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TIME-DEPENDENT DAMAGE BASED TENSION STIFFENING IN REINFORCED CONCRETE

A Thesis in

Civil Engineering

by

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ABSTRACT

Deflection control is an important criterion for design of reinforced concrete structures. Deflection control becomes critical for lightly reinforced members. Behavior of concrete members at service load levels involves complex interactions of cracking, creep, shrinkage and bond between concrete and embedded reinforcing bars. The creep and shrinkage phenomena are well established but the deterioration of tension stiffening with time is not fully understood. Current tension stiffening models consider short term effects of cracking. The primary focus of this research is to examine the effect of time on tension stiffening.

An analytical model is developed to examine the time-dependent effects of creep, shrinkage and bond on tension stiffening. The model is evaluated by comparison with the available experimental data. The sensitivity of the model to various parameters is examined through parametric study. The results of the research can be extended to flexural members for incorporation in design.
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LIST OF SYMBOLS

\( A_c \) = Area of concrete

\( A_s \) = Area of reinforcing steel bar

\( b \) = Width of beam

\( C(t, \tau) \) = Creep Compliance function at time \( t \), due to stress increment at time \( \tau \)

\( C_u \) = Ultimate Creep Coefficient

\( D \) = Diameter of cylinder

\( d \) = Depth of beam

\( E_c(t) \) = Modulus of elasticity of concrete at time \( t \)

\( E_s \) = Modulus of elasticity of steel reinforcing bar

\( E_c^r(t_{i+1}) \) = Reduced elastic modulus for time step \( t_{i+1} \)

\( f_{bt} \) = Modulus of rupture

\( f_{ct} \) = Direct tensile strength of concrete

\( f'_c(t) \) = Compressive strength of concrete at time \( t \)

\( f_t \) = Maximum tensile stress in concrete

\( f'_t \) = Maximum Tensile strength of concrete

\( f_{ct, eff} \) = Effective tensile strength of concrete after shrinkage effect

\( I_g \) = Moment of Inertia of the uncracked concrete section

\( I_{cr} \) = Moment of Inertia of the cracked concrete section

\( L \) = Length of cylinder

\( M \) = Applied Moment

\( M_a \) = Applied Moment

\( M_{cr} \) = Cracking Moment
\[ n = \text{Modular ratio} \]
\[ P = \text{compressive load at failure} \]
\[ P_{\text{app}} = \text{Applied force} \]
\[ P_c = \text{Force in concrete} \]
\[ P_s = \text{Force in reinforcing steel bar} \]
\[ t = \text{Age of concrete in days} \]
\[ t_c = \text{Age of concrete when drying starts at end of moist curing in days} \]
\[ \bar{y}(t) = \text{Reduction factor for shrinkage strain} \]
\[ \alpha = \text{Strength reduction parameter (obtained from tension stiffening model)} \]
\[ \alpha = \text{Empirical constant (days) for calculating strength of concrete} \]
\[ \beta = \text{Empirical constant for calculating strength of concrete} \]
\[ \beta = \text{Tension stiffening parameter} \]
\[ \beta_1 = \text{Coefficient dependent on bond properties} \]
\[ \beta_2 = \text{Coefficient dependent on loading} \]
\[ \gamma_c = \text{Creep Correction factor for non-standard conditions} \]
\[ \gamma_{\text{LA}} = \text{Loading age correction factor} \]
\[ \gamma_{\text{RH}} = \text{Relative humidity correction factor} \]
\[ \gamma_s = \text{Slump correction factor} \]
\[ \gamma_{\text{sh}} = \text{Adjustment factor for non-standard conditions} \]
\[ \gamma_{\psi} = \text{Volume to surface ratio correction factor} \]
\[ \gamma_a = \text{Air content correction factor} \]
\[ \gamma_{\psi} = \text{Fine aggregate correction factor} \]
\[ \Delta \sigma_c(t_j) = \text{Applied stress increment at time } t_j \]
\[ \varepsilon_c(t_i) = \text{Elastic and creep strain in concrete at time } t_i \]
\[ \varepsilon_{cr}(t_i) = \text{Creep strain in concrete at time } t_i \]
\[ \varepsilon_e(t_i) = \text{Total elastic strain in concrete at time } t_i \]
\[ \varepsilon_l(t_i) = \text{Total inelastic strain in concrete at time } t_i \]
\[ \varepsilon_{sc} = \text{Strain in steel in cracked phase} \]
\[ \varepsilon_{sh} = \text{Free shrinkage strain prior to cracking} \]
\[ \varepsilon_{sh}(t) = \text{Shrinkage strain at time } t \]
\[ \varepsilon_{sh,\text{eff}}(t) = \text{Effective shrinkage strain at time } t \]
\[ \varepsilon_{shu} = \text{Ultimate shrinkage strain} \]
\[ \varepsilon_{suc} = \text{Strain in steel in uncracked phase} \]
\[ \varepsilon_T = \text{Total strain at time } t \]
\[ \varepsilon_t' = \text{Cracking strain} \]
\[ \varepsilon_t(t_i) = \text{Total strain in concrete at time } t_i \]
\[ \varepsilon_a(t_i) = \text{Total temperature strain in concrete at time } t \]
\[ \sigma_s(t_i) = \text{Stress in steel at time } t_i \]
\[ \lambda_d = \text{Long time multiplier} \]
\[ \xi = \text{Time dependent factor for sustained load} \]
\[ \rho = \text{Reinforcement ratio} \]
\[ \rho' = \text{Compression reinforcement ratio} \]
\[ \sigma_c(t_i) = \text{Stress in concrete at time } t_i \]
\[ \sigma_{ct,sh} = \text{Tensile stress developed in concrete due to shrinkage restraint} \]
\( \phi_1 \) = Curvature at the uncracked phase

\( \phi_2 \) = Curvature at the cracked phase

\( \varphi(t, \tau) \) = Creep coefficient at time \( t \) due to load at time \( \tau \)

\( \tau \) = Initial time of loading in days
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Chapter 1

Introduction

1.1 Background

Deflection control is an important criterion for design of reinforced concrete members. Deflection control considerations largely dominate lightly reinforced members. With the use of high strength material, construction of thinner slabs with longer spans is made possible leading to need of a reliable calculation of deflection. Assessment of effect of tension stiffening is important for calculating deflection in lightly reinforced members.

The term “tension stiffening” refers to the contribution of concrete between cracks to the overall stiffness of a member due to bond between concrete and steel reinforcement. The primary focus of this research is to examine the effect of time on tension stiffening. Current tension stiffening models consider short term effects of cracking. However, concrete in the tensile zone between cracks is subjected to creep and shrinkage effects as well as breakdown of bond over time. An analytical model is developed including these effects and results are compared with available experimental data.
1.2 Problem Statement

An analytical model is needed to examine the time-dependent effects of creep, shrinkage, and bond degradation on tension stiffening. Such a model is useful in understanding the behavior of tension zones in reinforced concrete members when subjected to long term sustained loads.

1.3 Objective & Scope

The objective of the research was to develop an analytical model for time dependent deformations in reinforced concrete members considering the combined effect of creep, shrinkage and tension stiffening.

The following tasks were conducted to achieve the objective:

i. Literature review was conducted on tension stiffening models.

ii. A method of analysis was developed to calculate time dependent deformations in cracked tension zones of reinforced concrete members. The analysis procedure includes effects of creep and shrinkage in the tension zone. The tension stiffening model for short term loading was extended to include sustained load effects.

iii. Analytical results were validated using available experimental data.

iv. A parametric study was conducted to determine the effect of different conditions on time dependent deformation.
v. Recommendations are made for additional research required to extend the model to flexural members and provide a theoretical basis for long term multipliers for deflection control considering the time dependent nature of tension stiffening.
Chapter 2

Literature Review

The proposed research builds on the recent work by Karschner (2012) who studied the problem by considering the limiting cases of fully cracked section and uncracked section. A similar time-stepping procedure was employed for the case between the two limits where the tension stiffening effect was incorporated in the analysis procedure in terms of a stress-strain diagram for concrete with a descending branch after cracking is initiated. The following is a brief review of relevant literature.

2.1 Time dependent deformation

A great deal of research has been conducted on creep and shrinkage of concrete. ACI Committee 209 has reported on the work that has been done and provides descriptions of available models for creep and shrinkage. The subsequent paragraphs present a brief summary of one of the simpler models recommended by ACI 209. The general approach to time dependent analysis proposed in this study can be modified if desired to accommodate other models.
2.1.1 Creep Function

When concrete is loaded, it deforms instantaneously. If the load is constant over a period of time, concrete members develop creep strains. Creep strains tend to increase the deflection with time even when the loading is constant. Creep strains are usually one to three times the instantaneous elastic strains and therefore, cannot be neglected.

Creep consists of basic creep and drying creep. During basic creep, there is no exchange of water between concrete and its surroundings. Drying creep is the additional creep that occurs due to drying of concrete. For calculation of long time deflection, both of these components are summed together as total creep. Creep is affected by strength of concrete, humidity, temperature, dimension of member and composition of mix. Concrete with high cement-paste content creeps more than concrete with large aggregate content.

ACI 209 creep function is used for the purpose of this thesis. The ACI 209 creep function is a simple function and contains parameters like compressive strength of concrete and age of loading that can be easily defined at the design stage. Moreover, this method is most commonly used in United States. The creep coefficient is the ratio of creep strain at a very long time to elastic strain. Under standard conditions, creep coefficient is given by:

$$\varphi (t, \tau) = \frac{(t - \tau)^{0.06}}{10 + (t - \tau)^{0.06}} C_u \gamma_c$$  \hspace{1cm} (2-1)

where:

$\varphi (t, \tau) = \text{creep coefficient at time } t \text{ due to load at time } \tau$;

$C_u = \text{ultimate creep coefficient}$;

$t = \text{age of concrete in days}$;
\( \tau \) = initial time of loading in days; and

\( Y_c \) = creep correction factor for non-standard conditions.

\[
Y_c = Y_{LA} Y_{RH} Y_{vs} Y_s Y_\phi Y_\alpha
\]  

(2-2)

where:

\( Y_{LA} \) = loading age correction factor;

\( Y_{RH} \) = relative humidity correction factor;

\( Y_{vs} \) = volume to surface ratio correction factor;

\( Y_s \) = slump correction factor;

\( Y_\phi \) = fine aggregate correction factor; and

\( Y_\alpha \) = air content correction factor.

