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DYNAMIC GAMES AND ROBUST MODELS FOR TRANSPORTATION AND SERVICE NETWORKS

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ABSTRACT

This dissertation reviews the theories and methods of dynamic games in traffic assignment and optimization under uncertainty, refines and applies them to address the following two specific research problems:

(1) Urban freight transportation planning. Urban freight transportation supports the economic and social development of an urban area. However, there are negative externalities associated with urban freight transportation such as emission, noise and congestion. We consider that there exists a metropolitan planning organization (MPO) that is responsible for reducing the congestion caused by freight transportation on the urban road network. In this study, based on different assumptions on the MPO's capability to influence freight truck traffic, we develop two dynamic game-theoretic models to support the MPO's goal of reducing congestion caused by freight transportation. Heuristic algorithms are developed to solve the proposed models. We conduct numerical analyses to derive optimal urban freight transportation plans. Our analyses show that the congestion caused by urban freight transportation can be effectively reduced by properly controlling the freight truck traffic. This can provide insights to the MPO into urban freight traffic regulation.

(2) Truckload service procurement under uncertainty. Truckload (TL) transportation is a necessary component of a shipper's logistics system and the associated procurement expenditure accounts for a significant portion of the shipper's overall cost. In this study, we argue that a shipper's TL service procurement cost can be reduced by appropriately handling the uncertainty during the procurement process. In particular, based on different assumptions on the availability of distributional information of the uncertain shipping demand, we propose two robust models to reduce the shipper's cost. We develop new solution approaches and conduct numerical tests on real-world sized instances of TL service procurement to demonstrate the applicability of the proposed robust models. Our analyses show that the solutions to robust models yield a lower procurement cost than the solution to a deterministic model. Insights into the design and operation of TL service procurement are drawn from the numerical analyses.

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Chapter 1

Introduction

Solving a complex real-world problem which is dynamic and also involves uncertainty is a challenging task. Many static and deterministic methods are created to accomplish this task. However, for simplicity and practicability, these methods inevitably involve simplifications and approximations of the real problem. This dissertation is dedicated to investigating dynamic game-theoretic models and robust models which are applicable to two specific engineering and management topics in transportation and service networks: urban freight transportation and truckload service procurement.

1.1 Urban freight transportation planning

Urban freight transportation, which is also referred to as city logistics, aims to reduce negative externalities associated with freight activity (e.g., emission, noise and congestion) while supporting the economic and social development of the urban area (Crainic et al., 2009). Surveys show that in Europe freight traffic accounts for 14% of total urban traffic (in vehicle-kilometers), 19% of energy consumptions and 20-30% of total vehicle emissions (Filippi et al., 2010; Schoemaker et al., 2006). Today the problem is even more challenging due to the growing number of private automobiles, soaring demand of urban freight transportation services and increased recognition of the need for a paradigm shift toward environmentally sustainable logistics and freight technologies.

Urban freight transportation has attracted lots of research efforts in the past decades from different perspectives including transportation regulation, emission estimation and reduction, transportation planning, etc. According to Crainic (2000), urban freight transportation planning (city logistics) is an operational (short-term) planning issue. It is different from the intercity freight transportation planning, also known as long-haul transportation planning, which is a tactical (medium-term) planning issue. Most operational transportation planning problems are modeled as vehicle routing problems (VRPs). Crainic et al. (2009) model urban freight transportation planning as a vehicle routing problem with time windows (VRPTW). They propose an integrated model and argue that a holistic logistics system that integrates shipments, firms and vehicles is necessary for today's urban freight system. Intuitively, the integrated urban freight system can improve the overall efficiency of urban freight transportation. Nevertheless, since such a system focuses only on freight activities while overlooks personal transportation, whether it can successfully reduce the associated negative externalities and improve the social welfare is still not clear. In fact, as Ambrosini and Routhier (2004) and Paglione (2006) note, among the urban freight transportation planning literature, there is a lack of behavioral models that characterize the interactions of private economic and transport agents.

Nonetheless, the study of competition and cooperation among different road users is not new. Yang et al. (2007) model the routing behaviors of system optimum (SO), user equilibrium (UE) and Cournot-Nash (CN) travelers using a static Stackelberg game. Specifically, the SO traveler is the leader and the UE and CN travelers are the followers. Yang et al. assume that given the SO traveler's routing decision, the UE and CN travelers' routing decisions satisfy a mixed equilibrium. The SO traveler optimizes its routes considering the potential reactions of UE and CN travelers. This model successfully characterizes the interactions among different groups of travelers. However, a dynamic game-theoretic model is more applicable for urban freight transportation planning which focuses on operational planning.

In this dissertation, we propose two dynamic Stackelberg game-theoretic models for urban freight transportation planning. Both models are formulated as dynamic mathematical program with equilibrium constraints (MPEC) which is a well-known computationally challenging problem. We propose two heuristics to solve the dynamic MPEC and conduct numerical tests to derive optimal urban freight transportation plan. Specifically, assuming that the movement of all freight trucks on an urban road network is controlled by a metropolitan planning organization (MPO) who wants to minimize the congestion caused by urban freight transportation, we model the MPO's problem as a dynamic Stackelberg game. Assuming that the movement of all freight trucks is controlled by a union of truck companies (UTC) who wants to minimize urban freight transportation delay, we model the UTC's problem as a different dynamic Stackelberg game which can serve as a simulator to support the MPO regulating urban freight transportation. Heuristic algorithms are developed to solve the proposed models. Our analyses show that the congestion caused by urban freight transportation can be effectively reduced by properly controlling the freight truck traffic. Managerial insights into urban freight transportation regulation for the MPO are drawn from the numerical analyses.

1.2 Truckload service procurement under shipment volume uncertainty

Truckload (TL) transportation is the movement of homogenous freight for a single shipper between a fixed origin and a fixed destination. It is a service provided by TL carriers. The shipper, who is the owner of the freight, can be an agent at any stage of a supply chain (e.g., suppliers, manufacturers, distributors, or retailers). TL transportation is a necessary component of a shipper's logistics system and the associated expenditure accounts for a significant portion of the shipper's overall cost. In many corporations, transportation expenditure accounts for about 30% of the overall cost of goods sold (Chen et al., 2009; Ballou, 1992). From a market perspective, truck transportation accounts for 29% of for-hire transportation services expenditure which is about 363.7 billion U.S. dollars in 2007 (RITA, 2009). TL accounts for over half of total trucking expenditure (Sheffi, 2004). Reducing TL service procurement cost can greatly benefit shippers.

Most shippers buy transportation service through a market-based mechanism known as combinatorial auction (Sheffi 2004). First, the auctioneer (i.e., the shipper) predicts the volume of shipments in the coming contract period[1], formulates a list of lanes[2], and invites a group of carriers to participate in the auction. Then, each carrier can bid on several packages of lanes of their interests. Finally, the shipper evaluates the bids and determines the winners of the auction.

Determining the winners of a combinatorial auction so that the total procurement cost to the shipper is minimized is nontrivial. It is well-known as the winner determination problem (WDP) which is an NP hard combinatorial optimization problem. Since exact volume of shipments in the coming year is not available at the time that the combinatorial auction is conducted for TL service procurement, the shipper solves the WDP with a forecast of the shipment volume. However, by this method, when the shipper determines the winners, they do not take into account the errors in the predicted shipment volume which can lead to two possible outcomes. The volume of shipments may be underestimated, resulting in needs for extra shipping capacity from other third-party carriers who did not participate in the auction. On the other hand, the volume of shipments may be overestimated, which can cause some carriers to be assigned less shipment than expected. Both outcomes incur extra procurement costs which are not accounted for in the deterministic WDP. Therefore, robust winner determination methods are needed that can characterize and mitigate the impact of shipment volume uncertainty on TL service procurement.

[1] A contract period in the transportation service market is usually a year according to Sheffi (2004).

[2] The term “lane” is used in Sheffi (2004) to refer to an “O-D-commodity-period specific flow” as first described by Jara-Diaz (1988).

Recently, a new bidding structure and a corresponding two-stage stochastic winner determination model for TL service procurement are proposed by Ma et al. (2010). Assuming that the distribution of uncertain shipment volume is exactly known to the shipper, Ma et al. (2010) is a remarkable first attempt to explicitly model shipment volume uncertainty and resolve the corresponding stochastic WDP. Based on the same modeling framework, assuming only the support for uncertainty is known, Remli and Rekik (2012) develop a two-stage robust winner determination model and solve it for worst-case optimal solution.

In this dissertation, we propose two approaches to minimize a shipper's cost associated with TL service procurement under shipment volume uncertainty. Specifically, assuming that the full distributional information (i.e., probability density function) of the uncertain shipment volume is available, we extend the study by Ma et al. (2010) and propose a sampling-based two-stage stochastic winner determination model. Assuming that only partial distributional information (i.e., mean and variance) of the uncertain shipment volume is available, we propose a two-stage robust winner determination model. We conduct numerical tests on real-world sized instances of TL service procurement to demonstrate the applicability of the proposed robust models. The analyses show that the solutions to the robust models yield a lower procurement cost than the solution to a deterministic model.

Note that the proposed two-stage robust winner determination model is distinct from the one in Remli and Rekik (2012) in the following perspectives. First of all, our model is based on a refinement of the deterministic model adopted by Remli and Rekik (2012). Our modified deterministic model is more general. Moreover, we assume that partial distributional information, that is mean and variance of the uncertain shipment volume is available and based on that we construct a general polyhedral uncertainty set. We develop a new reformulation solution method which, demonstrated by numerical tests, is much more efficient than the constraint generation algorithm that is adopted in most two-stage RO literature including Remli and Rekik (2012).

What is more, we evaluate the quality of our solution in a sample of general scenarios by Monte Carlo simulation and compare our robust solution with the benchmark deterministic solution. These distinct investigations provide more insights into the tractability and robustness of the solution to the WDP under uncertainty and the impact of robust solution on the costs of TL service procurement.

1.3 Research objectives

The goal of this dissertation is to fill the gap in the existing body of urban freight transportation and truckload service procurement literatures. Specifically, the objectives of the research are summarized as the following:

- Develop game-theoretic models for urban freight transportation planning to minimize the congestion caused by freight activities on urban road networks;
- Develop robust winner determination models to reduce shipper's cost associated with TL service procurement under uncertainty.

1.4 Summary of contributions

The main contributions of this dissertation are summarized as follows.

- In Chapter 3, we refine the link delay model (LDM) based dynamic network loading (DNL) to characterize the affects of freight transportation on network traffic assignment. We discuss the existence and uniqueness of solution to the refined DNL model and prove that the refined DNL model conforms to the first-in-first-out (FIFO) principle.
- In Chapter 4, we create the first dynamic Stackelberg game-theoretic model in this dissertation for urban freight transportation planning. We show that the proposed model

can be solved by simulated annealing and the solution can be used by the MPO to regulate urban freight transportation which can significantly reduce the associated network-wide congestion.

- In Chapter 5, we propose the second dynamic Stackelberg game-theoretic model in this dissertation for urban freight transportation planning. We develop a new heuristic to solve the model. Extensive numerical experiments are conducted and our results illustrate the interaction between urban freight transportation and personal transportation. The model can be used by the MPO as a simulator for testing the impact of urban freight transportation policies on urban road network traffic. Our numerical analyses reveal a dynamic Braess-like paradox which provides the MPO managerial insights into urban freight transportation regulation.
- In Chapter 6, we propose a refinement of the benchmark winner determination model which is more general and feasible under uncertainty. We analyze the feasibility of the refined model and its insights into the design and operations of TL service procurement.
- In Chapter 7, assuming that the exact probability density function of uncertainty is available, we develop a sampling-based two-stage stochastic winner determination model for TL service procurement. We use Monte Carlo simulation to demonstrate that the solution to our proposed stochastic model can yield lower procurement cost than the solution to the deterministic model.
- In Chapter 8, utilizing only partial distributional information, which is the mean and variance of the uncertainty, we develop a two-stage robust winner determination model. We develop a reformulation solution method that is much more efficient than the constraint generation algorithm which is widely adopted in two-stage robust optimization literature. In addition, we show that after reformulation our robust model is more numerically tractable than the deterministic model. We conduct Monte Carlo simulation

to demonstrate that the solution to our proposed model is more robust to uncertainty and yield lower procurement cost than the solution to the deterministic model when the variance of the uncertainty is large.

Three papers were produced and submitted to peer-reviewed journals based on the work in this dissertation. Specifically, we submit one paper on urban freight transportation planning based on the models and analyses presented in Chapter 3, Chapter 4 and Chapter 5 of this dissertation. The other two papers are regarding truckload service procurement under shipment volume uncertainty: one is based on the stochastic winner determination model presented in Chapter 7 and the other is based on the robust winner determination model presented in Chapter 8.

Chapter 2

Literature Review

In this chapter, we review the existing studies on urban freight transportation planning and truckload service procurement under uncertainty. In particular, we review the research methods, which have been or can be used to investigate the two research problems, including intercity freight assignment models, vehicle routing problem, dynamic traffic assignment, combinatorial auctions and the winner determination problem, Monte Carlo method and robust optimization.

2.1 Intercity freight forecasting models

Intercity freight transportation, also known as long-haul transportation, is a tactical (medium-term) planning issue for a transportation agency (Crainic, 2000). Different from transshipment network assignment models which are proposed to address carriers' transport operations (e.g., Guélat et al., 1990), intercity freight forecasting models focus on predicting the freight flows on the transportation network. Friesz et al. (1983) provide a survey of the predicative intercity freight network models and discuss the advantage of combined shipper-carrier models and spatial equilibrium models. Assuming perfect competition and explicit demand functions, Friesz et al. (1985) formulate a computationally tractable model of combined shipper-carrier freight network equilibrium. Friesz et al. (1986) create a sequential shipper-carrier freight assignment model and apply it to three detailed network data bases. Numerical tests show that the proposed model is a more accurate predictor of freight transportation flows than the benchmarks. Harker and Friesz (1986a, 1986b) propose a simultaneous shipper-carrier freight assignment model to predict freight transportation flows. Two alternative mathematical formulations are presented and the existence and uniqueness of the solution are discussed. Friesz et al. (1998) create a dynamic disequilibrium model to predict the interregional commodity flows. The model has steady states that are consistent with the traditional Samuelson-Takayama-Judge (STJ) static spatial price equilibrium model (Samuelson, 1952; Takayama and Judge, 1971). More recently, Agrawal and Ziliaskopoulos (2006) propose a dynamic shipper-carrier freight assignment model and develop an iterative approach to solve for an equilibrium solution.

2.2 Vehicle routing problems

The common objective of a vehicle routing problem (VRP) is to minimize the total cost/delay to the carriers or truck companies by designing optimal pick-up and delivery routes. It is an NP-complete combinatorial optimization problem first proposed by Dantzig and Ramser (1959). In the past decades, VRP has attracted lots of research efforts and its related research questions are still popular in the field of operations research and transportation science nowadays. Kulkarni and Bhave (1985) propose several assignment-based integer programming formulations of VRP. Golden and Assad (1986) review the state-of-the-art VRP studies by mid 1980s and predict that more exciting computational methods will be created with the development of computation techniques. Laporte (1992) survey the VRP literature with a focus on the solution approaches. Both exact and approximate algorithms are discussed. Besides the study in the numerical solution of VRP, several extensions of classical VRP have been proposed and developed, including dynamic vehicle routing problem (DVRP) which allows en route rerouting (Larsen, 2000; Pillac et al., 2011), stochastic vehicle routing problem (SVRP) and robust vehicle routing problem (RVRP) which consider optimal routing under uncertainty (see Bertsimas and

Simchi-Levi (1996) and Gendreau et al. (1996) for a review of SVRP; see Ordóñez (2010) for a tutorial of RVRP).

Vehicle routing problem with time windows (VRPTW) is a special type of VRP which incorporates time constraints for the pick-up and delivery of freights. Bräysy and Gendreau (2005a, b) survey the VRPTW literature that discusses the heuristic route construction methods, local search algorithms and metaheuristic methods. Kallehauge et al. (2005) discuss the mathematical modeling and numerical solutions of VRPTW. Column generation, as an efficient solution method, is discussed and tested. Gendreau and Tarantilis (2010) conduct a survey on the state-of-the-art in the field of heuristics for solving large-scale VRPTW. In the context of urban freight transportation planning, Taniguchi and Thompson (2002) develop a stochastic VRPTW which incorporates travel time variance. A simulation dynamic traffic assignment model is created to estimate the travel delay. Crainic et al. (2009) propose a VRPTW-based integrated model that addresses short-term scheduling of operations and resource management based on a two-tiered freight distribution structure. The two-tiered city logistics model is further extended to address shipping demand uncertainty in Crainic et al. (2011).

2.3 Dynamic traffic assignment

Dynamic traffic assignment (DTA) is the descriptive modeling of time-varying traffic flows on a network. A variety of mathematical formulations and solution approaches have been developed (see Peeta and Ziliaskoupolous (2001) for a survey). As defined in Friesz (2010): “Dynamic user equilibrium (DUE) is one type of DTA wherein the effective unit travel delay, including early and late arrival penalties, of travel for the same purpose is identical for all utilized path and departure time pairs.”

The early analytical DUE models are greatly influenced by the system optimum DTA models studied by Merchant and Nemhauser (1978a, b) which apply arc exit flow functions to model dynamics. Friesz et al. (1989) and Wie et al. (1995) employ the same dynamics and propose certain extensions of DUE. Due to the drawbacks of arc exit flow functions discussed by Carey (1986, 1987, 1992, 1995), the dynamics based on them are finally abandoned. Several modifications of the Merchant-Nemhauser arc dynamics are proposed later (see Bernstein et al. (1993), Ran et al. (1993), and Ran and Boyce (1996)). However, the incompressibility assumption made in these studies is incompatible with the arc delay model which is widely employed in DTA modeling.

Friesz et al. (1993) introduce exit time functions and use these functions and their inverse to model arc dynamics, which avoids the use of exit flow functions and the associated shortcomings. This new dynamics is later employed by Wu et al. (1998), Zhu and Marcotte (2000), and Bliemer and Bovy (2003) to study algorithm, theoretical properties, and modeling extensions of DUE, respectively. The formulation of DUE proposed in Friesz et al. (1993) is exact but cannot be solved by traditional methods since the employed path delay operators are non-analytic. To resolve this issue, Friesz et al. (2001) reformulate the DUE as an equivalent differential variational inequality (DVI) with state-dependent time shift. Friesz and Mookherjee (2006) propose a fixed point algorithm to solve the DVI reformulation with state-dependent time shifts. More recently, Friesz et al. (2011) approximate the time shifts using the second order Taylor expansion for computational efficiency.

A dynamic Stackelberg game which contains a DUE in the lower level can be formulated as a dynamic mathematical program with equilibrium constraints (MPEC). MPEC is an optimization problem with constraints that includes variational inequalities or equivalent complementarities (see Luo et al. (1996) for more details). It is a bi-level program where the upper level aims at optimizing a decision maker's utility and the lower level captures the feasible equilibrium. Dynamic MPEC is computationally challenging. Friesz et al. (2007) discussed two methods, namely descent in Hilbert space and a discrete time approximation, to solve a small-sized instance of dynamic optimal toll problem with equilibrium constraints (DOTPEC) which is a specific type of dynamic MPEC. Chung et al. (2012) introduced toll price uncertainty into the DOTPEC and proposed a bi-level metaheuristic to solve the resulting robust DOTPEC. The reported computation time for a small-sized instance of the robust DOTPEC is more than 3 hours.

2.4 Combinatorial auctions and the winner determination problem

A combinatorial auction is a mechanism in which bidders can bid on combinations of items. It is often conducted in the sale of assets such as Federal Communications Commission (FCC) spectrum, airport time slots, pollution permits, railroad segments, and network routing, etc. (see de Vries and Vohra (2003) and Ma (2008) for a survey). In transportation procurement, many large shippers (e.g., Wal-Mart Stores Inc., The Home Depot Inc., K-Mart Corporation) have been utilizing combinatorial auctions to procure TL services from carriers, which typically save them 3 to 15 percent of procurement cost. For instance, Home Depot used combinatorial auction to procure TL service which did not only lower the cost but also increased the service performance (Elmaghraby and Keskinocak 2004).

The use of combinatorial auctions in TL service procurement incurs many research questions. From a carrier's perspective, the evaluation of the true value of each combination of lanes and the optimal bidding strategy are the two most interesting questions (Song and Regan 2003). We refer the readers to Lee et al. (2007) and Chang (2009) for some discussions on these topics. From a shipper's perspective, the winner determination problem (WDP) associated with the combinatorial auction is critical. WDP refers to the process of allocating items to bidders in a combinatorial auction that optimizes the auctioneer's utility, which is an NP-hard integer program

(Lehmann et al., 2006). A variety of mathematical formulations and numerical solutions of WDP have been developed (Rothkopf et al., 1998; Sandholm, 1999; Andersson et al, 2000; Nisan, 2000; de Vries and Vohra, 2003). A detailed review of WDP can be found in Lehmann et al. (2006). In the context of transportation procurement, Caplice and Sheffi (2003) develop several integer programming formulations of WDP with side constraints that reflect business considerations from both shippers and carriers.

A few studies have attempted to address WDP under uncertainty. Boutilier et al. (2004) develop a min-max regret robust optimization approach to solve WDP that contains uncertainties in the auctioneer's utility function. Kameshwaran and Benyoucef (2008) propose a framework of robust models for bid evaluation under uncertainty. Ma et al. (2010) propose the first model that addresses WDP for TL service procurement under shipment volume uncertainty. In particular, a new bidding structure which takes into account the uncertain shipment volume on each O-D pair is proposed and a two-stage stochastic program with recourse is developed to minimize the expected procurement cost. The first-stage decision variable is binary and represents the allocation of lanes to the carriers, while the second-stage recourse variables are the assignments of freight to the winning carriers and thus are continuous. In Ma et al. (2010), it is assumed that the uncertain shipments will realize at 1 of the 3 levels (low, medium and high) with specified probability and volume. Therefore, the two-stage stochastic program is actually equivalent to a deterministic mixed integer program (MIP) and a moderately sized instance of such an MIP can be solved by commercial software packages like CPLEX.

2.5 Monte Carlo method

When the equivalent deterministic reformulation of a stochastic program is not available, Monte Carlo method can be used to derive an approximate formulation. Monte Carlo method, as first documented by Metropolis and Ulam (1958), is a computation method that makes use of random samples of data. The only requirement of implementing Monte Carlo method is the ability to draw samples. It has been widely adopted in various aspects of engineering, finance, physical science, etc. and is an especially powerful tool in stochastic programming. Rockafellar and Uryasev (2000) apply Monte Carlo simulation to minimize Conditional Value-at-Risk (CVaR) which is a risk measure in portfolio management. Calafiore and Campi (2005, 2006) investigate the appropriate sample size when applying Monte Carlo method to solve chance-constrained programs[3].

Using Monte Carlo method, a sample average approximation (SAA) of the expected value function in a stochastic program can be derived which reformulates the stochastic program as a deterministic optimization problem. This method is also known as sample path optimization method (Plambeck et al., 1996) and stochastic counterpart method (Rubinstein and Shapiro, 1990; Ahmed et al., 2002). Shapiro and Homem-de-Mello (1998) develop a SAA based approach to two-stage stochastic programming with recourse. Various variance reduction techniques are discussed and a statistical inference is developed to estimate the error. Norkin et al. (1998) propose to use stochastic upper and lower bounds to guide the partitioning process of a branch and bound algorithm when solving stochastic global optimization problems. They prove that the method converges almost surely. Mak et al. (1999) conduct statistical analyses and use the convergence results of the SAA method to construct confidence intervals on the optimality gap of a candidate solution of the true problem. Shapiro and Homem-de-Mello (2001) investigate the convergence rate of the SAA method and show that for stochastic linear programs with discrete probability distributions, the optimal solution of the SAA problem converges to an exact solution of the true problem as the sample size increases. The convergence results of the SAA approach

[3] We refer the readers to Nemirovski and Shapiro (2006) for more details about chance-constrained programs.

are further extended to two-stage stochastic programs with finite discrete feasible sets of first-stage decisions (Kleywegt et al., 2002). Moreover, the SAA method has been integrated with decomposition techniques to solve large-scale stochastic linear programs (Linderoth et al., 2002). Verweij et al. (2003) successfully apply the SAA method to stochastic routing problems which involve a large number of scenarios and first-stage integer variables. Ahmed et al. (2002) integrate the SAA method and a decomposition-based branch and bound algorithm to solve two-stage stochastic programs with integer recourse. Numerical tests are conducted and results show that the proposed method significantly outperforms the traditional SAA method.

