PREDICTION OF SAND PARTICLE TRAJECTORIES AND
SAND EROSION DAMAGE ON HELICOPTER ROTOR BLADES

A Dissertation in
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by
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ABSTRACT

Erosion damage on a helicopter rotor blade by the impact of solid particle causes serious material removal from the surface of the rotor blade. The material removal results in a serious decrease of its life-time and an increase of repair and replacement cost. A good understanding and prediction of erosion on a helicopter rotor blade is thus very important to reduce the damage and to develop erosion protection system (EPS). However, an erosion prediction on helicopter rotor blades is still challenge because of the complexity of the physical processes involved and the limitation of existing open literature.

Therefore, in this dissertation, accurate and time-efficient methodologies were developed for performing sand particle tracking and predicting sand erosion damage on actual helicopter rotor blades under realistic hover and vertical lift conditions. The development of these methodologies were performed based on an extensive literature review in the area of solid particle erosion in turbomachinery systems because flow physics and erosion phenomena in turbomachinery components and those around helicopter rotor blades.

In this dissertation, first, injection (release) conditions of solid particles with new injection parameter, sand particle mass flow rate (SPmFR), were specified to deal with the effect of non-uniform and unsteady flow conditions surrounding at each injection point from which solid particles are released. The SPmFR defines the number of solid particles released from the same injection position per unit time.

Secondly, a general definition of erosion rate, “mass or volume loss from the metal surface due to the impact of a unit “mass” of solid particles” was also modified by multiplying with SPmFR in order to solve the limitation for predicting erosion damage on actual helicopter rotor blade.
Next, a suitable empirical particle rebound model and an erosion damage model for spherical sand particles with diameters ranging from $10 \, \mu m$ to $500 \, \mu m$ impacting on the material Ti-6Al-4V, the material of helicopter rotor blade, were developed.

Finally, C++ language based codes in the form of User Defined Functions (UDFs) were developed and implemented into the commercially available multi-dimensional viscous flow solver ANSYS-FLUENT in order to develop and integrate with the general purpose flow solver, ANSYS-FLUENT, for a specific Lagrangian particle trajectory computing algorithm and rebound and erosion quantification purposes.

In the erosion simulation, a reasonably accurate fluid flow solution is necessary. In order to validate the numerical results obtained in this dissertation, computations for flow-only around 2D RAE2822 airfoil and 3D rotating rotor blade (NACA0012) without any sand particle were performed. In the comparison of these results with experimental results, it is found that the flow solutions are in good agreement with the experimental data. Next, second computational validation for flow around the SC1095 airfoil for various turbulence models were performed in order to select a suitable turbulence model. These results concluded that numerical results with $k - \omega$ SST model have a reasonably best accuracy.

By using developed methodologies for particle tracking, erosion prediction, and flow simulation, 2D erosion simulations for various inflow conditions, such as inflow Mach number, inflow angle of attack, and the diameter of solid particle, were performed in order to understand the details of erosion mechanism existing on helicopter rotor blades. These results indicate that the magnitude of erosion rate is affected by inflow Mach number and that erosion rate distribution, including maximum erosion rate position and the area of erosion damaged airfoil, is highly depended upon the diameter of solid particle and inflow angle of attack.

Relative inflow conditions to the blade section of helicopter rotor blades are highly dependent upon rotor blade geometric conditions and helicopter rotor operational conditions.
Therefore, in this dissertation, 3D erosion simulations for four different rotating blades with uniform airfoil profile (SC1095) were performed in order to understand the details of erosion mechanism. These results indicate that erosion patterns including maximum erosion rate position and the extent of erosion damaged area on the blade section were highly dependent upon a spanwise twist distribution. It is found that the magnitude of erosion damage on the blade section is affected by not a spanwise twist but a swept tip.

Next, in this dissertation, UH-60A helicopter rotor blades rotating in the computational domain for various collective pitch angles and climb velocities were simulated. These results indicate that overall erosion characteristics for helicopter rotor blades can be considered to be not dependent upon these operational parameters though there is a little difference in the magnitude of erosion damage and the maximum erosion rate position. These results concluded that a hover condition can be chosen as a reference operational condition for predicting erosion characteristics or for investigating erosion reduction methods.

The final phase of this research is a generalization for particle trajectories and erosion characteristics on 3D helicopter rotor blades in order to reduce very expensive erosion computational cost. The generalized results show that aerodynamic and erosion characteristics for a 3D rotor blade can be predicted by using the 2D airfoil results for corresponding relative inflow angle of attack with coefficient for inflow velocity magnitude and aerodynamic loss difference between 2D and 3D flow simulation results.

The present thesis provides a complete and unique methodology in predicting sand particle trajectories and their erosion damage on actual helicopter rotor blades under realistic hover and vertical lift conditions. This thesis describes completely detailed phenomena of sand particle trajectories and sand erosion damage on 2D airfoils and on 3D helicopter rotor blades. Finally, this thesis suggests a generalization for sand particle trajectories and sand erosion damage in order to reduce significantly computational cost.
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NOMENCLATURE

\( C_D \) Drag coefficient
\( C_P \) Pressure coefficient
\( C_T \) Thrust coefficient
\( D_{T,P} \) Thermophoretic force coefficient
\( D_p \) Solid particle diameter
\( dL \) Interval length between two adjacent injection points
\( E \) Young’s modulus
\( F \) External force acting on a solid particle
\( F_{add} \) Additional forces acting on a solid particle
\( F_{Bi} \) Brownian force
\( F_D \) Drag force coefficient
\( F_l \) Hydrodynamic force due to fluid flow
\( F_{lift} \) Saffman lift force
\( F_{mass} \) Added mass force
\( F_T \) Thermophoretic force
\( K_n \) Knudsen number, \( 2\lambda/d_p \)
\( L \) Injection domain length
\( L_{cell, min} \) Minimum cell size on wall condition
\( L_p \) Depth of plastic deformation
\( L_{tot} \) Total length of injection domain (plane)
\( M_i \) Inflow Mach number
\( M_{in} \) Mach number at inlet boundary domain
\( M_{pi} \) Mach number of solid particle at inflow boundary domain
\( M_0 \) Inflow Mach number
\( M_{tip} \) Rotor blade tip Mach number
\( N_p \) The number of injection points
\( \dot{N}_p \) The number of solid particles released from injection point
\( P \) Pressure
\( R \) Radial length of a rotor blade
\( Re \) Reynolds number of fluid flow, \( \rho d_p |\dot{V} - \dot{V}_p|/\mu \)
\( Re_p \) Reynolds number of solid particle
\( T \) Temperature
\( T_p \) Temperature of a solid particle
\( V \) Velocity of fluid flow
\( V \) Relative velocity
\( V_{\text{impact}} \) Impact velocity of a solid particle
\( V_{\text{inlet}} \) Solid particle velocity at injection domain
\( V_{N_1} \) Normal velocity of a solid particle before impact on metallic surface
\( V_{N_2} \) Normal velocity of a solid particle after impact on metallic surface
\( V_{N_{2M}} \) Normal velocity of a solid particle after impact on measurement plane
\( V_p \) Velocity of a solid particle
\( V_{p_1} \) Impact velocity of a solid particle
\( V_{p_2} \) Relative radial velocity to the blade section of a rotor blade
\( V_{R} \) Relative velocity of fluid flow in the radial direction
\( V_{T} \) Relative in-plane velocity to the blade section of a rotor blade
\( V_{T_1} \) Tangential velocity of a solid particle before impact on metallic surface
\( V_{T_2} \) Tangential velocity of a solid particle after impact on metallic surface
\( V_{T_{2M}} \) Tangential velocity of a solid particle after impact on measurement plane
\( V_y \) Relative velocity in y direction
\( V_z \) Relative velocity of fluid flow in the axial direction
\( V_{\theta} \) Relative velocity of fluid flow in the circumferential direction
\( V_1 \) Solid particle velocity before impact on metallic surface
\( Y(t) \) Coordinate of a solid particle suspended in domain at time step
\( Y_S \) Material yield strength at ambient temperature
\( Y_{SRT} \) Material yield strength at operating temperature
\( c \) Chord length of a 2D airfoil or the blade section of a rotor blade
\( c_d \) Section (local) drag coefficient
\( c_l \) Section (local) lift coefficient
\( c_1 \) Steady-state drag force coefficient
\( d \) Solid particle diameter
\( d \) Deformation tensor
\( d_p \) Solid particle diameter
\( e_{N} \) Normal velocity restitution ratio, \( V_{N_{2M}}/V_{N_1} \)
\( e_{V} \) Velocity restitution ratio, \( V_{2M}/V_1 \)
\( e_{T} \) Tangential velocity restitution ratio, \( V_{T_{2M}}/V_{T_1} \)
\( e_{\beta} \) Angle restitution ratio, \( \beta_{2M}/\beta_1 \)
\( f \) Dimensionless empirical factor for the steady-state force
\( g \) Gravity
s

Greek

$K$ Thermal conductivity ratio of fluid flow to a solid particle, $\kappa/\kappa_p$

$\Delta_A$ Added-mass force coefficient

$\Delta_H$ History integral force coefficient

$\phi$ Mass concentration of solid phase, $m_p/(m_p + m)$

$\Omega$ Rotational speed of a rotor blade

$\alpha$ Radius of solid particle

$\alpha$ Aerodynamic angle of attack

$\alpha$ Volume ratio of fluid phase to all phase, $V_f/(V_f + V_p)$

$\alpha_{eff}$ Effective ratio of fluid phase to all phase

$\alpha_0$ Inflow angle of attack

$\beta$ Volume ratio of solid phase to all phase, $V_p/(V_f + V_p)$

$\beta_A$ Interphase momentum transfer coefficient

$\beta_S$ Slip coefficient

$\beta_1$ Impingement angle

$\beta_1$ Moving angle of a solid particle before impact

$\beta_2$ Moving angle of a solid particle after impact

$\beta_{2M}$ Moving angle of a solid particle after impact on measurement plane
\( \delta \) Linear strain, \( \sigma_{max} L_p / E \)
\( \epsilon_G \) Mass erosion rate [mg/g]
\( \epsilon_{mass} \) Mass erosion rate (Erosion mass parameter) [mg/g]
\( \epsilon_V \) Volumetric erosion rate (Erosion mass parameter) [cm\(^3\)/g]
\( \zeta \) Gaussian random number
\( \theta \) Pitch angle of the blade section of a rotor blade
\( \theta_c \) Collective pitch angle
\( \theta_p \) Coordinate of a solid particle in \( \theta \) (angular) direction
\( \kappa \) Ratio of the dynamic viscosities, \( \mu_p / \mu \)
\( \kappa_B \) Boltzmann constant
\( \kappa_p \) Thermal conductivity of a solid particle
\( \lambda \) Dimensionless length scale of fluid flow, \( \sqrt{(s \alpha^2 / u)} \)
\( \lambda \) Mean free path of fluid flow
\( \lambda_i \) Local induce inflow ratio, \( v_i / (\Omega R) \)
\( \lambda_p \) Dimensionless length scale of a solid particle, \( \sqrt{(s \alpha^2 / \nu_p)} \)
\( \mu \) Dynamic viscosity
\( \rho \) Density of fluid phase
\( \rho_p \) Density of solid phase
\( \tau \) Shear stress for fluid phase
\( \tau_p \) Shear stress for solid phase
\( \sigma \) Dimensionless slip coefficient parameter
\( \sigma \) Solidity of rotor disk
\( \sigma_{max} \) Maximum stress in target
\( \phi \) Induce inflow angle
\( \omega \) Blade angular velocity

**Subscripts**

0 Inflow boundary condition
1 Component of a solid particle before impact
2 Component of a solid particle after impact
max maximum
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<td>Angle of attack</td>
</tr>
<tr>
<td>BL</td>
<td>Baldwin-Lomax turbulence model</td>
</tr>
<tr>
<td>ER</td>
<td>Erosion rate</td>
</tr>
<tr>
<td>EPS</td>
<td>Erosion protection system</td>
</tr>
<tr>
<td>Ref.</td>
<td>Reference</td>
</tr>
<tr>
<td>SA</td>
<td>Spallart-Allmaras turbulence model</td>
</tr>
<tr>
<td>SpmFR</td>
<td>Solid particle mass flow rate, the number of solid particles released from a injection point per unit time [EA/s]</td>
</tr>
<tr>
<td>UDF</td>
<td>User Defined Function</td>
</tr>
<tr>
<td>W.T.</td>
<td>Wind tunnel</td>
</tr>
<tr>
<td>κe</td>
<td>κ – e standard turbulence model</td>
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Chapter 1

Introduction

The phenomenon of material removal from a metal surface is called “wear”. In general, wear is mainly caused by four different phenomena (1). First, wear phenomena is mass or volume loss from a metal surface due to the impact of solid particles suspended in fluid flow on the surface. It is defined as “abrasive or erosive cutting wear”. The second mechanism for wear is “adhesive wear”, which means material removal from metal surfaces when one metal surface contacts with another metal surface and slides on the surface. Next is “chemical wear” which occurs by chemical action. Finally, “fatigue wear” is a result of repeated shocks and fatigue.

A helicopter, aircraft, or turbomachinery system inevitably encounters solid particles suspended in air or generated during their operation. Therefore, material removal from metal surfaces of these systems is caused by the impact of solid particles on a metallic surface, “abrasive-erosive cutting wear” or “erosion”.

Many analytical and experimental researches were performed over past decades (2), (3), (4). These results concluded that the erosion damage on the metallic surfaces is mainly affected by the properties of the surface material, and properties of impacting particles. The property of a surface material, i.e., hardness, roughness, elasticity, metallurgy (brittle or ductile) and solid particle size, shape, velocity, density, and impact angle are important parameters for predicting erosion.

The understanding and modeling of sand erosion mechanism on helicopter rotor blades is challenging. In the field of turbomachinery and rotorcraft, the erosion phenomena is a serious problem because it results in significant deterioration of component geometry, reduces
performance, shortens component's lifetime, and increases operating / repair cost, which is discussed in chapter 1.1. The erosion damage is more serious when aircraft or helicopters are operated at low attitudes or in sand-laden environment as shown in Figure 1-1 and Figure 1-2.

![Figure 1-1. Ingestion of the sand to a jet engine of aircraft](image1)

![Figure 1-2. Brown-out during a low attitude flight (left) and landing (right)](image2)

In the prediction of erosion phenomena, accurate solid particle trajectories, rebound characteristics and erosion rates are required. An effective prediction of rebound characteristics and erosion rates usually come from wind tunnel based erosion studies in the form of empirical corrections. Many experimental and numerical investigations were performed in the past understand the erosion mechanism and to predict erosion damage and its effect on aerodynamics.
and performance of turbomachinery systems, internal flow systems and air intakes. However, the understanding and prediction of erosion on a helicopter rotor blade is still a challenge because the flow near a rotor blade tip is highly 3D, compressible and complex. Tip leakage flows and shock / boundary layer interactions are common in this area. Flows are dominated by unsteadiness and turbulence. A good prediction of erosion damage also requires “time efficient” particle tracking algorithms, rebound models and empirical erosion models at the fluid / solid interface.

In this study, first, a review of the past studies in the area of “erosion in turbomachinery systems” was performed. Erosion prediction studies on helicopter rotor blade are very limited. Most analytical methods to track sand particles in air, rebound models and erosion rate models developed for turbomachinery systems in the past are directly applicable for erosion prediction studies on helicopter blades because of physics based similarities between the two flow problems. Next, accurate prediction methodologies for calculating sand particle trajectories and erosion damage on helicopter blades were developed and implemented into a general purpose viscous flow solver, ANSYS-FLUENT. Finally, 2-D and 3-D erosion predictions were obtained by using these methodologies.
1.1. Impact of Erosion on a Helicopter Rotor Blade and in Turbomachinery Components

In a helicopter system, erosion results in a significant deterioration of airfoil profile and damages surface of erosion protection system. This frequently occurring mode of damage shortens the lifetime of a rotor blade and increases operating / repair cost as shown in an investigation performed by Calvert et al. (7). They performed a numerical study about flow and performance characteristics for two different NACA 63-414 airfoil profiles in order to investigate the effect of sand erosion on the performance and aerodynamics of an AH-64 Apache attack helicopter tail-rotor blade. Figure 1-3 shows a base and a damaged airfoil section used in this research. As shown in these profiles, the greatest erosion damage occurs at between 5 percent and 35 percent of the chord length on the suction surface of the airfoil. It is also observed that another significant erosion damage area is between 8 percent and 22 percent of the chord length on the pressure surface. However, as shown in the figure, positive deviations (increase in thickness) are observed at 5 percent and 35 percent of the chord length on a suction side, which is caused by a problematic alignment of the baseline airfoil and the damaged airfoil near leading edge.

Figure 1-3. Geometry of the baseline airfoil and the erosion damaged airfoil from Ref. (7)
In turbomachinery systems, it is well known that erosion damage causes serious geometric changes in guide vanes and rotor blades, which eventually affects aerodynamics and performance of the turbomachinery system. Richardson et al. (8) investigated erosion damage within a JT9D high-pressure compressor. They showed that rotor-blade erosion was very serious in the outer 50 percent of the span. A significant reduction in blade chord and thickness was also observed. They also found that erosion blunted the leading and trailing edges and increased tip clearance. Kleinert (9) indicated that damaged tip of rotor blades reduces surge margin by 4
percent because of an increasing of tip leakage loss and also increased fuel consumption in modern turbofan engines. Roberts (10) concluded that geometric variations due to leading edge erosion in compressor airfoils may account for an increase of 3 percent or more on the thrust specific fuel consumption.

Tabakoff et al. (11) performed an experimental study about the effect of particle concentration on turbine performance. They concluded that injected solid particles damaged the surface of a rotor blade and increased total pressure loss, which eventually decreased turbine's rotational speed, efficiency and torque. According to Schmucker's full engine test (12), the efficiency of a compressor is decreased with increasing the total mass of sand particle injected into the compressor as shown in Figure 1-6. In this test, 25 percent loss in engine power is observed when a total of 35kg sand particle is injected. These results dealing with erosion damage on a conventional turbomachinery system indicated that solid particles ingested into a compressor or a turbine blunted leading edges, sharpened trailing edges, reduced blade chords and increased surface roughness and tip clearance of rotor blades, which resulted in total pressure loss, reduction in power, efficiency and surge margin.

Figure 1-6. Compressor efficiency versus total mass entering into an engine from Ref. (12)
1.2. Erosion Protection Method of a Helicopter Rotor Blade

Helicopter rotor blades are designed to generate high aerodynamic performance, to improve dynamic and structural stability, and to reduce noise and vibration. In addition, the rotor blade requires a good resistance to adverse environmental conditions such as sand erosion, icing, and lightning strikes. In order to satisfy these requirements, a rotor blade has a fiber-glass composite structure with a metallic shield or with a hard coating as shown in Figure 1-7 (13). The metallic shield or the hard coating must absorb high kinetic energy on the surface of rotor blade caused by collision with sand particles and rain drops. They also need to have strong erosion resistance to rain drops and sand particles. Metallic shields made from nickel alloy, titanium and stainless steel have excellent rain resistance but poor sand erosion performance. Conversely, hard polyurethane-based coatings have good sand erosion protection, but poor rain resistance. Research on an optimal erosion protection system was performed by Haynie et al. (14). They suggested a multilayer, non-metallic leading edge as erosion protection system (EPS) as shown in Figure 1-8. In this study, it is found that the multilayered EPS has more benign failure mode and can improve wear life by 138 percent longer than that of the single-layer EPS. Because the kinetic and wear energy are absorbed by the removal of metal from shield surface, the metal shield must be frequently changed. This reduces aerodynamic performance and increases maintenance and repair / replacement cost.

However, these erosion protection systems have an application problem because a metal shield or a hard coating increases the thickness of a helicopter rotor blade, which results in aerodynamic and performance loss. Therefore, a new methodology for reduction of sand erosion damage is required.
Figure 1-7. Erosion protection system for a helicopter rotor blade from Ref. (13)

Figure 1-8. Advanced erosion protection system for a helicopter rotor blade from Ref. (14)
Chapter 2

A Literature Review on Solid Particle Tracking and Erosion Damage Prediction

In the prediction of particle trajectories and resulting erosion, a good understanding and modeling of two interactive mechanisms is required. First is the interactive mechanism between a solid particle and fluid flow, which is required in order to determine solid particle trajectory of solid particles within flow field and their impact conditions on the metallic surface. The mechanism is dealt by either Eulerian-Eulerian or Eulerian-Lagrangian particle tracking algorithm. Next is the interactive mechanism between a solid particles and a metal surface, which is used in order to quantify erosion damage on the metal surface of interest. This mechanism is commonly represented by a rebound model and an erosion model which usually comes from wind tunnel based erosion studies in the form of empirical correlations.

Many experimental and numerical investigations were performed in order to understand these interactive mechanisms and to develop a particle tracking algorithm and a suitable rebound model and an erosion model. These erosion prediction studies were mostly focused in the field of turbomachinery and internal flow systems, and the open literatures for the erosion damage prediction on a helicopter rotor blade is very limited.

In this chapter, a review of past literature on solid particle tracking and sand damage prediction in the field of turbomachinery was performed in order to develop proper methodologies for sand particles impacting on a helicopter rotor blade. This is because a particle tracking algorithm and an erosion mechanism for sand particles impacting on a helicopter rotor blade is very similar to those on sand particles or fly ash impacting on turbomachinery system components.
2.1. Sand Particle Tracking in a Viscous Flow Environment

In general, there are two different approaches for solid particle tracking in a viscous flow environment. First approach is the Eulerian-Eulerian approach. In this approach, the fluid and particle phases are considered as a mixed continuum. Therefore, continuity equation and momentum equations for both continua must be solved in order to determine the motion of fluid flow and the motion of solid particles. Next approach is the Eulerian-Lagrangian approach. In this approach, a mixture can be considered as the combination of homogeneous continuum (only fluid phase) and the solid phase is treated as a dispersed flow. This approach is a reasonable when mass concentration of solid phase is much less than unity or the diameter of solid particle is very small because, in this condition, the effect of momentum of solid particles on the motion of fluid flow is negligible. The Eulerian-Lagrangian approach determines particle trajectories by solving the motion of equation based on fluid flow solution.

2.1.1. The Eulerian-Eulerian Approach

The Eulerian-Eulerian approach considers two phase flow (fluid flow containing suspended solid particles) as a mixture continuum. In this approach, solid particle trajectories are computed at the same time with fluid flow computation by solving these following governing equations. (15)

- Mass conservation for solid phase:
  \[
  \frac{\partial}{\partial t}(\beta \rho_p) + \frac{\partial}{\partial x_j}(\beta \rho_p u_{p,j}) = 0, \quad \text{for } j = 1,2,3
  \]
• Mass conservation for gas phase:

\[
\frac{\partial}{\partial t}(\alpha \rho) + \frac{\partial}{\partial x_j}(\alpha \rho u_j) = 0, \quad \text{for } j = 1,2,3 \tag{2}
\]

Where, \( \beta = \frac{V_p}{(V_p + V_f)} \) is a volume ratio (concentration) of solid phase to all phase and \( \alpha = 1 - \beta \) is a volume ratio of fluid phase to all phase. The subscript \( p \) and \( g \) represents solid particle phase and flow phase.

• Momentum equation for solid phase in \( i \) direction:

\[
\frac{\partial}{\partial t}(\beta \rho_p u_{p,i}) + \frac{\partial}{\partial x_j}(\beta \rho_p u_{p,i} u_{p,j}) = -\frac{\partial}{\partial x_j}(\beta \tau_{p,ij}) + \beta \rho_p g_i - \beta \frac{\partial P}{\partial x_i} + \beta_A(u_i - u_{p,i}) \tag{3}
\]

• Momentum equation for fluid phase in \( i \) direction;

\[
\frac{\partial}{\partial t}(\alpha \rho u_i) + \frac{\partial}{\partial x_j}(\alpha u_i u_j) = -\alpha \frac{\partial P}{\partial x_i} - \frac{\partial}{\partial x_j}(\beta_f \tau_{ij}) + \alpha \rho g_i - \beta_A(u_i - u_{p,i}) \tag{4}
\]

\( \tau_{ij} \) and \( \tau_{p,ij} \) are shear stresses for fluid and solid phase. The last term of Equation 3 and Equation 4 is a drag force generated by velocity difference between two phases. \( \beta_A \) is an interphase momentum transfer coefficient. These terms are also developed in experimental studies (16).

As shown in these equations, the governing equations in Eulerian-Eulerian approach are complicated. This approach is computationally expensive because of additional equations and additional terms.
2.1.2. The Eulerian-Lagrangian Approach

The complexity of two-phase flow in Eulerian-Eulerian approach can be simplified when the dispersed phase of solid particle is sufficiently dilute in the fluid phase. In this condition, the governing equation is controlled by the principle of low concentration limits (17). The principle states when mass concentration of solid particles ($\Phi$) is much less than unity, e.g. $\Phi = 0.000254$ for sand particles suspended in air at hovering condition of a helicopter, the two-phase mixture flow can be considered as a single phase flow (fluid phase), because solid particles do not affect the fluid motion for the low concentration flow or for the low relative density flow. Therefore, in the calculation of solid particle trajectories by using the Eulerian-Lagrangian approach, a two step procedure is required. First step is the determination of the flow-field for the single fluid phase that is very much slightly modified due to the action of the particles by solving the Navier-Stokes equations. The second step is the calculation of particle trajectory by solving the equation of motion of individual solid particle released from a known initial position.

In the second step, an understanding of forces acting on solid particles is required in order to determine solid particle trajectories within the fluid field, especially when there is a transient and unsteady motion of solid particles. Boussinesq (18) concluded that the motion of solid particles was determined by the combination of a steady-state drag force, an added mass force, and a history integral force due to transients. Therefore, the Boussinesq expression for the hydrodynamic force consists of three independent force terms – the steady-state drag force, the added mass force, and the history integral force. Basset (19) developed the independent history integral force term of a spherical solid particle under the creeping flow conditions, so the term is called as “Basset force” or “Basset integral”. Michaelides (20) shows the early form for the transient hydrodynamic force based on the Boussinesq / Basset mathematical expression as shown in Equation 5.
where, $\alpha$ is radius of a solid particle.

These three independent forces were suggested in terms of a basic force term and an empirical factor. Various experimental results were performed in order to determine these empirical factor and to develop an exact form for these force

First, the steady-state drag developed by early experiments was suggested in the literatures of Wallis (21), Clift et al. (22), and other researchers. These studies concluded that the steady-state drag can be defined as $0.5C_D\pi\alpha^2V_p^2$ at finite velocities. The steady-state drag is also reduced to $6\pi\mu V_p$ because drag coefficient at finite Reynolds numbers can be assumed to be $C_D = 24/Re$. Therefore, in terms of an alternative form, the steady-state drag force can be expressed as $(6\pi\mu V_p)f$. Where, $f$ is a dimensionless empirical factor for the steady-state force.

Odar and Hamilton (23) suggested an extended hydrodynamic force expression as shown in the Equation 6. They introduced two additional coefficients to account for the added mass force and the history integral force.

$$F = c_1(6\pi\mu V_p) + \Delta_A \left(m \frac{dV_p}{dt}\right) + \Delta_H \left(\alpha^2 \sqrt{\pi\rho\mu} \int_0^t \frac{dV_p}{\sqrt{t-\tau}} d\tau\right)$$  \hspace{1cm} (6)

where, $c_1$, $\Delta_A$, and $\Delta_H$ are the coefficient of the steady-state drag force, the added-mass force and history integral force as shown in Equation 7

$$\Delta_A = 1.05 - \frac{0.066}{0.12 + Ac^2} \quad \text{and} \quad \Delta_H = 2.88 + \frac{3.12}{(1 + Ac)^2}$$  \hspace{1cm} (7)

Where, $Ac = \frac{|V-V_p|^2}{(2\alpha^4 \frac{dV_p}{dt})}$

However, these coefficients are useful in the case of a rigid sphere starting from a stationary condition (zero velocity) in a quiescent fluid.
Michaelides et al. (24) suggested a developed expression for the equation of motion of a solid particle in a stationary frame of reference as shown in Equation 8.

\[
m_p \frac{dV_p(t)}{dt} = \left( m_p - m \right) g_i + m \left. \frac{DV_i(x_i,t)}{Dt} \right|_{Y(t)} + F_i
\]  

(8)

where, \( V_p(t) \) is the arbitrary velocity of the center of the viscous sphere, located at \( Y(t) \), \( V_i(x_i,t) \) is the velocity of the fluid and \( F_i \) is the hydrodynamic force exerted by the flow field. The total hydrodynamic force on a viscous sphere can be simplified in the Laplace (or Fourier) domain as shown in the Equation 9.

\[
F_i = -6\pi\mu \left[ V_p(s) - V(Y(t),s) \right] \cdot \left[ \frac{\lambda^2}{9} + \lambda + 1 - \theta(\lambda,\kappa) \right]
\]  

(9)

where,

\[
\theta(\lambda,\kappa) = \frac{(\lambda + 1)^2 \left[ k_p^2 - k_p \tanh k_p - 2f(k_p) \right] \kappa + f(k_p)}{[1 + \sigma(\lambda + 3)] \left[ k_p^2 - k_p \tanh k_p - 2f(k_p) \right] \kappa + (\lambda + 3) f(k_p)}
\]  

(10)

\( \kappa = \mu_p/\mu \) is the ratio of the dynamic viscosities and \( \sigma = \mu/\beta_s \alpha \) is a dimensionless parameter related to the slip coefficient, \( \beta_s \). The parameters \( \lambda \) and \( \lambda_p \) are the two dimensionless length scales of the fluid and the viscous sphere in the Laplace domain.

\[
\lambda = \sqrt{\frac{s \alpha^2}{\nu}} \text{ and } \lambda_p = \sqrt{\frac{s \alpha^2}{\nu_p}}
\]  

(11)

where, \( s \) is Laplace transform variable.

However, these equations are complex and a general analytical solution for the equation in the time-domain is unfeasible. Moreover, previous equations were developed under creeping-flow condition (very low Reynolds number flow condition). Therefore, Galindo et al. (25), Lovalenti et al. (26), other researchers performed to get an analytical solution for the equation of motion for a solid particle over wide Reynolds number ranges.
Equation 12 is a recently developed and simplified equation for the motion of a solid particle suspended in the fluid flow presented in Fluent manual (27).

\[
\frac{dV_p}{dt} = F_D(V - V_p) + \left(\frac{\rho_p - \rho}{\rho_p}\right)g + F_{add}
\]  

(12)

As shown in this equation, the motion of an individual solid particle is determined by three independent force terms. First force term is an induced drag force. The induced drag force is the aerodynamic force due to the motion of a solid particle relative to fluid motion. The force is dependent upon the size and the shape of a solid particle as well as the relative velocity between a solid particle and fluid flow. Drag force term, \(F_D(V - V_p)\), is as follows:

\[
F_D(V - V_p) = \frac{18\mu C_D Re}{\rho_p d_p^2} (V - V_p) = \frac{3}{4} \frac{\rho}{\rho_p d_p} |V - V_p|(V - V_p) C_D
\]

(13)

where, \(F_D\) is the drag force coefficient

Next term is buoyancy force. The force is generated due to the difference between fluid and solid density. The final force acting on a solid particle is the addition of all forces per unit mass of a solid particle. Past researches show that the dominant additional forces acting on a solid particle may be added mass force, thermophoretic force, Brownian force, and lift forces which can be dominant under certain conditions. Fortunately, for sand particles suspend in air, these additional forces can be neglected.

The force affecting dominantly the motion of sand particles is induced drag force which is caused by velocity difference between a solid particle and surrounding fluid flow.
2.2. Collision and Erosion Mechanism between a Solid Particle and Metal Surface

Erosion damage on a metallic surface of interest is calculated by using sand particle trajectories and impact conditions which are determined by interactive mechanism between solid particles and the metal surface. In this calculation, collision and erosion mechanisms between a solid particle and metal surface are required. In general, these mechanisms are represented by a rebound model and an erosion model.

