The Pennsylvania State University

The Graduate School

Department of Energy and Mineral Engineering

MODELING OF PROPPANT TRANSPORT THROUGH HYDRAULIC FRACTURE NETWORK

A Thesis in

Energy and Mineral Engineering

by

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Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

May 2014
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ABSTRACT

Hydraulic fracturing is one of the most efficient and effective methods to enable economic oil and gas production in shale gas plays. Hydraulic fracturing creates a fracture network system instead of a planar fracture in real cases. As a matter of fact, proppant transport in the fracture network is extremely important to long-term fracture conductivity. Several models are developed to simulate proppant placement in fracture networks. The models are coupled to a finite-difference based fracture network pressure propagation model developed by Chong Hyun Ahn (Ahn, Wang, 2012) which is able to simulate the propagation of fracture networks. Once the fracture geometry and pressure profile is obtained from the model, the proppant distribution can be generated from the proppant transport models. The four different models are linked to the fracture propagation model with an increasing degree of coupling. With more criteria added on and assumptions taken off, the models are able to generate accurate and trustable proppant distribution in fracture networks.

Screen out, one of the most common problem encountered in fields, can also be predicted by these models and engineers can utilize the models to monitor proppant distribution in the network and prevent screen out. Models in this thesis simulate different screen out behaviors and mechanisms, and also identify locations of high screen out potential during the treatment under different scenarios. In general, pressure gradient is a key factor during proppant transport in fracture network. With higher pressure gradient, proppant particles have enough energy to be carried further into the network. As a matter of fact, to maintain a smooth flow field, the pad
stage of the treatment is very critical that a larger pad zone ensures higher pressure gradient that the fluid has enough energy to transport proppant with higher velocity and to a further distance. Combined with the pressure fracture propagation model, several controlling factors that affect proppant transport can be obtained from the proppant distribution models. With the models, engineers can design treatments along with the logic and new findings to improve proppant displacement.
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NOMENCLATURE

\[ \pi \] \quad \text{pi}
\[ d \] \quad \text{radius of a proppant particle (ft)}
\[ C_d \] \quad \text{drag coefficient}
\[ \rho_f \] \quad \text{fluid density (lb/ft}^3\text{)}
\[ \rho_l \] \quad \text{liquid phase density (lb/ft}^3\text{)}
\[ V_s \] \quad \text{Settling velocity (ft/s)}
\[ N_{Rep} \] \quad \text{particle Reynolds number}
\[ \mu \] \quad \text{fluid viscosity (cp)}
\[ \phi_s \] \quad \text{slurry porosity}
\[ V_{original} \] \quad \text{original settling velocity (ft/s)}
\[ V_{con} \] \quad \text{settling velocity affected by concentration effect (ft/s)}
\[ w \] \quad \text{width of the fracture (ft)}
\[ f_w \] \quad \text{wall effect coefficient}
\[ V_{x,proppant}^n \] \quad \text{proppant velocity in flow direction (ft/s)}
\[ V_{x,fluid}^n \] \quad \text{fluid flow velocity (ft/s)}
\[ \text{flowrate}^n_{fluid} \] \quad \text{injection rate (bbl/day) (ft}^3\text{/s)}
\[ \text{cross section}^n_{yz} \] \quad \text{cross section area in y-z plane (ft}^2\text{)}
\[ V_{settles,proppant}^n \] \quad \text{proppant settling velocity (ft/s)}
\[ \Delta t \] \quad \text{a time step (s)}
\[ Z_{proppant}^n \] \quad \text{the top of the slurry (ft)}
\[ H_{fracture}^n \] \quad \text{the height of the fracture (ft)}
**PropCon**<sub>n+1</sub>\(_x+t,unit\) proppant concentration of a section (lb/ft<sup>2</sup>)

\( W^n_{fracture_x} \) width of a section of the fracture (ft)

\( X^n_{proppant} \) the front of the slurry (ft)

\( q^n_{leak} \) leak off volume (ft<sup>3</sup>)

\( C^{n+1}_{slurry} \) concentration of slurry (lb/gal) (lb/ft<sup>3</sup>)

\( \Delta L \) length of a fracture section (ft)

\( sumprop^n \) total proppant mass in a section (lb)

\( propmass^n \) settled proppant mass in a time step (lb)

\( \Delta V^{frac} \) volume of a section of the fracture (ft<sup>3</sup>)

\( \rho_{bed} \) density of settled proppant bank (lb/ft<sup>3</sup>)

\( bedH \) height of the settled proppant bank in a time step (ft)

\( sumbedH^n \) total height of the settled proppant bank (ft)

\( P_a \) pressure at point a (psi)

\( P_b \) pressure at point b (psi)

\( K \) permeability (md)

\( B_o \) volumetric factor of oil (RB/STB)

\( B_g \) volumetric factor of oil (RCF/SCF)

\( B_w \) volumetric factor of oil (RB/STB)

\( \phi \) porosity

\( S \) saturation

\( V_b \) block volume (ft<sup>3</sup>)

\( k_{rw} \) relative permeability of water

\( k_{ro} \) relative permeability of oil

\( k_{rg} \) relative permeability of gas

\( R_{so} \) ratio of gas dissolves in oil (SCF/STB)

\( R_{sw} \) ratio of gas dissolves in water (SCF/STB)
$P_{cow}$ capillary pressure between oil and water (psi)

$W_{wij}^{n+1k}$ transmobility term in west direction of water

$E_{wij}^{n+1k}$ transmobility term in east direction of water

$S_{wij}^{n+1k}$ transmobility term in south direction of water

$N_{wij}^{n+1k}$ transmobility term in north direction of water

$q_{wij}^{n+1k}$ water phase flow rate

$Q_{wij}^{n+1k}$ summation term of the transmobility terms

$Conppit_{block,i,j}^{n+1}$ volumetric proppant concentration of a block (lb/ft$^3$)

$ProppantMass_{sum,i,j}^{n+1}$ proppant mass in a block (lb)

$V_{block,i,j}$ block volume (ft$^3$)

$ProppantLoss_{block,i,j}^{n+1}$ proppant mass that settles out of the pay zone (lb)

$H_{block,i,j}$ height of a block (ft)
ACKNOWLEDGMENTS

I would like to thank my family at the first place that they have been supporting me to obtain my master degree. Also I want to thank the members of the Taiwanese student association for accompanying me during the years. Thanks for Dr. Wang, my advisor, for spending a lot of effort on me guiding me to finish my job and learn great knowledge and attitude during the period.
CHAPTER 1

INTRODUCTION

Hydraulic fracturing has been widely used in shale gas exploration. In order to make the development economically feasible, successful fracture treatment is a prerequisite to stimulate these low-permeability hydrocarbon bearing rocks. Significant increase in production rates and reserves could be obtained after the treatment.

In hydraulic fracturing, fracture fluid is pumped into the formation to generate fractures. The aperture of the fracture provides better conductivity and connectivity in the pay zone. When a fracture fluid creates enough width to accept proppant, usually around 10% to 50% percent of the total treatment volume, slurry with proppant will be introduced to the fracture. In shale gas, the pad volume is around 10% of the total treatment since the permeability of the reservoir is low. Proppant concentration of the slurry increases as the treatment continues. A common increase in industry of the concentration of the slurry is from 0.5 lbm/gal to 5 lbm/gal. Another reason for the increasing proppant concentration is to make sure that the proppant can be smoothly introduced to the fracture without a high risk of screen out. At the end of the treatment, engineers often introduce slurry with higher proppant concentration into the fracture that ensures the near-wellbore region maintains high fracture conductivity.
Proppant transport is one of the most important factors that determine the treatment is successful or not. A better treatment will create a higher and larger stimulated volume of the pay zone. Modeling proppant transport can provide proppant distribution and fracture conductivity, which enhances one to optimize the treatment.

Previous proppant transport research was focusing on the transport through the bi-wing fractures perpendicular to the minimal in-situ stress. However, in shale gas, proppant will be transported into fracture network system. Instead of a propped planar fracture only, a propped stimulated zone will be created. In order to simulate the proppant movement in the networks, several models are developed to capture the behavior of fracture fluid flow and proppant movement. The goal is to obtain an accurate distribution of proppant in the networks. With the above data, one is able to quantify the degree of stimulation, to evaluate the treatment is successful, and to optimize operations.
2.1 Basics of particle settlement – single particle settlement

Movement of proppant particles in a fracture is controlled by different forces that work on the proppant particles. In Cartesian coordinates, velocities in the X, Y and Z directions can be obtained by the following assumptions and analysis.

For a single particle in a Newtonian fluid, the terminal settling velocity can be calculated by the balance of the gravitational force, buoyant force and the drag force that work on the particle. The gravitational force and the buoyant force work in different directions. The sum of the two forces can be simply expressed by \( \frac{\pi d^3 g (\rho_p - \rho_l)}{6} \), and the drag force for a single particle in Newtonian fluid can be expressed by \( \frac{\pi d^2 \rho_l \mu C_d V^2}{8} \). However, the drag coefficient is not just a constant but a function of particle Reynolds number. Different flow regions are used to describe the settling of a single particle.

The Reynolds number of particles is expressed as follows:

\[
N_{Rep} = \frac{d_p V_p \rho_f}{\mu}
\]

For the Stoke-law region, \( N_{Rep} \leq 2 \), \( C_d = 24/ N_{Re} \)
For the intermediate region, \( 2 < N_{Re}p < 500, C_d = 18.5/ N_{Re}^{0.6} \)

\[
V_S = \frac{g d_p^2 (\rho_p - \rho_f)}{18 \mu} \]

And for the Newton’s law region, \( 500 \leq N_{Re}, C_d = 0.44 \)

In most of the situation during proppant settlement, the flow around the particle is considered laminar flow. It means once the proppant particle and the fluid reach a balanced state, the relative motion of the solid and the liquid is stable and at a slow pace. Under this circumstance, the equation of the Stoke-law region can be used to describe the settling velocity of a single particle. In this thesis only Newtonian fluid will be discussed for application in slick water treatment in shale gas system.

