OUTAGE PROBABILITY AND TRANSMITTER POWER
ALLOCATION FOR NC-BASED COOPERATIVE NETWORKS
OVER NAKAGAMI-M FADING CHANNEL

A Thesis in
Electrical Engineering
by
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Abstract

In previous work, we proposed a modified network coding (NC) cooperation system based on complementary code (CC)-code division multiple access (CDMA) technique. This technique not only consumes fewer time slots in either synchronous or asynchronous transmission but also alleviates multi-path interference. In that system, a multiplier is used at the base station where the received signal is multiplied with redundant signals coming from the relays, rather than performing a binary sum operation in the media access control (MAC) layer, which is normally used in traditional network coding. In this thesis, a further investigation is carried out in the proposed system in terms of outage probability and transmitter power allocation. Outage probability was obtained by setting some conditions on a modified mutual information equation. In addition, using the max-min optimization method, the optimal power allocation was obtained. Simulation results show that the proposed NC-based cooperative system reduces the overall system outage probability and provides a better transmitter power allocation as compared to the traditional cooperative communication system. These results are obtained by using a Nakagami-$m$ fading channel.

Index Terms—Bit error rate, network coding, complementary code-code division multiple access (CC-CDMA), cooperative, relay, diversity, outage probability, capacity, optimal power.
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<td>Third Generation</td>
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<td>ACF</td>
<td>Auto Correlation Function</td>
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<td>AF</td>
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<td>BER</td>
<td>Bit Error Rate</td>
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<td>BS</td>
<td>Base Station</td>
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<td>CCF</td>
<td>Cross Correlation Function</td>
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<tr>
<td>DF</td>
<td>Decode and Forward</td>
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<tr>
<td>MAI</td>
<td>Multiple Access Interference</td>
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<td>ML</td>
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**PG** Proccessing Gain

**SNR** Signal Noise Ratio

**TDMA** Time Division Multiple Access

- \( c_{i,m}(t) \): spreading signal or spreading sequence at transmitter \( i \)
- \( c_{r_j,m}(t) \): spreading signal or spreading sequence at relay node \( j \)
- \( f_m \): central frequency
- \( h_{i,r_j}(t) \): channel response via \( i \)-th transmitter and \( j \)-th relay
- \( n_m(t) \): \( m \)-th subcarrier noise
- \( K \): the number of users which the complementary code can support
- \( Z \): the number of relay nodes which the complementary code can support
- \( Q(x) \): \( Q \)-function
- \( s_{i}(t) \): transmitting signal at transmitter \( i \)
- \( s_{r_j}(t) \): transmitting signal at relay node \( j \)
- \( w_t \): wideband noise
- \( w_{t_i,d} \): weight of \( t_i \) to \( d \)
- \( w_{r_{j},d} \): weight of \( r_j \) to \( d \)
- \( x_{t_i}(t) \): local information at transmitter \( i \)
- \( x_{r_j}(t) \): local information at relay node \( j \)
- \( y_{t_i,r_j} \): received signals at \( r_j \) from \( t_i \)
- \( y_{t_i,d} \): received signals at \( d \) from \( t_i \)
- \( y_{r_{j},d} \): received signals at \( d \) from \( r_j \)
- \( \tilde{y}_{d,sum} \): total signals multiplied by the corresponding weight received at \( d \)
- \( \tilde{y}_{t_1,r_1} \): decision variable at \( r_1 \) from \( t_1 \)
$\alpha_{t_i,r_j,l}(t)$ Nakagami-$m$ distributed random variable
$\alpha_{t_i,d,l}(t)$ Nakagami-$m$ distributed random variable
$\alpha_{r_j,d,l}(t)$ Nakagami-$m$ distributed random variable
$\theta_{t_i,r_j,l}(t)$ phase of $i$-th transmitter to $z$-th relay
$\theta_{t_i,d,l}(t)$ phase of $i$-th transmitter to $d$
$\theta_{r_j,d,l}(t)$ phase of $j$-th relay to $d$
$\lambda_{t_i,r_j,l}(t)$ delay via $i$-th transmitter and $j$-th relay

$\delta$ Dirac-delta function
$\Gamma_{t_i,r_j}^{(DF)}$ useful signals via $t_i$ and $r_j$ in DF case
$\Gamma_{t_i,d}^{(DF)}$ useful signals via $t_i$ and $d$ in DF case
$\Gamma_{r_j,d}^{(DF)}$ useful signals via $r_j$ and $d$ in DF case
$\xi_{t_i,r_j}^{(DF)}$ noise via $t_i$ and $r_j$ in DF case
$\xi_{t_i,d}^{(DF)}$ noise via $t_i$ and $d$ in DF case
$\xi_{r_j,d}^{(DF)}$ noise via $r_j$ and $d$ in DF case
$\mathcal{E}$ the conditional expected value of $\tilde{y}_{d,sum}^{(t_1)}$ for relay one case
$\bar{\mathcal{E}}$ the conditional expected value of $\tilde{y}_{d,sum}^{(t_1)}$ for relay two case
$\check{\mathcal{E}}$ the conditional expected value of $\tilde{y}_{d,sum}^{(t_1)}$ for relay three case
$\hat{\mathcal{E}}$ the conditional expected value of $\tilde{y}_{d,sum}^{(t_1)}$ for relay four case
$\mathcal{E}$ the conditional expected value of $\tilde{y}_{d,sum}^{(t_1)}$ for general case
$\mathcal{V}$ the conditional variance of $\tilde{y}_{d,sum}^{(t_1)}$ for relay one case
$\bar{\mathcal{V}}$ the conditional variance of $\tilde{y}_{d,sum}^{(t_1)}$ for relay two case
$\check{\mathcal{V}}$ the conditional variance of $\tilde{y}_{d,sum}^{(t_1)}$ for relay three case
$\tilde{\mathcal{V}}$ the conditional variance of $\tilde{y}_{d,sum}^{(t_1)}$ for relay three case
\( \hat{\nu} \) the conditional variance of \( \tilde{y}_{d,\text{sum}}^{(t_1)} \) for relay four case

\( \hat{\nu} \) the conditional variance of \( \tilde{y}_{d,\text{sum}}^{(t_1)} \) for general case

\( \gamma_{b,1r} \) SNR at \( d \) for relay one case

\( \gamma_{b,2r} \) SNR at \( d \) for relay two case

\( \gamma_{b,3r} \) SNR at \( d \) for relay three case

\( \gamma_{b,4r} \) SNR at \( d \) for relay four case

\( P_{out} \) outage probability

\( \Psi(\cdot,\cdot) \) the lower incomplete gamma function

\( L \) Lagrangian

\( \lambda \) Lagrangian multiplier
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I lovingly dedicate this thesis to my parents, who supported me each step of the way.
Chapter 1

Introduction

1.1 Introduction

In this thesis, the outage probability performance and optimal power allocation, of a network coding cooperative communication system based on complementary code CDMA (NC-CC-CDMA), is investigated. In addition, our previous work in NC-CC-CDMA is further validated with several new MATLAB simulations.

1.2 Outage performance

1.2.1 Outage probability

There are two typical ways to improve and measure a communication system performance, which are bit error rate (BER: $P(e)$) and outage probability ($P_{out}$). In this thesis, we focus on reducing $P_{out}$ to improve our system performance. The outage performance method is based on channel capacity and mutual information [1]. Outage occurs when the channel capacity is less than the transmission rate $R$, i.e., $\log(1 + \text{SNR}|h|^2) < R$. Hence, the main idea of the performance improvement is increasing the channel capacity, which can be achieved by several transmission techniques such as CDMA [3]-[7], cooperative communication [8]-[10] and network coding [11] (see also Section 1.4). The essential ideas of these transmission techniques will be introduced in the following sections. The outage probability performance comparisons with others’ works are also depicted in Chapter 3.
1.2.2 Optimal power allocation

Optimization methods are categorized into two parts: standard max-min problem [12] (see Section 4.1.1) and ”Lagrangian problem [13]” (see Section 4.1.2). The optimization is always the main concept to achieve the optimal situation. In other words, we not only pursue the best system performance but also provide an optimal power allocation for mobile users.

Optimal power allocation can meet the limited radio resources requirements such as bandwidth and transmission power. The main objective of this research is to find solutions that can improve the channel capacity and utilization of the radio resources that are available to the service providers. We will show with numerical results in Chapter 4, that the outage probability can be further reduced with optimal power allocation.

1.2.3 System outage probability introduction

The outage probability performance analysis is performed on NC-CC-CDMA that was proposed in our previous work [2]. The NC-CC-CDMA system will be briefly introduced in the following section. Due to the cooperative nature of the system and its diversity, the outage probability can be dramatically reduced; while the network coding part improves the system’s channel capacity.

1.3 The system framework

In this Section, we briefly review the network-coding-based cooperation from our previous work [2], and introduce the main idea in our system model. In Chapter 2, mathematical analyses for the model will be presented.

In NC-based cooperative communication system, network coding is used as the second transmission phase into the traditional cooperative networks [15]-[25]. The network coding method improved the cooperation system capacity as compared to the conventional one, which is a significant enhancement for outage probability performance analysis. In addition, the traditional NC-based cooperative communication system achieved diversity, which is an important characteristic for system reliability. In [2], we offered an alternative solution for diversity by introducing a
multiplier at the base station (BS) as well as using CC-CDMA in the cooperation system. This method, NC-based CC-CDMA, provided a better BER performance as compared to the traditional cooperative communication in addition to saving time slots and channel bandwidth. Moreover, the system’s complexity was also reduced because there was no maximum likelihood detection used at the base station (no MAC layer decoding).

1.3.1 Cooperation

![Diagram of cooperative communication]

**Figure 1.1.** Traditional cooperative communication scheme. Mobile user 2 conveys user 1’s signal to the base station because of its better communication channel quality.

In this subsection, we briefly review how cooperative networks work. As shown in Fig. 1.1, cooperation is simply a process that mobile user 1 asks relay(s) – mobile user 2 – for help to convey their signals to the BS. The reason we use cooperation is because the communication link can have several barriers (see Fig. 1.1) on the line of sight (LOS). For example, the channel between mobile user 1 and the BS has couple of buildings, which can be regarded as barriers. Thus, this route may weaken the transmitted signal. Taking into account the uncontrollable conditions of the wireless transmission, another mobile user, who has a clear communicating
channel with the BS, can receive and retransmit signals. This simple idea improves the overall performance of the wireless communication system; however, it taxes the limited power resources of mobile users.

1.3.2 Network coding

![Diagram of network coding](image)

**Figure 1.2.** An example of a network-coding-based cooperative communication with two mobile users, one relay station, and a receiver.

In the traditional network coding mechanism, two or more digital signals are XOR-ed (in binary) at the MAC layer at the relay(s) and the BS (see Fig. 1.2). After passing through the cooperative user’s channel, the received messages, which have been XOR-ed at the relays, will be XOR-ed with the original signal at the BS to recover the mobile user’s signal. This idea is widely used in a cellular system as shown in Fig. 1.2, where, for simplicity, one relay with two mobile users and a receiver is depicted in [2].

1.3.3 Multiplier

The traditional network coding [11] uses an XOR operation throughout the communication link, as shown in Fig. 1.2. Our scheme used a multiplier at the BS that matches the operation of the traditional XOR, as shown in Table 1.1 and Fig. 1.3. However, the main advantage of using the multiplier is its use in the received signal, at the BS, thereby avoiding one extra decoding step in the protocol stack.
In order to show that the multiplier can accomplish the same function as the XOR operation, BPSK signals on the quadrant that are basically separated by positive and negative axes can be considered. The results of multiplication of $s_1$ and $s_2$ analog signals give either a positive or negative phase, which has equivalent results as the XOR logic gate (see Table 1.1). In the decode-and-forward (DF) case, decisions on the signals ($s_1$ and $s_2$) are made after despreading and demodulation to obtain the original analog signals from the transmitters. Therefore, if the channel conditions are such that the signal phases do not severely change, we can safely replace the XOR operation by a multiplier in the case of analog signals.

### 1.4 Benefits of the proposed system

In this Section, we briefly introduce the major differences between TDMA and CDMA and their spectral efficiency properties [4]-[7].
1.4.1 TDMA

1. Advantages of TDMA [7]:

There are plenty of advantages of TDMA in cellular communication systems.

- It can easily adapt to data transmission as well as voice communication.
- It is capable of carrying 64 kbps to 120 Mbps of data rates. This allows the operator to do services like fax, voice band data, and short message service (SMS) as well as bandwidth-intensive application such as multimedia and videoconferencing.
- TDMA separates users by time slots to ensure there will be no interference with transmissions.
- It provides users with an extended battery life, because it transmits only a portion of the time during conversations.
- It is the most cost-effective technology for upgrading analog to digital.

2. Disadvantages of TDMA [7]:

- Each user has a predefined time slot. When a user is moving from one cell to another, if all time slots in this cell are fully loaded, the user might be disconnected.
- It is subjected to multipath distortion. A message sent from a tower to a receiver might come from any one of multi-directions. It might have bounced off a couple of different buildings before arriving.

1.4.2 CDMA

1. Advantages of CDMA [7]:

- Failures happen only when the mobile device is at least twice as far from the base station. Hence, it is used in the rural areas where GSM (TDMA) cannot cover.
• It has a high spectral capacity that accommodates more users per MHz of bandwidth.

2. Disadvantages of CDMA [7]:

• The major problem in CDMA technology is channel pollution, i.e., signals from too many cells exist in the subscriber’s mobile device but none of them are dominant.

• Another drawback in this technology when compared to TDMA is the lack of international roaming capabilities.

1.4.3 Spectrum efficiency

Spectrum efficiency is defined as the maximum number of travel channels per MHz per cell. The spectrum efficiency depends upon the average bit error rate (BER) that is sufficient for a service quality. In a TDMA system, the channel capacity is fixed to a limited number of time slots and new users cannot be added when each of these slots is loaded. Due to GSM’s better error management and frequency hopping, the interference of the co-channel is reduced. This allows the frequency to be reused without ruining quality of service (QoS) [7]. CDMA spectral efficiency can be improved by radio resource management techniques, resulting in a higher number of simultaneous calls, and higher data rates can be achieved without adding more radio spectrum or more base station sites.

1.4.4 Benefits of complementary codes

CC-CDMA was introduced in [3] and [4] and offers several advantages. For example, the codes are perfectly orthogonal ones in a sense that they offer zero cross-correlation used between them in both synchronous and asynchronous transmission channels. Therefore, a CC-CDMA system can achieve multi-access interference (MAI)-free operation for both uplink and downlink transmissions. As it was mentioned in [3] ”The joint effect of ideal cross-correlation functions and ideal autocorrelation functions makes a CDMA system using them virtually interference-free, to make a truly noise-limited CDMA system!”
There are many different ways to characterize the CDMA codes, but nothing can be more intuitive and effective than the auto-correlation function (ACF) and cross-correlation function (CCF) [4].

