NEW METHODS IN ULTRA-WIDEBAND ARRAY DESIGN

AND

FINITE-DIFFERENCE TIME-DOMAIN MODELING OF MEMRISTIVE DEVICES

A Dissertation in
Electrical Engineering
by
Micah Dennis Gregory

© 2013 Micah Dennis Gregory

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

August 2013
The dissertation of Micah Dennis Gregory was reviewed and approved* by the following:

Douglas H. Werner  
Professor of Electrical Engineering  
Dissertation Advisor, Chair of Committee

Ram M. Narayanan  
Professor of Electrical Engineering

Douglas E. Wolfe  
Associate Professor of Materials Science and Engineering

Jack S. Brenizer  
Professor of Mechanical and Nuclear Engineering

Victor P. Pasko  
Professor of Electrical Engineering  
Graduate Program Coordinator

*Signatures are on file in the Graduate School.
Abstract

This dissertation covers two electromagnetics topics, the first is the design of ultra-wideband antenna array layouts. The second is the design of reconfigurable radio frequency devices with a newly discovered circuit element, the memristor. The two concepts are seemingly unrelated, however, reconfigurable devices are often used in antennas with multi-band abilities, sometimes switching between frequencies that can be octaves apart. In order to efficiently use these reconfigurable antennas in phased arrays they must be placed in a proper layout, the focus of the first topic.

The primary focus of the first topic is the elimination of grating lobes and minimization of peak sidelobe levels in the radiation pattern of antenna arrays. The occurrence of grating lobes in array factors is very similar to aliasing in a digital system when the sampling frequency is below twice the maximum frequency content (i.e. the Nyquist frequency) [1]. These lobes appear in the array factor or radiation pattern of periodic arrays when the distance between elements becomes greater than one wavelength (for an unsteered array, and less for a steered phased array) [2, 3], resulting in radiation and reception in undesired directions. Strong mutual coupling negatively affects antenna performance when elements are placed less than a half-wavelength apart, limiting the closest that common antenna elements can be placed [2]. These two phenomena generally limit the usable frequency bandwidth of conventional periodic array systems to about 2:1. As many emerging radio frequency systems that utilize multiple frequencies and ultra-wide bandwidths require high directivity, the need for capable array layout designs becomes apparent. The goal of this portion of research is the creation of design techniques which are capable of readily producing array layouts which yield no grating lobes and low sidelobe levels over vary large frequency bandwidths.
The second focus involves a new type of electronic device called the memristor. Their existence was speculated by Leon Chua in the 1970s as the fourth basic circuit element which relates flux to charge [4]. The result is a passive component with a charge or flux dependent resistance, allowing for design of interesting reconfigurable electromagnetic devices without active circuit elements such as transistors. It has received significant interest after a research team at Hewlett-Packard labs successfully fabricated a device using titanium dioxide and platinum which exhibits properties that can be modeled by a memristor [5]. Most excitement resides around their potential for computer memory applications [6], however, their properties are also useful for other devices [7,8]. Many frequency selective surface and antenna structures can have adjustable performance characteristics based on the values of embedded lumped resistor elements. Replacing the resistors with memristors allows reconfigurability without the use of significant controlling circuitry that is usually necessary with other reconfigurable devices such as active parts (transistors, varactors, etc.) or micro-electrical mechanical switches (MEMS). A specially tailored finite-difference time-domain (FD-TD) simulation code has been developed to analyze and design radio frequency structures with memristive elements [9]. The code incorporates a memristor model with non-linear dopant drift, an advancement in accuracy versus the conventional linear dopant drift models. The time-domain nature of FD-TD permits capturing the non-linear and transient behavior of memristors, where quasi-static approximations could only be possible with a frequency domain code.
# Table of Contents

List of Figures ........................................ viii
List of Tables ........................................ xi
Acknowledgments ........................................ xiii

Chapter 1
Antenna Array Background .......................... 1
  1.1 Antenna Array Principles .......................... 1
  1.2 Periodic Arrays .................................. 2
    1.2.1 Linear Arrays ................................ 3
    1.2.2 Planar Arrays ................................ 4
    1.2.3 Element Performance in Periodic Arrays ... 4
    1.2.4 Beam Steering ................................ 6
  1.3 Random Arrays ................................... 7
  1.4 Early Aperiodic Array Designs ................... 8
  1.5 Modern Ultra-Wideband Array Designs ........... 9
    1.5.1 Linear Arrays ................................ 9
    1.5.2 Planar Arrays ................................. 10
    1.5.3 Volumetric Arrays ............................. 10

Chapter 2
New Ultra-Wideband Array Design Techniques .... 12
  2.1 Introduction .................................... 12
  2.2 The Covariance Matrix Adaptation Evolutionary Strategy 13
  2.3 Compact Linear Arrays ........................... 21
  2.4 Planar Arrays .................................... 27
2.4.1 Optimized Rotationally Symmetric Arrays .......................... 30
2.4.2 Semi-Periodic Rotationally Symmetric Arrays ........................ 35
2.5 Effects of Element Position Error on Array Performance .............. 39
  2.5.1 Linear Arrays ......................................................... 41
  2.5.2 Planar Arrays ......................................................... 43
2.6 Summary ................................................................. 45

Chapter 3
The Memristor in Electromagnetic Devices ............................... 47
  3.1 Introduction ............................................................. 47
  3.2 Non-Linear Dopant Drift Memristor Model ............................. 50
  3.3 The Finite-Difference Time-Domain .................................. 55
  3.4 FD-TD Memristor Model ................................................. 59
  3.5 Example Designs ......................................................... 61
    3.5.1 Reconfigurable Periodic Absorber .............................. 61
    3.5.2 Dual-Band Reconfigurable Patch Antenna ....................... 63
    3.5.3 Polarization-Reconfigurable Patch Antenna ..................... 67
  3.6 Summary ................................................................. 70

Chapter 4
Conclusion ........................................................................... 78
  4.1 Summary of Contributions ............................................... 80
  4.2 Future Work .............................................................. 82

Appendix A
CMA-ES Performance Evaluation ......................................... 83
  A.1 Introduction ............................................................. 83
  A.2 Numerical Test Functions ............................................. 83
    A.2.1 Test Function Definitions ......................................... 84
      A.2.1.1 Sphere ......................................................... 84
      A.2.1.2 Ackley ....................................................... 85
      A.2.1.3 Rastrigin .................................................... 85
      A.2.1.4 Zakharov ................................................... 85
      A.2.1.5 Rotated Hyper-Ellipsoid .................................... 86
    A.2.2 Test Function Analysis Results .................................. 86
  A.3 Stacked-Patch Antenna Design ....................................... 86

Appendix B
FD-TD Specifics .................................................................... 93
  B.1 Field Update Equations ................................................ 93
B.1.1 Normal-Space Electric Field Update . . . . . . . . . . . . . . 93
B.1.2 Normal-Space Magnetic Field Update . . . . . . . . . . . . . 94
B.2 Current Through an Electric Field Location . . . . . . . . . . . . 95
B.3 Source Signals for FD-TD . . . . . . . . . . . . . . . . . . . . . . 96

Bibliography 99
# List of Figures

1.1 Geometry of linear and planar periodic arrays. .......................... 3
1.2 Array factors of a 10–element, linear periodic array. ................. 4
1.3 Array factors of square and hexagonal lattice periodic planar arrays. 5
1.4 Voltage standing wave ratio (VSWR) of dipole antennas ($Z_0 = 75\Omega$) in a 50–element array with varying element spacing. ............... 6
1.5 Visible and invisible regions of the array factor of a 20–element linear periodic array during scanning. .............................. 7

2.1 Flowchart of operation for CMA-ES. ............................... 14
2.2 Two-dimensional canyon test function. ............................... 17
2.3 Operation of CMA-ES on the two-dimensional canyon test function. 18
2.4 Coordinate system and array layout for the compact linear ultra-wideband arrays. ................................................. 22
2.5 Compact linear ultra-wideband array design summary. ............... 25
2.6 Layouts and array factors of select compact ultra-wideband arrays. 26
2.7 Bandwidths of the four arrays selected in Fig. 2.6. ................. 27
2.8 CMA-ES seed evolutions for the two array comparisons. ........... 28
2.9 Layouts of the optimized seeds for the two array comparisons. .... 28
2.10 Bandwidth of the two best array designs from the array comparisons. 29
2.11 Rotationally symmetric array geometry. ............................. 31
2.12 Geometry and array factor of the 220-element optimized array. .. 34
2.13 Cut of the array factor at $\phi = 0^\circ$ of the 220-element optimized array at $d_{\text{min}} = 4.72\lambda$. .................................................... 34
2.14 Geometry and array factor of the 600-element optimized array. ... 35
2.15 Cut of the array factor at $\phi = 0^\circ$ of the 600-element optimized array at $d_{\text{min}} = 2.68\lambda$. ..................................................... 35
2.16 Bandwidth of the optimized and semi-periodic rotationally symmetric array and a standard square lattice periodic array for broadside operation. ....................................................... 36
2.17 Geometry and array factor of the 6-fold, 90-element rotationally symmetric array at $d_{\text{min}} = 4\lambda$. ................................. 37
2.18 Geometry and array factor of the 7-fold, 70-element rotationally symmetric array at $d_{\text{min}} = 4\lambda$ ........................................ 38
2.19 Geometry and array factor of the 15-fold, 600-element rotationally symmetric array at $d_{\text{min}} = 2.5\lambda$. .................................. 39
2.20 Geometry and array factor of the 11-fold, 594-element rotationally symmetric array at $d_{\text{min}} = 2.5\lambda$. .......................... 40
2.21 Array factor cuts of the 600- and 594-element semi-periodic rotationally symmetric arrays at $\phi = 0^\circ$ and $d_{\text{min}} = 2.5\lambda$. 40

3.1 Circuit element relationships. ............................................. 49
3.2 Simplified memristor geometry. .......................................... 51
3.3 Window function value versus state for various values of $p$. ..... 53
3.4 Circuit used to illustrate the effects of the non-linear dopant drift window function. ..................................................... 53
3.5 Linear and non-linear dopant drift circuit simulation results. .... 54
3.6 The Yee cell with field components displayed. ...................... 56
3.7 Cut-plane of the field regions of the 2-D periodic FD-TD simulation code. ................................................................. 57
3.8 Reconfigurable absorber geometry. ..................................... 62
3.9 Reflection of the planar absorber in the two resistor states computed using a 2-D periodic FD-TD simulation tool. ............... 63
3.10 Results of the FD-TD simulation for the periodic reconfigurable planar absorber. ....................................................... 64
3.11 Reconfigurable dual-band patch antenna geometry. ............... 65
3.12 Scattering parameters of the reconfigurable antenna with fixed resistors in the two states. .................................................. 66
3.13 Realized gain of the reconfigurable antenna with fixed resistors in the two states. ........................................................... 66
3.14 Time-domain response of the dual-band patch antenna design. 71
3.15 Zoomed view of the probe feed voltages and current for the dual-band patch antenna. ....................................................... 72
3.16 Geometry of the polarization reconfigurable patch antenna. .... 73
3.17 Low-frequency circuit used to model the dual-polarization patch antenna memristor reconfiguration. ............................... 73
3.18 Time-domain circuit modeling of the memristor networks. ....... 74
3.19 Scattering parameters of the polarization reconfigurable patch antenna in the two polarization settings. ............................ 74
3.20 Realized gain at 6.2 GHz of the polarization reconfigurable patch antenna in the two polarization settings. .......................... 75
3.21 Time-domain results of the dual-polarization patch antenna design. 76
3.22 Zoomed view of the probe feed voltages and current of the dual-polarization patch antenna design.  

A.1 Geometry of the stacked-patch antenna.  
A.2 Evolutionary progress for the stacked-patch antenna problem.  
A.3 Antenna VSWR and gain for the stacked-patch designs  

B.1 Example pulses that may be used to reconfigure a memristor.  
B.2 Example sine-modulated pulses that may be used to characterize a configured device.
List of Tables

2.1 Symbols, values, and descriptions of the strategy parameters of CMA-ES. .................................................. 15
2.2 Symbols and descriptions of the internal operating parameters of CMA-ES. .................................................. 16
2.3 Number of iterations and function evaluations permitted for each optimization for both CMA-ES and PSO seeds. The optimization time is predominantly determined by the number of array evaluations. 24
2.4 Properties of the five seeds for each of the optimized designs. .... 27
2.5 Properties of the optimized and semi-periodic rotationally symmetric arrays. .................................................. 39
2.6 Properties of hexagonally-seeded semi-periodic arrays having various symmetry with approximately 300 elements and a minimum element spacing of 3λ. .................................................. 41
2.7 Effects of positional error on the peak sidelobe level of the example 25-element linear array. The ideal array possesses a peak sidelobe level of -9.77 dB. .................................................. 43
2.8 Effects of positional error on the peak sidelobe level of the example 100-element linear array. The ideal array possesses a peak sidelobe level of -15.44 dB. .................................................. 43
2.9 Effects of positional error on the peak sidelobe level of the example 220-element optimized planar array. The ideal array possesses a peak sidelobe level of -16.21 dB. .................................................. 44
2.10 Effects of positional error on the peak sidelobe level of the example 600-element semi-periodic rotationally symmetric planar array. The ideal array possesses a peak sidelobe level of -13.84 dB. ............. 45
3.1 Geometry parameters for the reconfigurable dual-band patch antenna. 64
A.1 Particle swarm optimization parameters used for the test function analysis. .................................................. 84
A.2 Test function analysis results for PSO and CMA-ES. ............... 87
A.3 Optimization parameter ranges and descriptions for the stacked-patch antenna. ........................................ 87
A.4 Particle swarm optimization parameters used for the stacked-patch antenna analysis. ........................................ 89
A.5 Statistical results of the stack-patch antenna problem. ............ 91
B.1 Parameters and definitions of the memristor tuning pulses. ........ 97
Acknowledgments

There are many people I would like to thank for their support and encouragement throughout my studies at Penn State. The first are my parents, Mary and Tom. My brothers Jonah, Jonathan, and Jess have been a much needed distraction at times. My grandparents Jenny and Dennis have been very helpful and I have always enjoyed sneaking away on the occasional weekend to visit them on the farm. My aunts and uncles, Lynn, Denise, Danny, and Jim have always somehow maintained interest.

This all would have of course been impossible without the many years of support and guidance from my advisor, Dr. Douglas Werner. Dr. Pingjuan Werner has also been very helpful with everything from research to the day-to-day requirements of getting things done in the lab. I would also like to thank my Ph.D. committee members, Drs. Brenizer, Narayanan, Pasko, and Wolfe for their very helpful advice concerning research and suggestions with the dissertation. Lastly, much appreciation is due to the Applied Research Lab (ARL) at Penn State for the Exploratory and Foundational (E&F) Research fellowship that has financially supported me through my Ph.D.
Chapter 1

Antenna Array Background

1.1 Antenna Array Principles

The antenna array was conceived near the turn of the 20th century mainly due to the works of Sydney George Brown, Dr. James Erskine-Murray, Lee De Forest, and Guglielmo Marconi [10, 11]. Experiments were basic and consisted of just a few antennas, although the principles remain the same for larger arrays. The basis for antenna array functionality is the constructive and destructive interference effects of $N$ individual antenna elements in the far-field radiation pattern given by

$$E_{\text{tot}}(\theta, \phi) = \sum_{n=1}^{N} E_n(\theta, \phi) e^{jk(x_n \sin \theta \cos \phi + y_n \sin \theta \sin \phi)},$$

(1.1)

where $k = 2\pi/\lambda$ is the free-space wavenumber, $E_n(\theta, \phi)$ is the electric far-field pattern of the $n^{th}$ element, and $(x_n, y_n)$ is the location of the element on the array plane [2,3]. An array can be formed from different elements, however, most arrays use identical elements for manufacturing advantages. The individual patterns of identical elements can vary depending on their amplitude and phase excitation, which is commonly the case for beam-steered and amplitude or phase-tapered arrays [12,13]. Only one and two-dimensional arrays are considered here; Eq. (1.1) can be expanded to three dimensions as is needed for some conformal and volumetric [14] array designs.

For most array designs, the full radiation pattern is ignored in favor of using the
array factor, which is a simplified form of Eq. (1.1) that assumes identical element patterns, given by

$$AF(\theta, \phi) = \sum_{n=1}^{N} I_n e^{jk(x_n \sin \theta \cos \phi + y_n \sin \theta \sin \phi)},$$

(1.2)

where $I_n$ is the complex excitation coefficient of the $n^{th}$ element. The final radiation pattern of the array can then be found by multiplying Eq. (1.2) by the electric field pattern of the element. Since the choice of antenna element can have a significant impact on the total radiation pattern in an array, the array factor in Eq. (1.2) is most commonly used instead of the complete pattern in Eq. (1.1) for comparing the performance of and analyzing array layouts. This is commonly referred to as using an array of isotropic radiators, which have $E_n(\theta, \phi) = 1$.

In the remainder of this chapter, the most common class of arrays, the periodic type, will be examined and its advantages and disadvantages discussed. Other early aperiodic array types will be covered as well, all leading up to the latest optimized aperiodic linear and planar array designs. This lays the foundation and motivation for the new array design methods introduced in the next chapter.

### 1.2 Periodic Arrays

The two most common reasons the periodic variety is the most prevalent type of array layout are manufacturing ease and performance predictability. A regular lattice of antenna elements lends itself to comparatively low-cost fabrication of large arrays, allowing the designer to achieve coverage of a large aperture and obtain narrow pattern beamwidths. The performance of these arrays is relatively easy to predict; half-power beamwidths, sidelobe levels, and directivity can easily found through simple equations or approximations [2]. The basic linear periodic array and some planar periodic array geometries are shown in Fig. 1.1. Since the designs are simple, they are mainly characterized by minimum element spacing ($d_{min}$), aperture size, and aperture shape. For the most basic linear array, only the number of elements and $d_{min}$ are needed to determine the array factor. The minimum element spacing of an array can be specified in either physical or electrical
distance, but is most often referred to here in electrical distance (wavelengths) in order to leave the designs in a generic form, leaving the end-user to select a final frequency or band of operation.

Figure 1.1: Geometry of a (a) linear periodic array, (b) square lattice periodic planar array, and (c) triangular lattice periodic planar array.

1.2.1 Linear Arrays

The periodic array is most appealing for single-band or narrow-band operation due to its low sidelobe level in the region of operation where elements are approximately $0.5\lambda$ apart. For an unsteered array, the electrical distance between elements must be limited to below one wavelength (for the linear and square lattice arrays) in order to avoid the occurrence of grating lobes, which are multiple maxima in the array factor. These lobes are undesirable for many direction finding and navigational applications due to the inability to determine the direction of peak reception. Grating lobes are illustrated for a linear periodic array in Fig. 1.2, where a frequency range of 4:1 is covered. At $d_{min} = 0.5\lambda$, the array factor is desirable with a peak sidelobe level of $-13.2$ dB relative to the main beam, however, at $d_{min} = 2.0\lambda$ there are several grating lobes with a peak sidelobe level of 0 dB.
4

Figure 1.2: Array factor of a linear periodic array of 10 elements when the elements are spaced (a) 0.5λ apart and (b) 2.0λ apart.

