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# MARKETS AND FIRMS: A SEARCH APPROACH TO SPOT-TRADING AND VERTICAL INTEGRATION

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# Abstract


> “Although economists claim to study the working of the market, in modern economic theory the market itself has an even more shadowy role than the firm.[1]”

In this thesis, we study both firms and markets, but in an environment in which much greater attention is placed on the workings of the market than has been the case heretofore.[2] Specifically, we analyze the make-or-buy decision within a dynamic environment, in which firms search for trading partners, and in which they can choose the optimal ownership form upon contact: either spot-trading or vertical integration.

All of the models considered in this thesis examine the following stylized production technology. During a given production run, a downstream asset produces a unit of final output that is demanded by a final customer. It does so by using a unit of a specialized intermediate input — a widget — that is produced by an upstream asset. The basic technological problem is that the type of widget varies from one production run to the next, and a given upstream asset is incapable of producing every conceivable widget. As a consequence, a downstream asset may be obliged to search on the market to locate a suitable widget. In broad outline, the objective of this thesis is to study the implications of this production arrangement for the make-or-buy decision described above, and hence for the equilibrium organization of the industry.

Chapter 1 begins by providing an overview of the problem, and by summarizing elements of the pertinent literature. Chapter 2 then analyzes market exchange


[1] Coase (1988) p. 7.

[2] The papers in this thesis are co-authored with Derek Laing.

versus vertical integration in the simplest environment: one composed of homogeneous upstream and downstream firms. In this setting, we assume that firms can enter the industry as either independent or vertically integrated entities (i.e., the simple model abstracts from mergers). An upstream asset — including one within a vertically integrated arrangement — cannot produce every type of widget that may be required by a given downstream asset. As noted above, this may force even vertically integrated firms to search for widgets produced by independent upstream firms. In contrast to other papers in the literature, we show that independent and integrated firms can co-exist in a non-knife-edge equilibrium.

In Chapter 3 we then extend the simple model presented in Chapter 2 to allow for asset trading — i.e., takeovers — between independent upstream and downstream firms. In this setting, firms can enter the industry as independent entities. The timing is such that new entrant downstream firms first wait for orders of their customers. After receiving an order, they search for a suitable upstream asset from which to procure a widget. Importantly, upon meeting a given upstream/downstream pair can either simply spot-trade (and go their separate ways) or else carry out a merger. As is in the previous Chapter, the upstream assets of vertically integrated firms cannot produce every conceivable type of widget that may be required by the downstream components. We fully characterize both the equilibrium widget and asset prices (which are determined via a Nash bargaining protocol with equal bargaining weights). Once again we show that independent and integrated firms generically coexist in equilibrium.

Finally in Chapter 4 we introduce heterogeneity into the basic model. The heterogeneity we consider takes the form of different types of firms, and idiosyncratic productivity shocks. With this extension, the co-existence of independent and integrated firms is no longer the primary issue. One of the main findings, in this richer environment, is the demonstration that search theory provides an appealing explanation for the observed "jumps" — both positive and negative — of an acquiring firm's stock-value before, and after integration. As we shall see, an appealing feature of the approach is that a decline in shareholder value is not symptomatic of managerial malfeasance — which is the usual interpretation advanced in the literature.

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# Dedication

To Eunmi with Love

Chapter 1

# Introduction

Although the major part of economic activity stems from an intricate and constantly shifting array of transactions organized within and across the respective boundaries of firms and markets, explaining these patterns is one of the most refractory problems in economic theory. Indeed, it is almost seventy years since Coase (1937) argued that in order to discover why firms emerge at all it is necessary to articulate the relative costs and benefits of conducting a transaction within each of these two spheres of governance. He identifies costs of using the market including search and information cost, and costs of using organization including decreasing returns and increasing governance cost. However, as pointed out by Hart (1989) and Williamson and Masten (1999), Coase's ideas remain to this day very hard to formalize and need elaboration to be operationalized.[1]

Yet, it is not until 1970s that some answers to Coase's question about the make-or-buy decision are seriously developed.[2] Williamson (1971) lays down key ideas that ignite the development of 'transaction cost economics.' He emphasizes the differential incentive and control properties of firms in relation to markets, which he calls "inherent structural advantages (of the firm)." He also considered market transaction costs, including contractual incompleteness, that can be attenuated by substituting firm for market exchange. Alcian and Demsetz (1972) offer another seminal contribution to address Coase's question. They point out that the firm

[1] Hart (1989) p. 1761, and Williamson and Masten (1999) p. xi.

[2] For excellent treatments on the theory of the firm, see Foss, et al (1999), Gibbons (2004), and Hart (1989). See also Holmström and Roberts (1998).

cannot be characterized by the power to settle issues by authority superior to that available in the market. They argue that the difference of a relationship between a grocer and his employee and that between a grocer and customer cannot be addressed based on the power. The grocer can fire or sue the employee on the breech of contract just as the customer can fire or sue the grocer. They propose, instead, that it is team production together with monitoring problem that shapes the firm. In particular, suppose that team production yields an output larger than the sum of separate productions and costs of organizing and monitoring team members. In this situation, one method of reducing shirking of team members is to provide the monitor the following bundle of rights: (i) to be a residual claimant; (ii) to observe input behavior; (iii) to be the central party common to all contracts with inputs; (iv) to alter membership of the team; (v) to sell these rights. This entire bundle of rights defines the ownership of the firm.

However, it is only recently, building upon the seminal contribution of Grossman and Hart (1986), that economists have begun to develop the requisite tools necessary to model these costs in a rigorous, but yet tractable manner. In doing so, the 'property rights' approach has furnished striking insights into the nature of the firm itself by resolving the question of how integration changes incentives.

## The Property Rights Approach

Grossman and Hart identify the firm with the (non-human) assets it owns. By emphasizing the practical importance of incomplete contracts along with the residual control rights conferred by virtue of ownership, the theory not only offers a cogent explanation of the optimal mode of transactions' governance, but also allows a precise articulation of what it means for one firm to be 'more integrated' than another. Vertical (or lateral) integration is equated with the acquisition of residual-rights of control over another agent's assets and it often matters greatly, in terms of investment incentives, which party in a transaction obtains these rights. The loss and surplus from changes of investment incentives relative to non-integration are defined to be the costs and benefits of integration.[3]

## The Market: Search Activity

But what of the "market"? Arguably, the property rights literature is less illuminating on this subject. For instance, Grossman and Hart equate the market (or

[3]Hart (1995) provides a superb introductory discussion of the property rights approach. See also Holmstöm and Roberts (1998) for a brief critical review of property rights approach.

'arms-length competition') with non-integration. While this is a fruitful methodological starting point for examining whether or not two firms will merge or remain separate, it abstracts from many features which appear to be germane to the conduct of actual market transactions. Thus, in Grossman and Hart's framework a market is presumably at work 'behind the scenes' in bringing firms together prior to any agreement on the optimal form of governance structure (whether it is a subsequent merger or arms-length competition). Moreover, in this framework firms that choose not to integrate can (implicitly) instantly and costlessly communicate with each other 'through the market.' Yet, practical market arrangements often entail significant costs in gathering information about the location, nature, and availability of potential trading partners.

On this ground, we claim that the property rights approach, at least so far, cannot be a full answer to Coase's question. In this thesis, we follow the property rights approach in analyzing incentives within and across firms, and in addition, we bring the market out of the shadow by explicitly modeling market transaction costs based on search theory. Firms have to search for trading partners before trade. These search activities incur cost and random delay. We stress that these cost and random delay incurred by search activities can be interpreted in different ways. We provide two interpretations here. Consider you and your friend are first to Washington DC and want to have a dinner at a restaurant on M street. Problem is that there are many restaurants there. Since you don't know which restaurant serves which kind of cuisine, you may first stroll up and down the street to check if they are Italian, French or something else. Luckily, almost all of them put the menu on the window so that people like you can have an idea about what kind of cuisine they serve. If you find something you would like to have, you may enter the restaurant. If you are not satisfied with the restaurants in M street, you may look for other restaurants in the downtown. This represents a search activity involving anonymity. You don't know them about the location and nature, except they are on M street. Suppose that you have found an Italian restaurant and you have really enjoyed the food and mood of the restaurant. Next time you visit Washington DC, you may go strait to that restaurant to have a dinner. However, this does not mean that they can serve you immediately. You may have to wait in a line to be served or they may be closed for a while undergoing a renovation. This represents a search activity involving availability. In either case, search activities incur cost and random delay.[4]

We employ search activities involving both of these two sources – anonymity and availability – that cause cost and random delay to model market ‘frictions.’ In particular, the intermediate input market populated by upstream and downstream firms is characterized by cost and random delays caused by search activities involving anonymity, while the relationship between a downstream firm and a customer is characterized by cost and random delays caused by search activities involving availability — each customer is associated with a downstream firm.[5]

In the following, we briefly discuss related literature.

## Related Literature

Our model is closely related to the recent contribution by Grossman and Helpman (2002). They analyze vertical integration and market transactions in a static matching environment. In their framework, they postulate that independent specialized upstream and downstream firms must search for each other, while vertically integrated firm can procure the intermediate input in-house. They show that if firms are homogeneous vertically integrated firms and independent firms cannot co-exist in a (non knife-edge) equilibrium.

Although their model explicitly incorporates elements of matching frictions, in terms of search costs, it is incongruent with the dynamic search models proposed by Diamond (1982, 1984), Mortensen (1982), and Pissarides (1984). More specifically, although they model search as “costly,” it is instantaneous. As a result, their approach is of little help in modeling the dynamic processes of takeovers; issues which represent the core of this thesis. To give a sampling of what is to come, one implication of our dynamic search approach is that we demonstrate that — even if firms are homogeneous — both vertically integrated and independent firms can co-exist in a (non knife-edge) equilibrium setting.

[4]For another example of availability, consider placing a delivery order for pizza. Even though you know the telephone number of your favorite pizza restaurant, the time interval till you get the delivery is a random variable, especially when there is a bowl game going on on TV.

[5]This is the case in Chapters 2 and 3. In Chapter 4, customers are no longer associated with downstream firms, and downstream firms and customers must search for each other in the final goods market. However, as will be discussed in detail, a matching between a downstream firm and a customer does not necessarily mean that the firm is readily available to fulfill the customer’s order immediately.

As a practical matter, the use of the anonymous market and formal vertical integration defines the two endpoints of a spectrum of possibilities that characterize the means of exchange between upstream and downstream assets. Informal networks (or in Grossman and Hart's terminology "arm's length competition") lie between these extremes, and constitute a common form of organization. In contrast to anonymous spot trading, firms that belong to the the same network know features of the nature of their trading partner and often maintain a long-term relationship with them. Moreover, in contrast to a vertically integrated firm, which controls both of its own upstream and downstream assets, the firms which belong to a network do not enjoy residual control rights over their partners' assets.

There is a small, but growing, literature on networks and vertical integration.[6] For example, McLaren (2000) compares vertical integration and an implicit bipartite complete network. (This means that each upstream firms has a "link" to every downstream firm and vice versa.) He shows that one firms decision on integration is influenced by other firms' decisions through market thickness.[7] Likewise, Kranton and Minehart (2000) compare vertical integration with an efficient network, which is defined to be the one in which those downstream firms with highest valuations obtain the scarce intermediate goods whenever possible. They show that in equilibrium, only one equilibrium industrial structure prevails: vertical integration or a network.

Both McLaren's and the Kranton-Minehart models are static in nature; they treat the network structure as exogenously "given;" moreover, just as in the Grossman Helpman framework, the equilibrium industry structure is characterized by a knife-edge: generically, it either exhibits exclusive vertical integration, or a network structure. This remark points to a serious weakness of the extant literature: in practice, most industries are simultaneously characterized by a dizzying array

[6]There is a considerable body of literature that exclusively focuses on either vertical integration or networks. See for example, Carlton (1979) and Perry (1989) for seminal articles on vertical integration, and Jackson (2003) and Jackson and Wolinsky (1996) for papers that deal with the formation of networks. In the following discussion we focus on the sparser literature that lies at the point of intersection between these literatures.

[7]The way the term 'market thickness' is used in McLaren (2000) is different from ours. Since our model is based on search theory, 'market thickness,' in our model, can be improved in either in upstream firm's perspective or in downstream firm's perspective, or both in upstream and downstream firms' perspectives while in McLaren, it can be improved only in both upstream and downstream firms' perspectives.

of organizational forms that include market exchange, networks, and vertical integration. As we shall see, it also points to a decisive advantage of the dynamic search approach advanced in this paper: search theory provides a simple tractable framework that can simultaneously accommodate a variety of organizational forms in a non-knife edge setting.

Compared to the substantial literature on vertical integration, the literature examining vertical disintegration – or separation – is rather modest in size. Lin (2006) shows that spin-offs can arise in the equilibrium under double Cournot model setting – that is, both the intermediate input price and final good price are determined by Cournot competition. He analyzes an integrated firms decision to enter the market as an integrated or separated entity and shows that spin-off can maximize joint profit if demand for the intermediate input exceeds threshold level. Likewise, Chen (2005) shows a similar result with Bertrand competition in an environment in which firm's decision to vertically disintegrate can be motivated by the presence of both economies of scale in its upstream production and the strategic purchasing behavior of its downstream competitors.

In both of these papers, vertical integration and spin-off are defined as joint profit maximization and individual profit maximization, hence, integration/spin-off does not involve asset trading. In our approach, integration does not mean joint profit maximization, rather it involves ownership structure and induces changes of incentives on search activities à la Grossman and Hart.

However, some papers have examined the integration and spin-off decision in a unified manner. For example, Fluck and Lynch (1999) appeal to the importance of imperfect capital markets. More specifically, they argue that "small" firms that are suffering from the effects of a negative (revenue) shock, may not have access to the means to finance "marginally" profitable projects. As a consequence, they may enter into a conglomerate merger to secure the appropriate financing. Thus, they view a merger as a means of circumventing financial constraints. Nevertheless, as the shock reverses itself, and as their profitability improves, the financing synergy ends and the acquiring firm then divests the assets under its control. Shleifer and Vishny (1989) argue that a manager may elect to acquire specific assets, diversifying the firm in the process, if this renders him indispensable to his employer. They argue that disintegration occurs if a change in the environment leads to a situation in which the acquired assets no longer provide him with entrenchment benefits.

Both of these models focus on the strategic aspects of the integration/spin-off decision. In Shleifer and Vishny these activities serve the strategic interests of managers; in Fluck and Lynch they allow agents to exploit a particular financing technology. Our approach, although lacking the spin-off part, is quite different. Specifically, we assume that firms are homogeneous and that the ownership structure does not change the technology per se (although it might affect the choices that agents make).

Another interesting approach has been proposed by Nöldeke and Schmidt (1998). They analyze an environment in which investments are made sequentially on an asset and are not contractible between the involved two firms. They show that an option-to-own contract — a contract that gives one party a right to buy the asset at a fixed price at some date — can induce efficient ex ante investments in this environment. As a consequence, the asset is owned by one firm and then by the other whoever performs the investment.

Although related to their model in the sense that ownership structure changes incentives, our approach focus on quite different issues. We analyze a make-or-buy decision in a dynamic environment, while they focus on whether a specific option contract can induce efficient investments.

## Stylized Facts

In practice, vertically integrated firms often sell/buy their products to/from other firms. For example, Samsung Electronics produces memory chips. Some of them are used for its own needs in order to produce mobile phones, MP3 players, notebook computers, and appliances, etc. However, it sells most of its chips to other companies. Recently, Samsung Electronics agreed to provide flash memory chips to Apple, Inc. which produces the MP3 player 'iPod nano.' Apple's MP3 player is in direct competition with Samsung Electronics' own MP3 player, 'Yapp.' LG-Phillips LCD produces TFT-LCD. LG Electronics, its mother firm, is one of its most important customers. Nevertheless, it sells most of the production of TFT-LCD to other firms: Dell, HP, Lenovo and Toshiba, to name but a few. Vertically integrated firms also use networks. For instance, they build networks or alliances for R&D purposes. The pharmaceutical industry is famous for its networks and alliances.[8] The Auto industry provides a par excellence illustration of alliances between companies for the purposes of: R&D, engine design and general manufacturing. Thus, GM and Toyota started a research collaboration in 1999. DaimlerChrysler, Hyundai and Mitsubishi have recently established a joint venture company, Global Engine Manufacturing Alliance for the design, development, and manufacturing of gasoline engines.

Generally speaking, vertically integrated firms often sell/buy their intermediate inputs — "widgets" to/from other firms. In this thesis, we will take these phenomena as "givens," by imposing the restriction that vertically integrated firms cannot produce every conceivable widget internally. As a consequence, sometimes they must procure certain widgets from other independent upstream firms. In broad overview, the primary goal of this thesis is to explore the implications of this restriction on the form of the industry's equilibrium organizational structure.

## The Production Technology

Although this is not the place to delve into the formal details of the models presented in this thesis, a brief overview of the production technology is in order to help providing the reader with a useful reference point. Throughout, we will have recourse to speak of assets, firms, markets, integration and so on and so forth, with a variety of extensions and embellishments developed on the fly. Nevertheless, the kernel of our argument is simple enough. Specifically, throughout we focus on a production technology in which:

- From time to time, final customers demand, and consume, a unit of a final product.
- The final product is manufactured by a downstream asset by using a unit of a specialized intermediate good — a "widget."
- The widget is produced by an upstream asset
- Both downstream and upstream assets require the labor services of a single manager.

[8] Baker, et. al. (2004) provides an analysis of nearly 12,500 strategic alliances in pharmaceutical and biotech industry.

Following the property-rights literature, we define a firm to be a collection of assets – upstream and downstream – over which an owner (or group of owners) enjoys residual control rights. Hart (1995, p.30) has defined these rights as:

> "[T]he owner of an asset has ***residual control rights*** over the asset: the right to decide all usages of the asset in any way not inconsistent with a prior contract, custom, or law. In fact, possession of residual control rights is taken virtually to be the definition of ownership." [emphasis in the original.]

The heart of our thesis is the focus on the precise mechanism whereby the downstream asset procures the widget that it requires to produce the final output.

At one extreme, we will consider anonymous spot-markets in which it must undertake a costly and time consuming process of search. At the other, we consider the case in which it vertically integrates with an upstream asset and may produce the asset "in house."

The reason that we view this as an interesting problem is that the equilibrium organization of the industry both influences, and is influenced by, the options available in the market. In a nutshell, on the one hand, the modern theory of the firm has by and large ignored the equilibrium volume of market transactions. On the other hand, the modern theory of markets — as exemplified by the search approach pioneered by Diamond, Mortensen, and Pissarides, has largely neglected issues pertaining to the boundaries of the firm. It is the primary goal of this thesis to advance some simple models that can bridge this gap, and hence bring both the firm, and indeed, the market out of the shadows.

## Overview of the Chapters of this Thesis

In this thesis, we study both 'firms' and 'markets,' but in an environment in which much greater attention is placed on the workings of the market than has been the case heretofore. In Chapters 2 and 3, we first analyze the role played by markets and vertical integration in facilitating production and exchange.

Chapter 2 itself begins by presenting a simple model which, for convenience, abstracts from asset trading (i.e., from mergers and takeovers). We assume that a downstream firm can enter the industry either as an independent entity, or as an integrated one, with an upstream asset in tow.

The environment is one in which final customers — at random intervals of time — place an order for a unit of output that is produced by the downstream asset. Customers' tastes are "specialized," and idiosyncratic. Accordingly, when a customer places an order for a product, they demand a particular type of good or service, rather than simply a generic one. Crucially, in order to manufacture this particular product, the downstream asset must locate a suitably specialized intermediate input — a "widget" — produced by an upstream asset. In contrast to previous static models in the property-rights tradition, in our framework the customer-firm relationship is enduring: a given firm may produce many times for its customer.

The basic technological problem is that each upstream asset can produce only a subset of conceivable widgets. (Indeed, the probability that it can produce a given randomly drawn widget, provides a natural metric of the asset's manufacturing flexibility.) It follows from this simple technological limitation that there are circumstances under which even a vertically integrated downstream/upstream asset pair may find itself in a "fix" and be unable to immediately produce the final customer's given order. Under these circumstances, it must use the market to search for an independent upstream firm that is capable of producing the desired widget. Once it finds such a producer, it can procure the widget through an arm's length transaction (i.e., spot-trading in the market).

The core result of this Chapter is the demonstration that both vertically integrated firms and independent downstream firms can co-exist in a non knife-edge equilibrium. This hybrid equilibrium arises for the following reason. Suppose, for the sake of argument, that it is optimal for all downstream assets to enter the industry as integrated units. In this case, it is also optimal for some independent upstream firms to enter the industry, since they may be able to produce widgets for integrated firms that cannot produce themselves. Therefore, pervasive integration does not 'dry up' the independent upstream firms in the widget market. Further, and this is the key, widget market is populated by relatively small mass of downstream assets since integrated assets that can process the order in-house stay out of the widget market. For some cases, this make the widget market 'thick' (in downstream firms perspective) to the extent that downstream assets have incentive to enter the industry as independent entities. This "breaks" the proposed equilibrium in which only integrated downstream units enter the industry. Suppose, on the contrary, that the industry exhibits pervasive spot-trading in which all downstream assets enter the industry as independent units. Then the widget market is populated by relatively large mass of downstream assets since every downstream with active order participates in the widget market. For some cases, this make the widget market thin (in downstream firms perspective) to the extent that downstream assets have incentive to enter the industry as integrated entities. The result of both of these considerations is the hybrid equilibrium, which is characterized by the coexistence of independent and integrated firms.

In Chapter 3 we extend the model in an important direction, by admitting the exchange of assets — i.e., mergers and takeovers — between independent upstream and downstream firms. In contrast to the previous Chapter, where firms could choose to enter the industry as either "pre-formed" integrated (or, indeed, independent) entities, in this Chapter both upstream and downstream assets enter as independent firms. Nevertheless, a downstream firm can elect to search for an independent upstream firm, and offer to purchase the upstream asset from the owner. (Incidently, in the interests of keeping the discussion manageable, the environment is constructed in a manner that it is only in the interests of downstream assets to acquire upstream assets. However, as we discuss in some detail, it is relatively simple to extend the model to encompass takeovers in which upstream assets acquire downstream assets as well.)

A downstream firm can only enter the industry as an independent firm. If it wants to integrate, it has to search for an independent upstream firm and buy out the upstream asset from the owner. We show, again, that (generically, and depending upon parameter values) there are three types of possible equilibria: (i) pure-market exchange (i.e., no integrated firms are formed in the market), (ii) pervasive integration (in which every independent downstream asset forms a vertically integrated firm upon meeting an upstream asset), and (iii) a hybrid equilibrium, which is characterized by the existence of both vertically integrated and non-integrated firms.

Our approach offers insights into recent interesting papers by Mullainathan and Scharfstein (2003) and Holmes (1995). Mullainathan and Scharfstein (2003) stress that the existing literature provides an inadequate explanation for why, within a given industry, some firms are integrated while others are not. Holmes (1995) finds a strong positive correlation between "localization" (i.e., proximate access to many upstream producers — California's Silicon Valley, being a prime case in point) and the extent of vertical disintegration in the industry. As we argue, our search model sheds some light on the reasons that an industry can exhibit these diverse ownership structures.

In Chapter 4 we present a significant extension of the model by introducing heterogeneity and uncertainty. More specifically, we consider an environment populated by different types of downstream assets, and posit idiosyncratic shocks that affect the quality of the final good. Using this richer setting we can explore the evolution of firms' (stock-market) values over time, and characterize the effects of mergers on them. Under conditions of heterogeneity, establishing the equilibrium coexistence of independent and integrated firms is no longer a major concern: the primary lacuna in the literature has been establishing this phenomena under conditions of homogeneity. In the presence of heterogeneity, firms optimally choose their ownership structures, after searching, based on their types and the realization of the idiosyncratic shock.

To begin with, we assume that independent downstream firms are one of two types: 'bad' for integration and 'good' for integration (the details are spelled out in the analysis). Every downstream asset enters the industry as the 'bad' type. Nevertheless, their type can switch from 'bad' to 'good' during the time they are searching for a customer. This formulation is arguably the simplest environment for characterizing a rudimentary learning environment. More specifically, downstream firms are initially inexperienced when they enter the industry. However, the longer the time that has elapsed since their birth, the greater is the likelihood they will switch to the "good" state; a state in which they at last have learned how to most effectively operate the asset. Independent downstream firms which are seeking to process a customer's specific order, search for an available upstream firm. Once the search process is resolved, the upstream and downstream firm learn the value of the idiosyncratic shock that characterizes their relationship. Based upon this realization, the two parties then agree to the ownership form — spot-trading or integration — that will govern their relationship, and the exchange of the widget.

