STRONG ENCOUNTERS WITH BLACK HOLES IN GLOBULAR CLUSTERS

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Astronomy & Astrophysics

by

Drew Reid Clausen

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The dissertation of Drew Reid Clausen was reviewed and approved* by the following:

Michael Eracleous  
Professor of Astronomy & Astrophysics  
Dissertation Co-Adviser  
Chair of Committee

Steinn Sigurdsson  
Professor of Astronomy & Astrophysics  
Dissertation Co-Adviser  
Head of the Astronomy & Astrophysics Graduate Program

Richard Wade  
Associate Professor of Astronomy & Astrophysics

Yuexing Li  
Assistant Professor of Astronomy & Astrophysics

Lee Samuel Finn  
Professor of Physics

*Signatures are on file in the Graduate School.
Abstract

As many as 1000 black holes could be produced when a globular cluster’s most massive stars end their lives. Following its formation, a globular cluster’s black hole population is transformed by the dynamical processes that occur within dense stellar systems. After decades of theoretical and observational study, the outcomes of this evolution remain uncertain. Outstanding questions include the fraction of clusters that retain black holes, the size of the retained population, and the mass distribution of these black holes. In this thesis, we model encounters between black holes and other cluster members to identify observable indicators of the presence and masses of black holes in globular clusters.

In one line of research, we model the formation of black hole–neutron star binaries via dynamical interactions in globular clusters. We find that in dense, massive clusters, many of the black hole–neutron star binaries formed by these encounters undergo gravitational radiation driven mergers. Black holes retained by the cluster after merging with a neutron star can acquire subsequent neutron star companions and undergo several mergers. However, the post-merger recoil is only suppressed below the globular cluster escape velocity for black holes with masses exceeding 30 M⊙. Thus, the merger rate is sensitive to the black hole mass distribution. Models with 35 M⊙ black holes predict Advanced LIGO detection rates in the range 0.04–0.7 yr⁻¹. Systems that do not merge may be observable as black hole–millisecond pulsar binaries. We discuss the distribution of orbital parameters in such binaries and the cluster properties that promote their formation. We find that the upper limit for the number of dynamically formed black hole–millisecond pulsar binaries in the Milky Way globular cluster system is ~ 10.

In a complementary study, we model the emission lines produced in the photoionized debris of tidally disrupted white dwarfs and horizontal branch stars. These emission lines can be used to investigate the intermediate mass black holes that may be present in some globular clusters. We find that bright, broad, asymmetric C iv λ1549 and [O iii] λ5007 emission lines can be used to identify white dwarf tidal disruption events. When a horizontal branch star is disrupted, the brightest optical emission lines are [N ii] λ6583 and [O iii] λ5007. We compare our models with two candidate white dwarf tidal disruption events that have been observed in globular clusters. We find several drawbacks to interpreting either of these sources as a white dwarf tidal disruption event. However, models of a red clump horizontal branch star undergoing mild disruption by a 50 – 100 M⊙ black hole yield an emission line spectrum that is in good agreement with one of these sources.
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Chapter 1

Introduction

Many significant processes in the dynamical evolution of a self gravitating system, including mass segregation, relaxation, and core collapse, occur in globular clusters within a Hubble time. As such, globular clusters offer an ideal testbed for our understanding of many-body gravitational dynamics. The dynamical evolution of the cluster as a whole is closely linked to the evolution of individual stars within the cluster. Much as the addition of a binary companion opens up evolutionary channels that are inaccessible to single stars, dynamical interactions between stars and binaries within the cluster enable evolutionary paths that cannot occur in the field. This interplay encodes the dynamical processes at play within the cluster on the stellar and binary populations. In the introduction we will outline important developments in the field of globular cluster dynamics and the role that these developments played in understanding observations of both the structure and the exotic stellar populations of globular clusters.

1.1 General Overview

Globular clusters are collections of $10^4 - 10^7$ stars found in the bulge and halo regions of the Galaxy. The clusters are nearly spherical with 75% having a projected ellipticity of less than 0.1. The stellar density in the clusters is much higher than in the solar neighborhood. A typical central value is $n = 10^4 \text{ pc}^{-3}$, but the density can be as high as $10^6 \text{ pc}^{-3}$. Compared to the Sun, globular clusters are metal poor. The metallicity distribution amongst the Milky Way globular clusters is bimodal with peaks at $\frac{[\text{Fe/H}]}{\text{H}} = -1.6$ and $-0.6$ (Harris 1996, 2010 version\textsuperscript{1}). Within a single cluster, however, $[\text{Fe/H}]$ does not vary much from star to star. The red giant branch (RGB) and lower main sequence are clearly visible in the color magnitude diagrams (CMDs) of globular clusters, but the blue end of the main sequence is absent. Stars with mass $\sim 0.8 \text{ M}_\odot$ can be seen “turning off” the main sequence and those more massive than this have already evolved up the RGB and beyond. Figure 1.1 shows

\textsuperscript{1}http://physwww.mcmaster.ca/~harris/mwgc.dat
a CMD for the globular cluster M3. These features of the CMD, along with the metallicity measurements, are the basis of the long held notion that globular clusters contain a single, coeval, chemically homogenous stellar population. The age of a globular cluster can, therefore, be determined by fitting its CMD with isochrones generated from stellar evolution models. As stellar evolution models became more sophisticated, and the input physics required for such models was better understood, the estimated age of the Milky Way globular cluster system evolved from more than 20 Gyr to the currently accepted value\(^2\) of 13 Gyr (Meylan & Heggie 1997; Carretta et al. 2000).

Recent technological advances have allowed for comprehensive, homogenous, high precision photometric studies of globular clusters that have solved long standing problems and challenged enduring ideas. Principal among these studies are observations of globular clusters carried out with the *Hubble Space Telescope (HST)*. HST has been especially useful in studying globular clusters outside of the Milky Way. Previous ground based studies had shown that early type galaxies possess much richer globular cluster populations than the Milky Way (Harris 1991; Brodie & Strader 2006), but HST made measurements of individual globular clusters within these populations possible. Surveys of the globular clusters associated with M31 and galaxies in the Fornax and Virgo clusters have shown that the distributions of structural parameters, luminosity, and metallicity in these clusters are similar to these distributions in the Milky Way globular cluster system (Jordán et al. 2005; Peng et al. 2006; Jordán et al. 2007; Madrid et al. 2009; Masters et al. 2010; Strader et al. 2011). These results suggest that lessons learned from dynamical models of the well studied Milky Way globular cluster system can be applied to globular clusters associated with other galaxies.

One component of a globular cluster’s stellar population that has been studied extensively are the horizontal branch stars. A globular cluster’s horizontal branch morphology is strongly influenced by its metallicity, with the horizontal branches in relatively metal rich globular clusters lying redward of the instability strip and those of most low metallicity clusters lying to the blue side. However, it has been known for some time that metallicity alone cannot account for the range of horizontal branch morphologies observed, and that horizontal branch morphology is determined by metallicity and at least a second parameter (e.g., Sandage & Wallerstein 1960; van den Bergh 1967). By the mid 1990s, a number of studies had established cluster age as a likely solution to the second parameter problem (e.g. Searle & Zinn 1978;

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\(^2\)Meissner & Weiss (2006) argued that isochrone fitting methods still need improvement because they cannot fit all age indicators (e.g., the difference in the brightness of the main sequence turn off and the bump on the red giant branch) simultaneously. One possibility for more robust age determination is to combine CMD isochrone fitting with constraints from precisely determined masses, radii, and luminosities of stars in detached, eclipsing binaries within the cluster (e.g. Thompson et al. 2001).
Lee et al. 1994). Utilizing HST data, Dotter et al. (2010) and Gratton et al. (2010) independently confirmed that the elusive second parameter of a globular cluster's horizontal branch morphology is the cluster's age. While these new studies presented the first robust determination of the second parameter, they disagreed on the third. Dotter et al. (2010) found that after the metallicity and age dependancies were removed, the central luminosity of the cluster correlated most strongly with horizontal branch morphology. Gratton et al. (2010) found the most likely third parameter to be the He abundance.

Metallicity and age determine the horizontal branch morphology of most globular clusters, but these parameters fail when confronted with the peculiar horizontal branch of NGC 2808. This globular cluster exhibits both the red clump and extended blue horizontal branch morphologies (Harris 1974). With deep HST photometry of the cluster, D’Antona et al. (2005) found that the main sequence was wider than
expected for a single population, and argued that the cluster had undergone self-enrichment. In this process, at least a second generation of stars was born out of the He enriched material shed by the first generation of stars as they evolved up the asymptotic giant branch (AGB). The presence of multiple generations of stars with different initial He abundances can account for the dual nature of the cluster’s horizontal branch, but this finding is contrary to the traditional assumption that globular clusters contain a single stellar population. Evidence of abundance anomalies in globular cluster stars had been mounting since the 1970s (Gratton et al. 2004), but the CMD of NGC 2808 was the first unambiguous sign that multiple stellar populations were present within some globular clusters\(^3\) (Piotto et al. 2007). Subsequently, several other clusters were shown to have multiple stellar populations, including NGC 1851 (Milone et al. 2008), NGC 6656 (Marino et al. 2009), 47 Tuc (Anderson et al. 2009), NGC 6752 (Milone et al. 2010), NGC 2419 (di Criscienzo et al. 2011), and NGC 6397 (Milone et al. 2012a). With mounting evidence that globular clusters contain multiple stellar generations, Downing & Sills (2007) examined whether such systems were plausible dynamically. They found that a two generation globular cluster was not only plausible, but similar to a single generation model both dynamically and structurally.

1.2 Structure and Dynamical Evolution

Below we will sketch some of the basic processes and equations that govern the dynamical evolution of a globular cluster following Spitzer (1987) and Binney & Tremaine (1987). Three characteristic radii are used to describe globular clusters. The core radius \(r_c\) is the distance at which the surface brightness drops to half its central value. The half-mass radius \(r_h\) is the radius of a circle containing half of the cluster’s mass. At the tidal radius \(r_t\), the gravitational field of the galaxy becomes more important than that of the cluster. The stellar density goes to zero at the tidal radius.

Much of the early work on globular clusters assumed that the clusters were static (Meylan & Heggie 1997). A globular cluster will undergo significant dynamical evolution during its lifetime, but static models are a valid representation of a cluster for times of order the half mass relaxation time, \(t_{rh}\). The value of \(t_{rh}\) varies significantly from cluster to cluster in the Milky Way system, but is \(\sim 1\) Gyr for many clusters. If, in addition to the cluster being static, one further assumes that the positions and velocities of stars in the cluster are well described by a smooth phase space distribution, \(f(r, v)\), where \(r\) and \(v\) are position and velocity vectors, and that the stars move under the influence of a smooth gravitational potential \(\phi\), then such a system

\(^3\)Lee et al. (1999) had previously shown that \(\omega\) Centauri had multiple RGBs, but \(\omega\) Centauri is likely the remnant core of a stripped dwarf galaxy and not a typical globular cluster (e.g., Bekki & Freeman 2003; Lee et al. 2009). It has also been suggested that NGC 2808 may be the remnant core of a dwarf galaxy (e.g., Forbes et al. 2004).
can be described by the collisionless Boltzmann equation:

\[ \frac{\partial f}{\partial t} - \nabla \phi \cdot \frac{\partial f}{\partial \mathbf{v}} + \mathbf{v} \cdot \nabla f = 0. \]  

(1.1)

The potential must satisfy Poisson’s equation:

\[ \nabla^2 \phi(\mathbf{r}) = 4\pi G \rho \]  

(1.2)

where \( G \) is the gravitational constant and \( \rho \) is the mass density.

Michie (1963) and King (1966) developed solutions to equation (1.1) that reproduce many features observed in globular clusters. These models use a lowered Maxwellian distribution function that limits the cluster to the tidal radius and an angular momentum distribution that permits velocity anisotropy in the cluster halo. Assuming that all stars in the cluster have the same mass, the distribution is given by:

\[ f(\epsilon, L) = \frac{\rho_0}{(2\pi v_m^2)^{3/2}} \exp \left( \frac{-L^2}{2r_a^2 v_m^2} \right) \left( e^{\epsilon/v_m^2} - 1 \right). \]  

(1.3)

Here, \( \epsilon \) is the relative energy per unit mass, \( v_m \) is the velocity dispersion of the stars, \( \rho_0 \) is the central mass density, \( L \) is the angular momentum per unit mass, and \( r_a \) is the radius at which the velocity distribution goes from isotropic to radial. The relative energy and angular momentum are related to \( \mathbf{r} \) and \( \mathbf{v} \) through:

\[ \epsilon = \Psi(r) - \frac{1}{2} v^2 \]  

(1.4)

\[ L = r v \sin \theta, \]  

(1.5)

where \( \theta \) is the angle between \( \mathbf{r} \) and \( \mathbf{v} \) and \( \Psi \) is the relative potential, \( \Psi(r) = \phi(r_t) - \phi(r) \). This distribution can be extended to include multiple mass groups with separate values for \( v_m \) and \( \rho_0 \) for each group. We will make extensive use of such models in later chapters.

In a globular cluster, the gravitational potential is not completely smooth and encounters between pairs of stars will drive the cluster’s evolution. A number of techniques are used to study how the cluster evolves as a result of these encounters. Perhaps the most intuitive approach is the N-body method, in which the gravitational interactions between all stars are are calculated directly (e.g., Aarseth 1999; Portegies Zwart et al. 2001). N-body simulations require few restrictive assumptions but are extremely expensive computationally. Fokker-Planck methods, on the other hand, require some simplifying assumptions but greatly reduce the computational burden. In such a model, the zero on the right hand side of equation (1.1) is replaced by a term that describes how the distribution function changes with time as a result of such encounters, \( (\partial f/\partial t)_{\text{enc}} = \Gamma(f) \). Under the local approximation, in which it is
assumed that encounters only produce small velocity perturbations and do not affect
the position of a star, one can derive an analytic approximation for the change in $f$
brought about by encounters:

$$\Gamma(f) = -\sum_{i}^{3} \frac{\partial}{\partial v_i} [fD(\Delta v_i)] + \frac{1}{2} \sum_{i,j=1}^{3} \frac{\partial^2}{\partial v_i \partial v_j} [fD(\Delta v_i \Delta v_j)].$$  

(1.6)

Here, $D(\Delta v_i)$ and $D(\Delta v_i \Delta v_j)$ are diffusion coefficients, which describe how stars
diffuse through phase space as a result of encounters. The first term in equation (1.6)
describes the drift in velocity space known as dynamical friction. The second term
accounts for diffusion in velocity space due to random kicks from stellar encounters.
Substituting the $\Gamma(f)$ given in equation (1.6) for the zero on the right hand side of
equation (1.1) produces the so called Fokker-Planck equation. In models of globular
cluster evolution, the Fokker-Planck equation can be numerically integrated directly
(e.g., Cohn 1979; Chernoff & Weinberg 1990) or, more commonly, integrated with
Monte Carlo methods (e.g. Spitzer & Hart 1971; Giersz 1998; Joshi et al. 2000; Freitag
& Benz 2001). Explorations of globular cluster evolution using either the N-body or
the Fokker-Planck method indicate that a number of interesting phenomena occur in
a cluster’s lifetime.

First, since globular clusters contain stars of different masses, over time interac-
tions between the stars drive the system towards energy equipartition. The more mas-
sive stars tend to slow down and sink into the cluster core and lower mass stars gain
kinetic energy and move to wider orbits. As a result, the cluster becomes mass seg-
regated. Mass stratification has been observed in Milky Way globular clusters (King
et al. 1995). N-body (Khalisi et al. 2007) and Fokker Planck (Watters et al. 2000)
calculations of multi-mass clusters confirm that the velocity dispersion of a group of
stars with mass $m$ will settle to an equilibrium value that is inversely proportional to
$m$. Spitzer (1969) showed that it is not always possible for a two component cluster
to reach equilibrium. If the total mass of the more massive component exceeds a
critical value, these stars will quickly sink to the center of the cluster and produce
a subcluster that undergoes a runaway collapse. This equipartition instability also
occurs in N-body and Fokker-Planck models, though each find a slightly different
criterion for instability (Watters et al. 2000; Khalisi et al. 2007).

Next, stellar dynamical systems are subject to the gravothermal catastrophe
(Lynden-Bell & Wood 1968). This instability arises because self gravitating systems
have a negative heat capacity. As the systems lose energy they become dynamically
hotter, which increases the flow of energy and drives the system away from equilib-
rium. N-body and Fokker-Plank models have explored a primary consequence of the
gravothermal catastrophe: core collapse. Models show that the central density of the
cluster approaches infinity after $\sim 15 t_{rh}$, although the value depends on the initial
conditions and physics included (e.g. Cohn 1979; Chernoff & Weinberg 1990; Joshi
et al. 2000; Watters et al. 2000; Khalisi et al. 2007). For example, core collapse is accelerated in multi-mass models that are also subject to the equipartition instability. It was soon realized that at such high densities, binaries could form through three-body interactions and act as a heat source to halt and reverse core collapse (Inagaki & Lynden-Bell 1983; Cohn & Hut 1984; Goodman 1984). If clusters were formed with a population of primordial binaries, then core collapse could be postponed (Hills 1975a).

1.3 Binaries in Globular Clusters

By the mid 1980s considerable evidence for the presence of binaries in globular clusters had accumulated, both observationally and on theoretical grounds (Hut et al. 1992b; Meylan & Heggie 1997). From a dynamical perspective, binaries are an important source of energy in the cluster. A study of the outcomes of three-body interactions between a binary and a single star by Heggie (1975) found that hard binaries become more tightly bound and transfer kinetic energy to the third body during an encounter. A binary is considered hard if its binding energy exceeds the kinetic energy of the surrounding stars. The degree of a binary’s hardness to an encounter is typically parameterized as the ratio of the single star’s relative velocity to a critical velocity $v_c$. A field star of mass $m_3$ interacting with a binary consisting of stars with masses $m_1$ and $m_2$ with semi-major axis $a$, must have a relative velocity

$$v_c^2 = \frac{Gm_1m_2(m_1 + m_2 + m_3)}{m_3(m_1 + m_2)a}$$

at infinity to disrupt the binary. Heggie (1975) found that a hard binary’s binding energy increased by an average of 40% after an encounter. The process of increasing the binary’s binding energy through encounters is often referred to as “hardening.” The liberated energy causes the binary and single star to recoil with increased kinetic energy, thereby heating the cluster. Alternatively, soft binaries tend to be disrupted as the result of a three body interaction.

Binary-single star interactions can be split into two categories, prompt and resonant. Prompt encounters occur in less than one binary orbital period. In a resonant encounter, a temporary three-body system that can survive for several orbital periods is formed. Furthermore, the outcome of these encounters can be described as a flyby, exchange, or disruption. In a flyby the membership of the binary does not change during the encounter. In an exchange, the single star ejects one member of the binary and becomes bound to the other. In a disruption all three stars are unbound at the end of the encounter. There is also the possibility of two of the stars involved in the encounter merging. Millions of binary-single interactions were simulated to determine the cross section for each of the above outcomes and the resulting energy exchange.
(e.g., Hut & Bahcall 1983; Sigurdsson & Phinney 1993). These cross sections can be used in Monte Carlo codes to study the effect of a population of primordial binaries on the evolution of a cluster.

Both N-body and Monte Carlo calculations for the evolution of a globular cluster with a primordial binary fraction of $5 - 20\%$ show a prolonged state of “binary burning.” During this phase, binding energy is extracted from the primordial binaries through dynamical encounters between these binaries and single stars or other binaries. The energy generated in these interactions can halt and reverse core collapse, allowing the cluster to remain in a quasi-equilibrium state for many $t_h$ (McMillan et al. 1990; Gao et al. 1991; Hut et al. 1992a; Fregeau et al. 2003). However, during the binary burning phase, the binary population is depleted. The binaries are ejected, driven to merger, or disrupted in encounters with other binaries. In a detailed study that included the effects of binary stellar evolution, cluster dynamics, and explicit integration of three and four body encounters involving binaries, Ivanova et al. (2005a) found that the binary population in a dense cluster’s core was depleted so rapidly that an initial binary fraction of 100% was required to yield the value of $\sim 5\%$ observed in some clusters. These authors also pointed out the importance of including both stellar evolution and dynamics in globular cluster simulations. For example, a binary can merge as the result of dynamically unstable mass transfer and common envelope evolution, but a purely dynamical model would not account for this process.

At the same time, through exchange and hardening, dynamical interactions drive binaries towards common envelope evolution. Hence, while it is clear that primordial binaries play an influential role in the dynamical evolution of a globular cluster, dynamical interactions also have a profound effect on the population of binaries within the cluster.

### 1.4 Neutron Stars in Globular Clusters

When X-ray sources were observed in globular clusters, it was proposed that these sources could be accreting neutron stars (NS) or black holes (BH; Clark et al. 1975). Katz (1975) noted that the number of the X-ray sources found in globular clusters was intriguingly large considering that the clusters make up less than 0.1% of the Galaxy’s mass. The increased X-ray-luminosity-to-mass ratio is evidence that the formation of such sources in globular clusters takes place through dynamical channels that are unavailable to systems in the field. Hills (1976) proposed that the enhanced production of X-ray sources in globular clusters was the result of BHs or NSs exchanging into primordial binaries. The first hint that the accretors in these systems were NSs and not BHs came when Grindlay et al. (1984) observed the positions of the X-ray sources within the clusters. Assuming a mass segregated cluster, the measured cluster radii of the X-ray sources suggested that they had masses of $\sim 1.5 M_\odot$. Today there are 15 known bright low mass X-ray binaries (LMXBs) in Milky Way globular
clusters (Benacquista 2006). Surprisingly, six of these systems have orbital periods of less than one hour and are classified as ultra-compact X-ray binaries (e.g Deutsch et al. 2000). All 15 undergo X-ray bursts, which confirms that these LMXBs have NS accretors (in’t Zand et al. 2003) because X-ray bursts are thermonuclear flashes on the NS's surface, and such flashes cannot occur on a BH.

Gravitational radiation will drive NS+NS binaries formed through exchange interactions in globular clusters to coalescence. Such mergers are thought to be the origin of short gamma ray bursts (GRBs). Models by Grindlay et al. (2006) and Ivanova et al. (2008) predict that 10 − 30% of short GRBs are due to NS+NS binaries formed by dynamical interactions in globular clusters. Another consequence of the high stellar density in the cores of globular clusters are physical stellar collisions. Ivanova et al. (2005b) showed that the rate of collisions between NS and RGB stars in globular clusters was large enough to produce the observed excess of ultra-compact X-ray binary systems. During the collision, the RGB star’s envelope is stripped leaving an eccentric NS+white dwarf (WD) binary. The binary then decays by emission of gravitational radiation, eventually becoming an ultra-compact X-ray binary.

In addition to the NSs in LMXBs, there are also 144 radio pulsars observed in 28 different globular clusters. The first pulsar discovered in a globular cluster was a millisecond pulsar (MSP) in M28 (Lyne et al. 1987). This discovery helped to confirm the origin of MSPs. The existence of a pulsar in a globular cluster is hard to reconcile with the pulsar’s short lifetime of $10^7$ yr and the cluster’s ancient stellar population, unless the pulsar had been recycled in a binary by accreting material from a companion. In recent years, improved instruments and search techniques have uncovered large populations of MSPs in globular clusters. For example, Ransom et al. (2005) reported the discovery of 21 MSPs in Terzan 5 using the Green Bank Telescope’s specialized Pulsar Spigot instrument. Another survey of globular clusters being carried out with the Parkes radio telescope has also identified a large population of MSPs, many of which are in exotic binary systems (e.g. D’Amico et al. 2001). While many of these observations confirmed predictions of dynamical models, Lynch & Ransom (2011) warn that some calculations over predict the MSP producing efficiency of globular clusters.

A serious challenge in modeling the NS populations in globular clusters is the strong kick that NSs receive at birth. Studies of the proper motions of pulsars in the galaxy showed that the average kick velocity was 200 − 450 km s$^{-1}$ (Lyne & Lorimer 1994; Hansen & Phinney 1997), much larger than the tens of km s$^{-1}$ escape speeds of globular clusters. This implies that only 1% of NSs are formed with a kick velocity far enough in the tail of the distribution to be retained by a globular cluster. By contrast, models of binary–single star interactions presented in Sigurdsson & Phinney (1995) produce MSP populations consistent with those observed but assume that 20% of NSs are retained. One way to increase the retention fraction is to assume
that some of the NSs were formed in binary systems. Pfahl et al. (2002) showed that under these conditions the retention fraction could be as large as 8%. Tuning coupled dynamical and binary evolution models to the observed globular cluster LMXB population, Ivanova et al. (2008) derived a NS retention fraction of $5 - 7\%$, corresponding to 220 NS for every $2 \times 10^5 M_\odot$. Most of the NS retained in these models were formed through electron capture supernovae, which resulted in kick velocities ten times smaller than core collapse supernovae. A portion of the electron capture supernovae occurred during the evolution of single stars, but many were the result of massive WDs undergoing accretion induced collapse (AIC). AIC had previously been proposed as a source of the globular cluster NSs (e.g., Bailyn & Grindlay 1990), and these new results indicate the channel’s importance. Even though the mechanism that keeps NSs in globular clusters remains uncertain, both observations and dynamical models indicate that globular clusters retain 100s of NSs.

1.5 Stellar and Intermediate Mass Black Holes in Globular Clusters

From a theoretical perspective, it is uncertain whether or not globular clusters contain BHs, because dynamical interactions can remove them from the cluster. Within a few hundred million years of a cluster’s formation, BHs will become the most massive component, sink to the core, and form a dense subcluster (Sigurdsson & Hernquist 1993; Kulkarni et al. 1993). Within the subcluster the BHs will interact, form binaries, and promptly eject one another from the cluster. N-body calculations by Portegies Zwart & McMillan (2000) confirmed the rapid depletion of a globular cluster’s BH population, finding that only 8% of BHs are retained. These authors also found that many dynamically formed BH+BH binaries will undergo a gravitational radiation driven merger within a Hubble time, and argued that dynamically formed binaries could be a dominant source for the Laser Interferometer Gravitational-Wave Observatory (LIGO; Abbott et al. 2009). These mergers will, however, occur after the binaries are ejected from the cluster. A recent Monte Carlo calculation presented in Downing et al. (2010) also found that all of the dynamically formed BH+BH binaries that merge will do so outside of the cluster. In an updated N-body simulation, Banerjee et al. (2010) noted that one or two BH–BH mergers can take place within the cluster before the entire BH population is ejected, but most occur in the field. Sadowski et al. (2008) avoided complete ejection of the cluster’s BH population by assuming that the BHs did not segregate into a subcluster, but instead remained in dynamical equilibrium with the other stars in the cluster. In this case, dynamically formed BH+BH binaries avoid disruption and ejection through encounters with other BHs, resulting in an almost steady cluster merger rate of $3 \text{ Gyr}^{-1}$.

Kalogera et al. (2004) questioned whether a BH that survived the ejection frenzy would be observable. Such a BH could acquire a companion through an exchange
interaction with a binary or by tidally capturing a single star. Kalogera et al. (2004) found that BH X-ray binaries formed through exchange interactions would rarely be observable because of their extremely low duty cycles. On the other hand, BH X-ray binaries formed when the BH tidally captures a single star should be persistent sources. Based on the absence of such a source in the Milky Way globular cluster system, Kalogera et al. (2004) concluded that either the clusters are devoid of BHs or, more likely, that a stable binary cannot be formed by tidal capture. In another study, Ivanova et al. (2010) considered the dynamical formation of BH X-ray binaries with WD donors. These authors found that any BH+WD binaries formed by exchange interactions would be too wide for mass transfer to occur. However, the authors identified two processes that could drive the binaries towards a mass transfer episode. First, the binaries could be hardened by subsequent encounters with other stars. Second, if the BH+WD binary were the inner binary in a hierarchical triple, then eccentricity pumping due to the Kozai mechanism could force the WD close enough to the BH that mass transfer occurs. Creating a BH+WD X-ray binary through either channel requires that 10% of the BHs formed within a globular cluster remain in dynamical equilibrium with the other stars and avoid sinking into a BH subcluster.

It has also been suggested that the BH subcluster could be the site of successive BH mergers, instead of ejections. These mergers could result in the growth of an intermediate mass black hole (IMBH; $10^2 - 10^4 M_\odot$). Miller & Hamilton (2002b) argued that a single, more massive BH ($M_{BH} \sim 50 M_\odot$) could resist ejection from the cluster and serve as the IMBH seed. Gültekin et al. (2004) tested this IMBH formation scenario with simulations of successive binary-single interactions and found that a $50 M_\odot$ BH could grow to $240 M_\odot$, but was usually ejected from the cluster before reaching this mass. Using an initial BH mass function computed in binary evolution models, O’Leary et al. (2006) found that 20-80% of globular clusters could grow BHs of more than $100 M_\odot$. Interestingly, these authors also found that mergers induced by the Kozai mechanism do not play a major role in the growth or retention of a BH, contrary to the hypothesis of Miller & Hamilton (2002a). O’Leary et al. (2006) also noted that the retention of IMBH seeds and growth of the black hole is sensitive to the magnitude of the post merger recoil. In models utilizing updated recoil velocities from numerical relativity calculations, Moody & Sigurdsson (2009) found that only $0.5 - 3.5\%$ of BH binaries were retained and their final masses were in the range $20 - 50 M_\odot$. Even if an IMBH cannot form through BH mergers, some simulations show that runaway stellar mergers early in a cluster’s evolution produce a very massive star that could collapse into an IMBH (Portegies Zwart et al. 2004; Freitag et al. 2006).