Following the statement as mentioned above, the new proposed CDMA system [4] (CC-CDMA) can be concluded in three main advantage points:

1. The significant property attribute of CC-CDMA system is that the "offset stacked" spreading modulation method is particularly beneficial for multi-rate data transmission in multimedia services.

2. This offset stacking spreading technique can show data transmission rate by simply shifting more than one chip (at most $L$ chips) between two neighboring offset stacked bits. When $L$ chips are shifted between two consecutive bits, the proposed system reduces to a traditional CDMA system, yielding the lowest data rate\(^1\) while the highest data rate is achieved if only one chip is shifted between two neighboring offset stacking bits.

3. By doing these steps, the highest spreading efficiency equivalent to one can be achieved, which implies that every chip is capable of carrying one bit information.

Thus, the proposed system – CC-CDMA – is capable of delivering a higher bandwidth efficiency than a traditional CDMA system under the same processing gain.

This thesis is organized as follows: in Chapter 1, we introduce the main topics of this thesis: outage probability and optimal power allocation. The outage performance system’s background is also described in Section 1.3. In Section 1.3, a brief review of network-coding-based cooperation is presented and the modified system model for a NC-based cooperative system is proposed. The outage and error probability’s system model is analyzed in detail in Chapter 2, Section 2.1, under the framework of CC-CDMA NC-based cooperative system. Diversity analysis is presented in Section 2.2. Numerical simulations using MATLAB and Nakagami-$m$ fading channel are provided in Section 2.3. In Chapter 3, outage probability is derived and simulation results, from the derived outage probability equations,

\(^1\)Note that the spreading efficiency (SE) of all conventional CDMA-based mobile communication systems, such as IS-95, CDMA2000, and W-CDMA, are equal to $1/N$, which is far less than 1, which implies that these systems can not offer a better bandwidth efficiency.
are presented and discussed. Similarly, in Chapter 4, optimal power allocation expression is derived, simulated and discussed. The numerical results are shown by MATLAB figures. Conclusions follow in Chapter 5. Supporting equations are included in Appendices A-C [2] and Appendix D, which is an extension analysis for the diversity gain in this thesis.
Chapter 2

Error rate probability

2.1 System model

In this Section, we will review the mathematical basis of the NC-based cooperative system [2] introduced in Section 1.3 where the *decode-and-forward* [9] mode is used. In the following subsections, the proposed method [2] was further analyzed by using maximum ratio combining (MRC) that leaded to error probability calculation. Note that the network coding is only used at the base station.

![Figure 2.1. Source $t_1$ transmitter block diagram.](image)
2.1.1 Signals received at relay nodes

According to [3], the complementary code (CC)-CDMA transmitted signal from $s_{t_1}$, as shown in Fig. 2.1 and Fig. 2.2, can be expressed as:

$$s_{t_1}(t) = \sum_{m=1}^{M} \sqrt{\frac{2E_b}{T_b}} c_{t_1,m}(t) \cos(2\pi f_m t) x_{t_1}(t),$$  \hspace{1cm} (2.1)

where

$$x_{t_1}(t) = \sum_{n=-\infty}^{\infty} a_{t_1}(n) g(t - nT_b), \quad a_{t_1}(n) \in \{+1, -1\},$$  \hspace{1cm} (2.2)
and \( c_{t_1,m}(t) \) is the \( m \)-th element spreading sequence \([3]\) assigned to \( s_{t_1} \), \( 1 \leq m \leq M \), \( c_{t_1,m} \) is the normalized chip waveform with unit energy.

\[
c_{t_1,m}(t) \in \left\{ \frac{1}{\sqrt{M}}, -\frac{1}{\sqrt{M}} \right\}, \quad nT_b \leq t \leq (n+1)T_b.
\] (2.3)

and \( g(t-nT_b) \) is the bit pulse waveform function. The rectangle function \( g(x) \) is a function that is 0 outside the interval \([0,T_b/2]\) and unity inside it. It is also called the gate function, pulse function, or window function. An L-tap transversal filter is used to model the multipath Nakagami-\( m \) fading channel. The complex low-pass impulse response of the channel for the \( t_i \)-th space \([2]\) can be expressed by

\[
h_{t_i,r_j}(t) = \sum_{l=0}^{L} h_{t_i,r_j,l} e^{-j\theta_{t_i,r_j}} \delta(t - \lambda_{t_i,r_j}), \quad i \leq j \leq M
\] (2.4)

where \( \alpha_{t_i,r_j}, \theta_{t_i,r_j}, \) and \( \lambda_{t_i,r_j} \) are the \( t_i \)-th space and \( r_j \)-th path’s envelope, phase, and delay, respectively. \( \delta(t) \) is a Dirac-delta function; \( \alpha_{t_i,r_j} \) is a Nagakami-\( m \) distributed random variable. Since the CC-CDMA system consists of \( M \) subcarriers, therefore, there are \( M \) channels between source and relay nodes. Then, the complete impulse response can be expressed as a matrix

\[
h_{t_i,r_j,l} = [h_{t_i,r_j,l}^{(1)}, \ldots, h_{t_i,r_j,l}^{(m)}, \ldots, h_{t_i,r_j,l}^{(M)}].
\] (2.5)

Therefore, signals received at the relay node, after transmission through a slow multipath Nakagami-\( m \) fading channel, can be written as:

\[
\zeta_{t_i,r_j}(t) = \sum_{m=1}^{M} \sum_{l=0}^{L} h_{t_i,r_j,l}^{(m)}(t) \ast s_{t_i,r_j,m}(t)
\]

\[
= \sum_{m=1}^{M} \sum_{l=0}^{L} \alpha_{t_i,r_j,l}^{(m)} e^{-jg_{t_i,r_j,l}^{(m)}} s_{t_i,r_j,m}(t - \lambda_{t_i,r_j,l}^{(m)}),
\] (2.6)

where \( \ast \) represents the convolution operation. The transmitted signals, after the convolution with the channels and without channel gain and phase, can be written as:

\[
s_{t_i,r_j,m}(t - \lambda_{t_i,r_j,l}^{(m)}) = \sqrt{\frac{2E_b}{T_b}} c_{t_1,m}(t - \lambda_{t_i,r_j,l}^{(m)}) \cos \left( 2\pi f_m(t - \lambda_{t_i,r_j,l}^{(m)}) \right)
\]
\[ x(t - \lambda^{(m)}_{t_i, r_j, l}) \times x_{t_i, r_j}(t), \quad (2.7) \]

where \( \lambda^{(m)}_{t_i, r_j, l} \) represents the \( m \)-subcarrier delay between \( t_i \) to \( r_j \) channel. Consequently, the received signals \( y_{t_i, r_j} \) at \( r_j \) from \( t_i \) are:

\[
y_{t_i, r_j}(t) = \zeta_{t_i, r_j}(t) + w(t) = \sum_{m=1}^{M} \sum_{l=0}^{L} \alpha^{(m)}_{t_i, r_j, l} e^{-j\theta^{(m)}_{t_i, r_j, l}} s_{t_i, r_j, m}(t - \lambda^{(m)}_{t_i, r_j, l}) + w(t), \quad (2.8)
\]

where \( w(t) \) is the wideband noise. By adding up the multipath signals, after despreading and demodulation, at the relay node as shown in Fig. 2.2,

\[
\tilde{y}^{(m)}_{t_i, r_j}(t), \ldots, \tilde{y}^{(m)}_{t_i, r_j}(t), \ldots, \tilde{y}^{(M)}_{t_i, r_j}(t),
\]

where \( \tilde{y}^{(m)}_{t_i, r_j}(t) \) is the \( m \)-th subcarrier signal from transmitter \( t_i \) to relay node \( r_j \)

\[
\tilde{y}^{(m)}_{t_i, r_j}(t) = \int_{0}^{T_b} y^{(m)}_{t_i, r_j}(t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_m t) c_{t_i, m}(t) dt, \quad (2.9)
\]

Similarly, the direct signal from a transmitter \( t_i \) to the destination can be expressed as:

\[
\tilde{y}^{(m)}_{t_i, d}(t) = \int_{0}^{T_b} y^{(m)}_{t_i, d}(t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_m t) c_{t_i, m}(t) dt, \quad (2.10)
\]

where

\[
y^{(m)}_{t_i, r_j}(t) = \sum_{l=0}^{L} \alpha^{(m)}_{t_i, r_j, l} e^{-j\theta^{(m)}_{t_i, r_j, l}} s_{t_i, r_j, m}(t - \lambda^{(m)}_{t_i, r_j, l}) + n_m(t), \quad (2.11)
\]

and

\[
y^{(m)}_{t_i, d}(t) = \sum_{l=0}^{L} \alpha^{(m)}_{t_i, d, l} e^{-j\theta^{(m)}_{t_i, d, l}} s_{t_i, d, m}(t - \lambda^{(m)}_{t_i, d, l}) + n_m(t), \quad (2.12)
\]

where \( \lambda^{(m)}_{t_i, r_j, l} \) represents a \( m \)-th subcarrier time delay between source \( t_i \) to relay \( r_j \) of the Nagakami-\( m \) channel. Hence, the detected signals, using CC-CDMA, at the relays can be written as:

\[
\tilde{y}_{t_i, r_j} = \sum_{m=1}^{M} \tilde{y}^{(m)}_{t_i, r_j}(t)
\]
\[
\begin{align*}
\sum_{m=1}^{M} \int_0^{T_b} \bar{y}_{t_i,r_j}^{(m)}(t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_m t) c_{t_1,m}(t) dt \\
= \sum_{m=1}^{M} \int_0^{T_b} \left( \sum_{l=0}^{L} a_{t_i,r_j,d}^{(m)} e^{-j\phi_{t_i,r_j,d}^{(m)}} s_{t_i,r_j,m}(t - \lambda_{t_i,r_j,l}^{(m)}) + n_m(t) \right)
\times \sqrt{\frac{2}{T_b}} \cos(2\pi f_m t) c_{t_1,m}(t) dt \\
= \sum_{m=1}^{M} \sum_{l=0}^{L} \int_0^{T_b} \sqrt{\frac{2}{T_b}} a_{t_i,r_j,d}^{(m)} e^{-j\phi_{t_i,r_j,d}^{(m)}} s_{t_i,r_j,m}(t - \lambda_{t_i,r_j,l}^{(m)}) \\
\times \cos(2\pi f_m t) c_{t_1,m}(t) dt \\
+ \sum_{m=1}^{M} \int_0^{T_b} \sqrt{\frac{2}{T_b}} n_m(t) \cos(2\pi f_m t) c_{t_1,m}(t) dt \\
= \Gamma^{(DF)}_{t_i,r_j} + \xi^{(DF)}_{t_i,r_j} \\
= \sqrt{E_b} a_{t_i}(n') \alpha_{t_i,r_j,0} e^{-j\phi_{t_i,r_j,0}} + \xi^{(DF)}_{t_i,r_j},
\end{align*}
\]

where \(DF\) represents decode-and-forward. Similarly, the detected signals at the base station are given by,

\[
\begin{align*}
\bar{y}_{t_i,d} &= \Gamma^{(DF)}_{t_i,d} + \xi^{(DF)}_{t_i,d} \\
&= \sqrt{E_b} a_{t_i}(n') \alpha_{t_i,d,0} e^{-j\phi_{t_i,d,0}} + \xi^{(DF)}_{t_i,d}.
\end{align*}
\]

Where \(\Gamma^{(DF)}_{t_i,d}\) and \(\xi^{(DF)}_{t_i,d}\) are signal and noise, respectively and are discussed in detail in Appendices A and B.

### 2.1.2 Signals received at the destination

Similar to the analysis shown above, the signals received at the destination from the relays can be written as:

\[
\begin{align*}
\tilde{y}_{r_j,d} &= \sum_{m=1}^{M} \tilde{y}_{r_j,d}^{(m)}(t) = \sqrt{E_b} a_{r_j}(n') \alpha_{r_j,d,0} e^{-j\phi_{r_j,d,0}} + \xi^{(DF)}_{r_j,d}.
\end{align*}
\]
2.2 DIVERSITY ANALYSIS

For diversity analysis, we will use the proposed method, which is shown in Fig. 2.3. The details of the boxed area in Fig. 2.3 is shown in Fig. 2.4. Note that in the proposed method, a multiplier is used before the decision device whereas in previous method the XOR operation is used after the decision device, therefore reducing system’s complexity and reducing probability of error.

![Diagram](image)

**Figure 2.3.** Received signals at the base station. Note that the boxed part of this block diagram will be depicted in detail in Fig. 2.4.

In the following analysis, we focus on the diversity properties of received signal from user 1, $t_1$, at the destination. Before diversity analysis, two issues need to be tackled, optimal weights and probability of error by first calculating the output of $\tilde{y}_{t_1,d} \times \tilde{y}_{r_1,d}$ which is a redundant signal containing $t_1$ information. $\tilde{y}_{t_1,d}$ will be multiplied by an optimal weight $w_{t_1,d}$, and $\tilde{y}_{t_2,d}$ will be multiplied by $\tilde{y}_{r_1,d}$ and
Figure 2.4. Detail diagram of the shaded area in Fig. 2.3 at the destination for DF case. \( \tilde{y}_{t_2,d} \) is multiplied by \( \tilde{y}_{r_1,d} \) in order to be a redundant signal for diversity achievement with \( \tilde{y}_{t_1,d} \).

another optimal weight \( w_{r_1,d} \) (see Fig. 2.4) as shown below:

\[
\tilde{y}_{d,\text{sum}}^{(t_1)} = w_{t_1,d}\tilde{y}_{t_1,d} + w_{r_1,d}(\tilde{y}_{t_2,d} \times \tilde{y}_{r_1,d}).
\]  

(2.16)

The expected value and variance of \( \tilde{y}_{t_1,d} \) are shown in Appendix C.
2.2.1 Sub-optimal weights

Let \( w_{t1,d} \) be the sub-optimal weight from transmitter one to the destination and \( w_{r1,d} \) be the optimal weight from relay one to the destination. Following the results described in the Appendix C, Eq. (C.14) to (C.18), the sub-optimal weights are given by:

\[
\begin{align*}
  w_{t1,d}^* &= \frac{\Psi \sqrt{\Upsilon}}{\sqrt{\frac{N_0}{2}} \sqrt{\phi^2 \frac{N_0}{2} + \psi^2 \Upsilon}} \\
  w_{r1,d}^* &= \frac{\Phi \sqrt{\frac{N_0}{2}}}{\sqrt{\sqrt{\phi^2 \frac{N_0}{2} + \psi^2 \Upsilon}}}
\end{align*}
\]  

(2.17)

The output signal to noise ratio is found in the Appendix C and is given by:

\[
(SNR)_o = \frac{w_{t1,d}^2 \Psi^2 + 2w_{t1,d} w_{r1,d} \Psi \Phi + w_{r1,d}^2 \Phi^2}{w_{t1,d}^2 \frac{N_0}{2} + w_{r1,d}^2 \Upsilon},
\]

(2.18)

