The Pennsylvania State University
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EFFECT OF GEOMETRIC PARAMETERS ON THE IN-PLANE CRUSHING BEHAVIOR OF HONEYCOMBS AND HONEYCOMBS WITH FACESHEETS

A Dissertation in
Aerospace Engineering

by
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ABSTRACT

In aerospace field, use of honeycombs in energy absorbing applications is a very attractive concept since they are relatively low weight structures and their crushing behavior satisfies the requirements of ideal energy absorbing applications. This dissertation is about the utilization of honeycomb crushing in energy absorbing applications and maximizing their specific energy absorption (SEA) capacity by modifying their geometry. In-plane direction crushing of honeycombs is investigated with the help of simulations conducted with ABAQUS. Due to the nonlinearity of the problem an optimization technique could not be implemented; however, the results of the trend studies lead to geometries with improved SEA.

This study has two objectives; the first is to obtain modified cell geometry for a hexagonal honeycomb cell in order to provide higher energy absorption for minimum weight relative to the regular hexagonal cell geometry which has 30° cell angle and walls at equal length. The results of the first objective show that by increasing the cell angle, increasing wall thickness and reducing vertical wall length it is possible to increase the SEA 4.8 times; where the honeycomb with modified geometry provided 3.3 kJ/kg SEA and with regular geometry 0.68 kJ/kg SEA.

The second objective considers integration of the energy absorbing honeycombs into the helicopter subfloor, possibly as the web section of a keel beam. In-plane direction crushing of a honeycomb core sandwiched between two facesheets is simulated. Effects of core and facesheet geometric parameters on the energy absorption are investigated, and modified geometries are suggested. For the sandwich structure with thin facesheets increasing cell angle, increasing wall thicknesses and decreasing the cell depth increase the SEA. For the ones with thick facesheet reducing vertical wall length, increasing wall thicknesses and reducing the cell depth increase the SEA. The results show that regular honeycomb geometry with thin facesheets has SEA of 7.24 kJ/kg and with thick facesheets 13.16 kJ/kg. When the geometries are modified the SEA increases to 20.5 kJ/kg for the core with thin facesheets and 53.47 kJ/kg for the core with thick facesheets.
The key finding of the dissertation is that the in-plane direction crushing of the honeycombs with facesheets has great potential to be used for the energy absorbing applications since their SEA levels are high enough to make them attractive for applications where high crash loads need to be absorbed such as helicopter crash.
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Chapter 1

Introduction

Vehicle collisions or crashes are unavoidable. While some researchers investigate the cause of accidents, many others focus on increasing the survivability after a crash by improving the design of the vehicle. The two main objectives of designing a crashworthy vehicle are eliminating or minimizing injuries and fatalities in mild impacts, and reducing the cost and weight of the vehicle. In order to avoid fatality, the deceleration loads of the crash which are transmitted to the occupants should be below the human tolerances. To improve the crashworthiness of a vehicle, the kinetic energy that the vehicle possessed prior to the impact should be absorbed by controlled deformation of the energy absorbing components.

In every vehicle there are several energy absorbing components. In a crashworthy aircraft; landing gears, aircraft structure and occupant seats must all contribute to the energy absorption. In vertical impact of a helicopter most of the energy is absorbed by structural deformation of the subfloor, which is the region of an aircraft between outer skin and the cabin floor. A typical subfloor structure consists of three main sections: longitudinal keel beams, lateral bulkheads and structural intersection elements. Longitudinal keel beams generally span the length of the fuselage and support the seat rails. Several researchers investigated the crashworthiness of various subfloor concepts which are capable of operating at normal flight and landing conditions and also providing energy absorption during crash.

In order to be used in energy absorbing applications, structures and materials should behave in a controlled manner while exhibiting a steady force level for a large stroke under compressive loading. The need for large stroke has led the researchers to focus on materials and structures that can undergo large deformations, such as honeycombs or foams [1]. The use of honeycombs in energy absorbing applications dates back to 1960s as the energy absorbing component of Apollo lunar module [2].
Since then honeycombs have extensively been used in the design of energy absorbing applications and crashworthy vehicles. Researchers still investigate the physical phenomena behind the crushing of honeycombs in order to maximize their energy absorption capability while keeping their weight at minimum. Similarly this dissertation aims to explain the cell deformation behavior that the honeycombs undergo during crushing, modify their geometry to increase the energy absorption, and exploit their high energy absorption capability by means of constraining them between facesheets while keeping the weight at minimum.

1.1 Motivation and methodology

Cellular honeycombs are attractive candidates for use as energy absorbing components of a crashworthy aircraft. To develop an application which uses honeycombs as energy absorbing components two areas need to be addressed. The first area is to understand how energy absorbed by the crushing of the cellular honeycombs can be maximized. The second is to determine how such energy-absorbing cellular honeycombs could be integrated into the aircraft as a part of a crashworthy structural design. Honeycombs are already in use in several commercial and military aircraft in different configurations; such as landing gear struts, energy absorbing devices used for seats, in sandwich configuration for the interior of the cabin, or interior of a longitudinal keel beam in the subfloor [3]. Commercially available honeycomb cells generally have regular cell geometry, where the cell angle is 30° and the cell walls are at equal length. Previous studies not specifically focused on honeycomb crushing have already shown that the cell geometry has a very large effect on the mechanical properties of the honeycombs; however, the effect of cell geometry has never extensively been studied in complex conditions such as non-linear crushing [4].

The motivation of this study is to investigate the effects of cell geometric parameters on the energy absorbed by the crushing of the honeycombs and integrate these energy absorbing honeycombs into the helicopter subfloor by suggesting a keel beam design. The purpose of this dissertation is to provide a background study for the design of
an energy absorbing keel beam to be used in helicopter subfloor which utilizes the honeycombs with improved energy absorption as the inner structure, providing a controlled crushing of the keel beam as well as increasing the bending load limits of the keel beam acting like the core of a sandwich structure between stiff load bearing facesheets. This dissertation reports the research efforts which also include an extensive background study.

The approach to this study was first to create a substantial literature survey on the crashworthiness of the rotary-wing aircraft in order to find a method to use the honeycombs in the design of an energy absorbing subfloor structure. Prior to the design of a keel beam, the complex behavior of the honeycomb crushing was studied computationally and modified cell geometry was suggested which provided higher energy absorption for minimum weight. This computational study focused on the in-plane compressive response of aluminum honeycombs. A keel beam design resembling a sandwich structure configuration with honeycomb core and facesheets was suggested, crushing simulations were conducted and the effect of geometric parameters on the crushing behavior was investigated. Commercially available ABAQUS finite element tool was used for the simulations. The in-plane direction energy absorption of the honeycombs with facesheets was compared to that of the honeycombs without facesheets. Out-of-plane direction crushing simulations of regular honeycombs were also conducted in order to obtain the energy absorption levels of honeycombs in this direction and these levels were also compared to energy absorption levels of the in-plane direction of honeycombs.

1.2 Thesis outline

Chapter 2 of this thesis summarizes several related previous studies which were gathered during an extensive literature survey. The chapter starts with introduction to the crash survivability, means of increasing crashworthiness and energy absorbing concepts of crashworthy helicopters. Special attention was given to the applications which use honeycombs as energy absorbing components. As a result of the literature survey it was
comprehended that the honeycombs are extensively used in energy absorbing applications, but mainly exploiting their crushing in out-of-plane direction. Sources on the in-plane crushing of sandwich structures were also surveyed. This chapter also discusses basis of the integral sandwich structure manufacturing techniques such as brazing.

The computational approach to both objectives is mentioned in Chapter 3: “Methodology”. The chapter initially explains several aspects regarding honeycombs such as general terminology for cell geometry, crushing directions and geometric parameters, etc. The chapter addresses the approach followed during this computational study. The 2 dimensional (2D) and 3 dimensional (3D) models used in the simulations are shown indicating nodal locations of loading and boundary conditions. Mesh convergence study results are presented in this chapter. Average dimensions for a keel beam were also suggested based on the keel beam dimensions given in literature sources.

In this dissertation two main objectives were considered: investigating the effect of cell geometric parameters on the crushing behavior of honeycombs and integrating these honeycombs into an energy absorbing keel beam structure. The results from each objective are presented in separate chapters. Chapter 4 is the “Results and discussions of the crushing of honeycomb cells without facesheets”. This chapter starts with explaining crushing of perfect and imperfect honeycomb cells, and typical stress vs. strain plots generated by crushing of these cells. In order to represent a real case, the cells had initial imperfections and the effect of initial imperfection on the crushing behavior is shown in this chapter. In order to establish the validity of the computational methods used in this research, results obtained from literature sources are compared to the computational results obtained using ABAQUS. In this section the crushing results of regular cells (cells with 30° cell angle and the equal length cell walls) were validated since the crushing behavior of the cells with different cell geometries are not covered in literature. In the simulations only a single cell was used after sufficiency of using a single cell in the simulations was established by comparing the single cell results to the results of cores with several rows and columns. Effect of each cell parameter on the energy absorption capability of the honeycomb cells are presented with stress vs. strain plots and the
specific energy absorption (energy absorption per unit mass and unit volume) of cells with modified geometry to regular cells were compared. The physical phenomenon behind the effect of each cell geometric parameter is also explained. Finally a preferred geometry was established considering the trends of the geometric parameters’ effects on the energy absorption and specific energy absorption of this modified geometry was compared to the regular cells. In order to obtain the difference of energy absorption between core-wise (out-of-plane) and edge-wise (in-plane) direction crushing, dynamic and quasi-static simulations of out-of-plane direction crushing are also conducted for regular honeycombs.

Chapter 5 covers simulation results related to the second sub-objective of the dissertation and it is called “Results and discussions of the crushing of honeycomb cells with facesheets”. It starts with validating the simulation method where results obtained from an experimental study compared to the single cell crushing simulation results which is conducted in ABAQUS. This is followed by comparing simulation results of a half cell, single cell and a single row structure. Further in the study a single cell core and facesheet structure is used in the simulations and effect of geometric parameters on the crushing behavior of this sandwich configuration is investigated. Physical understanding behind the behavior differences are explained, and if available, simulation results are compared to the analytical formulations provided in the literature. Finally keel beam dimensions were considered, and the effect of keel width and number of rows placed in the keel beam are investigated.

The whole thesis is summarized in Chapter 6 which is entitled: “Conclusion and future work suggestions”. Effect of each geometric parameter on the energy absorption and their physical interpretation are also listed. Contribution of this dissertation is clearly explained. Finally several future work ideas are suggested, some of which are related to the keel beam design.

Bibliography and appendix are also provided at the end of the thesis.
Chapter 2

Literature review

Crashing of a vehicle generates loads due to rapid deceleration, and these high loads should not be transmitted to the occupants. Energy absorbing components of the vehicle convert the kinetic energy of the impact to other forms of energy by undergoing large plastic deformations at steady force level. For occupant safety, the resisting force provided by the energy absorbing structure during crash is preferred to be in a steady form (no large initial peaks) where no high deceleration forces are transferred to the occupants [5]. A high peak load could also cause the whole structure to collapse [6]. In the 1960s researchers discovered that crushing of honeycombs generate this desirable form of energy absorption. Both in-plane (edge-wise) and out-of-plane (core-wise) crushing of honeycombs do not generate high initial peak loads and there is a plateau load corresponding to a large stroke [7]. When the loading is in the out-of-plane direction the honeycomb walls buckle and fold over progressively [2]. When the loading is in the in-plane direction, crushing of the honeycomb rows over each other provides the required continual form of deformation [8]. Therefore, cellular honeycombs are attractive candidates for use as energy absorbing components in crashworthy aircraft. To develop this technology further two areas need to be addressed. The first is to understand how energy absorption can be maximized in the crushing of cellular honeycombs. The second is to determine how such energy-absorbing cellular honeycombs could be integrated into the aircraft as part of a crashworthy structural design.

2.1 Design considerations for crashworthy aircraft

Crashworthiness is defined as ensuring the safety of occupants in case of potentially survivable impacts [9]. The crashworthiness can be improved by increasing the energy absorbed by the vehicle. Energy absorption of an aircraft is a complex concept
with consideration of human tolerance, crash conditions, seats and cargo restraints, cabin environment, post-crash fire, emergency exits, landing gear, and the airframe structure [5]. In a crashworthy design all of these issues should be addressed.

The main purpose of energy absorption is to bring the vehicle to a stop, and provide survivable conditions during and post crash [6]. Even though the type of aircraft affects the design approach, basic understanding of design requirements for crashworthiness is valid for both fixed-wing and VTOL type aircraft since similar impact conditions create similar structural loading and damage [5], [6]. Design approaches for different type of airplanes; transport, fixed-wing and rotary wing, can be seen on Fig. 2.1 [5]. Extra attention on the following features will increase the crash survivability; protective shell around occupants, tie-down strength for occupants, cargo and equipment, safe deceleration envelope considering the human tolerance, non-injurious occupant environment hazards, prevention against post-crash fire, adequate emergency escape and rescue supply [6]. In a severe crash, aircraft loses its functionality. At this point the only structural requirement is to provide occupant safety while providing structural integrity and livable space [5], [6]. Since helicopters generally fly at lower speeds and altitudes, helicopter accidents are often potentially survivable [9].

Figure 2.1: Energy absorption design approaches for different aircraft types [5]
Energy absorbing components of an aircraft can be seen in Fig. 2.2. For fixed-wing and VTOL aircraft, one of the major energy absorbing components is the landing gears [6]. Energy absorbed by the landing gears depends on the design criteria and the impact surface. In case of a crash, landing gears or the support system should not be driven into the occupied area or the fuel tanks. Landing gear crushing should not block the escape route or prevent the opening of the exit doors. Structural deformation such as compression, tension, bending, torsion, and shear also provide major energy absorption which is also affected by the materials properties, structure geometry and collapse mode [6], [10]. Breakaway of large-mass items provide an instantaneous reduction in kinetic and potential energy. The major problem involved is to ensure adequate clearance between free items and the occupied areas. Depending on the surface type ground friction and nose plowing may provide longitudinal deceleration. Displacement of earth (or sand, water, mud, etc.) is also another way of energy absorption. Energy absorbing seats are used to reduce the forces transferred to the occupants [6]. A large transport aircraft with considerable crushable subfloor depth may not need the energy absorbing seats or the landing gears, since the plastic deformation of a large subfloor may absorb most of the kinetic energy of the impact. However, for smaller fixed-wing aircraft and helicopters landing gears and seats play an important role in the energy absorption since the subfloor depth is not large [5]. Cargo restraints should be as light as possible, should occupy minimum storage space, be easy to install and remove, be adjustable for different cargo sizes, and should provide sufficient restraint of cargo in all directions [6].

First structural crashworthiness design requirements were established by military-MIL-STD-1290A in 1970s for military helicopters [11]. In 1972 NASA Langley Research Center started a crash safety joint project with FAA and industry which was aimed to develop new crashworthy design concepts for general aviation aircraft. A status report was published in 1980 [12]. According to the military standards, in a vertical impact with the landing gears extended and lift equal to the gross weight, the aircraft should withstand a 42 ft/sec vertical impact speed, without reducing the volume of the occupiable area more than 15% and transferring hazardous deceleration forces to the occupants. In case of retracted landing gears the aircraft should withstand a 26 ft/sec
vertical impact speed. In the design, an envelope of $+15^\circ$ to $-5^\circ$ of pitch and $\pm10^\circ$ of roll angles need to be considered in the vertical crash [6].

Main consideration while designing an airframe structure is to withstand airloads, ground handling loads and fatigue life. Effects of crash loads must be considered after basic structural layout is determined [6]. Crash resistance of an aircraft may be achieved efficiently during the design process or by retrofitting an existing aircraft with crashworthy components. Energy absorbing modification should be practical especially if crashworthiness of an existing aircraft is considered instead of a new design concept. Cost, weight, performance penalties should also be taken into account. Increasing strength is not the only way to improve the crash resistance. Excessively strong airframe

Figure 2.2: Design features for crash survival [6]
structure not only brings excessive weight but also increases the floor accelerations during a crash. The most important design requirement is to protect the shell around the occupant living space by minimizing the cabin penetration. Failed structural elements should not penetrate the occupied area. Inward buckling of sidewalls, bulkheads and floor should be prevented. Second, the forces and the deceleration loads transferred to the occupants should be in survivable levels, by designing the structure and seats accordingly. In order to maintain survivable conditions, acceleration magnitudes, durations, and rates of acceleration change experienced by the body should be controlled [6].

Predetermined relatively weaker points introduced in the design may help controlling the crushing response of the structure. Whenever possible multiple structural parts should be considered instead of a larger single piece, in order to avoid complete loss of post-crash integrity [6].

The most comprehensive source available on the crash survivability of fixed-wing and rotary wing vehicles is the US Army reports from 1989, which consists of five volumes [6]. Volume 1 is on the Design Criteria and Checklists which is about the information given in the other volumes, Volume 2 is on the Aircraft Design Crash Impact Conditions and Human Tolerances, Volume 3 is on the Aircraft Structural Crash Resistance, which is summarized in this literature survey, Volume 4 is Aircraft Seats, Restraints, Litters, and Cockpit/Cabin Delethalization and Volume 5 is on Aircraft Postcrash Survival [6].

Studies on the design of crashworthy vehicles can be grouped according to (1) the impact surfaces: such as impact to hard surfaces, soil or water, (2) the energy absorbing component: such as seats, landing gear, subfloor, cargo restraints, and (3) new concepts; such as deployable energy absorbing airbags.

According to the accident surveys in the last 18 years 25% of the helicopter accidents in UK and 11% of those in the US were water impacts or forced water landings and 29% of these were fatal [13]. The existing subfloor and landing gear concepts are not efficient in case of water impacts. One of the earliest studies on the water landing of helicopters was presented in 1953, which estimated the loads on the helicopter in case of
a water landing [14]. Literature survey shows that there is an increasing amount of interest on the water impact of helicopters starting in the 1990s. Several researchers included water impact in their studies of helicopter crash [15] - [21].

<table>
<thead>
<tr>
<th>Device Description</th>
<th>Energy Absorption Process</th>
<th>Tension or Compression</th>
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<tbody>
<tr>
<td>Strap/wire over die or roller</td>
<td>Metal bending and friction</td>
<td>T and C</td>
</tr>
<tr>
<td>Inversion tube</td>
<td>Hoop tension/compression and bending</td>
<td>T and C</td>
</tr>
<tr>
<td>Rolling torus</td>
<td>Cyclic compression and bending</td>
<td>T and C</td>
</tr>
<tr>
<td>Honeycomb compression</td>
<td>Buckling of membrane &quot;columns&quot;</td>
<td>C</td>
</tr>
<tr>
<td>Basic metal tube or plate</td>
<td>Elongation of metal</td>
<td>T</td>
</tr>
<tr>
<td>Basic stranded cable</td>
<td>Elongation of stainless steel</td>
<td>T</td>
</tr>
<tr>
<td>Tube Expansion</td>
<td>Hoop tension and friction</td>
<td>T and C</td>
</tr>
<tr>
<td>Tube Firing</td>
<td>Hoop tension, friction, and bending</td>
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<tr>
<td>Housed pulley</td>
<td>Shear and bending of sheave, housing</td>
<td>T</td>
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</table>

Figure 2.3: Comparison of load limiting devices used in seats [6]

In case of water or soft soil impact the loads are distributed along the skin of the subfloor as oppose to being concentrated on the frame and keel beams in case of hard surface impacts [16]. Soft soil impact was studied by researchers, such as by Fasanella et al. in their multi-terrain impact study [21], or by Bolukbasi who investigated the soft soil crash dynamics [22].
The most extensive review of crashworthy seats used in helicopters was done by Desjardins in 2006 where he traced the studies from 1960s to date [23]. Several types of energy absorbing mechanisms were explained, such as straps over rollers, inversion tubes, crushing of composite or honeycomb tubes. Some of the devices used for energy absorption in seats are compared in Fig. 2.3 [6]. In 2007 use of fluid dampers in helicopter seats for improved crashworthiness and vibration isolation is studied by Hiemenz et al [24], [25].

Landing gears are the first components of the aircraft which come into contact with the impact surface. The vertical velocities and the corresponding load on the landing gears can be seen in Fig. 2.4 [6]. For velocities less than 10 ft/sec landing gears perform normal landing, for velocities between 10 and 20 ft/sec (with roll or pitch) the goal of landing gear is to protect the fuselage, and for higher velocities its goal is to protect life [6].

One of the earliest papers on the energy absorption of landing gears is by Wharton from Lockheed Aircraft Corporation in 1960, where he presented an energy absorbing landing gear concept [26]. In 1989 Volume III of Aircraft crash survival design guide
provided information on the energy absorption of landing gears and different energy absorbing concepts. Some of these concepts can be seen on Fig. 2.5 [6].

In 1992 researchers in Eurocopter developed, manufactured and drop tested a composite fuselage section and landing gear [27]. In 1999 researchers from Bell Helicopter presented their study of landing gear simulations in case of hard landing conditions [28]. In 2004 Tho et al. presented the computational simulation results of helicopter skid landing gear dynamic structural behavior using LS-DYNA [18]. In 2007 same researchers presented utilization of optimization method for designing of crashworthy landing gears [29].

NASA Langley Research Center has a drop test facility, which is called The Impact Dynamics Research Facility (IDRF) with a 240 ft high structure. It was built in 1960s to be used as a Lunar Landing Research Facility [30]. In 1970s the facility is converted to a drop test facility, in order to study crushing of helicopters and light
aircraft. Karen Jackson and colleagues summarize the studies done between 1975 and 1999 in the drop test facility in a Journal of Aircraft paper [31]. The same team from NASA presented several papers related to occupant response in a crash, which is observed in the experiments in the drop test facilities and they also compared the experimental to computational results [32], [33], [34].

Due to the fact that there is not large amount of stroke available in the subfloor for energy absorption in helicopters, researchers recently focused on external deployable systems. In 2006 RAFAEL Armament Development Authority presented their study of “The Rotorcraft External Airbag Protection System” (REAPS), which inflates three external airbags with a help of a sensor [35]. Experimental studies show that the utilization of the airbag reduces the loads on the occupants significantly [35]. Another airbag concept is proposed by Boeing for UAVs [36]. The configuration, tests, modeling and simulations of this airbag are reported in 2007. In his paper Sotiris Kellas et al. present a deployable honeycomb energy absorbing concept [37]. They conclude that this system has the advantages over airbag system such as easy deployment, linear, radial or hybrid deployment directions and using honeycomb as crushing structure which provides a stable force regime.

2.1.1 Fuselage design

Limiting the loads by fuselage crushing is important since it reduces the deceleration forces transferred to the seats and the occupants [6]. Load carrying structural members of a typical helicopter with overhead mass are indicated on Fig. 2.6. Longitudinal beams have high bending and compressive strength; they also carry loads from cockpit to the cabin. Fuselage alteration should aim toward reducing soil scooping at initial contact to limit accelerations, reinforcing cabin and cockpit structure, modifying fuselage to provide controlled deformation, being resistive to penetration of failing parts, and providing breakaway of large mass items [6].
In longitudinal impact to flat surfaces most of the kinetic energy is dissipated by the friction between the aircraft and earth. Therefore, a lower percentage of the kinetic energy remains available for other forms of energy dissipation.
energy is absorbed by structural deformation. The main consideration in longitudinal impact is the post-crash integrity of the protective shell, minimizing the earth scooping and providing energy absorbing material in the forward of the occupied area. Longitudinal loadings are primarily responded by the subfloor beams which resist bending [6].

In vertical impact most of the energy is absorbed by structural deformation. The velocity change needs to be accomplished in a short period of time. The vertical crash conditions generally cause large structural deformations and high floor accelerations. Energy absorbing subfloor concept is incorporated in order to deal with these conditions [6].

![Figure 2.7: Idealization of fuselage structure with overhead mass [6]](image)

In order to investigate possible improvements in crashworthiness two types of idealized configurations are presented. One is to consider the fuselage with overhead mass where the aircraft mass is concentrated, as seen on Fig. 2.7. The fuselage acts as a nonlinear spring which behaves linear for small deformation and nonlinear beyond a critical value of force. Schematic view of fuselage with crushable regions in the subfloor and underneath the overhead mass is illustrated in Fig. 2.8. The idea is to initiate the
crushing of these regions earlier than the critical load so that the upper sidewalls do not deform [6].

Figure 2.8: Deceleration of overhead mass by energy absorbing concepts [6]

Controlled crushing of the helicopter subfloor contributes significantly to the energy absorption in vertical crush conditions. The subfloor is the region of a fuselage between the cabin floor and the outer skin. A typical subfloor structure consists of three main elements: longitudinal keel beams, lateral bulkheads and structural intersection elements, as seen on Fig. 2.9 [38]. Longitudinal keel beams generally span the length of the fuselage and support the seat rails. Subfloor crushing may help reducing the deceleration levels transferred to the occupants in case of vertical impacts by absorbing energy by large plastic deformations followed by instability failures or compressive collapse [5]. An important issue is that often other failure modes interfere and cause less than ideal crushing. Velocity effects are also another concern since dynamic effects may cause changes in the collapse mode [10]. Beside energy absorption, weight and airload issues, spacing for wiring and being practical and inexpensive are the determining design concerns [5]. Subfloor concepts which modify the existing structure are the most attractive since there is no additional weight introduced to the system.
If the subfloor loses its post-crash integrity this may cause the seats to come loose and create secondary impacts on the occupants [5]. The energy absorption of a subfloor is a function of the stroking distance between the cabin floor and the outer skin [6]. Figure 2.10 illustrates the principles of subfloor design which consists of a *strong structural floor* designed to carry loads and moments and sustain structural integrity, and *crushable zone* beneath it which is designed to crush in a controlled manner while absorbing as much energy as possible [5]. The crushable zone absorbs energy by large plastic deformations which come after instability failures or compressive collapse [5].
In order to provide efficient energy absorption, the load vs. deflection curve generated during the vertical impact should have a uniform crushing load as well as a high stroke-to-length ratio [6]. The crushing load of the subfloor should not exceed the design requirements in order not to transfer high deceleration loads to the occupants [5]. Existence of a high peak load at the initiation of stroking may cause the protective shell to collapse [6]. Highly reinforced subfloor which is designed to minimize the stroke will cause high load transfer to occupants. The desirable form of load-deflection behavior of the subfloor can be seen on Fig. 2.11. A “rectangular force response” is an efficient force-deflection curve, since an efficient system requires a constant load during the crushing and minimum spring back motion which creates a rectangular force response. The collapse loads are generally normalized with the weight or volume in order to compare the effectiveness. The “specific energy absorption” is the area under the force-deflection curve divided by the weight of the energy absorbing structure [10].

