A PROBABILISTIC ANALYSIS OF

AUTOCALLABLE OPTIMIZATION SECURITIES

A Thesis in

Statistics

by

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Abstract

We consider in this thesis some structured financial products, known as reverse convertible notes, which resulted in substantial losses to certain buyers of these notes in recent years. We shall focus on specific reverse convertible notes known as “Autocallable Optimization Securities with Contingent Protection Linked to the S&P 500 Financial Index” because these notes are representative of the broad spectrum of reverse convertibles notes. Therefore, the analysis provided in this thesis can be applied to many other reverse convertible notes.

In the early part of the thesis, we describe these reverse convertible notes. Further, we identify potential areas of confusion in the pricing supplement to the prospectus for these notes; hence, we deduce two possible interpretations of the payment procedure for the notes.

Next, we apply the Law of Total Expectation to obtain a probabilistic analysis for each interpretation of the payment procedure for these notes, and we determine the corresponding expected net payments to note-holders under various scenarios for the financial markets. In each interpretation of the payment procedure, we show that note-holders were highly likely to suffer substantial losses.

As a consequence, we infer that financial advisers who recommended purchases of these notes did not exercise *fiduciary duty* to their clients. Indeed, the prospectus is sufficiently complex that financial advisers generally lacked the mathematical knowledge and expertise to understand the prospectus completely, hence did not have the greater knowledge and expertise that is required by a fiduciary relationship. Therefore, financial advisers simply were unable to exercise fiduciary duty and ultimately misguided their clients.

We conclude that these reverse convertibles notes were designed by financial institutions to insure themselves, against significant declines in the equities markets, at the expense of note-holders.
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Chapter 1

Introduction

In July, 2011 the U.S. Financial Industry Regulatory Authority (FINRA) issued an alert concerning the sale of structured financial products known as “reverse convertible notes” [2]. These notes were issued by many financial firms and sold widely to clients who had an optimistic view of future market conditions and who were seeking to diversify their financial portfolios. Clients were promised high yields, and phrases such as “autocallable optimization” or “contingent protection” in the titles of the notes imbued clients with a high sense of confidence in the future of these notes. However, the notes turned out not to be as simple or as safe as customers had thought initially, and buyers of these notes eventually suffered enormous losses when the financial markets experienced a significant downturn in 2008.

The U.S. Securities Exchange and Commission (SEC) defines a reverse convertible note to be a financial product whose return is linked to the performance of a “reference asset” or a “basket of reference assets,” usually consisting of the stock price of an unrelated company or the level of a stock market index. Some common reference assets for such notes are the S&P 500 Financials Index, the EURO STOXX 50, and the common stock of JPMorgan Chase & Co. or other prominent corporations. In this thesis, we will investigate an autocallable note, a common type of reverse convertible note. Autocallable notes have one or more “call dates,” which are dates on which the note can
be redeemed prior to the maturity date and which are determined by specific market conditions. A wide variety of reverse convertible notes have been sold to the public since 2006, with sales continuing even today, and these notes can be examined in a manner similar to the analysis given in this thesis. Therefore, the analysis provided in this thesis serves as a template for the study of many reverse convertibles notes.

In this thesis, we will investigate a particular reverse convertible note known as “Autocallable Optimization Securities with Contingent Protection Linked to the S&P 500 Financials Index.” This financial product was issued by Lehman Brothers Holdings Inc. in early 2008, before the 2007-2008 financial crisis reached its nadir, and the prospectus for this note can be obtained from the SEC’s website. We will study the pricing supplement to the prospectus, and provide a description of the note and its key features. This pricing supplement is similar to the pricing supplements for many other reverse convertible notes issued by other financial companies.

At first glance, the pricing supplement to the prospectus, which we provide in an appendix, makes the note sound promising; however, after examining the pricing supplement in detail, we found some disturbing attributes. First, the term “contingent protection” in the title causes this financial product to appear more secure than it really is. The phrase “contingent protection” may suggest that there is some legal or financial protection on the notes, thereby causing some customers to become more confident about the returns on their purchases. Therefore, our first task will be to assess the weaknesses of the contingent-protection feature of these notes.

Another concern is that the details of the pricing supplement could have been confusing to a financially unsophisticated client and as a result, many clients would have
found it difficult to understand complete details of the note. Our second task, therefore, is to describe some vaguenesses that we have found in the prospectus regarding the payment procedure and redemption rules. In order to assess these details we will analyze the method used to determine payment and describe two interpretations of the payment procedure.

Next, we will provide a probabilistic analysis of the payment procedure and use the Law of Total Expectation to calculate the expected net returns to clients. We will show, based on the results derived from our analysis, that customers were at a substantial disadvantage from the very moment they purchased this note. We deduce from our analysis that the average client would have lost a significant proportion of their principal; in some situations, those losses could be as high as 50%.

At this point, we provide some remarks on the concept of fiduciary duty. Many state laws and the U.S. Investment Advisers Act of 1940 require that certain financial advisers act as fiduciaries in their business transactions with clients. West’s Encyclopedia of American Law [6] defines a fiduciary relationship as “one which encompasses the idea of faith and confidence and is generally established only when the confidence is given by one and accepted by another.” “A fiduciary has greater knowledge and expertise of the matters being handled and is held to a standard of conduct and trust above that of a casual business person” (Hill [4]). Fiduciary duty is “the obligation to act in the best interest of the beneficiary of the fiduciary relationship” (Rahaim [8]).

As a consequence, we will conclude that financial advisers who recommended purchases of reverse convertible notes in early 2008 did not exercise fiduciary duty to their clients. Indeed, the prospectus is sufficiently complex that financial advisers themselves
could not have understood every detail of the prospectus. Further, financial advisers lacked the mathematical knowledge and expertise necessary to determine the consequences of details in the prospectus. Therefore, financial advisers simply were unable to exercise fiduciary duty to their clients, and ultimately the advisers misguided their clients.

We remark that related probabilistic analyses of other types of financial derivatives were given by Richards [9] and Richards and Hundal [10]. Those papers analyzed structured products, such as, return optimization securities, yield magnet notes, reverse exchangeable securities, and principal-protected notes. In each case, it was shown that purchasing these structured products during the mid-2000’s would likely lead to substantial losses. In this thesis, a similar probabilistic analysis will be done on reverse convertible notes and our analysis will show that the purchase of these notes were likely to result in significant losses also.

Moreover, Richards [9] observed that certain structured financial products were designed “to insure the banks against substantial declines in the markets; such an arrangement allowed the banks to avoid direct stock sales on the open market, which could have triggered widespread market declines.” Likewise, the reverse convertible notes analyzed in this thesis provided a similar safety net to the financial institutions which issued them, but at the expense of note-holders.

We conclude the introduction with a summary of the results to follow in the remaining chapters of this thesis. In Chapter 2, we provide an assessment of the pricing supplement to the prospectus. We will point out that seemingly positive features of the payment procedure for the notes appear to provide overly optimistic views of future
market conditions seem to suggest that clients’ principal are protected against adverse market conditions, and that the chances of a loss of principal are low. Moreover, the pricing supplement presents positive features of the notes in the earlier part of the document, and in greater detail. On the other hand, negative aspects of the notes are described in a less prominent manner; those aspects are relegated to later sections which are less likely to be read by a typical client; and the supplement does not provide a comprehensive list of possible payout scenarios for the notes.

We shall also identify problems with the payment procedure for the notes. Indeed, the description of the payment procedure provided in the payment supplement seems incomplete and vague. Moreover, the conditions under which the note will be called is unclear, leading to at least two plausible interpretations of the payment procedure.

In Chapter 3, we describe and analyze a first interpretation of the payment procedure. We shall use the Law of Total Expectation to obtain mathematical formulas for the expected, or average, payout to a randomly chosen note-holder under a variety of market-outlook scenarios ranging from pessimistic to optimistic. By analyzing graphs of the expected net payment functions, we shall prove that for the expected net payment to be positive, the market outlook must necessarily be highly optimistic, and even in such a best-case scenario the note-holder is likely to be at a disadvantage to the issuing bank because of the call provisions of the note. As regards pessimistic scenarios, we show that the average return to note-holders can be as low as -50% in some instances.

In Chapter 4, we describe a second plausible interpretation of the payment procedure. We shall show that expected net payment under this interpretation can never be greater than the expected net payment under the interpretation analyzed in Chapter
Therefore, the outcomes for note-holders can be expected generally to be worse than those described in Chapter 3.

In Chapter 5, we present some concluding remarks on our analysis. We will conclude that these reverse convertible notes were simply clever marketing scheme used by financial institutions who needed to insure themselves against future market declines. Finally, we provide in an appendix the details of the actual outcome of the reverse convertible note analyzed in Chapter 2-5, and we provide in a second appendix a copy of the pricing supplement to the prospectus for that note.
Chapter 2

Assessment of the Pricing Supplement to the Prospectus

Reverse convertibles notes known as “Autocallable Optimization Securities with Contingent Protection Linked to the S&P 500 Financials Index” were sold to the public in multiples of $10. The pricing supplement to the prospectus, which is given an appendix, states that the trade date for this financial product was February 5, 2008, with a settlement date three days later on February 8, 2008. These notes had an 18-month “observation period,” from May 5, 2008 to August 5, 2009, with a total of six “observation dates”: May 5, 2008; August 5, 2008; November 5, 2008; February 5, 2008; May 5, 2009; and August 5, 2009. The final valuation date was on August 5, 2009, and the maturity date was five days later on August 10, 2009.

The pricing supplement also mentions information such as the advantages and disadvantages of the note and possible returns that clients might receive from their purchases. However, the information provided seem biased toward positive aspects of the note. For instance, the positive aspects are emphasized in a “Features Section” on the first page of the pricing supplement, and are highlighted in boldfaced font while the explanation of each feature is in a smaller font. The first feature listed states that the notes provide “positive call return in flat or bullish scenarios,” implying that clients will receive positive returns as long as the S&P 500 Financials Index remains above a certain level.
Another positive attribute listed in the “Features Section” is that of “contingent principal protection.” This feature suggests that clients have some protection against loss of principal; however, the specified market conditions under which the contingent protection applies is printed in a smaller font on the same page.

The last feature states that the notes “express a bullish view of the U.S. Financial Services Sector.” Given that the notes are linked to the S&P 500 Financials Index which comprises of 93 companies in the financial services sector of the S&P 500 Index, financial advisers recommending purchases of these notes should have provided their clients with an estimate of the likelihood that the S&P 500 Financials Index would continue on an upward trend. However, neither advisers nor the pricing supplement appear to have provided any such information.

Indeed, the features listed in the “Features Section” seem to promote optimistic views of future market conditions, leading clients to believe that their future gains will be high. These details imply that the clients’ principals are protected against adverse market conditions and their chances of a loss of principal are low. Moreover, buyers of these notes were more likely to read the first few sections in greater detail compared to the later sections. Thus, the positive aspects of these notes may have played a more influential role in clients' decisions to purchase the notes.