2.2.1. Particle Rebound Model during Impact on a Solid Surface

An evaluation of solid particle rebound characteristics at various impact conditions is important because the rebound condition determines the motion of a solid particle (velocity magnitude and direction) after the impact, which affects re-impact condition. In turbomachinery systems, most solid particles rebounded from the blades at first stage of a compressor or a turbine re-impact on the blades at next stage of these systems, which affects erosion characteristics on the aft-blades (11). In the helicopter, solid particles rebounded from advancing rotor blade also can impact a retreating blade, which affects overall erosion characteristics on helicopter rotor blades as well as a concentration of solid phase within the computational domain.

Moreover, the rebound condition is useful for evaluation of the kinetic energy exchange between a solid particle and metal surface during collision. The kinetic energy exchange between a solid particle and metal is directly proportional with the amount of material removed from the metallic surface. The rebound characteristics are useful to develop an empirical model for erosion damage. The rebound characteristics are usually represented as restitution ratios as shown in Equation 14 ~ Equation 17.
• Velocity restitution ratio:

\[ e_V = \frac{V_{2M}}{V_1} \] (14)

• Tangential velocity restitution ratio:

\[ e_T = \frac{V_{T2M}}{V_{T1}} \] (15)

• Normal velocity restitution ratio:

\[ e_N = \frac{V_{N2M}}{V_{N1}} \] (16)

• Angle restitution ratio:

\[ e_\beta = \frac{\beta_{2M}}{\beta_1} \] (17)

The restitution ratios are determined by the measurement of angles and velocities of solid particle before and after collision. Figure 2-1 shows statistical impact and rebound geometry for a single particle impacting a metallic surface. The particle impacts to a metallic surface at velocity \( V_1 \) and angle \( \beta_1 \) in the xz plane, and the particle rebounds at a velocity \( V_2 \) and angle \( \beta_2 \). In common experiments, velocity and angle of solid particles impacting on a metal surface and those of solid particles rebounding from the metal surface are measured on the 2 dimensional measurement plane which is parallel to the flow direction of a solid particle approaching to the metal surface and contains the impact position. In real condition, there is a difference between the measured condition (\( V_{2M} \) and \( \beta_{2M} \)) and the actual condition (\( V_2 \) and \( \beta_2 \)) of rebound particles because the rebounded particles have an out of plane velocity components due to random roughness of surface and irregular shape of solid particle. These differences can be neglected according to the numerical results obtained by Eroglu et al. (28)
Figure 2-1. A statistical impact and rebound geometry for the impact of a single solid particle from Ref. (29)

Eroglu et al. (28) measured 3-D restitution ratios for fly ash impacting on INCO718 and compared these values with 2-D restitution ratios in order to the effect of out-of-plane component of 3D restitution ratios. Figure 2-2 and Figure 2-3 are comparison between 2D and 3D tangential velocity restitution ratio distribution as a function for various impingement angles. In this study, only tangential velocity restitution ratios were considered because tangential restitution ratio is directly related to ductile material’s erosion damage (30). These results show that there was not much difference between the magnitude of 2-D and 3-D tangential restitution ratio in as wide impingement angle range. It is also found that the out-of-plane component of 3D restitution ratios can be neglected and that a 2-D restitution ratio is reasonable value in order to determine rebound condition.
Figure 2-2. Tangential velocity restitution ratio distribution as a function of impingement angle for INCO 718 from Ref. (28)

Figure 2-3. Tangential velocity restitution ratio distribution as a function of impingement angle for 2024 Aluminum from Ref. (28)

Next, Grant et al. (31) performed high speed experiments for sand (quartz) particles impacting on 2024 aluminum target to obtain basic erosion data for various impact conditions. Figure 2-4 shows high speed erosion experiment facility developed by Tabakoff at the University of Cincinnati. Figure 2-5 shows the normal and tangential restitution ratio distribution as a function of impingement angle, $\beta_1$. This set of data indicated that rebound characteristics are very
susceptible to impingement angle. Figure 2-6 is restitution ratio distributions as a function of impact velocity, \( V_{p1} \). This result shows that the only tangential restitution ratio is highly affected by impact velocity. According to Figure 2-6 and Figure 2-7, it was also found that tangential component of impact velocity causes erosive removal of material. This result also indicated that the normal velocity does not significantly affect the ductile erosion because the kinetic energy transformed by the normal component is dissipated by target's plastic deformation, which does not play a significant role in target's material loss. Equation 18 and Equation 19 are polynomial curve fits of the mean values of tangential and normal velocity restitution ratios for approximate 200 \( \mu \)m sand particles with 67~128 m/s of impact velocity impacting on 2024 Al. target. However, these results did not include the effect of solid particle size and material of a solid particle and the type of the metallic surface.

\[
e_N = 0.993 - 3.071779 e^{-2} \beta_1 - 4.752032 e^{-4} \beta_1^2 - 2.60512 e^{-6} \beta_1^3
\]  
\[
e_N = 0.988 - 2.897247 e^{-2} \beta_1 + 6.247428 e^{-4} \beta_1^2 - 3.562107 e^{-6} \beta_1^3
\] (18) (19)

Figure 2-4. Research facility for rebound and erosion test in the University of Cincinnati from Ref. (31)
Figure 2-5. Restitution ratio distribution as a function of impingement angle from Ref. (31)

Figure 2-6. Restitution ratio distribution as a function of impact velocity from Ref. (31)
Figure 2-7. Typical erosion behavior of ductile and brittle materials as a function of particle impact angle from Ref. (1)

Wakeman et al. (30) measured rebound characteristics for 165µm diameter silica sand particles impinging on INCO 718, Ti-6Al-4V, and 2024 Al targets. In this study, they investigated the effect of target material, target temperature (ambient ~ 1000°K), particle velocity (27 ~ 320 m/s), and impingement angle on rebound characteristics. They concluded that impingement angle is the most important factor to decide particle restitution ratios as shown in Figure 2-8 ~ Figure 2-10. It was also found that there is no significant change in restitution ratios in a function of target temperature, Figure 2-10 and Figure 2-11, but the type of the surface material affects the restitution ratio as shown in Figure 2-12. Finally, they suggested generalized empirical restitution ratio equations at various materials and various temperatures.

\[
e_N = 1.000 - 2.110e^{-2}\beta_1 + 2.280e^{-3}\beta_1^2 - 8.760e^{-7}\beta_1^3
\]  
\[
e_N = 0.953 - 4.460e^{-4}\beta_1 + 6.480e^{-6}\beta_1^3
\]
Figure 2-8. Restitution ratio distributions as a function of impingement angle for 2024 Al target material from Ref. (30)

Figure 2-9. Restitution ratio distributions for Ti-6Al-4V target material from Ref. (30)
Figure 2-10. Restitution ratio distributions according to materials temperature at $\beta_1=25^\circ$ from Ref. (30)

Figure 2-11. Restitution ratio distributions according to materials temperature at $\beta_1=90^\circ$ from Ref. (30)
Previous rebound models are an average value of restitution ratio data measured during a given period. As shown in the Figure 2-13, these experimental data include the deviation which is a normal bell shaped distribution from their average value. The deviations are caused by irregular shape of a solid particle and a roughened metal surface. Therefore, in order to improve the accuracy of the rebound model, a correction process is required.
Wakeman et al. (29) have conducted an analytical study to correct a rebound model with measured data. They analyzed the impact condition for an irregular rigid particle impacting a roughened target surface. In order to simplify the theory, the irregular particle is idealized as the spherical particle of which radius is the same as the radius of local spherical part of the irregular particle as shown in Figure 2-14. The solid particle penetrating into target become zero-velocity at maximum penetration depth, and then is pressed out of the target material as shown in Figure 2-15. At the maximum depth of penetration, the rebound kinetic energy of the particle is same with the stored elastic energy as shown in Equation 22 ~ Equation 25. Equation 25 shows that the rebound condition in normal direction is proportional to $\frac{\sigma_{\text{max}}}{E}$. Figure 2-16 and Figure 2-17 indicate that the tangential work and impulse follow the trend of ductile material's erosion rate and the trend of normal impulse and work is very much in agreement with the trend of brittle erosion rate. This result indicated that the work input function can be used in the development of erosion equations.

Figure 2-14. Simplification of an irregular solid particle impact condition from Ref. (29)
Figure 2-15. Particle normal impact contact geometry from Ref. (29)

\[ KE_2 = PE_S = \text{Work} \]  \hspace{1cm} (22)

- Kinetic energy of a solid particle and Elastic energy

\[ KE_2 = \frac{m_1 V_{N_2}^2}{2} \quad \text{and} \quad PE_S = \int_0^\delta PAdx \]  \hspace{1cm} (23)

where, \( \delta = \sigma_{\text{max}} L_P / E \) is the linear strain, \( \sigma_{\text{max}} \) = maximum strain stress in target, \( L_P \) = depth of plastic deformation, \( E \) = Young's modulus, the depth of plastic deformation, \( L_P \) can be approximated as a multiple of the depth of particle penetration, \( h \).

Therefore,

\[ \frac{m_1 V_{N_2}^2}{2} = \frac{\sigma_{\text{max}} nhA}{2E} \]  \hspace{1cm} (24)

where, \( h = V_{N_1} \sqrt{\frac{m_1}{2\pi R_{\sigma_{\text{max}}}}} \) and \( A = V_{N_1} \sqrt{\frac{2\pi R_{m_1}}{\sigma_{\text{max}}}} \)

Finally,

\[ \frac{V_{N_2}}{V_{N_1}} = \sqrt{\frac{h \sigma_{\text{max}}}{E}} \]  \hspace{1cm} (25)
In previous studies for rebound conditions, restitution ratios for $15^\circ$~$75^\circ$ of an impingement angle were measured with flat targets using 2-D LDV system because there were large deviations at the impingement angle of near $0^\circ$ or $90^\circ$. The restitution ratios that are out of this range were calculated by extrapolation. Therefore, in order to improve the accuracy of a rebound model, an accurate measurement of the restitution ratio for all range ($0^\circ$~$90^\circ$) of impingement angle are required.

Figure 2-16. Work and impulse in tangential direction for impingement angle from Ref. (29)

Figure 2-17. Work and impulse in normal direction for impingement angle from Ref. (29)
Tabakoff (32) measured restitution ratios at small impact angle (less than 15°). They also developed a generalized empirical-equation for restitution ratio by using experimental data for fly ash of mean diameter of 15µm, impacting on eight different materials - 410 stainless steel, 2024 Al, Ti-6Al-4V, INCO 718, RENE 41, AM355, L605 Cobalt, and Alumina. Equation 26 and Equation 27 are polynomial curves fitting the mean, the upper limit, and the lower limit of restitution ratios.

\[
e_N = 0.9 + 3.7874e^{-2}\beta_1 - 1.91e^{-3}\beta_1^2 + 3.4345e^{-5}\beta_1^3 - 2.0785e^{-7}\beta_1^4
\]  
(26)

\[
e_T = 0.93 - 8.7266e^{-4}\beta_1 - 8.5293e^{-5}\beta_1^2 - 2.8178e^{-6}\beta_1^3 + 6.4026e^{-8}\beta_1^4
\]  
(27)

As discussed in previous studies, rebound characteristics are highly depended upon impingement angle and velocity. Material of solid particles and a metallic surface are also important parameter to determine an erosion model. In this study, a rebound model for silica sand particles impacting on Ti-6Al-4V target surface, which is suggested in the study of Taslim et al. (33), is used.

\[
e_N = 0.993 - 3.07418e^{-2}\beta_1 + 4.752e^{-4}\beta_1^2 - 2.6051e^{-6}\beta_1^3
\]  
(28)

\[
e_T = 0.988 - 2.8972e^{-2}\beta_1 + 6.4274e^{-4}\beta_1^2 - 3.5621e^{-6}\beta_1^3
\]  
(29)

Throughout many experimental studies about rebound conditions, it is found that restitution ratio is highly depended upon the impingement angle. These results also indicate that rebound condition is varied with solid particle type and surface material.

Therefore, an accurate erosion damage prediction on a helicopter rotor blade requires a proper rebound model for sand particles impacting on a helicopter rotor blade which is corrected by measurement data for a wide range of impingement angles.
2.2.2. Erosion Model due to Particle Impact on a Solid Surface

In order to predict erosion damage on a metallic surface, a detailed understanding of the material loss mechanism for collisions between a solid particle and a metal surface is necessary. The loss mechanism is generally termed as an erosion model. Due to the existence of complicated mechanisms, general erosion models are empirical algebraic models based on experimental results which are average mass or volume loss from the surface after a certain amount of solid particles impact the target.

Grant et al. (31) investigated erosion damage for 2024 aluminum alloy impacted by 200µm sand (quartz) particles at various angle of attack and particle velocity. Erosion rates have been determined by measuring the weight loss of the target after test. Figure 2-18 shows that erosion is highly increased as the impact angle measured from the horizontal plane increases up to approximately 20° at which that maximum erosion is occurred, and then, the erosion rate slightly decreased. Figure 2-19 is the erosion rate distribution with respect to impact velocity at β₁ = 20° and β₁ = 90°. These results indicate that the erosion rate is proportional to the particle velocity taken to a constant power. (for β₁ = 20°, n is 3.8 and for β₁ = 90°, n is 4.0). Finally, they suggested erosion model for 2024 Aluminum alloy for 20~200µm diameter sand particles.

\[
\varepsilon_{\text{mass}} = K_1 f(\beta_1)V_1^2 \cos^2 \beta_1 [1 - R_T^2] + f(V_1) \ [mg/g] \tag{30}
\]

\[
R_T = 1 - 0.0016V_1 \sin \beta_1 \quad \text{and} \quad f(\beta_1) = [1 + C_K (K_{12} \sin 2\beta_0)]^2 \tag{31}
\]

\[
f(V_1) = K_3 (V_1 \sin \beta_1)^4 \tag{32}
\]

Where, \( K_1 = 3.67e^{-6}, \ K_{12} = 0.585, \) and \( K_3 = 6.0e^{-12} \)
Figure 2-18. Erosion rate distribution for impingement angle from Ref. (31)

Figure 2-19. Erosion rate distribution for impact velocity at $\beta_1 = 20^\circ$ and $90^\circ$ from Ref. (31) where, “Erosion mass parameter” defines mass removed from the metallic surface when a unit mass of solid particle impact on the metallic surface.
Wakeman et al. (34) investigated the effect of particle velocity, particle impingement angle, target temperature, and target material on erosion rate for 2024 Al., Ti-6A1-4V, and INCO 718. In the study, they investigated erosion rate for 150 ~ 180µm of quartz sand particles impacting on Ti-6A1-4V and INCO 718 target materials at various velocities (200~ 900 ft/s), target temperatures (ambient ~ 1300°F) and impingement angles (20 ~ 90°). Similar to many other researches, Figure 2-20 and Figure 2-21 show the erosion rate is proportional to the particle velocity taken to a constant power. The values of power are different for individual target materials. Figure 2-22 indicated that erosion rate is slowly increased or stays constant with increased temperature at low temperature, but at high temperature, the rate of change in erosion rate is increased suddenly. Figure 2-23 and Figure 2-24 are the erosion rate distribution as a function of impingement angle at various temperatures. As shown in this figure, erosion rate is highly dependent upon target temperature as well as impingement angle. Additionally, in the study, in order to improve the correlation of erosion rate with temperature, two normalization parameters (sample temperature / melting temperature, and sample temperature / annealing temperature) have been used. These figures show that material erosion rate increases rapidly as the yield strength decreases rapidly near the annealed temperature, which support the relation between material erosion rate and material yield strength.

Wakeman et al. (30) showed the effect of particle velocity and target temperature on erosion rate for 2024 Al., Ti-6A1-4V, and INCO 718 target material. In this study, it is also found that the erosion rate is highly dependent upon particle velocity, Figure 2-25, target material temperature, Figure 2-26, and impingement angle, Figure 2-27. Finally, in the study, Equation 33 and Equation 34 are semi-empirical erosion rate equations for INCO 718 material.
Figure 2-20. Erosion rate versus particle velocity for Ti-6Al-4V at $\beta_1=25^\circ$ and $\beta_1=45^\circ$ from Ref. (34)

Figure 2-21. Erosion rate versus particle velocity for INCO 718 4V at $\beta_1=45^\circ$ and $\beta_1=90^\circ$ from Ref. (34)
Figure 2-22. Erosion rate versus sample temperature for Ti-6Al-4V and INCO 718 at $\beta_1=25^\circ$ from Ref. (34)

Figure 2-23. Erosion rate versus impingement angle for Ti-6Al-4V from Ref. (34)
Figure 2-24. Erosion rate versus impingement angle for INCO 718 from Ref. (34)

Figure 2-25. Erosion rate versus particle velocity at $\beta_1=25^\circ$ and $\beta_1=90^\circ$ for INCO 718 from Ref. (30)
Figure 2-26. Erosion rate versus sample temperature at $\beta_i=25^\circ$, $45^\circ$ and $90^\circ$ for INCO 718 from Ref. (30)

Figure 2-27. Erosion rate versus Impingement angle for INCO 718 from Ref. (30)
\[ \varepsilon_G = \rho_{\text{target}} \cdot \varepsilon_V \]  

(33)

Where

\[ \varepsilon_V = 0.055e^{-4} \left[ \left( \frac{Y_{SRT}}{Y_s} \right)^{2.9} + \left( \frac{Y_s}{Y_{SRT}} \right) - 1 \right] \cdot \left[ \left( \frac{V_1}{100} \right)^{2.8} (1 - e_T^2) \cos^2 \beta_1 + 0.114 \left( \frac{V_1}{100} \right)^3 (1 - e_N^2) \sin^2 \beta_1 \right] \]  

(34)

In the Equation 33 and Equation 34, \( \varepsilon_G \) and \( \varepsilon_V \) are mass erosion rate \([\text{mg/g}]\) and volumetric erosion rate \([\text{cm}^3/\text{g}]\). \( Y_{SRT} \) and \( Y_s \) are material yield strength at ambient temperature and at operating temperature, respectively.

Wakeman et al. (29) investigated the impulse and the work imparted to the target material by impacting solid particles in order to understand the relation between the rebound restitution ratio and erosion rate. By several investigations, the work imparted to the material is useful to predict erosion rates. In addition, these investigations indicated that each velocity component of particle is related to different erosion mechanism. The normal velocity causes fatigue cracking of the surface and ductile cutting of the surface is caused by the tangential component of velocity. Elfeki et al. (35) investigated erosion damage from quartz solid particles (10, 20, 50 and 165\( \mu \)m) impacting on a compressor blade, hub and casing surfaces (410 stainless steel). This study shows that erosion rate is highly depended upon the size of solid particle because the size of solid particle is main parameter to decide impact conditions - position, velocity and impingement angle.
Chapter 3

Current Particle Tracking and Erosion Damage Prediction Scheme Developed for Helicopter Rotor Blades

A suitable particle tracking algorithm, rebound and erosion models are required in order to calculate erosion damage on metallic surfaces as described in the Chapter 2. Through flow physics and erosion phenomena in a helicopter system and in turbomachinery systems are very much similar from a fundamental point of view. However, previous methodologies for erosion prediction in turbomachinery systems have certain limitations when they are applied to erosion prediction on a helicopter rotor blade.

Unlike to the internal flow case such as a turbomachinery, internal flow systems and air intakes, in the external flow case, there is not a definite inlet boundary condition for solving fluid flow and for injecting solid particles in helicopter blade erosion problem. Moreover, flow characteristics around a 3D helicopter rotor blade vary tremendous in the span section of the rotor blade and for a given helicopter operational condition. Therefore, in this chapter, the development and modification of methodologies for sand particle tracking and sand erosion prediction on helicopter rotor blades.

First, a Lagrangian solid particle tracking algorithm is modified by the modified injection conditions, which will be discussed in the Chapter 3. Next, an empirical rebound and erosion model for a spherical sand particle impacting on Ti alloy surface are developed based on experimental data, which will be presented in the Chapter 3.3. The definition of erosion rate on the blade also is changed to deal with unsteady and non-uniform solid particles' injection properties, which will be mentioned in Chapter 3.2.
Finally, in order to control modified injection conditions and to model a rebound / erosion model, specific C++ based codes, in the forms of User Defined Function, are developed and integrated with a general flow solver, ANSYS-FLUENT as mentioned in Chapter 3.4

3.1. Injection Condition at Particle Release Location

In the determination of solid particle trajectory, a starting position (release position) of each solid particle as well as its properties – initial velocity, diameter, density, and mass flow rate - must be defined. This set of data is defined as “particle injection condition”.

In the erosion studies of turbomachinery, internal flow systems and air intakes, the injection position of solid particles is set at the inlet boundary of fluid flow because all particles released from the injection positions at the inlet boundary affect erosion characteristics in these systems. The properties of solid particles released from injection points can be assumed to be fixed because these systems are usually operated at design point. It is also assumed that these properties are uniform over the injection section.

For the case of sand particles suspended in air around a 3D rotating helicopter rotor blade, many sand particles impact on the rotor blade, which certainly cause erosion damage on the rotor blade and affect erosion characteristics. However, there are also other sand particles that pass away without a collision with the rotor surface. In order to predict accurate erosion damages on the blade, all sand particles suspended in a whole computation domain must be considered. However, it is very difficult to determine the motion of sand particles within the computational domain due to high computational cost. A proper number of sand particles considered in the determination of their trajectory and impact conditions must be chosen in order to improve the accuracy of sand particle tracking and erosion prediction in a helicopter system and to reduce the computational cost.
For a 2 dimensional sand erosion computation, the injection plane is commonly specified on the inlet boundary condition for the computation of fluid flow as shown in the Figure 3-1. However, in the 3 dimensional computations for sand erosion prediction on 3D rotating helicopter rotor blades, there is no definite inlet boundary condition for solving fluid flow, so a specific plane from that solid particles are released is required, which will be discussed in the Chapter 6.

Figure 3-1. Initial positioning of solid particles with velocity magnitude contours
Where, initial and final position defines the position of first (initial) and final inject point

In the erosion prediction on a 2D airfoil surface and on a 3D rotating helicopter rotor blade, the size of the injection plane may affect erosion characteristics on the surface. A narrow injection plane may cause the distortion of erosion characteristics and a wide injection plane requires large computation cost. Therefore, a suitable size of the injection plane must be also specified in order to improve the accuracy of sand erosion damage prediction and to reduce the computational time. Moreover, the injection plane size is directly relative to the distribution of injection points and the number of solid particles considered in this computations.

Helicopter rotor blades are operating at non-uniform and non-linear flow condition over the span of a rotor blade. Flow conditions around the rotor blade also vary with respect to azimuth angle of rotating blade. The properties of sand particles released from injection points are thus non-uniform in the span direction and with respect to the azimuth angle. Therefore, suitable
properties of released sand particles must be selected and implemented in order to predict accurate particle trajectories and erosion damage.

3.3.1. Injection Type of Solid Particles

In the computational viscous flow solver, ANSYS-FLUENT, used in this study, two different injection types are developed and implemented. First type is a “surface injection”. In this injection type, the released position of solid particles is defined by using the already existing computational grid used for flow only computation. All grid points in the inlet domain are utilized for particle injection as shown in the Figure 3-2. The “surface injection” is highly dependent upon the computational grid system. The clustering of grid points may cause the distortion of predicted erosion characteristics.

Next type is “group injection”. In this type, particles are released from “grid points on a pseudo-line” defined by two input position values as shown in the right side of the Figure 3-2. The position of injection points is calculated by the Equation 35. This injection type is not affected by the grid system. Properties of an individual solid particle can be defined by the only uniform or linear function, so this type has a limitation for 3D applications or special cases.

\[
\phi_i = \phi_1 + \frac{\phi_N - \phi_1}{N - 1} (i - 1) \tag{35}
\]

where, \(\phi_1\) and \(\phi_N\) : the properties at the first and final inject point

In this current study, a C language based code – which is called as UDF (user defined function) - is developed and linked to the general purpose viscous flow solver, ANSYS-FLUENT, in order to specify the position of an injection plane and to define properties of solid particles released from the injection plane.
3.3.2. The Properties of Released Solid Particles

In this chapter, certain assumptions are applied in order to define injection conditions of released sand particles such as initial velocity, diameter, density, and mass flow rate. First assumption is that all solid particles are released from specified points (injection point). Second is that properties of solid particles released from same injection point are same. Final assumption is that solid particles released from same injection point follow same trajectory.

First of these properties, injection points are distributed uniformly on a specified plane which is put at inlet boundary condition for fluid flow simulation. For a 2D sand erosion damage prediction on the airfoil, all injection points are positioned on the specified line which is put at the inlet boundary condition as shown in the Figure 3-3. As shown in this figure, the interval between two adjacent injection points is uniform.
where, \( L_{\text{tot}} \): Total length of injection section and \( N_p \): The total number of injection points

Secondly, released velocity of solid particles is set as the same as the fluid velocity at injection point \((V_{pi} = V_i)\). Next, density is a property of sand particle and the diameter of a sand particle is also input value which is come from measurement data as shown in Table 3-1. Finally, “mass flow rate” is defined as the mass of solid particles per a unit time released from the injection section, which can be calculated by using from Equation 36 to Equation 39. The “mass flow rate” of solid particles also determines “the number” of solid particles released from the injection section as shown in the Equation 40.

- Mass of fluid flow passing i-th section during time step \((\Delta t)\)

\[
M_i = \rho V_i L_i \Delta t
\]  \hspace{1cm} (36)

Where, \( M_{pi} \) and \( M_i \) are the total mass of solid particle and fluid injected from i-th section during operating time \((\Delta t)\) and \( \Phi \) is mass fraction of solid particle which are defined based on experimental data as shown in the Equation 37.
\[
\Phi = \frac{M_{pi}}{M_{pi} + M_i}
\]  

(37)

- Mass of solid particle passing i-th section during time step \((\Delta t)\)

\[
M_{pi} = \frac{\Phi}{1 - \Phi} M_i = \frac{\Phi}{1 - \Phi} (\rho V_i L_i \Delta t)
\]

(38)

- Mass flow rate of solid particle passing i-th section of injection domain

\[
\dot{m}_{pi} = \frac{M_{pi}}{\Delta t} = \frac{\Phi}{1 - \Phi} \frac{1}{\Delta t} (\rho V_i L_i \Delta t) = \frac{\Phi}{1 - \Phi} \dot{m}
\]

\[
\dot{m}_{pi} = \dot{N}_{pi} \left( \frac{\pi}{6} \rho_p D_p^3 \right)
\]

(39)

where, \(\dot{N}_{pi}\) is the number of solid particles released from i-th injection point. \(\rho_p\) and \(D_p\) are density and diameter of a solid particle.

<table>
<thead>
<tr>
<th>Helicopter Name</th>
<th>UH-1</th>
<th>CH-46</th>
<th>HH-60</th>
<th>CH-53</th>
<th>V-22</th>
<th>MH-53</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–15 µm</td>
<td>32.75%</td>
<td>32.41%</td>
<td>27.33%</td>
<td>24.77%</td>
<td>18.86%</td>
<td>22.55%</td>
</tr>
<tr>
<td>15–62 µm</td>
<td>31.47%</td>
<td>28.18%</td>
<td>21.37%</td>
<td>7.51%</td>
<td>18.39%</td>
<td>32.09%</td>
</tr>
<tr>
<td>62–125 µm</td>
<td>29.48%</td>
<td>31.73%</td>
<td>30.48%</td>
<td>27.93%</td>
<td>42.99%</td>
<td>42.34%</td>
</tr>
<tr>
<td>125–250 µm</td>
<td>6.30%</td>
<td>7.68%</td>
<td>20.82%</td>
<td>39.79%</td>
<td>19.76%</td>
<td>3.01%</td>
</tr>
</tbody>
</table>

3.3.3. The Number of Injection Points and the Size of Injection Plane

In this section, sand erosion damage distributions for the number of injection points \((N_p)\) and the size of injection plane \((L_{tot})\) were investigated in order to select a suitable condition for erosion damage predictions on helicopter rotor blades as shown in Figure 3-4.

In this computation, injection points were uniformly distributed on the injection plane (injection section) in order to remove the effect of their distribution. A released velocity of solid
particles is set as same as the fluid velocity at injection point \((M_{p_i} = M_l = 0.7)\). An angle of attack of inflow and released solid particles for zero-lift condition \((\alpha_0 = -0.07^\circ)\) is chosen as a reference value. Density \((\rho_p = 1650\text{kg/m}^3)\) and diameter \((D_p = 150\mu\text{m})\) of released solid particles are specified based on measurement data. Mass flow rate of solid particles released from each injection point is calculated by using Equation 36 ~ Equation 39.

Figure 3-4. Control parameters for injection condition with velocity magnitude contours

Figure 3-5. Initial positioning of solid particles with velocity magnitude contours
Firstly, the effect of the size (length) of an injection section on erosion characteristics on a SC1095 airfoil at zero-lift condition ($\alpha_0 = -0.07^\circ$) was investigated. Figure 3-6 and Figure 3-7 show area averaged volumetric erosion rate distributions on the pressure and the suction surface in terms of the maximum thickness of a 2D SC1095 airfoil ($T_{\text{max}} = 0.005\text{m}$). As shown in these figures, the erosion rate is sharply increased over the leading edge. The erosion rate reaches maximum at about 2 percent chord length on the pressure surface and at about 2.5 percent chord length on the suction side. The erosion rate is continuously decelerated up to approximately 15 percent chord length on the pressure surface and up to 20 percent chord length on the suction surface. These results show that erosion rate distributions over the airfoil surface including a maximum magnitude of erosion rate and an erosion damaged area very similar. These results concluded that the length of the injection domain at inlet boundary condition do not affect erosion rate characteristics when the length is greater than the maximum thickness of an airfoil.

Figure 3-8 shows the influence of the length of the injection domain on the impact velocity. As shown in the figure, solid particles impact the airfoil surface with the smaller velocity than initial (released) velocity because solid particles are decelerated by induced drag force. The reduction of solid particle velocity is increased as the impact position is closer to the leading edge of a SC1095 airfoil. The impact velocity distributions for various injection lengths over the airfoil have a little difference, which is caused by numerical errors. The effect of velocity difference can be neglected. It is concluded that the impact velocity of a solid particle is not depended upon the length.

Figure 3-9 is impingement angle distributions over the airfoil for various injection lengths. The impingement angle is sharply decreased near the leading edge of a SC1095 airfoil because of a huge change in the geometric angle over the leading edge. This distribution indicated that impingement angle distributions are also not affected by the injection length.
Figure 3-6. Influence of the length of an injection domain on erosion rate distribution over the pressure surface.

Figure 3-7. Influence of the length of an injection domain on erosion rate distribution over the suction surface.
Figure 3-8. Influence of the length of an injection domain on impact velocity distribution over the airfoil

Figure 3-9. Influence of the length of an injection domain on impingement angle distribution over the airfoil
These results indicated that erosion rate characteristics on the overall airfoil surface are not dependent upon the size of the injection domain when the injection plane is greater than the maximum thickness of airfoil. Therefore, in this current study, the size of an injection plane is chosen to be more than 10 times of the maximum thickness of the airfoil in order to perform 2D and 3D erosion damage predictions for various inflow angle of attacks.

Next, computations for the influence of the total number of injection points distributed on the injection plane were performed. Figure 3-11 and Figure 3-12 show area averaged volumetric erosion rate distributions in a function of the total number of injection points used at the inlet section. In these results, the number of injection points is represented as the interval size in terms of minimum grid cell size.

