DESIGN, ANALYSIS, AND CHARACTERIZATION OF LEAD MAGNESIUM NIOBATE – LEAD TITANATE (PMN-PT) SINGLE CRYSTAL ENERGY HARVESTERS FOR ROTORCRAFT WIRELESS SENSOR APPLICATIONS

A Thesis in
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by
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Abstract

The aerospace industry is quickly becoming extremely interested in structural health monitoring (SHM) and condition based maintenance (CBM). The current passive approach for aircraft maintenance and damage detection is time consuming and costly. This is especially true when time and resources are spent on inspections that reveal no damage of aircraft components. An active approach to SHM and CBM is highly desired to reduce costs and aircraft down-town. Unfortunately, current active approaches utilize conventional power sources such as batteries or draw from the aircraft’s main power. These methods require their own maintenance and even add large amounts of mass that may be unacceptable. Harvesting energy from aircraft surroundings offers a solution to power sources required by active SHM. Ambient sources include electromagnetic, electrostatic, thermal, and vibrational energy. This thesis focuses on vibrational energy harvesting that uses a relatively new piezoelectric material, Lead Magnesium Niobate – Lead Titanate (PMN-PT) single crystals.

Conventional vibrational energy harvesters use Lead Zirconate Titanate (PZT) ceramic as the active material. More recently single crystals have become an area of high interest due to their better piezoelectric properties. PMN-PT crystals have a higher strain limit, larger charge coefficients, and higher electromechanical coupling factors than conventional PZT ceramics. The combination of piezoelectric properties for PMN-PT results in better performance for vibrational energy harvester applications.

This thesis focuses on endurance, temperature, and performance characterization of PMN-PT single crystal energy harvesters. Endurance testing investigated long-term performance of single crystals under continuous use at lower vibration levels. Thermal testing characterized harvester performance at relatively low and elevated temperatures. Performance
testing examined harvester performance over a range of base acceleration amplitudes. Finite element models were created to predict harvester performance as well as to scale compact harvester devices to various resonance frequencies.

Single crystal harvesters offer high reliability and performance with respect to time. No electrical or mechanical failures were noted over 120 hours of continuous use. Performance testing showed a non-linear trend between voltage/power and base acceleration. At 1.0 g base acceleration, the harvester produced \( \sim 3.0 \text{ mW}_{\text{RMS}} \) which increased to \( \sim 45 \text{ mW}_{\text{RMS}} \) at 5.0 g acceleration. Thermal dependencies of single crystal piezoelectric materials cause loss of voltage at high temperature. Different chemical compositions of PMN-PT (percent PT in the sample) offer improved temperature performance that allows single crystal harvesters to provide adequate power. The energy harvesters are very scalable between 500 and 1500 Hz which allows for a wide range of applications.

A limited study into thermal energy harvesting was completed in order to compare thermal energy harvesters to vibrational energy harvesters. Thermal gradients and heat flow are used to generate electrical energy. Larger temperature gradients produce higher output power. Typically, thermal harvesters have a lower power to volume ratio than vibrational harvesters. Due to their simplistic design, however, a higher power to mass ratio is not unusual.

Overall, single crystal energy harvesters were shown to be reliable producers of usable electrical energy for short and extended periods of time. Performance was reduced at elevated temperature, however, different chemical compositions can abate this effect. A wide range of power was available for varying base accelerations. At all vibration levels, however, the PMN-PT harvester produced over twice the power as a PZT-based device.
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Chapter 1 – Introduction

1.1 Motivation

Applications of structural health monitoring (SHM) and condition based maintenance (CBM) within the aerospace industry have been increasing in demand in recent years. Aircraft of all types are subject to damage in many forms including metallic corrosion, delamination of laminate materials, and most commonly, fatigue-based damage. Some examples of damage on fixed- and rotary-wing aircraft are presented by Campbell [1] who explains that drivetrain/transmission failures were the most common failures for rotorcraft, and that 32% of these were fatigue-related.

Inspecting for the amount and severity of damage on rotorcraft requires scheduled inspections. This passive approach to rotorcraft maintenance does not allow for accurate prediction of fatigue damage and propagation [2], and only aids in finding damage after it has already begun. This approach may be detrimental to the safety of the rotorcraft. For example, if a crack found during routine maintenance is deemed safe for continued use, this damage may still cause catastrophic failure of its host component before the next scheduled maintenance.

Not only does scheduling maintenance at regular intervals introduce the possibility of failure occurring between maintenance, it introduces long down-times for the rotorcraft meaning extended periods of grounding the rotorcraft. This is detrimental to any missions that need to be completed. Also, this type of maintenance requires time and effort from maintenance personnel. Often maintenance is scheduled at intervals that are much shorter than the predicted lifecycle of the component of interest. This may lead to no damage being found in one, or even many, examinations. In this instance, time and resources are “wasted” on unnecessary examinations.
Therefore, part of the motivation behind structural health monitoring is to provide an active approach to damage detection in order to reduce maintenance down-time (which increases rotorcraft availability), and to reduce associated maintenance costs.

An active approach to SHM is desired to combat the problems found with passive damage detection. This allows for maintenance personnel to know the condition of the rotorcraft in near-real time, thus reducing the need for lengthy and possibly unnecessary maintenance procedures. A system of sensors would be embedded around the rotorcraft in order to approach SHM actively. These sensors still need power to operate and to transmit their data. This requires the use of lengthy wiring systems and power supplies, which add weight and complexity to the overall system.

Typically, aircraft power supplies or conventional batteries provide power to SHM components. However, batteries carry with them finite life spans and their own maintenance requirements (such as replacement of old/used batteries) [3, 4, and 5]. Many areas of interest (such as the rotorcraft drivetrain) are very complex and space-limited. Placing batteries in these areas require time-consuming and unnecessary disassembly of the rotorcraft, making maintenance very difficult.

A possible alternative to batteries is to convert ambient energy of the rotorcraft to usable power. Energy harvesting requires much less maintenance than reliance on battery power since its energy output has no finite lifetime. Many energy sources exist including electrostatic, electromagnetic, temperature gradients, and ambient vibrations [6]. Rotorcraft systems will, in particular, see temperature gradients, and vibrational energy under routine operation. These are
mostly in areas surrounding the engine, drivetrain, and gear boxes but also are found on rotor components and throughout the airframe in low frequency vibrations.

This work is focused on harvesting ambient vibrations from the rotorcraft and converting their energy into usable electrical power using single crystal piezoelectric materials.

1.2 Vibration-Based Energy Harvesting

All aircraft release energy through vibrations. However, this energy is often lost through heat dissipation and structural-acoustic sound radiation. Vibrational energy harvesting is a method that captures ambient vibrational energy and converts it to electrical energy. Often, this conversion utilizes a medium known as piezoelectric material. Piezoelectric materials have the property that when deformed, and thus strained, a charge (or voltage) is produced on its surfaces. This is known as the direct piezoelectric effect. Conversely, the converse piezoelectric effect has the property where applying a voltage to a piezoelectric material produces a strain on the material. This effect is useful for actuation purposes. Energy harvesting, however, utilizes the direct piezoelectric effect, illustrated in Figure 1-1.

Figure 1-1. Illustration of the direct piezoelectric effect. When deformed, a piezoelectric material produces a voltage between its surfaces.
Since energy harvesting relies on the direct piezoelectric effect, it is important to maximize the applied strain on the piezo material surfaces. Strain is very design-dependent and is impossible to predict without a known harvester design. Maximum strain in a system is known to occur when the system is driven at its resonance frequency. An energy harvester will therefore produce the most output power when the resonance frequency of the harvester matches the frequency of the host structure’s vibration [7].

A variety of piezoelectric materials exist. As early as 1880, Jacques and Pierre Curie discovered the direct piezoelectric effect in naturally occurring materials such as quartz, tourmaline, zinc blende, boracite, and topaz [8]. Other piezoelectric materials such as Lead Zirconite Titanate (PZT) and Barium Titanate (BaTiO₃) [9] are currently in use for sensing, actuating, and energy harvesting applications. PZT is by far the most commonly used piezoelectric material in today’s devices. PZT offers easy and low cost fabrication, and can be grown to virtually any size and shape [10]. More recent research focuses on a newer type of piezoelectric material known as single crystals. Examples of these include Lead Magnesium Niobate – Lead Titanate (PMN-PT) and Lead Zinc Niobate – Lead Titanate (PZN-PT). Single crystal materials are desirable because of their performance properties in relation to conventional ceramics such as PZT. They offer higher electromechanical coupling factors (k₃₁), lower loss factors (tanδ), and approximately five times larger charge properties (d₃₁) than PZT-based materials [11 and 12].
1.2.1 Comparison between PZT and PMN-PT Piezoelectric Materials

Piezoelectric energy harvesters are operable in different configurations. This work focuses on the 3-1 mode of operation depicted in Figure 1-2. This corresponds to a material producing a voltage in the 3-direction (through the thickness) due to a strain in the 1-direction (in plane). Most piezoelectric properties come with a subscript denoting the operating configuration. For example, $d_{31}$ is the charge coefficient that relates voltage in the 3-direction to strain in the 1-direction.

![Figure 1-2. Depiction of a piezoelectric 3-1 mode of operation. A stress in the 1-direction produces a voltage in the 3-direction.](image)

Piezoelectric charge coefficients relate the induced electric field to the applied stress/induced strain [13]. They carry the symbol $d_{31}$ for strain-based calculations and variables, and $g_{31}$ for stress-based applications. For single crystals, these values are larger than those for PZT ceramics.

Efficiencies of piezoelectric materials are often measured by the electromechanical coupling factor, denoted as $k_{31}$. In energy harvesting applications this value is the ratio of the electrical energy produced to the mechanical energy applied. Since it is impossible to obtain more energy than what is supplied, this value can never exceed one. Like the charge coefficient, this coupling factor is larger for single crystals than it is for PZT.
Single crystals tend to have higher elastic compliances than conventional ceramics. Since the elastic compliance is the inverse of Young’s Modulus, a higher compliance relates to a lower Modulus. Data presented by Ewart [14] show Young’s Modulus for PMN-PT is 75% lower than for Navy Type III PZT.

Ewart also presents strength properties for PMN-PT and a few for PZT. In general, PZT has a higher fracture toughness, flexural strength, and compressive strength than PMN-PT. These still are important mechanical properties that need to be carefully considered when designing energy harvesters.

Since energy harvesters are strain-based devices, the maximum strains of the active materials are important. Conventional PZT ceramics have a strain limit of approximately 500 – 550 με [7 and 15]. Since PMN-PT has higher elastic compliances, it stands to reason the maximum strain would be higher as well. In fact, under an electric field loading, the strain limit is 1400 με [16 and 17]. Also, using the tensile data from reference 14, the maximum strain is calculated as approximately 1600 με.

Many works exist that examine the relationship of stress in a PZT material to the number of cycles to failure. Examples include reference 18 which presents simple fatigue data for a compressively loaded PZT material, reference 19 that introduces temperature dependence on PZT fatigue life, reference 20 that examines effect of poling on PZT fatigue, and reference 21 which shows the effect of different electrode surfaces on fatigue. In contrast, the author has found no works for single crystals with the same analysis.

To the author’s knowledge, temperature dependencies of single crystal piezoelectric properties are not well documented as of this writing. Reference 22, however, does present data
on $k_{31}$ temperature dependence for PMN-PT and PZN-PT single crystals. Both materials show excellent stability of the electromechanical coupling factor for a large temperature range (-40 to ~75 °C for PMN-PT and -40 to ~110 °C for PZN-PT). At these temperature extremes, the phase of the crystalline structure begins to change and thus drastically reduces $k_{31}$. Upon cooling, however, the original value of $k_{31}$ is obtained. The piezoelectric charge factor, $d_{31}$, of PMN-PT is shown to increase overall until the phase transition temperature [23]. More works exist that investigate temperature dependence of piezoelectric properties of PZT than PMN-PT. Examples include references 19, 24, 25, and 26 which, in general, show a decrease in output voltage with an increase in temperature.