On the other hand, negative aspects of the note are portrayed in a less prominent manner and are postponed to later sections of the pricing supplement. For instance, the fact that the note is not insured by the Federal Deposit Insurance Corporation (FDIC) is stated in smaller font. The section “Key Risks,” which lists numerous risk factors, appears as one of the last sections in the pricing supplement. Also, the pricing
supplement provides no mention of the likelihood that downward trends in the market can occur, leading to a total loss of capital. Moreover, the prospectus implies that future market conditions will be positive but then warns later against the expectation of a “positive-return environment.” Further, based on the pricing supplement’s guidelines for net payment, the greatest possible return on the note is only 31.26%, whereas the greatest possible loss is 100% of capital. That is, profits on the note are limited to a specific percentage while there is no similar limit on the percentage of capital that clients might lose.

Another major concern is the description of the payment method given in the pricing supplement; in our view the description is incomplete. The pricing supplement provides only four examples of possible scenarios of the future market trends. In our view, the supplement should have provided a more comprehensive list of such scenarios. In two of the scenarios presented the client receives a positive return; in one scenario the client breaks even, and in only one scenario does the client lose part of their principal. Also, no example is given to demonstrate how a client could have suffered a loss of their entire capital. Therefore, these examples may have led some clients to believe that they were more likely to receive positive returns rather than losses.

Moreover, the details in the scenario analysis are vague. Specifically, the determination of the conditions under which the note will be called is unclear, and therefore we will now investigate the payment procedure in great detail.

In order to describe the payment method we define several terms. The Index Starting Level is the closing level of the S&P 500 Financials Index on February 5, 2008. The pricing supplement states the Index Starting Level to be 369.44. The Trigger Level
is 184.72, which is 50% of the Index Starting Level. The Index Ending Level is the closing level of the S&P 500 Financials Index on the corresponding observation or trade date. Having defined these terms, we can define the Index Return, denoted by $I$, to be

$$I = \frac{\text{Index Ending Level} - \text{Index Starting Level}}{\text{Index Starting Level}}.$$  

By means of this formula, we see that if the Index Ending Level on any trade date is at or above the Index Starting Level then the Index Return will be positive, i.e. $I \geq 0$, while if on any trade date the Index Ending Level is less than the Index Starting Level then the Index Return will be negative, i.e. $I < 0$. Also, if the Index Ending Level is less than the Trigger Level then $I < -0.5$ and conversely.

The section of the pricing supplement entitled “Payment at Maturity” contains a diagram which is used to illustrate the rules for calculating net payments to clients. The diagram presented here is typical, and hence representative of related diagrams appearing in the pricing supplements of many other autocallable notes found via the SEC’s website. This diagram is the primary source in the pricing supplement devoted to determining the payment procedure and it is reproduced as follows:
Was the closing level of the Index on any Observation Date at or above the Index Starting Level?

Yes

You will receive the applicable Call Price as described under “Indicative Terms—Return on Call Date” if the Notes have not been previously called.

No

Was the closing level of the Index below the Trigger Level on any trading day during the Observation Period?

Yes

Determine the Index Return

No

At maturity, you will receive your principal of $10 per Note.

In this scenario, you could lose some or all of your principal depending on how much the Index decreases.

Figure 2.1: Payment at Maturity as depicted in the pricing supplement
From Figure 2.1, we deduce that if $I > 0$ on an observation date, then the note might be called and redeemed for an amount stated in the pricing supplement. This redemption value is based on a rate of 20.84% per annum, with actual amounts stated in the “Final Terms” section of the supplement. This “Final Terms” section states each observation date and the corresponding percentage return if the note is called on that date.

However, if $I < 0$ on a particular observation date then the note is not called. Also, if $I < -0.5$ on any trading date during the observation period then there is a possibility that the client will receive a negative return at maturity. In particular, the diagram stated that the client can lose up to a 100% of their capital but provided no indication of the probability of this occurrence. Also, the diagram itself does not make it explicit when the note will be called, but as long as the notes are not called then the calculation of the Index Return will be done on every subsequent trading date until the final valuation date.

To determine the actual return that note-holders will receive on their notes is particularly vague. It appears that the notes could have been called before the final valuation date, and at the complete discretion of the sellers. Suppose also that the Index Return, when measured on the final valuation date, is substantially greater than zero, i.e., the Index Ending Level is higher than the Index Starting Level; then the language used in the pricing supplement appears to give the seller of the note the power to have called the note in a way so as to minimize the return to note holders. Therefore, it seems that the manner in which payments to clients are determined can be interpreted in different ways. In the following two chapters we will analyze two likely interpretations
of the payment procedure that could have been used and we will estimate the average return to clients under each interpretation.
Chapter 3

A First Interpretation of the Payment Procedure

In this chapter, we describe and analyze one of the possible interpretations of the payment procedure for calculating net payment. For this interpretation, we will show that even under optimistic scenarios, an average buyer will lose a substantial amount of their principal.

For each observation date within the 18-month period from February 5, 2008 to August 5, 2009, we define the corresponding index return as follows:

\[ I_1 : \text{Index return on May 5, 2008} \]
\[ I_2 : \text{Index return on August 5, 2008} \]
\[ I_3 : \text{Index return on November 5, 2008} \]
\[ I_4 : \text{Index return on February 5, 2009} \]
\[ I_5 : \text{Index return on May 5, 2009} \]
\[ I_6 : \text{Index return on August 5, 2009} \]

Let \( n \) denote the total number of trading days during the 18-month observation period. We have verified through Google Finance’s website on historical closing prices of the S&P 500 Financials Index, that for this note \( n = 381 \) trading days. For \( 1, \ldots, n, \ldots \),
define $d_i$ to be the cumulative index return for Day $i$; thus $d_i$ represents the total index return from Day 1, February 5, 2008, to Day $i$.

Let $d_{\text{min}} = \min (d_1, d_2, ..., d_n)$ be the smallest daily cumulative return over the entire observation period.

Using this notation, the steps for this interpretation of the payment procedure can now be described as follows:

Step 1: Calculate the Index Return, $I_r$ on the observation date for the $r$th quarter.

Step 2: If the Index Return on the observation date is non-negative, i.e. if $I_r \geq 0$, then the note is called and the client receives an amount as stated in the pricing supplement on the Final Valuation date.

Step 3: If the Index Return on the observation date is negative, i.e. if $I_r < 0$ then we calculate the cumulative returns, $d_i$, for each trading date during the quarter corresponding to the given observation date. If any of those cumulative returns are less than $-50\%$ then we record this occurrence for future reference.

Step 4: We return to Step 1 for the next observation date and repeat the process.
Steps 1-4 are carried out on every observation date until the final valuation date or until the note is called.

At the Final Valuation date, one of the following three cases can occur:

Case 1: If the Index Return is non-negative on the Final Valuation date i.e. if $I_6 \geq 0$, then the client receives the amount stated in the pricing supplement.

Case 2: If the Index Return on the Final Valuation date is negative and every daily Index Ending Level remained at or above the Trigger Level on each trading date during the entire observation period, i.e. if $I_6 < 0$ and $d_{\text{min}} \geq -50\%$, then the client receives $10$ at maturity.

Case 3: If on any trading date prior to the Final Valuation Date at least one daily Index Ending Level breached the Trigger Level, i.e. if $d_{\text{min}} < -50\%$, then the client receives a reduced payment equal to $10(1 + I_6)$, representing a negative return of $I_6$ to note-holders.

The following tree diagram describes this interpretation of the payment procedure:
Figure 3.1: Description of Interpretation I
Then according to the pricing supplement, given in an appendix, and this interpretation of the payment procedure, the net payment in U.S. dollars, to note-holders is given by the function:

\[
\text{Net Payment} =\begin{cases} 
0.52, & \text{if } I_1 \geq 0 \\
1.04, & \text{if } I_1 < 0, \text{ and } I_2 \geq 0 \\
1.56, & \text{if } I_j < 0, j = 1, 2 \text{ and } I_3 \geq 0 \\
2.08, & \text{if } I_j < 0, j = 1, 2, 3 \text{ and } I_4 \geq 0 \\
2.61, & \text{if } I_j < 0, 1 \leq j \leq 4 \text{ and } I_5 \geq 0 \\
3.13, & \text{if } I_j < 0, 1 \leq j \leq 5 \text{ and } I_6 \geq 0 \\
0, & \text{if } I_j < 0, 1 \leq j \leq 6 \text{ and } d_{\min} \geq \frac{1}{2} \\
10I_6, & \text{if } I_j < 0, 1 \leq j \leq 6 \text{ and } d_{\min} < \frac{1}{2}
\end{cases}
\]

(3.1)

- The first six cases occur if any of the index returns, \( I_1, \ldots, I_6 \) are non-negative, in which case, the note is called on the corresponding observation date, and the client receives a positive return on the Final Valuation date in those instances.

- The seventh case occurs if all of the index returns, \( I_1, \ldots, I_6 \) are negative and the minimum cumulative daily index return during the observation period, \( d_{\min} \) is at least \(-50\%\). In this case, the note matures and the client receives a net payment of zero on the Final Valuation date, i.e. the client breaks even.
• The eighth and last case occurs if the note is not called previously, and the minimum cumulative daily index return during the observation period, $d_{\text{min}}$ is less than $-50\%$ i.e. the smallest Index Ending Level fell below the Trigger Level. In this case, the note matures and on the Final Valuation date the client receives a negative net payment of $10I_6$.

In the function given by (3.1), $I_1, \ldots, I_6$ are continuous random variables whose probability density function (p.d.f) is strictly positive for the interval $(-1, 0)$ i.e. there is a non-zero probability of a downward market trend in the future. Thus,

$$P(d_{\text{min}} < -\frac{1}{2} | I_j < 0, 1 \leq j \leq 6) \neq 0$$

(3.2)

since the possibility of the market trends decreasing will always exist.

At this point, we recall the Law of Total Expectation (Ross [11], page 333, Equation (5.1b)): Let $X$ and $Y$ be jointly distributed random variables, where $Y$ is continuous with probability density function $f_Y(y)$, then

$$E(X) = \int_{-\infty}^{\infty} E(X|Y = y) f_Y(y) \, dy$$

(3.3)

We now apply the Law of Total Expectation to the net payment function in (3.1); i.e., we let $X$ be the net payment and $Y$ be the index return for the various periods. Then,
we obtain

\[ E(\text{Net Payment}) = 0.52 \times P(I_1 \geq 0) \]

\[ + 1.04 \times P(I_1 < 0, \text{ and } I_2 \geq 0) \]

\[ + 1.56 \times P(I_j < 0, j = 1, 2 \text{ and } I_3 \geq 0) \]

\[ + 2.08 \times P(I_j < 0, j = 1, 2, 3 \text{ and } I_4 \geq 0) \]

\[ + 2.61 \times P(I_j < 0, 1 \leq j \leq 4 \text{ and } I_5 \geq 0) \]

\[ + 3.13 \times P(I_j < 0, 1 \leq j \leq 5 \text{ and } I_6 \geq 0) \]

\[ + 0 \times P(I_j < 0, 1 \leq j \leq 6 \text{ and } d_{\min} \geq -\frac{1}{2}) \]

\[ + 10 \times E(I_6|d_{\min} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6) \]

\[ \times P(d_{\min} < -\frac{1}{2}|I_j < 0, 1 \leq j \leq 6) \]

\[ \times P(I_j < 0, 1 \leq j \leq 6). \]

In a real-world setting, the index return random variables \( I_1, \ldots, I_6 \) are unlikely to be independent. In fact, there are numerous research articles which have found evidence that momentum-trading “strategies which buy stocks that have performed well in the past and sell stocks that have performed poorly in the past generate significant positive returns over 3- to 12-month holding periods” (Jegadeesh and Titman [7]; Conrad and Kaul [1]); this phenomenon has been observed to hold not only for U.S. financial markets but also internationally (Hurn [5]). Therefore, if the financial index has recorded a negative return in a three-month period then it is natural to expect traders to be pessimistic about the ensuing three-month period. Therefore, we would expect to find
that, for example,

\[ P(I_2 \geq 0|I_1 < 0) \leq P(I_2 < 0|I_1 < 0) \]  \hspace{1cm} (3.5)

in a real-world setting.