For the purpose of this research, standard conditions have been considered and correction factors have been taken as 1.0. The creep compliance function given below expresses the total elastic and creep strain at time \( t \) due to the unit stress at time \( \tau \).

\[
C(t, \tau) = \frac{1}{E_c(\tau)} + \frac{\varphi(t, \tau)}{E_c(\tau)}
\]  

(2-3)

where:

\( C(t, \tau) \) = creep Compliance function at time \( t \), due to stress increment at time \( \tau \);

\( E_c(\tau) \) = elastic Modulus of concrete at time \( \tau \); and

\( \varphi(t, \tau) \) = creep function at time \( t \), due to stress increment at time \( \tau \).
2.1.2 Shrinkage Function

Shrinkage is the decrease in volume of the concrete with time. Shrinkage occurs because of the change in moisture content of the concrete and physio-chemical changes. Shrinkage consists of drying shrinkage, autogenous shrinkage, and carbonation shrinkage. Drying shrinkage is due to loss of moisture in concrete. Hydration of cement causes autogenous shrinkage. Carbonation shrinkage occurs when Carbon dioxide reacts in the presence of water with hydrated cement leading to formation of carbonated products. ACI 209 recommends following equations to calculate the shrinkage strain.

\[
\varepsilon_{sh}(t) = \frac{(t - t_c)}{35 + (t - t_c)} \cdot Y_{sh} \varepsilon_{shu}
\]

where:

\( \varepsilon_{sh}(t) \) = shrinkage strain at time \( t \);

\( \varepsilon_{shu} \) = ultimate shrinkage strain;

\( t \) = age of concrete in days;

\( t_c \) = age of concrete when drying starts at end of moist curing in days; and

\( Y_{sh} \) = adjustment factor for non-standard conditions.

\[
Y_{sh} = Y_{cp} Y_{RH} Y_{vs} Y_s Y_{\psi} Y_{\alpha}
\]

where:

\( Y_{cp} \) = moist curing correction factor;

\( Y_{RH} \) = relative humidity correction factor;

\( Y_{vs} \) = volume to surface ratio correction factor;

\( Y_s \) = slump correction factor;
\gamma_{\phi} = \text{fine aggregate correction factor}; \text{ and } 
\gamma_{\alpha} = \text{air content correction factor.}

2.2 Tension Stiffening Models

The flexural strength of concrete is calculated assuming that concrete carries only the compressive forces and the reinforcement carries the tensile forces. However, concrete has tensile strength which is roughly about one-tenth of the compressive strength. When the tensile load in concrete exceeds its tensile strength, cracks are formed. Concrete carries no tension at the crack location. As we move away from the crack, concrete continues to carry tensile forces. This phenomenon is known as tension stiffening. Due to tension stiffening concrete is able to contribute to stiffness even after cracking had occurred. As some of the tensile forces are carried by concrete, this results in reduced stress in reinforcement which in turn reduces the deformation in steel reinforcement. Thus, tension stiffening results in reduced deformation in the member. The member is stiffer than the case when it is assumed that concrete carries no tension. The stiffness of the member affects the deflection of the reinforced concrete member. The tension stiffening effect is shown in figure 2-1.

For a heavily reinforced member, tension stiffening is not prominent. However, for a lightly reinforced member like slabs, tension stiffening may have a significant effect. Neglecting tension stiffening effect results in over-estimation of deflection. Experiments have shown that at service level loads, contribution of tension stiffening is more than 50% of the instantaneous stiffness for lightly reinforce slabs (Gilbert, 2007).
Various models have been proposed to describe tension stiffening effect. Codes use interpolation functions to evaluate the response between cracked and uncracked section. The tension stiffening effect can also be modeled at the stress-strain level by using a descending stress-strain relationship after the tensile stress reaches the tensile strength. This approach was first introduced by Scanlon (1971), and Scanlon and Murray (1974). Since then many researchers have proposed similar models with varying treatment of the descending portion of the stress-strain diagram. Some of the tension stiffening models are described in the following paragraphs:
2.2.1 ACI 318

ACI 318 (2011) uses the effective moment of inertia approach proposed by Branson (1965) for flexural deformations. It is assumed that stiffness decreases with increase in cracks in the member. Hence, moment of inertia is assumed to be decreasing depending on the ratio of cracking moment and applied moment. The effective moment of inertia employs an interpolation function between the uncracked section and fully cracked section as shown in Figure 2-2. The flexural stiffness of the section is taken as the product of elastic modulus and moment of inertia of the section. For the uncracked section the moment of inertia is gross moment of the section. As moment exceeds the cracking moment, section cracks. After cracking has occurred, moment of inertia decreases, decreasing stiffness of the section. For a beam section, the moment changes along the length. However, for deflection calculations, average value of stiffness is used. ACI 318 uses effective moment of inertia which is calculated by the equation (2-6).

\[
I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr}
\]  

(2-6)

where:

\[ M_{cr} = \text{cracking Moment}; \]
\[ M_a = \text{applied Moment}; \]
\[ I_{cr} = \text{moment of Inertia of the cracked concrete section}; \] and
\[ I_g = \text{moment of Inertia of the uncracked concrete section}. \]

The response of concrete between the uncracked and the cracked conditions are determined by equation (2-6). For loads lower than the cracking moment, \( I_e \) is closer to
gross moment of inertia. The effective moment of inertia approaches \( I_{cr} \) with the increase in load. \( I_{cr} \) defines the lower limit of \( I_e \).

![Diagram](Figure 2-2. ACI 318 Model (Branson, 1965))

2.2.2 CEB MODEL

The Eurocode 2 (CEN-2004) employs an interpolation function similar to ACI 318 between deflection of the uncracked section and deflection calculated using the fully cracked section. It interpolates the curvature between the uncracked and the cracked phase (figure 2-3). A modification factor is applied for sustained loads. The transition from the uncracked section to the cracked section is employed by using the coefficient \( \zeta \). \( \zeta \) is given by equation (2-7)
\[ \zeta = 1 - \beta_1 \beta_2 \left( \frac{M_{cr}}{M} \right)^2 \]  
(2-7)

where:

\( \beta_1 \) = coefficient dependent on bond properties;
\( \beta_2 \) = coefficient dependent on loading;
\( M_{cr} \) = cracking Moment; and
\( M \) = applied Moment.

Average strain, \( \varepsilon_{sm} \) can be calculated by equation (2-8)
\[ \varepsilon_{sm} = (1 - \zeta) \varepsilon_{su} + \varepsilon_{sc} \]  
(2-8)

where:

\( \varepsilon_{su} \) = strain in steel in uncracked phase; and
\( \varepsilon_{sc} \) = strain in steel in cracked phase.

The curvature \( \phi_m \) is also calculated in the similar manner.
\[ \phi_m = (1 - \zeta) \phi_1 + \phi_2 \]  
(2-9)

where:

\( \phi_1 \) = curvature at the uncracked phase; and
\( \phi_2 \) = curvature at the cracked phase.
2.2.3 Scanlon and Murray Model

The concept of descending branch of stress-strain curve was first introduced by Scanlon (1971) and Scanlon and Murray (1974). This concept is simple yet powerful, hence it has been used extensively since its inception. Scanlon and Murray suggested that there is a gradual decrease in concrete stress due to formation of cracks. Further, it is suggested that the gradual decrease in stress can be modeled as stepwise reduction in stress. Figure 2-4 gives the details of the model.
2.2.4 Lin & Scordelis Model

Lin & Scordelis (1975) have described the tension stiffening effect by the linear descending branch of the stress-strain curve. The cracking occurs at the load $f_{ct}$. The strain value at that point is $\varepsilon_{ct}$. After the section has cracked, the tensile resistance decreases till the strain value is $a\varepsilon_{ct}$. When the strain in the section reaches $a\varepsilon_{ct}$, the tensile resistance of concrete is assumed to be zero. The figure 2-5 shows Lin & Scordelis Model.
2.2.5 Damjanic & Owen model

The Damjanic & Owen model (1984) is similar to the Lin & Scordelis Model. However, there is a sudden drop in the stress when the cracking occurs. The model proposed by Damjanic and Owen has been shown to provide good correlation with test results and has been extended to include time-dependent effects in the present study. The figure 2-6 shows the variation of stress and strain as suggested by Damjanic and Owen.
After cracking, maximum tensile stress in concrete at strain $\varepsilon_T$ is given by (2-10)

$$f_t = \alpha f'_t - \frac{\alpha E_c}{\beta - 1} (\varepsilon_T - \varepsilon'_t)$$

(2-10)

where:

- $f_t$ = maximum tensile stress in concrete at strain $\varepsilon_T$;
- $\alpha$ = strength reduction factor;
- $\beta$ = tension stiffening parameter;
- $f'_t$ = tensile strength of concrete; and
- $\varepsilon'_t$ = Cracking Strain.
2.3 Tensile Strength of Concrete

Tensile strength of concrete is very low as compared to the compressive strength. It falls between 8% to 15% of the compressive strength (MacGregor and Wight, 2009). This makes concrete vulnerable to cracking. The tensile strength of concrete is determined using indirect test methods. Both the tests give higher tensile value than the uniaxial tensile strength.