As discussed above, all of the LMXBs in Milky Way globular clusters show evidence of a NS accretor. With the exquisite angular resolution of Chandra, however, it is possible to resolve low mass X-ray binaries (LMXBs) in nearby galaxies and to associate these LMXBs with globular clusters (e.g., Sarazin et al. 2000). This capability
has enabled the discovery of several BH candidates in globular clusters outside of the Milky Way. Maccarone et al. (2007) presented the first strong candidate for a BH in a globular cluster, an ultraluminous\(^5\) X-ray source (ULX) in the globular cluster RZ 2109 associated with the elliptical galaxy NGC 4472. The object’s X-ray luminosity of \(L_X = 4 \times 10^{39}\) erg s\(^{-1}\) is too large to be explained by accretion onto a NS, and the source’s strong variability rules out the possibility of a superposition of multiple NSs. Subsequently, five additional ULXs have been identified as globular cluster BH candidates on the basis of their strong X-ray variability; one in NGC 1399 (Shih et al. 2010), two in NGC 3379 (Brassington et al. 2010, 2012), a second in NGC 4472 (Maccarone et al. 2011), and one in NGC 4649 (Roberts et al. 2012). Variable X-ray sources with luminosities above the Eddington limit of a NS offer the least ambiguous evidence of a BH accretor; however, Barnard et al. (2011) argued for BH accretors in three LMXBs located in M31 globular clusters that exhibit low/hard-state spectra at luminosities near a NS’s Eddington limit, and in a high luminosity recurrent transient in a fourth M31 globular cluster.

Irwin et al. (2010) discussed an additional BH candidate in an NGC 1399 globular cluster; however, the nature of this source is not as clear-cut as that of the other candidates. The object’s X-ray luminosity, \(L_X \sim 2 \times 10^{39}\) erg s\(^{-1}\), suggests a BH accretor, but the 35% decline in the luminosity between 2000 and 2008 does not convincingly rule out a superposition of sources. The X-ray spectrum can be fitted with a power law of photon index \(\Gamma = 2.5\), which is much softer than the slope of the power law used to fit the sum of several low \(L_X\) LMXBs (\(\Gamma \sim 1.6\)). This is also slightly softer than the spectra of other sources with \(L_X \sim 10^{39}\) erg s\(^{-1}\), which might indicate the presence of a more massive BH. Adding to the intrigue, optical spectroscopy of the globular cluster revealed bright \([\text{N} \text{ II}] \lambda 6583\) and \([\text{O} \text{ III}] \lambda 5007\) emission lines with luminosities \(\sim 10^{36}\) erg s\(^{-1}\), but no Balmer emission lines. This is not the only globular cluster BH candidate with optical emission lines, Zepf et al. (2008) found bright, broad \([\text{O} \text{ III}] \lambda 5007\) emission lines without accompanying Balmer lines in the optical spectrum of RZ 2109. Irwin et al. (2010) argued that these two sources could be evidence of the tidal disruption or detonation of a WD by an IMBH based on the lack of hydrogen emission lines in the spectra.

A BH in the core of a galaxy or a globular can tidally disrupt stars that pass too closely and then accrete a portion of the debris (Hills 1975b; Frank & Rees 1976; Sigurdsson & Rees 1997; Baumgardt et al. 2004; Ramirez-Ruiz & Rosswog 2009). For stars initially on parabolic orbits with the BH, roughly half of the disrupted star will become bound to the BH, while the other half remains unbound and streams back into the cluster (Lacy et al. 1982; Rees 1988). Roos (1992) first considered the emission lines produced when the unbound debris is photoionized by the UV/X-ray emission generated when the bound portion accretes onto the BH. Detailed studies of the emission lines produced when a main-sequence star is tidally disrupted by a

\(^5\)An ultraluminous X-ray source has an X-ray luminosity \(L_X > 10^{39}\) erg s\(^{-1}\)
massive BH have been made using numerical (Bogdanović et al. 2004) and analytic (Strubbe & Quataert 2009, 2011) models for the dynamical evolution of the debris. These studies found that the emission lines have substantial diagnostic power, as they can validate tidal disruption candidates and constrain properties of the black hole. Further study is required to determine if the sources in RZ 2109 and NGC 1399 are the result of a WD being tidally disrupted by a BH.

Although the recently identified ULXs in extragalactic globular clusters indicate that some globular clusters contain a BH, whether the accretors are stellar mass BHs, similar in mass to those found in the field, or IMBHs remains unclear. Shortly after X-ray sources were observed in globular clusters, it was also proposed that these sources were not X-ray binaries, but instead central $\sim 10^3 M_\odot$ BHs accreting the gas shed by stars during their evolution (Bahcall & Ostriker 1975; Silk & Arons 1975). Subsequently, Bahcall & Wolf (1976, 1977) calculated how the presence of an IMBH would impact a globular cluster’s structure and found that the IMBH would produce a $\rho \propto r^{-7/2}$ density cusp in the center of the cluster. Newell et al. (1976) measured the surface brightness profile of M15 and argued that the observed central cusp required the presence of a $800 \pm 300 M_\odot$ BH. The claim was quickly disputed by Illingworth & King (1977), who were able to reproduce the central excess with a central collection of binaries or NSs instead of a massive BH. The presence of an IMBH in M15 is still debated because high precision measurements of the central surface brightness and velocity dispersion profiles made with HST are also well explained by models with and without an IMBH (Gerssen et al. 2002; Murphy et al. 2011). A similar debate exists over whether or not a $10^4 M_\odot$ IMBH is needed to explain the surface brightness and kinematics observed in the core of G1, a globular cluster associated with M31 (Baumgardt et al. 2003; Gebhardt et al. 2005). Fitting observations with models is complicated because converting the observed surface brightness to a mass-density profile requires assumptions about the mass-to-light ratio in the cluster. Using N-body models, Baumgardt et al. (2005) found that the surface brightness profile would exhibit a much shallower cusp than previously expected. In the models, the core was dominated by the cluster’s most massive members, non-luminous remnants that would not impact the surface brightness. The models also showed that the velocity dispersion of the cluster stars should rise in the region where $r < (M_{BH}/M_{GC})r_h$, prompting the authors to argue that the definite detection of an IMBH would require observations of such a velocity dispersion profile. The surface brightness and velocity dispersion profiles of NGC 6388 are consistent with these predictions if the cluster harbors a $1.7 \pm 0.9 \times 10^3 M_\odot$ IMBH (Lanzoni et al. 2007; Lützgendorf et al. 2011).

While the core of a globular cluster may be the most obvious place to look for the effects of an IMBH, the dynamical imprint of a central IMBH can also be seen in the cluster’s outer regions. NGC 6752 is host to five MSPs, all of which exhibit peculiar properties that could be evidence of a central IMBH (D’Amico et al. 2002). Two of the MSPs were found on the cluster’s outskirts at radii of $1.4 r_h$ and $3.3 r_h$, and
the outermost MSP was determined to have a companion. Colpi et al. (2002, 2003) considered several scenarios to account for the location of the most distant MSP, and concluded that the binary could have been ejected from the core by interacting with a single IMBH or an IMBH–stellar mass BH binary. The three other MSPs reside in the core of the cluster and have $\dot{P}$ that cannot be explained by accelerations due to luminous matter in the cluster core, suggesting, again, the presence of a substantial, non-luminous mass. In addition to ejecting MSPs from the core, a binary IMBH could also interact with normal stars and create a population of a few hundred high-velocity stars (Mapelli et al. 2005). The spatial distribution of these high velocity stars would peak around $3r_c$. Furthermore, Mapelli et al. (2005) found that the angular momentum of many of these high velocity stars would be aligned with that of the binary IMBH, inducing an anisotropy in the stars’ angular momentum distribution.

Recent searches for IMBHs in globular clusters have been conducted in the radio band. These searches were motivated the fundamental plane of BH activity, which relates the radio luminosities, X-ray luminosities, and masses of BHs accreting at very low rates (Merloni et al. 2003; Falcke et al. 2004). According to this relation, the radio to X-ray luminosity ratio increases for more massive BHs and BHs with lower accretion rates. Furthermore, this relationship is observed in both stellar mass and supermassive BHs, suggesting that it is a universal property of accretion onto BHs. Based on the scaling relations implied by the fundamental plane of BH activity, Strader et al. (2012b) argued that radio observations may be more sensitive to IMBHs accreting at extremely low rates than X-ray observations. These authors carried out deep radio observations of three Milky Way globular clusters, M15, M19, and M22. No central sources were detected in these observations, which prompted Strader et al. (2012b) to conclude that IMBHs in globular clusters, with $M_{\text{BH}} \gtrsim 1000 M_\odot$, are either rare or accreting at extremely low rates.

Although these radio searches did not find the IMBHs that they were looking for, they did identify three stellar mass BH candidates in the Milky Way globular cluster system. Strader et al. (2012a) reported the discovery of two flat-spectrum radio sources near the center of M22. After ruling out several possible alternatives, the authors concluded that these sources were accreting BHs with masses of $M_{\text{BH}} \sim 15 M_\odot$. Strader et al. (2012a) further argued that the fact that this cluster harbors two observable BHs suggests that the cluster has a total BH population of 5–100 BHs. Chomiuk et al. (2013) discussed the discovery of a third BH candidate, which is in the cluster M62. The discovery of several BH candidates in the Milky Way has lead to a renewed interest in globular cluster BHs. Prompted by the discovery of these BHs, new N-body and Monte Carlo simulations found that it may be possible for globular clusters to retain a significant fraction of their BH populations, after all (Sippel & Hurley 2013; Morscher et al. 2013). These recent developments have motivated our study, which aims to identify additional means of probing the BHs in globular clusters.
1.6 Thesis Overview

Complementary theoretical and observational efforts have shown that dynamical interactions in globular clusters yield binary populations distinct from those in the field. Given their relatively small mass, globular clusters produce an excessive fraction of the observed X-ray binaries and short GRBs, and are thought to produce a disproportionately large fraction of merging BH+BH binaries. Observational evidence for the presence of BHs in globular clusters is mounting, but several questions about the nature of the globular cluster BH populations remain unanswered. What fraction of globular clusters retain BHs? How large are the retained BH populations? Furthermore, what is the mass distribution of BHs in globular clusters? In this thesis, we explore the production of BH+NS binaries through binary-single star interactions and the tidal disruption of evolved stars by IMBHs; both of which offer observational probes of globular cluster BH populations. We will investigate how these probes can aid in answering these open questions.

In Chapter 2, we study the formation of BH+NS binaries that will coalesce through emission of gravitational radiation within a Hubble time. Exchange interactions allow a single BH to merge with multiple NSs resulting in a merger rate that is significant when compared to the rate expected from binary evolution alone (e.g., Sipior & Sigurdsson 2002; Belczynski et al. 2007; O'Shaughnessy et al. 2010). Chapter 3 will discuss the formation of long lived BH+MSP binaries. We examine the distributions of these systems’ orbital parameters and the globular cluster properties that promote their formation. In Chapter 4, we present models of the emission-line spectra produced when WDs and horizontal branch stars are tidally disrupted by IMBHs. We will consider whether the models are consistent with the optical emission lines observed in globular clusters hosting ULXs, and determine if the models can place constraints on the nature of the BHs in these clusters. Finally, in Chapter 5 we summarize the results and assess prospects for future study.
Chapter 2

Black Hole–Neutron Star Mergers in Globular Clusters


2.1 Introduction

As was described in chapter 1, surveys of the Milky Way globular cluster system have discovered 15 low mass X-ray binaries (Deutsch et al. 2000; Sidoli et al. 2001) and 144 pulsars, many of which are recycled, millisecond pulsars (MSPs) and/or members of exotic binaries (Lynch & Ransom 2011; Freire et al. 2007; Hessels et al. 2006; Ransom et al. 2005; D’Amico et al. 2001). These observations reveal that 1) some fraction of the neutron stars (NSs) formed in globular clusters are retained despite their natal kicks, and 2) multibody interactions in the cores of globular clusters greatly enhance the formation rate of binary systems that contain a NS, resulting in an over abundance of low mass X-ray binaries and MSPs relative to the field (Katz 1975; Clark et al. 1975; Verbunt & Hut 1987). It has long been recognized that binary–single star interactions between NSs and primordial binaries can result in the NS exchanging into the binary by ejecting one member and becoming bound to the other (Hills 1976; Hut & Bahcall 1983; Sigurdsson & Phinney 1995; Ivanova et al. 2008). Such binary–single star interactions could also result in the formation of black hole (BH)+NS binaries (Sigurdsson 2003; Devecchi et al. 2007), which are expected to produce gravitational waves detectable by the ground-based interferometer LIGO if they coalesce (Abbott et al. 2009). Observations of a BH–NS merger would provide a test of General Relativity in the dynamical strong-field regime and could probe the NS equation of state (Kyutoku et al. 2009; Duez et al. 2010). Additionally, BH–NS mergers are also a likely progenitor of some short gamma-ray bursts (Nakar 2007).

Whether or not dynamical interactions in globular clusters can efficiently produce BH+NS binaries is unclear. Models suggest that the stellar mass BHs that form within a globular cluster will rapidly sink to the core of the cluster and interact with,
and eject one another, severely depleting the cluster’s BH population within 1 Gyr (Kulkarni et al. 1993; Sigurdsson & Hernquist 1993; O’Leary et al. 2006; Moody & Sigurdsson 2009; Banerjee et al. 2010; Aarseth 2012). Even though most of a globular cluster’s BHs are ejected early in its evolution, there are models (Miller & Hamilton 2002a) and mounting observational evidence that show some globular clusters retain at least one BH. Several extragalactic globular clusters harbor highly variable X-ray sources with luminosities \( \geq 10^{39} \text{erg s}^{-1} \), well in excess of the Eddington luminosity of an accreting NS, that are strong candidates for BHs in globular clusters (Maccarone et al. 2011). Even if a globular cluster only retains a single BH, it may be possible for this BH to merge with multiple NSs if the BH exchanges into another NS-containing binary after each merger.

What is the maximum rate for such successive mergers? If we assume that the BH and NS merge instantaneously, then the maximum merger rate is equal to the rate at which the BH can exchange into a NS-containing binary, \( R_{\text{ex}} = n_{\text{NS}} \sigma_{\text{ex}} v_{\text{rel}} \), where \( n_{\text{NS}} \) is the density of NS-binaries, \( \sigma_{\text{ex}} \) is the cross section for an exchange, and \( v_{\text{rel}} \) is the relative velocity of the BH and the binary. Ivanova et al. (2008) found that globular clusters tend to retain \( \sim 220 \) NS per \( 2 \times 10^5 \text{M}_\odot \). Due to mass segregation, NSs will be concentrated near the center of a globular cluster, and the central NS fraction, \( f_{\text{NS}} \), can be as high as 0.1. Assuming that all of the NSs at the center of the cluster are in a binary and that the collision cross section is dominated by gravitational focusing, we can approximate the maximum BH–NS merger rate as

\[
R_{\text{ex}} \sim 2 \times 10^{-10} \text{ yr}^{-1} \left( \frac{f_{\text{NS}}}{0.1} \right) \left( \frac{n}{10^5 \text{ pc}^{-3}} \right) \left( \frac{a}{1 \text{ au}} \right) \times \left( \frac{M_{\text{BH}}}{1 \text{ M}_\odot} \right) \left( \frac{v_{\text{m}}}{10 \text{ km s}^{-1}} \right)^{-1}.
\]  

(2.1)

Here, \( n \) is the total central stellar density, \( a \) is the typical semi-major axis of a NS binary, \( M_{\text{BH}} \) is the BH mass, and \( v_{\text{m}} \) is the mean central velocity dispersion.

In this chapter we present detailed models that investigate the rate at which the BHs and NSs retained by a globular cluster will form binaries that merge via the emission of gravitational radiation and the corresponding detection rate for LIGO. We will begin by discussing the methods used in our simulations in section 2.2. In section 2.3.1, we will describe the results of simulations that only allowed BHs to merge with a single NS, and in section 2.3.2 we will describe how the results change when the BH is allowed to undergo successive mergers as sketched above. We will conclude in section 2.4 by computing the LIGO detection rate and comparing our work with previous calculations.
2.2 Method

We modeled the formation of BH+NS binaries through binary–single star interactions using the method presented in Sigurdsson & Phinney (1995) and Mapelli et al. (2005). For each simulation, an ensemble of binaries was evolved in a static background cluster. The background clusters were multi-mass King models, with the stars of each mass group $\alpha$ following the isotropic distribution function

$$f_\alpha(\epsilon) = \begin{cases} \frac{n_0}{(2\pi v_{\text{m}_\alpha}^2)^{3/2}} (v_{\text{m}_\alpha}^2/\epsilon - 1) & \epsilon > 0 \\
0 & \epsilon \leq 0 \end{cases},$$

(2.2)

where $v_{\text{m}_\alpha}$ is the mass group’s core velocity dispersion, $n_0$ is a normalizing constant, and $\epsilon = \Psi(r) - v_\star^2/2$ is the relative energy per unit mass. Here, $v_\star$ is the velocity and $\Psi(r)$ is the relative potential given by $\Phi(r_t) - \Phi(r)$, where $\Phi(r)$ is the gravitational potential with respect to infinity at a distance $r$ from the cluster center and $r_t$ is the cluster’s tidal radius. To explore how the BH–NS merger rate is impacted by the structural properties of the globular cluster, we varied the central stellar number density, $n_c$, the density weighted mean central velocity dispersion, $\bar{v}_m$, and the King parameter $W_0 \equiv \Psi(0)/\bar{v}_m^2$ to generate clusters of different mass and concentration. The values of $n_c$, $\bar{v}_m$, and $W_0$ were chosen to create a set of background clusters with core radii $r_c$ and concentrations $c_{\text{GC}} = \log(r_t/r_c)$ similar to observed clusters. Furthermore, these clusters probe a range of core interaction rates $\Gamma \propto n_c^{1.5} r_c^2$, a quantity that is known to correlate with another product of dynamical interactions, the number of X-ray binaries present in a cluster (Pooley et al. 2003). We list the values used in our models in Table 2.1.

For each cluster, we used the initial stellar mass function

$$\xi(m) \propto \begin{cases} m^{-1.3} & m < 0.55 \, M_\odot \\
 m^{-2.35} & m > 0.55 \, M_\odot \end{cases}$$

(Kroupa 2001), and binned the evolved population into 10 mass groups. The main sequence (MS) turnoff mass, $m_{t_{\text{to}}}$, was set to 0.85 $M_\odot$ and stars with initial mass above the turnoff mass were assumed to be completely evolved. The evolved stars fell into one of three groups, those with initial mass $m_i < 8 \, M_\odot$ evolved into white dwarfs (WDs) with masses given by $m_{\text{WD}} = 0.38 + 0.12 \, m_i$ (Catalán et al. 2008). Stars with initial mass in the range $8 \, M_\odot \leq m_i < 25 \, M_\odot$ were assumed to form $1.4 \, M_\odot$ NSs and those with initial mass $> 25 \, M_\odot$ formed BHs.

We performed calculations using two different values for the BH mass. Recent statistical analyses of low mass X-ray binaries in the Galaxy have found that the masses of BHs are narrowly distributed around $7 \, M_\odot$ (Özel et al. 2010; Farr et al. 2011). Motivated by this, we have run models in which all BHs have a mass of $7 \, M_\odot$. 
Table 2.1. Background Globular Cluster Model Parameters

<table>
<thead>
<tr>
<th>Model name</th>
<th>( n_c ) (pc(^{-3}))</th>
<th>( \bar{v}_m ) (km s(^{-1}))</th>
<th>( W )</th>
<th>( M ) (M(_\odot))</th>
<th>( c_{GC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( 1 \times 10^4 )</td>
<td>6</td>
<td>6</td>
<td>( 1.0 \times 10^5 )</td>
<td>1.20</td>
</tr>
<tr>
<td>B</td>
<td>( 1 \times 10^5 )</td>
<td>10</td>
<td>10</td>
<td>( 5.2 \times 10^5 )</td>
<td>1.71</td>
</tr>
<tr>
<td>C</td>
<td>( 5 \times 10^5 )</td>
<td>11</td>
<td>13</td>
<td>( 7.2 \times 10^5 )</td>
<td>1.93</td>
</tr>
<tr>
<td>D</td>
<td>( 1 \times 10^6 )</td>
<td>12</td>
<td>15</td>
<td>( 1.1 \times 10^6 )</td>
<td>2.06</td>
</tr>
</tbody>
</table>

However, the progenitors of the field BHs considered in the above studies were stars with much higher metallicity than those found in globular clusters. Lower metallicity stars have reduced mass loss rates and could evolve to BHs with masses as high as \( 80 \; M_\odot \) (Fryer & Kalogera 2001; Fryer et al. 2002; Belczynski et al. 2010a). There is also observational evidence that BHs in globular clusters are more massive than those in the Galaxy. Properties of the ultraluminous X-ray sources observed in globular clusters associated with NGC 4472, M31, and NGC 1399 are consistent the presence of a BH of \( \gtrsim 30 \; M_\odot \) (Maccarone et al. 2007; Barnard et al. 2011; Irwin et al. 2010; Clausen et al. 2012). Accordingly, we also ran models with \( 35 \; M_\odot \) BHs to allow for the possibility that globular cluster BHs are more massive than those in the field.

We assumed that the clusters retained 20% of the NSs that were formed within them. We further assumed that nearly all BHs formed in the cluster were promptly ejected by self interaction, leaving 0, 1, or 2 BHs (Kulkarni et al. 1993; Sigurdsson & Hernquist 1993; O’Leary et al. 2006; Moody & Sigurdsson 2009; Banerjee et al. 2010; Aarseth 2012). We mimic this by truncating the high mass end of the initial stellar mass range so there is a single BH in the globular cluster model. We are, therefore, modeling the late evolution of the cluster, and hence recent times, so the merger rates calculated here apply to the local universe. In all of our simulations, we assumed that the remaining BH was in a binary. The properties of such a binary will have undergone extensive modification driven by both stellar and dynamical evolution. Consequentially, the initial configurations assigned to the BH binaries in our models represent the result of complicated processes that we do not attempt model.

### 2.2.1 Dynamics

For each simulation, we evolved 2000 binaries, one at a time, in one of the background clusters described above. Each binary contained a BH and a companion drawn at random from the cluster’s evolved mass distribution. The initial semi-major axis, \( a \), was drawn from a distribution that is flat in \( \log a \) between \( 10^{-3} \) au and \( a_{\text{max}} \) (Abt
We used a different value of $a_{\text{max}}$ for each background cluster, namely 100 au, 33 au, 15 au, and 10 au for clusters A, B, C, and D, respectively. These values of $a_{\text{max}}$ were chosen to cover the range of possibilities that might result from the complicated evolutionary histories of the BH binaries mentioned above. The initial eccentricity was drawn from $f(e) \propto 2e$ (Duquennoy & Mayor 1991).

Each binary was initially placed in the cluster core, with its position selected from the radial density distribution of the third most massive mass group. The results are insensitive to this somewhat arbitrary choice because the initial position and velocity of the binary are rapidly forgotten. The binaries were evolved in the cluster potential with dynamical friction and random kicks to account for scattering by individual stars. At each point along the binary’s trajectory, we computed the probability that it would experience a strong encounter with a single star using

$$P_{\text{enc}} = \Delta t \sum \int n_\alpha(r)\sigma(v, v_*) |v - v_*| f_\alpha(v_*) d^3 v_*,$$

where the subscripts $\alpha$ correspond to the mass groups, $\Delta t$ is the time step, $n$ is the number density, $f(v_*)$ is the velocity distribution of single stars given in equation (2.2), $v$ and $v_*$ are the velocity of the binary and a single star, respectively, and $\sigma$ is the encounter cross section. Throughout this chapter we are interested in encounters that alter the energy, angular momentum or components of the binary. Previous studies have shown that the bulk of these changes involve a background star approaching the binary center of mass closer than a few times the binary’s physical size and conversely that numerous, more distant encounters have relatively small effect. We choose the critical pericenter distance for simulated encounters to be $p = a[4 + 0.6(1 + e)]$ where $a$ is the binary semi-major axis and $e$ is its eccentricity (Hut & Bahcall 1983). The corresponding cross section for such encounters is

$$\sigma(v, v_*) = \pi p^2 + \frac{2\pi G(m_{\text{bin}} + m_\alpha)p}{|v - v_*|^2},$$

where $m_{\text{bin}}$ is the mass of the binary. An encounter took place if $P_{\text{enc}}$ was greater than a random number drawn from a uniform distribution between 0 and 1. Given that an encounter occurred, we randomly chose which particular mass group was involved based on the fractional encounter probabilities of all the groups. Then, the three-body interaction between the stars was integrated explicitly as described in Sigurdsson & Phinney (1993). There were many possible outcomes to the three-body encounter, including mergers, exchanges, and disruption of the binary, but if a binary still existed at the end of the interaction, then we continued to evolve the binary in the cluster.

We performed two types of simulations. In the first class of simulations we were only concerned with the fate of the initial BH binary, while in the second class we
sought to determine the ultimate fate of the BH. In the former set, a run continued until either the initial binary was disrupted, merged, or ejected, or the run had covered $10^{10}$ yrs. In the latter set of models, we continued a run when the merger or disruption of a binary resulted in an isolated BH (see section 2.2.3). These runs ended only after the BH was ejected from the cluster or the maximum run time of $10^{10}$ yrs was reached. In both simulation types, a small fraction of the runs ($\lesssim 5\%$) were aborted because integration of a three-body interaction exceeded the maximum number of allowed steps, $8 \times 10^7$.

2.2.2 Gravitational radiation effects on the evolution BH+NS binaries

Between interactions the eccentricity and semi-major axis of each BH+NS system were decreased to account for the emission of gravitational radiation using the relations given in Peters (1964). For the simulations in which we used a BH mass $M_{BH} = 7 M_\odot$, we stopped the calculation when a BH+NS binary coalesced. Numerical relativity simulations have found that the remnant formed by a BH–NS merger at this mass ratio will receive a kick from anisotropic emission of gravitational radiation that exceeds the globular cluster’s escape velocity (Shibata et al. 2009; Etienne et al. 2009; Foucart et al. 2011). In the models where we used $M_{BH} = 35 M_\odot$, the post-merger recoil velocity was suppressed because of the much smaller mass ratio ($q = M_{NS}/M_{BH}$). For the case of non-spinning point masses, the merger of a $1.4 M_\odot$ NS and a $35 M_\odot$ BH would result in a recoil velocity of $14.6$ km s$^{-1}$, well below the escape velocity for the clusters models that we considered (González et al. 2007). However, we allowed for spinning BHs and NSs, and computed the recoil velocity using the parameterization given in Campanelli et al. (2007, however, see Hirata 2011 for discussion of a resonant recoil, not included in this parameterization, that can dominate when the BH spin is nearly maximal and $q \ll 1$). At the beginning of each run, the BH spin parameter was randomly selected from a uniform distribution between 0 and 1. When a BH+NS binary merged, the NS spin was randomly selected from a uniform distribution between 0 and 0.7 (a spin parameter of 0.7 corresponds to a NS spinning at the break up frequency Miller et al. 2011) and the two spins were randomly oriented with respect to one another and to the orbital plane. The result was a range of kick velocities ($\sim 10 - 100$ km s$^{-1}$), some of which were low enough that the single BH was retained by the cluster after the merger.

2.2.3 Formation of a new binary from a single BH by stellar interactions

In some of our models, when the BH lost its binary companion, either through an interaction with a background star or as the result of a gravitational radiation driven
merger with a NS, we continued to evolve the now single BH in the cluster. As the BH moved through the cluster, we sought to determine the single BH’s next binary companion by generating binaries with a semi-major axis $a$, an eccentricity $e$, a primary of mass $m_1$, and a secondary of mass $m_2$. The semi-major axis and eccentricity were drawn from the same distributions as those of the initial BH binaries. The only difference is that we have chosen $a_{\text{max}} = 1$ au for these systems to ensure that the binaries were hard. Our choices for $m_1$ and $m_2$ are described below.

We determined whether or not the single BH would interact with the binary by temporarily replacing the secondary with the BH. We then calculated the probability that this temporary binary would interact with a background star of mass $m_2$ (the secondary), $P_{(1,\text{BH})+2}$, using equation (2.4). Next, $P_{(1,\text{BH})+2}$ was scaled to $P_{(1,2)+\text{BH}}$ by comparing the rates of two related exchange encounters, $[(1, \text{BH}) + 2 \rightarrow (2, \text{BH}) + 1]$ and $[(1, 2) + \text{BH} \rightarrow (1, \text{BH}) + 2]$. The rate for an arbitrary interaction between a binary $(a,b)$ and single star $c$ can be expressed in the form

$$R(X) = G m_a m_b (m_a + m_b + m_c) / m_c (m_a + m_b) a_{\text{in}} n \tilde{\sigma}(X) v,$$

where $X$ refers to the type of interaction (e.g., one of the desired exchange reactions), $v$ is the relative velocity of the binary and the single star, $a_{\text{in}}$ is the semi-major axis of the binary, $n$ is the density of single stars, $\tilde{\sigma}(X)$ is a numerically determined, dimensionless cross section, $G$ is the gravitational constant, and $m_i$ is the mass of star $i$. Using equation (2.6), we computed the relative rates of the above exchange interactions, assuming that $a_{\text{in}}$, $n$, and $v$ were the same in each case.

