OPTIMIZATION OF AN OUTBOUND LOGISTICS NETWORK:
A VEHICLE ROUTING PROBLEM

A Thesis in
Industrial Engineering

by
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ABSTRACT

Outbound logistics is one of the primary processes of logistics, concentrating on the delivery of finished products to the retailers. As a result, the responsiveness of the whole supply chain is closely related to the performance of the outbound logistics. Lean strategies such as cross docking, milkruns and freight consolidation have been considered in the distribution plan that help reduce total cost and improve customer satisfaction level.

This thesis builds an optimization model to aid a Manufacturing company in determining the best distribution plan for their retailers. The company currently uses direct shipping from plant to all their retailers. A Mixed Integer Linear Program (MILP) is developed to help determine one of the following options for each retailer:

1. Direct shipping from the plant to the retailers using LTL service.

2. Consolidate the shipment with other freight, shipping together from the plant to the crossdock using TL service and then shipping from cross dock to the retailers using LTL service.

3. Consolidate the shipment with other freight, shipping together from the plant to the crossdock using TL service and then shipping from crossdock to the retailers using milkrun.

The model determines the best distribution option for each retailer, to minimize the total cost, including transportation and inventory holding cost, while satisfying all the demand constraints, vehicle capacity constraints and delivery time constraints. A scenario analysis is also performed to examine the relationship between different delivery frequency and potential savings.
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CHAPTER 1 INTRODUCTION

1.1 Logistics Management: definition, mission and importance

Christopher, (1992) defines logistics as the process of strategically managing the procurement, movement and storage of materials, parts, finished inventory and the related information flows through the organization and its marketing channels in such a way that current and future profitability are maximized through the cost-effective fulfillment of orders. He also points out that the scope of logistics spans the organization, from the management of raw materials to the delivery of the final product. Figure1.1 shows the logistics management process from a total system point of view. Since the performance of each activity along the chain closely affects each other, the mission of the logistics management is to span and coordinate all those activities to achieve a desired delivered service and product quality at the lowest cost. Some specific targets include short delivery time, low inventory level, high utilization of capacity and a high due date reliability. There is always a trade-off between the total cost and customer service level. Therefore, a strategic logistics planning is crucial for every company.

Figure 1.1 Logistics Process
1.2 Outbound Logistics

1.2.1 Importance of outbound logistics

Outbound logistics is one of the primary processes of logistics, concentrating on the delivery of finished products to the retailers. As a result, the responsiveness of the whole supply chain is closely related to the performance of the outbound logistics. The main activities of the outbound logistics include order processing, warehousing and transportation. The use of distribution channels, transportation mode and distribution strategy are also considered in the outbound logistics.

1.2.2 Transportation network design for outbound Logistics

Transportation is a very important activity in the outbound logistics management. The design of a transportation network affects the performance of a supply chain by establishing the infrastructure within which operational transportation decisions regarding scheduling and routing are made. A well-designed transportation network can improve the responsiveness of the supply chain at a comparable low cost.

1.2.2.1 Transportation network option

- **Direct shipping network**: All shipments directly come from the manufacturer to each retailer location (Figure 1.2(a)).

- **Direct shipping with milkruns**: A manufacturer delivers directly to multiple retailer locations on a truck or a truck picks up deliveries destined for the same retailer location from many manufacturers. A milk run is a route on which a truck either delivers products from a single supplier to multiple retailers or goes from multiple suppliers to a single retailer location (Figure 1.2(b)).
• **All shipments via a Distribution Center (DC):** Manufacturer sends their shipments to a DC, which specifically supplies a group of retailers in the same region, and then DC forwards each shipment to each customer location (Figure 1.2(c)).

• **Shipping via DC using Milk Runs:** Manufacturer sends its shipments to a DC and a milk run is then used from the DC if the quantities delivered to each retailer location are small (Figure 1.2(d)).
• **Shipping via Line Haul with Cross-Docking:** Material is shipped from a DC in large trucks to a cross-dock point close to the retailer location. Deliveries are moved to smaller trucks for local delivery from the cross-dock (Figure 1.2(e)).

### 1.3 Problem Statement

#### 1.3.1 Motivation for the research

The research is motivated by an anonymous Manufacturing company’s desire to improve its outbound logistics operations. The company currently uses a centralized distribution center to store aftermarket parts to supply its retailers nationwide. The aftermarket parts, which refer to replacement parts for units that have failed at their business, are ordered on an emergency basis. With one-year’s worth of domestic less-than-truckload (LTL) shipment data for all their business units, the company wants to find any opportunity that may exist with regard to *a pooled distribution*. Pooled distribution represents a superior and cost-effective alternative compared with the traditional LTL shipments. Pooled distribution is the distribution of orders from a regional DC to numerous destination points within a particular geographic region (Figures 1.2(e) and 1.3). Instead of shipping direct from the origin shipping point to retailers, orders are shipped on consolidated truckload trailers (TL) direct to cross-dock point. There, the shipment is offloaded, segregated and sorted and then reloaded to local delivery trucks for delivery to the ultimate retailer destinations. In addition to the direct shipping cost savings, the company also believes pooled distribution can potentially reduce the damage related to LTL handling over long hauls.
1.3.2 Existing outbound logistics process

The manufacturing company has outsourced the business of transporting products from distribution centers to retail stores to a third party logistics (3PL) company. When an order comes, the company uses Oracle Transportation Management System and allocates the carrier to that shipment based on their pricing and transit times. Currently, they use LTL service for all the shipments without any pooling process.

1.3.2.1 Truckload (TL) carriers

TL carriers are dedicated trucks, carrying large shipments that usually can fill an entire container, travel directly from an origin to a destination. There may exist intermediate stops between the origin shipping point to the destination. The potential advantages of TL compared with LTL include:
Less transition time: Since they are not routed on a hub system, TL carrier travel directly to its destination instead of transferring goods at transshipment points.

Less damage: Since packages are only handled at the pick up and drop off point, TL carrier potentially reduce the damage related to LTL handling over long hauls.

Less unit cost: Due to economies of scale, the unit cost (cost per pound per mile) of TL carrier is less than that of LTL.

Potential disadvantages of TL compared with LTL include:

The shipping rates for different destinations are quite different. If you are shipping to a remote place, your shipping rates will be higher than shipping to a high volume destination even if the distance is the same. In addition, due to demand variations, the TL carrier may not be cost effective if the truck is not full.

1.3.2.2 Less than truck load (LTL) carriers

LTL carriers provide a shared service on the same truck and use an airline-type hub-and-spoke system with shipments. (Yildiz, 2010) LTL carriers collect freight from various locations and consolidate that freight for line haul to the next delivering terminal. The freight will be sorted and delivered at the terminal. The main advantage to using an LTL carrier is that the shipper will pay a fraction of the cost of hiring an entire trailer for an exclusive shipment. While the main disadvantage of LTL carrier is that transit times are longer than for TL carrier. LTL transit times are not directly related to the distance between shipping point and destination. Instead, LTL transit times are dependent upon the makeup of the network of terminals that are operated by a given carrier and interline partners. Since the transit times are longer, LTL carrier will increase the potential damage to the product compared with a TL carrier.
1.4 Proposed research

In order to select the best out bound logistics strategy for the manufacturing company, a network flow model combined with vehicle routing will be developed. The objective is to minimize the total cost, including transportation cost and inventory holding cost, while satisfying all the demand constraints, vehicle capacity constraints and delivery time constraints. Each retailer can either be served directly from centralized distribution center or from a cross-dock point. Both milkrun and cross-dock strategies will be considered in the model.

1.4.1 Essential features of the mathematical model

Objectives: To minimize the total cost including inventory holding cost and transportation cost

Cost Structure: Inventory holding cost will be based on the cost of the product and interest rate at the DC. It will not be considered at the cross-dock point. In transit inventory cost will be estimated at 85% of the inventory holding cost. Both TL and LTL costs will be based on the expertise of the logistics engineer. TL cost is a single fixed per mile cost, while the 3PL company or the local carrier gives the LTL cost.

Constraints

- All the retailer demands should be satisfied.
- Only one single transportation option is to be selected for each retailer.
- There is a limit on the total weight and total volume of the trucks.
- The maximum number of stops on a route to be at most 4.
- If the cross-dock point is on a milkrun, then it should be the last stop of the milkrun route.

Chapter 2 will give a literature review of the distribution planning models and vehicle routing problems (VRP), while Chapter 3 will give a detailed development of the optimization model with an illustrative
example. The model will be applied and analyzed in Chapter 4, with data collected from the anonymous manufacturing company. By changing several parameters in the model such as delivery frequency, the location of cross-dock point, number of retailers and variations in retailer demands, a scenario analysis will also be done. Potential areas for future work will be summarized in Chapter 5.
CHAPTER 2 LITERATURE REVIEW

2.1 Distribution Network Design

The decisions in a supply chain can be classified into three different types: strategic, tactical and operational. Examples are illustrated in the following table.

<table>
<thead>
<tr>
<th>Planning Period</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategic Decisions</strong></td>
<td>Supply chain network design</td>
</tr>
<tr>
<td>Long term, usually for several years</td>
<td></td>
</tr>
<tr>
<td><strong>Tactical Decisions</strong></td>
<td>Inventory policy, Price promotion, Discounts</td>
</tr>
<tr>
<td>Quarterly or yearly</td>
<td></td>
</tr>
<tr>
<td><strong>Operational Decisions</strong></td>
<td>Vehicle Routing Problem, Order scheduling</td>
</tr>
<tr>
<td>Weekly or daily</td>
<td></td>
</tr>
</tbody>
</table>

Distribution network design aims at finding the best distribution strategy to deliver products to customers. Most companies use the design that minimizes cost as the best distribution option while others may consider lead-time, customer responsiveness, product variety and order visibility. The design task includes the choice of facilities to be opened and satisfying the customer demand with minimum cost. Changing the distribution network design affects the supply chain costs, which include inventories, transportation, facilities and handling cost. The selection of optimal sites for warehouse location is a strategic decision, while distributing products to retail stores from a given set of warehouses is a tactical decision. An optimal vehicle routing plan is an operational decision.

2.1.1 Single objective distribution network design models.

Research in distribution network design mainly includes production and inventory levels in supply chain optimization model, the number of warehouses needed and their locations and optimal vehicle routing
Ambrosino et al. (2009) studies a distribution network that involves location study, fleet assignment and vehicle routing decisions. He proposed a two-phase heuristic method to solve this problem, which first determines an initial feasible solution and then improves it by using very large neighborhood search techniques. The goal is to minimize the total distribution costs and select the best distribution plan for each customer region. Similarly, Lee et al. (2008) develop a decomposition heuristic method to minimize total cost, which include replenishment cost, inventory holding cost as well as transportation cost, in a multi-level supply chain network. In their research, the manufacturing assembly line operation plan is integrated with the transportation plan. Altiparmak et al. (2009) have developed a heuristic approach based on generic algorithm for the single-source, multi-product, multistage supply chain network design problem.

