AN EXPERIMENTAL STUDY OF THE VELOCITY-FORCED FLAME RESPONSE OF A LEAN-PREMIXED MULTI-NOZZLE CAN COMBUSTOR FOR GAS TURBINES

A Dissertation in
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by
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Abstract

The velocity forced flame response of a multi-nozzle, lean-premixed, swirl-stabilized, turbulent combustor was investigated at atmospheric pressure. The purpose of this study was to analyze the mechanisms that allowed velocity fluctuations to cause fluctuations in the rate of heat release in a gas turbine combustor experiencing combustion instability. Controlled velocity fluctuations were introduced to the combustor by a rotating siren device which periodically allowed the air-natural gas mixture to flow. The velocity fluctuation entering the combustor was measured using the two-microphone method. The resulting heat release rate fluctuation was measured using CH* chemiluminescence.

The global response of the flame was quantified using the flame transfer function with the velocity fluctuation as the input and the heat release rate fluctuation as the output. Velocity fluctuation amplitude was initially maintained at 5% of the inlet velocity in order to remain in the linear response regime. Flame transfer function measurements were acquired at a wide range of operating conditions and forcing frequencies. The selected range corresponds to the conditions and instability frequencies typical of real gas turbine combustors. Multi-nozzle flame transfer functions were found to bear a qualitative similarity to the single-nozzle flame transfer functions in the literature. The flame transfer function gain exhibited alternating minima and maxima while the phase decreased linearly with increasing forcing frequency. Several normalization techniques were applied to all flame transfer function data in an attempt to collapse the data into a single curve. The best collapse was found to occur using a Strouhal number which was the ratio of the characteristic flame length to the wavelength of the forced disturbance. Critical values of Strouhal number are used to predict the shedding of vortical structures in shear layers. Because of the collapse observed when the flame transfer functions are plotted versus Strouhal number, vortical structures are thought to have a strong influence on the response of this multi-nozzle configuration.

The structure of heat release rate fluctuations throughout the flame is analyzed using CH* chemiluminescence acquired with a high speed camera. Flames with a similar level of flame transfer function gain are found to exhibit similarity in the spatial distribution of their heat release rate fluctuations, regardless of the operating condition. Flames with high gain are found to have high amplitude fluctuations near the downstream end of the flame, with weak fluctuations near the flame base. The phase of the downstream fluctuations changes minimally across the downstream region, indicating that they occur in-
phase. Flames with low gain exhibit stronger fluctuations near the flame base, but weak fluctuations in the downstream region. The phase of the fluctuations near the flame base changes continuously along the flame axis, indicating that parts of the flame will fluctuate out-of-phase. Accordingly, from a global perspective, destructive interference between heat release rate fluctuations in different parts of the flame can be expected. The behavior observed in the flame is ascribed to the interaction of acoustic velocity fluctuations, vortical disturbances and swirl fluctuations.

The response of the multi-nozzle flame to high amplitude velocity fluctuations was tested for a single operating condition. Based on the global flame response, most frequencies responded linearly over the tested range of amplitudes. Nonlinear effects were found to occur at three frequencies. The behaviors observed at these frequencies matched those observed in the literature and included flame response saturation and mode triggering. For conditions which responded linearly at all amplitudes, the structure of heat release rate fluctuations was found to remain nearly constant. For conditions with nonlinear behavior, the structure of the fluctuations was a function of the forcing amplitude, particularly in the downstream region.

The behavior of the multi-nozzle flame was compared directly to that of a single-nozzle flame of the same nozzle design. The multi-nozzle characteristic flame length was found to be on average 10% longer than for the single-nozzle flame. The flame transfer functions from the two cases were found to exhibit qualitative similarity, where the frequencies at which the extrema occur are similar. The actual value of gain for the same operating condition and frequency does, however, vary by more than a factor of two in some cases. The phase value can also vary by as much as $\pi$ radians. These differences indicate that single-nozzle flame transfer functions should not be used directly to predict the instability driving force of real gas turbine combustors.
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# Nomenclature

## Abbreviations & Terms

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<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Amplitude</td>
</tr>
<tr>
<td>c</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>Case</td>
<td>Unique combination of $U_{inlet}$, $T_{inlet}$, $\Phi$ and $F_{forcing}$</td>
</tr>
<tr>
<td>D</td>
<td>Diameter</td>
</tr>
<tr>
<td>DS</td>
<td>Downstream</td>
</tr>
<tr>
<td>F</td>
<td>Frequency</td>
</tr>
<tr>
<td>$F_{G_{\text{max}}}$</td>
<td>Frequency of maximum gain</td>
</tr>
<tr>
<td>$F_{G_{\text{min}}}$</td>
<td>Frequency of minimum gain</td>
</tr>
<tr>
<td>$f_{s}$</td>
<td>Sampling frequency</td>
</tr>
<tr>
<td>$F_{S_{\text{max}}}$</td>
<td>Frequency of maximum swirl</td>
</tr>
<tr>
<td>$F_{S_{\text{min}}}$</td>
<td>Frequency of minimum swirl</td>
</tr>
<tr>
<td>FTF</td>
<td>Flame transfer function</td>
</tr>
<tr>
<td>G</td>
<td>Gain or single sided power spectral density</td>
</tr>
<tr>
<td>i</td>
<td>Index variables</td>
</tr>
<tr>
<td>ICCD</td>
<td>Intensified charge capture device</td>
</tr>
<tr>
<td>IRZ</td>
<td>Inner recirculation zone</td>
</tr>
<tr>
<td>j</td>
<td>The imaginary unit $\sqrt{-1}$</td>
</tr>
<tr>
<td>k</td>
<td>Index variable or wavenumber</td>
</tr>
<tr>
<td>L</td>
<td>Length</td>
</tr>
<tr>
<td>LDV</td>
<td>Laser Doppler velocimetry</td>
</tr>
<tr>
<td>LES</td>
<td>Large eddy simulation</td>
</tr>
<tr>
<td>$L_{\text{flame}}$</td>
<td>The characteristic length of the flame</td>
</tr>
<tr>
<td>$L_{\text{SW-CB}}$</td>
<td>Length from the end of the swirler to the centerbody</td>
</tr>
<tr>
<td>n</td>
<td>Index for a sample</td>
</tr>
<tr>
<td>N</td>
<td>Total number of samples</td>
</tr>
<tr>
<td>Operating Condition</td>
<td>Unique combination of $U_{inlet}$, $T_{inlet}$, and $\Phi$</td>
</tr>
<tr>
<td>ORZ</td>
<td>Outer recirculation zone</td>
</tr>
<tr>
<td>P</td>
<td>Pressure</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle image velocimetry</td>
</tr>
<tr>
<td>PLIF</td>
<td>Planar laser induced fluorescence</td>
</tr>
<tr>
<td>Q</td>
<td>Heat release</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$R^2$</td>
<td>Coefficient of determination</td>
</tr>
<tr>
<td>RMS</td>
<td>Root mean squared</td>
</tr>
<tr>
<td>$S$</td>
<td>Swirl magnitude</td>
</tr>
<tr>
<td>$S_L$</td>
<td>Laminar flame speed</td>
</tr>
<tr>
<td>SSCSD</td>
<td>Single sided cross-spectral density</td>
</tr>
<tr>
<td>SSPSD</td>
<td>Single sided power spectral density</td>
</tr>
<tr>
<td>$S_T$</td>
<td>Turbulent flame speed</td>
</tr>
<tr>
<td>$St$</td>
<td>Strouhal number</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T$</td>
<td>Period</td>
</tr>
<tr>
<td>TMM</td>
<td>Two-microphone method</td>
</tr>
<tr>
<td>U</td>
<td>Axial velocity component</td>
</tr>
<tr>
<td>V</td>
<td>Azimuthal velocity component</td>
</tr>
<tr>
<td>US</td>
<td>Upstream</td>
</tr>
<tr>
<td>$x,y$</td>
<td>Indices for spatial coordinates in a 2D field</td>
</tr>
</tbody>
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**Greek Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nabla$</td>
<td>Gradient</td>
</tr>
<tr>
<td>$\gamma^2$</td>
<td>Coherence</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Increment</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Phase</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Viscous stress tensor</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Equivalence ratio</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
</tr>
</tbody>
</table>
### Superscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{X}$</td>
<td>The rate of $X$ with respect to time</td>
</tr>
<tr>
<td>$X^\circ$</td>
<td>$X$ degrees</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>Time-averaged value of $X$</td>
</tr>
<tr>
<td>$X'$</td>
<td>Fluctuating component of $X$</td>
</tr>
<tr>
<td>$X^*$</td>
<td>Excited species $X$</td>
</tr>
<tr>
<td>$\vec{X}$</td>
<td>The vector field $X$</td>
</tr>
</tbody>
</table>

### Subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{\text{mean}}$</td>
<td>Arithmetic mean of $X$</td>
</tr>
<tr>
<td>$X_{\text{inlet}}$</td>
<td>Magnitude of $X$ at the nozzle inlet</td>
</tr>
<tr>
<td>$X_{\text{min}}$</td>
<td>The minimum value of $X$</td>
</tr>
<tr>
<td>$X_{\text{max}}$</td>
<td>The maximum value of $X$</td>
</tr>
</tbody>
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Chapter 1

Introduction

1.1 Gas Turbine Combustion Instability

In 2011, natural gas sources provided nearly 25% of all electrical power produced in the United States [1]. Natural gas offers advantages compared to other fossil fuels included reduced harmful emissions and a vast domestic supply from the Gulf of Mexico and shale mining. One of the primary means for producing useful electrical power from natural gas is the gas turbine. The first gas turbines used diffusion flame combustors to burn natural gas or other higher order hydrocarbons. While such machines produced less carbon and sulfur compounds than their main competitor, the coal plant, problems remained with the high levels of oxides of nitrogen (NO\textsubscript{x}) production.

The generally accepted approach for meeting current and future NO\textsubscript{x} emissions regulations in land-based gas turbines is lean-premixed combustion. Through a reduction of the reaction zone temperature, the amount of NO\textsubscript{x} produced is substantially abated [2]. Lean-premixed gas turbine combustors, however, are more susceptible to combustion instabilities than conventional diffusion flame combustors. Combustion instability is the result of coupling between system acoustics and unsteady heat release, and the amplification of this process by feedback through one or more instability driving mechanisms. Figure 1-1 shows this in its most basic form, from Zinn and Lieuwen [3]. The resultant pressure and velocity fluctuations can cause large amplitude structural vibrations, increased heat fluxes at the system walls, flashback and flame blowoff. In the most extreme cases the outcome is catastrophic failure [4].

Many combustion systems, including industrial boilers, rocket engines and gas turbines are vulnerable to combustion instabilities. There are two primary reasons for this. The first is that most combustion systems have highly restricted geometries with little natural damping of acoustic fluctuations. The second is that only a small percentage of the energy released in the combustion process needs to remain in the system to drive pressure fluctuations [5]. Of these systems, lean-premixed gas turbines are
particularly vulnerable to combustion instability. Lean combustors operate close to the lean blowout limit. Due to pressure fluctuations, the fuel flow rate may decrease, allowing the equivalence ratio to drop below the lean blowout limit. This may extinguish or partially extinguish the flame. Once the equivalence ratio increases again, the flame can reignite, but a substantial perturbation to the heat release rate will have occurred [6]. In diffusion flame combustors, the liner that contains the combustion process is perforated to allow cooling air to flow along the liner walls. The cooling air prevents the liner from melting while the perforations provide acoustic damping for any pressure fluctuations that occur inside of the combustor. Liner cooling air is not used in lean systems because the reduced combustion temperature allows the liner temperature to remain within safe limits. The additional air would also increase the risk of inducing flame blowout. Without the damping effect of the cooling air perforations, instabilities are more likely to grow [7]. Another contributing factor is that gas turbine flames are typically short compared to longitudinal acoustic wavelengths, and therefore acoustically compact. This allows for easy coupling between heat release and system acoustics [5].

![Figure 1-1: Basic Thermoacoustic Coupling. Modified from Zinn [3]](image)

The process whereby fluctuations in the heat release rate produce pressure fluctuations was identified over a century ago by Rayleigh [8]. When pressure and heat release rate fluctuations occur in-phase, the unsteady heat release rate can perform work on the gas and the pressure fluctuations will increase in amplitude [3]. This completes the first segment of the feedback loop in Figure 1-1. Understanding the remainder of the loop and the mechanisms whereby acoustic pressure fluctuations cause fluctuations in the rate of heat release remains an area of active research. Studies in single-nozzle research combustors have provided valuable insights regarding the phenomenology of the flame’s response [3-7, 9-]
These studies have identified a number of mechanisms which cause fluctuations in the flame’s rate of heat release, including velocity fluctuations [3, 4, 9-11], vortex shedding [3, 4, 12-15], swirl fluctuations [16], equivalence ratio fluctuations [3, 4, 6, 17] and unsteady strain [18, 19]. Furthermore, many of these studies [9, 10, 16, 20-22] have yielded empirical and modeled flame transfer functions (FTF). The flame transfer function provides a quantitative relationship between an input perturbation, such as the velocity fluctuation, and the output perturbation, which is the fluctuation in the flame’s rate of heat release [23].

The findings from single-nozzle research have served as a first step to allow for the prediction of instabilities in real combustors. Unlike most research completed to date, however, industrial gas turbines employ multiple nozzles. The additional nozzles placed in close proximity increase the power output of the engine, as well as ensure that if one nozzle extinguishes, the other nozzles will reignite it. The nozzles are typically arranged in an annular or can configuration, as shown in Figure 1-2. In a multi-nozzle combustor there are phenomena that may affect the flame’s response which are not accounted for by single-nozzle flame transfer functions [24-26]. In a typical single-nozzle combustor, a cooled outer wall confines the flame. In a multi-nozzle combustor, in addition to a cooled outer wall, each flame is confined by neighboring flames. This allows for heat and mass transfer as well as flame interaction. Additionally, most industrial can combustor configurations place five or more nozzles in a combustor with a diameter only slightly larger than the combined diameter of the nozzles. In the current study, the ratio of combustor area to open nozzle area, known as the confinement ratio, is six. For a typical single-nozzle study, this value is usually 10 or higher [16, 20, 27]. Consequently, the space in which a multi-nozzle flame burns is more confined than for typical single-nozzle research scale flames.

The flame response of multi-nozzle lean-premixed gas turbine combustors has received little attention to date. Five studies of annular combustors have been reported [24, 25, 28-30] and none of can combustors. This study will focus on an isolated five nozzle can combustor and its response to velocity perturbations forced into the flow using a device called a siren. A comparison of that response to a similar single-nozzle configuration will also be performed. Unlike studies on naturally-occurring instabilities, this velocity-forced approach allows for the study of the flame’s response at any frequency or amplitude. Forced response studies have the additional benefit of reduced dependence of the system response on the
system geometry. The response of the flame to the forced velocity fluctuations is quantified using the flame transfer function. The flame transfer function will be described in detail in the literature review.

![Figure 1-2: Industrial Combustor Configurations. (A) 16 nozzle annular combustor, (B) 5 nozzle can combustor, (C) 5 nozzle can-annular combustor.](image)

Chapter 1 will conclude with a review of the existing literature on single-nozzle and multi-nozzle configurations and lay out the objectives of the current research. In Chapter 2, the experiment that has been designed for this study is examined with all of the data acquisition and processing techniques used. Additionally, the reasoning behind the selected range of operating conditions is described. In Chapter 3 the global flame response to low amplitude velocity fluctuations is analyzed at all operating conditions using the flame transfer function. In Chapter 4, spatially resolved heat release rate fluctuations from a selected range of conditions will be analyzed using a high speed camera. Evidence of flame response mechanisms that have been identified in single-nozzle studies is then presented. In Chapter 5 the response of the flame to high amplitude velocity perturbations will be analyzed and compared to the response at low forcing amplitude. In Chapter 6 the global response of the multi-nozzle combustor will be compared directly to the response of a single-nozzle combustor with the same nozzle design. Finally, in Chapter 7 the important findings from this study are summarized and topics for future work on multi-nozzle flame response are suggested.
1.2 Literature Review

There have been numerous flame response studies in single-nozzle research combustors which have provided valuable insights regarding the phenomenology of the flame’s response. To understand the differences between the single- and multi-nozzle configurations, literature pertaining to the single-nozzle configurations is first reviewed. Following this, the available literature on multi-nozzle flames is presented.

1.2.1 Early Studies in Combustion Instability

The first identification of the process that is now referred to as combustion instability is typically attributed to J.W.S. Rayleigh [8]. In 1878 he published an article postulating how small disturbances in heat release and pressure could couple and “encourage” the growth of both disturbances. The growth of the disturbances requires that the fluctuations of the two properties occur in-phase with each other, that is, less than 90 degrees separation in phase angle. The heat released by the flame raises the gas temperature and results in gas dilatation. If the combustor pressure is increasing in phase with the heat release, then the heat release rate fluctuation can perform work on the gas [3]. This allows energy to be added to the acoustic field. If the absolute phase separation between the two fluctuating quantities is between 90 and 180 degrees, i.e. out-of-phase, then energy is removed from the acoustic field and the fluctuations will not grow. In later publications this set of conditions has become known as the Rayleigh criterion.

Rayleigh’s criterion can be written as an integral, as shown in Eq. (1.1). In the equation, \( p' \) and \( \dot{Q}' \) represent the local pressure and heat release rate fluctuations. The product of the two quantities is integrated over one full period, represented by \( T \). If the result of the integral is negative, the fluctuations are out-of-phase and energy is removed from the acoustic field. If the result of the integral is positive, then the fluctuations are in-phase and energy will be added to the acoustic field. If the pressure and heat release rate fluctuations are in-phase, the Rayleigh criterion is said to have been satisfied.

\[
\int_{T} p'(\vec{x}, t)\dot{Q}'(\vec{x}, t)\,dt > 0 \tag{1.1}
\]

While satisfaction of the Rayleigh criterion is a necessary condition for coupling to occur, some systems that satisfy the criterion may still be stable. An additional requirement for coupling is the presence
of feedback mechanisms, such as velocity and equivalence ratio fluctuations, as shown in Figure 1-1. These and other mechanisms will be discussed below. Even in the presence of driving mechanisms however, a combustion system may still remain stable. Such a situation may be explained by the system’s ability to damp out a thermal or acoustic perturbation. Damping processes include acoustic radiation, viscous dissipation, heat transfer through the system walls and convection of acoustic energy out of the overall system [3]. The study at hand is concerned with the driving forces that allow instabilities to grow. As such, damping processes will not be discussed further.

The first field where combustion instability caused enough problems to warrant an engineering solution was in the design of early rocket engines in the 1950s. By design, rocket systems typically operate close to their structural and thermal safety limit. Because of the high energy density of these combustion systems, only a small percentage of that energy had to remain in the system through coupling to exceed the safe limits of pressure and temperature and create a catastrophic situation. Generations of rocket engines were constantly at risk to combustion instability before a solution, using metal baffles and out-of-phase pressure waves, was finally implemented. Reviews are available on the subject from Culick [31] and Yang [32]. Since the focus of this work is on lean-premixed gas turbine combustion instability, the early work in rockets will not be further examined.

1.2.1.1 The Flame Transfer Function

In 1957 H.J. Merk completed one of the first studies [23] addressing gas turbine combustion instability. The study focused on a simple model of a reacting system, divided into a supply system, a burner head and a combustor. Merk was the first to describe the response of a flame to a perturbation in terms of a flame transfer function (FTF). The flame transfer function quantifies the relationship between perturbations in the supply system and the resulting fluctuations in the combustion system. The concept of the flame transfer function has become common in modern combustion instability studies. Therefore, as mentioned in the introduction, determining the flame transfer function for a multi-nozzle rig will be a focus of this work.

The flame transfer function is made up of two components, a gain and a phase. The gain is the ratio of two quantities: the amplitude of the output of the system and the amplitude of the input of the system. In most forced response studies, including the current multi-nozzle study, the input to the flame
transfer function is the mean-normalized velocity fluctuation that occurs in the nozzle, represented by $U_{RMS}'/U_{mean}$. The output is the mean-normalized heat release rate fluctuation detected from the combustor, represented by $\dot{Q}_{RMS}'/\dot{Q}_{mean}$. Eq. (1.2) shows the calculation for gain. A high gain indicates that the flame, in a global sense, responds strongly to a perturbation at that frequency and may be more likely to experience a real combustion instability. The phase of the flame transfer function describes the amount of time between the occurrence of a peak in the input signal and a peak in the output signal.

Example flame transfer functions are shown in Figure 1-3, from Jones et al. [10], and Figure 1-4 from Palies et al. [16]. The transfer function gain is typically observed to alternate between local maximum response frequencies (such as 80 Hz and 300 Hz for $\Phi = 0.75$ in Figure 1-3) and local minimum response frequencies (such as 200 Hz for $\Phi = 0.75$ in Figure 1-3). Such points will henceforth be referred to as maxima and minima, or collectively as the extrema. At higher frequencies the gain is typically observed to decay towards zero, so the flame is said to exhibit low-pass filter behavior [10].

$$FTF(F, \frac{U_{RMS}'}{U_{mean}}) = \frac{\dot{Q}_{global}'(F)}{\dot{Q}_{mean} U_{RMS}'}$$

(1.2)

Figure 1-3: Flame transfer function gain and phase, $U_{mean} = 25$ m/s, $U' / U_{mean} = 5\%$, for varied equivalence ratio. [10]
Initially, phase values tend to increase or decrease linearly. If the heat release rate wave is assumed to lead the velocity wave, then phase values will be negative, as in Figure 1-3. If the velocity wave is assumed to lead the heat release rate fluctuation, then phase values will be positive, as in Figure 1-4. The former case is assumed in the current study. A discontinuity is typically observed to occur in the phase around the minima frequencies. This phenomenon has yet to be adequately explained, but will be addressed in Chapter 4.

![Figure 1-4: Flame transfer function gain and phase, $U_{\text{mean}} = 2.67 \text{ & } 4.13 \text{ m/s}$ for varied disturbance amplitude.][16]

For a particular set of inlet conditions, the flame transfer function primarily depends on frequency. At higher excitation amplitudes, the flame transfer function gain and phase begin to depend on the excitation amplitude [21, 33]. A range of forcing amplitudes that produces a constant value of gain is referred to as a linear regime. A range where gain varies with forcing amplitude is referred to as a nonlinear regime. The mechanisms controlling the nonlinear response may be different than those that occur in the linear regime and are of great interest to anyone trying to predict the maximum amplitude of combustion instability in a real combustor. The nonlinear regime will be discussed in greater detail in Chapter 6.

1.2.2 Flame Response Mechanisms in Single-Nozzle Combustors

Since the general process whereby pressure fluctuations couple with heat release rate fluctuations was identified more than a century ago, one of the main aims of instability studies has been to isolate and
study the specific mechanisms that allow the coupling to occur. Several detailed reviews have been completed that summarize all known and agreed upon instability mechanisms [3, 4, 34]. The following sections of this literature review detail each of the mechanisms described therein, with additions from other authors where appropriate.

1.2.2.1 Velocity Fluctuations

Longitudinal velocity perturbations are one frequently cited means by which pressure fluctuations can cause heat release rate fluctuations [9-11, 16, 35]. Pressure fluctuations occurring in a combustor cause compression and expansion in an incoming flow which in turn causes the velocity to fluctuate at the same frequency as the pressure fluctuations. The velocity fluctuation propagates through the flame at the acoustic velocity inside the combustor. Due to the high wave speed, the velocity fluctuation will have a wavelength much longer than the characteristic length of the flame for any frequency of interest to the current study. This means the flame will be ‘acoustically compact,’ and respond as a point source. A flame responding purely to these long wavelength acoustic velocity fluctuations is said to be responding in ‘bulk’ [16, 35].

One of the difficulties with studying the velocity fluctuation mechanism in a system with fixed geometry is that the operator has little control over when combustion instability occurs. Fortunately, it is possible to study a controlled velocity fluctuation by forcing it into the flow through mechanical means, such as a siren. A siren creates velocity fluctuations by periodically opening and closing a flow passage. A motor controls the frequency of fluctuations and a bypass line controls the amplitude. Studies which make use of a device to create perturbations are referred to as forced response studies, while the term “self-excited” is used to refer to studies on the naturally occurring type of instability. The current study is of the former type, and as such, the system is not actually experiencing combustion instability, but rather forced perturbations of velocity.

The forced modulation fixes the perturbation amplitude ($U'_{RMS}$) and frequency. Because the input perturbation is fixed, the response amplitude of the flame ($\dot{Q}'_{RMS}$) will be constant with respect to time. Additionally, the combustor in a forced study can be left open to the atmosphere to allow more acoustic energy to convect out of the system to increase damping. This decreases the likelihood of the occurrence of an unwanted self-excited instability. As a consequence of the fixed input and the open system, the feedback
loop described in **Figure 1-1** is broken. **Figure 1-5** shows how heat release rate oscillations will occur in this multi-nozzle study. Pressure fluctuations are still present in a forced response study; however, they are typically at least an order of magnitude smaller than the velocity fluctuations. Laminar flame speed and heat of reaction are both pressure dependent, but the pressure fluctuations are small enough that their values can be considered constant. As such, pressure fluctuations are considered to have no direct effect on the flame response in the current study [36].

![Figure 1-5: The velocity forced flame response.](image)

1.2.2.2 **Vortex Shedding**

Inlet velocity fluctuations can excite a number of other feedback mechanisms inside the combustor, one of which is vortex formation [15]. The interaction of vortices with the flame is the single most commonly cited disturbance mechanism across all gas turbine combustion instability studies. The vortices are formed at the interface of two parallel streams of different velocities and densities. At the interface of the streams there will be a substantial shear force acting on the flow. The vortices are formed by the destabilizing effect of this shear, which overcomes the stabilizing effect of stratification [37]. A small disturbance introduced to the interface will tend to grow into a large coherent structure as shown in **Figure 1-6**, from Turner [38]. These vortex structures exert a significant influence on the combustion process by modulating the mixing processes among fuel, air, and hot combustion product [5].

The effects of flame/vortex interaction have been studied for decades, and extensive reviews are available [14, 15]. Vortices affect the stability of a flame through several different means. The first mechanism occurs when the vortex is strong enough to roll up the flame. This generates extra flame area
and leads to heat release rate fluctuations. A second mechanism occurs when a vortex interacts with a wall or another vortex. The interaction results in the entrainment and ignition of fresh material and a corresponding perturbation to the heat release rate. It is also possible to generate a region of high vorticity though the breakdown of a coherent vortex structure, or from an inlet perturbation that was not strong enough to generate a coherent structure in the first place [10]. A region of high vorticity can disturb the heat release rate in a similar manner to coherent vortex structures through generation of fine scale mixing and additional flame area.

![Figure 1-6](image_url)

**Figure 1-6:** Nonlinear numerical calculation of the evolution of a shear layer that has been given a small sinusoidal displacement of wavelength $\lambda$. [38]

Spatially, a vortex or region of high vorticity originates downstream from a step in the flow path, as demonstrated in an elementary study by Hertzberg [39]. This could be the end of a centerbody, a bluff body or the edge of the dump plane, as seen in **Figure 1-7**, modified from Steinberg et al. The step in the flow path is the origin of a shear layer that exists between the high velocity reactant jet and the lower velocity hot recirculation zone. In the time domain, the vortex is shed when the velocity gradient across the shear layer is greatest. In a centerbody dump combustor, the configuration of the current study, it is
possible to have two hot recirculation zones, and therefore, two shear layers, as seen in Figure 1-7. One of the recirculation zones is stabilized in the wake of centerbody, and is typically referred to as the inner recirculation zone [5]. The second stabilizes immediately downstream of the dump plane outside of the nozzle inlet and is referred to as the outer recirculation zone.

![Diagram of flow field](image)

**Figure 1-7:** The flow field in a typical single-nozzle combustor. Flow field elements shown in blue, with nozzle components in gray and arrows to display velocity vectors. Modified from [5].

There is evidence to suggest that heat release in a shear layer has a tendency to attenuate the formation of vortical structures [40-42]. The likelihood of vortex formation is reduced due to the increased fluid dilatation, kinematic viscosity, and baroclinic production [43]. The theoretical basis for this comes from the vorticity transport equation, shown in Eq. (1.3).

\[
\frac{D\vec{\omega}}{Dt} = (\vec{\omega} \cdot \nabla)\vec{V} - \frac{\vec{\omega}(\nabla \cdot \vec{V})}{\rho} - \frac{\nabla \times \vec{\omega} \times \nabla \rho}{\rho^2} - \frac{\nabla \times \vec{V} \cdot \tau}{\rho}
\]  

(1.3)
In the equation $\vec{\omega}$ is the vorticity, $\vec{\nabla}$ is the Del operator, $\vec{V}$ is the three dimensional velocity field, $\rho$ is the local density, $p$ is the local pressure and $\tau$ is the viscous stress tensor. From the dilatation perspective, the flame produces significant expansion in the gases on its downstream side, increasing the magnitude of term (II). Due to the inclination of the flame with respect to the flow, as well as the pressure gradient, term (III) is also expected to grow in magnitude due to misaligned pressure and density gradients. Heat release in the inner shear layer would also cause a sharp rise in the kinematic viscosity of the gases, which would increase the magnitude of term (IV). With an increase to the magnitude of terms (II), (III) and (IV), the overall vorticity is expected to decrease as each term is of the opposite sign to vorticity generated in the wake of a centerbody.