We recognize that there can be exceptions to this inequality; if, for instance, the markets are in a euphoric state of mind, where any downturn is regarded optimistically as a sign of positive future developments then the above inequality might even be reversed at times. Since it is unrealistic and unwise to treat the markets as being in a permanent state of euphoria, we will treat this inequality as being valid generally. Therefore for the purposes of analyzing the expected net payment to note-holders, we will assume that the inequality (3.5) holds.

We remark that because

\[ P(I_2 \geq 0|I_1 < 0) + P(I_2 < 0|I_1 < 0) = 1, \]  \hspace{1cm} (3.6)

then the inequality (3.5) implies that

\[ P(I_2 \geq 0|I_1 < 0) \leq \frac{1}{2} \leq P(I_2 < 0|I_1 < 0). \]  \hspace{1cm} (3.7)

Also, (3.6) implies that if \( P(I_2 \geq 0|I_1 < 0) \) is large, i.e. close to one, then \( P(I_2 < 0|I_1 < 0) \) is small, i.e. close to zero.
At this point, it is worthwhile to provide historical context relating to the state of the financial markets in early 2008, at the same time when these reverse convertible notes were being sold to the public. Figure 3.2 was obtained from Google Finance’s website and is based on historical closing prices from September 20, 2002 to December 28, 2007 of the S&P 500 Financials Index.

As Figure 3.2 shows, the S&P 500 Financial Index had undergone a sustained increase during the period 2002 to late 2007. By early 2008, many investors were concerned that the bull market had gone too far and that stock prices might be due for substantial declines. Therefore, it was reasonable to expect that $P(I_r < 0)$ was large for the $r$th quarter, hence, $P(I_r \geq 0)$ was small for that quarter.

Given the statistical evidence that momentum-trading strategies are successful over 3 to 12-month periods, we also could expect that, in late 2007, $P(I_2 < 0|I_1 < 0)$
was high, hence that $P(I_2 \geq 0 | I_1 < 0)$ was small. Then it follows that

$$P(I_1 < 0, \text{ and } I_2 \geq 0) \equiv P(I_2 \geq 0 | I_1 < 0) P(I_1 < 0) \quad < \quad P(I_2 \geq 0 | I_1 < 0),$$

which proves that $P(I_1 < 0, \text{ and } I_2 \geq 0)$ also is small. By repeating this argument, we deduce that, in early 2008, each of the probabilities in the first six terms in (3.4) are very small. Therefore,

$$E(\text{Net Payment}) \simeq 10 E(I_6 | d_{\min} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6) \times P(d_{\min} < -\frac{1}{2} | I_j < 0, 1 \leq j \leq 6) \times P(I_j < 0, 1 \leq j \leq 6) \quad (3.8)$$

which, clearly, is negative. As a consequence, we deduce that clients who purchased these reverse convertible notes in early 2008 likely were destined to obtain negative net returns, on average.

Even more can be deduced from (3.8). By late 2007, $P(I_j < 0, 1 \leq j \leq 6)$ and $P(d_{\min} < -\frac{1}{2} | I_j < 0, 1 \leq j \leq 6)$ both were very large. Hence, by (3.8),

$$E(\text{Net Payment}) \simeq 10 \times E(I_6 | d_{\min} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6). \quad (3.9)$$

If it was believed in early 2008 that the financial markets were likely to have a precipitous, quarter-over-quarter, fall over the next 18 months, then it would mean that $I_6 \simeq d_{\min}$,
in which case it follows from (3.9) that

\[ E(\text{Net Payment}) \simeq 10 \times E(d_{\min}\mid d_{\min} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6) \]

\[ < 10 \times -0.5 \]

\[ = -5. \]

Consequently, the outcome of purchasing these reverse convertible notes in early 2008 was to undertake a substantial risk of least a 50% loss of capital, on average.

Admittedly, the analysis provided above is based on a pessimistic view in 2007-2008 of coming trends in the financial markets. To provide an analysis which is based on an unbiased approach toward future market trends, we will therefore treat the market as a “random walk” in which \( I_1, \ldots, I_6 \) are mutually independent and identically distributed random variables. These assumptions imply that medium-term momentum-based strategies will provide a trader with no advantages over the general market. Also, such an assumption will be more profitable for a client who purchased reverse convertible notes in 2008, for it means that the various probabilities appearing in the first six cases in the expected net-payment function are likely to be more favorable to the clients than would have been the case under the real-world assumption underlying the inequality (3.5).

Nevertheless, under the assumptions that \( I_1, \ldots, I_6 \) are mutually independent and identically distributed, we shall show that the expected net-payment to note-holders still remains significantly negative. Hence, we will deduce that even in a “random walk”
scenario an average note-holder would have faced losses except under highly optimistic environments.

Using the assumption that $I_1, \ldots, I_6$ are independent, we see that equation (3.4) becomes

$$E(\text{Net Payment}) = 0.52 \times P(I_1 \geq 0)$$

$$+ 1.04 \times P(I_1 < 0) \times P(I_2 \geq 0)$$

$$+ 1.56 \times \left[ \prod_{j=1}^{2} P(I_j < 0) \right] \times P(I_3 \geq 0)$$

$$+ 2.08 \times \left[ \prod_{j=1}^{3} P(I_j < 0) \right] \times P(I_4 \geq 0)$$

$$+ 2.61 \times \left[ \prod_{j=1}^{4} P(I_j < 0) \right] \times P(I_5 \geq 0)$$

$$+ 3.13 \times \left[ \prod_{j=1}^{5} P(I_j < 0) \right] \times P(I_6 \geq 0)$$

$$+ 10 \times E(I_6 \mid d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)$$

$$\times P(d_{\text{min}} < -\frac{1}{2} \mid I_j < 0, 1 \leq j \leq 6) \times \prod_{j=1}^{6} P(I_j < 0).$$

(3.10)

Denote $P(I_1 \geq 0)$ by $p$, because of the assumption that $I_1, \ldots, I_6$ are identically distributed, it follows that $p \equiv P(I_j \geq 0)$ for all $j = 1, \ldots, 6$. Note that, $p$ is the probability that the note is called, i.e. that the Index Return on any given observation
date was non-negative. Then, we obtain

\[ E(\text{Net Payment}) = 0.52p + 1.04(1 - p)p + 1.56(1 - p)^2p \]
\[ + 2.08(1 - p)^3p + 2.61(1 - p)^4p + 3.13(1 - p)^5p \]
\[ + 10 \times E(I_6|d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6) \]
\[ \times P(d_{\text{min}} < -\frac{1}{2}|I_j < 0, 1 \leq j \leq 6) \times (1 - p)^6. \]  

(3.11)

In order to derive explicit values for the terms

\[ E(I_6|d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6) \]

and

\[ P(d_{\text{min}} < -\frac{1}{2}|I_j < 0, 1 \leq j \leq 6), \]

we would need to make strong assumptions about the random variables \( I_1, \ldots, I_6 \), which we prefer to avoid doing. Therefore, we will assign values to these terms to reflect a variety of market conditions, and then we will study the resulting behavior of the function (3.11) for each set of values.

Define

\[ B_1 \equiv -E(I_6|d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6). \]  

(3.12)

Note that the expectation \( E(I_6|d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6) \) clearly is negative; hence, \( B_1 < 0 \). Also, because \( I_6 \geq -1 \), then \( B_1 \leq 1 \). Therefore, we have \( 0 < B_1 \leq 1 \).

Also, define

\[ B_2 \equiv P(d_{\text{min}} < -\frac{1}{2}|I_j < 0, 1 \leq j \leq 6); \]  

(3.13)
clearly, $0 \leq B_2 \leq 1$ because it is a probability. Moreover, by (3.2), $B_2 \neq 0$; therefore 
$0 < B_2 \leq 1$.

By (3.11), we obtain

$$E(\text{Net Payment}) = 0.52p + 1.04(1-p)p + 1.56(1-p)^2 p + 2.08(1-p)^3 p$$

$$+ 2.61(1-p)^4 p + 3.13(1-p)^5 p - 10B_1B_2(1-p)^6.$$  \hspace{1cm} (3.14)

For $p = 0$, we obtain $E(\text{Net Payment}) = -10B_1B_2 < 0$. Therefore, there will always be a small interval around $p = 0$ where the expected net payment is negative.

As regards an interpretation for values of $B_1$, we offer the following. Suppose that $B_1$ is small; then it follows from (3.12) that $E(I_6|d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)$ is close to zero \textit{and} is negative. So, for small values of $B_1$, a note-holder will receive a small, but negative, overall expected return despite the conditioning event that $I_1, \ldots, I_6$ all are negative. That is to say, although the index returns on all six observation dates were negative, and the minimum cumulative daily return is less than -50%, the final outcome was that the average loss to note-holders nevertheless turned out to be small.

Hence, small values of $B_1$ indicate that the financial markets are likely to have maintained an optimistic outlook throughout the 18-month observation period despite consecutive quarterly losses. This leads us to associate small values of $B_1$ with general optimism amongst market participants, and by a similar argument, we can associate large values of $B_1$ with broad market pessimism.

In the case of $B_2$, it follows from (3.13) that if $B_2$ is small then there is a small probability of any daily cumulative index return breaching the Trigger Level, conditional
on six consecutive quarterly losses. In a real-world situation, consecutive quarterly losses usually induces general gloom on market participants; so it would indeed be an optimistic assumption to believe that there remains a small probability of avoiding the Trigger Level. Therefore, we see that small values of $B_2$ can be associated with a generally optimistic market outlook; and by the same argument, large values of $B_2$ can be associated with broad market pessimism.

Consider the case in which, $B_1 = B_2 = 0.1$. Using these values, we plot (3.14) with respect to $p$, where $0 \leq p \leq 1$, to obtain the following graph of the expected net payment function:

![Graph of Expected Net Payments](image)

Figure 3.3: A Graph of Expected Net Payments, where $B_1 = 0.1$ and $B_2 = 0.1$
In this case, the maximum expected net payment is 1.15, approximately, i.e., an average return of about 11.5%. Also, the minimum expected net payment is \(-0.1\), approximately, i.e., an average loss of about 10%. Thus, even under this optimistic scenario, the average client could lose as much as 10% of their capital.