The value the customer places on the final good depends upon the realization of the idiosyncratic shock. We demonstrate that the optimal merger decision is characterized by a reservation property that varies between the two types — i.e., "good" and "bad" — of firm. If the value of the idiosyncratic shock exceeds this value, the firms integrate; if it does not, they remain independent entities.[9] Our model exhibits positive "jumps" in the acquiring firm's value before integration and both positive and negative jumps after integration. We show that search theory provides an appealing explanation for them.[10]

The intuition for this result is simple, and follows directly from the reservation property. The ex ante value of an acquiring firm depends upon the conditional expectation of the distribution of ex post merger valuations. Nevertheless, ex post it is perfectly conceivable that a pair of firms will draw a value of the idiosyncratic shock that lies above the reservation value, rendering merger optimal, but below the conditional mean of the return distribution, and as a result "disappoint the market." In this event, there is an ex post decline in shareholder value.

We regard this result as one of the principle findings of this thesis. Up to this point, explaining the loss in shareholder value precipitated by mergers has proven to be an especially refractory problem. Indeed, the few studies that we are aware of that have broached the problem have invariably regarded it as stemming from inefficient managerial "empire building."[11] Nevertheless, within the context of the dynamic search approach developed in this thesis, the reduction in shareholder value is perfectly consistent with ex ante optimal, and indeed proper managerial oversight.

Summarizing, our approach is complementary with and builds upon the property rights conception of the firm expostulated in Grossman and Hart (1986) and in Hart and Moore (1990). While incomplete contracts and residual control rights determine the firms' optimal choice of ownership structure from among: spot-trading and mergers, our key point of departure is that we view the 'market' through the

[9] Jennings and Mazzeo (1991) report that among 472 proposed acquisitions during 1979 - 1987, about a quarter (121 proposals) of them are canceled.

[10] As observed by Andrade, et. al (2001), during 1990-98 (using 1,864 observations on mergers), over a three day event window (i.e., from one day before, to one day after the merger announcement) the average abnormal return experienced by the acquiring firm is -1.0%. If the window is expanded to from 20 days prior to the announcement to the completion of the merger, the average return is -3.9%. Moreover, these returns are not statistically significant at 5% level. This implies that acquiring firms experience large variance in returns.

[11] See for example, Jordan [38].

conceptual lens of dynamic search theory as first promulgated in the pioneering works of Diamond (1982), Mortensen (1986), and Pissarides (1984).[12] To the best of our knowledge, the previous literature on the equilibrium industrial organization structure has been largely been conducted within static environments.[13]

The emphasis of this thesis is that the relationships between firms are on-going dynamic phenomena. Crucially in this richer dynamic environment, an industry can exhibit a diversity of firm structures in equilibrium.

In order to ease the burden for the reader in keeping track of the notation, below we provide a list of symbols that will be used in this thesis. Here, we provide only the refined notation, which is stripped of the subsequent potpourri of indices and superscripts and subscripts developed in the text. In Appendix B, we spell out all of the notation used in the thesis. Where possible, we have chosen standard notation from the search literature. Moreover, we have sought to ensure that the other notation we use is relatively transparent, and we hope easy to remember. (For example, we use g for the “good” state and b for the “bad” one.)

**Numbers**

| | |
|---|---|
| 0 | without customer order (as in subscript) |
| 1 | with customer order (as in subscript) |

**English Alphabet (lower case)**

| | |
|---|---|
| $b$ | value of a bad type independent downstream asset |
| $c$ | (integration) maintaining cost (flow) |
| $d$ | mass of independent downstream assets |
| $e$ | search cost (flow) |

[12] Search can be viewed as investment that is not contractible.

[13] Of course, there are numerous dynamic models in the industrial organization literature: the Jovanovic (1982) and Hopenhayn (1992) models of industry evolution are two prime cases in point. Nevertheless, these models have taken the firm’s boundaries as given. The focus of this thesis is understanding the nature of these boundaries in a dynamic general equilibrium framework.

| | |
|---|---|
| $g$ | value of a good type independent downstream asset |
| $i$ | integration (as in subscript) |
| $m$ | matching parameter |
| $n$ | number of types of upstream assets or non-integration (as in subscript) |
| $p$ | widget price |
| $q$ | quality of final good |
| $r$ | time preference |
| $u$ | mass of independent upstream assets |
| $z$ | proportion/composition of independent downstream assets or probability to choose spot-trading |

**English Alphabet (upper case)**

| | |
|---|---|
| $B$ | value of a bad type integrated asset |
| $C$ | mass of customers |
| $D$ | mass of integrated assets |
| $G$ | value of a good type integrated asset |
| $K$ | value of an integrated asset |
| $M(\cdot)$ | matching function |
| $R$ | reservation value for integration |
| $U$ | mass of actively searching upstream assets |
| $V$ | value of an independent downstream asset |
| $X$ | proportion of widget type that an upstream asset can produce |

**Greek (lower case)**

| | |
|---|---|
| $\alpha$ | rate at which an upstream asset meets a downstream asset |
| $\beta$ | rate at which a downstream asset meets an upstream asset |
| $\gamma$ | rate at which a bad type downstream asset changes to a good type downstream asset |
| $\delta$ | asset's evaporating/death rate |
| $\eta$ | rate at which a customer meets a downstream asset |
| $\theta$ | idiosyncratic shock |
| $\kappa$ | widget production cost |

| | |
|---|---|
| $\lambda$ | rate at which a customer retracts order or leaves/separates |
| $\mu$ | rate at which a downstream asset meets a customer or gets an order |
| $\nu$ | customer's valuation on final good |
| $\pi$ | defined as $(\nu - \kappa)/2$ |
| $\rho$ | price of an upstream asset |
| $\tau$ | market tightness in downstream asset's point of view |
| $\chi$ | indicator function |
| $\omega$ | type of widget/final good |

**Greek (upper case)**

| | |
|---|---|
| $\Pi$ | value of an upstream asset |

Chapter 2

# Market Exchange vs. Vertical Integration: A Simple Model

In this Chapter we present a simple "barebones" model that is designed to flesh out the intuition on the major elements of the search approach to the make-or-buy decision advanced in this thesis. To this end we consider an industry that is populated by (an endogenous mass of): (i) homogeneous independent upstream and downstream firms, and (ii) homogeneous vertically integrated firms.

We assume that a downstream firm can choose to enter the industry as an independent entity, or as a vertically integrated firm, with an upstream asset in tow. In other words, in this Chapter we consider a simple environment in which we abstract from ex post changes in asset ownership. (These and other related issues are the focus of the latter parts of this thesis.) Moreover, in the interests of simplicity, in this Chapter we posit that a vertically integrated firm incurs an exogenous flow cost; an independent firm does not. This is intended to capture the costs of ownership/governance — à la Grossman and Hart — in a rudimentary, and tractable, manner.[1]

The object of the exercise is to characterize the equilibrium industry structure, and, in particular, isolate the economic determinants of the "mix" between independent and vertically integrated firms.

The production process we consider is precisely that described on page 8. To recap, at random intervals of time a customer places an order for a (specialized)

[1]McLaren (2000) and Grossman and Helpman (2002) take a similar short cut.

single unit of a final product from a downstream asset/firm. In order to produce this output, the manager of the downstream asset requires a correspondingly specialized intermediate input — a “widget” from now on. Widgets are produced by upstream assets. The basic technological problem is that a given upstream asset is incapable of producing every conceivable type of widget.[2] The main “bite” of this restriction is that to satisfy the customer’s order, sometimes even vertically integrated firms must use the market to procure widgets from upstream firms through spot-trading.

One of our main findings is that although the firms are ex ante homogeneous, there exists a non-knife edge equilibrium in which independent downstream firms and integrated firms co-exist. Thus, the equilibrium is characterized by both the active market exchange of widgets, and by their internal production by integrated firms. In this sense, our simple framework captures the endogenous boundary between the market and the firm.

We regard the result of a non-knife edge equilibrium — under conditions of homogeneity — as an important one. Previous studies (vide supra page 4) have invariably uncovered knife-edge cases, in which, depending upon parameter values, all firms are either integrated or independent. Now, the reason this result is interesting is that it is indicative of the fact that the dynamic search framework considered in this thesis possessed certain inherent stabilizing forces that serve to bound the equilibrium away from these extreme points (viz., complete integration or market exchange.)

The intuition for the result is as follows. Suppose, for the sake of argument, that it is optimal for all downstream assets to enter the industry as integrated units. In this case, it is also optimal for some independent upstream firms to enter the industry, since they may be able to produce widgets for integrated firms that cannot produce themselves. Therefore, pervasive integration does not “dry up” the independent upstream firms in the widget market. Further, and this is the key, widget market is populated by relatively small mass of downstream assets since integrated assets that can process the order in-house stay out of the widget market. For some cases, this make the widget market ‘thick’ (in downstream firms

[2]Milgrom and Roberts (1990) propose an interesting model of flexible manufacturing. In our framework, the flexibility of an upstream asset’s production capabilities are characterized by the probability that it can produce a given, but randomly drawn, widget type.

perspective) to the extent that downstream assets have incentive to enter the industry as independent entities. This "breaks" the proposed equilibrium in which only integrated downstream units enter the industry. Suppose, on the contrary, that the industry exhibits pervasive spot-trading in which all downstream assets enter the industry as independent units. Then the widget market is populated by relatively large mass of downstream assets since every downstream with active order participates in the widget market. For some cases, this make the widget market thin (in downstream firms perspective) to the extent that downstream assets have incentive to enter the industry as integrated entities. In this situation, we will show that there must exist a hybrid equilibrium.

One of the attractive features of our general (search-) equilibrium structure is that it sheds new light on vertical foreclosure issues considered in the I.O literature. Many of the studies in the literature have constructed partial equilibrium models, in which (say) a given downstream asset purchases a given upstream firm for both strategic and "supply assurance" concerns.[3] Nevertheless, in the general-(industry) equilibrium considered in this thesis, the limitation of this argument is plain enough to see. The mass of upstream firms is not fixed; rather it is determined as part of the equilibrium. The reason that this is interesting is that the integration of upstream and downstream assets does not "dry up" the supply of independent upstream firms. On the contrary, it engenders conditions that actively encourage their entry (i.e., they face a higher widget price when they trade with integrated firms relative to independent downstream firms). As a corollary, the robust supply of independent upstream assets also encourages entry by independent downstream firms. The result is an industry populated, in equilibrium, by active masses of integrated and independent firms.

## Overview

Before presenting the formal details of our model, we present a simple overview of the framework that is designed to help the reader gain his or her bearings.

The primary elements of the framework are: (i) an industry populated by masses of independent and/or integrated upstream/downstream assets, (ii) entry/exit from the industry, (iii) a production technology (of the sort outlined on

[3]For example, Bolton and Whinston (1993), and Choi and Yi (2000).

page 8), (iv) consumers who demand the industry's final product, and (iv) a market wherein widgets are produced and sold by upstream assets to downstream assets that use them in production. The basic setup is one in which:

⧫ There is an ocean of potential entrepreneurs ("owners"), managers, and productive (upstream/downstream) assets which are external to the given industry.

⧫ Each owner can instantly enter the industry if it is profitable for him to do so. He can bring to the industry either: (a) an independent upstream or downstream asset, or (b) an integrated upstream/downstream asset pair.

Remark: In order to keep the analysis simple, and indeed tractable, we (endogenously) rule out cases in which an owner enters the market with (say) eight upstream and twenty two downstream assets. Our focus on independent and unified upstream-downstream asset pairs is the minimum requirement for any (sensible) analysis of the make-or-buy decision. More specifically, even within this simple framework, it is part of the action space for owners to enter the industry exclusively as independent or as integrated firms. In other words, the restriction neither precludes pure-market exchange among independent firms nor pure internal production by integrated ones.

⧫ Each downstream asset — whether integrated or independent — is associated with a single final customer. The customer places an order for a (specialized) unit of the final product at random intervals of time.

⧫ With an exogenous probability, customers retract their order, which leads to the secession of the firm's search efforts (at least until the customer places another order). In this case, the firm exits the industry to find another customer.

Remark: The customer's retraction rate provides a parameter that usefully captures the "urgency" with which a downstream asset must produce the given order.

⧫ Upon receiving a customer's order, an independent downstream firm must search for an upstream asset that is capable of producing the specialized widget. (The upstream asset may belong to an independent firm, or belong to an integrated one.) As for an integrated firm, with some (exogenous) probability, it can manufacture the widget internally. If it cannot, it must use the market to search for an upstream asset which can do so. Therefore, the time interval to fulfill an order is a random variable. This assumption is intended to capture, in a rudimentary fashion, the intuition that, while we can all use the "yellow pages" (or the Web) to locate a potential producer, we do not know ex ante whether a given producer is available to immediately produce the good or service we desire.

Remark: As a heuristic, think of calling a cab or placing a delivery order for pizza. The telephone numbers are easy enough to track down. Nevertheless, experience tells us that the time to service is a random variable.

⧫ The market search process is directed — but time consuming. In other words, downstream assets know the broad region to find the widget but not the exact place. Therefore, the time interval required to find an upstream asset is a random variable. As mentioned in Chapter 1, the widget market is characterized by cost and random delay caused by search activities involving anonymity.[4]

Figure 2.1 on the following page illustrates the widget market and the industry. The terms $\boldsymbol{u}$, $\boldsymbol{d}$ and $\boldsymbol{c}$ denote: upstream assets, downstream assets, and customers, respectively. The solid line between $\boldsymbol{u}$ and $\boldsymbol{d}$ denotes integration. The solid (dotted) line between $\boldsymbol{d}$ and $\boldsymbol{c}$ indicates that the customer has placed (not placed) an active order. The cut ("X") line between $\boldsymbol{d}$ and $\boldsymbol{c}$ indicates that the customer has abandoned the firm and has forced the asset to exit the industry.

So much for the preamble. In the next three Sections we present the formal details of the model. There are three categories of assumptions that we make: assumptions about the primitives ("tastes and technology"); the matching process; and the contractual framework/bargaining protocol. We consider each of them in turn.

[4] Note that availability also plays an important role within an integrated firm in the sense that the (integrated) upstream asset cannot produce every type of the widget.

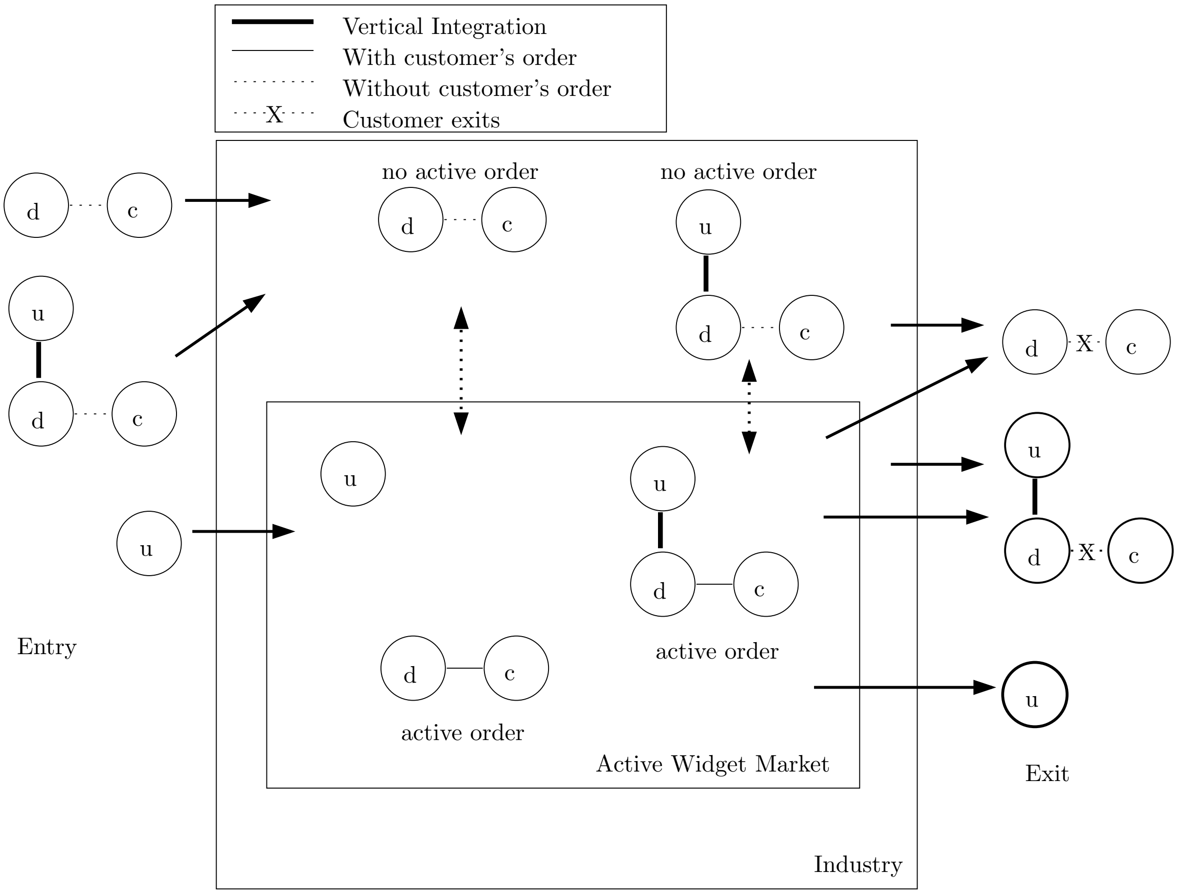


Figure 2.1. Widget Market and Industry – Simple Model

## 2.1 The Model

Time is continuous. The economy is populated by three types of infinitely lived agents: managers, owners and customers. Each type of agent is risk neutral and discounts the future with the common time preference rate $r > 0$, which equals the market rate of interest.

### 2.1.1 Goods, Assets, and Markets

There are two categories of indivisible goods: widgets and final goods, which are produced using upstream and downstream assets respectively.[5] Both widgets and final goods are of a specialized type, and are indexed by the (common) index: $\omega \in [0,1]$. In what follows, the generic widget $\omega_0 \equiv 0$ plays a special role. The widget market is populated by actively searching upstream, and downstream assets (in a variety of organizational configurations).

### 2.1.2 Customers

Each consumer is risk neutral, and is associate with one of the downstream assets (and each downstream asset is uniquely associated with one consumer). At a given point in time each consumer belongs to one of two states: "active" and "passive." When active he derives the common utility $v$ by consuming a unit of the appropriate final good (see below), and upon doing so enters the passive state. Each consumer switches from the passive to the active state with flow probability $\mu$. From the (downstream) firm's perspective $\mu$ captures the flow rate of demand for its product, and $v$ captures the consumer's willingness to pay for it.

Upon making the transition from the passive to the active state, each customer derives utility $v$ by only consuming a unit of a particular final product $\omega$. The type of final good desired by the consumer, $\omega$, is drawn randomly, from a uniform distribution, with support $(0,1]$. Consumers derive utility $v_0 = 0$, if they consume either the "generic" good $\omega_0$, or a good that is not to their liking. In this case, they abandon the firm that provided it.

Each consumer switches from the active to the passive state with flow probability $\lambda$. This implies that delays in production — and hence consumption — are potentially costly to firms, as the customer may change his state before they can complete his order. This generates some urgency on the part of suppliers to quickly meet the demands of their customers.

[5] Intuitively, an asset is the specialized capital (machines/buildings) used as cooperative factors to produce widgets and goods.

### 2.1.3 Owners and Managers

The economy is populated by a large number of infinitely lived homogeneous individuals, who are classified as either owners or managers. All individuals are risk neutral; they discount the future at the common rate $r$; they each possess an indivisible unit of labor; and are all equally talented. Indeed, the only economic distinction between a manager and an owner is that an owner is endowed with residual control rights over (i.e., owns) one or more assets; a manager is not. Owners can, if they so desire, choose to manage one of their own assets. They can even work as a manager for an asset owned by another individual.

Managerial labor is used to carry out search and production activities. All of the costs borne by a manager are in the form of the disutility of effort. Consider:

Assumption 2.1 For goods/widgets: $\omega \in (0,1]$:

(i) (Search.) The flow disutility borne by the manager of:

(a) an (u)pstream asset, who searches for a buyer of a (w)idget $\omega \in (0,1]$ is: $e^u_w > 0$.

(b) a (d)ownstream asset, who searches to buy a widget of type $\omega \in (0,1]$ is: $e^d_w > 0$

(ii) (Production.) The instantaneous disutility borne by a manager who uses his labor unit with the cooperation of:

(a) an upstream asset to produce a widget of type $\omega \in (0,1]$ is: $\kappa$.

(b) a downstream to produce a good of type $\omega \in (0,1]$ is (normalized) to zero.

(iii) (The generic good/widget.) The generic good/widget, $\omega_0$, imposes zero disutility either in search or in production. It can be instantly bought or sold on a competitive market, at a price that is normalized to zero.

Remark. The generic widget, $\omega_0$, plays a useful role in what follows by (in conjunction with the informational assumptions described below) endogenously precluding certain types of incentive mechanisms that undermine the thrust of the property rights arguments presented in this thesis.

Notice that all the costs incurred by managers are in the form of the disutility of effort. For simplicity, in this Chapter we consider the case in which $e_w^d \to 0$.[6] There exists a competitive market for managers and each manager can be replaced instantly, and without cost (i.e., employment is at will). Finally, manager's reservation utilities are normalized to zero.

### 2.1.4 The Technology

For simplicity, the only production costs are in the form of managerial disutility (see Assumption 2.1 on the previous page). The production technology is such that an upstream asset can be used to instantly transform one unit of managerial labor into one unit of a non-storable widget $\omega$. Nevertheless, an upstream asset is capable of producing from only among a restricted set of widget types. In other words, the production technology does not exhibit complete flexibility. To capture this in the simplest way, we assume that there are: $1,2,...,n$ different types of upstream assets. Each type is capable of producing only a measure (interval) of widget types $X \equiv 1/n \in \mathbb{Q}$. After arranging the widget types in a circle upstream assets of type 1 can produce specialized widgets in the interval: $(0,1/n]$; upstream assets of type 2 can produce widgets in the interval: $(1/n,2/n]$, and so on and so forth for $3,...,n$. We impose symmetry across upstream assets, in that an equal mass of upstream assets that can manufacture a given widget type $\omega'$ for all types $\omega \in (0,1]$. Finally, every upstream asset is capable of costlessly producing the generic widget $\omega_0$.

There is only one type of downstream asset. It can, by using a unit of managerial labor, instantly transform one unit of widget of type $\omega$ into one unit of non-storable final good of type $\omega$. This convention explains why both widgets and final goods can be indexed by the common index $\omega$. For simplicity, the production cost of final goods is normalized to zero.

We assume that a given owner can own at most two assets: an upstream asset, or a downstream asset, or an upstream asset and a downstream asset. The

[6] As we shall see, this assumption implies that non-owner managers do not actively search in the widget market. More specifically, the assumption that $e_w^d > 0$ implies that managers strictly prefer not to search. The assumption that $e_w^d \to 0$ merely simplifies the form of the Bellman equations presented below.

owner cannot own two (or more) upstream/downstream assets. Although this can easily be derived as a result (using an appropriate form of the arguments advanced by Grossman and Hart (1985)), we prefer to impose it as a primitive to avoid precipitating a discussion that is tangential to the arguments presented here. The effect of the assumption is to keep the state space (characterized by various organizational structures) manageable.

Since each asset requires the labor services of one manager, the owner of an integrated upstream/downstream asset pair can manage at most one of the assets he owns (of his choosing). Nevertheless, he requires the labor services of another manager to run the remaining asset. We assume that an owner of two assets cannot "chatter" back and forth between managing one asset for a while, and then the other. Instead, as noted above, owners select which (if any) asset they intend to manage, and are irrevocably bound to their choice.[7] Finally, we assume that the output produced by a given asset belongs to the owner of that asset.

Congruent with the tenets of the property right's literature, the technological properties of neither upstream nor downstream assets are affected by the type of ownership structure, whether of the integrated or non-integrated varieties.

#### 2.1.4.1 Vertically Integrated Firms

A vertically integrated firm comprises an upstream and a downstream asset. Both assets are owned by a single owner. There are no technological differences between vertically integrated assets and independent upstream and downstream assets, except that the owner of a vertically integrated asset must pay a flow cost $c_1^v$. This cost represents, in a rather simple way, some disadvantage of integration, such as: foregone investment opportunities, loss of managerial incentives, a governance cost, and so on.[8]

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[7]Heuristically, the owner in question might be thought of as acquiring ex ante the (costly) specific human capital that is required to run the asset of his choosing. Parameters are assumed to be such that it is optimal to acquire only one type of human capital. In passing, the motivation for being explicit on this point is that absent such a restriction, owners could dispense with managers altogether, and run the firm by themselves. In practice, this is not too implausible. The typical CEO of a large company could not fire everyone, and run the firm singlehandedly.

[8]A similar "reduced form" approach to modelling the costs of integration is adopted in McLaren (2000). See also Grossman and Helpman (2002).

