q_{14}) = PF_3 + \infty$, $\infty$ is the infinity value that will make this decision will not be chosen when the drug has failed in the FDA review.
1. Otherwise, no additional cost will be generated in the 2nd period. Hence the total cost from period 2 will be $C(Q_2, Q_3, ..., Q_{14}) = E(\sum_{n=3}^{14} C(Q_n))$.

2. Long-term time-flexible contract: if a drug fails to pass the FDA review under a time-flexible contract, although there is no detail demand forecasted in the certain month, a penalty will charge from the firm. The value will be the average total forecasted demand $S$ for every year after commercialized. So $C(Q_2, Q_3, ..., Q_{14}) = \frac{PS}{N-2}$. Otherwise, no additional cost will be charged at period 2 and $C(Q_2, Q_3, ..., Q_{14}) = E(\sum_{n=3}^{14} C(Q_n))$.

3. Short-term contract: In a situation when firm choose to outsource drugs in the short term, every time they decide to outsource in the next period, they spend a setup cost to make that order for the next period. Thus, no matter the drug succeed or failed after the FDA review, a constant setup cost $A$ will be charged from the firm. So $C(Q_2, Q_3, ..., Q_{14}) = A + E(\sum_{n=3}^{14} C(Q_n))$.

3.6 Main Algorithm

Based on the formulations in previous sections, the following implementation of the algorithm intuitively generates a long-term time inflexible contract by step. Assume the firm decides to sign a long term time inflexible contract with its CMO, and it has forecasted its demand during the next 14 period as $F = \{Q_1, Q_2, ..., Q_t, ..., Q_n\} = \{y_1, y_2, ..., y_t, ..., y_n\}$. As we know, the demand before the drug is commercialized is 0, so that $x_1, x_2 = 0$ in this case.
Figure 4 Process chart of the time inflexible contract

Step 1: Set $n = 1$, firm signed a contract with CMO and a setup cost was marked as $A$. Set value of $\gamma_1$, let the system have a probability $\gamma_1$ for the drug to pass the phase III review. If the drug fails to past the test, stop; otherwise go to step 2.

Step 2: Set $n = 2$, and set the value of $\gamma_2$, the drug will have a probability $\gamma_2$ to pass the FDA review test. If the drug fails to pass the test, stop; otherwise forecast the demand $Q_3$ and go to step 3.

Step 3: Set $n = n + 1$, $D_n \sim N \left( P_{n-1} \mu_n, \sqrt{P_n \sigma_n} \right)$. First, based on the quantity share formula and the forecasted demand, the firm is going to share the quantity with CMO. Hence the quantity $Q(n)$ will equal to either $D_n$ or $(1 + \alpha)Q_n$ or $(1 - \alpha)Q_n$. Then it going to share the unit price with the CMO, the price will be $g(x_n)$. Therefore, we can calculate the total price in period 3, then forecast the demand $Q_{n+1}$ and go to step 4.

Step 4: Still at period $n$. If $R_n = 0$, then go back to step 3 with $n = n + 1$. Otherwise, $R_n = 1$, go to step 5.
Step 5: Set \( n=4 \), since \( R_{n-1} = 1 \), then \( P_{n-1} = \rho^{R_{n-1}} \), so that the forecasted demand \( D_n \) will reduce with a demand reduce rate \( P_{n-1} \cdot D_n \sim N(P_{n-1}\mu_n, \sqrt{P_n\sigma_n}) \). In most situation, it will cause the demand unable to meet the least quantity sharing requirement, so that an over order will happen and firm will pay more to the CMO in order to reduce the CMO’s risk of lost sale. Then go back to step 4 with \( n=n+1 \).

Step 6: Repeat the process until \( n = 14 \).

The flow chart above intuitively shows the 14 periods during phase III clinical trial, FDA review and the rest of the periods after the drugs are commercialized.

The process of long-term time flexible contract shows little differences with the time inflexible contract; we have made the process chart below to show those differences:

![Figure 5 Process chart of the time flexible contract](image)
CHAPTER 4 ANALYSIS

In this section, we characterize the optimal policy for the short-term contract, long-term time flexible and long-term time inflexible contract.

4.1 Optimal policy for the short term contract

Consider Eq. (1) first, the following rule shows that, in each period t, the total order quantity $Q_t$ is depended on the penalty cost $p$ and the holding and disposal cost $h$. In this case, the firm can easily determine the order quantity each period depending on the estimated unit value of the penalty and holding and disposal cost. Let $Y(Q_t)$ be the expected value of penalty and disposal cost in period $t$ and $f(x_t)$ and $F(x_t)$ be the pdf and cdf of the demand normal distribution. To avoid triviality, we let $t>2$, otherwise no demand will occur on the drug before the drug receives the FDA approval.

Theorem 1. Leibnitz’s Rule:

\[
\frac{d}{dQ_t} \int_{f_1(Q_t)}^{f_2(Q_t)} f(Q_t, x_t) \, dx_t = \int_{f_1(Q_t)}^{f_2(Q_t)} \left[ \frac{d}{dQ_t} f(Q_t, x_t) \right] \, dx_t - f(Q_t, f_1(Q_t)) \frac{d}{dQ_t} f_1(Q_t) \\
+ f(Q_t, f_2(Q_t)) \frac{d}{dQ_t} f_2(Q_t)
\]

After adapting the Leibnitz’s Rule into Eq. (1), we got the optimal order quantity $Q_t^*$ with a probability $F(Q_t^*)$, where
\[ F(Q_t^*) = P(X \leq Q_t^*) = \int_{-\infty}^{Q_t^*} \frac{1}{\sqrt{2\pi} \sigma_t} e^{-\frac{(x_t - p_{t-1}\mu_t)^2}{2p_{t-1}\sigma_t^2}} \, dx_t = \frac{p}{h + p} \]

Using a standard format to represent this,

\[ \phi \left( \frac{Q_t^* - p_{t-1}\mu_t}{\sqrt{p_{t-1}\sigma_t}} \right) = \frac{p}{h + p} \]

Since we know the value of \( p_{t-1}, \mu_t, \) and \( \sigma_t, \) we can easily check the value of \( \frac{Q_t^* - p_{t-1}\mu_t}{\sqrt{p_{t-1}\sigma_t}} \) in the standard normal table by the value \( \frac{p}{h+p} \) and then solve the \( Q_t^* \).

So we can take the \( Q_t^* \) back to the formula, and get our minimum summation for the penalty cost and the disposal cost at period \( t \).

\[ Y(Q_t^*) = \int_{Q_t^*}^{\infty} [p(x_t - Q_t^*)]f(x_t) \, dx_t + \int_0^{Q_t^*} [h(Q_t^* - x_t)]f(x_t) \, dx_t \]

Repeat this process for 14 periods, and we can get the total cost, and the cost here should have been minimized.

4.2 Optimal policy for the long term time flexible contract

In section 4.1 we have discussed the optimal decision of the short term contract, which basically follow the traditional newsvendor rule, and could easily be determined under Leibnitz’s Rule.

This section studies the optimization of a more complex problem, the long term time flexible contract. There are three main factors that make the situation became more complex, the first one is that the demand distributions would be different from the short term contract. The time flexible contract requires a demand distribution to predict the summation demand for the whole
N periods so that the variance will be higher. Secondly, since the order quantity is very large, the CMO is always willing to offer a price discount and, therefore, the firm will be available to reduce their cost. Also, for a long term time flexible contract, there is a quantity sharing index between firm and CMO. Therefore, the solving process will be much more complex compared to the short term contract.

Considered Eq. (4), based on the same intuition, the optimal decision is S restricted by the value of unit price discount index \( \lambda \), regular unit price \( P \), upper (\( \bar{X} \)) and lower (\( \bar{X} \)) bound of the price discount model, penalty cost \( p \) and holding and disposal cost \( h \). Therefore, after simplify the model and we can determine the optimal decision much easier. However, one significant difference with the single period short-term model is the price discount will have an effect on the purchase cost for the long term time flexible contract. We let \( Y[S] \) be the sum of expected value of purchase cost, penalty and disposal cost. Also, we denote \( f(X) \) and \( F(X) \) are the pdf and cdf of the demand normal distribution. The Leibnitz’s Rule in the simplification process is still required in this problem.