We remark also that the expected net payment is maximized at \(p = 0.29\), approximately. Although it might have been presumed that consecutive quarterly increases in the financial markets, i.e., \(p = 1\), would be more beneficial to note-holders, such a scenario would have caused the note to be called early due to the sustained market increases. In such a case, paradoxically, average returns to note-holders would have been diminished.

Under a less-optimistic scenario for the financial markets, in which \(B_1 = B_2 = 0.5\), the graph of expected net payment function in (3.14) is given in the following figure:

![Figure 3.4: A Graph of Expected Net Payments, where \(B_1 = 0.5\) and \(B_2 = 0.5\)](image-url)
For this situation, the maximum expected net payment is 0.97, approximately, i.e., an average return of about 9.7%. Also, the minimum expected net payment is −2.5, approximately, representing an average loss of about 25% of principal. As in the previous scenario, moderate values of $p$ turn out to be more beneficial than high values of $p$ to note-holders.

If financial markets are expected to undergo significant downturns where the values of $B_1$ and $B_2$ are relatively high, say $B_1 = 0.7$ and $B_2 = 0.8$, then we obtain the following graph of the expected net payment function in (3.14)

![Graph of Expected Net Payments](image)

Figure 3.5: A Graph of Expected Net Payments, where $B_1 = 0.7$ and $B_2 = 0.8$

Under this pessimistic scenario, the highest expected net payment is 0.89, approximately, which means that maximum percentage return for the average note-holder was
only about 8.9%. On the other hand, the minimum expected net payment is −5.6, approximately, implying that the average note-holder could have suffered percentage losses of as much as 56% of their capital.

So far, we have been studying the exact values of the expected net payment function which, as we have seen, involves two unknown parameters, \( B_1 \) and \( B_2 \). We now derive upper bounds for the expected net payment function; these bounds will have the advantage of depending on only one unknown parameter.

We know that \( d_{\text{min}} \leq I_r \) for any \( r = 1, \ldots, 6 \); therefore,

\[
P(d_{\text{min}} < -\frac{1}{2} | I_j < 0, 1 \leq j \leq 6) \geq P(I_r < -\frac{1}{2} | I_j < 0, 1 \leq j \leq 6).
\] (3.15)

Also, the expectation \( E(I_6|d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6) < 0 \) because \( I_6 < 0 \) in the conditioning event. Therefore, by inequality (3.15), we see that

\[
E(I_6|d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)P(d_{\text{min}} < -\frac{1}{2} | I_j < 0, 1 \leq j \leq 6)
\]

\[
\leq E(I_6|d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)P(I_r < -\frac{1}{2} | I_j < 0, 1 \leq j \leq 6),
\] (3.16)

and then, by (3.16), we obtain

\[
E(\text{Net Payment}) \leq 0.52p + 1.04(1 - p)p + 1.56(1 - p)^2 p
\]

\[
+ 2.08(1 - p)^3 p + 2.61(1 - p)^4 p + 3.13(1 - p)^5 p
\]

\[
+ 10(1 - p)^6 E(I_6|d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)
\]

\[
\times P(I_r < -\frac{1}{2} | I_j < 0, 1 \leq j \leq 6).
\] (3.17)
By the definition of conditional probability and the assumptions that $I_1, \ldots, I_6$ are independent and identically distributed,

\[
P(I_r < -\frac{1}{2} | I_j < 0, 1 \leq j \leq 6) = P(I_r < -\frac{1}{2} | I_r < 0) \\
= \frac{P(I_r < -\frac{1}{2} \text{ and } I_r < 0)}{P(I_r < 0)} \\
= \frac{P(I_r < -\frac{1}{2})}{P(I_r < 0)} \\
= (1 - p)^{-1}P(I_r < -\frac{1}{2}).
\]

Thus, by (3.17) and (3.18),

\[
E(\text{Net Payment}) \\
\leq 0.52p + 1.04(1 - p)p + 1.56(1 - p)^2p \\
+ 2.08(1 - p)^3p + 2.61(1 - p)^4p + 3.13(1 - p)^5p \\
+ 10(1 - p)^5E(I_6 | d_{\min} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)P(I_r < -\frac{1}{2}).
\]

We can also obtain a lower bound on $E(I_6 | d_{\min} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)$. To that end, let

\[A_1 = \{I_r < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6\}\]

and

\[A_2 = \{d_{\min} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6\}.
\]

Because $d_{\min} \leq I_r$ for $r = 1, \ldots, 6$ then it follows that $A_1 \subseteq A_2$. 
Consider the cumulative index returns $d_1, \ldots, d_n$, viewed as continuous random variables. The sets $A_1$ and $A_2$ above clearly are events that depend on $d_1, \ldots, d_n$. Because $A_1 \subseteq A_2$ then it follows that

$$P(A_1) \leq P(A_2).$$  

(3.20)

We noted earlier in this chapter in (3.2) that $P(A_2) \neq 0$; by a similar argument, it follows that $P(A_1) \neq 0$. As mentioned previously, this condition holds because $I_1, \ldots, I_6$ are continuous random variables whose probability density function (p.d.f.) is strictly positive on the interval (-1,0), a condition which is equivalent to the assumption that there is a non-zero probability of the markets moving downward in any quarter.

Let $f(d_1, \ldots, d_n)$ denote the joint probability density function of $I_1, \ldots, I_6$. Since $I_6 \equiv I_6(x_1, \ldots, x_n)$, i.e., $I_6$ is a function of the random variables $d_1, \ldots, d_n$ then, by definition, we obtain

$$E(I_6|d_{\min} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)$$

$$\equiv E(I_6|A_2)$$

$$= \int_{A_2} I_6(x_1, \ldots, x_n) \cdot f(x_1, \ldots, x_n) dx_1 \cdots dx_n$$

$$\frac{P(A_2)}{P(A_2)}.$$  

(3.21)

Because $I_6 \equiv I_6(x_1, \ldots, x_n) < 0$ for all $(x_1, \ldots, x_n) \in A_2$ and because $A_1 \subseteq A_2$ then

$$\int_{A_2} I_6(x_1, \ldots, x_n) \cdot f(x_1, \ldots, x_n) dx_1 \cdots dx_n$$

$$\leq \int_{A_1} I_6(x_1, \ldots, x_n) \cdot f(x_1, \ldots, x_n) dx_1 \cdots dx_n.$$  

(3.22)
It now follows that from (3.20), (3.21) and (3.22) that

\[
E(I_6|A_2) = \frac{\int_{A_2} I_6(x_1, \ldots, x_n) \cdot f(x_1, \ldots, x_n)dx_1 \cdots dx_n}{P(A_2)} \leq \frac{\int_{A_1} I_6(x_1, \ldots, x_n) \cdot f(x_1, \ldots, x_n)dx_1 \cdots dx_n}{P(A_1)} \equiv E(I_6|A_1).
\]

(3.23)

This proves that

\[
E(I_6|d_{\min} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6) \leq E(I_6|I_r < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6).
\]

(3.24)

Hence for any \(r = 1, \ldots, 6,\)

\[
E(\text{Net Payment}) \leq 0.52p + 1.04(1 - p)p + 1.56(1 - p)^2p + 2.08(1 - p)^3p + 2.61(1 - p)^4p + 3.13(1 - p)^5p + 10(1 - p)^5E(I_6|I_r < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)P(I_r < -\frac{1}{2}).
\]

(3.25)

Also, note that the inequality (3.25) involves the random variables \(I_1, \ldots, I_6\) only, and does not depend on \(d_{\min}\). Thus, we can select \(r = 6\) to obtain

\[
E(\text{Net Payment}) \leq 0.52p + 1.04(1 - p)p + 1.56(1 - p)^2p + 2.08(1 - p)^3p + 2.61(1 - p)^4p + 3.13(1 - p)^5p + 10(1 - p)^5E(I_6|I_6 < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)P(I_6 < -\frac{1}{2}).
\]

(3.26)
It is clear that

\[ E(I_6 | I_6 < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6) < -\frac{1}{2}. \]

Introducing the notation \( \tau = P(I_6 < -\frac{1}{2}) \), then clearly \( p + \tau \leq 1 \). Therefore, it now follows from (3.26) that

\[
E(\text{Net Payment}) \leq 0.52p + 1.04(1 - p)p + 1.56(1 - p)^2p \\
\quad + 2.08(1 - p)^3p + 2.61(1 - p)^4p + 3.13(1 - p)^5p \\
\quad - 5(1 - p)^5\tau. \tag{3.27}
\]

Under an optimistic scenario, we expect there is a small probability of \( I_6 \) breaching the Trigger Level, i.e., \( \tau \) is small. For instance, let \( \tau = 0.1 \), then we get the following graph of an upper bound of the expected net payment function in (3.27)

Figure 3.6: Upper Bound for Expected Net Payments, where \( P(I_6 < -\frac{1}{2}) = 0.1 \)
In this case, an upper bound for the maximum expected net payment is 1.08, approximately, resulting in an upper bound for the average return of about 10.8%. On the other hand, an upper bound for the minimum expected net payment is $-0.5$, approximately, i.e., an average loss of about 5%.

Suppose we let $\tau = 0.5$; then we obtain the following graph of the upper bound for the expected net payment function in (3.27):

![Figure 3.7: Upper Bound for Expected Net Payments, where $P(I_6 < -\frac{1}{2}) = 0.5$](image)

Under this more pessimistic scenario, the highest value for an upper bound for expected net payment is 0.91, approximately. Also, an upper bound for the lowest expected net payment is $-2.5$, approximately. These values imply that an upper bound
for average return to note-holders is about 9.1%, whereas an upper bound for average loss to note-holders is about 25%.

Let us consider a highly pessimistic scenario for financial market conditions in which $\tau = 0.8$; then the following figure depicts an upper bound for expected net payment given by (3.27):

![Upper Bound for Expected Net Payments](image)

Figure 3.8: Upper Bound for Expected Net Payments, where $P(I_6 < -\frac{1}{2}) = 0.8$

For this scenario, an upper bound for the maximum expected net payment is 0.85, approximately. On the other hand an upper bound for the lowest expected net payment is $-4$, approximately. These values imply that an upper bound for the the return for the average client is about 8.5% while an upper bound for the possible loss is about 40%.

In conclusion, Figures 3.3–3.8 depict upper bounds for expected net payments under various scenarios. As a general rule, the highest values for expected net payments
occur at moderate values of $p$ and not at high values of $p$, as one would first presume. Under a scenario in which markets are expected to undergo significant upward trends and the value of $p$ is high, the note will be called early and therefore will limit returns to note-holders.

Based on our analysis of the six scenarios depicted in Figures 3.3–3.8, the best-case scenario leads to a possible maximum an average return of 11.5%, approximately, and a possible average loss of 10%, approximately. The worst-case scenario is an average return of 8%, approximately, and an average loss of 56%, approximately.

