The Pennsylvania State University
The Graduate School

WATER TUNNEL EXPERIMENTS ON A MODEL SCALE
HELICOPTER ROTOR HUB

A Thesis in
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by
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Abstract

A scaled model of a notional helicopter rotor hub was tested at hub-diameter-based Reynolds numbers of $Re_D = 2.45 \times 10^6$ and $Re_D = 4.9 \times 10^6$ based on the hub of a large helicopter at an advance ratio of 0.2 in the 48” Garfield Thomas Water Tunnel. The main objectives of the experiment were to understand the spatial- and temporal content of the unsteady wake downstream of a rotor hub up to a distance corresponding to the empennage. Primary measurements were the total hub drag and velocity measurements at three nominal downstream locations. Various flow structures were identified and linked to geometric features of the hub model. The most prominent structures were two-per-revolution (scissors) and four-per-revolution (main hub arms) vortices shed by the hub. Both the two-per-revolution and four-per-revolution structures persisted far downstream of the hub, but the rate of dissipation was greater for the four-per-revolution structures. A six-per-rev structure was also observed, which is conjectured to be from Strouhal shedding. This work provides a dataset for enhanced understanding of the fundamental physics underlying rotor hub flows and serves as validation data for future CFD analyses.
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$X$ coordinate system axis in the freestream direction

$Y$ coordinate system axis in the spanwise direction

$Z$ coordinate system axis in the vertical direction

$u$ fluid velocity in the X-direction, m/s

$v$ fluid velocity in the Y-direction, m/s

$w$ fluid velocity in the Z-direction, m/s

$U$ mean fluid velocity in the x-direction, m/s

$q$ dynamic Pressure, $q = \frac{1}{2} \rho U^2$

$D$ drag, N

$A$ reference area, m$^2$

$C_D$ drag coefficient, $C_D = D/(0.5 \rho_\infty U^2 A)$

$R$ hub radius, m

$r$ hub radial location, m

$\Psi$ hub azimuth angle, degrees

$\rho$ fluid density, kg/m$^3$

$\nu$ fluid kinematic viscosity, m$^2$/s

$Ma$ Mach Number
\(\Omega\)  hub angular velocity, rpm

\(\mu\)  advance ratio, \(\mu = U/((2\pi \Omega/60)R)\)

\(Re_D\)  Reynolds number based on hub diameter, \(Re = UD/\nu\)

\(f\)  frequency, Hz

\(L\)  length scale, m

\(St\)  Strouhal number, \(St = fL/U\)
List of Abbreviations

PSU  The Pennsylvania State University
ARL  Applied Research Laboratory
GTWT Garfield Thomas Water Tunnel
MS   model scale
FS   full scale
PIV  Particle Image Velocimetry
SPIV Stereo Particle Image Velocimetry
LDV  Laser Doppler Velocimetry
FFT  Fast Fourier Transform
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Introduction

1.1 Background and Motivation

The rotor hub assembly is the largest contributor to helicopter parasite drag, accounting for up to 30% of the total parasitic drag [1, 2, 3, 4]. Figure 1.1 illustrates the breakdown of parasite drag contributions from the data of [1]. Therefore, the ability to reduce rotor hub drag is critical to achieve increased maximum forward-flight speed, fuel efficiency, and vehicle payload capacity. In addition, the unsteady wake shed by the main rotor hub causes interference drag and associated flow separation on the engine pylon [3]. Further undesirable interactions occur when the rotor hub wake impinges on the empennage [5], which have been encountered during flight testing. Thus, there is a strong need for an improved understanding of rotor hub flows that can be of use in the early design phase. At present, the underlying physical mechanisms of rotor hub drag and wake flows combined with their unsteady aerodynamics interactions with other vehicle components lack a complete understanding and hence mitigation of the former. The problem of rotor hub flows is threefold: 1) Drag generated directly by the rotating main rotor hub,
2) Effect of unsteady hub wake on flow separation aft the engine pylon and 3) Adverse effects of the unsteady rotor hub wake on aerodynamic and stability/control characteristics of the empennage and tail. These three facets of the rotor hub aerodynamic problem are illustrated in Fig. 1.2.

![Figure 1.1. Helicopter parasite drag contributors. Data taken from [1]](image)

### 1.2 Literature Review

This section will summarize previous research conducted on the subject of helicopter main rotor hub aerodynamics. The section is divided into the three previously mentioned aspects of the main rotor hub aerodynamics problem: i) parasite drag, ii) wake interference with other aerodynamic bodies, and iii) stability and control issues.
1.2.1 Parasite Drag

Harrington first investigated the effect of parasite drag on helicopter maximum speed and range [6]. At that time, the effect of the rotor hub was unknown and greatly underestimated. However, it was found that parasite drag is a large factor in the maximum speed and range of a helicopter. Churchill and Harrington [7] conducted experiments to determine the drag characteristics of various rotor hub configurations, but did not assert the significance of rotor hub parasite drag in comparison to other parasite drag mechanisms. Felker [8] conducted a fullwind tunnel test of the XH-59A helicopter rotor hub and found that certain rotor hub fairing designs can significantly reduce rotor hub drag. The other notable finding of Felker was that Reynolds number affects the drag coefficient of the faired hubs bewtween Re = 1M and Re = 3M. Sung et al. [9] investigated drag reduction by using various rotor hub fairing design. The main finding was that while fairings have the potential for drag reduction, they also contribute to interference drag. Keys and Rosenstein [3] revealed the contribution of rotor hub drag to be 20-
30% of the total vehicle parasite drag. This is much higher than any other single contributor to parasite drag, including the fuselage. It follows, as noted earlier, that reducing the rotor hub drag is not only the most effective, but also critical to increase maximum speed and range of helicopters.

More recently, Ortega et al. [10] and Raghav et al. [11] conducted wind tunnel experiments on a simplified generic helicopter rotor hub. The hub was modeled to represent a 10 ton helicopter. Force measurements were conducted, as well as particle image velocimetry in the wake of the hub. Reynolds numbers (up to $\sim 8.5 \times 10^5$) were relatively low in relation to full scale flight speeds ($\sim 10 \times 10^6$). The authors found that there were significant effects of azimuthal position on the instantaneous drag, however little effect from rotation. Another important finding was that there was a sideward shift in the wake deficit due to the hub rotation. That is, rotational effects do not effect drag, but they do affect the unsteady wake shed by the hub, which in turn has effects on the pylon and fuselage flow.

1.2.2 Wake Interference with Pylon and Fuselage Flow

Graham et al. [12, 13] studied interactions between the rotor hub fairing and engine pylon. In particular, the effects of various fairing and pylon configurations on hub drag were investigated. The main finding of this study was that interactions between the hub and pylon are central in affecting the drag on similar pylon designs. Secondary findings include specific hub fairings (flat bottom) that minimize the effect on flow over the pylon.

Keys and Rosenstein [3] investigated specific interference effects between various hub and pylon designs. The interference drag from the hub on the pylon ranged from 15% to 30%, and the interference of the pylon on the hub ranged from 5%
to 20% of the total hub and pylon drag, depending on the specific hub and pylon designs and configurations. The mutual interference drag ranged from 14% to 35% of the total hub and pylon drag.

The results of [3, 12] indicate that interference drag is a crucial factor in the total hub drag, hence an area to be further investigated for hub and pylon drag reduction. The aforementioned studies have been limited to measuring the drag effects of interference between hub components. No studies have conducted optical diagnostics to quantify the exact flow-field effects of interference between hub components. This is the next logical step in understanding the fundamental physics and eventual mitigation of rotor hub drag.

1.2.3 Wake Effects on Stability and Control

Sheridan and Smith [14] investigated helicopter interactional aerodynamics, including the effect of the main rotor hub wake on the vibrations in the empennage and tail boom assembly. A wind tunnel test was conducted on a 1:4.85 scale model of the YUH-61 helicopter in the Boeing/Vertol 20-foot V/STOL wind tunnel. The main finding of this study was that the turbulent rotor hub wake causes the flow adjacent to the top of the fuselage to separate and excite natural structural modes of the tail boom. The flow quality downstream of the hub was found to be affected by the pitch links, blade root geometry, and the main rotor shaft, in addition to the slightly more obvious factors of hub height and fuselage design.

