NETWORK MODELING AND PREDICTION OF RETROGRADE GAS BEHAVIOR IN NATURAL GAS PIPELINE SYSTEMS

A Thesis in
Energy and Mineral Engineering
by
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ABSTRACT

As retrograde gases are transported through pipeline systems, retrograde condensation may occur. Gases expand during pipe transportation due to pressure losses, causing the gas to reach its saturation point and condensate liquids. The presence of this additional liquid phase is responsible for additional pressure losses in natural gas surface production systems. Gases and liquids have markedly different routing behavior when moving concurrently through a network system, and the determination of their distribution within the network is of vital importance for retrograde gas surface transportation operations.

This study is aimed at developing and testing an analytical tool able to predict phase behavior of retrograde gas in natural gas pipeline system so that the operator can make better decisions regarding system optimization and operation. It can help to make decisions regarding liquid phase removal from the network, pressure loss mitigation, design and location of surface production and separation equipments with an accurate map of the pressure, phase distribution and phase property inside the network. The study was undertaken in stages, starting with a model for a one-dimensional steady state flow of single-phase fluid (gas) in pipes. This model was then expanded with Beggs and Brill model (1999) to account for the additional pressure drop due to the appearance of liquid phase flow conditions in the system. In the third stage, tee junctions split models were incorporated for network-wide simulation. In the final stage, the phase behavior model is used to simulate the retrograde gas instead of gas and water in the stages before. As the first three stages are established, the products of the model were compared with prediction of commercial simulators and Essenfeld’s model (2010). The resulting model was used in case studies of retrograde gas transportation systems. The present work shows that the proposed model is capable of simulating the changes of properties of two-phase flow in systems of varying complexities in which two-phase hydrodynamically splits at tee junctions. As a result, the proposed network model for retrograde gas is able to handle realistic field situations and allows the users to effectively track the liquid and gas preferential routing within the network and provide guidance for the improvement the performance of the entire system.
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>Cross-sectional area</td>
<td>[ft²]</td>
</tr>
<tr>
<td>d</td>
<td>Pipe diameter</td>
<td>[in]</td>
</tr>
<tr>
<td>E_k</td>
<td>Kinetic energy</td>
<td>[lbm/ft s²]</td>
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<tr>
<td>f</td>
<td>Friction factor</td>
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</tr>
<tr>
<td>f_f</td>
<td>Fanning friction factor</td>
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<tr>
<td>f_n</td>
<td>Normalizing friction factor</td>
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<tr>
<td>f_m</td>
<td>Molar fraction of gas</td>
<td>[-]</td>
</tr>
<tr>
<td>f_l</td>
<td>Fugacity of liquid</td>
<td>[psi]</td>
</tr>
<tr>
<td>f_g</td>
<td>Fugacity of gas</td>
<td>[psi]</td>
</tr>
<tr>
<td>g</td>
<td>Gravity acceleration</td>
<td>[ft/s²]</td>
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<tr>
<td>g_c</td>
<td>Gravity correction factor</td>
<td>[32.174 lbm ft/(lbf s²)]</td>
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<tr>
<td>H</td>
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<tr>
<td>H_L</td>
<td>Hold up of liquid</td>
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</tr>
<tr>
<td>K_i</td>
<td>Vapor liquid equilibrium constant</td>
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<tr>
<td>KE</td>
<td>Kinetic energy ratio</td>
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<tr>
<td>L</td>
<td>Length of the pipe</td>
<td>[mile]</td>
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<td>L_e</td>
<td>Equilibrium length of the pipe</td>
<td>[mile]</td>
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<td>Molecular weight of gas</td>
<td>[lbm/lbmol]</td>
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<td>MW_air</td>
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<td>Gas mass flow rate in inlet pipe</td>
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<td>m_GR</td>
<td>Gas mass flow rate in run pipe</td>
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<tr>
<td>N_Lv</td>
<td>Liquid velocity number</td>
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<tr>
<td>n_c</td>
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<td>n_G</td>
<td>Molar flow rate of gas</td>
<td>[lbmol/d]</td>
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<td>n_L</td>
<td>Molar flow rate of liquid</td>
<td>[lbmol/d]</td>
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<td>P_c</td>
<td>Critical pressure</td>
<td>[psi]</td>
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<td>P_down</td>
<td>Pressure downstream</td>
<td>[psi]</td>
</tr>
<tr>
<td>P_sc</td>
<td>Pressure at standard conditions</td>
<td>[14.7 psia]</td>
</tr>
<tr>
<td>P_up</td>
<td>Pressure upstream</td>
<td>[psi]</td>
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<tr>
<td>p_r</td>
<td>Reduced pressure</td>
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<tr>
<td>q_gsc</td>
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<td>[SCFD]</td>
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<tr>
<td>q_g</td>
<td>Gas volumetric flow rate</td>
<td>[ft³/d]</td>
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<tr>
<td>R</td>
<td>Universal gas constant</td>
<td>[10.73 psi ft³/(lbmol R)]</td>
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<td>Reynolds number</td>
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<td>$T_{sc}$</td>
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<td>$V_c$</td>
<td>Critical volume</td>
<td>[ft$^3$]</td>
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<tr>
<td>$v_m$</td>
<td>Velocity of mixture</td>
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<tr>
<td>$v_{sg}$</td>
<td>Superficial velocity of gas</td>
<td>[ft/s]</td>
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<tr>
<td>$v_{sl}$</td>
<td>Superficial velocity of liquid</td>
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<tr>
<td>$\bar{v}$</td>
<td>Molar volume</td>
<td>[ft$^3$/lbmol]</td>
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<td>$x$</td>
<td>Molar fraction of component in liquid phase</td>
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<tr>
<td>$z$</td>
<td>Overall molar fraction of component</td>
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<tr>
<td>$Z_{av}$</td>
<td>Average compressibility factor</td>
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<td>$Z_g$</td>
<td>Gas compressibility factor</td>
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**Greek**

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<thead>
<tr>
<th>Symbol</th>
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<th>Unit(s)</th>
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<tr>
<td>$\rho_L'$</td>
<td>Molar density of liquid</td>
<td>[lbmol/ft$^3$]</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Gas density</td>
<td>[lbm/ft$^3$]</td>
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<td>$\rho_L$</td>
<td>Liquid density</td>
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<td>$\rho_n$</td>
<td>Non-slip mixture density</td>
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<td>$\rho_r$</td>
<td>Reduced density</td>
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<tr>
<td>$\rho_s$</td>
<td>Slip density</td>
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</tr>
<tr>
<td>$\rho_{sc}$</td>
<td>Density at standard conditions</td>
<td>[lbm/ft$^3$]</td>
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<td>$\gamma_g$</td>
<td>Gas specific gravity</td>
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<td>Branch to inlet mass intake ratio of gas</td>
<td>[-]</td>
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<td>$\lambda_L$</td>
<td>Branch to inlet mass intake ratio of liquid</td>
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<td>Liquid surface tension</td>
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<td>[in]</td>
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<td>$\sigma$</td>
<td>Pitzer's acentric factor</td>
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<td>$\mu_g$</td>
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<tr>
<td>$\mu^*$</td>
<td>Mixture viscosity</td>
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CHAPTER 1
INTRODUCTION

The presence of liquids in the pipelines will reduce the transportation capacity and cause corrosion problems in the pipelines and operating equipment. Therefore, the presence of liquids in pipelines is one of the most frequent problems in the gas industry. Nowadays, the tracking of liquid path in the gas-water systems has been studied for years. The pressure drops and liquid distribution in natural gas pipeline systems can be well predicted both for the flow in pipelines and tee junctions.

Currently available models and commercial simulators are used for dry gas networks and gas-water networks. Very few models have been established for retrograde gas transportation systems. In the pipeline networks for retrograde gas, the phase behavior will change with temperature and pressure, which makes the situation so complicated. It is also difficult to capture the pressure changes due to the appearance or disappearance of the second phase and the properties changes of the phases. Since it will affect the properties of the fluid in each pipe and then affect the performance of the pipeline networks, the phase behavior of retrograde gas is important in network simulation. In dry gas network, the fluid is all in gas phase so that there is no need for the phase behavior models. And in gas-water network, the liquid phase is all water while the gas phase is all dry gas, and the two phases will not react with each other, result in no need for the phase behavior models. Therefore, the phase behavior prediction for retrograde gas is the most different and essential part as compared with dry gas or gas-water pipeline network simulations.

In this study, the fluid flow and distribution are assumed to be very fast thus the operating temperature of the pipeline systems will not change significantly, which allows the whole process being considered as an isothermal process.

This study focus on the development of an isothermal model that allows mapping the phase distribution along the pipeline network, based on the combination of application of mass and energy balances under isothermal conditions with application of thermodynamic laws. The proposed model allows operators to: detect single-phase and
multiphase systems, track the phase properties of the fluid, calculate the mass flow rate of liquid phase and then select the route for liquid phase removal, thus maximizing the fraction of pipeline capacity available for gas transportation. A summary of the results of this comprehensive research study are presented in this thesis.

Objectives:

- To develop an isothermal model for a multiphase, multiple supplies retrograde gas network system simulation, which incorporates uneven T-junction split and phase behavior models.
- To verify the proposed model with GASNET developed by Essenfield (2010) for gas-water network system simulations.
- To test the proposed model with different retrograde gas networks and analyze the results.
CHAPTER 2
LITERATURE REVIEW

Multiphase flow in pipeline networks has been studied for several years and is an important issue in petroleum engineering. The foundation for the model and analysis of multiphase flow is single-phase flow in pipelines. In order to consider all the parameters governing gas flow, many empirical approaches had been investigated (Johnson and Berwald, 1935). Most of the equations were formulated based on experimental data and matching field data from operational gas pipeline systems such as Weymouth equation (Weymouth, 1912), Panhandle-A and Panhandle-B equation (Boyd, 1983). Weymouth, Panhandle-A and Panhandle-B equations are popular due to their simplicity and non-iterative properties. The difference in these equations lies in the relationship for friction factors.

For a natural gas network system, the problem is eventually expressed in terms of a set of highly non-linear equations for each component in the network. It must be solved simultaneously in terms of the desired target unknowns (Ayala, 2013). As the size of network grows, the more complex the resulting system of equations becomes. There have been many attempts to simplify the solution for the resulting non-linear equations. Nowadays, the application of the multivariate Newton-Raphson method is currently employed to solve the resulting simultaneous equations for the non-linear network systems. However, the most significant limitation of Newton-Raphson methods is their tendency to yield diverging results when the initial guess is not sufficiently close to the actual solution. To alleviate this problem, Ayala and Leong (2013) proposed a linear analog to convert these non-linear equations into linear equations so that it can simplify the solution.

For pressure gradient prediction of multiphase flow, empirical correlations are widely used. For the development of the empirical correlations, the correlations have no slip, no flow pattern consideration such as Poettmann and Carpenter (1952), Fancher and Brown (1963) were developed at first. The mixture density is calculated based on the input gas/liquid ration. That is, the gas and liquid are assumed to travel at the same velocity. The only correlation required is for the two-phase friction factor. Then, Gray (1978) and
Asheim (1986) considered slip and flow pattern effects on the multiphase flow. These correlations are required for both liquid holdup and friction factor. In the 1970s, Beggs and Brill not only drew slip and flow pattern into consideration but also proposed the methods for predicting the existing flow pattern which are necessary for correlations.

Treatment at junctions is of vital importance to the overall performance of network because uneven phase separation and liquid route preferences are governed by split mechanisms at junctions. T-junction is the junction where a second line is connected to the body of a straight main line. Based on the directions of flow through each arms, there are three kinds of T-junction including branching junction, combining junction and impaction junction.

![Figure 2-1 T-junction classification](image)

**Figure 2-1** T-junction classification, A) Splitting tee, B) Splitting impacting tee, C) Converging impacting tee, D) Converging tee (Essenfeld, 2010)

At the tee junction, a well-known phenomenon named fluid flow route selection which means that as the second phase (liquid) approaches the junction, it can either continue with the inertia of the original flow path or follow a different path based on the property of the junction and the fluids.
One-dimensional model for the analysis of two-phase flow split at T-junction mainly fall into four categories (Azzopardi, 1999; Peng and Shoukri, 1997; Lahey, 1986). They are empirical correlations such as Seeger et al. (1986), geometrical model such as Shoham et al. (1987), phenomenological-mechanistic models such as Azzopardi et al. (1989), and two-fluid model such as Hart et al (1990) and Otten et al. (2001).

Empirical correlations represent a much-simplified way to analyze phase separation at T-junction. Seeger et al. (1986) developed correlations for two-phase flow split in a T-junction with different branch inclinations. Empirical correlations are only valid for conditions where they are derived. Furthermore the accuracy of predictions mainly depends on the amount and quality of experimental data. Shoham et al. (1987) developed a geometrical model which was successful for a narrow range of experimental data. Phenomenological-mechanistic models are based on conservation principles and force balances according to physical understanding of the split mechanism. Azzopardi et al. (1989) developed a phenomenological-mechanistic model to predict the phase split in horizontal and vertical pipes. Particularly, phenomenological methods usually depend on junction type, geometry and flow pattern. Two-fluid models are based on the solution of one-dimensional mass and momentum conservation equations applied to each phase. Hart et al. (1990) proposed his double stream model (DSM) which is based on energy-balance equations for each phase. Later, Ottens et al. (2001) improved this model and renamed it as Advanced Double Stream Model (ADSM). In 2005, Shoham (2005) summarized the variables involved in two-phase flow as follows: mass and volumetric flowrates of liquid, gas and mixture; liquid holdup and gas void fraction; superficial velocities of each phase, mixture velocity, mass flux, slip velocity, drift velocity, drift flux, diffusion velocities, mass concentration, split holdup, slip density and average fluid properties. Then these parameters were incorporated to the T-junction analysis presented by Singh (2008), Alp (2009), and Fernandez -Luengo (2010). Margaris (2007) proposed a model for T-junction which has a horizontal run and a vertical branch. El-Shaboury et al. (2006) proposed a model for two-phase flow in a horizontal equal-sided impacting T-junction.

In this study, three two-fluid models for T-junction are mainly different due to the relationship between the liquid mass and gas mass intake fractions. They are even split model, kinetic energy ratio split and the dual stream model. These models have been used
for gas-water flow analysis in natural gas network (Essenfeld, 2010). The main difference between retrograde gas system and gas-water system is that the liquid phase of retrograde gas system is gas condensate which is liquified from gas and also can gasify back while that of gas-water system is water which will not react with gas. Thus, the phase behavior prediction and phase properties analyses are essential for retrograde gas system.

Currently, the phase behavior prediction models are usually based on Equations of State (EOS). EOSs are analytical expressions that provide functional relationships between pressure, temperature and volume of a system. Peng-Robinson EOS (Peng and Robinson, 1976) and the Soave RK EOS (Soave, 1972) are the most popular cubic EOS currently in use. And several authors, such as Martin (1979), Coats (1985) and Danesh (2001), have shown that all cubic EOS’s can be represented by single general formulations.

In 1990, Adewumi et al. developed a nonisothermal, 1D, compositional, two-fluid, multiphase hydrodynamic model to describe the formation and flow dynamics of condensate in horizontal natural gas pipelines. The results from them demonstrated the predictive and descriptive potential of the model in field applications. Mucharam et al. (1990) improved the model for natural gas pipelines with undulating topology and demonstrated the significant effect of inclination and inclination changes on the hydrodynamics of gas/condensate flow in transmission pipelines. At the same time, Vincent (1990) also developed a predictive compositional hydrodynamic model with a two-parameter equation of state. This model consists of a system of nonlinear algebraic and ordinary differential equations, which were solved numerically. These models were restricted to single-pipe analysis and network effects were not considered. In 1998, Zhou proposed a single-pipe, transient compositional multiphase hydrodynamic model for transient gas/condensate two-phase flow in pipelines. This model consists of a well-posed modified Soo’s partial pressure model in conservative form and the phase behavior model for natural gas mixtures.

Schouten et al. (2004) reported calculations of the amount of condensate in a natural gas mixed with 16.7% hydrogen in transmission pipelines. And they found that the problems are less in a mixture of natural gas with hydrogen than in the ‘pure’ natural gas. Hamersma (2006) studied the effect of hydrogen addition on the route preference in
natural gas flow in regular, horizontal T-junction. He found that the addition of hydrogen to a realistic natural gas/condensate system taking into account the calorific value of the gas, temperature and pressure will result in a slight increase in the route preference for the branch.
3.1 Single-phase Flow Equations

The governing equations for single-phase flow can be derived by analyzing the pressure losses which reflect the energy changes in pipelines. Total pressure losses can be calculated as the sum of the contributions of friction losses, elevation changes and acceleration changes as stated below (Ayala, 2013):

\[
\frac{dp}{dx}_{Total} = \frac{dp}{dx}_{friction} + \frac{dp}{dx}_{elevation} + \frac{dp}{dx}_{acceleration} \tag{3.1.1}
\]

Where:

\[
\frac{dp}{dx}_{friction} = -2f\frac{\rho v^2}{g_c d} \tag{3.1.2}
\]

\[
\frac{dp}{dx}_{elevation} = -\rho \frac{g dz}{g_c dx} \tag{3.1.3}
\]

\[
\frac{dp}{dx}_{acceleration} = -\frac{\rho v dv}{g_c dx} \tag{3.1.4}
\]

In pipeline flow, the contribution of the kinetic energy term to the overall energy balance is considered insignificant compared to the typical magnitudes of friction losses and potential energy changes. So, the final expression for pressure losses will be:

\[
\frac{dp}{dx}_{Total} = \frac{dp}{dx}_{friction} + \frac{dp}{dx}_{elevation} \tag{3.1.5}
\]

\[
\frac{dp}{dx}_{Total} = -\left(2f\frac{\rho}{g_c d}\right)\left(\frac{m}{\rho A}\right)^2 - \rho \frac{g dz}{g_c dx} \tag{3.1.6}
\]

Since the mass flow rate \(m\) and cross section area \(A\) can be replaced by the following equations:

\[
m = \rho v A \tag{3.1.7}
\]

\[
A = \frac{\pi d^2}{4} \tag{3.1.8}
\]
The rearranged equation to be prepare for integrations as shown below (Ayala, 2013).

\[ \int_{p_i}^{p_j} p dp = -\alpha \int_0^L dx - \beta \int_0^L \rho^2 dz \] \hspace{1cm} (3.1.9)

where \( \alpha = \frac{32 m^2 f}{\pi^2 g c d^5} \), \( \beta = \frac{g}{g c} \frac{\Delta H}{L} \). For the isothermal flow of gases, the fluid density depends on pressure (\( \rho = \varphi p \)) so that the flow rate also depends on pressure:

\[ p_i^2 - e^s p_j^2 = \frac{\alpha (e^s - 1)}{\eta^2} \beta \] \hspace{1cm} (3.1.10)

where \( \eta = \frac{\gamma_g MW_{air}}{Z_{av} RT_{av}} \), \( s = 2\eta \beta L \). Equation (3.1.10) states the well known fact that the driving force for gas flow through pipelines is the difference of the squared pressure. Replacing the gas mass flow rate by the volumetric gas flow rate using the equations below:

\[ m = \rho_{sc} q_{gsc} \] \hspace{1cm} (3.1.11)

\[ \rho_{sc} = \frac{P_{sc} \gamma_g MW_{air}}{RT_{sc}} \] \hspace{1cm} (3.1.12)

Finally, we can get the non-linear single phase gas flow equation:

\[ q_{gij} = C_{ij} (p_i^2 - e^s p_j^2)^{0.5} \] \hspace{1cm} (3.1.13)

where “\( C_{ij} \)” is the pipe conductivity defined as: (Ayala, 2013)

\[ C_{ij} = \left( \frac{\pi^2 g c R}{64 MW_{air}} \right)^{0.5} \frac{T_{sc}}{P_{sc} (\gamma_g Z_{av} T_{av})^{0.5} f^{0.5} L^{0.5}} \] \hspace{1cm} (3.1.14)

The pipe conductivity depends on the friction factor, pipe geometry and fluid properties on the flow capacity of the pipe. For horizontal flow, the equation becomes:

\[ q_{gij} = C_{ij} (p_i^2 - p_j^2)^{0.5} \] \hspace{1cm} (3.1.15)

Depending on the type of friction factor correlation used to evaluate pipe conductivity, the equations above can be rewritten into the different traditional forms of gas pipe flow equations mentioned in the literature such as Weymouth, Panhandle-A and Panhandle-B (Ayala, 2013).
Table 3.1-1 Summary of specialized equations for gas flow (adapted from Ayala, 2013)

<table>
<thead>
<tr>
<th>Gas Flow Equation</th>
<th>$f$</th>
<th>$r$</th>
<th>$m$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weymouth</td>
<td>$f_F = \frac{0.008}{d^\frac{3}{2}}$</td>
<td>$r = \frac{\gamma g Z_{av}T_{av}}{(433.391 \frac{p_{sc}}{p_{sc}})^2}$</td>
<td>5.3333</td>
<td>0.5000</td>
</tr>
<tr>
<td>Panhandle-A</td>
<td>$f_F = \frac{0.019231}{(\frac{q_{sc} \gamma g}{d})^{0.14}}$</td>
<td>$r = \frac{\gamma g^{0.8539} Z_{av}T_{av}}{(435.87 \frac{p_{sc}}{p_{sc}})^{1.0788} \frac{p_{sc}}{p_{sc}}^{1.854}}$</td>
<td>4.8540</td>
<td>0.5394</td>
</tr>
<tr>
<td>Panhandle-B</td>
<td>$f_F = \frac{0.003586}{(\frac{q_{sc} \gamma g}{d})^{0.03922}}$</td>
<td>$r = \frac{\gamma g^{0.961} Z_{av}T_{av}}{(737 \frac{p_{sc}}{p_{sc}})^{1.020} \frac{p_{sc}}{p_{sc}}^{1.96}}$</td>
<td>4.9600</td>
<td>0.5100</td>
</tr>
<tr>
<td>General Gas</td>
<td>Moody chart or Colebrook Equation</td>
<td>$r = \frac{\gamma g Z_{av}T_{av}}{(2818 \frac{p_{sc}}{p_{sc}})^2} f_F$</td>
<td>5.0000</td>
<td>0.5000</td>
</tr>
</tbody>
</table>

Unit: $d$ (in), $e$ (in), $L$ (ft), $\Delta H$ (ft), $T$ (R), $q_{sc}$ (SCF/D), $P$ (psia).

### 3.2 Single-phase Network Solver

#### 3.2.1 Linear Analog Algorithm

Since Newton Raphson method is prone to lack of convergence and sensitivity to initial guesses, presumption of appropriate initial guesses is the key for solving gas network equations for every existing method. Linear analog algorithm is one of the approaches to
get the appropriate initial guesses using a simpler method. The linear-pressure analog flow equation for horizontal pipes as follows (Ayala and Leong, 2013):

\[ q_{gij} = L_{ij} (P_i - P_j) \] (3.2.1)

Where, \( L_{ij} \) is the value of the linear pressure analog conductivity which is straightforwardly calculated as a function of actual pipe conductivity according to the following transformation rule:

\[ L_{ij} = T_{ij} C_{ij} \] (3.2.2)

Table 3.2.1-1 is the summary of linear pressure analog constitutive equations and expressions of analog conductivity transform for horizontal pipe and inclined pipe. And Figure 3.2.1-1 is the flow chart for linear analog implementation.

