ESSAYS ON MINING ECONOMICS AND SYSTEM MODELING

A Dissertation in

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by

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ABSTRACT

Investigating and promoting the understanding of economic, policy, management and supply chain issues associated with mines, mining economics is an important discipline that has been emphasized by the industry, relevant consulting firms, government agencies and research institutions. This dissertation is composed of four essential topics on mining economics, including evaluation of a gold mine with uncertainty in price and operational flexibility, the assessment of how critical the rare earth element-Yttrium is to the US’ economy, the determination of economic cutoff grade for a two-mineral metal mine, and a theoretical model for stockpiling and processing the intermediate grade ore.

Traditional evaluation technics, such as discounted cash flow approach, fails to address the uncertainties in commodity’s price and mining cost, as well as operational flexibility. In the first chapter, a real option model is set up to evaluate a gold mine incorporating the uncertainties, and determine the optimal price threshold to trigger the mining investment. By extension of the model, the research examines the effects of postponing the delivery of commodity to market. The results show that the real option value by virtue of operational flexibility can be significant and should not be ignored when valuing a mine.

The vulnerability of the US’ economy to restrictions in the supply of various rare earth elements have been of great concern in recent years. The second chapter examines how critical Yttrium is to US’ national economy, by considering its supply sources, China’s export quota and tax, relevant uses, possible substitution in use, and recycling. Its production, however, causes a variety of environmental harms, making the supply stream vulnerable to more stringent environmental regulation. Yttrium is difficult to replace in compact fluorescent lights (CFL), its largest use. The high short-run demand for CFLs increases the criticality of Yttrium. In the longer run, however, the demand for CFLs is expected to decline gradually.
Normally, there is variability in ore’s quality for metal mines. The third chapter introduces a net present value maximization algorithm for a by-product mining project. The algorithm accounts the time value of money effect for the cash flows. A rare earth case study is illustrated contingent on the algorithm. It is found the optimal economic cutoff grade of the primary product can be significantly impacted by the changes in economic variables related to the by-product.

The fourth chapter follows the problem of varied quality in ore, by establishing a theoretical two-stage model for a mining system with production from the stockpiled material. In a metal mine, the intermediate grade material between economic and breakeven cutoff grades is typically stockpiled for future processing after depletion. By deriving the first order condition for objective profit function and parameterized analysis, this research finds the profit from processing the stockpiled material contributes to a mine’s profit significantly; processing the stockpiled also shifts the optimal mining strategy; in addition, the research measures how responsive the optimal mining rate is to input variables’ change, and finds the impact of some of the variables is diminishing. The intrinsic advantages of the theoretical model compared to numerical approaches in interpreting the economic intuitive of stockpiling management and optimal mining strategy are discussed.

Keywords: Mining Economics, Real Option Evaluation, Mean-reverting of Commodity Price, Material Criticality, Yttrium, China’s Rare Earth Policy, Cutoff Grade Algorithm, Mining System Modeling, Stockpiling
TABLE OF CONTENTS

List of Figures ........................................................................................................................................... viii
List of Tables .................................................................................................................................................. xii
Acknowledgements ....................................................................................................................................... xiv

Chapter 1 ......................................................................................................................................................... 1
  Real Option Value of Mining Operations with Mean-reverting Commodity Price ......................... 1
  Abstract ..................................................................................................................................................... 1
  1. Introduction ........................................................................................................................................... 2
  2. A Comprehensive Review of Real Option’s Application in Mining Problems ......................... 5
      2.1 Introduction of Literature Review ................................................................................................. 5
      2.2 Longstaff and Schwartz (2001)’s Option Model ........................................................................... 5
      2.3 Literature based on the Lattice/Binomial Tree Method .............................................................. 12
      2.4 Review of the Black-Scholes Formulas and Application in Real Option Problems ............. 18
      2.5 Empirical Work ............................................................................................................................... 21
      2.6 Cutoff Grade and Real Option ....................................................................................................... 25
      2.7 Other Literature .............................................................................................................................. 25
      2.8 Conclusion of Literature Review ................................................................................................... 26
  3. Mean-reverting Process for Commodity Price ................................................................................. 28
      3.1 Mean-reverting Model ................................................................................................................... 28
      3.2 Estimation Results ........................................................................................................................... 29
      3.3 Half-life of Mean-reverting and Mean-reverting Speed .............................................................. 30
      3.4 Simulation of Price .......................................................................................................................... 31
  4. Real Option Model and Optimal Timing ......................................................................................... 32
      4.1 Model of Exercising the Option and Price Threshold ................................................................. 32
      4.2 Four Illustrative Price Paths and Expected Profit ...................................................................... 33
  5. Extensions of the Model ...................................................................................................................... 37
      5.1 Differentiated Mining Cost and Option-Exercising Probability ............................................... 37
      5.2 Differentiated Starting Price of Simulation ................................................................................. 40
      6. Stochastic and Declining Mining Cost ............................................................................................ 42
  7. Effects of Postponing Delivery of the Commodity Product ......................................................... 50
      7.1 Algorithm to Examine the Effect of Postponing the Delivery of the Commodity Product .... 50
      7.2 Results for Effects of Postponing the Delivery ........................................................................... 53
  8. Conclusion ............................................................................................................................................... 58

Reference ...................................................................................................................................................... 60

Chapter 2 ....................................................................................................................................................... 63
  A Strategic Approach for Criticality – Application to Yttrium ......................................................... 63
  Abstract ................................................................................................................................................... 63
  1. Introduction .......................................................................................................................................... 64
  2. Yttrium Resources .............................................................................................................................. 65
      2.1 China ............................................................................................................................................. 66
      2.2 Rare Earth Resource in India ....................................................................................................... 71
1. Introduction .......................................................................................................................... 151
2. A Two-stage Production Model ......................................................................................... 155
3. A Uniform Distribution of Grade ...................................................................................... 157
4. Derivatives Respect to Mining Rate .................................................................................. 160
5. First Order Condition for the Profit Functions in First and Second Stages ............. 162
6. Parameterization and Discussion ..................................................................................... 163
7. Capital Cost and Optimal Mining Rate ............................................................................. 169
8. Pseudo-Elasticity ............................................................................................................... 170
9. Discussion of the Results and Comparing to the Lane’s Cutoff Grade Model (1988) ......................................................................................................................... 174
10. Conclusion ........................................................................................................................ 176
Reference ................................................................................................................................ 178
LIST OF FIGURES

Figure 1 Normalized Real Gold Price from 1968 to 2012 (Price in December 2012 is set at 1) .......................................................... 4

Figure 2 Potential Future Coal Prices ($/ton) over a Ten-year Period (Hall and Nicholls, 2007) .......................................................... 14

Figure 3 Free Cash Flows ($ million) (Hall and Nicholls, 2007) .......................................................... 15

Figure 4 Discounted Cash Flow Valuation ($ million) of the Coal Mine over a Ten-year Period (Hall and Nicholls, 2007) .......................................................... 16

Figure 5 Real Option Valuations ($ million) of the Coal Mine over a Ten-year Period (Hall and Nicholls, 2007) .......................................................... 16

Figure 6 Discounted Cash Flow and Real Options Valuations ($ million) Including the Option to Expand in Year 5 (Hall and Nicholls, 2007) .......................................................... 17

Figure 7 Five Illustrative Paths of Simulated Gold Prices for the Next 35 Years ................. 32

Figure 8 Path A of Simulated Price and Profit by Exercising the Option vs. X .................. 34

Figure 9 Path B of Simulated Price and Profit by Exercising the Option vs. X .................. 34

Figure 10 Path C of Simulated Price and Profit by Exercising the Option vs. X .................. 35

Figure 11 Path D of Simulated Price and Profit by Exercising the Option vs. X .................. 36

Figure 12 Expected Profit vs. X by 10,000 Simulations .......................................................... 37

Figure 13 ROV and NPV of One Ounce of Gold (USD) .......................................................... 38

Figure 14 Flexibility Value ($/Oz.) for Different Mining Costs .......................................................... 39

Figure 15 Probabilities of Exercising the Option within 5 Years and 35 Years .................. 40

Figure 16 ROV and NPV for a Mine with Mining Cost of $1,000/Oz. by Differentiated Gold Price from $100/Oz. to $2,300/Oz. .......................................................... 41

Figure 17 Flexibility Value for a Mine with Mining Cost of $1,000/Oz. by Differentiated Gold Price from $100/Oz. to $2,300/Oz. .......................................................... 41

Figure 18 Three Illustrative Paths of Simulated Mining Cost for One Ounce of Gold (2% yearly decrease trend and 4% yearly standard deviation) ............................................. 43

Figure 19 Net Present Value (NPV), Real Option Value (ROV), and Effects of Decreasing Stochastic Mining Cost .......................................................... 45
Figure 20 Thresholds of Exercising the Option, for Constant Mining Cost and Decreasing Non-Stochastic Mining Cost .................................................................46

Figure 21 Real Option Value (ROV) for Mining Cost of 1,100$/Oz., 1,700$/Oz. and 2,300$/Oz. with Differentiated Standard Deviation of Mining Cost from 1% to 20% ....49

Figure 22 Thresholds for Mining Cost of 1,100$/Oz., 1,700$/Oz. and 2,300$/Oz. with Differentiated Standard Deviation from 1% to 20% ................................................50

Figure 23 “If-else” Statement to Generate the Matrix “profit” and “fullprofit” ........52

Figure 24 “If-else” Statement to Generate the Matrix “profit3” and “fullprofit3” ........52

Figure 25 NPV and Real Option Value for No Waiting, and Waiting Three Years and Five Years ........................................................................................................55

Figure 26 Value Lost by Waiting One Year, Three Years and Five Year ................55

Figure 27 Value Lost and Percent of Lost by Waiting Three Years for Differentiated Mining Cost ......................................................................................................56

Figure 28 Exercising Price Threshold by Differentiated Mining Cost .....................57

Figure 29 Exercising Probability by Differentiated Mining Cost ............................57

Figure 30 Distribution of China Rare Earth Operating Deposits ..............................67

Figure 31 Baotou (Inner Mongolia) Bastnaesite Concentrate ................................70

Figure 32 Longnan (Jiangxi) Ion Adsorption Clays .............................................70

Figure 33 An Example of Leaching Tanks Processing Ion Adsorption Clays ..........84

Figure 34 An Example of Large-scale Leaching Heap Processing Ion Adsorption Clays ....84

Figure 35 An Example of In-situ Leaching In Longnan, Jiangxi .........................85

Figure 36 Supply, Demand and Welfares of Rare Earths Export When Tax Binds More than Quota ..................................................................................................89

Figure 37 Supply, Demand and Welfares of Rare Earths Export When Quota Binds More than Tax ............................................................................................90

Figure 38 Number of Y2O3 Exporting Firms ..........................................................91

Figure 39 Number of Y2O3 Exporting Firms by Ownership Category ..................92

Figure 40 Number of Y2O3 Exporting Firms by Type ...........................................92

Figure 41 Y2O3 Export Average Price (RMB/kilogram) .........................................93
Figure 42 Y2O3 Total Export (kilogram) (Asian Metal Rare Earth Statistics, 2000 to 2009) ................................................................. 93

Figure 43 Total Value of Y2O3 Export (RMB) (Asian Metal Rare Earth Statistics, 2000 to 2009) ................................................................. 94

Figure 44 China's Yearly Yttrium Oxide Exports (Asian Metal, 2013) ................................................................. 95

Figure 45 Share of Yttrium Import from China (Jan 2007 to Jul 2012) by Country (Asian Metal, 2013) ................................................................. 95

Figure 46 US Yearly Yttrium Compound Import (Asian Metal, 2013) ................................................................. 96

Figure 47 Share of US Yttrium Import Compound by Countries (May 2006 to Jul 2011) (Asian Metal, 2013) ................................................................. 97

Figure 48 Percent of Price Decrease by Different Demand Elasticity ($\alpha$) and Supply Elasticity ($\beta$) due to an increase of 20% in supply from ROW Yttrium Prices .......... 100

Figure 49 Yttrium Metal and Oxide Prices in China’s Domestic Market and Export Market (Asian Metal, 2013) ................................................................. 101

Figure 50 Projected Domestic Compact Fluorescent Lighting Shipments under EISA 2007 Standards (Technology Metals Research, 2013) ................................................................. 107

Figure 51 Rare Earth Oxide Consumption (metric tons) in the Phosphor Market Sector in Metric Tons (Blade, 2010) ................................................................. 108

Figure 52 Short-term (Present-2015) Criticality Matrix (DOE, 2010 and 2011) ................. 121

Figure 53 Mid-term (2015-2025) Criticality Matrix (DOE, 2010 and 2011) ................. 121

Figure 54 An Example of Gold Mine’s Tonnage, Average Grade and Cutoff Grade Distribution (China Minmetals Corporation 2010) ................................................................. 130

Figure 55 Two-mineral By-product Algorithm for Maximization of the Value of a Mine ..... 143

Figure 56 Equivalent Cutoff for Combined Materials (Nd and Dy) and Nd’s Cutoff as the Mining Activity Proceeds for Scenario 1 of Dy’s Price ......................... 145

Figure 57 Nd’s Cutoff for Three Differentiated Dy’s Price Scenarios as the Mining Activity Proceeds ................................................................. 147

Figure 58 Value of the Mine’s Future Cash Flows by Years for Three Differentiated Dy’ Price Scenarios as the Mining Activity Proceeds ................................................................. 148

Figure 59 Economic Cutoff by Lane’s Model, Breakeven Cutoff and the Stockpiled Material ................................................................. 152
<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>A Two-Stage Production Model with Cutoff Grades and Processing the Stockpiled</td>
</tr>
<tr>
<td>61</td>
<td>Cutoff Grade for the First Stage Production ((x_1)) by Differentiated Mining Rate</td>
</tr>
<tr>
<td>62</td>
<td>Length of the First Stage Production ((T_1)) by Differentiated Mining Rate</td>
</tr>
<tr>
<td>63</td>
<td>Length of the Second Stage Production ((T_2 - T_1)) by Differentiated mining Rate</td>
</tr>
<tr>
<td>64</td>
<td>Present Value of the Profit in the 1st Stage ((\pi_1)) by Differentiated Mining Rate</td>
</tr>
<tr>
<td>65</td>
<td>Present Value of the Profit in the 2nd Stage ((\pi_2)) by Differentiated Mining Rate</td>
</tr>
<tr>
<td>66</td>
<td>Present Value of the Profit of Production for Two Stages ((\pi)) by Differentiated Mining Rate</td>
</tr>
<tr>
<td>67</td>
<td>Percent of the Profit of the 1st Stage Production in the Mine’s Total Profit by Differentiated Mining Rate</td>
</tr>
<tr>
<td>68</td>
<td>Percent of the Profit of the 2nd Stage Production in the Mine’s Total Profit by Differentiated Mining Rate</td>
</tr>
<tr>
<td>69</td>
<td>Profit of the Mine with Capital Cost by Differentiated Mining Rate</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 1 8 Paths of the Simulated Price relying on Geometric Brownian motion .......................... 7
Table 2 Cash Flows Realized at the Time 3 by the Option’s Owner.................................................. 7
Table 3 Vectors Y and X used to Estimate the Payoff Received at Time 3 if the Option is Not Exercised at Time 2 ........................................................................................................... 8
Table 4 Payoff of Exercising the Option at Time 2 and Conditional Estimation of Payoff for Continuation at Time 2 ........................................................................................................... 9
Table 5 Cash Flows Received if the Option is not Exercised before Time 2 ................................. 9
Table 6 Vectors Y and X used to Estimate the Payoff Received at Time 2 if the Option is Not Exercised at Time 1 ........................................................................................................... 10
Table 7 Payoff of Exercising the Option at Time 1 and Conditional Estimation of Payoff for Continuation at Time 1 ........................................................................................................... 10
Table 8 Optimal Exercising Rule ............................................................................................................. 11
Table 9 Cash Flows Received for Each Price Path ................................................................................. 11
Table 10 Copper Mines with Operational Flexibility in Canada from 1980 to 1993 (Slade, 2001) ........................................................................................................................................... 22
Table 11 Estimation Result of Least Square Regression ........................................................................ 30
Table 12 Real Option Values and Option Exercising Threshold for Decreasing (2% Yearly) Mining Cost of 1,100$/Oz., 1,700$/Oz., and 2,300$/Oz. by differentiated Standard Deviation from 1% to 20% ........................................................................ 48
Table 13 Yttrium Reserves in Thousand Metric Tons of Oxide by Countries (USGS, 1996-2013) ............................................................................................................................... 66
Table 14 Distributions of Ion Adsorption Deposits in Southern China (Ministry of Environmental Protection, 2010) .................................................................................................................. 68
Table 15 Northern and Southern China Yttrium and Other REEs’ Grades (Schuler, Liu and Merz, 2011) ........................................................................................................................... 69
Table 16 Top 10 Yttrium Projects (in terms of total tons of Y2O3) under Development Outside of China and India by March 8th, 2013 (Technology Metals Research, 2013) .......................................................................................................................... 75
Table 17 Production Cost of producing 1 Metric ton of Yttrium-Europium Oxide by Tank Leaching, Heap Leaching and In-situ Leaching (Zou, 2012) .......................................................... 82
Table 18 China's Rare Earth Production, Consumption and Export Quotas in Metric Tons of Rare Earth Oxide from 2000 to 2011 (Tse, 2011) .................................................................87

Table 19 China's Rare Earth Export Duty Rates (China Customs Import and Export Tariff Department, 2007-2011) .................................................................................................88

Table 20 Price Changes by Different Demand Elasticity ($\alpha$) and Supply Elasticity ($\beta$) due to an increase of 20% in supply from ROW ..................................................................................100

Table 21 Domestic Yttrium (Y2O3) Use by Categories from 1995 to 2010 in Metric Tons (USGS, 1996-2013) ..................................................................................................................104

Table 22 General Chemical Formulas for Phosphors and Share of Weight ................................105

Table 23 Relative Importance of End-Use Applications for Yttrium (Proportion of total US Yttrium for each application was determined from USGS Yttrium Summary, 2010) ................................................................116

Table 24 Tonnage-grade (tons) Distribution of Nd and Dy (China Minmetals Corporation 2010) .................................................................................................................................138

Table 25 Average Grade (%) of Nd at Different Nd and Dy Intervals (China Minmetals Corporation 2010) ..................................................................................................................138

Table 26 Average Grade (%) of Dy at Different Nd and Dy Intervals (China Minmetals Corporation 2010) ..................................................................................................................139

Table 27 Technical/Economic Parameters (China Minmetals Corporation 2010) ...............139

Table 28 Combined Tonnage-grade Inputs to Algorithm ........................................................140

Table 29 Scenarios Description .................................................................................................143

Table 30 Economic Input Parameters of the Gold Mine ..........................................................164

Table 31 Optimal Mining Rate when Each Input Variable Decreases -10%, -5%, -1%, and Increases 10%, 5%, 1% ..........................................................................................................171

Table 32 Change Percent of Mining Rate when Each Input Variable Decreases -10%, -5%, -1%, and Increases 10%, 5%, 1% ..........................................................................................171

Table 33 Pseudo-Elasticities for Input Variables on Optimal Mining Rate $M^*$ ......................172
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Chapter 1

Real Option Value of Mining Operations with Mean-reverting Commodity Price

Abstract

By determining the optimal price threshold for mining activation, this research aims at estimating a mine’s in-situ value by incorporating its real option value (ROV). The traditional discounted cash flow (DCF) method, the standard tool for economic feasibility studies in mining industry, can be problematic since it fails to address uncertainties and operational flexibilities (Trigeorgis 1990; Schwartz 1997; Slade 2001; Abdel Sabour and Dimitrakopoulos 2011). DCF normally results in under-evaluation when significant variability is present in commodity prices such as gold, silver, copper and recently rare earths. A mining project is more valuable in the expected value terms if it is activated following an appropriately chosen price threshold. In this research, the commodity price is modeled using a mean-reverting process, which is more relevant to commodity economics than the generally used Geometric Brownian motion process (Pindyck and Rubinfeld 1997). Optimal threshold for triggering a mine is then calculated. It is shown that the value of flexibility is significant and peaks when mining cost equals price; the threshold increases as average cost rises and probability of exercising the option is estimated; the stochastic pattern of the mining cost and the effects of postponing delivery of the product on real option value and threshold are also investigated. ROV method provides a tractable and realistic scheme to evaluate a mine’s in-situ value and a strategy to manage mining activities.
1. Introduction

Traditional evaluation of mining projects relies on the discounted cash flows (DCF), or the net present value (NPV) method. These methods are taken as standard tools and used broadly in mining feasibility studies. Nevertheless, these valuation methods fail to cover the uncertainties embedded in commodity price and operational cost, and often result in an under-evaluation of mine’s value. The other problem is that operational flexibility is not considered such as delaying the investment or expanding production rate under certain market conditions. In addition, DCF or NPV may not be relevant for valuation of a mining reserve that has not yet started to produce and is not generating any revenue (Botin et al. 2010).

The investment decision of mining activities involves the feature of irreversibility as 1) there is considerable amount of sunk cost involved in the mining processes, rendering it fairly costly to withdraw the investment; and 2) once mined and processed, the materials are gone; 3) the irreversibility can also arise from government regulation such as expiry of mining lease or permit, institutional arrangements such as investment in labor because of high costs of hiring, training and firing employees. Dixit and Pindyck (1994) suggest that an irreversible investment opportunity is like a financial call option, which offers the holder a right but not obligation, within a specified amount of time, to pay a strike price and in return, receive a financial asset. A mining firm with an investment opportunity has a similar option to invest immediately or in a future time, in return for the commodity products.

Evaluation methods covering timing flexibility or ROV method are generally believed to be more accurate and realistic than non-flexible evaluation techniques, i.e., DCF or NPV (Brennan and Schwartz 1985; Cortazar and Casassus 1998; Bengtsson 2001; Samis et al. 2006; Dimitrakopoulos and Sabour 2007). By taking advantage of understanding the likely trends in the commodity market, the owner of a mining operation can achieve additional expected profit by
setting an optimal price threshold. The real option method can assist the mine’s owner to 1) assess the value of a mine by considering the uncertainties and operational flexibility embedded in project; 2) making managerial decisions such as start mining, abandonment, expansion and contraction of production rate accordingly to changes in market. This research addresses the case of starting mining.

A substantial portion of mining project’s uncertainty comes from the volatility in commodity price (Hall and Nicholls 2007). In this research, a mean-reverting (MR) process is used to model gold price. MR process is based on microeconomic theory and argues that commodity prices are constrained by the average mining cost. Thus, the ups and downs of a commodity price are not permanent, as the price tends to revert toward a long-term equilibrium. By attaining a long-term equilibrium, MR model is considered to be more appropriate than the widely used Geometric Brownian motion (GBM) process to model commodity prices (Pindyck and Rubinfeld 1991).

Figure 1 presents the normalized real price of gold from 1968 to 2012 (with the price in December 2012 set at 1). In Figure 1, the nominal gold price is from London Bullion Market, and adjusted by U.S. Consumer Price Index from US Bureau of Labor Statistics to obtain the real gold price. It can be observed there has been considerable value associated with delaying investment for a gold mine during the years from 2000 to 2012, as the price skyrocketed during the time. Moderate volatility of gold price also implies an incentive for delaying investment and waiting for future price information to arrive. Thus, potential value can be achieved by waiting due to possible increase of commodity price. In contrast, waiting too long before start mining may result in a lower profit – even when the price rises – because of the time value of money effect; therefore, there is an optimal threshold for the commodity price beyond which it is optimal to exercise the option to start mining. Consequently, the relevant question to be asked is how to determine this price threshold.
This chapter is organized as following: In section 2, previous literature on this subject is reviewed and contribution of this research is outlined. In section 3, the mean-reverting process is adopted to model the price of gold. The estimated coefficients of the process are used to generate 10,000 simulated paths of gold prices for the following 35 years. Section 4 proposes the option-exercising model and four illustrative paths are selected to explicate the option-exercising mechanism. Section 5 is composed of two extensions of the real option method, comparing the ROV and NPV by 1) differentiated mining cost and 2) differentiated starting price used in the simulation. Section 6 extends the option model by examining the uncertainty in mining cost. Section 7 examines the effects of postponing delivery of the commodity product to market, on the real option value and price threshold. Section 8 concludes by discussion and implications of the results.
2. A Comprehensive Review of Real Option’s Application in Mining Problems

2.1 Introduction of Literature Review

There are five trends of literature that have addressed the real option problem in mining projects. They are Longstaff and Schwartz (2001)’s option model based on numerical method, the binomial tree/lattice method, Black and Scholes’ (1973) Formulas, empirical work, and applying the real option method in cutoff grade calculation. This section summarizes these trends of literature, discusses the limitations of the previous work, shows how this research is advanced over previous ones, and points out several future research directions.

2.2 Longstaff and Schwartz (2001)’s Option Model

Longstaff and Schwartz (2001)’s option model, or least square Monte Carlo model (LSM) has been applied to evaluate real option in mining problem extensively and a large portion of the relevant literature is based on this model. The model is relying on 1) least squares to estimate the conditional expected payoff to the option’s holder from continuation (or suspension) with simulation method. The intuition of this approach is that, the owner of an American option optimally compares the payoff from immediate exercising, with the expected payoff by continuing to keep the option alive, and exercises the option if the immediate payoff is higher. Longstaff and Schwartz (2001) regress the *ex post* realized payoffs by continuation on the underlying asset’s price.

Several pieces of literature use this approach, or make extension on it: Blais *et al.* (2005) extend the approach by considering a multi-mineral mine, by converting the secondary mineral to the metal equivalents of the primary mineral using an equivalent factor; Abdel Sabour and Poulin (2006) take advantage of the model in a case of a multi-mineral mine in Quebec, Canada,
producing nickel, copper, cobalt, gold, silver, platinum and palladium; Dimitrakopoulos and Abdel Sabour (2007) apply the approach to evaluate an Australian gold mine and making operational decisions using the Geometric Brownian motion process to model gold price; Abdel Sabour et al. (2008) use the approach along with Geometric Brownian motion to evaluate American style option of a mining operation, by considering the uncertainties from quantity and ore’s quality, metal prices, and foreign exchange; Abdel Sabour and Dimitrakopoulos (2011) apply the approach to select optimal limit for open-pit mine; Lemelin et al. (2006) adopt the model in a case study of complex mining project in Raglan, Canada, which consists of numerous mineralized zones and produces various minerals; Akbari et al. (2008) use the model to find the best ultimate pit limit for an open-pit mine. Following paragraphs use a numerical example to explicate Longstaff and Schwartz (2001)’s model by evaluating an American option. Disadvantages of the model are discussed after the demonstration.

2.2.1 A Numerical Example of Longstaff and Schwartz (2001)’s Option Model

This example inspects an American put option on a share of stock without paying dividend. The strike price is assumed to be $1.10 at time points 1, 2 and 3, and the time 3 is the mature date. The interest rate is assumed to be 6%, and there are 8 paths for the stock prices generated by Geometric Brownian motion as shown in formula (1.1),

\[ dS_t = \mu S_t dt + \sigma S_t dW_t \]

in which, \( dS_t \) is the incremental change of stock price; \( \mu \) is the drift term; \( \sigma \) is the volatility of the price; \( dW_t \) is a stochastic term, which is randomly drawn from normal distribution (Dixit and Pindyck 1994).
Following Table 1 shows 8 paths of simulated stock prices for the three time points. The option algorithm works backwards and Table 2 shows the cash flows realized by the optimal strategy at time 3.

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<td>1.00</td>
<td>1.09</td>
<td>1.08</td>
<td>1.34</td>
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</table>

Table 1 8 Paths of the Simulated Price based on Geometric Brownian motion

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<th>t=2</th>
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</tr>
</thead>
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<tr>
<td>2</td>
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<td>-</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>6</td>
<td>-</td>
<td>-</td>
<td>0.09</td>
</tr>
<tr>
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<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>8</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2 Cash Flows Realized at the Time 3 by the Option’s Owner

At the time 2, if the option is “in the money”, owner of the option need to decide whether to exercise the option immediately, or continue to active the option to the final expiry date in time 3. At the time 2, there are 5 paths for which the option is “in the money” (Path 2, 3, 4, 6, 8). For the five “in the money” paths at time 2, to make conditional estimates of the payoff received at time 3 (denoted Y) if the option is not exercised at time 2, the model denote the stock prices at time 2 for these 5 paths to be X. If the option is not “in the money”, the owner has only one choice – wait for the time 3 hoping the price will decrease to lower than the strike price (so that
the ordinary least squares (OLS) regression only includes the “in the money” paths at time 2 to estimate the payoff received at time 3 if the option is not exercised at time 2. Table 3 shows $X$ and $Y$ for the OLS regression (a discount factor of $e^{-0.06} = 0.94176$ is used to convert the payoff in time 3 to the value in time 2).

<table>
<thead>
<tr>
<th>Path</th>
<th>$Y$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.20*0.94176</td>
<td>0.77</td>
</tr>
<tr>
<td>3</td>
<td>0.18*0.94176</td>
<td>0.97</td>
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<td>4</td>
<td>0.07*0.94176</td>
<td>1.07</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.09*0.94176</td>
<td>0.84</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>0*0.94176</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Table 3 Vectors $Y$ and $X$ used to Estimate the Payoff Received at Time 3 if the Option is Not Exercised at Time 2

Exercised at Time 2

Regress $Y$ against a constant, $X$ and $X^2$ by following formula (1.2),

\[ Y = a_0 + a_1X + a_2X^2 \]

And the least square regression result is,

\[ E[Y|X] = -1.070 + 2.983X - 1.813X^2 \]

With formula (1.2), the conditional payoff in time 3 if the option is not exercised in time 2 can be estimated and compared to the payoff if the option is exercised in time 2. The comparing results are shown in Table 4 (optimal decisions are highlighted bold). Table 4 shows that it is optimal to exercise the option at time 2 for path 4, 6 and 7. Table 5 shows the cash flows received by the option’s owner conditional on not exercising the option before time 2.
It is necessary to examine whether to exercise the option at time 1. There are 5 paths of price “in the money” at time 1 (Path 2, 3, 5, 6 and 8). For these paths, $Y$ and $X$ needed to be defined. $X$ is defined as the stock prices at time 1 for the paths where the option is “in the money”. $Y$ is defined as the realized cash flows in Table 5.
Table 6 Vectors $Y$ and $X$ used to Estimate the Payoff Received at Time 2 if the Option is Not Exercised at Time 1

Based on the $Y$ and $X$ shown in Table 6, estimated conditional expectation function is obtained by formula (1.4),

\[ E[Y|X] = 2.038 - 3.335X + 1.356X^2 \]

Table 7 Payoff of Exercising the Option at Time 1 and Conditional Estimation of Payoff for Continuation at Time 1

By Table 7, it can be observed that it is optimal to exercise the option at time 1 for paths 2, 3, 5 and 6. Relying on Table 7 and Table 4 together, the optimal stopping rule can be obtained and shown in Table 8, where * denote the exercise time for each path. The corresponding option cash flows are given in Table 9.
Table 8 Optimal Exercising Rule

<table>
<thead>
<tr>
<th>Path</th>
<th>t=1</th>
<th>t=2</th>
<th>t=3</th>
</tr>
</thead>
<tbody>
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<td></td>
</tr>
<tr>
<td>2</td>
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<td>3</td>
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<td></td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
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<td></td>
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<tr>
<td>8</td>
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<td></td>
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</tbody>
</table>

Table 9 Cash Flows Received for Each Price Path

<table>
<thead>
<tr>
<th>Path</th>
<th>t=1</th>
<th>t=2</th>
<th>t=3</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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<td>0.07</td>
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</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

With the cash flows in Table 9, the value of the option in each path at time 0 can be calculated by discounting the future cash flows. The average of the option values of 8 paths is the expected option value, which is 0.1144 in this example.