![Figure 2.11: Load –deflection behavior of a subfloor crushing [5]](image)

Several researchers investigated the crashworthiness of various subfloor concepts which are capable of operating at normal flight and landing conditions and also providing energy absorption. Figure 2.12 illustrates some of these concepts which were proposed by helicopter manufacturers earlier than 1989. Even though these concepts are proposed to be effective under vertical, longitudinal and lateral impact conditions, the most effective direction is vertical impact since it provides progressive collapse. Concepts 1, 2, 3 and 6
are studied by Cronkhite from Bell Helicopter, Concept 4 is proposed for Hughes 500 helicopter, and Concept 5 is proposed for a composite helicopter by Sikorsky [6].

Five different subfloor concepts which have been investigated by Bell Helicopter for NASA and results are reported in the Contractor Report can be seen in Fig. 2.13. Formable keel beam concepts absorb energy by the plastic deformation of the keel beam, the corrugated web concept uses the deformation of the webs and the crushing of the filler foam, corrugated half-shell concept absorbs due to the bending of the shell and finally foam filled cylinders absorb energy by crushing the foam. Most of these concepts had foam filler, and these were later rejected due to the extra weight, and not much gain in the energy absorption [5].

Figure 2.12: Energy absorbing subfloor concepts from 1980s [6]

Composite materials are highly employed in aircraft design due to their cost and weight savings and resistance to corrosion. The composite “Kevlar” keel beam shown on Fig. 2.12 (concept 6) used as an energy absorbing keel beam which was designed under the ACAP (Army’s Composite Airframe Program) project and meets the military crashworthiness requirements. Bell Helicopter was responsible for the investigation of the composite airframe structure. Experimental and computational results of this investigation are present in literature. Composite materials have the advantage of
reduced weight over metallic materials. However, since they are brittle they can not stand to high strain values as ductile materials. Therefore, energy absorption should come from an innovative design [6]. Ubels et al. [13] also investigated the crushing of sine-wave composite carbon-aramid keel beam concept. In this design Aramid was used to provide post-crash integrity. In 2009 Ludin et al. [39] suggested tube based graphite-epoxy composite beam design to absorb energy in helicopter crash. This design consisted of tubes sandwiched between two c-channel facesheets. Preliminary results suggest that crushing of composite round tubes is a promising area; however, crushing of the tubes in bundled configuration provides relatively less specific energy absorption [39].

Figure 2.13: Energy absorbing subfloor concepts proposed by NASA [5]

2.2 Honeycombs in energy absorbing applications

In order to be used in energy absorbing applications structures and materials should behave in a controlled manner while exhibiting a steady force level for a large stroke under compressive loading. The need for large stroke has led the researchers to focus on materials and structures that can undergo large deformations, such as
honeycombs or foams [1]. Crushing of honeycombs generate this desirable form of energy absorption. Studies show that in-plane and out-of-plane crushing of honeycombs do not generate high initial peak loads and there is a plateau load along a large stroke [7]. As shown earlier in Fig. 2.3 and 2.5 honeycombs are already in use in several energy absorbing applications, such as devices for seats and in landing struts.

Cellular structures are array of geometric elements. They are highly common in nature; such as bones, wood, cork etc., and characterized by the material of the structure, density of cellular structure, the mean cell diameter, and having open or closed cells [7]. They can be two or three-dimensional, have uniform or non-uniform cell geometries, high or low density. Materials such as metals, polymers, paper can be used to manufacture cellular structures. Metallic honeycombs are highly used in aerospace industry due to their relatively high energy absorption capacity and high strength – to – weight ratio. Metallic honeycombs preserve their structural integrity after crush, which makes them preferable over composite material applications. There are several parameters which affect the crushing performance of the energy absorbing honeycombs such as the orientation of the cells, cell geometry, packing etc [7]. Cells of the cellular structure may be hexagonal, square, circular, and may have different packaging such as loose or close-packed with different cell bonding [1].

In the 1960s Brentjes [2] was one of the first to report benefits of using honeycombs in energy absorbing applications due to their high strength to weight ratio. Even though honeycombs were most commonly used in sandwich structure as the core material, it was later found out that crushing of honeycombs in out-of-plane direction provide progressive cell wall buckling and a controlled load behavior which is very attractive for energy absorbing applications. This realization yielded utilization of honeycombs for instrument protection in rocket nose-cones which was introduced by Sandia Corporation in 1954. Other early energy absorbing applications of honeycombs include landing struts of Surveyor, seat suspension struts for Apollo landing module and tail bumper of Boeing 727 [2], [7].

In 1966 Shaw and Sata [40] studied the crushing behavior of cellular materials under compressive loading. They calculated an upper and lower bound for the initial peak
that is observed on the stress-strain curve. They also suggested an expression for the slope of the crush band. In 1979 Abd El-Sayed [41] was the first to calculate the in-plane elastic properties; Poisson’s ratios and Young modulus of cellular honeycombs. However, Gibson et al. [42] claims that Abd El-Sayed et al. have numerous errors in their calculations for the collapse load of honeycombs made out of plastic materials. In 1981 Gibson et al. [43] studied the elastic properties of cork, and discussed using it for energy absorbing purposes. Using honeycombs or foams as filler material in energy absorption is also cited in Aircraft crash survival design guide, in 1989 [6]. In 1993 Reid et al. reviewed the literature sources on the behavior of cellular structures [1]. A more extensive review of cellular materials is in the “Cellular Solids” book by Gibson and Ashby, which had the first edition in 1988 and the second edition in 1997 [7].

There are three main loading directions of honeycombs, as shown on Fig. 2.14. Loading along $X_3$ direction is referred as out-of-plane direction (core-wise), along $X_1$ and $X_2$ direction are in-plane directions (edge-wise) [7]. In some sources W, L and T are used for $X_1$, $X_2$ and $X_3$, respectively. When compressed in in-plane direction (along $X_1$ or $X_2$), cell walls deform linearly at first, and then depending on the material and the geometric dimensions, walls collapse by elastic buckling, plastic yielding, creep or brittle fracture. When compressed in the out-of-plane direction (along $X_3$), the deformation mode of the cell walls is either extension or compression; therefore, the collapse stresses are much larger. For a honeycomb structure, the in-plane stiffness is lower than the out-of-plane stiffness, since extension or compression of the walls require more load than bending.

In some cases crushing may be multi-axial, which means that more than one loading direction may be involved during impact. In 1989 Gibson et al. studied the mechanical properties of cellular materials under biaxial loading [44], [45]. They investigated elastic buckling, plastic bending and brittle fracture. More recently, in 2002 Chung and Waas studied the biaxial loading of circular cellular honeycombs, and they compared simulation results to experiments [46], [47].
2.2.1 In-plane direction crushing of honeycombs

The typical behavior of honeycombs under in-plane direction compressive and tensile loading can be seen in Fig. 2.15 [7]. It can be seen that the elastomers (such as rubber), elastic-plastic (such as metals), and brittle materials (such as composites) behave in different ways.

Under compressive loading (shown on the left hand side of the figure) the honeycomb structures first exhibit linear elastic behavior when the cell walls bend. Beyond a critical load the cell walls of the elastomeric honeycomb exhibit elastic buckling, in the case of elastic-plastic material they form plastic hinges, and for brittle materials they exhibit brittle fracture. Finally at high strain values when the opposing cell walls come into contact stiffness increases rapidly and forms the steep portion of the stress-strain curve called densification. For tensile loading (shown on the right hand side of the figure) initially cell wall bending forms the linear elastic region which has the same slope as the linear region of the compressive loading curve. For elastomeric materials, in tensile loading cell walls do not buckle but rotate towards the loading axis, for elastic-plastic materials at the yield point plastic hinges form (as in the case for
compressive loading). Brittle materials fracture at a stress lower than the peak observed with the compression loading [7].

Figure 2.16 illustrates the effect of wall thickness on the in-plane crushing behavior of honeycombs. It is apparent that increasing cell wall thickness increases the crushing load; however, densification occurs earlier since the cell walls touch at lower strain values [7].

Figure 2.15: In-plane behavior of honeycombs in compression and extension [7]
A two-dimensional honeycomb structure has four independent elastic constants; two Young moduli $E_1$ and $E_2$, a shear modulus $G$, and Poisson ratio $\nu_1$ or $\nu_2$ [42]. In-plane properties for two-dimensional hexagonal cells were calculated by Gibson et al. in 1982 [42]. For the linear-elastic part they have used the beam formulas neglecting the shear deformation and the axial extension or compression of the beams. They also assumed small deformations. The expressions Gibson et al. derived and presented in their paper from 1982 and also Cellular Solids book show that the in-plane properties are highly sensitive to geometric parameters [42], [7].

In 1988 Klintworth and Stronge [48] from Cambridge University presented the derivation of the equations for elasto-plastic honeycombs which undergo large deformations during in-plane direction loading. In 1996 Masters and Evans [49] suggested models to obtain general expressions for the in-plane elastic properties of honeycombs with positive and also negative cell angles. They concluded that there are three different deformation types which may be observed on the cell walls as a result of deformation; those are flexure, hinging and stretching. They used these three mechanisms to produce a general model. In 2002 Honig and Stronge [50], [51] published two papers

![Figure 2.16: Effect of wall thickness on in-plane loading direction behavior [7]](image-url)
related to the in-plane dynamic crushing of the honeycombs, and in these papers they not only summarized the expressions that were derived by Klintworth in 1988, but also simulated the dynamic crushing of honeycombs to visualize the initiation of the crushing band. In 2002 Chung and Waas [52] studied the effect of imperfections on the in-plane crushing response of circular honeycombs. They observed that cell ellipticity and cell wall thickness variation affect the elastic properties of the honeycomb.

In 1994 Papka & Kyriakides [8] published their study on the in-plane crushing response of the honeycombs using ABAQUS finite element method (FEM) analysis and compared their computational results with experiments. They simulated crushing of a unit honeycomb cell, as well as a multi-cell core which consisted of 9 rows and 6 columns. They only investigated regular honeycombs, which is the case when the cell angle is 30° and cell walls are at equal length.

Energy absorbed by honeycomb crushing can be obtained by calculating the area under the force vs. deflection or stress vs. strain curve. Area under the force vs. deflection curve gives the total energy absorbed by the crushing, and the area under the stress vs. strain curve gives the energy absorbed by unit volume. In computational studies, stress is calculated by dividing the reaction forces on the constrained nodes to the effective cross sectional area [8]. Stress vs. strain plots of a single and a multi-cell honeycomb crushing under in-plane direction loading which are obtained by Papka and Kyriakides can be seen in Figs 2.17 and 2.18 [8]. The three regions explained earlier, which are the initial elastic region, following plateau region and the final densification region can be seen on these plots. Experimental and simulation results for the in-plane crushing of regular multi-cell honeycombs are shown in Fig. 2.18. Even though the experimental result is slightly lower than the simulation results the general behavior is very similar. These results show that in-plane crushing of honeycombs creates a stable crushing load level over a large amount of stroke, as required for an energy absorbing structure.
Figure 2.17: Crushing behavior of single cell and honeycomb [8]

Figure 2.18: Crushing behavior of multicell honeycomb [8]
2.2.2 Out-of-plane direction crushing of honeycombs

When honeycombs are loaded in the out-of-plane direction, the cell walls behave as thin flat columns and buckle when the compressive load exceeds the critical buckling load [2]. The studies show that the buckling starts generally at the bottom or top of the honeycomb and folds around a hinge and progresses further like an accordion. This behavior causes fluctuations on the load behavior. For bigger cores (honeycomb with more cells) the imperfections on cells, which may be due to the cell wall thicknesses, bonding, cell geometry differences, cause some of the cells to buckle earlier than the others; this effect smoothes the load behavior [2]. Buckling of the walls, as the loading in the out-of-plane progresses can be seen in Fig. 2.19 [53].

![Figure 2.19: Buckling of the cell walls loaded in out-of-plane direction [53]](image)

One of the earliest works on out-of-plane crushing of honeycombs is done by McFarland in 1963 who studied the derivation of the crushing stress of honeycombs compressed in out-of-plane direction [54]. He obtained an expression for an upper and lower load value. According to McFarland effect of dynamic loading is same as static loading for metal honeycombs as long as the impact velocity is less than 50 ft/sec [54]. In 1964 he conducted an extensive static and dynamic experimental study to determine the specific energy absorption capacity of metal honeycombs which are subjected to out-of-plane compressive loading [55]. In this study three different geometries were examined;
typical hexagon, square packed cylinders, loose packed cylinders. He observed that the material with higher yield stress and the honeycomb with greater unit weight provide higher energy absorption. Another valuable work for out-of-plane direction loading is by Wierzbicki presented in 1983 [56]. He suggested an analytical formula estimating the mean crushing load of honeycombs which successfully matched the experimental results.

As reported by Wu and Jiang in 1996 [57], the stress vs. strain behavior of out-of-plane crushing of honeycombs generally shows a sharp initial peak and following to the peak some oscillatory fluctuations giving a relatively stable plateau level load. These authors studied experimentally the crushing of different aluminum alloy honeycombs with different cell numbers, and compared their results to analytical results. They reported that the crushing strength does not depend on the cell number [57].

The effect of relative density, specimen height to cell size ratio and bonding type on compressive response of out-of-plane crushing of square-honeycombs are investigated experimentally and analytically by Cote et al. in 2004 [58]. They have reported a good agreement of experimental and analytical results, and also that peak strength is not sensitive to bonding or the height to cell size ratio.

In 2005 Yamashita and Gotoh [59] studied the crushing of Aluminum 5052 honeycombs in out-of-plane direction and investigated the effect of cell shape and foil thickness. They reported that for impact tests up-to impact speed of 10 m/sec (32.8 ft/sec) static experiments can replace dynamic ones. In their study, “Y” shaped cross-sectional column model was used and concluded that the crushing behavior of this cross sectional model represents the crushing of a honeycomb tube as long as proper boundary conditions are provided.

2.3 Sandwich structures

Sandwich structures are structural members made by two skins separated by a lightweight core. Separation of the facesheets by the core increases the moment of inertia with minimal increase in weight, similar to the idea with I-beam. Generally a sandwich structure consists of stiff facings and low-density core, so that they provide the benefits
of high strength and low weight. The strength of the sandwich structure comes from the combination of the properties of the facesheets, core and the interface [7]. The core stiffness against deformation in out-of-plane direction and in shear should be high [60]. The facesheets resist the in-plane and bending loads, and the core resists transverse shear loads [61]. In some cases the facesheet thicknesses or the material may vary; for example in an application where one face should withstand high temperatures, corrosive environment, etc. [61]. Sandwich structures have several application fields such as aviation, automotive, wind energy, civil engineering, etc. There are several advantages of using sandwich structures; such as up to 30% weight savings compared to conventional structures, high flexural stiffness-to-weight ratio, good thermal, fatigue, acoustical insulation properties, resistance to local deformations providing aerodynamic efficiency, and easy mass production [60].

They can carry both in-plane and out-of-plane loads. Several different failure modes may occur under these different loadings. These modes are summarized in Fig. 2.20. The modes under in-plane compressive loading are also shown in Fig. 2.21. The geometrical and material differences between the facings and the core cause instabilities at global and local scales. If compressed in the in-plane direction the sandwich structure may fail due to the skin buckling or core wrinkling. Wrinkling may occur in the form of symmetrical or antisymmetrical as shown in Fig. 2.21.

Figure 2.20: Sandwich structure failure modes [3]
In some studies deriving the analytical expressions to estimate the failure loads, the researchers consider the sandwich beams as equivalent homogeneous columns for the global buckling mode, and as beams (skin) resting on elastic foundation (core) for local buckling (wrinkling) mode [62]. Damage on any of the structural components affects the overall performance of the sandwich structure. Therefore, several researchers have focused on the initial failure of these widely used structures. Hoff and Mautner [63], [64] presented one of the earliest works on the failure of the sandwich structures in 1940s. In 1945 they presented the experimental buckling crushing results of sandwich panels and compared the experimental results to theory [63]. In 1948 they presented a similar study for the bending of the sandwich panels [64].

![Figure 2.21: In-plane direction failure modes [65]](image)

In the edge wise direction loading (in-plane) of sandwich structures, researchers mostly study the failure and instabilities of these structures. Buckling of the sandwich plates with different boundary conditions is studied by Jenkinson and Kuenzi [66] in 1965, analytically and computationally by Heder [67] in 1991, and more recently a 2D
elasticity analysis for predicting the local and global instabilities is studied by Ji and Waas [68] in 2008. In 1984 and 1997 Gibson presented analytical approach to weight optimization of sandwich structures [69], [7]. Stiffness analysis of sandwich structures in the in-plane, bending and transverse shear loading conditions is presented by Gibson and Ashby [7], Hohe [70]. A summary for the wrinkling analyses and design formulas can be found in the NASA contract report presented by Ley et al. in 1999 [65]. Local and global buckling analysis is also performed by Hu et al. [62] in 2009, where they solved set of nonlinear equations in order to detect the buckling and wrinkling bifurcations and compared it to analytical critical load estimations. Free vibration and buckling analysis of foam filled honeycombs with the facesheets are studied computationally by Burlayenko and Sadowski [71] in 2009, and they concluded that the foam causes reduction in the magnitude of the natural frequencies but increase in mass. Foam filled sandwich structures including honeycombs are also studied by Pitaressi et al.[72] in 2007. However, Ludin et al.[39] reports that the weight increase due to moisture entrapment is a major drawback of using foam. In 2009 effect of material properties and geometrical configuration on fatigue damage is studied by Belouettar et al. [73].

Most of the studies on the sandwich structures focus on crushing of the cells (or foam) in out-of-plane direction in impact conditions; such as Goldsmith and Sackman [74] in 1992, Hazizan and Cantwell [75] in 2002. Even though there are numerous studies on this subject a survey was not needed since out-of-plane direction loading of sandwich structures is beyond the scope of this thesis.

2.3.1 Sandwich structures without the adhesive layers

In commercial applications mostly adhesive bonding is used to manufacture honeycombs with facesheets due to its low cost and practicality. This bonding is a weak point of sandwich structures especially when they are loaded in bending or in-plane compression [76]. There are several advantages associated with perfect bonding between the core and the facesheet: (1) better post-crash integrity, (2) higher energy absorption (EA) if debonding is avoided, (3) higher loads reached before failure occurs. Post-failure
crushing of sandwich structures with foam core under in-plane direction loading studied by Stapleton and Adams [77] shows the significant drop in the stress levels if debonding failure occurs. Experiment conducted by these researchers and corresponding force vs. displacement plot can be seen in Fig. 2.22.

Several different approaches are considered to prevent or stop debonding of facesheets from the core, such as embedding peel stoppers [78], manufacturing foam with injection molding technique [79], by fusion bonding of continuously produced thermoplastic honeycombs [80], or by methods like resistance welding, brazing bond and transient alloy diffusion [81]. Hanebuth et al. [82] compares the laser beam welding, resistance welding and furnace welding techniques for steel skins and corrugated core. Additional to the studies where different techniques for bonding facesheets (skins) to the core are explained, there are already researchers who focus on novel individual hexagonal cells which are manufactured with skin or segments with different stiffnesses at skin locations [83], [84]. A foam sandwich structure manufactured using injection molding technique with varying foam densities can be seen in Fig. 2.23.
Amongst all these techniques the one which is offering a potential manufacturing method for the sandwich structure studied in this thesis is brazing technique. Jing et al. [81] used vacuum brazing technique to manufacture integral carbon steel sandwich panels and they studied experimental crushing of these panels under in-plane direction loading. As mentioned by Hanebuth et al. [82] furnace (vacuum) brazing has several advantages; possible to achieve multiple joints in one operation, low warping, low cost fixturing, no necessary post treatment, etc.

Another example of use of brazing method on sandwich structures is the stainless steel aircraft fan blade which is designed using steel foam core and facesheet [85]. This fan blade which is designed by NASA has foam core and facesheet made out of 17-4PH stainless steel and for brazing alloy BNi-6 (which is a good brazing filler material for high temperature- high stress application and also for thin materials such as honeycombs) was used [86]. In another study related to the same fan blade, ultrasonic spectroscopy method is used in order to evaluate the quality of the brazing [87]. Three different brazing qualities; poor, medium and good, respectively are shown in Fig. 2.24 [87]. In the best quality brazing the filler material should be integrated well between the facesheet and the core.

Figure 2.23: Denser foam close to the surfaces in order to create the I-beam effect [79]
2.4 Summary

This chapter summarizes several publications related to the crashworthiness of helicopters, use of honeycombs in energy absorbing applications, sandwich structure crushing under in-plane direction loading and use of different techniques for manufacturing integral sandwich structures. Papers focusing on the use of honeycomb crushing under in-plane direction are highlighted since the research efforts presented in this dissertation covers crushing behavior and energy absorption of honeycombs under in-plane direction loading.
Chapter 3
Methodology

One of the objectives of this research is to obtain honeycomb cell geometry providing maximum energy absorption for minimum weight. Geometric parameters related to the cell geometries for the cells with and without facesheets were varied, crushing of cells with different geometries was compared, and the effects of each geometric parameter on the energy absorption were observed. Since honeycomb crushing is a non-linear process due to large deformations, material nonlinearity and contact conditions, an optimization technique could not be implemented. Crushing simulations of honeycomb cells with different geometries were conducted using a commercially available finite element tool ABAQUS/Standard. Supplemental experimental results and analytical expressions obtained from literature were also presented to validate the simulation techniques and as supportive information for computationally obtained results. The simulations can be grouped under two main titles; crushing of honeycombs without facesheets and honeycombs with facesheets. The cell geometries used in these two sections were very similar, except in the second part facesheets were placed on either side of the honeycomb and the geometry resembled a sandwich structure. Quasi-static and dynamic simulations were conducted in ABAQUS/Standard, version 6.8. Dynamic simulations were only conducted for the out-of-plane direction crushing of honeycomb cores in order to visualize crushing at higher global strain levels. Quasi-static crushing of the cells was simulated for the rest of the study. In order to verify that ABAQUS simulation results successfully predicts the crushing behavior, validation studies were conducted for the honeycombs with and without facesheets by comparing the simulation results to previously published computational, analytical or experiments results. The Methodology chapter explains the simulation methods used in this study and provides general information about honeycomb crushing in order to prepare a base for the results chapter.
3.1 Honeycomb crushing

As provided in the Literature chapter, there are several studies which present the honeycomb crushing from different aspects; however, the most comprehensive source about cellular structures is the book by Gibson and Ashby [7]. In this dissertation the nomenclature related to the crushing directions presented in their book is also used, which is shown in Fig. 3.1. Crushing in $X_1$ and $X_2$ directions are called in-plane direction and in $X_3$ direction is called out-of-plane direction crushing. Both in-plane and out-of-plane crushing provide progressive form of crushing behavior; therefore, use of honeycomb is suitable for energy absorbing applications.

![Honeycomb core and directions related to a honeycomb](image)

Figure 3.1: Honeycomb core and directions related to a honeycomb [7]

The cells of a cellular structure may be square, rectangular, hexagonal etc. Honeycomb geometry considered in this study has hexagonal cells with varying cell geometric parameters. The cell geometric parameters related to a hexagonal cell can be seen in Fig. 3.2. The design parameters which determine the hexagon geometry are $h$: the vertical wall length, $l$: the inclined wall length, $t_h$: thickness of the vertical walls, $t_l$: thickness of the inclined walls, $\theta$: the cell angle, which is the angle between horizontal direction and the inclined walls, and $b$: the cell depth. The properties are often described by using non-dimensional set of parameters: $\alpha = h/l$ (cell aspect ratio), $\beta = t_l/l$ (inclined
wall non-dimensional thickness), \( \eta = \frac{t_h}{t_l} \) (vertical to inclined wall thickness ratio), \( \gamma = \frac{b}{l} \) (non-dimensional cell depth). The geometric parameters are also given in the Table 3-1. For the cells with facesheet, the thickness of the facesheet, \( t_f \) is also a geometric parameter.

![Cell geometric parameters of a hexagonal cell](image)

Figure 3.2: Cell geometric parameters of a hexagonal cell

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>Cell vertical wall height</td>
</tr>
<tr>
<td>( l )</td>
<td>Cell inclined wall length</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Cell angle – Angle between horizontal axis and inclined wall</td>
</tr>
<tr>
<td>( t_h )</td>
<td>Vertical wall thickness</td>
</tr>
<tr>
<td>( t_l )</td>
<td>Inclined wall thickness</td>
</tr>
<tr>
<td>( t_f )</td>
<td>Facesheet thickness</td>
</tr>
<tr>
<td>( \alpha = \frac{h}{l} )</td>
<td>Cell wall length ratio</td>
</tr>
<tr>
<td>( \beta = \frac{t_f}{l} )</td>
<td>Inclined wall thickness to length ratio</td>
</tr>
<tr>
<td>( \eta = \frac{t_h}{t_l} )</td>
<td>Vertical to inclined wall thickness ratio</td>
</tr>
<tr>
<td>( \gamma = \frac{b}{l} )</td>
<td>Cell depth to inclined wall length ratio</td>
</tr>
</tbody>
</table>
The most common metallic honeycomb manufacturing technique of “expansion” technique involves gluing sheets of virgin material at specific locations and expanding afterwards. Therefore, generally the vertical walls of the hexagonal cells have the double thickness of the inclined walls.