Roesch and Vuillet [15] discuss issues discovered during flight testing regarding tail shake and handling qualities related to the rotor hub and pylon flow on several models of Aerospatiale helicopters. Again, the turbulent rotor hub wake was found to excite natural modes in the tail boom and vertical stabilizer, causing unaccept-
able vertical and lateral vibrations in the cabin. The lateral shaking was the most problematic to the pilots. In addition to adding a rounded cap to the rotor hub, the pylon was re-designed to mitigate the aerodynamic interactions which caused the vibrations. The rounded cap was effective in reducing separation on the upper surface of the rotor hub. The pylon used sharp edges to create tip vortices, which enhanced mixing of the turbulent hub wake with the freestream. The downward sloping trailing edge of the pylon acted to draw the hub wake downwards and below the empennage.

1.3 Current Experimental and Computational Challenges

While all previously mentioned studies have characterized the drag associated with rotor hub and pylon flows, few have investigated the flow field far downstream (i.e. one rotor radius or 8-10 hub radii) of the hub [16]. Computational fluid dynamics (CFD) investigations of rotor hub flows to date (e.g. [17, 18]) have limited flow-field data available for code validation. Consequently, it is difficult to assess the accuracy of state-of-the-art turbulence models and advection schemes used to predict complex wake structures downstream where they would interact with the empennage (i.e. 1 rotor radius or 8-10 hub radii, which is at least 30-50 times the reference chord length of a hub component). The use of optical-based flow diagnostics can provide the needed data to improve the current understanding of the physics of rotor hub flows, which are a combination of multiple complex high Reynolds number bluff-body flow phenomena.

Small-scale wind tunnel tests are unable to match the Reynolds numbers of full-
scale applications. This influences flow properties such as transition, separation and wake features that interact with downstream vehicle components. Full-scale Reynolds numbers can be achieved in large wind tunnels, but they are expensive, and optical-based flow measurements can be challenging. Non-intrusive seeding of the flow is problematic due to difficulties associated with seed buoyancy. Water tunnel testing, on the other hand, directly addresses these problems, as it allows for adequate Reynolds number scaling and flow seeding. Since local velocities at the rotor hub are generally low-speed (Ma < 0.3), compressibility effects are negligible, allowing the use of water as the working fluid. Hollow glass spheres or titanium dioxide are nearly neutrally buoyant in water, which results in a uniform and dense seeding that accurately follows the flow. Consequently, modern non-invasive measurement techniques such as particle-image-velocimetry (PIV) and laser-Doppler-velocimetry (LDV) are more readily conducted in water tunnels than in wind tunnels.

1.4 Objectives

The goal of this project was to perform experiments to characterize the drag and flow field associated with a scale-model of an industry representative helicopter main rotor hub. The objectives of this study can be divided into three categories: drag, mean flow properties, and unsteady flow properties.

**Drag:** The first objective was to measure the drag on the scale-model rotor hub. This included average drag and unsteady drag as a function of azimuthal position as well as Reynolds number. This information is useful for knowing whether future testing can be conducted at lower Reynolds numbers, or if full-scale Reynolds numbers must be used to obtain relevant data. The other use of drag data will
be to serve as a benchmark for comparison to future drag reduction technology development.

**Mean Flow:** The second objective was to obtain mean flow properties downstream of the hub up to a distance representative of the location of the empennage. This information is valuable in determining where the greatest velocity deficits are located, hence which hub components are contributing most to the drag. This information can also be an indicator of overall hub drag.

**Unsteady Flow:** The final objective was to collect data on the unsteady flow downstream of the hub up to a distance representative of the location of the empennage. This information is highly valuable in relation to stability, control, and structural response (e.g. the tail shake phenomenon). The unsteady wake also gives insight into premature separation of the flow over the pylon and/or upper surface of the fuselage.
Chapter 2

Experimental Methods

2.1 Facility

The experiment was conducted in the Garfield Thomas Water Tunnel (GTWT) located at the Pennsylvania State University (PSU) Applied Research Laboratory (ARL). The GTWT, photographically shown in Figs. 2.1-2.2 and schematically shown in Fig. 2.3, is the second largest low-turbulence recirculating water tunnel in the United States. The GTWT is a closed circuit, closed jet facility. The total volume of the tunnel is 401,254 liters. The test section is 1.22 m in diameter and 4.27 m long. The maximum empty test section velocity is 18.3 m/s, and the pressure can be varied between 0 and 414 kPa (absolute). Two honeycombs in the nozzle reduce the freestream turbulence level to below 0.1%. The tunnel is powered by a 1500 kW variable-speed motor. The tunnel has numerous static pressure taps on the walls, as well as mounting points for other probes such as Kiel probes. Windows on the sides (see Fig. 2.2) and top hatch provide optical access within the test section.
Figure 2.1. Photograph of the 48” Garfield Thomas Water Tunnel.

Figure 2.2. The GTWT test section (flow direction is from left to right).
2.2 Experimental Design

2.2.1 Reynolds Scaling in a Water Tunnel

Water tunnels have an advantage compared to wind tunnels for Reynolds-scale testing due to the kinematic viscosity of water being nominally 1/15 that of ambient air. For this work, a large helicopter flying at an advance ratio of 0.2 and forward-flight speed of 42 m/s (138 ft/s, 82 knots) was taken as a reference. At these conditions, the Mach number is less than or equal to 0.18 anywhere on the hub, making the incompressible flow assumption reasonable. The full-scale rotor speed was assumed to be 233 rpm with a main hub radius of ~14% of the blade radius. A 1:4.25 scaled model was considered for the 48” GTWT. Full dynamic similarity for Reynolds number and advance ratio is achieved at a water tunnel speed of 13.0 m/s and a rotor speed of 304 rpm. At this condition, the hub-diameter-based Reynolds number is $7.35 \times 10^6$. These conditions can be easily achieved in the GTWT. However, the cyclic hydrodynamic loads at full-scale Reynolds number were too high to produce a sufficient factor of safety with respect to fatigue failure.
for the actual rotor hub model. This was a mere material restriction due to the
model being rapid-prototyped through Stereolithography (SLA). Thus, the actual
test speeds nominally correspond to one-third- and two-thirds of the Reynolds
numbers for a large helicopter, or full scale for smaller helicopters (see Fig. 2.4).
Table 2.1 details the physical parameters for various dimensional scales in air and
water. The following equations were used to compute scaling factors (FS = Full
Scale, MS = Model Scale)

\[ Re_{FS} = \frac{U_{FS}D_{FS}}{\nu_{FS}} \]  

\[ \frac{Re_{MS}}{Re_{FS}} = \frac{U_{MS}}{U_{FS}} \times \frac{D_{MS}}{D_{FS}} \times \frac{\nu_{FS}}{\nu_{MS}} \]  

\[ \Omega_{MS} = \frac{D_{FS}}{D_{MS}} \times \frac{U_{MS}}{U_{FS}} \]

where \( Re \) is the Reynolds number based on the hub diameter, \( U \) is the forward
flight or tunnel speed, \( \Omega \) is the rotor angular speed, and \( \nu \) is the fluid kinematic
viscosity.