### 2.2.2 Problems of the Longstaff and Schwartz’s Model (2001)

Longstaff and Schwartz (2001)’s model assumes that the market is efficient and the stock price can represent complete information, however this is not necessarily true for metal prices. In the model, decision at a time point (whether to exercise or continue the option) only depends on the price of the stock. However, in a real option mining problem, this is not necessary to be the
case. Whether to exercise or continue the option may also depend on other state variables such as ore quality, and mining cost (technology progress), which cannot be covered in the Longstaff and Schwartz (2001)’s model.

The option model proposed in this research can provide an optimal threshold for exercising the option, while Longstaff and Schwartz (2001)’s model fails to give specific direction on when the option should be exercised. In addition, it only applies to an option with a finite lifespan. This is in contrast to an actual mining option, which at least in theory, can have infinite length. The model can also be improved by using a mean-reverting process when simulating the commodity price, which is more suitable than the Geometric Brownian motion (Pindyck and Rubinfeld 1997).

The model assumes a constant strike price. However, for a real option problem such as a mining activation problem, the strike price (or mining cost) can also be a stochastic process. The model proposed in this research can be easily crafted to model the strike price (mining cost) as a stochastic process and incorporate its uncertainty.

2.3 Literature based on the Lattice/Binomial Tree Method

It is well known that, mineral assets are consistently traded at market values that are greater than their discounted cash flow values (Slade, 2001). Davis (1996), relying on surveying the results of 17 pieces of previous literature, attempts to empirically quantify the “option premium” associated with mineral assets’ value. The surveyed literatures include the work by Mardones (1993), Trigeorgis (1990), Bjerksund and Ekern (1990), and etc. It is found that managerial flexibility appears to add an average of 8 percent to the discounted cash flow value of the typical post-development mineral project, and about 41 percent to the discounted cash flow value of the typical undeveloped mineral project.
Taxes may be collected by some governments from the mining industry when mineral prices are higher than some presumed level. Samis et al. (2007) use discounted cash flow and real option theory with lattice to analyze the impact of the contingent taxes on mining projects. The real option value is calculated with lattice method by risk adjustment, which assumes that commodity price follows a Geometric Brownian motion. It is found that real option method is able to deal the risk associate with prices. Similar model is adopted in research by Davis and Samis (2006), Samis et al. (2007) and Hahn et al. (2007).

The binomial lattice formula for Geometric Brownian motion used in all above research is explained in following paragraphs. The probability of a risk-adjusted up-tick in price of a commodity that has a constant rate of convenience \( \delta \) is

\[
P_{RA,U} = \frac{S_0 e^{(r-\delta)\Delta t} - S_D}{S_U - S_D}
\]

In above formula, \( r \) is the risk-free interest rate; \( S_D \) is the level to which price can drop to; and \( S_U \) is the level to which price can rise to; the current price is \( S_0 \). The probability of a falling price is,

\[
P_{RA,D} = 1 - P_{RA,U}
\]

The risk-adjusted expected price for the next period is,

\[
E_{RA}(S) = P_{RA,U}S_U + P_{RA,D}S_D
\]

\[
= \frac{S_0 e^{(r-\delta)\Delta t} - S_D}{S_U - S_D}S_U + \left(1 - \frac{S_0 e^{(r-\delta)\Delta t} - S_D}{S_U - S_D}\right)S_D = S_0 e^{(r-\delta)\Delta t}
\]

which is simply the formula for the one-period-ahead forward price for a commodity. The price can either increase to \( S_U \) or drop to \( S_D \),

\[
S_U = S_0 e^{\sigma \sqrt{\Delta t}}
\]

\[
S_D = S_0 e^{-\sigma \sqrt{\Delta t}}
\]

where \( \sigma \) is a measure of the price’s volatility.
Above lattice model can be used to generate a tree of commodity prices for finite future years. Following explicates two examples of option valuation in a coal mine by Hall and Nicholls (2007). The two options considered are 1) it is possible to abandon the project with no cost and realize zero value, and 2) the mine’s owner can invest $200 million to initiate an expansion to increase production rate by incurring additional yearly cost. The yearly production rate and the operating/maintenance cost are assumed to be constant. The price fluctuation is the only uncertainty considered in the research, and coal price trees are generated by the lattice model (Figure 2). For each coal price, there is a corresponding Free Cash Flow (FCF) to the mine, which can be calculated by following formula,

\[
FCF_t = [(Price_t - Variable Costs) \times tons - Fixed Costs] \times (1 - tax\ rate)
\]

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<td>55.23</td>
<td>56.18</td>
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</tr>
</tbody>
</table>

Figure 2 Potential Future Coal Prices ($/ton) over a Ten-year Period (Hall and Nicholls, 2007)

The Free Cash Flows are shown in Figure 3. A discounted cash flow valuation assigns a probability to each of the potential cash flows. By discounting the expected cash flows and summing them together, the discounted cash flow values can be calculated using the formula (1.11) (where \( r \) is the interest rate),

\[
DCF = \sum_{t=1}^{n} \frac{FCF_t}{(1+r)^t}
\]
Figure 4 shows the discounted cash flow valuation. As shown in Figure 4, the discounted cash flow for the coal mining operation in the year 0 is $199 million.

In the lower right hand part of Figure 3, there is an area of negative cash flows. At certain time point, it will be optimal to abandon the mine instead of incurring the negative cash flows. The optimal abandon time is determined by following backward procedures: 1) calculate the value of the project by the end of the project’s life for each possible coal price (the most right-hand side column in Figure 4). If the value in the column is negative, it is better to abandon the mine than incurring the negative cash flow; 2) the value of the project in the second-last year can be calculated, given the information about the optimal strategy in last year; At each evaluation point in the 9th year, it is determined whether the mine is more valuable if the operation is continued, or is abandoned. The decision is made, by comparing the value of maintaining production, which is the sum of the discounted expected value at the end of year 10 and the cash flow achieved at year 9, versus the value of abandonment (which is assumed to be 0). Using this backward method, the real option value at each lattice point can be obtained (as shown in Figure 5). The operation’s real option value is $252 million, which is approximately 25% higher than the discounted cash flow value of $199 million.

<table>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<td>118</td>
<td>156</td>
<td>199</td>
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<td>237</td>
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</tr>
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<td></td>
<td>3</td>
<td>27</td>
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<td>-57</td>
<td>-57</td>
<td>-57</td>
<td>-57</td>
<td>-57</td>
<td>-57</td>
</tr>
</tbody>
</table>

| Expected FCFF             | 28.86| 31.31| 33.97| 36.85| 40.66| 44.03| 48.58| 52.55| 57.76| 62.43|

Figure 3 Free Cash Flows ($ million) (Hall and Nicholls, 2007)
The lattice method can be used to examine the expansion of the production rate (such as expanding to 10 million tonnes per year at the 5th year with an investment cost and increased yearly cost, and depleting the mining operation 2 years sooner). If the coal price increases dramatically, the expansion option adds value to the project. As shown in Figure 6, the mine’s value increases to $1013 million at the upper-most node of year 5, which means that it is attractive to expand production rate if the coal price realizes five consecutive increases. In other
cases, the expansion does not achieve higher value, and the total value is $253 million, which includes both the expansion and abandonment options.

Similar method has been used in several other literatures: Shafiee et al. (2009) use the lattice method to maximize the project value of the Century zinc mine in northwest Queensland, Australia; Dunbar et al. (1999) take advantage of the same method to evaluate a gold mine with constant production rate.

The lattice method shows that more value can be achieved when options are considered. However, there are two limitations of the methods: 1) the backward induction method is not suitable to examine the option of activating a mining operation, and 2) the expansion option can only be considered on a specific year, i.e. the 5th year, and the model cannot determine the best time to expand the production rate.

Figure 6 Discounted Cash Flow and Real Options Valuations ($ million) Including the Option to Expand in Year 5 (Hall and Nicholls, 2007)
2.4 Review of the Black-Scholes Formulas and Application in Real Option Problems

Relying on several assumptions, Black and Scholes (1973) derive a formula for the value of a European call option (formula (1.12)) and put option (formula (1.13)) at time 0 in terms of the underlying stock’s price.

\[
C = S_0 N(d_1) - Ke^{-rT} N(d_2)
\]

\[
P = Ke^{-rT} N(-d_2) - S_0 N(-d_1)
\]

Where,

\[
d_1 = \frac{\ln(S_0/K) + (r+\sigma^2/2)T}{\sigma\sqrt{T}}
\]

\[
d_2 = \frac{\ln(S_0/K) + (r-\sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}
\]

The function \(N(x)\) is the cumulative probability distribution function for a standardized normal distribution. \(\mu\) is the yearly expected return of the stock. \(\sigma\) is the yearly volatility of the stock price. \(T\) is the maturity time of the option. \(S_0\) is the stock price at time 0.

2.4.1 Comments on Black-Scholes’ Assumptions

Black-Scholes’ formula is based on following assumptions (Hull, 2010).

1) The stock pays no dividend during the option’s life. Some companies pay dividends to their shareholders, and higher dividend yields lower call option premiums. The real option model proposed in this research uses same assumption as the Black-Scholes’ formula.

2) The European option’s exercise terms is used. European exercise term dictates that the option can only be exercised on a determined date (the expiration date). American exercise term allows the option to be exercised at any time before the expiration date, rendering American options more valuable because of the greater flexibility. In practice, few call options are not
exercised during the last few days of their life because if one exercises a call early, he/she gives up the remaining time value of the option. So usually the price of American option is greater than the European option but the difference is slight. Black-Scholes formula can be used to evaluate a European option and approximate the value of an American option.

However, for a real option such as mining project activation, the model proposed in this research suggests that the real option can be exercised fairly early during the mine’s life, so the Black-Scholes formula cannot provides a reliable approximation of the real option value for a mining activation option.

3) Market is efficient. In an efficient market, one cannot consistently achieve returns in excess of average returns in market on a risk-adjusted basis, given the information is available at the time the investment is made.

4) No commission or transaction cost is charged. For stock options, usually market participants have to pay a commission to buy or sell options. Real options are usually not traded in market and no commission fees are applicable for real options. The real option model proposed in this research does not consider the commission or transition cost, either.

5) The short term interest is known and is constant. In this research, same assumption is used and the annual interest rate is assumed to be a constant of 2%.

6) The stock price follows a Geometric Brownian motion (in formula (1.1)) in continuous time with a variance proportional to the square of the stock price. Thus, the distribution of stock price is lognormal. The variance of return of the stock is assumed to be constant. The price model used in this research is a mean-reverting process (in formula (1.16)), which is advanced compared to the Geometric Brownian motion shown in formula (1.1) to model commodity price (Pindyck and Rubinfeld 1997), by adding a long run equilibrium term $\lambda$.

$$dS_t = \mu(S_t - \lambda)dt + \sigma S_t dW_t$$
The financial literature on commodity price modeling started with the pioneer work by Gibson and Schwartz (1990). Given the behavior of commodity price, Litzenberger and Rabinowitz (1995) introduce a mean-reverting drift and Schwartz (1997) and Pindyck (2001) keep the mean-reverting representation for commodity prices. The idea of mean-reverting comes from microeconomics theory. When price is below the long term equilibrium, demand for the product tends to increase while its production tends to diminish, since the producing firms can postpone investment and cease production. The opposite will happen if price is higher than the long term equilibrium. Pindyck and Rubinfeld (1991) have empirically shown that the prices of many commodities follow mean-reverting stochastic process, which is an advanced model than the Geometric Brownian motion used in many literatures such as the work by Cardin et al. (2008) and Sabour (1999).

2.4.2 Black-Scholes’ Problem in Choosing the Optimal Timing to Exercise the Option

The real option model proposed in this research provides a guideline for the mine’s owner on choosing an optimal gold price threshold to exercise the option, using large amount of simulated paths of future gold prices. However, Black-Scholes formula can only help to assess the option value, but does not provide any clue on the timing of option exercising; Black-Scholes formula assumes the price to be Geometric Brownian motion, which is considered to be inferior compared to mean-reverting process (Pindyck and Rubinfeld 1991); in addition, the real option model proposed can be extended to assess the impacts of postponing the delivery of the commodity product, on the price threshold and the real option value.
2.5 Empirical Work

Mean-reverting process assumes that commodity prices in general fluctuate around an estimated long run equilibrium that can change over time, only due to inflation. McCarthy and Monkhouse (2003) examine the application of real option method at a practical level, and suggest that the input assumptions to the real option approach must be realistic for better managerial decisions. One of the most important assumptions is the correct model for the commodity price. There are two pieces of empirical work on the real option analysis for mining operations by Slade (2001), and Moel and Tufano (2002), which will be discussed by following paragraphs.

Hustrulid et al. (2013) claim that there is little option value associated with the large modern metal mines such as Quebrada Blanca in Canada, because when the property are acquired either by discovery or purchase, they are developed as soon as possible as a result of the low cost of production. Empirical work suggests that this proposition is not true in industry. For instance, the large nickel deposit in Voisey’s Bay, Canada with low cost was postponed several years and downsized due to the depressed nickel market. Slade (2001) lists 21 mines with operational flexibility such as opening, closure, temporary closure, reopening, expansion and merger (Table 10). Using the operational activities of these 21 mines, Slade estimates four models: inflexible model without price uncertainty, inflexible model with price uncertainty, flexible model with uncertain stationary price (mean-reverting), and flexible model with uncertain non-stationary price (Geometric Brownian motion). The models are relying on discrete time contingent claims analysis and solved using the standard programming method.
It is found that the flexible models fit the operational activity data better than the inflexible mode. The most evident contrast is between the flexible models with stationary (mean-reverting) and non-stationary price (Geometric Brownian motion), and larger option value is found in non-stationary flexible model than the comparable stationary values. The non-stationary project value is almost twice the stationary value, and the non-stationary option value is almost 10 times the stationary value. The large difference found by Slade (2001) can be contrasted with the smaller difference found in financial call options (Lo and Wang, 1995), in which the mean-

<table>
<thead>
<tr>
<th>Mine</th>
<th>Proprietor</th>
<th>Location</th>
<th>Operational Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Afton</td>
<td>Teck</td>
<td>British Columbia</td>
<td>Temporary Closure; Reopening</td>
</tr>
<tr>
<td>Ansil/Lake Dufault</td>
<td>Inmet</td>
<td>Quebec</td>
<td>Closure; Opening; Closure</td>
</tr>
<tr>
<td>Bell</td>
<td>Noranda</td>
<td>British Columbia</td>
<td>Temporary Closure; Reopening; Reopening; Closure</td>
</tr>
<tr>
<td>Cedar Bay</td>
<td>Campbell</td>
<td>Quebec</td>
<td>Closure</td>
</tr>
<tr>
<td>Copper Rand/Portage</td>
<td>MSV</td>
<td>Quebec</td>
<td>Temporary Closure; Reopening</td>
</tr>
<tr>
<td>Craigmont</td>
<td>Craigmont</td>
<td>British Columbia</td>
<td>Closure</td>
</tr>
<tr>
<td>Gaspe/Murdock/Needle Mountain</td>
<td>Noranda</td>
<td>Quebec</td>
<td>Temporary Closure; Reopening; Temporary Closure</td>
</tr>
<tr>
<td>Gibraltar</td>
<td>Placer Dome</td>
<td>British Columbia</td>
<td>Temporary Closure</td>
</tr>
<tr>
<td>Goldstream</td>
<td>Cominco</td>
<td>Quebec</td>
<td>Open</td>
</tr>
<tr>
<td>Granduc</td>
<td>Esso</td>
<td>Quebec</td>
<td>Closure</td>
</tr>
<tr>
<td>Grandisle</td>
<td>Noranda</td>
<td>British Columbia</td>
<td>Closure</td>
</tr>
<tr>
<td>Highland Valley</td>
<td>Cominco/Teck /RionAlgom</td>
<td>British Columbia</td>
<td>Expansion</td>
</tr>
<tr>
<td>Highmont</td>
<td>Teck</td>
<td>British Columbia</td>
<td>Merger; Closure</td>
</tr>
<tr>
<td>Island Copper</td>
<td>BHP</td>
<td>British Columbia</td>
<td>Expansion</td>
</tr>
<tr>
<td>Lemoine</td>
<td>Northgate</td>
<td>Quebec</td>
<td>Closure</td>
</tr>
<tr>
<td>Louveum</td>
<td>Louveum</td>
<td>Quebec</td>
<td>Closure</td>
</tr>
<tr>
<td>Matagami/Isle Dieu/Norita</td>
<td>Noranda</td>
<td>Quebec</td>
<td>Closure; Opening</td>
</tr>
<tr>
<td>Oprimiska/Springer/Cooke/Perry</td>
<td>Inmet</td>
<td>Quebec</td>
<td>Closure; Closure; Closure</td>
</tr>
<tr>
<td>Selbaj</td>
<td>Billiton</td>
<td>Quebec</td>
<td>Opening; Opening; Closure</td>
</tr>
<tr>
<td>Similkameen/Simlico</td>
<td>Princeton</td>
<td>British Columbia</td>
<td>Temporary Closure</td>
</tr>
<tr>
<td>Valley</td>
<td>Cominco</td>
<td>British Columbia</td>
<td>Merger; Closure</td>
</tr>
</tbody>
</table>

Table 10 Copper Mines with Operational Flexibility in Canada from 1980 to 1993 (Slade, 2001)
reverting and GBM stock prices are used and approximately 5% of the difference is found. In contrast to a mine, the life of a financial option is typically under a year. When option life is short and mean-reverting effect is slow, less difference is found in the financial options.

In contrast to Slade’s result, Schwartz (1997) finds that the project value is higher when price is mean-reverting. Schwartz (1997)’s result relies on the argument that, if the price is low today, and the project owner decides to wait the situation that is expected to improve when prices are mean-reverting, but not when the price is non-stationary.

Slade (2001) also provides some insightful suggestions that why real option theory is not extensively used in mine’s practical investment decision making: 1) lack of data or/and the poor quality of the data; and 2) mineral assets are usually transacted by bilateral contracts, or the values of many mineral assets are contingent on the value of state variables not traded in market.

The empirical research by Slade (2001) provides a strong evidence of the existence of real option value in mining operations, and significance of flexibility value, as well as the importance of considering the mean-reverting pattern in the commodity price. However, Slade (2001)’s research can be improved by following aspects. The timing of entry considered in Slade (2001)’s real option model is fixed rather than flexible. Solving the model requires strong background in dynamic programming, and a straightforward model depending on simulation is needed if the real option model can be widely applied in the industry. In addition, when providing mining strategies, it is necessary to consider the probabilities of exercising the option, and the effects of postponing delivery of the product, which is considered in this research.

Moel and Tufano (1999) study empirically the real option theory and its application in mine’s openings and closings. A database tracking annual opening and closing decisions of 285 developed gold mines in North American from 1988 to 1997 and A Probit model are used to examine factors impacting the likelihood that a mine is open or suspended. A cost function (formula (1.17)) is estimated to decompose total mining costs to fixed and variable components.
The real option model of cost function is a “reduced form” model, in which any “non-economic” factor that leads firms not to close mines is recast as a “closing cost”.

\[ cq = a_0 + a_1 R + a_2 T + \beta_0 q + \beta_1 q T + \sum \beta_i D_i q \] (1.17)

In formula (1.17), \( c \) is the average cost of production, which is composed of fixed cost and marginal cost; \( a \) captures fixed cost that in independent of the production rate, while \( \beta \) captures the marginal mining costs for one unit of gold. Fixed cost is a function of the size of the remaining reserve \( R \) and technology of the mine \( T \) (a dummy variable equal to 1 if an underground mine, and equal to 0 if open-pit).

Marginal cost is a function of the production rate \( q \), the cumulative amount of mineral already extracted (which is inversely related to \( R \)), and the mining technology \( T \). To capture the effect of the remaining reserve on the marginal cost, a set of dummies \( D_R \) corresponding reserve characteristics, and their interactions with the production \( q \) are added in the model.

A Probit model for the probability of a mine will be open/closed in a given year is also estimated. The independent variables considered are prior state, gold price, volatility of gold price, operating costs, nominal and real discount rates, convenience yield, costs of shutting and reopening, costs of maintaining mine, and remaining reserves.

The research finds strong evidence of hysteresis in the mining operation data, indicating that operational flexibility is fairly common in mining. Although real option model can describe the mine-level decisions, they can fail to capture the mechanism of firm-level decision-making. Closing a mine can be a firm-level decision, when a firm has other mines in portfolio and lower operating cost can be found in these mines, the current mine is less likely to be closed. This means that decisions on one particular sector of a firm are impacted by the performance of the other sectors within the firm.

However, the research failed to consider the uncertainty in gold price, such as using Geometric Brownian motion or mean-reverting. In addition, since it is an empirical research, the
result does not provide any clue about the price thresholds or best time to make mining decisions. Also, the research assumes that the commodity is ready to be sold in the market as soon as the production decision is made, which is not necessarily to be the case in industry practice considering the production time.

2.6 Cutoff Grade and Real Option

As an extension to Lane’s theory (1988) of cutoff grades in deposits of two economic minerals, Asad (2005) develops a cutoff grade optimization algorithm with an option to stockpile. Mardones (1993) applies the option techniques to a copper project comprising mining, leaching and electro-winning processes, which shows that the flexibility to modify the cutoff grade strategy achieves additional value.

2.7 Other Literature

Topal (2008) reviews and compares mining project evaluation methods – Discounted Cash Flow (DCF), Decision Tree and Real Option, and concludes that DCF method does not allow for managerial flexibility; Decision Tree method analyses different managerial strategies and shows all the outcomes by the strategies, but can easily become complex when the number of variables increases (such as gold price, ore recovery rate, ore grades, and the initial investment). The Real Option method considers possible management choices and project uncertainties, and usually leads to a value of the project higher than DCF or Decision Tree. Real Option provides more reliable information for the evaluation of the project compared to other methods. Lilford and Minnitt (2005)’s research is a similar work of comparing different evaluation techniques. Mayer and Kazakidis (2007) introduce the common options in a mining project based on case
study such as capacity option, shutdown option and sequential decision option. Guj and Garzon (2007)’s work relies on the “stochastic price forecasting model”, which is indeed a stochastic process using the standard Geometric Brownian motion. A practical example is used to test the model and the authors find that project uncertainty (in particularly the price’s uncertainty) is particularly important. Any evaluation model based on a constant price can be unreliable when evaluating a mining project.

2.8 Conclusion of Literature Review

Five trends of literature have been identified and reviewed in this section. Longstaff and Schwartz (2001)’s model has been widely adopted, or extended to assess the real option value in a mining project. Methods based on Lattice/Binomial tree are also used in many pieces of research on the relevant topic. Another trend of research applies the Black-Scholes’ (1973) formulas to real option problems. Two literatures examine the significance of real option value in mining projects, and compare the models for commodity prices. There are also literatures applying the real option methods in problems such as cutoff grade calculation and optimal pit design.

Each trend of literature addressing the real option problem in mining industry has some limits: 1) Geometric Brownian motion is assumed to model commodity prices, which is can be improved by using a mean-reverting process; 2) the option exercising time considered is a fixed date i.e. the European type option. The option model in this research can be exercised at a flexible time; 3) for some literatures, it is fairly difficult or impossible to consider the uncertainty in strike price or the mining cost, and the mining cost in this research can be easily modeled using a stochastic process; 4) none of the literature considers the effects of postponing the delivery of the commodity product, which is examined in this research.
Although empirical work has shown the significance of real option value in a mining project, and several articles have provided insights on how to evaluate the value of such real options, there is still much room for improvement. For example, the widely adopted Longstaff and Schwartz (2001)'s model has two important limits when used to solve a real option problem. First, it fails to give specific direction on when the option should be exercised. Second, it only applies to an option with a finite lifespan. This is in contrast to an actual mining option, which at least in theory, has infinite length. In previous literature (Hall and Nicholls 2007), the lattice approach uses a backward induction method to solve an abandonment or expansion option problem. This approach, however, needs to be improved, since 1) the backward induction method is not suitable to examine the option of activating a mining operation; 2) the expansion option can only be considered for a specific year, i.e. the sixth year of the mining operation, and the model cannot determine which is the best year to expand the production rate. The approach in this research can make improvements by addressing a mining activation type option, as well as being more flexible in searching the optimal price threshold to exercise the option.

The real option approach proposed here uses a mean-reverting process price, which is more relevant to model the price of commodities than the Geometric Brownian motion process in previous literature. The approach derives an optimal price threshold for exercising the mining activation option. In addition, the approach can easily be extended to examine the uncertainty of the mining cost by modeling it as a stochastic process, instead of a constant mining cost as in the previous literature such as Hall and Nicholls (2007).
3. Mean-reverting Process for Commodity Price

3.1 Mean-reverting Model

Formula (1.18) shows the Geometric Brownian motion model (Dixit and Pindyck 1994). The variable $W_t$ is a Weiner process, and follows normal distribution $dW_t \sim N(0, \sqrt{dt})$. $S$ is the commodity price. $\alpha$ is the drift parameter, and $\sigma$ is the volatility or standard deviation of gold price.

\begin{equation}
(1.18) \quad dS = \alpha dt + \sigma dW_t
\end{equation}

However, it is possible that underlying prices modeled do not follow a stochastic process converging to GBM, because commodity price may revert to an average production cost in the long term. Pindyck and Rubinfeld (1991)’s empirical study shows that prices of many commodities follow MR stochastic processes. The one-factor MR process is widely used for modeling a commodity price and for valuing contingent options (Dixit and Pindyck 1994). The process can be written in formula (1.19),

\begin{equation}
(1.19) \quad dS = \lambda(\mu - S)dt + \sigma dW_t
\end{equation}

Or, in discrete terms,

\begin{equation}
(1.20) \quad S_t - S_{t-1} = \lambda(\mu - S)\Delta t + \sigma dW_t
\end{equation}

In formulas (1.19) and (1.20), $W_t$ and $S$ are defined same as in the equation (1.18); $\lambda$ measures the speed of mean-reverting. $\mu$ is the long-term mean, to which the process tends to revert. The term $\sigma$ represents the process volatility. Gillespie (1996) points out that above discrete specification is only valid when the time step $\Delta t$ is sufficiently small, implying the formula becomes,

\begin{equation}
(1.21) \quad S_t = e^{-\lambda \Delta t}S_{t-1} + (1 - e^{-\lambda \Delta t})\mu + \sigma \sqrt{\frac{1-e^{-\lambda \Delta t}}{2\lambda}}dW_t
\end{equation}
The logarithm of price is usually used for $S_t$ because commodity price is commonly assumed to be lognormally distributed. This specification also prevents the simulated path going negative. In this context, the one factor MR model based on formula (1.21) is used. Least square regressions and maximum likelihood are well known techniques for parameter estimation of the MR process. In the case of least square, formula (1.21) can be viewed as,

\[ y = a + bx + \varepsilon_t \text{ and } \varepsilon_t \sim N(0, \sigma^2) \]  

To estimate the MR process parameters $\mu, \lambda, \sigma, r$, use the least square regression for equation (1.21) (Wooldridge 2000) with variable derivations according to formula (1.23), (1.24), (1.25), (1.26) and (1.27),

\[ e^{-\lambda \Delta t} = b \]  
\[ \hat{\lambda} = -\frac{\ln(b)}{\Delta t} \]  
\[ \left(1 - e^{-\lambda \Delta t}\right)\hat{\mu} = a \]  
\[ \hat{\mu} = \frac{a}{1-b} \]  

Finally, the estimation of volatility is,

\[ \hat{\sigma} = sd(\varepsilon_t) \sqrt{\frac{2\lambda}{1 - e^{-\lambda \Delta t}}} \]  

where, $sd(\varepsilon_t)$ is the mean standard error of $\varepsilon_t$. The estimates of the parameters will be later used to simulate paths of price.

### 3.2 Estimation Results

The weekly gold prices in US dollar from Jan 1\textsuperscript{st}, 1964 to Dec 31\textsuperscript{st}, 2012 from London Bullion Market Association are employed to estimate the parameters in above formulas. In the
linear regression equation (1.21), \( \Delta t = \frac{21}{252} \), approximating one month, and there are 588 observations. The OLS the estimation results are shown in Table 11.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>b</strong></td>
<td>0.9958</td>
<td>252.53</td>
</tr>
<tr>
<td><strong>a</strong></td>
<td>0.0305</td>
<td>1.10</td>
</tr>
<tr>
<td><strong>R^2</strong></td>
<td>0.9902</td>
<td></td>
</tr>
<tr>
<td>Mean Standard Error</td>
<td>0.0489</td>
<td></td>
</tr>
</tbody>
</table>

Table 11 Estimation Result of Least Square Regression

Calculating the parameters for the MR model,

\[
\hat{\lambda} = -\frac{\ln(0.9958)}{21} = 0.0504
\]

(1.28)

\[
\hat{\mu} = \frac{0.03051}{1-0.9958} = 7.2759
\]

(1.29)

This implies that the long-term equilibrium price of gold is \( e^{7.2642} = 1428.24/\text{Oz.} \), and

\[
\hat{\sigma} = 0.0489 \times \frac{2+0.0504}{\sqrt{1-0.9958^2}} = 0.1696
\]

(1.30)

which implies that the annual volatility is 16.96% per year.

### 3.3 Half-life of Mean-reverting and Mean-reverting Speed

The half-life (denoted as \( H \)) of a variable \( S \) is defined as the time for the expected value of \( S(t) \) to reach halfway price between the current value \( S(0) \) and the long term equilibrium \( \mu \). The following shows the relationship between \( H \) and the speed of the mean-reverting \( \lambda \). In equation (1.19), when only the deterministic part is considered,

\[
dS = \lambda(\mu - S)dt
\]

(1.31)

or
(1.32) \[ \frac{1}{\mu-s} dS = \lambda dt \]

Integrating the left side from \( S_0 \) to \( S_1 \), and the right side from \( t_0 \) to \( t_1 \), gives,

(1.33) \[ \int_{S_0}^{S_1} \frac{1}{\mu-s} dS = \int_{t_0}^{t_1} \lambda dt \]

Implying,

(1.34) \[ \ln \left( \frac{S_1 - m}{S_0 - m} \right) = -\lambda (t_1 - t_0) \]

(1.35) \[ \ln \left( \frac{S_1 - m}{S_0 - m} \right) = -\lambda H \]

By definition of \( H \), the relationship \( \frac{S_1 - m}{S_0 - m} = 0.5 \), so

(1.36) \[ H = \frac{\ln(2)}{\lambda} \]

Plugging in the estimation from formula (1.28), \( \hat{\lambda} = 0.0504 \),

(1.37) \[ H = \frac{\ln(2)}{0.0504} = 13.75 \]

This implies that it takes 13.75 years for logarithm gold price to reach the halfway price between the current value \( S(0) \) and the long-term equilibrium \( \mu \).