In this study, as in most gas turbine combustion studies, the flame primarily attaches to the edge of the centerbody, the origin of the inner shear layer. As such, there is then a significant amount of heat release occurring in this inner shear layer. This means that vortex formation in the inner shear layer is disrupted through the mechanisms described above. The most likely location for the formation of vortical structures would then be the outer shear layer, where there is typically no heat release occurring. An example of this can be seen in Figure 1-8 from Durox et al. [44], where pockets of vorticity originate on the outer lip of a nozzle and convect downstream towards the flame, causing it to roll up at the extremity. These findings will be important later when trying to identify where in the combustor a vortex is most likely to disturb the flame surface.
The shedding of vortical structures or regions of high vorticity in the shear layer is a frequency dependent process \[14, 15\]. The strongest vortical structures are formed in the shear layer at a critical value of the Strouhal number, from Eq. (1.4). An analytical basis for this Strouhal number dependence can be found in the classic works of Freymuth \[45\] and Michalke \[46\]. The Strouhal number is the ratio of a characteristic length scale of the flow field to the wavelength of the disturbance. The frequency that provides the critical Strouhal number is referred to as the preferred shedding frequency, or $F_{\text{preferred}}$ \[15\]. If the flow is forced with a siren or loudspeaker at $F_{\text{preferred}}$, as discussed earlier, the largest vortical structures can be expected. The most appropriate value to use for the characteristic length will be discussed in Chapter 3. The response of the shear layer to inlet velocity disturbances for a range of Strouhal numbers is shown in Figure 1-9, from Crow et al. \[47\]. In this case the shear layer was found to experience the greatest downstream velocity fluctuations at a Strouhal number of 0.30, which was taken to be evidence of vortex shedding.
Several single-nozzle studies [9, 11, 22] have found that when the flame transfer function was plotted with respect to the Strouhal number, as opposed to the frequency, the curves collapsed around the extrema. This suggests that critical values of the Strouhal number can be used to predict the occurrence of conditions where instabilities are likely to be excited. Based on the findings in the preceding paragraphs on the importance of vortex shedding and its Strouhal number dependence, it is of interest to determine if the current multi-nozzle configuration exhibits similar behavior.

1.2.2.3 Interaction of Flames with Boundaries

The interactions of flames with boundaries constitute a source of heat release rate fluctuations [4]. If a turbulent flame is confined such that it will impinge upon a wall or an adjacent flame sheet, the surface of the flame must rapidly deform. This leads first to fluctuations in the surface area of the flame followed by fluctuations in the rate of heat release. In addition, noise is generated. Depending on the frequency of the noise generated, acoustic modes of the confined area may be excited. If the phase relationship between the heat release fluctuations and pressure fluctuations is such that the most heat is released at or near the highest pressure, the amplitude of both parameters will increase with each cycle.

$$St = \frac{L_{flow\,field}}{\lambda_{disturbance}} = \frac{L \cdot F_{preferred}}{U_{mean}}$$ (1.4)

**Figure 1-9:** Shear layer response as a function of the Strouhal number. [47]
1.2.2.4  **Equivalence Ratio Fluctuations**

Equivalence ratio fluctuations or combustion inhomogeneities are also capable of driving heat release rate fluctuations. There are two known pathways for this type of fluctuation to occur. The first occurs when pressure oscillations in a combustor interact with the fuel supply system and periodically change the fuel flow rate, and therefore, the equivalence ratio. The second occurs when the fuel supply is constant, but the amount of air entering the combustor fluctuates as a result of velocity fluctuations. In either case, the equivalence ratio perturbation convects to the flame front and results in a perturbed heat release rate. If the heat release rate perturbation is properly phased with the pressure fluctuations, energy may be added to the resonant acoustic mode and the perturbations will grow.

Equivalence ratio fluctuations continue to be a topic of interest for industry, but for the study at hand the focus will be on only velocity driven oscillations. Consequently, equivalence ratio fluctuations must be eliminated in the experiment. This is achieved by mixing the air and the fuel upstream of a choke so that they are not affected by the fluctuating acoustic field. Combustion in a configuration where no equivalence ratio fluctuations are possible is referred to as fully premixed.

1.2.2.5  **Unsteady Strain Field**

An unsteady strain rate field can be induced by the resonant acoustic motion acting on the flow [4]. If the strain rate exceeds a certain threshold, flame annihilation events begin to occur [48] and the heat release rate is perturbed. It is also possible for the strain rate to limit the rate of reaction through changing the species gradient in the flame, but this is more prevalent in cases where equivalence ratio fluctuations are possible. More in depth investigations are available from Lieuwen [19] and Clemens [18]. If the annihilation threshold is not exceeded, and there are no equivalence ratio fluctuations, then the unsteady strain field is typically not considered as a possible feedback mechanism [4].

1.2.2.6  **Swirl Fluctuations**

More recently a number of authors [16, 49-51] have studied a mechanism that can only exist in combustors which have a swirler located in the nozzle, referred to as swirl fluctuations. When a purely axial flow enters a swirler, the flow downstream of the swirler will have both axial and azimuthal velocity components. The swirling motion in the flow helps to stabilize the flame through strengthening of the
recirculation zones and increasing fine scale mixing [52, 53]. When the axial velocity of the flow entering the swirler is fluctuating, both axial and azimuthal components of the velocity downstream of the swirler will fluctuate. Using laser Doppler velocimetry measurements, Palies showed that the axial fluctuation propagates at the acoustic velocity while the azimuthal fluctuation propagates at the convective velocity of the flow. As a result, the crest of each wave arrives at the flame base at different times, and possibly out-of-phase. Their relative phase at the base of the flame determines the instantaneous swirl strength. The higher the instantaneous swirl value, the more the shear layer and the flame that is anchored there will deflect away from the flame axis. The deflection increases the flames radius and creates additional flame area, which then generates a heat release rate perturbation that convects downstream with the flow. The strength of the swirl fluctuation is a function of frequency and distance from the swirler and can be predicted by Eq. (1.5), from Palies and colleagues. In the equation, $S'$ is the swirl fluctuation strength, $a$ is the amplitude of the original axial velocity fluctuation, $\omega$ is the radial frequency, $x$ is the distance from the end of the swirler, $U$ is the inlet velocity, $t$ is time and $c$ is the acoustic velocity. Figure 1-10 shows how the amplitude of the swirl fluctuation from Eq. (1.5) varies with forcing frequency for a realistic operating condition. The fluctuations strength is only shown for the $x$ coordinate of the end of the centerbody, where the flame anchors, referred to as $L_{CB-SW}$.

$$\frac{S'}{S_{\text{mean}}} = a \cdot \left[ \exp \left( \frac{i \cdot \omega \cdot x}{U} \right) - \exp \left( \frac{i \cdot \omega \cdot x}{c} \right) \right] \cdot \exp (-i \cdot \omega \cdot t) \quad (1.5)$$

The Palies study also analyzes the extrema of flame transfer functions. The proposed explanation for the occurrence of the extrema in the flame transfer function is the constructive or destructive interference of two or more flame response mechanisms. If the mechanisms occur in-phase, then a local maximum in flame response will occur. If the feedback mechanisms occur out-of-phase, then a minimum will occur. In this paper, the first of the two main interacting mechanisms was the swirl fluctuations induced by the azimuthal velocity fluctuations mentioned earlier. The second was the generation of vortical structures by the axial acoustic perturbations forced into the flow. The swirl fluctuations were found to primarily affect the heat release rate near the flame base, while the vortical structures affected the heat release rate near the downstream end of the flame.
Kim et al. [9] studied the impact of the flame structure on the forced response of a flame for a swirled lean-premixed combustor. This research included both V flames and M flames, as shown in Figure 1-11. As the hydrogen mole fraction is increased, the flame grows shorter and exhibits stronger attachment to the outer diameter of the nozzle. A V-type flame is one that attaches entirely to the centerbody or bluff body, leaving the downstream ends of the flame free to move. An M flame has two attachment points, typically the centerbody and the edge of the dump plane, as in the right most image in the figure. Kim only observed an M flame in hydrogen enriched flames, which have a high turbulent flame speed compared to pure methane/air flames. As such, the onset of M-type attachment may be related to the increase in turbulent flame speed that occurs as the equivalence ratio, inlet temperature or hydrogen mole fraction is increased. In agreement with previous work [22, 54] the M flame was found to be less responsive to flow disturbances than V flames. With a reduced response, the gain of the M flame is lower and therefore, more stable. This has bearing upon the multi-nozzles flames of the current study, since the flames sometimes exhibit a secondary attachment to the dump plane.

**Figure 1-10:** Swirl fluctuation amplitude RMS at a fixed axial distance from the swirler for various frequencies. $U_{\text{mean}} = 20$ m/s, $c = 436$ m/s. Amplitude normalized by mean swirl value.
A second finding in the Kim paper was on the importance of the flame length in determining the gain. Regardless of operating conditions, if two flames had the same length, angle and Strouhal number, the flame transfer function gain would be the same. This has important implications because it would mean that all experimental data could be consistently quantified using only these three parameters ($L_{\text{flame}}$, $\alpha_{\text{flame}}$, $\text{St}$). It will be valuable to determine if the same three parameters provide the same gain value on a multi-nozzle can combustor.

1.2.2.8 Confinement Effects

A number of studies on single-nozzle flames found that the degree of confinement that a flame experiences can have an effect on the flame response [49, 55, 56]. Compared to single-nozzle flames, multi-nozzle can combustor flames typically have a higher degree of confinement. Therefore, it is valuable to know the effect the confinement will have on the flame response. Fu et al. [55] completed a laser Doppler velocimetry study on a swirled non-reacting flow. The length and width of the square duct attached to the nozzle was varied in 0.1 inch increments to determine the effect of confinement level on the flow field in the duct. Fu found that varied confinement had its greatest impact on the size and strength of the inner recirculation zones, and that the effects of the confinement are more pronounced farther downstream. Two distinct flow patterns were observed, as seen in Figure 1-12. For large ducts, two inner recirculation zones (IRZ) occupied the flow field and provided a negative centerline velocity for the majority of the duct length. This caused the bulk of the forward flow to occur near the wall at high axial

![Figure 1-11: Transition from a V flame to an M flame (with the addition of Hydrogen), $U_{\text{inlet}} = 60$ m/s, $T_{\text{inlet}} = 200\,^\circ\text{C}$, $\Phi = .6$. [9]](image)
velocities. For small ducts there is only one inner recirculation zone, after which the forward flow recovers and fills out across the duct cross section. The transition between the two flow regimes was found to occur abruptly. A discrete change to the flow field such as this will have a substantial impact of the flame response through changing the mean flame shape [9].

Figure 1-12: The effect of confinement ratio on the isothermal flow field for two different combustor sizes. The two inner recirculation zones are identified by IRZ1 and IRZ2. [55]

Birbaud et al. [56] completed a confinement study on a laminar flow, premixed, centerbody stabilized flame. An abrupt change was observed in the combustor flow field at a certain confinement ratio, ultimately leading to flame-wall interactions. In addition, the flame’s response to axial excitation under a range of different confinements was tested. The confinement was found to affect the response of the flame primarily through influencing the dynamics of the flame tip. With an unconfined flame the flame tip is free to respond to coherent flow structures such as vortices. Large motions can occur and a higher gain results at most forcing frequencies. For a low degree of confinement the vortices experience weak interaction with the walls and the flames movement is reduced, resulting in lower flame transfer function gain. For a high degree of confinement, strong interaction begins to occur with the wall and the flame actually contacts it.
The resulting gain is nearly constant over a range of Strouhal numbers, which indicated to the authors that the flame response was less dependent upon peripheral vortices.

It is also important to acknowledge that the difference a multi-nozzle flame experiences in confinement is more than just a matter of degree. Some of the flames in a multi-nozzle rig will experience a different type of boundary condition all together, that is, another flame. In other studies the single-nozzle flames are typically exposed to a relatively cold wall as their confinement, inevitably leading to heat transfer out of the system. Multi-nozzle flames, on the other hand, may experience nearly adiabatic boundary conditions on some or all sides due to the presence of another equally hot flame. Studies related to this topic will be covered in the following section on multi-nozzle literature.

1.2.3 Flame Response in Multiple Nozzle Combustors

It has been assumed that it is possible to extend the results from single-nozzle laboratory scale studies to industrial scale multi-nozzle combustors. If the continued existence of combustion instability problems is any indication, however, this is not entirely true. Multi-nozzle research is still in its early stages, but the first reports have shown that there are some distinct differences compared to single-nozzle systems.

1.2.3.1 Flame Interaction

An older study, with particular relevance to multi-nozzle research was completed by Poinsot et al. on a fairly simple laminar multiple inlet combustor [57]. The inlets were all connected to a common plenum, and a conical flame anchored on each. The purpose of the study was to describe vortex shedding, growth, burning and interaction in the context of an acoustically self-excited flame. The unique part of this experiment was the presence of vortex-vortex interaction in a reacting flow. The interaction of the vortices shed off of adjacent nozzles leads to intense mixing and the production of small scale turbulence in the interaction region and in the vortex wake. While the Spark Schlieren imaging technique in the paper could not be used to distinguish between burning and non-burning gas, it was still possible to infer that intense reactions were taking place in the interacting regions. The effect of the interaction is not quantified beyond this. The information is still useful, however, to provide some expectations for the flame response in a multi-nozzle configuration.
Another early study in multi-flame combustion was undertaken by Noiray et al. [58]. Their experiment was completed on an ensemble of conical premixed flames attached to a perforated plate. A longitudinal self-excited instability occurred when the heat release coupled with the acoustic modes of the upstream manifold. The length of the manifold could be varied to change the resonant frequency of the system. Coupling in this configuration is found to stem from flame annihilation effects that occur when adjacent flame sheets merge between periodic fresh releases of gas from the perforated plate. Noiray also found that that increasing the thickness of the perforated plate narrows the unstable combustion regimes.

Worth et al. [26] investigated how the level of flame interaction affected the forced response from a pair of nozzles by varying nozzle spacing. The inlet velocity perturbation and heat release rate disturbance were quantified using the two-microphone method and OH planar laser induced fluorescence (PLIF), respectively. Figure 1-13 shows an example of the results from this study. The authors found that the majority of heat release occurred in the interaction region, similar to when a flame impinges on a wall. Additionally, the merging of the two jets altered the response of the flame to forced velocity perturbations. In cases where the nozzles were relatively far apart (the far right image), vortices shed from the edges of the nozzles impinged upon each other in the interaction region, breaking up into small-scale turbulence. In cases where the nozzles were closer together (far left image), a single vortex structure could form in two adjacent shear layers, which would remain coherent until far into the downstream region. In the multi-nozzle can combustor of the current study the nozzle spacing was 1.36 nozzle diameters. Based on the findings of Worth, the shear layers in the can combustor would be expected to roll-up together and form a single vortical structure between them, as in the left and center image from Figure 1-13.

Figure 1-13: OF PLIF image of the forced response of a dual nozzle flame for various nozzle spacings (S = 1.14D, 1.43D & 2.00D), Phase averaged at 0° with respect to the velocity fluctuation. $U = 10$ m/s, $\Phi = 0.7$, $F_{\text{forcing}} = 160$ Hz. [26]
1.2.3.2 Large Eddy Simulation

One approach to the study of multi-nozzle combustion has been the use of Large Eddy Simulation (LES). Using a simulation saves the expense and difficulty of building a multi-nozzle combustor. LES reduces the computational expense by using a relatively simple model to represent the smallest scale eddies, rather than fully simulating them. The first researchers to report on this approach to multi-nozzle combustion were Staffelbach et al. [24]. They presented a self-excited study on a 15 nozzle, liquid-fueled helicopter annular combustion chamber. They found that the response of the flame was characterized by two superimposed counter rotating modes with different amplitudes. These acoustic modes are different than those studied in single-nozzle combustors because they are the result of acoustic resonance along the circumferential length of the combustor. They propagate transversely to the flow direction so will affect the flame differently than a longitudinal mode. In this example, the transverse excitation modulates the flow rate and causes the flames to oscillate back and forth downstream of each nozzle. Staffelbach’s LES demonstrated that the most significant effect of the rotating modes is to excite longitudinal pulsations in the flow rate through individual nozzles. Ultimately they conclude that the transfer function of all burners is the same, though no mechanism of flame interaction between the burners was identified. Accordingly, they state that a single burner is an accurate representation of multi-nozzle annular combustion and that there is no need to actually study such systems. Due to the lack of flame interaction, however, this is not likely to apply to multi-nozzle can combustors.

Fureby [29] studied an LES model of both a single-nozzle combustor and an 18 nozzle annular combustor. An experimental rig of only the single-nozzle configuration is presented for verification of the LES model. Specifically, this study focuses on the effect of combustor exit impedance and the influence of the swirler on the occurrence of self-excited instabilities. A parametric study was initially completed on the single-nozzle rig to account for these effects in the LES. The pressure at the exit of the reference nozzle was used to detect the presence of acoustic resonance. The gas temperature immediately downstream of the nozzle was used to detect heat release rate fluctuations. Good agreement in flame response was found between the LES code and the single-nozzle configuration. When the LES code was run on the multi-nozzle configuration, two separate self-excited instabilities were found to develop, one axial mode at 724 Hz and one azimuthal mode at 2124 Hz. Pressure fluctuations at each nozzle were found to be different but
follow a pattern, one possible explanation being the presence of the azimuthal mode. The pattern of low pressure and high pressure regions containing two or three of the nozzles each is thought to lead to burner to burner interactions, but no quantitative information is given. From a global response perspective the presence of the azimuthal mode leads to a difference in the flame response compared to the single-nozzle combustor. This leads to the conclusion that longitudinal single-nozzle studies are not enough to predict the flame response of a multi-nozzle annular rig.

1.2.3.3 **Phenomena from Annular Combustion Experiments**

Kunze et al. and Fanca et al. [25, 28] reported two experimental studies on annular combustion using a 2 MW annular combustor with twelve Alstom designed EV5 nozzles. Axial excitation of the flame was achieved using six downstream sirens which periodically increased the combustion chamber pressure by shutting off some of the combustor exit nozzles. The pressure fluctuations in the combustor excite longitudinal velocity fluctuations. The purpose of the first study was to characterize the differences in the flame transfer function between a single-nozzle and a multi-nozzle annular combustion chamber. Following this they worked to characterize the differences in the flow field.

Kunze et al. [28] reported on the response on the flame to incoming velocity perturbations. A hotwire installed in one nozzle measured the perturbation amplitude. A photo multiplier tube was used to measure OH chemiluminescence as an indicator of heat release rate. A comparison of variation in swirl, thermal power and mass flow along with steady state heat release rate distribution was completed. It was found that the effective swirl in the annular combustor was lower than for the single-nozzle burner. The decrease in swirl was the result of destructive interaction of azimuthal momentum. In between the co-swirling nozzles, the azimuthal momentum vectors were diametrically opposed, leading to their dissipation. The resulting lower swirl flames were longer and had a higher gain.

When the annular combustion chamber was modified to compensate for the difference in effective swirl, a discrepancy remained. The discrepancy was attributed to a second major difference between single and multi-nozzle rigs, circumferentially propagating acoustic waves, just as Fureby & Staffelbach proposed. In Kunze’s experiment the azimuthal waves were found to primarily affect the flame in the near field, that is, close to the attachment point. Fluctuations at the flame base were shown to be of greater amplitude than in cases where there were no azimuthal waves.
The second study on the 12 nozzle combustor by Fanaca et al. [25] attempted to quantify the differences in the flow field. Using particle image velocimetry (PIV) on a seeded flame, the flow field differences were quantified and used to explain the discrepancies in the flame response. The PIV revealed two different flow regimes occurring in single and multi-nozzle rigs. The observed flows were similar to the two regimes that Fu [55] had identified as the flame confinement was decreased. In the annular combustor a free swirling jet flow formed where the flow field resembled a swirling jet issuing into a quiescent chamber. In the single-nozzle combustor a wall jet flow formed, characterized by quick spreading of the swirling jet and high axial velocities close to the combustor wall. The difference in regime in this annular combustor was attributed to the reduced degree of confinement compared to the single-nozzle combustor. Despite the decrease in confinement in the annular combustor, evidence of dissipation of angular momentum was found. If this is the case for a minimally confined multi-nozzle combustor then it is possible that dissipation of angular momentum would be even more prevalent in a can configuration with much higher confinement.

1.2.3.4 Control Techniques for Combustion Instability

Additional studies have focused on active control in multi-nozzle combustion. Richards et al. [59] studied a dual nozzle 30 kW combustor for the purposes of using mismatched fuel system impedance to reduce self-excited combustion instabilities. Results demonstrated that a widely varied dynamic response is possible with combinations of fuel systems with different impedances. With this approach, in the best case, a 7% RMS self-excited pressure oscillation was reduced to 3%. Hermann et al. [60] took a similar approach for active control on a Siemens V94.3A stationary gas turbine. A combination of mismatched fuel system impedances and active control through periodic fuel injection was used to suppress instability. During in-field demonstrations these measures proved successful in suppressing a wide range of unstable conditions. Control measures such as these can be expensive and cause down-time in operation since they often must be implemented on site. For this reason it is the goal of gas turbine manufacturers to be able to understand the coupling mechanisms and predict the onset of instabilities before they occur, so as to avoid them all together.
1.3 Research Objectives

Through reviewing the available literature on combustion instability it becomes clear that there is a need to conduct research on realistic large scale combustion systems, particularly can combustors. A multitude of coupling mechanisms which could affect the flame’s response have been identified, although the manner in which these mechanisms interact and affect the response of a multi-nozzle flame is not fully understood. While computational models have made significant strides, their predictive capabilities will remain limited until all relevant mechanisms can be identified and accurately incorporated into the models.

Based on these deficiencies, the primary objective of the current study is to develop a mechanistic explanation for why lean-premixed swirled multi-nozzle flames respond in a particular way to velocity perturbations at a range of operating conditions. A secondary goal is the acquisition of a unique bank of flame transfer function data from a large-scale multi-nozzle can combustor. System designers can incorporate the mechanisms described here into their flame response models to improve their combustion instability predicting capability. The data bank created in this study is intended for use in validation of these models.

A long term goal of studies on multi-nozzle combustors is to determine if results acquired on single-nozzle research combustors can be used to predict the instability characteristics of real multi-nozzle combustors. The current study will address this issue in Chapter 6. This study is part of a larger effort across several combustion research centers to allow gas turbine designers to predict combustion instability during the design phase of gas turbine combustors and prevent any problems before the turbine is put into service.
Chapter 2

Experimental Methods

2.1 The Multi-Nozzle Can Combustor

The multi-nozzle can combustor test facility used in this study is illustrated schematically in Figure 2-1, Figure 2-2 and Figure 2-3. An air compressor system supplies air to the experiment at a maximum flow rate of up to 20 kg per minute at 2000 kPa. The flow rate to the combustor is controlled by a needle valve and monitored by a Sierra mass flow meter with a maximum flow rate of 10 kg/min. The pressure at the meter is controlled by a Powreactor dome regulator and set to 1400 kPa. To achieve the desired range of combustor inlet velocities, the flow rate is varied between 3 and 10 kg/min. Downstream of the valve the air is preheated by a 50 kW Tempco process air heater to achieve the desired combustor inlet temperature of between 50°C and 250°C. Downstream of the heater exit, natural gas is injected transversely into the cross flow of heated air through a multi-hole injector to achieve good mixing of the fuel and air. The flow rate of the natural gas is controlled by a needle valve and monitored by a Teledyne Hastings mass flow meter. The equivalence ratio can be set to 0.50 up to 0.70 corresponding to a flow rate range of 0.15 to 0.4 kg/min of natural gas. Additional mixing occurs as the fuel and air flow through a 38 mm inner diameter pipe with an L/D of 15 as well as several elbows. At the end of the pipe, the fuel-air mixture flows through a choked orifice. This ensures that the fuel injection and fuel-air mixing processes are not affected by pressure fluctuations downstream of the orifice. As a result, the mixture entering the combustor will be fully premixed and have a constant equivalence ratio.

Downstream of the choked orifice, the premixed fuel and air enters a siren device that produces periodic fluctuations in the velocity of the mixture entering the combustor. The siren primarily consists of a rotor with four holes and a fixed stator with a single open passage, as seen in Figure 2-2. The mixture of air and fuel can flow freely when a rotor hole aligns with the open passage of the stator, generating a sinusoidal perturbation in the mixture flow rate. The frequency of the perturbation is determined by the rotational speed of the siren rotor, which is controlled by a brushless DC motor. The amplitude of the fluctuation is controlled by varying the fraction of the fuel-air mixture that bypasses the siren. Each time
the rotor makes one full rotation an encoder device sends an electronic signal to the data acquisition computer. The signal is used to provide a trigger for the acquisition of pressure and heat release rate data.

Downstream of the siren the fuel-air mixture enters a manifold (Figure 2-3) which divides the flow into five separate streams, one for each of the five nozzles in the multi-nozzle combustor. The manifold has been designed such that each of the five flow paths is nearly identical, geometrically. In addition, a perforated plate is installed in each leg of the manifold to induce a pressure drop and help ensure that the mass flow rate to each nozzle is the same.

![Diagram](image)

**Figure 2-1**: The multi-nozzle combustion experiment, with important features labeled. The flow path is marked by boxes 1 – 7.
After exiting the manifold, the fuel-air mixture flows through five separate nozzles. They are connected directly to the combustor dump plane in a cross configuration, as seen in Figure 2-3. The nozzles are of typical industrial design. The nozzle tubes have an inner diameter of 50 mm and house a 30 mm diameter centerbody. The end of the centerbody is recessed 20 mm from the face of the dump plane. A 45° axial swirler with eight blades is located in each nozzle between the centerbody and the inside wall of the nozzle. The swirler imparts counter-clockwise motion to the flow. The downstream edge of the swirler is recessed 80 mm from the end of the centerbody. The spacing of the nozzles relative to the diameter of the combustor can is typical of industrial can combustors. The ratio of the dump area to the total nozzle open area, also known as the confinement ratio, is approximately six. The nozzles exhaust into the combustor can, which is a 257 mm inner diameter, 305 mm long quartz tube with a wall thickness of 5.0 mm. The downstream end of the quartz tube is open to the atmosphere. Therefore, combustion takes place at atmospheric pressure. A digital camera image of the flame is shown in Figure 2-4. The image was acquired at an angle offset from the optical axis by 45° to allow for easier differentiation between flames.
Figure 2-3: Multi-nozzle combustor downstream view, cross section AA and superimposed line-of-sight image in pseudo-color. Position of optics is shown at right. Pressure transducer locations marked by $P_1$-$P_6$, and $P_{comb}$.

Figure 2-4: Digital camera image of the multi-nozzle combustor showing the flame’s attachment to the centerbody. Red rings denote the end of the centerbody. The yellow ring highlights the outer wall of one of the nozzles.
An assortment of equipment regulates the environment in which the experiment is conducted. A cooling flow of air at 4.0 kg/min impinges on the outer surface of the quartz to keep its temperature from exceeding safe limits. An exhaust hood above the experiment draws in the products of combustion and air from the room at a rate of approximately 70 kg/min. An array of 14 nozzles (Hago M10) sprays a fine mist of water into the hot exhaust, which provides evaporative cooling. The water spray flow rate is set to maintain the room exhaust at a temperature less than 300 °C. Two portable industrial air conditioning units provide spot cooling to the data acquisition system and the high speed camera to maintain safe operating temperatures. The combined effect of the room exhaust and air conditioning units maintains the room temperature at 25 °C.
2.2 Instrumentation

2.2.1 Static Temperature and Pressure Measurements

K-type thermocouples are used to measure the temperature of the fuel-air mixture approximately 25 mm upstream of the swirler in each of the nozzles. For the conditions tested, the nozzle-to-nozzle variation in the mixture temperature was less than 10°C. The thermocouple in the middle nozzle serves as the control signal for the air heater. Additional thermocouples are located throughout the system to ensure that safe operating conditions are maintained.

Electronic differential pressure gauges are used to measure the pressure drop across the swirler in each nozzle. Using empirical calibration data, the pressure drop across the swirler provides the mean velocity through the nozzle. The accuracy of this measurement is ±1 m/s. For the test conditions reported in the results sections, the measured nozzle-to-nozzle variation in the mean velocity was less than ±3 m/s.

2.2.2 Pressure Fluctuation Measurements

Piezoelectric pressure transducer pairs (PCB Model 112A22) are used to measure fluctuations in the pressure in three of the five nozzles. The signals from the transducers, or microphones, are conditioned by amplifiers and digitalized by an analog-to-digital converter. Figure 2-3 shows the location of the each transducer at $P_1 - P_6$. The measurements are made as close to the combustor as possible in order to provide an accurate measurement of the perturbation that the flame experiences. The pressure transducers are spaced 38 mm apart. An additional pressure transducer, labeled $P_{comb}$, monitors the combustor pressure to detect the presence of self-excited instabilities. The transducers are temperature sensitive so they must be protected with circulating water jackets. The data acquisition system used for the pressure measurements has a sampling rate of 8192 Hz and provides a frequency resolution of 1.0 Hz. For each operating condition, 32 data sets are obtained to provide a statistically significant average and standard deviation.

With a sampling frequency of 8192 Hz, the maximum disturbance frequency, or Nyquist Frequency, that can be detected in the experiment is 4096 Hz [61]. This allows for the possibility of significant turbulent fluctuations at frequencies higher than 4096 Hz. Such fluctuations are considered to have a negligible effect on the forced response of the system for two reasons. First, the literature has
demonstrated that flames typically behave as low pass filters [9-11, 62]. The short wavelength of high frequency disturbances relative to the flame length scale prevents such disturbances from having a significant impact on how the flame responds. Second, in order for a naturally occurring turbulent fluctuation to affect the flame response, resonant coupling with the acoustic modes of the combustion chamber is required. Models which can predict the acoustic modes of chambers to within 10%, even under reacting conditions, have existed for some time [63]. Such models have been used to show that longitudinal modes of any realistic system that might experience combustion instability are well within the frequency range investigated in this experiment [5].

2.2.3 Chemiluminescence Measurements

During the oxidation reaction of natural gas and air, the excited chemical species OH*, CH* and CO₂* are formed, as seen in the reaction mechanisms shown in Figure 2-5 from Najm et al. [64]. As these radicals transition from the first excited state back to their ground state, the excited species emit light at a specific wavelength. The CH* radical emits light at 432 nm and the OH radical emits at 308 nm while CO₂ light emissions are broadband over the entire visible spectrum as shown in Figure 2-6, from Lee et al. [65]. This light emission is referred to as chemiluminescence. The increased chemiluminescence at each of the three wavelength bands is an indicator of an increased concentration of the appropriate excited species. As the fuel flow rate increases for a fixed equivalence ratio flame, it has been shown that the chemiluminescence at the wavelengths of CH* and OH* increases linearly [66]. Since the global rate of heat release is a function of only the fuel flow rate, this linear relationship implies that chemiluminescence can be used as a measurement of heat release rate for a flame with a fixed equivalence ratio.
There are a number of limitations with this technique that must be considered. First, CO\textsubscript{2} emissions are broadband, while CH* and OH* are narrowband. This means that without a calibration, the chemiluminescence signal measured at the center wavelength for OH* (308 nm) or CH* (432 nm) will be at least partially the result of CO\textsubscript{2} chemiluminescence. To accurately measure CH* or OH*, an independent measure of CO\textsubscript{2} background intensity should be acquired. However, this would require continuous use of a spectrometer, which was not available for the current study.