The required dimensionless cross sections were computed by Sigurdsson & Phinney (1993), who found $\tilde{\sigma}[(1, \text{BH}) + 2 \rightarrow (2, \text{BH}) + 1] \sim 2 \times \tilde{\sigma}[(1, 2) + \text{BH} \rightarrow (1, \text{BH}) + 2]$. In addition, we made the simplifying approximation that $m_1 = m_2 \ll M_{\text{BH}}$, because the BH was much more massive than either the primary or the secondary. This reduced the ratio of the mass factors to $m_2 / 2M_{\text{BH}}$. Combining these, we approximated the probability that the single BH would interact with a binary containing the primary and the secondary as

$$P_{(1,2)+\text{BH}} \sim \frac{f_b}{1 - f_b} \frac{m_2}{M_{\text{BH}}} P_{(1,\text{BH})+2},$$

where $f_b$ is the binary fraction and relates the densities of single and binary stars. Note that the right hand side of this expression diverges as $f_b \rightarrow 1$. Thus, for large values of $f_b$, use of equation (2.7) will overestimate the rate at which a single BH can exchange back into a binary. Furthermore, we have used the relative rates of two exchange encounters to scale the total encounter probability. The exchange cross section is a significant fraction of the total interaction cross section, but we will explore the consequences of this choice in section 2.4.1. We chose to compute the encounter probability using the scaling relation given in equation (2.7) because the
distribution of binaries in globular clusters is poorly understood and this technique offers an efficient and adequate estimation of the rates within the limitations of our code. When an encounter was deemed to have occurred, we restored the original binary and explicitly integrated the three-body interaction between the binary and the BH, accepting any outcome. We repeated this process until the BH exchanged into a binary, the BH was ejected from the cluster, or the maximum integration time was reached. If the BH exchanged into a binary, we continued the simulation as described in section 2.2.1.

We explored several different binary populations and values of $f_b$. In one set of models, we drew the primary’s mass from the cluster’s evolved mass distribution and required that $m_1 \geq m_{10}$, but we did not chose the secondary’s mass, initially. Instead, we used a binary consisting of the primary and the BH to compute the scaled probability that a BH would interact with a binary containing the primary and a member of any mass group. If an encounter occurred, then the secondary was chosen to belong to the mass group that interacted with the temporary binary. We then integrated the three-body interaction as described above. This method of selecting the secondary resulted in a binary population that is composed largely of systems that contain a NS. For the simulations in cluster D, more than 50% of the binaries that interacted with a single BH contained a NS. We have labeled the runs that use this binary population OPT, for optimized, because this method produces a binary population that is tuned to interact with the single BH. Furthermore, because many of the binaries in this population already contain a NS, the single BH can gain a NS companion immediately.

The second binary population that we considered was closely linked to observational constraints. A number of studies have used the fact that unresolved MS binaries appear brighter and redder than single MS stars to investigate binaries in globular clusters. The binary fraction can be measured by comparing the number of objects in the offset binary sequence to the number of stars that lie along the MS, with observed values typically found to be in the range $f_b = 0.05 - 0.3$ (e.g., Romani & Weinberg 1991; Sollima et al. 2007; Davis et al. 2008; Milone et al. 2012a). We generated a population that resembles these observed populations by independently drawing two stars from the cluster’s evolved stellar mass distribution and requiring that one of the stars have a mass $m \geq m_{10}$. The result was a collection of systems that consisted primarily ($\gtrsim 85\%$) of MS-MS binaries. We have labeled simulations that use this binary population OBS, and considered values for the binary fraction within the observed range: 0.05, 0.1, and 0.2.

We also considered a population that was motivated by detailed models of binary evolution in globular clusters. Analytic, Monte Carlo, and N-body models have indicated that over time the hard binary fraction will increase in the cluster core (Sollima 2008; Fregeau et al. 2009; Hurley et al. 2007). For such models to be consistent with the observations described above, the initial binary fraction in globular clusters would
have to have been quite low. Alternatively, as Fregeau et al. (2009) suggested, if many of the binaries in a globular cluster consisted of a MS star and a WD or two WDs, they would be “hidden” from observers. Following on this result, we have run simulations that use a binary population similar to that described in table 1 of Fregeau et al. (2009). This population consisted of 44% MS+WD binaries, 32% WD+WD binaries, 23% MS+MS binaries, and 1% binaries containing a NS. Runs that used this binary population were labeled FIR. Furthermore, because only 23% of this binary population would be measured by the observations described above, we have used larger values of $f_b$ with this population: 0.2, 0.5, and 0.75.

2.3 Results

In all of our simulations, binary–single star interactions produced BH+NS binaries. However, the number of BH+NS binaries formed and the properties of these systems were very sensitive to the structure of the globular cluster and the assumed binary population. Our simulations are summarized in Table 2.2. The initial conditions for each simulation, including the cluster model, the binary population (where applicable), the binary fraction (where applicable), and $M_{\text{BH}}$, are given. Also listed are $f_{\text{BHNS}}$, the fraction of runs that produced a BH+NS binary, $N_{\text{BHNS}}$, the total number of BH+NS binaries formed in all 2000 runs, $N_{\text{mrg}}$, the number of BH–NS mergers during the simulation, $\bar{t}_{\text{ex}}$, the average amount of time taken for a single BH to exchange back into a binary, $N_{\text{max}}$, the maximum number of mergers in a single run, and $R_{\text{GC}}$, the BH-NS merger rate for that model. We describe the results in detail below, beginning with models that assumed that the BH was ejected from the globular cluster after merging with a NS, followed by descriptions of models with more massive BHs, which allowed for a single BH to merge with multiple NS.

2.3.1 Single merger scenario

2.3.1.1 Without single BHs

In this section we describe simulations that ended if the binary containing the BH was disrupted or merged. These models did not require any assumptions about the globular cluster’s binary population as a whole because they were terminated as soon as the BH lost its companion. We have computed one such simulation in each of our background clusters, and in all of these simulations we assumed $M_{\text{BH}} = 7 M_\odot$.

In the lowest density cluster considered here, cluster A, only eight BH+NS binaries were formed in our 2000 runs, none of which merged during the $10^{10}$ yr simulations. In all of the other clusters, however, some fraction of the BH+NS binaries merged during the simulation. Figure 2.1 shows the initial semi-major axis and eccentricity of the BH+NS systems formed in simulations using cluster model D. Also plotted
Table 2.2: Globular Cluster Models and Merger Rates

<table>
<thead>
<tr>
<th>Cluster Model Population</th>
<th>$f_b$</th>
<th>$M_{BH}$ (M$_\odot$)</th>
<th>$f_{BHNS}$</th>
<th>$N_{BHNS}$</th>
<th>$N_{mrg}$</th>
<th>$t_{ex}$ (yr)</th>
<th>$N_{max}$</th>
<th>$R_{GC}$ (yr$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Single merger scenario without single black holes (section 2.3.1.1)</strong></td>
<td></td>
<td></td>
<td></td>
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<td><strong>Multiple merger scenario (section 2.3.2)</strong></td>
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<td>3</td>
<td>0</td>
<td>&gt; 10$^{10}$</td>
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<td>5266</td>
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</table>
Properties of the globular cluster models are given in Table 2.1, the binary populations are described in section 2.2.3, $f_b$ is the binary fraction, $M_{BH}$ is the mass of the BH, $f_{BHNS}$ is the fraction of runs in which a BH+NS binary was produced, $N_{BHNS}$ is the total number of BH+NS binaries formed in the 2000 run simulation, $N_{mrg}$ is the number of BH–NS mergers during the simulation, $\bar{t}_{ex}$ is the mean amount of time it takes for a single BH to exchange back into a binary, $N_{max}$ is the maximum number of BH–NS mergers in a single run, and $R_{GC}$ is the BH–NS merger rate.
are lines of constant gravitational radiation merger time. A majority of the systems lie above the right most line, which corresponds to a merger time $10^{10}$ yr. If these systems were to evolve in isolation, it would take more than a Hubble time for them to merge. However, in the massive globular clusters considered here subsequent interactions with single stars extracted energy from the binary’s orbit and drove some of these systems to high eccentricity and/or small separations, greatly accelerating the merger process. This mechanism is illustrated in Figure 2.2. BH+NS binaries with expected merger times, given their initial eccentricity and semi-major axis, $t_{\text{GW0}} > 10^7$ yr underwent several interactions with single stars after their formation, which altered the orbital parameters of the binary and reduced the actual amount of time between the formation of the BH+NS binary and the merger of the binary, $t_{\text{GW}}$, by several orders of magnitude. In these simulations, binary–single star encounters not only produced BH+NS binaries, but in some cases enhanced the rate at which these systems merged.

In other situations, subsequent interactions reduced the merger rate. In models that used cluster B, many BH+NS binaries were destroyed when a WD exchanged into the binary. In this relatively low concentration cluster, the density of WDs was much greater than the density of NSs, so multiple encounters with WDs overcame the low probability of a WD exchanging into the BH+NS binary. This property of low concentration clusters has implications for the formation of mass transfer binaries, and hence X-ray luminosity of such clusters, which we will investigate in a future study. At the end of most runs in cluster B, the BH remains in the cluster. The BH is ejected in 3% of the runs. Of those ejected, 42% are ejected after a BH–NS merger and 58% are ejected after a super-elastic collision between the binary and a background star. In the majority of runs (63%), the BH remains in the cluster with a non-NS binary companion. The BH loses its companion but remains in the cluster in the remaining 34%.

This over-density of WDs did not occur in the high concentration clusters, C and D. In these simulations, if a BH+NS binary was destroyed by an exchange, it was usually another NS exchanging into the system. The simulations suggest that in the clusters with $n \geq 5 \times 10^5$ pc$^{-3}$, there is a high likelihood that a BH retained by the cluster after the initial phase of self interaction will become part of a BH+NS binary that merges within a Hubble time. This, coupled with the high post-merger kicks predicted by numerical relativity, suggests that if globular cluster BHs have the same mass distribution as those in the field, BHs will only be retained in massive clusters with $n \leq 10^5$ pc$^{-3}$.

The distribution of BH–NS merger times, $t_{\text{mrg}}$, for the different globular clusters models are shown as solid histograms in Figure 2.3. We note that $t_{\text{mrg}} > t_{\text{GW}}$ because $t_{\text{mrg}}$ includes the time taken to produce the BH+NS binary. Encounters occurred frequently in simulations of the more massive, higher density clusters, so the BH+NS binaries were formed earlier and, as described above, encounters after their formation
accelerated the merger process. For each cluster we calculated the merger rate as \( R_{GC} = N_{mrg}/(N t_{\text{max}}) \), where \( N \) is the total number of binaries simulated and \( t_{\text{max}} = 10 \) Gyr is the length of each run. In each cluster, \( N \) was slightly less than 2000 because we discarded the runs where the initial binary was randomly chosen to contain a BH and NS. Although these simulations predicted many BH–NS mergers, a substantial faction of the runs ended prematurely because the BH lost its companion. For the runs in clusters C and D, nearly 40\% of simulations ended for this reason. In the next section we will examine how the merger rate changes if such single BHs are allowed to interact with and exchange back into a binary.
Figure 2.2 The actual amount of time it took a BH+NS binary to merge $t_{GW}$ vs. the amount of time expected for the BH+NS binary to merge, given the binary's initial configuration, $t_{GW_0}$ for the BH+NS binaries that merge in a simulation of cluster model D with $M_{BH} = 7 \, M_\odot$. Note that many systems formed with $t_{GW_0} \gg$ a Hubble time. Subsequent interactions between these binaries and single stars greatly reduced $t_{GW}$. We excluded systems where the initial binary was randomly chosen to contain a BH and NS in our merger rate estimates to avoid initial condition bias and because these would happen at high redshift and be unobservable.

2.3.1.2 With single BHs

Here, we describe a set of simulations that allowed BHs that lost their companions as the result of a binary–single interaction to continue their evolution in the cluster. Again, we have run one such simulation in each of our background cluster models and assumed that $M_{BH} = 7 \, M_\odot$. We note that in these simulations, each BH could only undergo one merger with a NS because with $M_{BH} = 7 \, M_\odot$ the post merger kick imparted to the BH exceeded the escape velocity of the cluster. In all four simulations, we used the FIR binary population with $f_b = 0.5$. 
These changes had little effect on simulations of cluster A. The interaction rate in cluster A was lower than the other clusters, so the initial BH binaries underwent far fewer encounters. Only 57 of 2000 runs in this simulation produced a single BH, and none of these single BHs exchanged back into a binary. On the other hand, allowing the single BHs to find new companions increased the number of BH+NS binaries that formed and the number of these systems that merged in clusters B, C, and D. The average amount of time that it took for a BH without a companion to exchange into a binary, $t_{\text{ex}}$, was 3.64, 3.31, and 1.88 Gyr for clusters B, C, and D, respectively. In most cases, a single encounter did not result in the BH exchanging into the binary, so $t_{\text{ex}}$ is longer than the interaction time scale. When the BH did exchange into the binary, the resulting systems (in all three cluster models) had an average semi-major axis of 2.8 au and average eccentricity of 0.8. Because the interaction cross section for such wide and eccentric binaries is large, the newly formed systems quickly interacted with additional background stars. In the simulations of cluster D, a BH+NS binary was formed in 87% of the runs, and 93% of the runs that produced such a binary ended with a BH–NS merger. A single run could produce multiple BH+NS binaries as the result of numerous exchanges, but only a single BH–NS merger. For example, in cluster D, the average number of BH+NS binaries produced in a single run was 2.1, but in one run the BH had ten different NS companions before merging. The average BH–NS merger rate in cluster B nearly doubled when we allowed single BHs to gain new companions. In clusters C and D the merger rate increased by more than a factor of three.

The dotted histograms in Figure 2.3 show the distribution of BH–NS merger times in this set of models. These dotted histograms are the combination of two $t_{\text{mrg}}$ distributions. One distribution, similar to the solid histograms, is due to BHs that merge with a NS before losing their binary companion. The second distribution is for mergers that occurred after the BH lost its binary companion and had to exchange back into a binary before finally gaining a NS companion and, eventually, merging. For many of these mergers, $t_{\text{mrg}}$ is much larger than in the previously discussed cases in which the BH merged before losing its original companion. The $t_{\text{mrg}}$ distributions are dominated by mergers that occurred after the BH became single. So, in addition to an increased average BH–NS merger rate, these runs also predict that the bulk of these mergers will occur at later times. However, many of the mergers, especially those in cluster D, still occurred within 3 Gyr of the start of the simulations. This suggests, again, that if globular cluster BHs are of similar mass to those found in the Galaxy and the large post-merger kicks predicted by numerical relativity are correct, most of the BHs that survived the initial phase of self-interaction will be ejected from the cluster after merging with a NS well before the current epoch.
Figure 2.3 Distribution of the merger times for BH+NS binaries in clusters D (left panel), C (center panel), and B (right panel) with $M_{\text{BH}} = 7 \, M_\odot$. These merger times include the time taken to form the BH+NS system through binary–single star interactions. The solid histogram is based on runs that extended only until the initial BH binary either merged or was disrupted; it shows the distribution of $t_{\text{merg}}$ when a merger occurred. The dotted histograms show the merger time distribution for runs that allowed BHs that lost their binary companions to exchange into a new binary. In these runs we have used the FIR binary population with $f_b = 0.5$. With this change, a larger number of runs produced a merging BH+NS binary, and the distribution shifts towards longer merger times. The larger merger times are due to the significant amount of time it takes for the single BH to exchange back into a binary. In all of the runs shown here, the BH is ejected from the cluster after the merger.
2.3.2 Multiple merger scenario

So far, we have considered scenarios in which the BH receives a post-merger kick that exceeds the escape velocity of the globular cluster. Under this constraint, each BH can undergo at most one merger. If globular cluster BHs are more massive than those found in field, then it is possible for the BH to remain bound to the cluster after the merger, regain a NS companion, and merge again. In this section we will discuss models that assumed $M_{BH} = 35 \, M_\odot$. For small values of the mass ratio $q$, the magnitude of the recoil velocity is approximately proportional to $q^2$. With a BH mass of $35 \, M_\odot$, $q$ is reduced to 0.04 and the recoil velocity becomes comparable to the globular cluster escape velocity. Thus, we explored the case in which the suppressed kick velocity helped but did not ensure that the post-merger BH would be retained by the cluster. The fraction of post-merger BHs retained in simulations using each background cluster depended on that cluster’s escape velocity $v_{esc} \approx \sqrt{2W} \bar{m}$. Among all the models that varied the binary population and binary fraction, in simulations using clusters A, B, C, and D, 50%, 23%, 20%, and 10% of the BHs that underwent a merger were ejected, respectively\(^1\). However, in these simulations many of the BHs remained bound to the cluster after merging with a NS. A fraction of these single BHs then gained new NS companions and underwent multiple mergers.

The number of times a BH could merge with a NS depended on how quickly the single BH could exchange into a binary after each merger. While the post merger kick was not always large enough to remove the BH from the cluster, the BH was expelled from the core after most mergers. Before it was able to exchange into a binary, it needed to sink back into the core where the interaction rates were highest. This process occurred rapidly in all of the simulations. The mass segregation time scale is proportional to $\bar{m}/M_{BH}$, where $\bar{m}$ is the average stellar mass in the cluster. The BHs in these models were $35 \, M_\odot$, which gave $\bar{m}/M_{BH}$ between 0.014–0.027. Typically, a kicked BH would return to the core on a time scale $O(10^7)$ yrs. As can be seen from Table 2.2, this time scale is short compared to the total time it takes the single BH to exchange into a binary. There was, therefore, little difference between the $t_{ex}$ distributions for BHs that became single because of a BH–NS merger and those that lost their companions after a binary–single interaction. We make no distinction between these distributions in the discussion below.

Figure 2.4 shows how the distribution of $t_{ex}$ is affected by the mass of the BH, the structural parameters of the globular cluster, and the binary fraction. All of the simulations shown here used the FIR binary population. Changing the mass of the BH had little impact on both the distribution of $t_{ex}$ and its average value. As expected, $\bar{t}_{ex}$ decreases with increasing cluster concentration. Increasing the binary fraction produced slightly narrower $t_{ex}$ distributions with peaks shifted to shorter times. The

\(^1\)Note that only two BH–NS mergers occurred in cluster A, so the ejection fraction derived from the models is smaller than expected.
t_{ex} distributions for simulations using the OBS binary population with \( f_b = 0.2 \) are shown in Figure 2.5. For each cluster model, the distributions tend towards longer \( t_{ex} \) when compared to the models using the FIR population shown in Figure 2.4. This is only a consequence of the larger \( f_b \) allowed in models that used the FIR population, because the bulk of the FIR binary population is unobservable. Models that used the FIR population with \( f_b = 0.2 \) had similar or slightly longer \( t_{ex} \) than models that used the OBS population at the same \( f_b \). The difference in composition between the FIR and OBS populations, on its own, did not impact the amount of time it takes a single BH to exchange into a binary.

Conversely, the composition of the OPT binary population directly impacted \( t_{ex} \) and \( t_{mrg} \). Models in cluster D that used the OPT population resulted in an average exchange time that was nearly a factor of two smaller than \( t_{ex} \) in models that used the OBS or FIR populations with the same \( f_b \). Furthermore, with the OPT population, in 27\% of the encounters that resulted in a single BH exchanging into a binary, the BH exchanged directly into a binary with a NS. Many of these highly eccentric BH+NS binaries quickly merged, and such mergers accounted for 10\% of all mergers in these runs. In models with the other binary populations, single BHs exchanging directly into a binary with a NS accounted for less than 0.3\% of all mergers.

In fact, the only simulation of cluster B that produced a run with multiple BH-NS mergers used the OPT population. In all other simulations that used cluster B, the post-merger BHs did not have enough time to exchange into a new binary, gain a NS companion, and merge. Figure 2.6 illustrates how the assumed binary population affects the merger rate. Each model shown used \( f_b = 0.2 \), cluster D, and \( M_{BH} = 35 \, M_\odot \). As was the case with the \( t_{ex} \) distributions, the \( t_{mrg} \) distributions for models using the FIR and OBS populations are similar. In each, the distribution of initial merger times is bimodal. The first peak corresponds to mergers that occurred before the initial BH binary was destroyed in a binary–single encounter. The second peak, at \( t_{mrg} > 3 \) Gyr is due to BHs that lost their companion in a three-body interaction and had to re-exchange into a binary system. The model using the OPT population does not display this bimodality. The OPT population is tuned to interact with the single BH, so BHs that lose their companions do not remain single for long. In models using all three populations, the distribution of subsequent merger times peaks in the largest \( t_{mrg} \) bin. This is expected because such mergers cannot occur until after the BH has undergone its first merger. Importantly, in addition to increasing the number of mergers that occurred, and the corresponding merger rate, \( R_{GC} \), the multiple merger scenario shifted the peak of all \( t_{mrg} \) distributions to much later times. This suggests that such mergers could be observed during the current epoch.

As with the single merger scenario, the more concentrated cluster models produced larger merger rates. The \( t_{mrg} \) distribution for models in clusters B, C, and D are shown in Figure 2.7. Each of the models shown used the FIR binary population with \( f_b = 0.75 \) and \( M_{BH} = 35 \, M_\odot \). Subsequent mergers dominated the \( t_{mrg} \) distribution at
Figure 2.4 Distribution of $t_{\text{ex}}$, the time for a single BH to exchange into a binary. Each panel shows the distribution for models using the FIR binary population in cluster B (solid), C (dashed), and D (dotted). Distributions are shown for simulations with $M_{\text{BH}} = 7 M_\odot$ and $f_b = 0.5$ (left), $M_{\text{BH}} = 35 M_\odot$ and $f_b = 0.5$ (center), and $M_{\text{BH}} = 35 M_\odot$ and $f_b = 0.75$ (right). Increasing $M_{\text{BH}}$ does not have a large effect on the distribution of $t_{\text{ex}}$. Increasing $f_b$, on the other hand, results in a narrower distribution that peaks at a lower $t_{\text{ex}}$. Note that the distributions shown in the center and right panels include $t_{\text{ex}}$ for BHs that became single after merging with a NS, while all such systems were promptly ejected from the cluster for the models shown in the left panel.
Figure 2.5 Distribution of $t_{ex}$ for models in cluster B (solid), C (dashed), and D (dotted). In these simulations we used the OBS binary population, $M_{BH} = 35 \, M_\odot$, and $f_b = 0.2$. These simulations tend towards much longer $t_{ex}$ despite having a similar observable binary population to the simulations shown in the right panel of Figure 2.4.

$t \gtrsim 1$ Gyr in cluster D. In the cluster C models, most initial mergers did not happen until $t \gtrsim 1$ Gyr, so the subsequent mergers were not a significant contribution to the total merger rate until late times. In the cluster B model, none of the BHs underwent multiple mergers, primarily because many of the initial mergers did not occur until the BH had evolved in the cluster for 6 Gyr.

Clusters with the highest concentrations and binary fractions permitted the largest number of mergers that a single BH could take part in. The maximum number of BH–NS mergers in a single run, $N_{\text{max}}$, is listed for each model in Table 2.2. In a run in cluster D using the FIR binary population and $f_b = 0.75$, and in a run in the same cluster model using the OPT binary population with $f_b = 0.2$, a single BH merged with 9 NS. These models also had the largest proportion of runs in which BHs underwent multiple mergers. In each, 73% of the BHs that underwent a single merger, underwent at least a second merger.
Figure 2.6 Merger time distributions for simulations with $f_b = 0.2$ using the FIR (left), OPT (center), and OBS (right) binary population. Each panel shows the distribution of $t_{\text{mrg}}$ for the initial merger during a run (dashed), subsequent mergers during that run (dotted), and the combination of the two (solid). The initial $t_{\text{mrg}}$ distributions for the simulations using the FIR and OBS populations show a clear bimodality. The peaks above $\log t_{\text{mrg}} = 9.5$ in these distributions are due to BHs that lost their binary companions before merging with a NS. The bimodality is amplified in the total merger time distributions for these runs by the distinct distributions for initial and subsequent mergers. The $t_{\text{mrg}}$ distribution in the OPT case is completely dominated by subsequent mergers.
Figure 2.7 Distribution of $t_{\text{mg}}$ for simulations using the FIR binary population with $f_b = 0.75$ in cluster D (left), C (center), and B (right). Each panel shows the distribution of $t_{\text{mg}}$ for the initial merger during a run (dashed), subsequent mergers during that run (dotted), and the combination of the two (solid). Subsequent mergers dominate in cluster D. In cluster B, there are no subsequent mergers.
2.4 Discussion

We have shown that binary–single star interactions in globular clusters can produce BH+NS binaries that will merge within a Hubble time. The rate at which the mergers occur is sensitive to the globular cluster structural parameters, the mass of the BH, and properties of the globular cluster’s binary population. If the BHs in globular clusters follow a similar mass distribution to those found in the disk of the Galaxy, then it is unlikely that any high concentration clusters could retain a BH until the current epoch. In such clusters, most BHs that managed to survive the early period of self-interaction, described in previous studies, would gain a NS companion, undergo a gravitational radiation driven merger, and be ejected from the cluster within $\sim 3$ Gyr. If, on the other had, globular cluster BHs are more massive than typical field BHs, many clusters could retain a BH and that BH could continue to merge with NSs until late times.

2.4.1 Uncertainties

The BH–NS merger rates calculated here depend on the uncertain distributions of several quantities. The distributions of some parameters are better constrained than others (e.g., $a$ Abt 1983 and $e$ Duquennoy & Mayor 1991 distributions, NS retention fraction Pfahl et al. 2002, and the structure of extragalactic globular clusters Strader et al. 2011). However, both the globular cluster BH mass function and the nature of globular cluster binary populations strongly influence the predicted rates and are not well constrained by observations. We have sampled several plausible scenarios for each, and the results suggest that the observed merger rate could be used to probe both the BH mass distribution and binary populations present in globular clusters.

Some of the simplifying assumptions that we made in the models also impacted the predicted merger rates. In our calculation of the probability that a single BH would exchange into a new binary, we related the densities of single and binary stars using an approximation that diverges for large binary fractions, $f_b/(1 - f_b)$. In the most extreme case considered here, $f_b = 0.75$, this term increases the encounter probability by a factor of 3. Any overestimate due to this term is of the same magnitude as errors introduced by other approximations made in our calculation. In addition to this approximation, in the calculations described above we used a relationship between the rates of specific exchange encounters to scale the total encounter rate. At the very least, this resulted in longer $t_{ex}$ because we only integrated the three-body interaction if the probability for a specific exchange $[(1, 2) + \text{BH} \rightarrow (1, \text{BH}) + 2]$ was large enough, but we allowed for any outcome of the three-body interaction.

We have run two additional simulations to explore how these assumptions affected our calculations. In these tests, we evolved 500 binaries in cluster D using the FIR binary population with $f_b = 0.5$. In one simulation, marked Retry, we scaled the
Table 2.3. Probability Scaling Tests

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<th>$R_{GC}$ (yr$^{-1}$)</th>
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</tbody>
</table>

encounter probability using equation (2.7), as before. However, in these runs if the probability of exchange was larger than the random number, we re-ran the three-body integrator until the desired exchange, $[(1, 2) + \text{BH} \rightarrow (1, \text{BH}) + 2]$, occurred. If the integration resulted in a different outcome (e.g., flyby or an exchange creating a binary (BH,2)), we changed the orbital phase of the initial binary and the orientation of the encounter, and then simulated the interaction again. In another simulation, marked Total, we scaled $P_{(1, \text{BH})+2}$ by the ratio of the total encounter cross sections, not just the single, desired exchange cross section. In the mass ratios considered here ($\sim 1 : 1 : 35$), the value of $\tilde{\sigma}(X)$ for the BH to exchange into a binary with the primary is roughly equal to $\tilde{\sigma}(X)$ for the BH to exchange into a binary with the secondary, and roughly half the value $\tilde{\sigma}(X)$ for a flyby (Sigurdsson & Phinney 1993). This results in a factor of 2-3 increase in the encounter probability over the probabilities used in the previous runs. For the Total runs, as in the runs presented in section 2.3.2, if an encounter occurred we accepted any outcome of the three-body integration.

The results of these tests are listed in Table 2.3. Both methods for correcting the probability scaling increased the exchange and merger rates. Furthermore, making the correction either of two ways gave consistent results, illustrating that the probability scaling argument used in our models is valid. The maximum number of mergers in each of the test runs increased from 7 to 8. The fact that this quantity did not change by a factor of two, as $\bar{t}_{ex}$ did, suggests that the time it takes a single BH to exchange back into a binary may not be the rate limiting step in the BH-NS merger process. Finally, these runs suggest that the factor of a few uncertainties in the exchange probability resulting from the approximations we made can lead to uncertainties of a similar magnitude in the BH–NS merger rates. As such, the rates given in Table 2.2 should be treated as having a factor of $\sim 2$ uncertainty.

In addition to uncertainties in the components of the models, physical processes and properties of globular clusters that we have omitted from the models could alter the BH–NS merger rates. We have not explicitly included an increase in the binary density at the cluster center. Since, next to the BH, NS-containing binaries are the
heaviest objects in the cluster they would likely be concentrated near the center. The situation could be similar to the models presented here that made use of the OPT binary population, which contained a large fraction of NS binaries. These models resulted in higher merger rates than models that used OBS or FIR binary populations with the same binary fraction. On the other hand, a high binary concentration could result in significant heating, expanding the radial profile expected for a population of thermalized binaries. A high concentration of binaries would also result in binary-binary interactions, which were not included in our models. Some binary–binary encounters could enhance the merger rate. For example, the interaction of a BH+MS binary with a NS+MS binary could produce a BH+NS binary. However, there is a high likelihood that a binary-binary encounter will result in the disruption of a binary. Because the rate of binary-binary encounters increases with $f_b$, this limits the binary fraction. So, binary-binary interactions could cause a reduction in the BH–NS merger rate, compared to the rates predicted here, by limiting $f_b$ and making it more difficult for a single BH to find a new binary companion. Detailed models that include binary-binary interactions are necessary to determine which process dominates.