Bearing in mind that only six scenarios were described above in detail, we also provide a more comprehensive analysis simply by graphing the upper bound function in (3.27) over all values of $(p,\tau)$ such that $0 \leq p, \tau \leq 1$ and $p + \tau \leq 1$. Then we obtain the following three-dimensional plot of expected net payments:
From this analysis, the best-case scenario is an expected net payment of at most 1.16, approximately, at $p = 0.28$ and $\tau = 0$. On the other hand, the worst-case scenario results in an expected net payment of at most $-5$, approximately, at $p = 0$ and $\tau = 1$. Thus, the net percentage return for note-holders ranges from $-50\%$ to $11.6\%$, approximately. This range implies that on average, the possible returns to note-holders are substantially lower than the potential losses. As a result, an average note-holder would have suffered a significant loss of capital.
Chapter 4

A Second Interpretation of The Payment Procedure

We now provide a second interpretation of the payment procedure and analyze it similarly to the way in which the previous interpretation was assessed. In the interpretation of the payment procedure studied in the previous chapter, if an Index Ending Level breached the Trigger Level during any quarter of the observation period but the Index Return was positive on a subsequent observation date, then the possibility of a positive return still existed. Because the prospectus was unclear on this issue, we will assume in this chapter that if an Index Ending Level breached the Trigger Level on any trading day during a quarter, i.e. if at least one cumulative daily index return was less than $-50\%$, then the client is destined to receive a negative return at the Final Valuation date.

Throughout this chapter the notation for the six observation dates and the daily cumulative index return will remain the same as in the previous chapter. We also need to define a collection of minimum daily cumulative index returns as a function of the various observation dates, viz.,

\[ d_{\text{min},1} : \text{Minimum cumulative daily index return on May 5, 2008} \]
\[ d_{\text{min},2} : \text{Minimum cumulative daily index return on August 5, 2008} \]
\[ d_{\text{min},3} : \text{Minimum cumulative daily index return on November 5, 2008} \]
\[ d_{\text{min},4} : \text{Minimum cumulative daily index return on February 5, 2009} \]
\( d_{\text{min},5} \): Minimum cumulative daily index return on May 5, 2009

\( d_{\text{min},6} \): Minimum cumulative daily index return on August 5, 2009

The last minimum cumulative daily index return, \( d_{\text{min},6} \), is equivalent to \( d_{\text{min}} \) in the analysis provided in Chapter 3. Also, from this notation it follows that:

\[
d_{\text{min},1} \geq d_{\text{min},2} \geq d_{\text{min},3} \geq d_{\text{min},4} \geq d_{\text{min},5} \geq d_{\text{min},6}.
\]  

(4.1)

With this notation, the steps for this interpretation of the payment procedure can now be described as follows:

Step 1: Calculate the Index Return, \( I_r \) on the observation date for the \( r \)th quarter.

Step 2: If the Index Return on the observation date is non-negative, i.e. if \( I_r \geq 0 \), then the note is called and the client receives an amount as stated in the pricing supplement on the Final Valuation date.

Step 3: Calculate the cumulative returns, \( d_i \), for each trading date during the quarter corresponding to the given observation date and determine the minimum cumulative daily index return, \( d_{\text{min},r} \) at this observation date.
Step 4: If the Index Return on the observation date is negative and the minimum cumulative daily return is greater than $-50\%$, i.e. if $I_r < 0$ and $d_{\text{min},r} \geq -50\%$, then we return to Step 1 for the next observation date and repeat the process.

Step 5: If the Index Return on the observation date is negative and minimum cumulative daily return is less than $-50\%$, i.e. if $I_r < 0$ and $d_{\text{min},t} < -50\%$, then no further action is taken until the Final Valuation date.

Steps 1-4 are carried out on every observation date until the Final Valuation date or until the note is called.

We remark that on the Final Valuation date if the note has not been called then all previous minimum cumulative daily index returns are greater than $-50\%$ and by equation (4.1) $d_{\text{min},5} \geq -50\%$. 
At the Final Valuation date, one of the following three cases can occur:

Case 1: If the Index Return on the observation date is non-negative i.e. \( I_6 \geq 0 \), then the note is called and the client receives an amount as stated in the pricing supplement on the Final Valuation date.

Case 2: If the Index Return is negative and the minimum cumulative daily index return is greater than \(-50\%\) on the Final Valuation date, i.e. if \( I_6 < 0 \) and \( d_{\text{min},6} \geq -50\% \), then the client receives $10 at maturity.

Case 3: If any minimum cumulative daily index return was less than \(-50\%\) i.e. if \( d_{\text{min},r} < -50\% \) for at least one \( r = 1, \ldots, 6 \), then the client receives a reduced payment equal to \( 10(1 + I_6) \), representing a negative return of \( I_6 \) to note-holders.

Note that on the final valuation date that if at least one \( d_{\text{min},r} < -50\% \) then by inequality (4.1) it follows that:

\[
d_{\text{min},6} \leq 50\%.
\]

The following tree diagram describes this interpretation of the payment procedure.
Figure 4.1: Description of Interpretation II

Trade Date (Feb. 5, 2008)

Go to Observation Date #1:
Is $I_1 \geq 0$ on May 5, 2008?
No: Then was any $d_i < -50\%$ during Feb. 5-May 5, 2008?
Yes: Then call the notes; pay note-holders $10.52$
No: Then go to Observation Date #2:
Is $I_2 \geq 0$ on Aug. 5, 2008?
No: Then was any $d_i < -50\%$ during May 6-Aug. 5, 2008?
Yes: Then call the notes; pay note-holders $11.04$
No: Then go to Observation Date #3:
Is $I_3 \geq 0$ on Nov. 5, 2008?
No: Then was any $d_i < -50\%$ during Aug. 6-Nov. 5, 2008?
Yes: Then call the notes; pay note-holders $11.56$
No: Then go to Observation Date #4:
Is $I_4 \geq 0$ on Feb. 5, 2009?
No: Then was any $d_i < -50\%$ during Nov. 6, 2008-Feb. 5, 2009?
Yes: Then call the notes; pay note-holders $12.08$
No: Then go to the Final Valuation Date:
Is $I_6 \geq 0$ on Aug. 5, 2009?
No: Then was any $d_i < -50\%$ during the observation period?
Yes: Then call the notes; pay note-holders $13.13$
No: Notes mature; pay note-holders $10$

Yes: Notes are not called

Notes mature; pay note-holders $10(1 + I_6)$

Notes mature; pay note-holders $10.52$
For this interpretation of the payment procedure, we apply the information provided in the pricing supplement to deduce that the net payment in US dollars is given by:

\[
\text{Net Payment} = \begin{cases} 
0.52, & \text{if } I_1 \geq 0 \\
1.04, & \text{if } I_1 < 0 \text{ and } I_2 \geq 0 \text{ and } d_{\text{min},1} \geq -\frac{1}{2} \\
1.56, & \text{if } I_j < 0, j = 1, 2 \text{ and } I_3 \geq 0 \text{ and } d_{\text{min},2} \geq -\frac{1}{2} \\
2.08, & \text{if } I_j < 0, j = 1, 2, 3 \text{ and } I_4 \geq 0 \text{ and } d_{\text{min},3} \geq -\frac{1}{2} \\
2.61, & \text{if } I_j < 0, 1 \leq j \leq 4 \text{ and } I_5 \geq 0 \text{ and } d_{\text{min},4} \geq -\frac{1}{2} \\
3.13, & \text{if } I_j < 0, 1 \leq j \leq 5 \text{ and } I_6 \geq 0 \text{ and } d_{\text{min},5} \geq -\frac{1}{2} \\
0, & \text{if } I_j < 0, 1 \leq j \leq 6 \text{ and } d_{\text{min},6} \geq -\frac{1}{2} \\
10I_6, & \text{if } I_j < 0, 1 \leq j \leq 6 \text{ and } d_{\text{min},6} < -\frac{1}{2} 
\end{cases} \]

(4.2)

- The first six cases occur if any of the index returns, \(I_1, \ldots, I_6\) are non-negative and the minimum cumulative daily index return of the previous quarter is at least \(-50\%\) i.e., \(I_r \geq 0\) and \(d_{\text{min},r-1} \geq -50\%\), in which case, the note is called on the corresponding observation date, and the client receives a positive return in these instances.

- The seventh case occurs if all of the index returns, \(I_1, \ldots, I_6\) are negative and every minimum cumulative daily index return is at least \(-50\%\). In this case, the note matures and the client receives a net payment of zero, i.e., the client breaks even.
• The eighth and last case occurs if any $d_{\text{min},r} \leq -50\%$, i.e. the minimum cumulative daily index return during some quarter of the observation period fell below $-50\%$. In this case, the note matures and the client receives a negative net payment of $10I_6$.

Based on our function for net payment above, we obtain:

\[
E(\text{Net Payment}) = 0.52 \times P(I_1 \geq 0)
\]

\[
+ 1.04 \times P(I_1 < 0 \text{ and } I_2 \geq 0 \text{ and } d_{\text{min},1} \geq -\frac{1}{2})
\]

\[
+ 1.56 \times P(I_j < 0, j = 1, 2 \text{ and } I_3 \geq 0 \text{ and } d_{\text{min},2} \geq -\frac{1}{2})
\]

\[
+ 2.08 \times P(I_j < 0, 1 \leq j \leq 3 \text{ and } I_4 \geq 0 \text{ and } d_{\text{min},3} \geq -\frac{1}{2})
\]

\[
+ 2.61 \times P(I_j < 0, 1 \leq j \leq 4 \text{ and } I_5 \geq 0 \text{ and } d_{\text{min},4} \geq -\frac{1}{2})
\]

\[
+ 3.13 \times P(I_j < 0, 1 \leq j \leq 5 \text{ and } I_6 \geq 0 \text{ and } d_{\text{min},5} \geq -\frac{1}{2})
\]

\[
+ 0 \times P(I_j < 0, 1 \leq j \leq 6 \text{ and } d_{\text{min},6} \geq -\frac{1}{2})
\]

\[
+ 10 \times E(I_6|d_{\text{min},6} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6) 
\]

\[
\times P(d_{\text{min},6} < -\frac{1}{2}|I_j < 0, 1 \leq j \leq 6)
\]

\[
\times P(I_j < 0, 1 \leq j \leq 6).
\]

Consider the probabilities appearing in the second thru sixth terms in (4.3), It is clear that for each $r = 1, \ldots, 6$,

\[
P(I_j < 0, j = 1, \ldots, r - 1, I_r \geq 0, d_{\text{min},r-1} \geq \frac{1}{2})
\]

\[
\leq P(I_j < 0, j = 1, \ldots, r - 1, I_r \geq 0). \tag{4.4}
\]
Also, as noted earlier, \( d_{\text{min},6} \equiv d_{\text{min}} \), the minimum cumulative index return encountered in Chapter 3. Therefore, it follows from (4.3) and (4.4) that

\[
E(\text{Net Payment}) \leq 0.52 \times P(I_1 \geq 0)
+ 1.04 \times P(I_1 < 0, \text{ and } I_2 \geq 0)
+ 1.56 \times P(I_j < 0, j = 1, 2 \text{ and } I_3 \geq 0)
+ 2.08 \times P(I_j < 0, j = 1, 2, 3 \text{ and } I_4 \geq 0)
+ 2.61 \times P(I_j < 0, 1 \leq j \leq 4 \text{ and } I_5 \geq 0)
+ 3.13 \times P(I_j < 0, 1 \leq j \leq 5 \text{ and } I_6 \geq 0)
+ 0 \times P(I_j < 0, 1 \leq j \leq 6 \text{ and } d_{\text{min}} \geq -\frac{1}{2})
+ 10 \times E(I_6|d_{\text{min}} < -\frac{1}{2} \text{ and } I_j < 0, 1 \leq j \leq 6)
\times P(d_{\text{min}} < -\frac{1}{2}|I_j < 0, 1 \leq j \leq 6)
\times P(I_j < 0, 1 \leq j \leq 6).
\]

The expression on the right hand side is precisely the expected net payment function which we derived in Chapter 3. Therefore, the average return to clients from the interpretation of the payment procedure considered here in Chapter 4 can be no greater than the average return from the interpretation in Chapter 3. Likewise, average losses for this interpretation will be greater than losses in the interpretation of the payment procedure in Chapter 3. Consequently, it is clear that clients are likely to suffer even greater losses under the new interpretation of the payment procedure considered in this latter chapter.
Chapter 5

Conclusions

In our analysis, we did not consider the effects of income taxes or commissions related to the sales of these reverse convertible notes. The inclusion of these expenses would have resulted in smaller returns and greater losses for note-holders.