Some other researchers deal with integrated logistical planning with a focus on coordinating inventory and transportation decisions. Specifically, two types of research emphasize the coordination between inventory and transportation decisions. One focuses on including transportation costs and capacities. However, transportation-related decisions such as vehicle routing and freight consolidation are not taken into consideration. Previous work on this topic includes Alp et al. (2003). They developed a model for a dynamic lot-sizing and vehicle-dispatching problem under dynamic deterministic demands and stochastic lead time. They devised a dynamic programming algorithm to compute the production quantities and lead time. Üster et al. (2008) have proposed a location-inventory model to help make an integrated distribution decision. The problem is about the integration of warehouse location and inventory replenishment decisions with the objective of minimizing transportation and inventory related cost. The other type of research addresses transportation related variables and aims at making operational decisions. The literature on this topic is also abundant (Bertazzi et al. (2008), Archetti et al. (2007), Cetinkaya et al. (2004), Baita et al. (1998)). This type of problem is known as inventory-routing problem. The inventory-routing problem, according to Campbell and Savelsbergh et al (2004), is a variation of the vehicle-routing problem in which the vendor can make decisions about the timing, sizing and routing of deliveries.
without allowing any shortages. Lei et al (2006) focus on a real-life application, which is concerned with coordinating production, inventory and delivery operations to meet customer demand. The delivery consolidation problem is formulated as a capacitated transportation problem with additional constraints and is solved heuristically.

2.1.2 Multi-Criteria optimization in supply chain design

Most distribution network models have focused only on minimizing cost; however some scholars believe that other criteria such as lead-time, customer service level and product quality should also be considered while minimizing cost. To achieve this goal, multi-criteria optimization model allows more than one objective function and allows decision makers to evaluate a greater number of alternative solutions. Some multi-criteria models exist for the location and allocation problem. Cintron, Ravindran and Ventura (2009) have proposed a multi-criteria mixed-integer linear programming for a consumer good company to design the best supply chain distribution network. The intent is to redesign the supply chain network with an objective of maximizing the profit by reducing the distribution costs. However, other criteria such as lead time, power, credit performance and distributor’s reputation are also considered in the model. The model selects the best distribution plan for each customer based on the above criteria. Similarly, Pokharel et al (2008) develop a two-objective decision making model to select suppliers and warehouses for a supply chain network. Their model intends to minimize costs and maximize customer service levels. Some other researchers have also considered inventory consolidation in their optimization model. A bi-criteria nonlinear stochastic integer programming model is developed to solve a risk pooling problem by Gaur and Ravindran et al. (2006). The problem is solved by a two stage optimization algorithm. The first stage of solution aims at finding a list of feasible warehouse locations and the amount of shipping units. The second stage of the solution focuses on finding the optimal inventory aggregation plan satisfying all the system constraints. The intent of the paper is to determine the optimal number of warehouses required to
meet the customer demand in different regions such that the cost is minimized while responsiveness is maximized. They also use the Analytic Hierarchy Process (AHP) to rank all the alternatives.

2.2 Research on outbound logistics

Most research work in outbound logistics design is related to real world case studies with the aim of lowering costs and improving customer responsiveness. Yildiz et al. (2011) proposed a mixed integer programming model to optimize the distribution network for Robert Bosch LLC. They selected a subset of customers and suppliers to consolidate their shipments on the same routes to reduce the transportation cost. Furthermore, they considered the use of a cross dock as a consolidation point and developed a network flow model combined with vehicle routing from the plant and the cross dock. Milkruns and pooled distribution were also considered in their model. They pointed out that the new system would result in reduced inventories, but they didn’t compute the direct inventory cost savings. The inventory considerations were integrated in the distribution optimization model in the paper written by Üster et al. (2009). They conducted research to improve Frito-Lay’s outbound supply chain activities by optimizing its inventory and transportation decisions simultaneously. In their model, inventory holding cost, production, storage, plant to plant and store to store shipping were all considered. An iterative solution approach was developed in which the overall model was decomposed into two sub problems that involved complementary inventory and routing components. Other researchers also considered assembly line sequencing in their model. Mingzhou et al (2007) addressed the production sequencing and outbound logistics planning decision problems with a real case study for an automotive company. The objective was to minimize the total operational cost that included the cost of utility work in production, inventory holding cost, shortage cost and transportation cost.
2.3 Use of cross-dock in distribution models

Use of cross-dock, in place of a traditional warehouse, was pioneered by Wal-Mart. Stalk et al (1992) have pointed out how cross-dock help Wal-Mart increase their market share and profitability. Galbreth et al (2008) describe cross-dock as the practice of transferring materials from an incoming shipment to an outgoing shipment without storing them at the transfer point. The potential advantage of cross-dock includes reducing the handling cost, operating cost, inventory-holding cost, short lead-time and centralized processing. Nowadays, more companies are turning to cross-dock which potentially bring them significant cost savings. Stalk et al (1992) pointed out how cross-dock help Wal-Mart increase their market share and profitability. Sung and Yang (2008) have proposed a branch-and-price algorithm as an exact algorithm for the cross-dock supply chain network design problem. The objective is to optimally locate the cross-dock center to satisfy a given set of freight demands at minimum cost. Similarly, Ross and Jayaraman (2008) have proposed a new heuristics solution procedure for the location of cross-dock and distribution centers in supply chain network design. In addition to the location planning for the cross-dock, they also evaluate the computational performance of the network design location model to better understand interactions effects with the sophisticated control parameter-temperature decrements, which is computed by a distance factor and a spread factor. Furthermore, Kreng and Chen (2008) developed two models where they coordinate both the manufacture production planning and the distribution center delivery policy.

2.4 Vehicle Routing Problem

2.4.1 Problem introduction

Vehicle Routing Problem (VRP) is an operation level problem in distribution network design and is first introduced by Dantzig et al. (1959) 50 years ago. In their paper, the authors proposed the first mathematical programming formulation and algorithmic approach to solve a real world problem, which is
about the delivery of gasoline to gas stations. According to Yildiz, et al (2010), VRP is defined as the process of selecting the paths in a transportation network along which to send physical traffic. VRP problem is akin to the famous Travelling Salesman Problem (TSP). It is a very difficult problem to solve and the difficulties increase along with the size of the problem. Among the most effective methods are heuristic algorithms, branch and bound method and set partitioning method. A broad summary of recent work on VRP is given in the book by Golden et al. (1988). The main components of VRP include road network, customers, depots, vehicles and drivers. The products should be distributed to the end customers in a given time period by a set of vehicles, which are located in one or more depots.

**Typical Objectives**

- Minimization of the global transportation cost
- Minimization of the number of vehicles required to serve all the customers
- Balancing the routes considering both the travel time and vehicle load

**Typical Operational Constraints**

- Items must be delivered or collected at the customer
- Retailers are served in a route within their time windows
- The current load of the associated vehicle cannot exceed the vehicle capacity
- Precedence constraints for the customers
- Maximum number of stops in a route

### 2.4.2 Basic models for VRP

The VRP may be described as the following graph theoretic problem. Let $G = (V, A)$ be a complete graph, where $V = \{0, \ldots, n\}$ is the vertex set and $A$ is the arc set. Vertices $i = 1, \ldots, n$ correspond to the customers, whereas vertex 0 corresponds to the depot. A nonnegative cost $c_{ij}$ is associated with each arc $(i,j) \in A$ and represents the travel cost from vertex $i$ to vertex $j$. Each customer $i = 1, \ldots, n$ is associated with a
known nonnegative demand $d_i$ to be delivered and the depot has a fictitious demand $d_0 = 0$. Given a customer vertex set $S \in V$, let $d(S) = \sum_{i \in S} d_i$ denote the total demand of the set. A set of $K$ identical vehicles, each with capacity $C$, is available at the depot. Given a set of $S' \in V\setminus\{0\}$, denoting all vertices in $V$ excluding the depot 0, we denote by $r\{S'\}$ the minimum number of vehicles needed to serve all the customers in $S'$.

**Integer Linear Programming Model**

The integer linear programming model is a two-index vehicle flow formulation that uses $O(n^2)$ binary variables to indicate if a vehicle traverses an arc in the optimal solution. Specifically, variable $X_{ij}$ takes a value 1 if arc $(i,j) \in A$ belongs to the optimal solution and takes a value 0 otherwise. The final result will show the optimal route consisting of arcs $(i,j)$ which take value 1. The objective is to minimize the transportation cost:

$$\min \sum_{i \in V} \sum_{j \in V} c_{ij} X_{ij}$$  \hspace{1cm} (2.1)

Subject to the following constraints:

(a) For each customer, represented by vertex $j \in S'$, exactly one arc $(i, j)$ enters vertex $j$.

$$\sum_{i \in V} X_{ij} = 1 \hspace{1cm} \forall \ j \in S',$$  \hspace{1cm} (2.2)

(b) For each customer represented by vertex $i \in S'$, exactly one arc $(i, j)$ leaves vertex $i$.

$$\sum_{j \in V} X_{ij} = 1 \hspace{1cm} \forall \ i \in S',$$  \hspace{1cm} (2.3)

(c) Let $K$ represent the number of vehicles and each vehicle serves one route. Each route starts and ends at the depot. Equations 2.4 and 2.5 denote that the number of routes that starts and ends at the depot is equal to the number of vehicles.

$$\sum_{i \in V} X_{io} = K,$$  \hspace{1cm} (2.4)
\[ \sum_{j \in V} x_{oj} = K, \quad (2.5) \]

(d) Since \( r \{S'\} \) denotes the minimum number of vehicles needed to serve all customers in \( S' \), Equation 2.6 imposes both the connectivity of the solution and the vehicle capacity requirements.

\[ \sum_{i \in S'} \sum_{j \in S'} x_{ij} \geq r(S'). \quad (2.6) \]

\[ x_{ij} \in \{0, 1\} \quad \text{for all } (i, j) \in A, i \neq j \]

**Commodity Flow Models**

Commodity Flow Models were first introduced by Garvin *et al* (1957) to solve oil delivery problems. In addition to the binary variables used in integer programming, a set of continuous variables is also considered in the model. Continuous variables represent the amount of flow in each arc. The formulation requires the extended graph \( G' = (V, A) \) obtained from \( G \) by adding vertex \( n+1 \), which is a copy of the depot node. Instead of starting and ending at vertex 0, routes are now paths from vertex 0 to vertex \( n+1 \). Two nonnegative flow variables \( y_{ij} \) and \( y_{ji} \) are associated with each edge \((i, j) \in A\), where \( A \) is the set of all arcs. If a vehicle travels from \( i \) to \( j \), then \( y_{ij} \) represents the vehicle current load and \( y_{ji} \) represents the residual capacity on the vehicle. The vehicle leaves vertex 0 with just enough product, delivering at every customer an amount equal to its demand and arrives empty at vertex \( n+1 \). Each customer \( i \in V \setminus \{0, n+1\} \) \((i = 1, \ldots, n)\) is associated with a known nonnegative demand \( d_i \), to be delivered. Let \( D = d(V \setminus \{0, n+1\}) = \sum_{i \in V \setminus \{0, n+1\}} d_i \) denote the total customer demand. A set of \( K \) identical vehicles, each with capacity \( C \), is available at the depot. The basic commodity flow model is described below and an example is given in Figure 2.1:

\[ \min \sum_{(i,j) \in A} c_{ij} x_{ij} \quad (2.7) \]