A second limitation of chemiluminescence is related to the fuel injection location of real gas turbine combustors. In these real systems, the fuel is typically injected close to the combustor where there may be pressure and velocity fluctuations present. The interaction between these fluctuations and the fuel supply can lead to equivalence ratio fluctuations. As such, that configuration is referred to herein as partially premixed. In the presence of an equivalence ratio fluctuation, the chemiluminescence does not accurately measure the rate of heat release [67]. The current study was limited to fully premixed combustion so that equivalence ratio fluctuations would not complicate the measurement of chemiluminescence. A concurrent study was completed to determine the effects of equivalence ratio fluctuations on the global flame response [68].
Thirdly, it has been assumed that since chemiluminescence has a linear relationship with the global heat release rate, that chemiluminescence can also be used to indicate spatially resolved or local heat release rate. There are other factors however, such as turbulence intensity, strain rate, and flame curvature, which influence the chemiluminescence [67]. These factors will vary over the area of a flame and may distort interpretations of the chemiluminescence as a measure of local heat release rate. While these limitations are not to be ignored, chemiluminescence still offers the best combination of accuracy and simplicity compared to other approaches, which are largely based on laser-diagnostic measurements.

2.2.3.1 Global Heat Release Rate Measurement

Photo multiplier tubes are used to quantify the global heat release rate from all five flames as a function of time. Two separate units (Hamamatsu Model H7732-10) measure OH* and CH* chemiluminescence using 308 nm and 432 nm bandpass filters, respectively. The units are located 2 meters away from the combustor, as seen in Figure 2-3, and are attached to a three way beam splitter. A series of lenses focuses all light from the combustor onto the beam splitter. The beam splitter then sends half the light towards the 308 nm filter and half the light towards the 432 nm filter. The optical axis of the beam splitter system is perpendicular to the combustor axis and the two axes intersect 75 mm downstream of the

Figure 2-6: The chemiluminescence spectrum for a lean natural gas flame. [65]
dump plane. This distance corresponds to the approximate location of the flame’s greatest rate of heat release. Chemiluminescence is acquired as a function of time at sampling rate of 8192 Hz for both PMTs, nearly simultaneously with the pressure fluctuation measurements. Each sample is one second long and a total of 32 samples are acquired at each forcing frequency.

A small amount of heat is released at the base of the flame where it attaches to the centerbody, upstream of the dump plane. This heat release is not captured by the chemiluminescence measurements as it is hidden from the view of the optics systems by the dump plane and nozzle walls. Due to the decreasing intensity of chemiluminescence near the dump plane, however, this region is not expected to provide a significant contribution to the global flame response.

2.2.3.2 Local Heat Release Rate Measurement

A Photron Fastcam SA4 high speed intensified charge capture device (or ICCD camera) with an attached Invisible Vision intensifier and 432 nm band pass filter captures the two dimensional distribution of the heat release rate from the five flames, or local heat release rate. The camera acquires images at a sampling frequency of 4000 Hz for one second, with a resolution of two pixels per millimeter. The unit is located adjacent to the PMT to the side of the combustor, at a distance of 2.0 meters from the combustor axis. The optical axis of the ICCD intersects the combustor axis 75mm downstream of the dump plane. A time-averaged CH* intensity image of the region sampled by the PMT and ICCD is shown inside the combustor in Figure 2-3. The image is shown in pseudo color, where the most intense regions of heat release are white and regions with no heat release are black. All imaging in this study is line-of-sight. Therefore, the chemiluminescence along the centerline in the image results from three overlapping flames. Line-of-sight imaging is not ideal, as details from the response of individual nozzles may be obscured. A new study on the multi-nozzle rig [69] is currently underway to rebuild the local heat release distribution in three dimensions using multi-angle imaging and radon transformations, The findings from that study are outside the scope of the current project.
2.3 Data Processing and Analysis Techniques

2.3.1 Signal Processing Tools

Throughout this study, the fast Fourier transform (FFT) and other signal processing tools will be employed. This section will discuss the mathematics behind these tools. The descriptions in the following sections draw from Mitra [61] and Peluso [70].

2.3.1.1 The Fast Fourier Transform

The fast Fourier transform (FFT) is used to convert a time-domain signal into the frequency-domain, and takes the form of Eq. (2.1). The result of the transform is referred to as the linear spectrum.

\[ A_k = \sum_{i=0}^{N-1} a_i e^{-j2\pi \frac{ik}{N}} \Delta t \]  (2.1)

In the equation, \( A \) is the linear spectrum, \( N \) is the total number of samples, \( a \) is the original time-domain signal, \( i \) is the index for a particular sample in the time domain, \( j \) is the imaginary unit, \( k \) is the index for an element of sample in the frequency domain, and \( \Delta t \) is the time step between samples. The linear spectrum is divided into \( N \) bins, each of which contains both real and imaginary components. The index \( k \) specifies a particular bin. The value of the first bin, i.e. \( A_{k=0} \), has a magnitude proportional to the mean of the time domain signal. For \( k = 1 \) up to \( k = N/2 \), the magnitude of each bin is a function of the fluctuating amplitude at the frequency associated with bin \( k \). Typically \( F \) is equal to \( k \). The maximum frequency detectable with an FFT is equal to \( N/2+1 \). For \( k = N/2+2 \) up to \( k = N \), the bins contain the complex conjugate of the value of each bin from \( k = N/2 \) back down to \( k = 1 \).

The linear spectrum is used to calculate the amplitude and phase of the fluctuating component at each frequency. The phase is determined by calculating the angle, in radians, between the real and the imaginary components at the frequency of interest. To calculate the amplitude, the single-sided power spectral density (SSPSD), represented by \( G_{aa} \), is first calculated using the following piecewise function:
\[
G_{aa} \equiv \frac{1}{T} |A_0|^2 \quad \text{for } k = 0
\]
\[
G_{aa} \equiv \frac{2}{T} |A_k|^2 \quad \text{for } 1 \leq k \leq \frac{N}{2} - 1
\]
\[
G_{aa} \equiv \frac{1}{T} |A_{N/2}|^2 \quad \text{for } k = \frac{N}{2}
\]

T is the period of the acquired signal, A is the linear spectrum, N is the number of samples and k is the frequency domain index. To calculate the amplitude of the time domain signal, Parseval’s theorem is employed:

\[
A_{rms}(k) = \sqrt{G_{aa}(k) \cdot \Delta f} \quad (2.3)
\]

In the equation, \(A_{rms}(k)\) is root mean squared amplitude of fluctuation at the frequency specified by index \(k\), and \(\Delta f\) is the frequency resolution. Parseval’s theorem allows for a signal of interest at particular frequency, such as a forced velocity fluctuation or heat release rate fluctuation, to be separated from the random noise present in any turbulent system. These steps will be used to calculate the amplitude and phase of forced velocity fluctuations, global heat release rate fluctuations and local heat release rate fluctuations.

### 2.3.1.2 Spectral Coherence and the Single Sided Cross Spectral Density

In order to prove that the two signals are related to one another, such as a forced velocity fluctuation and a heat release rate fluctuation, the coherence is used. Coherence provides quantification for the causality between two signals that have a constant phase relationship. Coherence is calculated using Eq. (2.4).

\[
\gamma^2_{ab} = \frac{\tilde{G}_{ab}^* \cdot \tilde{G}_{ab}}{\tilde{G}_{aa} \cdot \tilde{G}_{bb}} \quad (2.4)
\]

The term \(\gamma^2\) represents coherence between time-domain signal \(a\) and time-domain signal \(b\). \(G_{aa}\) and \(G_{bb}\) are the single-sided power spectral density of signal \(a\) and signal \(b\) respectively, from Eq. (2.2). The overscore (\(\tilde{\cdot}\)) on the characters in the equation indicates that the SSPSD value is a mean from many records. Multiple records are required to ensure that the phase relationship between the signals is steady. The star superscript
( * ) indicates that a value is a complex conjugate of the original value. $G_{ab}$ is the single-sided cross spectral density (SSCSD), determined from the piecewise function (2.5).

\[
G_{ab} \equiv \frac{1}{T} A^*_0 B_0 \quad \text{for } k = 0
\]
\[
G_{ab} \equiv \frac{2}{T} A^*_k B_k \quad \text{for } 1 \leq k \leq \frac{N}{2} - 1
\]
\[
G_{ab} \equiv \frac{1}{T} A^*_{N/2} B_{N/2} \quad \text{for } k = \frac{N}{2}
\]

In the function, T is again the period of the sample, A and B are the linear spectrum of signal $a$ and $b$, $k$ is the frequency domain index and N is the number of samples.

The stronger the causal relationship between signal $a$ and $b$, the closer the coherence value will be to unity. A value of 0.99 is used in later sections as the threshold above which two signals are considered coherent and acceptable for use in further calculations.

2.3.2 The Two-Microphone Method

The pressure fluctuation measurements in each nozzle are used as inputs to a two-microphone method (TMM) calculation [71]. This calculation is used to provide information regarding the time varying velocity in each nozzle. Calculation of the fluctuating velocity component is based on the Euler equation. By assuming that the amplitude of the fluctuations is small relative to the speed of sound, the second order terms in the Euler equation can be neglected to provide the linearized Euler equation [72], in Eq. (2.6).

\[
\frac{\partial U'}{\partial t} + \frac{1}{\rho} \frac{\partial p'}{\partial x} = 0
\]

In the equation, $U'$, $p'$ and $\rho$ represent the acoustic velocity fluctuation, the acoustic pressure fluctuation and the density, respectively. If Eq. (2.6) is discretized, the acoustic velocity fluctuation can be calculated from two pressure signals, using Eq. (2.7). The two-microphone method has been used successfully in many studies to quantify velocity fluctuations in forced flame environments [9, 26, 59, 73]. Figure 2-7 shows an example two-microphone calculation, starting with the power spectral density of the two pressure signals, followed by the result of the calculation and the coherence at each frequency.
The two-microphone method is only valid over a certain range of frequencies. As the forcing frequency decreases, the acoustic wavelength of the disturbance will increase. If the wavelength is significantly larger than the pressure transducer spacing (38 mm), the pressure differential measured

\[
U = \frac{P_{DS} - P_{US}}{\omega \cdot \rho \cdot \Delta x}
\]  

(2.7)

Figure 2-7: (A) Power spectral density of pressure at \( F_{\text{forcing}} = 200 \) Hz, (B) the velocity fluctuation spectrum and the (C) coherence at each frequency.
between the two transducer ports will be very small. When the pressure differential has a similar magnitude to the amplitude of random turbulent fluctuations, the measured fluctuations will have poor coherence [71]. If the coherence is low, the velocity fluctuation calculation will not be meaningful [74]. In the current study, the lowest forcing frequency which reliably provided a coherence value greater than 0.99 was 100 Hz. Accordingly, 100 Hz was adopted as the minimum forcing frequency.

An example of the $U'$ values calculated for each forcing frequency is shown in Figure 2-8 for a typical flame transfer function test. In Figure 2-8A and Figure 2-8B, gray lines display the gain and phase as measured by the three individual transducer pairs. The black line shows the averages from all three nozzles. In C and D of the figure, the coefficient of variation for each frequency is shown. The coefficient of variation is equal to the standard deviation of all the samples divided by the mean value of the samples. For individual microphone pairs the variation is relatively small, but across all three nozzles there is a large variation. For the phase, variation is much smaller than for the gain at all frequencies.

In order to explain the variation in gain, the theory of the two-microphone method and the transducers themselves were investigated. In the literature, it has been shown [75] that small non-idealities in the microphones result in small errors in phase, which can lead to large errors in the calculated amplitude of the velocity signal. Additionally, while the pressure fluctuation amplitude in the experiment is within the detectable range for the pressure transducers, this amplitude is at the extreme low end. In this range the transducers are less accurate, providing another possible source of error. Combined, these factors may explain the variation observed in the current experiment, but there may also be actual nozzle-to-nozzle differences in the velocity fluctuation. This is a non-negligible problem that will need to be addressed eventually. For the study at hand, however, a systematic approach has been taken to eliminate data where the velocity measurement exhibits a high degree of variation.
The approach begins by acquiring thirty-two data sets at each operating condition and forcing frequency. For each of these cases, pressure measurements are recorded by the three separate transducer pairs, totaling 96 samples. First, a coherence calculation was made between the two pressure signals from each nozzle for all 32 sets. Coherence alone does not ensure that the two-microphone calculation will be meaningful, but two pressure signals with a poor coherence value will invariably result in a meaningless calculation [74]. A good coherence is represented by a value close to one. Thus, all nozzles with pressure transducer pairs providing a 0.99 or higher coherence value for all 32 data sets are henceforth referred to as coherent. For any frequency where one or more nozzles do not meet this criterion, the data are discarded. For the coherent conditions, the velocity fluctuation amplitude and phase from the three nozzles are averaged together and a coefficient of variation (CV) is calculated. For any forcing frequency where the CV of the coherent nozzles is greater than 0.25, that frequency is also discarded. This threshold is represented by the red line in Figure 2-8. The remaining operating conditions are referred to as precise and

---

**Figure 2-8:** (A) $U_{RMS}^*$ fluctuation vs. frequency for three different two-microphone pairs, (B) $U^*$ phase with respect to an encoder signal, (C) The coefficient of variation of the RMS amplitude, (D) The coefficient of variation for the phase. The red line represents the threshold below which the variation is considered acceptable. $U_{inlet} = 20 \text{ m/s}$, $T_{inlet} = 200 \degree \text{C}$, $\Phi = 0.7$. 
only such conditions are presented in the chapters on results. Of the 860 unique combinations of operating condition and frequency that were initially acquired, 800 (93%) were found to be both coherent and precise.

2.3.3 Global Heat Release Rate Measurements

To quantify the flame response, chemiluminescence data in the time domain from the PMTs is subjected to a fast Fourier transform. The Fourier transform returns the signal in the frequency domain, also known as the linear spectrum. When the incoming flow is forced at a certain frequency, a peak appears in the heat release linear spectrum at the same frequency. This fluctuation is the result of the flame responding to the incoming perturbations through a number of feedback mechanisms, some of which have already been discussed. The amplitude and phase of the PMT signal at the forcing frequency are used as the output to the flame transfer function, as described in the literature review. Measurements from the OH* PMT and CH* PMT show good agreement as seen in Figure 2-9, so only CH* data will be presented in the results chapters.

![Figure 2-9: Forced heat release rate fluctuations as a function of forcing frequency as measured by CH* and OH* radicals.](image)

2.3.4 Time-Averaged Local Heat Release Rate

Line-of-sight CH* chemiluminescence images from the high speed camera are first used to provide time-averaged information on the heat release rate distribution. Such information is acquired by
averaging the 4000 images of a single high speed image set. An example of a time-averaged image is shown in Figure 2-10. Because all imaging is line-of-sight, the captured chemiluminescence is the combined emission from five flames, labeled A through E in the image. Flow direction in all images is bottom to top. The highest heat release rate occurs in the interaction region between the flames. Images such as these can be analyzed to determine structure of the flame and how it varies with operating conditions. The main purpose of time-averaged images is the determination of a characteristic length scale for the flame. This will be discussed in the next section.

Figure 2-10: Time-averaged CH* chemiluminescence image of a flame acquired using the high speed ICCD. Heat release rate intensity is shown in pseudo color. Radial and axial distances are in meters. A white oval identifies the base of each flame, labeled A through E. $U_{\text{inlet}} = 25$ m/s, $T_{\text{inlet}} = 200$ C, $\Phi = 0.6$. 
A small degree of asymmetry is detectable about the centerline of the combustor in the figure. This is a typical feature of most operating conditions. To quantify the asymmetry, the heat release is summed along the axis of the flame and plotted versus the radial position, as seen in Figure 2-11. The black lines show the radial profile of heat release rate for the left and right halves of the flame as a function of the distance from the centerline. The red line displays the percent difference between the two heat release rate profiles, normalized by the average value of heat release rate across the entire flame. In this example, the percent difference averages around 10%, a difference which is typical of all operating conditions at which high speed data were acquired. In later sections, the flames are assumed to be symmetric about the centerline of the images. To reduce processing time and the amount of space required to display an image, image arrays are averaged about the centerline and only the half image is displayed.

![Figure 2-11: Degree of asymmetry in local heat release rate. Percent difference is normalized by the maximum heat release rate intensity in the left half of the flame.](image)

**Figure 2-11**: Degree of asymmetry in local heat release rate. Percent difference is normalized by the maximum heat release rate intensity in the left half of the flame. $U_{\text{inlet}} = 25 \text{ m/s}$, $T_{\text{inlet}} = 200 \text{ C}$, $\Phi = 0.6$.

### 2.3.5 Axial Heat Release Profile and Multi-nozzle Flame Length

A challenge exists in the multi-nozzle combustor in determining a characteristic length for the flame. In past experiments [9, 27], the flame length was extracted from a planar image of the flame...
acquired through a technique called Abel deconvolution. If it is assumed that the flame is rotationally symmetric about the combustor axis, the technique can be applied to calculate the chemiluminescence intensity along an axial slice of the flame. From this slice, an accurate center of heat release could be determined in two-dimensional coordinates. The vector from the tip of the centerbody to these coordinates was then used as the flame length. In a multi-nozzle flame, the line-of-sight integrated images are highly asymmetric and so standard deconvolution techniques are not applicable. In addition, the ICCD images from the multi-nozzle combustor have three overlapping non-identical flames at the center, which further complicates the measurement. Therefore a multi-nozzle definition of flame length must be specified.

The means of identifying the flame length in the current study is based on the axial heat release rate profile of the flame. To acquire the axial profile, the image is first divided into bins along the flame axis, as shown in Figure 2-12. The chemiluminescence intensity at each pixel on the camera sensor in a particular bin is then summed and plotted as a function of the axial length. The characteristic flame length is then defined as the axial distance from the end of the centerbody to most intense radial band of heat release. The flame length parameter provides a length scale for the flame that is a function of the operating conditions ($U_{inlet}$, $T_{inlet}$ & $\Phi$). Additionally, the term helps to determine the amount of time it would take a disturbance entering the combustor to convect to the region of the flame where it would have the greatest effect on the global heat release rate fluctuation.

![Figure 2-12](image)

**Figure 2-12**: Calculation of axial heat release rate profile and characteristic flame length.
2.3.6 **Frequency Filtered Local Heat Release Rate**

Time-averaged flame images are useful for analyzing the mean shape of the flame. When the flame is excited, however, it is of interest to observe the local heat release rate of the flame at small increments through an excitation cycle. The high speed camera provides a sufficient sampling frequency (4000 Hz) to view a series of images during a single forcing period. However, due to the broadband noise always present in a turbulent combustion process, as well as the small amplitude of the forced perturbations, such images can be difficult to analyze. **Figure 2-13** displays a series of forced raw images over a single cycle at 400 Hz.

![Figure 2-13: A series of raw images from the high speed ICCD.](image)

In order to analyze the local heat release rate over a forcing cycle, a frequency filter is applied to the high speed ICCD set, using a method similar to Hauser et al.[76]. Each pixel on the camera sensor is treated as a separate detector that provides heat release rate data as a function of time. The time domain data from each pixel are subjected to a fast Fourier transform to provide the frequency spectra for each pixel. **Figure 2-14** displays the power spectral density at each frequency, for a single pixel of a flame forced at 400 Hz. The X axis is divided into bins. The bins contain an intensity value which is a function of the amplitude of the fluctuation at that frequency, per Section 2.3.1. Mean intensity information is stored in the ‘0 Hz’ bin. The intensity values are normalized with respect to the mean intensity of the time domain signal. The frequency filter is applied by extracting the amplitude and phase of fluctuations at the forcing frequency and inserting the values into Eq. (2.8). In the equation, $\dot{Q}_{x,y}$ is the instantaneous intensity value at pixel (x,y), t is the time, $F$ is the forcing frequency, $\bar{Q}_{x,y}$ is the mean intensity value, and $A_{x,y}$ and $\theta_{x,y}$ are the fluctuation amplitude and phase calculated from the linear spectrum.
A series of forced images can be reconstructed by applying Eq. (2.8) at every pixel in a window of the same size as the original sample. Figure 2-15 displays an example of this reconstruction technique applied to a 400 Hz case. The line-of-sight heat release rate is assumed to be symmetric about the centerline so only half images are shown. The white line is extracted from the time-averaged image and drawn at an intensity threshold of 10% of the maximum intensity. It is superimposed onto the instantaneous images to provide a fixed frame of reference for the mean location of the flame edge. The white text on each image marks the phase with respect to the velocity fluctuation. Compared to Figure 2-13, the images in Figure 2-15 are considerably easier to interpret. However, the forced fluctuations are only barely visible, as seen in the most intense regions of heat release.

\[
\dot{Q}_{x,y}(t,F) = \bar{Q} + \dot{Q}' = \bar{Q}_{x,y} + A_{x,y} \cos(2\pi F t + \theta_{x,y})
\]  

(2.8)
In order to more effectively visualize the forced fluctuations in the flow, the mean information must be removed from the images. This is achieved simply by removing the $\bar{Q}_{x,y}$ term from Eq. (2.8), to get Eq. (2.9).

$$\dot{Q}'_{x,y}(t, F) = A_{x,y} \cos(2\pi \cdot F \cdot t + \theta_{x,y})$$

Making use of the modified equation, Figure 2-16 displays an image showing only the fluctuating component of the heat release rate for the same conditions as Figure 2-15. Fluctuations in the heat release rate are now clearly visible. The fluctuation intensity at each pixel is normalized by the single most intense pixel in the time-averaged flame, with a scale shown in the bottom right of the image. In the images, the warm colors (red, yellow) represent above-mean fluctuations, while the cool colors (blue, cyan) represent below mean fluctuations. In order to provide a frame of reference, the 10% intensity threshold from the time-averaged flame image is superimposed on these images as well. The features of these fluctuation images will be discussed in detail in Chapter 4.
Another way to make use of the frequency domain data from the high speed images is amplitude and phase plots. Rather than using Eq. (2.8) to rebuild a set of instantaneous images, the amplitude and phase at each pixel can be displayed directly as an image. This approach allows all of the important information about the fluctuations to be displayed in a minimal amount of space and makes it easier to compare between different forcing frequencies. Figure 2-17A shows the amplitude of fluctuations for a $F_{\text{forcing}} = 120$ Hz case, where the amplitude at each pixel is normalized by the single most intense pixel in the time-averaged flame. Figure 2-17B contains the phase information. Regions with similar colors have heat release rate fluctuations that are occurring in-phase with each other. Phase values for pixels with small fluctuations are often unreliable and do not contribute significantly to the global heat release rate fluctuation. Accordingly, it is appropriate to mask out the phase of such fluctuations. Figure 2-17C shows the phase with pixels that experience a fluctuation smaller than 2% masked out. From such an image we can determine at what phase the most important pixels fluctuate. In this example, we can see that most of the high amplitude pixels fluctuate at nearly the same phase.
In order to quantitatively analyze heat release fluctuations, it is of interest to plot the fluctuations as a function of the axial position. The same technique described in Figure 2-12 can be applied to a series of fluctuation images, like those in Figure 2-16. This provides the axial heat release rate fluctuation as a function of distance from the centerbody for a particular phase angle. Figure 2-18 shows this type of axial profile, along with the line-of-sight images used to calculate it. The fluctuation profiles are normalized by the maximum intensity in the time-averaged profile. Note that the flame axis is horizontal in Figure 2-18A, while it is vertical in the half-flame images of Figure 2-18B.

Figure 2-17: (A) Amplitude and (B) phase of fluctuations at $F_{\text{forcing}} = 120 \, \text{Hz}$. (C) phase of fluctuations with small fluctuations masked out. $U_{\text{inlet}} = 25 \, \text{m/s}$, $T_{\text{inlet}} = 200 \, ^\circ\text{C}$, $\Phi = 0.60$, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$.

2.3.7 Axial heat Release Fluctuations

In order to quantitatively analyze heat release fluctuations, it is of interest to plot the fluctuations as a function of the axial position. The same technique described in Figure 2-12 can be applied to a series of fluctuation images, like those in Figure 2-16. This provides the axial heat release rate fluctuation as a function of distance from the centerbody for a particular phase angle. Figure 2-18 shows this type of axial profile, along with the line-of-sight images used to calculate it. The fluctuation profiles are normalized by the maximum intensity in the time-averaged profile. Note that the flame axis is horizontal in Figure 2-18A, while it is vertical in the half-flame images of Figure 2-18B.
Another way to present the axial fluctuation is an axial amplitude profile. A fast Fourier transform can be applied to the time signal of intensity at each axial location to extract the amplitude and phase at the forcing frequency. Figure 2-19A shows these amplitude and phase profiles. Amplitude is normalized with respect to the maximum intensity in the time-averaged axial profile. Phase is shown with respect to the forced velocity signal. It will occasionally be useful to compare axial fluctuation profiles for a wide range of forcing frequencies. To do this, the amplitude profiles are converted to pseudo-color, as seen in Figure 2-19B. In such an image, white regions represent the largest axial fluctuations, while black regions represent a lack of heat release rate fluctuations. A scale is not included with the pseudo-color bar below as the values match the black curve in Figure 2-19A. Whenever a set of such bars is presented later on, an appropriate scale will be included.

Figure 2-18: (A) Axial heat release fluctuation profile and (B) corresponding fluctuation images for 9 phase angles in a full period at $F_{\text{forcing}} = 120$ Hz. $U_{\text{inlet}} = 25$ m/s, $T_{\text{inlet}} = 200$ °C, $\Phi = 0.60$, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$. 
Figure 2-19: (A) Axial heat release rate fluctuation amplitude and phase as a function of distance from the centerbody (B) Axial amplitude converted to pseudo-color. $U_{\text{inlet}} = 25 \text{ m/s}$, $T_{\text{inlet}} = 200 \degree \text{C}$, $\Phi = 0.60$, $F_{\text{forcing}} = 120 \text{ Hz}$, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$. 
2.4 Operating Conditions

2.4.1 Inlet Velocity, Temperature and Equivalence Ratio

One advantage offered by gas turbines is their ability to quickly ramp-up or ramp-down the power output. To allow this, a wide range of operating conditions must be achievable. In the current experiment, an operating condition refers to a unique combination of the parameters \( U_{\text{inlet}} \), \( T_{\text{inlet}} \), and equivalence ratio (\( \Phi \)). These parameters were selected to match the range for which the GE15 nozzle was designed, and are shown in Table 1. The specific value of each parameter used for the tests in Chapter 3 and Chapter 4 is listed in the Appendix, along with a unique identification number.

Table 1: Operating Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tested Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet velocity (m/s)</td>
<td>15 – 30</td>
</tr>
<tr>
<td>Inlet temperature (°C)</td>
<td>50 – 250</td>
</tr>
<tr>
<td>Equivalence ratio</td>
<td>0.50 - 0.70</td>
</tr>
<tr>
<td>Reynolds number (Based on nozzle hydraulic diameter)</td>
<td>24,000-45,000</td>
</tr>
<tr>
<td>Forcing frequency (Hz)</td>
<td>100 – 450</td>
</tr>
<tr>
<td>Forcing amplitude</td>
<td>5% - 32%</td>
</tr>
</tbody>
</table>

At certain conditions within the specified operating range, the combustor may experience self-excited instability. The presence of a self-excited instability at one frequency could affect the way the flame responds to a forced perturbation at another frequency, so these conditions must be avoided. A self-excited condition is one in which any of the below criteria are met at a frequency other than the forcing frequency, or a harmonic frequency of the forcing. Operating conditions which meet any of these criteria are not presented in the current study.
2.4.2 Operating Pressure

The mean pressure inside the combustor is one parameter that is not within the range of typical industrial gas turbine combustors. Industrial gas turbines operate with combustor pressures as high as 20 atmospheres. Experimental considerations necessitated that the current multi-nozzle combustor operate at atmospheric pressure. These considerations included concerns for safety, the limited fuel supply and additional costs associated with building a high pressure combustor. Since it was not possible to account for the effect of elevated pressure experimentally, some literature is presented below explaining the expected effect.

Cheung et al. [77] investigated flame transfer functions at high pressure, high temperature conditions. Across a range of frequencies for two different combustor pressures (1 to 7 atm), the transfer function gain was found to be qualitatively similar. The minima and maxima for each operating condition occurred around the same frequency for all pressures. Discontinuities were observed in the transfer function phase at different frequencies for different pressures. This was attributed to a shorter flame length at high pressures. Reduced flame length is an indicator of increased turbulent flame speed, a parameter with a known pressure dependence [48, 78, 79]. Cheung concluded that it may be possible to obtain qualitative information on the flame response at real gas turbine operating conditions by extrapolating results from atmospheric pressure tests.

Freitag et al. [80] acquired flame transfer functions at a fixed combination of $U_{\text{inlet}}$, $T_{\text{inlet}}$ and $\Phi$, while varying the operating pressure from 1 to 5 atmospheres. At frequencies below 100 Hz, the transfer function was found to be independent of pressure, but, at higher frequencies, significant changes are observed. As the pressure increases, the gain tends to decrease asymptotically. The phase for the higher pressure transfer functions is found to exhibit more discontinuities than their low pressure counterparts. Freitag attributes the changes in the flame response to movement in the location of the greatest heat release.
rate fluctuation, which results from an elevated flame speed. As the pressure increases, the greatest fluctuations are found to move upstream, towards the base of the flame. Ultimately the flame transfer function is found to have power law scaling with pressure, with changes becoming smaller with equal sized steps towards higher pressure. This led Freitag et al. to conclude that full engine pressure should be approached for an accurate flame response measurement, but it need not be actually reached.

From the literature it becomes apparent that high pressure tests will eventually have to be conducted in a multiple flame environment. Given the qualitative similarity in the flame response at a range of pressures, it is likely that the mechanisms identified in atmospheric tests are still present at elevated pressure. As such, the current study will aim to identify these mechanisms so that future high pressure multi-nozzle studies have a framework to start with.

