ANISOTROPIC METAMATERIALS FOR MICROWAVE ANTENNAS
AND INFRARED NANOSTRUCTURED THIN FILMS

A Dissertation in

Electrical Engineering

by

Zhihao Jiang

© 2013 Zhihao Jiang

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

August 2013
The dissertation of Zhihao Jiang was reviewed and approved* by the following:

Douglas H. Werner
John L. and Genevieve H. McCain Chair Professor of Electrical Engineering
Dissertation Advisor
Chair of Committee

Theresa S. Mayer
Distinguished Professor of Electrical Engineering

Pingjuan L. Werner
Professor of Electrical Engineering

Brian Weiner
Professor of Physics

Kultegin Aydin
Professor of Electrical Engineering
Head of the Department of Electrical Engineering

*Signatures are on file in the Graduate School
ABSTRACT

Wave-matter interactions have long been investigated to discover unknown physical phenomena and exploited to achieve improved device performance throughout the electromagnetic spectrum ranging from quasi-static limit to microwave frequencies, and even at infrared and optical wavelengths. As a nascent but fast growing field, metamaterial technology, which relies on clusters of artificially engineered subwavelength structures, has been demonstrated to provide a wide variety of exotic electromagnetic properties unattainable in natural materials. This dissertation presents the research on novel anisotropic metamaterials for tailoring microwave radiation and infrared scattering of nanostructured thin films. First, a new inversion algorithm is proposed for retrieving the anisotropic effective medium parameters of a slab of metamaterial. Secondly, low-loss anisotropic metamaterial lenses and coatings are introduced for improving the gain and/or bandwidth for a variety of antennas. In particular, a quad-beam high-gain lens for a quarter-wave monopole, a low-profile grounded leaky metamaterial coating for slot antenna, and an ultra-thin anisotropic metamaterial bandwidth-enhancing coating for a quarter-wave monopole are experimentally demonstrated. In the infrared regime, novel nanostructured metamaterial free-standing thin-films, which are inherently anisotropic, are introduced for achieving exotic index properties and further for practical photonic devices. In particular, a low-loss near-infrared fishnet zero-index metamaterial, a dispersion-engineered optically-thin, low-loss broadband metamaterial filter with a suppressed group delay fluctuation in the mid-infrared, and a conformal dual-band near-perfectly absorbing coating in the mid-infrared are experimentally demonstrated. These explorations show the great promise anisotropic metamaterials hold for the flexible manipulation of electromagnetic waves and their broad applicability in a wide spectrum range.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>List of Figures</td>
<td>vii</td>
</tr>
<tr>
<td></td>
<td>Acknowledgements</td>
<td>xvi</td>
</tr>
<tr>
<td>1</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>Background</td>
<td>1</td>
</tr>
<tr>
<td>1.1.1</td>
<td>Anisotropic Metamaterials</td>
<td>4</td>
</tr>
<tr>
<td>1.1.2</td>
<td>Metamaterial-Enabled High-Gain and Broadband Microwave Antennas</td>
<td>6</td>
</tr>
<tr>
<td>1.1.3</td>
<td>Optical Metamaterial Nanostructures</td>
<td>7</td>
</tr>
<tr>
<td>1.2</td>
<td>Overview</td>
<td>9</td>
</tr>
<tr>
<td>1.3</td>
<td>Original Contributions</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>Effective Medium Parameter Retrieval for Anisotropic Metamaterials</td>
<td>14</td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>14</td>
</tr>
<tr>
<td>2.2</td>
<td>Anisotropic Retrieval Method</td>
<td>16</td>
</tr>
<tr>
<td>2.1.1</td>
<td>Scattering from a Homogeneous Anisotropic Slab</td>
<td>16</td>
</tr>
<tr>
<td>2.1.2</td>
<td>Retrieval Equations</td>
<td>18</td>
</tr>
<tr>
<td>2.3</td>
<td>Application to A Specific Metamaterial – A SRR-wire Composite Array</td>
<td>22</td>
</tr>
<tr>
<td>2.4</td>
<td>Overview</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>Multi-Beam Transformation Optics Lenses Using Anisotropic Metamaterials</td>
<td>29</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>29</td>
</tr>
<tr>
<td>3.2</td>
<td>Two-/Three-Dimensional Linear Transformation for Highly Directive Emission</td>
<td>33</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Two/Three Dimensional Linear Coordinate Transformations</td>
<td>33</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Numerical Validations</td>
<td>37</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Wave Propagation in the Transformed Medium</td>
<td>40</td>
</tr>
<tr>
<td>3.3</td>
<td>Lens Design Using Anisotropic Metamaterial and Associated Simulations</td>
<td>42</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Metamaterial Unit Cell Design</td>
<td>43</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Integrated Lens Simulation</td>
<td>44</td>
</tr>
<tr>
<td>3.4</td>
<td>Experimental Verification</td>
<td>49</td>
</tr>
<tr>
<td>3.5</td>
<td>Overview</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>Low-Profile High-Gain Anisotropic Metamaterial Coating for Slot Antennas</td>
<td>52</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>52</td>
</tr>
<tr>
<td>4.2</td>
<td>Leaky Modes of Grounded Anisotropic Slab</td>
<td>54</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Field Equations of a Grounded Anisotropic Slab</td>
<td>55</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Properties of the Leaky Modes Supported by the Grounded Anisotropic Slab</td>
<td>57</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Truncation Effect of the Anisotropic Low Index Slab on the Radiation Pattern</td>
<td>59</td>
</tr>
<tr>
<td>4.3</td>
<td>Coating Design Using Anisotropic Metamaterial and Associated Simulations</td>
<td>62</td>
</tr>
</tbody>
</table>
9.1 Introduction .................................................................................................................. 146
9.2 Electromagnetic Design Optimization ........................................................................ 148
9.3 Nanoresonator Array for Single- and Multi-Band Operation ................................... 150
  9.3.1 Dual-Band Designs ............................................................................................... 150
  9.3.2 Triple-Band Designs ............................................................................................. 153
9.4 Experimental Verification of Conformal Dual-Band Absorber ................................. 154
9.5 Protecting a Curved Surface with a Metamaterial Absorber Coating ...................... 157
9.5 Overview ..................................................................................................................... 158

Chapter 10 Summary and Suggestions for Future Work ................................................. 160

  10.1 Summary ................................................................................................................. 160
  10.2 Suggestions for Future Work .................................................................................... 161

Bibliography ....................................................................................................................... 163
LIST OF FIGURES

Figure 1.1. Various types of metamaterial unit cells for providing electric and magnetic resonances..........................4

Figure 2.1. Schematics of a homogeneous anisotropic slab placed in free space and illuminated by (a) TE polarized and (b) TM polarized normally and obliquely incident plane waves.................................16

Figure 2.2. Unit cells (a=2.6mm, d=1.82mm) of a composite SRR-wire array. (a) 3D isometric view of the three layer unit cell. (b) Top views of the wire and SRR structures with dimensions given by g=0.39mm, l=2.08mm, slot=0.13mm, t=0.13mm, and w=0.13mm. The dielectric slabs (FR4: εr=4.4, δtan=0.02) have thickness ds=0.13mm.............................................22

Figure 2.3. Magnitudes of the scattering parameters for the SRR-wire composite array. Curves are shown for (a)-(b) TE and (c)-(d) TM polarized waves at angles of incidence corresponding to 𝜃𝑖=0˚, 10˚, 20˚, 30˚, and 40˚............................................23

Figure 2.4. Retrieved effective parameters of the SRR-wire composite array: (a) εxx, (b) εyy, and (c) εzz using normal and oblique incidence angles; (d) εzz using two oblique angles of incidence; (e) μxx, (f) μyy, and (g) μzz using normal and oblique incidence angles; (h) μzz using two oblique angles of incidence. Note that for εxx, εyy, μxx and μyy, results retrieved from two oblique incidence angles and from only normal incidence are compared........................................26

Figure 3.1. Two dimensional directive emission coordinate transformation. (a) Geometry of the fan-shaped virtual space. (b) Geometry of the simplified triangular virtual space. (c) Geometry of the triangular physical space..................................................33

Figure 3.2. Configuration of multiple rotated lens segments to produce multi-beam radiation.................................................................35

Figure 3.3. Three-dimensional directive emission coordinate transformation. (a) Geometry of the spherical cone shaped virtual space. (b) Geometry of the simplified square pyramid shaped virtual space. (c) Geometry of the square pyramid shaped physical space........................................36

Figure 3.4. Snapshots of the z-directed near- and far-zone electric field determined via a 2D COMSOL simulation of the transformation optics lens at 3 GHz (a) with four radiated beams uniformly distributed, (b) with six radiated beams uniformly distributed (d) five radiated beams non-uniformly distributed in the x-y plane. Note that in both cases the corresponding μr′ parameter in the direction of the radiated beam has a magnitude of 0.01 for each segment of the lens. 2D COMSOL simulations of a similar lens to (a) but when the corresponding μr′ parameter in the direction of the radiated beam has a magnitude of (d) 0.1, (e) 0.2 and (f) 0.3 for each segment of the lens........................................39
Figure 3.5. Three dimensional coordinate transformation lens applied to a quasi-isotropic antenna proposed in [135]. (a) The three dimensional directive emission lens and the embedded quasi-isotropic antenna. The lens is designed to produce six highly directive beams; one normal to each face of the lens as indicated by the labels. (b) The HFSS simulated radiation pattern of the quasi-isotropic antenna without (left) and with (right) the lens.

Figure 3.6. (a) Conceptual configuration of the lens surrounding a monopole antenna. (c) Final HFSS model of the metamaterial lens with the monopole feed. The inset shows the monopole located at the middle of the lens.

Figure 3.7. Unit cell geometry. The dimensions are \( a=6mm \), \( d=0.76mm \), \( l=4.5mm \), \( g=0.5mm \), \( w=0.5mm \) and \( h=2.5mm \). Rogers RT/duroid 5880 was used as the substrate. Retrieved effective medium parameters (b) \( \varepsilon_x \), (c) \( \mu_y \) and (d) \( \mu_z \) using scattering parameters calculated at different angles of incidences. The legend indicates the two angles of incidence used for each set of curve.

Figure 3.8. Simulated (\(--\)--) and measured (\(\ldots\)) \( S_{11} \) of the monopole antenna (a) without the metamaterial lens, and (b) with the metamaterial lens. The simulated efficiency is also shown in (b).

Figure 3.9. Simulated and measured \( H \)-plane realized gain patterns of the monopole antenna with and without the metamaterial lens at (a) 4.25 GHz, (b) 4.50 GHz, (c) 4.85 GHz, (d) 5.10 GHz, and (e) 5.30 GHz. The legends represent simulation without the lens (\(--\)--), simulation with the lens (\(\ldots\)), measurement without the lens (\(--\)--), measurement with the lens (\(\ldots\))

Figure 3.10. Simulated and measured \( E \)-plane realized gain patterns of the monopole antenna with and without the metamaterial lens at (a) 4.25 GHz, (b) 4.50 GHz, (c) 4.85 GHz, (d) 5.10 GHz, and (e) 5.30 GHz. The legends represent simulation without the lens (\(--\)--), simulation with the lens (\(\ldots\)), measurement without the lens (\(--\)--), measurement with the lens (\(\ldots\))

Figure 3.11. Simulated 3D realized gain pattern of the monopole antenna with and without the metamaterial lens at 5.10 GHz.

Figure 3.12. Photograph of the fabricated lens. The inset is an enlarged photograph of the inner layers of the lens with monopole inside.

Figure 4.1. (a) Configuration of the grounded anisotropic slab with defined geometrical dimensions and material properties. (b) Equivalent transverse network for both TE and TM modes of the grounded anisotropic slab.

Figure 4.2. \( TM_z \) dispersion curves for a grounded anisotropic slab with \( \varepsilon_{ry} = 2.4, \mu_{rx} = 1 \) and varying \( \varepsilon_{rx} \) as a function of frequency. (a) \( \beta_{yTM}/k_0 \). (b) \( \alpha_{yTM}/k_0 \). (c) \( \beta_{0zTM}/k_0 \). (d) \( \alpha_{0zTM}/k_0 \). \( f_0 \) denotes the frequency at which the thickness of the grounded slab equals a wavelength, \( i.e. t = \lambda_0 \).
Figure 4.3. (a) Configuration of a 1-mm-wide slot source covered by the grounded anisotropic slab and backed by an absorber. The structure is infinite in the x direction. $t_s = 12mm$, $L_y = 120mm$ or $1200mm$, and the solving frequency is 5 GHz. (b) The normalized far-zone $E$-field magnitude in the $y$-$z$ plane for the structure with $L_y = 1200mm$ and (c) $L_y = 120mm$. (d) Direction of beam maximum for structures with $L_y = 1200mm$, $L_y = 120mm$, and an infinite slab using the equation $\theta_m = \sin^{-1}(\beta_{yTM}/k_0)$. .................................................................60

Figure 4.4. (a) Geometry of the end-loaded dipole unit cell for constructing anisotropic ZIM. The dimensions are $p = 6.5 \ mm$, $b = 5.35 \ mm$, $a = 0.7 \ mm$, $d = 0.508 \ mm$, and $g = 2.8 \ mm$. The substrate material is Rogers RT/duroid 5880 with a dielectric constant of 2.2 and a loss tangent of 0.009. (b) The retrieved effective medium parameters $\mu_{rx}$, $\varepsilon_{ry}$, and $\varepsilon_{rz}$. ..................................................63

Figure 4.5. (a) Configuration of infinite array simulations for an actual AZIM coating, a dispersive effective medium slab, and the slot alone. The structures are infinite in the $y$ direction with a periodicity of 6.5 mm. The finite sized PEC plane is 92 mm long in the $x$ direction (underneath an AZIM coating with 14 cells). A perfectly matched absorbing slab is placed underneath the slots in the simulations to absorb the radiation in the $z$ half space. (b) Snapshots of $E$-field distributions in the upper $x$-$z$ plane for the three cases. The blocks with dashed lines indicate the position of the metamaterial coating or the effective medium slab. (c) Normalized radiated power for the three cases at 5.4 GHz ($i.e.$ close to the effective plasma frequency of the metamaterial). All the curves are normalized to Case C at broadside ($\theta = 0^\circ$). (d) Normalized radiated power at broadside ($\theta = 0^\circ$) as a function of frequency. .................64

Figure 4.6. Schematic view of the SIW fed slot antenna. The dimensions are $L = 133$, $W = 92.5 \ mm$, $W_{ss} = 4.83 \ mm$, $L_{ss} = 19 \ mm$, $W_{pss} = 19.3 \ mm$, $L_{pss} = 23.5 \ mm$, $r = 1 \ mm$, dis = 0.5 mm, $L_{SW} = 86.75 \ mm$, $W_{SW} = 22.5 \ mm$, $W_{slot} = 1.33 \ mm$, $L_{slot} = 24.4 \ mm$, $L_x = 32.55 \ mm$, $W_{off} = 11.24 \ mm$. The substrate material is Rogers RT/duroid 5880 with a dielectric constant of 2.2 and a loss tangent of 0.009. The substrate thickness is 1.575 mm. The inset in the top right corner shows the SIW fed slot antenna symmetrically covered by the AZIM coating with 5 rows in the $y$ direction each containing 14 unit cells. .................................................................68

Figure 4.7. (a) Simulated and measured $S_{11}$ of the SIW fed slot antenna without the AZIM coating. (b) Simulated and measured $S_{11}$ of the SIW fed slot antenna with the AZIM coating, including the dispersive homogeneous effective medium AZIM coating. .................................................................69

Figure 4.8. (a) Simulated and measured normalized radiation patterns of the SIW fed slot antenna with and without the AZIM coating in the $E$-plane ($y$-$z$ plane) at 5.6 GHz, (b) 5.8 GHz, and (c) 6.0 GHz. (d) Simulated and measured normalized radiation patterns of the SIW fed slot antenna with and without the AZIM coating in the $H$-plane ($x$-$z$ plane) at 5.6 GHz, (e) 5.8 GHz, and (f) 6.0 GHz. The simulated $E$- and $H$-plane patterns for the SIW fed slot antenna without the AZIM coating on a ground plane infinite in the $y$ direction and the simulated $E$- and $H$-plane patterns for the
SIW fed slot antenna with the homogeneous dispersive effective medium AZIM coating are also shown for comparison.

Figure 4.9. (a) Simulated and measured gain at broadside (θ = 0°) for the SIW fed slot antenna with and without the AZIM coating. (b) Simulated and measured front-to-back ratio for the SIW fed slot antenna with and without the AZIM coating.

Figure 4.10. (a) Simulated efficiency for the SIW fed slot antenna with and without the AZIM coating. (b) Simulated aperture efficiency for the SIW fed slot antenna with the AZIM coating.

Figure 4.11. Photograph of the fabricated SIW fed slot antenna covered by the AZIM coating. Additional dielectric slabs cut with interlocking slits were used on both sides of the AZIM coating to provide better mechanical support. The inset shows the SIW fed slot antenna alone.

Figure 5.1. (a) Geometry and dimensions of the unit cells of the anisotropic metamaterial coating. All dimensions are in millimeters: a = 2.5, d_s = 0.051, d_c = 0.017, w = 2, b = 10, c = 1.5, g = 0.8 and l = 8. (b) Real and imaginary parts of the retrieved effective anisotropic permittivity tensor parameters (ε_x, ε_y, ε_z).

Figure 5.2. Configuration of (a) the quarter-wave monopole antenna and (b) the same monopole with ultra-thin flexible anisotropic metamaterial coating. All dimensions are in millimeters: d_a = 1, h_a = 28.5, d_i = 5, d_o = 2d_i, and h_l = 40. The dielectric is 51 μm thick Rogers Ultralam 3850 (ε_r = 2.9, δ_tan = 0.0025).

Figure 5.3. Simulated VSWR of monopole alone (───), monopole with actual metamaterial coating (───), and monopole with homogeneous anisotropic effective medium coating (───). The same ground plane size (32 cm × 32 cm) was used in all three simulations.

Figure 5.4. Simulated input impedance of the monopole antenna with and without the metamaterial coating. The insets plot the current magnitude distribution on the monopole at various frequencies.

Figure 5.5. Photographs of the fabricated metamaterial coated monopole.

Figure 5.6. Simulated and measured VSWR of the monopole antenna with and without the metamaterial coating.

Figure 5.7. Simulated and measured H-plane (x-y plane) and E-plane (y-z plane) radiation patterns of the MM coated monopole at (a) 2.2 GHz, (b) 3.3 GHz, and (c) 4.4 GHz. Red lines: simulated H-plane patterns. Black lines: measured H-plane patterns. Blue lines: simulated E-plane patterns. Gray lines: measured E-plane patterns.

Figure 5.8. Simulated VSWR of monopole alone, monopole with actual metamaterial coating, and monopole with homogeneous isotropic dielectric coating.
Figure 5.9. (a) Configuration of sleeve monopole with the same horizontal footprint. (b) Simulated VSWR of the sleeve monopole, monopole with actual metamaterial coating, and monopole with homogeneous anisotropic effective medium coating. 85

Figure 5.10. Simulated VSWR of monopole alone and monopole with actual metamaterial coating at C-band. 87

Figure 6.1. (a) Diagram of the optimized fishnet structure that produces a near zero refractive index at 1.55 μm. One unit cell is enclosed within the red dotted square. The inset shows a 3D view of the unit cell with w=365 nm, p=956 nm, and t=381 nm. The top and bottom Au layers (yellow) are 39 nm thick and the polyimide layer (red) is 303 nm thick. Top-view FESEM image of the fabricated ZIM is shown on the right. Scale bar, 500 nm. (b) FESEM image of the free-standing, flexible ZIM structure. Scale bar, 2 μm. 91

Figure 6.2. (a) Amplitude and (b) phase of the transmission (blue, light blue (resimulated with adjusted dimensions)) and reflection (red, light red (resimulated with adjusted dimensions)) coefficients. Real (blue, light blue (resimulated with adjusted dimensions)) and imaginary (red, light red (resimulated with adjusted dimensions)) parts of the inverted effective refractive index (c) and normalized effective impedance (d). Real (blue, light blue (resimulated with adjusted dimensions)) and imaginary (red, light red (resimulated with adjusted dimensions)) parts of the inverted effective permittivity (e) and effective permeability (f). 92

Figure 6.3. Distribution of volumetric current density on the top (a) and bottom (b) Au layers of the ZIM excited with a normally incident beam having the polarization shown. (c) Cross-sectional view of a snapshot of the electric field. The nearly identical field vectors throughout the fishnet structure confirm that the metamaterial has a near-zero phase delay with high transmission. 94

Figure 6.4. (a) Schematic of the Mach-Zehnder interferometer used to find the complex transmission coefficients of the ZIM. (b) Schematic of the Michelson interferometer used to find the complex reflection coefficients of the ZIM. 97

Figure 6.5. (a) The unit cell geometry of the same fishnet nanostructure shown in Figure 6.1(a), but with air holes that have sloped sidewalls. The dimensions are p=956 nm, w=365nm, t=381 nm, and θ=80°. The top and bottom Au layers (yellow) are 39 nm thick and the polyimide layer (red) is 303 nm thick. (b) Simulated amplitude of the transmission and reflection coefficients. The solid red line corresponds to the reflection amplitude for a wave normally incident on the top surface of the structure, while the dashed red line is for a wave normally incident on the back surface. The difference between the two reflection coefficients is shown in green. 99

Figure 6.6. (a) 16 × 16 pixel geometry for the metamaterial stack with two Ag screens. (a) Optimized geometry for the first design, with a unit cell size a₁=1.42μm, total thickness d₁=476nm, and a Ag screen thickness of 75nm. (b) 3D isometric view of the first ZIM design. (c) Optimized geometry for the second design, with a unit cell size a₂=1.58μm, total thickness d₂=735nm, and a Ag screen thickness of 75nm. (d) 3D isometric view of the second ZIM design. 102
Figure 6.7. Retrieved (a) $z$ component of the effective refractive index and (b) normalized wave impedance at 100 THz versus angle of incidence for both IR ZIM designs shown in Figure 6.6. ................................................................. 102

Figure 6.8. Retrieved effective parameters of IR ZIM design #1: (a) $\varepsilon_{xx}$, (b) $\mu_{yy}$, and (c) $\mu_{zz}$ using normal and oblique incidence angles; (d) $\mu_{zz}$ using two oblique angles of incidence. Note that for $\varepsilon_{xx}$ and $\mu_{yy}$, results retrieved from two oblique incidence angles and from only normal incidence are compared. ........................................ 104

Figure 7.1. (a) Ideal response of a band pass filter with a flat transmission window and a flat group delay within the pass band. (b) Real parts of the dispersive permittivity (red), permeability (blue) and refractive index (green) profiles of a theoretical material. (c) The transmission (red), reflection (blue) and group delay $\tau_e$ (green) of a slab of this theoretical material with a thickness of $0.15\lambda_0$. (d) Simulated field plots at the labeled wavelengths in (b) of a prism composed of the theoretical material with the electromagnetic properties shown in (b) and a 30° slope angle. The outer field plots show no transmission at the two magnetic resonances corresponding to $\lambda_{m1}$ and $\lambda_{m2}$, whereas the inner three plots show high transmission in the pass band with beam angles of 0°, 30°, and 60° where $n = 1$, 0, and -1, at $\lambda_p$, $\lambda_e$, and $\lambda_n$, respectively. ........................................ 108

Figure 7.2. The geometry and dimensions of a single unit cell of the dispersion engineered metamaterial. The optimized geometry dimensions are $p=2113$ nm, $w=1123$ nm, $g=198$ nm, $t=30$ nm and $d=450$ nm. ......................................................... 111

Figure 7.3. (a) Simulated and measured transmission (top, blue) and reflection (middle, red) magnitudes for normally incident radiation showing broadband transmission over the highlighted region from 3 μm to 3.5 μm. Simulated group delay $\tau_e$ (bottom) shows minimal variation over the transmissive window. (b) Real (top) and imaginary (bottom) parts of the effective index of refraction $n_{eff}$ (green), permittivity $\varepsilon_{eff}$ (red), and permeability $\mu_{eff}$ (blue) retrieved from the full-wave simulation of the metamaterial structure. The real parts of $\varepsilon_{eff}$ and $\mu_{eff}$ follow the same slope from 3 μm to 3.5 μm, indicating a matched impedance, whereas the imaginary parts are small, indicating low intrinsic losses. ........................................ 112

Figure 7.4. (a) Volumetric current density distribution on the top Au layer at 3.7 μm (left) and 2.85 μm (right). (b) Volumetric current density distribution on the bottom Au layer at 3.7 μm (left) and 2.85 μm (right). (c) Top-view of magnetic field distribution in the structure at 3.7 μm (left) and 2.85 μm (right). ........................................ 114

Figure 7.5. Numerical parametric analysis of the modified fishnet nanostructure with various nano-notch inclusion sizes. (a) Transmission amplitude. (b) Group delay. (c) Effective permittivity. (d) Effective permeability. .......................................................... 116

Figure 7.6. (a) Top-view SEM image of a portion of the fabricated modified fishnet metamaterial filter structure (inset, magnified top view). The accurate reproduction of the nano-notch features is critical to maintaining the designed properties in the
fabricated structure. (b) Tilted view SEM image of the free-standing fabricated metamaterial filter structure with vertical (89°) side walls.

Figure 7.7. (a) The 3D tilted view of the configuration of the prism and the orientation of the incident beam. The optimized dimensions for this structure are \( p = 1985 \text{ nm}, w = 1178 \text{ nm}, g = 372 \text{ nm}, t = 46 \text{ nm} \) and \( d = 76 \text{ nm} \). The inset shows the side view of the metamaterial prism. (b) The simulated reflection of the actual metamaterial prism for both TE and TM polarizations. (c) Snapshots of electric field distribution for TE polarization at different wavelengths. (d) Snapshots of electric field distribution for TM polarization at different wavelengths. Outside the pass band, no wave is transmitted through the prism. Within the pass band, the wave is refracted with the exiting beam at angles of 24°, 13° and 0° relative to the incident beam, corresponding to the \( n_{\text{eff}} = -1, 0, \) and 1 bands, respectively.

Figure 8.1. Schematics of the fishnet metamaterial on the substrate. Top view of a portion of the fishnet structure. \( d = 381 \text{ nm}, p = 956 \text{ nm}, \) and \( w = 365 \text{ nm} \).

Figure 8.2. (a) Simulated (left) and measured (right) scattering parameter amplitudes of the free-standing fishnet nanostructure. (b) Simulated (left) and measured (right) scattering parameter phases of the free-standing fishnet nanostructure.

Figure 8.3. (a) Simulated (left) and measured (right) scattering parameter amplitudes of the on-substrate fishnet nanostructure. (b) Simulated (left) and measured (right) scattering parameter phases of the on-substrate fishnet nanostructure.

Figure 8.4. (a) Simulated (left) and measured (right) scattering parameter amplitudes of the on-substrate fishnet nanostructure. (b) Simulated (left) and measured (right) scattering parameter phases of the on-substrate fishnet nanostructure.

Figure 8.5. (a) Simulated \( x \)-component of current density at 1.39 \( \mu \text{m} \) (top) and 1.55 \( \mu \text{m} \) (bottom) for fishnet without substrate. (b) Simulated \( x \)-component of current density at 1.39 \( \mu \text{m} \) (top) and (bottom) 1.55 \( \mu \text{m} \) for fishnet with a substrate.

Figure 8.6. (a) A multilayer fishnet metamaterial sandwiched between a superstrate and a substrate with finite thickness. Beneath the substrate and above the superstrate are the bottom and top half-spaces, respectively. (b) The unit cell geometry of the multilayer fishnet nanostructure composed of Ag and SiO\(_2\). The dimensions are \( p_x = 600, p_y = 600, w_x = 72, w_y = 336, t_d = 15, t_d = 20 \) (all in nanometers). (c) Retrieved bianisotropic effective medium parameters for the free-standing fishnet displayed in (b). (d) Evolution of the real part of the effective index as a function of the number of functional layers \( (N) \) for the free-standing fishnet in (b).

Figure 8.7. (a) Scattering parameters of the multilayer fishnet alone on an infinite substrate. (b) Retrieved effective permittivity, permeability and magnetoelectric coupling parameter corresponding to (a). (c) Extracted scattering parameters of the multilayer fishnet alone on a 5\( \mu \text{m} \) thick substrate. (d) Retrieved effective permittivity, permeability and magnetoelectric coupling parameter corresponding to (c). (e) Evolution of the maximum real and imaginary parts of the retrieved
magnetoelectric coupling parameter as a function of the thickness of the SiO$_2$ substrate. The point on the right edge corresponds to the semi-infinite substrate case.

Figure 8.8. (a) Magnitudes of the induced electric ($p$) and magnetic ($m$) dipole moments in the free-standing multilayer fishnet under an electric ($e$) or magnetic ($m$) excitation. (b) Magnitudes of the induced electric ($p$) and magnetic ($m$) dipole moments in the multilayer fishnet on a semi-infinite substrate under an electric ($e$) or magnetic ($m$) excitation.

Figure 8.9. (a) Evolution of the maximum real and imaginary parts of the retrieved magnetoelectric coupling parameter as a function of the superstrate thickness with a permittivity of 2.25. The point on the right edge corresponds to the semi-infinite SiO$_2$ superstrate case. (b) Retrieved magnetoelectric coupling parameter for the fishnet on a semi-infinite SiO$_2$ substrate and the fishnet underneath a 200nm SiO$_2$ superstrate. (c) Magnitudes of the induced electric ($p$) and magnetic ($m$) dipole moments in the multilayer fishnet sandwiched between a semi-infinite substrate and a 200nm superstrate under an electric ($e$) or magnetic ($m$) excitation.

Figure 8.10. (a) Evolution of the maximum real and imaginary parts of the retrieved magnetoelectric coupling parameter as a function of the superstrate thickness with a permittivity of 1.75 and 2.25, respectively. The point on the right edge corresponds to the infinite substrate case. (b) Optimum superstrate thickness as a function of the permittivity value of the superstrate.

Figure 9.1. (a) Unit cell configuration of dual-band absorber design #1. $p = 1475$ nm, $t_1 = 50$ nm, $t_2 = 50$nm, $d = 100$nm. (b) Unit cell configuration of dual-band absorber design #2. $p = 1420$ nm, $t_1 = 50$ nm, $t_2 = 50$nm, $d = 80$nm. (c) Unit cell configuration of triple-band absorber design. $p = 1730$ nm, $t_1 = 50$ nm, $t_2 = 50$nm, $d = 100$nm.

Figure 9.2. Simulated absorption of dual-band design #1 at various angles of incidence for (a) TE and (b) TM polarizations.

Figure 9.3. Simulated absorption of dual-band design #2 at various angles of incidence for (a) TE and (b) TM polarizations.

Figure 9.4. Simulated absorption of the triple-band design at various angles of incidence for (a) TE and (b) TM polarizations.

Figure 9.5. (a) FESEM image of a portion of the fabricated dual-band design #1. The dashed red square indicates the unit cell. Scale bar: 600 nm. (b) FESEM image of the fabricated conformal metamaterial absorber coating showing its mechanical flexibility and durability. Scale bar: 1800 nm.

Figure 9.6. Measured absorption of the dual-band design #1 at various angles of incidence for (a) TE and (b) TM polarizations.

Figure 9.7. (a) Schematics of a curved 50 nm Au ground plane producing strong reflection in multiple directions when illuminated by an incident beam. The inset shows a 3D view of the curved ground plane. (b) Schematics of a metamaterial
absorber coated curved ground plane (50 nm Au) with significantly reduced reflection when illuminated by an incident beam. The inset shows a 3D view of the metamaterial absorber coated curved ground plane. (c) Simulation of the electric field magnitude for the curved ground plane without metamaterial absorber coating at 3.3 µm (top) and 3.9 µm (bottom). (d) Simulation of the electric field magnitude for the curved ground plane with metamaterial absorber coating under TE incident radiation at 3.3 µm (top) and 3.9 µm (bottom). (e) Simulation of the electric field magnitude for the curved ground plane with metamaterial absorber coating under TM incident radiation at 3.3 µm (top) and 3.9 µm (bottom).
ACKNOWLEDGEMENTS

This dissertation would not have been possible without the support from numerous people who in one way or another contributed during the preparation and completion of this research. First of all, I would like to express my highest gratitude to my advisor and committee chair, Dr. Douglas H. Werner, for his invaluable support and guidance throughout my Ph.D. research. I would also like to thank Dr. Theresa S. Mayer, Dr. Pingjuan L. Werner, and Dr. Brian L. Weiner for serving on my committee and providing valuable feedback on my dissertation. I feel enjoyable when working with Dr. Theresa S. Mayer and Dr. Zhiwen Liu on NSF MRSEC projects from whom I have received a great deal of encouragement and guidance. I cherish the discussion and joint efforts that have been made with Seokho Yun, Lan Lin, Ding Ma, and Qian Xu on MRSEC projects - we have been a team with innovation, perseverance, and friendship.

I owe thanks to many of the students and postdoc researchers that I have collaborated with during my graduate research for their stimulating discussions, useful help, including members from or outside our research group: Seokho Yun, Lan Lin, Ding Ma, Qi Wu, Jeremy Bossard, Micah Gregory, Donovan Brocker, Xiande Wang, Anastasios Panaretos, Frank Namin, Peter Sieber, Jeremy Turpin, Clint Scarborough, Yong Zeng, Jason Ashbach, Phil Gorman, Spencer Martin, and Qian Xu. I also thank my friends, for those in State College as well as those back in Nanjing, that you made my Ph.D. life more colorful.

Finally, I would like to express my deepest love to my parents Yujun Jiang and Qiuping Jin, who have always been giving their endless love and spiritual support and sharing my happiness and sorrow on the road of my life.
To Yujun Jiang and Qiuping Jin, my beloved parents.

To the city of Nanjing, where I always belong to.
Chapter 1 Introduction

This chapter introduces the topic of the dissertation, which is the development of anisotropic metamaterials for microwave antennas and infrared nanostructured thin films. A brief background of metamaterials is given with a discussion of the limitations and challenges of the current technologies in the related sub-fields and a summary of the research goals of the dissertation. An overview of the subsequent chapters and original contributions of this dissertation is provided.

1.1 Background

The interaction between electromagnetic waves and matter has been the primary focus of research throughout the electromagnetic spectrum which can be dated back to ancient Greece when people began to study the how light passes through and reflected back from different materials [1]. Since then, it established that the geometrical shape and the electric and magnetic properties of materials determine the properties of wave propagation inside a medium and the way waves are reflected, refracted, and diffracted upon arriving at the interfaces between different media. Today, due to the booming development in material science and micro-/nano-fabrication technologies [2]-[3], materials with more distinct electric and/or magnetic properties are becoming available, including various alloys, semiconductors, and metal-oxides, which greatly expands the library of materials that can be used for tailoring the propagation, scattering, and radiation of electromagnetic waves.

