FLAME TRANSFER FUNCTION MEASUREMENTS AND MECHANISMS IN A SINGLE-NOZZLE COMBUSTOR

A Dissertation in
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by
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Abstract

The response of a fully-premixed flame to velocity fluctuations was experimentally measured in a single-nozzle, swirl-stabilized, model gas turbine combustor. Flame response was quantified in terms of the flame flame transfer function relating the input velocity fluctuations to the output heat release rate fluctuations. The velocity fluctuation was measured using the two-microphone method and the heat release rate fluctuation was measured using $CH^*$ chemiluminescence emission over the forcing frequency range of $100 - 500$ Hz with a fixed velocity fluctuation amplitude, $u'_{rms}/u$, of 5%. Measurements were conducted over a broad range of operating conditions encompassing varied combustor pressure, $0.1 - 0.4$ MPa, inlet temperature, $373 - 573$ K, average velocity, $15 - 35$ m/s, and equivalence ratio, $0.45 - 0.75$. A total of 47 flame transfer function measurements were acquired over this range of operating conditions. Time-averaged $CH^*$ chemiluminescence flame images were acquired at all operating conditions. At select operating conditions, the flame structure during forcing was characterized through high-speed $CH^*$ chemiluminescence flame imaging.

Flame transfer function gain at all operating conditions exhibited similar characteristics indicating that the same velocity fluctuation mechanisms may be present at all operating conditions. At low frequencies, flame transfer function gain decreased with increasing forcing frequency. After reaching a minimum, flame transfer function gain then increased with increasing forcing frequency. Once a maximum was reached the behavior repeated. Flame transfer function phase increased quasi-linearly with increasing forcing frequency. Deviation from the linear trend occurred in the form of inflection points at forcing frequencies corresponding to flame transfer function gain minima. The effect of each operating condition parameter on the flame transfer function was investigated independently.

Velocity fluctuation mechanisms were investigated from a global perspective by comparing the collapse of flame transfer function gain with different frequency
scaling parameters. Four frequency scaling parameters were compared: Strouhal number based on flame length \( (St_{L_f}) \), Strouhal number based on nozzle diameter \( (St_{D_{nozzle}}) \), phase between axial and azimuthal velocity fluctuations at the flame anchoring location \( (\theta_{u-v}) \), and phase between swirl number and axial velocity fluctuations at the flame anchoring location \( (\theta_{S-u}) \). It was found that \( (\theta_{u-v}) \) collapsed the flame transfer function gain best. Since this parameter is directly related to the swirl number fluctuation magnitude it indicates that swirl number fluctuations are an important velocity fluctuation mechanism. It was also found that the maximum flame transfer function gain decreased with increasing \( St_{L_f}/\pi \) which is related to the response time of the flame.

Velocity fluctuation mechanisms were then investigated on a local scale through analysis of phase-synchronized flame images. Root mean square fluctuation images showed that heat release fluctuations are equally distributed about the mean flame position at flame transfer function gain minima. Conversely, at flame transfer function gain maxima the largest heat release fluctuation occurred in the downstream region of the flame. A windowing analysis was applied to the phase-synchronized flame images to investigate the interference of axial velocity and swirl number fluctuations. It was found that interference between these two mechanisms was only present at flame transfer function gain minima, and then only for certain window divisions showing that interference between between the two mechanisms is not the cause of the flame transfer function gain extrema.

Swirl number fluctuations were then examined through their direct effect on the flame, movement of the flame base position. Flame base movement followed an inverse trend to flame transfer function gain, i.e. when flame transfer function gain increased flame base movement decreased and vice versa. This trend was shown for all but the shortest flames tested. This indicates that flame base movement acts to decrease global flame response and that the degree of flame wall interaction modifies flame response. Through examination of the vorticity equation it was shown how the flame could decrease the vorticity of the flow by gas expansion, baroclinic production of vorticity of opposite side, and increased viscous diffusion. Therefore it is proposed that when the swirl number fluctuation is largest the flame base movement is largest and the position of the flame relative to the shear layer changes causing decreased vorticity and in turn decreased flame transfer function gain. When the swirl number fluctuation is smallest the flame base does not move and the vorticity of the shear layer is not damped before interacting with the flame leading to high flame transfer function gain.
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List of Symbols

Latin symbols

A  Area \([m^2]\)

  a. Normalized amplitude of axial velocity fluctuation, \(\bar{u}/\bar{u}\)

  a.u. Arbitrary unit

CoHR  Center of heat release

CV  Coefficient of variation

c  Speed of sound \(\text{[m/s]}\)

D  Diameter \([m]\)

FTF  Flame transfer function

f  Frequency \([\text{Hz}]\)

G  Scalar function marking the flame surface

HFA  Hot-film anemometer

\(\Delta h_R\)  Heat of reaction \([\text{J/kg}]\)

  i  Imaginary unit, \(\sqrt{-1}\)

L  Length \([m]\)

Ma  Mach number

N  Number of samples
p Pressure [MPa]
q Heat release [J]
$R^2$ Coefficient of determination
$Re$ Reynolds number
r Radial coordinate
S Swirl number or flame speed [m/s]
SSPSD Single-sided power spectral density
$St$ Strouhal number
T Temperature [K]
t Time [s]
u Axial velocity [m/s]
v Azimuthal velocity [m/s]
x Axial coordinate

**Greek symbols**

$\alpha$ Flame angle
$\gamma^2$ Coherence
$\Theta$ Azimuthal coordinate
$\theta_{a-b}$ Phase of quantity a with respect to quantity b [deg]
$\lambda$ Wavelength [m]
$\mu$ Mean or viscosity [kg/s⋅m]
$\nu$ Kinematic viscosity [m²/s]
$\rho$ Density [kg/m³]
$\sigma$ Standard deviation
\( \phi \)  Equivalence ratio
\( \omega \)  Angular frequency \([\text{rad/s}]\) or vorticity \([1/\text{s}]\)
\( \nabla \)  Gradient

**Superscripts**

\( \vec{\cdot} \)  Vector field
\( \cdot' \)  Fluctuation
\( \overline{\cdot} \)  Average
\( \cdot' \)  Rate
\( \cdot^* \)  Excited species or complex conjugate

**Subscripts**

\( \cdot_{acs} \)  Acoustic
\( \cdot_{CB} \)  Centerbody
\( \cdot_{conv} \)  Convective
\( \cdot_c \)  Combustor
\( \cdot_{DS} \)  Downstream
\( \cdot_{fl} \)  Flame
\( \cdot_{Gmin1} \)  First flame transfer function gain minimum
\( \cdot_{Gmax} \)  Flame transfer function gain maximum
\( \cdot_{Gmin2} \)  Second flame transfer function gain minimum
\( \cdot_h \)  Hydraulic
\( \cdot_{in} \)  Inlet
\( \cdot_L \)  Laminar
\( \cdot_{LW} \)  Lower window
\( (\cdot)_{rms} \) Root mean square
\( (\cdot)_{sw} \) Swirler
\( (\cdot)_{T} \) Turbulent
\( (\cdot)_{US} \) Upstream
\( (\cdot)_{UW} \) Upper window
\( (\cdot)_{u} \) Unburned
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Chapter 1

Introduction

1.1 Background

1.1.1 Gas turbines

The world’s first industrial gas turbine was in a 4 MW power station in Neuchatel, Switzerland in 1939 and is shown schematically in Figure 1.1a [1]. Over the past 74 years industrial gas turbine technology has made massive strides as can be seen by comparing Figure 1.1b to Figure 1.1a. Industrial gas turbines now account for 26% of all electricity generated in the United States, second only to coal power plants [11]. During the 1970s, industrial gas turbines were primarily used for power generation in peaking and emergency situations due to their fast start-up capabilities [12]. These early gas turbines used diffusion flame combustors for their stability and reliable performance. While diffusion flame combustors are highly stable and reliable, they have high NOx emissions due to high flame temperatures. Early attempts at combating NOx emissions involved injecting water or steam to reduce combustion temperature however, this approach caused problems related to engine lifetime and provision of enough treated water [12].

Due to ever more stringent emissions standards, industrial gas turbines have transitioned to lean-premixed combustion. Although this transition translates into greatly reduced emissions of pollutant species such as NOx, it also increases the systems susceptibility to self-excited combustion instabilities [13,14]. The increase in the system’s susceptibility to self-excited combustion instabilities is due to two
main reasons. First, these systems operate at lean equivalence ratios to reduce the temperature of the flame zone and thereby reduce the amount of thermal NO\textsubscript{x} produced [15]. Lean operation means that the system is more susceptible to perturbations because of its closer proximity to the lean blowout limit as opposed to operating in the diffusion combustion mode [16]. Second, in diffusion combustors
a substantial portion of air enters the combustor through the liner. This dilution air is beneficial from the perspective that it decreases the temperature of the combustion products before entering the turbine and it also serves as a strong damper of pressure oscillations in the combustor [17]. Lean-premixed combustors with their lower combustion temperature do not use nearly as much dilution air and therefore have much less of the acoustic damping associated with it.

1.1.2 Combustion instabilities

Combustion instabilities result from the coupling between unsteady heat release from the flame and system acoustics which lead to sustained large amplitude pressure oscillations in the combustor [13, 14, 18, 19]. Detrimental consequences of combustion instabilities include decreased overall system efficiency, flame flashback or blow off, increased heat transfer, and severe damage to system components [9, 19, 20], see Figure 1.2. The combustion instability feedback loop is illustrated in Figure 1.3. In order to understand the entirety of the combustion instability process, it is necessary to understand each relationship that comprises it. Since the process is cyclic there is no true beginning or end, but one must start somewhere.

Figure 1.2: (a) New burner assembly. (b) Burner assembly damaged by combustion instability.

Figure 1.2: (a) New burner assembly and (b) burner assembly damaged by combustion instability [3].
Starting with a heat release rate fluctuation in the flame, an acoustic pressure fluctuation is generated. This can be easily understood by the fact that the addition of heat to a gas causes a rise in temperature due to the added energy. Because the pressure of the gas in the combustor is constant, the rise in temperature is accompanied by gas expansion [21]. Only a small fraction of the energy released from the combustion process is necessary to generate large pressure fluctuations in the combustor [13, 14]. The phase between the heat release fluctuations and the pressure fluctuations is also important in determining the growth and ultimate amplitude of the combustion instability. This phase relationship was first stated by Rayleigh [22] in what is known as the Rayleigh criterion. In order for the amplitude of the combustion instability to grow, the phase between the heat release and pressure fluctuations must be within ±90 degrees.

The pressure fluctuations can induce two types of fluctuations depending on the geometry of the system as well as the fuel injection scheme. Pressure fluctuations generate velocity fluctuations, i.e. mass flow rate fluctuations, through the acoustics of the system. The amplitude and phase of the induced velocity fluctuations is governed by the system’s acoustic impedance which primarily depends on geometry and boundary conditions [23]. If the fuel injection location is choked, equivalence ratio fluctuations will result due to the fluctuating mass flow rate of air caused by the pressure fluctuations. In addition, equivalence ratio fluctuations can be created by pressure fluctuations at the fuel injection location which modulate the fuel mass flow rate when the fuel injection location is not choked. However, the possibility of equivalence ratio fluctuations can be eliminated by injecting the
fuel upstream of a choked inlet.

Lastly, the velocity and/or equivalence ratio fluctuations perturb the flame causing heat release rate fluctuations and completing the cycle. The relationship between velocity and/or equivalence ratio fluctuations and heat release rate fluctuations is the most complicated in the combustion instability feedback loop [24]. As such, most of the research to understand combustion instabilities has focused on understanding this relationship. Flame response is the name commonly given to this area of research. Figure 1.4 shows the dominant processes whereby velocity, Figure 1.4a, and equivalence ratio, Figure 1.4b, fluctuations cause heat release rate fluctuations.

\[\dot{q} = \int_{A_{fl}} \rho_u S_L \Delta h_R dA_{fl}\quad (1.1)\]
where $\dot{q}$ is the heat release rate, $\rho u$ is the unburned gas density, $S_L$ is the laminar flame speed, $\Delta h_R$ is the heat of reaction, and $A_{fl}$ is the flame surface area. The dependence of laminar flame speed and heat of reaction on equivalence ratio are two of the processes whereby equivalence ratio fluctuations generate heat release rate fluctuations [15]. Laminar flame speed fluctuations also modulate the flame surface area, providing another means for equivalence ratio fluctuations to affect heat release rate. There is only one process connecting velocity fluctuations to heat release rate fluctuations. When the velocity fluctuates the mass flow rate of the reactants into the flame fluctuates. Flame area changes to accommodate the fluctuating mass flow rate and burn all of the fuel in the fuel lean case. In turn, the heat release rate fluctuates due to the fluctuating flame area.

The feedback loop nature of combustion instabilities means that each of the individual relationships shown in Figure 1.3 cannot be studied independently during a combustion instability. Therefore, the feedback loop must be broken and each relationship studied independently to gain a better understanding of combustion instabilities. This study will focus on the flame response relationship discussed previously. Specifically, it will focus on the response of fully premixed flames to velocity fluctuations. A companion study is being undertaken to determine the flame response due to equivalence ratio fluctuations [25]. The flame response can be characterized by defining a flame transfer function (FTF) which relates the input velocity fluctuations to the output heat release rate fluctuations. Merk [26] was the first to introduce the concept of a flame transfer function in 1957. In the case of fully premixed flame response to velocity fluctuations the flame transfer function is defined as

$$FTF \left( f, \frac{u'}{u} \right) = \frac{\dot{q}'(f)}{\dot{q}'(f)} \frac{u'(f)}{\bar{u}}$$  \hspace{1cm} (1.2)$$

where $u$ is the axial velocity, $\dot{q}$ is the heat release rate, $f$ is frequency, the prime denotes fluctuation, and the overbar denotes temporal averaging. The flame transfer function is dependent on both the frequency of the velocity fluctuation as well as its amplitude. As a complex quantity, the flame transfer function has both magnitude and phase. The magnitude is commonly referred to as the gain, indicating whether the flame dampens or amplifies incident velocity fluctuations. The phase represents the time delay between velocity fluctuations and the induced heat
release rate fluctuations.

The flame response is divided into two distinct regimes. The regime where the flame response, normalized heat release rate fluctuation amplitude, scales linearly with the normalized velocity fluctuation amplitude is known as the linear regime. In the nonlinear regime the flame response depends on the normalized velocity fluctuation amplitude. The difference between the two flame response regimes is shown in Figure 1.5. Since the flame transfer function gain is the slope of the data as plotted in Figure 1.5, it can be seen that the gain depends on normalized velocity fluctuation amplitude in the nonlinear regime but is independent of it in the linear regime.

![Figure 1.5: Linear and nonlinear flame response regimes.](image-url)

1.2 Literature review

Research regarding the flame’s response to imposed velocity fluctuations will be reviewed in the following sections. Laminar flame response research will be reviewed
first as this is where the flame response research area started. Subsequently, the more recent research on turbulent flame response will be reviewed. Flame response has been investigated analytically, numerically, and experimentally. A review of all three research methods will be given. Lastly the work regarding specific velocity fluctuation mechanisms will be reviewed.

1.2.1 Laminar flame response

In an early experimental study, Matsui [27] measured the pyro-acoustic amplification of fully-premixed laminar flames. It was found that over the frequency range tested, approximately 50-500 Hz,

“... the plot of amplification versus excitation frequency shows a jagged shape with peaks and valleys [27].”

A mathematical model was also proposed which incorporated several phase lags in order to predict the occurrence of maxima and minima, implying that several processes affect the flame response.

Baillot et al. [28] experimentally and analytically studied the response of a conical laminar fully-premixed flame to velocity fluctuations created by a loud speaker. They focused on the deformation of the flame front by using a laser tomography system. Flame area fluctuations of up to 2.5 times greater than the imposed velocity fluctuation were observed. Their research showed that the flame front oscillates at the perturbation frequency and that the oscillation was due to waves generated at the flame base and convected at a velocity proportional to the mean flow velocity.

Analytical studies progressed with the work of Fleifil et al. [29] who studied the dynamic response of a laminar fully-premixed flame stabilized on the rim of a tube to velocity oscillations. They started with the G-equation, first introduced by Markstein [30] in 1955, shown in Equation (1.3) where $G$ is a scalar function marking the flame surface, $\vec{u}$ is the velocity field, and $S_L$ is the laminar flame speed.

$$\frac{\partial G}{\partial t} + \vec{u} \cdot \nabla G - S_L |\nabla G| = 0 \quad (1.3)$$
They concluded that the flame is more sensitive to low frequency perturbations than to high frequency perturbations, i.e. the flame exhibits low-pass filter behavior.

Ducruix et al. [31] extended the model of Fleifil et al. to account for any flame angle. Their analytical model showed that the flame transfer function could be generalized in terms of a reduced frequency, $\omega^*$, defined as

$$\omega^* = \frac{\omega R}{S_L \cos(\alpha_0)}$$  \hspace{1cm} (1.4)$$

where $\omega$ is the angular frequency, $R$ is the burner radius, $S_L$ is the laminar flame speed, and $\alpha_0$ is the half angle of the conical flame. Model comparisons with experimental data were good up to a reduced frequency of 6. For $\omega^* > 6$, the model under predicts the flame response. Schuller et al. [32] identified the cause of the discrepancy between the model predictions and experimental data. Previous analytical models had assumed a spatially uniform velocity perturbation as input to the G-equation. By using a spatially non-uniform velocity perturbation in the model, the experimentally measured flame response could be predicted well over the entire range of reduced frequencies. The need to use a spatially non-uniform velocity perturbation was also noted by Preetham et al. [33]. The difference in velocity perturbation models used gives rise to the notion of compact, flame length $\ll$ wavelength of disturbance, and non-compact, flame length $\gg$ wavelength of disturbance, flames.

An analytical comparison between the dynamics of conical flames and V-flames was made by Schuller et al. [34]. The difference between conical and V-flame structure, as well as M-flame structure discussed later, is shown in Figure 1.6. V-flame dynamics were not only governed by a reduced frequency, similar to conical flames, but also by the flame angle with respect to the flow direction. In addition, whereas conical flames exhibited overall low-pass filter behavior, V-flames acted as amplifiers over a range of frequencies. It was noted that the amplification nature of V-flames may make them more susceptible to combustion instabilities. V-flame dynamics were studied experimentally by Durox et al. [35]. Flame transfer function gain measurements showed amplification over a certain frequency range in agreement with analytical model predictions. The phase of the flame transfer
(a) Conical flame.  (b) V-flame.  (c) M-flame.

Figure 1.6: Comparison of (a) conical, (b) V, and (c) M flame structures. The flame is indicated by red lines.

function increased linearly with forcing frequency indicating the convective nature of the velocity perturbation. Detailed particle image velocimetry measurements of the velocity field showed that the main wrinkling of the flame front is due to vortex structures generated in the shear layer. Durox et al. [36] then experimentally compared the response of M-flames to the previously studied conical and V-flames. Similar to V-flames, M-flames acted as an amplifier over certain frequency ranges and exhibited a linear increase of phase with frequency. However, the gain of M-flames in the amplification range was smaller than that of V-flames.