Further, any financial adviser who was required to exercise fiduciary duty to their clients, could not have done so by recommending that clients purchase these notes. The SEC has stated that a financial adviser has a fiduciary duty to make reasonable investment recommendations to clients. Graham [3] defines an investment operation as “one which upon thorough analysis promises safety of principal and an adequate return.” Had financial advisers carried out a thorough analysis of these reverse convertible notes then they would have realized that the notes promised neither safety of principal nor adequate return to clients. As a result, any financial adviser acting as a fiduciary, should have opposed the purchase of these notes.

We have seen recently an increase in the sales of similar notes by many financial firms; the prospectuses for these notes are available on the SEC’s website. This increase in sales perhaps is due to the fact that the financial markets have become increasingly bullish since 2009 and as a result these notes now appear more attractive to persons looking to expand their financial portfolio by purchasing fixed-income securities with higher
yields. However, we predict that if markets undergo substantial downward movements then clients are again likely to lose substantial capital from purchases of these notes.

Further, these notes were issued during a time when future market conditions were uncertain and direct sales of stock may accelerated further downturns. Thus, these notes were a clever marketing scheme used by financial institutions who wanted to insure themselves against future market declines.
Appendix A

An Actual Outcome for a Reverse Convertible Note

We now describe the actual outcome for the reverse convertible note analysed in this thesis. As the table below indicates, the final return to note-holders was −48.7%.

The Index Starting Level, Trigger Level, and Index Ending Level for each observation date are as follows:

Index Starting Level = 369.44

Trigger Level = 184.72

<table>
<thead>
<tr>
<th>Observation Date</th>
<th>Index Ending Level</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 5, 2008</td>
<td>365.48</td>
<td>Below Index Starting Level and Above Trigger Level; Securities NOT called</td>
</tr>
<tr>
<td>August 5, 2008</td>
<td>302.05</td>
<td>Below Index Starting Level and Above Trigger Level; Securities NOT called</td>
</tr>
<tr>
<td>November 5, 2008</td>
<td>201.77</td>
<td>Below Index Starting Level Above Trigger Level; Securities NOT called</td>
</tr>
<tr>
<td>Feb 5, 2008</td>
<td>121.51</td>
<td>Below Index Starting Level and Below Trigger Level; Securities NOT called</td>
</tr>
<tr>
<td>May 5, 2009</td>
<td>155.52</td>
<td>Below Index Starting Level and Below Trigger Level; Securities NOT called</td>
</tr>
<tr>
<td>August 5, 2009</td>
<td>189.37</td>
<td>Below Index Starting Level and Above Trigger Level; Securities NOT called</td>
</tr>
</tbody>
</table>

Settlement Amount (per $10) $5.13 (total return of -48.7%, or -35.9% per annum)
Note that the Index Return on the Final Valuation date equals

\[
\frac{\text{Index Starting Return} - \text{Index Ending Level}}{\text{Index Starting Level}} = \frac{189.37 - 369.44}{369.44} = -0.487,
\]

which represents a loss to the note-holder of 48.7\%. Thus, the settlement amount is

\[
$10 \times (1 + \text{Index Return on the Final Valuation date}) = $10 \times (1 - 0.487) = $5.13.
\]
Appendix B

The Pricing Supplement to the Prospectus

The following pages provide the pricing supplement to the prospectus for the reverse convertible notes analyzed in Chapter 1 – 5. This copy of the pricing supplement was downloaded on Saturday, July 06, 2013 from the SEC’s website.


### Calculation of the Registration Fee

<table>
<thead>
<tr>
<th>Title of Each Class of Securities Offered</th>
<th>Maximum Aggregate Offering Price</th>
<th>Amount of Registration Fee(1)(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notes</td>
<td>$48,310,620.00</td>
<td>$1,898.61</td>
</tr>
</tbody>
</table>

1. Calculated in accordance with Rule 457(r) of the Securities Act of 1933.
2. Pursuant to Rule 457(p) under the Securities Act of 1933, filing fees of $1,265,034.20 have already been paid with respect to unsold securities that were previously registered pursuant to a Registration Statement on Form S-3 (No. 333-134553) filed by Lehman Brothers Holdings Inc. and the other Registrants thereto on May 30, 2006, and have been carried forward, of which $1,898.61 is offset against the registration fee due for this offering and of which $1,263,135.60 remains available for future registration fees. No additional registration fee has been paid with respect to this offering.
Autocallable Optimization Securities with Contingent Protection Linked to the S&P 500® Financials Index

Tactical Strategies for Flat or Bullish Markets
Lehman Brothers Holdings Inc. $48,310,620 Securities linked to the S&P 500® Financials Index due August 10, 2009

Investment Description
These Autocallable Optimization Securities with Contingent Protection Linked to the S&P 500® Financials Index (the "Notes") are designed for investors who want to express a bullish view of the U.S. financial services sector through an investment linked to the S&P 500® Financials Index (the "Index"). If the closing level of the Index on any quarterly Observation Date is at or above the Index Starting Level, the Notes will be called for an annualized return of 20.84%. If the Notes are not called, at maturity you will receive your principal, unless the Index closes below the Trigger Level on any trading day during the Observation Period, in which case you will receive a payment equal to the principal amount of your Notes reduced by a percentage equal to the absolute value of the Index Return. Investors must be willing to risk losing up to 100% of their investment.

Features
- **Positive Call Return in Flat or Bullish Scenarios**—If the closing level of the Index on any Observation Date is at or above the Index Starting Level, the Notes will be called and you will receive a positive return on your investment.
- **Contingent Principal Protection**—If the Notes are not called, at maturity the contingent principal protection feature protects your principal if the Index closing level is not below the Trigger Level on any trading day during the Observation Period.
- **Express a Bullish View of the U.S. Financial Services Sector**—The Notes are linked to the Index, which as of January 31, 2008 consisted of 93 companies involved in the U.S. financial services sector and is designed to represent the sector’s diverse sub-sectors, such as banking, mortgage finance, consumer finance, specialized finance, investment banking and brokerage, asset management and custody, corporate lending, insurance and financial investment and real estate, including REITs.

Security Offerings
We are offering Autocallable Optimization Securities with Contingent Protection Linked to the S&P 500® Financials Index. The Notes are offered at a minimum investment of $1,000 in denominations of $10 and integral multiples of $10 in excess thereof.

See "Additional Information about Lehman Brothers Holdings Inc. and the Notes" on page 2. The Notes offered will have the terms specified in the base prospectus dated May 30, 2006, the MTN prospectus supplement dated May 30, 2006, product supplement no. 720-I dated January 28, 2008, underlying supplement no. 910 dated January 28, 2008 and this pricing supplement. See "Key Risks" on page 6, the
more detailed “Risk Factors” beginning on page SS-1 of product supplement no. 720-I for risks related to an investment in the Notes and “Risk Factors” beginning on page US-1 of underlying supplement no. 910 for risks related to the Index.

Neither the Securities and Exchange Commission nor any state securities commission has approved or disapproved of the Notes or passed upon the accuracy or the adequacy of this pricing supplement, the accompanying base prospectus, MTN prospectus supplement, product supplement no. 720-I, underlying supplement no. 910 or any other related prospectus supplements, or any other relevant terms supplement. Any representation to the contrary is a criminal offense. The Notes are not deposit liabilities of Lehman Brothers Holdings Inc. and are not FDIC-insured.

<table>
<thead>
<tr>
<th>Per Note</th>
<th>Price to Public</th>
<th>Underwriting Discount</th>
<th>Proceeds to Us</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10.00</td>
<td>$0.15</td>
<td>$9.85</td>
</tr>
<tr>
<td>Total</td>
<td>$48,310,620.00</td>
<td>$724,659.30.00</td>
<td>$47,585,960.70</td>
</tr>
</tbody>
</table>

UBS Financial Services Inc.  Lehman Brothers Inc.
Additional Information about Lehman Brothers Holdings Inc. and the Notes

Lehman Brothers Holdings Inc. has filed a registration statement (including a base prospectus) with the U.S. Securities and Exchange Commission, or SEC, for this offering. Before you invest, you should read this pricing supplement together with the base prospectus, as supplemented by the MTN prospectus supplement relating to our Series I medium-term notes, of which the Notes are a part, and the more detailed information contained in product supplement no. 910 (which describes the Index, including risk factors specific to it). Buyers should rely upon the base prospectus, the MTN prospectus supplement, product supplement no. 720-I, underlying supplement no. 910, this pricing supplement, any other relevant terms supplement and any other relevant free writing prospectus for complete details. This pricing supplement, together with the documents listed below, contains the terms of the Notes and supersedes all prior or contemporaneous communications concerning the Notes. To the extent that there are any inconsistencies among the documents listed below, this pricing supplement shall supersede product supplement no. 720-I, which shall, likewise, supersede the base prospectus and the MTN prospectus supplement. You should carefully consider, among other things, the matters set forth in “Risk Factors” in the accompanying product supplement no. 720-I and “Risk Factors” in the accompanying underlying supplement no. 910, as the Notes involve risks not associated with conventional debt securities. We urge you to consult your investment, legal, tax, accounting and other advisors before you invest in the Notes. You may get these documents and other documents Lehman Brothers Holdings Inc. has filed for free by searching the SEC online database (EDGAR®) at www.sec.gov with “Lehman Brothers Holdings Inc.” as a search term or through the links below, or by calling UBS Financial Services Inc. toll-free at 1-877-827-2010 or Lehman Brothers Inc. toll-free at 1-888-603-5847.