2.6 Robust optimization

Robust optimization (RO) derives optimal solutions that are immune to uncertainty. The idea of RO is first proposed by Soyster (1973). He considers a linear program with the column-wise constraint coefficients undetermined but restricted in a convex set. The idea of RO has not been further explored until late 1990s (El Ghaoui et al., 1997, 2003; Ben-Tal and Nemirovski, 1998, 1999, 2000; Bertsimas and Sim, 2003, 2004). Since then, RO has received lots of attentions from the operations research society (Ben-Tal and Nemirovski, 2002; Ben-Tal et al., 2004; Ben-Tal and Nemirovski, 2008, 2009). Now, it serves as an alternative to the classical stochastic programming methods that handles mathematical programs under uncertainty. Different from stochastic programming, RO requires partial distributional information which is usually the bounds of the uncertainty and seeks the optimal solution in the worst-case scenario. For a comprehensive review of methodology and applications of robust optimization, we refer the readers to Ben-Tal et al. (2009) and Bertsimas et al. (2011). More recently, Bandi and Bertsimas (2012) propose to apply RO to several stochastic systems including queueing networks, auction design, and option pricing. The authors demonstrate that their robust approach is numerically tractable, while the classical approach to these application areas that relies on probability theory-based analysis is not.

Among the RO methods, multi-stage RO, especially two-stage RO, has been successfully applied to a variety of problems including transportation planning, network design, inventory routing, and the design and operations of power grid systems. Specifically, Ben-Tal et al. (2011) propose to apply an affinely adjustable robust counterpart (AARC) approach for dynamic emergency logistics planning under demand uncertainty. Numerical tests are conducted to show that AARC solution significantly outperforms the deterministic solution and sampling-based stochastic solution in terms of reducing the emergency logistics cost. Thiele et al. (2009) propose a modeling and solution approach to two-stage robust linear program with recourse. Atamtűrk and Zhang (2007) develop a two-stage robust network design model under travel demand uncertainty. Distinct from Thiele et al. (2009), the first-stage decision variable in Atamtűrk and Zhang (2007) is required to be integer. Jiang et al. (2012) propose a two-stage robust unit commitment model, where the first-stage decision variable is binary, for wind power system under wind output uncertainty. A similar study has been conducted by Bertsimas et al. (2013). In the context of TL service procurement, Remli and Rekik (2012) propose a two-stage robust winner determination model based on the bidding structure and deterministic modeling framework originated by Ma et al. (2010). The first-stage decision variable is binary and represents the allocation of lanes to the carriers, while the second-stage recourse variables are the assignments of freight to the winning carriers and thus are continuous. Most of these two-stage RO studies adopt a constraint generation algorithm that is similar to Benders' decomposition to solve the problem.

Chapter 3

Dynamic Assignment of Inhomogeneous Traffic

In this chapter we consider two types of traffic[4] during peak hours of a day: freight transportation by freight trucks and personal transportation by private automobiles. We assume that a driver of a private automobile (DPA), who needs to travel during peak hours of a day, wants to minimize his or her personal travel delay. Since the travel delay is a non-decreasing function of the traffic volume on a road section, the travel delay experienced by a DPA is affected by the number of private automobiles and freight trucks traveling on his/her route. In another word, each DPA's travel delay is affected by other DPAs' travel decisions and the freight truck traffic.

In Section 3.1, we use dynamic user equilibrium (DUE) to describe the dynamic assignment of private automobile traffic on the urban road network. In Section 3.2, we illustrate how the DUE for private automobiles is affected by freight truck traffic.

3.1 Dynamic user equilibrium for private automobiles

We denote the within-day time interval of interest by $\left[t_0, t_f\right] \subset \Re_+^1$. We assume that for all travelers (freight trucks and private automobiles), there is an identical desired time to arrive at

[4] Note that public transportation by buses is ignored in this dissertation since the buses' travel pattern (departure time and route choice) is fixed and is independent of the traffic.

their destinations[5], T_A. For example, we may set t_0, T_A and t_f to be 8am, 9am and 10am respectively if we are interested in the dynamic traffic assignment during the morning peak hours everyday.

The delay to each traveler on a specific path is denoted by the effective unit path delay operator $\Psi_p\left(t,h^\chi,h^\eta\right)$, which is defined as the sum of travel delay and penalty:

$$\Psi_p\left(t,h^\chi,h^\eta\right)=D_p\left(t,h^\chi,h^\eta\right)+F\left[t+D_p\left(t,h^\chi,h^\eta\right)-T_A\right] \quad \forall p\in \mathrm{P},$$

where $D_p\left(t,h^\chi,h^\eta\right),\ \forall p\in \mathrm{P}$ denotes the unit delay on path p and P is the set of all paths that can be taken by travelers. t denotes the departure time and h^χ,h^η denote vectors of flows/departure rates (number of vehicles entering a path per unit time) of private automobiles and freight trucks, respectively. In this dissertation, we use superscript "η" and "χ" to represent parameters and variables for freight trucks and private automobiles, respectively. $D_p\left(t,h^\chi,h^\eta\right)$ is an explicit function of the traveler's departure time t and an implicit function of the inhomogeneous traffic flow h^χ and h^η. The quantification of $D_p\left(t,h^\chi,h^\eta\right)$ is discussed in detail in Section 3.2. $F\left[t+D_p\left(t,h^\chi,h^\eta\right)-T_A\right]$ denotes early/late arrival penalties and the following equations hold:

[5] Note that this assumption can be relaxed. We can assume a finite number of desired arrival times and classify travelers with the same desired arrival time into the same group. Then we will have a mixed DUE for private automobiles. Although playing with a mixed DUE will not change the structure of our dynamic Stackelberg game and the framework of the proposed solution approach, it will complicate the analyses significantly. So for simplicity, we do not relax this assumption in this dissertation but propose it as one direction of future research.

$$\begin{cases} \text{if} \quad t+D_p\left(t,h^\chi,h^\eta\right) > T_A, & F\left[t+D_p\left(t,h^\chi,h^\eta\right)-T_A\right] > 0 \\ \text{if} \quad t+D_p\left(t,h^\chi,h^\eta\right) < T_A, & F\left[t+D_p\left(t,h^\chi,h^\eta\right)-T_A\right] > 0 \\ \text{if} \quad t+D_p\left(t,h^\chi,h^\eta\right) = T_A, & F\left[t+D_p\left(t,h^\chi,h^\eta\right)-T_A\right] = 0 \end{cases} .$$

For simplicity, in this dissertation we assume a quadratic penalty function

$$F\left[x\right]=\alpha x^2$$

where α is a constant.

To satisfy the travel demand for all private automobiles, the following flow conservation law must hold:

$$\sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} h_p^\chi\left(t\right)dt = Q_{ij}^\chi \quad \forall\left(i,j\right)\in W ,$$

where $\left(i,j\right)$ is an O-D pair and W is the set of all O-D pairs. $Q_{ij}^\chi \in \Re_{++}^1$ is the fixed travel demand on O-D pair $\left(i,j\right)$ for private automobiles and $\mathrm{P}_{ij} \subset \mathrm{P}$ is a set of paths that connect O-D pair $\left(i,j\right)$.

The set of all feasible private automobile flows can then be defined as

$$\Lambda_0 = \left\{ h_p^\chi \geq 0 : \sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} h_p^\chi\left(t\right)dt = Q_{ij}^\chi \quad \forall\left(i,j\right)\in W \right\} \subset \left(L_+^2\left[t_0,t_f\right]\right)^{|\mathrm{P}|} .$$

Given that each DPA seeks to minimize the private travel delay, we can use DUE to describe the private automobile traffic in the within-day time interval of our interest.

Theorem 3.1 A vector of path flows $h^* \in \Lambda_0$ is a DUE if

$$h_p^*\left(t\right) > 0,\, p \in \mathrm{P}_{ij} \Rightarrow \Psi_p\left[t, h^*\left(t\right)\right] = v_{ij}$$

where

$$v_{ij} = ess\inf\left[\Psi_p\left(t,h\right) : p \in \mathrm{P}_{ij}\right], \quad \forall\left(i,j\right)\in W .$$

Proof: See Friesz et al. (1993). □

Let's denote this equilibrium by $DUE\left(\Psi,\Lambda_0,t_0,t_f\right)$.

Theorem 3.2 $DUE\left(\Psi,\Lambda_0,t_0,t_f\right)$ is equivalent to the following differential variational inequality (DVI):

$$\left.\begin{array}{l}\text{find } h^* \in \Lambda \text{ such that} \\ \sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} \Psi_p\left(t,h^*\right)\left(h_p - h_p^*\right)dt \geq 0 \\ \forall h \in \Lambda\end{array}\right\} DVI\left(\Psi,\Lambda,t_0,t_f\right)$$

where

$$\Lambda = \left\{ h \geq 0 : \frac{dy_{ij}}{dt} = \sum_{p\in \mathrm{P}_{ij}} h_p\left(t\right), y_{ij}\left(t_0\right) = 0, y_{ij}\left(t_f\right) = Q_{ij} \quad \forall \left(i,j\right) \in W \right\}$$

and $y_{ij}\left(t\right)$ stands for the volume of traffic that arrived at destination j at time t.

Proof: See Friesz et al. (2011). □

Using Theorems 3.1 and 3.2, we can derive a DVI formulation of the DUE for private automobiles:

$$\left.\begin{array}{l}\text{find } h^{\chi*} \in \Lambda \text{ such that} \\ \sum_{(i,j)\in W} \sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} \Psi_p\left(t,h^{\chi*},h^{\eta}\right)\left(h_p^{\chi} - h_p^{\chi*}\right)dt \geq 0 \\ \forall h^{\chi} \in \Lambda\end{array}\right\} DVI^{pr}\left(\Psi,\Lambda,t_0,t_f\right) \tag{3.1}$$

where

$$\Lambda = \left\{ h^{\chi} \geq 0 : \frac{dy_{ij}^{\chi}}{dt} = \sum\nolimits_{p \in \mathrm{P}_{ij}} h_p^{\chi}(t), y_{ij}^{\chi}(t_0) = 0, y_{ij}^{\chi}\left(t_f\right) = Q_{ij}^{\chi} \quad \forall (i, j) \in W \right\}$$

and $y_{ij}^{\chi}(t)$ stands for the volume of private automobiles that arrived at destination j at time t.

To solve (3.1), the evaluation of $\Psi_p\left(t, h^{\chi}, h^{\eta}\right)$ given h^{χ} and h^{η} is necessary and it requires the quantification of $D_p\left(t, h^{\chi}, h^{\eta}\right)$ through dynamic network loading (DNL).

3.2 Link delay model based dynamic network loading of inhomogeneous traffic

Dynamic network loading (DNL) refers to "the determination of arc-specific volumes, arc-specific exit rates and experienced path delay when departure rates are known for each path" (Friesz et al., 2011). In this section we use a link delay model based DNL to quantify the path delay $D_p\left(t, h^{\chi}, h^{\eta}\right)$.

We represent each path by a sequence of connected arcs:

$$p = \left\{a_1, a_2, \ldots, a_i, \ldots, a_{m(p)}\right\}$$

where $m(p)$ denotes the number of arcs contained in path p. Flows of freight trucks and private automobiles that travel along path p exiting arc a_i are denoted by $g_{a_i}^{p,\eta}$ and $g_{a_i}^{p,\chi}$, respectively. Volume of freight trucks and private automobiles traveling along path p on arc a_i are denoted by $x_{a_i}^{p,\eta}$ and $x_{a_i}^{p,\chi}$, respectively.

It is straightforward to derive the following arc dynamics:

$$\left.\begin{aligned}
&\frac{dx_{a_1}^{p,\chi}(t)}{dt} = h_p^{\chi}(t) - g_{a_1}^{p,\chi}(t) \quad \forall p \in \mathrm{P} \\
&\frac{dx_{a_i}^{p,\chi}(t)}{dt} = g_{a_{i-1}}^{p,\chi}(t) - g_{a_i}^{p,\chi}(t) \quad \forall p \in \mathrm{P}, i \in [2, m(p)] \\
&x_{a_i}^{p,\chi}(t_0) = x_{a_i 0}^{p,\chi} \quad \forall p \in \mathrm{P}, i \in [1, m(p)] \\
&\frac{dx_{a_1}^{p,\eta}(t)}{dt} = h_p^{\eta}(t) - g_{a_1}^{p,\eta}(t) \quad \forall p \in \mathrm{P} \\
&\frac{dx_{a_i}^{p,\eta}(t)}{dt} = g_{a_{i-1}}^{p,\eta}(t) - g_{a_i}^{p,\eta}(t) \quad \forall p \in \mathrm{P}, i \in [2, m(p)] \\
&x_{a_i}^{p,\eta}(t_0) = x_{a_i 0}^{p,\eta} \quad \forall p \in \mathrm{P}, i \in [1, m(p)]
\end{aligned}\right\} \tag{3.2}$$

where $x_{a_i 0}^{p,\chi}$ and $x_{a_i 0}^{p,\eta}$ are initial volumes of private automobiles and freight trucks traveling along path p on arc a_i.

Let's define

$$\delta_{a_i p} = \begin{cases} 1, & \text{if arc } a_i \text{ belongs to path } p \\ 0, & \text{otherwise} \end{cases}.$$

The volume of private automobiles on a specific arc a_i at time t is

$$x_{a_i}^{\chi}(t) = \sum_{p \in \mathrm{P}_{ij}} \delta_{a_i p} x_{a_i}^{p,\chi}(t).$$

The volume of freight trucks on a specific arc a_i at time t is

$$x_{a_i}^{\eta}(t) = \sum_{p \in \mathrm{P}_{ij}} \delta_{a_i p} x_{a_i}^{p,\eta}(t).$$

The unit path delay equals the accumulated delays on arcs that compose the path. We denote the delay that a vehicle will experience when it enters arc a_i at time t by $D_{a_i}\left(x_{a_i}(t)\right)$, which is a function of the traffic volume on arc a_i at time t. For simplicity, we assume a linear delay function, that is,

$$D_{a_i}(x) = A_{a_i} + B_{a_i} x$$

where A_{a_i} and B_{a_i} are two positive constants. Considering the fact that a freight truck and a private automobile may have different impacts on congestion due to their different sizes and speeds, we assume that

$$x_{a_i}(t) = x_{a_i}^{\chi}(t) + \beta x_{a_i}^{\eta}(t)$$

where β is a constant and is greater than or equal to 1.

If a vehicle enters path p at time t, we denote its exit time from arc a_i of path p by $\tau_{a_i}^{p}(t)$ and we have

$$\tau_{a_1}^{p}(t) = t + D_{a_1}\left[x_{a_1}(t)\right] \quad \forall p \in P$$

$$\tau_{a_i}^{p}(t) = \tau_{a_{i-1}}^{p}(t) + D_{a_i}\left[x_{a_i}\left(\tau_{a_{i-1}}^{p}(t)\right)\right] \quad \forall p \in P, i \in \left[2, m(p)\right].$$

By applying the chain rule, we can derive the following flow propagation constraints based on the above exit time functions (see Friesz et al. (2001) for more details):

$$\left.\begin{array}{l}
g_{a_1}^{p,\chi}\left(t + D_{a_1}\left[x_{a_1}(t)\right]\right)\left(1 + D_{a_1}'\left[x_{a_1}(t)\right]\dot{x}_{a_1}\right) = h_p^{\chi}(t) \quad \forall p \in \mathrm{P} \\
g_{a_1}^{p,\eta}\left(t + D_{a_1}\left[x_{a_1}(t)\right]\right)\left(1 + D_{a_1}'\left[x_{a_1}(t)\right]\dot{x}_{a_1}\right) = h_p^{\eta}(t) \quad \forall p \in \mathrm{P} \\
g_{a_i}^{p,\chi}\left(t + D_{a_i}\left[x_{a_i}(t)\right]\right)\left(1 + D_{a_i}'\left[x_{a_i}(t)\right]\left(\dot{x}_{a_i}\right)\right) = g_{a_{i-1}}^{p,\chi}(t) \quad \forall p \in \mathrm{P}, i \in \left[2, m(p)\right] \\
g_{a_i}^{p,\eta}\left(t + D_{a_i}\left[x_{a_i}(t)\right]\right)\left(1 + D_{a_i}'\left[x_{a_i}(t)\right]\left(\dot{x}_{a_i}\right)\right) = g_{a_{i-1}}^{p,\eta}(t) \quad \forall p \in \mathrm{P}, i \in \left[2, m(p)\right]
\end{array}\right\}$$

where as defined earlier

$$x_{a_i}(t) = \sum_{p \in \mathrm{P}_{ij}} \delta_{a_i p}\left(x_{a_i}^{p,\chi}(t) + \beta x_{a_i}^{p,\eta}(t)\right).$$

In the above flow propagation constraints, the superscript "$'$" denotes differentiation with respect to the associated function argument and the overdot "·" refers to differentiation with respect to time.

The time shifts ($t+D_{a_i}\left[\cdot\right]$) in the above flow propagation constraints complicate the solution of DNL. We adopt the approximation proposed by Friesz et al. (2011) to simplify the network loading procedure. By defining

$$r_{a_i}^{p}\left(t\right)=\frac{dg_{a_i}^{p}\left(t\right)}{dt} \quad \forall p\in \mathrm{P},\, i\in\left[1,m\left(p\right)\right],$$

the flow propagation constraints can be approximated by

$$\left.\begin{aligned}
&\frac{dr_{a_1}^{p,\chi}\left(t\right)}{dt}=R_{a_1}^{p,\chi}\left(x,g,r,h\right) \quad \forall p\in \mathrm{P}\\
&\frac{dr_{a_i}^{p,\chi}\left(t\right)}{dt}=R_{a_i}^{p,\chi}\left(x,g,r\right) \quad \forall p\in \mathrm{P}, i\in\left[2,m\left(p\right)\right]\\
&\frac{dr_{a_1}^{p,\eta}\left(t\right)}{dt}=R_{a_1}^{p,\eta}\left(x,g,r,h\right) \quad \forall p\in \mathrm{P}\\
&\frac{dr_{a_i}^{p,\eta}\left(t\right)}{dt}=R_{a_i}^{p,\eta}\left(x,g,r\right) \quad \forall p\in \mathrm{P}, i\in\left[2,m\left(p\right)\right]
\end{aligned}\right\} \tag{3.3}$$

where

$$R_{a_1}^{p,\chi}\left(x,g,r,h\right)=\frac{2h_p^{\chi}\left(t\right)}{\left(D_{a_1}\left[x_{a_1}\left(t\right)\right]\right)^2\left(1+D_{a_1}'\left[x_{a_1}\left(t\right)\right]\dot{x}_{a_1}\right)}-\frac{2\left(g_{a_1}^{p,\chi}\left(t\right)+r_{a_1}^{p,\chi}\left(t\right)D_{a_1}\left[x_{a_1}\left(t\right)\right]\right)}{\left(D_{a_1}\left[x_{a_1}\left(t\right)\right]\right)^2}$$
$$\forall p\in \mathrm{P}$$

$$R_{a_i}^{p,\chi}\left(x,g,r\right)=\frac{2g_{a_{i-1}}^{p,\chi}\left(t\right)}{\left(D_{a_i}\left[x_{a_i}\left(t\right)\right]\right)^2\left(1+D_{a_i}'\left[x_{a_i}\left(t\right)\right]\dot{x}_{a_i}\right)}-\frac{2\left(g_{a_i}^{p,\chi}\left(t\right)+r_{a_i}^{p,\chi}\left(t\right)D_{a_i}\left[x_{a_i}\left(t\right)\right]\right)}{\left(D_{a_i}\left[x_{a_i}\left(t\right)\right]\right)^2}$$
$$\forall p\in \mathrm{P}, i\in\left[2,m\left(p\right)\right]$$

$$R_{a_1}^{p,\eta}\left(x,g,r,h\right)=\frac{2h_p^{\eta}\left(t\right)}{\left(D_{a_1}\left[x_{a_1}\left(t\right)\right]\right)^2\left(1+D_{a_1}'\left[x_{a_1}\left(t\right)\right]\dot{x}_{a_1}\right)}-\frac{2\left(g_{a_1}^{p,\eta}\left(t\right)+r_{a_1}^{p,\eta}\left(t\right)D_{a_1}\left[x_{a_1}\left(t\right)\right]\right)}{\left(D_{a_1}\left[x_{a_1}\left(t\right)\right]\right)^2}$$
$$\forall p\in \mathrm{P}$$

$$R_{a_i}^{p,\eta}\left(x,g,r\right)=\frac{2g_{a_{i-1}}^{p,\eta}\left(t\right)}{\left(D_{a_i}\left[x_{a_i}\left(t\right)\right]\right)^2\left(1+D_{a_i}'\left[x_{a_i}\left(t\right)\right]\dot{x}_{a_i}\right)}-\frac{2\left(g_{a_i}^{p,\eta}\left(t\right)+r_{a_i}^{p,\eta}\left(t\right)D_{a_i}\left[x_{a_i}\left(t\right)\right]\right)}{\left(D_{a_i}\left[x_{a_i}\left(t\right)\right]\right)^2}$$
$$\forall p\in \mathrm{P}, i\in\left[2,m\left(p\right)\right]$$

Theorem 3.3 There exists a unique solution to (3.2) and (3.3) if (3.2) and (3.3) satisfy the following regularity conditions:

(a). the arc exit time functions $\tau_{a_i}^{p}(t)$ for all *i* are strictly monotonic;

(b). the arc delay functions $D_{a_i}\left(x_{a_i}(t)\right)$ for all *i* are bounded and strictly positive;

(c). $D''_{a_i}\left(x_{a_i}(t)\right)$ exists and is continuous;

(d). $h_p^{\eta}(t)$ and $h_p^{\chi}(t)$ are continuous.

Proof: Per Walter (1988), a unique solution to (3.2) and (3.3) exists if the right-hand side of each differential equation in (3.2) and (3.3) is continuously differentiable with respect to $x_{a_i}^{\chi}$, $x_{a_i}^{\eta}$, $g_{a_i}^{p,\chi}$, $g_{a_i}^{p,\eta}$, $r_{a_i}^{p,\chi}$ and $r_{a_i}^{p,\eta}$. It is obvious that the differential equations in (3.2) have this property. The remaining effort is on proving those in (3.3) have the same property, that is, $R_{a_i}^{p,\chi}$, $R_{a_i}^{p,\eta}$ are differentiable with respect to $x_{a_i}^{\chi}$, $x_{a_i}^{\eta}$, $g_{a_i}^{p,\chi}$, $g_{a_i}^{p,\eta}$, $r_{a_i}^{p,\chi}$ and $r_{a_i}^{p,\eta}$. For simplicity, here we just show the closed-form expressions for those derivatives of $R_{a_1}^{p,\chi}$ and $R_{a_1}^{p,\eta}$, similar results can be established for $R_{a_i}^{p,\chi}$ and $R_{a_i}^{p,\eta}$ for $i \in \left[2, m(p)\right]$.