Table 3.2-1 Summary of linear pressure analog equations (adapted from Ayala and Leong, 2013)

<table>
<thead>
<tr>
<th>Network Type</th>
<th>Linear pressure analog equation</th>
<th>Analog conductivity transform, ( T_{ij} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Pipes</td>
<td>( q_{gij} = L_{ij} (P_i - P_j) )</td>
<td>( T_{ij} = \sqrt{1 + \frac{2}{r_{ij} - 1}}, \text{where } r_{ij} = \frac{P_i}{P_j} )</td>
</tr>
<tr>
<td>(for a symmetric characteristic matrix)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inclined Pipes – I</td>
<td>( q_{gij} = L_{ij} (P_i - e^{\frac{s_{ij}}{2}} P_j) )</td>
<td>( T_{ij} = \sqrt{1 + \frac{2}{r_{ij} - 1}}, \text{where } r_{ij} = \frac{P_i}{e^{\frac{s_{ij}}{2}} P_j} )</td>
</tr>
<tr>
<td>(for an asymmetric characteristic matrix)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inclined Pipes – II</td>
<td>( q_{gij} = L_{ij} (P_i - P_j) )</td>
<td>( T_{ij} = \sqrt{\frac{r_{ij}^2 - e^{\frac{s_{ij}}{2}}}{(r_{ij} - 1)^2}}, \text{where } r_{ij} = \frac{P_i}{P_j} )</td>
</tr>
<tr>
<td>(for a symmetric characteristic matrix)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.2.2 Newton Raphson Protocol

The Newton Raphson protocol is built to solve a nonlinear system of equations, by determining the derivative of each function of the system and will try to find the roots that will satisfy all the equations representing the system under study. The Newton Raphson protocol can be summarized as follows (Essenfeld, 2010):

1) Assign an initial value of P for each node.

2) Enter the Newton Raphson Method.
a. Calculate the initial residual:

\[ q_{sc} = C(P_{up}^2 - P_{dn}^2)^n \]  \hspace{1cm} (3.2.3)

b. Introduce perturbation, node by node:

\[ P_{pert} = P + \Delta P \]  \hspace{1cm} (3.2.4)

c. Calculate the new residual:

\[ q_{pert} = C(P_{up,pert}^2 - P_{dn}^2)^n \]  \hspace{1cm} (3.2.3)

d. Determine the entry to the Jacobian:

\[ Jac = \frac{q_{pert} - q_{sc}}{\Delta P} \]  \hspace{1cm} (3.2.4)

A Jacobian matrix is a square matrix whose main entries are composed by all the derivatives with respect to pressure of the equations that conforms the system. After iteration, improvements are applied to the residual matrix, and the iteration process will continue until convergence.

e. Remove the perturbation for the node and move on to the next node.

f. Check that the main diagonal of the Jacobian is not zero, and the Jacobian normalization.

g. Solve the equation.

h. Upgrade the value of P for each node.

3) If convergence is achieved, meaning that the nodal pressure was found and the residual value is close to zero. If convergence has not been achieved, step 2 is repeated with the upgraded pressure values.

### 3.3 Pressure Gradient in Multiphase Flow System

The Beggs and Brill method was the first one to predict flow behavior at all inclination angles. Beggs and Brill proposed the following pressure gradient equation for inclined pipe (Brill, 1999):
\[
\frac{dp}{dL} = \frac{f \rho_n v_m^2}{2d} + \rho_s g \sin \theta \frac{1 - E_k}{1 - E_k}
\]  
(3.3.1)

Where, \(E_k\) is represent kinetic energy and the expression is below:

\[
E_k = \frac{v_m v_s \rho_n}{\rho}
\]  
(3.3.2)

\[
\rho_s = \rho_L H_{L(\theta)} + \rho_g \left[1 - H_{L(\theta)}\right]
\]  
(3.3.3)

Since Beggs and Brill method is a method with flow pattern dependency, flow pattern prediction is of vital importance. Figure 3.3-1 illustrates the horizontal flow patterns considered by Beggs and Brill (gas phase is in white while liquid phase is in blue).

**Figure 3.3-1** Beggs and Brill horizontal flow patterns (Essenfeld, 2010)

The flow patterns not only depend on the volumetric ratio of two phases, but also affected by the velocities, physical properties of each phase and geometry of the pipes. Figure 3.3-2 shows the flow pattern map for horizontal pipes. At low gas and liquid velocities, the flow pattern tends to be stratified flow. And as the liuquid velocity increases, it will become bubble or wavy flow. While as the gas velocity increases, the annular flow will occurs. If both gas and liquid velocities are high, distributed flow will happen in the pipes.
Beggs and Brill chose to correlate flow-pattern transition boundaries with no slip liquid holdup and mixture Froude number, given by

\[ N_{Fr} = \frac{v_m^2}{gd} \]  

(3.3.4)

The equations for the modified flow-pattern transition boundaries are

\[ L_1 = 316\lambda_L^{0.302} \]  

(3.3.5)

\[ L_2 = 0.000925\lambda_L^{-2.468} \]  

(3.3.6)

\[ L_3 = 0.10\lambda_L^{-1.452} \]  

(3.3.7)

\[ L_4 = 0.5\lambda_L^{-6.738} \]  

(3.3.8)

The following inequalities are used to determine the flow pattern that would exist if the pipe were horizontal.

Segregated:

\[ \lambda_L < 0.01 \text{ and } N_{Fr} < L_1 \]

or

\[ \lambda_L \geq 0.01 \text{ and } N_{Fr} < L_2 \]

Transition:
\[ \lambda_L \geq 0.01 \text{ and } L_2 \leq N_{Fr} \leq L_3 \]

Intermittent:

\[ 0.01 \leq \lambda_L < 0.4 \text{ and } L_3 < N_{Fr} \leq L_1 \]

or

\[ \lambda_L \geq 0.4 \text{ and } L_3 < N_{Fr} \leq L_1 \]

Distributed:

\[ \lambda_L < 0.4 \text{ and } N_{Fr} \geq L_1 \]

or

\[ \lambda_L \geq 0.4 \text{ and } N_{Fr} > L_4 \]

Different correlations for liquid holdup prediction were developed for three horizontal flow patterns. The liquid holdup that would exist if the pipe were horizontal is first calculated and then corrected for the actual pipe inclination angle. The same equations are used to calculate liquid holdup for all flow patterns. However, the empirical coefficients are different for each flow pattern.

The liquid holdup for horizontal pipe is calculated from the following equation with the restriction that \( H_{L(0)} \geq \lambda_L \).

\[ H_{L(0)} = \frac{a \lambda_L^b}{N_{Fr}^c} \quad (3.3.9) \]

The coefficients \( a, b \text{ and } c \) are determined from table 3.3-1 for the appropriate horizontal flow pattern.

**Table 3.3-1** Beggs and Brill empirical coefficients for horizontal liquid holdup (Brill, 1999)

<table>
<thead>
<tr>
<th>Flow Pattern</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segregated</td>
<td>0.980</td>
<td>0.4846</td>
<td>0.0868</td>
</tr>
<tr>
<td>Intermittent</td>
<td>0.845</td>
<td>0.5351</td>
<td>0.0173</td>
</tr>
<tr>
<td>Distributed</td>
<td>1.065</td>
<td>0.5824</td>
<td>0.0609</td>
</tr>
</tbody>
</table>
The equations below give the liquid holdup corrected for the effect of inclination angle.

\[ H_L(\theta) = H_L(0) \Psi \]  \hspace{1cm} (3.3.10)

\[ \Psi = 1.0 + C [\sin(1.8\theta) - 0.333\sin^3(1.8\theta)] \]  \hspace{1cm} (3.3.11)

Where \( \theta \) is the actual angle of the pipe from horizontal and \( C \) is determined by

\[ C = (1.0 - \lambda_L) \ln\left( e \lambda_L N_{Lv}^g N_{Fr}^h \right) \]  \hspace{1cm} (3.3.12)

\[ N_{Lv} = \frac{v_{sl}^4}{g \sigma_L} \]  \hspace{1cm} (3.3.13)

with the restriction that \( C \geq 0 \). The coefficients \( e, f, g \) and \( h \) are determined from table 3.3-2 for the appropriate horizontal flow pattern.

**Table 3.3-2** Beggs and Brill empirical coefficients for \( C \) (Brill, 1999)

<table>
<thead>
<tr>
<th>Flow Pattern</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segregated uphill</td>
<td>0.011</td>
<td>-3.7680</td>
<td>3.5390</td>
<td>-1.6140</td>
</tr>
<tr>
<td>Intermittent uphill</td>
<td>2.960</td>
<td>0.3050</td>
<td>-0.4473</td>
<td>0.0978</td>
</tr>
<tr>
<td>Distributed uphill</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All patterns downhill</td>
<td>4.700</td>
<td>-0.3692</td>
<td>0.1244</td>
<td>-0.5056</td>
</tr>
</tbody>
</table>

When the flow pattern falls in the transition region, the liquid holdup must be interpolated between the segregated and intermittent liquid holdup values as

\[ H_{L(\theta),Tr} = AH_{L(\theta),Seg} + (1 - A)H_{L(\theta),Int} \]  \hspace{1cm} (3.3.14)

Where,

\[ A = \frac{L_3 - N_{Fr}}{L_3 - L_2} \]  \hspace{1cm} (3.3.15)

The two-phase friction factor is calculated from the following relationship:

\[ f = f_n \left( \frac{f}{f_n} \right) \]  \hspace{1cm} (3.3.16)

The normalizing friction factor, \( f_n \), is determined from Chen’s friction model.
The ratio of the two-phase friction factor to the normalizing friction factor was correlated with the Beggs and Brill experimental data, resulting in

\[
\frac{f}{f_n} = e^s
\]  

(3.3.18)

Where,

\[
s = \ln y - 0.0523 + 3.182 \ln y - 0.8725 \ln y^2 + 0.01853 \ln y^4
\]  

(3.3.19)

And

\[
y = \frac{\lambda_L}{(H_L(\theta))^2}
\]  

(3.3.20)

For \( s \) that is used when \( 1 < y < 1.2 \), Beggs and Brill introduced this correlation

\[
s = \ln(2.2y - 1.2)
\]  

(3.3.21)

3.4 Phase Behavior Model

3.4.1 Equation of State

The volumetric behavior of multi-component systems is described by equations of state (EOS). Peng-Robinson EOS (Peng and Robinson, 1976) and the Soave RK EOS (Soave, 1972) are, by far, the most popular cubic EOS currently in use (Ayala, 2012):

\[
PR \ EOS: \quad p = \frac{RT}{\tilde{v} - b_m} - \frac{(aa)_m}{\tilde{v}^2 + 2b_m \tilde{v} - b_m^2}
\]  

(3.4.1)

\[
SRK \ EOS: \quad p = \frac{RT}{\tilde{v} - b_m} - \frac{(aa)_m}{\tilde{v}^2 + b_m \tilde{v}}
\]  

(3.4.2)

Several authors, such as Martin (1979), Coats (1985) and Danesh (2001), have shown that all cubic EOS’s can be represented by single general formulations. One of the most useful generalized formulations is the one presented by Coats, which is shown below.

Z-factor form of the generalized EOS:
\[ Z^3 + [(m_1 + m_2 - 1)B - 1]Z^2 + [A + m_1 m_2 B^2 - (m_1 + m_2)B(B + 1)]Z \\
- [AB + m_1 m_2 B^2(B + 1)] = 0 \quad (3.4.3) \]

Where:

\[ A = \sum_{i}^{n_c} \sum_{j}^{n_c} c_i c_j A_{ij} \quad (3.4.4) \]

\[ A_{ij} = (1 - \delta_{ij}) (A_i A_j)^{0.5} \quad (3.4.5) \]

\[ A_i = \Omega_{ai}^0 [1 + m_i (1 - T_{ri}^{0.5})] \frac{P_{ri}}{T_{ri}^2}, \text{(SRK and PR only)} \quad (3.4.6) \]

\[ B = \sum_{i=1}^{n_c} c_i B_i \quad (3.4.7) \]

\[ B_i = \Omega_{bi}^0 \frac{P_{ri}}{T_{ri}} \quad (3.4.8) \]

Whenever needed, the different equations of state are obtained by substituting the proper definitions of \( m_1, m_2, \Omega_{ai}^0, \) and \( \Omega_{bi}^0 \) for the particular EOS. These definitions are given below for some of the most traditional EOS.

**Table 3.4-1** Values of Parameters for different EOS

<table>
<thead>
<tr>
<th>Equation of State</th>
<th>( m_1 )</th>
<th>( m_2 )</th>
<th>( \Omega_{ai}^0 )</th>
<th>( \Omega_{bi}^0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RK EOS Redlich Kwong</td>
<td>0</td>
<td>1</td>
<td>0.4274802</td>
<td>0.08664035</td>
</tr>
<tr>
<td>SRK EOS Soave RK</td>
<td>0</td>
<td>1</td>
<td>0.4274802</td>
<td>0.08664035</td>
</tr>
<tr>
<td>PR EOS Peng Robinson</td>
<td>( 1 + \sqrt{2} )</td>
<td>( 1 - \sqrt{2} )</td>
<td>0.457235529</td>
<td>0.077796074</td>
</tr>
</tbody>
</table>

SRK EOS:

\[ m_i = 0.48 + 1.574 \sigma_i - 0.176 \sigma_i^2 \quad (3.4.9a) \]

PR EOS:
\[ m_i = \begin{cases} 0.374640 + 1.54226\sigma_i - 0.26992\sigma_i^2, & \text{if } \sigma_i \leq 0.49 \\ 0.379642 + 1.48503\sigma_i - 0.164423\sigma_i^2 + 0.016666\sigma_i^3, & \text{if } \sigma_i > 0.49 \end{cases} \] (3.4.9b)

3.4.2 Vapor Liquid Equilibrium

The equilibrium constant \( K_i \) play a key role during two-phase splitting prediction problems. The equilibrium constant, \( K_i \), is defined as the ratio of the mole fraction of the i-th component in the gas phase \( y_i \) to the mole fraction of the same component in the liquid phase \( x_i \), which is shown below (Ayala, 2012).

\[ K_i = \frac{y_i}{x_i} \] (3.4.10)

From a molar material balance applied to a two-phase system in equilibrium and the definition of \( K_i \), we can derive the following expression (Rachford and Rice, 1952):

\[ g(f_{ng}) = \sum_{i=1}^{n_c} \frac{c_i(K_i - 1)}{1 + f_{ng}(K_i - 1)} = 0 \] (3.4.11)

Equation (3.4.11) is widely used and known as the Rachford-Rice Objective Function. Note that if all the \( K_i \)'s values were known, the only unknown left to solve for is \( f_{ng} \).

3.4.3 The Iso-Fugacity Criteria

As mentioned in the previous section, reliable values for the equilibrium constants \( K_i \)'s must be obtained before we can solve the Rachford-Rice Objective Function. For a system to be in equilibrium, any net transfer must be zero. For this, all potentials must be the same in all the phases. Therefore, the temperature and pressure of both phases are the same and a zero net transfer for all components in the mixture results when all chemical potential are the same (Ayala, 2012). This is, the thermodynamic phase equilibrium equation is written as:

\[ f_{li} = f_{gi} \] (3.4.12)

Since “fugacity” is a measure of the potential for transfer of a component between two phases, equal fugacities of a component in both phases results in a zero net transfer. The fugacity coefficient is defined as the ratio of the fugacity of a material to its partial pressure. For a two-phase system which consists of N-components, the fugacity
coefficient of component ‘i’ for both vapor and liquid phases can be written, respectively, as:

\[
\phi_{gi} = \frac{f_{gi}}{y_i P} (i = 1,2, ..., n_c)
\]  

(3.4.13)

\[
\phi_{li} = \frac{f_{li}}{x_i P} (i = 1,2, ..., n_c)
\]  

(3.4.14)

For the case of the generalized cubic EOS, the fugacity coefficient of the i-th component in a mixture is given by: (Ayala, 2012)

\[
\ln \phi_i = -\ln(Z - B) + \frac{A}{(m_1 - m_2)B} \left( \frac{2 \sum_{j=1}^{n_c} A_{ij} c_j}{A} - \frac{B_i}{B} \right) \ln \left[ \frac{Z + m_2 B}{Z + m_1 B} + \frac{B_i}{B} (Z - 1) \right]
\]  

(3.4.15)

Equation (3.4.15) shows that the fugacity of the components in each phase can be introduced to develop a criterion for thermodynamic equilibrium. This is the fact that SSM (Successive Substitution Method) takes advantage of. It can be shown that Ki-values are related to fugacity coefficients as follows:

\[
K_i = \frac{\phi_{li}}{\phi_{gi}} = \frac{y_i}{x_i} \left( \frac{f_{li}}{f_{gi}} \right)
\]  

(3.4.16)

Using the above equation, the correction step formulation to find K-values in SSM is written as:

\[
K_i^{n+1} = \left( \frac{y_i}{x_i} \right)^n \left( \frac{f_{li}}{f_{gi}} \right)^n
\]  

(3.4.17)

\[
K_i^{n+1} = K_i^n \left( \frac{f_{li}}{f_{gi}} \right)^n
\]  

(3.4.18)

What SSM does is to basically update all previous equilibrium ratios using the fugacities predicted by the equation of state. This iteration method requires an initial estimation of Ki-values, where the Wilsons’s correlation is used as following:

\[
K_i = \frac{1}{P_{ri}} EXP \left[ 5.37 (1 + \omega_i) \left( 1 - \frac{1}{T_{ri}} \right) \right]
\]  

(3.4.19)
It can be easily concluded that the convergence criteria will be satisfied whenever the fugacity ratios of all components in the system are close to unity. Such condition is achieved when the following inequality is satisfied:

\[
\sum_{i}^{n} \left( \frac{f_{li}}{f_{gi}} - 1 \right)^2 < 10^{-14}
\]

**3.4.4 Phase Determine Model**

The most difficult aspect of two-phase equilibrium calculation is to determine whether or not a mixture will actually split into two phases for a pressure and temperature condition. Several approaches may be used here: bring-back technique outlined by Risnes et al. (1981), phase stability criteria introduced by Michelsen (1982), among others.

The procedure for Michelsen’s stability test is below.

1. Calculate the mixture fugacity using overall composition.
2. Create a vapor-like second phase.
   a. Use Wilson’s correlation to obtain initial Ki-values.
   b. Calculate second-phase mole numbers:
      \[
      Y_i = z_i K_i
      \]  
      (3.4.20)
   c. Obtain the sum of the mole numbers:
      \[
      S_v = \sum_{i}^{n} Y_i
      \]  
      (3.4.21)
   d. Normalize the second-phase mole numbers to get mole fractions:
      \[
      y_i = \frac{Y_i}{S_v}
      \]  
      (3.4.22)
   e. Calculate the second-phase fugacity using the corresponding EOS and the previous composition.
   f. Calculate corrections for the Ki-values:
\[ R_i = \frac{f_{zi}}{f_{yi} S_v} \]  \hspace{1cm} (3.4.23)

\[ K_i^{(n+1)} = K_i^{(n)} R_i \]  \hspace{1cm} (3.4.24)

**g. Check if:**

**g.1) Convergence is achieved:**

\[ \sum_{i}^n (R_i - 1)^2 < 10^{-10} \]

**g.2) A trivial solution is approached:**

\[ \sum_{i}^n (\ln K_i)^2 < 10^{-4} \]

If a trivial solution is approached, stop the procedure. If convergence has not been attained, use the new Ki-values and go back to step (b).

**3. Create a liquid-like second phase.**

Follow the same previous steps by replacing equations (3.4.20), (3.4.21), (3.4.22), and (3.4.23) by (3.4.25), (3.4.26), (3.4.27) and (3.4.28) respectively.

\[ Y_i = \frac{z_i}{K_i} \]  \hspace{1cm} (3.4.25)

\[ S_l = \sum_{i}^n Y_i \]  \hspace{1cm} (3.4.26)

\[ x_i = \frac{Y_i}{S_l} \]  \hspace{1cm} (3.4.27)

\[ R_i = \frac{f_{xi}}{f_{zi}} S_l \]  \hspace{1cm} (3.4.28)

**4. Single-phase/two-phase conditions detection.**

Mixture is stable and single-phase condition prevails if: both tests yields \( S \leq 1 \) (\( S_v < 1 \) and \( S_l < 1 \)), or both tests converge to trivial solution, or one test converges to trivial solution and the other gives \( S \leq 1 \). Only one test indicating
$S > 1$ is sufficient to determine that the mixture is unstable and that the two-phase condition prevails.

### 3.4.5 Phase Property Prediction

1. Molecular weight of the liquid and gas phases.

   The molecular weight of each of the phases is calculated as a function of the molecular weight of the individual components, once the composition of the gas and the liquid are known:

   \[ MW_g = \sum_{i=1}^{n} y_i MW_i \]  
   \[ MW_l = \sum_{i=1}^{n} x_i MW_i \]

2. Density of the liquid and gas phases.

   The density of the phase is calculated using its compressibility factor as predicted by the corresponding equation of state. From the real gas law, the density is expressed as:

   \[ \rho_g = \frac{P}{RT} \left( \frac{MW_g}{Z_g} \right) \]  
   \[ \rho_l = \frac{P}{RT} \left( \frac{MW_l}{Z_l} \right) \]


   Lee, Gonzalez and Eakin (1966) presented the following correlation for the calculation of the viscosity of a natural gas:

   \[ \mu_g = 10^{-4} k_v EXP \left( x_v \left( \frac{\rho_g}{62.4} \right)^{y_v} \right) \]

   Where,

   \[ k_v = \frac{(9.4 + 0.02 MW_g)T^{1.5}}{209 + 19 MW_g + T} \]
\[ y_v = 2.4 - 0.2x_v \]  
\[ x_v = 3.5 + \frac{986}{T} + 0.01MW_g \]

In this expression, temperature is given in (R), the density of the fluid in (lbm/ft\(^3\)), and the resulting viscosity is expressed in centipoises (cp).


Lohrenz, Bray and Clark (1964) proposed an empirical correlation for the prediction of the viscosity of a liquid hydrocarbon mixture from its composition.

\[ \mu_l = \mu^\ast + \xi_m^{-1}[(0.1023 + 0.023364\rho_r + 0.058533\rho_r^2 - 0.040758\rho_r^3 \\
+ 0.0093324\rho_r^4)^4 - 10^{-4}] \]  
(3.4.37)

For the viscosity of the mixture at atmospheric pressure, Lohrentz et al. (1964) suggested to use the Herning & Zipperer equation:

\[ \mu^\ast = \frac{\sum_i x_i \mu^\ast_i \sqrt{MW_i}}{\sum_i x_i \sqrt{MW_i}} \]  
(3.4.38)

\[ \mu^\ast_i = \frac{34 \times 10^{-5}T_{ri}^{0.94}}{\xi_i} \text{ for } T_{ri} \leq 1.5 \]  
(3.4.39a)

\[ \mu^\ast_i = \frac{17.78 \times 10^{-5}(4.58T_{ri} - 1.67)^{0.625}}{\xi_i} \text{ for } T_{ri} > 1.5 \]  
(3.4.39b)

While for the mixture viscosity parameter, Lohrentz et al. (1969) presented the following expression:

\[ \xi_i = \frac{5.4402T_{ci}^{1/6}}{\sqrt{MW_i p_{ci}^{2/3}}} \]  
(3.4.40)

And for the reduced density of the liquid mixture, from Lohrentz et al. (1969) work it is derived:

\[ \rho_r = \frac{\rho_l}{\rho_{pc}} = \left(\frac{\rho_l}{MW_i}\right)V_{pc} \]  
(3.4.41)

Where,
\[ T_{pc} = \sum x_i T_{ci} \quad (3.4.42) \]
\[ P_{pc} = \sum x_i P_{ci} \quad (3.4.43) \]
\[ V_{pc} = \sum x_i V_{ci} \quad (3.4.44) \]

### 3.5 Tee Junction Split Model

In a typical T-junction, the different pipe sections making up the tee junction can be identified as inlet pipe, run pipe and the branch pipe as shown in Figure 3.5-1.