1.2.2 Turbulent flame response

Research regarding the response of fully premixed turbulent flames to velocity fluctuations has been mixed in terms of the behavior of the flame transfer function observed. The literature can be broken down into two main categories based on the behavior observed. The first category will be termed low-pass filter like behavior and the second will be termed alternating extrema behavior. These two names are derived from the behavior of the gain of the flame transfer function because
the behavior of the phase of the flame transfer function for the two categories is fairly similar. An example flame transfer function gain and phase showing low-pass filter like behavior is shown in Figure 1.7a and Figure 1.7b shows an example flame transfer function gain and phase showing alternating extrema behavior. These two categories will be reviewed separately starting with the low-pass filter like behavior.

(a) Gain and phase difference of FTF versus forcing frequency at $V'/V_{mean} = 0.100$. Operating conditions: $T_{in} = 200 \, ^\circ C$; $V_{mean} = 60 \, m/s$; $\phi = 0.60$ and premixed; and $X_{H2} = 0.00$ and 0.15 [5].

(b) Flame describing function based on hot wire anemometry for flame B [6].

Figure 1.7: Example flame transfer functions showing (a) low pass filter behavior [5] and (b) alternating extrema behavior [6].
One of the first experimental investigations of the response of turbulent flames was done by Kulsheimer and Buchner [37]. They compared the response of turbulent axial jet flames to that of turbulent swirling flames. It was found that while both flames exhibited low-pass filter like behavior, the gain of the swirling flame showed a higher amplification than that of the axial jet flame. The difference was attributed to the presence of swirl but not fully explained. Balachandran et al. [38] also noted amplification behavior for a bluff-body-stabilized flame without swirl. Gentemann et al. [39] compared computational fluid dynamics reconstruction of flame transfer functions to experimental results for a swirling flame. Flame transfer function gain values in excess of unity at low frequencies were found in both the experimental and computational fluid dynamics results. In order for the gain to exceed unity energy must be transferred from the mean flow. It was stated that this transfer of energy occurred because

“Swirl number fluctuations traveling into the flame cause accumulation of unburned gases and their periodic release [39].”

The effect of flame structure on the flame transfer function was examined by Kim et al. [40]. For a fixed frequency, as flame length increased the gain of the flame transfer function decreased. Flame transfer function phase was found to increase with increasing flame length regardless of frequency. Similar to the comparison made with laminar flames, Kim et al. [5] compared the response of turbulent V and M flames. In order to achieve the M-flame structure, a blended fuel of hydrogen and natural gas was used. M-flame amplification was observed to be less than that of V-flames. Lower amplification of M-flames was attributed to them being ’stiffer’ due to attachment at two locations, see Figure 1.6c compared to Figure 1.6b. The flame transfer function gain for both V and M flames could be empirically fitted well by a second order oscillator model and the flame transfer function phase could be approximated by a linear fit. Kim et al. [41] then went on to explain the physical processes responsible for the amplification behavior using an analysis of the relevant length scales involved. The relevant length scales are: flame length $L_{fl}$, convective wavelength $\lambda_{\text{conv}} = \frac{\pi}{f}$, acoustic wavelength $\lambda_{\text{acs}} = \frac{c}{f}$, flame preheat zone thickness, and reaction zone thickness. Flame length is an important length scale because its variation is proportional to variations in flame surface area which
through Equation (1.1) directly correlate with heat release variations. Over the range of conditions tested the only other relevant length scale of the same order of magnitude as flame length is the convective wavelength. The flame preheat zone and reaction zone thicknesses being several orders of magnitude smaller and the acoustic wavelength at least an order of magnitude greater. A nondimensional parameter, \( X = \frac{0.5\lambda_{\text{conv}}}{L_{\text{fl}}} \), was defined. Flame transfer function gain measurements from 26 different conditions were shown to collapse when plotted against \( X \). This shows that the ratio of the flame length to the convective wavelength is the dominant parameter governing the flame’s response in their experiments.

One of the earliest studies to show evidence of the alternating extrema behavior is that of Bellows et al. [42]. Three gain minima and two gain maxima were found in the frequency range of 10-550 Hz. The phase exhibited a linear frequency dependence up to approximately 400 Hz indicating a constant time delay between velocity and heat release. They noted that the frequency at which the linear phase dependence ceased coincided with the highest frequency gain minimum. Results showing this behavior were only presented for one operating condition which limits the amount of conclusions that can be drawn.

One factor that all of the aforementioned research shares is that the work was done at atmospheric pressure. In reality combustors in land-based gas turbines operate at pressures in the range of 12-20 atmospheres [43]. There have been few reported investigations of the flame transfer function at elevated combustor pressure. One study does not systematically evaluate the effect of pressure [44]. The second is related to liquid fuel partially-premixed aero-turbines and shows qualitative agreement between the low pressure and high pressure tests but cautions drawing further conclusions based on the limited amount of data [45]. The third notes major changes in the flame transfer function for increasing combustor pressure [46]. Changing combustor pressure was found to change the frequencies at which the flame amplified flow disturbances which indicates that pressure has an effect on combustor stability. This fact calls into question the validity of incorporating flame transfer function measurements made at atmospheric pressure into analytical models used to predict the stability characteristics of combustors at full engine pressure.
1.2.3 Velocity fluctuation mechanisms

1.2.3.1 Vortex shedding

Vortex shedding has long been, circa 1970s [47], considered a dominant velocity fluctuation mechanism. In most gas turbine combustor geometries there exists one or more shear layers within the flow field of the combustor. These shear layers are the source of vorticity generation [48]. During a combustion instability [49] or when forced [50] it has been observed that the vortex shedding in the shear layer synchronizes with the instability or forcing frequency respectively. When the vortex shedding synchronizes with a resonant acoustic mode or forcing frequency the vortices generated are of a much larger scale compared to the natural shear layer instability [51]. In an experimental study of a dump combustor, Poinsot et al. [52] found that “vortex shedding occurs when the velocity fluctuation at the inlet plane is positive and reaches its maximum” meaning that vortex shedding is synchronized with the velocity fluctuation.

As discussed in Section 1.1.2, the process whereby a velocity fluctuation generates a heat release rate fluctuation is through a modification of flame area. Ducruix et al. [20] separated the flame-vortex interaction into two mechanisms. The first is a change in flame area due to vortex induced flame wrinkling or roll up. Lee et al. [53] experimentally determined that the degree to which flame area was generated by a vortex depended more on the size of the vortex rather than its speed, with larger vortices being more effective in generating flame area. The second is vortex interaction with a wall “inducing a sudden ignition of fresh material [20]” which was enclosed in the vortex. This behavior was experimentally observed in a laboratory-scale swirl-stabilized lean premixed combustor undergoing a self-excited combustion instability by Gonzalez-Juez et al. [54]. A review on the dynamics of a flame’s interactions with vortices was given by Renard et al. [55].

1.2.3.2 Swirl number fluctuations

Swirling flow is a common means of flame stabilization in gas turbine combustors [56]. The swirling flow is typically generated by an axial/radial swirler or injecting the fuel and/or air tangentially. A common feature of swirling flows is vortex breakdown, first observed by Peckham and Atkinson in 1957 [57], which induces
a central recirculation zone in the flow [58]. The central recirculation zone aids in flame stabilization by: recirculating hot products back to the flame front to increase flame speed, creating a low velocity region where the flame can anchor, and increasing turbulence which also leads to increased flame speed [59]. Research has transitioned from investigating the effects of swirl on flame stabilization to investigating the effects of swirl on flame dynamics.

One of the first studies to note the importance of the swirler in regards to flame dynamics was that of Straub et al. [60]. It was shown that changing the axial location of the swirler affected the amplitude of self-excited combustion instabilities in a partially-premixed combustor. Straub et al. showed that the axial location of the swirler affected the velocity profile in the injector and concluded that this lead to different distributions of fuel-air ratio which was the cause of the variation in combustion instability amplitude.

Stone and Menon [61] computationally investigated the effect of varying swirl number on the stability of a model swirl-stabilized, lean-premixed gas turbine combustor using large-eddy simulation. At low swirl numbers, large-scale coherent vortices were observed which interacted with the flame to produce large flame area modulation and pressure fluctuations. As swirl number increased, the strength of the vortices decreased which lead to a more stable flame with lower pressure fluctuation amplitudes. Huang and Yang [9] also computationally investigated the effect of swirl on combustion dynamics in a lean-premixed swirl-stabilized combustor using large-eddy simulation. Their findings were similar to those of Stone and Menon with the additional result that higher swirl numbers led to shorter flames.

Komarek and Polifke [62] investigated the effects of fluctuating swirl number on the heat release of a perfectly premixed swirl stabilized burner. Similar to Straub et al. the axial location of the swirler could be varied. It was shown that while the flame structure was not affected by the axial location of the swirler the flame transfer function was. As the distance from the swirler to the flame increased the frequency of minimum flame response decreased and

"... the modulation of the phase increased [62]."

Computational fluid dynamics simulations were performed to gain an understanding of the changes in the flame response. These unsteady Reynolds-averaged
Navier-Stokes simulations showed that the impingement of the axial velocity fluctuations on the swirler generated azimuthal velocity fluctuations. The axial velocity fluctuation propagates at the speed of sound while the azimuthal velocity fluctuation is convected at the mean flow speed. The additional convective time delay introduced by increasing the distance from the swirler to the flame was used to explain the changes observed in the flame transfer function.

1.2.3.3 Interaction of multiple mechanisms

Preetham et al. [63] used analytical and computational solutions of the nonlinear G equation to show that:

“the flame dynamics are controlled by the superposition of two waves propagating along the flame sheet: those originating at the flame-anchoring point and from flow non uniformities along the flame [63].”

Constructive interference of the two waves caused the flame to act as an amplifier at certain frequencies, and destructive interference caused a null in flame response in the linear regime. It was also shown that the flame response in the nonlinear regime at a given frequency depends on whether constructive or destructive interference occurs at that frequency in the linear regime.

Jones et al. [64] explained their experimental observations of the flame’s response to inlet velocity fluctuations:

“in terms of the interaction of two flame perturbation mechanisms, one originating at flame-anchoring point and propagating along the flame front and the other from the vorticity field generated in the outer shear layer ... [64].”

They argued that the phasing between these two perturbations dictated when the flame response was amplified or damped. An attempt was made to individually extract the heat release fluctuations due to these two perturbations from phase-synchronized flame images in order to examine their phase difference. Heat release fluctuations due to the two perturbations were found to be in phase when flame transfer function gain was high and somewhat out of phase when flame transfer function gain was low at one operating condition. However, it is important to
note that quantitative comparisons between the magnitudes of the heat release fluctuations could not be made due to the different manners in which they were determined.

Palies et al. [6] recently investigated the interaction of multiple velocity fluctuation mechanisms in an experimental study. They stated that the flame response was the product of the interaction of two mechanisms referred to as:

“It is also known from previous studies that when the axial velocity perturbation reaches a maximum a vortex is shed from the injector lips. This vortex is convected by the flow and eventually rolls up the flame tip. This is a first mechanism of unsteady heat release rate which induces large flame surface area fluctuations ... The second mechanism is here associated with the combined axial and azimuthal velocity perturbations. This induces fluctuations in swirl number ... which in return give rise to a breathing motion of the central recirculation region resulting in an angular deflection of the flame [6].”

They applied a windowing analysis to phase-synchronized flame images at two operating conditions to investigate the interaction of these two velocity fluctuation mechanisms. The windowing analysis will be discussed in much greater detail in Section 7.2.2. Palies et al. [65] also investigated the interaction of these two velocity fluctuation mechanisms through computational fluid dynamics simulations.

1.3 Motivation and objectives

Based on the amount of research still ongoing it is evident that combustion instabilities remain a serious problem. Flame transfer functions of turbulent flames in conditions approaching those of realistic combustor configurations have been measured. However, from the literature review it is apparent that an understanding of how specific inlet parameters effect flame response is not available. Several velocity fluctuation mechanisms have been proposed as the physical link between heat release rate fluctuations and velocity fluctuations. This raises the question as to the ubiquity of these velocity fluctuation mechanisms in different gas turbine combustors. Even in studies where specific velocity fluctuation mechanisms were
validated, e.g. Jones et al. [64] and Palies et al. [6], the validation was only shown for a small number of conditions, 1 and 2 respectively. These two areas which have been identified as needing further research were used to set the objectives of this study.

There are two primary objectives to this study. The first is to experimentally measure the heat release response due to velocity fluctuations of a single-nozzle, turbulent, swirling, fully-premixed flame and investigate the effects of inlet parameters on flame response. The second objective of this study is to identify the mechanism(s) whereby velocity fluctuations cause heat release fluctuations and quantify their effects over a broad range of conditions. Recent studies have begun to look at how these velocity fluctuation mechanisms affect the flame. This work will serve as a starting point to determine what mechanisms are evident in this study.
Chapter 2

Experimental methods

An overview of the experimental facility used for this study is given in Section 2.1 and detailed explanations of the individual components are given in Sections 2.1.1 to 2.1.5. Measurement techniques are discussed in Section 2.2 and data acquisition is discussed in Section 2.3. Section 2.4 details data analysis procedures for time domain signals while Section 2.5 explains analysis procedures applied to flame images.

2.1 Experimental facility

The lean fully-premixed, swirl-stabilized, single-nozzle gas turbine combustor used in this study consists of a siren, nozzle, combustion chamber, and exhaust as shown in Figure 2.1. The flow direction is from left to right. From the most upstream point of the siren to the exit of the exhaust section is approximately 3 meters.

2.1.1 Fuel and air supply

High pressure air is supplied to the experiment by a dual compressor system. The maximum output of the compressor system is 0.14 $m^3/s$ at 2.17 MPa. The air supply line is split into two separate flow passages, one for main air and one for cooling air. Both flow passages have filters to remove liquids and particulates. Cooling air pressure is regulated to 0.70 MPa using a domeloaded pressure regulator and flow rate is measured using a thermal mass flow meter (CDI Meters model CDI-
Similarly, main air pressure is regulated to 1.4 MPa using a domeloaded pressure regulator and flow rate is measured using a thermal mass flow meter (Sierra Instruments model FlatTrak 780S). The main air can be preheated by an 88 kW electric heater to a maximum temperature of 573 K.

Natural gas (approximately 95% methane) is used as the fuel for this study. The flow rate of natural gas is measured using a thermal mass flow meter (Teledyne Hastings model HFM-D-301). Natural gas is injected into the main air far upstream of the inlet to the experiment to ensure that the mixture entering the experiment is fully-premixed. Additionally, a choke plate is located at the inlet to the experiment to prevent any pressure oscillations in the system from affecting the mixture homogeneity. These two factors prevent any equivalence ratio fluctuations in the experiment.

2.1.2 Siren

The first section of the experiment is the siren which is used to generate the velocity fluctuations. Premixed natural gas and air enter the experiment through the vertical pipe in the bottom right of Figure 2.2 after passing through a choke plate. The flow passage then splits into a left and right branch. The left branch contains the siren which is comprised of a stator and rotor driven by a variable speed brushless DC motor (VEXSTA model BLFM6400-A). A digital motor driver (VEXSTA model BLFD400S2) controls the rotation speed to within one revolution per minute, which, when taking into account the gear ratio between the motor and
siren and the number of holes in the rotor, corresponds to a ±0.13 Hz control over the frequency of the velocity fluctuation. Maximum motor rotation speed yields a maximum forcing frequency of 500 Hz.

The right branch is a bypass line. The amplitude of the velocity fluctuation can be controlled by adjusting the fraction of the premixed natural gas and air mixture which passes through the siren and that which bypasses it. This is accomplished by adjusting the siren and bypass valves. The maximum velocity fluctuation amplitude, \( \frac{u'_{\text{rms}}}{u} \), possible is frequency dependent with increasing frequency leading to decreasing maximum velocity fluctuation amplitude. In general, a velocity fluctuation amplitude of 30% can be achieved at all frequencies.

![Figure 2.2: Model of siren.](image)

### 2.1.3 Nozzle

The nozzle used in this study is industry scale and swirl-stabilized and can be seen in Figure 2.3. It consists of an outer tube and inner centerbody, recessed from the nozzle exit by 19 mm, held in place by an axial swirler. The swirler consists of 8 vanes with a geometry that yields a swirl number, which is the ratio of the azimuthal to axial momentum flux as defined in Equation (2.1) [66], of approximately 0.70.

\[
S = \frac{\int_0^R \rho uv 2\pi r^2 \, dr}{\int_0^R \rho u^2 2\pi r \, dr}
\]  

(2.1)
In Equation (2.1) \( R \) is the annulus height, \( \rho \) is the mixture density, \( u \) is the axial velocity, and \( v \) is the azimuthal velocity. The swirling flow generated by the swirler is a common means of flame stabilization [56]. The downstream edge of the swirler is located 98 mm upstream of the nozzle exit. The diameter of the outer tube of the nozzle is 55 mm while the diameter of the centerbody is 32 mm. The exit of the nozzle is commonly refereed to as the dump plane indicating the location at which the premixed natural gas and air mixture dumps into the combustor.

There are several additional internal flow passages in the nozzle which are primarily used for fuel injection when the nozzle is operated in a partially-premixed configuration. However, there is one internal flow passage that is used when the nozzle is operated in a fully-premixed configuration. Air is injected into the centerbody and exits through the face of it into the combustion chamber. This air serves to cool the face of the centerbody and prevent warping. The amount of centerbody cooling air is less than 1% of the total air used for the experiment. Additionally, experiments were conducted with varying amounts of centerbody cooling air and it was found the cooling air did not effect the flame response.

![Figure 2.3: Cross-section of model of nozzle. Flow direction is left to right.](image)

### 2.1.4 Combustion chamber

The upstream section (left side) of the combustion chamber shown in Figure 2.4 consists of a 305 mm long, 150 mm inner diameter, 156 mm outer diameter fused quartz tube (type GE 214) enclosed in a stainless steel rectangular prism with dimensions 203 mm x 203 mm x 349 mm. Upstream and downstream ends of the fused quartz tube are secured to the dump plane and a water-cooled transition
piece respectively using high-temperature RTV-silicone. A small port in the dump plane contains the ignitor and a pilot fuel line used for ignition only. The dump plane is also water-cooled.

The left and right sides of the stainless steel box contain 141 mm x 245 mm x 19 mm thick fused quartz windows to allow optical access to the flame. Cooling air flows through the gap between the fused quartz tube and stainless steel box. This air is used to cool the outer surface of the tube and to maintain a small pressure difference across the 3 mm thick fused quartz cylinder. Since the fused quartz cylinder is stronger in compression than expansion, the pressure on the outside is always maintained higher than on the inside.