2.3.1 Split Cylinder Test (ASTM C 496)

In this test a standard 6 in. x 12 in. test cylinder is placed horizontally between the loading surface of compression testing machine. The load is applied diametrically and uniformly along the length of the cylinder. Tensile stress is generated due to Poisson's effect and concrete cylinder splits into two halves along the vertical diameter. The tensile strength of concrete can be calculated by equation (2-11)

\[
f_t = \frac{2P}{\pi DL}
\]  

(2-11)

where:

\( f_t \) = tensile strength of concrete;

\( P \) = compressive load at failure;

\( L \) = length of cylinder; and

\( D \) = diameter of cylinder.
2.3.2 Flexure Test (ASTM C78 or C293)

In this test, a simple concrete beam of size 6 in. x 6 in. x 30 in. is loaded at one-third span points using the equal load. The load is increased till the point fracture occurs within middle one-third portion of the beam. The middle one-third portion of the beam is subjected to pure bending. The maximum tensile strength determined in this case is called modulus of rupture and is calculated by the formula (2-12)

\[ f_{bt} = \frac{6M}{bd^2} \]  \hspace{1cm} (2-12)

where:

- \( f_{bt} \) = modulus of rupture;
- \( M \) = moment;
- \( d \) = depth of beam; and
- \( b \) = width of beam.

2.4 Decay of tension stiffening with time

The concrete stress reduces to a constant level in a relatively short time. The results from Beeby & Scott (2005) have shown that this reduction in the tensile stress can occur from six hours to thirty days. The specimen with higher reinforcement ratio has shorter decay time. The mechanisms that influence the decay of tension stiffening are:

i. Creep of Concrete in tension

ii. Shrinkage
iii. Deterioration of bond
   a. Formation of further cracks
   b. Internal failure

2.4.1 Creep in Tension

Even after cracking, concrete surrounding the reinforcement is supporting the
tensile stresses. Tensile force causes the material to expand and water is adsorbed from
the larger capillary pores to gel pores. In drying conditions, the adsorbed water layers are
affected both by evaporation and tensile loading (Bissonnette et al, 2007). This causes
some degree of creep in concrete. Since creep is proportional to applied stress, and
concrete carries low tensile stress, the amount of creep would be low. After cracking, as
the tensile capacity of concrete decreases, creep would further decrease. Hence, the effect
of creep on tension concrete members is small. For the purpose of this study, it is
assumed that tensile creep is same as the compressive creep. It has been shown
experimentally by Bissonnette et al (2007) that parameters like drying conditions and age
of loading have the same effect both in compression and tension. However, tensile creep
is more sensitive to fiber reinforcement and increases with the decrease in paste content,
unlike compressive creep.

Creep causes reduction in the peak stress. The test results indicate that loss of
tension stiffening cannot be attributed entirely to creep (Beeby and Scott 2005).
2.4.2 Shrinkage in Tension

Concrete shrinkage is restrained by reinforcement. Hence, shrinkage induces compressive stress in reinforcement and tensile stress in concrete. If the beam is allowed to shrink before the application of load, initial tensile stress develops in concrete. Due to this initial tensile stress, concrete will tend to crack at lower load than as predicted on the basis of tensile strength. Hence, in this case shrinkage causes reduction in the effective tensile strength of concrete. Beeby and Scott (2006) have suggested following relationship for effects of shrinkage in tension:

\[
f_{ct,\text{eff}} = f_{ct} - \sigma_{ct,sh} = f_{ct} - \frac{E_s \rho \varepsilon_{sh}}{(1 + n\rho)}
\]  

(2-11)

where:

- \(f_{ct,\text{eff}}\) = effective tensile strength of concrete after shrinkage effect;
- \(f_{ct}\) = direct tensile strength of concrete;
- \(\sigma_{ct,sh}\) = tensile stress developed in concrete due to shrinkage restraint;
- \(\varepsilon_{sh}\) = free shrinkage strain prior to cracking;
- \(\rho\) = reinforcement ratio; and
- \(n\) = modular ratio.

If the initial shortening of the member is not accounted, it leads to underestimation of tension stiffening (Bischoff, 2001). If shrinkage develops after application of load in a cracked member, its effect is coupled with bond-slip mechanism and its deterioration (Zanuy, 2010). It is assumed that with cracking the effect of shrinkage diminishes.
Hence, concept of effective shrinkage is used in the model. The procedure is described in figure 2-7. The effective shrinkage is calculated using the equation (2-12).

\[ \varepsilon_{sh\, eff}(t) = \tilde{y}(t) \times \varepsilon_{sh}(t) \quad (2-12) \]

where:

- \( \varepsilon_{sh\, eff}(t) = \) effective shrinkage strain at time \( t \);
- \( \varepsilon_{sh}(t) = \) shrinkage strain at time \( t \); and
- \( \tilde{y}(t) = \) reduction factor for shrinkage strain.

The coefficient \( \tilde{y} \) describes reduction in shrinkage strain. Before cracking occurs, \( \tilde{y} \) is equal to one, indicating that there is no reduction in shrinkage in uncracked concrete. After cracking, \( \tilde{y} \) is interpolated depending on the limiting strain obtained from tension stiffening model.

\[ \tilde{y} = \begin{cases} \frac{\beta \varepsilon_t' - \varepsilon_T}{\beta \varepsilon_t' - \varepsilon_t} & \varepsilon_T \leq \varepsilon_t' \Rightarrow \tilde{y} = 1 \\ \frac{\varepsilon_T - \varepsilon_t}{\varepsilon_T - \varepsilon_t} & \varepsilon_T \geq \beta \varepsilon_t' \Rightarrow \tilde{y} = 0 \end{cases} \quad (2-13) \]

where:

- \( \beta = \) tension stiffening parameter (obtained from tension stiffening model);
- \( \varepsilon_T = \) strain at time \( t \); and
- \( \varepsilon_t' = \) cracking strain.
2.4.3 Bond Deterioration

It is observed that deformation in concrete is more in tension than in compression (Kristiawan, 2006). The main cause of decay in concrete tensile strength is deterioration of bonds. As the stress increases, surface cracks are formed. With time and increase in load, the existing crack lengthens and new cracks are formed. The width of these primary cracks is largest at the surface and very small at the bar-concrete interface. Internal cracking also leads to reduction of stiffness. Reduction in tensile stress was noticed at Durham experiments without any visible surface cracks on the specimen (Beeby and Scott, 2005). This is attributed to formation of internal cracks. These internal cracks
initiate from the ribs of the reinforcement and lead to loss of adhesion along the bar between the ribs. With increase in load, these cracks propagate along the bar until the midpoint between two surface cracks is reached. Hence, reduction in the stress in concrete is also caused by secondary cracks and loss of bond between steel and concrete.
Chapter 3
Method of Analysis

3.1 Development of Algorithm - Axial Prism

A method of analysis is developed for computing time dependent deformation in an axial prism. The algorithm calculates strains and stresses in concrete and steel at each time step. The tensile capacity of concrete, even after cracking, is taken into account.

3.1.1 Assumptions

The algorithm is developed based on the following assumptions:

- plane sections remain plane after deformation;
- cracking is assumed using a "smeared" model with reduced stiffness;
- there is perfect bonding between concrete and reinforcement such that strain compatibility is valid;
- applied stress remains constant throughout the time-step;
- axial prism is uniformly loaded;
- shrinkage is considered as negative in the analysis;
- shrinkage strain is constant throughout the section;
- the concept of effective shrinkage (reduced shrinkage due to cracking) is used;
- creep superposition holds true;
- tensile creep is same as compressive creep; and
- tension stiffening stress-strain model is based on limiting strain due to elastic and creep.

### 3.1.2 Material Properties

The compressive strength and elastic modulus of concrete varies with time. The variation is modeled by ACI 209 function:

\[
f'_{c}(t) = \frac{t}{\alpha + \beta t} (f'_{c})_{28}
\]

\[
E_{c}(t) = 57,000 \sqrt{f'_{c}(t)}
\]

where:

\( f'_{c}(t) \) = compressive strength of concrete at time \( t \) (psi);

\( E_{c}(t) \) = time-dependent elastic modulus of concrete at time \( t \) (psi);

\( \alpha \) = empirical constant (days);

\( \beta \) = empirical constant; and

\( t \) = age of concrete (days).

Yoshitake et al (2012) observed that within the first week of age of concrete, the tensile moduli of concrete are 1.0-1.3 times larger than the compressive moduli of concrete. In this research, tensile modulus is assumed to be same as compressive modulus.
3.1.3 Computation of strain

The total strain at any point in time in the specimen can be subdivided into elastic strain and inelastic strain. The elastic strain is the instantaneous strain which is reversible on the removal of the loading. The inelastic strain is the sum of creep and shrinkage strains.

**Creep Compliance Function**

Creep and elastic strain due to each stress increment is determined using the creep compliance function at every time step. Creep compliance function expresses the total elastic and creep strain due to unit stress. Total strain at any time is calculated by summing all the strain effects. In this case concrete will be subjected to varying stress history and the total elastic and creep strain in concrete can be calculated by the following formula:

\[
\varepsilon_c(t_i) = \Delta\sigma_c(\tau_1)C(t_i, \tau_1) + \Delta\sigma_c(\tau_2)C(t_i, \tau_2) + \cdots + \Delta\sigma_c(\tau_{i-1})C(t_i, \tau_{i-1})
\]

\[
\varepsilon_c(t_i) = \sum_{j=1}^{i-1} \Delta\sigma_c(\tau_j)C(t_i, \tau_j)
\]

\[
\varepsilon_c(t_i) = \sum_{j=1}^{i-1} \Delta\sigma_c(\tau_j) \left( \frac{1}{E_c(\tau)} + \frac{\varphi(t, \tau)}{E_c(\tau)} \right)
\]

where:

\(C(t_i, \tau_j)\) = creep compliance function at time \(t_i\) due to stress increment at time \(\tau_j\);  
\(\Delta\sigma_c(\tau_j)\) = applied stress increment at time \(\tau_j\); and  
\(\varepsilon_c(t_i)\) = elastic and creep strain in concrete at time \(t_i\).
Creep Strain

Creep strain can be expressed as the difference between total time dependent concrete strain due to stress and the summation of elastic strain at time, $t_i$.

$$
\varepsilon_{cr}(t_i) = \sum_{j=1}^{i-1} \Delta \sigma_c(t_j) \left( \frac{1}{E_c(t_j)} + \frac{\varphi(t_o, t_j)}{E_c(t_j)} \right) - \sum_{j=1}^{i-1} \Delta \sigma_c(t_j) \left( \frac{1}{E_c(t_j)} \right)
$$

(3-6)

Shrinkage Strain

Shrinkage also induces time dependent strains in concrete. Shrinkage strain is taken as following:

$$
\varepsilon_{sh}(t_i) = \frac{(t_i - t_c)}{35 + (t_i - t_c) + \gamma_{sh} \varepsilon_{shu}}
$$

(3-7)

Where:

- $\varepsilon_{sh}(t_i)$ = shrinkage strain at time $t_i$;
- $\varepsilon_{shu}$ = ultimate shrinkage strain;
- $t_i$ = age of concrete in days;
- $t_c$ = age of concrete when drying starts at end of moist curing in days; and
- $\gamma_{sh}$ = adjustment factor for non-standard conditions.