![Figure 3.5-1 T-junction for split factor description](image)

A convenient way to evaluate the split phenomenon at tees is through the definition of split factor. Split factors are defined as the ratio of the amount of phase leaving one of the outlets to the amount of phase that originally entered the T-junction. Obviously, the split factor can range from 0 to 1 and can be defined for both the liquid and gas phases. The definition formula for split factors at the branch and run pipe in a mass basis are given as follows (Essenfeld, 2010):

\[ \lambda_{LB, mass} = \frac{m_{LB}}{m_{Lin}} , \quad \lambda_{GB, mass} = \frac{m_{GB}}{m_{Gin}} \quad (3.5.1, 3.5.2) \]
\[ \lambda_{LR, mass} = \frac{m_{LR}}{m_{Lin}} , \quad \lambda_{GR, mass} = \frac{m_{GR}}{m_{Gin}} \quad (3.5.3, 3.5.4) \]

Note that:

\[ \lambda_{LB, mass} + \lambda_{LR, mass} = 1 \quad (3.5.5) \]
\[ \lambda_{GB, mass} + \lambda_{GR, mass} = 1 \quad (3.5.6) \]
But

\[ \lambda_{LB,\text{mass}} + \lambda_{GB,\text{mass}} \neq 1 \]
\[ \lambda_{LR,\text{mass}} + \lambda_{GR,\text{mass}} \neq 1 \]

### 3.5.1 Even Split Model

The even split model can be expressed to mean that the liquid mass intake fraction in the branch pipe will be equal to the gas mass intake fraction in the branch pipe, as implied by the following expression:

\[ \lambda_{LB,\text{mass}} = \lambda_{GB,\text{mass}} \]  \hspace{1cm} (3.5.7)
\[ \lambda_{LR,\text{mass}} = \lambda_{GR,\text{mass}} \]  \hspace{1cm} (3.5.8)

This scenario implies that the T-junction split are uniform and that they always occur at equal proportions, neglecting phase inertia and well-established uneven distribution phenomena at T-junction.

### 3.5.2 Kinetic Energy Ratio Split Model

The kinetic energy ratio split model is established based on the experimental observations that the tendency of the flow path of the heavy phase (liquid) is dominated by the relative inertia of the phases. The kinetic energy ratio quantifies this relative inertial effect as shown below (Essenfeld, 2010):

\[ KE = \frac{\frac{1}{2} \rho_g v_g^2}{\frac{1}{2} \rho_l v_l^2} \]  \hspace{1cm} (3.5.9)

The split model indicates that the liquid split factor of the branch pipe is related to the gas split factor of the branch pipe and the kinetic energy ratio of the inlet pipe.

\[ \lambda_{LB,\text{mass}} = \lambda_{GB,\text{mass}} * KE_{\text{inlet}} \]  \hspace{1cm} (3.5.10)

### 3.5.3 Double Stream Model

The two-phase flow split is affected by at least the following physical quantities and geometrical parameters in Hart’s study (1991):

1. Mass flow rates of gas and liquid in the inlet of the junction
2. Densities and viscosities of gas and liquid
3. Surface tension and gas-liquid-solid contact angle
4. Diameters of inlet, run and branch
5. Inclination angles of main pipe and side-arm
6. Angle of the junction
7. Radius at connection between main pipe and branch.

The double stream model is based on the assumption that in the inlet pipe, both gas flow and liquid flow arriving at the T-junction can be split into two streams. Therefore, in the area of the junction the following four streams can be distinguished:

1. Inlet-to-run gas flow
2. Inlet-to-run liquid flow
3. Inlet-to-branch gas flow
4. Inlet-to-branch liquid flow

![Figure 3.5.3-1](image)

**Figure 3.5.3-1** Schematic representation of four streamlines in the gas-liquid flow through a regular dividing T-junction elucidating the double stream model. Streamlines refer to: (A) inlet-to-run gas flow. (B) inlet-to-run liquid flow. (C) inlet-to-branch gas flow. (D) inlet-to-branch liquid flow.

Also, in the double stream model, the characteristic equation that governs the uneven split phenomenon is developed based on the Bernoulli equation for single-phase flow which is called “extended Bernoulli equation” as following:
For the inlet-to-run flow as:

\[(P_1 - P_2) + \frac{1}{2} \rho (w_1^2 - w_2^2) + \rho g(z_1 - z_2) = k_{12} \frac{1}{2} \rho w_1^2 \]  \hspace{1cm} (3.5.11)

For the inlet-to-branch flow as:

\[(P_1 - P_3) + \frac{1}{2} \rho (w_1^2 - w_3^2) + \rho g(z_1 - z_3) = k_{13} \frac{1}{2} \rho w_1^2 \]  \hspace{1cm} (3.5.12)

where, the subscripts 1, 2 and 3 represent the inlet pipe, run pipe and the branch pipe respectively.

Subtraction of equation (3.5.11) from equation (3.5.12) and write for gas phase and liquid phase separately results in:

\[(P_2 - P_3)_G + \frac{1}{2} \rho_G (w_{G2}^2 - w_{G3}^2) + \rho_G g(z_{G2} - z_{G3}) = \frac{1}{2} \rho_G w_{G1}^2 (k_{13} - k_{12}) \]  \hspace{1cm} (3.5.13)

\[(P_2 - P_3)_L + \frac{1}{2} \rho_L (w_{L2}^2 - w_{L3}^2) + \rho_L g(z_{L2} - z_{L3}) = \frac{1}{2} \rho_L w_{L1}^2 (k_{13}' - k_{12}') \]  \hspace{1cm} (3.5.14)

Another assumption in double stream model is that the pressure differences between the branch pipe and run pipe are equal for both gas phase and liquid phase:

\[(P_2 - P_3)_G = (P_2 - P_3)_L \]  \hspace{1cm} (3.5.15)

Using the definition and formulas for kinetic energy ratio (Equ. 3.5.9), mass intake fraction of gas and liquid in branch and run (Equ. 3.5.1 – 3.5.4), Froude number (Equ. 3.3.4), and the approximation below:

Under low liquid-holdup conditions:

\[\epsilon_{L1} = \epsilon_{L2} = \epsilon_{L3} \]  \hspace{1cm} (3.5.16)

\[z_{G2} = z_{G3} \]  \hspace{1cm} (3.5.17)

\[z_{L3} = z_{L2} + \frac{1}{2} (D_1 - D_3) \]  \hspace{1cm} (3.5.18)

\[k_{13}' - k_{12}' = k_{13} - k_{12} \]  \hspace{1cm} (3.5.19)

We can get the general expression for the double stream model:
\[(a^4 - 1)\lambda_L^2 + 2\lambda_L - KE[(a^4 - 1)\lambda_G^2 + 2(\lambda_G - \lambda_0)] + \frac{1}{\beta_L F_{r_{13}}} - 2\lambda_0 = 0 \quad (3.5.20)\]

where, \(a = \frac{D_1}{D_3}\), \(F_{r_{13}} = \frac{v_{11}^2}{g(D_1 - D_3)}\).

\[k_{12} = 0.03(1 - \lambda_G)^2 + 0.35\lambda_G^2 - 0.2\lambda_G(1 - \lambda_G) \quad (3.5.21)\]

\[k_{13} = 0.95(1 - \lambda_G)^2 + \lambda_G^2 \left\{1.3\tan\left(\frac{\phi}{2}\right) - 0.3 + (0.4a^2 - 0.1)a^2 \right\}(1 - 0.9a\sqrt{r})\]

\[+ 0.4\lambda_G(1 - \lambda_G)(1 + a^2)\tan\left(\frac{\phi}{2}\right) \quad (3.5.22)\]

\[\lambda_0 = \frac{1}{2}(1 + k_{12} - k_{13}) \quad (3.5.23)\]

For a regular T-junction:

\[\lambda_L = \lambda_0 + KE(\lambda_G - \lambda_0) \quad (3.5.24)\]

### 3.6 Mass Balance Equations for Retrograde Gas

Based on the definition of the split factor of the branch pipe,

\[
\lambda_{LB,\text{mass}} = \frac{m_{LB}}{m_{Lin}} \quad (3.6.1)
\]

\[
\lambda_{GB,\text{mass}} = \frac{m_{GB}}{m_{Gin}} \quad (3.6.2)
\]

We may apply the following relationship between mass rate and molar rate into the mass intake fraction.

\[
m_G = n_G \cdot MW_G \quad (3.6.3)
\]

\[
m_L = n_L \cdot MW_L \quad (3.6.4)
\]

So, we can get the equations for molar intake fraction:

\[
\lambda_{LB,\text{mole}} = \frac{n_{LB}}{n_{Lin}} = \lambda_{LB,\text{mass}} \frac{MW_{Lin}}{MW_{LB}} \quad (3.6.5)
\]

\[
\lambda_{GB,\text{mole}} = \frac{n_{GB}}{n_{Gin}} = \lambda_{GB,\text{mass}} \frac{MW_{Gin}}{MW_{GB}} \quad (3.6.6)
\]

By definition, the composition is given as:
\[ z_{ij} = \frac{\text{Mole of liquid into the pipe}}{\text{Total mole into the pipe}} x_i^{in} + \frac{\text{Mole of gas into the pipe}}{\text{Total mole into the pipe}} y_i^{in} \quad (3.6.7) \]

where, \( i \) represent the component \( i \), and \( j \) represent the pipe \( j \).

The mole fraction of liquid in the branch is defined as:

\[ \frac{\text{Mole of liquid into branch}}{\text{Total mole into branch}} = \frac{n_{LB}}{n_{LB} + n_{GB}} = \frac{\lambda_{LB,\text{mole}} n_{Lin}}{\lambda_{LB,\text{mole}} n_{Lin} + \lambda_{GB,\text{mole}} n_{Gin}} \quad (3.6.8) \]

Substituting equation 3.6.5, 3.6.6 and 3.6.8 into equation 3.6.7 given:

\[ z_{ij} = \frac{\lambda_{LB,\text{mole}} n_{Lin}}{\lambda_{LB,\text{mole}} n_{Lin} + \lambda_{GB,\text{mole}} n_{Gin}} x_i^{in} + \frac{\lambda_{GB,\text{mole}} n_{Gin}}{\lambda_{LB,\text{mole}} n_{Lin} + \lambda_{GB,\text{mole}} n_{Gin}} y_i^{in} \quad (3.6.9) \]

Since, for the branch pipe in the T-junction,

\[ n_j^T = \lambda_{LB,\text{mole}} n_{Lin} + \lambda_{GB,\text{mole}} n_{Gin} \quad (3.6.10) \]

\[ n_{ij}^T = n_j^T * z_{i,j} = \left( \lambda_{LB,\text{mole}} n_{Lin} + \lambda_{GB,\text{mole}} n_{Gin} \right) \]

\[ \times \left( \frac{\lambda_{LB,\text{mole}} n_{Lin}}{\lambda_{LB,\text{mole}} n_{Lin} + \lambda_{GB,\text{mole}} n_{Gin}} x_i^{in} + \frac{\lambda_{GB,\text{mole}} n_{Gin}}{\lambda_{LB,\text{mole}} n_{Lin} + \lambda_{GB,\text{mole}} n_{Gin}} y_i^{in} \right) \quad (3.6.11) \]

\[ n_{ij}^T = \lambda_{LB,\text{mole}} * n_{Lin} * x_i^{in} + \lambda_{GB,\text{mole}} * n_{Gin} * y_i^{in} \quad (3.6.12) \]

According to the material balance, we can get,

\[ n_j^T = \sum_{i=1}^{n_c} n_{i,j}^T \quad (3.6.13) \]

For the whole system, we can write mass balance equations of each component for each node,

\[ n_{i,in} - n_{i,out} + n_{i,\text{Supply}} - n_{i,\text{Demand}} = 0 \quad (3.6.14) \]

Combine with the equations we get from the split model for T-junction, we can solve for the \( n_i \) for each pipe. Figure 3.6-1 is an example of the equation for solving \( n_i \)'s of each pipe for case #1 network.
3.7 Multiple supplies of retrograde gas

For a network system with multiple supplies of retrograde gas which means the composition of the gas injected into the system are different at different injected nodes, to determine the points where the mix happens and to calculate the composition of the mixture is a new task for the network simulation. In this study, after the single-phase calculation, we use the known flow directions to evaluate single supply or multiple supplies for each pipe starting with the injected pipes. Then, for single supply pipes, the compositions of the fluids are the same with those of the supply retrograde gas. While, for multiple supplies pipes, the mixing rule will be applied in the composition calculation. The model developed for a given pipe is shown below:

\[
z_i = \sum_{j=1}^{n_{\text{supply}}} \left( \frac{C_j (P_{up}^2 - P_{dn}^2)^n}{q_{sc}} \right) z_{i,j}
\]  

(3.7.1)

where \(z_i\) is the molar fraction of the component \(i\), \(C_j\) is the conductivity of the supply gas \(j\), \(z_{i,j}\) is the molar fraction of the component \(i\) in supply gas \(j\), \(q_{sc}\) is the overall volumetric flow rate of the pipe at standard condition. This model is only used to get the initial overall composition of each pipe after single-phase calculation. After the split model selection, the overall composition will be recalculated using the mass balance equations combined with the split factors. The difference between single supply and multiple supplies systems simulation is that the overall compositions should be updated for multi-
supply system while those are the same for single-supply system when even split model is selected.

3.8 Model Integration

The main goal for this study is to combine the phase behavior models to the multiphase pipeline system simulation, converting the gas-water system simulation into a retrograde gas system model. By incorporating proper split model and material balance equations, the model would be capable to predict the phases’ distribution in the entire network.

First part of the proposed model is the information of the definition of the network connectivity and the properties of the inlet retrograde gas (as required by the input file and shown in APPENDIX B). Then, we determine the initial nodal pressures using the linear analog algorithm and apply them into the Weymouth gas flow equations and the iterative protocol of Newton Raphson. After the tee junction identification, a multiple supplies model is selected based on the input information. Then, with the data of the fluid properties in each pipe and the nodal pressure, phase behavior model is solved and the phases’ distribution is accomplished. In general, we calculate the flow rates of each phase in each pipe with Even Split model and apply them into Beggs and Brill module to determine the two-phase pressure drop and the new pipe conductivities used for mass flow rate calculation. After iterative calculation using Newton Raphson protocol, new pressure distribution is obtained. Then, three options are made available to handle the split at the tee junction: Even Split, Kinetic Energy Split and Double Stream Model (DSM) that will run according to the user’s input. Note that for the Even Split model, the overall compositions of the fluid in each pipe are the same for single-supply system, but for multi-supply system, the overall compositions should be recalculated using the updated nodal pressures. While, for Kinetic Energy Split model and Double Stream Model, the overall compositions will be solved using the mass balance equations both for single-supply and multi-supply systems. Once the phases’ distribution and properties are invoked, Beggs and Brill model is used to get the pressure drop and the new pipe conductivities. The resulting system of equations is solved using Newton Raphson subroutine and an iterative process. Each time new pressures are found, the system will recalculate fluid properties and pipe conductivities until pressure convergence is
achieved. The whole program flow chart is shown in the Figure 3.8-1. With the entire model integrated and compared with GASNET in Essenfeld’s thesis (2010) as verification reference for gas-water system simulation in Chapter 4, the capabilities of the proposed model were tested using the cases discussed in Chapter 5.
Figure 3.8-1 Flow chart for the proposed model
CHAPTER 4

MODEL VERIFICATION

Since experimental data and model simulation results for retrograde gas pipeline systems are limited, and in order to demonstrate the reliability and accuracy of this study, we choose GASNET from Essenfeld’s thesis (2010) as reference benchmark for model verification. GASNET is a program for tracking liquid preferential routes in natural gas pipeline systems. The cases in this chapter are for dry gas and water distribution in the network.

4.1 Case #1

This case is the network 1 in Essenfeld’s thesis (2010). The network is depicted in Figure 4.1-1 and the operating conditions for this network are presented in Table 4.1-1.

![Figure 4.1-1 Network for Case #1 (12 nodes 13 pipes 2 loops)](image)

**Table 4.1-1 Operating conditions for Case #1 (adapted from Essenfeld, 2010)**

<table>
<thead>
<tr>
<th>Node</th>
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<th>Dem. qₖ</th>
<th>Dem. qₖ</th>
<th>P Specify</th>
<th>Demand Information</th>
</tr>
</thead>
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<td>Supply qₖ</td>
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<td>50 [MMSCFD]</td>
<td>25 [MMSCFD]</td>
<td>400 [psia]</td>
</tr>
<tr>
<td></td>
<td>Load qₖ</td>
<td>100 [STBD]</td>
<td>50 [MMSCFD]</td>
<td>25 [MMSCFD]</td>
<td>400 [psia]</td>
</tr>
<tr>
<td></td>
<td>Demand qₖ</td>
<td>100 [MMSCFD]</td>
<td>50 [MMSCFD]</td>
<td>25 [MMSCFD]</td>
<td>400 [psia]</td>
</tr>
</tbody>
</table>

| Number of Pipes | 13 | T average | 82 [F] |
| Number of Nodes | 12 | Flow efficiency | 1 |
| Number of Loops | 2 | Pipe Roughness | 0.0006 [in] |
| Supply and Demand Information | Gas Comp. Factor (Z) | 0.89 |
| Node | 5 | Supply qₖ | 100 [MMSCFD] | Gas Specific Gravity | 0.62 |
| Node | 10 | Dem. qₖ | 50 [MMSCFD] | Gas Viscosity | 0.0127 [cp] |
| Node | 11 | Dem. qₖ | 25 [MMSCFD] | Liquid Density | 62.4 [lb/ft³] |
| Node | 12 | Dem. qₖ | 25 [MMSCFD] | Liquid Viscosity | 0.8566 [cp] |
| Node | 10 | P Specify | 400 [psia] | Surface Tension | 59.6667 [dynes/cm] |
Table 4.1-1 continued Operating conditions for Case #1 (adapted from Essenfeld, 2010)

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</tbody>
</table>

Network performance results using program in this study and GASNET are presented in Figure 4.1-2 and Figure 4.1-3. Figure 4.1-2(a) shows the performance comparison when single-phase conditions are assumed throughout the network and incoming liquid rates are ignored. In the figures, the plots show pressure predictions of the network. For the single phase case, the maximum difference in pressures between this study (in black) and GASNET (in red) is found to be 0.32%. In Figure 4.1-2(b), both programs assume Even Split at all junctions, and the maximum difference between this study (in blue) and GASNET (in red) is found to be 0.03% for the pressures.

![Figure 4.1-2](image-url)

**Figure 4.1-2** Results for Single-phase gas network (a) and two-phase gas network with Even Split Model (b). Run with program of this study, GASNET and PIPESIM

In Figure 4.1-3(a), both programs assume Kinetic Energy Split at all junctions, and the maximum difference in pressures between this study (in blue) and GASNET (in red) is found to be 0.09%. In Figure 4.1-3(b), both programs apply Double Stream Model at all junctions, and the maximum difference between this study (in black) and GASNET (in red) is found to be 4.50% for the pressures.
4.2 Case #2

This case is revised from network 2 in Essenfeld’s thesis. The network is depicted in Figure 4.2-1 and the operating conditions for this network are presented in Table 4.2-1.

**Figure 4.2-1** Network for Case #2 (22 nodes 25 pipes 4 loops)
Table 4.2-1 Operating conditions for Case #2

<table>
<thead>
<tr>
<th>Operating conditions</th>
<th>Value</th>
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<tbody>
<tr>
<td>Number of Pipes</td>
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<tr>
<td>Number of Nodes</td>
<td>22</td>
</tr>
<tr>
<td>Number of Loops</td>
<td>4</td>
</tr>
<tr>
<td>T average</td>
<td>70 [F]</td>
</tr>
<tr>
<td>Flow efficiency</td>
<td>1</td>
</tr>
<tr>
<td>Pipe Roughness</td>
<td>0.0006 [in]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Supply and Demand Information</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply q&lt;sub&gt;s&lt;/sub&gt;</td>
<td>25 [MMSCFD]</td>
</tr>
<tr>
<td>Dem. q&lt;sub&gt;g&lt;/sub&gt;</td>
<td>5000 [STBD]</td>
</tr>
<tr>
<td>Dem. q&lt;sub&gt;g&lt;/sub&gt;</td>
<td>1 [MMSCFD]</td>
</tr>
<tr>
<td>Dem. q&lt;sub&gt;g&lt;/sub&gt;</td>
<td>1 [MMSCFD]</td>
</tr>
<tr>
<td>Dem. q&lt;sub&gt;g&lt;/sub&gt;</td>
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</tr>
<tr>
<td>Dem. q&lt;sub&gt;g&lt;/sub&gt;</td>
<td>6 [MMSCFD]</td>
</tr>
<tr>
<td>Dem. q&lt;sub&gt;g&lt;/sub&gt;</td>
<td>2 [MMSCFD]</td>
</tr>
<tr>
<td>Dem. q&lt;sub&gt;g&lt;/sub&gt;</td>
<td>13 [MMSCFD]</td>
</tr>
<tr>
<td>P Specify</td>
<td>80 [psia]</td>
</tr>
<tr>
<td>Gas Comp. Factor (Z)</td>
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</tr>
<tr>
<td>Gas Specific Gravity</td>
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<tr>
<td>Gas Viscosity</td>
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<tr>
<td>Liquid Density</td>
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</tr>
<tr>
<td>Liquid Viscosity</td>
<td>1 [cp]</td>
</tr>
<tr>
<td>Surface Tension</td>
<td>8.41 [dynes/cm]</td>
</tr>
</tbody>
</table>

Network performance results using program in this study and GASNET are presented in Figure 4.2-2 and Figure 4.2-3. Figure 4.2-2(a) shows the performance comparison when single-phase conditions are assumed throughout the network and incoming liquid rates are ignored. For the single phase case, the maximum difference in pressures between this study (in blue) and GASNET (in red) is found to be 0.38%. In Figure 4.2-2(b), both programs assume Even Split at all junctions, and the maximum difference between this study (in black) and GASNET (in red) is found to be 0.04% for the pressures.
Figure 4.2-2 Results for Single-phase gas network (a) and two-phase gas network with Even Split Model (b). Run with program of this study and GASNET

In Figure 4.2-3(a), both programs assume Kinetic Energy Split at all junctions, and the maximum difference in pressures between this study (in black) and GASNET (in red) is found to be 0.04%. In Figure 4.2-3(b), both programs apply Double Stream Model at all junctions, and the maximum difference between this study (in black) and GASNET (in red) is found to be 4.55% for the pressures.