**Table 2.1.** Reynolds number scaling for various flow conditions in air and water (\( Re_{FS} = 7.35 \times 10^6 \)).

<table>
<thead>
<tr>
<th></th>
<th>Air</th>
<th>Water</th>
<th>Water</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( Re_{FS} )</td>
<td>( Re_{FS} )</td>
<td>( \frac{2}{3} Re_{FS} )</td>
<td>( \frac{1}{3} Re_{FS} )</td>
</tr>
<tr>
<td>D (m)</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>( U_\infty ) (m/s)</td>
<td>225</td>
<td>13.0</td>
<td>6.50</td>
<td>3.25</td>
</tr>
<tr>
<td>( \Omega ) (rpm)</td>
<td>5030</td>
<td>304</td>
<td>152</td>
<td>76</td>
</tr>
</tbody>
</table>

**2.2.2 Mechanical Design**

Figure 2.5 illustrates the experimental setup. A 17.6 horsepower hydraulic motor
was mounted below the tunnel. The rapid-prototyped SLA model rotor hub (ma-
terial: Watershed XC11122) was mounted on top of the drive shaft, which was
attached to a specifically designed seal plate on the tunnel floor. The nominal position of the assembly had a forward shaft angle of 5 degrees in reference to the full-scale flight condition at an advance ratio of 0.2. The motor-driveshaft-hub assembly pivoted about a point just above the motor in order to provide compliance for a load cell measurement. A button load cell was placed between the bearing mounting plate and the support bracket, see Figs. 2.5 and 2.15. A bellows coupling was used to allow the drive shaft to be sealed on the tunnel floor (through the seal plate) while still allowing compliance for load cell measurements. A fairing (NACA0025) was mounted around the assembly to isolate the hub wake from any unsteady wake shed by the drive shaft, seal, and bellows. The fairing was designed with a flat top with the intent of causing wake flow structures to advect straight downstream. A photograph of the final test assembly taken from a downstream viewpoint is shown in Figure 2.6.
Figure 2.5. Cut-away view of the experimental setup in the GTWT including hydraulic motor, driveshaft, seals, and rotor hub model.
Figure 2.6. Complete hub assembly in the GTWT. View is from downstream of the hub, looking towards the test section inlet.

2.2.2.1 Hub Model Design

The test model was based on a large commercial helicopter. Simplifications to the complex geometry were made in order to obtain a more canonical test case, as well as to reduce structural stresses on the rapid prototyped SLA hub. In order to transfer torque from the steel driveshaft to the SLA hub, the driveshaft transitioned from round to square, and the square cross section was inserted into the SLA hub and epoxied in place. For safety, a bolt on top fastened the driveshaft to the hub. Figure 2.7 shows the hub after the driveshaft was epoxied in place, but before the safety bolt was fastened.
Figure 2.7. Top view of the model rotor hub before safety bolt and washer were installed.

A summary of the hub components is shown in Figure 2.8. The model rotor hub includes the features: upper spider, main hub arms, lower spider, scissors, and swashplate. The decision to not include pitch links was made based on an earlier computational study that suggested the pitch links merely contribute additional four-per-rev harmonic content [5].
2.2.2.2 Finite Element Analysis

Dynamic Finite-Element Analyses (FEA) were conducted on the SLA model rotor hub and steel drive shaft assembly to ensure structural integrity. Unlike in wind tunnel testing, water tunnel test models incur high dynamic pressures (and therefore forces) due to the higher density of water compared to air. ABAQUS/CAE was used to create the model and mesh as well as solve for the stresses. For reference, the material properties are listed in Table 2.2. Time-varying forces were applied to elemental sections of all hub components based on local dynamic pressure and drag coefficients.

Figure 2.10 details the method for calculating these time varying loads along each hub component. Each hub component (labeled in Fig. 2.8) was partitioned into radial elements in Abaqus. The local fluid velocity of each element was then calculated based on the freestream velocity and tangential velocity due to rotation,
Table 2.2. Model hub and shaft material properties

<table>
<thead>
<tr>
<th></th>
<th>Watershed XC11122</th>
<th>17-4 PH Stainless Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ($kg/m^3$)</td>
<td>1120</td>
<td>7780</td>
</tr>
<tr>
<td>Modulus of Elasticity (MPa)</td>
<td>200,000</td>
<td>2,650-2,880</td>
</tr>
<tr>
<td>Tensile Strength (MPa)</td>
<td>47.1-53.6</td>
<td>1276</td>
</tr>
</tbody>
</table>

Projected onto the surface of the element. The equation for local velocity at each radial location is as follows.

$$U_{local} = (U_\infty + \Omega r)\sin(\psi)$$  \hspace{1cm} (4)

Then the local dynamic pressure was calculated using the fluid density and calculated local velocity. The equation for local dynamic pressure is as follows.

$$q_{local} = \frac{1}{2} \rho U_{local}^2$$  \hspace{1cm} (5)

The elemental drag is then calculated using the local dynamic pressure and drag coefficient from [19] as follows:

$$D = qC_D$$  \hspace{1cm} (6)

Figure 2.9 shows total drag estimate based on these loads as a function of rotor azimuth. Only 180° of hub rotation are shown due to symmetry.
Figure 2.9.

Figure 2.10. Schematic of a four bladed helicopter rotor hub for use with calculating local time varying loads.

The Abaqus automatic mesh generation feature was used to create the com-
putational mesh. Figure 2.11 shows an overview of the mesh that was used to compute the stresses in the model. The mesh was comprised of approximately 450,000 unstructured, tetrahedral elements. Refinement was required in two locations where stress concentrations were observed. These locations included the corners of the square shaft hole in the SLA hub and one of the corners where the scissors met the lower spider. A dynamic, implicit solver was used to compute the stresses in the hub assembly.

Figure 2.11. Computational mesh for finite element analysis.

At full-scale Reynolds number ($V = 13 \text{ m/s}, Re_D = 7.35 \times 10^6$), the peak cyclic stress in the SLA hub was about 6.9 MPa (1000 psi) (located at the intersection of the scissors and lower spider), or roughly the endurance limit of ABS-like plastics. Conditions at two-thirds scale Reynolds number resulted in a safety factor of four or greater for the SLA model rotor hub. Figure 2.12 shows an instantaneous distribution of von-Mises stresses on the SLA hub at two-thirds scale Reynolds
number.

Figure 2.12. Von Mises stress on the surface of the Watershed XC11122 hub model, units in psi.

2.3 Test Conditions and Measurement Locations

The experiment focused on two test conditions, (i) $U_\infty = 3.25 \pm 0.03$ m/s, $\Omega = 76$ rpm (1.27 Hz) and (ii) $U_\infty = 6.5 \pm 0.06$ m/s, $\Omega = 152$ rpm (2.53 Hz). Here $U_\infty$ is the free-stream speed at the streamwise hub location and $\Omega$ is the rotor speed. The free-stream uncertainty was determined from twice the standard deviation of at least 20 individual samples. The average temperature during testing was 25.7 °C with a maximum deviation of ± 1.5 °C variation throughout testing. In order
to avoid cavitation at the tips of the blade stubs on the model rotor hub, the water tunnel was pressurized at 25 psi throughout testing. The corresponding average density and kinematic viscosity ($\nu$) of the water during testing was 995 kg/m$^3$ and $9.4 \times 10^{-6}$ ft$^2$/s, respectively. The corresponding Reynolds numbers ($Re = U_\infty D/\nu$) were $2.45 \times 10^6$ and $4.90 \times 10^6$ for conditions (i) and (ii), respectively, where $D$ is the hub diameter. The ratio of test Reynolds number to full-scale Reynolds number was one-third and two-thirds for conditions (i) and (ii), respectively. The advance ratio for both test conditions was fixed at 0.191, which is slightly below the 0.2 target.

Measurements included the drag on the hub assembly, time-resolved point measurements of velocity (laser-Doppler velocimetry), 2D and 3D phase-averaged velocity vector flow-fields (particle image velocimetry and stereo particle image velocimetry), hub shaft angular frequency, and several tunnel monitoring measurements. Optical flow diagnostics were performed at three main downstream locations, termed near wake, mid wake, and far wake, which were approximately 2, 4, and 7 hub radii downstream of the center of the hub, respectively, as illustrated in Figures 2.13 and ?? . The coordinate system origin is defined as the point of intersection of the blade stub tip path plane and the driveshaft centerline. This coordinate system will be referenced for the remainder of this paper and is shown in Figure 2.13. Positive Y-direction corresponds to the advancing side of the rotor hub.
2.4 Instrumentation and Measurement Techniques

2.4.1 Drag

The total hub drag was measured with a standard 4450 N load cell (LC305-1K, Omega) located below the tunnel between the bearing mounting plate and the
support bracket as shown in Fig. 2.15. The drag data were low passed filtered at 250 Hz and recorded at 1 kHz. See section 2.2.2 of this document for details on the drag measurement mechanism.