Subject to the following constraints:
The difference between the sum of the commodity flow variables associated with arcs entering and leaving each vertex $i$ is equal to twice the demand of node $i$, denoted by $d_i$. 

$$\sum_{j \in V} (y_{ij} - y_{ji}) = 2d_i \quad \forall i \in V \setminus \{0, n + 1\}, \quad (2.8)$$

(b) The amount of products carried by a vehicle, when leaving a depot, is equal to the total customer demand $D$.

$$\sum_{j \in V \setminus \{0, n + 1\}} y_{oj} = D, \quad (2.9)$$

(c) The residual capacity of a vehicle, when leaving at a depot, is equal to the total vehicle capacity minus the total customer demand.

$$\sum_{j \in V \setminus \{0, n + 1\}} y_{jo} = KC - D, \quad (2.10)$$

(d) The residual vehicle capacity, when arriving at a depot, is equal to the total vehicle capacity

$$\sum_{j \in V \setminus \{0, n + 1\}} y_{n+1j} = KC - D, \quad (2.11)$$

(e) The relationships among vehicle flow, commodity flow variables are given below:

$$y_{ij} + y_{ji} = CX_{ij} \quad \forall (i, j) \in A, \quad (2.12)$$

$$\sum_{j \in V} (X_{ij} + X_{ji}) = 2 \quad \forall i \in V \setminus \{0, n + 1\}, \quad (2.13)$$

$$y_{ij} \geq 0 \quad \forall (i, j) \in A, \quad (2.14)$$

$$X_{ij} \in \{0, 1\} \quad \text{for all } (i, j) \in A \quad i \neq j \quad (2.15)$$

Figure 2.1 gives an example of flow paths on a route. For any feasible solution, the flow variable $y_{ij}$ defines two directed paths. One from vertex 0 to $n+1$, whose variables represent the vehicle load. Another route is from vertex $n+1$ to vertex 0, whose variables represent the residual capacity on the vehicle. For example, in fig 2.1, the route starts from the depot 0, $y_{0,a} = 25$, $y_{a,0} = 5$. The vehicle goes
from 0 to n+1, leaving vertex 0 with just enough product, delivering at every customer an amount equal to its demand, and arriving empty at vertex n+1. Along the route, \( y_{ji} = C - y_{ij} \). Therefore, the equation \( y_{ij} + y_{ji} = C \) holds for each arc.

![Figure 2.1 Example of flow paths on a route(C=30).](image)

**Set-Covering Models**

The set-covering method was first suggested by Balinski and Quandt (1964) to solve a capacitated vehicle routing problem (CVRP). According to Ravindran et al (2012), a set covering matrix \((m \times n)\) is considered, whose elements \( a_{ij} \) are 0 or 1. If \( a_{ij} = 1 \), column ‘j’ “covers” row ‘i’. If not, \( a_{ij} = 0 \). The set covering problem is to select the minimum number of columns such that every row is covered by at least one column. They define a binary variable for each column such that

\[
X_j = \begin{cases} 
1 & \text{if column ‘}j\text{‘ is selected} \\
0 & \text{otherwise}
\end{cases}, \quad \text{for } j = 1, 2, \ldots, m \quad (2.16)
\]

The Integer Programming (IP) model is given by:

\[
\text{Minimize } Z = \sum_{j=1}^{n} X_j, \quad (2.17)
\]
Subject to

\[ \sum_{j=1}^{n} a_{ij} x_j \geq 1 \text{ for } i = 1,2, \ldots, m, \quad (2.18) \]

\[ x_j \in (0,1) \]

Applying the set covering model to CVRP, the rows correspond to the customers and columns correspond to all feasible routes to supply the customers.

2.5 Summary

This thesis mainly deals with the Vehicle Routing Problem (VRP), with a specific emphasis on coordinating inventory and transportation decisions. A large body of research exists to address different types of VRP. Jayaraman et al (2008) and Alp et al (2002) study problems that retailers are served in a route within different time windows. Yildiz et al (2011) and Mingzhou et al (2007) studied a pick-up and delivery problem, where products must be collected and delivered at the same time. Altiparmak et al (2009) consider backhauls in their model. Most of these studies do not include milkrun and only a few of them investigate the interaction between inventory and transportation decisions. In this thesis, we look at selecting a subset of customers and suppliers and combining their shipments on the same routes. We also consider the possibility to use milkrun and the cost incurred by inventory. Due to the difficulty of VRP, most of the techniques developed are heuristics, which can be found in Ambrosino et al (2009), Bianchessi (2005) and Jayaraman (2008). Although heuristics methods generate quality solutions very fast, they do not guarantee optimality. In this thesis, we use a Mixed Integer Programming model to get the optimal vehicle routing plan for each retailer. In addition, a case study is presented, which aims at making an operational decision for an automotive company. Despite the growing interest in the VRP, our specific problem, which considers milkrun and inventory cost, has not been previously analyzed in the literature. Table 2.2 gives a summary of related research and the contribution of this thesis for comparison.
<table>
<thead>
<tr>
<th>Windows</th>
<th>Backhauls</th>
<th>Delivery</th>
<th>Inventory</th>
<th>Heuristic</th>
<th>MIP</th>
<th>Decision</th>
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Table 2.2 Key Papers Summary
CHAPTER 3 MODEL DEVELOPMENT FOR VEHICLE ROUTING

3.1 Problem Description

The problem is to find the best vehicle routing plan for a manufacturing company. We begin by developing an optimization model for an example and then present the general model. In the example problem, we consider one manufacturing plant, a cross-dock point and five retailers. Table 3.1 gives the average retailer demands:

Table 3.1 Average retailer demands

<table>
<thead>
<tr>
<th>Retailer</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>In total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave</td>
<td>752</td>
<td>2120</td>
<td>2797</td>
<td>2443</td>
<td>1401</td>
<td>9513</td>
</tr>
<tr>
<td>Demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The distance matrix among plant, cross dock and retailers is given in Table 3.2:

Table 3.2 Distance matrix (In miles)

<table>
<thead>
<tr>
<th>In miles</th>
<th>plant</th>
<th>crossdock</th>
<th>1Hunstville</th>
<th>2Birmingham</th>
<th>3Jacksonville</th>
<th>4Orlando</th>
<th>5Sarasota</th>
</tr>
</thead>
<tbody>
<tr>
<td>plant</td>
<td>0</td>
<td>739</td>
<td>215</td>
<td>239</td>
<td>728</td>
<td>810</td>
<td>875</td>
</tr>
<tr>
<td>crossdock</td>
<td>739</td>
<td>0</td>
<td>546</td>
<td>512</td>
<td>92</td>
<td>79</td>
<td>158</td>
</tr>
<tr>
<td>1</td>
<td>215</td>
<td>546</td>
<td>0</td>
<td>103</td>
<td>535</td>
<td>617</td>
<td>683</td>
</tr>
<tr>
<td>2</td>
<td>239</td>
<td>512</td>
<td>103</td>
<td>0</td>
<td>501</td>
<td>583</td>
<td>648</td>
</tr>
<tr>
<td>3</td>
<td>728</td>
<td>92</td>
<td>535</td>
<td>501</td>
<td>0</td>
<td>131</td>
<td>261</td>
</tr>
<tr>
<td>4</td>
<td>810</td>
<td>79</td>
<td>617</td>
<td>583</td>
<td>131</td>
<td>0</td>
<td>126</td>
</tr>
<tr>
<td>5</td>
<td>875</td>
<td>158</td>
<td>683</td>
<td>648</td>
<td>261</td>
<td>126</td>
<td>0</td>
</tr>
</tbody>
</table>

In the example problem, each retailer can be served in three ways:

4. Direct shipping from the plant to the retailer using LTL service.

5. Consolidate the shipment with other freight, shipping together from the plant to the crossdock using TL service and then shipping from cross dock to the retailer using LTL service.
6. Consolidate the shipment with other freight, shipping together from the plant to the crossdock using TL service and then shipping from crossdock to the retailer using milkrun.

The final result will show what is the best option for each retailer that minimizes cost.

The cost structure for each option is as follows:

It is assumed that per mile LTL cost is $p^m = 5 \text{ dollar/mile}$, one-way TL from plant to crossdock is $p^{TL} = 3 \text{ dollar/mile}$, milkrun cost from crossdock to retailer is $p^{LTL}_{xd,i} = 8 \text{ dollar/mile}$. By multiplying the per mile cost by the distance matrix, we get the following cost table:

<table>
<thead>
<tr>
<th>Plant</th>
<th>1 Huntsville</th>
<th>2 Birmingham</th>
<th>3 Jacksonville</th>
<th>4 Orlando</th>
<th>5 Sarasota</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>1075</td>
<td>1195</td>
<td>3640</td>
<td>4050</td>
<td>4375</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Crossdock</th>
<th>1 Huntsville</th>
<th>2 Birmingham</th>
<th>3 Jacksonville</th>
<th>4 Orlando</th>
<th>5 Sarasota</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>2730</td>
<td>2560</td>
<td>460</td>
<td>395</td>
<td>790</td>
</tr>
</tbody>
</table>
In addition to the transportation cost, we also consider the inventory holding cost and in-transit inventory cost in the model. The objective function is to minimize the total transportation cost as well as the inventory holding cost and the in-transit inventory cost. The value of each product is 100 dollars. The delivery frequency is 3 days, the maximum number of stops in a milk run is 3. The vehicle capacity is 25000 units. The optimization problem is formulated as follows:

### 3.2 Model Formulation

#### 3.2.1 Data and Parameters

- \( pl \): the plant
- \( xd \): the cross-dock
- \( N \): set of all retailers
- \( N' = N \cup \{xd\} \): set of all retailers and the cross-dock
- \( dist_{i,j} \): distance between nodes \( i, j \in N' \cup pl \)
- \( d_i \): average demand for retailer \( i \in N \) in units
maxstops maximum number of stops allowed on a route excluding the cross-dock
maxvol maximum volume capacity allowed on a TL in units
Q total quantity of products to be delivered \( Q = \sum_{i=1}^{N} d_i \)
F delivery frequency in days
M an arbitrary large number

**Costs**

\( p^m \) per mile cost for milkrun trucks
\( p^{TL} \) per mile cost for one-way TL trucks
\( p^{LTL}_{xd,i} \) direct shipping cost between the cross-dock and retailer \( i \in N \)
\( p^{LTL}_{pl,i} \) direct shipping cost between the plant and retailer \( i \in N \)
\( H \) Inventory holding cost of one unit of product held at the plant per year including cost of capital, cost of physically storing inventory and cost of labor. \( H \) is estimated by using \( H = IC \) where \( I = 10\% \) per dollar per year, \( C = \) product value.