Figure 4.2-3 Results for two-phase gas network: (a) Kinetic Energy Split (b) Double Stream Model. Run with program of this study and GASNET
4.3 Case #3

This case is an adaption of case #1. The main difference from case #1 is that there are two supply nodes in the network. The network is depicted in Figure 4.3-1 and the operating conditions for this network are presented in Table 4.3-1.

**Figure 4.3-1** Network for Case #3 (13 nodes 14 pipes 2 loops)

**Table 4.3-1** Operating conditions for Case #3

<table>
<thead>
<tr>
<th>Number of Pipes</th>
<th>14</th>
<th><strong>T average</strong></th>
<th>82 [F]</th>
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</thead>
<tbody>
<tr>
<td>Number of Nodes</td>
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<td>Flow efficiency</td>
<td>1</td>
</tr>
<tr>
<td>Number of Loops</td>
<td>2</td>
<td>Pipe Roughness</td>
<td>0.0006 [in]</td>
</tr>
<tr>
<td><strong>Supply and Demand Information</strong></td>
<td></td>
<td><strong>Gas Comp. Factor (Z)</strong></td>
<td>0.89</td>
</tr>
<tr>
<td>Node</td>
<td>5</td>
<td><strong>Supply qₙₜ</strong></td>
<td>50 [MMSCFD]</td>
</tr>
<tr>
<td>Node</td>
<td>5</td>
<td><strong>Supply qₙₜ</strong></td>
<td>50 [STBD]</td>
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<tr>
<td>Node</td>
<td>13</td>
<td><strong>Supply qₙₜ</strong></td>
<td>50 [MMSCFD]</td>
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<tr>
<td>Node</td>
<td>13</td>
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<td>50 [STBD]</td>
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<tr>
<td>Node</td>
<td>10</td>
<td><strong>Dem. qₙₜ</strong></td>
<td>50 [MMSCFD]</td>
</tr>
<tr>
<td>Node</td>
<td>11</td>
<td><strong>Dem. qₙₜ</strong></td>
<td>25 [MMSCFD]</td>
</tr>
<tr>
<td>Node</td>
<td>12</td>
<td><strong>Dem. qₙₜ</strong></td>
<td>25 [MMSCFD]</td>
</tr>
<tr>
<td>Node</td>
<td>10</td>
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<td>400 [psia]</td>
</tr>
<tr>
<td><strong>Pipe</strong></td>
<td>Diameter [in]</td>
<td>Length [ft]</td>
<td><strong>Pipe</strong></td>
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<td>1</td>
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<td>8</td>
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<tr>
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</tbody>
</table>

Network performance results using program in this study and GASNET are presented in Figure 4.3-2 and Figure 4.3-3. Figure 4.3-2(a) shows the performance comparison when single-phase conditions are assumed throughout the network and incoming liquid rates are ignored. In the figures, the plots show pressure predictions of the network. For the
single phase case, the maximum difference in pressures between this study (in blue) and GASNET (in red) is found to be 0.32%. In Figure 4.3-2(b), both programs assume Even Split at all junctions, and the maximum difference between this study (in blue) and GASNET (in red) is found to be 0.03% for the pressures.

**Figure 4.3-2** Results for Single-phase gas network (a) and two-phase gas network with Even Split Model (b). Run with program of this study and GASNET

In Figure 4.3-3(a), both programs assume Kinetic Energy Split at all junctions, and the maximum difference in pressures between this study (in blue) and GASNET (in red) is found to be 0.03%. In Figure 4.3-3(b), both programs apply Double Stream Model at all junctions, and the maximum difference between this study (in blue) and GASNET (in red) is found to be 3.41% for the pressures.

**Figure 4.3-3** Results for two-phase gas network: (a) Kinetic Energy Split (b) Double Stream Model. Run with program of this study and GASNET
4.4 Discussion

In this chapter, the proposed two-phase model has been tested and validated with GASNET program (Essenfeld, 2010) for the case of gas-water networks. For case #1, the results of single phase and even split model are also compared with those from PIPESIM which are shown in Essenfeld’s thesis. For the pressure distribution profile, the single phase results show the range of maximum differences was found from 0.32 to 0.38%, and the even split results show the maximum differences range from 0.03 to 0.04%. The maximum differences of kinetic energy split range from 0.03 to 0.09%, and those of double stream model range from 3.41 to 4.55%. The results for single phase, even split and kinetic split are perfect to illustrate the reliability of the proposed model. For the double stream model, the differences between the diameters of three pipes joint at T-junction were ignored in GASNET while those were considered in this study. Therefore, this may explain the difference in the results.
CHAPTER 5

CASE STUDIES AND DISCUSSIONS

Several case studies, detailed below, have been deployed for study as part of the testing and validation process of the proposed model.

5.1 Case #1: Single-pipe two-phase flow analysis

For the purpose of comparison between multi-phase flow with a single pipe and multi-phase flow in multiple pipes discussed later, case #1 represents two-phase flow in a single pipe system. The scheme for the system is shown in Figure 5.1-1.

![Scheme of the single pipe system](image)

**Figure 5.1-1** Scheme of the single pipe system

The first condition analyzed here is that the effect of gas flow rate and inlet liquid flow rate on pressure drop (Figure 5.1-2). The fluids in this condition are gas \( \gamma_g = 0.62 \) and water. As the results shown in Figure 5.1-2, with the increasing inlet liquid flow rate, the pressure drop increases at constant inlet dry gas flow rate. This figure corroborates...
natural intuition, which dictates that the pressure drop should increase if liquid flow rate increases for a given amount to gas volume. Also, the same trend can be found for different inlet gas flow rate: for a given amount of liquid flow, pressure drop increase as gas rate increases. It must be noted that the total molar rate inside the pipe is not being constrained here; that is, when either liquid or gas flow increases as the other remains constant, total mass/molar rate inside the pipe is being increased.

A second condition of interest is the case when total mass/molar amount of fluid inside the system is constrained but gas \( \gamma_g = 0.62 \) and liquid rates are allowed to change at each other’s expense. Figure 5.1-3 shows the molar flow rate of gas and liquid at different liquid-gas molar ratios \( (n_L/n_G) \) with the total molar amount of inlet fluid being maintained at 10,000 lbmol/d. Clearly, gas molar rate decreases while that of liquid increases as the liquid-gas molar ratio increases, which is exactly what would happen during retrograde condensation in a pipe. During retrograde condensation, total mass/molar rate in the pipe remains the same as condensation conditions develop but molar liquid rate would increase (as the expense of a decreasing gas molar rate) inside the pipe.

Figure 5.1-3 Molar flowrate vs. liquid-gas molar ratio at \( n_{\text{total}} = 10000 \) lbmol/d

Figure 5.1-4 shows the changes of mixture velocity with increasing liquid-gas molar ratio for liquid with different molar densities. As liquid rate increases, the mixture velocity will decrease which we can found in this figure. Figure 5.1-5 shows the changes of
mixture density with increasing liquid-gas molar ratio for liquid with different molar densities. Mixture density keeps increasing as liquid rate increases.

\[ \rho' = \text{constant} \]

\[ \frac{n_L}{n_G} \]

Figure 5.1-4 Mixture velocity vs. liquid-gas molar ratio for all given liquid molar densities at \( n_{\text{total}} = 10,000 \) lbmol/d

\[ \rho' = \text{constant} \]

\[ \frac{n_L}{n_G} \]

Figure 5.1-5 Mixture density vs. liquid-gas molar ratio for all given liquid molar densities at \( n_{\text{total}} = 10,000 \) lbmol/d (\( \rho_L = 180 \text{ lbm/ft}^3 \))
According to the multiphase pressure gradient equation developed by Beggs and Brill (1973), for a single pipe with no inclination, the pressure drop is calculated following the equation below.

\[ P_1 - P_2 = \frac{f \rho_n v_m^2}{2d} L \]  

(5.1.1)

As we found in Figure 5.1-4 and Figure 5.1-5, the mixture no-slip density will increase while the mixture velocity will decrease as liquid rate increases while friction factor changes less significantly. Therefore, the changes of two-phase pressure drop will result as a competition between mixture no-slip density and mixture velocity, resulting in the behavior displayed in Figure 5.1-6.

**Figure 5.1-6** Pressure drop (P1-P2) vs. liquid-gas molar ratio for all given liquid molar densities at \( n_{\text{total}}=10,000 \text{ lbmol/d} \)

Figure 5.1-6 shows the changes of pressure drop with increasing liquid-gas molar ratio for liquid with different molar densities. In this case, the gas molar density is not a constant, but the average gas molar density is about 0.50 lbmol/ft\(^3\). Here we study the effect that the appearance of a second-phase (“liquid”) densities ranging from 0.67 to 10 lbmol/ft\(^3\) would have on pipe pressure response. From the figure, two important observations can be found:
1. If the molar density of liquid is much larger than gas molar density (e.g., gas density of 0.50 lbmol/ft\(^3\) vs. liquid density of 10 lbmol/ft\(^3\)), **pressure drop will decrease** as more liquid is formed (i.e., liquid-gas molar ratio increases) at constant total molar rate. This is an important observation, which can be counter-intuitive at first, given the findings in Figure 5.1-2. A crucial difference is the constant mass/molar constraint being applied here. Note for that retrograde gases or near-critical fluids, liquid densities tend to be close but still larger than gas densities.

2. If the molar density of the second phase (“liquid”) is close or little larger than gas molar density (e.g., gas density of 0.50 lbmol/ft\(^3\) vs. liquid density of 0.67 lbmol/ft\(^3\)), the pressure drop will increase at low \(n_L/n_G\) values but will still decrease at high \(n_L/n_G\) ratios as liquid molar rate increases.

From the results of the second condition, both the liquid-gas molar ratio and the difference between the liquid molar density and gas molar density will affect the pressure drop of the pipe.

### 5.2 Case #2: Retrograde gas transport - 1

The retrograde gas for case #2 is from Ayala (2001). The composition and critical properties of the retrograde gas are shown in the Table 5.2-1. The binary interaction coefficients in the gas are shown in Table 5.2-2, while the phase envelope is given in Figure 5.2-1.

<table>
<thead>
<tr>
<th>Comp.</th>
<th>(z_i)</th>
<th>(T_{ci} (\text{R}))</th>
<th>(P_{ci} (\text{psia}))</th>
<th>(MW_i)</th>
<th>(w_i)</th>
<th>(V_{ci} (\text{lbm/ft}^3))</th>
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<tr>
<td>Lumped-1</td>
<td>76.58</td>
<td>341.80</td>
<td>665.47</td>
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<td>0.09817</td>
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<tr>
<td>Lumped-2</td>
<td>22.40</td>
<td>630.80</td>
<td>659.73</td>
<td>39.368</td>
<td>0.13420</td>
<td>0.07460</td>
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<tr>
<td>Lumped-3</td>
<td>1.02</td>
<td>1300.00</td>
<td>230.00</td>
<td>130.000</td>
<td>0.19000</td>
<td>0.06683</td>
</tr>
</tbody>
</table>

Table 5.2-1 Composition and critical properties of the retrograde gas (Vincent, 1988)
**Table 5.2-2** Binary interaction coefficients in the retrograde gas (Ayala, 2001)

<table>
<thead>
<tr>
<th>$k_{ij}$</th>
<th>Lumped-1</th>
<th>Lumped-2</th>
<th>Lumped-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumped-1</td>
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<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td>Lumped-2</td>
<td>0.028</td>
<td>0.000</td>
<td>0.028</td>
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<tr>
<td>Lumped-3</td>
<td>0.028</td>
<td>0.028</td>
<td>0.000</td>
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</table>

**Figure 5.2-1** Phase envelope of the retrograde gas

This case is similar to the first case in chapter 4, the only difference being the feed into the network. In chapter 4, the feed was dry gas and water while in this case, the feed is retrograde gas. The first network tested is depicted in Figure 5.2-2 while the operating conditions given in Table 5.2-3.

**Figure 5.2-2** Network for Case #2 (12 nodes 13 pipes 2 loops)
Table 5.2-3 Operating conditions for Case #2

<table>
<thead>
<tr>
<th>Number of Pipes</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Nodes</td>
<td>12</td>
</tr>
<tr>
<td>Number of Loops</td>
<td>2</td>
</tr>
<tr>
<td>Flow efficiency</td>
<td>1</td>
</tr>
<tr>
<td>Pipe Roughness</td>
<td>0.0006 [in]</td>
</tr>
</tbody>
</table>

Supply and Demand Information

<table>
<thead>
<tr>
<th>Node</th>
<th>Supply $q_g$</th>
<th>Dem. $q_g$</th>
<th>Dem. $q_g$</th>
<th>P Specify</th>
</tr>
</thead>
</table>

<table>
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</tr>
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<td>8</td>
<td>99000</td>
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<td>1</td>
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<td></td>
</tr>
</tbody>
</table>

Figure 5.2-3 Total mass flow rate at different temperatures

Figure 5.2-3 shows the total mass flow rate at different temperature. In this figure, as temperature increases, the total inlet (or out) gas flow rate will increase while the total inlet (or out) liquid flow rate will decrease. This phenomenon represents the retrograde gas phase behavior between the critical temperature and the temperature at cricondenbar. It is important to note that the total mass flow rate out of the network is a constant and equal to the total mass flow rate into the network which is a constant as well, which illustrates that this study follows the material balance.
Figure 5.2-4(a) shows the pressure distribution for each node at different temperature when even split model is applied to the T-junction. Figure 5.2-4(b) shows the molar fraction of gas phase (fng) in each pipe at different temperature. Mass flow rate of liquid phase for each pipe at different temperature are presented in Figure 5.2-5. When even split model is applied to the T-junction, as the temperature increases, the nodal pressure will increase except the fixed node, the mass flow rate of gas phase for each pipe will increase while that of liquid phase will decrease.

Figure 5.2-4(a) Pressure distribution for each node obtained from two-phase program of this study at different temperature. (b) Molar fraction of gas phase (fng) of each pipe at different temperature. (Even Split Model)

Figure 5.2-5 Mass flow rate of liquid phase distribution for each pipe obtained from two-phase program of this study at different temperature. (Even Split Model)
Figure 5.2-6(a) shows the pressure distribution for each node at different temperature when kinetic energy split model is applied to the T-junction. Figure 5.2-6(b) shows the fng of each pipe at different temperature with kinetic energy split model. Mass flow rate of liquid phase for each pipe at different temperature are presented in Figure 5.2-7. As the temperature increases, the nodal pressure will increase except the fixed node when the kinetic energy split model is applied.

Figure 5.2-6(a) Pressure distribution for each node obtained from two-phase program of this study at different temperature. (b) Molar fraction of gas phase (fng) of each pipe at different temperature. (Kinetic Energy Split Model)

Figure 5.2-7 Mass flow rate of liquid phase distribution for each pipe obtained from two-phase program of this study at different temperature. (Kinetic Energy Split Model)
Figure 5.2-8(a) shows the pressure distribution for each node at different temperature when double stream model is applied to the T-junction. Figure 5.2-8(b) shows the fng of each pipe at different temperature as double stream model applied. Mass flow rate of liquid phase for each pipe at different temperature are presented in Figure 5.2-9. As the temperature increases, the nodal pressure will increase except the fixed node when the double stream model is applied.

**Figure 5.2-8(a)** Pressure distribution for each node obtained from two-phase program of this study at different temperature. **(b)** Molar fraction of gas phase (fng) of each pipe at different temperature. (Double Stream Model)

**Figure 5.2-9** Mass flow rate of liquid phase distribution for each pipe obtained from two-phase program of this study at different temperature. (Double Stream Model)
In this case, at low temperature (T=20F), the pressure distribution for each node obtained from the two-phase program with kinetic energy split model is similar to that obtained from the program with double stream model, which is much different to that obtained from the program with even split model. But at higher temperature (T=40F, 60F, 80F), the pressure distributions for each node are similar to each other with different split models.

In Figure 5.2-4(b), Figure 5.2-6(b) and Figure 5.2-8(b), the molar fraction of gas phase (fng) of each pipe as kinetic energy split model and double stream model applied are dramatically different to that as even split model applied. And the differences between the fng with different split model are increasing as the temperature decreases.

From the comparison of pressure distribution and molar fraction of gas phase, the results from double stream model are similar to those from kinetic energy split model which illustrate that double stream model is developed based on kinetic energy approach.

As indicated before, the fng represents the liquid-gas molar ratio in each pipe which can affect the pressure drop of the pipe. As the temperature increases, the fng is increasing and the liquid-gas molar ratio will decrease. And for retrograde gas, the molar density of liquid phase is close to that of gas phase. According to the observations in case #1, the pressure drop should be increasing as temperature increases which can be found from Figure 5.2-4(a), Figure 5.2-6(a) and Figure 5.2-8(a).

In this case, the mass flowrates of liquid in the pipe #12 and pipe #13 are almost the same no matter which split model is selected. Therefore, there should be very little liquid flow in the pipe #2. As even split model applied, the mass flowrate of liquid in pipe #2 is about 50-100 Mlbm/d in Figure 5.2-5, while that is about 25-70 Mlbm/d with kinetic energy split model which is shown in Figure 5.2-7. Figure 5.2-9 shows that the mass flowrate of liquid in pipe #2 is almost zero as double stream model applied. Thus, double stream model predicts the results which are close to the reality mostly. The double stream model is considered as better approximations to the real world than the even split model and kinetic energy model.

Figure 5.2-10 and Figure 5.2-11 show the overall composition of the fluid in each pipe at different temperature. The split model in Figure 5.2-10 is kinetic energy split model
while that in Figure 5.2-11 is double stream model. Since the overall composition are the same as the composition of inlet fluid when the even split model applied, the overall composition distribution for even split model is not shown in this report.

Through the comparison between Figure 5.2-10 and Figure 5.2-11, the overall compositions of the fluid in each pipe change with increasing temperature, which are also different using different split models. Since these changes are reasonable and necessary in the real operations, the model in this study can capture the changes of the fluid composition which illustrate the reliability of this model to some extent. Also, the compositions of the phases can be predicted from this model which is important for retrograde gas pipeline operation.
Figure 5.2-10 Overall composition of fluid in each pipe obtained from two-phase program of this study at different temperature.
(Kinetic Energy Split Model)

Figure 5.2-11 Overall composition of fluid in each pipe obtained from two-phase program of this study at different temperature.
(Double Stream Model)
5.3 Case #3: Retrograde gas transport - 2

This case is adapted from the second case in chapter 4. The differences between them are the demand flow rates at the outlet nodes as well as the fixed nodal pressure. Additionally, the substances injected into the network are the same as those in the case #1. The network of this case is shown in Figure 5.3-1. And the operating conditions for this network are presented in Table 5.3-1.

![Network for Case #3](image)

**Figure 5.3-1** Network for Case #3 (22 nodes 25 pipes 4 loops)

**Table 5.3-1** Operating conditions for Case #3

<table>
<thead>
<tr>
<th>Number of Pipes</th>
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<tbody>
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<td>Pipe Roughness</td>
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</table>

<table>
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Table 5.3-1 continued Operating conditions for Case #3

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</table>

**Figure 5.3-2** Total mass flow rates at different temperatures

Figure 5.3-2 shows the total mass flow rate at different temperature. In this figure, as temperature increases, the total inlet (or out) gas flow rate will increase while the total inlet (or out) liquid flow rate will decrease. This phenomenon represents the retrograde gas phase behavior between the critical temperature and the temperature at cricondenbar. It is important to note that the total mass flow rate out of the network is a constant and equal to the total mass flow rate into the network which is a constant as well, which illustrates that this study follows the material balance.

Figure 5.3-3(a) shows the pressure distribution for each node at different temperature when even split model is applied to the T-junction. Figure 5.3-3(b) shows the fng in each pipe at different temperature. Mass flow rate of liquid phase for each pipe at different temperature are presented in Figure 5.3-4. When even split model is applied to the T-
junction, as the temperature increases, the nodal pressure will increase except the fixed node, the mass flow rate of gas phase for each pipe will increase while that of liquid phase will decrease.

![Graph](image)

**Figure 5.3-3(a)** Pressure distribution for each node obtained from two-phase program of this study at different temperature. **(b)** Molar fraction of gas phase (fng) of each pipe at different temperature. (Even Split Model)

![Graph](image)

**Figure 5.3-4** Mass flow rate of liquid phase distribution for each pipe obtained from two-phase program of this study at different temperature. (Even Split Model)

Figure 5.3-5(a) shows the pressure distribution for each node at different temperature when kinetic energy split model is applied to the T-junction. Figure 5.3-5(b) shows the fng of each pipe at different temperature with kinetic energy split model. Mass flow rate of liquid phase for each pipe at different temperature are presented in Figure 5.3-6. As the
temperature increases, the nodal pressure will increase except the fixed node when the kinetic energy split model is applied.

Figure 5.3-5(a) Pressure distribution for each node obtained from two-phase program of this study at different temperature. (b) Molar fraction of gas phase ($f_{ng}$) of each pipe at different temperature. (Kinetic Energy Split Model)

Figure 5.3-6 Mass flow rate of liquid phase distribution for each pipe obtained from two-phase program of this study at different temperature. (Kinetic Energy Split Model)

Figure 5.3-7(a) shows the pressure distribution for each node at different temperature when double stream model is applied to the T-junction. Figure 5.3-7(b) shows the $f_{ng}$ of each pipe at different temperature as double stream model applied. Mass flow rate of liquid phase for each pipe at different temperature are presented in Figure 5.3-8. As the
temperature increases, the nodal pressure will increase except the fixed node when the double stream model is applied.

Figure 5.3-7(a) Pressure distribution for each node obtained from two-phase program of this study at different temperature. (b) Molar fraction of gas phase (fng) of each pipe at different temperature. (Double Stream Model)

Figure 5.3-8 Mass flow rate of liquid phase distribution for each pipe obtained from two-phase program of this study at different temperature. (Double Stream Model)

In this case, the pressure distribution for each node obtained from the two-phase program with kinetic energy split model is similar to that obtained from the program with double stream model, which is a few different to that obtained from the program with even split model, especially at the inlet node (node #1).
In Figure 5.3-3(b), Figure 5.3-5(b) and Figure 5.3-7(b), the molar fraction of gas phase (fng) of each pipe as kinetic energy split model and double stream model applied are dramatically different to that as even split model applied. And the differences between the fng with different split model are increasing as the temperature decreases.

From the comparison of pressure distribution and molar fraction of gas phase, the results from double stream model are similar to those from kinetic energy split model, which are more realistic than the results from even split approach.

Same as that found in case #2, as the temperature increases, the fng is increasing and the liquid-gas molar ratio will decrease. The pressure drop is increasing as temperature increases since the molar density of liquid phase is similar to that of gas phase, which are shown in Figure 5.3-3(a), Figure 5.3-5(a) and Figure 5.3-7(a).

5.4 Case #4: Handling of multiple retrograde gases

This case is similar to the third case in chapter 4. The only difference between them is the substances injected into the network. In chapter 4, the injection fluid is dry gas and water as that is retrograde gas in this chapter. In addition, there are two supply nodes for this system, and the injection fluids at the nodes are different. The composition of the supply fluids are shown in Table 5.4-1 and the phase behavior envelopes of these two fluids are presented in Figure 5.4-1. The network tested is depicted in Figure 5.4-2. And the operating conditions for this network are presented in Table 5.4-2.