Pre- and post-test calibrations of the load cell were performed (in the drained tunnel) by loading the stationary hub-shaft assembly with weights applied through a pulley mounted on the tunnel floor. The uncertainty in the drag measurements was found from the difference between the back-calculated loads and known applied calibration loads, typically termed residual loads. This provides a measure of both the random and bias errors, but the primary limitation is that it does not account for error in the calibration procedure (e.g. load alignment). Friction in the pulley is an additional uncertainty source, but is typically only a significant contributor when measuring drag on a body with very low drag coefficient, such as a flat plate. The final uncertainty in the drag measurement was approximately ±8%. Note that this uncertainty is for the steady or mean drag measurements, which does not account for additional unsteady noise associated with the long moment arm and the periodic loading. For a more detailed discussion on the assessment of steady and quasi-steady force measurement accuracy, see Ref. [20]. This additional noise was minimized by filtering out the 1-per-revolution frequency content due to shaft imbalance and 7-per-revolution frequency content due to structural resonance in the assembly.
2.4.2 Particle Image Velocimetry and Stereo

Particle Image Velocimetry

Two-dimensional particle image velocimetry (PIV) was acquired in double-frame, double-pulse mode. The 2D measurements (streamwise and vertical velocity components) were acquired within two streamwise oriented planes aligned along the tunnel centerline (Y = 0 m). The PIV planes were illuminated with a light sheet that entered the flow from above the test section, was centered in the tunnel (Y = 0 m), and aligned with the streamwise direction. The sheet was formed with the beam of a pulsed Nd:YAG laser (Gemini PIV, New Wave Research) operating at 532 nm wavelength. The maximum repetition rate of the laser was 15 Hz. The beam was formed into a sheet with the use of a cylindrical lens attached directly to the laser. The illuminated plane was imaged with the use of hollow glass
sphere particles (Sphericel 110P8, Potters Industries), which had a nominal diameter of 11.7 $\mu$m. The illuminated plane was imaged with a 1280×1024 pixel CCD camera (SensiCam, Cooke Corp.) fitted with a 28 mm lens (AF Nikkor, Nikon) and positioned on a Scheimpflug mount (see Figure 2.16). The resulting nominal field-of-view (FOV) was 180 mm (streamwise) $\times$ 175 mm (vertical). The images were spatially calibrated by imaging a 3D calibration target aligned with the laser sheet. The target consisted of a 180×180 mm rectangular grid of dots spaced 10 mm apart, and the dots were located in one of two planes spaced 1 mm apart. Only dots in a single plane were used to calibrate the 2D PIV, but the other plane was needed subsequently for the stereo-particle-image-velocimetry (SPIV) calibration.

![Figure 2.16. Particle image velocimetry camera with cloth over the windows to remove noise-causing ambient light.](image)

Since the laser pulse repetition rate was insufficient to fully resolve the relevant physics of the hub wake, the PIV system was set up to acquire multiple images at a given hub rotation position using the shaft encoder, schematically shown at the
bottom of Figure 2.5, as a triggering source. For each test condition, a minimum of half a revolution (180°) was acquired in 4.5° or 9° increments. The full revolution was not acquired due to time limitations. For a given phase increment, 500 image pairs were acquired. The camera and laser timing were controlled with a laser pulse synchronizer (model 610035, TSI) and commercial software (INSIGHT, TSI). The recorded images were then imported into a separate commercial PIV software processing package (DaVis 8, LaVision), which applied the image calibration and computed the velocity vector field. The vectors were computed using standard cross-correlation methods with multiple passes. Each pass reduced the interrogation window, which had 50% overlap, until the final interrogation window size (32 × 32 pixels) was achieved. This produced a final vector spacing of 3 mm.

SPIV was also acquired in a single plane located at X/R = 7.8 and oriented normal to the axial tunnel flow. Due to processing limitations the SPIV FOV was fixed based on the size of the calibration plate (180×180 mm). The SPIV plane position had a small bias towards the advancing side in order to capture flow structures that originated from the hub scissors and spiders. These structures were expected to be at this location due to the Magnus effect induced by the hub flow [18]. The only differences from the PIV setup are that there were two cameras (one on each side of the tunnel) and, due to optical limitations, the laser sheet also entered from the side of the tunnel. Figure 2.17 shows one of the SPIV cameras mounted on the side of the tunnel. A prism was used to access the desired field-of-view. Table 2.3 summarizes the PIV and SPIV plane locations and ranges.
Figure 2.17. Stereo particle image velocimetry camera on one side with prism on the tunnel window to access the field of view.

Table 2.3. Phase averaging and location information for PIV measurements.

<table>
<thead>
<tr>
<th>Plane Details</th>
<th>Ω (rpm)</th>
<th>$U_\infty$ (m/s)</th>
<th>$\Psi$ (deg.)</th>
<th>$\Delta\Psi$ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PIV Plane 1 (X/R= 1.74 to 2.36, Y/R= 0, Z/R= -0.39 to 0.06)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>76</td>
<td>3.25</td>
<td>0 to 180</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>152</td>
<td>6.50</td>
<td>0 to 180</td>
<td>4.5</td>
</tr>
<tr>
<td><strong>PIV Plane 2 (X/R= 6.83 to 7.38, Y/R=0, Z/R= -0.49 to 0.08)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>76</td>
<td>3.25</td>
<td>0 to 189</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>152</td>
<td>6.50</td>
<td>0 to 180</td>
<td>4.5 (0 to 27), 9 (27 to 180)</td>
</tr>
<tr>
<td><strong>SPIV Plane 3 (X/R= 7.8, Y/R= -0.17 to 0.39, Z/R= -0.18 to 0.08)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>152</td>
<td>6.5</td>
<td>0 to 180</td>
<td>4.5</td>
</tr>
</tbody>
</table>
2.4.3 Laser-Doppler Velocimetry

Laser Doppler velocimetry (LDV) is an irregularly sampled point measurement of two velocity components. The irregular sampling is because the frequency at which tracer particles pass through the measurement volume determines the acquisition rate. The nominal sampling rate achieved during testing was 100 Hz, though the random sampling allows for the Nyquist frequency to be exceeded. The tracer particles used during testing were 11.7 µm diameter hollow glass spheres (Sphericel 110P8, Potters Industries), which have a specific gravity of 1.10. LDV was performed at all three measurement locations, and was mounted on a 3-axis linear traversing table, as shown in Figure 2.18.

![Figure 2.18. Experimental setup for LDV measurements in the far wake.](image)

At each measurement location the measurement probe was traversed over a range of 11.4 cm in the X direction, 33.8 cm in the Y direction, and 12.8 cm in the Z direction. The argon ion laser (Innova 70C-5, Coherent) has a nominal beam
diameter of 1.5 mm and is cooled with a liquid heat exchanger (Laserpure 20, Coherent). The laser beam was then separated into blue and green beams with a multicolor beam generator (fiberlight 450200, TSI), which also coupled the beams to fiber optic cables. Alignment of the beams with the fiber optic couplers was accomplished with a power meter (Nova, Ophir Optronics) measuring the outputted beam power. The fiber optic cables were then connected to a fiber optic probe head (9832, TSI), which emitted four beams (2 green and 2 blue beams) through a 750 mm ($\kappa = 1.905$°) lens (9167-750, TSI). Light reflected from tracer particles within the measurement volume back towards the fiber optic head was coupled to a separate receiving fiber optic cable. The received light was then separated by wavelength and converted to electrical signals with a photodetector (PDM1000, TSI). The electrical signals are then processed with a multi-bit digital burst processor (FSA 35500-3, TSI). The burst processor band pass filters the signals and then uses both the Fast Fourier Transform (FFT) and autocorrelation techniques to determine the burst frequency, which is correlated to the tracer particle velocity. The burst processor also emitted the power to control the Bragg cell in the beam separator. The measured two components of velocity were then recorded with an LDV acquisition software package (Flowsizer, TSI) along with the traverse position and the rotor shaft position. Typically, about 36,000 velocity measurements were acquired at each measurement position. Uncertainty was calculated as the sum of setup, random, and bias. The setup uncertainty was the sum of the uncertainties in calibration, clock accuracy, laser stability, flow alignment, and laser positioning. The bias uncertainty is a result of velocity bias corrections to the data. LDV data are inherently biased to higher velocities, since when the velocity is higher, more particles flow through the probe volume. The data acquisition software applies a gate time weighting to correct for this bias. The uncertainty introduced by
this correction is modeled using the local turbulence intensity as detailed in [21]. The maximum overall uncertainty from all LDV measurements was ± 3% of the freestream speed in u-velocity and ± 0.3% of the freestream speed in w-velocity. Locations of LDV measurements are provided in Table A.1 of the appendix. Points 1-15 are in the near wake, points 16-30 are in the mid wake, and points 31-45 are in the far wake, as shown in Figure 2.19.
Figure 2.19. LDV probe numbers and positions at (a) X/R = 2, (b) X/R = 4, and (c) X/R = 7.
Figure 2.20 provides a view of the LDV probe positions in relation to the rotor hub. The probe locations ranged from $Y/R = -0.66$ to $Y/R = 0.45$, and $Z/R = -0.46$ to $Z/R = 0.14$.