To classify the electromagnetic properties of materials from a macroscopic perspective, permittivity ($\varepsilon$) and permeability ($\mu$) are usually adopted to quantify how the materials respond
to the electric and magnetic field components of the illuminating waves, respectively [4]. Based on the signs of the real parts of the permittivity and permeability, the electromagnetic materials can be separated into four categories. The first class is for the case when both the permittivity and permeability are positive ($\epsilon > 0$, $\mu > 0$). For example, naturally occurring dielectric materials have positive relative permittivity with a value no less than unity ($\epsilon_r \geq 1$) and a unity valued relative permeability ($\mu_r = 1$). When external fields are applied to this material class, the bound negative and positive charges will shift to create numerous electric dipoles, which give rise to a macroscopic electric polarization. In addition to dielectric materials, certain diamagnetic and paramagnetic materials also fall into this category. The index of refraction of this type of materials is positive ($n > 0$), such that the waves can propagate through. The second class of materials corresponds to when the permittivity is negative but the permeability remains positive ($\epsilon < 0$, $\mu > 0$). This property can be found in several types of noble metals such as silver and gold when operating below their plasma frequencies. Materials in the third category have positive permittivity and negative permeability values ($\epsilon > 0$, $\mu < 0$), which are manifested in some ferrite materials. Inside these two classes of materials, waves are exponentially attenuated and thus cannot propagate through. The last category of materials, is for the case when both permittivity and permeability are negative ($\epsilon < 0$, $\mu < 0$) which corresponding to a negative index of refraction ($n < 0$). This unprecedented electromagnetic property does not exist in natural materials, but can be realized by artificially structured electromagnetic materials; so called metamaterials [5-7].

Electromagnetic materials with both negative permittivity and permeability were first hypothesized by Veselago in 1968 from a sense of symmetry in nature [8]. It was found that a number of interesting electromagnetic wave phenomena can be achieved in the presence of negative index materials (NIMs), such as negative refraction [8, 9], inverse Doppler effect [8, 10], reversed Cherenkov radiation [8, 11], and negative Good-Hänchen shift [12]. Actually, the
earliest discussion on negative phase velocity dates back to 1904 by Sir Author Schuster and H. Lamb [13]. Simultaneous negative permittivity and permeability was not made possible until Pendry et al. proposed the wire mesh array for achieving negative permittivity [14] and split-ring resonator (SRR) arrays for providing negative permeability [15] from an effective medium point of view. It was further proposed that a planar NIM lens with $\varepsilon = -1$ and $\mu = -1$ can accomplish focusing beyond the diffraction limit due to its ability in amplifying the evanescent fields [16]. Since then, many other different types of subwavelength metamaterial building blocks have also been proposed for providing desirable electric and magnetic properties [17-20]. It has also been demonstrated that these artificially structured electromagnetic materials can provide distinct anisotropy and even bianisotropy [21-23], which significantly expands the capability of metamaterials in manipulating electromagnetic wave propagation, radiation, and scattering.

The field of metamaterials has witnessed a fast development throughout the electromagnetic spectrum from direct current (DC), microwave range, to terahertz regime and even optical wavelengths [5-7]. The electromagnetic metamaterials have been applied in diverse fields including super lensing [24-26], miniaturized microwave circuits [27-29], electrically small antennas [30-32], high-gain microwave antennas [33-35], ultra-thin microwave and optical absorbers [36-38], microwave and optical wave front shaping [39-41], tunable microwave and terahertz components [42-44], electromagnetic cloaks [45-47], optical artificial mirrors [48-50], optical phase-delay lines and films [51-52], subwavelength tunneling [53], optical hologram [54], molecule/biological sensors [55-57], long distance plasmonic optical wave guiding [58], and so on.

In this dissertation, we present the development of anisotropic metamaterials for two subfields - microwave antennas with improved performance and infrared nanostructured thin-films with extraordinary optical properties. First, a new effective medium parameter inversion algorithm is proposed for retrieving the anisotropic effective material parameters by taking the
angular response of a metamaterial into account. In the second thrust of research, several novel broadband, low-loss anisotropic metamaterial-based lenses and coatings are proposed to tailor the radiation pattern and/or the input impedance of conventional antennas for superior gain and impedance bandwidth performance. The third thrust of the research in this dissertation focuses on the design and optimization of anisotropic infrared nanostructures, in the final form of freestanding thin-films, with extraordinary optical properties. The design methodologies presented in this dissertation overcome the shortcomings of current state-of-the-art optical metamaterials, and thus will allow for the exploration of more diverse optical functionalities and enrich optical applications of metamaterials.

Figure 1.1. Various types of metamaterial unit cells for providing electric and magnetic resonances.

1.1.1 Anisotropic Metamaterials

Many structures have been proposed as unit cells for constructing metamaterials both in microwave frequencies and at optical wavelengths. Several of the most common examples are
displayed in Figure 1.1, including wire mesh [14], cut wire dipole, SRR [15], electric LC resonator [17], omega-type resonator [18], and tri-layer fishnet [19-20]. Each type of unit building block functions as an electric, magnetic, or hybrid resonator. It can be observed that these structures are inherently asymmetric, leading to anisotropy in their electromagnetic responses governed by the orientation of the unit cells. This anisotropy, which is rare in natural materials, can be easily obtained with metamaterials. The anisotropic material parameters, \( i.e. \) the permittivity and permeability, can be described in a tensor form as

\[
\begin{align*}
\bar{\varepsilon} &= \varepsilon_0 \bar{\varepsilon} = \varepsilon_0 diag[\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}] \\
\bar{\mu} &= \mu_0 \bar{\mu} = \mu_0 diag[\mu_{xx}, \mu_{yy}, \mu_{zz}],
\end{align*}
\]

(1.1)

where only diagonal parameters are included for simplicity. Actually, symmetric anisotropic permittivity and permeability tensors can always be expressed as diagonal tensors through proper rotation of the optical axis in the material. More importantly, the anisotropy of the metamaterials can be engineered by assigning the value of each of these tensor parameters to be positive, near-zero, or even negative. Under such circumstances, exotic dispersion relations can be achieved, such as elliptical, hyperbolic, or even quasi-linear in addition to the circular one for the conventional dielectric materials [21]. With these exotic permittivity and permeability tensor parameters and the tailored anisotropy, unprecedented flexibility is available for scientists and engineers to manipulate how waves propagate inside a medium and how they are reflected and refracted on the air-medium interface.

Anisotropic metamaterials have been extensively investigated to achieve new wave phenomena in the fields of imaging, waveguiding, antenna engineering, and nanophotonics, such as perfect lensing [24-26], in-phase reflection [59], high-efficient wave bending [60], highly-directive emission [33], and so on. They have also been served as the basic building blocks for transformation optics devices [46, 61], which often require materials with spatially dependent anisotropy to facilitate a more powerful and comprehensive control of wave propagation.
1.1.2 Metamaterial-Enabled High-Gain and Broadband Microwave Antennas

In the antenna engineering community, metamaterials have been employed to manipulate the radiation properties of antennas, including metamaterial coatings for electrically small antennas [30] which enhances the radiation efficiency and gain of the radiator inside, transmission line metamaterial for leaky-wave antennas [62] which extend the scanning range to include the backward directions, broadband negligible-loss metaliners for horn antennas that support low sidelobes and low cross-polarization [63], inhomogenous transformation optics metamaterial lenses for broadband highly directive antennas [64,65], electromagnetic bandgap or artificial magnetic ground plane for realizing high-gain conformal antennas [66], partial reflecting metamaterial surfaces for high directive radiation and many others. In addition, metamaterials have also been utilized to enhance the impedance bandwidth of planar monopoles and microstrip antennas by proper loading with split ring resonators [67] or through the use of negative refractive index transmission lines [68].

However, these techniques exhibit drawbacks in several aspects. First, in terms of generating high-gain radiated beams, most of the proposed devices can provide only a single beam and at the same time have electrically large form factors. Secondly, the direction of beam is not stable as a function of frequency. Beam squinting is observed due to the leaky nature of transmission line metamaterial-based antennas and metamaterial Fabry-Pérot cavity antennas. Also, when these metamaterial structures are applied to the practical antenna, the input impedance is greatly degraded, resulting in a narrow bandwidth or strong port reflection. In terms of bandwidth broadening techniques using metamaterials, the reported works are limited to planar monopole antennas using transmission line metamaterial loading [67, 68]. In Chapter 3, Chapter 4, and Chapter 5 of the dissertation, we will explore the applicability of utilizing metamaterial
anisotropy for synthesizing microwave antennas with improved radiation and/or bandwidth performances.

1.1.3 Optical Metamaterial Nanostructures

Nanophotonics is another hot area where metamaterial nanostructures have been investigated to enable new optical functionalities, with the hope that these meta-photonic components can one day be integrated into photonic systems to improve system performance. The most fundamental aim in this sub-field is to construct low-loss three-dimensional nanostructures with exotic index properties, such as negative, zero/low, and even high indices of refraction.

The most widely used type of nanostructure to achieve this goal in the optical range is the fishnet [69-71], which is illustrated in Figure 1.1(f). The magnetic resonance in between the paired nanostrips aligned along the magnetic field can be directly excited from the incident light with a high coupling efficiency; thus, the limitations of the planar split ring resonator can be overcome. The non-resonant strips in the orthogonal direction, along the electric field, are responsible to give a Drude-like electric response. The capability to control both the permittivity and permeability simultaneously in a fishnet nanostructure allows the realization of negative index materials at optical wavelengths. Since its introduction, this fishnet structure has served extensively as a basic geometry in synthesizing various kinds of optical metamaterials. Recently, a loss-compensated metamaterial has been demonstrated based on this geometry [72].

Compared with NIMs, zero index metamaterials (ZIMs) have received less attention in recent years, but have a growing repertoire of possible practical applications [73,74]. A zero refractive index condition can be achieved by three different cases of the permittivity and permeability. The first case, when the permittivity approaches zero, results in a large value for the effective impedance and a corresponding reflection coefficient approaching +1, meaning that
reflected wave is in-phase with the incident wave. Such an epsilon-near-zero material can be used as either an artificial magnetic conducting surface [59, 75] or in subwavelength channels, tunneling electromagnetic energy [76]. In the second case, when the permeability approaches zero, the material acts like a perfect electric conductor, with the reflection coefficient approaching -1. Hence, in the first two cases the ZIM acts as either a perfect magnetic mirror (in-phase reflection) or a perfect electric mirror (180 degrees out-of-phase reflection). The final, and perhaps most interesting, case is when the permittivity and permeability simultaneously approach zero at the same rate, resulting in a ZIM that is impedance matched to free space. Another important property of ZIMs is their ability to act as an effective collimator, i.e. to convert cylindrical or spherical waves emanating from a source embedded in the metamaterial to plane waves at the interface between the metamaterial and free space. Thus, ZIMs can be utilized as flat lenses to achieve highly directive far-field radiation from embedded antennas, as extremely convergent microlenses and in other imaging applications [73].

In addition to the fact that the less-studied zero- and low-index values require more investigation, current state-of-the-art metamaterials have several limitations. First, they suffer from significant reflection loss originated from impedance mismatch at the air-metamaterial interface [69, 70], which limits the applicability of these optical metamaterials in transmissive devices. Secondly, from the perspective of fabrication and configuration imperfection, undesired bianisotropy is introduced into the system as a result of the tapered sidewalls in the layers [77] and the presence of the supportive substrate [78]. This broken-symmetry-induced bianisotropy degrades the performance of the metamaterial, leading to lower transmission amplitude and possibly higher absorption loss. The third drawback of previously demonstrated index-engineered optical metamaterials is their narrow operating bandwidth. Due to the difficulty of balancing the permittivity and permeability, engineering the values of the refractive index with high
transmission has been restricted within a narrow frequency band, restricting the optical metamaterials to operate in the broad range of frequencies.

In order to overcome the stated drawbacks of current technologies and diversify the applications of metamaterials, in this research, we explore free-standing nanostructures in macroscopic forms as thin films. We investigate the possibility of achieving extraordinary index properties and high transmission simultaneously. In particular, a narrow band near-infrared fishnet ZIM with matched impedance and a broadband dispersion engineered mid-infrared modified fishnet for filtering applications are proposed. Theoretical and experimental studies on substrate-induced bianisotropy and their associated compensation are also carried out. Finally, for more practical device applications, metamaterial absorber nanostructures with multiple absorption bands are demonstrated. The proposed metamaterial devices, together with the general design and synthesis approach, are expected to pave the way towards diversifying and improving optical metamaterials and consequently the practical systems into which they can be incorporated.

1.2 Overview

This dissertation introduces novel approaches for the design of various anisotropic metamaterial lenses/coatings for microwave antennas and the synthesis of a variety of metamaterials for the infrared wavelengths. This section provides an overview of the concepts covered in each of the following chapters along with a brief summary.

In Chapter 2, a new inversion algorithm is proposed for retrieving the anisotropic effective material parameters metamaterial slab. In contrast to the popularly used retrieval method where only scattering parameters at normal incidence are considered, this algorithm accounts for scattering parameters at oblique incidences, thereby providing an improved picture of the metamaterials’ electromagnetic properties and how they can be exploited in practical device
applications. The proposed method is validated by applying it to a well-known SRR-wire composite array. The proposed algorithm is subsequently employed for the metamaterial designs in several of the later chapters of this dissertation.

In Chapter 3, a transformation-optics enabled multi-beam high-gain antenna lens is proposed and realized using anisotropic zero-index metamaterials (AZIMs). Different from most of the transformation optics lens designs, which relies on both the inhomogeneity and anisotropy of metamaterials, the proposed coordinate transformation fully exploits the material anisotropy and suppresses the inhomogeneity. These properties result in a much simpler lens with a much broader bandwidth that can be readily fabricated. More importantly, this technique can generate an arbitrary number of beams each pointing at a pre-defined direction. A specific microwave quad-beam lens is designed, and experimentally demonstrated with a monopole feed at the center.

Inspired by the work described in Chapter 3, in Chapter 4 a low-profile microwave AZIM coating is proposed for achieving unidirectional radiation with slot antenna. In contrast to the previously reported metamaterial lens designs, which are bulky, the leaky modes of the grounded AZIM slab are utilized. Theoretical and numerical investigations on the leaky modes of the grounded AZIM slab, as well as its truncation effect on the radiation pattern of an embedded slot are presented. The AZIM is applied to a low-profile substrate-integrated waveguide fed slot antenna, experimentally showing a much improved gain and front-to-back ratio.

Different from the metamaterials presented in Chapter 3 and Chapter 4, which use the low-index band, Chapter 5 reports an ultra-thin antenna coating comprised of non-resonating anisotropic metamaterials with a high effective permittivity. This coating creates another resonating mode at a frequency higher than that of the fundamental mode of the monopole, thus greatly broadening the antenna’s impedance bandwidth. It is also compared to conventional methods, such as solid dielectric coated monopole and sleeve monopoles, showing a superior electromagnetic performance and extremely light weight.
Chapter 6 discusses the design, fabrication, and characterization of a free-standing optical ZIM that is symmetric in the direction of wave propagation. The fishnet nanostructure was optimized to achieve a near-zero phase delay with low absorption loss and an impedance matched to free space. The complex transmission and reflection coefficients of the fabricated ZIM were characterized using spectral holography, showing a strong agreement with simulation predictions. The resulting free-standing ZIM thin-film overcomes the limitations such as substrate and sidewall angle induced bianisotropy in the previously reported optical metamaterials, representing a new state-of-the-art in high performance optical metamaterials.

Using a similar synthesis method, a free-standing broadband dispersion engineered flat-top band-pass optical metamaterial filter is presented in Chapter 7, exhibiting a negative-zero-positive index behavior and suppressed group delay variation over the 3.0 - 3.5 μm transmission band. In contrast to previously reported index engineered optical metamaterials which only have a narrow transmission band, the dispersive properties of the metamaterial are tailored over a broadband wavelength range to fulfill the targeted device performance. This was enabled by introducing deep-subwavelength inclusions into the air-holes of a conventional fishnet nanostructure, which significantly changes the wavelengths and strength of the magnetic gap surface plasmonic polariton (SPP) resonances. The performance of the fabricated metamaterial filter was verified by measuring its transmission and reflection using a Fourier Transform Infrared (FTIR) spectrometer.

Chapter 8 discusses the effects of a substrate on optical metamaterials, as well as techniques for compensating the associated magnetoelectric coupling of the system. Previously, it has only been theoretically proposed that the substrate introduces bianisotropy in the optical metamaterials. Here, we experimentally demonstrate this effect by characterizing a fishnet nanostructure with and without the substrate. The vertical sidewalls of the fishnet sample eliminate the tapered sidewall induced bianisotropy, therefore the contribution from the substrate
alone can be evaluated. In addition, a technique to compensate the substrate-induced bianisotropy is proposed and numerically validated. It is shown that by adding an ultra-thin superstrate coating with a properly chosen thickness and dielectric constant, the entire system appears bianisotropy-free to outside observers.

Instead of minimizing the loss of the structured metamaterials, Chapter 9 introduces near-perfectly absorbing nanostructured thin-films that exploit the loss of the metamaterials which can be applied for infrared signature control, thermal imaging, and so on. Several infrared metamaterial absorber designs are presented with multiple absorption bands at pre-defined wavelengths. Particularly, a conformal metamaterial absorber with a narrow band, polarization-independent absorptivity centered at mid-infrared wavelengths of 3.3 μm and 3.9 μm is designed and demonstrated experimentally. The highly efficient nanoresonators were identified by using a genetic algorithm, exhibiting an effective electric and magnetic response that maximizes absorption in each wavelength band. The structure was patterned on an Au thin film backed flexible Kapton substrate and characterized by collecting angle-resolved reflection using FTIR.

To summarize this work and propose possible directions of future research, in Chapter 10 the accomplishments and contributions presented in each chapter are listed to point out what has been accomplished and identifies what needs to be done. Possible routes for synthesis of more agile microwave and optical metamaterials and metasurfaces are suggested for future work.

1.3 Original Contributions

The research that I have performed during my Ph.D. studies at Penn State has led to several new developments in the field of metamaterials, microwave antennas, and nanophotonics.

- Development of a new inversion algorithm to retrieve the anisotropic effective medium parameters of metamaterials.
• Development of transformation optics lenses for generating an arbitrary number of beams each pointing in an arbitrary direction. Design, fabrication and characterization of quad-beam lens for a G band monopole antenna.

• Design, fabrication, and characterization of a low-profile leaky grounded anisotropic metamaterial coating for WLAN slot antenna.

• Development of a new type of ultra-thin anisotropic metamaterial coating to greatly broaden the bandwidth of wire antennas. Design, fabrication, and characterization of an S band metamaterial coating for a quarter-wave monopole.

• Synthesis of a near-infrared free-standing ZIM with matched impedance and low loss that was fabricated by E-beam lithography and characterized by spectral holography technique.

• Development of a dispersion engineered modified fishnet nanostructure for broadband filters in the mid- and near-infrared. The mid-infrared metamaterial filter was fabricated by E-beam lithography and characterized using FTIR spectrometer.

• Design and optimization of multi-band mid-infrared optical metamaterial absorbers with wide field-of-views. The conformal dual-band absorber was fabricated by E-beam lithography and characterized using FTIR spectrometer.

• Demonstration of substrate-induced bianisotropy by characterizing the fishnet free from sidewall angle induced bianisotropy with and without the substrate.

• Development of a substrate-induced bianisotropy compensation technique for multilayer optical metamaterials by adding designed superstrate coatings.
Chapter 2

Effective Medium Parameter Retrieval for Anisotropic Metamaterials

2.1 Introduction

The artificial, structured metamaterials are most commonly synthesized by means of a periodic arrangement of resonant or non-resonant elements, such as arrays made of metallic wires, electric LC resonators, SRRs, cut wire dipoles, and so on. As the sizes of these constitutive building blocks and the periodicity are smaller than the wavelength of the electromagnetic fields propagating through the structure, effective medium parameters (including the effective permittivity and permeability) can be defined through homogenization to describe the macroscopic behavior of the metamaterials. Various methods have been proposed for retrieving the effective parameters of metamaterials. One method is to calculate the ratios of the electromagnetic fields inside the metamaterial structure, which is straightforward in numerical simulations but not practical to apply in the case of experimental measurements [15, 79]. Another method is to estimate the effective parameters by using approximate analytical models. Although this method provides more physical insight into how the effective properties arise from the subwavelength geometrical features that comprise the metamaterials, it is difficult to generalize this approach from simple to more complicated structures [22, 80]. Apart from the above two methods, a more commonly used scattering parameter retrieval method compares the calculated or measured transmission and reflection coefficients (or scattering parameters) of a metamaterial slab with a certain thickness to the analytical expression representing a homogeneous slab of the same thickness [81, 82]. This method has the advantage that it can be applied to both simple and complicated structures, and it can also be used in both simulation as well as experiment.
Several reports have been published discussing the application of scattering parameter retrieval methods under various situations [81-85]. Most of them assume that the metamaterial has isotropic effective parameters (*e.g.*, a scalar permittivity and permeability) [81-84]. However, it has been shown that many metamaterials have inherent anisotropic properties governed by the orientation of their unit cell structures, such as SRRs and wire dipoles. In addition, conventional SRRs have been shown to possess bianisotropy, due to the magnetoelectric coupling induced by the structural asymmetry of the metamaterial unit cell [22, 80]. Inversion procedures capable of handling bianisotropic material properties have been proposed recently [85, 86]. However, these retrieval methods [85, 86] require that scattering parameters in three orthogonal directions be collected to obtain all effective material parameters, which is difficult to achieve in experiment, especially for measurements performed in the infrared or optical wavelength regimes, where the metamaterials are thin films. Furthermore, the angular dependent response of metamaterials to obliquely incident waves, an important characteristic of metamaterials, has only been considered in a few papers [87, 88]. The authors of Ref. 87 attempted to retrieve the effective wave parameters for metamaterials at oblique incidence, but they used a conventional isotropic material model (*i.e.* isotropic effective permittivity and permeability), which was unable to correctly describe the properties of the fishnet structures considered in their paper. However, they successfully addressed the angular dependence of the retrieved effective wave parameters due to the intrinsic spatial dispersion and inhomogeneity of the metamaterials under consideration. The method proposed in Ref. 88 measures the amplitudes of a set of scattering parameters at varying angles of incidence and fits parts of the anisotropic effective electromagnetic parameters to the measurement using certain pre-assumed frequency dependent forms for the effective permittivity and permeability.

In this chapter, we propose a methodology for retrieving the anisotropic effective permittivity and permeability of a metamaterial slab [89]. This method is based on a combination
of transmission and reflection coefficients calculated or measured at several angles of incidence with respect to only one face of the metamaterial slab. We will present the analytical retrieval expressions used to determine the constitutive parameters of a homogeneous anisotropic slab. Two retrieval procedures are described with and without the use of normal incidence scattering parameters. This approach is validated by applying it to analyze a composite SRR-wire array. The physical relevance of the retrieved parameters is also discussed.

2.2 Anisotropic Retrieval Method

In order to retrieve the constitutive parameters of a slab of homogeneous anisotropic material, in this section, we first solve the forward problem by deriving the analytical expressions for the scattering from an anisotropic slab. Then, considering the scattering parameters for both polarizations at different angles of incidences as known variables, a set of inversion equations are developed for obtaining the effective anisotropic permittivity and permeability of a metamaterial slab. Considerations that must be taken into account when implementing the procedure are also given.

2.1.1 Scattering from a Homogeneous Anisotropic Slab

![Figure 2.1](image)

Figure 2.1. Schematics of a homogeneous anisotropic slab placed in free space and illuminated by (a) TE polarized and (b) TM polarized normally and obliquely incident plane waves.
As previously mentioned in the introduction, a periodic metamaterial can be approximated as a homogeneous medium under the long wavelength condition. Therefore, in this section we present the calculation of the scattering parameters based on a simplified model of a homogeneous anisotropic material slab which has diagonal constitutive permittivity and permeability tensors given by

\[
\bar{\varepsilon} = \varepsilon_0 \bar{\varepsilon}_r = \varepsilon_0 \text{diag} \left[ \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz} \right] \tag{2.1}
\]

\[
\bar{\mu} = \mu_0 \bar{\mu}_r = \mu_0 \text{diag} \left[ \mu_{xx}, \mu_{yy}, \mu_{zz} \right] , \tag{2.2}
\]

where \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of free space, respectively. In our model, the harmonic time dependence is assumed to be \( e^{i \omega t} \). Figure 2.1 shows the schematics of a homogeneous anisotropic slab with thickness \( d \) illuminated by a plane wave at an angle \( \theta_i \) with respect to the free-space slab interface normal \( \hat{z} \). Without loss of generality in the case of a homogeneous slab, we assume that the incident plane wave vectors are in the \( y-z \) plane for both transverse electric (TE) and transverse magnetic (TM) polarized waves. The TE waves satisfy the conditions \( \vec{k} \cdot \vec{E} = 0 \) and \( E_z = 0 \), whereas the TM waves satisfy \( \vec{k} \cdot \vec{H} = 0 \) and \( H_z = 0 \). The dispersion relations inside the material for TE and TM polarizations are

\[
\frac{\beta_y^2}{\mu_{zz}} + \frac{\beta_{\text{TE}}^2}{\mu_{yy}} = k_0^2 \varepsilon_{xx} \tag{2.3}
\]

\[
\frac{\beta_y^2}{\varepsilon_{zz}} + \frac{\beta_{\text{TM}}^2}{\varepsilon_{yy}} = k_0^2 \mu_{xx} , \tag{2.4}
\]

where the \( y \) component of the wave number satisfies \( \beta_y = k_y = k_0 \sin \theta_i \) and \( k_0 \) is the free space wave number [90]. The general expression is the same regardless of whether the dispersion relation is elliptical or hyperbolic [91]. By assigning boundary conditions on both interfaces of the slab, the scattering parameters of the slab can be calculated for an illuminating plane wave with an arbitrary incident angle less than 90 degrees. The expressions for the scattering parameters corresponding to TE and TM waves can be written as
\[
S_{11} = \frac{\Gamma_{TE(TM)}(1-e^{-j2\beta x TE(TM)d})}{1-\Gamma_{TE(TM)}^2 e^{-j2\beta x TE(TM)d}}
\]
\[
S_{21} = \frac{(1-\Gamma_{TE(TM)}^2)e^{-j\beta z TE(TM)d}}{1-\Gamma_{TE(TM)}^2 e^{-j2\beta x TE(TM)d}},
\]
where
\[
\Gamma_{TE(TM)} = \frac{Z_{TE(TM)}-1}{Z_{TE(TM)}+1}
\]
is the reflection coefficient from the top interface. The normalized wave impedances for TE and TM waves are given by
\[
Z_{TE} = \frac{k z \mu_{yy}}{\beta_{TE}}
\]
\[
Z_{TM} = \frac{\beta_{TM}}{k z \epsilon_{yy}},
\]
respectively, where
\[
k_z = k_0 \cos \theta_i
\]
is the \(z\) component of the free space wave number. Because the structures considered in this paper possess vertical symmetry, the scattering parameters will be reciprocal, such that \(S_{11} = S_{22}\) and \(S_{21} = S_{12}\).

### 2.1.2 Retrieval Equations

Here we illustrate the procedure for retrieving all the constitutive tensor parameters of an equivalent homogeneous anisotropic slab using scattering parameters corresponding to a set of plane wave illuminations at different angles of incidence. In this retrieval method, the six tensor parameters are divided into two groups: \(\epsilon_{xx}, \mu_{yy}, \) and \(\mu_{zz}\), which are active when the slab is illuminated by TE waves, and \(\epsilon_{yy}, \mu_{xx}, \) and \(\epsilon_{zz}\), which are active when the slab is illuminated by TM waves.
First, let us consider the scattering parameters for two TE incident waves with different angles of incidence, $\theta_{i1}$ and $\theta_{i2}$, which provide four equations ($S_{TE11-1}$, $S_{TE21-1}$, $S_{TE11-2}$, $S_{TE21-2}$) given by Eqs. (2.5) and (2.6). These four equations are then used to solve for three unknowns ($\varepsilon_{xx}$, $\mu_{yy}$, and $\mu_{zz}$). During the solution process, we first invert the $z$ components of the refractive indices of the slab ($n_{zTE-1}$, $n_{zTE-2}$) and the wave impedances ($Z_{TE-1}$, $Z_{TE-2}$) for both incidence angles by the following two equations:

$$\cos(n_{zTE-l}k_0d) = \frac{1-S_{TE11-l}^2+S_{TE21-l}^2}{2S_{TE21-l}}$$  (2.11)

$$Z_{TE-l} = \pm \sqrt{\frac{(1+S_{TE11-l}^2) - S_{TE21-l}^2}{(1-S_{TE11-l}^2) + S_{TE21-l}^2}}, \quad l = 1,2.$$  (2.12)

Proper care should be exercised to select the correct branch for the real part of the $z$ components of the refractive indices. Similar to approaches that have been employed for isotropic inversion procedures, the imaginary parts of both $n_{zTE-1}$ and $n_{zTE-2}$ must obey the conditions $n_{zTE-1}'' \leq 0$, $n_{zTE-2}'' \leq 0$. Likewise, for passive materials, the real parts of $Z_{TE-1}$ and $Z_{TE-2}$ must satisfy $Z_{TE-1}' \geq 0$ and $Z_{TE-2}' \geq 0$. Then, making use of the four inverted parameters and the dispersion relation Eq. (2.3) as well as the wave impedance Eq. (2.8), the three tensor parameters active for the TE case can be retrieved by using Eqs. (2.13) - (2.15):

$$\mu_{yy} = n_{zTE-l} \cdot Z_{TE-l} / \cos \theta_{il}, \quad l = 1,2.$$  (2.13)

$$\varepsilon_{xx} = \frac{n_{xTE-1} \cdot \theta_{i1} \cdot \sin^2 \theta_{i2} - n_{xTE-2} \cdot \theta_{i1} \cdot \sin^2 \theta_{i2}}{\sin^2 \theta_{i1} - \sin^2 \theta_{i2}} \frac{\cos \theta_{i2} \cdot \sin^2 \theta_{i1}}{Z_{TE-2}}$$  (2.14)

$$\mu_{zz} = \frac{n_{xTE-1} \cdot \theta_{i1} \cdot \sin^2 \theta_{i2}}{\sin^2 \theta_{i1} \cdot \cos \theta_{i2} \cdot \cos \theta_{i2}} \frac{Z_{TE-2}}{Z_{TE-1}} $$  (2.15)

Analogous to the above retrieval process for TE case, we can utilize the four scattering parameters ($S_{TM11-1}$, $S_{TM21-1}$, $S_{TM11-2}$, $S_{TM21-2}$) for two TM waves with different angles of incidence to obtain the other three constitutive tensor parameters of the slab ($\varepsilon_{yy}$, $\mu_{xx}$, and $\varepsilon_{zz}$). As before, the $z$ components of the refractive indices of the slab ($n_{zTM-1}$, $n_{zTM-2}$) and the wave
impedances \((Z_{TM-1}, Z_{TM-2})\) associated with both incidence angles are inverted first by using Eqs. (2.16) - (2.17):

\[
\cos(n_{zTM-l}k_0d) = \frac{1-S_{TM11-l}^2+S_{TM21-l}^2}{2S_{TM21-l}} \tag{2.16}
\]

\[
Z_{TM-l} = \pm \sqrt{\frac{(1+S_{TM11-l}^2)-S_{TM21-l}^2}{(1+S_{TM11-l}^2)+S_{TM21-l}^2}}, \quad l = 1, 2. \tag{2.17}
\]

Once again, the signs for the refractive index and wave impedance are chosen such that the imaginary parts of both \(n_{zTM-1}\) and \(n_{zTM-2}\) satisfy \(n_{zTM-1}'' \leq 0\), \(n_{zTM-2}'' \leq 0\), and the real parts of \(Z_{TM-1}\) and \(Z_{TM-2}\) satisfy \(Z_{TM-1}' \geq 0\) and \(Z_{TM-2}' \geq 0\). Then, substituting the four inverted parameters into the dispersion relation Eq. (2.4) and the wave impedance Eq. (2.9), the three remaining constitutive tensor parameters are found to be

\[
\varepsilon_{yy} = \frac{n_{zTM-l}/(Z_{TM-l} \cdot \cos \theta_{il})}{1} \tag{2.18}
\]

\[
\mu_{xx} = \frac{n_{zTM-1}Z_{TM-1} \cos \theta_{i1} \sin^2 \theta_{i2} \sin^2 \theta_{i1} - n_{zTM-2}Z_{TM-2} \cos \theta_{i1} \sin^2 \theta_{i1}}{1-\sin^2 \theta_{i2}} \tag{2.19}
\]

\[
\varepsilon_{zz} = \frac{\sin^2 \theta_{i2} - \sin^2 \theta_{i1}}{n_{zTM-1}Z_{TM-1} \cos \theta_{i1} - n_{zTM-2}Z_{TM-2} \cos \theta_{i2}}. \tag{2.20}
\]

It should be noted that the main difference between this method and the conventional retrieval methods that assume isotropic effective parameters is that here the calculated or measured scattering parameters for both polarizations at two different angles of incidence are utilized for the anisotropic constitutive parameter extraction. This is necessary for the anisotropic case because an electromagnetic constitutive wave directly incident on such a slab cannot sense the longitudinal electric or magnetic response. There are two procedures for implementing the anisotropic retrieval method with the above generalized equations. One procedure makes use of the scattering parameters at normal incidence to retrieve the four active permittivity and permeability tensor components in the \(x\)-\(y\) plane, which are \(\varepsilon_{xx}\) and \(\mu_{yy}\) for TE polarization and \(\varepsilon_{yy}\) and \(\mu_{xx}\) for TM polarization. The two remaining tensor parameters in the \(z\) direction, \(\varepsilon_{zz}\) and \(\mu_{zz}\), which are only active under TM and TE oblique incident wave illumination, respectively.
can be retrieved by including the scattering parameters for both polarizations calculated at another oblique angle. The second procedure retrieves all six electromagnetic tensor quantities of the slab directly from the scattering parameters collected at two different oblique angles. The first procedure can be regarded as a simplified case of the second procedure. However, considering that many metamaterial structures are periodic along the metamaterial surface, the transverse components of obliquely incident waves experience more spatial dispersion and inhomogeneity along the direction tangential to the interface of the metamaterial slab than in the normal direction [87]. Thus, the effective parameters of the metamaterial retrieved using normal incidence and one oblique incidence angle should be more accurate than those using two oblique waves. It should be noted that when applying this algorithm to a metamaterial slab, the six effective medium parameters can also be retrieved using equivalent equations for incident plane wave vectors limited in the $x$-$z$ plane. However, for cases when the metamaterial structure is asymmetric, the two sets of effective medium parameters retrieved using incident plane wave vectors in the $y$-$z$ and $x$-$z$ planes may be different if spatial dispersion is seen in one plane but not the other.