These results show that for less than 5000 injection points along the inlet section, these erosion rate distributions have a visible deviation from the general pattern because a relatively
small number of particles used at the inlet imply the use of a smaller number of “larger” bins on the airfoil surface. A larger segment of the airfoil surface receives a single erosion rate value assigned to each bin. The spatial resolution of the erosion rate computation suffers at a great rate when a small number of sand particles are used in the current computations. Erosion rate distributions are not affected when more than 6000 sand particles are released at the inlet section. In other words, a reasonable distance between the two subsequent sand particles at the inlet is typically less than of the distance for 6000 sand particles are used at the injection section.

Therefore, in this current study, the number of injection points is chosen for the interval size to be less than 10 times of the size of a minimum grid cell \( L_{cell,\text{min}} = 1.0 \times 10^{-5} \text{ m} \) in order to improve the erosion prediction accuracy.

![Figure 3-11. Influence of the number of injection points distributed at inlet boundary condition on the erosion rate on the pressure surface](image)
3.2. Definition of Erosion Rate on the Metallic Surface

In a general erosion experiment, a known mass of erosive particles are released from the inlet over a very long time period (or over a relatively shorter time period). The exact amount of removed mass (or removed volume) from metallic surface is then measured. Therefore, an erosion rate prediction model is developed from the measured average damage for a certain impact condition because it is very difficult to measure erosion damages due to just single solid particle impact on the metal surface. Moreover, it is also difficult to control impact conditions of each solid particle. The impact condition includes impact velocity, impingement angle, mass flow rate, diameter and density of a solid particle. Therefore, the previous definition of “erosion rate” determines “the amount of mass or volume removed from the metal surface when unit mass of solid particles hit the surface of interest”.

Figure 3-12. Influence of the number of injection points distributed at inlet boundary condition on the erosion rate on the suction surface
Throughout these general experiments, the general erosion rate also indicated that as long as the total sand mass released and the injection conditions are kept the same, each experimental or numerical investigation should yield to a similar cumulative erosion damage characterized as removed mass of volume from the surface of interest. This definition is a reasonably accurate one when the fluid flow and the released solid particle are in a steady-state and when a total mass of solid particles released from a specific area are exactly measured. However, in the erosion problem on typical helicopter rotor blades, the injection condition of released particles is not steady. Moreover, it is very difficult to measure mass of individual solid particle released from the injection points. Therefore, a “specific” definition of erosion rate in terms of time has been implemented in this study to predict the erosion characteristics on helicopter blades with good accuracy.

According to the assumptions applied in this study (which is mentioned in Chapter 3.1), solid particles released from the same injection point follow the same trajectory within computation domain and impact on the same position of a metal surface with the same impact conditions. The number of solid particles released from each injection point is represented by “mass flow rate”. In this current study, the mass flow rate of impacting particles is used to modify the definition of erosion rate.

The multiplication of “mass flow rate of each particle”, \( m_{pi} \), and “the removed mass when a unit mass of solid particle impacts on the target”, \( f(\beta_i, \nu_{pi}, D_p) \), results in the erosion rate for this specific particle per a unit time as shown in Equation 41. When a summation for all particles hitting the surface in a specified area is performed, the erosion rate as “removed mass per unit time in “a specified area” will be obtained in \([mg/s]\) (or \(cm^3/s\)).
Erosion Rate

\[
ER = \sum_{i=1}^{n} \frac{M_{pi}}{\Delta t} \cdot f(\beta_1, V_{p1}, D_p) = \sum_{i=1}^{n} m_{pi} \cdot f(\beta_1, V_{p1}, D_p)
\]  

(41)

Where, \( n \) is the number of solid particles impacting on the metallic surface of interest.

\[
ER = \sum_{i=1}^{n} \frac{M_{pi}}{\Delta t} \cdot f(\beta_1, V_{p1}, D_p)
\]

(42)

Where,

\[
m_{pi} = \frac{M_{pi}}{\Delta t} = \frac{\Phi \cdot M_i}{1 - \Phi \cdot (\rho V L_i)} = \frac{\Phi \cdot m_i}{1 - \Phi}
\]

(43)

\( f(\beta_1, V_{p1}, D_p) \) function comes from experimental studies performed in wind tunnels where a selected sand particle type is impacted on a target surface in a controlled environment. The \( f(\beta_1, V_{p1}, D_p) \) function strongly depends on impingement angle \( \beta_1 \), impact velocity \( V_{p1} \) and the particle diameter \( D_p \). Figure 3-13 shows the definition of the impingement angle \( \beta_1 \), impact velocity \( V_{p1} \) and rebound velocity \( V_{p2} \) in a typical sand particle and target surface interaction, including the restitution ratios \( e_N \) and \( e_T \). Therefore, a more comprehensive definition of this \( f(\beta_1, V_{p1}, D_p) \) function can be expressed as shown in Equation 44. An accurate value of \( f(\beta_1, V_{p1}, D_p) \) can be evaluated from the three functions termed as \( g_1 \), \( g_2 \) and \( g_3 \). Each one of these \( g \) functions are also expressed in functions of impingement angle \( \beta_1 \), impact velocity \( V_{p1} \) and the particle diameter \( D_p \), respectively. There is usually a coefficient \( c_1 \) termed as erosion rate coefficient.

\[
f(\beta_1, V_{p1}, D_p) = c_1 \cdot g_1(\beta_1) \cdot g_2(V_{p1}) \cdot g_3(D_p)
\]

(44)
3.3. Rebound and Erosion Model for Sand Particle and Ti-6Al-4V

As mentioned in Chapter 2, a rebound model is highly depended upon the impingement angle and an erosion model consists of three independent parameters such as impingement angle, the impact velocity of solid particle, and solid particle diameter. These models are varied with the material of metal surface and the type of solid particle. Moreover, these models are empirical equations which are obtained from wind tunnel experimental data. Therefore, a rebound model and an erosion model for sand particles impacting on Titanium alloys are necessary for the prediction of erosion damages on helicopter rotor blades.

In this study, a rebound model for silica sand particles impacting on Ti-6Al-4V target surface suggested in the study of Taslim et al. (33) was used.

\[
e_N = 0.993 - 3.07418e^{-2} \beta_1 + 4.752e^{-4} \beta_1^2 - 2.6051e^{-6} \beta_1^3
\]

(45)

\[
e_T = 0.988 - 2.8972e^{-2} \beta_1 + 6.4274e^{-4} \beta_1^2 - 3.5621e^{-6} \beta_1^3
\]

(46)
Next, an empirical erosion equation was developed by using erosion experimental results from Bahadur et al. (36). They measured the volume change in the target (25mm×19mm×16mm rectangular block) after a total of 200 gram of silicon carbide particles hit the target as shown in Figure 3-14. Figure 3-15 is the measured erosion rate distribution as a function of impingement angle. In this measurement, the erosion rate is defined as "the average mass loss when a unit mass of solid particle is released from the inlet". Likely to other erosion experimental results, this result shows that the measured erosion rate is linearly increased with increased impingement angle for up to $30^\circ$ and that after the magnitude reaches a maximum value at $30^\circ$ of impingement angle, the erosion rate is gradually decreased.

First, the specific $g_1$ function is developed by using multi-section polynomial curves in function of impingement angle based on Bahadur’s experimental results, Figure 3-15 as shown in Equation 47 ~ Equation 49

\[
g_1(\beta_1) = 0.0 + 5.2324e^{-2}\beta_1 + 2.4913e^{-5}\beta_1^2 - 2.1912e^{-5}\beta_1^3
g_1(\beta_1) = -7.9495 + 9.2678e^{-1}\beta_1 - 3.4063e^{-2}\beta_1^2 + 5.2455e^{-3}\beta_1^3 - 2.9421e^{-4}\beta_1^4
g_1(\beta_1) = 2.1042 - 6.2304e^{-2}\beta_1 + 7.1168e^{-4}\beta_1^2 - 2.8912e^{-6}\beta_1^3
\]

for $\beta_1 \leq 25.5$ for $25.5 \leq \beta_1 \leq 51.3$ for $51.3 \leq \beta_1 \leq 90.00$ (47) (48) (49)

Next, experimental results for erosion rate distribution function $g_2$ in function of impact velocity $V_{p_1}$ as shown in indicated that erosion rate for sand particles impacting on Ti target is proportional to $(V_{p_1}/60)^{2.35}$. Therefore, in this study, the function for impact velocity, $g_2$, has been suggested as Equation 50.

\[
g_2(V_{p_1}) = \left(\frac{V_{p_1}}{60}\right)^{2.35}
\]
Finally, the experimental results in a function of the diameter of a solid particle, as shown in the Figure 3-17, shows that the function of erosion rate for solid particle diameter has three different characteristics section. One section for particle diameter of less than 40µm shows the erosion rate is linearly proportioned with the particle diameter. Second section for particle diameter between 40µm and 70µm shows that the erosion rate is proportional with the particle diameter, but the change rate is decreased with the particle diameter. Final section for greater than 70µm, the erosion rate is constant, which is not dependent upon the particle size.

Equation 51 is a specific function for solid particle diameter, g₃.

\[
\begin{align*}
g_3(D_p) &= \begin{cases} 
2.321e^{-3}D_p + 2.149e^{-2} & \text{for } D_p \leq 40\mu m \\
4.735e^{-2} + 2.369e^{-3}D_p - 1.705e^{-5}D_p^2 & \text{for } 40\mu m < D_p \leq 70\mu m \\
1.000 & \text{for } 70\mu m < D_p 
\end{cases}
\end{align*}
\] (51)

Finally, Equation 52 is the final form to predict erosion rate for Ti-6Al-4V:

- The final form of erosion equation for Ti-6Al-4V:

\[
ER_{mass} = 0.13 \left( \frac{V_p}{60} \right)^{2.35} \cdot g_1(\beta_1) \cdot g_3(D_p) \quad [\text{mg/g}]
\] (52)

Figure 3-14. Experimental system for sand erosion on the Ti-6Al-4V target from Ref. (36)
Figure 3-15. Erosion rate distribution with impingement angle for Ti-6A1-4V from Ref. (37)

Figure 3-16. Erosion rate distribution with impact velocity for Ti-6A1-4V from Ref. (37)

Figure 3-17. Erosion rate distribution with particle size for Ti-6A1-4V from Ref. (37)
3.4. User Defined Function (UDF)

As described in Chapter 2, in order to predict sand erosion damage for helicopter rotor blades, a Lagrangian particle tracking algorithm including modified injection conditions as well as a suitable empirical rebound and erosion equations obtained from experimental results for spherical sand particles impacting on Ti-6Al-4V material surface are required. However, the general purpose viscous flow solver, ANSYS-FLUENT, does not have a specific option for dealing with these schemes.

Therefore, in this study, C++ language based specific codes, which is called as User defined functions (UDFs), were developed and linked into the general purpose viscous flow solver, ANSYS-FLUENT, in order to determine proper injection conditions and to apply the rebound and erosion model presented in Appendix C.

Figure 3-18 shows the flow chart of a numerical approach including UDFs for determining particle trajectories and predicting erosion damage on helicopter rotor blade.
Chapter 4

Validation of 2D Flow-Only Simulation Results

An accurate solution of fluid flow in computational domain is required for an erosion damage calculation because the motion of solid particle is determined by their inertia and the relative motion of surrounding fluid flow. In this section, first, computations for fluid flow-only around a 2D airfoil were performed and these results were compared with experimental results and other numerical results in order to validate 2D flow simulation results. Next, turbulent flow field influence on the final erosion calculations was investigated. Therefore, in this chapter, influence of 5 different turbulence models on flow-only solution was investigated and these results obtained in these studies were compared to experimental data measured in various wind tunnels in order to select a proper turbulence model using typical helicopter airfoil cross sections.

4.1. Flow Computation around a 2D Airfoil

The first 2D computational effort is the prediction of the flow around a RAE 2822 airfoil for an inlet Mach number of 0.729, 2.31 degree of angle of attack, and 6.5 million of Reynolds number based on airfoil chord. The computations were then compared to experimental results from Cook et al. (38) and Sitaraman et al. (39) for validation assessment. Figure 4-1 shows a multi-block grid system used in this study to simulate 2D airfoil flow with a high subsonic inlet flow condition at $M_{\text{inlet}} = 0.729$. Based on grid independence study presented in Appendix B, a 280×80 H-type structured grid was used to mesh each block of the computational domain. The H-type structured grid system has a merit to capture shock and wake flow observed over the airfoil.
The size of the first grid cell (first wall spacing) was chosen such that $y^+$ was less than 1.0. 2D computations performed in this study took approximately 4 hours on $4 \times 2$ CPU parallel computer.

Figure 4-2 shows Mach number contours around a RAE 2822 airfoil. As shown in this figure, flow suddenly accelerates over the leading edge on the suction surface and the flow eventually become supersonic flow. The velocity of flow passed over the suction surface is suddenly dropped due to a normal shock generated at about 55 percent of chord length. The flow passing the normal shock is eventually decelerated. A separated flow zone is also observed starting from 60 percent chord length on the suction side.

Figure 4-3 is the comparison of negative pressure coefficient distributions over a RAE 2822 airfoil surface. The negative coefficient distribution over the airfoil more clearly shows the velocity distribution of flow passing near the airfoil surface. As shown in this result, the flow passing on the suction surface is sharply accelerated over the leading edge and a little velocity reduction at about 3 percent chord length on the suction side occurs due to weak shock generated at the end of the leading edge. On the suction side, the sudden velocity drop is also observed at approximately 55 percent chord length, which is caused by normal shock.

The figure indicates that the numerical results (pressure coefficient, $c_p$) obtained in this study and other numerical analysis (Cook et al. and Sitaraman et al.) have negligible difference in the pressure distributions over the airfoil against experimental result. The difference may be caused due to a turbulence model and a grid system. The comparison concluded that the numerical result obtained in this current study is in very good more agreement with experimental data. These results concluded that the numerical solver with this grid system can predict a reasonably accurate flow fluid solution for sand erosion studies.

Therefore, a modeling for sand erosion simulation on a 2D airfoil surface and on a 3D rotating helicopter rotor blade was performed based on this grid system.
Figure 4-1. Multi H-Grid system for 2D simulation around airfoil

Figure 4-2. Mach number contours around RAE 2822 airfoil
4.2. Validation of Turbulence Model

A selection of a suitable turbulence model has a significant impact in the simulation of a compressible, turbulent and viscous flow at high Reynolds number. It is well known that turbulence model affects simulated flow conditions near airfoil due to different modeling approaches for turbulent viscosity and that the effect of turbulence model is dependent upon the grid system (e.g. the size of first grid cell from the wall) and final local strain rates. The selected turbulence model can affects particle trajectories and erosion characteristics.

In this section, computations about the flow around SC1095 airfoil using various turbulence models, which is presented in Appendix A.2, were performed in order to select a suitable turbulence model for the flow around helicopter rotor blade. The computational results
were then compared to the numerical result calculated by Mayda et al. (40) and experimental result performed by Bousman (41) to analyze the effect of turbulence model on aerodynamics around the airfoil and to select a proper turbulence model for the prediction of sand erosion on helicopter rotor blades. Table 4-1 shows numerical conditions at the inlet of the domain in this computational study for the assessment of the role of turbulence models. Bousman (41) analyzed aerodynamic performance characteristics for a SC1095 airfoil and a SC1094R8 airfoil against various wind tunnel experiments as shown in the Table 4-2.

Table 4-1 Inlet flow conditions used for the assessment of the role of turbulence models

<table>
<thead>
<tr>
<th>Turbulence model</th>
<th>Mach number of inlet flow</th>
<th>Angle of attack of inlet flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spalart-Allmaras (SA)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k-ε Standard (kε-Standard)</td>
<td>0.1, 0.4, 0.6</td>
<td>1.0, 0.7, 0.5, 0.0</td>
</tr>
<tr>
<td>k-ε RNG (kε-RNG)</td>
<td>0.8, 0.85, 0.9</td>
<td>-0.5, -0.7, -1.0</td>
</tr>
<tr>
<td>k-ω Standard (kω-Standard)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k-ω SST (kω-SST)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-2 Wind tunnels for the experimental datasets

<table>
<thead>
<tr>
<th>Test</th>
<th>Wind tunnel</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. 1</td>
<td>UTRC large subsonic wind tunnel</td>
<td>Griffin, 1973 (42)</td>
</tr>
<tr>
<td>Exp. 2</td>
<td>UTRC large subsonic wind tunnel</td>
<td>Vogt, 1975 (43)</td>
</tr>
<tr>
<td>Exp. 3</td>
<td>OSU 6by 22in transonic wind tunnel</td>
<td>Lednicer et al., 1985 (44)</td>
</tr>
<tr>
<td>Exp. 4</td>
<td>NRC 12 by 12in icing wind tunnel</td>
<td>Flemming et al., 1985 (45)</td>
</tr>
<tr>
<td>Exp. 5</td>
<td>NSRDC 7 by 10ft transonic wind tunnel</td>
<td>Flemming, 1984 (49)</td>
</tr>
<tr>
<td>Exp. 6</td>
<td>Langley 6 by 28in transonic wind tunnel</td>
<td>Noonan et al, 1980 (47)</td>
</tr>
<tr>
<td>Exp. 7</td>
<td>Ames 2 by 2ft transonic wind tunnel</td>
<td>Hicks et al., 1985 (48)</td>
</tr>
<tr>
<td>Exp. 8</td>
<td>Ames 11 by 11ft transonic wind tunnel</td>
<td>Flemming, 1984 (49)</td>
</tr>
<tr>
<td>Exp. 9</td>
<td>Ames 7 by 10ft subsonic wind tunnel</td>
<td>McCroskey et al., 1982 (50)</td>
</tr>
<tr>
<td>Exp.10</td>
<td>UMd 8 by 11ft subsonic wind tunnel</td>
<td>Robinson et al, 1998 (51)</td>
</tr>
</tbody>
</table>

Figure 4-4 shows distribution of lift-curve slope at zero-lift (Hovering condition) as a function of inlet Mach number for various turbulence models. In these results, lift-curve slope at
subsonic inflow condition (for less than 0.7 of inlet Mach number) increased slowly with increasing inlet Mach number. For the transonic inflow condition (at from 0.75 to 0.85 of Mach number), the lift-curve slope is suddenly increased as inlet Mach number is increased. In this ranges of Mach number (for less than 0.85 of inlet Mach number), the lift is increased with increased inlet Mach number. The slope reaches maximum value at approximately 0.85 of inlet Mach number, and the slope is steeply dropped due to weak and normal shocks and increase in the size of separation.

Though an overall lift-curve slope obtained the numerical simulations is greater than the slope measured in these experiment due to finite wing effect and the slope value measured in experiment 1 and 2 is much smaller due to subsonic wind tunnel system, these results concluded that the computation results performed in this study are in good agreement with experimental results. It is also found that the numerical result have more accuracy than ARC2D results. ARC2D is a 2D flow solver used in past aerodynamic studies (40). The comparison of the current computations with other measured data and computations shown in Figure 4-4 that $k-\omega$ SST model performs better than the Spalart-Allmaras model, the $k-\epsilon$ standard model, the $k-\epsilon$ RNG model and the $k-\omega$ standard model.

Drag coefficient at zero lift condition is shown in Figure 4-5. All experimental and numerical data indicated that the drag is slightly reduced with increasing inflow Mach numbers at subsonic flow (for less than 0.7 of inlet Mach number) because the effect of a boundary layer and a separation flow on the airfoil surface is reduced. For a transonic flow condition, (greater than 0.75 of inlet Mach number), drag is sharply increased with increasing inlet Mach number due to the aerodynamic loss including an interaction between shock and boundary layer as shown in the Figure 4-5. It is observed that the results measured in experiment 4 wind tunnel have great difference with other results due to very small wind tunnel size.
This figure indicated that the numerical results obtained in this current computation are in good agreement with experimental data. It is also found that computational aerodynamic results with kω SST turbulence model are the most successful computations in the current effort. One should note that experiment 4 has a significant deviation from the computations was obtained from a wind tunnel where test section has a very limited flow area (NRC 12 inch by 12 inch).

Figure 4-6 is the plot of pitching moment coefficient versus inflow Mach number. Pitching moment does not vary significantly when the inlet Mach number is less than 0.6. Pitching moment coefficient is reduced visibly between 0.6 and 0.85 of inlet Mach number. There is a significant increase just after 0.85 of inlet Mach number because of a huge increase in drag force. The peak of the pitching moment coefficient is at about 0.88 of inlet Mach number. Pitching moment drops suddenly between 0.88 and 0.92 of inlet Mach number, because the lift is suddenly reduced due to a normal shock generated on the suction side. Unlike previous results, the deviations of pitching moment at zero-lift coefficient at various experimental data are observed over a wide range of inlet Mach number. It is well known that an accurate measurement for pitching moment is challenging. However, numerical results obtained in this current simulation is very similar to experimental results.

From Figure 4-4 to Figure 4-6 show that numerical results performed in this study have reasonably good accuracy and that computational aerodynamic results with k − ω SST turbulence model are the most successful computations in the current effort. Therefore, k − ω SST turbulence model is selected as main turbulence model in all computations performed in this current study.
Figure 4-4. Lift-curve slope at zero-lift versus Mach number of inflow

Figure 4-5. Drag coefficient at zero-lift versus Mach number of inflow
Figure 4-6. Pitching moment coefficient at zero-lift versus Mach number of inflow
Chapter 5

Particle Trajectories and Erosion Damages on a 2D SC1095 Airfoil

As discussed in the previous section, an erosion damage on a metallic surface due to the impact of single solid particle depends upon three different variables:

- Impingement angle
- Impact velocity
- Diameter of a solid particle

These variables are defined as “impact conditions”. These impact conditions are determined by using a solid particle tracking algorithm. The particle tracking algorithm determines the Lagrangian motion of solid particles within a computational domain by calculating forces acting on the solid particle. In the flow including sand particles around a helicopter rotor blade, the induced drag generated due to the velocity difference between a solid particle and fluid flow is only dominant force, so the solid particle motion is affected by flow conditions within computation domain including inflow boundary condition. The injection conditions of solid particles are commonly determined by the inlet boundary conditions of fluid flow. It is concluded that the overall erosion characteristics on metal surfaces are thus decided by the inlet (inflow) boundary conditions of fluid flow.

In addition, it is time consuming to analyze flow characteristics and erosion characteristics near 3D rotating rotor blades due to complex aerodynamics around the rotor blade and very high computational cost. In the general analysis for the 3D flow around a 3D rotating helicopter rotor blade, the simulation for 3D flow around the rotor blade rotor blade can be alternatively performed by 2D relative flow simulations for the cross section (2D airfoil) in the
span direction of the rotor blade. This is reasonable because a spanwise component of resultant velocity of relative flow, $V_R$, to the cross-section of a rotor blade is usually ignored by independence principle (52). Therefore, the investigation of sand erosion damage on a 2D airfoil surface can be used in order to understand erosion phenomena on a rotating 3D rotor blade.

Figure 5-1 shows a sketch of relative inflow conditions to the blade section of the rotor blade. As shown in this figure, the resultant local inflow velocity ($V$) is consists of three different components – an in-plane velocity ($V_T$), an out-of-plane velocity ($V_P$), and a radial velocity ($V_R$). Moreover, an inflow angle of attack to the leading edge of the blade section is defined as an aerodynamic angle of attack (or an effective angle of attack) as shown in Equation 53.

$$\alpha_{eff} = \theta - \phi$$

(53)

where, $\theta$ is the pitch angle at the blade section and $\phi$ is a relative inflow angle (or induced angle of attack) at the blade section.

$$\phi = \tan^{-1}\left(\frac{V_P}{V_T}\right) \approx \frac{V_P}{V_T}$$

(54)
5.1. The Effect of Inflow Boundary Conditions (Injection Condition of a Sand Particle) on Particle Trajectories and Sand Erosion Damages

In internal flow cases such as compressors, turbines, or pipes, the inlet conditions of fluid and injection condition of solid particles are fixed because these systems are usually operated at a well known design point. Moreover, the inlet condition can be assumed to be uniform. The particle diameter is thus a unique parameter to decide erosion patterns and rates within the system as shown in the erosion prediction study performed by Mazur et al. (53). They investigated trajectories of solid particles for different sand particle diameters within a turbine stage. These results determined that large size particles do multi-impact on stators and do not enter into the rotor channels after rebounding at rotor leading edges while small particles follow the flow motion with continuous multi-impact on a whole turbine system.

On the other side, in the case about the flow around a helicopter rotor blade, each section of the rotor blade in the span direction is operated at various relative flow conditions. Moreover, the relative flow conditions are nonlinearly varied in the span direction of the rotor blade, which indicates that injection conditions of released sand particles are also varied with the position of injection section.

Therefore, in this study, numerical predictions about particle trajectory and erosion damage on a 2D airfoil for various inflow boundary conditions (injection conditions solid particle) were performed in order to investigate the effect of these conditions on erosion characteristics on a 2D airfoil surface. A SC1095 airfoil is chosen as a reference 2D airfoil shape because this airfoil is a main airfoil shape of Sikorsky UH-60A Helicopter. Computational conditions used in this analysis are specified based on a real hover condition of a UH-60A helicopter because erosion damage on the rotor blades dominantly occurs when the helicopter is operated at hover,
vertical climb and descend conditions near the ground. In this study, inflow Mach number, inflow angle of attack, and the diameter of solid particles are selected as a control parameter as shown in the Table 5-1.

<table>
<thead>
<tr>
<th>Released condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflow Mach number</td>
<td>$M_0 = 0.4, 0.5, 0.6, 0.7$</td>
</tr>
<tr>
<td>Inflow angle of attack</td>
<td>$\alpha_0 = 5.0, 2.0, 0.0, -0.7, -2.0, -5.0$ [degree]</td>
</tr>
<tr>
<td>Particle diameter</td>
<td>$D_p = 10, 25, 50, 75, 100, 150, 250, 500$ [$\mu$m]</td>
</tr>
</tbody>
</table>

In the general erosion predictions, the accumulated mass or volume loss from the metallic surface of interest is determined. This erosion rate may depend upon the grid size for wall boundary condition, because the grid cell size for wall boundary condition is highly relative to the number of solid particles impacting on the metallic surface. The large grid cell for a wall boundary condition has a high probability for solid particles impact on the metallic surface, which indicates that the accumulated erosion rate can be over-estimated. The small grid cell for a wall boundary condition has a low probability for solid particle impact on the metallic surface, which indicates that the accumulated erosion rate can be under-estimated. Therefore, in this study, “area averaged volumetric (mass) erosion rate” is used as a major definition of erosion rate in order to exclude the effect of grid cell size. The “area averaged volumetric (mass) erosion rate” means volume (mass) loss from the metal surface of interest per unit area due to the impact of a group of solid particles released from the injection plane per unit time. In this study, another definition for erosion rate is used. This is “local erosion rate” defined as mass or volume loss from the specific area of metallic surface due to the impact of a single solid particle.
5.1.1. The Effect of Inflow Mach Number

First of all, the effects of inflow Mach number on sand particle trajectories and erosion damage on a 2D SC1095 airfoil were investigated. In the calculations, the inflow angle of attack and particle diameter is fixed. Inflow Mach number is chosen as a control variable.

Figure 5-2 and Figure 5-3 are area averaged volumetric sand erosion rate distribution over airfoil surface for $D_p = 50\mu m$ and $\alpha_{pa} = -0.07^\circ$ and Figure 5-4 and Figure 5-5 are area averaged volumetric erosion rate over airfoil surface for $D_p = 250\mu m$ and $\alpha_{pa} = -0.07^\circ$. As shown in the erosion rate distribution on the pressure side, the erosion rate is sharply increased over the leading edge of the airfoil. The magnitude of erosion rate becomes a maximum at about 1 percent chord length, and then the erosion rate is sharply decreased over the airfoil. The erosion rate distributions on the suction side are almost same as those on the pressure side, but the maximum erosion rate on the suction side is observed at approximately 2 percent chord length. It is also shown that the damaged area on suction side is wider than the area on the pressure side. The airfoil surface after 10 percent chord length on the pressure side and after 15 percent chord length on the suction surface do not have any damage due to the impact of solid particles.

Figure 5-12 to Figure 5-19 are trajectories of sand particles around a SC1095 airfoil surface. These figures show that sand particle trajectories around the airfoil with respect to inflow Mach number are similar although the trajectory of solid particles rebounding from the airfoil surface is varied with the inflow Mach number. This shows that the location at which solid particles impact the airfoil is not a function of inflow Mach number.

These results concluded that the magnitude of erosion rate is proportioned with the inflow Mach number of solid particles, but the pattern of erosion rate over the airfoil including
position and the size of airfoil at which maximum erosion rate occurs is not a function of inflow Mach number.

The relation between the magnitude of erosion rate and the inflow Mach number can be discussed in more detail in the Figure 5-6 and Figure 5-7. These results are nondimensional impact velocity distribution for \( D_p = 50\mu m \) and \( D_p = 250\mu m \) at \( \alpha_{pa} = -0.07^\circ \). As shown in particle trajectories, Figure 5-12 ~ Figure 5-19, solid particles are decelerated when they approach the airfoil surface.

The reduction of solid particles velocity is more pronounced when impact position is closer to the leading edge. This is because static pressure of fluid flow is increased when the flow is near to the leading edge (stagnation point), which causes a great velocity difference between a solid particle and fluid flow at the leading edge. These figures also show that the impact velocity near the leading edge is slightly reduced as the inflow Mach number increases. This is because static pressure of fluid flow near the airfoil is also increased with an increasing the inflow Mach number, which results in an increase of induce drag force acting on solid particles. However, absolute magnitude of the impact velocity is linearly proportional with the inflow Mach number.

At a section of the airfoil after 10 percent chord length, the impact velocity is large enough but the impingement angle is very small as shown in Figure 5-8 and Figure 5-9, which cause negligible erosion damage. In addition, it is shown that the impact velocity on the suction side is greater than that of pressure side, which indicates that the erosion damage on the suction side is greater than the erosion damage on the pressure side. These results show also that the impact velocity over the airfoil for \( D_p = 250\mu m \) is much greater than the impact velocity for \( D_p = 50\mu m \) because the large sand particle gets the smaller effects from momentum of fluid due to its high inertia.

Figure 5-8 and Figure 5-9 show impingement angle distributions over the airfoil for inflow Mach number. Impingement angle distributions for various inflow Mach numbers are
similar because the trajectory of solid particles before the collision with the metal surface is not
dependent upon the release velocity of solid particles. However, the impingement angle of solid
particles is affected by its diameter because of the inertia of each solid particle. As mentioned
previously, the motion of small solid particles can be changed more easily by the motion of fluid
flow due to small inertia, but large solid particles keep their trajectory until they impact on the
metal surface.

The effect of impact velocity distribution and impingement angle distribution can be
shown in the local erosion rate distributions, Figure 5-10 and Figure 5-11. These are local erosion
rate distributions in function of inflow Mach number. As shown in these results, the maximum
magnitude of local erosion rate on the suction side is same as that on the pressure side, but the
overall magnitude of local erosion rate on suction side is greater than the magnitude on the
pressure side.

These computations of particle tracking and sand erosion damages on a 2D SC1095
airfoil for various inflow Mach numbers indicate that the magnitude of erosion damage is linearly
proportional with the inflow Mach number. However, the pattern of erosion rate distribution in
chordwise direction is not dependent upon inflow Mach number. These results also concluded
that the impact velocity can be reduced by an induced aerodynamic drag force. The erosion
damage on the airfoil surface can be reduced by using the induced drag force.
Figure 5-2. Area averaged volumetric erosion rate distributions on the pressure surface for inflow Mach number at $D_p = 50\mu m$ and $\alpha_0 = -0.07^\circ$.

Figure 5-3. Area averaged volumetric erosion rate distributions on the suction surface for inflow Mach number $D_p = 50\mu m$ and $\alpha_0 = -0.07^\circ$. 
Figure 5-4. Area averaged erosion rate distributions on the pressure surface for inflow Mach number at $D_p = 250\mu m$ and $\alpha_0 = -0.07^\circ$.