To simplify the solving process, we also denote \( Y[S]_1, Y[S]_2 \) and \( Y[S]_3 \).

\[
Y[S]_1 = \int_{(1+\alpha)S}^{\infty} \left\{ (1+\alpha)S[P - \lambda ((1+\alpha)S - \bar{X})] + p[X - (1+\alpha)S] \right\} f(X) dX
\]

\[
Y[S]_2 = \int_{0}^{(1-\alpha)S} \left\{ (1-\alpha)S[P + \lambda (\bar{X} - (1-\alpha)S)] + h[(1-\alpha)S - X] \right\} f(X) dX
\]

\[
Y[S]_3 = \int_{(1-\alpha)S}^{\bar{X}} X[P + \lambda (\bar{X} - X)] f(X) dX + \int_{\bar{X}}^{\bar{X}} XP f(X) dX
\]

\[
+ \int_{\bar{X}}^{(1+\alpha)S} X[P - \lambda (X - \bar{X})] f(X) dX
\]
In order to simplify this problem we take the most restricted situation for the contract into consideration, that is, to set the value of $\alpha=0$. In this case,

$$F(S^*) = P\{X \leq S^*\} = \frac{P - 2\lambda S^* + \lambda \bar{X} - p}{\lambda \bar{X} - \lambda \bar{X} - p - h}$$

Here we notice that in the expression of $F(S^*), S^*$ is not eliminated during the simplification. The traditional way to solve this problem is the linear interpolation method. Use a standardized format to represent the expression:

$$\phi\left(\frac{S^* - (12\mu_t + 72\mu_D)}{\sqrt{12V_t + 72V_D}}\right) = \frac{P - 2\lambda S^* + \lambda \bar{X} - p}{\lambda \bar{X} - \lambda \bar{X} - p - h}$$

And denote:

$$Z_1 = \phi\left(\frac{S^* - (12\mu_t + 72\mu_D)}{\sqrt{12V_t + 72V_D}}\right) - \frac{P - 2\lambda S^* + \lambda \bar{X} - p}{\lambda \bar{X} - \lambda \bar{X} - p - h}$$

Since we know the value of $P, \lambda, \mu_t, \text{ and } \sigma_t$, we can easily check the value of $\frac{S^* - (12\mu_t + 72\mu_D)}{\sqrt{12V_t + 72V_D}}$ in the standard normal table. Whenever the value of $Z_1$ reaches 0, the optimal $S^*$ is meeting.

So we can take $S^*$ back to the formula, and get our minimum summation for the total purchase cost, the penalty cost and the disposal cost.
\[ Y [S^*] = \int_{(1+\alpha)S^*}^{\infty} \left\{ (1 + \alpha)S^* \left[ P - \lambda d \left( (1 + \alpha)S^* - \bar{X} \right) \right] + p[X - (1 + \alpha)S^*] \right\} f(X) dX \]

\[ + \int_{(1-\alpha)S^*}^{\bar{X}} X[P + \lambda d (\bar{X} - X)] f(X) dX + \int_{\bar{X}}^{\infty} X P f(X) dX \]

\[ + \int_{(1-\alpha)S^*}^{(1+\alpha)S^*} X[P - \lambda d (X - \bar{X})] f(X) dX \]

\[ + \int_{0}^{(1-\alpha)S^*} \left\{ (1 - \alpha)S^* [P + \lambda d (\bar{X} - (1 - \alpha)S^*)] + h[(1 - \alpha)S^* - X] \right\} f(X) dX \]

To make it comparable with the short term contract cost, we select penalty and disposal cost to show the extra cost of the time flexible contract.

\[ Y [S^*] = \int_{(1+\alpha)S^*}^{\infty} \left\{ p[X - (1 + \alpha)S^*] \right\} f(X) dX + \int_{(1-\alpha)S^*}^{(1-\alpha)S^*} \left\{ h[(1 - \alpha)S^* - X] \right\} f(X) dX \]

4.3 Optimal policy for the long term time inflexible contract

The advantages of the contracts above are obvious. For the short term contract, it is very easy to handle the demand risk caused by the PR-crisis. Firms can quickly react to the market fluctuation and change its order quantity respectively. For the time flexible contract, the price will be discounted because of the large quantity. However, when consider the defects of these two contracts, the short term contract will not be given a quantity discount, while the time flexible contract is stable at beginning but cannot handle the potentially lost sale at the end of the periods. This section studies the contract that eliminated all the problems that the other two contracts have—the long term time inflexible contract. For a long term time inflexible contract, firm can forecast the demand quantity for the every next period and could immediately react to the market situation. Moreover, since it is a long-term contract, unit price would be discounted by a given order quantity.
Considered Eq. (6), based on the same intuition, the optimal decision is $Q_t$ based on the value of unit price discount index $\lambda$, regular unit price $P$, upper ($\bar{D}$) and lower ($\underline{D}$) bound of the price discount model, penalty cost $p$ and holding and disposal cost $h$. Therefore, after simplify the model and we can determine the optimal decision much easier. Here the unit purchase price is available to be discounted, which means, same with a long term time flexible contract, the total cost model has taken purchase cost into consideration, thus, we let $Y[Q_t]$ be the sum of expected value of purchase cost, penalty and disposal cost at period $t$. Also, we denote $f(x_t)$ and $F(x_t)$ are the pdf and cdf of the demand normal distribution. We still need to adapt the Leibnitz’s Rule in the simplification process. The solving process is similar with the Long term time flexible contract, in order to simplify this problem we take the most restricted situation for the contract into consideration, that is, to set the value of $\alpha=0$. In this case,

$$F(Q_t^*) = P\{X \leq Q_t^*\} = \frac{P - 2\lambda Q_t + \lambda \bar{D} - p}{\lambda \bar{D} - \lambda \underline{D} - p - h}$$

Use the standardized format to represent this

$$\phi \left( \frac{Q_t^* - P_{t-1}\mu_t}{\sqrt{P_{t-1}\sigma_t}} \right) = \frac{P - 2\lambda Q_t + \lambda \bar{D} - p}{\lambda \bar{D} - \lambda \underline{D} - p - h}$$

And denote

$$Z_2 = \phi \left( \frac{Q_t^* - P_{t-1}\mu_t}{\sqrt{P_{t-1}\sigma_t}} \right) - \frac{P - 2\lambda Q_t + \lambda \bar{D} - p}{\lambda \bar{D} - \lambda \underline{D} - p - h}$$

Whenever $Z_1$ reaches 0, the value of $Q_t^*$ is optimized.

So we can take the $Q_t^*$ back to the formula, and get our minimum summation for the penalty cost and the disposal cost at period $t$. 
\[ Y[Q_t^*] = \int_{\frac{1}{1+\alpha}Q_t^*}^{\infty} \{p[x_t - (1 + \alpha)Q_t^*] + g(x_t)(1 + \alpha)Q_t^*\} f(x_t) dx_t + \int_{(1-\alpha)Q_t^*}^{(1+\alpha)Q_t^*} g(x_t)x_t f(x_t) dx_t \]

\[ + \int_0^{(1-\alpha)Q_t^*} \{h[(1 - \alpha)Q_t^* - x_t] + g(x_t)(1 - \alpha)Q_t^*\} f(x_t) dx_t \]

To make it comparable with the short term contract cost, we select penalty and disposal cost to show the extra cost of the time flexible contract.

\[ Y[Q_t^*] = \int_{\frac{1}{1+\alpha}Q_t^*}^{\infty} \{p[x_t - (1 + \alpha)Q_t^*]\} f(x_t) dx_t + \int_0^{(1-\alpha)Q_t^*} \{h[(1 - \alpha)Q_t^* - x_t]\} f(x_t) dx_t \]
CHAPTER 5   NUMERICAL STUDIES

We conduct numerical studies in this section. The first purpose is to introduce the steps of how to select the right contract for the firm. We all know that the values of the parameters are usually not constant when the factors, such as target markets, economic conditions and size of the pharmaceutical firms, are different. Therefore, different firms may choose the strategy which is best fit for their company. The numerical studies we demonstrated below are the guidance of how to use the parameters value that firms collected from market to select the right contract strategy for their actual situation. Second, we will exam the advantages of different types of contracts methods under different situations.

In order to meet the purpose of the numerical studies, we suggest two ways of numerical study methods in this chapter, the analytical calculation and the Monte Carlo simulation. For the analytical calculation, we consider an ideal situation where PR-crisis in not going to happen before the maturity of the drug. That is because, although PR-crisis is considered in the long term time inflexible contract and short term contract analytically, it is not possible to consider them in the time flexible contract due to the analytical complexity of the intertwined distributions between the demand distribution and the undetermined probability of PR-crisis. The Monte Carlo simulation is a supplement for the analytical calculation, because we could consider the PR-crisis under Monte Carlo simulation, and then compare with the analytical method and find out the problem with the analytical calculation, so that we could make our contract choosing process more comprehensive.