2.2 Effect of proppant concentration (multiple proppant particles)

In the previous section the behavior of a single particle settlement can be described by the given equations. However, in real cases, the settling velocity of the slurry, in other words, settling the velocity of the particles, is the one engineers want to capture. Concentration effect puts the settling velocity into a larger scale to describe a group of particles settling in a fluid not only a single one. Steinour (1944) and Richardson and Zaki (1954) stated the relations between slurry concentration and settling velocities in Newtonian fluid (Richardson, 1954; Steinour, 1944). The
researches have been cooperated with the three different flow regions stated in the previous section. Exponential relations have been found between the flow regions and slurry porosity. Slurry porosity $\varnothing s$ is the volumetric ratio of the slurry. In hydraulic fracturing treatment, fluid will loss into the formation and somehow increases the concentration of the slurry and also decreases the slurry porosity. Richardson and Zaki (1954) explained that because of the viscosity and density increases, the relative velocity around the particles increases correspondingly (Richardson, 1954). As a matter of fact, with the increase of the relative velocity, the drag force working on the particles will increase and finally cause the decrease of the settling velocity. The summary of the concentration effect can be expressed as:

For the Stoke-law region, $N_{Rep} \leq 2$, $C_d = 24/ N_{Rep}$

$$V_{con} = V_{original} \times \varnothing s^{5.5}$$

For the intermediate region, $2 < N_{Rep} < 500$, $C_d = 18.5/ N_{Rep}^{0.6}$

$$V_{con} = V_{original} \times \varnothing s^{3.5}$$

For the Newton’s law region, $500 \leq N_{Rep}$, $C_d = 0.44$

$$V_{con} = V_{original} \times \varnothing s^{2}$$

2.3 Wall effect

In hydraulic treatment, the settlement of proppant particles will be hindered by the fracture wall. The random, unsmoothed fracture wall gives a resistance to the particles near to it. Fidleris (1961) and Sutterby (1973) provided some trustable correlations for the settling velocity between the
fracture walls (Sutterby, 1973; Whitmore, 1961). Wall effect coefficient $f_w$ is used to calibrate the terminal velocity and can be expressed as:

For the region, $N_{Rep} \leq 1$

$$f_w = 1 - 0.6526 \cdot \left( \frac{d}{w} \right) + 0.147 \cdot \left( \frac{d}{w} \right)^3 - 0.131 \cdot \left( \frac{d}{w} \right)^4 - 0.0644 \cdot \left( \frac{d}{w} \right)^5$$

For the region, $N_{Rep} \geq 100$

$$f_w = 1 - \left( \frac{d}{2w} \right)^\frac{3}{2}$$

For the regions in between, a linear interpolation can be used to obtain the wall effect coefficient. In most of the situations, the Reynolds number is lower than 1 and the first equation is applicable.

### 2.4 Proppant transport with low viscosity fluid

Proppant depositional rate in low viscosity fluid is higher. In other words, the settling velocity in a low viscosity environment is higher. The controlling equation of the terminal settling velocity shows that if the viscosity is lower, the terminal velocity increases. The mechanism behind is that the drag force working on the particles is lower due to the thinner agent. Finally the shear forces around the particles need longer distance to balance the momentum that causes a higher terminal velocity. According to the above reasons, proppant transport in a thinner fluid will lead to a bank of proppant that accumulates proppant falling from the suspending slurry rapidly at the bottom of the fracture. According to the researches done by Babcock et al, several flow regions can be characterized for different types of particle movements (Babcock, Prokop, & Kehle).
From the top to the bottom, the first region is the blank region that contains no proppant inside. All the proppants have left the region due to the settling. The second region is the zone of the main slurry body where the particles are suspended by the turbulence and viscosity. The third one is a zone that the particles roll instead of suspend, it has a higher density than the slurry but not totally settled and packed, called the fluidized region. The last one is the region that the particles are totally settled down at the bottom of the fracture. A stationary bed of proppant is formed. Based on the result of the experiment, the fluidized bed is so thin that can be neglected in the thin fluid based transportation. Medlin et al and Biot et al stated that a perfect proppant transport is very seldom in real cases. It is reasonable that because of fractures in reality have ununiformed and rough walls that could decrease the transmissibility of the particles so much. Also the ununiformity can capture particles during the suspension without settling down to the bottom (Blot & Medlin; Medlin, Sexton, & Zumwalt).

2.5 Proppant transport with high viscosity fluid

Proppant transport in a high viscosity fluid is another story. Due to the very high viscosity, particles can be nearly entirely suspended in slurry with very little settling. The fracture temperature now affects the settling more than the thin fluid. During the treatment, the temperature along the fracture usually increases from the wellbore to the tip. The increasing temperature starts to decrease the viscosity of the fluid. The settling along the fracture then increases. As a matter of fact, during the simulation of the using a high viscosity fluid, the leak off at the tip area has the most severe leakoff.
2.6 Screen out

Screen out is one of the most common problems encountered during the treatment. When a well could no more intake any slurry during injection, a screen out or a sand out are then taking places. Several mechanisms can cause the well to limited slurry injectivity. It happens very often during the treatment. According to *Recent Advances In Hydraulic Fracturing 2008 chapter 8* (Gidley, 1989). There are two kinds of screen out that the industries often come across with. The first one is the rapid screen out, and the second one is the gradual screen out.

2.7 The rapid screen out

The rapid screen out often occurs without much preliminary signs. The pressure builds up rapidly and then the well cannot intake proppant as the designed schedule. According to *Recent Advances In Hydraulic Fracturing, 2008, chapter 8* (Gidley, 1989), the rapid screen out often takes place around the near well bore area. Especially at the perforated area, the particles easily clots the through causing the fluid and the particles can no more be carried away from the zone and transported along the fracture. Once the situation happens at every perforated area, the blocking mechanism then brings about the screen out. The particle size also participates the mechanism to a certain degree. If the perforation hole is less than three times wider than the particle diameter, the bridging of proppant would occur and block the through and avoid further injection. Since once the small perforated pass way is easily blocked, the pressure can build up rapidly without any prior signals.
2.8 The gradual screen out

The gradual screen out is the one that the pressure builds up steadily and finally achieves the maximum power of the pump. Generally, the mechanism of the gradual screen out is that the proppant particles accumulate in the fracture and piles up to form a bed. In other situations the proppant don’t really have to form a bank of particles for the screen out to occur. The proppant particles can be trapped by the rough fracture face and accumulate all along the fracture. When the accumulation is enough to block the fluid flow and particle movement, the screen out takes place. The proppant capturing and banking can cause a decrease in fracture permeability. The fracture then has insufficient force to propagate the fracture. Once the fracture does not propagate, the pressure then could not be released.

2.9 Fracture permeability

The relationship between permeability and porosity can be derived by the Kozeny-Carman equation. The equation was developed by a series of capillary tube experiment. The equation was firstly developed by Kozeny et al., 1927. The original equation is stated as:

\[ \nu = \gamma (I/\mu) c (\phi^3 / \sigma_1^2) \]

Where \( \nu \) is the Darcy’s velocity, \( \gamma \) is the weight of the fluid and \( I \) is the hydraulic gradient. \( \phi \) is the porosity, \( \mu \) is the dynamic viscosity of the fluid, \( \sigma \) is the specific area and \( c \) is the geometry constant. The geometry factor varies with different shape of the cross section (Kozeny, 1927).

After the equation was stated, Carman (1937) then put some modification into the equation. Carman introduced the notion of hydraulic radius and then expressed the ratio of specific area
per mass will not change by the porosity. Another notion stated by Carman was that the non-linear pass of the water flow. Water in a capillary media has to conquer the tortuosity of the pass way. In the article, Carman stated that an average 45 degree of the tortuosity was used. The main idea of the Carman’s reinforcement was that the previous capillary system is now extended to a porous media-tube system. After the Navier-Stoke equation was taken into place, the capillary system was considered an assemblage of a continuous but tortuous porous media system (Carman, 1937). The modified equation can be derived as:

For Darcy’s Law:

\[ U = \frac{-k \, dP}{\mu \, dl} \]

Where the \( U \) term is the expression of the whole equation.

For a general Poiseuille equation:

\[ U = -\frac{A \, dP}{\alpha \mu \, dl} \]

The term \( \alpha \) is the dimensionless geometrical factor. For a cylindrical tube, the factor is \( 8\pi \). \( A \) is the general cross section area of a capillary micro channel system.

Combined the two equations we have:

\[ U = \frac{-c \, R^2 \, \phi \, dP}{8\mu \, \tau \, dl} \]

Where the \( c \) is introduced as a geometrical parameter, \( \phi \) is the porosity and \( \tau \) is the tortuosity defined as the square of the ratio of the effective length and the body length of the media.
The hydraulic radius $R_h$ is defined as $\frac{\varnothing}{a_v(1-\varnothing)}$, which means the ratio of pore volume to solid-liquid interfacial area. The term $a_v$ is the specific internal area. The equation can be rewritten into:

$$k = C_{kc} \frac{\varnothing^3}{(1 - \varnothing)^3}$$

For a more complex vertical cylindrical form, the equation is then expended to:

$$k = C \frac{g}{u_w \rho_w} \frac{e^3}{S^2 D_R^2 (1 + e)}$$

Where $k$ is the permeability or the hydraulic conductivity, $g$ is the gravitational constant, $u_w$ is the dynamic viscosity of water, $\rho_w$ is the density of water, $S$ is the specific area, $D_R$ is the specific weight of the solid ($\rho_s/\rho_w$) and the $e$ is the void ration. In some cases the void ration can be directly assumed to be the porosity of the media ($\frac{\varnothing^3}{(1-\varnothing)^3} = \frac{e^3}{(1-e)^3}$).

For a bed packed by particles, which may mostly fulfill the need of the scenario, the equation can be modified into:

$$\frac{\Delta p}{L} = \frac{180 V_0 \mu (1 - e)^2}{\varnothing^2 D^2 e}$$

The equation can be then again couple with the Darcy’s Law and derive the permeability of the porous-capillary media.
The following figure 2.1 shows the fracture permeability with different sizes of Ottawa frac sand proppant agents under different closure stresses. During the injection, since the fracture is still sustained by the pressure of fluid larger than the closure stress, the proppant are packed without fracture walls applying closure force on. As a matter of fact, the permeability can be obtained by reading the value under zero effective stress.

The KC equation generates similar results with figure 2.1 by calculating the porosity and permeability relationship. For 20/40 proppant, the average particle diameter is 630 µm. If the proppant bed is compacted with a hexagonal close-compaction, and the particles are assumed to be perfect spherical bodies, the porosity of the packed bed is 24.95%. By introducing the notion of hydraulic radius, correlation between porosity and permeability is more strongly bonded.

During the treatment, the fracture permeability will change along with the proppant filling the void volume within the aperture. With the change of permeability in fracture, models in this thesis try to quantify this dynamic procedure during the injection and characterize some phenomenon observed from the field experience.