### 3.4 Simulation of Price

The simulations start from Jan 1\(^{st} \), 2013, when price of gold is $1,687.34/Oz., and 10,000 paths of the future 35 years of price of gold are generated by simulation. In equation (1.21), the time step \( \Delta t \) is a month, or 21/252 of a year. In each time step, the factor \( W_t \) is randomly drawn from normal distribution \( N(0, \sqrt{\Delta t}) \) by Monte Carlo simulation. In equation (1.21), the deterministic part of the price difference between two consecutive steps can be calculated by plugging the estimated results in equations (1.28), (1.29) and (1.30). Thus, based on Monte Carlo simulation and starting from an original gold price, paths of gold price for the following 35 years
can be generated. Figure 7 shows an example of five paths of simulated price for 35 years (420 months) since Jan 1st, 2013.

Figure 7 Five Illustrative Paths of Simulated Gold Prices for the Next 35 Years

4. Real Option Model and Optimal Timing

4.1 Model of Exercising the Option and Price Threshold

Assume that the average production costs for the gold is $900/Oz. and the annual risk-free interest rate is 2%. In this context, the mining cost can be viewed as the strike price for a financial option. The gold spot price on Jan 1st, 2013, $1,687.32/Oz is used as the initial price for simulation. Given the uncertainty in gold price, the goal is to determine the “optimal exercise gold price threshold” to activate the mining operation. For each ounce of gold, the current value of the amount that gold price above the average mining cost is the profit by exercising the option.
Denote the spot price of gold as $S$. For each ounce of gold that has been produced from the mine, not including transportation and marketing cost, the mine’s owner can realize profit $S - 900$. On Jan 1st, 2013, although gold price is $1,687.32/Oz., and far above the strike price ($900/Oz.), it does not mean it is optimal to activate the project immediately. The gold price may continue to climb and a higher NPV may be achieved.

Determining the optimal price threshold to start mining is equivalent to determine the optimal $X$ that maximizing the expected profit. For instance, if $X = 800$ and 2.4 years later, gold price reaches $1,701/Oz., which is greater than the threshold price $1,700/Oz., then the option could be exercised. The discounted profit per ounce of gold by exercising this option is,

\[(1,701 - 900) \times e^{-0.02 \times 2.4} = 763.46\]

Note that the option will not be exercised across the 35-year time span if the price never reaches a level that is greater than the strike price by the threshold $X$. The goal is to find the optimal $X$ which maximizes the expected profit based on 10,000 simulations of price paths. For each simulated price path, there is a corresponding deterministic graph of profit by exercising the option vs. $X$. By 10,000 simulations, for each $X$ from $500/Oz.$ to $2,000/Oz.$ (with interval of $10/Oz.$), 10,000 profits can be obtained. The expected profit for each $X$ is the average of 10,000 profit graphs.

### 4.2 Four Illustrative Price Paths and Expected Profit

In this session, four selected paths of simulated price (Path A, B, C and D) are used to illustrate the option-exercising mechanism. Each simulated price has a corresponding graph of expected profit by exercising the option vs. the threshold $X$.

Figure 8 shows path A of simulated prices. Based on this path, during the next 35 years, the price maximum is $3,150/Oz., which is above the average mining cost (strike price) per ounce.
by $3,150 − $900 = $2,250. Therefore, the mine activation option will not be exercised unless $X$ is greater than $2,250/Oz.

Figure 8 Path A of Simulated Price and Profit by Exercising the Option vs. $X$

Figure 9 presents path B of simulated price. During the next 35 years, the maximum price in this path is around $4,000/Oz., which is above the average mining cost (strike price) per ounce by $4,000 − $900 = $3,100. The mine activation option will not be exercised unless $X$ increases to $3,100/Oz., and the profit from exercising the mining activation option is zero when $X$ is greater than $3,100/Oz. The profit by exercising the option shows an increasing trend until $X$ reaches $3,100/Oz.

Figure 9 Path B of Simulated Price and Profit by Exercising the Option vs. $X$
Figure 10 shows path C of simulated price. Based on this path, during the next 35 years, the price maximum is $5,000/Oz., which is above the average mining cost (strike price) per ounce by $5,000 \(-\$900 = \$4,100\). The mine activation option is not exercised until \(X\) is above $4,100/Oz., and the profit by exercising the option is zero when \(X\) is greater than $4,100/Oz. The profit trend is not always increasing due to the time value of money effect.

Figure 10 Path C of Simulated Price and Profit by Exercising the Option vs. \(X\)

Figure 11 presents the path D of simulated price. Based on this path, during the following 35 years, the price maximum is $1687.32/Oz., which is the spot price on Jan 1\(^{st}\), 2013. This implies that it is optimal to start mining immediately with no hesitation. The mine activation option is not exercised during the following 35 years based on this path of simulated price. In this case, since mine activation the option is not exercised during the following 35 years, the “profit by exercising the option” is zero for any \(X\) that is greater than $887.32/Oz. (right hand side of Figure 11)
For each path of simulated price, there is a corresponding deterministic graph of profit by exercising the mine activation option vs. \( X \). Based on 10,000 simulations, for each \( X \) from $500/Oz. to $2,000/Oz., 10,000 profits can be generated. The expected profit for each \( X \) is the average of these 10,000 profit graphs, which is shown in Figure 12. A peak of expected profit by exercising the mine activation option can be observed in Figure 12, where \( X \) is $884/Oz. The results imply that it is optimal to exercise the option when the price is above the average mining cost by $884/Oz., which implies the optimal option-exercising threshold for gold price is $1,784/Oz.
5. Extensions of the Model

5.1 Differentiated Mining Cost and Option-Exercising Probability

This section investigates and compares the ROV and NPV of a gold mine based on extensions of the option model to differentiated mining costs. In previous section, the case of $900/Oz. for mining cost is studied. A similar procedure can be applied to differentiated mining costs. Figure 13 shows the ROV and NPV for mines with average mining cost from $100/Oz. to $2,500/Oz. Note that when calculating the ROV and NPV, the gold price of $1,687.32/Oz. (gold price on January 1st, 2013) is used. When mining cost is higher than price, the NPV is zero because the mine’s owner will decide to suspend production; when mining cost is lower than the price, the NPV of one ounce of gold is the price at the time minus the mining cost.
The ROV of a mine is composed of two parts – the net present value (NPV) and the flexibility value. When the mining cost is relatively low, almost all of its value comes from the NPV and there is slight flexibility value, for the mine’s owner has little incentive to defer the mining activation. Figure 14 shows the flexibility value for differentiated mining costs, which is obtained using the ROV minus the NPV.

As Figure 14 indicates, the flexibility value peaks when mining cost is equal to the gold price, as the mine’s owner has maximum operational flexibility in this situation. When the mining cost is greater than the price, the NPV is zero, and the mine’s owner is more likely to choose to wait and not to start mining immediately (because of the negative profit if the option is exercised); when the mining cost is lower than the price, as mining cost decreases, there is less incentive to wait, because the option is increasingly “in the money” and will be exercised soon. These two scenarios imply that the flexibility value peaks when mining cost and price are equal, when the mine’s owner has full flexibility.
In Figure 14, when the average mining cost is lower than the price, as the average mining cost declines, the owner of the mine has less incentive to wait for a later price; when the average mining cost is higher than the price, as mining cost increases, the mine’s flexibility drops, for the likelihood that a high-cost mine is activated becomes lower.

From Figure 13, it can be observed that the ROV is always higher than NPV, because waiting for a possibly higher price in future is an option but not an obligation. If there is no additional value can be achieved by waiting, the mine’s owner can always decide to activate the mine immediately to realize the NPV.

![Figure 14 Flexibility Value ($/Oz.) for Different Mining Costs](image)

For each mining cost, the probability of exercising the option is examined based on simulation. Figure 15 shows the probabilities that a mine is started within the following 5 and 35 years. The possibility of exercising the option declines as mining cost rises, for less profit can be realized. $500/Oz. is believed to be a reasonable approximation of average mining cost for all
gold mines around the world (DOE, 2007). If a gold mine has average mining cost of $500/Oz., it can be observed from Figure 15 that the probability that this mine is started within the next 5 and 35 years are 65% and 90%, respectively. In the extreme, for a high-cost mine such as $3,000/Oz. to be triggered, it is unlikely that the mine is started in short term (in 5 years). However, there still is a 20% possibility that the mine can be activated within the next 35 years, if gold price skyrockets during the period.

![Figure 15](image)

**Figure 15 Probabilities of Exercising the Option within 5 Years and 35 Years**

### 5.2 Differentiated Starting Price of Simulation

Figure 16 exhibits the ROV and NPV for a mine with average mining cost of $1,000/Oz. with differentiated starting prices of simulation. Figure 17 shows the corresponding flexibility value, obtained by using the ROV minus the NPV. The results are fairly similar to results of differentiated mining cost: 1) the ROV increases as the price increases; 2) when the price is in the lower or higher end, the flexibility is low since the owner of the mine has less incentive to wait or
operate the mine flexibly; 3) the flexibility value peaks when the price is equal to the mining cost, since the mine’s owner holds maximum operation flexibility in this situation.

Figure 16 ROV and NPV for a Mine with Mining Cost of $1,000/Oz. by Differentiated Gold Price from $100/Oz. to $2,300/Oz.

Figure 17 Flexibility Value for a Mine with Mining Cost of $1,000/Oz. by Differentiated Gold Price from $100/Oz. to $2,300/Oz.
6. Stochastic and Declining Mining Cost

Decreasing stochastic mining cost with differentiated level of fluctuation is examined in this session. Specifically, the effects of decreasing mining cost (such as due to technology progress) and uncertainty of mining cost on the real option value of one ounce of gold and the option exercising threshold are examined. A mining activation option (like financial call option) is considered. The threshold is defined as the optimal amount that gold price is higher than the mining cost when the option is exercised.

Assume there is a yearly 2% decrease and a standard deviation (yearly volatility) of 4% for the mining cost. Denote the monthly decrease of mining cost by \( r \), such that
\[
(1 - r)^{12} = 0.98
\]

Solve for \( r \),
\[
(1.40) \quad r = 0.168\%
\]

The monthly volatility is,
\[
(1.41) \quad 4\% \times \frac{1}{\sqrt{12}} = 1.15\%
\]

Thus, the monthly decrease rate and standard deviation for mining cost is 0.168% and 1.15%, which will be used for the simulation of monthly mining cost. The change of mining cost is shown in following stochastic process,
\[
(1.42) \quad \frac{dc_t}{c_t} = \lambda_c dt + \sigma_c \varepsilon
\]
Where \( \lambda_c \) is -0.168%. The error term \( \varepsilon \) complies to the normal distribution \( N(0, 1) \), and the monthly volatility \( \sigma_c \) is 1.15%.

Assume the mining cost on Jan 1\(^{st} \), 2013 is $900/Oz., and the gold price is $1,687.3/Oz. Following Figure 18 shows three paces of simulated mining cost for the following 35 years after Jan 1\(^{st} \), 2013. Note that the processes of mining cost and gold price are assumed to be independent.
To examine the effects of decreasing mining cost (or technology progress) with uncertainty, following three cases are explored and compared: Case 1) the mining cost is assumed to be constant, $900/Oz., during the following 35 years; Case 2) the mining cost is assumed to be decreasing by a constant rate 2% yearly, and there is no stochastic pattern (formula (1.43)); Case 3) the mining cost is assumed to be decreasing by a rate 2% yearly, and meanwhile, a stochastic term (4% standard deviation) is considered (formula (1.44)).

\[
\frac{dc_t}{c_t} = \lambda_c dt
\]

(1.43)

\[
\frac{dc_t}{c_t} = \lambda_c dt + \sigma_c \varepsilon
\]

(1.44)

For each case, the real option value (ROV) for one ounce of gold and option exercising threshold are explored depending on simulation method and extending the proposed real option
model. Figure 19 shows the net present value and real option value for one ounce of gold and the effects of decreasing/stochastic mining cost, for differentiated mining cost from $200/Oz. to $3,100/Oz. Figure 19 is based on the three cases described in previous paragraph.

Figure 19 provides several insights about the decreasing and stochastic mining cost’s impacts: 1) ROV is always higher than the NPV; 2) When the mining cost is low, i.e. lower than $1,000/Oz., no significant effect of decreasing mining cost (due to technology progress) and stochastic pattern can be observed. This is for the reason that, when the mining cost is much lower than the spot price ($1,684.3/Oz.), the mine’s owner will determine to start mining immediately and does not wait for the mining cost to decrease, which may take decades. In addition, the mine’s owner chooses to start mining immediately and does not wait the stochastic pattern of mining cost to make impacts in the long term. Because the option is exercised in the early months before the mining cost begins to decrease and fluctuate, the effects of decreasing mining cost and the stochastic pattern of mining cost on ROV are little.

3) For mining cost that is higher than $1,000/Oz, the decreasing mining cost and stochastic pattern exert significant positive impacts on the ROV. Because of mining cost decreases in the long run, the real option value of one ounce of gold is higher than the case of constant mining cost. The fluctuation in mining cost also increases the real option value compared to the “decreasing, non-stochastic” case, as a result of the added uncertainty in mining cost. If the mining cost decreases quickly and dramatically, when the option is exercised, more value can be realized; while if the mining cost increases due to its uncertainty, the mine’s owner will choose to suspend the option and wait. Both scenarios result in higher real option value compared to the case without fluctuation in mining cost.
Figure 19 Net Present Value (NPV), Real Option Value (ROV), and Effects of Decreasing Stochastic Mining Cost
Figure 20 shows the thresholds of exercising the option for the cases of constant and decreasing non-stochastic mining cost. Two points can be inferred from the figure. 1) By comparing the two lines in the figure, it is found that a non-fluctuating decreasing mining cost will increase the threshold of exercising the option. Because when the mine’s owner expects there is a decreasing trend in mining cost, he/she will decide to wait longer to exercise the option when larger profit can be achieved, which reflects into a higher threshold.

2) Additionally, it can be observed that both lines in the Figure 20 show a decreasing trend, which means that the threshold decreases as the mining cost increases. This is for the reason that as the mining cost increases, it is less likely the option will be exercised; to achieve profit by exercise the option, the mine’s owner has to set a lower threshold to raise the possibility that the option is exercised.
The impacts of differentiated volatility (standard deviation) in mining cost ROV and the option exercising threshold are explored. Three cases of mining cost are explored: 1,100$/Oz., 1,700$/Oz., and 2,300$/Oz. The mining cost has a yearly decreasing rate of 2%. The gold price in market is 1,687.3$/Oz. Table 12 shows ROV and the option exercising thresholds for the three cases, by differentiated yearly standard deviation in mining cost from 1% to 20%. Figure 21 and Figure 22 show the graphs of ROV and the thresholds by differentiated yearly standard deviation.

From Figure 21, several implications can be drawn: 1) ROV decreases as the mining cost increases; 2) ROV increases as the standard deviation of mining cost increases. This is for the reason that as the standard deviation of mining cost increases, there is larger possibility for the situation of low mining cost to happen. If low mining cost is more likely to happen, there is higher possibility that higher option value is achieved. 3) The ROV for high mining cost (2,300$/Oz.) is more sensitive to the change in mining cost’s standard deviation compared to low mining cost (1,100$/Oz.). This is for the reason that the mine’s owner with low mining cost (1,100$/Oz.) will decide to start mining soon before the fluctuation in mining cost exerts impacts; the gold mine with high mining cost (2,300$/Oz.) is not “in the money” and will be exercised in the relative longer term, when the fluctuation in mining cost exerts more significant impacts.

Figure 22 shows the thresholds for exercising the option for mining cost of 1,100$/Oz., 1,700$/Oz. and 2,300$/Oz., respectively, for differentiated standard deviation from 1% to 20%. By focusing on one single line of the three, it can be observed that, for a given mining cost, the threshold increases as the volatility in mining cost increases. This is for the reason that as uncertainty in mining cost rises, there is higher possibility that the mining cost can be low, which means that both the threshold and the ROV can be high.
<table>
<thead>
<tr>
<th>Yearly Standard Deviation</th>
<th>Monthly Standard Deviation</th>
<th>Mining Cost=1,100$/Oz.</th>
<th>Mining Cost=1,700$/Oz.</th>
<th>Mining Cost=2,300$/Oz.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ROV</td>
<td>Threshold</td>
<td>ROV</td>
</tr>
<tr>
<td>1%</td>
<td>0.29%</td>
<td>733.4</td>
<td>1,231</td>
<td>453.2</td>
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<tr>
<td>2%</td>
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<td>733.8</td>
<td>1,200</td>
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<td>1,189</td>
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</tr>
<tr>
<td>5%</td>
<td>1.44%</td>
<td>740.5</td>
<td>1,222</td>
<td>470.3</td>
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<tr>
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<td>1.73%</td>
<td>746.5</td>
<td>1,192</td>
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<td>2.02%</td>
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<td>1,301</td>
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<td>1,287</td>
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<td>15%</td>
<td>4.33%</td>
<td>815.9</td>
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<td>597.8</td>
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<td>1,407</td>
<td>665.9</td>
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<td>20%</td>
<td>5.77%</td>
<td>882.6</td>
<td>1,370</td>
<td>688.4</td>
</tr>
</tbody>
</table>

Table 12 Real Option Values and Option Exercising Threshold for Decreasing (2% Yearly)

Mining Cost of 1,100$/Oz., 1,700$/Oz., and 2,300$/Oz. by differentiated Standard Deviation from 1% to 20%
Figure 21 Real Option Value (ROV) for Mining Cost of 1,100$/Oz., 1,700$/Oz. and 2,300$/Oz. with Differentiated Standard Deviation of Mining Cost from 1% to 20%
7. Effects of Postponing Delivery of the Commodity Product

7.1 Algorithm to Examine the Effect of Postponing the Delivery of the Commodity Product

When inspecting the optimal timing in a mining operation, one important issue rises—the time gap between starting mining and the product is sold in market. Usually, it takes months or even years for the commodity product to be available to be sold in the market after the mining decision is made. This section is aiming at examining the effects of the “the time of production” on the real option decision and the price threshold given the mine’s owner knows the length of
production time. Although the literature reviewed in the session 2 all contributes to the knowledge on real option’s application in mining projects, no literature has addressed the issue of production time in a mining real option problem, and the time related issue is always important in the industry where the price can be highly volatile.

To investigate the effects of delaying three years by the time of production (five years is similar), 10,000 simulations of price paths starting from $1687.32/Oz. (gold price on Jan 1st, 2013) up to the following 35 years are generated. The objective of the algorithm is generating four values: 1) NPV without waiting; 2) ROV without waiting; 3) NPV by waiting three years; 4) ROV by waiting three years, for average mining costs of $100/Oz., $110/Oz., $120/Oz., $130/Oz…. $3,100/Oz. The algorithm is described in following six steps.

1) For a mining cost, i.e. $500/Oz., 10,000 simulations of price paths are generated for the following 35 years from Jan 1st, 2013. The “price” matrix’s dimension is 420 multiplied by 10,000.

2) $X$ is defined as the threshold amount that price excesses the average mining cost at which the mine activation decision is made. The optimal $X$ can be found by following step 3 to step 5.

3) For each $X$ starting from 0 to $3,000/Oz. with step $10/Oz.$ (specifically, $X=0$, $10/Oz.$, $20/Oz.$, $30/Oz.$, $40/Oz.$…$3,000/Oz.$), two matrices will be generated, “profit” with dimension 1 by 10,000 and “profit3” with dimension 1 by 10,000. Each element in “profit” matrix is the profit by exercising the mining activation option based on one simulated price path. To obtain the matrix “profit”, a matrix denoted “fullprofit” with dimension 420 by 10,000 needs to be generated by following “if-else” statement in Figure 23.
In the matrix of \( \text{price}(i,j) \), \( i \) is the index for month, which is from 1 to 420; \( j \) is the index for simulation, which is from 1 to 10,000.

If,

\[
\text{price}(i,j) - \text{Mining Cost} \geq X
\]

then,

\[
\text{fullprofit}(i,j) = \{\text{price}(i,j) - \text{Mining Cost}\} * e^{-0.02^{*}\frac{21}{12}\cdot i}
\]

else,

\[
\text{fullprofit}(i,j) = 0
\]

So \( \text{fullprofit}(i,j) \) is a matrix with dimension 420 by 10,000. The matrix \( \text{profit}(1,10000) \) is the first non-zero element in each column of the matrix \( \text{fullprofit}(420,10000) \). Note for each \( X \), a matrix \( \text{profit}(1,10000) \) has been generated.

Figure 23 “If-else” Statement to Generate the Matrix “\text{profit}” and “\text{fullprofit}”

4) Each element in “\text{profit3}” matrix is the profit by exercising the mining activation option based on the simulated price path and the exercising threshold \( X \) when the mine’s owner knows that it takes three years to produce the commodity. To obtain the matrix “\text{profit3}”, a matrix denoted “\text{fullprofit3}” with dimension 384*10,000 needs to be generated by following “if-else” statement in Figure 24.

In a matrix of \( \text{price}(i,j) \), \( i \) is the month index, from 1 to 384; \( j \) is the simulation index from 1 to 10,000.

If,

\[
\text{price}(i + 36,j) - \text{Mining Cost} \geq X
\]

then,

\[
\text{fullprofit3}(i,j) = \{\text{price}(i + 36,j) - \text{Mining Cost}\} * e^{-0.02^{*}3 - \text{Mining Cost}} * e^{-0.02^{*}\frac{21}{12}\cdot i}
\]

else,

\[
\text{fullprofit3}(i,j) = 0
\]

So \( \text{fullprofit3}(i,j) \) is a matrix with dimension 384*10,000. Then, the matrix \( \text{profit3}(1,10000) \) is the first positive element in each column of matrix \( \text{fullprofit3}(i,j) = 0 \). For each \( X \), a matrix \( \text{profit3}(1,10000) \) has been obtained.

Figure 24 “If-else” Statement to Generate the Matrix “\text{profit3}” and “\text{fullprofit3}”
5) There are totally 3,001 \( X \)'s from 0 to $3,000/Oz., so based on matrices “profit” and “profit3”, the matrices “finalprofit” with dimension 3,001 by 10,000 and “finalprofit3” with dimension 3,001 by 10,000 are generated. By taking average across columns of “finalprofit” and “finalprofit3”, matrices “expected” and “expected3” with dimension 3,001 by 1 are generated. The “expected” matrix is the expected profit by exercising the start-mining option based on 10,000 simulations. The “expected3” matrix is the expected profit by exercising the mining activation option based on 10,000 simulations when the mine’s owner knows that the delivery is postponed by three years. If graphed, expected profit and expected profit by waiting three years shows a parabolic pattern. Thus, the optimal \( X \) for the expected profit without waiting and optimal \( X3 \) for postponing three years are found. The expected profit without waiting and the expected profit by waiting three years are the ROV and the ROV by postponing three years.

6) Finally, NPV without waiting is,

\[
NPV = 1687.32/Oz. - Mining\ Cost
\]

With above 10,000 simulations of expected price path, an expected price of the future three years after Jan 1st, 2013 can be obtained, which is denoted \( E(price\ in\ 3\ years) \). NPV by waiting three years can be calculated by

\[
NPV\ by\ waiting\ 3\ years = E(price\ in\ 3\ years) \cdot e^{-0.02\cdot3} - mining\ cost
\]

Similar algorithm is applied to the case of postponing five years.

7.2 Results for Effects of Postponing the Delivery

This session shows the results for the effects of postponing the delivery on the real option value and the thresholds. Figure 25 shows the NPV and ROV without waiting, waiting three years and five years. ROV is always higher than the corresponding NPV for scenarios of no waiting, waiting three years and five years. The postponing decreases not only the NPV but the ROV.
Postponing three years decreases the ROV because the optimal threshold may occur during the postponed time. And when the mining operation is producing, the mine’s owner may miss the optimal price threshold. The difference between the ROV and the ROV by waiting three years reflects the value lost by waiting. The value lost by waiting and the percent of lost value can be written as,

\[ \text{Value Lost} = \text{Real Option Value} - \text{Real Option Value by Postponing} \] \hspace{1cm} (1.47)

\[ \text{Percent of Value Lost} = \frac{\text{Value Lost}}{\text{Real Option Value}} \] \hspace{1cm} (1.48)

Figure 26 compares the value lost for differentiated mining cost for cases of waiting one year, three years and five years. It can be observed that longer waiting leads to higher lost value, because longer waiting time results in a larger possibility that the mine’s owner misses the optimal price to exercise the option during the waiting time.

Figure 27 shows the value lost and the percent of lost value by waiting three years compared to the case of no waiting. The percent of value lost shows an increasing pattern as the mining cost increases. As the mining cost increases, the portion of the contribution from the flexibility value to the total ROV of a mine increases. The postponing exerts a negative effect on this portion, but does not impact the NPV. Thus, the percent of lost value increases as the mining cost increases.

In Figure 27, it’s also shown that the lost value decreases as the mining cost increases, and the decreasing trend is more evident when mining cost is higher than the gold price, 1687.32$/Oz. This result unveils a future direction of this research, to explore how the lost value changes, as the average mining cost varies.
Figure 25 NPV and Real Option Value for No Waiting, and Waiting Three Years and Five Years

Figure 26 Value Lost by Waiting One Year, Three Years and Five Year
Figure 27 Value Lost and Percent of Lost by Waiting Three Years for Differentiated Mining Cost

Figure 28 shows the exercising price by mining cost for scenarios of no waiting, and waiting for five years. It can be seen that the price threshold increases as average mining cost increases. This is for the reason that for lower mining cost, the operation is deeply “in the money” and there is less incentive for longer waiting and the threshold price is lower; mines with higher mining cost need a higher price threshold to exercise the option to guarantee a positive profit.

In addition, the postponing renders the exercising price threshold increases steeper (starting from a lower threshold and ending at a higher threshold). To interpret this result, the price threshold can be regarded to reflect the length of waiting time for a future higher price. When the mining cost is low, as the postponed time can be taken as part of the waiting, the price threshold is lower. By a lower threshold, the mine’s owner hopes during the postponed time, the price will climb.
Figure 28 Exercising Price Threshold by Differentiated Mining Cost

Figure 29 Exercising Probability by Differentiated Mining Cost
When the mining cost is high, the postponing effect raises the price threshold. This is for the reason that the delaying of delivery adds uncertainty to the “delivering price” (because the price will not always increase and mean-reverting model guaranties that price decreases in a longer time span). Thus, a higher threshold is necessary to insure the “deliver price” is high enough to cover the possibly high mining cost, as well as considering the price may decrease during the postponed time.

Figure 29 shows the option exercising probabilities for the cases of no postponing, and postponing for 5 years. Because by postponing, there is a lower threshold for the low-cost mine and a higher threshold for the high-cost mine, the option exercising probabilities shifts accordingly: the postponing increases the possibility that a low-cost mine is activated, and curtails the possibility that a high-cost mine is activated.

8. Conclusion

Mining firms hold large amount of mineral reserve that can be particularly valuable if mined at the right time. This flexibility value, which has not been considered in much of the previous literature, can be quite significant. The flexibility value peaks when average mining cost equals to the spot price. This research’s main contributions are: 1) providing a tractable and realistic method of determining the optimal timing of irreversible decisions incorporating a mean-reverting model for commodity price; 2) presenting several extensions of the model showing how timing considerations to be important. It is found that flexibility value peaks when average mining cost equals to the spot price. The stochastic pattern of the mining cost and the effects of postponing delivery of the commodity product on real option value, as well as the price threshold are examined.
The real option method provides a tractable and realistic scheme to evaluate the in-situ value of a mining project and a strategy to manage the timing of mining activities. The materials mined can be much more valuable if the mine is activated following an appropriately chosen price threshold. This research implies that the real option value due to operational flexibility can be significant and should not be overlooked when valuing mining investment properties.
Reference


Akbari, Afshin Dehkharghani, Morteza OSANLOO, and Mohsen Akbarpour SHIRAZI. "Reserve estimation of an open-pit mine under price uncertainty by real option approach." Mining Science and Technology (China) 19, no. 6 (2009): 709-717.


Chapter 2

A Strategic Approach for Criticality – Application to Yttrium

Abstract

The vulnerability of the US economy to restrictions in the supply of various rare earth elements have been of great concern over the last five years. In this chapter, the “criticality” of Yttrium, the most commonly used rare earth element, is studied. Criticality depends on sources of supply, relevant uses, relevant substitutes and recycling possibility for mineral in question. Yttrium currently comes largely from only a few areas in southern China, increasing the economic vulnerability in the short term. In the medium term, however, the supply threat for Yttrium may decrease as more sources outside China become available.

In south China, which is the major source of world Yttrium supply, Yttrium is produced with other rare earth elements. Since Yttrium’s content is high in the ion adsorption clays in south China, its supply is not likely to be disturbed by the price changes of other minerals. In north China (Baiyun Ebo), where the content of Yttrium is low, rare earth elements are usually produced as by-product of iron. Its production, however, causes a variety of environmental harms, making the supply stream vulnerable to more stringent environmental politics. Yttrium is employed in a variety of uses. Yttrium is difficult to replace in compact fluorescent lights (CFL), its largest use. With the short-run demand for CFLs high, this increases the criticality of Yttrium. In the longer term, however, the demand for CFLs is expected to fall. The criticality of Yttrium in other uses is mixed.
1. Introduction

Rare earth materials – rare earth ores, oxides, metals and alloys – are used in a variety of high-tech, renewable and defense related applications such as computer displays, wind turbines, refractory and wear-resistant tools. However, worldwide availability of these materials that are economically exploitable may be limited to a few sources – primarily in China. In 2009, China produced about 97 percent of the world’s rare earth oxides (REO). China’s market dominance may affect future availability of rare earth materials in the US.

Since 2007, China has taken measures to affect global rare earth supply and prices. These restrictive actions include domestic quotas for rare earth production and export, and raising export taxes on rare earth materials. During the rare earth crisis in August 2009, China proposed to ban exports of unprocessed critical REEs. Exports were quickly resumed, partly because of the fear of World Trade Organization’s action. In addition, China is vertically integrating its rare earth production procedures and trying to form large rare earth companies. Industry officials believe that in future, China will only export finished rare earth material products with high value added rather than raw products.

The Department of Energy (2011) defines a “critical material” in terms of its importance to the clean energy economy and the risk of supply disruption. Rare earth materials are important not merely for clean energy, but also for other high-tech and military uses. China’s quota and taxes on rare earth materials places the rare earth supply to the US at risk. Thus, an examination of the “criticality” of rare earth is necessary to make sure that sufficient, reliable and low-priced rare earth supply is available for US in the future.
This research provides a framework for addressing material “criticality” focusing on one of the heavy rare earth elements – Yttrium. In the DOE report, Yttrium is listed as a critical material in both short term (2011 to 2014) and medium term (2015 to 2025). By examining the resources, policies, environmental vulnerability and rare earth processing, uses and projections of demand, this framework presents a rigorous and quantitative analysis of Yttrium availability. The framework can also be applied to other materials.