The parameter retrieval method described above can be used to fully determine the effective permittivity and permeability tensor quantities of an anisotropic metamaterial slab illuminated by incident plane waves at different angles. In contrast to the conventional isotropic retrieval methods where only scattering parameters at normal incidences are considered, this method takes the response of the metamaterial to obliquely incident waves into account. This not only gives us a true picture of the inherently anisotropic metamaterials but also sheds light on the angular dependence of the retrieved effective parameters. In light of the fact that most of the existing metamaterial designs only take into account responses to normally incident waves, this method is particularly relevant to the design of metamaterials with a wide field-of-view, where the response to obliquely incident waves is an important design consideration. A design example for this kind of metamaterial will be presented in Chapter 6 of this dissertation.
2.3 Application to A Specific Metamaterial – A SRR-wire Composite Array

The described anisotropic retrieval method can be applied to both lossless and lossy metamaterials. In fact, for this method there is no significant difference between the retrieval processes for either type of metamaterial. Considering that most metamaterials are lossy, the validity of the anisotropic retrieval method will be demonstrated for an important type of lossy metamaterial: a composite SRR-wire array.

![Diagram of SRR-wire composite array](image)

Figure 2.2. Unit cells (a=2.6mm, d=1.82mm) of a composite SRR-wire array. (a) 3D isometric view of the three layer unit cell. (b) Top views of the wire and SRR structures with dimensions given by $g=0.39mm$, $l=2.08mm$, $slot=0.13mm$, $t=0.13mm$, and $w=0.13mm$. The dielectric slabs (FR4: $\varepsilon_r=4.4$, $\delta_{tan}=0.02$) have thickness $d_z=0.13mm$.

The unit cell geometry for the metamaterial under consideration is illustrated in Figure 2.2(a). Periodic boundary conditions are assigned to the lateral walls in both $x$ and $y$ directions. A plane wave (contained in the $y$-$z$ plane) is assumed to be incident from the upper half-space at an angle $\theta_i (0^\circ \leq \theta_i \leq 90^\circ)$ with respect to the free-space metamaterial interface normal $\hat{z}$. Three layers of unit cells, rather than only a single layer, are used in the $z$ direction in order to take into account the coupling between adjacent unit cells, thus enabling the acquisition of more accurate
Figure 2.3. Magnitudes of the scattering parameters for the SRR-wire composite array. Curves are shown for (a)-(b) TE and (c)-(d) TM polarized waves at angles of incidence corresponding to $\theta_i = 0^\circ$, $10^\circ$, $20^\circ$, $30^\circ$, and $40^\circ$.

effective medium parameters. For each layer, the infinite wires are sandwiched by two dielectric slabs of thickness $d$. A pair of broad-side coupled SRRs are printed on each side of the dielectric substrate as shown in Figure 2.2(a), thus eliminating the biaxial nature associated with more conventional arrangements of SRRs, as suggested by Marques et al. [22, 80]. The geometrical parameters of the SRRs and wires are defined in Figure 2.2(b). Since the unit cells are much smaller than the wavelength of interest, these two metamaterials can be approximated as homogeneous anisotropic materials under the effective medium theory with diagonal effective
permittivity and permeability tensors as described by Eq. (2.1), provided the geometrical axes of the metamaterials coincide with the principal axes of the effective parameter tensors [91]. Under this condition, which is typically the case for most metamaterials, the effective electromagnetic parameters can then be retrieved using the method proposed in the previous section.

Before proceeding with a discussion of the retrieved effective medium tensor parameters, we will consider the scattering parameters corresponding to the composite SRR-wire array calculated for both polarizations and at different angles of incidence. The magnitudes of the scattering parameters are shown plotted in Figure 2.3 (only $S_{11}$ and $S_{21}$ are shown since $S_{11} = S_{22}$ and $S_{21} = S_{12}$). High frequency structure simulator (HFSS) was employed to predict the scattering parameters of the SRR-wire array. First let us consider Figure 2.3(a)-(b), in which the scattering parameters of the SRR-wire array illuminated by TE waves are shown. In both sets of spectra, one predominant feature can be identified: a narrow, angle-dependent pass band for oblique incidence between 8.5 GHz and 9 GHz. This pass band is absent for $\theta_i = 0^\circ$, where the applied magnetic field is in-plane, but grows with increasing angle of incidence. There is also a broad trend of increasing transmission with frequency observed in each curve. These features are attributed to the metamaterial anisotropy, in that the additional resonance at oblique incidence is due to the out-of-plane magnetic response to the longitudinal components of $\mathbf{H}$, and the increasing transmission at higher frequencies is due to the electric response of the infinite-wire array. This narrow pass band at oblique incidence is considered to be a negative index band\(^{32}\), where the corresponding effective in-plane permittivity and out-of-plane permeability tensor parameters are simultaneously negative. As a comparison, for TM incident waves, the scattering parameters of both structures have similar behavior for normal and obliquely incident waves in which a weak, angle-independent stop band is observed in each curve as shown in Figure 2.3(c)-(d). From the above qualitative analysis of the scattering parameters collected at different incident angles, it is evident that the differences between the sets of data in Figure 2.3 indicate the existence of
anisotropy in the metamaterials, *i.e.* distinct characteristics for each effective permittivity and permeability tensor parameter. In the following discussion, the effective properties of the structure will be analyzed in detail based on the quantitative retrieved effective parameters using the anisotropic extraction technique presented in the previous section.

(a)  (b)  (c)  (d)
Figure 2.4. Retrieved effective parameters of the SRR-wire composite array: (a) $\varepsilon_{xx}$, (b) $\varepsilon_{yy}$, and (c) $\varepsilon_{zz}$ using normal and oblique incidence angles; (d) $\varepsilon_{zz}$ using two oblique angles of incidence; (e) $\mu_{xx}$, (f) $\mu_{yy}$, and (g) $\mu_{zz}$ using normal and oblique incidence angles; (h) $\mu_{zz}$ using two oblique angles of incidence. Note that for $\varepsilon_{xx}$, $\varepsilon_{yy}$, $\mu_{xx}$, and $\mu_{yy}$, results retrieved from two oblique incidence angles and from only normal incidence are compared.

Figure 2.4 shows the retrieved effective medium parameters of the composite SRR-wire array. It can be observed that the retrieved results using scattering parameters calculated at different incidence angles agree well with each other. Figure 2.4(a) shows a Drude-type response in $\varepsilon_{xx}$ due to the infinite-wire array. It is also expected that a Lorentz-type electric resonance in $\varepsilon_{xx}$ exists at higher frequencies caused by the edges of the SRRs parallel to the wire array. The curves in Figure 2.4(b) imply that $\varepsilon_{yy}$ has a resonance at higher frequencies originating from the
sides of the SRRs along the $y$ direction, which act like cut-wire dipole antennas. The axial effective permittivity $\varepsilon_{zz}$, which is active only under obliquely incident TM polarized waves, is expected to vary around a constant value since there are no vias or vertical metal strips to be excited. The retrieved $\varepsilon_{zz}$ shown in Figure 2.4(c)-(d) confirms the non-resonant behavior, except for small disagreements attributed to the anti-resonance in $\mu_{yy}$ within the same narrow frequency band. The retrieved $\mu_{xx}$ and $\mu_{yy}$ have no strong resonances except for a small anti-resonance in $\mu_{yy}$ within the same frequency region as the magnetic resonance observed in $\mu_{zz}$. The anti-resonance phenomenon have been widely discussed in the literature, and are attributed to the intrinsic periodicity of the metamaterial [92, 93]. The weak resonance observed in $\mu_{xx}$ at 9 GHz accounts for the coupling between adjacent unit cells. The longitudinal magnetic resonance excited by the component of the incident $H$-field, which is perpendicular to the plane of the SRRs, occurs at around 7.4 GHz. This resonance can be easily characterized by the inversion method described here with the utilization of obliquely incident waves. The region where both $\varepsilon_{xx}$ and $\mu_{zz}$ are negative forms a negative refractive index band, which is consistent with the analysis of the pass band in the scattering parameters of the SRR-wire array for oblique TE waves (Figure 2.3(a)-(b)).

The good agreement of the retrieved effective electromagnetic tensor parameters for the composite SRR-wire array confirms the validity of the retrieval method as well as the particular homogeneous anisotropic model assumed for the metamaterials (i.e., diagonal effective permittivity and permeability tensors). Anti-resonances found in the retrieved parameters are attributed to the inhomogeneity and the intrinsic periodicity of the metamaterial, and the minute disagreements among the retrieved results with different angles of incidence are caused by the narrow band anti-resonance. For comparison, we also retrieved the effective parameters with incident plane waves limited in the $x$-$z$ plane (not shown here) and found that $\varepsilon_{xx}$ varies slightly
as a function of the angle of incidence of the plane waves used in the inversion. Such variation is due to the presence of spatial dispersion because a component of the incident wave vectors is parallel to the infinite wires, corroborating the results presented in Refs. 94 and 95. The anisotropic retrieval method presented in the previous section thus provides a useful tool for retrieving the effective anisotropic tensor parameters of metamaterials with the angular response taken into account.

2.4 Overview

In this chapter, we have proposed a new approach to retrieve the effective electromagnetic parameters of a slab of anisotropic metamaterial from reflection and transmission coefficients (or scattering parameters). In this retrieval method, calculated or measured scattering parameters are employed for a plane wave incident obliquely on a metamaterial slab at different angles. Useful analytical expressions are derived for extracting the homogeneous anisotropic medium parameters of a metamaterial. To validate the method, the effective permittivity and permeability tensor parameters for a composite SRR-wire array are retrieved and shown to be consistent with observations previously reported in the literature. This technique can facilitate the designs of both microwave and optical metamaterials for diverse applications, as will be illustrated in the following chapters.
Chapter 3

Multi-Beam Transformation Optics Lenses Using Anisotropic Metamaterials

3.1 Introduction

The field of transformation optics/electromagnetics [45, 96, 97], has witnessed dramatic development ever since it was proposed by Pendry et al. and Leonhardt in 2006. This technique provides engineers and scientists with unprecedented ability to manipulate the propagation and radiation of electromagnetic waves. The design approach was made possible by the fact that Maxwell's equations can be written in a form-invariant manner under coordinate transformations, where only the permittivity and permeability tensors are changed [98, 99]. In this way, the electromagnetic waves in one coordinate system \((x, y, z)\) can be described as if propagating in another different coordinate system \((x', y', z')\) with the coordinate transformation function \(x'=x'(x, y, z), y'=y'(x, y, z), z'=z'(x, y, z)\) applied to the permittivity and permeability tensors. To design an electromagnetic transformation, one first defines the desired wave propagation characteristics in a virtual space \((x, y, z)\) with the constitutive parameter tensors denoted as \(\varepsilon(x, y, z)\) and \(\mu(x, y, z)\). Next, the designer finds a physical space \((x', y', z')\) with the constitutive parameter tensors denoted as \(\varepsilon'(x', y', z')\) and \(\mu'(x', y', z')\) which are unknown. In order to keep the same wave propagation characteristics with that of the virtual space, the material parameter tensors of the physical space can be calculated in the last step by

\[
\varepsilon' = \frac{A \varepsilon A^T}{\det(A)} \quad \text{and} \quad \mu' = \frac{A \mu A^T}{\det(A)},
\]

where \(A\) is the Jacobian transformation matrix between the virtual space and the physical space.
Over the past five years, the transformation optics approach has been extensively exploited to create a wide variety of novel, and otherwise unattainable, electromagnetic devices. The most well-known one of these is the electromagnetic invisibility cloak [45-47], which is designed to bend the waves around a region of space so that the object hidden inside the region is exempt from being detected by the scattered fields. Other intriguing transformation optics examples have also been theoretically proposed and numerically examined including field rotators [100, 101], polarization splitters [101, 102], electromagnetic concentrators [103, 104], wave collimators [105-109], beam benders [107, 110], illusion devices [111, 112], flat-reflectors [113], and many others. Moreover, the rapid progress of metamaterial technology provides various sub-wavelength resonating/non-resonating metamaterial unit cells with exotic anisotropic/isotropic effective material parameters required by most of these transformation optics designs. As a result, a few of these transformation optics devices have been implemented and experimentally demonstrated [45, 100, 112]. However, the majority of these designs are still constrained to the realm of purely mathematical constructions with associated numerical verifications, and for most experimentally demonstrated examples, are limited to operate within a fairly narrow frequency range. This is primarily due to two limitations, the first of which is inhomogeneous material parameters with a high degree of sensitivity to the spatially dependent permittivity and permeability tensors on a sub-wavelength scale. The second is strong anisotropy of the material parameters which sometimes requires permittivity and permeability tensors with extreme values.

Several efforts have been carried out in recent years that attempt to circumvent these shortcomings by sacrificing certain types of functionality. One major achievement was the employment of quasi-conformal mappings, eliminating the material anisotropy and allowing implementation using purely isotropic dielectrics [114]. Although this has the disadvantage of inhomogeneous material parameters and a degraded impedance match at the interface between
free space and the transformed medium, this approach enables devices with broader bandwidth and low losses, such as the carpet cloaks [115, 116], Luneburg lenses [117, 118], as well as beam benders [119]. More recently, two works on macroscopic cloaks have suggested alternative methods for design simplification, which involve the use of coordinate transformations that require anisotropic yet spatially invariant material parameters [120, 121]. Although these cloaks operate over a limited range of incident azimuthal angles, they demonstrate the potential for using simple embedded coordinate transformations to design devices that have straightforward implementation.

With the vital growth of wireless networks and the increasing complexity of their environments, highly directive antennas with multiple main beams and a single feed are found to be useful for various wireless communication systems such as base station antennas, multiple-input multiple-output (MIMO) systems, automotive radar systems, point/multipoint-to-point radios, and so on [122-124]. Instead of omnidirectional or unidirectional radiation patterns, customized multiple main beams can provide more flexibility in avoiding obstacles in urban areas while providing stronger signal intensity in directions where receiver clusters are located. Having the ability to produce multiple highly collimated beams with a single feed antenna is also desirable for maintaining ease of implementation. Thus far, several different techniques have been reported for improving the directivity and/or increasing the number of beams radiated by an antenna. Several two dimensional coordinate mappings based on the transformation optics approach were investigated that are capable of effectively converting a cylindrical wave into a collimated plane wave [105-109]. However, these designs all require inhomogeneous anisotropic material parameters with extreme values, thus not leading themselves to practical realization or broadband applications. Other methodologies such as the Fabry-Pérot cavities [125, 126] and artificial magnetic conducting (AMC) substrates [127] have also been used to increase the directivity of an embedded source. The operation of the FP cavities is based on the constructive
phase addition of the transmitted and reflected waves from the multilayer structures, whereas the advantage of the AMC substrates is their unique near-zero reflection phase. These AMC substrates can also be used to replace the ground plane in a FP cavity for miniaturization purposes [128]. Alternatively, electromagnetic band gap (EBG) structures can be applied to increase the directivity of low profile patch and wire antennas as a result of the suppressed surface waves propagating on these EBG structures [129], [130]. Recently, it has been proposed to use bulk zero-index metamaterials [33, 73] and anisotropic low-index metamaterials [64, 131-133] to achieve directive emission from an embedded antenna. Nevertheless, most of these approaches, when applied to an actual antenna, either have only a narrow bandwidth with improved directivity or suffer from a high return loss which deteriorates the realized gain. At the same time, a majority of these approaches are also limited to providing only one or two radiated beams and thus are not suitable for multi-beam radiation.

In this chapter, we extend the idea of a simple embedded coordinate transformation which requires only anisotropic homogeneous material parameters for the design of highly directive multi-beam antenna lenses. Notably, the technique proposed here is capable of converting the radiation from an embedded omnidirectional source into any number of highly directive beams, each radiating in an arbitrary direction within a broad bandwidth. We first describe the polarization independent two- and three-dimensional embedded coordinate transformations that require only simple material parameters for directive emission, along with several numerical examples, in Section 3.2. The wave propagation properties inside the transformed medium are also studied by examining the equations that govern the dispersion relation. In Section 3.3, a broadband transformation optics metamaterial lens is designed to generate a quad-beam radiation pattern from a simple embedded monopole with efficient operation over a broad bandwidth. The experimental measurements of the monopole with and without the lens will be presented therein. The results using effective medium model are also
presented as a comparison. The effects of changing the lens dimensions are discussed in Section 3.4.

### 3.2 Two-/Three-Dimensional Linear Transformation for Highly Directive Emission

In this section, the analytical expressions of the proposed embedded coordinate transformations for directive emission in both two and three dimensions are presented, along with corresponding full-wave simulations using homogeneous lenses as numerical validations. The dispersion relation and the isofrequency curves of the resulting transformed medium are also discussed.

![Figure 3.1](image.png)

**Figure 3.1.** Two dimensional directive emission coordinate transformation. (a) Geometry of the fan-shaped virtual space. (b) Geometry of the simplified triangular virtual space. (c) Geometry of the triangular physical space.

#### 3.2.1 Two/Three Dimensional Linear Coordinate Transformations

We first consider a two-dimensional directive emission transformation, where the fields are restricted to be invariant along the $z$-direction. The schematic of the embedded coordinate transformation is shown in Fig.1 where an air-filled, fan-shaped virtual space with a central angle
of $\Phi_1$ (see Figure 3.1(a)) is mapped to an isosceles triangle with a vertex angle of $\Phi_2$ in the physical space (see Figure 3.1(c)) which has material parameters denoted by $\overline{\epsilon}_r(x', y')$ and $\overline{\mu}_r(x', y')$. An isotropic line source is located at the center (point $O$), with either the electric field or magnetic field along the $z$-direction, representing the TMz and TEz polarization, respectively. Note that the value of $\Phi_1$ should be much smaller than the value of $\Phi_2$ to correspond to high directivity in the virtual space. Since the equi-amplitude, equi-phase lines of the virtual space are a set of parallel arcs while those of the physical space are a set of parallel straight lines, a direct mapping from the fan region to the triangular region will inevitably result in spatial dependent anisotropic transformed material parameters. This is caused by the nonlinearity of the point-to-point mapping relation between the virtual and physical spaces as shown in several previously reported cylindrical-to-plane wave source transformation designs [105-107].

To reduce the unwanted spatial dependency of the transformed material parameters in the physical space, a geometrical simplification can be applied to the virtual space geometry. Since the value of $\Phi_1$ is small, the fan region can be approximated by an isosceles triangle with the same central angle as shown in Figure 3.1(b). Thus, an intermediate space which possesses a linear geometrical similarity with the physical space can be inserted into the transformation process to simplify the transformed material parameters. Now, the transformation can be written as

$$\begin{cases} x' = L_2 x / L_1 \\ y' = W_2 y / W_1 \\ z' = z \end{cases} \quad (3.2)$$

where $(x, y, z)$ and $(x', y', z')$ denote the coordinates of the virtual and physical space, respectively. By virtue of the metric invariance of Maxwell's equations, the relative permittivity and permeability tensors of the transformed medium in the physical space can be expressed as

$$\begin{bmatrix} L_2 W_1 / L_1 W_2 \\ 0 \\ L_1 W_2 / L_2 W_1 \end{bmatrix}$$

or

$$\begin{bmatrix} L_1 W_1 / L_2 W_2 \end{bmatrix}.$$
These tensor parameters can be divided into two groups ($\mu'_{rx}, \mu'_{ry}, \varepsilon'_{rz}, \varepsilon'_{rx}, \varepsilon'_{ry}, \mu'_{rz}$) which are active under the TM$_z$ and TE$_z$ polarization, respectively. As Maxwell's equations show, the dispersion relations and the wave trajectory in the physical space remain the same provided that $\mu'_{rx}\varepsilon'_{rz}, \mu'_{ry}\varepsilon'_{rz}, \mu'_{rz}\varepsilon'_{rx},$ and $\mu'_{rz}\varepsilon'_{ry}$ are held constant [89]. Thus, the material tensor expressions can be scaled to

$$\bar{\varepsilon}'_r = \bar{\mu}'_r = \text{diag}[(W_1/W_2)^2, (L_1/L_2)^2, 1].$$  \hspace{1cm} (3.4)

Since the radiated beam is highly directive in the $x$-direction which is perpendicular to the interface between the transformed medium and free space, we further let $L_1 = L_2$ so that the impedance is matched on the interface. The relative permittivity and permeability tensors can then be written as

$$\bar{\varepsilon}'_r = \bar{\mu}'_r = \text{diag}[(W_1/W_2)^2, 1, 1]$$  \hspace{1cm} (3.5)

where each requires only one parameter possessing a low value.

It should be noted that a previously reported conformal mapping enabled collimating lens [133] is actually a special case of the coordinate transformation proposed here when we set $L_1W_1 = L_2W_2$ in (3). In this sense, (3) is a general expression for achieving directive emission with homogenous anisotropic medium and can be simplified to transformed media with different anisotropy.

Figure 3.2. Configuration of multiple rotated lens segments to produce multi-beam radiation.
In addition, by applying the coordinate rotation transformation [134] to the above directive emission transformation, more advanced highly directive multi-beam lenses can be synthesized by surrounding the embedded isotropic source with several triangular segments. Importantly, this type of lens can provide an arbitrary number of collimated beams, each radiating in a prescribed direction. As shown in Figure 3.2, the material tensors of each lens segment can be expressed as

\[
\begin{bmatrix}
\frac{w_1^2}{w_2^2} \cos^2 \phi_n + \sin^2 \phi_n & (\frac{w_1^2}{w_2^2} - 1) \cos \phi_n \sin \phi_n & 0 \\
(\frac{w_1^2}{w_2^2} - 1) \cos \phi_n \sin \phi_n & \frac{w_1^2}{w_2^2} \sin^2 \phi_n + \cos^2 \phi_n & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (3.6)

Even though the mathematical expression contains non-zero off-diagonal tensor parameters, the medium can be implemented by rotating the directions of the subwavelength metamaterial building block resonators.

Figure 3.3. Three-dimensional directive emission coordinate transformation. (a) Geometry of the spherical cone shaped virtual space. (b) Geometry of the simplified square pyramid shaped virtual space. (c) Geometry of the square pyramid shaped physical space.

In certain applications, however, multi-beam radiation is desired with beams pointing at specific directions in three dimensions. To fulfill this potential design requirement, the three-dimensional coordinate transformation and associated numerical validations are also provided.
As shown in Figure 3.3(a), the virtual space is an air-filled, spherical cone with a cone angle of \( \Theta_1 \). It is mapped to a square pyramid with an apex angle of \( \Theta_2 \) in the physical space (see Figure 3.3(c)) which has material parameters denoted by \( \varepsilon_r'(x',y',z') \) and \( \mu_r'(x',y',z') \). Similar to the two-dimensional case, the direct mapping between the virtual and physical spaces is not a linear transformation, thus leading to inhomogeneous anisotropic material parameters with extreme values. Here, we can also make a geometrical approximation by using a square pyramid with an apex angle \( \Theta_1 \) the same as the virtual space (see Figure 3.3(b)). The coordinate transformation can thus be expressed as

\[
\begin{align*}
x' &= \frac{L_2 x}{L_1} \\
y' &= \frac{W_2 y}{W_1} \\
z' &= \frac{W_2 z}{W_1}
\end{align*}
\]  

(3.7)

where \((x, y, z)\) and \((x', y', z')\) are the coordinates of the virtual and physical space, respectively. Like the two-dimensional case, simplification can also be made to match the impedance of the transformed medium to free space for the directive radiating beam. The resulting relative permittivity and permeability tensors of the transformed medium are again

\[
\varepsilon_r' = \mu_r' = \text{diag}[(W_1/W_2)^2, 1, 1].
\]  

(3.8)

Similarly, a highly directive multi-beam transformation optics lens can be formed by applying the 3-D coordinate rotation mapping [134] to the material tensors of each lens segment. This type of lens can convert the radiation from an embedded isotropic source to a customized radiation pattern in three dimensions.

### 3.2.2 Numerical Validations

To validate the concept, two- and three-dimensional full-wave simulations were carried out using COMSOL and HFSS finite element solvers, respectively. For the two-dimensional lens examples, only TM\(_z\) polarization with a \( z \)-directed \( E \)-field was used in the simulations for
simplicity. The multiple lens segments are arranged following the concept shown in Figure 3.2. Considering the outer radiation boundary is 5λ away from the source, these simulations exhibit both the near- and far-field behaviors of the lenses. The first lens has four collimated beams uniformly distributed in the x-y plane pointing at \( \phi = \{0^\circ, 90^\circ, 180^\circ, 270^\circ\} \) as shown in Figure 3.4(a), while the second lens has a hexagonal beam uniformly distributed in the x-y plane pointing at \( \phi = \{30^\circ, 90^\circ, 150^\circ, 210^\circ, 270^\circ, 330^\circ\} \) as shown in Figure 3.4(b). To demonstrate the flexibility of controlling the radiated beams, a second lens is displayed in Figure 3.4(c) having five customized collimated beams, each radiating at the desired angles of \( \phi = \{30^\circ, 90^\circ, 165^\circ, 247.5^\circ, 322.5^\circ\} \). All three lenses have a low-value \( \overline{\mu}_r' \) parameter with a magnitude of 0.01 for each of the segments. From the electric field distribution, it is observed that the waves radiated from the central isotropic source are well-collimated, even in close proximity to the source. To study the impact of the variations in the material parameter values on the lens performance, three additional quad-beam lenses similar to the one shown in Figure 3.4(a) were simulated with the magnitude of the \( \overline{\mu}_r' \) parameter in the direction of the radiated beams set to be 0.1, 0.2, and 0.3 for each lens segment. As presented in Figure 3.4(d)-2(f), the beam width of the lenses broadens as the magnitude of the corresponding \( \overline{\mu}_r' \) parameter increases, however, the lenses are still able to maintain highly directive beams in the four desired directions. Since most metamaterial realizations of effective media are less dispersive in the low index band which is on the resonance tail [63], the insensitivity of material parameters makes this type of transformation optics lens suitable for broadband applications.

For the three-dimensional example, we surrounded a previously reported quasi-isotropic radiator [135] shown in Figure 3.5(a) with six lens segments, each having low-value \( \overline{\varepsilon}_r \) and \( \overline{\mu}_r \) parameters with a magnitude of 0.01 in the direction of the radiated beam. Figure 3.5(b) shows the 3D radiation patterns with and without the presence of the transformation optics lens simulated by Ansoft HFSS™ finite element solver. It can be observed that the antenna alone has a
near-isotropic radiation pattern, whereas with the lens present, the radiation pattern exhibits six highly directive beams. It should be noted that not all of the six directive beams have the same linear polarization, which is due to the quasi-isotropic source antenna employed here and not the transformation optics lens since its response is polarization independent.

Figure 3.4. Snapshots of the z-directed near- and far-zone electric field determined via a 2D COMSOL simulation of the transformation optics lens at 3 GHz (a) with four radiated beams uniformly distributed, (b) with six radiated beams uniformly distributed (d) five radiated beams non-uniformly distributed in the x-y plane. Note that in both cases the corresponding $\mu_r'$ parameter in the direction of the radiated beam has a magnitude of 0.01 for each segment of the lens. 2D COMSOL simulations of a similar lens to (a) but when the corresponding $\mu_r'$ parameter in the direction of the radiated beam has a magnitude of (d) 0.1, (e) 0.2 and (f) 0.3 for each segment of the lens.
3.2.3 Wave Propagation in the Transformed Medium

To obtain a better understanding of the characteristics of wave propagation inside the transformed medium and why a medium with one low-value permittivity and permeability tensor parameter would lead to directive emission, it is helpful to consider the dispersion relationship in the medium. Suppose we assume the transformed anisotropic medium has diagonal-only constitutive permittivity and permeability tensors given by

$$\overline{\varepsilon_{tr}} = \varepsilon_0 \text{diag}[\varepsilon_{trx}, \varepsilon_{try}, \varepsilon_{trz}] \quad (3.9)$$

$$\overline{\mu_{tr}} = \mu_0 \text{diag}[\mu_{trx}, \mu_{try}, \mu_{trz}], \quad (3.10)$$

where $\varepsilon_0$ and $\mu_0$ are the permittivity and permeability of free space, respectively. A two-dimensional problem can be decomposed into TM$_z$ polarization with electric field along the $z$-direction and TE$_z$ polarization with magnetic field along the $z$-direction. Under these conditions the dispersion relations can be separated for each polarization and written as

$$\beta_{TMz}^2 / \mu_{trx} + \beta_{TMx}^2 / \mu_{try} = k_0^2 \varepsilon_{trz} \quad (3.11)$$
\[
\beta_{TEz}^2 / \varepsilon_{tx} + \beta_{TEz}^2 / \varepsilon_{ty} = k_0^2 \mu_{trz},
\]

(3.12)

where \( k_0 \) is the free space wave number and \( \beta_{TMz}(TEz)_{x} \) and \( \beta_{TMz}(TEz)_{y} \) are the \( x \)- and \( y \)-components of the wave number in the material for TMz and TEz polarizations, respectively. When \( \varepsilon_{tx} \) and \( \mu_{tx} \) have small near-zero magnitudes while other tensor parameters possess positive values equal to or larger than unity (as dictated by the coordinate transformations), \( \beta_{TMz,y} \) and \( \beta_{TEz,y} \) are limited to be small in order to satisfy the two dispersion relations given in Eq. (3.11) and (3.12). This indicates that inside the transformed medium, the waves are forced to propagate close to the \( x \)-direction with a very small \( y \)-component in the wave vector. The anisotropic zero-index medium has a slim elliptical shaped isofrequency curve with its long axis parallel to the direction of the low-index permittivity/permeability tensor component. The behavior is confirmed in Figure 3.4, where the waves are collimated not only in the far-field, but also in the near-field region inside each lens segment, even in close proximity to the source.

Figure 3.6. (a) Conceptual configuration of the lens surrounding a monopole antenna. (c) Final HFSS model of the metamaterial lens with the monopole feed. The inset shows the monopole located at the middle of the lens.
Figure 3.7. Unit cell geometry. The dimensions are $a=6\text{mm}$, $d=0.76\text{mm}$, $l=4.5\text{mm}$, $g=0.5\text{mm}$, $w=0.5\text{mm}$ and $h=2.5\text{mm}$. Rogers RT/duroid 5880 was used as the substrate. Retrieved effective medium parameters (b) $\varepsilon_x$, (c) $\mu_y$ and (d) $\mu_z$ using scattering parameters calculated at different angles of incidences. The legend indicates the two angles of incidence used for each set of curve.

### 3.3 Lens Design Using Anisotropic Metamaterial and Associated Simulations

To experimentally verify the proposed multi-beam directive emission coordinate transformation, a quad-beam metamaterial lens was designed that tailors the radiation of a G-band quarter-wavelength monopole which nominally radiates omnidirectionally in the $H$-plane around
4 to 5 GHz [122]. Because the radiated electric fields are nearly perpendicular to the ground plane in the \(H\)-plane, the lens needs only to work for the TM\textsubscript{p} polarization. Following the configuration of the two-dimensional quad-beam lens example in Figure 3.4(a), the monopole is surrounded with four triangular anisotropic metamaterial lens segments as presented in Figure 3.6(a). Segments 1 and 3 have a low value of effective \(\mu_{rx}\), and segments 2 and 4 have a low value of effective \(\mu_{ry}\).

### 3.3.1 Metamaterial Unit Cell Design

To design an anisotropic metamaterial with one specific effective permeability tensor parameter having a low value in the G-band, broadside coupled capacitor loaded ring resonators (CLRRs) made of copper [127] are utilized as the building blocks. The geometry of the unit cell is shown in Figure 3.7(a). The CLRRs are printed on each side of the dielectric substrate. The openings of the CLRRs are oriented in opposite directions to eliminate any bi-anisotropy that might be caused by the structural asymmetry [22].

The metamaterial building block was designed using Ansoft HFSS\textsuperscript{TM}, where the scattering parameters were calculated through the application of periodic boundary conditions assigned to the lateral walls in both \(x\)- and \(y\)-directions. A TE polarized plane wave (contained in the \(y\)-\(z\) plane with the electric field along the \(x\)-direction) is assumed to be incident from the upper half-space at an angle \(\theta_i(0° \leq \theta_i \leq 90°)\) with respect to the axis of the metamaterial \(\hat{z}\). Three layers of unit cells were used in the \(z\)-direction in order to take into consideration the coupling between adjacent layers, thus enabling the acquisition of more accurate effective medium parameters. The three material tensor parameters active under TE polarized illumination (\(\mu_y\), \(\mu_z\) and \(\varepsilon_x\)), which can be retrieved using the anisotropic inversion algorithm presented in Chapter 2, are shown plotted in Figure 3.7(b)-(d). It is observed that the inverted metamaterial parameters
extracted from the scattering parameters calculated at different angles of incidence agree well with each other, indicating an angularly independent anisotropic effective medium property. Figure 3.7(b) and Figure 3.7(c) show that $\varepsilon_x$ and $\mu_y$ are weakly dispersive and do not resonate in the band of interest. This provides a stable normalized impedance for waves propagating in the $z$-direction, i.e. the direction of the outgoing beam in the directive emission coordinate transformation. Additionally, the lens is nearly matched to free space ($\sqrt{\mu_y/\varepsilon_x} \approx 0.7$), ensuring low reflection at the lens-air interface. In contrast, the longitudinal tensor parameter $\mu_z$ has a strong Lorentz-shaped resonance at 3.8 GHz and maintains a low value ($0 \leq |\mu_z| \leq 0.4$) throughout the resonance tail over a broad bandwidth (4.2 GHz ~ 5.3 GHz) with a weakly dispersive profile. This low $\mu_z$ region, located away from the resonance band, is very low loss with the magnitude of $\text{Im}\{\mu_z\}$ less than 0.04.

### 3.3.2 Integrated Lens Simulation

The configuration of the final quad-beam lens with the monopole feed and actual metamaterial layers is shown in Figure 3.6(b). The constructed lens is composed of nine concentric anisotropic capacitor loaded ring resonator (CLRRs) layers. The square-shaped layers are each five unit cells tall and decrease in edge length by two unit cells for each successive layer starting from the outside of the lens and moving inward. The length of the outer edge of the lens is 102 mm or about $1.6 \lambda$ at 4.7 GHz, while the height of the lens is 30 mm, about $0.5 \lambda$ at 4.7 GHz. For segments 1 and 3, the axis of the CLRRs is along the $x$-direction, thereby providing a low value of effective $\mu_{rx}$. Conversely, for segments 2 and 4, the axis of the CLRRs is along the $y$-direction, which yields a low value of effective $\mu_{ry}$. A monopole with a length of 14.5 mm is located in the middle of the lens as shown in the inset of Figure 3.7(b). A 14 cm by 14 cm (~2 $\lambda$ ×
2 \lambda) brass plate is used as the ground plane. Copper is used for the metallic patterns to account for the loss resulting from finite conductivity.

![Graph](image)

Figure 3.8. Simulated (— — — —) and measured (—) $S_{11}$ of the monopole antenna (a) without the metamaterial lens, and (b) with the metamaterial lens. The simulated efficiency is also shown in (b).