The permeability change will be quantified and put into the program. A dynamic fracture permeability and porosity prediction can be obtained. However in this model, since it is semi-coupled that the permeability change could not be simulated. The change in permeability can be eventually built into the fracture propagation model to improve the capture of the flow behavior.
2.10 Fracture fluid characteristics

There are several fracturing fluids being used in the industry. In this thesis, two major types of liquid will be studied for the simulation. A high viscosity fluid and a low viscosity fluid, such as a linear gelled system and a slick water system, will give very different proppant transportability and also forms of particle transportation. Before we go into the discussion of the real fracturing fluid, general understandings of fluid systems have to be carefully understood. Basically, fluids

Figure 2.1: Effect of Ottawa sand mesh size on pack permeability 
(*Proppants, second edition, 1984*).
can be categorized into the Newtonian and non-Newtonian fluid based on different responses when subjected to shear forces. Figure 2.2 shows the categories of the fluids and the relation between shear stress and shear rate. As we can the Newtonian fluid maintains a constant value of the slope, which means the viscosity of the fluid will not change with the shear rate. Thinner fluid such as water would be considered Newtonian fluid. If the fluid viscosity increases with shear rate, the fluid is called the shear-thickening fluid or the dilatant fluid. In the opposite, the fluid with the thinning behavior with the increase of shear rate is called the pseudo-plastic fluid. The Bingham fluid has a character that the viscosity remains constant once the kick off point of the shear stress is achieved. An example is demonstrated and shown in figure 2.3. A viscosity vs. shear rate plot is measured under fixed temperature condition. We can observe that the viscosity of the HPG fluid doesn’t change so much at lower share rate, and once the shear rate passes 1 s⁻¹, the viscosity starts to decrease dramatically. In other words, the 1040 HPG fluid behaves like a Newtonian fluid at low shear rate region.

Not only shear rate can cause viscosity change but also temperate. In some situation, the temperature effect affects the viscosity even more than the shear rate. Figure 2.4 show the viscosity vs. temperature plot with different concentration of HPG additives. As we can see the more the HPG is added, the more the change in viscosity. For a 1040 HPG fluid, the viscosity decreases from 40cp to 20cp with a 60-170 F° temperature increase. A 20cp difference in viscosity is observed for this linear high viscosity gel system.

Pressure is another affecting factor changing the viscosity of the fluids. However in a liquid system, the pressure effect is so minimal that we could generally neglect its effect.
In this thesis, only water based fluid and linear system will be discussed. Two kinds of fluid as mentioned at the beginning of the section will be built into the models. For the low viscosity fluid, water is used to represent its characteristics. For the high viscosity one, the 1040 and 1080 HPG fluid are selected to emphasize the effect of viscosity change during the injection.

However, in real field cases, slick water is the most common treating fluid for shale gas reservoirs. Since slick water has low viscosity, immediate proppant settling could be expected. In this thesis, only thin fluid would be simulated due to the reality. Proppant transport using high viscosity fluid would be put into future researches.

![Fluid type classification base on viscosity.](image)
Figure 2.3: Viscosity vs. shear rate plot for a 1040 HPG fracture fluid (Guillot, 1985).
Figure 2.4: High viscosity HPG, viscosity vs. temperature (Gidley, 1989).
2.11 Characteristics of shale gas reservoirs

Shale gas is a kind of unconventional resources that the gas is stored in a very fine-grained shale formation. The formation is basically composed of clay minerals bearing high organic matters. The sedimentary environment of black shale, containing higher total organic matter, is basically formed under very deep basin, a relatively tranquil surroundings. As a matter of fact, these fine-grained particles can accumulate and also the organic matter can be preserved under an anoxic condition. In general, shale gas formation contains very high water saturation due to its schistic structure of clay minerals forming cage-typed pore spaces. There is no traditional gas trap formation in shale gas systems. Hydrocarbons are directly stored and explored within the shale formation which was often treated as the source under the traditional oil system concept. The concept of resource triangle was stated by Gray (1977) and Maters (1979), showing the relations between the abundance, reservoir quality and engineering conditions. Shale gas is located at the lower left corner of the triangle (Gray, 1977). Figure 2.5 indicates that shale gas is a large amount of resource; however, the permeability and reservoir quality are almost the lowest among all the resources. Therefore, the increasing technology challenge is inevitable.

![Figure 2.5: Resource triangle concept (Gray, 1977).](image-url)
During the maturation of the hydrocarbons, the pressure starts to build up due to the generation of lighter components of the hydrocarbon family. Once the formation rock could not sustain the pressure, the rock starts to crack down to create some more space for the newly generated hydrocarbons. Hence the shale gas plays usually have a system of natural fractures. In the fractures, the permeability and the porosity are significantly higher than the matrix. Not only the fracture network helps the connection and acquisition, a significant amount of hydrocarbon is also stored in the natural fractures. During the treatment, the natural fracture system sometimes helps the hydraulic fracturing to enhance its stimulated area. The permeability of the matrix, according to geological samples and experiments, is around 0.001md, which is extremely low.

Fracture fluid of shale gas treatment should be carefully selected. The goal of the treatment is to reduce the damage caused by fracture fluid as much as possible. Water with a little additive may be the most common treating fluid for shale gas reservoirs. If the fracture fluid has a higher content of gelling agent, the breaker schedule must be accurately executed. Otherwise the gel could block the pores and result in serious damage. According to Kundert and Mullen, 2009, if the shale formation is brittle, slick water is a suitable choice for the treatment. Due to its low viscosity, the portability of proppant is lower so the slurry concentration should be fewer. Also the leakoff of the low-viscosity fluid is more significant than a gel system. As a matter of fact, the total volume of the treating fluid is higher. However if the formation is too ductile, probably caused by higher clay or organic contents, a thicker gel system is appropriate for the treatment. Less fluid is used because the fluid has higher viscosity and also the fluid is able to carry more proppant per volume of the slurry. Similarly the damage control should be precisely executed. Else the formation rock might be successfully fractured but the pores and natural fracture network are clotted, leading the stimulation to failure (Kundert & Mullen).
Table 2.1: fracture fluid selection based on different reservoir properties (Kundert and Mullen, 2009).

<table>
<thead>
<tr>
<th>Rock Properties</th>
<th>Frac Fluid</th>
<th>Fluid Volume</th>
<th>Prop Concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brittle</td>
<td>Slickwater frac</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Laminated</td>
<td>Hybrid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ductile</td>
<td>Linear or XL gel</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>

If underpressured, use foam or gas assist.
CHAPTER 3

SIMPLE T-JUNCTION MODEL

3.1 Model description

The T-junction model is our first simplest model which has a T-shaped fully symmetric fracture. The velocity of the fluid flow can be derived by the ratio of flow mass rate and cross section area. In a symmetric case, the flow velocity in the branched can be easily obtained by the division of the flow mass and the crosssection area. In this case, the proppant transport module is linked to a fracture network pressure propagation model. Once the fracture geometry and pressure profile are available, the proppant transport module is able to predict proppant distribution along the fracture network systems. Several assumptions are made in this module. First one is that the pressure profile is assumed to be unchanged under the simulation of proppant transport. Secondly, velocity of proppant is the same as the fluid velocity in flow direction. An assumption of no relative velocity between proppant and fracture fluid is made. Proppant settling is modeled by the settling velocity stated in the previous section. The module tracts the very top proppant particle until it touches the very bottom of the pay zone. Proppant is assumed to be injected into the pay zone though the whole section of the fracture initial point. Based on the module, the proppant particles move out of the pay zone is considered to be lost and will not be calculated for proppant concentration in pay zone. A cartoon is shown in figure 3.1 and figure 3.2 indicating the main idea of the injection model. In the figure, the yellow part is the pay zone, and the blue
part is the non-pay zone confining formation. The green part is the slurry that stays in the pay zone.

Figure 3.1: Concept of the T-junction model.

Figure 3.2: Proppant transport model without accumulation.
The red part is the part that lost to the formation and will not be calculated into the concentration. The slurry is injected from the left hand side. The slurry is assumed to enter the pay zone the entire cross section. No particle interaction and relative motion are assumed in the module. For each time step, the slurry front progrades for a certain distance. The distance can be calculated by the product of the velocity of the slurry front and the time step. The velocity can be decreased by leak off since the cross section has less flow rate to come in to maintain the velocity. Effects of leak off and fracture wall effect are shown in the coming sections for different scenarios.

The injection continues following two conditions are reached. The first one is once the very top particle of the slurry descends beneath to the bottom of the pay zone. This circumstance means that the furthest distance that the particle can be transported. Further injection will not result in a larger propped length. As a matter of fact, the treatment stops to prevent ineffective injection. The second condition is the assigned schedule is reached. For the two constrains stated above, the model is able to optimize or simulate a designed injection. Cases will be demonstrated in the next section.

The controlling equations of the model:

\[ V_{x, \text{proppant}}^n = V_{x, \text{fluid}}^n = \text{flowrate}_{\text{fluid}}^n / \text{cross section}_{yz}^n \]

\[ V_{\text{settle}, \text{proppant}}^n = V_S = \frac{g a_p^2 (\rho_p - \rho_f)}{16 \mu} \]

For most of the situation. Settling velocities for different Reynolds number can be corrected by equations in the literature review section.

\[ X_{\text{proppant}}^n = V_{\text{proppant}}^n \Delta t \]
For a new time step:

If the leak off is taken into place:

\[ q_{fluid}^{n+1} = \text{flowrate}_{fluid}^{n+1} = q_{fluid}^{n+1} - q_{leak} \]

\[ V_{x,prop}^{n+1} = V_{x,fluid}^{n+1} = (q_{fluid}^{n+1} - q_{leak}) / \text{cross section}_{yz}^{n+1} = q_{fluid}^{n+1} / \text{cross section}_{yz}^{n+1} \]

\[ C_{slurry}^{n+1} = C_{slurry}^{n} \times \frac{q_{fluid}^{n}}{q_{fluid}^{n+1}} \]

If other effects such as the concentration effect and the wall effect take place, the equations can be modified by the given corrections listed in the critical literature review. The original and corrected results for different scenarios are shown in the next paragraph.