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<td>Lumped-3</td>
<td>0.1020</td>
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Figure 5.4-1 Phase envelope of the injected fluids
**Figure 5.4-2** Network for Case #4 (13 nodes 14 pipes 2 loops)

**Table 5.4-2** Operating conditions for Case #4

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<th>Number of Pipes</th>
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<td>Pipe Roughness</td>
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</table>

**Figure 5.4-3** Total mass flow rate at different temperature
Figure 5.4-3 shows the total mass flow rate at different temperature. In this figure, as temperature increases, the total inlet gas flow rate and the total out gas flow rate will increase while the total inlet liquid flow rate and the total out liquid flow rate will decrease. Also, the total mass flow rate out the network is a constant and equal to the total mass flow rate into the network which illustrates that this study follows the material balance.

Figure 5.4-4(a) shows the pressure distribution for each node at different temperature when even split model is applied to the T-junction. Figure 5.4-4(b) shows the fng in each pipe at different temperature. Mass flow rate of liquid phase for each pipe at different temperature are presented in Figure 5.4-5. When even split model is applied to the T-junction, as the temperature increases, the nodal pressure will increase except the fixed node, the mass flow rate of gas phase for each pipe will increase while that of liquid phase will decrease.

Figure 5.4-4(a) Pressure distribution for each node obtained from two-phase program of this study at different temperature. (b) Molar fraction of gas phase (fng) of each pipe at different temperature. (Even Split Model)
Figure 5.4-5 Mass flow rate of liquid phase distribution for each pipe obtained from two-phase program of this study at different temperature. (Even Split Model)

Figure 5.4-6(a) shows the pressure distribution for each node at different temperature when kinetic energy split model is applied to the T-junction. Figure 5.4-6(a) shows the fng of each pipe at different temperature with kinetic energy split model. Mass flow rate of liquid phase for each pipe at different temperature are presented in Figure 5.4-7. As the temperature increases, the nodal pressure will increase except the fixed node when the kinetic energy split model is applied.

Figure 5.4-6(a) Pressure distribution for each node obtained from two-phase program of this study at different temperature. (b) Molar fraction of gas phase (fng) of each pipe at different temperature. (Kinetic Energy Split Model)
**Figure 5.4-7** Mass flow rate of liquid phase distribution for each pipe obtained from two-phase program of this study at different temperature. (Kinetic Energy Split Model)

Figure 5.4-8(a) shows the pressure distribution for each node at different temperature when double stream model is applied to the T-junction. Figure 5.4-8(b) shows the fng of each pipe at different temperature as double stream model applied. Mass flow rate of liquid phase for each pipe at different temperature are presented in Figure 5.4-9. As the temperature increases, the nodal pressure will increase except the fixed node when the double stream model is applied.

**Figure 5.4-8(a)** Pressure distribution for each node obtained from two-phase program of this study at different temperature. (b) Molar fraction of gas phase (fng) of each pipe at different temperature. (Double Stream Model)
Figure 5.4-9 Mass flow rate of liquid phase distribution for each pipe obtained from two-phase program of this study at different temperature. (Double Stream Model)

In this case, the pressure distribution for each node obtained from the two-phase program with kinetic energy split model is similar to that obtained from the program with double stream model, which is much different to that obtained from the program with even split model.

In Figure 5.4-4(b), Figure 5.4-6(b) and Figure 5.4-8(b), the molar fraction of gas phase ($f_{ng}$) of each pipe with different split model are dramatically different to each other. In Figure 5.4-5, Figure 5.4-7 and Figure 5.4-9, the liquid phase distribution for each pipe with different split model are also dramatically different to each other, especially in pipe #2, pipe #3, pipe #8 and pipe #14.

Since this case is a multiple supplies of retrograde gas system, the phase behavior and phase distribution will be much more complicated than those of single supply systems. Therefore, the uneven split models such as kinetic energy split model and double stream model are much more realistic than the even split model.

In this case, the overall composition are almost the same at different temperature when the even split model applied, which is shown in Table 5.4-3. Figure 5.4-10 and Figure 5.4-11 show the overall composition of the fluid in each pipe at different temperature. The split model in Figure 5.4-10 is kinetic energy split model while that in Figure 5.4-11 is double stream model.
Table 5.4-3 Overall composition of fluid in each pipe obtained from two-phase program of this study at different temperature. (Even Split Model)

<table>
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<tbody>
<tr>
<td>Lumped-1</td>
<td>0.7113</td>
<td>0.7120</td>
<td>0.7159</td>
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Through the comparison between Figure 5.4-10 and Figure 5.4-11, the overall compositions of the fluid in each pipe change with the increasing temperature, which are also different using different split models. Since these changes are reasonable and necessary in the real operations, they illustrate the realibility of uneven split models as well.
Figure 5.4-10 Overall composition of fluid in each pipe obtained from two-phase program of this study at different temperature. (Kinetic Energy Split Model)

Figure 5.4-11 Overall composition of fluid in each pipe obtained from two-phase program of this study at different temperature. (Double Stream Model)
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

A comprehensive network model has been completed with the successful development and testing, which is capable of handling complex retrograde gas pipeline networks with the flow splits at T-junctions. The developed code is able to handle and effectively model single-phase and two-phase flows in pipeline networks with single supply or multiple supplies of retrograde gas. Both even split model and uneven split models such as kinetic energy split and double stream model are incorporated in this study. For retrograde gas, the phase behavior and phase distribution are important for the network simulation. Thus, the phase behavior model is also incorporated in this study.

Developed model captures the expected pressure rise with the temperature increasing and predicts different amounts of liquid phase separation at T-junction. In addition, the model in this study can capture the changes of overall composition of the fluid in each pipe which illustrate the reliability of the developed model.

6.1 Conclusions

- The proposed two-phase model has been tested and validated with GASNET program (Essenfeld, 2010) for gas-water networks. Among three cases, the maximum differences are less than 0.4% as single-phase approach, even split approach and kinetic energy split approach, which illustrate the validation of the developed model for gas-water systems simulation. While the maximum differences range from 3.41% to 4.55% as double stream model is applied. It is mainly due to the different treatments for the pipeline diameter changes at T-junction between this model and GASNET.

- Among all the cases, the total mass flow rate out of the network is a constant and equal to the total mass flow rate into the network which is a constant as well, which illustrates that this study follows the material balance.

- Double stream model is developed based on kinetic energy split approach. Both kinetic energy split model and double stream model are considered as better
approximations to the real world than the even split model. In addition, double stream model is much more reliable than kinetic energy split model.

- Both the liquid-gas molar ratio and the difference between the liquid molar density and gas molar density will affect the pressure drop of the pipe.
- For single supply systems, the compositions of the fluids in each pipe are the same with those of the supply retrograde gas.

### 6.2 Recommendations

- Field data deployment is recommended for further tuning and validation of the proposed model.
- The T-junction split models used in this study are even split, kinetic split and double stream model. For further study, the advanced double stream model and some other newest models for impacting T-junction are recommended to be considered.
- In order to simulate the real operation conditions for the retrograde gas transmission pipelines, the operation temperatures are always between the critical point and cricondenbar point. More case studies for the temperatures between the cricondenbar point and cricondentherm point are recommended.
- This model is isothermal and only accounts for the pressure changes along the pipelines. It recommended that this model to be improved for nonisothermal systems incorporated with thermal simulation models for retrograde gas transmission networks.
BIBLIOGRAPHY


Appendix A---Code

Main code:
clear all
clc

%% Importing Data

data1=xlsread('pipe1.xlsx');
data2=xlsread('pipe2.xlsx');
data3=xlsread('gas.xlsx');

pipe=data1(:,1);  % read the number of the pipe
D=data1(:,2);      % read the diameter(inch) of the pipes
L=data1(:,3)/5280; % read the length(mile) of the pipes
angle=data1(:,4);  % read the angle(degree) of the pipes
inp=data1(:,5);    % read the number of the inlet node of each pipe
outp=data1(:,6);   % read the number of the outlet node of each pipe
node=data2(:,1);   % read the number of the node

N90=data2(:,2);    % read the number of 90 degree pipe
type=data2(:,3);   % read the type of the node (0-nonfixed, 1-fixed)

Supply=data2(:,4); % read the support flowrate(SCFD) at each node
Demand=data2(:,5); % read the demand flowrate(SCFD) at each node
Pi=data2(:,6);     % read the initial pressure(psia) at each node
int=xlsread('inletpipe.xlsx');  % read the number of the inlet pipe of each node
outt=xlsread('outletpipe.xlsx'); % read the number of the outlet pipe of each node

NPIPES=size(pipe,1);    % total number of pipes
NNODES=size(node,1);    % total number of nodes
NLOOPS=NPIPES-NNODES+1; % total number of loops

%% Specific Condition (Prompt Input)

PI=3.1415926535897;    % pi
g=32.174;               % acceleration ft/s^2
gc=32.174;             % unit conversion of force
Psc=14.7;              % Pressure (standard condition-psia)
Tsc=520;               % Temperature (standard condition-R)
fe=1.0;                % efficient

Tav=460+80;            % average temperature R
e=0.0006;              % total roughness inch
NFIX=1;                % number of fixed nodal pressure
RR=10.731;             % thermo number
MWair=28.9644;         % molecular weight of air

%% properties of gas (single-supply)

nc=3;         % number of components
load data C;  % molecular composition of the gas
load data MW; % molecular weight of the gas

MWi=C.*MW;     % molecular composition of the gas
MWt=sum(MWi);  % total molecular weight

rg=MWt/MWair;  % gas gravity
Zav=0.9;      % compressibility factor
sigma=80;     % dyne/cm^2, surface tension

%% properties of gas (multi-supply)

gas=zeros(NPIPES,1);
gas(6)=1;
gas(14)=2;
nc=3;         % number of components
C1=data3(:,1);% molecular composition of the gas
MW1=data3(:,3);% molecular weight of the gas
C2=data3(:,2); % molecular composition of the gas
MW2=data3(:,4);% molecular weight of the gas
MWi1=C1.*MW1;
MWi2=C2.*MW2;
MWi=(MWi1+MWi2)/2; % molecular weight of each component
MWt1=sum(MWi1);
MWt2=sum(MWi2);
MWt=sum(MWi); % total molecular weight
rg=MWt/MWair; % gas gravity
rg1=MWt1/MWair; % gas gravity of supply 1
rg2=MWt2/MWair; % gas gravity of supply 2
Zav=0.9; % compressibility factor
sigma=80; % dyne/cm^2, surface tension

%%%%% model selection %%%%%%
model1=1; % single phase approach: 1 for weymouth; 2 for Chen's
model2=1; % multi-supply approach: 1 for single-supply; 2 for multi-supply
model3=3; % split type: 1 for even split; 2 for kinetic energy ratio split; 3 for double stream model

%%%%% Single phase Calculation %%%%%%
S=zeros(NPIPES,1);
Leq=zeros(NPIPES,1);
for i=1:NPIPES
    if angle(i)~=0
        S(i)=0.0375*rg*L(i)*sin(angle(i))*5280/Tav/Zav; % surface factor
        Leq(i)=L(i)*(exp(S(i))-1)/S(i);
    else
        S(i)=0;
        Leq(i)=L(i);
    end
end
if model1==1; % weymouth
    n=0.5;
    m=16/3;
    if model2==1; % single-supply
        r=rg*Zav*Tav/(433.391*Tsc/Psc)^(1/n);
        CC=d.^(m*n)./(r.*Leq).^n;
        R=(r.*Leq)/d.^m;
        end
        If model2==2; % multi-supply
            r1=rg1*Zav*Tav/(433.391*Tsc/Psc)^(1/n);
            r2=rg2*Zav*Tav/(433.391*Tsc/Psc)^(1/n);
            CC1=d.^(m*n)./(r1.*Leq).^n;
            CC2=d.^(m*n)./(r2.*Leq).^n;
            end
end

%%%%% Linear Analog Algorithm %%%%%%
pratio=ones(NPIPES,1);T=zeros(NPIPES,1);
ff=zeros(NPIPES,1);Cw=zeros(NPIPES,1);
kw=0.008; % d in inch for Weymouth
for i=1:NPIPES
    ff(i)=kw/d(i)^(1/3); % Weymouth
    Cw(i)=38.784*fe*Tsc/Psc/(rg*Tav*Zav)^0.5*d(i)^2.5/ff(i)^0.5/Leq(i)^0.5;
end
p=Pi;
residual=zeros(NNODES,1);
for i=1:NPIPES
    T(i)=1;
end
error=1;
while error>0.00001
    matrix=zeros(NNODES);
    for i=1:NPIPES
        matrix(inp(i),outp(i))=T(i)*Cw(i);
        matrix(outp(i),inp(i))=T(i)*Cw(i);
        if abs(S(i))>0
            matrix(inp(i),outp(i))=exp(S(i)/2)*matrix(inp(i),outp(i));
        end
    end
    for row=1:NNODES
        for loop=1:NPIPES
            if inp(loop)==row
                matrix(row,row)=(-T(loop)*Cw(loop))+matrix(row,row);
            end
            if abs(S(loop))>0 && outp(loop)==row
                matrix(row,row)=(-exp(S(loop)/2)*Cw(loop)*T(loop))+matrix(row,row);
            elseif outp(loop)==row
                matrix(row,row)=(-Cw(loop)*T(loop))+matrix(row,row);
            end
        end
        residual(row)=(-Supply(row)+Demand(row));
        for j=1:NNODES
            if type(j)==0 && row==j % fixed node
                matrix(row,:)=0;
                matrix(row,row)=1;
            end
        end
    end
    for i=1:NNODES
        if type(i)==0  % fixed node
            residual(i)=p(i);
        end
    end
    pold=p;
    p=matrix\residual;
    deltap=p-pold;
    error=max(abs(deltap));
end
for i=1:NPIPES
    if p(inp(i))>=p(outp(i)) && p(outp(i))<0
        pratio(i)=(abs(p(inp(i))-p(outp(i)))+14.7)/14.7;
    end
    if p(inp(i))<=p(outp(i)) && p(inp(i))>=0
        pratio(i)=p(outp(i))/p(inp(i));
    end
    if p(inp(i))>=p(outp(i)) && p(outp(i))>=0
        pratio(i)=p(inp(i))/p(outp(i));
    end
    if p(inp(i))<=p(outp(i)) && p(inp(i))<0
        pratio(i)=(abs(p(outp(i))-p(inp(i)))+14.7)/14.7;
T(i)=(1+2/(pratio(i)-1))^0.5;

pLA1=p;

end

%%% Newton Raphson Analog %%%
q=zeros(NPIPES,1);NR_residual=zeros(NNODES-NFIX,1);
NR_residual2=zeros(NNODES-NFIX,1);Jac=zeros(NNODES-NFIX);
err=1;
for i=1:NNODES % if only one nodal pressure fixed
  if type(i)==0
    fixn=i;
  end
end
nin=zeros(NNODES,1);nout=zeros(NNODES,1);
for i=1:NNODES
  nin(i)=sum(inn(i,:)~=0,2);
  nout(i)=sum(outn(i,:)~=0,2);
end
while err>0.00001
  pav=zeros(NPIPES,1);
  for i=1:NPIPES
    pav(i)=2/3*(p(inp(i))+p(outp(i))-p(inp(i))*p(outp(i))/(p(inp(i))+p(outp(i)))); % psia
  end
  if model1==2; % generalized with Chen's
    n=0.5;
    m=5;
    Rhon=MWt/(Zav*RR*Tav/pav); % lbm/ft^3
    kv=(9.4+0.02*MWt)*Tav^1.5/(209+19*MWt+Tav);
    xv=3.5+986/Tav+0.01*MWt;
    yv=2.4-0.2*xv;
    Mun=0.0001*kv*exp(xv*(Rhon/62.4)^yv);
    Um=80000000/Tsc*Tav/pav/Psc*Zav*Pl^4/(d/12)^2/24/3600; % ft/sec
    Re_ns=Rhon*Um*d/12*1488.16/Mun; % nonslip Reynolds Number
    if e==0
      f=1/(2*log10(Re_ns/(4.5223*log10(Re_ns)-3.8215)))^2; % nonslip friction factor
    else
      f=4*0.008/d^(1/3);
    end
    if Re_ns<2100 && Re_ns~=0
      f=4*16/Re_ns;
    end
    if Re_ns>=2100
      T1=(e/d)/3.7065;
      T2=(e/d)^1.1098/2.8257;
      T3=5.8506/(Re_ns^0.8981);
      f=4*-(4*log10(T1-(5.0452/Re_ns)*log10(T2+T3)))^(-2);
    end
    r=rg*Zav*Tav.*f/4/(38.774^2)*(Psc/Tsc)^2;
    CC=d.^(m*n)/(r.*Leq).^n;
  end
end
for i=1:NPIPES
if abs(p(inp(i))) >= abs(p(outp(i)))
q(i) = CC(i) * (p(inp(i))^2 - p(outp(i))^2)^n;
else
q(i) = -CC(i) * (p(outp(i))^2 - p(inp(i))^2)^n;
end
end
for i=1:fixn-1
  if nin(i)==2 && nout(i)==1
    NR_residual(i)=Demand(i)-Supply(i)-q(inn(i,1))-q(inn(i,2))+q(outn(i));
  end
  if nin(i)==1 && nout(i)==2
    NR_residual(i)=Demand(i)-Supply(i)+q(outn(i,1))+q(outn(i,2))-q(inn(i));
  end
  if nin(i)==0 && nout(i)==1
    NR_residual(i)=Demand(i)-Supply(i)+q(outn(i));
  end
  if nin(i)==1 && nout(i)==0
    NR_residual(i)=Demand(i)-Supply(i)-q(inn(i));
  end
  if nin(i)==1 && nout(i)==1
    NR_residual(i)=Demand(i)-Supply(i)-q(inn(i))+q(outn(i));
  end
end
for i=fixn+1:NNODES
  if nin(i)==2 && nout(i)==1
    NR_residual(i-1)=Demand(i)-Supply(i)-q(inn(i,1))-q(inn(i,2))+q(outn(i));
  end
  if nin(i)==1 && nout(i)==2
    NR_residual(i-1)=Demand(i)-Supply(i)+q(outn(i,1))+q(outn(i,2))-q(inn(i));
  end
  if nin(i)==0 && nout(i)==1
    NR_residual(i-1)=Demand(i)-Supply(i)+q(outn(i));
  end
  if nin(i)==1 && nout(i)==0
    NR_residual(i-1)=Demand(i)-Supply(i)-q(inn(i));
  end
  if nin(i)==1 && nout(i)==1
    NR_residual(i-1)=Demand(i)-Supply(i)-q(inn(i))+q(outn(i));
  end
end
for j=1:fixn-1
  p(j)=p(j)+0.1; % perturb=0.1 psia
  for i=1:NPIPES
    if abs(p(inp(i))) >= abs(p(outp(i)))
      q(i) = CC(i) * (p(inp(i))^2 - p(outp(i))^2)^n;
    else
      q(i) = -CC(i) * (p(outp(i))^2 - p(inp(i))^2)^n;
    end
  end
end
for i=1:fixn-1
  if nin(i)==2 && nout(i)==1
    NR_residual2(i)=Demand(i)-Supply(i)-q(inn(i,1))-q(inn(i,2))+q(outn(i));
  end
  if nin(i)==1 && nout(i)==2
    NR_residual2(i)=Demand(i)-Supply(i)+q(outn(i,1))+q(outn(i,2))-q(inn(i));
  end
  if nin(i)==0 && nout(i)==1
    NR_residual2(i)=Demand(i)-Supply(i)+q(outn(i));
  end
  if nin(i)==1 && nout(i)==0
    NR_residual2(i)=Demand(i)-Supply(i)-q(inn(i));
  end
  if nin(i)==1 && nout(i)==1
    NR_residual2(i)=Demand(i)-Supply(i)-q(inn(i))+q(outn(i));
  end
end
NR_residual2(i)=Demand(i)-Supply(i)+q(outn(i,1))+q(outn(i,2))-q(inn(i));
end
if nin(i)==0 & nout(i)==1
NR_residual2(i)=Demand(i)-Supply(i)+q(outn(i));
end
if nin(i)==1 & nout(i)==0
NR_residual2(i)=Demand(i)-Supply(i)-q(inn(i));
end
if nin(i)==1 & nout(i)==1
NR_residual2(i)=Demand(i)-Supply(i)-q(inn(i))+q(outn(i));
end
diff
for i=fixn+1:NNODES
if nin(i)==2 & nout(i)==1
NR_residual2(i-1)=Demand(i)-Supply(i)-q(inn(i,1))-q(inn(i,2))+q(outn(i));
end
if nin(i)==1 & nout(i)==2
NR_residual2(i-1)=Demand(i)-Supply(i)+q(outn(i,1))+q(outn(i,2))-q(inn(i));
end
if nin(i)==0 & nout(i)==1
NR_residual2(i-1)=Demand(i)-Supply(i)+q(outn(i));
end
if nin(i)==1 & nout(i)==0
NR_residual2(i-1)=Demand(i)-Supply(i)-q(inn(i));
end
if nin(i)==1 & nout(i)==1
NR_residual2(i-1)=Demand(i)-Supply(i)-q(inn(i))+q(outn(i));
end
end
for k=1:NNODES-NFIX
Jac(k,j)=(NR_residual2(k)-NR_residual(k))/0.1; % jacobian matrix
end
p(j)=p(j)-0.1; % remove pterb
end
for j=fixn+1:NNODES
p(j)=p(j)+0.1; % perturb=0.1 psia
for i=1:NPIPES
if abs(p(inp(i)))>=abs(p(outp(i)))
q(i)=CC(i)*(p(inp(i))^2-p(outp(i))^2)^n;
else
q(i)=-CC(i)*(p(outp(i))^2-p(inp(i))^2)^n;
end
end
for i=1:fixn-1
if nin(i)==2 & nout(i)==1
NR_residual2(i)=Demand(i)-Supply(i)-q(inn(i,1))-q(inn(i,2))+q(outn(i));
end
if nin(i)==1 & nout(i)==2
NR_residual2(i)=Demand(i)-Supply(i)+q(outn(i,1))+q(outn(i,2))-q(inn(i));
end
if nin(i)==0 & nout(i)==1
NR_residual2(i)=Demand(i)-Supply(i)+q(outn(i));
end
if nin(i)==1 & nout(i)==0
NR_residual2(i)=Demand(i)-Supply(i)-q(inn(i));
end
if nin(i)==1 & nout(i)==1
NR_residual2(i)=Demand(i)-Supply(i)-q(inn(i))+q(outn(i));
end
end