1.2.2 Commercially Available Energy Harvesters

The Midé Technology Corporation offers an excellent example of piezoelectric vibrational energy harvesters. The particular appeal of Midé’s device is their marketing for helicopter applications. They offer frequency-tuned versions of their Volture Piezoelectric Vibration Energy Harvesters for frequencies between 40 and 150 Hz as well as untuned versions. A frequency-tuned device is sold for $399.00 and the untuned device is $299.00 at the time of this writing. For oscillations at 50 Hz and a base acceleration level of 1.0 g, the Volture device produces 8 mW of power. All data was taken from the Midé website, reference [27].

KCF Technologies also offers a bandwidth-tuned energy harvester suited for rotating machinery. It operates between 200 and 1600 Hz [28]. This device costs around $500 and produces 3.3 V around 500 Hz. [29].
Perpetuum, Ltd. sells stand-alone electromagnetic vibrational energy harvesters for industrial applications. Tuned versions are available for 50, 60, 100, and 120 Hz. It produces 5 mA of current at 5 V when driven at 0.5 g$_{rms}$. [30].

1.3 Previous Research on PMN-PT Energy Harvesters

Work by Song, et. al. [4] presents data on a PMN-PT energy harvester from design through experimentation and power optimization. However, this work is only focused on performance of the energy harvester with respect to base acceleration and load resistance. It also offers comparisons with various other energy harvesting devices including PZT-based, electromagnetic, and electrostatic energy harvesters. The study also looked at the effect of various resistances of the harvesting circuitry. In this regard, output voltage and current increases with increasing resistance when driven at 0.05 g. Also, the optimal resistances for maximum voltage, current, and power are different from each other. For example, this study shows that maximum output voltage occurs at a high resistance of 1 MΩ, the maximum current rises from a resistance of 5 kΩ, and maximum power appears at 50 kΩ. Therefore, circuit design is dependent on which electrical property is of most importance.

KCF Technologies has furthered the research of PMN-PT energy harvesters by conducting experiments on endurance properties of resonance and output voltage with respect to time, temperature dependence of the resonance and output voltage of the harvester, and a performance experiment of output power with respect to base acceleration [31]. They investigate harvester performance under a 0.5 g base acceleration for a continuous 60 hours. Temperature tests ranged from 20 to 50 °C at 0.25 g vibration amplitude. Finally, they conducted an
amplitude dependence test from 0.1 g to 1.0 g base acceleration. The device presented in reference 31 consists of the energy harvester, associated circuitry, as well as a sensor and wireless transmitter. While this data is excellent for insight into single crystal harvesters, design properties such as strain in the piezoelectric material are not discussed.

1.4 Objectives and Technical Approach

This work focuses on expanding the knowledge base of available PMN-PT energy harvester performance data. This will be accomplished by extending endurance data in terms of time and acceleration level, the temperature tests by increasing the temperature range and amplitude of vibration, and the performance tests by increasing the range of base acceleration. It will also include predicted bond-layer shear stresses and normal strain on the piezoelectric material using a validated finite element model. These design parameters are important for engineers to explore and to design future high-performance energy harvesters.

The overall objective of this work is to understand more clearly the application of single crystal piezoelectric materials to the rotorcraft operational environment. To accomplish this goal, an investigation into the open-circuit voltage and resonance frequency with respect to time will be completed for 120 hours of continuous use driven at 0.5 g and 1.0 g base acceleration, as well as with respect to temperature from 0 to 70 °C. Performance will be characterized by relating the open-circuit voltage and output power to base acceleration up to 5.0 g. In addition, finite element models will be created for prediction of energy harvester performance and design parameters such as stresses and strains. In general, the goal is to provide performance and design data for single crystal energy harvesters to aid future designers.
Previous research [3, 4, and 5] utilizes cantilever beam geometry due to its ease of fabrication and analysis. A piezoelectric patch is placed at the root to maximize the amount of strain on its surfaces. This research uses a thickness-tapered cantilever beam which creates constant strain on the top surface of the beam and the piezoelectric material. One example of this geometry is offered by Mehraeen in reference 32.

1.5 Thesis Outline

This thesis presents measured data and model simulation results for a single crystal PMN-PT energy harvester in order to further the available knowledge on these devices. It begins in Chapter 2 by giving an introduction to piezoelectric theory and describing methods used in the design of a single crystal energy harvester. It presents generalized voltage and power equations for a generic energy harvester, and details the design process of the prototype harvester and the compact harvester. It also will explain the codes and processes used to obtain a design and the preliminary verification of the design with a finite element model. The testing setup and experimental methods are discussed in Chapter 3. It explains the design requirements determined through literature reviews and industry representatives, the programs used for data acquisition, and testing procedures. Endurance, temperature, and performance results for the prototype and compact harvester are presented in Chapter 4. Discussions and short conclusions are included after each section of results. In addition to the experimental results, Chapter 4 will also describe the method of obtaining, and results of, the bond layer shear stress and piezoelectric material surface normal strain. Next, a scalability study on energy harvesters was completed and is explained in Chapter 5. For Chapter 6, the focus changes from vibrational
harvesters to thermal harvesters. A quick background of thermal harvesters is presented along with comparisons of vibrational and thermal harvester performance. Finally, a short and overall conclusions of the single crystal energy harvester project as well as ideas for future work that may be a continuation of this project are offered in Chapter 7.
Chapter 2 – Piezoelectric Theory and Energy Harvester Design

2.1 – Background on the Piezoelectric Effect

In 1880, the Curie brothers discovered that certain naturally occurring materials produce an electrical charge that is proportional to a mechanical stress [33]. This phenomenon is known as the \textit{direct piezoelectric effect} and has a variety of uses. Technical applications utilizing this effect include various sensors, including many accelerometers and common household products such as automotive airbags and ignition sources for handheld lighters and for propane barbecues.

Piezoelectric materials also exhibit the reverse phenomenon called the \textit{converse piezoelectric effect}. This property was first explained through mathematics by Gabriel Lipmann [33] in 1881. This effect corresponds to the deformation produced on the piezoelectric material caused by applying a voltage across its surfaces. Applications involving actuation of structures heavily utilizes this effect. Sonar transducers operate in this fashion to produce and to emit their characteristic “chirp”. Research performed at the Pennsylvania State University even uses piezoelectric materials in an effort to eliminate ice accumulation on the leading edge of rotor blades [34]. Some earlier loudspeakers also rely on the converse effect to produce their sound. Piezoelectric devices are already abundant in and important for everyday life, yet their uses continue to increase.

Both piezoelectric effects can be mathematically explained by the piezoelectric constitutive equations. These relate the stress ($\sigma$) and strain ($\delta$) to the electric field ($E$) and electrical displacement ($D$) through the piezoelectric charge coefficient, $d$. 
Regardless of which effect is desired for operation, piezoelectric constants are important for defining all properties of piezoelectric materials. They define the relation between stress/strain and voltage, the efficiency of energy conversion, and the electric displacement related to applied electric field.

2.2 Piezoelectric Parameters and Constants with Comparisons between PZT and PMN-PT

Piezoelectric properties differ among each material. In general, magnitudes of the coupling properties for single crystals (such as PMN-PT) are larger than those for conventional ceramics (such as PZT). Polarization of a piezoelectric material allows the material to act as an electrical component. Typically polarization is in only one direction such as through the thickness. It realigns the crystalline structure so the average positions of the positive and negative particles in a molecule are not symmetric. Poling is accomplished by imposing a high electric field to a piezoelectric material for a short period of time. This aligns dipole moments generally in the same direction as the applied field. Polarization is the process which piezoelectric materials require in order to operate. All piezoelectric properties have specific directions in which they act meaning the material is orthotropic and typically does not have the same piezoelectric properties in all directions. Subscripts i and j on the constants denote these directions, where i represents the direction of the electrical action, and j represents the direction of mechanical action. For example, $d_{ij}$ could represent the induced electric field in the i-direction.

\[
\delta = \frac{\sigma}{\nu} + dE, \tag{1}
\]

\[
D = \varepsilon E + d\sigma, \tag{2}
\]
produced from a mechanical strain in the j-direction. Figure 2-1 illustrates the directions for typical piezoelectric materials [13].

One of the most common combinations of subscripts is $i = 3$, and $j = 1$ that often corresponds to beam bending. These subscripts refer to a mechanical strain in the 1 direction that produces a voltage in the 3 direction. In this mode, the 1 and 2 directions would be equivalent. Another commonly used combination is $i = 3$, and $j = 3$ which corresponds to electrical and mechanical action in the same direction (usually through the thickness). Finally, materials can be operated in a shear mode that typically corresponds to $i = 1$, and $j = 5$.

2.2.1 Piezoelectric Charge Coefficients

Two parameters exist that describe the relationship between strain/stress and the output voltage. These are the piezoelectric charge constant ($d_{ij}$) and the piezoelectric voltage constant ($g_{ij}$). The charge coefficients specifically relate the polarization (or charge) generated per unit stress applied, or the strain produced by an applied unit electric field, while the voltage coefficients relate the electric field generated by application of mechanical strain, or the strain
produced to a unit electrical displacement [13]. In energy harvester equations, however, the piezoelectric charge coefficient \( d_{ij} \) is used almost exclusively over the voltage coefficient. The charge coefficient and voltage coefficient are related by

\[
g_{ij} = \frac{d_{ij}}{\varepsilon_{ii}^{T}},
\]

where \( \varepsilon_{ii}^{T} \) is the permittivity at constant stress [13]. The units of these coefficients are Volt-Meter per Newton for \( g \), Meters per Volt (or Coulomb per Newton) for \( d \), and Farad per Meter for \( \varepsilon \).

The piezoelectric charge coefficient can be measured by using a product called a Piezoelectric \( d_{33}/d_{31} \) Meter. One such instrument is offered by KCF Technologies of State College, Pennsylvania. However, these instruments are not very common and are expensive. It is possible to determine these values as

\[
d_{ij} = -k_{ij} \sqrt{\varepsilon_{ii}^{T} s_{jj}^{E}},
\]

where \( k_{ij} \) is the electromechanical coupling factor, and \( s_{jj}^{E} \) is the elastic compliance (or the inverse of Young’s Modulus) [35]. Often, however, values for \( d_{31} \) or \( d_{33} \) are given by the supplier of the piezoelectric material.

Typical \( d_{31} \) magnitudes for single crystals are larger than those of conventional ceramics. Kim [36] tabulates and presents \( d_{33} \) data for a variety of conventional ceramics and single crystals. Generally, piezoelectric ceramics exhibit \( d_{33} \) values between 150 and 700 pC/N, while single crystal values lie between 1500 and 3500 pC/N. Ivan [12] compares \( d_{33} \) and \( d_{31} \) values of soft and hard PZT to PMN-PT. This research shows \( d_{31} \) values for soft PZT between -140 to -
280 pC/N and for hard PZT between -42 and -140 pC/N. In comparison, PMN-PT has $d_{31}$ values between -800 and -1500 pC/N. Numerous other examples (references 35, 37, 38) also show larger charge coefficients for single crystal materials than ceramic-based materials.

<table>
<thead>
<tr>
<th></th>
<th>$d_{31}$ (pC/N)</th>
<th>$d_{33}$ (pC/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMN-PT</td>
<td>-800 to -1500</td>
<td>150 to 700</td>
</tr>
<tr>
<td>Soft PZT</td>
<td>-140 to -280</td>
<td></td>
</tr>
<tr>
<td>Hard PZT</td>
<td>-42 to -140</td>
<td>1500 - 3500</td>
</tr>
</tbody>
</table>

### 2.2.2 Electromechanical Coupling Factor

With any electromechanical device, efficiency is important. In piezoelectric theory, the efficiency of energy conversion is denoted by the square of *electromechanical coupling factor*, $k_{ij}$. The efficiency is described as the ratio of the electrical energy produced to the mechanical energy input, or vice versa [39]. Since the coupling factor is a measure of conversion efficiency, this value cannot exceed one and is unitless.