![Figure 3.3](image.png)

Figure 3.3 : Proppant transport model with accumulation and banking.
Previous assumption was that once proppant particles reached the lower boundary, the proppant is lost to the pay zone. The proppant concentration in fracture is thoroughly depending on the slurry concentration and the slurry height. No proppant accumulation and deposition take place in this model. However, if a fracture is well monitored and the height growth is also well predicted, accumulation of proppant can be calculated. In a slick water treatment, a proppant bed forms immediately during the injection since the viscosity of the fluid is very low. Another proppant depositional method is developed to simulate proppant bed accumulation process. Figure 3.3 shows the proppant accumulation model. The method is a little different from the previous model. This model is constructed under a section-completing algorism, which is that the model determines a segment of a fracture network first, and then calculates the time for the slurry to complete the section, instead of determining the treatment time at first. The reason why this algorism is developed is because of the ability to calculate the deposited proppant mass and the change in flow velocities for every section. In order to minimize the deviation of the model, the segment length is very small, that the time and space errors can be decreased. Unlike the previous model, the accumulation model will not stop once the top of the slurry meets the lower boundary of the pay zone. If the slurry meets the lower limit, the proppant particles are then still accumulating in the fracture. Once the proppant bed is high enough to create a higher velocity of the flow, the slurry front will be able to prograded further into the fracture. As shown in figure 3.3, once the slurry hits the boundary at time T1, the accumulation continues. An assumption is made that once proppant particles settle down, the particles are immobile, since the liquefied layer is very trivial in slick water system. For the packed bed, a very loose packing is assumed, since the proppant bed is not imposed to strong compacting force. Several time steps later, the proppant bed grows higher deducing the cross sectional area to create higher velocity and the
slurry moves forward to reach the next section S2 at T2, S3 at T3 and so on. The process stops when the scheduled treatment time is achieved or the screen out takes place.

The controlling equations of the model:

The segment length is determined first and the time to complete the section can be calculated.

\[ V_{proppant}^n = \frac{q_{fluid}}{\text{cross section}_{yz}} = \frac{q_{fluid}}{(H_{fracture}^n \times W_{fracture}^n)} \]

\[ \Delta t = \Delta L / V_{proppant}^n \]

The accumulation of the proppant mass is then calculated:

\[ \text{propmass} = C_{slurry} \times \Delta L \times W_{fracture}^n \times (V_{settle}^n \times \Delta t) \]

\[ \text{sumprop}^n = \text{sumprop}^{n-1} + \text{propmass}^n \]

\[ \text{propcon} = \text{sumprop} / \Delta V_{frac} \]

Since the settled proppant accumulates to form a bed, the height of the bed is expressed as:

\[ \text{bedH} = \text{propmass} / \rho_{bed} / W_{fracture}^n / \Delta L \]

\[ \text{sumbedH}^n = \text{sumbedH}^{n-1} + \text{bedH}^n \]

The proppant bed accumulates reducing the cross sectional area through time, so the velocity of the fluid change in response. Also the slurry height will increase since the velocity is higher causing the time to pass a section is shorter.

\[ \text{cross section}_{yz}^n = \text{cross section}_{yz}^{n-1} - \text{sumbedH}^n \times W_{fracture}^n \times \Delta L \]
\[ V_{x_{\text{proppant}}}^n = \frac{\text{flowrate}_{\text{fluid}}^n}{\text{cross section}_{yz}^n} \]
\[ \Delta t = \frac{\Delta L}{V_{x_{\text{proppant}}}^n} \]
\[ Z_{\text{proppant}}^n = Z_{\text{proppant}}^n - V_{\text{settle}_{\text{proppant}}}^n \times \Delta t \]

Of course, leak of effect, wall effect and concentration effect in the previous method can be all applied to this method.

### 3.2 Results of the T-Junction model without porppant accumulation

![Fracture geometry of the case study.](image)

**Figure 3.4:** Fracture geometry of the case study.

Figure 3.4 shows the geometry of the reservoir. A simple two-turning pointed reservoir model is constructed for simulation. The first segment of the major fracture (point A to point B) is 50 feet long, 0.1 inch wide and the second segment (point B to point C) is 30 feet long, 0.06 inch wide.
The second fracture that is perpendicular to the main fracture from point C to point D is 40 feet long 0.05 and inch wide. The third consecutive fracture (point D to point E) that is parallel to the main fracture is 120 feet long and 0.055 inch wide. The height of the fracture is 200 feet all along the fracture. The geometry of the fracture can be tuned by available data for different scenarios.

![Graph showing the concentration of proppant (lb/ft²) in the fracture complex from point A to point D’](image)

**Figure 3.5: Result of the base case of the T-junction model without accumulation.**

Figure 3.5 shows that the concentration of proppant (lb/ft²) in the fracture complex from point A to point D’. A ladder-shaped profile of decreasing proppant concentration is observed at the end of a 1936 sec treatment simulation. In the simulation, a 5 lb/gal constant injection slurry concentration is determined. For the stairs-shaped concentration profile we could see that for
each joint of the steps is located at a changing point of the fracture geometry. In addition we could observe that when the slurry enters a narrower section of the fracture complex, the gradient of the descending concentration decreases. Higher fluid velocity in the narrower section causes a “jetting” effect. Due to the decrease in cross section, the velocity in the horizontal direction increases to compensate the mass balance under a perfect transporting situation. In this zone, proppant can be carried through the fracture without losing too much volume into the non-pay zone and gives a better concentration. Another observation is that at the tailend of the concentration profile a large descending slope is obtained. In the setting of the reservoir parameters, local stresses distribution and directions affect the fracture width. In this case, the maximum in-situ stress is in E-W direction and the minimum in-situ stress is orthogonal to the maximum in-situ stress. The fracture width, in response to the mechanical condition, has a thinner width in the first diverging branches.

The ratio of the fluid volume and the cross section determines the velocity. Once the ratio is decreased by the splitting of the fluid volume, in this case the fluid is divided evenly, and the by the increase of the width, caused by the fracture branches that propagate into a zone of lower in-situ stress, the fluid velocity decreases. Proppant losses in to the formation significantly in this case at the tailend because the fracture turns into the low-stress direction that increases the width and also the fluid has been evenly divided twice that magnifies the loss of the velocity.

For a certain fracture geometry, the model predicts the proppant concentration along the fracture complex once the slurry properties, such as proppant type and concentration, and injection rate are determined. In the demonstrated case, the proppant is predicted to be losing into the
formation entirely for 35 minutes injection. Further treatment will not increase the proppant concentration nor the stimulated length. Once a designed ideal stimulated area is determined, the model gives a maximum limit to the treatment time. If the treatment time exceeds the limit, the treatment is considered to be ineffective that the proppant concentration stays unchanged. In this study, no leak off, concentration effect or wall effect takes place. This case is designed to be a base case that will be compared to the other cases where these effects are applied to the model.

![Graph showing concentration profile with points A, B, C, D, D', and D''].

**Figure 3.6: Result of leakoff effect is applied vs. base case.**

Figure 3.6 shows that the leak off effect is applied to the model. As shown in the figure, the concentration profile starts to bend into a curve which is different from the base case where the line of the profile stays straight. The leak off decreases the flow velocity because of the loss of the fluid volume. However the slurry concentration also increases as a response to the loss of the
liquid phase. In the case we can observe that the concentration profile is below to a horizontal line, which means the effect of the velocity loss outweighs the increase of concentration. It means if the leak off is very significant and the settling is minute, the concentration can increase dramatically that the concentration profile appears to have a positive slope. Another thing is that except for the increase of the concentration, the terminal point that the proppant can reach is closer than the base case. This observation is reasonable because the fluid loss cause the decrease in velocity so in a certain time step the proppant will be applied to the settling velocity more significantly. As a matter of fact the proppant will reach the bottom for the same duration. However, the prograded distance is shorter. The leak off in this case is a pressure dependent behavior controlled by The Darcy’s Law. The pressure profile is also already determined by the propagation model. As a matter of fact, the leak off for every segment of the fracture can be determined. A boundary layer is designed to give a reasonable quantification of the leak off since the permeability is too different from the fracture blocks to the matrix blocks that could potentially predicts incorrect leak off. After all the leak off function can be coupled to the proppant propagation protocol and the simulation can be done.
Figure 3.7 shows the concentration profiles that the wall effect is taken into place (the red line) versus the base case (the blue line). The concentration profile now has a slope of slighter angle for every segment. The wall effect hinders the proppant settlement during the injection and can be quantified by equations stated in the critical literature review. The effect has relatively larger influence on the thinner part of the fracture due to the smaller ratio of the proppant size and the width. Also in the narrower part of the fracture, the jetting effect is acting on the settling simultaneously. The ultimate result of the two effects is a near horizontal line at the narrower part, keeping the fracture to maintain higher proppant concentration.
Figure 3.8 shows that all the effects are applied to the model. We can observe that the trend of the figure is very similar to figure 3.6, which is the case with only leak off effect is applied. Leak off effect seems to be a more dominant effect during proppant transport the wall effect.

### 3.3 Results of the T-junction model with proppant accumulation

In this model, engineers are available to observe the growth of deposited sand bed, instant fluid flow velocity at every section of a fracture, instant suspending slurry height and also the concentration profile after the treatment. In addition, the model is able to simulate the screen out timing, which is one of the most important things that engineers would like to understand and
predict. The settings of the fracture geometry used in the model are listed below. A simple two-turning pointed reservoir model is constructed for simulation. The first segment of the major fracture (point A to point B) is 100 feet long, 0.1 inch wide and the second segment (point B to point C) is 50 feet long, 0.08 inch wide. The second fracture that is perpendicular to the main fracture from point C to point D is 100 feet long 0.06 inch wide. The third consecutive fracture (point D to point E) that is parallel to the main fracture is 800 feet long and 0.02 inch wide. The height of the fracture is 200 feet all along the fracture. The fracture geometry is shown in figure 3.9. Again these settings are adjustable for different fracture geometries.

![Fracture geometry of the case study.](image)

**Figure 3.9:** Fracture geometry of the case study.
Figure 3.10 shows the result of a 30 minute injection with a constant slurry concentration of 2 lb/gal of 4070 frac sand. In the figure we can observe that proppant particles deposit near the well bore to form higher proppant concentration in the fracture. Using slick water as a fracture fluid, proppant particles couldn’t be carried far away from the well bore but settle down and accumulate rapidly. The result fits with field experiences and lab observations.

Figure 3.10: proppant concentration of the base case.
In addition, in figure 3.11 we can observe the height of the proppant bed. At the well bore is the highest achieving 16 feet. The height decreases along the fracture smoothly which is reasonable that the section away from the well bore receives less proppant due to the less exposure time to the slurry. Also in figure 3.12, we could observe that the instantaneous velocity profile in every segment of the fracture compared to the initial velocity before the treatment, since the settled proppant bed height is calculated. The red line represents the original velocity and the blue line represents the flow velocity after the particles settle down to form a bed. We obtained higher velocity in every section before point at 540 feet. This result is expected because the cross section area before point at 261 ft is decreased by the accumulated proppant particles causing the flow velocity to increase. In addition, due to the higher bed height, the velocity difference is larger.