NR_residual2(i)=Demand(i)-Supply(i)-q(inn(i));
end
if nin(i)==1 && nout(i)==1
    NR_residual2(i)=Demand(i)-Supply(i)+q(outn(i));
end
for i=fixn+1:NNODES
    if nin(i)==2 && nout(i)==1
        NR_residual2(i-1)=Demand(i)-Supply(i)-q(inn(i,1))-q(inn(i,2))+q(outn(i));
    end
    if nin(i)==1 && nout(i)==2
        NR_residual2(i-1)=Demand(i)-Supply(i)+q(outn(i,1))+q(outn(i,2))-q(inn(i));
    end
    if nin(i)==0 && nout(i)==1
        NR_residual2(i-1)=Demand(i)-Supply(i)+q(outn(i));
    end
    if nin(i)==1 && nout(i)==0
        NR_residual2(i-1)=Demand(i)-Supply(i)-q(inn(i));
    end
    if nin(i)==1 && nout(i)==1
        NR_residual2(i-1)=Demand(i)-Supply(i)-q(inn(i))+q(outn(i));
    end
end
for k=1:NNODES-NFIX
    Jac(k,j-1)=(NR_residual2(k)-NR_residual(k))/0.1;
end
p(j)=p(j)-0.1;
end
dp=Jac\(-NR_residual);
err=max(abs(dp));
for i=1:fixn-1
    p(i)=p(i)+dp(i);
end
for i=fixn+1:NNODES
    p(i)=p(i)+dp(i-1);
end
end
pNR1=p;
%%%%%%% Tee junction identification %%%%%%%% qg=zeros(NPIPES,1);
NPN=zeros(NNODES,1);UPN=zeros(NNODES,1);DNN=zeros(NNODES,1);
nn=zeros(NNODES,1);jun=zeros(NNODES,1);tee=zeros(NNODES,1);
for i=1:NPIPES
    if p(inp(i))>=p(outp(i))
        qg(i)=CC(i)*(p(inp(i))^2-p(outp(i))^2)^n;
    else
        qg(i)=-CC(i)*(p(outp(i))^2-p(inp(i))^2)^n;
    end
end
for i=1:NPIPES
    if qg(i)<0
        nin(inp(i))=nin(inp(i))+1;
        nout(inp(i))=nout(inp(i))-1;
        nin(outp(i))=nin(outp(i))-1;
    end
end
nout(outp(i))=nout(outp(i))+1;
end
end
for i=1:NNODES
  NPN(i)=nin(i)+nout(i); % number of neighbor pipes to a node
  UPN(i)=nin(i);        % number of upstream flows
  DNN(i)=nout(i);       % number of downstream flows
end
for i=1:NPIPES
  if p(inp(i))<p(outp(i))
    nn(i)=inp(i);
    inp(i)=outp(i);
    outp(i)=nn(i);
  end
  qg(i)=CC(i)*(p(inp(i))^2-p(outp(i))^2)^n;
end
for i=1:NNODES
  if NPN(i)==3
    jun(i)=1; % T-junction
    if UPN(i)==1 && DNN(i)==2
      tee(i)=1; % split tee
      for j=1:NPIPES
        if outp(j)==i
          inn(i,1)=j;
          inn(i,2)=0;
        end
      end
      if inp(j)==i && N90(i)~=j
        outn(i,1)=j;
        outn(i,2)=N90(i);
      end
    end
  end
  if UPN(i)==2 && DNN(i)==1
    tee(i)=2; % converging tee
    inn(i,1)=0;
    inn(i,2)=0;
    for j=1:NPIPES
      if inp(j)==i
        outn(i,1)=j;
        outn(i,2)=0;
      end
    end
    if outp(j)==i
      if inn(i,1)==0
        inn(i,1)=j;
      else
        inn(i,2)=j;
      end
    end
  end
else
  jun(i)=2; % non T-junction
  tee(i)=0;
end
if model2==2  % multi-supply
for j=1:100
  product=1;
  for k=1:NPIPES
    product=product*gas(k);
  end
  if product==0
    for i=1:NNODES
      if UPN(i)==1 && DNN(i)==1
        if gas(inn(i))~=0
          gas(outn(i))=gas(inn(i));
        end
      end
      if UPN(i)==2 && DNN(i)==1
        if gas(inn(i,1))~=gas(inn(i,2)) && gas(inn(i,1))~=0 && gas(inn(i,2))~=0
          gas(outn(i))=3;
        end
      end
    end
  end
  else
    break
  end
end
qg1=zeros(NPIPES,1);qg2=zeros(NPIPES,1);C=zeros(nc,NPIPES);
for i=1:NPIPES
  if gas(i)==1
    qg1(i)=qg(i);
    qg2(i)=0;
    for j=1:nc
      C(j,i)=C1(j);
    end
  end
  if gas(i)==2
    qg1(i)=0;
    qg2(i)=qg(i);
    for j=1:nc
      C(j,i)=C2(j);
    end
  end
  if gas(i)==3
    qg1(i)=CC1(i)*(p(inp(i))^2-p(outp(i))^2)^n;
    qg2(i)=CC2(i)*(p(inp(i))^2-p(outp(i))^2)^n;
    for j=1:nc

\( C(j,i) = C1(j) \times qg1(i)/(qg1(i) + qg2(i)) + C2(j) \times qg2(i)/(qg1(i) + qg2(i)) \); 

end
end
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PBM Phase Identification%%%%%%%%%%%%%%%%
pav=zeros(NPIPES,1);
for i=1:NPIPES
  pav(i)=2/3*(p(inp(i))+p(outp(i)) - p(inp(i))*p(outp(i))/(p(inp(i))+p(outp(i)))) % psia
end
load data Tc  % critical temperature for each components
load data pc  % critical pressure for each components
model4=1;     % 1 for PR EOS; 2 for SRK EOS
Tr=zeros(nc,1);pr=zeros(nc,NPIPES);
for i=1:nc
  Tr(i)=Tav/Tc(i);    % dimensionless temperature
  for j=1:NPIPES
    pr(i,j)=pav(j)/pc(i);     % dimensionless pressure
  end
end
Phase=zeros(NPIPES,1);fng=zeros(NPIPES,1);MWg=zeros(NPIPES,1);
MWl=zeros(NPIPES,1);rhog=zeros(NPIPES,1);rhol=zeros(NPIPES,1);
ug=zeros(NPIPES,1);ul=zeros(NPIPES,1);zg=zeros(NPIPES,1);
zl=zeros(NPIPES,1);x=zeros(NPIPES,nc);y=zeros(NPIPES,nc);
for i=1:NPIPES
  Phase(i)=Model5(Tr,pr(:,i),Tc,pc,C(:,i),nc,pav(i),Tav,model4);
  [fng(i),MWg(i),MWl(i),rhog(i),rhol(i),ug(i),ul(i),zg(i),zl(i),x(i,:),y(i,:)]=Model4(Tr,pr(:,i),Tc,pc,nc,pav(i),Tav,C(:,i),model4);
end
ratiom=zeros(NPIPES,1);
for i=1:NPIPES
  if Phase(i)==1;
    ratiom(i)=1;
  else
    ratiom(i)=1+(1-fng(i))/fng(i)*MWl(i)/MWg(i);
  end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%% Even Split %%%%%%%%%%%%%%%%%
ntot=zeros(NPIPES,1);nL=zeros(NPIPES,1);nG=zeros(NPIPES,1);
for i=1:NPIPES
  ntot(i)=qg(i)/379.56; % total mole in each pipe after single phase calculation (lbmol)
  nL(i)=ntot(i)*(1-fng(i)); % total mole of liquid in each pipe (lbmol)
  nG(i)=ntot(i)*fng(i); % total mole of gas in each pipe (lbmol)
end
nn=zeros(nc,NPIPES);
for i=1:NPIPES
  for j=1:nc
    nn(j,i)=nG(i)*y(i,j)+nL(i)*x(i,j));
  end
end
mg=zeros(NPIPES,1);ml=zeros(NPIPES,1);
for i=1:NPIPES
  mg(i)=nG(i)*MWg(i);
  ml(i)=nL(i)*MWl(i);
end
qg=zeros(NPIPES,1);qw=zeros(NPIPES,1);
for i=1:NPIPES
qg(i)=mg(i)/rhog(i); % ft^3/D
if fng(i)==1
ml(i)=0;
qw(i)=0;
else
qw(i)=ml(i)/rhol(i); % ft^3/D
end
end

%%%%%% Beggs and Brill relation %%%%%%
Um=zeros(NPIPES,1);Usg=zeros(NPIPES,1);Usl=zeros(NPIPES,1);
deltapp=zeros(NPIPES,1);
for i=1:NPIPES
if qw(i)==0
deltaP(i)=abs(p(inp(i))-p(outp(i)));
else
Usg(i)=qg(i)/PI*4/(d(i)/12)^2/24/3600; % ft/sec
Usl(i)=qw(i)/PI*4/(d(i)/12)^2/24/3600; % ft/sec
Um(i)=Usg(i)+Usl(i); % ft/sec
deltaP(i)=BB(e,d(i),rhol(i),rhog(i),Usl(i),Usg(i),Um(i),sigma,ul(i),ug(i),angle(i),Leq(i),pav(i)); % psia
end
end

%%%%% Gas Conductivity %%%%%%
CCC=zeros(NPIPES,1);
for i=1:NPIPES
% CCC(i)=mg(i)/(abs((p(outp(i))+deltaP(i))^2-p(outp(i))^2))^n;
CCC(i)=mg(i)/(abs(p(inp(i))^2-(p(inp(i))-deltaP(i))^2))^n;
end

%%%%% Newton Raphson Analog %%%%%%
m=zeros(NPIPES,1);NR_residual1=zeros(NNODES-NFIX,1);
NR_residual2=zeros(NNODES-NFIX,1);Jac=zeros(NNODES-NFIX);
err=1;
p=pNR1;
while err>0.00001
for i=1:NPIPES
if abs(p(inp(i)))>=abs(p(outp(i)))
m(i)=CCC(i)*(p(inp(i))^2-p(outp(i))^2)^n;
else
m(i)=-CCC(i)*(p(outp(i))^2-p(inp(i))^2)^n;
end
end
for i=1:fixn-1
if nin(i)==1 & nout(i)==2 % split
NR_residual1(i)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-m(inn(i))*ratiom(inn(i));
end
if nin(i)==0 & nout(i)==1 % inlet
NR_residual1(i)=Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 & nout(i)==0 % outlet
NR_residual1(i)=Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
end
if nin(i)==2 && nout(i)==1 % converge
    NR_residual1(i)=m(inn(i,1))*ratiom(inn(i,1))-m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 && nout(i)==1 % in series
    NR_residual1(i)=m(outn(i))*ratiom(outn(i))-m(inn(i))*ratiom(inn(i));
end
end
for i=fixn+1:NNODES
    if nin(i)==1 && nout(i)==2 % split
        NR_residual1(i-1)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-m(inn(i))*ratiom(inn(i));
    end
    if nin(i)==0 && nout(i)==2 % outlet
        NR_residual1(i-1)=m(outn(i,2))*ratiom(outn(i,2))-m(outn(i,1))*ratiom(outn(i,1));
    end
    if nin(i)==1 && nout(i)==1 % in series
        NR_residual1(i-1)=m(outn(i))*ratiom(outn(i))-m(inn(i))*ratiom(inn(i));
    end
end
for i=1:fixn-1
    p(i)=p(i)+0.1;% perturb=0.01 psia
    for i=1:NPIPES
        if abs(p(inp(i)))>=abs(p(outp(i)))
            m(i)=CCC(i)*(p(inp(i))^2-p(outp(i))^2)^n;
        else
            m(i)=-CCC(i)*(p(outp(i))^2-p(inp(i))^2)^n;
        end
    end
end
for j=1:fixn-1
    if nin(i)==1 && nout(i)==2 % split
        NR_residual2(i)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-m(inn(i))*ratiom(inn(i));
    end
    if nin(i)==0 && nout(i)==2 % outlet
        NR_residual2(i)=m(outn(i,2))*ratiom(outn(i,2))-m(outn(i,1))*ratiom(outn(i,1));
    end
    if nin(i)==1 && nout(i)==1 % in series
        NR_residual2(i)=m(outn(i))*ratiom(outn(i))-m(inn(i))*ratiom(inn(i));
    end
end
for i=1:NNODES
    if nin(i)==2 && nout(i)==1 % converge
        NR_residual1(i)=m(inn(i,1))*ratiom(inn(i,1))-m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
    end
    if nin(i)==1 && nout(i)==1 % in series
        NR_residual1(i)=m(outn(i))*ratiom(outn(i))-m(inn(i))*ratiom(inn(i));
    end
end
for i=fixn+1:NNODES
    if nin(i)==1 & nout(i)==2 % split
        NR_residual2(i-1)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-
m(inn(i))*ratiom(inn(i));
    end
    if nin(i)==0 & nout(i)==1 % inlet
        NR_residual2(i-1)=-Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
    end
    if nin(i)==1 & nout(i)==0 % outlet
        NR_residual2(i-1)=Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
    end
    if nin(i)==2 & nout(i)==1 % converge
        NR_residual2(i-1)=-m(inn(i,1))*ratiom(inn(i,1))-
m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
    end
    if nin(i)==1 & nout(i)==1 % in series
        NR_residual2(i-1)=m(outn(i))*ratiom(outn(i))-
m(inn(i))*ratiom(inn(i));
    end
end
for k=1:NNODES-NFIX
    Jac(k,j)=(NR_residual2(k)-NR_residual1(k))/0.1; % jacobian matrix
end
p(j)=p(j)-0.1;
end
for j=fixn+1:NNODES
    p(j)=p(j)+0.1;% perturb=0.1 psia
for i=1:NPIPES
    if abs(p(inp(i)))>=abs(p(outp(i)))
        m(i)=CCC(i)*(p(inp(i))^2-p(outp(i))^2)^n;
    else
        m(i)=-CCC(i)*(p(outp(i))^2-p(inp(i))^2)^n;
    end
end
for i=1:fixn-1
    if nin(i)==1 & nout(i)==2 % split
        NR_residual2(i)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-
m(inn(i))*ratiom(inn(i));
    end
    if nin(i)==0 & nout(i)==1 % inlet
        NR_residual2(i)=-Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
    end
    if nin(i)==1 & nout(i)==0 % outlet
        NR_residual2(i)=Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
    end
    if nin(i)==2 & nout(i)==1 % converge
        NR_residual2(i)=-m(inn(i,1))*ratiom(inn(i,1))-
m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
    end
    if nin(i)==1 & nout(i)==1 % in series
        NR_residual2(i)=m(outn(i))*ratiom(outn(i))-
m(inn(i))*ratiom(inn(i));
    end
end
for i=fixn+1:NNODES
if nin(i)==1 & nout(i)==2 % split
    NR_residual2(i-1)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-
m(inn(i))*ratiom(inn(i));
end
if nin(i)==0 & nout(i)==1 % inlet
    NR_residual2(i-1)=-Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 & nout(i)==0 % outlet
    NR_residual2(i-1)=-Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
end
if nin(i)==2 & nout(i)==1 % converge
    NR_residual2(i-1)=-m(inn(i,1))*ratiom(inn(i,1))-
m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 & nout(i)==1 % in series
    NR_residual2(i-1)=m(outn(i))*ratiom(outn(i))-
m(inn(i))*ratiom(inn(i));
end
end
for k=1:NNODES-NFIX
    Jac(k,j-1)=(NR_residual2(k)-NR_residual1(k))/0.1;
end
p(j)=p(j)-0.1;
end
dp=Jac\(-NR_residual1);
err=max(abs(dp));
for i=1:fixn-1
    p(i)=p(i)+dp(i);
end
for i=fixn+1:NNODES
    p(i)=p(i)+dp(i-1);
end
end
pNR2=p;
DPi=abs(p-pNR1);
DP=abs(max(DPi));
% Main Iteration % Main Iteration
while DP>1
    pppold=p;
pav=zeros(NPIPES,1);
    for i=1:NPIPES
        pav(i)=2/3*(p(inp(i))+p(outp(i))-p(inp(i))*p(outp(i))/(p(inp(i))+p(outp(i)))) ; % psia
    end
% Split model selection % Split model selection
% Even Split % Even Split
if model3==1
    if model2==1 % single-supply
        end
    if model2==2 % multi-supply
        Flb=zeros(NNODES,1);Flr=zeros(NNODES,1);mgb=zeros(NNODES,1);mgin=zeros(NNODES,1);
        Fgb=zeros(NNODES,1);Fgr=zeros(NNODES,1);
        for i=1:NNODES
            if tee(i)==1 && Phase(inn(i))==2
                mgb(i)=mg(N90(i));
                mgin(i)=mg(inn(i));
Fgb(i)=mgb(i)/mgin(i);
Fgr(i)=1-Fgb(i);
Flb(i)=Fgb(i);
if Flb(i)>1
    Flb(i)=1;
end
if Flb(i)<0
    Flb(i)=0;
end
Flr(i)=1-Flb(i);
end
for j=1:nc
    matrix2=zeros(NPIPES);
    residual2=zeros(NPIPES,1);
    for i=1:NNODES-1
        if UPN(i)==1 && DNN(i)==1
            matrix2(i,outn(i))=-1;
            matrix2(i,inn(i))=1;
            residual2(i)=0;
        end
        if UPN(i)==2 && DNN(i)==1
            matrix2(i,outn(i))=-1;
            matrix2(i,inn(i,1))=1;
            matrix2(i,inn(i,2))=1;
            residual2(i)=0;
        end
        if UPN(i)==1 && DNN(i)==2
            matrix2(i,outn(i,1))=-1;
            matrix2(i,outn(i,2))=-1;
            matrix2(i,inn(i))=1;
            residual2(i)=0;
        end
        if Supply(i)~=0 && gas(outn(i))=1
            matrix2(i,:)=0;
            matrix2(i,outn(i))=1;
            residual2(i)=Supply(i)/379.56*C1(j);
        end
        if Supply(i)~=0 && gas(outn(i))==2
            matrix2(i,:)=0;
            matrix2(i,outn(i))=1;
            residual2(i)=Supply(i)/379.56*C2(j);
        end
        if Demand(i)~=0
            matrix2(i,:)=0;
            matrix2(i,inn(i))=1;
            residual2(i)=Demand(i)/379.56*y(inn(i),j)+nL(inn(i))*x(inn(i),j);
        end
    end
end
for k=1:NNODES
    for i=NNODES:NPIPES
        if tee(k)==1
            if sum(matrix2(i,:))~==0,NPIPES)==0
matrix2(i,N90(k))=1;
residual2(i)=Flb(k)*MWl(inn(k))/MWl(N90(k))*nL(inn(k))*x(inn(k),j)+Fgb(k)*MWg(inn(k))/MWg(N90(k))*nG(inn(k))*y(inn(k),j);
    break
end
if sum(matrix2(NPIPES,:))~=0,NPIPES)~=0
    break
end
end
end
ni=matrix2/residual2;
for i=1:NPIPES
    nn(j,i)=ni(i);
end
end
ntot=zeros(NPIPES,1);
for i=1:NPIPES
    ntot(i)=sum(nn(:,i)); % total mole in each pipe after single phase calculation (lbmol)
end
comp=zeros(nc,NPIPES);
for i=1:NPIPES
    for j=1:nc
        comp(j,i)=nn(j,i)/ntot(i);
    end
end
Phase=zeros(NPIPES,1);fng=zeros(NPIPES,1);MWg=zeros(NPIPES,1);
MWl=zeros(NPIPES,1);rhog=zeros(NPIPES,1);rhol=zeros(NPIPES,1);
ug=zeros(NPIPES,1);ul=zeros(NPIPES,1);zg=zeros(NPIPES,1);
zl=zeros(NPIPES,1);x=zeros(NPIPES,nc);y=zeros(NPIPES,nc);
for i=1:NPIPES
    Phase(i)=Model5(Tr,pr(:,i),Tc,pc,comp(:,i),nc,pav(i),Tav,model4);
    [fng(i),MWg(i),MWl(i),rhog(i),rhol(i),ug(i),ul(i),zg(i),zl(i),x(i,:),y(i,:)]=Model4(Tr,pr(:,i),Tc,pc,nc,pav(i),Tav,comp(:,i),model4);
end
nL=zeros(NPIPES,1);nG=zeros(NPIPES,1);
mg=zeros(NPIPES,1);ml=zeros(NPIPES,1);
for i=1:NPIPES
    nL(i)=ntot(i)*(1-fng(i)); % total mole rate of liquid in each pipe (lbmol/day)
    nG(i)=ntot(i)*fng(i); % total mole rate of gas in each pipe (lbmol/day)
    mg(i)=nG(i)*MWg(i); % mass rate of gas in each pipe (lbm/day)
    ml(i)=nL(i)*MWl(i); % mass rate of liquid in each pipe (lbm/day)
end
qg=zeros(NPIPES,1);qw=zeros(NPIPES,1);
for i=1:NPIPES
    qg(i)=mg(i)/rhog(i); % ft^3/D
    if fng(i)==1
        ml(i)=0;
        qw(i)=0;
    else
        qw(i)=ml(i)/rhol(i); % ft^3/D
    end
end
if model3==2
Usg=zeros(NPIPES,1);Usl=zeros(NPIPES,1);
for i=1:NPIPES
Usg(i)=qg(i)/PI*4/(d(i)/12)^2/24/3600; % ft/sec
Usl(i)=qw(i)/PI*4/(d(i)/12)^2/24/3600; % ft/sec
end
Hangle=zeros(NPIPES,1);vgin=zeros(NNODES,1);vlin=zeros(NNODES,1);
Flb=zeros(NNODES,1);Flr=zeros(NNODES,1);mgb=zeros(NNODES,1);mgin=zeros(NNODES,1);
Fgb=zeros(NNODES,1);Fgr=zeros(NNODES,1);KE=zeros(NNODES,1);
for i=1:NNODES
if tee(i)==1 && Phase(inn(i))==2
mgb(i)=mg(N90(i));
mgin(i)=mg(inn(i));
Fgb(i)=mgb(i)/mgin(i);
Fgr(i)=1-Fgb(i);
Hangle(inn(i))=BBB(d(inn(i)),rhol(inn(i)),Usl(inn(i)),Um(inn(i)),sigma,angle(inn(i)));
vgin(i)=Usg(inn(i))/(1-Hangle(inn(i))); % average axial velocity of gas in the inlet, ft/sec
vlin(i)=Usl(inn(i))/Hangle(inn(i)); % average axial velocity of liquid in the inlet, ft/sec
KE(i)=rhog(inn(i))/rhol(inn(i))*vgin(i)^2/vlin(i)^2;
Flb(i)=Fgb(i)*KE(i);
if Flb(i)>1
    Flb(i)=1;
end
if Flb(i)<0
    Flb(i)=0;
end
Flr(i)=1-Flb(i);
end
end
nn=zeros(nc,NPIPES);
for j=1:nc
matrix2=zeros(NPIPES);
residual2=zeros(NPIPES,1);
for i=1:NNODES-1
if UPN(i)==1 && DNN(i)==1
    matrix2(i,outn(i))=-1;
    matrix2(i,inn(i))=1;
    residual2(i)=0;
end
if UPN(i)==2 && DNN(i)==1
    matrix2(i,outn(i))=-1;
    matrix2(i,inn(i,1))=1;
    matrix2(i,inn(i,2))=1;
    residual2(i)=0;
end
if UPN(i)==1 && DNN(i)==2
    matrix2(i,outn(i,1))=-1;
    matrix2(i,outn(i,2))=-1;
    matrix2(i,inn(i))=1;
    residual2(i)=0;
end
if Supply(i)~=0 && gas(outn(i))==1
    matrix2(i,:)=0;
end
end
if UPN(i)==2 && DNN(i)==2
    matrix2(i,outn(i))=-1;
    matrix2(i,inn(i,1))=-1;
    matrix2(i,inn(i,2))=-1;
    residual2(i)=0;
end
if UPN(i)==1 && DNN(i)==2
    matrix2(i,outn(i,1))=-1;
    matrix2(i,outn(i,2))=-1;
    matrix2(i,inn(i))=1;
    residual2(i)=0;
end
if Supply(i)~=0 && gas(outn(i))==1
    matrix2(i,:)=0;
end