By substituting Eq. (2.17) (sub-optimal weights) into Eq. (2.18) (output SNR equation), the maximum output signal-to-noise ratio is given by:

\[
(SNR)_o = \frac{\left( \frac{\Psi \sqrt{T}}{\sqrt{\frac{N_0}{2}} \sqrt{\phi^2 \frac{N_0}{2} + \psi^2 T}} \right)^2 + \left( \frac{\Phi \sqrt{\frac{N_0}{2}}}{\sqrt{T} \sqrt{\phi^2 \frac{N_0}{2} + \psi^2 T}} \right)^2 \Upsilon}{\frac{1}{\phi^2 \frac{N_0}{2} + \psi^2 T} \left[ \left( \frac{\Psi \sqrt{\frac{T}{N_0}}}{\sqrt{\frac{N_0}{2}}} \right)^2 + \left( \frac{\Phi \sqrt{\frac{N_0}{2}}}{\sqrt{\Upsilon}} \right)^2 \right]} = \frac{\left( \frac{\Psi \sqrt{T}}{\sqrt{\frac{N_0}{2}}} \right)^2 + \left( \frac{\Phi \sqrt{\frac{N_0}{2}}}{\sqrt{\Upsilon}} \right)^2 \Upsilon}{\Psi^2 \Upsilon + \Phi^2 \frac{N_0}{2} \Upsilon},
\]

(2.19)

where \( \Psi, \Upsilon, \) and \( \Phi \) are defined in the Appendix C.
2.2.2 Probability of error at destination

Considering a BPSK CC-CDMA-based transmission technique, and following Fig. 2.3, the probability of error of the received signal, after the decision device at the base station, from transmitter \( t_1 \) is analyzed (extension to other received signals can be similarly done). Assuming that signals are broadcasted from all transmitters and relays, and noting that the output of \( \tilde{y}_{t_2,d} \times \tilde{y}_{r_1,d} \) is a redundant signal containing \( t_1 \) information, the error probability is given by:

\[
P(e) = \int_0^\infty \int_0^\infty \int_0^\infty P(e|\alpha_{t_1,d,0},\alpha_{r_1,d,0}) p_{\alpha_{t_1,d,0}}(\alpha_{t_1,d,0}) p_{\alpha_{t_2,d,0}}(\alpha_{t_2,d,0}) \]
\[
p_{\alpha_{r_1,d,0}}(\alpha_{r_1,d,0}) d\alpha_{t_1,d,0} d\alpha_{t_2,d,0} d\alpha_{r_1,d,0},
\]

(2.20)

where the conditional error probability in Eq. (2.20), given the channel responses to transmitter \( t_i \) and relay \( r_1 \) depicted by \( \alpha_{t_i,d,0} \) and \( \alpha_{r_1,d,0} \), respectively, is as follows:

\[
P(e|\alpha_{t_i,d,0},\alpha_{r_1,d,0}) = \int_0^\infty p(\tilde{y}_{d,sum}^{(t_1)})|_{\alpha_{t_1} = -1, \alpha_{t_1,d,0}, \alpha_{r_1,d,0}} dy_{d,sum}^{(t_1)}.
\]

(2.21)

and the conditional probability distribution of the decision variable \( \tilde{y}_{d,sum}^{(t_1)}(t) \), given also the channel responses, is:

\[
p(\tilde{y}_{d,sum}^{(t_1)})|_{\alpha_{t_1} = -1, \alpha_{t_1,d,0}, \alpha_{r_1,d,0}} = \frac{e^{-\left(\tilde{y}_{d,sum}^{(t_1)} + \varepsilon\right)^2/2\nu}}{\sqrt{2\pi\nu}},
\]

(2.22)

where \( \varepsilon \) and \( \nu \) are shown in the Appendix C. The Nakagami-\( m \) PDF is in essence a central chi-square distribution and is given by:

\[
p_{\alpha}(\alpha) = \frac{2m^m}{\Gamma(m)} \alpha^{2m-1} e^{-m\alpha^2}, \quad \alpha \geq 0.
\]

(2.23)

Where \( m \) is the Nakagami-\( m \) fading parameter, which ranges from 0.5 to \( \infty \). Combining Eqs. (2.20) and (2.23), provides the following error probability:

\[
P(e) = \int_0^\infty \int_0^\infty \int_0^\infty Q\left(\sqrt{\frac{\varepsilon^2}{\nu}}\right) \frac{2m_{t_1}^{m_{t_1}}}{\Gamma(m_{t_1})} \frac{2m_{r_1}^{m_{r_1}-1}}{\Gamma(m_{r_1})} e^{-m_{t_1} \alpha_{t_1,d,0}^2}.
\]
Therefore, the error probability can be rewritten as:

\[
\begin{align*}
\frac{2m_{t_2}^{m_{t_2}}\alpha_{t_2,0}^2}{\Gamma(m_{t_2})} e^{-m_{t_2}\alpha_{t_2,0}^2} \frac{2m_{r_1}^{m_{r_1}}\alpha_{r_1,0}^2}{\Gamma(m_{r_1})} e^{-m_{r_1}\alpha_{r_1,0}^2} \\
\times d\alpha_{t_1,0} d\alpha_{t_2,0} d\phi.
\end{align*}
\tag{2.24}
\]

where the Q-function in Eq. (2.24) is given by:

\[
Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp \left( - \frac{x^2}{2 \sin^2 \phi} \right) d\phi, \quad x \geq 0.
\tag{2.25}
\]

Therefore, the error probability can be rewritten as:

\[
P(e) = \frac{1}{\pi} \int_0^{\pi/2} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \exp \left( - \frac{\mathcal{E}^2}{\mathcal{V}(1R) \sin^2 \phi} \right) \\
\times \frac{2m_{t_1}^{m_{t_1}}\alpha_{t_1,0}^2}{\Gamma(m_{t_1})} e^{-m_{t_1}\alpha_{t_1,0}^2} \frac{2m_{t_2}^{m_{t_2}}\alpha_{t_2,0}^2}{\Gamma(m_{t_2})} e^{-m_{t_2}\alpha_{t_2,0}^2} \\
\times \frac{2m_{r_1}^{m_{r_1}}\alpha_{r_1,0}^2}{\Gamma(m_{r_1})} e^{-m_{r_1}\alpha_{r_1,0}^2} \frac{2m_{r_2}^{m_{r_2}}\alpha_{r_2,0}^2}{\Gamma(m_{r_2})} e^{-m_{r_2}\alpha_{r_2,0}^2} \\
\times \frac{2m_{r_3}^{m_{r_3}}\alpha_{r_3,0}^2}{\Gamma(m_{r_3})} e^{-m_{r_3}\alpha_{r_3,0}^2} \frac{2m_{r_4}^{m_{r_4}}\alpha_{r_4,0}^2}{\Gamma(m_{r_4})} e^{-m_{r_4}\alpha_{r_4,0}^2} \\
\times d\alpha_{t_1,0} d\alpha_{t_2,0} d\phi d\alpha_{r_1,0} d\alpha_{r_2,0} d\alpha_{r_3,0} d\alpha_{r_4,0} d\phi.
\tag{2.26}
\]

The above integrations do not have closed form solutions\(^1\); therefore, a numerical integration, in MATLAB, is performed on Eq. (2.26). The system’s performance can improve dramatically when the number of relays increases, as discussed in the next sections. For instance, Eq. (2.26) can be extended from a single relay equation to a more general form including several relays (see Fig. 2.5). For simplicity, Eq. (2.27) shows the probability of error using a four-relay NC-based CC-CDMA cooperative communication system:

\[
P(e) = \frac{1}{\pi} \int_0^{\pi/2} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \exp \left( - \frac{\mathcal{E}^2}{\mathcal{V}(1R) \sin^2 \phi} \right) \\
\times \frac{2m_{t_1}^{m_{t_1}}\alpha_{t_1,0}^2}{\Gamma(m_{t_1})} e^{-m_{t_1}\alpha_{t_1,0}^2} \frac{2m_{t_2}^{m_{t_2}}\alpha_{t_2,0}^2}{\Gamma(m_{t_2})} e^{-m_{t_2}\alpha_{t_2,0}^2} \\
\times \frac{2m_{r_1}^{m_{r_1}}\alpha_{r_1,0}^2}{\Gamma(m_{r_1})} e^{-m_{r_1}\alpha_{r_1,0}^2} \frac{2m_{r_2}^{m_{r_2}}\alpha_{r_2,0}^2}{\Gamma(m_{r_2})} e^{-m_{r_2}\alpha_{r_2,0}^2} \\
\times \frac{2m_{r_3}^{m_{r_3}}\alpha_{r_3,0}^2}{\Gamma(m_{r_3})} e^{-m_{r_3}\alpha_{r_3,0}^2} \frac{2m_{r_4}^{m_{r_4}}\alpha_{r_4,0}^2}{\Gamma(m_{r_4})} e^{-m_{r_4}\alpha_{r_4,0}^2} \\
\times d\alpha_{t_1,0} d\alpha_{t_2,0} d\phi d\alpha_{r_1,0} d\alpha_{r_2,0} d\alpha_{r_3,0} d\alpha_{r_4,0} d\phi.
\tag{2.27}
\]

\(^1\)For a special case, we have a closed form as shown in subsection Appendix D.
Figure 2.5. A general case of a NC-based cooperative communication system with $K$ sources and $Z$ relays.

2.3 Numerical analyses

2.3.1 NC-based cooperative by using a multiplier

In this Section, we analyze the proposed system’s performance using the probability of error described in Eq. (2.26), where $P(e)$ is plotted against the SNR denoted by $\gamma_{b,1r}$, in Figs. 2.6, 2.7, and 2.8 under different Nagakami-$m$ parameters. In these figures, $m_{t_i}$ represents the Nagakami-$m$ parameter fading channel from transmitter $t_i$ to relay $r_j$ and the base station $d$; $m_{r_j}$ represents the Nagakami-$m$ parameter fading channel from relay $r_j$ to the base station $d$. Notice that the system’s performance dramatically improves when the $m_{t_i}$ value increases (see Figs. 2.6 - 2.8).

In Fig. 2.6, a single relay NC-based cooperative communication system, $m_{t_2}$ and $m_{r_j}$’s Nakagami-$m$ parameters are fixed at 1, while $m_{t_1}$ changes from 1, 2, and 3 (note that when $m = \infty$, the channel has no fading). Similarly, in Figs. 2.7 and
The proposed system & SNR $\approx 5$ dB & SNR $\approx 10.5$ dB & SNR $\approx 15$ dB & SNR $= 20$ dB \\
Distributed phase steering ($0^\circ$) [21] & 2.39e-02 & 2.846e-03 & 3.808e-04 & 2.18e-05 \\
Distributed phase steering ($90^\circ$) [21] & 1.727e-01 & 9.725e-02 & 5.629e-02 & 2.799e-02 \\
NCCD [22] & 1.041e-01 & 3.916e-02 & 1.6e-02 & 4.926e-03 \\
ML detection [23] & 8.025e-02 & 7.109e-03 & 1.023e-03 & 9.059e-05 \\
\hline

Table 2.1. BER comparisons vary from SNR 5 to 20 dB.

$2.8$, $m_t$ and $m_r$’s Nakagami-$m$ parameters are fixed at 1, the system performance improves when $m_t$ value increases.

In Fig. 2.10, we categorize multi-relay system performance into a graph to show diversity property. When $m_t$ and $m_r$’s Nakagami-$m$ parameters are fixed at 1, system performance improves when the number of relays increase.

### 2.3.2 Multiplier network coding vs. ML distributed phase steering

Considering the worst case ($\theta = 0^\circ$) and the best case ($\theta = 90^\circ$) [21] in Rayleigh channel fading, which is equivalent to Nakagami-$m$ with $m_{t_1} = 1$, comparisons are made with a similar scheme [21], using decode-and-forward with maximum likelihood detection method. The proposed system performs better as depicted in Fig. 2.11. In addition, in [21], it is assumed a distributed phase precoding, which is not the case in the proposed method. A BPSK constellation, i.e., $x_j \in \{\pm 1\}$, the received waveform is $\sum_{j=1}^{2} \pm h_j$ gives four possibilities for any fixed set of fading coefficients. The distributed phase steering with the maximum likelihood (ML) detection [21], when $\theta = 0^\circ$, the received signals map onto two different quaternary pulse amplitude modulation constellations (4-PAM); while $\theta = 90^\circ$, the constellation becomes a 4-QAM. By the BER analysis in Fig. 2.11, when SNR $= 20$ dB, their performance improves to $BER = 2.799 \times 10^{-2}$ when $\theta = 0^\circ$, which is the worst case; and the performance improves to $BER = 4.926 \times 10^{-3}$ when $\theta = 90^\circ$, which is the best case while the proposed method has a better performance, $P(e) = 2.18 \times 10^{-5}$. However, this performance is achieved using multi-relays with a simple multiplier at the destination, while [21] uses a more advanced demodulator at the MAC layer. This comparison shows that there is a
trade-off between performance and receiver complexity. The BER performance in Fig. 2.11 [21] is shown as:

1. For the best case ($\theta = 90^\circ$):

$$P_{b_{\theta=90}}^b = \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma}{2 + \gamma}} \right), \quad (2.28)$$

where $\gamma = \frac{1}{\sigma^2_n}$ is the average received SNR.