\( K \) In transit inventory cost is estimated at 85% of \( H \).

**Decision Variables**

**Binary Variables**

\( Y_i = 1 \) if retailer \( i \) uses LTL from the plant, 0 otherwise, \( \forall i \in N \).
\[ W_i = 1 \quad \text{if retailer } i \text{ is on a milrun from the cross-dock, } 0 \text{ otherwise, } \forall i \in N. \]

\[ X_i = 1 \quad \text{if retailer } i \text{ uses LTL from the crossdock, } 0 \text{ otherwise, } \forall i \in N. \]

\[ Dock_i^d = 1 \quad \text{if retailer } i \text{'s demand goes through the cross dock and } 0 \text{ otherwise, } \forall i \in N. \]

\[ Visit_j^f = 1 \quad \text{if retailer } i \text{'s demand goes through node } j, \text{ and } 0 \text{ otherwise } \forall i \in N, \forall j \in N'. \]

**General Integer Variables**

\[ M_{u,v} \quad \text{number of times the milrun arc } (u, v) \text{ is used, } \forall u, v \in N'. \]

\[ OW_{pl,xd} \quad \text{number of times the one way arc from the plant to the crossdock is used.} \]

**Continuous Variables**

\[ F_{u,v}^{TL} (i) \quad \text{flow of retailer } i \text{'s demand on the milrun arc } (u, v), \forall u, v \in N', \forall i \in N. \]

\[ F_{pl,xd}^{OW} (i) \quad \text{flow of retailer } i \text{'s demand from the plant to the crossdock on one way TL trucks, } \forall i \in N. \]

\[ F_{pl,i}^{LTL} (i) \quad \text{LTL flow of retailer from plant to retailer } i \in N. \]

\[ F_{xd,i}^{LTL} (i) \quad \text{LTL flow of retailer from cross dock to retailer } i \in N. \]

**Objective Function**

Minimize \[ Z = p^m * \left( \sum_{u,v \in N' \cap pl} dist_{u,v} * M_{u,v} \right) + P_{pl,xd}^{TL} * dist_{pl,xd} * OW_{pl,xd} + \sum_{i \in N} X_i * P_{i,xd}^{LTL} \]

\[ + \sum_{i \in N} Y_i * P_{i,pl}^{LTL} + Q * H * F/365 + K \] (3.1)
Where,

Milkrun cost

\[
\text{Milkrun cost} = 4368 \cdot M_{x,d,1} + 4096 \cdot M_{x,d,2} + 736 \cdot M_{x,d,3} + 632 \cdot M_{x,d,4} + 1264 \cdot M_{x,d,5} + 4368 \cdot M_{1,x,d} + 824 \cdot M_{1,2} + 4280 \cdot M_{1,3} + 4936 \cdot M_{1,4} + 5464 \cdot M_{1,5} + \ldots + 1264 \cdot M_{5,x,d} + 5464 \cdot M_{5,1} + 5184 \cdot M_{5,2} + 2088 \cdot M_{5,3} + 1008 \cdot M_{5,4}
\]

Oneway TL cost

\[
\text{Oneway TL cost} = 3 \cdot 739 \cdot OW_{pl,x,d}
\]

LTL cost at crossdock

\[
\text{LTL cost at crossdock} = 2730 \cdot X_1 + 2560 \cdot X_2 + 460 \cdot X_3 + 395 \cdot X_4 + 790 \cdot X_5
\]

LTL Cost at Plant

\[
\text{LTL Cost at Plant} = 1075 \cdot X_1 + 1195 \cdot X_2 + 3640 \cdot X_3 + 4050 \cdot X_4 + 4375 \cdot X_5
\]

Inventory holding cost

\[
\text{Inventory holding cost} = 9513 \cdot 0.1 \cdot 100 \cdot \frac{3}{365} = 782
\]

In-transit Inventory holding cost

\[
\text{In-transit Inventory holding cost} = 0.85 \cdot 782 = 665
\]

Constraints

Consider retailer 1:

\[
\text{Dock}_1^d + Y_1 = 1, \quad (3.2)
\]
\[ W_1 + X_1 = Dock^d_1, \] (3.3)

\[ F_{\text{T}1}(1) + F_{\text{T}2}(1) + F_{\text{T}3}(1) + F_{\text{T}4}(1) + F_{\text{T}5}(1) = 752 \cdot W_1, \] (3.4)

\[ F_{\text{L}1}(1) = 752 \cdot X_1, \] (3.5)

\[ F_{\text{p}1}(1) = 752 \cdot Y_1, \] (3.6)

\[ F_{\text{pl}1}(1) \leq M \cdot Dock^d_1 \] (3.7)

\[ F_{\text{T}1}(1) = 0, F_{\text{T}2}(1) = 0, F_{\text{T}3}(1) = 0, F_{\text{T}4}(1) = 0, F_{\text{T}5}(1) = 0, \] (3.8)

\[ M_{1,1} + M_{1,2} + M_{1,3} + M_{1,4} + M_{1,5} + M_{1,6} \leq 1, \ldots, M_{5,1} + M_{5,2} + M_{5,3} + M_{5,4} + M_{5,5} + M_{5,6} \leq 1 \]

\[ M_{1,1} + M_{1,2} \leq 1, M_{2,1} + M_{2,2} \leq 1, M_{3,1} + M_{3,2} \leq 1, M_{4,1} + M_{4,2} \leq 1, M_{5,1} + M_{5,2} \leq 1 \]

\[ M_{2,1} + M_{2,2} \leq 1, M_{3,1} + M_{3,2} \leq 1, M_{4,1} + M_{4,2} \leq 1, M_{5,1} + M_{5,2} \leq 1 \]

\[ M_{3,1} + M_{3,2} \leq 1, M_{4,1} + M_{4,2} \leq 1, M_{5,1} + M_{5,2} \leq 1 \]

\[ M_{4,1} + M_{4,2} \leq 1, M_{5,1} + M_{5,2} \leq 1 \]

\[ M_{5,1} + M_{5,2} \leq 1, M_{5,3} + M_{5,4} + M_{5,5} + M_{5,6} \leq 1 \]

\[ M_{5,1} + M_{5,2} + M_{5,3} + M_{5,4} + M_{5,5} + M_{5,6} = M_{1,5} + M_{2,5} + M_{3,5} + M_{4,5} + M_{5,5} + M_{5,6}, \] (3.10)

\[ 752 \cdot Y_1 + F_{\text{pl}1}(1) = 752, \] (3.11)

\[ F_{\text{T}2}(1) + F_{\text{T}3}(1) + F_{\text{T}4}(1) + F_{\text{T}5}(1) + F_{\text{T}6}(1) - F_{\text{T}2}(1) - F_{\text{T}3}(1) - F_{\text{T}4}(1) - F_{\text{T}5}(1) - F_{\text{T}6}(1) = 0, \] (3.12)
\[ F_{pl}^{ow} (1) - 752 * \text{Dock}_{1}^{d} = 0, \quad (3.13) \]

For example, take milkrun arc (1, 2):

\[ F_{1,2}^{TL} (1) + F_{1,2}^{TL} (2) + F_{1,2}^{TL} (3) + F_{1,2}^{TL} (4) + F_{1,2}^{TL} (5) \leq 25000 * M_{1,2}, \quad (3.14) \]

\[ F_{pl}^{ow} (1) + F_{pl}^{ow} (2) + F_{pl}^{ow} (3) + F_{pl}^{ow} (4) + F_{pl}^{ow} (5) \leq 25000 * OW_{pl}, \quad (3.15) \]

Take node \( i = 1 \) and node \( i = 2 \) for example,

\[ F_{2,1}^{TL} (1) + F_{2,1}^{TL} (2) + F_{2,1}^{TL} (3) + F_{2,1}^{TL} (4) + F_{2,1}^{TL} (5) \leq \text{Visit}_{1}^{1} * M, \quad (3.16) \]

\[ \text{Visit}_{1}^{2} + \text{Visit}_{1}^{3} + \text{Visit}_{1}^{4} + \text{Visit}_{1}^{5} + \text{Visit}_{2}^{1} \leq 3, \quad (3.17) \]

### 3.3 Model Solution

The problem is solved by GAMS. This is a small size problem, with 5 retailers, 1 cross-dock and a manufacturing plant. The model contains 180 constraints, 278 single variables and 82 discrete variables. The final results are given in Figure 3.1.

![Figure 3.1 Final results](image)

From Figure 3.1, we can see that retailer 1 and retailer 2 choose LTL service shipping from plant to their locations, while retailer 3, retailer 4 and retailer 5 combine their shipment together, and ship from plant to
crossdock first and then use LTL service to deliver to their locations. The total cost is $7579, where the inventory holding cost is $782, in-transit inventory cost is $667 and transportation cost is $6130. No milkrun is used in the final result since the milk run cost is much higher than the other two options. The model is closely related to the cost structure of each option and the distance among the retailers. To examine the model solution, suppose we change the milk run cost from 8 to 1 dollar per mile and other cost remains the same, the results will be as shown in Figure 3.2.

![Figure 3.2 Final results with milkrun cost reduced from $8 to $1/km mile](image)

From Figure 3.2, we can see that when the milkrun cost decreases from 8 to 1 dollar per mile, all the shipments are combined together and shipped from plant to the cross dock first. Then they are shipped by two different milkruns to the retailers. Now, we shall present the general model.

### 3.4 General Model Formulations

The model is formulated as a Mixed Integer Linear Programming (MILP). The detailed formulation of the model is given below:
3.4.1 Assumptions

- Single mode of transport is considered
- Capacities of the trucks are known in advance
- There is no breakdown of the truck
- Retailers demand are deterministic and known in advance
- Inventory holding costs at the cross dock are not considered
- Locations of all entities are known
- The manufacturer has only one type of product.
- Carrier unloading time and waiting time at the cross-dock and the retailer location are not considered

3.4.2 Data and Parameters

\( p_l \) the plant

\( x_d \) the cross-dock

\( N \) set of all retailers

\( N' = N \cup \{x_d\} \) set of all retailers and the cross-dock

\( dist_{i,j} \) distance between nodes \( i, j \in N' \cup p_l \)

\( d_i \) average demand for retailer \( i \in N \) in units

maxstops maximum number of stops allowed on a route excluding the cross-dock

maxvol maximum volume capacity allowed on a full truck load in units

\( Q \) total quantity of products to be delivered
F delivery frequency in days

M an arbitrary large number

**Costs**

\[ p_m \] per mile cost for milkrun trucks

\[ p_{TL} \] per mile cost for one-way TL trucks

\[ p_{xd,i}^{LTL} \] LTL cost between retailer \( i \in N \) and the cross-dock

\[ p_{pl,i}^{LTL} \] LTL cost between retailer \( i \in N \) and the plant

\( H \) Inventory holding cost of one unit of product held at the plant per year including cost of capital, cost of physically storing inventory and cost of labor, where \( H = IC, I = 10\% \) per dollar per year, \( C \) = product value.