You may access these documents on the SEC website at www.sec.gov as follows (or if such address has changed, by reviewing our filings for the relevant date on the SEC website):

- Product supplement no. 720-I dated January 28, 2008:
  [http://www.sec.gov/Archives/edgar/data/806085/000119312508013525/d424b2.htm](http://www.sec.gov/Archives/edgar/data/806085/000119312508013525/d424b2.htm)
- Underlying supplement no. 910 dated January 28, 2008:
  [http://www.sec.gov/Archives/edgar/data/806085/000119312508013504/d424b2.htm](http://www.sec.gov/Archives/edgar/data/806085/000119312508013504/d424b2.htm)
- MTN prospectus supplement dated May 30, 2006:
  [http://www.sec.gov/Archives/edgar/data/806085/000104746906007785/a2170815z424b2.htm](http://www.sec.gov/Archives/edgar/data/806085/000104746906007785/a2170815z424b2.htm)
- Base prospectus dated May 30, 2006:
  [http://www.sec.gov/Archives/edgar/data/806085/000104746906007771/a2165526zs-3asr.htm](http://www.sec.gov/Archives/edgar/data/806085/000104746906007771/a2165526zs-3asr.htm)

References to “Lehman Brothers,” “we,” “our” and “us” refer only to Lehman Brothers Holdings Inc. and not to its consolidated subsidiaries. In this document, unless the context otherwise requires, “Notes” refers to the Autocallable Optimization Securities with Contingent Protection Linked to the S&P 500® Financials Index that are offered hereby.

Investor Suitability

The Notes may be suitable for you if, among other considerations:

- You believe the Index will not close below the Trigger Level on any trading day during the Observation Period
- You believe the Index will close at or above the Index Starting Level on an Observation Date, including the Final Valuation Date
- You are willing to hold Notes that will be called on any Observation Date on which the Index closes at or above the Index Starting Level, or you are otherwise willing to hold the Notes to maturity
- You believe the Index will remain stable for the term of the Notes and will close at or above the Index Starting Level on the Final Valuation Date
- You are willing to make an investment whose return is limited to the pre-specified Return on Call Date, a

The Notes may not be suitable for you if, among other considerations:

- You believe the Index will close below the Trigger Level on at least one trading day during the Observation Period and that at maturity the Index Return will be negative
- You believe stock prices of companies involved in the U.S. financial services sector will decrease during the Observation Period
- You seek an investment that is 100% principal protected
- You are not willing to make an investment in which you could lose up to 100% of your principal amount
- You seek an investment whose return is not limited to the pre-specified Return on Call Date, a total return based upon an annualized return of 20.84%
total return based upon an annualized return of 20.84%

* You do not seek current income from this investment and are not seeking an investment for which there will be an active secondary market
* You seek an investment for which there will be an active secondary market
* You are unable or unwilling to hold Notes that will be called on any Observation Date on which the Index closes at or above the Index Starting Level, or you are otherwise unable or unwilling to hold the Notes to maturity
* You prefer the lower risk, and therefore accept the potentially lower returns, of fixed income investments with comparable maturities and credit ratings
* You seek current income from your investment

The suitability considerations identified above are not exhaustive. Whether or not the Notes are a suitable investment for you will depend on your individual circumstances, and you should reach an investment decision only after you and your investment, legal, tax, accounting and other advisors have carefully considered the suitability of an investment in the Notes in light of your particular circumstances. You should also review carefully “Key Risks” on page 6, “Risk Factors” in product supplement no. 720-I, underlying supplement no. 910 and the MTN prospectus supplement for risks related to an investment in the Notes.
<table>
<thead>
<tr>
<th>Observation Date</th>
<th>Return on Call Date</th>
<th>Call Price (per $10.00)</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 5, 2008</td>
<td>5.21%</td>
<td>$10.52</td>
</tr>
<tr>
<td>August 5, 2008</td>
<td>10.42%</td>
<td>$11.04</td>
</tr>
<tr>
<td>November 5, 2008</td>
<td>15.63%</td>
<td>$11.56</td>
</tr>
<tr>
<td>February 5, 2009</td>
<td>20.84%</td>
<td>$12.08</td>
</tr>
<tr>
<td>May 5, 2009</td>
<td>26.05%</td>
<td>$12.61</td>
</tr>
<tr>
<td>Final Valuation Date (August 5, 2009)</td>
<td>31.26%</td>
<td>$13.13</td>
</tr>
</tbody>
</table>

If the Notes are not called and the Index closing level is not below the Trigger Level on any trading day during the Observation Period, you will receive a cash payment on the Maturity Date equal to $10 per $10 principal amount Note.

If the Notes are not called and the Index closing level is below the Trigger Level on any trading day during the Observation Period, you will receive a cash payment on the Maturity Date equal to:

$$10 \times (1 + \text{Index Return})$$

In this case, you may lose all or a substantial portion of your principal depending on how much the Index decreases.
Lehman Brothers Holdings Inc. is rated A+ by Standard & Poor’s, A1 by Moody’s and AA- by Fitch. A credit rating reflects the creditworthiness of Lehman Brothers Holdings Inc. and is not a recommendation to buy, sell or hold securities, and it may be subject to revision or withdrawal at any time by the assigning rating organization. Each rating should be evaluated independently of any other rating. The creditworthiness of the issuer does not affect or enhance the likely performance of the investment other than the ability of the issuer to meet its obligations.

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index Return</td>
<td>Index Ending Level - Index Starting Level</td>
</tr>
<tr>
<td>Trigger Level</td>
<td>184.72, which is 50% of the Index Starting Level</td>
</tr>
<tr>
<td>Observation Period</td>
<td>The period commencing on (and including) the Trade Date and ending on (and including) the Final Valuation Date.</td>
</tr>
<tr>
<td>Index Starting Level</td>
<td>369.44, which was the closing level of the Index on the Trade Date</td>
</tr>
<tr>
<td>Index Ending Level</td>
<td>The closing level of the Index on the Final Valuation Date</td>
</tr>
<tr>
<td>CUSIP</td>
<td>52522L657</td>
</tr>
<tr>
<td>ISIN</td>
<td>US52522L6570</td>
</tr>
</tbody>
</table>

Your principal, depending on how much the index declines.
What are the tax consequences of the Notes?

There is no statutory, judicial or administrative authority that directly addresses the proper U.S. federal income tax characterization and treatment of securities similar to the Notes. No ruling is being sought from the Internal Revenue Service as to the proper U.S. federal income tax characterization and treatment of the Notes. You should also be aware that our special tax counsel, Sidley Austin LLP, has not provided us with an opinion regarding the proper characterization of the Notes for U.S. federal income tax purposes. Therefore, the proper U.S. federal income tax characterization and treatment of the Notes is uncertain. Notwithstanding the foregoing, Lehman Brothers Holdings Inc. intends to treat, and by purchasing a Note, for all tax purposes, you agree to treat, a Note as a cash-settled financial contract, rather than as a debt instrument.

Recent Tax Law Developments. On December 7, 2007, the Internal Revenue Service released a Notice indicating that the Internal Revenue Service and the Treasury Department are considering and seeking comments as to whether holders of instruments similar to the Notes should be required to accrue income on a current basis over the term of the Notes, regardless of whether any payments are made prior to maturity. In addition, the Notice provides that the Internal Revenue Service and the Treasury Department are considering related issues, including, among other things, whether gain or loss from such instruments should be treated as ordinary or capital, whether foreign holders of such instruments should be subject to withholding tax, whether the tax treatment of such instruments should vary depending upon the nature of the underlying asset, and whether such instruments should be subject to the special “constructive ownership rules” contained in Section 1260 of the Internal Revenue Code of 1986, as amended. It is not possible to predict what changes, if any, will be adopted, or when they will take effect. Any such changes could affect the amount, timing and character of income, gain or loss in respect of the Notes, possibly with retroactive effect. Holders are urged to consult their tax advisors concerning the impact of the Notice on their investment in the Notes. Subject to future developments with respect to the foregoing, Lehman Brothers Holdings Inc. intends to continue to treat the Notes for U.S. federal income tax purposes in accordance with the treatment described in the accompanying product supplement no. 720-I under the headings “Risk Factors” and “Certain U.S. Federal Income Tax Consequences.”


Scenario Analysis and Examples at Maturity

The following scenario analysis and examples reflect an Index Starting Level of 369.44, a Return on Call Date of 20.84% per annum and a Trigger Level of 184.72 (50% of the Index Starting Level). The actual Index Starting Level, Return on Call Date and Trigger Level will be set on the Trade Date.

**EXAMPLE 1: Notes are Called One Year after Trade Date**

<table>
<thead>
<tr>
<th>Observation Date</th>
<th>Index Level</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 5, 2008</td>
<td>304.40</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>August 5, 2008</td>
<td>332.50</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>November 5, 2008</td>
<td>384.20</td>
<td>Above Index Starting Level and Above Trigger Level, Securities are Called</td>
</tr>
</tbody>
</table>

Call Price (per $10.00) $11.56 (return of 20.84% per annum)

**EXAMPLE 2: Notes are Called on the Final Valuation Date**

<table>
<thead>
<tr>
<th>Observation Date</th>
<th>Index Level</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 5, 2008</td>
<td>348.50</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>August 5, 2008</td>
<td>316.40</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>November 5, 2008</td>
<td>347.30</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>February 5, 2009</td>
<td>284.50</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>May 5, 2009</td>
<td>332.50</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
</tbody>
</table>

Final Valuation Date (on or about August 5, 2009) 392.50 Above Index Starting Level and Above Trigger Level, Securities are Called
Call Price (per $10.00) $13.13 (return of 31.26%, or 20.84% per annum)

**EXAMPLE 3:** Notes are NOT Called and the Index never closes below the Trigger Level on any trading day during the Observation Period

<table>
<thead>
<tr>
<th>Observation Date</th>
<th>Settlement Amount</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 5, 2008</td>
<td>348.50</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>August 5, 2008</td>
<td>304.40</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>November 5, 2008</td>
<td>316.40</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>February 5, 2009</td>
<td>347.30</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>May 5, 2009</td>
<td>284.50</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>Final Valuation Date (on or about August 5, 2009)</td>
<td>332.50</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
</tbody>
</table>

Settlement Amount (per $10.00) $10.00 (return of 0.00% per annum)
### EXAMPLE 4: Notes are NOT Called and the Index closes below the Trigger Level on one or more trading days during the Observation Period

<table>
<thead>
<tr>
<th>Observation Date</th>
<th>Index Level</th>
<th>Securities Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 5, 2008</td>
<td>350.90</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>August 5, 2008</td>
<td>166.20</td>
<td>Below Index Starting Level and Below Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>November 5, 2008</td>
<td>351.00</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>February 5, 2009</td>
<td>332.50</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>May 5, 2009</td>
<td>325.10</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
<tr>
<td>Final Valuation Date (on or about August 5, 2009)</td>
<td>258.60</td>
<td>Below Index Starting Level and Above Trigger Level, Securities NOT Called</td>
</tr>
</tbody>
</table>

**Settlement Amount (per $10.00)**

\[
\text{Settlement Amount} = \$10.00 \times (1 + \text{Index Return on the Final Valuation Date})
\]

\[
\text{Settlement Amount} = \$10.00 \times (1 - 30.00\%)
\]

\[
\text{Settlement Amount} = \$7.00 \text{ (return of } -30.00\%, \text{ or } -20.00\% \text{ per annum)}
\]
Key Risks

An investment in the Notes involves significant risks. Investing in the Notes is not equivalent to investing in any of the stocks included in the Index. These risks are explained in more detail in the "Risk Factors" section of the accompanying product supplement no. 720-I and in the "Risk Factors" section of the accompanying underlying supplement no. 910. You should reach an investment decision only after you have carefully considered with your advisors the suitability of an investment in the Notes in light of your particular circumstances.