#### 2.1.4.2 The Entry and Exit of the Firms

We assume that upstream firms can costlessly and instantly enter the industry (i.e., there is a perfectly elastic supply of upstream assets).[9] The total (exogenous) mass of downstream assets is normalized to one. The owners of downstream assets can enter the industry as either an independent downstream firm, or as an vertically integrated firm (with an upstream asset in tow). We assume that upstream and downstream assets are perfectly durable. (It is simple enough to relax this assumption, and to admit depreciation).

## 2.2 Matching

We assume, as is common in the search literature, that all meetings between upstream and downstream assets are pairwise and anonymous.[10] Let $U_w$ and $D_w$ denote the mass of upstream and downstream assets actively searching in the widget market. The upstream/downstream assets in question may belong to either integrated or non-integrated firms.

The search process in the intermediate goods market is represented by a standard homogeneous of degree one matching function.

$$\alpha U_w = \beta D_w = m_w \cdot M_w(U_w, D_w) : \quad \text{(SS)}, \tag{2.1}$$

where $m_w > 0$ is a parameter that captures the efficacy of the matching process. The flow probabilities $\alpha$ and $\beta$ play an important role in what follows, and will be determined as part of the equilibrium of the model. Here, $\alpha$ is the flow probability that each upstream asset locates a downstream asset. Similarly, $\beta$ is the flow rate at which each downstream asset meets an upstream asset.[11]

Since meetings are pairwise, the first equality on the left of Equation (2.1) says

[9] Of course, this does not imply that the industry will be inundated by firms that include an upstream asset. Recall that operating the asset is costly, so that entry may not be profitable.

[10] The anonymity assumption precludes the emergence of explicit — or indeed implicit — long-term relationships between independent firms.

[11] More formally, $\alpha$ and $\beta$ are the parameters of respective Poisson processes that govern the matching between the two asset groups. Accordingly (for example), over small time interval $dt$, the probability that an upstream asset meets a downstream asset is: $\alpha \cdot dt$. Likewise the probability that an upstream asset meets a downstream asset is $\beta \cdot dt$.

that the mass of upstream assets that meet downstream assets must equal the mass of downstream assets that meet upstream assets. The second equality says that the flow contact rate is governed by a matching technology. We impose the following (standard) restrictions on $M_w(\cdot)$.

Assumption 2.2

1. $M_w(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable
2. $M_w(\cdot)$ is homogeneous of degree one
3. $M_w(\cdot)$ satisfies the Inada conditions:

$$\lim_{W\to 0} \partial M_w/\partial W = \infty, \lim_{W\to\infty} \partial M_w/\partial W = 0 \text{ for } W \in \{U_w, D_w\}$$

   and boundary conditions:

$$M_w(0, D_w) = M_w(U_w, 0) = 0.$$

It is important to stress that equation (2.1) describes the contact rate between actively searching assets. The substance of this remark is that: (i) non-searching assets do not locate trading partners, and (ii) conditional upon search, the flow probability with which an asset locates a potential trading partner does not depend upon whether the asset belongs to an independent or integrated firm. This is, of course, important, and is congruent with the basic tenets of the property rights literature. Specifically, the nature of the governance relation should not affect deep technological parameters — and hence the feasible action space; rather it should affect choices made from this action space.

Finally, recall that there are $n \in \mathbb{N}$ different types of upstream asset. We assume that each type is clustered in a specific "region," so that a searching downstream firm knows where to find an appropriate widget. (Nevertheless, the time to service is a random variable, and is governed by the matching process described above.)

Equation (2.1) and Assumption ?? impose limitations on the admissible set of contact rates $\alpha$ and $\beta$ that are consistent with any steady-state outcome (equilibrium or not). Accordingly, Lemma 2.1 presents one of the most important constructs in this thesis: the SS locus. Consider, Lemma 2.1 Equation (2.1) and Assumption 2.1 imply that $\alpha$ and $\beta$ are related according to the SS locus:

$$\alpha = \alpha^{SS}(\beta;\ m_w) \tag{2.2}$$

where: (i) $\partial\alpha^{SS}/\partial\beta < 0$; (ii) $\alpha^{SS}(\cdot)$ is strictly convex to the origin in $\beta$; (iii) $lim_{\beta\to 0}\alpha^{SS} = \infty$, $lim_{\beta\to\infty}\alpha^{SS} = 0$; and (iv) $\partial\alpha^{SS}/\partial m_w > 0$.

Proof. See Appendix A ■

The SS locus characterizes the $(\alpha, \beta)$ pairs that are consistent with steady-state behavior. A glance ahead at Figure 2.2 depicts the main substance behind this Lemma for the canonical representation of the SS locus. The parameter $m_w$ is used to capture exogenous changes in the extent of trade frictions in the market. An increase in $m_w$ shifts the SS locus out from the origin, and corresponds to an improvement in matching efficacy.

## 2.3 Contracts and Bargaining

The types of contracts (more generally incentive mechanisms) that can be enforced hinge upon the nature of the information that is verifiable in a court of law. In turn, the feasible class of incentive mechanisms play a pivotal role in the property rights literature, by circumscribing the costs and benefits of acquiring residual control rights.

As a general proposition, the basic issue that all papers in the literature on the modern theory of the firm must broach is: (i) restricting the class of feasible incentive mechanisms in a manner that ownership still matters, and (ii) doing so in a rather "minimalist" way so that it is possible to transparently identify the role played by alternative governance structures (as opposed to detecting the effects of arbitrary restrictions on the set of feasible contracts). However, because of the complexity of our dynamic search framework, we have a third criterion that must be satisfied. It is (iii) limiting the class of incentive schemes to readily exclude "certain types" of behaviors (described below) in order to ensure that the analysis remains tractable. With this preamble in mind, the first assumption is harmless enough:

Assumption 2.3 (Info. 1)

The courts can verify that: (i) the production, and the delivery of a widget or a final good from seller to buyer took place, and (ii) that cash was paid by the buyer to the seller

This is a rather rudimentary requirement. Without access to this basic verifiable information, then even market exchange is problematic. After receiving payment, the seller of a widget/good could refuse to supply it. Alternatively, after receiving a good/widget the buyer could refuse to pay for it.

Our next Assumption concerns the type of widget/final good that is produced or traded.

Assumption 2.4 (Info. 2)

(i) The widget/final good type, $\boldsymbol{\omega} \in [0,1]$, is neither describable ex ante nor verifiable in a court of law ex post.

(ii) The type $\boldsymbol{\omega} \in [0,1]$ is observable only by the managers who are directly involved in the transaction (whether in production or exchange).

(iii) Although the final customer can articulate the type of good that provides him with utility, he learns the type of good sold to him only after he consumes it.

Following the property rights literature, part (i) can be justified by assuming that the firm cannot anticipate every type of product that its consumer will subsequently desire. The lack of verifiability can be justified by appealing to the practical difficulties of writing down the clauses of an appropriate contract that is useful to a court of law. (For example, just what is "service with a smile.")

Parts (ii) and (iii) are essential. If both the courts and managers are completely in the dark, then it is (generically) impossible for the firm to manufacture/procure a specific type of good/widget. Similarly, (in part (iii)) it is essential that the consumer can describe to the firm the particular good or service he desires, if he is to have any hope of obtaining it.

Notice in part (ii) that a non-manager owner cannot observe precisely what his employee managers are up to. In anticipation, taken as a whole, the three parts of Assumption 2.3 will lead to a situation in which it is optimal for owners to manage one of their assets in general, and (in the case of an integrated firm) manage the downstream asset in particular. (The reason is that a manager-employee has an incentive to produce and provide the final customer with the "shoddy" generic good $\omega_0$, which leads – from the owner's perspective – to the disastrous loss of the customer).

The final informational assumption concerns the manager's search activities.

Assumption 2.5 (Info. 3)

The manager's search activities are (nonverifiable) private information.

The manager obviously knows whether he is searching to buy (in the case of a downstream asset) or sell (in the case of an upstream one) an appropriate widget. As a practical matter, managers enjoy considerable discretion in how they spend their time. Thus, the endeavour to establish (i.e., verify) that the manager was not attempting to search to buy/or sell a suitable widget is no simple legal matter.

The force of this assumption is that the lack of verifiability implies that his search activities are a non-contractible investment. It further implies that the owner who manages an asset cannot write an incentive contract that directly compensates his employee-manager for carrying out costly search. (And this is for the simple reason that prior search activity is a sunk investment. Consequently, after buying or selling a widget, the owner could claim that the manager did not search, and lacking any verifiable information the manager would lack any redress in court.)

Nevertheless, rather than basing the manager's pay on his "inputs" (i.e., his search efforts) one might suspect that suitable incentives can be engendered by basing his pay on his "output" (i.e., his performance in buying/selling widgets). Yet, this type of scheme is foiled by the assumptions that: the type of widget is non-verifiable, and the ability of the manager to costlessly acquire the generic widget $\omega_0$. As a result, a manager who is paid by the piece, has an incentive to amass (or sell) inferior (and in our framework disastrous) generic widgets, so as to maximize his earnings. Because of these considerations, we assume there are no feasible incentive mechanisms that owners can employ to encourage the managers they employ to undertake costly search on the market. (Of course, owners can – and indeed will – avoid this problem by managing the asset themselves, and searching on the widget market.)

Recall, from subsection 2.1.2 that once a customer switches to the "active" consumption state, he will switch back to the inactive one only after he consumes his desired product or else rescinds his order. Since each downstream asset is associated with a single customer, it follows that it is not (strictly) optimal for the downstream asset to refuse to try and comply with its customer's demands. The reason is that it must do so in order to get the consumer to switch back to the "passive" state, which primes him to (at some point in the future) to switch back to the active one and (possibly) demand a product that is more to the firm's liking.

This formulation captures – in an admittedly rather rudimentary manner – the notion that, as a practical matter, firms place great store in maintaining the goodwill of their customers. It also implies that if a vertically integrated firm cannot produce the specific widget type internally, then it must use the market to search for an appropriate upstream asset that can do so.

By virtue of his residual control rights, the owner of a particular asset can, by fiat, order the asset's manager to produce a specific type of good/widget. If the manager is recalcitrant, then the owner can fire him, and hire someone that is willing to produce it. Nevertheless, since the manager's effort is costly, the owner's power is not absolute: the owner must compensate the manager for his efforts.[12] The force of this observation is that the owner of a vertically integrated firm can (provided it is technologically feasible) produce the appropriate final good/widget by fiat (subject to the caveat noted above). In this case, the internal transfer price of the widget, denoted $p_0$, is simply the minimum amount required to compensate

[12]Recall that the courts can verify that production has or has not taken place, and that the good/widget did or did not change hands. The problem is that they cannot ascertain the type of good/widget involved in the transaction. This structure provides sufficient leverage to ensure that an owner can agree to a contract with a manager, wherein the manager has an incentive to carry out his production duties. By assumption, the cost of producing the final good is normalized to zero. Consequently, it is trivial to induce the manager to produce (or find one who will produce) the proper final good. The production of the widget entails a managerial disutility of $\kappa$. A simple incentive mechanism is one in which: (a) the owner requests that the manager produce a specific widget $\omega$, (b) the owner agrees to pay the manager $\kappa$ upon delivery of the widget, and (c) the owner has the right to refuse the widget, in which case it is not transferred from the manager to the owner. In this setting, it is (weakly) optimal for the manager to produce and to deliver the proper widget. Clearly, the manager will not produce the wrong widget – as he then incurs the disutility of effort $\kappa$, and the owner will refuse to accept it. He then has the option of producing no widget at all, or the proper one and is indifferent between these options. In this case, we assume that he complies with the owner's request.

the manager for his disutility of effort: $p_0 = \kappa$.

However, matters are quite different when it comes to those firms which meet on the spot market. In this case, the owner (or manager) of a downstream asset cannot dictate that the owner (or manager) of the upstream asset produce "this or that" widget. The reason, of course, is that the owner of the upstream asset enjoys residual control rights over the asset he owns, and can use it in any (legal) manner he sees fit. In particular, in accordance with the property rights literature, the owner of the upstream asset can deny access to his asset, and refuse to produce a widget. This option gives the owner of an upstream firm economic "power," in his face to face negotiations with the owner of a downstream firm. Similar remarks apply, of course, to the owner of a downstream asset, who can refuse to purchase an upstream asset's widget.

In what follows, we assume that between independent firms the market widget price, $p$, is determined via a (bilateral) Nash bargaining protocol with (for convenience) equal bargaining weights. This is one of the simplest, and most common, bargaining frameworks used in the literature; it also possesses the virtue of accommodating in a rather transparent manner both parties relative "power," characterized by the value of their outside options.

Finally, we (harmlessly) assume that the final goods price is determined by a take-it-or-leave-it offer that is made to the consumer by the manager of the downstream asset. As an immediate corollary, the final good's price equals the consumer's valuation, $v$, indicating that the firm captures all of the consumer's surplus.

## 2.4 States and Value Functions

In this Chapter, we consider three alternative organizational structures: (i) independent upstream firms, (ii) independent downstream firms, and (iii) vertically integrated upstream-downstream pairs.

Generically, independent (i.e., non-integrated) upstream firms occupy only one state: they are unmatched and are (may be) searching for a downstream asset with which to trade.[13] In what follows, let $\Pi$ and $u_0$ represent the value, and mass of

[13] The match between an upstream and downstream asset, is transient. If it can, it instantly produces the intermediate good and again enters the unmatched state.

unmatched upstream assets that belong to the industry.

An independent downstream firm also (generically) belongs to one of two states: its customer either has (indexed "1") or has not (indexed "0") placed an active order for a specialized final good.[14] Let $V_0$ and $d_0$ denote the value and mass of non-integrated downstream assets without a customer's order, and let $V_1$ and $d_1$ denote the value and mass of independent downstream asset with one.

Finally, an integrated firm also generically belongs to two possible states: it either has a customer's order that it cannot produce internally, or it has no order at all, and is awaiting one from its customer.[15] The values of an integrated firm in each of these two states are denoted $K_0$ and $K_1$, respectively. The corresponding masses of integrated firms in each state are denoted by $D_0$ and $D_1$.[16]

As noted earlier, the market widget price, $p$, is determined via a Nash bargaining protocol (with equal bargaining weights). However, because of differences in the value of their outside options, the price that is agreed will, in general, depend upon the type of the agents who are involved in the bargain (viz., independent and/or integrated firms). Accordingly, let $p_{\text{n}}$ denote the price of the widget agreed between (N)on-integrated upstream and downstream firms, and let $p_{\text{i}}$ denote the widget price that is agreed between an (I)ntegrated firm, and an independent (non-integrated) upstream firm.[17]

In this Chapter, the setup is such that — other than the time taken to locate a trading partner — there is no uncertainty. The value functions (Bellman equations) are:

$$r\Pi = -e_w^u + \alpha\left[\frac{d_1}{d_1+D_1}\cdot p_{\text{n}} + \frac{D_1}{d_1+D_1}\cdot p_{\text{i}} - \kappa\right] \tag{2.3}$$

[14]Just as in the case of independent upstream firms, the process of matching and trade is instantaneous.

[15]In this case, there are two transient states. In the first of them, the firm can produce the order internally (in which case it does so instantly). The second state is the point of contact on the market with an upstream firm that can produce the desired widget. Agreement, production, and consumption occur immediately, and the firm again immediately enters the state of having no order in hand (and awaiting one).

[16]Recall that the mass of downstream assets is normalized to equal unity. Moreover, since each integrated firm consists of a single downstream asset, it follows that: $1 \equiv D_0 + D_1 + d_0 + d_1$.

[17]There is even a third market price which is the one agreed between an integrated firm and an non-integrated downstream firm. Nevertheless, as we show below, the managers of integrated upstream assets do not search for trading partners. As a consequence, we can inconsequentially set this price to zero without loss of generality.

$$rV_0 = \mu(V_1 - V_0) \tag{2.4}$$

$$rV_1 = \beta(\nu - p_{\mathrm{n}} + V_0 - V_1) + \lambda(V_0 - V_1) \tag{2.5}$$

$$rK_0 = -c_1^v + \mu X(\nu - \kappa) + \mu(1 - X)(K_1 - K_0) \tag{2.6}$$

$$rK_1 = -c_1^v + \beta(\nu - p_{\mathrm{i}} + K_0 - K_1) + \lambda(K_0 - K_1) \tag{2.7}$$

It is instructive to discuss the intuition captured by each of these equations. To recap: $\Pi$ is the value of a non-integrated upstream firm; $V$ is the value of a non-integrated downstream firm; and $K$ is the value of an integrated firm (which comprises an upstream and downstream asset). Finally, $\{0,1\}$ indicates whether the downstream asset (whether as part of an integrated or non-integrated firm) does (1) or does not (0) have a customer's order in hand.

To begin with, consider a non-integrated upstream firm. The term $r \cdot \Pi$ represents the flow value of searching for a downstream asset that is willing to purchase its particular widget. (Were the asset a consol, then $r \cdot \Pi$ would equal the yield on the bond). According to equation (2.3) this flow value consists of two parts: $-e_w^u$ is the flow cost of searching for a potential trading partner.

The term in square brackets is the expected capital gain accruing from search. Thus, with flow probability $\alpha$ the firm meets a downstream asset that desires its widget. Now, downstream assets come in two varieties: the non-integrated sort, and those that are part of an integrated firm. Crucially, the agreed widget price generally depends upon the type of downstream firm that is contacted. Thus, with probability: $d_1/(d_1 + D_1)$ the firm meets a non-integrated downstream firm (and the agreed widget price is $p_{\mathrm{n}}$); with complementary probability $D_1/(d_1 + D_1)$ it meets a downstream asset that is part of an integrated firm (in which case the price is $p_{\mathrm{i}}$). Hence, the bulk of the term in square brackets is simply the price that the upstream asset expects to receive for its widget (suitably weighted by the proportion of each type of downstream asset). Last, but not least, $\kappa$ is the cost of producing the widget. In summary, the term in square brackets is the expected profits that accrue to the upstream asset when it meets another firm, and produces and then sells a widget.

Now consider equations (2.4)–(2.5) which characterize the values ($V$) of a non-integrated downstream firm. The left hand side of equation (2.4) $r \cdot V_0$ is the firm's flow value, as it waits for its customer to place an order. The right-hand side shows what this value equals. With flow probability $\mu$ an inactive customer switches to the active state, and demands a specialized product. In this event, the firm in question enjoys the capital gain: $[V_1 - V_0]$. In equation (2.5) $r \cdot V_1$ is the (non-integrated) downstream firm's flow value as it searches in the widget market. The expression is intuitive. With flow probability $\beta$ it locates a suitable widget, and accrues profits of: $(\nu - p_{\mathrm{n}})$, where $p_{\mathrm{n}}$ is the widget price that is pertinent for non-integrated assets.[18] Nevertheless, the opportunity cost of satisfying the customer is that the firm exits the state $V_1$ and enters the state $V_0$, where it once again must await for the customer to place an order. Thus, the firm also suffers the capital loss $(V_0 - V_1)$, by producing for its customer. Finally, since search is time consuming, the customer may change his mind — once again entering the passive state — during the time that the firm is trying to meet his order. The flow probability of this event is $\lambda$, and if it occurs the firm suffers the capital loss: $(V_0 - V_1)$. The paramater $\lambda$ is useful for capturing the "urgency" with which a downstream asset must locate a suitable widget to produce for its customer.

Since the Bellman equations for integrated assets $(K)$ have broadly similar interpretations to those given above, we will be brief in our descriptions of them. First, consider equation (2.6), which describes the flow value of an integrated firm that is waiting for its customer to place an order $(r \cdot K_0)$. The integrated firm incurs the flow governance cost $c_1^{\nu}$. With flow probability $\mu$ the customer places an order. For an integrated firm, there are two possible outcomes: (a) with probability $X$ it can (instantly) produce the specialized widget internally (in which case it accrues instantaneous profits: $(\nu - \kappa)$), and (b) with complementary probability $(1 - X)$ it cannot do so, and must procure the widget from the spot market. In the former case, the firm makes the immediate return transition: $K_0 \rightarrow K_0$. In the latter, it makes the transition from an integrated firm without an order, to an integrated firm with an order that is seeking to locate a suitable widget. In this case, it enjoys the capital gain: $(K_1 - K_0)$.

As for equation (2.7) — which describes the flow value of an integrated firm with a customer's order in hand — this possesses broadly similar properties to equation (2.5). Briefly: as an integrated firm, it incurs the flow governance cost $c_1^{\nu}$; with flow probability $\beta$ it meets an appropriate independent upstream widget producer.

[18]Recall that the consumer pays $\nu$ for the final good.

Upon doing so, agreement, production, and consumption occur immediately. The firm accrues instantaneous profits of $(v - p_{\mathrm{i}})$ — where $p_{\mathrm{i}}$ is the appropriate widget price for this class of meeting — and makes the return transition to the state of a firm waiting for a customer to place an order (leading to the capital loss $(K_0 - K_1)$.) Finally, just as in equation (2.5) delay is costly, in that with flow probability $\lambda$ the customer switches back to the passive state, which results in the capital loss: $(K_0 - K_1)$.

## 2.5 Preliminary Results

In this Section we present some elementary results that help clear the way for the presentation of the more major ones in the next Section. We begin by considering managerial search effort.

### 2.5.1 The Management of Assets

Suppose that a manager devotes effort towards seeking to buy or sell a widget on the market, incurring disutility in the process. Ex post, after finding a widget, the owner will not pay him for his troubles. Recall, managers can be replaced instantly on the competitive market, and that search is a non contractible investment. Anticipating this, managers will not devote any efforts towards search. Hence,

Lemma 2.2 (Non-owner) managers do not devote any effort towards search.

The force of this Lemma is that, since (employee) managers do not search, each owner must manage (one of) the assets he owns, if any search activity is to be carried out. Moreover, since it is a feature of the technology that the owner of an integrated firm can manage only one of the assets, he must choose whether to manage the upstream or the downstream asset. The cost of managing one of the assets is that he forgoes the opportunity of managing the other; the benefit is that he can carry out costly search and (depending upon his choice) either buy or sell widgets.

If the owner of an integrated firm were to manage the upstream asset, then he can use its capacity to sell widgets on the open market. The problem, and it is a serious one, is that if he manages this asset, then the manager of the downstream one does not search. As a consequence if the customer becomes active and places an order; and if the upstream asset is incapable of producing the widget (which occurs with probability $(1-X)$ ); then the firm cannot satisfy the customer's order (which requires search and successfully procuring a widget from the spot market.) As a consequence, the firm must wait until the customer retracts his order, by switching back to the inactive state (which occurs with flow probability $\lambda$.) Only then, can the customer place another order, which the upstream asset can satisfy (with probability $(X)$). Nevertheless, the owner can avoid all of these production holdups by managing the downstream asset himself. Under these circumstances, as residual claimant, if the customer places an order that the firm cannot satisfy, then he has an incentive to search for a suitable widget on the spot market.[19] In summary:

Lemma 2.3 Each owner also:

(a) manages the asset he owns in non-integrated firms, and

(b) manages the downstream asset of an integrated firm.

Together Lemmas 2.2 and 2.3 point to a rather simple owner-managerial structure. Indeed, managers are hired as employees only to run the production department of upstream assets within integrated firms.

Moreover, integrated firms are characterized by a form of "vertical foreclosure," in the sense that they remove an upstream asset from the market and use it exclusively for their own production purposes (i.e., they do not exploit the upstream asset's capacity to produce widgets for other firms).

In much of the industrial organization literature, vertical foreclosure is often viewed as a "mischievous" activity, in which a firm attempts to strategically hurt its rivals by hindering their ability to acquire an essential input.[20] Yet, in the context of our framework, their is no mischief at all intended when a firm acquires

[19]As noted above, the "cost" is that the manager loses the opportunity of using the upstream asset to produce and sell widgets on the spot market. Nevertheless, as we will shortly demonstrate, the ex ante value of running an upstream asset is zero – a result which follows from our assumption of free entry into this sector.

[20]For example, Ordover, et. al (1990), Hart and Tirole (1990), and Bolton and Whinston (1993). Indeed, one reading of this literature is that it arose to explain the tendency for the upstream divisions of integrated firms to rarely use their capacity to supply intermediate inputs to other firms.

an upstream asset. Indeed, the owner of the firm would be delighted were the manager to exert costly effort and to use its capacity to produce and sell widgets to other firms. The snag is that (although an admittedly extreme case), the owner is incapable of providing the employee manager with a credible incentive scheme that compensates him for his search efforts.

### 2.5.2 Free entry

There is an abundance of available upstream assets that are external to the industry. The owners of upstream assets can costlessly enter the industry. Similarly, the owner of a downstream asset can costlessly enter the industry as either an independent firm, or as an integrated one (with an upstream asset in tow). The posited free entry condition immediately implies that $\Pi$ — the ex ante asset value of a non-integrated upstream asset is zero. However, downstream assets are "scarce," in that there is an exogenous unit mass of them. Consequently, although they also enjoy free entry into the industry, the ex ante value of either non-integrated downstream firms, $V_0$, or integrated firms (which include a downstream asset) $K_0$ may be strictly positive.