\( K \) In transit inventory cost is estimated as 85\% of \( H \).

**Decision Variables**

**Binary Variables**

\[ Y_i = 1 \text{ if retailer } i \text{ uses } LTL \text{ from the plant, } 0 \text{ otherwise, } \forall i \in N. \]

\[ W_i = 1 \text{ if retailer } i \text{ is on a milkrun, } 0 \text{ otherwise, } \forall i \in N. \]

\[ X_i = 1 \text{ if retailer } i \text{ uses } LTL \text{ from the crossdock, } 0 \text{ otherwise, } \forall i \in N. \]

\[ Dock_i^d = 1 \text{ if retailer } i \text{'s demand goes through the cross dock and } 0 \text{ otherwise, } \forall i \in N. \]
\[ \text{Visit}_i^j = 1 \quad \text{if retailer } i \text{'s demand goes through node } j, \text{ and } 0 \text{ otherwise } \forall i \in N, \forall j \in N'. \]

**General Integer Variables**

\[ M_{u,v} \quad \text{number of times the milkrun arc } (u,v) \text{ is used, } \forall u,v \in N'. \]

\[ OW_{pl,xd} \quad \text{number of times the one way arc from the plant to the crossdock is used.} \]

**Continues Flow Variables**

\[ F_{u,v}^{TL}(i) \quad \text{flow of retailer } i \text{'s demand on the milkrun arc } (u,v), \forall u,v \in N', \forall i \in N \]

\[ F_{pl,xd}^{OW}(i) \quad \text{flow of retailer } i \text{'s demand from the plant to the crossdock on one way TL trucks, } \forall i \in N. \]

\[ F_{pl,i}^{LTL}(i) \quad \text{flow of retailer } i \text{'s demand direct from plant } \forall i \in N. \]

\[ F_{xd,i}^{LTL}(i) \quad \text{flow of retailer } i \text{'s demand direct from crossdock } \forall i \in N. \]

**Objective Function**

Minimize \[
\text{Milkrun Cost} + \text{Oneway TL Cost} + \text{LTL Cost at Cross-dock} + \text{LTL Cost at Plant} + \]

\[ \text{Inventory Holding Cost} + \text{In-transitInventory Cost} \]

Minimize \[
\begin{align*}
Z &= p_{m}^m \times (\sum_{u,v \in N'} \text{dist}_{u,v} \times M_{u,v}) + p_{pl}^\text{TL} \times \text{dist}_{pl,xd} \times OW_{pl,xd} + \sum_{i \in N} X_i \times P_{i,xd}^{\text{LTL}} + \\
&\quad \sum_{i \in N} Y_i \times P_{i,pl}^{\text{LTL}} + Q \times H \times F/365 + K
\end{align*}
\]

(3.18)

Where,
**Milkrun Cost:** We use a fixed per mile milkrun cost multiplied by the total length of milkrun arcs to calculate the milkrun cost.

**Oneway TL Cost:** $P^\text{TL} \times \text{dist}_{\text{plxd}}$, is the fixed full truck load per mile cost multiplied by the distance between the plant and crossdock to calculate the oneway full truck load cost.

**LTL Cost at Cross-dock:** The shipping cost $P^{\text{LTL}}_{i\text{xd}}$ between retailer $i \in N$ and the crossdock is predetermined. If retailer $i$ uses LTL service at cross dock, $X_i$ is set to 1, otherwise, $X_i$ is set to 0.

**LTL Cost at Plant:** The shipping cost $P^{\text{LTL}}_{i\text{pl}}$ between retailer $i \in N$ and the plant is predetermined. If retailer $i$ uses LTL service at plant, $Y_i$ is set to 1, otherwise, $Y_i$ is set to 0.

**Inventory Holding Cost:** We use $0.1 \times \text{product price/unit}$ to estimate yearly inventory holding cost/unit. Note that $Q^*H/365$ represent the inventory holding cost for the total amount of products to be shipped per day, multiplied by the delivery frequency represent the total inventory holding cost incurred in the delivery time window.

**In-transit Inventory:** It is estimated at 85% of the inventory holding cost.

**Constraints**

(1) We consider three types of distribution options that include using LTL from the plant, using one way TL shipping to crossdock and then shipping locally either by milk run or LTL carrier. Only one mode of transportation can be used by each retailer $i \in N$.

\[
\text{Dock}_i^d + Y_i = 1, \ \forall i \in N
\]  
(3.19)

\[
W_i + X_i = \text{Dock}_i^d, \ \forall i \in N
\]  
(3.20)

(2) If retailer $i \in N$ is on a milkrun, the amount of retailer $i$'s demand flow $F^{\text{TL}}_{u,i}(i)$ from node $u \in N'$ into retailer $i$ should be equal to retailer $i$'s demand $d_i$. 

33
\[ \sum_{u \in N} F_{u,i}^{TL} = d_i \cdot W_i, \forall i \in N \]  
(3.21)

(3) If retailer \( i \in N \) uses LTL carrier from crossdock, \( X_i = 1 \), the amount of retailer \( i \)'s demand flow \( F_{xd,i}^{LTL} \) from crossdock into retailer \( i \) should be equal to retailer \( i \)'s demand \( d_i \).

\[ F_{xd,i}^{LTL} = d_i \cdot X_i, \forall i \in N \]  
(3.22)

(4) If retailer \( i \in N \) uses LTL carrier from plant, \( Y_i = 1 \), the amount of retailer \( i \)'s demand flow \( F_{pl,i}^{LTL} \) from plant into retailer \( i \) should be equal to retailer \( i \)'s demand \( d_i \).

\[ F_{pl,i}^{LTL} = d_i \cdot Y_i, \forall i \in N \]  
(3.23)

(5) If retailer \( i \)'s flow passes through the crossdock, then \( \text{Dock}_i^d = 1 \), for \( i \in N \), the demand flow on one way full truck load from plant to crossdock is less than an arbitrary large number.

\[ F_{pl,xd}^{OW} \leq M \cdot \text{Dock}_i^d, \forall i \in N \]  
(3.24)

(6) Unallowed flows are set to zero. Demand flow of retailer \( i \) from retailer to node \( u, i \in N, u \in N' \) or a round trip starts and ends at node \( u, u \in N' \) on a milkrun are not allowed

\[ F_{i'u}^{TL} = 0, \forall i \in N, \forall u \in N' \]  
(3.25)

\[ F_{iu,i}'^{TL} = 0, \forall i \in N, \forall u \in N' \]  
(3.26)

(7) For each retailer, \( u \in N \), the sum of times the milkrun arc \( (u, v), v \in N' \) used is no more than one time, which means at most one milkrun truck can leave each retailer.

\[ \sum_{v \in N'} M_{u,v} \leq 1, \forall u \in N \]  
(3.27)

(8) For the milkrun arcs \( (u, v) \) and \( (v, u) \) between node \( u \) and node \( v, u, v \in N' \), only one of the two arcs can be used in one milkrun between the two nodes.

\[ M_{u,v} + M_{v,u} \leq 1, \forall u, v \in N' \]  
(3.29)
(9) For each node u, if a milkrun arc (u,v) is used, then a truck leaves node u to node v. Similarly, if a milkrun arc (v,u) is used, it means a truck enters node u from node v. Since the number of milkrun trucks entering a retailer should be equal to that leaving it, the sum of the number of times milkrun arc (u,v) and (v,u), \( u \in N, v \in N' \) used should be the same.

\[
\sum_{v \in N'} M_{u,v} = \sum_{v \in N'} M_{v,u} \quad \forall u \in N'
\] (3.30)

(10) For each retailer, its demand is either satisfied by LTL carrier shipping from the plant or by full truck load carrier shipping from the plant to the cross dock.

\[
Y_i * d_i + F_{\text{plxd}}^\text{OW}(i) = d_i, \forall i \in N
\] (3.31)

(11) For each retailer i, namely node i \( \in N \), its demand flow on milkrun arc (u, j) and (j, u), \( i, j \in N: i \neq j, u \in N' \), are the same. Since we set \( F_{i,j,i}^\text{TL}(i) = 0 \), \( F_{j,i,j}^\text{TL}(i) \neq F_{i,j,i}^\text{TL}(i) \), thus \( i \neq j \).

\[
\sum_{u \in N'} F_{i,j,i}^\text{TL}(i) - \sum_{u \in N'} F_{j,i,j}^\text{TL}(i) = 0, \forall i, j \in N: i \neq j
\] (3.32)

(12) Demand flow balance at the crossdock. For each node i \( \in N \), the demand of node i entering crossdock is equal to that leaving crossdock, the demand flow passes by the one way arc (pl,xd) is equal to \( d_i \) if a crossdock is considered.

\[
F_{\text{plxd}}^\text{OW}(i) - X_i * d_i = 0, \forall i \in N
\] (3.33)

(13) The total demand flow of all retailers \( i \in N \) on each milkrun arc (u,v), \( u,v \in N' \) cannot exceed the vehicle capacity.

\[
\sum_{i \in N} F_{u,v}^\text{TL}(i) \leq M_{v,u} * \text{maxvol} \quad \forall u, v \in N'
\] (3.34)

(14) The total demand flow of all retailers \( i \in N \) on one way arc (pl,xd) cannot exceed the vehicle capacity.

\[
\sum_{i \in N} F_{\text{plxd}}^\text{OW}(i) \leq \text{OW}_{\text{plxd}} * \text{maxvol}
\] (3.35)
(15) If retailer $i$’s demand goes through node $j$ in a milkrun, then $Visit_j^i = 1, \forall i \in N, \forall j \in N'$, which means node $i$’s flow visits node $j$. The sum of node $i$’s demand on an milkrun arc $\{(j,k) \in N' : i \neq j, k \in N'\}$, is less than an arbitrary large number.

\[
\sum_{k \in N'/F_{ik}^j} Visit_j^i \leq Visit_j^i * M \forall i \in N, j \in N' : i \neq j
\]  

(3.36)

(16) For each retailer $i \in N$, the number of nodes, which its demand flow passes by, cannot exceed the maximum number of stops.

\[
\sum_{j \in N' i \neq j} Visit_j^i \leq \text{max stops} \forall i \in N
\]  

(3.37)

In the next chapter, we will apply the general model to an actual case study and discuss the results.
CHAPTER 4 CASE STUDY

This section is a real application of the model presented in Chapter 3. The case study is about to find the best vehicle routing plan for an automotive company. The network considered in the case study consists of one manufacturing plant, one cross-dock point and eleven retailers. The problem to be solved is very similar to the numerical example. In the case study, each retailer can be served in three ways:

1. Direct shipping from the plant to the retailer using LTL service.
2. Consolidate the shipment with other freight, shipping together from the plant to the crossdock using TL service, and then shipping from cross dock to the retailer using LTL service.
3. Consolidate the shipment with other freight, shipping together from the plant to the crossdock using TL service and then shipping from crossdock to the retailer using milkrun.