**Figure 3.11: Accumulated proppant bank height of the base case.**
near the well bore which may help the proppant particles to be carried further into the inner part of the fracture. Once the slurry reaches the part with no proppant accumulation the velocity begins to converge with the original velocity. In figure 3.13, the model shows that the top of the slurry front hasn’t tough the bottom of the fracture. The slurry front still maintains a 5 feet height at a 30-min injection. Hence, the slurry front hasn’t achieved its maximum distance that the stimulated area can be larger than this point of time if further treatment continues.

![Graph showing average flow velocity comparison](image)

**Figure 3.12: Average flow velocity of the base case vs. no-accumulation case.**

Figure 3.14 shows the results of 10, 30 and 60 minutes treatments. We could observe that the proppant bed accumulated further into the fracture and higher with the treatment continues.
Figure 3.14: Results of 10, 30 and 60 minutes treatment results.
As mentioned, this model is able to monitor screen out timing. As the treatment continues, the settled sand bed grows higher and higher. Once the accumulated proppant bed blocks the entire cross section, screen out takes place. Figure 3.15 shows the screen out timing for a case which is a constant slurry concentration (2.5 lb/gal) and pumping rate (10 bbl/min) treatment and also with the same fracture geometry with the previous case. In the figure we can observe that the total treatment time is 4474 sec and the fracture encounters the screen out. The concentration at the initial part of the fracture is 5lb/gal, reaching the highest proppant concentration in this scenario. The proppant concentration decreases rapidly along the fracture and reaches zero at 375.

Figure 3.15: proppant concentration of the base case at the moment that screen out takes place.
In figure 3.16 we could also see that the whole cross section at the beginning of the fracture is zero, totally blocked by the piled up proppant particles. Based on the model, the treatment will terminate at 4474 sec, further injection would be impracticable.

Figure 3.16: Accumulated settled proppant bed height of the base case at the moment that screen out takes place.
Figure 3.17 shows a comparison between using 40/70 and 20/40 frac sand. The red line represents the treatment using 2040 frac sand and the blue line is 4070 for a 1400 sec treatment. In the figure we could observe that the case using 2040 frac sand has a higher proppant concentration at the front section of the fracture. However after 150ft, the case using 4070 has a higher concentration. In addition we could observe that the 4070 case has a longer propped total length of 212 ft compared to the 4070 case reaching only 170 ft. One reason causes a shorter propped length is that the settling velocity of 2040 frac sand is larger than 4070 frac sand due to its larger particle diameters. As a matter of fact 2040 proppant particles will hit the bottom of the fracture sooner than 4070 sand particles. Also we can see that the 2040 case has achieved its maximum proppant concentration of 5 lb/ft^2 at the well bore area causing screen out. The packed bed
height in figure 3.18 also indicates that the total 50 feet cross section near the well bore is blocked. In contrast, the 4070 case has reached only 1.7 lb/ ft$^2$ at the initial section of the fracture. Further injection is executable for the 4070 case.

![Graph](image)

**Figure 3.18:** Settled proppant bed height of using 40/70 and 20/40 proppant for 1400 seconds.
Variable proppant concentration can also be modeled in this program. Figure 3.19 shows the result of a scheduled variable slurry concentration treatment. The schedule is listed in table 3.1. A base case is also compared to the variable concentration case. In the figure we could observe that along with the increase of slurry concentration, the proppant concentration in the fracture increases as a response. If the treatment continues, effects of slurry concentration may appear. In figure 3.20, we can observe that with increasing slurry concentration, the former part of the fracture may have higher proppant concentration. However the concentration at the latter part does not have higher proppant concentration. The total injection time for the variable concentration case is 2528 sec, and the total injection time of the base case is 3600 sec. the variable concentration case ends earlier since the screen out takes place, and also resulting in a shorter propped length. The screen out causes the fluid flow could not carry proppant into the
inner part of the fracture, causing the deeper part bearing low proppant concentration. As a matter of fact, with increasing proppant slurry concentration, the sooner the fracture will screen out with a shorter propped length. However the near well bore area could embrace high proppant concentration.

![Graph](image)

**Figure 3.20:** Proppant concentration of the base case and the adjustable slurry concentration case. Note that in the adjustable case, screen out takes place faster than the base case due to its high concentration at the later time of the treatment.

**Table 3.1:** The treatment schedule of the adjustable slurry concentration case.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Slurry concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-200</td>
<td>1 lb /gal</td>
</tr>
<tr>
<td>200-500</td>
<td>2.5 lb /gal</td>
</tr>
<tr>
<td>500-800</td>
<td>3.5 lb /gal</td>
</tr>
<tr>
<td>800-1400</td>
<td>5 lb /gal</td>
</tr>
</tbody>
</table>
CHAPTER 4
MULTI-BRANCHED JUNCTION MODEL

4.1 Model description

For the multi- branched junction model, the slurry that enters into the branches is controlled by the pressure of each branch. The pressure will act like a barrier that limits the entering flow volume. In this model, the flow in the fractures is assumed to have a similar behavior of a pipe flow on the pressure selective influx. In this model, the proppant transport logic without accumulation is selected for the model. The controlling equations and a demonstration cartoon figure 2.1 and figure 2.2 for two scenarios are shown as follows:

For multi-branches condition:

\[ q_{branch\ i} = q_{total} \times \left( \frac{A_{branch\ i}}{P_{branch\ i}} \right) \frac{A_{branch\ 1}}{P_{branch\ 1}} + \frac{A_{branch\ 2}}{P_{branch\ 2}} + \frac{A_{branch\ 3}}{P_{branch\ 3}} + \ldots + \frac{A_{branch\ n}}{P_{branch\ n}} \]

Where \( n = 1, 2, 3 \ldots\ n \)

\[ V_{branch\ i} = \frac{q_{branch\ i}}{A_{branch\ i}} \]
Figure 4.1: Concept of multi-channel model I.

Figure 4.2: Concept of multi-channel model II.
Same as the previous T-junction model, the multi-junction model is firstly demonstrated by a simple case. The fracture geometry is determined by the fracture propagation model that a 60-minute injection has been applied to the reservoir previously to the slurry injection. The geometry is enlarged by a certain degree that the model can work better to characterize some effects and chapter some phenomena. The settling velocity and the other effects are also able to be implemented into the program. The algorithm of applying those effects are basically the same as the previous simple T junction model. The fracture geometry and pressure is shown in figure 4.3.

![Fracture Geometry](image)

**Figure 4.3:** Fracture geometry of the case study.

### 4.2 Results of the Multi-branched Junction Model

In this case, the fracture geometry is not always symmetric. The first segment of the major fracture (point A to point B) is 50 feet long, 0.1 inch wide and the second segment (point B to point C) is 30 feet long, 0.06 inch wide. The second fracture that is perpendicular to the main
fracture from point C to point D is 20 feet long 0.05 inch wide. Point D to point E is 20 feet long and 0.02 inch wide. The third consecutive fracture has two wings. One is from point E to F, which is a thinner segment with a width of 0.035 inches. The other one is from point (E to point G), which is 0.045 in width. The height of the fracture is 200 feet all along the fracture. The fracture information is listed in table 4.1.

Figure 4.4: Results of the multi-channel model.

Figure 4.4 shows the result of treatment. The total injection time is 2054.49 sec, and the leak off, the concentration effect and the wall effect are all considered in the model. Before point D, the model works like a simple T-junction model because of the symmetric geometry. However after the point D, the branches diverge from each other and they both have different entering pressures and aperture widths. As the result shows, the branch E-F has a thinner entry causing a higher
fluid velocity. In the other hand, the branch E-G is wider in the aperture that gives a higher initial proppant quantity in the same fracture wall area and the velocity is then inversely lower. In the branch E-G, because of the higher pressure, several elements are working phenomenally on keeping the velocity lower. One is that the leak off is higher that leads to larger decrease in velocity. The second one is that the smaller the wall effect hindering the settling. The last one is the water head barrier effect that allows less fluid to come into the fracture. The ultimate impact on the proppant transport is that the concentration will have a higher value at the beginning and them it drops faster than the thinner branch resulting in a shorter length.

![Figure 4.5: Result of variable proppant concentration of slurry.](Image)

Figure 4.5 shows the result of variable slurry concentration under a constant pumping rate.
Figure 4.6: Results of different injection periods.

Figure 4.6 show the results of different injection duration. The green line shows a 10 minute injection. The red line gives a 20 minute treatment. The blue line is the result of a full time injection.
Figure 4.7 show the results of all the effects combined and increasing slurry concentration. The schedule is shown below in Table 4.1.
Table 4.1: The injection schedule of the case.

<table>
<thead>
<tr>
<th>Schedule time (min)</th>
<th>Slurry concentration (lb/gal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T = 0 --- 5</td>
<td>2 lb / gal</td>
</tr>
<tr>
<td>T = 5 ---10</td>
<td>2.4 lb / gal</td>
</tr>
<tr>
<td>T = 10 --- 15</td>
<td>2.6 lb / gal</td>
</tr>
<tr>
<td>T = 15 --- 25</td>
<td>3.0 lb / gal</td>
</tr>
<tr>
<td>T = 25 --- 30</td>
<td>3.4 lb / gal</td>
</tr>
<tr>
<td>T = 30 --- 35</td>
<td>3.6 lb / gal</td>
</tr>
<tr>
<td>T = 35 --- End</td>
<td>4 lb / gal</td>
</tr>
</tbody>
</table>
5.1 model description

In chapter three, a finite difference model is constructed to capture the movement of the injected flow. The model is constructed as a body-centered system consisting of the matrix blocks and the fracture blocks. The fracture geometry is also obtained from the fracture network pressure propagation model. After the geometry is obtained, the fracture is meshed into grids for simulation runs. In this model, fracture geometry remains unchanged, which means the fracture does not propagate during the treatment. Initial injection pressure is assumed to be the pressure obtained by the fracture propagational model, which also means the pressure at the pad zone stage.