The simulated input port reflection $S_{11}$ of the monopole with and without the lens is shown in Figure 3.8(a)-(b). For the monopole alone, it has a single resonance at 4.8 GHz with a -10 dB bandwidth of around 0.78 GHz (4.42 GHz ~ 5.20 GHz). However, with the lens present, the -10 dB $S_{11}$ bandwidth is increased to 1.35 GHz (4.20 GHz ~ 5.55 GHz), an increase of about 70%. This can be attributed to two causes, the first being an effective $\varepsilon_{rz}$ (i.e. $\varepsilon_x$ shown in Fig. 3.7(a)) having a value weakly growing from 2.3 to around 2.7, dropping the quality factor of the monopole resonance. Hence, the $S_{11}$ curve is not as sharp as that of the monopole alone at higher frequency. Secondly, the near-field coupling between the monopole and the unit cells on the inner layers of the lens is tuned such that multiple resonances can be observed at 4.3 GHz, 4.75 GHz and 5.4 GHz. These resonances pull the $S_{11}$ below -10 dB over a much broader bandwidth.
Figure 3.9. Simulated and measured $H$-plane realized gain patterns of the monopole antenna with and without the metamaterial lens at (a) 4.25 GHz, (b) 4.50 GHz, (c) 4.85 GHz, (d) 5.10 GHz, and (e) 5.30 GHz. The legends represent simulation without the lens (- - - - - -), simulation with the lens (———), measurement without the lens (- - - - - -), measurement with the lens (———).

Figure 3.9(a)-(e) (blue curves) show the simulated realized gain patterns in the $H$-plane of the monopole with and without the lens at 4.25 GHz, 4.50 GHz, 4.85 GHz, 5.10 GHz and 5.30 GHz. Without the lens, the monopole exhibits an omni-directional pattern in the $x$-$y$ plane with a variation of about 0.7 dB due to the finite ground plane size. The maximum realized gain varies within the range from -1.5 dB to -0.3 dB as the frequency increases. With the lens, however, four highly directive beams can be observed at $0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$. Within the frequency band of 4.20 GHz to 5.30 GHz, the peak realized gain of the four beams grows from 4.3 dB to 5.8 dB, about 5.8 ~ 6.1 dB higher than that of the monopole alone. The half-power beam widths (HPBW) of the four directive beams are approximately $35^\circ$, $34^\circ$, $32^\circ$, $30^\circ$ and $32^\circ$ at the five frequencies.
Figure 3.10. Simulated and measured E-plane realized gain patterns of the monopole antenna with and without the metamaterial lens at (a) 4.25 GHz, (b) 4.50 GHz, (c) 4.85 GHz, (d) 5.10 GHz, and (e) 5.30 GHz. The legends represent simulation without the lens (-----), simulation with the lens (----), measurement without the lens (- - - - -), measurement with the lens (-----).

The realized gain pattern in the E-plane (the elevation plane) is also investigated. In Figure 3.10(a)-(e) (blue curves), the simulated gain patterns in the y-z ($\phi=90^\circ$) plane with and without the lens are plotted. Without the lens, the monopole has maximum radiation at around $40^\circ$ from the horizon ($\theta = 90^\circ$) and nulls along the z-direction ($\theta = 0^\circ$) due to the finite ground plane. The metamaterial lens, on the other hand, redirects the maximum of the radiated beam in a direction closer to the horizon (i.e. only about 6°-10° above the horizon in the range of 4.25-5.30 GHz) where receivers are usually located. The E-plane HPBW is also narrowed from 51-60° to 35-41° in the range of 4.25-5.30 GHz. This effect shows that the lens not only effectively transforms a two dimensional cylindrical wave into four collimated waves in the H-plane, but also pushes the radiated beam towards the horizon in the third dimension. It is noted that the
ground plane size, which is about 2\(\lambda\) by 2\(\lambda\), is the same for comparisons with and without the metamaterial lens.

The simulation results using an effective medium model for the lens are also shown in Figure 3.9 and Figure 3.10, where the anisotropic material parameters retrieved from the unit cell simulations are used for the effective medium lens. It can be seen that the \(S_{11}\) and the radiation patterns agree well between the two models, thus confirming that the discrete metamaterial implementation of the lens indeed behaves as an effective homogeneous anisotropic medium with certain permeability tensor parameters having a low, near-zero value.

The simulated 3D radiation patterns at 5.10 GHz (see Figure 3.11) clearly show that the bowl shaped pattern is transformed into four well-collimated beams pointing in directions near the horizon. The simulated efficiency within the band of interest (4.25-5.30 GHz) is plotted in Figure 3.8(b). Since the metamaterial lens is operating away from resonance, it does not introduce a significant amount of loss, thus maintaining an efficiency above 90% throughout the band.

Figure 3.11. Simulated 3D realized gain pattern of the monopole antenna with and without the metamaterial lens at 5.10 GHz.
3.4 Experimental Verification

A lens prototype and a monopole antenna platform were fabricated and characterized to validate the theory and numerical design of the transformation optics lens. The fabricated sample is presented in Figure 3.12 with an enlarged inset figure showing the inner monopole antenna. An Agilent E8364B network analyzer was used to characterize the reflection coefficient magnitude of the monopole with and without the lens. As the results in Figure 3.8(a)-(b) show, good agreement can be found between simulations and measurements not only in terms of the -10dB bandwidth but also the resonance positions. The measured $S_{11}$ of the monopole alone has a resonance at 4.75 GHz, with a -10 dB bandwidth of 0.76 GHz (4.42 GHz ~ 5.18 GHz). When the monopole is surrounded by the metamaterial lens, the measured $S_{11}$ is below -10 dB from 4.22 GHz to 5.60 GHz with three resonances located at 4.35 GHz, 4.7 GHz and 5.45 GHz, exhibiting strong correspondence with the simulated results. Even at low frequencies, good agreement is observed in the minor resonances around 3.7 GHz and 3.95 GHz, which can be attributed to the near field coupling to the lens.
The realized gain patterns of the monopole antenna with and without the lens were characterized in an anechoic chamber with an automated antenna movement platform. In Figure 3.9 and Figure 3.10 (black curves), the measured gain patterns in both the $H$-plane and $E$-plane at 4.25 GHz, 4.50 GHz, 4.85 GHz, 5.10 GHz and 5.30 GHz are shown, where good agreement is seen with the simulated results. The discrepancies are primarily due to minor inaccuracies in fabrication, non-ideal effects of the test setup, and noise in measurement. For the case without the metamaterial lens, the monopole has a nearly omni-directional radiation pattern in this band. The maximum measured realized gain values in the $H$-plane vary between -1.6 dB to -0.3 dB. With the lens present, four high gain beams are located at 0°, 90°, 180° and 270° with a measured realized gain varying between the range from 4.3 dB to 5.9 dB, yielding about 5.9 dB to 6.2 dB of realized gain improvement. The measured average HPBW's of the four directive beams are approximately 36°, 37°, 30°, 31° and 34° at the five frequencies considered. The measured $E$-plane patterns confirm the beam bending effect in the $\theta$-direction. Without the lens, the monopole alone has a measured beam maxima moving from 45° to 40° off horizon as the frequency increases. With the lens present, the beam maxima is maintained at approximately 8° to 12° from the horizon. In all, the experiment verifies the 3D collimating effect of the metamaterial transformation optics lens in reshaping the radiation of the embedded monopole antenna into multiple highly directive radiated beams.

3.5 Overview

In this chapter, we have proposed two- and three-dimensional embedded coordinate transformations that generate multiple highly directive radiated beams from an embedded isotropic source, each pointing in a desired direction. The coordinate transformation introduced here offers anisotropic but spatially invariant material parameters that can be readily implemented
by metamaterial building blocks. In addition, the performance of the coordinate transformation is not sensitive to small variations in the material parameter values, thus enabling broad operating bandwidth. A microwave frequency metamaterial lens fed by a simple monopole antenna was designed, fabricated and tested, exhibiting strong agreement between simulation and measurement. The metamaterial lens achieves a quad-beam radiation pattern over a 1.26:1 bandwidth with around 5.9 to 6.2 dB of realized gain improvement in the $H$-plane compared to the monopole alone. Additionally, it broadens the impedance bandwidth of the monopole by 70%. The simple coordinate transformation suggests possibilities for the design and synthesis of more practical broadband metamaterial devices, including lenses that are able to tailor the radiation properties of microwave/millimeter wave wireless antennas and even perhaps optical nano-antennas [136].
Chapter 4
Low-Profile High-Gain Anisotropic Metamaterial Coating for Slot Antennas

4.1 Introduction

To date, metamaterial technologies have been applied to a variety of different antennas yielding a wealth of functionality improvements, including reducing the thickness of Fabry-Pérot cavity antennas [125, 128], extending the angular steering range of leaky wave antennas [62, 137], broadening the impedance bandwidth of monopole [138], microstrip [67], and patch [68] antennas, reducing the sidelobe level and cross polarization of horn antennas [63], enhancing the directivity of monopole [64, 139], dipole [140, 141], patch [142, 143], and planar Vivaldi [144] antennas, and so on.

Among these applications, achieving unidirectional radiation with high gain at broadside from a single compact antenna is of particular interest for point-to-point communications, wireless power transfer, radar systems, and various other wireless systems [122, 145]. Conventional approaches frequently employ FP cavities [126, 146] to obtain high directivity within a narrow bandwidth due to the high quality factors of the resonating cavities. When they are constructed with a ground plane, the device thickness is usually on the order of one half of the operating wavelength. Recent advanced FP cavity designs employing AMC surfaces allow the total device thickness to be reduced to around one quarter of the wavelength [128]. Further reduction in the antenna profile can be achieved, e.g. down to $\lambda/9$, by using metamaterial surfaces designed with customized reflection phases for both the top and bottom covers of the cavity [147]. However, most of these FP cavity antennas exhibit an enhanced directivity and matched input impedance only over a very narrow bandwidth, which considerably limits their utility.
Apart from the FP cavity related technique, another metamaterial approach for increasing the directivity has recently been theoretically proposed and experimentally demonstrated by embedding the electromagnetic source within a volumetric ZIM/LIM lens [33, 73]. The working principle is based on the well-known Snell's law of refraction that states when light passes through from a low index medium to a high index medium, it is bent towards directions that are normal to the interface between the two media. It has also been demonstrated that volumetric anisotropic ZIMs (AZIMs), with one of the permittivity and/or permeability tensor parameters approaching zero, can give rise to directive radiation [133, 139, 148]. In contrast to their isotropic counterparts, these AZIMs are capable of directing the propagation of electromagnetic waves not only at interfaces, but also inside the metamaterial as enforced by their unique dispersion properties, allowing for enhanced wavefront control. It has also been shown that such AZIMs can be used as a volumetric AMC antenna backing to confine the radiation energy into a narrower angular range [127]. However, such ZIM or AZIM lenses are usually electrically large in all three dimensions, unavoidably resulting in increased net size and weight of the device.

More recently, it has been proposed that thin single- and bi-layer grounded metamaterial slabs with either isotropic negative or zero index of refraction can give rise to novel electromagnetic properties, such as surface wave guidance [149] and suppression [150], and frequency dependent directive radiation [151-154]. In particular, it has been shown theoretically that the directive radiation is caused by leaky wave modes of the metamaterial slab, which facilitates the realization of a conical and/or a pencil beam. The structures considered thus far are generally infinite or finite but electrically long, and support leaky waves with phase velocities slightly higher than that in free-space. As a result, the direction of beam maximum drifts away from broadside and varies as a function of frequency, rendering them unsuitable for maintaining broadband high gain unidirectional radiation at broadside [62, 137, 151-154].
In this chapter, we propose and experimentally realize a low-profile, high-gain, compact-footprint antenna that is composed of a subwavelength thick AZIM coating and a substrate-integrated waveguide (SIW) [155] fed slot antenna. The SIW is another emerging technology that has been widely applied in microwave and millimeter wave components [156, 157] and antennas [158, 159] due to its planar topology which can be readily connected to other planar guided-wave transmission lines. In Section 4.2, we study the leaky modes supported by a grounded anisotropic slab. Particularly, the effect of non-ideal values for the z component of the material tensor parameter on the leaky mode propagation constant is examined; this analysis is important for addressing metamaterial dispersion and fabrication tolerances. The AZIM slab truncation effects on the radiation patterns are also investigated to obtain stable unidirectional radiation at broadside over a reasonably wide bandwidth. In Section 4.3, a metamaterial is developed to realize the required anisotropic zero/low index properties and a SIW fed slot antenna is designed to operate at around 5.8 GHz. Full-wave simulations are carried out in order to compare the antenna performance of the slot with and without the AZIM coating. A dispersive homogeneous effective medium slab is also simulated for comparison with the results using the actual discrete metamaterial. In Section 4.4, experimental results are presented, which are found to be in strong agreement with numerical simulations.

4.2 Leaky Modes of Grounded Anisotropic Slab

In this section, we first study the leaky modes supported by a grounded anisotropic slab with analytical expressions of the field equations and numerical calculations of the leaky mode propagation constants. Then, with the help of the full-wave electromagnetic solver HFSS, we investigate the effects of a truncated AZIM slab on the radiation properties of an infinite magnetic line source.
4.2.1 Field Equations of a Grounded Anisotropic Slab

As shown in Figure 4.1(a), the grounded anisotropic slab considered here is infinite in the $y$ direction with a thickness of $t$ and anisotropic permittivity and permeability tensors expressed as

\[
\vec{\varepsilon} = [\varepsilon_x, \varepsilon_y, \varepsilon_z] = \varepsilon_0 [\varepsilon_{rx}, \varepsilon_{ry}, \varepsilon_{rz}], \quad (4.1)
\]

\[
\vec{\mu} = [\mu_x, \mu_y, \mu_z] = \mu_0 [\mu_{rx}, \mu_{ry}, \mu_{rz}], \quad (4.2)
\]

The $y$-component of the complex propagation constant ($k_y$) of the leaky wave is studied under the following conditions. A time-harmonic dependence $e^{j\omega t}$ is assumed and used throughout the paper. Since no field variation is assumed in the $x$ direction, the two dimensional nature of the configuration allows us to separate the problem into TE$_z$ and TM$_z$ modes. For the TE$_z$ mode, the set of material tensor parameters ($\varepsilon_{rx}$, $\mu_{ry}$, and $\mu_{rz}$) are pertinent, while for the TM$_z$ mode, the other set of material tensor parameters ($\mu_{rx}$, $\varepsilon_{ry}$, and $\varepsilon_{rz}$) are responsible for the wave propagation properties. For simplicity, only the TM$_z$ mode is considered here since we will employ a slot antenna, which represents a quasi-TM$_z$ source, in the later sections. To write out the field equations in both regions inside the anisotropic slab and in the free-space above the
grounded slab, two vector potentials, $A^{fs}_z$ and $A^{an}_z$, are defined for these two regions, respectively, which are expressed as

$$A^{fs}_z = 2A^{fs}_z = A_{fs}e^{-jk_0xz}e^{-jk_yTM_y}$$  \hspace{1cm} (4.3)

$$A^{an}_z = 2A^{an}_z = A_{an}\sin(k_zz)e^{-jk_yTM_y}.$$  \hspace{1cm} (4.4)

The Sine function is selected for the vector potential in the grounded anisotropic slab due to the vanishing electric field on the PEC ground plane. The propagation constant $k^TM_y$ is in the $y$ direction, while $k^TM_{0z}$ and $k^TM_z$ are the propagation constants in the $z$ direction in free space and in the grounded anisotropic slab, respectively. The field equations may then be written as

$$H^{an}_x = \frac{1}{\mu_x} \frac{\partial A^{an}_y}{\partial y} = \frac{-A_{an}k^TM_y}{\mu_x} \cos(k^TM_z z)e^{-jk^TM_y y},$$  \hspace{1cm} (4.5)

$$E^{an}_y = -\frac{1}{\omega\varepsilon_0\mu_x} \frac{\partial^2 A^{an}_y}{\partial z \partial y} = \frac{-j(k^TM_y)^2A_{an}}{\omega\varepsilon_0\mu_x} \sin(k^TM_z z)e^{-jk^TM_y y},$$  \hspace{1cm} (4.6)

$$E^{an}_z = \frac{1}{\omega\varepsilon_0\mu_x} \frac{\partial^2 A^{an}_z}{\partial y^2} = \frac{-k^TM_yk^TM_zA_{an}}{\omega\varepsilon_0\mu_x} \cos(k^TM_z z)e^{-jk^TM_y y},$$  \hspace{1cm} (4.7)

$$H^{fs}_x = \frac{1}{\mu_0} \frac{\partial A^{fs}_z}{\partial y} = \frac{jA_{fs}k^TM_{0z}}{\mu_0} \cos(k^TM_z z)e^{-jk^TM_y y},$$  \hspace{1cm} (4.8)

$$E^{fs}_y = -\frac{1}{\omega\varepsilon_0\mu_0} \frac{\partial^2 A^{fs}_y}{\partial z \partial y} = \frac{-jA_{fs}(k^TM_{0z})^2}{\omega\varepsilon_0\mu_0} \cos(k^TM_z z)e^{-jk^TM_y y},$$  \hspace{1cm} (4.9)

$$E^{fs}_z = \frac{1}{\omega\varepsilon_0\mu_0} \frac{\partial^2 A^{fs}_z}{\partial y^2} = \frac{jA_{fs}k^TM_{0z}k^TM_z}{\omega\varepsilon_0\mu_0} \cos(k^TM_z z)e^{-jk^TM_y y}.$$  \hspace{1cm} (4.10)

By enforcing the boundary conditions at $z=t$ in Eq. (4.5) – Eq.(4.10) and utilizing the TM$_2$ mode dispersion relations for free space and the anisotropic medium, which are given by

$$(k^TM_y)^2 + (k^TM_{0z})^2 = k^2_0,$$  \hspace{1cm} (4.11)

$$\frac{(k^TM_y)^2}{\varepsilon_{rx}} + \frac{(k^TM_z)^2}{\varepsilon_{ry}} = k^2_0\mu_{rx},$$  \hspace{1cm} (4.12)

the dispersion equation of the propagation constant $k^TM_y$ for the TM$_2$ mode can be expressed as

$$\sqrt{k^2_0 - (k^TM_y)^2} + j \sqrt{k^2_0\varepsilon_{rx} - (k^TM_y)^2}\tan \left( \sqrt{k^2_0\varepsilon_{rx} - (k^TM_y)^2} \frac{\varepsilon_{ry}}{\varepsilon_{rx}} t \right) = 0.$$  \hspace{1cm} (4.13)
This equation can also be arrived at using a transverse equivalent network in the $z$ direction [146], as shown in Figure 4.1(b). The dispersion equation and field expressions for the TE$_z$ mode can be readily obtained using the duality theorem.

4.2.2 Properties of the Leaky Modes Supported by the Grounded Anisotropic Slab

Figure 4.2. TM$_z$ dispersion curves for a grounded anisotropic slab with $\varepsilon_{ry} = 2.4$, $\varepsilon_{rx} = 1$ and varying $\varepsilon_{rz}$ as a function of frequency. (a) $\beta_{yTM}/k_0$. (b) $\alpha_{yTM}/k_0$. (c) $\beta_{0zTM}/k_0$. (d) $\alpha_{0zTM}/k_0$. $f_0$ denotes the frequency at which the thickness of the grounded slab equals a wavelength, i.e. $t = \lambda_0$.

The propagation constant $k_{yTM} = \beta_{yTM} + j\alpha_{yTM}$ of the leaky modes supported by the grounded anisotropic slab can be calculated numerically using Eq. (4.13). Figure 4.2(a) and Figure 4.2(b) show the real and imaginary parts of $k_{yTM}$ (normalized to $k_0$), as a function of
frequency for different $\varepsilon_{rz}$ values. The values of $\varepsilon_{ry}$ and $\mu_{rx}$ are fixed to be 2.4 and 1, while $f_0$ denotes the frequency at which the thickness of the slab ($t$) is equal to one wavelength. The corresponding values of the wave number along the $z$ direction in the free-space region, which is expressed as $k_{0z}^{TM} = \beta_{0z}^{TM} + ja_{0z}^{TM}$, are also displayed in Figure 4.2(c) and Figure 4.2(d).

To confirm the accuracy of the numerically solved solutions, the ordinary isotropic case is first considered where $\varepsilon_{rz} = \varepsilon_{ry} = 2.4$. The results corroborate previously published works on leaky modes in a grounded slab with isotropic material [146], [154]. Specifically, multiple solutions can be found including the physical surface wave (SW), leaky wave (LW), and improper nonphysical SW (IN), all of which are labeled in the figures. The leaky wave band is narrow and appears to be highly dispersive. The value of $\beta_{y}^{TM}$ is only slightly smaller than that of $k_0$. When the value of $\varepsilon_{rz}$ decreases to 0.8, the characteristics of the propagation constant change drastically. First, the second branch cut point disappears due to the fact that the slab no longer supports the propagation of a higher order surface wave mode in the frequency range being considered. The improper surface wave modes also do not exist and transform completely into the extension of the leaky mode. Secondly, the leaky mode band of operation is broadened from $0.088 \sim 0.244f_0$ in the isotropic case to $0.094 \sim 0.8f_0$ for the AZIM case, (i.e. nearly a 450% increase). The smallest value of $\beta_{y}^{TM}$ is also lowered from $0.86k_0$ to $0.75k_0$. Most importantly, $\beta_{y}^{TM}$ exhibits low dispersion over a much broader frequency range, i.e. less than a 10% change in the range from $0.29f_0$ to $0.8f_0$, which indicates the possibility of achieving a stable direction of beam maximum over an ultrawide bandwidth. Finally, the fundamental physical surface wave mode transforms into an improper (i.e. non-physical) surface wave mode since $\varepsilon_{rz}\mu_{rx} < 1$. Despite similar line shapes for $\beta_{y}^{TM}$, the $a_{0z}^{TM}$ parameters of the two cases develop into curves with opposite signs. The negative sign indicates an evanescently decaying wave, which is
physical, while the positive sign represents an evanescently growing wave, which is non-physical, in the free-space region.

As the value of $\varepsilon_{rz}$ is further decreased, the grounded anisotropic slab can be referred to as a grounded anisotropic low index slab. Several interesting and useful characteristics can be identified in $k_{y}^{TM}$ and $k_{0z}^{TM}$. First, the value of $\beta_{y}^{TM}$ for the improper nonphysical SW mode at frequencies larger than even 0.05$f_0$ is dramatically increased; these values are not included in the display range of the figure. For instance, $\beta_{y}^{TM}$ reaches 27.01$k_0$ at $f > 0.05f_0$ with $\varepsilon_{rz} = 0.417$. Secondly, as $\varepsilon_{rz}$ approaches zero, both $\beta_{y}^{TM}$ and $\alpha_{y}^{TM}$ of the leaky modes become smaller and their profiles also get flatter. This is beneficial for a stable radiated beam near broadside across a wide frequency range, since the radiated beam direction is given by $\theta_m = \sin^{-1}(\beta_{y}^{TM}/k_0)$.

Correspondingly, the curves of $\beta_{0z}^{TM}$ and $\alpha_{0z}^{TM}$ also become less dispersive with the value of $\beta_{0z}^{TM}$ approaching unity and the value of $\alpha_{0z}^{TM}$ approaching zero. This means that the wave in the free-space region above the grounded anisotropic low index slab has a propagation constant very close to that of the free space with a small attenuation. In theory, when $\varepsilon_{rz}$ is extremely close to zero, $k_{y}^{TM}$ is forced to be zero in order to satisfy the dispersion relation (12) which has an elliptical isofrequency curve with a near-vanishing short axis [21]. Physically, near-zero-$\varepsilon_{rz}$ means quasi-infinite phase velocity in the $y$ direction and vanishing $z$ component of the electric displacement ($i.e. D_z$) in the grounded slab, which forces the $z$ component of the electric field ($E_z$) in the free-space region to approach zero due to the boundary condition at the slab-air interface.

4.2.3 Truncation Effect of the Anisotropic Low Index Slab on the Radiation Pattern

Following the previous discussions of the leaky mode propagation constants supported by a grounded anisotropic (low index) slab, we now investigate its effect on the radiation from an
embedded source. Truncated structures are considered here since, in practice, a compact antenna footprint is often desired.

Figure 4.3. (a) Configuration of a 1-mm-wide slot source covered by the grounded anisotropic slab and backed by an absorber. The structure is infinite in the \( x \) direction. \( t_s = 12\, \text{mm}, L_y = 120\, \text{mm} \) or \( 1200\, \text{mm} \), and the solving frequency is 5 GHz. (b) The normalized far-zone \( E \)-field magnitude in the \( y-z \) plane for the structure with \( L_y = 1200\, \text{mm} \) and (c) \( L_y = 120\, \text{mm} \). (d) Direction of beam maximum for structures with \( L_y = 1200\, \text{mm} \), \( L_y = 120\, \text{mm} \), and an infinite slab using the equation \( \theta_m = \sin^{-1}(\beta_y^M/k_0) \).

As illustrated in Figure 4.3(a), a 1-mm-wide slot that is infinitely long in the \( x \) direction is employed as a source which produces a near omnidirectional TM\(_y\) wave in the \( y-z \) plane. In the simulation domain of HFSS, the infinite slot is excited by placing an infinitesimal current source across the slot in the \( x \) direction. A perfectly matched absorbing layer is placed underneath the slot to absorb the direct radiation from the slot to the \( -z \) half space. On top of the ground plane, a
thin anisotropic slab having the same width in the y direction as that of the ground plane is employed. The material parameters of the grounded slab are set to be $\varepsilon_{ry} = 2.4$ and $\mu_{rx} = 1$, with varying $\varepsilon_{rz}$ values. This configuration corresponds well to the structure discussed in the previous sub-section on the leaky mode analysis. Two values of slab widths in the y direction are considered: 1200 mm and 120 mm, which are $20\lambda$ and $2\lambda$ at 5 GHz, respectively. The slab in the second case can be regarded as a strongly truncated leaky slab [160], which will be shown to yield radiation properties very different from the one with a width of $20\lambda$ in the first case.

Figure 4.3(b) and Figure 4.3(c) show the normalized far-zone $E$-field magnitude in the y-z plane radiated from the $20\lambda$ and $2\lambda$ wide grounded anisotropic slabs excited by the infinite 1-mm-wide slot, respectively. Due to the mirror symmetry of the structure about the $x$-$z$ plane, only the patterns in the range of $0^\circ \leq \theta \leq 180^\circ$ are plotted for clarity. It can be seen from Figure 4.3(b) that, when the structure is electrically long, the beam steers as a function of the value of $\varepsilon_{rz}$, which determines the complex propagation constant of the leaky mode in the slab. Only when $\varepsilon_{rz}$ has an extremely small value does the beam point exactly at broadside. As $\varepsilon_{rz}$ increases, the beam starts to deviate off broadside towards end-fire, which is commonly seen in conventional leaky wave antennas [146]. The normalized far-zone $E$-field magnitude for the same slot without the anisotropic coating is also shown in Figure 4.3(b). The normalized radiation field magnitude remains larger than 0.8 for $\theta \leq 80^\circ$ with multiple ripples and the beam maximum is located at $\theta = 74^\circ$ due to the diffraction at the edge of the slab.

In contrast, when the slab has an electrical length of only $2\lambda$, the radiation patterns exhibit distinctly different properties, as shown in Figure 4.3(c). The beam maximum maintains at broadside with a narrow beamwidth for a wide range of $\varepsilon_{rz}$ values (i.e. up to 0.3). Only at large values does the beam drift away from broadside. This indicates that a strongly truncated, thin leaky slab can produce stable unidirectional radiation at broadside within a wide bandwidth, which corroborates previous findings on the truncation effects of conventional leaky wave
antennas [160, 161]. Such properties can give rise to low-profile, small footprint, and light weight unidirectional high-gain antenna at broadside with a reasonable bandwidth. As a comparison, the normalized far-zone E-field magnitude for the same slot without the anisotropic coating is also shown in Figure 4.3(c). Due to diffraction caused by the compact ground plane size, the beam maximum occurs at $\theta = 32^\circ$. This further illustrates the significance of the anisotropic coating, which is capable of removing the commonly encountered edge diffraction caused by a finite ground plane.

The direction of beam maximum as a function of the value of $\varepsilon_{rz}$ for the three cases, namely the 1200 mm and 120 mm long slab, and the infinite slab, are shown in Figure 4.3(d). For the infinite slab, the direction of beam maximum is calculated according to $\theta_m = \sin^{-1}(\beta_y^{TM}/k_0)$. The results obtained for the 1200 mm long slab correspond well to those of the infinite slab, especially when $\varepsilon_{rz}$ is relatively small. The discrepancy which develops as $\varepsilon_{rz}$ increases is primarily caused by the edge diffraction of the finite ground plane. For the case of the 120 mm long slab, the beam maximum maintains at broadside for $\varepsilon_{rz}$ smaller than 0.3. Considering the weakly dispersive properties inherent to the low-index band compared to the negative-index band of practical metamaterial designs, the strongly truncated grounded AZIM coating can provide a broad operational bandwidth.

4.3 Coating Design Using Anisotropic Metamaterial and Associated Simulations

In this section, we first provide the metamaterial design to realize the desired anisotropic properties of the coating. A SIW fed slot antenna is then designed as the radiator to be coated. Finally, the AZIM coating and the SIW fed slot antenna are integrated and tuned to generate a low-profile, high-gain antenna at the 5.8 GHz WLAN band.
4.3.1 Metamaterial Unit Cell Design

Figure 4.4. (a) Geometry of the end-loaded dipole unit cell for constructing anisotropic ZIM. The dimensions are \( p = 6.5 \, \text{mm} \), \( b = 5.35 \, \text{mm} \), \( a = 0.7 \, \text{mm} \), \( d = 0.508 \, \text{mm} \), and \( g = 2.8 \, \text{mm} \). The substrate material is Rogers RT/duroid 5880 with a dielectric constant of 2.2 and a loss tangent of 0.009. (b) The retrieved effective medium parameters \( \mu_{rx} \), \( \varepsilon_{ry} \), and \( \varepsilon_{rz} \).

To realize the anisotropic zero/low index property mentioned above for the 5.8 GHz band, periodic end-loaded dipole resonators (ELDRs) are employed. The self-inductance of the wire and the capacitance provided by the gaps between the meandered end-loaded arm traces give rise to an electric resonance. The unit cell geometry and dimensions are shown in the inset of Figure 4.4(a). To retrieve the effective medium parameters \( (\varepsilon_{ry}, \varepsilon_{rz}, \mu_{rx}) \), an infinite doubly periodic array of ELDRs was simulated in HFSS, with periodic boundary conditions and Floquet ports properly assigned to the boundaries of the simulation domain, together with an anisotropic parameter inversion algorithm presented in Chapter 2. The retrieved \( \varepsilon_{ry}, \varepsilon_{rz}, \mu_{rx} \) are shown in Figure 4.4(b), among which \( \varepsilon_{ry} \) and \( \mu_{rx} \) are virtually non-dispersive with values around 2.4 and 1, respectively. \( \varepsilon_{rz} \) exhibits a Lorentz-shaped electric resonance with an effective plasma frequency of 5.38 GHz. Furthermore, because the resonance tail is weakly dispersive, the value of \( \varepsilon_{rz} \) remains positive and below 0.15 within the broad frequency range of 5.4 - 6.1 GHz. In this frequency band, the practical dispersive metamaterial has \( \beta_{y}^{TM} \) that varies from a near-zero value to 0.35. Simulations with multiple numbers of unit cells in the direction of wave propagation...
were also conducted, showing consistency in the retrieved effective medium parameter values (not shown here) compared to those obtained from a single layer structure, which corroborates results previously reported on weakly coupled metamaterial structures [162]. This geometry yields a broader low index bandwidth compared to that of commonly used subwavelength electric LC resonator [17] due to the lower quality factor provided by the large capacitance inherent in the meandered arms.

Figure 4.5. (a) Configuration of infinite array simulations for an actual AZIM coating, a dispersive effective medium slab, and the slot alone. The structures are infinite in the y direction with a periodicity of 6.5 mm. The finite sized PEC plane is 92 mm long in the x direction (underneath an AZIM coating with 14 cells). A perfectly matched absorbing slab is placed underneath the slots in the simulations to absorb the radiation in the -z half space. (b) Snapshots of E-field distributions in the upper x-z plane for the three cases. The blocks with dashed lines indicate the position of the metamaterial coating or the effective medium slab. (c) Normalized radiated power for the three cases at 5.4 GHz (i.e. close to the effective plasma frequency of the
metamaterial). All the curves are normalized to Case C at broadside ($\theta = 0^\circ$). (d) Normalized radiated power at broadside ($\theta = 0^\circ$) as a function of frequency.

Before applying the discrete structured metamaterial coating to a practical antenna, full wave simulations were carried out to investigate the radiation properties of an infinite slot covered by a finite AZIM coating. The setup is similar to the simulations presented in Section 4.2.3. As illustrated in Figure 4.5(a), three cases were considered: (A) a 1mm wide slot covered by a single layer MM slab consisting of 14 cells in the $x$ direction, (B) a 1mm wide slot covered by a dispersive effective medium slab with the retrieved $\varepsilon_r$ and $\mu_r$ tensors, and (C) a 1mm wide slot alone. The structures are infinitely long in the $y$ direction with only the field components $H_y$, $E_x$, and $E_z$ existing in the far-zone, corresponding to the TM mode. The infinite slot, excited by an infinitesimal current source in the $x$ direction, sufficiently approximates a TM line source. The far-field patterns (normalized to the value of Case C at broadside) in the upper $x$-$z$ plane are presented in Figure 4.5(c) for the three cases at 5.4 GHz. It can be observed that for the slot alone, the radiated wave is maximum close to $\theta = \pm 40^\circ$ due to diffraction caused by the finite size of the ground plane in the $x$ direction. However, with the actual metamaterial or the effective medium slab the beam maximum is located at broadside and exhibits a significant 9-fold enhancement. The radiated beam is sharpened and the radiation in other directions is greatly suppressed compared to that generated by the slot alone. The enhancement of radiated power at broadside as a function of frequency is shown in Figure 4.5(d). The maximum enhancement occurs at around 5.4 GHz, corresponding well to the effective plasma frequency predicted from the unit cell simulation. Within a broad frequency range from 5.2 - 6.4 GHz, the broadside radiation enhancement remains above 4.5-fold. The drop in the enhancement factor is caused by the dispersive nature of the leaky metamaterial slab, as discussed previously. However, compared to conventional directive leaky-wave antennas, stable unidirectional radiation at broadside is achieved within a much broader bandwidth. The drop in the enhancement at frequencies below
the peak is attributed to the fact that a partially negative anisotropic slab no longer supports a leaky mode with small tangential wave number. Overall, the numerical simulations show that the proposed subwavelength AZIM coating provides an efficient way of suppressing commonly encountered edge diffraction. In addition, the good agreement between the simulation results using the actual metamaterial and the effective medium slab justifies the homogenization approximation employed here, which is assumed valid due to the subwavelength size of the unit cells.