As given in Equation 4, one can calculate the coupling factor knowing the charge coefficient, permittivity, and Young’s Modulus. Another method of calculating the coupling factor uses the resonance and anti-resonance of the piezoelectric material. Equations 5 and 6 [13] respectively show these two methods as,
where \( \omega_n \) refers to the anti-resonance frequency taken at maximum impedance, and \( \omega_m \) refers to the resonance taken at minimum impedance of the bare piezoelectric material.

Two different values of the coupling factor exist. The first, described above, is only acceptable for a lone piece of piezoelectric material while the second is an effective coupling factor for an entire built-up system, denoted as \( k_{sys}^2 \). This factor is calculated easily using the system’s open-circuit (\( \omega_{OC} \)) and short-circuit (\( \omega_{SC} \)) resonances [6],

\[
k_{sys}^2 = \frac{\omega_{OC}^2 - \omega_{SC}^2}{\omega_{OC}^2},
\]

and is lower than the coupling factor of the piezoelectric material itself.

Lesieutre [40] asks an important question regarding the system coupling and electromechanical coupling coefficients. Lesieutre asks if it is possible for a system coupling factor to be greater than that of the active material. The work focuses on explanations of the coupling factor, various methods of measuring it, and a method of increasing the overall system coupling factor. Increases of the system coupling factor of up to 40% occur when using compressive preloads on a clamped-clamped bimorph bender. In general, the coupling factor was shown to increase toward 1.0 as the compressive preload was increased.

The equations found in reference 40 show that the system coupling factors may increase through decreasing the host structure’s stiffness, as well as increasing the frequency separation.
between the open circuit frequency (maximum impedance) and the short circuit frequency (minimum impedance). Increasing the coupling factor of a system increases its efficiency at which the device converts mechanical energy to electrical energy.

Similar to the piezoelectric charge coefficient, the electromechanical coupling factor is larger for single crystals than it is for ceramic materials. Ewart, 2007 [14] presents $k_{33}$ data for PZT8 of 0.65 and 0.91 for PMN-PT. Kim [37] compares a $k_{33}$ PZT-5A coupling factor of 0.71 to a PMN-PT value of 0.92. Also, APC International [13] shows $k_{31}$ values of PZT around 0.35 for their conventional ceramics and 0.55 for their PMN-PT material. Typically, the coupling factor in the 3-3 direction is larger than its counterpart in the 3-1 direction.

<table>
<thead>
<tr>
<th></th>
<th>$k_{31}$</th>
<th>$k_{33}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMN-PT</td>
<td>.55</td>
<td>.92</td>
</tr>
<tr>
<td>PZT8</td>
<td>.35</td>
<td>.65</td>
</tr>
<tr>
<td>PZT-5A</td>
<td>.35</td>
<td>.71</td>
</tr>
</tbody>
</table>

### 2.2.3 Permittivity and Dielectric Constant

Another important property of piezoelectric theory is the permittivity, $\varepsilon_{ii}$. This constant’s first subscript represents the direction of dielectric displacement and the second is the direction of the electric field [13]. Piezo theory defines two types of permittivities. A superscript of T denotes the permittivity at constant stress (meaning mechanically free) and a superscript of S denotes permittivity at constant strain (mechanically clamped) [13, 39]. The units are Farad per Meter, or equivalently, Coulomb per Volt-Meter.
Permittivity is the ability of a material to store electrical energy within an electric field. For energy harvesting applications, a material with high value of permittivity is preferred. A higher permittivity correlates to lower source impedances. A lower impedance is preferred since piezoelectric materials have inherently high impedances [41]. The permittivity of a piezoelectric material relates to the capacitance of the material through,

$$\varepsilon_{33}^T = \frac{C_p z}{xy},$$  \hspace{1cm} (8)

where $C_p$ is the capacitance of the material, and $x$, $y$, and $z$ are the dimensions along those same axes [35] with $z$ the thickness of the crystal.

In addition to permittivity itself, sometimes theory uses a relative permittivity called the dielectric constant, symbolized by $K^T$. This constant is the ratio of the material permittivity ($\varepsilon_{33}^T$) to the constant permittivity of free space ($\varepsilon_0 = 8.85\times10^{-12}$ F/m) and is given as [13]

$$K^T = \frac{\varepsilon_{33}^T}{\varepsilon_0}.$$  \hspace{1cm} (9)

Dielectric constants for PMN-PT single crystals are larger than that of PZT ceramics. Kim [37] describes dielectric constant values of 5500 for PMN-PT and of 2000 for PZT-5A. APC International [13] relays similar values of 5000 for PMN-PT and between 1250 and 4100 for their PZT ceramics.

<table>
<thead>
<tr>
<th>Table 2-3. Dielectric constant comparison between PMN-PT and PZT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>K^T</strong></td>
</tr>
<tr>
<td>PMN-PT</td>
</tr>
<tr>
<td>PZT</td>
</tr>
</tbody>
</table>
2.2.4 Dissipation/Loss Tangent

Vibrating systems always lose energy to heat. In electromechanical systems, some electrical energy is also lost to heat. The dielectric dissipation factor, or the loss tangent, \( \tan(\delta) \), measures the electrical energy lost to heat in a piezoelectric material.

Various methods of explaining damping and losses are present between the electrical and structural engineering disciplines. Loss factor (\( \eta \)), fraction of critical damping (\( \zeta \)), quality factor (\( Q \)), 3dB method, and the loss tangent are just a few. Table 2-4 relates these commonly used values to each other.

<table>
<thead>
<tr>
<th></th>
<th>( \eta )</th>
<th>( \zeta )</th>
<th>( Q )</th>
<th>( \tan(\delta) )</th>
<th>3dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta )</td>
<td>-</td>
<td>2( \zeta )</td>
<td>1/Q</td>
<td>( \tan(\delta) )</td>
<td>( \Delta \omega_{3dB}/\omega_n )</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>( \eta/2 )</td>
<td>-</td>
<td>1/(2Q)</td>
<td>( \tan(\delta)/2 )</td>
<td>( \Delta \omega_{3dB}/(2\omega_n) )</td>
</tr>
<tr>
<td>( Q )</td>
<td>1/( \eta )</td>
<td>1/(2( \zeta ))</td>
<td>-</td>
<td>1/ ( \tan(\delta) )</td>
<td>( \omega_n/\Delta \omega_{3dB} )</td>
</tr>
<tr>
<td>( \tan(\delta) )</td>
<td>( \eta )</td>
<td>2( \zeta )</td>
<td>1/Q</td>
<td>-</td>
<td>( \Delta \omega_{3dB}/\omega_n )</td>
</tr>
</tbody>
</table>

Values of the dissipation loss tangent are very low for single crystals, typically being less than 1%. Kim [36] states the loss tangent of PMN-PT of 0.8% and APC International [13] claims a loss tangent of 0.56%. APC also indicates a range of loss tangents for their PZT materials ranging between 0.35% and 1.4%.

| PMN-PT         | 0.5 to 0.8 |
| PZT            | 0.35 to 1.4 |
2.2.5 Energy Density

The energy per unit volume, or energy density, is the metric energy storage and harvesting devices use for various comparisons [41]. A larger energy density refers to the ability to store more energy in a given volume and is therefore preferred for rotorcraft applications. For piezoelectric energy harvesters, Roundy [41] presents the theoretical maximum energy density equation as

\[ E_{\text{max}} = \frac{\sigma_{Y}^{2}k^{2}}{2Y}, \]  

(10)

where \( \sigma_{Y}^{2} \) denotes the yield stress of the material, \( k \) is the electromechanical coupling factor, and \( Y \) is Young’s Modulus. Inspection of Equation 10 shows that a piezoelectric material with higher electromechanical coupling factors will yield larger energy densities given similar yield strengths and Young’s Modulus.

Using Equation 10, Roundy shows that the theoretical maximum energy density is 335 mJ/cm\(^3\) for a sample of PZN-PT (Lead Zirconate Niobate – Lead Titanate) single crystal, and 35.4 mJ/cm\(^3\) for PZT-5H. Also using Equation 10 with PMN-PT parameters for mode 3-1 from Ewart [14], the theoretical maximum energy density of that specific PMN-PT single crystal is 104 mJ/cm\(^3\).

<table>
<thead>
<tr>
<th>Energy Density</th>
<th>(mJ/cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZN-PT</td>
<td>335</td>
</tr>
<tr>
<td>PMN-PT</td>
<td>104</td>
</tr>
<tr>
<td>PZT-5H</td>
<td>35.4</td>
</tr>
</tbody>
</table>

Table 2-6. Energy Density comparison between single crystals and PZT

21
2.2.6 Phase Transition Temperature and Curie Temperature

Crystalline materials, including piezoelectric materials, have various phases that correspond to different crystalline orientations. According to the IEEE Standard on Piezoelectricity [42], piezoelectric crystals have seven distinct phases. However, for energy harvesting applications, only three are of importance: rhombohedral, tetragonal, and cubic phases. These are the phases present at room as well as higher temperatures, little is known regarding low-temperature phases of these materials. Across the phase transitions, the properties of the material change, sometimes drastically.

The phase transition from a rhombohedral to a tetragonal orientation (R-T) is the first transition for single crystals. Typically these temperatures are below 100 °C, for PMN-PT but may be higher due to chemical composition and their effects are reversible. For instance, changing the percentage of PT in a PMN-PT sample changes the phase transition temperature. TRS Technologies, State College, PA [43] offers three types of PMN-PT crystals. They differ in percentage PT including 28, 30, and 32 percent. As the TRS datasheet shows, as the percentage of PT is decreased, the phase transition temperature increases. For example, a sample with 28% PT has an R-T phase transition temperature of ~95 °C, but for a sample with 32% PT, the temperature is only 75 °C. A second phase transition occurs at higher temperatures from the tetragonal to cubic (T-C) orientations. The transition to cubic structure is a special temperature which is called the Curie Temperature. Above this temperature, the crystalline structure is unable to produce piezoelectric effects. The Curie Temperature is the temperature at which a piezoelectric material depoles [13] and does not regain polarization on its own. Typically these values are less than 200 °C, but change with chemical composition. The TRS samples show Curie Temperatures of 145 °C for PMN-.28PT and 160 °C for PMN-.32PT.
If piezoelectric material is heated and cooled while never passing the Curie temperature, all effects on the properties are reversible [13]. However, once the Curie Temperature is reached, the effects on piezoelectric properties are irreversible, due to depoling, and the material must be poled again.

Phase diagrams in Guo, [44] show the various phases of PMN-PT as functions of temperature and percentage of PT in the sample. The work shows there exists some concentrations of PT where the materials only undergoes a T-C transition, and some that only undergo a R-T transition. Most applications of PMN-PT place the percentage PT within 25 and 35, where there the three main phase transitions exist. It is therefore possible to change the phase transition temperature of a PMN-PT sample by altering the percentage of PT in the sample. Doing so will change the piezoelectric properties and cause them to differ among various compositions.

<table>
<thead>
<tr>
<th>Table 2-7. Phase transition temperature comparison between different compositions of PMN-PT and PZT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase Transition (°C)</td>
</tr>
<tr>
<td>Curie Temperature (°C)</td>
</tr>
</tbody>
</table>

2.2.7 Strength Property Comparisons between PZT and PMN-PT

Generally, Young’s Modulus values for single crystal PMN-PT are lower than PZT ceramics. Measurements from Ewart [14] on a PMN-PT sample poled through the thickness and operated in the 31 mode show a Young’s Modulus of 18 GPa. That compares to a value of 74 GPa for a PZT-8 sample. Other references such as Ivan [12], Lee [35], and APC International [13] present larger Young’s Moduli (or a smaller elastic compliance) for PZT than PMN-PT.
Ewart also presents strength properties for PMN-PT and a few for PZT. A poled sample of PMN-PT has a compressive strength of 740 MPa, a fracture toughness of 0.23 Mpa-m$^{1/2}$, flexural strength of 62 MPa, and a tensile strength of 30 MPa. An unpoled PMN-PT sample shows a compressive strength of 725 MPa, a fracture toughness of 0.25 Mpa-m$^{1/2}$, flexural strength of 83 MPa, and a tensile strength of 35 MPa. In comparison, a poled sample of PZT has a fracture toughness of 0.98 Mpa-m$^{1/2}$, and a flexural strength of 108 MPa while an unpoled sample of PZT shows a compressive strength of 1310 MPa.