In some situation, a discount rate needed to be applied to the model because of the time value of the money. The same amount of money will be more valuable in current time than in future due
to the fluctuation of the inflation rate and market interest rate of money. Therefore, we denote the
discount factor as $\omega$, $0 < \omega < 1$. However, as the parameters we used are designated in present
values without considering their fluctuation in the future, so we assume that the discount
rate $\omega = 0$.

5.1 Parameter setting

Many parameters are involved in our model, and those parameters are already demonstrated in
the table below. However, due to the property of our models, there are several constraints should
be incorporated into the model, which are $\frac{P+\lambda X_t + h}{2\lambda} > S^*$ and $\frac{P+\lambda \bar{D} + h}{2\lambda} > Q_t^*$. These two
constraints are very easy to be satisfied because $\lambda$ should be very small compare with the unit
price and holding cost.

As we have mentioned before, we take a typical planning horizon of $N=14$ periods. The periods
could represent years but in some levels different through years. Typically, it was because the
phase III clinical trial and the FDA review always take more than 1 year, but since it is not a
consideration in our model, we can still view them as one period. We have noticed that the three
different strategies have different effects to the total spending for the firm due to different
situations. In this case, we need to set several values for a single parameter to simulate the
different situations. A preliminary study have indicated the success rate of the phase III clinical
trial as $\gamma_1 = \{0.46, 0.66, 0.86\}$, represented the minimum, mean and maximum trial success rate.

And the success rate of the FDA review is $\gamma_2 = \{0.67, 0.89\}$, represent the minimum and mean
FDA review success probabilities. The public relation crisis risk is one of the main
considerations in our model, as we are discussing the advantages of the risk sharing performance
of the three contracts under various kind of risk. PR-crisis is not happening frequently, so we
take \( p = \{0.01, 0.05\} \). The initial increase rate of the demand is set to be \( \mu_\text{D} = \{12.5, 25, 37.5\} \) due to the research by Lichtenberg and Duflos (2009). Moreover, penalty and holding cost are very important parameters in our problem solving, we set the penalty cost \( p = 0.2c \) and disposal and holding cost \( h = 0.05c \), where \( c \) is equal to the unit cost. The parameter values we used in our numerical study have been listed in Table 2.

Table 4 Parameter Values in Numerical Analysis

<table>
<thead>
<tr>
<th>N</th>
<th>14</th>
<th>( \gamma_1 )</th>
<th>{0.46, 0.66, 0.86}</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>20</td>
<td>( \gamma_2 )</td>
<td>{0.67, 0.89}</td>
</tr>
<tr>
<td>h</td>
<td>5%</td>
<td>( P_{t-1} )</td>
<td>1 or 0.2</td>
</tr>
<tr>
<td>p</td>
<td>20%</td>
<td>( \mu_\text{D} )</td>
<td>{12.5, 25, 37.5}</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.1</td>
<td>( \sigma_\text{D} )</td>
<td>{10, 20, 30}</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0</td>
<td>PR-crisis rate</td>
<td>0.95 or 1</td>
</tr>
<tr>
<td>( \mu_t )</td>
<td>100</td>
<td>( \sigma_t )</td>
<td>20</td>
</tr>
</tbody>
</table>

5.2 Analytical calculation

In this section, we will calculate the optimal order quantity and their expected cost under different contract. The calculation process follows the models we have derived in Chapter 3 and Chapter 4. As we have mentioned before, we assume the PR-crisis rate is 1 here in both long term and short term contract. Therefore, the analytical calculation follows an ideal situation where the real demand only follows the normal distribution.

5.2.1 Short term contact

Given the parameters we have set above, in each period \( \Phi\left(\frac{Q^*_t - P_{t-1} \mu_t}{\sqrt{P_{t-1} \sigma_t}}\right) = \frac{p}{h + p} = 0.8 \). Check the value on the standardized normal table and we can easily know that \( \frac{Q^*_t - P_{t-1} \mu_t}{\sqrt{P_{t-1} \sigma_t}} = 0.85 \).

Therefore, in a normal situation without considering the PR-crisis, the order quantity during the
14 periods could like the data listed in the table below (Consider when the mean is 25 and standard deviation is 20):

<table>
<thead>
<tr>
<th>Period</th>
<th>Order Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>0</td>
</tr>
<tr>
<td>Q2</td>
<td>0</td>
</tr>
<tr>
<td>Q3</td>
<td>117</td>
</tr>
<tr>
<td>Q4</td>
<td>149</td>
</tr>
<tr>
<td>Q5</td>
<td>179.4</td>
</tr>
<tr>
<td>Q6</td>
<td>209</td>
</tr>
<tr>
<td>Q7</td>
<td>238</td>
</tr>
<tr>
<td>Q8</td>
<td>266.6</td>
</tr>
<tr>
<td>Q9</td>
<td>295</td>
</tr>
<tr>
<td>Q10</td>
<td>323.1</td>
</tr>
<tr>
<td>Q11</td>
<td>351</td>
</tr>
<tr>
<td>Q12</td>
<td>378.8</td>
</tr>
<tr>
<td>Q13</td>
<td>406.4</td>
</tr>
<tr>
<td>Q14</td>
<td>433.9</td>
</tr>
</tbody>
</table>

Figure 6 Forecasted order quantity of the short term contract

It is linear here because it is more close to a traditional newsvendor model. Therefore, we could now calculate the expected cost based on the order quantity we have calculated above. First, transit our expected cost function into the integrated form, for example,

\[
Y(Q_3) = \int_{Q_3}^{\infty} [4(x_t - Q_t)] f(x_t) dx_t + \int_{0}^{Q_3} [(Q_3 - x_t)] f(x_t) dx_t
\]

\[
= \left. -10\sqrt{2\pi}(Q_3 - 100) \text{erf}\left(\frac{x - 100}{20\sqrt{2}}\right) - 400e^{-\frac{1}{800}(x-100)^2} \right|_{Q_3}^{\infty} + \left. 10\sqrt{2\pi}(Q_3 - 100) \text{erf}\left(\frac{x - 100}{20\sqrt{2}}\right) + 400e^{-\frac{1}{800}(x-100)^2} \right|_{0}^{Q_3}
\]

Like we have denoted in Chapter 4, \(Y(Q_3)\) is the notation of the expected extra cost value of the period 3. Then, we can calculate the total expected cost by period in Matlab.
\[ \sum_{t=1}^{14} Y(Q_t) = 321.8 \]

Figure 7 the expected cost of the short term contract under analytical calculation

5.2.2 Long term time flexible contract

Here we are introducing the bisection method to solve the problem because in each side of the function there is a decision variable S. The intuition of this method is not complex. We first define the upper and lower start value of the method, as we denote \( S_u = 10000 \) and \( S_l = -10000 \). Step 2 will be a simple average of the value we denoted, and we will get \( S_m = \frac{S_u + S_l}{2} = 0 \). Then, the step 3 we should bring \( S_m \) to the function \( Z_1 = \phi \left( \frac{S^*-(12\mu_t+66\mu_D)}{\sqrt{12V_t+66V_D}} \right) - \frac{P - 2\lambda S^* + \lambda X_t + p}{\lambda X_t - \lambda X_t - p - h} \), if \( Z_1 > 0 \) we set the value of \( S_m \) to \( S_u \), if \( Z_1 < 0 \), \( S_m \) will be set to \( S_l \). Repeat the process until we find the S that make \( Z_1 = 0 \), and then stop. With the help of the bisection method we find that when S= 2835 units during the 14 periods, the optimal will meet. Therefore, we could now calculate the expected cost based on the order quantity we have calculated above. First, transit our expected cost function into the integrated form,
\[ Y[S] = \int_S^\infty [4(X - S)] f(X) dX + \int_0^S [(S - X)] f(X) dX \]

\[ = \frac{300\sqrt{39\pi} \text{erf} \left( \frac{X - 2850}{40\sqrt{39}} \right) - 31200e^{-\frac{1}{62400}(X - 2850)^2}}{10\sqrt{39\pi}} \bigg|_0^S \]

\[ - \frac{300\sqrt{39\pi} \text{erf} \left( \frac{X - 2850}{40\sqrt{39}} \right) - 31200e^{-\frac{1}{62400}(X - 2850)^2}}{40\sqrt{39\pi}} \bigg|_0^S \]

Then, we can calculate the expected cost by period in Matlab.

\[ Y[S] = 374.8 \]

5.2.3 Long term time inflexible contract

Section 5.2.1 and 5.2.2 have discussed the short term contract and the time flexible long term contract. Then, in this section, we will calculate the optimal order quantity and the optimal expected cost.