Figure 5-5. Area averaged erosion rate distributions on the suction surface for inflow Mach number at $D_p = 250\mu m$ and $\alpha_0 = -0.07^\circ$. 

Figure 5-6. Nondimensional impact velocity distributions over the airfoil surface for inflow Mach number at $D_p = 50 \mu m$ and $\alpha_0 = -0.07^\circ$

Figure 5-7. Nondimensional impact velocity distributions over the airfoil surface for inflow Mach number at $D_p = 250 \mu m$ and $\alpha_0 = -0.07^\circ$
Figure 5-8. Impingement angle distributions over the airfoil surface for inflow Mach number at $D_p = 50\mu m$ and $\alpha_0 = -0.07^\circ$

Figure 5-9. Impingement angle distributions over the airfoil surface for inflow Mach number at $D_p = 250\mu m$ and $\alpha_0 = -0.07^\circ$
Figure 5-10. Local volumetric erosion rate distributions over the airfoil surface for inflow Mach at $D_p = 50\mu m$ and $\alpha_0 = -0.07^\circ$

Figure 5-11. Local volumetric erosion rate distributions over the airfoil surface for inflow Mach at $D_p = 250\mu m$ and $\alpha_0 = -0.07^\circ$
Figure 5-12. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $D_p = 50\mu m$, $\alpha_0 = -0.07^\circ$ and $M_0=0.4$

Figure 5-13. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $D_p = 50\mu m$, $\alpha_0 = -0.07^\circ$ and $M_0=0.5$
Figure 5-14. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $D_p = 50\mu m$, $\alpha_0 = -0.07^\circ$ and $M_0=0.6$

Figure 5-15. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $D_p = 50\mu m$, $\alpha_0 = -0.07^\circ$ and $M_0=0.7$
Figure 5-16. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $D_p = 250\mu$m, $\alpha_0 = -0.07^\circ$ and $M_0$=0.4.

Figure 5-17. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $D_p = 250\mu$m, $\alpha_0 = -0.07^\circ$ and $M_0$=0.5.
Figure 5-18. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $D_p = 250\mu m$, $\alpha_0 = -0.07^\circ$ and $M_0=0.6$

Figure 5-19. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $D_p = 250\mu m$, $\alpha_0 = -0.07^\circ$ and $M_0=0.7$
Next, the investigation of the effect of inflow Mach number at high inflow angle of attack was performed. Figure 5-20 and Figure 5-21 are area averaged volumetric erosion rate distributions over airfoil surface for \( D_p = 50\mu m \) and \( \alpha_{p_0} = 5.0^\circ \). Figure 5-22 and Figure 5-23 are area averaged volumetric erosion rate distributions over airfoil surface for \( D_p = 250\mu m \) and \( \alpha_{p_0} = 5.0^\circ \). Compared with the results for \( \alpha_{p_0} = -0.07^\circ \), these results for positive inflow angle of attack shows that the erosion rate on the pressure side is increased, which causes maximum erosion damage on the pressure side as same as that on the suction side. The damage area on the pressure side is also more widen. These results also indicated that erosion rate distributions over the airfoil are a little changed for different inflow angle of attack. The effect of the inflow angle of attack will be discussed in Chapter 5.1.3.

These results about the effect of inflow Mach number at different inflow angle of attack concluded that when the inflow angle of attack is same, the magnitude of erosion rate is linearly proportioned with the inflow Mach number and erosion distributions rate except the magnitude is not dependent upon the inflow Mach number.
Figure 5-20. Area averaged volumetric erosion rate distributions on the pressure surface for inflow Mach number at $D_p = 50\mu m$ and $\alpha_0 = 5.0^\circ$

Figure 5-21. Area averaged volumetric erosion rate distribution on the suction surface for inflow Mach number at $D_p = 50\mu m$ and $\alpha_0 = 5.0^\circ$
Figure 5-22. Area averaged volumetric distributions on the pressure surface for inflow Mach number at $D_p = 250\mu m$ and $\alpha_0 = 5.0^\circ$

Figure 5-23. Area averaged volumetric erosion rate distributions on the suction surface for inflow Mach number at $D_p = 250\mu m$ and $\alpha_0 = 5.0^\circ$
5.1.2. The Effect of the Diameter of a Solid Particle

In this section, the effects of the diameter of solid particles on particle trajectories and erosion damage on a 2D SC1095 airfoil were investigated. In the analysis, the computations of particle tracking and erosion damage predictions for various particle diameters (\(D_p = 10 \sim 250\mu m\)) were performed at constant inflow angle of attack (\(\alpha_0 = -0.07^\circ\)) and constant inflow Mach number (\(M_0 = 0.4\) and 0.7)

Figure 5-24 and Figure 5-25 are area averaged volumetric erosion rate distribution over the airfoil surface for particle diameter at \(M_0 = 0.4\) and \(\alpha_0 = -0.07^\circ\). Figure 5-26 and Figure 5-27 are area averaged erosion rate distribution for particle diameter at \(M_0 = 0.7\) and \(\alpha_0 = -0.07^\circ\). As shown in from Figure 5-24 to Figure 5-27, the erosion damage is sharply increased over the leading edge of a 2D SC1095 airfoil, and the erosion damage has a maximum value at less than approximate 2 percent chord length, and the erosion damage is then steeply reduced over the airfoil. At the airfoil surface after about 20 percent chord length, the sand particles can never impact the surface because sand particles go downstream without any interaction with the airfoil.

As shown in these results, for less than \(100\mu m\) of solid particle diameter, as the diameter of released solid particles is decreased, erosion damage over the airfoil surface are hugely reduced and maximum erosion damage is occurred at much closer to the leading edge. However, for greater than \(100\mu m\) of solid particle diameter, the difference in the magnitude of erosion rate over the airfoil surface and maximum erosion damage position can be negligible. Therefore, for more than \(100\mu m\) of particle diameter, the erosion rate distribution over the airfoil surface can be assumed to be constant.
As shown in particle trajectories around the airfoil (Figure 5-34 ~ Figure 5-37) and impact velocity distributions (Figure 5-28 ~ Figure 5-29), the impact velocity is highly dependent upon the impact position and the diameter of solid particle. For small size solid particles, their velocity is eventually decelerated as they approach the airfoil surface, so they impact on the metal surface with huge reduced velocity. It is also shown that, for large size solid particles, they are little reduced by the induced drag force and they collide with the metallic surface with almost same velocity as their released velocity. The difference of impact velocity between large and small solid particles is due to their inertia. These results also concluded that the impact velocity distribution for solid particles, of which diameter is more than 150µm, can be also assumed to be constant.

Figure 5-30 and Figure 5-31 are impingement angle distribution over the airfoil for various diameters of a sand particle. As shown in these figure, the change rate of a decrease of impingement angle over the airfoil surface is increased as the particle diameter gets smaller. Because the motion of a small sand particle is more easily changed by the motion of fluid flow due to its small inertia, the small sand particle impact on the airfoil surface with great change in the direction while a large sand particle keeping initial direction can collide with the surface. It is also found that the impingement angle distributions for more than 150µm are not dependent upon the diameter of a solid particle.

The effect of impact velocity distributions and impingement angle distributions over the overall airfoil surface can be shown in the local volumetric erosion rate distribution, Figure 5-32 and Figure 5-33. For up to 100µm, the local erosion rate is dependent upon the diameter of a sand particle. However, for greater than 150µm of sand particle diameter, the erosion characteristics including the magnitude of erosion rates and its position are not dependent upon the diameter of sand particle.
Figure 5-24. Area averaged volumetric erosion rate distributions on the pressure surface for the diameter of a solid particle at $M_0 = 0.4$ and $\alpha_0 = -0.07^\circ$.

Figure 5-25. Area averaged volumetric erosion rate distributions on the suction surface for the diameter of a solid particle at $M_0 = 0.4$ and $\alpha_0 = -0.07^\circ$. 
Figure 5-26. Area averaged volumetric erosion rate distributions on the pressure surface for the diameter of a solid particle at $M_0 = 0.7$ and $\alpha_0 = -0.07^\circ$.

Figure 5-27. Area averaged volumetric erosion rate distributions on the suction surface for the diameter of a solid particle at $M_0 = 0.7$ and $\alpha_0 = -0.07^\circ$. 
Figure 5-28. Nondimensional impact velocity distributions for the diameter of a solid particle at $M_0 = 0.4$ and $\alpha_0 = -0.07^\circ$.

Figure 5-29. Nondimensional impact velocity distributions for the diameter of a solid particle at $M_0 = 0.7$ and $\alpha_0 = -0.07^\circ$. 
Figure 5-30. Impingement angle distributions for the diameter of a solid particle at \( M_0 = 0.4 \) and \( \alpha_0 = -0.07^\circ \)

Figure 5-31. Impingement angle distributions for the diameter of a solid particle at \( M_0 = 0.7 \) and \( \alpha_0 = -0.07^\circ \)
Figure 5-32. Local volumetric erosion rate distributions for the diameter of a solid particle at $M_0 = 0.4$ and $\alpha_0 = -0.07^\circ$

Figure 5-33. Local volumetric erosion rate distributions for the diameter of a solid particle at $M_0 = 0.7$ and $\alpha_0 = -0.07^\circ$
Figure 5-34. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $M_0 = 0.7$, $\alpha_0 = -0.07^\circ$ and $D_p = 10\mu m$

Figure 5-35. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $M_0 = 0.7$, $\alpha_0 = -0.07^\circ$ and $D_p = 75\mu m$
Figure 5-36. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $M_0 = 0.7$, $\alpha_0 = -0.07^\circ$ and $D_p=150\mu m$.

Figure 5-37. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $M_0 = 0.7$, $\alpha_0 = -0.07^\circ$ and $D_p=250\mu m$. 
5.1.3. The Effect of the Inflow Angle of Attack

Finally, the effect of inflow angle of attack on solid particle trajectories and sand erosion characteristics on the 2D SC1095 airfoil was investigated. In these computations, the diameter of a solid particle \( D_p = 50 \) and \( 250 \mu m \) and inflow Mach number \( M_0 = 0.4 \) and \( 0.7 \) are fixed variables. The inflow angle of attack \( \alpha_0 = \pm 5.0, \pm 2.0, -0.07, \text{and } 0.0^\circ \) is a control variable.

Figure 5-38 and Figure 5-39 are area averaged erosion rate distributions over the airfoil for inflow angle of attack at \( M_0 = 0.4 \) and \( D_p = 50 \) & \( 250 \mu m \). Figure 5-40 and Figure 5-41 are area averaged erosion rate distributions over the airfoil for inflow angle of attack at \( M_0 = 0.7 \) and \( D_p = 50 \) & \( 250 \mu m \).

Comparing area averaged volumetric erosion rate distributions on the pressure side for various inflow angle of attacks, the erosion rate is more sharply increased over the leading edge of the airfoil as inflow angle of attack is decreased. After the erosion damage reaches maximum, the value is more steeply reduced with a decreasing inflow angle of attack. As shown in these figures, the maximum erosion rate is observed at further downstream position as the inflow angle of attack is decelerated. It is also found that the size of an erosion damaged airfoil surface is inversely proportional with inflow angle of attack. These results also show that the maximum value of erosion rate is not dependent upon inflow angle of attack (released angle of solid particle).

As shown in area averaged volumetric erosion rate distributions on the suction surface, the erosion distributions have opposite characteristics with those on the pressure side. The change rate in the increase of erosion rate distribution and that in the decrease of erosion rate distribution
over the airfoil surface are increased as inflow angle of attack is increased. It is found that the maximum erosion damage position moves to downstream as the angle of attack is increased.

While there is a little difference in the magnitude of erosion rate at the same position on the airfoil, the magnitude of maximum erosion rate and its position is slightly changed for inflow angle of attack because erosion damages over the airfoil surface are mainly observed at the front round section near the leading edge of the airfoil. Because geometric angle is suddenly changed at the round section, the effect of inflow angle of attack is reduced.

Figure 5-42 and Figure 5-43 are nondimensional impact velocity distribution over the airfoil surface for inflow angle of attack at $M_0 = 0.4$ and $D_p = 50$ & 250 $\mu m$. These results show that the impact velocity over the airfoil is a little depended upon the inflow angle of attack. The impact velocity on the suction surface is linearly increased as inflow angle of attack is increased, but the impact velocity on the pressure side is linearly decreased. The impact velocity change is caused because the stagnation point on the airfoil is also changed when inflow angle of attack changes.

Figure 5-44 and Figure 5-45 are impingement angle distributions over the airfoil surface for inflow angle of attack. As shown in these results, the impingement angle distribution is seen to be shifted from the pressure side to the suction side as the inflow angle of attack is increased due to the change of geometric angle of the airfoil surface relative to inflow. These results are also found that the impingement angle is mainly changed at less than 10 percent chord length on the pressure and suction side, which is little depended upon the inflow angle of attack.
Figure 5-38. Area averaged volumetric erosion rate distributions on the pressure surface for inflow angle of attack at $M_0 = 0.4$ and $D_p = 50\mu m$

Figure 5-39. Area averaged volumetric erosion rate distributions on the suction surface for inflow angle of attack at $M_0 = 0.4$ and $D_p = 50\mu m$
Figure 5-40. Area averaged volumetric erosion rate distributions on the pressure surface for inflow angle of attack at $M_0 = 0.4$ and $D_p = 250\mu m$.

Figure 5-41. Area averaged volumetric erosion rate distributions on the suction surface for inflow angle of attack at $M_0 = 0.4$ and $D_p = 250\mu m$. 
Figure 5-42. Nondimensional impact velocity distributions for inflow angle of attack at \( M_0 = 0.4 \) and \( D_0 = 50\mu m \)

Figure 5-43. Nondimensional impact velocity distributions for inflow angle of attack at \( M_0 = 0.4 \) and \( D_0 = 250\mu m \)
Figure 5-44. Impingement angle distributions for inflow angle of attack at \( M_0 = 0.4 \) and \( D_p = 50\mu m \)

Figure 5-45. Impingement angle distributions for inflow angle of attack at \( M_0 = 0.4 \) and \( D_p = 250\mu m \)
Figure 5-46. Local volumetric erosion rate distributions for inflow angle of attack at $M_0 = 0.4$ and $D_p = 50\mu m$

Figure 5-47. Local volumetric erosion rate distributions for inflow angle of attack at $M_0 = 0.4$ and $D_p = 250\mu m$
Figure 5-48. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours attack at $\text{M}_0 = 0.4$, $\text{D}_p = 250\mu$m and $\alpha_0 = -5.0^\circ$.

Figure 5-49. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours attack at $\text{M}_0 = 0.4$, $\text{D}_p = 250\mu$m and $\alpha_0 = -2.0^\circ$. 
Figure 5-50. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $M_0 = 0.4$, $D_p = 250\mu m$ and $\alpha_0 = -0.07^\circ$

Figure 5-51. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $M_0 = 0.4$, $D_p = 250\mu m$ and $\alpha_0 = 0.0^\circ$
Figure 5-52. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $M_0 = 0.4$, $D_p = 250\mu m$ and $\alpha_0 = 2.0^\circ$

Figure 5-53. Sand particle trajectories around a SC1095 airfoil and their velocity magnitude contours at $M_0 = 0.4$, $D_p = 250\mu m$ and $\alpha_0 = 5.0^\circ$
Chapter 6

3D Sand Particle Release Conditions and Domain Definition around a Helicopter Rotor Blade

6.1. Injection Condition for a 3D Erosion Simulation

As described in Chapter 3, the selection of a suitable injection condition is very important to predict particle trajectory and erosion on a helicopter rotor blade. Unlike 2D erosion simulations, 3D erosion simulations on helicopter rotor blades have more complicated injection conditions due to non-uniform flow conditions and rotor downwash. Moreover, in the simulation of flow around 3D rotating blades, there are no definite planes for specifying inlet boundary conditions for fluid flow and for injecting solid particles.

In the determination of injection conditions for a 3D erosion simulation, first, an injection plane must be defined. In this current study, two different planes are considered as the injection plane from which sand particles are released as shown in Figure 6-1. One is the upper plane of rotor blade disk to consider erosion damage of sand particles moving down due to downwash. The other is the rotation (side) plane of rotor blade disk in order to compute the erosion damage of solid particles suspended in the rotation area of helicopter rotor blades.

Secondly, distribution of injection points on the injection plane must be defined. All injection points are distributed uniformly over the plane in order to exclude the effect of the clustering of these injection points. As mentioned in Chapter 3.1.3, on the side plane of rotor disk, erosion characteristics can be affected by the interval size between adjacent injection points. In this study, the interval in y (height) and z (span) direction is defined as 10 times of minimal cell size of grid used in this 3D flow and erosion prediction. However, on the upper plane of rotor
disk, the interval size between two injection points in x and z direction did not highly affect on the erosion characteristics on the rotor blade. Therefore, in this study, the interval size is set as 25 times of interval size for rotation plane of disk.

Next step is about determining properties of solid particles released from injection points set on the injection plane. The properties include velocity, density, diameter, and flow rate. Density and diameter of solid particles are specified values inputted from the input data file using UDFs. The release velocity of solid particles is set as the same as the fluid velocity in the moving reference frame. Mass flow rate of solid particles defines the number of solid particles released from the injection point per unit operating time. The injection point represents the injection area.

Figure 6-1. Sand particle injection planes for erosion simulation of helicopter rotor blade

Figure 6-2 shows the injection conditions and trajectory of solid particles released from the side plane of rotor disk. The velocity and flow rate of solid particles released from the side plane of rotor disk are calculated by using the three equations as follows:
\[ \dot{m}_{p,side} = \frac{\phi}{1 - \phi} \dot{m}_{side} \]  
(55)

\[ \dot{m}_{side} = \rho V_z A_{xy} \]  
(56)

where,

\[ V_x = \Omega z + v_t \quad \text{and} \quad A_{yz} = dy \cdot dz \]  
(57)

where, \( v_t \) is induced inflow velocity (downwash velocity)

---

Figure 6-2. Injection condition and trajectory of sand particles released from the side plane of rotor disk

Figure 6-3 shows injection locations and trajectory of solid particles released from the upper plane of rotor blade disk. The injection velocity, \( V_y \), of solid particles is an induced velocity caused by an induced flow (down wash). The flow rate of each solid particle is calculated by using the three equations as follows:
\[
\dot{m}_{p,\text{upper}} = \frac{\Phi}{1 - \Phi} \dot{m}_{\text{upper}}
\]

\[
\dot{m}_{\text{upper}} = \rho V_y A_{xz}
\]

where,

\[
V_y \approx v_i \quad \text{and} \quad A_{xz} = dx \cdot dz
\]

where, \(v_i\) is induced inflow velocity (downwash velocity)

---

**Figure 6-3.** Injection condition and trajectory of sand particles released from the upper plane of rotor disk
6.2. Modification of 3D Impact Conditions

Suitable rebound and erosion equations for sand particle impacting on Ti-6A1-4V metallic surface are necessary in order to investigate exact erosion characteristics on an actual helicopter rotor blade. These models are developed based on experimental results as discussed in Chapter 2. In common experiment, rebound condition and material removal from metallic surface for various impact conditions were measured in the normal plane of the metallic surface as shown in Figure 6-4. Therefore, in this study, the normal component of velocity vector is considered during erosion damage prediction. This is reasonable because the tangential component of velocity vector does not affect the erosion damage on the metallic surface due to zero impingement angle.

Figure 6-4. A statistical impact and rebound geometry for the impact of a single solid particle from Ref. (29)
Figure 6-5. A 3D impact condition on a helicopter rotor blade
Chapter 7

Validation of 3D Flow-Only Simulation

7.1. Validation of Simulation Results for Flow-Only around 3D Rotating Blade (NACA 0012)

An accurate fluid field solution is necessary in order to calculate sand particle trajectories and erosion damage on a 3D rotating helicopter rotor blade.

In this section, 3D viscous and turbulent flows around a 3D rotating rotor blade without any sand particle injection were simulated. These results were then compared to experimental results performed by Caradonna et al. (54) in order to validate the current 3D flow simulations. Caradonna et al. (54) measured static pressure distributions at various collective pitch angles \( \theta = 0^\circ \sim 12^\circ \) and various rotor speed \( \Omega = 650 \sim 2400 \text{RPM} \) by using a hover test facility as shown in Figure 7-1.

In this study, flows around a 3D “rotating rotor” blade were calculated for two different rotational speed conditions termed as “low” and “high”. The rotor blade used in this computation is an untwisted, non-tapered and non-swept as shown in the Figure 7-2. The airfoil profile of the rotor blade is NACA 0012. The radius of rotor blade is 1.143m and the aspect ratio is 6.

The rotational speed in this experimental study is 1250 rpm corresponding to a tip Mach number equal to 0.44, at three different collective pitch angles, 5, 8, and 12 degrees. This chapter also includes computations of high speed experimental data from Caradonna et al. A computation for high rotating speed is chosen based on the tip velocity of a real UH-60A helicopter rotor blade, \( M_{\text{tip}} \approx 0.8 \). The high rotating speed condition is set as 2280 rpm of rotating blade speed and 8 degree of collective pitch angle.
Figure 7-1. Hover test facility in Army Aeromechanics Laboratory from Ref. (54)

(a) Upper view

(b) Side view

Figure 7-2. Geometry of the 3D rotor blade used in the computation and the experiment
Figure 7-3 ~ Figure 7-6 show a hybrid grid system for 3D flow validation computations. The grid system for 3D flow simulations consists of two main parts - main blade part and outfield part.

In the main blade part (near blade section), multi-block H-type grid system is used to model the computational domain. As discussed in Chapter 4 and Appendix B, an optimized 2D grid system, a $280 \times 80$ multi-block H-type structured grid system, is stacked uniformly in the span direction as shown in Figure 7-4. The structured grid systems (the total cell number for main blade part domain : 3.5 millions) usually have merit to reduce the computational cost and to capture wake and shocks generated on the blade surface.

Next, the computational domain around the tip of rotor blade is also meshed by using a hybrid grid system as shown in Figure 7-5. The expansion section of the tip domain is meshed by using H-type structured grid system and the expansion part from the tip surface of the rotor blade is meshed by a Hex unstructured grid system – in order to capture circulatory flow around the tip of the rotor blade and tip vortices emanating from the tip.

Finally, in this outfield grid system, Hex unstructured grids (the total cell number : 2.5 millions) are used to reduce required grid cell number and computational time as shown in the Figure 7-6.
Figure 7-4. Grid system for main blade part (near the rotor blade)

Figure 7-5 Grid system for the domain of tip section

Figure 7-6. 3D grid system for validation computations (Outfield part)
In this study, overall computations for predicting aerodynamics and erosion characteristics on 3D rotating blades were performed by using 8 × 2 CPU parallel computer, which spent 30 hours CPU times.

Figure 7-7 ~ Figure 7-11 show pressure coefficient distributions numerically obtained in this study and comparison to experimental results at r/R = 0.50, 0.68, 0.80 0.89 and 0.96 for a pitch angle of 5 degree. As shown in these results, the flow near the leading edge on the suction surface is suddenly accelerated, the velocity of flow reaches maximum magnitude at about 5 percent chord length, and the flow is then eventually decelerated over airfoil surface. On the pressure surface, the flow is accelerated gradually up to 20 percent chord length and the flow is slowly decelerated. The flow characteristics over the span of the rotor blade are similar.

A comparison of negative pressure coefficient (near leading edge) at r/R = 0.89 and r/R = 0.96 show that the local velocity on the suction side of the tip section is somewhat smaller than that of the mid-section of the rotor blade because of the circulatory flow characteristics around the tip as shown in Figure 7-14.

As shown in Figure 7-15, the maximum value of the negative pressure coefficient on suction side is increased as the position of the cross section approaches to the tip of the rotor blade because the blade velocity, \( V = \Omega \cdot r \), is linearly proportional with the radius of the cross section. However, the maximum negative pressure coefficient is suddenly dropped near the tip of the rotor blade, which is caused by the circulatory flow generated at the tip of the rotor blade.

These results indicate that the flow-only computational results obtained in this study are in good agreement with experimental results although there are small differences between computationally simulated pressure distributions obtained in this study and pressure distributions measured by Caradonna and Tung. It is found that 3D computations performed in this study can reasonably predict flow-only characteristics around rotating 3D rotor blade.
Figure 7-7. Negative pressure coefficient distributions at $r/R = 0.50$ for $\theta = 5^\circ$ and $\Omega=1250\text{RPM}$ ($M_{\text{ip}} = 0.44$)

Figure 7-8. Negative pressure coefficient distributions at $r/R = 0.68$ for $\theta = 5^\circ$ and $\Omega=1250\text{RPM}$ ($M_{\text{ip}} = 0.44$)

Figure 7-9. Negative pressure coefficient distributions at $r/R = 0.80$ for $\theta = 5^\circ$ and $\Omega=1250\text{RPM}$ ($M_{\text{ip}} = 0.44$)
Figure 7-10. Negative pressure coefficient distributions at $r/R = 0.89$ for $\theta = 5^\circ$ and $\Omega=1250\text{RPM}$ ($M_{tip} = 0.44$)

Figure 7-11. Negative pressure coefficient distributions at $r/R = 0.96$ for $\theta = 5^\circ$ and $\Omega=1250\text{RPM}$ ($M_{tip} = 0.44$)
Figure 7-12. Velocity magnitude contours over rotor blade for $\theta = 5^\circ$ and $\Omega = 1250$RPM ($M_{tip} = 0.44$)

Figure 7-13. Velocity magnitude contours near leading edge over rotor blade for $\theta = 5^\circ$ and $\Omega = 1250$RPM ($M_{tip} = 0.44$)
Figure 7-14. “Tip-loss” effect at the blade tip caused by circulatory flow (tip vortex)

Figure 7-15. Maximum negative pressure coefficient versus radial position for $\theta = 5^\circ$ and $\Omega = 1250\text{RPM}$ ($M_{tip} = 0.44$)
Comparisons between numerical solutions at various collective pitch angles and corresponding experimental results are required in order to validate the accuracy of flow-only solution.

In this section, the results for collective pitch angles of 8° and 12° will be discussed. It is well known that the flow velocity over the suction side of the airfoil is increased and the velocity of the flow near the pressure side of the airfoil is typically decreased as the flow angle of attack is increased, while the lift generated at the airfoil is increased. This fact can be observed in Figure 7-16 ~ Figure 7-20.

Figure 7-16 ~ Figure 7-20 show negative pressure coefficient distributions for the collective pitch angle of 8°. Negative pressure coefficient distributions at collective pitch angle of 8° have similar patterns with those at 5 degree of collective pitch angle. However, the magnitudes of the negative pressure coefficient on the suction side for collective pitch angle of 8° is greater than those for 5°, and the magnitude of negative pressure coefficient on the pressure side for 8° is smaller than those for 5°. Moreover, the maximum value of the negative pressure coefficient on suction side is increased as the position of the cross section is much closer to the tip of the rotor blade. A change rate of maximum negative pressure coefficient with respect to radius of the blade section for 8 degree is greater than that of collective pitch angle of 5 degree. The magnitude of the negative pressure coefficient over the airfoil is slightly reduced due to circulatory flow at the tip of the rotor blade near the tip section of rotor blade, r/R = 0.96,

These results indicate that the computational results performed in this study are in good agreement with experimental results although there are small differences between simulated pressure distributions obtained in this study and pressure distributions measured by Caradonna and Tung.
Figure 7-16. Negative pressure coefficient distribution at $r/R = 0.50$ for $\theta = 8^\circ$ and $\Omega = 1250$ RPM ($M_{tip} = 0.44$)

Figure 7-17. Negative pressure coefficient distribution at $r/R = 0.68$ for $\theta = 8^\circ$ and $\Omega = 1250$ RPM ($M_{tip} = 0.44$)

Figure 7-18. Negative pressure coefficient distribution at $r/R = 0.80$ for $\theta = 8^\circ$ and $\Omega = 1250$ RPM ($M_{tip} = 0.44$)
Figure 7-19. Negative pressure coefficient distribution at \( r/R = 0.89 \) for \( \theta = 8^\circ \) and \( \Omega = 1250 \text{RPM} \) \( (M_{in} = 0.44) \)

Figure 7-20. Negative pressure coefficient distribution at \( r/R = 0.96 \) for \( \theta = 8^\circ \) and \( \Omega = 1250 \text{RPM} \) \( (M_{in} = 0.44) \)
Figure 7-21 ~ Figure 7-25 show negative pressure coefficient distributions for the collective pitch angle of 12 degree. When these results are compared to those with smaller collective pitch angles ($\Theta = 5^\circ$ and $8^\circ$), the resulting flow condition is somewhat different. The negative pressure coefficient reaches a maximum value near the leading edge on the suction side, and the pressure coefficient is reduced at a faster rate over the suction surface. The negative pressure coefficient increases at a much slower rate over the pressure side. The coefficients have a maximum value at about 40 percent chord length on pressure side, and the coefficient is eventually decreased. This is caused due to a separated flow which starts at a location closer to the leading edge of the airfoil when the angle of attack is high as shown in the Figure 7-26 ~ Figure 7-29.

These results also indicated that the numerical results for high collective pitch angle obtained in this current study are in very good agreement with experimental results.
Figure 7-22. Negative pressure coefficient distribution at r/R = 0.68 for $\theta = 12^\circ$ and $\Omega=1250\text{RPM}$ ($\text{M}_{\infty} = 0.44$)

Figure 7-23. Negative pressure coefficient distribution at r/R = 0.80 for $\theta = 12^\circ$ and $\Omega=1250\text{RPM}$ ($\text{M}_{\infty} = 0.44$)
Figure 7-24. Negative pressure coefficient distribution at $r/R = 0.89$ for $\theta = 12^\circ$ and $\Omega = 1250\text{RPM} \ (M_{ip} = 0.44)$

Figure 7-25. Negative pressure coefficient distribution at $r/R = 0.96$ for $\theta = 12^\circ$ and $\Omega = 1250\text{RPM} \ (M_{ip} = 0.44)$
Figure 7-26. Velocity magnitude contours over rotor blade for $\theta = 8^\circ$ and $\Omega = 1250\text{RPM}$ ($M_{tip} = 0.44$)

Figure 7-27. Velocity magnitude contours near leading edge over rotor blade for $\theta = 8^\circ$ and $\Omega = 1250\text{RPM}$ ($M_{tip} = 0.44$)
Figure 7-28. Velocity magnitude contours over rotor blade for $\theta = 12^\circ$ and $\Omega = 1250$RPM ($M_{tip} = 0.44$)

Figure 7-29. Velocity magnitude contours near leading edge over rotor blade for $\theta = 12^\circ$ and $\Omega = 1250$RPM ($M_{tip} = 0.44$)
The final validation case is with a high rotating speed of the rotor. As mentioned previously, a high rotating speed is chosen based on the tip velocity of a real UH-60A helicopter rotor blade, \( M_{tip} = 0.8 \). The rotational speed condition is set at 2280 rpm and 8 degree of collective pitch angle.

Unlike the flow at low rotational speed, the flow conditions at a high rotational speed are varied strongly over the spanwise direction as shown in Figure 7-32 ~ Figure 7-34. In especial, the flow on the suction surface is more accelerated with increasing inflow velocity and increasing inflow angle of attack. For \( r < 0.700R \), the flow on the suction side is subsonic. The negative pressure coefficient distribution at \( r = 0.500R \) is similar to that of \( \Omega = 1250\text{RPM} \). For \( r > 0.800R \), the flow on the suction side is changed to supersonic flow. A sudden drop in negative pressure coefficient on the suction surface is observed. This is caused by the shock-boundary layer interaction as shown in Figure 7-37 ~ Figure 7-39. As shown in these results, shock occurs at more downstream location and its strength is more weakened with increasing radius of the blade section. This is because the aerodynamic losses and a decrease in inflow angle of attack which is caused by the circulatory flows near the tip of the rotor blade. Strong separated flow on the suction side at \( r = 0.800R \) is also observed after the shock as shown in the Figure 7-37. The separated flow is also more weakened with increasing radius because of a decrease in shock strength as shown in Figure 7-38 and Figure 7-39.