### 2.5.3 The Widget Price

As noted earlier, the internal transfer price of a widget within an integrated firm is $p_0 = \kappa$, which is the payment made by the owner to the manager which is just sufficient to compensate the manager for the disutility of his production efforts.

The widget price determined on the spot market is determined via a Nash bargain. First, consider a meeting between an independent upstream and an independent downstream firm (implying the relevant widget price is $p_N$). Upon production and trade, the owner (manager) of the upstream asset accrues the payoff:

$$p_{\mathrm{n}} - \kappa + \Pi. \tag{2.8}$$

He receives the widget price $p_{\mathrm{n}}$; incurs the production cost (disutility) $\kappa$; and he enters the state of an upstream asset that is searching for a widget buyer.

The payoff to the downstream asset is:

$$\nu - p_{\mathrm{n}} + V_0. \tag{2.9}$$

The owner receives $\nu$ from his customer; pays for the widget $p_{\mathrm{n}}$; and enters the state of a non-integrated downstream firm that is waiting for its customer to place an order ($V_0$).

Of course, both parties have the option of not trading. In this case, they separate. The upstream firm continues searching for another widget buyer (which is worth $\Pi$) and the downstream firm, with customer's order in hand, searches for a widget supplier (which is worth $V_1$).

According to the Nash bargaining protocol, the agreed widget price, $p_{\mathrm{n}}$, solves the following maximization problem:

$$\max_{\kappa \leq p_{\mathrm{n}} \leq \nu} (\nu - p_{\mathrm{n}} + V_0 - V_1)(p_{\mathrm{n}} - \kappa) \tag{2.10}$$

Broadly similar considerations describe a meeting between a non-integrated upstream firm and an integrated firm that has an active order from its customer. Upon production and trade, the upstream asset accrues:

$$p_{\mathrm{i}} - \kappa + \Pi \tag{2.11}$$

where $p_{\mathrm{i}}$ is the widget price pertinent for this class of meeting. The integrated asset accrues:

$$\nu - p_{\mathrm{i}} + K_0. \tag{2.12}$$

The only distinction between equations (2.11)-(2.12) and (2.8)-(2.9) is the presence of the pertinent widget price (viz., $p_{\mathrm{i}}$ and $p_{\mathrm{n}}$). If they fail to reach agreement, the upstream asset enters accrues $\Pi$. The downstream asset enters the state of an integrated firm, with customer's order in hand, that is looking for a widget (which is worth $K_1$). By the Nash bargaining solution, $p_{\mathrm{i}}$ is the solution to the following maximization problem:

$$\max_{\kappa \leq p_{\mathrm{i}} \leq \nu} (\nu - p_{\mathrm{i}} + K_0 - K_1)(p_{\mathrm{i}} - \kappa) \tag{2.13}$$

Lemma 2.4 characterizes the solutions to these problems.

Lemma 2.4 The price of the widget agreed in the spot trade between:

- independent upstream and downstream firms is:

  $p_{\mathrm{n}} = \frac{1}{2} \cdot (\nu + \kappa + V_0 - V_1)$,

- an independent upstream firm and an integrated firm is:

  $p_{\mathrm{i}} = \frac{1}{2} \cdot (\nu + \kappa + K_0 - K_1)$

The transfer price of a widget produced internally is $p_0 = \kappa$.

## 2.6 Steady State Equilibria

We first define the steady state equilibrium.

Definition 1 A steady state equilibrium is the population distribution of all the assets $(u_0, d_0, d_1, D_0, D_1)$, the value functions $(\Pi, V_0, V_1, K_0, K_1)$, and the matching rates $(\alpha, \beta)$ that satisfy the following conditions:

1. Lemmas 2.2–2.4 are satisfied.

2. Value functions satisfy equations (2.3)–(2.7).

3. Owners of the downstream assets optimally choose whether to enter the industry as independent or integrated firms.

4. The free entry of upstream assets: $\Pi = 0$.

5. The mass of downstream asset is one: $d_0 + d_1 + D_0 + D_1 = 1$

6. Steady state matching rates $\alpha$ and $\beta$ satisfy the SS locus (Equation (2.2)): $\alpha = \alpha^{SS}(\beta;\ m_w)$.

We present the following typology which classifies the potential types of equilibria that may arise.

Definition 2 There are three classes of (pure strategy) steady state equilibria: spot-trading, pervasive integration, and hybrid.

- In the spot-trading equilibrium, all of the owners of downstream assets enter the industry as independent firms.

- In the (pervasive) integration equilibrium, all the downstream asset owners choose to enter the industry as integrated firms.

- In the hybrid equilibrium, positive measures of downstream assets enter the industry as independent and integrated firms.

The steady state population distribution can be easily derived using the condition that inflow to a certain state should be the same as the outflow from the state. Let $z$ denote the proportion of downstream assets that enter the industry as independent assets. In other words, $z$ denote the composition of independent downstream assets. That is, $zD_0 = (1-z)d_0$. Note $z = 1$ in the spot-trading equilibrium, $z = 0$ in the pervasive integration equilibrium, and $z \in (0,1)$ in the hybrid equilibrium.

Lemma 2.5 describes the steady-state populations.

Lemma 2.5 In a steady state equilibrium, the population distribution is represented by the following:

$$
\begin{aligned}
d_0 &= \frac{z(\beta+\lambda)}{\beta+\lambda+\mu(1-X(1-z))}, \quad d_1 = \frac{z\mu}{\beta+\lambda+\mu(1-X(1-z))},\\
D_0 &= \frac{(1-z)(\beta+\lambda)}{\beta+\lambda+\mu((1-X(1-z))}, \quad D_1 = \frac{(1-z)\mu(1-X)}{\beta+\lambda+\mu((1-X(1-z))},\\
u_0 &= \frac{\beta\mu(1-X(1-z))}{\alpha[\beta+\lambda+\mu(1-X(1-z))]}.
\end{aligned}
$$

In a steady state spot-trading equilibrium, $z = 1$, in a steady state integration equilibrium, $z = 0$, and in a steady state hybrid equilibrium, $0 < z < 1$.

Proof. See Appendix A. ■

It is helpful to define $\pi \equiv (\nu - \kappa)/2$, which is half of the ex post value of production. The free-equilibrium entry (EE) of upstream assets, and the resulting condition $\Pi = 0$, imposes restrictions upon the admissible matching rates $\alpha$ and $\beta$. Indeed, it leads to another of the most important analytic relations — the EE locus — considered in this thesis. Consider, Lemma 2.6 In a steady state spot-trading equilibrium,

$$\alpha_s = \frac{e_w^u(2r+\beta+2\lambda+2\mu)}{2(r+\lambda+\mu)\pi}: \qquad (EE_s)$$

In a steady state pervasive integration equilibrium,

$$\alpha_{\mathrm{i}} = \frac{e_w^u(2r+\beta+2\lambda+2\mu(1-X))}{2(r+\lambda+\mu)\pi}: \qquad (EE_{\mathrm{i}})$$

In a steady state hybrid equilibrium,

$$\alpha_h = \frac{e_w^u(1-X(1-z))(2r+\beta+2\lambda+2\mu(1-X))(2r+\beta+2\lambda+2\mu)}{2(r+\lambda+\mu)[(1-X(1-z))(2r+\beta+2\lambda)+2\mu(1-X)]\pi}: \qquad (EE_h)$$

Furthermore,

$$\lim_{z\to 1}\alpha_h = \alpha_s, \quad \lim_{z\to 0}\alpha_h = \alpha_{\mathrm{i}}, \text{ and } \frac{\partial\alpha_h}{\partial z} > 0$$

Proof. See Appendix A. ■

The EE loci described in Lemma 2.6 carve out regions of $(\alpha,\beta)$ space that are: (i) consistent with the free entry of upstream assets (and the condition $\Pi = 0$) and (ii) are conditional upon a given organizational form for the industry (whether spot trading, integration, or hybrid.) Furthermore, Lemma 2.6 states that $EE_h$ locus lies between $EE_{\mathrm{i}}$ and $EE_s$ loci.

The "trick" in establishing the existence and class of equilibrium that emerges is to use the SS locus and each of the EE loci to first determine candidate values of $(\alpha,\beta)$ for each class of equilibrium. These candidate values are conditional upon a posited organizational form; they are consistent with zero profits; and are consistent with the model's steady-state. One then checks (verifies) which (if any) of the equilibria are the pertinent ones, by showing that given the matching rates the posited organizational form results from individually rational behavior.

Notice that the $EE_s$ land $EE_{\mathrm{i}}$ loci are strictly increasing and are linear in beta. Therefore, together with the negatively sloped steady-state, $SS$, locus, we have the following.

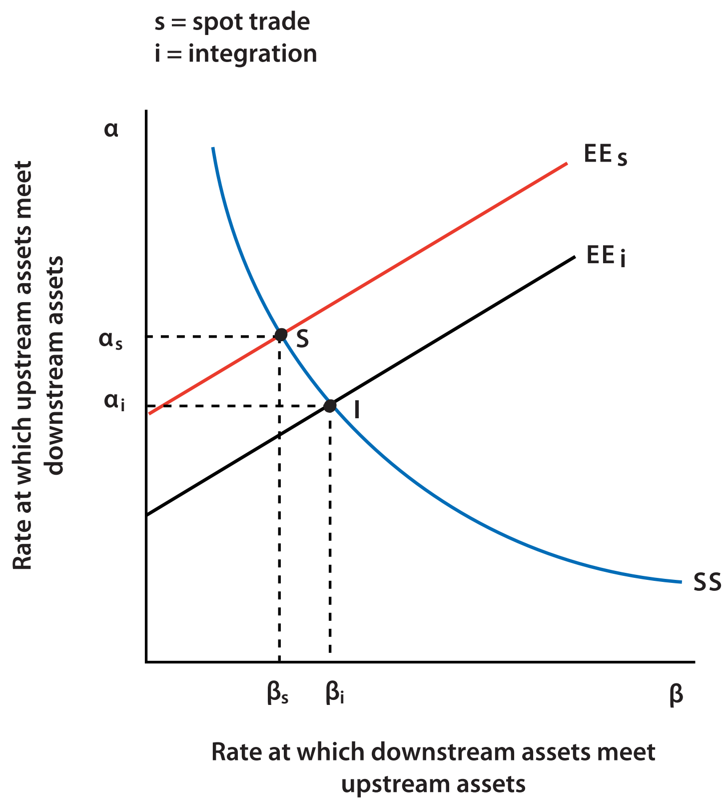


Figure 2.2. Equilibrium Matching Rates

Lemma 2.7 If a steady state spot-trading or pervasive integration equilibrium exists, it is unique. That is, two spot trading equilibria or two integration equilibria do not exist.

Note that Lemma 2.7 does not rule out the possibility of multiple equilibria. More specifically, it is conceivable that for a common set of parameters both the spot-trading and pervasive integration equilibria may exist.

Lemma 2.8 Let $E \equiv \{u_0, d_0, d_1, D_0, D_1; \Pi, V_0, V_1, K_0, K_1; \alpha, \beta\}$. Then $E$ is:

- a spot-trading equilibrium if and only if $E$ is an equilibrium with $V_0 \geq K_0$ and $D_0 = D_1 = 0$
- a (pervasive) integration equilibrium, if and only if $E$ is an equilibrium with $K_0 \geq V_0$ and $d_0 = d_1 = 0$.

- a hybrid equilibrium, if and only if $E$ is an equilibrium with $V_0 = K_0$ and $d_0, d_1, D_0, D_1 > 0$.

Proof. See Appendix A. ■

As is clear from Lemma 2.8 the category of equilibrium that arises hinges upon the sign of the difference between $V_0$ and $K_0$. In turn, this difference depends upon the parameters of the model. Fortunately, a little algebra can be used to show that:

Lemma 2.9 The class of equilibrium that arises is governed by the condition:

$$\begin{aligned} & V_0 \geq (\leq, =) K_0 \\ & \Leftrightarrow T_s(\beta) \equiv c_1^v \beta^2 + \left[2c_1^v(2r + 2\lambda + \mu(2 - X)) - 4\mu X(r + \lambda + \mu)\pi\right]\beta \\ & \qquad\qquad + 4(r + \lambda + \mu)\left[c_1^v(r + \lambda + \mu(1 - X)) - 2\mu X(r + \lambda)\pi\right] \\ & \qquad\qquad \geq (\leq, =) 0 \end{aligned}$$

Proof. See Appendix A. ■

The function $T_s(\beta)$ plays an important role in the analysis. Intuitively, it incorporates all of the economic constructs developed earlier, and presented in the model. Crucially, $sign T_s(\beta)$ can be used to check whether the "candidate" values of $(\alpha, \beta)$ determined from the SS and EE loci (which to reiterate are derived on an assumption about optimal behavior) are indeed consistent with this posited optimal behavior.

To see how the $T_s(\cdot)$ locus is used, let $(\alpha_s, \beta_s)$ satisfy $EE_s$ and $SS$ and $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ satisfy $EE_{\mathrm{i}}$ and $SS$. Ultimately, the equilibrium that prevails depends upon specific parameter values, and in the case of multiple equilibria agents' expectations. For example, evaluated at a candidate equilibrium, if $T_s(\beta) > 0$ for $\beta_s$ and $\beta_{\mathrm{i}}$, spot-trading equilibrium prevails. In this case, downstream firms do not have incentive to "deviate" and to enter the industry as integrated firms. In contrast, if $T_s(\beta) < 0$ for $\beta_s$ and $\beta_{\mathrm{i}}$, the pervasive integration equilibrium prevails in the industry. Here, downstream firms do not have incentive to deviate to enter the industry as independent firms.

More specifically, the existence of a particular equilibrium or equilibria, depends on the number of positive roots of the equation $T_s(\beta) = 0$, and the relationship of $\beta_s$ and $\beta_{\mathrm{i}}$ to these roots. Let the real roots of quadratic equation $T_s(\beta) = 0$ be represented by $\beta_0$ and $\beta_1$ ($\beta_0 \leq \beta_1$). Suppose $\beta_y = \beta_z$, where $y \in \{s, i\}$, $z \in \{0, 1\}$. Then a hybrid equilibrium cannot exist since all the owners of downstream assets choose to enter the industry either as independent or integrated. If $c_1^v = 0$, it is easy to see that $T_s(\beta) < 0$ for all $\beta > 0$ so that a unique integration equilibrium exists and other equilibria do not exist.

The following proposition summarizes the existence of equilibria (see Figures 2.3–2.5).

Proposition 2.1 If $T_s(\beta_s) \geq 0$ then there exists a unique spot-trading equilibrium. If $T_s(\beta_{\mathrm{i}}) \leq 0$ then there exists a unique integration equilibrium. If either $\beta_s < \beta_0 < \beta_{\mathrm{i}}$ or $\beta_s < \beta_1 < \beta_{\mathrm{i}}$ then there exists a unique hybrid equilibrium. If $\beta_s < \beta_0 < \beta_{\mathrm{i}}$ and $\beta_s < \beta_1 < \beta_{\mathrm{i}}$ then there exist two hybrid equilibria.

Proof. Note that $(\alpha_s, \beta_s)$ satisfies $EE_s$ and $SS$ and that $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ satisfies $EE_{\mathrm{i}}$ and $SS$. Obviously, $0 < \alpha_{\mathrm{i}} < \alpha_s$ and $0 < \beta_s < \beta_{\mathrm{i}}$. By Lemma 2.8, $(\alpha_s, \beta_s)$ and $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ is a steady state spot-trading and integration equilibrium matching rates if $T_s(\beta_s) \geq 0$ and $T_s(\beta_{\mathrm{i}}) \leq 0$, respectively. Therefore the first two results follows.

Suppose $\beta_s < \beta_0 < \beta_{\mathrm{i}}$. Let $(\alpha_0, \beta_0)$ satisfy $SS$. Obviously $\alpha_s > \alpha_0 > \alpha_{\mathrm{i}}$. Since $T_s(\beta_0) = 0$, to prove the existence of a hybrid equilibrium, We must show that there exists a $z_h \in (0, 1)$ such that $z_h$ satisfies $EE_h$ with $\alpha_h = \alpha_0$ and $\beta_h = \beta_0$.

From $EE_h$

$$z_h = \frac{\frac{(1-X)[(\alpha_0-\alpha_{\mathrm{i}})(2r+\beta_0+2\lambda+2\mu)]}{(\alpha_s-\alpha_0)(2r+\beta_0+2\lambda+2\mu(1-X))}}{1+\frac{(1-X)[(\alpha_0-\alpha_{\mathrm{i}})(2r+\beta_0+2\lambda+2\mu)]}{(\alpha_s-\alpha_0)(2r+\beta_0+2\lambda+2\mu(1-X))}}$$

Note, since $\alpha_s > \alpha_0 > \alpha_{\mathrm{i}}$,

$$(\alpha_s - \alpha_0)(2r + \beta_0 + 2\lambda + 2\mu(1 - X)) > 0$$
$$(\alpha_0 - \alpha_{\mathrm{i}})(2r + \beta_0 + 2\lambda + 2\mu) > 0$$

Therefore, there exists a unique $z_h \in (0, 1)$.

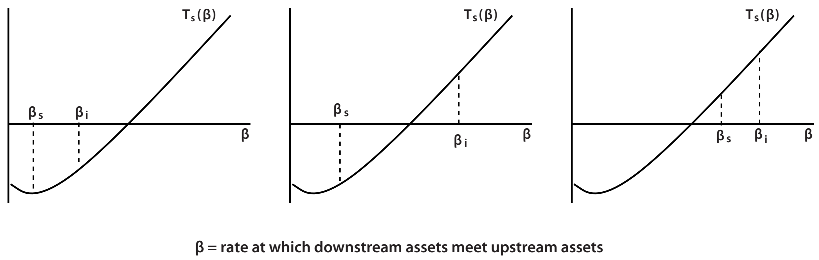


Figure 2.3. Equilibria: $\beta_0 \leq 0 < \beta_1$

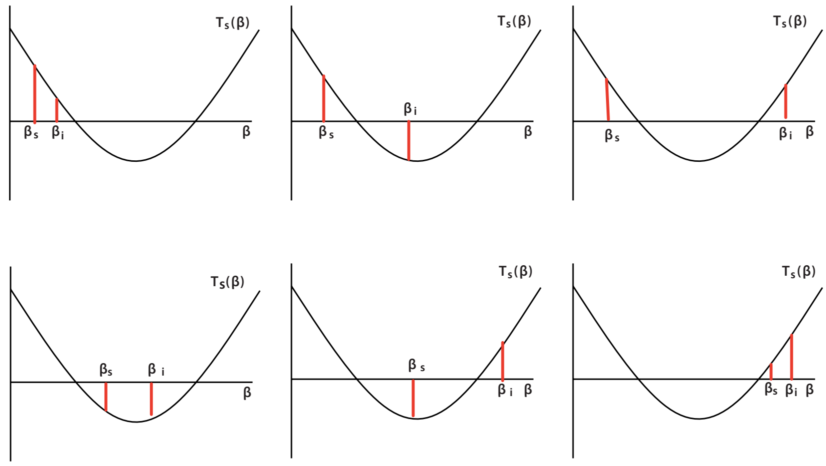


Figure 2.4. Equilibria: $0 < \beta_0 < \beta_1$

Similarly for $\beta_s < \beta_1 < \beta_{\mathrm{i}}$, there exists a unique $z_h' \in (0,1)$ that corresponds to $(\alpha_1, \beta_1)$. ■

Proposition 2.2 There exists at least one equilibrium.

Proof. Given $\beta_s$ and $\beta_{\mathrm{i}}$, if $T_s(\beta_s) \geq 0$ then spot-trading equilibrium exists and if $T_s(\beta_{\mathrm{i}}) \leq 0$ then integration equilibrium exists. Otherwise, hybrid equilibrium exists. ■

Note that there may exist multiple equilibria – three at maximum: one spot trading, one integration and one hybrid for some parameter values ($0 < \beta_s < \beta_0 < \beta_{\mathrm{i}} < \beta_1$), and one spot trading and two hybrid equilibria for some parameter values ($0 < \beta_s < \beta_0 < \beta_1 < \beta_{\mathrm{i}}$).

Clearly, the hybrid equilibrium is not a knife-edge equilibrium. Parameter values may change, but as long as parameter values still satisfy the certain criteria,

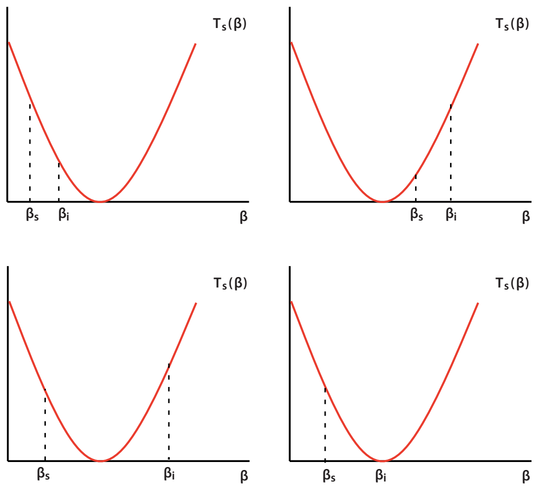


Figure 2.5. Equilibria: $0 < \beta_0 = \beta_1$

there exists a hybrid equilibrium. The industry adjusts to the new parameter values by changing $z$, the composition of independent downstream assets and integrated assets. The mass of upstream assets entering the industry depends on $z$. Hence, the adjustment in the composition will affect the market thickness, and in turn, the payoffs of independent and integrated assets.

Note that an independent downstream asset and integrated asset face different outside options. Therefore, they face different prices of widget in the market. Suppose, for the sake of argument, all the downstream assets are integrated. It is conceivable that integrated assets may face higher prices for some parameter values. This drives more upstream assets to enter the industry. Further, integrated firms that can procure the widget internally would stay out of the widget market, which makes the widget market 'thick' for downstream assets. Hence, for some parameter values, downstream assets may deviate to enter the industry as independent.Suppose, on the contrary, all the downstream assets are independent. For some parameter values, upstream assets could face lower price, and less of them would enter the industry making the widget market thin for downstream assets. Further, the widget market is populated by relatively large mass of downstream assets since every downstream with active order participates in the widget market. This makes the widget market 'thin' for downstream assets. Therefore, downstream assets may deviate to integrate to avoid costly and time consuming search process. In this situation neither spot-trading nor integration can be an equilibrium and, we have shown, there must exist a hybrid equilibrium in which independent downstream assets and integrated assets co-exist. This mechanism is in sharp contrast with McLaren (2000) in which integrations of other downstream assets dry up the upstream assets and make the market thin for remaining independent downstream assets.

## 2.7 Comparative Statics

Our model is amenable to a variety of comparative static exercises. In this Section, we focus on what we regard as the most interesting of them. We first examine how improvements in search efficacy affect the equilibrium volume of vertical integration. We then examine the effects of a reduction in the search cost. Finally we consider a decline in the order rate, which captures a "shock" that reduces the demand for the industry's product.

### 2.7.1 Improvements in Search Efficacy

We first show how improvements in search efficiency, parameterized by $m_w$ affect the industry equilibrium. Recall that, by Lemma 2.8 on page 44, $T_s(\beta)$ plays an important role in determining the nature of the equilibrium. We can use this to perform comparative statics. Note $T_s(\beta)$ is independent of $m_w$.

Suppose $\beta_0 \leq 0 < \beta_1$. Consider an industry with very low search activity. That is, $\beta_s < \beta_{\mathrm{i}} < \beta_1$. In this situation, the industry exhibits an integration equilibrium —all the downstream asset owners choose to enter the industry as integrated units. As the search efficiency improves, the industry then moves to a hybrid equilibrium and is populated by independent downstream and integrated assets. Eventually, if the search efficiency becomes large enough, the industry exhibits exclusive spot-trading. In this case, it is populated only be independent downstream.

Now suppose $0 < \beta_0 < \beta_1$. It is interesting to note that if the search efficacy is low enough, the industry exhibits spot-trading, rather than integration. Note that owners of independent downstream assets pay no search or maintaining costs and hence are entitled to enjoy positive surplus even if search efficiency and hence the matching rate, $\beta$ is very small. Integrated assets sometimes cannot procure the widget internally and must search for independent upstream assets while paying the maintaining cost. If the matching rate, $\beta$ is very low, it is hard to avoid this costly situation. Hence, in anticipation of the high governance cost (relative to the benefits) firms prefer not to integrate. Nevertheless, at higher matching rates, the integration cost declines in relative importance since firms anticipate that they can rapidly serve the customer.

It is interesting to see how improvements in search efficiency change the composition of independent and integrated assets. The following proposition states that the improvements in search efficiency induce more downstream assets to choose to enter the industry independent.

Proposition 2.3 Improvement in search efficiency $m_w$ increases the composition of independent downstream assets in case the industry exhibits hybrid equilibrium before and after the improvement in search efficiency.