![Figure 2.4: Cross-section of model of combustion chamber.](image)

The downstream section (right side) of the combustion chamber is a double-walled stainless steel pipe with an inner pipe diameter of 128 mm and an outer pipe diameter of 154 mm. Combustion products flow through the inner pipe to a restriction plate at the end of this section and then into the exhaust. The fused quartz tube cooling air flows in the annular gap between the inner and outer pipes, additionally cooling this section, and then exits through a small pipe perpendicular to the main flow direction. A valve (Masoneilan 35002 series Camflex rotary control valve) is located at the cooling air exit to allow the cooling air passage to be pressurized. A muffler is located after the valve to minimize the noise when the cooling air exits the experiment. The overall length of the combustion chamber, defined as the distance between the nozzle exit and the restriction plate, is 612 mm. The blockage ratio, defined as the ratio of the closed area of the restriction plate to the open area of the combustor, is 91%.
2.1.5 Exhaust

The exhaust section is a single 102 mm diameter stainless steel pipe 1100 mm long. Combustion products flowing through the pipe are cooled by distilled water injected through nine water spray nozzles (Hago model M15). A pump was used to raise the water pressure so that boiling did not occur in the nozzles. Distilled water was used to prevent the buildup of impurities within the nozzles and exhaust. A valve (Masoneilan 35002 series Camflex rotary control valve) is located at the end of the exhaust section, the far right of Figure 2.5, which allows the combustion chamber to be pressurized up to 0.4 MPa. After passing through the valve, the combustion products flow through a muffler to minimize noise and then exit the experiment.

![Figure 2.5: Cross-section of model of exhaust.](image)

2.2 Measurement techniques

2.2.1 Pressure

Fluctuating pressure is measured at five locations, marked as P1-P5 in Figure 2.6, in the experiment using dynamic pressure transducers (PCB model 112A05). All of the pressure transducers are water cooled and recessed mounted. The most downstream pressure transducer, P1, is used to measure the combustor pressure which is used to assess the stability of a particular operating condition to determine if flame transfer function measurements can be taken, see Section 3.1. The four remaining pressure transducers are divided into two pairs, P2-P3 and P4-P5. Pressure signals from these pairs of pressure transducers are used to calculate the velocity fluctuation downstream and upstream of the swirler using the two-microphone method.
discussed in Section 2.2.2. Since the dynamic pressure transducers do not measure static pressure, two static pressure transducers (Omega model PX219) are used to measure the static pressure in the combustor and outside the fused quartz cylinder.

![Image of dynamic pressure measurement locations](image)

Figure 2.6: Dynamic pressure measurement locations.

### 2.2.2 Velocity

There are several ways to measure the velocity fluctuation in the nozzle such as laser doppler anemometry, particle image velocimetry, hot-film anemometry, and the two-microphone method. Both laser doppler anemometry and particle image velocimetry require optical access to the location at which velocity measurements are to be made. This is impossible in this experiment because the nozzle is made of metal. Hot-film anemometry is an attractive means to measure the velocity fluctuation because of the small probe size and fast response time. However, the probes are not capable of surviving at high temperatures. Additionally, the velocity fluctuation measured by the hot-film anemometer varies depending on the radial location of the probe. A detailed set of experiments proving this is shown in Appendix A. Appendix A also shows that the two-microphone velocity fluctuation measurement is equivalent to area-averaging the hot-film anemometer velocity fluctuation over the nozzle annulus. Since the flame transfer function as defined in Equation (1.2) is a single input system, the two-microphone method is used to
measure the velocity fluctuation in the nozzle. In addition, the flame transfer function input, i.e. the velocity fluctuation, should be measured as close to the flame as possible, therefore the pressure signals from the P4-P5 pressure transducer pair are used to calculate the velocity fluctuation. The derivation of the two-microphone method presented here closely follows that of Waser [67]. It begins with Euler’s equation which gives a relationship between velocity, $u$, and pressure, $p$, as shown in Equation (2.2) where $\rho$ is the fluid density and $\vec{g}$ is the body force vector.

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho} \nabla p + \vec{g}$$  (2.2)

Implicit in the use of Euler’s equation are the assumptions of an incompressible and inviscid flow. The validity of the incompressible assumption can be assessed by determining the Mach number, $Ma \equiv u/c$, of the flow. For $Ma < 0.3$, the maximum density variation is less than 5 percent and thus the flow can be treated as incompressible [68]. The maximum Mach number in this experiment is 0.09 which is within the range required for the validity of the incompressible assumption. An inviscid flow implies that the viscosity of the fluid is zero which is never the case in reality. However, an inviscid flow can be approximated in cases where the viscous forces are small compared to the inertial forces. The ratio of inertial forces to viscous forces is quantified in terms of the Reynolds number defined as

$$Re \equiv \frac{\rho u D_h}{\mu}$$  (2.3)

where $\rho$ is the fluid density, $D_h$ is the hydraulic diameter, and $\mu$ is the fluid viscosity. For an annulus, the hydraulic diameter is given by $D_h = D_o - D_i$ where $D_o$ and $D_i$ are the outer and inner diameters of the annulus respectively. The minimum Reynolds number in this experiment is 7525 which exceeds the criterion of 4000 for turbulent pipe flow [68]. Therefore, inertial forces are much greater than viscous forces and the flow can be approximated as inviscid validly.

Equation (2.2) is transformed into Equation (2.4) by linearization, neglecting body forces, and restricting analysis to the flow direction. Body forces can be neglected because the only body force, gravity, acts perpendicular to the flow.
direction.

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \tag{2.4}
\]

Applying a Fourier transform to Equation (2.4) and a finite difference approximation to the pressure gradient yields Equation (2.5) where \( U \) is the velocity linear spectrum, \( i \) is the imaginary unit, \( \omega \) is the angular frequency, \( \Delta x \) is the spacing between the pressure measurement locations, and \( P_{DS} \) and \( P_{US} \) are the pressure linear spectra of the downstream and upstream pressure measurements respectively.

\[
U \approx \frac{i}{\omega \rho \Delta x} (P_{DS} - P_{US}) \tag{2.5}
\]

The resultant velocity linear spectrum is complex, containing both magnitude and phase information. Transformation back to the time domain is accomplished through an inverse Fourier transform. The two-microphone method is incapable of calculating the mean axial velocity since Equation (2.5) is undefined for zero frequency. Therefore, the average nozzle velocity based on nozzle open area, fluid density, and total mass flow rate is added to the calculated velocity time domain signal.

There exists both a low and high frequency limit to the two-microphone method. The low frequency limit is due to the neglect of attenuation between the pressure transducers [69]. At low frequencies the wavelength of the pressure wave is large and therefore the measured pressure difference between the two microphones is small. As a result, any attenuation of the pressure wave between the two measurement locations will have a large effect on the approximated pressure gradient. Additionally, pressure fluctuations associated with noise in the system, i.e. turbulent velocity fluctuations, can have a large effect on the approximated pressure gradient when the acoustic wavelength is large.

The finite difference approximation of the pressure gradient is also the cause of the high frequency limit of the two-microphone method [70]. Although the spacing between the pressure measurement locations, \( \Delta x \), is fixed, the wavelength of the pressure perturbation decreases with increasing frequency. If the acoustic wavelength is on the order of the microphone spacing, or smaller, the measured pressure gradient will be incorrect [67]. In this experiment the microphone spacing is 31.75 mm and the acoustic wavelength can be calculated using the known frequency and
speed of sound which is dependent on inlet temperature. Figure 2.7 compares the transducer spacing and the acoustic wavelength for three different inlet temperatures. Over the range of frequencies shown, 0-500 Hz, which is the maximum rotation speed of the siren, the acoustic wavelength is always much much greater than the transducer spacing. Even in the worst case scenario, the highest frequency and lowest inlet temperature, the transducer spacing is only 4% of the acoustic wavelength. Therefore, over the range of frequencies tested in this experiment, the high frequency limit of the two-microphone method is not encountered.

![Graph showing transducer spacing and acoustic wavelength comparison](image)

Figure 2.7: Comparison of transducer spacing and acoustic wavelength for three different inlet temperatures.

Amplitude and/or phase mismatch between the pressure transducers can be a source of error for the two-microphone method [67,69,71,72]. Therefore, the pressure transducers used for the two-microphone method were tested in an impedance tube to determine if there was any amplitude or phase mismatch between them. The impedance tube is a 502 mm long, 76 mm diameter pipe terminated on one end by a speaker and the other by a rigid plate. Both pressure transducers were mounted in the rigid plate so that they would be exposed to the same pressure and ideally measure the same amplitude and phase. The response of the pressure transducers was determined by varying the frequency of the excitation signal sent to the speaker and measuring the fluctuating pressure seen by both transducers. Results of the impedance tube tests are shown in Figure 2.8. Figure 2.8a shows the pressure fluctuation amplitude ratio and Figure 2.8b shows the phase difference.
between the pressure transducers used for the two-microphone method both as a function of frequency. As can be seen from Figure 2.8a both pressure transducers measure the same pressure fluctuation amplitude within ±5%. Additionally the phase difference between the pressure transducers is ±1 degree, or 0.5% of the maximum possible phase difference of 180 degrees. These two factors indicate that this pair of pressure transducers is well suited for use with the two-microphone method.

Figure 2.8: (a) Amplitude ratio and (b) phase difference between pressure transducers used for the two-microphone method.
2.2.3 Global heat release rate

Three photomultiplier tubes (Hamamatsu model H7732-10) with narrow band-pass filters are used to measure flame chemiluminescence emission from $CH^*$ (432±5 nm), $OH^*$ (307±5 nm), and $CO_2^*$ (365±5 nm). It has been shown that under fuel lean fully-premixed conditions, as is the case for this study, the chemiluminescence emission intensity provides a measure of the heat release rate of the flame [73–75]. Chemiluminescence emission has been used as an indicator of heat release rate in many other fuel lean fully-premixed flame response studies [6, 35, 39, 46, 76–78]. In a comparison with other heat release rate measurement techniques, $OH$ and $CH_2O$ planar laser-induced fluorescence, Balachandran [38] concluded that $OH^*$ and $CH^*$ chemiluminescence agreed well and are much less complicated measures of heat release rate.

2.2.4 Flame structure

Flame structure is characterized by taking chemiluminescence images of the flame. Two types of images are acquired, time-averaged and phase-synchronized images. Time-averaged images are long exposure time images of either unforced or forced flames. In contrast, phase-synchronized images are short exposure time images of forced flames only. Time averaged images are acquired using an intensified change coupled device (ICCD) camera (Princeton Instruments PI-MAX model 7467-0008). Phase-synchronized images are acquired using a high-speed camera (Phantom model 7.1) and intensifier (Video Scope model VS4-1845HS). A high-speed camera was used to acquire the phase-synchronized images because it allows image acquisition in real time which is much faster than the phase-resolved acquisition required by the ICCD camera. Both cameras were equipped with a $CH^*$ (430±5 nm) band-pass filter and a 60 mm f/2.8 Nikkor lens. Camera settings for the two types of images, time-averaged and phase-synchronized, are shown in Table 2.1. In the case of time-averaged images, the intensifier gain was adjusted to the minimum value possible for each condition to minimize noise amplification. This is possible because of the long exposure time used for the time-averaged image which allowed additional signal to be collected. Conversely, the short exposure time used for phase-synchronized images necessitated the use of the maximum intensifier gain.
possible in order to collect as much signal as possible. Images acquired using both cameras are integrated line of sight, or projection, images. This means that the intensity value of each pixel within the image is the integral of the intensity across a chord of the truncated cone shape of the flame.

Table 2.1: Camera settings for time-averaged and phase-synchronized images.

<table>
<thead>
<tr>
<th>Image type</th>
<th>Time-averaged</th>
<th>Phase-synchronized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camera</td>
<td>PI-MAX</td>
<td>Phantom 7.1</td>
</tr>
<tr>
<td>Exposure time</td>
<td>100 ms</td>
<td>230 $\mu s$</td>
</tr>
<tr>
<td>Number of exposures</td>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>Number of images</td>
<td>1</td>
<td>4000</td>
</tr>
<tr>
<td>Sampling frequency [Hz]</td>
<td>NA</td>
<td>4000</td>
</tr>
<tr>
<td>Intensifier gate</td>
<td>100 ms</td>
<td>200 $\mu s$</td>
</tr>
<tr>
<td>Intensifier gain</td>
<td>variable</td>
<td>42,500</td>
</tr>
</tbody>
</table>

2.3 Data acquisition

A data acquisition board (National Instruments model BNC-2110) in conjunction with a LabVIEW program is used to acquire the pressure and chemiluminescence signals with 8192 data points at a sampling frequency of 8192 Hz yielding a frequency resolution of 1 Hz. Signals from the pressure transducers were run through a signal conditioner (PCB model 482A16) which amplified them by a factor of ten before entering the data acquisition board. Chemiluminescence signals ran directly into the data acquisition board. For each measurement 32 sequential measurements are acquired so that a mean and standard deviation can be calculated to assess the uncertainty in the measurement. Temperature is also recorded at the beginning and end of acquiring the pressure and chemiluminescence signals using the LabVIEW program. After acquiring the data with the LabVIEW program, subsequent data processing and analysis is done in a MATLAB program.
2.4 Pressure, velocity, and global chemiluminescence data analysis

Time-domain signals are converted to the frequency-domain using a discrete Fourier transform, commonly implemented as a fast Fourier transform [79], shown in Equation (2.6):

\[ A_m = \sum_{n=0}^{N-1} a_n e^{-i2\pi \frac{nm}{N} \Delta t} \]  

(2.6)

where \( A \) is the linear spectrum, \( a \) is the time domain signal, \( i \) is the imaginary unit, \( N \) is the number of data points, \( \Delta t \) is the time between data points, and \( m \) and \( n \) are the frequency and time domain indices respectively. The resultant linear spectrum is complex and contains both the amplitude and phase information of the time domain signal as a function of frequency. Phase values are determined by calculating the angle between the real and imaginary parts of \( A \) at the frequency of interest.

Determination of the amplitude of time-domain signals is done by calculating the single-sided power spectral density from the linear spectrum through use of Equation (2.7) where \( G_{AA} \) is the single-sided power spectral density, and \( T \) is the period of the time-domain signal.

\[
G_{AA} \equiv \begin{cases} 
\frac{1}{T}|A_m|^2 & \text{for } m = 0 \\
\frac{2}{T}|A_m|^2 & \text{for } 1 \leq m \leq \frac{N}{2} - 1 \\
\frac{1}{T}|A_m|^2 & \text{for } m = \frac{N}{2}
\end{cases} 
\]  

(2.7)

Parseval’s theorem in discrete form, Equation (2.8), relates the root mean square of a signal in the time-domain, \( a_{rms} \), and the integral of its corresponding single-sided power spectral density over all frequencies:

\[
a_{rms} = \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} a_n^2} = \sqrt{\sum_{m=0}^{N/2} G_{AA}(m) \times \Delta f} 
\]  

(2.8)

where \( \Delta f \) is the frequency resolution. Although Parseval’s theorem only applies to the root mean square of the entire time-domain signal and the integral of the
single-sided power spectral density over all frequencies, a modified form can be written to allow determination of the root mean square of a time-domain signal at a single frequency. This form of Parseval’s theorem, shown in Equation (2.9), is most relevant to this study because measurements are taken at discrete frequencies. The notational difference of $a_{rms}$ versus $a'_{rms}$ denotes that mean information is included in Equation (2.8) but not in Equation (2.9) and therefore only the root mean square fluctuation, $a'_{rms}$, is calculated when using Equation (2.9).

$$a'_{rms}(f) = \sqrt{G_{AA}(f) \times \Delta f} \quad (2.9)$$

An important parameter in the context of transfer function measurements is the coherence, $\gamma^2$. Coherence is a measure of how well correlated two signals are and is defined in Equation (2.10) where $G_{AB}$ is the single-sided cross spectral density and $G_{AA}$ and $G_{BB}$ are the single-sided power spectral densities of the input and output respectively. The single-sided cross spectral density is defined similarly to the single-sided power spectral density as shown in Equation (2.11). In Equation (2.10), the over bar denotes averaging over multiple calculations of the single-sided cross spectral density and single-sided power spectral density. In Equation (2.11) $(\cdot)^*$ denotes the complex conjugate. Coherence can range from 0 for uncorrelated signals to 1 for perfectly correlated signals.

$$\gamma^2 \equiv \frac{|G_{AB}|^2}{G_{AA}G_{BB}} \quad (2.10)$$

$$G_{AB} \equiv \begin{cases} \frac{1}{T} A^* B & \text{for } m = 0 \\ \frac{2}{T} A^* B & \text{for } 1 \leq m \leq N/2 - 1 \\ \frac{1}{T} A^* B & \text{for } m = N/2 \end{cases} \quad (2.11)$$

### 2.5 Flame image analysis

Initial image processing of both time-averaged and phase-synchronized images is the same. Background images, acquired with the same camera settings but without the flame present, are subtracted from the raw projection images. After background subtraction, projection images are smoothed using median (3 x 3) and
moving average (11 x 11) filters and then mirrored to yield perfectly symmetric images. Regions outside of the fused quartz combustor are cropped from the background subtracted, smoothed, and mirrored projection images. Projection images are deconvoluted to yield emission images using a discrete Hankel-Fourier inverse Abel transform shown in Equation (2.12) [80]:

\[ \epsilon(r_j) = \frac{1}{2\pi[(2N + 1)\Delta x]^2} \sum_{i=-N}^{N} I(x_i) \sum_{k=0}^{N} \cos \left[ \left( \frac{i}{2N + 1} \right) k \right] k J_0 \left( \frac{jk}{2N + 1} \right) \]

(2.12)

where \( \epsilon(r_j) \) is the emission intensity at radial location \( r_j \), \( N \) is one half of the number of pixels in a column of the projection image, \( \Delta x \) is the spacing between projection intensity values, \( I(x_i) \) is the projection intensity at \( x_i \), \( i, j, k \) are index variables which range from \(-N\) to \( N \), \( 0 \) to \( N \), and \( 0 \) to \( N \) respectively, and \( J_0 \) is the zero-order Bessel function of the first kind.

Emission images are infinitesimally thin two-dimensional slices of the flame. To account for the three-dimensional nature of the flame, the intensity values in the emission image are weighted by \( 2\pi \) times their radial distance from the centerline of the combustor. This so called “r-weighting” yields an image which will be referred to as a revolved image. An example projection image and its corresponding emission and revolved images are shown in Figure 2.9. Only the top halves of the images are shown because they are axisymmetric. Chemiluminescence intensity is displayed by a linear pseudo color scale with white being the highest and black the lowest intensity. The gray area to the left and above the images is the nozzle/dump plane and combustor respectively.

![Figure 2.9: Example (a) projection, (b) emission, and (c) revolved images.](image)

Two metrics, flame length \( L_{fl} \) and flame width \( W_{fl} \), shown in Figure 2.10 are extracted from the revolved images. Flame length is defined as the distance from the edge of the centerbody to the center-of-heat release of the flame, marked
by a + in Figure 2.10a. The center of heat release is calculated exactly as the center of mass would be for a solid body except intensity, rather than density, is used as the weighting function as shown in Equation (2.13):

\[
x_{CoHR} = \frac{\sum_{i=1}^{N} \tilde{\epsilon}_i x_i}{\sum_{i=1}^{N} \tilde{\epsilon}_i}
\]

\[
r_{CoHR} = \frac{\sum_{i=1}^{N} \tilde{\epsilon}_i r_i}{\sum_{i=1}^{N} \tilde{\epsilon}_i}
\]

where \(x\) and \(r\) are the axial and radial coordinates respectively, and \(\tilde{\epsilon}\) is the top 10% of the intensity values in the revolve image. Flame length represents the distance a disturbance must travel before interacting with the portion of the flame where the majority of the heat release occurs. Flame width is defined as the full width at half maximum of the axial heat release distribution as shown in Figure 2.10b. The axial heat release distribution is the radial summation of the intensity in the revolve image at each axial location. The degree to which the flame interacts with the wall of the combustor is indicated by flame width, with increasing flame width indicating increasing interaction with the combustor wall.