These results indicate the two results are in good agreement although there is small difference in pressure distribution near the shock between numerical results and experimental results. It is found that computations performed in this study can predict reasonable flow-only condition around the 3D rotating rotor blade.
Figure 7-30. Negative pressure coefficient distribution at \( r/R = 0.50 \) for \( \theta = 12^\circ \) and \( \Omega = 2280 \text{RPM} \) (\( M_{ip} = 0.80 \)).

Figure 7-31. Negative pressure coefficient distribution at \( r/R = 0.68 \) for \( \theta = 12^\circ \) and \( \Omega = 2280 \text{RPM} \) (\( M_{ip} = 0.80 \)).

Figure 7-32. Negative pressure coefficient distribution at \( r/R = 0.80 \) for \( \theta = 12^\circ \) and \( \Omega = 2280 \text{RPM} \) (\( M_{ip} = 0.80 \)).
Figure 7-33. Negative pressure coefficient distribution at $r/R = 0.89$ for $\theta = 12^\circ$ and $\Omega = 2280$ RPM ($M_{\text{tip}} = 0.80$)

Figure 7-34. Negative pressure coefficient distribution at $r/R = 0.96$ for $\theta = 12^\circ$ and $\Omega = 2280$ RPM ($M_{\text{tip}} = 0.80$)
Figure 7-35. Velocity magnitude contours over rotor blade for $\theta = 12^\circ$ and $\Omega = 2280$RPM ($M_{tip} = 0.80$)

Figure 7-36. Velocity magnitude contours near leading edge over rotor blade for $\theta = 12^\circ$ and $\Omega = 2280$RPM ($M_{tip} = 0.80$)
Figure 7-37. Velocity magnitude contours on the suction surface at $r/R = 0.96$ for $\theta = 12^\circ$ and $\Omega = 2280$RPM ($M_{tip} = 0.80$)

Figure 7-38. Velocity magnitude contours on the suction surface at $r/R = 0.89$ for $\theta = 12^\circ$ and $\Omega = 2280$RPM ($M_{tip} = 0.80$)

Figure 7-39. Velocity magnitude contours on the suction surface at $r/R = 0.80$ for $\theta = 12^\circ$ and $\Omega = 2280$RPM ($M_{tip} = 0.80$)
Chapter 8

Particle Tracking and Erosion Damage Prediction for a 3D Rotating Rotor Blade at Hover Condition (SC1095)

As shown in Chapter 2, local erosion damage due to the impact of a single solid particle is dependent upon impact conditions such as the impact velocity, impingement angle, and particle diameter. It is also found that the impact conditions of solid particles are highly dependent upon the flow conditions within the computational domain including of inlet boundary conditions and release conditions of solid particles. Therefore, an erosion damage distribution over a 2D airfoil surface are affected by inflow Mach number, inflow angle of attack (the direction of the released solid particles), and particle diameter as discussed in Chapter 5. These results also indicated that the relative flow condition with respect to the cross section in the span of a 3D rotating rotor blade is very important for sand particle trajectories and erosion characteristics on the rotor blade.

It is also known that relative flow conditions over a 3D rotating helicopter rotor blade vary with operating conditions such as hover, vertical flight, or forward flight. Moreover, most helicopter rotor blades have complicated geometric conditions such as spanwise twist, swept tip or tapered tip, and non-uniform airfoil contours. These geometric conditions results in highly complicated flow conditions, which affect solid particle trajectory and erosion characteristics. It is challenging to analyze the erosion characteristics of an actual helicopter rotor blade by using particle tracking and sand erosion damage prediction on the rotor blade.

In this current study, four different 3D rotor blades were designed based on real helicopter rotor blade geometric conditions. Comprehensive 3D computational analysis is presented using well known helicopter blade airfoil SC1095.
It is well known that most erosion damage on an actual helicopter rotor blades occur during low altitude flight condition including hover flight condition. In this study, a hover flight condition is chosen as a reference flight condition. Therefore, flow and erosion damage simulations for four different rotor blades at hover condition were performed. The geometric definitions and the 3D grid system for this actual hover simulation are provided in Section 8.1. Section 8.2 explains the flow and erosion simulations for the actual rotor blade system defined in the previous paragraph.

8.1. Geometric Conditions and Grid System for 3D Uniform Rotor Blades

In this simulation, a spanwise twist angle distribution and swept tip are chosen as control parameters. Four different rotor blades – “a non-twist and non-swept rotor blade”, “a linear-twist and non-swept rotor blade”, “a non-twist and swept rotor blade”, and “a linear twist and swept rotor blade” – are used in the current computational simulation.

These rotor blades are designed based on an actual main rotor blade of UH-60A helicopter rotor blade. The helicopter rotor blade of UH-60A consists mainly of two different airfoils, SC1095 and SC1094R8. The rotor blade platform of UH-60A has non-uniform and non-linear twist. However, in this simulation, the airfoil contour over the rotor blade planform is fixed in order to exclude the effect of airfoil profile distribution. A SC1095 airfoil is thus chosen as main the airfoil shape. The radius of the rotor blade is fixed as $R = 8.1788 \text{ m (322 inch)}$. The chord length of airfoil section is also constant, $c = 0.527304 \text{ m (20.76 inch)}$. Figure 8-1 shows geometric condition of four different rotor blades.
Figure 8-1. Geometric condition for four different rotor blades
In this section, only two types of the spanwise twist distribution, non-twist and \(-16^\circ\) linear-twist, are considered. For a swept, the starting position of swept is the same as that for a UH-60A main rotor blade. \(r_{0, \text{swept}} = 0.9286R\) and the angle is also fixed as \(\Lambda = 20^\circ\).

All grid systems for flow simulation and erosion damage prediction on the 3D rotor blade are made based on the grid system validated in the Chapter 7. Figure 8-2 ~ Figure 8-7 show the hybrid grid system for 3D erosion simulations. These grid systems also consist of two main parts - main blade part and outfield part as shown in Figure 8-7.

In the main blade part, multi-block H-type structure grid system is used to model computational domains for the cross section in the spanwise direction as shown in Figure 8-2. The H-type structured grid system is meshed based on the validated grid system for 2D airfoil flow around the airfoil validated in Chapter 4. The structured grid systems are rotated with the rotor blade twist angle and stacked in the spanwise direction as shown in Figure 8-3. The structured grid systems have merit to reduce the computational cost. They can also effectively capture wake and shocks generated on the blade surface. In the domain near the tip of a rotor blade, a hybrid grid system is used as shown in the Figure 8-4. The expansion section of the tip domain is meshed by using H-type structured grid system and the expansion part from the tip surface of the rotor blade is meshed by a triangular unstructured grid system. This unstructured grid system can effectively capture more clearly circulation flow observed around the tip of a rotor blade and tip vortices generated from the tip. In this outfield grid system, triangular unstructured grids are used to reduce required grid cell number and computational time as shown in Figure 8-7.
Figure 8-2. H-type grid system for the cross section of a rotor blade

Figure 8-3. Grid system in the span direction of a rotor blade

Figure 8-4. Grid system of the domain near the tip of rotor blade
Figure 8-5. Grid system for computation domain around rotor blade (Main part: non-twist and non-swept rotor blade)

Figure 8-6. Grid system of computation domain around rotor blade (Main part: -16° linear-twist and swept rotor blade)

Figure 8-7. Grid system for computation domain of a outside part
8.2. Flow and Erosion Rate Prediction Results for 3D Uniform Rotor Blades

8.2.1. Flow and Erosion Characteristics for the Baseline Rotor Blade

In this section, a non-twist & non-swept rotor blade is selected as the baseline planform for a 3D uniform rotor blade. The simulation for flow and erosion damage for this baseline rotor blade was performed in order to determine reference flow and erosion characteristics. As mentioned in previous parts, an airfoil profile for this rotor blade is SC1095 airfoil with constant chord length.

Figure 8-9 and Figure 8-10 show relative velocity contours around the baseline rotor blade at zero degree of collective pitch angle. SC1095 airfoil contour used in this rotor blade is almost symmetric to the mean chord line and its zero-lift angle of attack is -0.07 degree. The lift force generated at zero degree of angle of attack is thus very small. As shown in these figures, the relative velocity contours around the cross section of the rotor blade (blade section) show a symmetric pattern. There figures also show that the flow characteristics in the spanwise direction of the rotor blade are quite similar expect for the velocity magnitude over the blade section. The magnitude of velocity is slightly increased with an increasing radial position on blade section.

Figure 8-11 ~ Figure 8-14 present negative pressure coefficient distributions at \( r = 0.575R, 0.725R, 0.900R, \) and \( 0.965R \) for the baseline rotor blade. As shown in negative pressure coefficient distributions on the suction side, the coefficient is suddenly increased near the leading edge and the coefficient reaches a maximum value at approximately 5 percent chord length \( (x = 0.05c) \). After \( x = 0.05c \) on the suction side, the pressure coefficient is slowly reduced. On the pressure surface, the negative pressure coefficient reaches a maximum at \( x = 0.05c \), the
coefficient is quickly reduced up to $x = 0.10c$, and then the coefficient is slowly reduced over the airfoil surface. It is found that, for less than approximately $x = 0.10c$, the negative pressure coefficient on the pressure side is greater than that on the suction side. This feature generates negative lift force on this section. This is caused by negative relative inflow angle of attack with respect to the blade section. Though local pitch angle at each blade section is zero degree, the relative inflow angle of attack to the rotor blade is reduced due to the downwash as shown in Figure 8-16. Pressure coefficient distributions on the blade section are similar in the spanwise direction of the rotor blade. This observation indicates that the velocity magnitude of relative inflow with respect to blade section is linearly proportional with respect to radial position on the blade section.

Figure 8-15 shows average relative inflow angle distribution in the spanwise direction of the rotor blade. As shown in the figure, relative in-plane velocity is linearly increased with increasing radius of the blade section, $V_F = \Omega r$.

In a blade element momentum theory, the local induced inflow ratio for a non-twisted rotor blade is linearly proportional with the radius of the span section. The numerical results about the local induced inflow ratio for a non-twist rotor blade indicate that the inflow ratio is also linearly increased with increasing radius up to 80 percent span length ($r = 0.80R$), but the induced inflow ratio is suddenly dropped near the rotor blade tip due to circulatory flow (tip vortex) as shown in Figure 8-19. Moreover, the magnitude of induced inflow ratio is very small over the span of the rotor blade because the lift force generated on the rotor blade is very small.

Figure 8-17 is a relative inflow angle distribution over the span of a baseline rotor blade. It is found that from $r = 0.3R$ to $r = 0.8R$, the relative inflow angle is linearly increased as the radius of the blade section is increased. At greater than $r = 0.8R$, the inflow angle is slightly reduced. Between $r = 0.3R$ and $r = 0.8R$, the deviation in the relative inflow angle in the spanwise direction is very small, so the relative inflow angle can be considered a constant.
Figure 8-18 is the aerodynamic angle of attack (aerodynamic AoA) distribution in the spanwise direction of the baseline rotor blade. Though pitch angles to blade section over the span are set to be zero degree ($\theta_c = 0.0^\circ$), blade section at less than $r = 0.20R$ is operated at positive aerodynamic AoA and the aerodynamic AoA of the blade section at great than $r = 0.20R$ is a small negative angle. This decrease in aerodynamic AoA is caused due to the induced inflow (down wash). The effective angle distribution is very important because flow and erosion characteristics for each rotor blade are highly dependent upon the effective angle. However, the aerodynamic AoA over the span can be considered to be constant, $\alpha_{\text{eff}} \approx -0.25^\circ$. Almost constant angle of attack introduces similar flow condition over the span of a rotor blade as shown in Figure 8-9 and Figure 8-10.

Figure 8-8. Distribution of inflow for an ideally twisted blade as predicted by the BEMT compared to an untwisted blade from Ref. (55)
Figure 8-9. Relative velocity contours around a baseline rotor blade

Figure 8-10. Relative velocity contours around the tip of a baseline rotor blade
Figure 8-11. Negative pressure coefficient distribution at $r = 0.575R$ for a baseline rotor blade

Figure 8-12. Negative pressure coefficient distribution at $r = 0.725R$ for a baseline rotor blade
Figure 8-13. Negative pressure coefficient distribution at $r = 0.900R$ for a baseline rotor blade.

Figure 8-14. Negative pressure coefficient distribution at $r = 0.965R$ for a baseline rotor blade.
Figure 8-15. Relative in-plane velocity distribution at the front plane of a rotor blade in the span direction of a baseline rotor blade

Figure 8-16. Local induced inflow ratio distribution at the front plane of a rotor blade in the span direction of a baseline rotor blade
Figure 8-17. Relative inflow angle distribution at the front plane of a rotor blade in the span direction of a baseline rotor blade.

Figure 8-18. Aerodynamic (effective) angle distribution at the front plane of a rotor blade in the span direction of a baseline rotor blade.
Figure 8-19. Velocity vectors near the tip of a baseline rotor blade at \( x = 0.5c \) in terms of magnitude of \( V_y \).

Figure 8-20 ~ Figure 8-22 show area averaged volumetric erosion contours on the surfaces of a baseline rotor blade that is “non-twist” & “non-swept” rotor blade. As shown in these results, the magnitude of volumetric erosion rate is increased with radius of blade section in the span direction because the magnitude of relative inflow velocity is almost linearly proportional with blade radius. The maximum erosion damage is observed at about \( r = 0.975R \). The magnitude of erosion damage is slightly decreased when \( r > 0.975R \) in spanwise direction.

The erosion rate distribution in the chord-wise direction is shown in Figure 8-23. The maximum erosion damage at each blade section is observed at approximately 2 percent chord length (\( x = 0.02c \)), which is good agreement with the erosion results obtained for a 2D airfoil as presented in Chapter 5. These results also show that erosion damaged area on the blade section is similar in the spanwise direction because of nearly constant aerodynamic AoA (effective AoA) as shown in Figure 8-18. This result also verifies the effect of inflow angle of attack on erosion.
characteristics presented on the airfoil. The size of erosion damaged area is highly depended upon the inflow angle of attack. Figure 8-23 is area averaged volumetric erosion rate distribution over the blade section from \( r = 0.500R \) to \( r = 0.975R \) for a baseline rotor blade. As shown in overall erosion rate distribution on the pressure surface of the blade section, the magnitude of erosion rate suddenly increases up to 1.0 percent chord length \( (x = 0.010c) \), and then the erosion rate steeply drops before \( x = 0.05c \). As shown in erosion rate distribution on the suction surface, the magnitude of erosion rate is greater than that of the pressure side. The maximum erosion damage occurs at about \( x = 0.015c \).

The erosion patterns at each blade section are almost similar because the aerodynamic AoA can be considered to be constant as shown in Figure 8-18. The magnitude of erosion rate is linearly proportional with a radius of blade section, which is caused by an increasing in-plane velocity magnitude as shown in Figure 8-15.

![Figure 8-20. Area averaged volumetric erosion rate contours on the suction side of a baseline rotor blade \( \text{[m}^3/\text{m}^2\text{s}] \)](image-url)
Figure 8-21. Area averaged volumetric erosion rate contours on the pressure side of a baseline rotor blade [m³/m²s]

Figure 8-22. Area averaged volumetric erosion rate contours on the leading edge of a baseline rotor blade [m³/m²s]
Figure 8-23. Area averaged volumetric erosion rate distribution on each blade sections of a baseline rotor blade

Figure 8-24. Trajectories of solid particles released from the upper disk plane in terms of relative velocity magnitude
8.2.2. The Effect of Rotor Blade Twist on Flow and Erosion Characteristics

It is well known that the application of spanwise twist on helicopter rotor blade can reduce the variation of local lift force distribution and induced inflow (down wash) distribution over the span of the rotor blade. The spanwise twist also helps to reduce the induced power as shown in Figure 8-25 and Figure 8-26. Therefore, many helicopter rotor blades were designed with some amount of linear twist because the proper blade twist can significantly improve the figure of merit in hover flight condition (56). However, when the rotor blade with a high negative spanwise twist is operated at forward flight condition, relative aerodynamic angles of attack are highly reduced. This results in the reduction of lift or even negative lift production on the advancing blade tip. The distortion of lift force near the tip of an advancing rotor blade causes the loss of rotor thrust and propulsive force. Therefore, a nonlinear twist or double linear blade twist is used in most rotor blade shapes.

In general studies regarding to helicopter aerodynamics at hover flight condition, the effect of a linear rotor twist on aerodynamic and performance characteristics is similar to that of a non-linear rotor twist. Therefore, in the analysis for the effect of rotor blade twist on aerodynamic and erosion characteristics, only a linear spanwise rotor twist was considered. As mentioned in the previous part, -16 degree of a twist angle is chosen as a reference twist angle. In this part, the computations for a linear-twist & non-swept rotor blade were performed. These results were then compared to the result for a baseline planform of a rotor blade.
Figure 8-25. Local induced inflow ratio distributions on a rotor blade for different linear twist angle rate predicted by blade element momentum theory from Ref. (55)

Figure 8-26. Local lift coefficient distributions on a rotor blade for different linear twist angle rate predicted by blade element momentum theory from Ref. (55)
Figure 8-27 ~ Figure 8-28 are relative velocity contours around a linear-twist & non-swept rotor blade at zero degree of collective pitch angle. Though relative inflow velocity is increased with an increasing radial position of blade section, local pitch angle at blade section for a linear-twist & non-swept rotor blade is decreased as the radius of blade section is increased. This results in similar velocity contours over the suction surface at \( r > 0.725R \). However, the maximum velocity near the leading edge on the pressure surface is linearly proportional with the radius of blade section because the inflow angle of attack is also increased as the radius of blade section is increased.

Figure 8-29 shows negative pressure coefficient distributions over the blade section at \( r = 0.527R \). As shown in this figure, the magnitude of negative pressure coefficient on the suction surface for a linear-twist rotor blade is greater than that of the baseline rotor blade. While the negative lift force for a non-twist & non-swept rotor blade is generated on the blade section at less than \( x < 0.1c \), the positive lift force for a linear-twist & non-swept rotor blade is generated over the whole blade section due to positive inflow angle. Negative pressure coefficient distribution at \( r = 0.725R \) for a linear-twist & non-swept rotor blade is very much similar to the distribution for a baseline rotor blade as shown in Figure 8-30. This result is agreement with the results obtained by Gessow et al. They concluded that a “linear-twist” rotor blade has the same thrust coefficient as the one of constant pitch when the collective pitch angle for a non-twist rotor blade is set to the pitch angle for the twisted blade defined at 3/4-radius (57).

As shown in Figure 8-31 and Figure 8-32, at \( r > 0.900R \), the pressure coefficients are much greater over the suction side and the pressure coefficients on the pressure side are smaller. This observed characteristics increases negative lift force on the blade section. These results also indicated that negative lift force is reduced near the tip of a linear-twist rotor blade due to the circulatory flow of which rotational direction is opposite to that of a non-twist rotor blade as shown in Figure 8-37.
Figure 8-33 shows average relative inflow velocity distribution in the spanwise direction. This result shows that relative inflow velocity for a linear & non-twist rotor blade is also linearly proportional with the radius of the blade section location. However, a reduction in velocity is observed from \( r = 0.300R \) to \( r = 0.900R \). This may be caused by high static pressure near the leading edge of the rotor blade. Figure 8-34 is a local induced inflow ratio distribution over the span of the rotor blade. In a blade element momentum theory as shown in Figure 8-25, the local induced inflow ratio for a linear-twist rotor blade is suddenly increased and the increase rate of induced inflow ratio is also decreased with increasing radius. As shown in the computational result for a -16 degree linear-twist & non-swept rotor blade, the local induced inflow ratio increases up to approximately \( r = 0.400R \), and then the induced inflow ratio suddenly drops. It is also observed that the local induced flow ratio for a linear-twist & non-swept rotor blade is greater than that of a non-twist & non-swept rotor blade for \( r < 0.700R \). While for \( r > 0.700R \), the local induced inflow ratio of a linear-twist and non-swept rotor blade is smaller. It is also found that negative induced inflow ratio for a linear-twist rotor blade is generated on the section at \( r = 0.700R \).

While a baseline rotor blade (non-twist & non-swept rotor blade) generates small positive induced inflow (down wash) over the span of the rotor blade, a linear-twist rotor blade produces high positive induce inflow up to \( r = 0.7R \). However, at greater than \( r = 0.7R \), an induced inflow is changed to opposite direction due to negative aerodynamic AoA as shown in Figure 8-34. The induced inflow ratio distribution affects relative inflow angle distribution over the span of a rotor blade as shown in Figure 8-35. Figure 8-36 shows aerodynamic AoA distributions in the spanwise direction. The aerodynamic AoA with respect to the leading edge of a linear-twist and non-swept rotor blade is linearly decreased as the radius of the blade section is increased. Moreover, at greater than \( r = 0.7R \), the change rate in the aerodynamic AoA is also reduced due to the circulatory flow as shown in Figure 8-37.
These results indicate that local lift force generated on the blade section results in induced inflow. The induced inflow changes aerodynamic AoA and resultant velocity magnitude, which influences the erosion characteristics on the rotor blade. It is also found that the slope of aerodynamic AoA with respect to radial position is smaller than that of pitch angle due to induced inflow.

Figure 8-27. Relative velocity contours around a linear-twist & non-swept rotor blade

Figure 8-28. Relative velocity contours around the tip of a linear-twist & non-swept rotor blade
Figure 8-29. Negative pressure coefficient distribution at $r = 0.575R$ for a non-twist & non-swept rotor blade.

Figure 8-30. Negative pressure coefficient distribution at $r = 0.725R$ for a non-twist & non-swept rotor blade.
Figure 8-31. Negative pressure coefficient distribution at $r = 0.900R$ for a non-twist & non-swept rotor blade

Figure 8-32. Negative pressure coefficient distribution at $r = 0.965R$ for a non-twist & non-swept rotor blade
Figure 8-33. Average relative inflow angle distributions at the front plane of a rotor blade in the span direction of a non-twist and non-swept rotor blade.

Figure 8-34. Average local induced inflow ratio distributions at the front plane of a rotor blade in the span direction of a non-twist and non-swept rotor blade.
Figure 8-35. Average local induced inflow ratio distributions at the front plane of a rotor blade in the span direction of a non-twist and non-swept rotor blade.

Figure 8-36. Average relative inflow angle distributions at the front plane of a rotor blade in the span direction of a non-twist and non-swept rotor blade.
Figure 8-37. Velocity vectors around the tip of a linear-twist & non-swept rotor blade at $x = 0.5c$ in terms of magnitude of $V_y$

From Figure 8-38 to Figure 8-40 show area averaged volumetric erosion contours on the surfaces of a linear-twist & non-swept rotor blade. Likely to the erosion result for a baseline rotor blade, the magnitude of volumetric erosion rate for a linear-twist & non-swept is increased with increased radius in the spanwise direction. This is because the magnitude of relative inflow velocity is linearly proportional with the radius of blade section. Moreover, a little reduction in the erosion damage at greater than $r = 0.975R$ is observed. This is caused by tip vortex. In the erosion rate distribution at each blade section, the maximum erosion damage on pressure and suction surface at each blade section occurs at approximately 2 percent chord length ($x = 0.02c$).

As shown in Figure 8-36, the relative inflow angle of attack (aerodynamic AoA) distribution for a linear-twist & non-swept rotor blade linearly decreases with increasing radius. The relative inflow angle of attack for a baseline rotor blade is almost constant.
Figure 8-40 and Figure 8-41 show area averaged volumetric erosion rate distributions over the blade section between mid-section and blade tip for a linear-twist & non-swept rotor blade and a baseline rotor blade. When $r < 0.800R$, aerodynamic AoA of a linear-twist and non-swept rotor blade is greater than that of a baseline rotor blade. On the suction side of blade section, the magnitude of erosion rate of a linear-twist rotor blade is a little greater than that of a non-twist rotor blade and the extent of erosion damage area of a linear-twist rotor blade is widened. On the pressure side of blade section, erosion rate of a linear-twist rotor blade is smaller and the erosion damage area of this rotor blade is also smaller than those of a non-twist rotor blade. When $r > 0.800R$, aerodynamic AoA of a baseline rotor blade is greater than that of a linear-twist & non-swept rotor blade. The magnitude of erosion rate and the extent of erosion damage area on the suction side of a baseline rotor blade is greater than those of a linear-twist & non-swept rotor blade. However, on the pressure side, the magnitude of erosion rate and the extent of erosion damage area on the suction side of a linear-twist & non-swept rotor blade is greater.

These differences in erosion rate distribution for a linear-twist & non-swept rotor blade and a baseline rotor blade are caused by aerodynamic AoA distributions. As presented in the results for erosion characteristics for an inflow angle of attack, as shown in the Chapter 5.1.3, the magnitude of erosion rate on the suction surface is increased and the size of erosion damaged section on suction side is more widened as the inflow angle of attack is increased.
Figure 8-38. Relative velocity contours around a linear-twist & non-swept rotor blade

Figure 8-39. Relative velocity contours around a linear-twist & non-swept rotor blade
Figure 8-40. Area averaged volumetric erosion rate distribution on each blade sections of a baseline rotor blade

Figure 8-41. Area averaged volumetric erosion rate distribution on each blade sections of a baseline rotor blade
8.2.3. The Effect of Rotor Blade Sweep on Flow and Erosion Characteristics

It is well known that a rotor blade with swept tip has inherent advantage of high forward flight speed flight. The sweep of a rotor blade can reduce the relative inflow velocity near the leading edge, thus the sweep allows much higher forward speed flight with small compressibility effect which causes an increase in sectional drag and required power of a helicopter. However, the use of rotor blade sweep change the tip vortex characteristics, which can increase induced. Figure 8-42 shows the results of an optimization study for minimizing induced drag by using three different swept tip geometries.

In the analysis of the effect of swept tip on aerodynamic and erosion characteristics, the result for a non-twisted and swept rotor blade were compared with the result for a baseline planform of a rotor blade. As mentioned in the previous section, 20 degrees of sweep angle is chosen as a reference sweep angle.

Figure 8-42. Equivalent lift-to-drag ratios for a rotor with different tip shapes from Ref. (58)
Figure 8-27 ~ Figure 8-28 show relative velocity contours around a non-twist & swept rotor blade at zero degrees of a collective pitch angle. As shown in Chapter 8.1, the non-twist & swept rotor blade has the same profile as the profile for a non-twist & non-swept rotor blade. Therefore, flow conditions over the blade section for $r < 0.9286R$ are similar to those of a baseline (non-twist & non-swept) rotor blade. However, for $r > 0.9286R$, the magnitude of velocity over the cross section is reduced because the relative velocity near the leading edge of the rotor blade is decreased. The flow conditions near the tip of the rotor blade are also affected by tip vortex as shown in Figure 8-45. As shown in Figure 8-46, additional vortices are observed along the swept tip section. Tip vortices affect relative inflow conditions to the blade section between $r = 0.925R$ and $r = 1.000R$.

Figure 8-46 ~ Figure 8-49 show negative pressure coefficient distributions at $r = 0.575R$, $0.725R$, $0.900R$, and $0.965R$ for a non-twist & swept rotor blade. For $r < 0.925R$, pressure coefficient distribution for a swept rotor blade is the same as that of a baseline rotor blade. These figures also show that relative inflow characteristics do not change by swept tip implementation (d by a swept tip of rotor blade. However, for $r > 0.925R$, the swept rotor blade affects pressure coefficient distributions over the cross section. The additional changes near the tip may be caused by additional vortices generated at a swept section.

Figure 8-50 shows relative in-plane velocity distribution in the spanwise direction of a rotor blade. The magnitude of the relative in-plane velocity is proportional with the radius of local cross section. A small variation in in-plane velocity distribution at approximately $r = 0.94R$ is observed. This variation is also caused by the circulatory flow generated at the swept section.

Figure 8-51 is a local induced inflow ratio distribution in the spanwise direction. This result shows that the magnitude of an induced inflow ratio over the non-twist & swept rotor blade is smaller than that of a baseline rotor blade. This is caused by the reduction in total lift force generated on the rotor blade. A maximum difference in the induced inflow ratio is observed at
about \( r = 0.80R \) at which the induced inflow velocity is a maximum. In addition, for \( r > 0.95R \), a deviation in the induce inflow ratio is observed.

As shown in Figure 8-52, a relative inflow angle of a non-twist & swept blade is smaller than that of a baseline rotor blade. Figure 8-53 is aerodynamic AoA distributions for a non-twist & swept rotor blade and a baseline rotor blade. This result shows that aerodynamic AoA of a non-twist & swept rotor blade is greater than that of a baseline rotor blade.

These results indicate that a swept tip causes in the reduction of total lift force generated on the rotor blade as well as local lift force generated on the sweep section. It is found that local lift force highly affects induced inflow and aerodynamic AoA. It is also found that the decrease of total lift force results in the reduction of the induce inflow velocity and the change in aerodynamic AoA. The change rate of the decreases is proportional with increasing radius of the blade section for \( r < 0.800R \). For \( r > 0.800R \), the difference of the induce inflow velocity is decreased due to tip losses.

Figure 8-43. Relative velocity contours around a non-twist & swept rotor blade
Figure 8-44. Relative velocity contours around the tip of a non-twist & swept rotor blade

Figure 8-45. Velocity vectors around the tip of a non-twist & swept rotor blade at $x = 0.5c$ in terms of magnitude of $V_y$
Figure 8-46. Negative pressure coefficient distribution at $r = 0.500R$ for a non-twist & swept rotor blade and a baseline rotor blade

Figure 8-47. Negative pressure coefficient distribution at $r = 0.680R$ for a non-twist & swept rotor blade and a baseline rotor blade
Figure 8-48. Negative pressure coefficient distribution at $r = 0.800R$ for a non-twist & swept rotor blade and a baseline rotor blade

Figure 8-49. Negative pressure coefficient distribution at $r = 0.960R$ for a non-twist & swept rotor blade and a baseline rotor blade
Figure 8-50. Relative in-plane velocity distributions at the front plane of a rotor blade in the span direction of a non-twist & swept and a baseline rotor blade

Figure 8-51. Local induced inflow ratio distributions at the front plane of a rotor blade in the span direction of a non-twist & swept and a baseline rotor blade
Figure 8-52. Relative inflow angle distributions at the front plane of a rotor blade in the span direction of a non-twist & swept and a baseline rotor blade

Figure 8-53. Aerodynamic AoA distributions at the front plane of a rotor blade in the span direction of a non-twist & swept and a baseline rotor blade
Figure 8-54 ~ Figure 8-55 show area averaged volumetric erosion contours on the surfaces of a non-twist & swept rotor blade. These results show that the magnitude of volumetric erosion rate is linearly proportional with the radius of the blade section. However, a maximum erosion damage is observed at $r = 0.925R$. This location is near the beginning position of the swept rotor blade. It is found that the size of erosion damage section is not varied in the spanwise direction of the rotor blade.

Figure 8-40 presents average volumetric erosion rate distributions over the blade section between $r = 0.500R$ and $r = 0.900R$ for a non-twist & swept rotor blade and a baseline rotor blade. As shown in these results, erosion damage is proportional with radial position of the rotor blade for $r < 0.900R$. The erosion characteristics of two rotor blades are same. One can conclude that a swept tip can not affect aerodynamic and erosion characteristics on the blade section for $r < 0.900R$.

On the swept tip section for $r > 0.900R$, erosion rate of a swept rotor blade is decreased with an increasing radial position. This is caused by the decrease in impact velocity. The reduction of impact velocity is due to circulatory flows and sweep of the leading edge of the rotor blade. The circulatory flows generated near the tip increase aerodynamic losses of relative inflow and the sweep of the rotor blade reduces the Mach number normal to the leading edge of the rotor blade.