Strains within a piezoelectric material are important. Energy harvesters utilize the strain to produce a voltage showing a higher strain yields higher voltage. However, all materials have a strain limit that causes failure. Single crystals have a higher strain limit than PZT. Kumar, [16] and Zhao [17] present PMN-PT strain limits of 1400 με. In addition, data from Ewart [14] yields a calculated maximum strain of 1600 με. Alternatively Roundy [7] and Fett [15] give a maximum strain limit of 500 to 550 με for PZT ceramics.

<table>
<thead>
<tr>
<th></th>
<th>Maximum Strain (με)</th>
<th>Tensile Strength (MPa)</th>
<th>Compressive Strength (MPa)</th>
<th>Young’s Modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMN-PT</td>
<td>1400 to 1600</td>
<td>30</td>
<td>740</td>
<td>18</td>
</tr>
<tr>
<td>PZT</td>
<td>500 to 550</td>
<td>-</td>
<td>1310</td>
<td>74</td>
</tr>
</tbody>
</table>

### 2.3 Temperature Dependence of Piezoelectric and Elastic Properties

The charge coefficient displays dependence on ambient temperature. Previous research by Kim [26] and Wang [25] show the magnitude of the charge coefficient of PZT ceramics increases with increasing temperature. Kim presents data from ~25 to 150 °C that shows $d_{31}$ for soft PZT ceramics increasing almost 50% and hard ceramics staying relatively constant. Wang
presents data from -120 to 80 °C that shows $d_{31}$ increasing 150% for PZT-5H. However, work by Schulz [24] indicates the piezoelectric properties return when cooled to room temperature, provided the tests do not exceed the Curie Temperature. Sulc [23] investigates the temperature dependence of PMN-PT single crystals. His research also shows an increase in charge coefficient magnitude of ~300% from 20 to 90 °C where a crystalline phase transition occurs. At this point, the magnitude drastically reduces.

Research by Kim [26] shows the coupling coefficient of soft PZT ceramics decreases by ~66% and hard PZT ceramics decreases by ~45% over a temperature range from 25 to 150 °C. In contrast, work by Benayad [22] shows the coupling factor of PZN-PT and PMN-PT single crystals as relatively constant with temperature until the phase transition temperature. At this temperature, the coupling factor significantly reduces. Upon cooling, the original values are regained.

As shown with the previous piezoelectric constants, the dielectric constant also exhibits temperature dependence. Kim [26] shows the dielectric constant of soft PZT increases more rapidly as temperature increases while hard ceramics stay relatively constant. The dielectric constant increases ~275% from 25 to 150 °C. Wang [25] also presents the same rapid increase of the dielectric constant for PZT-5H. This work illustrates a 300% increase from -120 to 150 °C. Luo [45] presents data on the dielectric constant of PMN-0.29PT with respect to temperature from 30 to over 200 °C. The dielectric constant increases until ~80 °C, where the magnitude drops ~50% from its high value. It then begins to increase more rapidly until around 125 °C, where the magnitude sharply increases 250% before it decreases up to 200 °C. Luo’s research also explores the effect of Mn doping (1% and 3%) on the dielectric constant and its dependence
on temperature. The trends remain the same between the Mn-doped samples, however, magnitudes of the Mn-doped samples’ dielectric constants are lower than a pure sample.

The temperature dependence of the piezoelectric properties affects the output voltage and power output. In general, more research is present that investigates PZT-based harvesters than single crystal harvesters. However, all data exhibit the same trend: a decrease in output voltage and power with an increase in temperature. Work by Schulz [24] suggests the output voltage of a PZT harvester decreases nearly 100% from 10 to 260 °C. Over a more usable range of 10 to 70 °C, the voltage decreases 20%.

The temperature dependencies of the charge coefficient, dielectric constant, and coupling factor shown in [26] relate as expected through Equation 5. The overall coupling factor decreases as temperature increases. Although the magnitude of the charge coefficient increases, the dielectric constant increases more rapidly, thus contributing to the overall reduction of the coupling factor. An increase in the dielectric constant (and therefore a decrease in the coupling factor) is the main reason power output decreases with increasing temperature [26]. The soft PZT experiments show a 46% loss in power with a 125 °C increase and a 16% loss for hard PZT.

The elastic properties of piezoelectric materials are also subject to change with temperature. Zhang [46] presents data on the elastic compliance and stiffness for a PMN-PZT single crystal sample at three discrete temperatures: 30, 50, and 100 °C. As the ambient temperature increases, so does the elastic compliance. This relates to a decrease in Young’s Modulus. Young’s Modulus for the 1-1 direction decreases 27.4% from 30 to 100 °C while the stiffness remains relatively constant.
2.3.1 Piezoelectric Properties Across the Phase Transition and Curie Temperatures

The piezoelectric and elastic properties of piezoelectric materials change when the material passes through the phase transition. Section 2.3 discussed temperature dependencies of material properties, however, did not detail what happens at the phase transition. This section will explain what changes the material properties undergo through phase transitions.

Sulc [23] investigates the relationship between the piezoelectric charge coefficients $d_{33}$ and $d_{31}$ and temperature from 20 to 130 °C of a PMN-PT single crystal material. This temperature range includes a phase transition from rhombohedral to tetragonal (R-T) at 101 °C and a transition between tetragonal and cubic at 124 °C. Sulc found that the piezoelectric coefficients are higher in the rhombohedral phase than they are in the tetragonal phase. When crossed from the tetragonal into the cubic phase (T-C), the piezoelectric coefficients become very low, in fact they are near zero. This is in agreement with the fact that the cubic phase is where the material crystalline domains shift and the material loses its piezoelectric properties.

<table>
<thead>
<tr>
<th>Temperature Range (°C)</th>
<th>$d_{31}$</th>
<th>$k_{31}$</th>
<th>$K^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMN-PT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-20 to 20</td>
<td>-</td>
<td>Constant</td>
<td>-</td>
</tr>
<tr>
<td>20 to 70</td>
<td>+43%</td>
<td>+2.5%</td>
<td>+55%</td>
</tr>
<tr>
<td>70 to 90</td>
<td>+60%</td>
<td>-</td>
<td>+72%</td>
</tr>
<tr>
<td>-120 to 20</td>
<td>+60%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Soft PZT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 to 70</td>
<td>+12.5%</td>
<td>-20%</td>
<td>+17%</td>
</tr>
<tr>
<td>70 to 150</td>
<td>+20%</td>
<td>-50%</td>
<td>+130%</td>
</tr>
<tr>
<td>-120 to 20</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hard PZT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 to 70</td>
<td>Constant</td>
<td>-19%</td>
<td>+2%</td>
</tr>
<tr>
<td>70 to 150</td>
<td>Constant</td>
<td>-31%</td>
<td>+2%</td>
</tr>
</tbody>
</table>
Upon approaching the R-T phase transition, the piezoelectric coefficients exhibit a large increase, of approximately 300% over their room temperature values.

As discussed earlier, the electromechanical coupling factor ($k_{31}$) is constant with temperature until the rhombohedral to tetragonal phase transition occurs. Benayad [22] reports this effect on PMN-0.33PT and PZN-0.045PT single crystals with phase transitions near 70 and 110 °C respectively. At these temperatures, the coupling factors respectively decrease ~40 and 31 percent from their room temperature values. Benayad also confirms that upon cooling of the material back below the phase transition temperature the original values are regained, however, this change is not immediate. The return temperature for PMN-PT is 20 °C and for PZN-PT is near 80 °C.

Kumar [16] presents data on the dielectric constant through the Curie Temperature for a sample of PMN-0.32PT. The dielectric constant appears to have little change through the rhombohedral to tetragonal phase transition (around 75 °C for 32% PT). However, there is a large increase when approaching the Curie Temperature (~190 °C for this sample) followed by a large decrease. The dielectric constant increases 600% from room temperature for the sample when measured at 100 Hz. When measured at 1 MHz, the increase is 400%.

Kumar also illustrates the dependence of the loss tangent with temperature and what happens as the Curie Temperature is approached. Similarly to the other piezoelectric properties, the loss tangent increases with temperature and exhibits a larger increase at the Curie Temperature. Measurements taken at 100 Hz show approximately a 100% increase while measurements at 1 MHz show a 500% increase with little effect at the R-T phase transition.
Research by Cao [47] presents data for elastic and piezoelectric properties of PMN-PT for varying percentages of PT. His work shows a significant increase in the elastic compliance for a 42% PT sample over a 33% PT sample. As shown in Section 2.2.6, increasing the PT content in the composition decreases the R-T phase transition temperature. Therefore, it stands to reason the higher PT content places the crystalline structure in the tetragonal phase at room temperature. The elastic compliance is higher at the 42% PT composition than at 33% PT. Thus, Young’s Modulus decreases as the material passes through the R-T phase transition.

<table>
<thead>
<tr>
<th>Material</th>
<th>-d_{31} (pC/N)</th>
<th>d_{33} (pC/N)</th>
<th>K'</th>
<th>tanδ</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMN-.29PT</td>
<td>500</td>
<td>1000</td>
<td>.78</td>
<td>2500</td>
</tr>
<tr>
<td>20 °C</td>
<td></td>
<td></td>
<td></td>
<td>.025</td>
</tr>
<tr>
<td>PMN-.29PT</td>
<td>700</td>
<td>1250</td>
<td>.8</td>
<td>3000</td>
</tr>
<tr>
<td>50 °C</td>
<td></td>
<td></td>
<td></td>
<td>.025</td>
</tr>
<tr>
<td>PMN-.33PT</td>
<td>2000</td>
<td>3750</td>
<td>.55</td>
<td>3500</td>
</tr>
<tr>
<td>100 °C</td>
<td>0</td>
<td>0</td>
<td></td>
<td>.03</td>
</tr>
<tr>
<td>PMN-.32PT</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>2750</td>
</tr>
<tr>
<td>130 °C</td>
<td>-</td>
<td>-</td>
<td></td>
<td>.03</td>
</tr>
<tr>
<td>PMN-.32PT</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5000</td>
</tr>
<tr>
<td>150 °C</td>
<td></td>
<td></td>
<td></td>
<td>.035</td>
</tr>
</tbody>
</table>

**2.4 Comparison of Available Energy Harvesters**

A variety of research presents data on representative vibrational energy harvesters using PZT and PMN-PT materials. Two examples of PZT-based harvesters are found in Roundy [3] and Glynne-Jones [48]. Roundy shows that a PZT energy harvester occupying 1 cm³ produces 375 μW of power from a 0.25 g base acceleration at 120 Hz. The Glynne-Jones harvester occupies 0.125 cm³ producing 2 μW of power at 0.23 g base acceleration at 80.1 Hz. This correlates to power densities of 0.375 mW/cm³ and .016 mW/cm³ respectively.
One example of PMN-PT based harvesters is shown in [4] where the harvester is 26.4 cm$^3$ and produces 19 mW of power at 0.2 g base acceleration oscillating at 60 Hz. KCF Technologies, State College, PA, also presents data for a PMN-PT harvester. Their device size is 8.19 cm$^3$ and produces 6.4 mW of power at 700 Hz driven at 1.0 g base acceleration [31]. The power densities here are 0.72 mW/cm$^3$ and 0.78 mW/cm$^3$. Overall, the power densities of PMN-PT based devices are larger than the PZT devices. Note that lower frequencies would require larger masses (or less stiff host structures) to tune. Therefore, strain in the piezoelectric device with a lower frequency would be higher than in a device with a higher resonance frequency. This may inflate the power densities of the low frequency devices. More energy harvesters and device properties are found in reference [4].