The corresponding $E$-field distributions at 5.4 GHz (see Figure 4.5(b)) in both the near- and far-field show well-collimated beams emanating from the slot covered with the actual metamaterial or the effective medium slab. The symmetric field tapering along the $x$ direction, observable in both the actual metamaterial and the effective medium slab coatings, reveals the leaky nature of the structure. This also differentiates its physical operation from the volumetric AZIM lenses. Since 5.4 GHz is very close to the effective plasma frequency, the quasi-infinite phase velocity gives rise to a nearly ideal in-phase response over the entire aperture. Notably, the weak fields near the ends of the coating ensure the possibility of shrinking the footprint and lowering the profile of the device without sacrificing performance.

4.3.2 SIW Fed Slot Antenna Design

In order to design a feeding antenna for the AZIM coating, a half-wave slot fed by a SIW is adopted here due to its low profile compared to cavity backed or conventional waveguide fed slots [146]. The schematic view of the SIW fed slot antenna is shown in Figure 4.6. It is comprised of a $50\Omega$ microstrip and a shorted SIW with a longitudinal slot cut on its broad wall. The SIW is equivalent to a conventional rectangular waveguide filled with dielectric that only
supports $TE_{00}$ modes due to its subwavelength height. The following empirical formula can be employed to calculate the equivalent waveguide width [163]

$$W_{\text{eff}} = W_{\text{SIW}} - 1.08 \frac{4r^2}{2r + ds} + 0.1 \frac{4r^2}{W_{\text{SIW}}}.$$  \hspace{1cm} (4.14)

The characteristic impedance of the SIW can then be calculated by

$$Z_{\text{SIW}} = \frac{h}{W_{\text{eff}}} \frac{377}{\sqrt{1 - (\lambda_0/2W_{\text{eff}}\varepsilon_r)^2}}$$  \hspace{1cm} (4.15)

where $h$ is the substrate thickness and $\varepsilon_r$ is the substrate dielectric constant. Because the value of $h$ is very small, the impedance of SIW is smaller than that of the feeding microstrip. Hence, a tapered microstrip is used for impedance matching between the 50Ω feedline and the SIW. In order to achieve a magnetic dipole mode in the slot for efficient radiation, the length of the slot is chosen to be around $\lambda/2$ at 5.8 GHz. The distance between the center of the slot to the shorted wall of the SIW in the $x$ direction is set to be about $3\lambda/4$ at 5.8 GHz, which allows the standing wave peak to be located at the center of the slot. The slot also has a slight offset from the center line of the SIW in the $y$ direction, following the design considerations for the slot antenna fed by a conventional rectangular waveguide [146].

All the geometrical dimensions were finely tuned (see Figure 4.6 captions) for good impedance matching and radiation properties. The simulated $S_{11}$ of the SIW fed slot antenna is shown in Figure 4.7(a). The $S_{11}$ is below -10 dB from 5.58 to 6.03 GHz. The normalized $E$-plane ($y$-$z$ plane) and $H$-plane ($x$-$z$ plane) radiation patterns at 5.6, 5.8, and 6.0 GHz are presented in the top parts of Figure 4.8(a)-(f). As a comparison, the normalized radiation patterns for the same antenna on a ground plane infinite in the $y$ direction are also plotted. It can be seen that, for the infinite ground plane case, omnidirectional radiation exists in the $E$-plane, while the beam maximum is directed at broadside in the $H$-plane. However, for a finite ground plane with a size of $1.7\lambda$ in the $y$ direction, significant edge diffraction can be observed in the $E$-plane, showing two peaks located at around $40^\circ$-$45^\circ$ off broadside. Such double-peak radiation in the $E$-plane,
caused by the finite ground plane, is undesired for unidirectional radiation due to the gain drop occurring at broadside. Different from the contrast exhibited in the E-plane patterns, the H-plane patterns are similar for the two cases since the ground plane is maintained at the same size in the x direction. The simulated broadside gain of the antenna on the finite and infinite (in the y direction) ground planes are shown in Figure 4.9(a). For the infinite ground plane case, the broadside gain is maintained at around 5 dBi while for the finite ground plane case the gain varies from 3 to 3.8 dBi. The simulated efficiency of the antenna is plotted in Figure 4.10(a), exhibiting a value higher than 95% from 5.6 to 6 GHz, where the $S_{11}$ is below -10 dB.

Figure 4.6. Schematic view of the SIW fed slot antenna. The dimensions are $L = 133$, $W = 92.5$ mm, $W_{ms} = 4.83$ mm, $L_{ms} = 19$ mm, $W_{mat} = 19.3$ mm, $L_{mat} = 23.5$ mm, $r = 1$ mm, $dis = 0.5$ mm, $L_{SIW} = 86.75$ mm, $W_{SIW} = 22.5$ mm, $W_{slot} = 1.33$ mm, $L_{slot} = 24.4$ mm, $L_{ox} = 32.55$ mm, $W_{off} = 11.24$ mm. The substrate material is Rogers RT/duroid 5880 with a dielectric constant of 2.2 and a loss tangent of 0.009. The substrate thickness is 1.575 mm. The inset in the top right corner shows the SIW fed slot antenna symmetrically covered by the AZIM coating with 5 rows in the y direction each containing 14 unit cells.
Figure 4.7. (a) Simulated and measured $S_{11}$ of the SIW fed slot antenna without the AZIM coating. (b) Simulated and measured $S_{11}$ of the SIW fed slot antenna with the AZIM coating, including the dispersive homogeneous effective medium AZIM coating.

4.3.3 Integrated Simulations

The designed AZIM coating is added directly on top of the SIW fed slot antenna, as shown in the inset of Figure 4.6(a). The single-layer metamaterial coating consists of 5 strips in the $x$ direction, each having 14 unit cells in the $y$ direction. The 14 cells are oriented symmetrically in the $y$ direction to maintain a mirror symmetry, which helps to reduce the beam squinting of the AZIM coated slot. It should be noted that the thickness of the AZIM coating is only 6.5 mm, i.e. $\sim 0.12\lambda$ at 5.8 GHz, which is much thinner than conventional FP cavities [126, 146], or recently proposed bulky ZIM lenses [33, 133, 139, 148].
Since the AZIM coating has the near-zero permittivity tensor parameter only in the $z$ direction, the impedance of the slot is well maintained when it is covered by the AZIM. As shown in Figure 4.7(b), the simulated $S_{11}$ is below -10 dB from 5.6 to 6 GHz, which is very similar to that of the slot alone. This robust input impedance behavior ensures that the AZIM coating can be readily added onto or taken away from the slot to achieve different radiation properties without any additional modification to the slot antenna itself. The normalized $E$-plane ($y$-$z$ plane) and $H$-plane ($x$-$z$ plane) radiation patterns at 5.6, 5.8, and 6.0 GHz are presented in Figure 4.8(a)-(f) (bottom sections). Distinctly different from the two-peak patterns for the slot without the AZIM coating, a well-defined single beam at broadside is observed in the $E$-plane with a beam squint less than 2° off broadside and a 3 dB beamwidth of about $35^\circ \sim 40^\circ$. In contrast from the radiation properties in the $E$-plane, the $H$-plane patterns have beams with similar beamwidth compared to those for the slot without the AZIM coating. Notably, with the presence of the AZIM coating, the broadside gain is significantly increased to $10.2 \sim 10.6$ dBi, which indicates an improvement of about 7 dB. It should also be noted that the front-to-back ratio is greatly reduced by about 10 dB throughout the entire frequency range, which is shown in Figure 4.9(b). Both the broadside gain increase and the front-to-back ratio drop are primarily attributed to the reduction of fields at the edges of the ground plane. This is facilitated by the presence of the AZIM coating, resulting in much weaker diffracted fields at the back side of the antenna. From an alternative perspective, the ultrathin AZIM coating can be considered as a highly efficient radiating aperture made of an array of subwavelength radiators fed by the slot. This is manifested by the aperture efficiency plotted in Figure 4.10(b) showing a value larger than 87%, which is normalized to the theoretical limit $4\pi A/\lambda^2$ where $A$ is the effective area of the AZIM coating. The radiation efficiency of the antenna with the AZIM (see Figure 4.10(a)) is slightly lower than that without the AZIM due to the loss introduced by the metamaterial, but nevertheless maintains a value higher than 92% within the 5.6 - 6.0 GHz range.
Figure 4.8. (a) Simulated and measured normalized radiation patterns of the SIW fed slot antenna with and without the AZIM coating in the E-plane (y-z plane) at 5.6 GHz, (b) 5.8 GHz, and (c) 6.0 GHz. (d) Simulated and measured normalized radiation patterns of the SIW fed slot antenna with and without the AZIM coating in the H-plane (x-z plane) at 5.6 GHz, (e) 5.8 GHz, and (f) 6.0 GHz. The simulated E- and H-plane patterns for the SIW fed slot antenna without the AZIM coating on a ground plane infinite in the y direction and the simulated E- and H-plane patterns for the SIW fed slot antenna with the homogeneous dispersive effective medium AZIM coating are also shown for comparison.
Figure 4.9. (a) Simulated and measured gain at broadside ($\theta = 0^\circ$) for the SIW fed slot antenna with and without the AZIM coating. (b) Simulated and measured front-to-back ratio for the SIW fed slot antenna with and without the AZIM coating.

Figure 4.10. (a) Simulated efficiency for the SIW fed slot antenna with and without the AZIM coating. (b) Simulated aperture efficiency for the SIW fed slot antenna with the AZIM coating.

As a comparison, the actual metamaterial coating structure is replaced by a homogeneous slab with the dispersive effective anisotropic material parameters shown in Figure 4.4(b). It can be observed that the $S_{11}$ and the front-to-back ratio of the slot covered with the effective medium slab correspond well to those of the slot covered with the actual discrete metamaterial. The radiation patterns in both the $E$-plane and $H$-plane also show good agreement, especially in the
main beam; the greatest disagreement can be found in the angular range near the back lobes. The correspondence seen from this comparison justifies the effective medium approximation employed here, which is assumed valid due to the subwavelength size of the unit cells.

Figure 4.11. Photograph of the fabricated SIW fed slot antenna covered by the AZIM coating. Additional dielectric slabs cut with interlocking slits were used on both sides of the AZIM coating to provide better mechanical support. The inset shows the SIW fed slot antenna alone.

4.4 Experimental Verification

The SIW fed slot antenna and the AZIM coating structure were fabricated and assembled, as shown in Figure 4.11. An Agilent E8364B network analyzer was used to characterize the $S_{11}$ of the slot with and without the AZIM coating. As shown in Figure 4.7(a) and Figure 4.7(b), good agreement can be found between simulations and measurements not only in terms of the $-10$ dB bandwidth but also the resonance positions. The measured $S_{11}$ of the slot with and without the AZIM coating has a $-10$ dB band from 5.52 to 6.03 GHz and from 5.54 to 6.01 GHz, respectively. They are both slightly broader than the simulations predict due to the minor frequency shift of the resonance at 5.6 GHz and the lower quality factors of both resonances within the $-10$ dB
bandwidth. The lower quality factors are primarily caused by the increased Ohmic loss originating from the copper and the solder of the fabricated prototype, which gives a higher level of background absorption at frequencies away from the resonances.

The radiation patterns and the gain of the slot antenna with and without the AZIM coating were characterized in an anechoic chamber with an automated antenna movement platform, as shown in Figure 4.8(a)-(f), respectively. In all, the measured radiation patterns in both the $E$-plane and $H$-plane agree well with the simulated results, conforming the proposed antenna design. Specifically, patterns exhibiting a double-peak can be seen for the slot alone, while a single sharp beam pointing at broadside can be observed for the slot with the AZIM coating. The measured 3 dB beamwidth in the $E$-plane is about $40^\circ - 50^\circ$, which is slightly broader than the simulated beamwidth, especially in the high frequency band. This is mainly attributed to fabrication and assembly imperfections which result in a non-ideal symmetry in the actual metamaterial structure. The measured broadside gain for the slot with and without the AZIM coating is in the range of 9.8 - 10.4 dBi and 2.9 - 3.5 dBi, respectively, which indicates an improvement of about 6.9 dB. The measured front-to-back ratio is displayed in Figure 4.9(b), showing a more than 10 dB reduction throughout the band, which is in good agreement with simulation results.

4.5 Overview

In this chapter, we have proposed and demonstrated a low-profile, high-gain, compact-footprint antenna that is comprised of a SIW fed slot covered by an ultrathin AZIM coating. The leaky modes of the grounded AZIM slab and the radiation properties of the truncated structure were exploited for achieving stable unidirectional radiation at broadside with a total device thickness of only $0.12\lambda$. A prototype was fabricated and characterized, showing a strong
agreement between measurement and simulation. The AZIM coating provides \( \sim 7 \) dB broadside gain enhancement and \( \sim 10 \) dB front-to-back ratio reduction over the SIW fed slot antenna alone.

The proposed AZIM-coated antenna achieves a \(-10\) dB impedance bandwidth of more than 9\%, a broadside gain higher than 10 dBi and a front-to-back ratio larger than 26 dB, which represents a promising candidate for applications ranging from point-to-point communications and wireless power transfer, to various other wireless systems.
Chapter 5

Ultra-Wideband Monopole Antenna Using Ultrathin Metamaterial Coating

5.1 Introduction

Broadband antennas have been widely used in modern communication system for higher data transfer rate and radar systems for achieving shorter pulse duration. Several techniques have been proposed since the early 70s to broaden the impedance bandwidth of vertical polarized wire and planar antennas. The conventional approaches includes placing conducting sleeves around the main antenna to provide a second resonance at a higher frequency [146, 164] or inserting lumped series resistor-inductor circuits into a long wire monopole [146, 165]. The planar version of the sleeve monopole has been realized by Spence et al. using printed planar sleeves beside an end-loaded microstrip monopole on the same substrate board [166]. Apart from using the conducting sleeves to give the second resonance, bulk dielectric resonators of various shapes have also been employed to broaden the impedance bandwidth of wire antennas [167, 168]. Other techniques involving the reshaping the periphery of the planar monopole to achieve a traveling wave like antenna have also been intensively investigated [170, 171]. The gradually tapered slot between the edge of the planar monopole and the ground plane enables the impedance matching over a broad bandwidth. More recently, artificial electromagnetic metamaterials have been proven to extend the bandwidth of planar monopoles and microstrip antennas by proper loading with split ring resonators or negative refractive index transmission lines [67, 68].

In this chapter, we exploit the strong anisotropy that can be provided by metamaterials to greatly enhance the impedance bandwidth of a quarter-wave wire-type monopole antenna by surrounding it with an ultra-thin flexible anisotropic metamaterial coating. The coating gives rise
to another higher frequency resonance, which enables a similar current distribution on the monopole to that of the fundamental mode through the metamaterial’s broadband anisotropic property. In contrast to previously reported broadband vertical polarized planar monopoles which typically develop multiple lobes in their radiation patterns as frequency increases [169, 170], the new octave bandwidth metamaterial-enabled monopole has stable vertically polarized radiation patterns over the entire frequency band of operation. Moreover, compared to broadband open sleeve dipoles/monopoles [164] and broadband dielectric resonator antennas fed by monopoles [167, 168], the proposed metamaterial coated monopole antenna is more compact and extremely lightweight, suggesting possible applications ranging from broadband arrays to portable wireless devices.

5.2 Anisotropic Metamaterial Coating Design For S-Band Monopole

In this section, anisotropic metamaterial coating designs are presented for an S-band (2 GHz – 4 GHz range) quarter-wave monopole antennas. The mechanisms of operation is discussed by studying the input impedance and current distribution of the coated and uncoated monopole. The results are also compared to sleeve monopole and dielectric coated monopole, showing the superiority of the anisotropic metamaterial coating. The antenna prototype was fabricated and measured, showing strong agreement with simulation predictions.

5.2.1 Unit Cell Design

The unit cell of the metamaterial coating is comprised of two identical I-shaped copper patterns printed on both sides of a Rogers Ultralam 3850 substrate (see Figure 5.1(a)). The thicknesses of the substrate \(d_s\) and the copper \(d_c\) are 51 μm and 17 μm, respectively. Using this
thin flexible substrate, the nominally planar metamaterial structure can be formed into a cylindrical configuration. The effective medium properties of the metamaterial are obtained according to the diagram shown in Figure 5.1(a), where periodic boundary conditions are assigned to the walls in the y- and z-directions. A TE/TM polarized plane wave, with the $E$-field/$H$-field oriented along the z-direction, is incident from the left half-space at an angle of $\varphi$ ($0^\circ \leq \varphi \leq 90^\circ$) with respect to the x-axis. An anisotropic inversion technique presented in Chapter 2 was employed to extract all six effective permittivity and permeability tensor quantities from the scattering parameters calculated at different angles of incidence using HFSS™.

Figure 5.1. (a) Geometry and dimensions of the unit cells of the anisotropic metamaterial coating. All dimensions are in millimeters: $a = 2.5$, $d_s = 0.051$, $d_c = 0.017$, $w = 2$, $b = 10$, $c = 1.5$, $g = 0.8$ and $l = 8$. (b) Real and imaginary parts of the retrieved effective anisotropic permittivity tensor parameters ($\varepsilon_x, \varepsilon_y, \varepsilon_z$).

The retrieved effective permittivity tensor parameters are shown in Figure 5.1(b). It can be seen that none of the parameters exhibit a resonant response in the band of interest as a result of the subwavelength sized I-shaped elements, which corroborates previously reported results on I-shaped unit cells utilized for broadband microwave transformation optics devices [115]. The retrieved $\varepsilon_x$ and $\varepsilon_y$ have non-dispersive values near unity, whereas $\varepsilon_z$ exhibits a large value which is attributed to the inductance provided by the central microstrip in the I-shaped elements and capacitance associated with the gaps between the stubs of adjacent unit cells in the z-
direction. Controlling the series inductance and capacitance enables manipulation of the value of $\varepsilon_z$ across the band. The three effective permeability tensor parameters (not shown here) have non-dispersive values equal to unity with very low loss, indicating that the metamaterial does not have any effect on the radiated magnetic field.

### 5.2.2 Monopole Antenna Coated with Metamaterial

The configuration of the monopole antenna with and without the metamaterial coating is shown in Figure 5.2. The monopole is 28.5 mm long and resonates at 2.5 GHz. The cylindrical metamaterial coating is composed of two concentric layers of metamaterial cells as illustrated in Figure 5.3(b). The inner and outer layers contain eight and sixteen unit cells along their circumference, respectively, in order to approximate a circular outer periphery to minimize its impact on the monopole's omnidirectional radiation patterns in the $H$-plane. The outer radius of the metamaterial is 5 mm or about $\lambda/24$ at 2.5 GHz, ensuring that the ultra-thin subwavelength coating is compact in the radial direction.

![Figure 5.2](image)

Figure 5.2. Configuration of (a) the quarter-wave monopole antenna and (b) the same monopole with ultra-thin flexible anisotropic metamaterial coating. All dimensions are in millimeters: $d_a = 1$, $h_a = 28.5$, $d_i = 5$, $d_o = 2d_i$, and $h_i = 40$. The dielectric is 51 μm thick Rogers Ultralam 3850 ($\varepsilon_r = 2.9$, $\delta_{tan} = 0.0025$).
When applying this metamaterial to the monopole antenna, only a finite number of unit cells can be utilized; and instead of a planar structure used in the S-parameter simulations, a curved configuration is adopted to achieve a uniform coating surrounding the monopole. The radius and the effective \( \varepsilon_z \) were carefully chosen during the design process in order to generate the optimal antenna performance. To examine the effect of the metamaterial coating on the impedance bandwidth of the monopole and the efficacy of the anisotropic effective medium model, we compare the simulated VSWR for three cases: the monopole alone, the monopole with the actual metamaterial coating, and the monopole with the anisotropic effective medium coatings (Figure 5.3).

![Figure 5.3. Simulated VSWR of monopole alone (—), monopole with actual metamaterial coating (—), and monopole with homogeneous anisotropic effective medium coating (—). The same ground plane size (32 cm × 32 cm) was used in all three simulations.](image)

The monopole alone yields a VSWR<2 bandwidth of 0.4 GHz (2.3~2.7 GHz) with a single resonance at 2.5 GHz, whereas with the actual metamaterial coating present, the VSWR<2 bandwidth is remarkably broadened to 2.3 GHz (2.1~4.4 GHz). The main resonance shifts down slightly to 2.35 GHz and a new resonance is enabled at 3.85 GHz. When a homogeneous anisotropic effective medium coating with the retrieved material parameters is used, the VSWR exhibits a similar behavior to that of the actual metamaterial coating. The VSWR<2 bandwidth is
2.2 GHz (2.1~4.3 GHz) with the first and the second resonance located at 2.32 and 3.65 GHz, respectively, indicating that the assumed homogeneous anisotropic effective medium model is a valid approximation for the actual curved metamaterial. This is primarily because a sufficient number of unit cells are used to form the cylindrical coating such that the metamaterial still possesses a reasonably good local flatness. In addition, considering that the effective medium parameters extracted from the scattering parameters calculated at different angles of incidence remain essentially unchanged (not shown here), the effective $\varepsilon_\rho$ and $\varepsilon_\phi$ components of the cylindrical metamaterial coating can therefore be represented by the retrieved effective $\varepsilon_x$ and $\varepsilon_y$.

![Figure 5.4](image)

Figure 5.4. Simulated input impedance of the monopole antenna with and without the metamaterial coating. The insets plot the current magnitude distribution on the monopole at various frequencies.

### 5.2.3 Principle of Operation

To gain a better understanding of the principle of operation of the anisotropic metamaterial coating, the input impedance of the monopole with and without the coating is provided in Figure 5.4, along with the current distributions on the monopole at certain critical frequencies. Without the metamaterial coating, a distinct resonance can be identified around 3.3
GHz with the best matching frequency (for \( Z_0 \) of 50 \( \Omega \)) at 2.5 GHz. By coating the monopole with the metamaterial, both the real and imaginary parts of the input impedance are flattened in the band of interest, with the real part fairly close to 50 \( \Omega \) and imaginary part varying between –10 to –35 \( \Omega \). The current plots on the metamaterial coated monopole at 2.35 GHz and 3.85 GHz (see Figure 5.4) demonstrate that the fundamental and the metamaterial-introduced resonances have a very similar current distribution which ensures stable in-band radiation patterns. Without the metamaterial, the current at 3.85 GHz has its maxima located nearly \( h_a/3 \) up from the base of the monopole, resulting in a large reactance for the input impedance. Further examination reveals that the currents on the metamaterial coating are significantly weaker than those on the monopole, indicating that the performance of the coating is expected to be robust with respect to fabrication and assembly tolerances since it is a non-resonant structure.

![Figure 5.5. Photographs of the fabricated metamaterial coated monopole.](image)

**5.2.4 Experimental Verification**

The metamaterial coating was realized by first fabricating two planar metamaterial sheets for the inner and outer layers. The two sheets were then curled to form the inner and outer layers of the metamaterial coating as shown in Figure 5.2(b). Four polypropylene washers were used as
a frame for the coating and to ensure correct inner and outer layer diameters. The inner substrate layer is held in place by friction and the outer layer is held in place with thin strips of polyimide tape around the outside of the coating. The structural rings and thin strips of tape were positioned at the centers of the I-shaped metallic structures to avoid influencing the capacitances in the gaps. The final antenna prototype is presented in Figure 5.5.

VSWR measurements were carried out using an Agilent E8364B network analyzer. Figure 5.6 compares the simulated and measured VSWR curves of the monopole with and without the metamaterial coating on a 32 cm × 32 cm ground plane. The measured VSWR of the monopole alone is almost identical to the simulated results with VSWR<2 from 2.3 GHz to 2.7 GHz. With the metamaterial present, a 2.14:1 ratio bandwidth (2.15~4.6 GHz) of VSWR<2 is obtained. Frequency shifts of 0.05 GHz and 0.2 GHz were found in the lower and higher ends of the band, respectively, possibly resulted from a slight tilt between the monopole and the coating, as well as fabrication imperfections.

Figure 5.6. Simulated and measured VSWR of the monopole antenna with and without the metamaterial coating.

The radiation patterns of the metamaterial coated monopole were also measured using an anechoic chamber. Figure 5.7 presents the simulated and measured $E$-plane and $H$-plane patterns
at 2.2 GHz, 3.3 GHz, and 4.4 GHz. The $H$-plane patterns exhibit stable omni-directional radiation characteristics throughout the entire band. The gain variations are around 0.5 dB and 1.2 dB for simulation and measurement, respectively. The increased measured gain variation as a function of the azimuthal angle is primarily caused by the imperfection of assembly and noise, as well as the antenna rotation platform. In the $E$-plane, characteristic ear-shaped patterns can be observed which are very similar to the patterns for the monopole without the metamaterial, indicating that the added metamaterial coating has negligible impact on the spatial distribution of the radiated energy of the monopole. The maximum gain of the metamaterial coated monopole varies from 3.75 dBi to 5.46 dBi in the VSWR<2 band with the direction of maximum gain moving from $32^\circ$ to $26^\circ$ off horizon due to the finite sized ground plane used in both simulation and measurement. The measured gain is 0.3 dB to 0.8 dB smaller than the simulated values. In both simulations and measurements, we observe radiation efficiency above 97%, substantiating the broadband low-loss nature of the metamaterial coating. The overall very good agreement between simulation and measurement confirms the expected performance of the proposed metamaterial antenna coating.

Figure 5.7. Simulated and measured $H$-plane (x-y plane) and $E$-plane (y-z plane) radiation patterns of the metamaterial coated monopole at (a) 2.2 GHz, (b) 3.3 GHz, and (c) 4.4 GHz. Red lines: simulated $H$-plane patterns. Black lines: measured $H$-plane patterns. Blue lines: simulated $E$-plane patterns. Gray lines: measured $E$-plane patterns.
5.2.5 Comparison to Sleeve and Dielectric Coated Monopoles

In order to appreciate the performance of the anisotropic metamaterial coating, simulations for monopole antennas coated with conventional isotropic dielectric materials or surrounded with parasitic conducting sleeves are carried out. When the monopole is loaded with a homogeneous isotropic coating (which can be considered as a dielectric ring resonator) with permittivity equal to the value of the retrieved $\varepsilon_x$ of the metamaterial given in Figure 5.2(b). It can be seen from Figure 5.8(a) that the isotropic coating shifts the main resonance to a much lower frequency due to loading with a high isotropic permittivity. It has two additional resonances, one at 3.6 GHz and another near 5 GHz; however, they do not serve to reduce the
antenna's VSWR below 2. Further studies (not included here) reveal that when the radius of the isotropic coating is increased to about $\lambda/4$ and the gap between the dielectric ring and the monopole is carefully tuned, the bandwidth can be increased to over an octave [168]. However, the structure becomes more bulky and heavier; thus it is not as compelling as the proposed metamaterial coating for use in portable devices. When the parasitic conducting sleeves are employed, as shown in Figure 5.9(b), a monopole-like resonance mode can be excited on the sleeves, thereby extending the impedance bandwidth of the original antenna. As a fair comparison, we kept the footprint of the sleeve monopole to be the same as that of the metamaterial coated monopole and optimized for the largest possible bandwidth. It can be seen from Figure 5.8(c) that the sleeve monopole achieves a VSWR < 2 bandwidth from 2.3 GHz to 4.15 GHz, which is about 21% narrower than that accomplished using the metamaterial coated monopole.

5.3 Anisotropic Metamaterial Coating Design For C-Band Monopole

To show the flexibility of the proposed technique, In this section, anisotropic metamaterial coating designs are presented for an C-band (4 GHz – 8 GHz range) quarter-wave monopole antennas. It will be shown that a VSWR < 2 bandwidth of more than an octave can be achieved using the proposed technique.

The design procedure is similar to that for the S-band metamaterial coating. The unit cell of the metamaterial coating is comprised of two identical I-shaped copper patterns printed on both sides of a Rogers Ultralam 3850 substrate. The thicknesses of the substrate ($d_s$) and the copper ($d_c$) are 51 μm and 17 μm, respectively. The other dimensions are (all in millimeter): $a = 2.5$, $d_s = 0.051$, $d_c = 0.017$, $w = 1.9$, $b = 3.9$, $c = 1.1$, $g = 0.5$ and $l = 2.6$. Using this thin flexible substrate, the nominally planar metamaterial structure can be formed into a cylindrical
configuration. The configuration of the monopole antenna with the metamaterial coating is the same as that of the S-band design. The monopole is 15 mm long and resonates at 4.5 GHz. The cylindrical metamaterial coating is composed of two concentric layers of metamaterial cells as illustrated in Figure 5.3(b). The inner and outer layers contain eight and sixteen unit cells along their circumference, respectively, in order to approximate a circular outer periphery to minimize its impact on the monopole's omnidirectional radiation patterns in the \textit{H}-plane. The outer radius of the metamaterial is 3.7 mm or about $\lambda/20$ at 4.5 GHz, ensuring that the ultra-thin subwavelength coating is compact in the radial direction.

![Figure 5.10. Simulated VSWR of monopole alone and monopole with actual metamaterial coating at C-band.](image)

To examine the effect of the metamaterial coating on the impedance bandwidth of the monopole, we compare the simulated VSWR for the cases of the monopole alone and the monopole with the metamaterial coating (see Figure 5.10). The monopole alone yields a VSWR$<2$ bandwidth of 0.5 GHz (4.5~5.0 GHz) with a single resonance at 4.75 GHz, whereas with the actual metamaterial coating present, the VSWR$<2$ bandwidth is remarkably broadened to 4.35 GHz (3.85~8.2 GHz). The main resonance shifts down slightly to 4.55 GHz and a new resonance is enabled at 7.0 GHz.
5.4 Overview

In summary, in this chapter, a novel impedance broadening technique utilizing compact (ultra-thin) flexible anisotropic metamaterial coatings has been proposed and verified both numerically and experimentally. It has been demonstrated that such metamaterial coating can greatly enhance the impedance bandwidth of a quarter-wave wire-type monopole to over an octave in both S-band and C-band. Through the engineered anisotropy of the metamaterial, the coating provides two resonating modes for the antenna with similar current distributions, thus ensuring stable radiation patterns over the entire band. Measurements are shown to be in good agreement with simulated results, confirming the desired performance of the proposed metamaterial-enabled broadband antenna design. This type of flexible metamaterial coating is compact in size, extremely lightweight, low in cost, and can be easily scaled to different frequency bands, thereby paving the way for widespread use as radiating elements in, for example, broadband arrays and portable wireless devices.
Chapter 6
Free-Standing Optical Metamaterials With Near-Zero Phase Delay

6.1 Introduction

Metamaterials provide a unique opportunity to create highly customizable refractive index-engineered thin films that will enable optical devices with entirely new physical properties, optical functions, and form factors. By optimizing the geometry of artificially engineered metallodielectric nanostructures to give a specific electric and magnetic response to incident electromagnetic waves, the effective refractive index of the composite materials can be varied from negative, through zero, to large positive values. Compared to NIM, far less research has focused on the equally important class of ZIMs/LIMs, which provide quasi-infinite phase velocity and infinite wavelength for the light propagating inside [52, 171]. These optical properties can be exploited to produce a variety of new and exciting electromagnetic devices with unique functionalities, as introduced in Chapter 1.

Along with providing the required exotic refractive index values, many metamaterial-enabled devices are required to have nearly ideal optical transmission, with minimal reflection and absorption loss. To minimize reflection loss, the impedance of the metamaterial should be well matched to the impedance of its surroundings. Previous experiments on optical metamaterials primarily investigated structures with specific negative index or high index values, without controlling the interfacial impedance matching. The effective refractive index \( n_{\text{eff}} \) and interfacial impedance values \( Z_{\text{eff}} \) of a metamaterial are given by,

\[
\begin{align*}
  n_{\text{eff}} &= \sqrt{\varepsilon_{\text{eff}} \mu_{\text{eff}}} , \\
  Z_{\text{eff}} &= \sqrt{\mu_{\text{eff}}/\varepsilon_{\text{eff}}} 
\end{align*}
\] (6.1)
where $\varepsilon_{\text{eff}}$ and $\mu_{\text{eff}}$ are its effective permittivity and permeability, respectively. Consequently, the metallo-dielectric nanostructures must be optimized to give an electromagnetic response that simultaneously balances $\varepsilon_{\text{eff}} \mu_{\text{eff}}$ and $\mu_{\text{eff}} / \varepsilon_{\text{eff}}$ to produce the desired refractive index and impedance match across the wavelength band of interest. For example, the metamaterial structure is surrounded by air, its effective impedance should be matched to that of free-space \cite{73}, which is given by:

$$Z_{\text{eff}} = Z_0 = \sqrt{\mu_0 / \varepsilon_0} \quad (6.2)$$

where $\varepsilon_0$ is the permittivity and $\mu_0$ is the permeability of free-space. This condition can only be satisfied by tailoring both the $\varepsilon_{\text{eff}}$ and the $\mu_{\text{eff}}$ of the metamaterial structure.

In this chapter, we present the design, fabrication, and characterization of a polarization-insensitive optical ZIM with near-perfect transmission in the form of a mechanically flexible, free-standing thin film. This was achieved by optimizing the geometry of the metallo-dielectric fishnet structure to give both permittivity and permeability approaching zero at a wavelength of 1.55 $\mu$m, which provides a matched impedance with free space. The nanofabricated ZIM samples accurately replicated the optimized, axially symmetric fishnet design, thereby minimizing degradation in optical properties (increased loss) due to bianisotropy from substrate loading \cite{78} and/or from feature asymmetry \cite{77}. The experimentally retrieved effective medium parameters obtained by inverting the complete set of scattering parameters (amplitude and phase) measured using spectral holography confirmed the targeted design properties. In addition, we also propose a ZIM design in the mid-infrared with a wide field-of-view by employing the anisotropic inversion algorithm during the design optimization process.
6.2 Optical ZIM Design and Numerical Results

6.2.1 ZIM Design Optimization

The impedance-matched ZIM operating at 1.55 μm was designed using a free standing symmetric fishnet nanostructure consisting of two gold (Au) screens separated by a polyimide spacer with square air holes perforating all three layers (see Figure 6.1(a)). The air holes were aligned in a doubly periodic arrangement with identical periodicity in both the x- and y-directions, thus making the structure polarization insensitive to incoming light at normal incidence. With the combination of the resonant magnetic paired strips and non-resonant electric strips, the fishnet structure can be regarded as being comprised of an array of magnetic dipoles embedded inside a diluted metal, which yields a resonant \( \mu_{\text{eff}} \) and a Drude-like \( \varepsilon_{\text{eff}} \). By carefully tailoring the nanostructure geometry to give \( \mu_{\text{eff}} \rightarrow 0 \) and \( \varepsilon_{\text{eff}} \rightarrow 0 \) at the same wavelength, the desired impedance-matched ZIM can be obtained.