As mentioned in the Literature chapter, for the impact velocities less than 50 ft/sec quasi-static results are accurate for metallic honeycomb crushing. Therefore, in the simulations quasi-static cases were conducted. For out-of-plane direction crushing, dynamic simulations converged for higher global strain values; therefore, the results for quasi-static and dynamic simulations are presented.

Mesh convergence study was conducted for each case in order to find the required number of elements. It is important to have sufficient elements; however, having more than necessary number of elements lengthens the simulation time.

Crushing of a single cell honeycomb without any interaction property defined determines which walls need contact properties. This means initially in the simulations the penetration of the opposing walls through each other was allowed, and this does not represent a real crushing case. When contact properties are applied to the walls, realistic contact conditions occur between the walls where no penetration occurs. For a single cell simulation seven self-contact conditions were defined; one was for the contact of the inner 6 walls of the hexagon, two of the conditions were defined for the right and left outer sides of the walls, and the last four were defined for the contact of the short vertical walls to the inclined walls. The normal behavior of the contact has a pressure-overclosure modification, which allows the pressure to increase exponentially as the surfaces come into contact [88]. Similar to the single cell honeycomb, multi-cell honeycomb cells also had self-contact properties defined for the interior of each cell. In the dynamic simulation of out-of-plane direction crushing, general contact conditions were defined, which also helped with convergence of the simulations.

The resisting force that the structure generates over a stroke when it is crushed can be presented with a stress vs. global strain curve, and the area under the curve gives the absorbed energy per unit volume. Stress (or global stress) is the total reaction force per effective cross section area and the global strain is the crushed displacement of the
honeycomb compared to the total length of the uncrushed honeycomb. Therefore, higher plateau stress for a longer stroke provides higher energy absorption. In this thesis the crushing behaviors of single and multi-cell honeycomb cores are presented with stress vs. global strain plots. Specific energy absorption (SEA) is the energy absorbed per unit mass, and it is simply calculated by dividing the total energy absorption to the mass of the crushing structure. For the study where no facesheets were involved charts are used in order to compare SEA. In some applications, energy absorbed per unit volume might be needed, and therefore, these quantities are compared as well. More information on how to calculate the total energy absorption and the specific energy absorption can be found in Appendix A.1. For the crushing of the cells with the facesheets the results are presented in stress vs. global strain plots and mass-normalized stress vs. global strain plots in order to observe the effect of mass.

3.1.1 In-Plane crushing of honeycombs

As a result of an extensive literature survey on the use of honeycomb crushing in energy absorption a journal paper by Papka and Kyriakides [8] was chosen as the reference paper for the first part of this study where honeycomb crushing was simulated (no facesheets). They have conducted the in-plane direction crushing simulations of hexagonal honeycombs using ABAQUS and compared their results to experiments. In their simulations and experiments they considered regular honeycombs (30° cell angles and equal length cell walls). Most of the commercially available honeycombs have 30° cell angles and the cell walls have equal lengths. This specific geometry is called regular cells and crushing of the regular cells was taken as the baseline in this study.

In the present dissertation crushing simulations of regular honeycombs were conducted and validated against Papka and Kyriakides’ paper [8]. The material properties and the baseline geometric values were based on this reference paper. The material used in all of the simulations was considered to be Al-5052-H39. Even though the Young’s modulus and the yield stress values of this specific material were used in the simulations, any elastic-plastic material would give similar behavioral results. The stress-strain
behavior of Al-5052-H39 was assumed to be bilinear with pre-yield modulus of 68.97 GPa, yield stress of 292 MPa and the post yield modulus is 0.69 GPa. Density of Aluminum is also required for dynamic analysis and the mass calculations and it is 2680 kg/m³. In the regular geometry, vertical and inclined cell walls had equal length of \( h = l = 5.5 \) mm. The inclined walls had thickness of \( t_l = 0.145 \) mm, the vertical walls had double thickness of inclined walls, \( t_h = 0.290 \) mm and the cell depth was \( b = 10 \) mm.

A two dimensional model was used in the in-plane direction study with B22 beam elements. These elements follow the Timoshenko beam theory and have 3 element-nodes; 1 internal and 2 end nodes, each of them having 3 degrees of freedom; horizontal displacement, vertical displacement, and rotation.

![Mesh convergence study for the 2D cell](image)

Figure 3.3: Mesh convergence study for the 2D cell; Number of elements varying along a) the vertical wall, b) the inclined wall
For the single cell, 2, 4 and 20 equal length elements were respectively used along the short vertical walls, long vertical walls, and along the inclined walls. This selection was based on the results of a convergence study; results can be seen on Fig. 3.3 (a) and (b). The vertical wall mesh sensitivity results are shown in Fig. 3.3 (a) and the inclined wall mesh sensitivity results are shown in Fig. 3.3 (b). It is apparent that for the number of elements that were studied the results did not vary. For the inclined walls, 20 elements were considered to be sufficient since most of the cell deformation occurred on the inclined walls. For the Vertical walls 4 elements were selected since the vertical walls go into a rigid body deformation, and these were sufficient to visualize the deformation. Selecting 20 elements for the inclined walls instead of lower number of elements did not cause any significant increase in the computation time.

In the single and multi-cell model some boundary and symmetry conditions are required to suppress the rigid body motion. Vertical displacements of nodes a, b, c, and horizontal displacement of b were constrained (Fig. 3.2). Rotations of nodes d, e and f were matched to those at nodes a, b and c, respectively. Loading was created by imposing downward vertical displacements of equal magnitudes on nodes d, e, and f. Nodes are shown in Fig. 3.2.

![Figure 3.4: Full scale honeycomb core geometrical parameters and boundary conditions](image-url)
Besides single cell, crushing of multi cell honeycombs was also simulated. An undeformed 9x6 (9 row x 6 column) honeycomb core can be seen in Fig. 3.4 with applied boundary conditions. As with the single cell simulations, the vertical displacements of the bottom nodes were constrained, in order to suppress the rigid body motion. Horizontal displacements of two nodes; one bottom and one top, were constrained in order to model the effect of friction along external contact surfaces. Honig and Stronge [50] used these boundary conditions in order to generate similar boundary conditions to dynamic crushing conditions. They concluded that the static loading study with these defined conditions is nearly identical to a model with contact surfaces. In the multi cell models B22 elements were also used. Similar to the single cell simulations, interaction properties were defined for the walls which were anticipated to contact.

Some type of imperfection, such as wall misalignments, thickness or length variations is inevitable in a manufactured honeycomb. As explained in the Results chapter existence of an imperfection changes the crushing mode from symmetric to asymmetric. In the symmetric mode the vertical walls do not rotate and hinges occur at the both ends of the inclined walls. In the asymmetric mode vertical walls rotate, hinge at one end, and local stress reduction occurs at the other end of the inclined wall. An imperfection was also considered in the simulations by imposing a small 0.2° initial wall misalignment. The imperfection on the single cell honeycomb was at the long vertical walls (walls between nodes mo and np) and in the full size honeycomb core it is imposed on the vertical wall of the middle cell (3rd row, vertical wall common to 3rd and 4th cells in case of 9x6 honeycomb core). In Chapter 4 (results of the crushing of the cells without facesheets) the effect of imperfection on the crushing behavior is shown for imperfection varying between 0.1° and 4° misalignment.

For the cells without facesheets, initially results for honeycombs with regular cell geometry were validated. These include effect of material (elastic and elastic-plastic materials), and post-yield modulus on the crushing response. Following to the validation, simulation results for honeycomb with single cell and multi-cell cores were compared in order to investigate the sufficiency of simulating a single cell with periodic boundary conditions. Results with single-cell and multi-cell cores for different cell angles were
also compared. When it was assessed that simulating crushing of a single cell honeycomb was sufficient, the parametric study was conducted where the effect of non-dimensional geometric parameters on the energy absorption and crushing behavior of hexagonal cells was studied while $\theta$, $\alpha$, $\beta$ and $\eta$ values were varied. The baseline for $\theta$ was $30^\circ$; additional to the baseline $15^\circ$, $45^\circ$ and $60^\circ$ cell angles were studied. The baseline for $\alpha$ was $1$; additional to the baseline $2$, $0.5$ and $0.25$ cell wall length ratios were studied. The baseline for $\beta$ was $\beta^* = 0.026$ since the ratio of $t = 0.145$ to $l = 5.5$ mm gives this value; additional to the baseline $2\beta^*$ and $0.5\beta^*$ were studied. The baseline for $\eta$ was $2$; additional to the baseline $4$, $1$ and $0.25$ cell wall length ratios were studied. The results of the in-plane direction crushing of the cells without facesheets are given in Chapter 4.

Crushing of the honeycombs without facesheets can also be modeled in 3D, using shell elements. However, simulations using 2D model with beam elements are more cost effective compared to the 3D models since they take less amount of computational time. Further discussion on the comparison of the results of 2D beam and 3D shell elements for the crushing of honeycombs without facesheets is given in Appendix A.2.

### 3.1.2 Out-of-plane crushing of honeycombs

Analytical, computational, experimental studies on the out-of-plane direction crushing of honeycombs and utilization of this crushing mode in energy absorbing applications are often studied by researchers. In the present study the main crushing direction was considered to be the in-plane direction. The reason for the out-of-plane direction crushing simulations which are conducted within this study was to compare their specific energy absorption capacity to the in-plane direction crushing of honeycombs and honeycombs with facesheets. Therefore, only baseline cell geometry was used in the out-of-plane direction simulations; where the cells had $30^\circ$ cell angle and $5.5$ mm cell wall length. Cell depth was varied to observe the effect of height of the cell on the crushing behavior. Dynamic simulations were conducted for this type of crushing as well as quasi-static simulations. Results are presented in Chapter 4 (honeycombs without facesheets).
3.1.3 Honeycombs with facesheets – Sandwich configuration

The main objective of the dissertation was to employ energy absorbing honeycombs in a crashworthy helicopter subfloor application. One way to use honeycombs in the subfloor is to use them as a part of a crushable keel beam. In this study the in-plane direction crushing of honeycombs was proposed; and therefore, they were placed in the keel beam as shown in Fig. 3.5. It may be considered as the web section of an I-beam, but the flanges were not considered in the simulations. This configuration is similar to sandwich structures where the honeycomb core is placed between two stiff facesheets. The results of the crushing of the cells with facesheet are presented in Chapter 5 (honeycombs with facesheets).

Figure 3.5: Keel beam with crushable honeycombs, a) web section, b) I-beam

In literature there are several computational, experimental and/or theoretical studies on sandwich structures with honeycomb or foam core, some of which focus on the in-plane direction crushing of them. These studies generally present ways of predicting the failure load of sandwich structures under different types of loadings and the mode of failure [63] - [65], [89]. Computational studies are generally solely buckling analyses and do not include post-yield behavior of these structures [62], [83], [90]. As explained in more detail in the Literature chapter, different failure modes related to in-plane direction loading are defined; such as dimpling (intracellular buckling), wrinkling,
facesheet debonding, core shear etc. If an adhesive layer is used in order to attach the facesheet to the core then debonding might be expected as one of the possible failure modes. However, recent sources also report different methods of facesheet-core bonding such as brazing for metallic core and facesheets [81]. A method for brazing of steel sandwich structures with honeycomb core is patented in 1984 in US [91]. In that case it is possible to create an integral structure where debonding of the facesheets is not one of the possible failure modes; therefore, the sandwich structure can carry much higher loads and the structure preserves its post-failure integrity which is an important requirement in crashworthy applications. The present study also assumes an integral structure and as a result only dimpling and wrinkling modes are observed in the simulations. Similar to the part where no facesheets were involved, a validation study was also conducted for the cells with facesheets. In this part the reference paper, which is used in the validation study was Jing et al. [81]. They have conducted experiments for the crushing of carbon steel honeycomb and facesheet sandwich structure under compressive loading where the honeycombs were placed in the in-plane direction. Their paper provides the material properties for the carbon steel and geometric properties for the facesheet and honeycombs. In order to validate the simulation techniques used for the present study, the crushing simulation of a single cell honeycomb with facesheets which possessed the same geometric and material properties of the samples used in Jing et al. study was conducted, and the simulation results were successfully compared to their experimental results.

In-plane crushing of the honeycomb with facesheet sandwich structure was modeled in 3D using the single cell model shown in Fig. 3.6. In this model shell elements called S4R were used, since one-dimension (thickness) is smaller compared to other two dimensions. These are conventional 4-node, quadrilateral shell elements with reduced integration and large strain formulation. Reduced integration means that these elements use reduced integration to form element stiffness matrix, but regular integration for mass and distributed loading matrices. Reduced integration provides more accurate results in less computational time [92]. They have displacement and rotational degrees of freedom. S4R type elements are in the group of elements called “General purpose, conventional shell elements” which provide robust and accurate solution for most
applications [92]. These elements allow transverse shear deformation; they use “thick shell theory” for thick elements and become “Kirchhoff thin shell elements” as the thickness decreases. Shear deformation becomes very small as the element thickness decreases. They account for finite membrane strains and large rotations; therefore, suitable for large-strain analysis. In geometrically nonlinear analysis the thickness of the elements changes as a function of membrane strain (more information can be found in Appendix A.3 ) [92].

In the baseline case the cell core had the same geometry as in the part of the study where no facesheets were considered. Therefore, the inclined and the vertical cell wall lengths were equal to 5.5 mm, the cell angle was 30°. The facesheet thickness was varied; the minimum value for the facesheet thickness was 0.145 mm, which was the same as the inclined wall thickness, and the maximum value for the facesheet thickness was $t_f = 1.16$ mm which was eight times the inclined wall thickness. In the regular case $\eta = 2$, meaning that the core vertical walls had twice the thickness of the inclined walls.

![Figure 3.6: The unit cell of a sandwich structure with variable dimensions](image)

Only half of the model was used in the simulations with symmetric boundary conditions. Boundary conditions can be seen on Fig. 3.7 (a) and the mesh of this model can be seen on Fig. 3.7 (b). The edges at the middle-core plane had symmetric boundary condition in Z-direction (meaning displacement in Z, rotations around X and Y were constrained), the edges on the bottom of the structure had symmetric boundary conditions
in Y-direction (meaning displacement in Y, rotations around X and Z were constrained), the vertical left and right edges of the facesheet and the core had symmetric boundary conditions in X-direction (meaning displacement in X, rotations around Y and Z were constrained). The middle edge of the core at the bottom and at the top were constrained to move in X-direction, in order to prevent rigid body movements. The edges highlighted with red lines show the location of the displacement loading. Facesheet was slightly longer than the core in the y-direction and the displacement of the top edge on this piece was constrained not to move in Z-direction in order to provide the directionality of the loads and to create the shear effect of an existing loading surface (such as flanges). Additional to these boundary conditions the nodes along the facesheet on the right and left side were constrained to have same amount of displacement in Z-direction.

Figure 3.7: Model for the keel beam study; a) boundary conditions, b) Generated mesh
As a result of a mesh convergence study the following numbers of elements were chosen for the corresponding edges: the cell depth had 10 elements (for the half model), the inclined walls had 16 elements along its length, the vertical walls and the central line of the hexagon facesheet had 16 elements (can be seen in Fig. 3.7 (b)). Mesh convergence study results for the sandwich cell model can be seen in Fig. 3.8. Each plot includes results for the cells with thin and thick facesheets.

The effect of number of elements on the core depth is shown in Fig. 3.8 (a), along the centerline of the facesheet is shown in Fig. 3.8 (b), and along the inclined walls is shown in Fig. 3.8 (c). Number of elements along the cell depth affects the behavior of the cells with thick facesheets, but increasing number of elements provides much better results. For the inclined wall and the facesheet, increasing number of elements did not change the results, which means that the numbers of elements chosen for the mesh convergence study are high enough to provide successful simulations.

The effect of geometric parameters on the crushing behavior of sandwich cells was also studied. Initially the effect of facesheet thickness, $t_f$ on the failure modes and the crushing behavior were studied. In this study cell depth (or the keel beam width) was 10 mm, and the facesheet thickness was varied as multiples of the cell inclined wall thickness. Effect of the core geometric parameters $\theta$, $\alpha$, $\beta$, $\eta$, and $\gamma$ on the in-plane direction crushing behavior of sandwich structure was also studied. The baseline value for cell angle was $30^\circ$, and additionally crushing of the cells with $15^\circ$ and $45^\circ$ cell angle was studied. The baseline value for wall length ratio was $\alpha=1$, where the vertical wall length was equal to the inclined wall length. In addition crushing of cells with $\alpha=0.5$ and $\alpha=2$ was studied. The baseline value for $\beta$ was $\beta^*=0.026$. Additionally cases with $2\beta^*$ and $0.5\beta^*$ were studied. The baseline value for the wall thickness ratio was $\eta=2$, where the vertical wall thickness was twice the thickness of the inclined walls. Additionally crushing of the cells with $\eta=1$ and $\eta=4$ was studied. $\gamma$ is the ratio of cell depth, $b$ to the inclined wall length, $l$ and variation of $\gamma$ means variation of cell depth which is covered in the keel width study as explained further in the chapter.
Figure 3.8: Mesh convergence study for the 3D cell; Number of elements varying along a) the cell depth, b) the facesheet centerline, c) the inclined wall
In order to study the effect of numbers of rows of cells (fewer rows of larger cells vs. many rows of smaller cells) in a keel beam, specific dimensions were considered for the keel beam. Keel beam dimensions were selected after reviewing literature sources. An average value for height and width of the keel beam were chosen. Figure 3.9 is taken from a study which investigates the crushing of Army’s Advanced Composite Airframe Program (ACAP) cabin section composite subfloor structure [93]. For this cabin section the crushable portion of the keel beam height is considered to be 254mm (10’’)[93]. In another study, where sinusoidal web and bulkheads are studied the keel beam height was considered to be 225mm (8.85’’)[94], and for a crushing of a riveted intersection study the height was 195mm (7.6’’)[38]. Similar to these sources in this study the height of the keel beam was considered to be 291mm (~11.5’’). There is a wider range of keel beam width given in the sources: Taher et al. [95] assume a keel beam width of 150mm (5.9’’), Cronkhite et al. [5] take width of 25.4 mm (1’’) for the web section of the formable keel beam, and Bisagni [38] assumes web section width of 70mm (2.75’’). In the present study the keel beam width (cell depth) was assumed to be 101.6mm (4’’). These dimensions represent the web section, and the flanges were not taken into account in this study.

Figure 3.9: Composite cabin test section of ACAP helicopter [93]
3.1.3.1 Effect of cell depth – Keel width study

The baseline core geometric values for the cells with facesheets were same as the study where no facesheets were considered. Therefore, the baseline for cell depth (keel beam width) was 10mm. As explained earlier, when specific dimensions were considered for the web section of the keel beam, the keel beam width was assumed to be 101.6mm. These two sections of the study can be considered as lower and upper bound for the width of the web section.

In the first part where the width was assumed as 10 mm, a keel width study was conducted where cells with 20, 10 and 5 mm depth were modeled and simulated. Results showed the effect of the increasing or decreasing depth on the crushing behavior of the honeycombs with facesheets.

3.1.3.2 Effect of number of rows

Since a specific dimension was considered for the keel beam height, changing the cell wall length changes the number of cells (or number of rows) which can be fitted into the web section of the keel beam. Instead of simulating a multi-cell honeycomb and facesheet model crushing which would be computationally expensive, a single cell was used. Since a single cell model was used in the simulations the only way to consider different numbers of rows is to change the cell wall length. Therefore, different cell wall lengths were used in this study, to create the effect of varying numbers of rows. In order to keep the other non-dimensional parameters unchanged at $\theta = 30^\circ$, $\alpha = 1$, and $\eta = 2$, other cell dimensions were also varied with varying $l$. In order to keep $\beta = 0.026$ constant, wall thicknesses had to vary accordingly with varying wall length.

Both thin and thick facesheets were considered where the thin case facesheet had the thickness of the inclined wall and the thick facesheet had four times the thickness of the facesheet. Beyond this point two different paths were followed for the cell depth. One method is to keep the keel beam width constant and the other is to keep $\gamma = b/l$ constant. If a fixed value is considered for the keel beam width while the wall length varies these
results in two changing variables; number of rows and \( \gamma \). In this method the baseline cell wall depth (or the keel beam depth) was taken as 101.6 mm. In the other method the cell depth also varied with varying cell length providing constant \( \gamma \). For the baseline sandwich structure where the wall length, \( l \) was 5.5 mm and cell wall depth, \( b \) was 101.6 mm the \( \gamma=b/l \) value can be calculated as 18.47. When the \( \gamma \) value was kept constant, the cell depth had to vary.

It is possible to derive an equation which relates the total height of the keel beam and the cell wall length. A schematic figure for the height of the keel beam or the multi-cell core, number of rows and the wall length is illustrated in Fig. 3.10. The total height of the structure (in this case the keel beam), \( H \) can be found by using Eq. 3.1 which uses the number of rows, \( N \), the wall length, \( l \), the ratio of the vertical wall length to the inclined wall length, \( \alpha \), and the cell angle, \( \theta \). For the regular cell where the cell angle is \( \theta=30^\circ \), \( \alpha=1 \) Eq. 3.2 gives a simplified form.

\[
H = l(N\alpha + (N + 1)\sin\theta) \quad \text{Eq. 3.1}
\]

\[
H = \frac{3N + 1}{2} l \quad \text{Eq. 3.2}
\]
For the baseline study the cell wall length was chosen to be 5.5 mm and cell angle was 30°. Therefore, for a keel beam which has around 291 mm of height, 35 cell rows can be fitted to the web section. Increasing the cell wall length will reduce the number of rows to be placed in a keel beam with fixed height. The effect of number of rows on the crushing behavior was studied by changing the wall length (and keeping the other parameters constant). The wall length and the corresponding number of rows are shown in the following table, Table 3-2.

<table>
<thead>
<tr>
<th>Number of rows, N</th>
<th>Cell wall length, l (mm)</th>
<th>b (mm)</th>
<th>t_l (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.936</td>
<td>35.76</td>
<td>0.05</td>
</tr>
<tr>
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<td>5.5</td>
<td>101.6</td>
<td>0.145</td>
</tr>
<tr>
<td>5</td>
<td>36.44</td>
<td>672.86</td>
<td>0.96</td>
</tr>
</tbody>
</table>

### 3.1.3.3 Initial Imperfection

Crushing of honeycombs with facesheets is a nonlinear post-buckling problem which makes it a complex case. In ABAQUS the way to do post-buckling analysis is to introduce imperfections to the model. There are several ways to do this. Similar to the study where no facesheets were involved one can introduce geometric imperfection to any node at the step where the part geometry was established. The other way, which is preferred for more complex problems, is to first run a buckling analysis and store the deformation of the modes, and introduce this buckling mode to the initial geometry of the post-buckling analysis. When the crushing of the honeycomb with the facesheets analysis was conducted the second method was followed. First the buckling modes were extracted with a buckling analysis using ABAQUS keyword *nodefile, then initial modes were embedded into the static crushing simulation with the keyword *imperfection.
The mode and the amplitude of the nodal displacements have great importance in the simulations. The amplitude of the introduced nodal displacement should not be too high; otherwise the initial problem changes into a different geometry. If the amplitude of the deformation is too low the imperfection on the geometry will be too low to start a post-buckling analysis. The way which was followed in this study included an effect of imperfection study, where for every simulation case the effect of imperfection was studied by comparing the results for every imperfection case. The amplitude of the imperfection was chosen small so that there was not any effect of imperfection on the general crushing response of the structure. The only reason of introducing the imperfection is to initiate the post-buckling analysis.

The imperfection might be introduced using any mode which appears as results of the buckling (eigenvalue) analysis. In this study the first few modes were introduced to the initial geometry of the post-buckling analysis study, where the deformation was only on the facesheets. For the cells with thin or thick facesheets, same buckling modes were introduced to the structure in the imperfection step. Even though the facesheet central nodes (and the nodes at the vicinity of the facesheet center) were displaced slightly due to these imperfections, this did not bias the post-buckling crushing mode of the whole cell.

### 3.2 Discussion on the maximum strain of the material

The material properties used in this dissertation was for Aluminum 5052- H38 since the same material was used by Papka and Kyriakides [8] and their paper was selected to be used in the validation study for the first part. In literature for material standards the maximum elongation of this material is reported to be 7% elongation [96]. However, this maximum elongation strain was not assumed to be a failure point. The maximum strain values were observed in simulations for the cells with facesheets and they are greater than 7% but lower than 30%. For the cells without facesheet maximum strain were lower than 7%. “Metal handbooks” [97] lists the material properties for several different aluminum alloys. There are alloys with Yield strength of 290 MPa and Young’s modulus of 69 GPa and with elongation of 35% strain. This shows that even
though for this study Aluminum 5052-H38 was chosen, it is possible to assume any other material with higher elongation capability. For example Aluminum 5056-O has Young’s modulus of 71 GPa, Tensile strength of 290 MPa and elongation capacity of 35%. This means that high strain locations detected as a result of the simulations are not material rupture locations since strains were never greater than ~30%.
Chapter 4

Crushing of honeycombs without facesheets: Results and discussions

The main objective of this thesis was to develop a high performance crashworthy cellular structure with significantly increased energy absorption capability and utilize the concept in helicopter subfloor. The first sub-objective was to study the in-plane direction crushing of hexagonal honeycombs, to understand the effect of geometric parameters on the crushing behavior and to suggest cell geometry with improved energy absorption. The second sub-objective was to integrate the energy absorbing honeycombs to an application which can be used in helicopter subfloor. The results chapter includes the results from these two sub-objectives. Initially the quasi-static in-plane direction crushing of hexagonal honeycombs was computationally studied using ABAQUS/ Standard. Out-of-plane direction crushing of regular honeycombs was also simulated and the energy absorption was compared to that of in-plane direction crushing. The keel beam study results follow this section.