It is not difficult to derive the following closed-form expressions for derivatives of $R_{a_i}^{p,\chi}$ and $R_{a_i}^{p,\eta}$:

$$\frac{\partial R_{a_1}^{p,\chi}}{\partial x_{a_1}^{\chi}} = \frac{\partial R_{a_1}^{p,\chi}}{\partial x_{a_1}^{\eta}} = X_{a_1}^{\chi} - \frac{2 r_{a_1}^{p,\chi}(t) D'_{a_1}\left[x_{a_1}(t)\right]}{\left(D_{a_1}\left[x_{a_1}(t)\right]\right)^2} + \frac{4\left(g_{a_1}^{p,\chi}(t) + r_{a_1}^{p,\chi}(t) D_{a_1}\left[x_{a_1}(t)\right]\right) D_{a_1}\left[x_{a_1}(t)\right] D'_{a_1}\left[x_{a_1}(t)\right]}{\left(D_{a_1}\left[x_{a_1}(t)\right]\right)^4} \quad \forall p \in \mathrm{P}$$

$$\frac{\partial R_{a_1}^{p,\eta}}{\partial x_{a_1}^{\chi}} = \frac{\partial R_{a_1}^{p,\eta}}{\partial x_{a_1}^{\eta}} = X_{a_1}^{\eta} - \frac{2r_{a_1}^{p,\eta}(t) D'_{a_1}\left[x_{a_1}(t)\right]}{\left(D_{a_1}\left[x_{a_1}(t)\right]\right)^2} + \frac{4\left(g_{a_1}^{p,\eta}(t) + r_{a_1}^{p,\eta}(t) D_{a_1}\left[x_{a_1}(t)\right]\right) D_{a_1}\left[x_{a_1}(t)\right] D'_{a_1}\left[x_{a_1}(t)\right]}{\left(D_{a_1}\left[x_{a_1}(t)\right]\right)^4} \quad \forall p \in \mathrm{P}$$

$$\frac{\partial R_{a_1}^{p,\chi}}{\partial g_{a_1}^{p,\chi}} = \frac{2h_p^{\chi}(t)}{\left(D_{a_1}\left[x_{a_1}(t)\right]\right)^2} \frac{D'_{a_1}\left[x_{a_1}(t)\right]}{\left(1 + D'_{a_1}\left[x_{a_1}(t)\right]\left(h_p^{\chi}(t) - g_{a_1}^{p,\chi}(t)\right)\right)^2} - \frac{2g_{a_1}^{p,\chi}(t)}{\left(D_{a_1}\left[x_{a_1}(t)\right]\right)^2} \quad \forall p \in \mathrm{P}$$

$$\frac{\partial R_{a_1}^{p,\eta}}{\partial g_{a_1}^{p,\eta}} = \frac{2h_p^{\eta}(t)}{\left(D_{a_1}\left[x_{a_1}(t)\right]\right)^2} \frac{D'_{a_1}\left[x_{a_1}(t)\right]}{\left(1 + D'_{a_1}\left[x_{a_1}(t)\right]\left(h_p^{\eta}(t) - g_{a_1}^{p,\eta}(t)\right)\right)^2} - \frac{2g_{a_1}^{p,\eta}(t)}{\left(D_{a_1}\left[x_{a_1}(t)\right]\right)^2} \quad \forall p \in \mathrm{P}$$

$$\frac{\partial R_{a_1}^{p,\chi}}{\partial r_{a_1}^{p,\chi}} = \frac{\partial R_{a_1}^{p,\eta}}{\partial r_{a_1}^{p,\eta}} - \frac{2}{D_{a_1}\left[x_{a_1}(t)\right]} \quad \forall p \in \mathrm{P}$$

where

$$X_{a_1}^{\chi} = \frac{-2h_p^{\chi}(t)\left(Y_{a_1}^{\chi} + Z_{a_1}^{\chi}\right)}{\left(D_{a_1}\left[x_{a_1}(t)\right]\right)^4 \left(1 + D'_{a_1}\left[x_{a_1}(t)\right]\left(h_p^{\chi}(t) - g_{a_1}^{p,\chi}(t)\right)\right)^2} \quad \forall p \in \mathrm{P}$$

$$X_{a_1}^{\eta} = \frac{-2h_p^{\eta}(t)\left(Y_{a_1}^{\eta} + Z_{a_1}^{\eta}\right)}{\left(D_{a_1}\left[x_{a_1}(t)\right]\right)^4 \left(1 + D'_{a_1}\left[x_{a_1}(t)\right]\left(h_p^{\eta}(t) - g_{a_1}^{p,\eta}(t)\right)\right)^2} \quad \forall p \in \mathrm{P}$$

$$Y_{a_1}^{\chi} = 2D_{a_1}\left[x_{a_1}(t)\right] D'_{a_1}\left[x_{a_1}(t)\right]\left(1 + D'_{a_1}\left[x_{a_1}(t)\right]\left(h_p^{\chi}(t) - g_{a_1}^{p,\chi}(t)\right)\right) \quad \forall p \in \mathrm{P}$$

$$Y_{a_1}^{\eta} = 2D_{a_1}\left[x_{a_1}(t)\right] D'_{a_1}\left[x_{a_1}(t)\right]\left(1 + D'_{a_1}\left[x_{a_1}(t)\right]\left(h_p^{\eta}(t) - g_{a_1}^{p,\eta}(t)\right)\right) \quad \forall p \in \mathrm{P}$$

$$Z_{a_1}^{\chi} = \left(D_{a_1}\left[x_{a_1}(t)\right]\right)^2 \left(D''_{a_1}\left[x_{a_1}(t)\right]\left(h_p^{\chi}(t) - g_{a_1}^{p,\chi}(t)\right)\right) \quad \forall p \in \mathrm{P}$$

$$Z_{a_1}^{\eta} = \left(D_{a_1}\left[x_{a_1}(t)\right]\right)^2 \left(D''_{a_1}\left[x_{a_1}(t)\right]\left(h_p^{\eta}(t) - g_{a_1}^{p,\eta}(t)\right)\right) \quad \forall p \in \mathrm{P}$$

Thus the theorem is proven. □

So given h^χ and h^η fixed, solving (3.2) and (3.3) which is also known as the differential algebraic equation (DAE) system, we can evaluate $x_{a_i}(t)$, $D_{a_i}\left[x_{a_i}(t)\right]$ and $\tau_{a_i}^p(t)$ for all $p \in \mathrm{P}, i \in \left[2, m(p)\right]$, and then quantify unit path delay using the following function

$$D_p(t) = \sum_{i=1}^{m(p)} \left[\tau_{a_i}^p(t) - \tau_{a_{i-1}}^p(t)\right] = \tau_{a_{m(p)}}^p(t) - t\,.$$

It is proven by Friesz et al. (1993) that a closed-form formulation of DUE can preserve first-in-first-out (FIFO) rule under mild assumptions. In the remaining of this section, we prove that a similar result holds for the proposed DNL of inhomogeneous traffic.

First we need to introduce the following lemma.

Lemma 3.1 If function $f: \Re \to \Re$ is invertible and differentiable with a derivative f':

$$\left[f^{-1}\right]'(z) = 1 \big/ f'\left[f^{-1}(z)\right]$$

Proof: See Friesz et al. (1993). □

Now we can demonstrate the following theorem.

Theorem 3.4 For the arc delay function

$$D_{a_i}\left(x_{a_i}(t)\right) = A_{a_i} + B_{a_i}\left(x_{a_i}^\chi(t) + \beta x_{a_i}^\eta(t)\right),$$

the resulting arc exit time function τ_{a_i} is strictly increasing thus invertible. Consequently, the FIFO rule is satisfied.

Proof: We partition the time into appropriate intervals and assume without loss of generality that

$$x_{a_i}^{pr}(0) = x_{a_i}^{tr}(0) = 0\,.$$

We in addition assume that the first vehicle enters arc a_i at time 0 and it does not matter whether it is a private automobile or a freight truck. We denote the time that the first vehicle exits arc a_i by t_1 and we have by definition

$$t_1 = D_{a_i}(0) = A_{a_i}$$

and

$$x_{a_i}(t) = x_{a_i}^{\chi}(t) + \beta x_{a_i}^{\eta}(t) = \int_0^t u_{a_i}^{\chi}(s)\,ds + \beta\int_0^t u_{a_i}^{\eta}(s)\,ds, \quad \forall t \in [0, t_1]$$

where

$$u_{a_i}^{\chi}(t) = \sum_{p\in P} h_p^{\chi}(t)\,\delta_{a_i p}$$

and

$$u_{a_i}^{\eta}(t) = \sum_{p\in P} h_p^{\eta}(t)\,\delta_{a_i p}\ .$$

$u_{a_i}^{\chi}(t)$ and $u_{a_i}^{\eta}(t)$ denote the flow of private automobiles and freight trucks entering arc a_i , respectively.

Hence for any $t \in [0, t_1]$, we denote the exit time function by $\tau_{1,a_i}(t)$, and we have

$$\tau_{1,a_i}(t) = t + D_{a_i}(t) = t + A_{a_i} + B_{a_i}\left[\int_0^t u_{a_i}^{\chi}(s)\,ds + \beta\int_0^t u_{a_i}^{\eta}(s)\,ds\right].$$

Note that

$$t_1 = \tau_{1,a_i}(0)$$

and $\tau_{1,a_i}(t)$ is differentiable with

$$\tau_{1,a_i}'(t) = 1 + B_{a_i}\left[u_{a_i}^{\chi}(t) + \beta u_{a_i}^{\eta}(t)\right] \tag{3.4}$$

which is strictly positive since B_{a_i} is positive and $u_{a_i}^{\chi}(t)$ and $u_{a_i}^{\eta}(t)$ are nonnegative. Hence, $\tau_{1,a_i}(t)$ is increasing on $[0,t_1]$ and has a well defined inverse function $\tau_{1,a_i}^{-1}(t)$ on $[\tau_{a_1}(0),\tau_{a_1}(t_1)]$.

Let $t_2=\tau_{a_1}(t_1)$

and

$$[\tau_{a_1}(0),\tau_{a_1}(t_1)]\equiv[t_1,t_2].$$

The commuters traveling on arc a_i at time $t\in[t_1,t_2]$ are those who entered the arc during the interval $\left[\tau_{1,a_i}^{-1}(t),t\right]$. Hence, if we denote the exit time for commuters entering arc a_i during the interval $[t_1,t_2]$ by $\tau_{2,a_i}(t)$, then for all $t\in[t_1,t_2]$,

$$\tau_{2,a_i}(t)=t+A_{a_i}+B_{a_i}\left[\int_{\tau_{1,a_i}^{-1}(t)}^{t}u_{a_i}^{\chi}(s)ds+\beta\int_{\tau_{1,a_i}^{-1}(t)}^{t}u_{a_i}^{\eta}(s)ds\right]$$

and

$$\tau_{2,a_i}(t_1)=\tau_{1,a_i}(t_1)=t_2.$$

The derivative of $\tau_{2,a_i}(t)$ is given by

$$\begin{aligned}
\tau_{2,a_i}'(t)&=1+B_{a_i}\left[u_{a_i}^{\chi}(t)-u_{a_i}^{\chi}\left[\tau_{1,a_i}^{-1}(t)\right]\left[\tau_{1,a_i}^{-1}\right]'(t)+\beta u_{a_i}^{\eta}(t)-\beta u_{a_i}^{\eta}\left[\tau_{1,a_i}^{-1}(t)\right]\left[\tau_{1,a_i}^{-1}\right]'(t)\right]\\
&=1+B_{a_i}u_{a_i}^{\chi}(t)-\frac{B_{a_i}u_{a_i}^{\chi}\left[\tau_{1,a_i}^{-1}(t)\right]}{\tau_{1,a_i}'\left[\tau_{1,a_i}^{-1}(t)\right]}+B_{a_i}\beta u_{a_i}^{\eta}(t)-\frac{B_{a_i}\beta u_{a_i}^{\eta}\left[\tau_{1,a_i}^{-1}(t)\right]}{\tau_{1,a_i}'\left[\tau_{1,a_i}^{-1}(t)\right]}\\
&=1+B_{a_i}u_{a_i}^{\chi}(t)-\frac{B_{a_i}u_{a_i}^{\chi}\left[\tau_{1,a_i}^{-1}(t)\right]}{1+B_{a_i}\left(u_{a_i}^{\chi}\left[\tau_{1,a_i}^{-1}(t)\right]+\beta u_{a_i}^{\eta}\left[\tau_{1,a_i}^{-1}(t)\right]\right)}\\
&\qquad+B_{a_i}\beta u_{a_i}^{\eta}(t)-\frac{B_{a_i}\beta u_{a_i}^{\eta}\left[\tau_{1,a_i}^{-1}(t)\right]}{1+B_{a_i}\left(u_{a_i}^{\chi}\left[\tau_{1,a_i}^{-1}(t)\right]+\beta u_{a_i}^{\eta}\left[\tau_{1,a_i}^{-1}(t)\right]\right)}\\
&=B_{a_i}\left(u_{a_i}^{\chi}(t)+\beta u_{a_i}^{\eta}(t)\right)-\frac{1}{1+B_{a_i}\left(u_{a_i}^{\chi}\left[\tau_{1,a_i}^{-1}(t)\right]+\beta u_{a_i}^{\eta}\left[\tau_{1,a_i}^{-1}(t)\right]\right)}\\
&>B_{a_i}\left(u_{a_i}^{\chi}(t)+\beta u_{a_i}^{\eta}(t)\right)\geq 0
\end{aligned}$$

The second and third equalities hold because of Lemma 3.1 and equation (3.4), respectively.

Thus, we can again conclude that $\tau_{2,a_i}(t)$ is increasing on $[t_1, t_2]$ and has a well defined inverse function $\tau_{2,a_i}^{-1}(t)$ on $\left[\tau_{a_1}(t_1), \tau_{a_1}(t_2)\right]$.

We proceed by induction. For $n = 2$, we have already shown that

$$\tau_{2,a_i}(t) = t + A_{a_i} + B_{a_i}\left[\int_{\tau_{1,a_i}^{-1}(t)}^{t} u_{a_i}^{\chi}(s)\,ds + \beta\int_{\tau_{1,a_i}^{-1}(t)}^{t} u_{a_i}^{\eta}(s)\,ds\right]$$

$$\tau_{2,a_i}(t_1) = \tau_{1,a_i}(t_1) = t_2$$

$$\tau_{2,a_i}'(t) > B_{a_i}\left(u_{a_i}^{\chi}(t) + \beta u_{a_i}^{\eta}(t)\right) \geq 0$$

Then we choose any $k > 2$. Suppose that for $n = k$, the exit time function $\tau_{k,a_i}(t)$ is invertible and the following conditions hold:

$$\tau_{k,a_i}(t) = t + A_{a_i} + B_{a_i}\left[\int_{\tau_{k-1,a_i}^{-1}(t)}^{t} u_{a_i}^{\chi}(s)\,ds + \beta\int_{\tau_{k-1,a_i}^{-1}(t)}^{t} u_{a_i}^{\eta}(s)\,ds\right] \tag{3.5}$$

$$\tau_{k,a_i}(t_{k-1}) = \tau_{k-1,a_i}(t_{k-1}) = t_k \tag{3.6}$$

$$\tau_{k,a_i}'(t) > B_{a_i}\left(u_{a_i}^{\chi}(t) + \beta u_{a_i}^{\eta}(t)\right) \text{ for all } t \in [t_{k-1}, t_k] \tag{3.7}$$

We wish to show that for $n = k + 1$, if we let

$$t_{k+1} = \tau_{k,a_i}(t_k),$$

the exit time function $\tau_{k+1,a_i}(t_k)$ satisfies the following conditions:

$$\tau_{k+1,a_i}(t) = t + A_{a_i} + B_{a_i}\left[\int_{\tau_{k,a_i}^{-1}(t)}^{t} u_{a_i}^{\chi}(s)\,ds + \beta\int_{\tau_{k,a_i}^{-1}(t)}^{t} u_{a_i}^{\eta}(s)\,ds\right] \tag{3.8}$$

$$\tau_{k+1,a_i}(t_k) = \tau_{k,a_i}(t_k) = t_{k+1} \tag{3.9}$$

$$\tau_{k+1,a_i}'(t) > B_{a_i}\left(u_{a_i}^{\chi}(t) + \beta u_{a_i}^{\eta}(t)\right) \text{ for all } t \in [t_k, t_{k+1}] \tag{3.10}$$

By definition, it is easy to show that equation (3.8) is satisfied. So we only need to derive equations (3.9) and (3.10) in the rest of this proof.

First, similar to the derivation of $\tau'_{2,a_i}(t)$ we can show equation (3.10) by the following:

$$
\begin{aligned}
\tau'_{k+1,a_i}(t) &= 1 + B_{a_i}\left[u_{a_i}^{\chi}(t) - u_{a_i}^{\chi}\left[\tau_{k,a_i}^{-1}(t)\right]\left[\tau_{k,a_i}^{-1}\right]'(t) + \beta u_{a_i}^{\eta}(t) - \beta u_{a_i}^{\eta}\left[\tau_{k,a_i}^{-1}(t)\right]\left[\tau_{k,a_i}^{-1}\right]'(t)\right] \\
&= 1 + B_{a_i}u_{a_i}^{\chi}(t) - \frac{B_{a_i}u_{a_i}^{\chi}\left[\tau_{k,a_i}^{-1}(t)\right]}{\tau'_{k,a_i}\left[\tau_{k,a_i}^{-1}(t)\right]} + B_{a_i}\beta u_{a_i}^{\eta}(t) - \frac{B_{a_i}\beta u_{a_i}^{\eta}\left[\tau_{k,a_i}^{-1}(t)\right]}{\tau'_{k,a_i}\left[\tau_{k,a_i}^{-1}(t)\right]} \\
&= B_{a_i}\left(u_{a_i}^{\chi}(t) + \beta u_{a_i}^{\eta}(t)\right) - \frac{1}{1 + B_{a_i}\left(u_{a_i}^{\chi}\left[\tau_{k,a_i}^{-1}(t)\right] + \beta u_{a_i}^{\eta}\left[\tau_{k,a_i}^{-1}(t)\right]\right)} \\
&> B_{a_i}\left(u_{a_i}^{\chi}(t) + \beta u_{a_i}^{\eta}(t)\right) \geq 0
\end{aligned}
$$

Then by equation (3.6) and the assumed invertibility of $\tau_{k,a_i}(t)$ and $\tau_{k-1,a_i}(t)$, we have

$$t_{k-1} = \tau_{k,a_i}^{-1}(t_k)$$

and

$$t_{k-1} = \tau_{k-1,a_i}^{-1}(t_{k-1}).$$

This together with equation (3.8) yields

$$
\begin{aligned}
\tau_{k+1,a_i}(t_k) &= t_k + A_{a_i} + B_{a_i}\left[\int_{\tau_{k,a_i}^{-1}(t_k)}^{t_k} u_{a_i}^{\chi}(s)ds + \beta\int_{\tau_{k,a_i}^{-1}(t_k)}^{t_k} u_{a_i}^{\eta}(s)ds\right] \\
&= t_k + A_{a_i} + B_{a_i}\left[\int_{t_{k-1}}^{t_k} u_{a_i}^{\chi}(s)ds + \beta\int_{t_{k-1}}^{t_k} u_{a_i}^{\eta}(s)ds\right] \\
&= t_k + A_{a_i} + B_{a_i}\left[\int_{\tau_{k-1,a_i}^{-1}(t_{k-1})}^{t_k} u_{a_i}^{\chi}(s)ds + \beta\int_{\tau_{k-1,a_i}^{-1}(t_{k-1})}^{t_k} u_{a_i}^{\eta}(s)ds\right] \\
&= \tau_{k,a_i}(t_k)
\end{aligned}
$$

Therefore, equation (3.9) holds.

Consequently, the exit time function $\tau_{a_i}(t)$ is everywhere continuous and increasing on $\Re_+$ which indicates that

$$\tau_{a_i}(t_1) > \tau_{a_i}(t_2) \text{ if } t_1 > t_2.$$

Thus the FIFO rule holds. □

Chapter 4

A Dynamic Stackelberg Game-Theoretic Model for Urban Freight Transportation Planning: Model I

In this chapter, we consider that there is a metropolitan planning organization (MPO) who is responsible for reducing traffic congestion on the urban road network and has the power to control urban freight transportation. Also, there are drivers of private automobiles (DPAs) who need to travel on the urban road network. We assume that each DPA aims at minimizing the individual travel delay and use dynamic user equilibrium (DUE) to describe the dynamic assignment of private automobile traffic on the urban road network. Since the freight truck traffic can affect the travel delay experienced by DPAs, the DUE of private automobiles is dependent on the freight truck traffic. Therefore, the MPO wants to find the optimal freight truck transportation plan which can satisfy the freight transportation demand and minimize the total delay experienced by all travelers (freight trucks and private automobiles). We model urban freight transportation planning as a dynamic Stackelberg (leader-follower) game where the leader is the MPO and the followers are the DPAs. In Section 4.1, we formulate the problem as a dynamic mathematical program with equilibrium constraints (MPEC). The solution approach is discussed in Section 4.2. Finally in Section 4.3, we conduct numerical tests to derive the optimal urban freight transportation plan that minimizes the associated congestion while satisfying the freight transportation demand.

4.1 Problem formulation

We model the problem as a dynamic Stackelberg game. Specifically, the leader is the MPO who controls the freight truck traffic. The followers are DPAs whose reactions to the leader's action lead to a DUE of private automobile traffic. Note that although the interaction between freight truck traffic and private automobile traffic is not clear in reality, we do observe in the real world that on an urban road network the DPAs may not travel on a route because the route is mainly occupied by freight trucks. Also truck drivers may select a route because the route does not pass through the regions with high population density and thus are less crowded with private automobiles. Since the DPAs do not cooperate, the impact of each private automobile's travel behavior (route and departure time) on the network-wide traffic is not comparable to that of the freight trucks. While, the MPO, as a player who can control the movement of all freight trucks on the urban road network, clearly has the power to affect DPAs' travel behaviors by controlling freight truck traffic. Moreover, user equilibrium is a well-recognized model that characterizes the network traffic assignment of independent drivers. Since the MPO can access traffic data and has the ability to estimate the equilibrium, it has the ability to predict the reactions of DPAs to a specific freight transportation schedule. What is more, the MPO, whose goal is to increase social welfare, has the motivation to act as a leader in this game. As a result, the assumption of the leader-follower relation between the MPO and the DPAs is valid.

In the proposed Stackelberg game, the leader (the MPO) aims at minimizing the total delay while satisfying the freight shipping demand over the time frame of interest, which can be represented by (4.1) and (4.2), respectively:

$$\min_{h^{\eta}} \sum_{(i,j)\in W} \sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} \Psi_p\left(t, h^{\chi*}, h^{\eta}\right)\left(h_p^{\chi*} + h_p^{\eta}\right) dt \tag{4.1}$$

$$\text{s.t.} \quad \sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} h_p^{\eta}\left(t\right) dt = Q_{ij}^{\eta},\ h_p^{\eta} \geq 0 \quad \forall\left(i,j\right)\in W \tag{4.2}$$

where

$$\Psi_p\left(t,h^{\chi},h^{\eta}\right)=D_p\left(t,h^{\chi},h^{\eta}\right)+F\left[t+D_p\left(t,h^{\chi},h^{\eta}\right)-T_A\right] \quad \forall(i,j)\in W,\ p\in P_{ij}\,.$$

Then the Stackelberg game-theoretic model can be represented by {(3.1)-(3.3), (4.1), (4.2)} which is a dynamic mathematical program with equilibrium constraints (MPEC). We denote this dynamic MPEC by D-MPEC-I$\left(t,h^{\chi^*},h^{\eta}\right)$.

In D-MPEC-I$\left(t,h^{\chi^*},h^{\eta}\right)$, h^{χ^*} stands for the vector of equilibrium flows of private automobiles and h^{η} is the vector of freight truck flows. (4.1) minimizes the total effective delay experienced by all travelers on the network in time interval $\left[t_0,t_f\right]$. Equation (4.2) is the demand consumption constraint for freight trucks. (4.1) and (4.2) compose the upper level of D-MPEC-I$\left(t,h^{\chi^*},h^{\eta}\right)$. (3.1)-(3.3) characterize the equilibrium flows of private automobiles and compose the lower level of D-MPEC-I$\left(t,h^{\chi^*},h^{\eta}\right)$.

Whether there exists a solution to D-MPEC-I$\left(t,h^{\chi^*},h^{\eta}\right)$ is a crucial question. It will be addressed if we can show that for any assignment of freight truck flows $h_p^{\eta}(t)$, $\forall p$ that are feasible to (4.2), the DVI (3.1) has a solution h^{χ^*}. If $\Psi_p\left(t,h^{\chi},h^{\eta}\right)$ for all p is continuous and the feasible set Λ in (3.1) is compact, the fixed point theory of multi-valued mappings in topological vector spaces discussed by Browder (1968) can be applied to show that there exists a solution to (3.1). So, existence results are general if the DUE is based on the formulation (3.1). However, a rigorous proof of existence is still challenging and remains to be addressed by future studies.