These results concluded that a rotor blade sweep causes additional aerodynamic losses, which decreases the relative inflow velocity. It is also found that a swept tip can reduce the erosion damage.
Figure 8-54. Relative velocity contours around a linear-twist & swept rotor blade

Figure 8-55. Relative velocity contours around a linear-twist & swept rotor blade
Figure 8-56. Area averaged volumetric erosion rate distribution on each blade sections of a non-twist & swept rotor blade and a baseline rotor blade

Figure 8-57. Area averaged volumetric erosion rate distribution on each blade sections of a non-twist & swept rotor blade and a baseline rotor blade
8.2.4. The Effect of Rotor Blade Twist and Sweep on Flow and Erosion Characteristics

Most helicopter rotor blades include a spanwise twist and a swept tip in order to improve their performance. Their combination effect as well as their individual effect on aerodynamic and especially on erosion characteristics must be investigated. Therefore, in this section, flow and erosion damage for a linear-twist & swept rotor blade was simulated. These results for the rotor blade were then compared to the results for a baseline rotor blade.

Figure 8-58 and Figure 8-59 are relative velocity contours around a linear-twist & swept rotor blade at zero degree of a collective pitch angle. As shown in these results, flow characteristics around the rotor blade can separate into two major parts. First part is a main planform section for \( r < 0.925R \). In this section, the effect of a linear-twist is a dominant feature. Second part is a swept tip section for \( r > 0.925R \). A swept tip is the main characteristics in this section. In the first section, velocity contours around the blade sections of a linear-twist & swept rotor blade looks like to that of a baseline rotor blade. Although in-plane velocity is increased with an increasing radius of blade section, the velocity contours around the blade section are similar except for velocity magnitude, which is caused by the negative linear-twist angle. However, in the second section, the velocity contours over the blade section have significant differences compared to that of the first section. This is caused by the drop in the normal velocity and tip vortex generated at the tip of the rotor blade.

Figure 8-60 ~ Figure 8-63 show the comparison of negative pressure coefficient distributions at \( r = 0.575R, 0.725R, 0.900R, \) and \( 0.965R \) for a non-twist & swept rotor blade and a base. These results clearly show that the flow characteristics for a linear-twist and swept rotor blade consist of two major characteristics part for a spanwise twist and a swept tip.

As discussed in the result for a linear-twist & non-swept rotor blade, in the first part \( (r < 0.900R) \), the rotor blade twist rearranges local pitch angle distribution in spanwise direction.
The rotor blade twist also results in the change of relative inflow condition and local induced inflow condition at each blade section. Therefore, for \( r < 0.900R \), pressure coefficient distributions over the blade section for a linear-twist & swept are same as those for a linear-twist & non-swept blade shown in Figure 8-60 ~ Figure 8-62. However, in the second part, negative pressure coefficient over the blade section for a linear-twist & swept rotor blade is smaller when compared to those for a linear-twist & non-swept rotor blade. Unlikely to the results for a non-twist rotor blade, the blade section near the tip of a linear-twist rotor blade is operated at high negative angle of attack, resulting in high negative lift force. Therefore, an application of the swept tip in a linear-twist rotor blade causes the decrease of normal velocity magnitude to the blade section, resulting in a decrease in the difference of pressure coefficient between the suction side and the pressure side as shown in the in Figure 8-63. The reduction in pressure difference on both surfaces results in a decreases of negative lift force.

Figure 8-66 shows relative in-plane velocity distribution in the spanwise direction of the rotor blade, the magnitude of a relative in-plane velocity is proportional with the radius of blade section. A small variation in in-plane velocity distribution for a linear-twist blade is also observed. This variation may be caused by the circulatory flows generated on the leading edge of a swept tip section.

Figure 8-68 shows local induced inflow ratio distributions with respect to radius of the blade section for a baseline, linear-twist & non-swept, and linear-twist & swept rotor blades. Figure 8-69 shows aerodynamic AoA distributions for three different rotor blades. These results indicate that a spanwise twist changes local lift force and induced inflow velocity distribution. The change in the induced inflow velocity results in re-arrangement of relative inflow angle and aerodynamic AoA in the spanwise direction of the rotor blade. This result also shows that local induced inflow velocity of a linear-twist & swept rotor blade is greater than the velocity of a linear-twist & non-swept rotor blade. This is caused by the increase in total lift generated on the
rotor blade because the use of swept tip can reduce the negative lift force generated on the swept section for zero degree of collective pitch angle. The increase of total lift force results in an increase of the induced inflow velocity and a decrease of aerodynamic AoA.

Figure 8-58. Relative velocity contours around a linear-twist & swept rotor blade

Figure 8-59. Relative velocity contours around the tip of a linear-twist & swept rotor blade
Figure 8-60. Negative pressure coefficient distribution at $r = 0.500R$ for a non-twist & non-swept rotor blade

Figure 8-61. Negative pressure coefficient distribution at $r = 0.680R$ for a non-twist & non-swept rotor blade
Figure 8-62. Negative pressure coefficient distribution at $r = 0.890R$ for a non-twist & non-swept rotor blade

Figure 8-63. Negative pressure coefficient distribution at $r = 0.960R$ for a non-twist & non-swept rotor blade
Figure 8-64. Velocity vectors around the tip of a linear-twist & swept rotor blade at $x = 0.5c$ in terms of magnitude of $V_y$. 

Figure 8-65. Velocity vectors around the tip of a linear-twist & swept rotor blade at $x = 0.5c$ in terms of magnitude of $V_y$. 

Figure 8-66. Average relative inflow angle distributions at the front plane of a rotor blade in the span direction of a non-twist and non-swept rotor blade

Figure 8-67. Average local induced inflow ratio distributions at the front plane of a rotor blade in the span direction of a non-twist and non-swept rotor blade
Figure 8-68. Average local induced inflow ratio distributions at the front plane of a rotor blade in the span direction of a non-twist and non-swept rotor blade

Figure 8-69. Average relative inflow angle distributions at the front plane of a rotor blade in the span direction of a non-twist and non-swept rotor blade
Figure 8-70 ~ Figure 8-71 show area averaged volumetric erosion contours on the surfaces of a non-twist & swept rotor blade. These results show that the magnitude of volumetric erosion rate is linearly proportional with increasing radius of the blade section. However, the maximum erosion damage is observed at $r = 0.925R$. The position is near the beginning position of the swept rotor blade. It is found that the extent of erosion damage area on the suction surface is more widened than that of the pressure surface.

Figure 8-40 is the comparison of averaged volumetric erosion rate distributions with respect to radius of the blade section between $r = 0.500R$ and $r = 0.900R$ for a linear-twist & swept rotor blade, a linear-twist & non-swept and a baseline rotor blade.

The erosion rate distribution for a linear-twist & swept rotor blade is the same as that of a linear-twist & non-swept rotor blade for $r < 0.925R$. This result also indicated that the effect of a swept tip on the erosion characteristics on this section is negligible small.

It is found that the erosion characteristics on this section are affected mainly by a spanwise twist of the rotor blade. At the swept tip section ($r > 0.900R$), erosion rates for a swept rotor blade is much smaller than that of a baseline rotor blade. This is because of the reduction in the normal relative velocity to the blade section.

These results show that a rotor blade sweep causes a decrease in aerodynamic performance in hover condition, but can reduce the erosion damage.
Figure 8-70. Area averaged volumetric erosion rate contours on the suction surface for a linear-twist & swept rotor blade

Figure 8-71. Area averaged volumetric erosion rate contours near the leading edge of a linear-twist & swept rotor blade
Figure 8-72. The comparison of area averaged volumetric erosion rate distributions at the main planform section for a linear-twist & swept rotor blade.

Figure 8-73. The comparison of area averaged volumetric erosion rate distributions at the swept tip section for a linear-twist & swept rotor blade.
Chapter 9

Particle Tracking and Erosion Damage Prediction for a UH-60A Helicopter Rotor Blade

9.1. Rotor Blade Geometry and 3D Grid System

As mentioned in Chapter 8, rotor twist distribution and swept tip shape affects significantly aerodynamic characteristics around a 3D rotating rotor blade. The change in aerodynamic characteristics near the rotor blade influences sand particle trajectory and sand erosion characteristics on the rotor blade. As discussed in Chapter 5 and Chapter 8, the magnitude of erosion rate is linearly proportional with radial position of the blade section. The spanwise twist controls the aerodynamic AoA distribution to the blade section of the rotor blade. The specific flow incidence at each radial position controls the erosion pattern on the pressure and suction surface. The swept tip of the rotor blade reduces the normal velocity to the blade section. Although the reduction in relative normal velocity decreases total lift force, this also reduces the erosion damage.

In this section, particle tracking and erosion damage predictions for an actual 3D helicopter rotor blade were performed. A UH-60A helicopter rotor blade is chosen as the main test case because there are many numerical and experimental investigations for the specific rotor blade including real flight test results. The UH-60A Black Hawk is a four-bladed helicopter, which is the primary troop-carrying transport developed in 1970s. The helicopter rotor blade of UH-60A consists mainly of two different airfoils, SC1095 and SC1094R8 as shown in Figure 9-1. The rotor blade platform of UH-60A is a non-uniform airfoil as shown in Figure 9-2. Table 9-1 is airfoil profile distribution in the spanwise direction.
Table 9-1  Airfoil distribution over UH-60A blade planform

<table>
<thead>
<tr>
<th>Part</th>
<th>Range</th>
<th>Airfoil</th>
<th>Chord Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root Cutout</td>
<td>42.00° (0.1304R) ~ 62.00° (0.1925R)</td>
<td>SC1095</td>
<td>20.760° Constant</td>
</tr>
<tr>
<td>Inboard</td>
<td>62.00° (0.1925R) ~ 150.00° (0.4658R)</td>
<td>SC1095</td>
<td>20.760° Constant</td>
</tr>
<tr>
<td>Linear Transition 1</td>
<td>150.00° (0.4658R) ~ 160.00° (0.4969R)</td>
<td>SC1095</td>
<td>20.760° ~ 20.965°</td>
</tr>
<tr>
<td>Midboard</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Beginning of Midboard ~ Beginning of tab inner</td>
<td>SC1094R8</td>
<td>20.965° ~ 22.317°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Beginning of Tab ~ Beginning of Linear Transition 2</td>
<td>SC1094R8</td>
<td>22.317° Constant</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Linear Transition ~ End of linear transition 2</td>
<td>SC1094R8</td>
<td>22.317° ~ 22.112°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- End of linear transition 2 ~ End of Tab</td>
<td>SC1095</td>
<td>22.112° Constant</td>
</tr>
<tr>
<td>Outboard</td>
<td>265.00° (0.8230R) ~ 299.00° (0.9286R)</td>
<td>SC1095</td>
<td>20.760° Constant</td>
</tr>
<tr>
<td>Swept</td>
<td>299.00° (0.9286R) ~ 322.00° (1.000R)</td>
<td>SC1095</td>
<td>20.760° ~ 22.092°</td>
</tr>
</tbody>
</table>
It is well known that the use of spanwise linear twist on a helicopter rotor blade improves the uniformity of local lift force distribution and induced inflow velocity distribution over the span of the rotor blade. The linear twist also reduces the induced power in hover condition. Moreover, the most helicopter rotor blades use a nonlinear or double linear twist instead of a high linear twist because the non-linear or double-linear (two different sections of linear spanwise twist) twist can reduce the distortion of local lift force near the tip and improves rotor thrust and propulsive force. Therefore, the UH-60A helicopter rotor blade has non-linear twist (washout) as shown in Figure 9-3. The UH-60A helicopter rotor blade also has a swept tip shape in order to improve the forward flight speed and to reduce the induced drag force. Blade tip section sweep angle is 20 degrees measured from spanwise direction near the leading edge of the tip section.

![Figure 9-3. Aerodynamic twist of UH-60A blade based on mean chordline from Ref. (41)](image)

It is well known that helicopter aerodynamics is strongly influence from operating conditions, which can affect the erosion characteristics and erosion damage on a helicopter rotor blade. Significant erosion damage occurs near ground flight such as hover, take-off, or landing. Therefore, in this current study, collective pitch angles ($\theta_c = 0, 3, \text{ and } 6^\circ$) and vertical flight velocities ($V_z = 0, -25, \text{ and } 25 \text{ m/s}$) are chosen as typical operating parameters. Therefore, flow simulations and erosion damage predictions for these operating parameters were performed...
in order to investigate their effects on rotor aerodynamics and erosion characteristics for a UH-60A helicopter rotor blade.

Figure 9-4 ~ Figure 9-6 show a hybrid grid system for flow simulation and erosion damage predictions on a UH-60A helicopter rotor blade. Similar to previous computations, the grid system consists of two parts - main blade part and outfield part. In the main blade part (near blade section), a multi-block H-type structure grid system is used to model 2D-cross section computation domain in the spanwise direction. The 2D H-type structured grid system is stacked in the spanwise direction and rotated based on the twist angle of the rotor blade as shown in Figure 9-5. Moreover, in order to capture complex flows around the hub and the tip of rotor blade including circulatory flows and tip vortices, the computational domain in the specific areas is designed and meshed by using hybrid grid systems. The expansion section of the tip domain is meshed by using H-type structure grid system and the expansion part from the tip surface of the rotor blade is meshed by triangular unstructured grid system. In this outfield grid system, triangular unstructured grids are used to reduce required grid cell number and computational time as shown in the Figure 9-6

Figure 9-4. H-type grid system for the cross section of a rotor blade
Figure 9-5. Grid system in the spanwise direction of a rotor blade

Figure 9-6. Grid system for computation of the domain away from the blade
9.2. Flow and Erosion Prediction Results for a UH-60A Helicopter Rotor Blade

9.2.1. Flow and Erosion Characteristics for Various Collective Pitch Angles

Figure 9-7 show relative velocity contours around a UH-60A helicopter rotor blade for zero degree of collective pitch angle, $\theta_c = 0^\circ$. Similar to the results for the flow around a linear-twist 3D SC1095 rotor blade, the velocity magnitude distribution over the suction surface of a UH-60A rotor blade is slightly increased with increasing radius of the blade section due to nose-down spanwise twist. This is because the nose-down twist increases relative inflow angle of attack (effective angle) to the leading edge of the blade section.

Figure 9-8 are relative velocity contours for $\theta_c = 3^\circ$ and Figure 9-9 are relative velocity contours for $\theta_c = 6^\circ$. These results indicate that flow velocity over the suction surface is increased with increasing collective pitch angle because relative inflow angle of attack over the span of the rotor blade is a linear function of the collective pitch angle. This result also shows that the change rate of velocity magnitude with collective pitch angle is also proportional at the same radial position of the rotor blade.

It is also found that the maximum velocity position is moved from the leading edge on the pressure surface to the suction surface because of an increase in the relative inflow angle of attack to each blade section. The relative angle of attack is highly dependent upon the collective pitch angle and the radial position over the rotor blade.

Moreover, the flow becomes a supersonic flow on the section at greater than $r = 0.900R$ for $\theta_c = 6^\circ$. The weak shocks are observed at the section as shown in Figure 9-8.
Figure 9-10 and Figure 9-11 are velocity vector distributions around a UH-60A rotor blade at 50 percent chord length \((x = 0.5c)\) for zero degree of a collective pitch angle \((\theta_c = 0.0^\circ)\). As shown in these results, a tip vortex generated at the tip of a UH-60A rotor blade circulates from the pressure surface to the suction surface. The tip vortex flow affects the flow around the rotor blade up to \(r = 0.9R\). Additional vortices generated at the leading edge of a swept tip part are observed. This vortex flow affects strongly the relative inflow conditions near the leading edge of the swept tip part.

Figure 9-12 and Figure 9-15 are velocity vector distributions in terms of the magnitude of \(y\) component velocity for \(\theta_c = 3^\circ\) and \(\theta_c = 6^\circ\). It is found that the tip vortex is much stronger as the collective pitch angle is increased. When the collective pitch angle is large, the pressure difference between the two surfaces is significant, resulting in a high lift force and strong tip vortex. The stronger the tip vortex flow, the wider the flow around the rotor blade is affected by the tip vortex. The additional vortices generated at the leading edge on the swept tip are also stronger as the collective pitch angle is increased.

Figure 9-16 is negative pressure coefficient distributions for three different collective pitch angles, \(\theta_c = 0.0^\circ, 3.0^\circ\) and \(6.0^\circ\) at mid-span position, \(r = 0.575R\). As shown in this figure, the negative pressure coefficient is suddenly increased over the leading edge on the suction side, and the pressure coefficient becomes a maximum at \(x = 0.05c\) on the suction side. The magnitude of negative pressure coefficient over the suction surface is increased as the collective pitch angle is increased.

This occurs because the relative inflow angle of attack (aerodynamic AoA) to blade section of the rotor blade is also linearly increased with collective pitch angle. However, the negative pressure coefficient distribution on pressure side for \(\theta_c = 0.0^\circ\) is different from that of \(\theta_c = 3.0^\circ\) and \(6.0^\circ\). The relative inflow angle of attack at \(r = 0.575R\) for \(\theta_c = 0.0^\circ\) is very small positive specify angle while the aerodynamic AoA for \(\theta_c = 3.0^\circ\) and \(6.0^\circ\) is a large positive
specify angle. As mentioned in the previous parts, pitch angle at $r = 0.75R$ is the same as the absolute value of collective pitch angle, so the blade section at $r = 0.725R$ is operated at small positive angle of attack as shown in the Figure 9-17.

Figure 9-18 ~ Figure 9-19 show negative pressure coefficient distributions near the rotor blade tip, at $r = 0.900R$ and $r = 0.965R$, for three different collective pitch angles. Unlikely to a linear-twist rotor blade, as presented in Chapter 8.2.2, the pitch angle of a UH-60A rotor blade suddenly varies to have a positive value, as shown in Figure 9-3, in order to complement the tip lift loss.

Therefore, the sections near the rotor tip are operated at high positive angle attack. As shown in these results, the difference in the pressure coefficient between both sides is increased as the collective pitch angle is increased.
Figure 9-8. Relative velocity contours around the tip of a UH-60A rotor blade for $\theta_c = 3.0^\circ$

Figure 9-9. Relative velocity contours around the tip of a UH-60A rotor blade for $\theta_c = 6.0^\circ$
Figure 9-10. Velocity vectors around the tip of a UH-60A rotor blade at \( x = 0.5c \) in terms of magnitude of \( V_y \) for \( \theta_c = 0.0^\circ \)

Figure 9-11. Velocity vectors around a UH-60A rotor blade at \( x = 0.075c \) in terms of magnitude of \( V_y \) for \( \theta_c = 0.0^\circ \)
Figure 9-12. Velocity vectors around the tip of a UH-60A rotor blade at $x = 0.5c$ in terms of magnitude of $V_y$ for $\theta_c = 3.0^\circ$.

Figure 9-13. Velocity vectors around a UH-60A rotor blade at $x = 0.075c$ in terms of magnitude of $V_y$ for $\theta_c = 3.0^\circ$. 
Figure 9-14. Velocity vectors around the tip of a UH-60A rotor blade at x = 0.5c in terms of magnitude of $V_y$ for $\theta_c = 6.0^\circ$

Figure 9-15. Velocity vectors around a UH-60A rotor blade at x = 0.075c in terms of magnitude of $V_y$ for $\theta_c = 6.0^\circ$
Figure 9-16. Negative pressure coefficient distribution for various collective pitch angles at $r = 0.575R$ for a UH-60A rotor blade

Figure 9-17. Negative pressure coefficient distribution for various collective pitch angles at $r = 0.725R$ for a UH-60A rotor blade
Figure 9-18. Negative pressure coefficient distribution for various collective pitch angles at $r = 0.900R$ for a UH-60A rotor blade

Figure 9-19. Negative pressure coefficient distribution for various collective pitch angles at $r = 0.965R$ for a UH-60A rotor blade
Figure 9-20 shows relative in-plane velocity distributions for three different collective pitch angles in the spanwise direction of a rotor blade. The result shows that the magnitude of a relative in-plane velocity is linearly proportional with the radial position of blade section. Unlike a non-linear or linear-twist rotor blade, a little variation in the relative in-plane velocity for a non-linear twist rotor blade is observed at approximately $r = 0.40R$ and $r = 0.80R$. These variations are caused by the discontinuity of geometric condition, a non-linear twist angle.

Figure 9-21 presents the comparison of local induced inflow ratio distributions for the collective pitch angle. This result shows that local induced inflow for $\theta_c = 0.0^\circ$ is highly dependent upon the local pitch angle distribution of a UH-60A rotor blade as shown in the Figure 9-3. When the pitch angle is positive (negative), the local induced inflow ratio is also positive (negative).

A local induced inflow ratio for $\theta_c = 3.0^\circ$ is almost linearly proportional with the radius of blade section until the beginning of the swept tip section. The induced inflow ratio distribution for $\theta_c = 0.3^\circ$ can be considered as constant. When the collective pitch angle is 6 degrees, the most blade sections are operated at a large positive pitch angle, resulting in large positive induced inflows over the span. Near the tip of the rotor blade for all collective pitch angles, the induced inflow velocity is steeply reduced because of the reduction of the lift force caused by tip losses.

As shown in Figure 9-21, as collective pitch angle is increased, the sudden reduction of the induced inflow velocity starts further away from the rotor blade tip and the change rate of induced inflow velocity is also increased.

This occurs because circulatory flows are much stronger as collective pitch angle is increased. Small deviations from mean local induced inflow ratio are also observed at about $r = 0.4R$ and $r = 0.8R$. These deviations are occurred by the discontinuity of blade geometry as shown in Figure 9-3.
The change in induced inflow velocity over the rotor blade causes the re-arrangement of relative inflow angle distribution and aerodynamic AoA distribution over the rotor blade as shown in Figure 9-22 and Figure 9-23.

As shown in Figure 9-22, the relative inflow angle is inversely proportional with the radial position up to $r = 0.65R$. For $r > 0.65R$, the relative angle can be considered as zero. The inflow angle for $\theta_c = 3.0^\circ$ is almost constant. As shown in the relative inflow angle distribution for $\theta_c = 6.0^\circ$, the inflow angle is gradually reduced as increasing radius of the blade section. Moreover, for $r > 0.85R$, the inflow angle is reduced due to the reduction of lift force caused by tip losses.

Figure 9-23 shows aerodynamic AoA distributions for various collective pitch angles. Because a UH-60A rotor blade has the nose-down twist, aerodynamic AoA for $r < 0.82R$ is inversely proportional with increasing radius of the blade section. However, $r > 0.82R$, aerodynamic AoA is suddenly increased and reaches maximum value at approximately $r = 0.925R$. After reaching maximum value, aerodynamic AoA is steeply reduced. This is because of a sudden change of twist angle and the effect of the tip vortex.
Figure 9-20. Comparison of relative in-plane velocity distributions at the front plane of a rotor blade for a collective pitch angle

Figure 9-21. Comparison of local induced inflow ratio distributions at the front plane of a rotor blade for a collective pitch angle
Figure 9-22. Comparison of relative inflow angle distributions at the front plane of a rotor blade for a collective pitch angle.

Figure 9-23. Comparison of aerodynamic AoA (effective AoA) distributions at the front plane of a rotor blade for a collective pitch angle.
Figure 9-30 and Figure 9-31 show area averaged volumetric erosion rate distributions at the linear part of a UH-60A rotor blade for \( r < 0.9286R \). These results indicated that the magnitude of erosion rate is linearly increased as the radial position of blade section. However, at the swept tip section (for \( r > 0.9286R \)), the erosion rate is decreased due to the effect of a rotor blade sweep and the effect of the tip vortex. These results show that the influence of aerodynamic AoA for a real 3D rotor blade is stronger on erosion characteristics than that of a 2D airfoil. This is caused by an increased resultant velocity magnitude.

Although of erosion rate difference between a 3D rotor blade and a 2D airfoil, erosion characteristics on the 3D rotor blade for aerodynamic AoA (effective AoA) are very agreement with erosion characteristics on the 2D airfoil for inflow angle of attack. For negative or small positive aerodynamic AoA, the magnitude of erosion damage of the suction surface is greater than that of the pressure surface and wider extent on the suction side is damaged by sand erosion. For \( \alpha > 5.0^\circ \), the magnitude of erosion rate and the extent of erosion damage area on the pressure side are much greater than that of the suction surface. It is found that the magnitude of erosion damage and the extent of erosion damage area on two surfaces are highly dependent upon aerodynamic AoA.

Figure 9-32 ~ Figure 9-35 present comparison of area averaged volumetric erosion rate distribution at \( r = 0.6R \sim 0.98R \) for different collective pitch angles. As mentioned in previous parts, the aerodynamic AoA over the span of the rotor blade is linearly proportioned with an increasing collective pitch angle as shown in Figure 9-23. The change in aerodynamic AoA distribution in the spanwise direction affects erosion characteristics on the rotor blade. At less than \( r = 0.900R \), the magnitude of erosion rate on suction surface and the size of erosion damage area are increased as the collective pitch angle is increased. However, at the region near \( r = 0.9R \), the opposite patterns for the collective pitch angle are observed due to the existence of large aerodynamic AoA.
Figure 9-24. Area averaged volumetric erosion rate contours on the suction side of a UH-60A rotor blade for $\theta_c = 0.0^\circ$.

Figure 9-25. Area averaged volumetric erosion rate contours on the pressure side of a UH-60A rotor blade for $\theta_c = 0.0^\circ$.
Figure 9-26. Area averaged volumetric erosion rate contours on the suction side of a UH-60A rotor blade for $\theta_c = 3.0^\circ$.

Figure 9-27. Area averaged volumetric erosion rate contours on the pressure side of a UH-60A rotor blade for $\theta_c = 3.0^\circ$. 
Figure 9-28. Area averaged volumetric erosion rate contours on the suction side of a UH-60A rotor blade for $\theta_c = 6.0^\circ$.

Figure 9-29. Area averaged volumetric erosion rate contours on the pressure side of a UH-60A rotor blade for $\theta_c = 6.0^\circ$. 

Figure 9-30. Area averaged volumetric erosion rate distribution at the linear section for a UH-60A rotor blade for $\theta_c = 0.0^\circ$.

Figure 9-31. Area averaged volumetric erosion rate distribution at the swept tip section for a UH-60A rotor blade for $\theta_c = 0.0^\circ$. 
Figure 9-32. Comparison of area averaged volumetric erosion rate distributions at the linear section of a UH-60A rotor blade for a collective pitch angle

Figure 9-33. Comparison of area averaged volumetric erosion rate distributions at the swept tip section of a UH-60A rotor blade for a collective pitch angle
Figure 9-34. Comparison of area averaged volumetric erosion rate distributions at the linear section of a UH-60A rotor blade for a collective pitch angle.

Figure 9-35. Comparison of area averaged volumetric erosion rate distributions at the swept tip section of a UH-60A rotor blade for a collective pitch angle.
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9.2.2. Flow and Erosion Characteristics for a Climb Velocity Ratio

It is well known that erosion damage on a helicopter rotor blade usually occurs at a vertical flight condition (take-off and landing) as well as at a hover condition. Therefore, in order to reduce erosion damage on the rotor blade, an analysis about flow and erosion characteristics for these operation conditions must be performed. In a real vertical flight condition, the vertical velocity is a function of helicopter’s total load and total lift force. When total weight of a helicopter is constant, a climb or descend velocity of a helicopter is directly dependent upon the collective pitch angle. However, in this simulation of flow and erosion damage on the rotor blade for a climb velocity, two assumptions are considered. One assumption is that a constant collective pitch angle is kept during the operation in order to analyze only the effect of vertical flight velocity. The other assumption is that a flight condition is steady, which indicates that the climb velocity is constant.

As shown in the Figure 9-36, a vertical motion of a helicopter is usually analyzed in terms of a climb velocity ratio. Therefore, in this section, computations for flow and erosion damage on a UH-60A for three different climb velocity ratios, $V_c/V_l = -1.0$, 0.0, and 1.0, were performed in order to investigate the effect of the climb velocity (the vertical velocity) on flow and erosion characteristics. The reference collective pitch angle is chosen as 6 degrees in this section.
Figure 9-36. Induced velocity distribution for climb velocity ratio from Ref. (55)

Figure 9-37. Total power required distribution for climb velocity ratio from Ref. (55)
Figure 9-38 presents relative velocity contours around a UH-60A helicopter rotor blade for \( \theta_c = 6^\circ \) and \( V_c/V_i = 0.0 \). As mentioned in Chapter 9.2, although a UH-60A rotor blade includes a nose-down non-linear twist, a large collective pitch angle causes a high aerodynamic AoA to blade section over a rotor blade. This high aerodynamic AoA generates a high lift force, producing a strong tip vortex as shown in Figure 9-38.

Figure 9-39 show the results for \( \theta_c = 6^\circ \) and \( V_c/V_i = 1.0 \). Figure 9-40 show the results for \( \theta_c = 6^\circ \) and \( V_c/V_i = -1.0 \). As shown in these results, the climb velocity changes flow conditions over a rotor blade including the aerodynamic AoA distribution and tip vortex strength.

A descend flight condition, \( V_c/V_i < 0.0 \), results in an increase of aerodynamic AoA with respect to a rotor blade, causing a separated flow on the suction side as shown in Figure 9-40 and Figure 9-42. As shown in these results, when the descend velocity is small, the separated flow is observed at the suction surface of blade section near the rotor blade hub because the blade section near hub has a high pitch angle. When the descend velocity is increased, the separated flow can be observed at the suction surface over the whole surface, resulting in stall. A climb flight condition also decreases aerodynamic AoAs to a rotor blade, but the flow condition around a rotor blade is almost similar to that for a hover condition as shown in Figure 9-39 and Figure 9-41 except for the tip vortex characteristics. Similar phenomena are shown in the past researches, Figure 9-36 and Figure 9-37. These results show that a descending motion of a helicopter rotor blade causes very different flow and performance characteristics compared to climb motion.

Figure 9-43 and Figure 9-44 show negative pressure coefficient distributions for various climb velocity ratios at \( r = 0.575R \) and \( r = 0.725R \). As mentioned previously, an operational condition with a negative climb velocity (a descend condition) increases aerodynamic AoA (effective angle) to a rotor blade, especially near the hub. Therefore, the pressure difference between both surfaces for \( V_c/V_i = -1.0 \) is the greatest in these cases. Moreover, the pressure...
difference for $V_c/V_l = 1.0$ is also greater than that for $V_c/V_l = 0.0$ due to a high aerodynamic 
AoA at the blade section.

Figure 9-45 and Figure 9-46 present negative pressure coefficient distributions near the 
tip of the rotor blade for three different climb velocity ratios. These results also show that the 
pressure difference between both surfaces for $V_c/V_l = -1.0$ is the greatest in these cases. 
However, it is found that the flow conditions around the blade section of $V_c/V_l = 1.0$ is almost 
the same as that of $V_c/V_l = 0.0$. These results conclude that an aerodynamic AoA near the rotor 
blade tip is not highly dependent upon the climb velocity as shown in Figure 9-50.

Figure 9-38. Relative velocity contours around a UH-60A rotor blade for $\theta_c = 6.0^\circ$ and $V_c/V_l = 0.0$
Figure 9-39. Relative velocity contours around a UH-60A rotor blade for $\theta_c = 6.0^\circ$ and $V_c/V_i = 1.0$

Figure 9-40. Relative velocity contours around a UH-60A rotor blade for $\theta_c = 6.0^\circ$ and $V_c/V_i = -1.0$
Figure 9-41. Relative velocity contours around a UH-60A rotor blade for $\theta_c = 6.0^\circ$ and $V_c/V_i = 5.0$

Figure 9-42. Relative velocity contours around a UH-60A rotor blade for $\theta_c = 6.0^\circ$ and $V_c/V_i = -5.0$
Figure 9-43. Negative pressure coefficient distribution for various climb velocity ratios at $r = 0.575R$ for a UH-60A rotor blade.