![Forecasted Order Quantity](image)

Figure 8 the optimal order quantities of the time inflexible contract.
Therefore, we could now calculate the expected cost based on the order quantity we have calculated above. First, transit our expected cost function in to integrated form.

\[ Y[Q_t^*] = \int_{Q_t^*}^{\infty} \{4[x_t - Q_t^*]f(x_t)dx_t + \int_{0}^{Q_t^*} [(Q_t^* - x_t)]f(x_t)dx_t \]  

Then, we can calculate the total expected cost by period in Matlab.

\[ \sum_{t=1}^{14} Y[Q_t^*] = 818 \]

Figure 9 the expected cost of time inflexible contract under analytical calculation

5.2.4 Analytical calculation comparison

Compare the sum of expected cost for the three contracts above, we can find that the short term contract have a relatively lowest expected cost when we have not taken the PR-Crisis into consideration. It is because that the order quantity calculated from the short term contract is based on the normal newsvendor rule.
We also noticed that, compare the expected cost between short term contract and the long term time inflexible contract at the beginning of the commercialized periods, the long term time inflexible contract have a relatively lower cost. However, when the time moves forward, the increase rate of the time inflexible contract is significantly lower that the short term contract. That is because, when we consider the model of the time flexible and time inflexible contract, we also considered the risk of the PR-crisis that could reduce the demand. Due to the penalty cost is always larger that the disposal cost under this situation, set fewer order quantity from the CMO will potentially reduce the cost when the PR-crisis occurs in the future.

![Expected Cost Chart]

Figure 10 the expected costs of different contracts under analytical calculation

In order to observe the real performance of these three contracts, a simulation numerical study that has taken the PR-crisis into consideration is needed.

5.3 Monte Carlo simulation
Simply input the model into Matlab. Run the simulation for hundreds of replications and use the simulated demand to calculated expected cost. Therefore, we can observe three histograms of the expected cost.

Figure 11 the expected cost of time short term contract under simulation
The first output is from the short term contract, with the mean=838.5 and standard deviation = 242.4.

Figure 12 the expected cost of time flexible contract under simulation
The second output is from the long term time flexible contract, with the mean = 712.2 and standard deviation = 643.

Figure 13 the expected cost of time inflexible contract under simulation

The third output is from the long term time inflexible contract, with the mean = 695.908 and standard deviation=273.

Compare these three outputs above and we can easy notice that the expected cost is reduced based on the level of risk sharing. The long term time inflexible contract have the lowest expected cost and relatively similar standard deviation with the short term contract, while for the time flexible contact, since its order quantity is forecasted in an aggregated way, the standard deviation is larger than the time inflexible contract although its expected cost is lower that the short term contract. This shows that under the given circumstance, long term time flexible contract will have a better performance compare with the other two contracts.
5.4 Numerical study conclusion

The numerical study here shows us that the optimal contract strategy under the given parameters is a long term time flexible contract because it has relatively lower extra cost and standard deviation of the expected extra cost. Although short term contract and long term time inflexible contract are more precisely forecasted the demand in every period, once the demand variance is large, understock or over inventory could happen and then cause the unexpected extra cost. However, since the time flexible contract have more flexibility to face the demand fluctuation, it is reasonable that it incurs less cost. In order to have a better use of this study, firm could also use the time flexible contract to strategically forecast the order it going to purchase from CMO, and modify the order yearly by the long term time inflexible contract or short term contract.

However, the numerical study is not a demonstration that the long term time flexible contract is always outstanding compare to the other two strategies. There are so many parameters that need to be considered in the numerical study. Parameters like penalty cost and holding cost could
heavily influence the solution as they become larger or smaller, so did the other parameter. Thus, every time when firm has received the parameter data, which collected from market, it needs to recalculate the cost and then decide the optimal decision again.

Moreover, for the time flexible contract, since it forecasted an aggregate demand at the beginning of the period when the final unit price is not determined. We here denote a way in the real world that the firm initially pays a front money to the CMO, and later, when the drug reaches its maturity, pay the rest of the money which have considered the real price.
CHAPTER 6 CONCLUSION

This thesis studies the pharmaceutical outsourcing contract structures, which are always facing by the firm when deciding to commercialize its newly proved drugs. As we know, the pharmaceutical industry is a risky industry due to its uncertainty of both demand and the drug’s performance during FDA test. To handle the risks, many research are working on various aspects of this problem. Contract strategy is one of those aspects. Our study is focusing on the share of risks and cost between pharmaceutical firms and CMO. There are three different types of contracts methods we have mentioned in this thesis, and they all have their advantages in certain circumstances. In this study, we analyzed the most common situation under regular market pattern. For the short term contract, it always has a fast reaction to the market change while receive no discount due to its small quantity. For a long term time flexible contract, order quantity is very large and CMO will glad to offer a quantity discount to the firm, but forecasting should be made at the beginning period to predict the demand during the whole life cycle. Whereas, the long term time inflexible contract shows the advantages much more than the two above, and can either reduce the potential risk effect or share the cost.

From an academic research perspective, we develop a stochastic newsvendor model that captures the key factors including demand risk for a new drug with a test uncertainty and a growing random demand; and the supply risk with an upper bound of the supply capacity from CMO. We have investigated that unit purchase price can be reduced with the increase of quantity, and on the other side it would increase when the order decreased. Also, contracts can offer quantity flexibility to firm and then firm can reduce the risk of lost sale and over order. We have
developed an efficient computation method based on a newsvendor and stochastic process, which enables numerical solutions for the problem.

For a practice perspective, our study is following a growing trend in the global pharmaceutical outsourcing industry. We provide a simple structure for a total cost analysis and give an easier way to forecast the output of the contract. Although this work is initiated by the pharmaceutical application, the modeling framework, the managerial insights and the solution techniques can still be applied to many other industries with the closed characteristics as in the pharmaceutical industry.

Finally, there are several further researches in this area that required being strength in the future. For example, in order to simplify the solving process, we denoted the quantity sharing index is 0, where having a quantity sharing will further reduce the cost. Also, we considered a single-source situation in our study; while some of the pharmaceutical firms have dual sources include outsourcing and in house capacity. Furthermore, we have combined the various kinds of risks that defect the firm reputation as PR-crisis risk in our study, but in the real world, there still some other risk that have different effects on the drugs’ market performance.
References


APPENDIX A:

MATLAB Codes

The MATLAB sample codes provided in the appendix are for the analysis of the optimal decision of the order quantity of short term contract, long term time flexible and long term time inflexible contract.