However, after cracking it is anticipated that the effect of shrinkage will decrease. Hence, effective shrinkage strain is employed after the section has cracked. The effective shrinkage strain is calculated by equation (3-8).

It is assumed that a reduction in shrinkage follows the same curve as the tension stiffening stress-strain curve. A reduction factor, $\tilde{\gamma}(t_i)$, is introduced that calculates the
decrease in shrinkage. When the section is uncracked, \( \hat{y}(t_i) \) is equal to one. As the crack forms, reduction occurs as per equation (3-9):

\[
\epsilon_{sh\,Eff}(t_i) = \hat{y}(t_i) \times \epsilon_{sh}(t_i)
\]  

\[
\hat{y}(t_i) = \frac{(\beta \epsilon'_t - \epsilon_T(t_i))}{(\beta \epsilon'_t - \epsilon'_T)} \left\{ \begin{array}{ll}
\epsilon_T \leq \epsilon'_t & \hat{y}(t_i) = 1 \\
\epsilon_T \geq \alpha \epsilon'_t & \hat{y}(t_i) = 0
\end{array} \right.
\]  

where:

\( \epsilon_{sh\,Eff}(t_i) \) = effective shrinkage strain at time \( t_i \);

\( \epsilon_{sh}(t_i) \) = shrinkage strain at time \( t_i \);

\( \beta \) = tension stiffening parameter (obtained from tension stiffening model);

\( \hat{y}(t_i) \) = reduction factor for shrinkage strain;

\( \epsilon_T(t_i) \) = total strain at time \( t \); and

\( \epsilon'_t \) = cracking strain.

**Inelastic Strain**

Creep, shrinkage and temperature strain can be lumped together to constitute inelastic strain:

\[
\epsilon_I(t_i) = \epsilon_{cr}(t_i) + \epsilon_{sh\,Eff}(t_i) + \epsilon_(t_i)
\]  

where:

\( \epsilon_I(t_i) \) = total inelastic strain in concrete at time \( t_i \)

Temperature strain is neglected in this study so the inelastic strain will consist of creep and effective shrinkage strains.
Total Strain

The total strain in concrete is given by the following equation:

\[ \varepsilon_t(t_i) = \varepsilon_e(t_i) + \varepsilon_{cr}(t_i) + \varepsilon_{sh\hspace{1pt}Eff}(t_i) + \varepsilon_a(t_i) \]  

(3-11)

where:

\[ \varepsilon_t(t_i) = \text{total strain in concrete at time } t_i; \]
\[ \varepsilon_e(t_i) = \text{total elastic strain in concrete at time } t_i; \]
\[ \varepsilon_{cr}(t_i) = \text{total creep strain in concrete at time } t_i; \]
\[ \varepsilon_{sh\hspace{1pt}Eff}(t_i) = \text{total effective shrinkage strain at time } t_i; \] and
\[ \varepsilon_a(t_i) = \text{total temperature strain in concrete at time } t_i. \]

The total strain can be written as:

\[ \varepsilon_T(t_i) = \varepsilon_e(t_i) + \varepsilon_i(t_i) \]  

(3-12)

Elastic strain can be calculated as

\[ \varepsilon_e(t_i) = \varepsilon_T(t_i) - \varepsilon_i(t_i) \]  

(3-13)

Under a perfect bond assumption, strain in the concrete will be equal to strain in the steel which will be equal to the total strain in the specimen. Hence,

\[ \varepsilon_s(t_i) = \varepsilon_c(t_i) = \varepsilon_T(t_i) \]  

(3-14)

3.1.4 Computation of stresses

According to Hooke's law, stress is equal to the product of elastic modulus and elastic strain. The stress in the axial prism can be calculated as following:

\[ \sigma_c(t_i) = E_c(t_i)\varepsilon_e(t_i) = E_c(t_i)[(\varepsilon_T(t_i) - \varepsilon_i(t_i)] \]  

(3-15)
\[ \sigma_s(t_i) = E_s\varepsilon_s(t_i) = E_s\varepsilon_T(t_i) \]  

(3-16)

where:

\[ E_c(t_i) = \text{modulus of elasticity of concrete at time } t_i; \]

\[ E_s = \text{modulus of elasticity of steel reinforcing bar}; \]

\[ \sigma_c(t_i) = \text{stress in concrete at time } t_i; \text{ and} \]

\[ \sigma_s(t_i) = \text{stress in steel at time } t_i. \]

When force is applied in an axial prism, concrete will creep and the steel bar will restrain it. This will increase stress in the steel and decrease stress in the concrete over time. This change in stresses is not known, and hence, direct solution for strain cannot be obtained from equation (3-5). To maintain the equilibrium at each time step, the applied force must be equal to the internal forces in steel and concrete. Hence, the following relations should hold true to maintain the equilibrium:

\[ P_{app} = P_c + P_s \]

(3-17)

\[ P_{app} = A_c\sigma_c + A_s\sigma_s \]

(3-18)

where:

\[ P_c = \text{force in concrete}; \]

\[ P_s = \text{force in reinforcing steel bar}; \]

\[ P_{app} = \text{applied force}; \]

\[ A_c = \text{area of concrete}; \text{ and} \]

\[ A_s = \text{area of reinforcing steel bar}. \]

Substituting equation (3-15) and (3-16) into (3-18) we get

\[ P_{app} = A_c E_c(t_i)[(\varepsilon_T(t_i) - \varepsilon_i(t_i)) + A_s E_s\varepsilon_T(t_i)] \]

(3-19)
Total strain in the prism can be obtained by rearranging equation (3-19)

\[ \varepsilon_T(t_i) = \frac{P_{app} + A_c E_c(t_i) \varepsilon_I(t_i)}{A_c E_c(t_i) + A_s E_s} \]  

(3-20)

3.1.5 Introduction of Tension Stiffening

The tensile stress in concrete cannot exceed its tensile strength. The tensile strength of concrete is assumed to vary according to the load-strain distribution given by Damjanic and Owen (1984). When the partial cracking of the concrete member has occurred, the maximum tensile stress in concrete at strain \( \varepsilon_{TS}(t_i) \) is given by (3-21)

\[ f_t(t_i) = \alpha f'_t(t_i) - \frac{\alpha E_c}{\beta - 1} (\varepsilon_{TS}(t_i) - \varepsilon'_t) \]  

(3-21)

where:

- \( f_t \) = maximum tensile stress in concrete at strain \( \varepsilon_T \);
- \( \alpha \) = strength reduction factor;
- \( \beta \) = tension stiffening parameter; and
- \( f'_t \) = tensile strength of concrete.

\[ f'_t = 7.5 \sqrt{f'_c} \]  

(3-22)

\( \varepsilon'_t \) = Cracking Strain.

\[ \varepsilon'_t = \frac{f'_t}{E_c(28)} \]  

(3-23)

\[ \varepsilon_{TS}(t_i) = \varepsilon_T(t_i) - \varepsilon_{sh \, eff}(t_i) \]  

(3-24)

With increase in tensile loading on the specimen, the section will crack and the elastic modulus will reduce because of cracking. The tension stiffening effect is
incorporated by computing the reduced elastic modulus. This reduced modulus is
calculated as secant modulus of the stress-strain curve given by Damjanic and Owen
Model. If strain calculated by equation (3-20) is greater than the cracking strain, the
reduced elastic modulus will calculated by equation (3-25).

\[
E'_c(t_{i+1}) = \frac{f_i(t_i)}{\varepsilon_{TS}(t_i)}
\]  

(3-25)

Figure 3-1. Modeling Tension Stiffening

For computing strain in the next time step, the reduced elastic modulus will be
employed. Hence, the total strain value will be calculated as following:

\[
\varepsilon_{t}(t_i) = \frac{P_{app} + A_cE'_c(t_i)\varepsilon_i(t_i)}{A_cE'_c(t_i) + A_sE_s}
\]  

(3-26)

Stress in concrete will be calculated by equation (3-27)
\[ \sigma_c(t_i) = E'_c(t_i)\varepsilon_c(t_i) = E'_c(t_i)[(\varepsilon_{\tau}(t_i) - \varepsilon_l(t_i))] \]  

(3-27)

3.1.6 Methodology

Total inelastic strain and total strain is calculated by solving (3-10) and (3-20) respectively for time step at time \( t_i \) using the stress increment for the time step at time \( t_{i-1} \). The stress in concrete and steel for the time step at time \( t_i \) can be calculated using the equation (3-15) and (3-16). The applied stress increment at time \( t_i \) is the difference between the concrete stress at time \( t_i \) and time \( t_{i-1} \) as given by equation (3-28)

\[ \Delta\sigma_c(t_i) = \sigma_c(t_i) - \sigma_c(t_{i-1}) \]  

(3-28)

The above process is repeated by using the stress increment \( \Delta\sigma_c(t_i) \) to find out the strains for the time step at time \( t_{i-1} \). At each time step, total strain is compared with the cracking strain. If total strain exceeds the cracking strain, tension stiffening correction is applied. The reduced elastic modulus is calculated by equation (3-25) and used for the calculation of stress and strain in the next time step.