Figure 9-44. Negative pressure coefficient distribution for various climb velocity ratios at $r = 0.725R$ for a UH-60A rotor blade.
Figure 9-45. Negative pressure coefficient distribution for various climb velocity ratios at $r = 0.900R$ for a UH-60A rotor blade.

Figure 9-46. Negative pressure coefficient distribution for various climb velocity ratios at $r = 0.965R$ for a UH-60A rotor blade.
Figure 9-47 shows relative in-plane velocity distributions for three different climb velocities in the spanwise direction of the rotor blade. The result shows that the magnitude of the relative in-plane velocity is linearly proportional with the radial position of blade section. It is also found that small variation in the relative in-plane velocity occurs at approximately $r = 0.40R$ and $r = 0.80R$. These variations are caused by the discontinuity of the geometric condition, a non-linear twist angle.

Figure 9-48 shows local induced inflow ratio distributions for a climb velocity ratio. The vertical climb motion of a helicopter rotor blade, for $V_c/V_i = 1.0$, generates weaker induced inflows than that of a hover flight. The decrease in induce inflow velocity results in the decrease in aerodynamic AoA as shown in the Figure 9-50. However, a descend flight of the helicopter rotor blade results in strong induced inflow in opposite direction. As shown in Figure 9-50, the strong induced inflow increases aerodynamic AoA over the rotor blade, which occurs the strong separation flow on the suction surface as shown in Figure 9-40. These results are in good agreement with previous results as shown in Figure 9-36.
Figure 9-47. Comparison of relative in-plane velocity distributions at the front plane of a rotor blade at $\theta_c = 6.0^\circ$ for climb velocity ratio

Figure 9-48. Comparison of local induced inflow ratio distributions at the front plane of a rotor blade at $\theta_c = 6.0^\circ$ for climb velocity ratio
Figure 9-49. Comparison of relative inflow angle distributions at the front plane of a rotor blade at $\theta_c = 6.0^\circ$ for climb velocity ratio.

Figure 9-50. Comparison of aerodynamic AoA (effective AoA) distributions at the front plane of a rotor blade at $\theta_c = 6.0^\circ$ for climb velocity ratio.
Similar to the results for the collective pitch angle, the climb velocity of a helicopter rotor blade affects the flow conditions over the rotor blade including aerodynamic AoA distribution and resultant velocity magnitude which is normal to the blade section.

Figure 9-51 and Figure 9-52 show area averaged volumetric erosion rate distribution at the linear part of a UH-60A rotor blade, for \( r < 0.9286R \). These results indicate that the magnitude of erosion rate is linearly increased as the radius of blade section increases. It is also found that great erosion damage near the leading edge is observed due to high angle of attack. However, at the swept tip section, the magnitude of erosion rate is decreased because of rotor blade sweep and circulatory flows generated on the leading edge of swept tip section and near the rotor blade tip as shown in Figure 9-53 and Figure 9-54. The rotor blade sweep reduces normal velocity of relative inflow to the blade section, which highly affects impact velocity of solid particles. The circulatory flows causes aerodynamic losses, which increases the induced drag force acting on a solid particle.

These results conclude that the magnitude of erosion rate for a climb or descend flight condition is greater than erosion rate magnitude for a hover condition because the climb velocity increases in the magnitude of resultant flow velocity.

It is also found that the erosion pattern except for the erosion rate magnitude at each blade section is not dependent upon climb velocity. These result also conclude that a hover flight condition can be selected as a reference operating condition in order to predict erosion characteristics on a 3D helicopter rotor blade or to develop a new method for minimizing erosion damage.
Figure 9-51. Comparison of area averaged volumetric erosion rate distributions of a UH-60A rotor blade at $r = 0.600R$ and $\theta_c = 6.0^\circ$ for a climb velocity ratio

Figure 9-52. Comparison of area averaged volumetric erosion rate distributions of a UH-60A rotor blade at $r = 0.800R$ and $\theta_c = 6.0^\circ$ for a climb velocity ratio
Figure 9-53. Comparison of area averaged volumetric erosion rate distributions of a UH-60A rotor blade at \( r = 0.940R \) and \( \theta_c = 6.0^\circ \) for a climb velocity ratio.

Figure 9-54. Comparison of area averaged volumetric erosion rate distributions of a UH-60A rotor blade at \( r = 0.980R \) and \( \theta_c = 6.0^\circ \) for a climb velocity ratio.
Chapter 10

Generalization of Solid Particle Trajectory and Erosion Damage on a Helicopter Rotor Blade

As discussed in Chapter 2, in order to predict erosion damage on a 3D helicopter rotor blade, first, the flow around the rotor blade must be simulated, and a trajectory and an impact condition of solid particles are then determined. Finally, erosion damage is calculated by using the impact condition obtained from the flow-only solution. This effort requires a large scale computation due to complicated flow condition around a helicopter rotor blade. The number of released solid particles used in this simulation is also excessive. A typical optimization effort for minimizing erosion damage involving the general design of a helicopter rotor blade, many constraints such as forward flight performance, noise and structure dynamics problem must be considered. Therefore, a new time efficient methodology is necessary in order to reduce the computational costs for erosion damage prediction on helicopter rotor blades.

As presented in Chapter 5 and Chapter 9, the trajectory of solid particles and their resulting erosion damage on a 2D airfoil surface or on a 3D helicopter rotor blade require relatively accurate relative inflow conditions to the surface when particle size, particle type, and a material of metallic surface are specified. The relative inflow conditions affects dominantly particle path, the magnitude of sand particle velocity and impingement angle.

It is well known that the flow simulation around a 3D rotor blade can be alternatively performed by using 2D relative flow simulation with respect to blade section in the span direction. This is reasonable because the effect of a spanwise component of the resultant flow velocity to the blade section on aerodynamics and performance is negligibly small (52).
Therefore, in this current study, a generalization of particle trajectory and erosion characteristics on 2D airfoil was performed in order to reduce computational costs. Next, erosion damage on a 3D rotor blade was predicted by using generalization results, and these results were then compared to 3D erosion damage results in order to verify generalization results.

10.1. Generalization for Particle Trajectory and Erosion Damage in 2D Airfoil Sections

Two different variables were selected in this generalization effort. The first variable is a “Characteristic length, S”. The characteristic length was introduced by Clevenger et al. (59). They suggested the variable in order to match inertia and drag forces acting on a solid particle as shown in Equation 61.

\[
S = \frac{10 \rho_p d_p}{3 \rho}
\]  

(61)

where, \(\rho_p\) and \(\rho\) are the density of solid particle and gas and \(D_p\) is the diameter of solid particle.

The other variable is an “Inertia number, \(\lambda_L\)” introduced by Laitone (60). He concluded that particle motions throughout the entire flow domain are dependent upon the initial (injection) condition, so he suggested the non-dimensional parameter as shown in Equation 62. The inertia number is based on the properties of a solid particle and inflow condition at inlet boundary condition.

\[
\lambda_L = \frac{2 \rho_p}{\delta} \left( \frac{d_p}{L} \right)^2 \left( \frac{V_L}{v} \right) = \frac{2}{9} \delta_L (AR)^2 Re
\]  

(62)

Where, \(\delta_L\) is the ratio of particle density to gas density, and AR : Aspect ratio is particle radius / reference length, L.
When properties of a solid particle and fluid are fixed, the characteristic length is a function of only diameter of solid particle and the inertia number is a function of solid particle diameter and inflow velocity as shown in these equations.

These parameters are used in the generalization for tracking of solid particles which have a different size and a different density. However, the sand particle density can be taken as constant in the current study. Particle trajectories and erosion characteristics on a 2D airfoil surface or on a 3D rotor blade are affected by inflow Mach number, inflow angle of attack, and the diameter of a solid particle. Therefore, in this section, the computation about particle tracking and erosion damage prediction for only characteristics length and inflow Mach number were performed.

10.1.1. Particle Trajectory and Erosion Characteristics using a Generalization Parameter

In this section, first, solid particle trajectories around a 2D SC1095 airfoil surface and erosion damage on the airfoil surface at the same characteristic length for various inflow Mach numbers were investigated.

Figure 10-1 ~ Figure 10-4 show trajectories for various Mach numbers at the same characteristic length. For small characteristic length, the orientation and the speed of solid particles can be changed more easily by the induced aerodynamic force which is generated directly by the momentum difference. In this case, a group of solid particles impact airfoil surface, but most of the solid particles go downstream without collision with metallic surface due to their small inertia. For high characteristic length, many more particles impact the airfoil because they collide with the airfoil surface without great change in their orientation and velocity.
These results indicated that the trajectory of solid particles with the same characteristics length and without collision with the airfoil surface is not depended upon the inflow Mach number. The motion of solid particles colliding and rebounding from the airfoil surface has a difference according to inflow Mach number. The difference is much increased as the value of characteristic length is increased.

However, in the prediction of sand erosion damage on a 2D airfoil or a 3D rotor blade, the effect of solid particles rebounding from the airfoil or rotor blade on the erosion characteristics of a aft-airfoil or a aft-rotor blade is negligible small. Therefore, it is found that a generation of particle trajectory by using “characteristic length” is reasonable.

Figure 10-5 and Figure 10-6 show area averaged volumetric erosion rate distributions over the airfoil for various inflow Mach numbers at same characteristics length. These results conclude that the solid particles with the same characteristic length results in the same erosion patterns over the 2D airfoil surface.

These results also show that inflow Mach number affects only the magnitude of erosion rate over the airfoil. It is found that erosion rate distribution (except for the magnitude) is dependent upon not the inflow Mach number but the characteristic length. Therefore, it is conclude that a generation of erosion damage prediction by using “characteristics length” is also reasonable when the coefficient for erosion rate magnitude is introduced.
Figure 10-1. Trajectory of solid particles for various Mach number around the airfoil at $S = 0.047$

Figure 10-2. Trajectory of solid particles for various Mach number near the leading edge of the airfoil at $S = 0.047$
Figure 10-3. Trajectory of solid particles for various Mach number around the airfoil at $S = 1.167$

Figure 10-4. Trajectory of solid particles for various Mach number near the leading edge of the airfoil at $S = 1.167$
Figure 10-5. Area averaged erosion rate distributions on the pressure surface for inflow Mach number at $S = 1.167$

Figure 10-6. Area averaged erosion rate distributions on the suction surface for inflow Mach number at $S = 1.167$
Solid particle trajectories around a 2D SC1095 airfoil and erosion damage on the airfoil surface at same inertial number for various inflow Mach numbers were investigated. In these computations, sand particle size was chosen based on the simulations for characteristic length.

Figure 10-7 ~ Figure 10-10 presents solid particle trajectories for various Mach numbers at the same inertia number. These results indicate that solid particles with the same inertia number before and after collision moves on a similar trajectory, which is not depended upon the inflow Mach number.

It is also found that solid particles with same inertia number, which is rebounded from the airfoil, moves on a similar trajectory. Therefore, it is also concluded that a generation of particle trajectory by using the “inertia number” is reasonable.

Figure 10-11 and Figure 10-12 show area averaged volumetric erosion rate distributions over the airfoil for various inflow Mach number at same inertia number. It is also found that the solid particles with the same inertia number results in the same erosion patterns over the 2D airfoil surface, which is not depended upon the inflow Mach number. An inflow Mach number is relative to only the magnitude of erosion rate over the airfoil.

Therefore, it is concluded that a generation of erosion damage prediction by using “inertia number” is also reasonable when the coefficient for erosion rate magnitude is introduced. However, in the generalization with an inertia number, the particle size must be also considered as a control parameter.
Figure 10-7. Trajectory of solid particles for various Mach number around the airfoil at $\lambda = 0.5$

Figure 10-8. Trajectory of solid particles for various Mach number near the leading edge of the airfoil at $\lambda = 0.5$
Figure 10-9. Trajectory of solid particles for various Mach number around the airfoil at $\lambda = 100.0$.

Figure 10-10. Trajectory of solid particles for various Mach number near the leading edge of the airfoil at $\lambda = 100.0$. 
Figure 10-11. Area averaged erosion rate distributions on the pressure surface for inflow Mach number at $\lambda = 100.0$.

Figure 10-12. Area averaged erosion rate distributions on the suction surface for inflow Mach number at $\lambda = 100.0$. 
According to the results obtained in this chapter, it is found that the characteristic length is a good non-dimensional parameter to generalize solid particle trajectory and erosion damage on a 2D airfoil surface.

As presented in Chapter 5.1.2, an erosion damage on 2D airfoil by large size sand particles (\(D_p > 100\mu m\)) is much greater than that by small size sand particles (\(D_p < 75\mu m\)). Moreover, a weight ratio of small sand particles (\(D_p < 75\mu m\)) is very small as shown in Table 10-1. Therefore, an actual erosion damage on a 2D airfoil and a 3D helicopter rotor blade due to the collision with small size sand particles (\(D_p < 75\mu m\)) is negligible small comparing with the erosion damage by large size sand particles (\(D_p > 100\mu m\))

Figure 10-13 shows area-averaged erosion rate distributions for “all ranges of sand particle sizes (\(D_p = 10\sim 250\mu m\))” and “only one sand particle size (\(D_p = 150\mu m\))”. At this figure, the erosion rate distribution for “only one sand particle size” represents the erosion damage on the airfoil surface by sand particles when all sand particles suspended in air are considered to be uniform size (\(D_p = 150\mu m\)). The erosion rate for “all ranges of sand particle sizes” indicates that accumulate erosion damage on the surfaces due to various size sand particles with weight ratio. The particle size and its weight ratio are selected based on measurement data as shown in Table 10-1. Moreover, total mass flow rate of two cases are specified to be same in order to remove the effect of weight on the erosion characteristics. As shown in erosion simulation results for various sand particle sizes, which is presented in Chapter 5.1.2, the erosion distribution for \(D_p > 100\mu m\) is very much similar. Therefore, in this study, 150 \(\mu m\) is chosen as a reference particle size.

Figure 10-13 shows that erosion rate distribution for “only one sand particle size” has reasonably small difference against that for all ranges of sand particle size. It is conclude that the effect of sand particle size on the erosion characteristics can be negligible when a reference particle size is reasonable.
Table 10-1 Sand particle size and its weight ratio distribution at helicopter flight test (Cowherd, 2007)

<table>
<thead>
<tr>
<th>Particle size</th>
<th>Size distribution</th>
<th>Weight ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0~15 µm</td>
<td>27.33%</td>
<td>0.000891</td>
</tr>
<tr>
<td>15~75 µm</td>
<td>21.37%</td>
<td>0.025806</td>
</tr>
<tr>
<td>75~125 µm</td>
<td>30.48%</td>
<td>0.294401</td>
</tr>
<tr>
<td>125~250 µm</td>
<td>20.82%</td>
<td>0.678841</td>
</tr>
</tbody>
</table>

In conclusion, the erosion characteristics over a 2D airfoil can be presented as a function of only inflow angle of attack as shown in Equation 63 and Equation 64.

\[
ER_{2D,airfoil} = f(M_0, \alpha_0, D_p) = C_{V_0} \cdot g(\alpha_0)
\]  

(63)

where, \(C_{V_0}\) is erosion coefficient relative to inflow velocity. This coefficient is the same as the function of impact velocity, \(g_2\), presented in Chapter 3.3.

For sand particle impacting on Ti-6Al-4V surface,

\[
C_{V_0} = g_2 \left( \frac{V_{p_1}}{60} \right)^{2.35}
\]  

(64)

Figure 10-13. Comparison of area averaged erosion rate distributions on a SC1095 airfoil for all range of particle size and for one particle size.
10.2. Generalization of Erosion Damage Prediction on a 3D Helicopter Rotor Blade

In this section, flow condition and sand erosion damage on a 3D helicopter rotor blade were compared to the results obtained from flow and erosion damage simulations of a 2D airfoil surface in order to evaluate the application of the generalization concept for flow simulation and erosion damage predictions.

Inflow boundary condition for flow and erosion damage simulation of a 2D airfoil is specified as the same as the relative inflow conditions to the blade section of interest.

Figure 10-14 presents the comparison between negative pressure coefficient distribution over the blade section at \( r = 0.875R \) of a 3D helicopter rotor blade and negative pressure coefficient distribution over the 2D SC1095 airfoil surface. As shown in this figure, the pressure coefficient distribution over the blade section of a 3D rotor blade is in good agreement with that of the 2D airfoil surface. This result concludes that the relative inflow condition measured at the plane in front of the leading edge of the 3D rotor blade is a reasonable reference condition for the analysis of aerodynamic and erosion characteristics. It is also found that when relative inflow condition for a 3D rotor blade is same as the inlet boundary condition for the 2D airfoil, flow characteristics on the 3D rotor blade can be predicted by using the 2D flow simulation results.

Figure 10-15 shows the comparison between the area averaged volumetric erosion rate distribution over the blade section at \( r = 0.875R \) of a 3D helicopter rotor blade and erosion rate distribution over 2D SC1095 airfoil surface. It is observed that the maximum erosion rate for the 2D airfoil surface is greater than that of the 3D rotor blade. This result shows also that the maximum erosion rate position for 2D airfoil is observed at more downstream region when compared to that of the 3D rotor blade. The change rate in the decrease of erosion damage after
reaching a maximum value for a 3D rotor blade is much greater than the change rate for a 2D airfoil surface. This difference may be caused by the difference in induced aerodynamic drag force near the 2D airfoil and 3D rotor blade. Flow over a 2D airfoil surface generates less induced aerodynamic force compared to that of the flow over a 3D rotor blade. Smaller aerodynamic forces affect the impact velocity of sand particles and the magnitude of erosion rate distribution. The difference of erosion characteristics between on the 3D rotor blade and 2D airfoil is also caused by the down wash generated by the rotation of 3D rotor blade, which affects continuously the motion of sand particles. The down wash causes the change in impingement angle and results in a visible drop of erosion damage over the 3D rotor blade.

Although of the difference between the magnitude and position of maximum erosion rate between a 2D airfoil and a 3D rotor blade, the overall erosion rate distributions on the 3D rotor blade are very similar to that of the 2D airfoil.

Figure 10-16 shows the comparison between negative pressure coefficient distribution over the blade section at \( r = 0.925R \) of a 3D helicopter rotor blade and that over a 2D SC1095 airfoil surface. The relative inflow velocity to the leading edge of the 3D rotor blade is increased with increasing radial position. Therefore, the supersonic flow is observed on the suction surface of the 2D airfoil. However, the flow over the blade section of the 3D rotor blade does not reach sonic flow condition due to aerodynamic losses. However, the flow characteristics on the 2D airfoil can be considered to be similar to the result on the 3D rotor blade.

Figure 10-17 presents the comparison between the area averaged volumetric erosion rate distribution over the blade section at \( r = 0.925R \) of the 3D helicopter rotor blade and that over a 2D SC1095 airfoil surface. Although high relative inflow velocity causes an increase in the difference of pressure coefficient between a 2D airfoil and a 3D rotor blade, the high inflow velocity generates high aerodynamic losses of the flow approaching to the 2D airfoil surface, which decreases the induced aerodynamic forces and sand erosion damage over the 2D airfoil.
This result also indicates that overall erosion characteristics on the 3D rotor blade can be predicted from the erosion damage results of the 2D airfoil (except for an absolute erosion rate magnitude).

Although there is a little difference of flow condition and erosion damage between the 2D airfoil and the 3D rotor blade, the aerodynamic and erosion characteristics for a 3D rotor blade can be predicted by using flow simulation and erosion damage methods over a 2D airfoil. These results also concluded that an application of the generalization for sand erosion damage prediction for a 3D rotor blade with a correction parameter, $C_{3D}$.

$$ER_{3D,\text{blade section}} = C_{3D} \cdot f(M_0, \alpha_0, D_p) = C_{3D} \cdot C_{V_0} \cdot g(\alpha_0)$$

(65)

where, $C_{V_0}$ is erosion coefficient relative to inflow velocity. This coefficient is the same as the function of impact velocity, $g_2$, presented in the Chapter 3.3

For sand particle impacting on Ti-6Al-4V surface,

$$C_{V_0} = g_2(V_p) = \left(\frac{V_p}{60}\right)^{2.35}$$

(66)

As shown in these generalization results, sand erosion characteristics of a 3D helicopter rotor blade can be predicted by using 3D flow-only simulation result and 2D flow and erosion damage simulation results. This method can reduce significant computational cost.
Figure 10-14. Comparison of negative pressure coefficient distribution on a 3D UH-60A rotor blade at \( r = 0.875R \) and that on a 2D SC1095 airfoil at corresponding condition.

Figure 10-15. Comparison of area averaged erosion rate distribution on a 3D UH-60A rotor blade at \( r = 0.875R \) and that on a 2D SC1095 airfoil at corresponding condition.
Figure 10-16. Comparison of negative pressure coefficient distribution on a 3D UH-60A rotor blade at $r = 0.925R$ and that on a 2D SC1095 airfoil at corresponding condition.

Figure 10-17. Comparison of area averaged erosion rate distribution on a 3D UH-60A rotor blade at $r = 0.925R$ and that on a 2D SC1095 airfoil at corresponding condition.
Chapter 11

Influence of Blade Contour Change on Flow and Erosion Damage

In most erosion damage predictions, only “accumulated erosion rate distributions” on “the fixed profile” of metal surfaces due to the impact of a unit mass of solid particles were investigated. However, in real erosion applications, erosion damage continuously changes the rotor blade profile. The geometric changes result in a significant distortion in aerodynamic contour and alteration in the overall helicopter performance. Moreover, the change of aerodynamic characteristics near the rotor blade surface may affect the most recent particle trajectories and erosion characteristics.

In the case of sand erosion phenomena on a helicopter main rotor blade, a slight alteration in the rotor blade profile can lead to a significant aerodynamic loss because of highly complex and 3-dimensional flow around a helicopter rotor blade including shock waves at the advancing blade and stall/separated flow at the retreating blade.

Therefore, in this current study, the changes in the profile of a SC1095 airfoil—a main airfoil profile of a UH-60A helicopter main rotor blade—due to erosion damage for reference particle injection conditions were investigated. The effect of the airfoil contour change on aerodynamics and erosion characteristics were analyzed and presented.
11.1. **The Schematic of a Geometric Change**

Figure 11-1 shows the flow chart for the investigation of the interaction between the airfoil profile change and the deterioration of aerodynamics & erosion rate distribution over the airfoil. In current procedure, first, an erosion damage distribution over the airfoil per unit time is calculated by using the general purpose viscous flow field solver, ANSYS-FLUENT, Lagrangian particle tracking algorithm, and a suitable rebound and erosion model. Next, the step time is calculated until the maximum erosion damage over the airfoil reaches step damage value. In this study, step damage value is specified as 0.005 inches (5 mils). Finally, the new airfoil profile damaged during the selected step time is determined.

![Flow chart showing the procedure for the continuous alteration of airfoil shape due to erosion damage](image-url)
Figure 11-2 defines the overall geometrical change procedure the airfoil contour due to erosion damage. In this procedure, three assumptions are applied to calculate material removal on each grid cell of the airfoil. The first assumption is that the deformation of each cell occurs uniformly over the whole area of each cell. Next, the geometric deformation occurs only in the vertical direction of airfoil geometry. Final assumption is that amounts of material removal from each cell per unit time are constant during time step. In this current study, a specific numerical procedure based on Fortran-language was developed for the surface erosion modification model.

In order to determine the new grid position altered by erosion damage, the line $[y_i = mx_i + n]$, which is normal to the original airfoil geometric line and the crossing point at the i-th cell point, $(x_i, y_i)$, must be defined. These coefficients of the normal line, $m$ and $n$, can be defined by equations from Equation 67 to Equation 69. Next calculation is for the magnitude of deformation at each grid cell occurring at the end of an operating time when the maximum damage on any cell reaches a proscribed erosion level of 5 mils. The deformation magnitude at each grid cell is calculated by Equation 70. Finally, a new position of grid cell, $(x_{ni}, y_{ni})$, is determined by Equation 70 ~ Equation 73.

- Line equation normal to the airfoil geometry at grid cell : $[y_i = mx_i + n]$  

\[
dx_i = \frac{L_{i+1} dx_i + L_i dx_{i+1}}{L_i + L_{i+1}} \quad (67)\]

\[
dy_i = \frac{L_{i+1} dy_i + L_i dy_{i+1}}{L_i + L_{i+1}} \quad (68)\]

where, $L_i$ and $L_{i-1}$ the length of (i) th and (i-1) th grid cell, $dx_i$ and $dy_i$ are the change coordinate in $x$ & $y$ direction

\[
\therefore \quad m = -\frac{dx_i}{dy_i}, \quad n = y_i - mx_i \quad (69)\]
• Magnitude of the deformation due to erosion damage at each grid cell:

\[ z_i = \epsilon_v \cdot m_p \cdot t \quad (70) \]

where, \( \epsilon_v \): volumetric erosion rate due to impact of a unit mass of solid particle \([\text{m}^3/\text{g}]\),

\( m_p \): mass flow rate of solid particle \([\text{g/s}]\), \( t \): operating total time \([\text{s}]\)

• New coordinate of a grid point, \((x_{n_i}, y_{n_i})\):

\[
z_i = \sqrt{(x_{n_i} - x_i)^2 + (y_{n_i} - y_i)^2}
= \sqrt{(x_{n_i} - x_i)^2 + (m x_{n_i} + n - m x_i - n)^2}
\quad (71)

\[ z_i^2 = (m^2 + 1)(x_{n_i} - x_i)^2 \quad (72) \]

\[ x_{n_i} = x_i \pm \frac{z_i}{\sqrt{m^2 + 1}}, \quad y_{n_i} = m x_{n_i} + n \quad (73) \]

Figure 11-2. Schematic of geometric change model due to erosion damage
11.2. Rotor blade Contour Change Calculation due to Sand Erosion

In order to investigate the material removal on the airfoil profile due to erosion damage and the change in aerodynamic and erosion characteristics caused by a geometry change, the initial computations have been performed under reference conditions defined in Table 11-1. In general, it is currently accepted that the main rotor blade of a helicopter fails when the maximum erosion damage reaches 30 mils. Therefore, the step variable (the maximum deformation) is set to 5mils (0.005 inches) and the number of damage steps is set to 6. In this analysis, the same inflow boundary condition and injection conditions of released solid particles were applied at all six “damage steps”.

Table 11-1 Computational conditions for airfoil surface change study

<table>
<thead>
<tr>
<th>Flow condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_0 = 0.7$ and $\alpha_0 = 0.0^\circ$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Particle released condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{p_0} = 0.7, \alpha_{p_0} = 0.0^\circ, D_p = 150\mu m$, and $\Phi = 0.002543498$</td>
</tr>
</tbody>
</table>

Figure 11-3 and Figure 11-4 show the airfoil profile at each damage step. Figure 11-5 and Figure 11-6 are material removal distribution on pressure and suction surfaces at each damage step. As shown in these results, the maximum material removal over the airfoil occurs at 2.5 percent chord length on the suction surface. On pressure surface, the maximum material removal occurs at approximately 1.5 percent chord length. The section up to 10 percent chord length of airfoil near the leading edge was damaged for the current computational conditions. These results also indicate that material removal at each damage step is quite similar, but the erosion becomes more pronounced as the airfoil is further damaged.
Figure 11-3. Airfoil profile at each damage step

Figure 11-4. Zoom in near leading edge of Airfoil profile at each damage step
Figure 11-5. Material removal distribution around leading edge on the pressure side

Figure 11-6. Material removal distribution around leading edge on the suction side
Figure 11-7 and Figure 11-8 present the absolute static pressure distributions on the pressure and suction surfaces. The change in static pressure is maximum at about 2.5 percent chord length on the suction side. These results indicated that erosion damage may change aerodynamic characteristics measurably.

Figure 11-9 ~ Figure 11-10 show the volumetric erosion rate on the pressure and suction surface at each damage step. These results show the incremental erosion damage after each step. The maximum change in accumulative erosion occurs at the location of maximum erosion rate. Moreover, the erosion rate is slightly increased as the airfoil is more damaged. The change in erosion rate magnitude is caused by the change of local impact velocity and impingement angle as shown in the Figure 11-11 ~ Figure 11-13.
Figure 11-7. Static pressure distribution on the pressure side at each damage step

Figure 11-8. Static pressure distribution on the suction side at each damage step
Figure 11-9. Erosion rate distribution on pressure side

Figure 11-10. Erosion rate distribution on suction side
Figure 11-11. Impingement angle distribution near leading edge at each damage step

Figure 11-12. Impact velocity distribution near leading edge at each damage step
These results indicated that the erosion damage causes measurable geometric change of the airfoil. The deformation of the airfoil shape leads to a distortion of aerodynamic characteristics around the airfoil. This aerodynamic change affects the local impact conditions such as impact velocity and impingement angle. The change of impact conditions affects the erosion characteristics. Therefore, a geometric change of the airfoil is an important effect to predict in helicopter rotor airfoil erosion studies.
Chapter 12

Summary and Conclusions

12.1. Summary

In this dissertation, first, a comprehensive review of past open literature on the quantification of erosion damage on metallic surfaces, especially in helicopter rotor blade and turbomachinery system, was performed in order to develop a computational methodology for predicting sand erosion damage on an actual helicopter rotor blade.

- The trajectory of sand particles within the computational domain is determined by Eulerian-Lagrangian particle tracking algorithm because the volume fraction of sand particle suspend in air is very small. In the Eulerian-Lagrangian particle tracking algorithm, the motion of an individual sand particle can be calculated by using previous-solved flow simulation result.

- The impact conditions can be also determined by the Eulerian-Lagrangian particle tracking algorithm. The impact conditions represent collision position, impact velocity, impingement angle, and particle’s properties.

- By using the impact conditions, erosion damage on the surface of interest is calculated by using a proper rebound and erosion models. The rebound model determines the motion of a solid particle, velocity and direction, after the impact. The erosion model indicates the amount of mass or volume removed from the metallic surface due to the collision with sand particles. The rebound model and erosion model are highly depended upon the material of the solid particle and the metallic surface material.
Next, the methodologies for sand particle tracking and sand erosion damage prediction on a helicopter rotor blade were developed in order to solve the limitation of previous methods.

- Suitable and time-efficient injection (release) conditions of sand particles were developed with a new injection parameter, “solid particle mass flow rate (SPmFR)” in order to deal with the effect of non-uniform and unsteady relative inflow condition to blade section of a 3D rotor blade and in order to reduce the computation cost.
- The definition of erosion rate was modified by multiplication with “SPmFR” in order to predict overall erosion characteristics on a 3D rotor blade in terms of an operating time.
- New erosion and rebound models for sand particles impacting on Ti-6Al-4V were developed by using existing experimental data
- C++-language based specific codes (User Defined Functions) were developed and linked to ANSYS-FLUENT in order to control injection conditions and to perform erosion damage modeling.

Next, in this dissertation, validation for flow-only simulation results on a 2D airfoil and 3D rotating rotor blades were performed. In addition, in order to select suitable turbulence model, flow and erosion damage simulations for various turbulence models were performed.

- The flow-only simulation results for a 2D stationary airfoil (RAE2822) and 3D rotating rotor blade (NACA0012) obtained in this dissertation have negligible small difference against experimental results.
- It is also found that the computational aerodynamic results with $k-\omega$ SST turbulence model are the most successful computations.
Flow characteristics of a 3D rotor blade can be usually analyzed by using a 2D flow simulations results because the effect of spanwise inflow velocity on aerodynamics and performance can be neglected. Therefore, in this dissertation, an investigation of aerodynamic and erosion characteristics on a 2D airfoil surface was performed in order to understand a detailed erosion mechanism for an actual helicopter rotor blade.

- Erosion characteristics for a 2D airfoil is mainly dependent upon inflow velocity, inflow angle of attack and diameter of a solid particle. The data set is defined as inflow boundary condition.
- Inflow velocity affects dominantly the magnitude of erosion rate on both surfaces of airfoil. However, erosion pattern including maximum erosion rate position and the extent of erosion damaged area on both surfaces is not dependent upon the inflow velocity
- Erosion patterns, except for erosion rate magnitude, are highly dependent upon inflow angle of attack and particle diameter.