2. For the worst case ($\theta = 0^\circ$):

$$P_{b_{\theta=0}}^b \approx \frac{1}{4} - \frac{\sqrt{\gamma}}{4\sqrt{4 + \gamma}} + \frac{1}{24 + 12\gamma} + \frac{\gamma \arctan \sqrt{1 + \gamma}}{24(1 + \gamma)^{1.5}}$$

$$- \frac{\gamma}{24(1 + \gamma)(2 + \gamma)} + \frac{3}{24 + 16\gamma}$$

$$+ \frac{\sqrt{3}\gamma \arctan \sqrt{1 + \frac{4}{3}\gamma}}{2(3 + 4\gamma)^{1.5}} - \frac{3\gamma}{4(3 + 4\gamma)(3 + 2\gamma)}$$

$$+ \frac{1}{12(1 + 2\gamma)(4 + 9\gamma)} - \frac{3}{12(2 + 5\gamma)^{1.5}}$$

$$+ \frac{\gamma(1 + 3\gamma)}{6(2 + 5\gamma)(4 + 17\gamma + 18\gamma^2)} + \frac{3}{8(3 + 8\gamma)(2 + 6\gamma)}$$

$$- \frac{\sqrt{3}\gamma \arctan \frac{\sqrt{3 + 10\gamma}}{\sqrt{(1 + 4\gamma)}}}{(3 + 10\gamma)^{1.5}} + \frac{3\gamma(1 + 4\gamma)}{2(3 + 10\gamma)(6 + 34\gamma + 48\gamma^2)}$$

$$- \frac{\sqrt{2}\gamma \arctan \frac{\sqrt{2 + 5\gamma}}{\sqrt{(1 + \gamma)}}}{12(2 + 5\gamma)^{1.5}} - \frac{9}{16(3 + 8\gamma)(3 + \gamma)}$$

$$+ \frac{\gamma(1 + \gamma)}{6(2 + 5\gamma)(4 + 9\gamma + 2\gamma^2)} - \frac{\sqrt{3}\gamma \arctan \frac{\sqrt{3(3 + 10\gamma)}}{(3 + 4\gamma)}}{2(3 + 10\gamma)^{1.5}} +$$

$$+ \frac{3\gamma(3 + 4\gamma)}{4(3 + 10\gamma)(9 + 27\gamma + 8\gamma^2)} - \frac{1}{12(1 + 2\gamma)(4 + \gamma)}, \quad (2.29)$$

where $\gamma = \frac{1}{\sigma^2_n}$ is the average received SNR.
2.3.3 Multiplier network coding vs. Network-Coded Cooperative Diversity (NCCD)

Considering the same fading channel (Rayleigh which is equivalent to Nakagami-\(m\) with \(m_t = 1\)), comparisons are made with a similar scheme [22], using decode-and-forward method. The proposed system performs better as depicted in Fig. 2.11. Note that [22] uses only only transmitter \((N_s = 1)\) and one relay, which typically offers better performance than using two transmitters and one relay as in the proposed scheme. Consider the cooperation scheme with a single relay with DF [22], when \(\text{SNR} = 20\ \text{dB}\), their performance improves to \(\text{BER} = 4.52 \times 10^{-5}\) when \(N_s = 1\), and \(\text{BER} = 9.059 \times 10^{-5}\) when \(N_s = 2\), while the proposed method has a better performance, \(P(e) = 2.18 \times 10^{-5}\) with two transmitters \((N_s = 2)\) and a single relay. Note that [22] always uses cooperative maximum ratio combining (C-MRC) as the proposed system does; however, they use a more advanced demodulator at the MAC layer. This comparison shows that there is a trade-off between performance and receiver complexity. The BER performance in Fig. 2.11 [22] is shown as:

\[
P_s^{i,\mathcal{X}} = \frac{1}{\bar{\gamma}_f} \left( C_{\mathcal{X}}^1 \sum_{i=1}^{N_s} \frac{1}{\bar{\gamma}_g_i} + C_{\mathcal{X}}^2 \left[ \sum_{j=1}^{N_s} \frac{1}{\bar{\gamma}_f_j} + \frac{1}{\bar{\gamma}_R} \right] \right),
\]

(2.30)

where \(\mathcal{X} \in \{\text{BPSK}\}\). \(C_{\mathcal{X}}^1\) and \(C_{\mathcal{X}}^2\) are \(\frac{45 + \sqrt{5}}{160}\) and \(\frac{3}{16}\), respectively.

2.3.4 Multiplier network coding vs. ML detection network coding

Considering the same fading channel (Rayleigh which is equivalent to Nakagami-\(m\) with \(m_t = 1\)), comparisons are made with a similar scheme [23], using decode-and-forward with maximum likelihood detection method. The proposed system performs better or equivalent to as depicted in Fig. 2.11. In addition, in [23], it is assumed an error/delay-free control channel, which is not the case in the proposed method. With the DF maximum likelihood detection [23], when \(\text{SNR} = 20\ \text{dB}\), their performance improves to \(\text{BER} = 3.35 \times 10^{-5}\) for one user and one
relay \(^2\), and BER = 6.95 \times 10^{-5} for two users and one relay; while the proposed method has a better performance, \( P(e) = 2.18 \times 10^{-5} \), still with two transmitters. However, this performance is achieved using multi-relay with a simple multiplier at the destination, while [23] uses a more advanced demodulator at the MAC layer. This comparison shows that there is a trade-off between performance and receiver complexity. The BER performance in Fig. 2.11 is shown as:

\[
\begin{align*}
    r_{D,N_s+1} &= h_{RD} \hat{x}_C + n_{D,N_s+1} \\
    &= \frac{h_{RD}}{\sqrt{N_s}} \sum_{i=1}^{N_s} \hat{x}_i + n_{D,N_s+1} \\
    &= \frac{h_{RD}}{\sqrt{N_s}} \sum_{i=1}^{N_s} (x_i + e_i) + n_{D,N_s+1} \\
    &= \frac{h_{RD}}{\sqrt{N_s}} \sum_{i=1}^{N_s} x_i + \frac{h_{RD}}{\sqrt{N_s}} \sum_{i=1}^{N_s} e_i + n_{D,N_s+1}.
\end{align*}
\]

(2.31)

where \( N_s \) represents the number of users.

### 2.4 Summary of the advantages of using a multiplier network coding

Comparisons between a multiplier XOR and a conventional XOR operation are made as follows:

1. The first advantage of using a multiplier XOR is that considering maximum ratio combining (MRC) at the destination \( d \) for both multiplier and conventional XOR operation, the sum of weighted signals at \( d \) from the proposed system are much stronger than from the traditional XOR system. At the destination in the proposed system, each received relay signal is multiplied by received signal from \( t_2 \) to generate a \( t_1 \)-like signal, which is considered as a \textit{copy} for \( t_1 \) signal to achieve diversity performance. The detailed mathematics
Figure 2.6. Error probability when changing $m_{t_1}$ value with 1, 2, and 3, under the conditions of setting $m_{t_2} = 1$, $m_{r_1} = 1$.

for the multiplier XOR is shown as:

$$
\tilde{y}_{d, \text{sum}}^{(t_1)} = w_{t_1,d}\tilde{y}_{t_1,d} + \bar{y}_{t_2,d} \sum_{j=1}^{4} w_{r_j,d}\bar{y}_{r_j,d} \\
= \tilde{y}_{s}^{(t_1)} + \bar{y}_{d}^{(t_1)},
$$

(2.32)

where the valued signal term is:

$$
\tilde{y}_{s}^{(t_1)} = w_{t_1,d}\sqrt{E_b}a_{t_1}(n')\alpha_{t_1,d,0}e^{-j\theta_{t_1,d,0}} \\
+ \tilde{y}_{t_2,d}\sqrt{E_b} \sum_{j=1}^{4} w_{r_j,d}a_{r_j}(n')\alpha_{r_j,d,0}e^{-j\theta_{r_j,d,0}}
$$
Figure 2.7. Error probability when changing $m_{t_1}$ value with 1, 2, and 3, under the conditions of setting $m_{t_2} = 1, m_{r_1} = 1, m_{r_2} = 1$.

\[
E_{t_1,d} = w_{t_1,d}\sqrt{E_b}a_{t_1}(n')\alpha_{t_1,d,0}e^{-j\theta_{t_1,d,0}} + \sqrt{E_b}a_{t_2}(n')\alpha_{t_2,d,0}e^{-j\theta_{t_2,d,0}} \\
\times \sqrt{E_b}\sum_{j=1}^{4}w_{r_j,d}a_{r_j}(n')\alpha_{r_j,d,0}e^{-j\theta_{r_j,d,0}}.
\]  

(2.33)

Note that $a_{r_j,d}(n')$ is a conventional XOR signal at each relay node, which is $a_{t_1} \oplus a_{t_2} = a_{r_j}$. Following the digital signals simplification in Eq. (2.33), the channel weight for a multiplier XOR operation depends on each $t_2 \times r_j$ channel, so it can generate a stronger output SNR (see Eq. (2.19)) by larger weights (see Eq. (2.17)); while a conventional XOR can be regarded as an equal gain channel because $t_1$ signals is recovered by $t_2$ and $r_j$’s signals after
Figure 2.8. Error probability when changing $m_{t_1}$ value with 1, 2, and 3, under the conditions of setting $m_{t_2} = 1$, $m_{r_1} = 1$, $m_{r_2} = 1$, $m_{r_3} = 1$.

decoding, so the weights are exactly the same. In other words, the output SNR cannot be greater than the output SNR in the multiplier XOR method. Therefore, BER performance comparison results in Fig. 2.11 can clearly show that system performance for a multiplier XOR is better.

2. A multiplier operation is simpler than using any signal detection method like maximum likelihood (ML) detection [23]-[21], or multiuser detection (MUD) [19] method. The error rate probability in the numerical analysis clearly shows that using a simple multiplier method cannot only improve system performance, but also reduce the system’s complexity.
Figure 2.9. Error probability when changing $m_{t_1}$ value with 1, 2, and 3, under the conditions of setting $m_{t_2} = 1$, $m_{r_1} = 1$, $m_{r_2} = 1$, $m_{r_3} = 1$, $m_{r_4} = 1$. 
Figure 2.10. Error probability when fixing all $m_t_i$ and $m_r_j$ values with 1, under the conditions of changing the number of relays.
Comparison of the proposed method vs. other methods

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>Error probability (P_e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method with 2-Tx, 1-Relay, m=1</td>
<td></td>
</tr>
<tr>
<td>DF ML method with 2-Tx, 1-Relay, m=1</td>
<td></td>
</tr>
<tr>
<td>Distributed phase method (θ=90°), 2-Tx, 1-Relay, m=1</td>
<td></td>
</tr>
<tr>
<td>Distributed phase method (θ=0°), 2-Tx, 1-Relay, m=1</td>
<td></td>
</tr>
<tr>
<td>NCCD with 2-Users (Tx or Relay), m=1</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.11. BPSK error probability performance.
Outage probability

In this Chapter, we introduce an outage probability analysis to determine the system’s performance. In Section 3.1, we build up channel mutual information and channel capacity equations, respectively. Following the channel capacity equation, we can derive a closed form outage probability based on Nakagami-\(m\) fading channel. Next, in Section 3.2, we show the outage performance in our work and three significant comparisons with results of other researchers.

3.1 Outage probability

Outage happens when the signals cannot be reliably decoded at the destination. From an information theoretic point of view, given the information rate \(R\) and the channel gain \(h\), outage occurs if the channel capacity is less than \(R\), i.e., \(\log(1 + \text{SNR}|h|^2) < R\). Therefore, outage probability under the channel gain \(h\) can be expressed as a function of the transmission rate as given below:

\[
P_{\text{out}}(R) \triangleq \Pr(\log(1 + \text{SNR}|h|^2) < R) = \Pr(\text{SNR}|h|^2 < 2^R - 1).
\] (3.1)

We expect that our system shall have a better outage probability performance because of CC-CDMA and network-coding-based system. As we mentioned in the introduction, network coding helps characteristics of the proposed system save bandwidth. Therefore, the greater the channel bandwidth, the better the outage
performance.

<table>
<thead>
<tr>
<th>Phase 1</th>
<th>Phase 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**Figure 3.1.** Illustration of a cooperative relay network with $N$ parallel potential relays [35].

In [8], [32], and [36], Laneman et al. proposed the maximum average mutual information [1] for repetition-coded decode-and-forward as shown in Fig. 3.1, which is given by,

$$I_{DF} = \frac{1}{2} \min \left\{ \log (1 + \text{SNR} |\alpha_{s,r}|^2), \right.$$  
$$\log (1 + \text{SNR} |\alpha_{s,d}|^2 + \text{SNR} |\alpha_{r,d}|^2) \right\},$$  

as a function of the fading random variables. In addition, outage event for spectral efficiency $R$ was given by $I_{DF} < R$ and is equivalent to the event,

$$\min \{|\alpha_{s,r}|^2, |\alpha_{s,d}|^2 + |\alpha_{r,d}|^2\} < \frac{2^{2R} - 1}{\text{SNR}}.$$  

Therefore, the overall outage probability can be written as:

$$P^{out}_{DF}(\text{SNR}, R) := \Pr[I_{DF} < R]$$  
$$= \Pr \left[ |\alpha_{s,r}|^2 < \frac{2^{2R} - 1}{\text{SNR}} \right] + \Pr \left[ |\alpha_{s,r}|^2 \geq \frac{2^{2R} - 1}{\text{SNR}} \right]$$  
$$\times \Pr \left[ |\alpha_{s,d}|^2 + |\alpha_{r,d}|^2 < \frac{2^{2R} - 1}{\text{SNR}} \right].$$  

(3.4)
Based on the above method in [8], the mutual information in our work can be shown:

\[
I_{DF} = \frac{1}{2} \log \left( 1 + \frac{2}{S} \sum_{i=1}^{K} \text{SNR}_{t_i,r_j} |\alpha_{t_i,r_j}|^2 \right) \\
+ \frac{1}{2} \log \left( 1 + \frac{2}{S} \sum_{i=1}^{K} \text{SNR}_{t_i,d} |\alpha_{t_i,d}|^2 + \frac{2}{S} \sum_{j=0}^{Z} \text{SNR}_{r_j,d} |\alpha_{r_j,d}|^2 \right), \tag{3.5}
\]

where \(K\) represents the number of users, \(Z\) represents the number of relay nodes, and \(S\) represents the spreading sequence. Therefore, the outage probability can be written as:

\[
P_{DF}^{\text{out}}(\text{SNR}, R) := \Pr[I_{DF} < R] = \Pr \left[ \sum_{i=1}^{K} |\alpha_{t_i,r_j}|^2 < \frac{(2^R - 1)S}{2\text{SNR}_{t_i,r_j}} \right]
\]
\begin{equation}
+ \Pr \left[ \sum_{i=1}^{K} |\alpha_{t_i,r_j}|^2 \geq \frac{(2^2R - 1)S}{2\text{SNR}_{t_i,r_j}} \right]
\times \Pr \left[ \frac{1}{S} \sum_{i=1}^{K} |\alpha_{t_i,d}|^2 + \frac{1}{S} \sum_{n=0}^{Z} |\alpha_{r_j,d}|^2 < \frac{2^2R - 1}{2\text{SNR}_{t_i,d}} \right], \quad (3.6)
\end{equation}

where SNRs are defined as:

- SNR at relay \( j \) from \( t_i \) can be written as:

\begin{equation}
(SNR)_{t_i,r_j} = \frac{P_{t_i,r_j}}{\sigma_n},
\end{equation}

- SNR at \( d \) from \( t_i \) can be written as:

\begin{equation}
(SNR)_{t_i,d} = \frac{P_{t_i,d}}{\sigma_d},
\end{equation}

- SNR at \( d \) from relay \( j \) can be written as:

\begin{equation}
(SNR)_{r_j,d} = \frac{P_{r_j,d}}{\sigma_d}.
\end{equation}

Knowing that \(|h_{i,j}|^2\) is a Nakagami-\(m\) distribution with parameters \( m, \Omega, \) and \( \theta. \) \( \theta \) is assumed to be uniformly distributed over \((0, 2\pi] \). Hence, the PDF of \(|h_{i,j}|\) is given by,

\begin{equation}
\begin{align*}
\frac{2}{\Gamma(m)} \left( \frac{m}{\Omega} \right)^m x^{2m-1} e^{-\frac{m}{\Omega} x^2} U(x).
\end{align*}
\end{equation}