5.2 Main controlling equations and factors

Darcy’s law is the major controlling equation for the three phases through the whole program. Darcy’s law is a consecutive equation that phenomenologically derived by injecting flow though a porous media. The equation is expressed as:

\[ q = -\frac{KA (P_b - P_a)}{\mu L} \]
The equation can be expanded to two dimensions and written into differential form:

$$\frac{\partial}{\partial x} A_{x} k_{x} \frac{\partial \phi_{x}}{\partial x} \Delta x + \frac{\partial}{\partial y} A_{y} k_{y} \frac{\partial \phi_{y}}{\partial y} \Delta y + \frac{\partial}{\partial z} A_{z} k_{z} \frac{\partial \phi_{z}}{\partial z} \Delta z + q = \frac{V_{b}}{5.615 \mu t} \frac{\partial}{\partial z} \left( \frac{\phi S_{B}}{B} \right)$$

Then expend the equation into three phases form:

For the water phase (w):

$$\frac{\partial}{\partial x} A_{x} k_{x} k_{rw} \frac{\partial \phi_{wx}}{\partial x} \Delta x + \frac{\partial}{\partial y} A_{y} k_{y} k_{rw} \frac{\partial \phi_{wy}}{\partial y} \Delta y + \frac{\partial}{\partial z} A_{z} k_{z} k_{rw} \frac{\partial \phi_{wz}}{\partial z} \Delta z + q_{w} = \frac{V_{b}}{5.615 \mu t} \frac{\partial}{\partial z} \left( \frac{\phi S_{w}}{B_{w}} \right)$$

For the oil phase (o):

$$\frac{\partial}{\partial x} A_{x} k_{x} k_{ro} \frac{\partial \phi_{ox}}{\partial x} \Delta x + \frac{\partial}{\partial y} A_{y} k_{y} k_{ro} \frac{\partial \phi_{oy}}{\partial y} \Delta y + \frac{\partial}{\partial z} A_{z} k_{z} k_{ro} \frac{\partial \phi_{oz}}{\partial z} \Delta z + q_{o} = \frac{V_{b}}{5.615 \mu t} \frac{\partial}{\partial z} \left( \frac{\phi S_{o}}{B_{o}} \right)$$

For the gas phase (g):

$$\frac{\partial}{\partial x} \left( R_{sw} A_{x} k_{x} k_{rw} \frac{\partial \phi_{w}}{\partial x} + R_{so} A_{x} k_{x} k_{ro} \frac{\partial \phi_{o}}{\partial x} + A_{x} k_{x} k_{rg} \frac{\partial \phi_{g}}{\partial x} \right) \Delta x +$$

$$\frac{\partial}{\partial y} \left( R_{sw} A_{y} k_{y} k_{rw} \frac{\partial \phi_{w}}{\partial y} + R_{so} A_{y} k_{y} k_{ro} \frac{\partial \phi_{o}}{\partial y} + A_{y} k_{y} k_{rg} \frac{\partial \phi_{g}}{\partial y} \right) \Delta y +$$

$$\frac{\partial}{\partial z} \left( R_{sw} A_{z} k_{z} k_{rw} \frac{\partial \phi_{w}}{\partial z} + R_{so} A_{z} k_{z} k_{ro} \frac{\partial \phi_{o}}{\partial z} + A_{z} k_{z} k_{rg} \frac{\partial \phi_{g}}{\partial z} \right) \Delta z +$$

$$R_{sw} q_{w} + R_{so} q_{o} + q_{g} = \frac{V_{b}}{5.615} \frac{\partial}{\partial t} \left( R_{sw} \frac{\phi S_{w}}{B_{w}} R_{so} \frac{\phi S_{o}}{B_{o}} + \frac{\phi S_{g}}{B_{g}} \right)$$

Where

$$P_{w} + P_{cow} = P_{o}$$

$$S_{w} + S_{o} + S_{g} = 1$$
5.3 Main constructive concepts and algorism

For this model, a body-centered system is constructed and finite-difference scheme is also used. First of all, we have to determine what the variables are for iterations. Po, Sw and Sg are selected for the model. Since we can’t choose three saturations at the same time, otherwise it is not going to achieve a sum of 1, which can bring out unreasonable outcomes. For the kro construction I used stone’s second method. For krw and krg selection I used single point up stream weighted method.

The transmobility term is the first one should be constructed. After the construction, the values calculated by the transmobility function could be transferred into Residue functions to obtain the Residue derivatives for Po, Sw and Sg in every direction and center. For the derivative form in Po, we can have a tolerance of 0.01, however for saturations we have to use smaller tolerance of 0.001 in order to obtain more accurate capillary related terms.

The Jacobian residue matrix will be constructed after the residue terms output. The main point is how to reallocate the derivative arrays into the right place into the matrix. By using the function “reshape” and “diag” built in MATLAB and setting a 3x3 cycle configuration, we could obtain a tri-tri-hepta diagonal Jacobian matrix.

Last step is the implementation of the solver. Bi-conjugate solver is chosen for this project and also built in in MATLAB. By using “bicg” function, a [A][X]=[B] type solver, we could set up a
tolerance value, $10^{-5}$, in this case, we could iterate the residue values and numerically solve it, for every iteration level $k$. For every time step $n$ we also have iterations, means $k$, to achieve the final balanced residue value 0.

### 5.4 Interaction among phases

The model is a 2-D, 3-phases system that monitors the water phase, the oil phase and the gas phase. For a shale gas reservoir, the oil saturation is reduced to a very minimal limit of the irreducible oil saturation. In this case, a value of 0.005 is given to be the lowest limit. Instead of constructing a 2-phases system, a 3-phases system is more flexible that can simulate a wetter reservoir that has a certain amount of oil, or even an oil dominated reservoir.

### 5.5 Fracture permeability

For a shale gas reservoir, the matrix permeability is assumed to be 0.001. The fracture permeability is designed to be a very high value since the fracture is nearly void. 20000 darcy is assigned to the fracture for a porous media system.

### 5.6 Proppant distribution

The model assumes the proppant particles have the same velocity of the injecting flow in the horizontal directions. In a certain time step, the net flux that enters the block can be expressed in the SIP notation as:
The $Q_w$ term is the summation of the net water flux that enters a specific block. In this case the water term is assumed to be the injection slurry. The $Q_w$ term will vary along with the increase of the injection time. The term $\Delta Q_w$ is the discrepancy between the $Q_w$ in the present time step and the previous time step. However, for a specific block, flows come in and out from different surrounding blocks processing various proppant concentrations. The Mass control then can be expressed as:

$$W_{w,j}^{n+1} (p_{w-1,j}^{n+1} - p_{w,j}^{n+1}) + E_{w,j}^{n+1} (p_{w+1,j}^{n+1} - p_{w,j}^{n+1}) + N_{w,j}^{n+1} (p_{w,j-1}^{n+1} - p_{w,j}^{n+1}) + S_{w,j}^{n+1} (p_{w,j+1}^{n+1} - p_{w,j}^{n+1}) + q_{w,j}^{n+1} = Q_{w,j}^{n+1}$$

Also the proppant losing from the

$$Conppt_{block,ij}^{n+1} = ProppantMass_{sum,ij}^{n+1} / V_{block,ij}$$

$$ProppantMass_{rate,ij}^{n+1} = Conppt_{block,ij-1}^{n+1} \times W_{w,j}^{n+1} (p_{w-1,j}^{n+1} - p_{w,j}^{n+1}) + Conppt_{block,ij+1}^{n+1} \times E_{w,j}^{n+1} (p_{w+1,j}^{n+1} - p_{w,j}^{n+1}) + Conppt_{block,ij-1}^{n+1} \times N_{w,j}^{n+1} (p_{w,j-1}^{n+1} - p_{w,j}^{n+1})$$

$$+ Conppt_{block,ij+1}^{n+1} \times S_{w,j}^{n+1} (p_{w,j+1}^{n+1} - p_{w,j}^{n+1}) + Conppt_{slurry,ij+1}^{n+1} \times q_{w,j}^{n+1}$$

$$ProppantLoss_{block,ij}^{n+1} = ProppantMass_{sum,ij}^{n+1} \times (V_{settle,ij}^{n+1} \times \Delta t / H_{block,ij})$$

Proppant mass transport from each surrounding block can be calculated for each time step. Base on the above equations, the influx and outflux of proppant that enters and leaves can be calculated and then renews the concentration of each block. Also the proppant losing from the
pay zone is calculated by the settling velocity. In a certain time step, a volumetric percentage of proppant settles out of the pay zone.

For each time step, proppant will follow the flow and be carried into the fracture. Along with the treatment continues, proppant concentration profile can be constructed more accurately. Predictions for different treatment time and designs can also be made by tuning parameters and settings of the model.
5.7 Flow chart

Figure 5.1: Flow chart.
5.8 Results and analysis

Same as the previous model, the fracture geometry is generated by the fracture propagation model after a 60-minute treatment. The reservoir is gridded into a body-centered system for the finite-difference simulation. In order to expedite the simulation, the reservoir is reduced to its half since the geometry is symmetric by the main fracture. The pressure profile of the initial condition will be loaded into the program as a starting pressure distribution.

Figure 5.2: Fracture geometry and initial pressure profile.
Figure 5.3: Proppant concentration at 5 mins injection.

Figure 5.4: Proppant concentration at 5 mins injection.
In the results we can observe that the proppant concentration decreases from the well bore to the inner part of the branches, especially the after the junction point. Very few amount of proppant

**Figure 5.5:** Proppant concentration at 25 mins injection.

**Figure 5.6:** Pressure profile at 25 mins injection.

In the results we can observe that the proppant concentration decreases from the well bore to the inner part of the branches, especially the after the junction point. Very few amount of proppant
enters the branches. One reason is because the flow is not moving smoothly since the fracture geometry is fixed and the leak off through the matrix is low. The only effective outlet of the flow is the natural fracture fissures at the end of the fracture. Due to limited discharge of the flow and restricted fracture geometry, the pressure builds up very quickly although the injection rate is lower than 5 bbl/min. As a matter of fact, the growth of fracture during the treatment is very important to release the pressure in the fracture by creating more space in low perm reservoirs.

Figure 5.7 is another pressure profile at 18 minutes of the injection. In this run, another natural fracture (fissure) is activated at the junction point. Proppant concentration showed in figure 5.8 indicates that during the proppant transport, if the slurry encountered a relatively high-leak off point such as a fissure, proppant concentration increases due to the fluid loss from the fissure. If the treatment continues, proppant concentration rises and could finally have a chance to block the entire fracture cross-section and then a screenout may occur. As a matter of fact that the rise

![Figure 5.7: Pressure profile at 18 mins injection.](image-url)
of proppant concentration at a high-leak off point, especially at the junction points where the natural fracture is not opened yet, may have potential to cause a screen out.