Mass, and power per unit mass, are other key properties for energy harvesters. Since single crystals offer better piezoelectric properties, it is possible to create smaller single crystal devices to produce the same power as a larger PZT device. The two PMN-PT energy harvesters from references 4 and 31 have masses of 198 grams and 37 grams, respectively. This calculates to power mass densities of 0.096 mW/gram and 0.173 mW/gram.

The Midé Technology Corporation sells an energy harvester commercially. One can purchase their device pre-tuned between 40 and 150 Hz for $399, or their untuned device for $299. The harvester is a complete package consisting of the harvester structure, PZT active material, and associated harvesting circuitry. It has dimensions of 9.2 x 4.45 x 0.99 centimeters, giving a volume of 40.5 cm$^3$ and has a mass of 85 grams. For oscillations at 50 Hz and a base acceleration level of 1.0 g, the Volture device produces 8 mW of power. This shows an energy density of 0.197 mW/cm$^3$ and a mass power density of 0.094 mW/gram. All data was taken from the Midé website, reference 27.
KCF Technologies also offers a commercial energy harvester. Their device, the VH-1 [28] is an untuned harvester designed for rotating machinery that operates between 200 and 1600 Hz. The dimensions of the entire enclosure are 48.5 x 27.2 x 27.1mm yielding a volume of 35.7 cm$^3$. It has a mass of 134 grams and an operational temperature of -40 to 80 °C. The VH-1 energy harvester is tuned roughly to 500 Hz and produces approximately 3.3 V. It costs around $500 and is used mostly as a demonstration of KCF’s capabilities [29].

In addition, Perpetuum Ltd. produces various configurations of their energy harvester, the PMG FSH. They offer devices that are not hazardous and that are hazardous certified for industrial applications. Four tuned versions are available at 50, 60, 100, and 120 Hz. It is fairly large at 63.3mm tall with a diameter of 68mm and heavy with a mass of 1.075 kg. Operational temperature is between -40 and 85 °C. At 0.5 g$_{rms}$ the harvester produces ~5 mA of current at less than 5 Volts. All data is from the PGM FSH Data Sheet [30]. Table 2-11 summarizes the available energy harvesters.

<table>
<thead>
<tr>
<th>Reference #</th>
<th>Type</th>
<th>Frequency (Hz)</th>
<th>Acceleration (g)</th>
<th>Power (mW)</th>
<th>Volume (cm$^3$)</th>
<th>Power Density (mW/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>PZT</td>
<td>120</td>
<td>0.25</td>
<td>0.375</td>
<td>1</td>
<td>0.375</td>
</tr>
<tr>
<td>27</td>
<td>PZT</td>
<td>50</td>
<td>1.0</td>
<td>8</td>
<td>40.5</td>
<td>0.094</td>
</tr>
<tr>
<td>30</td>
<td>PZT</td>
<td>500</td>
<td>0.5</td>
<td>25</td>
<td>856</td>
<td>0.03</td>
</tr>
<tr>
<td>48</td>
<td>PZT</td>
<td>80</td>
<td>0.23</td>
<td>.002</td>
<td>0.125</td>
<td>0.016</td>
</tr>
<tr>
<td>4</td>
<td>PMN-PT</td>
<td>60</td>
<td>0.2</td>
<td>19</td>
<td>26.4</td>
<td>0.72</td>
</tr>
<tr>
<td>31</td>
<td>PMN-PT</td>
<td>700</td>
<td>1.0</td>
<td>6.4</td>
<td>8.19</td>
<td>0.78</td>
</tr>
</tbody>
</table>
2.5 PMN-PT Energy Harvester Available Data

To date, few works exist that focus on single crystal energy harvester properties such as endurance and strength, and temperature effects on the harvester. Research by Song [4] delves into single crystal energy harvester performance that produces a variety of data on output voltage and power with respect to frequency and vibration acceleration. It also shows the effect of various load resistances on the energy harvester. In addition to the novel research, the author compares their results with a variety of other types of energy harvesters including PZT-based, single crystal based, electromagnetic, and electrostatic. Song furthers the available data by measuring his own data and compiling other data on power densities, device volume, and information on the vibration environment.

KCF Technologies [31] also expands available knowledge on single crystal harvesters using a compact, commercially-inspired design. They measure energy harvester performance in three areas: endurance of the harvester, temperature dependencies of the system resonance frequency and output voltage, and performance of the harvester based on output power at varying base accelerations. Their endurance test relates the open-circuit voltage and resonance frequency of their energy harvester to 60 hours of continuous use driven sinusoidally at the resonance frequency with 0.5 g base acceleration. There is a slight start-up transience in the first few hours; however, both voltage and resonance exhibit very stable behavior at approximately 680 Hz and 25 volts. KCF also examines temperature dependencies of resonance and voltage from 25 to 50 °C at 0.25 g base acceleration. Over this range, resonance drops from 700 Hz to 690 Hz (loss of 1.4%) and open-circuit voltage drops from ~7.5 volts to 6 volts (a 21.4% decrease). However, the voltage does not drop continuously. There are relative increases in voltage at 25 °C and 35 °C. Output power of the harvester increases steadily from near 0 mW at
0.1 g acceleration to ~6.4 mW at 1.0 g acceleration. This increase is nonlinear and tends to increase more slowly with increasing base acceleration when driven at resonance.

Both of these sources offer outstanding information for single crystal energy harvesters, yet they omit stress and strain measurements/predictions in the bond layer and piezoelectric material, which are critical pieces of information for the design of new energy harvesters.

2.6 Vibrational Energy Harvester Electromechanical Model

Mechanical structures and electrical systems have constitutive equations, such as Hooke’s Law and Gauss’s Law. Energy harvesters combine mechanical and electrical systems which therefore have coupled constitutive equations. The coupling occurs by adding an electrical term to Hooke’s Law and by adding a mechanical term to Gauss’s Law. Equations 1 and 2 (reproduced here as Equations 11 and 12) show the constitutive equations for a piezoelectric energy harvester [41],

\[
\delta = \frac{\sigma}{\gamma} + dE, \tag{11}
\]

\[
D = \varepsilon E + d\sigma, \tag{12}
\]

where \(\delta\) and \(\sigma\) are the mechanical strain and stress, \(E\) is the electric field, \(D\) is electrical displacement (or charge density), and here, \(\varepsilon\) is the dielectric constant. From these equations, it is easy to see when no piezoelectric materials exist in the system (i.e. \(d = 0\)), the equations reduce to the well-known mechanical and electrical constitutive equations.
Roundy [41] offers derivations for the output voltage and power using the coupled equations. First, he offers a simple relationship between the open circuit voltage and mechanical stress in a piezoelectric energy harvester as

\[ V_{OC} = \frac{-dt_c}{\varepsilon} \sigma, \]  

with \( d \) being the piezoelectric constant, \( t_c \) the thickness of the material, and \( \varepsilon \) the dielectric constant.

He then analogizes the system to an equivalent circuit for the mechanical system coupled to an electrical system (for the piezo and output voltage) by a transformer. Through these methods, Roundy develops an analytical representation of open circuit voltage rate of change for a cantilever bimorph energy harvester,

\[ \dot{V} = \frac{-Y_c d t_c}{\varepsilon} \dot{\delta}, \]  

where, \( t_c \) is the thickness, \( Y_c \) is the Young’s Modulus of the piezoelectric material, and \( \dot{\delta} \) is the time rate of change of strain. When using energy harvesters, a resistive load (in the form of sensors perhaps) is attached. The resulting change in the voltage rate expression is minimal becoming,

\[ \dot{V} = \frac{-Y_c d t_c}{\varepsilon} \dot{\delta} - \frac{1}{RC_p} V, \]  

with \( V \) being the output voltage, \( R \) the load resistance, and \( C_p \) the piezoelectric capacitance. However, this form of the equations is not helpful as it contains a time derivative of voltage.
Through Laplace transformations and assuming the energy harvester is driven at the resonance frequency, an expression for output voltage with a simple resistive load develops.

\[
V = \frac{-\gamma_c dt c b^*}{2 \zeta \omega^2 + \left[ \omega^2 k^2 + \frac{2 \zeta \omega}{RC_p} \right]} A_{in},
\]

where \(\zeta\) is the damping ratio, \(\omega\) is the driving (and resonance) frequency, and \(A_{in}\) are acceleration terms from the Laplace transform of the input vibrations. The variable \(b^*\) is what Roundy calls a “geometric constant” that relates bending stress to the force at the beam's end. For a cantilever bimorph harvester, the \(b^*\) constant is

\[
b^* = \frac{3b (2l_b + l_m - l_c)}{l_b^2 \left( 2l_b + \frac{3}{2}l_m \right)}.
\]

This equation creates a geometric factor that considers the dimensions of the beam and piezoelectric material. In it, \(b\) is the distance between the neutral axes of the beam and of the piezoelectric material, \(l_b\) is the beam length to the beginning of the tip mass, \(l_m\) is the length of the tip mass, and \(l_c\) is the length of the piezoelectric material. Figure 2-2 illustrates the definition of each dimension (adapted from Roundy [41]).

![Figure 2-2. Illustration of beam and piezo dimensions for Roundy's voltage and power derivations. Note the piezo does not have to have the same length as the beam.](image)
From basic electronic theory, usable power delivered to a resistive load is simply $V^2/R$. Therefore, squaring Equation 16 and dividing by the load resistance, an expression for theoretical power develops as,

$$P = \frac{1}{\omega^2} \frac{RC_p \left( \frac{ycdb^*}{\varepsilon} \right)^2}{(4\zeta^2 + k^4)(RCP_\omega)^2 + 4\zeta k^2 (RCP_\omega) + 2\zeta^2 A_{in}^2}.$$  

(18)

Simple optimization of load resistance occurs when the derivative of Equation 18 equals 0 while solving for $R$. Doing so obtains the optimized load resistance for the energy harvester,

$$R_{opt} = \frac{1}{\omega C_p} \frac{2\zeta}{\sqrt{4\zeta^2 + k^4}}.$$  

(19)

Optimizing the load resistance is not the only power optimization technique. In terms of the mechanical system, it is possible to optimize the design parameters using volume, weight, or power constraints [41]. Creating self-tuning devices or devices that have wide resonance bandwidth allows the energy harvester to adapt to the vibration environment and achieve higher power levels [7]. Electrically, creating circuitry whose impedance matches that of the mechanical system is an efficient method of power optimization [49].

2.7 Design of a Single Crystal Energy Harvester

The present work focused on two energy harvesters: the first was a prototype harvester that was fabricated for ease of handling and preliminary testing, and the second was a compact, commercially-inspired harvester that more accurately represents a finalized device. Both
harvesters initially were designed using a finite element code written in Matlab (Appendix A) using the Euler beam assumptions. The compact harvester then went through detailed design using Abaqus FEA software to accurately achieve the desired design parameters.

The prototype harvester design was only on a tapered cantilever beam and used bulky clamps and mounting fixtures. The compact design took into account the need to eliminate bulky fixtures and replaced these with a harvester frame system.

A few constraints were imposed when designing each energy harvester. First, a maximum strain limit on the piezoelectric material of 300 με at 3.0 g was selected for the lab asset. At the time of initial design, strain limits for PMN-PT were unknown. However, Roundy [7] gave a limit of 500 με for PZT. Thus, 300 με was chosen for the maximum strain. A strain limit of 1200 με at 1.0 g acceleration was imposed when designing the compact harvester. Also, a resonance frequency of 800 Hz was used to correspond to typical rotorcraft drivetrain and gear mesh frequencies. Finally, in order to increase the output voltage and power, constant strain across the piezoelectric material was desired. This required a thickness-tapered beam to achieve. Also, size considerations were taken into account. For example, Song [4] used a rather lengthy 60 mm beam. This work constrained the length to 40 mm for the prototype design and 20 mm for the compact harvester. Table 2-12 condenses the design constraints for the energy harvesters.