Each of the channel gain, i.e. \(|h_{i,j}|^2\) follows a Gamma distribution with parameters \( \alpha_i = m_i > 0 \) and \( \beta_i = \Omega_i/m_i > 0 \). Therefore, the PDF of \(|h_{i,j}|^2\) is given as:

\begin{equation}
\begin{align*}
\frac{x^{\alpha_i-1} e^{-x/\beta_i}}{\Gamma(\alpha_i) \beta_i^{\alpha_i}} U(x).
\end{align*}
\end{equation}

For example, when \( K = 2 \) users, \( Z = 1 \) relay node, and the spreading sequence is \( S = 4 \), the mutual information is given by,

\begin{equation}
I_{DF} = \frac{1}{2} \log \left( 1 + \frac{2}{4} \sum_{i=1}^{K} \text{SNR}_{t_i,r_j} |\alpha_{t_i,r_j}|^2 \right)
\end{equation}
\[ + \frac{1}{2} \log \left( 1 + \frac{2}{4} \sum_{i=1}^{K} \text{SNR}_{t_i,d} |\alpha_{t_i,d}|^2 + \frac{2}{4} \sum_{j=0}^{Z} \text{SNR}_{r_j,d} |\alpha_{r,j,d}|^2 \right). \]  

(3.12)

Assuming that SNR_{t_i,d} = SNR_{r_j,d}, the overall outage probability can be written as:

\[
P_{\text{out}}^{DF}(\text{SNR},R) = \Pr \left[ |\alpha_{t_i,r_j}|^2 < \frac{2^{2R} - 1}{\text{SNR}_{t_i,r_j}} \right] \\
+ \Pr \left[ |\alpha_{t_i,r_j}|^2 \geq \frac{2^{2R} - 1}{\text{SNR}_{t_i,r_j}} \right] \\
\times \Pr \left[ |\alpha_{t_i,d}|^2 + |\alpha_{r_j,d}|^2 < \frac{3(2^{2R} - 1)}{2\text{SNR}_{t_i,d}} \right]. \tag{3.13}
\]

Let \(|h,s,r|^2 = |h,s,d|^2 = |h,r,d|^2 = X_i\) be independent Gamma variates with parameter \(\alpha\) and \(\beta\), therefore, the PDF of \(Y_{s,r} = X_{s_1,r} + X_{s_2,r}\) can be expressed as:

\[
f_Y(y = x_{s_1,r} + x_{s_2,r}) = f_X(x_1) * f_X(x_2) = \frac{y^{2\alpha-1}e^{-y/\beta}}{\Gamma(2\alpha)\beta^{2\alpha}}U(y). \tag{3.14}
\]

Thus, \(Y_{s,d,r,d} = X_{s_1,d} + X_{s_2,d} + X_{r,d}\) can be expressed as:

\[
f_Y(y) = f_X(x_1) * f_X(x_2) * f_X(x_3) = \frac{y^{3\alpha-1}e^{-y/\beta}}{\Gamma(3\alpha)\beta^{3\alpha}}U(y). \tag{3.15}
\]

According to [34],

\[
\int_0^u x^{v-1}e^{-\mu x}dx = \mu^{-v}\Psi(v,\mu u), \quad \text{for } \Re\{v > 0\}, \tag{3.16}
\]

where \(\Psi(\cdot, \cdot)\) is the lower incomplete gamma function defined as:

\[
\Psi(\kappa, x) = \int_0^x e^{-t\kappa}dt. \tag{3.17}
\]

Therefore,

\[
F(x) = \frac{\beta^{2\alpha}}{\Gamma(2\alpha)\beta^{2\alpha}}\Psi \left( 2\alpha, \frac{x}{\beta} \right), \tag{3.18}
\]

where \(F(x) = \int_0^x f_Y(y)dy\). Hence, the identity \(\Psi(\alpha, x) = \alpha^{-1}x^{\alpha-1}F_1(\alpha; 1 + \alpha; -x)\)
Therefore,

\[ \Psi \left( 2\alpha, \frac{x}{\beta} \right) = (2\alpha)^{-1} \left( \frac{x}{\beta} \right)^{2\alpha} _1F_1 \left( 2\alpha; 1 + 2\alpha; -\frac{x}{\beta} \right). \]  

(3.19)

Finally, Eq. (3.13) can be written as

\[
p_{DF}^{out}(SNR,R) = \left( \int_0^{x_1} \frac{y^{2\alpha-1}e^{-y/\beta}}{\Gamma(2\alpha)\beta^{2\alpha}} dy \right)^L + \left( 1 - \int_0^{x_1} \frac{x_2^{2\alpha-1}e^{-y/\beta}}{\Gamma(3\alpha)\beta^{3\alpha}} dy \right) \left( \int_0^{x_2} \frac{y^{3\alpha-1}e^{-y/\beta}}{\Gamma(3\alpha)\beta^{3\alpha}} dy \right)
\]

\[= \left( \frac{x_1^{2\alpha}}{\Gamma(2\alpha)\beta^{2\alpha}} _1F_1 \left( 2\alpha; 1 + 2\alpha; -\frac{x_1}{\beta} \right) \right)^L + \left( 1 - \frac{x_1^{2\alpha}}{\Gamma(2\alpha)\beta^{2\alpha}} _1F_1 \left( 2\alpha; 1 + 2\alpha; -\frac{x_1}{\beta} \right) \right)^L \times \frac{x_2^{3\alpha}}{\Gamma(3\alpha)\beta^{3\alpha}} _1F_1 \left( 3\alpha; 1 + 3\alpha; -\frac{x_2}{\beta} \right), \]

(3.20)

where \( L = 1, 2, 3, 4 \) represent the number of transmission paths between users and relays. Since \( x_1 = \gamma_1 = (2^{2R} - 1)/SNR_{t_i,r_j} \) and \( x_2 = \gamma_2 = (2^{2R} - 1)/SNR_{t_i,d} \), so that Eq. (3.20) can be rewritten as:

\[
p_{DF}^{out}(SNR,R) = \left( \frac{\gamma_1^{2\alpha}}{\Gamma(2\alpha)\beta^{2\alpha}} _1F_1 \left( 2\alpha; 1 + 2\alpha; -\frac{\gamma_1}{\beta} \right) \right)^L + \left( 1 - \frac{\gamma_1^{2\alpha}}{\Gamma(2\alpha)\beta^{2\alpha}} _1F_1 \left( 2\alpha; 1 + 2\alpha; -\frac{\gamma_1}{\beta} \right) \right)^L \times \frac{\gamma_2^{3\alpha}}{\Gamma(3\alpha)\beta^{3\alpha}} _1F_1 \left( 3\alpha; 1 + 3\alpha; -\frac{\gamma_2}{\beta} \right). \]

(3.21)


3.2 Numerical analyses

3.2.1 Outage probability for NC-based cooperative by using a multiplier

In our work, under the same consideration and fading channel, the outage probability results of the proposed system (Figs. 3.3-3.14) are better than those in [38], [40], and [39]. The detailed of the performance analysis are shown in the following subsections, which are based on Fig. 3.15.

3.2.2 Multiplier network coding vs. Two users cooperative network coding

Considering the same fading channel (Rayleigh which is equivalent to Nakagami-$m$ with $m_{t_1} = 1$), the reference network coding cooperative system uses users’ cooperation – all sources can be transmitters and relays, while the proposed system restricts sources to be only transmitters but cannot be relays. The proposed system performs better as depicted in Fig. 3.15. When the spectral efficiency is fixed at $R = 0.5$ [38] at SNR = 18.95 dB, their performance improves to $P_{out} = 6.201 \times 10^{-5}$, while the proposed method has a better performance, $P_{out} = 2.93 \times 10^{-9}$. However, this performance is achieved using multiple relays with CC-CDMA transmission technique at the destination (see Fig. 3.15), while [38] uses TDMA, yet, it consumes more time and affect the system’s capacity even though TDMA can facilitate orthogonal transmission. For completeness, the mutual information and outage probability in [38] are shown as:

$$I_{NC} = \frac{2}{3} \log \left(1 + \gamma |h_{u_1,d}|^2\right) + \frac{2}{3} \log \left(1 + \gamma |h_{u_1,d}|^2 + \gamma |h_{u_2,d}|^2\right),$$  \(3.22\)

and

$$P[R] = P[I_{NC} < R]$$

$$= P\left[\frac{2}{3} \log \left(1 + \gamma |h_{u_1,d}|^2\right) + \frac{2}{3} \log \left(1 + \gamma |h_{u_1,d}|^2 + \gamma |h_{u_2,d}|^2\right)\right].$$  \(3.23\)
3.2.3 Multiplier network coding vs. Network-coding-based hybrid AF and DF system

Considering the same fading channel, the proposed system performs better as depicted in Fig. 3.15. When the spectral efficiency is fixed at $R = 0.5$ [39] at SNR = 25 dB, their performance improves to $P_{out} = 2.987 \times 10^{-9}$ in case 2, and $P_{out} = 1.495 \times 10^{-9}$ in case 4, while the proposed method has a better performance, even when SNR = 20 dB, $P_{out} = 1.111 \times 10^{-9}$. However, this performance is achieved using multi-relays with CC-CDMA transmission technique at the destination Fig. 3.15. The reference network coding system [39] proposed a hybrid Amplify-and-Forward and Decode-and-Forward for the network coding system. They show that the proposed scheme outperforms conventional protocols on outage probability and prove that it has full diversity at relatively high-SNR regime. The outage probability for the best two cases out of four in [39] are shown as:

$$P_{out}^{Case2} = Pr(\gamma_{1,2} < 2^{2R} - 1) \cdot Pr(\gamma_{2,1} < 2^{2R} - 1) \cdot Pr\left(\gamma_{1,d} + \frac{\gamma_{1,2}\gamma_{2,d}}{\gamma_{1,2} + \gamma_{2,d} + 1} < 2^{2R} - 1\right), \quad (3.24)$$

and

$$P_{out}^{Case4} = Pr(\gamma_{1,2} < 2^{2R} - 1) \cdot Pr(\gamma_{2,1} > 2^{2R} - 1) \cdot Pr(\gamma_{AF}^2 < 2^{2R} - 1) \cdot \left(Pr(\gamma_{2,d} < 2^{2R} - 1) + Pr(\gamma_{2,d} > 2^{2R} - 1)Pr(\gamma_{1,d} < 2^{2R} - 1)\right), \quad (3.25)$$

where $\gamma_{AF}^2 = \gamma_{1,d} + \frac{\gamma_{1,2}\gamma_{2,d}}{\gamma_{1,2} + \gamma_{2,d} + 1}$.

3.2.4 Multiplier network coding vs. Distributed orthogonal Space-Time block codes

Considering the same fading channel, the reference cooperative system uses distributed extended orthogonal space-time-block code (D-EO-STBC). The proposed system performs better as depicted in Fig. 3.15. When the spectral efficiency is fixed at $R = 0.5$ [40] at SNR = 20 dB, their performance improves to $P_{out} = \ldots$
5.824 × 10^{-2} for one relay scheme, and $P_{out} = 2.643 \times 10^{-3}$ for two relays scheme; while the proposed method has a better performance, $P_{out} = 1.668 \times 10^{-5}$ for only one relay system. However, this performance is achieved using multi-relays with CC-CDMA transmission technique at the destination Fig. 3.15, while [40] uses time slots. It requires two time slots to complete the transmission between the source and destination and achieve a minimum outage probability threshold. The outage probabilities for one relay and two relays in [40] are shown as:

$$P_{out}(\gamma) = F_{\Gamma_1}(\gamma) = 1 - e^{-\frac{\gamma}{\lambda_1}},$$

and

$$P_{out}(\gamma_{th}) = F_{\Gamma_2}(\gamma_{th}) = \frac{\frac{1}{\lambda_1} \left( 1 - e^{-\frac{\gamma_{th}}{\lambda_2}} \right) - \frac{1}{\lambda_2} \left( 1 - e^{-\frac{\gamma_{th}}{\lambda_1}} \right)}{\frac{1}{\lambda_1} - \frac{1}{\lambda_2}}.$$  

(3.27)
Figure 3.3. Outage probability vs. SNR with $R = 0.5$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$.

### 3.3 Summary of the advantages of using a CC-CDMA cooperative network coding

Comparisons between a CC-CDMA and a TDMA cooperative network coding system are made as follows:

1. As we mentioned at subsection 1.4.3 in Chapter 1, CDMA provides a better spectrum efficiency than TDMA. The results are shown in outage probability performance in Fig. 3.15. The proposed system uses CDMA rather than TDMA, which uses time slots for transmission while providing orthogonal
### Table 3.1. Outage performance comparisons vary from SNR 5 to 20 dB.

<table>
<thead>
<tr>
<th>System</th>
<th>SNR ≈ 6.3 dB</th>
<th>SNR ≈ 10.5 dB</th>
<th>SNR ≈ 16.8 dB</th>
<th>SNR ≈ 20 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>The proposed system</td>
<td>3.067e-04</td>
<td>6.824e-06</td>
<td>2.037e-08</td>
<td>1.11e-09</td>
</tr>
<tr>
<td>2-Users coop-NC [38]</td>
<td>1.814e-02</td>
<td>2.857e-03</td>
<td>1.627e-04</td>
<td>4.28e-05</td>
</tr>
<tr>
<td>NC Hybird AF-DF (case 4) [39]</td>
<td>0.461e-04</td>
<td>3.181e-05</td>
<td>3.473e-07</td>
<td>5.659e-08</td>
</tr>
<tr>
<td>D-EO-STBC (1-Relay) [40]</td>
<td>7.538e-01</td>
<td>4.123e-01</td>
<td>1.168e-01</td>
<td>5.824e-02</td>
</tr>
<tr>
<td>D-EO-STBC (2-Relay [40]</td>
<td>4.547e-01</td>
<td>1.318e-01</td>
<td>1.06e-02</td>
<td>2.643e-03</td>
</tr>
</tbody>
</table>

**Figure 3.4.** Outage probability vs. SNR with $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$.

transmission, i.e., multi-users interference-free communication.

2. Due to the channel pollution, the disadvantage of CDMA technology, we introduce CC-CDMA into our NC-based cooperation system to efficiently
Figure 3.5. Outage probability vs. SNR with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$.

eliminates multiple access interference (MAI). CC-CDMA technology not only improves outage capacity, but also provides a higher bandwidth efficient than a traditional CDMA system. Fig. 3.15 shows that the proposed system outperforms a conventional TDMA system and a modified TDMA system – hybrid AF system [39].
Figure 3.6. Outage probability vs. SNR with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 3.7. Outage probability vs. SNR with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 3.8. Outage probability vs. SNR with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 

Figure 3.9. Outage probability vs. SNR with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 3.10. Outage probability vs. SNR with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 3.11. Outage probability vs. SNR with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 3.12. Outage probability vs. SNR with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 3.13. Outage probability vs. SNR with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 3.14. Outage probability vs. SNR with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega / m$. 
Figure 3.15. Outage probability comparisons.
Optimal power allocation

In this Chapter, we introduce two optimization solutions for the outage probability and the optimal power allocation outage performance. Due to the limited radio resources such as bandwidth and transmission power, the optimal power allocation strategy references can reduce outage probability while optimizing power allocation. In Section 4.1.1, we use a standard maximum-minimum method to discuss the optimization solution approaches. Next, in Section 4.1.2, we use a Lagrangian multiplier method to solve the optimization problem.