4.1 In-plane direction crushing of honeycombs

Figures 4.1 and 4.2 show the crushing behavior of a honeycomb unit cell and deformed cell shapes corresponding to various strain locations. Typically, when honeycombs made out of ductile materials, such as aluminum, are crushed three regions appear on the stress vs. strain plot; the initial linear elastic region, followed by the plateau region and finally the steep “densification” region [7]. Gibson and Ashby explain these regions as following: In the initial linear region, until a critical stress is reached, stress increases linearly with increasing strain. When plastic hinges develop on the inclined walls the linear elastic behavior ends, and the plateau region starts. The region called densification follows the plateau region. In the densification region, stress increases rapidly due to the contact of the opposing walls. Even though this explains the general
compressive behavior of honeycomb crushing, changes in the material properties and geometric parameters result in differences in the output, such as higher or lower plateau stresses, or smaller or larger deformation (stroke).

Imperfections in the geometry cause a different mode of crushing where rotations of vertical walls take place, causing delay of the densification region, as shown in Fig. 4.2. Imperfections in the simulations were introduced by misalignment of the vertical walls. The deformation in the perfect case (where no initial imperfection was involved) (Fig. 4.1) is uniform until the end of the densification and the upper and lower walls

Figure 4.1: Compressive behavior of a perfect 30° honeycomb single cell model

Figure 4.2: Compressive behavior of an imperfect 30° honeycomb single cell model
come in contact around 70% global strain value. In the case of an imperfect cell crushing, uniform crushing ends when the plateau region starts, the vertical walls rotate, and densification occurs at higher global strains (Fig. 4.2). Experiments show that in manufactured honeycombs there is always a type of imperfection which causes the asymmetric mode of deformation, where the vertical walls tilt causing the rows to fold over each other. If there was no imperfection on the geometry the crushing would be uniform which means that the vertical walls do not tilt but compress.

Effect of initial misalignment of the vertical walls on the stress-strain plot was studied for 0.1°, 0.2°, 1°, 2°, and 4° misalignment angles. The stress vs. global strain curves of crushing of imperfect cells with different initial vertical wall misalignments are presented in Fig. 4.3. There is no significant effect of the misalignment angle on the plateau stress levels. The remaining of the imperfect simulations has 0.2° of vertical wall initial misalignment, which is the value used in the reference paper (Papka & Kyriakides [8]).

Figure 4.3: Crushing behavior of single cell model 30° imperfect honeycomb with different misalignment angles
4.1.1 Validation of crushing results of regular honeycombs

Papka & Kyriakides [8] presented in-plane direction crushing experimental and computational results of regular honeycombs (cells with 30° cell angle and equal wall lengths). The approach to the first sub-objective of this dissertation starts with conducting several simulations, where the cell geometry, material properties, loading, boundary conditions etc. perfectly matched to what Papka & Kyriakides have considered. The results were compared, which are shown in the following figures.

Figure 4.4 and 4.5 show the stress vs. global strain (stroke) plot of the crushing behavior for elastic and elastic-plastic material using a single cell model (30° cell angle) up to 25% global strain. Figure 4.4 is from Papka & Kyriakides’ research (computational results) and Fig. 4.5 is the ABAQUS results obtained during this dissertation. They also show the differences between a perfect honeycomb crushing (no-misalignment) and an imperfect honeycomb crushing (0.2° misalignment). In the perfect case the cell with elastic material follows the 0-A-B’ line while in the imperfect elastic case it follows the 0-A-B line. For elastic-plastic material, perfect crushing follows the curve 0-a-b-c’ and for imperfect elastic-plastic it follows the 0-a-b-c curve. In case of elastic-plastic material the plateau region starts at the same stress value for both perfect and imperfect cell; however, there is a stress reduction in the imperfect case. The same stress reduction in the imperfect simulation is also noticeable on Fig. 4.2 when compared to Fig. 4.1. Good correlation between Figs. 4.4 and 4.5 establish the validity of the current simulations.

Papka & Kyriakides studied the effect of post-yield modulus on the compressive response of honeycomb crushing. For validation purposes same study was also conducted, and the results are presented in this dissertation. The ratio of post yield modulus to pre-yield modulus is denoted by $a$, which is varied between 1 and 1000. For $a$ =1 the pre-yield modulus was equal to the post-yield modulus similar to an elastic material. For $a$ =1000 the material softens 1000 times beyond yield stress. Papka & Kyriakides’ [8] results are shown in Fig. 4.6 and the current simulation results are in Fig. 4.7. Again, the results are in good agreement. In the rest of the study $a$ is taken to be 100, providing a post-yield modulus of 0.69 GPa.
Figure 4.4: Crushing behavior of an elastic and elastic-plastic 30° single cell honeycomb with and without misalignment, Papka & Kyriakides’ results [8]

Figure 4.5: Crushing behavior of an elastic and elastic-plastic 30° single cell honeycomb with and without misalignment, current simulations
Figure 4.6: Effect of post-yield modulus on crushing response of 30° single cell regular honeycomb, Papka & Kyriakides’ results [8]

Figure 4.7: Effect of post-yield modulus on crushing response of 30° single cell regular honeycomb, current simulations
Figures 4.4 through 4.7 verify the simulation technique used during this study for in-plane direction crushing of a single honeycomb cell.

Figure 4.8: Crushing of a regular hexagon 9x6 cell honeycomb core, a) Papka & Kyriakides’ results [8]

Figure 4.9: Crushing of a regular hexagon 9x6 cell honeycomb core, current simulations
The simulation results of crushing of a 9x6 multi-cell regular hexagonal honeycomb are presented in Figs. 4.8 and 4.9. Figure 4.8 corresponds to Papka & Kyriakides [8], and Fig. 4.9 corresponds to the current simulation results. These figures are plotted to show that the crushing of the full honeycomb has the same general trend of band initiation and propagation (collapse of successive rows of cells). Figures 4.4 through 4.9 establish the validity of the current simulation results by comparing them to the results previously published by Papka & Kyriakides [8]. Papka & Kyriakides validated their computational results by comparing to the experimental results.

4.1.2 Effect of core size on the crushing response

Papka & Kyriakides [8] stated that using a single honeycomb cell in the simulations is sufficient to capture the behavior of a multi-cell honeycomb core. Using a single cell in the simulations instead of a multi-cell honeycomb core saves great deal of computation time. Therefore, it is important to prove the sufficiency of using a single cell. For this purpose, crushing simulations of single and multi-cell honeycomb cores were conducted, and the crushing results were compared. In these simulations different cell angles were also considered. Figures 4.10, 4.11, 4.12 and 4.13 show the stress vs. global strain plots of 15°, 30°, 45° and 60° honeycomb simulations. Each figure contains results for five different core sizes; single cell, 1x6 (1 row 6 columns), 3x6, 5x6, and 9x6 core sizes. Cores with multiple rows generate fluctuations on the plateau region, since rows collapse progressively as shown in Figs. 4.8 and 4.9. For all cell angles, up to the densification region the crushing behavior of the cores with different numbers of rows and columns match. This validates the use of a single cell in the simulations. Therefore, for the rest of the parametric study only single cell analogs were used.
Figure 4.10: Single cell, 1x6 (1 row 6 columns), 3x6, 5x6 and 9x6 results for a) 15° honeycomb

Figure 4.11: Single cell, 1x6 (1 row 6 columns), 3x6, 5x6 and 9x6 results for a) 30° honeycomb
Figure 4.12: Single cell, 1x6 (1 row 6 columns), 3x6, 5x6 and 9x6 results for a) 45° honeycomb

Figure 4.13: Single cell, 1x6 (1 row 6 columns), 3x6, 5x6 and 9x6 results for a) 60° honeycomb
4.1.3 Effect of cell angle, $\theta$

The stress vs. global strain crushing behavior of honeycomb cells with different cell angles are illustrated in Fig. 4.14. These simulations were conducted using a single-cell model since the sufficiency of modeling a single cell instead of a full size honeycomb is verified as shown in Fig. 4.10 through 4.13. In all cases densification starts around 90% global strain. Apparently the plateau stress for the 15° honeycomb is very close to that of the 30° honeycomb; whereas 45° and 60° honeycombs have higher plateau stresses. An initial load peak starts to appear on the stress curve for larger cell angles.

The area under the stress vs. global strain curve gives the total energy absorbed per unit volume (the total energy absorbed divided by the volume occupied by the undeformed honeycombs). Energy absorbed up to 70% global strain values are shown on Figs. 4.15 and 4.16. Figure 4.15 compares the energy absorbed per unit volume, and indicates that the 45° honeycomb can absorb 23% more and 60° honeycomb can absorb 75% more energy per unit volume compared to the 30° honeycomb (baseline). The energy absorbed by the 15° honeycomb is 10% less than in the 30° case. The total energy absorbed per unit mass (specific energy absorption, SEA) is shown in Fig. 4.16. These values are obtained by dividing the total energy absorbed by crushing a single honeycomb cell by the mass of a single honeycomb cell. Note that the mass of all the honeycombs in Fig. 9a are the same. According to these results, 45° honeycomb absorbs 14.5% more and 60° honeycomb absorbs 26% more energy per unit mass than the 30 honeycomb. 15° honeycomb absorbs 15.7% less energy (per unit mass) than the 30° honeycomb. The 60° honeycomb appears to be the best with the highest absorbed energy per unit volume and with the highest absorbed energy per unit mass. However, the high initial peak stress of the 60° honeycomb could potentially be a cause for concern.
Figure 4.14: Single imperfect cell crushing results for 15°, 30°, 45° and 60° honeycombs

Figure 4.15: Energy absorbed per unit volume up to 70% global strain
4.1.4 Observations and underlying physics of effects of varying $\theta$

Deformed and undeformed configurations of honeycombs with 3x6 core sizes for $15^\circ$ and $60^\circ$ cell angles, respectively can be seen in Figs. 4.17 and 4.18. It is observed that during crushing, the inclined walls of $60^\circ$ honeycomb develop much more bending than the $15^\circ$ honeycomb. In the case of $15^\circ$ honeycomb a localized hinge develops at the location of maximum bending, and the rest of the inclined wall rotates around that hinge as a rigid body. However, in the case of $60^\circ$ honeycomb, bending in the inclined wall is not localized, but distributed. Therefore, the inclined walls do not behave as a rigid body undergoing pure rotation about a hinge.
Figure 4.17: Crushing of 3x6 honeycomb core with 15° cell angle

Figure 4.18: Crushing of 3x6 honeycomb core with 60° cell angle
Vertical wall rotations are also different for these two cases. For the same global strain value the rotation of the vertical walls of 60° honeycomb are much larger than 15° honeycomb vertical walls, which is also clearly seen in Fig. 4.19. The local stresses in the element A and B which are located at the ends of the inclined walls, for 15° and 60° single cell honeycombs are also shown in Fig. 4.19. From this figure several observations can be deduced. When the vertical walls start to rotate (at 2.8% global strain for 60° honeycomb and 7.5% global strain for 15° honeycomb) local stress in element A decreases while that in element B increases. Stress reduction in element A is much sharper for 60° honeycomb than the 15° honeycomb. Also note that at the point where the vertical walls start to rotate and element A starts to show stress reduction behavior, a plastic hinge starts to develop around element B. This point corresponds to the initial peak on the stress vs. global strain curve (as shown on Fig. 4.14).

![Figure 4.19: Local stresses at the ends of inclined walls (element A and B), and rotation of the vertical wall for 15° and 60° honeycombs](image)

Figure 4.20 presents the stress vs. global strain for the 15° and 60° honeycombs, up to 18% global strain. The blue curve corresponds to the 60° honeycomb cell and the red line corresponds to 15° honeycomb. Deformations of the cells at several global strain locations are shown in Figs. 4.21 and 4.22. These locations are indicated with circled...
numbers on Fig. 4.20. Figures 4.21 and 4.22 also show the Mises stresses along the cell walls of a single cell for the 15° and 60° honeycombs, respectively. Point 3 on Figs. 4.20 through 4.22 corresponds to the initiation of the vertical wall rotation. At this point the local stress in element A has peaked and stress reduction begins (seen in Figs. 4.19, 4.21 and 4.22). The sharper stress reduction for the 60° case (compare Fig. 4.21 and 4.22, also observed on Fig. 4.19), corresponds to a prominent initial peak followed by stress reduction on the global stress/strain curve in Fig. 4.20. Such a prominent peak and subsequent stress reduction is absent for the 15° case where the local stress reduction in element A is more gradual. Point 3 also corresponds to the initiation of the plastic deformation in element B resulting in the development of a hinge or a flexure.

Figure 4.20: Global stress vs. strain curve for 60° and 15° single cells, up to 18% global strain. The circled numbers correspond to the deformed configurations on Figs 4.21 and 4.22.
The energy output of ABAQUS shows that the internal energy of the cell has two contributing components: the plastic dissipation energy and the strain energy. Plastic dissipation energy and the strain energy generated due to the deformation of an inclined wall (only one inclined wall) for 15° (shown with purple curves) and 60° cells (shown in red).

Figure 4.21: Local stresses along the cell walls for 15°

Figure 4.22: Local stresses along the cell walls for 60°
with red curves) as a function of the global strain are shown in Fig. 4.23. For both 15° and 60° honeycombs the plastic dissipation curves are very close to the internal energy, indicating that most of the internal energy is in the form of plastic dissipation energy, which is caused by the plastic deformation of the inclined walls. 

Figure 4.23: Internal energy of an inclined wall of a single cell and contributing energies of Plastic Dissipation and Strain Energy for 15° and 60° cell angles

Figure 4.24 presents the plastic energy for an entire inclined wall (20 elements) as well as for the section closest to the junction (4 elements starting from element B (marked on Figs. 4.19, 4.21 and 4.22) and moving toward the center of the wall). For the 15° honeycomb dashed line is very close to the solid line, indicating that the plastic energy absorbed by the entire inclined wall is very close to the plastic energy absorbed by the elements near the junction. This implies that there is a lot of localized curvature (formation of a concentrated hinge) at the end of the inclined wall for the 15° case. On the other hand, for 60° honeycomb, the energy in the entire inclined wall is substantially larger than that in the end elements. The curvature is more distributed for the 60° case implying development of an extended flexure. This phenomenon is also observed on Fig. 4.18.
4.1.5 Effect of vertical to inclined wall length ratio, $\alpha$

The effect of cell wall length on the energy absorption was studied by simulating crushing of 30° and 60° unit cells with different $\alpha$ values. In a regular honeycomb $\alpha$ is 1 since the vertical and inclined wall lengths are equal. The geometry of the hexagonal cells for five different $\alpha$ values are shown in Fig. 4.25; for $\alpha = 2$ the vertical wall length is twice that of the inclined wall, $\alpha = 1$ shows a regular honeycomb, for $\alpha = 0.5$ the vertical wall length is half of the inclined wall lengths, and for $\alpha = 0.25$ the vertical wall length is quarter of the inclined wall lengths. $\alpha = 0$ corresponds to a diamond shape, where the vertical walls disappear. $\beta$ and $\eta$ values are not varied for these simulations.
The stress vs. global strain curves up to 80% global strain at various $\alpha$ values for $30^\circ$ are shown in Fig. 4.26, and Fig. 4.27 is for $60^\circ$ unit cells.

Figure 4.25: Cell geometry at various $\alpha$ values ($\beta = \beta^*$, $\eta = 2$)

Figure 4.26: Global stress vs. strain curves for $30^\circ$ hexagonal unit cell at various $\alpha$ values
For hexagonal cells the plateau stress decreases with increasing $\alpha$ for both $30^\circ$ and $60^\circ$ unit cells; however, when $\alpha = 0$, the diamond shape, the cell crushing generates a stress vs. strain curve which does not follow this trend. For $30^\circ$ cell, the diamond cells’ plateau level is around the regular ($\alpha = 1$) cell levels; however, for $60^\circ$ cell the diamond cells’ plateau level falls around the $\alpha = 0.5$ levels. It is also observed that the initial peak load observed for hexagonal cell crushing disappears in case of diamond cell crushing.

Figures 4.28 and 4.29 show the energy absorbed per unit mass, SEA (Fig. 4.28), and the energy absorbed per unit volume (Fig. 4.29), for global strains up to 70%. For $30^\circ$ cells $\alpha = 0.25$ absorbs 2.4% more energy per unit mass than $\alpha = 1$. For $60^\circ$ cells $\alpha = 0.25$ absorbs 45% more energy per unit mass than $\alpha = 1$. The $60^\circ$, $\alpha = 0.25$ cell absorbs 88.6% more specific energy than the regular honeycomb cell ($\theta = 30^\circ$, $\alpha = 1$). The energy per unit volume absorbed by the $60^\circ$ cell with $\alpha = 0.25$ is 2.68 times higher than the regular honeycomb cell.
Figure 4.28: Energy absorbed per unit mass for 30° and 60° hexagonal unit cell at various $\alpha$ values (up to 70% strain)

![Energy absorbed per unit mass](image)

Figure 4.29: Energy absorbed per unit volume for 30° and 60° hexagonal unit cell at various $\alpha$ values (up to 70% strain)

![Energy absorbed per unit volume](image)
4.1.6 Observations and underlying physics of effects of varying $\alpha$

Total energy absorbed by a 30° honeycomb cell is shown in Fig. 4.30 for different $\alpha$ values. The energy absorbed due to the plastic deformation by a single inclined wall (solid line, 20 elements) and the last 4 elements on the hinge end of the inclined wall (dashed lines) are shown in Fig. 4.31. Increasing $\alpha$ causes the dashed lines separate from the solid lines, which implies that for the cases with high $\alpha$ there is a more distributed deformation over the length of the inclined wall, rather than the deformation concentrating around a virtual hinge point.

![Figure 4.30: Total energy absorption by a 30° honeycomb cell with different $\alpha$ values](image)

Figure 4.30: Total energy absorption by a 30° honeycomb cell with different $\alpha$ values

The curves on Figs. 4.30 and 4.31 indicate that the actual energy absorption increases when $\alpha$ increases. Based on Fig. 4.30 this could be attributed to the rotation in the vertical walls and the initiation of the plastic deformation in the inclined walls starting at lower global strain for higher $\alpha$. The lower energy per unit volume for higher $\alpha$, then is due to the larger volume associated with the higher $\alpha$ cells. The variation in mass and volume of the cells with varying $\alpha$ can be seen in Table 4-1.
The initial slopes of the curves on Fig. 4.26 and 4.27 vary, showing that the Young’s modulus in the elastic region changes with $\alpha$. The slope of these curves can be compared to analytical results which can be calculated using the formula provided by Ashby and Gibson [7]. The formula which calculates the Young’s modulus of the honeycomb cell in the y-direction is given in Eq. 4.1. Table 4-2 compares the slopes on Fig. 4.26 and 4.27 to the Young’s modulus calculated using Eq. 4.1. The variation of $\alpha$

![Figure 4.31: Variation of plastic energy absorption along the elements of an inclined wall with varying $\alpha$ for a 30° cell](image)

<table>
<thead>
<tr>
<th>30° Cell Angle</th>
<th>$\alpha =0.5$</th>
<th>$\alpha =1$</th>
<th>$\alpha =2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume/Volume of regular cell</td>
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<td>1</td>
<td>1.66</td>
</tr>
<tr>
<td>Mass/Mass of regular cell</td>
<td>0.75</td>
<td>1</td>
<td>1.5</td>
</tr>
</tbody>
</table>
affects the Young’s modulus, and simulation results match well to the formula. Increasing $\alpha$ increases the effective Young’s modulus of the honeycomb in the elastic region.

\[ E_y = E_s \times \left( \frac{t_l}{l} \right)^3 \frac{h/l + \sin \theta}{\cos^3 \theta} \tag{Eq. 4.1} \]

Table 4-2: Comparison of Young’s modulus (MPa) calculated and simulation results

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Cell angle = 30°</th>
<th>Cell angle = 60°</th>
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<tr>
<td></td>
<td>Analysis, Eq. (4.1)</td>
<td>ABAQUS</td>
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<tr>
<td>2</td>
<td>4.23</td>
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<tr>
<td>0.5</td>
<td>1.69</td>
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The local stresses at the end elements (left and right) of an inclined wall for a 30° honeycomb cell at various $\alpha$ values are shown in Fig. 4.32. Thin solid line corresponds to the diamond shape ($\alpha = 0$), dashed lines correspond $\alpha = 0.5$ and line with symbols correspond to $\alpha = 2$. Blue lines show left end where there is no hinge; indicated with a stress reduction beyond the initial linear region, and the green lines show the right end element surface stresses where a hinge is generated; indicated with a stress increase observed beyond the initial linear region. In case of the diamond shape cell, stress curves overlap since both ends of an incline wall exhibit same kind of deformation due to symmetric mode of deformation, and increasing stress beyond the linear region indicates that hinge forms at both ends unlike a non-uniform cell deformation. As expected, since the deformation is uniform, the local stresses on the left and right elements coincide for the diamond shape cell (thin solid lines).

For a hexagonal cell (non-zero $\alpha$) as the vertical walls get shorter (decreasing $\alpha$), the local stress variation between left and right end of an inclined wall is observed to lessen as seen on Fig. 4.32. The local stress reduction on the no-hinge side of the
inclined wall is much more for the high $\alpha$ values compared to small $\alpha$ values. The global stress vs. strain curves on Fig. 4.26 and 4.27 indicate that the global stress reduction beyond the initial peak stress is higher for high $\alpha$ values. As the deformed cell shapes shown on Fig. 4.32 at 20% and 40% global strains for $\alpha = 2$ and $\alpha = 0.5$ indicate, at the same global strain location the vertical wall rotation is higher for cells with high $\alpha$ values. This higher vertical wall rotation is related to the higher local and global stress reduction. From these observations it can be concluded that the reductions on the global stress vs. strain curve are linked to the local stress reductions on the no-hinge end which are caused by the vertical walls rotations.

![Figure 4.32: Local stresses at the Right (green lines) and Left (blue lines) end elements of an inclined wall for a 30° cell for different $\alpha$ values](image)

4.1.7 Effect of wall thickness to wall length ratio, $\beta$

$\beta$ is the ratio of the inclined wall thickness to the wall length. For the regular honeycomb simulations these values were $t = 0.145$ mm and $l = 5.5$ mm; which gives the
ratio of 0.026. This value is denoted as $\beta^*$ and the variation of the $\beta$ is given by multiples of $\beta^*$. In this part of the study two different $\beta$ values are investigated additional to the $\beta^*$ for 30° and 60° single cells. Figure 4.33 illustrates how unit cell geometry looks like for different $\beta$ values. The other geometric non-dimensional parameters are not changed; $\alpha = 1$ and $\eta = 2$.

![Figure 4.33: Cell geometry at various $\beta$ values ($\alpha = 1$, $\eta = 2$)](image)

The effect of $\beta$ on the global stress vs. strain plot for 30° and 60° unit cells is shown in Fig. 4.34. The blue lines correspond to the 60° cells and the red lines correspond to the 30° honeycomb cells. Since the plateau stress levels increase with increasing $\beta$, the trend shows that increasing wall thicknesses increases the energy absorption. Figure 4.35 and 4.36 show the corresponding SEA (Fig. 4.35) and energy per unit volume (Fig. 4.36), which are calculated up to 70% global strain. The increase in the energy absorption can be found by comparing the values to the regular honeycomb which is the cell with 30° cell angle and $\beta = \beta^*$. The specific energy absorption (energy per unit mass) of 60° with $2\beta^*$ is 3.02 times higher compared to the regular 30° honeycomb with $\beta^*$. The energy per unit volume of 60° with $2\beta^*$ is 8.57 times higher compared to the regular 30° honeycomb with $\beta^*$. 
Figure 4.34: Global stress vs. strain curves for 30° and 60° hexagonal unit cell at various $\beta$ values

Figure 4.35: Energy absorbed (a) per unit mass for 30° and 60° hexagonal unit cell at various $\beta$ values (up to 70% strain)
4.1.8 Observations and underlying physics of effects of varying $\beta$

The energy absorbed by honeycomb cell deformation has two components; energy absorbed due to the plastic deformation of the cell walls and the strain energy. It is explained earlier that the most of the energy absorption is due to the plastic dissipation energy caused by the plastic deformation of the inclined walls. The ratio of the energy absorbed by the plastic deformation to the total energy absorption for different $\beta^*$ values is presented in Table 4-3. Comparison of these percentages for different $\beta^*$ values indicate that increasing wall thicknesses increases the plastic deformation energy.

Increasing wall thickness increases the surface strains and stresses on the cell walls causing more plastic deformation and therefore, more energy absorption. The cell wall thickness can be increased until the surface strains exceed the strain levels at which the material fractures or ruptures.

Figure 4.36: Energy absorbed per unit volume for 30° and 60° hexagonal unit cell at various $\beta$ values (up to 70% strain)
4.1.9 Effect of vertical to inclined wall thickness ratio, $\eta$

$\eta$ is the ratio of vertical to inclined wall thickness. The effect $\eta$ on the energy absorption is studied by simulating crushing of 30° and 60° unit cells. In a regular honeycomb in order to represent the most common manufacturing method of expansion technique, vertical walls are assumed to have twice the thickness of the inclined walls, giving $\eta = 2$. In this study several $\eta$ values are investigated. The cell geometries corresponding $\eta = 1$, 2 and 3 are illustrated in Fig. 4.37. In this study other geometric parameters are kept constant; $\alpha = 1$, $\beta = \beta^*$. 