### 2.4.2 LIGO detection rate

We estimated the LIGO detection rate following Belczynski et al. (2007) and Banerjee et al. (2010). We computed the detection rate with

$$ R_{\text{LIGO}} = \frac{4\pi}{3} \rho_{\text{GC}} \left( D_0 \left( \frac{m_{\text{ch}}}{m_{\text{ch,NSNS}}} \right)^{5/6} \right)^3 R_{\text{GC}}, $$

where $\rho_{\text{GC}}$ is the space density of globular clusters, $m_{\text{ch}}$ is the chirp mass of the binary ($m_{\text{ch}} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$), and $m_{\text{ch,NSNS}}$ is the chirp mass of a binary with two $1.4$ M$\odot$ NSs. $D_0$ is the maximum distance from which the gravitational waves from a merging NS+NS binary can be detected, and has a value of 18 and 300 Mpc for LIGO and Advanced LIGO (aLIGO), respectively. Using the galaxy luminosity function parameters estimated in Croton et al. (2005) and the specific frequency of globular clusters per $8.5 \times 10^7$ L$\odot$ of galaxy luminosity presented in Brodie & Strader (2006), we calculated $\rho_{\text{GC}} = 8$ Mpc$^{-3}$. Our calculations above showed that $R_{\text{GC}}$ is strongly dependent on the cluster mass. To compute a merger rate that is averaged over all globular clusters, $R_{\text{GC}}$, we used the globular cluster mass function presented in McLaughlin & Pudritz (1996).

Although we have shown that the bulk of such mergers occur quickly, and therefore at high redshift, we compute the aLIGO detection rate for models with 7 M$\odot$ BHs for illustrative purposes. For these BH+NS binaries, $m_{\text{ch}} = 2.6$ M$\odot$. For the models that do not allow single BHs, we found $R_{\text{GC}} = 3.4 \times 10^{-12}$ yr$^{-1}$ GC$^{-1}$. To make a conservative estimate for the aLIGO detection rate, we assumed that 10% of clusters
retain one BH and arrived at the rate of $2 \times 10^{-3} \text{ yr}^{-1}$. Using instead the rates from runs that did allow single BHs to exchange back into binaries, we found an aLIGO detection rate of $10^{-2} \text{ yr}^{-1}$. Even if mergers between 7 $M_\odot$ BHs and NS were occurring in globular clusters at the current epoch, the event rates are so low that they would not be detected by aLIGO. For example, the rates calculated for cluster models B and C may be applicable to massive intermediate age clusters. With ages of 1-5 Gyr, these clusters are young enough that any retained BHs would only now be merging with a NS. However, the space density of massive intermediate age clusters clusters is even lower than that of globular clusters, so the conclusion remains the same. If globular cluster BHs follow the same mass distribution as field BHs, BH–NS mergers resulting from binary-single star interactions in clusters cannot be detected by aLIGO.

The detection rate could be much higher than this conservative estimate if globular cluster BHs are more massive than typical field BHs. Higher mass BHs enhance the detection rate in two ways. First, more massive binaries can be detected out to larger distances. Second, since the post-merger kick is suppressed for high mass ratio inspirals, the BH remains in the cluster and can undergo multiple mergers with NSs. Models with the FIR binary population and $f_b = 0.75$ produced the largest averaged merger rate, $\bar{R}_{GC} = 4.2 \times 10^{-11} \text{ yr}^{-1} \text{ GC}^{-1}$. Assuming that 10% of globular clusters retain a 35 $M_\odot$ BH, this corresponds to a aLIGO detection rate of 0.14 yr$^{-1}$. If 50% of globular cluster retain a BH the rate would approach 0.7 yr$^{-1}$. The FIR binary population with $f_b = 0.75$ represents an optimistic, yet physically motivated population. On the low merger rate side, models with the OBS population and $f_b = 0.05$ would yield an aLIGO detection rate of 0.04 yr$^{-1}$ if 10% of clusters retained a 35 $M_\odot$ BH. Although the aLIGO detection rates are not promising, the predicted merger rates are large enough that BH–NS mergers in globular clusters will be seen in the next generation of gravitational wave detectors (e.g., the Einstein Telescope Punturo et al. 2010).

2.4.3 Comparison with previous studies

Previous studies have estimated the BH–NS merger rate in the field and in globular clusters. The field rates are typically calculated using binary population synthesis models that are subject to uncertainties in common envelope evolution, wind mass loss from high mass stars, and natal kicks, resulting in a wide range of predicted aLIGO detection rates, $0.68 - 42.8 \text{ yr}^{-1}$ (Sipior & Sigurdsson 2002; Pfahl et al. 2005; Belczynski et al. 2007; O’Shaughnessy et al. 2010; Belczynski et al. 2010b). However, a case study of the likely BH+NS binary progenitor Cyg X-1 presented in Belczynski et al. (2011) avoids many of the poorly constrained components of binary population synthesis and predicts much lower field BH–NS merger detection rates in the range $(0.4 - 2.8) \times 10^{-2} \text{ yr}^{-1}$.
Our results are consistent with previous studies of the globular cluster merger rate that conclude the cluster merger rate for both NS+NS and BH+NS binaries is much lower than the field rate for corresponding pessimistic, realistic, or optimistic predictions (Phinney 1991; Grindlay et al. 2006; Sadowski et al. 2008). However, if the uncertainties in the field merger rate push it towards the pessimistic estimates, which is suggested by the models presented in Belczynski et al. (2011) that circumvent many of these uncertainties, and multiple mergers increase the globular cluster rate, then the BH–NS mergers detected by aLIGO might be dominated by, or consist entirely of, systems that form through multibody interactions in globular clusters.
Chapter 3

Dynamically Formed Black Hole+Millisecond Pulsar Binaries in Globular Clusters

3.1 Introduction

Radio pulsars in binary systems provide constraints on the processes that drive binary stellar evolution and unparalleled tests of general relativity in the strong-field regime. In most cases, the pulsars in these binaries are “recycled.” That is, the neutron star (NS) has been spun-up by accreting mass and angular momentum from its companion (Alpar et al. 1982). Compared to “normal” pulsars, recycled pulsars exhibit greater stability and have much shorter spin periods ($P_S \lesssim 100$ ms), both of which facilitate high precession measurements of the pulse arrival times (Lorimer 2008). The outcomes of binary evolution can be probed by using the recycled pulsar in such a system as a stable clock to precisely determine the binary’s Keplerian orbital parameters and the properties of its component stars. If the recycled pulsar’s companion is another neutron star, it is possible to measure post-Keplerian orbital parameters in a model-independent fashion and then compare these measurements with the predictions of various theories of gravity (Stairs 2004). The post-Keplerian parameters measured in the double pulsar binary PSR J0737-3039 offer the best test of gravity in the strong field limit, to date, and are in excellent agreement with the predictions of general relativity (Kramer et al. 2006). An even better test of general relativity may be possible by applying these techniques to a binary comprising a black hole (BH) and a recycled pulsar, however such a system has yet to be discovered.

It is possible to produce a BH+recycled pulsar binary through standard evolutionary processes in an isolated, high-mass binary. The scenario requires that the primary (the initially more massive member of the binary, which evolves faster than its companion) produces a NS at the end of its lifetime and that the secondary produces a BH. This can occur if the primary transfers enough material to its companion to drive the companion’s mass above the threshold for BH production. The NS created by the primary could then be recycled by accreting material from the companion before it evolves to become a BH. Under the assumption that these recycled pulsars would
have lifetimes longer than $10^{10}$ yr, Narayan et al. (1991) placed an empirical upper limit on the formation rate of such BH+recycled pulsar binaries of $10^{-6}$ yr$^{-1}$ within the Galaxy. Population synthesis models by Lipunov et al. (1994) found a comparable formation rate, while similar studies by Sipior & Sigurdsson (2002), Voss & Tauris (2003), Sipior et al. (2004), and Pfahl et al. (2005) favored lower BH+recycled pulsar binary formation rates of $\sim 10^{-7}$ yr$^{-1}$. Additionally, Pfahl et al. (2005) argued that the NSs in these systems would only be mildly recycled. Due to the rapid evolution of its massive companion, the NS, accreting at the Eddington limit, would only have time to accrete $10^{-3} - 10^{-4}$ $M_\odot$ of material before the secondary collapsed into a BH. These mildly recycled pulsars would only have lifetimes of $10^8$ yr. With these revised formation rates and lifetimes the number of BH+recycled pulsar binaries expected to exist in the Milky Way is only $\sim 10$.

As we have discussed in chapters 1 and 2, in a globular cluster, a BH+recycled pulsar binary need not form through the evolution of a primordial binary. The high stellar density in globular clusters leads to dynamical encounters between cluster members, which open a wide array of evolutionary pathways that are inaccessible to isolated binaries. For example, a single NS in a globular cluster can gain a companion by exchanging into a primordial binary during a three-body interaction. Subsequent evolution of these newly created binaries can result in the NS being spun-up into a recycled, millisecond pulsar (MSP; Hills 1976; Sigurdsson & Phinney 1995; Ivanova et al. 2008). These encounters are evidenced by the enhanced formation rates of low mass X-ray binaries and their progeny, MSPs, observed in globular clusters (e.g. Katz 1975; Verbunt & Hut 1987; Pooley et al. 2003; Camilo & Rasio 2005). In fact, three-body interactions in globular clusters produce MSPs so efficiently that nearly two-thirds of all known MSPs are found in globular clusters (e.g., Manchester et al. 1991; Ransom et al. 2005). Any BHs present in the cluster could acquire a MSP companion through similar interactions (Sigurdsson 2003). However, uncertainties in the size and nature of the BH population present in globular clusters complicate investigations of this formation channel.

Kulkarni et al. (1993) and Sigurdsson & Hernquist (1993) argued that the stellar mass BHs formed in a globular cluster would rapidly sink to the center of the cluster and eject one another in a phase of intense self-interaction. The frenzy of ejections results in a substantial depilation of the cluster’s stellar mass BH population during the first Gyr of evolution. The fact that a firm BH candidate had not been identified in a globular cluster during decades of observational study was inline with this theoretical picture. Given the meager BH populations implied by these investigations, the dynamical formation of BH+MSP binaries in globular clusters has received little attention. After all, this channel closes if there is not a population of BHs present in globular clusters. Nevertheless, the production of BH+MSP binaries through multi-body interactions has been considered in dense stellar environments, analogous to globular clusters, that are likely to harbor BHs. Faucher-Giguère & Loeb (2011)
showed that a few dynamically formed BH+MSP binaries should be present in the Galactic Center, where a cluster of $\sim 10^4$ stellar mass BH is expected to exist. This result indicates that BH+MSP binaries could be produced in globular clusters if the clusters retained some of their stellar mass BHs.

Recent observational efforts have shown that there are BHs present in some globular clusters, prompting a renewed interest in the nature of globular cluster BH populations. A number of promising BH candidates have been discovered in X-ray observations of extragalactic globular clusters (Maccarone et al. 2007, 2011; Irwin et al. 2010; Shih et al. 2010; Brassington et al. 2010, 2012; Roberts et al. 2012; Barnard et al. 2012). Furthermore, three BH candidates have been identified in deep radio observations of Milky Way globular clusters; two candidates reside in M22 and one candidate is in M62 (Strader et al. 2012a; Chomiuk et al. 2013). There is also a growing body of theoretical work suggesting that it may be possible for globular clusters to retain a substantial fraction of their stellar mass BH populations, under certain circumstances (Mackey et al. 2008; Sippel & Hurley 2013; Morscher et al. 2013). Motivated by these new results, we set out to explore how efficiently three-body exchanges produce BH+MSP binaries in globular clusters.

It has also been suggested that globular clusters may harbor IMBHs. Previous studies have considered the consequences of interactions between MSPs and these IMBHs. The encounters could result in a MSP being significantly displaced from the globular cluster core (Colpi et al. 2003), produce a IMBH+MSP binary (Devecchi et al. 2007), or populate the Milky Way halo with several high velocity MSPs (Sesana et al. 2012). We will not include IMBHs in the models presented here, and instead focus on stellar mass BHs.

This chapter is organized as follows. We describe the features of our models and the motivate the range of initial conditions that our simulations explore in section 3.2. In section 3.3 we discuss the orbital parameters of the BH+MSP binaries produced in our models. We discuss the size of the BH+MSP binary population and the possibility of detecting such a binary in section 3.4. Finally, in section 3.5, we summarize and discuss our findings.

### 3.2 Method

Several characteristics of a globular cluster can influence the nature of the BH+MSP binaries produced within it. The observed diversity in globular cluster structure combined with uncertainties in the clusters’ binary and BH populations yield a vast range of plausible starting points for our models. Exploring this parameter space efficiently necessitates the use of fast, approximate methods. The goals of the parameter exploration presented here are to motivate and inform the observations required to constrain the parameter space, and identify interesting regions in this space for followup with detailed N-body or Monte Carlo calculations.
We simulated the dynamical formation of BH+MSP binaries by evolving a variety of BH binaries in fixed background globular cluster models using the method described in section 2.2. In fact, many of the simulations discussed in this chapter were also discussed in chapter 2 in the context of BH–NS mergers. In all of the simulations presented in chapter 2, we assumed that globular clusters only retained one stellar mass BH. In this chapter we will present additional simulations that considered larger BH populations. We have also improved the treatment of single BHs (BHs that do not have a binary companion) in the new models discussed in this chapter. Specifically, these additional simulations used the “Retry” method, described in section 2.4.1, to model the process of a single BH exchanging into a background binary. In all other respects, these new models follow the procedure outlined in section 2.2.

As we have discussed in the preceding chapters, the BH populations in globular clusters are poorly constrained. Therefore, we considered a range of BH masses, $M_{BH}$, and population sizes. By analogy to the BHs found in the Galaxy, we used $M_{BH} = 7 M_\odot$ (Ozel et al. 2010; Farr et al. 2011). A second value of $M_{BH} = 15 M_\odot$ was motivated by Strader et al. (2012a), who used mass segregation arguments to estimate the masses of the BH candidates in M22 to be $\sim 15 M_\odot$. We also used $M_{BH} = 35 M_\odot$ in some of our runs, because observations of the BH candidates in extragalactic globular cluster indicate that these BHs may have masses $\gtrsim 30 M_\odot$ (Maccarone et al. 2007; Barnard et al. 2012; Irwin et al. 2010; Clausen et al. 2012). The use of such massive BHs was also motivated by theoretical studies that have shown that more massive BHs are produced in low metallicity environments, and that massive BHs are more likely to be retained by globular clusters (Belczynski et al. 2010a; Miller & Hamilton 2002b). Again, we controlled the number of BHs present in each simulation by truncating the high-mass end of the initial mass function. In a subset of our runs, we assumed that nearly every BH formed in the cluster was ejected, as described in chapter 1, leaving a lone BH. In other runs, we allowed the clusters to retain several BHs, with the number of BHs in the range $N_{BH} = 7 - 191$. Including these large BH populations in our static background globular clusters, which were assumed to be in mass-segregated equilibrium, had a substantial impact on the mass and structure of these clusters. To facilitate comparisons between models with different values of $N_{BH}$, we adjusted the free parameters in these models, slightly, to maintain similar values of $n_c$, $\bar{v}_m$, $c_{GC}$, and $M_{GC}$ when $N_{BH}$ was varied. The ranges of these parameters are given in Table 3.1.

### 3.3 BH+MSP Binary Orbital Parameters

The results of our simulations are summarized in Table 3.2. The first column lists an identification number for each simulation. The next six columns describe the initial conditions used for the simulations, noting the background cluster, binary population, binary fraction ($f_b$), NS retention fraction ($f_{ret}$), the BH mass ($M_{BH}$),
Table 3.1. Background Globular Cluster Model Parameter Ranges

<table>
<thead>
<tr>
<th>Cluster Name</th>
<th>$n_c$ ($10^5$ pc$^{-3}$)</th>
<th>$\bar{v}_m$ (km s$^{-1}$)</th>
<th>$M_{GC}$ ($10^5$ M$_\odot$)</th>
<th>$c_{GC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.1</td>
<td>6</td>
<td>1.0 – 1.2</td>
<td>1.20 – 1.35</td>
</tr>
<tr>
<td>B</td>
<td>1.0 – 1.2</td>
<td>8.1 – 10</td>
<td>5.2 – 6.2</td>
<td>1.71 – 1.79</td>
</tr>
<tr>
<td>C</td>
<td>5.0 – 5.2</td>
<td>10 – 11</td>
<td>7.2 – 8.5</td>
<td>1.93 – 2.02</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>11-13</td>
<td>11 – 14</td>
<td>2.06 – 2.15</td>
</tr>
</tbody>
</table>

and the number of BHs ($N_{BH}$). The final five columns list the number of BH+NS binaries produced in that simulation ($N_{BH+NS}$), the median orbital period ($P_B$) of these binaries, the standard deviation of the orbital period distribution, the fraction of time that a BH+NS binary exists ($\tau_{BH+NS}$), and the probability that a BH+NS is present in the cluster ($p_{BH+NS}$). We note that our code did not track whether or not a NS had been recycled into a MSP. Thus, in the discussion that follows we will examine the nature of all of the BH+NS binaries produced in our simulations. We will consider the fraction of NSs that are MSPs in section 3.4.

In our simulations, there were two processes that drove the evolution of a BH+NS binary's orbital parameters. Encounters between the binary and background stars changed the semi-major axis ($a$) and eccentricity ($e$) of the binary impulsively. In most cases, an encounter resulted in the binary becoming more tightly bound or “hardened.” The emission of gravitational radiation also modified the BH+NS binaries’s orbital parameters, driving $a$ and $e$ towards zero. Since the orbital parameters of the binaries were constantly changing, we resampled the simulation output in even time intervals to ensure that each orbital configuration was properly weighted. We chose a resampling time step of $10^7$ yr. We checked that our choice of $10^7$ yr intervals did not bias the resampled orbital parameter distributions by repeating the analysis of a subset of our simulations with finer ($10^5$ yr) time resolution. A coarser resampling, with steps of $10^8$ yr, failed to capture the wings of the semi-major axis distribution, where rapid evolution occurs. Examples of the resampled data from three representative simulations are shown in Figure 3.1. The figure shows the joint distribution for the eccentricity and semi-major axis of the BH+NS binaries produced in simulations using background clusters B, C, and D.

### 3.3.1 Eccentricity distribution

As expected, in nearly all of our simulations, the eccentricity distribution of the BH+NS binaries was roughly thermal for $e \lesssim 0.9$. At higher eccentricity the distri-
Figure 3.1 Joint $e - a$ distribution for BH+NS binaries produced in simulations 42 (left panel), 26 (center panel), and 10 (right panel). The eccentricity distribution is similar in all of these simulations. The semi-major axis distribution shifts to larger $a$ as the cluster density decreases from $n_0 = 10^6$ in the left panel to $n_0 = 10^5$ in the right panel.
bution flattened out and turned over. That is to say, there were fewer binaries with \( e \gtrsim 0.9 \) than expected given the \( f(e) = 2e \) distribution. This is because the emission of gravitational radiation acted to quickly circularize such high eccentricity systems. Given these results, we expect that the mean \( e \) of dynamically formed BH+NS binaries in globular clusters will be in the range 0.6–0.7. The only simulations that did not result in a thermal eccentricity distribution were those run in background cluster A. As can be seen in Table 3.2, very few BH+NS binaries were produced in this low density cluster. We were, therefore, unable to study the eccentricity distributions in these poorly sampled cases.

### 3.3.2 Orbital separation distribution

Of all of the parameters varied in our study, the background cluster’s structural properties had the strongest impact on the semi-major axes of the BH+NS binaries. For this reason, we will describe the distributions of semi-major axes and orbital periods cluster-by-cluster. Once we have described the results for each cluster, we will investigate the origin of the observed trends. Globular cluster A was the least massive cluster considered in our study. Furthermore, because of its low density, this cluster also had the lowest encounter rate. Only 45 of the \( 1.8 \times 10^4 \) BHs that we evolved in this cluster gained a NS companion. The semi-major axes of these rare BH+NS binaries spanned a range of nearly two orders of magnitude, from 1.2–113 au. The BH+NS binaries produced in this cluster were extremely wide because neither of the hardening mechanisms described above were effective. The binaries underwent very few encounters because of the cluster’s low density, and energy losses through gravitational radiation were negligible at such large orbital separations. Accordingly, we predict that any BH+MSP binaries present in low density clusters are likely to have orbital periods of several decades.

The dynamically formed BH+NS binaries in globular cluster B had significantly smaller orbital separations. In this higher density cluster, frequent encounters hardened the binaries to small enough \( a \) that gravitational radiation effects became important. Figure 3.3 shows the cumulative distributions of \( a \) and \( P_B \) amongst present day BH+NS binaries for all of our simulations in cluster B (i.e., simulations 10-25). We constructed a present day population by only selecting binaries that exist at \( t > 8 \times 10^9 \) yr. The median values of \( a \) in these distributions fell in the range 0.42–1.6 au. It is evident from Figure 3.3 that the orbital separation is related to \( M_{BH} \). Simulations with higher mass BHs produced tighter BH+NS binaries. This trend is amplified in the orbital period distributions because the semi-major axis and orbital period are linked by an additional factor of \( \sim M_{BH}^{-1/2} \), specifically \( P_B = \sqrt{a^3/(M_{BH} + M_{NS})} \). At a given value of \( M_{BH} \), the orbital period distributions are fairly similar to one another, despite significantly different assumptions about the number of BHs in the clusters and the clusters’ binary populations. This suggests that these properties do not strongly im-
Figure 3.2 Cumulative distributions of the semi-major axis (left) and orbital period (right) for a subset of the BH+NS binaries produced in each of the simulations in cluster B. Each curve shows the distribution of \( a \) or \( P_B \) in a particular simulation. The distributions shown only include systems that exist at \( t > 8 \times 10^9 \) yr, so they correspond to the present day. The color of each curve denotes the mass of the BH(s) used in a particular simulation, with \( 35 \, M_\odot \), \( 15 \, M_\odot \), and \( 7 \, M_\odot \) BHs denoted by red, green, and blue, respectively. Note the clear relationship between \( M_{BH} \) and orbital separation: simulations with more massive BHs produced BH+NS binaries with smaller semi-major axes. This is the opposite of the trend seen in higher density clusters. Many of these distribution functions appear to be under sampled.

impact the orbital periods of the BH+NS binaries. The median BH+NS binary orbital periods in all of the simulations in cluster B fell between 16 days and 260 days. The standard deviation of the \( P_B \) distribution within a particular simulation was much wider (see Table 3.2).

For simulations in cluster C, the BH+NS binary orbital separations were smaller still. The cumulative distributions of the semi-major axis and orbital periods for the BH+NS binaries formed in this cluster are shown in Figure 3.3. The median values of \( a \) in these simulations ranged between 0.15 au and 0.43 au. Simulation 32 resulted in the largest median \( a \). In this simulation, only 167 BH+NS were produced, so it is likely that the “true” semi-major axis distribution was not well sampled. From Figure 3.3 it is clear that the cumulative \( a \) and \( P_B \) distributions for simulation 32 deviate from distributions seen in other simulations. If we exclude simulation 32 from our analysis, the range of median semi-major axes reduces to 0.15–0.24 au. This range is much narrower than that observed in our simulations in cluster B. It seems that semi-major axes of the BH+NS binaries produced in cluster C only depend
Figure 3.3 Cumulative distributions of the semi-major axis (left) and orbital period (right) for a subset of the BH+NS binaries produced in each of the simulations in cluster C. Each curve shows the distribution of $a$ or $P_B$ in a particular simulation. The distributions shown only include systems that exist at $t > 8 \times 10^9$ yr, so they correspond to the present day. The color of each curve denotes the mass of the BH(s) used in a particular simulation, with 35 M$_\odot$, 15 M$_\odot$, and 7 M$_\odot$ BHs denoted by red, green, and blue, respectively. Note the clear relationship between $M_{\text{BH}}$ and orbital separation: simulations with more massive BHs produce BH+NS binaries with larger semi-major axes. This is contrary to the trend seen in the lower density cluster B. The distributions are poorly sampled for simulations that used 15 M$_\odot$ BHs because BH+NS binary production was inefficient in these simulations.

weakly on many of the input parameters, including $M_{\text{BH}}$, $N_{\text{BH}}$, and $f_b$. In addition to being a much smaller effect, the relationship between BH mass and semi-axis seen in cluster C is the reverse of what was seen in cluster B. Here, the simulations with the lowest mass BHs produced the BH+NS binaries with the smallest semi-major axes. Most of the BH+MSP binaries in globular clusters with structures similar to cluster C will have orbital periods shorter than 10 days.

Finally, the shortest period BH+NS binaries observed in our simulations were produced in cluster D. The encounter rate was highest in cluster D, so BH-binaries were rapidly hardened to small orbital separations in the simulations that used this background cluster. Figure 3.4 shows the present day cumulative distributions of $a$ and $P_B$ for each of the simulations performed in cluster D (i.e., simulations 42-51). As was seen in the simulations that used cluster C, the simulation with $M_{\text{BH}} = 7$ M$_\odot$ produced BH+NS binaries with the smallest semi-major axes. However, the median value of $a$ only varied slightly from simulation to simulation in cluster D. Simulation
Figure 3.4 Cumulative distribution of the semi-major axis (left) and orbital period (right) for a subset of the BH+NS binaries produced in each of the simulations in cluster family D. Each curve shows the distribution of $a$ or $P_B$ in a particular simulation. The distributions shown only include systems that exist at $t > 8 \times 10^9$ yr so they correspond to the present day. Note that simulations with 7 M$_\odot$ BHs produce binaries with smaller semi-major axes than the runs that used 35 M$_\odot$ BHs. This is contrary to the trend seen in the lower density cluster B. The distributions are poorly sampled for simulations that used 15 M$_\odot$ BHs because BH+NS binary production was inefficient in these simulations.

42 had the smallest median $a = 0.11$ au. The median value of $a$ was largest in simulations 47 and 48, both of which had median $a = 0.17$ au. We expect that many BH+MSP binaries present in densest globular clusters will have $P_B \lesssim 5$ days.

To review, we have shown that the BH+NS binaries produced in high density clusters have smaller semi-major axes than those produced in low density clusters. Furthermore, in the high density globular clusters, the binaries with lower mass BHs have smaller orbital separations. However, the size of a BH+NS binary’s semi-major axis only depends weakly on the mass of the BH. When we changed $M_{\text{BH}}$ by a factor of 5, the median semi-major axis only changed by a factor of $\sim 1.5$. The opposite trend is seen in lower density clusters (e.g., cluster B). In such clusters, binaries with higher mass BHs tend to have smaller semi-major axes than those with low mass BHs. Additionally, the mass of the BHs had a stronger impact on the orbital separations of the binaries produced in these simulations (compare Figure 3.3 and Figure 3.4). It is clear that the BH+NS binaries present in high density clusters (similar to clusters C and D) are in a different evolutionary phase than those in lower density clusters (similar to cluster B).
This behavior can be understood by comparing the time scales for the two evolutionary processes described above: encounters with single stars and the emission of gravitational radiation. If we assume that the interaction cross section is dominated by gravitational focusing, we can approximate the encounter time scale as

\[ t_{\text{enc}} \sim \frac{\bar{v}_m}{2\pi G(M_{\text{BH}} + M_{\text{NS}})n_c a}. \]  

(3.1)

Here we have used the central values of the stellar density and velocity dispersion, \( n_c \) and \( \bar{v}_m \). The BHs were the most massive members of the globular clusters that we modeled, so they spent most of their lifetimes deep in the cluster cores. Thus, using the central values gives a reasonable approximation of the encounter rate. Peters (1964) gives the time scale for a BH+NS binary to merge via the emission of gravitational radiation as:

\[ t_{\text{GW}} = \frac{15}{304 G^3 M_{\text{BH}} M_{\text{NS}} (M_{\text{BH}} + M_{\text{NS}})} a^4 c^5 g(e), \]  

(3.2)

where \( c \) is the speed of light and \( g(e) \) is a function of the binary’s eccentricity. For the typical eccentricity of a binary in our simulations \( g(e = 0.65) = 0.0572 \).

Now let us compare these time scales. For wide binaries, \( t_{\text{enc}} \ll t_{\text{GW}} \) and encounters with single stars will be the dominant hardening mechanism. However, each encounter will decrease the binary’s semi-major axis, resulting in an increase in \( t_{\text{enc}} \). Eventually, \( a \) will become so small that encounters cannot efficiently harden the binary. At this point, the emission of gravitational radiation becomes the dominant mechanism for orbital evolution and the semi-major axis begins to shrink more slowly. Accordingly, binaries will spend a large fraction of their lifetimes in this transition between the encounter and gravitational wave dominated phases. Therefore, the semi-major axis distributions will be dominated by BH+NS binaries with orbital configurations that fall in this transition.