2.4.3 **Forcing Frequency and Amplitude**

The lower limit of the forcing frequency is set by the two-microphone method, at 100 Hz, as detailed in Section 2.3.2. The upper frequency limit is set by the maximum safe rotational frequency of the siren, at 450 Hz. At forcing frequencies higher than 450 Hz, the bearings wear out too quickly and the excessive torque alarm on the siren motor is frequently activated. Fortunately, the range allowed by these factors, of 100 – 450 Hz, coincides with the range of typical longitudinal combustion instabilities experienced in real gas turbine engines [3, 60]. A unique combination of $U_{\text{inlet}}$, $T_{\text{inlet}}$, $\Phi$, $F_{\text{forcing}}$ and forcing amplitude is referred to as a case.

In all tests except for those in Chapter 5, the forcing amplitude was set to 5% of the mean velocity using the siren bypass valve. This allowed for the study of flame response mechanisms in the linear response regime. The value of 5% was selected as it provides a robust enough velocity fluctuation to provide a coherence value greater than 0.99 at all frequencies, while remaining in the linear regime (constant gain value). In order for a high amplitude nonlinear response to occur in a real combustor, coupling at low amplitude through linear regime mechanisms must first occur. For this reason, the linear regime is the main focus of this study. In Chapter 5, the nonlinear response is analyzed for a single operating condition and suggestions for future nonlinear studies are made.
Chapter 3

Global Flame Response

3.1 Introduction

The global response of the flame to an imposed velocity perturbation provides a means to measure the driving force, in terms of heat release rate, for combustion instability at a particular operating condition and frequency [23]. In this chapter, the global response is quantified by the flame transfer function, from Eq. (1.2). The RMS amplitude of the imposed velocity fluctuation is maintained at a level of 5% of the mean velocity at each forcing frequency. This ensures that the flame response remains in the linear response regime [21, 33] for all cases presented in this chapter. Analysis of the nonlinear response regime will be reserved for Chapter 5.

The next section of this chapter will analyze the unforced, or stable, structure of the flame at all operating conditions. This is included first because such a study on multi-nozzle can combustors has yet to be published. Secondly, the unforced flame structure has a substantial impact on how the flame will respond to a perturbation [22, 81]. In Section 3.3, the features of a typical flame transfer function from the multi-nozzle rig will first be analyzed. Following this, sets of flame transfer functions for varied inlet velocity, varied inlet temperature and varied equivalence ratio are analyzed. In Section 3.4, flame transfer functions are plotted with respect to the Strouhal number and another normalization parameter to look for trends. The findings there are also compared to previous studies.

3.2 Stable Flame Structure

3.2.1 The Effect of Operating Conditions

While the main focus of the current study is on the forced response of the flame, several studies [9, 22] have shown that the unforced or stable flame structure can have a pronounced impact on the flame response. Since little work has been done on the stable flame structure of multi-nozzle flames, it is useful to analyze this first before analyzing the forced response. Chemiluminescence imaging is used to provide qualitative descriptions of the differences between flames at various combinations of $U_{inlet}$, $T_{inlet}$ and $\Phi$. 
The flame length ($L_{\text{flame}}$) parameter will allow for quantitative comparisons. Line-of-sight chemiluminescence is assumed to be symmetric about the combustor centerline, as discussed in Section 2.3.4, so only half of each image is shown.

**Figure 3-1** shows a series of time-averaged heat release rate images for a range of inlet velocities from 15 to 25 m/s. The inlet temperature and equivalence ratio are fixed at 150°C and 0.55, respectively. The intensity of all five images is scaled to the same reference value to allow for comparisons between each image. This is possible because chemiluminescence is proportional to the fuel flow rate for a flame of fixed equivalence ratio and inlet temperature [65]. For all velocities, a measurable amount of heat is released in the outer recirculation zones as well as the regions between nozzles. These regions are indicated by the red arrows in image A. As the velocity increases, the flame brush spreads farther downstream away from its attachment point. Additionally, the maximum intensity in the image increases. Both of these observations are the result of increasing flame area, which occurs as the flow rate of reactants to the combustor increases [48]. The quantitative change in flame length will be discussed below.

**Figure 3-2** displays a series of images for a range of inlet temperatures from 100 to 250 °C for an inlet velocity and equivalence ratio of 20 m/s and 0.65. Note that each flame in this set is shown on its own intensity scale. This is because the relationship between heat release rate and inlet temperature is nonlinear [66]. As such, quantitative comparisons between the heat release rates for flames with different inlet
temperatures may result in erroneous findings. Instead, image sets such as this are presented primarily to allow for qualitative comparisons in the distribution of the heat release rate. In this image set, the length of the flames is small compared to the previous condition. This is a result of the elevated turbulent flame speed which comes with increased inlet temperature [48]. An additional feature of these flames is the initiation of M-type flame attachment. It is just barely visible at the left most part of flames D & E near the base, as indicated by the gray arrows. The attachment becomes more obvious from visual inspection, or a high resolution digital camera image like the one shown earlier, in Figure 2-4. M-type attachment occurs when the outer recirculation zone contains a large amount of thermal energy from the hot products of combustion. The recirculation zone supplies this energy to the incoming flow near the lip of the dump plane and allows a flame to anchor there. M-flames are of particular interest for stability analysis as studies have determined that they have favorable stability characteristics [9, 22, 82].

Figure 3-2: Stable flame structure for a range of inlet temperatures from 50 to 250 °C (50 °C increment), for \( U_{\text{inlet}} = 20 \text{ m/s} \) and \( \Phi = 0.65 \). Each image is shown on an independent intensity scale.

Figure 3-3 shows a set of heat release rate images where the equivalence ratio was varied from 0.50 to 0.65 in a 0.05 increment, for \( U_{\text{inlet}} = 20 \text{ m/s} \) and \( \Phi = 0.65 \). As in the previous figure, each image is self-scaled due to the nonlinear dependencies of the heat release rate. The first flame, in Figure 3-3A, shares features with the flames in Figure 3-1. This includes a long flame length as well as combustion in the outer recirculation zone (indicated by the red arrow). These features are characteristic of a flame near blowoff, which occurs when the turbulent flame speed is less than the flow velocity throughout the combustor [48]. If the equivalence ratio is decreased any further at this condition, the flame does in-fact blowoff. Increasing the equivalence ratio results in a shortening of the flame and strengthening of flame attachment, as evidenced by the sharper flame boundaries in Figure 3-3C and D. This is again the result of
an increase in the turbulent flame speed, which occurs as the equivalence ratio increases [48]. The M-type flame attachment is also just barely visible in the outer shear layer (indicated by gray arrows).

![Increasing Equivalence Ratio](image)

**Figure 3-3:** Stable flame structure for a range of equivalence ratios from 0.50 to 0.65 (0.05 increment), for $U_{\text{inlet}} = 20$ m/s, $T_{\text{inlet}} = 250$ °C. Each image is shown on an independent intensity scale.

With the qualitative flame descriptions addressed, the one quantifiable parameter that will be extracted from all stable flame images is now analyzed. **Figure 3-4** shows the characteristic flame length for all operating conditions on the multi-nozzle rig. These lengths are calculated with the technique described in Section 2.3.5. The data are divided into sets where the color indicates the inlet temperature and the marker type indicates the equivalence ratio. The following trends are observable in the figures:

1) Flame length increases with increasing inlet velocity, at a decaying rate
2) Flame length decreases with increasing equivalence ratio
3) Flame length decreases with increasing inlet temperature

Each of these findings is consistent with turbulent combustion theory [48]. The length of the flame is largely determined by the turbulent flame speed, which depends on the laminar flame speed and the turbulence intensity [83]. The laminar flame speed increases with both increasing equivalence ratio (For $\Phi < \Phi_{\text{stoichiometric}}$) and increasing inlet temperature [84, 85]. If the velocity in the combustor is fixed, an increase in the flame speed will result in a shorter flame length. The turbulence intensity increases with inlet velocity which leads to a higher turbulent flame speed. Based on this alone, the flame length might be expected to shorten for increasing inlet velocity. Counteracting the effect of the elevated flame speed, however, is the increased flow rate of reactants to the combustor. In order to oxidize the additional reactants, the flame area must increase, which results in a longer flame.
3.2.2 Empirical Model of the Flame Length

Through an empirical fit it is possible to find the dependence of flame length on the operating conditions in the current configuration. Nonlinear regression was used to fit the acquired flame lengths to an exponential model of the form given in Eq. (3.1).

**Figure 3-4:** Axial flame length as a function of inlet velocity for all inlet temperatures and equivalence ratios.
The MatLab built-in statistical function \textit{nlinfit} was used to perform the regression. This approach yielded the exponents shown in Eq. (3.2). \textbf{Figure 3-6} shows a comparison of the original experimental value compared to the value calculated using the exponential fit. All cases agree within 10\% \left( \frac{\Delta \text{MODEL-EXP}}{\text{L}_{\text{MEAN}}} \right), as shown by the red points in the figure.

\begin{equation}
L_{\text{flame}} = A \cdot U_{\text{inlet}}^{0.219} \cdot T_{\text{inlet}}^{-0.971} \cdot \Phi^{-0.765} \text{ [meters]} \tag{3.2}
\end{equation}

\textbf{Figure 3-5}: Axial flame length for all operating conditions and the corresponding value from Eq. (3.2). The percent difference between the model and experimental value is shown by the red points. \(T_{\text{inlet}}, \Phi\) and \(U_{\text{inlet}}\) for each operating condition number are listed in the Appendix.

These exponents show that, over the range of operating conditions for this multi-nozzle flame, the flame length is most sensitive to changes in \(T_{\text{inlet}}\). With an exponent of -0.971, \(L_{\text{flame}}\) is inversely proportional to \(T_{\text{inlet}}\). Equivalence ratio also displays the expected inverse relationship, but the dependence is not as strong as with \(T_{\text{inlet}}\). The sensitivity of the flame length to changes in the inlet velocity is comparatively small, with an exponent of only 0.219.
3.3 Flame Transfer Function Analysis

3.3.1 Example Flame Transfer Function

Figure 3-6 displays the flame transfer function gain and phase acquired for a typical operating condition on the multi-nozzle combustor. Error bars on the gain and phase represent +/- one standard deviation of 96 samples. Thirty-two samples are provided by each of the three two-microphone pairs as described in Section 2.3.2. The two-microphone measurements of velocity are combined with PMT measurements of heat release rate fluctuations in Eq. (1.2) to provide the flame transfer function gain and phase.

At low frequencies the gain is high, with a value greater than 1.0. If the gain is greater than unity, it means that the output fluctuation (heat release rate) has greater amplitude than the corresponding input perturbation (velocity). In this case the flame acts as an amplifier of the disturbance. While the gain could not be acquired for forcing frequencies below 100 Hz due to limitations of the two-microphone method, the expected behavior can be inferred from a previous study. Polifke and Lawn showed numerically that as the forcing frequency draws closer to zero, the gain should tend towards a value of unity [86]. In the example case, this boundary condition would result in a local maximum at a frequency less than 100 Hz, followed by decay to a gain value of 1.0.

As the forcing frequency is increased, the gain initially decreases, indicating a weakened global flame response. The minimum response is achieved at 180 Hz. From a design perspective this is an interesting condition as it is the frequency that provides the smallest driving force for combustion instability. Previous work has suggested that a minimum, response such as this, arises from the destructive interference of multiple flame response mechanisms all present in the flame at the same time [10, 16]. Following the minimum response the gain begins to rebound, reaching a value of 0.79 at a frequency of 280 Hz, a local maximum. Literature suggests that a local maximum such as this is caused by the same mechanisms present at the minimum response case, only now they interfere constructively [10, 16].
Beyond the maximum response frequency, the transfer function experiences another local minimum response at 340 Hz, followed by another rebound. The gain at the highest forcing frequency acquired (400 Hz) is 1.23, which is an overall maximum in the response for this operating condition. This is an unusual finding since most single-nozzle flame transfer functions tend to exhibit low pass filter behavior\[9-11, 62\]. Low pass filter behavior requires that the gain of the maxima decay towards zero with increasing frequency, as discussed in Chapter 1. In the current operating condition, as well as many of the other conditions that will be analyzed later in this chapter, this is clearly not the case. In order to shed more light on this phenomenon, it will be discussed further in Chapter 4 with additional insights from high speed chemiluminescence imaging.

The phase of the flame transfer function represents the time delay between a peak in the forced velocity, measured at the upstream pressure transducers, and a peak in the heat release rate fluctuation, measured by the PMT. In Figure 3-6, the phase initially decreases nearly linearly with increasing forcing frequency. This linear behavior is common to many operating conditions when forced at low frequency, and will be observed regularly in this chapter. After 180 Hz, the frequency of minimum response, the phase exhibits a discontinuity, followed by return to linear behavior. Similar discontinuities have been observed in the literature [3, 5, 16, 27], but a complete explanation has not yet been offered. The behavior could be

**Figure 3-6**: The flame transfer function gain (black) and phase (gray). \(U_{\text{inlet}} = 20\) m/s, \(T_{\text{inlet}} = 250\) °C, \(\Phi = 0.55\).
explained by a sudden change in the axial location of the greatest heat release rate fluctuation. In Chapter 4 high speed imaging will be employed to search for this sudden change.

3.3.2 The Flame Transfer Function as a Function of Inlet Velocity

In this section, a set of several flame transfer functions will be compared. Of the three inlet parameters that can be varied, the inlet temperature and the equivalence ratio are held constant. The inlet velocity is then varied over the design range for the experiment, in increments of 2.5 m/s. The purpose of this analysis is to identify the effect of the inlet velocity on the velocity-forced response of a multi-nozzle flame. This set of flame transfer functions corresponds to the image set in Figure 3-1.

Figure 3-7A shows the gain for five different inlet velocities while $T_{\text{inlet}}$ and $\Phi$ are held constant at 150 °C and 0.55, respectively. As in Figure 3-6, each flame transfer function starts out with relatively high gain, followed by a local minimum, then a rebound up to a local maximum. With increasing inlet velocity (gray $\rightarrow$ black lines), an increase in the frequency of the extrema can be observed. This indicates that inlet velocity is a controlling parameter for the occurrence of flame transfer function extrema. In Section 3.4, this dependence will be utilized in an attempt to generalize the flame transfer function data.

A noteworthy qualitative difference that these operating conditions exhibit compared to the one shown in Figure 3-6 is the gain value at the frequency of the local maximum and beyond. Unlike the example case, for the range of ~200 Hz – 400 Hz, the gain is significantly lower than at frequencies less than 200 Hz. These conditions are more consistent with the low pass filter behavior proposed by some single-nozzle studies [9-11, 62]. Compared to Figure 3-6, all of the operating conditions in Figure 3-7 have relatively low inlet temperature and equivalence ratio. Both of these parameters have a significant effect on the turbulent flame speed, which helps determine the flame structure. The flame structure in-turn has a substantial effect on the way the flame responds to a velocity disturbance [9, 22]. As such, the direct effect of changes in inlet temperature and equivalence ratio on the forced response of the flame should also be examined. The mechanism through which the decreased flame speed affects the gain of the flame will be discussed in the next section.
The flame transfer function phase for this same set of operating conditions is shown in Figure 3-7B. Like the example transfer function in Figure 3-6, the phase exhibits an initially linear decrease with increasing forcing frequency. Near the frequency of minimum response, the phase experiences a discontinuity, followed by a return to linear behavior. For the conditions that experience two local minima, such as 15 m/s (at 140 Hz and 280 Hz), two discontinuities are present in the phase.

Based on the findings from Figure 3-7, it can be concluded that varying only the inlet velocity to the combustor will change the frequencies of the extrema and the frequencies of the discontinuities in
phase. Qualitatively however, the flames respond similarly over the entire range of inlet velocities tested in this study.

### 3.3.3 The Flame Transfer Function as a Function of Inlet Temperature

**Figure 3-8** shows the gain and phase for a set of flame transfer functions in which only the inlet temperature is varied. The inlet velocity and the equivalence ratio are set to 20 m/s and 0.65, respectively. These cases correspond to the flame images shown in Figure 3-2. Like the previous transfer function set in Figure 3-7, four of the five gain curves in Figure 3-8A begin with high gain at low frequency, followed by a local minimum. As the inlet temperature increases, the frequency of minimum response shifts to slightly higher frequencies. Additionally, the gain at higher frequencies (> 250 Hz) typically increases with increasing inlet temperature. As discussed in Section 3.2.1, a major difference in these flames is found in the flame length as a result of increasing turbulent flame speed. The flames with lower inlet temperatures have a longer length and thicker flame brush. This will make them less vulnerable to high frequency disturbances, since the wavelength of the disturbance would be small relative to the flame length. From this data we can conclude that in a multi-nozzle setting the inlet temperature will have a small impact on the frequency of the extrema, but a more substantial impact on the gain at high frequencies.

The flame transfer function phase for cases with varied inlet temperature is shown in **Figure 3-8B**. These plots are qualitatively similar to the phase from previous cases. For low forcing frequencies, the phase is initially linear. Near the gain minimum there is, again, a discontinuity in the phase. For any particular frequency, a consistent decrease in the value of the phase is observed as the inlet temperature is increased. This is a result of the upstream movement of the bulk of the heat release as the turbulent flame speed is increased, as seen in **Figure 3-2**. With a greater portion of the heat release occurring near the base of the flame, the convective time for a disturbance is decreased. The phase value tracks this convective time delay.
Finally, the flame transfer function for cases with varied equivalence ratio is analyzed. Figure 3-9 shows the gain and phase for an equivalence ratio range of 0.50 up to 0.65 for $U_{\text{inlet}} = 20$ m/s and $T_{\text{inlet}} = 250$ °C. Heat release rate images for these conditions are shown in Figure 3-3. Each case demonstrates the alternating minima and maxima observed in other flame transfer functions. The frequency of both the local minima and the local maxima are observed to increase with increasing equivalence ratio. The gain at high frequencies (>250 Hz) is also observed to increase. From this perspective, the flame transfer function gain

**Figure 3-8:** The flame transfer function (A) gain and (B) phase for varied inlet temperature. $U_{\text{inlet}} = 20$ m/s, $T_{\text{inlet}} = 250$ °C, $\Phi = 0.65$. **The Flame Transfer Function as a Function of Equivalence Ratio**

Finally, the flame transfer function for cases with varied equivalence ratio is analyzed. Figure 3-9
responds to changes in the equivalence ratio in much the same way as to changes in the inlet temperature. Phase information for this set of operating conditions exhibits the same behavior observed previously. This includes an initially linear dependence on forcing frequency followed by a discontinuity near the gain minimum and then a return to linear behavior.

![Figure 3-9: The flame transfer function (A) gain and (B) phase for varied equivalence ratio. $U_{inlet} = 20$ m/s, $T_{inlet} = 250$ °C, $\Phi = 0.50$-0.65.](image)

3.3.5 **Empirical Model of the Extrema Frequencies**

In the same manner as the flame length, nonlinear regression can be used to fit the frequencies of the extrema to a function of the form in Eq. (3.1). Such an approach determines the statistical sensitivity of
the flame transfer function to the operating conditions examined here. Ideally, a single equation would be used to predict the gain at any frequency or operating condition, but an accurate set of exponents could not be determined because the nonlinear fit algorithm would not converge. Equation (3.3) shows the empirical fit of the minimum response frequency as a function of the operating conditions.

**Figure 3-10** shows a comparison between the experimentally determined frequencies and the prediction by the model. The percent difference between model and data are shown in red. For all cases, the difference is less than 10%. The equation predicts that the frequency of the minimum response will increase with an increase in $U_{inlet}$, $T_{inlet}$ or $\Phi$. This finding is in agreement with the data examined in the previous three sections. The sensitivity of the minima to changes in $U_{inlet}$ or $\Phi$ is very similar, with respective exponents of 0.85 and 0.89. With the smallest exponent of 0.465, the minimum response frequency is found to be the least sensitive to changes in inlet temperature.

$$F_{G_{min}} = 1.298 \cdot U_{inlet}^{0.850} \cdot T_{inlet}^{0.465} \cdot \Phi^{0.890} \text{ [Hz]} \tag{3.3}$$

The sensitivity of the maxima to the operating conditions is shown in Eq. (3.4). A comparison between the model and the experimental data is shown in **Figure 3-11**. The disagreement between the model and the experiment is worse than the previous model, but it is still less than 10% in most cases. The frequency of the maxima is found to be the most sensitive to the inlet velocity. $T_{inlet}$ provides the weakest sensitivity, while the sensitivity to equivalence ratio falls between the two. For $T_{inlet}$ and $\Phi$, the exponent values in the maxima model vary by 75% and 30% compared to the values in the minima model. This large variation indicates that different mechanisms may be responsible for causing the minimum response versus the maximum response.

$$F_{G_{max}} = 8.660 \cdot U_{inlet}^{0.834} \cdot T_{inlet}^{0.211} \cdot \Phi^{0.658} \text{ [Hz]} \tag{3.4}$$
Figure 3-10: The frequency of the first local minimum of gain for all operating conditions and the corresponding value from Eq. (3.3). The percent difference is shown in red. $T_{\text{inlet}}$, $\Phi$ and $U_{\text{inlet}}$ for each operating condition number are listed in the Appendix.

Figure 3-11: The frequency of the first local maximum of gain for all operating conditions and the corresponding value from Eq. (3.4). The percent difference is shown in red. The actual operating conditions are listed in the Appendix.
3.4 Normalization of the Flame Transfer Function

3.4.1 Normalization Techniques

In this section the goal is to identify a parameter which can be used to normalize all flame transfer functions acquired in this multi-nozzle study. Literature suggests that the Strouhal number can be used for this purpose, but the definition of this quantity varies from study to study [11, 15, 16]. In all cases, the Strouhal number is the ratio of a characteristic length from the flow field to the wavelength of a convective disturbance. Critical values of the Strouhal number are tied to the formation of vortical structures that can disturb the heat release rate, as discussed in Section 1.2.2.2.

Three possible normalization techniques will be discussed in this study. The first will be referred to as the classic Strouhal number, which is defined in Eq. (3.5). The preferred vortex shedding frequency, \( F \), is ultimately a function of the thickness of the shear layer, which depends on the length of the jet issuing from the nozzle [15]. Since flow field measurements are rarely available to quantify how far the jet penetrates into a combustion chamber, the nozzle diameter is typically used as an approximation [15]. For this study, a hydraulic diameter must be calculated for the nozzle, shown in Eq. (3.6). The definition of the classic Strouhal number, Eq. (3.5), states that when the ratio of the characteristic length of the shear layer to the disturbance wavelength equals a critical value between 0.25 and 0.50, the strongest shedding of vortices occurs [15]. Strong vortices can interact with the flame front and induce a significant fluctuation to the flame area and rate of heat release. If the vortical disturbance is the primary means through which the velocity disturbance creates a disturbance in the heat release rate, it is expected that such a fluctuation would correspond to a high gain value.

\[
S_{\text{classic}} = \frac{\text{Shear Layer Length scale}}{\text{Disturbance Wavelength}} = \frac{D_{\text{nozzle}}}{U/F} \approx 0.25 \text{ to } 0.50 \tag{3.5}
\]

\[
D_{\text{hydraulic}} = 4 \cdot \frac{\text{Open Area}}{\text{Wetted Perimeter}} = \frac{4\pi \cdot (R_{\text{outer}}^2 - R_{\text{inner}}^2)}{\pi \cdot (R_{\text{outer}} + R_{\text{inner}})} = 0.236 \tag{3.6}
\]

The classic Strouhal number has been used successfully in several studies to collapse flame transfer function data at a range of operating conditions [16, 28, 35].
One shortcoming of the $St_{\text{classic}}$ term is that it does not account for the effect of the flame on the flow field. Changing the equivalence ratio or inlet temperature of the combustor changes the distribution of the heat release rate and length scale of the flame, as the analysis of Section 3.2.1 demonstrated. This, in turn, will affect the length scale of the flow field. To take into account the effects of operating conditions on the flow field, in place of $D_{\text{nozzle}}$, the term $L_{\text{flame}}$ can be used in the Strouhal number [11].

$$St_{L_{\text{flame}}} = \frac{L_{\text{flame}} \cdot F}{U}$$  \hspace{1cm} (3.7)

No studies have been found that explicitly relate the flame length to the thickness of a reacting shear layer. However, from a collection of studies it can be inferred that the shear layer thickness and flame length respond to changes in operating condition in a similar manner. First, with regards to increasing inlet velocity, in Section 3.2.1 the flame length in this multi nozzle configuration was shown to increase. The rate of increase decays with increasing velocity. Similarly, from classical studies in fluid dynamics, the shear layer thickness is known to grow at a decaying rate with increasing velocity gradient across the shear layer $[87-89]$, at a rate of $\sim U^{5}$.

For increasing equivalence ratio, the flame length will decrease due to an increase in turbulent flame speed. Multiple studies have shown that as the heat release rate in the shear layer increases, the shear layer growth rate will decrease due to a reduction in turbulent shear stress $[42, 90]$. Hence, for a fixed downstream location in the combustor, a flame with a higher equivalence ratio will have a smaller shear layer thickness than one with a lower equivalence ratio.

As the temperature of the reactants at the combustor inlet increases, the flame speed will also increase, resulting in a shorter flame length. With a larger heat release rate occurring farther upstream than in a lower $T_{\text{inlet}}$ case, the initial growth rate of the shear layer will be abated, as in Hermanson and Dimotakis$[90]$. As a result, the shear layer thickness for a fixed downstream distance will be smaller for flames with higher $T_{\text{inlet}}$.

The direct relationship between $L_{\text{flame}}$ and the shear layer thickness is not likely to be linear. However, given the significant effect that the operating conditions have on the shear layer, $L_{\text{flame}}$ should provide a better quantification of the shear layer thickness than $D_{\text{nozzle}}$. 
There is an ambiguity that arises when using the $L_{\text{flame}}$ definition of Strouhal number. The primary purpose of the Strouhal number in the current study, as well as those included in the literature review was to identify a critical value for the formation of the strongest vortical structures. Because the definition of $St_{L_{\text{flame}}}$ is the length of the flame divided by the wavelength of a disturbance, the $L_{\text{flame}}$ Strouhal number also indicates how many convective disturbance waves will fit into the length of the flame. A Strouhal number of 1.0 means that one full sinusoidal disturbance fits into the flame, while a $St_{L_{\text{flame}}}$ of 0.5 indicates that half a disturbance wave fits into the flame. From the perspective of a PMT capturing global heat release rate, it might be expected that whenever a full disturbance wave is present in the flame, the above-mean and below-mean halves of the wave cancel each other out and result in a small global response. If only half of a wave or 1.5 waves are present in the flame, a high global response might be expected, as the half wave is not canceled out. This behavior would lead to the low pass filter behavior commonly observed in single-nozzle studies. Such an explanation, however, cannot explain why some operating conditions exhibit high global response at higher Strouhal numbers, as is the case in the current study. Additionally, the analysis of Chapter 4 will demonstrate that the flame’s response is too complicated to be the result of a single sinusoidal perturbation. Accordingly, the purpose of $St_{L_{\text{flame}}}$ in the current study remains as the identification of a critical point at which the strongest shedding of vortical structures occurs.

After $St_{\text{Classic}}$ and $St_{L_{\text{flame}}}$, the third normalization term that will be considered takes into account the effect of swirl fluctuations. When an axial velocity perturbation interacts with a swirler, an azimuthal velocity fluctuation is generated. The axial perturbation propagates at the speed of sound while the azimuthal fluctuation propagates at the flow velocity [16]. The phase difference of the two fluctuations at the base of the flame determines the instantaneous swirl strength, as discussed in Section 1.2.2.6. Equation (3.8) defines this Axial/azimuthal phase difference.

$$\Delta \theta_{V',U'} = 2\pi \cdot F \cdot \left( \frac{L_{SW-CB}}{U} - \frac{L_{SW-CB}}{c} \right)$$  \hspace{1cm} (3.8)

In the equation, F is the forcing frequency, $L_{SW-CB}$ is the length from the swirler to the end of the centerbody, U is the inlet velocity and c is the speed of sound. When $\Delta \theta_{V',U'}$ is equal to 0 or 2pi, the azimuthal and axial velocity fluctuations are in-phase and a small swirl fluctuation would be expected. When $\Delta \theta_{V',U'}$ is equal to pi, the velocity fluctuations are exactly out-of-phase which results in a large swirl
fluctuation. If the swirl has a controlling influence on the response of the flame, the gain data should collapse around these points.

3.4.2 Comparison between Normalization Techniques

In order to select the most effective normalization technique for the multi-nozzle configuration, the standard deviation of the gain as a function of the normalization parameter is analyzed. As a basis for comparison, the gain data are first plotted versus frequency in Figure 3-12. Fifty-seven flame transfer functions for a wide range of operating conditions are displayed, for a total of 807 unique combinations of $F_{\text{forcing}}$, $U_{\text{inlet}}$, $T_{\text{inlet}}$ and $\Phi$. The standard deviation of all gain values at a particular frequency is calculated to provide the red line. The blue line represents the typical standard deviation for a single flame transfer function, as discussed in Section 3.3.1, with a value of 0.06. This value comes from the average of the standard deviation for 96 samples individually calculated at all 807 cases.

Across the range of forcing frequencies, there are no discernible trends in the global flame response. This indicates that, as expected, the global flame response is a function of more than just the forcing frequency. Accordingly, the standard deviation (red line) calculated for all gain data at each frequency represents the worst case scenario, i.e., a normalization parameter that fails to actually collapse the data. The single case standard deviation, shown by the blue line, serves as the best case scenario, i.e. perfectly collapsed data.
With boundaries established to indicate good collapse versus poor collapse, the aforementioned normalization parameters can be analyzed. Figure 3-13 displays all 57 flame transfer functions plotted versus (A) the classic Strouhal number, (B) the \( L_{\text{flame}} \) Strouhal number and (C) the axial/azimuthal phase difference. The standard deviation of all gain values as a function of the x-axis parameter is shown by the red line.