![Cell geometry at various $\eta$ values](image)

Figure 4.37: Cell geometry at various $\eta$ values ($\alpha = 1$, $\beta = \beta^*$) – baseline is $\eta = 2$

The global stress vs. strain plots for the crushing of cells with different $\eta$ values are shown in Fig. 4.38. The blue lines correspond to the 60° and the red lines correspond
to the $30^\circ$ honeycomb cell. For both cases it is seen that $\eta$ does not have any significant effect on the crushing behavior for the three $\eta$ values ($\eta = 1.5, 2, 3$) that are presented in Fig. 4.38.

Figure 4.38: Global stress vs. strain curves for $30^\circ$ and $60^\circ$ hexagonal unit cell at various $\eta$ values

Figure 4.39: SEA for a $30^\circ$ and $60^\circ$ hexagonal unit cell at various $\eta$ values
The SEA corresponding to these three \( \eta \) values are presented in Fig. 4.39. Even though the plateau stresses overlap on the global stress vs. strain curve, SEA for smaller \( \eta \) values is greater than the higher \( \eta \) values. Because, as \( \eta \) increases, the energy absorption in the inclined walls remains unchanged but the thicker vertical walls increase the mass. Energy absorption per unit volume does not change for these values of \( \eta \) (\( \eta = 1.5, 2, 3 \)) since the curves overlap, and the volume of the cells remain the same.

4.1.10 Observations and underlying physics of effects of varying \( \eta \)

Figure 4.38 contains results for three different \( \eta \) values (\( \eta = 1.5, 2, 3 \)). For a 30° honeycomb cell, simulation results of cells with six different \( \eta \) values varying between 4 and 0.25 are presented in Fig. 4.40. On this figure it is observed that for high \( \eta \) values, such as \( \eta =4 \) or 2, there is no significant effect on the stress vs. strain behavior. However, for smaller \( \eta \) values crushing starts much earlier compared to the high \( \eta \) values and the plateau stress is much lower, causing a significant reduction in the energy absorption. Cell deformation showed that if \( \eta \) is reduced below a critical value, buckling of the vertical walls of the honeycomb cell initiates the crushing of the cell, and the inclined walls do not exhibit any significant deformation. However, for the cells with thicker vertical walls, the inclined walls deform and the vertical walls perform rigid body rotation. Since the premature buckling of the vertical walls result in a significant reduction in the energy absorption, the ideal cell geometry should not have very thin vertical walls. The optimum \( \eta \) value should be high enough to result in crushing due to the bending of the inclined walls and low enough not to provide unnecessary weight. For example for a 30° honeycomb cell, the curves on Fig. 4.40 indicate that \( \eta =1 \) can be used for ideal cell geometry with higher SEA.
Due to the nonlinearity of the problem, an optimization method could not be implemented for the current work. Instead, crushing of cells with different geometric parameters was simulated. The results indicated trends in the responses which yield to a cell geometry with increased SEA. Variation of cell angle, cell vertical wall to inclined wall length ratio, cell wall thickness to length ratio and vertical to inclined wall thickness ratio showed that:

- Increasing cell angle increases the specific energy absorption
- Decreasing vertical wall length increases the specific energy absorption
- Increasing wall thicknesses increase the specific energy absorption
- Vertical wall thickness should not be too small not to cause buckling and early initiation of crushing.

These observations provide a modified design where the cell has relatively bigger cell angle, shorter and thinner vertical walls and thicker inclined walls. Energy absorption
of a cell with $\theta = 60^\circ$, $\alpha = 0.25$, $\beta = 2\beta^*$, and $\eta=1$ can be compared to SEA of a regular cell which has $\theta = 30^\circ$, $\alpha = 1$, $\beta = \beta^*$, and $\eta=2$. The cell shapes can be seen on Fig. 4.41.

![Figure 4.41: Honeycomb cell geometry a) Regular honeycomb, b) Modified cell geometry with higher SEA](image)

The crushing behaviors of the cells with modified and regular geometries (geometries are shown in Fig. 4.41) are shown in Fig. 4.42. In this figure, case 1 corresponds to the cell geometry with $\eta =1$ and case 2 corresponds to the modified geometry with $\eta =2$. As explained earlier, smaller $\eta$ value provides higher SEA; however, the expansion manufacturing technique results in $\eta =2$. If the gain in the SEA by changing $\eta$ from 2 to 1 is significant, then another method should be followed to manufacture cells with $\eta =1$.

On Fig. 4.42 the dark solid line and the symbols overlap showing that the modified cells with $\eta =2$ and 1 provide same crushing response. The plateau level of the modified cells crushing is much higher compared to the regular honeycomb, which is shown in gray solid line. Area under the curves up to 70% global strain show that the modified cell geometries (case 1 or 2) absorbs 0.98 MJ/m$^3$ energy per unit volume which is 13 times higher compared to a regular cell (0.075 MJ/m$^3$). Cells with $\eta=1$ and $\eta=2$ have the same energy absorption per unit volume; however, the SEA of the cells with $\eta =1$ is greater compared to the geometry with $\eta =2$. Case 1 (geometry with $\eta=1$) has 3.3 kJ/kg energy absorption per unit mass (SEA), and case 2 (geometry with $\eta=2$) has 3.08 kJ/kg SEA. (Case 1 has 7% greater SEA compared to Case 2.) The preferred geometry (case 1) has 4.8 times higher SEA than regular cell geometry (0.68 kJ/kg).
4.2 Out-of-plane direction crushing of honeycombs

Out-of-plane direction crushing of hexagonal cells was conducted in order to compare the energy absorption and crushing response of the hexagonal cells to in-plane direction crushing. The crushing tube was considered to have fixed boundary conditions at both ends. There was no symmetry boundary conditions applied to the sides of the tubes, providing that these simulations correspond to crushing of a single hexagonal tube. Quasi-static and dynamic simulations were conducted for honeycombs and same boundary and loading conditions were used in these two cases. In ABAQUS, “General contact” option is available for the interaction of the faces during the crash in the dynamic simulations which provides convergences. This type of contact is not available in static simulations, and therefore, the simulations did not provide any results for large strains (greater than 20% global strain). Computationally, dynamic simulations are more expensive since they require significantly more amount of time.

Different cell heights were also considered in the out-of-plane direction crushing. The dynamic crushing simulation results for cells with 10 mm, 20 mm and 50 mm heights, at several global strain values are presented in Fig. 4.43 (the number under each

Figure 4.42: Global stress vs. strain curves for regular and modified hexagonal unit cells
figure shows the corresponding strain value in percentage). The cell geometry for the tubes with varying cell height was same; the cell walls had 5.5 mm wall length, 0.145 mm wall thickness (all faces had the same thickness) and 30° cell angle.

As seen on Fig. 4.43, the deformation starts at the center of the core and propagates to the rest of the tube (the location for buckling initiation depends on the boundary conditions). This deformation mode is called local buckling. Stress vs. global strain response corresponding to the dynamic and static crushing simulations of the cells

Figure 4.43: Deformed and undeformed configuration of the cells in out-of-plane direction loading; heights of the undeformed core are 10 mm, 20mm and 50 mm
with different core heights in out-of-plane directions are presented in Fig. 4.44. Solid lines correspond to dynamic simulations and dashed lines correspond to static simulation results. As mentioned earlier, dynamic simulations continue to larger global strains (greater than 20%). Even though the static simulation results had interaction problem and did not provide any result for large strain values (no greater than 20%), the average crushing stress provided with dynamic and static simulations match. Comparison of solid lines also indicates that the initial peak loads and the mean crushing loads for different core heights are similar.

![Figure 4.44: Comparison of dynamic and static crushing simulation results of cells with different core heights in out-of-plane direction, and analytical mean crushing load (Eq. 4.2)](image)

Both quasi-static and dynamic crushing behavior of honeycomb tubes in out-of-plane direction showed an initial peak load and it is followed by local peaks with a nearly constant mean load value. Theoretical expression by Wierzbicki [7], which is given by Eq. 4.2 to calculate the out-of-plane direction crushing mean stress for regular hexagonal tubes, is compared to the simulation results and it is observed that the theoretical value closely predicts the mean crushing load (as shown in Fig. 4.44.). Wu et al. also report similar results [57]. This theoretical mean load value was calculated for $t = 0.145$ mm, $l$
=5.5 mm and $\sigma_{ys}=290$ MPa. The mean crushing load estimated by dynamic and static simulation results match very well with the theoretical formula.

$$\sigma_{pl} = \sigma_{ys} 6.6 \left( \frac{l}{t} \right)^{5/3}$$

Eq. 4.2

4.3 Comparison of out-of-plane direction to in-plane direction energy absorption

The regular honeycomb crushing under in-plane direction loading provided a mean crushing stress of 0.1 MPa and the modified honeycomb provided around 1.4 MPa (refer to Fig. 4.42). When the honeycomb with regular cell geometry is crushed in out-of-plane direction the mean crushing stress is around 4 MPa (refer to Fig. 4.44). This shows that the out-of-plane crushing of hexagonal cells absorbs around 40 times more energy compared to in-plane direction crushing.

The specific energy absorption (SEA) of the out-of-plane direction crushing of regular honeycomb can also be calculated by using the area under the stress vs. strain curve and the mass and volume of the cell. If it is assumed that the cells are crushed up to 80 % global strain the SEA is calculated as 30 kJ/kg. In literature, crushing of aluminum empty and foam filled tubes have already been reported. According to Zarei and Kroger [98] crushing of Al 6060 square tubes in out-of-plane directions provides a SEA of 26 kJ/kg and a foam filled aluminum square tube has SEA of 31 kJ/kg. These values are very close to the ABAQUS results provided in this chapter.

It is reported in this chapter that the in-plane direction crushing of the regular honeycombs has SEA of 0.68 kJ/kg and cells with modified geometry has SEA of 3.3 kJ/kg. Crushing of the modified cell in in-plane direction absorbs 10 times less energy, and regular honeycombs in in-plane direction has 50 times less energy compared to out-of-plane direction crushing of regular honeycombs. Even though modifying the cell geometry significantly increases the SEA of cells under in-plane direction loading, the SEA of out-of-plane crushing of regular honeycombs is much higher.
4.4 Summary

Validation study showed that the ABAQUS crushing simulation results match to the previously published results for regular honeycomb crushing. The sufficiency of simulating crushing of only single cell analogs was proven since the simulation results of several different core sizes provided the same stress vs. strain behavior. The effect of geometric parameters on the crushing behavior and energy absorption showed that increasing cell angle, decreasing vertical wall length, increasing wall thicknesses increase the energy absorption. The thickness of the vertical walls should be large enough so that the cell crushing is triggered by the rotation of the vertical walls instead of buckling of them. Optimum cell geometry can be achieved by changing the regular cell geometry following these trends. The mass specific energy absorbed by the modified cell crushing is 4.8 times greater compared to the cell with regular geometry crushing. Out-of-plane crushing of regular honeycombs was also simulated and it has the specific energy absorption of 30 kJ/kg. This is 10 times greater compared to the specific energy absorption of the modified cell, which is 3.3 kJ/kg. Even though modifying the cell geometry significantly increases the SEA of cells under in-plane direction loading, the SEA of out-of-plane crushing of regular honeycombs is much higher. Results show that with such SEA values, in-plane direction crushing of honeycombs without facesheets is not comparable to out-of-plane direction crushing for energy absorption point of view. There are some disadvantages associated with the out-of-plane crushing of honeycomb; for example it has been reported that if L/D (tube length/cell diameter) ratio of the crushing tube is greater than 3, the global buckling occurs instead of progressive local buckling [98]. Global buckling of tubes is not desired for energy absorbing applications, since with this failure mode they do not absorb high amount of energy. For energy absorbing applications such as interior of an I-beam with a narrow web section, this disadvantage might cause an issue. At this point use of honeycomb crushing in in-plane direction might be preferable, providing comparable energy absorption levels. As presented in the following chapter, use of facesheets with honeycomb core is a promising suggestion to increase the SEA of in-plane direction crushing of honeycombs.
Crushing of honeycomb cells with facesheets: Results and discussions

The main objective of this dissertation was to integrate energy absorbing hexagonal honeycombs into a keel beam in order to provide a crashworthy structure for helicopter subfloor. As explained in the Methodology chapter honeycombs can be used as the interior of an I-beam. Drawing of an I-beam profile, with possible placement of the honeycombs in the web section is illustrated in Fig. 5.1.

Considering that the hexagonal honeycomb core is oriented in the in-plane direction, the simulation of the keel beam crushing (or the web section of the keel beam) can be modeled as crushing of a honeycomb cell with facesheets as shown in Fig. 5.2. The line at the center of the core corresponds to the symmetry axis, and in the simulations only half of the cell was modeled with the symmetry boundary conditions. This configuration resembles a sandwich structure where generally stiff facesheets are separated with a cellular or foam core in order to increase the bending loads that the structure can carry.
In this dissertation an integral sandwich structure was considered where the bond between the facesheets and the core was assumed to be perfect. There are several advantages associated with perfect bonding between the core and the facesheet: (1) better post-crash integrity, (2) higher energy absorption (EA) if debonding is avoided, (3) higher loads reached before failure occurs. Brazing technique is one of the methods to manufacture integral sandwich structures. The use of brazing for honeycomb sandwich structures is patented in the US in 1980s [91], and more recently Jing et al. [81] presented an experimental study of the in-plane crushing of an integral steel sandwich structure.

Figure 5.2: A unit cell model of honeycomb with facesheet structure

Assuming an integral structure in the simulation models means that the debonding of the facesheets from the core was not considered as a failure mode. Since use of adhesive bonding is cheaper and practical, most of the existing sandwich structures use this technique to bond the facesheets to the core. If other methods such as brazing were used, it would create an integral structure, increase the stiffness in post-failure, and provide a better solution for energy absorbing or load carrying structure. In the validation study, a research paper which uses brazing technique to bond the facesheets to the core in order to manufacture an integral steel sandwich structure was considered. Crushing test results of an integral steel sandwich honeycomb structure were compared to ABAQUS
simulation results, and they agreed perfectly. In this chapter the validation study is followed by the core size study. Three different core sizes with the same geometric and material properties were studied and sufficiency of using a single cell in the simulations was established. The core size study is followed by cell geometric parameters study, where the effect of each geometric parameter on the crushing behavior was studied, including facesheet thickness, \( t_f \), cell angle, \( \theta \), cell wall thickness, \( t_h \) and \( t_l \), wall lengths, \( h \) and \( l \), cell depth, \( b \) or \( \gamma \) and number of rows, \( N \), for a given keel beam height (or cell size). The crushing behavior is presented using stress vs. global strain plots as well as mass normalized energy absorption plots. The area under the curve for stress vs. strain plots give the energy absorption per unit volume and under the mass normalized energy absorption curve gives the energy absorption per unit mass. The underlying physics of the crushing behavior is explained with supplemental plots, such as nodal displacements, energy absorption (EA) of each component compared to the total EA of the cell.

5.1 Validation study

As explained in the Literature chapter sandwich structures are widely employed in energy absorbing applications. Since they are relatively lightweight, they are commonly used in the aerospace applications where they are exposed to several different types of loads such as bending, tension, compression, and shear. There are several different failure modes that sandwich structures exhibit under different loadings. Critical failure loads of these different modes can be calculated using theoretical methods or buckling analysis (for global and local buckling failure), and this issue has been studied and presented in several papers [60]-[68]. The bonding method of the facesheets to the core changes the failure load of sandwich structures significantly, especially when they are loaded under in-plane loading or bending. If adhesive bonding method is used, debonding is one of the failure modes in case of compression or in bending load. Jing et al. [81] suggest and experimentally prove that the debonding failure mode can be avoided if methods such as resistance welding, brazing, transient alloy diffusion are used to attach the skins to the core. They use brazing for the fabrication of carbon steel sandwich structure and test the
crushing behavior in several directions. Therefore, the paper by Jing et al. [81] was used as the baseline paper in the validation study for the honeycomb with facesheets.

A picture of a carbon steel sandwich honeycomb structure, taken from Jing et al. [81] is shown in Fig. 5.3, in the undeformed and crushed configurations. The specimen used in the tests had stiff facesheets which were attached to honeycomb core using brazing method. As can be seen in Fig. 5.3, the core is not deformed along the core depth, and the facesheets are deformed at each cell. This corresponds to the dimpling failure mode which is also observed in ABAQUS simulations for the sandwich structures with thin facesheets as shown later in this chapter (on Figure 5.9 (a) and (b)).

![Figure 5.3: Carbon steel sandwich structure a) undeformed and b) deformed [81]](image)

Jing et al. [81] specify the properties of the carbon steel as follows: Young’s modulus, $E = 202$ GPa, and yield strength, $\sigma_{ys} = 213$ MPa. Cell wall length was 5 mm, wall thickness was 0.49 mm and facesheet thickness was 0.45 mm. Core depth was 15 mm, height was 147.49 mm (19 rows) and width was 38.48 mm. In order to investigate if ABAQUS successfully predicts the crushing behavior and mean crushing stress of such sandwich structure, crushing of a single cell model was simulated with the given material and geometric properties. The results are given in Fig. 5.4. The solid line in Fig. 5.4 corresponds to the experimental results from Jing et al. [81] and the dashed line represents the ABAQUS simulation results of a single cell model. Since their
experimental specimen has several rows and cells, buckling (or dimpling) of each row creates fluctuations on the “plateau” of the stress vs. strain curve. However, only a single cell was considered in the ABAQUS simulations, therefore, the stress vs. strain curve does not have the same fluctuations in the plateau region. The mean crushing stress values and the general crushing response observed in the experiments and ABAQUS simulations are very close. ABAQUS successfully predicts the mean crushing stress.

5.2 Effect of core size on the crushing response

Similar to the honeycomb cores without facesheets, a core size study was also conducted for the crushing of honeycombs with the facesheets. In this study three different core sizes were considered; a half cell, a single cell and a single row. Convergence for the cores with multiple rows was not achieved beyond the crushing of one row due to the complexity of the problem caused by excessive interaction of the core and the facesheets; therefore, a maximum of 1 row was used. All cells had the same geometric and the material properties, which are given in the Methodology chapter (except for the validation study). The front view of the models used in the simulations

![Validation result for honeycomb with facesheets simulations](image)
with thick (0.58 mm) facesheets in deformed configuration and the Mises stresses are shown in Fig. 5.5. Deformation on the facesheets, cell inclined and vertical walls look very similar on all these core sizes.

![Figure 5.5: Deformation at 30% strain of a) half cell, b) single cell, and c) single row](image)

The stress vs. strain plots corresponding to these three different core sizes with thin (0.145 mm) and thick (0.58) facesheets are shown in Fig. 5.6. Solid lines correspond to the cells with thick facesheet and dashed lines correspond to cells with thin facesheets. For models with both thin and thick facesheets the results for the single cell and single row match; however, the half cell results do not match the rest (difference is more pronounced for the ones with thick facesheets). Since the half cell model exhibited some differences on the stress vs. strain curve, the smallest model used in rest of the study is the single cell. Cell geometric effects, keel beam height and depth studies were conducted using single cell.

The difference of the stress vs. strain behavior may be due to the following reason; the half cell model simulation does not include the smallest unit of a hexagonal cell, where the cell vertical walls are loaded and the load path is divided to two, to each inclined wall. However, single cell and single row sizes have at least one unit cell and 2
half cells. As explained in the Methodology chapter the outer vertical walls do not have symmetry boundary conditions in the x-direction in order to be able to capture any buckling of these walls.

5.3 Geometric parameters study

There are several geometric parameters which might affect the crushing behavior of the honeycomb with facesheets sandwich structure such as; facesheet thicknesses, core cell geometric parameters, honeycomb core depth (the distance between the two facesheets), and number of rows of cells (for a specific keel beam height). Each of these parameters is examined and the physical understanding of the behavior differences discussed in this section.

The simulation results are shown using stress vs. strain plots which provide the mean crushing stress, and the area under the curves give the energy absorption (per unit volume). The failure mode (dimpling or wrinkling), the energy absorbed by each deformed cell component (facesheets, inclined walls or the vertical walls of the core) are
also shown in order to highlight the differences in the crushing with varying parameters. Mass normalized energy absorption curves are also plotted. Area under mass normalized energy absorption vs. strain curve gives the energy absorbed per unit mass which is the specific energy absorption (SEA).

### 5.3.1 Effect of facesheet thickness, \( t_f \)

As explained by Pahr and Rammerstorfer [90], facesheet thickness affects the failure mode of the sandwich structures. For sandwich structures under in-plane loading with relatively thin facesheets the failure mode is called dimpling or intracellular buckling, for the ones with thicker facesheet the failure mode is called wrinkling. Critical failure load for sandwich structures can be predicted analytically or computationally. In Military Handbook 23-A [99] Norris suggests Eq. 5.1 for the critical failure load calculation (load per unit length of the facesheet) for dimpling mode.

\[
F_{\text{dimp}} = \frac{2E_f t_f}{(1-v_f^2)\left(\frac{t_f}{l\cos\theta}\right)^2}
\]

Eq. 5.1

This analytical expression can be compared to the buckling loads estimated by ABAQUS simulations. A linear eigenvalue analysis provides the critical failure loads of the structure. In this case only a quarter-cell model with proper boundary conditions is sufficient (Quarter cell model crushing is not suitable for post-buckling analysis; however, it is sufficient for linear buckling analysis.). A buckling analysis has been conducted for a quarter cell model (Fig. 5.7) in ABAQUS, and the critical loads obtained by ABAQUS are plotted on Fig. 5.8 with symbols. Norris’ analytical expression is plotted with the dashed line on Fig. 5.8, and it can be observed that the critical failure load increases with increasing facesheet thickness. This plot shows that the ABAQUS results of the buckling analysis and the analytical formula Eq. 5.1 given by Norris [99] successfully match. Pahr and Rammerstorfer [90] also compared Eq. 5.1 to their ABAQUS simulation results and they provided matching critical load estimations.
In order to study the crushing of the cells with thin and thick facesheets beyond the failure point, nonlinear crushing analyses are conducted. Single cells with two different facesheet thicknesses were crushed and the results were compared. The Mises stresses on the single cell model with two different facesheet thickness values (core has 15° cell angle) are shown in Fig. 5.9. This figure corresponds to 30% deformation.

The cells in Fig. 5.9 (a) and (b) have thin facesheets ($t_f = 0.145$ mm). In this case the deformation mode is called *dimpling*, where the face buckles individually at each cell. Figure 5.9 (b) corresponds to the side view of a cross section taken at the center of the
sandwich cell. The core vertical walls do not have much deformation, and the facesheets buckle locally at each cell. In this mode the honeycomb core governs the deformation. The cells in Fig. 5.9 (c) and (d) have thick facesheets (\( t_f = 0.58 \text{mm} \)). In this case the deformation is called wrinkling. Figure 5.9 (d) corresponds to the side view of a cross section taken at the center of the sandwich cell. The core vertical walls are deformed significantly. In this mode the facesheet governs the deformation.

Figure 5.9: Mises stresses on the 30% deformed shape of sandwich structures with different wall thicknesses: a) Cell with thin facesheet, dimpling mode, b) Dimpling mode side view, c) Cell with thick facesheet, wrinkling mode, d) Wrinkling mode side view

In order to observe the differences of the deformation on the vertical and the inclined cell walls Fig. 5.10 can be referred. The half models of the same single cells in Fig. 5.9 are shown in this figure. In addition to the observations above, it can be seen that the inclined cell wall deformation for the cells with thin facesheet does not vary through the core depth, however, for the cells with thicker facesheets the deformation of the inclined cell wall varies through the cell depth. For the dimpling mode (Fig. 5.10 (a))
vertical walls do not deform; however, for the wrinkling mode (Fig. 5.10 (b)) they are substantially deformed.

The stress vs. strain curves for the crushing of the honeycomb cores with facesheets, for different facesheet thicknesses can be seen in Fig. 5.13. The failure mode for the thin facesheet is dimpling and for thick ones is wrinkling; however, there is no distinctive behavior changes on the stress curves between these modes (except higher facesheet thickness provides higher mean crushing stress). When the deformed configurations are investigated carefully, it can be seen that the crushing mode for the thin facesheets is dimpling, for the thick ones is wrinkling. When the dimpling occurs, the largest deformation is on the center of the facesheet, and when wrinkling occurs the core vertical walls deform as well as the facesheets. Looking at the deformations of the vertical walls and the facesheets the critical failure mode can easily be captured from the simulations. Locations for two nodes on the undeformed single cell model of the sandwich configuration are shown in Fig. 5.11; one node is located at the center of the facesheet and the other is on the vertical wall.
The out-of-plane (z) direction displacements of the nodes for the sandwich structures with different facesheet thicknesses are compared in Fig. 5.12. In this figure the central node of the facesheet is denoted by Face node and the displacement (in z-direction) of these nodes are shown by the dashed lines, and the node on the vertical wall is denoted by Vertical wall node and the displacement (in z-direction) of these nodes are shown by the solid lines. For \( t_f = 0.4 \) mm the z-direction displacement of the facesheet starts and increases rapidly, where the vertical walls do not deform initially, but beyond 1\% global strain, there seems to be some vertical wall deformation. It means that the critical failure mode for the sandwich structure with \( t_f = 0.4 \) mm is dimpling; however, post-buckling crushing also involves some vertical wall deformation. Similar observations can be done for the sandwich structures with \( t_f = 0.6 \) mm and \( t_f = 1 \) mm. Another important observation is that, the deformation of the vertical wall starts sooner as the facesheets get thicker. This shows the change of critical failure mode from dimpling to wrinkling. Displacement of the nodes for \( t_f = 5 \) mm is also plotted, and in this case the mode is wrinkling for the failure and the post-failure since the dashed line and the solid line for this mode overlaps. The analytical way to calculate the facesheet thickness at which point the failure mode changes from dimpling to wrinkling is shown in Appendix A.4.
Effect of facesheet thickness on the stress vs. strain curves can be seen on Figure 5.13. The facesheet thickness significantly affects the initial and the mean stress levels. The initial stress value where the plateau region starts mainly depends on the thickness of the facesheet thickness. For the four different facesheet thicknesses plotted on Fig. 5.13, cells with thinnest facesheet ($t_f = 0.145$ mm) had dimpling mode, and with the thick facesheets ($t_f = 0.29$ mm, $t_f = 0.58$ and $t_f = 1.16$ mm) had wrinkling mode.