Figure 5.8: Proppant concentration at 18 mins injection.
CHAPTER 6

DYNAMIC SEMI-COUPLED MODEL

6.1 Model description

During the hydraulic fracturing, very high flow rate of fluid is injected into the formation. Not only just pad stage causes the fracture growth but also the stages of the treatment that contains proppant. In order to capture the dynamic procedure of the proppant transport simultaneously with the fracture propagation, a semi-coupled protocol is constructed.

The fracture propagation model generates a series of pressure and fracture geometry maps from the beginning to the end of the treatment in an interval of one minute. Once the pressure profile is obtained, mass balance equation of the Darcy’s Law can be applied to the fracture and then the flow flux to every block can be calculated. In the model, the equation is written in a single water phase mass balance form since the fracture is totally dominated by the injecting water. The main controlling equations are shown as below:

\[
W_{wi,j}^{n+1k} (P_{wi,j}^{n+1k} - P_{wi,j}^{n+1k}) + E_{wi,j} (P_{wi,j+1}^{n+1k} - P_{wi,j}^{n+1k}) + N_{wi,j} (P_{wi,j-1}^{n+1k} - P_{wi,j}^{n+1k}) + S_{wi,j}^{n+1k} (P_{wi,j+1}^{n+1k} - P_{wi,j}^{n+1k})
\]

\[\quad - p_{wi,j}^{n+1k}) + q_{wi,j}^{n+1k} = Q_{wi,j}^{n+1k}\]
Concentration of each fracture block is controlled by the following equations:

\[\text{Connt}_{\text{block}_{i,j}}^{n+1} = \frac{\text{ProppantMass}_{\text{sum}_{i,j}}^{n+1}}{V_{\text{block}_{i,j}}^{n+1}}\]

\[\text{ProppantMass}_{\text{rate}_{i,j}}^{n+1} = \text{Connt}_{\text{block}_{i-1,j}}^{n+1} \times W_{w_{i,j}}^{n+1}(P_{w_{i-1,j}}^{n+1} - P_{w_{i,j}}^{n+1}) + \text{Connt}_{\text{block}_{i+1,j}}^{n+1}\]

\[\times E_{w_{i,j}}^{n+1}(P_{w_{i+1,j}}^{n+1} - P_{w_{i,j}}^{n+1}) + \text{Connt}_{\text{block}_{i,j-1}}^{n+1} \times N_{w_{i,j}}^{n+1}(P_{w_{i,j-1}}^{n+1} - P_{w_{i,j}}^{n+1})\]

\[+ \text{Connt}_{\text{block}_{i+1,j}}^{n+1} \times S_{w_{i,j}}^{n+1}(P_{w_{i+1,j}}^{n+1} - P_{w_{i,j}}^{n+1}) + \text{Connt}_{\text{sturry}_{i,j+1}}^{n+1} \times q_{w_{i,j}}^{n+1}\]

\[\text{ProppantLoss}_{\text{block}_{i,j}}^{n+1} = \text{ProppantMass}_{\text{sum}_{i,j}}^{n+1} \times (V_{\text{settle}_{i,j}}^{n+1} \times \Delta t / H_{\text{block}_{i,j}}^{n+1})\]

\[\text{ProppantMass}_{\text{sum}_{i,j}}^{n+1} = \text{ProppantMass}_{\text{sum}_{i,j}}^{n+1} - \text{ProppantMass}_{\text{rate}_{i,j}}^{n+1}\]

\[\text{ProppantMass}_{\text{sum}_{i,j}}^{n+1} = \text{ProppantMass}_{\text{sum}_{i,j}}^{n+1} - \text{ProppantLoss}_{\text{block}_{i,j}}^{n+1}\]

6.2 Results and analysis

The dynamic model heritages several pressure maps and fracture geometry profiles from the fracture propagational model. For every 5 minute the pressure profile and fracture geometry renew once to enter the next stage of the treatment. In this thesis, a 45-minute injection is demonstrated to observe this dynamic process. An exception is made that for the last 10 minutes of the treatment, from 35 to 45 minutes, the fracture geometry stays the same. The model enables engineers to observe dynamic phenomenon during the treatment, and also to obtain the final designed result after the treatment. In this 45-minute run, 20/40, 40/70 and 100 mesh sand, three different sizes of proppant are used to observe their impacts on the treatment.
From figure 6.1 to figure 6.15, a series of proppant concentration distributions are obtained for every 5 minutes for a case using 20/40 frac sand, with the coupled fracture geometries and pressure profiles. The two sets of maps are put together to give an indication that how far and much of the proppant is transported into the fracture network. In general, we can observe that when the treatment is finished, proppants do not fill up the entire fracture networks, but leaving a very large blank area without being propped. The reason causes this phenomenon is that the proppant transport is entirely controlled by the fluid flow in this model. When the pressure gradient starts to decrease caused by the separation of branched flow or leakoff, the flow velocity starts to decrease. As a matter of fact the proppants are unable to be carried further into the networks, resulting in a large unpropped area. Also we can observe that till the end of the treatment, proppant concentration reaches a certain kind of balance that the concentration will not rise up constantly during the injection. A balanced concentration is maintained along the fracture branches once the balanced state is achieved. In addition, the balanced state is likely to be achieved when the slurry concentration leaving a specific block receives similar concentrated slurry from its previous block or from the source. Of course, the balanced concentration is dependent on the pressure profile and the slurry concentration schedule of each time step of the treatment.

In figures we could also observe a very important phenomenon is that the proppant particles accumulate at the junction points. In the figure we see the proppant concentration rises up at the segregation point of the branches and increases with the treatment continues in a specific pressure-map stage. This junction-accumulating effect is due to the pressure gradient decrease, causing the proppant particles to accumulate at the connecting points without being entirely
carried away by the flow. In other words, the flow is carrying slurry with higher proppant concentration from the near well bore blocks into the junction block, and due to the decrease of pressure gradient and the separation of fluid flows and also the leakoff, the flow that leaves the block is carrying less proppant, leaving proppant particles to stay in the junction block. Based on this clotting mechanism, the model is able to predict potential screen out events caused by over high proppant concentration in the fracture channels that blocks the passing flow. If a critical concentration of proppant can be determined that it is going to block the section causing a screen out, then the model becomes really helpful to decide the schedule of proppant concentration to prevent screen out. However, this specific concentration has to be determined from laboratory work and no such publication has been published yet. In a similar way, the results show that the proppant concentration accumulation happens not only at the junction points but also the near well bore area. In figure 6.13, at the moment that the fracture starts to increase its width, the average velocity decreases due to the increase of the cross-sectional area, the proppant accumulates at the near well bore area since the pressure gradient is too low to transport proppant further into the fracture. In the other hand, once the treatment continues the pressure gradient builds up again and the proppant particles are able to be carried away form the area. Also we can observe a similar effect that at the junction points, when the treatment continues and the pressure gradient changes, usually increases, with the propagation of the fracture network, the accumulated proppant starts to move with the flow again, reviving the mobility of the proppant with higher pressure gradient. This phenomenon relieves the risk of proppant clotting during the injection to a certain degree. There might be still a certain level of accumulation, which is totally dependent on the pressure profile, but usually less proppant is clotted than the previous time step. Based on this observation, the proppant transport and the propagation of fracture are closely
engaged together. A successful proppant transport is dependent on the change of pressure profiles with increasing pressure gradient that bring more proppant into the deeper part of the network and also relieves the accumulation among the junction points and other portions with low pressure gradient.

In figure 6.16 to figure 6.23, a series of proppant concentration profiles are generated to quantify the difference between using 20/40 and 40/70 mesh of proppant. To compare the two sets of figures, we could observe that the proppant concentration is higher by using 40/70 mesh sand than using 20/40. The explanation is that the settling velocity of the 40/70 particles is slower than the 20/40 particles. Hence, the overall proppant concentration is higher in the 40/70 case than the 20/40 case. Especially in the branches of the network, in much higher degree the minor branches are propped. According to the equation of terminal velocity, the settling velocity is decreased by the square of the radius of the particles. As a matter of fact, a significant decrease in settling velocity is observed in smaller sized particles. In this case, the total loss of proppant is less than the 20/40 case. However, the amount of proppant that accumulated at the junctions is also larger than the 20/40 case, since less proppant is lost. This phenomenon could potentially cause the treatment to encounter screen out earlier if the fracture does not propagate in time to release the accumulation, or the concentration of the slurry is too high. Also we can observe the importance of fracture propagation that relieves the accumulation by increasing pressure gradient in the deeper part of the fracture more significantly in 40/70 case. This series of figures indicate that the proppant is firstly trapped at the junction point. In the next time step, the fracture propagates out and revives the transport again. We can obviously observe that the trapped particles start to move along the fracture again, resulting in the proppant concentration to have a bell-shape
distribution during the re-moving moment. And the bell shape will move forward into the network and the peak decreases to reach its balanced state. We could also observe this phenomenon in the 20/40 case. However with the less loss of proppant, the more significant the phenomenon is. In conclusion, the fracture propagation is very important to proppant transport, especially when the slurry concentration is high or the proppant is less settling out of the pay zone.
Figure 6.1: Proppant concentration at 5 minutes.

Figure 6.2: Fracture geometry and pressure at 5 minutes.
Figure 6.3: Proppant concentration at 10 minutes.

Figure 6.4: Fracture geometry and pressure at 10 minutes.
Figure 6.5: Proppant concentration at 15 minutes.

Figure 6.6: Fracture geometry and pressure at 15 minutes.
Figure 6.7: Proppant concentration at 20 minutes.

Figure 6.8: Fracture geometry and pressure at 20 minutes.
Figure 6.9: Proppant concentration at 25 minutes.

Figure 6.10: Fracture geometry and pressure at 25 minutes.
Figure 6.11: Proppant concentration at 30 minutes.

Figure 6.12: Fracture geometry and pressure at 30 minutes.
Figure 6.13: Proppant concentration at 35 minutes.

Figure 6.14: Fracture geometry and pressure at 35 minutes.
Figure 6.15: Proppant concentration at 45 minutes.
Figure 6.16: Proppant concentration at 5 minutes using 40/70 proppant.

Figure 6.17: Proppant concentration at 10 minutes using 40/70 proppant.
Figure 6.18: Proppant concentration at 15 minutes using 40/70 proppant.

Figure 6.19: Proppant concentration at 20 minutes using 40/70 proppant.
Figure 6.20: Proppant concentration at 25 minutes using 40/70 proppant.

Figure 6.21: Proppant concentration at 30 minutes using 40/70 proppant.
Figure 6.22: Proppant concentration at 35 minutes using 40/70 proppant.