It is desirable to determine the position of the flame within the combustor because flame position is intimately linked to certain velocity fluctuation mechanisms as will be discussed in Section 7.2.3. Due to the turbulent nature of the flow in this experiment and the imaging technique used, a flame brush rather than a thin flame sheet is shown in the revolved flame images, as can be seen in Figure 2.9c. However, a mean flame position can be determined from the flame brush. This is done by finding the maximum intensity column-wise in the flame image. An example of this mean flame position is shown by the pink dashed line in Figure 2.11.

An additional processing step is taken with the phase-synchronized images. Since the images are taken in a time resolved manner, a time series for the intensity value of each pixel is obtained. Therefore, the image can be analyzed pixel by pixel using the same procedure applied to the pressure and global chemiluminescence data.
Figure 2.10: Example (a) revolved image with flame length ($L_{fl}$) indicated and (b) corresponding axial heat release distribution with flame width ($W_{fl}$) indicated.

Figure 2.11: Example revolved image with $r_m$ indicated by pink line.
as outlined in Section 2.4. After applying this analysis procedure a series of images is obtained where the intensity value of each pixel fluctuates purely sinusoidally at the forcing frequency. Additionally, images are created where the numerical value of each pixel corresponds to its root mean square fluctuation amplitude, as seen in Figure 2.12a, or its phase with respect to the velocity fluctuation, as seen in Figure 2.12b, and will be refereed to as rms and phase images respectively. Regions outside of the flame brush have been blacked out in the phase image. This region corresponds to pixels which have less than 10% of the maximum intensity in the time-averaged image. Freitag et al. [46] applied a similar technique to their phase-synchronized images. Subsequent processing of phase-synchronized images is used to determine the presence and importance of different velocity fluctuation mechanisms. These analysis procedures will be discussed in Chapter 7.
Figure 2.12: Example (a) root mean square fluctuation and (b) corresponding phase image.
Determination of stable operating conditions

3.1 Stable operating conditions

In this experiment an operating condition consists of four parameters: combustor pressure \( P_c \), inlet temperature \( T_{in} \), average velocity \( \bar{u} \), and equivalence ratio \( \phi \). Combustor pressure is limited to a maximum of 0.4 MPa by the design of the combustion chamber shown in Figure 2.4. The maximum heater sheath temperature limits the inlet temperature to a maximum of 573 K. Average velocity is limited to the range of 15-35 m/s based on the design specifications of the nozzle shown in Figure 2.3. Lastly, the equivalence ratio is limited at the low end by the lean blowout limit, which varies with the other operating condition parameters. The high end limit for equivalence ratio is set at 0.75 based on the desire to study combustion dynamics under fuel lean conditions.

The physical limits stated above are not the only limitations on what operating conditions can be run in this experiment. As mentioned in Section 1.1.2, flame response measurements cannot be taken during a combustion instability. For the purposes of this study, a combustion instability is defined as a normalized root mean square fluctuation of average velocity, \( \frac{u_{\text{rms}}}{\bar{u}} \), combustor pressure, \( \frac{(P_c)_{\text{rms}}}{P_c} \), or chemiluminescence intensity, \( \frac{I_{\text{rms}}}{I} \), greater than 3%. Operating conditions with normalized root mean square fluctuations of these quantities less than 3% are
defined as stable. The operability range of the experiment was tested to find stable operating conditions where flame transfer function measurements could be taken.

As discussed in Section 2.4, the root mean square fluctuation amplitude of time domain signals is calculated through a modified form of Parseval’s theorem, Equation (2.9). This approach is valid in the case of the forced response measurements because all of the signal energy is concentrated in the frequency bin corresponding to the forcing frequency as can be seen in Figure 3.1a. In contrast, during stability measurements, the signal energy is spread over several frequency bins due to the stochastic nature of the system, see Figure 3.1b. Therefore, in order to accurately calculate the root mean square fluctuation amplitude for stability measurements the single-sided power spectral density must be integrated over several frequency bins rather than picking a single frequency bin in order to capture all of the signal energy. This is accomplished by modifying Equation (2.9) into Equation (3.1) where the single-sided power spectral density is integrated over $\pm (5 \times \Delta f)$ Hz around the frequency of maximum fluctuation. Figure 3.1b illustrates this graphically where the frequency bins over which the single-sided power spectral density is integrated in order to calculate the root mean square fluctuation amplitude for stability measurements are highlighted in red.

$$a'_{\text{rms}}(f) = \sqrt{\sum_{m=f-5\times\Delta f}^{f+5\times\Delta f} G_{AA}(m) \times \Delta f}$$

(3.1)

### 3.2 Stable flame structure

Previous research has shown that the flame structure, in terms of shape, length, and flame attachment location, has an effect on the gain and phase of the flame transfer function [5, 18, 36, 40]. Therefore, during the determination of stable operating conditions discussed in Section 3.1, time-averaged images of stable flames were also acquired using the procedure outlined in Section 2.2.4. The raw projection images from the intensified charge-coupled device camera were deconvoluted and the center of heat release in the revolved images was determined to calculate the
Figure 3.1: Comparison of combustor pressure SSPSD’s during (a) forced response and (b) stability measurements.

Flame length and flame width as defined in Figure 2.10. Axial and radial locations of the center of heat release from 98 stable flames are shown in Figure 3.2. Also shown are example “short” and “long” flames. Throughout the entire range of operating conditions tested, the flame retained a V-flame structure similar to the examples shown. It can be seen that the location of the center of heat release follows a well defined curve. Similar behavior has been observed by Peluso [81] for varied inlet temperature, mean velocity, and equivalence ratio and, in the case of Kim [82] and Figura et al. [83], also for varied hydrogen enrichment percentage. These results are the first to show that the location of the center of heat release follows a well defined curve for varied combustor pressure as well.
A nonlinear fit to the flame length data shown in Figure 3.2, of the form of Equation (3.2), was performed to determine the effect of each operating condition parameter on flame length. The nonlinear regression was performed using the nlinfit function in the MATLAB Statistics Toolbox [84].

$$L_{fl} = a \times T_{in}^b \times \phi^c \times \bar{u}^d \times P_c^e$$  \hspace{1cm} (3.2)

The resulting curve fit parameters are summarized in Table 3.1. The exponent for $\bar{u}$, $d$, is positive meaning that an increase in average velocity will cause an increase in flame length. This is to be expected because the flame sits at a position where the flame speed is balanced by the approach velocity of the reactant mixture. Thus, holding all other parameters constant, i.e. fixing the flame speed, and increasing/decreasing the average velocity should also increase/decrease the flame length. The effect of the other three operating condition parameters, $T_{in}$, $\phi$, and $P_c$, can be understood by examining their effect on flame speed. Increased inlet temperature is known to increase flame speed due to the strong temperature dependence of reaction rates, hence the negative sign of the $b$ fit parameter. It is known that the primary effect of equivalence ratio on flame speed is through the effect of equivalence ratio on flame temperature. Increasing the flame temperature increases the reaction rate and in turn the flame speed [15]. The negative sign of
the $\phi$ fit parameter, $c$, agrees with this explanation. Measurements of the turbulent flame speed of a Bunsen-type flame in a high pressure chamber by Kobayashi et al. [85] showed an increase of turbulent flame speed with increasing pressure over the range of 0.1-1 MPa. This explains the negative sign of the fit parameter for $P_c$, $e$.

Table 3.1: Values of curve fit parameters in flame length curve fit, Equation (3.2).

<table>
<thead>
<tr>
<th>Operating condition parameter</th>
<th>Curve fit parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-a$</td>
<td>$-b$</td>
<td>509.55</td>
</tr>
<tr>
<td>$T_{in}$</td>
<td>$-c$</td>
<td>-0.9267</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$-d$</td>
<td>-1.1788</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>$-e$</td>
<td>0.0658</td>
</tr>
<tr>
<td>$P_c$</td>
<td></td>
<td>-0.1457</td>
</tr>
</tbody>
</table>

The percent difference between the measured and curve fit flame lengths is shown in Figure 3.3. Over the range of flame lengths measured the percent difference is at most 10% indicating that the flame length from the curve fit accurately predicts the measured flame length. Therefore, the curve fit shown in Equation (3.2) and coefficients shown in Table 3.1 could be used to interpolate flame lengths for other operating conditions within the range of the measurements. Extrapolation of flame lengths for operating conditions outside the range of the measurements is cautioned due to the fact that the curve fit is not conditioned on these values.

Flame width, defined in Figure 2.10b, was also calculated during the determination of stable operating conditions. Figure 3.4 shows flame width versus flame length for all stable operating conditions. While flame width does increase with flame length it does not do so in a linear manner as shown by the low value of the coefficient of determination of the linear fit applied to the data. This contradicts the result found by Peluso [81] where flame width was shown to increase linearly ($R^2 = 0.97$) with flame length. The most likely cause of the discrepancy is that while all of the flames investigated by Peluso impinged on the combustor wall, flames in this experiment do not always do so, for example see the “short” flame in Figure 3.2. This is a first indication that the behavior of the flame, in terms of
its heat release distribution, is affected by interaction with the combustor wall.

Figure 3.3: Percent difference between measured and curve fit, Equation (3.2), flame lengths.

Figure 3.4: Flame width ($W_{fl}$) versus flame length ($L_{fl}$) for all stable operating conditions.
Chapter 4

Flame transfer function measurements

4.1 Overview of flame transfer function measurements

After determining which operating conditions were stable, flame transfer function measurements were taken. For these measurements the velocity fluctuation amplitude was fixed at 5%. This was done to ensure that the flame response remained in the linear regime. Numerous researchers have investigated the velocity fluctuation amplitude necessary for nonlinear flame response [37, 38, 40, 81, 82, 86–89]. In summary their results show that the minimum velocity fluctuation amplitude necessary for nonlinear flame response decreases with increasing forcing frequency in a hyperbolic manner. Their work also shows that even at high frequencies, $f > 400 \text{ Hz}$, the flame response remained linear with 5% velocity fluctuation amplitude forcing. The flame response was measured from 100 Hz, the low frequency limit of the two-microphone method, to 500 Hz, the high frequency limit based on maximum motor rotation speed. A total of 47 flame transfer function measurements were acquired at the operating conditions listed in Table 4.1.

A representative flame transfer function gain, phase, and coherence are shown in Figure 4.1. Error bars on the gain and phase are $\pm$ two standard deviations of the 32 measurements taken for each data point. The flame transfer function can be
Table 4.1: Operating conditions of flame transfer function measurements.

<table>
<thead>
<tr>
<th>Case</th>
<th>$P_c$ [MPa]</th>
<th>$T_{in}$ [K]</th>
<th>$\bar{u}$ [m/s]</th>
<th>$\phi$</th>
<th>Case</th>
<th>$P_c$ [MPa]</th>
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<th>$\bar{u}$ [m/s]</th>
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<td>573</td>
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</table>

calculated using any of the three measured species chemiluminescence intensities as an indicator of heat release rate. The results for all three species are shown in Figure 4.1. Since all three species show nearly identical behavior, only the flame transfer function calculated using CH* chemiluminescence intensity as an indicator of heat release rate will be shown in subsequent figures.

There are several important characteristics of the flame transfer function. First, starting at low frequency, the gain shows alternating regions of decreasing gain with increasing frequency followed by regions of increasing gain with increasing frequency. This alternating behavior gives rise to gain extrema. A single local gain
Figure 4.1: Example flame transfer function (a) gain, (b) phase, and (c) coherence: $P_c = 0.1 \text{ MPa}, T_{in} = 373 \text{ K}, \bar{u} = 25 \text{ m/s}, \phi = 0.75$. 
maximum and two local gain minima can be seen in Figure 4.1a. The relationship between the extrema frequencies is nearly linear with the first gain minimum occurring at 180 Hz, the gain maximum occurring at 260 Hz \( \approx 1.5 \times 180 \text{ Hz} \), and the second gain minimum occurring at 380 Hz \( \approx 2 \times 180 \text{ Hz} \). Also of note is that at certain frequencies the flame transfer function gain is above unity, meaning that the flame amplifies the incoming flow disturbances. Due to this amplifying behavior, self-excited combustion instabilities are more likely to occur at these frequencies for this operating condition. In contrast, self-excited combustion instabilities are less likely to occur at frequencies where the flame transfer function gain is less than unity, i.e. where the flame acts as a damper.

Overall, the magnitude of the flame transfer function phase increases with increasing frequency, see Figure 4.1b. The trend is linear over nearly the entire frequency range. Deviation from the linear trend occurs in the form of inflection points at two frequencies, 180 and 380 Hz for this operating condition. These two frequencies where the phase diverts from its linear trend are also the frequencies where the flame transfer function gain exhibits local minima.

Lastly, the coherence between the input, velocity fluctuation, and output, heat release rate fluctuation, of the flame transfer function is shown in Figure 4.1c. It is essential to have good coherence between the input and output of the flame transfer function to ensure a high quality measurement [42]. As can be seen from Figure 4.1c the coherence is above 0.99 at all forcing frequencies.

The repeatability of flame transfer function measurements was assessed by repeating measurements at the same operating condition on different days. It is important to assess the day to day variation of flame transfer function measurements at a single operating condition before attempting to make comparisons between flame transfer function measurements at different operating conditions. The day to day variation in flame transfer function measurements at a single operating condition must be less than the variation between flame transfer function measurements at different operating conditions in order for comparisons to be meaningful. Figure 4.2 shows the repeatability of flame transfer function gain, phase, and coherence for one operating condition. Flame transfer function gain is highly repeatable as is evidenced by Figure 4.2a. As can be seen in Figure 4.2b, the day to day variation in flame transfer function phase is within experimental uncertainty, i.e. the error
bars overlap, at all but two forcing frequencies. This indicates that the day to
day variation in flame transfer function phase is within the ability to measure it.
Additionally the coherence values for all three tests are above 0.90, see Figure 4.2c.

All of the aforementioned qualitative trends are shown in the flame transfer
functions measured for each of the operating conditions listed in Table 4.1. This
fact is shown in Figure 4.3 where the flame transfer function gain, phase, and co-
herence are plotted versus forcing frequency normalized by the frequency at which
the first flame transfer function gain minimum occurs for each operating condi-
tion listed in Table 4.1. This normalized frequency parameter is given the name
$f^*$. Normalization of forcing frequency by the frequency at which the first gain
minimum occurs is done to highlight the qualitative similarity of flame transfer
function gain and phase at all operating conditions. However, quantitative differ-
ences do exist in terms of the actual numerical values of gain and phase as well as
the frequencies of the gain extrema and phase inflection points.

The relationship between the first gain minimum frequency and the frequency
at which the phase inflection point occurs also holds true for all operating condi-
tions as can be seen in Figure 4.4. A one to one relationship exists between
these two frequencies for all operating conditions and is shown by the high value
of the correlation coefficient of the linear fit applied to the data. Additionally
the linear relationship between gain extrema frequencies discussed earlier holds
for all operating conditions. Figure 4.5 shows that the gain maximum and first
gain minimum frequencies are related by a factor of 1.5 and the first and second
gain minima frequencies are related by a factor of 2. The effect of each operating
condition parameter; average velocity, equivalence ratio, inlet temperature, and
combustor pressure on the flame transfer function will be discussed in Sections 4.2
to 4.5. Additionally, the variation of flame images and flame metrics with each of
these parameters is shown because flame structure has been shown to affect the
flame transfer function as discussed in Sections 1.2.1 and 1.2.2.
Figure 4.2: Repeatability of flame transfer function (a) gain, (b) phase, and (c) coherence: $P_c = 0.1 \text{ MPa}$, $T_{in} = 373 \text{ K}$, $u = 25 \text{ m/s}$, $\phi = 0.75$. 
Figure 4.3: Flame transfer function (a) gain, (b) phase, and (c) coherence versus $f^*$: All operating conditions.
4.2 Effect of average velocity on flame transfer function

The flame transfer function gain and phase for five different average velocities with fixed combustor pressure, inlet temperature, and equivalence ratio are shown in Figure 4.6. As the average velocity increases, the frequency at which the first gain minimum occurs increases. Not only has the first gain minimum frequency increased, the entire gain curve has translated to higher frequency with increasing average velocity. This means that at low and high frequencies the gain increases with increasing average velocity, while in an intermediate frequency range the gain decreases with increasing average velocity. The phase curves, shown in Figure 4.6b, show a translation to higher frequency with increasing average velocity. At a given frequency, as average velocity increases, phase decreases. This is most readily seen
Figure 4.5: Frequencies of flame transfer function gain maximum and second gain minimum versus frequency of first flame transfer function gain minimum: All operating conditions.

at low frequencies because the phase in Figure 4.6b is wrapped, i.e. restricted to the range of -180 to 180 degrees. Flame transfer function phase has been linked to the time it takes for a perturbation to travel the length of the flame at approximately the average velocity [27, 29, 31, 32, 35]. Therefore, either an increase in average velocity or a decrease in flame length would cause a decrease in flame transfer function phase. Figure 4.7 shows qualitatively and Table 4.2 shows quantitatively that flame length increases with increasing average velocity. This means that for this group of operating conditions there are two parameters changing which should have opposite effects on flame transfer function phase. For this group of operating conditions, a 50% increase in average velocity only leads to a 5% increase in flame length. This indicates that the dominant factor affecting flame transfer function phase for these measurements is the increase in average velocity.
Figure 4.6: Effect of average velocity on flame transfer function (a) gain and (b) phase: $P_c=0.1$ MPa, $T_{in}=473$ K, $u=20, 22.5, 25, 27.5,$ and $30$ m/s, $\phi=0.65$.

Table 4.2: Effect of average velocity on flame metrics.

<table>
<thead>
<tr>
<th>$P_c$ [MPa]</th>
<th>$T_{in}$ [K]</th>
<th>$u$ [m/s]</th>
<th>$\phi$</th>
<th>$L_{fl}$ [m]</th>
<th>$W_{fl}$ [m]</th>
</tr>
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<td>0.65</td>
<td>0.0892</td>
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<td>0.65</td>
<td>0.0928</td>
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</table>
Figure 4.7: Effect of average velocity on flame structure: $P_c = 0.1$ MPa, $T_{in} = 473$ K, $u = (a) 20$, (b) 22.5, (c) 25, (d) 27.5, and (e) 30 m/s, $\phi = 0.65$. + symbol indicates center of heat release.

4.3 Effect of equivalence ratio on flame transfer function

The effect of equivalence ratio on the flame transfer function is shown in Figure 4.8. In this figure the gain and phase are shown for fixed combustor pressure, inlet temperature, average velocity, and equivalence ratio varied from 0.60 to 0.75 in increments of 0.05. Equivalence ratio does not affect the gain of the flame transfer function in the low frequency regime, i.e. frequencies less than 175 Hz, see Figure 4.8b. The magnitude of the first gain maximum is affected by equivalence ratio. However, the trend with increasing equivalence ratio is not consistent, i.e. the magnitude of the first gain maximum generally increases with decreasing
equivalence ratio but not always. Equivalence ratio also changes the frequency at which the second gain minimum occurs, but again not in a systematic manner. The phase decreases with increasing equivalence ratio except at frequencies greater than 300 Hz, see Figure 4.8b. From Figure 4.9 and Table 4.3 it is evident that flame length decreases with increasing equivalence ratio meaning that the decrease in phase is linked with a shortening of the flame.