This chapter is organized as follows: Section 2 reviews Yttrium resources across the world. In section 3, rare earth processing and the environmental vulnerability from rare earth mining are introduced. Section 4 explores China’s rare earth policy, while section 5 examines the economic impacts of rare earth export taxes and quotas. Section 6 covers the value and the prices of China’s rare earth export and US’s import. Section 7 examines the criticality of Yttrium in several important uses. The research concludes by section 8.

**2. Yttrium Resources**

The rare earth group comprises 15 Lanthanides in the periodic table with atomic numbers 57 to 71 and another two elements-Scandium (atomic number 21) and Yttrium (atomic number 39). Because of similar physical and chemical characteristics, the lanthanide elements (especially the heavy REEs with atomic number 64-71) and Yttrium often occur together in nature (Spooner, 2005). These physical properties include that, most of them are silver-metallic and lustrous. These elements are highly crystalline transition metal and stable in air due to oxide film that forms on their surfaces. Yttrium’s average concentration in the earth’s crust is 33 part per million (ppm) and it is the second most abundant rare earth in the earth’s crust after Cerium (Hedrick, 1999).
Principal commercial sources of REEs are: 1) bastnaesite, a fluoro-carbonate which occurs in carbonatites and igneous rocks; 2) xenotime, an Yttrium phosphate commonly found in mineral sand deposits; 3) loparite, a titanate related to perovskite and which occurs in alkaline igneous rocks; and 4) ion-adsorption clays, which are only found in southern China. Monazite is no longer a significant commercial source of REEs because it commonly contains Thorium, and the resulting intermediate products are radioactive (Spooner, 2005). There is no worldwide reserve estimation for individual REE elements except for Yttrium (Schuler, Liu and Merz, 2011). The estimations for Yttrium by country are presented in Table 13. Although China produces more than 95% of the global production, its share of reserves is much lower, at 41%. Large rare earth deposits are also found in US, Australia and India, from which significant amount of Yttrium can potentially be recovered.

<table>
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<tr>
<th></th>
<th>Thousand Metric Tons</th>
<th>Percentage of World Total</th>
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</thead>
<tbody>
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<td>China</td>
<td>220</td>
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<td>13</td>
<td>2.41%</td>
</tr>
<tr>
<td>Brazil</td>
<td>2.2</td>
<td>0.41%</td>
</tr>
<tr>
<td>Sri Lanka</td>
<td>0.24</td>
<td>0.0004%</td>
</tr>
<tr>
<td>Other countries</td>
<td>17</td>
<td>3.15%</td>
</tr>
<tr>
<td>World Total</td>
<td>540</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 13 Yttrium Reserves in Thousand Metric Tons of Oxide by Countries (USGS, 1996-2013)

2.1 China

In China, there are two groups of producers: 1) the Northern group based principally in Inner Mongolia, Gansu and Sichuan with production from bastnaesite, and 2) the Southern group in Guangdong, Fujian, Hunan, Jiangxi and Jiangsu with production from adsorption clays.
In the northern group the Yttrium content is quite low (such as in Baotou and Sichuan, the grades of bastnaesite are 0.2% and 0.5%, respectively). Near Baotou, Inner Mongolia, bastnaesite is a by-product of iron mining, while in Baiyun Ebo, Inner Mongolia, Gansu and Sichuan, bastnaesite is the primary product. Northern China rare earth deposits are dominated by light rare earths such as Cerium, Lanthanum and Neodymium (Haxel, Hedrick and Orris, 2002).

Significant Lanthanides, along with Scandium and Yttrium are found in the southern ion adsorption clays. Grades in Guangdong high-Europium clay and Jiangxi high-Yttrium clay are 22% and 62%, respectively. Figure 30 shows the geographic distribution of China’s rare earth reserves. The ion adsorption clays of southern China amount for most of the current world’s Yttrium supply. Table 14 shows the share of ion adsorption deposits in the southern China by province. Jiangxi, Guangdong, Fujian and Guangxi account for 92% of ion adsorption deposits in southern China. Yttrium supplied to the world is primarily produced from these provinces.

Figure 30 Distribution of China Rare Earth Operating Deposits
<table>
<thead>
<tr>
<th>Province</th>
<th>Jiangxi</th>
<th>Guangdong</th>
<th>Fujian</th>
<th>Guangxi</th>
<th>Hunan</th>
<th>Yunnan and Zhejiang</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share in %</td>
<td>36</td>
<td>33</td>
<td>15</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 14 Distributions of Ion Adsorption Deposits in Southern China (Ministry of Environmental Protection, 2010)

Table 15 compares the grades for Yttrium and other REEs in northern and southern China. Figure 31 and Figure 32 are the corresponding pie graphs. Figure 31 shows that Baotou’s rare earth deposit contains low concentration of Yttrium. Baotou’s deposit is primarily composed of REEs such as Cerium, Lanthanum, Scandium, Praseodymium, Neodymium, Samarium, Promethium, Gadolinium and Europium. These nine elements are known as light rare earth elements (LREEs) or the Cerium group. As Figure 32 shows, ion adsorption clay contains significant amount of Terbium, Dysprosium, Holmium, Erbium, Thulium, Ytterbium, Lutetium and Yttrium. These elements are known as heavy rare earth elements (HREEs) or the Yttrium group. Usually, HREEs are more expensive than LREEs in the market due to their relative low abundance, difficulty of processing, and more important applications. (Humphries, 2012)
<table>
<thead>
<tr>
<th></th>
<th>Baotou, Inner Mongolia, Bastnaesite Concentrate (Northern)</th>
<th>Longnan, Jiangxi, Ion Adsorption Clays (Southern)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Percentage in Total Rare Earth Ore</td>
<td>Weight Percent of the Ore</td>
</tr>
<tr>
<td>Total Rare Earth Ore</td>
<td>100%</td>
<td>50%</td>
</tr>
<tr>
<td>Lanthanum</td>
<td>23%</td>
<td>11.5%</td>
</tr>
<tr>
<td>Cerium</td>
<td>50.10%</td>
<td>25.05%</td>
</tr>
<tr>
<td>Praseodymium</td>
<td>5%</td>
<td>2.5%</td>
</tr>
<tr>
<td>Neodymium</td>
<td>18%</td>
<td>9%</td>
</tr>
<tr>
<td>Samarium</td>
<td>1.60%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Europium</td>
<td>0.20%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Gadolinium</td>
<td>0.80%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Terbium</td>
<td>0.30%</td>
<td>0.15%</td>
</tr>
<tr>
<td>Dysprosium</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Erbium</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Yttrium</td>
<td>0.20%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Holmium-Thulium-Ytterbium-Lutetium</td>
<td>0.80%</td>
<td>0.4%</td>
</tr>
</tbody>
</table>

Table 15 Northern and Southern China Yttrium and Other REEs’ Grades (Schuler, Liu and Merz, 2011)
Figure 31 Baotou (Inner Mongolia) Bastnaesite Concentrate

Figure 32 Longnan (Jiangxi) Ion Adsorption Clays
Baiyun Ebo is another important rare earth site in northern China. The mine is located in the Wulanchabu prairie of central Inner Mongolia, over 100 kilometers south of Mongolia and over 140 kilometers north of Baotou City. The mine was built in 1957 and after expansion in 2000, the open pit mine has an annual output of more than 12 million metric tons of ore, and employing more than 6,000 people. The Baiyun Ebo iron ore resources include 26 elements with proven iron ore reserves of 1.4 billion metric tons. It is also the second largest Niobium reserve in the world. (Baiyun Ebo Iron Mine, 2012)

Production from bastnaesite in Inner Mongolia in 2010 generated 71,000 metric tons of REO. Using a 0.1% grade for Yttrium implies that 71 metric tons of Yttrium Oxide were produced in Inner Mongolia. 31,000 metric tons of REO are produced in 2010 from ion adsorption clays found in Southern China (Chen, 2011). Using an Yttrium grade of 40%, there are around 12,400 metric tons of Yttrium produced in Southern China in the year 2010.

In addition, geologists have discovered a large reserve of rare earths in central China’s Hubei Province at the foot of Mountain Laoyin in Shiyan City in October 2010. The mineral deposit appear to contain large and high quality resources, containing 12 different REEs. The Hubei Land Reserve Committee is conducting estimation of the reserve scale. This deposit is not included in above statistics (The Media Office of the State Council of China, 2012).

2.2 Rare Earth Resource in India

India is the second largest producer of Yttrium (from monazite) after China. Monazite can be produced from heavy mineral sand deposit in Kerala, Tamil Nadu and Orissa. Since Thorium is co-produced with monazite (Spooner, 2005), which may cause a radioactivity

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1 From the China People’s Daily (October 9th, 2010) (http://english.peopledaily.com.cn/90001/90778/90860/7160942.html)
problem. India’s Yttrium Oxide production has been maintained at 50 metric ton per year since 1994. A 10,000-ton-per-year monazite processing plant is expected to kick off in India by the end of 2013. (USGS, 1996-2013)

### 2.3 Rare Earth Resource in Former Soviet Union Area (Russia and Central Asian Countries)

Relying on loparite from Lovozero massif in the Marmansk region, Yttrium concentrate is produced and processed by the Russian rare earth company Solikamsk Magnesium Works. There is an extra cost of handling chlorides due to their natural radioactivity. The radioactive elements need to be removed on-site. Irtysh Rare Earth Co Ltd produces several separated REOs, metals and other products up to purity of 99.99% in eastern Kazakhstan. In Kyrgyzstan, there is rare earth metal reported to be produced in Orlovka. (Technology Metals Research, 2013)

### 2.4 Rare Earth Resource in US

In the Bastnasite deposit of Mountain Pass, California, Yttrium’s content is very low with a grade of 0.1%. Yttrium was not recovered as a separate element during processing before 2002 (USGS, 1996-2013). As US environmental standards tightened, the costs of producing rare earths increased. There has been no Yttrium produced in the US since 2002 when the Mountain Pass mine was closed after a pipeline spill and in response to lower prices of REEs from China. Essentially, all purified Yttrium in the US is derived from imported compound (Hedrick, 1999). The Monazite reserve in Green Cove Springs, FL has a relatively higher grade of Yttrium (3.2%) (Hedrick, 1999) but there has been no production reported there so far.

Since 1999, almost all (more than 95%) of the separated REE in the US has been imported either directly from China or from countries that imported their plant feed materials
from China. Several factors have led to the situation of complete dependence on imports from a single country: 1) Much lower labor, regulatory and environmental costs in China; 2) The advantageous number, size and heavy REEs content of the China’s deposits, especially from southern China; 3) Ongoing environmental regulatory problems in the US such as at Mountain Pass (Haxel, Hedrick and Orris, 2002).

2.5 New Yttrium Projects outside China

The rare earth crisis in 2009 has caused an increase in rare earth exploration outside of China. This part examines the status of Yttrium exploration in the “rest of the world” (ROW) such as Yttrium resources under development and the potential supply. Technology Metals Research (2013) reports that within the last few years there has been an explosion in the number of new rare earth exploration and development projects across the world.

Table 16 shows the top 10 rare earth projects that are under development or operation outside China and India. Whether China’s ability to affect RRE markets can be reduced depends on whether these projects can achieve successful commercialization. According to Technology Metals Research (2013), existing ROW resources are dominated by the presence of Light REEs. Projects are ranked by the total potential metric tons of Yttrium Oxide. Canada, Greenland, Australia, Brazil and South Africa are the major potential Yttrium resources outside China. Most of these projects currently do not have developed complete mineral resource estimates or profitable mining activities.

The top five resources with in-situ Y2O3 in total rare earth ore by weight percent are Round Top (USA, 44.44%), Norra Karr (Sweden, 34.55%), Kutessay II (Kyrgyzstan, 27.17%), Strange Lake (Canada, 25.34%) and Bokan (USA, 25.5%), as described in Table 16. According
to Texas Rare Earth Resource, to a number of geological and metallurgical studies had been completed by 2011 with preliminary economic assessment completed in mid-2012. Texas Rare Earth Resource finished a pre-feasibility study on the Round Top project in the spring of 2013 and will conduct a full feasibility study in late summer 2014.

In March 2011, Stans Energy released a technical report including mineral resource estimate for REOs remaining below the Kutessay II open pit mine and posted detailed 2012 drilling results recently. Kutessay’s field program was started in 2013 and Stans Energy has plans for development and production at Kutessay II. In February 2013, Quest Rare Minerals Ltd updated its pre-feasibility study for the heavy REE project in Strange Lake, Quebec, Canada. The study shows that Strange Lake is a very large rare earth project with high concentrations of heavy REEs as well as by-products such as zirconium and niobium. The final demonstration scale pilot plant began operating in the second quarter of 2013.

According to the Preliminary Economic Assessment study by Ucore (Ucore, 2013), the Bokan project requires approximately two additional years of construction activities before complete. Temporary construction facilities will be built in summer 2014, including the batch plant and aggregate plant. The concrete for the main process building, truck shop and powerhouse

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3 Stans Energy’s Kutessay II Heavy Rare Earth Element Mine: http://www.stansenergy.com/projects/kutessay-ii

4 Strange Lake: http://questrareminerals.com/

building will be constructed by the summer of 2014. The plant start-up and commissioning are expected to happen in the first quarter of 2016.

<table>
<thead>
<tr>
<th>Project</th>
<th>Country</th>
<th>Owner</th>
<th>Measured Reserve (Mt)</th>
<th>Total Rare Earth Oxide (wt%)</th>
<th>TREO (Mt)</th>
<th>In-Situ TREO (%)</th>
<th>Basket Price ($/kg)</th>
<th>Grade of In-Situ Y_2O_3 (wt%)</th>
<th>Grade of In-Situ TREO (wt%)</th>
<th>Weight of Y_2O_3 in TREO (wt%)</th>
<th>Total tons of Y_2O_3</th>
<th>% of Light REO Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strange Lake Granite</td>
<td>Canada</td>
<td>Quest Rare Minerals Ltd.</td>
<td>472.46</td>
<td>0.87</td>
<td>4.119</td>
<td>500</td>
<td>57</td>
<td>0.21</td>
<td>0.87</td>
<td>23.68</td>
<td>8649.9</td>
<td>4.09</td>
</tr>
<tr>
<td>Kvanefjeld</td>
<td>Greenland</td>
<td>Greenland Minerals and Energy Ltd.</td>
<td>619</td>
<td>1.06</td>
<td>6.547</td>
<td>374</td>
<td>35</td>
<td>0.08</td>
<td>1.06</td>
<td>7.7</td>
<td>5237.6</td>
<td>1.82</td>
</tr>
<tr>
<td>Nechalacho Basal</td>
<td>Canada</td>
<td>Avalon Rare Metals Inc.</td>
<td>125.72</td>
<td>1.43</td>
<td>1.795</td>
<td>850</td>
<td>60</td>
<td>0.16</td>
<td>1.43</td>
<td>11.1</td>
<td>2872</td>
<td>1.48</td>
</tr>
<tr>
<td>Sørensen</td>
<td>Greenland</td>
<td>Greenland Minerals and Energy Ltd.</td>
<td>242</td>
<td>1.1</td>
<td>2.662</td>
<td>371</td>
<td>34</td>
<td>0.09</td>
<td>1.1</td>
<td>8.12</td>
<td>2395.8</td>
<td>2.88</td>
</tr>
<tr>
<td>Nechalacho Upper</td>
<td>Canada</td>
<td>Avalon Rare Metals Inc.</td>
<td>177.73</td>
<td>1.32</td>
<td>2.353</td>
<td>619</td>
<td>47</td>
<td>0.06</td>
<td>1.32</td>
<td>4.5</td>
<td>1411.8</td>
<td>0.47</td>
</tr>
<tr>
<td>Ashram Main</td>
<td>Canada</td>
<td>Commerce Resources Corp.</td>
<td>239.71</td>
<td>1.9</td>
<td>4.549</td>
<td>703</td>
<td>37</td>
<td>0.03</td>
<td>1.9</td>
<td>1.58</td>
<td>1364.7</td>
<td>0.02</td>
</tr>
<tr>
<td>Strange Lake Enriched</td>
<td>Canada</td>
<td>Quest Rare Minerals Ltd.</td>
<td>20.02</td>
<td>1.44</td>
<td>0.288</td>
<td>941</td>
<td>65</td>
<td>0.47</td>
<td>1.44</td>
<td>32.62</td>
<td>1353.6</td>
<td>0.07</td>
</tr>
<tr>
<td>Zone 3</td>
<td>Greenland</td>
<td>Greenland Minerals and Energy Ltd.</td>
<td>95.3</td>
<td>1.16</td>
<td>1.106</td>
<td>386</td>
<td>33</td>
<td>0.1</td>
<td>1.16</td>
<td>8.55</td>
<td>1106</td>
<td>0.02</td>
</tr>
<tr>
<td>Mount Weld Duncan</td>
<td>Australia</td>
<td>Lynas Corporation Ltd.</td>
<td>8.99</td>
<td>4.84</td>
<td>0.435</td>
<td>2474</td>
<td>51</td>
<td>0.25</td>
<td>4.84</td>
<td>5.17</td>
<td>1087.5</td>
<td>0.66</td>
</tr>
<tr>
<td>Mount Weld CLD</td>
<td>Australia</td>
<td>Lynas Corporation Ltd.</td>
<td>14.95</td>
<td>9.73</td>
<td>1.454</td>
<td>3723</td>
<td>38</td>
<td>0.07</td>
<td>9.73</td>
<td>0.76</td>
<td>1017.8</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Table 16 Top 10 Yttrium Projects (in terms of total tons of Y_2O_3) under Development Outside of China and India by March 8th, 2013 (Technology Metals Research, 2013)
2.6 By-product and Co-product Analysis of Yttrium and Rare Earths

The Society of Mining Engineers Mining Handbook (2011) provides the following definitions for main product, co-product and by-product: A main product is defined if it alone determines the economic viability of a mine. When two metals must be produced to make a mine economic, both influence output, and they are co-products. A by-product is produced in association with a main product or with co-products. Its price has no influence over the mine’s ore output. How Yttrium fits into this taxonomy may have important effect of the economics of its supply.

The difference in the supply of by-product and supply of individual/main product is that by-product’s supply is limited by the output of the primary product, such as iron in Baiyun Ebo. Thus, when the output of ore has reached some capacity, a higher by-product price does not increase the output of ore or of main product. Then, at the output capacity, supply may become inelastic with respect to further increases in by-product price.

However, the high price of by-product, i.e. rare earth in Baiyun Ebo, may still exert impacts on the production activities. A high by-product price may encourage new technology in the long run that allows a greater recovery rate of the by-product, which increases the output of the by-product without changing the output of the main product. (Darling, 2011)

The price change of the main product can affect the supply of by-products because if the demand of the main product rises, this causes the price and output of main product to rise, increasing the amount of ore from which by-product can be recovered. Therefore, besides its own price, the supply function for a co-product is also determined by the price (or output) of the main product. (Wilkinson, 1983)

Another difference between by-product and main product supply is that only costs specific to by-product processing affect by-product supply. While joint costs, which are necessary
for the processing of the main product, does not influence by-product supply because the joint costs have to be spent no matter whether by-product is produced or not. This means that the by-product supply curve simply reflects the marginal cost of by-product production exclusive of all joint costs. Therefore, the metal from by-product supply is often, but not always, at lower price than the same metal from the main product supply, due to the co-products low marginal cost. (Darling, 2011)

The major iron deposits at Baiyun Ebo in Inner Mongolia contain substantial light REEs recovered as by-products of iron ore mining (Humphries, 2012). In southern China, REEs are produced as the main product in ion adsorption clays and Yttrium is co-produced with the other 16 elements in REE group.

The co-product status of Yttrium depends on where it is produced. In the northern mines of China, Yttrium is a very small part of the total production. In the southern mines, however, Yttrium constitutes over half by weight of the desired mineral. Indeed, the vast majority of China’s Yttrium comes from the southern mines. Thus, at least compared to other rare earths, Yttrium faces little economic challenges of a co-product.

3. Yttrium Processing and Environmental Vulnerability

Rare earth mining in China causes serious environmental issues such as wastewater with acid, alkali and radioactive materials, air emissions with fluorine and sulfur, and soil erosion (North Carolina Wind Turbines, 2012). For China’s rare earth exports are all in the forms of metal, alloy or oxide, it can be inferred that all the REEs minerals sold commercially that have been extracted in China are processed within China. The processing procedure uses toxic chemicals, acids, sulfates and ammonia. This section outlines the processing procedures of Yttrium and their environmental impacts and costs, especially in China.
Before 2010, for most rare earth enterprises in China, there were not sufficient facilities to treat the waste. For some small operations, there were not even facilities for environmental protection. In contrast, in the US, REE operations are required to meet environmental quality standards include air, water and soil qualities and some individual states have more stringent standards. Reclamation under state authority is required (Castor and Hedrick, 2004). The Chinese government formulated more stringent emission standards in 2010 for rare earth industry including detailed concentration of pollutant content (Fluoride, Phospide, Nitride and Zinc, etc.) and the pH value in waste gas, water and residue (Chen, 2011).

3.1 Rare Earth Processing from Bastnaesite

The Great Western Minerals Group (2012) has provided a general description of the processing procedures, which include exploration, mining, mineral processing (includes crushing, grinding and sometimes leaching steps to bring elements into solution), separation, metal making and end-use alloy making. Following provides the processing procedures for bastnaesite and ion adsorption clays, respectively. Since almost all supplies of Yttrium originate from the clays, the major environmental impacts of producing Yttrium are discussed in the part for the ion adsorption clays.

The production of REEs from bastnaesite contains three stages—beneficiation, smelting and separation (Baiyun Ebo Iron Mine, 2012).

Beneficiation produces a mineral concentrate and consists of crushing, milling, and concentrating technologies such as gravity, magnetic and “floatation” techniques. The quality of rare earth ore in bastnaesite is usually below 10%, which is quite low, so beneficiation is necessary before smelting. In addition, because in Baiyun Ebo, rare earths are by-products of iron production, another goal of beneficiation is to increase the grade of Fe₂O₃ to above 55%. After
milling, the minerals are transported to beneficiation plant in Baotou, Inner Mongolia. By repeating the “floatation” method and magnetic separating, iron ore concentrate with a Fe₂O₃ grade of 45% are generated. The materials containing REE are embedded in the “floatation foam” with REO grade between 10% and 15%. The material is further treated by shakers to generate REO concentrate with grade around 30%. After further treatment, an REO concentrate with grades greater than 60% is obtained (Baiyun Ebo Iron Mine, 2012).

Hydrometallurgy and Pyrometallurgy are two methods used in the smelting stage after beneficiation. The processes of hydrometallurgy occur in solution. The processes include sedimentation, crystallization, redox reaction and solvent extraction. After these processes, the rare earth concentration can be purified to be REOs or compounds. Presently, the extraction method is widely used, employing organic solvent to obtain individual REO. Hydrometallurgy can generate REOs with highly concentrated final products (Habashi, 2009).

There are fewer steps involved in Pyrometallurgy. The process may adopt silicon-thermal reduction, molten salt electrolyzing or metal-thermal restoring. All three approaches generate rare earth metals or alloys under high temperature (Ojeda et al., 2009).

Rare earth concentrate composed of carbonate, phosphate, oxide or silicate is generally insoluble. The pre-treatment converts the REEs to the compounds that are soluble in water or inorganic acid (Misawa et al., 2000). After dissolution, separation, purification and burning, a mixture of rare earth compounds is obtained. After the pre-treatment, three methods can be used to separate REEs: the multiple-step method, ion exchanging and solvent extraction.

The multiple-step method takes advantage of differentiated solubility of individual compound to generate and purify each element (Max-Hansen et al., 2011). In the solvent extraction method (Luo and Byrne, 2004) organic solutions are used in this method to extract individual REE from aqueous solution. The method is widely used in REEs separation in large-scale operations.
In the ion exchange method (Hooker et al., 1975), rare earth concentrates are leached to generate complex compounds that are dissolved in leaching solution. The complex compound for individual REE flows at different speeds so that the mixture of complex compound can be separated. In large-scale rare earth production such as in Baiyun Ebo, the ion exchange method is not used as often as solvent extraction method because of above two disadvantages. However, when certain high purity of the product is required (99.999%-99.9999%), the ion exchange method is used (Zhang, et al., 2007). The advantage of this method is that high purity products are generated. Disadvantages include that it takes longer time for rare earth complex compounds to flow and separate and that the cost of recycling is high.

### 3.2 Processing Yttrium from Ion Adsorption Clays

Bastenaesite in China’s Northern provinces can be produced in vertically integrated large-scale operations (MOSCPRC, 2012). Since the ion adsorption clays within the five provinces (Jiangxi, Jiangsu, Hunan, Fujian and Guangdong) in southern China are widely dispersed, it is more difficulty to produce REEs in large-scale in these areas and the producing operations are relatively small. Beside the operational scale, the methods used to process bastenaesite and ion adsorption clays are also different because ionic REEs are absorbed in the soil rather than in the form of oxide. Usually, the cost of producing REEs from the clays is much higher than that of bastenaesite.

According to Zou (2012), three different methods have been adopted to process Yttrium from ion adsorption clays. In the order of adoption time, they are tank leaching, heap leaching, and in-situ leaching. Producing Yttrium by each method imposes different impacts on mining and ecological environment. In-situ leaching is believed to be the most advanced in protecting resources and environment and has been used in 20% of total clay operations by 2011 (Li, 2011).
Major costs of producing REEs from clays in China include the rare earth resource tax, and the restoration cost for the ecological plant cover and the environment in the mining area (Zou, 2012). The current estimated production cost is 49,000 USD (300,000 RMB) per metric ton of REO. Assuming the price of Ammonium Sulfate is 130 USD (800 RMB) per metric ton and the price of Ammonium Carbonate is 98 USD (600 RMB) per metric ton, Yuan (2009) estimates the production cost for heap leaching is between 4,083 USD (25,000 RMB) to 4,900 USD (30,000 RMB) per metric ton of REO and for in-situ leaching is about 4,083 USD (25,000 RMB) per metric ton of REO. Zou estimates the production cost of 1 metric ton of Yttrium-Europium oxide for tank leaching, heap leaching and in-situ leaching, respectively. Zou’s estimates are shown in Table 17.

In Table 17, the first column lists three chemical inputs to produce rare earth, ammonium sulfate, ammonium carbonate and sulfuric acid. The second column lists the unit price of each chemical input. The third, fifth and seventh column lists the amount of chemical inputs needed to produce rare earth in each method. The fourth, sixth and eighth column lists the total cost of each chemical input for each method, as well as the fixed cost for each method such as costs of electricity, maintenance and construction/tax. By summing up the fixed cost and chemical input cost, the total cost of producing one metric ton of Yttrium-Europium Oxide is provided in the last row. It can be seen from Table 17 that, due to less expenditure on construction and tax cost, in-situ leaching is not significantly more expensive than heap leaching and is less expensive than tank leaching. Meanwhile, in-situ leaching does not use much more chemical inputs than tank leaching or heap leaching and consequently does not significantly increase environmental harms.
<table>
<thead>
<tr>
<th>Unit Price of Chemical Inputs (USD/ton)</th>
<th>Cost of Tank Leaching</th>
<th>Cost of Heap Leaching</th>
<th>Cost of In-situ Leaching</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tons of Chemical Inputs used</td>
<td>Fixed and Total Cost (USD)</td>
<td>Tons of Chemical Inputs used</td>
</tr>
<tr>
<td>Ammonium Sulfate</td>
<td>139</td>
<td>4.4</td>
<td>611</td>
</tr>
<tr>
<td>Ammonium Carbonate</td>
<td>98</td>
<td>2.2</td>
<td>216</td>
</tr>
<tr>
<td>Sulfuric Acid</td>
<td>106</td>
<td>0.4</td>
<td>43</td>
</tr>
<tr>
<td>Electricity</td>
<td></td>
<td></td>
<td>41</td>
</tr>
<tr>
<td>Maintenance</td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>Construction Cost and tax</td>
<td></td>
<td></td>
<td>1,094</td>
</tr>
<tr>
<td>Total Cost to Produce 1 Metric Ton of Y-Eu Oxide</td>
<td></td>
<td></td>
<td>2,030</td>
</tr>
</tbody>
</table>

Table 17: Production Cost of producing 1 Metric ton of Yttrium-Europium Oxide by Tank Leaching, Heap Leaching and In-situ Leaching (Zou, 2012)

The main stages of the tank leaching method (Li, 2011) include topsoil stripping; extraction and moving the ore body; adding leaching solution to the leaching tank to leach the ore body; exchanging the active ions in the solution with the REE so that REE ions are active in the solution; adding “top water” into the tank to concentrate the REEs; transporting the solution to a sedimentation tank and eliminating the solid waste; adding sedimentation and impurity agents into the sedimentation tank so that mixed rare earth sediments can be obtained. The solution can be recycled after treatment and transported to the leaching tank; by burning the mixed rare earth sediments in high temperature a mixture of REOs with purity greater than 92% can be obtained. The tailings are transported off-site and treated. Compared to traditional mining and metallurgy
methods, the leaching method is advanced, because it generates mixed REO with high purity
directly instead of lower quality mineral ore (Liu, 2002).

The leaching method, however, may cause severe environmental damages. Since ion
adsorption clays are on the ground surface in broad areas, considerable amounts of earth need to
be moved (Zou, 2012). Additionally, the plant cover within the area is required to be removed for
the production. It is estimated that to produce one ton of mixture of REOs, approximately 1,200
to 2,000 metric tons of ore are processed. At the same time, approximately 1,200 to 2,000 tons of
tailings are generated. Other disadvantages of leaching tank methods include (Zou, 2012): the
waste generated contains ammonia and heavy metals, which can pollute water resources if not
treated appropriately; removing surface earth may lead to mudslides or landsides; the cost of a
leaching tank; the production can be affected by weather (i.e., production needs to be suspended
when raining).

Heap leaching is quite similar to tank leaching except that the mixed rare earth sediments
are obtained in the leaching heap. One advantage of heap leaching method is that it makes it
possible to process clays in large-scale operations. There are several small leaching tanks in
Figure 33 and one large-scale leaching heap in Figure 34. Besides the disadvantages of tank
leaching, the heap leaching suffers from other problems including the cost of land use; the flood
and drainage control of the heap area; and only the high-quality resources can be processed while
the poorer resources need to be abandoned.
In-situ leaching does not require elimination of surface earth and plant cover. Instead, the leaching solution is injected underground where the clays are processed. The rare earth ions are then collected by a “liquid-collecting ditch.” Compared to the tank/heap leaching method, in-situ
leaching requires much less work on removing earth and surface plant cover, produces at higher recovery rate (more than 70%) and generates less tailing.

Figure 35 shows an example of in-situ leaching operation in Longnan, Jiangxi. It can be seen that the plant cover and the earth on the surface are protected. Yet, in-situ leaching still has several disadvantages. In-situ leaching has technical difficulties and there is uncertainty in controlling the high-technology operations. In-situ leaching also requires special ore and rock features such as certain hardness and strength and that may not be applicable to all clays. With this method, the recovery rate of REEs can be significantly reduced for mining operations with poor geological conditions, and it may also lead to unexpected mudslides and landsides.