In each of the three plots, there is clearly a degree of collapse for the lower frequencies such that some trends are observable. Like the example case in Section 3.3.1, the gain is initially high and decreases with an increase of the normalization parameter. After a minimum response is achieved, the gain rebounds. For higher frequencies and Strouhal numbers, the gain is scattered to such a degree that there is no apparent trend. It should also be noted from Figure 3-13A that the range of Strouhal numbers over which tests were conducted is in the vicinity of the preferred frequency of vortex shedding. As mentioned earlier in the current section, classical studies in fluid dynamics [91] identified a Strouhal number of between 0.25 and 0.50 as being the most likely to experience strong vortical fluctuations in the jet/recirculation zone shear layer.
Figure 3-13: The flame transfer function gain as a function of (A) the Classic Strouhal Number, (B) the L_{flame} Strouhal Number and (C) Δθ_{v'-u'}. The standard deviation of the gain for each x-axis value is shown by the red line.
To determine the most successful normalization parameter, the coefficient of variation (CV) of the gain values for the various parameters can be compared. The coefficient of variation takes the standard deviation of all gain values at a single x-axis value (F, St, St\textsubscript{\text{Classic}} or Δθ\textsubscript{V'-U'}) and normalizes it by the average of the gain values for that x-axis value, as shown in Eq. (3.9).

\[ CV_G(x) = \frac{1}{N} \sum_{i=1}^{N} \left( G_i(x) - G_{\text{mean}}(x) \right)^2 \]

(3.9)

Where x represents the x-axis parameter (F, St, St\textsubscript{\text{Classic}} or Δθ\textsubscript{V'-U'}), N is the number of cases (57 for this study), G\textsubscript{i} refers to the ith gain value (from 1 to 57) and G\text{mean} is the mean value of the gain. Figure 3-14 shows the CV versus each of the three parameters. The range of values along each of the x-axes corresponds to the minimum and maximum values of the normalization parameters over all 807 points. The all-cases CV and single-case CV, calculated from the standard deviations in Figure 3-12, are shown by the red and blue lines to provide a frame of reference. Over the low frequency range (F\text{forcing} < 250 Hz), and corresponding normalized ranges, the L\textsubscript{flame} Strouhal number (black curve) provides the smallest CV for the majority of the range. A small CV value implies that the flame transfer functions exhibit a degree of self-similarity when plotted versus L\textsubscript{flame} Strouhal Number, i.e. the curves collapse. Self-similarity between the transfer functions indicates that the strength of the vortex shedding mechanism associated with the L\textsubscript{flame} Strouhal number may have a strong influence on the global response of the flame for frequencies less than ~250 Hz. For higher frequencies, the CV of the gain for the normalization parameters is actually worse than the frequency based CV. This lack of collapse versus St\textsubscript{\text{Lflame}} demonstrates that the ability of the vortices to disturb the global heat release rate may be reduced at high frequencies. This could be due to the decreasing size of the vortices relative to the flame brush thickness. Alternatively, other mechanisms that did not influence the flame at low frequency, such as swirl fluctuations or strain rate fluctuations, may have strengthened. Unfortunately, global heat release rate measurements do not allow for visual confirmation of vortex/flame interaction or any other flame response mechanisms. To this end, high speed heat release rate imaging will be employed in the next chapter.
Another way to select the parameter that provides the best collapse is to analyze only the extrema cases. The frequency of the minima and the maxima, $F_{G_{\text{min}}}$ and $F_{G_{\text{max}}}$ can be extracted from each of the 57 flame transfer functions and compared to the corresponding $St_{\text{flame}}$, $St_{\text{Classic}}$ and $\Delta \theta V' \cdot U'$ values. The normalization parameter with the least scatter in its value at the extrema would be the best-fit parameter. The minima conditions are shown in

**Figure 3-14.** The coefficient of variation of gain as a function of different normalization parameters, including $St_{\text{flame}}$, $St_{\text{Classic}}$ and $\Delta \theta V' \cdot U'$.

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**Figure 3-14.** The coefficient of variation of gain as a function of different normalization parameters, including $St_{\text{flame}}$, $St_{\text{Classic}}$ and $\Delta \theta V' \cdot U'$.

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**Figure 3-15.** The y-axis range is selected based on the minimum and maximum values of the normalization parameters over all 807 cases. The coefficient of variation for the 57 points is shown in the
top left corner of each plot and provides a means to quantify the degree of scatter. If the flame transfer functions were perfectly self-similar for one of the parameters, the set of points would be expected to provide a flat line with a CV of zero. This is not the case for any of the three parameters, but the St\textsubscript{L\textsubscript{flame}} does provide a slightly improved collapse compared to St\textsubscript{Classic} or Δθ\textsubscript{V'-U'}. The maxima conditions are shown in Figure 3-16. Again, none of the parameters provide a perfect fit, but based on the CV values, the L\textsubscript{flame} Strouhal number provides the best fit. These figures agree with the findings from Figure 3-14 that suggest that the gain has the strongest dependence on St\textsubscript{L\textsubscript{flame}}.

**Figure 3-15:** The Frequency, St\textsubscript{L\textsubscript{flame}}, St\textsubscript{Classic} and Δθ\textsubscript{V'-U'} at the minimum response condition for all 57 flame transfer functions. T\textsubscript{inlet}, Φ and U\textsubscript{inlet} for each operating condition number are listed in the Appendix.
If the L\textsubscript{flame} Strouhal number is indeed a controlling parameter in the global flame response, evidence of collapse should also be present in the flame transfer function phase. **Figure 3-17A** shows the phase data for all 57 operating conditions versus phase. For comparison, **Figure 3-17B** shows the same phase data plotted versus the L\textsubscript{flame} Strouhal Number. There is a clear degree of collapse when the normalization is performed. Over the low frequency range the phase initially decreases linearly. Near the Strouhal number of minimum gain (~0.75), there is a discontinuity in the phase. The phase then returns to linear behavior up to a Strouhal number of ~1.3, beyond which the collapse begins to degrade. These phase plots provide further evidence that the L\textsubscript{flame} Strouhal number is a controlling parameter for the lower frequency range of the global flame response.

**Figure 3-16**: The Frequency, St\textsubscript{flame}, St\textsubscript{Classic} and Δθ\textsubscript{V'U'} at the maximum response condition for all 57 flame transfer functions. T\textsubscript{inlet}, Φ and U\textsubscript{inlet} for each operating condition number are listed in the Appendix.
While the $L_{\text{flame}}$ Strouhal number may provide the best collapse, the collapse is not perfect, especially at higher frequencies. This can be explained by the fact that the Strouhal number only takes into account the strength of the vortex formation mechanism. From the literature review Section 1.2.2, it is clear that there are multiple flame response mechanisms, such as swirl fluctuations, flame interaction and flame strain. The global response of the flame is controlled by the amplitude of these mechanisms as well as their phase relative to one another. Due to this, it may be unreasonable to expect that a single normalization term could perfectly collapse the entire flame response. The manner in which these various mechanisms interact will be discussed in the next chapter.

**Figure 3-17:** The flame transfer function phase versus (A) forcing frequency and (B) $L_{\text{flame}}$ Strouhal number.
3.5 Conclusions

The effects of operating condition on a multi-nozzle flame were analyzed for a wide range of adopted parameters. The characteristic length of the flame was found to increase with increasing inlet velocity, or decreasing inlet temperature or equivalence ratio. Flames with higher equivalence ratio or inlet temperature were found to have M-type flame attachment. An empirical relationship between the operating conditions and the flame length was calculated using an equation of exponential form. The flame was found to have the strongest dependence on inlet temperature, followed by equivalence ratio, then inlet velocity.

The global response of the multi-nozzle flame to inlet velocity perturbations was analyzed using the flame transfer function. Velocity disturbances were limited to 5% RMS amplitude in order to remain in the linear response regime. The multi-nozzle flame transfer function was found to exhibit alternating minima and maxima in a manner similar to single-nozzle flame transfer functions in the literature. Some operating conditions displayed the low-pass filter behavior common in literature. At other conditions, the high frequency gain was as large as the gain at low forcing frequency.

An empirical fit of exponential form was applied to all flame transfer function data. It was not possible to use a single equation to predict the gain at any frequency or operating condition. It was possible, however, to predict the frequency of the minima and maxima as a function of operating condition using two separate equations. The minimum response frequency exhibited the strongest dependence on equivalence ratio followed by inlet velocity then inlet temperature. The maximum response frequency had the strongest dependence on inlet velocity, followed by equivalence ratio, then inlet temperature. Because the dependencies of the minima are not the same as those of the maxima, the mechanisms which cause one versus the other may be different.

Three different normalization techniques were applied to the forcing frequency in attempt to collapse all flame transfer function data into a single curve. Each technique focused on a single flame disturbance mechanism as the primary source of heat release rate fluctuations. These mechanisms included vortical structures and swirl fluctuations. Shear layer vortical structures were characterized by the Classical Strouhal number as well as a Strouhal number based on the characteristic flame length. Swirl fluctuations were characterized by the theoretical phase difference between axial and azimuthal velocity fluctuations. The Strouhal number based on characteristic flame length was found to provide the best collapse over the
widest range of frequencies. The collapse indicates that the Strouhal number, or the ratio of a characteristic length scale from the flow field to the wavelength of the disturbance, is a controlling parameter in the response of the flame. The fact that the Strouhal number does not fully collapse the data, particularly at high frequency, indicates that the response of the flame is dependent upon more than a single mechanism. In the following chapter, local heat release rate measurements will be employed to look for evidence of each of the mechanisms so that a more comprehensive understanding of the flame response can be attained.
Chapter 4

Local Heat Release Rate Fluctuations

4.1 Introduction

In Chapter 3 the global response of the flame was analyzed over a wide range of operating conditions to identify trends in the response related to changes in those conditions. In this chapter, spatially resolved high speed images showing the heat release rate are analyzed for evidence of the flame response mechanisms discussed in Chapter 3. High speed image sets will be analyzed and discussed for a subset of the operating conditions in the previous chapter, mostly at extreme response frequencies. Such conditions have been selected based on unique features exhibited in the global flame response.

4.2 Extrema Frequency Analysis

4.2.1 High Gain Frequencies

The first set of forced response flame images presented is from a low frequency, high gain condition, shown in Figure 4-1. The flame has an inlet velocity of 22.5 m/s, an inlet temperature of 150 °C and an equivalence ratio of 0.55. The operating condition matches the time-averaged image in Figure 3-1D. The corresponding flame transfer function is shown in Figure 3-7. With a forcing frequency of 100 Hz, the gain for this set is relatively high at a value of 1.0. To reiterate, a gain of 1.0 implies that the global heat release rate fluctuations are of the same amplitude as the inlet velocity perturbations. It may be that a case such as this is actually a local maximum in the response. Because the gain below 100 Hz cannot be resolved, however, such cases are referred to as low frequency, high gain conditions.

This set of images was produced with the frequency filtering technique described in Section 2.3.6. The mean heat release rate information has been removed so that only the fluctuating component at each phase angle is shown, from Eq. (2.9). Warm colors (red, yellow) in the image represent an above-mean heat release rate, while cool colors (blue, cyan) represent a below-mean heat release rate. The white contour represents the approximate boundary of the time-averaged flame, as discussed in Section 2.3.6. The legend of the image shows the amplitude of the fluctuations in comparison to the single most intense point in the time-averaged flame. The white text on each image is the phase with respect to the velocity signal.
For this case, the majority of points in the flame at each phase angle have a heat release rate that is either below-mean (such as 280° or 320°) or above-mean (such as 120° or 160°). Note the amplitude of the fluctuations, with a maximum value of around 15%. This type of response is to be expected from a disturbance which has a wavelength that is considerably longer than the length scale of the flame. At the current operating condition the characteristic flame length is 0.094 m, while the expected convective disturbance wavelength is 0.225 m. As a result of the relatively long wavelength of perturbation, most regions of the flame respond in-phase, so that, for a given phase angle, either all regions are above-mean or all regions are below-mean. When summed across the entire flame, the result is a very large global response, and corresponding high gain.

![Figure 4-1: Fluctuation image set for $U_{\text{inlet}} = 22.5$ m/s, $T_{\text{inlet}} = 150$ °C, $\Phi = 0.55$, $F_{\text{forcing}} = 100$ Hz, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$, Gain = 1.0.]

A set of images from another operating condition is displayed in Figure 4-2. Like the previous set, these images have been acquired at a low frequency, high gain case, with a gain value of 1.2. The flame transfer function corresponding to this case is shown in Figure 4-3. Over the forcing period, the images are again, mostly above-mean or mostly below-mean. The maximum fluctuation intensity is also high, at approximately 14%. These observations are consistent with those made of Figure 4-1, and explain the high flame transfer function gain.

An additional noteworthy point for both of these high gain sets is the location of the maximum fluctuation. Throughout the forcing period, the location of the maximum fluctuation remains fixed near the
downstream end of the flame. As will be seen throughout this chapter, this behavior is common to all high gain conditions.

![Figure 4-2: Fluctuation image set for a high gain frequency. With \( U_{\text{inlet}} = 25.0 \text{ m/s}, T_{\text{inlet}} = 200 \text{ °C}, \Phi = 0.60 \). \( F_{\text{Forcing}} = 120 \text{ Hz}, U'_{\text{RMS}}/U_{\text{MEAN}} = 5\% \), Gain = 1.2. Fluctuation magnitude normalized by the maximum intensity in the time-averaged flame.](image)

![Figure 4-3: Flame transfer function for \( U_{\text{inlet}} = 25 \text{ m/s}, T_{\text{inlet}} = 200 \text{ °C}, \Phi = 0.60 \). The blue highlights indicate the frequencies analyzed in this section at this operating condition. The gain from 100, 140 and 160 Hz is excluded due to unacceptable standard deviation per Section 2.3.2.](image)

Another, more quantitative way, to analyze the same data is to employ axial heat release rate fluctuation profiles. Fluctuation profiles are acquired by summing the fluctuation images from left to right and plotting the profile versus axial distance from the centerbody, as described in Section 2.3.7. As shown
in Figure 4-4, the axial fluctuation data can be presented either as sets of instantaneous profiles (in blue), an amplitude profile (in red), or as a pseudo-color bar (also described in Section 2.3.7). Fluctuations are normalized by the maximum intensity in the time-averaged heat release rate profile at the same conditions. From any form of the data, it is clear that the large fluctuations in the downstream region dominate the flame response, with a magnitude as high as 14%. Fluctuations occurring near the base of the flame are considerably smaller, with a maximum value of around 2%. At this operating condition, the characteristic flame length is 0.087 m, while the expected convective disturbance wavelength is 0.208 m. As in Figure 4-1, the perturbation wavelength is much longer than the characteristic length, so that the disturbance can be expected to be largely above-mean, or below-mean, at any phase angle. The possible causes of this high amplitude fluctuation will be discussed at the end of this chapter, in Section 4.4.

![Figure 4-4: Axial heat release rate fluctuations and amplitude for a high gain frequency. With \( U_{\text{inlet}} = 25.0 \) m/s, \( T_{\text{inlet}} = 200 ^\circ \text{C}, \Phi = 0.60, F_{\text{forcing}} = 120 \) Hz, \( U'_{\text{RMS}}/U_{\text{inlet}} = 5\% \), Gain = 1.2. Fluctuation magnitude normalized by the maximum axial intensity in the time-averaged flame profile.](image)

4.2.2 Local Minimum Response Frequencies

In this section a set of images from a local minimum in the flame transfer function will be analyzed. Figure 4-5 shows an image set for a flame forced at 240 Hz at the same operating condition as that of Figure 4-2. The distribution of the fluctuations in this case is distinctly different from that of the previous high gain sets. Here the largest fluctuations originate near the base of the flame, as shown by the
black cross in the 0° phase image. As the phase angle increases, the largest fluctuation propagates downstream away from the flame base. Near the downstream flame boundary, this high amplitude region finally dissipates. Unlike the high gain cases, the amplitude of the fluctuation near the end of the flame is small compared to fluctuations at the flame base.

![Fluctuation image set for a minimum response frequency.](image)

**Figure 4-5:** Fluctuation image set for a minimum response frequency. With $U_{\text{inlet}} = 25.0$ m/s, $T_{\text{inlet}} = 200$ °C, $\Phi = 0.60$. $F_{\text{Forcing}} = 240$ Hz, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$, Gain = 0.08. Fluctuation magnitude normalized by the maximum intensity in the time-averaged flame.

Also of interest here is the maximum fluctuation amplitude, with a value of around 7%. This is considerably lower than the peak experienced in the high gain cases (14%). This decrease is not, however, enough to explain the order of magnitude difference in gain from

(1.20 compared to 0.08). The minimum in global flame response here is instead caused by cancelation between equal regions of above-mean and below-mean heat release rate. The perturbation wavelength has grown shorter as a result of the increased forcing amplitude. The flow field and disturbance length scales are now of a similar magnitude ($\lambda_{\text{perturbation}} = 0.104$ m compared to $L_{\text{flame}} = 0.088$ m), such that one full perturbation wavelength nearly fits into the characteristic length of the flame.

As the peak of a convective disturbance wave passes through the combustor, an increase in the rate of heat release relative to the mean is expected. Similarly, as the trough of the wave passes through the combustor, a decrease in heat release rate relative to the mean should occur. The specific mechanisms that allow the heat release rate to be disturbed will be discussed in Section 4.4. With one full perturbation wavelength
present in the flame, from a global perspective, the above-mean and below-mean halves of the perturbation wave cancel each other out, resulting in a very small global response.

**Figure 4-6** shows the axial heat release rate profiles for this minimum response case. The scale of the y-axes has been changed to better display the details of the axial profiles. The large downstream fluctuation observed in the high gain case is no longer present, while the fluctuations near the flame base reach a magnitude as high as 4%. The profiles exhibit a roughly sinusoidal form where a full wavelength is present in the flame at any particular phase angle. Peaks and troughs in the profile clearly propagate downstream, away from the flame base, as seen **Figure 4-7**. From a global heat release rate perspective, the positive and negative parts of the perturbation are similar in magnitude and will cancel each other out, resulting in a very small gain.

![Pseudo-color Axial Heat Release Rate RMS Profile](image)

**Figure 4-6**: Axial heat release rate fluctuations for a minimum response frequency. With \(U_{\text{inlet}} = 25.0 \text{ m/s}, T_{\text{inlet}} = 200 ^\circ \text{C}, \Phi = 0.60, F_{\text{Forcing}} = 240 \text{ Hz}, U'_{\text{RMS}}/U_{\text{inlet}} = 5\%, \text{ Gain } = 0.08\). Fluctuation magnitude normalized by the maximum axial intensity in the time-averaged flame profile.
4.2.3 Local Maximum Response Frequencies

A set of images from the local maximum flame response is presented in Figure 4-8. The flame has the same operating conditions as the previous two cases, but now the flame is forced at 320 Hz. According to the flame transfer function in Figure 4-3, 320 Hz is a local maximum in the flame response with a gain of 0.86. The flame response is now more complex than the previous cases. Approximately 1.5 disturbance waves are visible along the flame axis, such that, at each phase angle there is either two above-mean regions and one below-mean region, or two below-mean regions and one above-mean region. Like the high gain case at low frequency, the largest fluctuation occurs near the downstream flame boundary and has a magnitude of about 11%. The fluctuation near the flame base is small by comparison. The axial heat release rate profiles shown in Figure 4-9 confirm this behavior.

The distribution of the local heat release rate fluctuations for the local maximum case at 320 Hz shares several similarities with the high gain case of 120 Hz. This indicates that the mechanism causing the flame to respond may be the same in the two cases, varying only by the wavelength of the disturbance. The fluctuations visible in the minimum response case at 240 Hz are very different from those observed at either 120 or 320 Hz. This suggests that the flame response mechanisms there are different, or that their relative strengths have changed.

Figure 4-7: The axial location of peaks and troughs in the heat release rate fluctuation profile from Figure 4-6.
Figure 4-8: Fluctuation image set for a minimum response frequency. With $U_{\text{inlet}} = 25.0$ m/s, $T_{\text{inlet}} = 200^\circ C$, $\Phi = 0.60$. $F_{\text{Forcing}} = 320$ Hz, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$, Gain = 0.86. Fluctuation magnitude normalized by the maximum pixel intensity in the time-averaged flame.

Figure 4-9: Axial heat release rate fluctuations for a minimum response frequency. With $U_{\text{inlet}} = 25.0$ m/s, $T_{\text{inlet}} = 200^\circ C$, $\Phi = 0.60$. $F_{\text{Forcing}} = 320$ Hz, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$, Gain = 0.86. Fluctuation magnitude normalized by the maximum axial intensity in the time-averaged flame profile.
4.3 Comparisons at Different Forcing Frequencies

4.3.1 Local Heat Release Fluctuation Comparison

Image sets like those presented in the previous sections are useful for identifying features of the flame response at a particular forcing frequency. For comparisons between frequencies, however, amplitude and phase images are better suited, as they can be analyzed side-by-side. Such images are acquired through the technique described in Section 2.3.6. Figure 4-10 shows a set of these types of images for \( U_{\text{inlet}} = 22.5 \text{ m/s}, T_{\text{inlet}} = 150 \degree \text{C}, \Phi = 0.55 \). These conditions match the 22.5 m/s flame transfer function in Figure 3-7. Also note that the 100 Hz case corresponds to the image sequence in Figure 4-1. The dashed white line is included in the images as a reference for the characteristic length of the flame from the time-averaged image. It is used in this chapter to differentiate between the upstream and downstream regions of the flame.

From the analysis in Section 3.3.2, it is known that the 100 Hz case has a high gain, with a value of 1.0. In the image for this frequency, there is a large downstream fluctuation dominating the response of the flame, as was observed in Figure 4-2 and Figure 4-8. The phase image shows that these high amplitude fluctuations in the downstream region are occurring nearly in-phase with each other. Combined, these factors will generate a high global flame response, which is reflected in the high gain at this frequency.

At 180 Hz, the current operating condition experiences a local minimum in the global flame response with a gain of 0.29. In the amplitude figure, the large downstream fluctuation is no longer present, while the upstream fluctuation has grown in amplitude. The phase image shows that the phase of the fluctuation changes continuously along the axis of the flame, with a negative gradient. This indicates that the disturbance propagates downstream through the flame, perturbing the heat release rate as it moves. This is in contrast to the behavior observed at the 100 Hz high gain condition where the disturbance was primarily fixed in the downstream region of the flame.

At 260 Hz, the gain is at a local maximum, with a value of around 0.55. This value is low relative to the gain at local maxima for other flame transfer functions, which average 0.80. In the amplitude image of Figure 4-10 the high amplitude downstream region has returned, albeit, at a lower amplitude than in the 100 Hz case. The fluctuation near the flame base is also still present, unlike the other maximum response cases in Figure 4-2 and Figure 4-8. The phase image shows values which vary less throughout the flame.
than in the 180 Hz case, but more than in the 100 Hz case. This will lead to more spatial cancelation of fluctuations at 260 Hz than at 100 Hz, but less spatial cancelation compared to 180 Hz. From this combination of factors, the global response of the flame at 260 Hz would be expected to exceed the gain at 180 Hz, while falling short of the gain at 100 Hz. In Figure 3-7, where forcing frequencies of 100, 180 and 260 Hz provide gain values of 1.00, 0.29 and 0.55 respectively, this is indeed the case.

Figure 4-10: (A) Amplitude and (B) phase of heat release rate fluctuations for a flame forced at several different frequencies. $U_{\text{inlet}} = 22.5$ m/s, $T_{\text{inlet}} = 150$ °C, $\Phi = 0.55$, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$. The white line indicates the characteristic flame length. The phase of pixels with fluctuation amplitude less than 2% is masked out.
The 400 Hz case exhibits a fluctuation structure which is qualitatively similar to the 180 Hz case. The strongest fluctuations occur in the upstream flame region while weak fluctuations occur in the downstream region. The phase values also decrease regularly along the axis of the flame. Because of the widely varied phase across the flame, there will be spatial cancelation effects between heat release rate fluctuations in different regions of the flame. The cancelation effects combine with the relatively low amplitude fluctuations so that a low global response is expected. From the flame transfer function, the gain matches this expectation with a value of 0.18, which is the global minimum observed for this operating condition.

4.3.2 **Axial Fluctuation Amplitude Comparison**

In order to conduct a quantitative comparison between the flame responses at various frequencies, axial heat release rate fluctuation amplitude profiles can be employed. Such information is obtained through the technique described in Section 2.3.7 and shown separately for several cases in Section 4.2. **Figure 4-12** shows the amplitude and phase of the axial heat release rate fluctuations for three different forcing frequencies which correspond to extrema in the flame transfer function. The flame transfer function for this operating condition is shown in **Figure 3-9**. Black lines in the figure correspond to high gain case while the red line corresponds to a minimum gain case.
In the figure, there is a distinct difference in the distribution of heat release rate fluctuations between the high and low gain cases. The two high gain cases exhibit similar distributions of heat release rate fluctuations to the high gain amplitude images in Figure 4-10. Their profiles are characterized by large downstream fluctuations and relatively small upstream fluctuations. The low gain case exhibits the strongest upstream fluctuations. Compared to the downstream fluctuations in the other cases, however, those upstream fluctuations are still relatively weak.

The phase profiles in Figure 4-11B show that for the low-frequency high-gain case, the phase changes by less than 0.5 radians across the region of high amplitude fluctuation. The small change means that most of the high amplitude fluctuations occur in-phase with each other so that a large global response can be expected. For the local maximum at 320 Hz, the change in phase is more significant, but still less...
than π radians. This explains why the gain is lower here than at 100 Hz (1.08 at 320 Hz compared to 1.22 at 100 Hz). At the 200 Hz minimum response case, changes in phase are the most significant of the three profiles. At this condition not only is the amplitude of the fluctuation small, but the fluctuations occur at many different phase angles. This will lead to cancelation effects between above- and below-mean regions of the flame as discussed in Section 4.2.2.

4.3.3 Axial Fluctuation Comparisons at All Frequencies and Operating Conditions

The previous two sections presented small sets of data for a few different operating conditions and frequencies. As mentioned earlier however, gas turbines need to operate safely at a wide range of conditions. To this end, one of the advantages provided by the high speed chemiluminescence approach is that similar data can be acquired for a wide range of conditions in a relatively small amount of time compared to other techniques. To display this wide range of conditions, pseudo-color bars representing the axial heat release rate fluctuation amplitude are employed. Pseudo-color profiles allow for the flame response over the entire frequency range to be displayed side-by-side. The colors bars are compared directly to their corresponding flame transfer functions to show the relationship between gain and the location of the largest fluctuations.

Figure 4-12 shows the axial fluctuation amplitude at all frequencies for the first operating condition discussed in Section 4.3.1. Fluctuation profiles are shown for 16 different frequencies, where each is aligned vertically. The bottom of the figure is the base of the flame and the top of the figure is the downstream end of the combustor. The x-axis shows the frequency of the fluctuation profile displayed above it. Only every other profile is labeled. The y-axis displays the axial distance from the centerbody for each profile. The color scale at the right shows the amplitude to which a particular color corresponds. The largest fluctuations are shown by white regions in the figure. For this operating condition the largest fluctuations have an RMS amplitude of approximately 9% of the maximum intensity of the time-averaged axial heat release rate profile. A white line is shown on the figure to denote the characteristic flame length. This location is used as a boundary to differentiate between the upstream and downstream regions. Overlaid on top is the flame transfer function gain for this condition. The scale for the transfer function is shown by the orange bar on the right side of the figure. In this case, the maximum gain, occurring at 100 Hz, is 1.00. The minimum gain for this frequency range occurs at 400 Hz, with a value of 0.18.
Figures like these allow an enormous amount of information to be displayed at once for the purpose of comparison. Previous sections looked at only extrema cases, but using this technique, the transition cases between the extrema can be simultaneously analyzed. As discussed in Section 4.3.1, the high gain case at 100 Hz exhibits a strong fluctuation in the downstream region and a relatively weak upstream fluctuation. As the forcing frequency is increased, the amplitude of the downstream fluctuation begins to decay while the upstream fluctuation grows stronger. A local minimum in gain occurs at 180 Hz, coinciding with the strongest disturbance observed in the upstream region. The downstream fluctuation continues to decay up to 200 Hz, before rebounding. For higher frequencies, this process repeats itself, where the downstream fluctuation grows to a maximum at 300 Hz, and then decays. The dependence of the upstream fluctuation on the frequency varies from that of the downstream fluctuation and achieves its smallest amplitude at 280 Hz before rebounding. The amplitude of the upstream fluctuations never reaches the high levels experienced in the downstream region, but the occurrence of larger upstream fluctuations does generally coincide with low gain, regardless of the operating condition.

**Figure 4-12**: The axial heat release rate amplitude for all forcing frequencies of a single flame transfer function. The white line denotes the characteristic flame length, while the orange line denotes the gain at each frequency. With $U_{\text{inlet}} = 22.5$ m/s, $T_{\text{inlet}} = 150$ °C, $\Phi = 0.55$. $F_{\text{Forcing}} = 320$ Hz, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$. Fluctuation amplitude normalized by the maximum axial intensity in the time-averaged flame profile.
Figure 4-13 displays the axial fluctuation amplitude for four additional operating conditions. Again, the white line displays the characteristic flame length while the orange line shows the gain. Like the previous figure, high gain cases are generally found to exhibit high amplitude downstream fluctuations and weak upstream fluctuations. Low gain cases exhibit stronger upstream fluctuations and relatively weak downstream fluctuations. Cases that have mid-range gain (~.5) tend to exhibit a mix of upstream and downstream fluctuations of approximately equal strength, such that neither is dominant.