<table>
<thead>
<tr>
<th></th>
<th>Prototype Harvester</th>
<th>Compact Harvester</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Acceleration (g)</td>
<td>3.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Strain Limit (με)</td>
<td>300</td>
<td>1200</td>
</tr>
<tr>
<td>Length limit (mm)</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>800</td>
<td>800</td>
</tr>
</tbody>
</table>

Table 2-12. Comparison of prototype and compact energy harvester design constraints.
2.7.1 Initial Design of Prototype Harvester using Matlab Code

The analysis was iterated manually using the tip mass, root thickness, ending thickness, and beam length as variables. A beam width of 20 mm was chosen for ease of handling and fabrication. Thus, a piezoelectric patch 20 mm long and wide by 1.5 mm thick was used. The analysis began by setting the beam length to 40 mm and arbitrarily choosing a tip mass. Using these values and the aforementioned constraints, a root thickness was determined to obtain 300 με at the root by iterating the thickness until strain was within a particular percent error of the design strain. Next, the value of the tip thickness was manually varied until a constant strain profile was achieved on the top of the piezoelectric surface. Finally, the resonance frequency for a variety of masses was calculated for these beam dimensions. The mass that gives 800 Hz was then chosen as the new tip mass. This process was repeated changing beam length, tip mass, and thicknesses until a reasonable design was obtained as is illustrated in Figure 2-3.
Figure 2-3. Flowchart illustrating iterative method for preliminary energy harvester design.
The final dimensions for the prototype harvester were as follows. The beam was 40mm long, 20mm wide, and had a tapered thickness beginning at 9.4mm and ending at 6mm. The tip mass to give 800 Hz then was 64 grams. The active material was a PMN-.32PT patch grown by APC International of Mackeyville, PA. Table 2-13 shows the piezoelectric properties of the crystal. Figure 2-4 illustrates the dimensions of the beam, and Figure 2-5 displays a CAD representation of the energy harvester with associated clamping and mounting fixtures with the final fabricated harvester.

Table 2-13. Piezoelectric and elastic properties of American Peizo's PMN-0.32PT single crystal material.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>8200</td>
</tr>
<tr>
<td>Coupling Factor, k₃₁</td>
<td>.62</td>
</tr>
<tr>
<td>Elastic Compliance, S₁₁ₑ (pm²/N)</td>
<td>50.50</td>
</tr>
<tr>
<td>Piezoelectric Constant, d₃₁ (pC/N)</td>
<td>-920</td>
</tr>
<tr>
<td>Loss Tangent, Tanδ (%)</td>
<td>.42</td>
</tr>
</tbody>
</table>

Figure 2-4. Prototype energy harvester beam showing dimensions of the usable beam portion.
A compact harvester was designed as a more representative device for rotorcraft gearbox applications. The preliminary design of the harvester utilized the same Matlab code to obtain a starting point for detailed design. Most of the design work was completed in Abaqus CAE finite element software. The compact design placed a beam length constraint of 20mm and width constraint of 10mm using a piezoelectric material (from APC International) of 10x10x0.5 mm. This analysis gave preliminary beam dimensions of 20 mm long, 10 mm wide, and included a taper from 3.3 mm to 1.2 mm with a tip mass of 5.2 grams to tune to 800 Hz.

### 2.7.1.1 Validation of Matlab Code

A validation of the finite element Matlab code was conducted against a known analytical solution for a simple cantilever beam with a concentrated tip mass. The parameters for the validation beam were dimensions of 30x20x5 mm, Young’s Modulus of 69 GPa, density of 2750 kg/m³ (an aluminum beam), and a tip mass of 50 grams. The Matlab code used in this work used
500 elements and predicted a resonance frequency of 854 Hz. This was compared to a finite element code given in a course in the Penn State Graduate Program for Aerospace [50], which gave a frequency of 859 Hz. This correlates to a 0.56% error. The tip mass was also removed for comparison to a known analytical solution. The written Matlab code produced 4606 Hz and an analytical solution from Blevins [51] yields 4495 Hz. This is a 2.5% error. Therefore, the Matlab code was considered to be a valid analysis for preliminary design.

2.7.2 Detailed Design of the Compact Harvester

Since the compact energy harvester did not include large clamps to ensure a cantilever condition, a commercially available finite element program was desired to accurately predict the resonance of the system. The chosen design was a tapered cantilever beam that was attached to a wall of an encompassing frame. The beam and frame were one piece to reduce effects of energy losses which will decrease mechanical damping and increase overall performance. Like the preliminary design, the detailed design was iterative. The main area of focus, however, was the tip mass. It was changed to accurately predict the resonance including the effects from the frame. The final tip mass needed was 6.1 grams. Figure 2-6 shows a CAD rendering of the compact energy harvester and Figure 2-7 shows the fully fabricated device. To closely replicate a “single package” energy harvester, space for rectifying circuitry was designed into the device.
Figure 2-6. CAD Illustration of the compact harvester. The beam is 20x10x3.3 to 1.2 mm in dimension with a 6.1 gram tip mass. It is entirely encompassed by the harvester frame.

Figure 2-7. Compact energy harvester without covers illustrating the beam, piezoelectric material, and harvesting circuitry. An accelerometer shows the scale of the device.
Since this device was supposed to emulate a true harvester design, a cover for the device was fabricated and is illustrated in Figure 2-8. For preliminary research, sheet metal was screwed into the frame in order to add additional stiffness and allow easy assembly/disassembly. The covers included a hole for access to the circuitry and a hole for testing using a laser vibrometer. With circuitry and covers, the whole system was only 40 grams (0.09 lbs) and was 18 cm³.

![Compact energy harvester with sheet metal covers](image)

**Figure 2-8.** Compact energy harvester with sheet metal covers. The hole on top is for the laser vibrometer to use for testing, and the hole on the side exposes the circuitry for measurements.
2.7.2.1 Compact Energy Harvester Circuitry

Energy harvester circuitry is relatively simple. The oscillatory behavior of a vibrational energy harvester produces an alternating voltage and current. Therefore, to be considered for use in powering sensors and other devices, the voltage must be rectified to a direct voltage and current. A small capacitor (10 μF) was included to smooth the DC signal after rectification. The green area in Figure 2-6 is the allotted space for the circuitry. Figure 2-9 shows the circuit diagram for the harvesting circuitry. The final circuitry is illustrated in Figure 2-10 and was only ~13 mm long. A load resistance could be easily attached to the circuitry in order to determine output powers. Electrical circuitry optimization was not pursued in this work but is an area for future research.

![Diagram of AC/DC Rectifier and Capacitor](image)

*Figure 2-9. Circuit diagram for energy harvesting circuitry showing the AC/DC rectifier, capacitor, and load resistance.*

![Compact Circuitry](image)

*Figure 2-10. Energy harvesting circuitry consisting of an AC/DC rectifier and a small capacitor.*
Chapter 3 – Test Setup and Experimental Methods

3.1 Testing Requirements and Criteria

Measured data from industry [52] were collected for a rotorcraft tail rotor gearbox to begin compiling test requirements for the energy harvester. These data show an average vibration level of 2.5 g driven between 1600 and 1800 Hz on the gearbox. They also report that the lifetime requirement for a rotorcraft component is up to 10,000 hours of non-failing operation for an airframe application. Survivability qualification requirements were also included from Military Standard 810G [53]. This standard is used for worst-case environment testing during component flight qualifications. Typically, the qualification standard is 2500 hours of total testing time, however, this is difficult to achieve. It aggressively time compresses the testing from 2500 hours to 12 hours (4 hours per axis) of sinusoidally-driven vibrations using a fatigue exponent of \( m = 6 \). Therefore the time compression is a method of which the amplitude of testing is increased in order to decrease the time to a more reasonable value. The standard allows for the calculation of various test parameters based on the specific application. For 800 Hz (a typical gear mesh frequency), the time compressed acceleration is 13 g while the real-time value is \(~4.5\) g. The standard also gives a temperature range of -54 to 60 °C when using any combination of two criteria from the following: temperature, altitude, vibration, and humidity.

In addition to measured data and requirements, test criteria for two other energy harvesting works were compiled. KCF Technologies [31] performed tests at different acceleration levels for each test category. KCF used 1.0g for power measurements, 0.5 g for continuous-use endurance testing, and 0.25 g for temperature effects on their harvester. All KCF tests were sinusoidally driven at a resonance of 700 Hz. Song [4] presented performance data for
accelerations between 0.05 g and 0.2 g for a 60 Hz sinusoidal vibration. Table 3-1 summarizes the gathered environmental test data.

The data in Table 3-1 helps define the test criteria. This work doubled the amplitude and duration of the KCF endurance test, increased the temperature range by three times while quadrupling the base acceleration, and increased the power measurement test base acceleration by five times. The endurance test ran at 1.0 g for 120 hours giving open-circuit voltage, resonance, and capacitance as a function of time. The temperature test was expanded to 0 – 70 °C while driving at 1.0 g and yielded resonance, open-circuit voltage, and strain versus temperature. Finally, the performance test was expanded to 0.5 g to 5.0 g base acceleration while trending the open-circuit voltage and power of the energy harvester.
Table 3-1. Summary of compiled measured, testing, and requirement data from various sources.

<table>
<thead>
<tr>
<th></th>
<th>Industry</th>
<th>KCF</th>
<th>Techno-Sciences</th>
<th>MIL-STD-810G</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input Acceleration (g)</strong></td>
<td>2.5</td>
<td>1 g Power Measurement</td>
<td>0.5 g Endurance</td>
<td>0.05 to 0.2</td>
</tr>
<tr>
<td><strong>Duration (hours)</strong></td>
<td>5,000 (drivetrain) 10,000 (airframe)</td>
<td>60 hours (Endurance Testing Only)</td>
<td>4 per axis (2500 full)</td>
<td></td>
</tr>
<tr>
<td><strong>Frequency (Hz)</strong></td>
<td>1600 to 1800</td>
<td>700</td>
<td>60</td>
<td>800</td>
</tr>
<tr>
<td><strong>Testing Method</strong></td>
<td>Sinusoidal</td>
<td>Sinusoidal</td>
<td>Sinusoidal</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td><strong>Temperature (°C)</strong></td>
<td>25 to 50</td>
<td></td>
<td></td>
<td>-41 to 71</td>
</tr>
<tr>
<td><strong>Application</strong></td>
<td>Drivetrain/Tail Rotor</td>
<td>Gear Mesh Frequencies</td>
<td></td>
<td>Drivetrain</td>
</tr>
<tr>
<td><strong>Comments</strong></td>
<td>Lifetime Requirements</td>
<td>Continuous Test for Endurance, Checked Resonance Daily</td>
<td>Performance Data Only, No Durations</td>
<td>13g from 800 Hz driving frequency for 2500-hour time compression to 4 hours per axis using fatigue exponent m = 6</td>
</tr>
</tbody>
</table>
3.2 Data Acquisition Setup and Signal Analysis

Data was acquired through the use of a National Instruments data card, and software written by Penn State’s Applied Research Laboratory (ARL). Details on the setup and use of the software is presented in Appendix B. Various vibration shakers, laser vibrometers, and single-axis accelerometers were used for testing. Figure 3-1 illustrates the signal flow diagram of the test setups.

![Diagram of test setup showing general signal flows. A signal generator provides input signals to the shaker while the sensors output signals that are read by the DAQ card and recorded by a computer.](image)
The data acquisition card used was the USB-4431 produced by National Instruments shown in Figure 3-2. It has four analog input channels with a range of +/- 10 Volts and one analog output channel with a range of +/- 3.5 Volts. Cables were connected via a BNC connector and data communicates with a computer through a USB 2.0 interface. All data was taken from the NI USB-443x specifications data sheet [54].