4.2 Solution approach

D-MPEC-I$\left(t, h^{\chi^*}, h^{\eta}\right)$ is nonconvex and contains a nondifferentiable objective function even after time-discretization. Classical gradient-based algorithm may not apply for solving D-MPEC-I$\left(t, h^{\chi^*}, h^{\eta}\right)$. In this chapter, we adapt the simulated annealing (SA) algorithm in Friesz et al. (1992) to solve D-MPEC-I$\left(t, h^{\chi^*}, h^{\eta}\right)$.

SA algorithm is a metaheuristic for global optimization. It is motivated by the similarity between combinatorial optimization and statistical mechanics (Kirkpatrick et al., 1983). The algorithm works as the annealing of a physical system. In order to obtain a crystal, we need to heat the system to a temperature at which the energy of the system is high and many atomic rearrangements occurs. Then we cool the system (lower the energy of the system) carefully so that at each temperature stage a thermal equilibrium is reached. Finally the material will freeze into a crystal. We can consider the solution and the associated objective value of an optimization problem as the state and the associated energy of a physical system, respectively. We start with an initial solution and a high "temperature". In each iteration, we randomly search the neighborhood of the current solution. We accept a new solution at 100% probability if it yields a better objective value (lower system energy). Otherwise, we will accept the new solution at a percentage which decreases with the "temperature". The essence of SA algorithm is that it seeks good candidate solutions globally at first and finally converges to the "local optimum" around one of the candidate solutions. Hence, by using SA algorithm we will not be stuck at a neighborhood of the initial solution while searching for global optimum.

The main structure of the adapted SA algorithm for solving D-MPEC-I$\left(t, h^{\chi^*}, h^{\eta}\right)$ is illustrated as follows.

Algorithm 4.1 SA

Step 0 Set $temp = 10^6$ (the initial "temperature"). Set $k = 0$. Choose an initial solution $h^{\eta,k}$ ("system state"). Solve (3.1) and obtain $h^{\chi^*,k}$. Calculate the total delay experienced by all travelers and denote it by C^k ("system energy").

Step 1 If $temp < temp_{\min}$ or $k > k_{\max}$ where $temp_{\min}$ and $k > k_{\max}$ are predetermined and represent respectively the minimum "temperature" and the maximum number of major iterations, stop the algorithm and $h^{\eta,k}$ is the optimal freight truck flow, $h^{\chi^*,k}$ is the equilibrium private automobile flow and C^k is the minimal total delay. Otherwise, set $k = k + 1$, $C^k = C^{k-1}$, $tk = 0$ and go to Step 2.

Step 2 Generate a random number in [0, 2], multiply it by $h^{\eta,k}$ and denote the product by $\bar{h}^{\eta,k}$. Adjust $\bar{h}^{\eta,k}$ such that constraint (4.2) is satisfied. Solve (3.1) and obtain $\bar{h}^{\chi^*,k}$. Calculate the total delay to all travelers and denote it by $\bar{C}^k$.

Step 3 If $\bar{C}^k < C^k$ or $\exp\left(\frac{C^k - \bar{C}^k}{temp}\right)$ is greater than a random number within [0, 1], let $h^{\eta,k} = \bar{h}^{\eta,k}$, $h^{\chi^*,k} = \bar{h}^{\chi^*,k}$ and $C^k = \bar{C}^k$. Set $tk = tk + 1$ and go to Step 4.

Step 4 If $tk > tk_{\max}$ where $tk_{\max}$ is also predetermined and represents the maximum number of iterations at each "temperature" stage, set $temp = temp * \rho$ where ρ is a constant within [0, 1], and go to Step 1. Otherwise, go to Step 2.

Note that in Algorithm 4.1, we need to solve a DUE that is represented by (3.1) given the freight truck flow $h^{\eta,k}$ when initializing the problem and also in each iteration k. Inspired by the following theorem, in this chapter, we apply the fixed point algorithm proposed by Friesz et al. (2011) to solve the DUE.

Theorem 4.1 Assume that the effective delay operator $\Psi_p\left(t,h^{\chi},h^{\eta}\right)$ is measurable for all $p \in \mathrm{P}$. The fixed point problem

$$h^{\chi} = \arg\min_{h^{\chi}} P_{\Lambda}\left[h^{\chi} - \sigma\Psi\left(t,h^{\chi},h^{\eta}\right)\right].$$

is equivalent to DVI (3.1) where $P_{\Lambda}[\cdot]$ is the minimum norm projection onto Λ, $\Psi\left(t,h^{\chi},h^{\eta}\right)$ is the vector of effective delay operators on all paths and σ is a positive constant.

Proof: See Friesz et al. (2011). □

Algorithm 4.2 Fixed Point Algorithm

Step 0 Choose an initial solution $h^{\chi,0}$. Select a rule for updating a coefficient γ^k which represents the step size in the *k*th iteration. Set $k = 0$.

Step 1 Calculate

$$h^{\chi,k+1} = \gamma^k h^{\chi,0} + \left(1-\gamma^k\right)P_{\Lambda}\left[h^{\chi,k} - \sigma\Psi\left(t,h^{\chi,k},h^{\eta}\right)\right].$$

Step 2 If $\left\|h^{\chi,k+1} - h^{\chi,k}\right\| \le \varepsilon$ where ε is a small positive constant, we stop the algorithm and $h^{\chi*} = h^{\chi,k+1}$. Otherwise, we set $k = k + 1$ and go to step 1.

In Step 1 of Algorithm 4.2, we need to compute $P_{\Lambda}\left[h^{\chi,k} - \sigma\Psi\left(t,h^{\chi,k},h^{\eta}\right)\right]$ which is equivalent to the following optimal control problem:

$$\min_{h^{\chi}} \sum_{(i,j)\in W} \sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} \frac{1}{2}\left[h_p^{\chi,k} - \sigma\Psi_p\left(t,h^{\chi,k},h^{\eta}\right) - h_p^{\chi}\right]dt \tag{4.3}$$

$$\text{s.t.} \quad \frac{dy_{ij}^{\chi}}{dt} = \sum_{p\in P_{ij}} h_p^{\chi}(t) \quad \forall (i,j)\in W \tag{4.4}$$

$h^{\chi} \geq 0$ (4.5)

Since D-MPEC-I$\left(t, h^{\chi^*}, h^{\eta}\right)$ is a continuous-time optimization model that contains differential equations, we apply finite difference method before implementing the algorithms. So while implementing the above algorithms, in each iteration, we actually solve the discretized subproblems.

4.3 Numerical analyses

The algorithm is tested on Nguyen-Dupuis network which contains 13 nodes and 19 links as shown by Figure 4.1. We set $t_0 = 100, t_f = 175$ and define a desired arrival time $T_A = 140$ for the freight trucks as well as private automobiles. This planning horizon is discretized into 300 time intervals and each of which is 0.25 time units long.

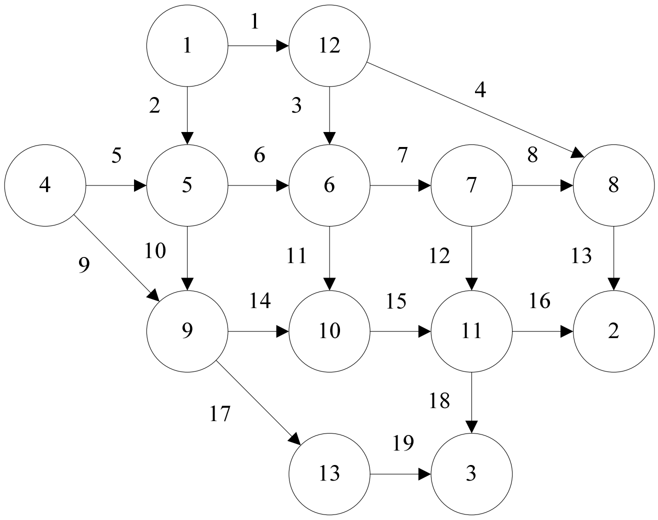

Figure 4.1 Nguyen-Dupuis network

Specifically, we consider $W=\{(1,2),(1,3),(4,2),(4,3)\}$ as the set of O-D pairs. We set a fixed travel demand for each O-D pair:

$$Q_{ij}^{\chi}=5000, Q_{ij}^{\eta}=1000 \quad \forall (i,j)\in W$$

There are 25 paths that connect these O-D pairs and we take all of them into account in the model.

We assume that in $D_{a_i}(x)=A_{a_i}+B_{a_i}x$, $A_{a_i}=1.5$ for i = 1, 7, 13, 15 and 19, $A_{a_i}=2.5$ for i = 4, and $A_{a_i}=2$ for remaining arcs. We set $B_{a_i}=6.67\times 10^{-4}$ for all arcs. We also assume that a freight truck and a private automobile have the same impacts on congestion and set $\beta=1$ in $x_{a_i}(t)=x_{a_i}^{\chi}(t)+\beta x_{a_i}^{\eta}(t)$. We set $\alpha=0.5$ in early/late arrival penalty $F[x]=\alpha x^2$.

Algorithm 4.1 embedded with the fixed point algorithm is coded in MATLAB 7 and GAMS. Specifically, for SA, we set $k_{\max}$ = 10, $tk_{\max}$ = 5, $temp_{\min}$ = 1000 and $\rho=0.9$.

We consider the following 3 test scenarios:

1. The MPO does not control the urban freight traffic. In this case, we assume that the total traffic on the urban road network (freight trucks and private automobiles) is described by a DUE.
2. The MPO uses an arbitrary rule to control urban freight traffic: the freight truck flow on each path is constant over time.
3. The MPO uses the solution to our proposed model D-MPEC-I$\left(t,h^{\chi *},h^{\eta}\right)$ to control freight truck traffic.

We calculate and compare the total delay experienced by all travelers in these scenarios.

We run the tests on the Penn State Lion-XJ system, which is a computational cluster run by the Penn State High Performance Computing Group, with the following attributes: Intel Xeon E5472 Quad-Core 3.0 GHz and 32GB RAM.

The computation time for solving D-MPEC-I$\left(t, h^{\chi^*}, h^{\eta}\right)$ is 17355 seconds. In all 51 DUEs are solved, thus the average time of solving one DUE is about 340 seconds.

The results are summarized in Table 4.1.

Table 4.1 Comparison of results in the 3 scenarios

	Scenario 1		Scenario 2		Scenario 3	
	Delay	Ratio[6]	Delay	Ratio	Delay	Ratio
Freight trucks	1.437×10^5	99.95%	1.113×10^6	774.49%	1.438×10^5	100%
Private automobiles	7.185×10^5	123.66%	5.740×10^5	98.79%	5.810×10^5	100%
Total	8.622×10^5	118.96%	1.687×10^6	232.83%	7.247×10^5	100%

From Table 4.1, we can see that the solution to our proposed model yields the lowest total delay. Specifically, the total delays in Scenario 1 (no control of freight truck traffic) and Scenario 2 (arbitrary control of freight truck traffic) are 18.96 % and 132.83% higher respectively than that in Scenario 3 (optimal control of freight truck traffic). Although the arbitrary control of freight truck traffic reduces the delay experienced by all DPAs, it immensely increases the freight truck delay. As a result, the arbitrary control of freight truck traffic significantly increases the total delay. Our optimal control of freight truck traffic can significantly reduce the delay experienced by all DPAs while increasing freight truck delay by only 0.05%, thus it effectively reduces the total delay.

Since there are in total 25 paths that connect the four O-D pairs, without loss of generality we only illustrate the time-varying traffic flows on Path 1 (Arcs 1, 4 and 13 that connect O-D pair (1, 2)) and Path 6 (Arcs 1, 3, 7, 8 and 13 that connect O-D pair (1, 2)) in each scenario in Figure 4.2.

[6] The percentage in the column "Ratio" is the ratio of delay in the corresponding scenario to the delay in Scenario 3.

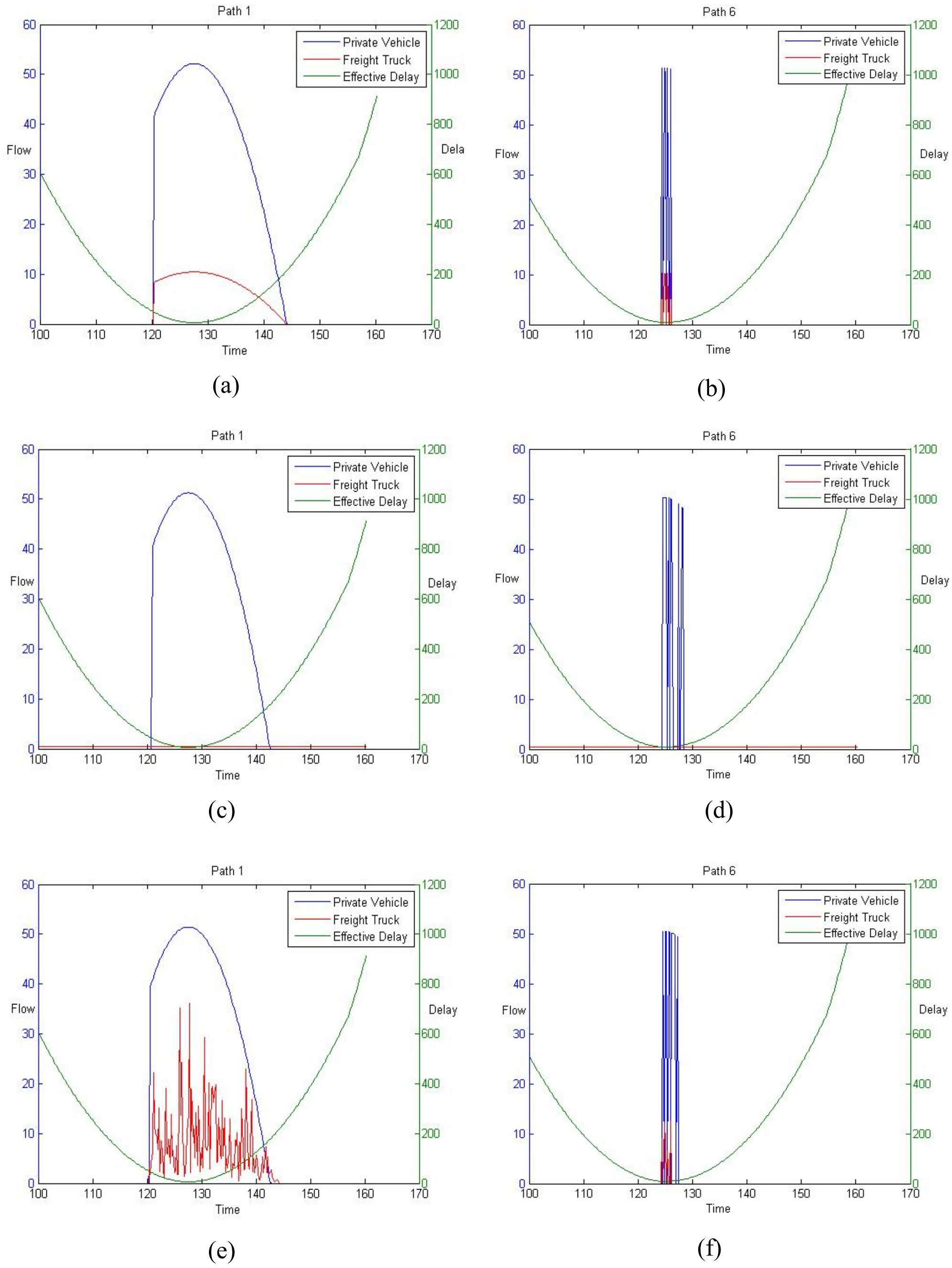

Figure 4.2 Dynamic traffic flows on two selected paths in the 3 scenarios

In Figure 4.2, (a), (c) and (e) show the dynamic traffic flows and the effective delay on Path 1 in Scenarios 1, 2 and 3, respectively. (b), (d) and (f) show the dynamic traffic flows and the effective delay on Path 6 in Scenarios 1, 2 and 3, respectively. We can see that on Path 1, the shape of the private automobile flow does not change significantly in different scenarios. The private automobile flow on Path 1 starts at around time 120, reaches the peak at around time 130 and vanishes after the desired arrival time 140. On Path 6, the private automobile flow appears only in a short time period and has multiple peaks in all 3 scenarios. Note that compared to the freight truck flow in other scenarios, the optimal freight truck flow identified in Scenario 3 does not have a specific pattern. So the MPO may need to use interpolation or other smoothing techniques to derive an implementable freight truck schedule that effectively reduces the total delay.

Chapter 5

A Dynamic Stackelberg Game-Theoretic Model for Urban Freight Transportation Planning: Model II

In this chapter, we relax the assumption made in the previous chapter that the metropolitan planning organization (MPO) is capable of controlling the movement of all freight trucks on the urban road network. Instead, we consider that there is a union of truck companies (UTC) who systematically control the movement of all freight trucks on an urban road network to minimize total urban freight transportation cost/delay. Similarly, we model the UTC's optimization problem as a dynamic Stackelberg game where the leader is the UTC and the followers are the drivers of private automobiles (DPAs). Although the solution to this model optimizes the utility of the UTC, a metropolitan planning organization (MPO) can still use this model as a simulator to investigate the impact of urban freight transportation policies on the urban road network traffic. We formulate the problem in Section 5.1 and develop a new solution approach in Section 5.2. At last, in Section 5.3, we conduct numerical analyses to demonstrate the efficiency of the proposed solution approach and derive managerial insights into urban freight transportation regulation for the MPO.

5.1 Problem formulation

In this chapter, again we consider two types of traffic: freight transportation by trucks and personal transportation by private vehicles. Trucks are controlled by the UTC who aims at minimizing the total freight transportation delay while satisfying the travel demand (i.e., the total number of trucks required to travel between each O-D pair to transport freight). The DPAs want to minimize the personal travel delay in the same time horizon. Thus, the two types of traffic compete for the limited road capacity. Since the DPAs do not cooperate, the impact of each DPA's travel behavior on the network traffic is not comparable to that of the UTC.

So we model the problem as a new Stackelberg game where the UTC is the leader and DPAs are the followers. The UTC aims at minimizing the total freight transportation delay while satisfying the travel demand over the time frame of interest, which can be represented by (5.1) and (4.2), respectively:

$$\min_{h^{\eta}} \sum_{(i,j)\in W} \sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} \Psi_p\left(t, h^{\chi^*}, h^{\eta}\right) h_p^{\eta} dt \tag{5.1}$$

The new Stackelberg game-theoretic model can be represented by {(3.1)-(3.3), (4.2), (5.1)} which is again a dynamic mathematical program with equilibrium constraints (MPEC). We denote this dynamic MPEC by D-MPEC-II$\left(t, h^{\chi^*}, h^{\eta}\right)$. The notations in D-MPEC-II$\left(t, h^{\chi^*}, h^{\eta}\right)$ are defined the same as those in D-MPEC-I$\left(t, h^{\chi^*}, h^{\eta}\right)$. Similarly, a rigorous proof of the existence of solution to D-MPEC-II$\left(t, h^{\chi^*}, h^{\eta}\right)$ is challenging and remains to be addressed by future studies.

5.2 Solution approach

Although a rigorous proof is not available, we still have a general property of existence of solutions to the dynamic MPEC. Different from section 4.2, in this section we develop a gradient-based heuristic to solve D-MPEC-II$\left(t, h^{\chi^*}, h^{\eta}\right)$.

We first reformulate the problem as a dynamic mathematical program with complementarity constraints (MPCC).

Theorem 5.1 The DVI represented by (3.1) is equivalent to the following nonlinear complementarity problem:

$$\left.\begin{aligned}&\left(\Psi_p\left(t,h^\chi,h^\eta\right)-\mu_{ij}\right)\perp h_p^\chi \quad (i,j)\in W,\, p\in P_{ij},\, t\in\left[t_0,t_f\right]\\&\Psi_p\left(t,h^\chi,h^\eta\right)-\mu_{ij}\geq 0 \quad (i,j)\in W,\, p\in P_{ij},\, t\in\left[t_0,t_f\right]\\&h^\chi\geq 0\end{aligned}\right\} \tag{5.2}$$

where $h^\chi \in \Lambda$, and $\mu := \left(\mu_{ij} : ij \in W\right)$ is a dual variable that is constant over time.

Proof: The equilibrium characterized by equation (3.1) is the solution to the following optimal control problem:

$$\begin{aligned}\min J_0 &= \sum_{(i,j)\in W}\mu_{ij}\left[Q_{ij}^\chi - y_{ij}^\chi\left(t_f\right)\right]+\sum_{(i,j)\in W}\sum_{p\in P_{ij}}\int_{t_0}^{t_f}\Psi_p\left(t,h^{\chi*},h^\eta\right)h_p^\chi\,dt\\ \text{s.t.}\quad &\frac{dy_{ij}^\chi}{dt}=\sum_{p\in P_{ij}}h_p^\chi(t)\quad \forall(i,j)\in W\\&y_{ij}^\chi\left(t_0\right)=0\quad \forall(i,j)\in W\\&h^\chi\geq 0\end{aligned}$$

where μ_{ij} is a dual variable for terminal conditions in Λ. The Hamiltonian for this optimal control problem is

$$H=\sum_{(i,j)\in W}\sum_{p\in P_{ij}}\Psi_p\left(t,h^{\chi*},h^\eta\right)h_p^\chi+\sum_{(i,j)\in W}v_{ij}\sum_{p\in P_{ij}}h_p^\chi$$

The adjoint equations are

$$\frac{dv_{ij}}{dt}=-\frac{\partial H}{\partial y_{ij}^\chi}=0,\quad \forall(i,j)\in W,\, t\in\left[t_0,t_f\right] \tag{5.3}$$

The transversality conditions are

$$v_{ij}\left(t_f\right)=\frac{\partial\sum_{(i,j)\in W}\mu_{ij}\left[Q_{ij}^\chi-y_{ij}^\chi\left(t_f\right)\right]}{\partial y_{ij}^\chi\left(t_f\right)}=-\mu_{ij},\quad \forall(i,j)\in W,\, p\in P_{ij} \tag{5.4}$$

Equations (5.3) and (5.4) imply that

$$\nu_{ij}(t) = -\mu_{ij}, \quad \forall (i,j) \in W, t \in [t_0, t_f]$$

The Pontryagin minimum principle is a necessary condition for the optimal control problem.

Hence,

$$h^{\chi*} = \arg\min_{h^{\chi} \in \Lambda} H$$

for which the necessary condition (Kuhn-Tucker condition) is

$$\nabla_{h_p^{\chi}} H \left(h_p^{\chi} - h_p^{\chi*} \right) \geq 0 \quad \forall p \in \mathrm{P}, h_p^{\chi} \in \Lambda \tag{5.5}$$

Since

$$\nabla_{h_p^{\chi}} H = \Psi_p \left(t, h^{\chi*}, h^{\eta} \right) + \nu_{ij} = \Psi_p \left(t, h^{\chi*}, h^{\eta} \right) - \mu_{ij}, \quad \forall (i,j) \in W, p \in P_{ij}$$

(5.5) is equivalent to

$$\left(\Psi_p \left(t, h^{\chi*}, h^{\eta} \right) - \mu_{ij} \right) \left(h_p^{\chi} - h_p^{\chi*} \right) \geq 0, \quad \forall (i,j) \in W, p \in P_{ij}, h_p^{\chi} \in \Lambda \tag{5.6}$$

From (5.6), we can derive the following conditions of a DUE

$$\begin{aligned} &h_p^{\chi*} > 0, (i,j) \in W, p \in P_{ij} \Rightarrow \Psi_p \left(t, h^{\chi*}, h^{\eta} \right) - \mu_{ij} = 0, (i,j) \in W, p \in P_{ij}, t \in [t_0, t_f] \\ &\Psi_p \left(t, h^{\chi*}, h^{\eta} \right) > \mu_{ij}, (i,j) \in W, p \in P_{ij}, t \in [t_0, t_f] \Rightarrow h_p^{\chi*} = 0, (i,j) \in W, p \in P_{ij} \end{aligned}$$

which can be represented by (5.2) . □

With complementarity constraints (5.2) substituting for (3.1), the MPCC reformulation of the proposed urban freight model can be represented by $\{(3.2), (3.3), (4.2), (5.1), (5.2)\}$.