Figure 35 An Example of In-situ Leaching In Longnan, Jiangxi
(Source: gzsdpc.gov.cn)
4. China’s Rare Earth Policy

4.1 Production and Export Quotas

Since the early 1990s, China has been developing production plans for strategic rare earth products including overall production quotas for individual provinces and export quotas. Provincial governments have been allocating production quotas to individual mining companies in the province. The output of rare earths and other commodities, however, had been much higher than the Government’s allowed output before 2011 (this can be seen from Table 18). A significant amount of the over-quota production was by miners without rare earth mining licenses. These miners often used outdated mining technology that led to significant environmental damages, especially in Guangdong, Jiangxi, and Sichuan. From Table 18, it can also be seen that rare earth production peaked in 2009 and declined 20% since then.

In addition to setting production quotas, the Chinese government has set quotas on the rare earth exports since 1998. Separate export quotas are set for domestic producers and for joint-venture producers. Because of the increase in domestic demand, the Chinese government has gradually reduced the export quota during the past several years (Tse, 2011). The total export quota peaked in 2007 and decreased to around 30,000 metric tons per year in 2010. Since then, the export quota has remained at that level.

The Chinese government has encouraged the export of high-value added products and discouraged the export of rare earth raw materials. This policy is reflected by a change in the way the export quota was released. Initially, the export quota was released as REO equivalent. Since 2006 the government has specified the tonnage limit of specific rare earth products that the joint-ventures are allowed to export (China Ministry of Commerce, 2006, 2008, 2009, 2010).
Table 18 China’s Rare Earth Production, Consumption and Export Quotas in Metric Tons of Rare Earth Oxide from 2000 to 2011 (Tse, 2011)

<table>
<thead>
<tr>
<th>Year</th>
<th>Production quota</th>
<th>Production</th>
<th>Consumption</th>
<th>Domestic producers and traders</th>
<th>Sino-foreign joint ventures</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>55,000</td>
<td>73,000</td>
<td>19,000</td>
<td>47,000</td>
<td>NA</td>
</tr>
<tr>
<td>2001</td>
<td>NA</td>
<td>81,000</td>
<td>20,000</td>
<td>45,000</td>
<td>NA</td>
</tr>
<tr>
<td>2002</td>
<td>NA</td>
<td>88,000</td>
<td>22,000</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2003</td>
<td>NA</td>
<td>92,000</td>
<td>30,000</td>
<td>40,000</td>
<td>NA</td>
</tr>
<tr>
<td>2004</td>
<td>NA</td>
<td>98,000</td>
<td>34,000</td>
<td>45,000</td>
<td>NA</td>
</tr>
<tr>
<td>2005</td>
<td>NA</td>
<td>119,000</td>
<td>52,000</td>
<td>48,010</td>
<td>17,570</td>
</tr>
<tr>
<td>2006</td>
<td>86,620</td>
<td>133,000</td>
<td>63,000</td>
<td>45,000</td>
<td>16,070</td>
</tr>
<tr>
<td>2007</td>
<td>87,020</td>
<td>120,000</td>
<td>73,000</td>
<td>43,574</td>
<td>16,069</td>
</tr>
<tr>
<td>2008</td>
<td>119,500</td>
<td>125,000</td>
<td>67,700</td>
<td>34,156</td>
<td>15,832</td>
</tr>
<tr>
<td>2009</td>
<td>110,700</td>
<td>129,000</td>
<td>73,000</td>
<td>31,310</td>
<td>16,845</td>
</tr>
<tr>
<td>2010</td>
<td>89,200</td>
<td>120,000</td>
<td>77,000</td>
<td>22,512</td>
<td>7,746</td>
</tr>
<tr>
<td>2011</td>
<td>93,800</td>
<td>96,800</td>
<td>NA</td>
<td>30,184</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>30,996</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>31,001</td>
<td></td>
</tr>
</tbody>
</table>

4.2 Export Duty Rates

The Chinese government uses taxes to regulate its rare earth industry and raise revenue. In the 1990s, the government encouraged firms to export REEs by refunding the value-added tax that producers had paid on exported REEs. In early 2000, owing to increased domestic consumption, the government reduced the export rebate for many strategic commodities including rare earths. In 2005, the rebate on exported rare earths was eliminated. China has imposed tax on rare earth minerals that are exported since June 1st, 2007 (Table 19). In 2007, the export tax for Yttrium compounds and metals was 10%, and increased to 25% in the year 2008.
<table>
<thead>
<tr>
<th>Commodity</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yttrium oxide</td>
<td>10</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Lanthanum oxide</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Cerium oxide, hydroxide, carbonate and others</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Neodymium oxide</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Europium and its oxide</td>
<td>10</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Terbium and its oxide, chloride and carbonate</td>
<td>10</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Dysprosium oxide, chloride and carbonate</td>
<td>10</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Other rare earth oxide</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Mixed rare earth chlorides and fluorides</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Mixed rare earth carbonates</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Mixed rare-earth, Yttrium and Scandium compounds and metals</td>
<td>10</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Non-mixed rare earth carbonates</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Rare earth ore</td>
<td>10</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td><strong>Metals</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lanthanum</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>25</td>
</tr>
<tr>
<td>Cerium</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>25</td>
</tr>
<tr>
<td>Neodymium</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Dysprosium</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Other mixed metals</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 19 China’s Rare Earth Export Duty Rates (China Customs Import and Export Tariff Department, 2007-2011)

4.3 Economics of Rare Earth Export Quotas and Taxes

By imposing export quotas and taxes, the economic welfares of rare earth producers, consumers and the government will be changed. In this section, these effects are analyzed using basic microeconomics.

Consider the case where the government sets an export tax on REEs and that tax is binding on product quantity. In addition, the government sets an export quota that is not binding the product quantity (or the government does not set export quota). Figure 36 shows the supply,
demand and welfares changes of rare earth export in this case. $Q_{\text{Tax}}$ is the quantity exported in this case, which is less the quantity without quotas or taxes.

Thus, when the export tax binds, the consumers face a higher price and the producers face a lower price compared to the unconstrained equilibrium. The consumer price is higher than the producers’ price by the amount of tax. In Figure 36, the area 1 is the new producer’s surplus and the area 3 is the new consumer’s surplus. Both producers and consumers’ welfares are reduced by export tax. The area 2 is collected by government as tax revenue. Society’s total welfare is reduced by area 4, which is referred to as the dead weight loss (Dahl, 2004).

![Figure 36 Supply, Demand and Welfares of Rare Earths Export When Tax Binds More than Quota](image)

Alternatively, it is possible that the export quota binds the quantity. Assume $Q^*$ is the quantity sold in the market when the quota binds, and $Q^*$ is less than the $Q_{\text{Tax}}$ in the previous case. Figure 37 shows how the supply, demand and welfares change in this case.
When the export quota binds the product quantity, the consumers face a higher price and the producers face a lower price compared to the unconstrained equilibrium. Area 1 is the producer’s surplus and area 4 is the consumer’s surplus. Both producers and consumers’ welfare are reduced. Area 2 is collected by the government as tax revenue. It is unclear who gets area 3. Perhaps area 3 goes to the rare earth exporting firms that may (or may not) be related to the rare earth producing firms. These exporting firms are not necessarily the rare earth producing firms, and so the welfare in area 3 are collected by the exporting firms that have the political connections necessary to obtain the export quotas.

![Figure 37 Supply, Demand and Welfares of Rare Earths Export When Quota Binds More than Tax](image)

4.4 Who Exports? Production and Exporting Firms in China

In the previous section, it has been shown that the profits of producing firms can be reduced by the exporting firms when the product quantity is bind by the export quota. This section examines who have been exporting Yttrium from China during the last several years.
Figure 38 shows the number of exporting firms from 2000 to 2006 in China. As the figure shows, the number of Y2O3 export firm decreases from 2000 to 2006. This may occur because 1) the government assigned the quota to fewer firms; or 2) the firms merged into large-scale firms. Figure 39 shows the number of exporting firms in the ownership categories of state-owned, collective-owned, domestic-private and joint-venture. In 2006, both the total number and the state-owned number decreased significantly. However, during a 7-year period, the numbers of domestic private and joint-ventures firms increased. Figure 40 shows the number of export entities that are producing firms, international-trade firms, research institutes, producing-trade-joint firms and firms from other industries. During the 7-year period, firms from other industries and research institutes were eliminated. The number of producing firms increased. The number of international trade firms decreased significantly in the year 2005 because the tax rebate was eliminated at the time. (China Ministry of Commerce, 2006-2010)

![Figure 38 Number of Y2O3 Exporting Firms](image)

(Source: China Ministry of Commerce, 2006-2010)
Figure 39 Number of Y2O3 Exporting Firms by Ownership Category
(Source: China Ministry of Commerce, 2006-2010)

Figure 40 Number of Y2O3 Exporting Firms by Type
(Source: China Ministry of Commerce, 2006-2010)
5. Export Price, Total Export Kilograms and Total Value

Figure 41 shows the export average price for Y$_2$O$_3$ in RMB/kilogram. Figure 42 shows total Y$_2$O$_3$ export amount in kilogram. Figure 43 shows total value of Y$_2$O$_3$ export in RMB. The tax rebate was eliminated in 2005 and export tax was imposed on 2007. Total exports decreased in 2005 and 2007. Since 2007, exports (both in kg and value) have decrease significantly. (The data in Figure 41, 42 and 43 are from Asian Metal’s rare earth statistics from 2000 to 2009.)

**Figure 41** Y$_2$O$_3$ Export Average Price (RMB/kilogram)

(Asian Metal Rare Earth Statistics, 2000 to 2009)

**Figure 42** Y$_2$O$_3$ Total Export (kilogram) (Asian Metal Rare Earth Statistics, 2000 to 2009)
Figure 43 Total Value of Y2O3 Export (RMB) (Asian Metal Rare Earth Statistics, 2000 to 2009)

5.1 Exports from China

Figure 44 shows China’s yearly Yttrium oxide export from 2007 to 2012 (seven months of data is used for 2012). Yttrium oxide exports peak in 2007 and 2010. China’s export is high during the middle of each year and is low during the beginning/end of each year. Figure 45 shows the share of each destination country for export of China. Japan purchases almost half of China’s exports. Italy, US and Hong Kong each accounts for approximately one eighth of Chinese exports.
Figure 44 China's Yearly Yttrium Oxide Exports (Asian Metal, 2013)

Figure 45 Share of Yttrium Import from China (Jan 2007 to Jul 2012) by Country (Asian Metal, 2013)
5.2 Imports to the United States

The United States also import Yttrium compounds from Canada, Germany, France, UK, Japan, South Korea, Russia, Austria, Spain, Liechtenstein and Switzerland. Figure 46 shows US yearly total Yttrium Compound Import. US’ imports peaked in 2010 (note the data for 2006 is from May to December). Figure 47 shows the share of each country for total US import of Yttrium Compound by country. More than 75% of imports are from China. Japan is also supplying significant amount of Yttrium compound to US. Other major suppliers include France, the United Kingdom, Austria and Germany.

Figure 46 US Yearly Yttrium Compound Import (Asian Metal, 2013)
6. Economic Analysis of Extra Yttrium Supply outside China

6.1 A Theoretical Approach

Given the increased prices of Yttrium and the risk of supply from China, the emerging rare earth producers outside of China will impose an impact on the Yttrium market. In this section, the impact of extra Yttrium supply from ROW (Rest of World) on the world Yttrium price is analyzed through economic analysis. For modeling purposes, exponential supply and demand curves are used here, and it is assumed that the demand and supply elasticities are not changed when emerging supply flows into the market.

Suppose that demand elasticity is constant and world demand for Yttrium in power function of price,

\[ Q = Ap^α \]
Where $\alpha$ is the constant demand elasticity (which is negative), and $A$ is a positive constant. The Yttrium supply curve before the emergence of ROW supply is

\[(2.2)\quad Q = B_1 p^\beta\]

The Yttrium supply curve after the emergence of ROW supply is

\[(2.3)\quad Q = B_2 p^\beta\]

where, $\beta$ is the supply elasticity and $\beta \in (0, \infty)$. In above specifications, $B_2 > B_1 > 0$.

Before the emergence of ROW supply, when the market is in equilibrium,

\[(2.4)\quad \text{Demand} = Ap^\alpha = B_1 p^\beta = \text{Supply}\]

The price before the emergence of ROW supply is

\[(2.5)\quad p_{\text{before}} = (\frac{B_1}{A})^{1\over \alpha-\beta}\]

The corresponding quantity is

\[(2.6)\quad Q_{\text{before}} = B_1^{\alpha-\beta} A^{-\beta}\]

By similar procedure, the price after emerging of ROW supply is

\[(2.7)\quad p_{\text{after}} = (\frac{B_2}{A})^{1\over \alpha-\beta}\]

The corresponding quantity is

\[(2.8)\quad Q_{\text{after}} = B_2^{\alpha-\beta} A^{-\beta}\]

Given that the supply curve shifts up 20% due to the Yttrium supply outside China, which means that,

\[(2.9)\quad \frac{B_2}{B_1} = 1.2\]

Following will derive how the Yttrium price changes given the shift of supply curve. Use formula (2.7) to be divided by (2.5) results,

\[(2.10)\quad \frac{p_{\text{after}}}{p_{\text{before}}} = (\frac{B_2}{B_1})^{1\over \alpha-\beta} = 1.2^{1\over \alpha-\beta}\]
The resulting quantity change is

\[
\frac{Q_{after}}{Q_{before}} = 1.2^{\frac{\alpha}{\alpha - \beta}}
\]

Equation (2.10) provides some implications of impact of extra Yttrium from ROW:

1) Normally, the demand elasticity \( \alpha \) is negative and the supply elasticity \( \beta \) is positive, which implies that \( \frac{1}{\alpha - \beta} < 0 \). Thus, extra Yttrium from ROW will lead to a declined Yttrium price in the market.

2) The percent of price decline depends on both the demand elasticity and the supply elasticity.

Some sensitivity analysis results are given in Table 20 and Figure 48, when the supply elasticity (\( \beta \)) changes from 0.25 to 2 and demand elasticity (\( \alpha \)) changes from -0.25 to -2. From the results, it is found that, due to the shift of supply curve, the price is very sensitive when the absolute values of \( \alpha \) and \( \beta \) are small, i.e., the demand and supply are relative inelastic. When one or both of \( \alpha \) and \( \beta \) are very far from zero (one or both of the demand and supply are elastic), the price change is very small (less than 5%).
<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>-0.25</th>
<th>-0.5</th>
<th>-0.75</th>
<th>-1</th>
<th>-1.25</th>
<th>-1.5</th>
<th>-1.75</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>30.6%</td>
<td>21.6%</td>
<td>16.7%</td>
<td>13.6%</td>
<td>11.4%</td>
<td>9.9%</td>
<td>8.7%</td>
<td>7.8%</td>
<td>7.8%</td>
</tr>
<tr>
<td>0.5</td>
<td>21.6%</td>
<td>16.7%</td>
<td>13.6%</td>
<td>11.4%</td>
<td>9.9%</td>
<td>8.7%</td>
<td>7.8%</td>
<td>7.8%</td>
<td>7.0%</td>
</tr>
<tr>
<td>0.75</td>
<td>16.7%</td>
<td>13.6%</td>
<td>11.4%</td>
<td>9.9%</td>
<td>8.7%</td>
<td>7.8%</td>
<td>7.0%</td>
<td>6.4%</td>
<td>6.4%</td>
</tr>
<tr>
<td>1</td>
<td>13.6%</td>
<td>11.4%</td>
<td>9.9%</td>
<td>8.7%</td>
<td>7.8%</td>
<td>7.0%</td>
<td>6.4%</td>
<td>5.9%</td>
<td>5.9%</td>
</tr>
<tr>
<td>1.25</td>
<td>11.4%</td>
<td>9.9%</td>
<td>8.7%</td>
<td>7.8%</td>
<td>7.0%</td>
<td>6.4%</td>
<td>5.9%</td>
<td>5.5%</td>
<td>5.5%</td>
</tr>
<tr>
<td>1.5</td>
<td>9.9%</td>
<td>8.7%</td>
<td>7.8%</td>
<td>7.0%</td>
<td>6.4%</td>
<td>5.9%</td>
<td>5.5%</td>
<td>5.1%</td>
<td>5.1%</td>
</tr>
<tr>
<td>1.75</td>
<td>8.7%</td>
<td>7.8%</td>
<td>7.0%</td>
<td>6.4%</td>
<td>5.9%</td>
<td>5.5%</td>
<td>5.1%</td>
<td>4.7%</td>
<td>4.7%</td>
</tr>
<tr>
<td>2</td>
<td>7.8%</td>
<td>7.0%</td>
<td>6.4%</td>
<td>5.9%</td>
<td>5.5%</td>
<td>5.1%</td>
<td>4.7%</td>
<td>4.5%</td>
<td>4.5%</td>
</tr>
</tbody>
</table>

Table 20 Price Changes by Different Demand Elasticity ($\alpha$) and Supply Elasticity ($\beta$) due to an increase of 20% in supply from ROW.

Figure 48 Percent of Price Decrease by Different Demand Elasticity ($\alpha$) and Supply Elasticity ($\beta$) due to an increase of 20% in supply from ROW Yttrium Prices.
Figure 49 shows Yttrium metal (with purity above 99.9%) and oxide (with purity above 99.999%) prices in USD/kg in China’s domestic and export markets. FOB export prices include the tax imposed on the products. Even including the tax, the export is almost the twice or even more of the domestic prices. Before 2010, prices are relatively stable. However, in mid-August 2010, prices skyrocketed when China’s plans become public to reduce or ban rare earth exports. Since 2010, China’s rare earth exports have decreased year by year.

As ore grades become lower and China’s environmental and social cost and labor cost rise, the cost of producing rare earth or Yttrium is expected to increase. China is not likely to increase production significantly to drive prices down because of higher costs, internal demand for domestic consumption and its desire for more the value-added exports. (Humphries, 2012)
7. Introduction to Yttrium Uses

Minerals usually are employed as inputs for production of final goods that are desired by consumers. Yttrium has value because it provides certain qualities in final products. This type of demand is known as a “derived demand.” The Department of Energy (2010 and 2011) studied the derived demand of Yttrium from clean energy technologies such as phosphors in energy-efficient lighting, fuel cells and coating of gas turbine blades. In the short run, two characteristics concerning final products lead to shifts in Yttrium’s derived demand.

One issue is the change in consumer demand for the final product. If the demand for Yttrium is ultimately derived from the demand for the final product, there will be a direct and positive relationship between consumers’ demand and the associated derived demand.

The other issue is the relative importance of Yttrium’s using cost to the total cost of manufacturing the final product. If the portion of final cost associated with Yttrium is high, the demand for Yttrium tends to be price elastic because if Yttrium is a large part of total production costs, any change in Yttrium price will have a significant impact on total costs and the reactions to it will be substantial. In contrast, if the share of Yttrium in the product is low, the demand for Yttrium from producers of that product will be inelastic.

Both in the short run and long run, there are two other important factors. The first is the availability of substitutes for Yttrium in these products. If the available substitution options are limited, the likelihood of a change in the use of Yttrium is also limited; Second is the possibility of recycling Yttrium. If “new” Yttrium can be created from “old” Yttrium, it may reduce any threat of supply reductions. In the long run, a fifth factor will become important: technological changes in Yttrium uses.

Table 21 provides the percent of each individual subgroup demand by category in the US in each year from 1995 to 2010. There has been a generally increasing trend in phosphors demand
while other applications have generally increased in volume. International Yttrium market has been competitive, although China was the source of most of the world’s supply.

As shown in Table 21, the Yttrium demand in US peaked in 1998. The increase in domestic Yttrium demand is primarily the result of US dollar strength and the recessionary Asian economies minimizing inflation and undercutting commodity prices. After 1998, Yttrium demand in US decreased because prices increased. Then, the US demand declined in 2002 as the US economy experienced a recessionary period and the continued strength of US dollar against many foreign currencies. The demand increased in 2003 and 2004 because the US economy experienced growth. The demands in 2005 to 2008 remained high, since the US required Yttrium for uses in phosphors and electronics, especially those used in defense applications. In 2009, the demand declined significantly due to the economic crisis and the demand started to climb up as the economy recovers. (USGS Yttrium Commodity Summary, 1996 to 2012)

Japan is playing an important role in US Yttrium supply. According to USGS Yttrium Commodity Summary 2013, on a gross weight basis, about 95% of imported Yttrium-bearing materials and compounds containing Y\textsubscript{2}O\textsubscript{3} were sourced from China (35%) and Japan (60%), although the leading source of Yttrium metal was from China.
<table>
<thead>
<tr>
<th>Year</th>
<th>Lamp and Cathode Ray Tube Phosphors</th>
<th>Structural Ceramics and Abrasives</th>
<th>Oxygen Sensors, Laser Crystals and Miscellaneous</th>
<th>Alloys/Metallurgy</th>
<th>Total Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>248.2</td>
<td>105.85</td>
<td>10.95</td>
<td>0</td>
<td>365</td>
</tr>
<tr>
<td>1996</td>
<td>202.86</td>
<td>0</td>
<td>4.14</td>
<td>0</td>
<td>207</td>
</tr>
<tr>
<td>1997</td>
<td>119.72</td>
<td>61.32</td>
<td>110.96</td>
<td>0</td>
<td>292</td>
</tr>
<tr>
<td>1998</td>
<td>350.88</td>
<td>67.08</td>
<td>98.04</td>
<td>0</td>
<td>516</td>
</tr>
<tr>
<td>1999</td>
<td>299.6</td>
<td>38.52</td>
<td>68.48</td>
<td>21.4</td>
<td>428</td>
</tr>
<tr>
<td>2000</td>
<td>317.8</td>
<td>36.32</td>
<td>77.18</td>
<td>22.7</td>
<td>454</td>
</tr>
<tr>
<td>2001</td>
<td>387.86</td>
<td>23.65</td>
<td>61.49</td>
<td>0</td>
<td>473</td>
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<tr>
<td>2002</td>
<td>263.86</td>
<td>33.4</td>
<td>36.74</td>
<td>0</td>
<td>334</td>
</tr>
<tr>
<td>2003</td>
<td>292.6</td>
<td>34.2</td>
<td>34.2</td>
<td>19</td>
<td>380</td>
</tr>
<tr>
<td>2004</td>
<td>606.62</td>
<td>0</td>
<td>6.19</td>
<td>6.19</td>
<td>619</td>
</tr>
<tr>
<td>2005</td>
<td>547.08</td>
<td>0</td>
<td>5.82</td>
<td>34.92</td>
<td>582</td>
</tr>
<tr>
<td>2006</td>
<td>623.28</td>
<td>51.94</td>
<td>51.94</td>
<td>14.84</td>
<td>742</td>
</tr>
<tr>
<td>2007</td>
<td>601.64</td>
<td>67.6</td>
<td>0</td>
<td>6.76</td>
<td>676</td>
</tr>
<tr>
<td>2008</td>
<td>535.92</td>
<td>61.6</td>
<td>6.16</td>
<td>12.32</td>
<td>616</td>
</tr>
<tr>
<td>2009</td>
<td>378</td>
<td>49.5</td>
<td>6.75</td>
<td>15.75</td>
<td>450</td>
</tr>
<tr>
<td>2010</td>
<td>542.7</td>
<td>80.4</td>
<td>13.4</td>
<td>33.5</td>
<td>670</td>
</tr>
<tr>
<td>2011</td>
<td>241.6</td>
<td>202.3</td>
<td>33.7</td>
<td>71.4</td>
<td>549</td>
</tr>
</tbody>
</table>

Table 21 Domestic Yttrium (Y2O3) Use by Categories from 1995 to 2010 in Metric Tons (USGS, 1996-2013)

### 7.1 Yttrium in Phosphors

Phosphors convert incident radiation into the light of a designated color based on properties of the elements included in the phosphor (Heyes, 1998). To convert ultraviolet radiation into visible light, manufacturers coat the inside of a lamp’s glass with powder of blended elements, or phosphors. Manufacturers use two groups of phosphor powders: blends of only “halo-phosphor” and blends of “rare earth phosphors” (shown in Table 22). While it can vary, a standard blended phosphor is composed of approximately 55% red, 35% green and 10% blue phosphors.
The amount of rare earth material in each of these phosphor colors can vary by manufacturer and application requirements. Some REEs are critical to high-performance General Service Fluorescent Lamps (GSFL). Ultraviolet radiation (UV) is generated during the operation of GSFL. To convert UV radiation into visible light, manufacturers coat the inside of the lamp’s glass with the powder of blended phosphors. Rare earth phosphors are more efficient and produce a higher quality light with “halo-phosphors” (OSRAM Sylvania, 2011). In high-performance GSFL, a blend of three rare earth phosphors (tri-phosphor) is used: one emits green light; one emits red light; and one emits blue light. Three key REEs are commonly used in tri-phosphor blends: Yttrium, Europium and Terbium.

Table 22 provides chemical formulas for each color phosphor (there are two common options for blue). Each manufactured phosphor is blended together for use in tri-phosphor lamps. Halo-phosphors blends are much less costly than rare earth but also less efficient (in terms of color quality and lumen maintenance) and produce lower quality light. Manufactures have expressed two concerns about phosphors: 1) increasingly stringent energy conservation standards and 2) whether there were sufficient and reliable supplies of the crucial REEs, particularly with regards to Yttrium, Terbium and Europium. (DOE, 2011)

<table>
<thead>
<tr>
<th>Phosphor Color</th>
<th>Chemical Formula</th>
<th>Share of Tri-phosphor Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>Y₂O₃; Eu</td>
<td>55%</td>
</tr>
<tr>
<td>Green</td>
<td>LaPO₄; Ce; Tb</td>
<td>35%</td>
</tr>
<tr>
<td>Blue 1</td>
<td>BaMgAl₁₀O₁₇; Eu</td>
<td>10%</td>
</tr>
<tr>
<td>Blue 2</td>
<td>(Sr,Ca,Ba)₅(PO₄)₃Cl; Eu</td>
<td>10%</td>
</tr>
</tbody>
</table>

Elements full names: Yttrium (Y), Europium (Eu), Lanthanum (La), Phosphorus (P), Ce (Cerium), Terbium (Tb), Barium (Ba), Magnesium (Mg), Aluminum (Al), Strontium (Sr), Calcium (Ca).

Table 22 General Chemical Formulas for Phosphors and Share of Weight
Yttrium’s principal uses are in phosphors for color televisions, computer monitors, flat-panel displays, trichromatic fluorescent lights, light-emitting diodes (LED), pigments and x-ray-intensifying screens (USGS, 1996-2013). In the coming year there will likely be additional lighting technology transitions to LEDs (Technology Metals Research, 2013). Cerium-doped Yttrium aluminum garnet (YAG: Ce) crystals are used as phosphors to make white LEDs. In addition, fiber optic cables need Erbium, Europium, Terbium and Yttrium. The red phosphor is almost entirely composed of Yttrium and Europium.

Lighting accounts for approximately 18% of electricity uses in US buildings, second after space heating (DOE, 2007). Traditional incandescent light bulbs are considered non-efficient and use considerable more energy than fluorescent lighting, light-emitting diodes (LEDs), organic light-emitting diodes (OLEDs) and halogen incandescent. Over the past 20 years, the Energy Policy Act of 1992 and the Energy Independence and Security Act of 2007 have resulted in a substantial increase in production of new lighting technologies. (DOE, 2009)

Compact fluorescent lamp (CFL) is a large demander of Yttrium. The Energy Policy Act of 1992 regulates a move from traditional halo-phosphors to tri-phosphors, which contain rare earths (DOE, 2010 and 2011). The EISA 2007 limits the maximum wattage use for general service lighting. The CFLs meet the standard from EISA 2007, while most common incandescent lamps do not (DOE, 2010 and 2011). The US Energy Information Administration (2011) indicates that CFLs will soon meet the bulk of mandated energy-efficient residential lighting demand and thus the demand for CFLs will peak in 2014. According to the EIA, CFL demand will decline further in the next 5 to 10 years, when a larger number of halogen-type incandescent bulbs that will be able to meet the standards. The projected CFL demand in the outlook is shown in Figure 50, with CFL demand declining after 2012.

Figure 50 shows one possible transition scenario under the standard. The transition could also be slow or partially reversed by higher efficiency halogen incandescent bulbs. Halogen
incandescent bulbs’ cost is competitive with CFLs. However, CFLs are three times more energy efficient than the new halogen incandescent lights and designed to last much longer (DOE, 2010 and 2011).

Figure 50 Projected Domestic Compact Fluorescent Lighting Shipments under EISA 2007 Standards (Technology Metals Research, 2013)

Figure 51 shows the distribution of REO consumption within the phosphor market sector in 2008. The data for Cerium Oxide, Europium Oxide, Gadolinium Oxide, Lanthanum Oxide and Yttrium Oxide are derived from Blade (2010). Yttrium accounts for 69% of total phosphor consumption globally and no other element has been found to substitute Yttrium within red phosphors.
Besides the transition from incandescent to fluorescent or halogen incandescent lighting, there is also a potential subsequent transition to other technologies, including LEDs and OLEDs. LED bulbs are already available for residential, but their price is quite high compared to incandescent or CFL bulbs (although LEDs’ longevity and efficiency make them more competitive on a total life cycle cost basis). LEDs are expected to become increasingly competitive as unit price drops. (Technology Metals Research, 2013)

Demand for fluorescent lighting (both compact and linear) is expected to grow sharply in the next few years in the US, which depends on phosphors made from Yttrium, Terbium and Europium. Within several years, demands for LEDs and halogen incandescent are expected to grow to replace demand of fluorescent lighting (DOE, 2010 and 2011). LEDs use much less REEs than fluorescent light bulbs, while OLEDs and halogen incandescent use no REE (DOE, 2010 and 2011). As the transition occurs, lighting-related demands for Yttrium, Terbium and Europium will decline.
7.2 Yttrium in Ceramics

YSZ is a zirconium oxide based ceramic in which Yttrium serves as an additive. According to Zircar (2012), products made of Yttria Stabilized Zirconia fiber exhibit exceptional resistance to most corrosive environments. Such products resist attack by molten alkali metal chlorides and carbonates at temperatures as high as 700°C (1300°F) and to aqueous solutions of alkali metal hydroxides at temperatures as high as 230°C (450°F). In addition to being able to withstand short term exposure to mineral acids at their boiling point, fibrous YSZ has strong resistance to oxidizing, reducing and vacuum atmospheres at high temperatures (Yanagida et al., 1996). These products are also used in abrasives, bearings and seals, high-temperature gemstones, wear-resistant and corrosion-resistant cutting tools, tooth crowns, refractory in jet engines, thermal barrier coating in gas turbines, material for non-metallic knife blades (Yanagida et al., 1996) (Minh, 1993). The alumina-zirconia abrasive is used in defense applications such as jet engines and energy applications such as thermal barrier coating in gas turbines and solid oxide fuel cells (SOFC).