Based on these axial heat release rate fluctuations, an interesting observation can be made with respect to the phase of the flame transfer function. In almost every transfer function analyzed, the minimum in gain was found to coincide with a discontinuity in the phase (see Figure 3-6 or Figure 4-3 for an example). From Figure 4-13, the minimum in gain also typically coincides with a distinct upstream shift in the location of the strongest heat release rate fluctuation. Therefore, a discontinuity in phase should occur at the same frequency as this upstream shift in the location of maximum fluctuation. In Figure 4-14, the transfer function phase, and the location of the maximum fluctuation are compared directly for two cases. As expected, the frequency of the location shift and the frequency of the phase discontinuity are the same. This phenomenon can be explained by the nature of a convective disturbance. If the largest fluctuation experiences a discrete shift from a downstream location to one close to the flame base, the time required for a convective disturbance to travel from its origin to the point where it will have the greatest impact on the global flame response should also display a discrete shift. While only two cases are shown, consistent behavior was found at all ten operating conditions where high speed video was acquired.
Figure 4-13: The axial heat release rate fluctuation amplitude for all forcing frequencies at several operating conditions. The white line denotes the characteristic flame length, while the orange line denotes the gain at each frequency. With $[U_{inlet}, T_{inlet}, \Phi]$, (A) $[25 \text{ m/s}, 200 ^\circ C, 0.60]$, (B) $[20.0 \text{ m/s}, 250 ^\circ C, 0.65]$, (C) $[15 \text{ m/s}, 200 ^\circ C, 0.60]$, (D) $[17.5 \text{ m/s}, 150 ^\circ C, 0.55]$. $U'?_{RMS}/U_{inlet} = 5\%$. Fluctuation amplitude normalized by the maximum axial intensity in the time-averaged flame profile.

Figure 4-14: The location of the maximum axial heat release rate fluctuation (black) compared to the flame transfer function phase (red). With $[U_{inlet}, T_{inlet}, \Phi]$, (A) $[25 \text{ m/s}, 200 ^\circ C, 0.60]$, (B) $[20.0 \text{ m/s}, 250 ^\circ C, 0.65]$. 
Images like those in Figure 4-13 provide information about where the largest fluctuations occur in a flame, but the phase of the fluctuations is left out. In order to understand the extent to which upstream and downstream fluctuations contribute to the global response on the flame, the phase must be analyzed. Axial phase plots like those in Figure 4-11B can be difficult to interpret, so a different approach is presented here. The upstream and downstream regions of the flame can be treated as separate windows, where the total intensity in each is summed to provide a single intensity value that varies with time. A Fourier transform can be applied to the time series from each window to extract the phase of fluctuations in the window at the forcing frequency. If the phase difference between the two windows is close to zero radians (or -2π, -4π, etc.), the upstream and downstream regions would be fluctuating in-phase, and so would interfere constructively. If the phase difference is close to -π radians (or -3π, -5π, etc.), the two regions are fluctuating out-of-phase, and would interfere destructively. It has been suggested in literature [16] that the former case should provide a maximum in the overall flame response, while the latter case provides a minimum.

Figure 4-15 displays the phase difference between the upstream and downstream regions of the flame for the same operating conditions shown in the previous image. Phase differences for each frequency are shown by the white squares and plotted on the right y-axis. Around y values of -π radians and -3π radians, the plot area is grayed-out. This indicates that destructive interference between the upstream and downstream region is expected for frequencies that exhibit a phase difference in this range. The gain for each frequency is also included using the black squares and plotted on the left y-axis for the sake of comparison.

If the gain was controlled purely by the phase relationship between the upstream and downstream fluctuations, as suggested by several studies [10, 11, 16, 92], a maximum gain would be expected any time the phase nears 0 or -2π radians. Conversely, a minimum gain would be expected any time the phase difference nears -π radians or -3π radians. This is certainly the case for a few of the frequencies, as indicated by the blue arrows. But for almost as many frequencies, the gain extrema do not align with the expected phase differences (red arrows). At a case like 200 Hz in Figure 4-15B, the phase difference of -0.02 indicates that the two regions fluctuate in-phase so that a maximum gain might be expected. From the gain plot however, this case is shown to be a local minimum. The discrepancy can be explained by
examining the axial fluctuation amplitude for 200 Hz in Figure 4-13B. The figure shows that the fluctuation amplitude along the entire flame axis, including both the upstream and the downstream region, is low compared to other frequencies. The fact that the two regions fluctuate in-phase is insignificant given that the fluctuations are small from the beginning. As such, the gain would be expected to be low for this case. For a case like 120 Hz in Figure 4-15A, the phase difference of -2.64 radians (nearing -π) indicates that windows fluctuate out-of-phase and that there should be a minimum gain. Despite the phase difference, the gain plot shows the case to be a local maximum with a gain of 1.20. Again, an explanation can be found in the axial fluctuation amplitude of Figure 4-13A. There, the fluctuations in the upstream window are nearly negligible when compared to the downstream window. So, while the two windows are fluctuating out-of-phase, the upstream fluctuation is too small to have a significant impact on the global flame response. This finding suggests that in order to predict the global response of the flame based on local fluctuations in the heat release rate, it is not only the relative phase of the upstream and downstream regions that is important. The level of response in the two regions must also be taken into account.

**Figure 4-15**: The phase difference between fluctuations in the upstream and downstream regions of the flame. The black squares represent the gain while the white squares represent the upstream-downstream phase difference. With \([U_{\text{inlet}}, T_{\text{inlet}}, \Phi]\), (A) \([25 \text{ m/s}, 200 ^\circ \text{C}, 0.60]\), (B) \([20.0 \text{ m/s}, 250 ^\circ \text{C}, 0.65]\), (C) \([15 \text{ m/s}, 200 ^\circ \text{C}, 0.60]\), (D) \([17.5 \text{ m/s}, 150 ^\circ \text{C}, 0.55]\). \(U'_{\text{RMS}}/U_{\text{inlet}} = 5\%\).
4.4 Mechanisms of the Linear Response Regime

This section will make use of the observations throughout the rest of the chapter to attribute particular aspects of the local flame response to mechanisms identified in the literature. It begins with the first mechanism that is initiated at the siren, that is, the fluctuation in the inlet velocity. Second, the means by which a vortical structure could interact with the flame is discussed. Third, fluctuations in the swirl number induced by the interaction of the axial velocity fluctuation and the swirler area are analyzed. Following this discussion, the overall reaction mechanism which takes into account the three aforementioned disturbance mechanisms is proposed. The section closes with a model created to simulate the effects of the three separate mechanisms on the axial heat release rate.

4.4.1 Acoustic Fluctuations in the Inlet Velocity

The siren introduces a small acoustic pressure fluctuation into the experiment that propagates at the speed of sound through the combustor. The fluctuation in pressure generates a fluctuation in the velocity entering the combustor which also propagates at the speed of sound [4, 16, 93]. Because of the high wave speed of the disturbance, the wavelengths associated with the tested frequency range are very long. Typically they are at least an order of magnitude greater than the length scale of the flame. As a result, the flame responds to these velocity fluctuations in an acoustically-compact manner. Heat release rate fluctuations caused by the acoustic inlet velocity fluctuation in an acoustically compact flame occur at essentially the same phase angle throughout the flame.

The expected impact of an acoustic velocity fluctuation on a flame can be modeled using only stable flame images and an assumption of quasi-steady behavior [35]. The quasi-steady assumption states that a flame responds to an acoustic velocity fluctuation by altering its heat release structure to match the stable flame structure of a flame at the inlet velocity at that instant. Take, for example, a flame with an inlet velocity of 25 m/s, similar to the the one discussed in Section 2.3.4. If a 5% fluctuation in the mean velocity was forced into the combustor, at the peak of the disturbance wave, the flame was expected to have the same structure as a stable flame with inlet velocity of 26.25 m/s (25 m/s + 5% of 25 m/s). At the trough of the disturbance wave, the flame structure would match that of a 23.75 m/s flame.
To create the model, two unforced chemiluminescence images were acquired at inlet velocities of 23.75, and 26.25 m/s (flames A and B). A set of nine images was then generated by assigning each pixel in each of the images with the sinusoidal form shown in Eq. (4.1).

\[
I_{x,y}(n) = \frac{1}{2} \cdot (B_{x,y} - A_{x,y}) \cdot \sin(\theta(n) + \varphi_{x,y})
\]  

(4.1)

The variable \( n \) is equal to the frame number, from 1 to 9 in this case. The amplitude of the wave was set equal to half the difference between the A and B flame at each pixel. The variable \( \theta \) is the phase angle for the \( n \)th frame in the set, where \( \theta(n) = n \cdot 2\pi/9 \). The phase offset for each pixel is represented by \( \varphi_{x,y} \). In the case of an acoustically compact flame, the entire flame fluctuates in-phase, so \( \varphi_{x,y} \) is set equal zero for all \( x \) and \( y \) coordinates. Figure 4-16 shows an example of the application of the model.

The model can now be compared to fluctuation image sets acquired experimentally. The model bears a distinct resemblance to the 120 Hz case in Figure 4-2. The axial heat release rate fluctuations also exhibit a similar form. Both the model and the data exhibit high amplitude downstream fluctuations and relatively weak fluctuations near the flame base. The main difference in the two sets is the relative phase across the flame. Because of the acoustically compact assumption, the model shows a constant phase angle across the flame. In the experimental data, the upstream fluctuations are found to be out-of-phase with the downstream fluctuations. Therefore, in the 120 Hz experimental case, as well as every other condition analyzed, the change in phase across the flame is much greater than would be expected for a flame responding to a purely acoustic disturbance.

An additional requirement for a flame responding to purely acoustic disturbances is that the global flame response amplitude can never exceed the amplitude of the inlet velocity perturbation. Due to the absence of secondary mechanisms, constructive interference is not possible. In this case, the maximum gain value is expected to be unity. Contradicting this expectation, the gain of the 120 Hz case (gain = 1.2) as well as many other cases, as seen in Figure 3-12, exceeds unity. This implies that the observed heat release rate fluctuations cannot be the result of purely acoustic disturbances in the multi-nozzle combustor. Instead, as many studies have concluded [3-5, 10, 92], the acoustic disturbance is always present in the
combustor, but through its interaction with features of the combustor, secondary disturbance mechanisms are created. These mechanisms are discussed in the following sections.

Figure 4-16: Model of an acoustic velocity fluctuation propagating through the flame. $U_{\text{inlet}} = 25 \text{ m/s}$, $T_{\text{inlet}} = 200 \ ^\circ\text{C}$, $\Phi = 0.60$.

The multi-nozzle nature of the current configuration should not have a significant effect on heat release fluctuations generated through the acoustic velocity mechanism. Since the acoustic velocity fluctuation is mainly dependent upon the speed of sound and the forcing frequency, the addition of four or more nozzles will produce no major changes in the disturbance introduced to the combustor. As a result of the long wavelength of such disturbances, the flame always responds in a compact or “point-source” manner, so that the response is independent of the flame shape.

4.4.2 Coherent Vortex Structures and Regions of High Vorticity

From the findings in the previous section, the heat release rate fluctuations observed in the combustor do exhibit similarities to purely acoustic velocity fluctuations. Additional mechanisms must be
present in the flame, however, to fully explain the response of the flame to forced velocity fluctuations. Based on the propagation speed and wavelength observed for some of the disturbances, these additional mechanisms must be of a convective nature, i.e. they move at the speed of the flow through the combustor. One mechanism constrained by the speed of the flow and cited frequently in literature is that of coherent vortex structures [4, 5, 10], as discussed in the literature review, Section 1.2.2.2.

To help identify the most likely location for vortex structure formation, it is helpful to first analyze the time-averaged flow field in the combustor. Figure 4-17 shows an illustration depicting the expected features of the flow field in a two-dimensional cross-section of the multi-nozzle combustor. The depiction is an extrapolation based on experimental data from single-nozzle studies [5, 55, 94]. In the figure, a high speed jet (dark blue) containing the unburned reactants exhausts from the three nozzles (labeled A, B & C) into a large chamber where combustion takes place. Recirculation zones (lighter blue), of relatively low velocity, form on either side of the jet. In keeping with single-nozzle nomenclature, they are referred to as the inner and outer recirculation zones (IRZ and ORZ). The regions between the jet and the recirculation zones are referred to as the inner and outer shear layers.

Recirculation zones are particularly important in turbulent combustion as they supply high temperature combustion products to the base of the flame. The energy supplied by these products to the incoming reactants increases the stability of the flame and prevents it from blowing out [52, 53]. The drawback in this flow configuration is that the shear layers created by the recirculation zones are fluid dynamically unstable structures. Under certain conditions, a small disturbance introduced at the base of the shear layer can grow into a large structure capable of producing significant distortion in the shear layer [45-47]. Distortion of the shear layer can lead to fluctuations in flame area and the rate of heat release. The conditions under which vortex structure formation can occur are associated with a critical value of the Strouhal number of between 0.25 and 0.50, as discussed in Section 1.2.2.2. The frequency associated with the critical Strouhal number is referred to as the preferred shedding frequency. If the flow is forced at the preferred shedding frequency \( F_{\text{preferred}} \), or a harmonic frequency of \( F_{\text{preferred}} \), particularly strong vortical structures can be expected. Based on the analysis of Section 3.4, the range of operating conditions and forcing frequencies clearly places this experiment within the range of the critical Strouhal number and the preferred shedding frequency.
In turbulent combustion, the flame typically propagates upstream until it reaches a location where there is equilibrium between the turbulent flame speed and the local flow field velocity [48]. The shear layers present a sharp gradient in the flow field velocity. On the recirculation zone side, the velocity is very low, while on the jet side, the velocity is very high. Based on this fact, chemiluminescence imaging and first hand observation of visible light, as shown in Figure 4-18, the flame (cyan line in Figure 4-17) is assumed to stabilize in the inner shear layer. This shear layer forms between the nozzle jet and the inner recirculation zone. Farther downstream, the nozzle jet impinges upon either the wall or another jet, generating significant turbulence. This region is referred to as the interaction region. The interaction label applies to both the flow field perspective and the combustion perspective, as adjacent flame sheets appear to have merged in chemiluminescence imaging. Owing to enhanced turbulent flame speed caused by the turbulence, the location of the flame surface will move rapidly around this interaction region.

Figure 4-17: An approximation of the multi-nozzle flow field. The dark blue regions show the high velocity nozzle jets. Lighter blue regions display the inner and outer recirculation zones (IRZ and ORZ). Lines representing the instantaneous flame position are shown in the shear layers (cyan). Elements of the combustor are shown in gray.
As discussed in Section 1.2.2.2 of the literature review, the presence of the flame in a shear layer has a detrimental effect on the formation of vortical structures in the shear layer [40, 41, 90]. This means that it is less likely, but not impossible, that a vortex strong enough to disturb the heat release rate will form here. The outer shear layer is the more likely location for the formation of such structures, owing to the relatively small amount of heat release that occurs there. In this case, the vortex structure is expected to form at the dump plane edge at the end of the nozzle, as shown by the red dot in Figure 4-19. The vortical structure propagates downstream along the outer shear layer, growing in strength as it moves. Eventually, it impinges on the wall or another vortex from an adjacent shear layer. The interaction of the structures generates turbulence and fine scale mixing [57] that eventually convects to the flame surface. The interaction of the vortex structure, turbulence and flame generates a large amount of flame area and a disturbance in the heat release rate.

**Figure 4-18:** Digital camera image of the multi-nozzle combustor showing the flames attachment to the centerbody. Red rings denote the end of the centerbody, the origin of the inner shear layer. The yellow ring highlights the outer wall of one of the nozzles, the origin of the outer shear layer.
In the case of vortical disturbances, the multi-nozzle nature of the current configuration may play a significant role in determining how the flame responds. The additional nozzles greatly complicate the flow field. For the middle flame, which is bound on all sides by other flames, the outer shear layer will be hotter since the adjacent recirculation zones do not contact the cold combustor wall. The elevated temperature of the shear layer reduces the strength of vortices formed there [41, 90]. It is also possible, however, that the close proximity of the nozzles may allow for the formation of vortical structures between two adjacent shear layers, as discussed by Dawson and Worth [26]. Such structures are large compared to single-nozzle vortical structures and may create a very large disturbance in the heat release. To validate which behavior is more prevalent, flow field measurements must be completed.

Many of the observations made of the local heat release rate fluctuations in this chapter are consistent with the structure of fluctuations that are expected to result from flame-vortex interaction. The large amplitude downstream fluctuations observed at most high gain conditions may be the result of outer shear layer vortical structures (or pockets of elevated turbulence resulting from vortex breakdown) finally impinging on the flame surface. As discussed in Section 4.4.1, however, such fluctuations may also be the result of acoustic velocity fluctuations. The upstream fluctuations observed for many low gain conditions may be the result of smaller weaker vortical structures shed in the inner shear layer. As the next section demonstrates, these fluctuations may also be the result of swirl fluctuations. Unfortunately, it is not possible to positively identify the presence of a vortical structure in a low forcing amplitude environment using only line-of-sight chemiluminescence data. For the time being, the current study must rely on

**Figure 4-19:** The shedding of vortical structures in a centerbody dump plane combustor. The red dot indicates the edge of the dump plane, which is the origin of the outer shear layer. The red bursts represent vortical structures.
findings in literature [15, 35, 44] for proof of the existence of vortical structures in the combustion chamber. An important goal for future studies on the multi-nozzle combustor must then be the acquisition of time-resolved flow field measurements under reacting conditions. Until such measurements are available, our understanding of how vortex shedding affects the response of a multi-nozzle flame will remain deficient.

4.4.3 Swirl Strength Fluctuations

Recently a number of studies [49, 50, 92] have identified a mechanism unique to swirl flames, as discussed in Section 1.2.2.6 of the literature review. Fluctuations in the strength of the swirl have been shown to occur as a result of interaction between acoustic velocity fluctuations and the swirler. For a fixed axial location in the combustor near the base of the flame, a swirl fluctuation is expected to create periodic widening and contracting of the shear layer position attached to the centerbody. The flame sheet is typically stabilized in the shear layer, so that fluctuations in the shear layer position produce fluctuations in the flame radius. When the radius of the flame expands, the heat release rate is expected to increase as a result of the increased flame area. When the radius of the flame contracts, the heat release rate is expected to diminish as a result of the decreased flame area. Heat release rate perturbations created in this manner are expected to propagate downstream at the velocity of the flow, referred to as the convective velocity. As shown in Eq. (1.5), derived by Palies et al [16], the strength of the swirl fluctuation varies with forcing frequency, inlet velocity and the speed of sound, which depends on the inlet temperature. Figure 4-20 shows an illustrated example of the expected fluctuations in the swirl angle and corresponding changes in the shear layer/flame sheet location.
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4.4.3.1 The Presence of Swirl Fluctuations

Using chemiluminescence imaging, it is possible to confirm the presence of swirl fluctuations by measuring movement in the location of the flame. Using an approach similar to Bunce et al. [43], a threshold value for the heat release rate gradient is used to mark the edge of the flame. Periodic movement of the flame boundary is then tracked across a fluctuation cycle and used as evidence for the level of flame angle and swirl fluctuations. This approach is shown in Figure 4-21A for the outermost flame only. The position of the flame edge for nine phase angles is shown by nine white lines superimposed onto a time-averaged flame image. The degree to which the lines deviate over the forcing cycle is indicative of the strength of swirl fluctuations. The figure shows clear evidence of swirl fluctuations in the multi-nozzle configuration, the strength of which varies with frequency. For the condition shown, 280 Hz has the

Figure 4-20: The expected swirl strength as a function of frequency and corresponding movement of the inner shear layer/flame sheet. (A) Large fluctuations in swirl strength, (B) small fluctuations in swirl strength. For $U_{\text{inlet}} = 20 \text{ m/s}$, and $T_{\text{inlet}} = 100 \degree \text{C}$.
smallest deviation in the flame sheet location. Movement in the position of other flame sheets in the combustor is also of interest, but due to line-of-sight nature of the images and the high degree of flame interaction, it was not possible to use the threshold identification technique. Future studies on the multi-nozzle combustor which provide planar flame images will correct this shortcoming.

To quantify the deviation of the outer flame sheet’s position, the standard deviation of the flame sheet location can be acquired at each point along the combustor axis. The mean value of this standard deviation along the entire flame axis provides a single number quantifying the degree to which the flame angle fluctuates at each forcing frequency. Figure 4-21B displays this value for the operating condition discussed in Figure 4-21A (red curve). The expected strength of swirl fluctuations based on Eq. (1.5) is included (in black) for comparison. The red curve clearly indicates that the strength of the swirl fluctuations varies with forcing frequency. When compared to the expected swirl fluctuations from the model, the experimental data match the trends well.
Figure 4-22 shows comparisons between the modeled swirl fluctuation (black lines) and the experimentally determined deviation in the flame sheet position (red lines) for several additional operating conditions. The two curves on each plot have been normalized between their own maximum and minimum values. The main purpose of these figures is to show that the trend in the data match the expectation based on Eq. (1.5). For the eight cases that were analyzed using this approach, the trend in the two curves shows good agreement over the range of forcing frequencies. Of the eight operating conditions that were analyzed, four are shown in the figure. From this analysis, it can be concluded that the swirl fluctuation is, in fact, present in the combustor and that it is having the expected effect on the flow field and position of the flame sheet.
4.4.3.2 The Impact of Swirl Fluctuations on the Flame Transfer Function

With the existence of swirl fluctuations in the multi-nozzle rig confirmed, their impact on the global flame response must now be identified. Based on literature [43, 49, 50, 92] there are two primarily effects that can be expected from large swirl fluctuations. The first is the generation of heat release rate perturbations near the base of the flame from the widening and contracting of the flame angle. Strong fluctuations near the base of the flame were observed throughout sections 4.2 and 0, which may be evidence of the effect of swirl fluctuations on heat release. To confirm this, the amplitude of the flame base fluctuations can be compared directly to the predicted swirl fluctuations from Eq. (1.5). This is shown in Figure 4-23 for several different operating conditions. The black lines show the expected strength of the swirl fluctuation from Eq. (1.5), while the red lines show the relative strength of the upstream fluctuations near the base of the flame. The two curves on each plot have, again, been normalized between their own maximum and minimum values to see if they exhibit similar trends.
For this comparison, the agreement between the swirl prediction and the actual heat release rate fluctuations occurring at the flame base is poor. Some operating conditions, such as plot (D), exhibit a similar trend in the two curves, but, in general, the two curves appear to be mostly independent of one another. This indicates that the upstream perturbation is not purely the result of swirl fluctuations, or that Eq. (1.5) does not accurately predict the swirl fluctuation. In the former case, the response of the flame in the upstream region may be complicated by the presence of vortical structures in the inner shear layer[5], an unsteady strain field [18] or local flame extinction [3].

The second means by which a swirl fluctuation can affect the global response of the flame is through the disruption of the formation of vortical structures. When the swirl fluctuation is large, the flame may move closer to the outer shear layer [43], the most likely location for the formation of vortical structures. The presence of the flame in the outer shear layer generates vorticity of the opposite sign to that
produced by the velocity gradient, as discussed in Section 1.2.2.2. The opposite sign vorticity is generated by increased kinematic viscosity, fluid dilatation and baroclinic production [40, 41, 43, 90]. The vorticity generated by the velocity gradient and the flame offset each other so that likelihood of coherent vortex formation is reduced.

As discussed in the previous section, regions of high vorticity or coherent vortex structures are regularly cited as being one of the primary means through which velocity fluctuations generate heat release rate fluctuations. If the swirl fluctuations grow large enough to offset vorticity production, then it might be expected that the global flame response would diminish under such conditions. Fortunately, this can be tested by comparing the flame transfer functions from Chapter 3 to the predicted swirl fluctuations from Eq. (1.5). To do this, the frequency of the minimum gain and maximum gain \( F_{G_{\text{min}}} \) and \( F_{G_{\text{max}}} \) is extracted from all 57 flame transfer functions. They are compared to the predicted frequency of minimum and maximum swirl fluctuation \( F_{S_{\text{min}}} \) and \( F_{S_{\text{max}}} \). If the swirl fluctuation is the only factor controlling the level of vorticity production, then \( F_{G_{\text{min}}} \) is expected to coincide with \( F_{S_{\text{max}}} \). In the absence of a swirl fluctuation at \( F_{S_{\text{min}}} \), the flame is expected to experience large heat release rate perturbations from the large vortical structures that can form there. Accordingly, \( F_{S_{\text{min}}} \) would be expected to coincide with \( F_{G_{\text{max}}} \).

**Figure 4-24** shows this comparison. In both plots, the x-axis corresponds to the operating condition (out of 57 cases) and the y-axis corresponds to the frequency of the most extreme response of either the gain (white squares) or the swirl (red dashes). The (A) plot compares the minima in gain to the maxima in swirl, while the (B) plot compares the maxima in gain to the minima in swirl. Contrary to expectation, for all of the operating conditions in the (A) plot, the gain minimum and the swirl maximum do not occur at the same frequencies. The swirl fluctuation maximum consistently occurs at lower frequency than the gain minimum. In the (B) plot, the two extrema do occasionally occur at the same frequency (30% of the conditions), but in general the maximum global flame response and the swirl minimum occur at different frequencies. Reasons for this disagreement include again that Eq. (1.5) may not accurately predict the swirl fluctuation in a real combustor. It could also mean that the length scale used in the equation, \( L_{SW-CB} \) (swirler to centerbody), is inaccurate. Still another possibility is that in a multi-nozzle combustor, the influence of the swirling flow on the combustion process is reduced due to
interaction between the nozzles. As Kunze et al. [28] found on their annular multi-nozzle system, azimuthal momentum vectors between adjacent flames are diametrically opposed and tend to cancel each other out.

![Figure 4-24](image_url)

**Figure 4-24:** (A) Comparison between experimentally determined minima in the gain and predicted maxima in the swirl fluctuations, (B) Comparison between experimentally determined maxima in the gain and predicted minima in the swirl fluctuations. Shown for all 57 operating conditions.

The importance of the two preceding figures is that in this multi-nozzle configuration, the swirl fluctuations do not appear to exhibit control over the global response of the flame. They are still present, as shown in the analysis of Section 4.4.3.1, but they coexist in the combustor with the other flame response mechanisms discussed in Sections 4.4.1 and 4.4.2. Unfortunately, using only chemiluminescence, it may be impossible to separate the effects of one flame response mechanism from another. For this reason, in future
work on multi-nozzle combustors, it is critical that researchers have access to velocity and vorticity field measurements as well as planar heat release measurements.

4.4.4 The Overall Response Mechanism

The purpose of Section 4.4 was to identify the flame response mechanisms that may be present in the multi-nozzle combustor under velocity forced conditions. Evidence of acoustic velocity fluctuations, vortical structures and swirl fluctuations was identified and discussed. In Figure 4-25 a flow chart for the overall response of the flame is presented, based on the evidence in the preceding section.

Starting from the top of the figure, the process begins with an acoustic velocity fluctuation created at the swirler. This approach assumes that the disturbances move purely along the combustor axis, so that no significant variations occur across the radius of the flame. The velocity fluctuation is controlled by a siren, which fixes the amplitude of the velocity disturbance for all forcing frequencies. The acoustic velocity disturbance generates heat release rate fluctuations directly by disturbing the shape of the flame. The flame responds to acoustic fluctuations in a quasi-steady manner. As discussed in Section 4.4.1, quasi-steady heat release rate fluctuations should occur in-phase at all locations in the flame. This is because the flame is acoustically compact. Because the resulting global heat release rate perturbation amplitude is set purely by the acoustic velocity disturbance amplitude, the multi-nozzle nature of the current experiment will have no effect on this mechanism.

The acoustic velocity fluctuations can generate both convective vorticity fluctuations and convective swirl fluctuations. The amplitude of the vortical perturbation is a function of forcing frequency, the length scale of the flame and the inlet velocity, i.e. the Strouhal number. The vortical perturbation distorts the shear layer and generates fine scale mixing and turbulence, all of which generates flame area. This in-turn creates heat release rate fluctuations. Vortices generated in the outer shear layer are expected to have the largest effect on the downstream region. The weaker vortices that may be generated in the inner shear layer are expected to have their largest effect on the heat release rate at the base of the flame. This is because the flame is anchored in the shear layer. As such, the amplitude of heat release fluctuations caused by vortical disturbances will also be a function of the axial location in the combustor, labeled x in the diagram. The vortex formation mechanism will behave differently in a multi-nozzle environment because
the additional nozzles change the shape of the flame and the length scales associated with the flow. This is accounted for in the model through the dependence of flame area generation on the characteristic length scale of the flow, $L_{\text{flow}}$.

The amplitude of swirl fluctuations created by the acoustic velocity fluctuation can be predicted with Eq. (1.5). Based on this equation, $A_{\text{swirl}}$ is a function of the forcing frequency, the inlet velocity, the distance from the swirler to the centerbody ($L_{\text{SW-CB}}$) and the inlet temperature (through the speed of sound). Swirl fluctuations are expected to mainly affect the heat release rate near the base of the flame, where the flame position is still a strong function of the swirl number. Accordingly, $A_{\text{swirl}}$ must then also be a function of $x$. The other important impact of swirl fluctuations is through the disruption of vortex generation, which is expected to reduce $A_{\text{vort}}$. To take this into account, $L_{\text{SW-CB}}$ and $T_{\text{inlet}}$ dependence must also be included for the $A_{\text{vort}}$ term.

**Figure 4-25**: The overall reaction mechanism in a multi-nozzle can combustor.
It is through these three pathways that a multi-nozzle flame primarily experiences heat release rate fluctuations caused by forced velocity fluctuations. Other factors discussed in the literature, such as unsteady strain fields, nozzle spacing and confinements affects, may also influence the flame response. In the current configuration, however, it was not possible to quantitatively test the effects of these other factors. Therefore, it is essential that future studies in this area develop the means to analyze the three dimensional flow field of the combustor as well as vary the geometry of the combustor (e.g. number of nozzles, nozzle spacing and confinement).

4.5 Wave Model of Flame Response

Through applying the framework laid out in Section 4.4, it is possible to build a simple model of how the flame should respond in a one-dimensional or axial frame-of-reference. In this approach, the heat release rate is assumed to vary only along the axis of the combustor. Heat release rate images have shown that there are variations across the radius of the flame, but incorporating these variations into the model would require detailed knowledge of the velocity vectors inside the combustor. In the absence of these details, the velocity at which fresh reactants are delivered to the flame is assumed to equal the inlet velocity throughout the combustor. The profile of heat release rate is based on the time-averaged profile acquired experimentally, as shown in Figure 2-12, referred to as $\dot{Q}_{\text{air}}$. Each of the three main disturbances is treated as a cosine wave that propagates along the axis of the combustor, disturbing the heat release rate as is moves. The purpose of such a model is to allow operators to predict the level of flame response based on only the operating conditions and the time-averaged, stable axial heat release rate profile.