3.2 Development of Matlab Code & Time Step Analysis

A program code in Matlab is developed to execute the following algorithm and calculate stress, strain at each time step.

i. The following values are entered as constants in the program script

- area of concrete
- reinforcement ratio
- elastic modulus of steel
- 28-day concrete strength
- concrete strength material constants
- applied load at time $t_1$
- time-steps
- constants for tension stiffening model

ii. Calculation of area of steel from the reinforcement ratio

iii. Calculation of time dependent strength of concrete (3-1) and elastic modulus of concrete (3-2) for each time step

iv. Calculation of creep strain (3-6)
   - Creep strain for the first time step is zero. From the second time step, creep strain is calculated using the stress increment and the creep coefficient corresponding to the previous time step.

v. Calculation of shrinkage strain (3-7) and effective shrinkage strain (3-8)
   - It is assumed that curing is done till the time of loading and hence the shrinkage strain for the first time step is zero.

vi. Calculation of total inelastic strain (3-10)

vii. Calculation of total strain (3-20)
   - For the first time step total strain will be equal to elastic strain

viii. Calculation of strain for tension stiffening model (3-24)

ix. Calculation of stresses in concrete and steel (3-15) & (3-16)

x. Comparison of strain with the cracking strain.
• If the calculated strain is less than cracking strain, proceed to step xi

• If the calculated strain is more than cracking strain, following applies
  
i. Calculation of permissible stress in concrete from Damjanic and Owen Model (3-21)
  
ii. Calculation of reduced elastic modulus for the next time step (3-25)

xi. Calculate the stress increment (3-28)
  
• The stress increment for the first time step is equal to the stress in the concrete at the first time step. For the subsequent time-steps it is calculated by equation (3-28).

xii. Repeat the algorithm for 100 days with an increment of 1 day.
Chapter 4
Validation of Results

4.1 Test Setup

Test results reported by Beeby & Scott (2002) are used to evaluate the assumptions in the model and adjustments are made as needed. Beeby and Scott have conducted a series of tension tests on 1200 mm long concrete specimen with square cross-section of width 120 mm. These specimens were reinforced with a strain gauged central bar and the diameter of the bar varied from 12 mm to 20 mm. The concrete specimens were of the strength 30, 70, 100 MPa.

Concurrent testing of pairs of specimens was carried out at two test rigs. For accurate measurement of strain, each bar contained 85 electric resistance strain gauges installed in a central duct of 4x4 mm running throughout the length of the specimen. To one set of specimen, loads were applied in increments. These loads were of magnitude 43, 58 and 72 KN; whereas for the other set, load of 72 KN was applied and maintained till three to four months. The loads were calculated so as to give average stress of 3, 4 and 5 MPa.

During the experiment average strains were noted. The measured strains consisted of reinforcement strains as well as surface strains measured using demec strain gage. Using the average strain, average tension force was calculated in the reinforcement from the stress-strain curve. The average force in concrete was calculated by deducing the
tension force in steel from the applied load. The force was then converted to stress by dividing it by the concrete cross-sectional area. The concrete drying shrinkage was not measured both before and during the period of sustained loading.

4.2 Comparison of Results

The analysis was carried out for the same parameters as used for experimental setup and results were compared with the experimental data. The details of the specimens used for the validation of model are as given in Table 4-1.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Bar Dia (mm)</th>
<th>Length of Test (Days)</th>
<th>Concrete Compressive Cube strength MPa (Start/End of test)</th>
<th>Concrete Tensile strength MPa (Start/End of test)</th>
<th>No of Load stages</th>
</tr>
</thead>
<tbody>
<tr>
<td>T16B1</td>
<td>16</td>
<td>127</td>
<td>23.5/27.6</td>
<td>1.9/2.2</td>
<td>1</td>
</tr>
<tr>
<td>T16R1</td>
<td>16</td>
<td>127</td>
<td>23.5/27.6</td>
<td>1.9/2.2</td>
<td>3</td>
</tr>
<tr>
<td>T16B3</td>
<td>16</td>
<td>119</td>
<td>123.4/135.2</td>
<td>5.5/5.2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4-1. Test Specimens (Beeby & Scott, 2002)

The tension stiffening model was calibrated to match the experimental results. The comparisons for strain vs. time and stress in concrete vs. time for specimen T16R1 are given in figure 4-1 and 4-2 respectively. The comparison for stress in concrete vs. time for specimen T16B3 is given in figure 4-3.
The measured strains vary along the length of the specimen. Figure 4-1 compares the strain in steel reinforcement at four different locations in the specimen with the analysis results. The strain in reinforcement at the crack location is highest and it decreases with increase in distance from the crack. The graph shows two of the strains measured at the crack location whereas two others measured at the mid-point of cracks.

The stress values for two specimens are compared in figure 4-2 and 4-3. The analysis results match reasonably well with the experimental data.
Figure 4-2. Comparison of experimental data (T16R1) with analysis results

Figure 4-3. Comparison of experimental data (T16RB3) with analysis results
As the strain measurements were taken both at the concrete surface and reinforcement bar, comparison of both these values were carried out (figure 4-4). It was noticed that strain measured using the demec strain gage were lower than the average strain measured in the steel reinforcement.

![Figure 4-4. Comparison of measured strains (T16R1)](image)

An effort was made to calibrate the model as per strains measured at the concrete surface. When the model was calibrated using the demec strain values, it gave close results while comparing the demec strain for the other specimens. Figure 4-5 gives the result for specimens T16R1 and T16B1. Figure 4-6 compares the stress in concrete for specimen T16R1 with the analysis results.
Figure 4-5. Comparison of experimental data (T16R1 & T16B1) with analysis results

Figure 4-6. Comparison of experimental data (T16R1) with analysis results
However, the stress in concrete calculated by the algorithm was very high than the calculated experimental value. Hence, model calibrated as per demec strains was discarded and model calibrated using the steel reinforcement strains was used for parametric analysis as it matched both stress and strain reasonably well.
Chapter 5

Parametric Studies

The main objective of parametric study is to observe the effect of various parameters on time-dependent deformation. The parameters selected for parametric study are reinforcement ratio, strength of concrete, ultimate creep coefficient, ultimate shrinkage coefficient, tension stiffening parameter, strength reduction parameter and applied load. The selection of values is based on typical design range. The section used for the parametric study is the same as the one used for validation of results. The applied load and the strength parameters are also the same as reported by Beeby and Scott (2002).

5.1 Input Parameters

The applied load, \( P_{APP} = 16.65 \) kips is used for single load application and \( P_{APP} = 9.67 \) kips, 13.04 kips and 16.65 kips is used for multiple load application. These loads will cause stresses 433 psi, 584 psi and 746 psi respectively. The area of the concrete section is 4.72" x 4.72" and the diameter of the reinforcing bar is 0.63". For the tension stiffening model, \( \alpha \) is taken as 0.35 and \( \beta \) is taken as 50, unless they are used as variable parameters. The compressive strength of concrete is 3706 psi and tensile strength of concrete is 304 psi. It is assumed that the specimen has been cured till the time just before the application of load. Hence, no initial shrinkage has been considered. The reinforcement ratio has been taken as 0.01. The ultimate creep coefficient is taken as 2.35
and ultimate shrinkage strain is taken as 300 micro strains. Table 5-1 presents the variables examined:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Reinforcement Ratio</td>
<td>$\rho = 0.002, 0.005, 0.01,$ $0.02, 0.04, 0.08$</td>
<td>$C_u = 2.35$ &lt;br&gt;$\varepsilon_{shu} = 300 \times 10^{-6} \text{ in./in.}$ &lt;br&gt;$t_c = 28 \text{ days}$ &lt;br&gt;$\alpha = 0.35$ &lt;br&gt;$\beta = 50$ &lt;br&gt;$f' _c = 3706 \text{ psi}$</td>
</tr>
<tr>
<td>2) Strength of concrete</td>
<td>$f' _c = 3000 \text{ psi, 4000 psi, 5000 psi, 6000 psi}$</td>
<td>$C_u = 2.35$ &lt;br&gt;$\varepsilon_{shu} = 300 \times 10^{-6} \text{ in./in.}$ &lt;br&gt;$t_c = 28 \text{ days}$ &lt;br&gt;$\alpha = 0.35$ &lt;br&gt;$\beta = 50$ &lt;br&gt;$\rho = 0.01$</td>
</tr>
<tr>
<td>3) Ultimate Creep Coefficient</td>
<td>$C_u = 1.3, 2.35, 4.15$</td>
<td>$\varepsilon_{shu} = 300 \times 10^{-6} \text{ in./in.}$ &lt;br&gt;$t_c = 28 \text{ days}$ &lt;br&gt;$\alpha = 0.35$ &lt;br&gt;$\beta = 50$ &lt;br&gt;$\rho = 0.01$ &lt;br&gt;$f' _c = 3706 \text{ psi}$</td>
</tr>
<tr>
<td>4) Ultimate Shrinkage</td>
<td>$\varepsilon_{shu} = 200 \times 10^{-6} \text{ in./in.}$</td>
<td>$C_u = 2.35$</td>
</tr>
<tr>
<td>Coefficient</td>
<td>300 x 10^-6 in./in., 400 x 10^-6 in./in., 600 x 10^-6 in./in., 800 x 10^-6 in./in.</td>
<td>$t_c = 28$ days</td>
</tr>
<tr>
<td>5) Tension stiffening parameter</td>
<td>$\beta = 3, 20, 60, 100, 150$</td>
<td>$C_u = 2.35$</td>
</tr>
<tr>
<td>6) Strength reduction parameter</td>
<td>$\alpha = 0.2, 0.4, 0.6, 0.8, 1.0$</td>
<td>$C_u = 2.35$</td>
</tr>
<tr>
<td>7) Tension Stiffening Parameter and Strength reduction parameter</td>
<td>$\alpha = 1.0, \beta = 18$</td>
<td>$\alpha = 0.8, \beta = 23$</td>
</tr>
</tbody>
</table>
\( \alpha = 0.2, \beta = 87 \quad \rho = 0.01 \)