1.2.2 Planar Arrays

Similar issues occur with planar arrays as well. The array factors of 100–element square lattice and regular hexagonal lattice arrays are given in Fig. 1.3. The grating lobes just begin to enter the array factor at $d_{\text{min}} = 1.0\lambda$ for the square lattice. For the hexagonal lattice, the occurrence begins at approximately $d_{\text{min}} = 1.15\lambda$, yielding a slightly larger potential bandwidth. For this reason, hexagonal arrays can be used to cover a larger aperture (compared to a square lattice array) with the same number of elements, which can reduce costs when seeking narrow beamwidths. The sidelobe level and other characteristics of the array factor of planar periodic arrays can tailored by customizing the shape of the aperture that the lattice is truncated to [15,16].

1.2.3 Element Performance in Periodic Arrays

Another issue that simultaneously occurs when grating lobes first appear is strong mutual coupling between nearby antenna elements, leading to degraded input impedance (poor voltage standing wave ratio, VSWR). As an example, the VSWR of a simple linear array of $\lambda/2$ dipoles is given in Fig. 1.4, computed via FEKO [17]. The regular spacing of antennas leads to strong field interference at element locations when distances are multiples of a wavelength. Periodic arrays are nominally designed for 0.5λ spacing, where VSWR is low. A VSWR of less than 2 is generally
desired for efficient antenna operation (approximately 90% of the power delivered to antenna is radiated).

Figure 1.3: Array factors of unsteered 100-element planar periodic arrays with (a) \( d_{min} = 0.5\lambda \), square lattice (b) \( d_{min} = 1.0\lambda \), square lattice, (c) \( d_{min} = 2.0\lambda \), square lattice, (d) \( d_{min} = 0.5\lambda \), hexagonal lattice, (e) \( d_{min} = 1.0\lambda \), hexagonal lattice, and (f) \( d_{min} = 2.0\lambda \), hexagonal lattice.
1.2.4 Beam Steering

One of the primary reasons for choosing to use an array over a large aperture antenna such as a dish is the ability to electronically steer the beam in one (for a linear array) or two (for a planar or volumetric array) dimensions by adjusting the excitation phase of the elements ($I_n$ in Eq. (1.2)). When concerned with the peak sidelobe level of an array design, steering the beam of an array has an effect similar to increasing the minimum element spacing. Increasing the minimum element spacing (by increasing the operating frequency of a fixed array) introduces new information at the ends of the array factor ($\theta = \pm 90^\circ$). Steering the beam introduces information on one end of the array factor while removing it from the opposite end. Since array factors possess symmetry about the main beam, the same information that is introduced while increasing frequency is introduced during scanning, and the peak sidelobe level is similarly affected. To illustrate this, the visible ($-1 \leq \sin \theta \leq 1$) and invisible ($\sin \theta \leq -1$, $\sin \theta \geq 1$) regions of the array factor are examined for a linear periodic array, shown in Fig. 1.5. Although the invisible regions are non-real, computing them serves as a method to predict array performance over an extended bandwidth and during scanning. In Fig. 1.5, the grating lobe that lies in the invisible region at $\sin \theta = -1.333$ moves into the visible region at $\sin \theta = -0.831$ ($\theta = -56.4^\circ$) when the main beam is steered to $\theta_0 = 30^\circ$. Increasing the spacing of the elements (by increasing frequency) simply compresses the invisible region in to the visible region.
Figure 1.5: Visible and invisible regions of the array factors of a 20-element linear periodic array with $d = 0.75\lambda$ when the main beam is steered to (a) $\theta_0 = 0^\circ$ and (b) $\theta_0 = 30^\circ$ ($\sin 30^\circ = 0.5$). The invisible regions of the array factors are shaded gray.

With this information, a direct correlation between minimum spacing and steering can be found (with respect to the peak sidelobe level generated by the array factor), leading to the relation

$$d_{\text{steered}} = \frac{d_{\text{min}}}{1 + \sin \theta_0},$$ \hspace{1cm} (1.3)

where $d_{\text{steered}}$ is the effective element spacing of the array (with respect to peak SLL) at $d_{\text{min}}$ while steered $\theta_0$ from broadside. For this reason, the scanning performance of many of the arrays presented is not explicitly examined, rather, the peak sidelobe level performance can be predicted by computing the array factor over a large bandwidth (minimum element spacing).

### 1.3 Random Arrays

Random arrays are formed by placing elements randomly over an aperture, either a line, plane, or volume. They were of particular interest in early array design due to the lack of grating lobes in their array factors, even when the arrays are thinned.
and span large apertures (effectively operating over a large bandwidth) [18–21]. This is due to the aperiodic nature of the designs, which avoid the grating effects of a regular lattice. Although random arrays do not possess grating lobes in their array factor, they generally have high peak sidelobe levels. In addition, typical random array methods can generate layouts with small interelement spacings that leave insufficient room for typical antenna sizes and can lead to strong mutual coupling between elements. The peak sidelobe level of random arrays can be statistically predicted (with a given certainty) according to the distribution used to create the array, the number of elements, and the aperture size. For example, a linear array with 1000 elements uniformly distributed over an aperture (line) of $10^3 \lambda$ has a 96% probability of having a peak sidelobe level less than -19 dB [19].

### 1.4 Early Aperiodic Array Designs

There have been many different aperiodic array geometries proposed since array design accrued significant research interest in the 1960s. Part of this was due to the growth of computers and computational resources, allowing array factors of non-trivial arrays to be computed in a reasonable amount of time. Most of these early aperiodic array designs relied on mathematical expressions to determine element positions [22–29]. The fractal-random array was a very interesting outcome of applying fractal theory to array design, and has laid the groundwork for other very powerful modern array topologies [30]. Planar array designs using concentric rings and space tapering has resulted in reasonably good performance over moderate array bandwidths [31].

Performance of array layouts began to significantly improve when optimization tools such as the genetic algorithm (GA) came about in the 1970s [32, 33]. Some early designs that used a GA to optimize the locations of elements in an array for reduced peak sidelobe levels during scanning have created a basis for many new ultra-wideband array layouts introduced later in this chapter [34]. Since the basic genetic algorithm operates on binary chromosomes, it is ideally suited to problems with cost functions that take the form of multiple decisions or on/off
operations [35]. With this in mind, it was effectively applied to turn on and off the elements that are located on a periodic lattice in order to obtain very low peak sidelobe levels in the array factor [11,36].

1.5 Modern Ultra-Wideband Array Designs

With the performance capabilities of pairing up optimization strategies and array design techniques well established, the logical next steps involved determining the best methods for creating array geometries from basic sets of optimizable parameters. Several techniques have been introduced in the past few years for designing linear, planar, and even volumetric ultra-wideband array layouts exhibiting frequency bandwidths beyond 10:1 with peak sidelobe levels below -10 dB.

1.5.1 Linear Arrays

Linear ultra-wideband arrays experienced significant gains when polyfractal arrays were introduced [37–39]. The polyfractal array design technique is based on the fractal-random method introduced in [30], except the fractal trees are connected together according to an optimized set of connection factors instead of at random. This array design technique is capable of effectively creating layouts with thousands of elements due to the rapid beamforming technique made possible by the fractal nature of the designs. One 1924–element design in [39] maintains a peak sidelobe level below -16 dB over a 40:1 frequency bandwidth ($d_{\text{min}} = 20\lambda$). Polyfractal arrays have also applied to interleaved designs as well in [40,41], made possible by the arrays’ ability to operate with large element spacings while not exhibiting grating lobes. Some validations of small, 32–element polyfractal arrays have been performed using fabricated microstrip antennas in [42,43].
1.5.2 Planar Arrays

The polyfractal design technique has been applied to planar geometries as well as linear arrays [44]. The fractal tree structures are now expanded to two dimensions to allow population of a plane with elements. The same rapid beamforming technique can be applied for quick computation of the array factor, allowing for fast optimization of large arrays. In [44], a planar array with 319 elements was created that yielded a peak sidelobe level less than -13 dB over greater than a 60:1 frequency bandwidth.

In addition to planar polyfractal arrays, another class of planar arrays has received a great deal of interest. A new type of array based on aperiodic tiling geometries has been introduced in [45]. By placing elements at the vertices of aperiodic tiling sets such as the Penrose [46] or Danzer [47], a naturally aperiodic lattice is formed that is well suited for array design. In addition, placing elements at optimized locations inside of each of the proto-tiles that make up the lattice can result in array layouts with low peak sidelobe levels over very large bandwidths [43,45,48]. In this manner, only the locations of a small set of elements need to be optimized to create any arbitrarily large array. For example, a 431–element array based on the Danzer tiling set exhibits a 22:1 frequency bandwidth with peak sidelobe levels below -10 dB.

Lastly, another planar array design technique has been proposed that is based on space-filling curves. In [49], elements are placed at optimized locations along a Peano-Gosper space filling curve. The curves can be applied recursively and tiled to create very large arrays while only requiring optimization of a few element locations. This makes them particularly attractive from a manufacturing standpoint, where multiple identical tiles can be fabricated and assembled into full array.

1.5.3 Volumetric Arrays

Volumetric arrays are often created by conforming a planar array to a non-planar surface [50–53]. In contrast to linear and planar arrays which form main beams
when all the elements have zero phase, the elements in volumetric arrays must have proper (and varying) phase excitation for a coherent, efficient main beam to form. In addition, unlike linear and planar arrays, volumetric arrays do not have monotonically increasing peak sidelobe levels as minimum element spacing (frequency) increases. For this reason, any attempt to design an ultra-wideband volumetric array layout must consider the array factor over the entire bandwidth rather than just at the highest minimum spacing (or frequency).

In spite of these issues, a powerful design technique using three-dimensional aperiodic tilings has been proposed, similar to those used in planar arrays. In [14, 48], a four–tetrahedron Danzer tiling set is used where the location of elements inside each tetrahedron are optimized using a genetic algorithm. In one example, the GA is applied to optimize the number and position of elements inside each proto-tile with a cost function aimed at minimizing the peak sidelobe level over 20 minimum element spacing (frequency) points spanning \(0.5\lambda \leq d_{\text{min}} \leq 15\lambda\) for a frequency bandwidth of 30:1. The array is truncated to a circular aperture with 720 elements and a maximum peak sidelobe level of -10.3 dB over the entire bandwidth is obtained.
New Ultra-Wideband Array Design Techniques

2.1 Introduction

In Chapter 1, a brief background on array layout design with special focus on ultra-wideband techniques was given. This foundation illustrates the great leaps that various methods in ultra-wideband array layout design have made in the past decade and leads to the motivations for the new methods presented in this chapter. For example, the linear polyfractal array design technique can be used to create very large arrays with impressive performance, however, the problem reduction method can lead to insufficient flexibility when designing small arrays. In this case, a more direct approach can be extremely useful, as is demonstrated by the technique introduced in Section 2.3, which uses the relatively new and powerful optimization algorithm covered in Section 2.2.

In Section 2.4, some new ultra-wideband planar array techniques are introduced. The first type uses the same optimization strategy as the compact linear array design method to create arrays based on rotationally symmetric tiles. The second is a non-optimized variation that came about as a replacement for large aperture antennas, where high directivity is desired along with electronic beam steering capability. This technique also has many manufacturing advantages, as will be
shown in Section 2.4.2.

2.2 The Covariance Matrix Adaptation Evolutionary Strategy

Global optimization strategies are a very popular tool not only for the design of aperiodic antenna array layouts, but for many different electromagnetics devices in general [33,35,36,54–78]. They are particularly useful for minimization of functions that are noisy, discontinuous, poorly conditioned, badly scaled, and are unable to be effectively or efficiently solved with conventional techniques such as direct search or Newton’s method. Often, electromagnetics design problems generate cost functions with many of these properties. The recently developed covariance matrix adaptation evolutionary strategy (CMA-ES) has stood out as a very powerful, efficient, and easy to use algorithm for these types of problems [79–84]. An overview of the algorithm is given here as it used extensively for the design of linear and planar ultra-wideband array layouts.

Evolutionary strategy parameters such as mutation and crossover rates with the commonly used genetic algorithm (GA), or nostalgia or social constants with particle swarm optimization (PSO) are typically chosen beforehand and remain fixed during the course of optimization. However, not only does this leave the decision up to a “best guess” for the user (although there are usually suggested values for each algorithm), but the ideal set of strategy parameters is likely problem dependent and may also change throughout the optimization. Modern self-adaptive strategies such as CMA-ES automatically adjust mutation rates according to progress, accounting for changing function landscapes and attempting to make the most progress in the fewest number of algorithm iterations. CMA-ES operates by moving and reshaping a multi-variate Gaussian search distribution about the search space in an attempt to find the global function minimum. Several internal strategy parameters, such as evolution paths, are utilized to give the algorithm its self-adaptive properties. In addition, the algorithm makes use of cumulation to dampen the adaptation of the covariance matrix to effectively work with small
CMA-ES uses several operating constants, vectors, and matrices for its operation. The strategy parameters and their symbols and descriptions are given in Table 2.1. Although there seems to be many strategy parameters to choose, all but the population size $\lambda$ are determined by the properties of the problem, and many are then determined by the choice of $\lambda$. The internal operating parameters are given in Table 2.2. These contain the information about the search distribution, evolution paths to determine past properties of the distribution, and information about population members.

For illustration purposes, a simple canyon test function shown in Fig. 2.2 will be used to demonstrate operation of the algorithm. The test function was specifically designed to have diagonal trenches to show how CMA-ES effectively traverses an
irregular terrain along inseparable search spaces. The algorithm is initialized by setting the initial distribution position \( \mathbf{m} \) and shape \( \sigma^2 \mathbf{C} \) as well as choosing a population size. The initial distribution position is usually selected randomly, with \( \sigma^2 \mathbf{C} \) chosen such that the standard deviation of the multi-dimensional distribution is one-third of the range of each optimizable parameter. After the problem is formulated, the initial position is typically set randomly inside the search boundary, although, like most evolutionary strategies, it can be preset to a specific position if information is known about the problem. The initial shape is usually set such that for each parameter, the distribution has a standard deviation of one-third of its range. This leaves the user to choose only the population size, for which

\[
\lambda \geq 4 + \lfloor 3 \ln(n) \rfloor
\]  

(2.1)
is recommended [80], with larger populations resulting in increased robustness and

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)</td>
<td>Determined by problem</td>
<td>Number of optimizable parameters</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>(\lambda \geq 4 + \lfloor 3 \ln(n) \rfloor)</td>
<td>Populations size or number of children</td>
</tr>
<tr>
<td>(\mu)</td>
<td>(\lfloor \lambda/2 \rfloor)</td>
<td>Number of parents, typically (\lambda/2)</td>
</tr>
<tr>
<td>(w_i)</td>
<td>(\frac{\ln(\mu' + 0.5) - \ln i}{\sum_{j=1}^{\mu} \ln(\mu' + 0.5) - \ln j}) where (\mu' = \lambda/2)</td>
<td>Selection and recombination weights</td>
</tr>
<tr>
<td>(\mu_{eff})</td>
<td>((\sum_{i=1}^{\mu} w_i^2)^{-1})</td>
<td>Variance effective selection mass</td>
</tr>
<tr>
<td>(c_\sigma)</td>
<td>(\frac{\mu_{eff} + 2}{n + \mu_{eff} + 3})</td>
<td>Learning rate for cumulation of the step size control</td>
</tr>
<tr>
<td>(d_\sigma)</td>
<td>(1 + c_\sigma + 2 \max(0, \sqrt{\frac{\mu_{eff} - 1}{n+1}} - 1))</td>
<td>Damping parameter for the step-size update</td>
</tr>
<tr>
<td>(c_c)</td>
<td>(\frac{4 + \mu_{eff}/n}{4 + n + 2 \mu_{eff}/n})</td>
<td>Learning rate for cumulation of the rank-one update of the covariance matrix</td>
</tr>
<tr>
<td>(c_1)</td>
<td>(\frac{2}{(n+1.3)^2 + \mu_{eff}})</td>
<td>Learning rate for the rank-one update of the covariance matrix</td>
</tr>
<tr>
<td>(c_\mu)</td>
<td>(\min(1 - c_1, \frac{2 \mu_{eff} - 4 + 2 \mu_{eff}}{(n+2)^2 + \mu_{eff}}))</td>
<td>Learning rate for the rank-(\mu) update of the covariance matrix</td>
</tr>
</tbody>
</table>
global search capacity at the cost of slower optimization (in the form of more function calls). Once $\lambda$ is selected, all of the strategy parameters in Table 2.1 can then be computed. The evolution paths in Table 2.2 are both set to 0 upon initialization as well. The initial distribution configured to operate on the test function in Fig. 2.2 is shown in Fig. 2.3 (at iteration 0). A small initial step-size is used in this example since the function is simple and it better illustrates the ability of CMA-ES to easily traverse the search space. Additionally, a large population size is again used for illustrative purposes, as it tends to generate a more regular movement of the distribution. Smaller population sizes tend to have more sporadic movement of the mean about the search space and require more iterations, yet often result in a fewer number of function evaluations (NFE) being required.

With the initial distribution and evolution paths configured, the algorithm is ready to begin the first round of sampling. In order to sample from the distribution $\mathcal{N}(\mathbf{m}, \sigma^2 \mathbf{C})$, the covariance matrix must first be broken up into its eigenvectors $\mathbf{B}$ and eigenvalues $\mathbf{D}$. This is commonly done through principle component analysis (PCA), also called eigen-decomposition. For optimizations where cost function calls are very fast, PCA can consume a significant percentage of the total CPU time. However, for electromagnetics design problems, cost function calls are usually the primary time consumer due to the need for computation-intensive simulations.

Table 2.2: Symbols and descriptions of the internal operating parameters of CMA-ES.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma \in \mathbb{R}^+$</td>
<td>Step-size</td>
</tr>
<tr>
<td>$\mathbf{m} \in \mathbb{R}^{n \times n}$</td>
<td>Distribution mean</td>
</tr>
<tr>
<td>$\mathbf{C} \in \mathbb{R}^{n \times n}$</td>
<td>Covariance matrix</td>
</tr>
<tr>
<td>$\mathbf{B} \in \mathbb{R}^{n \times n}$</td>
<td>Columns of $\mathbf{B}$ are eigenvectors of $\mathbf{C}$</td>
</tr>
<tr>
<td>$\mathbf{D} \in \mathbb{R}^{n \times n}$</td>
<td>Diagonal elements of $\mathbf{D}$ are eigenvalues of $\mathbf{C}$</td>
</tr>
<tr>
<td>$\mathbf{p}_\sigma \in \mathbb{R}^n$</td>
<td>Conjugate evolution path</td>
</tr>
<tr>
<td>$\mathbf{p}_e \in \mathbb{R}^n$</td>
<td>Evolution path</td>
</tr>
<tr>
<td>$\mathbf{z}_k \in \mathbb{R}^n$</td>
<td>A sample from the distribution $\mathcal{N}(0, 1)$</td>
</tr>
<tr>
<td>$\mathbf{y}_k \in \mathbb{R}^n$</td>
<td>A sample from the distribution $\mathcal{N}(0, \mathbf{C})$</td>
</tr>
<tr>
<td>$\mathbf{x}_k \in \mathbb{R}^n$</td>
<td>An offspring or potential candidate solution</td>
</tr>
</tbody>
</table>
Figure 2.2: Canyon test function to be used for illustrating the operation of CMA-ES. The minimum function value and goal is located at \((x, y) = (0.764, 0.724)\).