We explored the transition between the encounter dominated phase and the gravitational radiation dominated phase by examining the ratio \( t_{\text{enc}}/t_{\text{GW}} \). Using equations (3.1) and (3.2), we calculated the ratio for each point in the resampled output. The cumulative distribution of \( t_{\text{enc}}/t_{\text{GW}} \) from a subset of our simulations is shown in Figure 3.5. Here we have only plotted simulations in clusters C and D. As discussed above, the semi-major axis distributions in these clusters exhibit similar trends. We have further restricted ourselves to simulations with \( N_{\text{BH+NS}} > 350 \) to ensure that statistical noise does not hinder our analysis. The left panel shows how \( t_{\text{enc}}/t_{\text{GW}} \) is distributed at \( t < 1 \times 10^9 \) yr. From left to right, the curves in this panel form a sequence in decreasing \( t_{\text{enc}} \) for a given value of \( a \). The center panel shows how these distributions have changed after \( \sim 5 \) Gyr of evolution. Notice that the distribution of \( t_{\text{enc}}/t_{\text{GW}} \) for simulations of cluster D that used 35 \( M_\odot \) BHs has hardly changed.
Figure 3.5 Cumulative distributions of $t_{\text{enc}}/t_{\text{GW}}$, the ratio of the encounter time scale to the gravitational wave merger time scale (given in equations (3.1) and (3.2), respectively). Each curve shows the distribution of $t_{\text{enc}}/t_{\text{GW}}$ in a particular simulation in cluster C (dashed lines) or cluster D (solid lines). The three panels illustrate how the distribution evolves over time. The distributions shown are for $t < 1 \times 10^9$ yr (left panel), $4 \times 10^9 < t < 6 \times 10^9$ yr (center panel), and $t > 8 \times 10^9$ yr (right panel). The color of each curve denotes the mass of the BH(s) used in a particular simulation, with 35 $M_\odot$, 15 $M_\odot$, and 7 $M_\odot$ BHs denoted by red, green, and blue, respectively. The binaries evolve towards a steady distribution in $t_{\text{enc}}/t_{\text{GW}}$. Simulations with the largest encounter rates (those in cluster D and those in cluster C with $M_{\text{BH}} = 35 M_\odot$) have reached the steady configuration by the current epoch. The simulations with lower encounter rates are still approaching this configuration, from the left, but have not reached this state after $10^{10}$ yr of evolution.
The distributions in all of the other simulations, with lower encounter rates, evolved towards these stationary curves. The right panel shows how $t_{\text{enc}}/t_{\text{GW}}$ is distributed at the present day, after an additional $\sim 3$ Gyr of evolution. All of the $t_{\text{enc}}/t_{\text{GW}}$ distributions from simulations in cluster D (solid lines) lie on top of one another. Simulations in cluster C (dashed lines) that used $M_{\text{BH}} = 35 \, M_\odot$ also closely follow this trend, while the simulations with lower mass BHs exhibit smaller values of $t_{\text{enc}}/t_{\text{GW}}$.

We interpret this evolution as follows. Given sufficient time, the distribution of $t_{\text{enc}}/t_{\text{GW}}$ in all clusters will approach the steady configuration seen in the right panel of Figure 3.5. At this stage, the orbital evolution of most of the BH+NS binaries has slowed as they make the transition from the encounter dominated phase to the gravitational radiation dominated phase. The median value of $t_{\text{enc}}/t_{\text{GW}}$ in this steady configuration is $\sim 2 \times 10^{-3}$. Even though $t_{\text{enc}}$ is still a few orders of magnitude smaller than $t_{\text{GW}}$ at this point, this ratio corresponds to the beginning of transition between the encounter and gravitational wave dominated phases. Given the steep dependance of $t_{\text{GW}}$ on $a$, such binary is only about one encounter away from an orbit in which $t_{\text{enc}} \sim t_{\text{GW}}$. We note that despite the similarities illustrated in Figure 3.5, the $t_{\text{enc}}/t_{\text{GW}}$ distributions from each simulation are formally distinct from one another, in most cases. We performed the two-sample Kolmogorov-Smirnov test between each pair of distributions and found that only those from simulations 44, 45, and 46 were consistent with coming from the same parent distribution. However, our interpretation does not require that the distributions be identical, it only requires that each distribution is dominated by systems that are in the transition phase.

Since the binaries evolve towards a constant configuration in $t_{\text{enc}}/t_{\text{GW}}$, we can use equations (3.1) and (3.2) and derive a scaling relation for the median semi-major axis of the BH+NS binaries during this phase:

$$a \propto \left( \frac{M_{\text{BH}} M_{\text{NS}} \bar{v}_{i m}}{n_c} \right)^{1/5}.$$  

(3.3)

In deriving this expression we have also assumed that $g(e)$ does not vary from cluster to cluster, which is justified because the eccentricity distributions in every simulation were similar (see section 3.3.1). Many facets of this scaling relation are seen in the semi-major axis distributions from simulations computed in clusters C and D. Most importantly this scaling relation accounts for the rather weak dependance of the these semi-major axis distributions on many of the input parameters. The relation also accounts for the fact that the runs with lower mass BHs produced BH+NS binaries with smaller semi-major axes.

Most of our simulations did not evolve to the steady $t_{\text{enc}}/t_{\text{GW}}$ distribution described above. In clusters with low encounter rates ($1/t_{\text{enc}}$), many of the BH+NS binaries will not be hardened fast enough to reach the transition from the encounter dominated regime to the gravitational radiation dominated regime within a Hubble
time. Therefore, the orbital parameters of most BH+NS binaries in these clusters are determined by binary–single encounters alone. Accordingly, we expect that the semi-major axes of the binaries in these clusters will follow a different scaling relationship than those in the high density cluster. Since each encounter will reduce \( a \), binaries with shorter encounter times will have smaller semi-major axes. This is what we observed in simulations in cluster B; the binaries with higher mass BHs had smaller \( t_{\text{enc}} \), and were hardened to smaller \( a \) than the binaries with low mass BHs.

### 3.4 BH+MSP Binary Population Size

The 51 simulations presented in this chapter investigated how several parameters impact the dynamical formation of BH+NS binaries in globular clusters. Below we will describe how each of these traits affects the likelihood that a BH+NS binary exists within a cluster. We will use two metrics to characterize the likelihood that a cluster harbors a BH+NS binary. The first metric is the fraction of time that the BHs in our simulations had a NS companion, \( \tau_{\text{BH+NS}} = t_{\text{BH+NS}} / (N_{\text{runs}} t_{\text{max}}) \). Here \( t_{\text{BH+NS}} \) is the sum of the time that a BH had a NS companion during all of the runs in a particular simulation, \( t_{\text{max}} = 10^{10} \) yr is the duration of each run, and \( N_{\text{runs}} \) is the number of runs in a simulation. As was the case in chapter 2, \( N_{\text{runs}} \) is slightly less than 2000 because we rejected a small number of runs in which the initial BH-binary was randomly selected to contain a NS. The second metric that we use is the probability that a cluster contains a BH+NS binary \( p_{\text{BH+NS}} = N_{\text{BH}} \tau_{\text{BH+NS}} \). In simulations with multiple BHs, \( \tau_{\text{BH+NS}} \) corresponds to the probability that any one of these BHs has a NS companion, and must therefore be scaled by \( N_{\text{BH}} \) to find the probability that a BH+NS binary exists in the cluster. The values of these metrics for each of our simulations are listed in Table 3.2.

As was the case with the orbital parameter distributions, the structure of the background globular cluster had a large impact on the values of \( \tau_{\text{BH+NS}} \). In cluster A \( (n_c = 10^4 \text{ pc}^{-3}) \), the BH+NS binaries had a mean lifetime of \( 7 \times 10^9 \) yr. However, because of the low encounter rate, these binaries were produced so rarely that \( \tau_{\text{BH+NS}} \begin{array}{c} \lesssim \\ \end{array} 2 \times 10^{-3} \) in every simulation in this cluster. Simulations in cluster D \( (n_c = 10^6 \text{ pc}^{-3}) \) had the opposite problem. The BH+NS binaries in this cluster were rapidly driven to coalescence because of the high encounter rate. The intermediate encounter rates in clusters B \( (n_c = 10^5 \text{ pc}^{-3}) \) and C \( (n_c = 5 \times 10^5 \text{ pc}^{-3}) \) struck a balance between the production and destruction of BH+NS binaries. All other parameters held equal, simulations in clusters B or C had the largest values of \( \tau_{\text{BH+NS}} \). The BHs in these clusters typically spent a few percent of their lifetimes with a NS companion.

The amount of time that the BHs in our simulations had NS companions decreased with increasing \( M_{\text{BH}} \). Both of the evolutionary processes that drive the BH+NS binaries to merge speed up when the mass of the BH is increased. A more massive BH leads to a larger gravitational focusing cross-section, which increases the encounter
Table 3.2: Globular Cluster Models and BH+MSP Binary Properties

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<th>$f_{ret}$</th>
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<th>$N_{BH+NS}$</th>
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rate. Furthermore, orbital energy is lost more rapidly through gravitational radiation when $M_{\text{BH}}$ increases.

The assumed background binary population did not influence $\tau_{\text{BH+NS}}$ in clusters A and B. In these clusters mergers and disruptive encounters rarely occurred. Accordingly, there were few single BHs in these clusters that needed to interact with the background binary population to acquire a new companion. In clusters C and D, on the other hand, the nature of the background binary population had a substantial influence on $\tau_{\text{BH+NS}}$. In both clusters it appears that there is a threshold around $f_b \sim 0.2$. Simulations with $f_b < 0.2$ all exhibit similar values of $\tau_{\text{BH+NS}}$. At larger $f_b$, $\tau_{\text{BH+NS}}$ increases as $\sim f_b^2$. The composition of the binary population does not seem to impact the results much. As we explained in section 2.3.2, the main advantage of the FIR population over the OBS population is that it allows for larger binary fractions within observational constraints. At the same value of $f_b$, simulations using the FIR population and the OBS population produce BH+NS binaries with equal efficiency. Use of the OPT population did, however, lead to larger $\tau_{\text{BH+NS}}$ in clusters B, C, and D.

The runs presented in this chapter also explored how changing the size of the NS and BH populations impacted the formation of BH+NS binaries. The value of $\tau_{\text{BH+NS}}$ responded linearly to changes in the NS retention fraction ($f_{\text{ret}}$). In every run with $N_{\text{BH}} > 1$, the increased size of the BH population lead to a reduction in the value of $\tau_{\text{BH+NS}}$. When there was more than one BH in the simulation, it was common for the BH-binaries that we were evolving to interact with the other BHs in the cluster. Nearly half of all three-body encounters between a BH+star binary and a second BH will result in the formation of a BH+BH binary (Sigurdsson & Phinney 1993). Once a BH+BH binary formed, it was nearly impossible for a NS to exchange into the system. These BHs were essentially locked up for the rest of the run, leading to the reduction in $\tau_{\text{BH+NS}}$. In addition to preventing the formation of BH+NS binaries, the presence of several BHs can also result in the destruction of BH+NS binaries. In our runs with multiple BHs, 10-50% of BH+NS binaries were destroyed when another BH exchanged into the binary. For relatively small BH populations the value of $p_{\text{BH+NS}}$ increased compared to simulations with a single BH. In these simulations the small decline in $\tau_{\text{BH+NS}}$ was outpaced by increase in $N_{\text{BH}}$. However, $p_{\text{BH+NS}}$ is subject to the law of diminishing returns. Comparing simulations 14, 15, and 16, we see that $p_{\text{BH+NS}}$ increases as we add more BHs to the cluster. Extrapolating the trend seen in $\tau_{\text{BH+NS}}$ to larger values of $N_{\text{BH}}$, we find that increasing $N_{\text{BH}}$ to 100 will only boost the probability that this cluster harbors a BH+NS binary from 0.051 to 0.06. Of course, one should use caution in drawing conclusions from such an extrapolation, but it seems unlikely that the presence of a substantial BH population would increase the size of a cluster’s BH+NS binary population. In fact, such a large number of BHs could reduce the size of the BH+NS binary population. This behavior is seen in some of our simulations. In simulations 12 & 13 and 31 & 32 an increase in $N_{\text{BH}}$ resulted
in a reduction of $p_{\text{BH+NS}}$. When there were many BHs in a cluster, the production of BH+BH binaries was favored over the production of BH+NS binaries.

Combining all of the effects described above, we conclude that the probability of finding a BH+MSP binary is highest in massive globular clusters with $n_c \sim \text{few} \times 10^5 \text{ pc}^{-3}$, $f_b \gtrsim 0.2$, and BH populations that comprise a few dozen $\sim 10 M_\odot$ BHs. We can estimate the number of BH+MSP binaries in the Milky Way globular cluster system as:

$$N_{\text{BH+MSP}} = N_{\text{GC}} \ f_{\text{GC}} \ f_{\text{MSP}} \ f_{\text{BH}} \ p_{\text{BH+NS}},$$

where, following Narayan et al. (1991), we have implicitly assumed that the lifetime of a MSP is $> 10^{10}$ yr. Here $N_{\text{GC}} = 150$ is the number of globular clusters in the Milky Way. The fraction of globular clusters with structural properties similar to those used in our simulations is denoted $f_{\text{GC}}$. Approximately $15 - 20\%$ of the Milky Way globular clusters have $n_c \sim 10^5 \text{ pc}^{-3}$, $M_{\text{GC}} = \text{several} \times 10^5 M_\odot$, and $1.7 < c_{\text{GC}} < 2.0$ (Harris 1996; Gnedin & Ostriker 1997). We gauged the fraction of NSs that have been recycled into MSPs, $f_{\text{MSP}}$, using observational constraints on the total number of MSPs in 47 Tuc and Terzan 5. Abdo et al. (2010) used the integrated gamma-ray flux emitted by these clusters to estimate the number of MSPs, finding that 47 Tuc harbors $33^{+15}_{-12}$ MSPs and Terzan 5 contains $180^{+100}_{-100}$. Using radio measurements, the total numbers of MSPs in 47 Tuc and Terzan 5 have been estimated to be $163^{+108}_{-70}$ and $294^{+224}_{-130}$, respectively (Chennamangalam et al. 2013). Assuming that each of these clusters retains a total of 500-1000 NSs, we estimate that $f_{\text{MSP}}$ is between 5\% and 30\% (Pfahl et al. 2002; Ivanova et al. 2008). The fraction of globular clusters that retain at least one BH, $f_{\text{BH}}$, is poorly constrained by observations. However, we can place an upper limit on the number of BH+MSP binaries by assuming that every massive globular cluster retains a BH population. If we further assume the maximum reasonable value for every factor in equation (3.4), we find that the upper limit for the number of BH+MSP binaries in the Milky Way globular cluster system is 10. Here we have used the value of $p_{\text{BH+NS}} = 1.1$ computed in simulation 28. The number of detectable BH+MSP binaries is a factor of 2–3 smaller due to beaming effects. The upper limit on the number of dynamically formed BH+MSP binaries presented here is similar to the number of BH+MSP binaries expected to form through the evolution of isolated binaries (Sipior et al. 2004; Pfahl et al. 2005).

We also estimated the total number of BH+MSP binaries by generating several Monte Carlo realizations of the Milky Way globular cluster population and counting the number of BH+MSP binaries in each realization. In these models we used $N_{\text{GC}} = 150$. The mass of each cluster was drawn from the globular cluster mass function presented in McLaughlin & Pudritz (1996). We then assigned each cluster a $p_{\text{BH+NS}}$ by randomly selecting one of our simulations that was done in a globular cluster of similar mass. Next, we chose $f_{\text{MSP}}$ for each cluster from a normal distribution with a mean of 0.13 and a standard deviation of 0.07. Based on $10^4$ realizations, we found $N_{\text{BH+MSP}} = f_{\text{BH}} 0.6 \pm 0.2$. In computing this number we assumed that the size of
each globular cluster’s BH population was random. It is possible that the sizes of
the BH populations are correlated, i.e., the globular clusters that retain BHs either
all retain several or all retain ~ 1. If we recompute our Monte Carlo realizations
and require that \( N_{\text{BH}} > 1 \), then we find \( N_{\text{BH}+\text{MSP}} = f_{\text{BH}} \times 1.3 \pm 0.3 \). If we only
consider simulations that used \( N_{\text{BH}} = 1 \), the size of the BH+MSP binary population
is reduced to \( N_{\text{BH}+\text{MSP}} = f_{\text{BH}} \times 0.2 \pm 0.1 \). We have not accounted for the fact that
many components of these estimates, (e.g., \( N_{\text{BH}}, f_{\text{BH}}, \) and \( f_{\text{MSP}} \)) are likely to be
functions of the globular clusters’ structural parameters. However, it is unlikely that
including these dependancies will significantly alter our conclusion that \( N_{\text{BH}+\text{MSP}} \lesssim 1 \).
Unfortunately, it appears that dynamically formed BH+MSP binaries in globular
clusters may be even rarer than those produced through standard binary evolution
in the disk of the Galaxy.

### 3.5 Discussion

We have presented a study of the dynamically formed BH+MSP binaries in globular
clusters. We found that in the highest density clusters \( (n_c \gtrsim 5 \times 10^5 \text{ pc}^{-3}) \), the
semi-major axis distribution of the BH+MSP binaries is nearly independent of all
of the parameters that we varied in our study. This property of the BH+NS binary
populations is beneficial for observers who hope to identify such systems. Regardless
of the nature of many uncertain characteristics, including the globular cluster BH
and binary populations, the vast majority of BH+MSP binaries produced in dense
globular clusters will have \( 2 < P_B < 10 \) days. In lower density clusters, \( M_{\text{BH}} \) does
influence the expected orbital periods of the BH+MSP binaries. In clusters with
\( n_c \sim 10^5 \text{ pc}^{-3} \) BH+MSP binaries with massive stellar mass BHs \( (M_{\text{BH}} = 35 \text{ M}_\odot) \)
will typically have orbital periods around 20 days. For BH+MSP binaries with 7 \text{ M}_\odot
BHs, the expected orbital periods are much longer, with typical periods in the 150 to
250 day range.

Unfortunately, we have also found that dynamically formed BH+MSP binaries are
quite rare. We estimated that the maximum number of detectible BH+MSP binaries
in the Milky Way globular cluster system is 5. This result is not a consequence of the
assumption that most stellar mass black holes are ejected from the cluster early in
its evolution. The presence of a large BH population will also reduce the probability
that a cluster harbors a BH+MSP binary. BH+MSP binary formation can be stifled
by as few as 19 BHs. If there are several BHs in the cluster the BHs will preferentially
interact with each other and not the NSs. Furthermore, any BH+NS binaries that are
formed may be destroyed when another BH exchanges into the binary. This behavior
has also been seen in models that considered the evolution of the BH population as a
whole. Sadowski et al. (2008) and Downing et al. (2010) found that very few BH+NS
binaries were produced in their simulations, which included several hundred to over
one thousand BHs. We expect dynamically formed BH+MSP binaries to be rare
regardless of the size of the retained BH population.

Some of the limitations of our method will impact the results of our simulations. Because our simulations do not include binary–binary encounters, they do not capture several processes that affect the formation of BH+MSP binaries. As discussed above, binary–binary interactions open up additional BH+NS binary formation channels. Furthermore, in clusters with multiple BHs, collisions between pairs of BH+BH binaries could eject or disrupt many BH+BH binaries (e.g. O’Leary et al. 2006; Banerjee et al. 2010; Downing et al. 2010). Reducing the number of BHs in the cluster and freeing BHs from otherwise impenetrable BH+BH binaries would increase the likelihood that BH+MSP binary is produced. However, binary–binary interactions will also disrupt and eject BH+MSP binaries. As we concluded in section 2.4.1, models that include binary–binary interactions are needed to see which processes dominate. Finally, as a consequence of our assumption that the background cluster was static, we forced the BHs to remain in equilibrium with the rest of the cluster. If we had allowed for the dynamical evolution of the BHs, they might have decoupled from the rest of the cluster. This would have further reduced the number of encounters between BHs and NSs.

Although the number of BH+MSP binaries in the Milky Way globular cluster system is expected to be small, searching for these binaries is still feasible. We know that these binaries are in globular clusters, and our models make specific predictions about the types of globular clusters that are likely to harbor BH+MSP binaries. Given the potential scientific payoff, deep radio observations of the cores of the ∼20 Milky Way globular clusters with appropriate structural properties may be justified. Likely BH+MSP binary hosts include 47 Tuc, Terzan 5, NGC 1851, NGC 6266, and NGC 6441. Finally, even though there might not be any BH+MSP binaries in the Milky Way globular cluster system, such binaries could be detected in extra-galactic globular clusters with the Square Kilometer Array (SKA). SKA should be able to detect most pulsars within 10 Mpc (Cordes 2007), and our models predict there could be ∼100 dynamically formed BH+MSPs binaries within this volume.
Chapter 4

Emission Lines From the Photoionized Debris of Tidally Disrupted, Evolved Stars


4.1 Introduction

A star is tidally disrupted when its orbit brings it too close to a black hole (BH). If the two bodies are separated by less than the tidal disruption radius, \( R_T \sim R_\star (M_{BH}/M_\star)^{1/3} \), where \( R_\star \) is the radius of the star and \( M_{BH} \) and \( M_\star \) are the masses of the BH and star, respectively, the star is ripped apart because the tidal force across its diameter exceeds its self gravity. Frank & Rees (1976) calculated the rate at which a BH in the core of a globular cluster or galaxy would disrupt and ingest main sequence (MS) stars and suggested that the associated ultraviolet (UV)/X-ray flare would mark the presence of an otherwise undetectable BH. This is possible because after the star is disrupted, about half of the debris becomes bound to the BH. As this bound gas falls back towards the BH, shocks and accretion onto the BH produce a luminous flare that peaks in the UV/X-ray (Lacy et al. 1982; Rees 1988; Ulmer 1999). A number of analytic (e.g., Phinney 1989; Kochanek 1994; Khokhlov & Melia 1996; Strubbe & Quataert 2009) and numerical (e.g., Evans & Kochanek 1989; Laguna et al. 1993; Khokhlov et al. 1993; Bogdanović et al. 2004; Guillochon & Ramirez-Ruiz 2013) studies have explored the behavior of the debris after a MS star is tidally disrupted and the feasibility of using the luminosity and evolution of the flare to search for and determine properties of the BH. Others have proposed flares and possible detonation of the star due to tidal compression as an additional observable consequence of tidal disruptions (Carter & Luminet 1982, 1983; Bicknell & Gingold 1983; Luminet & Marck 1985; Kobayashi et al. 2004). Recent work along these lines suggests that the compression will produce shocks capable of raising the star’s surface temperature to that of its core, resulting in a prompt flare (Guillochon
Detailed models have also been employed to predict the rate at which MS stars are tidally disrupted by supermassive BHs. However, uncertainties in the supermassive BH mass spectrum, the stellar distribution function in galactic bulges, and the relation between the two complicate calculations of this rate. For example, improved measurements of the mean ratio of BH mass to bulge mass made between the studies conducted by Magorrian & Tremaine (1999) and Wang & Merritt (2004) resulted in a factor of ten increase in the predicted event rate, raising it to $\sim 10^{-5}$ yr$^{-1}$ Mpc$^{-3}$. This rate is constrained by systematic surveys for tidal disruption flares. Historically, around ten candidate tidal disruptions were first identified by serendipitous observations of UV/X-ray flares in surveys (e.g., Grupe et al. 1995; Brandt et al. 1995; Komossa & Bade 1999; Komossa & Greiner 1999; Gezari et al. 2006; Cappelluti et al. 2009; Maksym et al. 2010). Follow up observations of the variability of these sources ruled out other explanations and confirmed that the luminosity of the sources declined with the same $t^{-5/3}$ time dependence that models predict for the decline in the mass fallback rate (Halpern et al. 2004; Esquej et al. 2008). Subsequent studies have discovered additional tidal disruption candidates and, by accounting for the efficiency and sky coverage of the surveys, determined that supermassive BHs tidally disrupt MS stars at a rate consistent with $10^{-5} - 10^{-4}$ yr$^{-1}$ galaxy$^{-1}$ (Donley et al. 2002; Gezari et al. 2008, 2009; Esquej et al. 2008; Luo et al. 2008). The high energy accretion flares are the brightest and most immediate sign of a MS star being tidally disrupted by a BH, but they are not the only sign.

In addition to the UV/X-ray emission, Roos (1992) proposed that the unbound portion of the debris can be photoionized and produce optical emission lines. The nature of the optical emission lines was explored in detail in the numerical models of Bogdanović et al. (2004) and the analytic models developed by Strubbe & Quataert (2009, hereafter SQ09). In each of these studies, the source of the ionizing photons is the accretion flare. The prospect of optical emission from tidal disruptions is exciting for the following reasons. First, SQ09 showed that tidal disruptions are bright enough in the optical band that they are potential sources for large transient surveys (e.g. The Large Synoptic Survey Telescope, The Palomar Transient Factory, and Pan-STARS), which suggests that the number of detected candidate events will increase in the near future. Second, the emission-line signature of such an event helps us identify its nature and distinguish it from other, more common transients that can occur near the nuclei of galaxies, such as supernovae.

While flares from tidally disrupted MS stars might be a more frequent probe of dormant BHs in dense stellar systems, Sigurdsson & Rees (1997) pointed out that these BHs can capture compact objects as well because this region is also populated with the products of stellar evolution. In particular, white dwarfs (WDs) are also subject to the processes that drive MS stars into orbits around the central BH with pericenter distances $R_p < R_T$, and can therefore be tidally disrupted as well. However,
since the mass of a WD is comparable to that of a low mass MS star and its radius is smaller by a factor of $\sim 100$, $R_T$ is much closer to the BH for a WD than for a MS star. Consequently, if $M_{\text{BH}} \gtrsim 5 \times 10^5 M_\odot$, $R_T$ for a WD is inside of the BH’s Schwarzschild radius, $R_S = 2GM_{\text{BH}}/c^2$. When $R_T \leq R_S$, the WD plunges into the BH horizon before being tidally disrupted, making such events undetectable by electromagnetic observations. This makes WD tidal disruptions an excellent indicator of the presence of an IMBH in the center of a galaxy or globular cluster, because if the central BH were more massive than $\sim 10^5 M_\odot$ we would not observe anything.

Like the MS case, using WD tidal disruption events to probe IMBHs requires prior theoretical studies to determine how the observed flares are related to the properties of the WD-IMBH system. While this scenario has not received as much attention as the MS case, the early stages of the tidal disruption of a WD in an unbound orbit around an IMBH were studied numerically by Frolov et al. (1994), who found that material flowed away from the WD in supersonic jets as it was disrupted. Rosswog et al. (2009) followed the evolution of the debris further and found that gas returns to the BH at a rate that is sufficient to power an accretion flare at the Eddington luminosity, $L_{\text{Edd}} = 1.3 \times 10^{41} (M_{\text{BH}}/10^5 M_\odot) \text{ erg s}^{-1}$, for up to one year. Additionally, since it is supported by degeneracy pressure, when $R_p \lesssim 0.5 R_T$ the tidal compression of the WD can lead to explosive nuclear burning (Luminet & Pichon 1989; Rosswog et al. 2009). The latter authors found that, in the most favorable cases, the energy generated by thermonuclear burning during the compression phase exceeds the binding energy of the WD and is similar to that of a Type Ia supernova. However, if the WD does not penetrate too deeply within $R_T$, the case is similar to that of the tidally disrupted MS stars described above, in which a portion of the debris is accreted by the BH and the rest of the material remains unbound and flows away from the BH. Sesana et al. (2008) calculated the light curves of optical and near-UV emission lines produced when the unbound debris is photoionized by the accretion flare and proposed these lines might be an electromagnetic counterpart to the gravitational radiation emitted as a WD initially on a bound orbit spirals into the IMBH. Here, we revise and update these results by considering the case of a WD initially on an unbound orbit and using a more sophisticated model for the behavior of the debris.

Interestingly, there are two extragalactic globular clusters that exhibit both hallmarks of a tidal disruption event, a bright X-ray source and bright optical emission lines. One of these sources, in the globular cluster RZ 2109, associated with NGC 4472, undergoes large luminosity variations that suggest that the accretor is a BH (Maccarone et al. 2007). Optical spectroscopy of the globular cluster hosting this source revealed an extremely broad $[\text{O III}] \lambda 5007$ emission line with a luminosity $L = 1.4 \times 10^{37} \text{ erg s}^{-1}$, but did not detect any hydrogen emission lines (Zepf et al. 2008). The other source, in a globular cluster associated with NGC 1399, has a softer spectrum than typical ultraluminous X-ray sources, which suggests that it is also likely to be an accreting BH (Irwin et al. 2010). An optical spectrum of the host
globular cluster showed [O III] \( \lambda 5007 \) and [N II] \( \lambda 6583 \) emission lines, each with \( L \sim \text{few} \times 10^{36} \text{ erg s}^{-1} \). Intriguingly, there are no hydrogen lines in this spectrum either. Irwin et al. (2010) suggested that the high X-ray luminosities and peculiar emission line spectra seen in these globular clusters could be evidence of WD tidal disruption events. The absence of hydrogen emission lines in each source may indicate that the material producing the lines has the same composition as an evolved star. However, models are necessary to determine if these lines could be produced in the debris of a tidally disrupted star.

In this chapter we will model the emission lines produced when a WD or horizontal branch (HB) star is tidally disrupted by an IMBH, and then compare the results with the emission lines observed in two candidate WD tidal disruption events. We will begin by looking at the tidal disruption of a WD in detail. The dynamical model for the debris, which is an adaptation of the model derived by SQ09, is described in section 4.2. In section 4.3 we will discuss how the emission line luminosities and line-profiles are computed. The observational signatures of WD tidal disruption events will be presented in section 4.4. In section 4.5 and section 4.6 we will extend our models to study the emission lines produced in the photoionized debris of tidally disrupted HB stars. We will discuss some strengths and weaknesses of the models in section 4.7. Finally, we will confront the results of the models with the observations in section 4.8.

### 4.2 The Dynamical Model for the Debris

We adopted the analytic prescriptions of SQ09 to model the evolution of the debris after a WD is tidally disrupted by an IMBH. Their model describes a MS star approaching a supermassive BH on a parabolic orbit with pericenter distance \( R_p \leq R_T \). However, since all of the relevant quantities depend only on the BH mass, the pericenter distance, and the mass and radius of the star, the model can also be applied to the case of a WD being tidally disrupted by an IMBH. Furthermore, even though the WD equation of state is stiffer than that of a MS star, the deviations in the tidal disruption process and the behavior of the debris that arise from this difference are small, so the approximate formulae developed in SQ09 give a reasonable description of the WD case. The model assumes that half of the material becomes bound to the BH and forms an accretion disk after the WD is disrupted, and that the other half of the debris is ejected on hyperbolic orbits. The details of these components in the SQ09 model and the adaptations we have made are described below. For a full derivation of the model, we refer the reader to SQ09.