Also, we consider inventory holding cost and in-transit inventory cost and the final result will show what is the best option for each retailer that minimizes total cost. The data used for this case study is a combination of real inputs from an automotive company in North America and some assumed values.

4.1 Data Collection

The index sets used are:

\[ p_l \] the plant
\[ x_d \] the cross-dock
\[ N \] set of all retailers
\[ N' = N \cup \{x_d\} \] set of all retailers and the cross-dock

The data used in the case study is listed below:

- \( D_i \) Annual demand of retailer \( i, i \in N \)
Table 4.1 Annual Demand for each retailer in pounds

<table>
<thead>
<tr>
<th>Retailer</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>119351</td>
<td>453478</td>
<td>120009</td>
<td>303757</td>
<td>119828</td>
<td>204737</td>
</tr>
<tr>
<td>Retailer</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>312558</td>
<td>515896</td>
<td>288795</td>
<td>119200</td>
<td>233061</td>
<td></td>
</tr>
</tbody>
</table>

The company has used direct shipping from the plant to retailer by using LTL service every day (option 1). In this case study, we consider consolidated freight with different delivery frequency (option 2 and 3) to examine the potential cost savings. From the annual demand given in Table 4.1, we get the periodic demand by using \( \frac{\text{annual demand}}{365} \times \text{delivery frequency in days} \). In this case study, we consider 5 different delivery frequencies. Table 4.2 presents the periodic demands for each retailer with different delivery frequency.

- Delivery frequency (in days): \( f=3,4,5,6,7 \)
- Periodic Demand for retailer \( i \): \( d_i = \frac{D_i}{365} \times f, \ i \in N \)

Table 4.2 Periodic Demand for each retailer with different delivery frequency in pounds

<table>
<thead>
<tr>
<th>Retailer</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 days</td>
<td>981</td>
<td>3727</td>
<td>986</td>
<td>2497</td>
<td>985</td>
<td>1683</td>
<td>2569</td>
<td>4240</td>
<td>2374</td>
<td>980</td>
<td>1916</td>
</tr>
<tr>
<td>4 days</td>
<td>1308</td>
<td>4970</td>
<td>1315</td>
<td>3329</td>
<td>1313</td>
<td>2244</td>
<td>3425</td>
<td>5654</td>
<td>3165</td>
<td>1306</td>
<td>2554</td>
</tr>
<tr>
<td>5 days</td>
<td>1635</td>
<td>6212</td>
<td>1643</td>
<td>4162</td>
<td>1642</td>
<td>2805</td>
<td>4282</td>
<td>7067</td>
<td>3957</td>
<td>1633</td>
<td>3193</td>
</tr>
<tr>
<td>6 days</td>
<td>1962</td>
<td>7454</td>
<td>1972</td>
<td>4994</td>
<td>1970</td>
<td>3366</td>
<td>5138</td>
<td>8480</td>
<td>4748</td>
<td>1960</td>
<td>3832</td>
</tr>
<tr>
<td>7 days</td>
<td>2289</td>
<td>8696</td>
<td>2301</td>
<td>5826</td>
<td>2298</td>
<td>3927</td>
<td>5994</td>
<td>9893</td>
<td>5539</td>
<td>2287</td>
<td>4471</td>
</tr>
</tbody>
</table>
The cross-dock is at Retailer 9's location. The distance matrix among plant, cross dock and retailers is given in Table 4.3:

Table 4.3 Distance Matrix

<table>
<thead>
<tr>
<th>In miles</th>
<th>plant</th>
<th>crossdock</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>plant</td>
<td>0</td>
<td>803</td>
<td>775</td>
<td>697</td>
<td>486</td>
<td>786</td>
<td>780</td>
<td>993</td>
<td>971</td>
<td>1012</td>
<td>803</td>
<td>802</td>
<td>818</td>
</tr>
<tr>
<td>crossdock</td>
<td>803</td>
<td>0</td>
<td>143</td>
<td>218</td>
<td>375</td>
<td>95</td>
<td>80</td>
<td>267</td>
<td>238</td>
<td>281</td>
<td>0</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>1</td>
<td>775</td>
<td>143</td>
<td>0</td>
<td>80</td>
<td>350</td>
<td>55</td>
<td>66</td>
<td>252</td>
<td>229</td>
<td>270</td>
<td>143</td>
<td>140</td>
<td>160</td>
</tr>
<tr>
<td>2</td>
<td>697</td>
<td>218</td>
<td>80</td>
<td>0</td>
<td>271</td>
<td>128</td>
<td>139</td>
<td>325</td>
<td>302</td>
<td>343</td>
<td>217</td>
<td>213</td>
<td>233</td>
</tr>
<tr>
<td>3</td>
<td>486</td>
<td>375</td>
<td>271</td>
<td>0</td>
<td>359</td>
<td>354</td>
<td>55</td>
<td>545</td>
<td>586</td>
<td>373</td>
<td>377</td>
<td>326</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>786</td>
<td>95</td>
<td>55</td>
<td>128</td>
<td>359</td>
<td>0</td>
<td>17</td>
<td>232</td>
<td>210</td>
<td>251</td>
<td>95</td>
<td>92</td>
<td>112</td>
</tr>
<tr>
<td>5</td>
<td>780</td>
<td>80</td>
<td>66</td>
<td>139</td>
<td>354</td>
<td>17</td>
<td>0</td>
<td>220</td>
<td>196</td>
<td>238</td>
<td>80</td>
<td>76</td>
<td>96</td>
</tr>
<tr>
<td>6</td>
<td>993</td>
<td>267</td>
<td>252</td>
<td>325</td>
<td>55</td>
<td>232</td>
<td>220</td>
<td>0</td>
<td>26</td>
<td>21</td>
<td>266</td>
<td>256</td>
<td>261</td>
</tr>
<tr>
<td>7</td>
<td>971</td>
<td>238</td>
<td>229</td>
<td>302</td>
<td>545</td>
<td>210</td>
<td>196</td>
<td>26</td>
<td>0</td>
<td>43</td>
<td>239</td>
<td>225</td>
<td>270</td>
</tr>
<tr>
<td>8</td>
<td>1012</td>
<td>281</td>
<td>270</td>
<td>343</td>
<td>586</td>
<td>251</td>
<td>238</td>
<td>21</td>
<td>43</td>
<td>0</td>
<td>279</td>
<td>268</td>
<td>273</td>
</tr>
<tr>
<td>9</td>
<td>803</td>
<td>0</td>
<td>143</td>
<td>217</td>
<td>373</td>
<td>95</td>
<td>80</td>
<td>266</td>
<td>239</td>
<td>279</td>
<td>0</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>802</td>
<td>10</td>
<td>140</td>
<td>213</td>
<td>377</td>
<td>92</td>
<td>76</td>
<td>256</td>
<td>225</td>
<td>268</td>
<td>10</td>
<td>0</td>
<td>27</td>
</tr>
<tr>
<td>11</td>
<td>818</td>
<td>18</td>
<td>160</td>
<td>233</td>
<td>326</td>
<td>112</td>
<td>96</td>
<td>261</td>
<td>270</td>
<td>273</td>
<td>18</td>
<td>27</td>
<td>0</td>
</tr>
</tbody>
</table>

(Note: \(dist_{i,j}\) denotes the distance between nodes \(i,j \in N' \cup pl\))

Other supplemental data are given in Table 4.4:

- **maxstops**: maximum number of stops allowed on a route excluding the cross-dock
- **maxvol**: maximum volume allowed on a full truck load in units
- **\(p^m\)**: per mile cost for milkrun trucks
- **\(p^{\text{TL}}\)**: per mile cost for one-way TL trucks
- **\(p^{\text{LTL}}_{xd,i}\)**: LTL cost between retailer \(i \in N \) and the cross-dock
- **\(p^{\text{LTL}}_{pl,i}\)**: LTL cost between retailer \(i \in N \) and the plant
- **\(C_{\text{stop}}\)**: per stop cost in a milkrun route
- **\(M\)**: an arbitrary large number
Table 4.4 Supplemental data

<table>
<thead>
<tr>
<th>$P_{\text{LTL}}^{\text{pl}}$</th>
<th>1.9 dollar/mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{LTL}}^{\text{xd}}$</td>
<td>3.0 dollar/mile</td>
</tr>
<tr>
<td>$p_{\text{m}}$</td>
<td>2.45 dollar/mile</td>
</tr>
<tr>
<td>$P_{\text{LTL}}$</td>
<td>2.8 dollar/mile</td>
</tr>
<tr>
<td>$C_{\text{stop}}$</td>
<td>50 dollar</td>
</tr>
<tr>
<td>$\text{maxstps}$</td>
<td>5</td>
</tr>
<tr>
<td>$\text{Maxvol}$</td>
<td>45000 pounds</td>
</tr>
<tr>
<td>$M$</td>
<td>1000000</td>
</tr>
</tbody>
</table>

Since $P_{\text{LTL}}^{\text{pl}} = 1.9$ dollar per mile, the LTL cost from plant to retailer $i$, $P_{\text{LTL}}^{\text{pl},i}$ is given in Table 4.5.

Table 4.5 LTL cost from plant to retailers

<table>
<thead>
<tr>
<th>Retailer</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{dist}_{\text{pl},i}$</td>
<td>775</td>
<td>697</td>
<td>486</td>
<td>786</td>
<td>780</td>
<td>993</td>
<td>971</td>
<td>1012</td>
<td>803</td>
</tr>
<tr>
<td>LTL Cost($)</td>
<td>1473</td>
<td>1324</td>
<td>923</td>
<td>1493</td>
<td>1482</td>
<td>1887</td>
<td>1845</td>
<td>1923</td>
<td>1526</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Retailer</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{dist}_{\text{pl},i}$</td>
<td>802</td>
<td>818</td>
</tr>
<tr>
<td>LTL Cost($)</td>
<td>1524</td>
<td>1554</td>
</tr>
</tbody>
</table>

Since $P_{\text{LTL}}^{\text{xd}} = 3.0$ dollar per mile, the LTL cost from crossdock to retailer $i$, $P_{\text{LTL}}^{\text{xd},i}$ is given in Table 4.6.

Table 4.6 LTL cost from crossdock

<table>
<thead>
<tr>
<th>Retailer</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{dist}_{\text{xd},i}$</td>
<td>143</td>
<td>218</td>
<td>375</td>
<td>95</td>
<td>80</td>
<td>267</td>
<td>238</td>
<td>281</td>
<td>0</td>
</tr>
<tr>
<td>LTL Cost($)</td>
<td>429</td>
<td>654</td>
<td>1125</td>
<td>285</td>
<td>240</td>
<td>801</td>
<td>714</td>
<td>843</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Retailer</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{dist}_{\text{xd},i}$</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>LTL Cost($)</td>
<td>30</td>
<td>54</td>
</tr>
</tbody>
</table>
In the case study, we not only consider transportation cost, we also consider inventory holding cost and in transit inventory cost to minimize total cost. The related data is given in table 4.7

- \( C = 5.5 \) dollar per pound: Average value of the product per pound

- \( H \): Inventory holding cost of one pound of product held at the plant for one year including cost of capital, cost of physically storing inventory and cost of labor, by using \( H = IC \) to estimate it, where \( I = 15\% \) per dollar per year.