Figure 6.1. (a) Diagram of the optimized fishnet structure that produces a near zero refractive index at 1.55 μm. One unit cell is enclosed within the red dotted square. The inset shows a 3D view of the unit cell with \( w=365 \) nm, \( p=956 \) nm, and \( t=381 \) nm. The top and bottom Au layers (yellow) are 39 nm thick and the polyimide layer (red) is 303 nm thick. Top-view FESEM image of the fabricated ZIM is shown on the right. Scale bar, 500 nm. (b) FESEM image of the free-standing, flexible ZIM structure. Scale bar, 2 μm.
Figure 6.2. (a) Amplitude and (b) phase of the transmission (blue, light blue (resimulated with adjusted dimensions)) and reflection (red, light red (resimulated with adjusted dimensions)) coefficients. Real (blue, light blue (resimulated with adjusted dimensions)) and imaginary (red, light red (resimulated with adjusted dimensions)) parts of the inverted effective refractive index (c) and normalized effective impedance (d). Real (blue, light blue (resimulated with adjusted dimensions)) and imaginary (red, light red (resimulated with adjusted dimensions)) parts of the inverted effective permittivity (e) and effective permeability (f).

Due to the challenging design criteria for an impedance-matched ZIM, a powerful stochastic GA technique [172] was employed to achieve the fishnet nanostructure to satisfy the performance goals – a near-zero effective refractive index and an impedance matched to free space. The optimizer determined the geometric dimensions of the fishnet, including the unit cell
size, air hole size, and layer thicknesses, to provide optical properties that best satisfied the design criteria. The measured optical properties of the constituent materials (*i.e.*, Au and polyimide) were incorporated into the optimizer to accurately model the material dispersion and to ensure that the resulting structure could be readily fabricated without further modification. In the numerical simulations, Au was modeled as a dispersive dielectric with complex permittivity that was determined by spectroscopic ellipsometry measurements of the deposited films. For each candidate design in the GA optimization, the complex transmission and reflection coefficients were predicted using a full-wave periodic finite-element boundary-integral (PFEBI) solver. The effective refractive index \( n_{\text{eff}} \) and normalized effective impedance \( Z_{\text{eff}}/Z_0 \) were then inverted from the scattering parameters using an established retrieval method [81] and compared with an ideal ZIM response to determine the candidate’s cost defined by:

\[
\text{Cost} = |n_{\text{eff}} - n_{\text{tar}}|^2 + \left|Z_{\text{eff}}/Z_0 - Z_{\text{tar}}/Z_0\right|^2
\]

where \( n_{\text{tar}} = 0 + 0i \) and \( Z_{\text{tar}}/Z_0 = 1 + 0i \) are the target effective refractive index and normalized effective impedance, respectively. The GA minimized the cost and evolved a ZIM structure with a cost (deviation) of only 0.008 from the design criteria specified in Eq. (6.3).

### 6.2.2 Numerical Results and Field Properties at ZIM Band

The geometry and dimensions of the optimized ZIM design are shown in Figure 6.1(a). The width and length of the three-layered crossed strips in the unit cell are 365 nm and 956 nm, respectively. A 303 nm thick polyimide layer separates the 39 nm thick top and bottom Au screen layers. The predicted transmission and reflection coefficients of this GA optimized fishnet nanostructure are shown in Figure 6.2(a). Because the effective impedance is well-matched to free space and the absorption loss in the metamaterial is low, the transmission amplitude remains remarkably high at ~ 95% around the target ZIM band at 1.55 μm. Considering the physical
thickness of the structure, which is approximately $\lambda/4$, this near-zero absolute transmission phase provides confirmation that a near-zero optical path length can be achieved using this metamaterial.

The inverted theoretical effective index of refraction and impedance are shown in Figure 6.2(b). A zero-index band with an extremely small imaginary part ($n_{\text{eff}} = 0.072 + 0.051i$), which is indicative of the low absorption losses in the structure, is observed exactly at 1.55 $\mu$m. The near-unity normalized effective impedance with small imaginary part ($Z_{\text{eff}}/Z_0 = 1.009 - 0.021i$) substantiates the low reflection loss as exhibited in the transmission and reflection amplitudes. To quantitatively evaluate the ZIM performance, two figures-of-merit (FOMs) are defined for the zero-index band:

$$FOM_n = 1/|n_{\text{eff}}|, \quad FOM_z = 1/|Z_{\text{eff}}/Z_0 - 1|,$$

(6.4)

where $FOM_n$ evaluates how closely the effective refractive index approaches zero, and $FOM_z$ evaluates how closely the normalized effective impedance approaches unity. For this specific optimized design, the achieved FOMs have high values of 11.3 and 43.8, respectively. This represents a new benchmark in the state-of-the-art for optical ZIMs.

Figure 6.3. Distribution of volumetric current density on the top (a) and bottom (b) Au layers of the ZIM excited with a normally incident beam having the polarization shown. (c) Cross-sectional view of a snapshot of the electric field. The nearly identical field vectors throughout the fishnet structure confirm that the metamaterial has a near-zero phase delay with high transmission.
To acquire a clear understanding of the optical response of this ZIM, the current distributions on the top and bottom Au layers at a wavelength of 1.55 μm are plotted in Figure 6.3(a) and Figure 6.3(b) with an incident electric field linearly polarized along the x-direction. The array of long Au metal strips, which is aligned to the x-axis along the incident electric field, provides a diluted Drude-type response (see Figure 6.2(c)). The anti-parallel currents induced by the incident magnetic field along the y-axis produce a resonant magnetic response, resulting in a Lorentzian line-shaped resonance in the effective permeability (see Figure 6.2(c)). In this design, the geometry of the fishnet was optimized to give near-zero values of $\varepsilon_{\text{eff}} = 0.068 + 0.051i$ and $\mu_{\text{eff}} = 0.072 + 0.049i$ at 1.55 μm, leading to a low-loss, impedance-matched ZIM (see Figure 6.2(b)). The near-zero phase delay of this ZIM is clearly shown in the snapshot of the field evolution in Figure 6.3(c). Specifically, the amplitude and phase of the electric fields are nearly identical on the top and bottom interfaces as well as everywhere inside the metamaterial.

6.3 Fabrication and Characterization of ZIM Sample

6.3.1 Nanofabrication of the ZIM

The optimized low-loss ZIM properties are only valid for an ideal symmetric nanostructure, which has air-holes with perfectly vertical (90°) sidewalls, and is surrounded by air on the top and bottom surfaces. Previously reported fishnet-based optical metamaterials [69-72] were fabricated by defining the nm-scale air holes in multilayer metal/dielectric stacks deposited on optically transparent substrates using either focused ion beam (FIB) etching or lift-off techniques. These structures suffer from process-induced non-idealities that degrade their optical properties. Specifically, the physical FIB etching process causes the air holes in the top-most stack layers to widen, and the sputtered materials to redeposit onto the etched sidewalls. The lift-
off process limits the sidewall angle to < 80° for multi-layer stacks that are in the range of 100 nm to 300 nm thick. The slight deviation in the sidewall angle of 10° from vertical leads to asymmetry that introduces coupling between $\varepsilon_{\text{eff}}$ and $\mu_{\text{eff}}$, which is not present in the symmetric fishnet structure. As will be analysed later in Section 6.4, this coupling would result in a significant decrease in the transmission amplitude from 95% to 77% in the optimized ZIM structure. This underscores the importance of fabricating nanostructures that approach the ideal designed geometry to experimentally realize the theoretically predicted properties.

In this work, a top-down nanofabrication process that employed high-aspect-ratio dry etching to form air holes in the thick metal/dielectric stack was developed to overcome the inherent limitations of the FIB and lift-off processes. The optimized ZIM design was made by first sequentially depositing the 3-layer stack using electron-beam evaporation and spin-coating for the Au and polyimide layers, respectively. The individual evaporation, coating, and thermal treatment steps were optimized to produce films with high optical quality [173]. To create the fishnet nanostructure, a sacrificial silicon dioxide (SiO$_2$) hard etch mask was added on top of the Au-polyimide stack using low-temperature plasma enhanced chemical vapour deposition (PECVD), and the air hole array pattern was defined in the hard mask using electron-beam lithography and fluorine-based reactive ion etching (RIE). This pattern was transferred through the Au and the polyimide layers by anisotropic dry etching using in an inductively coupled chlorine-based high-density plasma process. In the final step, the SiO$_2$ hard mask was removed by low-power dry etching, and the Au-polyimide-Au fishnet structure was released from the Si handle substrate to give the symmetric ZIM structure.

Figures 6.1(a) show field-emission scanning electron microscope (FESEM) images of the fabricated fishnet structure. These images demonstrate that this process accurately reproduced the optimized ZIM design with air hole dimensions of 590 nm $\times$ 590 nm (measured on top and backside), nearly ideal vertical sidewalls (>89°), and no visible redeposition of Au on the etched
polyimide sidewalls. Additionally, because the constituent Au and polyimide materials are ductile, the freestanding fishnet structure is flexible and can be deformed without fracturing as shown in Figure 6.5(b).

![Diagram of interferometers](image)

**Figure 6.4.** (a) Schematic of the Mach-Zehnder interferometer used to find the complex transmission coefficients of the ZIM. (b) Schematic of the Michelson interferometer used to find the complex reflection coefficients of the ZIM.

### 6.3.2 Optical Measurement of the Fabricated ZIM Sample

The zero index properties of the nanofabricated fishnet structure were verified by experimentally measuring the amplitude and phase of the transmission and reflection coefficients using a spectral holography technique [174, 175]. First, transmission measurements were made using a Mach-Zehnder interferometer with a white-light supercontinuum input source (see Figure 6.4(a)). Interferograms were collected over a broad range of wavelengths from 1.2 to 1.7 µm before and after inserting the freestanding ZIM in one of the two optical beam paths. The complex transmission coefficient $T$ of the ZIM was calculated by taking the ratio of the reconstructed signal terms of the two measurements. Next, reflection measurements were made on the same sample using a Michelson interferometer (see Figure 6.4(b)). Interferograms were collected for the ZIM as well as the adjacent reference mirror, and the complex reflection coefficient $R$ was determined from the ratio of their reconstructed signals.
The measured transmission and reflection coefficients are compared to the simulated values in Figure 6.2(a). The distinguishing spectral features present in simulation are blue-shifted by 50 nm in the fabricated structure, and the prominent increase in the simulated reflection phase near the designed resonance is missing. Despite these differences, the average values of measured transmission and reflection amplitude remain above 95% and below 18% within the zero-index band around 1.50 µm. In addition, the transmission phase passes through zero at 1.50µm, which results in a near-zero optical path length, i.e., nearly infinite phase velocity. This confirms that the fabricated ZIM has a nearly ideal impedance match to free space (low reflection loss), and has a low absorption loss with the desired near-zero index value.

The small discrepancies between experiment and theory was explained by resimulating the optical properties of the ZIM structure with several different polyimide layer thicknesses around the optimized values. This analysis revealed that the best agreement is obtained for a structure having a slightly thicker polyimide layer of 321 nm, which is within the ~10% variation in polyimide layer thicknesses measured on a series of planar test samples. As shown in Figure 6.2(a) and Figure 6.2(b), all of the simulated transmission and reflection coefficient features now match the measured values, and have negligible shift over the entire wavelength range. This further demonstrates that the impedance matched condition of this fishnet structure is robust against small variations in dielectric thickness because the infinite wavelength in the zero-index band is preserved. This is in sharp contrast to the strong dependence on air hole sidewall angle, which resulted in a significant decrease in transmission amplitude.

The effective medium properties of the fabricated ZIM that were retrieved using the measured transmission and reflection coefficients are plotted in Figure 6.6(c)-(f). As expected from simulation, the real and imaginary parts of $\varepsilon_{\text{eff}}$ and $\mu_{\text{eff}}$ are close to zero within the zero-index band of the structure. The dielectric properties give a complex effective refractive index of $n_{\text{eff}} = 0.121 + 0.032i$ and a normalized effective impedance of $Z_{\text{eff}}/Z_0 = 0.861 - 0.049i$ at the zero-phase
crossing wavelength of 1.50 μm. These values agree well with the target design values of $n_{\text{eff}} = 0.072 + 0.051i$ and $Z_{\text{eff}}/Z_0 = 1.009 - 0.021i$, and give a measured $FOM_n = 8.1$ and $FOM_Z = 17.2$ for this structure. The measured data verifies theory and shows that high-performance optical metamaterials can be experimentally realized with proper control over the nanofabrication process.

Figure 6.5. (a) The unit cell geometry of the same fishnet nanostructure shown in Figure 6.1(a), but with air holes that have sloped sidewalls. The dimensions are $p=956$ nm, $w=365$nm, $t=381$ nm, and $\theta=80^\circ$. The top and bottom Au layers (yellow) are 39 nm thick and the polyimide layer (red) is 303 nm thick. (b) Simulated amplitude of the transmission and reflection coefficients. The solid red line corresponds to the reflection amplitude for a wave normally incident on the top surface of the structure, while the dashed red line is for a wave normally incident on the back surface. The difference between the two reflection coefficients is shown in green.

6.4 Comparison to ZIM with Non-Vertical Sidewalls

The transmission and reflection coefficients of the ZIM were simulated for the optimized fishnet structure in Figure 6.1(a), but with air holes that have an $80^\circ$ sidewall angle (Figure 6.5(a)) [69-72] instead of the ideal $90^\circ$ angle. As shown in Figure 6.5(b), the transmission coefficient amplitude drops from ~95% for the ideal structure to ~77% for the structure with the
sloped sidewall angle. This is due a significant increase in the reflection coefficient amplitude from 1% to 55% (from a degraded impedance match to free space). In contrast to the ideal structure, the reflection amplitude is different for a wave that is normally incident on the top side (side 1, solid red line) as compared to the back side (side 2, dashed red line) of the fishnet structure. The non-zero difference in the reflection coefficients shows that the non-ideal ZIM has slight bianisotropy (i.e., magnetoelectric coupling) such that the transmitted wave has a polarization component orthogonal to that of the incident wave.

6.5 Mid-Infrared ZIM with a Wide Field-Of-View

In many applications (e.g. flat lenses), metamaterials must be capable of properly responding to illumination by obliquely incident waves in addition to those that are normally incident. Hence, in such cases, the ability to customize the metamaterial properties via optimization is of significant practical importance. In this section, the retrieval method presented in Chapter 2 is coupled with a robust GA optimizer in order to synthesize a ZIM with a wide FOV at the mid-infrared wavelengths. In order to demonstrate the superiority of the presented retrieval method, the ZIM optimized by a GA coupled with the anisotropic retrieval technique is compared with a second GA optimized design based on using the conventional isotropic retrieval method.

The first ZIM design was optimized to have a field-of-view of ±30° when illuminated with oblique TE polarized waves. The objective was to minimize the \( z \) component of the effective refractive index and match the normalized wave impedance to free space within a range of incident angles using the previously described anisotropic retrieval method. In order to keep the computational demands on the GA optimization process to a minimum, four sample angles (\( \theta^\circ = \ldots \))
$0^\circ$, $10^\circ$, $20^\circ$, and $30^\circ$) were considered. The preceding goals are combined in a cost function given by

$$Cost_1 = \sum_{\theta_i} \left[ |n_{zTE,\theta_i} - n_{z-tar}| + |Z_{TE,\theta_i} - Z_{tar}| \right], \quad (6.5)$$

where $n_{z-tar} = 0 + 0i$ is the desired $z$ component of the effective refractive index and $Z_{tar} = 1 + 0i$ is the desired normalized wave impedance. The $z$ component of the effective refractive index was also constrained to be positive because in most transformation optics applications, such as electromagnetic cloaks, the prescribed permittivities and/or permeabilities encompass a range of non-negative real values including zero. This restriction was performed by assigning a poor cost value to any design possessing a negative value of $n_{zTE,\theta_i}$. The target frequency was also chosen to be 100 THz (3 μm in wavelength). The optimized unit cell geometry is shown in Figure 6.6(a).

As a comparison to the first design, a second ZIM was synthesized considering only its response to normally incident waves. The goal was to minimize the effective refractive index and match the wave impedance to free space at the target frequency of 100 THz. Because the normal incidence inversion can only determine the $z$ component of the effective refractive index, the notation $n_z$ will be used to represent the retrieved refractive index. This leads to a cost function which is given by

$$Cost_2 = [n_{zTE} - n_{z-tar}] + [Z_{TE} - Z_{tar}] \quad (6.6)$$

where $n_{z-tar} = 0 + 0i$ is the desired effective refractive index, and $Z_{tar} = 1 + 0i$ is the desired normalized wave impedance. As with the previous design, the real part of the refractive index is restricted to be positive. During the optimization process, the conventional isotropic retrieval method is used, but the anisotropic retrieval method is applied to the final design in order to study the angular response of the metamaterial slab. The optimized unit cell geometry for this design is shown in Figure 6.6(b).
Figure 6.6. (a) 16 × 16 pixel geometry for the metamaterial stack with two Ag screens. (a) Optimized geometry for the first design, with a unit cell size \( a_1 = 1.42 \mu m \), total thickness \( d_1 = 476 nm \), and a Ag screen thickness of 75 nm. (b) 3D isometric view of the first ZIM design. (c) Optimized geometry for the second design, with a unit cell size \( a_2 = 1.58 \mu m \), total thickness \( d_2 = 735 nm \), and a Ag screen thickness of 75 nm. (d) 3D isometric view of the second ZIM design.

Figure 6.7. Retrieved (a) \( z \) component of the effective refractive index and (b) normalized wave impedance at 100 THz versus angle of incidence for both IR ZIM designs shown in Figure 6.6.

The retrieved \( z \) component of the effective refractive index and normalized wave impedance at the target frequency as a function of angle of incidence are shown in Figure 6.7. It can be observed from Figure 6.7(a)-(b) that for the first design a near-zero \( z \) component of the effective refractive index with low loss and a normalized wave impedance matched to free space are achieved throughout the targeted range of incidence angles from 0° to 30°, thus ensuring good
transmission properties. The imaginary part of the wave impedance has a very small value ($|Z_{TE}''| < 0.25$) from $0^\circ$ to $23^\circ$ and then increases with incidence angle. Compared to the first design, the second design has smaller effective refractive index and better matched effective impedance for normal incidence but an inferior angular response. The $z$ component of the effective refractive index at large oblique angles also drops below zero, which is undesired. The impedance match for the second design deteriorates from $17^\circ$ to $30^\circ$ due to drastic changes in the real and imaginary parts of its normalized wave impedance. The three retrieved effective anisotropic tensor parameters for the first design that are active for the TE polarization are shown in Fig 6.8. It can be observed that, for all the three parameters, the results retrieved using scattering parameters calculated at different incidence angles agree well with each other. $\varepsilon_{xx}$ shows a Drude-like response, which is produced by the metal patches that are fully connected across the screen, similar to structures that exploit the inherent Drude property of a metal film (e.g. metallic meshes). The Lorentz-shaped magnetic response in $\mu_{yy}$ is attributed to the parallel metal plates oriented along the $H$-field components transverse to the surface of the metal patches. These $H$-field components excite a circulating current on the neighboring metal patches at resonance, which in turn produces another $H$-field opposing the incident wave. The effective parameter $\mu_{zz}$ shows an almost flat curve, implying that this structure does not have any resonant magnetic response to the normal components of the $H$-field. The small differences occurring in $\mu_{zz}$ among the retrieval results using scattering parameters at different angles of incidence are attributed to the intrinsic periodicity of the metamaterial 'seen' by the transverse components of the incident waves [87], especially at higher frequencies where the metamaterial no longer possesses good effective homogeneous properties.
Figure 6.8. Retrieved effective parameters of IR ZIM design #1: (a) $\varepsilon_{xx}$, (b) $\mu_{yy}$, and (c) $\mu_{zz}$ using normal and oblique incidence angles; (d) $\mu_{zz}$ using two oblique angles of incidence. Note that for $\varepsilon_{xx}$ and $\mu_{yy}$, results retrieved from two oblique incidence angles and from only normal incidence are compared.

6.6 Overview

In conclusion, this chapter reports the design, fabrication, and characterization of a polarization-insensitive, impedance-matched ZIM with nearly perfect transmission at optical wavelengths. This was achieved by optimizing a metaldielectric fishnet structure to simultaneously balance the electric and magnetic response to give $\varepsilon_{\text{eff}} \to 0$ and $\mu_{\text{eff}} \to 0$ at a
wavelength of 1.55 µm, which resulted in $n_{\text{eff}} \sim 0$ and $Z_{\text{eff}}/Z_0 \sim 1$. The nanofabricated free-standing, Au-polyimide-Au ZIM accurately reproduced the designed geometry, and provided an axially symmetric structure that eliminated bianisotropy induced degradation in optical properties. Measurements of the complex transmission and reflection coefficients acquired using spectral holography agreed well with the theoretical predictions, and verified the near-zero index and highly transmissive properties of the fabricated metamaterial. This demonstration of a low-loss ZIM paves the way to more advanced metamaterial-enabled devices that require a range of low index values (e.g., $0 \leq n_{\text{eff}} \leq 1$). The verification of substrate-induced bianisotropy and its compensation will be discussed in Chapter 8. In addition, we have also proposed that, by employing the anisotropic inversion algorithm which takes the angular response of metamaterials into account, a ZIM can be designed to have a wide field-of-view.
Chapter 7

Dispersion Engineered Broadband Optical Metamaterials

7.1 Introduction

Fundamentally, the functionalities of metamaterials rely on their effective medium properties [176], which arise from sub-wavelength inclusions typically aligned in a periodic lattice. However, it is well-recognized that, due to the resonant properties of these inclusions, their effective refractive index and group delay are frequency dependent. This dispersive behaviour results in signal distortion and leads to narrow operational bandwidths, which have been major roadblocks limiting the widespread integration of metamaterials into practical devices. Recent efforts have been directed at overcoming these limitations. One approach is to design metamaterials for operation at frequencies away from the resonant band to avoid strong dispersion. This method allows metamaterial devices to possess broad bandwidths, and has been applied in the design of ground plane cloaks [115, 116] and Luneburg lenses [117, 118]. However, by avoiding highly dispersive regions, these particular devices exclude negative and zero/low index properties, which are arguably the most important and potentially rich regimes associated with metamaterials. In contrast to avoiding strongly resonant bands, dispersion engineering exploits the frequency-dependent properties of the metamaterial by tailoring them to the specific device needs, thereby improving existing components or leading to new optical functionalities [137, 177-178].

Although broadband dispersion engineering has been employed in the microwave regime to facilitate the design of certain planar guided-wave devices [27-29] and radiated-wave components [63, 65, 153], it has not yet been applied to the synthesis of optical metamaterials, which are of more "ultimate" interest. Tailoring the dispersive properties of optical metamaterials
promises enormous potential benefits to optical devices, but more challenges than those encountered at low frequencies in terms of both design and fabrication need to be overcome in order to effectively utilize the wavelength-dependent response of optical metamaterials. These obstacles are primarily attributed to the limitations in the complexity and precision of structures, which are determined by currently available nanofabrication techniques. Such limitations further reduce flexibility in the selection of metamaterial geometric features and the arrangement of these "meta atoms". Therefore, it is of great importance to find a path to effectively take advantage of dispersion engineering for optical metamaterials with a broad operational bandwidth by exploiting new kinds of nanostructures that are compatible with current design and fabrication techniques.

In this chapter, we discuss the synthesis of large-area, freestanding dispersion engineered photonic metamaterial devices. We begin by using this technique to create an optically-thin metamaterial band pass filter operating in the mid-infrared (IR) from 2.5 to 4.0 μm, with a low-loss seamless negative-zero-positive index transition and reduced group delay variation over the 3.0 to 3.5 μm transmission band. This goal was achieved by introducing a new variant of the conventional metal-dielectric-metal fishnet structure to include strategically placed sub-wavelength nano-notches that provide additional degrees of freedom in tailoring the effective metamaterial dispersion over a broad bandwidth. More specifically, the added nano-notches offer the designer more control over the resonating mode patterns on the metamaterial structure, which can be manipulated to shape both the permittivity and permeability profiles. Fabrication and measurement of this dispersion engineered mid-IR band-pass filter confirms its broadband, low-loss performance. We then show that the nano-notched negative-zero-positive index metamaterial filter structure can be extended to form an impedance-matched, low-loss polarization independent beam steering prism. These examples illustrate the great potential for developing new broadband devices by controlling and exploiting the dispersive properties of photonic metamaterials.
Figure 7.1. (a) Ideal response of a band pass filter with a flat transmission window and a flat group delay within the pass band. (b) Real parts of the dispersive permittivity (red), permeability (blue) and refractive index (green) profiles of a theoretical material. (c) The transmission (red), reflection (blue) and group delay $\tau_g$ (green) of a slab of this theoretical material with a thickness of $0.15\lambda_0$. (d) Simulated field plots at the labeled wavelengths in (b) of a prism composed of the theoretical material with the electromagnetic properties shown in (b) and a $30^\circ$ slope angle. The outer field plots show no transmission at the two magnetic resonances corresponding to $\lambda_{m1}$ and $\lambda_{m2}$, whereas the inner three plots show high transmission in the pass band with beam angles of $0^\circ$, $30^\circ$, and $60^\circ$ where $n = 1, 0, \text{ and } -1$, at $\lambda_p$, $\lambda_e$, and $\lambda_n$, respectively.

### 7.2 Dispersion Engineering for Broadband Optical Metamaterials

For the first demonstration of dispersion engineering, we target high performance optical band pass filters, which are critical components that are widely used throughout the photonics community. Figure 7.1(a) shows the specifications for an ideal metamaterial band pass filter. The transmitted signal of this ideal filter should have uniform transmission intensity and group delay ($\tau_g$) within the pass band to ensure minimal distortion in its frequency and time domain response. Conventional techniques for realizing optical band pass filters include stacking evaporated thin films with alternating high/low indices [179] or combining multiple birefringent half-wave plates...
[180]. However, for mid- and near-IR wavelength ranges, these devices are too thick to be integrated into nano- or micro-scale systems, and they require the integration of many different materials or components. Frequency selective surface (FSS) filters, which consist of a single metallic screen patterned on a thin dielectric substrate, have also been demonstrated in the far- and mid-infrared wavelength regimes [181, 182]. These FSS filters offer optically thin structures and more design flexibility, but have relatively narrow transmission windows.

Here, we employ dispersion engineering to realize an optical band pass filter by controlling the effective medium properties of a theoretical material, which determine the refractive index, intrinsic impedance and group delay. To realize the filter function, the real parts of the desired material parameters for the band pass filter designed using this approach are displayed in Figure 7.1(b). This plot shows a Drude permittivity and a permeability profile with two Lorentzian resonances; one on each side of the plasma wavelength \( \lambda_e \) associated with the permittivity. Assuming a time dependence of \( e^{j\omega t} \), the dispersive permittivity (\( \varepsilon \)) and permeability (\( \mu \)) models can be expressed in the form

\[
\varepsilon(\lambda) = \varepsilon_0 \left( \varepsilon_\infty - \frac{c_0^2 / \lambda_e^2}{c_0^2 / \lambda^2 - j c_0 \gamma_e / \lambda} \right)
\]

(8.1)

\[
\mu(\lambda) = \mu_0 \left( 1 - \sum_{i=1}^{2} \frac{F_i c_0^2 / \lambda_e^2}{c_0^2 / \lambda^2 - c_0^2 / \lambda_{mi}^2 - j c_0 \gamma_{mi} / \lambda_{mi}} \right)
\]

(8.2)

where \( c_0 \) is the speed of light in free space, \( F_1 \) and \( F_2 \) are the filling factors, \( \gamma_e \), \( \gamma_{m1} \) and \( \gamma_{m2} \) are the damping factors, and \( \lambda_{m1} \) and \( \lambda_{m2} \) are the wavelengths associated with the two magnetic resonances.

By properly tuning the damping factors, plasma wavelength, and magnetic resonance wavelengths, this theoretical material is designed to exhibit a gradually changing refractive index from negative to positive values with decreasing wavelength. Between the wavelengths corresponding to a negative unity and positive unity refractive index (i.e., between \( \lambda_n \) and \( \lambda_p \)) the permittivity and permeability are balanced such that the effective impedance is matched to free
space throughout the band, thereby achieving a pass band with near-uniform transmission intensity. Outside of this pass band, the imbalanced permittivity and permeability produce a mismatched impedance that effectively blocks the transmission of waves. Additionally, the slopes of the permittivity, permeability, and the resulting refractive index must be carefully controlled to minimize group delay fluctuations within the pass band. The group delay can be calculated by

\[
\tau_G = \frac{L}{v_g} = \frac{L(\text{Re}(n)+\omega \frac{d(\text{Re}(n))}{d\omega})}{c_0},
\]

where \(v_g\) is the group velocity, \(c_0\) is the speed of light in free space, and \(L\) is the total thickness of the material slab. Figure 7.1(c) shows the scattering parameter magnitudes and the group delay for a uniform \(\sim 0.15\lambda\) thick slab of this hypothetical medium. As the figure indicates, the filter achieves a highly transmissive, flat pass band and a near-constant group delay between \(\lambda_n\) and \(\lambda_p\).

The theoretical material introduced here was further used to construct a prism with a 30° slope as shown in Figure 7.1(d). Similar prisms have been demonstrated previously for the microwave and optical regimes as qualitative evidence of negative refraction. However, these previous demonstrations were limited to a single polarization, exhibited high reflection due to impedance mismatch, experienced significant losses due to an evanescent mode gap near the index zero crossing, and did not focus on tailoring the filtering properties of the device. The prism constructed from this material exhibits excellent out-of-band signal rejection and high transmission within the pass band with wavelength-dependent beam angles. The three pass band field plots in Figure 7.1(d) illustrate beam angles of 0°, 30°, and 60° corresponding to refractive index values of 1, 0, and -1, respectively. Within the homogeneous prism, the backwards propagating wave can be identified when \(n = -1\) at \(\lambda_n\), and the wave expands to have an infinite wavelength inside the prism when \(n = 0\) at \(\lambda_e\). This prism simulation demonstrates that the
theoretical material can simultaneously serve as a filter and perform additional functionality such as wavelength dependent beam steering.

7.3 Broadband Metamaterial Filter

Following this theoretical dispersive model, an optical metamaterial filter was designed with a pass band between 3.0 µm and 3.5 µm and stop bands on either side extending to 2.5 µm and 4.0 µm, respectively. The structure for this metamaterial, which we refer to as a modified fishnet, consists of a tri-layer metal-dielectric-metal stack perforated with doubly periodic notch-loaded square air holes as shown in Figure 7.2. The infinite metallic strip arrays parallel to the incident electric and magnetic fields are primarily responsible for the effective permittivity and permeability profiles, respectively. The addition of sub-wavelength notch loads to the square air holes allows the distance between the fundamental and higher order magnetic resonances as well as the metallic strip permittivity dilution percentage to be controlled; thereby providing further design flexibility and allowing for finer manipulation of both the effective permittivity and permeability profiles.

Figure 7.2. The geometry and dimensions of a single unit cell of the dispersion engineered metamaterial. The optimized geometry dimensions are $p=2113$ nm, $w=1123$ nm, $g=198$ nm, $t=30$ nm and $d=450$ nm.
Achieving the aforementioned dispersive properties within a broad wavelength range still requires fine adjustment of the geometric dimensions. To optimize the metamaterial structure for the desired effective medium parameter profiles, a GA was employed, coupled with an efficient full-wave electromagnetic solver and an inversion algorithm used to extract the effective medium parameters. The single unit cell geometry with its optimized dimensions is shown in Figure 7.2.

The period $p$ of the modified fishnet is 2113 nm with 990 nm square air holes perforating the lattice. Each corner of the square hole is loaded with two sub-wavelength $198 \times 198$ nm$^2$ notches. The total thickness of the tri-layer stack is 510 nm, which is less than $\lambda/6$ at the centre wavelength of the pass band (i.e. $3.25 \mu$m). Such a remarkably thin device could easily be integrated into nano- or micro-scale optical systems.

Figure 7.3. (a) Simulated and measured transmission (top, blue) and reflection (middle, red) magnitudes for normally incident radiation showing broadband transmission over the highlighted region from 3 $\mu$m to 3.5 $\mu$m. Simulated group delay $\tau_g$ (bottom) shows minimal variation over the transmissive window. (b) Real (top) and imaginary (bottom) parts of the effective index of refraction $n_{\text{eff}}$ (green), permittivity $\varepsilon_{\text{eff}}$ (red), and permeability $\mu_{\text{eff}}$ (blue) retrieved from the full-wave simulation of the metamaterial structure. The real parts of $\varepsilon_{\text{eff}}$ and $\mu_{\text{eff}}$ follow the same slope from 3 $\mu$m to 3.5 $\mu$m, indicating a matched impedance, whereas the imaginary parts are small, indicating low intrinsic losses.
The simulated scattering profile of the designed metamaterial filter exhibits a pass band between 3.0 µm and 3.5 µm with an average insertion loss of 0.9 dB and a maximum of 1.1 dB over the band (Figure 7.3(a)). Notably, this pass band has a variation of less than 0.4 dB within the transmission window. Outside the pass band, the average transmission coefficient is ~10.2 dB, indicating that more than 90% of the incident light is blocked by the metamaterial filter. A sharp transition can be seen between the pass band and the stop band with a roll-off of ~93 dB/µm on the shorter wavelength side and ~101 dB/µm on the longer wavelength side. As predicted from theory, the filter characteristics arise from the custom designed effective medium properties of the metamaterial (Figure 7.3(b), top), with the refractive index changing from negative unity to positive unity with decreasing wavelength across the pass band. Moreover, the imaginary part of the refractive index has a magnitude less than 0.15 across the entire pass band (Figure 7.3(b), bottom), ensuring low absorption loss in this range. Notably, the transition from the negative to positive index band is impedance matched, eliminating the evanescent mode gap that is commonly observed in other negative index metamaterials [183, 184]. Hence, the zero index band possesses a non-vanishing group velocity and is an optical analogue to the balanced transmission-line metamaterial in the microwave range [137, 177]. The group delay of this metamaterial filter, which is inherently dispersive, can be calculated from Eq. (8.3). Owing to the careful manipulation of the profiles of the effective medium parameters throughout the pass band during the design process, the slope of the effective refractive index is well controlled such that the group delay shows only a small variation (Figure 7.3(a), bottom). Within the pass band from 3.0 µm to 3.5 µm, the group delay varies from 15 fs to 27 fs, which has a fluctuation of about 1 period within a 20% bandwidth. This fluctuation is much lower than has been previously reported for photonic metamaterials [70, 185]. The negative index metamaterials presented in Ref. 70 and Ref. 187 both have a group delay fluctuation of approximately 3 periods within a 5% bandwidth. In addition, the simultaneously negative phase and group velocity can also be identified within
the range from 3.65 µm to 3.70 µm, which numerically corroborates the previously reported result [185]. We expect that even better control over the effective parameter profiles and resulting filter properties could be achieved given additional geometric design flexibility, at the expense of more challenging nanofabrication requirements.