Figure 4.8: Effect of equivalence ratio on flame transfer function (a) gain and (b) phase: $P_c=0.1$ MPa, $T_{in}=373$ K, $\bar{u}=25$ m/s, $\phi=0.60, 0.65, 0.70$, and 0.75.
Figure 4.9: Effect of equivalence ratio on flame structure: $P_c = 0.1 \text{ MPa}$, $T_{in} = 373 \text{ K}$, $\bar{u} = 25 \text{ m/s}$, $\phi =$ (a) 0.60, (b) 0.65, (c) 0.70, and (d) 0.75. + symbol indicates center of heat release.

Table 4.3: Effect of equivalence ratio on flame metrics.

<table>
<thead>
<tr>
<th>$P_c \text{ [MPa]}$</th>
<th>$T_{in} \text{ [K]}$</th>
<th>$\bar{u} \text{ [m/s]}$</th>
<th>$\phi$</th>
<th>$L_{fl} \text{ [m]}$</th>
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### 4.4 Effect of inlet temperature on flame transfer function

Flame transfer function gain and phase for fixed combustor pressure, mean velocity, and equivalence ratio and varied inlet temperature are shown in Figure 4.10. Figure 4.10a shows that all three flame transfer function gain extrema frequencies are affected by inlet temperature in a consistent manner. When the inlet temperature increases the extrema frequency increases. While the increase in gain minima frequency is approximately 40 Hz over the range of inlet temperatures tested, the increase in gain maximum frequency is approximately double that. The magnitude
of the flame transfer function gain decreases with increasing inlet temperature. From Figure 4.11 and Table 4.4 it is clear that flame length decreases with increasing inlet temperature. Therefore, it can be concluded that longer flames have higher flame transfer function gain maxima than shorter flames. Flame transfer function phase also decreases with increasing inlet temperature, i.e. decreasing flame length, as can be seen in Figure 4.10b. Since the average velocity is fixed in this set of data, the decrease in phase is associated with the disturbance traveling a shorter distance before interacting with the part of the flame where the majority of the heat release occurs.

Figure 4.10: Effect of inlet temperature on flame transfer function (a) gain and (b) phase: $P_c = 0.1$ MPa, $T_m = 373$, 423, and 473 K, $\overline{u} = 25$ m/s, $\phi = 0.75$. 

(a) Flame transfer function gain.

(b) Flame transfer function phase.
Figure 4.11: Effect of inlet temperature on flame structure: $P_c = 0.1$ MPa, $T_{in} =$ (a) 373, (b) 423, (c) 473 K, $\bar{u} = 25$ m/s, $\phi = 0.75$. + symbol indicates center of heat release.

<table>
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<th>$P_c$ [MPa]</th>
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<th>$\bar{u}$ [m/s]</th>
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4.5 Effect of combustor pressure on flame transfer function

The gain and phase of the flame transfer function for three combustor pressures with fixed inlet temperature, average velocity, and equivalence ratio are shown in Figure 4.12. The corresponding operating condition at a combustor pressure of 0.4 MPa was unstable based on the criteria established in Section 3.1 so a complete set of transfer functions from 0.1-0.4 MPa could not be presented. Increasing combustor pressure does not affect the first gain minimum frequency. It may appear that the first gain minimum frequency for the $P_c = 0.2$ MPa operating
condition is higher than the other two operating conditions, however this is purely due to the manner in which the data was acquired. The forcing frequency increment for the \( P_c = 0.2 \, MPa \) operating condition is 10 Hz while the forcing frequency increment for the other two operating conditions is 25 Hz. Therefore the gain minimum frequency for these operating conditions can only be know within ±5 Hz and ±12.5 Hz respectively which means they overlap within the resolution of the measurement. Figure 4.13 and Table 4.5 show that increasing combustor pressure leads to decreased flame length. Therefore, based on the trends regarding flame length and flame transfer function phase seen in Sections 4.3 and 4.4, it would be expected that at a given forcing frequency, flame transfer function phase would decrease with increasing combustor pressure. Figure 4.12b shows that this is the case in going from \( P_c = 0.1 \, MPa \) to \( P_c = 0.2 \, MPa \) but not from \( P_c = 0.2 \, MPa \) to \( P_c = 0.3 \, MPa \). One possible explanation for this is that increasing the combustor pressure from 0.2 to 0.3 MPa causes the flame to shorten and no longer impinge on the combustor wall as can be seen by comparing Figure 4.13b and Figure 4.13c. It has been proposed that the flame’s interaction with the combustor wall modifies the mechanisms responsible for flame dynamics [77,90,91].

Table 4.5: Effect of combustor pressure on flame metrics.

<table>
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<tr>
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<th>( T_{in} ) [K]</th>
<th>( u ) [m/s]</th>
<th>( \phi )</th>
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Figure 4.12: Effect of combustor pressure on flame transfer function (a) gain and (b) phase: $P_c = 0.1, 0.2, \text{ and } 0.3 \text{ MPa}$, $T_{in} = 473 \text{ K}$, $\bar{u} = 25 \text{ m/s}$, $\phi = 0.65$. 
Figure 4.13: Effect of combustor pressure on flame structure: $P_c =$ (a) 0.1, (b) 0.2, and (c) 0.3 MPa, $T_{in} = 473$ K, $\bar{u} = 25$ m/s, $\phi = 0.75$. + symbol indicates center of heat release.
Chapter 5

Generalization of flame transfer function measurements

The effect of each operating condition parameter on the flame transfer function gain and phase was shown in Sections 4.2 to 4.5. An attempt to condense all of these effects by identifying the controlling parameters, i.e. non-dimensional numbers, of the flame’s response to velocity fluctuations is pursued in Section 5.1 while the collapse of flame transfer function gain is investigated in Section 5.2.

5.1 Frequency scaling parameters

Before discussing the frequency scaling parameters it is important to establish the baseline case to determine if any of the scaling parameters is an improvement. The baseline case is the unnormalized forcing frequency and the flame transfer function gain and phase of all measurements versus forcing frequency are shown in Figures 5.1a and 5.1b respectively. As is clear from Figure 5.1, the individual characteristics of the flame transfer function gain and phase discussed in Section 4.1 are not evident. Figure 5.1 serves as a reference when the flame transfer function gain and phase of all measurements are shown as a function of the various frequency scaling parameters in Sections 5.1.1 to 5.1.4.
Figure 5.1: Flame transfer function (a) gain and (b) phase versus forcing frequency: All operating conditions.

### 5.1.1 Strouhal number based on nozzle diameter

Strouhal number, $St$, is a dimensionless number describing oscillating flow mechanisms. As with all dimensionless numbers, it represents the ratio of two competing quantities and is defined in Equation (5.1).

$$St_{D_{nozzle}} = \frac{\omega D_{nozzle}}{U}$$  \hspace{1cm} (5.1)

The two competing quantities are the oscillation time, $\frac{1}{\omega}$, and the residence time, $L/U_0$, where $L$ is a characteristic length scale and $U_0$ is a characteristic velocity scale.
Traditionally, in the field of flame transfer function research, the characteristic length scale is chosen as the nozzle diameter, $D_{\text{nozzle}}$, and the characteristic velocity scale is chosen as the average velocity in the nozzle, $\bar{u}$. Strouhal number based on nozzle diameter is equivalent to the parameter $\omega^*$ defined in Equation (1.4) noting that $\bar{u} = S_L \cos(\alpha_0)$. The physical basis for using $St_{DNozle}$ to generalize flame transfer function measurements stems from fluid mechanics research on shear layer stability. Within the combustor there exist two shear layers, one anchored on the edge of the centerbody and the other anchored on the edge of the dump plane. An example flow field from particle image velocimetry measurements showing the two shear layers from a combustor with similar geometry is shown in Figure 5.2 [7].

Dependent on frequency, these shear layers can exhibit hydrodynamic instability in the form of roll-up into large scale coherent structures [50, 51, 93, 94]. In a review on the preferred mode of jets, Gutmark and Ho [95] found that the Strouhal number corresponding to maximum instability ranged from 1.51 to 4.02. Based on the discussion of the flame’s interaction with vortices in Section 1.2.3 it would be expected that when the shear layer response is large the flame response would also be large, i.e. the flame transfer function would exhibit high gain. Therefore, the Strouhal numbers of minimum and maximum flame transfer function gain are given by:

$$St_{DNozle} = \begin{cases} m \times 2.76 & \text{for } m = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots & \text{Gain minima} \\ m \times 2.76 & \text{for } m = 1, 2, 3, \ldots & \text{Gain maxima} \end{cases} \quad (5.2)$$

Since the Strouhal number corresponding to maximum instability was found to
vary in the literature, an average value over the range reported was chosen. Figure 5.3a shows the flame transfer function gain for all operating conditions versus $St_{D_{\text{nozzle}}}$. Highest gain is expected to occur at $St_{D_{\text{nozzle}}} \approx 2.76$ but gain increases for $St_{D_{\text{nozzle}}} < 2.76$. Moreover, a second gain maximum would be expected to occur at $St_{D_{\text{nozzle}}} \approx 5.52$, the first harmonic of the Strouhal number of maximum shear layer response. Figure 5.3a shows that this is not the case with the majority of gain maxima occurring in the range $3.25 \leq St_{D_{\text{nozzle}}} \leq 3.75$. Although numerous researchers have used the Strouhal number based on nozzle diameter to generalize their flame transfer function measurements [6, 37, 62, 77] or analytical models [29, 31–33] with varying degrees of success, none of them have quantified the degree of collapse with this parameter. The degree of collapse of flame transfer gain and phase with Strouhal number based on nozzle diameter is qualitatively shown in Figures 5.3a and 5.3b respectively while a quantitative discussion regarding the degree of collapse is reserved until the other scaling parameters are discussed.

### 5.1.2 Strouhal number based on flame length

Strouhal number can be alternatively defined using the flame length, $L_{fl}$, as the characteristic length scale as shown in Equation (5.3) and has also been used by several researchers to generalize their flame transfer function results [5, 8, 40, 81, 96, 97]. This definition comes from the physical basis of a single convective velocity fluctuation mechanism perturbing the flame.

$$St_{L_{fl}} = \frac{L_{fl}}{\lambda_{\text{conv}}} = \frac{L_{fl}}{\pi/\omega} \tag{5.3}$$

The Strouhal number defined thusly can be interpreted as the ratio of the flame length to a convective wavelength, $\lambda_{\text{conv}}$. If the Strouhal number is an integer there are an integer number of wavelengths of the velocity perturbation within the flame at a given time. This means that portions of the flame are experiencing a high velocity fluctuation while other portions are experiencing a low velocity fluctuation. In this situation, cancellation effects can occur between different portions of the flame. If the Strouhal number is an odd integer multiple of a half some cancellation may occur but there is always an additional half wavelength of the velocity
fluctuation left to perturb the flame. This discussion is summarized in Equation (5.4) which shows the expected values of \( St_{L_{fl}} \) corresponding to gain minima and maxima. In the limiting case of \( St_{L_{fl}} \leq \pi \) the flame is convectively compact, meaning that the entire flame experiences a high or low velocity fluctuation at a given time [98].

\[
St_{L_{fl}} = \begin{cases} 
  m \times 2\pi & \text{for } m = 1, 2, 3, \ldots \quad \text{Gain minima} \\
  m \times \pi & \text{for } m = 1, 3, 5, \ldots \quad \text{Gain maxima}
\end{cases}
\]  

(5.4)

The flame transfer function gain and phase for all of the operating conditions
listed in Table 4.1 are plotted against Strouhal number based on flame length in Figure 5.4. Contrary to the extrema limits shown by Equation (5.4), Figure 5.4a shows that the first and second gain minima occur near $St_{Lfl} = \pi$ and $St_{Lfl} = 2\pi$ while the gain maxima occur in the range $\pi \leq St_{Lfl} \leq \frac{3\pi}{2}$. Failure of the Strouhal number based on flame length generalization can be explained by the fact that in the definition of Strouhal number based on flame length, Equation (5.3), only a single convective flame perturbation mechanism is taken into account. The fact that the data do not collapse when plotted against Strouhal number based on flame length indicates that more than one mechanism plays a role in the flame response.

Figure 5.4: Flame transfer function (a) gain and (b) phase versus $St_{Lfl}$: All operating conditions.
Since all of the flames exhibit a V structure the failure of the Strouhal number based on flame length generalization cannot be attributed to differences in flame structure as suggested by Kim et al. [5]. Another possible source of discrepancy is a difference between the time-averaged flame length of an unforced flame and the time-averaged flame length of a forced flame at the same operating condition. Figure 5.5 shows the time-averaged flame lengths of forced flames normalized by their corresponding time-averaged unforced flame length for all 47 operating conditions listed in Table 4.1. This figure shows that the maximum difference in time-averaged flame lengths is 5% which is not large enough to account for the discrepancy between measured and expected values of $St_{L_{fl}}$ at gain extrema.

![Figure 5.5: Time-averaged forced flame length normalized by time-averaged unforced flame length versus forcing frequency: All operating conditions.](image)

5.1.3 Phase between azimuthal and axial velocity fluctuations at the flame anchoring location

The velocity fluctuation generated by the siren is axial. In complex exponential notation, its waveform can be written as:

$$\frac{u'}{\bar{u}} = a \times \exp \left( \frac{i \omega x}{c} - i \omega t \right)$$

where $a$ is the amplitude of the axial velocity fluctuation and $c$ is the speed of sound and the wave propagation speed of the axial velocity fluctuation. Although the
velocity fluctuation generated by the siren is axial, upon interaction with the swirler
an azimuthal velocity fluctuation wave is generated [99–101]. The swirler acts as
a mode conversion unit, transforming the incident acoustically propagating axial
velocity fluctuation into a convectively propagating azimuthal velocity fluctuation
whose waveform is given by Equation (5.6). Therefore, on the downstream side of
the swirler, both an axial and an azimuthal velocity fluctuation exist.

\[ \frac{v'}{u} = a \times \exp \left( \frac{i \omega x}{u} - i \omega t \right) \] (5.6)

An important distinction between these two perturbations is that while the
axial velocity fluctuation propagates at the speed of sound, the azimuthal velocity
fluctuation propagates much slower at the local average axial velocity. The differ-
ence in propagation speeds means that the interference pattern of these two waves
has spatial dependence. Since Equations (5.5) and (5.6) are only valid within the
annulus of the burner tube, the most relevant location at which to examine the
interference of these two perturbations is the flame anchoring location, i.e. the
end of the centerbody. Equations (5.5) and (5.6) are not valid in the combustor
because the speed of sound is no longer constant due to the temperature rise as-
associated with the flame and the average velocity also varies spatially due to the
area change and density gradient associated with the flame. The phase difference
between the axial and azimuthal velocity fluctuations at the flame base is given
by:

\[ \theta_{v-u} = \left( 1 - \frac{1}{c} \right) \omega L_{SW-CB} \] (5.7)

where \( L_{SW-CB} \) is the distance from the downstream edge of the swirler to the end
of the centerbody. Equation (5.7) shows that there is no initial phase difference
between the two velocity fluctuations at the exit of the swirler. This has been
verified computationally [65,100] and experimentally [100,101].

This frequency scaling parameter has been previously applied to self-excited
instability data [102] but Figure 5.6 represents the first time that it has been
applied to flame transfer function measurements. Based on the understanding
that gain extrema are caused by the interference of these two velocity fluctuations
it would be expected that gain minima and maxima occur at the following values
of $\theta_{v-u}$:

$$\theta_{v-u} = \begin{cases} 
  m \times \pi & \text{for } m = 1, 3, 5 \ldots \text{ Gain minima} \\
  m \times \pi & \text{for } m = 2, 4, 6 \ldots \text{ Gain maxima}
\end{cases} \quad (5.8)$$

Figure 5.6a shows that while the first gain minima are clustered near $\theta_{v-u} = \pi$ the gain maxima do not occur near $\theta_{v-u} = 2\pi$. Additionally, the second gain minima are not near their expected value of $\theta_{v-u} = 3\pi$.

Figure 5.6: Flame transfer function (a) gain and (b) phase versus $\theta_{v-u}$: All operating conditions.
5.1.4 Phase between swirl number and axial velocity fluctuations at the flame anchoring location

Palies et al. have suggested that the flame response is controlled by the interaction of swirl number and axial velocity fluctuations, specifically their phasing [6,65,103]. Equation (5.6) was previously given for the axial velocity fluctuation wave and Equation (2.1) was previously given for the swirl number at a given location. Equation (2.1) can be used to calculate instantaneous values of the swirl number or a linearized version of this equation given by:

\[
\frac{S'}{S} = \frac{v'}{v} - \frac{u'}{u} = a \times \left[ \exp \left( \frac{i\omega x}{u} - i\omega t \right) - \exp \left( \frac{i\omega x}{c} - i\omega t \right) \right]
\]  

(5.9)

can be used [6]. Although Equation (5.9) is approximate it has been argued that this form of the swirl number equation is more physical and easier to use [65].

Using Equations (5.6) and (5.9) the phase difference between swirl number and axial velocity fluctuations can be derived and is given by:

\[
\theta_{S-u} = \arctan \left( \frac{\sin \left( \frac{\omega L_{SW-CB}}{u} \right) - \sin \left( \frac{\omega L_{SW-CB}}{c} \right)}{\cos \left( \frac{\omega L_{SW-CB}}{u} \right) - \cos \left( \frac{\omega L_{SW-CB}}{c} \right)} \right) - \frac{\omega L_{SW-CB}}{c}
\]

(5.10)

Again, the axial location at which to examine this phase difference is chosen to be the flame anchoring location at the end of the centerbody. In Equation (5.10) the first term, the phase of the swirl number fluctuation at the end of the centerbody, is itself the combination of two waves since the swirl number fluctuation given by Equation (5.9) is the combination of the axial and azimuthal velocity fluctuation waves. Since this parameter is derived based on the interference of two mechanisms its expected values at gain extrema are:

\[
\theta_{S-u} = \begin{cases} 
    m \times \pi & \text{for } m = 1, 3, 5 \ldots \text{ Gain minima} \\
    m \times \pi & \text{for } m = 2, 4, 6 \ldots \text{ Gain maxima}
\end{cases}
\]

(5.11)

Figure 5.7 shows the flame transfer function gain and phase of all operating
conditions versus $\theta_{S-u}$. While the first flame transfer function gain minimum does occur near $\pi$, the second gain minimum does not occur near $3\pi$. Furthermore, the gain maximum does not occur near $2\pi$ as would be expected.

![Flame transfer function gain and phase](image)

(a) Flame transfer function gain.

(b) Flame transfer function phase.

Figure 5.7: Flame transfer function (a) gain and (b) phase versus $\theta_{S-u}$: All operating conditions.