<table>
<thead>
<tr>
<th>8) Applied Load</th>
<th>( P = 0.5 , P_{cr}, )</th>
<th>( C_u = 2.35 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P = 0.75 , P_{cr}, , 0.5 , P_{cr}, )</td>
<td>( \varepsilon_{shu} = 300 \times 10^{-6} \text{ in./in.} )</td>
<td></td>
</tr>
<tr>
<td>( P = 1.0 , P_{cr}, , 1.5 , P_{cr}, )</td>
<td>( t_c = 28 \text{ days} )</td>
<td></td>
</tr>
<tr>
<td>( P = 3.0 , P_{cr} )</td>
<td>( \alpha = 0.35 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \beta = 50 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( f'_{c} = 3706 \text{ psi} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \rho = 0.01 )</td>
<td></td>
</tr>
</tbody>
</table>

Table 5-1. Variables for parametric study

### 5.2 Study Results

The results of the study are given in the subsequent sections. The results are plotted considering each parameter in turn. Four plots, total strain vs. time, stress in steel vs. time, stress in concrete vs. time and ratio of force in steel bar to force in concrete bar with time are plotted for all considered parameters. A sensitivity study was carried out to examine the effect of number and size of time-steps before moving on to other parameters. The results are compared to the actual test results to decide on the size and number of time-steps.
5.2.1. Sensitivity to the Numbers of Time-steps

It was observed that the response of the algorithm at the instance of load application depends on the number of time-steps. The algorithm uses the reduced elastic modulus, and at the application of load there is a sudden decrease in the elastic modulus. Hence, it takes few time-steps to converge to the reduced value. It was noticed that at least six time-steps are required to replicate the actual test results. The size of the initial time-step is kept very small as cracking is an instantaneous process. The analysis result with size of time-step as one day is shown in figure 5-1. It can be noticed that in this case the model takes a few days to reach the reported strain values. The result obtained from the modified algorithm is given in figure 5-2. In this case, at least ten time-steps are run before day one. The response of the model matches quite well with the experimental data.

Figure 5-1. Analysis results with 1 day time-step
Figure 5-2. Analysis results with increased # of time-steps at load application

The results verify the sensitivity of the model to the number of time-steps at the application of load. It was noticed that 1 day time-step works well for the rest of the analysis.

### 5.2.2. Reinforcement Ratio, $\rho_g$

The reinforcement ratio is varied from 0.2 % to 8.0%. The gross reinforcement ratio is used as given by equation 5-1.

$$\rho_g = \frac{A_s}{A_g}$$  \hspace{1cm} (5-1)
where,

\[ \rho_g = \text{gross reinforcement ratio}; \]

\[ A_s = \text{area of steel}; \] and 

\[ A_g = \text{gross area of the section}. \]

The results of the variation of reinforcement ratio are presented in Figures 5-3 to 5-8.
Figure 5-4. Varying $\rho$ - Total Strain

Figure 5-5. Varying $\rho$ - Stress in Concrete
Figure 5-6. Varying $\rho$ - Stress in steel

Figure 5-7. Varying $\rho$ - Force ratio
The analysis shows that members with 0.2% and 0.5% reinforcement ratio are failing under the applied load. The members with high reinforcement ratio have higher stiffness and hence, strain values are lower. The strain is increasing with reduction in the stiffness of the member. The stresses in concrete for reinforcement ratio 1.5% to 8.0% are close. The decrease in stress in concrete is gradual in heavily reinforced members whereas it is high in lightly reinforced members. This is so because the strain values are high in lightly reinforced members due to low stiffness and this leads to lower tensile capacity of concrete as per the tension stiffening model. Moreover, the algorithm assumes a long branch of the tension stiffening model and it also leads to gradual decrease in stress in concrete.

The force ratio for $\rho = 1.5\%$ is close to the force ratio for $\rho = 8.0\%$ indicating that the contribution of concrete in load sharing is higher in lightly reinforced member.
Hence, it can be concluded that tension stiffening effect is more pronounced in the lightly reinforced members.

At the first application of load, the ratio of force in steel to force in concrete is low and it increases with increase in loading. Hence, it can be inferred that tension stiffening effect is low at higher loads and concrete is contributing less. The tension stiffening effect is diminishing with time and with increase in load.

From the strain pattern it can be noticed that time-dependent strains are not prominent. This is apparent from the straight branch of the strain vs. time curve. As the load transfers to steel from concrete, the creep strains decrease. The members are cracking at the first application of load reducing the shrinkage strain in the member.

5.2.3. Compressive strength of concrete, $f'c$

The compressive strength of concrete is varied from 3000 psi to 6000 psi. The results are depicted in figure 5-9 to 5-12.
Figure 5-9. Varying $f'_{c}$ - Total strain

Figure 5-10. Varying $f'_{c}$ - Stress in concrete
The variation in strain value is not sensitive to the compressive strength of concrete. This indicates that steel reinforcement is taking a major portion of the load. In
addition to this, tensile strength is about one-tenth of the compressive strength and hence, the effect of compressive strength is not pronounced and reduces to one-tenth for the tension members. The general trend shows that when the strength of concrete is low, strain is high and steel reinforcement carries the major portion of the load leading to lower stress in concrete. The concrete with compressive strength of 6000 psi is taking 1.5 times more load than the concrete with compressive strength of 3000 psi.

5.2.4. Ultimate Creep Coefficient, \( C_u \)

The plots for variation of \( C_u \) are given in figure 5-13 to 5-16. Three values of \( C_u \) are used in the analysis.

![Figure 5-13. Varying \( C_u \) - Total strain](image)
Figure 5-14. Varying Cu - Stress in concrete

Figure 5-15. Varying Cu - Stress in steel
Analysis results indicate that the effect of creep is not prominent. This result is also supported by theory. The tensile stress in concrete is low and creep depends on the stress in the member and hence, creep is less.

5.2.5. Ultimate Shrinkage Strain, $\varepsilon_{shu}$

Results for ultimate shrinkage strain varying from $200 \times 10^{-6}$ to $800 \times 10^{-6}$ are shown in Figures 5-17 to 5-22.
Figure 5.17. Varying $\varepsilon_{shu}$ - Total strain

Figure 5.18. Varying $\varepsilon_{shu}$ - Stress in concrete
Figure 5-19. Varying $\varepsilon_{shu}$ - Stress in steel

Figure 5-20. Varying $\varepsilon_{shu}$ - Force ratio
The plots show that the model is not sensitive to the ultimate shrinkage strain. The shrinkage strain is not prominent as it is assumed that specimen is cured till the application of load. The effective shrinkage also reduces as the cracks are formed, further decreasing shrinkage strain in the specimen. As ultimate shrinkage strain is increasing, the rate of force ratio is also increasing. This indicates that higher shrinkage is reducing the load carrying capacity of concrete at a higher rate.

5.2.6. Tension Stiffening Parameter, $\beta$

The results of the analysis are depicted in figure 5-21 to 5-24.
Figure 5-22. Varying $\beta$ - Stress in concrete

Figure 5-23. Varying $\beta$ - Stress in steel
\( \beta \) defines the inclination of the descending branch of the tension stiffening curve or the extent to which tension stiffening will be present in the member. For larger values of \( \beta \), inclination of the descending branch will be less and tension stiffening will be present for larger strain values. The same results are depicted in the analysis plots. When \( \beta \) is high, stress in concrete is more and stress in steel is low, decreasing the force ratio. The results depict that for \( \beta = 3 \) and 20, the specimen has surpassed the tension stiffening zone and hence, concrete is taking no tensile force. For these two cases the strain in specimen is the bare bar response.

Also, increase in \( \beta \) signifies gradual decrease in the stiffness of concrete. Hence, stress in concrete decreases gradually with time for higher \( \beta \) values.
5.2.7. Strength reduction factor, $\alpha$

The values of $\alpha$ varying from 0.2 to 1.0 with an increment of 0.2 are used in the analysis. The results are given in figure 5-25 to 5-28.

![Graph showing varying strain with time](image-url)
Figure 5-26. Varying $\alpha$ - Stress in concrete

Figure 5-27. Varying $\alpha$ - Stress in steel
The factor $\alpha$ specifies remaining tensile strength of concrete after the formation of first crack. The value $\alpha = 1$ signifies that there is no sudden drop in strength when the crack forms. For higher values of $\alpha$, contribution of concrete in load sharing is higher. Whereas, for lower values of $\alpha$, strength of concrete reduces to large extent after formation of crack and contribution of concrete in load sharing decreases.

Analysis results show that stress in concrete is directly proportional to $\alpha$ value. When $\alpha$ reduces from 1.0 to 0.2, the stress in concrete also reduces increasing the stress in steel reinforcement. The creep, shrinkage and tension stiffening effects are diminishing for lower values of $\alpha$ because of decrease in load carrying capacity of concrete. The effect of $\alpha$ can be understood by the fact that for $\alpha = 0.2$ steel is taking about 27 times the force taken by concrete whereas for $\alpha = 1.0$, steel is taking about 3.5 times the force of concrete.
5.2.8. Strength Reduction Parameter $\alpha$ & Tension Stiffening Parameter, $\beta$

The combination of $\alpha$ and $\beta$ was derived by keeping the area constant under the tension stiffening stress-strain curve. The five combinations used for the parametric study are:

- $\alpha = 1.0, \beta = 18$
- $\alpha = 0.8, \beta = 23$
- $\alpha = 0.6, \beta = 29$
- $\alpha = 0.4, \beta = 44$
- $\alpha = 0.2, \beta = 87$

The results are given in figures 5-29 to 5-32.
Figure 5-30. Varying $\alpha$ & $\beta$ - Stress in concrete

- $\alpha=1.0, \beta=18$
- $\alpha=0.8, \beta=23$
- $\alpha=0.6, \beta=29$
- $\alpha=0.4, \beta=44$
- $\alpha=0.2, \beta=87$

Figure 5-31. Varying $\alpha$ & $\beta$ - Stress in steel

- $\alpha=1.0, \beta=18$
- $\alpha=0.8, \beta=23$
- $\alpha=0.6, \beta=29$
- $\alpha=0.4, \beta=44$
- $\alpha=0.2, \beta=87$
The study illustrates the importance of both $\alpha$ and $\beta$. At first instance, $\alpha$ value appears to be more important as it is directly affecting the strength of concrete whereas $\beta$ seems secondary as it dictates the path of decrease in stiffness of concrete. However, $\beta$ is also important as it defines the tension stiffening zone. If strain is more than $\beta$ times cracking strain there will be no tension stiffening. The same concept is depicted in the results. For lower $\beta$ values concrete has well crossed the tension stiffening zone and no strength in tension is available. Hence, for $\beta = 18$, 23 and 29, concrete is not contributing and the response is of the bare bar. When the tension stiffening is available (for $\alpha = 0.4$ and $\alpha = 0.2$), the pair of $\alpha$ and $\beta$ give very close results. Torres et al (2004) have presented methodology for selecting $\alpha$ and $\beta$ and have concluded that values of $\alpha$ and $\beta$ can be obtained independently with accuracy, in spite of their relationship.
5.2.9. Cracking Load, $P_{cr}$

The applied load is varied and a factor of 0.5, 0.75, 1.0, 1.5 and 3.0 is applied to the cracking load to obtain the applied load. The results of the analysis are given in figures 5-33 to 5-36.