Relative inflow conditions to the blade section of a 3D rotating blade is influenced by geometric conditions of the rotor blade as well as operating conditions of the helicopter. Therefore, in this dissertation, the effect of geometric conditions of a rotor blade on aerodynamics and erosion characteristics was analyzed.

- The magnitude of erosion rate on the blade section of a rotor blade is linearly proportional with the radius of the blade section. This is because relative inflow velocity is increased as increasing radial position. However, near the rotor blade tip, the erosion damage is decreased due to the circulatory flow generated at the rotor tip. The circulatory flow results in aerodynamic losses, which increases the induced drag force acting on solid particle.
• A spanwise twist of the rotor blade changes effective angle of attack distribution in spanwise direction, which affects erosion pattern on the blade section including maximum erosion damage position and the extent of erosion damaged area on the blade section. However, the magnitude of erosion rate is not affected by the spanwise twist.

• The swept tip does not affect the erosion characteristics on the linear part of the rotor blade. However, the swept tip reduces erosion damage on the swept tip section of the rotor blade because of a reduction of normal velocity to the blade section.

• The erosion characteristics on a 3D rotor blade is a function of relative inflow condition which is also depended upon rotor blade geometric condition

Next, flow and erosion damage simulations on an actual rotor blade, a main rotor blade of UH-60A helicopter, for operating conditions was performed.

• As the collective pitch angle is increased, the effective angle of relative inflow to blade section is increased and the magnitude of induced inflow velocity is also increased. These increases result in higher erosion damage and move maximum erosion damage position to a downstream location.

• The vertical flight motion of a helicopter alters the resultant velocity distribution of the relative inflow and effective angle distribution over the rotor blade, which affects the erosion characteristics

• Although there is a little change in the magnitude of erosion rate and the maximum erosion damage for operating conditions, Hover condition can be chosen as a reference operation condition for minimizing erosion damage, optimizing the rotor blade geometric condition, or evaluating sand erosion damage reduction method
Particle tracking and erosion damage prediction on a 3D rotor blade required high computational cost. Therefore, in this dissertation, a generalization of particle trajectory and erosion characteristics on a 3D rotor blade was performed.

- As shown in the erosion simulation results for generalization parameters, erosion pattern by same size sand particles is not dependent upon initial (injection) velocity of sand particles. The injection velocity of a sand particle affects the magnitude of erosion damage.

Sand particles suspend in air can be considered to be uniform size sand particle (\(D_p > 100\mu m\)) because erosion damage by sand particles for \(D_p < 100\mu m\) is negligible small.

- The erosion characteristics over the 2D airfoil can be presented as a function of only inflow angle of attack with a coefficient for inflow velocity (injection velocity of a solid particle).

- Although there is small difference of erosion damage between a 2D airfoil and a 3D rotor blade, erosion damage on a 3D rotor blade can be predicted by generalized 2D erosion simulation results and a correction constant.
12.2. Final Conclusion

The present dissertation provides a complete and unique methodology for predicting sand particle trajectories and their erosion damage on a 2D stationary airfoil and an actual rotating helicopter rotor blade under realistic hover, climb and descent flight conditions.

In this dissertation, specific particle trajectory and rebound & erosion calculation scheme were implemented into a 3D RANS based general fluid dynamic solver, ANSYS-FLUENT. The sand erosion prediction methods developed in this dissertation are able to simulate the erosion rate and accumulative erosion damage of helicopter rotor blades sufficiently and accurately for hover, climb and descent conditions.

This dissertation describes completely detailed phenomena of sand particle trajectories and sand erosion damage on 2D airfoils and on 3D helicopter rotor blades. In addition, this dissertation suggests advanced methods for sand erosion damage reduction such as an optimal swept tip against sand erosion and performance and a blowing system.

Finally, this dissertation suggests a new method for generalization of sand particle trajectory and sand erosion damage on helicopter rotor blades. This method can reduce a significant computational cost and time.
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Appendix A

Governing Equation and Numerical Procedure

A.1 Governing Equation

In this dissertation, the time-dependent compressible and turbulent Navier-Stokes equations were solved by finite-difference approximation in order to simulate a reasonably accurate flow around a stationary 2D airfoil and rotating 3D blades. Properties of the turbulent flow in the exact Navier-Stokes equations can be separated into a sum mean (time-averaged) part and a fluctuation part as shown in Equation A-1 and Equation A-1 because local quantities of the turbulent flow vary randomly with time.

\[ u_i(x, t) = \overline{u_i(x, t)} + u'_i(x, t) \] (A-1)

where, \( u_i(x, t) \) is an instantaneous velocity, \( \overline{u_i(x, t)} = U_i(x, t) \) is a sum of mean velocity, and \( u'_i(x, t) \) is a fluctuating velocity.

\[ \phi = \bar{\phi} + \phi' \] (A-2)

where, \( \phi \) is a scalar quantities such as density and pressure.

The governing equations with applying these Reynolds averaging in to Navier-Stokes equations, which is called Reynolds-averaged Navier-Stokes (RANS) equations, are follows.

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho U_i) = 0 \] (A-3)

\[ \frac{\partial}{\partial t}(\rho U_i) + \frac{\partial}{\partial x_i}(\rho U_i U_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial U_k}{\partial x_k} \right) \right] + \frac{\partial}{\partial x_j} \left( -\rho u'_i u'_j \right) \] (A-4)
The Reynolds averaged Navier-Stokes equations (RANS) is almost same as the time-dependent Navier-Stokes equations with all dependent variables representing time-averaging values except for an additional term, $-\rho \bar{u}_i \bar{u}_j$. The additional term is called Reynolds stresses and presents the effects of turbulence. Therefore, in order to obtain a complete flow simulation result, a modeling of Reynolds stresses are required.

### A.2 Turbulence Model

As mentioned in Appendix A.1, the Reynolds-averaged Navier-Stokes equations require a modeling for the Reynolds stresses in Equation A-4. Various turbulence models were developed in order to deal with Reynolds stress terms. In a common turbulence modeling, Boussinesq hypothesis (61) is employed to simplify the Reynolds stresses by using mean velocity gradients as shown in Equation

$$-\rho \bar{u}_i \bar{u}_j = \mu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \left( \rho k + \mu_t \frac{\partial U_k}{\partial x_k} \right) \delta_{ij}$$  \hspace{1cm} (A-5)

where, $\mu_t$ is turbulent viscosity

The Boussinesq hypothesis has a advantage for computing turbulent viscosity, which reduces the computational cost. Therefore, in this dissertation, turbulence models using this approach were used.

#### A.2.1 The Spalart-Allmaras Turbulence Model

The Spalart-Allmaras turbulence model (62) is an empirical equation for modeling a turbulent viscosity. A main advantage of the Spalart-Allmaras model is the simplicity in imposing the free-stream and wall boundary conditions, so this model needs the least expensive turbulence
model. This turbulence model introduces turbulent viscosity solving a scalar transport equation with a diffusive term, destruction and production terms as shown in Equation A-6. Moreover, this empirical model can be treated for free shear flows, near-wall region with high or finite Reynolds number, and laminar regions.

\[
\frac{\partial \nu}{\partial t} + \nabla \cdot \nu \nabla - \frac{1}{\sigma} \left( \nabla \cdot \left( (\nu + \nu) \nabla v \right) + c_{D1} |\nabla v|^2 \right) - c_{D2} \bar{v} \nabla \nu - \bar{v} \nabla \nu = 0 \quad (A-6)
\]

where, \( \nu \) is fluid velocity, \( \bar{v} \) is a working variable and \( \nu \) is the molecular viscosity

\[
\bar{v} = \nu + \frac{\nu}{k^2 d^2} f_{v_2} \left( \frac{\nu}{\bar{v}} \right) \quad (A-7)
\]

\[
f_{v_2} (x) = 1 - x / [1 + x f_{v_1} (x)] \quad (A-8)
\]

\[
f_{v_1} (x) = x^3 / [x^3 + c_{v_1}^3] \quad (A-9)
\]

where, \( \omega = || \nabla \times \nabla || \) and \( c_{v_1} = 7.1 \)

\[
f_{\omega} (\nu) = g \left( \frac{1 + c_{\omega_3}^6}{g^6 + c_{\omega_3}^6} \right)^{1/6} \quad (A-10)
\]

\[
g (h) = h + c_{\omega_2} (h^6 - h) \quad (A-11)
\]

\[
h (\nu) = \nu / (\bar{v} \nu) \quad (A-12)
\]

\[
c_{\omega_1} = \frac{c_{D1}}{k^2} + \frac{1 + c_{D2}}{\sigma} \quad (A-13)
\]

where, \( c_{D1} = 0.1355, c_{D2} = 0.622, \sigma = 2/3, c_{\omega_2} = 0.3, \) and \( c_{\omega_3} = 2.0 \)

Finally, the turbulent viscosity \( \mu_t \) can be calculated by Equation A-14.

\[
\mu_t = \nu f_{v_1} \left( \frac{\nu}{\bar{v}} \right) \quad (A-14)
\]

\[
f_{v_1} (x) = \frac{x^3}{x^3 + c_{v_1}^3} \quad (A-15)
\]

where, \( \chi = u / \bar{v} \)
A.2.2 The $k - \epsilon$ (Standard) Turbulence Model

The $k - \epsilon$ (standard) model was first suggested by Launder et al. (63). The $k - \epsilon$ standard model is a semi-empirical model based on two dependent transport equations for the turbulence kinetic energy ($k$) and its dissipation rate ($\epsilon$) as shown in Equation A-16 and Equation A-17.

\[ \frac{\partial k}{\partial t} + \rho U_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial U_i}{\partial x_j} - \rho \epsilon + \frac{\partial}{\partial x_j} \left[ \left( \frac{\mu + \mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] \]  
\[ \frac{\partial \epsilon}{\partial t} + \rho U_j \frac{\partial \epsilon}{\partial x_j} = C_{\epsilon1} \frac{\epsilon}{k} \tau_{ij} \frac{\partial U_i}{\partial x_j} - C_{\epsilon2} \rho \frac{\epsilon^2}{k} + \frac{\partial}{\partial x_j} \left[ \left( \frac{\mu + \mu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right] \]  

where, $C_{\epsilon1} = 1.44$, $C_{\epsilon2} = 1.92$, $\sigma_k = 1.0$, and $\sigma_\epsilon = 1.3$

Next, turbulent viscosity, $\mu_t$, can be determined by turbulence kinetic energy and it dissipation rate as follows:

\[ \mu_t = C_\mu \frac{\rho k^2}{\epsilon} \]  

where, $C_\mu = 0.09$

As mentioned, the $k - \epsilon$ (standard) model is a semi-empirical model dependent highly upon phenomenological considerations and experimental results. This model can not predict accurately turbulent flow in viscous sub-layer due to the deficiency of a suitable viscous damping functions to model zero and adverse pressure gradient boundary layers. However, although of the shortcoming, the $k - \epsilon$ (standard) model is used for a wide range of turbulent flow and heat transfer simulations because of its robustness, economy, and reasonable accuracy.
A.2.3 The \( k - \epsilon \) RNG Turbulence Model

The \( k - \epsilon \) RNG turbulence model introduced by Yakhot et al. (64) was derived using a rigorous statistical technique [RNG (renormalization group) theory]. The RNG theory provides an analytically-derived differential formula for turbulent Prandtl numbers and effective viscosity that accounts for low-Reynolds-number effects. The RNG model has an additional term in its \( \epsilon \) equation that significantly improves the accuracy for rapidly strained flows. Therefore, this turbulence model improves significantly the accuracy of near wall flows, separated flows and flows in curved geometries. Moreover, the effect of swirl on turbulence is included in the RNG model, enhancing accuracy for swirling flows.

Transport equation for \( k \) and \( \epsilon \) are follows:

\[
\rho \frac{\partial k}{\partial t} + \rho U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_i} \left( \alpha_k \mu_{eff} \frac{\partial k}{\partial x_i} \right) + \mu_t S^2 - \rho \epsilon \quad (A-19)
\]

\[
\rho \frac{\partial \epsilon}{\partial t} + \rho U_j \frac{\partial \epsilon}{\partial x_j} = \frac{\partial}{\partial x_i} \left( \alpha_e \mu_{eff} \frac{\partial \epsilon}{\partial x_i} \right) + C_{\epsilon 1} \frac{\epsilon}{k} \mu_t S^2 - C_{\epsilon 2} \frac{\epsilon^2}{k} - R \quad (A-20)
\]

where, \( C_{\epsilon 1} = 1.44, \ C_{\epsilon 2} = 1.92, \ \sigma_k = 1.0, \ \text{and} \ \sigma_\epsilon=1.3 \)

The additional term \( R \) in RNG \( k - \epsilon \) turbulence model is given by

\[
R = \frac{C_\mu \eta^3 (1 - \frac{\eta}{\eta_0}) \epsilon^2}{1 + \beta \eta^3} \quad (A-21)
\]

Where, \( \eta = Sk/\eta_0, \eta_0 = 4.38, \beta = 0.012, \)

This \( R \) term can improve significantly the accuracy for rapidly strain flows which can show in rearrange form of \( \epsilon \) equation. The inverse effective Prandtl numbers \( \alpha_k \) and \( \alpha_\epsilon \) can be obtained from analytical formula, which derived from RNG theory.
A.2.4 The $k - \omega$ (Standard) Turbulence Model

The $k - \omega$ (standard) turbulence model introduced by Kolmogorov (65) is being developed and modified in many studies. The $k - \omega$ (standard) turbulence model used in recent turbulent flow simulations is an optimized model performed by Wilcox (66). This turbulence model incorporates modifications for low-Reynolds-number effects, compressibility, and shear flow spreading. Therefore, computation with $k - \omega$ (standard) turbulence model can predicts wall-bounded flows and free shear flows, which are in good agreement with experimental results.

The $k - \omega$ standard turbulence model is an empirical model based on model transport equations for the turbulence kinetic energy ($k$) and the specific dissipation rate ($\omega$) as shown in the Equation A-22 and Equation A-23.

\[
\rho \frac{\partial k}{\partial t} + \rho U_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial U_i}{\partial x_j} - \beta^* \rho k \omega + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma^* \mu_t) \frac{\partial k}{\partial x_j} \right] \tag{A-22}
\]

\[
\rho \frac{\partial \omega}{\partial t} + \rho U_j \frac{\partial \omega}{\partial x_j} = \alpha \frac{k}{\omega} \tau_{ij} \frac{\partial U_i}{\partial x_j} - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma \mu_t) \frac{\partial \omega}{\partial x_j} \right] \tag{A-23}
\]

where, $\alpha = 5/9$, $\gamma^* = 1$, $\beta = 3/40$, $\beta^* = 9/100$, $\sigma = 1/2$, $\sigma^* = 1/2$

In this turbulence model, a turbulence viscosity is calculated by Equation A-24.

\[
\mu_t = \gamma^* \frac{\rho k}{\omega} \tag{A-24}
\]

Compared with the $k - \epsilon$ standard turbulence model, the $k - \omega$ standard turbulence model is in better agreement with measurements of turbulence boundary layers for all pressure gradient conditions. This turbulence model can be integrated though the viscous sublayer without
applying complicated damping function. However, the $k - \omega$ model which is likely to other two-equation model does not predict correctly the asymptotic behavior of the flow as it approaches to the wall.

A.2.5 The $k - \omega$ SST (Shear Stress Transport) Turbulence Model

The $k - \omega$ SST turbulence model was developed by (67). This turbulence model is modified by the blending function. The function is designed to ensure that the model equations behave appropriately in both the near-wall and far-field zones.

The SST model incorporates a damped cross-diffusion derivative term in the $\omega$ equation in order to account for the transport of the turbulent shear stress. Therefore, the $k - \omega$ SST turbulence model has more accurate and reliable for a wider class of flows.

\[
\frac{\partial k}{\partial t} + \rho U_j \frac{\partial k}{\partial x_j} = \rho P_k - \beta^* \rho k \omega + \frac{\partial}{\partial x_j} \left( \mu + \sigma_k \mu_t \right) \frac{\partial k}{\partial x_j} \quad (A-25)
\]

\[
\frac{\partial \omega}{\partial t} + \rho U_j \frac{\partial \omega}{\partial x_j} = \alpha \rho S^2 - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left( \mu + \sigma_\omega \right) \frac{\partial \omega}{\partial x_j} + 2(1 - F_1) \sigma_{\omega^2} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_i} \quad (A-26)
\]

where, $\alpha_1 = 5/9$, $\alpha_2 = 0.44$, $\beta_1 = 3/40$, $\beta_2 = 0.0828$, $\beta^* = 0.09$, $\sigma_{k_1} = 0.85$, $\sigma_{k_2} = 1.0$, $\sigma_{\omega_1} = 0.50$, and $\sigma_{\omega_2} = 0.856$.

\[
F_2 = \tanh \left[ \max \left( \frac{2\sqrt{k}}{\beta^* \omega y^2} \frac{500v}{y^2} \right) \right] \quad (A-27)
\]

\[
P_{\kappa} = \min \left( \frac{\partial U_l}{\partial x_i}, 10 \beta^* k \omega \right) \quad (A-28)
\]

\[
F_1 = \tanh \left[ \min \left( \frac{\sqrt{k}}{\beta^* \omega y^2} \frac{10}{\omega y^2} \omega^2, \frac{4 \sigma_{\omega^2} k}{\omega y^2} \right) \right] \quad (A-29)
\]

\[
CD_{k\omega} = \max \left( 2 p \sigma_{\omega^2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}, 10^{-10} \right) \quad (A-30)
\]
A.3 Numerical Procedure

A.3.1 Density-Based Solver

In this dissertation, the density-based solver was used because this solver has accuracy advantage for high-speed compressible flow. In the Fluent, a general purpose viscous solver used in this dissertation, there are two different formulations in the density-based solver – implicit and explicit. In this study, the implicit formulation was used due to its stability and quick-convergence.

The governing equation in vector form is follows.

\[
\frac{\partial}{\partial t} \int_V \mathcal{W} dV + \int_V [\mathcal{F} - \mathcal{G}] \cdot dA = \int_V \mathcal{H} dV
\]

(A-31)

where, \(\mathcal{W}, \mathcal{F},\) and \(\mathcal{G}\)

\[
\mathcal{W} = \begin{pmatrix} \rho u \\ \rho v \\ \rho w \\ \rho E \end{pmatrix}, \quad \mathcal{F} = \begin{pmatrix} \rho v \\ \rho vu + p_i \\ \rho vw + p_j \\ \rho vE + pv \end{pmatrix}, \quad \text{and} \quad \mathcal{G} = \begin{pmatrix} 0 \\ \tau_{xi} \\ \tau_{yi} \\ \tau_{zi} + q \end{pmatrix}
\]

(A-32)

where, \(\rho, v, E, p, \tau,\) and \(q\) are the density, velocity, total energy per unit mass, pressure, viscous stress tensor and heat flux.

\[
E = H - \frac{p}{\rho} = h + \frac{|v^2|}{2} - \frac{p}{\rho}
\]

(A-33)

A.3.2 Roe Flux-Difference Splitting Scheme

In order to calculate flux vector \(\mathbf{F}\) at each control surface on a specific grid cell, Roe flux-difference splitting scheme, as shown in Equation A-34.
\[ F = \frac{1}{2} (F_R + F_L) - \frac{1}{2} J \delta Q \]  
\[ \delta Q = Q_R - Q_L \]

where, \( \delta Q \) is the spatial difference, \( F_R = F(Q_R) \) and \( F_L = F(Q_L) \) are flux vector at the left and the right side on a specific face, and \( Q_R \) and \( Q_L \) are solution vector at the left and the right side on the face.

\[ \left| \vec{A} \right| = M|\lambda|M^{-1} \]  
\[ \lambda_\mp \] is the diagonal matrix of eigenvalues and \( M = \Gamma^{-1} \) is the modal matrix, and 
\[ \lambda = \partial F / \partial Q \] is the inviscid flux jacobian.

**A.3.3 Upwind Spatial Discretization**

Upwind scheme in spatial discretization means flux and scalar quantities at each face are calculated by quantities in the cell upstream ("upwind") relative to the direction of the normal velocity, \( v_n \). In this dissertation, 2nd-order upwind scheme was used because high-order finite difference scheme for spatial derivative can improve solution accuracy while the high-order scheme requires high computation cost.

The value of velocity at each face, \( U^- \) or \( U^+ \), can be calculated by following equations.

\[ U^- = \frac{3U_i^n - 4U_{i-1}^n + U_{i-2}^n}{2\Delta x} \]  
\[ U^+ = \frac{-U_{i+2}^n + 4U_{i+1}^n - 3U_i^n}{2\Delta x} \]

Moreover, the plus (minus) Jacobian matrix has only positive (negative) eigenvalues and is computed by Equation A-38 and Equation A-39.

\[ A_\pm = X_1 AX_1^{-1} \]  
\[ A_\mp = \frac{1}{2} (A_1 \pm |A_1|) \]
A.3.4 Implicit Formulation

In this dissertation, an Euler implicit scheme combined with a Newton-type linearization was used in order to discretize in time of the governing equations.

\[
\begin{bmatrix}
\mathbb{D} + \sum_{j}^{N_{\text{faces}}} S_{j,k}
\end{bmatrix} \Delta Q^{n+1} = -R^n
\]  \hspace{1cm} (A-40)

where, \(R^n\) is residual vector.

\[
\mathbb{D} = \frac{\mathbb{V}}{\Delta t} \Gamma + \sum_{j}^{N_{\text{faces}}} S_{j,i}
\]  \hspace{1cm} (A-37)

\[
S_{j,k} = \left( \frac{\partial F_j}{\partial Q_k} - \frac{\partial G_j}{\partial Q_k} \right) A_j
\]  \hspace{1cm} (A-41)
Appendix B

Grid Sensitivity Analysis

It is well known that flow simulation results are highly dependent upon grid system. For the grid system with small number of grid cells, computations with the grid system perform low accuracy of simulation results due to numerical error. Although computations with large number of grid cells can improve flow solution’s accuracy, the computations required high computational cost. Therefore, in this dissertation, in order to select a proper grid system for reasonably accurate and time-efficient flow simulation, grid sensitivity analysis for a 2D airfoil were performed.

Figure B-1 show the Multi-block H-type structure grid system for predicting flow around a 2D airfoil. In this section, a $280 \times 80$ structured grid system for each computation domain is chosen as a reference grid system. In this analysis, four difference grid systems were considered. The height, which is a normal length to the airfoil surface, of first grid cell from the surface for all grid systems is same.

Figure B-1. Multi-block H-type structured grid system for 2D simulation around airfoil
Computational condition for the grid sensitivity analysis is inflow Mach number $M_0 = 0.75$ and inflow angle of attack $\alpha_0 = 2.5^\circ$. Figure B-2 show negative pressure coefficient distribution for four different grid systems. As shown in these results, a sudden drop in negative pressure coefficient is observed at about 35 percent chord length on the suction surface, which is caused by a normal shock. A small reduction of negative pressure coefficient is also observed at 7.5 percent chord length on the pressure surface, which is caused by expansion wave generated at the end of a leading edge.

As shown in Figure B-3, simulation results with $120\times40$ and $200\times60$ structured grid systems have differences in pressure coefficient distribution with the results with with $280\times80$ and $340\times100$ structured grid systems. This is because numerical smoothing due to small grid cell number (large cell size) as shown in Figure B-4.

These results indicated that computation with $280\times80$ and $340\times100$ structured grid systems can predict accurate flow characteristics, in especial, shock, expansion wave, and separated flow. Therefore, in this dissertation, a $280\times80$ H-type structured grid system is selected optimal grid system for predicting flow and erosion characteristics on a 2D airfoil and a 3D helicopter rotor blade.
Figure B-3. Negative pressure coefficient distributions for different grid systems (zoom)

Figure B-4. Mach number contours around SC1095 airfoil
Appendix C

User Defined Function (UDF)

C.1 UDF for Injection Conditions

```c
/* User Define Function for Inject Initialization of Solid particle */
/* For 2D case */
/* Ver 1.20 - To add option to select rebound and erosion model */
/* Modified by Bong Gun Shin on Oct. 29. 2008 */
/* Ver 2.00 - To add injection option for 3D Non- and Rotating Rotor Blade */

#include "udf.h"
#include <stdio.h>

DEFINE_DPM_INJECTION_INIT(dpm_ini, I)
{
    Particle *p;
    double x1,x2,y1,y2,z1,z2;
    double dx, dy, dz, u, v, w;
    double ddx,ddy,ddz;
    double rhog, rhop, omega, Vinlet, phi, rpm;
    double diap, pi, pre;
    double x,y,z, mg;
    int j, k, ns, nh, ns1, nh1, k1, iref, nlimit, icount;
    FILE *fp1;

    fp1=fopen("output_dpm_injecion.out","a");

    /* Reading input parameters */
    pi = 4.*atan(1.) ;

    /* Calculating control variables */
    rhog=1.176608;
    rhop=1650.0;
    phi=0.002543498;
    rpm=238.0;
    omega=2.*pi*rpm/60.;
    diap=150.0e-06;
    ns=501;
    nh=2001;
    nlimit=ns*nh;
    x1=0.35;
```
y1=-0.15;
z1=0.750;
x2=0.35;
y2=0.15;
z2=8.250;
u=0.0;
v=0.0;
w=0.0;

ns1=ns-1;
h1=nh-1;

ddz=z2-z1;
dz=(ddz)/((double) ns1);
ddy=y2-y1;
dy=(ddy)/((double) nh1);
ddx=x2-x1;
dx=(ddx)/((double) ns1);

x=x2;
y=y2;
z=z2;

j=0;
k=0;
icount=0;

iref=nh;

/* Loop for each particle */
loop(p,I->p)
{
    k=k+1;
k1=k-1;
icount=icount+1;
    /* x=x2-((double) k1)*dx; */
x=x2;
y=y2-((double) k1)*dy;

    if(k>iref)
    {
        j=j+1;
y=y2;
x=x2;
z=z2-((double) j)*dz;
k=1;
    }

    Vinlet=omega*z;
    pre=phi/(1.-phi);
    mg=rhog*Vinlet*dy*dz*pre*1.0e03;

    P_POS(p)[0]=x;
P_POS(p)[1]=y;
P_POS(p)[2]=z;
P_VEL(p)[0]=u;
P_VEL(p)[1]=v;
P_VEL(p)[2]=w;
P_DIAM(p)=diap;
}
fclose(fp);
}

C.2 UDF for Erosion Model

/* User Define Function for Erosion of Solid particle */
/* For 3D case */

#include "udf.h"
#include <stdio.h>
DEFINE_DPM_EROSION(dpm_cluster, p, t, f, normal, alpha, Vmag, Mdot)
{
  real A[ND_ND];
  int ier; /* ier=1 (mass) 2(avg. mass) 3(volume) 4(avg. volume) */
  double pi;
  double ER;
  double Vn, Vt;
  double beta_rad, beta_deg, beta, alpha_deg;
  double cn0,cn1,cn2,cn3,cn4;
  double ct0,ct1,ct2,ct3,ct4;
  double et, en;
  double pre1, pre2, pre3, pre4;
  double pre, new, mp;
  double dia, xi, yi, zi, xf, yf, zf, xf1, sign;
  double den_ti, ckl, at;
  double a0,a1,a2,a3,a4;

  FILE *fp;
  fp=fopen("output_dpm_erosion.out","a");
  pi = 4.*atan(1.); 
  dia=P_DIAM(p);
  xi=P_INIT_POS(p)[0];
  yi=P_INIT_POS(p)[1];
  zi=P_INIT_POS(p)[2];
  Vn= NV_DOT(P_VEL(p),normal);
\[
V_t = \sqrt{V_{mag}^2 - V_n^2}; \\
\text{pre} = \arctan(V_n/V_t); \\
alpha_{deg} = \alpha \cdot 180./\pi; \\
\beta = \alpha_{deg}; \\
\beta_{deg} = \alpha_{deg}; \\
\beta_{rad} = \alpha; \\
\]

\[
\begin{align*}
\text{cn0} &= 1.00; \\
\text{cn1} &= -7.258824 \times 10^{-3}; \\
\text{cn2} &= -1.521259 \times 10^{-4}; \\
\text{cn3} &= 1.55244 \times 10^{-6}; \\
\text{cn4} &= 0.0; \\
\text{ct0} &= 1.0; \\
\text{ct1} &= -3.700098 \times 10^{-2}; \\
\text{ct2} &= 9.375515 \times 10^{-4}; \\
\text{ct3} &= -5.848235 \times 10^{-6}; \\
\text{ct4} &= 0.0; \\
\end{align*}
\]

\[
\begin{align*}
\text{pre} &= \text{cn0} + \text{cn1} \cdot \beta + \text{cn2} \cdot \beta^2; \\
\text{en} &= \text{pre} + \text{cn3} \cdot \beta^3 + \text{cn4} \cdot \beta^4; \\
\text{pre} &= \text{ct0} + \text{ct1} \cdot \beta + \text{ct2} \cdot \beta^2; \\
\text{et} &= \text{pre} + \text{ct3} \cdot \beta^3 + \text{ct4} \cdot \beta^4; \\
\end{align*}
\]

\[
\begin{align*}
\text{if}(\text{ier} > 2) \\
&\{ \\
&\quad \text{den}_t = 4.428784 \times 10^6; \\
&\} \\
\text{else} \\
&\{ \\
&\quad \text{den}_t = 1.0; \\
&\} \\
\end{align*}
\]

\[
\begin{align*}
\text{if}(\beta_{deg} < 25.4824) \\
&\{ \\
&\quad a_0 = 0.; \\
&\quad a_1 = 5.232377 \times 10^{-2}; \\
&\quad a_2 = 2.491313 \times 10^{-5}; \\
&\quad a_3 = -2.1912 \times 10^{-5}; \\
&\quad a_4 = 0.0; \\
&\} \\
\text{else if}(\beta_{deg} < 51.29) \\
&\{ \\
&\quad a_0 = -7.94953; \\
&\quad a_1 = 9.26776 \times 10^{-1}; \\
&\quad a_2 = -3.406321 \times 10^{-2}; \\
&\quad a_3 = 5.245457 \times 10^{-4}; \\
&\quad a_4 = -2.942052 \times 10^{-6}; \\
&\} \\
\text{else} \\
&\{ \\
&\quad a_0 = 2.104237; \\
&\quad a_1 = -6.230386 \times 10^{-2}; \\
&\}
\end{align*}
\]
a2=7.116803*10.\(^{-4}\);
a3=-2.891157*10.\(^{-6}\);
a4=0.0;
}

pre1=a0+a1*beta_deg;
pre2=a2*pow(beta_deg,2);
pre3=a3*pow(beta_deg,3);
pre4=a4*pow(beta_deg,4);
pre=pre1+pre2+pre3+pre4;
ckl=0.13*10.\(^{-3}\);
pre1=Vmag/60.0;
pre2=pow(pre1,2.35);
ER=ckl*pre2*pre/den_ti*mp*1.0e-3;

if(ier==2)
{
    F_AREA(A,f,t);
    at = NV_MAG(A);
}
else if(ier==4)
{
    F_AREA(A,f,t);
    at = NV_MAG(A);
}
else
{
    at=1.0;
}

xf=P_POS(p)[0];
yf=P_POS(p)[1];
zf=P_POS(p)[2];

if(yf<0.0)
{
    sign=-1.0;
}
else
{
    sign=1.0;
}

xf1=xf*sign;

fprintf(fp,"%e,%e,%e,%e,%e,%e,%e,%e,%e,%e,%e\n",xf1,xf,yf,zf,at,Vmag,beta_deg,alpha_deg,mp,ER,dia);
fclose(fp);
VITA

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   - Doctoral Dissertation : “Prediction of Sand Particle Trajectories and Sand Erosion Damage on Helicopter Rotor Blades”
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