1. For the short term and time inflexible contract:

```matlab
x = {0.95, 20, 4, 1, 0.2, 100, 400, 25, 400, 1, 1, 100, 3, 1, 0.1};
[Pcrisis, p, Penalty, Holding, P, MuDtInitial, VarDtInitial, MuD, VarD, ProbPhaseIII, ProbFDA, SetupShort, dUpperIndex, dLowerIndex, Nambda] = deal(x{:});
MuDt = zeros(100, 14);
VarDt = zeros(100, 14);
Sigm = zeros(100, 14);
dUpper = zeros(100, 14);
dLower = zeros(100, 14);
Demand = zeros(100, 14);
SharedP = zeros(100, 14);
Q2 = zeros(100, 14);
Q3 = zeros(100, 14);
C3 = zeros(100, 14);
beroulli = zeros(1, 14);
v = zeros(100, 14);
YQ = zeros(100, 14);
YQ2 = zeros(100, 14);
l = zeros(100, 14);
```
Sup=zeros(100,14);
Slow=zeros(100,14);
Smiddle=zeros(100,14);
S=zeros(100,14);
for r=1:100
    bernoulli = rand(1,14) > Pcrisis; % Generate 0 or 1 represent happen or not happen for the PR crisis
    bernoulli(1)=0;
    bernoulli(2)=0;
    for k=1:14
        pt(r,k)=P^bernoulli(k); % demand reduce rate for each year.
    end
    % the probability that the drug pass the clinic trial
    PhaseIII = rand(1)>ProbPhaseIII;
    FDAreview = rand(1)>ProbFDA;
    % Modeling the demand distribution
    MuDt(r,1)=0;
    VarDt(r,1)=0;
    MuDt(r,2)=0;
    VarDt(r,2)=0;
    MuDt(r,3)=MuDtInitial;
    VarDt(r,3)=VarDtInitial;
    Sigmat(r,3)=sqrt(VarDt(r,3));
    for k=4:14
        t=k-1;
        MuDt(r,k)= pt(r,k-2)*MuDt(r,t)+MuD;
        VarDt(r,k)= pt(r,k-2)*VarDt(r,t)+VarD;
        Sigmat(r,k)=sqrt(VarDt(r,k));
    end;
% Period 1 and 2
% short term contract
for k=2:14
    Demand(r,k) = normrnd(pt(r,k-1)*MuDt(r,k),sqrt(pt(r,k-1))*Sigmat(r,k));
    v(r,k) = invNormCDF(Penalty/(Penalty+Holding));
    Q3(r,k) = v(r,k)*sqrt(pt(r,k-1))*Sigmat(r,k)+pt(r,k-1)*MuDt(r,k);
    YQ(r,k) = max(0, Penalty*(Demand(r,k)-Q3(r,k)))+max(0, Holding*(Q3(r,k)-Demand(r,k)));
end
% long term inflexible contract
for k=2:14
    dUpper(r,k) = MuDt(r,k)+dUpperIndex*sqrt(VarDt(r,k));
    dLower(r,k) = MuDt(r,k)-dLowerIndex*sqrt(VarDt(r,k));
    Sup(r,k) = -10000;
    Slow(r,k) = 10000;
    for i=1:100
        Smiddle(r,k) = (Sup(r,k)+Slow(r,k))/2;
        if normcdf((Smiddle(r,k)-pt(r,k-1)*MuDt(r,k))/sqrt(pt(r,k-1)*VarDt(r,k)))-(p-2*Lambda*Smiddle(r,k)+Lambda*dUpper(r,k)-Penalty)/(Lambda*dUpper(r,k)-Lambda*dLower(r,k)-Penalty-Holding)<0
            Sup(r,k) = Smiddle(r,k);
        elseif normcdf((Smiddle(r,k)-pt(r,k-1)*MuDt(r,k))/sqrt(pt(r,k-1)*VarDt(r,k)))-(p-2*Lambda*Smiddle(r,k)+Lambda*dUpper(r,k)-Penalty)/(Lambda*dUpper(r,k)-Lambda*dLower(r,k)-Penalty-Holding)>0
            Slow(r,k) = Smiddle(r,k);
        else
            end
    end
end
l(r,k)=p-2*Nambda*Smiddle(r,k)+Nambda*dUpper(r,k)-Penalty;
Q2(r,k)=Smiddle(r,k);
YQ2(r,k)=max(0,Penalty*(Demand(r,k)-(1+0.1)*Q2(r,k)))+max(0,Holding*((1-0.1)*Q2(r,k)-Demand(r,k)));
end
end

2. For the time flexible contract:

x={0.95,20,4, 1, 0.2,100, 400, 25, 400, 1,
1, 100, 3400, 2600, 0.007 };
[Pcrisis,p,Penalty,Holding,P,MuDtInitial,VarDtInitial,MuD,VarD,ProbPhaseIII,P
robFDA,SetupShort,dUpper,dLower,Nambda]=deal(x{:});
MuDt=zeros(100,14);
VarDt=zeros(100,14);
Sigmat=zeros(100,14);
Demand=zeros(100,1);
SharedP=zeros(100,14);
Q3=zeros(100,14);
C3=zeros(100,14);
beroulli=zeros(1,14);
v=zeros(100,14);
YQ=zeros(100,1);
MuS=zeros(100,1);
VarS=zeros(100,1);
Sup=zeros(100,1);
Slow=zeros(100,1);
Smiddle=zeros(100,1);
S=zeros(100,1);
l=zeros(100,3);
for r=1:100

  % Period 1 and 2
  % Time-Flexible long term contract
  MuS(r,1)=10*MuDtInitial+56*MuD;
  VarS(r,1)=10*VarDtInitial+56*VarD;
  z=(Nambda*dUpper-Nambda*dLower-Penalty-Holding);
  Sup(r,1)=-10000;
  Slow(r,1)=10000;
  for i=1:100
     Smiddle(r,1)=(Sup(r,1)+Slow(r,1))/2;
     if normcdf((Smiddle(r,1)-MuS(r,1))/sqrt(VarS(r,1))-(p-
     2*Nambda*Smiddle(r,1)+Nambda*dUpper-Penalty)/(Nambda*dUpper-Nambda*dLower-
     Penalty-Holding)<0
        Sup(r,1)=Smiddle(r,1);
     elseif normcdf((Smiddle(r,1)-MuS(r,1))/sqrt(VarS(r,1))-(p-
     2*Nambda*Smiddle(r,1)+Nambda*dUpper-Penalty)/(Nambda*dUpper-Nambda*dLower-
     Penalty-Holding)>0
        Slow(r,1)=Smiddle(r,1);
     else
         return
     end
  end
S(r,1)=Smiddle(r,1);
l(r,1)=normcdf((Smiddle(r,1)-MuS(r,1))/sqrt(VarS(r,1)));
Demand(r,1)= normrnd(MuS(r,1),sqrt(VarS(r,1)));
YQ(r,1) = \max(0, \text{Penalty} \cdot (\text{Demand}(r,1) - S(r,1))) + \max(0, \text{Holding} \cdot (S(r,1) - \text{Demand}(r,1)))
end

3. The calculation code of the analytical calculation.

x1 = \text{Inf}(1);
x2 = 0;
E = \exp(1);
Pi = \pi;
x = (117, 149, 179.4, 209, 238, 266.6, 295, 323.1, 351, 378.8, 406.4, 433.9);
y = (117, 149, 179.4, 209, 238, 266.6, 295, 323.1, 351, 378.8, 406.4, 433.9);
z1_1 = \frac{-400}{E} \cdot \left(\frac{-100 + x1}{2} - 800\right) - 10 \cdot \sqrt{2\pi} \cdot \left(-100 + y\right) \cdot \text{erf}\left(\frac{-100 + x1}{20\sqrt{2}}\right) / (5\sqrt{2\pi});
z1_2 = \frac{-400}{E} \cdot \left(\frac{-100 + y}{2} - 800\right) - 10 \cdot \sqrt{2\pi} \cdot \left(-100 + y\right) \cdot \text{erf}\left(\frac{-100 + y}{20\sqrt{2}}\right) / (5\sqrt{2\pi});
c_1 = z1_1 - z1_2;
z2_1 = \left(\frac{400}{E} \cdot \left(-100 + x2\right)^2 / 800\right) + 10 \cdot \sqrt{2\pi} \cdot \left(-100 + y\right) \cdot \text{erf}\left(\frac{-100 + x2}{20\sqrt{2}}\right) / (20\sqrt{2\pi});
z2_2 = \left(\frac{400}{E} \cdot \left(-100 + y\right)^2 / 800\right) + 10 \cdot \sqrt{2\pi} \cdot \left(-100 + y\right) \cdot \text{erf}\left(\frac{-100 + y}{20\sqrt{2}}\right) / (20\sqrt{2\pi});
c_2 = z2_1 + z2_2;
C = c_1 + c_2;

z1_1 = \frac{-800}{E} \cdot \left(-125 + x1\right)^2 / 1600 - 20 \cdot \sqrt{6\pi} \cdot \left(-125 + y\right) \cdot \text{erf}\left(\frac{-125 + x1}{40\sqrt{6}}\right) / (10\sqrt{6\pi});
z1_2 = \frac{-800}{E} \cdot \left(-125 + y\right)^2 / 1600 - 20 \cdot \sqrt{6\pi} \cdot \left(-125 + y\right) \cdot \text{erf}\left(\frac{-125 + y}{40\sqrt{6}}\right) / (10\sqrt{6\pi});
c_1 = z1_1 - z1_2;
z2_1 = \left(\frac{800}{E} \cdot \left(-125 + x2\right)^2 / 1600\right) + 20 \cdot \sqrt{6\pi} \cdot \left(-125 + y\right) \cdot \text{erf}\left(\frac{-125 + x2}{40\sqrt{6}}\right) / (40\sqrt{6\pi});
z2_2 = \left(\frac{800}{E} \cdot \left(-125 + y\right)^2 / 1600\right) + 20 \cdot \sqrt{6\pi} \cdot \left(-125 + y\right) \cdot \text{erf}\left(\frac{-125 + y}{40\sqrt{6}}\right) / (40\sqrt{6\pi});
c_2 = z2_1 + z2_2;
C = c_1 + c_2;