Figure 6.23: Proppant concentration at 45 minutes using 40/70 proppant
By using smaller proppant, less proppant loss could be controlled. In figure 6.24, a plot of proppant remaining ratio (proppant remaining in the fracture / total injected proppant) versus time with different sizes is presented. The figure indicates that after a 45-minute treatment, the ratio of using 40/70 is 0.4948, and the ratio of using 20/40 is 0.1839. Significant proppant loss is observed by using 20/40 proppant compared to using 40/70 proppant. Also with the increase of time, the proppant is increasingly losing from the pay zone. In order to have better proppant loss control, the ratio can give a better understand of proppant loss versus time during the treatment. The model can be cooperated with the economy analysis and the concentration profile presented in the previous section to make a better treatment schedule and prevent unnecessary proppant loss with further injection is not giving a better proppant distribution within the network.

![Figure 6.24: Proppant remaining ratio vs. time](image)

**Figure 6.24: Proppant remaining ratio vs. time**
6.3 The absolute fracture conductivity and the healed width

In hydraulic fracturing, one of the most important factors to determine the treatment is successful or not is the fracture conductivity. Desired designed fracture conductivity can lead to better hydrocarbon recovery and less propping agent loss. The proppant transport model in this thesis can simulate proppant distribution within fracture network. Once the proppant concentration is captured, fracture conductivity can then be calculated.

Fracture conductivity can be calculated by the proppant concentration, proppant particle size and proppant type within the fracture. Several assumptions are made for the calculation of fracture conductivity.

Figure 6.25: Fracture permeability vs. closure stress of Ottawa-typed proppant (Proppants, second edition, 1984).
Figure 6.25 shows the conversion of fracture permeability within a specific range of concentration of proppant per area under different effective stress value. In proppant transport stage, the fracture is open to maintain fluid flow to enter. As a matter of fact that the effective stress is 0. In this case the model assumes the proppant is Ottawa-type sand. Other kinds of proppant can be simulated also by change proppant character parameters. However, the results of the proppant transport model shown in the previous chapters indicate that proppant concentration could be higher and also lower to the suitable range of the plot. As a matter of fact, an assumption is made here that, no matter how the proppant concentration value is, the fracture permeability conversion is applicable for every proppant concentration. In addition, for proppant concentration higher than the suitable range, the estimated fracture permeability could be more accurate than the one bellows the lower limit of the range. The reason is that fracture permeability is very high once reaching a certain concentration. However, if the proppant concentration is low, significant difference in fracture permeability could appear. Hence, the error of the converted fracture permeability from higher proppant concentration is more accurate than from the lower values. Once the fracture permeability is obtained, fracture conductivity can be calculated. The following equation shows the conversion from fracture permeability to the absolute fracture conductivity.

\[ Cd = k_f w_f \]
Figure 6.26 shows the healed fracture width versus different proppant concentration of Ottawa-typed sand. The figure also indicates that the closure stress will not significantly affect the width of the propped fracture. If proppant concentration is lower than the lower limit of the plot, the width of the fracture is considered to be zero, which means the section is unsuccessfully propped. From this plot, the propped width can be obtained by the proppant concentration simulated by the models.

Figure 6.26: Healed fracture width vs. proppant concentration under different effective stress (Hannah, 1985).
Figure 6.27 shows the fracture width of the 20/40 case. Apparently we can observe that even after a 45-minute injection, effectively-propped area is still very limited. 20/40 proppant is too large that causes higher settling velocity, leading proppant to settle out of the pay zone. Also, a mono-layer of 20/40 Ottawa-typed proppant is 0.28 lb/ft². Due to the higher proppant-losing rate that less fracture blocks are able to maintain a higher proppant concentration to create even a single layer of 20/40 Ottawa-typed proppant. If the treatment is entirely treated by large-size proppant, only the near well bore area is effectively propped and the branched remains unpropped or ineffectively propped although the pad zone area is large.
Figure 6.28 shows the fracture width of the 40/70 case. Compared to the previous 20/40 case, the effective propped area is larger. Smaller proppant is able to be transported to deeper part of the fracture network due to its lower settling velocity. A mono-layer of 40/70 Ottawa-typed proppant requires 0.17 lb/ft$^2$, which is lower than the 20/40 case. The above two reasons let the 40/70 to have a larger effectively propped area in the fracture network. However compared to the 20/40 case at the near well bore area, the width is approximately the same. 20/40 proppant gives better fracture permeability that creates better fracture conductivity. As a matter of fact, at the later stage of the treatment, larger sized proppant can be introduced to the fracture to maintain a better fracture conductivity. At the early stage of the treatment smaller sized proppant, can be even as smaller as 100 mesh sized proppant, is suggested to be used to create a larger propped fracture network area.
Figure 6.29 shows the fracture conductivity of a 45–minute treatment using 20/40 Ottawa-typed sand proppant before the fracture closes. Within the propped area, we could observe that the absolute fracture conductivity is high. It is high up to 5000 md-ft at the very near well bore area. However, due to small effectively propped area, the high-conductivity zone is restricted. Using 20/40 sized proppant can create high fracture conductivity, but restricted to the very near well bore area.
Figure 6.30 shows the fracture conductivity of a 45–minute treatment using 40/70 Ottawa-typed sand proppant before the fracture closes. The absolute fracture conductivity is lower than the case using 20/40 sized proppant. It has its highest at the very near well bore area with a value of 1000 md-ft. However, in the figure we could see that a significant larger stimulated zone is created. Note that fracture conductivity in the branches is still very low by using 40/70 proppant. 100 mesh sized proppant is suggested to be used at the early stage of the treatment so that more proppant can be transported to the deeper part of the fracture network. In conclusion, smaller sized proppant is suggested at the early stage of the treatment and then increases the size of proppant to maintain better fracture conductivity nearing the wellbore. Finally large sized proppant and high concentration is suggested to finish up the treatment to maintain high proppant volume and high fracture conductivity at the near wellbore area.
6.3 Limitations of the models

The models in this thesis simulate proppant distribution under different scenarios. However the basics of these models are built on mass balance equations. In particle movement prediction, momentum balance is one of the most important elements. For example, at the turning point of a fracture, proppant particles firstly stopped and then are dragged by fluid flow, into a direction perpendicular to the previous segment of the fracture. Velocity variations of particles can be predicted by using discrete-element method. However in a large scale simulation, the momentum change of each particle is unrealistic to be predicted. As a matter of fact, a middle-sized method should be developed to solve the problem. Energy balance is consideration. During the injection, cooler (assumed to be the same temperature as the surface temperature) treatment fluid is introduced to formations. Fluid viscosity could change along with fractures, which causes different settling velocity. Hence the proppant transportability is different. Once the cooler fracturing fluid leaks into the formation, the temperature for both matrix and natural fractures will also decrease as a response. As a matter of fact, the stress and pore pressure changes to keep the energy balanced. This temperature effect could significantly affect the treatment and also the post treatment period.
CHAPTER 7

CONCLUSIONS

7.1 Conclusions

In this thesis, four models of proppant transport are stated to predict the proppant distribution in fracture network in shale gas plays. Each model is suitable for different conditions based on different assumptions, logics and methodologies. Different results and phenomenon, during or after the treatment, are obtained; however, some similar trend can be captured in the fracture network system. Several conclusions can be made from the results generated by these models.

The simple T-junction model without proppant accumulation:

- Proppant concentration within fracture network is entirely controlled by the slurry concentration since no accumulation occurs.
- Higher injection rate creates higher fluid flow velocity that transports proppant to a further distance in fracture and creates a larger stimulated zone.
- Leak off is more dominant than wall effect in fracture.
- Once the slurry top reaches the bottom of the pay zone, further injection will not create better fracture conductivity because proppant is transported to its maximum distance.

The simple T-junction model with proppant accumulation:
- Proppant concentration in fracture depends on the slurry concentration and the accumulated proppant bank at the bottom.
- Fluid flow velocity increases with accumulation of proppant bank, since the cross section area decreases.
- Screen out occurs when the near well bore area is entirely blocked by accumulated proppant bank.
- In the same fracture geometry with same injection rate, proppant accumulation helps to create a longer propped length by the increase of fluid flow velocity.

**The multi-channel model:**

- Pressure barrier at each entry of a fracture branch controls the entering volume of fluid.
- Pressure distribution is extremely important in this model since the flow is entirely dependent on it.
- The ratio of entering fluid volume and cross section area of each branch play an important role that controls the flow velocity within.
- It is a flexible model that most of the situations are multi-channeled cases.

**The finite-difference model:**

- In addition to proppant concentration, dynamic pressure distribution can be also monitored during the treatment.
- Pressure dependent leak off can be monitored to calculated fluid loss.
Fracture propagation is essential to proppant transport. Once the injectivity of the fracture is achieved, it takes very high pressure to inject a very little volume of fluid.

Fracture propagation creates enough pad zone space to intake proppant and also to ensure the fluid receives enough space to flow in.

The semi-coupled dynamic model:

- A very important phenomenon is observed that the proppant particles accumulate at the junction points of fracture network.
- The clotting effect is caused by low pressure gradient that creates insufficient fluid flow velocity at the junction points.
- The accumulation at junction points could be one of the reasons that cause screen out.
- Once the fracture propagates, immobile proppant at the junction points is remobilized again to move further into the fracture network.
- Creating a larger pad zone gives a larger higher-pressure-gradient area that ensures better proppant transport.
- Proppant concentration is significantly higher in the main fracture than the branches, because of the clotting effect at the junctions.
- Proppant loss using 20/40 proppant is significantly higher than using 40/70 proppant.
- Conductivity by using 20/40 at the main fracture is higher than using 40/70, however, in the branches using 40/70 proppant has better conductivity since more proppant remain in the pay zone.
• Using smaller-sized proppant particles at the early stages of the treatment to prevent severe proppant loss in branches and use larger-sized proppant at the later stage to maintain high fracture conductivity at the near well bore area.

7.2 Future work

• Dynamic fracture permeability during injection is neglected in the models in this thesis. More researches can be done in this part to obtain a more accurate pressure distribution.

• Proppant transport by different fluid would be simulated to understand their behaviors.

• Models in this thesis can be written into modules that can be directly built into the fracture propagational model.

• Experiments and field data are needed to verify the proppant transport models.

• When cooperated with the fracture propagational model, the proppant transport models can be utilized to estimate explorability of naturally-fractured hydrocarbon reservoirs especially shale gas plays.

• Long term recovery simulation can be studied and to history match some real world cases to justify the results and also to obtain a more thorough understanding about characteristics and behaviors of hydraulic fracture network systems during production.
REFERENCES