Proof. Suppose that $m_w$ increases to $m_w'$ and the industry shows hybrid equilibrium for both $m_w$ and $m_w'$. Let $(\alpha_h, \beta_h)$ and $(\alpha_h', \beta_h')$ denote the hybrid equilibrium matching rates under $m_w$ and $m_w'$, respectively. Let $(\alpha_s, \beta_s)$ and $(\alpha_{\text{i}}, \beta_{\text{i}})$ satisfy $EE_s$ and $SS$ and $EE_{\text{i}}$ and $SS$, respectively under $m_w$, and $(\alpha_s', \beta_s')$ and $(\alpha_{\text{i}}', \beta_{\text{i}}')$ satisfy $EE_s$ and $SS$ and $EE_{\text{i}}$ and $SS$, respectively under $m_w'$. Note $\alpha_s' > \alpha_s$ and $\alpha_{\text{i}}' > \alpha_{\text{i}}$ because $\alpha$ increases along $EE_s$ and $EE_{\text{i}}$ as $m_w$ improves to $m_w'$. Note that $\beta_h$ is independent of $m_w$. Therefore $\alpha_h' > \alpha_h$, and $\alpha_h' - \alpha_h > \alpha_s' - \alpha_s$ and $\alpha_h' - \alpha_h > \alpha_{\text{i}}' - \alpha_{\text{i}}$ since in this case, $\alpha_h$ increase to $\alpha_h'$ along a vertical line $\beta = \beta_h$ (Figure 2.6). Rearranging these inequalities yields $\alpha_s - \alpha_h > \alpha_s' - \alpha_h'$ and $\alpha_h' - \alpha_{\text{i}}' > \alpha_h - \alpha_{\text{i}}$. Recall that:

$$
\begin{aligned}
z_h &= \frac{A}{1+A}, \\
&\text{where } A \equiv \frac{(1-X)[(\alpha_h - \alpha_{\text{i}})(2r+\beta_h+2\lambda+2\mu)]}{(\alpha_s - \alpha_h)(2r+\beta_h+2\lambda+2\mu(1-X))} \\
z_h' &= \frac{A'}{1+A'}, \\
&\text{where } A' \equiv \frac{(1-X)[(\alpha_h' - \alpha_{\text{i}}')(2r+\beta_h+2\lambda+2\mu)]}{(\alpha_s' - \alpha_h')(2r+\beta_h+2\lambda+2\mu(1-X))}
\end{aligned}
$$

It follows that $z_h' > z_h$ since $A' > A$. ■

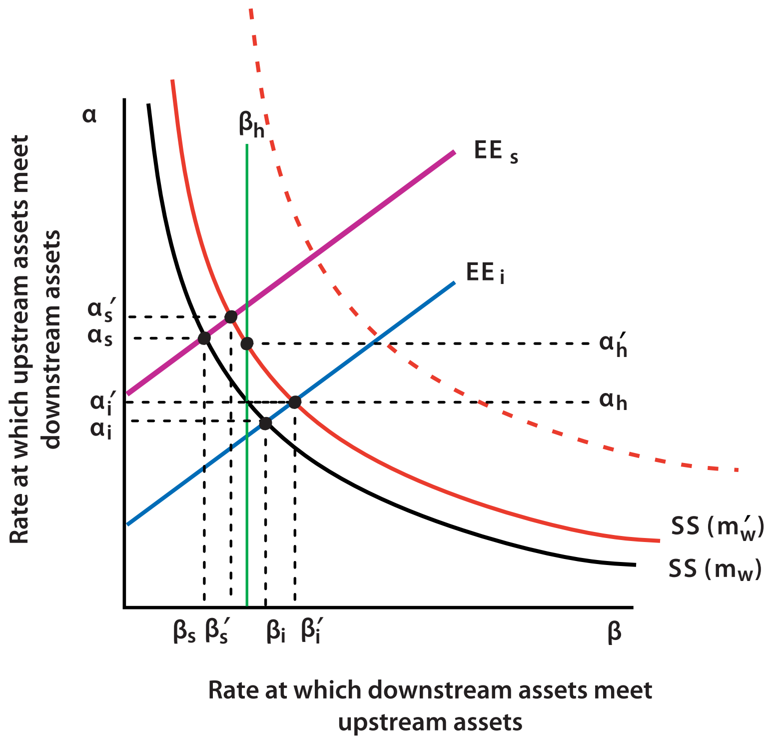


Figure 2.6. Improvements in Search Efficiency

### 2.7.2 A Decrease in the Upstream Asset's Search Costs

Next, we analyze the impact of decrease in the upstream asset's search cost $e_w^u$. Again, $T_s(\beta)$ is independent of the search cost. $EE_s$ and $EE_{\mathrm{i}}$ both shift downward and the slope of both loci get flatter as the search cost decreases. This implies that $\beta_s$ and $\beta_{\mathrm{i}}$ both increase while $\alpha_s$ and $\alpha_{\mathrm{i}}$ both decrease. This implies that the widget market becomes less tight in downstream asset's point of view. This is because as search cost decreases, more and more upstream assets would enter the industry. Therefore, the decrease in the search cost have similar effects on the industry as improvement in search efficiency. With low enough search cost, the industry would show spot-trading equilibrium. With high enough search cost, the industry would show spot-trading equilibrium or integration equilibrium depending on the number of positive real solutions of $T_s(\beta) = 0$ as shown above in the improvement in search efficiency.

### 2.7.3 Decrease in the Customer's Order Rate

It is easy to see that if the customer's order rate is low enough, the industry show spot-trading equilibrium: $\lim_{\mu \to 0} T_s(\beta) > 0$ for all $\beta > 0$. It is not profitable to pay the flow maintaining cost waiting for the customer's order that are rarely placed.

If the customer's order rate is very high and approaches infinity, the sign of $\lim_{\mu \to \infty} T_s(\beta)$ depends on the sign of $c_1^v(1-X) - \pi X(2r+\beta+2\lambda)$. If $c_1^v(1-X) - (2r+\beta+2\lambda)X\pi > 0$, the industry shows spot-trading equilibrium. Otherwise ($c_1^v(1-X) - (2r+\beta+2\lambda)X\pi < 0$), integration equilibrium would appear. If $c_1^v(1-X) - \pi X(2r+\beta+2\lambda) = 0$, which equilibrium would appear depends on the equilibrium matching rate $\beta$ (see Proposition 2.1).

Chapter 3

# Market Exchange vs. Vertical Integration: An Extended Model with Takeovers

In this Chapter, we extend the model presented in the previous one, by allowing owners to trade productive assets – i.e., we allow takeovers and mergers. More specifically, a downstream asset can enter the industry only as part of a non-integrated firm, rather than entering it as an integrated upstream-downstream unit. If it wants to integrate with an upstream asset, it must first search for one and buy it out. With the exception of this modification, the major assumptions used in this Chapter are the same as in the last one.

Figure 3.1 depicts the main features of the industry. Once again, $u$, $d$ and $c$ denote upstream assets, downstream assets, and customers, respectively. The heavy solid line between $u$ and $d$ denotes integration. The solid (dotted) line between $d$ and $c$ shows that the customer is active (inactive) and has placed (has not placed) an order. Finally, the dotted line between $d$ and $c$, that is cut by "X," indicates that the customer has abandoned the firm, forcing it to exit the industry.

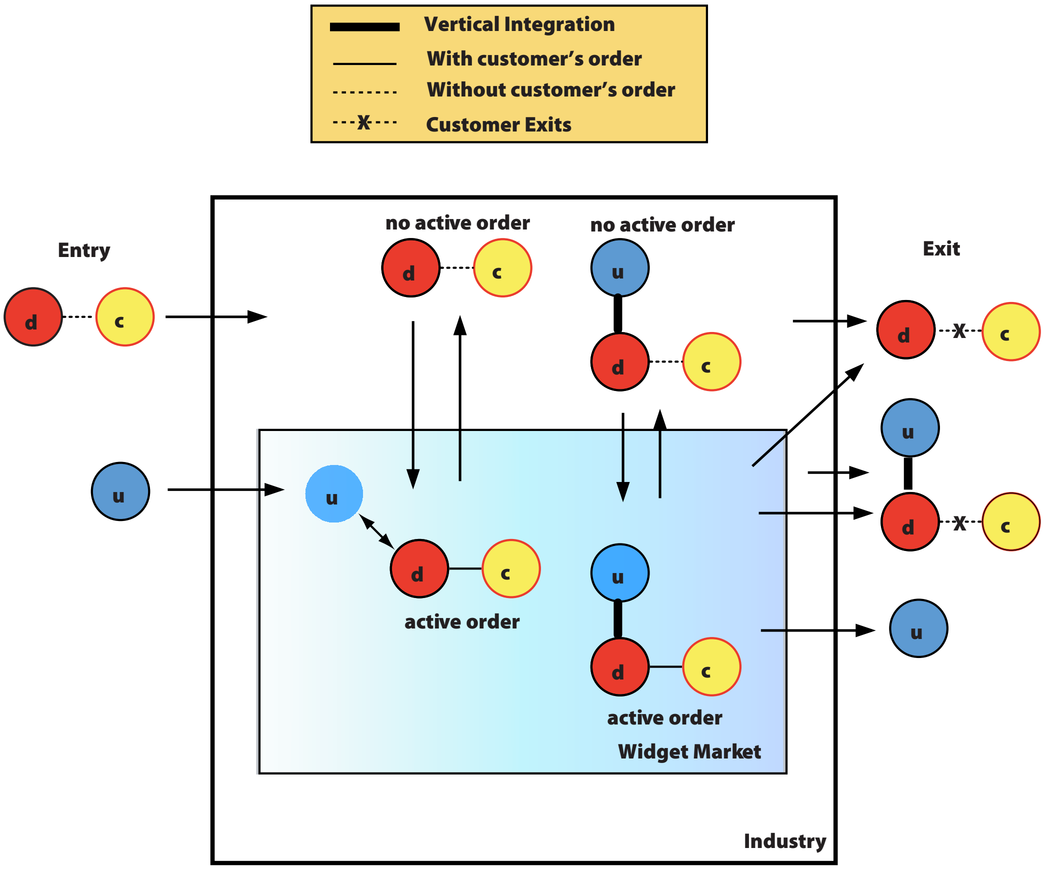


Figure 3.1. Widget Market and Industry – Extended Model

The goal of this Chapter is to examine the determinants of the equilibrium merger rate and to characterize the resulting industrial structure. In the next Section we outline the main assumptions of the model. Since we have striven to provide a structure that inherits the main properties of the one set out in the previous Chapter, our discussion will largely focus on the differences between the two models.

## 3.1 The Environment

In the previous Chapter, we assumed that the owner of a downstream asset could enter the industry as either a non-integrated firm, or instantly acquire an upstream asset and enter it as an integrated upstream/downstream unit. In this Chapter we model a process of takeovers, in which the owner of a downstream firm pays for, and acquires residual control rights, over an upstream asset.

This additional dimension necessitates that some minor modifications are carried out on the timing of events. For simplicity, we assume that new-entrant downstream assets must wait until their customers first place an order before they commence the integration process.[1]

The motivation for this restriction is that it eliminates some (tedious) complexity in the dimension of the strategy space, which would force us to separately analyse the cases in which the owner-manager of a downstream asset must decide whether to: (i) first search for – and integrate with – an upstream asset, and then wait for its customer to place an order, and (ii) (the case considered here) waits for his customer to place an order, and then search for an upstream firm that can produce the appropriate widget and take it over.

We assume that when a non-integrated upstream-downstream pair contact each other on the market, they first choose the governance structure. After this is determined, the upstream asset is used to produce the widget, which is then used by the downstream asset to produce the customer's order.

If integration occurs, the price tendered on the upstream asset is determined cooperatively in a manner that equally divides net surplus from integration between the owners of the upstream and downstream assets. (The integrated unit then produces the widget and final good internally). In contrast, if they choose not choose to integrate, they trade a unit of widget at a price determined by Nash bargaining with equal bargaining weights. Given that the net surplus is divided equally, the owners choose integration if the net surplus from integration exceed that from continuing spot trading (and vice versa).

If a non-integrated upstream firm contacts an integrated one meet, there are no changes in ownership. First, all upstream assets (of a given type) are homogeneous (so there is nothing to be gained by exchanging one upstream asset for another). Second, the technology is such that only two assets can belong to a given integrated firm. Nevertheless, the two parties can agree to trade a widget at a price that is

[1]Suppose that customers differ according to the type of goods that potentially provide them with utility, and that their types are revealed to de novo downstream firms only after they place their first orders. In this setting, it would never — for a large enough number of customer types — be optimal for the downstream owner to randomly acquire a (specialized) upstream asset before he learns his customer's proclivities.

again determined via a Nash bargaining procedure with equal bargaining weights.

In this Chapter, spinoffs play no role. An integrated firm could spin-off its upstream subsidiary. Nevertheless, by the free-entry condition, doing so cannot be strictly optimal as the price of the upstream asset is zero.

## 3.2 States and Value Functions

The states are similar to the simplified model, except that the downstream assets must enter the industry as independent assets.

$\Pi$ and $u_0$ represents the value and mass of unmatched upstream asset, respectively. Let $V_0$ denote the value of an independent downstream asset without order, and $d_0$ denote the mass of them. Let $V_1$ denote the value of an independent downstream asset with order, and $d_1$ denote the mass of them. The value of an integrated asset without and with customer's order that cannot be produced internally is denoted by $K_0$ and $K_1$, respectively, and mass is denoted by $D_0$ and $D_1$, respectively.

Let $p_{\mathrm{n}}$ denote the price of one unit of specialized widget that independent downstream assets face, and $p_{\mathrm{i}}$ denote the price of widget that integrated assets face, in the spot-trading. Let $\rho_0$ denote the asset price.

Value functions can be represented as the following.

$$\begin{aligned}
r\Pi = -e_w^u + \alpha\Big[&\frac{d_1}{d_1+D_1}\big[z(p_{\mathrm{n}}-\kappa)+(1-z)(\rho_0-\Pi)\big] \\
&+\frac{D_1}{d_1+D_1}(p_{\mathrm{i}}-\kappa)\Big] \qquad (3.1)
\end{aligned}$$

$$rV_0 = \mu\,(V_1-V_0) \qquad (3.2)$$

$$\begin{aligned}
rV_1 = \beta\max\{&\nu-p_{\mathrm{n}}+V_0-V_1,\, K_0-\rho_0+\nu-\kappa-V_1\} \\
&+\lambda(V_0-V_1) \qquad (3.3)
\end{aligned}$$

$$rK_0 = -c_1^v+\mu X(\nu-\kappa)+\mu(1-X)(K_1-K_0)+\lambda(V_0-K_0) \qquad (3.4)$$

$$rK_1 = -c_1^v+\beta(\nu-p_{\mathrm{i}}+K_0-K_1)+\lambda(V_0-K_1) \qquad (3.5)$$

An upstream asset pays flow search cost, $e_w^u$ and when an upstream asset meets an independent downstream asset, it may sell off its asset for $\rho_0$, or it may produce a unit of widget at cost $\kappa$, and trades the widget for $p_{\mathrm{n}}$ depending on the trading partner. When an upstream asset meets an integrated asset, it may produce a unit of widget, at cost $\kappa$, and trade the widget for $p_{\mathrm{i}}$. $d_1/(d_1+D_1)$ denotes the probability that the trading partner is an independent downstream asset. $z$ and $1-z$ denotes the probability that an independent downstream wants to spot-trade and to integrate, respectively. $D_1/(d_1+D_1)$ denotes the probability that the trading partner is an integrated asset.

A downstream asset, with order in hand, searches for an upstream asset. Upon meeting an upstream asset, the downstream asset may buy out the upstream asset and produce the widget internally, or remain independent and trade $p_{\mathrm{n}}$ for a unit of widget. After the procurement of a unit of widget, the downstream asset produces a unit of final good instantly and sell it to the customer for $v$.

An integrated asset pays maintaining flow cost $c_1^v$ and waits for an order. As mentioned earlier in the previous section, this flow cost represents some disadvantage of integration: foregone investment opportunity, loss of incentives, governance cost, etc. If the order can be fulfilled internally, it produces the widget and final good instantly at a cost $\kappa$ and sells it to the customer for $v$. If it's not the case, the integrated firm must search for an independent upstream asset to procure the widget through spot-trading. While searching, an independent downstream and integrated asset face a risk of losing the customer. Note when an integrated asset loses a customer it should re-enter the industry as an independent downstream asset.

## 3.3 Preliminary Results

Note, as before, managers will not devote any search efforts. The owner of independent upstream asset or downstream asset always manages her own asset. The owner of vertically integrated firm always manages downstream asset.

Consider the widget prices. Since the owner of vertically integrated assets can order to produce a widget, if it is of type that can be produced by the integrated upstream asset, the price of widget that is internally produced is $\kappa$. The widget price between independent upstream and downstream assets is the same as in the simplified model. If they trade, upstream asset gets $p_{\mathrm{n}}-\kappa+\Pi$ and downstream asset gets $\nu - p_{\mathrm{n}} + V_0$. If they do not trade, upstream firm gets $\Pi$ and downstream firm gets $V_1$. Therefore, equation (2.10) still determine the widget price, $p_{\mathrm{n}}$. Now consider the price of widget between independent upstream and integrated assets. If they trade, upstream firm gets $p_{\mathrm{i}} - \kappa + \Pi$ and integrated firm gets $\nu - p_{\mathrm{i}} + K_0$. If they don't trade, upstream firm gets $\Pi$ and downstream firm gets $K_1$. Again, equation (2.13) still determine the widget price, $p_{\mathrm{i}}$.

Therefore, the price of a unit of widget is $\frac{\nu+\kappa+V_0-V_1}{2}$ in a spot-trading between independent upstream and independent downstream assets, and $\frac{\nu+\kappa+K_0-K_1}{2}$ in a spot-trading between independent upstream and integrated assets. If an integrated asset can procure the widget internally, the price of widget is $\kappa$.

Now consider the asset price. It is determined in a way such that the net surplus from the integration is divided equally between upstream and downstream asset owners. When the downstream asset integrates with the upstream asset, the net surplus that arise from the integration is $K_0 + \nu - \kappa - V_1$. Note that the integrated asset is currently with an active order which will be fulfilled right after the integration. Therefore, the upstream asset price is $(K_0 + \nu - \kappa - V_1)/2$.

Lemma 3.1 The price of upstream asset is $\frac{\nu-\kappa+K_0-V_1}{2}$.

## 3.4 Steady State Equilibria

We first define the steady state equilibrium.

Definition 3 A steady state equilibrium is the population distribution of all the assets $(u_0, d_0, d_1, D_0, D_1)$, the value functions $(\Pi, V_0, V_1, K_0, K_1)$, and the matching rates $(\alpha, \beta)$ that satisfy the following conditions:

1. Lemmas 2.2–2.4 and 3.1 are satisfied.
2. Value functions satisfy equations (3.1)–(3.5).
3. Owners of the downstream assets optimally choose whether to remain independent or to integrate when they meet upstream assets.
4. Free entry of upstream assets: $\Pi = 0$.
5. Mass of downstream asset is one: $d_0 + d_1 + D_0 + D_1 = 1$

6. Steady state matching rates satisfy matching functions:

   $\alpha u_0 = \beta(d_1 + D_1) = m_w M_w(u_0, d_1 + D_1)$.

Definition 4 There are three classes of (pure strategy) steady state equilibria: spot-trading, pervasive integration, and hybrid.

- In spot-trading equilibrium, all the independent downstream asset owners choose to remain independent when they meet upstream assets.
- In the (pervasive) integration equilibrium, all the independent downstream asset owners choose to integrate when they meet upstream assets.
- In the hybrid equilibrium, independent downstream asset owners are indifferent to remain independent or to integrate when they meet upstream assets such that some positive mass of independent downstream assets choose to remain independent and the other positive mass choose to integrate.

The steady state population distribution for three types of equilibria can be easily derived. Let $z$ denote the probability that independent upstream and downstream assets choose spot-trading. Then, $zD_0 = (1-z)d_0$. Note $z = 1$ in a spot-trading equilibrium, $z = 0$ in an integration equilibrium, and $z \in (0,1)$ in a hybrid equilibrium.

Lemma 3.2 In a steady state equilibrium,

$$\begin{aligned}
u_0 &= \frac{\beta\mu\left[\lambda(\beta+\lambda+\mu(1-X)\big(\beta(1-z)+\lambda\big)\right]\beta(1-X)]}{\alpha[\beta+\lambda+\mu(1-X)][\beta(\lambda+\mu(1-z))+\lambda(\lambda+\mu)]},\\
d_0 &= \frac{\lambda(\beta+\lambda)}{\beta(\lambda+\mu(1-z))+\lambda(\lambda+\mu)},\\
d_1 &= \frac{\mu\lambda}{\beta(\lambda+\mu(1-z))+\lambda(\lambda+\mu)},\\
D_0 &= \frac{(1-z)\mu\beta(\beta+\lambda)}{[\beta+\lambda+\mu(1-X)][\beta(\lambda+\mu(1-z))+\lambda(\lambda+\mu)]},\\
D_1 &= \frac{(1-z)\beta\mu^2(1-X)}{[\beta+\lambda+\mu(1-X)][\beta(\lambda+\mu(1-z))+\lambda(\lambda+\mu)]},
\end{aligned}$$

In a steady state spot-trading equilibrium, $z = 1$, in a steady state integration equilibrium, $z = 0$, and in a steady state hybrid equilibrium, $z \in (0,1)$.

Proof. See Appendix A. ■

We define $\pi \equiv (\nu-\kappa)/2$. It is convenient to begin with the free entry condition for the upstream assets.

Lemma 3.3 In a steady state spot-trading equilibrium,

$$\alpha_s = \frac{e_w^u(2r+\beta+2\lambda+2\mu)}{2(r+\lambda+\mu)\pi}: \ EE_s^e$$

In a steady state integration equilibrium,

$$\alpha_{\mathrm{i}} = \frac{e_w^u(\beta+\lambda)(2r+2\beta+2\lambda)(\lambda+\mu(1-X))}{\begin{array}{c}\{-c_1^v\lambda(\beta+\lambda+\mu(1-X))[2r+\beta+2(\lambda+\mu(1-X))] \\ +2(\beta+\lambda)(2r+\beta+2\lambda)(r+\lambda+\mu)(\lambda+\mu(1-X))\pi\}\end{array}}: \ EE_{\mathrm{i}}^e$$

In a steady state hybrid equilibrium,

$$\alpha_h = \frac{\begin{array}{c}\{e_w^u(2r+\beta+2\lambda)(2r+\beta+2\lambda+2\mu) \\ \times[\lambda(\beta+\lambda)+\mu(1-X)(\beta(1-z)+\lambda)]\}\end{array}}{\begin{array}{c}\{c_1^v(1-X)(1-z)\beta\mu(2r+\beta+2\lambda+2\mu) \\ +2(r+\lambda+\mu)(2r+\beta+2\lambda)[\lambda(\beta+\lambda)+\mu(1-X)(\beta(1-z)+\lambda)]\pi\}\end{array}}: \ EE_h^e$$

Further, $\partial\alpha_h/\partial z > 0$.

Proof. See Appendix A. ■

The condition that spot-trading equilibrium matching rates should satisfy, that is $EE_s^e$, does not change from that of simple model $EE_s$ in Lemma 2.6 on page 43. However, the condition that integration matching rates should satisfy, $EE_{\mathrm{i}}^e$, is not the same as from that of simple model $EE_{\mathrm{i}}$. Now it reflects the fact that all the downstream assets must enter the industry independent.

Lemma 3.4 Let $E \equiv \{u_0, d_0, d_1, D_0, D_1; \Pi_0, V_0, V_1, K_0, K_1; \alpha, \beta\}$. $E$ is a spot-trading equilibrium if and only if $E$ is an equilibrium with $V_0 \geq K_0$ and $D_0 = D_1 = 0$, $E$ is an integration equilibrium, if and only if $E$ is an equilibrium with $K_0 \geq V_0$ and $d_0 = d_1 = 0$. $E$ is a hybrid equilibrium, if and only if $E$ is an equilibrium with $V_0 = K_0$ and $d_0, d_1, D_0, D_1 > 0$.

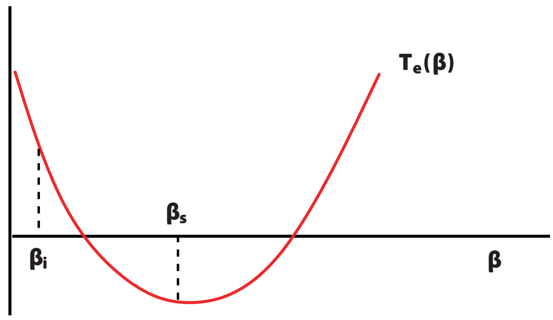


Figure 3.2. No Equilibrium Case

Further,

$$\begin{aligned} V_0 &\geq (\leq, =) K_0 \\ \Leftrightarrow T_e(\beta) &\equiv c_1^v \beta^2 + \left[2c_1^v(2r+2\lambda+\mu(2-X)) - 4\mu X(r+\lambda+\mu)\pi_d\right]\beta \\ &\quad + 4(r+\lambda+\mu)\left[c_1^v(r+\lambda+\mu(1-X)) - 2\mu X(r+\lambda)\pi_d\right] \\ &\geq (\leq, =) 0 \end{aligned}$$

Proof. See Appendix A. ■

Note that $T_e(\beta)$ is exactly the same as $T_s(\beta)$ in Lemma ?? on page ?? despite the fact that all the downstream assets must enter the industry independent. Let $(\alpha_s, \beta_s)$ satisfy $EE_s^e$ and $SS$, $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ satisfy $EE_{\mathrm{i}}^e$ and $SS$. By Lemma 3.4, $(\alpha_s, \beta_s)$ and $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ is a steady state spot-trading and integration equilibrium matching rates if $T_e(\beta_s) \geq 0$ and $T_e(\beta_{\mathrm{i}}) \leq 0$, respectively.