Once \(B\) and \(D\) are obtained, the distribution can be sampled by first sampling from a standard Normal distribution (a simple procedure for computers)

\[
z_k \sim \mathcal{N}(0, 1) \quad (2.2)
\]

and then transforming to the desired distribution through

\[
y^g_k = BDz_k \quad (2.3)
\]

and

\[
x^g_k = m + \sigma y^g_k \quad (2.4)
\]

giving our first set of candidate solutions (or population members) at iteration \(g = 0\). Now, the population is evaluated according to the user-defined cost function to return a single cost value for each member. It is not uncommon here for a large cluster of computers to each share in the burden of computing the cost of one or more of the population members, thus saving the user a significant amount of total optimization time. After the entire population is evaluated, the members \(y^g_k\) and \(x^g_k\) are sorted (where the sorted members are identified by \(y^g_{1:\lambda}\) and \(x^g_{1:\lambda}\)) according
to cost value and are used to form the new mean using

$$\langle y \rangle_w = \sum_{i=1}^{\mu} w_i y_{1,\lambda}^g$$

(2.5)

and

Figure 2.3: Operation of CMA-ES on the two-dimensional cavern test function shown in Fig. 2.2. A population size of $\lambda = 30$ is used with the algorithm initialized to $m = (0.2, 0.7)$, $\sigma = 0.1$, and $C = I$. The iteration number is given in the upper-left corner of each square. The dashed ellipse represents a contour of equal likelihood of selection. The $\mu$ selected children from each iteration are represented by a green circle; the $(\lambda - \mu)$ discarded children with a red $\times$. The arrow represents the movement of the mean at each iteration. A smaller than nominal initial step-size is used to highlight the ability of the algorithm to traverse valleys of low cost in inseparable search spaces.
\[ m^{g+1} = m^g + \sigma^g \langle y \rangle_w, \]  
(2.6)

which is also equivalent to simply adding a weighted average of the \( \mu \) best members in

\[ m^{g+1} = \sum_{i=1}^{\mu} w_i x_i^g. \]  
(2.7)

After the new mean is computed, the conjugate evolution path and evolution path are updated using

\[ p_{\sigma}^{g+1} = (1 - c_\sigma)p_\sigma^g + \sqrt{c_\sigma(2 - c_\sigma)\mu_{eff}} (C^g)^{-1/2}\langle y_w \rangle \]  
(2.8)

and

\[ p_{c}^{g+1} = (1 - c_c)p_c^g + \sqrt{c_c(2 - c_c)\mu_{eff}} \langle y_w \rangle, \]  
(2.9)

respectively. These contain normalized (2.8) and non-normalized (2.9) distribution movement history that is used for updating the step size and covariance matrix, respectively. Note that \( C^{-1/2} \) can be found through the identity

\[ C^{-1/2} = BD^{-1}B^T, \]  
(2.10)

for which \( D^{-1} \) can be computed easily as it is a diagonal-only matrix. Next, the step size is updated using

\[ \sigma^{g+1} = \sigma^g e^{\frac{c_\sigma}{2\alpha}} \left( \frac{\|p_{\sigma}^g\|\|N(0, 1)\|}{\|p_{\sigma}^g\|\|N(0, 1)\|} \right)^{-1} \]  
(2.11)

where \( E\|N(0, 1)\| \) is the expected value of the \( N \)-dimensional standard normal distribution given by the approximation

\[ E\|N(0, 1)\| \approx \sqrt{N} \left( 1 - \frac{1}{4N} + \frac{1}{21N^2} \right). \]  
(2.12)

Lastly, the covariance matrix is updated using
\[
C^{g+1} = ((1 - c_1 - c_\mu)C^g + c_1 D D^T) + c_\mu \sum_{i=1}^{\mu} w_i Y_{i;\lambda} Y_{i;\lambda}^T,
\]

where the first term is the historical contribution (cumulation), the second term is the rank-one update (elongation of distribution along the direction of search), and the third term is the rank-\(\mu\) update (formation of distribution from successful search steps). Note that the update signals \(h_\sigma\) and \(\delta(h_\sigma)\) are omitted for simplicity from (2.9) and (2.13) in the aforementioned implementation of CMA-ES. With the updated mean, step-size, and covariance matrix, the next round of sampling can begin and the process repeats until the desired function value is reached, the algorithm converges, or time is expired. Convergence is usually indicated when the average population cost is equal to the cost of the best population member to within several significant digits. If the algorithm has not converged in a specified number of iterations, it can be easily resumed without any loss of optimization performance.

The remaining blocks in Fig. 2.3 show the algorithm progressing along the search space, eventually finding the desired function goal of 0.001 over 16 iterations. It is easy to observe how the algorithm effectively operates by extending the search distribution along the path of movement, thus ensuring that future steps will yield solutions with lower cost. Conversely, when the movement begins to slow, the step-size shrinks, and the algorithm begins to search more locally.

While implementing CMA-ES is much more complex compared to a simple genetic algorithm or particle swarm technique, these challenges are balanced by the considerable advantage that CMA-ES offers in terms of performance improvement over the simpler strategies [82–84]. For common electromagnetics problems, where cost function calls can take minutes or longer per evaluation, the optimization time can be reduced to a fraction of what was previously required, sometimes subtracting hours or days from the optimization. For example, in [84] the time to design a stacked-patch antenna with CMA-ES was reduced to 38% of what was required with PSO (18 hours versus 47 hours). In addition to being a fast algorithm, CMA-
ES also tends to be robust even with the smallest recommended population size given by Eq. (2.1). The stacked-patch antenna optimization in [84] was carried out reliably with a population size of 10 compared to the population of 40 particles needed by PSO to be reliable. More detail on this design problem and a comparison using test functions can be found in Appendix A.

2.3 Compact Linear Arrays

A great deal of attention has been recently given to the optimization of aperiodic antenna array layouts that do not exhibit the grating lobes associated with periodic arrays operated over an extended bandwidth. Most design methods incorporate an optimization strategy with a technique for parameter reduction to allow creation of large arrays as mentioned in Section 1.5. For example, the polyfractal arrays in [37–39] use fractal tree generator structures to build large arrays from relatively small sets of defining parameters. In this manner, arrays with thousands of elements can be created without overwhelming the optimization tool (in this case, the genetic algorithm). One drawback to these parameter reduction methods is that with much smaller size arrays, the solutions become limited due to the inadequate number of describing parameters. For these smaller arrays, optimizing the location of each element individually can be beneficial if the optimizer can successfully handle the large set of parameters (approximately one parameter per antenna element). This direct optimization of antenna element locations within a linear array has been previously attempted with up to 24 elements in [34], yielding fruitful aperiodic low-sidelobe designs for steered arrays (beam steering is analogous to an increase in bandwidth).

A similar array design technique will be applied here using many more elements at a much greater bandwidth, allowing the potential creation of better performing UWB arrays. One portion of the research conducted involves comparison of CMA-ES to other optimization strategies when applied to electromagnetics problems. For several of the array designs, the particle swarm optimization technique is also applied to observe the performance differences of the arrays generated with the
two algorithms. The proposed array geometry for these compact linear array is shown in Fig. 2.4. The design is relatively simple, and has the benefit of implicitly controlling the minimum element spacing of the array, which is of principle concern for control of the array’s bandwidth. The size of an antenna element is typically around 0.5λ at the lowest operating frequency, so they must be placed no closer than this in a proposed array geometry. In addition, this spacing is enforced to reduce mutual coupling to levels that do not significantly degrade antenna input impedance. At higher frequencies, the electrical distance in wavelengths between elements proportionally increases. Array factors can be computed using

$$AF(\theta, \phi) = \sum_{n=1}^{N} e^{j2\pi x_n \sin \theta} \text{ where } x_n = \begin{cases} 0 & n = 1 \\ \sum_{m=1}^{n-1} d_m & n \geq 2 \end{cases}$$ \hspace{1cm} (2.14)$$

where $x_n$ is in wavelengths, hence it is relatively straightforward to simply scale the element positions to emulate a higher frequency. Since the peak sidelobe level is the primary interest in the array design and this sidelobe level monotonically increases as frequency increases, the array factor and peak sidelobe level of the array need only be computed at the upper frequency of the array. For example, to design an array with a 10:1 frequency bandwidth, the peak sidelobe level of the array needs to be computed using (2.14) at a minimum element spacing of 5λ. Using the geometry in Fig. 2.4, the elements must be arranged such that $\min(d) \geq 5\lambda$, a task easily accomplished by setting parameter bounds in the optimization.

Figure 2.4: Coordinate system and array layout for the compact linear ultrawideband arrays.

Although most array designs will be tailored to their specific application, some generic designs can be created to determine the array design capabilities. Arrays with 25, 50, 75, and 100 elements are optimized at a minimum element spacing of
$4\lambda$ for an 8:1 frequency bandwidth. In order to gather some statistical significance to the array performance, five optimizations (seeds) were conducted with each strategy and array size. The optimizations were truncated according to Table 2.3, since no specific function goal is desired but rather the lowest function value possible in the allotted amount of time. Fig. 2.5 shows the results of the optimizations, indicating that for any equivalent optimization time (number of array evaluations), CMA-ES always produces a better UWB array design. In these cases, the worst performing seed of CMA is always better than the best performing seed of PSO. Fig. 2.5 also highlights the progress of each algorithm though the initial mean best indicators, illustrating how PSO yields only marginal improvements over the initial populations with the larger size arrays. CMA-ES, however, produces significant performance improvements over the initial populations for all of the array sizes.

The largest optimizations of 100 elements, requiring a significant amount of time (nearly 400 hours per seed on a single process for $2 \times 10^6$ array evaluations), were initially only run with 17 and 100 population members at the iteration counts given in Table 2.3. However, in the interest of determining if PSO could surpass CMA-ES in array performance with a larger scale optimization, a population size of 500 with 5,000 iterations was permitted for PSO. Despite the potential advantage, no benefit over CMA-ES was observed, even compared to the smallest optimization (17 population members).

The layouts and arrays factors of some of the designs optimized with CMA-ES in Fig. 2.3 are given in Fig. 2.6. A random seed (with arbitrary performance) from each set of optimization trials with a population size of 100 is selected to observe the array factor properties. It can be seen that many of the sidelobes of the array factor for each of the designs are equal in magnitude and equal to the peak sidelobe level, an indication that the designs are well optimized. The bandwidths of these four array designs over a frequency range of 20:1 are also give in Fig. 2.7. Even though the arrays were optimized for an 8:1 frequency bandwidth, the designs possess low sidelobe levels over the illustrated range. Both the 75-element and the 100-element designs maintain a peak sidelobe level below -10 dB up to a 20:1 bandwidth.
In addition to creating these generic ultra-wideband antenna arrays, several designs were optimized to compete with array layouts found in the literature. Small example designs were selected to best match the capabilities of the method introduced in this section. One example is based on a 46-element polyfractal array found in [37], which has been optimized for minimum peak sidelobe level at a minimum element spacing of $0.5\lambda$. In [37], the optimized design has a resulting peak sidelobe level of -19.17 dB. For comparison, the direct optimization technique developed here is applied to design a 46-element aperiodic array subject to $0.5\lambda \leq d_m \leq 5\lambda$. A moderate population size of 50 members was used with a maximum of 1000 iterations. The evolution of the five seeds is shown in Fig. 2.8a, with the array design layouts given in Fig. 2.9a. The final performance of the seeds is given in Table 2.4.

Another example was selected for comparison, the 55-element raised-power series (RPS) array reported in [85]. This array was optimized at a minimum element spacing of $10\lambda$, yielding a typical frequency bandwidth of 20:1 at the resulting peak

Table 2.3: Number of iterations and function evaluations permitted for each optimization for both CMA-ES and PSO seeds. The optimization time is predominantly determined by the number of array evaluations.

<table>
<thead>
<tr>
<th>Array Size (Elements)</th>
<th>Population Size</th>
<th>Maximum Iterations</th>
<th>Maximum Array Evaluations</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>13</td>
<td>2,500</td>
<td>32,500</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>2,500</td>
<td>250,000</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>3,000</td>
<td>750,000</td>
</tr>
<tr>
<td>50</td>
<td>15</td>
<td>5,000</td>
<td>75,000</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>5,000</td>
<td>500,000</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>2,500</td>
<td>625,000</td>
</tr>
<tr>
<td>75</td>
<td>16</td>
<td>25,000</td>
<td>400,000</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>10,000</td>
<td>1,000,000</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>5,000</td>
<td>2,500,000</td>
</tr>
<tr>
<td>100</td>
<td>17</td>
<td>50,000</td>
<td>850,000</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>20,000</td>
<td>2,000,000</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>5,000</td>
<td>2,500,000</td>
</tr>
</tbody>
</table>
sidelobe level of -9.55 dB. Again, five seeds with a population size of 50 members were used, except the element spacing ranges were changed to $10\lambda \leq d_m \leq 30\lambda$. The evolution results for the five seeds are shown in Fig. 2.8b with the corresponding array layouts given in Fig. 2.9b. The final performance of the seeds is provided in Table 2.4.

For all of these examples, the worst seed of each CMA-ES array design yields better performance than their literature counterpart. Hence, for these relatively small array designs, this method proves very useful for obtaining the best performance with a limited set of elements. At their optimized minimum element spacings, the worst of the 46- and 55-element CMA-ES optimized designs produce a respective 4.4 dB and 2.15 dB reduction in peak sidelobe levels versus their polyfractal and

Figure 2.5: Summary of the CMA-ES and PSO ultra-wideband array optimizations for various array and population sizes according to Table 2.3. For seed trials where the population and element counts are the same, CMA-ES and PSO have the same run time (number of array evaluations), yielding a fair comparison between the two algorithms. The initial mean best indicators show the progress of the evolutionary strategies by pointing out where the algorithm begins. Each point is computed by finding the best result of the initial arrays of each seed (the best of the randomly selected initial populations) and taking the average of the five seeds.
Figure 2.6: Layouts and normalized array factors for several of the designs shown in Fig. 2.3. The layout and array factor (at $d_{\text{min}} = 4\lambda$) of the first CMA-ES optimized seed with a population size of 100 is given for array sizes of (a) 25 elements, (b) 50 elements, (c) 75 elements, and (d) 100 elements.

RPS counterparts. In Fig. 2.10, the performance of the two best arrays of the five 46- and 55-element designs are examined over a large bandwidth as in [37] and [85].
Figure 2.7: Bandwidths of the four arrays selected in Fig. 2.6 over a 20:1 frequency range.

Table 2.4: Properties of the five seeds for each of the optimized designs.

<table>
<thead>
<tr>
<th>Array</th>
<th>( d_{\text{min}} )</th>
<th>Mean Avg. Spacing</th>
<th>Peak Sidelobe Level (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>46-element</td>
<td>0.5( \lambda )</td>
<td>0.70( \lambda )</td>
<td>-25.73 -24.49 -23.57</td>
</tr>
<tr>
<td>55-element</td>
<td>10( \lambda )</td>
<td>19.05( \lambda )</td>
<td>-11.83 -11.77 -11.70</td>
</tr>
</tbody>
</table>

2.4 Planar Arrays

In addition to the design of linear arrays, some attention was brought to planar array design to determine if the technique in Section 2.3 could be similarly be applied to improve performance of these arrays. Both of the planar array design techniques introduced in this chapter exploit the use of rotational symmetry to simplify the structure for optimization and/or manufacturing. Rotational symmetry in lattice structures, particularly crystals, has recently gained attention due to the ability to synthetically fabricate quasicrystals [86] in aluminum and magnesium alloys in [87], for which Shechtman was awarded a Nobel Prize in Chemistry in 2011.
(a) 46-element arrays: All of the seeds surpass the performance of the polyfractal array design after the 155th iteration (7,750 array evaluations).

(b) 55-element arrays: All of the seeds surpass the performance of the RPS array design after the 67th iteration (3,350 array evaluations).

Figure 2.8: CMA-ES seed evolutions for the two array comparisons.

(a) 46-element arrays: Array layouts of the five designs at the 1,000th iteration.  
(b) 55-element arrays: Array layouts of the five designs at the 5,000th iteration.

Figure 2.9: Layouts of the optimized seeds for the two array comparisons.

The optical properties of quasicrystals have been a topic of immense interest in recent years [86-90]. Aperiodic tilings or quasicrystals were initially proposed by mathematicians in the 1960s. While these structures do not exhibit periodicity, they are completely deterministic and possess long-range order and display orders of rotational symmetry (8, 10, 12, etc.) which are not mathematically possible in periodic structures. The diffraction patterns of these aperiodic structures display the same order of the rotational symmetry as the underlying geometry. The diffraction pattern is much denser than that of a periodic structure in that there are many more sidelobes of lower amplitude rather than regularly spaced peaks of equal amplitude (i.e. grating lobes). Due to the lack of translational symmetry, the diffraction pattern is devoid of Bragg diffraction points, however it possesses a
Figure 2.10: Peak sidelobe level performance of the two best seeds shown in Figs. 2.8 and 2.9 over an extended bandwidth. Although the 46- and 55-element arrays are optimized at minimum element spacings of $0.5\lambda$ and $10\lambda$, respectively, they yield good sidelobe suppression well beyond.

much denser pattern of local discrete resonances which can be attributed to different spacings in the structure. This lack of Bragg diffraction peaks (grating lobes) makes aperiodic element layouts ideal for broadband antenna arrays. Furthermore, it has been shown that the value of local resonances, which from an array theory point of view correspond to sidelobes, can be further reduced by optimization [44].

Planar arrays based on certain classes of quasicrystalline aperiodic tilings have been demonstrated in [45, 48]. The tilings consist of a set of prototiles that when arranged properly, can fill a plane with no gaps. Elements are placed either at the vertices of tiles, or in the case of [45, 48], they are positioned in optimized locations inside the tiles. In this manner, large arrays can be generated by simply optimizing the positions of elements inside a small set of tiles (i.e. prototiles). The full array geometry is determined by truncation of the tiling set to the desired aperture size (or number of elements) and shape. Volumetric arrays can similarly be formed as in [14, 48] by using three-dimensional aperiodic tiling sets.

Rotational symmetry is an interesting property of many of these aperiodic tiling or
quasicrystal configurations. One of the more common aperiodic tiling sets known as the Danzer tiling [86, 91], which has been used in [43, 45, 48] is seven-fold rotationally symmetric and produces radiation patterns naturally void of grating lobes when antenna array elements are placed at tile vertices. Rotational symmetry has also been utilized in the design of thinned arrays with no grating lobes in [92]. The natural lack of translational symmetry in rotationally symmetric structures lends itself to an excellent framework for the design of aperiodic antenna arrays with broadband properties.

The new design concepts here aim to further exploit rotational symmetry to aid in achieving optimal ultra-wideband performance in large planar arrays. The first technique utilizes CMA-ES to directly optimize the locations of antenna elements in an $n$-fold rotationally symmetric planar array. The second method uses simple periodic lattices inside of slices of an $n$-fold rotationally symmetric structure to realize ultra-wideband array layouts which retain most of the manufacturing ease of standard narrowband fully populated periodic arrays.