Fuel cells are promising clean energy technology for vehicle propulsion, auxiliary power and distributed power generation. Yttrium is used as electrolyte in SOFCs. The National Energy Technology Laboratory (2013) estimates that commonly designed SOFC requires 21 grams of Yttrium oxide per kilowatt of fuel cell capacity. Yttrium can also be used as an electro-ceramic due to ion-conducting properties, e.g., to determine oxygen content in exhaust gases or to measure pH in high-temperature water. Another application of YSZ is cementing to seal water in high temperature.
7.3 Alloys and Metallurgy

According to American Elements (2012), copper Yttrium is used as 1) superconductor precursor powder; 2) any application where high surface areas are desired such as water treatment and in fuel cell and solar application. A Chromium Aluminum Yttrium alloy powder is introduced to be used in high performance coating, preparation of pressed and bonded sputtering targets and Chemical Vapor Deposition (CVD) and Physical Vapor Deposition (PVD) processes. Powders are also useful in applications where high surface areas are desired such as water treatment and in fuel cell and solar applications. Chromium Yttrium Alloy’s primary applications include bearing assembly. Yttrium is used to make the high-temperature superconductor YBCO (Yttrium Barium Copper Oxide). (Knizhnik et al., 2003)

The introduction of small amounts of Yttrium into steel makes its structure fined-grained and improves its mechanical, electrical and magnetic properties (Naumov, 2008). If a small amount (hundredths of percent) is added to cast iron, its hardness will increase by a factor of two and wear resistance will increase by a factor of four. Light Mg-9% Yttrium alloy has high corrosion resistance and is used for fabricating various details and units of aircrafts. In commercial uses, abrasives produced with Yttrium can be used as a material for knife blades. Yttrium oxide is also used to make Yttrium iron garnets ($Y_3Fe_5O_{12}$), which are very effective microwave filters. Yttrium-aluminum garnets ($Y_3Al_5O_{12}$ or YAG) are synthetic crystals that are widely used as an active laser medium in solid-state lasers, dopant and gemstone (simulated diamond). A magnesium alloy containing Lanthanides, Yttrium, and Zirconium can substitute for magnesium-thorium alloys in aerospace applications.

Gas turbines for stationary power generation use Yttrium for the thermal barrier coatings of turbine blades. Lesser amounts are used in coating of bond, high-temperature overlay, substrate
and structural support. These coatings are applied during initial manufacture then reapplied periodically during an average 30-year turbine life (DOE, 2010 and 2011).

The National Energy Technology Laboratory (2006) estimates that existing turbines require approximately 150 tons of Yttrium for initial installation and another 3,350 tons for refurbishments over the life of the fleet. The new turbines will require an additional 1.3 to 1.7 tons of Yttrium for initial installation and 30-40 tons for additional refurbishments, depending on the mix of turbine sizes used to meet forecast generation capacity requirements. Using an average life of 30 years for gas turbines, this means about 119 tons of demand per year, which is about 1 percent of 2010 global production. Based on above analysis, US Yttrium demand in gas turbines is not expected to be a significant driver of global demand. However, expansions in global generation capacity may have a greater impact on the demand in the future.

### 7.4 Other Uses

Others uses for Yttrium include electronics, laser, temperature/oxygen sensor and etc. catalyst in ethylene polymerization; high-temperature superconductor Yttrium Barium Copper Oxide; Yttrium aluminum garnet, Yttrium lithium fluoride and Yttrium vanadate are used in combination with dopants such as Neodymium or Erbium in infrared lasers. (Yang et al, 2005)

In Molten Carbonate Fuel Cells (MCFC), fibrous YSZ’s chemical stability and resistance to attack make it useful where other materials fall short. In this application the mechanically interlocked zirconia fiber is exposed to high concentrations of hot LiCO₃ without any signs of degradation. YSZ’s compressibility and flexibility enable it to complies irregular surfaces and accommodate thermal expansion of various fuel cell components. In luxury automobiles and heavy trucks, YSZ is also used as the burner mantle in diesel and gasoline by performing proper uniform porosity for even distribution of the fuel and resistance to attack by lead oxide.
Yttrium Aluminum Garnet (YAG, with chemical formula Y₃Al₅O₁₂, 45% Yttrium in weight) is used to produce laser host and popular as a substrate material for optical components, particularly for high power lasers. According to RP Photonics (2012), YAG has the advantage that with careful optimization, laser operation with moderate energy inputs (moderate excitation levels and pump intensities) can obtain substantial output powers. The laser generated by YAG has been widely used in scientific, industrial, military and medical domains. (RP Photonics, 2012)

Additional applications of YAG lasers include cutting, welding and drilling of metals for the automobile industry, marking and repairing semiconductors and medical and dental treatments. These applications utilize lasers operating from the ultraviolet to the mid-infrared generated by substitution of other rare earth and transition metal ions, such as Holmium (Ho³⁺), Thulium (Tm³⁺), Erbium (Er³⁺), Ytterbium (Yb³⁺) and Chromium (Cr³⁺) (Weber, 2000; Kaminski, 1981). Together with YAG, Yttrium Lithium Fluoride and Yttrium Vanadate are used in combination with dopants such as neodymium or erbium in infrared lasers. Researchers also report electronic applications of the YSZ film such as a buffer layer of the superconducting film and a gate dielectric of the integrated circuit (Shin et al., 2010).

7.5 Substitution Possibilities

According to the USGS Yttrium Summaries (2012), in most uses including phosphors, electronics and lasers, Yttrium cannot be substituted for by other elements. However, as a stabilizer in zirconia ceramics, Yttria (Yttrium Oxide) may be substituted for by Calcia (Calcium Oxide) or Magnesia (Magnesium Oxide), but the final product is generally not as resilient and has lower toughness (USGS, 1996-2013). According to Thijssen (2011), Yttrium oxide is used as a dopant to stabilize the YSZ commonly used for electrolyte or electrodes. Yttria stabilizes the particular crystal structure that provides the ionic conductivity for electrolyte or electrodes. It has
been commonly considered that an alternative to YSZ as an electrolyte is Scandia-Stabilized Zirconia or Lanthanum Strontium Gallate Magnesite and each of them contains significant fractions of Scandium or Lanthanum, which are less “critical” than Yttrium. (DOE, 2010 and 2011)

7.6 Recycling of Yttrium

In the long run, a portion of rare earth phosphor demand may also be supported by recycling. However, according to United Nations Environment Program (2011), Yttrium’s global average end-of-life (post-consumer) functional recycling is less than 1 percent.

Yet, the low rate of recycling also implies a great potential for recycling rate progress. DOE’s Critical Material Strategy report (2011) discusses the importance of improving the recycling and reuses of rare earth materials-particularly from phosphors and large scale magnets.

8. The Criticality of Yttrium

The National Research Council (NRC, 2008) reviewed the definitions used for “critical mineral” from its historical meaning at the time of World War II to the Strategic and Critical Materials Stockpiling Act (2005). The report suggests that a material is critical if only if 1) it performs an essential function for which few or no satisfactory substitutes exist; and 2) if an assessment indicates a high probability that the supply of the material may become limited, leading either to physical unavailability or substantially higher prices for that material.

Similarly, in reports by the California Institute of Technology (Fromer et al., 2011) and in DOE’s Critical Materials Strategy (DOE, 2010 and 2011), a “critical material” is defined in terms of its importance to the clean energy and the risk of supply interruption. In the report,
critical materials have one or more properties that are physically essential for the performance of the system and there is uncertainty or risk in the material’s supply (Fromer et al., 2011).

The European Commission (EU, 2010) labels a raw material as “critical” when the risks of supply limit and their impacts on the economy are higher compared with most of the other raw materials. The whole rare earth group (including Yttrium) is labeled as “critical raw materials” by the EU.

8.1 The Criticality Matrix from NRC (2008)

The NRC (2008) proposed a two-dimensional criticality matrix based on an element’s importance in applications and availability.

The first dimension applies the importance of the element in particular uses. The importance in uses or applications is based on the ideas some minerals may be technically difficult or particularly expensive to replace. NRC (2008) assesses the importance of a mineral by examining how that mineral is more fundamental for specific uses than other minerals. The greater the difficulty, expense, or time to find a suitable substitute for a given mineral, the greater will be the impact of a restriction in the mineral supply.

The second dimension of criticality, potential supply restrictions, reflects a number of considerations composing of geological, technical, social, environmental, political and economic factors.

The degree of a mineral’s importance may vary across one end use to another. Thus, the NRC evaluates the degree of importance of a specific mineral for each important end-use. For each end-use, three indicators for the mineral are considered.
1) The estimated value of US consumption of the mineral, giving an indication of the economic size of the sector; 2) The percentage of US consumption in existing uses for which substitution is difficult or impossible; 3) The committee’s professional judgment about the importance of growth in emerging uses that could overwhelm existing raw material production capacity in the short term. (NRC, 2008)

In 2010, 670 metric tons of Yttrium Oxide was imported to US (USGS Yttrium Summaries, 2011). Using an average import price of 120 USD per kilogram, the total value of US consumption of Yttrium Oxide is 80.4 million USD. In 2006, US total rare earth oxide import worth in excess of 1 billion USD (NRC, 2008) and this number may have reached 2 billion USD in 2010 due to the increased price of rare earth. Thus, the value of imported Yttrium accounts for around 4% of the total US value of imported rare earth elements.

With the criterions of determining importance in uses by NRC (2008), a score from “1” to “4” for each use can be chosen. Table 23 shows the relative importance of end-use applications for Yttrium. In Table 23, each use of Yttrium is given a score based on the use’s importance. After weighted by each use’s proportion of total US market share, a weighted score for each use is obtained in the fourth column of the table. By summing up the weighted score of each use, the composite weighted score in the last row of the table is obtained.

With respect to the uses of Yttrium phosphors, substitution by other elements in electronics and lasers are considered to be difficult (USGS Yttrium Summaries, 2012). These non-substitutable uses account 86% of US total Yttrium uses in 2010, which is a relative higher level compare to other REEs. In the short and medium terms (before the transition to LED or other lighting technologies that do not require rare earth), the use of Yttrium as phosphors in fluorescent lighting and color display screens are not substitutable or the performance of substitutes is not desirable. This amounts to 81% of total Yttrium consumption in US (USGS Yttrium Summaries, 2011). Given that 1) there is not sufficient substitutes for Yttrium in lighting,
and 2) the increasing importance of efficient lighting in the US society, and 3) all Yttrium comes from one place, it is suggested that, the highest level of importance “score 4” is applied to the first application category, phosphors.

The second most important use of Yttrium is in Ceramics and Abrasives. The consumption in this sector decreased from 1995 until 2006 and the consumption after 2006 has been stable (Table 21). Because of the limited amount of demand in the US market and some possible substitutes, a moderate score of “2” is assigned to this use. The use of Yttrium in alloys or metallurgy has been increasing since 1999, and a higher score “3” is given to this use due to the increasing trend in demand in this sector and the limited possible substitutes.

The remaining uses of Yttrium such as in lasers, oxygen sensors and electronics, account 2% of total uses. Although the volumes of demand are low compared to the categories of phosphors, ceramics and alloys, Yttrium in these uses is difficult to substitute for. In particular, the performance of lasers produced by substitutes for Yttrium is significantly reduced (USGS, Yttrium Summaries, 2011). In addition, the amount of Yttrium consumed in this sector has been increasing since 2007 (Table 21). Thus, the highest score, “4” is given for criticality to this use.

<table>
<thead>
<tr>
<th>Application Group</th>
<th>Proportion of Total US Market</th>
<th>Impact of Supply Restriction</th>
<th>Weighted Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phosphors</td>
<td>0.81</td>
<td>4</td>
<td>3.24</td>
</tr>
<tr>
<td>Structural Ceramics and Abrasives</td>
<td>0.12</td>
<td>2</td>
<td>0.24</td>
</tr>
<tr>
<td>Alloys or Metallurgy</td>
<td>0.05</td>
<td>3</td>
<td>0.15</td>
</tr>
<tr>
<td>Lasers, Oxygen Sensors and Miscellaneous</td>
<td>0.02</td>
<td>4</td>
<td>0.08</td>
</tr>
<tr>
<td>Composite, Weighted Score</td>
<td></td>
<td></td>
<td>3.71</td>
</tr>
</tbody>
</table>

Table 23 Relative Importance of End-Use Applications for Yttrium (Proportion of total US Yttrium for each application was determined from USGS Yttrium Summary, 2010)

Five attributes of availability for Yttrium’s supply risk over the long term (more than ten years) are considered in NRC (2008):
Geologic availability covers geologically characteristics for a given mineral such as mineral associations, depths, grade, tonnage and geometry of the reserve.

Technical availability considers the state of technology and knowledge to exploit, extract and process the mineral.

Environmental and social availability accounts for the factors of the environment in which the mineral is found or processed such as endangered species, water, air quality and scenic beauty. Social factors include community authorization of the resource development or referred as license.

Political availability represents the predictability of regulation, independence of judiciary, limits on litigation, land right and willingness of the host country to allow or facilitate the resource development.

Economic availability considers the cost of exploiting, extracting, processing, concentrating and purifying the minerals compared with the market value of the product.

NRC (2008) considers the supply risk for the whole rare earth group to be high for all applications, which is scored “4” in this category.

The supply risk related geological factor for Yttrium is higher than that of the whole rare earth group since Yttrium comes primarily from southern ion adsorption clay resources and Yttrium’s content in Bastenaesite of northern China is quite low.

Assume that the supply risk of Yttrium related to the other four factors to be the same as the whole rare earth group, Yttrium’s total supply risk cannot be lower than the whole rare earth group, which scores “4” in the vertical axis. Thus, Yttrium is scored “4” for this category.

Depending on the above considerations for availability of Yttrium, the supply risk for Yttrium is considered to be high for all applications. The composite weighted vertical score of importance of use is 3.71 (Table 23) and the overall score of supply risk for Yttrium is 4. Thus,
Yttrium should be considered as a critical mineral based on the limited supply from China and the importance of its uses.

8.2 Yttrium’s Importance to Clean Energy (DOE, 2011)

DOE (2011) presents detailed information on the “importance to clean energy” of Yttrium. The report focuses on Yttrium application as phosphors linear fluorescent lamps (LFLs) and compact fluorescent lamps (CFLs), television displays and liquid crystal display (LCD) screens. Additionally, Yttrium is used to increase the strength of aluminum and magnesium structural alloys. In DOE’s report, “short term” is defined as from the present to three years out and “medium term” is 4 to 14 years out. Figures 52 and Figure 53 show the criticality matrix for Yttrium and other materials in the short and medium terms. DOE’s assessment methodology is described by following:

“Importance to clean energy” is composed of two attributes for each material over the short and medium terms. 1. Clean energy demand (weight 75%) covers the importance of the material in magnets, batteries, photovoltaic (PV) and phosphors used in clean energy technologies; 2. Substitutability limitation (25%) involves estimating the constraints on practically substitute possibilities for the material in clean energy technologies.

Demand for lighting phosphors is expected to spike between 2012 and 2014, because of implementation of new US lighting efficiency standards; US demand for CFLs is growing and the new US federal minimum efficiency standards for general service lighting will increase CFL demand significantly; the EU and other regions have also implement similar lighting standards to largely eliminate traditional incandescent lamps from the market; Clean energy demand in phosphors accounts for roughly one half of global Yttrium demand; Yttrium demand may increase more rapidly if high-temperature superconductors begin to capture market share from
permanent magnets used in other applications such as wind turbines. Thus, Yttrium’s criticality score for clean energy demand is 4 in short term and 3 in the medium term. (DOE, 2011)

In terms of substitutability limitations, Yttrium’s score is 4 both in short term and medium term because: No effective substitute for Yttrium as phosphor in fluorescent lamps, television or LCD screens has been found; There are new technologies using significantly less or no REEs that have emerged in recently years, such as LEDs as lighting equipment. As the utilization of such technology expands, the use of Yttrium as phosphors is expected to decline in the medium term (after 2015).

Yttrium’s supply risk is discussed in this paragraph. In DOE (2011)’s view, the overall supply risk for each material depends on five risk attributes in the short and medium terms. For each attributes, key materials were assigned scores from one (least critical) to four (most critical). Following are the descriptions of the attributes: 1. Basic availability (40%): the extent to which global supply will be able to meet demand. Short-term availability examines production capacity relative to demand. Medium-term availability examines the potential for other mines to trigger production relative to anticipated increases in demand; 2. Competing technology demand (10%): whether the non-energy sector’s demand is expected to grow rapidly, and constrain the supply for the energy sector; 3. Political, regulatory and social factors (20%): The risk associated with political, social and regulatory factors within producing countries, including political instability in a country that could threaten mining and processing operations; the countries could impose export quotas or other restrictions; or social pressures, licensing or regulatory processes could delay the startup of new mining projects; 4. Codependence on other markets (10%): possibilities where a mineral is a co-product or by-product of other minerals. Codependence can be an advantage or a disadvantage, depend on which mineral is driving production capacity. In general, co-products with lower value will have higher scores since they are less likely to drive production than co-
products with higher value; 5. Producer diversity (10%): Market risks resulted by lack of diversity in producing countries or firms (e.g., monopoly or oligopoly).

The overall scores of supply risk of Yttrium in the short and medium term are both 3 because Yttrium is not mined or refined currently in the US. All suppliers are imported, predominately from China; Chinese export tax and quotas can significantly constrain supply in the short terms. Yet several possible sources outside China can relieve this problem in the medium term.

In terms of availability, Yttrium’s score is 4 in short term and 3 in the medium term because significant productions are available worldwide in adsorption clays and significant potential resources from monazite and xenotime. Thus producers out of China are likely to emerge in the medium term, although supply may rise only modestly.

In the attribute of competing technologies, Yttrium’s score is 2 both in short and medium term because 1. Yttrium is not likely to be substituted for in ceramics, electrodes, electrolytes, electronic filters, lasers, lightweight alloys, superconductors and advanced medical application. These non-phosphor applications, however, are not expected to increase drastically. 2. In contrast, there are competing technologies in phosphor sector that perhaps can reduce the use of Yttrium substantially.

In the political, regulatory and social attributes, Yttrium’s score is both 4 in short and medium term. China has imposed export quotas and tax on all REEs. China clearly has the ability to disrupt the market for Yttrium in the short and medium terms.

In the attribute of codependence on other markets, Yttrium’s score is 2 both in short and medium term because 1. Yttrium is found in varying abundance with other REEs, most predominantly in ion adsorption clays and potential resources in monazite and xenotime and 2. Yttrium is the most abundant of the heavy REEs, with a relatively high revenue stream. In the category of producer diversity, Yttrium has score 4 both in short and medium term since China
currently produces almost all Yttrium and will continue to be the dominant producer in the short and medium term.

Figure 52 Short-term (Present-2015) Criticality Matrix (DOE, 2010 and 2011)

Figure 53 Mid-term (2015-2025) Criticality Matrix (DOE, 2010 and 2011)
9. Conclusion

This research reviews current Yttrium supplies from southern and northern China, which accounts the overwhelming majority of total world supply in short term, as well as the potential resources outside China, which may reduce the supply risk of Yttrium in the long term. In southern China, Yttrium together with other rare earth elements determines the production capacity as co-products. Because the content of Yttrium is high in the southern China’s ion adsorption clays, Yttrium’s supply is less likely to be impacted by the price changes of other minerals. The mining and processing of Yttrium can cause serious environmental issues and the supply of Yttrium may be significantly impacted by more stringent environmental regulations by the Chinese government.

The Chinese government has been imposing a series of policy such as production quota and export quota on rare earth oxide and products, aiming at regulating its domestic market, controlling the environmental damages, limiting raw material export and promoting exports of high value-added products. The impacts of these policies on Yttrium supply are also analyzed in this research.

In US, there are four major categories of Yttrium uses, phosphors, ceramics, alloys and metallurgy, and electronics. Yttrium is generally not easy to substitute in these uses. Phosphor is the most important of the uses in volume and the high demand of compact fluorescent lights in the short term increases the criticality of Yttrium. As the commercial utilization of new technology expands, such as light-emitting diodes, the demand for compact fluorescent lights is expected to decline. Currently, less than 1 percent of Yttrium is recycled.

Two methods, adopted in NRC (2008)’s report and DOE (2011)’s report, respectively, are used to assess the criticality of Yttrium. Based on Yttrium’s supply risk and importance to its
uses, both methods suggest that Yttrium should be considered as critical mineral for US’ economy.

The framework in this research can be applied to other materials by examining following aspects of the target material: current resources, potential resources, environmental issues, policy and regulation issues, material’s uses, technology substitution and recycling possibility. Based on the results of above aspects, the supply risk and importance to uses can be assessed. The criticality of the materials can be expressed and compared in a diagram showing the values of supply risk in one axis and importance to uses in another.


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Chapter 3

Economic Cutoff Grade of a By-Product Mining Operation – A Rare Earth Study

Abstract

In a two-mineral mining operation, the optimal economic cutoff grade of the primary product can be significantly impacted by changes in economic variables of the by-product. Economic variables such as market price, mining and processing costs, metallurgical recovery rate, and tonnage-grade distribution in deposit must be considered in order to maximize the net present value (NPV) of the mine. In this chapter, an algorithm maximizing the NPV of a rare earth mining project is introduced, considering the extraction and processing of a by-product. The algorithm is an extension of Lane (1988)’s cutoff grade model using an equivalent factor to determine cutoff grade for each of the two mineral products. The mining strategy, including optimal mine’s life, cutoff grade, and corresponding optimal NPV are determined by the algorithm. A rare earth case study is demonstrated to verify the algorithm. The research finds the by-product (Dysprosium, Dy) price’s change significantly impacts the primary product’s cutoff grade (Neodymium, Nd).
1. Introduction to Breakeven Cutoff Grade, Opportunity Cost and Lane’s Model (1988)

In the mining industry, cutoff grade is used to distinguish ore and waste in a given mineral deposit (Lane 1988). Within the mineral reserve, if the material content is above the cutoff, it is classified as ore and then mined and processed. Otherwise, the mineral content is treated as waste (the possibility of stockpiling intermediate content material is examined in the Chapter 4). The cutoff grade policy allows a mine’s owner to adjust its extraction strategy respect to change in financial objective. Figure 54 shows an example of relationship between cutoff grade and the tonnage distribution. In any given time during the production stage, accordingly to the tonnage-grade distribution in Figure 54, the average grade of the produced material rises as the cutoff increases.

Figure 54 An Example of Gold Mine’s Tonnage, Average Grade and Cutoff Grade Distribution

(China Minmetals Corporation 2010)
Because of the time value of money effect, cash flows in the future are more aggressively discounted than earlier cash flows when calculating the mine’s NPV. So the extraction logic leads to a tendency to extract as much content as possible in the earlier years. However, the production rate is usually constrained by capacity at some production stage such as mining or processing, which makes it unrealistic to extract all minerals in the early years (Nieto and Bascetin 2011). In this research, the case of constrain in processing capacity is considered, and there is no constrains in mining capacity. This means that the processing plant is designed that it always produces in full capacity during the mine’s life.

In order to produce more mineral content in the earlier years, the logistics of the operation aims to select higher cutoff in early years, and decrease the cutoff systematically until the mining operation reaches depletion of the reserve. Higher cutoff renders that more metal content is produced if the production is constrained by processing capacity.

Higher cutoff in early years results in higher content, and therefore, higher revenues for the overall project. In addition, the cutoff grade also controls how much ore is mined in each year, and how much waste is dumped or stockpiled for later processing (Lane 1988).

In mining practice, stripping ratio is defined as the quantity of waste need to be handled in order to produce some quantity of ore, over the quantity of the ore. Higher cutoff leads to an increased stripping ratio in a given year during the mine’s life.

Other mining indicators, such as mine’s life, production rates, fleet size, and equipment capacity can also be impacted by cutoff grade policy. Therefore, in practice, cutoff grade has been one of the most important economic and operational indicators used to determine the optimal mining strategy, which can be adjusted to maximize the NPV of a mining project (Nieto 2007; Asad 2007).

For decades, optimal cutoff grade determination has been an important operational and planning factor to the industry. Traditionally, the breakeven or heuristic cutoff grade shown in
equation (3.1) is used in practice (Taylor 1972), which equates the total value of the commodity material contained within the mineral block and the costs incurred to produce the mineral in the block.

\[(3.1) \quad Breakeven\:cutoff = \frac{Pr_o + f}{(P-M)y}\]

In equation (3.1), \(Pr_o\) is the cost of processing one unit of ore; \(P\) is the commodity’s price; \(M\) is the marketing/sale cost incurred for one unit of metal; \(y\) is the metallurgical recovery rate.

Some denotations are given in following to derivate the breakeven cutoff. \(x\) is defined as the average grade; \(M_o\) and \(M_{waste}\) are the mining costs for one unit of extracted ore and waste; \(C\) is the processing capacity; \(\delta\) is the interest rate; \(f_a\) is the annual maintenance cost.

Assume a mine’s only economic choice variable is the cutoff grade, here denoted \(x_c\), and profit of any mining decision is determined by the choice variable. Other economic variables, such as recovery rate, price of commodity, mining/processing/marketing costs are assumed to be constant and do not affected the choice variable. The decision to be made is one metric ton of material with average grade \(x\) is either 1) treated as ore and processed, or 2) treated as waste and not processed. When the time value of money effect is not considered, the economic profit from processing one metric ton of material with average grade \(x\) is,

\[(3.2) \quad U_{ore}(x) = x \cdot y \cdot (P - M) - (M_o + Pr_o + f)\]

The economic profit from wasting one metric ton of material with average grade \(x\) is (note that this metric ton of material still need to be mined together with the ore),

\[(3.3) \quad U_{waste}(x) = -M_{waste}\]

The breakeven cutoff grade \(x_c\) is determined at a margin that the profit of processing one metric ton of material and selling the recovered metal in market, is equal to the profit of wasting it,

\[(3.4) \quad U_{ore}(x_c) = U_{waste}(x_c)\]
The equation (3.4) can be interpreted that, if \( U_{\text{ore}}(x) \) exceeds \( U_{\text{waste}}(x) \) for the \( x \) greater than \( x_c \), then all material for which \( x \) is higher than \( x_c \) should be processed and sold in the market; the material with average grade lower than \( x_c \) has a lower \( U_{\text{ore}}(x) \) than \( U_{\text{waste}}(x) \), and the material should be wasted. The breakeven grade is determined at the margin where the cost of production can be just recovered by the revenue from selling the metal in market.

Plug in equation (3.2) and (3.3) into (3.4),

\[
(3.5) \quad x_c \cdot y \cdot (P - M) - (M_o + Pr_o + f) = -M_{\text{waste}}
\]

If the mining costs for one unit of ore and waste are equal,

\[
(3.6) \quad M_o = M_{\text{waste}}
\]

Plug in equation (3.6) into equation (3.5), the breakeven cutoff can be calculated by,

\[
(3.7) \quad x_c = \frac{Pr_o + f}{(P - M) \cdot y}
\]

Which is exactly the breakeven cutoff grade given in equation (3.1). When the breakeven cutoff grade is considered at the limit (edge) of the pit, where the waste are not mined and left in the pit, the profit of wasting one metric ton of material is zero,

\[
(3.8) \quad U_{\text{waste}}(x) = 0
\]

Then, by using the equation (3.5), the breakeven cutoff grade considering the limit (edge) of the mining pit is,

\[
(3.9) \quad x_c = \frac{M_o + Pr_o + f}{(P - M) \cdot y}
\]

In equation (3.7) and (3.9), the annual maintenance cost to produce one unit of ore can be calculated using the annual maintenance cost divided by the processing capacity,

\[
(3.10) \quad f = \frac{f_a}{c}
\]

The breakeven cutoff grade is an economic indicator that assesses the minimum cutoff grade that is followed before the overall operation stops generating profits, and above derivation does not consider the effect of “time value of money”. Thus, it does not provide a potential
maximum of discounted total mining profits. A maximization algorithm covering “time value of money” or “opportunity cost” has yet to be implemented to calculate the economic cutoff grade to provide the maximized total discounted profits.

Lane (1988)’s model is adapted to maximize the total value of the discounted cash flows. The maximization is carried out by taking into account the opportunity cost, which equals to the loss of interest dollars resulting from postponing planned production by adding one unit of new material to a capacity constrained facility. Equation (3.11) shows how the economic cutoff is formulated at the year $i$ when the system is constrained by the processing capacity. The term $F_i$ in equation (3.12) is the opportunity cost in a given year $i$ of all the discounted profit which is not yet accessible due to the capacity constrain. Opportunity cost can be viewed as decrease in the value of the mine resulted by postponing currently scheduled production as a result of adding one metric ton of material to the capacity-constrained facility.

\begin{align}
  x_c(i) &= \frac{Pr_0 + F_i}{(P-M)\cdot y} \\
  F_i &= \frac{\delta \psi_i}{c}
\end{align}

Lane (1988) argues that the opportunity cost must be added to cost of the procedure (mining or processing) in which the capacity is constrained. When the mining operation is constrained by the processing capacity, the opportunity cost is added directly to the processing cost $Pr_0$.

Following provides the intuitive of the opportunity cost given in equation (3.12). Consider a mining project with estimate of total future cash flows, $V_i$ from any given year $i$ through the end of the mine’s life based on the currently planned cutoff grade strategy $x_c(i)$ for $i = 1, 2, \ldots N$. According to the plan, the processing plant has no spare capacity. If one metric ton of material is added to the capacity-constrained plant, processing of the originally scheduled material is postponed by the time needed to process the additional one metric ton, which is $1/C$. 


Thus, receiving the value $V_i$ is postponed by time $1/C$, and this incurs a cost, which is defined as opportunity cost in a given the year $i$ (which is same as formula (3.12)),

$$F_i = \delta \cdot V_i \cdot 1/C = \frac{\delta V_i}{c}$$

The opportunity cost thus decreases as the mine’s life approaches depletion (as $V_i$ decreases). From equation (3.11), it can be observed that the opportunity cost $F_i$ in a year $i$ depends on the value of the estimates of future cash flows of the mine, $V_i$; while the cutoff grade in equation (3.11) is a function of opportunity cost, and affects the $V_i$ at the time. Thus, in any given year $i$, the cutoff grade $x_c(i)$ and the value of the mine $V_i$ are interdependent, and they can be solved by an iterative trial-and-converge algorithm.

The Lane’s model starts with a zero value for $V_i$, and is entered to equation (3.12), then equation (3.11) and a series of cutoff grade strategy can be obtained in the iteration 1. Using the cutoff grade strategy from iteration 1, a new value $V_i$ can be calculated, and then the new $V_i$ is plugged in equation (3.12), and then equation (3.11). Another series of cutoff grade strategy can be obtained, which concludes the iteration 2. This iteration process continues until convergence of the $V_i$ (when $V_i$ stops to grow as the iteration proceeds). In the convergence point, the $V_i^*$ is put into equation (3.12) and then equation (3.11), leading to a series of cutoff grade strategy, and this series of cutoff grade will result in the same value of $V_i^*$. More details about the algorithm are discussed in the section 2.