Since the complementarity constraints do not satisfy certain constraint qualifications that are necessary to guarantee convergence of the solution (Rodrigues and Monteiro, 2006), to solve the MPCC, we penalize these constraints and obtain the augmented objective function as:

$$
\begin{aligned}
&\Theta\left(h^{\chi}, h^{\eta}, \mu, M\right) \\
&:= \sum_{(i,j)\in W} \sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} \Psi_p\left(t, h^{\chi}, h^{\eta}\right) h_p^{\eta} dt \\
&\quad + M \sum_{(i,j)\in W} \sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} \left[\left(\Psi_p\left(t, h_p^{\chi}, h_p^{\eta}\right) - \mu_{ij}\right) h_p^{\chi}\right]^2 dt \\
&\quad + M \sum_{(i,j)\in W} \sum_{p\in \mathrm{P}_{ij}} \int_{t_0}^{t_f} \left[\max\left\{\mu_{ij} - \Psi_p\left(t, h_p^{\chi}, h_p^{\eta}\right), 0\right\}\right]^2 dt
\end{aligned}
\tag{5.7}
$$

where M is a large number.

By substituting (5.7) for (5.2), we have finalized the urban freight transportation planning problem as a single-level nonlinear program $\{(3.2), (3.3), (4.2), (5.1), (5.7)\}$. We design the following projected gradient algorithm to solve this nonlinear program.

Algorithm 5.1 Projected Gradient algorithm

Step 0 Identify an initial feasible solution $z^k := \left(h^{\chi,k}, h^{\eta,k}, \mu_{ij}^k\right)^T$ and set $k = 0$;

Step 1 Solve the optimal control subproblem:

$$
\begin{aligned}
\min_{v} \int_{t_0}^{t_f} \frac{1}{2}\Big(&\left\|h^{\chi,k}(t) - \alpha F_1 - h^{\chi}(t)\right\|_2^2 + \left\|h^{\eta,k}(t) - \alpha F_2 - h^{\eta}(t)\right\|_2^2 + \left\|\mu^k - \alpha F_3 - \mu\right\|_2^2\Big) dt \\
s.t.\quad & \frac{dy_{ij}^{\chi}(t)}{dt} = \sum_{p\in P_{ij}} h_p^{\chi}(t) \\
& \frac{dy_{ij}^{\eta}(t)}{dt} = \sum_{p\in P_{ij}} h_p^{\eta}(t) \\
& y_{ij}^{\chi}(t_0) = 0, y_{ij}^{\chi}\left(t_f\right) = Q_{ij}^{\chi} \\
& y_{ij}^{\eta}(t_0) = 0, y_{ij}^{\eta}\left(t_f\right) = Q_{ij}^{\eta}
\end{aligned}
$$

Denote the solution by $z^{k+1} := \left(h^{\chi,k+1}, h^{\eta,k+1}, \mu^{k+1}\right)^T$.

Step 2 Stop if $\left|z^{k+1} - z^k\right| \le \varepsilon_1$, where $\varepsilon_1 \in \Re_{++}^1$ is a predetermined scalar. Otherwise, set $M = CM$ where C is a constant that are great than 1, and go to Step 1.

In the optimal control subproblem, we define

$$F_1 := \left.\frac{\partial \Theta}{\partial h^{\chi}}\right|_{\left(t, h^{\chi,k}(t), h^{\eta,k}(t), \mu^k\right)},$$

$$F_2 := \left.\frac{\partial \Theta}{\partial h^{\eta}}\right|_{\left(t, h^{\chi,k}(t), h^{\eta,k}(t), \mu^k\right)},$$

$$F_3 := \left.\frac{\partial \Theta}{\partial \mu}\right|_{\left(t, h^{\chi,k}(t), h^{\eta,k}(t), \mu^k\right)}.$$

Since the problem is nonconvex in general, by the proposed solution approach, a global solution can not be guaranteed.

5.3 Numerical analyses

The algorithm is tested on Nguyen-Dupuis network (Figure 4.1). We set $t_0 = 100, t_f = 175$ and define a desired arrival time $T_A = 160$ for the freight trucks as well as private automobiles. This planning horizon is discretized into 100 time intervals and each of which is 0.75 time units long.

We still focus on the following O-D pairs: $W = \{(1,2), (1,3), (4,2), (4,3)\}$. However, in this section, four test scenarios are considered and the travel demands in these scenarios are different. They are:

$$\begin{cases} \text{Scenario 1: } Q_{ij}^{pr} = 20000,\ Q_{ij}^{tr} = 6667 \\ \text{Scenario 2: } Q_{ij}^{pr} = 20000,\ Q_{ij}^{tr} = 3333 \\ \text{Scenario 3: } Q_{ij}^{pr} = 20000,\ Q_{ij}^{tr} = 667 \\ \text{Scenario 4: } Q_{ij}^{pr} = 20000,\ Q_{ij}^{tr} = 0 \end{cases}, \quad \forall (i,j) \in W .$$

We consider all the 25 paths that connect these O-D pairs and use the same settings for all parameters as those in Section 4.3.

Algorithm 5.1 is coded in MATLAB 7 and GAMS. We run the tests on the Penn State Lion-XC system, which is a computational cluster run by the Penn State High Performance Computing Group, with the following attributes: Intel Xeon 3160 Dual-Core 3.0 GHz and 8GB RAM.

The "optimal" freight truck flows and equilibrium private automobile flows are computed by the proposed algorithm. Without loss of generality we analyze the time-varying traffic flows on one of the 25 paths, Path 10 (Arcs 9, 14, 15 and 18 that connect O-D pair (4, 3)), in each scenario as illustrated in Figure 5.1.

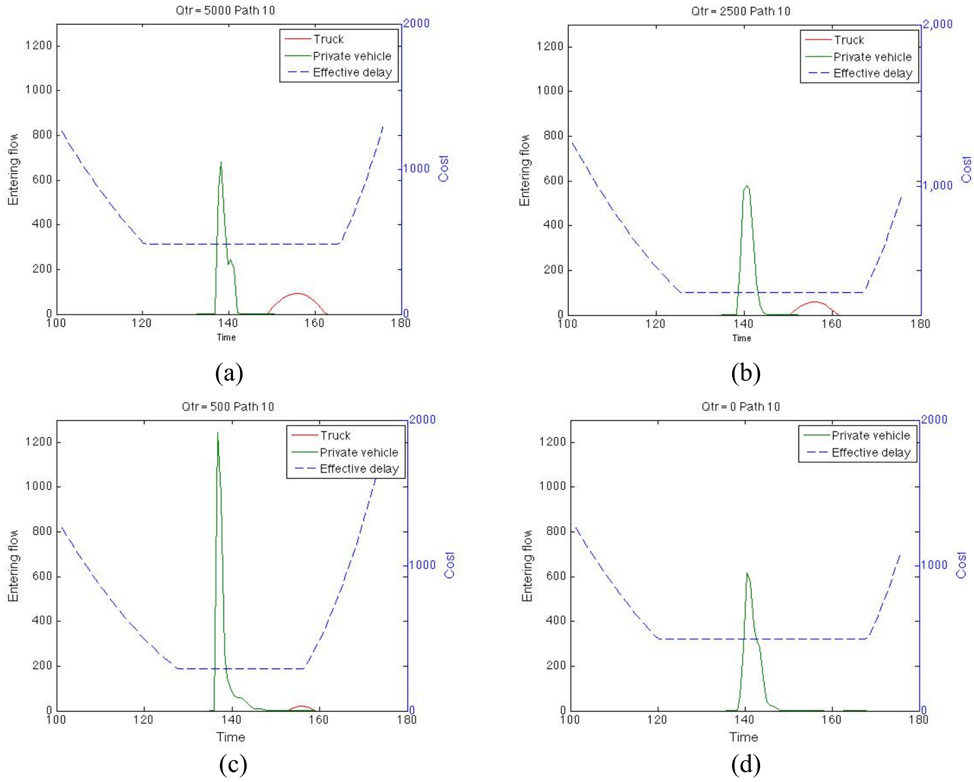

Figure 5.1 Dynamic traffic flows on Path 10 in 4 scenarios

In Figure 5.1, (a), (b), (c) and (d) show the dynamic traffic flows and the effective delay operators on Path 1 in Scenario 1, 2, 3 and 4, respectively. We can see that in all of these 4 scenarios, traffic flows exist only when the effective delay is at its minimum and the peak of the private automobile flows occurs at around time 140 which can be considered as the appropriate departure time for private automobiles in order to arrive at the destination at the desired time T_a. From Figure 5.1, we can clearly identify the interaction of freight and personal transportation. As an example, in Scenario 3 (Figure 5.1(c)) when there are in total 667 freight trucks traveling on the network, the private automobile flow significantly differs from that in Scenario 4 (Figure 5.1(d)) when there is no freight truck at all. What's more, again in Scenario 3, the peak of private automobile flow occurs earlier than that in Scenario 4.

By Figure 5.1, we have demonstrated that freight transportation can affect personal transportation even when the number of freight trucks is very small compared to that of the private automobiles. We would like to further investigate the impact of considering such an interaction when planning freight transportation on the total transportation delay. Numerical tests are conducted in the following two cases. In Case 1, the UTC ignores the interaction and simply assumes that the DUE of private automobile traffic will not change by freight truck flows. The UTC estimates the effective delay operator under the regular DUE and uses it as the cost coefficient to optimize freight truck flows. Let's mark the optimal solution in this case as $\left(\widehat{h}_p^\eta\right)$. However, as we know, the freight truck flow will influence the private automobile flow so that given $\left(\widehat{h}_p^\eta\right)$, the real DUE for private automobiles is different from the estimation and thus the real effective delay operator is not the same as estimated. Thus in Case 1, the real total freight transportation delay should be calculated using the real effective delay operator. Let's mark the real total delay as z^0. In Case 2, the UTC considers the interaction while optimizing freight truck flows. The optimal solution will be exactly the solution to the MPCC {(3.2), (3.3), (4.2), (5.1), (5.2)}. The total freight transportation delay is the optimal objective value and we mark it as z^1. We then compare z^0 and z^1 in Scenario 1, 2 and 3 in Table 5.1. We can see that in all 3 scenarios, considering the interaction between freight and personal transportation while planning urban freight transportation can help reduce the total transportation delay significantly, especially when freight truck traffic accounts for a considerable portion of total traffic.

Table 5.1 Comparison of freight transportation delay in 3 scenarios

	Scenario 1	Scenario 2	Scenario 3
z^0	1.42×10^7	4.52×10^6	7.99×10^5
z^1	2.58×10^7	7.30×10^6	9.09×10^5
Reduction	44.90%	38.16%	12.12%

The computation time is summarized in Table 5.2. The computation time in row "Case 1" is the time required to solve a DUE and that in row "Case 2" corresponds to the time required to solve the MPCC. We can see that the proposed heuristic works more efficient when the travel demand for freight trucks is larger.

Table 5.2 Computation time (in second)

	Scenario 1	Scenario 2	Scenario 3
Case 1	2180.46	1760.42	1851.65
Case 2	27408.29	29994.63	63580.18

So far in this chapter, we have demonstrated how freight transportation and personal transportation interact in an urban road network. Now, we want to address the concern of the MPO whose objective is to minimize delay experienced by all travelers, while ensuring that the freight transportation demand is satisfied. A practical approach for the MPO to influence the freight truck traffic is to make a policy to restrict freight trucks from entering a certain part of the network. Note that the MPO knows exactly that the UTC and the DPAs play a Stackelberg game, the MPO's problem is actually a tri-level optimization problem. To simplify the formulation and solution of the problem, we explore the existence of such a policy by a trial-and-error approach. We arbitrarily choose one arc at a time from the network and block it for freight trucks. Then we conduct numerical test to check whether blocking that arc for freight trucks can reduce the total delay: if yes, then we find the solution and we can stop; otherwise, we choose another arc and repeat the numerical test and the judgment of result. After several iterations, we find that blocking all the freight trucks from entering Arc 12 during the time interval of our interest can achieve the MPO's goal (see Figure 5.2). More details of the impact of this policy on the urban road network traffic are summarized in Table 5.3.

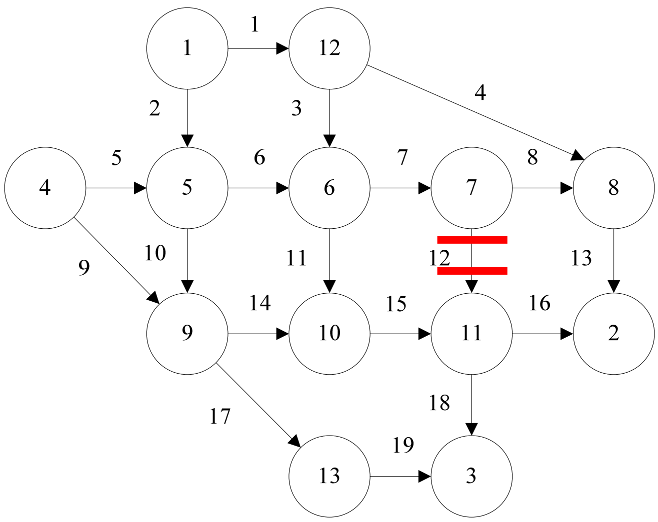

Figure 5.2 Nguyen-Dupuis network with Arc 12 blocked for all trucks

Table 5.3 Dynamic Braess-like Paradox

	All DPAs' delay	UTC's delay	Total delay
All arcs are accessible	2.71×10^{7}	4.52×10^{6}	3.16×10^{7}
Arc 12 blocked for trucks	2.61×10^{7}	5.02×10^{6}	3.11×10^{7}
Improvement	3.7%	-11.1%	1.6%

In Table 5.3, we show all DPAs' delay, the UTC's delay and the total delay (sum of all DPAs' delay and the UTC's delay). From Table 5.3, we see that the MPO can block freight trucks from entering Arc 12 to effectively reduce all DPA's delay. Although it may significantly increase the UTC's delay, the total delay is still reduced. As a result, we find a dynamic Braess-like Paradox: reducing capacity of a network for partial road users who selfishly select their routes can increase overall performance (see Akamatsu and Heydecker (2003) and Lin and Lo (2009) for more details about the dynamic extensions of Braess Paradox). This paradox indicates that before the MPO implement such a policy, they should consider the possible obstruction from the truck companies and try to balance the increase in social welfare and the loss in the truck companies' profit. As an example resolution, the MPO may compensate the truck companies for restricting their travel rights.

Chapter 6

Winner Determination Problem in Truckload Service Procurement

In this chapter, we investigate the modeling of a deterministic winner determination problem (WDP) in truckload service procurement. Specifically, in Section 6.1, we introduce a benchmark formulation. A refined model is presented in Section 6.2. In Section 6.3, we discuss a variety of shipper's business considerations and how they can be incorporated into WDP formulations.

All the notations used in the rest of the dissertation for WDP in truckload service procurement are summarized in Table 6.1.

Table 6.1 Notations

Symbol	Description
I	set of lanes;
J	set of carriers/bidders;
K_j	set of package bids submitted by carrier *j*;
d_i	volume of shipments on lane *i*;
a_{ijk}	indicator which equals 1 if carrier *j*'s package *k* includes lane *i* and 0 otherwise;
$N_{\min}$	minimum number of carriers who can win the auction;
$N_{\max}$	maximum number of carriers who can win the auction, $N_{\max} \geq N_{\min}$;
LS_{jk}	minimum volume of shipments on all lanes in package *k* required by carrier *j* if carrier *j* wins package *k*;
US_{jk}	maximum volume of shipments on all lanes in package *k* required by carrier *j* if carrier *j* wins package *k*, $US_{jk} \geq LS_{jk}$;
LQ_j	minimum volume of shipments specified by the shipper for carrier *j*;
UQ_j	maximum volume of shipments specified by the shipper for carrier *j*;
e_i	cost of shipping 1 unit of freight by other third-party carriers who are not invited to the auction (unit extra service cost);
p_i	cost of lacking 1 unit of shipment on lane *i* to meet carriers' requirement (unit penalty cost);
q_{jk}	the penalty cost of being in short of 1 unit of shipment on every lane in winner *j*'s

	package k
c_{jk}	cost of shipping 1 unit of freight on all lanes in carrier j's package k (unit shipping cost);
h_{jk}	cost of shipping all shipments on all lanes in carrier j's package k;
x_{jk}	binary decision variable which equals 1 if carrier j wins their package k and 0 otherwise
y_{jk}	volume of shipments assigned to carrier j on all lanes in their package k;
z_{jk} :	the shortage in required shipment volume on all lanes in carrier j's package k;
φ_i	volume of shipments on lane i that is assigned to other third-party carriers who are not invited to the auction;
θ_i	volume of shipments lacked on lane i to meet carriers' requirement

6.1 A benchmark formulation of WDP

The simplest formulation of a deterministic WDP for TL service procurement is like the formulation of a set-covering problem (SCP):

$$\text{(SCP)}\ \min \sum_{j\in J}\sum_{k\in K_j} h_{jk}x_{jk} \tag{6.1}$$

$$\text{s.t.}\quad \sum_{j\in J}\sum_{k\in K} a_{ijk}x_{jk} \ge 1, \quad i\in I \tag{6.2}$$

$$x_{jk} = \{0,1\}, \quad j\in J, k\in K_j \tag{6.3}$$

The objective function (6.1) is the minimization of shipper's procurement cost. Constraint (6.2) ensures that each lane is served by at least one carrier. Equation (6.3) is an integrality constraint.

There are two underlying assumptions made for formulating a WDP as a SCP: (A1) every carrier has enough capacity to transport all shipments on every lane in each awarded package; (A2) the shipper only buys TL service from the carriers in set J. Both assumptions may not hold in practice. They can be relaxed in a new bidding scheme proposed by Ma et al. (2010): when a carrier bids on a package of lanes, in addition to the price, they also inform the shipper a range of the volume of shipments on all lanes in that package that they are willing to accept; if they win the package, the shipper will assign shipments that satisfy their specified requirements; shipper is allowed to procure TL service from other third-party carriers who are not invited to the auction. As a result of this new bidding scheme, the shipper needs to determine not only the winners of the auction but also the volume of shipments that should be assigned to each winner and other third-party carriers.

The benchmark formulation of WDP under the new bidding scheme is as follows:

$$\text{(b-WDP)} \quad \min \sum_{j \in J} \sum_{k \in K_j} c_{jk} y_{jk} + \sum_{i \in I} e_i \varphi_i \tag{6.4}$$

$$\text{s.t.} \quad \sum_{j \in J} \sum_{k \in K_j} a_{ijk} y_{jk} + \varphi_i \geq d_i, \quad i \in I \tag{6.5}$$

$$LS_{jk} x_{jk} \leq y_{jk}, \quad j \in J, k \in K_j \tag{6.6}$$

$$US_{jk} x_{jk} \geq y_{jk}, \quad j \in J, k \in K_j \tag{6.7}$$

$$y_{jk} \geq 0, \quad j \in J, k \in K_j \tag{6.8}$$

$$\varphi_i \geq 0, \quad i \in I \tag{6.9}$$

constraint (6.3)

The objective function (6.4) is the minimization of the shipper's total procurement cost. That is, the cost of purchasing TL service from carriers who are invited to the auction and those who are not. Equation (6.5) is the shipment assignment constraint which guarantees that the shipments on each lane are either assigned to the winning carriers or other third-party carriers. Constraints (6.6) and (6.7) restrict that the volume of shipments assigned to a winning carrier is within the range specified by the carrier. Constraints (6.8) and (6.9) are nonnegativity constraints.

If we denote a solution to b-WDP by $\left[(x_{jk}), (y_{jk}), (\varphi_i)\right]$, it is trivial that $[(0), (0), (d_i)]$ is always a feasible solution. Therefore, b-WDP is always a feasible mathematical program.

6.2 Refined formulations of WDP

The benchmark formulation b-WDP takes into account the possibility that the shipping demand exceeds the capacity of all the carriers who participate in the auction and employs other third-party carriers to resolve this issue. However, constraint (6.5) in b-WDP may not be always tight. For example, if e_i is immensely large, φ_i will vanish. If in addition $d_i < LS_{jk}$ for all j, k, the left-hand-side of constraint (6.5) will be strictly greater than the right-hand-side, which indicates that the shipper assign to carriers more shipments than they actually have. We call this phenomenon that makes no physical sense "imaginary shipment assignment". Note that although changing (6.5) to an equality constraint may resolve this issue when the shipment volume is deterministic, it may make the problem infeasible when the shipment volume becomes uncertain. What is more, as pointed out by Caplice and Sheffi (2003), there are always negotiations between the shipper and the carriers. A good winner determination model should be more general and feasible.

We propose two refined winner determination models, r1-WDP and r2-WDP, which allow but penalize shortages in carriers' required shipment.

Specifically, we can replace constraint (6.5) by the following constraint:

$$\sum_{j\in J}\sum_{k\in K_j} a_{ijk} y_{jk} + \varphi_i = d_i + \theta_i, \quad i \in I \tag{6.10}$$

where θ_i is the volume of shipments lacked on lane i to meet carriers' requirement and

$$\theta_i \geq 0, \quad i \in I \tag{6.11}$$

By replacing equation (6.5) with (6.10), imposing nonnegativity constraint (6.11), and adding penalty cost to the objective function (6.4), we can transform b-WDP to r1-WDP:

$$\text{(r1-WDP)} \ \min \sum_{j\in J}\sum_{k\in K_j} c_{jk} y_{jk} + \sum_{i\in I}\left(e_i\varphi_i + p_i\theta_i\right) \tag{6.12}$$

$$\text{s.t.} \quad (6.3), (6.6)-(6.11).$$

Note that in r1-WDP, y_{jk} in fact denotes the volume of shipments that are supposed to be assigned, which is not necessarily equal to that are actually assigned to carrier *j* on all lanes in their package *k*.

In r1-WDP, the shortage on each lane is modeled. However, shortage occurs because the carriers' requirements on the volume of shipments that are assigned to their certain packages are not met. Moreover, in fact the penalty on the shipper is paid to the carriers. So a measure of shortage on each carrier's each package of lanes is more appropriate. We can replace constraint (6.6) by the following constraint:

$$LS_{jk}x_{jk} \le y_{jk} + z_{jk}, \quad j \in J, k \in K_j \tag{6.13}$$

where z_{jk} is a decision variable which represents the shortage in required shipment volume on all lanes in carrier *j*'s package *k* and

$$z_{jk} \ge 0, \quad j \in J, k \in K_j \tag{6.14}$$

By changing constraint (6.5) to equality, replacing constraint (6.6) by (6.13), imposing nonnegativity constraint (6.14) and adding the sum of penalty paid to each carrier to the objective function (6.4), we can formulate r2-WDP:

$$\text{(r2-WDP)} \ \min \sum_{j \in J} \sum_{k \in K_j} \left(c_{jk} y_{jk} + q_{jk} z_{jk} \right) + \sum_{i \in I} e_i \varphi_i \tag{6.15}$$

$$\text{s.t.} \quad \sum_{j \in J} \sum_{k \in K_j} a_{ijk} y_{jk} + \varphi_i = d_i, \quad i \in I \tag{6.16}$$

constraints (6.3), (6.7)-(6.9), (6.13), (6.14)

With the volume of shortages explicitly modeled, both r1-WDP and r2-WDP are immune to the "imaginary shipment assignment" problem identified above.

6.3 Modeling of other business considerations

In practice, while determining the winners of the auction, the shipper always has specific business considerations (see Caplice and Sheffi (2003) for a full list and more detailed discussions of shipper's business considerations) and some of them can be explicitly modeled as follows:

- XOR bids[7]:

$$\sum_{k \in K} x_{jk} \le 1, \quad j \in J \tag{6.17}$$

- Minimum/maximum number of winners:

$$\sum_{j \in J} w_j \le N_{\max}$$

$$\sum_{j \in J} w_j \ge N_{\min}$$

$$x_{kj} \le w_j, \quad j \in J, k \in K_j$$

$$\sum_{k \in K_j} x_{kj} \ge w_j, \quad j \in J$$

$$w_j = \{0,1\}, \quad j \in J$$

where $N_{\min}$ and $N_{\max}$ are respectively the minimum and maximum number of carriers that are allowed to win the package. w_j is a new binary variable that is not shown in Table 6.1. It equals 1 if carrier *j* wins any package and 0 otherwise. Note that if XOR bids are adopted in the auction, we have

$$\sum_{k \in K_j} x_{kj} = w_j, \quad j \in J$$

and w_j is redundant. Consequently, the constraints on the number of winners reduce to:

[7] The bidder can submit multiple bids but obtain at most one of them (Nisan 2006).

$$\sum_{j \in J} \sum_{k \in K_j} x_{jk} \le N_{\max} \tag{6.18}$$

$$\sum_{j \in J} \sum_{k \in K_j} x_{jk} \ge N_{\min} \tag{6.19}$$

- Minimum/maximum coverage:

$$\sum_{k \in K_j} y_{jk} \le w_j UC_j, \quad j \in J$$

$$\sum_{k \in K_j} y_{jk} \ge w_j LC_j, \quad j \in J$$

where LC_j and UC_j denote respecively the minimum and maximum volume of shipments that can be assigned to carrier *j* if carrier *j* wins. If XOR bids are adopted, these constraints overlap constraints (6.6) and (6.7).