z1_1 = \frac{-1200}{E} \cdot \left(-150 + x1\right)^2 / 2400 - 10 \cdot \sqrt{2\pi} \cdot \left(-150 + y\right) \cdot \text{erf}\left(\frac{-150 + x1}{20\sqrt{2}}\right) / (5\sqrt{2\pi});
z1_2 = \frac{-1200}{E} \cdot \left(-150 + y\right)^2 / 2400 - 10 \cdot \sqrt{2\pi} \cdot \left(-150 + y\right) \cdot \text{erf}\left(\frac{-150 + y}{20\sqrt{2}}\right) / (5\sqrt{2\pi});
c_1 = z1_1 - z1_2;
z2_1 = \left(\frac{1200}{E} \cdot \left(-150 + x2\right)^2 / 2400\right) + 10 \cdot \sqrt{2\pi} \cdot \left(-150 + y\right) \cdot \text{erf}\left(\frac{-150 + x2}{20\sqrt{2}}\right) / (20\sqrt{2\pi});
z2_2 = \left(\frac{1200}{E} \cdot \left(-150 + y\right)^2 / 2400\right) + 10 \cdot \sqrt{2\pi} \cdot \left(-150 + y\right) \cdot \text{erf}\left(\frac{-150 + y}{20\sqrt{2}}\right) / (20\sqrt{2\pi});
c_2 = z2_1 + z2_2;
C = c_1 + c_2;

z1_1 = \frac{-1600}{E} \cdot \left(-175 + x1\right)^2 / 3200 - 20 \cdot \sqrt{2\pi} \cdot \left(-175 + y\right) \cdot \text{erf}\left(\frac{-175 + x1}{40\sqrt{2}}\right) / (10\sqrt{2\pi});
z1_2 = \frac{-1600}{E} \cdot \left(-175 + y\right)^2 / 3200 - 20 \cdot \sqrt{2\pi} \cdot \left(-175 + y\right) \cdot \text{erf}\left(\frac{-175 + y}{40\sqrt{2}}\right) / (10\sqrt{2\pi});
c_1 = z1_1 - z1_2;
\[ z_{2_1} = \frac{1600}{E^{\left(\frac{-175 + y}{2}\right)^2/3200} + 20\sqrt{2\pi}\left(-175 + y\right)\text{erf}\left(\frac{-175 + x2}{40\sqrt{2}}\right)}/(40\sqrt{2\pi})/20\sqrt{2\pi}; \\
\]
\[ z_{2_2} = \frac{1600}{E^{\left(\frac{-175 + x}{2}\right)^2/3200} + 20\sqrt{2\pi}\left(-175 + x2\right)\text{erf}\left(\frac{-175 + y}{40\sqrt{2}}\right)}/(40\sqrt{2\pi})/20\sqrt{2\pi}; \\
\]
\[ c_2 = z_{2_1} + z_{2_2}; \]
\[ C = c_1 + c_2; \]

\[ z_{1_1} = \frac{-2000}{E^{\left(\frac{-200 + x1}{2}\right)^2/4000} - 10\sqrt{10\pi}\left(-200 + y\right)\text{erf}\left(\frac{-200 + x1}{20\sqrt{10}}\right)}/(5\sqrt{10\pi})/10\sqrt{10\pi}; \\
\]
\[ z_{1_2} = \frac{-2000}{E^{\left(\frac{-200 + y}{2}\right)^2/4000} - 10\sqrt{10\pi}\left(-200 + y\right)\text{erf}\left(\frac{-200 + y}{20\sqrt{10}}\right)}/(5\sqrt{10\pi})/10\sqrt{10\pi}; \\
\]
\[ c_1 = z_{1_1} - z_{1_2}; \]
\[ z_{2_1} = \frac{2000}{E^{\left(\frac{-200 + y}{2}\right)^2/4000} + 10\sqrt{10\pi}\left(-200 + y\right)\text{erf}\left(\frac{-200 + y}{20\sqrt{10}}\right)}/(20\sqrt{10\pi})/10\sqrt{10\pi}; \\
\]
\[ z_{2_2} = \frac{2000}{E^{\left(\frac{-200 + x}{2}\right)^2/4000} + 10\sqrt{10\pi}\left(-200 + y\right)\text{erf}\left(\frac{-200 + x}{20\sqrt{10}}\right)}/(20\sqrt{10\pi})/10\sqrt{10\pi}; \\
\]
\[ c_2 = z_{2_1} + z_{2_2}; \]
\[ C = c_1 + c_2; \]

\[ z_{1_1} = \frac{-2400}{E^{\left(\frac{-225 + x1}{2}\right)^2/4800} - 20\sqrt{3\pi}\left(-225 + y\right)\text{erf}\left(\frac{-225 + x1}{40\sqrt{3}}\right)}/(10\sqrt{3\pi})/20\sqrt{3\pi}; \\
\]
\[ z_{1_2} = \frac{-2400}{E^{\left(\frac{-225 + y}{2}\right)^2/4800} - 20\sqrt{3\pi}\left(-225 + y\right)\text{erf}\left(\frac{-225 + y}{40\sqrt{3}}\right)}/(10\sqrt{3\pi})/20\sqrt{3\pi}; \\
\]
\[ c_1 = z_{1_1} - z_{1_2}; \]
\[ z_{2_1} = \frac{2400}{E^{\left(\frac{-225 + y}{2}\right)^2/4800} + 20\sqrt{3\pi}\left(-225 + y\right)\text{erf}\left(\frac{-225 + y}{40\sqrt{3}}\right)}/(40\sqrt{3\pi})/20\sqrt{3\pi}; \\
\]
\[ z_{2_2} = \frac{2400}{E^{\left(\frac{-225 + x}{2}\right)^2/4800} + 20\sqrt{3\pi}\left(-225 + y\right)\text{erf}\left(\frac{-225 + x}{40\sqrt{3}}\right)}/(40\sqrt{3\pi})/20\sqrt{3\pi}; \\
\]
\[ c_2 = z_{2_1} + z_{2_2}; \]
\[ C = c_1 + c_2; \]

\[ z_{1_1} = \frac{-2800}{E^{\left(\frac{-250 + x1}{2}\right)^2/5600} - 40\sqrt{14\pi}\left(-250 + y\right)\text{erf}\left(\frac{-250 + x1}{20\sqrt{14}}\right)}/(80\sqrt{14\pi})/5\sqrt{14\pi}; \\
\]
\[ z_{1_2} = \frac{-2800}{E^{\left(\frac{-250 + y}{2}\right)^2/5600} - 40\sqrt{14\pi}\left(-250 + y\right)\text{erf}\left(\frac{-250 + y}{20\sqrt{14}}\right)}/(80\sqrt{14\pi})/5\sqrt{14\pi}; \\
\]
\[ c_1 = z_{1_1} - z_{1_2}; \]
\[ z_{2_1} = \frac{2800}{E^{\left(\frac{-250 + y}{2}\right)^2/5600} + 10\sqrt{14\pi}\left(-250 + y\right)\text{erf}\left(\frac{-250 + y}{20\sqrt{14}}\right)}/(20\sqrt{14\pi})/10\sqrt{14\pi}; \\
\]
\[ z_{2_2} = \frac{2800}{E^{\left(\frac{-250 + x}{2}\right)^2/5600} + 10\sqrt{14\pi}\left(-250 + y\right)\text{erf}\left(\frac{-250 + x}{20\sqrt{14}}\right)}/(20\sqrt{14\pi})/10\sqrt{14\pi}; \\
\]
\[ c_2 = z_{2_1} + z_{2_2}; \]
\[ C = c_1 + c_2; \]

\[ z_{1_1} = \frac{-3200}{E^{\left(\frac{-275 + x1}{2}\right)^2/6400} - 40\sqrt{6\pi}\left(-275 + y\right)\text{erf}\left(\frac{-275 + x1}{80}\right)}/(80\sqrt{6\pi})/20\sqrt{6\pi}; \\
\]
\[ z_{1_2} = \frac{-3200}{E^{\left(\frac{-275 + y}{2}\right)^2/6400} - 40\sqrt{6\pi}\left(-275 + y\right)\text{erf}\left(\frac{-275 + y}{80}\right)}/(80\sqrt{6\pi})/20\sqrt{6\pi}; \\
\]
\[ c_1 = z_{1_1} - z_{1_2}; \]
\[ z_{2_1} = \frac{3200}{E^{\left(\frac{-275 + y}{2}\right)^2/6400} + 40\sqrt{6\pi}\left(-275 + y\right)\text{erf}\left(\frac{-275 + y}{80}\right)}/(80\sqrt{6\pi})/20\sqrt{6\pi}; \\
\]
\[ z_{2_2} = \frac{3200}{E^{\left(\frac{-275 + x}{2}\right)^2/6400} + 40\sqrt{6\pi}\left(-275 + y\right)\text{erf}\left(\frac{-275 + x}{80}\right)}/(80\sqrt{6\pi})/20\sqrt{6\pi}; \\
\]
\[ c_2 = z_{2_1} + z_{2_2}; \]
\[ C = c_1 + c_2; \]
z_1_1 =(-3600/E^((-300 + x1)^2/7200) - 30*sqrt(2*Pi)* (-300 + y)*erf((-300 + x1)/(60*sqrt(2))) - (15*sqrt(2*Pi)));
z_1_2 =(-3600/E^((-300 + y)^2/7200) - 30*sqrt(2*Pi)* (-300 + y)*erf((-300 + y)/(60*sqrt(2))))/(15*sqrt(2*Pi));
c_1=z_1_1-z_1_2;
z_2_1 = (3600/E^(((-300 + y)^2/7200) + 30*sqrt(2*Pi)* (-300 + y)*erf((-300 + y)/(60*sqrt(2))))/(60*sqrt(2*Pi));
z_2_2 = (3600/E^(((-300 + y)^2/7200) + 30*sqrt(2*Pi)* (-300 + y)*erf((-300 + y)/(60*sqrt(2))))/(60*sqrt(2*Pi));
c_2=z_2_1+z_2_2;
C=c_1+c_2;