Two shakers were utilized during testing of the energy harvester. They were a Vibration Test Systems VTS 300 shaker (Figure 3-3) and a Wilcoxon F4 Shaker (Figure 3-4). The VTS 300 is a large floor shaker mounted in a trunnion base. It produces a peak force of 300 lbf and a maximum acceleration of 70 g. The shaker’s usable frequency range is 2 to 5000 Hz [55]. It was powered by a Techron 7541 Power Supply Amplifier. This shaker was used for performance testing that extends to 5.0 g base acceleration. The one main disadvantage of this shaker is it heats up during prolonged use which affects the data. This problem was abated by using a Wilcoxon F4 shaker. At 700 to 800 Hz, the shaker produces approximately 12 lbf of
force. It has a usable frequency of 10 to 7500 Hz and can only handle 1.4 A_{RMS} unless it is air cooled at 25 psi. This limited the base acceleration to approximately 1.0 g. The shaker was powered by a Crown XTI2000 shaker amplifier. This shaker was used in tests requiring long periods of use and us in the temperature chamber.

Figure 3-3. Vibration Test Systems VTS 300 floor shaker with Techron amplifier power supply used for performance tests.

Figure 3-4. Wilcoxon F4 shaker system used with a Crown XTI2000 amplifier for endurance and temperature tests.
Few sensors were needed for data collection. They were PCB single-axis accelerometers and two Polytec laser vibrometers. The accelerometers measured base accelerations while the laser measured tip velocity at the end of the system’s tip mass in order to determine the resonance frequency.

The accelerometer used for base acceleration measurements was a PCB 352C66 single-axis accelerometer. It had a nominal sensitivity of 100 mV/g and uses 2 mA of ICP power. This accelerometer was used as the base acceleration sensor and was always connected to input channel one on the DAC.

Two laser vibrometers were used. The first, a Polytec OFV-534 Compact Sensor Head with a Polytec OFV-2500-3 Controller (Figure 3-5) was initially used for endurance testing until it was discovered the tip velocity of the compact energy harvester exceeded its maximum velocity of 0.5 m/s at high base acceleration levels. When in use, the laser was set to 50
mm/s/mV sensitivity and used a low pass filter set at 20 kHz. To obtain data at larger magnitude velocities, a Polytech OFV-505 sensor head coupled with an OFV-5000 controller was used, shown in Figure 3-6. The sensitivity on this laser was set at 125 mm/s/mV with no low pass filter. Regardless of which laser was in use, the velocity output was always attached to input channel two on the NI USB-4431 DAC.

The DAC software allowed changes in sampling parameters to better optimize data acquisition performance. For this work, the sampling parameters were set to 5000 Hz bandwidth, 65536 blocksize, and a sampling frequency of 12800 Hz. This correlated to a sampling time of 5.12 seconds and frequency resolution of 0.2 Hz. A Hanning window and RMS averaging were used for data collection. Table 3-2 summarizes the sampling parameters.
A 5000 Hz bandwidth was more than adequate to capture the first bending mode of the harvesters. A sampling time of 5 seconds and differential frequency of 0.2 Hz allowed for high frequency resolution in the FRF and accurate resonance determination.

<table>
<thead>
<tr>
<th>Bandwidth (Hz)</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocksize</td>
<td>65536</td>
</tr>
<tr>
<td>Sampling Frequency (Hz)</td>
<td>12800</td>
</tr>
<tr>
<td>Time (s)</td>
<td>5.12</td>
</tr>
<tr>
<td>dF (Hz)</td>
<td>0.195</td>
</tr>
<tr>
<td>Sampling Window</td>
<td>Hanning</td>
</tr>
<tr>
<td>Averaging Type</td>
<td>RMS</td>
</tr>
</tbody>
</table>

### 3.3 Experimental Setups

Three tests were run including an endurance test, a temperature test, and a performance test. The following sections describe the objectives, the desired outcomes, and procedures for each test. Table 3-3 describes the environmental conditions for each test.

<table>
<thead>
<tr>
<th>Environment</th>
<th>Acceleration (g)</th>
<th>Temperature (°C)</th>
<th>Compared to Previous Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibration Endurance</td>
<td>0.5 and 1.0</td>
<td>Room (~20)</td>
<td>2x amplitude</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2x total time (120 hours)</td>
</tr>
<tr>
<td>Temperature Dependency</td>
<td>0.25 and 1.0</td>
<td>0 to 70</td>
<td>4x amplitude</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3x temperature range</td>
</tr>
<tr>
<td>Vibration Performance</td>
<td>0.5 to 5.0</td>
<td>Room (~20)</td>
<td>5x amplitude</td>
</tr>
</tbody>
</table>
3.3.1 Endurance Testing

A continuous use test was conducted at room temperature to assess the stability of the lab asset’s resonance frequency and open-circuit voltage for 120 hours of use at 0.5 g and 1.0 g base acceleration. Previous studies [31] ran at 0.5 g only. This work doubled the previously available data on single crystal energy harvesters and lends insight into prolonged operation of these harvesters at higher acceleration levels. This test produced data regarding resonance shifts and voltage changes based on long periods of constant use.

An F4 shaker was used for the endurance test to reduce effects of shaker heating on the measured data. Figure 3-7 shows the test specimen setup for endurance testing. The Polytec OFV-534 sensor head and OFV-2500-3 controller were used for measuring tip velocity on the laboratory asset. Tests were conducted on a vibration-isolated table.

![Figure 3-7. Setup of endurance test using lab asset to characterize stability of resonance frequency and output voltage over prolonged use.](image)

The procedure for these tests began with checking the capacitance of the crystal under static conditions. A finite value of capacitance was desired, indicating the connections to the piezoelectric patch were secure, and that the piezo patch was not damaged. Before each data
point, a resonance frequency test was required. Band limited white noise between 10 and 5000 Hz was output to the F4 shaker system. The DAC software then recorded the sensor data for 20 averages and saved it to a text file for analysis. A Matlab code (also written by ARL) was used to determine the resonance frequency of the system. Once found, the resonance frequency was used as a sinusoidal input signal to the shaker at 0.5 g and 1.0 g base acceleration as measured by the base accelerometer. Data was then recorded for five averages. Output voltage of the harvester was measured using a Fluke multimeter. This process was repeated every 30 minutes until voltages and resonance stabilize, then every hour, and finally every two or three hours until the full 120 hours have been reached.

3.3.2 Temperature Testing

Temperature tests were run to characterize the sensitivity of the lab asset energy harvester’s natural frequency and open-circuit voltage in a temperature range of 0 to 70 °C at 0.25 g and 1.0 g. A previously conducted temperature study [31] tested a single crystal harvester at 0.25 g in a temperature range of 25 to 50 °C. Results presented in this work represent three times the temperature range and four times the excitation amplitude. Two different single crystal compositions were tested to assess high temperature performance. The materials were the standard PMN-.32PT used in the harvester design and the other tests, and a piece of PMN-.28PT. Lower percentage of PT content in the composition yields a higher phase transition temperature of the crystal which may offer better high temperature performance. The temperature tests provided data on resonance and open-circuit voltage with respect to temperature as well as gave overall insight into high temperature behavior of PMN-PT energy harvesters.
Tests were administered in a TPS T20C-1.5 temperature chamber capable of 200 °C, illustrated in Figure 3-8. The harvester was placed inside the chamber facing a side-mounted access hole. The hole was used to shine the laser vibrometer (same as endurance test) on the harvester tip during testing. Figure 3-9 illustrates the harvester as seen through the access hole.

Figure 3-8. TPS Tenney temperature chamber used in temperature tests. The lab asset can be seen through the chamber's window.
The harvester was placed in the temperature chamber and allowed to soak to 0 °C overnight to ensure the temperature of the piezoelectric material was the same as the environment. When the device was at temperature, a static capacitance measurement was taken to ensure leads were secured and the piezo was undamaged. The chamber was then shut off to eliminate the chamber vibrations interfering with the tests. A resonance test was run with the same method as the endurance test. Band-limited white noise was input to the harvester and a resonance was obtained through use of the laser and DAQ system. The harvester was then driven at this resonance for a base acceleration of 0.25 g and 1.0 g and open-circuit voltages were recorded. One set of temperatures tests also included a foil strain gage to measure strain on the top surface of the piezo. This value was recorded by the DAQ at the same time as voltage readings were taken. When completed, the chamber was set to the next temperature and turned
on to soak for at least four hours. This procedure was repeated for all temperatures from starting at 0 °C, going to 70 °C at 10 degree increments. After the 70 °C point was completed, the same procedure was applied to return to 0 °C.

3.3.3 Performance Testing

Amplitude-dependent performance tests were conducted in order to characterize the behavior of the lab asset and compact energy harvesters with various base accelerations. The testing ranged from 0.5 g to 5.0 g excitation levels. This corresponds to five times the levels of previous research that investigated single crystal harvester performance between 0.125 g and 1.0 g accelerations [31]. The lab asset was tested only for open-circuit voltage conditions. The compact energy harvester was used to test open-circuit voltages and power for a 0.5 mm thick and a 1.0 mm thick PMN-.32PT piezoelectric material, and power for a PZT-5A based harvester. These experiments gave data on resonance frequency, voltage, and power as a function of amplitude. It also investigated and obtained optimal resistances for the compact harvester to use for power measurements.

Performance tests were run on a Vibration Test Systems floor shaker in order to achieve the higher acceleration levels. The same laser system as endurance and temperature tests were used for the lab asset. A Polytech OFV-505 laser sensor head and OFV-5000 controller were used for the compact harvester in order to measure the faster tip velocities. The setup for lab asset tests is shown in Figure 3-10 and for the compact harvester in Figure 3-11.
Figure 3-10. Performance test setup for lab asset showing VTS floor shaker and Polytec laser vibrometer.

Figure 3-11. Performance test setup for compact harvester. The laser vibrometer is set up as shown in Figure 3-10.
The procedure for the performance tests was nearly the same for the lab asset and the compact harvester. First, the resonance frequency of the system was found by using band-limited white noise and measuring the resultant tip velocity. Resonance was determined through the FRF response of the devices. To measure open-circuit voltages, the device was sinusoidally driven at resonance with a base acceleration of 0.5 g. A multimeter connected to the piezoelectric material showed the AC voltage output. The driving frequency was then tuned to obtain the maximum output voltage. This was done since previous research shows that device resonance changes with respect to driving amplitude [31, 56, 57]. The compact harvester was used to characterize output voltage and output power. Voltage was found in the same manner as the lab asset. For power, first the optimal resistance had to be found. This was done by driving the harvester at 1.0g and testing a variety of resistances between 1 kΩ and 1000 kΩ. The experimentally determined optimal resistance was then used for power measurements. Power was obtained by connecting the harvester to simple harvesting circuitry (Figure 2-6) and the optimal load resistance. Power was then calculated as the voltage across the resistor squared divided by the resistance.

Damping of the compact harvester was also investigated. It was calculated by the log-decrement method using the time history of the laser vibrometer signal. Time signals of tip velocity were measured by driving the harvester slightly above the acceleration of interest and turning the shaker off during data recording. Damping was of interest due to the effects of the clamping fixture for the lab asset. The compact harvester had inherently low damping due to the monoblock design and therefore, damping was an important variable to characterize.
Chapter 4 – Energy Harvester Results

This chapter presents finite element model (FEM) results and results gathered from the various energy harvesters in different environmental conditions. The finite element model was validated and may be used for future designs. Time-dependent tests were run to look at the behavior of single crystal energy harvesters under continuous use; temperature tests were conducted to investigate piezoelectric properties at varying temperatures; and amplitude tests were performed to characterize harvester performance at varying base accelerations. This data can be used to design and to develop new harvesters that are compact, lightweight, and produce high power magnitudes.