Unlike in the simple model, it is not guaranteed $\alpha_s > \alpha_h > \alpha_{\mathrm{i}}$ for all $\beta$ because of the presence of $c_1^v$. This can make a problematic situation in which there does not exist an equilibrium. Consider a situation in which $\lim_{\beta\to 0} T_e(\beta) > 0$. Suppose $\alpha_s < \alpha_{\mathrm{i}}$. Then $\beta_s < \beta_{\mathrm{i}}$ since $(\alpha_s, \beta_s)$ and $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ satisfy $SS$. Suppose, further, $T_e(\beta_s) < 0$ and $T_e(\beta_{\mathrm{i}}) > 0$ so that neither spot-trading nor integration equilibrium exists (see Figure 3.2). Does there exist a hybrid equilibrium in this situation? The answer is negative. Suppose there exists a hybrid equilibrium. Then some downstream assets integrate while some remain independent. We need $\alpha_s < \alpha_h < \alpha_{\mathrm{i}}$. However, since $\partial\alpha_h/\partial z > 0$, $\alpha_h < \alpha_s$ always. Hence, we need $\alpha_{\mathrm{i}} < \alpha_h < \alpha_s < \alpha_{\mathrm{i}}$, a contradiction.

To avoid this situation, we can make either a regularity condition that guarantees $\alpha_s > \alpha_{\mathrm{i}}$ for all $\beta$, or we can make a condition under which $\lim_{\beta\to 0} T_e(\beta) < 0$. We follow the latter approach for simplicity.

Condition 1 We assume that the parameter values satisfy the following conditions.

$$\begin{aligned}&(r+\lambda+\mu(1-X))c_1^v < 2\mu X(r+\lambda)\pi, \text{ and}\\ &\lambda c_1^v < 2(r+\lambda+\mu)(\lambda+\mu(1-X))\pi\end{aligned}$$

Note that Condition 1 implies two things: the existence of $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ and $\lim_{\beta\to 0} T_e(\beta) < 0$, which we show below.

Lemma 3.5 Under Condition 1, $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ exists and $\lim_{\beta\to 0} T_e(\beta) < 0$.

Proof. See Appendix A. ■

Note that $T_e(\beta_0) = (\geq, \leq) 0$ can also be represented as the following:

$$c_1^v = (\geq, \leq) \frac{4\mu X(r+\lambda+\mu)(2r+\beta_0+2\lambda)\pi}{(2r+\beta_0+2\lambda+2\mu)(2r+\beta_0+2\lambda+2\mu(1-X))} \equiv c_s.$$

We can get rid of $c_1^v$ from $EE_h^e$ using $T_e(\beta_h) = 0$.

Lemma 3.6 Let $(\alpha_h, \beta_h)$ be the hybrid equilibrium matching rates. Suppose $\alpha_{\mathrm{i}} < \alpha_s$. Then,

$$\alpha_{\mathrm{i}} \leq \alpha_{\mathrm{i}}|_{c_1^v \to c_s} = \lim_{z\to 0} \alpha_h < \alpha_h < \lim_{z\to 1} \alpha_h = \alpha_s.$$

Proof. See Appendix A. ■

Condition 1 ensures that $T_e(\beta) = 0$ has only one positive real root. Let it be denoted by $\beta_0 > 0$. Suppose $\beta_y = \beta_0$, where $y \in \{s, i\}$. Then hybrid equilibrium cannot exist since all the owners of upstream and downstream assets choose to remain independent or to integrate. If $c_1^v = 0$, it is easy to see that $T_e(\beta) < 0$ for all $\beta > 0$ so that a unique integration equilibrium exists and other equilibria do not exist.

The following proposition summarizes the existence of equilibria.

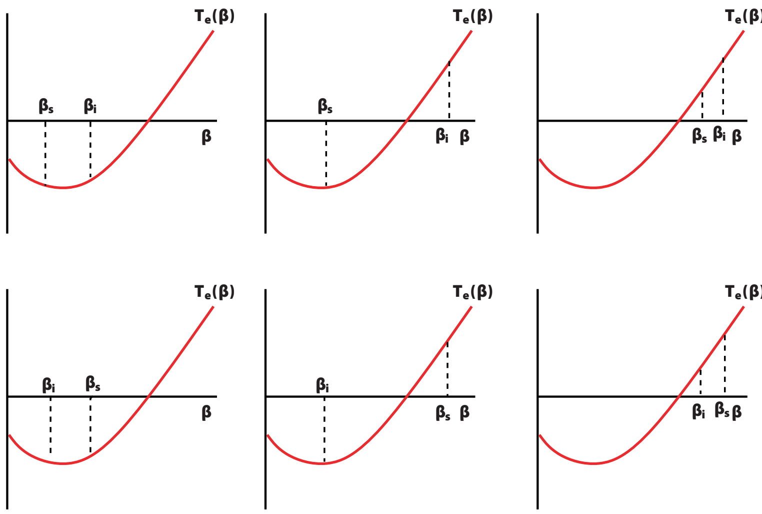


Figure 3.3. Equilibria

Proposition 3.1 Let Condition 1 hold. Let $(\alpha_s, \beta_s)$ satisfy $EE_s^e$ and $SS$, $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ satisfy $EE_{\mathrm{i}}^e$ and $SS$. Then, if $T_e(\beta_s) \geq 0$, there exists a unique spot-trading equilibrium. If $T_e(\beta_{\mathrm{i}}) \leq 0$, there exists an integration equilibrium. If $\beta_s < \beta_0 < \beta_{\mathrm{i}}$, there exists a unique hybrid equilibrium.

Proof. Note that $(\alpha_s, \beta_s)$ satisfies $EE_s^e$ and $SS$ and that $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ satisfies $EE_{\mathrm{i}}^e$ and $SS$ . By Lemma 3.4, $(\alpha_s, \beta_s)$ and $(\alpha_{\mathrm{i}}, \beta_{\mathrm{i}})$ is a steady state spot-trading and integration equilibrium matching rates if $T_e(\beta_s) \geq 0$ and $T_e(\beta_{\mathrm{i}}) \leq 0$, respectively. Therefore the first two results follows.

Suppose $\beta_s < \beta_0 < \beta_{\mathrm{i}}$. Let $(\alpha_0, \beta_0)$ satisfy $SS$. Obviously $\alpha_s > \alpha_0 > \alpha_{\mathrm{i}}$. Since $T_s(\beta_0) = 0$, to prove the existence of a hybrid equilibrium, we must show that there exists a $z_h \in (0,1)$ such that $z_h$ satisfies $EE_h^e$ when $\alpha_h = \alpha_0$ and $\beta = \beta_0$. Note $\partial \alpha_h / \partial z > 0$. By Lemmas 3.3 and 3.6, given $\beta_0$ and $T_e(\beta_0) = 0$, there exist a unique $z \in (0,1)$ that satisfies $\alpha_h = \alpha_0$. ■

Proposition 3.2 There exists at least one equilibrium under Condition 1.

Proof. Given $\beta_s$ and $\beta_{\mathrm{i}}$, if $T_e(\beta_s) \geq 0$ then spot-trading equilibrium exists and if $T_e(\beta_{\mathrm{i}}) \leq 0$ then integration equilibrium exists. Otherwise, hybrid equilibrium exists. ■

Figure 3.3 presents equilibria for some interesting cases. The cases in which only a unique integration equilibrium exists are depicted in upper left and lower left graphs. Spot-trading equilibrium, in these cases, cannot exist since $T_e(\beta_s) < 0$. Hybrid equilibrium cannot exist, either, because $\beta_s, \beta_{\mathrm{i}} < \beta_0$. The cases in which only a unique spot-trading equilibrium exists are drawn in upper right and lower right graphs. Note in these cases, integration or hybrid equilibrium cannot exist since $T_e(\beta_{\mathrm{i}}) > 0$ and $\beta_s, \beta_{\mathrm{i}} > \beta_0$. Upper middle graph shows the case in which only hybrid equilibrium exists. Spot-trading and integration equilibrium don't exist in this case, since $T_e(\beta_s) < 0$ and $T_e(\beta_{\mathrm{i}}) > 0$. Hybrid equilibrium exists since $\beta_s < \beta_0 < \beta_{\mathrm{i}}$. Lower middle graph depicts the case in which both spot-trading and integration equilibria exist. Note $T_e(\beta_s) > 0$ and $T_e(\beta_{\mathrm{i}}) > 0$ so that spot-trading and integration equilibrium exist. Hybrid equilibrium does not exist since $\beta_s > \beta_0 > \beta_{\mathrm{i}}$. Note that some other cases might happen in which either $\beta_{\mathrm{i}}$ or $\beta_s$ equals $\beta_0$. In that case, hybrid equilibrium does not exist since all the downstream assets would choose to integrate or spot-trade.

Clearly, as noted before, the hybrid equilibrium is not a knife-edge equilibrium. Parameter values may change but as long as parameter values still satisfy the above environment, there exists a hybrid equilibrium. The industry adjusts to the new parameter values by changing $z$, proportion of downstream assets that choose to remain independent in meetings with upstream assets. The mass of upstream firms entering the industry depends on $z$. Hence, the adjustment in the composition will affect the market thickness, and in turn, the payoffs of independent and integrated assets.

Now a downstream firm can only enter the industry as an independent firm. If it wants to integrate, it has to search for an independent upstream firm and buy out the upstream asset from the owner. Yet, even if the industry is characterized by pervasive vertical integration, the market for spot-trading between independent upstream firms and vertically integrated firms remains active since the integrated firm cannot procure all the type of widgets internally. The industry can exhibit pervasive integration or pervasive spot-trading or both in equilibrium. However, in some case, the pervasive integration or pervasive spot-trading are not sustainable as an equilibrium. We have shown that, in this case, vertically integrated firms and independent downstream firms co-exist in a non knife-edge equilibrium.

## 3.5 Applications

In this section, we present two examples that our model can shed light onto in understanding the phenomena.

According to Mullainathan and Scharfstein (2003), vinyl chloride monomer (VCM) and polyvinyl chloride (PVC) industry shows two thirds of VCM producers in their sample are integrated downstream into PVC. Their main focus is to analyze whether integrated producers show different behavior from nonintegrated producers. They state:

> We focus on the producers of the vinyl chloride monomer (VCM), the sole use of which is in the production of the widely used waterproof plastic, polyvinyl chloride (PVC). VCM is a homogenous commodity and is traded in relatively liquid markets. Moreover, there is no obvious production link between VCM and PVC other than that one is an input into the other. For example, unlike some chemicals, PVC is not a by-product of VCM, but rather requires a separate production process. Nevertheless, two thirds of VCM producers in our sample are integrated downstream into PVC. The existing literature would ask why we observe this degree of integration — and might well come up short given the focus of that literature on incomplete contracts, lock-in, opportunistic behavior, and relationship-specific investments, none of which appear to be important in this setting.

Our approach can explain why some firms in the industry are integrated while others remain independent. The industry exhibits hybrid equilibrium in our terminology.

Holmes (1999) presents some evidence of a link between localization of industry and vertical disintegration based on Stigler's idea that concentration of industry may encourage vertical disintegration. He reports a positive correlation between localization and vertical disintegration. In our point of view, localization of a industry increases search efficiency and lowers search cost, and leads to higher composition of independent firms.

## 3.6 Concluding Remarks for Chapters 2 and 3.

Our simple model in Chapter 2 expands the determinants of boundary of firms from individual firm level to industry level using a search theory. We have explicitly modeled the market friction and examined an industrial structure in an environment in which one firm's decision on the ownership structure is affected by other firms' choices on the ownership structure through the changes in market thickness. We have analyzed the determinants of an industrial structure under this environment and shown that independent downstream and integrated firms can co-exist in a non-knife edge equilibrium even if firms are homogeneous. We have also provided how an industry would evolve according to the changes in search efficiency, search cost, and customer's arrival rate.

In the simple model, firms' decisions on the ownership structure are made before they enter the industry. In Chapter 3, we have relaxed this assumption and introduced asset trading into the model. Firms are assumed to enter the industry independent and choose ownership when they meet a trading partner. We have shown that the same results can be obtained when we consider asset trading explicitly.

Chapter 4

# Heterogeneous Downstream Assets and Jumps of the Asset Values Before and After the Integration

In this Chapter, we present a significant extension of the model by introducing heterogeneity and uncertainty. More specifically, we consider an environment populated by different types of downstream assets, and posit idiosyncratic shocks that affect the quality of the final good. Using this richer setting we can explore the evolution of firms' (stock-market) values over time, and characterize the effects of mergers on them. Under conditions of heterogeneity, establishing the equilibrium coexistence of independent and integrated firms is no longer a major concern: the primary lacuna in the literature has been establishing this phenomena under conditions of homogeneity. In the presence of heterogeneity, firms optimally choose their ownership structures, after searching, based on their types and the realization of the idiosyncratic shock. To focus on the changes of asset values, we reduce one dimension of the model by assuming that upstream assets can produce every type of widget so that integrated assets don't search for upstream assets.[1] To keep the model simple, following Wasmer and Weil (2004), we assume that downstream

[1]Note that the assumption that integrated asset cannot procure all the type of widgets internally was one of the key assumptions in deriving the co-existence of independent and integrated assets in a non-knife-edge equilibrium with homogeneous assets in the previous two Chapters. With heterogeneity, we don't need it.

assets have to search for customers and that upstream and downstream assets can freely enter the industry. This means that all the matching rates are determined endogenously.

We start with an illustration. Consider an industry composed of upstream assets, downstream assets, and customers. Downstream assets are of two types: bad and good. All the downstream assets must enter the industry as bad type. A bad type downstream asset turns to a good type downstream asset while searching in the final goods market. A good type downstream asset does not turn to a bad type while searching in the final goods market. However, a good type changes to a bad type in two occasions: (i) Upon separation without integration in the widget market, and (ii) Upon the resolution of an integrated firm. Therefore, the good type downstream asset has only one shot of chance to integrate before turning back to a bad type. Bad type downstream assets perform as equivalently well as good downstream assets in spot-trading but they perform badly if they integrate in terms of flow maintaining cost of integration. A downstream asset of either type searches for a customer first. With a customer's order in hand, the downstream asset searches for an upstream asset. Note that downstream assets don't change types while searching for the other assets. The idiosyncratic shock is realized upon a meeting of downstream asset and an upstream asset. Given the realization of the idiosyncratic shock, upstream and downstream asset owners determine ownership: spot-trading or integration. When they choose integration under the case in which the realization of the shock is $\boldsymbol{\theta}_0$, the shock becomes fixed within the integrated asset in the sense that the realization of the idiosyncratic shock is always $\boldsymbol{\theta}_0$ whenever the integrated asset produces a unit of final good.

Figure 4.1 illustrates this environment. $\boldsymbol{u}$, $\boldsymbol{b}$, $\boldsymbol{g}$ and $\boldsymbol{c}$ denotes upstream asset, bad and good type downstream asset, and customer, respectively. The solid line between $\boldsymbol{u}$ and $\boldsymbol{g}$ denotes integration.[2] The solid lines between $\boldsymbol{b}$ and $\boldsymbol{c}$ and $\boldsymbol{g}$ and $\boldsymbol{c}$ denote that the downstream assets meet customers and have active orders. The dotted line with $\boldsymbol{X}$ on it between $\boldsymbol{u}$ and $\boldsymbol{g}$ denotes the resolution of integration and assets are forced to exit the industry, and upon the resolution the downstream asset changes the type from good to bad.

The customer's valuation is solely dependent on the realization of the idiosyn-

[2] In the following, we will make a cost assumption such that the bad type never integrates.

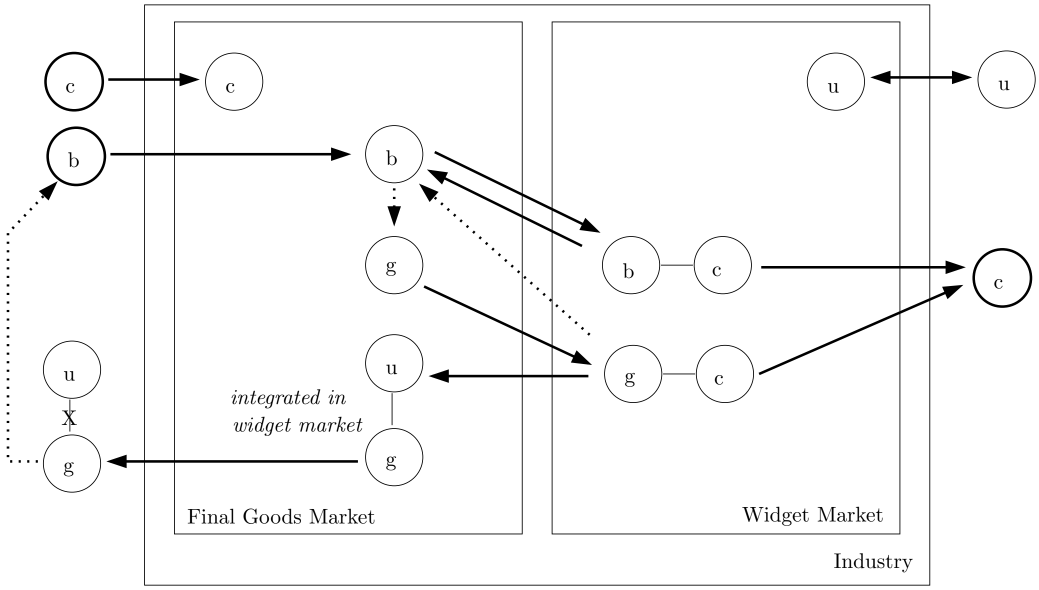


Figure 4.1. Markets and Industry

cratic shock regardless of the choice of the ownership, and the flow maintaining cost of integration is dependent on the type of downstream asset.

When they choose spot-trading, price is determined by Nash bargaining with equal power, and they separate right after the trade. The asset price, when they choose integration, is determined by take-it-or-leave-it offer by the downstream asset. Upstream asset can accept or reject the offer. When it accept the offer, an integration takes place. When it rejects the offer, a spot-trading takes place.

Note that there exist reservation values of idiosyncratic shock for integration and that the reservation values are different across the types of the downstream assets.

## 4.1 Technological Assumptions

For a clarification, we present the assumptions in full and in detail in this section although many of them overlap with those in the previous Chapters. However, for brevity, we don't lay down primitive informational assumptions in this Chapter. We just state that the search activity is not contractible as opposed to derive the result from primitive informational assumptions as in Chapter 2.[3]

[3]See Section 2.3 on page 29 for the primitive informational assumptions.

As before, all the agents — owners, managers, and customers — are risk neutral and share a common time preference $r$.

### 4.1.1 Goods and Markets

There are two goods: widget and final good. The widget and final goods are of type $\omega \in [0,1]$. The final goods of the same type can have different quality $q$. The quality will be specified later. Widget market is populated by upstream assets and downstream assets while final goods market is populated by downstream assets and customers.

### 4.1.2 Upstream and Downstream Assets

The upstream asset instantly transforms one unit of managerial labor into one unit of non-storable widget of specific type with no production cost ($\kappa = 0$). An upstream asset evaporates at an exogenous rate $\delta$ while searching for a downstream asset in the widget market.[4] Downstream asset instantly transforms one unit of widget of type $\omega$ and one unit of managerial labor into one unit of non-storable final good of type $\omega$ with no production cost. Downstream assets are of two types: bad and good. Downstream assets changes the types at a rate when they are in the specific states. The details will be provided later. Bad downstream assets and good downstream assets are not distinguishable in spot-trading. However, they perform differently upon integration with upstream assets. Bad downstream assets pay more in maintaining cost relative to good downstream assets. The specification of the cost will be provided later. The properties of upstream and downstream assets are not affected by the ownership.

### 4.1.3 Customers

A customer wants one unit of specific type $\omega$ of final good. We assume that $\omega$ follows a uniform distribution with support $[0,1]$. A customer searches in the final goods market with no cost. When he/she meets an independent downstream asset or an integrated asset, he/she place an order and the specific type $\omega$ is revealed.

[4] An integrated upstream asset does not evaporate while it does not search for an downstream asset.

The independent downstream asset must search for an upstream asset to procure a unit of widget of type $\omega$ while integrated asset can produce the final good instantly.

We assume that customers do not leave the matched downstream assets while the downstream assets are searching for upstream assets to procure the specific types of widgets to fulfill the orders. However, customers leave the matched downstream assets when bargaining between upstream and downstream assets involved breaks down.[5]

When a trade takes place, the customer consumes the final good, exits the market and is replaced by a new customer who wants one unit of specific type $\omega'$ of final good.[6]

Customer's common valuation on one unit of final good of type $\omega$ is $v(q;\omega)$, which depends on the type $\omega$ and quality $q$ of the final good:

$$v(q;\omega) = \chi(\omega) \cdot v(q),$$

where $\chi(\omega) = 1$ if the type of the final good is of type $\omega$, and $\chi(\omega) = 0$ otherwise. $v(q)$ will be specified later.

### 4.1.4 Spot-Trading and Vertical Integration

There are two forms of widget procurement: through spot-trading and through vertical integration.

When upstream and downstream assets meet, they are subject to a idiosyncratic shock, $\theta$, which determines the customer's valuation of the final product.

$$v(q) = \theta$$

We assume that $\theta$ follows a uniform distribution on $[0,1]$. The type of a downstream asset is irrelevant in the sense that it does not affect the customer's valuation. Customer's valuation, $v(q)$, is solely dependent on the idiosyncratic shock.

[5]This assumption, together with the free entry condition, make the outside options of upstream and downstream assets – independent or integrated – play no role in the bargaining.

[6]This assumption prevents downstream assets – independent or integrated – from forming a long term relationship with a customer.

In a spot-trading, after realization of the shock, upstream and downstream assets trade a unit of proper type of widget and separate. Now we explain about the vertical integration. A bad (good) type integrated asset is composed of an upstream asset and a bad (good) type downstream asset. When assets integrate, we assume that the realization of the shock $\theta$ becomes fixed within the integrated asset in the sense that the realization of the idiosyncratic shock is always $\theta$ whenever the integrated asset produces a unit of final good. Hence, if the realization of the idiosyncratic shock is high enough, the downstream asset would buy out the upstream asset to exploit the high valuation of the customer. A vertically integrated asset incurs flow maintaining cost upon the owner of the integrated asset. Let $c_b$ ($c_g$) denote the flow maintaining cost for a bad (good) type integrated asset. Vertical integration is subject to resolve at an exogenous rate $s$. When a vertical integration resolves upstream asset evaporates and downstream asset changes to bad type.[7] We impose a restriction that only the owner of the downstream asset can buy out the upstream asset, not vice versa.[8]

### 4.1.5 Managers and Owners

Managers are needed in searching and production. Managers are equally talented at operating either type of asset. All the costs paid by the managers are in the form of disutility for them. Managers of upstream assets may search in the widget market paying flow cost of $e_w^u > 0$. Managers of downstream assets may search in the widget market and final goods market paying flow cost of $e_w^d$ and $e_g^d$, respectively. We set $e_w^d \to 0$ and $e_g^d > 0$ in this section.

There exists a competitive market for managers and managers can be replaced instantly.[9]

Each owner can own at most two assets: an upstream asset, a downstream asset or an upstream asset and a downstream asset but not two upstream assets nor two downstream assets. An owner can manage at most one asset in the strict

[7]Equivalently, together with free entry conditions for upstream and downstream assets, we can assume that both assets must exit the industry.

[8]The restriction simplifies our analysis without the loss of the generality since, as will be shown later, the owner of the integrated assets always manage the downstream asset in equilibrium.

[9]This strong assumption prevents managers who do not own the assets from searching and making investments.