4.1 Optimal power allocation

4.1.1 Standard max-min problem

First of all, we can use a capacity maximization problem to examine the optimal power allocation of the proposed system, as a special case (See. Fig. 4.1) with the DF cooperative strategy. The capacity of the source to relay link is given [12]:

\[ C_{SR} = \frac{1}{2} \log \left(1 + \frac{2}{S} \sum_{i=1}^{K} \text{SNR}_{t_i,r_j} |\alpha_{t_i,r_j}|^2 \right) \]

\[ = \frac{1}{2} \log \left(1 + \frac{2}{S} \sum_{i=1}^{K} \frac{P_{t_i}}{\sigma_r^2} |\alpha_{t_i,r_j}|^2 \right), \quad (4.1) \]
where $K$ represents the number of users, $Z$ represents the number of relay nodes, and $S$ represents the spreading sequence. In the second phase, the destination combines the signals from both source and relay systems by using maximum ratio combining (MRC), so that the capacity can be written as:

$$C_{SR:RD} = \frac{1}{2} \left( 1 + \frac{2}{S} \sum_{i=1}^{K} \text{SNR}_{t_i,d} |\alpha_{t_i,d}|^2 + \frac{2}{S} \sum_{j=0}^{Z} \text{SNR}_{r_j,d} |\alpha_{r_j,d}|^2 \right)$$

$$= \frac{1}{2} \left( 1 + \frac{2}{S} \sum_{i=1}^{K} \frac{P_t}{\sigma_d^2} |\alpha_{t_i,d}|^2 + \frac{2}{S} \sum_{j=0}^{Z} \frac{P_r}{\sigma_d^2} |\alpha_{r_j,d}|^2 \right). \quad (4.2)$$

The channel capacity in the DF cooperative framework is limited by the minimum of these two phases ($K = 2$, $Z = 1$, and $S = 4$):

$$C_{DF} = \min \left\{ \frac{1}{2} \log \left( 1 + \frac{2 \frac{P_t}{4 \sigma_r^2} |\alpha_{t,r}|^2 \right), \right. \right.$$

$$\left. \frac{1}{2} \left( 1 + \frac{2 \frac{P_t}{4 \sigma_d^2} |\alpha_{t,d}|^2 + \frac{2 \frac{P_r}{4 \sigma_d^2} |\alpha_{r,d}|^2} \right) \right\}. \quad (4.3)$$
Hence, the optimization problem turns into a standard max-min problem [12] as shown:

\[
\begin{align*}
\text{maximize} & \quad P_{t_i} \times P_{r_j}, \forall i, j \\
\text{min} & \quad \min \left\{ \frac{1}{2} \log \left( 1 + \frac{P_{t_i}}{\sigma_r^2} |\alpha_{t_i,r}|^2 \right), \right. \\
& \quad \left. \frac{1}{2} \left( 1 + \frac{P_{t_i}}{\sigma_d^2} |\alpha_{t_i,d}|^2 + \frac{P_{r_j}}{2 \sigma_d^2} |\alpha_{r_j,d}|^2 \right) \right\} \\
\text{subject to} & \quad K \sum_{i=1}^{K} P_{t_i} + Z \sum_{j=1}^{Z} P_{r_j} \leq P_{sum}, \quad \text{and} \quad P_{t_i,d}, P_{r_j,d} \geq 0, \forall i, j.
\end{align*}
\]

If $|\alpha_{r_j,d}|^2 \geq |\alpha_{t_i,d}|^2$ and $|\alpha_{t_i,r}|^2 \geq |\alpha_{t_i,d}|^2$, the capacity is maximized with equal capacity for the two phases, i.e.,

\[
\frac{1}{2} \log \left( 1 + \frac{P_{t_i}}{\sigma_r^2} |\alpha_{t_i,r}|^2 \right) = \frac{1}{2} \left( 1 + \frac{P_{t_i}}{\sigma_d^2} |\alpha_{t_i,d}|^2 + \frac{P_{r_j}}{2 \sigma_d^2} |\alpha_{r_j,d}|^2 \right).
\] (4.4)

Given the total power constraint $2P_{t_i} + P_{r_j} = 3P \leq P_{sum}$. Therefore, we can show that the optimal power allocation is:

\[
P_{t_i}^* = \frac{3P}{2} \frac{|\alpha_{r_j,d}|^2}{\sigma_r^2} - \frac{|\alpha_{t_i,d}|^2}{\sigma_d^2} + \frac{|\alpha_{r_j,d}|^2}{\sigma_d^2},
\] (4.5)

and

\[
P_{r_j}^* = 3P \frac{|\alpha_{t_i,r}|^2}{\sigma_r^2} - \frac{|\alpha_{t_i,d}|^2}{\sigma_d^2} + \frac{|\alpha_{r_j,d}|^2}{\sigma_d^2}.
\] (4.6)

Following Eqs. (4.5) and (4.6), we can see that there is more power allocated to source $t_i$ than to relay $r_j$.

Using the optimal power, Eqs. (4.5) and (4.6), we can rewrite the outage probability (Eq. (3.13)) as:

\[
P_{DF}^\text{out}(\text{SNR,R}) = \Pr \left[ |\alpha_{t_i,r}|^2 < \frac{2^{2R} - 1}{\text{SNR}_{t_i,r}} \right] + \Pr \left[ |\alpha_{t_i,r}|^2 \geq \frac{2^{2R} - 1}{\text{SNR}_{t_i,r}} \right]
\times \Pr \left[ \text{SNR}_{t_i,d}|\alpha_{t_i,d}|^2 + \frac{1}{2}\text{SNR}_{r_j,d}|\alpha_{r_j,d}|^2 < 2^{2R} - 1 \right]
\]
\begin{align*}
&= \Pr \left[ |\alpha_{t,r}|^2 < \frac{2^{2R} - 1}{P^*_t/\sigma_d^2} \right] + \Pr \left[ |\alpha_{t,r}|^2 \geq \frac{2^{2R} - 1}{P^*_t/\sigma_d^2} \right] \\
&\times \Pr \left[ \frac{P^*_t}{\sigma_d^2} |\alpha_{t,d}|^2 + \frac{P^*_r}{2\sigma_d^2} |\alpha_{r,d}|^2 < 2^{2R} - 1 \right]. \tag{4.7}
\end{align*}

Assuming that \( \alpha_{t,d} = \alpha_{r,d} \) for simplicity, hence, the optimal power allocation outage probability can be rewritten as:

\begin{align*}
P^{\text{out}}_{\text{DF}} (\text{SNR}, R) &= \Pr \left[ |\alpha_{t,r}|^2 < \frac{2^{2R} - 1}{P^*_t/\sigma_d^2} \right] + \Pr \left[ |\alpha_{t,r}|^2 \geq \frac{2^{2R} - 1}{P^*_t/\sigma_d^2} \right] \\
&\times \Pr \left[ |\alpha_{t,d}|^2 < \frac{2^{2R} - 1}{(P^*_t + P^*_r)/\sigma_d^2} \right] \\
&= \Pr \left[ |\alpha_{t,r}|^2 < \frac{2^{2R} - 1}{P^*_t/\sigma_d^2} \right] + \Pr \left[ |\alpha_{t,r}|^2 \geq \frac{2^{2R} - 1}{P^*_t/\sigma_d^2} \right] \\
&\times \Pr \left[ |\alpha_{t,d}|^2 < \frac{2^{2R} - 1}{P^{\text{sum}}/\sigma_d^2} \right]. \tag{4.8}
\end{align*}

### 4.1.2 Lagrangian problem

Huang proposed that the optimal power allocation \([13]\) can be found by maximizing the achievable rate (or, equivalently, the effective SNR) at the destination. Given the total power constraint \( \sum_{i=1}^{K} P_{t_i,d} + \sum_{j=1}^{Z} P_{r_j,d} \leq P^{\text{sum}} \), the problem can be formulated equivalently as a linear programming given as follows:

\begin{align*}
\text{maximize} & \quad \log \left( 1 + \frac{2}{S} \sum_{i=1}^{K} \text{SNR}_{t_i,d} |\alpha_{t_i,d}|^2 + \frac{2}{S} \sum_{j=0}^{Z} \text{SNR}_{r_j,d} |\alpha_{r_j,d}|^2 \right) \\
\text{subject to} & \quad \sum_{i=1}^{K} P_{t_i,d} + \sum_{j=1}^{Z} P_{r_j,d} \leq P^{\text{sum}}, \quad \text{and} \quad P_{t_i,d}, P_{r_j,d} \geq 0, \forall i, j.
\end{align*}

Where \( S = 4 \) represents the spreading sequence, \( \text{SNR}_{t_i,d} = P_{t_i,d}/\sigma_d^2 \), and \( \text{SNR}_{r_j,d} = P_{r_j,d}/\sigma_d^2 \). For example, when \( K = 2 \) (source), \( Z = 1 \) (relay), using the Lagrangian function to find the optimal solution gives,

\begin{align*}
L(2P_{t_i,d} + P_{r_j,d}; \lambda) &= \log \left( 1 + \frac{2}{4} \frac{2P_{t_i,d} |\alpha_{t_i,d}|^2}{\sigma_d^2} + \frac{2}{4} \frac{P_{r_j,d} |\alpha_{r_j,d}|^2}{\sigma_d^2} \right) \\
&\quad - \lambda (2P_{t_i,d} + P_{r_j,d} - P^{\text{sum}}). \tag{4.9}
\end{align*}
For the special case, when $P_{t_i,d} = P_{r_j,d}$ and $\alpha_{t_i,d} = \alpha_{r_j,d}$, Eq. (4.9) can be simplified as:

$$\mathcal{L}(3P_{t_i,d}, \lambda) = \log \left(1 + \frac{P_{t_i,d}|\alpha_{t_i,d}|^2}{2\sigma_d^2}\right) - \lambda(3P_{t_i,d} - P_{sum}).$$

Taking the first derivative with respect to $P_{t_i,d}$, thus,

$$\frac{\partial \mathcal{L}(3P_{t_i,d}, \lambda)}{\partial P_{t_i,d}} = \log e \cdot \left(1 + \frac{P_{t_i,d}|\alpha_{t_i,d}|^2}{2\sigma_d^2}\right)^{-1} \cdot \frac{|\alpha_{t_i,d}|^2}{2\sigma_d^2} - 3\lambda \leq 0.$$  \hspace{1cm} (4.11)

Therefore, following the Karush-Kuhn-Tucker (KKT) conditions \[44\], the optimal transmission power $P_{t_i,d}^*$ is as shown:

$$P_{t_i,d}^* = \begin{cases} \frac{\log e}{3\lambda} - \frac{2\sigma_d^2}{|\alpha_{t_i,d}|^2} & \text{otherwise} \\ 0 & \text{if } \frac{|\alpha_{s,d}|^2}{\sigma_d^2} \geq \eta. \end{cases}$$

$$= \left(\frac{1}{3}\eta - \frac{2\sigma_d^2}{|\alpha_{t_i,d}|^2}\right)^+. \hspace{1cm} (4.12)$$

Where $\eta = \frac{\log e}{\lambda}$ is a constant chosen to satisfy the sum power constraint, and $(x)^+ = \max(x, 0)$. With the optimal pre-coder and the optimal decoder, the
channel capacity is given by,

\[ C = \frac{1}{2} \log \left( \frac{\eta |\alpha_{t_i,d}|^2}{\sigma_d^2} \right). \]  

(4.14)

4.2 Summary of the advantages of using optimal power allocation

Comparisons between optimal power allocation and equal power strategies give us the following:

1. Optimal power allocation can meet the limited radio resources requirements such as bandwidth and transmission power. The main objective of this research is to find solutions that can improve the channel capacity and utilization of the radio resources that are available to the service providers.

2. As depicted in Eqs. (4.5) and (4.6), we can see that there is more power allocated to sources than to relays. Due to this optimization property, Eq. (4.8) shows a optimization solution for the outage probability (See Figs. 4.3-4.14). The numerical results prove that a optimal power allocation strategy maximizes the channel capacity; thus, its outage performance outperforms the equal power outage probability.
4.3 Numerical analyses

Figure 4.3. Optimal power vs. equal power with $R = 0.5$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 4.4. Optimal power vs. equal power with $R = 1 \text{ bit/sec/Hz}$, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 

Graph showing the comparison between optimal power and equal power for different values of $m$ with $R = 1 \text{ bit/sec/Hz}$. The graph plots outage probability against SNR in dB for various optimal and equal power scenarios.
Figure 4.5. Optimal power vs. equal power with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 4.6. Optimal power vs. equal power with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 4.7. Optimal power vs. equal power with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 4.8. Optimal power vs. equal power with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 4.9. Optimal power vs. equal power with $R = 0.5$ and $R = 1 \text{ bit/sec/Hz}$, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 4.10. Optimal power vs. equal power with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 4.11. Optimal power vs. equal power with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 4.12. Optimal power vs. equal power with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Figure 4.13. Optimal power vs. equal power with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 

Optimal power vs. Equal power (m=0.5)
Figure 4.14. Optimal power vs. equal power with $R = 0.5$ and $R = 1$ bit/sec/Hz, while the Gamma distribution with parameters $\alpha = m$, $\beta = \Omega/m$. 
Conclusions

5.1 Conclusions

In our previous work [2], we proposed a modified method of combining CC-CDMA cooperative communication system and network coding, hence minimizing the time slots problems and alleviating multi-path interference. We also introduced a multiplier operation at the destination, instead of using the traditional XOR operation. In Chapter 2, we refined the analysis of CC-CDMA by providing new simulation results showing that the modified system can improve diversity, i.e. reliability, without deterioration of the average probability of error. In Chapter 3, the outage probability of the proposed CC-CDMA system is analyzed. Simulation results shows that the proposed system outperforms other similar systems. In Chapter 4, we further prove that by using an optimization method, it is possible to maximize the system performance, i.e., minimize the outage problem. Consequently, the proposed system can improve communication between mobile consumer electronic devices. Based on the analysis presented and comparisons made with other similar systems, we can conclude that the proposed method performs better not only in reducing time slots, but also channel capacity, without an advanced demodulator or the assumption of an error/delay-free control channel. Moreover, the more relays used in the two-transmitters NC-based cooperative system the better the performance of detecting signals. It confirms the assumption that the redundant signals can improve the overall diversity and hence a system’s performance.
5.2 Future work

In our current research, we have used maximum ratio combining (MRC) with decode-and-forward (DF) strategy; however, there are still a couple of other coding strategies that we have not discussed yet, such as amplify-and-forward, selection relay, repetition relay, etc. This could be included in further research. The diversity-multiplexing-tradeoff is also an important system performance approach to determine the cooperative system diversity gain, which can be also analyzed in future work. Following Chapters 2 and 3, error probability and outage probability can approach a high SNR regime to get a diversity order. Finally, further analysis using other technique such as a Monte-Carlo simulation, could be used to show that the numerical analyses are accurately calculated so that the BER [45] and outage probability performance [46]-[47] results are reliable.
Received signal at relay $r_j$ or destination $d$