Hence, \( H = 0.15 \times 5.5 = 0.825 \) pound/year

- \( K \): In transit inventory cost is estimated as 85% of \( H \).

- **Total Inventory holding cost** \( S = H \times \frac{\sum D_t \times f}{365} \)

Inventory holding cost and in-transit inventory cost varies with different delivery frequency \( f \) and the data is given in Table 4.7.

<table>
<thead>
<tr>
<th>Delivery Frequency</th>
<th>( f=3 ) days</th>
<th>( f=4 ) days</th>
<th>( f=5 ) days</th>
<th>( f=6 ) days</th>
<th>( f=7 ) days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periodic total demand(lb)</td>
<td>22938</td>
<td>30583</td>
<td>38231</td>
<td>45876</td>
<td>53521</td>
</tr>
<tr>
<td>Inventory holding cost($)</td>
<td>156</td>
<td>277</td>
<td>432</td>
<td>622</td>
<td>847</td>
</tr>
<tr>
<td>Intransit Inventory cost($)</td>
<td>133</td>
<td>235</td>
<td>367</td>
<td>529</td>
<td>720</td>
</tr>
<tr>
<td>Total cost($)</td>
<td>289</td>
<td>512</td>
<td>799</td>
<td>1151</td>
<td>1567</td>
</tr>
</tbody>
</table>

### 4.2 Mathematical Model

**Assumptions**

- Single mode of transport is considered
- Capacities of the trucks are known in advance
- There is no breakdown of the truck
- Retailers demands are deterministic and known in advance
• Inventory holding costs at the cross dock are not considered
• Locations of all entities are known
• The manufacturer has only one type of product.
• Carrier unloading time and waiting time at the cross-dock and the retailer location are not considered

**Decision Variables**

All the decision variables are the same as defined in Chapter 3.

**Objective Function**

Minimize \[ Z = p^m \left( \sum_{u,v \in N'} \text{dist}_{u,v} \cdot M_{u,v} \right) + C_{\text{stop}} \cdot \sum_{i \in N} W_i \cdot P^\text{TL} \cdot \text{dist}_{\text{pl}, \text{xd}} + 0 \cdot W_{\text{pl}, \text{xd}} + \sum_{i \in N} X_i \]

\[ \cdot P^\text{LTL}_{\text{lx}, \text{xd}} + \sum_{i \in N} Y_i \cdot P^\text{LTL}_{\text{pl}, \text{pl}} + S + 0.85 \cdot S \]

**Constraints**

All the constraints remain the same as defined in Chapter 3.

### 4.3 Model Solution

The problem is solved in GAMS. The problem contains one manufacturing plant, one cross dock and 11 retailers. The model has 654 constraints and 1928 continuous variables and 310 discrete variables. The optimal solution is given in Table 4.8 for a delivery frequency of 3 days.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Total Inventory Cost</th>
<th>Transportation</th>
<th>DS from XD</th>
<th>DS from PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5,837</td>
<td>$289</td>
<td>$5,548</td>
<td>R9,R10,R11</td>
<td>0</td>
</tr>
</tbody>
</table>

**Milrun routes**

1 xd→R5→R2→R1→R4→xd  2 xd→R7→R8→R6→R3→R11→xd

Note: DS stands for Direct shipping.
The company had used direct shipping to all their retailers before. Hence the total transportation cost was estimated as $0.37 /lb * total periodic demand=$0.37/lb*22938lb=$8487. Instead of using direct shipping from the plant, the company will use pooled distribution to ship their products to the crossdock first and then either use milkrun or direct shipping for the delivery. With the new distribution plan, the cost saving is up to 31% for a delivery frequency of 3 days.

### 4.4 Scenario Analysis

We used $f = 3$ days for delivery frequency in Table 4.8. We also want to examine the routes with different delivery frequencies. By comparing the solution results, we can actually examine the shipping option for each customer under different delivery frequency as well as the effect on the inventory cost.

We solve the model with $f = 3$ days, 4 days, 5 days, 6 days and 7 days and kept other input data and constraints the same. The optimal solutions are summarized in Table 4.9.

<table>
<thead>
<tr>
<th>Delivery Frequency</th>
<th>DS from XD</th>
<th>DS from pl</th>
<th>Milkrun Routes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F=3days</strong></td>
<td>R9, R10, R11</td>
<td>None</td>
<td>1 xd→R5→R2→R1→R4→xd, 2 xd→R7→R8→R6→R3→xd</td>
</tr>
<tr>
<td><strong>F=4days</strong></td>
<td>R9, R10, R11</td>
<td>None</td>
<td>1 xd→R3→R6→R8→R7→xd, 2 xd→R5→R2→R1→R4→xd</td>
</tr>
<tr>
<td><strong>F=5days</strong></td>
<td>R10, R9, R11</td>
<td>None</td>
<td>1 xd→R5→R2→R1→R4→xd, 2 xd→R7→R8→R6→R3→xd</td>
</tr>
<tr>
<td><strong>F=6days</strong></td>
<td>R11, R9, R10</td>
<td>R3</td>
<td>1 xd→R5→R2→R1→R4→xd, 2 xd→R7→R6→R8→xd</td>
</tr>
<tr>
<td><strong>F=7days</strong></td>
<td>R11, R9, R10</td>
<td>R2</td>
<td>1 xd→R7→R8→R6→R3→xd, 2 xd→R1→R4→R5→xd</td>
</tr>
</tbody>
</table>

From Table 4.9, we can see that the routes are very similar under different delivery frequencies. For retailers R9, R10, R11, since their distance to crossdock is less than 20 miles, it is more cost-effective for them to choose direct shipping from the crossdock since there is an extra $50 per stop cost for the milkrun. The model identifies two routes with the maximum number of stop at 5. They are route 1:
xd→R5→R2→R1→R4→xd and route 2: xd→R7→R8→R6→R3→xd. When the delivery frequency is less than 6 days, there is no direct shipping from the plant. However, when \( f=6 \) days and \( f=7 \) days, the one way truckload cannot fulfill all the retailer’s demand because of the capacity restriction of the vehicle. In that case, R3 and R2 choose direct shipping from the plant.

The total cost, inventory cost, transportation cost and potential cost savings are given in Table 4.10. The current cost is calculated based on their current operations, which is shipping directly from the manufacturing plant on a daily basis. We used 3.7 dollar/lb to estimate their current cost.

<table>
<thead>
<tr>
<th>Delivery Frequency</th>
<th>( f=3 ) days</th>
<th>( f=4 ) days</th>
<th>( f=5 ) days</th>
<th>( f=6 ) days</th>
<th>( f=7 ) days</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inventory Cost</strong></td>
<td>$289.00</td>
<td>$512.00</td>
<td>$799.00</td>
<td>$1,151.00</td>
<td>$1,567.00</td>
</tr>
<tr>
<td><strong>Transportation cost</strong></td>
<td>$5,548.00</td>
<td>$5,620.00</td>
<td>$5,550.00</td>
<td>$6,085.00</td>
<td>$6,596.00</td>
</tr>
<tr>
<td><strong>Total cost</strong></td>
<td>$5,837.00</td>
<td>$6,132.00</td>
<td>$6,349.00</td>
<td>$7,236.00</td>
<td>$8,163.00</td>
</tr>
<tr>
<td><strong>Current cost</strong></td>
<td>$8,487.00</td>
<td>$11,316.00</td>
<td>$14,145.00</td>
<td>$16,974.00</td>
<td>$19,803.00</td>
</tr>
</tbody>
</table>

A comparison line chart for different cost under different delivery frequency is given in Figure 4.1. We can see that with \( f \) increasing from 3 days to 7 days, the inventory cost and transportation cost are all increasing. Inventory cost, which include inventory holding cost and the in-transit inventory cost, only takes up a small portion of the total shipping cost. Although it will increase with increasing \( f \), the cost savings for freight consolidation is still very huge. Therefore, we recommend that the company consolidate freight with a longer delivery frequency as long as the lead-time meets the retailer’s requirements.
We can also conclude that the company will have more cost savings when they choose a long delivery frequency, as can be seen in Figure 4.2. We can see by using 7-day frequency, pooled distribution and milkrun routing, the company can save as much as 60% of the total cost.
CHAPTER 5 CONCLUSION AND FUTURE WORK

A strategic outbound logistics planning is crucial for every company since the performance closely affects the delivered service, product quality, inventory level and total cost. The main activities of the outbound logistics include order processing, warehousing, transportation mode and distribution strategy. A well-designed transportation network can improve the responsiveness of the supply chain at a comparable low cost. In recent years, more and more lean strategies such as cross docking, milkruns and freight consolidation have been considered in the distribution plan that help reduce total cost and improve customer satisfaction level.

In this thesis, we developed an optimization model in order to select the best out bound logistics strategy for the manufacturing company. The transportation network includes a manufacturer, a crossdock and retailers. The objective is to minimize the total cost, including transportation cost and inventory holding cost, while satisfying all the demand constraints, vehicle capacity constraints and delivery time constraints. Each retailer can either be served directly from centralized distribution center or from a cross-dock point. Both milkrun and crossdock were considered.

The optimization problem is modeled as a Mixed Integer Linear Programming (MILP) model. We considered three different distributions for each retailer:

1. Direct shipping from the plant to the retailers using LTL service.
2. Consolidate the shipment with other freight, shipping together from the plant to the crossdock using TL service and then shipping from cross dock to the retailers using LTL service.
3. Consolidate the shipment with other freight, shipping together from the plant to the crossdock using TL service and then shipping from crossdock to the retailers using milkrun.
We illustrated the model by solving a numerical example, which included a manufacturer, a cross-dock and 5 retailers. The problem was solved in GAMS.

We then applied the model to solve an actual case study for a manufacturing company involving a manufacturer plant, a cross-dock and 11 retailers. The company currently uses direct shipping from the plant to all their retailers. The data used for this case study was a combination of real inputs from an automotive company in North America and some assumed values. The problem was solved by GAMS software. The solution indicated that instead of using direct shipping from the plant, the company should use pooled distribution to ship their products to the crossdock first and then either use milkrun or direct shipping for delivery to the retailers. With the new distribution plan, the cost saving was at least 31%. A scenario analysis with 5 different delivery frequencies was also presented. From the results, we concluded that by using pooled distribution and milkrun routing, the company could save as much as 60% of the total cost with a longer delivery frequency.