### 5.1.5 Comparison of frequency scaling parameters

Each of the frequency scaling parameters is derived from a different physical basis as to the velocity fluctuation mechanisms perturbing the flame. The degree of collapse of the flame transfer function when plotted against these frequency scaling parameters gives insight into which velocity fluctuation mechanisms are most
important in this combustor. Qualitatively, this collapse was shown in Figures 5.3, 5.4, 5.6 and, 5.7. In order to quantify the degree of collapse, the coefficient of variation (CV) of each of the frequency scaling parameters at flame transfer function gain extrema frequencies was calculated. The coefficient of variation is defined as the standard deviation divided by the mean of each frequency scaling parameter at extrema frequencies as shown in Equation (5.12). Due to the fact that flame transfer function measurements were taken with different forcing frequency increments, each flame transfer function was interpolated in 5 Hz increments using a cubic spline. Then the gain extrema frequencies were determined from these interpolated flame transfer functions before the frequency scaling parameters were calculated and the coefficients of variation determined.

\[
CV = \frac{\sigma}{\mu} \tag{5.12}
\]

The degree of collapse of the frequency scaling parameters discussed in Sections 5.1.1 to 5.1.4 at flame transfer function gain extrema frequencies is shown in Figure 5.8. Also shown in Figure 5.8 is the coefficient of variation for frequency for comparison purposes in order to determine if any of the frequency scaling parameters is an improvement. \(\theta_{S-u}\) can immediately be discounted as it does not provide an improvement over the coefficient of variation for frequency at two of the extrema frequencies. Since the coefficient of variation for \(St_{Lfl}\) at the second gain minimum frequency is larger than that for forcing frequency it must also be discounted. Amongst the remaining frequency scaling parameters, \(St_{D\text{nozzle}}\) and \(\theta_{v-u}\), the maximum difference in the coefficients of variation is 1%. Therefore, using the coefficient of variation alone it is not possible to determine the dominant velocity fluctuation mechanisms.

Also of importance is how close the values of these frequency scaling parameters fall to their expected values at gain extrema frequencies. In Sections 5.1.1 to 5.1.4 the expected numerical values of each of the frequency scaling parameters at flame transfer function gain extrema frequencies were established as well as the physical basis for each. Figure 5.9 shows the percent difference between the average of the measured and expected values of the frequency scaling parameters at gain extrema frequencies. The large percent difference between the measured and expected
values of $\theta_{S-u}$ further supports the rejection of $\theta_{S-u}$ as the proper frequency scaling parameter. Comparing the percent difference for $St_{D_{\text{nuzz}}}$ and $\theta_{v-u}$ it can be seen that $\theta_{v-u}$ has the smaller percent difference at two of the three extrema frequencies. At the second gain minimum frequency, the percent difference for these two scaling parameters differs by only 3%. The degree of collapse with $\theta_{v-u}$ and the fact that the values of $\theta_{v-u}$ at gain extrema frequencies agree with their expected values from a physical perspective indicates that the dominant velocity fluctuation mechanisms in this combustor are the axial and azimuthal velocity fluctuations.

Figure 5.8: Comparison of coefficient of variation of frequency scaling parameters at flame transfer function gain extrema frequencies: All operating conditions.

Figure 5.9: Percent difference between measured and expected values of frequency scaling parameters at flame transfer function gain extrema frequencies: All operating conditions.
While the previous results indicate that the dominant velocity fluctuation mechanisms in this combustor are the axial and azimuthal velocity fluctuations, recent research has shown that the net effect of an azimuthal disturbance on an axisymmetric flame is zero \cite{104}. This fact then precludes the azimuthal velocity fluctuation as a dominant velocity fluctuation mechanism. Although this apparently contradicts the use of $\theta_{v-u}$ as the correct frequency scaling parameter it can be shown that $\theta_{u-v}$ also relates directly to another velocity fluctuation mechanism, the swirl number fluctuation. Equation (5.9) shows that the swirl number fluctuation wave is the difference of the axial and azimuthal velocity fluctuation waves. Therefore, the magnitude of the swirl number fluctuation is directly related to the phase between the axial and azimuthal velocity fluctuations, $\theta_{v-u}$. Because the swirl number fluctuation is the difference of the two waves, $\theta_{v-u} = \pi$ corresponds to a maximum swirl number fluctuation and $\theta_{v-u} = 2\pi$ corresponds to a minimum swirl number fluctuation. Consequently, the values of $\theta_{v-u}$ previously established at flame transfer function gain extrema frequencies can be amended to include the variation of swirl fluctuation magnitude as:

$$\theta_{v-u} = \begin{cases} 
 m \times \pi & \text{for } m = 1, 3, 5 \ldots \text{ Gain minima, and } S' \text{ maxima} \\
 m \times \pi & \text{for } m = 2, 4, 6 \ldots \text{ Gain maxima, and } S' \text{ minima}
\end{cases}$$

(5.13)

The relationship between flame transfer function gain minima/maxima and swirl number fluctuation maxima/minima will be discussed further when the effect of each velocity fluctuation mechanism on the flame is examined through analysis of phase-synchronized flame images in Chapter 7.

### 5.2 Flame transfer function gain scaling

In Section 5.1.5 it was determined that $\theta_{v-u}$ is the correct parameter to scale the frequency axis. However, there still exists significant variation in the flame transfer function gain at a given value of $\theta_{v-u}$. Figure 5.6a shows that the variation of flame transfer function gain is a function of $\theta_{v-u}$. In particular the vertical spread in flame transfer function gain is smallest before the first gain minimum ($\theta_{v-u} < \pi$) and largest after the first gain minimum ($\theta_{v-u} > \pi$). Analytical \cite{18, 63} and
experimental [5] research has shown that the maximum flame transfer function gain has a dependence on flame length. Kim et al. [5] showed that the relationship between maximum gain and flame length is linear as shown by Figure 5.10 for fixed combustor pressure and inlet temperature and varied average velocity, equivalence ratio, and hydrogen mole fraction.

![Figure 5.10: Dependence of maximum flame transfer function gain ($G_{max}$) upon the flame length ($L_{CH^*_{max}}$). Operating conditions: $T_{in} = 200 °C$; $V_{mean} = 60, 70,$ and $80 \text{ m/s}$; $\phi = 0.55, 0.60, 0.65, 0.70,$ and premixed; and $X_{H2} = 0.00, 0.15,$ and $0.30$ [5].]

Within the data set of flame transfer function measurements shown in Table 4.1 there are four groupings of measurements that have fixed combustor pressure and inlet temperature and varied average velocity and equivalence ratio. The maximum flame transfer function gain for these four groupings is shown in Figure 5.11 versus flame length. Figures 5.11b to 5.11d show that maximum flame transfer function gain does increase with increasing flame length. However, Figure 5.11a also shows that maximum gain appears to decrease for the longest flame. The decrease of maximum flame transfer function gain for long flames could not be confirmed.
because flames longer than $L_f = 0.116 \text{ m}$ could not be stabilized in this combustor.

![Graphs](image)

(a) Group 1.  
(b) Group 2.  
(c) Group 3.  
(d) Group 4.

Figure 5.11: Maximum flame transfer function gain versus flame length for four groupings of measurements with fixed combustor pressure and inlet temperature and varied average velocity and equivalence ratio.

Based on the results shown in Figure 5.11 it would be expected that the maximum flame transfer function gain of all operating conditions would scale linearly with flame length. However, Figure 5.12 shows that this is not the case. In Figure 5.12 all four operating condition parameters, combustor pressure, inlet temperature, average velocity, and equivalence ratio, were varied simultaneously while in Figure 5.11 only average velocity and equivalence ratio were varied. The behavior seen in Figure 5.12 suggests a more complex relationship between flame transfer function gain maxima and flame length. One factor that is closely related to flame length is flame speed as discussed in Section 3.2. Combustor pressure, inlet temperature, and equivalence ratio each individually affect flame speed in a well described manner but when these parameters change simultaneously the effect
on flame speed is a combination of the relative change in each parameter as well as the sensitivity of flame speed to each parameter.

Several correlations have been developed to allow calculation of the laminar flame speed based on inlet conditions. One such correlation is that of Boyde et al. [105]. Their correlation is based on both experimental and computational data over the following inlet parameter ranges: $P_c = 0.05 - 0.6 \, MPa$, $T_{in} = 293.15 - 593.15 \, K$, and $\phi = 0.6 - 1.5$. The inlet parameter range of the correlation overlaps with the operating condition parameter range of the experiments in this study. Using the correlation, the laminar flame speed for each operating condition was calculated. The maximum flame transfer function gain is shown versus laminar flame speed normalized by average velocity in Figure 5.13. Comparing Figures 5.12 and 5.13 it can be seen that the maximum flame transfer function gain is related to a change in flame speed rather than a change in flame length. Birbaud et al. [77] identified $S_L/\pi$ as an important dimensionless group in regards to laminar flame response. Figure 5.13 shows that $S_L/\pi$ is equally an important dimensionless group in regards to turbulent flame response.
Figure 5.13: Flame transfer function gain maximum versus laminar flame speed normalized by average velocity ($S_L/u$): All operating conditions.
Chapter 6

Flame transfer function measurements in modified nozzle geometry

Based on the collapse of flame transfer function measurements with the parameter $\theta_{v-u}$ it is evident that swirl number fluctuations have a large effect on flame dynamics. Therefore a modification was made to the nozzle geometry in order to change the convective time delay associated with the swirl number fluctuation to verify its effect on flame dynamics.

6.1 Modification of experimental setup to investigate importance of swirl number fluctuations

Figure 6.1 compares the original and modified nozzle geometries. Modifications to the original nozzle geometry are indicated in green in Figure 6.1b. The modifications consist of a 0.019 m extension to the centerbody, dump plane, and burner tube. Both the original and extended centerbodies have the same face profile and cooling air holes discussed in Section 2.1.3. The distance from the most downstream edge of the swirler to the flame anchoring location, the end of the center-
body, was increased from 0.077 m to 0.096 m, or an increase of 25%, while the original centerbody recess is preserved.

![Original and Modified Nozzle Geometries](image)

Figure 6.1: Comparison of (a) original and (b) modified nozzle geometries. Modifications are indicated in green.

## 6.2 Comparison of time-averaged flame structure in original and modified nozzle geometries

Before comparing the flame transfer function measurements in the two nozzle geometries, a comparison between the time-averaged flame structure in the original and modified nozzle geometries at the same operating condition was made. Table 6.1 shows the operating condition at which a time-averaged flame image was acquired in the modified nozzle geometry. Figure 6.2 compares the time-averaged flame images. Figure 6.2a is from the original nozzle geometry while Figure 6.2b is from the modified nozzle geometry. Qualitatively, the flames appear to be very similar in terms of their center-of-heat release location and impingement on the combustor wall. This observation isquantitatively substantiated by Table 6.2 which compares the axial ($x_{CoHR}$) and radial ($r_{CoHR}$) locations of the center of heat release and the flame widths in the original and modified nozzle geometries. The axial and radial locations of the center-of-heat release differ by only a mil-
limeter. Other researchers have also shown that flame structure is unaffected by the axial location of the swirler [62,102].

Table 6.1: Operating condition of time-averaged flame image and flame transfer function measurement in modified nozzle geometry.

<table>
<thead>
<tr>
<th>$P_c$ [MPa]</th>
<th>$T_{in}$ [K]</th>
<th>$u$ [m/s]</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>473</td>
<td>30</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Figure 6.2: Comparison of time-averaged images in (a) original and (b) modified nozzle geometries: $P_c = 0.2$ MPa, $T_{in} = 473$ K, $u = 30$ m/s, $\phi = 0.60$. + symbol indicates the center of heat release.
Table 6.2: Comparison of flame metrics in original and modified nozzle geometries.

<table>
<thead>
<tr>
<th>Flame metric</th>
<th>Original nozzle geometry</th>
<th>Modified nozzle geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{CoHR}$ [m]</td>
<td>0.082</td>
<td>0.083</td>
</tr>
<tr>
<td>$r_{CoHR}$ [m]</td>
<td>0.050</td>
<td>0.049</td>
</tr>
<tr>
<td>$W_{fl}$ [m]</td>
<td>0.038</td>
<td>0.038</td>
</tr>
</tbody>
</table>

6.3 Flame transfer function in modified nozzle geometry

Figure 6.3a compares flame transfer function gain in the original and modified nozzle geometries. The values of minimum and maximum gain are unaffected by the change in nozzle geometry. All flame transfer function gain extrema occur at lower frequencies in the modified nozzle geometry. The decrease of flame transfer function gain extrema frequencies is consistent with the physical basis established in Section 5.1.3 for the occurrence of extrema. Flame transfer function gain minima should occur at $\theta_{v-u} = \pi, 3\pi, 5\pi, \ldots$ and gain maxima should occur at $\theta_{v-u} = 2\pi, 4\pi, 6\pi, \ldots$ and in Section 5.1.5 this was indeed shown to be true. Therefore, for the flame transfer function gain extrema to occur at the same value of $\theta_{v-u}$ in the modified nozzle geometry where $L_{SW-CB}$ was increased the gain extrema frequency would have to decrease accordingly because of the relationship between $L_{SW-CB}$ and $\omega$ in the definition of $\theta_{v-u}$, Equation (5.7). Given the change in $L_{SW-CB}$ introduced by the change in nozzle geometry the expected change in the frequency of the first gain minimum can be calculated using Equation (5.7). For an increase in $L_{SW-CB}$ from 0.077 m to 0.096 m it would be expected that the first gain minimum frequency would decrease by 40 Hz. Figure 6.3a shows that the first gain minimum frequency decreases by 50 Hz when $L_{SW-CB}$ is increased. Since the forcing frequency resolution of the measurement is 25 Hz, the first gain minimum frequency can only be measured within ±12.5 Hz. Therefore the expected and measured decreases in the first gain minimum frequency coincide. Komarek and Polifke [62] also observed that the first gain minimum frequency decreasing with increasing distance from the swirler to the centerbody end.

Figure 6.3b compares the flame transfer function phase in the original and
modified nozzle geometries. Phase values for the two nozzle geometries do not overlap. Differences in phase are largest near the frequencies corresponding to flame transfer function gain minima. In general the phase difference increases with increasing forcing frequency. The flame transfer function phase of the modified geometry is always larger than that of the original geometry. Both these facts are consistent with the additional time delay introduced by increasing the distance from the swirler to the centerbody. These results confirm the importance of swirl number fluctuations as a dominant velocity fluctuation mechanism.

Figure 6.3: Comparison of flame transfer function (a) gain and (b) phase in original and modified nozzle geometries: $P_c = 0.2$ MPa, $T_{in} = 473$ K, $u = 30$ m/s, $\phi = 0.60$. 
Chapter 7

Phase-synchronized flame image analysis

7.1 Introduction

Flame response was investigated through measurement of the flame transfer function in Chapter 4. In Chapter 5, the velocity fluctuation mechanisms responsible for the observed flame response were investigated on a global scale. In this chapter, velocity fluctuation mechanisms will be investigated on a local scale through analysis of phase-synchronized flame images. The operating conditions at which phase-synchronized flame images were acquired are shown in Table 7.1. For these 11 operating conditions, phase-synchronized flame images were acquired at all forcing frequencies.

Figure 7.1 shows the axial and radial locations of the center of heat release of the time-averaged flame images at the operating conditions shown in Table 7.1. Comparing Figures 3.2 and 7.1 shows that phase-synchronized flame images were acquired at operating conditions which span the range of possible flame lengths in this combustor. The range of flame lengths over which phase-synchronized flame images were acquired will allow it to be determined if the degree to which the flame interacts with the wall has an effect on the velocity fluctuation mechanisms.
Table 7.1: Operating conditions of phase-synchronized images.

<table>
<thead>
<tr>
<th>Number</th>
<th>Case</th>
<th>$P_c$ [MPa]</th>
<th>$T_{in}$ [K]</th>
<th>$\bar{u}$ [m/s]</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45</td>
<td>0.1</td>
<td>373</td>
<td>25</td>
<td>0.75</td>
</tr>
<tr>
<td>2</td>
<td>46</td>
<td>0.1</td>
<td>373</td>
<td>20</td>
<td>0.75</td>
</tr>
<tr>
<td>3</td>
<td>47</td>
<td>0.1</td>
<td>373</td>
<td>30</td>
<td>0.75</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>0.1</td>
<td>373</td>
<td>30</td>
<td>0.60</td>
</tr>
<tr>
<td>5</td>
<td>49</td>
<td>0.1</td>
<td>373</td>
<td>25</td>
<td>0.60</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>0.1</td>
<td>373</td>
<td>25</td>
<td>0.65</td>
</tr>
<tr>
<td>7</td>
<td>51</td>
<td>0.1</td>
<td>373</td>
<td>25</td>
<td>0.70</td>
</tr>
<tr>
<td>8</td>
<td>52</td>
<td>0.1</td>
<td>473</td>
<td>25</td>
<td>0.75</td>
</tr>
<tr>
<td>9</td>
<td>53</td>
<td>0.1</td>
<td>423</td>
<td>25</td>
<td>0.75</td>
</tr>
<tr>
<td>10</td>
<td>54</td>
<td>0.1</td>
<td>423</td>
<td>20</td>
<td>0.75</td>
</tr>
<tr>
<td>11</td>
<td>55</td>
<td>0.1</td>
<td>423</td>
<td>30</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Figure 7.1: Center of heat release location for operating conditions at which phase-synchronized images were acquired. Numbers in legend are case numbers shown in Table 7.1.

7.2 Phase-synchronized image analysis techniques

Several analysis techniques will be applied to the phase-synchronized flame images acquired in order to determine the important velocity fluctuations in this combustor. Rather than presenting these analysis techniques applied to flame images at all forcing frequencies for the operating conditions shown in Table 7.1, 198 in total, analysis of a representative long, medium, and short flame will be shown. Long, medium, and short refer to the flame length as shown by the center of heat release
locations in Figure 7.1. The operating conditions of the long, medium, and short flames are shown in Table 7.2.

**Table 7.2: Long, medium, and short flame operating conditions.**

<table>
<thead>
<tr>
<th>Designation</th>
<th>Case</th>
<th>$P_c$ [MPa]</th>
<th>$T_{in}$ [$K$]</th>
<th>$\bar{u}$ [m/s]</th>
<th>$\phi_{L_{fl}}$</th>
<th>$L_{fl}$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (Long)</td>
<td>49</td>
<td>0.1</td>
<td>373</td>
<td>25</td>
<td>0.60</td>
<td>0.1163</td>
</tr>
<tr>
<td>B (Medium)</td>
<td>45</td>
<td>0.1</td>
<td>373</td>
<td>25</td>
<td>0.75</td>
<td>0.0994</td>
</tr>
<tr>
<td>C (Short)</td>
<td>52</td>
<td>0.1</td>
<td>473</td>
<td>25</td>
<td>0.75</td>
<td>0.0807</td>
</tr>
</tbody>
</table>

### 7.2.1 Root mean square fluctuation and phase flame images

The methodologies of creating the root mean square fluctuation and phase flame images and determining the mean flame position were discussed in Section 2.5. Figure 7.2 shows the root mean square fluctuation and phase flame images at all forcing frequencies for the medium length flame operating condition. Each root mean square fluctuation image is scaled individually, meaning that quantitative intensity comparisons cannot be made between images at different forcing frequencies. Also shown in Figure 7.2 are the center of heat release and mean flame position of the time-averaged flame image denoted by a + symbol and a dashed line respectively. There are two important points to be made in regards to the root mean square fluctuation images. The first point is that the center of heat release does not correspond to the location of maximum heat release fluctuation, i.e. the + symbol does not fall within the white region of the root mean square fluctuations in Figure 7.2a. This indicates that analyses based on single point metrics of the flame, such as $St_{fl}$, are incorrect because while the center of heat release captures the location of maximum heat release, it does not coincide with the location of maximum heat release fluctuation which is more important to the overall flame dynamics. The second point is that there is a well defined region of zero heat release fluctuation which runs down the centerline of the flame. Comparing the regions of zero heat release and the dashed lines in Figure 7.2a near the flame base shows that these two coincide. This validates the methodology used to
determine the mean flame position because by definition the mean location of the
flame should correspond to the location of zero heat release fluctuation.