z_1_1 =(-4000/E^((-325 + x1)^2/8000) - 20*sqrt(5*Pi)* (-325 + y)*erf((-325 + x1)/(40*sqrt(5))) - (10*sqrt(5*Pi)));
z_1_2 =(-4000/E^((-325 + y)^2/8000) - 20*sqrt(5*Pi)* (-325 + y)*erf((-325 + y)/(40*sqrt(5))))/(10*sqrt(5*Pi));
c_1=z_1_1-z_1_2;
z_2_1 = (4000/E^(((-325 + y)^2/8000) + 20*sqrt(5*Pi)* (-325 + y)*erf((-325 + y)/(40*sqrt(5))))/(40*sqrt(5*Pi));
z_2_2 = (4000/E^(((-325 + y)^2/8000) + 20*sqrt(5*Pi)* (-325 + y)*erf((-325 + y)/(40*sqrt(5))))/(40*sqrt(5*Pi));
c_2=z_2_1+z_2_2;
C=c_1+c_2;

z_1_1 =(-4800/E^((-375 + x1)^2/9600) - 20*sqrt(6*Pi)* (-375 + y)*erf((-375 + x1)/(40*sqrt(6))) - (10*sqrt(6*Pi)));
z_1_2 =(-4800/E^((-375 + y)^2/9600) - 20*sqrt(6*Pi)* (-375 + y)*erf((-375 + y)/(40*sqrt(6))))/(10*sqrt(6*Pi));
c_1=z_1_1-z_1_2;
z_2_1 = (4800/E^(((-375 + y)^2/9600) + 20*sqrt(6*Pi)* (-375 + y)*erf((-375 + y)/(40*sqrt(6))))/(40*sqrt(6*Pi));
z_2_2 = (4800/E^(((-375 + y)^2/9600) + 20*sqrt(6*Pi)* (-375 + y)*erf((-375 + y)/(40*sqrt(6))))/(40*sqrt(6*Pi));
c_2=z_2_1+z_2_2;
C=c_1+c_2;
APPENDIX B

Short term contract simplification:

\[ Y(Q_t) = \int_{Q_t}^{\infty} [p(x_t - Q_t)]f(x_t)dx_t + \int_0^{Q_t} [h(Q_t - x_t)]f(x_t)dx_t \]

Optimal Solution: Take the derivative of \( Y(Q_t) \) with respect to \( Q_t \), setting it equal to zero, and solving yields:

\[ 0 = \frac{d}{dQ_t} \left[ \int_{Q_t}^{\infty} [p(x_t - Q_t)]f(x_t)dx_t + \int_0^{Q_t} [h(Q_t - x_t)]f(x_t)dx_t \right] \]

According to Leibnitz’s Rule:

\[
\frac{d}{dQ_f} \int_{f_1(Q_t)}^{f_2(Q_t)} f(Q_t, x_t) dx_t
\]

\[ = \int_{f_1(Q_t)}^{f_2(Q_t)} \left[ \frac{d}{dQ_t} f(Q_t, x_t) \right] dx_t - f(Q_t, f_1(Q_t)) \frac{d}{dQ_t} f_1(Q_t) + f(Q_t, f_2(Q_t)) \frac{d}{dQ_t} f_2(Q_t) \]

Hence:

\[ 0 = \frac{d}{dQ_t} \left[ \int_{Q_t}^{\infty} [p(x_t - Q_t)]f(x_t)dx_t + \int_0^{Q_t} [h(Q_t - x_t)]f(x_t)dx_t \right] \]

\[ = h \left[ \int_0^{Q_t} (1)f(x_t)dx_t + 0 \right] + p \left[ \int_{Q_t}^{\infty} (-1)f(x_t)dx_t + 0 \right] \]

\[ = hF(Q_t) - p[1 - F(Q_t)] = (h + p)F(Q_t) - p \]

Therefore:
That is,

\[ F(Q_t^*) = P\{X \leq Q_t^*\} = \frac{p}{h + p} \]

Using the standard format to represent this,

\[ \phi \left( \frac{Q_t^* - P_{t-1}\mu_t}{\sqrt{P_{t-1}\sigma_t^2}} \right) = \frac{p}{h + p} \]

Since we know the value of \( P_{t-1}, \mu_t, \) and \( \sigma_t, \) we can easily check the value of \( \frac{Q_t^* - P_{t-1}\mu_t}{\sqrt{P_{t-1}\sigma_t^2}} \) in the standard normal table by the value \( \frac{p}{h + p}, \) and then solve the \( Q_t^*. \)

So we can take the \( Q_t^* \) back to the formula, and get our minimum summation for the penalty cost and the disposal cost at period \( t. \)

\[ Y(Q_t^*) = \int_{Q_t^*}^{\infty} [p(x_t - Q_t^*)]f(x_t)dx_t + \int_0^{Q_t^*} [h(Q_t^* - x_t)]f(x_t)dx_t \]

Repeat this process for 14 periods, and we can get the total cost, and the cost here should have been minimized.
Long term time flexible contract:

\[
g(X) = \begin{cases} 
  P - \lambda((1 + \alpha)S - \bar{X}), & \text{if } X > (1 + \alpha)S \\
  P - \lambda(X - \bar{X}), & \text{if } (1 + \alpha)S > X > \bar{X}, \\
  P, & \text{if } \bar{X} \leq X \leq \bar{X}, \\
  P + \lambda(\bar{X} - X), & \text{if } (1 - \alpha)S < X < \bar{X}, \\
  P + \lambda(\bar{X} - (1 - \alpha)S), & \text{if } X < (1 - \alpha)S 
\end{cases}
\]

\[
Y[S] = \int_{(1+\alpha)S}^{\infty} \left\{ (1+\alpha)S g(X) + p[X - (1 + \alpha)S] \right\} f(X) dX + \int_{(1-\alpha)S}^{(1+\alpha)S} X g(X) f(X) dX \\
+ \int_{0}^{(1-\alpha)S} \left\{ (1 - \alpha)S g(X) + h[(1 - \alpha)S - X] \right\} f(X) dX
\]

According to Leibnitz’s Rule:

\[
\frac{d}{dQ_t} \int_{f_1(Q_t)}^{f_2(Q_t)} f(Q_t, x_t) dx_t \\
= \int_{f_1(Q_t)}^{f_2(Q_t)} \left[ \frac{d}{dQ_t} f(Q_t, x_t) \right] dx_t - f(Q_t, f_1(Q_t)) \frac{d}{dQ_t} f_1(Q_t) \\
+ f(Q_t, f_2(Q_t)) \frac{d}{dQ_t} f_2(Q_t)
\]
\[
Y [S] = \int_{(1+\alpha)S}^{\infty} \left\{ (1 + \alpha)S[P - \lambda d( (1 + \alpha)S - \overline{X})] + p[X - (1 + \alpha)S] \right\} f(X) \, dX \\
+ \int_{(1-\alpha)S}^{\overline{X}} X[P + \lambda d(\overline{X} - X)]f(X) \, dX + \int_{\overline{X}}^{\infty} XPf(X) \, dX \\
+ \int_{\overline{X}}^{(1+\alpha)S} X[P - \lambda d(X - \overline{X})]f(X) \, dX \\
+ \int_{0}^{(1-\alpha)S} \left\{ (1 - \alpha)S[P + \lambda d(\overline{X} - (1 - \alpha)S)] + h[(1 - \alpha)S - X] \right\} f(X) \, dX
\]