matrix2(i,outn(i))=1;
residual2(i)=Supply(i)/379.56*C1(j);
end
if Supply(i)~=0 && gas(outn(i))==2
matrix2(i,:)=0;
matrix2(i,outn(i))=1;
residual2(i)=Supply(i)/379.56*C2(j);
end
if Demand(i)~=0
matrix2(i,:)=0;
matrix2(i,inn(i))=1;
if Phase(inn(i))==1
residual2(i)=Demand(i)/379.56*C(j);
else
residual2(i)=Demand(i)/379.56*y(inn(i),j)+nL(inn(i))*x(inn(i),j));
end
end
end
for k=1:NNODES
for i=NNODES:NPIPES
if tee(k)==1
if sum(matrix2(i,:)~=0,NPIPES)==0
matrix2(i,N90(k))=1;
residual2(i)=Flb(k)*MWl(inn(k))/MWl(N90(k))*nL(inn(k))*x(inn(k),j)+Fgb(k)*MWg(inn(k))/MWg(N90(k))*nG(inn(k))*y(inn(k),j));
break
end
if sum(matrix2(NPIPES,:)~=0,NPIPES)~=0
break
end
end
end
ni=matrix2\residual2;
for i=1:NPIPES
nn(j,i)=ni(i);
end
end
ntot=zeros(NPIPES,1);
for i=1:NPIPES
ntot(i)=sum(nn(:,i)); % total mole in each pipe after single phase calculation (lbmol)
end
comp=zeros(nc,NPIPES);
for i=1:NPIPES
for j=1:nc
comp(j,i)=nn(j,i)/ntot(i);
end
end
Phase=zeros(NPIPES,1);fng=zeros(NPIPES,1);MWg=zeros(NPIPES,1);
MWl=zeros(NPIPES,1);rhog=zeros(NPIPES,1);rhol=zeros(NPIPES,1);
ug=zeros(NPIPES,1);ul=zeros(NPIPES,1);zg=zeros(NPIPES,1);
zl=zeros(NPIPES,1);x=zeros(NPIPES,nc);y=zeros(NPIPES,nc);
for i=1:NPIPES
Phase(i)=Model5(Tr,pr(:,i),Tc,pc,comp(:,i),nc,pav(i),Tav,model4);
[fng(i),MWg(i),MWl(i),rhog(i),rhol(i),ug(i),ul(i),zg(i),zl(i),x(i,:),y(i,:)]=Model4(Tr,pr(:,i),Tc,pc,nc,pav(i),Tav,comp(:,i),model4);
end

nl=zeros(NPIPES,1);nG=zeros(NPIPES,1);
mg=zeros(NPIPES,1);ml=zeros(NPIPES,1);
for i=1:NPIPES
nL(i)=ntot(i)*(1-fng(i)); % total mole rate of liquid in each pipe (lbmol/day)
nG(i)=ntot(i)*fng(i); % total mole rate of gas in each pipe (lbmol/day)
mG(i)=nG(i)*MWg(i); % mass rate of gas in each pipe (lbm/day)
ml(i)=nL(i)*MWl(i); % mass rate of liquid in each pipe (lbm/day)
end

qg=zeros(NPIPES,1);qw=zeros(NPIPES,1);
for i=1:NPIPES
qg(i)=mg(i)/rhog(i); % ft^3/D
if fng(i)==1
ml(i)=0;
qw(i)=0;
else
qw(i)=ml(i)/rhol(i); % ft^3/D
end
end

if model3==3
Usg=zeros(NPIPES,1);Usl=zeros(NPIPES,1);KE=zeros(NNODES,1);
for i=1:NPIPES
Usg(i)=qg(i)/PI*4/(d(i)/12)^2/24/3600; % ft/sec
Usl(i)=qw(i)/PI*4/(d(i)/12)^2/24/3600; % ft/sec
end
rd=zeros(NNODES,1);k12=zeros(NNODES,1);k13=zeros(NNODES,1);F0=zeros(NNODES,1);
H=zeros(NPIPES);
rr=0; % R/D of junction
A=PI/2; % T-junction angle of the junction
Re_slin=zeros(NNODES,1);X=zeros(NNODES,1);Hlin=zeros(NNODES,1);
vgin=zeros(NNODES,1);vlin=zeros(NNODES,1);Fr_lin=zeros(NNODES,1);
Fwall0=zeros(NNODES,1);Fwall_in=zeros(NNODES,1);Fr_l13=zeros(NNODES,1);
Re_lin=zeros(NNODES,1);betal=zeros(NNODES,1);betag=zeros(NNODES,1);
Flb=zeros(NNODES,1);Flr=zeros(NNODES,1);Fgb=zeros(NNODES,1);
aa=zeros(NNODES,1);bb=zeros(NNODES,1);cc=zeros(NNODES,1);
for i=1:NNODES
if tee(i)==1 && Phase(inn(i))==2
rd(i)=d(inn(i))/d(N90(i)); % ratio of diameters of inlet to branch
if rd(i)==1
Re_slin(i)=1488.16*(d(inn(i))/12)*Usl(inn(i))/ul(inn(i)); % superficial liquid Reynolds number in the inlet
X(i)=Usl(inn(i))/Usg(inn(i))*1+10.4*Re_slin(i)^(-0.363)*(rhog(inn(i))/rhol(inn(i)))^0.5;
Hlin(i)=X(i)/(1+X(i)); % liquid holdup in the inlet
vgin(i)=Usg(inn(i))/(1+Hlin(i)); % average axial velocity of gas in the inlet, ft/sec
vlin(i)=Usl(inn(i))/Hlin(i); % average axial velocity of liquid in the inlet, ft/sec
Fr_lin(i)=vlin(i)^2/g/(d(inn(i))/12)*rhog(inn(i))/(rhol(inn(i))-rhog(inn(i))); % Froude number in the inlet
Fwall0(i)=0.52*Hlin(i)^0.374; % wetted wall fraction
Fr_l13(i)=vlin(i)^2/g/(d(inn(i))-d(N90(i)))/12); % modified Froude number
Fwall_in(i)=Fwall0(i)+0.26*Fr_lin(i)^0.58; % wetted wall fraction in the inlet
end
end

%%%%%%%%%% Double Stream Model Split %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if Fwall_in(i)>1
    Fwall_in(i)=1;
end
Re_lin(i)=Re_slin(i)/Fwall_in(i); % liquid film Reynolds number in the inlet
if Re_lin(i)<2000
    betai(i)=1.54;
else
    betai(i)=1;
end
betag(i)=1;
KE(i)=betag(i)/betal(i)*rhog(inn(i))/rhol(inn(i))*vgin(i)^2/vlin(i)^2; % Kinetic energy ratio at inlet
mgb(i)=mg(N90(i));
mgin(i)=mg(inn(i));
Fgb(i)=mgb(i)/mgin(i);
Fgr(i)=1-Fgb(i);
k12(i)=0.03*(1-Fgb(i))^2+0.35*Fgb(i)^2-0.2*Fgb(i)*(1-Fgb(i));
k13(i)=0.95*(1-Fgb(i))^2+Fgb(i)^2*((1.3*tan(A/2)-0.3+(0.4*rd(i)^2-0.1)*rd(i)^2)*(1-0.9*rd(i)^2*rr^0.5))+0.4*Fgb(i)*(1-Fgb(i))*(1+rd(i)^2)*tan(A/2);
F0(i)=0.5*(1+k12(i)-k13(i));
aa(i)=rd(i)^4-1;
bb(i)=2;
cc(i)=KE(i)*((aa(i)*Fgb(i)^2+2*(Fgb(i)-F0(i)))+1/betal(i)/Fr_l13(i)-2*F0(i));
Flb(i)=(-bb(i)+(bb(i)^2-4*aa(i)*cc(i))^0.5)/2/aa(i);
if Flb(i)<0
    Flb(i)=0;
end
if Flb(i)>1
    Flb(i)=1;
end
Flr(i)=1-Flb(i);
else
mgb(i)=mg(N90(i));
mgin(i)=mg(inn(i));
Fgb(i)=mgb(i)/mgin(i);
Fgr(i)=1-Fgb(i);
k12(i)=0.03*(1-Fgb(i))^2+0.35*Fgb(i)^2-0.2*Fgb(i)*(1-Fgb(i));
k13(i)=0.95*(1-Fgb(i))^2+Fgb(i)^2*((1.3*tan(A/2)-0.3+(0.4*rd(i)^2-0.1)*rd(i)^2)*(1-0.9*rd(i)^2*rr^0.5))+0.4*Fgb(i)*(1-Fgb(i))*(1+rd(i)^2)*tan(A/2);
F0(i)=0.5*(1+k12(i)-k13(i));
Flb(i)=F0(i)+KE(i)*(Fgb(i)-F0(i));
if Flb(i)<0
    Flb(i)=0;
end
if Flb(i)>1
    Flb(i)=1;
end
Flr(i)=1-Flb(i);
end
end
nn=zeros(nc,NPIPES);
for j=1:nc
    matrix2=zeros(NPIPES);
residual2=zeros(NPIPES,1);
for i=1:NNODES-1
    if UPN(i)==1 && DNN(i)==1
        matrix2(i,outn(i))=-1;
        matrix2(i,inn(i))=1;
        residual2(i)=0;
    end
    if UPN(i)==2 && DNN(i)==1
        matrix2(i,outn(i))=-1;
        matrix2(i,inn(i,1))=1;
        matrix2(i,inn(i,2))=1;
        residual2(i)=0;
    end
    if UPN(i)==1 && DNN(i)==2
        matrix2(i,outn(i,1))=-1;
        matrix2(i,outn(i,2))=-1;
        matrix2(i,inn(i))=1;
        residual2(i)=0;
    end
    if Supply(i)~=0 && gas(outn(i))==1
        matrix2(i,:)=0;
        matrix2(i,outn(i))=1;
        residual2(i)=Supply(i)/379.56*C1(j);
    end
    if Supply(i)~=0 && gas(outn(i))==2
        matrix2(i,:)=0;
        matrix2(i,outn(i))=1;
        residual2(i)=Supply(i)/379.56*C2(j);
    end
    if Demand(i)~=0
        matrix2(i,:)=0;
        matrix2(i,inn(i))=1;
        if Phase(inn(i))==1
            residual2(i)=Demand(i)/379.56*C(j);
        else
            residual2(i)=Demand(i)/379.56*y(inn(i),j)+nL(inn(i))*x(inn(i),j);
        end
    end
end
for k=1:NNODES
    for i=NNODES:NPIPES
        if tee(k)==1
            if sum(matrix2(i,:)~=0,NPIPES)==0
                matrix2(i,N90(k))=1;
                residual2(i)=Flb(k)*MWl(inn(k))/MWl(N90(k))*nL(inn(k))*x(inn(k),j)+Fgb(k)*MWg(inn(k))/MWg(N90(k))*nG(inn(k))*y(inn(k),j);
                break
            end
        end
    end
    if sum(matrix2(NPIPES,:))~=0
        break
    end
end
for i=1:NPIPES
  ni(matrix2); residual2;
  for i=1:NPIPES
    nn(j,i)=ni(i);
  end
end
ntot=zeros(NPIPES,1);
for i=1:NPIPES
  ntot(i)=sum(nn(:,i)); % total mole in each pipe after single phase calculation (lbmol)
end
comp=zeros(nc,NPIPES);
for i=1:NPIPES
  for j=1:nc
    comp(j,i)=nn(j,i)/ntot(i);
  end
end
Phase=zeros(NPIPES,1); fng=zeros(NPIPES,1); MWg=zeros(NPIPES,1);
MWl=zeros(NPIPES,1); rhog=zeros(NPIPES,1); rhol=zeros(NPIPES,1);
ug=zeros(NPIPES,1); ul=zeros(NPIPES,1); zg=zeros(NPIPES,1);
zl=zeros(NPIPES,1); x=zeros(NPIPES,nc); y=zeros(NPIPES,nc);
for i=1:NPIPES
  Phase(i)=Model5(Tr,pr(:,i),Tc,pc,comp(:,i),nc,pav(i),Tav,model4);
  [fng(i),MWg(i),MWl(i),rhog(i),rhol(i),ug(i),ul(i),zg(i),zl(i),x(i,:),y(i,:)]=Model4(Tr,pr(:,i),Tc,pc,nc,pav(i),Tav,comp(:,i),model4);
end
nL=zeros(NPIPES,1); nG=zeros(NPIPES,1);
mg=zeros(NPIPES,1); ml=zeros(NPIPES,1);
for i=1:NPIPES
  nL(i)=ntot(i)*(1-fng(i)); % total mole rate of liquid in each pipe (lbmol/day)
  nG(i)=ntot(i)*fng(i); % total mole rate of gas in each pipe (lbmol/day)
  mg(i)=nG(i)*MWg(i); % mass rate of gas in each pipe (lbm/day)
  ml(i)=nL(i)*MWl(i); % mass rate of liquid in each pipe (lbm/day)
end
qg=zeros(NPIPES,1); qw=zeros(NPIPES,1);
for i=1:NPIPES
  qg(i)=mg(i)/rhog(i); % ft^3/D
  if fng(i)==1
    ml(i)=0;
    qw(i)=0;
  else
    qw(i)=ml(i)/rhol(i); % ft^3/D
  end
end
ratiom=zeros(NPIPES,1);
for i=1:NPIPES
  if Phase(i)==1;
    ratiom(i)=1;
  else
    ratiom(i)=1+(1-fng(i))/fng(i)*MWl(i)/MWg(i);
  end
end
%%%%%%%%%%%%%%%%%%%%%%
Um=zeros(NPIPES,1); Usg=zeros(NPIPES,1); Usl=zeros(NPIPES,1);
deltaP=zeros(NPIPES,1); deltapp=zeros(NPIPES,1);
for i=1:NPIPES
if qw(i)==0
    deltaP(i)=abs(p(inp(i))-p(outp(i)));
else
    Usg(i)=qg(i)/PI*4/(d(i)/12)^2/24/3600; % ft/sec
    Usl(i)=qw(i)/PI*4/(d(i)/12)^2/24/3600; % ft/sec
    Um(i)=Usg(i)+Usl(i); % ft/sec
    deltaP(i)=BB(e,d(i),rhol(i),rhog(i),Usl(i),Usg(i),Um(i),sigma,ul(i),ug(i),angle(i),Leq(i),pav(i)); % psia
end
end

%%%%%%%%%%%%% Gas Conductivity %%%%%%%%%%%%%%%
CCC=zeros(NPIPES,1);
for i=1:NPIPES
    CCC(i)=mg(i)/(abs((p(outp(i))+deltaP(i))^2-p(outp(i))^2))^n;
end

%%%%%%%%%%%%% Newton Raphson Analog %%%%%%%%%%%%
m=zeros(NPIPES,1); NR_residual1=zeros(NNODES-NFIX,1); NR_residual2=zeros(NNODES-NFIX,1); Jac=zeros(NNODES-NFIX);
err=1;
p=pNR1;
while err>0.0001
    for i=1:NPIPES
        if abs(p(inp(i)))>=abs(p(outp(i)))
            m(i)=CCC(i)*(p(inp(i))^2-p(outp(i))^2)^n;
        else
            m(i)=-CCC(i)*(p(outp(i))^2-p(inp(i))^2)^n;
        end
    end
    for i=1:fixn-1
        if nin(i)==1 && nout(i)==2 % split
            NR_residual1(i)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-m(inn(i))*ratiom(inn(i));
        end
        if nin(i)==0 && nout(i)==1 % inlet
            NR_residual1(i)=-Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
        end
        if nin(i)==1 && nout(i)==0 % outlet
            NR_residual1(i)=Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
        end
        if nin(i)==2 && nout(i)==1 % converge
            NR_residual1(i)=-m(inn(i,1))*ratiom(inn(i,1))-m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
        end
        if nin(i)==1 && nout(i)==1 % in series
            NR_residual1(i)=m(outn(i))*ratiom(outn(i))-m(inn(i))*ratiom(inn(i));
        end
    end
    for i=fixn+1:NNODES
        if nin(i)==1 && nout(i)==2 % split
            NR_residual1(i-1)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-m(inn(i))*ratiom(inn(i));
        end
        if nin(i)==0 && nout(i)==1 % inlet
            NR_residual1(i)=-Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
        end
    end
end
NR_residual1(i-1)=Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 && nout(i)==0 % outlet
NR_residual1(i-1)=Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
end
if nin(i)==2 && nout(i)==1 % converge
NR_residual1(i-1)=m(inn(i,1))*ratiom(inn(i,1))-m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 && nout(i)==1 % in series
NR_residual1(i-1)=m(outn(i))*ratiom(outn(i))-m(inn(i))*ratiom(inn(i));
end
end
for j=1:fixn-1
p(j)=p(j)+0.1;% perturb=0.01 psia
for i=1:NPIPES
if abs(p(inp(i)))>=abs(p(outp(i)))
m(i)=CCC(i)*(p(inp(i))^2-p(outp(i))^2)^n;
else
m(i)=-CCC(i)*(p(outp(i))^2-p(inp(i))^2)^n;
end
end
for i=1:fixn-1
if nin(i)==1 && nout(i)==2 % split
NR_residual2(i)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-m(inn(i))*ratiom(inn(i));
end
if nin(i)==0 && nout(i)==1 % inlet
NR_residual2(i)=-Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 && nout(i)==0 % outlet
NR_residual2(i)=Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
end
if nin(i)==2 && nout(i)==1 % converge
NR_residual2(i)=-m(inn(i,1))*ratiom(inn(i,1))-m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 && nout(i)==1 % in series
NR_residual2(i)=m(outn(i))*ratiom(outn(i))-m(inn(i))*ratiom(inn(i));
end
end
end
for i=fixn+1:NNODES
if nin(i)==1 && nout(i)==2 % split
NR_residual2(i-1)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-m(inn(i))*ratiom(inn(i));
end
if nin(i)==0 && nout(i)==1 % inlet
NR_residual2(i-1)=-Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 && nout(i)==0 % outlet
NR_residual2(i-1)=Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
end
if nin(i)==2 && nout(i)==1 % converge
NR_residual2(i-1)=Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
end
if nin(i)==2 && nout(i)==1 % converge
NR_residual2(i-1) = -m(inn(i,1))*ratiom(inn(i,1))-m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 & nout(i)==1 % in series
NR_residual2(i-1) = m(outn(i))*ratiom(outn(i))-m(inn(i))*ratiom(inn(i));
end
end
for k=1:NNOD
Jac(k,j)=(NR_residual2(k)-NR_residual1(k))/0.1; % jacobian matrix
end
p(j)=p(j)-0.1;
end
for j=fixn+1:NNODES
p(j)=p(j)+0.1; % perturb=0.1 psia
for i=1:NPIES
if abs(p(inp(i)))>=abs(p(outp(i)))
m(i)=CCC(i)*(p(inp(i))^2-p(outp(i))^2)^n;
else
m(i)=-CCC(i)*(p(outp(i))^2-p(inp(i))^2)^n;
end
end
for i=fixn-1
if nin(i)==1 & nout(i)==2 % split
NR_residual2(i)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-m(inn(i))*ratiom(inn(i));
end
if nin(i)==0 & nout(i)==1 % inlet
NR_residual2(i)=-Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 & nout(i)==0 % outlet
NR_residual2(i)=Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
end
if nin(i)==2 & nout(i)==1 % converge
NR_residual2(i)=-m(inn(i,1))*ratiom(inn(i,1))-
m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
end
end
for i=fixn+1:NNODES
if nin(i)==1 & nout(i)==2 % split
NR_residual2(i-1)=m(outn(i,1))*ratiom(outn(i,1))+m(outn(i,2))*ratiom(outn(i,2))-m(inn(i))*ratiom(inn(i));
end
if nin(i)==0 & nout(i)==1 % inlet
NR_residual2(i-1)=-Supply(i)/(RR*Tsc/Psc)*MWt+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 & nout(i)==0 % outlet
NR_residual2(i-1)=Demand(i)/(RR*Tsc/Psc)*MWg(inn(i))-m(inn(i));
end
if nin(i)==2 & nout(i)==1 % converge
NR_residual2(i-1)=-m(inn(i,1))*ratiom(inn(i,1))-m(inn(i,2))*ratiom(inn(i,2))+m(outn(i))*ratiom(outn(i));
end
if nin(i)==1 && nout(i)==1 % in series
    NR_residual2(i-1)=m(outn(i))*ratiom(outn(i)) - m(inn(i))*ratiom(inn(i));
end
for k=1:NNODES-NFIX
    Jac(k,j-1)=(NR_residual2(k)-NR_residual1(k))/0.1;
end
p(j)=p(j)-0.1;
dp=Jac\(-NR_residual1);
err=max(abs(dp));
for i=1:fixn-1
    p(i)=p(i)+dp(i);
end
for i=fixn+1:NNODES
    p(i)=p(i)+dp(i-1);
end
DP=max(abs(p-pppold))
End