The mass of the structure also increases with increasing facesheet thickness. Therefore, it is also practical to compare the energy absorption normalized with the total weight of the cell. Figure 5.14 corresponds to the mass normalized energy absorption vs. the global strain plot of the cells with varying facesheet thicknesses. It can be seen that the crushing mean stress values are higher for the cells with thicker facesheets but when normalized by the weight of the cell the magnitude of the effect of facesheet thickness decreases.
Figure 5.13: Stress vs. strain plot for the crushing of honeycombs with facesheets

Figure 5.14: Mass normalized energy absorption plot for the effect of facesheet thickness
5.3.2 Effect of cell angle, $\theta$

The baseline for the cell angle study was the sandwich configuration with a regular cell core (30° cell angle, equal wall lengths). Additional to the baseline, cells with 15° and 45° cell angles are considered in this section. Both thick ($t_f = 0.58$ mm) and thin ($t_f = 0.145$ mm) facesheets are examined. The stress vs. strain behaviors are shown in Fig. 5.15. The solid lines correspond to the models with thick facesheets and the dashed lines correspond to the ones with thin facesheets. For the cells with thin facesheets increasing cell angle increases the plateau level. For the cells with thick facesheets change in the cell angle does not improve the energy absorption.

![Figure 5.15: Stress vs. strain behavior for the cells with varying core cell angle](image)

Mass normalized energy absorption curves are plotted against the global strain in Fig. 5.16. For the mass normalized plot it can be seen that the trend is same as the stress vs. strain plot. It should be noted that when the energy absorption (EA) per unit mass is considered (Fig. 5.16), the honeycombs with thin facesheets perform almost as well as those with the thick facesheets. Even though the energy absorbed per unit volume is greater for thick facesheets (Fig. 5.15) there is a penalty associated with greater mass. In
addition, for honeycomb core with thin facesheets the existence of a relatively flat plateau (no initial high peak) is an advantage for crashworthy applications. For the core with thin facesheets, the area under the curve for the 45° cell angle is 1.6 times greater compared to the 30° cell (11.2 kJ/kg vs. 7.1 kJ/kg).

![Figure 5.16: Mass normalized energy absorption plot for varying core cell angle](image)

The cells with the thin facesheets (dashed lines) follow a significant trend; increasing cell angle increases the initial stress value and the plateau stress level. The behavior in the initial linear region depends mainly on the facesheets and the post-yield behavior is governed by the core crushing. The initial stress value increase is also supported by the equation which predicts the dimpling failure load, Eq. 5.1; increasing cell angle increases the critical dimpling load. Increasing plateau level might be related to the Young’s modulus of the core in $X_2$ (y) direction ($E_2$ is given by Eq. 5.2); increasing cell angle increases the Young’s modulus. For the cells without facesheets increasing cell angle also increased the plateau level.

For the cells with thick facesheets the increase in the initial load can be explained with the increase of the Young’s modulus of the core in $X_2$ (y) direction with increasing cell angle ($E_2$ is given by Eq. 5.2). However, since the failure mode is wrinkling for the honeycomb with thick facesheets, the vertical walls also deform. Deformation of the
vertical walls affects the Young’s modulus, $E_1$, of the core in $X_1$ (x) direction (given by Eq. 5.3). Increasing cell angle decreases $E_1$. Therefore, the drop on the plateau level is much greater for 45° cells compared to 30° which is greater compared to 15° cells.

$$\frac{E_2}{E_s} = \left(\frac{t}{l}\right)^3 \frac{(h/l + \sin\theta)}{\cos^3\theta} \quad \text{Eq. 5.2}$$

$$\frac{E_1}{E_s} = \left(\frac{t}{l}\right)^3 \frac{\cos \theta}{(h/l + \sin\theta)\sin^2\theta} \quad \text{Eq. 5.3}$$

As presented in the previous results chapter, the increase of the cell angle of the honeycombs without facesheets also caused a significant increase in the plateau level however, a drop on the stress level in the plateau region was also observed which was explained with the increase in the rotation of the inclined walls.

The energy absorption (EA) of the facesheets, inclined walls and the vertical walls also vary with varying cell angle. Table 5-1 presents the EA of each component for the cells with thin and thick facesheets (percentages to the EA of the whole cell). For the cells with thick facesheets most of the energy is absorbed by the facesheets (greater than 70%), this is followed by the vertical walls and the inclined walls. However, for the cells with thin facesheets inclined walls absorb more energy compared to the vertical walls since vertical walls do not deform significantly. For both cells with thin and thick facesheets increasing cell angle increases the deformation and the EA of the inclined walls, similar to the cells without facesheets.

<table>
<thead>
<tr>
<th>Table 5-1: Percentages of the plastic dissipation energy absorption with varying $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Thick facesheet</strong></td>
</tr>
<tr>
<td>$\theta = 15^\circ$</td>
</tr>
<tr>
<td>Facesheet</td>
</tr>
<tr>
<td>Vertical walls</td>
</tr>
<tr>
<td>Inclined walls</td>
</tr>
</tbody>
</table>
5.3.3 Effect of vertical wall length to the inclined wall length ratio, $\alpha$

The baseline for $\alpha$ is 1 since for a regular honeycomb vertical wall length is equal to the inclined wall length. The effect of $\alpha$ was studied for three different values; 0.5, 1 and 2, for the honeycombs with thick ($t_f = 0.58$ mm) and thin ($t_f = 0.145$ mm) facesheets. In this study the other parameters were kept constant; $\theta = 30^\circ$, $\beta = 0.026$ and $\eta = 2$. The stress vs. strain behaviors of these cells are shown in Fig. 5.17. In this figure the dashed lines are for the cells with thin facesheet and the solid lines are for the ones with thick facesheets.

![Figure 5.17: Stress vs. strain behavior of the honeycomb with facesheets structure with varying $\alpha$](image)

The initial loads and the plateau stress levels are same for the dashed lines, showing that $\alpha$ does not have a significant effect for the cells with thin facesheets, since in dimpling mode vertical walls do not deform. For cells with thick facesheets the initial load does not vary with varying $\alpha$. However, the plateau stress levels vary significantly. Increasing $\alpha$ decreases the plateau stress level, therefore, providing much less energy absorption (EA). However, decreasing $\alpha$, (decreasing vertical wall length), increases the plateau level. The same trend was also observed for the cells without facesheets.
It is also important to compare the mass normalized energy absorption levels since varying $\alpha$ also changes the weight of the sandwiched honeycomb. The mass normalized EA vs. strain levels for the cells with thin and thick facesheets are shown in Fig. 5.18. For honeycomb with thick facesheets, reducing $\alpha$ increases the EA per unit mass. The best performance is observed by the cells with short vertical walls. Cells with thick facesheets and $\alpha = 0.5$ absorbs (17.3 kJ/kg - up to 30% global strain) 25% more energy (per unit mass) compared to the regular cells with thick facesheets, $\theta = 30^\circ$, $\alpha = 1$, $t_f = 0.58$ mm (13.8 kJ/kg - up to 30% global strain).

The total plastic dissipation energy absorbed by each cell (with varying $\alpha$, thick and thin facesheets) can be seen on Fig. 5.19. The total energy absorbed by cells with higher $\alpha$ is greater for honeycombs with thin and thick facesheets. However, the specific energy absorption where the volume and the weight of the cells come into the equation has the opposite trend; and the cells with shorter vertical walls (smaller $\alpha$) become better solution. A similar observation was also made for the cells without facesheets.
The plastic dissipation energies absorbed by the components of a regular honeycomb with thick facesheets; the whole model, facesheets, vertical walls and the inclined walls are shown in Fig. 5.20. Summation of the energies absorbed by the facesheet, vertical walls and the inclined walls give the energy absorbed by the whole model. According to this chart facesheets absorb most of the energy, followed by the vertical walls and the inclined walls. Similar charts can be plotted for different $\alpha$ values; however, in order to see the variation of the percentages of the energies absorbed by each component tables are sufficient (Table 5-2, data corresponding to 30% of global strain). It can also be seen in Fig. 5.20 that the facesheets start to plastically deform first, followed by the inclined walls and finally the vertical walls. For the cells with thin facesheets plastic deformation of the vertical walls delayed more compared to the cells with thick facesheets, and they significantly absorb much less energy.

Figure 5.19: Total plastic dissipation energy of cells with varying $\alpha$
Table 5-2 presents the percentages of the energies absorbed by each cell component (facesheet, vertical and inclined walls) with varying $\alpha$ values. It was mentioned earlier that for the cells with thick facesheets the vertical walls deform significantly. The table supports this observation; for the cells with thick facesheets vertical walls absorb significant amount of energy however, for the cells with thin facesheets their EA contribution to the total EA of the unit cell is less than 5%. The variation of the vertical wall length does not affect the energy absorbed by those walls in case of cells with thin facesheets.

Table 5-2: Percentages of the plastic energy absorption with varying $\alpha$

<table>
<thead>
<tr>
<th></th>
<th>Thick facesheet</th>
<th></th>
<th>Thin facesheet</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 2$</td>
<td>68</td>
<td>72.8</td>
<td>76</td>
<td>76</td>
</tr>
<tr>
<td>$\alpha = 1$</td>
<td>72.8</td>
<td>76</td>
<td>76</td>
<td>76</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
<td>76</td>
<td>76</td>
<td>73.4</td>
<td></td>
</tr>
<tr>
<td><strong>Facesheet</strong></td>
<td><strong>68</strong></td>
<td><strong>72.8</strong></td>
<td><strong>76</strong></td>
<td><strong>76</strong></td>
</tr>
<tr>
<td><strong>Vertical walls</strong></td>
<td><strong>27</strong></td>
<td><strong>18</strong></td>
<td><strong>13.5</strong></td>
<td><strong>3.9</strong></td>
</tr>
<tr>
<td><strong>Inclined walls</strong></td>
<td><strong>4.5</strong></td>
<td><strong>8.7</strong></td>
<td><strong>9.2</strong></td>
<td><strong>19</strong></td>
</tr>
</tbody>
</table>
The deformation (at 18% global strain) on the cells with thick facesheets and with varying $\alpha$ values can be seen in Fig. 5.21. It can be seen on this figure that for the cells with thick facesheets the deformation of the vertical walls decreases with decreasing $\alpha$ and therefore, the total EA of cells with longer vertical walls are greater. The drop on the stress values which are seen on Fig. 5.17 and 5.18 for the cells with greater $\alpha$ and thick facesheets is also related to the Young’s modulus of the core in $X_1$ ($x$) direction, $E_1$ (given by Eq. 5.3). Increasing $\alpha$ decreases $E_1$. Therefore, the cells with greater $\alpha$ has sharper drop on the stress levels.

Figure 5.21: Deformation of the cells with thick facesheets with varying $\alpha$

5.3.4 Effect of the cell wall thickness to wall length ratio, $\beta$

In this section the wall thicknesses are varied by varying the parameter $\beta$ (ratio of $t_l$ to $l$). All other parameters were kept constant; $\alpha = 1$, $\theta = 30^\circ$, $\eta = 2$. In the baseline cell the cell wall length, $l = 5.5$ mm, and the inclined wall thickness, $t_l = 0.145$ mm; therefore, the ratio is 0.026 which is denoted by $\beta^*$. In this study three values of $t_l$ was used giving; $0.5 \beta^*$, $\beta^*$ and $2 \beta^*$. The stress vs. strain responses of the crushing of the cells with thin and thick facesheets and with varying $\beta$ are shown in Fig. 5.22. The thickness of the facesheets for “thin” cases was same as the inclined wall and for the “thick” cases it was four times the inclined wall thickness. Same trend was observed for the cells with thin
and thick faces; increasing $\beta$ increases the plateau stress and therefore, the energy absorption. This behavior was expected since increasing wall thicknesses increase the strains and the stresses on the elements increasing the plastic deformation and the energy absorption. Increasing wall thickness also increase the mass of the cell.

![Stress vs. strain behavior of the honeycomb with facesheets structure with varying $\beta$](image)

**Figure 5.22:** Stress vs. strain behavior of the honeycomb with facesheets structure with varying $\beta$

The mass normalized energy absorption values can be seen on Fig. 5.23. The same trend – increasing $\beta$ increases EA - is valid for mass normalized values too. However, EA of honeycomb with thin facesheets, per unit mass, are now comparable to levels observed for honeycomb with thick facesheets. The increase in the plateau levels can also be explained by increasing Young’s modulus of the core with increasing $\beta$, which can be seen in Eq. 5.2 and Eq. 5.3. The areas under the mass normalized EA curves can be compared for the cells with higher EA and the regular cells. For the cells with thin facesheets cells with $2\beta^*$ absorbs 76% more energy (per unit mass) compared to the regular cell. For the cells with thick facesheets $2\beta^*$ absorbs 2.13 times more energy (per unit mass) compared to the regular cell.
5.3.5 Effect of inclined wall thickness to wall length ratio

In the previous section the thickness of all the walls was varied. In this section $\beta$ is varied but $\beta, \eta$ is kept constant to examine the effect of varying $\beta$ on the inclined wall thickness, $t_l$ (leaving the vertical wall thickness unchanged at its baseline value). The other parameters; $\alpha, \theta, \text{ and } \gamma$ were kept constant. The crushing behaviors of these cells with thin and thick facesheets are shown in Fig. 5.24. Similar to the previous section facesheet thickness also had varying values since $t_l$ was varying; for thin facesheets $t_f = t_l$ and for thick facesheets $t_f = 4 t_l$. It can be seen that the cells with thin and thick facesheets follow the same trend, increasing $\beta$ while keeping $\beta, \eta$ constant increases the EA. However, when compared to Fig. 5.22 it can be seen that the plateau level for $\beta=2\beta^*$ and $\eta=2$ is higher when compared to the $\beta=2\beta^*$ and $\eta=1$ plot on Fig. 5.24. This is expected since increase in EA with thicker walls (thicker vertical and inclined walls) is greater compared to when only the inclined walls are thickened. For the cells with thick facesheets increasing wall thickness increases the EA as shown earlier in the $\beta$ study.

Figure 5.23: Mass normalized energy absorption plot for varying $\beta$
Additional to this observation increasing $\eta$ also increases the plateau level for the cells with thick facesheets.

![Stress vs. strain behavior of the honeycomb with facesheets structure with varying $\beta$, $\eta$.](image)

**Figure 5.24:** Stress vs. strain behavior of the honeycomb with facesheets structure with varying $\beta$, $\eta$.

### 5.3.6 Effect of vertical wall to inclined wall thickness ratio, $\eta$

Effect of vertical wall thickness on the crushing behavior of the cells with facesheets was examined by changing the $\eta$ value for the $30^\circ$ cells with thin ($t_f = 0.145$ mm) and thick ($t_f = 0.58$ mm) facesheets. In this study the thickness of the vertical walls was varied, giving $\eta=4$, $\eta=2$ and $\eta=1$. The stress vs. strain plots corresponding to the crushing behavior of these cells are shown in Fig. 5.25. In this figure solid lines show the cells with thick facesheets and the dashed lines show the ones with thin facesheets. There is a significant trend in the behavior of the cells with thick facesheets; increasing $\eta$ increases the plateau level. This is expected since thicker vertical walls increase the stiffness of the core in out-of-plane direction (direction of the keel beam width). However, in case of thin facesheets there are other concerns. For $\eta=2$ and 4 the plateau
stress values do not change significantly, however, for $\eta=1$ plateau level is lower. When the deformation of the cells were observed it was seen that the crushing mode for $\eta=2$ and 4 was dimpling however, it was wrinkling for $\eta=1$. When the failure mode is dimpling increasing $\eta$ did not change the stress response of the cells, since the vertical walls do not deform in this case. However, reducing $\eta$ beyond a critical point changed the failure mode from dimpling to wrinkling, and causing a reduction in the stress levels, similar to the case without facesheets.

![Stress vs. strain behavior of the honeycomb with facesheets structure with varying vertical wall thickness](image)

**Figure 5.25:** Stress vs. strain behavior of the honeycomb with facesheets structure with varying vertical wall thickness

Varying $\eta$ also changes the mass of the cells; therefore, it is important to compare mass normalized energy absorption values. The mass normalized EA vs. strain plot for varying $\eta$ can be seen in Fig. 5.26. For the cells with thick facesheets the trend does not change and higher $\eta$ provides better EA. For the cells with thin facesheets the curves for $\eta=2$ and 4 do not overlap anymore since cell with $\eta=4$ has higher mass (therefore, lower mass normalized EA). Curve for $\eta=1$ has higher initial stress value, however, since the mode is wrinkling there is a drop in the plateau level as was visible on Fig. 5.25. Cells with thick facesheet and $\eta=4$ absorbs 45 % more energy per unit mass compared to the regular cell with thick facesheets. Therefore, if the cells with thick facesheets are
involved in the design of an energy absorbing application the vertical walls should be thick so that the plateau level is higher and therefore, the energy absorption.

5.3.7 Keel beam width

For the cell geometric parameters study, the baseline cell assumed to have 10 mm cell depth. The core depth effect on the energy absorption or the crushing behavior can be studied by changing the cell depth. In this study three different core depth was considered; \( b = 5 \text{ mm} \), \( b = 10 \text{ mm} \) and \( b = 20 \text{ mm} \). The stress vs. strain response of crushing of cells with varying cell depth can be seen in Fig. 5.27. Decreasing core depth increases the initial peak load and the plateau stress level. Same trend was observed for the cells with thin and thick facesheets.

Increasing core depth also increases the mass and volume of the cell. Therefore, it is important to compare the mass normalized energy absorption. Figure 5.28 indicates that the trend stays the same; cells with shorter core depth have higher energy absorption. The specific energy absorption of cells with thin facesheets is comparable to the ones

Figure 5.26: Mass normalized energy absorption plot for varying \( \eta \)
with thick facesheets. The magnitude of the difference between the curves comes smaller when the effect of mass and volume is considered.

Figure 5.27: Effect of core depth on the stress vs. strain behavior

Figure 5.28: Mass normalized energy absorption plot for varying $b$
Honeycombs without facesheet do not have variation in the stiffness through the core depth; however, when there is facesheet the sandwiched honeycomb stiffness depends on the cell depth. This phenomenon was analytically studied by Becker [100], who plotted the variation of the core stiffness with respect to the cell depth, (shown in Fig. 5.29). On this chart $C_{xx}$, $C_{xy}$ and $C_{yy}$ are the effective stiffnesses in x, shear and y-direction, respectively. $h$ is the core thickness (cell depth). The solid lines correspond to the analytical results and the symbols to the simulation results. This plot indicates that the stiffness of the core in y-direction is high when the core height, $h$ (cell depth) is small, meaning that the core has higher stiffness closer to the facesheets. The ABAQUS simulation results presented in Fig. 5.27 support Becker’s results.

Figure 5.29: Variation of the effective core stiffnesses with varying cell depth [100]

It can be concluded from this study that the sandwich structure with shorter core has higher SEA. The stiffening effect of the short core on the sandwich structure can be benefited by using multiple sandwich structures stacked together (or using intermediate sheets along the core depth).
5.3.8 Number of rows in a fixed keel beam height

In the geometric parameters study, the cell depth for the baseline cell was 10 mm. This value was chosen to match the dimensions of Papka and Kyriakides’ paper [8] (the paper used in the validation study for the cells without facesheets). However, for a keel beam, longer width might be considered. As explained in the Methodology chapter, in the cell number of rows study the height of the keel beam is chosen to be 291 mm (≈11.5”) and the width was 101.6 mm (4”). These values were accepted as an average of the dimensions given in several literature sources for helicopter keel beams. For a keel beam with fixed height increasing number of rows also means decreasing cell wall length (or decreasing cell size). For the number of rows study the simulations of the unit cell crushing were conducted where the cells had different cell wall lengths corresponding to different numbers of rows. As explained in more detail in the Methodology chapter, the cell dimensions that are used in the core without facesheet crushing study correspond to 35 rows of cells for a keel beam with 291 mm height. In order to study a keel beam with more number of rows cell wall length should be decreased, and vice-versa. For example, considering only 5 rows along the core height requires cell geometry with wall length of 36.44 mm, and similarly considering 100 rows requires wall length of 1.936 mm. The geometries with many and few numbers of rows can be seen in Fig. 5.30.

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Figure 5.30: Varying numbers of rows along the keel height with specific dimensions; a) many numbers of rows, \( N = 35 \), b) core with fewer numbers of rows, \( N = 5 \)
While changing the cell wall length, the other non-dimensional geometrical parameters were kept constant in order to observe the effect of only number of rows (or cell number) on the crushing behavior. Therefore, $\theta$, $\alpha$, $\beta$, and $\eta$ was kept constant at their base value of 30°, 1, 0.026 and 2, respectively. $\gamma$ is the ratio of the cell depth, $b$ to cell wall length, $l$. Two different approaches were followed related to $\gamma$; one method was keeping $b$ constant which corresponds to a fixed keel beam width and the other method was keeping $\gamma$ constant which corresponds to a keel beam with varying depth with varying cell numbers (or cell wall length). For the first method the keel beam width was taken as 101.6 mm, for the second method $\gamma$ was calculated for the baseline of $b=101.6$ mm and $l=5.5$, $\gamma=b/l=18.47$. For varying $l$, $b$ was calculated for each $l$ to keep $\gamma$ constant. In this study thin and thick facesheets were considered. In the case of thin facesheets the facesheet thickness was equal to the inclined wall thickness ($t_f=t_l$) and in the case of thick facesheets the facesheet thickness was four times the thickness of the inclined wall thickness ($t_f=4t_l$).

Figure 5.31 presents the effect of number of rows on the crushing behavior for the cells with thin facesheet ($t_f=t_l$, varying) and thick facesheet ($t_f=4t_l$, varying) while $\gamma$ was kept constant, meaning that the keel beam width was varying with number of rows of cells (less rows mean larger cells with larger wall length and greater keel beam width for same $\gamma$). When all the non-dimensional parameters were kept constant for the cells with thin facesheet and thick facesheets, changing the number of rows did not affect the crushing response as can be seen on Fig. 5.31. This is expected since all the cell parameters stay the same including $\gamma$. Increasing or decreasing the number of rows in the keel beam is equal to scaling up or down the cell geometric dimensions. The crushing responses of the cores with different numbers of rows while the keel width (cell depth, $b$) was kept constant are shown in Fig. 5.32. Earlier in this chapter the cell depth study showed that increasing the cell depth for the same cell properties caused diminishing effect of the facesheet on the crushing behavior and consequently, lower mean stress values for the cells with higher cell depth. Similar to that observation, in Fig. 5.32 it is observed that increasing cell wall length (therefore, creating the effect of decreased number of rows) while keeping the cell depth (keel width) constant increases the effect of
facesheet on the crushing behavior. However, reducing the cell wall length (or the cell size) for a constant cell depth causes reduction in the stress response of the crush.

Figure 5.31: Effect of cell number of rows if $\gamma$ was kept constant, $\gamma=18.47$ ($b$ varying)

Figure 5.32: Effect of cell number of rows if $b$ was constant, $b=101.6$ mm ($\gamma$ varying)
5.4 Optimizing the geometry of the honeycomb cell with facesheets

Results presented in this chapter revealed that some of the non-dimensional parameters affect the cells with thin or thick facesheets in different fashion. Therefore, the optimized geometry is different for the cells with thin and thick facesheets. For the cells with thin facesheets (or when the initial crushing mode is dimpling) it was observed that increasing core cell angle, $\theta$, increasing wall thicknesses, $\beta$ and decreasing $\gamma$ (by decreasing cell depth, $b$) increase the energy absorption. On the other hand, $\alpha$ and $\eta$ did not have any significant effect (assuming $\eta$ does not cause change in the crushing mode from dimpling to wrinkling). Therefore, an optimum geometry for the cells with thin facesheets can be obtained by implementing these observations to the baseline cell.

Table 5-3: Dimensions for the modified geometry for the cell with thin facesheets

<table>
<thead>
<tr>
<th></th>
<th>Baseline geometry</th>
<th>Case 1 ($\theta$ and $\beta$ modified)</th>
<th>Case 2 – Preferred ($\theta$, $\beta$ and $\gamma$ modified )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$ (mm)</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>$h$ (mm)</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>$t_l$ (mm)</td>
<td>0.145</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>$t_h$ (mm)</td>
<td>0.29</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>$t_f$ (mm)</td>
<td>0.145</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>$b$ (mm)</td>
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<td>10</td>
<td>5</td>
</tr>
<tr>
<td>$\theta$ (°)</td>
<td>30</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.026 ($\beta^*$)</td>
<td>0.052 ($2\beta^*$)</td>
<td>0.052 ($2\beta^*$)</td>
</tr>
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<td>$\eta$</td>
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<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.818</td>
<td>1.818</td>
<td>0.909</td>
</tr>
</tbody>
</table>

Table 5-3 presents the geometric dimensional and non-dimensional values for the baseline and the modified geometries for the cell with thin facesheets. For Case 1 the cell angle was increased from 30° (baseline geometry) to 45° and $\beta$ was increased from $\beta^*$ to
For Case 2, additional to these parameters, $b$ was decreased to 5 mm from 10 mm (baseline). The change in the geometry from baseline to Case 2 can be seen in Fig. 5.33.