The evolution of the debris was modeled for a range of initial conditions. To explore how the optical emission lines are affected by the mass of the IMBH, we computed models with \( M_{BH} = 10^2, 10^3, \) and \( 10^4 \, M_\odot \). Even though a WD of the mass used in the models would be tidally disrupted outside of the event horizon of
Table 4.1. Model Parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_{\text{BH}}$ (M$_{\odot}$)</th>
<th>$\beta$</th>
<th>$R_{\text{LSO}}/R_S$</th>
<th>$t_{\text{Edd}}^a$ (days)</th>
<th>$\Delta \phi^b$ (rad)</th>
<th>$v_{\text{max}}^c/c$</th>
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</tr>
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<td>2.7</td>
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</tr>
<tr>
<td>C</td>
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<td>3</td>
<td>440</td>
<td>0.99</td>
<td>0.052</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.5</td>
<td>440</td>
<td>0.99</td>
<td>0.052</td>
</tr>
<tr>
<td>F</td>
<td>$10^4$</td>
<td>1</td>
<td>3</td>
<td>170</td>
<td>0.68</td>
<td>0.082</td>
</tr>
</tbody>
</table>

$^a$The duration of the super-Eddington mass fallback phase

$^b$The azimuthal dispersion of the unbound debris in the orbital plane

$^c$The maximum velocity in the unbound debris cloud as a fraction of the speed of light

a 10$^5$ M$_{\odot}$ IMBH, we do not consider this case because the tidal disruption radius is within the last stable orbit around the IMBH. We also constructed models for two different values of the penetration factor $\beta = R_T/R_p$. In one set we used $\beta = 1$. For comparison, we also computed a second set of models with $\beta = 3$, even though Rosswog et al. (2009) reported that the WD would explode in this case. Table 4.1 lists the initial conditions for each model we considered. In each of the models, we set $M_{\text{WD}} = 0.55$ M$_{\odot}$ because the mass distribution of WDs observed in the Sloan Digital Sky Survey is strongly peaked at this value (Madej et al. 2004). We calculated the WD’s radius to be $R_{\text{WD}} = 8.6 \times 10^8$ cm using the mass-radius relation of Nauenberg (1972).

### 4.2.1 The accretion disk

As the bound material falls back to pericenter, it shocks on itself and the orbits of the debris circularize to form an accretion disk. In the model, the disk extends from the last stable orbit, $R_{\text{LSO}}$, out to approximately 2$R_p$. Like SQ09, we considered both Schwarzschild BHs with $R_{\text{LSO}} = 3R_S$ and maximally spinning BHs with $R_{\text{LSO}} = R_S/2$. The rate at which the bound material falls back towards the BH is set by the time it takes the debris to return to pericenter and the mass of the WD, and was computed
The mass fallback rate decreases as $t^{-5/3}$, and at early times it can exceed the Eddington mass accretion rate, $\dot{M}_{\text{Edd}} = 10L_{\text{Edd}}/c^2$. We capped the rate at which material is accreted by the BH, $\dot{M}$, at $\dot{M}_{\text{Edd}}$ and, following SQ09, we assumed that the excess mass is blown off in a radiation driven wind. Because the nature of these outflows is uncertain, we did not consider emission from the outflows or how the outflows might affect the emission lines produced in the unbound material. SQ09 suggested that the material in the outflows can also be photoionized and produce additional line luminosity, making the predictions of this model lower limits. In a followup paper, Strubbe & Quataert (2011) considered the photoionization of the out-flowing gas that is outside the photosphere of the outflow and calculated the resulting absorption-line spectrum. Furthermore, the outflows could alter the radiation field that photoionizes the unbound material and, thus, change the ionization state of the unbound debris. Since these effects were not accounted for in our model, the emission line luminosities predicted during the super-Eddington phase are uncertain. The outflows subside after a time $t_{\text{Edd}}$, given in equation (3) of SQ09 as

$$t_{\text{Edd}} \sim 0.1 \left( \frac{M_{\text{BH}}}{10^6 M_\odot} \right)^{2/5} \left( \frac{R_p}{3 R_S} \right)^{6/5} \left( \frac{M_{\text{WD}}}{M_\odot} \right)^{3/5} \left( \frac{R_{\text{WD}}}{R_\odot} \right)^{-3/5} \text{yr.}$$

In Table 4.1 we list the value of $t_{\text{Edd}}$ for each of our models.

### 4.2.2 The unbound debris

As the star approaches the BH, it is tidally stretched along the direction of its motion and then spun up, resulting in a spread in the specific energy of the debris (Rees 1988). SQ09 calculated how this range of specific energies leads to an expanding, curved wedge of unbound debris spanning a range of azimuthal angles and radial distances from the BH (see Figure 4.1). The accretion flare illuminates the inside face of this wedge, and the gas along this edge reprocesses the radiation and produces the optical emission lines we are interested in. SQ09 modeled the extent of this face in terms of $R_{\text{max}}$, $\Delta \phi$, and $\Delta \iota$, the maximum radial distance from the BH, the azimuthal dispersion of the debris in the orbital plane, and the angular dispersion of the debris
perpendicular to the orbital plane, respectively. We adopt their equations for the angular dispersion of the unbound material:

$$\Delta \phi \sim \left( \frac{12 R_{\text{WD}}}{R_p} \right)^{1/2}$$  \hspace{1cm} (4.4)

and

$$\Delta i \sim 2 \frac{R_{\text{WD}}}{R_p}.$$  \hspace{1cm} (4.5)

However, we make a minor modification to calculate the distance between the BH and material along the debris arc. Since we considered several points along the arc, not just the most distant material, we calculated the distance to the BH at any point along the arc with

$$R(\phi) \sim v_p t \cot \left( \frac{\phi}{2} \right),$$  \hspace{1cm} (4.6)

where $v_p$ is the pericenter velocity of the WD, $t$ is the time since pericenter passage, and $\phi$ is the angle in the orbital plane between pericenter and the point in the debris cloud being considered and is in the range $(\pi - \Delta \phi) < \phi < \pi$. The WD is initially on a parabolic orbit, so $v_p = (2GM_{\text{BH}}/R_p)^{1/2}$. SQ09 calculated $R_{\text{max}}$ in a similar manner and our equation (4.6) is consistent with the derivation of Evans & Kochanek (1989) for the values of $\phi$ considered here. In Table 4.1, we list $\Delta \phi$ and $v_{\text{max}}/c$, the maximum velocity in the unbound debris cloud as a fraction of the speed of light, for each set of initial conditions. In addition to $R$ and $\phi$, we define the $\ell$-curve, which begins at the IMBH and lies along the illuminated face of the debris cloud.

Furthermore, because we computed photoionization models at several points along the inside face of the cloud of unbound debris, we also had to deviate from SQ09’s model when we calculated the density of the unbound debris, $n$. Kochanek (1994) found that the density in the stream of unbound material was not uniform and decreased with distance from the BH as $n \propto R^{-3}$ because the material is undergoing free expansion. We used this density profile in our model and normalized it using the fact that the mass of the wedge of unbound material was half the mass of the WD.

### 4.3 Modeling The Emission Lines

Predicting the emission lines we will observe when a WD is tidally disrupted by an IMBH requires a model for the emission from both the bound and unbound portions of the debris. To calculate the luminosity and spectrum of the accretion flare generated by the disk of bound gas, SQ09 assumed a “slim disk” model (Abramowicz et al. 1988) and derived an expression for the effective temperatures as a function of radius
Figure 4.1 The spatial distribution of the unbound debris from model C, projected into the orbital plane. The BH sits at the origin and the WD approached it from the \(-x\) direction on a parabolic orbit. The debris is shown one, three, and seven years after the tidal disruption. The shading shows the density of the debris, which goes as \(R^{-3}\). The \(\ell\)-curve begins at the BH and lies along the illuminated face of the debris cloud. The ionizing radiation from the disk encounters the cloud at a different angle of incidence for each point along \(\ell\) and travels along the corresponding \(\vec{h}\) into the cloud. The length of \(\vec{h}\) shown here is greatly exaggerated, the incident radiation only reaches depths \(\lesssim 10^{-3}R\).

in the disk

\[
[T_{\text{eff}}(R_d)]^4 = \frac{3GM_{\text{BH}}\dot{M}f}{8\pi R_d^3\sigma_{\text{SB}}} \left\{ \frac{1}{2} + \left[ \frac{1}{4} + \frac{3}{2}f \left( \frac{10\dot{M}}{M_{\text{Edd}}} \right)^2 \left( \frac{R_d}{R_S} \right)^{-2} \right]^{1/2} \right\}^{-1}, \tag{4.7}
\]

where \(\sigma_{\text{SB}}\) is the Stefan–Boltzmann constant, \(R_d\) is the disk radius, and

\[
f = 1 - \left( \frac{R_{\text{LSO}}}{R_d} \right)^{1/2}. \tag{4.8}
\]
Then, using the range of temperatures throughout the disk, they modeled the spectrum as a multicolor blackbody. We considered this spectral energy distribution (SED) for the disk as well as two others. We also used a SED that consisted of the multicolor blackbody and an additional X-ray component from 0.1 – 100 keV with a power law of the form $f_E \propto E^{-1}$, where $f_E$ is the flux density per unit energy and $E$ is the photon energy. We normalized the intensity of the X-ray power law using the intensity of the multicolor blackbody at 2500 Å and the $\alpha_{OX}$ parameter based on the findings of Steffen et al. (2006). The optical-to-X-ray spectral index, $\alpha_{OX}$, is the exponent of a power-law connecting the monochromatic luminosity densities at 2500 Å and 2 keV. If $L_\nu \propto \nu^{-\alpha_{OX}}$, then

$$\alpha_{OX} \equiv \frac{- \log L_\nu(2500 \text{ Å}) - \log L_\nu(2 \text{ keV})}{\log \nu(2500 \text{ Å}) - \log \nu(2 \text{ keV})} = 1 + 0.384 \left[ \log \left( \nu L_\nu \right)_{2500 \text{ Å}} - \log (\nu L_\nu)_{2 \text{ keV}} \right].$$

(4.9)

The third SED we considered was an empirically constructed, multi-component model for the SED of an active galactic nucleus (AGN) that was first used in Korista et al. (1997) and explored in greater detail by Casebeer et al. (2006). This model is parameterized by the temperature of the peak of the model’s thermal component, $\alpha_{OX}$, and the slope of the X-ray component. We set these parameters to the temperature of the peak of the multicolor blackbody, $-1.4$, and $-1$, respectively. The accretion disk-IMBH system is similar to an AGN, so we used these three SEDs to sample the range of UV/X-ray and UV/infrared ratios observed in AGN. This is necessary because free-free heating of the free electrons in the unbound material by infrared radiation can change its ionization structure. Additionally, irradiating this material with X-rays can lead to significant heating per ionization, which would affect the luminosities of collisionally excited lines. In all three cases, the time evolution of the SED is governed by the declining mass accretion rate, which decreases the bolometric luminosity of the SED and causes its peak to shift to longer wavelengths as time goes on. Figure 4.2 shows each SED and its evolution. These SEDs were used as one of the inputs to the photoionization models described in the next section.

### 4.3.1 Photoionization models

The unbound material is assumed to radiate because it is illuminated by the accretion flare described above. Predicting the emission lines generated by this material requires photoionization models to determine how the gas reprocesses the incident radiation. We performed the photoionization calculations with version 08.00 of Cloudy, last described by Ferland et al. (1998). Since the debris cloud consists of material from a tidally disrupted WD, its composition is different from that of a MS star. The composition of WDs is predicted by the yields of nuclear reactions and confirmed
Figure 4.2 The parameterized AGN (solid line), multicolor blackbody (dashed line), and the multicolor blackbody plus X-ray power law (dotted line) SEDs incident on an azimuthal segment of the debris cloud considered in model C. The segment has $R = 1.9 \times 10^{16}$ cm and $R = 1.6 \times 10^{17}$ cm at $t = 300$ days and $t = 2500$ days, respectively. Note that over time the position of the peak moves towards longer wavelengths as the temperature in the disk decreases and the total flux decreases as the accretion rate declines and the segment moves further away from the ionizing source.
with asteroseismology (Metcalfe 2003). Following Madej et al. (2004), we assumed mass fractions of 67% O, 32% C, and 1% He and all other elements with their relative abundances scaled from the solar values. Hydrogen is assumed to make up 0.001% of the WD’s mass. For each set of initial conditions (i.e., models A-F in Table 4.1), we calculated the state of the accretion flare SED and the wedge of unbound debris at a series of times after the tidal disruption. For each time step, we split the wedge into six azimuthal segments. With six segments, we were able smoothly interpolate quantities along the $\ell$ direction. Next, we calculated the density of the gas and the intensity of the ionizing radiation at the center of each segment. Several considerations went into the calculation of the latter. First, each segment is at a different distance from the accretion disk and has a correspondingly different geometric attenuation factor. Next, since the segments lie along an arc, the ionizing radiation’s angle of incidence is also different for each segment. Finally, in addition to changes in the intensity of the incident radiation, the shape of the SED incident on each segment also varies. The light travel time, $t_{lt}$, to the unbound debris furthest from the BH is comparable to the timescale on which the accretion flare evolves, so the SED of the radiation incident on these portions of the wedge was emitted at $t - t_{lt}$, when the peak was at a higher energy. Each of these considerations has a significant effect on our models. Geometric attenuation and light travel time dominate for more distant regions, but the material nearest the disk is illuminated at nearly glancing incidence, significantly reducing the intensity of the incident radiation despite the proximity to the disk. With these parameters, we are able to calculate how the unbound debris responds to the ionizing radiation emitted by the accretion disk.

The ionization state of the gas is conventionally described by the ionization parameter $U_H = Q_H/(4\pi n_H R^2 c)$, where $Q_H$ is the emission rate of photons with energy greater than 1 Ry, $n_H$ is the hydrogen number density, and $R$ is the distance to the ionizing source. However, since hydrogen is depleted in the material considered here, it is advantageous to use a more abundant reference element when calculating the ionization parameter. We chose to use oxygen because its first ionization potential is very near that of hydrogen so the ratio of the two ionization parameters is $U_O/U_H = X_O/X_H$, where $X_O$ and $X_H$ are the oxygen and hydrogen abundances by number, respectively. Thus, $U_O$ is the ratio of the density of photons capable of singly ionizing oxygen to the total density of oxygen atoms. Figure 4.3 shows how $U_O$, the total number density of all elements, and the radial distance to the IMBH change along the illuminated face of the debris cloud, $\ell$, for model C at 600 and 1200 days after tidal disruption. Since the density drops faster with distance from the accretion disk than the flux of ionizing photons does, $U_O$ increases along the $\ell$-curve. While the fallback rate is super-Eddington, the value of $U_O$ increases linearly with time at all points along the $\ell$-curve because the luminosity of the ionizing source remains constant at $L_{Edd}$, while the density of the expanding debris cloud decreases. We find numerically that $U_O$ declines approximately as $t^{-0.2}$ after the super-Eddington accretion phase.
Figure 4.3 The oxygen ionization parameter $U_O$ (dashed line), radial distance from the IMBH $R$ (dotted line), and the number density $n$ (solid line) along $\ell$, the illuminated face of the cloud (see Figure 4.1). These quantities are plotted at 600 (blue) and 1200 (red) days after the tidal disruption for model C. At these times, the super-Eddington outflows have subsided so $U_O$ declines as $t^{-0.2}$. Note that $R$ is nearly proportional to $\ell$. The horizontal, dot-dashed line shows the critical density of $[\text{O III}] \lambda 5007$. At later times, a larger portion of the cloud is below $n_{cr}$ and the region of maximum emissivity, the portion of the cloud with $n = n_{cr}$, shifts to a part of the cloud moving with a lower velocity with respect to the IMBH.

ends. This agrees with a simple analytic estimate, $U_O$ is the ratio of the density of ionizing photons to the density of oxygen atoms so $U_O \propto L_d/(T_d R^2 n) \propto t^{-1/4}$ where $L_d$ and $T_d$ are the bolometric luminosity and maximum temperature in the accretion disk, respectively. The time dependance is slightly steeper than in our models because this estimate does not adequately account for the broad peak in the SEDs.

The fractions of the first four ions of oxygen and carbon along $\vec{h}$ from a representative photoionization calculation are shown in Figure 4.4. In nearly all of our photoionization models with $\beta = 1$ (i.e., models A, C, E, and F), the dominant ion of oxygen is $O^+$ and the dominant ion of carbon is $C^+$. The exceptions are the az-
imuthal segment nearest the IMBH at times less than 300 days after tidal disruption, which consists mostly of neutral oxygen and carbon, and the segment farthest from the IMBH at times greater than 2500 days after tidal disruption, whose dominant ion are O$^{++}$ and C$^{++}$. As the density and ionization parameter change, the relative thickness of the zone dominated by each ion of oxygen does not change significantly between 300 and 2500 days after tidal disruption, so the relative abundances of the ions of oxygen do not vary significantly in these models. In the case of carbon, however, the abundance of C$^{++}$ relative to C$^{+3}$ decreases during the first 600 days after tidal disruption, before leveling off. During this time some of the C$^{++}$ is photoionized into C$^{+3}$. For the models with $\beta = 3$ (i.e., models B and D), O$^+$ and C$^+$ are the dominant ions of oxygen and carbon, respectively, throughout the debris tail for the first 300 days. In these models, the density of the unbound debris is much lower, so $U_O$ is much higher. Therefore, at later times, higher ionization species ($O^{+3}$, $O^{+4}$, $C^{+3}$, and $C^{+4}$) dominate in different portions of the cloud.

In the photoionization models, we make the simplifying assumption that the density of the debris is constant along $\vec{h}$, the path the ionizing radiation follows into the debris cloud (see Figure 4.1). This assumption is valid because the thickness of the ionized layer is very small compared to the distance to the BH, so the density in this skin only deviates from the surface value by 0.1% in the most extreme cases. In addition, we assumed that the ionized gas remains in photoionization equilibrium during the 4000 days probed by our models. This requires that the density of the gas and flux of ionizing radiation change on timescales greater than the recombination time $t_{\text{rec}} \sim (n_e \alpha_{\text{rec}})^{-1}$ of the ions of interest. Since the cloud consists mostly of carbon and oxygen, there are many electrons per ion, which keeps the electron density reasonably large even at late times when the density of the debris cloud is low. Furthermore, the range of conditions encountered in the models yielded $O^{+3}$, $O^{++}$, $O^+$, $C^{+3}$, and $C^{++}$ recombination coefficients that, in combination with the electron density, kept the recombination time well below the timescales on which the conditions in the wedge of unbound debris changed (the ranges of recombination rates are $6.7 \times 10^{-7} - 1.9 \text{ s}^{-1}$, $1.3 \times 10^{-7} - 0.52 \text{ s}^{-1}$, $4.4 \times 10^{-8} - 0.15 \text{ s}^{-1}$, $1.3 \times 10^{-7} - 0.97 \text{ s}^{-1}$, and $3.8 \times 10^{-7} - 0.98 \text{ s}^{-1}$ for the above ions at the lowest and highest electron densities, respectively). Therefore, the cloud can remain in photoionization equilibrium for over ten years.

### 4.3.2 Line profiles

Combining the dynamical and photoionization models allows us to calculate the observed profile of the emission lines in the [O iii] $\lambda\lambda 4959, 5007$ doublet. The dynamical model we have adopted gives us an analytic expression for the velocity of the material along the illuminated edge of the debris cloud ($\ell$) in the orbital plane, $v_R(\phi) \sim v_p \cot(\phi/2)$. This is the dominant component of the debris cloud’s velocity,
Figure 4.4 The fractions of the first four ions of oxygen (left panel) and carbon (right panel) along $\vec{h}$. These values are from a photoionization calculation for an azimuthal segment near the middle of the debris arc with $R = 1.8 \times 10^{17}$ cm, $n = 7.5 \times 10^4$ cm$^{-3}$, and $U_O = 0.08$, 1500 days after tidal disruption in model C, using the multicolor blackbody plus X-ray power law SED. Highly ionized species are present in the surface layers of the cloud and singly ionized carbon and oxygen are the dominant ions in deeper layers. In most of our photoionization models the dominant ions of oxygen and carbon are $O^+$ and $C^+$. 
so we neglected the motion of the material perpendicular to the orbital plane, which is smaller by a factor of $R_{WD}/R_p$ (SQ09). The velocity was projected onto the observer’s line of sight, which is defined by $i_0$ and $\theta_0$, the angles between the line of sight and the orbital plane and the pericenter direction, respectively. Figure 4.1 shows the $i_0 = \theta_0 = 0$ axis. We interpolated the [O III] $\lambda 5007$ flux computed in our photoionization models along the $\ell$-curve to determine the rest-frame flux at each velocity along $\ell$. We took the flux at 4959 Å to be one third of this value. We considered Doppler boosting of this emissivity, but the effect is insignificant at most points along the arc and produces a maximal change of 1.2%. Finally, at each point, we took into account the relativistic Doppler shift, the gravitational redshift, and local broadening. We found that the gravitational redshift was negligible in all of our models. The local broadening was assumed to have a Gaussian profile and included terms for the thermal motions of the gas, the range of velocities within each bin along the $\ell$-curve, and the range of velocities in the ionized skin, along $\vec{h}$. Since the conditions along the $\ell$-curve varied significantly, each source of local broadening considered in our models dominates in a different portion of the illuminated face of the cloud at any given time. Furthermore, the conditions in the unbound debris change substantially over time, so the dominant source of local broadening in a given portion of the debris cloud also changes over time. In the next section, we discuss the results of the photoionization and emission-line profile calculations.

4.4 The Emission Line Signature of WD Tidal Disruptions

Our photoionization calculations predict an emission-line spectrum that is dominated by lines from carbon and oxygen. The light curves of some of the strongest emission lines are shown in Figure 4.5 and the luminosities of the six strongest features at two different times are listed in Table 4.2. The values given in Figure 4.5 and Table 4.2 are from calculations performed with the multicolor blackbody plus X-ray power law SED, but calculations with the multicolor blackbody SED produced similar results because the ionization state of the gas is largely determined by the flux of UV photons, which is nearly identical in these two SEDs. In the case of the multicomponent, empirical AGN SED, the emission-line luminosities we calculated were consistent with those calculated using the other two SEDs, only varying by a factor of $\sim 2$ in the most extreme cases. The AGN SED has much stronger optical and near-IR continuum emission, which allows the continuum to outshine the emission lines, greatly reducing their equivalent widths. However, these results indicate that heating of the free electrons in the unbound material by IR radiation from the IMBH-accretion disk system does not have a strong effect on the emission line luminosities. Thus the main difference between the results from the AGN SED and the other two cases is that the optical continuum is higher and equivalent widths of the optical emission lines are lower, even though their luminosities are approximately the same. Our discussion
Figure 4.5 The light curves of five of the strongest emission lines produced in the photoionized debris of a tidally disrupted WD. These light curves come from dynamical model C and were calculated using the multicolor blackbody plus X-ray power law SED. The dotted portion of each curve shows the line luminosity while the fallback rate is super-Eddington and our model is uncertain. The dashed line illustrates the $t^{-5/3}$ dependence of the mass fallback rate. The normalization of the dashed line is arbitrary and is shown for comparison with the slopes of the emission line light curves.

will focus on the spectra predicted by calculations using the other two SEDs, which are nearly identical in the optical and UV bands.

For each model with $\beta = 1$ (i.e., models A, C, E, and F), the UV lines C iv $\lambda 1549$ and C iii $\lambda 977$ have the highest luminosity early on. The luminosities of these and other permitted lines decline over time. The luminosity of the C iii $\lambda 977$ line declines faster than the C iv $\lambda 1549$ line because of the reduction in the amount of C$^{++}$ relative to C$^{+3}$ discussed in §3.1. After 100 days, the [O iii] $\lambda 5007$, [O iii] $\lambda 4363$, and [O ii] $\lambda 7325$ lines have the largest equivalent widths. These are forbidden transitions, which are collisionally excited and then de-excite radiatively as long as the electron density is below the threshold for collisional de-excitation (i.e., the critical density for the transition $n_{cr}$, listed in Table 4.2 for reference). In the early phases
Table 4.2. Predicted Line Luminosities

<table>
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<tr>
<th>Model</th>
<th>Luminosity of [O III] λ5007 (erg s(^{-1}))</th>
<th>Luminosities of Selected Lines Relative to [O III] λ5007</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7 × 10(^5)</td>
<td>3 × 10(^7)</td>
</tr>
<tr>
<td>B</td>
<td>1.2 × 10(^37)</td>
<td>0.094</td>
</tr>
<tr>
<td>C</td>
<td>1.1 × 10(^36)</td>
<td>0.37</td>
</tr>
<tr>
<td>D</td>
<td>5.6 × 10(^36)</td>
<td>0.15</td>
</tr>
<tr>
<td>E</td>
<td>5.9 × 10(^35)</td>
<td>0.044</td>
</tr>
<tr>
<td>F</td>
<td>2.5 × 10(^37)</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>2.7 × 10(^36)</td>
<td>0.17</td>
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600 days

<table>
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<tr>
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<th>Luminosity of [O III] λ5007 (erg s(^{-1}))</th>
<th>Luminosities of Selected Lines Relative to [O III] λ5007</th>
</tr>
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</tr>
<tr>
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<td></td>
<td>8.6 × 10(^35)</td>
<td>0.015</td>
</tr>
</tbody>
</table>

2500 days

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\(^{a}\)The critical densities of the forbidden transitions were taken from the following sources: [O III] lines from De Robertis & Osterbrock (1986) [O II] λ7325 from Tsvetanov & Iankulova (1989). The value for [C I] λ8727 was estimated from the data provided by Nussbaumer & Rusca (1979) and Pequignot & Aldrovandi (1976).
of the evolution of the debris (the super-Eddington phase), the luminosity of these forbidden lines rises with time and the lines reach their maximum luminosities at 600, 250, and 300 days, respectively, when the outer tip of the ionized debris tail is at the corresponding critical density. After the peak, the line luminosities begin to decline, like the permitted lines. To understand this trend, we note that, at densities below the critical density of a transition the line emissivity per unit volume increases linearly with density and peaks at the critical density. As the density increases above the critical density, the line emissivity per unit volume drops by a factor of several (with the depletion of the upper level population by collisions) and then levels off (as the upper level is re-populated by collisions and radiative decays from higher levels). We verified this behavior by solving the population equations for the [O \textsc{iii}] \(\lambda 5007\) transition and then combining the resulting level populations with the luminosity per unit density of ions in the lower level of the transition. To carry out this exercise, we assumed a 5-level ion and followed the methodology in chapter 3 of Osterbrock & Ferland (2006). Thus, the initial rise in the luminosity of the forbidden lines is the result of the expansion of the debris while the density is above the critical density of the transition. This expansion occurs during the super-Eddington phase, when the ionizing luminosity is constant. During this expansion, the peak luminosity in the light curves corresponds to the time when the density at the outer tip of the debris tail reaches the critical density. After that time the density of an increasing portion of the debris drops below the critical density (causing a reduction in the emissivity per unit volume) and the total luminosity declines. We can quantify the rate of the initial rise in a forbidden emission line’s luminosity if it occurs while the ionizing luminosity is constant and the density in the debris cloud is above the critical density. As the debris expands, the column density of the ionized skin remains constant because the decline in the flux of ionizing photons resulting from the increasing distance from the IMBH is balanced by the increase in the cloud’s surface area. This column sustains a constant emission line flux at the surface of the cloud, so the rise in the forbidden emission line luminosity is driven by the cloud’s expanding emitting area. This area grows as \(t^2\), and this evolution can be seen in Figures 4.5 and 4.6.

We show the [O \textsc{iii}] \(\lambda 5007\) and C \textsc{iv} \(\lambda 1549\) light curves for each of the models in Figure 4.6. These curves come from calculations performed with the multicolor blackbody plus X-ray power law SED and we note that calculations that exclude the X-ray component of the incident radiation predict luminosities within a few percent of those shown. We calculated the luminosities by multiplying the emergent flux computed by Cloudy in each azimuthal segment by that segment’s emitting area and then summing the luminosities across the entire illuminated face. The light curves indicate how the evolution of the [O \textsc{iii}] \(\lambda 5007\) line depends on the properties of the WD-IMBH system and how its evolution can be used to identify such a tidal disruption event. First, the luminosity of the line at any given time increases as the mass of the IMBH decreases. This is because both the initial, maximal mass accretion...
Figure 4.6 The [O III] λ5007 (left panel) and C IV λ1549 (right panel) light curves for each model given in Table 4.1. The photoionization calculations that produced these light curves used the multicolor blackbody plus X-ray power law SED; calculations that exclude the X-ray component produce nearly identical results. The dotted portion of each curve shows the line luminosity while the fallback rate is super-Eddington and our model is uncertain. The curves in the left panel differ from those in the right because [O III] λ5007 is a forbidden emission line and the density in the debris cloud must drop to the critical density for this transition for this emission line to form. Curves B and D in the left panel are different from the others because they correspond to models with $\beta = 3$. Similarly, the curves in the right panel corresponding to models with $\beta = 3$ differ from all the others by their steep decline at early times. The dashed line illustrates the $t^{-5/3}$ dependence of the mass fallback rate. The normalization of the dashed line is arbitrary and is shown for comparison with the slopes of the emission line light curves.
rate and the emitting area of the cloud of unbound debris decrease as the mass of the IMBH increases. Next, the line reaches its peak luminosity at earlier times for large values of $\beta$ (i.e., models B and D). This is primarily because the spread in the debris is larger for smaller $R_p$ and the critical density of the transition is reached earlier. The steeper decline of the $[\text{O III}] \lambda 5007$ luminosity in models B and D is also a result of the lower density in these debris tails. The less dense debris becomes highly ionized, reducing the amount of $\text{O}^{++}$ in the cloud. In any case, the decline of the line luminosity after it reaches its maximum value is less step than the $t^{-5/3}$ decline of the mass fallback rate. Finally, if the IMBH is spinning, the inner radius of the accretion disk is much closer to the IMBH and therefore has a maximum temperature that is 2.7 times higher than that of a disk around a Schwarzschild BH. This leads to an increase in the flux of ionizing photons and a corresponding overall increase in the luminosity of the $[\text{O III}] \lambda 5007$ line.