### 2.4.1 Optimized Rotationally Symmetric Arrays

The first planar array design method uses CMA-ES to optimize the locations of isotropic antenna elements inside of a single slice of an $n$-fold rotationally symmetric structure such that the peak sidelobe level is minimized and the minimum spacing requirements are met. Fig. 2.11 illustrates the geometry with a seven-fold example, where $\Delta \phi$ is determined according to the simple relation

$$\Delta \phi = \frac{2\pi}{n}. \hspace{1cm} (2.15)$$

Configuration of the element positions in the pie-shaped wedge defined by $(r_{\text{min}} \leq r_p \leq r_{\text{max}}, 0 \leq \phi_p \leq \Delta \phi)$ requires two optimizable parameters per element. Depending on the number of folds in the symmetry, it is easy to create large arrays without resulting in an unreasonable number of optimizable parameters. Element spacings are typically constrained to $0.5\lambda$ at the lowest operating frequency of the array to avoid overlapping elements and undesirably large mutual coupling.
For arrays that are able to operate effectively with no grating lobes and low side-lobes at a minimum element spacing of $b\lambda$ (where $b \geq 0.5$), their resulting frequency bandwidth is $2b : 1$. In the case considered here, where limits are only placed on the range of $r$ and $\phi$ for element positions, there is no intrinsic method for ensuring that elements are not placed closer than is desired ($d_{\text{target}}$). Simply scaling the array to meet a requirement can result in an extremely large aperture if the optimization strategy happens to place two elements very close together. Therefore, the ability to enforce element spacing is included as part of the cost function, which typically would only include a targeted value for a single peak sidelobe level at a specific minimum element spacing (or frequency bandwidth).

![Figure 2.11: Geometry of the $n$-fold rotationally symmetric planar array, where $n = 7$. For each element at $(r, \phi)$, there lies an element at $(r, m\Delta \phi + \phi)$ for $m = 1, \ldots, (n - 1)$.](image)

Two example arrays were optimized to determine the capabilities of the design technique. A cost function of

$$f_{\text{cost}} = SLL^{dB} + c(d_{\text{target}} - \min(d_{\text{min}}, d_{\text{target}}))^2$$

was employed that aimed to keep the minimum element spacing above $d_{\text{target}}$ and minimize the peak sidelobe level of the array factor. Alternatively, a multi-objective approach could be employed to design a range of arrays with various sidelobe performances versus bandwidth [93] instead of fixing the tradeoff via the constant $c$ in (2.16). To create the array, the parameter set generated by CMA-ES
for each population member is split into the $r_p$-values and $\phi_p$-values for each element, and isotropic sources are placed at those locations. Then, $n$-fold rotational symmetry is applied to form the full array. Next, the minimum spacing between the array elements ($d_{\text{min}}$) is determined for the second portion of the cost function defined in (2.16). Lastly, the array factor for an array with $P$ elements is computed using [2,3]

\[
AF(\theta, \phi) = \sum_{p=1}^{P} e^{j2\pi r_p \cos \phi_p \sin \theta \cos \phi} e^{j2\pi r_p \sin \phi_p \sin \theta \cos \phi} \tag{2.17}
\]

which reduces to

\[
AF(\theta, \phi) = \sum_{p=1}^{P} e^{j2\pi r_p \sin \theta \cos(\phi_p - \phi)} \tag{2.18}
\]

where $r_p$ is in wavelengths, after which the peak sidelobe level can be determined. A very useful property of the $n$-fold rotationally symmetric arrays are that their associated array factors also possess similar symmetry when the array is steered to broadside. For arrays where $n$ is even, the array factor possesses $\phi$-symmetry of $\Delta \phi$. When $n$ is odd, the symmetry is $\Delta \phi/2$. Therefore, at the very most, the array factor needs only to be computed for $(0 \leq \theta \leq \pi/2, 0 \leq \phi \leq \Delta \phi)$, and $(0 \leq \theta \leq \pi/2, 0 \leq \phi \leq \Delta \phi/2)$ when $n$ is odd. This can save significant amounts of computational time and result in greatly reduced total optimization times, especially if $n$ is large.

Although the design techniques introduced here consider only broadside arrays (to take advantage of the aforementioned pattern symmetry), it is a straightforward modification to extend them to include broadband wide-angle scanning. The visible region of the array factor for an array steered to $(-90^\circ \leq \theta_0 \leq 90^\circ, 0^\circ \leq \phi_0 \leq 360^\circ)$ with a minimum element spacing of $d$ wavelengths lies inside the visible region of the unsteered array at a minimum spacing of $2d$ wavelengths [3]. Therefore, ultra-wideband phased arrays with low sidelobe levels can be easily created using this design technique by appropriately extending the optimization bandwidth. Usually, the performance of these array designs is still acceptable beyond the optimization bandwidth so this extension is not necessary.
The first optimized design is chosen to have 11-fold symmetry with 20 elements per fold, requiring a total of 40 parameters. The elements are limited to the region defined by \((3\lambda \leq r_p \leq 60\lambda, 0 \leq \phi_p \leq 2\pi/11)\) for a 120\(\lambda\) circular aperture. A \(d_{\text{target}}\) of 5\(\lambda\) was chosen for a desired 10:1 frequency bandwidth, with the fitness constant \(c\) \((\text{in (2.16)})\) set to unity. A population size of 16 was used with a limit of 1000 CMA-ES iterations, eventually reaching a cost value of -16.13 with a peak sidelobe level of -16.21 dB and a minimum element spacing of 4.72\(\lambda\). The number of iterations was selected to ensure the algorithm is fully converged.

Increasing the constant \(c\) in Eq. (2.16) will likely yield a result where the targeted bandwidth is achieved at the cost of a slightly increased peak sidelobe level. Increasing the range of \(r\) will also likely allow the minimum spacing (bandwidth) requirement to be satisfied at the expense of a larger aperture. These designs often yield low peak sidelobe levels beyond their optimized design bandwidth, however, making these adjustments unnecessary. The optimization required approximately 30 hours on six 2.4 GHz Intel Xeon processor cores. The optimized element locations and array factor are shown in Fig. 2.12. A cut of the array factor is given in Fig. 2.13 and the performance of the array over an extended bandwidth is demonstrated in Fig. 2.16.

The second optimized design is chosen to have 15-fold symmetry with 40 elements per fold, requiring 80 parameters. The elements are limited to the region defined by \((2\lambda \leq r_p \leq 100\lambda, 0 \leq \phi_p \leq 2\pi/15)\) for a 200\(\lambda\) circular aperture and a \(d_{\text{target}}\) of 2.5\(\lambda\) was chosen for a desired 5:1 frequency bandwidth. A population size of 18 was used with a limit of 1500 CMA-ES iterations, eventually reaching a cost value of -18.85 with a peak sidelobe level of -18.85 dB and a minimum element spacing of 2.68\(\lambda\). The optimization required approximately 270 hours on six 2.4 GHz Intel Xeon processor cores. The optimized element locations and array factor are shown in Fig. 2.14. A cut of the array factor is given in Fig. 2.15 and the array performance over an extended bandwidth is shown in Fig. 2.16. Even though the array was optimized for only a 5:1 bandwidth (a 5.4:1 bandwidth was achieved), it is observed that the design yields very low peak sidelobe levels over a much broader
bandwidth.

Figure 2.12: Geometry of the 220-element optimized array (a) and one hemisphere of the normalized array factor of the 220-element optimized array at $d_{\text{min}} = 4.72\lambda$ (b). Note that the array factor has 22-fold symmetry, whereas the array has 11-fold symmetry.

Figure 2.13: Cut of the array factor at $\phi = 0^\circ$ of the 220-element optimized array at $d_{\text{min}} = 4.72\lambda$. 
Figure 2.14: Geometry of the 600-element optimized array (a) and one hemisphere of the normalized array factor for the 600-element optimized array at $d_{\text{min}} = 2.68\lambda$ (b).

Figure 2.15: Cut of the array factor at $\phi = 0^\circ$ of the 600-element optimized array at $d_{\text{min}} = 2.68\lambda$.

2.4.2 Semi-Periodic Rotationally Symmetric Arrays

In addition to optimizing the locations of elements inside the $\Delta \phi$ slices shown in Fig. 2.11, it is also possible to insert sections of typically narrowband periodic arrays to yield a wider bandwidth. Arrays can be generated with triangular or square lattices, however, care must be placed in selecting the number of folds in the symmetry so as to not create any regular periodic arrays (e.g. four-fold symmetry with
Figure 2.16: Bandwidth of the optimized and semi-periodic rotationally symmetric array and a standard square lattice periodic array for broadside operation.

a square lattice seed array). There are two immediate advantages to arrays of this variety. The first is an increase in directivity over the usually very thinned optimized types, due to the denser element packing in equivalent aperture sizes. Secondly, the arrays can be fabricated much more easily due the periodic nature of the array slices. An example of the resulting performance benefit achieved by changing from a six-fold to a seven-fold symmetric array with a minimum element spacing of $4\lambda$ can be seen in Figs. 2.17 and 2.18. Here, a regular hexagonal lattice is used to seed each array wedge section, hence, a six-fold rotationally symmetric array will result in full translational symmetry and grating lobes for large element spacings. The seven-fold array does not possess full translational symmetry of the elements and consequently has some desirable wideband characteristics, yielding a peak sidelobe level of -8.4 dB over an 8:1 frequency bandwidth.

For comparison to the large, 600-element optimized rotationally symmetric designs, two semi-periodic rotationally symmetric arrays (SPRSA) were created. The first utilizes the same symmetry, 15-fold, with a regular hexagonal seed and a minimum element spacing of $2.5\lambda$ (see Fig. 2.19). The second is similar with 11-fold symme-
try, leading to more regularity in the array layout (see Fig. 2.20). The aperture sizes were selected to yield approximately the same number of elements as in the optimized array of Fig. 2.14. A cut of the array factor for each array is shown in Fig. 2.21. Although the directivity of the arrays is approximately the same as that of the optimized design, the semi-periodic arrays use only about 12\% of the aperture area, as represented in Table 2.5. The extended bandwidth of the arrays is shown in Fig. 2.16, demonstrating that they are useful beyond their intended design frequency (minimum spacing), although no longer possessing the extremely low peak sidelobe levels of the optimized arrays.

![Array layout and normalized array factor](image)

Figure 2.17: Array layout (a) and normalized array factor (b) of the six-fold rotationally symmetric array at a minimum element spacing of 4\(\lambda\). With six-fold symmetry, the array forms a regular hexagonal lattice (with the central element omitted), resulting in 0 dB grating lobes for the large element spacings. The array possesses 90 elements with a minimum element spacing of 4\(\lambda\) and a circular aperture constrained to a diameter of 40\(\lambda\).

In addition to creating two semi-periodic arrays with element counts similar to one optimized design, several more hexagonally-seeded arrays were created to examine the effects of different symmetries. Arrays of approximately 300 elements were creating using 5-fold to 21-fold symmetry (including 6-fold symmetry leading to a periodic array) targeting a minimum element spacing of 3\(\lambda\). Table 2.6 gives the
performance of the arrays, where peak sidelobe level, directivity, and normalized aperture area were computed. The area is normalized to the most dense variation, the fully periodic 6-fold array (with an area of $2290\lambda^2$). It is interesting to note that the designs with a number of folds equal to a prime number lend themselves to low sidelobe levels compared to the other designs. The arrays with folds equal to an integer multiple of the seed lattice (which is 6-fold symmetric) had the highest peak sidelobe levels due to increased regularity of the total array lattice.

Generally, aperture area increases with the number of folds due to the vacant central area of the array growing to maintain the minimum element spacing and an approximately fixed number of elements. In addition, elements are more sparse near the fold intersections and, with a larger number of folds, those areas become more frequent. Most of the arrays have a directivity of approximately 25 dB, with the 6-fold periodic design having the greatest directivity of 26.8 dB.

![Array layout (a) and normalized array factor (b) of the seven-fold rotationally symmetric array at a minimum element spacing of $4\lambda$. With seven-fold symmetry, the array yields a peak sidelobe level of -8.4 dB. The array possesses 70 elements with the same aperture size as the previous design. With increasing aperture sizes, the percent change in elements when symmetry is adjusted is reduced.](image)
Figure 2.19: Array layout (a) and normalized array factor (b) of the 15-fold, 600-element semi-periodic rotationally symmetric array.

Table 2.5: Properties of the optimized and semi-periodic rotationally symmetric arrays.

<table>
<thead>
<tr>
<th>Array</th>
<th>$d_{\text{min}}$</th>
<th>Peak SLL</th>
<th>Max. Directivity</th>
<th>Aperture Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>600-element opt., 15-fold</td>
<td>$2.68\lambda$</td>
<td>-18.9 dB</td>
<td>27.8 dB</td>
<td>$\sim 31400\lambda^2$</td>
</tr>
<tr>
<td>220-element opt., 11-fold</td>
<td>$4.72\lambda$</td>
<td>-16.2 dB</td>
<td>23.4 dB</td>
<td>$\sim 11300\lambda^2$</td>
</tr>
<tr>
<td>600-element unopt., 15-fold</td>
<td>$2.5\lambda$</td>
<td>-13.8 dB</td>
<td>27.7 dB</td>
<td>$\sim 3850\lambda^2$</td>
</tr>
<tr>
<td>594-element unopt., 11-fold</td>
<td>$2.5\lambda$</td>
<td>-15.4 dB</td>
<td>27.7 dB</td>
<td>$\sim 3630\lambda^2$</td>
</tr>
</tbody>
</table>

### 2.5 Effects of Element Position Error on Array Performance

In all of the arrays presented thus far, the designs assume perfect element placement with respect to their optimized locations. In realistic fabricated designs, however, this will usually not be the case. For this reason, it is of interest to determine how robust the array performance is when the arrays contain some positional error, similar to the analysis performed in [44]. For high frequency arrays, the antenna elements could potentially all lie on a single fabricated printed circuit board, leading to very small positional errors. For low frequency designs, each antenna
Figure 2.20: Array layout (a) and normalized array factor (b) of the 11-fold, 594-element semi-periodic rotationally symmetric array.

Figure 2.21: Array factor cuts of the 600- and 594-element semi-periodic rotationally symmetric arrays at $\phi = 0^\circ$ and $d_{\text{min}} = 2.5\lambda$.

may reside on its own fixture and positions may be adjusted by hand, leading to a greater positional error. Whatever the case may be, element placement errors will likely be small relative to the operating wavelength due to modern positional and fabrication techniques. In this section, several linear and planar ultra-wideband arrays layouts that have been created in the previous sections will have positional
Table 2.6: Properties of hexagonally-seeded semi-periodic arrays having various symmetry with approximately 300 elements and a minimum element spacing of $3\lambda$.

<table>
<thead>
<tr>
<th>Folds</th>
<th>Elements</th>
<th>Max. Directivity</th>
<th>Peak SLL</th>
<th>Aperture Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>300</td>
<td>25.2 dB</td>
<td>-12.8 dB</td>
<td>1.12</td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td>26.8 dB</td>
<td>0 dB</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>301</td>
<td>24.8 dB</td>
<td>-13.9 dB</td>
<td>1.15</td>
</tr>
<tr>
<td>8</td>
<td>304</td>
<td>24.9 dB</td>
<td>-10.2 dB</td>
<td>1.15</td>
</tr>
<tr>
<td>9</td>
<td>306</td>
<td>24.7 dB</td>
<td>-9.3 dB</td>
<td>1.20</td>
</tr>
<tr>
<td>10</td>
<td>300</td>
<td>24.5 dB</td>
<td>-11.3 dB</td>
<td>1.23</td>
</tr>
<tr>
<td>11</td>
<td>308</td>
<td>25.0 dB</td>
<td>-13.2 dB</td>
<td>1.23</td>
</tr>
<tr>
<td>12</td>
<td>300</td>
<td>25.1 dB</td>
<td>-5.6 dB</td>
<td>1.15</td>
</tr>
<tr>
<td>13</td>
<td>312</td>
<td>25.0 dB</td>
<td>-13.2 dB</td>
<td>1.37</td>
</tr>
<tr>
<td>14</td>
<td>308</td>
<td>24.5 dB</td>
<td>-11.5 dB</td>
<td>1.37</td>
</tr>
<tr>
<td>15</td>
<td>300</td>
<td>25.0 dB</td>
<td>-13.7 dB</td>
<td>1.27</td>
</tr>
<tr>
<td>16</td>
<td>304</td>
<td>24.0 dB</td>
<td>-10.8 dB</td>
<td>1.27</td>
</tr>
<tr>
<td>17</td>
<td>306</td>
<td>25.4 dB</td>
<td>-13.7 dB</td>
<td>1.37</td>
</tr>
<tr>
<td>18</td>
<td>306</td>
<td>24.8 dB</td>
<td>-8.9 dB</td>
<td>1.37</td>
</tr>
<tr>
<td>19</td>
<td>304</td>
<td>25.1 dB</td>
<td>-12.7 dB</td>
<td>1.49</td>
</tr>
<tr>
<td>20</td>
<td>300</td>
<td>24.9 dB</td>
<td>-10.8 dB</td>
<td>1.37</td>
</tr>
<tr>
<td>21</td>
<td>315</td>
<td>25.1 dB</td>
<td>-12.3 dB</td>
<td>1.49</td>
</tr>
</tbody>
</table>

errors applied to them and the resulting performance impacts will be observed. For each design type, the positional error will only be applied according to the dimensionality of the array designs (i.e. along an axis for linear arrays and in a plane for planar arrays).

### 2.5.1 Linear Arrays

Here, the 25-element and 100-element array designs shown in Fig. 2.6 are selected for the positional error analysis. This will allow observation of the effects with relatively small and relatively large array designs produced by the compact linear ultra-wideband design technique. For the 25-element design, the array possesses a peak sidelobe level of -9.77 dB at a minimum element spacing of $4\lambda$. For the
100-element design, the array possesses a peak sidelobe level of -15.44 dB at the same minimum element spacing. It is conceivable that the most likely source of error will be along the axis of a linear array due to capability of fashioning elements along a beam. Therefore, positional error along only the axis of the array ($x$-axis) will be applied according to

$$x_i^{\text{real}} = \mathcal{N}(x_i^{\text{ideal}}, \sigma^2) = x_i^{\text{ideal}} + \sigma \mathcal{N}(0, 1), \quad (2.19)$$

where $x_i^{\text{real}}$ is the realistic location of the $i^{th}$ element (with error), $x_i^{\text{ideal}}$ is the ideal location (optimized), and $\sigma$ is the standard deviation of the element positional error. Statistically, 95.4% of the elements will lie within $2\sigma$ of their intended location. For the analysis with these arrays, the performance effects of several choices of $\sigma$ will be considered. In addition, 100 trials with each array and choice of error standard deviation are performed to gather some statistical significance since the process in Eq. (2.19) is random. In Table 2.7, the effects of three different positional error standard deviations are given which coincides with excellent ($\sigma = 0.01$), good ($\sigma = 0.05$), and poor ($\sigma = 0.25$) fabrication practices. The same is given for the 100-element design in Table 2.8. The indicated mean positional error is given by

$$MPE(\%) = 100 \times \frac{1}{d_{\text{min}}} \frac{1}{N_{\text{element}}} \sum_{i=1}^{N_{\text{element}}} |x_i^{\text{real}} - x_i^{\text{ideal}}|, \quad (2.20)$$

where $d_{\text{min}}$ is the original minimum element spacing of the array (here, $4\lambda$ for both array sizes). Eq. (2.20) can be estimated for arbitrary arrays by

$$MPE(\%) = 100 \times \frac{\sigma}{d_{\text{min}}} \sqrt{\frac{2}{\pi}}, \quad (2.21)$$

which uses the expected value of a normal distribution, $E[|\sigma \mathcal{N}(0, 1)|]$.