![Figure 5-33. Varying $P_{cr}$ - Total strain](image-url)
Figure 5-34. Varying $P_{cr}$ - Stress in concrete

Figure 5-35. Varying $P_{cr}$ - Stress in steel
As expected, the strains are higher for larger applied load. For lower loads, time-dependent strains are prominent. This is indicated by gradual increase in the strain values. For higher loads, stress in concrete is low and force ratio is low depicting less contribution of concrete in load sharing and less tension stiffening. The results clearly indicate that tension stiffening effect is prominent at lower loads and decreases as the load increases.
Chapter 6

Conclusions & Recommendations

The behavior of concrete under tensile force was studied and factors affecting the tensile deformation were investigated. A model for calculation of long term deflection in a tension member was developed. A literature review on the current state of research was conducted and the findings were applied in the development of algorithm. The model incorporates the effects of creep, shrinkage and tension stiffening. Damjanic and Owen (1984) tension stiffening model has been used for calculating the reduced elastic modulus of concrete. The model has been calibrated as per the experimental results (Beeby and Scott, 2002). The theory of creep superposition is employed and strain at each time-step is computed. The concept of effective shrinkage has been used for calculation of shrinkage strains at each time-step. At each time-step, the model calculates the reduced elastic modulus of concrete according to the tension stiffening model. After calibration of the model, sensitivity of different parameters on deformation of tension member has been evaluated. Conclusions are drawn based on the results of the parametric study and recommendations are made for future research and design.

6.1. Conclusions

The conclusions drawn from the study are as follows:

- Even after cracking concrete is able to carry significant tensile stresses.
• The reinforcement ratio primarily defines the behavior of tension member. At lower reinforcement ratios, tension stiffening effect is more significant and contribution of concrete in load sharing is major.

• At sustained loads, tensile capacity of concrete decreases gradually.

• The study shows that after cracking, creep and shrinkage have minor influence on tension stiffening under increasing loads.

• The tensile behavior of concrete is very sensitive to the strength reduction parameter, $\alpha$. Hence, prediction of $\alpha$ needs careful evaluation.

• For long term tension-stiffening effect, the tension stiffening parameter, $\beta$, tends to be a high value.

6.2. Recommendations

• Further research needs to be carried out for evaluation of $\alpha$ and $\beta$ for the tension stiffening model. These two parameters define the tension stiffening model and accuracy of the result depends on their precise evaluation.

• The bilinear model has its limitations in predicting the behavior of concrete in tension. The author suggests investigating the use of non-linear models for better prediction of tension stiffening effect.

• This study has been conducted on a pure tension member to understand the behavior of concrete in tension. The proposed model calculates deformation in an axial prism under tension force. The model can be
extended to analyze beams and slabs, which are common construction elements.
References

ACI Committee 209, (1997). *Prediction of creep, shrinkage and temperature effects in concrete structure (ACI 209R-92)*. American Concrete Institute, Farmington Hills, MI.

ACI committee 318, (2011). *Building code requirements for structural concrete (ACI 318-11)*. American Concrete Institute, Farmington Hills, MI.


A.1. Program for calculating tension stiffening stress-strain curve

alpha=0.35;
gamma=50;
a=4;
b=0.85;
f_c28=3706;
f_t28=7.5*sqrt(f_c28);
Ep_t=7.5/57000;
datano1=1000;
increment=gamma*Ep_t/datano1;
datano=2+datano1;
ft=zeros(datano,1);
Ept=zeros(datano,1);
f_c=zeros(datano,1);
Ec=zeros(datano,1);
Ept(1)=0;
t=1:1:(datano);
for i=1:1:datano
    f_c(i)=t(i)/(a+b*t(i))*f_c28;
    Ec(i)=57000*sqrt(f_c(i));
    if Ept(i)<Ep_t
        ft(i)=Ept(i)*Ec(i);
    end
    if Ept(i)>Ep_t
        ft(i)=Ept(i)*Ec(i);
        ft(i+1)=Ept(i)*Ec(i);
        Ept(i+1)=Ept(i);
        Ept(i+2)=Ept(i+1)+increment;
        m=i+2;
        break
    end
    if i<datano
        Ept(i+1)=Ept(i)+increment;
    end
end
for j=m:1:datano
    if Ept(i)>Ep_t && Ept(i)<gamma*Ep_t
        f_c(j)=t(j)/(a+b*t(j))*f_c28;
        Ec(j)=57000*sqrt(f_c(j));
if $E_{pt}(j) > E_{pt}$
    \[ f_t(j) = \alpha * f_{t28} - (\alpha * E_c(j) * (E_{pt}(j) - E_{pt})/(\gamma - 1)); \]
if $f_t(j) < 1$
    \[ f_t(j) = 0; \]
end
end
end
if $j < \text{datano}$
    \[ E_{pt}(j+1) = E_{pt}(j) + \text{increment}; \]
end
end
plot(E_{pt}, f_t);

A.2. Program for calculating total strain

% INPUT PARAMETERS%
Ac=22.32;                 %inch square > Area of Concrete
Es=29000000;              %psi > Elastic Modulus of Steel
a=4;
b=0.85;
f_{c28}=3706;               %psi > Strength of Concrete
Ec_{28}=57000*sqrt(f_{c28}); %psi > Elastic Modulus of Concrete
Po1=16657.68;             %lbs > Applied Force
LD1=100;                  %First Loading
Increments=1:1:LD1;       %time interval <days
Ep_{shu}=-300*10^(-6);      %Ultimate Shrinkage Strain in/in
gamma_sh=1;               %Adjustment factor for non-standard
conditions
T_c=28;                   %age of concrete when drying starts at end of
moist curing in days
As=0.014*Ac;

%Tension Stiffening Parameters
alpha=.35;
gamma=50;
tsf_{t28}=304;
tsf_{Ep_t}=tsf_{t28}/Ec_{28};

% VARIABLES
\text{time\_step}=\text{length}(\text{Increments});       %Calculating the loop
i=\text{time\_step}+8;
E_{p_{cr}}=\text{zeros}(i,1);       %Creep Strain
E_{p_{t}}=\text{zeros}(i,1);      %Total Strain
\Sigma_{c}=\text{zeros}(i,1);      %Stress in Concrete
\Sigma_{s}=\text{zeros}(i,1);      %Stress in Steel
Del\_sigma\_c=zeros(i,1); % Stress increment
t=zeros(i,1); % days < time
f\_ct=zeros(i,1); % Creep Function
Ec=zeros(i,1);
Ep\_sh=zeros(i,1); % shrinkage strain at time t
Ep\_i=zeros(i,1); % Total Inelastic strain at time t
Ep\_ts=zeros(i,1); % strain for plotting tension stiffening
force\_c=zeros(i,1);
force\_s=zeros(i,1);
tsft=Zeros(i,1);
actualSigma\_c=zeros(i,1);
Ects=zeros(i,1);
y\_sh=zeros(i,1);

% For t=0
f\_ct(1)=(0+28)/(a+b*(0+28))*f\_c28;
Ec(1)=57000*sqrt(f\_ct(1));
Ects(1)=Ec(1);

% Calculations

% Initial Application of Load at t=0.1
  t(1)=0;
  for i=2
    t(i)=0.01;
    f\_ct(i)=(t(i)+28)/(a+b*(t(i)+28))*f\_c28;
    Ec(i)=57000*sqrt(f\_ct(i));
    Ects(i)=Ec(i);
    y\_sh(i)=1;
    
    Ep\_cr(i)=0;
    Ep\_sh(i)=y\_sh(i)*(t(i)+28-t\_c)/(35+t(i)+28-t\_c)*gamma\_sh*Ep\_shu;
    Ep\_i(i)=Ep\_cr(i)+Ep\_sh(i);
    Ep\_t(i)=(Po1+Ac*Ects(i)*Ep\_i(i))/(Ac*Ects(i)+As*Es);
    
    Sigma\_c(i)=Ects(i)*(Ep\_t(i)-Ep\_i(i));
    force\_c(i)=Sigma\_c(i)*Ac;
    Sigma\_s(i)=Es*Ep\_t(i);
    force\_s(i)=Sigma\_s(i)*As;
    Ep\_ts(i)=Ep\_t(i)-Ep\_sh(i);
    actualSigma\_c(i)=Sigma\_c(i);
  
  if Ep\_ts(i)<tsp
    tsft(i)=f\_ct(I);
  end
  
  if Ep\_ts(i)>tsp & Ep\_ts(i)<(gamma*tsp)
    tsft(i)=alpha*tsf\_t28-((alpha*Ec(i)*(Ep\_ts(i)-tsp)))/(gamma-1));
  end
if Ep_ts(i)>(gamma*tsEp_t)
    tsft(i)= 0;
end