The approach begins by assigning each of three identified disturbance mechanisms to an equation in the form of a traveling cosine wave, shown in Eq. (4.2).

$$I(x, t) = A \cdot \cos(kx - \omega t + \theta) \quad (4.2)$$

In the equation I is the magnitude of the axial heat release rate, A is the amplitude of the disturbance wave, k is the wavenumber $(2\pi \cdot \frac{F}{U})$, x is the axial distance from the base of the flame, $\omega$ is the angular frequency, t is the time and $\theta$ is the phase relative to a reference signal.
The acoustic wave in this model has already been discussed in Section 4.4.1. The disturbance will affect the flame in bulk due to its high propagation speed. The equation is modified slightly for this section, to fit the form in Eq. (4.2), which is a function of time, as opposed to phase angle. It is shown in Eq. (4.3). In the equation, \( c \) is the speed of sound, which will be very large. As a result, the spatial (first) term inside the cosine function will be negligible compared to the temporal (second) term. The phase term will be zero in this case because the acoustic disturbance is the reference signal. An example of the result of this model for a single phase angle is shown in red in Figure 4-26. The fluctuations are normalized by the maximum intensity of the time-averaged axial heat release rate profile.

\[
I_{\text{bulk}}(x, t) = U'_{\text{RMS}}/U_{\text{Inlet}} \cdot Q_{\text{AHR}}(x) \cdot \cos(2\pi F/c x - 2\pi F \cdot t + \theta)
\]  

(4.3)

Figure 4-26: Individual contributions by acoustic wave (red), vortical wave (blue) and swirl wave (green) to the total axial disturbance (black) for a single phase angle (0 degrees). For \( U_{\text{inlet}} = 25 \text{ m/s}, T_{\text{inlet}} = 200 \text{ °C}, \Phi = 0.60, F_{\text{forcing}} = 320 \text{ Hz}, U'_{\text{RMS}}/U_{\text{inlet}} = 5\%. \) Y axis normalized by maximum axial heat release rate along axis of time-averaged flame.

The vortical disturbance that issues from the unstable shear layer between the nozzle jet and the recirculation zones is represented by Eq. (4.4). \( A_{\text{vort}}(\text{St}) \) is a reference function used to specify the amplitude of vortical fluctuations at the Strouhal number of the operating condition. Its values range from 0 for no vortical fluctuations to 1 for large vortical fluctuations. Based on classical literature [15, 45, 46, 90], large vortical disturbances are expected at \( \text{St} = 0.3 \) or any integer multiple of 0.3. Similarly, minimal
vortical disturbances are expected at \( St = 0.6 \) or any integer multiple of 0.6. The phase term, \( \theta_{vort} \), is calculated from the amount of time it takes an acoustic velocity perturbation to propagate from the base of the flame to the edge of the dump plane where the majority of vorticity is created. An example of the application of this equation is shown by the blue line in Figure 4-26.

\[
l_{vort}(x, t) = \frac{U'_{RMS}}{U_{inlet}} \cdot \sqrt{\bar{Q}_{AHR}(x)} \cdot A_{vort}(St) \cdot \cos \left( 2\pi \frac{F}{U_{inlet}} x - 2\pi \cdot F \cdot t + \theta_{vort} \right) \quad (4.4)
\]

The influence of swirl is introduced to the model through Eq. (4.5). The unique terms in this equation include: \( A_{swirl} \), \( L_{SW-CB} \), and \( \theta_{swirl} \). The term \( A_{swirl} \) is the expected amplitude of swirl fluctuations and comes from Eq. (1.5), which is discussed in Section 1.2.2.6. The length term, \( L_{SW-CB} \) is again the distance from the swirler to the end of the centerbody, an important value in determining the swirl strength. The phase term, \( \theta_{swirl} \) is determined by the amount of time it takes a disturbance originating at the swirler to propagate to the base of the flame. The swirl contribution to the overall flame response is shown in green in Figure 4-26.

\[
l_{swirl}(x, t) = \frac{U'_{RMS}}{U_{inlet}} \cdot \sqrt{\bar{Q}_{AHR}(x)} \cdot A_{swirl}(F, U_C, U_A, L_{SW-CB}) \cdot \cos \left( 2\pi \frac{F}{U_{inlet}} x - 2\pi \cdot F \cdot t + \theta_{swirl} \right) \quad (4.5)
\]

Finally, the overall flame response is calculated by adding together the three individual contributions, as shown in Eq. (4.6). The black line in Figure 4-26 represents the combined response of the three disturbance mechanisms.

\[
l_{total}(x, t) = l_{bulk}(x, t) + l_{vort}(x, t) + l_{swirl}(x, t) \quad (4.6)
\]

Using this model, the axial flame response as a function of phase can be calculated at any forcing frequency and operating condition. The model was applied to the operating condition discussed in Section
4.2 for validation. **Figure 4-27** shows a comparison between the axial profile from the model and an axial profile acquired experimentally for $F_{\text{forcing}} = 320 \, \text{Hz}$ at six different phase angles. At each phase angle, the behavior of the forced axial profile is well represented by the model in both magnitude and shape. This indicates that, for this frequency, the model is successful at predicting the flame response.

**Figure 4-27**: Comparison between the wave model (red) and the experimentally determined axial fluctuation profile (blue). For $U_{\text{inlet}} = 25 \, \text{m/s}$, $T_{\text{inlet}} = 200 \, \text{C}$, $\Phi = 0.60$, $F_{\text{forcing}} = 320 \, \text{Hz}$, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$. 
Another case is shown in Figure 4-28, this time for a forcing frequency of 240 Hz. Here, the model and experimental data are quite different. The amplitude in the two cases varies as well as the location of peaks along the flame axis. This is a situation where the model fails to accurately represent the response of the flame. Of the sixteen cases that were analyzed using this model, five were found to be well represented by the model while the remaining eleven were poorly represented. While this does indicate that there are missing factors in the model, given its simplicity, this is an encouraging result. Possible reasons for the inaccurate predictions are discussed below.

![Wave Model vs Exp Data](image1.png)

**Figure 4-28:** Comparison between the wave model (red) and the experimentally determined axial fluctuation profile (blue). For $U_{\text{inlet}} = 25$ m/s, $T_{\text{inlet}} = 200$ C, $\Phi = 0.60$, $F_{\text{forcing}} = 240$ Hz, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$.

A model such as this can be used to predict a global level of flame response. This is done by summing the axial components into a single value that varies with phase. The RMS amplitude of the fluctuations can be normalized by the inlet disturbance amplitude to acquire a theoretical gain value. This value can then be compared directly to the flame transfer function gain, as shown in Figure 4-29. Again,
the prediction does not match the experimental data well, but they do share qualitative similarities. In the model, the flame response is initially very high, followed by a local minimum at 160 Hz. The flame response then rebounds and remains high up to the highest tested frequency of 400 Hz. These are all features which are observed in the flame transfer functions discussed in Chapter 3.

![Figure 4-29: Comparison between predicted flame response from the wave model (red) and experimental data (black). For $U_{\text{inlet}} = 25 \text{ m/s}$, $T_{\text{inlet}} = 200 \text{ C}$, $\Phi = 0.60$, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$.](image)

Based on comparisons with experimental data, the model is not ready for use in predicting the response of a flame on its own. It does however demonstrate that the axial flame response can be represented by a model consisting of traveling waves. This model could be improved through several methods. First, several physical phenomena discussed in Section 4.4 were not included in the equations. These include the absence of swirl fluctuations in the downstream region, absence of vortical fluctuations in the upstream region and swirls disruption of vorticity generation. These features were excluded because no functional dependence existed in the literature and not enough evidence was uncovered in the current study to generate such a function. A study focused on the individual effects of each mechanism could allow for their incorporation into the model.

Second, the propagation speed of the disturbances in the model is an estimate based on only inlet conditions. Both the acoustic velocity and flow velocity will vary throughout the combustor. The model is
sensitive to these terms, particularly the convective velocity. As such, improvements to the accuracy of the velocity terms could substantially improve the model.

The last area that needs improvement is the relative phase of the three disturbances. The values used in the model are only rough estimates based, again, on inlet conditions. The overall response of the flame is strongly dependent on these values. A study using advanced flow measurement techniques could be used to determine phase relationships between the three mechanisms. This would substantially improve the accuracy of terms input into the model.

### 4.6 Conclusions

Fluctuations in the local rate of heat release for a multi-nozzle combustor were analyzed for several operating conditions. The investigated range of forcing frequencies was selected based on the range at which real combustors experience combustion instability. Qualitative similarities were found in the structure of fluctuations for flames experiencing a similar level of gain, regardless of operating condition. High gain cases were consistently found to exhibit high amplitude fluctuations in the downstream flame region, near the end of the flame. Low gain cases were found to exhibit comparatively weak disturbances in the downstream region. In the upstream region however, low gain cases displayed the strongest disturbances. The amplitude of upstream disturbances was never as large as the largest downstream fluctuations, though they were larger than might be expected based on the gain alone. The full explanation for the minimum in gain was found in phase images. In low gain cases the phase values throughout the flame changed continuously so that individual parts of the flame fluctuated out-of-phase with each other. This resulted in cancelation from a global fluctuation perspective, so that the global heat release rate fluctuation was small. In high gain cases, not only was the amplitude of local fluctuations higher, but the phase of fluctuations was more consistent across the flame. Therefore, most of the flame fluctuated in-phase, resulting in a high global response.

The observed heat release rate disturbances were ascribed to flame disturbance mechanisms identified in the literature. Evidence of acoustic velocity fluctuations, coherent vortical structures and swirl fluctuations was identified and discussed. Acoustic velocity fluctuations were expected to affect the flame in a quasi-steady manner along the entire flame length, increasing or decreasing the global heat release rate
of the flame by the same percentage as the forcing amplitude. Vortical disturbances are thought to originate in the outer shear layer and convect downstream. When the structure impinges on the flame surface, flame area is generated and the heat release rate is disturbed. Evidence of swirl fluctuations was found near the base of the flame that matched well with theoretical predictions. Swirl fluctuations can affect the heat release rate by perturbing the radius of the flame near its base or by disrupting the formation of vortical structures in the shear layers.

An overall reaction mechanism for the current multi-nozzle configuration is proposed in Section 4.4.4. Therein, the disturbances are assumed to propagate through the combustor in a purely axial fashion. The mechanism suggests that overall response of the flame is the result of acoustic velocity fluctuations, vortical disturbances and swirl fluctuations. The strength of the three disturbances varies with operating condition, forcing frequency, swirler to centerbody length, and axial location in the flame. The global response of the flame is determined by the individual strength of the three disturbance mechanisms as well as the relative phase between them.

Finally, a model was designed to predict the global flame response based on only stable flame images and operating conditions. Each disturbance mechanism was treated as a traveling wave that propagated along the combustor axis. When compared, the model resembled the experimental data in only 30% of cases. Given the simplicity of the model however, this was considered enough to warrant further investigation in future work.
Chapter 5

The Nonlinear Response Regime

5.1 Introduction

In the preceding chapters, the velocity forcing amplitude was maintained a level of 5% of the inlet velocity. A combustor suffering from self-excited instability may experience fluctuations of much higher amplitude. As such, it is of interest to investigate the flame response at higher disturbance amplitudes. For a fixed disturbance frequency, the gain of a velocity-forced flame will be nearly constant over a range of low forcing amplitudes. This is to say that the global disturbance amplitude of the heat release rate increases proportionally with the forced velocity disturbance amplitude. At some higher forcing amplitude the gain ceases to be constant, and begins to exhibit a dependence on the forcing amplitude. This is referred to as the transition to the nonlinear regime [21, 36, 73]. Studies on the linear regime are useful for identifying the frequencies at which a real combustor might experience combustion instability. But they provide no information regarding the highest amplitude that the identified instability may achieve. It is the flame response mechanisms of the nonlinear regime that control the maximum amplitude that a combustion instability can reach. This maximum fluctuation amplitude is referred to as the limit cycle.

A system experiencing combustion instability achieves the limit cycle amplitude in a matter of seconds. Subsequently the combustor operates in an effectively quasi-steady environment, where the amplitude of the fluctuations is fixed, as seen in Figure 5-1 from Kim [81]. The combustor remains at the limit cycle until the operating condition is changed, counter measures are enacted to stop the instability, or the system is destroyed. Accordingly, the mechanisms allowing the flame to respond at limit cycle conditions are of great interest to gas turbine operators, modelers and experimentalists.
Figure 5-1: Initiation of combustion instability and achievement of limit cycle oscillations. $U_{\text{inlet}} = 60$ m/s, $T_{\text{inlet}} = 200$ °C and $\Phi = 0.60$.

Figure 5-2 shows the hypothetical growth of combustion instability with curves corresponding to the driving ($H(A)$) and damping ($D(A)$) forces, from Zinn and Lieuwen [3]. At the origin, the system is stable. The introduction of any small disturbance will create a situation in which the driving forces are larger than the damping forces, so that the instability amplitude will grow. If the system is only to grow to some finite amplitude, either the driving or the damping curve must grow nonlinearly. The literature [3, 5, 36, 95, 96] indicates that the most probable source of the nonlinearity is the driving forces and not the damping forces. The limit cycle occurs when the driving forces have reached equilibrium with the damping forces. Knowledge of the limit cycle behavior is critical for determining whether an operating condition will experience fluctuations with sufficient amplitude to be destructive [21].
In a forced response study, such as the one at hand, the combustor never actually reaches the limit cycle, since there is no feedback loop at work. Under forced conditions, limit cycle mechanisms can be studied by forcing the flame at very high amplitude in order to achieve nonlinear flame response. Some studies have found that the flame may respond nonlinearly for velocity forcing amplitudes as low as 10% of the mean velocity [73, 81, 97]. It is the nonlinear flame response mechanisms that determine the final limit cycle amplitude in a combustor. Therefore it is essential that the nonlinear mechanisms are fully understood to allow for the design of accurate prediction tools [97].

5.2 Identifying Nonlinear Response

The amplitude at which a flame begins to respond nonlinearly is a function of the operating conditions as well as forcing frequency [33]. Accordingly, an exhaustive study of all conditions will eventually have to be undertaken to reach a complete understanding of multi-nozzle nonlinear behavior. Until the resources are available for such a study, the following will serve as an exploratory investigation of nonlinear behavior in a multi-nozzle configuration. For this initial investigation, a single operating condition was chosen for the focus of the study. With an inlet velocity of 25 m/s, inlet temperature of 200 °C, and an equivalence ratio of 0.60, this condition has already received considerable attention throughout

Figure 5-2: Hypothetical driving (H(A)) and damping (D(A)) curves showing the low amplitude linear regime, the nonlinear regime and the final limit cycle amplitude. Modified from Zinn [3].
the preceding chapters. Unless otherwise noted, all figures in this section were acquired at this operating condition.

The linearity of the flame response was tested by exposing the flame to a range of velocity forcing amplitudes. The maximum forcing amplitude in the range was limited by what the siren was capable of producing. Additional constraints were imposed by safety concerns, since, at higher forcing amplitudes, the sound level in the room could have exceeded 120 dB. For the safety of the operators and the instruments, such conditions were avoided. As shown by Figure 5-3, at most frequencies, the siren produced a velocity disturbance amplitude of at least 15% of the inlet velocity. Ideally, much higher forcing amplitudes would be included, but this was not possible with the current configuration.

![Figure 5-3: The maximum forcing RMS amplitude acquired at each forcing frequency.](image)

The response of the flame was quantified by the fluctuation amplitude of global CH* chemiluminescence. Figure 5-4 shows an example of such a test. The forcing frequency for this figure was 120 Hz. According to the analysis of the previous chapters, the gain in the linear regime was very high, with a value greater than unity. In this case, the flame was forced with an amplitude as high as 21%. The point farthest to the left was acquired with the siren completely closed, and was effectively unforced. It was, therefore, an indication of the noise in the measurement. At a low frequency such as this, the noise level was large relative to the perturbation amplitude due to the limitations of the two-microphone method,
as discussed in Section 2.3.2. At frequencies well beyond the limits imposed by the two-microphone method, the relative noise level would have been much lower. The error bars on the measured velocity were the result of uncertainty in the two-microphone method and nozzle-to-nozzle differences. The corresponding heat release rate fluctuation error bars were very small in comparison, so they were not shown. From the figure, the 120 Hz response was an example of a frequency at which the flame responded linearly over the entire forcing amplitude range. For the sake of quantifying the degree of linearity, a line was fit to the data using linear regression. A set of points over which a linear regression provided an $R^2$ value (the coefficient of determination) of 0.99 or greater was henceforth considered to be responding linearly.

![Graph](image)

**Figure 5-4:** The forced flame response at a range of inlet amplitudes. $F_{\text{forcing}} = 120$ Hz.

Another useful quantification for the linearity of the flame response is the gain and phase of the flame transfer function. Rather than plotting these values versus frequency, here, it is more appropriate to use forcing amplitude as the independent variable. **Figure 5-5** displays such a plot with the gain on the left $y$-axis and the phase on the right $y$-axis. The unforced measurement mentioned in the previous figure is not included here as the gain and phase value at an unforced condition is not meaningful.
For a frequency to be considered linear over a range, the gain should be constant. The range of gain values in the figure is found to be 23.4% of the mean gain value, with the farthest outlier occurring at the lowest forcing amplitude. This variation is greater than expected, but the low amplitude outlier can be explained by the relatively low signal to noise ratio found there. If the lowest amplitude point is excluded, the range of gain values decreases to 11% of the mean gain. This level of variation in the gain is comparable to the variation seen at a single forcing amplitude case across 32 data samples. Accordingly, the gain here is considered constant enough to label the condition linear over the entire range.

The phase values in the figure are nearly constant for all forcing amplitudes. This indicates that it takes the same amount of time for a flame to respond to an incoming perturbation, regardless of the amplitude. The constant phase value is further evidence of the linearity of the 120 Hz case. A linear response over the entire amplitude range indicates that the same flame response mechanisms are present in the flame at all forcing amplitudes. As the amplitude increases, the strength of the individual mechanisms simply increases proportionally with the velocity fluctuation.

Moving on from this single forcing frequency, Figure 5-6 displays the global flame response as a function of the forcing amplitude for all forcing frequencies in the tested range. The response curves are
divided between three separate plots purely for ease of viewing. Note that the y-axis scales are different for the three plots in order to show the details of each curve. A frequency which exhibits a steeper slope is a case which has a relatively strong response to the incoming perturbation. Cases with steep slopes should, therefore, have high gain. This can be confirmed by a comparison between the slopes of the flame response shown here and the flame transfer function gain, which is shown in Figure 4-3.

At first glance, most of the frequencies in Figure 5-6 appear to respond linearly over the entire forcing range. As the forcing amplitude is increased, the flame responds proportionally so that the slope of the curve is constant. In order to quantify the degree of linearity in each curve, linear regression is used. The R$^2$ value for each frequency is shown in Figure 5-7. Here, it can be seen more clearly that there is a significant deviation from linear behavior starting at 220 Hz. With an R$^2$ value of 0.9897, 220 Hz is on the cusp of being considered nonlinear. At 240 Hz, the R$^2$ value decreases to 0.9490 and is exhibiting strong nonlinear features. At 260 Hz, the R$^2$ value reaches its minimum value at 0.7657 and is responding in a highly nonlinear fashion. At higher frequencies the flame returns to linear behavior.

The lack of observed nonlinear transitions at other frequencies is not proof that they remain linear for all forcing amplitudes. If this were the case, an instability initiated at one of those frequencies would grow to be infinitely strong. The more likely scenario is that the tests did not achieve sufficient forcing amplitude to reach the transition to the nonlinear regime. Literature in this area [3, 97] indicates that nonlinear transition may not occur at some frequencies until a velocity fluctuation amplitude as high as 60% is achieved. For this reason, it is recommended that future studies on the nonlinear response regime of multi-nozzle flames possess the capability to force a high amplitude velocity perturbation into the flow. This perturbation should have an amplitude of at least 50% of the inlet velocity at all frequencies.
Figure 5-6: The heat release rate fluctuation amplitude for all forcing frequencies versus the forcing amplitude.
Figure 5-7: The coefficient of determination from linear regression performed on the forced response of each separate frequency.

Figure 5-9 shows the phase difference between the heat release rate fluctuations and the velocity fluctuations. Some conditions have been shifted down by $2\pi$ for ease of viewing and to reflect the phase of the linear flame transfer function (Figure 4-3). As expected, the frequencies identified as linear in the previous figure exhibit a nearly constant phase value for all forcing amplitudes. For the frequencies that have low $R^2$ values (red markers), the phase is not as constant. The most significant change in phase is observed at 240 Hz. At higher amplitudes, the phase values for 240 Hz level out to a constant value. This indicates that, at higher amplitudes, the 240 Hz case may have returned to linear behavior.

Figure 5-8: The flame transfer function phase as a function of the forcing amplitude for all forcing frequencies.
From the three preceding figures, the frequencies of 240 Hz and 260 Hz can be identified as frequencies of interest for further study. The flame response at 240 Hz is weak for forcing amplitudes up to around 10%. For amplitudes higher than 10%, a nonlinear transition occurs and the flame response increases in strength relative to the velocity perturbation. For the points from 13% up to 32%, the R$^2$ value from linear regression is 0.9981. This is further evidence that, across the higher amplitude region, the flame response actually returns to linear behavior.

In the 260 Hz case, the flame response begins to decrease, or saturate, for forcing amplitudes greater than 14%. If the damping curve is linear for all disturbance amplitudes, like the hypothetical curve in Figure 5-2 suggests, then the limit cycle amplitude could be expected to occur soon after the transition. The mechanisms observed to affect the local heat release rate fluctuations around this amplitude might then be a better indication of how the flame behaves under limit cycle conditions. Figure 5-9 presents the 240 Hz and 260 Hz flame response compared to a hypothetical damping curve like the one in Figure 5-2. The 260 Hz case behaves similarly to the original hypothetical instability driving curve, where a small disturbance creates a driving force (blue) with greater magnitude than the damping force (red). Under such conditions, the instability grows until the driving and damping forces reach equilibrium, i.e. the limit cycle amplitude. In this theoretical situation, that limit cycle is shown to occur at a velocity fluctuation amplitude of around 18%. In the 240 Hz case, a small disturbance results in a situation where the damping is greater than the driving force so that the disturbance is quickly damped out. If a large disturbance is introduced, on the order of 25% of the mean velocity, the driving force will exceed the damping force so that the instability can grow. The final fluctuation amplitude is not reached until the curves intersected again at some amplitude beyond the range of the siren’s capacity. The minimum disturbance amplitude required to initiate the growth of an instability (25% in this example) is known as the triggering amplitude [3].

The other important parameter that must be measured to accurately predict the limit cycle amplitude is the damping force. Unfortunately, with the instruments available on the multi-nozzle combustor, it was not possible to measure the actual damping forces at work. Accordingly, the dependence of acoustic and thermal damping on system geometry and operating conditions is an area that should receive greater attention in the future.
Local Heat Release Rate Fluctuations in the Nonlinear Regime

It is also of interest to analyze how the distribution of local heat release rate fluctuations changes with forcing amplitude. In this section three frequencies are selected for analysis based on the different types of response observed in the previous section. In Figure 5-10 the response of a flame forced at 120 Hz is shown for two different forcing amplitudes. Five different phase angles are shown, labeled in white text, with each contained inside a green rectangle. The left half of each rectangle shows the low amplitude response, while the right half of each rectangle shows the high amplitude response, as indicated by the orange text. Each set of five images for a particular forcing amplitude is shown on its own pseudo-color scale. The purpose of this is to allow for comparisons between the structure of the heat release rate fluctuations, rather than the absolute amplitude of fluctuations. The actual magnitude of fluctuations is included on the legend at the bottom right of the figure with the low amplitude legend on the left and the high amplitude legend on the right.

As mentioned previously, the case being discussed has a very high gain, with a value of 1.20. Like other high gain cases, the fluctuation images exhibit their largest amplitude in the downstream region, regardless of the forcing amplitude. When the forcing amplitude is increased by a factor of five, the

Figure 5-9: The forced response of two frequencies exhibiting nonlinear behavior compared to a hypothetical damping curve.
magnitude of the largest heat release rate disturbances also changes by a factor of five. Thus, the magnitude of the largest fluctuations can be said to increase proportionally with the forcing amplitude. The white line indicating the edge of the time-averaged flame does move downstream slightly with the increase in forcing amplitude. So the flame shape does appear to be affected by the level of forcing. Otherwise, the structure of the heat release rate fluctuations appears to be independent of the forcing amplitude. This supports the hypothesis that same flame response mechanisms are present throughout the linear regime.

Figure 5-10: Heat release rate fluctuation images for two different forcing amplitudes (orange text) at the same frequency. Five phase angles (white text) are shown for each amplitude. $U_{\text{RMS}}/U_{\text{inlet}} = 4\% \text{ & } 20\%$, $F_{\text{forcing}} = 120$ Hz.

Next, in Figure 5-11, a comparison is shown between two forcing amplitudes for a flame forced at 240 Hz, a frequency which experiences a nonlinear transition. The two amplitudes shown, 6% and 22%, are acquired on either side of the nonlinear transition, which occurs at around 10% forcing amplitude. Comparing side-by-side images at the same phase angles reveals some similarity, but in general, the structure of the fluctuations is quite different. The discrepancy is particularly visible in the downstream region. In the low amplitude images, on the left in each frame, the downstream region is consistently above-mean or below-mean. The amplitude of the fluctuation is also generally decreasing as it propagates downstream. In the high amplitude images, on the right of each frame, the downstream disturbance is more complex. Both above-mean and below-mean regions are simultaneously present here. Additionally, the magnitude of these downstream fluctuations is relatively strong compared to the rest of the flame, unlike the low amplitude case. High amplitude downstream fluctuations such as this may be the result of vortex
shedding in the shear layer. As previously discussed, an operating condition will have a preferred mode at which the strongest vortices are generated [15]. It is possible, however, to generate vortices at frequencies other than the preferred mode. The difference is, formation of these “non-preferred mode” vortices requires significantly higher forcing amplitudes.

Figure 5-11: Heat release rate fluctuation images for two different forcing amplitudes (orange text) at the same frequency. Five phase angles (white text) are shown for each amplitude. $U'_{\text{RMS}}/U_{\text{inlet}} = 6\%$ & 22\%, $F_{\text{forcing}} = 240$ Hz.

Figure 5-12 shows a comparison between two amplitudes (4\% & 20\%) of a flame forced at 260 Hz, the other frequency which exhibits nonlinear behavior. Here, the fluctuations in the upstream region are very similar in both structure and phase. This indicates that the mechanisms causing the upstream fluctuations may be the same. The downstream region shows significant differences between the two forcing amplitudes. The structures in the high amplitude case are similar to those observed at 240 Hz. Again, this may be evidence of vortices which are only generated under high amplitude forcing.

Through the preceding analysis, it became evident that the transition to nonlinear response produced a conspicuous change in the structure of heat release rate fluctuations. Unfortunately, the nonlinear frequencies analyzed were both relatively low gain conditions. As such, the risk of an actual combustion instability occurring here should be low. Under ideal circumstances, studies on the nonlinear regime would focus on cases that have high gain and the corresponding high likelihood of self-excited instability. Since the high gain frequencies in the current study did not exhibit nonlinear behavior in the tested range, such an examination was not possible. For this reason, future studies will need the ability to
achieve high forcing amplitude at all frequencies. This will allow for the mechanisms of the nonlinear regime at high gain frequencies to be properly analyzed.

**Figure 5-12:** Heat release rate fluctuation images for two different forcing amplitudes (orange text) at the same frequency. Five phase angles (white text) are shown for each amplitude. \( U_{\text{RMS}}/U_{\text{inlet}} = 4\% \) & 20\%, \( F_{\text{forcing}} = 260 \) Hz.

### 5.4 Mode Switching at High Frequency

For forcing frequencies higher than 260 Hz, the response of the flame was generally linear at all forcing amplitudes. An exception occurred at 320 Hz. At a glance, the condition did not display any nonlinear behavior. The flame response increased proportionally with forcing amplitude and the phase was nearly constant. The local heat release rate fluctuations, shown in **Figure 5-13**, exhibited similar structures. This anomaly was not identified until the frequency spectrum of the global heat release rate was analyzed.

**Figure 5-13:** Heat release rate fluctuation images for two different forcing amplitudes (orange text) at the same frequency. Five phase angles (white text) are shown for each amplitude. \( U_{\text{RMS}}/U_{\text{inlet}} = 4\% \) & 20\%, \( F_{\text{forcing}} = 320 \) Hz.
Figure 5-15 shows the frequency spectrum (x-axis) of global chemiluminescence for a flame forced at 320 Hz, for a range of forcing amplitudes (z-axis). Red lines mark the frequencies with the greatest disturbance amplitude. As expected, a large peak appears at the fundamental forcing frequency of 320 Hz. For forcing amplitudes of 14% and higher, significant peaks also appear at 520 Hz and 200 Hz. These frequencies are neither harmonic nor subharmonic to the fundamental frequency. The 200 Hz disturbance reaches a maximum chemiluminescence fluctuation of about 2%, but the 520 Hz disturbance goes as high as 5%. The frequency spectrum of pressure fluctuations is shown in Figure 5-14. Here, the 200 Hz disturbance is barely measureable, while the 520 Hz disturbance remains quite strong. A strong peak also appears at 640 Hz, but this is assumed to be the result of harmonic leakage caused by the siren.

Figure 5-14: The frequency spectrum of chemiluminescence for a flame forced at 320 Hz. Spectra are shown as a function of forcing amplitude along the Z axis. Red lines denote frequencies which experience significant fluctuations.

The anomaly uncovered in the frequency spectra is not unique to the current experiment. Boudy et al. [96] found that when experiencing velocity fluctuations on the order of 60%, a flame can exhibit coherent disturbances at frequencies other than the forcing frequency. They label this behavior as mode triggering, although the precise mechanism that allows it to occur is not fully understood. The inherent danger in mode triggering is that it cannot be predicted by linear stability analysis. For this reason, it is
particularly important to account for the mechanisms of the nonlinear regime when conducting an analysis of the stability of a new combustor.

![Figure 5-15](image.png)

**Figure 5-15**: The frequency spectrum of pressure for a flame forced at 320 Hz. Spectra are shown as a function of forcing amplitude along the Z axis. Red lines denote frequencies which experience significant fluctuations.