In our previous work [2], the detected signal shown in Eq. (2.14) are explained in detail below:

\[
\Gamma_{t_i,r_j}^{(DF)} = \sum_{m=1}^{M} \sum_{l=0}^{L} \int_0^{T_b} \sqrt{\frac{2}{T_b}} \alpha_{t_i,r_j,l}^{(m)} e^{-j\theta_{t_i,r_j,l}^{(m)}} s_{t_i,r_j,m}(t - \lambda_{t_i,r_j,l}^{(m)}) \cos(2\pi f_m t) c_{t_i,m}(t) dt
\]

\[
= \sum_{m=1}^{M} \sum_{l=0}^{L} \int_0^{T_b} \sqrt{\frac{2}{T_b}} \alpha_{t_i,r_j,l}^{(m)} e^{-j\theta_{t_i,r_j,l}^{(m)}} \sqrt{\frac{2E_b}{T_b}} c_{t_i,m}(t - \lambda_{t_i,r_j,l}^{(m)})
\]

\[
\times \cos\left(2\pi f_m (t - \lambda_{t_i,r_j,l}^{(m)})\right) x_{t_i,r_j}(t - \lambda_{t_i,r_j,l}^{(m)}) \cos(2\pi f_m t) \cos A \cos B dt, \quad (A.1)
\]

where the cosine product in Eq. (A.1) can be simplified as shown:

\[
\cos A \cos B = \frac{1}{2} \left( \cos \left(2\pi f_m (t - \lambda_{t_i,r_j,l}^{(m)}) + 2\pi f_m t\right) \right.
\]

\[
+ \cos \left(2\pi f_m (t - \lambda_{t_i,r_j,l}^{(m)}) - 2\pi f_m t\right) \right)
\]

\[
= \frac{1}{2} \left( \cos \left(4\pi f_m t - 2\pi f_m \lambda_{t_i,r_j,l}^{(m)}\right) + \cos \left(2\pi f_m \lambda_{t_i,r_j,l}^{(m)}\right) \right). \quad (A.2)
\]

Because $\cos \left(4\pi f_m t - 2\pi f_m \lambda_{t_i,r_j,l}^{(m)}\right)$ is filtered out by a low pass filter since the center frequency $4\pi f_m t$ is two times higher than $2\pi f_m \lambda_{t_i,r_j,l}^{(m)}$. Therefore, Eq. (A.1) can
be rewritten as:

\[
\Gamma_{t_i,r_j}^{(DF)} \approx \sum_{m=1}^{M} \sum_{l=0}^{L} \int_0^{T_b} \frac{1}{T_b} \sqrt{E_b} \alpha_{t_i,r_j}^{(m)} e^{-j\theta_{t_i,r_j}^{(m)}} c_{t_i,m}(t - \lambda_{t_i,r_j}^{(m)}) x_{t_i,r_j}(t - \lambda_{t_i,r_j}^{(m)}) \times \cos(2\pi f_{m}\lambda_{t_i,r_j}^{(m)}) c_{t_i,m}(t) dt
\]

\[
= \sum_{m=1}^{M} \int_0^{T_b} \frac{1}{T_b} \sqrt{E_b} \alpha_{t_i,r_j}^{(m)} e^{-j\theta_{t_i,r_j}^{(m)}} c_{t_i,m}(t - \lambda_{t_i,r_j}^{(m)}) x_{t_i,r_j}(t - \lambda_{t_i,r_j}^{(m)}) \times \cos(2\pi f_{m}\lambda_{t_i,r_j}^{(m)}) c_{t_i,m}(t) dt + \sum_{m=1}^{M} \sum_{l=1}^{L} \int_0^{T_b} \frac{1}{T_b} \sqrt{E_b} \alpha_{t_i,r_j}^{(m)} e^{-j\theta_{t_i,r_j}^{(m)}} \times c_{t_i,m}(t - \lambda_{t_i,r_j}^{(m)}) x_{t_i,r_j}(t - \lambda_{t_i,r_j}^{(m)}) \cos(2\pi f_{m}\lambda_{t_i,r_j}^{(m)}) c_{t_i,m}(t) dt. \quad (A.3)
\]

Define \( \lambda_{t_i,r_j}^{(m)} = \varepsilon_{t_i,r_j}^{(m)} T_c \), where the range of \( \varepsilon_{t_i,r_j}^{(m)} \) is:

\[
\varepsilon_{t_i,r_j}^{(m)} = \begin{cases} 
0, & \text{for } l = 0 \\
\nu, & \nu \in \{1, 2, \ldots, N, \ldots\}, \text{for } l \neq 0
\end{cases} \quad (A.4)
\]

Assume the channels are fully correlated, in other words the signal bandwidth is narrower than the coherence channel’s bandwidth, thus:

\[
\alpha_{t_i,r_j}^{(1)} = \alpha_{t_i,r_j}^{(2)} = \cdots = \alpha_{t_i,r_j}^{(M)} = \alpha_{t_i,r_j} \quad (A.5)
\]

and

\[
\theta_{t_i,r_j}^{(1)} = \theta_{t_i,r_j}^{(2)} = \cdots = \theta_{t_i,r_j}^{(M)} = \theta_{t_i,r_j} \quad (A.6)
\]

and

\[
\lambda_{t_i,r_j}^{(1)} = \lambda_{t_i,r_j}^{(2)} = \cdots = \lambda_{t_i,r_j}^{(M)} = \lambda_{t_i,r_j} \quad (A.7)
\]

Now define:

\[
\int_0^{T_b} c^2(t) dt = \frac{1}{M}. \quad (A.8)
\]

Using the property of complementary code, which are Eqs. (A.5), (A.6), and (A.7), thus, the first part of \( \Gamma_{t_i,r_j}^{(DF)} \) can be rewritten as:

\[
\text{Part I. } = \sum_{m=1}^{M} \int_0^{T_b} \alpha_{t_i,r_j,0}^{(m)} e^{-j\theta_{t_i,r_j,0}^{(m)}} c_{t_i,m}(t - \lambda_{t_i,r_j,0}^{(m)}) c_{t_i,m}(t) dt
\]
Consequently, by Eq. (A.9) and Eq. (A.10), hence, Eq. (A.3) can be rewritten as:

\[
\Gamma_{t_i,r_j}^{DF} = \sum_{m=1}^{M} \int_{0}^{T_b} c_{t_i,m}(t - \lambda_{t_i,r_j,0}^{(m)}) c_{t_i,m}(t) dt
\]

\[
= \begin{cases} 
\alpha_{t_i,r_j,0} e^{-j\theta_{t_i,r_j,0}} T_b, & \text{for } \lambda_{t_i,r_j,0}^{(m)} = 0 \\
0, & \text{for } \lambda_{t_i,r_j,0}^{(m)} \neq 0
\end{cases}
\]  

(A.9)

and the second part of \(\Gamma_{t_i,r_j}^{DF}\) is as shown:

\[
\text{Part II. } = \sum_{m=1}^{M} \sum_{l=1}^{L} \int_{0}^{T_b} \alpha_{t_i,r_j,l}^{(m)} e^{-j\theta_{t_i,r_j,l}^{(m)}} c_{t_i,m}(t - \lambda_{t_i,r_j,l}^{(m)}) c_{t_i,m}(t) dt
\]

\[
= \alpha_{t_i,r_j,l} e^{-j\theta_{t_i,r_j,l}^{(m)}} \sum_{m=1}^{M} \sum_{l=1}^{L} \int_{0}^{T_b} c_{t_i,m}(t - \lambda_{t_i,r_j,l}^{(m)}) c_{t_i,m}(t) dt
\]

\[
= 0.
\]  

(A.10)

Consequently, by Eq. (A.9) and Eq. (A.10), hence, Eq. (A.3) can be rewritten as:

\[
\Gamma_{t_i,r_j}^{DF} \approx \frac{\sqrt{E_b}}{T_b} \alpha_{t_i,r_j,0} e^{-j\theta_{t_i,r_j,0}^{(m)}} \left( \sum_{m=1}^{M} \int_{0}^{T_b} c_{t_i,m}(t - \lambda_{t_i,r_j,0}^{(m)}) x_{t_i,r_j}(t - \lambda_{t_i,r_j,0}^{(m)}) dt \right)^{M/L} \sum_{n=-\infty}^{\infty} a_t(n) g(t-nT_b) \]

\[
\times \cos(2\pi f_m \lambda_{t_i,r_j,0}^{(m)}) c_{t_i,m}(t) dt
\]

\[
= \frac{\sqrt{E_b}}{T_b} \alpha_{t_i,r_j,0} e^{-j\theta_{t_i,r_j,0}^{(m)}} \left( \sum_{m=1}^{M} \int_{0}^{T_b} c_{t_i,m}(t - \lambda_{t_i,r_j,0}^{(m)}) c_{t_i,m}(t) dt \right)
\]

\[
\times \left( a_t(n') g(t-n'T_b) + \sum_{n=-\infty}^{\infty} a_t(n) g(t-nT_b) \right) \cos(2\pi f_m \lambda_{t_i,r_j,0}^{(m)})
\]

(A.11)

out of the window

If \(\lambda_{t_i,r_j,0} = 0\), thus Eq. (A.11) can be rewritten as:

\[
\Gamma_{t_i,r_j}^{DF} = \sqrt{E_b} a_t(n') \alpha_{t_i,r_j,0} e^{-j\theta_{t_i,r_j,0}^{(m)}}.
\]  

(A.12)

Similarly,

\[
\Gamma_{t_i,d}^{DF} = \sqrt{E_b} a_t(n') \alpha_{t_i,d,0} e^{-j\theta_{t_i,d,0}^{(m)}}.
\]  

(A.13)
Received noise at relay $r_j$ or destination $d$

In our previous work [2], consider that

$$\xi^{(DF)} = \sum_{m=1}^{M} \int_{0}^{T_b} \sqrt{2 T_b} c(t) n(t) \cos(2\pi f_m t) dt, \quad (B.1)$$

where $n(t)$ is a zero mean, variance $N_0/2$ Gaussian procedure, $c(t)$ is a spreading code.

Let $\xi_m^{(DF)} = \int_{0}^{T_b} \sqrt{2 T_b} c(t) n(t) \cos(2\pi f_m t) dt$, thus,

$$E[\xi_m^2] = \int_{0}^{T_b} \frac{2}{T_b} c(t) n(t) \cos(2\pi f_m t) dt$$

$$= \int_{0}^{T_b} \frac{2}{T_b} c(t) n(t) \cos(2\pi f_m t) \int_{0}^{T_b} \sqrt{2 T_b} c(t') n(t) \cos(2\pi f_m t') dt'$$

$$= \int_{0}^{T_b} \int_{0}^{T_b} \frac{2}{T_b} c(t') c(t') n(t) n(t') \cos(2\pi f_m t) \cos(2\pi f_m t') dt dt'$$

$$= \int_{0}^{T_b} \int_{0}^{T_b} \frac{2}{T_b} E[n(t) n(t')] c(t') \cos(2\pi f_m t) \cos(2\pi f_m t') dt dt'$$

$$= \int_{0}^{T_b} \int_{0}^{T_b} \frac{2}{T_b} \frac{N_0}{2} \delta(t - t') c(t') \cos(2\pi f_m t) \cos(2\pi f_m t') dt dt'$$

$$= \int_{0}^{T_b} \frac{N_0}{T_b} c^2(t') \cos^2(2\pi f_m t') dt'$$
\[ \frac{N_0}{2M}, \quad (B.2) \]

Therefore, \( \xi_m^{(DF)} \) is a zero mean, variance \( N_0/2M \) Gaussian distribution. Since

\[ \xi^{(DF)} = \sum_{m=1}^{M} \xi_m^{(DF)}, \quad (B.3) \]

and \( \xi_1^{(DF)}, \xi_2^{(DF)}, \ldots, \xi_M^{(DF)} \) are i.i.d., thus,

\[ E[\xi^2] = \sum_{m=1}^{M} E[\xi_m^2] = \sum_{m=1}^{M} \frac{N_0}{2M} = \frac{N_0}{2}, \quad (B.4) \]

definition, \( \xi^{(DF)} \) is a zero mean, variance \( N_0/2 \) Gaussian distribution.
Appendix C

Maximum output SNR analysis

In our previous work [2], following by Eq. (2.16) in Section 2.2, a decision variable value $\tilde{y}_{d, \text{sum}}^{(t_1)}$ can be written as:

$$
\tilde{y}_{d, \text{sum}}^{(t_1)} = w_{t_1,d} \tilde{y}_{t_1,d} + w_{r_1,d} [\tilde{y}_{t_2,d} \times \tilde{y}_{r_1,d}]
$$

$$
= w_{t_1,d} \sqrt{E_b a_{t_1}(n') \alpha_{t_1,d,0} e^{-j \theta_{t_1,d,0}} + w_{t_1,d} \xi_{t_1,d}^{(DF)}}
$$

$$
+ w_{r_1,d} \left[ E_b a_{r_1}(n') a_{t_2}(n') \alpha_{r_1,d,0} \alpha_{t_2,d,0} e^{-j (\theta_{r_1,d,0} + \theta_{t_2,d,0})} 
$$

$$
+ \sqrt{E_b a_{t_2}(n') \alpha_{t_2,d,0} e^{-j \theta_{t_2,d,0}} + w_{t_2,d} \xi_{t_2,d}^{(DF)}}
$$

$$
+ \xi_{r_1,d}^{(DF)} + \xi_{r_1,d}^{(DF)} \right]
$$

$$
= \sqrt{E_b} \left[ w_{t_1,d} a_{t_1}(n') \alpha_{t_1,d,0} e^{-j \theta_{t_1,d,0}} 
$$

$$
+ w_{r_1,d} \sqrt{E_b a_{r_1}(n') a_{t_2}(n') \alpha_{r_1,d,0} \alpha_{t_2,d,0} e^{-j (\theta_{r_1,d,0} + \theta_{t_2,d,0})}} \right]
$$

$$
+ \left[ w_{t_1,d} \xi_{t_1,d}^{(DF)} + w_{r_1,d} \sqrt{E_b a_{r_1}(n') \alpha_{r_1,d,0} e^{-j \theta_{r_1,d,0}} + w_{r_1,d} \xi_{r_1,d}^{(DF)}} \right]
$$

$$
= \tilde{y}_s^{(t_1)} + \tilde{y}_n^{(t_1)},
$$

where $\tilde{y}_s^{(t_1)}$ and $\tilde{y}_n^{(t_1)}$ represent signal part and noise part, respectively. If the channel responses $\alpha_{t_i,d,0}$ and $\alpha_{r_j,d,0}$ are fixed, for transmitters $i = 1, 2$, and relay $j = 1.$
The conditional expected value of $\tilde{y}_{d,\text{sum}}$ is given by,

$$\text{Exp}_{\text{cond}} = E[\tilde{y}_{d,\text{sum}}|\alpha_{t_k,d,0},\alpha_{r_1,d,0}] = \sqrt{E_b} [w_{t_1,d}a_{t_1}(n')\alpha_{t_1,d,0}e^{-j\theta_{t_1,d,0}} + w_{r_1,d}\sqrt{E_b}a_{r_1}(n')a_{t_2}(n')\alpha_{r_1,d,0}\alpha_{t_2,d,0}e^{-j(\theta_{r_1,d,0}+\theta_{t_2,d,0})}] = \tilde{y}_s^{(t_1)}. \quad (C.2)$$