The performance of the trials given in Tables 2.7 and 2.8 illustrate that positional errors can have a significant impact on the performance of the arrays. For the small, 25-element array, the impact with poor fabrication practices can lead to designs with unacceptable peak sidelobe levels, averaging -6.39 dB. For the large 100-element design, even with poor fabrication tolerances the array may still produce
Table 2.7: Effects of positional error on the peak sidelobe level of the example 25-element linear array. The ideal array possesses a peak sidelobe level of -9.77 dB.

<table>
<thead>
<tr>
<th>σ</th>
<th>Mean Pos. Error</th>
<th>Best SLL</th>
<th>Mean SLL</th>
<th>Worst SLL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.2 %</td>
<td>-9.67 dB</td>
<td>-9.46 dB</td>
<td>-9.10 dB</td>
</tr>
<tr>
<td>0.05</td>
<td>1.0 %</td>
<td>-9.21 dB</td>
<td>-8.51 dB</td>
<td>-6.78 dB</td>
</tr>
<tr>
<td>0.25</td>
<td>5.0 %</td>
<td>-7.62 dB</td>
<td>-6.39 dB</td>
<td>-4.52 dB</td>
</tr>
</tbody>
</table>

Table 2.8: Effects of positional error on the peak sidelobe level of the example 100-element linear array. The ideal array possesses a peak sidelobe level of -15.44 dB.

<table>
<thead>
<tr>
<th>σ</th>
<th>Mean Pos. Error</th>
<th>Best SLL</th>
<th>Mean SLL</th>
<th>Worst SLL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.2 %</td>
<td>-15.27 dB</td>
<td>-15.10 dB</td>
<td>-14.68 dB</td>
</tr>
<tr>
<td>0.05</td>
<td>1.0 %</td>
<td>-14.51 dB</td>
<td>-13.81 dB</td>
<td>-12.66 dB</td>
</tr>
<tr>
<td>0.25</td>
<td>5.0 %</td>
<td>-12.73 dB</td>
<td>-11.37 dB</td>
<td>-9.42 dB</td>
</tr>
</tbody>
</table>

acceptable peak sidelobe levels, averaging -11.37 dB. Maintaining tight tolerances clearly yields the best performance for both designs, however, more care must be taken when constructing the smaller arrays in order to maintain acceptable peak sidelobe level performance.

### 2.5.2 Planar Arrays

For the planar array designs, the optimized 220-element and unoptimized 600-element, 15-fold semi-periodic rotationally symmetric arrays are considered. Since these arrays require considerably more time to compute array factors, only 10 trials for each case are performed. Here, a similar error configuration is applied where element locations are defined by

\[
\begin{align*}
x_i^{real} &= \mathcal{N}(x_i^{ideal}, \sigma^2) = x_i^{ideal} + \sigma \mathcal{N}(0,1) \\
y_i^{real} &= \mathcal{N}(y_i^{ideal}, \sigma^2) = y_i^{ideal} + \sigma \mathcal{N}(0,1). \tag{2.22}
\end{align*}
\]

In this case, the mean (radial) positional error is given by
Table 2.9: Effects of positional error on the peak sidelobe level of the example 220-element optimized planar array. The ideal array possesses a peak sidelobe level of -16.21 dB.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>Mean Pos. Error</th>
<th>Best SLL</th>
<th>Mean SLL</th>
<th>Worst SLL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0075</td>
<td>0.2 %</td>
<td>-16.09 dB</td>
<td>-16.03 dB</td>
<td>-15.97 dB</td>
</tr>
<tr>
<td>0.0375</td>
<td>1.0 %</td>
<td>-15.41 dB</td>
<td>-15.18 dB</td>
<td>-14.85 dB</td>
</tr>
<tr>
<td>0.1875</td>
<td>5.0 %</td>
<td>-13.54 dB</td>
<td>-12.85 dB</td>
<td>-12.09 dB</td>
</tr>
</tbody>
</table>

$$MPE(\%) \approx 100 \times \frac{\sigma}{d_{\text{min}}} \sqrt{2 \left(1 - \frac{1}{8} + \frac{1}{84}\right)},$$

which makes use of the expected value of a two-dimensional Normal distribution given in Eq. (2.12). To maintain the same positional tolerances (0.2 %, 1.0 %, and 5.0 %), error standard deviations of 0.0075, 0.0375, and 0.1875 are now used for the 220-element design. This array provides a peak sidelobe level of -16.21 dB at a minimum element spacing of $4.71\lambda$. As with the linear arrays, the performance is increasingly affected with growing positional error, demonstrated by Table 2.9. Even with 5.0 % error, however, the arrays still maintain peak sidelobe levels below approximately -12 dB.

The 600-element, 15-fold SPRSA provides a peak sidelobe level of -13.84 dB at a minimum element spacing of $2.5\lambda$ and requires positional standard deviations of 0.004, 0.02, and 0.1 at this minimum spacing for error tolerances of 0.2 %, 1.0 %, and 5.0 %, respectively. Interestingly, the errors do not significantly impact the performance of this array as is shown in Table 2.10. At least one of the designs within each error class produces a peak sidelobe level that is less than the original ideal design. Unlike the optimized arrays, these arrays possess no fine tuning of element positions to reduce peak sidelobe level, and are therefore minimally impacted when elements are slightly adjusted from their original locations.

The error analysis presented in this section indicates what performance impacts can be expected with varying degrees of element placement error. For the optimized linear and planar arrays, greater error clearly leads to higher peak sidelobe levels. Larger arrays with more elements can usually accept more error before
Table 2.10: Effects of positional error on the peak sidelobe level of the example 600-element semi-periodic rotationally symmetric planar array. The ideal array possesses a peak sidelobe level of -13.84 dB.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>Mean Pos. Error</th>
<th>Best SLL</th>
<th>Mean SLL</th>
<th>Worst SLL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>0.2 %</td>
<td>-13.88 dB</td>
<td>-13.86 dB</td>
<td>-13.84 dB</td>
</tr>
<tr>
<td>0.020</td>
<td>1.0 %</td>
<td>-13.87 dB</td>
<td>-13.72 dB</td>
<td>-13.63 dB</td>
</tr>
<tr>
<td>0.100</td>
<td>5.0 %</td>
<td>-14.04 dB</td>
<td>-13.64 dB</td>
<td>-13.20 dB</td>
</tr>
</tbody>
</table>

array performance becomes unacceptable (peak sidelobe level greater than -10 dB, for example). The unoptimized designs do not ideally posses the low peak sidelobe levels of the optimized designs, however, their performance is not significantly impacted by placement error. For any of the designs, more placement error can lead to a decrease in the minimum element spacing, which leads to a decrease in array bandwidth and potential increases in mutual coupling between elements. If error is expected in a fabricated design, however, a layout with increased bandwidth can be designed to account for this.

2.6 Summary

In this Chapter, several new methods for the design of ultra-wideband antenna array layouts were presented. The first is the compact linear ultra-wideband array design technique. The combination of the direct element spacing method in Fig. 2.4 and the powerful CMA-ES can result in array layouts with performance not possible with other array design techniques. This array design technique effectively bridges the design gap of ultra-wideband array layouts with a small to moderate number of elements left with the methods in [37] and [85]. Further exploration with larger arrays (100+ elements) and increased bandwidth (beyond 10:1) may possibly lead to still impressive performance.

The optimized and semi-periodic rotationally symmetric array design techniques are well suited for the creation of ultra-wideband planar array layouts with the added benefit of geometrical characteristics that make them desirable from a man-
ufacturing standpoint. In addition to the example array layouts presented here, many more arrays can be created which target different performance goals, aperture sizes, directivities, and so on. Exploiting rotational symmetry not only provides a way to simplify fabrication and add modularity to assembly, but it also serves to expedite the optimization of large arrays. The problem size is reduced by only requiring that the optimal placement of elements be determined in a small slice of the aperture, and only a correspondingly small portion of the total array factor needs to be computed to ascertain the full radiation pattern.

Both the optimized designs and the semi-periodic designs yield array factors with low sidelobe levels over extended bandwidths, however, some performance is sacrificed in the case of the semi-periodic arrays that may be acceptable if manufacturing ease is a priority. In this case, the semi-periodic variety provides a very distinct ultra-wideband performance advantage compared to a similarly sized and similarly constructed periodic array. The optimized rotationally symmetric array design method is shown to be capable of generating planar array layouts with bandwidth performance beyond other reported ultra-wideband design techniques.
Chapter 3

The Memristor in Electromagnetic Devices

3.1 Introduction

Electrical or mechanical reconfiguration of antennas, filters, frequency selective surfaces, and other radio frequency devices has become popular for enabling devices with functionality such as band-switching and frequency tuning [94–104], beam-steering [104–106], polarization adjustment [107–109], and many other capabilities. The ultra-wideband array design techniques introduced in Chapter 2 can make effective use of such frequency-reconfigurable antennas with operating bands that are separated by octaves and beyond. Reconfigurability is typically accommodated by use of varactors, PIN diodes, micro-electrical mechanical switches (MEMS), photo-conductive devices, and variable capacitors, relays, transistors, and even dielectric adjustment through fluidic movement. Intrinsic device properties such as capacitances and inductances, as well as switching times, actuation waveforms, on/off resistances and power handling are the primary concerns when choosing a switching or tuning device for a specific application. Each device usually has advantages and disadvantages. For example, MEMS require only a small footprint and can be designed to operate up to the millimeter bands with near zero distortion, but offer limited power handling compared to other devices [110].
One characteristic of many of the aforementioned switching or tuning devices is that their controlling signals must be continually applied for the devices to retain their properties. These must have maintained bias voltages or currents for the switch to remain in its present state, with the exception of latching relays and some interesting bistable, “buckled-beam” MEMS [111,112]. For simple circuits this is usually of little concern, however, for large scale design the quiescent power requirements may become appreciable. MEMS typically require low power to remain in the actuated state due to the electrostatic nature of the device, however, they require large actuation voltages which may not be desirable in certain applications. PIN diodes typically require significant forward current in order to maintain low RF resistance, whereas RF field effect transistors merely require a moderate gate bias with a very small leakage current at the expense of reduced power handling. The non-volatile switching or tuning devices typically possess other drawbacks such as slow switching speed. Due to the demand from modern microwave antenna and communications systems, research has been ongoing to design or discover devices with improved properties such as faster switching times, higher power handling, and reduced switching power.

A new “fourth” circuit element that possesses a relationship between flux and charge was purported to exist in the 1970s by Chua [4] and has received recent attention due to the fabrication of a device with very similar properties by engineers at Hewlett-Packard laboratories [5, 6]. The device, or rather a class of devices, bridges the relationship between flux and charge as shown in Fig. 3.1 [113,114]. The new device has been called the “memristor” (not to be confused with “memistor” [115]) due to the resistance of the component which exhibits a memory effect, controlled by the accumulated charge or flux. The memristor has great potential for high density solid-state non-volatile memory due to the small footprint required by the relatively simple Pt-TiO₂-Pt structure and straightforward, fast state switching. Although the devices do not actually operate by accumulating flux or charge, they can be modeled as such within a set of operating conditions. For microwave applications the non-volatile nature of the memristor is particularly attractive, requiring no constant voltage or current bias to remain in a particular resistance state.
Figure 3.1: After [4], the relationship between circuit quantities and component values of traditional elements and the memristor. Note that since the unit of flux is V-s and charge is A-s, the unit of memristance is also ohms. For the memristor to be non-trivial (equivalent to a basic resistor), M must be a function of flux or charge.

Alternative materials for device realization are constantly being explored as well [114,116–118]. Most resistance-based memory devices are based on an oxide layer sandwiched between two metal electrodes. For memory applications, the state of the two-terminal device is programmed and read from the same electrical connections (much like dynamic RAM without the volatility), simplifying trace routing and circuit design. Some experimentation with various materials and geometries will likely be needed to determine the configurations most suited for radio frequency switches. Power handling will become a concern that has not thoroughly been explored yet, as most studies have focused on memory applications. In most cases, however, series and parallel application of memristors can be used to increase power handling, further customize resistance ranges, and so on.

Very little exploration has been done on how the unique properties of memristors could be exploited by integrating them into electromagnetic (radio frequency) devices. In [7], a passive switching scheme that employed linear dopant drift memristors was proposed for reconfigurable frequency selective surfaces. A memristor-
modulated antenna was presented in [8], where a narrowband patch antenna is modified using edge-terminating memristors to achieve an expansion of bandwidth at the expense of efficiency. An interesting low-power, ultra-wideband receiver comprised of memristors has also been theorized in [119]. Additionally, some investigations have been performed in the terahertz regime using split-ring resonators and memcapacitive materials [120].

The memristor is explored here as a microwave switching component for reconfigurable devices. Modeling of the device could be potentially performed with traditional frequency domain electromagnetic simulation tools and quasi-static resistance (state) values, however, this technique does not capture the transient, non-linear behavior of the memristor. It also does not capture any effects that the circuit in operation may have on the state of the device. For this reason, a finite-difference time-domain tool is utilized with a memristor model to observe the RF and transient switching characteristics. Here, additional microwave devices are designed and examined to explore the prospect of the memristor in future applications. A non-linear dopant drift memristor model is implemented to allow for capturing the state boundary effects seen with realistic devices. A reconfigurable planar absorber, a band-switching patch antenna, and a polarization switchable patch antenna are designed using one or more memristive elements to achieve reconfigurability.

3.2 Non-Linear Dopant Drift Memristor Model

Although it is possible to externally connect an FD-TD cell to a spice simulation engine [9], for simplicity the memristor is directly incorporated into the FD-TD update equations at specific Yee cell locations, similar to the approach taken for conventional lumped elements such as capacitors and inductors [121]. The model is implemented as a lumped resistance that varies according to the current integrated over time at the element location and a set of initial conditions. Although many forms of memristors have been postulated [122], one model that is based on the physics of the device created by HP labs [5] is employed here as it seems to represent
Figure 3.2: Device model of the memristor reported in [5].

the most likely realization of any future commercially fabricated components. The static resistance of the device is determined by the total series resistance of the doped TiO$_{2-x}$ and undoped regions of TiO$_2$ (or other proposed material) shown in Fig. 3.2 and expressed as

$$R = \frac{w}{D} R_{ON} + \left(1 - \frac{w}{D}\right) R_{OFF}. \quad (3.1)$$

When the device is completely undoped, the memristor has a resistance of $R_{OFF}$. When it is fully doped, it has a resistance of $R_{ON}$. Positive voltage across the device causes the barrier between the two regions to migrate towards the positive symbol, and vice versa. The velocity of this migration can be approximated by linear or non-linear dopant drift techniques, the latter being more accurate at the expense of complexity [123]. The linear dopant drift approximation has been used in the past for memristor modeling in electromagnetic devices [7] and is given by

$$\frac{dw(t)}{dt} = \frac{\mu_D R_{ON}}{D} i(t), \quad (3.2)$$

where $\mu_D$ is the average ion mobility. Eq. (3.2) can be integrated to determine the state as a function of charge [5], or alternate forms can be found that are functions of flux [7]. The basic form of (3.2) will be kept here since it is relatively straightforward to determine FD-TD cell current (as in Sec. B.2). For simplicity, the ratio $\frac{w}{D}$ is represented by $x$ and is called the device state. The state of the memristor must be bounded to $[0, 1]$ so that $0 \leq w \leq D$. For the linear dopant drift case in (3.1), this simply means boundary limiting.

Although the linear dopant drift model gives a great deal of insight into the functionality and usage of memristors in EM devices [121], more accurate models of
memristors have been introduced based on the characteristics of fabricated samples [5]. These non-linear drift models are based on window functions that make the drift velocity a function of device state and in some cases device current. The non-linear dopant drift model is represented by

\[ \frac{dx}{dt} = \frac{\mu_D R_{ON}}{D^2} i(t)f(x), \]  

(3.3)

where \( f(x) \) is the non-linear window function. In this case, the drift velocity is affected by the location of the ion barrier as has been experimentally demonstrated in [5]. Depending on the form of the window function, it may be used to enforce the state boundaries for the non-linear dopant drift case. The non-linear window function can take several forms; the most common function that is used for the designs here is given by

\[ f(x) = 1 - (2x - 1)^2p, \]  

(3.4)

where \( p \) is the non-linear exponent constant [123,124]. The choice of \( p \) determines the effect of the window function when the state approaches the boundaries as shown in Fig. 3.3. For all of the simulations here, \( p = 2 \) is chosen since it has a significant impact on switching performance. This value (or complete window function), along with other memristor properties, will likely need to be adjusted once commercial devices become available. Alternate window functions exist that can additionally depend on the direction of state movement (current flow) that aim to allow for easy state movement away from boundaries but increased difficulty in moving the state towards a boundary. Each of these window functions are a system of approximating the non-linear field effects when the doped or undoped regions become very small.

To illustrate the differences between the linear and non-linear dopant drift memristor models, the simple circuit in Fig. 3.4 is used. The circuit is simple enough that a discrete time form of (3.2) and (3.3) can be easily used with MATLAB to quickly determine their responses. In addition, this will be very similar to the cell update that will be used in the FD-TD implementation. Two simultaneous simulations are performed, one with a linear dopant drift (as in (3.2)), and one with a
non-linear dopant drift (as in (3.3)). Both memristors have properties $R_{ON} = 1\Omega$, $R_{OFF} = 100\Omega$, and $k = 10^3 A^{-1}s^{-1}$. The non-linear dopant drift memristor has $p = 2$. A SPICE simulation using the model in [125] with an identical stimulus is also performed to validate the results of the non-linear dopant drift model (discrete update), as shown in Fig. 3.5.

![Figure 3.3: Window function value versus state for various values of $p$.](image)

Figure 3.3: Window function value versus state for various values of $p$.

![Figure 3.4: Circuit used to illustrate the effects of the non-linear dopant drift window function. The black band indicates the current must flow into this terminal in order to increase the state (decrease the resistance) of the memristor.](image)

Figure 3.4: Circuit used to illustrate the effects of the non-linear dopant drift window function. The black band indicates the current must flow into this terminal in order to increase the state (decrease the resistance) of the memristor.

A ramped-sine excitation is applied to first observe the similarities when the devices are in the mid-state region, then the differences when they are near the boundaries. It is observed that the non-linear model shows resistance to the state to moving towards the boundaries (compared to the linear drift model) and, once
Figure 3.5: Simulation results for the circuit of Fig. 3.4 for the linear and non-linear dopant drift memristor models, as well as the SPICE transient analysis. Source excitation (a), device currents (b), states (c), and resistances (d).

near a boundary, resistance to moving away from it, as is suggested by the window function in (3.4). For this reason, it may result in better total device performance (compared to the linear drift model) in electromagnetics designs as it will tend to remain at a boundary in spite of any radio frequency currents that may occur during normal device operation.
3.3 The Finite-Difference Time-Domain

Some details on the finite-difference time-domain simulation tool that is used for design of the electromagnetics devices in the following sections are given here. The custom simulation code created for these designs is based on the standard algorithm with the Yee cell shown in Fig. 3.6 [9, 126, 127]. The algorithm operates by staggered updates of the electric and magnetic fields based on discrete-time and discrete-space evaluation of Maxwell’s equations given in Appendix B. Being a time-domain code, any type of input signal can be used to excite the simulation space; this is particularly useful for the types of signals required to switch memristors and observe their behavior.