![Figure 2.20. LDV Measurement locations, viewed from downstream of the hub.](image)

### 2.4.4 Monitoring of Flow Conditions

The hub drag and rotor speed were acquired simultaneously at 1 kHz along with water temperature, tunnel pressure (total and static), and the tunnel impeller settings (frequency and blade pitch). The water temperature, which had minimal variation during testing, was used to determine the water density, viscosity, and vapor pressure at each test condition. A Kiel-probe mounted in the upstream contraction measured the total pressure of the flow. Static pressure was acquired at six downstream positions ($X = 0.02, 0.63, 0.99, 1.5, 2.1, 3.0$ m) with the use of flush mounted ports and digital pressure transducers. The difference between the total and static pressure measurements provided a measure of the local free-stream speed. The closest measurement to the hub location was used as the reference velocity for this experiment. The local free-stream velocity was influenced by
blockage effects associated with boundary-layer growth on the tunnel walls and
the hub wake downstream of the test model. Free-stream velocity measurements
spanning the test section downstream of the hub indicate that the blockage effects
result in a deviation of less than 5% downstream of the hub. For a consistent
reference velocity the measurement at the hub location is used as the free-stream
velocity for a given test condition. The blockage due to the hub varies dependent
on the azimuthal position of the hub arms, and ranges between 7.1% and 7.4%.
See [22, 23] for additional GTWT details.

2.4.5 Frequency Analysis of the Experimental Setup

In order to quantify the vibrational characteristics of the model rotor hub and
support structure assembly, a modal analysis was conducted by using an impulse
excitation technique. Several accelerometers were mounted on the model rotor hub
and the hub was then struck with an impact hammer. The data from the impact
hammer and accelerometer was then used to calculate the natural frequencies of the
model rotor hub and support structure assembly. Table 2.4 summarizes the first
fifteen natural frequencies of the system. It is important to note that the test was
only able to be conducted with the tunnel empty. This impacts the fluid damping
on the model hub. Due to higher density of water than air, the natural frequencies
of the assembly will be lower in water than air. This would reduce the frequencies
of the natural modes of the system. Since there is no readily available technique
for easily accounting for the affect of water instead of air, no conversion was made.
The information from the impulse excitation test was still useful, however. There
was a 17.7 Hz signal observed in the drag measurements, which was markedly
absent from the flow field measurements. It was surmised that this was the first
mode of the model hub and support assembly, since the addition of water has the effect of adding damping and therefore lowering the natural frequencies, as stated earlier.

**Table 2.4.** Natural Frequencies of the Model Rotor Hub Test Assembly in GTWT

<table>
<thead>
<tr>
<th>f (Hz)</th>
<th>f (1/rev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.02</td>
<td>8.703</td>
</tr>
<tr>
<td>22.84</td>
<td>9.026</td>
</tr>
<tr>
<td>53.79</td>
<td>21.26</td>
</tr>
<tr>
<td>132.3</td>
<td>52.31</td>
</tr>
<tr>
<td>137.5</td>
<td>54.35</td>
</tr>
<tr>
<td>150.8</td>
<td>59.60</td>
</tr>
<tr>
<td>190.4</td>
<td>75.27</td>
</tr>
<tr>
<td>262.6</td>
<td>103.8</td>
</tr>
<tr>
<td>288.4</td>
<td>114.0</td>
</tr>
<tr>
<td>465.9</td>
<td>184.2</td>
</tr>
<tr>
<td>532.7</td>
<td>210.6</td>
</tr>
<tr>
<td>696.2</td>
<td>275.2</td>
</tr>
<tr>
<td>805.1</td>
<td>318.2</td>
</tr>
<tr>
<td>1148</td>
<td>453.8</td>
</tr>
<tr>
<td>1511</td>
<td>597.2</td>
</tr>
</tbody>
</table>
3.1 Drag

3.1.1 Average Drag and Reynolds Number Effects

Comparing drag measurements from both test conditions in Table 3.1, there is a difference in drag coefficient (or alternatively, D/q, drag divided by the freestream dynamic pressure). The drag coefficient is based on the maximum projected frontal area of the hub (0.039 m²). These results show that the drag coefficient increases between the low-speed and high-speed test conditions. The analytical model drag coefficient remains unchanged since a constant sectional drag coefficient was used. Based on the drag behavior for blunt bodies, it is conjectured that this observation is a Reynolds number effect. For example, the drag coefficient of a cylinder is relatively constant until a critical Reynolds number is achieved, at which drag sharply decreases and then gradually increases again with increasing Reynolds number (Ref. [24, 25]). This Reynolds number range of increasing drag coefficient is called the supercritical flow regime. Figure 3.1 compares the two data points acquired from this test with non-rotating cylindrical rod data from the literature.
(Ref. [24]). The Reynolds number ranges shown are for the advancing blade stub from the cylinder-like shape where the chord length transitions from approximately 7.5 cm at 75% hub radius to 2.5 cm at the blade stub tip. It is apparent from this figure that flow over some of the rotor hub components is expected to be supercritical. In addition, plotting the drag coefficient versus the hub-diameter-based Reynolds number (black symbols in Fig. 3.1) shows a similar trend as observed for a supercritical cylinder. Thus, the average drag values measured in the experiment indicate that at least part of the model rotor hub is operating in the supercritical flow regime. This suggests that supercritical Reynolds numbers must be considered in model-scale tests of rotor hub flows with reference to large helicopters. Additional testing over a larger range of Reynolds numbers is needed to obtain a complete curve and identify the exact Reynolds number dependence as well as the sensitivity to hub features.

<table>
<thead>
<tr>
<th>Table 3.1. Average Drag Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp.</td>
</tr>
<tr>
<td>Low-speed</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>$U_\infty$ (m/s)</td>
</tr>
<tr>
<td>D (N)</td>
</tr>
<tr>
<td>$D/q$ ($m^2$)</td>
</tr>
<tr>
<td>$C_D$</td>
</tr>
</tbody>
</table>

The average drag values measured in the experiment indicate that at least part of the model rotor hub is operating in the region of supercritical flow. This information is valuable for future testing, namely that Reynolds number effect is an essential consideration. Additional testing over a large range of Reynolds numbers is needed to obtain a complete curve and identify the exact Reynolds number dependence.
Figure 3.1. Reynolds number effect on drag coefficient. The solid line represents non-rotating cylinder data, black circles are measured results with a rotating hub where the Reynolds number is based on the hub diameter. Vertical dashed lines denote the Reynolds number range of the advancing blade stub for 1/3, 2/3, and full scale conditions.

3.1.2 Unsteady Drag

Phase-averaged unsteady drag is shown in Figure 3.2, comparing the analytical model to experimental results. Experimental data were filtered using a 250Hz low-pass filter. Two specific frequencies were filtered out using a band stop filter. The first was a 1-per-rev (2.53 Hz) signal due to imbalance in the hub model and driveshaft assembly. The second was a 7-per-rev (17.7 Hz) that was attributed to structural resonance based on the impulse response test described earlier. Only 180 degrees of azimuth are shown due to the geometric symmetry of the hub as mentioned earlier.