- Each lane is allocated to at most one carrier:

$$\sum_{j \in J} \sum_{k \in K_j} a_{ijk} x_{jk} \le 1, \quad i \in I \tag{6.20}$$

- Restrictions on carriers:

$$\sum_{k \in K_j} y_{jk} \le UQ_j, \quad j \in J \tag{6.21}$$

$$\sum_{k \in K_j} y_{jk} \ge LQ_j, \quad j \in J \tag{6.22}$$

where UQ_j and LQ_j are respectively maximum and minimum volume of shipments that can be assigned to carrier *j*. In practice, this restriction is usually proposed by the shipper to make sure that certain carriers win a target level of business (or, conversely, restrict certain carriers from winning too much business).

Another important consideration to the shipper is service level (see Sheffi (2004) for a list of measures). The simplest way to incorporate service level into WDP is to introduce to the carriers' cost coefficient either an additve or a multiplicative factor which reflects carriers' service levels. However, in practice, service level is usually used before the auction as a criterion to judge whether a carrier is qualified to participate in the auction (Caplice and Sheffi 2003). Therefore, in this dissertation, we do not account for service level in our winner determination model.

When introducing the above side constraints into either r1-WDP or r2-WDP, the shipper should carefully evaluate the parameters in each constraint to make sure that the problem has a feasible solution.

Proposition 6.1 Assume we have incorporated constraints (6.17)-(6.22) into both r1-WDP and r2-WDP. Let

$$J^+ \equiv \left\{ j \mid LQ_j > 0, \quad j \in J \right\}.$$

If the maximum number of winners is not specified carefully such that

$$N_{\max} < \left| J^+ \right|,$$

neither r1-WDP or r2-WDP is feasible.

Proof: Given the definition of J^+ and constraints (6.7), (6.17) and (6.22), for both r1-WDP and r2-WDP, we have

$$\sum_{k \in K} x_{jk} = 1, \quad j \in J^+.$$

Hence,

$$\sum_{j \in J^+} \sum_{k \in K} x_{jk} = \left| J^+ \right|.$$

If the maximum number of winners is not specified carefully such that

$$N_{\max} < \left| J^+ \right|,$$

constraint (6.18) will be violated and both r1-WDP or r2-WDP become infeasible. □

Proposition 6.1 indicates that when a shipper wants to favor a certain group J^+ of carriers by assigning each of them a considerable volume of shipments and at the same time restrict the maximum number of winners N_{max}, they should set N_{max} at least equal to the number of carriers in group J^+. That is, $N_{max} \geq |J^+|$. Moreover, when the shipper set $N_{max} = |J^+|$, the winners of the auction are implicitly determined and they are those carriers in group J^+.

Chapter 7

A Two-Stage Sampling-Based Stochastic Winner Determination Model for Truckload Service Procurement under Shipment Volume Uncertainty

In this chapter, we consider that the shipment volume on each lane in truckload service procurement is uncertain. We model this uncertainty as a random variable and assume that we know the exact probability density function of it. We introduce this stochasticity into the following deterministic WDP and propose a two-stage sampling-based stochastic winner determination model.

$$\text{(c1-WDP)}\ \min \sum_{j\in J}\sum_{k\in K_j} c_{jk} y_{jk} + \sum_{i\in I}\left(e_i\varphi_i + p_i\theta_i\right) \tag{6.12}$$

s.t. $(6.3), (6.6)-(6.11), (6.17)-(6.22).$

Specifically, in Section 7.1, we formulate the stochastic WDP and use Monte Carlo method to derive a deterministic approximation. We propose a solution approach in Section 7.2 and conduct numerical analyses in Section 7.3.

7.1 Problem formulation

As we mentioned in Chapter 1, during an auction for TL service procurement, exact volume of shipments on each lane is usually unknown. The shipper has to determine the winners of the auction subjected to the shipment volume uncertainty. After the shipment volume is realized, the shipper also needs to determine the volume of shipments assigned to the carriers who won the auction. Apparently, a WDP under uncertainty intrinsically possesses a two-stage optimization framework. We denote by $\tilde{d}_i$ the random volume of shipments on lane *i* and assume that we know the probability distribution it follows. Let

$$\tilde{\mathbf{d}} \equiv \left[d_1, d_2, \ldots, d_{|I|}\right]^{\mathrm{T}} \subset \mathbf{R}^{|I|},$$

we can formulate a two-stage stochastic winner determination model as follows:

$$\text{(s-WDP)}\ \min \underset{\tilde{\mathbf{d}}}{E}\left[\sum_{j\in J}\sum_{k\in K_j} c_{jk} y_{jk}\left(\tilde{\mathbf{d}}\right) + \sum_{i\in I}\left(e_i\varphi_i\left(\tilde{\mathbf{d}}\right) + p_i\theta_i\left(\tilde{\mathbf{d}}\right)\right)\right] \tag{7.1}$$

$$\text{s.t.}\quad \sum_{j\in J}\sum_{k\in K_j} a_{ijk} y_{jk}\left(\tilde{\mathbf{d}}\right) + \varphi_i\left(\tilde{\mathbf{d}}\right) = \tilde{d}_i + \theta_i\left(\tilde{\mathbf{d}}\right), \quad i \in I, \tilde{d}_i \in \Delta_i \tag{7.2}$$

$$LS_{jk} x_{jk} \le y_{jk}\left(\tilde{\mathbf{d}}\right), \quad j \in J, k \in K_j \tag{7.3}$$

$$US_{jk} x_{jk} \ge y_{jk}\left(\tilde{\mathbf{d}}\right), \quad j \in J, k \in K_j \tag{7.4}$$

$$\sum_{k\in K_j} y_{jk}\left(\tilde{\mathbf{d}}\right) \le UQ_j, \quad j \in J \tag{7.5}$$

$$\sum_{k\in K_j} y_{jk}\left(\tilde{\mathbf{d}}\right) \ge LQ_j, \quad j \in J \tag{7.6}$$

$$y_{jk}\left(\tilde{\mathbf{d}}\right) \ge 0, \quad j \in J, k \in K_j \tag{7.7}$$

$$\varphi_i\left(\tilde{\mathbf{d}}\right) \ge 0, \quad i \in I \tag{7.8}$$

$$\theta_i\left(\tilde{\mathbf{d}}\right) \ge 0, \quad i \in I \tag{7.9}$$

constraints (6.3), (6.17)-(6.20)

In s-WDP, we add the term $\left(\tilde{\mathbf{d}}\right)$ to the notations of second-stage decision variables to indicate that these decisions are made dependently with the realized shipment volume.

Proposition 7.1 The feasibility of s-WDP is not dependent on the realized shipment volume. That is, as long as a feasible first-stage solution exists, there are always feasible second-stage solutions to s-WDP.

Proof: Let's denote the first-stage solution that is feasible to s-WDP by $\bar{x}_{jk}, j \in J, k \in K_j$. Note that although constraints (7.3)-(7.6) contain a second-stage decision variable, Proposition 6.1 shows that a feasible first-stage solution may not exist if the parameters in these constraints are not carefully evaluated. Therefore, if there exists a feasible first-stage solution to s-WDP, we should also have

$$y_{jk}\left(\tilde{\mathbf{d}}\right) = \bar{y}_{jk}, j \in J, k \in K_j$$

so that constraints (7.3)-(7.7) are satisfied. Then it is trivial that the following solution is always feasible to s-WDP:

$$x_{jk} = \bar{x}_{jk}, \quad j \in J, k \in K_j$$

$$y_{jk}\left(\tilde{\mathbf{d}}\right) = \bar{y}_{jk}, \quad j \in J, k \in K_j$$

$$\varphi_i\left(\tilde{\mathbf{d}}\right) = \left(\tilde{d}_i - \sum_{j \in J} \sum_{k \in K_j} a_{ijk} \bar{y}_{jk}\left(\tilde{\mathbf{d}}\right)\right)^+, \quad i \in I, \tilde{d}_i \in \Delta_i$$

$$\theta_i\left(\tilde{\mathbf{d}}\right) = \left(\sum_{j \in J} \sum_{k \in K_j} a_{ijk} \bar{y}_{jk}\left(\tilde{\mathbf{d}}\right) - \tilde{d}_i\right)^+, \quad i \in I, \tilde{d}_i \in \Delta_i$$

where

$$(u)^+ = \begin{cases} u, & \text{if } u \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

The proposition is proven. □

In order to solve s-WDP, it is necessary to evaluate the expected procurement cost in equation (7.1). Note that if the probability distribution of the random shipment volume is discrete and finite, the expected procurement cost has a closed-form linear expression and s-WDP has an equivalent mixed-integer programming (MIP) formulation (Ma et al., 2010). In this chapter, we assume a continuous probability distribution for the random shipment volume and propose a sampling-based method to solve s-WDP.

Assume that we have a sample of $\tilde{\mathbf{d}}$, $\left(\hat{\mathbf{d}}^1, \hat{\mathbf{d}}^2, \ldots, \hat{\mathbf{d}}^N\right)$, where

$$\hat{\mathbf{d}}^n \equiv \left[d_1^n, d_2^n, \ldots, d_{|I|}^n \right]^{\mathrm{T}}$$

and N is the sample size[8]. By law of large numbers, we have

$$\begin{aligned}
&\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \left[\sum_{j\in J} \sum_{k\in K_j} c_{jk} y_{jk}\left(\hat{\mathbf{d}}^n\right) + \sum_{i\in I}\left(e_i \varphi_i\left(\hat{\mathbf{d}}^n\right) + p_i \theta_i\left(\hat{\mathbf{d}}^n\right)\right)\right] \\
&\qquad = E_{\tilde{\mathbf{d}}}\left[\sum_{j\in J} \sum_{k\in K_j} c_{jk} y_{jk}\left(\tilde{\mathbf{d}}\right) + \sum_{i\in I}\left(e_i \varphi_i\left(\tilde{\mathbf{d}}\right) + p_i \theta_i\left(\tilde{\mathbf{d}}\right)\right)\right]
\end{aligned}$$

So we can use the sample average approximation of the expected procurement cost in equation (7.1) and derive a deterministic approximation of s-WDP:

$$\text{(m-WDP)} \ \min \frac{1}{N} \sum_{n=1}^{N} \left(\sum_{j\in J} \sum_{k\in K_j} c_{jk} y_{jk}^n + \sum_{i\in I}\left(e_i \varphi_i^n + p_i \theta_i^n \right)\right) \tag{7.10}$$

$$\text{s.t.} \quad \sum_{j\in J} \sum_{k\in K_j} a_{ijk} y_{jk}^n + \varphi_i^n = d_i^n + \theta_i^n, \quad i \in I, n = 1, \ldots, N \tag{7.11}$$

$$LS_{jk} x_{jk} \le y_{jk}^n, \quad j \in J, k \in K_j, n = 1, \ldots, N \tag{7.12}$$

$$US_{jk} x_{jk} \ge y_{jk}^n, \quad j \in J, k \in K_j, n = 1, \ldots, N \tag{7.13}$$

$$\sum_{k\in K_j} y_{jk}^n \le UQ_j, \quad j \in J, n = 1, \ldots, N \tag{7.14}$$

[8] Note that the superscript n for variables does not denote the power but an index.

$$\sum_{k \in K_j} y_{jk}^n \geq LQ_j, \quad j \in J, n = 1, \ldots, N \tag{7.15}$$

$$y_{jk}^n, \quad j \in J, k \in K_j, n = 1, \ldots, N \tag{7.16}$$

$$\varphi_i^n \geq 0, \quad i \in I, n = 1, \ldots, N \tag{7.17}$$

$$\theta_i^n \geq 0, \quad i \in I, n = 1, \ldots, N \tag{7.18}$$

constraints (6.3), (6.17)-(6.20)

7.2 Solution approach

According to law of large numbers, the larger the sample size N is, the better m-WDP is an approximation of s-WDP. The solution to m-WDP converges to a solution of s-WDP as N tends to infinity. However, as N increases, m-WDP, which is a mixed-integer program, will become more computationally prohibitive. Therefore, we need to balance the solution quality and the computation efficiency. Inspired by Ahmed et al. (2002), we propose the following algorithm to solve s-WDP.

Algorithm 7.1 Monte Carlo Approximation (MCA)

Step 0 According to the distribution of $\tilde{\mathbf{d}}$, generate M samples of size N. Denote the shipment volume in sample m by $\left[d_i^{m,n} \mid i \in I, n = 1, \ldots, N \right]$.

Step 1 For m = 1, …, M, solve m-WDP in which we let $d_i^n = d_i^{m,n}$ and denote the optimal solution by

$$\mathbf{x}^m \equiv \left[x_{jk}^m \mid j \in J, k \in K_j \right].$$

After we obtain all the solutions $\left\{ \mathbf{x}^1, \ldots, \mathbf{x}^M \right\}$, go to Step 2.

Step 2 According to the distribution of $\tilde{\mathbf{d}}$, generate a large sample of size N^* where $N^* \gg N$. Denote the shipment volume in this large sample by

$$\mathbf{d}^* \equiv \left[d_i^{*n} \mid i \in I, n = 1, \ldots, N^* \right].$$

For m = 1, …, M, we let $d_i^n = d_i^{*n}$ and $x_{jk} = x_{jk}^m$ in m-WDP and solve it to find the optimal second-stage decisions. Record the objective value as z^m. Let

$$\bar{m} = \arg\min_m \left\{ z^m \mid m = 1, \ldots, M \right\}.$$

$\mathbf{x}^{\bar{m}}$ is the optimal first-stage solution.

In MCA, the greater the N is, the better solution we are likely to obtain from solving each m-WDP; the greater the N^* is, the more accurate is our evaluation of solutions. Moreover, the more the number of samples we use (the larger the m is), the more likely we will find a near-optimal solution. However, the greater value those parameters take, the more computation time is required by MCA. Since the investigation of the efficiency of MCA is not an emphasis of this dissertation, in next section we evaluate these parameters arbitrarily.

7.3 Numerical analyses

An alternative to solve WDP under shipment volume uncertainty ($\tilde{\mathbf{d}}$) is to solve c1-WDP in which $d_i = E\left(\tilde{d}_i\right)$. In this section, we refer to this approach as nominal approach and compare it with MCA. Specifically, we solve the problem using the two approaches and record the solutions. We denote the optimal first-stage solutions obtained from nominal approach and MCA by

$$\mathbf{x}^{\text{nominal}} = \left[x_{jk}^{\text{nominal}} \mid j \in J, k \in K_j \right]$$

and

$$\mathbf{x}^{\mathrm{MCA}} = \left[x_{jk}^{\mathrm{MCA}} \mid j \in J, k \in K_j \right],$$

respectively. Then we evaluate the quality of each solution by the following algorithm based on Monte Carlo simulation.

Algorithm 7.2 Solution Evaluation (SE)

Step 0 According to the distribution of uncertain shipment volume $\tilde{\mathbf{d}}$, generate a large sample of size N^{test} where $N^{\mathrm{test}} \gg N^*$. We denote by $\left[d_i^{\mathrm{test},n} \mid i \in I \right]$ the shipment volume in scenario n where $n = 1, \ldots, N^{\mathrm{test}}$. Let $w^{\mathrm{nominal}} = w^{\mathrm{MCA}} = 0$, $z^{\mathrm{nominal}} = z^{\mathrm{MCA}} = 0$.

Step 1 For $n = 1, \ldots, N^{\mathrm{test}}$,

(a) let $d_i = d_i^{\mathrm{test},n}$ and $x_{jk} = x_{jk}^{\mathrm{nominal}}$, solve c1-WDP for optimal second-stage decisions and denote the objective value by $z^{\mathrm{nominal},n}$;

(b) let $d_i = d_i^{\mathrm{test},n}$ and $x_{jk} = x_{jk}^{\mathrm{MCA}}$, solve c1-WDP for optimal second-stage decisions and denote the objective value by $z^{\mathrm{MCA},n}$;

(c) if $z^{\mathrm{nominal},n} \geq z^{\mathrm{MCA},n}$, $w^{\mathrm{nominal}} = w^{\mathrm{nominal}} + 1$; if $z^{\mathrm{nominal},n} \leq z^{\mathrm{MCA},n}$, $w^{\mathrm{MCA}} = w^{\mathrm{MCA}} + 1$.

Step 2 Compute

$$z^{\mathrm{nominal}} = \frac{1}{N^{\mathrm{test}}} \sum_{n=1}^{N^{\mathrm{test}}} z^{\mathrm{nominal},n}$$

and

$$z^{\mathrm{MCA}} = \frac{1}{N^{\mathrm{test}}} \sum_{n=1}^{N^{\mathrm{test}}} z^{\mathrm{MCA},n}.$$

In SE, w^{nominal} denotes the number of scenarios in which the procurement cost associated with the nominal solution is lower than or equal to that associated with the MCA solution.

Similarly, w^{MCA} denotes the number of scenarios in which the procurement cost associated with the MCA solution is lower than or equal to that associated with the nominal solution. Note that the sum of w^{nominal} and w^{MCA} may be greater than N^{test} since in the case when $z^{\text{nominal},n} = z^{\text{MCA},n}$, we add 1 to both w^{nominal} and w^{MCA}. $\bar{z}^{\text{nominal}}$ and $\bar{z}^{\text{MCA}}$ are the average of procurement costs in all N^{test} scenarios associated with nominal solution and MCA solution, respectively.

7.3.1 An illustrative numerical example

In this section, we provide an illustrative example which is modified based on the example in Ma et al. (2010). Specifically, we assume that there are three lanes and four carriers/bidders and each carrier submits three package bids as summarized in Table 7.1. For instance, by submitting bid 2, carrier 3 indicates that they are willing to accept at least 1 unit of shipment (LS_{32} = 1) and at most 5 units (US_{32} = 5) of shipments on lanes 1 and 3 ($a_{132} = a_{332}$ = 1, a_{232} = 0) at a price of 14 per unit shipment (c_{32} = 14). The other cost coefficients are shown in Table 7.2. In addition, we assume that there are at least 1 winner ($N_{\min}$ = 1) and at most 3 winners ($N_{\max}$ = 3) of this auction. Each bidder will be assigned at least 0 unit (LQ_j = 0 for all j) and at most 20 units (UQ_j = 20 for all j) of shipment. The volume of shipments on lane 1, 2 and 3 are assumed to be uniformly distributed in [1, 7], [0, 8], and [4, 10] respectively. So the mean volumes of shipments on lane 1, 2 and 3, which are the input to the deterministic WDP solved by the nominal approach, are 4, 4, and 7, respectively.

Table 7.1 Summary of all bids

Carrier	Package bid	Lane	Required volum	Unit price
1	1	1 and 2	[2, 4]	13
	2	2 and 3	[1, 5]	20
	3	2	[2, 4]	10
2	1	2	[3, 5]	9

	2	1 and 2	[2, 4]	14
	3	1 and 3	[2, 4]	16
3	1	3	[2, 4]	11
	2	1 and 3	[1, 5]	14
	3	1	[2, 4]	5
4	1	1 and 3	[1, 7]	15
	2	2	[2, 5]	9
	3	2 and 3	[1, 4]	18

Table 7.2 Unit extra service cost and penalty cost

	lane 1	lane 2	lane 3
Unit extra service cost	$e_1 = 9.0$	$e_2 = 17.5$	$e_3 = 12.5$
Unit penalty cost	$p_1 = 5.0$	$p_2 = 10.0$	$p_3 = 11.0$

When we apply MCA to solve this example, we just generate one sample ($M = 1$) of size $N = 10$. Therefore, MCA actually stops after Step 1. We use a sample of size $N^{\text{test}} = 100$ in SE. The problem is coded in GAMS 23.6.5 and solved by CPLEX 12.4 MIP solver. We run the test on a personal computer with the following attributes: Intel Core2 1.20GHz and 1.0GB RAM. The result in one test scenario (Scenario 100) is summarized in Table 7.3.

Table 7.3 Result in Scenario 100 of the illustrative example

	nominal approach	MCA
Allocation of lanes	$x_{33} = x_{43} = 1$	$x_{32} = x_{42} = 1$
Shipments assigned to winners	$\begin{cases} y_{33} = 3.496 \\ y_{43} = 3.499 \end{cases}$	$\begin{cases} y_{32} = 3.496 \\ y_{42} = 3.499 \end{cases}$
Shipments assigned to other third-party carriers	$\varphi_3 = 4.486$	$\varphi_3 = 4.490$
Shortage	0	0
Shipping cost	80.462	80.435
Extra service cost	56.075	56.125
Penalty cost	0	0
Total procurement cost	136.537	136.560

In Scenario 100, the realized shipment volumes are:

$d_1^{\text{test},100} = 3.496,\ d_2^{\text{test},100} = 3.499,\ d_3^{\text{test},100} = 7.985\,.$

If the shipper adopts the nominal solution, carrier 3's bid 3 (lane 1) and carrier 4's bid 3 (lanes 2 and 3) will be awarded. Carrier 3 will be assigned 3.496 units of shipments on lane 1 and carrier 4 will be assigned 3.499 units of shipments on lanes 2 and 3. The shipping cost, which is the amount of money paid to carrier 3 and carrier 4 is 80.462. Extra shipping capacity from other third-party carriers is needed to handle the remaining 4.490 units of shipments on lane 3. This extra service costs 56.075. Since there are no shortage and the associated penalty cost, the total procurement cost is: 80.462 + 56.075 = 136.537. Note that in this scenario, the nominal solution is better since its associated total procurement cost is lower. However, if we consider the performance of both solutions in general, we can easily find that MCA is better. Table 7.4 summarizes the evaluation result of the illustrative example. On average, MCA solution yields a procurement cost that is 3.45%[9] lower. Moreover, MCA solution yields a procurement cost that is lower than or equal to the cost associated with the nominal solution in 71 out of the 100 scenarios. The problem can be solved by either MCA algorithm or nominal approach in several seconds.

Table 7.4 Evaluation result of the illustrative example

	nominal approach	MCA
Average of procurement costs in all scenarios	$z^{\text{nominal}} = 144.447$	$z^{\text{MCA}} = 139.466$
"Number of wins"	$w^{\text{nominal}} = 37$	$w^{\text{MCA}} = 71$
Computation time (seconds)	2.750	1.547

7.3.2 Tests on moderately sized instances

In this section, we solve moderately sized instances of WDP under shipment volume uncertainty. Specifically, we consider problem instances at five different scales in [number of

[9] $\left(z^{\text{nominal}} - z^{\text{MCA}}\right) / z^{\text{nominal}} = 3.45\%$.

lanes, number of carriers/bidders, number of bids submitted per carrier]: [60, 10, 10], [120, 15, 10], [180, 20, 10], [240, 25, 10] and [300, 30, 10]. In order to ensure that a feasible first-stage solution exists, the shipper may specify the combination of lanes in a certain portion of bids. We set this portion to 80% in our test. The unit shipping costs, extra service costs and penalty costs are all randomly generated. Specifically, the unit shipping cost c_{jk} is evaluated by multiplying a number that is uniformly distributed in [1/60, 1/12] by the number of lanes included in carrier *j*'s bid *k*. The unit extra service costs e_i and unit penalty costs p_i for each lane are uniformly distributed in [1/12, 1/6] and [1/3, 5/12], respectively. We assume that the volume of shipments on each lane falls in the range [50, 150]. The lower bound LS_{jk} and upper bound US_{jk} on the acceptable shipment volumes are uniformly distributed in [25, 75] and [125, 175], respectively. The minimum and maximum volume of shipments that can be assigned to carriers (LQ_j and UQ_j) are set to 0 and 1000, respectively. The restriction on the number of winners in different instances is summarized in Table 7.5.