Due to the cuboidal Yee cell, the geometry must similarly follow the rectangular lattice. For many common RF fabrication and design techniques, this is not an issue as many already follow linear profiles and constant thicknesses. For example, a standard planar dielectric substrate can be accurately modeled with a rectangular slab and many patch antennas and frequency selective surfaces follow a rectangular profile. The lattice parameters ($\Delta x, \Delta y, \Delta z$) must simply be adjusted to conform to the given geometry. For curvilinear objects and profiles, approximation techniques can be used with stair-casing (for metallic objects) or cell permittivity averaging (for dielectrics). All of the information that defines the geometry is contained in the update constants $C^{(1)}_s$, $C^{(2)}_s$, $G^{(1)}_s$, and $G^{(2)}_s$ given in Sec. B.1. Typically, the dielectric and metallic geometry is interpreted before the timestepping process begins in order to reduce total computational time [9].

Two variations of the basic FD-TD code are used for the following designs. The first is a three-dimensional code with two-dimensional periodicity, applying periodic boundary conditions on the opposing $x - y$ vertical walls. The domain is separated into three regions as shown in Fig. 3.7. At the very top and bottom are the perfectly matched layers (PML) which are a split-field formulation (12 field components instead of six) of lossy absorbing layers introduced in [128]. A lattice truncation method such as the PML must be implemented to prevent the fields from being reflected back into the domain were simple truncation ($i.e. \ E = 0$ or...
Figure 3.6: One Yee cell with the facial field components displayed and the origin at located at \((i, j, k)\). Each computational cell has six field components: \(E_x, E_y, E_z, H_x, H_y, H_z\).

\(H = 0\) at the boundary) to be used. The Berenger-PML [128] has been found to provide excellent absorption of fields at normal and wide-angle incidence with a great deal of flexibility in terms of number of cells required, etc., allowing for accurate analysis of devices with little concern of the effects of boundary reflection. In the following designs, a minimum of 8 PML layers (cells) are used for very low boundary reflection.

To excite the simulation domain, a normally-incident, uniform plane wave (or pulse, etc.) is inserted at the top total field / scattered field (TF/SF) interface with propagation in the \(-z\)-direction. Since the incident field can be easily computed for all regions in the domain, the pure scattered field can be determined outside the total field region by subtraction at the TF/SF interfaces. The scattered field is used to compute the reflection and transmission through the device under test (DUT). For devices with a lower ground plane (as is common with frequency selective sur-
faces), the region below the plane can be ignored since no fields will exist, saving significant computational time and resources. Fourier transforms can be used to determine the frequency-domain transmission and reflection properties of the DUT according to the spectral properties of the incident waveform. The most common excitation is a Gaussian pulse which allows the broadband properties of structures to be computed via FFT [9,127].

Figure 3.7: Cut-plane of the field regions of the 2-D periodic FD-TD simulation code.

The second variation of the code is a generic three-dimensional type with PMLs on all six boundary surfaces. These additional boundary layers add a significant number of cells to the total lattice, requiring greatly increased computational requirements compared to the previous configuration. For the radiating problems that are to be analyzed, there is no scattered field region, only a total field region. The antenna structures are excited by a resistive voltage source at an electric field location according to (for a $z$-oriented example) the update
\( E_z^{n+\frac{1}{2},i+\frac{1}{2},j+\frac{1}{2},k} = \frac{1}{1 + \frac{\Delta t \Delta z}{2R \Delta x \Delta y}} E_z^{n-\frac{1}{2},i+\frac{1}{2},j+\frac{1}{2},k} + \frac{\Delta t}{1 + \frac{\Delta t \Delta z}{2R \Delta x \Delta y}} \times \left( \frac{H_y^{n,i+1,j+\frac{1}{2},k} - H_y^{n,i,j+\frac{1}{2},k}}{\Delta x} - \frac{H_x^{n,i+\frac{1}{2},j+1,k} - H_x^{n,i+\frac{1}{2},j,k}}{\Delta y} \right) \) (3.5)

\[ + \frac{\Delta t}{1 + \frac{\Delta t \Delta x \Delta y}{2R \Delta x \Delta y}} u^{n+\frac{1}{2}}, \]

where \( R \) is the source resistance and \( u^{n+\frac{1}{2}} \) is the source voltage signal at timestep \( n + \frac{1}{2} \). The source impedance is most commonly 50\( \Omega \) with typical RF systems. To calculate the input impedance of the antenna (typically done over a large bandwidth), the frequency-domain developed voltage (at the source field location) and current must be computed via FFT; then \( Z(\omega) = V(\omega)/i(\omega) \). Current is easily computed through an electric field location by the line integral of the magnetic fields around the axis of the electric field as given in Sec. B.2 [9].

To calculate the far-field gain or directivity, an imaginary box must be defined just inside the PML, where equivalent tangential electric and magnetic currents are computed from the fields at those lattice locations according to

\[ J_S = \hat{n} \times H \] (3.6)

\[ M_S = -\hat{n} \times E, \] (3.7)

where \( \hat{n} \) is the vector normal to the appropriate face of the imaginary box. These currents are transformed to the frequency domain via FFT and then transformed to magnetic and electric vector potentials

\[ A = \frac{\mu_0}{4\pi} \oint_S J_S e^{-jkR/R} ds' \] (3.8)
\[ F = \frac{\varepsilon_0}{4\pi} \int \int_S \mathbf{M}_S \frac{e^{-jkR}}{R} ds', \]  

(3.9)

where \( s' \) is a point on the imaginary surface. In most cases the \( e^{-jkR}/R \) term is ignored since we are only interested in field magnitude versus \((\theta, \phi)\) and not distance from the radiator. Once the vector potentials are computed, the far-field electric fields are computed for each desired \((\theta, \phi)\) according to

\[
E_\theta = -j\omega (A_\theta + \eta_0 F_\phi),
\]

(3.10)

\[
E_\phi = -j\omega (A_\phi + \eta_0 F_\theta).
\]

(3.11)

Directivity can be found by computing the entire radiation sphere, or realized gain (including effects of material losses) can be computed by determining the far-field electric field magnitude relative to the input power at the source [9, 129]. In the case of designs here, realized gain is desired to observe any losses that occur with the memrisitive switching elements.

### 3.4 FD-TD Memristor Model

The memristor is implemented at an electric field location of the Yee cell. During the electric field update, the current opposing the electric field direction (going into the memristor) is first computed using (as an example for an \( x \)-oriented memristor)

\[
i_x|_{i,j+\frac{1}{2},k+\frac{1}{2}} = \left( H_z|_{i,j+1,k+\frac{1}{2}}^{n} - H_z|_{i,j,k+\frac{1}{2}}^{n} \right) \Delta z + \left( H_y|_{i,j+\frac{1}{2},k+1}^{n} - H_y|_{i,j+\frac{1}{2},k}^{n} \right) \Delta y.
\]

(3.12)

The current used in Eq. (3.2) or (3.3) may need to be reversed depending on the orientation (polarity) of the memristor. After computing the current, the window function value is evaluated using the state,
Next, the state is updated according to the discrete time form of (3.3),

\[
x^{n+1/2} = x^{n-1/2} + \Delta t k f(x^{n-1/2}),
\]

where \( k = \frac{\mu D R_{ON}}{D} \). For the discrete time version of (3.3) given in (3.14), it is possible to overshoot a boundary depending on the value of current, even with the window function. To eliminate this issue and the possibility of zero-state (where the device would lock up due to the window function), the state is limited to \( \delta \leq x_{n+1/2} \leq (1 - \delta) \) after the update in (3.14), where \( \delta \) is a small value (\( \delta = 10^{-8} \) for simulations in Section 3.5). The new resistance is then computed using the discrete time form of (3.1),

\[
R^{n+1/2} = x^{n+1/2} R_{ON} + \left(1 - x^{n+1/2}\right) R_{OFF}.
\]

Lastly, the lumped resistor update equation proposed in [9] is used (for an \( x \)-oriented example),

\[
E_x \bigg|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+1/2} = \frac{1 - \frac{\Delta t \Delta x}{2R^{n+1/2} \epsilon_0 \Delta y \Delta z}}{1 + \frac{\Delta t \Delta x}{2R^{n+1/2} \epsilon_0 \Delta y \Delta z}} \frac{\Delta \epsilon}{\epsilon_0} \times \left( \frac{H_z}{\Delta y} \bigg|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n} - H_z \bigg|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n} \right) - \frac{H_y}{\Delta z} \bigg|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n} \bigg|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n}.
\]

To design a device using a memristor, one must choose \( R_{ON}, R_{OFF} \) and \( k \). The first two values are generally determined by the desired functionality and are fairly easy to select. \( \frac{R_{OFF}}{R_{ON}} \) ratios of beyond \( 10^4 \) have been reported in [117,118], allowing for competition with other RF switching devices. The parameter \( k \) (which is now primarily a function of \( \mu_D \) and \( D \)) must be sufficiently small so that the state of the memristor is not significantly affected by the peak current encountered during normal operation of the system, and must be large enough that reasonable switching times can be achieved with the available control current. The parameter
can be adjusted by changing the channel length or material, leading to different average ion mobilities.

Note that no series inductance or parallel capacitance is included in the model (or FD-TD cell updates) as device packaging can substantially affect these values. They can, however, be included when commercially available products or usable prototypes are available for testing. In addition, more appropriate ionic drift models and memcapacitive effects can be included for the given memristor structure and materials to allow for more accurate simulations, since these are generally not firmly established yet [124].

3.5 Example Designs

Several examples of electromagnetic designs that incorporate memristors as reconfigurable switching elements are presented here. The finite-difference time-domain tool is used to characterize the devices under transient switching conditions. For all of the designs, the memristors begin in one condition (state) and are reconfigured using Gaussian or quasi-square pulses. The RF functionality is observed simultaneously or in between reconfiguration cycles. By using FD-TD to characterize the device, any effects that the operating signals have on the memristor state can be observed and accounted for.

3.5.1 Reconfigurable Periodic Absorber

The first application utilizing memristive elements for reconfigurability is a switchable planar absorber. The structure is first designed for absorption at a single frequency using a periodic array of square patches on a thin substrate backed by a metallic ground plane as in [130]. Connecting the patches with resistors as depicted in Fig. 3.8 with values that match the impedance of free space will result in near perfect absorption of normally incident plane waves at the resonant frequency of the patches. On the other hand, using resistors of much higher value causes nearly
perfect reflection, as shown in Fig. 3.9.

![Diagram](image)

Figure 3.8: One unit cell of the periodic planar absorber structure for operation at 2.91 GHz (a) and an illustration of an array of interconnected unit cells and memristors as seen from the top (b).

Reconfigurability is achieved by employing a variable resistance in place of the fixed resistor; in this case a memristor is utilized. A series of control lines could potentially be implemented into the structure, however, one easy and practical method for controlling all of the memristors is to place a source at the edges of the absorber when it is in its final, truncated configuration. The current flowing in series through the memristors serves to adjust all of the devices simultaneously. This is emulated in the 2-D periodic FD-TD simulation tool by placing the tuning voltage source in series with the memristor. The source is an effective electrical short circuit at RF, resulting in no impact on the high-frequency performance of the absorber.

To test the theory and observe the switching performance of the reconfigurable absorber, it is excited with an $x$-polarized plane wave of 1 V/m at 2.91 GHz while the tuning voltage source changes the state of the memristor, as shown in Fig. 3.10. The tuning signal takes the form of

$$f_{tune}(t) = 2.5e^{-\frac{(t-0)^2}{2\sigma^2}} - 2.5e^{-\frac{(t-\tau_{off})^2}{2\sigma^2}},$$ (3.17)
where $t_{on} = 36.8$ ns, $t_{off} = 129$ ns, and $\sigma = 4.1$ ns. A memristor with $R_{ON} = 377\Omega$, $R_{OFF} = 10k\Omega$, and $k = 3.02 \times 10^{12} A^{-1}s^{-1}$ is used to replace the resistor in Fig. 3.8a. The memristor is initially configured to the $R_{OFF}$ state ($x \approx 0$). The positive Gaussian pulse is used to switch the memristor to the $R_{ON}$ position ($x \approx 1$), enabling the absorber. Some of the energy in the enabling pulse is radiated via the patches at about 35 ns; a tuning pulse with a lower bandwidth would alleviate this at the cost of slower switching speed. The absorber is turned off at approximately 120 ns via the negative tuning pulse, allowing near total reflection again. In addition to simple switching, this type of setup could also conceivably be used for reflection amplitude modulation in communications applications.

### 3.5.2 Dual-Band Reconfigurable Patch Antenna

Another reconfigurable design uses memristors to achieve switching between two bands of operation. Functionality is fairly straightforward; resistive switches connect additional portions of metal to the ends of a patch, lowering its resonant frequency when the switches are of low resistance. The resistors must be of low enough or high enough value in their respective states to ensure that absorptive losses remain low during normal antenna operation. The geometry of the reconfigurable antenna is shown in Fig. 3.11 with geometrical parameters in Table. 3.1.
Figure 3.10: Excitation and properties of the memristor for the FD-TD simulation of the reconfigurable absorber. The incident (a) and reflected (b) field, tuning voltage (c) and instantaneous resistance (d) are given. The magnitude of the reflected field during the on-state is 34.9 mV/m, yielding 29.1 dB of attenuation.

Table 3.1: Geometry parameters for the reconfigurable dual-band patch antenna.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>5 mm</td>
</tr>
<tr>
<td>$L_2$</td>
<td>30 mm</td>
</tr>
<tr>
<td>$L_3$</td>
<td>3 mm</td>
</tr>
<tr>
<td>$W$</td>
<td>32 mm</td>
</tr>
<tr>
<td>$d$</td>
<td>1.57 mm</td>
</tr>
<tr>
<td>$P$</td>
<td>9 mm</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Six memristors are used, three in each gap connecting the metallic structures. The memristors have properties $R_{ON} = 0.2 \Omega$, $R_{OFF} = 2k\Omega$, and $k = 3 \times 10^{10} A^{-1} s^{-1}$. The structure is first simulated in the two states to determine the resonant frequencies and the effects of the resistive switches on the gain of the antenna in the two bands. Again, a FD-TD simulation tool is used to obtain the wideband
performance of the antenna. The scattering parameters of the patch antenna with fixed resistors in the two states are demonstrated in Figs. 3.12 and 3.13, yielding a frequency shift of about 31%. Gain in both the low-band and high-band is approximately 6.3 dB and 5.9 dB, respectively, which includes the effects of resistor losses.

Figure 3.11: Geometry of the band-switching reconfigurable antenna, designed for operation at 2.308 GHz and 3.143 GHz. The gaps between the patch extensions and the central patch are 1 mm. The ground plane is 70 mm on an edge. Inductors (450 nH) are placed between the patch extensions and the ground plane to form a DC path for controlling the memristors. A resistive probe feed source of 50 Ω is used.

To observe the transient operation of the antenna system, the memristors are initially set to the \( R_{OFF} \) state \((x \approx 0)\), thus beginning in the high-band configuration shown in Fig. 3.14. To modulate the state of the memristors in the following designs, a discrete-time tuning pulse is used that is of the form found in Appendix B.3. This form allows for realistic, band-limited switching pulses to be recreated with user-defined properties according to Table B.1. Additionally, the pulses closely match that of signals generated by realistic circuit devices.

The antenna is first excited with a sine-modulated pulse at the high frequency of operation using a sine-burst pulse (from Eq. (B.18)) with properties
Figure 3.12: Scattering parameters of the reconfigurable antenna with fixed resistors in the two states.

Figure 3.13: Realized gain of the reconfigurable antenna with fixed resistors in the two states.

\[ \text{sineburst}_1 = \begin{cases} 
  t_{\text{on}} = 10 \text{ ns} \\
  t_{\text{width}} = 25 \text{ ns} \\
  t_{\text{trans}} = 3 \text{ ns} \\
  a = 0.5 \text{ Volts} \\
  f_{\text{modulation}} = 3.143 \text{ GHz}. 
\end{cases} \]

Then, a tuning pulse with properties
pulse = \begin{cases} 
  t_{on} = 56 \text{ ns} \\
  t_{width} = 50 \text{ ns} \\
  t_{trans} = 7 \text{ ns} \\
  a = 5 \text{ Volts} 
\end{cases}

originating from the same probe source is used to switch the memristors to the \( R_{ON} \) configuration, after which a sine-modulated pulse with properties

\[
sineburst_2 = \begin{cases} 
  t_{on} = 155 \text{ ns} \\
  t_{width} = 25 \text{ ns} \\
  t_{trans} = 3 \text{ ns} \\
  a = 0.5 \text{ Volts} \\
  f_{modulation} = 2.308 \text{ GHz.} 
\end{cases}
\]

is used to observe the antenna performance in the low-band. A closer inspection of the developed voltage and current at steady-state in Fig. 3.15 demonstrates that the current is in phase with the voltage and the impedance is well matched at \( 42.3 - j1.1 \Omega \) (\( S_{11} \) of -21.6 dB) in the high-band and \( 41.8 + j8.0 \Omega \) (\( S_{11} \) of -18.1 dB) in the low-band. The memristors require approximately a 50 ns positive tuning pulse of 5 volts to switch completely from the off-state to the on-state; different pulse lengths and amplitudes may be required with different memristor properties.

### 3.5.3 Polarization-Reconfigurable Patch Antenna

The final reconfigurable electromagnetics design is a patch antenna that can switch between two orthogonal linear polarizations. The structure is a simple square patch that has two edge-coupled feeds in perpendicular directions [131]. Selection of feeds is determined by the resistance values of a set of four memristors, which determine which transmission line and feed to use as shown in Fig. 3.16. In this case, a pair of memristors determines the attenuation of the line. The low-frequency circuit model in this case becomes a bit more complex, suggesting some simulations of the memristors in a circuit analysis scenario before beginning FD-TD simulations.
The circuit in Fig. 3.17 is used to model a low frequency approximation of the memristor network. After some experimentation, a suitable set of memristor properties were found, those being $R_{ON} = 1\, \Omega$, $R_{OFF} = 500\, \Omega$, and $k = 2.5 \times 10^{10}\, A^{-1}\, s^{-1}$. This results in an ideal insertion loss of about 0.5 dB in the on-configuration and an isolation of about 49 dB in the off-configuration. A positive-amplitude tuning pulse with properties

$$\text{pulse}_1 = \begin{cases} 
  t_{on} = 10 \, \text{ns} \\
  t_{width} = 25 \, \text{ns} \\
  t_{trans} = 3 \, \text{ns} \\
  a = 5 \, \text{Volts}
\end{cases}$$

activates the transmission line which feeds the $x$-polarization and a negative tuning pulse with properties

$$\text{pulse}_2 = \begin{cases} 
  t_{on} = 70 \, \text{ns} \\
  t_{width} = 25 \, \text{ns} \\
  t_{trans} = 3 \, \text{ns} \\
  a = -5 \, \text{Volts}
\end{cases}$$

selects the transmission line which feeds the $y$-polarization. In the circuit simulation shown in Fig. 3.18, the memristors are initialized in the $y$-polarized state ($\mathbf{x} = \{0, 1, 1, 0\}$), after which a positive pulse activates the M1–M2, $x$-polarization line. The pulse ends while the line stays active, after which a negative pulse disables the M1–M2 switch and enables the M3–M4, $y$-polarization line again.