Average drag values are relatively close between the analytical model and experiment, however the amplitude of the oscillations is much greater in the experimental results than in the analytical model. The analytical model does not account for any oscillations higher than four-per-rev, while the experiment shows many high frequency oscillations. It is also important to note that the drag measurement setup is limited by added noise and uncertainty due to the distance from the hub to the load cell (nominally 0.75 m).
In order to quantify the spectral content of the hub drag, a Fast Fourier Transform (FFT) was performed on the drag data from the analytical and experimental results. Figure 3.3 shows the drag amplitudes at chosen per-rev intervals. There is significantly higher four-per-rev energy in the experiment than in the analytical model. The experiment also shows significant energy in the six-per-rev harmonic, which is unexpected. The six-per-rev mode is conjectured to be an effect of natural vortex shedding, which is explained later in the laser-Doppler velocimetry results section.
Figure 3.3. Unsteady drag harmonics.

3.2 Particle Image Velocimetry and Stereo Particle Velocimetry

3.2.1 Velocity Field

Panoramas of vertical velocity over 180 degrees of hub rotation are shown for the near- and far-wake locations in Figure 3.4. Six phase-averaged contour plots of vertical velocity, separated by 27 degrees of azimuth are overlapped to show spatial distribution of shed structures in the far wake. The far-wake panorama clearly illustrates two-per-rev (blue) and four-per-rev (red) structures shed from the hub, while the near-wake panorama is not as clear due to high frequency content not yet being dissipated. In the far wake, the first four-per-rev structure is followed closely by the two-per-rev structure, which is consistent with the orientation of the scissors with respect to the main hub arms. The scissors lag two of the main hub arms by 30 degrees of azimuth and lead the other two main hub arms by 60 degrees of azimuth. Thus, the source of these structures can be attributed
to the scissors and main hub arms and spiders, respectively. It is notable that the two-per-rev structure, although produced by a relatively geometrically smaller hub component, is still prominent and comparable in magnitude to the four-per-rev structure in the far wake, which corresponds to approximately 35 reference chord lengths downstream of the scissors. Also note in the far wake that the two-per-rev structure is spatially oriented above the four-per-rev structure, which is unexpected when considering the geometric locations of the scissors (two-per-rev) and main hub arms and spiders (four-per-rev). This indicates that the respective structures have changed their vertical positions in the wake flow. Note that in the near wake, the harmonic content is not as clear, and the red and blue cannot be simply attributed to four-per-rev and two-per-rev oscillations.

The SPIV uncertainty was relatively large compared to the LDV and 2D PIV uncertainty. This was due to limited optical access, which resulted in optical distortions and poor signal to noise ratio. The required position of one of the two cameras relative to the laser sheet resulted in a significantly reduced amount of scattered light collected by that camera. This effectively reduced the seeding density, and limited the final vector spacing resolution as well as increased the noise and drop-outs present in the instantaneous images. The optical distortions primarily affected the edges of the images, which results in a reduced effective field of view. In spite of these limitations, physical features of the flow are still present in the phase-averaged vector-fields. Figure 3.5 shows a representative SPIV result from an azimuthal angle of 12°. In this plane, all three components of velocity were acquired, though only the v and w velocity components are visible in the vectors, and the contour colors correspond to the vertical velocity component (w). In this figure, it is apparent that there is a counterclockwise vortical structure (denoted by black arrows) centered at approximately Y/R = 0.27 and Z/R = 0.10 (denoted by
white X). This position nominally corresponds to the location of the intersection of the scissors with the lower spider, which would suggest that the structure seen in the figure is the two-per-revolution structure observed in the 2D PIV and LDV results. This assumption is further supported by the strong downwash near the centerline observed in the 2D PIV results. Inspection of the 2D results at the same angular position (12°) shows that the two-per-revolution downward structure had just passed through the 2D PIV FOV. This is in agreement with the SPIV results since the SPIV plane is further downstream (X/R = 7.8) than the far-wake 2D PIV plane (X/R = 7).
Figure 3.5. Representative phase-averaged vector-field from the SPIV measurements at an azimuthal angle of 12°. The contour colors correspond to the vertical velocity component ($w$) in m/s.

3.2.2 Spectral Analysis

Although temporal resolution is relatively low, spectral analysis of the PIV data can give some insight into the unsteadiness of the flow. Five locations were chosen each in the near and far wake, see Figure 3.4 for locations, and an FFT was performed on the $w$ (vertical) velocity. Figure 3.6 shows the amplitudes of each harmonics up to 10-per-rev. In the near wake, the two-per-rev signal is highly concentrated at probe location 3, which spatially corresponds to the location of the scissors. In the far wake, the two-per-rev signal is spread across all probes, which is consistent with the growth behavior of the wake. The four-per-rev signal is highly concentrated in probe number 1, which spatially corresponds to the main hub arms. All harmonics are dissipated as they are convected downstream into the far wake.
Figure 3.6. PIV harmonics at X/R = 2 (left) and X/R = 7 (right) at the high-speed test condition ($Re_D = 4.90 \times 10^6$). Numbers on the right indicate probe positions shown in Fig. 3.4

3.3 Laser Doppler Velocimetry

3.3.1 Averaged Velocity Measurements

Spanwise profiles of the average streamwise velocity for all three wake measurement locations are shown in Fig. 3.7. In general, the velocity deficit is greater on the advancing side (positive Y direction) and varies little with radial location over the measurement range. The retreating side has an overall lower velocity deficit that becomes weaker with increasing radial location, which is a result of decreased relative velocity on the retreating side due to the hub rotation. At X/R = 7,
the differences are less pronounced between advancing and retreating side velocity deficits except for the measurement at $Y/R = -0.67$. As shown in Fig. 2.20, this location is outside the center hub region, and thus initially has less deficit generated due to smaller frontal area and lower relative velocity.

Figure 3.8 shows vertical profiles of the streamwise velocity component at $X/R = 2$, $X/R = 4$, and $X/R = 7$. Of interest is the location of maximum velocity deficit, which occurs around $Z/R = -0.33$. This vertical position corresponds to the location of the swashplate. It is notable that the velocity deficit is higher there than in the wake of the main hub arms and spiders. The near-field ($X/R = 2$) PIV images in Fig. 3.4 reveal that the flow field is very complex with large gradients in both the streamwise and vertical directions, where coherent structures are not easily identifiable. This is different in the far wake ($X/R = 7$) in Fig. 3.4.

**Figure 3.7.** Streamwise velocity profiles in the spanwise direction at $X/R = 2$ and $X/R = 7$. Low speed: $Re_D = 2.45 \times 10^6$, High speed: $Re_D = 4.9 \times 10^6$
Figure 3.8. Streamwise velocity profiles in the vertical direction at $X/R = 2$ and $X/R = 7$. Low speed: $Re_D = 2.45 \times 10^6$, High speed: $Re_D = 4.9 \times 10^6$

Extensive LDV data can be found in Appendices A and B. Table A.1 provides locations of all LDV probes. Tables A.2 and A.3 provide average velocities and uncertainties at all LDV probe locations for both flow conditions. Appendix B provides average velocity profiles of the vertical ($w$) velocity. Tables B.1 and B.3 show vertical ($w$) velocity along the spanwise ($Y$) direction. Tables B.2 and B.4 show vertical ($w$) velocity along the vertical ($Z$) direction.