Table 7.5 Restrictions on the number of winners

	[60, 10, 10]	[120, 15, 10]	[180, 20, 10]	[240, 25, 10]	[300, 30, 10]
$N_{\min}$	1	1	1	1	1
$N_{\max}$	8	12	15	19	23

We assume that the volume of shipments on all lanes follows an identical continuous distribution which is supported on [50, 150]. Specifically, in our test we adopt beta distribution which is a family of continuous distributions on [0, 1] with shapes controlled by two parameters, α and β. We test the problem using 5 different beta distributions: beta(5, 5), beta(1, 1), beta(0.5, 0.5), beta(2, 5), and beta(1, 5). To ensure that the sampled shipment volumes follow beta distributions on [50, 150], we set

$$d_i^{\text{test},n} = 100 z_i^n + 50$$

where $z_i^n \sim \text{beta}(\alpha, \beta)$. The mean volume of shipments on all lanes is summarized in Table 7.6. The probability density functions (PDFs) of the 5 beta distributions are illustrated in Figure. 7.1.

Table 7.6 Mean volume of shipments on all lanes

	beta(5, 5)	beta(1, 1)	beta(0.5, 0.5)	beta(2, 5)	beta(1, 5)
Mean	100	100	100	78.57	66.67

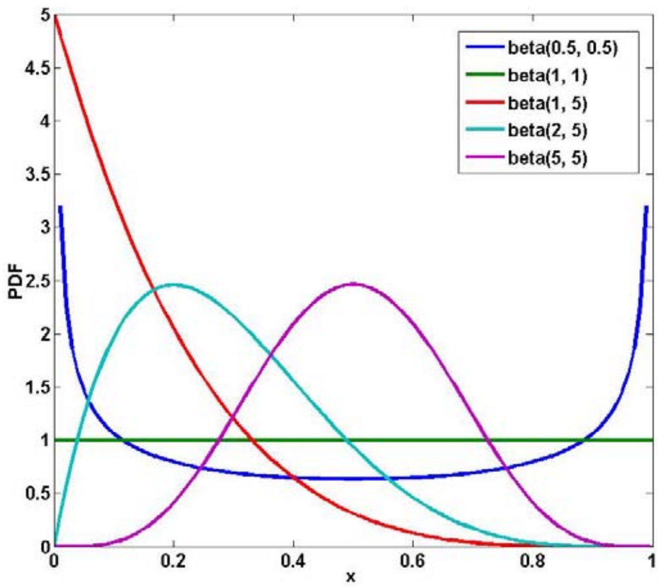

Figure 7.1 PDFs of the five beta distributions

We set M = 10, N = 10 and N^* = 100 for MCA and N^{test} = 10000 for SE. The problem is coded in GAMS 23.6.5 and MATLAB (R2010b). Specifically, the major loops of MCA and SE are coded in MATLAB. To solve a specific mathematical program in MCA or SE, e.g., m-WDP or c1-WDP, we call GAMS from MATLAB and use CPLEX 12.4 MIP solver. We conduct the test on Penn State Lion-XC system.

The evaluation results are summarized in Table 7.7.

Table 7.7 Evaluation results

Distribution	Scale	Case	nominal approach			MCA		
			RAPC(%)	w^{nominal}	CT(s)	RAPC(%)	w^{MCA}	CT(s)
beta(5, 5)	60	1	100.00	10000	0.22	100.00	10000	94.88
	120	2	100.00	10000	0.19	100.00	10000	153.48
	180	3	100.00	10000	1.50	100.00	10000	238.97
	240	4	100.00	10000	0.59	100.00	10000	343.24
	300	5	100.25	4882	1.85	100.00	10000	877.91
beta(1, 1)	60	6	101.14	4187	0.22	100.00	5823	98.02
	120	7	106.10	521	0.19	100.00	9480	168.79
	180	8	107.18	75	1.50	100.00	9925	274.78
	240	9	100.00	10000	0.59	100.00	10000	480.63
	300	10	102.48	128	1.85	100.00	9882	1761.50
beta(0.5, 0.5)	60	11	107.08	500	0.22	100.00	9501	98.98
	120	12	116.25	10	0.19	100.00	9990	161.60
	180	13	116.30	0	1.50	100.00	10000	321.10
	240	14	100.00	10000	0.59	100.00	10000	468.77
	300	15	109.64	1	1.85	100.00	9999	966.94
beta(2, 5)	60	16	117.90	6	0.11	100.00	9994	107.77
	120	17	100.00	10000	0.23	100.00	10000	175.41
	180	18	103.09	430	0.37	100.00	9571	264.32
	240	19	100.00	10000	0.57	100.00	10000	487.63
	300	20	100.55	3870	1.17	100.00	6136	1044.30
beta(1, 5)	60	21	100.16	4746	0.13	100.00	5258	93.45
	120	22	100.00	10000	0.25	100.00	10000	153.53
	180	23	122.86	0	0.38	100.00	10000	264.17
	240	24	122.75	0	0.54	100.00	10000	558.01
	300	25	100.00	10000	1.46	100.00	10000	987.04

In Table 7.7, in the second column “Scale”, each number denotes the number of lanes in that problem. For instance, “180” corresponds to the scale [180, 20, 10]. We consider five beta distributions and five different problem scales, so we have in all twenty-five cases. The simulation time for different cases ranges from 1.5 hours to 5.5 hours. “RPAC” denotes the ratio of average procurement cost associated with each solution over the cost associated with the MCA solution. Specifically, the numbers in column “RPAC” under “nominal approach” are calculated by: $\frac{z^{\text{nominal}} - z^{\text{MCA}}}{z^{\text{nominal}}} \times 100\%$. "CT" denotes the computation time. For example, in Case 20 where the shipment volume follows distribution beta(2, 5) and the problem scale is [300, 30, 30], the MCA solution yields a procurement cost that is 0.55% lower than the cost associated with the nominal solution. In the same case, the MCA solution yields a procurement cost that is lower than or equal to the cost associated with the nominal solution in 6136 out of the 10000 scenarios.

There is no guarantee that MCA always yields a solution that is better than the nominal solution when solving WDP under shipment volume uncertainty. We can see from Table 7.7 that in some cases like Case 1 the two approaches yield the same solution. However, it is apparent that MCA can derive a solution that yields a lower average procurement cost in most of the 25 cases. In particular, in some cases, the MCA solution outperforms the alternative significantly. For instance, in Case 23, the MCA solution yields an average procurement cost that is 22.86% lower than the cost associated with the nominal solution. Moreover, the MCA solution yields a lower cost in more than half of the 10000 simulation scenarios in every case. Especially in Cases 13, 23 and 24, the MCA solution outperforms the nominal solution in all 10000 simulation scenarios. Although MCA consumes more time than the nominal approach to compute the solution, it can solves the largest test problem that involves 300 lanes, 30 carriers and 300 bids within half an hour which is a reasonable computation time.

It is worth noting that when the uncertainty follows beta(5, 5), the two approaches yield the same solution in Cases 1-4. In Case 5, MCA solution only yields an average procurement cost that is 0.25% lower. This is because when the uncertainty follows beta(5,5), the mean is a fairly good characterization of the uncertainty. From Figure 7.1, we can see that the probability density function of beta(5, 5) reaches its peak at the mean point. However, as the variance of the uncertainty increases (e.g., when the uncertainty follows beta(1, 1) and beta(0.5, 0.5)), the MCA solution outperforms the nominal solution more significantly. What is more, the MCA solution outperforms the nominal solution more significantly when the uncertainty follows asymmetric beta distributions (i.e., beta(2, 5) and beta(1, 5)). We can conclude that when mean is no longer a good characterization of the uncertainty, MCA which makes use of more distributional information of the uncertainty can provide a significantly better solution than the nominal approach.

Chapter 8

A Two-Stage Robust Winner Determination Model for Truckload Service Procurement under Shipment Volume Uncertainty

As we mentioned earlier, the volume of shipments on each lane is usually unknown to the shipper at the time of auction. In this chapter, we assume that only partial distributional information, mean and variance of the uncertain shipment volume on each lane in truckload service procurement is available. In this case, the stochastic programming approach introduced in the previous chapter may not be applicable. We robustify the following deterministic WDP and propose a two-stage robust winner determination model.

$$\text{(c2-WDP)} \ \min \sum_{j \in J} \sum_{k \in K_j} \left(c_{jk} y_{jk} + p_{jk} z_{jk} \right) + \sum_{i \in I} e_i \varphi_i \tag{6.15}$$

s.t. $(6.3), (6.7)-(6.9), \ (6.13), \ (6.14), (6.16)-(6.19).$

Note that c2-WDP is different from c1-WDP that it does not contain the constraint that prohibits multiple carriers winning the same single lane. This will greatly enlarge the feasible region of the shipper's decision pool which leads to an optimal solution that guarantees a lower procurement cost. So compared to c1-WDP, c2-WDP is more general but more challenging to solve.

Utilizing mean and variance of the uncertain shipment volume which can be obtained from historical data, we construct a polyhedral uncertainty set for shipment volume in Section 8.1 and formulate a two-stage robust winner determination model in Section 8.2. We develop solution approaches in Section 8.3 and conduct numerical analyses in Section 8.4.

8.1 Construction of uncertainty set

We denote the uncertain volume of shipments on lane *i* by $\tilde{d}_i$ and assume that $\tilde{d}_i, \quad i \in I$ are independent and identically distributed[10] random variables. Different from Ma et al. (2010), in this chapter we assume that only limited information regarding the distribution of $\tilde{d}_i$, namely mean μ and variance σ, is known, which usually holds in practice. With respect to the classical central limit theorem, the random variable $\sum_{i=1}^{|I|} \tilde{d}_i$ follows a standard normal distribution as $|I| \to \infty$. That is,

$$\lim_{|I|\to\infty} \Pr\left(\frac{\sum_{i=1}^{|I|} \tilde{d}_i - |I|\mu}{\sigma\sqrt{|I|}} \le t \right) = \Pr\left(Z \le t\right)$$

where $Z \sim N(0,1)$.

Instead of describing $\tilde{d}_i, \quad i \in I$ as random variables, we can assume that they take values in the following uncertainty set:

$$\Delta = \left\{ \left(\tilde{d}_1, \ldots, \tilde{d}_{|I|}\right) \middle| \left| \sum_{i=1}^{|I|} \tilde{d}_i - |I|\mu \right| \le \Gamma\sigma\sqrt{|I|} \right\}.$$

where Γ is a constant that usually takes the value of 2 or 3 to make a good fit empirically (Bandi and Bertsimas, 2012). Moreover, since $\tilde{d}_i$ stands for shipment volume, it is required to be nonnegative. As a result, we have the following polyhedral set for the uncertain shipment volume:

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[10] This assumption can be relaxed as "independent but not necessarily identically distributed" and we can still construct a polyhedral uncertainty set using Lyapunov central limit theorem. In this chapter, we keep the original assumption for simplicity.

$$\bar{\Delta} = \left\{ \left(\tilde{d}_1, \ldots, \tilde{d}_{|I|} \right) \middle| \; \left| \sum_{i=1}^{|I|} \tilde{d}_i - |I|\mu \right| \le \Gamma\sigma\sqrt{|I|}, \quad \tilde{d}_i \ge 0, \forall i \in I \right\}.$$

8.2 Problem formulation

With the shipment volume described by the uncertainty set $\bar{\Delta}$ specified in last section, we can formulate the following two-stage robust winner determination model:

$$\text{(RO-WDP)} \ \min_{\mathbf{x} \in X} \max_{\tilde{\mathbf{d}} \in \bar{\Delta}} \min_{(\mathbf{y},\mathbf{z},\boldsymbol{\varphi}) \in Y(\mathbf{x},\tilde{\mathbf{d}})} f_0\left(\mathbf{y},\mathbf{z},\boldsymbol{\varphi}\right) \tag{8.1}$$

where

$$f_0\left(\mathbf{y},\mathbf{z},\boldsymbol{\varphi}\right) = \sum_{j \in J} \sum_{k \in K_j} \left(c_{jk} y_{jk} + p_{jk} z_{jk} \right) + \sum_{i \in I} e_i \varphi_i \,,$$

$$X = \left\{ \left(x_{jk}, j \in J, k \in K_j \right) \mid (6.3), (6.17) - (6.19) \right\},$$

$$\bar{\Delta} = \left\{ \left(\tilde{d}_1, \ldots, \tilde{d}_{|I|} \right) \middle| \; \left| \sum_{i=1}^{|I|} \tilde{d}_i - |I|\mu \right| \le \Gamma\sigma\sqrt{|I|}, \quad \tilde{d}_i \ge 0, \forall i \in I \right\},$$

$$Y\left(\mathbf{x}, \tilde{\mathbf{d}}\right) = \left\{ \left(y_{jk}, z_{jk}, \varphi_i, i \in I, j \in J, k \in K_j \right) \mid (6.7) - (6.9), (6.13), (6.14), (6.16) \right\}.$$

In RO-WDP, $\mathbf{x}$ and $\left(\mathbf{y},\mathbf{z},\boldsymbol{\varphi}\right)$ are respectively the vectors of first-stage and second-stage decision variables. In practice, at the time of the auction, the shipper needs to allocate lanes to carriers when the shipping demand on each lane is still uncertain and $\mathbf{x}$ represents this allocation decision. Once the shipment volume is realized, the shipper needs to determine the volume of shipments that are assigned to each winning carrier ($\mathbf{y}$) and third-party carriers ($\boldsymbol{\varphi}$). In addition, the shortage ($\mathbf{z}$) in shipments that are required by winning carriers is also revealed. By solving RO-WDP, we are actually seeking the most robust allocation of lanes to carriers such that the total procurement cost in the worst-case scenario, or in other words the maximum total procurement cost that might be incurred, is minimized.

Note that RO-WDP is a min-max-min problem which is difficult to handle. A model simplification is necessary. Because the inner minization problem is a linear program given that the winners are determined and the volume of shipments on each lane is known, we can formulate the dual of the inner minimization problem $\min_{(\mathbf{y},\mathbf{z},\boldsymbol{\varphi})\in Y(\mathbf{x},\tilde{\mathbf{d}})} f_0(\mathbf{y},\mathbf{z},\boldsymbol{\varphi})$ in RO-WDP:

$$\text{(P1)} \quad f_1(\mathbf{x},\tilde{\mathbf{d}}) = \max\left[\sum_{i\in I}\tilde{d}_i\lambda_{1,i} + \sum_{j\in J}\sum_{k\in K_J}\left(LS_{jk}x_{jk}\lambda_{2,jk} - US_{jk}x_{jk}\lambda_{3,jk}\right)\right] \tag{8.2}$$

$$\text{s.t.} \quad \sum_{i\in I} a_{ijk}\lambda_{1,i} + \lambda_{2,jk} - \lambda_{3,jk} \le c_{jk}, \quad j\in J, k\in K_j \tag{8.3}$$

$$\lambda_{1,i} \le e_i, \quad i\in I \tag{8.4}$$

$$\lambda_{2,jk} \le p_{jk}, \quad j\in J, k\in K_j \tag{8.5}$$

$$\lambda_{2,jk}, \lambda_{3,jk} \ge 0 \quad j\in J, k\in K_j \tag{8.6}$$

where $\lambda_{1,i}, \lambda_{2,jk}, \lambda_{3,jk}$ are the dual variables corresponding to constraints (6.16), (6.13), and (6.7), respectively. The optimal objective value of (P1) is equal to that of the inner minimization problem $\min_{(\mathbf{y},\mathbf{z},\boldsymbol{\varphi})\in Y(\mathbf{x},\tilde{\mathbf{d}})} f_0(\mathbf{y},\mathbf{z},\boldsymbol{\varphi})$ due to strong duality theory. Therefore, we can reformulate $\max_{\tilde{\mathbf{d}}\in\tilde{\Delta}} \min_{(\mathbf{y},\mathbf{z},\boldsymbol{\varphi})\in Y(\mathbf{x},\tilde{\mathbf{d}})} f_0(\mathbf{y},\mathbf{z},\boldsymbol{\varphi})$ as a max-max problem. Combining the two maximization problem, we have

$$\text{(P2)} \quad f_2(\mathbf{x}) = \max f_3(\mathbf{d},\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2,\boldsymbol{\lambda}_3) = \left[\sum_{i\in I}\tilde{d}_i\lambda_{1,i} + \sum_{j\in J}\sum_{k\in K_J}\left(LS_{jk}x_{jk}\lambda_{2,jk} - US_{jk}x_{jk}\lambda_{3,jk}\right)\right] \tag{8.7}$$

$$\text{s.t.} \quad \sum_{i=1}^{|I|}\tilde{d}_i \le |I|\mu + \Gamma\sigma\sqrt{|I|} \tag{8.8}$$

$$-\sum_{i=1}^{|I|}\tilde{d}_i \le \Gamma\sigma\sqrt{|I|} - |I|\mu \tag{8.9}$$

$$\tilde{d}_i \ge 0 \quad i\in I \tag{8.10}$$

constraints (8.3)-(8.6)

Now, RO-WDP can be reduced to $\min_{\mathbf{x}\in X} f_2(\mathbf{x})$ which is a bi-level min-max problem.

Proposition 8.1 (P2) is feasible and bounded.

Proof: It is trivial that (P2) is feasible. To show (P2) is bounded, it is sufficient to show that: (a) in (P2) $\tilde{d}_i$ is bounded for all $i \in I$; and (b) given fixed $\tilde{d}_i$, (P1) is bounded. (a) is true since $\tilde{d}_i \in \bar{\Delta}$ and $\bar{\Delta}$ is a polyhedral set. Due to the duality theory, a sufficient and necessary condition of (b) is that the inner minimization problem $\min_{(\mathbf{y},\mathbf{z},\boldsymbol{\varphi})\in Y(\mathbf{x},\tilde{\mathbf{d}})} f_0(\mathbf{y},\mathbf{z},\boldsymbol{\varphi})$ is feasible. So, we just need to show that $Y(\mathbf{x},\tilde{\mathbf{d}}) \neq \varnothing$.

It is evident that for any given $\mathbf{x} \in X$ and $\tilde{\mathbf{d}} \in \bar{\Delta}$,

$$\begin{cases} y_{jk} = 0, \quad j \in J, k \in K_J \\ z_{jk} = LS_{jk} x_{jk}, \quad j \in J, k \in K_J \\ \varphi_i = \tilde{d}_i, \quad i \in I \end{cases}$$

is always feasible to constraints (6.7)-(6.9), (6.13), (6.14) and (6.16).

Therefore, $Y(\mathbf{x},\tilde{\mathbf{d}}) \neq \varnothing$. □

Given Proposition 8.1, if in addition $X \neq \varnothing$, there exists an optimal solution to RO-WDP.

Proposition 8.2 There always exists an optimal solution to (P2) which is obtained at an extreme point of the feasible region of (P2).

Proof: Due to the term $\tilde{d}_i \lambda_{1,i}$ in equation (8.7), (P2) is bilinear. We denote the feasible region for all $\lambda_{1,i}, \lambda_{2,jk}, \lambda_{3,jk}$ by Λ, that is,

$$\Lambda \equiv \left\{ \left(\lambda_{1,i}, \lambda_{2,jk}, \lambda_{3,jk}, i \in I, j \in J, k \in K_j \right) \middle| (8.3)-(8.6) \right\}.$$

It is apparent that (P2) is always feasible. $\tilde{\mathbf{d}}$ and $(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3)$ are separately constrained. That is, the value of $\tilde{\mathbf{d}}$ will not affect the feasible region of $(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3)$ and vice versa. Let's denote by $\tilde{\mathbf{d}}^*$ and $(\boldsymbol{\lambda}_1^*, \boldsymbol{\lambda}_2^*, \boldsymbol{\lambda}_3^*)$ an optimal solution to (P2) and we have

$$f_3\left(\tilde{\mathbf{d}}^*, \boldsymbol{\lambda}_1^*, \boldsymbol{\lambda}_2^*, \boldsymbol{\lambda}_3^*\right) \geq f_3\left(\tilde{\mathbf{d}}, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3\right)$$

for all $\tilde{\mathbf{d}} \in \bar{\Delta}$ and all $(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3) \in \Lambda$. Note that the existence of such an optimal solution to (P2) is guaranteed given Proposition 8.1. If we fix $(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3)$ as $(\boldsymbol{\lambda}_1^*, \boldsymbol{\lambda}_2^*, \boldsymbol{\lambda}_3^*)$ and solve (P2) for $\tilde{\mathbf{d}}$, the problem reduces to a linear program and there must exist an optimal solution $\tilde{\mathbf{d}}^{**}$ which like $\tilde{\mathbf{d}}^*$ is also a basic feasible solution of $\bar{\Delta}$. As a result, we have

$$f_3\left(\tilde{\mathbf{d}}^{**}, \boldsymbol{\lambda}_1^*, \boldsymbol{\lambda}_2^*, \boldsymbol{\lambda}_3^*\right) = f_3\left(\tilde{\mathbf{d}}^*, \boldsymbol{\lambda}_1^*, \boldsymbol{\lambda}_2^*, \boldsymbol{\lambda}_3^*\right).$$

We then fix $\tilde{\mathbf{d}}$ as $\tilde{\mathbf{d}}^{**}$ and solve (P2) for $(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3)$ which is again a linear program and has an optimal solution $(\boldsymbol{\lambda}_1^{**}, \boldsymbol{\lambda}_2^{**}, \boldsymbol{\lambda}_3^{**})$ that is a basic feasible solution of Λ. So, we have

$$f_3\left(\tilde{\mathbf{d}}^{**}, \boldsymbol{\lambda}_1^{**}, \boldsymbol{\lambda}_2^{**}, \boldsymbol{\lambda}_3^{**}\right) = f_3\left(\tilde{\mathbf{d}}^{**}, \boldsymbol{\lambda}_1^*, \boldsymbol{\lambda}_2^*, \boldsymbol{\lambda}_3^*\right) = f_3\left(\tilde{\mathbf{d}}^*, \boldsymbol{\lambda}_1^*, \boldsymbol{\lambda}_2^*, \boldsymbol{\lambda}_3^*\right) \geq f_3\left(\tilde{\mathbf{d}}, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3\right)$$

for all $\tilde{\mathbf{d}} \in \bar{\Delta}$ and all $(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3) \in \Lambda$. Consequently, $\left(\tilde{\mathbf{d}}^{**}, \boldsymbol{\lambda}_1^{**}, \boldsymbol{\lambda}_2^{**}, \boldsymbol{\lambda}_3^{**}\right)$ is an extreme point of the feasible region ($\bar{\Delta} \times \Lambda$) of (P2) and also the optimal solution to (P2). □

Based on Proposition 8.2, we can derive two different equivalent reformulations of $\min_{\mathbf{x} \in X} f_2(\mathbf{x})$, (P3) and (P4), both of which are single-level optimization problems.

(P3) $\quad \min A$ (8.11)

$$\text{s.t.} \quad A \geq \sum_{i \in I} \tilde{d}_i^l \lambda_{1,i}^l + \sum_{j \in J} \sum_{k \in K_j} \left(LS_{jk} x_{jk} \lambda_{2,jk}^l - US_{jk} x_{jk} \lambda_{3,jk}^l \right), \quad l \in \Omega \tag{8.12}$$

constraints (6.3), (6.17)-(6.19)

In P(3), Ω is the index set of all the extreme points of $\bar{\Delta} \times \Lambda$ and A is a dummy decision variable.