### 5.5 Conclusions

The high amplitude velocity forced response of a multi-nozzle system was investigated for one exploratory operating condition. The system was found to exhibit linear response over all achievable amplitudes for the majority of forcing frequencies. For these frequencies, the slope of the heat release rate fluctuation amplitude versus velocity fluctuation amplitude, as well as the phase between the two remained nearly constant. Nonlinear flame response was found to occur at 240 and 260 Hz. For 240 Hz, the flame response exhibited nonlinear transition at approximately 10% forcing amplitude before returning to linear behavior. For 260 Hz, the flame response was found to saturate and decrease for forcing amplitudes higher than 14%.

Fluctuation images were used to compare the response of flames at different forcing amplitudes. For cases which responded linearly at all amplitudes, local heat release rate fluctuations were found to exhibit similar structures in the upstream and downstream regions regardless of forcing amplitude. This
was taken as evidence that the same mechanisms are present at all amplitudes as long as the forcing amplitude remains in the linear response regime. For the frequencies experiencing nonlinear response, the local heat release rate fluctuations at different amplitudes exhibit measurable differences. The differences were most obvious in the downstream flame region. This finding suggested that the mechanisms present in the nonlinear regime may be different than those in the linear regime. Alternatively, the relative phase of the mechanisms may have changed.

Mode triggering, as described by Boudy [96], was identified at a forcing frequency of 320 Hz. Velocity forcing at this frequency was found to excite disturbances at 200 and 520 Hz. Mode triggering is unique to the nonlinear regime and cannot be predicted by linear stability analysis. Because of anomalies like mode triggering and the nonlinear behavior of 240 and 260 Hz, the nonlinear regime of multi-nozzle systems should receive greater attention in the future.
Chapter 6
Multi-Nozzle to Single-Nozzle Comparisons

6.1 Introduction

One of the original goals of the current study was to determine if flame transfer function data acquired on single-nozzle experiments is indicative of the response of a realistic multi-nozzle system. A single-nozzle combustor with the same nozzle design was used to provide data for comparison. This single-nozzle configuration and the data acquired from it has already been the focus of several studies by Bunce and colleagues [27, 43, 95]. Over the course of the current study it became apparent that there were a number of fundamental differences between the two configurations that would complicate any comparisons between them. The first major difference is the degree to which the flame is confined. In the single-nozzle system, combustion takes place in a quartz tube with an inner diameter of 150mm. This gives the single-nozzle system a confinement ratio (combustor area / open nozzle area) of nearly 11. The multi-nozzle flame burns in a combustor with an inner diameter of 257 mm, giving it a confinement ratio of only ~6. Several single-nozzle studies have shown that varying only the degree of confinement can have a significant impact on the response of the flame [49, 98], as discussed in Section 1.2.2.8. For example, if a single-nozzle system with a confinement ratio of 6 was compared to another single-nozzle system with confinement ratio of 11, the responses would be expected to vary.

The second major complication for comparing the single-nozzle results to multi-nozzle results is the level of flame interaction. Several studies [24, 26, 28] have shown that the degree to which the flames interact has a strong impact on how the flames respond to a forced perturbation. For flames that are spaced apart by several nozzle diameters, the flame tends to respond like a single-nozzle exhausting into a quiescent chamber [28]. For flames with spacing only slightly greater than the nozzle diameter, adjacent shear layers can roll up into large coherent vortical structures that produce elevated flame response [26].

In the current experiment, it is not possible to independently test the effects of flame confinement versus the level of flame interaction, as the geometry of the combustor is fixed. As such, it is not possible to attribute the observed differences in the flame response to one feature versus the other. These differences
notwithstanding, there still remains a good reason to compare between the single- and multi-nozzle cases. Namely, the multi-nozzle combustor was designed to match the features of a real industrial can combustor. Compared to a single-nozzle, the multi-nozzle configuration is therefore a better representation of how a real combustor responds to disturbances in the inlet velocity. The differences observed here between the two cases should be representative of the differences between single-nozzle experimental studies and real industrial combustors.

6.2 Time-Averaged Flame Structure Comparison

Before analyzing the forced response of the flame, it is of interest to make some comparisons between the unforced time-averaged characteristics of the two flame configurations. Figure 6-1 shows a comparison of the time-averaged flame structure between a single-nozzle (SN) flame and a multi-nozzle (MN) flame. The dotted white line denotes the most intense axial location, i.e. the characteristic flame length. In the single-nozzle configuration, the greatest heat release rate occurs near where the flame interacts with the wall. In the multi-nozzle configuration the greatest heat release rate occurs in the flame interaction region. Another difference can be found in the characteristic flame length, where the $L_{\text{flame, MN}}$ is almost 10% longer than $L_{\text{flame, SN}}$. The multi-nozzle flame also appears to have M-type attachment, while the single-nozzle flame is purely V-type (see Section 1.2.2.7 for description). While only one operating condition is shown in the figure, the differences between the flames are typical of all operating conditions studied. Based on findings in the literature review (Section 1.2), such differences are expected to have an impact on the way the flames respond to a forced velocity disturbance.
Figure 6-2 shows the characteristic flame length for nine different operating conditions from both configurations compared side-by-side. Single-nozzle cases are shown in black and multi-nozzle cases are shown in gray. In general, the multi-nozzle flame is longer, by an average of 9.3%. This may be the result of the decreased confinement ratio in the multi-nozzle configuration. With a reduced confinement ratio, the space in which the flame can burn per axial unit is reduced. In order to produce enough flame area to burn all of the reactants, the flame must then extend downstream, resulting in an elevated flame length. This is important because it means that the convective time scale of the flame ($L_{\text{flame}}/U_{\text{inlet}}$) will increase by 9.3%. As such, it can be expected that the phase of convective mechanisms in the flame with respect to the acoustic disturbance will change by approximately the same percentage.
The differences observed in the time-averaged flame structure for the two cases may be the result of changes to the flow field caused by the introduction of additional nozzles or the decreased confinement ratio. These changes could include the strength and position of the recirculation zones, the angles of the jets issuing from the nozzle, and the location where the jet impinges on the combustor wall. Unfortunately in the current experiment it is not possible to characterize any features of the flow field, or how they change in the presence of additional nozzles. Therefore, it is of the utmost importance that future studies on multi-nozzle combustors have the capability to quantify the velocity field inside of the combustor.

### 6.3 Flame Transfer Function Comparison

The next point of inquiry is on the differences in the forced response of the flame. The first comparison condition is shown in **Figure 6-3**. The operating condition shown has received considerable attention in the preceding chapters, with $U_{\text{inlet}} = 25$ m/s, $T_{\text{inlet}} = 200$ °C, and $\Phi = 0.60$. The gain plot in figure A shows the response of the multi-nozzle system (MN) with red squares, and the response of the single-nozzle (SN) system with black squares. From a qualitative standpoint, the two configurations respond similarly, with alternating minima and maxima. The single-nozzle flame transfer function exhibits many of the same features as the multi-nozzle transfer function, but shifted to lower frequencies. With regards to the uncertainty in the gain, it is generally higher for the multi-nozzle conditions. This can be

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**Figure 6-2**: Comparison of time-averaged characteristic flame length between single-nozzle (SN) and multi-nozzle (MN) configurations.

<table>
<thead>
<tr>
<th>Op. Con.</th>
<th>$V$</th>
<th>$T$</th>
<th>$\Phi$</th>
<th>% Dif.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.0</td>
<td>200</td>
<td>0.60</td>
<td>11.9%</td>
</tr>
<tr>
<td>2</td>
<td>25.0</td>
<td>200</td>
<td>0.65</td>
<td>10.4%</td>
</tr>
<tr>
<td>3</td>
<td>15.0</td>
<td>150</td>
<td>0.60</td>
<td>3.3%</td>
</tr>
<tr>
<td>4</td>
<td>25.0</td>
<td>200</td>
<td>0.55</td>
<td>11.9%</td>
</tr>
<tr>
<td>5</td>
<td>20.0</td>
<td>200</td>
<td>0.65</td>
<td>1.5%</td>
</tr>
<tr>
<td>6</td>
<td>25.0</td>
<td>100</td>
<td>0.60</td>
<td>15.8%</td>
</tr>
<tr>
<td>7</td>
<td>22.5</td>
<td>150</td>
<td>0.60</td>
<td>7.9%</td>
</tr>
<tr>
<td>8</td>
<td>25.0</td>
<td>100</td>
<td>0.65</td>
<td>11.0%</td>
</tr>
<tr>
<td>9</td>
<td>30.0</td>
<td>200</td>
<td>0.55</td>
<td>9.9%</td>
</tr>
</tbody>
</table>
attributed to the use of three different microphone pairs for the two-microphone measurements, as discussed in Section 2.3.2.

From a quantitative perspective, the frequencies at which the extrema occur are not the same. This is an important difference for modelers trying to predict the frequency at which the largest instability driving force will occur. The frequency of the minimum response occurs at 175 Hz for the single-nozzle system and 240 Hz for the multi-nozzle system. The frequency of the local maximum at high frequency shifts from 300 Hz up to 320 Hz. For this operating condition, the change from single-nozzle to multi-nozzle combustion corresponds to an 11.9% increase in the flame length. The direction of the frequency shift observed at both extrema is consistent with the shift observed for increasing flame length created by increasing the inlet velocity, as discussed in Section 3.3.2.

The phase data for this operating condition are shown in Figure 6-3B. Like the gain data, the two plots are qualitatively similar. Initially the phase of both the single-nozzle and the multi-nozzle systems decreases linearly with increasing frequency. Around the frequency of minimum gain, there is a discontinuity, followed by a return to linear decline. After the slopes of the two curves have stabilized (\(F_{\text{forcing}} > 250 \text{ Hz}\)), there is consistent difference between each point on the two curves of around \(-\frac{1}{2}\pi \) (or \(\frac{3}{2}\pi\)). This indicates that global fluctuations in heat release rate for the multi-nozzle case occur nearly 90° earlier during a forcing cycle of 360°, compared to the single-nozzle case. Such a difference is important when trying to predict whether self-excited instability will occur. While single-nozzle experiments may predict a phase relationship that would lead to energy loss in the acoustic field, the real multi-nozzle combustor may have a phase relationship that could add energy to the acoustic field and create an unstable feedback loop.
Next, in Figure 6-4, comparisons are shown for flame transfer function gain at a number of operating conditions. All conditions exhibit qualitative similarity with alternating minima and maxima. In images A through D, the gain values for each forcing frequency are quite similar, varying by less than 20% in most cases. These flames are generally well attached to the centerbody in both the single- and multi-nozzle configurations. In the E through H images, the trends in the two curves as well as the frequencies of the extrema, are still similar. The actual value of gain, however, varies by more than 100% in some cases. The E through H images are all acquired at operating conditions close to blowoff in the multi-nozzle case. This means that if the velocity is increased, or the inlet temperature or equivalence ratio is decreased, the flame would either extinguish or become a lifted flame. Under such conditions the
flame was observed to have weaker attachment near the flame base and spread downstream. This behavior is confirmed by the characteristic flame length in Figure 6-2 (operating conditions 6 – 9). The single-nozzle flame in these cases remains well attached to the centerbody. At near blowoff conditions, the multi-nozzle flame responds with low gain at most frequencies. The extrema that are present at higher inlet temperatures and equivalence ratios are not as obvious here. The fact that the gain is so different under the latter conditions (E – H) may be an indication that it is not the flame’s phenomenological response to forced perturbations that changes from single-nozzle to multi nozzle conditions. Rather, it is the stable flame structure and the flow field that are altered by the switch from single-nozzle to multi-nozzle. The changes in the flame’s response are then the result of the changes that occur in the stable flame structure as a result of the switch from single-nozzle to multi-nozzle configurations.

Figure 6-5 displays the phase for the same operating conditions as the previous figure. For most cases, the single-nozzle phase (black squares) exhibits qualitative similarity to the multi-nozzle phase (red squares). Initially, the phase in both configurations declines linearly. At the frequency of minimum gain there is a discontinuity in the phase. As described in Section 4.3.3, this continuity is related to the upstream movement of the axial location of greatest heat rate fluctuation. Following the discontinuity, the phase returns to linear behavior. While the trend is similar, the actual phase values vary by as much as π radians in some cases. As mentioned previously, an issue is raised because the use of single-nozzle phase data may lead to erroneous predictions about the stability of a real combustor.

From this analysis, it becomes clear that, from a global perspective, multi-nozzle flames do not respond to inlet velocity perturbations in exactly the same manner as single-nozzle flames. The trends between the two cases are generally similar and the frequencies of the extrema vary by an average of only 10%. The main difference between the two configurations is found in the actual value of gain, which varies by more than 100% in some cases. The difference in the flame’s response may be caused by the changes to the flow field, the shape of the flame, or flame to flame interactions. Using the current experimental setup, however, it is not possible to differentiate between the impacts of each of these factors. In order to accurately predict the behavior of a multi-nozzle flame from single-nozzle data, the effect of each factor must be quantified separately.
Figure 6-4: Comparison of single-nozzle (black squares) and multi-nozzle (red squares) flame transfer function gain for several different operating conditions. The operating condition is shown in the top right corner of each plot.
Figure 6-5: Comparison of single-nozzle (black squares) and multi-nozzle (red squares) flame transfer function phase for several different operating conditions. The operating condition is shown in the bottom left corner of each plot.
6.4 Local Heat Release Rate Fluctuation Comparison

The next logical step would be to compare the local heat release rate fluctuations in the multi-nozzle combustor to those in the single-nozzle combustor. Unfortunately, none of the high speed data acquired on the two experiments matched exactly in operating condition. While the operating conditions may not be the same, it is still useful to see if the trends in the single-nozzle configuration match those observed in the multi-nozzle configuration. The flame transfer function for the single-nozzle operating condition that will be analyzed is shown in Figure 6-6. Blue highlights indicate the frequencies that will be analyzed further. Figure 6-7 shows the amplitude of heat release rate fluctuations at the four highlighted forcing frequencies. Such images are acquired through the technique described in Section 2.3.6. The dotted white line indicates the characteristic flame length. The four frequencies were selected based on the extreme nature of their response, in the same manner as Figure 4-10.

![Figure 6-6: Single-nozzle flame transfer function gain and phase for U_{inlet} = 25.0 m/s, T_{inlet} = 100 °C, Φ = 0.75, U’_{RMS}/U_{inlet} = 5%. Blue highlights indicate frequencies examined in the proceeding fluctuation images.](image_url)

The purpose of Figure 6-7 is to identify where in the flame the largest fluctuations occur. For the high gain case at 100 Hz, the downstream region dominates the flame response while the upstream fluctuations are much smaller. A particularly intense pocket of heat release rate fluctuation occurs near the downstream end of the approximate location of the inner shear layer. This pocket may be the result of vortex-flame interaction. At 200 Hz, a local minimum in the flame response occurs. The upstream
fluctuations have grown in strength while the downstream fluctuations have diminished. At this frequency, a pocket of intense fluctuation occurs near the expected location of the outer shear layer. Around 280 Hz, the gain has increased to 0.70 and then leveled off. The fluctuations are spread throughout the flame, so that neither region can be said to dominate the flame response. At 380 Hz the gain decreases to a local minimum. At this frequency, the upstream fluctuation near the outer shear layer has grown to an amplitude of 15%. A disturbance such as this may be evidence of a strong vortical structure formed in the outer shear layer that interacts with the flame.

![Figure 6-7](image)

**Figure 6-7:** Amplitude of heat release rate fluctuations for several different forcing frequencies in a single-nozzle combustor. With $U_{\text{inlet}} = 25.0$ m/s, $T_{\text{inlet}} = 100$ °C, $\Phi = 0.75$, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$.

Phase information for the amplitude plots in the preceding figure is shown in Figure 6-8. Regions with fluctuations smaller than 3% have been blacked out since they do not have a significant impact on the overall response of the flame. In the 100 Hz case, the phase across the high amplitude downstream region changes by only $\frac{1}{4}\pi$. This indicates that the fluctuations there occur nearly in-phase, so that the global fluctuation can be expected to be high, in agreement with the 100 Hz gain in Figure 6-6. The phase for the minimum response conditions at 200 and 380 Hz varies continuously from the base to the downstream end of the flame. This indicates that the fluctuations originate at the flame base and convect downstream from there. Because the phase angle changes so quickly, there will be cancelation effects between different parts of the flame. This should result in a low overall response, which is reflected in the low value of gain. At
280 Hz, a local maximum frequency, the phase variation is greater than in the 100 Hz case, but not as large as the minimum response cases. As expected, the gain for 280 Hz falls between the gain at 100 Hz and the gain at 200 Hz.

![Figure 6-8: Phase of heat release rate fluctuations for several different forcing frequencies in a single-nozzle combustor. With $U_{\text{inlet}} = 25.0$ m/s, $T_{\text{inlet}} = 100$ °C, $\Phi = 0.75$, $U'_{\text{RMS}}/U_{\text{inlet}} = 5\%$.](image)

In **Figure 6-9A** the axial heat release rate fluctuation amplitude is displayed for all frequencies at the current operating condition. This figure is acquired through the technique described in Section 2.3.7. Intensity is indicated by pseudo-color on the scale shown to the right of the plot. The white line shows the characteristic flame length. The orange curve is the flame transfer function gain. **Figure 6-9B** shows a set of multi-nozzle profiles for comparison. Note that the operating conditions in the two cases are not the same. The two are presented together here mainly to look for similar trends in relation to the gain.

In the single-nozzle figure, at 100 Hz, the downstream fluctuation is large and corresponds to a high gain. As the frequency is increased, the amplitude of the downstream fluctuation decreases while the upstream amplitude grows. The maximum upstream amplitude is achieved at 180 Hz, while the minimum downstream amplitude is achieved at 200 Hz, the minimum gain frequency. Beyond the minimum response frequency, the downstream fluctuation again begins to grow, reaching its maximum amplitude at 340 Hz. The upstream fluctuation reaches its minimum strength at 240 Hz and rebounds to a maximum at 360 Hz. The maximum downstream fluctuations do not always correspond to maximum gain and the maximum
upstream fluctuations do not always correspond to minimum gain. But, in general, low gain cases have large upstream fluctuations and high gain cases have large downstream fluctuations.

**Figure 6-9**: The axial heat release rate fluctuation amplitude for (A) a single-nozzle flame with $U_{\text{inlet}} = 25.0$ m/s, $T_{\text{inlet}} = 100$ °C, $\Phi = 0.75$ and (B) a multi-nozzle flame with $U_{\text{inlet}} = 25.0$ m/s, $T_{\text{inlet}} = 200$ °C, $\Phi = 0.60$.

Compared to the multi-nozzle axial fluctuations in **Figure 6-9B**, the trends observed in the axial fluctuations for the single-nozzle combustor are similar. High gain cases tend to have large downstream fluctuations while low gain cases tend to have large upstream fluctuations. This may be evidence that the two different flame configurations are responding to the same heat release disturbance mechanisms. As discussed in Section 4.4 and literature [43, 95], these mechanisms include acoustic velocity fluctuations, inner and outer shear layer vortical structures and swirl fluctuations.

### 6.5 Conclusions

A comparison was conducted between the multi-nozzle combustor and a single-nozzle combustor that utilized the same nozzle design. The distribution of heat release rate in the time-averaged flame structure was found to vary significantly. In the multi-nozzle flame the highest heat release rate occurred in the interaction region between flames. In the single-nozzle system the highest heat release rate occurred near where the flame impinges on the combustor wall. Additionally, the multi-nozzle flame exhibited M-type flame attachment, while the single-nozzle flame maintained V-type attachment. Significant differences were also found in the flame length, where the multi-nozzle flames were found to be an average of 10%
longer. The unique flame shape observed in the multi-nozzle combustor indicates that the flow field therein is substantially different than that of single-nozzle combustors. Given the importance of the flow field structures in the response of the flame, it is essential that future studies develop the capability to examine the flow field of multi-nozzle combustors in a quantitative manner.

Flame transfer functions, at nine different operating conditions from the two configurations, were compared directly. From the comparison it was be concluded that single- and multi-nozzle systems exhibit many similarities in their response to forced velocity perturbations. These similarities included: the general shape of the flame transfer function, the occurrence of minima, maxima and discontinuities in the phase. From flame imaging, similarities were found in the distribution of local heat release rate fluctuations at particular magnitudes of gain. For high gain values, the flame typically exhibited high amplitude heat release rate fluctuations in the downstream region. For low gain cases, the flames exhibited stronger fluctuations near the flame base. The difference between the actual gain values for particular forcing frequencies, however, did exceed 100% in some cases. The phase difference between the two configurations showed significant variation as well, reaching π radians (180°) in some cases. This means that substantial inaccuracies may result if predictions of the instability driving force at a particular frequency in a multi-nozzle system are based only on single-nozzle data. An inaccurate prediction of the instability driving force could have catastrophic results for a real gas turbine system and must be avoided. To correct for such inaccuracies, the quantitative effect of the following factors must be understood: confinement ratio, flame interaction and swirl dissipation.
Chapter 7

Summary and Future Work

7.1 Summary

The effect of forced velocity fluctuations on a lean-premixed multi-nozzle can combustor was studied at a wide range of operating conditions and forcing frequencies. The purpose of such forcing studies is to simulate part of the feedback cycle in a system experiencing combustion instability. This study is unique from previous work in that the combustor contains multiple nozzles in a can configuration. Previous work has focused on single-nozzle combustors only. It was assumed that their findings would be applicable to the more realistic multi-nozzle configuration utilized in real gas turbine engines. This work demonstrates that such an assumption is not always valid. The ultimate purpose of forced response studies is to elucidate the mechanisms which allow the flame to respond to disturbances in the inlet conditions. With a thorough understanding of the mechanisms, models can be developed which allow for the prediction of unstable behavior in combustors during the design stage of new gas turbine engines.

Chapter 1 introduced the concept of combustion instability and outlined why it is an important area of research. Through a review of the literature, deficiencies in the current understanding of the feedback cycle were identified. The chapter ended with an outline of the objectives for the current study. In Chapter 2 the experimental methods utilized in this study were introduced. The multi-nozzle combustor utilized herein was described in detail. The reasoning behind the selected range of operating conditions was also discussed.

Chapter 3 focused on the global rate of heat release from the flame. CH* chemiluminescence was used as an indicator of the rate of heat release. The time-averaged shape of the multi-nozzle flame was quantified using the characteristic flame length, based on the most intense axial location along the flame. The characteristic flame length was found to vary systematically with inlet velocity, temperature and equivalence ratio so that it could be predicted with an empirically derived formula. The flame transfer function was defined using the velocity fluctuation as the input and the heat release rate fluctuation as the output. The flame transfer function was calculated over a range of frequencies for 57 different operating
conditions. For most operating conditions, the multi-nozzle combustor provided flame transfer functions with a qualitatively similar form to those observed in literature, with alternating minima and maxima. Several normalization techniques were applied to the flame transfer functions to test the data for collapse. The best collapse occurred when the flame transfer function was plotted versus a Strouhal number based on the characteristic flame length. This was taken as evidence that vortical disturbances have a strong influence on the extent to which a flame responds to a particular forcing frequency. At frequencies greater than 250 Hz, the collapse of the data was poor, indicating that the Strouhal number is no longer the sole controlling parameter for the gain. Accordingly, other coupling mechanisms must be present.

In Chapter 4 the distribution of local heat release rate fluctuations was examined. Line-of-site CH* chemiluminescence intensity was acquired using a high speed camera and intensifier and used as a measure of the heat release rate. Random turbulent fluctuations were filtered from the images in the frequency domain, leaving only the fundamental component of heat release rate fluctuation at the forcing frequency. Local fluctuations in the rate of heat release were found to have a consistent behavior depending on the gain value from Chapter 3. For cases with high gain, the flames exhibit high amplitude fluctuations in the downstream region, near the end of the flame. The fluctuation structure there bears a resemblance to those that would be expected from the interaction of an acoustic velocity disturbance and the flame. It is also possible, however, that the observed fluctuations are the result of vortices shed in the outer shear layer that impinge on the flame, generating a heat release rate fluctuation. In order to differentiate between the two, velocity field measurements will have to be acquired. For low gain cases, the amplitude of heat release rate fluctuations in the downstream region is significantly reduced. Instead, stronger fluctuations are found in the upstream region near the base of the flame. These fluctuations are found to originate at the base of the flame and propagate downstream at a velocity on the order of the convective velocity. Such disturbances may be the result of fluctuations in the swirl angle. Swirl fluctuations would be expected to produce their own heat release rate fluctuations through periodically increasing and decreasing the radius of the flame. They may also offset the formation of vortical structures through the production of vorticity with an opposite sign to that generated by the flow. To confirm the existence and the importance of swirl fluctuations, velocity field measurements are required. The chapter closes with a proposal for an overall reaction mechanism including the effects of acoustic velocity fluctuations, vortical disturbances and swirl...
fluctuations. A model based on the overall reaction mechanism is applied to a range of forcing frequencies and compared to experimental data. The data matches the model for 30% of the analyzed cases, which indicates that additional factors still need to be incorporated.

Chapter 5 analyzed the response of the multi-nozzle combustor to high amplitude velocity forcing. The aim of the study was to look for differences in the flame response as the forcing amplitude was increased, i.e. nonlinear flame response. Over the achievable range of forcing amplitudes, most frequencies did not exhibit any nonlinear behavior. Exceptions occurred at 240, 260 and 320 Hz. At 240 Hz the flame response was initially very low, but for forcing frequencies greater than 10%, the flame began to respond to a greater extent and the gain value increased. At 260 Hz, the flame response was initially mid-level, but for amplitudes greater than 14%, the response was found to saturate and then decrease. For both of these frequencies, the phase between global heat release rate fluctuations and velocity fluctuations was also found to change significantly. At 320 Hz the flame responded linearly for all amplitudes at the fundamental frequency. At higher forcing amplitudes the velocity forcing was found to generate disturbances at 200 Hz and 520 Hz. This behavior has been labeled mode triggering in other studies and is a highly nonlinear process that cannot be predicted by linear stability analysis. The findings in Chapter 5 suggest that a more lengthy study into the nonlinear regime of multi-nozzle combustors must eventually be undertaken.

The final analysis in Chapter 6 was focused on comparing the response of the multi-nozzle combustor to a single-nozzle system that used the same nozzle design. The flame structure was found to vary significantly between the two cases, with the multi-nozzle flame being longer by an average of 10%. A direct comparison of the flame transfer function gain and phase for nine different conditions revealed a surprising amount of similarity given the differences in the two systems. The gain in each system exhibited the typical alternating minima and maxima behavior. The frequency values of the extrema matched at most operating conditions as well. The primary difference between the two configurations was found in the value of gain for certain frequencies, varying by as much as 100% in some cases. Flame transfer function phase comparisons between the two flames also showed a similar behavior, though the actual value varied by \( \pi \) radians at times. The differences between the response of the two configurations are thought to be the result of the following: changes to the confinement ratio, the introduction of flame-to-flame interaction and changes to the boundary conditions that each flame experiences. This degree of variation between cases
indicates that single nozzle flame response measurements should not be used directly to predict the stability characteristics of full scale multi-nozzle gas turbine combustors.

7.2 Future Work & Recommendations

Throughout the course of the current study, it became apparent that the understanding of the velocity field inside the combustor was deficient, particularly under unstable reacting conditions. Flame transfer function measurements and even high speed imaging treat the entire velocity field as a black box. A controlled velocity disturbance is the system input and the global heat release rate fluctuation is the system output, but no details of the intermediate processes are revealed. Using this approach, the features of the flow field such as vortical structures, the position of the recirculation zones and shear layers as well as swirl fluctuation amplitude cannot be quantified. Given the importance of these features in determining the forced response of a flame, the ability to measure the velocity vectors of the flow field must be acquired. Particle image velocimetry and laser Doppler velocimetry are both capable of providing such data. Any future study on excited flames, forced or self-excited, should, therefore, include one of these techniques.

With regards to future multi-nozzle studies, a degree of variable geometry must be attained. Literature has shown that the confinement level [49, 98], swirl angle [5, 16] and the level of flame interaction [26] can all have an impact on the response of the flame. In the current study, it was not possible to vary any of these parameters because the combustor geometry was fixed. This limitation made it was impossible to attribute the changes observed in the flame response to a particular parameter. Accordingly, future studies on multi-nozzle systems should include the ability to vary the spacing between nozzles, the number of nozzles, the position of the swirler and the size of the combustor.

The current study allowed only for velocity fluctuations to occur inside the combustor. It is well known, however, that equivalence ratio fluctuations are also likely to be present in a real combustor experiencing self-excited instability. Equivalence ratio fluctuations act as one more coupling mechanism that can interact with the mechanisms introduced by the velocity fluctuation to generate a heat release rate disturbance. These two primary disturbance pathways have been studied largely in isolation, thus far. However, to fully understand combustion instability in real combustors they must be brought together in a single experiment and studied thoroughly.
A final factor which future studies should focus on is the pressure inside of the combustor. Because the current study was the first of its kind, as well as the inherent difficulty in constructing high pressure combustion facilities, all tests were completed at atmospheric pressure. Real gas turbines utilize combustor pressures as high as 20 atmospheres. A small number of studies have analyzed the effect that elevated pressure might have on the flame response, but there is still much to be learned. Future studies in combustion dynamics and forced flame response need to determine if the findings in atmospheric pressure studies are applicable to gas turbine combustors at full engine pressure.
Appendix

Operating Conditions

The below list includes all operating conditions at which flame transfer function data was acquired. Bold, blue conditions indicate that high speed video was acquired for all forcing frequencies.

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Michael Szedlmayer graduated from Lehigh University in 2007 with a Bachelor’s degree in Mechanical Engineering. During his years as an undergraduate he spent summers as an intern for Boeing, the US Navy and Lehigh Valley Plastics. He began coursework for his Ph.D. at Penn State in the fall of 2007 and was funded as a teaching assistant for Dynamic Systems and then Fluid Dynamics. He joined the Turbulent Combustion Lab in the summer of 2008 to design a new experimental facility incorporating multiple gas turbine nozzles and study how the interaction of flames affected combustion instability. In 2011 he was awarded a two year NSF Fellowship to teach science and engineering to underprivileged schools at the middle school grade level. Following graduation, Michael intends to pursue a career in combustion research in industry or at a national lab.