To test the RF properties of the antenna in the two desired states, fixed resistors are first used with a standard FD-TD excitation (e.g. Gaussian pulse) to determine the resonant frequency and scattering parameters of the antenna with each transmission line selected. Since the devices in the low-frequency scenario are ideal, gain simulations using less than full-on and full-off resistances ($R_{ON} = 3\, \Omega$, $R_{OFF} = 495\, \Omega$) are performed to determine the worst case to be expected during
the final FD-TD simulation and in practical use. Scattering parameters and gain are computed and given in Figs. 3.19 and 3.20. Demonstrating the polarization switching of the design, during the $x$-polarization setting, the antenna should exhibit a large $G_\theta$ and small $G_\phi$ at $\phi = 0^\circ$ and a large $G_\phi$ and small $G_\theta$ at $\phi = 90^\circ$, as can be seen in Fig. 3.20. The opposite can be observed for the $y$-polarization setting as well. The computed gain includes the effects of the resistor losses, realizing a minimum of 5.8 dB at broadside for each desired setting. The patterns are slightly asymmetric due to the relatively small, offset ground plane and minor feed line radiation. The patch (with appropriate matching transmission line) resonates in both polarization settings at approximately 6.2 GHz with a 2% frequency bandwidth. More complex antennas could potentially be utilized to achieve a larger bandwidth.

The transient behavior of the entire microstrip and patch antenna system is captured using an identical set of tuning pulses with a set of 6.2 GHz signals placed in the portions of time where the memristors are in their desired states. The first modulated pulse has properties

$$sineburst_1 = \begin{cases} 
  t_{on} = 47 \text{ ns} \\
  t_{width} = 10 \text{ ns} \\
  t_{trans} = 2 \text{ ns} \\
  a = 1 \text{ Volt} \\
  f_{modulation} = 6.2 \text{ GHz}
\end{cases}$$

and the second is defined by

$$sineburst_2 = \begin{cases} 
  t_{on} = 105 \text{ ns} \\
  t_{width} = 10 \text{ ns} \\
  t_{trans} = 2 \text{ ns} \\
  a = 1 \text{ Volt} \\
  f_{modulation} = 6.2 \text{ GHz}
\end{cases}$$

The excitation and resulting voltage, current, and memristor properties are pro-
vided in Fig. 3.21. The ability to predict this type of behavior using FD-TD allows for full characterization and tuning of the system thereby avoiding potentially costly fabrication cycles. In this case, the resistance of the series element in the on-state stays very close to 1Ω due to the effects of the non-linear window function, permitting efficient use and high gain. A zoomed in view of the signals during RF excitation at steady-state is given in Fig. 3.22. The input impedance (and $S_{11}$) of the system during the $x$-polarization setting and the $y$-polarization setting is $62.4 + j25.6\Omega$ (-12.1 dB) and $44.5 + j24.0\Omega$ (-12.0 dB), respectively.

### 3.6 Summary

The memristor has been demonstrated to be a useful and easily implemented microwave switching component for many different types of electromagnetic devices. It has been successfully utilized with FD-TD modeling as the switching element in a reconfigurable absorber, a dual-band patch antenna, and a polarization switching patch antenna. Furthermore, the core functionality of the third example design is a simple microstrip transmission line switch, which can be a very useful component for many microwave systems.

The switching behavior of the memristors, which includes all of the effects of the accompanying electromagnetic structure, has been carefully observed by using the aforementioned FD-TD simulation tool. The groundwork for use of the memristor in practical reconfigurable microwave devices is laid such that when commercial devices or prototypes become available, implementation will be greatly simplified. At that time, more appropriate material parameters and drift models can be easily included to allow for more accurate simulations with the devices at hand.
Figure 3.14: Time-domain memristor resistance (a), probe source current (b), developed voltage (c), and source voltage (d) of the dual-band patch design. Simulation domain of $100 \times 100 \times 33$ Yee cells required $1.26 \times 10^5$ timesteps and 41 hours of CPU time on a single 2.4 GHz processor core.
Figure 3.15: Zoomed view of the waveforms in the two modes of operation at steady-state. Low-band source current (a), developed voltage (c), and source voltage (e). High-band source current (b), developed voltage (d), and source voltage (f).
Figure 3.16: Geometry of the polarization reconfigurable patch antenna. M1 and M3 lie flat with the printed circuit substrate while M2 and M4 are connected from the top metal layer to the ground plane underneath. The inset at the bottom right highlights the orientation of the memristors, where dark bands on the slabs or cylinders indicate the positive terminal. The substrate has a permittivity of $\epsilon_r = 2.2$ and a thickness of 0.787 mm.

Figure 3.17: Low-frequency circuit used to model the dual-polarization patch antenna memristor reconfiguration. M1 and M2 control the $x$-polarization, M3 and M4 control the $y$-polarization. Non-ideal properties such as parasitic capacitances and inductances are ignored for simplicity.
Figure 3.18: Time-domain circuit modeling of the memristor networks. Memristor resistance (a), memristor state (b), estimated port RF attenuation (c), and tuning voltage (d). RF attenuation is estimated when a 50Ω load at the microstrip input is used.

Figure 3.19: Scattering parameters of the polarization reconfigurable patch antenna in the two polarization settings.
Figure 3.20: Realized gain at 6.2 GHz of the polarization reconfigurable patch antenna in the two polarization settings.
Figure 3.21: Memristor resistance (a), probe source current (b), probe source developed voltage (c), and source excitation signal (d). The probe source has a characteristic impedance of 50Ω. Simulation of $110 \times 110 \times 33$ Yee cells with $\Delta t = 0.668$ ps required $1.8 \times 10^5$ timesteps and 76.5 hours of CPU time on a single core of a 2.4 GHz Xeon processor.
Figure 3.22: Zoomed view of the waveforms in the two modes of operation at steady-state. $x$-pol source current (a), developed voltage (c), and source voltage (e), $x$-pol source current (b), developed voltage (d), and source voltage (f).
Chapter 4

Conclusion

In the new work presented in Chapters 2 and 3, two research topics were investigated. The first is the design of high-performance linear and planar antenna array layouts. The compact linear ultra-wideband array design technique originated to cover the performance gaps created by previous modern ultra-wideband array design techniques which work best with medium to large sized arrays (hundreds to thousands of elements) [132–135]. For example, the polyfractal method can be constrictive when applied to smaller array sizes (less than 100 elements), hence, a method with a great deal of flexibility in the array representation technique is desired. The compact UWB array method, combined with the covariance matrix adaptation evolutionary strategy, effectively bridges the availability of high-performance array design from just a few to approximately 100 antenna elements. In the course of this design work, CMA-ES was effectively introduced to the electromagnetics community and shown to be very fast, robust, and easy to use compared to the popular particle swarm and genetic algorithm optimization algorithms [84,136–138].

In addition to small linear arrays, a similar technique was developed for planar arrays based on rotational symmetry [139–141]. Any type of geometric symmetry is usually desirable from a manufacturing standpoint, however, translational symmetry that is commonly used in periodic arrays is the cause of its narrowband properties. Rotational symmetry in an array layout does not lead to grating lobes as has been demonstrated in designs which utilize aperiodic tilings, making it an
attractive foundation for array design. Here, the position of elements in a slice of an $N$-fold rotationally symmetric structure are optimized directly using CMA-ES. The implemented cost function includes portions from peak sidelobe level as well as minimum element spacing in order to enforce a desired bandwidth. In addition to reducing the problem scale (number of elements versus number of parameters), the resulting array factors of these designs also possess rotational symmetry. This allows the performance (peak sidelobe level) of the arrays to be quickly computed for large scale optimizations.

A similar technique can be applied where sections of periodic arrays are placed in the rotationally symmetric tiles instead of optimized element locations. This has the added benefit of even more regularity in the array layout, leading to an even more attractive design from a manufacturing perspective. The performance of this class of arrays is not as impressive as that of the optimized designs, however, it does have the added benefit of having a much larger directivity when compared to optimized designs with similar aperture sizes. This is due to the dense nature of the arrays, compared to the optimized designs which are fairly sparse. Additionally, this type of array is less sensitive to positional error than the optimized variety, another desirable property for manufacturing.

The second research topic in Chapter 3 is the investigation of a new type of circuit component for use in reconfigurable electromagnetic devices [142]. In 1971, the existence of a fourth circuit element element called the memristor was hypothesized by Leon Chua. It bridges the relationship between flux and charge, and possesses a unit of ohms. The device is effectively a charge-dependent variable resistor, where the quasi-static resistance is a function of the integral of current. The memristor is appealing for reconfigurable devices because of the ability to switch from a low-resistance state to a high-resistance state by short-duration current pulses. Additionally, the device is stable and will maintain its state (resistance) in the absence of any controlling signal, unlike many switching elements such as PIN diodes or transistors which require bias currents or voltages to maintain their state. The memristor has received recent attention due to the fabrication of a device by scientists at Hewlett-Packard Labs which exhibits characteristics that can be modeled
with memristive systems. This nano-scale device is created by sandwiching an oxide layer in between two metal electrodes. Many different oxide and electrode materials have been explored and each can yield different device properties.

In the studies performed, a discrete-time non-linear dopant drift memristor model was created to allow for implementation in a custom finite-different time-domain simulation tool. The non-linear drift model allows the device to exhibit state boundary-region effects similar to those observed by fabricated samples. An FD-TD tool was selected due to the ability to observe the transient characteristics of the device, the switching behavior of the memristor, and any effects the radio-frequency signals may have on the memristor. Since there is a great deal of flexibility in excitation signals with FD-TD, the memristor control and radio frequency signals can be simultaneously applied to the device, giving the ability to reconfigure the memristor(s) and observe the RF performance in a single simulation. Two variations in the FD-TD code were created, a periodic version and a generic 3D version. The periodic version allows for design of 2D periodic structures such as frequency selective surfaces and antenna arrays. In this case, a reconfigurable periodic planar absorber was created and modeled, demonstrating the viability of the memristor as the reconfiguring element in the design. With the generic 3D FD-TD code, band-switching and polarization-reconfigurable patch antennas were designed using memristors as the switching elements. The design technique could also be used for creation of multi-band antennas with operating frequencies that span very large ranges (octaves or more), which would be well-suited for use in the ultra-wideband array layouts introduced in Chapter 2. The demonstrated designs have successfully illustrated that memristors can be an effective component in the design of reconfigurable radio frequency devices.

4.1 Summary of Contributions

The contributions to the fields of ultra-wideband antenna array design, reconfigurable antenna design, and the electromagnetics community in general can summarized by the following:
• Effectively introduced the covariance matrix adaptation evolutionary strategy (CMA-ES) to the electromagnetics community.

• Applied CMA-ES to a simple linear array geometry framework to generate very high performance-to-size ratio array designs for small arrays.

• Created a new method for planar array design which uses slices of a rotationally symmetric aperture and has the following benefits:
  – Reduced number of parameters required to define the locations of all of the elements in the array (compared to controlling the location of every element).
  – Symmetry in the array factor which reduces the computational times required to determine array performance, an especially desirable characteristic for optimization purposes.
  – Geometric regularity in the form of rotationally symmetric tiles. The arrays possess some desirable properties in terms of fabrication compared to fully aperiodic designs.

• Demonstrated the ability of the planar array design technique to produce arrays with exceptional ultra-wideband performance using:
  – An optimized configuration where positions of elements in one slice of the aperture are determined by CMA-ES.
  – A unoptimized array type where each rotationally symmetric slice of the aperture is seeded with a standard periodic array lattice, allowing for great simplification in array fabrication.

• Investigated the effect of element position error on the peak sidelobe level performance of the optimized linear arrays, optimized rotationally symmetric arrays, and semi-periodic rotationally symmetric arrays.

• Integrated a discrete-time non-linear memristor model into two specialized finite-difference time-domain simulation tools.

• Demonstrated the viability of the memristor as an effective switching element using three reconfigurable electromagnetic designs:
– Periodic planar absorber.
– Dual-Band patch antenna.
– Dual-polarization patch antenna.

4.2 Future Work

The work that has been completed thus far leaves some interesting possibilities for future research. These primarily include:

• Evaluation of the abilities of the compact ultra-wideband linear array design technique with larger arrays (more elements).

• Evaluation of the abilities of the rotationally symmetric ultra-wideband planar array design techniques with larger arrays (more elements).

• Further investigations into available device properties (i.e. $R_{ON}$, $R_{OFF}$, $\mu_D$, $D$) of recently fabricated memristor samples.

• Inquiries to semiconductor fabrication facilities to have memristors fabricated for testing in basic reconfigurable RF devices.
Appendix A

CMA-ES Performance Evaluation

A.1 Introduction

When the covariance matrix adaptation evolutionary strategy was first encountered, it was run through rigorous tests to determine if it was a suitable algorithm to be used for commonly encountered electromagnetics design problems. As such, it was compared to the other most commonly used real-valued optimization algorithm in electromagnetics, particle swarm optimization (PSO). Before the algorithms were applied to array designs, they were applied to a set of numerical test functions and a stacked-patch antenna design. These comparisons are given here to illustrate the performance benefits to be had by using CMA-ES over other commonly used algorithms.

A.2 Numerical Test Functions

The two evolutionary strategies (CMA-ES and PSO) were first compared using several unimodal and multimodal test functions with nine dimensions. This was selected because the following antenna design problem required nine parameters to define the geometry. The operating parameters for PSO are given in Table A.1. Since PSO is well documented and widely used, its internal operation will be not be covered. Only the population size needs to be chosen for CMA-ES, for which several values are chosen along with PSO. The selected functions are given in
Sec. A.2.1 and the performance results are given in Sec. A.2.2. The sphere [143], Zakharov [144], and rotated hyper-ellipsoid [145] functions are unimodal, while the Ackley [146] and Rastrigin [147] are multimodal. Multimodal functions are generally much more difficult for optimization algorithms, as is illustrated in Sec. A.2.2. The tendency to get caught in local minima must be mitigated to achieve success (reach the global optima), some algorithms perform better than others in this respect. Success is declared if the algorithm reaches the function value goal \(10^{-6}\) within \(10^4\) iterations.

Table A.1: Particle swarm optimization parameters used for the test function analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_1)</td>
<td>1.5</td>
</tr>
<tr>
<td>(c_2)</td>
<td>1.5</td>
</tr>
<tr>
<td>(g_{\text{expected}})</td>
<td>(10^4)</td>
</tr>
<tr>
<td>(\omega_{\text{min}})</td>
<td>0.2</td>
</tr>
<tr>
<td>(\omega_{\text{max}})</td>
<td>0.7</td>
</tr>
<tr>
<td>(v_{\text{max}})</td>
<td>0.35</td>
</tr>
</tbody>
</table>

### A.2.1 Test Function Definitions

For each of the following test functions, \(x\) is the set of input parameters to be optimized and \(x^*\) is the optimal set that yields the lowest function value given by \(F(x^*)\). All of the test functions chosen for the analysis here have a global minimum value of 0 located at 0.

#### A.2.1.1 Sphere

\[
F_{\text{SPH}}(x) = \sum_{i=1}^{N} x_i^2 \quad (A.1)
\]

where

\[x_i \in [-10, 10]\]
A.2.1.2 Ackley

\[
F_{ACK}(\mathbf{x}) = 20 + e - 20\exp\left(-0.2\sqrt{\frac{1}{N} \sum_{i=1}^{N} x_i^2}\right) - \exp\left(\frac{1}{N} \sum_{i=1}^{N} \cos(2\pi x_i)\right) \quad (A.2)
\]

where

\[
x_i \in [-32, 32]
\]

\[
\mathbf{x}^* = \mathbf{0}
\]

\[
F_{ACK}(\mathbf{x}^*) = 0
\]

A.2.1.3 Rastrigin

\[
F_{RAS}(\mathbf{x}) = 10N + \sum_{i=1}^{N} \left(x_i^2 - 10\cos(2\pi x_i)\right) \quad (A.3)
\]

where

\[
x_i \in [-5, 5]
\]

\[
\mathbf{x}^* = \mathbf{0}
\]

\[
F_{RAS}(\mathbf{x}^*) = 0
\]

A.2.1.4 Zakharov

\[
F_{ZAK}(\mathbf{x}) = \left(\sum_{i=1}^{N} x_i^2\right) + \left(\sum_{i=1}^{N} 0.5ix_i\right)^2 + \left(\sum_{i=1}^{N} 0.5ix_i\right)^4 \quad (A.4)
\]

where

\[
\mathbf{x}^* = \mathbf{0}
\]

\[
F_{ZAK}(\mathbf{x}^*) = 0
\]
\[ x_i \in [-5, 10] \]
\[ x^* = 0 \]
\[ F_{ZAK}(x^*) = 0 \]

A.2.1.5 Rotated Hyper-Ellipsoid

\[ F_{RHE}(\mathbf{x}) = \sum_{i=1}^{N} \left( \sum_{j=1}^{i} x_j \right)^2 \quad (A.5) \]

where

\[ x_i \in [-65.535, 65.535] \]
\[ x^* = 0 \]
\[ F_{RHE}(x^*) = 0 \]

A.2.2 Test Function Analysis Results

The resulting success rates and mean NFE requirements of the analysis are given in Table A.2. Significant reductions in mean NFE for CMA-ES can be observed for the population scenarios where the algorithms begin to approach a 100% success rate.

A.3 Stacked-Patch Antenna Design

The two algorithms were used to optimize a stacked-patch antenna for operation from 1.1 GHz to 1.3 GHz. This was done by coupling the algorithms to the FEKO software package [17], which is based on a full-wave method of moments analysis technique. The geometry of the stacked-patch antenna is shown in Fig. A.1. The dielectric substrates were treated by layered media Green’s functions, while the metallic patches were specified as perfect electric conductors. Nine optimization parameters were required to represent the antenna design, with their description
Table A.2: Performance results for the CMA-ES and PSO when applied to the selected test functions using nine dimensions \((N = 9)\). For each trial, 100 seeds were run. Mean and variance statistics only account for successful seeds. A seed is considered successful if it reaches a function value of \(10^{-6}\) or better within \(10^4\) iterations.

<table>
<thead>
<tr>
<th>Function</th>
<th>(N_{\text{pop}})</th>
<th>Particle Swarm Optimization</th>
<th>Covariance Matrix Adaptation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean NFE</td>
<td>Var. NFE</td>
</tr>
<tr>
<td>(F_{\text{SPH}})</td>
<td>10</td>
<td>1630</td>
<td>(2.51 \times 10^4)</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>2597</td>
<td>(2.99 \times 10^4)</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>5518</td>
<td>(7.04 \times 10^4)</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>9964</td>
<td>(2.49 \times 10^5)</td>
</tr>
<tr>
<td>(F_{\text{ACK}})</td>
<td>10</td>
<td>2948</td>
<td>(9.44 \times 10^4)</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>4769</td>
<td>(8.01 \times 10^4)</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>10140</td>
<td>(2.71 \times 10^5)</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>18355</td>
<td>(4.31 \times 10^5)</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>18873</td>
<td>(3.64 \times 10^5)</td>
</tr>
<tr>
<td>(F_{\text{ZAK}})</td>
<td>10</td>
<td>9669</td>
<td>(2.53 \times 10^7)</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>8416</td>
<td>(9.53 \times 10^7)</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>12073</td>
<td>(2.03 \times 10^7)</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>19648</td>
<td>(2.48 \times 10^7)</td>
</tr>
<tr>
<td>(F_{\text{RAS}})</td>
<td>50</td>
<td>12438</td>
<td>(9.51 \times 10^7)</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>38475</td>
<td>(3.74 \times 10^8)</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>77485</td>
<td>(7.66 \times 10^8)</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>48050</td>
<td>(9.60 \times 10^8)</td>
</tr>
<tr>
<td>(F_{\text{RHE}})</td>
<td>10</td>
<td>5109</td>
<td>(8.98 \times 10^5)</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>6345</td>
<td>(3.03 \times 10^5)</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>11645</td>
<td>(6.55 \times 10^5)</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>19624</td>
<td>(5.03 \times 10^6)</td>
</tr>
</tbody>
</table>

and ranges provided in Table A.3. Aspect ratios were imposed to constrain the dimensions of the metallic patches to practical ranges so that thin strips are excluded from the parameter space. This, however, introduces some parametric inseparability into the problem, making it more challenging for the optimization strategy (as opposed to specifying values for \(y_U\) and \(y_L\) directly).