We can also simplify Eq. (C.2) as follows:

$$\text{Exp}_{\text{cond}} = \mathcal{E}_{(1r)} \sqrt{E_b} = \mathcal{E}, \quad (C.3)$$

where

$$\mathcal{E}_{(1r)} = w_{t_1,d}a_{t_1}(n')\alpha_{t_1,d,0}e^{-j\theta_{t_1,d,0}} + w_{r_1,d}\sqrt{E_b}a_{r_1}(n')a_{t_2}(n')\alpha_{r_1,d,0}\alpha_{t_2,d,0}e^{-j(\theta_{r_1,d,0}+\theta_{t_2,d,0})}. \quad (C.4)$$

By letting

$$\sqrt{E_b}a_{t_1}(n')\alpha_{t_1,d,0}e^{-j\theta_{t_1,d,0}} = \Psi, \quad (C.5)$$

and

$$E_b a_{r_1}(n')a_{t_2}(n')\alpha_{r_1,d,0}\alpha_{t_2,d,0}e^{-j(\theta_{r_1,d,0}+\theta_{t_2,d,0})} = \Phi, \quad (C.6)$$

so, Eq. (C.2) can be rewritten as:

$$\tilde{y}_s^{(t_1)} = w_{t_1,d}\Psi + w_{r_1,d}\Phi. \quad (C.7)$$

Similarly, consider the same conditions as in Eq. (C.2), then the conditional variance of $\tilde{y}_{d,\text{sum}}^{(t_1)}$ is given by,

$$\text{Var}_{\text{cond}} = \text{Var}(\tilde{y}_{d,\text{sum}}^{(t_1)}|\alpha_{t_k,d,0},\alpha_{r_1,d,0}) = w_{t_1,d}^2 N_0 + w_{r_1,d}^2 \left[ E_b a_{r_1}^2(n')\alpha_{r_1,d,0}^2 e^{-2j\theta_{r_1,d,0}} \frac{N_0}{2} + E_b a_{t_1}^2(n')\alpha_{t_1,d,0}^2 e^{-2j\theta_{t_1,d,0}} \frac{N_0}{2} + \frac{N_0^2}{4}\right] = E\left[\tilde{y}_{n}^{(t_1)}\right]^2|\alpha_{t_k,d,0},\alpha_{r_1,d,0}] = (\tilde{y}_n^{(t_1)})^2. \quad (C.8)$$
Also, we can simplify Eq. (C.8) as follows:

\[
\text{Var}_{\text{cond}} = \mathcal{V}_{(1r)} \frac{N_0}{2} = \mathcal{V},
\]

where

\[
\mathcal{V}_{(1r)} = w_{t_1,d}^2 + w_{r_1,d}^2 \left[ E_b \alpha_r^2 (n') \alpha_{t_1,d}^2 e^{-2j\theta_{t_1,d,0}} + E_b \alpha^2 (n') \alpha_{t_2,d}^2 e^{-2j\theta_{t_2,d,0}} \right]
\]

\[
+ \frac{N_0}{2}.
\]

By letting \( E_b \alpha_r^2 (n') \alpha_{t_1,d}^2 e^{-2j\theta_{t_1,d,0}} \frac{N_0}{2} + E_b \alpha^2 (n') \alpha_{t_2,d}^2 e^{-2j\theta_{t_2,d,0}} \frac{N_0}{2} + \frac{N_0^2}{4} = \Upsilon \), therefore, Eq. (C.8) can be rewritten as:

\[
\left( \hat{y}^{(t_1)}_n \right)^2 = w_{t_1,d}^2 \frac{N_0}{2} + w_{r_1,d}^2 \Upsilon.
\]

The output signal-to-noise ratio (SNR) \( \sigma \) can be written as:

\[
\text{(SNR)}_o = \frac{\left( \hat{y}^{(t_1)}_s \right)^2}{\left( \hat{y}^{(t_1)}_n \right)^2} = \frac{\left( \hat{y}^{(t_1)}_s \right)^2}{w_{t_1,d}^2 \frac{N_0}{2} + w_{r_1,d}^2 \Upsilon}.
\]

By Eqs. (C.7) and (C.11), thus, (C.12) can be rewritten as:

\[
\text{(SNR)}_o = \frac{\left( w_{t_1,d} \Psi + w_{r_1,d} \Phi \right)^2}{w_{t_1,d}^2 \frac{N_0}{2} + w_{r_1,d}^2 \Upsilon} = \frac{w_{t_1,d}^2 \Psi^2 + 2w_{t_1,d}w_{r_1,d} \Psi \Phi + w_{r_1,d}^2 \Phi^2}{w_{t_1,d}^2 \frac{N_0}{2} + w_{r_1,d}^2 \Upsilon}.
\]

Let \( f(w_{t_1,d}, w_{r_1,d}) = \text{(SNR)}_o \), hence, after performing partial derivative of \( f(w_{t_1,d}, w_{r_1,d}) \) and let it equal to 0 to obtain the maximum SNR.

\[
\frac{\partial f(w_{t_1,d}, w_{r_1,d})}{\partial w_{t_1,d}} = 0,
\]

and

\[
\frac{\partial f(w_{t_1,d}, w_{r_1,d})}{\partial w_{r_1,d}} = 0.
\]
By Eqs. (C.14) and (C.15) yield

\[
\left[\frac{\left(2 w_{t_1,d} \Phi + 2 w_{r_1,d} \Psi \Phi \right) \left(w_{t_1,d}^2 \frac{N_0}{2} + w_{r_1,d}^2 \Upsilon \right)}{-\left(w_{t_1,d} \Phi + w_{r_1,d} \Psi \right)^2 \left(w_{t_1,d} N_0\right)} \right] = 0, \quad (C.16)
\]

and

\[
\left[\frac{\left(2 w_{t_1,d} \Psi \Phi + 2 w_{r_1,d} \Phi^2 \right) \left(w_{t_1,d}^2 \frac{N_0}{2} + w_{r_1,d}^2 \Upsilon \right)}{-\left(w_{t_1,d} \Phi + w_{r_1,d} \Psi \right)^2 \left(2 w_{r_1,d} \Upsilon \right)} \right] = 0. \quad (C.17)
\]

By simplifying Eq. (C.16) and (C.17) yields,

\[
w_{r_1,d}(w_{r_1,d} \Phi + w_{t_1,d} \Psi)(-N_0 w_{t_1,d} \Phi + 2 w_{r_1,d} \Upsilon \Psi) = 0, \quad (C.18)
\]

and

\[
w_{t_1,d}(w_{r_1,d} \Phi + w_{t_1,d} \Psi)(N_0 w_{t_1,d} \Phi - 2 w_{r_1,d} \Upsilon \Psi) = 0. \quad (C.19)
\]
Error probability closed form solution for a special case and diversity gain analysis

In this Section, an extension diversity gain analysis from our previous work [2] is performed, using similar set up as Section 2.2. For this analysis, the moment-generating function (MGF) of a Nakagami-$m$ fading channel is needed which is defined as:

$$
M_{\gamma_{b,1R}}(s) = \langle e^{s\gamma_{b,1R}} \rangle \\
= \int_{-\infty}^{\infty} \exp(s\gamma_{b,1R}) p_{\gamma_{b,1R}}(\gamma_{b,1R}) d\gamma_{b,1R} \\
= \left(1 - \frac{s\bar{\gamma}_{b,1R}}{m}\right)^{-m}, \tag{D.1}
$$

where $\gamma_{b,1R}$ is defined in Eq. (2.19) and $\bar{\gamma}_{b,1R}$ represents the average SNR; while $\langle e^{s\gamma_{b,1R}} \rangle$ denotes the expectation value of $e^{s\gamma_{b,1R}}$ and $p_{\gamma_{b,1R}}$ is the probability density function (pdf) of a Nakagami-$m$ fading channel, which is:

$$
p_{\gamma_{b,1R}} = \frac{m^m \gamma_{b,1R}^{m-1}}{\Gamma(m)\bar{\gamma}_{b,1R}} \exp\left(-\frac{m\gamma_{b,1R}}{\bar{\gamma}_{b,1R}}\right). \tag{D.2}
$$
For independent random variables $X$ and $Y$, the moment-generating function satisfies:

$$M_{x+y}(s) = \langle e^{s(x+y)} \rangle = \langle e^{sx}e^{sy} \rangle = \langle e^{sx} \rangle \langle e^{sy} \rangle = M_x(s)M_y(s). \quad (D.3)$$

Using the above results and when $m_{t_1} = m_{t_2} = m_{r_1}$ (i.e., two transmitters and single relay case), Eq. (2.27) can be written as:

$$P(e) = \frac{1}{\pi} \int_0^{\pi/2} \int_0^{\infty} \exp \left( -\frac{\mathcal{E}_{1R}^2 \gamma_{b,1R}}{V_{1R} \sin^2 \phi} \right) (p_{\gamma_{b,1R}})^3 d\gamma_{b,1R} d\phi$$

$$= \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_{b,1R}} \left( -\frac{\mathcal{E}_{1R}^2}{V_{1R} \sin^2 \phi} \right)^{-3m} d\phi$$

$$= \frac{1}{\pi} \int_0^{\pi/2} \left( 1 + \frac{\mathcal{E}_{1R}^2}{V_{1R} \sin^2 \phi} \frac{\gamma_{b,1R}}{m} \right)^{-3m} d\phi. \quad (D.4)$$

Eq. (D.4) can be approximated to:

$$P(e) \approx \sqrt{\frac{\gamma_{c,1r}}{1 + \gamma_{c,1r}} \frac{(1 + \gamma_{c,1r})^{-3m}}{2\sqrt{\pi}} \frac{\Gamma(3m + \frac{1}{2})}{\Gamma(3m + 1)}} 2F_1 \left( 1, 3m + \frac{1}{2}; 3m + 1; \frac{1}{1 + \gamma_{c,1r}} \right). \quad (D.5)$$

where $\gamma_{c,1r} = \frac{\mathcal{E}_{1R}^2}{mV_{1R}} \bar{\gamma}_{b,1R}$, and $2F_1(\cdot; \cdot; \cdot)$ [43] is the Gauss hypergeometric function. By definition,

$$2F_1(1, m + \frac{1}{2}; m + 1; x) = \sum_{i=0}^{\infty} \frac{(1)_i (m + \frac{1}{2})_i x^i}{(m + 1)_i i!}, \quad (D.6)$$

where

$$x = \frac{1}{1 + \gamma_{c,t_1,r_1}}, \quad (D.7)$$

and

$$(a)_i = \Gamma(a + i)/\Gamma(a) = a(a + 1) \cdots (a + i - 1). \quad (D.8)$$

From (D.8), it is seen that $(1)_i = i!$, $(m + 1)_i = (m + i)!/m!$ and $(m + 1/2)_i =$
Thus, (D.6) can be rewritten as

\[ 2F_1\left(1, m + \frac{1}{2}; m + 1; x\right) = \sum_{i=0}^{\infty} \frac{\left(\frac{1}{2}\right)_{m+i} m!}{\left(\frac{1}{2}\right)_m (m+i)!} x^i \]

\[ = \frac{m!}{(\frac{1}{2})_m} \sum_{i=0}^{\infty} \frac{(\frac{1}{2})_{m+i} m^m}{(m+i)!} x^{m+i} \]

\[ = \frac{m!}{(\frac{1}{2})_m} x^{-m} \left[ \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)_i \frac{x^i}{i!} - \sum_{i=0}^{m-1} \left(\frac{1}{2}\right)_i \frac{x^i}{i!} \right]. \tag{D.9} \]

For the case where we have \( L \) links (from transmitters to destination and relays to destination) and when all the Nakagami-\( m \) links’ parameters are equal, then we can rewrite Eq. (D.4) as:

\[
P(e) = \frac{1}{\pi} \int_0^{\pi/2} \int_0^\infty \exp\left(-\frac{\mathcal{E}^2_{(L-2)R_1} \bar{\gamma}_{b,(L-2)r}}{\mathcal{V}_{(L-2)R_1} \sin^2 \phi}\right) (p_{\gamma_{b,(L-2)r}})^L \ d\gamma_{b,(L-2)r} \ d\phi
\]

\[
= \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_{b,(L-2)r}} \left(-\frac{\mathcal{E}^2_{(L-2)R_1}}{\mathcal{V}_{(L-2)R_1} \sin^2 \phi}\right)^{-Lm} \ d\phi
\]

\[
= \frac{1}{\pi} \int_0^{\pi/2} \left(1 + \frac{\mathcal{E}^2_{(L-2)R_1}}{\mathcal{V}_{(L-2)R_1} \sin^2 \phi} \frac{\bar{\gamma}_{b,(L-2)r}}{m}\right)^{-Lm} \ d\phi
\]

\[
\approx \sqrt{\frac{\bar{\gamma}_{c,(L-2)r}}{1 + \bar{\gamma}_{c,(L-2)r}}} \left(1 + \frac{\bar{\gamma}_{c,(L-2)r}}{1 + \bar{\gamma}_{c,(L-2)r}}\right)^{-Lm} \frac{\Gamma(Lm + \frac{1}{2})}{\Gamma(Lm + 1)} \ x_2F_1\left(1, Lm + \frac{1}{2}; Lm + 1; \frac{1}{1 + \bar{\gamma}_{c,(L-2)r}}\right), \quad L > 0 \tag{D.10}
\]

where \( \bar{\gamma}_{c,(L-2)r} = \frac{\mathcal{E}^2_{(L-2)R_1}}{m \mathcal{V}_{(L-2)R_1}} \bar{\gamma}_{b,(L-2)r} \).

The definition of diversity gain is given by [33],

\[ D = -\lim_{\text{SNR} \to \infty} \frac{\log P_e(\text{SNR})}{\log \text{SNR}}. \tag{D.11} \]

For example, when using two transmitters, \( L - 2 = Z \) (where \( Z \) is the number of relays used in the scheme), and under the Nakagami-\( m \) distribution when \( m = 1 \) (Rayleigh distribution), the diversity gain in the proposed system, using Eq.
(D.11), reduces to:

\[ D = - \lim_{\gamma_e \to \infty} \frac{\log P(e)}{\log \gamma_e} = Z. \]  

(D.12)
Bibliography