The emission-line profile of the $[\text{O III}] \lambda \lambda 4959, 5007$ doublet depends on the properties of the WD-IMBH system. Qualitatively, the full width at half maximum (FWHM) of each line is relatively small shortly after the tidal disruption. At these times, the debris cloud is above the critical density and the line profile is dominated by emission from the outer tip of the debris cloud where the emitting area is maximal. The spread of velocities in this small segment of the cloud is comparatively narrow, resulting in an emission line with a small FWHM. This material has the greatest velocity with respect to the IMBH, so the peak of the emission line will be offset from its rest value. Eventually, the density of the material at the outer tip of the debris tail drops to $n_{cr}$ and the emission line reaches its maximum luminosity. As time goes on, material at smaller $\ell$, that is moving at slower velocities relative to the IMBH, reaches $n_{cr}$ for the $[\text{O III}] \lambda \lambda 4959, 5007$ doublet and the emissivity per unit volume increases. At the same time, the increasing volume of material in the more distant regions of the cloud partially compensates for the decreasing emissivity per unit volume of this material. As a result, material at a large range of velocities produces the $[\text{O III}] \lambda \lambda 4959, 5007$ doublet at comparable luminosities and the FWHM of each line increases. The broadening is asymmetric because the velocity of the gas along the illuminated edge increases monotonically with distance from the IMBH. The orientation of the observer determines the degree of asymmetry and whether the wing blueward or redward of the peak is broader. Furthermore, the position of the peak of each line also shifts as the debris cloud expands and the region of maximum emissivity moves toward the IMBH along the $\ell$-curve. The emissivity is largest when $n = n_{cr}$, and as the cloud expands, regions near the IMBH traveling at lower velocities relative to the IMBH reach this density and become brighter at 5007 Å. Later, when most of the unbound debris is below the critical density, the brightest region can shift away from the IMBH towards a region with a larger volume, and in some cases a larger fraction of $\text{O}^{++}$. Again, whether the peak shifts towards the red or the blue is determined by the orientation of the observer. The range of velocities along the inner edge of the debris
cloud is broader for higher mass BHs\(^1\), but since the FWHM of observed line profiles depends on the velocity along the line of sight, broader lines do not necessarily mean more massive BHs.

To illustrate these points, Figures 4.7 and 4.8 show the time evolution of the \([\text{O} \text{ III}] \) \(\lambda 5007\) line for model C, as observed from \(\theta_0 = -5^\circ\) and \(\theta_0 = 30^\circ\), respectively, with \(i_0 = 0\) in both figures. In the former, the observer is positioned such that the spread in line of sight velocities along the brightest part of the cloud is quite large. In the latter, the velocity of the material is mostly transverse to the line of sight, so the range of observed velocities in the brightest part of the cloud is relatively small. These figures do not include the \([\text{O} \text{ III}] \) \(\lambda 4959\) line, so the distinct “components” in the line profile result from the distribution of \([\text{O} \text{ III}] \) \(\lambda 5007\) emitted flux as a function of projected velocity. Unfortunately, since changes in the position of the observer result in such drastic changes in the line profile, it is difficult to extract detailed information about the nature of the WD-IMBH system from it. However, as we discuss in section 4.7, the general properties of the emission-line profiles are useful in identifying candidate WD tidal disruptions.

4.5 Extending the Model to Horizontal Branch Stars

As we discussed in section 4.2, the post-disruption dynamics are described reasonably well by prescriptions that depend only on the BH mass, the pericenter distance, and the mass and radius of the disrupted star. The star’s exact density profile does affect the rate at which material returns to pericenter at early times, however the detailed models presented Lodato et al. (2009) showed that the evolution of the debris at late times is well described by the simple, yet broadly applicable, analytic approximations presented in section 4.2. Since we are concerned with the late time evolution of tidal disruption events, we can apply our technique to a broad range of stellar types, including HB stars.

We modeled the emission-line spectrum produced when a HB star on an unbound orbit is disrupted by a BH, using the method outlined in section 4.2 and section 4.3, with a few minor changes. First, we determined the mass, radius, and composition of the HB stars used in our models with the stellar evolution code MESA star (Paxton et al. 2011). Second, in the photoionization models we only used the multicolor blackbody SED. We have not added an X-ray power law to the SEDs, as we did in some of the WD models. Photoionization by X-rays can result in significant heating of the unbound material and affect the luminosities of collisionally excited emission lines. However, we showed above that the luminosities of the optical emission lines

\(^1\)SQ09 showed that the maximum velocity of the unbound material is \(v_{\text{max}} \propto v_p R_p^{-1/2}\). For tidal disruption, \(R_p \lesssim R_T \propto M_{\text{BH}}^{1/3}\). Since \(v_p \propto (M_{\text{BH}}/R_p)^{1/2}\), the maximum velocity of the unbound material increases as \(M_{\text{BH}}^{1/6}\).
Figure 4.7 The time evolution of the $[\text{O iii}]$ $\lambda 5007$ emission-line profile as seen by an observer at $i_0 = 45^\circ$ and $\theta_0 = -5^\circ$. Time increases from bottom to top and the time for each profile is written on the plot. The dotted lines show the $F_\lambda = 0$ level for each profile. The horizontal axis shows $z = (\lambda - 5007\text{Å})/5007\text{Å}$. Each profile has been normalized so that it has a maximum height of 1 and the profiles are offset by one unit for clarity. Note that in addition to becoming broader with time, the peak of the line profile shifts to lower velocity. Once most of the cloud is below $n_{cr}$, the emissivity per unit volume decreases throughout the cloud and the peak shifts back towards higher velocities that correspond to regions in the tail with larger volume and a larger fraction of $\text{O}^{++}$. 
Figure 4.8 The time evolution of the [O III] $\lambda 5007$ emission-line profile as seen by an observer at $i_0 = 45^\circ$ and $\theta_0 = 30^\circ$. Time increases from bottom to top and the time for each profile is written on the plot. The dotted lines show the $F_\lambda = 0$ level for each profile. The horizontal axis shows $z = (\lambda - 5007\text{Å})/5007\text{Å}$. Each profile has been normalized so that it has a maximum height of 1 and the profiles are offset by one unit for clarity. Note that in addition to becoming broader with time, the peak of the line profile shifts to lower velocity. Once most of the cloud is below $n_{cr}$, the emissivity per unit volume decreases throughout the cloud and the peak shifts back towards higher velocities that correspond to regions in the tail with larger volume and a larger fraction of O$^{++}$. 
of interest here were not sensitive to this additional component. We modeled the emission lines produced with 20 different sets of initial conditions to explore the parameter space in BH mass \((30 \, M_\odot \leq M_{BH} \leq 10^4 M_\odot)\), HB star structure and composition (red clump and extreme horizontal branch), and penetration factor \(\beta\) \((1 \leq \beta \leq 10)\).

### 4.5.1 Horizontal branch star structure and composition

A globular cluster’s horizontal branch morphology is strongly influenced by its metallicity, with the horizontal branches in most metal rich globular clusters lying redward of the instability strip and those of most low metallicity clusters lying to the blue side. The globular clusters hosting the tidal disruption candidates discussed in section 4.1 have drastically different metallicities. RZ 2109 has \([\text{Fe/H}] = -1.7\), suggesting that the cluster has a very blue horizontal branch (Maccarone et al. 2011). The cluster associated with NGC 1399, on the other hand, is quite red with \(B - I = 2.25\), which implies \([\text{Fe/H}] = +0.5\) (Kundu et al. 2007). Therefore the cluster likely has a horizontal branch consisting primarily of red clump stars, as is typical for high metallically globular clusters. There is also evidence that some metal rich globular clusters harbor extreme horizontal branch (EHB) and even blue hook stars (e.g. Dalessandro et al. 2008; Rey et al. 2007; Peacock et al. 2010). To bracket the range of possibilities we considered the tidal disruption of both red clump and EHB stars.

Using MESA star, we evolved a 1 M_\odot star with \(Z = Z_\odot/2\) to determine the mass, radius, and composition of the red clump star. The star evolved to the horizontal branch after \(~ 9.9\) Gyr, at which point \(M_\star = 0.67\, M_\odot\) and \(R_\star = 6.2\, R_\odot\) (see Figure 4.9). Furthermore, CNO cycle burning had enriched the core of the star with nitrogen at the expense of carbon and oxygen. Leading up to the HB phase, the total mass fraction of nitrogen rose and those of oxygen and carbon fell as the star’s relatively carbon and oxygen rich envelope was shed. The ratio of nitrogen to oxygen is highest right after helium ignition, at the start of the HB phase. We used the mass, radius, and composition of the HB star at this point as our fiducial model. At this point the star was made up of 15% H, 84% He, 0.1% C, 0.5% N, and 0.2% O, by mass, with all other elements present in their initial mass fraction. The mass and radius of our adopted HB star is consistent with previous models (e.g., Dorman 1992; Charbonnel et al. 1996). Furthermore, measurements of the surface abundances in HB stars have found nitrogen enhancements, but the surface abundance of nitrogen, which depends on a star’s mixing history, is highly variable from star to star (e.g., Behr et al. 1999; Gratton et al. 2000; Tautvaišienė et al. 2001).

We found \(M_\star = 0.49\, M_\odot\) and \(R_\star = 0.12\, R_\odot\) for an EHB star by running a MESA star model of a 1 M_\odot star with \(Z = 0.1\, Z_\odot\). This combination of \(M_\star\) and \(R_\star\) are in good agreement with the asteroseismology measurements of the masses and radii of hot subdwarfs (Charpinet et al. 2008; van Grootel et al. 2010). The composition of
Figure 4.9 Mass fractions (top) of carbon (solid), nitrogen (dotted), and oxygen (dashed) vs. time and radius (bottom) vs. time from the MESA star model of a 1 M$_\odot$ star with $Z = Z_\odot/2$.

The EHB star was 5.2% H, 94.5% He, $5 \times 10^{-3}$% C, 0.1% N, and 0.03% O.

4.6 The Emission Line Signature of HB Star Tidal Disruptions

The light curves for the emission lines produced in the photoionized debris of a tidally disrupted HB star are qualitatively similar to the light curves expected in the WD case. The light curves for some of the brightest emission lines produced when a red clump HB star is tidally disrupted by a 1000 M$_\odot$ IMBH are shown in Figure 4.10. Just as in the WD case, the spectra are dominated by bright recombination lines at early times. As the unbound debris tail expands and the density of the material drops, the luminosities of these permitted lines fade and emission lines produced by forbidden transitions become brighter. However, when a HB star is tidally disrupted a different set of emission lines are expected to shine for a much longer times, owing
Figure 4.10 Light curves produced in the photoionized debris of a tidally disrupted HB star. In this model we used a red clump star with $M_\star = 0.67 \, M_\odot$ and $R_\star = 6.2 \, R_\odot$ and an IMBH with $M_{BH} = 1000 \, M_\odot$. The dotted portions of the curves denote the super-Eddington fallback phase. Note the qualitative similarities to the WD case shown in Figure 4.5.

to differences in composition and structure between a HB star and a WD. Bright H$\alpha$ and He I $\lambda 5876$ lines may be observable shortly after disruption, before they become too faint to stand out above the continuum emission. At late times, in addition to the same [O III] doublet expected in the WD tidal disruptions, HB tidal disruptions can also emit a bright [N II] $\lambda \lambda 6548, 6583$ doublet. The luminosities of these emission lines evolve over much longer time scales than in the WD case, because the tidal disruption radius for a HB star is much further from the BH. Accordingly, the unbound debris is launched at much lower velocities and expands relatively slowly. Since changes in the density of the debris tail drive the evolution in the emission line luminosities, we expect the HB star emission lines will remain bright for over one hundred years. A more detailed discussion of the emission lines produced in HB star tidal disruptions will be presented when we compare these models to observations in section 4.8.1.
4.7 Discussion of the Models

We have presented calculations of the line emission produced when the unbound debris of a WD that has been tidally disrupted by an IMBH is illuminated by the associated accretion flare. We found that the two strongest emission lines, by far, are C iv $\lambda$1549 at early times and [O iii] $\lambda$5007 at late times. Furthermore, the emission lines formed in the illuminated face of the debris cloud will have broad, asymmetric profiles with velocity widths of several hundred to a few thousand km s$^{-1}$. Our results suggest that UV and optical spectra whose strongest features are broad C iv $\lambda$1549 and [O iii] $\lambda$5007 emission lines can serve to identify WD tidal disruption events, when they are detected in addition to an X-ray flare in the center of a globular cluster or dwarf elliptical galaxy. Figure 4.6 shows how the characteristic timescales and luminosities of these emission lines scale with $R_p$ and the mass of the IMBH. The luminosities of the C iv $\lambda$1549 and [O iii] $\lambda$5007 lines decline more slowly if $\beta = 1$ than they do if $\beta = 3$. For a constant $\beta$, the shapes of the emission-line light curves do not change much when the mass of the IMBH is changed, however, systems with lower mass IMBHs produce brighter C iv $\lambda$1549 and [O iii] $\lambda$5007 emission. The emission line luminosities are sensitive to the density of the photoionized debris, and since this density is only weakly dependent on $M_{BH}$, the line luminosities evolve on similar timescales despite a factor of 100 increase in $M_{BH}$. In these models $n = n_0/R^3$, where the normalization factor

$$n_0 = \frac{M_{WD}^{5/6}M_{BH}^{1/6}}{8\mu(3\beta)^{1/2}} \left\{ \frac{\tan \left( \frac{\pi - \Delta \phi}{2} \right)}{\sin \Delta i \left[ \log \left( \frac{\sin \frac{\Delta \phi}{2}}{2} \right) + 6 \right]} \right\}$$  \hspace{1cm} (4.10)$$

ensures that the mass of the unbound debris tail is 0.5 $M_{WD}$. Here we have used the mean molecular mass $\mu$. Numerical fits to the quantity in the curly brackets show that it increases with the mass of the IMBH as $M_{BH}^{0.54}$ when $\beta$ is held constant. Altogether, we find that the density of the unbound debris is only weakly dependent on the mass of the IMBH, $n \propto M_{BH}^{0.21}$, so it is difficult to constrain $M_{BH}$ with the [O iii] $\lambda$5007 light curve.

We have also presented models of HB star tidal disruptions. We found that the emission lines produced in the photoionized debris of a disrupted HB star will remain bright for hundreds of years. During these late stages of the disruption, we expect the optical spectra emitted by the HB star debris to be similar to that emitted in the late stages of a WD tidal disruption. In both cases the spectra exhibit bright [O iii] doublets and are devoid of hydrogen Balmer lines. However, the presence of a bright [N ii] doublet, unique to HB disruptions, can be used to discriminate between the two scenarios.

Our work drew heavily on the SQ09 model for the tidal disruption of a MS star
by an supermassive BH, so we summarize the uncertainties in the model described there. First, since the accretion disk becomes geometrically thin after $M$ drops below the Eddington rate, it becomes susceptible to viscous instabilities that impede the steady flow of material through the disk and this could result in a deviation from the smooth $t^{-5/3}$ dependance of the ionizing radiation. Material will accumulate in the disk rather than accreting onto the BH when disk’s viscous timescale becomes comparable to the time since tidal disruption. This occurs at $t \sim 8 - 12$ yr for the disks in our WD tidal disruption models and at $t \sim 200$ yr in the HB star disruption models. Furthermore, the model is no longer valid when the mass accretion rate drops below $\sim 0.01 \dot{M}_{\text{Edd}}$ and the disk becomes radiatively inefficient. The final time steps in WD models B-F are nearing this limit, and this change will lead to a reduction in the number of UV photons not accounted for in this model. Since the photoionization cross-sections for O and C decline significantly from UV to X-ray energies, the ionization state of the unbound debris depends sensitively on the flux of UV photons whose energies are near the ionization potentials of C and O. Next, a significant amount of initially bound material can be driven away from the IMBH in the super-Eddington outflows. The impact that the outflows have on emission from the unbound material is uncertain. SQ09 pointed out that the outflows could also be photoionized by the accretion flare and generate additional line luminosity. Additionally, since we did not account for how the outflows affect the radiation that illuminates the unbound debris, the emission line luminosities predicted for $t < t_{\text{Edd}}$ are uncertain. The impact of the super-Eddington fallback phase is especially strong in our HB star tidal disruption models, where this phase lasts between 90 and 155 years.

The modifications we made to the SQ09 model allowed us to test how some of their simplifications affect the predictions of the model. In their photoionization calculations, they simplify the geometry of the wedge of unbound debris by approximating it as a cloud separated from the ionizing source by $R_{\text{max}}$ (i.e. the maximum of equation (4.6) or $R(\pi - \Delta \phi)$). Furthermore, they assume that the density of the unbound material is uniform and evaluate it by placing half the mass of the star in a region of volume $R_{\text{max}}^2 \Delta R \Delta i \Delta \phi$. In our calculations, on the other hand, we have accounted for the changing ionization state of the material along the illuminated edge of the cloud by allowing the density and intensity of the ionizing radiation to vary along this edge, as discussed in section 4.3.1. Both methods produce similar emission-line light curves, with the SQ09 model predicting slightly higher luminosities for most lines. This was true for both the WD-IMBH case and the MS-supermassive BH case. Our models also show that the choice of SED for the accretion flare does not have a strong affect on the emission line luminosities. We conclude, therefore, that their simpler method is sufficient to predict the luminosities and light curves of optical emission lines from tidally disrupted stars.

One interesting facet of our WD tidal disruption models that deserves further
exploration involves the composition of the unbound material. The models of Luminet & Pichon (1989) and Rosswog et al. (2009) found that compression of the WD as it passes through pericenter resulted in nuclear burning. The amount of energy released by these reactions and how significantly they alter the composition of the debris depends on the initial composition of the WD, $M_{\text{BH}}$, and $R_p$. To explore how a change in the abundances brought about by nuclear burning would influence the emission lines emitted by the unbound debris, we also computed photoionization models using abundances given in Rosswog et al. (2009) for a 1.2 $M_\odot$ WD ($R_{\text{WD}} = 2.7 \times 10^8$ cm) that is tidally disrupted by a 500 $M_\odot$ IMBH with $\beta = 3.2$. For the first year, the brightest lines are the UV lines C iv $\lambda 1549$, Si iii $\lambda 1888$, and O vi $\lambda 1035$. Later however, the emission-line spectrum is dominated by forbidden iron lines rather than carbon and oxygen lines. The most luminous lines include [Fe x] $\lambda 6373$, [Fe vi] $\lambda 6807$, [Si vii] $\lambda 1446$, [Fe vii] $\lambda 3759$, [Fe v] $\lambda 3892$, and [Fe vii ]$\lambda 5271$. While, this scenario represents an extreme case in which the energy released by nuclear burning is greater than the binding energy of the WD, the presence of these iron and silicon lines in a spectrum that is otherwise similar to that described above could be evidence of nuclear burning during the tidal disruption.

4.8 Comparison to Observations

The next question to consider is how often we should expect to see emission lines from tidally disrupted WDs or HB stars. Ramirez-Ruiz & Rosswog (2009) estimate that the rate of $L_{\text{Edd}}$ flares from MS stars being tidally disrupted by $10^3 - 10^4 M_\odot$ IMBHs in globular clusters is $\sim 4000$ yr$^{-1}$ Gpc$^{-3}$. Following these authors, we assume all globular clusters have an IMBH in their center and adopt a globular cluster space density of $n_{\text{GC}} \sim 6$ Mpc$^{-3}$ (Brodie & Strader 2006). We take the WD tidal disruption rate of $10^{-8}$ yr$^{-1}$ (globular cluster)$^{-1}$ from Sigurdsson & Rees (1997) and arrive at a WD tidal disruption rate of $\sim 60$ yr$^{-1}$ Gpc$^{-3}$, 67 times lower than the MS rate. This rate is a very optimistic estimate because it is unlikely that every globular cluster has an IMBH in its center and the actual rate could be much lower. The likelihood of observing such an event is further reduced by its relatively short lifetime of $\sim 10$ yr.

As was mentioned in section 4.1, despite the meager rate and limited observing window, Irwin et al. (2010) proposed the tidal disruption of a WD by an IMBH as an explanation for two recent observations of X-ray and optical emission in the centers of globular clusters. We will review the observations in greater detail now. The first candidate is in the center of globular cluster RZ 2109 associated with NGC 4472. Maccarone et al. (2007) measured the source’s X-ray luminosity to be $L_X = 4 \times 10^{39}$ erg s$^{-1}$. In follow up optical observations, Zepf et al. (2008) found that the [O iii] $\lambda 5007$ line luminosity was $1.4 \times 10^{37}$ erg s$^{-1}$ and that the line’s FWHM was 1500 km s$^{-1}$. Irwin et al. (2010) reported on the second candidate, the ultraluminous X-ray source CXOJ033831.8-352604, which lies in a globular cluster associated with
NGC 1399. The source has a $0.3 - 10$ keV luminosity of $\sim 2 \times 10^{39}$ erg s$^{-1}$ as well as $\text{[O iii]} \lambda 5007$ and $\text{[N ii]} \lambda 6583$ emission lines with luminosities of $\text{few} \times 10^{36}$ erg s$^{-1}$ and FWHMs of 140 km s$^{-1}$. Balmer lines were not detected in the spectrum, but the following lower limits were placed on the emission line luminosity ratios: $\text{[N ii]} \lambda 6583/\text{H}$α $> 7$ and $\text{[O iii]} \lambda 5007/\text{H}$β $> 5$.

The X-ray luminosity of the source in RZ 2109 is consistent with the WD tidal disruption models that assumed a multicolor blackbody plus X-ray power law SED for the accretion flare. The optical emission lines observed by Zepf et al. (2008) are also consistent with these models in terms of both luminosity and FWHM. The observed luminosity of the $\text{[O iii]} \lambda 5007$ line falls within the range of luminosities predicted for systems with IMBH masses between 100 - 1000 $M_\odot$. Figure 4.11 shows a synthesized $\text{[O iii]} \lambda \lambda 4959, 5007$ emission-line profile that has been smoothed to the same spectral resolution as the observations of Zepf et al. (2008). The line profile was synthesized for model C with the observer at $i_0 = 89^\circ$ and $\theta_0 = -12^\circ$. We have chosen these values to match the shape of the observed emission-line profile. The first similarity between the model and the observations is that the lines are so broad that they are blended. The FWHM of the model line profile is 1700 km s$^{-1}$. Also, both the observed and modeled profiles are asymmetric, with the blue wing much broader than the red. This suggests that we are oriented with the flow such that $v_{\text{max}}$ is nearly parallel to our line of sight.

Even though there seems to be good agreement between our models and the observations, there are several drawbacks to interpreting the source in RZ 2109 as a WD tidal disruption. The first involves the duration of the X-ray flare and the observed $\text{[O iii]} \lambda 5007$ luminosity. An X-ray luminosity of $8.5 \times 10^{39}$ erg s$^{-1}$ was measured for this source with ROSAT in 1992 (Colbert & Ptak 2002). The change in luminosity between the ROSAT observation and the observation reported in Maccarone et al. (2007) is consistent with the model for the X-ray flare, if the ROSAT observation detected the flare $\sim 1$ year after the WD was tidally disrupted. However, this means that the $\text{[O iii]} \lambda 5007$ luminosity was still $1.4 \times 10^{37}$ erg s$^{-1}$ more than 15 years after the tidal disruption, and long after the peak in $\text{[O iii]} \lambda 5007$ emission predicted by this model (see Figure 4.6). The model can only explain this $\text{[O iii]} \lambda 5007$ luminosity if the IMBH in this globular cluster is $100 M_\odot$ and maximally spinning. Even though it is possible to tune the model to explain the observed X-ray and $\text{[O iii]} \lambda 5007$ luminosities, because the high emission line luminosity persists for many years longer than it would in all but the most favorable scenario, it is unlikely that this source is a WD that has been tidally disrupted by a IMBH, according to the model. Next, the large amplitude X-ray variability of the source reported in Maccarone et al. (2010) is inconsistent with the steady luminosity decline expected from a tidal disruption event. Finally, improved measurements have revealed that the photoionized debris of a tidally disrupted WD is an unlikely origin of $\text{[O iii]} \lambda \lambda 4959, 5007$ doublet seen RZ 2109. Using a higher spectral resolution and signal-to-noise ratio observation of
the doublet, Steele et al. (2011) found that the emission line profile was well fit by a two component model with a bipolar conical outflow and a lower velocity Gaussian component. Peacock et al. (2012) performed spatially resolved spectroscopy on the cluster and discovered that the [O m] emitting region has a half-light radius of 3-7 pc. This is two orders of magnitude larger than the tidal debris tail produced after a WD is disrupted (see Figure 4.1).

Alternate explanations for the source of these emission lines have been proposed. Maccarone et al. (2010) suggested that the source is a hierarchical triple system with an inner, mass transferring BH+WD binary. Such a system is in line with the theoretical work of Ivanova et al. (2010), who suggested that a triple system may be necessary to form a mass transferring BH+WD binary. Ripamonti & Mapelli (2012) showed that nova ejecta that have been photoionized by a ULX could produce bright, broad [O m] emission lines similar to those observed in RZ 2109.

If, in spite of the above discrepancies, the source in RZ 2109 is a WD that has been tidally disrupted by a IMBH, then the strongest line in a UV spectrum will be C iv λ1549. Unlike the [O m] λ5007 emission line, the C iv λ1549 flux is fairly
constant across the illuminated face of the cloud so most of this line’s luminosity is
generated in the outer portions of the debris tail where the emitting area is largest.
This results in an emission-line profile consisting of a redshifted core with a FWHM
of several hundred km s$^{-1}$ and a broad blue shoulder. Furthermore, if the X-rays
decline dramatically in the future, the [O III] luminosity should also dim over the
course of one year according to this model. The delay in the decline is due to the
light travel time from the IMBH to the distant portions of the unbound debris that
generate most of the [O III] luminosity.

4.8.1 HB star tidal disruption candidate

Although Irwin et al. (2010) proposed that CXOJ033831.8-352604 and the spatially
coincident [O III] and [N II] emission lines were the result of a WD tidal disruption,
the observational properties of this source are better explained by our HB star tidal
disruption models. In fact, each combination of $M_{\text{BH}}$, HB star type, and $\beta$ that we
considered in our suite of HB star tidal disruption models predicted [N II] $\lambda$6583 and
[O III] $\lambda$5007 emission lines with luminosities $\sim 10^{36}$ erg s$^{-1}$ at some point during the
post tidal disruption evolution. Beyond merely identifying this source as a HB star
tidal disruption candidate, we can use our models to probe the BH present in this
globular cluster. Only models with a red clump HB star, a BH with $M_{\text{BH}} \lesssim 200 M_\odot$, and $\beta \sim 1$ were able to reproduce the [N II] $\lambda$6583/H$\alpha$, [O III] $\lambda$5007/H$\beta$, and [N II]
$\lambda$6583/[O III] $\lambda$5007 emission line luminosity ratios reported in Irwin et al. (2010).

We illustrate this fact in Figure 4.12, which shows the time evolution of the line
ratios in a subset of our models. In the models plotted here, we have used our
fiducial red clump HB star and assumed that $\beta = 1$, but varied $M_{\text{BH}}$. In each of
these models, the Balmer lines outshone the [O III] and [N II] lines initially. Then,
as the debris cloud expanded and $n$ decreased, the luminosity of the Balmer lines
diminished because the volume emissivity of these permitted transitions is proportional
to the product of the electron and ionized hydrogen number densities. At the same
time, the [O III] $\lambda$5007 and [N II] $\lambda$6583 lines became brighter. These forbidden lines
are produced when ions are collisionally excited into a metastable state and then
de-excite radiatively. As the density in the unbound debris decreased, the collision
rate declined, allowing a larger fraction of the excited ions to decay radiatively, as
opposed to collisionally, and the forbidden emissions lines grew brighter. As a result,
all of the models shown in Figure 4.12 can satisfy the [O III]/H$\beta$ constraint, provided
we are observing the debris more than 40 years after tidal disruption. The models
with $M_{\text{BH}} = 10^3$ and $10^4 M_\odot$ were unable to reproduce the observed [N II]/H$\beta$ ratio.
These more massive BHs permitted higher accretion rates that led to an increased
flux of ionizing photons. By the time the density in the debris was low enough to
produce a significant [N II] luminosity, [N II] emission was suppressed because the
cloud had become so highly ionized that much of the N$^+$ was further ionized to N$^{++}$. 
Figure 4.12 Emission-line luminosity ratios over time from several tidal disruption models with varying $M_{\text{BH}}$. The long-dashed line in the top panel shows the constraint on the $\text{[N II]}/\text{[O III]}$ ratio measured by Irwin et al. (2010). Models that fall near this line are consistent with observations. In the lower two panels, the shaded region indicates line ratios ruled out by the observations of Irwin et al. (2010), only models that reach into the unshaded region are consistent with observations. For each model shown here, we considered a red clump star on an orbit with $R_p = R_T$. 
Figure 4.13 Emission-line luminosity ratios over time from several tidal disruption models with varying $M_{BH}$, tidal disruption parameter $\beta$, and stellar type. The long-dashed line in the top panel shows the constraint on the $[\text{N II}] / [\text{O III}]$ ratio measured by Irwin et al. (2010). Models that fall near this line are consistent with observations. In the lower two panels, the shaded region indicates line ratios ruled out by the observations of Irwin et al. (2010), only models that reach into the unshaded region are consistent with observations.
This over-ionization of the unbound debris drove the evolution of the $\frac{[N \, II]}{[O \, III]}$ ratio. In all of the models, the ratio steadily declined as an increasing portion of the oxygen and nitrogen in the cloud became doubly ionized. With $M_{\text{BH}} = 10^3$ or $10^4 M_\odot$, the unbound debris became too highly ionized for the $\frac{[N \, II]}{[O \, III]}$ ratio to be $\sim 1$ within ten years of tidal disruption (see Figure 4.12). We found that in models with $M_{\text{BH}} < 200 M_\odot$, the decline is slow enough that the luminosities of the two forbidden emission lines can differ by less than a factor of two for up to 200 years.