Subroutine---Beggs and Brill Approach

function deltaP=BB(e,d,densityl,densityg,Usl,Usg,Um,sigma,visl,visg,angle,L,pav)
g=32.174; % ft/s^2; standard gravity
gc=32.174;
Nfr=Um^2/g/(d/12); % Froude number
Fl=Usl/Um; % liquid fraction
% check for single phase flow
if Fl>0.99999 % liquid single phase
    Hangle=Fl;
end
if Fl<0.00001 % gas single phase
    Hangle=Fl;
end
if Fl=0.00001 && Fl<=0.99999 % two phase
    Nlv=1.938*Usl*(densityl/sigma)^0.25; % liquid velocity number
    L1=316*Fl^0.302;
    L2=0.0009252*Fl^(-2.46842);
    L3=0.1*Fl^(-1.45155);
    L4=0.5*Fl^(-6.738);
    %%%%%%%%%%%Flow Pattern Determination%%%%%%%%%%%%%%%%%%%%%%%%%
    if (Fl<0.01 && Nfr<L1) || (Fl>=0.01 && Nfr<L2)
        pattern=1; % segregated flow
    end
    if (Fl>=0.01 && Fl<0.4 && Nfr>L3 && Nfr<=L1) || (Fl>=0.4 && Nfr>L3 && Nfr<=L4)
        pattern=2; % intermittent flow
    end
    if (Fl<0.4 && Nfr>=L1) || (Fl>=0.4 && Nfr>L4)
        pattern=3; % distributed flow
    end
end
if Fl=0.01 && Nfr>L2 && Nfr<=L3
pattern=4; % transition flow
end
if pattern==1 % segregated flow
    a=0.98;
    b=0.4846;
    c=0.0868;
    if angle>0 % uphill
        ee=0.011;
        ff=-3.768;
        gg=3.539;
        hh=-1.614;
        \( H_0 = a \cdot Fl^b / Nfr^c \); % holdup
        if \( H_0 < Fl \)
            \( H_0 = Fl \);
        end
        C = (1-Fl) \cdot \log(ee \cdot Fl^ff \cdot Nlv^gg \cdot Nfr^hh);
        if C<=0
            Phi=1;
        else
            Phi=1+C*(sin(1.8*angle)-0.333*(sin(1.8*angle))^3);
        end
        \( H_{angle} = H_0 \cdot Phi \); % holdup
    end
    if angle<0 % downhill
        ee=4.7;
        ff=-0.3692;
        gg=0.1244;
        hh=-0.056;
        \( H_0 = a \cdot Fl^b / Nfr^c \); % holdup
        if \( H_0 < Fl \)
            \( H_0 = Fl \);
        end
        C = (1-Fl) \cdot \log(ee \cdot Fl^ff \cdot Nlv^gg \cdot Nfr^hh);
        if C<=0
            Phi=1;
        else
            Phi=1+C*(sin(1.8*angle)-0.333*(sin(1.8*angle))^3);
        end
        \( H_{angle} = H_0 \cdot Phi \); % holdup
    end
    if angle==0 % horizontal
        \( H_0 = a \cdot Fl^b / Nfr^c \); % holdup
        if \( H_0 < Fl \)
            \( H_0 = Fl \);
        end
        \( H_{angle} = H_0 \); % holdup
    end
end
if pattern==2 % intermittent flow
    a=0.845;
    b=0.5351;
    c=0.0173;
    if angle>0 % uphill
        ee=2.96;
ff=0.3050;
gg=-0.4473;
hh=0.0978;
H0=a*Fl^b/Nfr^c; % holdup
if H0<Fl
H0=Fl;
end
C=(1-Fl)*log(ee*Fl^ff*Nlv^gg*Nfr^hh);
if C<=0
Phi=1;
else
Phi=1+C*(sin(1.8*angle)-0.333*(sin(1.8*angle))^3);
end
Hangle=H0*Phi; % holdup
end
if angle<0 % downhill
ee=4.7;
ff=-0.3692;
gg=0.1244;
hh=-0.056;
H0=a*Fl^b/Nfr^c; % holdup
if H0<Fl
H0=Fl;
end
C=(1-Fl)*log(ee*Fl^ff*Nlv^gg*Nfr^hh);
if C<=0
Phi=1;
else
Phi=1+C*(sin(1.8*angle)-0.333*(sin(1.8*angle))^3);
end
Hangle=H0*Phi; % holdup
end
if angle==0 % horizontal
H0=a*Fl^b/Nfr^c; % holdup
if H0<Fl
H0=Fl;
end
Hangle=H0; % holdup
end
end
if pattern==3 % distributed flow
a=1.065;
b=0.5824;
c=0.0609;
if angle<0 % downhill
ee=4.7;
ff=-0.3692;
gg=0.1244;
hh=-0.056;
H0=a*Fl^b/Nfr^c; % holdup
if H0<Fl
H0=Fl;
end
C=(1-Fl)*log(ee*Fl^ff*Nlv^gg*Nfr^hh);
if C<=0
Phi=1;
else
Phi=1+C*(sin(1.8*angle)-0.333*(sin(1.8*angle))^3);
end
Hangle=H0*Phi; % holdup
end
if angle==0 % horizontal
H0=a*Fl^b/Nfr^c; % holdup
if H0<Fl
H0=Fl;
end
Hangle=H0; % holdup
end
end
if pattern==3 % distributed flow
a=1.065;
b=0.5824;
c=0.0609;
if angle<0 % downhill
ee=4.7;
ff=-0.3692;
gg=0.1244;
hh=-0.056;
H0=a*Fl^b/Nfr^c; % holdup
if H0<Fl
H0=Fl;
end
C=(1-Fl)*log(ee*Fl^ff*Nlv^gg*Nfr^hh);
if C<=0
Phi=1;
else
Phi=1+C*(sin(1.8*angle)-0.333*(sin(1.8*angle))^3);
end
Hangle=H0*Phi; % holdup
end
if angle>=0 % horizontal & uphill
H0=a*Fl^b/Nfr^c; % holdup
if H0<Fl
H0=Fl;
end
Hangle=H0; % holdup
end
end
if pattern==4 % transition flow
A=(L3-Nfr)/(L3-L2);
B=1-A;
% segregated flow
a1=0.98;
b1=0.4846;
c1=0.0868;
if angle>0 % uphill
ee1=0.011;
ff1=-3.768;
gg1=3.539;
hh1=-1.614;
H01=a1*Fl^b1/Nfr^c1; % holdup
if H01<Fl
H01=Fl;
end
C1=(1-Fl)*log(ee1*Fl^ff1*Nlv^gg1*Nfr^hh1);
if C1<=0
Phi1=1;
else
Phi1=1+C1*(sin(1.8*angle)-0.333*(sin(1.8*angle))^3);
end
Hangle1=H01*Phi1; % holdup
end
if angle<0 % downhill
ee1=4.7;
ff1=-0.3692;
gg1=0.1244;
hh1=-0.056;
H01=a1*Fl^b1/Nfr^c1; % holdup
if H01<Fl
H01=Fl;
end
C1=(1-Fl)*log(ee1*Fl^ff1*Nlv^gg1*Nfr^hh1);
if C1<=0
Phi1=1;
else
Phi1=1+C1*(sin(1.8*angle)-0.333*(sin(1.8*angle))^3);
end
Hangle1=H01*Phi1; % holdup
end
if angle==0 % horizontal
H01=a1*Fl^b1/Nfr^c1; % holdup
if H01<Fl
H01=Fl;
end
Hangle1=H01; % holdup
end
% intermittent flow
a2=0.845;
b2=0.5351;
c2=0.0173;
if angle>0 % uphill
ee2=2.96;
ff2=0.3050;
gg2=-0.4473;
hh2=0.0978;
H02=a2*Fl^b2/Nfr^c2; % holdup
if H02<Fl
H02=Fl;
end
C2=(1-Fl)*log(ee2*Fl^ff2*Nlv^gg2*Nfr^hh2);
if C2<=0
Phi2=1;
else
Phi2=1+C2*(sin(1.8*angle)-0.333*(sin(1.8*angle))^3);
end
Hangle2=H02*Phi2; % holdup
end
if angle<0 % downhill
ee2=4.7;
ff2=-0.3692;
gg2=0.1244;
hh2=-0.056;
H02=a2*Fl^b2/Nfr^c2; % holdup
if H02<Fl
H02=Fl;
end
C2=(1-Fl)*log(ee2*Fl^ff2*Nlv^gg2*Nfr^hh2);
if C2<=0
Phi2=1;
else
Phi2=1+C2*(sin(1.8*angle)-0.333*(sin(1.8*angle))^3);
end
Hangle2=H02*Phi2; % holdup
end
if angle==0 % horizontal
H02=a2*Fl^b2/Nfr^c2; % holdup
if H02<Fl
H02=Fl;
end
Hangle2=H02; % holdup
end
% transient flow
  Hangle = A*Hangle1 + B*Hangle2; % holdup
end
% check Hangle
if Hangle > 1
  Hangle = 1;
end
if Hangle < 0
  Hangle = 0.00001;
end
correction = 1; % 0 for no correction, 1 for Palmer correction
% apply Palmer correction factors if desired
if correction == 1
  if angle < 0
    Hangle = Hangle * 0.685;
  end
  if angle > 0
    Hangle = Hangle * 0.924;
  end
end
end
Rhos = densityl * Hangle + densityg * (1 - Hangle); % slip density
Rhon = densityl * Fl + densityg * (1 - Fl); % nonslip density
Mun = visl * Fl + visg * (1 - Fl); % nonslip density
deltap_ele = Rhos * g * sin(angle) / gc / 144; % psi/ft; pressure drop due to hydrostatic head
Re_ns = Rhon * Um * d / 12 * 1488.16 / Mun; % nonslip Reynolds Number
% Chen’s friction factor calculation
if e == 0
  Fns = 1 / (2 * log10((4.5223 * log10(Re_ns) - 3.8215)))^2; % smooth pipes friction factor
else
  if Re_ns == 0
    Fns = 4 * 0.008 / d^(1/3);
  end
  if Re_ns < 2100 && Re_ns ~= 0
    Fns = 4 * 16 / Re_ns;
  end
  if Re_ns > 2100
    T1 = (e/d) / 3.7065;
    T2 = (e/d)^(1.1098 / 2.8257);
    T3 = 5.8506 / (Re_ns^0.8981);
    Fns = 4 * (-4 * log10(T1 - (5.0452 / Re_ns) * log10(T2 + T3)))^(-2);
  end
end
y = Fl / Hangle^2;
if y > 1 && y < 1.2
  s = log((2.2 * y - 1.2); % Beggs and Brill Coefficient
else
  s = log(y) / (0.0523 + 3.182 * log(y) - 0.8725 * (log(y))^2 + 0.01853 * (log(y))^4);
end
Ftp = Fns * exp(s); % nonslip friction factor
deltap_fric = Ftp * Um^2 / Rhon / 2 / gc / (d / 12) / 144; % psi/ft; pressure gradient due to friction
Ek = Rhos * Um * Usg / gc / pav / 144; % acceleration term
ICRIT = 0;
if Ek > 0.95
ICRIT=1;
end
if ICRIT==1
Ek=0.95;
end
deltaP=(deltap_ele+deltap_fric)/(1-Ek)*L*5280; % psi
end

Subroutine---PBM approach
function [Phase]=Model5(Tr,pr,C,nc,p,model4)
load data w; % load Pitzer’s acentric factor
for i=1:nc;
Ki(i)=1/pr(i)*exp(5.37*(1+w(i))*(1-1/Tr(i)));
end;
if model4==1
    [z,phi]=Model1_PR(C,Tr,pr,nc);
else
    [z,phi]=Model1_SRK(C,Tr,pr,nc);
end;
fz=phi.*C.*p;
K=Ki';
trail_v=0;
Y=[];
flag1=1;
while flag1
    Y=C.*K;
    Sv=sum(Y);
y=Y./Sv;
    if model4==1
        [zg,phig]=Model1_PR(y,Tr,pr,nc);
    else
        [zg,phig]=Model1_SRK(y,Tr,pr,nc);
    end;
fy=phig.*y.*p;
R=fz./fy/Sv;
K=K.*R;
flag1=sum((R-1).^2)>=0.1^10;
    if sum(log(K).^2)<0.1^4
        trail_v=1;
        break;
    end
end
K=Ki';
trail_l=0;
Y=[];
flag2=1;
while flag2
    Y=C./K;
    Sl=sum(Y);
x=Y./Sl;
    if model4==1
        [zl,phil]=Model1_PR(x,Tr,pr,nc);
    else
        [zl,phil]=Model1_SRK(x,Tr,pr,nc);
    end;
end;
f_{x}=p_1.*x*p;
R=f_{x}/f_2*Sl;
K=K.*R;
flag2=sum((R-1).^2)>=0.1^10;
if sum(log(K).^2)<0.1^4
  trail_l=1;
  break;
end
end
if Sv>1 || Sl>1 % two-phase
  Phase=2;
elseif Sv<=1&&Sl<=1 || trail_l==1&&trail_v==1 || trail_l==1&&Sv<=1 || trail_v==1&&Sl<=1
  Phase=1;
end
end

function [fng,MWg,MWl,rhog,rhol,ug,ul,zg,zl,x,y]=Model4(Tr,pr,Tc,pc,nc,p,T,model4)
load data MW; % load molecular weight
load data Vc; % load critical volume (ft^3/lbm)
load data s; % load "si" values
R=10.73;
% calculate molecular weight of each component of the liquid and gas phase
[x,y,ommiga,zl,zg,fng]=Model3(Tr,pr,nc,p,model4);
mwg=y.*MW;
mwl=x.*MW;
% calculate molecular weight of the liquid and gas phase
MWg=sum(mwg);
MWl=sum(mwl);
% calculate component-dependent volume-shift parameters ci
cdvs=s(:,model4).*bc;
xc=x.*cdvs;
yc=y.*cdvs;
% calculate viscosiy parameter ξi of each component
xi=5.4402*Tc.^(1/6)./(sqrt(MW).*pc.^(2/3));
% calculate viscosiy at low pressure μ*i of each component
for i=1:nc;
  if Tr(i)>1.5
    ustar(i)=0.0001778*(4.58*Tr(i)-1.67)^0.625/xi(i);
  else
    ustar(i)=0.00034*Tr(i)^0.94/xi(i);
  end
end;
% calculate xi*μ*iMWi^0.5;
xustarMW=x.*ustar'.*sqrt(MW);
% calculate xiMWi^0.5;
xMW=x.*sqrt(MW);
% calculate composition of pseudocritical temperature & pressure & volume of each component in liquid phase
Tpc=x.*Tc;
ppc=x.*pc;
vpc=x.*Vc.*MW;
% calculate for EOS-calculated molar volume vEOSl & vEOSv
vEOSl=zl*R*T/p;
vEOSv=zg*R*T/p;
% calculate vl & vv
vl=vEOSl-sum(xc);
vv=vEOSv-sum(yc);
% calculate pl & pv (before volume shift is applied)
rhoEOSg=MWg/vEOSv;
rhoEOSl=MWl/vEOSl;
% calculate pl & pv (after volume shift is applied)
rhog=MWg/vv;
rhol=MWl/vl;
% calculate viscosity of the gas μg
kv=(9.4+0.02*MWg)*T^1.5/(209+19*MWg+T);
xv=3.5+986/T+0.01*MWg;
yv=2.4-0.2*xv;
ug=0.0001*kv*exp(xv*(rhog/62.4)^yv);
% calculate viscosity of the mixture at atmospheric pressure μ*
ustar=sum(xustariMW)/sum(xMW);
% calculate mixture viscosity parameter ξm
xim=5.4402*sum(Tpc)^(1/6)/(sqrt(MWl)*sum(ppc)^(2/3));
% calculate reduced liquid density ρr
rhor=sum(vpc)*rhoEOSl/MWl;
% calculate viscosity of the liquid μl
ul=ustar+((0.1023+0.023364*rhor+0.058533*rhor^2-
0.040758*rhor^3+0.0093324*rhor^4)^4-
0.0001)/xim;

function [x,y,ommiga,zl,zg,fng,K,fl,fg]=Model3(Tr,pr,nc,p,model4)
load data w;  % load Pitzer’s acentric factor
e=10^(-14); % SSM accuracy condition
sigma=0;
for i=1:nc;
    % Use Wilson’s correlation to initialize Ki
    K(i)=1/pr(i)*exp(5.37*(1+w(i))*(1-1/Tr(i)));
end;
[x,y,fl,fg,zl,zg,fng,ommiga]=Model2(K,Tr,pr,nc,p,model4);
for i=1:nc;
    sigma=sigma+(fl(i)/fg(i)-1)^2;
end;
while sigma>e
    for i=1:nc;
        K(i)=K(i)*fl(i)/fg(i);
    end;
    [x,y,fl,fg,zl,zg,fng,ommiga]=Model2(K,Tr,pr,nc,p,model4);
    sigma=0;
    for i=1:nc;
        sigma=sigma+(fl(i)/fg(i)-1)^2;
    end;
end;

function [x,y,fl,fg,zl,zg,fng,ommiga]=Model2(K,Tr,pr,nc,p,model4)
load data C;  % load composition (molar)
fngold=0.5;
g=0; G=0;
g0=0; g1=0; fngu=0; fngl=1; fg=[]; fl=[]; E=10^(-10); for i=1:nc; g0=g0+C(i)*(K(i)-1); g1=g1+C(i)*(K(i)-1)/K(i); end; if g0>0 && g1<0 for i=1:nc; gg=C(i)*(K(i)-1)/(1+fngold*(K(i)-1)); g=g+gg; GG=-C(i)*((1-K(i))^2/(1+fngold*(K(i)-1))^2; G=G+GG; end fngnew=fngold-g/G; if (1/(1-max(K)))<fngnew && fngnew < (1/(1-min(K))) while abs(fngnew-fngold)>E fngold=fngnew; g=0; G=0; for i=1:nc; gg=C(i)*(K(i)-1)/(1+fngold*(K(i)-1)); g=g+gg; GG=-C(i)*((1-K(i))^2/(1+fngold*(K(i)-1))^2; G=G+GG; end fngnew=fngold-g/G; end else fngu=0; fngl=1; fngnew=0.5*(fngl+fngu); fngold=fngnew; for i=1:nc; gg=C(i)*(K(i)-1)/(1+fngnew*(K(i)-1)); g=g+gg; end if g>0 fngu=fngnew; else fngl=fngnew; end g=0; fngnew=0.5*(fngl+fngu); while abs(fngnew-fngold)>E for i=1:nc; gg=C(i)*(K(i)-1)/(1+fngnew*(K(i)-1)); g=g+gg; end if g>0 fngu=fngnew; else fngl=fngnew; end g=0;
\begin{verbatim}
function \[z,phi,ommiga\]=Model1_PR(C,Tr,pr,nc)
load data w;  % load Pitzer's acentric factor
load data delta;   % load binary interaction coefficient
m=[1+sqrt(2),1-sqrt(2)];
ommiga=[0.457235529,0.077796074];
mi=[ ]; % mi for PR EOS
A=0;
A0=zeros(nc,1);
Ai=[ ]; Aij=[ ];
\end{verbatim}
B=0;
Bi=[];
for i=1:nc;
  % calculate mi for PR EOS
  if w(i)<=0.49
    mi(i)=0.374640+1.54226*w(i)-0.26992*w(i)^2;
  else
    mi(i)=0.379642+1.48503*w(i)-0.164423*w(i)^2+0.016666*w(i)^3;
  end;
  % calculate Ai for PR EOS
  Ai(i)=ommiga(1)*(1+mi(i)*(1-(Tr(i))^0.5))^2*pr(i)/Tr(i)^2;
end;
for i=1:nc;
  for j=1:nc;
    % calculate Aij for PR EOS
    Aij(i,j)=(1-delta(i,j))*(Ai(i)*Ai(j))^0.5;
  end;
  % calculate A
  A=A+C(i)*C(j)*Aij(i,j);
  % calculate the sum of Aij*Cj
  A0(i)=A0(i)+Aij(i,j)*C(j);
end;
% calculate Bi
Bi(i)=ommiga(2)*pr(i)/Tr(i);
% calculate B
B=B+C(i)*Bi(i);
end;
a=(m(1)+m(2)-1)*B-1;
b=A+m(1)*m(2)*B^2-(m(1)+m(2))*B*(B+1);
c=-(A*B+m(1)*m(2)*B^2*(B+1));
Q=(a^2-3*b)/9;
R=(2*a^3-9*a*b+27*c)/54;
M=R^2-Q^3;
if M<=0  % three real roots
  theta=acos(R/sqrt(Q^3));
  x1=-(2*sqrt(Q)*cos(theta/3))-a/3;
  x2=-(2*sqrt(Q)*cos((theta+2*pi)/3))-a/3;
  x3=-(2*sqrt(Q)*cos((theta-2*pi)/3))-a/3;
  zmax=max([x1,x2,x3]);
  zmin=min([x1,x2,x3]);
  if zmin<B
    z=zmax;
  else
    if (zmax-zmin+log((zmin-B)/(zmax-B))-A/(B*(m(2)-m(1))))*log((zmin+m(1)*B)*(zmax+m(2)*B)/((zmin+m(2)*B)*(zmax+m(1)*B)))>0
      z=zmin;
    else
      z=zmax;
    end
  end
else  % only one real root
  S1=sign(R)*(abs(R)+sqrt(M))^1/3;
  if S1==0
    T1=0;
  else
    T1=Q/S1;
  end
end
113
end
z=S1+T1-a/3;
end
% calculate the fugacity coefficient of each component
for i=1:nc;
    phi(i)=exp(-log(z-B)-A/(B*(m(2)-m(1)))*2*A0(i)/A-
        Bi(i)/B)*log((z+m(2)*B)/(z+m(1)*B))+Bi(i)/B*(z-1));
end
end

function [z,phi,ommiga]=Model1_SRK(C,Tr,pr,nc)
load data w;  % load Pitzer's acentric factor
load data delta;   % load binary interaction coefficient
m=[0,1];
ommiga=[0.4274802,0.08664035];
a=[];b=[];c=[];
mi=[]; % mi for SRK EOS
A=0;
A0=zeros(nc,1);
Ai=[]; Aij=[];
B=0;
Bi=[];
theta=[];
x1=[]; x2=[]; x3=[];
zmin=[]; zmax=[]; z=[];
S1=[]; T1=[];
for i=1:nc;
    % calculate mi for SRK EOS
    mi(i)=0.48+1.574*w(i)-0.176*w(i)^2;
    % calculate Ai for SPK EOS
    Ai(i)=ommiga(1)*(1+mi(i)*(1-(Tr(i))^0.5))^2*pr(i)/Tr(i)^2;
end;
for i=1:nc;
    for j=1:nc;
        % calculate Aij for SPK & PR EOS
        Aij(i,j)=(1-delta(i,j))*(Ai(i)*Ai(j))^0.5;
    end;
end;
% calculate Bi
    Bi(i)=ommiga(2)*pr(i)/Tr(i);
% calculate B
    B=B+C(i)*Bi(i);
end;
a=(m(1)+m(2)-1)*B-1;
b=A+m(1)*m(2)*B^2-(m(1)+m(2))*B*(B+1);
c=-A*B+m(1)*m(2)*B^2*(B+1);
Q=(a^2-3*b)/9;
R=(2*a^3-9*a*b+27*c)/54;
M=R^2-Q^3;
if M<=0  % three real roots
    theta=acos(R/sqrt(Q^3));
end
\begin{align*}
x_1 &= -(2\sqrt{Q}\cos(\theta/3)) - a/3; \\
x_2 &= -(2\sqrt{Q}\cos((\theta+2\pi)/3)) - a/3; \\
x_3 &= -(2\sqrt{Q}\cos((\theta-2\pi)/3)) - a/3; \\
z_{\text{max}} &= \max([x_1, x_2, x_3]); \\
z_{\text{min}} &= \min([x_1, x_2, x_3]); \\
\text{if } z_{\text{min}} < B & \quad z = z_{\text{max}}; \\
\text{else } & \quad z_{\text{min}} > z_{\text{max}} - \log((z_{\text{min}} - B)/(z_{\text{max}} - B)) - A/(B*(m(2) - m(1)))*\log((z_{\text{min}} + m(1)*B)*(z_{\text{max}} + m(2)*B)/(z_{\text{min}} + m(2)*B)*(z_{\text{max}} + m(1)*B))) > 0 \\
& \quad \begin{cases} 
z = z_{\text{min}}; \\
z = z_{\text{max}}; & \text{end} 
\end{cases} \\
\text{else } & \quad \begin{cases} 
\text{only one real root} \\
S_1 &= \text{sign}(R) \cdot (|R| + \sqrt{M})^{1/3}; \\
\text{if } S_1 = 0 & \quad T_1 = 0; \\
\text{else } & \quad T_1 = Q/S_1; \\
& \quad \begin{cases} 
z = S_1 + T_1 - a/3; & \text{end} 
\end{cases} 
\end{cases} \\
\text{end} \\
\text{end} \\
\text{calculate the fugacity coefficient of each component} \\
& \quad \begin{cases} 
\text{for } i = 1: nc; & \text{phi(i)} = \exp(-\log(z - B) - A/(B*(m(2) - m(1)))*(2*A_0(i)/A - B(i)/(z + m(2)*B))) + B(i)/B*(z - 1)); 
\end{cases} \\
\text{end} 
\end{align*}
# Appendix B---Input File

The tables below are input files 'pipe1.xlsx' in the code.

## Case #1

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