Figure 7.4. (a) Volumetric current density distribution on the top Au layer at 3.7 µm (left) and 2.85 µm (right). (b) Volumetric current density distribution on the bottom Au layer at 3.7 µm (left) and 2.85 µm (right). (c) Top-view of magnetic field distribution in the structure at 3.7 µm (left) and 2.85 µm (right).
The gradual index transition and the low-loss, flat top pass band can be better understood by examining the effective permittivity and permeability of the metamaterial (Figure 7.3(b)). The effective permittivity varies from negative unity to positive unity with decreasing wavelength in the band of interest, with two small anti-resonances caused by the corresponding magnetic resonance modes. The effective permeability shows a primary resonance at 3.7 µm and a weaker, secondary resonance at 2.85 µm, which correspond to the two transmission minima. Within the pass band, the permeability also changes from negative unity to positive unity with a sharper slope on the longer wavelength side. The main resonance has been widely exploited in the literature as a prerequisite for the formation of a negative index band, but the second resonance mode has often been overlooked because it is typically weaker than the primary mode, and its position is generally too far from the main resonance to be useful. In this design, the effective excitation of this second resonance arises from the perturbed current distribution produced by the nano-notches, providing the possibility of shaping the permeability and, thus, the dispersive profile in the shorter wavelength region.

The volumetric current density distributions on the top and bottom metal layers in Figure 7.4 show that both of the magnetic resonance modes can be attributed to the anti-parallel currents. These currents, driven by the incident magnetic field, induce another magnetic field that either enhances or reduces the total magnetic field strength as characterized by Lorentz-shaped resonances. For the primary resonance at 3.7 µm, the currents flow predominantly in the x direction, with a phase difference of 180º between the central area and the outer areas of the unit cell. This results in the thin rectangular charge accumulation/depletion regions near the boundary of the central square with different current phases, which further gives rise to displacement currents in the regions between the two metal layers. The magnetic field distributions show a resonating dipole mode (Figure 7.4(c), left) in close correlation with the current distributions. The fields are pointing mainly in the y direction and exhibit a 180º phase difference in the central area.
compared with the other areas. In contrast to the primary resonance, the current distribution at 2.85 µm is more complicated with the charge accumulating/depleting areas reduced to two small, point-like regions. This leads to a curl-type distribution of the magnetic fields in the corresponding regions due to the existence of a linear displacement current between the metal layers (Figure 7.4(c), right). Two quadrapole modes can be identified near the top and bottom edges of the central area in the magnetic field distribution. When compared to the primary mode at 3.7 µm, these quadrapole modes are weaker in terms of the magnetic field intensity, resulting in a weaker resonance in the permeability with a lower quality factor as seen in Figure 7.3(b).

![Numerical parametric analysis](image)

**Figure 7.5.** Numerical parametric analysis of the modified fishnet nanostructure with various nano-notch inclusion sizes. (a) Transmission amplitude. (b) Group delay. (c) Effective permittivity. (d) Effective permeability.

As illustrated in Figure 7.4, the deep subwavelength nano-notch inclusions perturb the current distribution and magnetic field of the conventional fishnet structure. A detailed parametric
study that analyzes the effect of the nano-notch inclusions on the broadband metamaterial properties and the effective medium parameters is provided in Figure 7.5. These results show that removing the nano-notch inclusions from the modified fishnet structure converts the high transmittance 1 dB pass-band window into a much wider stop-band, with a high average reflected power of 85% (-0.7 dB) across the 3.0 µm to 3.5 µm wavelength range. This dramatic difference in the optical properties of the metamaterial is due to the large change in resonance wavelength and slope of the effective permittivity and permeability parameter profiles. Further study re-optimizing the dimensions of the conventional fishnet nanostructure to meet the metamaterial filter design criteria reveals that the lack of subwavelength nano-notches places appreciable limits on the best filter performance that can be obtained.

The effect of varying the nano-notch sidewall length from $g = 138$ to $258$ nm in 30 nm increments around the optimized value of $g = 198$ nm is also presented in Figure 8.5. Notably, increasing or decreasing critical feature size of the nano-notches by only ± 30 nm (equivalent to $\sim\lambda_p/100$) from the optimized value results in a significant reduction in the average pass-band transmission of 82% down to 69% for $g = 168$ nm and 64% for $g = 228$ nm (Figure 7.5(a)). In addition, the 0.5 µm bandwidth of the 1 dB pass-band window increased by 1000 nm for $g = 168$ nm and decreased by 800 nm for $g = 228$ nm. Even larger deviations from the optimized transmission, reflection, and bandwidth are observed for the smallest and largest nano-notch dimensions studied here. Similar trends in variability and bandwidth are observed in the group delay of the metamaterial (Figure 7.5(b)).

Figure 7.5(c) and Figure 7.5(d) show that variations in nano-notch critical feature size equivalent to $\sim\lambda_p/100$ have a large effect on the effective permittivity and permeability dispersion across the wavelength band of interest. This is because the air-hole waveguide array cut-off wavelength and the gap-SPP propagation are both strongly dependent on the deep subwavelength inclusions. Specifically, the effective plasma wavelength of the modified fishnet structure
increases from a value of \( \lambda_{\text{plasma}} = 3.0 \, \mu m \) when \( g = 138 \, \text{nm} \) to \( \lambda_{\text{plasma}} = 3.6 \, \mu m \) when \( g = 258 \, \text{nm} \). Additionally, the anti-resonance associated with the fundamental magnetic resonance mode becomes weaker with increasing \( g \), which reduces the wavelength dependence in the effective permittivity for structures with larger nano-notches. In contrast, the primary magnetic resonance becomes significantly stronger, while the secondary resonance becomes weaker and shifts to longer wavelengths, with increasing \( g \). Thus, the effective permeability becomes more dispersive as the nano-notch size is increased. The dependence of structural resonances on small geometrical variation revealed here also corroborates previous study on the resonance positioning of plasmonic particles based on geometrical perturbation [186].

![Image of fabricated metamaterial structure](image)

Figure 7.6. (a) Top-view SEM image of a portion of the fabricated modified fishnet metamaterial filter structure (inset, magnified top view). The accurate reproduction of the nano-notch features is critical to maintaining the designed properties in the fabricated structure. Scale bar: 3 \( \mu m \). (b) Tilted view SEM image of the free-standing fabricated metamaterial filter structure with vertical (89°) side walls. Scale bar: 1 \( \mu m \).

A prototype of the metamaterial pass band filter was fabricated by patterning the optimized notch-loaded square air hole array in a deposited tri-layer Au-polyimide-Au stack using electron-beam lithography and high-aspect-ratio dry etching. Recently, it has been reported that a supportive substrate under the metamaterial breaks the symmetry of the structure, resulting
in bianisotropy [78]. As a result of this coupling, the transmitted light will have significant components of undesirable polarization orthogonal to the incident light. The substrate also causes increased reflection loss due to the impedance mismatch at the metamaterial interfaces. To prevent these nonidealities in our experimental measurements, the optically thin metamaterial structure was released from the sacrificial handle substrate used during fabrication prior to characterization. A field emission scanning electron microscope (FESEM) image captured from the top of the freestanding filter (see Figure 7.6(a)) shows that the modified fishnet structure replicates the design geometry almost perfectly. Importantly, the critical dimensions of the nanonotchtes are 200×200 nm² (measured from both the top and backside). The vertical sidewalls with a measured sidewall angle of 89° (see Figure 7.6(b)) also ensure identical dimensions for all three layers of the structure. This prevents sidewall angle induced magneto-electric coupling, which is another source of bianisotropy present in most metallo-dielectric optical metamaterials [77].

The freestanding metamaterial filter was characterized using a Fourier transform infrared spectrometer equipped with an additional custom optical setup to obtain transmission and reflection at normal incidence. The measured scattering parameter amplitudes shown in Figure 7.3(a) agree well with the simulation results, showing high flat in-band transmission and high out-of-band reflection, with only slight discrepancies in filter bandwidth and roll-off rates. Specifically, the measured bandwidth of the transmission window is 0.15 µm wider than the designed range, extending from 2.95 µm to 3.60 µm as compared with 3.0 µm to 3.5 µm. The roll-off rates of the short and long wavelength edges of the pass band are decreased from the predicted 93 dB/µm and 101 dB/µm to the measured 76 dB/µm and 91 dB/µm. The bandwidth broadening and the decreased roll-off rates are caused by the lower quality factor of the resonances within the band (see Figure 7.3(a)). The transmission window of the fabricated metamaterial filter has a flattened shape with a variation of only 0.5 dB over the band, which shows that the metamaterial impedance match is maintained in the band as the index traverses
zero. The average in-band insertion loss of the fabricated filter is 1.0 dB with a worst case value of 1.3 dB over the band, while the transmission outside the pass band remains below -10 dB at longer wavelengths and slightly higher than -10 dB in the shorter wavelength region, with an overall average stop band amplitude of -10.1 dB. The measured device performance is slightly reduced from the predicted 0.9 dB average in-band insertion loss and -10.2 dB average out of band rejection, but it still meets the design target of 1.0 dB insertion loss in the pass band and -10 dB stop band transmission (less than 10% transmission intensity).

Figure 7.7. (a) The 3D tilted view of the configuration of the prism and the orientation of the incident beam. The optimized dimensions for this structure are \( p=1985 \) nm, \( w=1178 \) nm, \( g=372 \) nm, \( t=46 \) nm and \( d=76 \) nm. The inset shows the side view of the metamaterial prism. (b) The
simulated reflection of the actual metamaterial prism for both TE and TM polarizations. (c) Snapshots of electric field distribution for TE polarization at different wavelengths. (d) Snapshots of electric field distribution for TM polarization at different wavelengths. Outside the pass band, no wave is transmitted through the prism. Within the pass band, the wave is refracted with the exiting beam at angles of 24°, 13° and 0° relative to the incident beam, corresponding to the \( n_{\text{eff}} = -1, 0, \text{ and } 1 \) bands, respectively.

7.4 Polarization-Independent Beam-Steering Metamaterial Prism

Next, we numerically demonstrate that this dispersion engineered modified (notched) fishnet structure, when constructed in a prism configuration, can be utilized to create a polarization-independent wavelength-dependent beam steering device. By once again employing a GA optimization scheme, the best design dimensions for the notched fishnet structure with eleven alternating layers of Au and polyimide were found to be \( p=1985 \text{ nm}, w=1178 \text{ nm}, g=372 \text{ nm}, t=46 \text{ nm}, \text{ and } d=76 \text{ nm} \). A prism was then constructed from the multilayer stack with a slope angle of approximately 13° across four periods as illustrated in Figure 7.7(a). For the simulation of the metamaterial prism structure, the HFSS finite element solver was employed. In the simulation domain, a portion of the metamaterial prism with a width of one unit cell in the \( x \) direction and a length of four unit cells in the \( y \) direction was considered. The prism has a step every half unit cell with a slope angle of around 13°. The thinnest end has nine layers while the thickest end has thirty seven layers. To mimic a 1D infinite structure, perfect electric conducting boundary conditions were assigned to the front and back walls in the \( x \) direction for TE polarization and perfect magnetic conducting boundary conditions for TM polarization. A waveguide and a wave port were employed to produce the incident waves impinging normally on the prism. Dispersive optical constants for the Au and Kapton layers measured by variable angle spectroscopic ellipsometry were also included in the electromagnetic model to enhance its
accuracy. Similar to the previously presented filter, this prism has a pass band from 3.0 μm to 3.7 μm with the effective index of refraction ranging from negative unity through zero to positive unity for both TE and TM polarizations at normal incidence. Outside the pass band, the incident light is strongly reflected (see Figure 7.7(b)), which is to be expected due to the >12.2 dB insertion loss from the metamaterial alone. By virtue of the tilted interface of the prism and the changing index in the pass band, the transmitted beam is directed at different angles depending on the wavelength of the incident light. Figure 7.7(b) shows the reflection properties of the prism for both TE and TM polarizations, which demonstrate that the reflection amplitude is smaller than -12 dB only within the desired beam steering pass band, signifying a good out-of-band band rejection by the prism. Snapshots of the electric field distribution at different wavelengths for both polarizations are shown in Figure 7.7(c) and Figure 7.7(d) indicating that outside the pass band, there is almost no wave passing through the prism due to the as-designed strong impedance mismatch between the prism and free space. Within the pass band, at 3.66 μm where the $n_{\text{eff}}$ is close to negative unity, the beam is directed at 24°. At 3.41 μm, where the $n_{\text{eff}}$ is close to zero, the beam is directed at 13° (normal to the interface of the prism), and at 3.06 μm, where the $n_{\text{eff}}$ is close to positive unity, the beam is in the same direction as that of the incident wave. It should be noted that this represents the first example of an impedance matched beam-scanning metamaterial prism with negative-zero-positive effective refractive indices that works for both TE and TM polarizations.

7.5 Overview

In summary, the concept of broadband dispersion engineering was demonstrated for the first time in the optical regime and successfully applied to synthesize a photonic metamaterial band pass filter that operates in the mid-IR wavelength range from 2.5 μm to 4.0 μm. This
optically thin, freestanding metamaterial filter exhibits a flat pass band from 3.0 µm to 3.5 µm along with suppressed group delay fluctuation. The filtering functionality was achieved by adding sub-wavelength nano-notches to the air-holes of a conventional fishnet structure, thereby enabling the manipulation of the resonant modes and the tailoring of both the permittivity and permeability profiles of the photonic metamaterial over a wide bandwidth. Measurements of the fabricated proof-of-concept device confirmed the theoretical predictions of the filter’s broadband low-loss performance. This broadband metamaterial was further applied in a prism configuration to realize the first impedance matched, polarization independent beam steering metamaterial lens with strong out-of-band rejection. The dispersion engineering approach introduced here opens up the doors to an entirely new array of broadband optical metamaterial devices such as coatings with user-specified index profiles, wavelength-dependent beam steering lenses, and other devices with extraordinary optical functionalities.
Chapter 8

Verification and Compensation of Substrate-induced Bianisotropy

8.1 Introduction

Recently, significant attention has been devoted to realizing self-symmetric multilayer structures that approximate an ideal isotropic/anisotropic medium under illumination at normal incidence. Metamaterials have been demonstrated at infrared and optical wavelengths by patterning nanoscale air holes in a stack of alternating metal and dielectric layers deposited on optically transparent substrates using either lift-off techniques or focused ion beam (FIB) etching. However, most of these structures possess unwanted broken symmetry that is caused by the process-induced sidewall angles [87] and the presence of a supporting substrate [88]. This broken symmetry invalidates the previously used anisotropic effective medium model because the electric and magnetic polarizations are now determined by both the electric and magnetic field components. Thus, the bianisotropic effective medium model must be adopted to accurately describe the effective material properties of these optical metamaterials.

Particularly, it has been experimentally demonstrated that tapered sidewalls induce bianisotropy due to the asymmetric field distribution on the top and bottom metal layers of the self-asymmetric fishnet structure [87]. It has also been proposed theoretically that the presence of a substrate influences the plasmonic resonances of metamaterials and metasurfaces by introducing an anti-symmetric mode that gives another source of bianisotropy to the self-symmetric fishnet [88]. The sidewall-angle and/or substrate induced bianisotropy not only complicates the effective medium description, but may also inhibit the desired index properties and introduce more reflection and absorption loss to the system. However, the previous
experimental study on the sidewall angle induced bianisotropy characterized metamaterial structures that were supported by a substrate. This hence makes it difficult to determine if the observed bianisotropy was due to the tapered sidewalls or the substrate, or both of them. Therefore, it is necessary to optically characterize metamaterials with only one of the two sources of bianisotropy to determine the contribution from each source independently.

In this chapter, we first present an experimental demonstration of substrate-induced bianisotropy in an optical metamaterial. By fabricating a fishnet nanostructure free from the broken symmetry that results from tapered sidewalls, the contribution of the bianisotropy from the substrate alone is evaluated. This bianisotropy was characterized by measuring the complex scattering parameters for a fishnet metamaterial mounted on a substrate from both sides using spectral holography, which are then compared to those for the free-standing fishnet. The effective medium parameters were then extracted, showing a magnetoelectric coupling parameter with non-zero values at the resonances. The measured results agree well with numerical simulations, showing asymmetric reflections for the on-substrate fishnet but not for the free-standing one, thus confirming the bianisotropy introduced by the substrate. In addition, we also propose an effective way to suppress and even eliminate the substrate-induced bianisotropy (as seen by an outside observer) of the system containing a finite-thickness metamaterial. We study the effect of a substrate with varying finite thickness values, showing that the induced bianisotropy is a near-field effect. The effect of introducing a superstrate with various thickness and permittivity values is investigated by full-wave simulations, where the substrate is considered to be semi-infinite, closely approximating the practical situation. It is shown that by properly choosing the thickness and permittivity values of the superstrate, the substrate-induced bianisotropy of the system can be greatly suppressed or even completely cancelled out. Furthermore, to confirm that the bianisotropy compensation is a real physical effect that does not rely on the homogenizability of the structure, the induced electric dipole moment (EDM) and magnetic dipole moment (MDM)
excited by approximately pure electric or magnetic fields are calculated from volumetric microscopic fields. The results exhibit a non-zero magnetoelectric coupling only for the case with a substrate, but near-zero values for the cases without substrate and with both substrate and properly designed superstrate. The studies presented in this paper will be particularly beneficial, for metamaterial and/or metasurface associated experimental validations and practical implementations.

Figure 8.1. Schematics of the fishnet metamaterial on the substrate. Top view of a portion of the fishnet structure. $d = 381$ nm, $p = 956$ nm, and $w = 365$ nm.

8.2 Experimental Verification of Substrate-Induced Bianisotropy

The specific optical metamaterial used here for verifying the substrate-induced bianisotropy is a fishnet nanostructure (see Figure 8.1(a)), which is the same one that has been presented in Chapter 6. For the numerical simulation of the fishnet, a full-wave electromagnetic solver HFSS, based on finite element method, was employed with periodic boundary conditions assigned to a unit cell and Floquet ports at the top and bottom of the simulation domain to excite a normally incident plane wave. To accurately account for the constituent material dispersion, measured optical properties of the metallic and dielectric materials were incorporated. It was confirmed in our previous study that this structure has a zero index band in the vicinity of 1.5 $\mu$m with an impedance matched to that of free space because both the effective permittivity and permeability are approaching zero at the same rate. These properties ensures a near-zero phase
delay and high transmission for the light passing through the metamaterial nanostructure at 1.5 \( \mu \)m. It can be seen from Figure 8.2(b) that the complex reflection and transmission coefficients measured from both sides of the nanostructure are almost the same, indicating that the fabricated sample is truly symmetrical in the direction of wave propagation, i.e. having near-perfectly vertical sidewalls. For completeness, the simulated scattering parameters are also displayed in Figure 8.2(a), which are in strong agreement with the measured results.

Figure 8.2. (a) Simulated (left) and measured (right) scattering parameter amplitudes of the free-standing fishnet nanostructure. (b) Simulated (left) and measured (right) scattering parameter phases of the free-standing fishnet nanostructure.
To examine the substrate effect, this symmetric metamaterial was mounted on a clean fused silica substrate by adding a drop of water. The capillary action of water flattened the metamaterial film and reduced the gap in between the metamaterial film and substrate. As water evaporates between the fishnet and the substrate in a desiccator, the mounted metamaterial film was further smoothed and eventually stuck to the substrate. The substrate-induced bianisotropy in the mounted optical metamaterial was verified by measuring the complex transmission and reflection coefficients using a spectral holography technique with a white-light supercontinuum input source, as described in Chapter 6.

Figure 8.3. (a) Simulated (left) and measured (right) scattering parameter amplitudes of the on-substrate fishnet nanostructure. (b) Simulated (left) and measured (right) scattering parameter phases of the free-standing fishnet nanostructure.
Figure 8.3(a) and Figure 8.3(b) present the simulated and measured scattering parameters for the on-substrate fishnet. The calculated scattering parameters were transformed to be referenced to the characteristic impedance of free space. Figure 8.3(a) shows the amplitudes of the calculated and measured scattering parameters. The transmission coefficients seeing from both sides are almost identical in terms of both amplitude and phase due to reciprocity. The calculated transmission curves, which contain two stop-bands located at 1.39 µm and 1.55 µm, are consistent with the measured transmission amplitudes and phases from both the air and substrate sides. The two stop-bands correspond to the excitation of symmetric and antisymmetric plasmonic resonance modes. It should be noted that the stop-band at 1.39 µm is not seen in the free-standing fishnet with the same dimensions, due to the lack of excitation of the antisymmetric mode, while the stop-band at 1.57 µm for the free-standing fishnet is shifted to 1.55 µm for the on-substrate fishnet caused by the substrate loading. In contrast, the reflection amplitudes and phases seen from the air and substrate sides show noticeable differences, especially around the two resonances, due to the broken symmetry, indicating that these resonances undergo the substrate induced effect which produces asymmetric field profiles. Because of measurement noise, small discrepancies are present between the measured and simulated scattering parameter curves. Nonetheless, the bianisotropy effect introduced by the substrate can still be clearly observed.

To quantitatively analyze the bianisotropy of the mounted optical fishnet, measured and calculated scattering parameters were inverted to obtain the effective medium properties, including the effective permittivity ($\varepsilon_{\text{eff}}$), the effective permeability ($\mu_{\text{eff}}$) and the magnetoelectric coupling coefficient ($\xi_{\text{eff}}$). Because the isotropic/anisotropic retrieval methods used for a homogeneous symmetric structure fail to account for the bianisotropy in the extraction, a bianisotropic retrieval algorithm [86] was employed to take the coupled electric and magnetic fields into consideration. The $\varepsilon_{\text{eff}}$, $\mu_{\text{eff}}$, and $\xi_{\text{eff}}$ retrieved from both simulated and measured
scattering parameters are shown in Figure 8.4. For clarity, only the real parts are plotted. It can be seen that the effective permittivity has a strong resonance around 1.39 μm, contributing to the shorter wavelength resonance observed in the transmission coefficient. At 1.55 μm, additional resonances in both the effective permeability and permittivity combine to produce a second dip in the transmission coefficient. The magnetoelectric coupling coefficient is significant at both of these resonance wavelengths with apparent negative values, which corroborates the results presented previously in theoretical works [87].

Figure 8.4. (a) Simulated (left) and measured (right) scattering parameter amplitudes of the on-substrate fishnet nanostructure. (b) Simulated (left) and measured (right) scattering parameter phases of the on-substrate fishnet nanostructure.

To understand the impact of the substrate on the current distribution on the fishnet metamaterial, the $x$-component current densities at two resonance wavelengths of the fishnet with and without the substrate are plotted in Figure 8.5. The incident electric field is also along the $x$-direction; thus the relevant bianisotropic constitutive relations are $D_x = \varepsilon_0 \varepsilon_x E_x - i \xi H_y / c$, $B_y = \mu_0 \mu_y H_y + i \xi E_x / c$, where $c$ is the speed of light. When $\xi$ is zero, the medium reduces to an anisotropic one, so the electric/magnetic polarization is only related to the electric/magnetic field. As shown in Figure 8.5(a), the fishnet without a substrate has almost equal current distribution on the top and bottom Au layers, meaning that no electric/magnetic dipoles are induced by the
magnetic/electric field. However, with the substrate present, asymmetric current distributions can be observed at the two resonances where $|\xi_{\text{eff}}|$ is most significant. In response to the $y$-directed incident magnetic field, an uneven current distribution arises on the two Au layers which further induces a non-zero net electric dipole moment in the $x$-direction. At the same time, the unequal currents in the $x$-direction induced by the incident electric field result in a magnetic dipole in the $y$-direction. Therefore, the electric and magnetic fields simultaneously affect each other. From another perspective, since the substrate has a higher index than air, it provides stronger field confinement which in turn results in a stronger current on the bottom Au layer. This difference becomes more significant as the index of the substrate gets larger, leading to a larger $|\xi_{\text{eff}}|$ as pointed out in the literature.

Figure 8.5. (a) Simulated $x$-component of current density at 1.39 µm (top) and 1.55 µm (bottom) for fishnet without substrate. (b) Simulated $x$-component of current density at 1.39 µm (top) and (bottom) 1.55 µm for fishnet with a substrate.
8.3 Compensating Substrate-Induced Bianisotropy Using Ultrathin Superstrates

Having experimentally verified the existence of substrate-induced bianisotropy and its effect on the metamaterial electromagnetic properties, in this section, we propose a technique to compensate for this undesired side effect by adding a properly designed superstrate coating on the multilayer metamaterials.

8.3.1 On-Substrate Multilayer Fishnet Metamaterial

Here we consider a specific example of the experimentally important fishnet nanostructure. Instead of a single functional layer, a thirteen-layer (i.e. six functional layers) fishnet structure is employed. The general setup under consideration is displayed in Figure 8.6(a), which includes a multilayer fishnet sandwiched between a superstrate and a substrate each with a certain thickness \( (h_{sup}, h_{sub}) \) and relative permittivity value \( (\varepsilon_{sup}, \varepsilon_{sub}) \). Below the substrate and above the superstrate are two semi-infinite half-spaces each having a relative permittivity \( \varepsilon_b \) and \( \varepsilon_t \), respectively. A time dependence of \( \exp(-i\omega t) \) is assumed throughout the paper. In order to retrieve the effective parameters of the fishnet, first, the scattering matrix of the metamaterial alone \( (S_{MM}) \), which cannot be measured directly in experiment, must be extracted from the measurable total scattering matrix \( (S_{tot}) \).

The procedure starts by obtaining the total transmission matrix from the total scattering matrix \((S_{tot})\) as [187]

\[
T_{tot} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{S_{12}} \begin{bmatrix} -|S_{tot}| & S_{22} \\ -S_{11} & 1 \end{bmatrix}
\]

(8.1)
Then, because the entire system can be regarded as several cascaded two-port networks separated by infinitesimally thin free-space layers, its total transmission matrix \( T_{tot} \) can be readily expressed as the product of all the two-port transmission matrices in free-space [188]

\[
T_{tot} = (T_{ths})(T_{sub})(T_{MM})(T_{sup})(T_{bhs}),
\]

where \( T_{ths(bhs)} \) and \( T_{sup(sub)} \) are the transmission matrices for the top (bottom) half space interface and the superstrate (substrate), respectively. Hence, the transmission matrix of the metamaterial alone \( (T_{MM}) \) is

\[
T_{MM} = (T_{sub})^{-1} (T_{ths})^{-1} (T_{tot})^{-1} (T_{bhs})^{-1},
\]

and the scattering matrix of the metamaterial alone can then be calculated from \( T_{MM} \) by

\[
S_{MM} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} = \frac{1}{T_{22}} \begin{bmatrix}
-T_{21} & 1 \\
1 & T_{12}
\end{bmatrix}.
\]

In regards to the substrate and the superstrate layers, their transmission matrices can be written in the form of the transmission matrix of a slab with a known relative permittivity \( \varepsilon_s \), and thickness \( d \) in free space, as expressed in Eq. (5):

\[
T_{slab} = \frac{1}{2} \begin{bmatrix}
1 + \sqrt{\frac{\varepsilon_s}{\varepsilon_s}} & 0 \\
1 - \sqrt{\frac{\varepsilon_s}{\varepsilon_s}} & e^{j \sqrt{\varepsilon_s} k d}
\end{bmatrix},
\]

The first and the last terms represent the transmission matrices for the two interfaces of the slab, \textit{i.e.} the interface from free-space into the slab and the interface from the slab into free-space, while the matrix in the middle contains the phase delay and amplitude attenuation information within the homogenous slab. The transmission matrices for the top and bottom half-space interfaces are simply in the form of the first and the last matrices in Eq. (8.5), respectively, with properly chosen values for the permittivities. Following the above procedure, the scattering matrix of the metamaterial alone can be obtained, which may then be used for retrieving the corresponding effective medium parameters.
The effective medium parameters of the on-substrate multilayer fishnet nanostructure can be retrieved from its complex transmission and reflection coefficients. Due to the substrate-induced bianisotropy, the electric displacement \( \mathbf{D} \) and the magnetic flux density \( \mathbf{B} \) are expressed as \( D_i = \varepsilon_0 \varepsilon_i E_i + i \xi_{ij} H_j / c \) and \( B_j = \mu_0 \mu_i H_i - i \xi_{ji} E_i / c \) where \( \xi_{ij} = \xi_{ji} = \xi \). In addition, the impedances seen by waves propagating in opposite directions differ with \( z^+ = \mu / (\sqrt{\mu \varepsilon - \xi^2} + i \xi) \) and \( z^- = \mu / (\sqrt{\mu \varepsilon - \xi^2} - i \xi) \), resulting in different reflection coefficients on opposite sides. A bianisotropic retrieval method is adopted here [86], which utilizes the reflection coefficients obtained from the two interfaces of the structure, denoted as \( S_{11} \) and \( S_{22} \) \( (S_{11} \neq S_{22}) \) and the transmission coefficient \( S_{21} \) or \( S_{12} \) \( (S_{21} = S_{12}) \), to retrieve the effective permittivity \( \varepsilon \), permeability \( \mu \), and magnetoelectric coupling parameter \( \zeta \). This method can also be applied for a free-standing symmetrical fishnet nanostructure, where the magnetoelectric coupling should be zero.

Figure 8.6. (a) A multilayer fishnet metamaterial sandwiched between a superstrate and a substrate with finite thickness. Beneath the substrate and above the superstrate are the bottom and top half-spaces, respectively. (b) The unit cell geometry of the multilayer fishnet nanostructure.
composed of Ag and SiO$_2$. The dimensions are $p_x = 600$, $p_y = 600$, $w_x = 72$, $w_y = 336$, $t_a = 15$, $t_d = 20$ (all in nanometers). (c) Retrieved bianisotropic effective medium parameters for the free-standing fishnet displayed in (b). (d) Evolution of the real part of the effective index as a function of the number of functional layers ($N$) for the free-standing fishnet in (b).

The geometry of the multilayer fishnet nanostructure unit cell considered here is shown in Figure 8.6(b), along with its dimensions. The HFSS finite element method full-wave solver was used to calculate the scattering matrix with periodic boundary conditions applied to the lateral walls of the simulation domain. A fifth order Drude-Lorentz model was employed to fit the measured permittivity of silver (Ag) [189] and the permittivity of SiO$_2$ was chosen to be 2.25. The normally incident plane wave has its $E$-field polarized in the $x$ direction. Before investigating the effects of the added substrate and/or superstrate, we first retrieve the effective medium parameters of the free-standing multilayer fishnet, as displayed in Figure 8.6(c), where for clarity only the real parts are shown. A negative index band can be identified between 180 and 210 THz ($i.e.$ 1.67 and 1.43 $\mu$m), while the magnetoelectric coupling is zero throughout the entire band due to the structural symmetry. As previous studies have shown, at normal incidence, the zeroth-order Bloch mode dominates light propagation inside the fishnet nanostructure so that it can be considered effectively homogeneous [190]. An indicator of such a property is the convergence of the effective refractive index with an increasing number of functional layers [191, 192]. In Figure 8.6(d), the effective index as a function of the number of functional layers ($N$) is presented, showing that the effective index begins to converge with $N=6$, $i.e.$ thirteen layers, which corroborates previous findings in the literature [191]. In the remainder of the paper, the thirteen-layer fishnet nanostructure will be employed for studying the effect and compensation of substrate-induced bianisotropy.

In previously reported work on substrate-induced bianisotropy in metamaterials, the substrate was considered as a semi-infinite half-space [193], which is a practical approximation
because the thickness of the supporting wafer is usually on the order of more than 200 wavelengths. In Figure 8.7(a) and Figure 8.7(b), the scattering parameters and the retrieved effective parameters of the multilayer fishnet nanostructure on a semi-infinite substrate made of SiO$_2$ are plotted, respectively. It is shown that both the amplitudes and phases of $S_{11}$ and $S_{22}$ are different, with the most significant differences occurring near the plasmonic resonances of the nanostructure due to resonance hybridization [78, 194]. While the retrieved effective permittivity and permeability exhibit similar properties compared to those of the free-standing fishnet, there is a slight frequency shift and additional small resonances due to the substrate loading. The relevant component of magnetoelectric coupling ($\xi$), however, becomes non-zero and shows a strong resonance around the main magnetic resonance at 180 THz (1.67 µm). In particular, when comparing Figure 8.7(a) with Figure 8.7(b) it is observed that the reflection phase difference and the magnetoelectric coupling parameter both have a main peak in the 185~195 THz (i.e. 1.54~1.62 µm) range and two small peaks around 130 THz (2.31 µm) and 160 THz (1.87 µm); whereas, the differences in the magnitudes of $S_{11}$ and $S_{22}$ can be seen as having a main peak around 160 THz (1.87 µm) and two additional smaller peaks at 130 THz (2.31 µm) and 185 THz (1.62 µm). This indicates that the property of the magnetoelectric coupling is manifested more in the difference of the phases, rather than the amplitudes of $S_{11}$ and $S_{22}$ as commonly stated in literature. In all, both the scattering parameters and the retrieved effective parameters corroborate previous substrate-induced bianisotropy studies [78, 195].

In addition to the semi-infinite substrate scenario, we further investigated the dependence of the induced bianisotropy on the thickness of the substrate. First, a 5µm thick substrate was considered, which is about 3 wavelengths thick at the frequency of the magnetic resonance. Different from the semi-infinite substrate case, a Fabry-Perot effect appears in the total scattering parameters due to the multiple reflections at the fishnet-substrate and substrate-air interfaces [190, 196] (not shown here). However, it can be seen that the scattering parameters of the
metamaterial alone (see Figure 8.7(d)), both in amplitudes and phases, are very similar to the case where the metamaterial is on top of semi-infinite substrate. This results in a set of effective parameters (see Figure 8.7(d)) that are similar to those of the metamaterial on the semi-infinite substrate. Hence, this implies that the induced bianisotropy caused by the substrate is a near-field effect primarily attributable to the evanescent modes, which corroborates findings on the asymmetric reflection effect due to substrate loading for chiral metamaterials [197]. In order to confirm this, the strength of the magnetoelectric coupling induced by a substrate with different thickness values was studied. In Figure 8.7(e), the maximum real and imaginary parts of the retrieved magnetoelectric coupling parameter over the entire frequency range from 100 to 250 THz (i.e. 1.2 to 3.0 μm) as a function of the substrate thickness is plotted. As the substrate thickness increases from zero, the magnetoelectric coupling parameter first increases, and then starts to converge and maintain at a complex value of 1.85 + i3.75. The threshold thickness is around 200nm, which is on a subwavelength scale for this specific fishnet. Hence, in terms of the strength of the induced magnetoelectric coupling, a semi-infinite substrate yields a similar effect compared to a subwavelength-thick substrate. This result is important since it will be utilized when the bianisotropy compensation technique is discussed in the following sections.