A poly-metallic mining project consisting of two minerals can fall into two possible cases: a by-product or a co-product case, depending on the whether there is additional cost associated with producing the secondary commodity. In particular, if there is no additional cost associated to produce the secondary mineral, it applies to a by-product case; otherwise, it is a co-product case.
In this research, a by-product case is explored consisting of Neodymium (Nd) as the primary product and Dysprosium (Dy) as the secondary commodity, or by-product. To establish a cutoff model based on a two-mineral model, the Lane (1988)’s model is adapted to maximize the total value of the discounted cash flow from the production of the primary product and the by-product. This method incorporates an equivalent grade factor to transform the content of the by-product to the equivalent content of the primary product.

1.1 Rare Earth Case Study and the Equivalent Factor

An economic incentive to develop better method estimating cutoffs for rare earth deposit is the soaring prices from 2002 to 2013. In 2010, China supplied more than 95% of the total world demand of rare earth. Since 2005, China began to restrict rare earth supply by imposing production and export quotes, which leads the price to skyrocket, and increases the uncertainty in the rare earth market (Nieto 2012). In addition, rare earths are commonly produced as the by-product of iron, copper, and aluminum in both northern and southern China. Thus, a case study based on rare earth can provide a clear demonstration of the proposed cutoff grade algorithm. The main objective of this research is to explore how the price changes of the by-product affect the economic cutoff grade of the primary product.

A heavy rare earth element (HREE) and a light rare earth element (LREE) are used to present a hypothetical case study. Generally, LREEs are more abundant than HREEs, but HREEs usually experience higher demand levels and unit prices (Castor 2006). A Nd and Dy by-product mining deposit is considered in the case study. Dy is one of the HREEs that generally occur with low grade in deposit with a relative higher price than Nd. When the ore of Nd is mined and processed, Dy is produced as a by-product in the Baiyun Erbo rare earth mine in northern China (Hurst 2010).
The tonnage-grade distribution for Nd and Dy and the corresponding average Nd and Dy grades are given in Table 24, Table 25 and Table 26. Table 24 shows the tonnage distribution of the material by different intervals of Nd and Dy’s grades. Table 25 shows the average grade of Nd at different Nd and Dy’s grade Intervals. Table 26 shows the average grade of Dy at different Nd and Dy’s grade Intervals. The tonnage and grade distribution data as well as the economic factors shown in Table 27 are from the database of China Minmetals Corporation (2010).

An equivalent factor (Liimatainen 1998) is adopted to convert the by-product grade distribution into the primary mineral’s grade. The equivalent factor is defined,

$$F_{eq} = \frac{(p_2-s_2)y_2}{(p_1-s_1)y_1}$$

In equation (3.14), $p_1$ and $p_2$ are the two minerals’ market prices, $s_1$ and $s_2$ are the two minerals’ sales/marketing costs, and $y_1$ and $y_2$ are the two minerals’ metallurgical recovery factors. When using the equivalent factor in equation (3.14), the subscript 1 refers to the primary mineral, and subscript 2 refers to the by-product mineral.

Nd is considered as the primary product, and by the equivalent factor, Dy’s tonnage-grade distribution in Table 26 is transformed and added to the equivalent Nd tonnage-grade distribution in Table 25. With the transformed tonnage-grade distribution, the problem is reduced to a combined one mineral problem. The combined tonnage-grade distribution is shown in Table 28, which will be used later as the inputs to Lane (1988)’s algorithm. A new tonnage-grade distribution of deposit based on Nd as the primary resource has been developed, as shown in the last two columns in Table 28. The tonnage distribution does not change when compared to the original Nd tonnage distribution. However, the grade value and distribution of Nd is increased due to the contribution of the equivalent Dy grade.

A summary of economic and operation parameters used in the algorithm is provided in Table 27. These parameters are assumed to be constant during the mine’s life. A high interest rate
of 10% is adopted to make the effect of “time value of money” on the cutoff grade easier to observe (a low interest rate will result in a series of almost “flat” cutoff grade during mine’s life because of the aggressively reduced “time value of money” effect).

<table>
<thead>
<tr>
<th></th>
<th>Dy (%)</th>
<th>Nd (%)</th>
<th>0-0.025</th>
<th>0.025-0.05</th>
<th>0.05-0.075</th>
<th>0.075-0.1</th>
<th>&gt;0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-0.1</td>
<td>1,230,000</td>
<td>950,000</td>
<td>295,000</td>
<td>335,000</td>
<td>500,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1-0.2</td>
<td>350,000</td>
<td>290,000</td>
<td>250,000</td>
<td>125,000</td>
<td>70,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2-0.3</td>
<td>725,000</td>
<td>525,000</td>
<td>320,000</td>
<td>220,000</td>
<td>40,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3-0.4</td>
<td>1,210,000</td>
<td>580,000</td>
<td>395,000</td>
<td>135,000</td>
<td>40,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4-0.5</td>
<td>555,000</td>
<td>235,000</td>
<td>85,000</td>
<td>70,000</td>
<td>80,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5-0.6</td>
<td>490,000</td>
<td>290,000</td>
<td>230,000</td>
<td>115,000</td>
<td>40,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6-0.7</td>
<td>395,000</td>
<td>260,000</td>
<td>190,000</td>
<td>80,000</td>
<td>80,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt;0.7</td>
<td>625,000</td>
<td>800,000</td>
<td>590,000</td>
<td>505,000</td>
<td>380,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 24 Tonnage-grade (tons) Distribution of Nd and Dy (China Minmetals Corporation 2010)

<table>
<thead>
<tr>
<th></th>
<th>Dy (%)</th>
<th>Nd (%)</th>
<th>0-0.025</th>
<th>0.025-0.05</th>
<th>0.05-0.075</th>
<th>0.075-0.1</th>
<th>&gt;0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-0.1</td>
<td>0.03</td>
<td>0.02</td>
<td>0.03</td>
<td>0.02</td>
<td>0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1-0.2</td>
<td>0.11</td>
<td>0.18</td>
<td>0.17</td>
<td>0.08</td>
<td>0.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2-0.3</td>
<td>0.24</td>
<td>0.28</td>
<td>0.24</td>
<td>0.23</td>
<td>0.25</td>
<td></td>
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<tr>
<td>0.3-0.4</td>
<td>0.35</td>
<td>0.31</td>
<td>0.34</td>
<td>0.33</td>
<td>0.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4-0.5</td>
<td>0.44</td>
<td>0.48</td>
<td>0.44</td>
<td>0.49</td>
<td>0.45</td>
<td></td>
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</tr>
<tr>
<td>0.5-0.6</td>
<td>0.52</td>
<td>0.56</td>
<td>0.58</td>
<td>0.53</td>
<td>0.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6-0.7</td>
<td>0.68</td>
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<td>0.66</td>
<td>0.62</td>
<td>0.68</td>
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</tr>
<tr>
<td>&gt;0.7</td>
<td>0.99</td>
<td>1.05</td>
<td>1.03</td>
<td>1.08</td>
<td>1.03</td>
<td></td>
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</table>

Table 25 Average Grade (%) of Nd at Different Nd and Dy Intervals (China Minmetals Corporation 2010)
<table>
<thead>
<tr>
<th>Nd(%)</th>
<th>0-0.025</th>
<th>0.025-0.05</th>
<th>0.05-0.075</th>
<th>0.075-0.1</th>
<th>&gt;0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-0.1</td>
<td>0.003</td>
<td>0.025</td>
<td>0.053</td>
<td>0.078</td>
<td>0.114</td>
</tr>
<tr>
<td>0.1-0.2</td>
<td>0.016</td>
<td>0.032</td>
<td>0.07</td>
<td>0.086</td>
<td>0.115</td>
</tr>
<tr>
<td>0.2-0.3</td>
<td>0.013</td>
<td>0.027</td>
<td>0.065</td>
<td>0.090</td>
<td>0.135</td>
</tr>
<tr>
<td>0.3-0.4</td>
<td>0.032</td>
<td>0.041</td>
<td>0.055</td>
<td>0.095</td>
<td>0.139</td>
</tr>
<tr>
<td>0.4-0.5</td>
<td>0.009</td>
<td>0.032</td>
<td>0.056</td>
<td>0.091</td>
<td>0.120</td>
</tr>
<tr>
<td>0.5-0.6</td>
<td>0.013</td>
<td>0.038</td>
<td>0.08</td>
<td>0.083</td>
<td>0.153</td>
</tr>
<tr>
<td>0.6-0.7</td>
<td>0.013</td>
<td>0.03</td>
<td>0.063</td>
<td>0.084</td>
<td>0.13</td>
</tr>
<tr>
<td>&gt;0.7</td>
<td>0.01</td>
<td>0.034</td>
<td>0.065</td>
<td>0.088</td>
<td>0.129</td>
</tr>
</tbody>
</table>

Table 26 Average Grade (%) of Dy at Different Nd and Dy Intervals (China Minmetals Corporation 2010)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mine Capacity</td>
<td>Tons/year</td>
<td>Unlimited</td>
</tr>
<tr>
<td>Processing Capacity</td>
<td>Tons/year</td>
<td>600,000</td>
</tr>
<tr>
<td>Mining Cost</td>
<td>$/ton</td>
<td>1.08</td>
</tr>
<tr>
<td>Processing Cost</td>
<td>$/ton</td>
<td>3.67</td>
</tr>
<tr>
<td>Refining Cost (Nd)</td>
<td>$/ton</td>
<td>68</td>
</tr>
<tr>
<td>Refining Cost (Dy)</td>
<td>$/ton</td>
<td>195</td>
</tr>
<tr>
<td>Price (Nd)</td>
<td>$/ton</td>
<td>1,675</td>
</tr>
<tr>
<td>Price (Dy)</td>
<td>$/ton</td>
<td>2,568</td>
</tr>
<tr>
<td>Recovery (Nd)</td>
<td>%</td>
<td>82</td>
</tr>
<tr>
<td>Recovery (Dy)</td>
<td>%</td>
<td>80</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>%</td>
<td>10</td>
</tr>
<tr>
<td>Annual Maintenance Cost</td>
<td>$/year</td>
<td>835,000</td>
</tr>
</tbody>
</table>

Table 27 Technical/Economic Parameters (China Minmetals Corporation 2010)
### Table 28 Combined Tonnage-grade Inputs to Algorithm

<table>
<thead>
<tr>
<th>Nd Grades (Average, %)</th>
<th>Nd (Average, %)</th>
<th>Dy (Average, %)</th>
<th>Equivalent Grade (%)</th>
<th>Tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00-0.1</td>
<td>0.0306</td>
<td>0.0381</td>
<td>0.0855</td>
<td>3,310,000</td>
</tr>
<tr>
<td>0.1-0.2</td>
<td>0.1403</td>
<td>0.0471</td>
<td>0.2082</td>
<td>1,085,000</td>
</tr>
<tr>
<td>0.2-0.3</td>
<td>0.2504</td>
<td>0.0380</td>
<td>0.3052</td>
<td>1,830,000</td>
</tr>
<tr>
<td>0.3-0.4</td>
<td>0.3378</td>
<td>0.0434</td>
<td>0.4004</td>
<td>2,360,000</td>
</tr>
<tr>
<td>0.4-0.5</td>
<td>0.4533</td>
<td>0.0324</td>
<td>0.5001</td>
<td>1,025,000</td>
</tr>
<tr>
<td>0.5-0.6</td>
<td>0.5434</td>
<td>0.0441</td>
<td>0.6070</td>
<td>1,165,000</td>
</tr>
<tr>
<td>0.6-0.7</td>
<td>0.6610</td>
<td>0.0418</td>
<td>0.7213</td>
<td>1,005,000</td>
</tr>
<tr>
<td>0.7-2</td>
<td>1.0356</td>
<td>0.0569</td>
<td>1.1176</td>
<td>2,900,000</td>
</tr>
</tbody>
</table>

2. **Maximization Algorithm of Mine’s Value $V_i$**

As shown in equations (3.11) and (3.12), the economic cutoff $x_c(i)$ depends on the $V_i$ (value of the total future cash flows from year $i$ to the end of mine’s life); meanwhile, $V_i$ is estimated based on the cutoff grade $x_c(i)$. A potential total maximum $V_i$ is found once the cutoff grade $x_c(i)$ for every year within the mine’s life is calculated. An iterative approach is adopted to solve this interdependency problem, where $V_i$ is guessed at first (or start from zero) and at each iteration they are improved until the solution converges to a stable optimal value. The approach is explained in the following steps and shown in Figure 55.

1) Tabulate tonnage-grade distributions of both minerals (as Table 24, Table 25 and Table 26) from a preliminary geological survey. The by-product tonnage-grade distribution is transformed to an equivalent primary product tonnage-grade distribution by the equivalent factor as shown in equation (3.14). Read mining cost, processing cost, mining capacity, processing capacity, interest rate, metallurgical recovery rate, prices of the minerals, annual maintenance cost, and sale costs from Table 27 to the algorithm.
2) Based on equation (3.14), transform and generate a combined primary and by-product tonnage-grade distribution (like Table 28). This combination of tonnage-grade distribution is denoted as “equivalent” tonnage-grade, and the optimal cutoff is denoted as “equivalent” cutoff in later results.

3) Calculate the cutoff grade in equation (3.11) by setting initial \( V_i = 0 \) in equation (3.12) if the \( V_i \) is not yet known for \( i = 1, 2, 3 \ldots N \).

4) From the most current grade tonnage curve of the deposit, calculate ore tonnage \( T_o \) and the average grade \( g_{ave} \) for the material above cutoff grade \( x_c(i) \) and waste tonnage \( T_w \) that is below the cutoff grade \( x_c(i) \) from the tonnage-grade distribution. \( T_o \) is the quantity of the ore with grade above the cutoff, and \( T_w \) the quantity of waste with grade below the cutoff. Stripping ratio is then calculated by \( SR = T_w/T_o \).

5) If the ore tonnage \( T_o \) is greater than processing capacity, set the quantity processed in year \( i \), \( Q_{ic} = C \), which means set the ore processed in year \( i \) equal to processing capacity. At the same time, set mining quantity as \( Q_{im} = Q_{ic}(1 + SR) \) and final product quantity \( Q_{ip} = Q_{ic} \cdot g_{ave} \cdot y \).

6) Find the annual profit by following equation,

\[
Profit_i = (P - M) \cdot Q_{ip} - (Pr_o + f) \cdot Q_{ic} - M_o \cdot Q_{im}
\]

7) Adjust the tonnage-grade distribution of the deposit by subtracting ore tons \( Q_{ic} \) from the grade distribution blocks above optimal cutoff \( x_c(i) \) and the waste \( Q_{im} - Q_{ic} \) from the blocks below optimal cutoff \( x_c(i) \), such that the shape of the distribution is not changed.

8) Check whether \( Q_{ic} \) is less than the processing capacity \( C \) for the year \( i \). If so, set mine’s life \( N = i \) and go to step 9; Otherwise, set year footnote \( i = i + 1 \) and go back to step 3.

9) Based on profits calculated in step 6 for each year \( i \) from 1 to \( N \), calculate the accumulated future \( V_i \) by following equation (3.16),
\( V_i = \sum_{j=1}^{N} \frac{\text{Profit}_j}{(1+\delta)^{j-i+1}} \)

10) If the total value of future profits for the mine does not converge to a stable maximum solution, go back to step 3; otherwise, the algorithm is completed. The cutoff \( x_c(i) \) for years \( i \) to \( N \) is the optimal cutoff grade to maximize the value of future profits from the mine.

The algorithm and the programming steps indicated above are realized in Matlab. The steps are explained in the flow chart shown in Figure 55.

**3. Results for Low, Moderate and High Dysprosium’s Prices**

Three scenarios of Dy’s price are established to examine the effect of changes in Dy’s price on Nd’s cutoff grade. These three scenarios use low, medium and high levels of Dy’s price, and keep the price of Nd constant. How the escalated Dy’s price in scenarios 2 and 3 compared to scenario 1 (Dy’s price in January 2009 is used as the low price level) is shown in Table 29. Meanwhile, the equivalent factor in the third column of Table 29 calculated by equation (3.14) rises as Dy’s price increases.
Figure 55 Two-mineral By-product Algorithm for Maximization of the Value of a Mine

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>2,568</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>5,136</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>7,704</td>
</tr>
</tbody>
</table>

Table 29 Scenarios Description
3.1 Scenario 1: Low Price of Dy

The first scenario is based on production constrained by the processing capacity and an unlimited mining capacity; no extra costs are incurred to produce Dy as a by-product; it is assumed that prices of Nd and Dy remain unaltered during the mine’s life. In this scenario, processing and mining costs are associated with Nd. A fixed cost of $835,000/year is assumed. Nd’s price is $1,675/ton (Nd’s price in January 2009), and Dy’s price is $2,568/ton (price in January 2009); Equivalent factor is determined by equation (3.14),

\[ F_{eq} = \frac{(2,568-195)+0.8}{(1,675-68)+0.82} = 1.44 \]

In equation (3.17), an equivalent factor of 1.44 is used to adjust and combine the tonnage-grade distributions of Dy and Nd. A new combined tonnage-grade input table is generated as shown in Table 28. The Dy’s average grade is multiplied by the equivalent factor (1.44 in this scenario) and added to the Nd’s average grade at each grade interval to generate the new tonnage-grade distribution (Table 28).

Based on this combined equivalent tonnage-grade distribution, the optimal cutoff grades can be calculated using the proposed algorithm. Figure 56 shows the results for the combined cutoff and Nd’s cutoff; Figure 57 compares the Nd’s cutoffs based on three scenarios of Dy’s price; Figure 58 shows value of the mine based on the three scenarios as the mine approaches to depletion. The details of the other two scenarios are discussed in session 3.2 and 3.3.
This case scenario is based on a Dy’s price at $2,568/ton and a corresponding equivalent factor of 1.44, which leads to an increase in the optimal cutoff grade of the operation when considering the Dy as a by-product (Figure 56) compared to the single mineral case. In addition, due to the opportunity cost and time discount effect, both lines of cutoff grade decrease as production approaches total depletion. The discrepancy between the combined cutoff and the Nd cutoff is due to the added contribution of the Dy’s equivalent content.

3.2 Scenario 2: Moderate Price of Dy

The second scenario assumes that Dy’s price has increased moderately to $5,136/ton. It is also presumed that prices of Nd and Dy remain constant during the mine’s life. The production is still constrained by the processing capacity, and the mining capacity is assumed unlimited. There
is no extra cost associated to produce Dy. Except for the Dy price, the other economic parameters and the tonnage-grade distribution for Nd and Dy are same as the scenario 1. Using Nd’s price ($1,675/ton) and Dy’ price ($5,136/ton), the equivalent factor is determined,

\[ F_{eq} = \frac{(5,136-195) + 0.8}{(1,675-68) + 0.82} = 3.00 \]

Using the proposed algorithm, the economic cutoff grade and the value of the total future cash flows of the mine by years are calculated and shown in Figures 57 and Figure 58. Results indicate there is an increase in value of future cash flows compared to scenario 1 due to the increased value generated by the soaring price of the Dy (Figure 58). Compared to scenario 1, in scenario 2, a higher cutoff grade is found in the initial years but drops more sharply to a lower cutoff grade in the depleting years. Similar results are found in scenario 3.

### 3.3 Scenario 3: High Price of Dy

In scenario 3, the price of the Dy is increased dramatically while other conditions are assumed same as the previous two scenarios; Dy’s price is increased to $7,704/ton; and there is no additional cost to produce Dy. It is also presumed that price of Nd remains unchanged. The resulting equivalent factor under scenario 3 is,

\[ F_{eq} = \frac{(7,704-195) + 0.8}{(1,675-68) + 0.82} = 4.56 \]

Per the result of scenario 3, the overall economic cutoff grade is again increased in the initial years due to Dy’s higher prices, but is lowered in the ending years; the mine’s life does not change. When compared to scenarios 1 and 2, a certain amount of previously defined low-grade mineral is now economic to be mined and processed thanks to the added content and value provided by the Dy’s equivalent, which leads to a higher value for future cash flows in Figure 58.
By comparing scenario 2 and 3 to scenario 1, it can be seen that the increased Dy’s price will raise the primary product’ cutoff in the initial years and lower the cutoff in the ending years. This is for the reason that in the initial years, increasing the price of Dy means increasing the content value of the material, and a higher cutoff grade for the primary product is used to generate more cash flows in the early years; in the ending years, because of the increased price of Dy, the low quality material that should have been treated as waste generates positive cash flows and should not be treated as waste, which leads to a lowered cutoff.

Figure 57 Nd’s Cutoff for Three Differentiated Dy’s Price Scenarios as the Mining Activity Proceeds
4. Conclusion

In this chapter, an algorithm has been developed to determine the economic cutoff grade by maximizing yearly discounted total profits of a by-product mine. In order to determine a series of annual economic cutoff grade for the primary product, a tonnage-grade equivalent transformation of the by-product is carried out. To convert the by-product’s tonnage-grade distribution to the primary mineral’s tonnage-grade distribution, an equivalent factor depending on market price, sales cost, and metallurgical recovery rates is adopted.

The economic cutoff grade is raised in the two-mineral case compared to the single mineral case. This is due to the contribution of by-product’s added content and the resulted increased value of the mineral deposit, which also increases the opportunity cost throughout the

Figure 58 Value of the Mine’s Future Cash Flows by Years for Three Differentiated Dy’ Price Scenarios as the Mining Activity Proceeds
mine’s life. Considering the contribution of by-product generates an increase in the economic cutoff grade for the primary product when compared to the single product mine.

To examine the effect of price changes of the by-product (Dy) on the economic cutoff grade of the primary mineral (Nd), three scenarios of by-product’s price are established: low, moderate, and high prices. The results indicate that, due to the escalation in by-product’s price, cutoff of the primary product is increased during the initial years and lowered in the depleting years. The higher cutoff in the initial years is attained to generate higher cash flows due to the added content by increase in by-product’s price; lowered cutoff in the depleting years is introduced for the reason that the increase in the price of by-product makes it economic to mine and process the material should have been wasted.

Based on the by-product algorithm used in this research and three case scenarios, it is found that fluctuation of the by-product’s price significantly affects the economic cutoff grade of the major product, thus affects the overall mining strategy.

Other economic parameters associated with the equivalent factor such as by-product’s sale cost and the recovery rate also impact the economic cutoff strategy. Therefore, in order to successfully maximize the total net present value of a by-product operation, the economic parameters of both the primary product and the by-product must be carefully considered.
Reference


Chapter 4

A Two-Stage Model for Metal Mining and Stockpiling Management

Abstract

For a metal mine, the intermediate grade ore between economic cutoff grade and breakeven cutoff grade is stockpiled for future processing after the mine is depleted. This research establishes a theoretical two-stage mining model with production from the stockpiled ore. By deriving the first order conditions of objective profit function and parameterized analysis, this research finds that the profit from processing the stockpiled material can significantly boost a mine’s profit; inspecting the stockpiled material shifts the choice of the optimal production rate and cutoff grade strategy; the research also measures how responsive the optimal mining rate is to the changes in input variables such as metal price, total material quantity, processing capacity, etc.; the impacts of some variables diminish as the percent of change increase. The intrinsic advantages of the theoretical model compared to numerical methods in explaining the economic intuition of stockpiling management and optimal mining strategy are discussed.

1. Introduction

The Lane’s model (1988) has been adopted in the mining industry for decades to determine the optimal cutoff grade for metal mines. By maximizing the net present value, the Lane’s model suggests a series of economic cutoff grade strategy that is usually higher than the breakeven or heuristic cutoff grade, which is the minimum grade in the deposit that is economic to be processed. As shown in Figure 59, the portion of the materials with the highest quality (the
part with dark color in Figure 59) is mined and processed early in production. This strategy leaves a portion of the ore not processed, which may still provide positive cash flows. These intermediate grade materials (the middle part in Figure 59) are generated each year during the mine’s life, and usually stockpiled beside the mine for the possibility to be processed after depletion. Although mined during the production, the lowest grade portion of the deposit (the part with light color in Figure 59), is regarded as waste and never processed.

![Economic Cutoff by Lane’s Model, Breakeven Cutoff and the Stockpiled Material](image)

Figure 59 Economic Cutoff by Lane’s Model, Breakeven Cutoff and the Stockpiled Material

The industry and practitioners have been interested in, and discussing about the stockpiled intermediated grade ore, such as that how valuable the stockpiled material is and its impact on the optimal mining strategy, when is the best time to process the stockpiled material, and how will the ore’s quality changes as the stockpiled material is processed. However, there is barely previous work related to the stockpiled material, which is an important part of the deposit and may significantly contribute to the mine’s profit.

One piece of research addressing the relevant issue is by Asad (2005), which models the grade-tonnage distribution of a deposit as a series of contours. Determining a cutoff grade strategy means specifying an intercept on the grade axes. Asad (2005) provides an algorithm to solve the economic cutoff grade by numerical and iterative method, as well as processing the
intermediate grade material after exhausting the mine. The stockpiled material is treated as a new mine without mining cost, and assessed in the same manner as ore from the original deposit. It is found that stockpiles, with the advantage of no mining cost, can generate a significant amount of additional cash flows. As the stockpiled material is examined as a new mining deposit, Asad’s model mistakenly ignore the processing strategy of the stockpiled may affect the mining strategy of the original deposit. The interaction of mining the original mine and processing the stockpiled is examined in this research.

Although not studying about the stockpiled intermediate grade material, there are multiple pieces of literature have explored the issue of a metal mine with varied quality in deposit by numerical algorithms. Franco-Sepulveda and Velilla-Avilez (2014) review the relevant work on metal mine’s cutoff using a hypothetical gold mine to explicate that, the Lane’s algorithm leaves the intermediate grade material stockpiled. Based on Lane’s model, Nieto and Zhang (2013) study the cutoff grade for a by-product mine based on a rare earth case of Dysprosium and Neodymium. Nieto and Zhang find that the changes in the by-product’s price can lead to a significant shift in the primary product’s cutoff.

Krautkraemer (1988)’s theoretical work uses cylinder model and optimal control method to explore the optimal strategy of mining rate and how changes in mining cost and metal price impact the strategy. It is found that, if the price increases more quickly than the discount rate, the mine is depleted sooner with a higher cutoff grade. The research’s limit is that, the stockpiling issue is not considered, and the impacts of other input variables such as the highest grade in deposit, capital cost, processing cost, and processing capacity are not examined. Therefore, a theoretical model is necessary to be established to bring in insights on choosing a mine’s optimal production strategy with the possibility of stockpiling, as well as how the strategy responses to the changes in mine’s input variables.
Based on a two-stage model, this research scrutinizes the production of stockpiled material after exhausting the mine. The profit functions in both stages, and their first order conditions are explicitly formulated. By a parameterized analysis, the optimal mining rate is found corresponding to the assumption in capital cost; profit from the stockpiled material proves to be worthwhile, which may account more than 20% of the mine’s total profit; to examine how responsive the optimal mining rate is to input variables, the pseudo-elasticities are assessed; total material quantity, metal price, recovery rate, highest grade in deposit, processing capacity and discount rate are found to positively impact the optimal mining rate; processing cost is perfectly inelastic; mining cost and the capital cost of the mining equipment negatively impact the optimal mining rate; the effects of several inputs diminish as the percent of their change increases. In addition, the theoretical two-stage model in this research has its intrinsic advantages compared to numerical methods, in explaining some intuitive results, such as the grade change pattern when processing the stockpiled material, which does not depend on the mining rate in the first stage.

This chapter is organized as following: session 2 describes a general two-stage production model considering the possibility of stockpiled intermediate grade material; session 3 applies the two-stage production model to a metal mine with uniform grade distribution and formulates explicitly the profit functions for both stages; session 4 prepares some derivatives respect to the mining rate, and explores that how the grade changes during the second stage production; session 5 derives the first order conditions for the profit functions respect to the mining rate, and provides some insights on choosing the optimal mining rate; session 6 analyzes the two-stage production model, the profit function, as well as the first order condition by a parameterized method; session 7 shows how to set an optimal mining rate depending on an assumed pattern of capital cost; session 8 examines how the optimal mining strategy responses to the changes in input variables by calculating each variable’s pseudo-elasticity; session 9 discusses
the results and compares the two-stage model with the Lane’s model; conclusions and implications of the research are given in the session 10.

2. A Two-stage Production Model

In this session, a model of two-stage production of a metal deposit with variety in quality is established. There are two stages of production (shown in Figure 60): the first stage production includes mining the material in deposit with a stable rate $M$, processing the part of the mined material with quality higher than a cutoff $x_1$ in each year before the depletion; the other part of the mined material with quality lower than $x_1$ (the intermediate grade material) is stockpiled for the second stage production after the first stage; the second stage production processes the stockpiled intermediate grade material until all such material is processed. $x_2$ is the breakeven cutoff grade for the final period of the second stage processing, which is determined by the metal price and the processing cost. The processing capacity $C$ is assumed to be constant during all the years of production. The total quantity of material can be mined is $Q$, here measured in tons. This session describes a general model, and a deposit with uniform grade distribution based on the general model is assessed in the next session.