![Figure 5.33: Geometry for the honeycomb with thin facesheets; a) baseline, b) preferred geometry (Case 2 in Table 5-3; $\theta$, $\beta$, $\gamma$ modified)](image)

The stress vs. global strain plots for the baseline and the geometries with improved energy absorption for the cell with thin facesheets are shown in Fig. 5.34. The energy absorption plots per unit mass of the cells with thin facesheets for the baseline geometry and the preferred geometry are shown in Fig. 5.35.

![Figure 5.34: Stress vs. strain plot for modified geometries for cells with thin facesheet](image)
The areas under the curves in Fig. 5.35 give the energy absorbed per unit mass (SEA). The SEA for the baseline is 7.24 kJ/kg (up to 30% strain), for Case 1 it is 15.4 kJ/kg and for Case 2 it is 20.5 kJ/kg. Therefore, if $\theta$ is increased from 30° to 45°, $\beta$ is increased from 0.026 to 0.053 (by increasing $t_l$ from 0.145 to 0.29) and $\gamma$ is changed from 1.818 to 0.909 (by decreasing $b$ from 10 mm to 5 mm) the SEA can be increased 2.86 times (keeping $\alpha$ =1 and $\eta$ =2).

Figure 5.35: Mass normalized energy absorption for cells with thin facesheet

![Figure 5.35: Mass normalized energy absorption for cells with thin facesheet](image)

Figure 5.36: Geometry for the honeycomb with thin facesheets (effect of number of rows included); a) baseline, b) preferred geometry (Case 3 in Table 5-4; $\theta$, $\beta$, $\gamma$, $N$ modified)
It was also observed from the results that decreasing number of rows in a keel beam increases the energy absorption (for a keel beam with fixed width). In order to include the effect of the number of rows on the optimum geometry study, a keel beam with fixed keel width should be considered. In this case the baseline geometry, which has regular cell geometry with 101.6 mm keel width, is same as the one used in Fig. 5.32. The regular and the preferred geometries can be seen in Fig. 5.36.

Table 5-4: Dimensions for the modified geometry for the cell with thin facesheets (number of rows effect is included)

<table>
<thead>
<tr>
<th></th>
<th>Baseline geometry</th>
<th>Case 1 ( (\gamma, N) ) modified</th>
<th>Case 2 ( (\theta, \gamma, N) )</th>
<th>Case 3 ( (\theta, \beta, \gamma, N) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l ) (mm)</td>
<td>5.5</td>
<td>36.43</td>
<td>31.486</td>
<td>31.486</td>
</tr>
<tr>
<td>( h ) (mm)</td>
<td>5.5</td>
<td>36.43</td>
<td>31.486</td>
<td>31.486</td>
</tr>
<tr>
<td>( t_l ) (mm)</td>
<td>0.145</td>
<td>0.96</td>
<td>0.819</td>
<td>1.637</td>
</tr>
<tr>
<td>( t_h ) (mm)</td>
<td>0.29</td>
<td>1.92</td>
<td>1.638</td>
<td>3.274</td>
</tr>
<tr>
<td>( t_f ) (mm)</td>
<td>0.145</td>
<td>0.96</td>
<td>0.819</td>
<td>1.637</td>
</tr>
<tr>
<td>( b ) (mm)</td>
<td>101.6</td>
<td>101.6</td>
<td>101.6</td>
<td>101.6</td>
</tr>
<tr>
<td>( \theta ) (°)</td>
<td>30</td>
<td>30</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.026</td>
<td>0.026</td>
<td>0.026</td>
<td>0.052</td>
</tr>
<tr>
<td>( \eta )</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>18.473</td>
<td>2.789</td>
<td>3.227</td>
<td>3.227</td>
</tr>
<tr>
<td>( N )</td>
<td>35</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5-4 presents the baseline and the modified geometry dimensions for the cells with thin facesheets. The baseline and the modified cases are comparable since they both assume fixed keel beam width. Case 1 was already studied, and the results were included in Fig. 5.32. Additional to lower number of rows, Case 2 also includes the modified cell angle. Compared to the baseline geometry, Case 3 has four modified parameters, \( N, \gamma, \theta \) and \( \beta \). The fact that wall length changes with the cell angle is related
to the keel beam height. As the cell angle increases the height of a cell row also increases, therefore, in order to fit $N$ number of rows in a keel beam with fixed height, cell wall length should be reduced if the cell angle is increasing.

Figure 5.37 compares the stress vs. strain behavior of the cells with improved geometry to the baseline geometry. The geometric values for the baseline, Case 1, Case 2 and Case 3 are given in Table 5-4. It can be seen that $\beta$ and $\gamma$ significantly affect the plateau level; however, the effect of cell angle is relatively less (the difference between Case 1 and Case 2 only depends on the cell angle). According to this plot the mean crushing stress levels for Case 3 is around 22 times greater (12.8 MPa vs. 0.56 MPa). The mean crushing stress levels for Case 2 is around 8 times greater (4.34 MPa vs. 0.56 MPa). Increasing cell angle and wall thicknesses also change the volume and the mass of the cell. Therefore, it is important to compare the mass normalized energy absorption. The mass normalized energy absorption of the cells with thin facesheets is presented in Fig. 5.38. The area under the curves show that Case 3 has 10.8 times more SEA compared to the baseline (11.1 kJ/kg vs. 1.02 kJ/kg) and Case 2 has around 7 times more SEA compared to the baseline (7.1 kJ/kg vs. 1.02 kJ/kg).

![Figure 5.37: Stress vs. strain plot for modified geometries for cells with thin facesheet (number of rows effect included)](image-url)
For the cells with thick facesheets (or when the initial crushing mode is wrinkling) it was observed that decreasing $\alpha$, increasing $\beta$, increasing $\eta$ and decreasing $\gamma$ (by decreasing cell depth, $b$) increase the energy absorption. On the other hand $\theta$ does not have any significant effect on the energy absorption, and therefore, in the modified geometry $\theta$ was kept 30°. Table 5-5 presents the dimensions for the baseline and the modified geometries for the cells with thick facesheets. The regular geometry and the preferred geometry can be seen in Fig. 5.39.

The stress vs. global strain plots for the baseline and the geometries with improved energy absorption for the cell with thick facesheets are shown in Fig. 5.40. The geometric values for the baseline and the modified cases can be seen in Table 5-5. Additionally cell geometry with improved $\alpha$ ($\alpha = 0.5$) is also shown, which was previously presented in Fig. 5.17. The plateau stresses significantly increase with addition of each modified dimension. Since mass and volume also changes with changing wall thickness and length, it is also important to compare the mass normalized energy absorption. The mass normalized energy absorption of the cells with improved geometry can be seen in Fig. 5.41. It can be seen that Case 1 (42.68 kJ/kg) and Case 2 (44.65 kJ/kg)

Figure 5.38: Mass normalized energy absorption for cells with thin facesheet (number of rows effect included)
have around same amount of specific energy absorption (SEA) which is 3.21 times higher compared to the baseline (13.16 kJ/kg). SEA for Case 3 is 4.06 times higher (53.47 kJ/kg) compared to the baseline (13.16 kJ/kg).

Figure 5.39: Geometry for the honeycomb with thick facesheets; a) baseline, b) preferred geometry (Case 3 in Table 5-5; $\alpha$, $\beta$, $\eta$, $\gamma$ modified)

<table>
<thead>
<tr>
<th></th>
<th>Baseline geometry</th>
<th>Case 1 ($a$, $\beta$ modified)</th>
<th>Case 2 ($a$, $\beta$, $\eta$)</th>
<th>Case 3 ($a$, $\beta$, $\eta$, $\gamma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$ (mm)</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>$h$ (mm)</td>
<td>5.5</td>
<td>2.75</td>
<td>2.75</td>
<td>2.75</td>
</tr>
<tr>
<td>$t_l$ (mm)</td>
<td>0.145</td>
<td>0.29</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>$t_h$ (mm)</td>
<td>0.29</td>
<td>0.58</td>
<td>1.16</td>
<td>1.16</td>
</tr>
<tr>
<td>$t_f$ (mm)</td>
<td>0.58</td>
<td>1.16</td>
<td>1.16</td>
<td>1.16</td>
</tr>
<tr>
<td>$b$ (mm)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>$\theta$ (°)</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>$a$</td>
<td>1</td>
<td><strong>0.5</strong></td>
<td><strong>0.5</strong></td>
<td><strong>0.5</strong></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.026</td>
<td><strong>0.052</strong></td>
<td><strong>0.052</strong></td>
<td><strong>0.052</strong></td>
</tr>
<tr>
<td>$\eta$</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.82</td>
<td>1.82</td>
<td>1.82</td>
<td><strong>0.909</strong></td>
</tr>
</tbody>
</table>
The effect of number of rows can also be included to the geometry optimization. In this case different baseline geometry is required in order to compare the geometries for

Figure 5.40: Stress vs. strain plot for modified geometries for cells with thick facesheet

Figure 5.41: Mass normalized energy absorption for cells with thick facesheet
a keel beam with fixed width. The regular and the preferred geometries can be seen in Fig. 5.42.

![Figure 5.42: Geometry for the honeycomb with thin facesheets (effect of number of rows included); a) baseline, b) preferred geometry (Case 4 in Table 5-6; $\alpha, \beta, \eta, \gamma, N$ modified)](image)

<table>
<thead>
<tr>
<th>Table 5-6: Dimensions for the modified geometry for the cell with thick facesheets (number of rows effect is included)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline geometry</strong></td>
</tr>
<tr>
<td>$l$ (mm)</td>
</tr>
<tr>
<td>$h$ (mm)</td>
</tr>
<tr>
<td>$t_l$ (mm)</td>
</tr>
<tr>
<td>$t_h$ (mm)</td>
</tr>
<tr>
<td>$t_f$ (mm)</td>
</tr>
<tr>
<td>$b$ (mm)</td>
</tr>
<tr>
<td>$\theta$ (°)</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
<tr>
<td>$\eta$</td>
</tr>
<tr>
<td>$\gamma$</td>
</tr>
<tr>
<td>$N$</td>
</tr>
</tbody>
</table>
The dimensions for the cells with thick facesheets (regular and with modified geometry) considering a keel beam with fixed width are given in Table 5-6. As for the cells with thin facesheets, each geometric parameter was introduced to the baseline geometry one by one, and the cases presented in the table correspond to modified geometries with higher energy absorption. The increase in the energy absorption can be observed by comparing the areas under the stress vs. strain plots. The stress vs. strain plots for the geometries presented in Table 5-6 are shown in Fig. 5.43. The mean crushing stress values for Case 3 and 4 are significantly higher relative to the other cases and the baseline geometry. Figure 5.44 corresponds to the mass normalized energy absorption of the cells with improved geometries. When areas under the mass normalized energy absorption vs. global strain curves are compared, it can be seen that the geometries shown in Case 3 and 4 have similar energy absorption capabilities. Up to 30% global strain, the modified geometry in Case 4 absorbs 41.8 kJ/kg energy, which is 9.9 times greater compared to the baseline geometry (4.2 kJ/kg).

Figure 5.43: Stress vs. strain plot for modified geometries for cells with thick facesheet (number of rows effect included)
5.5 Comparison of the in-plane direction energy absorption of cells with and without facesheets

The specific energy absorption (SEA) of the in-plane direction crushing of honeycombs with thin and thick facesheets are calculated by using the area under the mass normalized energy absorption vs. strain curves. The results show that the regular honeycomb cells with thin facesheets (baseline geometry in Table 5-3) have SEA of 7.24 kJ/kg and with thick facesheets (baseline geometry in Table 5-5) have SEA of 13.16 kJ/kg (up to 30% global strain). Results of the trend studies presented in this chapter showed that changing geometric dimensions in a specific manner increases the SEA. For the honeycomb cells with thin facesheets the structure with modified core and facesheet geometry (Case 2 in Table 5-3) have SEA of 20.5 kJ/kg and with thick facesheets the modified geometry (Case 3 in Table 5-5) have SEA of 53.47 kJ/kg.

As presented in the previous chapter, when crushed under in-plane direction loading, honeycombs with regular geometry and without facesheets can absorb 0.68
kJ/kg energy (when crushed up to 70% global strain), and with modified geometry they can absorb 3.3 kJ/kg. These results indicate that adding facesheets to the honeycomb core significantly increases the energy absorbed by the honeycomb crushing under in-plane direction.

5.6 Summary

In this chapter crushing simulation results for the honeycombs with facesheets were presented. Initially simulations for steel integral sandwich structure under in-plane loading were conducted and the results were compared to the previously published experimental results. The matching results indicated the validity of the ABAQUS model and the technique which was used in the simulations. Further in the research, crushing simulations of aluminum honeycombs with facesheets were conducted in order to observe the effect of facesheet on the in-plane direction crushing behavior of hexagonal honeycombs. Two different failure modes were observed; dimpling and wrinkling. Cores with thin facesheets exhibited dimpling mode; in which case the vertical walls of the cells were not deformed and the deformation of the cells were governed by the honeycomb core. Cores with thick facesheets exhibited wrinkling mode; in which case the vertical walls of the core deformed as well as the inclined walls and the facesheets and the deformation was governed by the facesheets.

The effect of geometric parameters on the energy absorption was also investigated by systematically changing the honeycomb core and facesheet geometric dimensions. Results showed that the non-dimensional parameters had different effects on the cells with thin or thick facesheets. For the cells with thin facesheets increasing cell angle, vertical and inclined wall thickness and decreasing the core depth increased the SEA. For the cells with thick facesheets increasing inclined and vertical wall thicknesses and decreasing the vertical wall length and the core depth increased the SEA. Furthermore, geometric dimensions of web section of a keel beam were also considered. In this case, the effect of number of cell rows in the web section on the energy absorption was also investigated. In the cell level (since only a single cell was considered in the ABAQUS
model), this effect was simulated by changing the cell size. Increasing the cell size leads to fewer numbers of rows and decreasing cell size leads to many numbers of rows in a keel beam. The results of this study showed that core with fewer numbers of rows (or bigger cells) has higher SEA.

The results of these trend studies lead to the modified geometries which provided relatively higher energy absorption when crushed. The modified geometry for the honeycomb core with thin facesheets provides 20.5 kJ/kg SEA which is 2.8 times higher compared to the structure with regular cell geometry and thin facesheets (20.5 kJ/kg vs 7.24 kJ/kg). The modified geometry for the honeycomb core with thick facesheets provides 53.47 kJ/kg SEA which is 4 times higher compared to the structure with regular cell geometry and thick facesheets (53.47 kJ/kg vs. 13.16 kJ/kg).

For the predetermined web section dimensions, the cells with thin facesheets and the modified geometry had the SEA of 11.1 kJ/kg (up to 30% global strain) which is 10.8 times more compared to the regular cell crushing (with thin facesheets). The cells with thick facesheets and the modified geometry had the SEA of 41.8 kJ/kg (up to 30% global strain) which is 9.9 times more compared to the regular cell crushing (with thick facesheets).

These results indicate that modifying the core and the facesheet geometric dimensions significantly increase the energy absorbed by the honeycombs with facesheets. When compared to the in-plane crushing results of the honeycombs without facesheets which were presented in the previous chapter, adding facesheets to the core increases the SEA more than 10 times for much less global strain values (compared to the in-plane crushing of the modified geometry). The SEA levels of in-plane crushing of honeycomb core and facesheet structure is very close or in excess of the SEA levels of the out-of-plane direction crushing of regular honeycombs (out-of-plane direction crushing SEA levels were presented in the previous chapter). When these facts are considered, use of in-plane direction crushing of honeycombs with perfectly bonded facesheets in energy absorbing applications seems a very promising concept.
Chapter 6

Conclusions and future work suggestions

6.1 Overall summary

Energy absorbing applications based on the crushing of cellular structures date back to 1960s. Their crushing behavior generates a rectangular shape load vs. displacement behavior which satisfies the requirements of an ideal energy absorbing structure; no high initial peak loads, steady crushing load levels for large stroke. Their low weight to strength ratio makes them attractive for energy absorbing applications in aerospace field.

The objective of this thesis was to develop a high performance crashworthy cellular structure with significantly increased energy absorption capability suitable for use in the helicopter subfloor. The first sub-objective was to study the in-plane direction crushing of hexagonal honeycombs, to understand the effect of geometric parameters on the crushing behavior and to suggest cell geometry with improved energy absorption. The second sub-objective was to integrate the energy absorbing honeycombs to an application which can be used in helicopter subfloor. Therefore, use of honeycomb cores with facesheets as the web section of an energy absorbing keel beam was suggested and in-plane direction crushing of sandwiched honeycomb cores was studied.

The energy absorbed by the honeycomb crushing under in-plane direction loading is highly influenced by the cell geometry. The approach to the first sub-objective involved investigating the effect of each geometric parameter on the energy absorption and suggesting a honeycomb cell geometry which provides improved energy absorption compared to the regular hexagonal geometry. The simulations of in-plane direction crushing of aluminum hexagonal cells were conducted using ABAQUS/Standard finite element tool with two-dimensional beam elements. In order to represent a manufactured honeycomb core, the vertical walls of the single cell honeycomb had an imperfection of
0.2° initial misalignment. First the validity of using a single-cell with symmetric boundary conditions in the simulations was established by comparing the single-cell crushing results to the results of cores with several rows and columns of cells. Crushing response of the cells was presented using stress vs. global strain plots. Three regions appear on these plots; initial linear region, followed by the plateau region and the final densification region. In the linear region cell deformation is symmetric and vertical walls do not deform. Due to the initial imperfection at some critical load, the vertical walls start to rotate and the plateau region starts on the stress vs. strain plot. Throughout the whole cell deformation the vertical walls do not undergo plastic deformation but only perform rigid body rotation. Finally when opposing cell walls come into contact the stiffness increases rapidly, seen in the densification region. The specific energy absorption (SEA) which is the energy absorbed per unit mass of the cell can be calculated using the area under the stress vs. global strain curves.

The design parameters which determine the hexagon geometry are $h$: the vertical wall length, $l$: the inclined wall length, $t_h$: thickness of the vertical walls, $t_i$: thickness of the inclined walls, $\theta$: the cell angle, which is the angle between horizontal direction and the inclined walls, and $b$: the cell depth. The properties are often described by using non-dimensional set of parameters: $\alpha = h/l$ (cell aspect ratio), $\beta = t_i/l$ (inclined wall non-dimensional thickness), $\eta = t_h/t_i$ (vertical to inclined wall thickness ratio), $\gamma = b/l$ (non-dimensional cell depth). Changing the cell geometry systematically and comparing the energy absorbed by the crushing of the cells with improved geometries to the energy absorption of the regular hexagonal geometry provided honeycomb geometry with relatively higher energy absorption under in-plane direction crushing. In-plane crushing of honeycomb cell with regular and the improved geometry provided SEA of 0.68 kJ/kg and 3.3 kJ/kg, respectively, showing a 4.8 times increase in the energy absorption.

In the literature it has been reported that the out-of-plane direction crushing of honeycombs absorb higher amount of energy compared to the one absorbed by the in-plane direction crushing. In this dissertation, limited simulations of the out-of-plane direction crushing of the cells were also conducted, and their SEA was compared to the SEA of in-plane direction crushing of honeycombs. Compared to the in-plane crushing of
honeycombs with regular cell geometry, out-of-plane crushing provided 50 times more SEA. Some disadvantages of out-of-plane crushing are mentioned in literature sources which might be a concern in the applications using this method for energy absorption. The failure mode which provides ideal form of energy absorption is the progressive local buckling mode. If the mode changes to global buckling mode, the energy absorption reduces significantly. It has been reported that if the height of the core is more than 3 times greater than the diameter of the cell, the global buckling failure mode occurs in out-of-plane direction crushing. For a given I-beam where the honeycombs are placed in a slender web section, avoiding global buckling might be a design challenge if out-of-plane direction crushing of tubes or honeycombs intended to be used. The use of in-plane direction crushing of honeycombs might solve this problem associated with the out-of-plane crushing of honeycombs. However, as mentioned earlier the SEA of in-plane crushing of honeycombs with regular or with improved geometry is significantly less compared to the SEA of the out-of-plane direction crushing. Constraining honeycomb core between two stiff facesheets and investigating the crushing of the sandwiched core under in-plane direction crushing is a possible way of increasing the SEA, which was the subject of the second part of the dissertation.

The approach to the second sub-objective of investigating the crushing of sandwiched honeycomb core under in-plane direction loading also included conducting crushing simulations of a single honeycomb cell with facesheets. This geometry which resembled sandwich structures was assumed to have perfect bonding between the facesheets and the honeycomb core. Since perfect bonding was considered, debonding of the facesheets was not one of the possible failure modes. Therefore, the sandwich structure can carry much higher loads and the structure preserves its post-failure integrity which is an important requirement in crashworthy applications. The challenges of this design come from the assumption of the perfect bond. In reality mostly the sandwich structures use adhesive layers to bond the core to the facesheet which reduce the load carrying capacity of the sandwich structure under in-plane and bending loads significantly. However, recent studies have shown that methods such as brazing, resistance welding, etc. provide integral sandwich structures where the facesheets are
perfectly attached to the core. Existence of such technology for manufacturing integral sandwich structures not only opens new areas of study but also provide very promising application design ideas for energy absorbing structures. Literature survey study showed that the out-of-plane direction crushing of sandwich structures in many different applications is studied by several researchers; however, the use of sandwich structures under the in-plane direction loading for energy absorbing applications has not been studied, because the adhesive bonding between the core and the facesheets fails prior to any significant energy absorption. One of the main contributions of this dissertation is that the crushing of integral sandwich structures under in-plane load beyond the initial failure provide significant amount of energy absorption.

Crushing simulations of the honeycomb core with facesheets were performed using ABAQUS with three dimensional shell elements. Only half of the model was used in the simulations with symmetric boundary conditions. Effect of the core and facesheet geometric parameters on the energy absorption capability of the sandwich configuration was investigated and the results were presented in this dissertation. Similar to the honeycombs without facesheets, investigation of the effect of geometric parameters was studied by varying the cell angle, wall thicknesses and lengths. In addition to these parameters, the energy absorption of the sandwiched cells was also influenced by the core depth and the facesheet thickness, $t_f$. Effect of numbers of rows, $N$ along the keel beam height was also studied by changing the cell sizes. For this part, specific dimensions for the web section of a keel beam were assumed. Smaller cells with short wall length meant many numbers of rows and larger cells with long wall length meant less numbers of rows in the keel beam.

The results of these studies suggested sandwiched honeycomb core geometries with relatively higher energy absorption compared to the baseline geometry (honeycomb core with regular cell geometry). Regular honeycomb geometry with thin facesheets had SEA of 7.24 kJ/kg and with thick facesheets 13.16 kJ/kg. When the geometries are improved the SEA increases to 20.5 kJ/kg for the core with thin facesheets and 53.47 kJ/kg for the core with thick facesheets.
The SEA of these sandwich structure geometries with improved energy absorption were also compared to the SEA of the in-plane direction crushing of regular honeycombs without facesheets. When compared to the in-plane direction crushing of regular cells without facesheets the increase in SEA varies between 10.6 times (7.24 kJ/kg vs. 0.68 kJ/kg) for thin facesheet and 20 times (13.16 kJ/kg vs. 0.68 kJ/kg) for thick facesheet.

When the SEA of honeycomb with facesheets are compared to the in-plane direction crushing of cells with improved geometries without facesheets, the increase in SEA varies between 6.2 times (20.5 kJ/kg vs. 3.3 kJ/kg) and 16.2 times (53.47 kJ/kg vs. 3.3 kJ/kg).

The SEA of honeycombs with facesheets was calculated up to 30% global strain, meaning that in order to experimentally observe these values of SEA a relatively small stroke is sufficient. For example, if a keel beam with 12” height is crushed 3.6” with given geometric dimensions (for example thick facesheets with modified geometry) SEA level of 50 kJ/kg can be reached. This is a very important feature of using in-plane crushing of the sandwiched honeycombs, since in a real crashing scenario only limited stroke might be achieved due to the varying impact conditions. In a similar scenario where only 30% of the stroke is achieved, the out-of-plane crushing of honeycombs would provide SEA less than 15 kJ/kg.

The SEA of in-plane direction crushing of the core without facesheets is not enough to absorb the high crash loads of helicopters even if the cell geometry is optimized. However, the SEA of the improved geometries with facesheets is comparable or greater to the out-of-plane crushing of the regular honeycombs. The high energy absorption levels of the sandwiched honeycomb crushing and their post-crash integrity depend on the perfect bonding conditions between the core and the facesheets. This leads to the key finding of this dissertation; crushing of the honeycombs with facesheets has great potential to be used for the energy absorbing applications since their SEA levels are high enough to make them attractive for every application where out-of-plane direction crushing of honeycombs were utilized, and in-plane crushing does not posses the disadvantages associated with out-of-plane crushing.
6.2 Summary of results

Geometric parameters had different effects on the energy absorption capability of the cells, but they all responded in a specific trend. Due to the nonlinearity of the problem an optimization technique could not be implemented; however, the results of the trend studies lead to geometries with improved energy absorption. Results can be grouped for the cells with and without facesheets. Some of the geometric parameters affect the cells with thin and thick facesheets in different fashion. The optimum geometry is different for the cells with thin or thick facesheets. Therefore, for the honeycomb with facesheets the results are sub-grouped for the ones with thin and thick facesheets. The effect of individual geometric parameter and underlying physics are presented in the following.