3.3.2 Spectral Analysis

Spectral analysis of the LDV data provides further insight into the unsteady components of the hub wake. Since the LDV data are sampled at irregular time intervals, it is not possible to simply perform an FFT on the data. To resolve this problem, the autospectrum was used for spectral analysis. It is possible to
compute the autocorrelation of unevenly spaced data, and then compute the FFT of that, which gives the autospectrum. The autocorrelation is given as:

$$C_{xx}(\tau) = \frac{1}{N-m} \sum_{n=1}^{N-m-1} x_n x_{n+m-1}$$  \hfill (7)

where

$$m = \pm 0, 1, 2, 3, \ldots M - 1, M$$

Here, $N$ is the number of points in the time series, and $2M+1$ is the number of points in the autocorrelation. This is trivial to compute for evenly spaced time series, but an extra step is needed to compute the autocorrelation for unevenly sampled data. The time lags used to compute the autocorrelation must be sorted into bins such that

$$\tau_m - \frac{\Delta \tau}{2} \leq \tau_m \leq \tau_m + \frac{\Delta \tau}{2}$$  \hfill (8)

where

$$\Delta \tau = \frac{t_N}{M}$$  \hfill (9)

In this study, $M$ was set at 500, and $N$ varied depending on the record length. Typical values of $N$ were around 25,000 to 30,000. The effective sampling rate nominally ranged from 150-400 Hz for one-third-scale Reynolds number and 400-1000Hz for two-thirds-scale Reynolds number. The autocorrelation is evaluated for the time series indices, $n$, that fit in the above bins. The FFT of the autocorrelation is then computed to give the autospectrum as follows:

$$G(m\Delta f) = \sum_{n=0}^{N-1} C_n e^{-i2\pi n m \Delta t}$$  \hfill (10)

$$S_{xx}(m\Delta f) = \frac{G(m)G^*(m)}{N} \Delta t$$  \hfill (11)
where

\[
\Delta f = \frac{N}{(2M+1)}
\]  \hspace{1cm} (12)

\(G\) is the FFT of the autocorrelation, and \(G^*\) is its complex conjugate. More information can be found in [26, 27, 28].

Figure 3.9 shows autospectra at selected points at \(X/R = 2\), \(X/R = 4\), and \(X/R = 7\). Of particular interest is the 2-per-rev signal, which is smaller than the 4-per-rev signal at \(X/R = 2\), however it maintains strength into the far wake, as opposed to the 4-per-rev signal, which is dissipated at a higher rate. Autospectra were calculated at higher frequencies, but only lower frequencies are plotted to highlight the harmonic content (2/rev, 4/rev, 6/rev, etc.) associated with the hub rotation. A line representing Kolmogorov’s -5/3 law is shown and suggests that the turbulence is locally isotropic (i.e. homogeneous) at small scales.

Also of interest is a 6-per-rev signal (15 Hz) as seen in Figs. 3.9 and 3.10, which is conjectured to be Strouhal shedding (only coincidentally aligned with 6-per-rev) from the intersection of the upper spider, main hub arm, and lower spider on the retreating side. The 6-per-rev signal is strongest at a spanwise and vertical location close to the aforementioned geometric feature, see the ”x” in Fig. 3.5 (the peak is slightly lower in Fig. 3.10, but the energy is spread out due to lower frequency resolution, a product of variable LDV sampling frequency). It is proposed that this is Strouhal shedding (\(St = fL/U\)), where \(f\) is the shedding frequency in Hertz, \(L\) is the characteristic length scale, and \(U\) is the reference velocity. Assuming a Strouhal number of 0.14 for a square cylinder (Ref. [29]), the 6-per-rev frequency corresponds to a bluff body with a characteristic length of 4.1 cm. The width of the
lower spider intersection with the main arms is about 3.8 cm, which corresponds to a Strouhal number of 0.13. The width of the upper spider intersection with the main hub arms is about 3.0 cm, which corresponds to a Strouhal number of 0.11. Since the actual shapes are not simply square cylinders, it can be surmised that the 6-per-rev signal is, within the uncertainty of the actual Strouhal number, natural vortex shedding from the bluff body generated by the intersection of the lower spider, main hub arm, and upper spider on the retreating side of the hub.

The 6-per-rev signal is not observed on the advancing side. It is believed that this is due to higher relative velocity causing the natural shedding frequency to be higher and dissipate more rapidly.

Figure 3.9. Autospectra of vertical velocity at the high-speed test condition of $Re_D = 4.90 \times 10^6$ (Y/R = -0.10, and Z/R = -0.19).
Figure 3.10. Autospectra of vertical velocity at the high-speed test condition of $Re_D = 4.90 \times 10^6$ ($Y/R = -0.44$, and $Z/R = -0.19$).
3.3.3 Effect of Reynolds Number on Wake Flow

Since it is known that Reynolds number has a significant effect on drag coefficient, it is important to examine the effects of Reynolds number on the shed wake of the rotor hub. Normalized velocity profiles are compared with little difference between the two flow conditions in both the near- and far- wake locations. Figures 3.11 and 3.12 illustrate this with u velocity Y- and Z- profiles. It can be seen, however, that larger velocity deficits occur at two-thirds scale Reynolds number. This is consistent with the measured drag coefficients in Table 3.1.

Figure 3.11. Velocity profiles of streamwise velocity in the Z direction for both flow speeds in the near and far wake.

Figure 3.13 compares autospectra between the two wake locations and the two flow speeds at a selected point. Although the levels are slightly different at high frequencies, the peaks match between the two flow speeds, and the shapes are
Figure 3.12. Velocity profiles of streamwise velocity in the Y direction for both flow speeds in the near and far wake.

comparable. At a given flow speed, the frequency content appears to be the same at each wake location, only lower magnitude in the far wake. Comparing flow speeds, the shapes are generally the same, with the exception of the absence of a six-per-rev peak in the low speed, near wake location.
Figure 3.13. Velocity profiles of streamwise velocity in the Y direction for both flow speeds in the near and far wake.
Summary and Conclusions

4.1 Summary

A 1:4.25 scaled model rotor hub was tested at one-third- and two-thirds-scale Reynolds number at an advance ratio of approximately 0.2 in the Garfield Thomas Water Tunnel at ARL Penn State. The model rotor hub is based on a large commercial helicopter. The main objective of this effort was to characterize the spatial and temporal content of complex flow structures emanating from a model rotor hub up to a distance in the wake that corresponds to the locations of empennage and tail. Water tunnel flow diagnostics including PIV, SPIV, and LDV were used in the experiment. The water tunnel is particularly suited for advanced and high-quality flow diagnostics due to homogeneous and dense flow seeding that cannot be easily obtained in wind tunnels at such high Reynolds numbers. Collected data in this experiment will enhance the physics-based understanding of rotor hub flows as a whole and provide the rotorcraft community with valuable insights and a wealth of high-quality data for subsequent computational model validation.
4.2 Conclusions

Drag measurements on a load cell confirmed that drag coefficient increased with Reynolds number. This indicates that the model rotor hub was operating, at least partially, in supercritical flow. Further comparisons of mean flow and autospectra at various downstream locations confirmed that consistent measurements were obtained between one-third- and two-thirds-scale Reynolds number with slightly larger velocity deficits at two-thirds-scale Reynolds number, which is consistent with the drag measurements. Furthermore, analyses of autospectra revealed that the strength of individual per-rev content was also consistent between both test Reynolds numbers. Hence it can be surmised that a one-third-scale Reynolds number may be sufficient for physics-based testing of model rotor hubs.

Particle Image Velocimetry (PIV) measurements in the near and far wake showed strong per-rev content of the flow. The strongest signals were two-per-rev (scissors), four-per-rev (hub arms), and a six-per-rev signal attributed to Strouhal shedding from the intersection between upper and lower spiders with the hub arm on the retreating side of the model rotor hub. While the four-per-rev and six-per-rev contents were seen to lose strength with further downstream distance from the hub, the two-per-rev signal generated by the scissors maintained its strength into the far wake.

Stereo Particle Image Velocimetry (SPIV) measurements were acquired in a cross-flow (YZ) plane in the far wake only. The SPIV system allowed to measure all three velocity components. The plane location had some bias towards the advancing side in hopes to capture the strong two-per-rev flow structure caused by the scissors that may have slightly deviated from its centerline position due to action of the Magnus effect. The two-per-rev structure was indeed detected in
the SPIV plane consistent in size and position to the PIV measurements slightly upstream.