- **Potentially Full Market Risk; You May Lose Some or All of Your Principal:** If the Index closes below the Trigger Level on any trading day during the Observation Period, you are fully exposed to any decrease in the level of the Index (as measured by the Index Return). In these circumstances, you will lose 1% of your principal at maturity for every 1% decrease in the Index up to 100% of your investment.

- **Limited Return on the Notes:** Your potential gain on a $10 principal amount Note will be limited to the Call Price applicable for an Observation Date (less the $10 initial investment), regardless of the appreciation in the Index, which may be significant. Similarly, because the determination of whether the Notes will be called will be based on the Index closing level on a limited number of Observation Dates prior to the Maturity Date, and because, if the Notes are not called, the Index Ending Level will be based on the Index closing level on the last Observation Date (i.e., the Final Valuation Date), your return may be adversely affected by a sudden or temporary decrease in the Index closing level on any or all of the Observation Dates. Conversely, you will not benefit from higher Index closing levels at any time during the term of the Notes other than on the Observation Dates. As a result, you may receive a lower return on the Notes than you would receive if you were to take a position in the stocks underlying the Index or in contracts relating to the Index.

- **No Assurances of a Positive-Return Environment:** While the Notes are structured to provide potentially enhanced returns in a positive-return environment, we cannot assure you of the economic environment during the term or at maturity of your Notes.

- **Dealer Incentives:** We, our affiliates and agents act in various capacities with respect to the Notes. Lehman Brothers Inc. and other of our affiliates may act as a principal, agent or dealer in connection with the Notes. Such affiliates, including the sales representatives, will derive compensation from the distribution of the Notes and such compensation may serve as an incentive to sell the Notes instead of other investments. We will pay such compensation of $0.15 per Note to the principals, agents and dealers in connection with the distribution of the Notes.

- **Lack of Liquidity:** The Notes will not be listed on any securities exchange. Lehman Brothers Inc. intends to offer to purchase the Notes in the secondary market but is not required to do so. Even if there is a secondary market, it may not provide enough liquidity to allow you to trade or sell the Notes easily. Because other dealers are not likely to make a secondary market for the Notes, the price at which you may be able to trade your Notes is likely to depend on the price, if any, at which Lehman Brothers Inc. is willing to buy the Notes. If you are an employee of Lehman Brothers Holdings Inc. or one of our affiliates, you may not be able to purchase the Notes from us and your ability to sell or trade the Notes in the secondary market may be limited.

- **Potential Conflicts:** We and our affiliates play a variety of roles in connection with the issuance of the Notes, including acting as calculation agent and hedging our obligations under the Notes. In performing these duties, the economic interests of the calculation agent and other affiliates of ours are potentially adverse to your interests as an investor in the Notes. Additionally, UBS Financial Services Inc. and its affiliates may engage in certain activities with companies whose stock is included in the Index, and such activities could potentially affect the level of the Index, and accordingly, could affect the value of the Notes.

- **Reinvestment Risk:** If your Notes are called early, the holding period over which you would receive the per annum return of 20.84% could be as little as three months. There is no guarantee that you would be able to reinvest the proceeds from an investment in the Notes at a comparable return for a similar level of risk following an early call.

- **No Interest or Dividend Payments or Voting Rights:** As a holder of the Notes, you will not receive interest payments, and you will not have voting rights or rights to receive cash dividends or other distributions or other rights that holders of stocks included in the Index would have.

- **Certain Built-in Costs are Likely to Adversely Affect the Value of the Notes Prior to Maturity:** While the payment at maturity described in this pricing supplement is based on the full principal amount of your Notes, the original issue price of the Notes includes the agent’s commission and the cost of hedging our obligations.
under the Notes through one or more of our affiliates, which includes our affiliates’ expected cost of providing such hedge as well as the profit our affiliates expect to realize in consideration for assuming the risks inherent in providing such hedge. As a result, the price, if any, at which Lehman Brothers Inc. will be willing to purchase Notes from you in secondary market transactions, if at all, will likely be lower than the original issue price and any sale prior to the Maturity Date could result in a substantial loss to you. The Notes are not designed to be short-term trading instruments. YOU SHOULD BE WILLING TO HOLD YOUR NOTES TO MATURITY.

- **We and our Affiliates and Agents May Publish Research, Express Opinions or Provide Recommendations that are Inconsistent with Investing in or Holding the Notes. Any Such Research, Opinions or Recommendations Could Affect the Level of the Index and, Consequently, the Value of the Notes:** We, our affiliates and agents publish research from time to time on matters that may influence the value of the Notes, or express opinions or provide recommendations that may be inconsistent with purchasing or holding the Notes. We, our affiliates and agents may publish or may have published research or other opinions that are inconsistent with an investment position in the S&P 500® Financials Index. Any research, opinions or recommendations expressed by us, our affiliates or agents may not be consistent with each other and may be modified from time to time without notice. Additionally, UBS Financial Services Inc. and its affiliates may publish or may have published research or other opinions that are inconsistent with an investment position in the S&P 500® Financials Index. Investors should make their own independent investigation of the merits of investing in the Notes.

- **We Cannot Control Actions by the Companies Whose Stocks or Other Equity Securities are Represented in the S&P 500® Financials Index:** We are one of the companies that are represented in the S&P 500® Financials Index, but we are not affiliated with any of the other companies whose stock is represented in the S&P 500® Financials Index. As a result, we will have no ability to control
the actions of such companies, including actions that could affect the value of the stocks underlying the Index or your Notes. None of the money you pay us will go to any of the companies represented in the S&P 500® Financials Index, and none of those companies will be involved in the offering of the Notes in any way. Neither those companies nor we will have any obligation to consider your interests as a holder of the Notes in taking any corporate actions that might affect the value of your Notes.

- **Many Economic and Market Factors Will Impact the Value of the Notes:** In addition to the level of the Index on any day, the value of the Notes will be affected by a number of economic and market factors that may either offset or magnify each other and which are set out in more detail in product supplement no. 720-I.

- **Uncertain Tax Treatment:** Significant aspects of the tax treatment of the Notes are uncertain. You should consult your own tax advisor about your own tax situation before investing in the Notes.

- **Creditworthiness of Issuer:** An investment in the Notes will be subject to the credit risk of Lehman Brothers Holdings Inc., and the actual and perceived creditworthiness of Lehman Brothers Holdings Inc. may affect the market value of the Notes.

- **Your Investment is Concentrated in the U.S. Financial Services Sector:** All of the securities included in the Index are issued by companies whose primary lines of business are directly associated with the U.S. financial services sector.

### The S&P 500® Financials Index

The S&P 500® Financials Index, a subset of the S&P 500® Index, is published by Standard & Poor’s ("S&P"), a division of The McGraw-Hill Companies, Inc. The Index is a float-adjusted, capitalization-weighted index, which as of January 31, 2008 was composed of 93 companies, designed to effectively represent the performance of the U.S. financial services sector. The companies that make up the Index account for a large percentage of the market capitalization of the U.S. financial services sector and represent the sector’s diverse sub-sectors, such as banking, mortgage finance, consumer finance, specialized finance, investment banking and brokerage, asset management and custody, corporate lending, insurance and financial investment and real estate, including REITs. A current list of the stocks that make up the S&P 500® Financials Index is available on the Standard & Poor’s website at www.standardandpoors.com.

You can obtain the level of the S&P 500® Financials Index at any time from the Bloomberg Financial Markets page "SSFINL <Index> <GO>" or from the S&P website at www.standardandpoors.com.

The information on the S&P 500® Financials Index provided in this document should be read together with the underlying supplement no. 910. Information contained in the Standard & Poor’s website referenced above is not incorporated by reference in, and should not be considered a part of, this free writing prospectus.

### Supplemental Plan of Distribution

We have agreed to sell to UBS Financial Services Inc. and Lehman Brothers Inc. (together, the "Agents"), and the Agents will agree to purchase, all of the Notes at the price indicated on the cover of this pricing supplement. UBS Financial Services Inc. may allow a concession not in excess of the underwriting discount to its affiliates.

We have agreed to indemnify the Agents against liabilities, including liabilities under the Securities Act of 1933, as amended, or to contribute to payments that the Agents may be required to make relating to these liabilities as described in the MTN prospectus supplement and the base prospectus. We have agreed that UBS Financial Services Inc. may sell all or a part of the Notes that it purchases from us to its affiliates at the price indicated on the cover of this pricing supplement.

Subject to regulatory constraints, Lehman Brothers Inc. has agreed to use reasonable efforts to make a market in the Notes for so long as the Notes are outstanding.

We have, or our affiliate has, entered into swap agreements or related hedge transactions with one or more of our other affiliates or unaffiliated counterparties in connection with the sale of the Notes, and the Agents and/or an affiliate may earn additional income as a result of payments pursuant to the swap or related hedge transactions.
Bibliography


Curriculum Vitae

Gilna K. Samuel

Education

The Pennsylvania State University State College, Pennsylvania 2011–Present
Master in Statistics, expected in August 2013
Area of Specialization: Analysis of structured financial products

Morgan State University Baltimore, MD 2007-2011
Bachelor of Science in Mathematics, summa cum laude

Research Experience

Graduate Research The Pennsylvania State University 2012–Present
Thesis Advisor: Prof. Donald St. P. Richards

I researched structured financial products known as reverse convertible notes which resulted in substantial losses to buyers of these notes. I analyzed different scenarios and determined expected net payment for an average buyer.

Teaching Experience

Teaching Assistant The Pennsylvania State University 2011–Present

I taught computer labs, held office hours, and graded for STAT 100, Statistical Concepts and Reasoning; STAT 319, Applied Statistics in Science; STAT 414, Introduction to Probability; and STAT 415, Introduction to Mathematical Statistics.

Work Experience

Summer Research Intern University of California, Los Angeles (UCLA) 2010

I worked on a collaborative research project between UCLA and Standard and Poor’s (S&P). I employed regression, time series analysis, stochastic optimization models to forecast electricity demand and determine the effect of imposing a cost on carbon dioxide emissions.

Summer Research Intern Massachusetts Institute of Technology (MIT) 2009

I conducted research involving data mining for clinical trials of cancer in order to determine optimal cancer regimens. I used regression models to determine how different drugs affect survivability and toxicity rates of various types of cancer.