Table A.3: Optimization parameter ranges and descriptions for the stacked-patch antenna.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min.</th>
<th>Max.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_{r_U})</td>
<td>0.6</td>
<td>1.4</td>
<td>Aspect ratio for the upper patch</td>
</tr>
<tr>
<td>(x_U)</td>
<td>20 mm</td>
<td>68 mm</td>
<td>(x)-dimension for the upper patch</td>
</tr>
<tr>
<td>(a_{r_L})</td>
<td>0.6</td>
<td>1.4</td>
<td>Aspect ratio for the lower patch</td>
</tr>
<tr>
<td>(x_L)</td>
<td>20 mm</td>
<td>68 mm</td>
<td>(x)-dimension for the lower patch</td>
</tr>
<tr>
<td>(t_U)</td>
<td>0.1 mm</td>
<td>10 mm</td>
<td>Thickness of the upper dielectric layer</td>
</tr>
<tr>
<td>(t_L)</td>
<td>0.1 mm</td>
<td>10 mm</td>
<td>Thickness of the lower dielectric layer</td>
</tr>
<tr>
<td>(\epsilon_U)</td>
<td>1.0</td>
<td>7.0</td>
<td>Relative permittivity of the upper dielectric layer ((\mu_r = 1.0))</td>
</tr>
<tr>
<td>(\epsilon_L)</td>
<td>1.0</td>
<td>7.0</td>
<td>Relative permittivity of the lower dielectric layer ((\mu_r = 1.0))</td>
</tr>
<tr>
<td>(x_{\text{feed}})</td>
<td>0.2 mm</td>
<td>10 mm</td>
<td>Distance of feed probe from the edge of patch</td>
</tr>
</tbody>
</table>
Figure A.1: Geometry of the stacked-patch antenna.

The broadside gain and S-parameters (VSWR) for each antenna were calculated at five equally spaced frequency points between 1.1 GHz and 1.3 GHz. Design cost to be minimized was computed using

$$F_{ANT} = 4 \max (VSWR_{1.1-1.3GHz}) - Gain_{1.2GHz}^{dB},$$  \hspace{1cm} (A.6)$$

containing contributions from the worst input VSWR across the frequency band and the broadside, mid-band (1.2 GHz) gain. A function goal of 3.0 was set for the designs, which typically yielded a 2:1 maximum VSWR and a gain of 5 dB. Approximate time required per function evaluation (5 frequency points) was 90 seconds on a single core of an Intel 2.4 GHz Xeon, quad-core processor running a Linux operating system.

Because of the lengthy function evaluation (simulation) time, it was necessary to use far fewer trials than what was possible for the test functions. Five seeds per algorithm were chosen as a compromise between statistical certainty of performance and total run time. The algorithm parameters in Table A.4 were selected for PSO. An initial population size of 20 particles was chosen, since it performed well on most of the test functions. This small population size, however, resulted in poor success rates for the antenna design. Out of five seeds, three failed to reach the goal after 100 iterations (2000 function evaluations). As shown in Fig. A.2a, the two...
that did succeed yielded fast results as expected, achieving the function value goal of 3.0 at 500 and 860 function evaluations. Each curve in Fig. A.2 represents, for one seed, the best function value achieved from the beginning of the optimization up until the number of function evaluations shown on the x-axis.

Table A.4: Particle swarm optimization parameters used for the stacked-patch antenna analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>1.8</td>
</tr>
<tr>
<td>$c_2$</td>
<td>1.8</td>
</tr>
<tr>
<td>$g_{expected}$</td>
<td>10$^2$</td>
</tr>
<tr>
<td>$\omega_{min}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\omega_{max}$</td>
<td>0.7</td>
</tr>
<tr>
<td>$v_{max}$</td>
<td>0.35</td>
</tr>
</tbody>
</table>

For many optimization problems, a success rate of 40% would be considered insufficient. It is generally desirable to use a population size that will achieve reasonable success rates (e.g. 80% or higher), therefore the PSO population was gradually increased until an acceptable success rate was achieved. At 30 particles, one additional seed was successful in the allotted 100 iterations, as shown in Fig. A.2b. Slower performance was exhibited, but at increased algorithm reliability. At a population of 40 particles, PSO gave acceptable success rates, having all five seeds meet the goal with a mean requirement of 1864 function evaluations (simulations).

A population size of 10 was chosen for the CMA-ES since it is the minimum suggested population size (from Eq. (2.1)) and it performed well on the selected test functions. No other algorithm parameters were required for the CMA-ES optimization. Yielding a success rate of 100% with five seeds at this small population size, there was no need to increase the population as with PSO. The evolution results for PSO and CMA-ES are shown in Fig. A.2c, while a statistical comparison is given in Table A.5. While both algorithms achieved a 100% success rate, the CMA-ES exhibited a 62% reduction in optimization time compared to PSO. At the rarely successful PSO population size of 20, even the seeds that did converge were only
Figure A.2: Evolutionary progress for the stacked-patch antenna problem using (a) PSO with a population side of 20, (b) PSO with a population size of 30 and (c) PSO with a population size of 40 and CMA-ES with a population size of 10. Black dashed lines indicate the cost function goal of 3.0.

comparable in speed to the cases where CMA-ES always yielded a successful result.

The resulting antenna performance for PSO (population size of 40) and CMA-ES (population size of 10) are shown in Fig. A.3. Each algorithm evolved antenna designs with a relatively diverse set of characteristics. Moreover, each design has $F_{\text{ANT}} \leq 3.0$, which leaves room for differences between VSWR and gain.

Through the test function analysis and optimization of a stacked-patch antenna design, the CMA-ES has shown itself to be a superior optimization strategy. Con-
Figure A.3: VSWR and gain for each of the five seeds optimized with (a) PSO at a population size of 40 and (b) CMA-ES with a population size of 10.

Table A.5: Optimization statistics for the stacked-patch antenna optimization problem (considering only successful seeds). Asterisks indicate runs with low success rates; results can only partially be compared to other runs with good success rates.

<table>
<thead>
<tr>
<th>Population Size</th>
<th>CMA-ES</th>
<th>PSO*</th>
<th>PSO*</th>
<th>PSO*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success Rate</td>
<td>100%</td>
<td>40%</td>
<td>60%</td>
<td>100%</td>
</tr>
<tr>
<td>Minimum NFE</td>
<td>570</td>
<td>500</td>
<td>1110</td>
<td>1480</td>
</tr>
<tr>
<td>Maximum NFE</td>
<td>900</td>
<td>860</td>
<td>2010</td>
<td>3120</td>
</tr>
<tr>
<td>Mean NFE</td>
<td>708</td>
<td>680</td>
<td>1640</td>
<td>1864</td>
</tr>
<tr>
<td>Variance NFE</td>
<td>$1.88 \times 10^4$</td>
<td>$6.48 \times 10^4$</td>
<td>$2.22 \times 10^5$</td>
<td>$4.98 \times 10^5$</td>
</tr>
<tr>
<td>Mean Time (hours)</td>
<td>17.7</td>
<td>17.0</td>
<td>41.0</td>
<td>46.6</td>
</tr>
<tr>
<td>Relative Mean Time</td>
<td>0.38</td>
<td>0.36</td>
<td>0.88</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Considering the five seeds for each algorithm and at comparable success rates, the CMA-ES reduced the optimization time by approximately 62% on average compared to a conventional PSO. Even with a small PSO population of 20 particles where it yielded fast time-to-success but a reduced success rate, the mean time-to-success (680 NFE), was only comparable to where CMA-ES achieved 100% success rates. It is apparent that for either algorithm, choosing the smallest population size will yield the fastest optimization times (for the successful seeds). This, however, can significantly affect the reliability of the algorithm, therefore a population large...
enough for reasonable success must be used. In the case of the CMA-ES, small populations yield fast performance in addition to extremely high success rates.
Appendix B

FD-TD Specifics

B.1 Field Update Equations

B.1.1 Normal-Space Electric Field Update

Electric field update equations for regions that do not contain a lumped element or source and are not part of the absorbing boundary layer:

\[
E_x|_{i,j+\frac{1}{2},k+\frac{1}{2}} = C_x^{(1)}|_{i,j+\frac{1}{2},k+\frac{1}{2}} E_x|_{i,j+\frac{1}{2},k+\frac{1}{2}} + C_x^{(2)}|_{i,j+\frac{1}{2},k+\frac{1}{2}} \times \left( \frac{H_z|_{i,j+1,k+\frac{1}{2}} - H_z|_{i,j,k+\frac{1}{2}}}{\Delta y} - \frac{H_y|_{i,j+\frac{1}{2},k+1} - H_y|_{i,j+\frac{1}{2},k}}{\Delta z} \right) \tag{B.1}
\]

\[
E_y|_{i+\frac{1}{2},j,k+\frac{1}{2}} = C_y^{(1)}|_{i+\frac{1}{2},j+\frac{1}{2},k} E_y|_{i+\frac{1}{2},j+\frac{1}{2},k} + C_y^{(2)}|_{i+\frac{1}{2},j+\frac{1}{2},k} \times \left( \frac{H_x|_{i+1,j,k+\frac{1}{2}} - H_x|_{i,j+\frac{1}{2},k}}{\Delta z} - \frac{H_z|_{i+1,j,k+\frac{1}{2}} - H_z|_{i,j,k+\frac{1}{2}}}{\Delta x} \right) \tag{B.2}
\]

\[
E_z|_{i+\frac{1}{2},j+\frac{1}{2},k} = C_z^{(1)}|_{i+\frac{1}{2},j+\frac{1}{2},k} E_z|_{i+\frac{1}{2},j+\frac{1}{2},k} + C_z^{(2)}|_{i+\frac{1}{2},j+\frac{1}{2},k} \times \left( \frac{H_y|_{i+1,j+\frac{1}{2},k} - H_y|_{i,j+\frac{1}{2},k}}{\Delta x} - \frac{H_x|_{i+\frac{1}{2},j+1,k} - H_x|_{i+\frac{1}{2},j,k}}{\Delta y} \right) \tag{B.3}
\]
where

\[ C_s^{(1)} = \frac{1 - \frac{\sigma_s \Delta t}{2C_*}}{1 + \frac{\sigma_s \Delta t}{2C_*}} \]  \quad \text{(B.4)}

\[ C_s^{(2)} = \frac{\Delta t}{1 + \frac{\sigma_s \Delta t}{2C_*}} \]  \quad \text{(B.5)}

Note: * indicates that this applies for \(x-, y-, \) and \(z-\)fields. The correct \(\epsilon_x, \epsilon_y, \) or \(\epsilon_z \) and \(\sigma_x, \sigma_y, \) or \(\sigma_z \) must be selected for the appropriate \(C_x, C_y \) or \(C_z, \) respectively. Location indices \((i, j, k)\) are omitted for simplicity.

### B.1.2 Normal-Space Magnetic Field Update

Magnetic field update equations for regions that do not contain a source and are not part of the absorbing boundary layer:

\[ H_x|_{i+\frac{1}{2},j,k}^{n+1} = G_x^{(1)}|_{i+\frac{1}{2},j,k} H_x|_{i+\frac{1}{2},j,k}^{n-1} + G_x^{(2)}|_{i+\frac{1}{2},j,k} \times \]

\[ \left( \frac{E_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_y|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta z} - \frac{E_x|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_x|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta y} \right) \]  \quad \text{(B.6)}

\[ H_y|_{i,j+\frac{1}{2},k}^{n+1} = G_y^{(1)}|_{i+\frac{1}{2},j,k} H_y|_{i+\frac{1}{2},j,k}^{n-1} + G_y^{(2)}|_{i+\frac{1}{2},j,k} \times \]

\[ \left( \frac{E_z|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_z|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta x} - \frac{E_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta z} \right) \]  \quad \text{(B.7)}

\[ H_z|_{i,j,k+\frac{1}{2}}^{n+1} = G_z^{(1)}|_{i,j,k+\frac{1}{2}} H_z|_{i,j,k+\frac{1}{2}}^{n-1} + G_z^{(2)}|_{i,j,k+\frac{1}{2}} \times \]

\[ \left( \frac{E_x|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_x|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta y} - \frac{E_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta x} \right) \]  \quad \text{(B.8)
where

\[ G_s^{(1)} = \frac{1 - \frac{\sigma^m \Delta t}{2\mu^*}}{1 + \frac{\sigma^m \Delta t}{2\mu^*}} \]  

(B.9)

\[ G_s^{(2)} = \frac{\Delta t}{1 + \frac{\sigma^m \Delta t}{2\mu^*}} \]  

(B.10)

Note: * indicates that this applies for \( x \), \( y \), and \( z \) fields. \( \sigma^m \) is magnetic conductivity. The correct \( \mu_x \), \( \mu_y \), or \( \mu_z \) and \( \sigma^m_x \), \( \sigma^m_y \), or \( \sigma^m_z \) must be selected for the appropriate \( G_x \), \( G_y \) or \( G_z \), respectively. Location indices \((i, j, k)\) are omitted for simplicity.

### B.2 Current Through an Electric Field Location

\[
i_x|_{i+\frac{1}{2},j,k+\frac{1}{2}} = \left( H_z|_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_z|_{i,j,k+\frac{1}{2}} \right) \Delta z + \left( H_y|_{i+\frac{1}{2},j,k} - H_y|_{i,j+\frac{1}{2},k+1} \right) \Delta y
\]  

(B.11)

\[
i_y|_{i+\frac{1}{2},j,k+\frac{1}{2}} = \left( H_x|_{i+\frac{1}{2},j,k+1} - H_x|_{i+\frac{1}{2},j,k} \right) \Delta x + \left( H_z|_{i,j+\frac{1}{2},k} - H_z|_{i,j,k+\frac{1}{2}} \right) \Delta z
\]  

(B.12)

\[
i_z|_{i+\frac{1}{2},j,k+\frac{1}{2}} = \left( H_y|_{i+1,j+\frac{1}{2},k} - H_y|_{i,j+\frac{1}{2},k} \right) \Delta y + \left( H_x|_{i+\frac{1}{2},j+1,k} - H_x|_{i+\frac{1}{2},j,k} \right) \Delta x
\]  

(B.13)
B.3 Source Signals for FD-TD

Since a variety of signals are needed to control the memristor and evaluate the RF properties of the configured device, a simple modified-square pulse is used that takes the form of

\[ f_{\text{pulse}}[n] = \frac{a \Delta t}{\sigma \sqrt{2\pi}} \sum_{m=1}^{n} \left( e^{-\frac{(m\Delta t-t_{\text{on}})^2}{2\sigma^2}} - e^{-\frac{(m\Delta t-t_{\text{off}})^2}{2\sigma^2}} \right) \]  \hspace{1cm} (B.14)

where

\[ \sigma = \frac{1}{\text{erf}^{-1}(0.8)} \approx 0.5933 t_{\text{trans}}, \]  \hspace{1cm} (B.15)

\[ t_{\text{off}} = t_{\text{on}} + t_{\text{width}}, \]  and \( n \) is the current timestep. Eq. (B.14) is basically an area-compensated discrete integration of two Gaussian pulses; one to turn on, and one to turn off. The constants in (B.14) and (B.15) are defined in Table B.1. Multiple pulses can be combined as in Fig. B.1, which incorporates two pulses with properties

\[ \text{pulse}_1 = \begin{cases} t_{\text{on}} = 10 \text{ ns} \\ t_{\text{width}} = 15 \text{ ns} \\ t_{\text{trans}} = 1 \text{ ns} \\ a = 4.5 \text{ Volts} \end{cases} \]  \hspace{1cm} (B.16)

and

\[ \text{pulse}_2 = \begin{cases} t_{\text{on}} = 60 \text{ ns} \\ t_{\text{width}} = 20 \text{ ns} \\ t_{\text{trans}} = 5 \text{ ns} \\ a = -3 \text{ Volts} \end{cases} \]  \hspace{1cm} (B.17)

In addition, the pulse in Eq. (B.14) may be modulated to allow for characterization of the RF properties of the device by way of

\[ f_{\text{sineburst}}[n] = \sin (2\pi n \Delta t f_{\text{modulation}}) f_{\text{pulse}}[n] \]  \hspace{1cm} (B.18)
where $f_{\text{modulation}}$ is the modulation frequency. An example of two sine-burst signals is given in Fig. B.2 which have properties

$$sineburst_1 = \begin{cases} 
  t_{\text{on}} = 15 \text{ ns} \\
  t_{\text{width}} = 30 \text{ ns} \\
  t_{\text{trans}} = 6 \text{ ns} \\
  a = 2 \text{ Volts} \\
  f_{\text{modulation}} = 300 \text{ MHz} 
\end{cases} \tag{B.19}$$

and

$$sineburst_2 = \begin{cases} 
  t_{\text{on}} = 70 \text{ ns} \\
  t_{\text{width}} = 15 \text{ ns} \\
  t_{\text{trans}} = 3 \text{ ns} \\
  a = 1 \text{ Volt} \\
  f_{\text{modulation}} = 800 \text{ MHz} 
\end{cases} \tag{B.20}$$

Table B.1: Parameters and definitions of the memristor tuning pulses.

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{\text{on}}$</td>
<td>Pulse start time (at 50% pulse magnitude).</td>
</tr>
<tr>
<td>$t_{\text{width}}$</td>
<td>Pulse width (50% to 50%).</td>
</tr>
<tr>
<td>$t_{\text{trans}}$</td>
<td>Pulse transition time (20% to 80% pulse magnitude).</td>
</tr>
<tr>
<td>$a$</td>
<td>Pulse magnitude.</td>
</tr>
</tbody>
</table>
Figure B.1: Example pulses that may be used to reconfigure a memristor.

Figure B.2: Example sine-modulated pulses that may be used to characterize a configured device.


Vita

Micah Dennis Gregory

Micah Dennis Gregory was born in Williamsport, PA on 13 April 1984. He graduated Liberty Junior/Senior High School in 2002, then attended undergraduate studies at Bucknell University. After graduating with a Bachelor’s degree in Electrical Engineering and a minor in Mathematics in 2006, he pursued a Masters of Science at the Pennsylvania State University until completion in 2009. He has worked in the Computational Electromagnetics and Antennas Research Lab (CEARL) at Penn State while pursuing his Ph.D. until 2013. While there, he was granted the A. J. Ferraro Graduate Research Award in 2009 and the Dr. Nirmal K. Bose Dissertation Excellence Award in 2013.