(P4) $\quad \min B$ (8.13)

$$\text{s.t.} \quad B \geq \sum_{j \in J} \sum_{k \in K_j} \left(c_{jk} y_{jk}^l + p_{jk} z_{jk}^l \right) + \sum_{i \in I} e_i \varphi_i^l, \quad l \in \bar{\Omega} \tag{8.14}$$

$$\sum_{j \in J} \sum_{k \in K_j} a_{ijk} y_{jk}^l + \varphi_i^l = \tilde{d}_i^l, \quad i \in I, l \in \bar{\Omega} \tag{8.15}$$

$$LS_{jk} x_{jk} \leq y_{jk}^l + z_{jk}^l, \quad j \in J, k \in K_j, l \in \bar{\Omega} \tag{8.16}$$

$$US_{jk} x_{jk} \geq y_{jk}^l, \quad j \in J, k \in K_j, l \in \bar{\Omega} \tag{8.17}$$

$$y_{jk}^l, z_{jk}^l, \varphi_i^l \geq 0 \quad i \in I, j \in J, k \in K_j, l \in \bar{\Omega} \tag{8.18}$$

constraints (6.3), (6.17)-(6.19)

In P(4), $\bar{\Omega}$ is the index set of all the extreme points of $\bar{\Delta}$, B is a dummy decision variable, $y_{jk}^l, z_{jk}^l, \varphi_i^l$ are new recourse variables corresponding to d_i^l. If we denote by $w_{\bar{\Delta}}$ and w_Λ the number of extreme points of $\bar{\Delta}$ and Λ, respectively, then $|\Omega| = w_{\bar{\Delta}} \cdot w_\Lambda$ and $|\bar{\Omega}| = w_{\bar{\Delta}}$.

8.3 Solution approaches

As analyzed in the previous section, RO-WDP can be reduced to two equivalent single-level mathematical programs, (P3) and (P4), which however can not be solved directly until all the extreme points of $\bar{\Delta} \times \Lambda$ and those of $\bar{\Delta}$ are enumerated, respectively. In most two-stage RO studies (Thiele et al., 2009; Jiang et al., 2012; Remli and Rekik, 2012; Bertsimas et al., 2013), the following Benders' decomposition type constraint generation algorithm is applied to solve (P3).

Algorithm 8.1 Constraint Generation

Step 0 Set $\Omega = \varnothing$, $LB = -\infty$, $UB = +\infty$, $t = 1$. Go to Step 1.

Step 1 Solve (P3). Denote the optimal solution and the corresponding objective value as $\mathbf{x}^t$ and A^t, respectively. Set $LB = A^t$. Go to Step 2.

Step 2 Solve (P2) with first-stage decisions fixed as $\mathbf{x}^t$. Denote by $\left(d_i^t, \lambda_{1,i}^t, \lambda_{2,jk}^t, \lambda_{3,jk}^t\right)$ the optimal solution. Set

$$UB = \min\left\{UB,\quad \sum_{i\in I}\tilde{d}_i^t\lambda_{1,i}^t + \sum_{j\in J}\sum_{k\in K_J}\left(LS_{jk}x_{jk}^t\lambda_{2,jk}^t - US_{jk}x_{jk}^t\lambda_{3,jk}^t\right)\right\}.$$

If $LB = UB$, stop the algorithm and $\mathbf{x}^t$ is the optimal first-stage solution. Otherwise, add t to the index set Ω, let $t = t + 1$ and go back to Step 1.

Note that in Step 2 of Algorithm 8.1, we need to solve the bilinear program (P2) which in general can only be solved to local optimum by classical gradient-based methods. However, a local solution to (P2) will underestimate UB and may result in an early termination of Algorithm 8.1 with a less robust solution. Therefore, in order to guarantee the algorithm stops when the true solution to (P3) is found, we have to globally solve (P2) in every iteration of Algorithm 8.1.

Assuming a budget uncertainty set[11] and utilizing its special structure, the bilinear subproblem of a two-stage robust model can be reformulated as a mixed-integer program (MIP) which at a moderate scale can be globally solved by commercial MIP solvers such as CPLEX (Thiele et al., 2009; Gabrel et al., 2011; Remli and Rekik, 2012). However, this approach can not apply to (P2) since in (P2) a general polyhedral uncertainty set is assumed.

[11] Budget uncertainty set is a special case of a polyhedral uncertainty set. We refer the readers to Bertsimas and Sim (2004) for more details.

Note that $\bar{\Delta}$ has $2|I|$ extreme points which can be explicitly expressed as:

$$\left.\begin{array}{ll} \text{for } m=1,\ldots,|I|, & \tilde{d}_i^m = \begin{cases} |I|\mu + \Gamma\sigma\sqrt{|I|}, & i=m \\ 0, & i \neq m \end{cases} \\ \text{for } m=|I|+1,\ldots,2|I|, & \tilde{d}_i^m = \begin{cases} \max\left(|I|\mu - \Gamma\sigma\sqrt{|I|}, 0\right), & i=m-|I| \\ 0, & i \neq m-|I| \end{cases} \end{array}\right\} \tag{8.19}$$

Inspired by Proposition 8.2 and equation (8.19), we can reformulate (P2) as

$$\text{(P5)} \quad f_2(\mathbf{x}) = \max_{m=1,\ldots,2|I|}\left[\sum_{i\in I}\tilde{d}_i^m\lambda_{1,i} + \sum_{j\in J}\sum_{k\in K_J}\left(LS_{jk}x_{jk}\lambda_{2,jk} - US_{jk}x_{jk}\lambda_{3,jk}\right)\right] \tag{8.20}$$

s.t. constraints (8.3)-(8.6)

(P5) can be solved by the following algorithm.

Algorithm 8.2 Extreme Point Enumeration

Step 0 Set $z_0 = -\infty$, $count = 0$, $\tilde{d}_i^* = 0, \lambda_{1,i}^* = 0, i \in I$, $\lambda_{2,jk}^*, \lambda_{3,jk}^* = 0, j \in J, k \in K_j$. Go to Step 1.

Step 1 For $m = 1,\ldots,2|I|$, evaluate $\tilde{d}_i^m$ using equation (8.19), then solve

$$\text{(P6)} \quad \max\left[\sum_{i\in I}\tilde{d}_i^m\lambda_{1,i} + \sum_{j\in J}\sum_{k\in K_J}\left(LS_{jk}x_{jk}\lambda_{2,jk} - US_{jk}x_{jk}\lambda_{3,jk}\right)\right] \tag{8.21}$$

s.t. constraints (8.3)-(8.6)

Denote the objective value by z and the solution by $\left(\lambda_{1,i}^m, \lambda_{2,jk}^m, \lambda_{3,jk}^m\right)$. If $z \geq z_0$, let

$\left(d_i^*, \lambda_{1,i}^*, \lambda_{2,jk}^*, \lambda_{3,jk}^*\right) = \left(d_i^m, \lambda_{1,i}^m, \lambda_{2,jk}^m, \lambda_{3,jk}^m\right)$.

Step 2 Output $\left(d_i^*, \lambda_{1,i}^*, \lambda_{2,jk}^*, \lambda_{3,jk}^*\right)$ as the optimal solution to (P2).

By Algorithm 8.2, instead of solving the bilinear program (P2) at Step 2 of Algorithm 8.1, we actually solve $2|I|$ linear programs (P6), which guarantees obtaining a global optimal solution in polynomial time. Given that Λ is also a polyhedron and thus contains a finite number of extreme points, we have the following proposition.

Proposition 8.3 If (P3) can be globally solved at Step 1 in each iteration of Algorithm 8.1, RO-WDP can be globally solved by Algorithm 8.1 embedded with Algorithm 8.2 in a finite number of iterations.

Due to equation (8.19), we can also reformulate (P4) as:

(P7) $\min B$ (8.22)

s.t. $B \geq \sum_{j\in J}\sum_{k\in K_j}\left(c_{jk}y_{jk}^m + p_{jk}z_{jk}^m\right) + \sum_{i\in I} e_i\varphi_i^m, \quad m = 1,\ldots,2|I|$ (8.23)

$\sum_{j\in J}\sum_{k\in K_j} a_{ijk}y_{jk}^m + \varphi_i^m = \tilde{d}_i^m, \quad i\in I, m = 1,\ldots,2|I|$ (8.24)

$LS_{jk}x_{jk} \leq y_{jk}^m + z_{jk}^m, \quad j\in J, k\in K_j, m = 1,\ldots,2|I|$ (8.25)

$US_{jk}x_{jk} \geq y_{jk}^m, \quad j\in J, k\in K_j, m = 1,\ldots,2|I|$ (8.26)

$y_{jk}^m, z_{jk}^m, \varphi_i^m \geq 0 \quad i\in I, j\in J, k\in K_j, m = 1,\ldots,2|I|$ (8.27)

constraints (6.3), (6.17)-(6.19)

where the value of $\tilde{d}_i^m$ is determined by equation (8.19). Note that (P7) is a single-level mixed-integer program, a moderately sized instance of which can be solved by commercial MIP solvers.

Now, we have two methods to solve RO-WDP: (a) applying a Benders' decomposition type constraint generation algorithm embedded with Extreme Point Enumeration; and (b) solving an equivalent MIP reformulation of RO-WDP. For simplicity, in the rest of this paper, we call method (a) constraint generation (CG) and method (b) solving reformulation (SR). Both methods involve solving mixed-integer programs and can converge to a global optimum in a finite number of iterations if those mixed-integer programs can be globally solved. We will compare the performance of these two methods in next section.

8.4 Numerical analyses

In this section, we conduct numerical experiments to demonstrate the tractability of our proposed robust winner determination model and test the quality of the robust solution. Specifically, in Section 8.4.1, we compare the efficiency of the two solution methods, CG and SR, developed in the previous section. In Section 8.4.2, we compare numerical tractability of the robust WDP (P7) and c2-WDP. In addition, we demonstrate the quality of the robust solution by Monte Carlo simulation.

All the tests are coded in GAMS 23.6.5 and solved using CPLEX 12.4 solver. We run the tests on Penn State Lion-XC system.

8.4.1 Comparison of CG and SR

Both CG and SR can converge to a global solution if the mixed-integer programs involved are globally solved. So we are interested only in comparing the numerical tractability of these two methods. Numerical tests are conducted on WDPs at five different scales in [number of lanes, number of bidders, number of packages submitted per bidder]: [10, 5, 5], [30, 6, 10], [60, 10, 10], [120, 15, 10] and [180, 20, 10]. In each scenario, the bids submitted by each bidder, which are reflected by the value of a_{ijk}, are randomly generated. The unit shipping cost quoted on each package (c_{jk}) is evaluated by multiplying the number of lanes included in the package by a random number in [0, 1/(total number of lanes in the auction)] so that the average unit shipping cost on every single lane has the same distribution. Penalty cost p_{jk} is set to double the corresponding unit shipping cost. The unit extra service costs e_i are also randomly generated so that on a lane the average extra service cost is 10/(total number of lanes in the auction) more than the average unit shipping cost. We assume that the volume of shipments on each lane $\tilde{d}_i$ is i.i.d. with mean 100 and variance 100. The lower bound and upper bound on acceptable shipment volume proposed by each carrier on each bid, LS_{jk} and US_{jk}, are uniformly distributed in [25, 75] and [125, 175], respectively. The minimum and maximum number of winners of the auction are 2/5 and 4/5 of the total number of bidders, respectively[12].

The result is summarized in Table 8.1.

Table 8.1 Comparison of CG and SR in tractability

	10		30		60		120		180	
	gap	time	gap	time	gap	time	gap	time	gap	time
CG	0.0%	34.28	0.0%	5218.80	3.0%	NA	12.3%	NA	9.2%	NA
SR	0.0%	0.18	0.0%	0.15	0.0%	0.66	0.0%	3.80	0.0%	12.38

In Table 8.1, “gap” denotes the relative gap between the best possible objective value and the objective value obtained when the solver terminates. In column “time”, the number refers to the computation time which is counted in seconds. “NA” indicates that the solver terminates because of running out of memory. Given Table 8.1, it is apparent that SR is much more efficient than CG in solving the two-stage robust winner determination model. For instance, CG can not solve a problem with more than 60 lanes while SR only uses 12.38 seconds to solve the 180-lane problem to its global optimum. It takes CG about 1.5 hours to solve the 30-lane problem while SR solves the same problem in 0.15 seconds. Therefore, in the following tests we adopt SR as the solution approach to the proposed two-stage robust winner determination model.

[12] If the number is not an integer, we will round it to the smallest following integer.

8.4.2 Comparison with deterministic solution

Given mean and variance of the uncertain shipment volume, an alternative solution to the WDP is to solve a deterministic model c2-WDP using the mean as an estimate for shipment volume. We compare our proposed two-stage robust optimization approach with this deterministic approach in this section. For simplicity, in the rest of this section, we denote the former approach by "RO-SR" and the later one by "DS".

8.4.2.1 Tractability

We first compare the tractability of RO-SR and DS. Again, we test on the same problems discussed in Section 8.4.1. The result is summarized in Table 8.2.

Table 8.2 Comparison of DS and RO-SR in terms of tractability

	10		30		60		120		180	
	gap	time	gap	time	gap	time	gap	time	gap	time
DS	0.0%	0.19	0.0%	1.06	0.0%	16.82	0.0%	5755.01	13.2%	NA
RO-SR	0.0%	0.18	0.0%	0.15	0.0%	0.66	0.0%	3.80	0.0%	12.38

From Table 8.2, we can see that both RO-SR and DS can solve the smallest 10-lane problem in around 0.18 seconds. But as the problem scale increases, the computation time required by DS increases dramatically and more significantly than that required by RO-SR. To solve the 120-lane problem, the time required by DS and RO-SR are 5755.01 seconds and 3.80 seconds, respectively. DS can not solve the 180-lane problem due to a lack of memory while RO-SR can solve the problem in 12.38 seconds. To sum up, RO-SR is a much more efficient method than DS. This is a surprising result because although RO-SR and DS both solve single-level mixed-integer programs, the one solved by RO-SR, which is (P7), has more demand consumption constraints and second-stage decision variables than that solved by DS, which is c2-WDP. We conjecture this is because most demand consumption constraints in (P7) are redundant due to zero right-hand-sides and (P7) is more sparse than c2-WDP.

8.4.2.2 Solution quality

It is widely accepted that RO is a method which provides optimal solutions that are immune to uncertainty. However, RO is also recognized as a conservative method because it focuses on optimizing the objective value only in the worst-case scenario. Moreover, in our two-stage robust winner determination model, the uncertainty set constructed using mean and variance of the shipment volume is a general polyhedral set. Thus, intuitively, our model tends to provide conservative solutions. In this section, we test the quality of solutions derived by RO-SR using Monte Carlo simulation and compare it with the solution derived by DS.

The experiments are designed as follows. Given the mean and variance of the volume of shipments on each lane, we solve the WDP by DS and RO-SR and we get two different solutions for the first-stage decision variables. Then we generate 100 random volumes of shipments on each lane and obtain 100 simulated objective values (procurement costs) by solving the second-stage of WDP. The average of 100 simulated objective values is used to compare the quality of solution derived by DS and RO-SR.

The experiments are conducted on the 60-lane problem which is introduced in Section 8.4.1. The mean of uncertain shipment volume is again assumed to be 100. We test the quality of two solutions at different levels of variance. We assume Gamma distribution for the simulation. The results when the variance is small (600) and large (9000) are summarized in Table 8.3.

Table 8.3 Comparison of DS and RO-SR in terms of solution quality

variance	DS	RO-SR	Improvement
600	930.16	1210.36	-30.1%
9000	1485.28	1239.95	16.5%

According to Table 8.3, when the variance is small, DS is a better approach than RO-SR since the average simulated cost associated with DS is less. Indeed, RO-SR is conservative compared to DS when the variance is small. However, when the variance is large, RO-SR provides a better solution than DS. As shown by Figure 8.1, as the variance of the uncertainty increases, the average simulated cost associated with DS solution increases significantly while that associated with RO-SR solution does not change significantly with the variance. As a conclusion, the RO-SR solution is more robust to uncertainty and yield lower procurement cost than DS solution when the variance of the uncertain shipment volume is large.

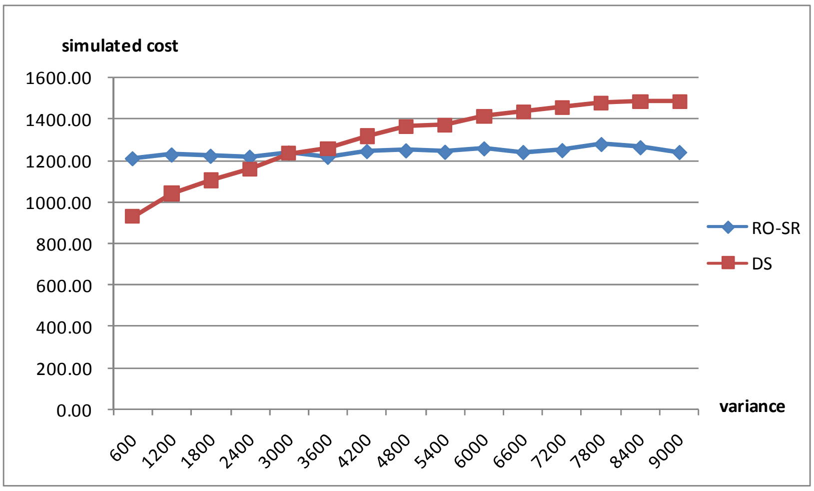

Figure 8.1 Simulated result at different levels of variance

Chapter 9

Conclusion

In this dissertation, we investigate dynamic game-theoretic models and robust models for two specific engineering and management topics in transportation and service networks: urban freight transportation and truckload service procurement. Specifically, we want to reduce the congestion associated with freight activities on urban road networks and reduce shipper's cost associated with TL service procurement under uncertainty. We use extensive numerical experiments to demonstrate that our proposed models successfully resolve the problems and help achieve our goal. The results can provide insights into the planning and regulation of urban freight transportation and the operations and management of truckload service procurement. We conclude our research on urban freight transportation planning and truckload service procurement in details in Section 9.1 and Section 9.2, respectively

9.1 Urban freight transportation planning

In this dissertation we model urban freight transportation planning by dynamic Stackelberg games. Specifically, the problems can be formulated as bi-level dynamic mathematical programs with equilibrium constraints (MPECs). The lower level of both models is a dynamic user equilibrium (DUE) that describes the dynamic assignment of private automobile traffic. The modeling of upper level problem depends on who controls the movement of freight trucks. If it is a metropolitan planning organization (MPO), we call the associated dynamic MPEC "Model I" and its upper level is an optimal freight traffic control problem that minimizes the total delay experienced by all travelers while satisfying the freight transportation demand. If it is a union of truck companies (UTC), we call the associated dynamic MPEC "Model II" and its upper level is an optimal freight traffic control problem that minimizes the total delay experienced by all freight trucks while satisfying the freight transportation demand. Both models can provide the MPO insights into urban freight transportation regulation.

To model the interaction between freight trucks and private automobiles on an urban road network, we refine the link delay model (LDM) based dynamic network loading (DNL). In particular, we create traffic dynamics and flow propagation constraints for both freight trucks and private automobiles and integrate them into a system of differential algebraic equations (DAEs). We prove that there exists a unique solution to the DAE under some regularity conditions. Moreover, we demonstrate that the new DNL model preserve first-in-first-out (FIFO) rule under mild assumptions.

We apply simulated annealing (SA) algorithm, which is embedded with a fixed point algorithm for solving DUE, to solve Model I. Numerical tests are conducted to derive optimal urban freight transportation plan. Results show that the solution to Model I can significantly reduce the total congestion associated with freight transportation on an urban road network. Our model can serve as a decision support tool for an MPO to regulate urban freight activities. For solving Model II, we develop a gradient-based heuristic. Numerical results show that the interaction between freight and personal transportation exists and is nonnegligible even when the amount of freight trucks compared to that of private automobiles is small. Moreover, by conducting extensive numerical tests we find a dynamic Braess-like paradox when solving Model II. It implies that the MPO may increase social welfare by proper urban freight transportation regulations such as restricting freight trucks from entering specific sections of the network during the peak hours of a day. At the same time, since the restriction may significantly increase the truck companies' cost, the MPO could compensate the truck companies in order to smooth the implementation of the restriction.

Some interesting future research directions can base on the relaxation of assumptions made in this dissertation. In the current models, we assume that travel demand for both freight trucks and private automobiles are fixed and independent of congestion on the road network. It will be interesting to investigate the problem with elastic travel demand. What is more, we consider only two types of traffic in both models: one is controlled by a central planner and the other is described by DUE. Extending the models to account for more types of traffic or road users is worth investigating. A mixed DUE is needed to describe the dynamic assignment of this more inhomogeneous traffic. In fact, the derivation of a mixed DUE itself can be a significant contribution to DUE literature. Moreover, although the assumption that the MPO is capable of controlling the movement of all freight trucks on the urban road network is relaxed in Model II, we still consider that only one agent (the UTC) controls the movement of all freight trucks. In practice, the freight trucks belong to several truck companies who can schedule their own freight transportation and compete for service. In this case, we can replace the UTC by multiple truck companies and introduce them as a new group of players to the current game. As a result, the optimization problem for the MPO becomes tri-level or even quad-level. Lastly, besides relaxing the assumptions made in this study, we can extend the study by robustifying the problem: we can take into account the travel demand uncertainty. In addition, we can replace the upper level problem in the current model by a service network design problem which can directly derive implementable truck schedules on an urban road network.

9.2 Truckload service procurement under shipment volume uncertainty

In this dissertation we propose two robust winner determination models for truckload service procurement under shipment volume uncertainty. Specifically, assuming that the full distributional information (i.e., probability density function) of the uncertain shipment volume is available, we propose a sampling-based two-stage stochastic winner determination model. We propose a solution method called Monte Carlo Approximation (MCA) which can solve moderately sized instances of winner determination problem (WDP) under uncertainty in a reasonable time via a commercial mixed-integer programming (MIP) solver, CPLEX. We use Monte Carlo simulation to demonstrate that the MCA solution yields a lower procurement cost than the nominal solution. The numerical results also reveal that MCA performs particularly well when the uncertainty can not be well-characterized by its mean. We conclude that the proposed stochastic winner determination model is a practical tool due to its numerical tractability and its capability of constraining procurement cost under uncertainty.

When only partial distributional information (i.e., mean and variance) of the uncertain shipment volume is available, stochastic programming may not be applicable to solve the associated WDP. In this case, we construct a general polyhedral uncertainty set utilizing mean and variance of the uncertainty and formulate a two-stage robust winner determination model. Although robust optimization is recognized as a conservative method which provides solution only optimal in the worst-case scenario, we use simulation-based numerical tests to show that the solution to our robust WDP yields low procurement cost in general scenarios too, especially when the variance of the uncertainty is large. In particular, the simulated procurement cost associated with the robust solution does not change significantly with the variance of the shipment volume uncertainty, while the cost associated with the deterministic solution increases dramatically as the uncertain shipment volume becomes more variant. Although WDP is an NP-hard problem and is challenging to solve even without uncertainty, we develop a solutiom method which is more efficent than the widely adopted Benders' type constraint generation algorithm. Moreoever, we demonstrate that our robust WDP has better numerical tractablity than a deterministic WDP, especially when the problem scale is large. We conjecture this is because the reformulation of the robust WDP is more sparse than a deterministic WDP. Because of the excellent numerical tractability, the proposed two-stage robust winner determination model is applicable to truckload service procurement in practice.

Besides developing robust models, we also comprehensively discuss the deterministic MIP formulations of WDP for TL service procurement and illustrate how a variety of business considerations can be modeled as side constraints. In particular, we propose a refined formulation in which shortage in shipments and the associated penalty cost are explicitly modeled. We demonstrate that the shipment volume uncertainty will not affect the feasibility of the refined model which is thus more general and more feasible under uncertainty than the benchmarks. Moreover, by analyzing the problem feasibility, we find that some of the shipper's business considerations may conflict with each other. Therefore, the shipper needs to carefully evaluate these constraints and properly impose them on the auction.

An interesting topic for future research is to better balance tractability and solution quality. On one hand, when we solve the stochastic WDP, we sacrifice the solution quality for computational efficiency: we only use a few small samples in MCA. In the future study, we can try more larger samples and investigate the optimal design of samples for MCA. On the other hand, when we solve the robust WDP, although the proposed model is more numerically tractable, it is conservative when the variance of uncertainty is small. Assuming more knowledge of the uncertainty will reduce the conservativeness of the robust model. However, its impact on problem tractability remains to be investigated. Moreover, since both stochastic and robust winner determination models finally reduce to an MIP problem, we can try to integrate more efficient MIP algorithms involving decompostion or relaxation into the numerical solution methods of WDP under uncertainty. Lastly, incorporating game-theoretic behaviors of shippers and carriers in truckload service procurement under uncertainty is another interesting research direction. In particular, new bidding structures for the auction or new mechanisms which can further reduce shippers' truckload transportation procurement cost is worth investigating.

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