Figure 4.13, shows a number of alternative models for strong disruption and/or blue HB stars. These models fail to reproduce the observed $\frac{[N \, II]}{[O \, III]}$ ratio for an extended period of time, in contrast to our models of weakly disrupted red HB stars. Like the models involving high mass BHs discussed above, the unbound debris cloud becomes too highly ionized to produce significant $[N \, II]$ emission. Using a simple estimate of the ionization parameter $U_H$ (defined in section 4.3.1), we can illustrate why some debris clouds become over ionized and others do not. Recasting the ionization parameter in terms of our model parameters, we find $U_H \propto R_T^{-1/2} \beta^3 \dot{M}^{3/4} t$. Debris from stars with a larger kinetic energy at pericenter passage, due to a more compact star with smaller $R_T$, an orbit with $\beta > 1$, or a more massive BH, spanned a wider range in velocity and quickly expanded to a low density, resulting in a large $U_H$. As mentioned above, more massive BHs lead to a higher flux of ionizing photons, also contributing to increased $U_H$. During the super-Eddington phase, $\dot{M} \propto M_{\text{BH}}$ was constant and the ionization parameter increased with time. The $N^+$ fraction reached a maximum\(^1\) when $U_H \sim 0.06$, and at larger values of $U_H$, the luminosity of the $[N \, II]$ line declined. Keeping the ionization state in the unbound debris low enough to produce significant $[N \, II]$ emission after the super-Eddington phase had ended required the relatively large radius of the red clump star and a BH of relatively low mass, $M_{\text{BH}} < 200 M_\odot$. After the super-Eddington phase, $U$ declined as $t^{-1/4}$ and N$^++$ recombined into N$^+$. This drove the $\frac{[N \, II]}{[O \, III]}$ ratio back towards unity in some cases (see models with $\beta > 1$ in Figure 4.13). However, at this point the density in the cloud is so low that the luminosities of the lines are well below the observed values.

Finally, by combining the dynamical model with the photoionization model, we synthesized the emission line profiles of the $[N \, II]$ and $[O \, III]$ lines. Figure 4.14 shows the range of observed FWHM of the $[N \, II]$ line produced in a the model of a red clump star on an orbit with $\beta = 1$ being tidally disrupted by a $100 M_\odot$ BH, 125 years after tidal disruption. The angles $\theta_0$ and $i_0$ describe the orientation of the observer, and are the angle between the line of site and the direction of pericenter and the orbital plane, respectively (see Figure 4.1). The black line shows the observed FWHM $= 140 \text{ km s}^{-1}$. For a range of observer orientations, the FWHM of the synthesized

\(^1\)The value of $U_H$ at which this maximum occurs is abundance dependent. The value given is for the red clump star composition. The maximum occurs at a much smaller value of $U_H$ if solar abundances are used.
Figure 4.14 FWHM of the [N II] λ6583 emission line 125 years after tidal disruption, synthesized from a model using the red clump star, $\beta = 1$, and $M_{BH} = 100 M_\odot$. The angles $\theta_0$ and $i_0$ describe the orientation of the observer (see Figure 4.1). The black curve marks the observed FWHM. Any pair of $\theta_0$ and $i_0$ along the black line produces a FWHM that is consistent with observations.

emission lines is consistent with the observations.

4.8.1.1 Analysis of the HB star tidal disruption scenario

We have shown that the photoionized debris of a tidally disrupted HB star can account for the emission lines observed in optical spectra presented in Irwin et al. (2010). Reproducing the line ratios within the context of this model requires $M_{BH} \lesssim 200 M_\odot$. We did not consider spinning BHs in our study of HB star tidal disruption. The last stable orbit around a spinning BH is closer to the event horizon, allowing the accretion disc to extend to smaller radii than the discs considered here. Accretion discs around spinning BHs produce a larger flux of ionizing photons with an SED that peaks at a higher photon energy because the maximum temperature in these discs is larger than that of accretion discs around non-spinning BHs. However, unlike
the supermassive BHs found in the cores of galaxies, BHs in GCs have not gained an appreciable portion of their mass though disc accretion. Belczynski et al. (2008) found that any accretion that does occur while the BH is in a mass transferring binary does not substantially increase its spin. However, it is possible that the GC BHs are born with a large spin. If this is the case, the HB tidal disruption scenario remains a plausible explanation for the observed emission lines, but the BH mass inferred from the models would change by a factor of a few.

The \([\text{N} \text{ II}] / [\text{O} \text{ III}]\) ratio is sensitive to the abundance of nitrogen and oxygen in the HB star. In the models discussed above, we used the composition of the HB star at the onset of helium burning, when the ratio of nitrogen to oxygen is largest. Helium burning destroys nitrogen and creates oxygen, so \(X_N / X_O\) declines as the star evolves on the HB. Figure 4.15 shows how \([\text{N} \text{ II}] / [\text{O} \text{ III}]\) responds to changes in composition of the HB star. In these models, we used the composition of the red clump star at later times, as computed in the MESA star models, corresponding to points where the nitrogen and oxygen abundance are equal, and where the nitrogen abundance is half that of oxygen. The mass and radius of the star did not change appreciably in this time, so we used the same values as in our fiducial model; and we used \(\beta = 1\). When \(M_{\text{BH}} = 50 \, M_\odot\), the models with both abundance sets are consistent with the observed \([\text{N} \text{ II}] / [\text{O} \text{ III}]\). The \([\text{N} \text{ II}] / [\text{O} \text{ III}]\) ratio drops too quickly to be consistent with observations when \(M_{\text{BH}} = 100 \, M_\odot\) and the oxygen abundance is twice that of nitrogen. This sets the period during which the HB star can be disrupted by the BH and produce the observed emission line spectrum. For the 100 \(M_\odot\) BH, the HB star must be disrupted within 30 Myr of helium ignition to satisfy the \([\text{N} \text{ II}] / [\text{O} \text{ III}]\) constraint, while a 50 \(M_\odot\) BH can disrupt the HB star within 80 Myr of helium ignition and meet the constraint.

It is appealing that the tidal disruption models require a red horizontal branch star to meet the observational constraints on the emission line ratios. The host globular cluster is red and likely exhibits the red horizontal branch morphology typical of a metal rich cluster. However, given the predicted rate at which globular cluster BHs tidally disrupt stars, it is surprising that such an event might have been observed. Employing the tidal disruption rate equation computed by Baumgardt et al. (2004), with the mass and radius of a red clump HB star, \(M_{\text{BH}} = 100 \, M_\odot\), a core density of \(5 \times 10^5 \, \text{pc}^{-3}\), a core velocity dispersion of 10 km s\(^{-1}\), and assuming that 5% of the stars in a globular cluster’s core are HB stars, we find a HB tidal disruption rate of \(1.2 \times 10^{-10} \, \text{yr}^{-1} \, (\text{globular cluster})^{-1}\). Given that the bright \([\text{N} \text{ II}]\) and \([\text{O} \text{ III}]\) emission lines persist for 200 years, and that the space density of globular clusters is \(\sim 6 \, \text{Mpc}^{-3}\) (Brodie & Strader 2006; Croton et al. 2005), at any time there should only be \(5 \times 10^{-3}\) observable HB tidal disruptions within the 20 Mpc distance to NGC 1399. Furthermore, if the NGC 1399 globular cluster source is the aftermath of a HB tidal disruption, it is also puzzling that several main sequence tidal disruptions have not been observed because they should occur more frequently (Baumgardt et al.
Figure 4.15 Evolution of \([\text{N II}]/[\text{O III}]\) for HB stars of different composition. We show two sets of models – young HB stars 30 Myr after He ignition when the nitrogen and oxygen abundance are equal \((M_{\text{BH}} = 50M_\odot \text{ dashed}; M_{\text{BH}} = 100M_\odot \text{ dotted})\) and the same models but with the HB star older so that 80 Myr of He burning has made oxygen twice as abundant as nitrogen \((M_{\text{BH}} = 50M_\odot \text{ dot-dashed}; M_{\text{BH}} = 100M_\odot \text{ solid})\). For a 100 M_\odot BH, the observed \([\text{N II}]/[\text{O III}]\) ratio cannot be reproduced once helium burning has created an oxygen abundance that is twice the nitrogen abundance.

The low disruption rate and the dearth of main sequence tidal disruptions are only an issue if the tidal disruption occurs through the conventional loss cone orbit channel. One possibility that would result in the preferential disruption of post main sequence stars is for the disrupted star to be bound to the BH. If this is the case, tidal disruption could be triggered in two different scenarios; 1) a scattering induced merger or 2) a triple decay. In a scattering induced merger, the HB star progenitor is bound to the BH, expands to fill its Roche Lobe while on the giant branch, and transfers material to the BH. During mass transfer, the semi-major axis of the binary expands, greatly increasing the likelihood that the binary will undergo an encounter with another star in the globular cluster. Such an encounter could impulsively increase the eccentricity.
and decrease the semi-major axis of the binary and result in the tidal disruption of
the bound star (Davies 1995; Sigurdsson & Phinney 1993, 1995). Successive distant
encounters can also to drive the eccentricity high enough for \( R_p = R_T \). We have
estimated the timescale for this process using the expression given in Maccarone
(2005), which uses the encounter cross sections computed by Heggie & Rasio (1996).
In a binary with a post-mass-transfer semi-major axis of 1 AU containing a 100 M\(_\odot\) BH,
a 0.65 M\(_\odot\) HB star will be driven to the tidal disruption radius in 10 Myr. This
is well within the 30 Myr window required to produce [N ii] and [O iii] emission lines
of similar luminosity. Here we have assumed that the field stars have mass 0.8 M\(_\odot\), a
cluster density of \( 5 \times 10^5 \) pc\(^{-3} \), and a velocity dispersion of 10 km s\(^{-1}\). A scattering
induced merger would be more likely in a dense globular cluster.

In the triple decay, the BH and HB star progenitor are initially the inner binary
in a stable, hierarchal triple system. Again, evolution of the HB progenitor results
in a phase of mass transfer and expansion of the inner binary’s semi-major axis. In
this case, expansion of the inner binary drives the triple to instability and triggers
the tidal disruption. A similar scenario was discussed by Perets & Kratter (2012),
only these authors invoked mass loss, not mass transfer, to drive the expansion of
the inner binary. The triple decay scenario would require a medium density globular
cluster, because triple systems are unstable to external perturbations in dense globular
clusters. Both the scattering induced merger and triple decay scenarios increase the
odds that tidal disruptions are linked to post main sequence stars, but determining
if disruption through either scenario would occur more frequently than the loss cone
tidal disruptions discussed above is beyond the scope of this chapter.

Other models have been proposed to explain the emission lines observed in this
globular cluster. Porter (2010) argued that the emission lines are produced in an
accretion disc around an IMBH of mass \( \gtrsim 100 M_\odot \). Alternatively, Maccarone et al.
(2011) showed that the lines observed in the NGC 1399 globular cluster could be
formed in the photoionized winds of R Corona Borealis stars. Fortunately, UV spec-
troscopy offers a means of determining whether any of these proposed scenarios can
explain the source. Our models predict several strong UV emission lines. These
lines could be used to test the tidally disrupted HB star hypothesis for the ULX in
NGC 1399 and to further constrain the mass of the BH. The strongest lines are N \( \nu \)
\( \lambda \lambda 1239, 1243, \) O \( \nu \) \( \lambda \lambda 1032, 1038, \) N \( \nu \) \( \lambda 1486, \) and C \( \nu \) \( \lambda \lambda 1548, 1550. \) These lines
should have FWHM similar to the observed optical lines. The bright C \( \nu \) doublet
could be used to constrain the composition of the disrupted star and narrow the allowed range in \( M_{BH} \). The light curves for the bright UV emission lines are shown in
Figure 4.16 along with the X-ray luminosity of the accretion flare, scaled down by a
factor of 2000 for plotting purposes. In contrast, Maccarone et al. (2011) do not
report any strong UV lines from their models of photoionized RCB star winds.
Figure 4.16 UV Light curves from a model using the fiducial red clump HB star, $\beta = 1$, and $M_{BH} = 100 \, M_\odot$. Emission line light curves are shown for N v $\lambda 1240$, C iv $\lambda 1548$, O vi $\lambda 1035$, N iv $\lambda 1486$, C iii] $\lambda 1909$, and Lyman-$\alpha$. The dashed curve shows $L_X$ scaled down by a factor of 2000. These UV lines can be used to test the HB star tidal disruption hypothesis and to better constrain the mass of the IMBH. The dotted portion of each curve shows the line luminosity while the fallback rate is super-Eddington and our model is uncertain.

4.9 Conclusion

We have presented models for the emission lines generated in the photoionized debris of tidally disrupted WDs and HB stars. We have compared our models with observations of two WD tidal disruption candidates, and we disfavor the interpretation of either sources as a WD tidal disruption event. However given the capabilities of current and planed optical and X-ray transient surveys, the number of candidate WD tidal disruption events should increase in the near future. These candidates can be vetted with follow up optical and UV spectroscopy that searches for strong, broad [O iii] $\lambda 5007$ and C iv $\lambda 1549$ lines that evolve as shown if Figure 4.6. Finally, within the assumptions made in our models, the emission line spectrum produced when a red clump star is disrupted by a BH with $M_{BH} \lesssim 200$ is consistent with the optical emission lines observed by Irwin et al. (2010) in the ULX hosting globular cluster in NGC 1399. The models also make testable predictions about the source’s UV-emission line spectrum. Observations of the UV-emission lines could significantly improve our understanding of this source and are likely possible with the Hubble Space Telescope.
Chapter 5

Summary and Outlook

As a consequence of the high stellar density in globular clusters, any BHs present in these clusters are likely to undergo strong encounters with stars and other stellar remnants within the cluster. The goal of this thesis was to investigate how the observable outcomes of such interactions can be used to probe the nature of the BHs harbored by globular clusters. Since little is known about the mass distribution and size of a globular clusters’s BH population, we considered a wide range of possibilities. Our models studied encounters involving both stellar and intermediate mass BHs, and considered populations ranging from a single BH up to nearly 200 BHs. In each case, we determined how the properties of these populations are imprinted on sources of electromagnetic and/or gravitational radiation in the cluster. We will briefly summarize our findings and suggest some directions for future study below.

We explored the dynamical formation of BH+NS binaries through exchange interactions between binary and single stars. In chapter 2, we discussed the prospect of detecting the gravitational waves emitted by these binaries as they merge. The merger rate depends critically upon whether or not the BH is retained by the cluster after the merger. If the BH is retained, it can acquire and merge with additional NS companions, thereby greatly enhancing the merger rate. General relativistic simulations indicate that the magnitude of the kick imparted to a $\sim 7 M_\odot$ BH after it merges with a NS will exceed a globular cluster’s escape velocity (e.g. Shibata et al. 2009). For mergers involving more massive BHs, the recoil speed is suppressed and the BH can remain in the cluster after each merger. In addition to increasing the merger rate per cluster, the coalescence of a NS with a more massive BH can be detected out to much larger distances. Thus, the rate of detectable mergers is extremely sensitive to the mass of the BH. However, the BH mass function is not the only poorly understood characteristic of a globular cluster that strongly impacts the BH+NS merger rate. Our models show that the rate is also sensitive to the nature of a cluster’s binary population. A post-merger BH is more likely to gain a subsequent NS companion if there is a large reservoir of background binaries for this newly single BH to exchange into. Given the uncertainty in these inputs to our models, we predict
a broad range of aLIGO detection rates, $0 - 0.7 \text{ yr}^{-1}$.

Only a fraction of the BH+NS binaries formed through exchange interactions in globular clusters will merge within a Hubble time. However, if the NS had been spun up into a MSP before exchanging into a binary with a BH, then these BH+MSP binaries could be observed in the electromagnetic band. We explored this possibility extensively in chapter 3. The orbital period distributions of these dynamically formed BH+MSP binaries are most sensitive to the structural properties of the globular cluster in which they are produced, and not the uncertain nature of the underlying BH or binary populations. In dense, massive clusters, where the encounter rate is large, the BH+MSP binaries are rapidly hardened to small orbital separations. The median orbital period of the BH+MSP binaries in such clusters is around 5 days. On the other hand, in lower density clusters, where binaries are not hardened as quickly, the typical orbital period of a BH+MSP binary is over 100 days. The more gradual hardening process in these clusters also results in longer lifetimes for the BH+MSP binaries, which increases the probability that such a system can be observed. However, the probability that a BH+MSP binary exists in a globular cluster also depends on the BH mass function, the number of BHs present in the cluster, and properties of the cluster's binary population. BH+MSP binaries are produced most efficiently in clusters with a few dozen low-mass BHs and high binary fractions. Our optimistic estimates predict that there could be as many as 10 BH+MSP binaries in the Milky Way globular cluster system. Clusters with structures similar to 47 Tuc, NGC 1851, M62, and Terzan 5 are most likely to harbor a BH+MSP binary.

We have investigated two ways in which the BH+NS binaries produced in globular clusters might reveal themselves, as gravitational wave sources when they coalesce and as radio sources while they are BH+MSP binaries. Interestingly, these two observable manifestations are sensitive to different ends of the BH mass function. BH–NS mergers will only be detected if the stellar mass BHs in globular clusters are substantially more massive than the stellar mass BHs observed in the field. Conversely, the formation and survival of a BH+MSP binary is most likely if the stellar mass BHs in globular clusters are of similar mass to those in the field. Unfortunately, the dynamical production of BH+NS binaries may be too inefficient for either manifestation to be detected using the current generation of instruments. However, the Einstein Gravitational Wave Telescope is expected to detect BH+NS mergers out to $z \gtrsim 6$ (Sathyaprakash et al. 2012). Furthermore, the Square Kilometer Array will be able to detect most pulsars within 10 Mpc (Cordes 2007). With the much larger volumes probed by these next generation detectors, the discovery of a dynamically formed BH+NS binary is expected in all but the most pessimistic cases investigated in our models.

In a complementary study, we investigated a mechanism that probes the IMBHs that might be lurking in the centers of globular clusters. In chapter 4, we presented models of the emission lines produced in the photoionized debris of WDs and HB stars.
that have been tidally disrupted by an IMBH. We found that bright, broad C\textsc{iv} $\lambda 1549$ and [O\textsc{iii}] $\lambda\lambda 4959,5007$ emission lines can be used to identify WD tidal disruption events, provided that there are no hydrogen lines observed along side these features in the spectrum, and that the emission lines are seen in concert with an UV/X-ray flare near the center of a globular cluster or a galaxy. The peak luminosity of the [O\textsc{iii}] $\lambda 5007$ emission line depends weakly on the mass of the IMBH, but this mass dependance is degenerate with changes in the peak luminosity produced by varying the IMBH’s spin. Additional uncertainties arise if we consider the possibility that the assumptions made in our models do not adequately describe what occurs in nature. For example, little is understood about the structure of the accretion flow and the efficiency at which the bound debris’s potential energy is converted into ionizing radiation following a tidal disruption event. Therefore, we concluded that it is difficult to determine the IMBH’s mass to better than an order of magnitude given observations of the [O\textsc{iii}] $\lambda 5007$ line alone.

Our HB star tidal disruption models showed that the emission lines produced in these events can remain bright for more than a century. Our models suggest that the photoionized debris of a tidally disrupted HB star may emit detectable H$\alpha$ and H$\beta$ lines shortly after disruption. At late times, however, the spectra should be devoid of hydrogen lines. In addition to the [O\textsc{iii}] doublet expected in WD tidal disruptions, the photoionized debris of a tidally disrupted HB star will also emit a bright [N\textsc{ii}] $\lambda\lambda 6548,6583$ doublet. The [N\textsc{ii}] doublet is produced because the core of a HB star is enriched with nitrogen as a result of CNO-cycle burning.

We compared our modeled emission line luminosities and profiles to two WD tidal disruption candidates in extragalactic globular clusters. Both candidates exhibit a bright X-ray source and an [O\textsc{iii}] $\lambda 5007$ emission line luminosity that is comparable to that predicted by our models. However, there are drawbacks to interpreting either source as a WD tidal disruption. This is not surprising because the predicted rate of WD tidal disruption events is low enough that it is unlikely that one should have been observed (Ramirez-Ruiz & Rosswog 2009).

Interestingly, the optical spectrum of the WD tidal disruption candidate in a globular cluster associated with NGC 1399 exhibits both of the bright emission features predicted by our HB star tidal disruption models. Beyond identifying this source as a HB star tidal disruption candidate, we can use the observed [N\textsc{ii}]/[O\textsc{iii}] emission line luminosity ratio, and the limits on the [N\textsc{ii}]/H$\alpha$ and [O\textsc{iii}]/H$\beta$ ratios, to constrain the mass of the BH in this cluster. Reproducing the observed [N\textsc{ii}]/[O\textsc{iii}] $\sim 1$ requires that $M_{\text{BH}} < 200$ $M_\odot$. This places the BH on the low mass end of the IMBH mass spectrum. Again, though, given the predicted rate for HB star tidal disruption events, it is surprising that such an event has been observed. It is also puzzling that we have yet to observe evidence of a main sequence star’s tidal disruption in a globular cluster because this should occur 20 times more often than the disruption of an evolved star.
Given its enigmatic nature, the candidate HB star tidal disruption event warrants further investigation. There are alternative explanations for the emission lines observed in this cluster. The emission lines could be produced if the radiation emitted by the accreting BH photoionizes material ejected by an unrelated source within the cluster (Maccarone & Warner 2011; Ripamonti & Mapelli 2012). It is also possible to generate these emission lines in the IMBH’s accretion disk itself (Porter 2010). One way to distinguish between these varied explanations is to obtain a UV spectrum of the source, as each model predicts a very distinctive spectral features in the UV. If the observed UV spectrum supports the HB star tidal disruption hypothesis, then the mass of the BH can be better constrained based upon the luminosities of the UV emission lines.

5.1 Suggestions for Future Study

Now that several promising BH candidates have been discovered in globular clusters, it is obvious that at least a few clusters are able to retain some of the BHs that form within them. A clear question to ask is whether there are common traits among these clusters that promote BH retention. Unfortunately, it is difficult to measure the structural parameters of many of these clusters because most of the BH candidates are in distant, extragalactic clusters. By comparing the luminosities and colors of the extragalactic globular clusters that contain BH candidates to those of clusters that do not, Maccarone et al. (2011) argued that BHs are more likely to be found in massive, metal-rich clusters. It is not clear, however, whether this is because massive, metal-rich clusters are more likely to retain BHs (e.g. Moody & Sigurdsson 2009) or because X-ray binary formation is more efficient in high metallicity clusters (Ivanova et al. 2012).

The structure of the Milky Way globular cluster that contains two BH candidates, M22, can be studied in detail. In fact, at 1.24 pc the core radius of M22 is peculiarly large (Strader et al. 2012a). As was discussed in chapter 1, the binaries in a cluster can serve as a source of energy and halt a cluster’s collapse. Miller & Davies (2012) argued that the hard binary fraction alone may determine whether or not a dense stellar system will collapse. Figure 5.1 shows a plot of the core radius, $r_c$, vs. the photometric binary fraction, $f_b$, for globular clusters in the Milky Way (data from Milone et al. 2012b). It does appear that clusters with higher binary fractions do have larger core radii. It is conceivable that in addition to supporting the entire cluster against collapse, the presence of a substantial population of hard binaries may also prevent the formation of the dense BH subcluster. There is evidence of a bimodality in the distribution of core radii in massive star clusters associated with the Large Magellanic Cloud. Elson et al. (1989) noted that the $r_c$ evolution of these clusters appears to bifurcate; the cores of some clusters expand with time while the cores of others contract. Perhaps the clusters with a large enough binary population to
drive this core radius expansion are also the clusters that are able to retain BHs. However, Mackey et al. (2008) showed that this core expansion could be caused by a population of BHs. Thus, the large core radii observed in some globular clusters may be a product of a retained BH population, not a trait that fosters their retention. Clearly, more work is needed to understand why some globular clusters retain stellar mass BHs.

In addition to learning which types of globular clusters retain BHs, it may be possible to leverage these recent observational results to constrain the nature of the globular cluster BH population, as a whole. This could be achieved by modeling how different BH populations reveal themselves as they evolve in a globular cluster. The goal is to compare the growing sample of globular cluster BHs with the formation rates and lifetimes of potentially observable systems, including X-ray binaries, gravitational wave sources, and tidal disruptions, expected for various BH populations. With slight modification, the Fokker–Planck technique that we used to study the formation of

![Figure 5.1 Core radius vs the photometric binary fraction for a sample of Milky Way globular clusters. Clusters with higher binary fractions tend to have larger core radii. The data are from Milone et al. (2012b).](image)
BH+NS binaries would be an excellent tool to address this problem. Adapting the code to make the background cluster dynamic and adding distribution functions that describe the globular cluster’s binary population would greatly improve the models. Adding binaries is essential to understanding the production of observable BH systems because interactions between two binaries can result in the formation of a hierarchical triple. Evolution of these triple systems may result in the formation of an X-ray binary or a tidal disruption, either of which could explain the bright X-ray sources seen in some globular clusters.

To better understand the nature of BHs in globular clusters, the evolution of BH+WD X-ray binaries should also be investigated. Previous work suggests that these systems might be the dominant means by which globular cluster BHs reveal themselves (e.g. Ivanova et al. 2010; Maccarone et al. 2010). Using the stellar evolution code MESA star it is possible to determine, in detail, how the WDs in these systems respond to mass loss, which in turn governs the rate and stability of mass transfer. By computing a grid of detailed binary evolution models with MESA star it is possible to characterize the stability, rate, and timescale of mass transfer in such systems. These results can then be used to establish the X-ray luminosity and the outburst properties of the BH+WD binaries. Furthermore, as the BH accretes material shed by the WD, emission lines can be produced in a wind and/or in the accretion disk itself. The BH’s mass might be encoded in these emission lines, as it is in the tidal disruption case described in chapter 4. Modeling the electromagnetic signatures of these binaries is vital to comparing the outcomes of dynamical models with the observed BH candidates, thereby constraining the nature of the globular cluster black hole population, and shedding light on an enduring astrophysical problem.
Bibliography

Bahcall, J. N., & Ostriker, J. P. 1975, Nat, 256, 23
Belczynski, K., Bulik, T., & Baily, C. 2011, ApJL., 742, L2

109
Benacquista, M. J. 2006, Living Reviews in Relativity, 9, 2
Carter, B., & Luminet, J. P. 1982, Nat, 296, 211
Cordes, J. M. 2007, SKA Memo, 97
Hills, J. G. 1975a, AJ, 80, 809
—. 1975b, Nat, 254, 295
MNRAS, 386, 553
ApJL, 621, L109
Katz, J. I. 1975, Nat, 253, 698
Lorimer, D. R. 2008, Living Reviews in Relativity, 11, 8
Nakar, E. 2007, Physics Reports, 442, 166
Punturo, M., Abernathy, M., Acernese, F., et al. 2010, Classical and Quantum Gravity, 27, 194002
Rees, M. J. 1988, Nat, 333, 523
Sigurdsson, S., & Hernquist, L. 1993, Nat, 364, 423
Spitzer, Jr., L. 1969, ApJL., 158, L139
Stairs, I. H. 2004, Science, 304, 547
Strader, J., Chomiuk, L., Maccarone, T., Miller-Jones, J., & Seth, A. 2012a, Nat, 490, 71
van den Bergh, S. 1967, AJ, 72, 70
of Neutron Stars, ed. D. J. Helfand & J. H. Huang, 187
Vita

Drew Reid Clausen

**EDUCATION**

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<tr>
<td>The Pennsylvania State University</td>
<td>Ph.D. in Astronomy &amp; Astrophysics</td>
<td>University Park, PA</td>
<td>expected August 2013</td>
</tr>
<tr>
<td>University of Colorado</td>
<td>B.A. in Astronomy, magna cum laude</td>
<td>Boulder, CO</td>
<td>2004</td>
</tr>
<tr>
<td></td>
<td>B.F.A. Film Production, with distinction</td>
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**HONORS & AWARDS**

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<td>Zaccheus Daniel Fellowship</td>
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<td>2010</td>
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<td>2009</td>
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**RESEARCH EXPERIENCE**

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<td>Graduate Research Assistant</td>
<td>Department of Astronomy &amp; Astrophysics</td>
<td>May 2008 – present</td>
</tr>
<tr>
<td></td>
<td>The Pennsylvania State University</td>
<td></td>
</tr>
<tr>
<td>Undergraduate Research Assistant</td>
<td>Department of Astrophysical and Planetary Sciences</td>
<td>Sep 2003 – Dec 2004</td>
</tr>
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<td></td>
<td>University of Colorado</td>
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**TEACHING EXPERIENCE**

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<td>Instructor</td>
<td>ASTRO 001: Astronomical Universe</td>
<td>Summer 2010</td>
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<td>The Pennsylvania State University</td>
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<tr>
<td>Teaching Assistant</td>
<td>ASTRO 11: Elementary Astronomy Lab</td>
<td>Fall 2007 – Spring 2009</td>
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