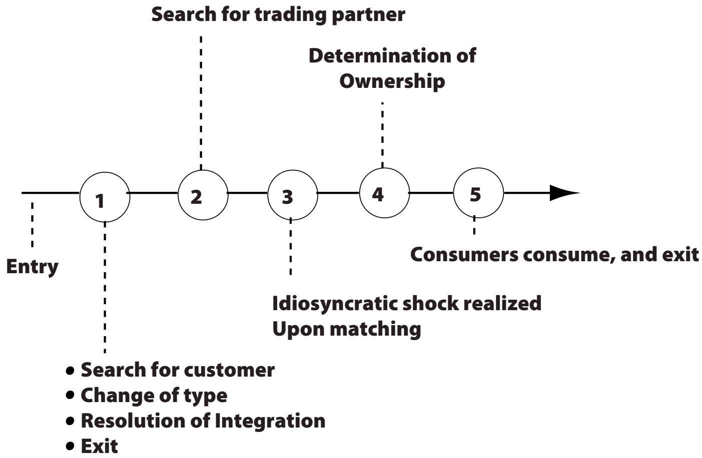


Figure 4.2. Timing of the Model

sense that an owner of two assets cannot manage upstream asset for a while and then downstream asset for a while.

The owners can hire/fire managers, and pay the (flow) maintaining costs, $c_b$ and $c_g$. Upstream asset owners can sell out their assets to downstream asset owners upon meeting. Integrated asset owners cannot sell off its upstream assets.[10]

### 4.1.6 Entry of the Assets and Change of the Type

Upstream and downstream assets can enter the industry without any cost. The mass of customers is fixed and normalized to one. Upstream and downstream assets must enter the industry independent and downstream assets must enter the industry as bad type. Bad type downstream assets without order, in the middle of searching for customers, can change their type to good at an exogenous rate $\gamma$. Good type downstream assets do not change their type until the resolution of the integration, which happens at an exogenous rate $s$.

### 4.1.7 Timing

Timing of the model is illustrated in Figure 4.2. Note that 3, 4 and 5 in Figure 4.2 take place at the same time.

[10] That is, we do not allow spin-offs in this Chapter.

1. Managers of independent downstream assets without order and integrated assets search for customers in the final goods market. While searching for customers, bad type independent downstream assets can change to good type at a rate $\gamma$, and integrated assets resolve at a rate $s$.

   Managers of upstream assets, and downstream assets with order in hand search in the widget market.

2. When they meet in the final goods market, the type of final good $\omega$ is revealed. An independent downstream asset must search for an upstream asset while integrated asset can produce the final good instantly.

3. When they – independent upstream and downstream assets – meet, idiosyncratic shock, $\theta$, is realized upon the meeting.

4. They choose the ownership structure: spot-trading or vertical integration. Asset trade may take place. When they choose spot-trading, bad type downstream asset remains bad type while good type downstream asset changes to bad type.

5. Given the ownership structure, prices are determined, and production (of the widget and final good) and consumption (of the final good) take place instantly.

## 4.2 Matching

All meetings in the widget market are bilateral involving an upstream asset and a downstream asset. Let $U_w$ and $D_w$ denote the mass of upstream and downstream assets actively searching in the widget market. The widget market is represented by a standard homogeneous of degree one matching function.

$$\alpha U_w = \beta D_w = m_w M_w(U_w, D_w),$$

where $m_w > 0$.

All the meetings in the final goods market are also bilateral involving a customer and a downstream asset. Let $C_g$ and $D_g$ denote the mass of customers and downstream assets actively searching in the final goods market. The final goods market is represented by a standard homogeneous of degree one matching function.

$$\eta C_g = \mu D_g = m_g M_g(C_g, D_g),$$

where $m_g > 0$.

We assume the following assumptions on $M_i(\cdot)$, $i \in \{w, g\}$.

1. $M_i(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable

2. $M_i(\cdot)$ is of homogeneous of degree one

3. $M_i(\cdot)$ satisfies the Inada conditions:

$$\lim_{W_i \to 0} \partial M_i / \partial W_i = \infty, \lim_{W_i \to \infty} \partial M_i / \partial W_i = 0$$
$$\text{for } i \in \{w, g\}, \text{ and } W_w \in \{U_w, D_w\}, W_g \in \{C_g, D_g\},$$

and boundary conditions:

$$M_i(0, \cdot) = M_i(\cdot, 0) = 0.$$

## 4.3 Contracts and Bargaining

The managerial search efforts are not ex-ante contractible between owners and managers.

The prices of widget between independent upstream and downstream assets are determined by Nash Bargaining with equal power after the realization of the idiosyncratic shock.

The price of upstream asset is determined by a take-it-or-leave-it offer by the downstream asset manager. Upstream asset manager can accept or reject the offer. When he/she accepts the offer, a vertical integration takes place. When he/she rejects the offer, they perform a spot-trading.

The price of the final good is determined by a take-it-or-leave-it offer by the downstream asset manager. The customer can accept or reject the offer. When the customer accepts the offer trade takes place, and when the customer rejects the offer the downstream asset and the customer separates without production and consumption, and they keep searching.[11]

## 4.4 States and Value Functions

Depending on the realization of the shock, independent upstream and independent downstream firms may integrate or spot-trade. Note that integrated firms do not have to search in the widget market since its upstream asset can now produce every type of widget.

$\Pi$ and $u_0$ represents the value and mass of unmatched upstream firm, respectively. Let $b_0$ and $b_1$ denote the value of an independent bad downstream asset without order and with order, respectively, and $d_0^b$ and $d_1^b$ denote the mass of them. Let $g_0$ and $g_1$ denote the value of an independent good downstream asset without order and with order, respectively, and $d_0^g$ and $d_1^g$ denote the mass of them. Let $B_0(\theta)$ denote the value of integrated upstream and bad downstream assets, with idiosyncratic shock realization of $\theta$, and $D_0^b$ denote the mass of them. Let $G_0(\theta)$ denote the value of integrated upstream and bad downstream assets, with idiosyncratic shock realization of $\theta$, and $D_0^g$ denote the mass of them.

Since the values of customers are all zero, we define mass of the customers only. Let $C_0$ denote mass of unmatched customers. $C_1^b$ and $C_1^g$ denotes the mass of customers matched with a bad and a good independent downstream asset, respectively.

Let $p_b(\theta)$ and $p_g(\theta)$denote the price of one unit of specialized widget that independent bad and good downstream assets face in the spot-trading. Let $\rho_b(\theta)$ and $\rho_g(\theta)$ denote the asset price that independent bad and good downstream assets face, respectively. The value functions for the problem are

$$\begin{aligned} r\Pi = -e_w^u + \alpha \int_\theta \Big[ & \frac{d_1^b}{d_1^b + d_1^g}[z_b p_b(\theta) + (1 - z_b)\rho_b(\theta)] \\ & + \frac{d_1^g}{d_1^b + d_1^g}[z_g p_g(\theta) + (1 - z_g)\rho_g(\theta)] \Big] d(\theta) \end{aligned} \tag{4.1}$$

[11]Hence, customer's valuation can be fully extracted by downstream/integrated firm.

$$
\begin{aligned}
rb_0 &= -e_g^d + \mu(b_1 - b_0) + \gamma(g_0 - b_0) && (4.2)\\
rb_1 &= \beta \int_\theta \max\{\theta - p_b(\theta) + b_0 - b_1, \\
&\qquad B_0(\theta) - \rho_b(\theta) + \theta - b_1\} d\theta && (4.3)\\
rB_0(\theta) &= -c_b - e_g^d + \mu\theta + s[b_0 - B_0(\theta)] && (4.4)\\
rg_0 &= -e_g^d + \mu(g_1 - g_0) && (4.5)\\
rg_1 &= \beta \int_\theta \max\{\theta - p_g(\theta) + b_0 - g_1, \\
&\qquad G_0(\theta) - \rho_g(\theta) + \theta - g_1\} d\theta && (4.6)\\
rG_0(\theta) &= -e_g^d + \mu\theta + s[b_0 - G_0(\theta)], && (4.7)
\end{aligned}
$$

where $z_b$ and $z_g$ denote the probability that bad and good independent downstream assets choose spot-trading.

An upstream asset pays search cost, $e_w^u$ and when it meets an independent bad (good) type downstream asset, it may sell off its asset for $\rho_b(\theta)$ ($\rho_g(\theta)$), or it may trade the widget for $p_b(\theta)$ ($p_g(\theta)$).[12]

An independent downstream asset first search for a customer paying $e_g^d$. The bad type can change to good type at a rate $\gamma$ while searching.

An independent downstream asset, with order in hand, searches for an upstream asset. The idiosyncratic shock is realized upon meeting, and after observing the realization, $\theta$, an independent downstream asset may buy out the asset at the price $\rho_b(\theta)$ or $\rho_g(\theta)$ depending on the type or execute a spot-trading at the price $p_b(\theta)$ or $p_g(\theta)$, then it produces a unit of final good instantly and sell it to the customer for the customer's valuation.

An bad integrated asset pays maintaining flow cost $c_b$ and search cost $e_g^d$, and search for a customer. Upon meeting a customer, it produces widget and final good instantly and sells it to the customer for the valuation $\theta$. A good integrated asset does not pay maintaining flow cost but pays search cost $e_g^d$, and search for a customer. Upon meeting a customer, it produces widget and final good instantly and sells it to the customer for the valuation $\theta$. Note when an integrated asset resolves, it is forced to exit the industry and the upstream asset evaporates and good type downstream asset changes its type to bad while bad type downstream

[12] Note since the production cost is zero, $\kappa = 0$, assets will not separate without a trade.

asset maintains its type.

## 4.5 Preliminary Results

Suppose a manager has devoted search effort and, consequently, some disutility, to find an upstream asset or a downstream asset. The owner will not pay for the effort of the manager since managers can be replaced instantly. Knowing this, managers will not devote any search efforts.

Lemma 4.1 (Non-owner) managers do not devote any efforts towards search.

Note, since managers would not search, owners of independent upstream or downstream assets should manage their own assets and search for trading partners. The owner of integrated assets can manage only one asset. Note that (integrated) downstream asset involves search while (integrated) upstream asset does not.

Lemma 4.2 The owner of independent upstream asset or downstream asset always manages her own asset. The owner of vertically integrated asset always manages downstream asset.

Consider a meeting between an independent upstream and a bad or good independent downstream firm. Suppose that the realization of the idiosyncratic shock is $\theta$. Let $j \in \{b, g\}$. When they trade, upstream firm gets $p_j(\theta) + \Pi$ and downstream firm gets $\theta - p_j(\theta) + b_0$. When they do not trade, upstream firm gets $\Pi$ and downstream firm gets $b_0$.[13] Therefore, by Nash bargaining with equal power together with free entry conditions for both assets, $p_j(\theta)$ is the solution to the following maximization problem:

$$\max_{0 \le p_j \le \theta} (\theta - p_j(\theta)) p_j(\theta)$$

[13]Note that the good type downstream asset changes to bad type upon separation without integration, and that the customer leaves when the bargaining breaks down.

Note in case the widget can be procured internally, the widget can be produced by order of the owner and the price should be zero. The following lemma summarizes the widget price determination.

Lemma 4.3 Given the realization of the shock, $\theta$, The price of a unit of widget is $\frac{\theta}{2}$ in a spot-trading between independent upstream asset and independent downstream asset, regardless of the type of the downstream asset. If an integrated firm can procure the widget internally, the price of widget is zero.

Now consider the asset price given the realization of the shock, $\theta$. It is determined by the take-it-or-leave-it offer by the downstream asset manager. The downstream asset manager offers the net surplus the upstream asset could have if they would do a spot-trade.

Lemma 4.4 Given the realization of the shock, $\theta$, the price of upstream asset is $\frac{\theta}{2}$, regardless of the type of the downstream asset.

Given the widget prices and asset prices, we can determine the optimal ownership.

Lemma 4.5 Given the realization of the shock, $\theta$, A bad independent downstream asset integrates if $B_0(\theta) \geq b_0$ and do a spot-trading if $B_0(\theta) \leq b_0$. A good independent downstream asset integrates if $G_0(\theta) \geq b_0$ and do a spot-trading if $G_0(\theta) \leq b_0$.

Proof. See Appendix A. ■

Note that when the inequalities hold with equalities, they are indifferent to integrate or to do a spot-trading. We assume that they integrate if the inequalities hold with equalities.

## 4.6 Steady State Equilibrium

We first define the steady state equilibrium.

Definition 5 A steady state equilibrium is

- the population distribution of all the assets and customers:

  $u_0, d_0^b, d_1^b, D_0^b, d_0^g, d_1^g, D_0^g, C_0, C_1^b, C_1^g,$
- the value functions:

  $\Pi, b_0, b_1, B_0(\theta), g_0, g_1, G_0(\theta)$ for all $\theta$,
- and the matching rates: $\alpha, \beta, \mu, \eta$

that satisfy the following conditions:

1. Lemmas 4.1–4.5 are satisfied.
2. Value functions satisfy equations (4.1)–(4.7).
3. Free entry of upstream and downstream assets: $\Pi = 0$ and $b_0 = 0$.
4. Mass of customer is one: $C_0 + C_1^b + C_1^g = 1$
5. Steady state matching rates satisfy matching functions:

$$\alpha u_0 = \beta(d_1^b + d_1^g) = m_w M_w(u_0, d_1^b + d_1^g)$$
$$\mu(d_0^b + d_0^g + D_0^b + D_0^g) = \eta C_0 = m_g M_g(C_0,\, d_0^b + d_0^g + D_0^b + D_0^g).$$

Let $R_b$ and $R_g$ denote the reservation value of $\theta$ that a bad and good independent downstream asset integrates. We assume, from now on, that $c_b$ is large enough that bad type downstream assets never integrate and that good type downstream assets pay $c_g = 0$ when they integrate.

Assumption 4.1 1. $c_b$ is large enough that bad type downstream assets never integrate: $B_0(\theta) < b_0$ for all $\theta \in [0,1]$. In particular, $B_0(1) < b_0$.

2. $c_g = 0$.

Assumption 4.1 implies $R_b = 1$ but still $R_g > 0$ since the good type integrated asset still has to pay the search cost. In particular, $G(0) < 0$.

Under Assumption 4.1, and using the free entry conditions of upstream and downstream assets, $\Pi = b_0 = 0$, we can derive explicit solutions for value functions.

Lemma 4.6 In a steady state equilibrium,

$$\begin{aligned}
\Pi &= b_0 = 0 \\
b_1 &= \frac{\beta}{4(r+\beta)} \\
B_0(\theta) &= \frac{-c_b - e_g^d + \mu\theta}{r+s} \\
g_0 &= \frac{-4e_g^d[(r+s)(r+\beta)+\beta\mu(1-R_g)]+\beta\mu[r+s+2\mu(1-R_g^2)]}{4(r+s)(r+\beta)(r+\mu)} \\
g_1 &= \frac{\beta[-4e_g^d(1-R_g)+s+2\mu(1-R_g^2)]}{4(r+s)(r+\beta)} \\
G_0(\theta) &= \frac{-e_g^d + \mu\theta}{r+s}
\end{aligned}$$

Proof. See Appendix A. ■

Using the solutions above, we can get the good type downstream asset's reservation value of integration, $R_g$, explicitly.

Lemma 4.7 The optimal good type downstream asset's reservation value of integration is:

$$R_g^* = \frac{e_g^d}{\mu}.$$

Proof. See Appendix A. ■

The $R_g^*$ presents a nice interpretation of the reservation value. Larger the search cost, and smaller the rate at which the integrated asset meets the customer, larger the reservation value. To compensate the large search cost, the reservation value must be big while if the customer arrives more frequently, integrated asset can enjoy the realization of surplus more often instantly, which brings down the reservation value of integration. Also, we need $\mu > e_g^d$ so that $R_g^* \in (0,1)$.[14] We will later show that under some restriction, this holds always.

We can, now, use the free entry conditions for upstream and downstream assets to derive steady state equilibrium matching rates.

Lemma 4.8 Under Assumption 4.1, the steady state equilibrium matching rates satisfy the following:

$$\begin{aligned}
&\alpha^* = 4e_w^u \ : \ EE_u \\
&\beta^* = \frac{4e_g^d r(r+s)(r+\gamma+\mu^*)}{\begin{Bmatrix}(r+s+2\gamma)\mu^{*2} + [(r+s)(r+\gamma) - 4e_g^d(r+s+\gamma)]\mu^* \\ +2e_g^d[e_g^d\gamma - 2(r+s)(r+\gamma)]\end{Bmatrix}} \ : \ EE_d
\end{aligned}$$

Proof. See Appendix A. ■

$EE_u$ implies that the widget market tightness in downstream point of view, $\tau_w \equiv u_0/(d_1^b + d_1^g)$, should satisfy a certain value. Therefore, $\beta$ must also satisfy a certain value.

$$\alpha^* = \frac{m_w M_w(\tau_w, 1)}{\tau_w} = 4e_w^u$$

$EE_d$ is a little complex object. Note that the numerator of $EE_d$ is positive. Therefore the shape of $EE_d$ in $(\mu, \beta)$ is dependent on the number of positive real roots of $\mu$ of the following equation:

$$\begin{aligned}
&(r+s+2\gamma)\mu^2 + [(r+s)(r+\gamma) - 4e_g^d(r+s+\gamma)]\mu \\
&\quad + 2e_g^d[e_g^d\gamma - 2(r+s)(r+\gamma)] = 0
\end{aligned} \tag{4.8}$$

The left hand side of Equation (4.8) denotes the denominator of $EE_d$. For example, the shape of $EE_d$ is drawn in Figure 4.3. Left graph shows $EE_d$ if there exist two positive real roots, and right graph shows $EE_d$ if there exists only one positive real root. Here, we make the following condition on the parameter values.

[14] Recall that $\theta \in [0,1]$.

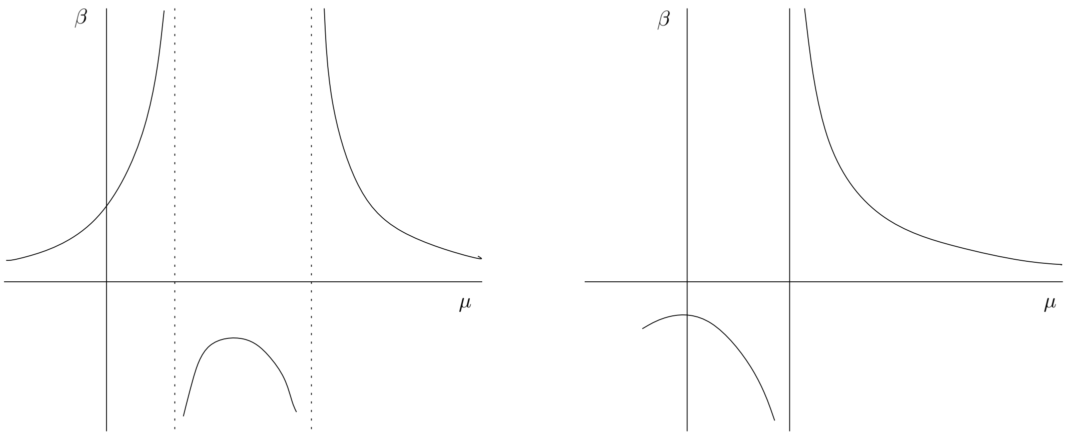


Figure 4.3. Some Possible Shapes of $EE_d$

Assumption 4.2

$$\frac{(r+s)(r+\gamma)}{4(r+s+\gamma)} < e_g^d < \frac{2(r+s)(r+\gamma)}{\gamma}.$$

Consider the left hand side of Equation (4.8). Assumption 4.2 makes the coefficient of $\mu$ and constant term both negative so that Equation (4.8) has one real positive root. Let $\mu_0$ denote the unique positive real root of Equation (4.8).

$$\mu_0 \equiv \frac{\begin{aligned}&\{-(r+s)(r+\gamma)+4e_g^d(r+s+\gamma)\\ &\quad+\big([(r+s)(r+\gamma)-4e_g^d(r+s+\gamma)]^2\\ &\quad+8e_g^d(r+s+2\gamma)[(r+s)(r+\gamma)-e_g^d\gamma]\big)^{\frac{1}{2}}\}\end{aligned}}{2(r+s+2\gamma)} > 0$$

Lemma 4.9 Under Assumption 4.2, $EE_d$ satisfies the following properties.

$$\lim_{\mu\to\mu_0}\beta=\infty, \quad \lim_{\mu\to\infty}\beta=0, \quad \frac{d\beta}{d\mu}<0$$

Proof. See Appendix A. ■

Further, we can show $\mu^* > e_g^d$ and hence the reservation value is always less than the highest possible realization of the idiosyncratic shock, $R_g^* < 1$.

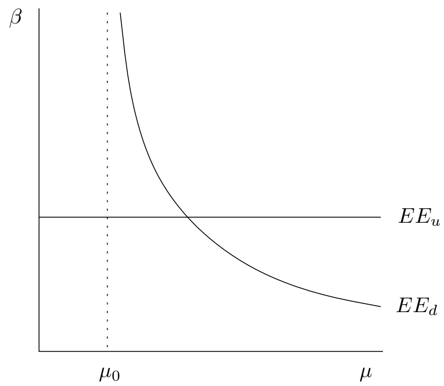


Figure 4.4. Equilibrium Matching Rates

Lemma 4.10 Under Assumption 4.2,

$$\mu^* > \mu_0 > \frac{4e_g^d(r+s+\gamma)}{2(r+s+2\gamma)} > e_g^d$$

Proof. See Appendix A. ■

Since $EE_u$ and $EE_d$ intersect only once, we have a unique steady state equilibrium.

Lemma 4.11 There exists a unique steady state equilibrium.

## 4.7 Jumps of the Value of Downstream Assets Before and After the Integration

Now we are ready to trace the downstream asset value in each state. Using $EE_d$, and the solution for the reservation value, $R_g^*$, we can get rid of $\beta$ and $R_g$ from the solutions of the value functions in Lemma 4.6.

$$g_0 = \frac{2e_g^d(\mu - e_g^d)^2}{(r+s+2\gamma)\mu^2 + [-4e_g^d\gamma + (r+s)(r+\gamma)]\mu + 2e_g^{d^2}\gamma}$$

$$g_1 = \frac{e_g^d(r+\lambda+\mu)[2e_g^{d^2} - 4e_g^e\mu + \mu(r+s+2\mu)]}{\mu\{(r+s+2\gamma)\mu^2 + [-4e_g^d\gamma + (r+s)(r+\gamma)]\mu + 2e_g^{d^2}\gamma\}}$$

$$G_0(\theta) = \frac{-e_g^d + \mu\theta}{r+s}$$

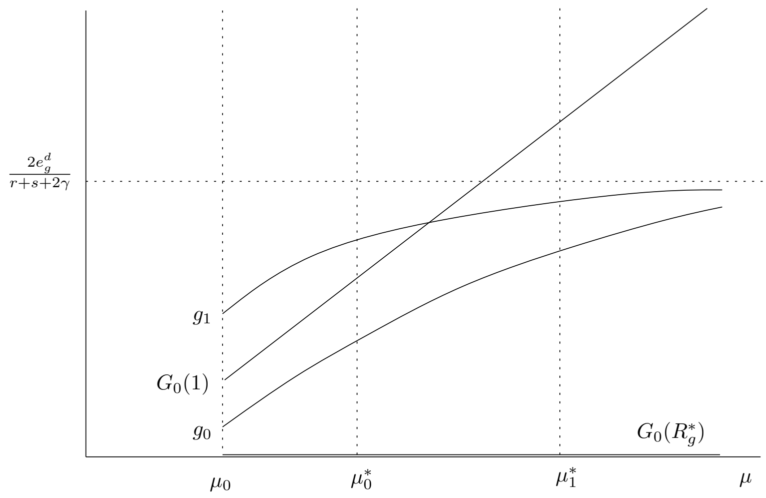


Figure 4.5. Asset Values

Also note that $G_0(\theta)$ for $\theta \in [R_g, 1]$ has a value between the following two extreme:

$$G_0(R_g^*) = 0$$
$$G_0(1) = \frac{-e_g^d + \mu}{r+s}.$$

Figure 4.5 shows how the asset values change according to equilibrium rate at which a downstream asset meets a customer, $\mu^*$. Note that, by definition, $g_0 < G_0(1)$, however, for some low $\mu^*$, it is possible that $g_1 > G_0(1)$. This is because $G_0(1)$ represents the value of the state without the order, and $g_1$ with the order. Note also if $\mu^*$ is small enough, as $\mu^* = \mu_0^*$ in Figure 4.5, depending on the parameter values, the downstream asset value always exhibits negative jump after integration, while if $\mu^*$ is large enough, for example $\mu^* = \mu_1^*$ in Figure 4.5, downstream asset value exhibits both positive and negative jump after integration.