While the retrieved effective parameters reproduce the original scattering matrix, they are nonlocal in nature since they are closely related to the surface-averaged transverse fields rather than the volume-averaged fields [23, 198]. Thus, volumetric microscopic fields inside the metamaterial must be probed directly in order to confirm that the induced bianisotropy is a real physical effect. In other words, we need to quantify the electric displacement ($D_x$) induced by the magnetic field ($H_y$) and the magnetic flux density ($B_y$) induced by the electric field ($E_x$) over the entire volume of the multilayer fishnet. With reference to Figure 8.6(a), two plane waves were used to illuminate the structure at normal incidence from both top and bottom half-spaces simultaneously. By adjusting the magnitudes and phases of the incident waves, a standing wave
pattern can be formed such that the center of the fishnet nanostructure is positioned at a zero of the magnetic or electric field [199]. Due to the fact that the total thickness of the fishnet is much less than a wavelength, it is reasonable to consider that the nanostructure is excited by a pure electric or magnetic field, respectively. For each excitation, the induced EDM and MDM were evaluated using volume integrals on the microscopic fields predicted by HFSS, which are expressed by [200]

\[
\bar{p}_e = \int (\varepsilon_d - 1)\bar{E}_e d^3a, \quad \bar{m}_e = \frac{j\omega}{2} \int (\varepsilon_d - 1)\bar{n} \times \bar{E}_e d^3a,
\]

where \(\varepsilon_d\) and \(\varepsilon_m\) denote the relative permittivity of the dielectric and metal, respectively. Hence, this gives the EDM induced by the electric field \((p_e)\) or the magnetic field \((p_m)\), and the MDM induced by the electric field \((m_e)\) or the magnetic field \((m_m)\). The four quantities calculated for the fishnet in free-space and the fishnet on a semi-infinite substrate, which are all normalized to the unit cell volume of the fishnet and the strength of the incident field, are shown in Figure 8.8. Hence, they are dimensionally equivalent to the polarizabilities \(\chi_{ee}, \chi_{em}, \chi_{mm}\) and \(\chi_{me}\). Comparing the free-standing and on-semi-infinite-substrate fishnet nanostructures, the quantities \(p_e\) and \(m_m\) show similar properties except for small frequency shifts and strength differences. The EDM induced by the electric field \((p_e)\) has its minimum value at 235 THz (1.28 \(\mu m\)) and 220 THz (1.36 \(\mu m\)) for the free-standing and on-substrate fishnets, respectively, which agree well with the \(\varepsilon \approx 1\) frequencies in the retrieved effective permittivity. The MDM induced by the magnetic field \((m_m)\) has a main peak around 195 THz (1.54 \(\mu m\)) and another minor peak at 130 THz (2.31 \(\mu m\)) for both the free-standing and on-substrate fishnets. These are also in accordance with the resonant frequencies of the retrieved effective permeability. Additionally, it can be seen that in the case of the free-standing fishnet structure, \(p_m\) and \(m_e\) are zero throughout the entire band, meaning that no magnetoelectric coupling exists. However, with the semi-infinite substrate present, \(p_m\) and \(m_e\) both exhibit near-zero values at low frequencies and peaks around 190 THz (1.58 \(\mu m\)), again in
close proximity to the frequency of the magnetic resonance. Furthermore, the similarities in the line shape, peak value and peak spectral position of $p_{m}$ and $m_{e}$ provide mutual confirmation of the validity of the calculated quantities because they are obtained with different excitations - one electric and one magnetic. In summary, the induced EDM and MDM calculated from the microscopic fields in the entire volume of the fishnet nanostructure clearly demonstrate that the substrate indeed induces magnetoelectric coupling even in a multilayered structure, with the strongest effect at the resonance of plasmonic modes.

Figure 8.7. (a) Scattering parameters of the multilayer fishnet alone on an infinite substrate. (b) Retrieved effective permittivity, permeability and magnetoelectric coupling parameter corresponding to (a). (c) Extracted scattering parameters of the multilayer fishnet alone on a 5μm thick substrate. (d) Retrieved effective permittivity, permeability and magnetoelectric coupling parameter corresponding to (c). (e) Evolution of the maximum real and imaginary parts of the
retrieved magnetoelectric coupling parameter as a function of the thickness of the SiO$_2$ substrate. The point on the right edge corresponds to the semi-infinite substrate case.

Figure 8.8. (a) Magnitudes of the induced electric ($p$) and magnetic ($m$) dipole moments in the free-standing multilayer fishnet under an electric ($e$) or magnetic ($m$) excitation. (b) Magnitudes of the induced electric ($p$) and magnetic ($m$) dipole moments in the multilayer fishnet on a semi-infinite substrate under an electric ($e$) or magnetic ($m$) excitation.

### 8.3.2 On-substrate Multilayer Fishnet with Superstrate

In many applications, the parasitic effect of bianisotropy due to the unit cell geometry can be eliminated by employing subwavelength resonators with mirror symmetry [22, 65]. However, the bianisotropy inevitably reintroduced by the substrate is not easily removed. In this section, we consider how adding a superstrate to an on-substrate metamaterial can affect the induced bianisotropy. It will be shown that by judiciously choosing the permittivity and thickness values of the superstrate, the bianisotropy of the entire system can be greatly suppressed. First, a superstrate having the same permittivity ($\varepsilon_r=2.25$) with that of the substrate, but different thickness values, is considered. In the extreme case when the superstrate is semi-infinite, i.e. a half space, the entire system becomes symmetrical. Under such circumstance, the reflection coefficients obtained from both sides of the metamaterial and the impedances seen by waves travelling in opposite directions are the same; thus, no magnetoelectric coupling exists for the
entire system, which has been previously noticed in Ref. [193]. Figure 8.9(a) shows the maximum real part of the retrieved magnetoelectric coupling parameter as a function of superstrate thickness. It can be seen that the strength of the magnetoelectric coupling monotonically drops and converges to zero as the superstrate increases in thickness. A threshold thickness, defined as the thickness of the superstrate that causes the $\text{Max}(\text{real}(\zeta))$ and $\text{Max}(\text{imag}(\zeta))$ to be below 0.02, is observed around 200nm. This corresponds well with the finite-thickness substrate study presented in previous sections, demonstrating that the strength of the bianisotropy induced by the substrate converges at a thickness of 200nm. Figure 8.9(b) shows the retrieved magnetoelectric coupling parameters for the fishnet on a semi-infinite substrate and the fishnet when placed under a 200nm thick superstrate. The magnetoelectric coupling parameters of these two systems exhibit approximately the same magnitudes but opposite signs, meaning that the magnetic (electric) fields induced by the incident electric (magnetic) fields of the two systems are complex conjugate to each other. For the multilayer fishnet with only the semi-infinite substrate present, in response to a $y$-directed incident magnetic field, uneven currents arise on the metal layers of the fishnet, which further induces a non-zero net electric dipole moment in the $x$-direction. At the same time, unequal currents in the $x$-direction induced by the incident electric field also result in a non-zero magnetic dipole in the $y$-direction. When the superstrate is present, the net electric dipole moment formed by the superstrate-induced uneven currents has similar amplitude but opposite direction compared to that caused by the substrate. The same behavior occurs with the magnetic dipole moment originated from the unequal currents in the $x$-direction produced by the superstrate alone as well. When the two systems are combined into one - a fishnet sandwiched between an infinite substrate and a designed finite superstrate - the magnetoelectric coupling of the entire system that can be seen by outside observers is cancelled out. It should be noted that at the region near the interface between the metamaterial and the substrate, the bianisotropy cannot be eliminated. However, for the entire system, the addition of the superstrate controllably introduces another
source of bianisotropy to compensate that induced by the substrate. To verify that this is a real physical phenomenon, the induced EDM and MDM were calculated, as shown in Figure 8.9(c). Similar to what we have observed from the retrieved effective parameters, the EDM induced by the magnetic excitation \( p_m \) and the MDM induced by the electric excitation \( m_e \) have near-zero values throughout the entire frequency range. The MDM induced by the magnetic excitation has a main peak at around 185 THz (1.62 μm) and a small peak at 130 THz (2.31 μm), while the EDM induced by the electric excitation has a near-zero value at 200 THz (1.50 μm). Compared to the free-standing fishnet, the induced dipole moments have slight shifts in their line-shapes and variations in their values, which are attributed to the joint loading effect of the substrate and superstrate; but in all, they maintain similar properties.

Figure 8.9. (a) Evolution of the maximum real and imaginary parts of the retrieved magnetoelectric coupling parameter as a function of the superstrate thickness with a permittivity of 2.25. The point on the right edge corresponds to the semi-infinite SiO\(_2\) superstrate case. (b) Retrieved magnetoelectric coupling parameter for the fishnet on a semi-infinite SiO\(_2\) substrate and the fishnet underneath a 200nm SiO\(_2\) superstrate. (c) Magnitudes of the induced electric \( p \)
and magnetic \((m)\) dipole moments in the multilayer fishnet sandwiched between a semi-infinite substrate and a 200nm superstrate under an electric \((e)\) or magnetic \((m)\) excitation.

Further studies were carried out by considering superstrates having different permittivity values from that of the substrate. In Figure 8.10(a), the maximum real parts of the retrieved magnetoelectric coupling parameters as a function of the superstrate thickness with a permittivity value of 1.75 and 2.75 are shown. For the case where the superstrate has a permittivity of 1.75, which is smaller than that of the substrate, the strength of the magnetoelectric coupling monotonically decreases as the superstrate gets thicker, and finally converges at a certain value around 0.63 + i1.61. It is important to note that, in this case, the magnetoelectric coupling parameter is reduced, but never eliminated. This is because when the superstrate has a permittivity smaller than that of the substrate, stronger field confinement always occurs on the substrate side, regardless of the superstrate thickness. Hence, the bianisotropy and the resulting asymmetry in the reflection coefficients seen from opposite sides can only be reduced. In contrast, when the permittivity of the superstrate is 2.75, which is larger than that of the substrate, the strength of magnetoelectric coupling varies very differently with superstrate thickness. As shown in Figure 8.10(a), the maximum real part of the magnetoelectric coupling parameter first drops and then grows. Finally, it converges at a value of around 0.83 + i1.58. Interestingly, when the superstrate is around 74nm thick, the magnetoelectric coupling reaches its minimum with a value of less than 0.01, meaning that the magnetoelectric coupling of the system is almost eliminated, resulting in a symmetric scattering matrix of the fishnet, despite being sandwiched inside of an asymmetric electromagnetic environment. It should be noted that, in the extreme case where the superstrate becomes an infinite half-space, the magnetoelectric coupling parameter for these two cases has different signs, which is attributed to the permittivity values either being larger or smaller compared to that of the substrate. Actually, at the optimum thickness when the superstrate permittivity is 2.75, the line shape of the magnetoelectric coupling parameter switches
from a Lorentz-shaped profile to a flipped Lorentz-shaped profile. Furthermore, a superstrate with several different permittivity values larger than that of the substrate were employed to determine the optimum thickness corresponding to the elimination of the substrate-induced bianisotropy of the entire system. In Figure 8.10(b), it is shown that as the permittivity value of the superstrate increases, the optimum thickness decreases. The line shape for the optimum superstrate thickness as a function of relative permittivity follows an exponentially decaying profile. It should be noted that any superstrate permittivity value greater than or equal to the substrate relative permittivity is capable of suppressing the magnetoelectric coupling magnitude down to less than 0.01, all while possessing a subwavelength thickness. This is particularly important from the experimental viewpoint because a subwavelength-thick superstrate coating can be easily fabricated on top of a metamaterial to eliminate the bianisotropy of the entire system. Moreover, as an added practical benefit, this ultrathin dielectric superstrate can also serve as a protective layer to keep the otherwise exposed top metallic layer from oxidizing and corroding over time.

Figure 8.10. (a) Evolution of the maximum real and imaginary parts of the retrieved magnetoelectric coupling parameter as a function of the superstrate thickness with a permittivity of 1.75 and 2.25, respectively. The point on the right edge corresponds to the infinite substrate case. (b) Optimum superstrate thickness as a function of the permittivity value of the superstrate.
8.4 Overview

In conclusion, we have proposed an effective and practical way to fully compensate for the undesirable substrate-induced bianisotropy commonly encountered in optical metamaterials. As an example, the experimentally important multilayer fishnet nanostructure was considered. First, with the aid of a cascaded transmission matrix approach, the effect of a finite substrate with varying thickness values on the induced bianisotropy was studied, showing that substrate-induced bianisotropy is a near-field effect. Superstrates with different permittivities and thickness values were then investigated for the on-substrate metamaterial. It was shown that by properly choosing the permittivity value for the superstrate, which needs to be larger than that of the substrate, along with an appropriate subwavelength thickness, the substrate-induced bianisotropy of the entire system can be greatly suppressed or even eliminated. In addition to the retrieved effective medium parameters, induced EDM and MDM were calculated using the volumetric microscopic fields, confirming the bianisotropy compensation to be a real physical effect. The ultrathin superstrate not only provides a means for system bianisotropy suppression seen by outside observers, but also can serve as a protective coating layer. This work will be beneficial to achieving future optical metamaterial designs with bianisotropic-free properties facilitating their implementation into practical devices.
Chapter 9

Near-Perfect Multi-Band Optical Metamaterial Absorbers

9.1 Introduction

As most of the research in optical metamaterials focus on obtaining low-loss nanostructures with exotic electromagnetic properties, another far less studied class of metamaterials relies on creating structures that are tailored for complete absorption of the incident light in one or more wavelength bands independent of polarization and incidence angle [36]. Thus metamaterial absorber designs must balance the electric and magnetic resonances within each wavelength band to simultaneously minimize reflection and transmission, and hence maximize absorption, for both the incident electric and magnetic fields. The availability of such metamaterial absorbers could provide significant performance improvements for diverse applications including microwave-to-infrared signature control [36-38], bio-chemical spectroscopy [201-204], and thermal imaging [205-207].

Rapid progress has been made in demonstrating metamaterial absorbers that operate in the microwave and terahertz (up to 1.6 THz) regimes. A number of single-band device designs were reported with measured polarization independent absorptivity greater than 90% for angles of incidence from 0° to 60° at frequencies up to 1.6 THz [37]. More advanced multi-band terahertz metamaterial absorber with up to three well defined absorption bands have also been designed and fabricated. Currently, the best performing dual-band metamaterial absorber has a maximum absorptivity of 85% at 1.41 THz and 94% at 3.02 THz measured at a 30° incidence angle [208]. Several of the terahertz devices incorporated a flexible Kapton film as the dielectric spacer to enable their use in conformal coatings [205]. A new type of absorber with a 2D omnidirectional broadband response was designed using transformation electromagnetics approaches [209].
Fewer high-performance infrared-to-visible metamaterial absorbers have been experimentally verified because the aggressively scaled nanoresonators needed to reach these shorter wavelengths impose even stricter constraints on the metamaterial design and fabrication process [38, 207]. Recently, an angularly-tolerant single-band metamaterial absorber designed for near-IR operation at 1.6 μm was implemented by patterning a periodic array of simple circular Au features on a planar magnesium difluoride and Au thin film stack [38]. The absorptivity of this device measured at normal incidence using unpolarized light was 99%. In the mid-IR at 6 μm, a metamaterial absorber design composed of an array of nanoscale Au crosses on a planar alumina and Au thin film stack achieved a near-unity absorptivity of 97% at normal incidence [207]. In contrast, multi-band metamaterial absorber require nanostructures with more sophisticated geometries to induce an additional electric and/or magnetic resonance for each absorption band, which forces additional restrictions on the design. Moreover, mechanically flexible constituent materials such as Kapton must be integrated to realize metamaterial absorber coatings for curved surfaces. This added complexity has hindered progress in developing metamaterial absorbers with highly customized multi-band IR response to complement existing single-band [38, 207] and broad-band absorbers [210].

In this chapter, we report on the electromagnetic design optimization of multi-band ultra-thin metamaterial absorbers in the mid-infrared. A GA was used to identify an array of nanoresonators on a flexible Kapton and Au thin film stack that excite the appropriate electric and magnetic resonances for strong absorption in each band. The resulting dual-band and triple-band designs all have wide field-of-view – greater than 90% absorption over a ±50° angular range in each band. To validate the proposed designs, the dual-band design was fabricated and characterized. Measurements of a fabricated metamaterial absorber are in strong agreement with theoretical predictions, showing polarization-independent absorptivity greater than 90% over a ±50° angular range in both of the targeted wavelength bands. Full wave simulations that
illuminate a curved metal surface protected with this conformal metamaterial absorber coating confirm that nearly all of the reflected light is eliminated within the two absorption bands, which is in sharp contrast to the unprotected surface. This content of this chapter represents a significant step toward realizing high efficiency metamaterial absorber coatings that suppress the reflection from curved surfaces at multiple bands at the mid-infrared wavelengths.

9.2 Electromagnetic Design Optimization

The multi-band mid-infrared metamaterial absorber employs a three-layer metallodielectric stack composed of two gold (Au) layers—a doubly periodic array of electrically isolated nanoresonators at the top and a solid ground plane at the bottom—separated by a thin dielectric layer. Kapton was chosen for our dielectric layer because it is a highly durable and flexible polymer that can easily conform to the topography of most practical curved surfaces. The array of Au nanostructures on the top screen create a resonant electric response, while the Au ground plane functions together with the top screen to produce strong coupling to the magnetic component of the incident light radiation. The continuous Au ground plane, which is thicker than the penetration depth of light in the mid-IR wavelength regime, prevents transmission of incident radiation through the structure. Therefore, strong absorption is achieved by minimizing the in-band reflection. Importantly, the Au ground plane also decouples the electromagnetic properties of the metamaterial absorber coating from the surface it protects, allowing integration onto curved surfaces of arbitrary materials.

The specific goal was to design a periodic array of nanoresonators that gives polarization independent absorption bands centered at pre-selected wavelengths with absorptivity greater than 90%, i.e., at least 10 dB attenuation, over a ±50° field-of-view. The absorption band wavelengths were selected arbitrarily within the 3 to 5 μm atmospheric window for the proof-of-concept
designs. To achieve the desired multi-band performance using the described three-layer structure, a robust GA coupled with a full-wave electromagnetic solver was employed to optimize the geometry and dimensions of the structure to best satisfy the user-defined requirements. For each of the starting candidate nanoresonator designs, the unit cell of the periodically patterned top Au layer was divided into a $14 \times 14$ grid of pixels that were randomly assigned a binary value corresponding to the presence “1” or absence “0” of Au on the pixel. The Au pattern was further constrained to possess eight-fold symmetry to achieve a polarization independent absorber response. During the optimization, the unit cell size and the Kapton thickness were allowed to take values over a predefined range up to a maximum value of $\lambda/2$ to suppress higher order diffraction. To accurately account for the material dispersion in the design, the measured optical constants of the Au and Kapton thin film were used in the optimization. Furthermore, nanofabrication design constraints were incorporated to ensure that the optimized structure could be easily fabricated without modifying the nanoresonator geometry.

During the GA evolution, the wavelength-dependent scattering parameters of each candidate design was calculated using the Ansoft High Frequency Structure Simulator (HFSS) full-wave finite-element solver with appropriate boundary conditions assigned to approximate a TEM wave incident on the structure at different angles. The absorptivity was calculated by $A_{\text{TE,TM}}=1-T_{\text{TE,TM}} R_{\text{TE,TM}}$, where $R_{\text{TE,TM}}=|S_{11}|^2$ and $T_{\text{TE,TM}}=|S_{21}|^2$ represent the TE and TM reflectivity and transmittance, respectively. The absorptivity was evaluated against an ideal dual-band absorber response to determine its cost, given by:

$$\text{Cost} = \sum_{\lambda} \sum_{\theta_i} [ (1 - A_{\theta_i,\text{TE}}) + (1 - A_{\theta_i,\text{TM}}) ]$$

(1)

where $\lambda$ is the wavelength of the target bands and $\theta_i$ is the desired angle of incidence range ($0^\circ$ to $50^\circ$). $\lambda$ is chosen to be $3.3 \, \mu m$ and $3.9 \, \mu m$, $3.0 \, \mu m$ and $3.4 \, \mu m$, and $3.2 \, \mu m$, $3.5 \, \mu m$, and $4.0 \, \mu m$, for the first dual-band, the second dual-band, and the triple-band designs, respectively. The GA
evolved the top Au screen nanoresonator geometry, unit cell size, and Kapton thickness until it converged to a sufficiently low cost solution, i.e., the optimized design was achieved.

9.3 Nanoresonator Array for Single- and Multi-Band Operation

9.3.1 Dual-Band Designs

Figure 9.1. (a) Unit cell configuration of dual-band absorber design #1. \( p = 1475 \) nm, \( t_1 = 50 \) nm, \( t_2 = 50 \) nm, \( d = 100 \) nm. (b) Unit cell configuration of dual-band absorber design #2. \( p = 1420 \) nm, \( t_1 = 50 \) nm, \( t_2 = 50 \) nm, \( d = 80 \) nm. (c) Unit cell configuration of triple-band absorber design. \( p = 1730 \) nm, \( t_1 = 50 \) nm, \( t_2 = 50 \) nm, \( d = 100 \) nm.

The unit cell of the first GA-optimized dual-band metamaterial absorber design is displayed in Figure 9.1(a) including its geometry and dimensions. The top Au screen is made up of a doubly periodic array of orthogonal stub-loaded H-shaped (SLH) nanoresonators identified by the GA, which have a central connecting bar that is 630 nm long and 210 nm wide and two arms that are 840 nm long and 105 nm wide. Each arm is offset from the edge of the central bar by 105 nm and the arms are separated from one another by 210 nm. The total thickness of the three-layer structure is 200 nm, less than 1/15 of the operating wavelength. This design satisfies the nanofabrication design constraints imposed during design optimization, which ensures that the exact structure can be realized experimentally to minimize discrepancies that would degrade the resonant electromagnetic properties compared to theory.
The simulated absorption curves of the dual-band metamaterial absorber as a function of wavelength at different angles of incidence are shown in Figure 9.2 for both polarizations. Two strong absorption bands are clearly resolved at the target wavelengths of 3.3 μm and 3.9 μm. At normal incidence, both bands have a −10 dB bandwidth of ~0.1 μm with a maximum absorptivity of 94.7% at 3.3 μm and of 99.6% at 3.9 μm. At oblique incidences, the two absorption peaks remain centered at 3.3 μm and 3.9 μm over a broad range of incidence angles for both polarizations. The absorption in both bands remains >92% over a wide field-of-view of ±50˚ due to the efficient excitation of both electric and magnetic resonances. Further investigation shows that this metamaterial absorber still achieves absorptivity >60% for TE polarization and >85% for TM polarization in both bands at an incident full-angle of 160˚.

To show the versatility of the design approach, another dual-band metamaterial absorber is designed targeting at different wavelengths. The unit cell geometry and dimensions of the second GA-optimized dual-band metamaterial absorber design are shown in Figure 9.1(b). The top Au screen is made up of a doubly periodic array of orthogonal square ring nanoresonators identified by the GA, which have an outer edge length of 609 nm. The inner rectangular slot has a size of 406 nm by 203 nm. Four small rectangular parasitic patches are located near the center of
the unit cell, which are 203 nm long and 102 nm wide. The total thickness of the three-layer structure is 180 nm, less than 1/15 of the operating wavelength. This design satisfies the nanofabrication design constraints imposed during design optimization, which ensures that the exact structure can be realized experimentally to minimize discrepancies that would degrade the resonant electromagnetic properties compared to theory.

Figure 9.3. Simulated absorption of dual-band design #2 at various angles of incidence for (a) TE and (b) TM polarizations.

The simulated absorption curves of the dual-band metamaterial absorber as a function of wavelength at different angles of incidence are shown in Figure 9.3 for both polarizations. Two strong absorption bands are clearly resolved around the target wavelengths of 3.0 μm and 3.4 μm. At normal incidence, both bands have a −10 dB bandwidth of ~0.1 μm with a maximum absorptivity of 99.3% at 2.95 μm and of 99.6% at 3.38 μm. At oblique incidences, the two absorption peaks remain centered at 2.95 μm and 3.38 μm over a broad range of incidence angles for both polarizations. The absorption in both bands remains >88% over a wide field-of-view of ±50° due to the efficient excitation of both electric and magnetic resonances.
Figure 9.4. Simulated absorption of the triple-band design at various angles of incidence for (a) TE and (b) TM polarizations.

9.3.2 Triple-Band Designs

The unit cell of the GA-optimized triple-band metamaterial absorber design is displayed in Figure 1(c) including its geometry and dimensions. The top Au screen is made up of a doubly periodic array of three types of isolated nanoresonators identified by the GA distributed near the center, on the edges, and in the corners of the unit cell, respectively. The total thickness of the three-layer structure is 200 nm, less than 1/15 of the operating wavelength. This design satisfies the nanofabrication design constraints imposed during design optimization, which ensures that the exact structure can be realized experimentally to minimize discrepancies that would degrade the resonant electromagnetic properties compared to theory.

The simulated absorption curves of the triple-band metamaterial absorber as a function of wavelength at different angles of incidence are shown in Figure 9.4 for both polarizations. Three strong absorption bands are clearly resolved at the target wavelengths of 3.2 μm, 3.5 μm, and 4.0 μm. At normal incidence, all three bands have a 90% absorption bandwidth of ~0.12 μm with a maximum absorptivity of 99.2% at 3.2 μm, of 99.6% at 3.5 μm, and of 95.6% at 4.0 μm. At oblique incidences, the three absorption peaks, remain centered at 3.2 μm, 3.5 μm and 4.0 μm.
over a broad range of incidence angles for both polarizations. The absorption in all three bands remains >90% over a wide field-of-view of ±40° due to the efficient excitation of both electric and magnetic resonances.

Moreover, in the course of design optimization, we observed a design tradeoff between the absorbing efficiency and the desired number of absorption bands, which is similar to the findings in other studies [211, 212]. In spite of these tradeoffs, the optimized dual-band and triple-band metamaterial absorber coatings achieve all of the original performance goals.

![Figure 9.5](image.png)

Figure 9.5. (a) FESEM image of a portion of the fabricated dual-band design #1. The dashed red square indicates the unit cell. Scale bar: 600 nm. (b) FESEM image of the fabricated conformal MMA coating showing its mechanical flexibility and durability. Scale bar: 1800 nm.

### 9.4 Experimental Verification of Conformal Dual-Band Absorber

In order to verify the proposed absorbing nanostructures, the first dual-band metamaterial absorber design was fabricated by evaporating the Au ground plane layer and spin coating the thin Kapton dielectric layer on a handle substrate. The periodic array of H-shaped nanoresonators was patterned on top of the Kapton layer using electron beam lithography followed by a Au lift off procedure as shown in the field emission scanning electron microscope (FESEM) image in Figure 9.5. The three-layer metallodielectric structure was then removed from the handle substrate to demonstrate its mechanical flexibility and durability. The reflectivity of the fabricated
metamaterial absorber was measured using a Fourier Transform IR (FTIR) spectrometer with a custom optical setup. Since there is no energy transmitted through the structure, thus the absorption can be readily calculated from the measured reflectivity. The absorption of the fabricated metamaterial absorber is shown in Figure 9.6 as a function of wavelength at different angles of incidence. The two absorption peaks of the metamaterial absorber remain above 90% for an incidence angle up to 50° for both polarizations, with the 90% absorption bandwidth of both bands exhibiting a maximum broadening of 0.06 μm over the entire angular range compared to the simulated results.

Figure 9.6. Measured absorption of the dual-band design #1 at various angles of incidence for (a) TE and (b) TM polarizations.

The measured results confirm that the optical properties of the nanofabricated metamaterial absorber coating agree well with the simulations for all of the initial design parameters. The sources of the small differences between experiment and theory were identified through an in-depth study of the fabricated structures by FESEM. Simulations reveal that the primary cause of the wavelength shift is the slight rounding at the ends of the patterned Au nanoresonators, which changes the effective electrical length of the central connecting bar and the two arms. Two primary factors contribute to the degradation in bandwidth. First, the slightly
tapered sidewall profile (~80°) of the evaporated Au nanoresonators results in some degradation in the quality factor of the resonances. Second, variations in the Au nanoresonator dimensions across the large area metamaterial absorber (~5%) can give slightly shifted absorption bands, resulting in a broadened integrated line shape of the absorption peaks from the large area structure. Further optimization of the Au nanoresonator fabrication processes is expected to produce metamaterial absorber coatings with even better correspondence to theory.

Figure 9.7. (a) Schematics of a curved 50 nm Au ground plane producing strong reflection in multiple directions when illuminated by an incident beam. The inset shows a 3D view of the curved ground plane. (b) Schematics of a metamaterial absorber coated curved ground plane (50 nm Au) with significantly reduced reflection when illuminated by an incident beam. The inset shows a 3D view of the metamaterial absorber coated curved ground plane. (c) Simulation of the electric field magnitude for the curved ground plane without metamaterial absorber coating at 3.3
µm (top) and 3.9 µm (bottom). (d) Simulation of the electric field magnitude for the curved ground plane with metamaterial absorber coating under TE incident radiation at 3.3 µm (top) and 3.9 µm (bottom). (e) Simulation of the electric field magnitude for the curved ground plane with metamaterial absorber coating under TM incident radiation at 3.3 µm (top) and 3.9 µm (bottom).

9.5 Protecting a Curved Surface with a Metamaterial Absorber Coating

To investigate the performance of this conformal dual-band metamaterial absorber for applications such as mid-IR reflection suppression, we conducted a 3D full-wave simulation of the metamaterial absorber coating a curved metal surface. As the schematics in Figure 9.7(a) and Figure 9.7(b) illustrate, the field pattern of a curved Au ground plane is compared with that of the same ground plane protected by the metamaterial absorber when illuminated by an obliquely incident beam with a finite beamwidth. The insets show the 3D views of the structures for each case. For the simulation of the metamaterial absorber coated curved metal ground plane surface, the HFSS finite element solver was employed. In the simulation domain, a strip of the metamaterial absorber with a width of one unit cell in the x direction and a length of twenty-six unit cells was considered. To mimic a 1D infinite structure, perfect electric conducting boundary conditions were assigned to the front and back walls in the x direction for TE polarization and perfect magnetic conducting boundary conditions for TM polarization. A waveguide was connected to the semicircular air box as a feed for a finite width beam impinging obliquely on the structure. Dispersive optical constants measured by variable angle spectroscopic ellipsometry were also included to accurately model the Au and Kapton layers. Without the metamaterial absorber coating, the curved ground plane creates strong reflections in multiple directions for each band as shown in Figure 9.7(c). The field snapshots show that standing wave patterns arise due to the interference between the incident and reflected waves. However, when the metamaterial absorber covers the curved surface, reflection is nearly eliminated for both
polarizations in both bands, as displayed in Figure 9.7(d) and Figure 9.7(e). A slight reflection remains near the horizon, which is caused by the portion of the curved metamaterial absorber that is illuminated by waves at near-grazing incidence angles. This can be attributed to the reduction in absorptivity that occurs at large incidence angles, particularly for the TE polarization. The considerable reduction in reflection achieved by the metamaterial absorber coating confirms that its performance is retained when it protects a highly reflective curved surface. In addition, as is evident from Figure 9.7(d) and Figure 9.7(e), the field is strongly confined within the thin functional layer (only \( \sim 1/15\lambda \)) of the metamaterial absorber indicating a high absorptivity-over-thickness efficiency that is superior to conventional (non-metamaterial) absorbers. Moreover, no fields are observed behind the solid Au ground plane; accordingly, the performance will not be affected by the object that the metamaterial absorber covers, making it useful as a coating on various metal and dielectric surfaces.

### 9.5 Overview

In this chapter, we have shown the design optimization of polarization insensitive, multi-band metamaterial absorbers with an absorption efficiency higher than 90% over a wide field-of-view in the mid-infrared range. Particularly, a dual-band design was fabricated and characterized to validate the performance, showing a measured absorptivity greater than 90% over a \( \pm 50^\circ \) incidence-angle range in the mid-infrared at 3.3 \( \mu \)m and 3.9 \( \mu \)m. The highly conformal nature of these types of metamaterial absorber coatings was demonstrated experimentally, and 3D full-wave simulations demonstrate that the absorbing properties are maintained when it is placed on a highly reflective curved surface. Because the performance of the metamaterial absorber is independent of the material it protects, it is promising for a variety of applications, including
suppressing the mid-infrared reflection of curved metal surfaces in more than one wavelength band.
Chapter 10

Summary and Suggestions for Future Work

10.1 Summary

In this research, metamaterials that are structurally anisotropic were investigated in both microwave and infrared regimes. Particularly, they were exploited to improve the radiation and/or bandwidth properties of microwave antennas and enable exotic electromagnetic characteristics in nanostructured thin films. First, a new inversion algorithm was proposed which retrieves the anisotropic effective medium parameters of a slab of metamaterial using scattering parameters for plane waves illuminating on only a single interface of the metamaterial. This methodology provides an improved picture of the angular response of metamaterials which can be used for the design of various metamaterial unit cells in both microwave and infrared ranges.

To satisfy the growing demand of high performance wireless antennas, anisotropic metamaterials serve as promising candidates in terms of re-shaping the radiation of the electromagnetic waves, which facilitate the tailoring of both the far-field pattern and the impedance bandwidth of an antenna. On this topic, the anisotropy of the metamaterials were exploited and successfully applied to a variety of antennas to achieve high-gain radiation patterns with either a single or multiple beams and/or ultra-wide band impedance matching. Specifically, based on extreme anisotropy with near-zero permittivity and/or permeability tensor parameters, a type of transformation optics lenses that is able to generate arbitrary number of beams each pointing at an arbitrary direction from an embedded isotropic source and a grounded leaky antenna coating that is capable to provide a stable unidirectional beam were proposed, designed, and experimentally demonstrated. In addition, using anisotropy that involves certain permittivity and/or permeability tensor parameters having high values, an ultra-thin metamaterial coating was
introduced and experimentally realized which greatly broadens the impedance bandwidth of a conventional quarter-wave monopole antenna.

In the infrared ranges, several anisotropic nanostructured metamaterial thin films were proposed and investigated, giving exotic optical properties such as near-zero phase delay, smooth negative-zero-positive index transition, and multi-spectral electromagnetic absorption. Importantly, for transmissive metamaterial films, impedance matching and unnatural index properties are achieved simultaneously, representing a giant advance in the development of this field. Broadband dispersion engineering concept has been successfully transformed from microwave to optical range which greatly facilitates the design of broadband and low-loss optical metamaterials. The substrate-induced magnetoelectric coupling effect was carefully studied both numerically and experimentally. An efficient approach to compensate this commonly observed undesired effect was proposed and numerically validated. Finally, instead of suppressing the loss, multi-spectral highly efficient metamaterial absorbers were designed and experimentally demonstrated. These adventures suggest new pathways towards the design and realization of optical metamaterial components with diverse novel optical functionalities and broader operational bandwidth.

10.2 Suggestions for Future Work

Several directions can be envisioned in the future. First, in the microwave range, more advanced antenna lenses and coatings should be investigated. For example, while maintaining the performance, form factor reduction and lighter weight are desired for the integrated antenna systems. The possibility of applying the metamaterial lenses and/or coatings to other types of antennas are yet to be explored. In addition, introducing dynamic control into the device, such as adding tunability or making them reconfigurable, definitely expands the applicability of these
antennas. Similarly, at optical wavelengths, tunable devices that employ semiconductors or various oxide compound materials are worth of studying. Other challenges are reducing loss and making the nanostructures towards fully three dimensional which mimics real materials rather than thin films.
Bibliography


VITA

Zhihao Jiang

Born in Nanjing, China, 01/22/1986.

Educational Background

Ph.D. 2008 – 2013
Electrical Engineering
The Pennsylvania State University, University Park, PA, USA

B.S. 2004 – 2008
Electrical Engineering
Southeast University, Nanjing, China

Selected Journal Publications