As with the root mean square fluctuation images there are two points to be
made regarding the phase flame images shown in Figure 7.2b. First, based on the
variation of phase across the flame, it is evident that the heat release perturbation
is of a convective, rather than an acoustic nature. If the heat release perturbation
were of an acoustic nature the phase of the heat release fluctuation across the flame
would change by a few degrees at most because the acoustic wavelength is on the
order of 1 meter even at the highest forcing frequency as shown in Figure 2.7.
Since Figure 7.2b shows that this is not the case, this means that the propagation
speed of the heat release disturbance is convective and on the order of the average
velocity. The second point is that at frequencies near the gain minima, there is a
well defined line which runs down the centerline of the flame. This well defined
line also coincides with the mean flame position. On the two sides of this line the
phase of the heat release fluctuations differs by 180 degrees. Therefore the flame
oscillates about its mean position when the global flame response is low. This
behavior is less evident when the global flame response is high.

Several important characteristics regarding the root mean square fluctuation
and phase images for a medium length flame were discussed in the preceding para-
graphs. Now, root mean square fluctuation and phase images of short and long
flames at gain extrema frequencies will also be examined to determine if the be-
havior previously observed is modified when the flame interacts more or less with
the wall. Figures 7.3a, 7.3c, and 7.3e show that as the flame lengthens, the high-
est heat release fluctuation moves from the downstream side of the mean flame
position to the upstream side. However, the heat release fluctuation is still dis-
tributed on either side of the mean flame position. All of the phase images at first
gain minima frequencies, Figures 7.3b, 7.3d, and 7.3f, show the well defined line
down the centerline of the flame indicating movement of the flame about its mean
position at gain minima.

Figures 7.4a, 7.4c, and 7.4e show the root mean square fluctuation images at
gain maxima frequencies. In these images, the heat release fluctuation occurs
predominantly in one region of the flame. This contrasts the behavior observed
at the first gain minima where the heat release fluctuation was distributed about
Figure 7.2: (a) root mean square fluctuation and (b) phase images at all forcing frequencies of medium length flame: $P_c = 0.1$ MPa, $T_{in} = 373$ K, $u = 25$ m/s, $\phi = 0.75$. + symbol denotes center-of-heat release and dashed line indicates mean flame position.

The extent of the line of zero fluctuation along the centerline of the flame increases with decreasing flame length. Correspondingly the demarcation line down the centerline of the flame in the phase images, Figures 7.4b, 7.4d, and 7.4f, becomes longer as the flame shortens. Additionally the variation of the phase of the heat release fluctuation across the flame increases as the flame shortens.

Finally, the root mean square fluctuation and phase images at second gain minima frequencies are shown in Figure 7.5. The root mean square fluctuation images are similar to those at the first gain minimum frequency shown in Figure 7.3, however the heat release fluctuation is not as evenly distributed about the mean flame position. One marked difference is that in the case of the longest flame, Figure 7.5a, the line of zero fluctuation down the centerline of the flame is no longer visible. In regards to the phase images, Figures 7.5d and 7.5f closely resemble Figures 7.3d and 7.3f respectively while Figure 7.5b and Figure 7.3b differ.
Figure 7.3: (a, c, e) root mean square fluctuation and (b, d, f) phase flame images at first gain minimum frequency. + symbol denotes center-of-heat release and dashed line indicates mean flame position.

7.2.2 Windowing analysis

Palies et al. [6] argued that the effect of swirl number and axial velocity fluctuations perturbing the flame acted in different spatial locations. Swirl number fluctuations predominantly affected heat release at the base of the flame while vortex roll-up induced by the axial velocity fluctuation predominantly affected heat release at the tip of the flame. These differences in spatial location of effect allowed them to separate the flame into base and tip regions to investigate the effect of each mechanism separately. Phase-synchronized flame images were divided into a lower
window, where 30% of total heat release occurred, and an upper window, where
the remaining 70% of total heat release occurred. Heat release fluctuations in these
two interrogation windows were summed and normalized by their respective mean
values at flame transfer function gain minima and maxima. This normalization
by mean values which differ by a factor of 2.3 means that quantitative magnitude
comparisons cannot be made. It was found that at flame transfer function gain
minima heat release fluctuations in the two windows were out of phase indicating
destructive interference between the two regions of the flame, see the left hand side

Figure 7.4: (a, c, e) root mean square fluctuation and (b, d, f) phase flame images
at gain maximum frequency. + symbol denotes center-of-heat release and dashed
line indicates mean flame position.
(a) Long flame root mean square fluctuation image.

(b) Long flame phase image.

(c) Medium flame root mean square fluctuation image.

(d) Medium flame phase image.

(e) Short flame root mean square fluctuation image.

(f) Short flame phase image.

Figure 7.5: (a, c, e) root mean square fluctuation and (b, d, f) phase flame images at second gain minimum frequency. + symbol denotes center-of-heat release and dashed line indicates mean flame position.

of Figure 7.6. In contrast, heat release fluctuations in the two windows were in phase at flame transfer function gain maxima indicating constructive interference between the two regions of the flame, see the right hand side of Figure 7.6.

Ranalli and Ferguson [8] also applied this windowing analysis to phase-synchronized images from another combustor however, they chose to define the two windows based on a 50/50 split of total heat release. Heat release fluctuations in the two windows were shown to be out of phase at flame transfer function gain minima, see the top row of Figure 7.7, but heat release fluctuations in the two windows did not
Figure 7.6: Heat release rate signals reconstructed from the light emission detected in the upper and lower windows. (a) $f=60$ Hz, flame A. (b) $f=90$ Hz, flame A. (c) $f=100$ Hz, flame B. (d) $f=150$ Hz, flame B [6]. (a) and (c) correspond to flame transfer function gain minima and (b) and (d) correspond to flame transfer function gain maxima.

show in phase behavior at all flame transfer function gain maxima as seen in the bottom row of Figure 7.7. These differing results with different window divisions calls into question how sensitive the results and interpretation of the mechanisms perturbing the flame are to the choice of window division.

The sensitivity of the windowing analysis to the location of the window division was investigated by calculating the magnitude ratio and phase difference between the heat release fluctuations in both windows for all possible window divisions. Results of this calculation at the gain extrema frequencies of the medium length flame operating condition are shown in Figure 7.8. The magnitude ratio is shown on the left ordinate and the phase difference is shown on the right ordinate. Figure 7.8a shows the magnitude ratio of heat release fluctuations in the lower and upper windows at the first gain minimum frequency as a function of percentage of total heat release in the lower window, i.e. the window division location. As the percent-
Figure 7.7: Amplitude of oscillation of the flame split axially into upstream (US) and downstream (DS) halves. Left: HSI 100 l/min, Phi=0.80, no diluent at frequencies 110 and 160 Hz. Right: LSI 100 l/min, Phi=0.90, no diluent at frequencies 90 and 140 Hz. Frequencies correspond to the first notch and the first rebound peak after the notch for their respective transfer functions [8].

If the percentage of total heat release in the lower window tends toward 0%, the heat release fluctuation magnitude ratio tends toward infinity because all of the heat release occurs in the upper window. In contrast, as the percentage of total heat release in the lower window tends toward 100%, the heat release fluctuation magnitude ratio tends toward zero because all of the heat release occurs in the lower window. Therefore, observations regarding the heat release fluctuation magnitude ratio and phase difference are invalid at the extreme limits of window division location. Figure 7.8a also shows the phase difference between the heat release fluctuations in the upper and lower windows at the first gain minimum, also as a function of window division location. Both the amplitude and phase are necessary to determine the interference between two quantities. Figure 7.8a shows that the heat release fluctuation magnitude ratio is between one and two for the majority of window division locations, 20-85% of total heat release in lower window. Consequently, the heat release fluctuations in the two windows are of comparable magnitude and
destructive interference can occur given the right phase difference between the two windows. The phase between heat release fluctuations in the windows shown in Figure 7.8a is near 180±15 degrees over nearly the same range of window divisions, 25-65% of total heat release in lower window, where the fluctuation magnitudes are comparable. At the first gain minimum, the fact that heat release fluctuations in the two windows are out of phase and of comparable magnitude over a broad range of window divisions indicates that the results of the windowing analysis at the first gain minimum are insensitive to window division location.

The sensitivity of the windowing analysis to window division location at the gain maximum was also investigated. Results of this sensitivity analysis are shown in Figure 7.8b. Unlike the results at the first gain minimum shown in Figure 7.8a, the heat release fluctuation magnitude ratio is highly sensitive to the window division location at the gain maximum, see Figure 7.8b. The phase difference between heat release fluctuations in the two windows also varies greatly with window division location as can be seen in Figure 7.8b. At the gain maximum the heat release fluctuations in the two windows must be in phase for constructive interference to occur. Figure 7.8b shows that this occurs when the percentage of total heat release in the lower window is near 0% and near 100%. However, at 0% virtually all of the heat release is in the upstream window while at 100% nearly all is in the lower window, so there is no constructive interference. These results show that the windowing analysis at the gain maximum is very sensitive to the window division location and also that there is no constructive interference at the gain maximum. This is consistent with the different observations made by Palies et al. and Ranalli and Ferguson when using different window divisions.

Lastly, the sensitivity of the windowing analysis to window division location at the second gain minimum was investigated. Similar to the results at the first gain minimum, Figure 7.8c shows that the magnitude ratio between the heat release fluctuations in the two windows is between one and two for the majority of window division locations, 15-75% of total heat release in the lower window, at the second gain minimum frequency. Contrary to the results at the first gain minimum, Figure 7.8c shows that the phase difference between the heat release fluctuations in the two windows is only out of phase, i.e. 180±15 degrees, over a much smaller range of window divisions, 40-60% of total heat release in the lower
window. Therefore, destructive interference could only be concluded for window divisions of 40-60% of total heat release in the lower window. In summary, the windowing analysis is insensitive to the window division location at the first gain minimum frequency but very sensitive to the window division location at the gain maximum and second gain minimum frequencies.

Results of the window sensitivity analysis at gain extrema frequencies of the long length flame operating condition are shown in Figure 7.9. Criteria for destructive interference expected at gain minima, magnitude ratio between one and two and phase difference of $180\pm15$ degrees, and constructive interference expected at gain maxima, magnitude ratio between one and two and phase difference of $0\pm15$ degrees, were established in the previous paragraphs. Figure 7.9a shows that at the first gain minimum the criteria for destructive interference is only satisfied for window divisions of 40-55% total heat release in the lower window. The same criteria at the second gain minimum are not satisfied for any window division, see Figure 7.9c. At the gain maximum, the criteria for constructive interference are not satisfied for any window division as shown in Figure 7.9b. These results contradict the results of Palies et al. and Ranalli and Fergusson who concluded that gain minima/maxima are caused by destructive/constructive interference between different regions of the flame.

Finally, results of the window sensitivity analysis at gain extrema frequencies of the short length flame operating condition are shown in Figure 7.10. At the first gain minimum, Figure 7.10a, the heat release fluctuation magnitude ratio criterion is met over a broad range of window divisions but the phase criterion is not met. Figure 7.10b is very similar to Figure 7.8b in that the heat release fluctuations are only in phase when the majority of the heat release fluctuation occurs in one of the windows. Consequently, constructive interference between the heat release fluctuations in the two windows cannot be the cause of the flame transfer function gain maximum. At the second gain minimum, shown in Figure 7.10c, the magnitude ratio and phase criterion are not satisfied simultaneously which indicates that destructive interference between two regions of the flame is not the cause of the second gain minimum. Due to the subjectivity involved when using the windowing analysis, a different phase-synchronized image analysis technique was used to investigate the presence/importance of swirl number fluctuations starting
Figure 7.8: Window sensitivity analysis at (a) first gain minimum, (b) gain maximum, and (c) second gain minimum of medium length flame operating condition: \( P_c = 0.1 \) MPa, \( T_{in} = 373 \) K, \( \bar{u} = 25 \text{ m/s}, \phi = 0.75 \).
Figure 7.9: Window sensitivity analysis at (a) first gain minimum, (b) gain maximum, and (c) second gain minimum of long length flame operating condition: $P_c = 0.1 \text{ MPa}$, $T_{in} = 373 \text{ K}$, $\overline{u} = 25 \text{ m/s}$, $\phi = 0.60$. 
from a fundamental understanding of their effect on the flame in Section 7.2.3.

### 7.2.3 Mean flame position analysis

The effect of swirl number fluctuations is to change the dynamics of the vortex breakdown process which occurs in the inner recirculation zone \([9, 61, 106, 107]\). When the swirl number increases the inner recirculation zone moves upstream and vice versa. As the inner recirculation zone moves, so does the flame to adjust to the changing flow field. Therefore, evidence of swirl number fluctuations can be found by examining fluctuations in flame position. The determination of a mean flame position from the turbulent flame brush was discussed in Section 2.5.

The envelope of mean flame positions over one period of forcing at gain extrema frequencies of the medium length flame operating condition listed in Table 7.2 are shown in Figure 7.12. Figures 7.12a and 7.12c show that there is movement in the mean flame position near the flame base, defined as the area within the red box, when the flame transfer function gain is minimum. In contrast, Figure 7.12b shows that when the flame transfer function gain is maximum there is no movement in the mean flame position near the flame base. Since mean flame position movement is indicative of swirl number fluctuations, these results show that swirl number fluctuations exist at flame transfer function gain minima but do not exist at flame transfer function gain maxima.

All forcing frequencies at the medium length flame operating condition are now investigated and the envelope of mean flame positions near the flame base is shown in Figure 7.13. Gain extrema frequencies are highlighted in red. It can be seen that when flame transfer function gain increases, 180-260 Hz, the width of the envelope of mean flame position near the flame base decreases. In contrast, when flame transfer function gain decreases, 260-380 Hz, the width of the envelope of mean flame position near the flame base increases. Therefore there is an inverse trend between flame transfer function gain and the amplitude of flame base movement, i.e. when one increases the other decreases and vice versa. This observation indicates that flame base movement acts to decrease flame transfer function gain. The mechanism through which flame base movement could decrease flame transfer function gain is discussed in Section 7.3. Before this discussion, flame
Figure 7.10: Window sensitivity analysis at (a) first gain minimum, (b) gain maximum, and (c) second gain minimum of short length flame operating condition: $P_c = 0.1$ MPa, $T_{in} = 473$ K, $\bar{u} = 25$ $m/s$, $\phi = 0.75$. 
Figure 7.11: Mean temperature fields and streamline patterns for two different swirl numbers [9].

base movement for the long and short length flames whose operating conditions are given in Table 7.2 will be examined.

Figure 7.14 shows the envelope of mean flame positions near the flame base at all forcing frequencies for the long length flame operating condition. The trends seen in Figure 7.13 are also seen in Figure 7.14. From 200 to 280 Hz where flame transfer function gain increases, the amplitude of flame base movement decreases. Conversely, from 280 to 420 Hz where flame transfer function gain decreases, the amplitude of flame base movement increases. This confirms that flame base movement is also a subtractive effect for flames that impinge heavily on the combustor wall.

Finally, the envelope of mean flame positions near the flame base at all forcing
Figure 7.12: Envelope of mean flame position fluctuation over one period of forcing at (a) first gain minimum, (b) gain maximum, and (c) second gain minimum frequencies for medium length flame: $P_c = 0.1$ MPa, $T_{in} = 373$ K, $\bar{u} = 25$ m/s, $\phi = 0.75$. Flame base is defined as the area within the red box.
Figure 7.13: Envelope of mean flame position fluctuation near flame base at all forcing frequencies for medium length flame: $P_c = 0.1$ MPa, $T_{in} = 373$ K, $\bar{u} = 25$ m/s, $\phi = 0.75$.

Figure 7.14: Envelope of mean flame position fluctuation near flame base at all forcing frequencies for long length flame: $P_c = 0.1$ MPa, $T_{in} = 373$ K, $\bar{u} = 25$ m/s, $\phi = 0.60$.

frequencies for the short length flame operating condition are shown in Figure 7.15.
7.3 Interaction of vortex shedding and flame position fluctuations

It is widely accepted that the flame’s interaction with vortices is a dominant mechanism of fluctuating heat release [13, 14, 20, 35, 54, 55, 108–110]. It is then logical to consider that the mechanism through which flame angle fluctuations decrease flame transfer function gain is a modification of the interaction of the flame with vortices. In the case of combustor geometries similar to that shown in Figure 2.3 the dominant source of vortices is the Kelvin-Helmholtz instability of the mixing layer [9, 111, 112]. Several researchers have investigated the effect of a flame on the Kelvin-Helmholtz instability of a mixing layer [10, 47, 53, 113–120]. In order to understand the effect of a flame on the Kelvin-Helmholtz instability one can examine the vorticity equation written in the form of Equation (7.1) [10]. The vortex stretching term is unaffected by the presence of the flame and therefore the effect of the flame on terms II-IV in Equation (7.1) will be discussed in the subsequent paragraphs.

\[
\frac{D\vec{\omega}}{Dt} = \left( \overline{\vec{\omega}} \cdot \nabla \right) \vec{V} - \overline{\vec{\omega}} \left( \nabla \cdot \vec{V} \right) - \frac{\overleftarrow{\nabla} p \times \overrightarrow{\nabla} \rho}{\rho^2} + \kappa \nabla^2 \vec{V}
\]

\[\text{(7.1)}\]
The flame has a direct effect on vorticity through the gas expansion term due to the temperature rise associated with the flame. Gas expansion increases with increasing temperature because of the inverse relationship between density and temperature in the ideal gas equation of state, Equation (7.2).

\[
\frac{P}{\rho} = RT
\]  

(7.2)

The term \((\nabla \cdot \vec{V})\) is positive in an expanding flow. Therefore, the effect of gas expansion results in a decrease in the magnitude of vorticity [114]. This can be alternatively understood by angular momentum considerations. As a fluid element expands, the magnitude of its local rotation rate, and hence its vorticity, must decrease to conserve angular momentum [121]. Several researchers have cited this as a mechanism of decreasing vorticity in shear flows with heat release [47,115,118,120].