Note that a downstream enters the industry as a bad type. Figure 4.6 illustrates a sample time path of a downstream asset's value. A possible path of its value would be $b_0 \to b_1 \to b_0 \to g_0 \to g_1 \to G_0(0.5) \to b_0$. As the downstream asset changes the state the value also jumps. Sometimes the value jumps up and sometimes it jumps down. Upon the change of the type, the asset's value jumps up while upon the resolution of the integration, the asset's value jumps down. Upon integration, the value may jump up and down provided that the rate at which a

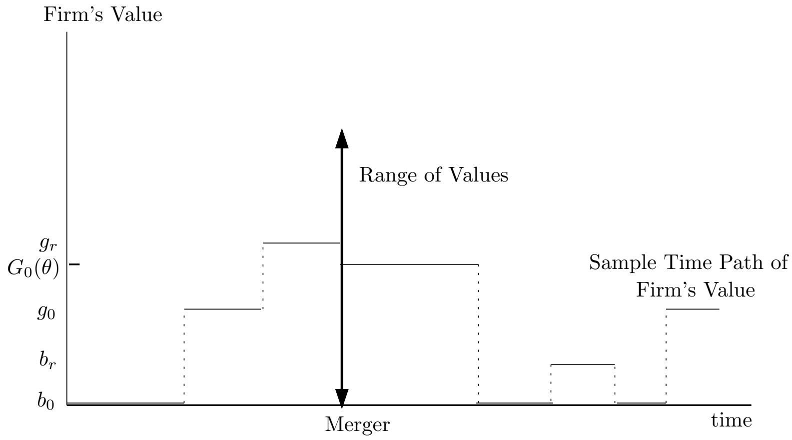


Figure 4.6. Sample Time Path of Asset's Value

downstream asset meets a customer, $\boldsymbol{\mu}^*$ is large enough.

The intuition for this result is simple, and follows directly from the reservation property. The ex ante value of an acquiring firm depends upon the conditional expectation of the distribution of ex post merger valuations. Nevertheless, ex post it is perfectly conceivable that a pair of firms will draw a value of the idiosyncratic shock that lies above the reservation value, rendering merger optimal, but below the conditional mean of the return distribution, and as a result "disappoint the market." In this event, there is an ex post decline in shareholder value.

## 4.8 Concluding Remarks

In this Chapter, we have introduce heterogeneity and uncertainty into the model in the form of types of downstream assets and idiosyncratic shock and shown that search theory can provide an appealing explanation regarding the jumps of the asset value before and after the integration. Uncertainties are naturally unveiled as assets search and meet. Specifically, asset's value jumps up before integration upon the change of type from ***bad*** to ***good***. This may explain the reason firm's value jumps up in the stock market when it announces considering an integration. The value tends to reflect some expectations about the 'fit' of the integration that will happen in the future. Upon the integration, this uncertainty is resolved and firm's value can jump up and down depending on whether the 'fit' of integration exceeds the expectation or not. Our model explains key features regarding the jumps of the value of firms after integration.

We regard this result as one of the principle findings of this thesis. Up to this point, explaining the loss in shareholder value precipitated by mergers has proven to be an especially refractory problem. Indeed, the few studies that we are aware of that have broached the problem have invariably regarded it as stemming from inefficient managerial "empire building." Nevertheless, within the context of the dynamic search approach developed in this thesis, the reduction in shareholder value is perfectly consistent with ex ante optimal, and indeed proper managerial oversight.

Appendix A

# Proofs

Proof of Lemma 2.1

We suppress the subscript $w$ in equations. Note

$$\alpha = mM(1, \alpha/\beta)$$

Total differentiation, given $dm = 0$, yields

$$\beta(\beta - mM_2)d\alpha = -mM_2\alpha d\beta$$

Note that the sign of $d\alpha/d\beta$ depends on the sign of $(\beta - mM_2)$. By Euler theorem,

$$\beta D = mM(U, D) = m[M_1(U, D)U + M_2(U, D)D]$$

Therefore, $(\beta - mM_2)D = mM_1(U, D)U$. This implies that $(\beta - mM_2) > 0$. Hence the result follows.

For the strict convexity, note

$$\frac{d}{d\beta}\left(\frac{d\alpha}{d\beta}\right) = \frac{d\alpha/d\beta \cdot (-mM_2) + mM_2\alpha(2\beta - mM_2)}{[\beta(\beta - mM_2)]^2} > 0.$$

The limiting properties follow directly from the Inada conditions. ■

Proof of Lemma 2.5

In a steady state, in which bilateral optimal ownership is integration and trilateral optimal ownership is partial integration, outflow from each state must be equal to inflow to the state.

$$\begin{aligned} &d_0, d_r : \mu d_0 = (\beta + \lambda)d_r \\ &D_0, D_r : \mu(1 - X)D_0 = (\beta + \lambda)D_r \end{aligned}$$

Also the number of upstream and downstream assets in match must be equal.

$$\alpha u_0 = \beta(d_r + D_r)$$

Finally, mass of downstream assets is one and $z$ proportion of downstream assets enter the industry independent while $(1-z)$ proportion integrated.

$$\begin{aligned}
&d_0 + d_r + D_0 + D_r = 1 \\
&(1-z)D_0 = zd_0
\end{aligned}$$

The steady state population distribution is a solution to the above equations. Note that in spot-trading equilibrium, all the downstream assets enter the industry as independent entities($z = 1$), and that in integration equilibrium, all the downstream assets enter the industry as integrated entities($z = 0$). ■

Proof of Lemma 2.6

In a steady state equilibrium, together with free entry conditions of upstream assets, value functions can be solved as the following:

$$\begin{aligned}
\Pi_0 &= 0 \\
V_0 &= \frac{2\beta\mu\pi}{r(2r+\beta+2\lambda+2\mu)} \\
V_1 &= \frac{2\beta(r+\mu)\pi}{r(2r+\beta+2\lambda+2\mu)} \\
K_0 &= -\frac{c_1^v}{r} + \frac{2(\beta\mu+2\mu X(r+\lambda))\pi}{r(2r+\beta+2\lambda+2\mu(1-X))} \\
K_1 &= -\frac{c_1^v}{r} + \frac{2((r+\beta)\mu+2\mu X\lambda)\pi}{r(2r+\beta+2\lambda+2\mu(1-X))}
\end{aligned}$$

Plug in these solutions to Equation (2.3), together with steady state population distribution given in Lemma 2.5, to get $EE_s$, $EE_i$, and $EE_h$. For the latter part, note

$$\begin{aligned}
\lim_{z_h \to 1} \alpha_h &= \frac{e_w^u(2r+\beta+2\lambda+2\mu)}{2(r+\lambda+\mu)\pi} = \alpha_s \\
\lim_{z \to 0} \alpha_h &= \frac{e_w^u(2r+\beta+2\lambda+2\mu(1-X))}{2(r+\lambda+\mu)\pi} = \alpha_i \\
\frac{\partial \alpha_h}{\partial z} &= \frac{e_w^u \mu X(1-X)(2r+\beta+2\lambda+2\mu)(2r+\beta+2\lambda+2\mu(1-X))}{(r+\lambda+\mu)[(1-X(1-z))(2r+\beta+2\lambda)+2\mu(1-X)]^2\pi} > 0
\end{aligned}$$

■

Proof of Lemma 2.8

The results come directly from Definitions 1 and 2. ■

Proof of Lemma 2.9

Use the solutions of value functions presented above in the proof of Lemma 2.6 to get the result.

$$V_0 = \frac{2\beta\mu\pi}{r(2r+\beta+2\lambda+2\mu)}$$
$$K_0 = -\frac{c_1^v}{r} + \frac{2(\beta\mu+2\mu X(r+\lambda))\pi}{r(2r+\beta+2\lambda+2\mu(1-X))}$$

■

Proof of Lemma 3.2

In a steady state, outflow from each state must be equal to inflow to the state.

$$\begin{aligned} d_0 &: \mu d_0 = (z\beta+\lambda)d_1 + \lambda(D_0+D_1) \\ d_1 &: (\beta+\lambda)d_1 = \mu d_0 \qquad\qquad D_0 : (\lambda+\mu(1-X))D_0 = (1-z)\beta d_1 \\ D_1 &: (\beta+\lambda)D_1 = \mu(1-X)D_0 \end{aligned} \tag{A.1}$$

Also the number of upstream and downstream assets in match must be equal and mass of downstream assets is one.

$$\frac{\alpha}{d_1+D_1}u_0 = \beta$$
$$d_0+d_1+D_0+D_1 = 1$$

The steady state population distribution is a solution to the above equations. ■

Proof of Lemma 3.3

In a steady state spot-trading equilibrium, the value functions, with free entry condition of upstream assets, can be solved as the following:[1]

$$\Pi_0 = 0$$
$$V_0 = \frac{2\beta\mu\pi}{r(2r+\beta+2\lambda+2\mu)}$$

[1]Note that the values $K_0$ and $K_1$ calculated in the spot-trading equilibrium represent the value of one-time deviation to integration when other firms choose spot-trading when they meet.

$$
\begin{aligned}
V_1 &= \frac{2\beta(r+\mu)\pi}{r(2r+\beta+2\lambda+2\mu)} \\
K_0 &= \frac{\begin{array}{l}\big\{-c_1^v r(2r+\beta+2\lambda+2\mu)(2r+\beta+2\lambda+2\mu(1-X)) \\ \quad +2\big[(2rX+\beta)(r+\lambda)(2r+\beta+2\lambda)\mu \\ \quad +2[2rX(r+\lambda)+\beta(r+\lambda(1-X))]\mu^2\big]\pi\big\}\end{array}}{r(r+\lambda)(2r+\beta+2\lambda+2\mu)(2r+\beta+2\lambda+2\mu(1-X))} \\
K_1 &= \frac{\begin{array}{l}\big\{-c_1^v r(2r+\beta+2\lambda+2\mu)(2r+\beta+2\lambda+2\mu(1-X)) \\ \quad +2\beta\big[r(r+\lambda)(2r+\beta+2\lambda) \\ \quad +(r+\lambda)(4r+\beta+2\lambda)\mu+2(r+\lambda(1-X))\mu^2\big]\pi\big\}\end{array}}{r(r+\lambda)(2r+\beta+2\lambda+2\mu)(2r+\beta+2\lambda+2\mu(1-X))}
\end{aligned}
$$

Plug in these solutions, together with steady state population distribution and widget price given in Lemma 2.4, to Equation (3.1). Then the result follows. Note in this case, $z = 1$.

In a steady state integration equilibrium, the value functions, with free entry condition of upstream assets, can be solved as the following:

$$
\begin{aligned}
\Pi_0 &= 0 \\
V_0 &= \frac{-c_1^v\beta\mu}{r(2r+\beta+2\lambda)} + \frac{2\beta\mu\pi}{r(2r+\beta+2\lambda+2\mu)} \\
V_1 &= \frac{-c_1^v\beta(r+\mu)}{r(2r+\beta+2\lambda)} + \frac{2\beta(r+\mu)\pi}{r(2r+\beta+2\lambda+2\mu)} \\
K_0 &= \frac{-c_1^v[r(2r+\beta+2\lambda+2\mu)+\beta\mu]}{r(r+\lambda+\mu)(2r+\beta+2\lambda)} + \frac{2(2rX+\beta)\mu\pi}{r(2r+\beta+2\lambda+2\mu(1-X))} \\
K_1 &= \frac{-c_1^v r(2r+\beta+2\lambda+2\mu)}{r(r+\lambda+\mu)(2r+\beta+2\lambda)} + \frac{2\beta(r+\mu)\pi}{r(2r+\beta+2\lambda+2\mu(1-X))}
\end{aligned}
$$

Plug in these solutions, together with steady state population distribution, and widget price and asset price given in Lemma 2.4, to Equation (3.1). Then the result follows. Note in this case, $z = 0$.

In a steady state hybrid equilibrium, $V_0 = K_0$. Using this, we have the following solutions:

$$
\begin{aligned}
\Pi_0 &= 0 \\
V_0 &= \frac{2\beta\mu\pi}{r(2r+\beta+2\lambda+2\mu)} \\
V_1 &= \frac{2\beta(r+\mu)\pi}{r(2r+\beta+2\lambda+2\mu)} \\
K_0 &= V_0
\end{aligned}
$$

$$K_1 = \frac{-2c_1^v}{2r+\beta+2\lambda} + \frac{2\beta(r+\mu)\pi}{r(2r+\beta+2\lambda+2\mu)}$$

Plug in these solutions, together with steady state population distribution, and widget price and asset price given in Lemma 2.4, to Equation (3.1). Then the result follows. Note in this case, $z \in (0,1)$. ■

Proof of Lemma 3.4

The first part comes directly from Definitions 3 and 4. Note that assets will integrate (spot-trade) if

$$\begin{aligned} K_0 - \rho_0 + \nu - \kappa &\geq (\leq) \nu - p_{\mathrm{n}} + V_0 \\ \Leftrightarrow K_0 &\geq (\leq) V_0 \end{aligned}$$

To get the latter part, use the solutions of value functions presented above in Proof of Lemma 3.3. Note that all three cases — spot-trading, integration and hybrid — yield exactly the same condition. ■

Proof of Lemma 3.5

Let $(\alpha_i^0, \beta_i^0)$ satisfy $EE_i^e$. Then,

$$\lim_{\beta_i^0 \to 0} \alpha_i^0 = \frac{2e_w^u(r+\lambda)(r+\lambda+\mu(1-X))}{-c_1^v(r+\lambda+\mu(1-X))+2(r+\lambda)(r+\lambda+\mu)\pi}.$$

Therefore, $0 < \lim_{\beta_i^0 \to 0} \alpha_i^0 < \infty$ since the denominator is positive by Condition 1. Note $\lim_{\beta_i^0 \to \infty} \alpha_i^0 = \infty$ since $\lambda c_1^v < 2(r+\lambda+\mu)(\lambda+\mu(1-X))\pi$ by Condition 1. Hence, since $EE_i^e$ is continuous, there exists $(\alpha_i, \beta_i)$ that satisfies both $EE_i^e$ and $SS$.

For the latter part, note:

$$\lim_{\beta \to 0} T_e(\beta) = (r+\lambda+\mu(1-X))c_1^v - 2\mu X(r+\lambda)\pi < 0$$

■

Proof of Lemma 3.6

In a hybrid equilibrium, $T_e(\beta_h) = 0$. Note $c_1^v$ can be removed from $\alpha_i$ and $\alpha_h$ using $T_e(\beta_h) = 0$.

$$\begin{aligned} \alpha_i &\leq \alpha_i|_{c_1^v \to c_s} \\ &= \frac{e_w^u(\beta_0+\lambda)(\lambda+\mu(1-X))(2r+2\beta_0+2\lambda+2\mu)(2r+2\beta_0+2\lambda+2\mu(1-X))}{\begin{array}{l}\big\{2(r+\lambda+\mu)\big[(\beta_0+\lambda)(\lambda+\mu(1-X))(2r+\beta_0+2\lambda+2\mu(1-X)) \\ \qquad -2\lambda\mu X(\beta_0+\lambda+\mu(1-X))\big]\pi\big\}\end{array}} \end{aligned}$$

$$= \lim_{z \to 0} \alpha_h < \alpha_h < \lim_{z \to 1} \alpha_h$$
$$= \frac{e_w^u(2r+\beta+2\lambda+2\mu)}{2(r+\lambda+\mu)\pi} = \alpha_s$$

■

Proof of Lemma 4.5

Given the realization of the idiosyncratic shock, $\theta$, a bad type downstream asset integrates if and only if

$$B_0(\theta) + \frac{\theta}{2} - b_1 \geq \frac{\theta}{2} + b_0 - b_1$$

A good type downstream asset integrates if and only if

$$G_0(\theta) + \frac{\theta}{2} - g_1 \geq \frac{\theta}{2} + b_0 - g_1$$

Therefore, the result follows. ■

Proof of Lemma 4.6

First, note that $\Pi = b_0 = 0$ by the free entry conditions for upstream downstream assets. Second, we can explicitly solve the integration parts in Equations (4.3) and (4.6). Note that Assumption 4.1 implies $R_b = 1$ and $R_g > 0$.

$$(r+\beta)b_1 = \frac{\beta}{4}$$
$$(r+s)B_0(\theta) = -c_b - e_g^d + \mu\theta$$
$$(r+\mu)g_0 = -e_g^d + \mu g_1$$
$$(r+\beta)g_1 = \beta\left(\int_0^{R_g} \frac{\theta}{2} d\theta + \int_{R_g}^1 G_0(\theta) + \frac{\theta}{2} d\theta\right)$$
$$(r+s)G_0(\theta) = -c_g - e_g^d + \mu(\theta)$$

Therefore,

$$\int_{R_g}^1 G_0(\theta) d\theta = \frac{1}{r+s}\left(-(c_g+e_g^d)(1-R_g) + \frac{\mu}{2}(1-R_g^2)\right)$$

Plug this into the above equation and solve for each variable to get the result.

■

Proof of Lemma 4.7

From free entry condition $b_0 = 0$, and $G_0(R_g)$ from Lemma 4.6,

$$G_0(R_g) = \frac{-e_g^d + \mu R_g}{r+s} = 0$$

Therefore, the result follows. ■

Proof of Lemma 4.8

First note that the upstream asset gets $\theta/2$ at every match regardless of the type of the downstream assets and regardless of selling a unit of widget or selling off the asset itself. Therefore, Equation (4.1) implies:

$$\Pi = -e_w^u + \alpha \int_\theta \frac{\theta}{2} d\theta$$

Therefore, $EE_u$ follows. For $EE_d$, plug in the solutions in Lemma 4.6 into Equation (4.2), and solve for $\beta$. ■

Proof of Lemma 4.9

The first two results are strait forward. Note

$$\frac{d\beta}{d\mu} = \frac{\begin{array}{c} 4e_g^d r(r+s)\left[2e_g^{d^2}\gamma - (r+\gamma)(-4e_g^d\gamma + (r+s)(r+\gamma))\right. \\ \left. -2(r+\gamma)(r+s+2\gamma)\mu - (r+s+2\gamma)\mu^2\right] \end{array}}{\begin{array}{c} \{(r+s+2\gamma)\mu^2 + [(r+s)(r+\gamma) - 4e_g^d(r+s+\gamma)]\mu \\ +2e_g^d[e_g^d\gamma - 2(r+s)(r+\gamma)]\}^2 \end{array}} \tag{A.2}$$

Note that the denominator is positive by Assumption 4.2. Hence, the sign of $d\beta/d\mu$ depends on the sign of numerator.

Note that, by Assumption 4.2 and by $\mu > \mu_0$,

$$\begin{aligned} &-(r+s+2\gamma)\mu^2 - [(r+s)(r+\gamma) - 4e_g^d(r+s+\gamma)]\mu \\ &\quad -2e_g^d[e_g^d\gamma - 2(r+s)(r+\gamma)] < 0 \end{aligned} \tag{A.3}$$

The result follows by noting that the numerator of $d\beta/d\mu$ in Equation (A.2) is less than the left hand side of Equation (A.3). ■

Proof of Lemma 4.10

Recall that

$$\mu_0 \equiv \frac{\begin{array}{l} \{-(r+s)(r+\gamma) + 4e_g^d(r+s+\gamma) \\ \quad +\left([(r+s)(r+\gamma) - 4e_g^d(r+s+\gamma)]^2\right. \\ \quad \left. +8e_g^d(r+s+2\gamma)[(r+s)(r+\gamma) - e_g^d\gamma]\right)^{\frac{1}{2}}\} \end{array}}{2(r+s+2\gamma)} > 0$$

It suffices to show that

$$\begin{aligned}&\big([(r+s)(r+\gamma)-4e_g^d(r+s+\gamma)]^2\\&\quad+8e_g^d(r+s+2\gamma)[(r+s)(r+\gamma)-e_g^d\gamma]\big)^{\frac{1}{2}}\\&>(r+s)(r+\gamma)\end{aligned}$$

Note that

$$\begin{aligned}&[(r+s)(r+\gamma)-4e_g^d(r+s+\gamma)]^2\\&\quad+8e_g^d(r+s+2\gamma)[(r+s)(r+\gamma)-e_g^d\gamma]\\&=8e_g^{d^2}[(r+s+\gamma)(2r+2s+\gamma)-\gamma^2]+8e_g^d r(r+s)(r+\gamma)+[(r+s)(r+\gamma)]^2\end{aligned}$$

Therefore the result follows. ■

Appendix B

# List of Symbols

| **English Alphabet (lower case)** | |
|---|---|
| $b_0$ | value of a bad type independent downstream asset without order |
| $b_1$ | value of a bad type independent downstream asset with order |
| $c_b$ | maintaining cost for a bad type integrated asset (flow) |
| $c_g$ | maintaining cost for a good type integrated asset (flow) |
| $c_1^v$ | maintaining cost for an integrated asset (flow) |
| $d_0$ | mass of independent downstream assets without customer's order |
| $d_1$ | mass of independent downstream assets with customer's order |
| $d_0^b$ | mass of bad type independent downstream assets without order |
| $d_1^b$ | mass of bad type independent downstream assets with order |
| $d_0^g$ | mass of good type independent downstream assets without order |
| $d_1^g$ | mass of good type independent downstream assets with order to an integrated asset |
| $e_g^d$ | downstream asset's final goods market search cost (flow) |
| $e_w^d$ | downstream asset's widget market search cost (flow) |
| $e_w^u$ | upstream asset's widget market search cost (flow) |
| $g_0$ | value of a good type independent downstream asset without order |
| $g_1$ | value of a good type independent downstream asset with order |
| $m_w$ | widget market matching parameter |
| $m_g$ | final goods market matching parameter |
| $n$ | number of types of upstream assets |

| | |
|---|---|
| $p_0$ | internal transfer price of widget |
| $p_b(\theta)$ | widget price between independent upstream and bad type independent downstream assets when idiosyncratic shock realization is $\theta$ |
| $p_g(\theta)$ | widget price between independent upstream and good type independent downstream assets when idiosyncratic shock realization is $\theta$ |
| $p_{\text{i}}$ | widget price between independent upstream and integrated assets |
| $p_{\text{n}}$ | widget price between independent upstream and downstream assets |
| $q$ | quality of final good |
| $r$ | time preference |
| $u_0$ | mass of independent upstream assets unmatched |
| $z$ | proportion of independent downstream assets |
| $z_b$ | probability that a bad type downstream asset chooses spot-trading |
| $z_g$ | probability that a good type downstream asset chooses spot-trading |
| $z_h$ | hybrid equilibrium composition of independent downstream assets |

**English Alphabet (upper case)**

| | |
|---|---|
| $B_0(\theta)$ | value of bad type integrated asset when idiosyncratic shock realization is $\theta$ |
| $C_0$ | mass of unmatched customers |
| $C_1^b$ | mass of customers matched with bad type independent downstream asset |
| $C_g$ | mass of customers actively searching in the final goods market |
| $C_1^g$ | mass of customers matched with good type independent downstream asset |
| $D_0$ | mass of integrated assets without customer's order |
| $D_0^b$ | mass of bad type integrated assets |
| $D_g$ | mass of downstream assets actively searching in the final goods market |
| $D_0^g$ | mass of good type integrated assets |
| $D_1$ | mass of integrated assets with customer's order that cannot be procured internally |
| $D_w$ | mass of downstream assets actively searching in the widget market |
| $G_0(\theta)$ | value of a good type integrated asset when idiosyncratic shock realization is $\theta$ |
| $K_0$ | value of an integrated asset without a customer's order |
| $K_1$ | value of an integrated asset with a customer's order that cannot be procured internally |

| | |
|---|---|
| $M_g(\cdot)$ | final goods market matching function |
| $M_w(\cdot)$ | widget market matching function |
| $R_b$ | bad type downstream asset's reservation value for integration |
| $R_g$ | good type downstream asset's reservation value for integration |
| $U_w$ | mass of upstream assets actively searching in the widget market |
| $V_0$ | value of an independent downstream asset without customer's order |
| $V_1$ | value of an independent downstream asset with customer's order |
| $X$ | proportion of widget type that an upstream asset can produce |

**Greek (lower case)**

| | |
|---|---|
| $\alpha$ | rate at which an upstream asset meets a downstream asset |
| $\beta$ | rate at which a downstream asset meets an upstream asset |
| $\gamma$ | rate at which a bad type downstream asset changes to a good type downstream asset |
| $\delta$ | asset's evaporating/death rate |
| $\eta$ | rate at which a customer meets a downstream asset |
| $\theta$ | idiosyncratic shock |
| $\kappa$ | widget production cost |
| $\lambda$ | rate at which a customer retracts orders or leaves/separates |
| $\mu$ | rate at which a downstream asset meets a customer or receive an order |
| $\mu_0$ | constant |
| $\mu_1$ | constant |
| $\nu$ | customer's valuation on final good |
| $\pi$ | defined as $(\nu - \kappa)/2$ |
| $\rho_0$ | price of an independent upstream asset |
| $\rho_b(\theta)$ | upstream asset price a bad type independent downstream asset faces when idiosyncratic shock realization is $\theta$ |
| $\rho_g(\theta)$ | upstream asset price a bad type independent downstream asset faces when idiosyncratic shock realization is $\theta$ |
| $\tau_g$ | final goods market tightness in downstream asset's point of view |
| $\tau_w$ | widget market tightness in downstream asset's point of view |
| $\chi(\omega)$ | indicator function |
| $\omega$ | type of widget/final good |

**Greek (upper case)**

| | |
|---|---|
| $\Pi$ | value of an upstream asset unmatched or with no links |

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