Figure 60 A Two-Stage Production Model with Cutoff Grades and Processing the Stockpiled
The material in the deposit is assumed to have homogeneous tonnage-grade distribution. Assume \( z \) represents the metal grade in percent, which is in the range 0 to \( G \) (\( G \) is the highest grade in the deposit),

\[
0 \leq z \leq G < 1
\]

Assume \( f(z) \) is the probability distribution function for \( z \), such that,

\[
\int_{0}^{G} f(z) dz = 1
\]

The relationship of \( x_1, x_2 \) and \( G \) is

\[
0 < x_2 < x_1 < G
\]

For one ton of material that has been mined, the metal content in the material for the first stage processing is

\[
\int_{x_2(M)}^{G} f(z) dz = F_1(M)
\]

The average grade of the ore that goes to the first stage of production is,

\[
\frac{M}{c} \int_{x_2(M)}^{G} f(z) dz = \frac{M}{c} F_1(M)
\]

Assume the price for per unit of metal is \( p \); \( y \) is the metallurgical recover rate (the portion of metal content that can be recovered in ore, over the total metal content in ore); \( R \) is the processing cost for per unit of ore; \( A \) is the mining cost for per unit of mined material; \( \delta \) is the discount rate. In each period of the first stage production, the profit function is,

\[
p yC * \frac{M}{c} \int_{x_2(M)}^{G} f(z) dz - AM - RC = p y F_1(M) - AM - RC
\]

The first stage of production will last \( T_1 = \frac{Q}{M} \) years, and the profit function in continuous time for the first stage production is,

\[
\int_{0}^{Q} \{ p y F_1(M) - AM - RC \} e^{-\delta t} dt
\]

To solve for \( x_2 \), consider the profit function for the final period in the second stage,
By setting the formula (4.8) equal to zero, the cutoff grade for the final period of production is

\[ x_2 = \frac{R}{p} \]

Assume the second stage ends at the year \( T_2 \). At a time point \( t \) in the second stage \( (T_1 \leq t \leq T_2) \), the quality of ore that goes to the processing plant can be a function of \( t \) and \( M \), such that, \( x = x(t, M) \) and

\[ x_2 = x(T_2) \leq x(t) \leq x(T_1) = x_1 \]

And \( x(t) \) is decreasing as time goes,

\[ \dot{x}(t) < 0 \]

So the profit function at a time point \( t \) in the second stage is

\[ p y C * x(t, M) - RC \]

And the present value of the total profits in the second stage in continuous time is

\[ \int_{T_1}^{T_2} [p y C * x(t, M) - RC] * e^{-\delta t} \, dt \]

### 3. A Uniform Distribution of Grade

Assume there is a uniform distribution of grade, such that

\[ f(z) = \frac{1}{G} \]

And the integral of the probability distribution function is,

\[ \int_0^G f(z)dz = \int_0^G \frac{1}{G} \, dz = \frac{1}{G} (G - 0) = 1 \]

In each time of the first stage production, the profit function is,

\[ p y M * \frac{G + x_1(M)}{2} - AM - RC = \frac{1}{2} p y M \left( G + x_1(M) \right) - AM - RC \]
Since the grade distribution is uniform, in the first stage, the portion of ore that is processed is,

\[ \frac{G - x_1}{G} = \frac{C}{M} \]  

(4.17)

So \( x_1 \) can be solved,

\[ x_1 = G \left(1 - \frac{C}{M}\right) \]  

(4.18)

The formula (4.16) can be written as,

\[ pyM \left(G - \frac{GC}{2M}\right) - AM - RC \]  

(4.19)

The present value of the profit in the first stage is,

\[ \pi_1(M) = \int_0^{T_1} (pyM \left(G - \frac{GC}{2M}\right) - AM - RC) e^{-\delta t} dt \]

\[ = \frac{1}{\delta} \left\{ pyM \left(G - \frac{GC}{2M}\right) - AM - RC \right\} \left( -e^{-\delta T_1} + 1 \right) \]

\[ = \frac{1}{\delta} \left\{ (pyG - A)M - \left(\frac{pyG}{2} + RC\right) \right\} (1 - e^{-\delta T_1}) \]

Then, the production’s total production time is examined. From the beginning of production to the end of the second stage, all the tonnage of material with grade higher than \( x_2 \) will be processed. The total processed tonnage is,

\[ Q \cdot \frac{G - x_2}{G} = Q \left(1 - \frac{x_2}{G}\right) \]  

(4.21)

The total production time is,

\[ T_2 = \frac{Q}{C} \left(1 - \frac{x_2}{G}\right) \]  

(4.22)

Then, the grade function \( x(t, M) \) is examined. Since the grade distribution is uniform and processing capacity is constant, it can be known that \( x(t, M) \) is a linear function of \( t \). So assume that,

\[ x(t, M) = x_1 + \frac{t-T_1}{T_2-T_1} (x_2 - x_1) \]  

(4.23)
Based on formula (4.23) and the relationship in (4.10), the cutoff grades at time points $T_1$ and $T_2$ are

\begin{align}
(4.24) & \quad x(T_1) = x_1 = G(1 - \frac{C}{M}) \\
(4.25) & \quad x(T_2) = x_2 = \frac{R}{py}
\end{align}

It is assumed that $x_1 > x_2$, i.e., $G(1 - \frac{C}{M}) > \frac{R}{py}$. In addition, assume

\begin{align}
(4.26) & \quad a(M) = \frac{x_2 - x_1}{T_2 - T_1} \\
(4.27) & \quad b(M) = \frac{x_1 T_2 - x_2 T_1}{T_2 - T_1}
\end{align}

The formula (4.23) can be written as,

\begin{align}
(4.28) & \quad x(t, M) = x_1 + \frac{t - T_1}{T_2 - T_1} (x_2 - x_1) \\
= & \frac{x_2 - x_1}{T_2 - T_1} t + \frac{x_1 T_2 - x_2 T_1}{T_2 - T_1} \\
= & at + b
\end{align}

With (4.26) and (4.27), the present value of the second stage in formula (4.13) can be written as,

\begin{align}
(4.29) & \quad \pi_2 = \int_{T_1}^{T_2} [pyC \ast x(t, M) - RC] \ast e^{-\delta t} dt \\
= & pyCa \int_{T_1}^{T_2} te^{-\delta t} dt + (pyb - R)C \int_{T_1}^{T_2} e^{-\delta t} dt \\
= & -pyCa \ast \frac{(\delta t + 1)e^{-\delta t}}{\delta^2} \bigg|_{T_1}^{T_2} - \frac{1}{\delta} (pyb - R)C \ast e^{-\delta t} \bigg|_{T_1}^{T_2} \\
= & -\frac{pyCa}{\delta^2} [(\delta T_2 + 1)e^{-\delta T_2} - (\delta T_1 + 1)e^{-\delta T_1}] - \frac{1}{\delta} (pyb - R)C(e^{-\delta T_2} - e^{-\delta T_1}) \\
= & \frac{pyCa}{\delta^2} [(\delta T_1 + 1)e^{-\delta T_1} - (\delta T_2 + 1)e^{-\delta T_2}] + \frac{1}{\delta} (pyb - R)C(e^{-\delta T_1} - e^{-\delta T_2})
\end{align}
Using formula (4.20) and (4.29), the present value of the total profit for the two stages \( \pi \) is,

\[
(4.30) \quad \pi(M) = \pi_1 + \pi_2
\]

\[
= \frac{1}{\delta} \left\{ (pyG - A)M - \left( \frac{pyG}{2} + R \right) C \right\} (1 - e^{-\delta T_1}) + \frac{pyCA}{\delta^2} \left[ (\delta T_1 + 1)e^{-\delta T_1} - (\delta T_2 + 1)e^{-\delta T_2} \right]
\]

\[
+ \frac{1}{\delta}(pyb - R)C \left( e^{-\delta T_1} - e^{-\delta T_2} \right)
\]

### 4. Derivatives Respect to Mining Rate

Since that \( T_1 \) and \( x_1 \) are functions of \( M \), and \( T_2 \) and \( x_2 \) are not, following derivatives can be obtained,

\[
(4.31) \quad \frac{\partial T_1}{\partial M} = -\frac{Q}{M^2}
\]

\[
(4.32) \quad \frac{\partial x_1}{\partial M} = \frac{GC}{M^2}
\]

\[
(4.33) \quad \frac{\partial T_2}{\partial M} = 0
\]

\[
(4.34) \quad \frac{\delta x_2}{\delta M} = 0
\]

Based on formula (4.26) and (4.27), the derivatives of \( a(M) \) and \( b(M) \) respect to \( M \) are,

\[
(4.35) \quad \frac{\partial a}{\partial M} = \frac{\partial (x_2 - x_1)}{\partial M}
\]

\[
= \frac{(x_2 - x_1)'(T_2 - T_1) - (x_2 - x_1)(T_1' - T_2)'}{(T_2 - T_1)^2}
\]

\[
= -\frac{GC}{M^2} \frac{(x_2 - x_1)Q}{(T_2 - T_1)^2}
\]

\[
(4.36) \quad \frac{\partial b}{\partial M} = \frac{\partial (x_2 - x_1)}{\partial M}
\]
In formulas (4.35) and (4.36), the derivatives are respect to \( M \). Following derivations prove that \( \frac{\partial a}{\partial M} = 0 \) and \( \frac{\partial b}{\partial M} = 0 \). Look at the numerator of the formula (4.35), and plug in \( x_1, x_2, \)

\[ T_2 \] from formulas (4.18), (4.9) and (4.22), and \( T_1 = \frac{Q}{M} \).

\[(4.37)\]

\[ -\frac{GC}{M^2} (T_2 - T_1) - \frac{(x_2 - x_1)Q}{M^2} \]

\[ = -\frac{GC}{M^2} \left[ \frac{Q}{C} \left( 1 - \frac{x_2}{G} \right) - \frac{Q}{M} \right] - \frac{(x_2 - x_1)Q}{M^2} \]

\[ = -\frac{GQ}{M^2} + \frac{GQ}{M^2} + \frac{GCQ}{M^3} - \frac{Qx_2}{M^2} + \frac{Qx_1}{M^2} \]

\[ = -\frac{GQ}{M^2} + \frac{GCQ}{M^3} + \frac{Qx_1}{M^2} \]

\[ = -\frac{GQ}{M^2} + \frac{GCQ}{M^3} + \frac{G}{M^2} \left( 1 - \frac{C}{M} \right) Q \]

\[ = -\frac{GQ}{M^2} + \frac{GCQ}{M^3} + \frac{GQ}{M^2} - \frac{GCQ}{M^3} = 0 \]

Look at the numerator the formula (4.36), and divide that by \( T_2 \).

\[(4.38)\]

\[ x'_1T_2 - x'_1T_1 - x'_2T'_1 + x_1T'_1 \]

Plug formulas (4.32), (4.31), (4.18), (4.22) and \( T_1 = \frac{Q}{M} \) into (4.38).

\[(4.39)\]

\[ x'_1T_2 - x'_1T_1 - x'_2T'_1 + x_1T'_1 \]
Thus, by formulas (4.37) and (4.39), the derivative of $\alpha$ and $b$ respect to $M$ is zero,

\begin{align}
\frac{\partial \alpha}{\partial M} &= 0 \\
\frac{\partial b}{\partial M} &= 0
\end{align}

It can be seen that in formula (4.30), only $T_1$ and $x_1$ are functions of $M$.

### 5. First Order Condition for the Profit Functions in First and Second Stages

This session derives the first order condition (FOC) for the profit function. The FOC for the profit in the first stage is (using formula (4.20) and (4.31)),

\begin{align}
\frac{\partial \pi_1}{\partial M} &= \frac{1}{\delta} \left\{ (pyG - A)(1 - e^{-\delta T_1}) + \delta e^{-\delta T_1} \frac{\partial T_1}{\partial M} \left[ (pyG - A)M - \left( \frac{pyG}{2} + R \right) C \right] \right\} \\
&= \frac{1}{\delta} \left\{ (pyG - A)(1 - e^{-\delta T_1}) - \delta e^{-\delta T_1} \frac{Q}{M^2} \left[ (pyG - A)M - \left( \frac{pyG}{2} + R \right) C \right] \right\}
\end{align}

In formula (4.29), there are two terms. The derivative of each of the two terms will be explored. The FOC for the first term in formula (4.29) is (note that $\frac{\partial \alpha}{\partial M} = 0$ and $T_1 = \frac{Q}{M}$),

\begin{align}
\frac{\partial \left\{ pyCa \left( e^{-\delta T_1 + 1} - e^{-2\delta T_1} \right) \right\}}{\partial M} &= \frac{pyCa}{\delta^2} \left\{ \delta \frac{\partial T_1}{\partial M} e^{-\delta T_1} + (\delta T_1 + 1)(-\delta) e^{-\delta T_1} \frac{\partial T_1}{\partial M} \right\} \\
&= \frac{pyCa}{\delta} \frac{\partial T_1}{\partial M} \left\{ e^{-\delta T_1} - \delta T_1 e^{-\delta T_1} - e^{-\delta T_1} \right\} \\
&= \frac{pyCa}{\delta} \left( \frac{Q}{M^2} \right) \left( -\delta T_1 e^{-\delta T_1} \right) \\
&= \frac{pyCa}{\delta} \frac{Q}{M^2} T_1 e^{-\delta T_1}
\end{align}
\[ \frac{\partial}{\partial M} \left[ \frac{Cp(yb-R)(e^{-\delta T_1}-e^{-\delta T_2})}{\delta} \right] \]

\[ = \frac{C}{\delta} (pyb - R) \left( -\delta e^{-\delta T_1} \frac{\partial T_1}{\partial M} \right) \]

\[ = C(pyb - R) \left( -e^{-\delta T_1} \left( -\frac{Q}{M^2} \right) \right) \]

\[ = \frac{CQ(pyb - R)}{M^2} e^{-\frac{\delta Q}{M}} \]

Based on formulas (4.43) and (4.44), it is known that the FOC for the profit function in the second stage is,

\[ \frac{\partial \pi_2}{\partial M} = \left[ \frac{pyCaQ^2}{M^3} + \frac{CQ(pyb-R)}{M^2} \right] e^{-\frac{\delta Q}{M}} \]

The sign of formula (4.42) is not clear to conclude. However, in formula (4.45), if the price \( p \) is high enough, then \( pyb - R > 0 \) and the formula (4.45) is positive. So the present value of the profit in the second stage increases to \( M \). This is because that if there is an increase in \( M \), the mine will be depleted in the first stage more quickly. This leaves more ore to be stockpiled and processed in the second stage, and raises the profit from the second stage. A parameterization method is then adopted to explore how the changes of \( M \) will impact the profit in first stage and the total profit.

**6. Parameterization and Discussion**

A gold mine instance and its economic parameters are adopted to examine how the changes in the mining rate in the first stage will affect the mine’s total profit based on the two-stage model. Assume there are 1 million tons of material underground that can be mined; the gold
price is $1,200/Oz., or $42.34/gram; the metallurgical recovery rate is 70%; the highest grade in the deposit is 20 gram/ton; discount rate is 2%; the cost for mining one ton of underground material is $50; the cost for processing one ton of ore is $100. The processing plant is built such that the processing capacity is 40,000 tons of ore each year. The summary of the economic parameters for the gold mine is given in following Table 30.

<table>
<thead>
<tr>
<th>Total Material Quantity (Q)</th>
<th>1,000,000</th>
<th>tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold Price (p)</td>
<td>42.34</td>
<td>$/gram</td>
</tr>
<tr>
<td>Recovery Rate (γ)</td>
<td>70%</td>
<td>%</td>
</tr>
<tr>
<td>Highest Grade in Deposit (G)</td>
<td>20</td>
<td>gram/ton</td>
</tr>
<tr>
<td>Processing Cost (R)</td>
<td>100</td>
<td>$/ton</td>
</tr>
<tr>
<td>Processing Capacity (C)</td>
<td>40,000</td>
<td>tons/year</td>
</tr>
<tr>
<td>Discount Rate (δ)</td>
<td>2</td>
<td>%</td>
</tr>
<tr>
<td>Mining Cost (A)</td>
<td>50</td>
<td>$/ton</td>
</tr>
<tr>
<td>Capital Cost Per Unit of Mining Rate (ω)</td>
<td>1,000</td>
<td>$/ton</td>
</tr>
</tbody>
</table>

Table 30 Economic Input Parameters of the Gold Mine

By above parameters, the cutoff grade by the end of the second stage $x_2$ is calculated to be 3.37 gram/ton by formula (4.9); the total production time length is 20.78 years, based on formula (4.22); $a$ is calculated to be -0.8; $b$ is calculated to be 20.

If the yearly mining rate in the first stage $M$ varies from 50,000 tons to 1 million tons per year, the graphs for $x_1, T_1, T_2 - T_1$, and profits for first stage, second stage and total production period can be obtained.

Based on formula (4.18), the graph for the cutoff grade in the first production stage is calculated (shown in Figure 61). The cutoff grade in the first stage increases as the mining rate expands, but the increase rate diminishes as the cutoff grade approaches the highest grade in the deposit – 20 gram/ton.
Based on $T_1 = \frac{Q}{M}$, the length of the first stage can be obtained and shown in Figure 62.

Figure 63 shows the length of the second stage, which can be calculated by $T_2 - T_1$. Since the total production time is 20.78 years, there is a trade-off between the lengths of the first stage and second stage. The deposit is depleted sooner as the mining rate increases, but this effect diminishes as the mining rate increases.

Figure 61 Cutoff Grade for the First Stage Production ($x_1$) by Differentiated Mining Rate
Figure 62 Length of the First Stage Production \((T_1)\) by Differentiated Mining Rate

Figure 63 Length of the Second Stage Production \((T_2 - T_1)\) by Differentiated mining Rate

Figure 64, Figure 65 and Figure 66 show present values for the profits in first stage, second stage, and two stages together based on formulas (4.20), (4.29) and (4.30). All three profit graphs show an increasing trend, but the increase rate is diminishing as the mining rate expands.
Figure 64 and Figure 65 show how much percent of the total profit is distributed to the first stage and second stage productions, respectively. As the mining rate increases, the weight of the first stage’s profit decreases and the weight of the second stage’s profit increases. The production of the stockpiled material in the second stage can provide at most 20% of the mine’s total profit.

Figure 64 Present Value of the Profit in the 1st Stage ($\pi_1$) by Differentiated Mining Rate

Figure 65 Present Value of the Profit in the 2nd Stage ($\pi_2$) by Differentiated Mining Rate
Figure 66 Present Value of the Profit of Production for Two Stages ($\pi$) by Differentiated Mining Rate

Figure 67 Percent of the Profit of the 1st Stage Production in the Mine’s Total Profit by Differentiated Mining Rate
From Figure 66, it can be observed that the total profit increases as the mining rate $M$ increases. However, this graph does not include the capital cost or initial investment to build up the mining operation i.e. the equipment to drill and trucks to haul the mined material to surface.

Assume the capital cost increases as the mining rate $M$ increases linearly. For instance, the capital cost increases $1,000 if the mining rate expands by 1 ton. Given the economic parameters in Table 30, the profit with considering the capital cost is obtained and shown in Figure 69.

From the figure, it can be seen the optimal mining rate $M^*$ is 179,210 tons per year. The cutoff grade for the first stage is 15.54 gram/ton. The first stage lasts 5.58 years, and the second stage lasts 15.20 years. About 20% of the mine’s profit comes from processing the stockpiled material. The maximum profit of production by considering the capital cost is $339.44 million.

Figure 68 Percent of the Profit of the 2nd Stage Production in the Mine’s Total Profit by Differentiated Mining Rate

7. Capital Cost and Optimal Mining Rate
To examine how responsive the optimal mining rate $M^*$ is to a change of input variable, this session assess the pseudo-elasticity for an input variable $v$, 

$$
\epsilon_v = \frac{\% \text{ of change in } M^*}{\% \text{ of change in } v} = \frac{\partial M^*}{\partial v} \frac{v}{M^*}
$$

(4.46)

To estimate the pseudo-elasticities of input variables in Table 30, six scenarios of change in each input variable are considered. They are the variable decreases -10%, -5%, -1% and increases 1%, 5%, 10%, respectively in each scenario. The corresponding percent of change in $M^*$ can be calculated.

Table 31 shows the optimal mining rate $M^*$ when each input variable decreases -10%, -5%, -1% and increases 1%, 5%, 10%. Table 32 shows the percent of change of $M^*$ when each input variable decreases -10%, -5%, -1% and increases 1%, 5%, 10%. Table 33 shows the pseudo-elasticities for the input variables.
### Table 31 Optimal Mining Rate when Each Input Variable Decreases -10%, -5%, -1%, and Increases 10%, 5%, 1%

<table>
<thead>
<tr>
<th>Each Input Variable’s Change</th>
<th>-10%</th>
<th>-5%</th>
<th>-1%</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Material Quantity ($Q$)</td>
<td>168,215</td>
<td>173,770</td>
<td>178,130</td>
<td>180,290</td>
<td>184,555</td>
<td>189,810</td>
</tr>
<tr>
<td>Gold Price ($p$)</td>
<td>168,515</td>
<td>173,940</td>
<td>178,170</td>
<td>180,250</td>
<td>184,350</td>
<td>189,360</td>
</tr>
<tr>
<td>Recovery Rate ($y$)</td>
<td>168,515</td>
<td>173,940</td>
<td>178,170</td>
<td>180,250</td>
<td>184,350</td>
<td>189,360</td>
</tr>
<tr>
<td>Highest Grade in Deposit ($G$)</td>
<td>168,515</td>
<td>173,940</td>
<td>178,170</td>
<td>180,250</td>
<td>184,350</td>
<td>189,360</td>
</tr>
<tr>
<td>Processing Capacity ($C$)</td>
<td>171,865</td>
<td>175,590</td>
<td>178,495</td>
<td>179,925</td>
<td>182,730</td>
<td>186,155</td>
</tr>
<tr>
<td>Discount Rate ($\delta$)</td>
<td>178,680</td>
<td>178,950</td>
<td>179,155</td>
<td>179,265</td>
<td>179,475</td>
<td>179,735</td>
</tr>
<tr>
<td>Processing Cost ($R$)</td>
<td>179,210</td>
<td>179,210</td>
<td>179,210</td>
<td>179,210</td>
<td>179,210</td>
<td>179,210</td>
</tr>
<tr>
<td>Mining Cost ($A$)</td>
<td>179,360</td>
<td>179,285</td>
<td>179,225</td>
<td>179,195</td>
<td>179,140</td>
<td>179,065</td>
</tr>
<tr>
<td>Capital Cost Per Unit of Mining Rate ($\omega$)</td>
<td>190,300</td>
<td>184,540</td>
<td>180,245</td>
<td>178,195</td>
<td>174,265</td>
<td>169,655</td>
</tr>
</tbody>
</table>

### Table 32 Change Percent of Mining Rate when Each Input Variable Decreases -10%, -5%, -1%, and Increases 10%, 5%, 1%

<table>
<thead>
<tr>
<th>Each Input Variable’s Change</th>
<th>-10%</th>
<th>-5%</th>
<th>-1%</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Material Quantity ($Q$)</td>
<td>-10,995</td>
<td>-5,440</td>
<td>-1,080</td>
<td>1,080</td>
<td>5,345</td>
<td>10,600</td>
</tr>
<tr>
<td>Gold Price ($p$)</td>
<td>-10,695</td>
<td>-5,270</td>
<td>-1,040</td>
<td>1,040</td>
<td>5,140</td>
<td>10,150</td>
</tr>
<tr>
<td>Recovery Rate ($y$)</td>
<td>-10,695</td>
<td>-5,270</td>
<td>-1,040</td>
<td>1,040</td>
<td>5,140</td>
<td>10,150</td>
</tr>
<tr>
<td>Highest Grade in Deposit ($G$)</td>
<td>-10,695</td>
<td>-5,270</td>
<td>-1,040</td>
<td>1,040</td>
<td>5,140</td>
<td>10,150</td>
</tr>
<tr>
<td>Processing Capacity ($C$)</td>
<td>-7,345</td>
<td>-3,620</td>
<td>-715</td>
<td>715</td>
<td>3,520</td>
<td>6,945</td>
</tr>
<tr>
<td>Discount Rate ($\delta$)</td>
<td>-530</td>
<td>-260</td>
<td>-55</td>
<td>55</td>
<td>265</td>
<td>525</td>
</tr>
<tr>
<td>Processing Cost ($R$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Mining Cost ($A$)</td>
<td>150</td>
<td>75</td>
<td>15</td>
<td>-15</td>
<td>-70</td>
<td>-145</td>
</tr>
<tr>
<td>Capital Cost Per Unit of Mining Rate ($\omega$)</td>
<td>11,090</td>
<td>5,330</td>
<td>1,035</td>
<td>-1,015</td>
<td>-4,945</td>
<td>-9,555</td>
</tr>
</tbody>
</table>

Increase 10%, 5%, 1%
From Table 33, it can be observed that total material quantity, gold price, recovery rate, highest grade in deposit, processing capacity, and discount rate have positive pseudo-elasticity. This means that an increase in each of these input variables will lead to an increase in the optimal mining rate.

Total material quantity has the highest value of pseudo-elasticity, and the elasticity is not constant. The elasticity decreases as the total material quantity’s change percent increases. There are some implications can be drawn: if the total material quantity happens to suddenly increase (due to a newly found deposit beside the original one), the mine’s owner will expand the investment and set a higher mining rate to produce. However, this expanding effect of total material quantity to the optimal mining rate is diminishing as more new deposits are detected.

Gold price, recovery rate and highest grade in deposit have the same value of pseudo-elasticity, and this also can be implied by the profit functions in formula (4.30) as these three variables are multiplied to each other. The elasticities for these input variables are not constant, and decrease as the percent of change increases for each variable. Similar to increase in total
material quantity, a boost in gold price, a raise in recovery rate due to technology progress, and an increment in the highest grade in deposit, respectively, can make the mine’s owner to set a higher mining rate. However, the positive effects from gold price, recovery rate or highest grade in deposit are diminishing as the change percent of each variable increases.

Processing capacity has a positive pseudo-elasticity. It is not constant and decreases as the processing capacity’s percent of change increases. So there is positive effect from processing capacity on the optimal mining rate (if the cost of ramping up is not considered), but the effect is diminishing as the processing capacity increases. Note that, the mine’s owner needs to choose a mining rate that is higher than the processing capacity, to make the processing plant in full capacity during all the years of production.

The pseudo-elasticity of discount rate is positive and close to zero (0.03). The elasticity for discount rate is constant. Higher discount rate will make the mine’s owner set a higher mining rate to deplete the mine sooner, avoid partially the discounting effect for the future cash flows.

The optimal production rate is perfectly inelastic to processing cost, as processing cost’s elasticity is zero. This means changes in processing cost will not affect the choice of the optimal mining rate, because the situation always holds that the gold price is high enough to cover the processing cost and keep the operation goes; in addition, as the mining rate is always higher than the processing capacity, the processing plant is assumed to be operating in full capacity during the production period, no matter how costly it is to process. This implication can also be derived mathematically. Look at the term that includes processing cost $R$ in formula (4.42),

$$
\frac{1}{\delta} \left[-\delta e^{-\delta T_1} \frac{Q}{M^2} (-RC) \right] = \frac{RCQ}{M^2} e^{\frac{-\delta Q}{M}}
$$

The term that includes processing cost $R$ in formula (4.45) is,

$$
- \frac{RCQ}{M^2} e^{\frac{-\delta Q}{M}}
$$
If the FOCs for the first and second stage’s profits are summed up together, term (4.47) and term (4.48) will be cancelled. Then, the FOC for the total profit has no term including the processing cost $R$. Thus, the processing cost is perfectly inelastic with the optimal mining rate. Capital cost per ton of mining rate has negative pseudo-elasticity, so an increase in capital cost per unit of mining rate will decrease the choice of optimal mining rate. In the extreme, at a point when the mining equipment or haul trucks are particularly expensive, the mine’s owner will set mining rate to be zero, which means that the mining investment and operation are suspended. This negative effect from capital cost is also diminishing, at least for the current set up of parameters.

The elasticity of mining cost is negative and close to zero (-0.01). The elasticity does not change as the mining cost’s percent of change varies. Higher mining cost for each ton of mined material (due to it is more difficult to mine) will make the mine’s owner use a lower mining rate. And this effect does not diminish as the mining cost for per ton of mined material increases.

9. Discussion of the Results and Comparing to the Lane’s Cutoff Grade Model (1988)

Lane (1988) establishes an approach to calculate the cutoff grade strategy across the life of a metal mine based on numerical techniques. The Lane’s model and the model introduced in this research address the similar problem about a metal mining operation with variability in ore’ quality and constrain in processing capacity. The theoretical work in this research can provide several insights that cannot be drawn from the Lane’s method.

The model in this research addresses the possibility of stockpiling by using a two-stage production model. Lane’s approach can calculate a cutoff grade strategy which is higher than the breakeven cutoff, the lowest grade in the deposit that is economic to process. The limit is that, it leaves the intermediated grade material, or the stockpiled that still may generate positive cash
flows, but does not consider this portion of the deposit when setting the optimal cutoff grade strategy and mining rate. The model set up in this research covers processing the stockpiled, then determines the optimal mining rate and cutoff grade strategy by considering the profits from the first stage production as well as processing the stockpiled.

The lane’s approach fails to incorporate the capital cost of building up the mine, and does not provide any clue on choosing the optimal mining capacity. The model introduced in this research explicitly formulates the profit function for production and its first order condition, and can resolve the optimal mining rate by adding a term of capital cost. Note that since the mine is constrained by processing capacity, choosing an optimal mining rate strategy is equivalent to choosing an optimal cutoff grade strategy.

In addition to determining the optimal mining rate, this research measures how responsive the optimal mining rate is to changes in input variables by calculating pseudo-elasticities for gold price, total quantity of ore, processing cost, highest grade in deposit, etc. Total quantity of ore, gold price, recovery rate, highest grade in deposit, processing capacity and discount rate prove to be positively impact the optimal mining rate; the variable mining cost and capital cost for per unit of mining rate negatively impact the optimal mining rate; processing cost proves to be perfectly inelastic with the optimal mining rate; the effects of several input variables are found to be diminishing. The Lane’s model, at best, can be helpful on determine the effects of input variables on the mine’s profit, but not on the optimal mining rate, or whether their effects are diminishing.

There are some intrinsic advantages can be learnt for a theoretical model compared to the Lane’s numerical approach, when used to explain the results intuitively. It is found that the processing cost is perfectly inelastic with the optimal mining rate. This can be clearly explained by the theoretical model as the terms of the processing cost are cancelled out in the first order
conditions of the profits in the first and second stages. However, it is fairly difficult to reach this conclusion based on the Lane’s model, let alone to explain.

This research finds that the grade function in the second stage \( x(t) = at + b \) is not a function of the mining rate, i.e. \( \frac{\partial a}{\partial M} = 0 \) and \( \frac{\partial b}{\partial M} = 0 \). Although not straightforward, this conclusion can be reached and proved by the theoretical model in this research. The Lane’s model does not provide any insights about processing the stockpiled, or how the stockpiled material’s grade changes in the second stage.

**10. Conclusion**

In mining practice, the Lane’s model has been used for decades as a standard tool to determine a metal mine’s cutoff grade policy. The Lane’s algorithm proposes a higher cutoff grade than the breakeven cutoff, which leaves a portion of the deposit with intermediate grade not processed. These materials are usually stockpiled, and can be processed to contribute to the mine’s profit. The stockpiled material has been discussed by the industry and practitioners for decades, but there is few pieces of literature addresses this issue by incorporate it with the mining strategy of the original deposit. This research has established a two-stage model, examining the processing of the stockpiled, and its impacts on the optimal mining rate, cutoff grade and the mine’s profit. Profit from the stockpiled material cannot be ignored, which may account as much as 20% of the mine’s total profit.

The mine is assumed to have variability in ore’s quality and is constrained by the processing capacity. It is assumed that the processing plant is always in full capacity across all the production years. By deriving explicitly the mine’s profit function and its first order condition, the optimal mining rate strategy is determined by the two-stage model with parameterized analysis.
The research measures how responsive the optimal mining rate is to the changes in the input variables, with the calculation of their pseudo-elasticities. Total material quantity, metal price, recovery rate, highest grade in deposit, processing capacity and discount rate are found to positively impact the optimal mining rate; as the mine’s processing plant is always operating in full capacity during the production years, the processing cost is perfectly inelastic with the optimal mining rate; mining cost and the capital cost of the mining equipment negatively impact the optimal mining rate; the effects of several variables are diminishing as their percent of change increases.

The two-stage model in this research has intrinsic advantages over numerical approach, such as the Lane’s model. By a theoretical model, it is straightforward to explain the reason that the processing cost is perfectly inelastic to the optimal mining rate, as well as why the change of ore’ quality is not a function of mining rate when processing the stockpiled.

This research implies that, by choosing an optimal mining strategy, the stockpiled material contributes to the mine’s profit by a great part, and should not be ignored. In addition, the consideration of the stockpiled material impacts the mining strategy (mining rate and cutoff grade) of the original deposit. The two-stage model is recommended to be used by policy makers and mineral property owners, when assessing the effects of a policy promoting the processing of the stockpiled material, as well as planning mining strategy.


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Kuangyuan’s research interests include applying real option models to natural resources evaluation and investment decision-making, material criticality assessment, cutoff grade algorithm for mining projects, mining system modeling, mineral stockpiling, China’s rare earth policy, and effects of energy efficiency policy.

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