Laser Doppler Velocimetry (LDV) measurements were performed at a total of 45 locations by traversing in all spatial directions at a near, mid, and far wake location. Integrated autospectra confirmed that the two-per-rev signal due to the scissors maintains its strength far into the wake, while accompanying four-per-rev (hub arms) and six-per-rev (Strouhal shedding) lose their strengths at a higher rate. Furthermore, the LDV measurements in the near wake suggested that the observed six-per-rev Strouhal shedding originated from the retreating side at the spanwise location where upper and lower spiders merge into the hub arms. Autospectra processed from collected LDV data illustrated a decrease in signal strength with downstream distance. A comparison of the autospectra to the classical -5/3 slope suggests that harmonics larger than 8-per-rev contain nearly homogeneous turbulence.

This work experimentally confirms that multiple harmonic flow structures generated by a helicopter main rotor hub persist for a long downstream distance. Hence these structures may affect handling qualities and vibrations on the empennage and tail of the aircraft.

4.3 Future Work

Experiments are planned for the ARL PSU GTWT 12 inch test section water tunnel to further investigate the rotor hub flow problem. These experiments will focus less on the far-field wake flow and more on Reynolds number effects as well as interactions between i) individual hub components and ii) the hub and pylon fairing. The 12 inch GTWT test will be able to match the low speed test condition
($Re_D = 2.45 \times 10^6$) of the 48 inch GTWT to show repeatability of results. Figure 4.1 shows a layout of the preliminary design of the future experiment with major components of the assembly labeled.

![Diagram](image.png)

**Figure 4.1.** Preliminary design layout of helicopter rotor hub test in the ARL PSU GTWT 12 inch water tunnel.

The rotor hub model will retain the same geometry with the addition of pitch links, however each hub component (upper spider, main hub arms, lower spider, scissors, and swashplate) will be separately attached to a central shaft. This will allow for individual component drag to be investigated, and more importantly, interference drag can be quantified. Investigation of the effect of individual hub
components on the wake flow will also be possible.

As in the previous experiment, drag will be measured using a similar pivoting shaft design. In addition, however, there will be a rotary torque transducer in-line with the hub driveshaft. This provides the ability to study the torque characteristics of the hub model, which ultimately relate to power required of the helicopter engine.

Flow diagnostics will be performed, including LDV to obtain velocity spectra and high-speed PIV. High-speed PIV was not available for the previous study. Flow diagnostics will focus on the near and mid wake regions (X/R = 2 to X/R = 4) in order to pinpoint the source of specific flow features. The LDV will be used as before to investigate velocity spectra downstream of the hub. High-speed PIV will provide much higher azimuthal resolution for phase-averaged planar flow field measurements. This will be highly valuable in visually identifying specific flow features as well as providing average flow properties such as wake deficit, which is directly related to drag.
## Laser-Doppler Velocimetry Data

Table A.1: LDV Probe Locations

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# Table A.2: Average LDV Velocities, Two-Thirds-Scale Reynolds Number Condition

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Table A.3: Average LDV Velocities, One-Third-Scale Reynolds Number Condition

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Figure B.1. Vertical velocity profile along the Y direction, $U_{\infty} = 6.5$ m/s.
Figure B.2. Vertical velocity profile along the $Z$ direction, $U_\infty = 6.5$ m/s.

Figure B.3. Vertical velocity profile along the $Y$ direction, $U_\infty = 3.25$ m/s.
Figure B.4. Vertical velocity profile along the Z direction, $U_\infty = 3.25$ m/s.
Matlab Code

C.1 Analytical Model For Hub Drag Estimation

function [Azimuth, Drag, Torque, Avg_Drag] = analyticalmodel(V,rpm)

%This code takes a freestream speed, V (in m/s) and a rotor speed, rpm (rpm) and computes
%the drag and torque on a helicopter rotor hub using blade element theory.

%Outputs are Azimuth: 0:360 degrees, Drag (Drag) at each azimuth, torque
%(Torque) at each azimuth, average drag over all azimuths (Avg_Drag), and
%average Drag divided by dynamic pressure (Doq)

Cd = 1.2;

rho = 997; %kg/m^3

omega = rpm*2*pi/60; %convert rotational speed from rev/min to rad/sec

%Calculate drag from square and round central shaft sections (hardcoded,
%specific to particular hub geometry)
Dcyl = 0.5*997*V^2*0.3*(8.27*0.83+2.8*1.28+0.57*1.8)*0.0254^2*0.2248;
Dsqr = 0.5*997*V^2*1.0*(((3.69*1.185+4.26*0.741+1.43*0.484+4*0.391))*0.0254^2);

%radial element locations (hard coded, specific to particular hub geometry)
r2 = {linspace(2.694,11.56,10)*0.0254 linspace(2.183,4.854,10)*0.0254 ...
linspace(1.43,4.029,10)*0.0254 linspace(2.183,4.379,10)*0.0254 ...
linspace(4.379,5.448,10)*0.0254};

%height of each radial element (hard coded, specific to particular hub geometry)
h1 = [1.185+2.13*sin(5*pi/180) 0.741+1.5*sin(5*pi/180) 0.468 0.391+1.67*sin(5*pi/180) 0.831]*0.0254;

inc = 1; %azimuth step size

offset = 0:90:270; %offset from zero azimuth for each hub arm

for l = 1:length(offset) %loop through each hub arm

  %set range of azimuth for particular hub arm
  psi = (offset(l):inc:360+offset(l))*pi/180;

  totalnewD = zeros(length(psi),1); %initialize drag vector
  totalnewT = zeros(length(psi),1); %initialize torque vector

  for q = 1:length(r2)
    r = cell2mat(r2(q));
    h = h1(q);
  end
for k = 1:length(psi)

    if h == h1(3) %check if calculating for scissors

        psi(k) = psi(k) + 30*pi/180; %scissors offset by 30 deg

    end

for ii = 1:length(r)-1

    for jj = 1:length(psi)

        %calculate radius at middle of the element
        r1 = (r(ii+1)+r(ii))/2;

        %calculate local velocity
        U = (omega*r1) + V*sin(psi(jj));

        %check if reversed flow
        if omega*r1 + V*sin(psi(jj)) < 0
            sign = -1; %reversed flow
        else
            sign = 1; %not reversed flow
        end

        %calculate dynamic pressure for the element
        D(jj) = sign*Cd*(0.5)*rho*(U^2);

        %calculate drag for the element
        D1(ii,jj) = D(jj)*(r(ii+1)-r(ii))*h;

    end

end
T1(ii,jj) = D(jj)*(r(ii+1)-r(ii))*h*...
        (r(ii+1)+r(ii))/(2*0.0254);
end

if h == h1(3) %check if scissors

    %only two scissors, so set drag for other two to zero
    if offset(l) == 90 || offset(l) == 270
        D(jj) = zeros;
        D1(ii,:) = zeros;
        T1(ii,:) = zeros;
    end
end

end

end

%sum the drag over all radial elements and project force in
%freestream flow direction based on azimuth angle
totalD(k) = (sum(D1(:,k)*sin(psi(k))))';

%sum the torque over all radial elements
totalT(k) = (sum(T1(:,k)));
end

%add to matrix of drag values, 1 column for each component (main
%arms, spiders, scissors, etc.)
totalnewD = [totalnewD totalD'];
%add to matrix of torque values, 1 column for each component (main arms, spiders, scissors, etc.)
totalnewT = [totalnewT totalT'];

end

for y = 1:length(totalnewD)
    %create matrix of drag values 1 column for each hub arm
    totalnew2D(y,1) = sum(totalnewD(y,:));

    %create matrix of drag values 1 column for each hub arm
    totalnew2T(y,1) = sum(totalnewT(y,:));
end

for y = 1:length(totalnewD)
    %add constant drag of square shaft and round shaft, sum over all 4 hub arms
    totalnew3D(y) = sum(totalnew2D(y,:))+Dcyl+Dsqr;

    %sum torque over all 4 hub arms
    totalnew3T(y) = sum(totalnew2T(y,:));
end

Azimuth = 0:inc:360;
Drag = totalnew3D;
Torque = totalnew3T;
Avg_Drag = mean(Drag);
Bibliography