For future research, we would like to point out four potentially beneficial methodological extensions. The first is the use of stochastic programming to deal with the uncertainty in retailer’s demands. Instead of the average customer demand for each delivery frequency period, it can be expanded to accommodate stochastic demand variations of each retailer. Second, the model can be extended to multiple crossdock locations and within different delivery time windows. The third extension is the use of a route-based restricted model. In reality, we certainly want to minimize the total cost; however, we also want to generate stable loads routings, which is beneficial to operation management as well as a strong partnership with the third party logistics company. For this, we can generate routes that have retailers with the positively and negatively correlated shipments on the same routes. The final extension could be not only consider to minimize cost in the distribution plan, we should also consider customer service level, lead time and product quality. For example, in the case study, the company not only wanted to minimize transportation cost, but also wanted milkrun to minimize the damage to the product due to the long haul by using LTL service. Company was willing to pay more money if the customer service level would
increase. For this, a bi-criteria model can be developed considering both the total cost and the customer service level.
REFERENCES


11. Çetinkaya, S., C.-Y. Lee. 2000. ‘Stock replenishment and shipment scheduling for vendor-


APPENDIX

GAMES programming for numerical example:

sets
i cities /pl, xd, n1, n2, n3, n4, n5/
n(i) all retailers / n1, n2, n3, n4, n5/

nx(i) retailers and xd /xd, n1, n2, n3, n4, n5/;
alias(nx, vx);
alias(i, j);
alias(n, u);

parameter
d(i) demand of node i in unit / n1 981

    n2 3727
    n3 986
    n4 2497
    n5 985

/;

parameter

Table dist(i,j) distance between two nodes in miles

    pl   xd   n1   n2   n3   n4   n5

pl  0  803  775  697  486  786  780

xd  803  0  143  218  375  95  80

n1  775  143  0  80  350  55  66
Scalars

ptl per mile cost for one way TL /2.8/

pm per mile cost for milkrun /0.0001/

pltls per mile cost for LTL from pl/20/

pltlxdr per mile cost for LTL from xd/1/;

Parameter

ltlx(i,n) ltl cost from xd to node i;
ltlxr('xd',n)=pltlxdr*dist('xd',n);

ltlpl(i,n) ltl cost from pl to node i;
ltlpl('pl',n)=pltls*dist('pl',n);

Parameter

Q total quantities to be shipped;
Q = sum(i,d(i));

Scalars

H inventory holding cost parameter/0.1/

F delivery frequency /3/

maxvol maxvolume/30000/
maxstops /5/
maxnum/10000000/;

Variables

Y(i) If node i uses ltl from pl
W(i) If node i is on a milkrun
X(i) If node i uses LTL from the cross dock
dock(i) If node i’s demand goes through the cross dock
visit(n,i) if node i’s demand goes through node j;

Binary Variables Y, W, X, dock, visit;

Variables

M(i,j) number of times the milkrun arc is used
OW number of times the one way arc from the plant to the crossdock used;

binary Variables M, OW;

Variables FTL(i,j,n), FOW(n), FLTL(n), FPLTL(n), z;

Positive Variables FTL, FOW, FLTL, FPLTL;

EQUATIONS

cost define objective function
cons1(n)
cons2(n)
cons3(n)
cons4(n)
cons5(n)
cons6(n)
cons7(u)
cons8(nx,nx)
cons9(vx)
*cons10(n)
cons11(n)
cons12(u,n)
cons13(n)
cons14(nx,vx)

cons15
cons16(n,u)
cons17(n)
cons18(n)
cons19(n);

cost.. z =e= pm*sum((nx,vx),dist(nx,vx)*M(nx,vx))+ptl*dist('pl','xd')*OW+sum(n,ltlx('xd',n)*X(n))+sum(n,ltlpl('pl',n)*Y(n))+Q*5.5+0.85*5.5*Q+sum(n,W(n)*50);

cons1(n).. dock(n)+Y(n)=e=1;
cons2(n).. sum((nx),FTL(nx,n,n))=e=d(n)*w(n);
cons3(n).. FLTL(n)=e=d(n)*x(n);
cons4(n).. FPLTL(n)=e= d(n)*y(n);
cons5(n)..sum((u),FTL(u,'xd',n))+sum((u),FTL('xd',u,n))+FOW(n)+FLTL(n)=l=maxnum*dock(n);
cons6(n)..sum((nx),FTL(nx,nx,n))+sum((nx),FTL(nx,nx,n))=e=0;
cons7(n)..sum((nx),M(nx,nx))=l=1;
cons8(nx,vx).. M(nx,vx)+M(vx,nx)=l=1;
cons9(vx).. sum((nx),M(nx,vx))=e= sum((nx),M(nx,vx));
*cons10(n).. sum((nx),M(nx,nx))=e=W(n);
cons11(n).. Y(n)*d(n)+ FOW(n)=e=d(n);
cons12(u,n) .. sum((nx),FTL(nx,u,n)$(ord(u)<>ord(n)))=e=sum((nx),FTL(u,nx,n)$(ord(u)<>ord(n))); 
cons13(n) .. FOW(n)*x(n)*d(n)=e=0;
cons14(nx,vx) .. sum(n,FTL(nx,vx,n))=l= M(nx,vx)* maxvol;
cons15 .. sum((n),FOW(n))=l= ow*maxvol;
cons16(n,u) .. sum((vx),FTL(n,vx,u)$(ord(n)<>ord(u)))=l= visit(n,u)$(ord(u)<>ord(n))*maxnum;
cons17(n) .. sum((nx),FTL("xd",nx,n))=l= visit(n,"xd")*maxnum;
cons18(n) .. sum((u),visit(n,u)$(ord(n)<>ord(u)))=l= maxstops;
cons19(n) .. W(n)+X(n)=e=dock(n);

Model transport /all/ ;
solve transport using mip minimizing  z;

GAMS Programming for case study:

sets
i cities  /pl,xd,n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,n11/
n(i) all retailers  / n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,n11/

nx(i) retailers and xd /xd,n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,n11/ 
alias(nx,vx);
alias(i,j);
alias(n,u);
parameter
d(i) demand of node i in unit / n1 1635
           n2 6212
           n3 1643
           n4 4162
           n5 1642


<table>
<thead>
<tr>
<th></th>
<th>n6</th>
<th>n7</th>
<th>n8</th>
<th>n9</th>
<th>n10</th>
<th>n11</th>
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<td></td>
</tr>
</tbody>
</table>

`parameter` Table dist(i,j) distance between two nodes in miles

<table>
<thead>
<tr>
<th></th>
<th>pl</th>
<th>xd</th>
<th>n1</th>
<th>n2</th>
<th>n3</th>
<th>n4</th>
<th>n5</th>
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<td>775</td>
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<td>803</td>
<td>802</td>
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<td>80</td>
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<td>66</td>
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<td>210</td>
<td>196</td>
<td>26</td>
<td>0</td>
<td>43</td>
<td>239</td>
<td>225</td>
</tr>
</tbody>
</table>
n8  1012  281  270  343  586  251  238  21  43  0  279  268
n9  803  0  143  217  373  95  80  266  239  279  0  10
n10  802  10  140  213  377  92  76  256  225  268  10  0
n11  818  18  160  233  326  112  96  261  270  273  18  27

+  n11
pl  818
xd  18
n1  160
n2  233
n3  326
n4  112
n5  96
n6  261
scalars

ptl per mile cost for one way TL /2.8/

pm  per mile cost for milkrun /2.45/

pltls per mile cost for LTL from pl/1.9/

pltlxn per mile cost for LTL from xd/3.0/;

parameter ltlxd(i,n) ltl cost from xd to node i;

ltlxn('xd',n)=pltlxn*dist('xd',n);

parameter ltlpl(i,n) ltl cost from pl to node i;

ltlpl('pl',n)=pltls*dist('pl',n);

Parameter

Q total quantities to be shipped;

Q= sum(i,d(i));

Scalars

H inventory hodling cost parameter/0.15/

F delivery frequency /5/
Variables

Y(i)  If node i uses LTL from pl
W(i)  If node i is on a milkrun
X(i)  If node i uses LTL from the cross dock
dock(i) If node i's demand goes through the cross dock
visit(n,i) if node i's demand goes through node j;
Binary Variables Y, W, X, dock, visit;

Variables

M(i,j) number of times the milkrun arc is used
OW number of times the one way arc from the plant to the crossdock used;

binary Variables M, OW;

Variables FTL(i,j,n), FOW(n), FLTL(n), FPLTL(n), z;

Positive Variables FTL, FOW, FLTL, FPLTL;

EQUATIONS

cost define objective function
cons1(n)
cons2(n)
cons3(n)
cons4(n)
cons5(n)
cons6(n)
cons7(u)
cons8(nx,nx)
cons9(vx)
*cons10(n)
cons11(n)
cons12(u,n)
cons13(n)
cons14(nx,vx)
cons15
cons16(n,u)
cons17(n)
cons18(n)
cons19(n);

cost..  

z =e=  

pm*sum((nx,vx),dist(nx,vx)*M(nx,vx))+ptl*dist('pl','xd')*OW+sum(n,ltlx('xd',n)*X(n))+sum(n,ltlpl('pl',n)*Y(n))+1.85*Q*5.5*H/365*F+sum(n,W(n)*50);

cons1(n).. dock(n)+Y(n)=e=1;
cons2(n).. sum((nx),FTL(nx,n,n))=e=d(n)*w(n);
cons3(n).. FLTL(n)=e=d(n)*x(n);
cons4(n).. FPLTL(n)=e= d(n)*y(n);
cons5(n)..sum((u),FTL(u,'xd',n))+sum((u),FTL('xd',u,n))+FOW(n)+FLTL(n)=l=maximum*dock(n);
cons6(n)..sum((nx),FTL(nx,nx,n))+sum((nx),FTL(nx,nx,n))=e=0;
cons7(n)..sum((nx),M(n,nx))=l=1;
cons8(nx,vx).. M(nx,vx)+M(vx,nx)=l=1;
cons9(vx).. sum((nx),M(nx,vx))=e= sum((nx),M(nx,vx));
*cons10(n)..<sum((nx),M(n,nx))=e=W(n);
cons11(n).. Y(n)*d(n)+ FOW(n)=e=d(n);
cons12(u,n).. sum((nx),FTL(nx,u,n)$(ord(u)<ord(n)))=e=sum((nx),FTL(u,nx,n)$(ord(u)<>ord(n)));
cons13(n).. FOW(n)-X(n)*d(n)=e=0;
cons14(nx,vx).. sum((n),FTL(nx,vx,n))=l= M(nx,vx)* maxvol;
cons15.. sum((n),FOW(n))=l= ow*maxvol;
cons16(n,u).. sum((vx),FTL(n,vx,u)$(ord(n)<>ord(u)))=l= visit(n,u)$(ord(u)<>ord(n))*maxnum;
cons17(n).sum((nx),FTL("xd",nx,n))=l=visit(n,"xd")*maxnum;
cons18(n).. sum((u),visit(n,u)$(ord(n)<>ord(u)))=l= maxstops;
cons19(n).. W(n)+X(n)=e=dock(n);

Model transport /all/ ;
solve transport using mip minimizing z;