Del_sigma_c(i)=Sigma_c(i)-Sigma_c(i-1);
y_sh(i+1)=(gamma*tsEp_t-Ep_ts(i))/(gamma*tsEp_t-tsEp_t);
Ects(i+1)=tsft(i)/Ep_ts(i);

del sigma c(i)=Sigma_c(i)-Sigma_c(i-1);
y_sh(i+1)=(gamma*tsEp_t-Ep_ts(i))/(gamma*tsEp_t-tsEp_t);
Ects(i+1)=tsft(i)/Ep_ts(i);

for i=3
    t(i)=.05;
    f_ct(i)=(t(i)+28)/(a+b*(t(i)+28))*f_c28;
    Ec(i)=57000*sqrt(f_ct(i));

    Ep_cr(i)=0;
    Ep_sh(i)=y_sh(i)*(t(i)+28-t_c)/(35+t(i)+28-t_c)*gamma_sh*Ep_shu;
    Ep_i(i)=Ep_cr(i)+Ep_sh(i);
    Ep_t(i)=(Po1+Ac*Ects(i)*Ep_i(i))/(Ac*Ects(i)+As*Es);

    Sigma_c(i)=Ects(i)*(Ep_t(i)-Ep_i(i));
    force_c(i)=Sigma_c(i)*Ac;
    Sigma_s(i)=Es*Ep_t(i);
    force_s(i)=Sigma_s(i)*As;
    Ep_ts(i)=Ep_t(i)-Ep_sh(i);
    actualSigma_c(i)=Sigma_c(i);

    if Ep_ts(i)<tsEp_t
        tsft(i)=f_ct(i);
    end

    if Ep_ts(i)>tsEp_t && Ep_ts(i)<(gamma*tsEp_t)
        tsft(i)=alpha*tsf_t28-{(alpha*Ec(i)*(Ep_ts(i)-tsEp_t)/(gamma-1))};
    end

    if Ep_ts(i)>(gamma*tsEp_t)
        tsft(i)= 0;
    end

    Del sigma c(i)=Sigma_c(i)-Sigma_c(i-1);
    Ects(i+1)=tsft(i)/Ep_ts(i);
    y_sh(i+1)=(gamma*tsEp_t-Ep_ts(i))/(gamma*tsEp_t-tsEp_t);
end

end

t(4)=.1;
for i=4:1:8
\[ f_{ct}(i) = \frac{(t(i) + 28)}{(a + b \cdot (t(i) + 28))} \cdot f_{c28}; \]
\[ Ec(i) = 57000 \cdot \sqrt{f_{ct}(i)}; \]

for \( n = 1: (i-1) \)
\[ \text{temp}_{cr\_ft} = 2.35 \cdot (\frac{(t(i) - t(n))^0.6}{10 + ((t(i) - t(n))^0.6)}); \]
\[ \text{temp}_{ep\_cr} = \text{Del}_{sigma\_c}(n) \cdot \text{temp}_{cr\_ft} / \text{Ec}(n); \]
\[ \text{Ep\_cr}(i) = \text{Ep\_cr}(i) + \text{temp}_{ep\_cr}; \]
\[ \text{clear temp}_{cr\_ft} \text{ temp}_{ep\_cr}; \]
\]end

\[ \text{Ep\_sh}(i) = y_{sh}(i) \cdot \frac{(t(i) + 28 - t_c)}{35 + t(i) + 28 - t_c} \cdot \gamma_{sh} \cdot \text{Ep\_shu}; \]
\[ \text{Ep\_i}(i) = \text{Ep\_cr}(i) + \text{Ep\_sh}(i); \]
\[ \text{Ep\_t}(i) = \frac{(P_{o1} + A_c \cdot E_{cts}(i) \cdot \text{Ep\_i}(i))}{(A_c \cdot E_{cts}(i) + A_s \cdot E_s)}; \]

\[ \text{Sigma\_c}(i) = E_{cts}(i) \cdot (\text{Ep\_t}(i) - \text{Ep\_i}(i)); \]
\[ \text{force\_c}(i) = \text{Sigma\_c}(i) \cdot A_c; \]
\[ \text{Sigma\_s}(i) = E_s \cdot \text{Ep\_t}(i); \]
\[ \text{force\_s}(i) = \text{Sigma\_s}(i) \cdot A_s; \]
\[ \text{Ep\_ts}(i) = \text{Ep\_t}(i) - \text{Ep\_sh}(i); \]
\[ \text{actualSigma\_c}(i) = \text{Sigma\_c}(i); \]

if \( \text{Ep\_ts}(i) < \gamma \cdot \text{tsEp\_t} \)
\[ \text{tsft}(i) = f_{ct}(1); \]
end

if \( \text{Ep\_ts}(i) > \gamma \cdot \text{tsEp\_t} \) && \( \text{Ep\_ts}(i) < (\gamma \cdot \text{tsEp\_t}) \)
\[ \text{tsft}(i) = \alpha \cdot \text{tsf\_t28} - \left( \frac{\alpha \cdot \text{Ec}(i) \cdot (\text{Ep\_ts}(i) - \gamma \cdot \text{tsEp\_t})}{\gamma - 1} \right); \]
end

if \( \text{Ep\_ts}(i) > (\gamma \cdot \text{tsEp\_t}) \)
\[ \text{tsft}(i) = 0; \]
end

\[ \text{Del}_{sigma\_c}(i) = \text{Sigma\_c}(i) - \text{Sigma\_c}(i-1); \]
\[ E_{cts}(i+1) = \text{tsft}(i) / \text{Ep\_ts}(i); \]
\[ y_{sh}(i+1) = \frac{(\gamma \cdot \text{tsEp\_t} - \text{Ep\_ts}(i))}{(\gamma \cdot \text{tsEp\_t} - \text{tsEp\_t})}; \]

if \( i < 8 \)
\[ t(i+1) = t(i) + .05; \]
end
end

for \( i = 9 \)
\[ t(i) = 1; \]
\[ f_{ct}(i) = \frac{(t(i) + 28)}{(a + b \cdot (t(i) + 28))} \cdot f_{c28}; \]
\[ Ec(i) = 57000 \cdot \sqrt{f_{ct}(i)}; \]
for n=1:(i-1)
    temp_cr_ft=2.35*((t(i)-t(n))^0.6)/(10 + ((t(i)-t(n))^0.6));
    temp_ep_cr=Del_sigma_c(n)*temp_cr_ft/Ec(n);
    Ep_cr(i)=Ep_cr(i)+temp_ep_cr;
    clear temp_cr_ft temp_ep_cr;
end

Ep_sh(i)=y_sh(i)*(t(i)+28-t_c)/(35+t(i)+28-t_c)*gamma_sh*Ep_shu;
Ep_i(i)=Ep_cr(i)+Ep_sh(i);

Ep_t(i)=(Pol+Ac*Ects(i)*Ep_i(i))/(Ac*Ects(i)+As*Es);
Sigma_c(i)=Ects(i)*(Ep_t(i)-Ep_i(i));
force_c(i)=Sigma_c(i)*Ac;
Sigma_s(i)=Es*Ep_t(i);
force_s(i)=Sigma_s(i)*As;
Ep_ts(i)=Ep_t(i)-Ep_sh(i);
actualSigma_c(i)=Sigma_c(i);

if Ep_ts(i)<tsEp_t
    tsft(i)=f_ct(1);
end

if Ep_ts(i)>tsEp_t & Ep_ts(i)<(gamma*tsEp_t)
    tsft(i)=alpha*tsf_t28-((alpha*Ec(i)*(Ep_ts(i)-tsEp_t)/(gamma-1)));
end

if Ep_ts(i)>(gamma*tsEp_t)
    tsft(i)= 0;
end

Del_sigma_c(i)=Sigma_c(i)-Sigma_c(i-1);
Ects(i+1)=tsft(i)/Ep_ts(i);
y_sh(i+1)=(gamma*tsEp_t-Ep_ts(i))/(gamma*tsEp_t-tsEp_t);

end

% Time Step from t=2 to t=100
for i=10:1:108
    t(i)=i-8;
    f_ct(i)=(t(i)+28)/(a+b*(t(i)+28))*f_c28;
    Ec(i)=57000*sqrt(f_ct(i));
for n=1:(i-1)
    temp_cr_ft=2.35*{(t(i)-t(n))^0.6}/(10 + ((t(i)-t(n))^0.6));
    temp_ep_cr=Del_sigma_c(n)*temp_cr_ft/Ec(n);
    Ep_cr(i)=Ep_cr(i)+temp_ep_cr;
    clear temp_cr_ft temp_ep_cr;
end

Ep_sh(i)=y_sh(i)*((t(i)+28-t_c)/(35+t(i)+28-t_c))*gamma_sh*Ep_shu;
Ep_i(i)=Ep_cr(i)+Ep_sh(i);

Ep_t(i)=(Pol+Ac*Ects(i)*Ep_i(i))/(Ac+Ects(i)+As*Es);

Sigma_c(i)=Ects(i)*(Ep_t(i)-Ep_i(i));
force_c(i)=Sigma_c(i)*Ac;
Sigma_s(i)=Es*Ep_t(i);
force_s(i)=Sigma_s(i)*As;
Ep_ts(i)=Ep_t(i)-Ep_sh(i);
actualSigma_c(i)=Sigma_c(i);

if Ep_ts(i)<tsEp_t
    tsft(i)=f_ct(1);
end

if Ep_ts(i)>tsEp_t && Ep_ts(i)<(gamma*tsEp_t)
    tsft(i)=alpha*tsf_t28-((alpha*Ec(i)*(Ep_ts(i)-tsEp_t)/(gamma-1)))/(gamma-1));
end

if Ep_ts(i)>(gamma*tsEp_t)
    tsft(i)= 0;
end

Del_sigma_c(i)=Sigma_c(i)-Sigma_c(i-1);

if i<108
    t(i+1)=t(i)+1;
    Ects(i+1)=tsft(i)/Ep_ts(i);
    y_sh(i+1)=(gamma*tsEp_t-Ep_ts(i))/(gamma*tsEp_t-tsEp_t);
end
end