\[
Y [S]_1 = \int_{(1+\alpha)S}^{\infty} \left\{ (1 + \alpha)S[P - \lambda ( (1 + \alpha)S - \overline{X})] + p[X - (1 + \alpha)S] \right\} f(X) \, dX \\
\frac{dY [S]_1}{dS} = \int_{(1+\alpha)S}^{\infty} \frac{d\left\{ (1 + \alpha)S[P - \lambda ( (1 + \alpha)S - \overline{X})] + p[X - (1 + \alpha)S] \right\} f(X)}{dS} \, dX \\
- (1 + \alpha)(1 + \alpha)S[P - \lambda ( (1 + \alpha)S - \overline{X})]f((1 + \alpha)S) \\
= \int_{(1+\alpha)S}^{\infty} \left\{ (1 + \alpha)[P - \lambda ( (1 + \alpha)S - \overline{X})] - (1 + \alpha)S\lambda (1 + \alpha) \\
n - p(1 + \alpha)]f(X) \, dX - (1 + \alpha)(1 + \alpha)S[P - \lambda ( (1 + \alpha)S - \overline{X})]f((1 + \alpha)S)
\]

\[
Y [S]_2 = \int_{0}^{(1-\alpha)S} \left\{ (1 - \alpha)S[P + \lambda (\overline{X} - (1 - \alpha)S)] + h[(1 - \alpha)S - X] \right\} f(X) \, dX \\
\frac{dY [S]_2}{dS} = \int_{0}^{(1-\alpha)S} \frac{d\left\{ (1 - \alpha)S[P + \lambda (\overline{X} - (1 - \alpha)S)] + h[(1 - \alpha)S - X] \right\} f(X)}{dS} \, dX \\
+ (1 - \alpha)(1 - \alpha)S[P + \lambda (\overline{X} - (1 - \alpha)S)]f(1 - \alpha)S \\
= \int_{0}^{(1-\alpha)S} \left\{ [(1 - \alpha)[P + \lambda (\overline{X} - (1 - \alpha)S)] - (1 - \alpha)S\lambda (1 - \alpha) \\
n + h(1 - \alpha)]f(X) \, dX + (1 - \alpha)(1 - \alpha)S[P + \lambda (\overline{X} - (1 - \alpha)S)]f(1 - \alpha)S
\]
\[
Y [S]_3 = \int_{(1-\alpha)S}^{\bar{X}} X[P + \lambda (\bar{X} - X)]f(X)\,dX + \int_{\bar{X}}^{X} XPf(X)\,dX \\
+ \int_{\bar{X}}^{(1+\alpha)S} X[P - \lambda (X - \bar{X})]f(X)\,dX \\
\]

\[
\frac{dY [S]_3}{dS} = \frac{d}{dS} \int_{(1-\alpha)S}^{\bar{X}} X[P + \lambda (\bar{X} - X)]f(X)\,dX + \frac{d}{dS} \int_{\bar{X}}^{(1+\alpha)S} X[P - \lambda (X - \bar{X})]f(X)\,dX \\
= \int_{(1-\alpha)S}^{\bar{X}} \frac{d[X[P + \lambda (\bar{X} - X)]f(X)]}{dS}\,dX \\
- (1 - \alpha)(1 - \alpha)S[P + \lambda (\bar{X} - (1 - \alpha)S)]f((1 - \alpha)S) \\
+ \int_{\bar{X}}^{(1+\alpha)S} \frac{d[X_{t}[P - \lambda (X - \bar{X})]f(X)]}{dS}\,dX \\
+ (1 + \alpha)(1 + \alpha)S[P - \lambda ((1 + \alpha)S - \bar{X})]f((1 + \alpha)S) \\
= (1 + \alpha)(1 + \alpha)S[P - \lambda ((1 + \alpha)S - \bar{X})]f((1 + \alpha)S) \\
- (1 - \alpha)(1 - \alpha)S[P + \lambda (\bar{X} - (1 - \alpha)S)]f((1 - \alpha)S) \\
\]

Hence,
\[
\frac{dY[S]}{dS} = \frac{dY[S]}{dS}^1 + \frac{dY[S]}{dS}^2 + \frac{dY[S]}{dS}^3
\]

\[
= \int_{(1+\alpha)S}^{\infty} [(1+\alpha)[P - \lambda ((1+\alpha)S - \bar{X})] - (1+\alpha)S\lambda (1+\alpha) - p(1+\alpha)] f((1+\alpha)S) \]

\[
- p(1+\alpha)]f(X)dX - (1+\alpha)(1+\alpha)S[P - \lambda ((1+\alpha)S - \bar{X})]f((1+\alpha)S) \]

\[
+ \int_{0}^{(1-\alpha)S} [(1-\alpha)[P + \lambda (\bar{X} - (1-\alpha)S)] - (1-\alpha)S\lambda (1-\alpha)] f((1-\alpha)S) \]

\[
+ h(1-\alpha)]f(X)dX + (1-\alpha)(1-\alpha)S[P + \lambda (\bar{X} - (1-\alpha)S)]f((1-\alpha)S) \]

\[
+ (1+\alpha)(1+\alpha)S[P - \lambda ((1+\alpha)S - \bar{X})]f((1+\alpha)S) \]

\[
- (1-\alpha)(1-\alpha)S[P + \lambda (\bar{X} - (1-\alpha)S)]f((1-\alpha)S) \]

\[
= \int_{(1+\alpha)S}^{\infty} [(1+\alpha)[P - \lambda ((1+\alpha)S - \bar{X})] - (1+\alpha)S\lambda (1+\alpha) \]

\[- p(1+\alpha)]f(X)dX \]

\[
+ \int_{0}^{(1-\alpha)S} [(1-\alpha)[P + \lambda (\bar{X} - (1-\alpha)S)] - (1-\alpha)S\lambda (1-\alpha)] f((1-\alpha)S) \]

\[
+ h(1-\alpha)]f(X)dX \]

\[
0 = [(1 + \alpha)[P - \lambda ((1 + \alpha)S - \bar{X})] - (1 + \alpha)S\lambda (1 + \alpha) - p(1 + \alpha)] (1 - F((1 + \alpha)S)) \]

\[
+ [(1 - \alpha)[P + \lambda (\bar{X} - (1 - \alpha)S)] - (1 - \alpha)S\lambda (1 - \alpha) \]

\[
+ h(1 - \alpha)]F((1 - \alpha)S) \]

To simplify this problem we take the most restricted situation in to consideration, that is, to set the value of \(\alpha = 0\). In this case,

\[
0 = [(P - \lambda (S - \bar{X})] - S\lambda - p](1 - F(S)) + [(P + \lambda (\bar{X} - S)] - S\lambda + h] F(S) \]
\[ F(S^*) = P\{X \leq S^*\} = \frac{P - 2\lambda S + \lambda X - p}{\lambda X - \lambda X - p - h} \]

Long term time inflexible contract:

\[ Y[Q_t] = \int_{(1+\alpha)Q_t}^{\infty} [p[x_t - (1 + \alpha)Q_t] + g(x_t)(1 + \alpha)Q_t] f(x_t) dx_t + \int_{(1-\alpha)Q_t}^{(1+\alpha)Q_t} g(x_t)x_t f(x_t) dx_t + \int_{0}^{(1-\alpha)Q_t} [h[(1 - \alpha)Q_t - x_t] + g(x_t)(1 - \alpha)Q_t] f(x_t) dx_t \]

\[ g(x_t) = \begin{cases} 
  P - \lambda((1 + \alpha)F_t - \bar{D}), & \text{if } x_t > (1 + \alpha)F_t, \\
  P - \lambda(x_t - \bar{D}), & \text{if } \bar{D} \leq x_t \leq (1 + \alpha)F_t, \\
  P, & \text{if } \bar{D} \leq x_t \leq \bar{D}, \\
  P + \lambda(\bar{D} - x_t), & \text{if } (1 - \alpha)F_t \leq x_t \leq \bar{D}, \\
  P + \lambda(\bar{D} - (1 - \alpha)F_t), & \text{if } x_t < (1 - \alpha)F_t, 
\end{cases} \]

\[ Q_t = \begin{cases} 
  D_t, & \text{if } (1 - \alpha)F_t \leq D_t \leq (1 + \alpha)F_t, \\
  (1 + \alpha)F_t, & \text{if } D_t \geq (1 + \alpha)F_t, \\
  (1 - \alpha)F_t, & \text{if } D_t \leq (1 - \alpha)F_t, 
\end{cases} \]

\[ F(Q_t^*) = P\{X \leq Q_t^*\} = \frac{P - 2\lambda Q_t + \lambda \bar{D} - p}{\lambda \bar{D} - \lambda \bar{D} - p - h} \]