6.2.1 Honeycomb geometry without facesheets

The in-plane direction crushing of honeycombs and investigation of the effects of geometric parameters on the energy absorption provided the following results:

- Simulating the crushing of a single cell honeycomb is sufficient to generate the crushing behavior of a honeycomb core with several rows and columns. This saves significant amount of computational time.
- Increasing cell angle increases the SEA.
- For the cells with smaller cell angles, the deformation of the inclined walls are in the form of rigid body rotation around plastic hinge points; however, for the cells with larger cell angles the inclined walls’ deformation is distributed towards the center of the inclined walls.
- Decreasing the vertical wall length increases the SEA.
- Similar to the phenomena observed with varying cell angle, increasing vertical wall length causes a more distributed deformation on the inclined walls as opposed to localized hinge formation in case of cells with short vertical walls. Therefore, the total
energy absorbed by the cells increased by increasing the vertical wall length. However, the density of honeycomb also increases with increasing vertical wall length. Therefore, the specific energy absorption is higher for the cells with short vertical walls.

- Increasing the cell wall thicknesses increases the SEA.
- Increasing the cell wall thicknesses also increases the deformation on the surfaces of the cell walls, and therefore, they absorb more plastic deformation energy.
- Vertical wall thickness should be high enough to initiate the crushing due to bending of the inclined walls and low enough not to cause excessive mass.
- Increasing the thickness of the vertical walls did not cause any change on the crushing behavior. However, increasing wall thicknesses also increase the mass of the cells; therefore, for the same amount of total energy absorption the specific energy absorption decreases. Decreasing the vertical wall thickness beyond a critical value causes significant change in the crushing modes of the honeycomb. If the vertical walls are too thin they start buckling, instead of rotating. Therefore, the crushing of the cells occurs prematurely causing a significant drop in the energy absorption.
- For improved energy absorption per unit mass, these results suggested a cell with bigger cell angle, shorter and thinner vertical walls and thicker inclined walls. Energy absorption of a cell with $\theta = 60^\circ$, $\alpha = 0.25$, $\beta = 2\beta^*$, and $\eta=1$ (modified geometry) was compared to the SEA of a regular cell which has $\theta = 30^\circ$, $\alpha = 1$, $\beta = \beta^*$, and $\eta=2$. The preferred cell geometry has SEA of 3.3 kJ/kg which is 4.8 times higher SEA than regular cell geometry (0.68 kJ/kg).

The out-of-plane direction crushing of honeycombs provided the following results:

- The out-of-plane crushing of regular cells with different core heights were simulated and it was observed that the mean crushing load provided by the cells with different heights are very close.
- The SEA of out-of-plane crushing of regular honeycombs was $\sim$30 kJ/kg, which is around 10 times greater compared to the in-plane crushing of honeycombs with improved geometry and $\sim$50 times greater compared to the in-plane direction crushing of honeycomb with regular cell geometry (without facesheets).
• It is presented in the literature that if the height to diameter ratio of a honeycomb is greater than 3, the failure mode of the out-of-plane crushing of honeycombs is global buckling (as opposed to local buckling for shorter cells). The global buckling failure mode does not provide a progressive crushing behavior; therefore, the energy absorption reduces significantly compared to the progressive local buckling failure. Another disadvantage of using out-of-plane crushing of honeycombs is the bouncing effect of trapped air in the crushed tubes.

6.2.2 Honeycomb core with facesheets

6.2.2.1 Honeycomb core with thin facesheets

• Effect of facesheet thickness on the failure mode of the sandwich honeycomb cell was investigated and it was presented that for cells with thin facesheets the failure mode is called “intracellular buckling” (or dimpling).
• In the dimpling mode the core vertical walls do not deform, and the deformation of the inclined walls does not vary through the cell depth. The deformation is governed by the core. In this mode ~ 70% of the energy is absorbed by the deformation of the facesheets, less than 5% of the energy is absorbed by the vertical walls.
• Increasing facesheet thickness increases the SEA.
• Increasing cell angle increases the SEA. This is explained with the effect of cell angle on the y-direction modulus of the core; increasing cell angle increases the modulus of the core along y-direction. The increase in the initial stress level is also supported by the analytical formula given in literature which calculates the failure loads at which dimpling occurs.
• Increasing inclined wall thickness increases the SEA. This is similar to the observation for the cells without facesheets. Increasing wall thickness increases the surface deformation and therefore, the energy absorption.
Increasing vertical wall thicknesses does not change the crushing behavior; however, adds unnecessary weight to the structure, reducing SEA. If the vertical wall thicknesses are reduced beyond a critical level, the failure mode changes from dimpling to wrinkling, and causes a reduction in the SEA.

Decreasing cell depth (or keel beam width) increases the SEA. It has already been reported that the in-plane modulus of the core is higher closer to the facesheets. The stiffening effect of the facesheet on the core diminishes towards the center of the core.

For a web section with specific dimensions fewer numbers of rows along the core height provide higher SEA relative to a denser core.

For the cells with thin facesheets the optimum geometry with higher SEA has bigger cell angle, thicker inclined cell walls and shorter cell depth. Energy absorption of a sandwiched core with thin facesheets and with $\theta = 45^\circ$, $\beta = 2\beta^*$, and $\gamma=0.909$ (modified geometry) was compared to the SEA of a regular cell which has $\theta = 30^\circ$, $\beta = \beta^*$, and $\gamma=1.818$ (baseline). SEA increased from 7.24 kJ/kg to 20.5 kJ/kg (2.8 times) when the geometry was modified.

For a web section with specific dimensions the modified geometry had $\theta = 45^\circ$, $\beta = 2\beta^*$, $\gamma=3.227$ and $N=5$ and SEA was compared to $\theta = 30^\circ$, $\beta = \beta^*$, $\gamma=18.473$ and $N=35$. SEA increased from 1.02 kJ/kg to 11.1 kJ/kg (10.8 times) when the geometry was modified.

### 6.2.2.2 Honeycomb core with thick facesheets

Effect of facesheet thickness on the failure mode of the sandwich honeycomb cell was investigated and it was presented that for cells with thick facesheets the failure mode is called wrinkling.

In the wrinkling mode vertical walls deform significantly and the deformation of the inclined walls varies through the cell depth. The deformation is governed by the facesheet. In this mode $\sim 70\%$ of the energy is absorbed by the deformation of the facesheets, $\sim 20\%$ of the energy is absorbed by the vertical wall deformation.
• Increasing facesheet thickness increases the SEA.
• Core cell angle did not show any significant effect on the energy absorption. The varying stress reduction with varying cell angle was related to the vertical wall deformation. Cells with higher cell angle had more deformation on the vertical walls compared to the cells with smaller cell angles. This can be explained with the x-direction modulus of the core, which is smaller for higher cell angles.
• Decreasing the vertical wall length increases the SEA. Similar to the cells without facesheets this is related to the volume and the mass of the cell which increases with increasing vertical wall length. Actually total energy absorption for the cells with longer walls are greater compared to the ones with shorter vertical walls since the deformation of the vertical walls is relatively higher. However, for specific energy absorption the trend shows that the cells with shorter vertical walls provide more SEA for the cells with thick facesheets.
• Increasing inclined wall thickness increases the SEA. This is similar to the observation for the cells without facesheets. Increasing wall thickness increases the surface deformation and therefore, the energy absorption.
• Increasing vertical wall thickness (relative to the inclined wall thickness) increases the SEA. This is related to the out-of-plane modulus of the core which is higher for cells with thicker vertical walls.
• Decreasing cell depth (or keel beam width) increases the SEA.
• For a web section with specific dimensions, fewer numbers of rows along the core height provide higher SEA compared to a denser core.
• The optimum geometry with higher SEA has shorter vertical walls, thicker inclined and vertical cell walls, shorter cell depth. Energy absorption of a sandwiched core with thick facesheets and with $\alpha =0.5$, $\beta = 2\beta^*$, $\eta =4$ and $\gamma=0.909$ (modified geometry) was compared to the SEA of a regular cell which has $\alpha =1$, $\beta = \beta^*$, $\eta =2$ and $\gamma=1.818$ (baseline). SEA increased from 13.16 kJ/kg to 53.47 kJ/kg (4.06 times) when the geometry was modified.
• For a web section with specific dimensions the modified geometry had $\alpha =0.5$, $\beta = 2\beta^*$, $\eta =4$, $\gamma=1.92$ and $N=5$ and SEA was compared to $\alpha =1$, $\beta = \beta^*$, $\eta =2$, $\gamma=18.473$ and $N$
=35. SEA increased from 4.2 kJ/kg to 41.8 kJ/kg (9.9 times) when the geometry was modified.

6.3 Contributions of the thesis

As mentioned earlier and reported extensively in the Literature chapter there are numerous studies on the use of honeycombs in energy absorbing applications. Therefore, it was highly important to conduct a literature survey in order to gather information of the issues which were not mentioned or studied extensively, or the areas where there seemed to be lack of attention. Crushing of honeycombs in the edge-wise direction (in-plane) is an area which is neglected since it was observed that the core-wise direction crushing (out-of-plane) absorbs more energy.

In this dissertation the effect of each cell geometric parameter on the energy absorption capability of honeycombs under in-plane direction loading was investigated and underlying physics were presented. As a result it was showed that by making simple changes to the cell geometry it is possible to increase the SEA of in-plane direction crushing of honeycombs by 4.8 times compared to the regular cell geometry which are generally used in the applications.

Even though the modifications in the geometry improved the SEA of the honeycomb cell under in-plane direction loading, the SEA levels were still not comparable to that of out-of-plane direction crushing honeycombs. Therefore, use of honeycomb core sandwiched between two facesheets was suggested which might also be considered as the web section of a keel beam to be used in a helicopter subfloor. The crushing of honeycombs with perfectly bonded facesheets provided significant increase in the SEA. When the core and facesheet geometry was modified it was observed that the SEA levels for in-plane crushing of honeycombs with facesheets were around or in excess of SEA levels of out-of-plane crushing of honeycombs. The novelty of in-plane direction crushing of sandwiched honeycombs came from the fact that a perfect bond between the core and the facesheet was assumed which allowed high levels of SEA and post-crash integrity.
6.4 Suggestions for future work

As mentioned earlier the key finding of this dissertation is that the SEA levels of the in-plane crushing of honeycombs with perfectly bonded facesheets is in the levels of out-of-plane crushing of honeycombs. This observation makes them a potential candidate for energy absorbing applications. With more research their reliability can be extended.

In this thesis static simulations were conducted, results of which are perfectly valid for lower vertical crash velocities. It was experimentally shown that for vertical crashing velocities up to 50 ft/sec the effect of dynamic loading is same as the static loading for metal honeycombs [55], and military standards require the energy absorbing applications to be tested at least up to 42 ft/sec vertical speeds [6]. For greater impact velocities dynamic simulations or experiments might be considered.

There are several studies in the literature where crushing of composite structures or sandwich structures with foam cores are studied. It has been reported that the addition of the foam to an energy absorbing structure does not increase the energy absorption; however, causes excessive mass [5]. It also has been reported that the foams might entrap the moisture and cause even more increase in the mass with time [39]. Therefore, if use of foams is considered in the future for an energy absorbing applications these drawbacks should be addressed. A preliminary study was conducted for the crushing of the foam core sandwich structures. The approach and results can be found in A.5. Composite materials have the advantage of reduced weight over metallic materials. However, since they are brittle they can not stand to high strain values as ductile materials. Therefore, energy absorption should come from an innovative design [6].

As mentioned in this thesis honeycomb cores with facesheets might be utilized as the web section of a keel beam, and be placed in a helicopter subfloor. The follow-up study related to the design of such a keel beam might consider a specific helicopter with specific dimensions for the keel beam. In that case there are several areas which need to be addressed:

- Number of keel beams to be placed in the subfloor: Depending to the number of the keel beams in the subfloor the crashing loads that each one need to absorb will change.
This will also affect the required energy absorbed by the sandwiched honeycomb core webs.

- Specific requirements for the keel beam design: Spacing for wiring, for fuel tank, etc. In some existing helicopter subfloor designs the keel beams do not have the same geometry through the length of the vehicle (even though they are continuous beams from the nose to the aft of the vehicle). The intersection of the keel beams with lateral bulkheads form a relatively stiffer section called cruciform which absorbs different amount of energy compared to the rest of the keel beam. This fact affects the total energy which needs to be absorbed by the keel beam section containing the honeycombs. As shown in Fig. 3.9 the energy absorbing areas of the keel beam are assigned sections instead of the whole length of the keel beam.

- Flight envelope: Depending to the type of the helicopter the loads on the keel beam during flight (such as bending moments and shear forces) and crash will vary. If the flight envelope involved high-maneuvers the keel beams will have high torsion, bending etc. loads and therefore, these high loads should be included in the keel beam design, which might change their geometry.

- Size of the helicopter: The purpose of the energy absorption is to absorb the kinetic energy that the vehicle possessed prior to the impact. This kinetic energy depends on the crash velocity and the mass of the helicopter. For the same crashing velocity a smaller helicopter will have much less kinetic energy to be absorbed.

- Crashing conditions: Similar to the mass of the helicopter, the crashing velocity changes the kinetic energy of the helicopter tremendously. Also depending to the landing gear condition during crash, the regulations require minimum vertical crashing velocity of 42 ft/sec (landing gear extended) or 26 ft/sec (landing gears retracted). Military standards also require the energy absorbing applications to be tested in an envelope of $+15^\circ$ to $-5^\circ$ of pitch and $\pm10^\circ$ of roll angles in the vertical crash. In order to satisfy these conditions the crushing and the SEA levels of the honeycomb cores should be evaluated at multidirectional loading conditions in addition to the perfectly directional loading (the case considered in this thesis).
• Impact surface conditions: Soft soil, hard surface or water impact cause different load paths on the subfloor. One causes distributed loading, the other causes concentrated load locations on the subfloor and the keel beam. In the design all these cases should be considered.

In addition to addressing each of these areas through computational research, experiments should be conducted in order to validate the findings.
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92. ABAQUS Version 6.8 Theory Manual


Appendix A

Supplemental information

A.1 Stress and energy absorption calculations

In this study the results are presented in two main groups; cells with and without facesheets. For both simulation models the y-direction movements of the nodes at the bottom edges were constrained in order to prevent rigid body movement. Displacement loading was applied to the edges on the top of the cells (and facesheets). The reaction forces on these nodes were gathered as the output of the ABAQUS simulations. It was previously reported that the stress that the honeycomb generates can be calculated by dividing the reaction forces to the effective cross section area of the honeycomb. This can be seen in Eq. A.1, where TRF stands for “Total Reaction Force” which is the summation of the reaction forces on the nodes in y-direction, and c is the distance between the vertical walls of the cell and b is the cell depth. Strain is found by dividing the displacement of the top nodes to the original length, which is seen in Eq. A.2 where l is the height of an undeformed cell.

\[
\text{Stress} = \frac{TRF}{2bc} \quad \text{Eq. A.1}
\]

\[
\text{Strain} = \frac{\Delta l}{l} \quad \text{Eq. A.2}
\]

The energy absorbed by the crushing can be calculated using force vs. displacement or stress vs. strain curves. The area under the force vs. displacement curve provides the total energy absorption. If the units are in Newton and meter, respectively the unit of the energy will be in Joule. The area under the stress vs. strain curve gives the energy absorbed per unit volume. Multiplication of stress and strain is shown in Eq. A.3
where $2bcl$ is the volume of an undeformed cell and the nominator is the multiplication of force and displacement which is the total energy absorption.

\[
Stress.Strain = \frac{TRF \cdot \Delta l}{2bc.l}
\]

In this study stress vs. strain plots were used. In the first results section where the behavior of the cells without facesheets were presented bar graphs showing the specific energy absorption were presented as well. Specific energy absorption is the total energy absorption per unit mass of the cell. It is also important to compare the energy absorption per unit volume of the cell. As mentioned before, the area under the curve gives the energy absorption per unit volume. The energy absorbed by unit mass is calculated by first finding the area under the stress vs. strain curve, than multiplying this value with initial volume of the cell and therefore, obtaining the total energy absorption and dividing the total energy absorption to the mass of the cell. This sequence is shown in Eq. A.4.

\[
Total\ Energy\ Absorption = Stress.Strain \times Volume = \frac{TRF \cdot \Delta l}{2bc.l} \times 2bc.l = TRF \cdot \Delta l
\]

\[
Specific\ Energy\ Absorption = \frac{TRF \cdot \Delta l}{Mass}
\]

In the second results section where behavior of cells with facesheets was presented, bar graphs were not used. Additional to the stress vs. strain plots, mass normalized energy absorption vs. strain plots were shown. These plots were prepared multiplying the stress with the volume of the cell and dividing with the mass of it, which can be seen in Eq. A.5. In order to find the total energy absorbed per unit mass, the area under this curve should be calculated.

\[
\frac{Stress.Volume}{Mass} = \frac{TRF}{2bc} \cdot \frac{1}{Mass} = \frac{TRF \cdot l}{Mass}
\]
A.2 Discussion on the 2D model with beam elements vs. 3D model with shell elements

As mentioned in the Methodology chapter, for the crushing of the cells without facesheets a 2D model was used in the simulations. For these models B22 beam elements were used based on the Papka and Kyriakides’ paper [8] which was selected for the validation study. Since for the cells without facesheets the depth of the core does not affect the crushing behavior the selection of 2D elements were sufficient to capture the behavior. However, it has been reported that the shell elements are more suitable choice for the simulations when in the model one-dimension is smaller compared to other two dimensions [92]. In the honeycomb crushing study, for each element the thickness of the core is significantly small compared to the depth and length.

In order to compare the response of 2D model with beam elements and 3D model with shell elements, simulations for the crushing of the honeycomb without facesheets were performed using the 2D and 3D models shown in Fig. A.1. The 2D model had B22 beam elements and the 3D model had S4R shell elements. The crushing response of these models can be seen in Fig. A.2. Cell angle of the models were varied between 15° and 60°. The 2D and 3D models provide very close response for the elastic region of the stress vs. strain curve. The plateau region of the stress response estimated using the 3D shell elements is around 10% higher compared to the 2D beam elements result. For the trend studies, 2D models with beam elements provide the same answer as the 3D models with shell elements. As explained earlier, increasing cell angle increases the energy absorption. The same trend was observed with 3D and 2D models as shown in Fig. A.2. The areas under the curves give the energy absorbed per unit volume and Fig. A.3 shows the areas under the curves for the simulations with 2D and 3D models with varying cell angle. For all the cases around 10% higher energy absorption was calculated for the simulations with 3D models compared to the 2D models. With 3D models the increase in the energy absorption for the cells with 60 cell angle is 80% greater, and with 2D models 76% greater compared to the 30 regular honeycombs. Use of 2D model with beam elements is more cost effective since the 2D simulations take relatively less computational time compared to the 3D models.
Figure A.1: Honeycomb cell without facesheets; a) 2D model, b) 3D model

Figure A.2: Comparison of the crushing response of the honeycomb model with 2D beam elements and 3D shell elements
In this dissertation the results of the effects of geometric parameters have been reported by comparing them to the baseline results. Therefore, the use of 2D or 3D models does not have any effect on the relative increase or decrease on the energy absorption.

A.3 Thickness change in S4R elements

S4R elements are used in 3D simulations; those are 4-node quadrilateral elements with transverse shear deformation. They account for change in the thickness during the analysis, using the Poisson’s ratio provided by the user. The relation between the element thickness and the Poisson’s ratio can be shown as following:

The in-plane stress $\sigma_{33} = 0$; linear elasticity gives Eq. A.6:

$$\epsilon_{33} = -\frac{\nu}{1-\nu}(\epsilon_{11} + \epsilon_{22}) \quad \text{Eq. A.6}$$

Figure A.3: Areas under the stress vs. strain curves up to 70% global strains
In terms of logarithmic strains Eq. A.7 gives:

\[
\ln\left(\frac{t}{t_o}\right) = -\frac{\nu}{1-\nu} \left(\ln\left(\frac{l_1}{l_1^o}\right) + n\left(\frac{l_2}{l_2^o}\right)\right) = -\frac{\nu}{1-\nu} \left(\ln\left(\frac{A}{A^o}\right)\right)
\]

Eq. A.7

where \(A\) is the shell’s reference area. Finally the relation is given by Eq. A.8 [92].

\[
\frac{t}{t_o} = -\frac{\nu}{1-\nu} \left(\frac{A}{A^o}\right)
\]

Eq. A.8

**A.4 The analytical method to calculate the facesheet thickness at mode change point**

Analytical formula for the stress on the facesheet at which point the wrinkling starts is given by Ley et al. [65], and is reproduced below in Eq. A.9. In this equation \(E_f\) is the facesheet Young’s modulus, \(E_c\) is the out-of-plane modulus of hexagonal honeycomb core, \(t_c\) is the core depth, \(t_f\) is the facesheet thickness and \(\nu_f\) is the poisons ratio of the facesheet. The out-of-plane Young’s modulus can be calculated by Eq. A.10 which is given by Gibson and Ashby [7] for the linear elastic deformation. In this equation \(E_s\) is the Young’s modulus of the solid material used for honeycomb.

\[
\sigma_f = \frac{0.82E_f}{\sqrt{1-\nu_f^2}} \left(\frac{E_c t_f}{E_f t_c}\right)
\]

Eq. A.9

\[
E_c = E_s \left\{ \frac{h/l + 2}{2(h/l + \sin\theta \cos\theta)}\right\} \frac{t}{l}
\]

Eq. A.10

For the regular honeycomb and the material and geometric properties given in the Methodology chapter, Eq. A.10 gives 2.1 GPa. Equating Eq. A.11 calculates force, dividing this equation by \(t_f\) provides stress) to Eq. A.9 and keeping \(t_f\) as unknown provides the facesheet thickness at which point the critical failure mode changes from
dimpling to wrinkling. For the regular honeycomb $t_f$ at which point the critical failure mode changes from dimpling to wrinkling can analytically be found as 0.602 mm. These equations assume that the in-plane loads are only carried by the facesheets. In the simulations the displacement loading was applied to facesheets and the core. Therefore, there expected to be some discrepancies between the facesheet thicknesses obtained by using the equations and the simulations at which the failure mode changes from dimpling to wrinkling.

$$F_{\text{dimp}} = \frac{2E_f t_f}{(1 - v_f^2)} \left( \frac{t_f}{l\cos \theta} \right)^2$$  \hspace{1cm} \text{Eq. A.11}

**A.5 Preliminary results for crushing of sandwich structure with foam core**

A preliminary study was conducted for a sandwich structure with foam core and facesheets. Similar to the study of the crushing of the honeycomb core with facesheets, a half model was also used for foam sandwich model. Applied boundary conditions and the generated mesh can be seen on the half model, which is shown in Fig. A.4. In this figure the facesheet thickness is $t_f = 1.16$ mm. The facesheet and the foam core are shown in different colors; lighter blue region shows the facesheet, the darker blue region shows the foam core. Symmetry conditions were applied to the left, right and the bottom faces of the foam and the facesheet and to the face of the foam to represent that this model is only half of a unit structure. Displacement loading was applied on the top surface, in the directions of arrows. The crushing response of the foam core sandwich structure can be seen in Fig. A.5. Foam core with two different facesheet thicknesses were modeled, $t_f = 0.58$ mm and $t_f = 1.16$ mm. For both of these structures wrinkling mode was observed.

The material data used in ABAQUS simulations was taken from Lu et al. paper [101]. They have modeled crushing of Aluminum foam (without facesheets) using ABAQUS and validated their results against experimental results. The material model for the foam core was “Crushable foam” with compressive Young’s modulus of $E = 200$ MPa and yield stress $\sigma = 4$ MPa. Foam density was 20.8% of solid aluminum density, which
gives $\rho_{\text{foam}}=540$ kg/m$^3$. Isotropic hardening was used, and the required data of true stress and logarithmic strain values are also given in Lu et al. paper [101].

First, the crushing of an aluminum foam block without facesheets was conducted, and the ABAQUS results were compared to the experimental results provided by Lu et al. [101]. They conducted experiments where they investigated crushing response of
aluminum foams with different densities. These experimental results can be seen in Fig. A.6 [101].

Figure A.6: Experimental results of crushing of aluminum foam without facesheets, with varying foam density [101]

Figure A.7: Comparison of ABAQUS and experimental results for foam crushing (without facesheets)
Similar to these experiments a foam block crushing was conducted in ABAQUS for the aluminum foam with 20.8% density of the solid Aluminum density. The experimental and computational results are compared in Fig. A.7. The blue dashed line on this figure corresponds to the dashed line (20.8% density) shown in Fig. A.6. ABAQUS successfully predicts the crushing of bare foam block with given properties, as shown in this figure.

Crushing behaviors of sandwich structure with foam core and honeycomb core are compared in Fig. A.8. In this figure the pink dashed and solid line are also plotted for comparison purpose and they correspond to the honeycomb without facesheets (regular and modified geometry, respectively). The peak stress levels are close for the sandwich structures for foam or honeycomb core with the same facesheet thicknesses. Beyond the initial stress peak, the sandwich structure with foam core exhibits a significant drop in the stress levels. Therefore, the energy absorbed by these structures is significantly less compared to the sandwich structure with honeycomb core (for the same facesheet thickness). The plateau region for the foam core sandwich structure is relatively more steady, which is an advantage for energy absorbing applications.

Figure A.8: Stress vs. global strain response of sandwich structure with honeycomb and foam core
The core densities of the foam and the honeycomb are different; therefore, the mass of the models are different. The mass normalized energy absorption curves corresponding to the sandwich structure with foam core and honeycomb core simulations are illustrated in Fig. A.9. The mass of the foam core sandwich structure has higher effect compared to the honeycomb core sandwich structure. For energy absorption point of view, relatively higher mass causes smaller SEA for the sandwich structure with foam cores. Crushing results of sandwich structure with foam core showed that the crushing of sandwich structure with a foam core has significantly lower SEA compared to the honeycomb crushing with facesheets, and the crushing behavior of foam structure exhibited an initial peak which is much higher compared to the mean crushing levels.

Figure A.9: Mass normalized energy absorption vs. global strain plot for sandwich structure crushing with foam and honeycomb core
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