Baroclinic production of vorticity occurs when pressure and density gradients are misaligned, as seen from the baroclinic production term in Equation (7.1) which involves the cross product between these two quantities. The flame introduces a density gradient into the flowfield because of the increase in temperature. If the flame is not aligned with the flow direction, i.e. the pressure gradient, vorticity can be produced [47,114,122]. The vorticity generated by baroclinic production is of opposite sign to that generated in the shear layer [10,47,114,115,121] as can be seen in Figure 7.16. These counter rotating vortices interact with the vortices generated in the shear layer in a destructive manner which decreases the overall vorticity of the flow.

Kinematic viscosity, \(\nu = \frac{\mu}{\rho}\), increases for two reasons with increasing temperature. The first is that density decreases with increasing temperature as discussed previously. The second is that for a gas, viscosity (\(\mu\)) increases with increasing temperature. The increased kinematic viscosity tends to suppress the Kelvin-Helmholtz instability of the mixing layer [10,47,122,123].

Since all of these effects are related to an increase in temperature, the flame position relative to the shear layer is very important [10,118–120]. It is proposed that when the mean flame position fluctuates, its position relative to the shear layer changes. As the flame moves closer to the shear layer the vorticity of the
Figure 7.16: Illustration of vorticity production by the baroclinic mechanism. The figure shows instantaneous vorticity field and flame edge at excitation amplitude $u'/u_{inf} = 0.29$ and phase $= 45^\circ$. The equivalence ratio is $\phi = 0.71$. Directions of pressure gradient, $\nabla p$ (green arrows) and density gradient, $\nabla \rho$ (crimson arrows) are indicated. The curved arrows indicate the direction of circulation of the vortex [10].

Flow is dissipated for the reasons discussed previously and the flame response is low. In contrast, when the mean flame position is stationary, the vorticity of the flow is not dissipated before interacting with the flame which leads to large flame response.

This point is further substantiated by Figure 7.17 which shows the mean normalized fluctuations in flame base, flame downstream, and total heat release at all forcing frequencies. Flame downstream heat release is defined as the heat release which occurs in the area outside of the red box in Figure 7.12 while flame base heat release is defined as the heat release which occurs inside the red box in Figure 7.12. Total heat release is the sum over the entire flame and is a measure of flame transfer function gain since the velocity fluctuation amplitude is the same at all forcing frequencies. Comparing the red and green bars in Figure 7.17, flame downstream and total heat release respectively, relative to the blue bar, flame base heat release, it can be seen that the total flame response is dominated by what occurs in the region defined as the flame downstream. Heat release fluctuations in the flame downstream region are primarily due to the flame’s interaction with
vortices [6,35,77]. Therefore, the changes in flame response with forcing frequency must be due to changes in the strength of the vortices that interact with the flame.
(a) Long length flame: $P_c = 0.1$ MPa, $T_{in} = 373$ K, $\bar{u} = 25$ m/s, $\phi = 0.60$.

(b) Medium length flame: $P_c = 0.1$ MPa, $T_{in} = 373$ K, $\bar{u} = 25$ m/s, $\phi = 0.75$.

(c) Short length flame: $P_c = 0.1$ MPa, $T_{in} = 473$ K, $\bar{u} = 25$ m/s, $\phi = 0.75$.

Figure 7.17: Comparison of flame base, flame downstream, and total normalized heat release fluctuations for (a) long, (b) medium, and (c) short flames at all forcing frequencies.
Summary and recommendations for future work

8.1 Summary

A review of combustion instabilities and the forced response of both laminar and turbulent flames to imposed velocity fluctuations was given in Chapter 1. The flame transfer function relating input velocity fluctuations to output heat release rate fluctuations was defined. Previous research regarding specific velocity fluctuation mechanisms which link input velocity fluctuations to output heat release rate fluctuations were summarized. Additionally, the few studies which have investigated the interaction of multiple velocity fluctuation mechanisms were reviewed. Based on the reviews presented in this chapter the objectives of this study were laid out.

In Chapter 2 an overview of the experimental facility used for this study was given and detailed explanations of the main components of the facility: the siren, nozzle, combustion chamber, and exhaust were given. Measurement techniques used for pressure, velocity, global heat release, and flame structure were explained. The data acquisition system was described and the analysis procedures applied to time-domain signals were discussed. Lastly, the analysis of flame images was explained.

Chapter 3 explained the determination of stable operating conditions for flame
transfer function measurements. Differences between the analysis techniques applied to self-excited and forced response measurements were discussed. A review of stable flame structure over a broad range of operating conditions was presented and a correlation for predicting flame length was developed. This correlation allows for prediction of flame lengths at operating conditions not tested but within the range of the measurements. A relationship between flame length and width was shown that was affected by the degree of flame-wall interaction.

The important characteristics of flame transfer function gain, phase, and coherence were discussed in Chapter 4. Flame transfer function gain exhibited oscillatory behavior giving rise to gain extrema. Flame transfer function phase showed quasilinear behavior with deviation from the linear trend occurring in the form of inflection points at frequencies corresponding to flame transfer function gain minima. The flame transfer function coherence was above 0.90 at all forcing frequencies indicating that the input and output of the flame transfer function are well correlated. All 47 flame transfer function measurements taken for this study exhibited the same characteristics.

The effect of each independent operating condition parameter: average velocity, equivalence ratio, inlet temperature, and combustor pressure on the flame transfer function was also investigated in Chapter 4. Increasing average velocity caused a horizontal translation of the flame transfer function gain curve to higher forcing frequencies and a vertical translation of the flame transfer function to lower phase values. Maximum flame transfer function gain and flame transfer function phase both decreased with increasing equivalence ratio. Increasing inlet temperature increased the frequencies of flame transfer function gain extrema and decreased the maximum gain. It also decreased flame transfer function phase. Combustor pressure did not affect the frequencies of flame transfer function gain extrema. Flame transfer function phase decreased with increasing combustor pressure except when the flame no longer impinged on the combustor wall.

In Chapter 5, an attempt was made to condense all of the effects observed in Chapter 4 by identifying the controlling parameters of the flame’s response to velocity fluctuations. Four frequency scaling parameters were investigated: Strouhal number based on nozzle diameter, Strouhal number based on flame length, the phase between azimuthal and axial velocity fluctuations at the flame anchoring lo-
cation, and the phase between swirl number and axial velocity fluctuations at the flame anchoring location. It was found that the phase between azimuthal and axial velocity fluctuations at the flame anchoring location, which is related to the magnitude of the swirl number fluctuation, collapsed the frequency axis best showing that swirl number fluctuations are an important velocity fluctuation mechanism. Maximum flame transfer function gain was shown to decrease with increasing ratio of the laminar flame speed to the average velocity ($\frac{S_L}{u}$). Flames with higher ($\frac{S_L}{u}$) are able to respond to the incident disturbance faster and therefore have lower global response.

Chapter 6 showed a modification that was made to the nozzle geometry to verify the importance of swirl number fluctuations on flame response. The distance from the swirler to the flame anchoring location was increased to change the convective time delay of the swirl number fluctuation. This modification to the nozzle geometry did not affect flame structure but did affect the flame transfer function. Flame transfer function gain extrema frequencies decreased in a manner consistent with the increased swirler to centerbody distance. Flame transfer function phase increased with increasing swirler to centerbody distance which is consistent with the additional convective time delay. These results further confirm that swirl number fluctuations are an important velocity fluctuation mechanism in this combustor.

Velocity fluctuation mechanisms were investigated on a local level through the analysis of phase-synchronized flame images in Chapter 7. A representative long, medium, and short length flame were analyzed out of the 11 flames for which phase-synchronized flame images were acquired. The first analysis technique was the creation of root mean square fluctuation and phase flame images. Root mean square fluctuation flame images showed that the location of the center of heat release does not always coincide with the location of maximum heat release fluctuation. They also showed that at flame transfer function gain minima, heat release fluctuations were equally distributed about the mean flame position while at flame transfer function gain maxima heat release fluctuations occurred predominantly in the downstream region of the flame indicating movement of the flame when the flame transfer function gain is low but not when it is high. Phase flame images showed a variation of phase across the flame consistent with a convective rather
than an acoustic heat release perturbation.

The second analysis method applied to phase-synchronized images is a windowing technique which was previously applied by other researchers. This analysis technique involved summing the heat release in different regions of the flame. It was found that the interpretation of the results of this analysis are highly sensitive to the window division location. Therefore, it was concluded that this analysis technique is not suitable to determine the presence/importance of different velocity fluctuation mechanisms.

The last analysis technique applied to phase-synchronized images was an investigation of mean flame position fluctuations. Swirl number fluctuations cause movement of the inner recirculation zone in the combustor. In response to the movement of the inner recirculation zone the flame also moves. Therefore flame position fluctuation is indicative of swirl number fluctuations. Analysis of phase-synchronized flame images showed that swirl number fluctuations exist at flame transfer function gain minima but not at maxima. The trends between these extrema frequencies indicate swirl number fluctuations, i.e. flame position fluctuations, act to decrease flame transfer function gain. It was proposed that the mechanism through which flame position fluctuations decrease flame transfer function gain is through a decrease of the vorticity in the shear layer. Through the vorticity equation it was shown that the flame, more specifically the temperature rise associated with it, acts to decrease the vorticity in the shear layer. Therefore, swirl number fluctuations are an important velocity fluctuation mechanism because they alter flame position which has a large effect on the vorticity of the flow field within the combustor which is one of the main velocity fluctuation mechanisms affecting flame response.

8.2 Recommendations for future work

This research presents a detailed study of the mechanisms responsible for fully-premixed flame response to only velocity fluctuations. Because industrial gas turbines typically operate in a partially-premixed configuration, during a combustion instability both the equivalence ratio and the velocity fluctuate. Flame response to purely equivalence ratio fluctuations was investigated in a separate study in this
combustor [25]. The results of these two studies, as well as an understanding of the interaction between velocity and equivalence ratio fluctuations could be incorporated into an analytical model in hopes of predicting combustion instabilities in a realistic partially-premixed combustor.

In Chapters 3 to 5 and 7 it was shown that the degree to which the flame interacts with the wall affects flame structure and flame response. In this study the degree to which the flame interacts with the wall could be changed but only by altering one of the operating condition parameters. These parameters were shown to have an effect on flame response. Therefore, the observed change in flame response with changing degree of wall interaction could be due to either a change in one of the operating condition parameters or a change in the degree of wall interaction. In the current experiment it is impossible to distinguish between the two. However, experiments could be done in a variable geometry combustor where the degree of wall interaction could be changed by changing the combustor diameter. This would allow only the degree to which the flame interacts with the wall to be investigated.

Not only can the flame interact with the combustor wall, but in a multi-flame combustor, which is the configuration of most industrial gas turbine combustors, a single flame can also interact with other flames. The boundary conditions of flame-wall and flame-flame interaction are completely different: impermeable versus permeable, no-slip versus slip, non-adiabatic versus adiabatic, etcetera. An understanding of how these changing boundary conditions affect flame response is necessary to understand combustion instabilities in multi-flame systems.

In Chapter 6 it was shown that the flame response can be affected by moving the axial location of the swirler. It was also shown that the change in flame response could be predicted based on the change in convective time delay of the swirl number fluctuation. These results demonstrate that axial location of the swirler could be used as a means to control combustion instabilities. If the acoustic mode of the combustor was known, the axial location of the swirler could be adjusted so that the largest swirl number fluctuations also occur at this frequency. Chapter 7 showed that large swirl number fluctuations lead to low flame response so this would effectively damp the combustion instability. Further measurements with varied axial swirler location are necessary to confirm this.
While linear regime flame response measurements are necessary to predict the frequency of self-excited combustion instabilities, nonlinear flame response measurements are necessary to determine the ultimate amplitude of the combustion instability [124]. The air siren used in this study is capable of generating normalized velocity fluctuation amplitudes of up to 50%. This high level of normalized velocity fluctuation amplitude should be sufficient to measure nonlinear flame response at all forcing frequencies in this experiment.
Appendix A

Comparison of two-microphone method and hot-film anemometry

A.1 Introduction

In Section 2.2.2 it was discussed that the two-microphone method is more appropriate than hot-film anemometry to measure the input function, i.e. the normalized velocity fluctuation amplitude, for flame transfer function measurements because in this context the flame is treated as a “black box” with a single input and a single output. This appendix discusses a series of measurements undertaken to prove this point. Section A.2 details the hot-film anemometer measurement location and Section A.3 explains the calibration of this device. Finally, the two methods are compared in Section A.4.

A.2 Hot-film anemometer measurement location

Measurements were made using a hot-film anemometer (TSI model 1210-20) and bridge circuit (TSI CTA model 1750). The hot-film anemometer was located 68.58 mm upstream of the dump plane and diametrically opposite to the pressure transducers used for the two-microphone method as shown in Figure A.1. Since the measurement location of the velocity fluctuation by the two-microphone method is the midpoint between the pressure transducers, both the hot-film anemometer
and two-microphone method measure the velocity fluctuation at the same axial location. Orientation of the hot-film anemometer was perpendicular to the page so that the probe measured the axial velocity. Fluctuating and average velocity were measured using the hot-film anemometer at 7 radial locations \((r_{HFA})\) in the annular gap between the centerbody and burner tube. Due to the fact that the hot-film anemometer was inserted through the outside wall of the burner tube the coordinate system is defined such that \(r_{HWA} = 0\ mm\) corresponds to the inside wall of the burner tube and \(r_{HWA} = 11.5\ mm\) corresponds to the outside wall of the centerbody.

![Figure A.1: Cross-section of model of nozzle with two-microphone method (TMM) and hot-film anemometer (HFA) locations indicated.](image)

A.3 Hot-film anemometer calibration

Calibration of the hot-film anemometer was done by inserting the probe at a fixed radial location and varying the average axial velocity, calculated based on total mass flow rate, fluid density, and annular area, and recording the corresponding hot-film anemometer voltage. The radial location of the probe was changed and the procedure repeated for the 6 other radial locations. A background hot-film anemometer voltage measurement was taken with no flow in the system. This background voltage was subtracted from the voltage measurement for each average axial velocity at each radial location. Results of the calibration procedure are shown in Figure A.2 where the average axial velocity is plotted against the background subtracted hot-film anemometer voltage for 7 different radial locations. The fact that the calibration points do not overlap for all radial locations indicates that the radial profile of axial velocity at this axial location is not uniform. This fact is more readily seen in Figure A.3 where the radial profiles of hot-film anemometer voltage are shown for 6 different average axial velocities. The black
dash-dot line indicates the midpoint of the annulus. Figure A.3 shows that the axial velocity profile is not symmetric about the midpoint of the annulus and does not exhibit the characteristic “top hat” profile of fully developed turbulent pipe flow. Therefore, even if the axial velocity fluctuation amplitude were constant across the annulus, the normalized axial velocity fluctuation amplitude \( \left( \frac{u'_{\text{rms}}}{u} \right) \) would differ depending on the radial location of the hot-wire anemometer due to the difference in local axial velocity.

Figure A.2: Hot-film anemometer calibrations at 7 different radial locations. The black dash-dot line indicates the midpoint of the annulus.

Figure A.3: Radial profiles of hot-film anemometer voltage for 6 different average axial velocities.
Table A.1: Curve fit parameters and coefficients of determination of quadratic curve fits applied to hot-film anemometer calibration data shown in Figure A.2.

<table>
<thead>
<tr>
<th>$r_{HFA \ [mm]}$</th>
<th>1.48</th>
<th>2.86</th>
<th>4.44</th>
<th>5.92</th>
<th>7.40</th>
<th>8.88</th>
<th>10.36</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.9998</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>1.0000</td>
<td>0.9994</td>
<td>0.9998</td>
</tr>
</tbody>
</table>

Conversion from hot-film anemometer voltage to velocity is accomplished by applying a curve fit to the data in Figure A.2. Then, for a measured hot-film anemometer voltage the corresponding velocity can be determined. In general, the relationship between voltage and velocity for a hot-film anemometer is quadratic. Therefore a quadratic curve fit of the form $y = Ax^2 + Bx$ was applied to the calibration data at each radial location. Since the hot-film anemometer voltage is background subtracted, the curve fits were forced through the origin. The curve fit coefficients and coefficients of determination of the quadratic curve fits are shown in Table A.1. For all radial locations the relationship between hot-film anemometer voltage and velocity is well characterized by a quadratic function.

A.4 Comparison of hot-film anemometer and two-microphone method normalized velocity fluctuation amplitude measurements

Figure A.4b shows the normalized velocity fluctuation amplitude as measured by the hot-film anemometer at 6 different radial locations. Error bars represent ± two standard deviations of the 32 measurements taken at each frequency. As can be seen from Figure A.4b the normalized velocity fluctuation amplitude across the annulus at a given forcing frequency exhibits complex behavior. The largest difference between radial locations is approximately a factor of three and occurs at 225 Hz between the location nearest to the burner tube wall and the location nearest to the centerbody. Differences that large in the input to the flame transfer function would lead to vastly different values of the flame transfer function.
gain. Furthermore, the differences in normalized velocity fluctuation amplitude at different radial locations are not consistent as a function of forcing frequency. For example, the inner and outermost radial location measurements agree within experimental uncertainty at 375 Hz while they differ by a factor of three at 225 Hz. The corresponding two-microphone measurements taken during the hot-film anemometer measurements are shown in Figure A.4a. These measurements were taken to ensure consistency of the velocity fluctuation amplitude while the hot-film anemometer was being moved between radial locations.

While Figure A.4 shows that the normalized velocity fluctuation amplitude measured by the two-microphone and hot-film anemometry at a single radial location do not agree, agreement between the two methods does occur if the hot-film anemometer measurements are area-averaged. Area-averaging is equivalent to the process used to calculate average velocity from a velocity profile, however the axial velocity fluctuation profile is integrated across the annulus rather than the axial velocity profile as shown by:

\[
(u'_{rms})_{area-average} = \frac{1}{A_{annulus}} \int u'_{rms} dA_{annulus}
\]  

(A.1)

Figure A.5 compares the area-averaged hot-film anemometer and two-microphone method velocity fluctuation measurements. Within experimental uncertainty, these two measurements of the velocity fluctuation agree at all frequencies. Agreement between the two measurement techniques indicates that the two-microphone method measures the average velocity fluctuation in the nozzle. Therefore, the two-microphone method will be used to measure the velocity fluctuation because the flame transfer function is a single input model.

### A.5 Conclusions

The two-microphone and hot-film anemometer methods of measuring the velocity fluctuation were compared. Both methods measured the velocity fluctuation downstream of the swirler at the same axial location. Measurements with the hot-film anemometer were performed at seven different radial locations across the annulus of the nozzle. Hot-film anemometer measurements showed that the time-averaged
axial velocity is not uniform across the nozzle annulus. These measurements also showed that the velocity fluctuation amplitude is not uniform across the nozzle annulus. A maximum difference of a factor of three was observed between the hot-film anemometer measurements at different radial locations. Therefore, if the hot-film anemometer were used to measure the flame transfer function input the flame transfer function gain would vary depending on the radial location of the probe. This indicates that hot-film anemometry is not well suited to measure the
velocity fluctuation amplitude. While the two-microphone and hot-film anemometer measurements at a single radial location did not agree, the two methods did agree when the hot-film anemometer measurements were area-averaged. This indicates that the two-microphone method measures the average velocity fluctuation in the nozzle. Since the flame transfer function is defined as a single input single output model, the two-microphone method is the best option to measure the transfer function input.
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