4.1 Finite Element Modeling of Energy Harvesters

A validated finite element model was desired allowing for predictions of mechanical behaviors and open-circuit voltages. This section discusses the methods used for FEM validation and their corresponding results. Given a validated model and correct input parameters (such as damping), designers will be able to design, optimize, and scale new energy harvesting devices.

4.1.1 Finite Element Models

The lab asset and compact energy harvesters were both modeled in the FEM software Abaqus. The models were used to predict important harvester dynamic response quantities including tip to base acceleration transmissibilities, stresses, and strains, and resonance frequencies.
The lab asset model (Figure 4-1) included the beam, piezo, and tip mass. The masses and piezo were modeled assuming a tied connection. There was no bond layer modeled between the piezo and the beam, and no bolts modeled through the tip mass. As Figure 4-1 shows, the clamp fixture was not modeled. Instead, the compliance of the clamp is taken into account through use of a spring in the 2-direction at the beam root with stiffness of $k = 7.11$ MN/m. All degrees of freedom at the beam root were rigidly tied to a single point on the spring. The spring introduced enough compliance in the system to match the FEM predicted resonance frequency to the experimentally measured first resonance frequency. An acceleration base motion, with amplitude controlled through the user interface, was applied to the fixed end of the spring which transferred motion to the beam root. Without the compliance spring, the FE model predicted the resonance around 790 Hz while the prediction with the spring was nearly 740 Hz. Inputs included the base acceleration amplitude and the damping ratio as a direct modal damping value.
The lab asset was meshed as shown in Figure 4-2. The piezoelectric material has 6000 elements (with 15 elements through its thickness). Elements used were standard 3D stress brick elements with 8 nodes per element.

![Figure 4-2. Mesh of the lab asset. Note the very fine mesh through the piezo thickness.](image)

Output for the lab asset included shear stress at the beam-piezo interface, strain on the top surface of the piezo, and acceleration at the tip of the system at the fundamental resonance for varying base accelerations inputs.

The compact harvester was fully modeled in the FEM software (Figure 4-3). It included the harvesting components, encompassing frame, and sheet metal covers. As with the lab asset, all surface-to-surface contacts were modeled with a tied connection and there were no bolts through the tip mass. The base acceleration was applied to the entire underside of the harvester to simulate base input motion.
Mesh of the compact harvester is illustrated in Figure 4-4. It was meshed with 2000 elements in the piezo with five elements through the piezo thickness. Elements were 3D 8-noded brick elements. The compact harvester model was used to predict tip accelerations and strains on the piezo surfaces.
The finite element software was also used to scale the compact harvester to 500 and 1500 Hz designs. Strain output on the piezo surface was required to remain the same in order to obtain the same output voltage/power. Dimensions of the beam length and thickness, as well as the tip mass, were systematically changed to find an appropriate design for each frequency.

The FE model predicts tip acceleration, stress at the beam-piezo interface, and strain on either surface of the piezoelectric patch. Data was extracted from the data file that Abaqus wrote during analysis. Two Matlab codes were used to extract first the transmissibility of the system, and second, the stresses and strains. Both Matlab codes can be found in Appendix C.

4.1.2 Validation of the Finite Element Model

The first step of validation was to compare the FE predicted transmissibilities to experimental transmissibilities. This was done on the lab asset before rigorous testing began. The analysis used 1.0 g as a base acceleration and the node that corresponded to where the laser vibrometer beam shone. Figure 4-5 shows the node used for transmissibility analysis. The compliance spring introduced at the root in the FE model was used to tune the resonance of the system. The springs’ stiffness was systematically changed until the FEM predicted resonance was approximately the same as the experimental resonance frequency. Over the frequency range that captured the first bending mode of the beam (from 0 to 1500 Hz), the FEM predicted transmissibility matches well with experimental data as shown in Figure 4-6. A true value of the system damping ratio was unknown during the analysis. This explains the amplitude discrepancy between the prediction and experimental data illustrated in Figure 4-6. However, an accurate prediction of the resonance frequency itself was of more concern than matching amplitudes. With an accurate transmissibility prediction, it was assumed the mechanical model
itself is accurate. The low frequency peaks shown below 250 Hz were associated with motion of the mounting fixture picked up by the accelerometer, and were of no concern for this analysis.

Figure 4-5. Lab asset mesh with node highlighted that was used in transmissibility analyses.
When modeled, the compact harvester FEM resonance frequency was within 5% of the experimentally measured resonance as shown in Table 4-1. Further validation on the compact harvester was achieved through comparing the FEM predicted tip acceleration to the experimental tip acceleration. Figure 4-7 illustrates the tip acceleration comparison.

<table>
<thead>
<tr>
<th></th>
<th>Predicted Resonance</th>
<th>Measured Resonance</th>
<th>Percent Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lab Asset</td>
<td>738.6</td>
<td>735.4</td>
<td>0.44</td>
</tr>
<tr>
<td>Compact Harvester</td>
<td>860.1</td>
<td>862.3</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Figure 4-6. Transmissibility comparison between FEM prediction and experimental data at 1.0 g base acceleration. The amplitude difference is due to inaccurate input damping, and the peak below 250 Hz is motion of the mounting fixture. Experimental data was taken from the lab asset harvester.
Overall, the compact harvester FE model was an accurate representation of the true harvester given the correct system damping. Therefore, the models may be used for future design and optimization.

4.1.2.1 Damping Calculations

The FE model required correct system damping ratio for complete validation. Damping for these tests is defined as the damping ratio (ζ) and is related to the loss factor and quality factor through ζ = η/2 = 1/2Q. The damping ratio was experimentally obtained by driving the harvester slightly harder than the desired base acceleration. During data capture, the output signal was shut off to allow the system to damp out naturally. The tip velocity was recorded before and after signal shut off to obtain the decay profile to calculate damping. A simple filtering code was used to filter out frequencies introduced by the shaker system during data capture. The log decrement method was used to calculate the system damping ratio

$$\zeta = \frac{1}{\omega_n} \frac{\ln(A_0/A_n)}{t_n-t_0},$$  (19)
Where $\omega_n$ is the system frequency, $A_0$ is the amplitude at the first data point, $A_n$ is the amplitude at the second data point, $t_0$ is the time at the first data point, and $t_n$ is time at the second data point on a time-dependent data plot. Figure 4-8 illustrates the log-decrement method variables for damping calculation.

![Figure 4-8. Simple damped oscillatory motion time trace to show log decrement variables in determining fraction of critical damping](image-url)
4.2 Endurance Testing Results

An endurance test was completed on the lab asset for 120 hours of continuous use at 0.5g and 1.0g base acceleration. This test provided resonance frequency and open-circuit voltage versus time. Three different methods of obtaining an electrical connection from the underside of the piezo were tested and are described in detail in Appendix D. These methods are listed below and summarized in Table 4-2.

1) Using the contact of the piezo to the aluminum beam while using the beam itself as the conducting material,

2) Attaching a small-diameter wire to the underside of the piezo, and

3) Epoxying a thin copper tab to the piezo bottom and soldering a wire to the copper outside of the piezo.

The upper piezo surface also had varying connections including:

1) Using single-sided copper tape with a soldered wire,

2) Using the single-sided tape again, and

3) Soldering a wire to a non-adhesive copper tab then taping/epoxying to the piezo surface.
### 4.2.1 Lab Asset Results – Design 1 – Electrical Connection through Piezo-Beam Contact

The first design utilized direct contact between the piezo and the beam to form an electrical connection. Testing proved this method was not reliable and should not be used in future designs. Tests were run at 0.5 g first for a full 120 hours. The resonance frequency of the system remained very stable at 740 Hz following an initial thermal transient (Figure 4-9). From the initial resonance test at time 0, the percent change in resonance frequency never exceeded 2%. After the start-up transient ended around 5 hours, the percent change dropped to less than 1% for the remaining 115 hours. The open-circuit voltage also exhibited a stable behavior around 13 V\textsubscript{RMS} following the same start-up transient as illustrated in Figure 4-10. The voltage had a maximum percent change of ~13%.
Figure 4-9. Resonance frequency as a function of time at 0.5 g base acceleration for the lab asset’s first design endurance test. Percentage change from the initial value is only ~2%.

Figure 4-10. Open-circuit voltage at 0.5 g for the lab asset’s first endurance test design. Maximum percent change is 13%.
Static capacitance of the system was used as a measure to determine the connectivity of the piezo to the beam. At 0.5 g acceleration, the capacitance remained constant after about 15 hours (Figure 4-11). This indicated that the piezo to beam connection is good. Low capacitance readings may be indicative of the piezo and beam surfaces not having enough contact. However, voltage was still obtained during times of low static capacitance. When vibrating at resonance, the piezoelectric patch made enough contact with the beam to establish a reliable connection.

In general, for low-amplitude vibrations, the simple harvester design that used the piezo to beam surface contact provided stable performance for long durations at low amplitude. Therefore, this simple design may be useful for very low amplitude applications.

Tests were then run at 1.0 g (2x the amplitude of previous studies [31]) to assess the performance at higher acceleration levels for prolonged usage. These tests showed many anomaly/failure points within the first 60 hours of testing. Resonance frequency, open-circuit voltage, and capacitance all displayed significant changes during testing. Figure 4-12 shows the resonance frequency overlaid with dashed lines which represent stopping and restarting of the

![Figure 4-11. Static capacitance for lab asset endurance testing design one. Constant capacitance corresponds to a good electrical connection.](image-url)
Within each time frame of continuous testing, the resonance did stabilize, signifying there is no mechanical degradation of the system. However, the open-circuit voltage showed distinctly abrupt reductions during testing. Figure 4-13 illustrates the voltages as a function of time with the same start/stop lines overlain. The abrupt loss of voltage coupled with a stable

Figure 4-12. Resonance frequency of the lab asset endurance test design one at 1.0 g base acceleration. Abrupt changes in resonance are shown when the harvester failed, and dashed line represent system start/stop times.

Figure 4-13. Open-circuit voltage of lab asset endurance test design one at 1.0 g. Abrupt voltage losses indicate an electrical failure occurred due to unreliable contact at the beam-piezo interface.
resonance indicates an electrical failure. The fact that the voltage returned to a higher value upon restarting means the piezoelectric chip itself had not been damaged. Figure 4-14 displays the static capacitance of the system over the same time period. This figure shows abrupt losses in capacitance that corresponded to the losses in voltage.

![Figure 4-14. Static capacitance of lab asset endurance test design one at 1.0 g. Abrupt capacitance changes further indicate electrical connection problems with the harvester design.](image)

A new harvester utilizing the same piezo to beam surface contact was created to investigate the failures at 1.0 g base acceleration. A theory stood that the single crystal material may have reached its fatigue life faster due to the 120 hours of use at 0.5 g before the 1.0 g test. Therefore, this next harvester was only tested at 1.0 g acceleration. Figure 4-15 displays the open-circuit voltage for this test. Only one true failure point existed around 30 hours with a sudden reduction of voltage. The capacitance dropped significantly at 30 hours, which agreed with the voltage reduction (Figure 4-16).
As shown in the 1.0g base acceleration data, using surface-to-surface contact to establish electrical connection was not a reliable design for energy harvesting. Losses in voltage are unpredictable and may cause losses of power to critical system components. This led to development of a new design that placed a wire directly underneath the piezo patch.
Stresses and strains were also predicted for the energy harvesting piezo at 1.0 g base acceleration. Shear stresses were predicted at the beam-piezo interface, and strains were predicted on the piezo top surface. The first and third designs yielded the same results due to the same assumption of a perfect bond line. The coordinate system for the stress and strain predictions is shown in Figure 4-1. Shear stress on the underside of the piezo would correspond to stress in the 1-2 or 2-3 direction. The strain of interest was the normal strain along the length of the piezo, or in the 1-1 direction.

Shear stress in the 1-2 direction was predicted to determine the integrity of the bond line. Figure 4-17 shows the shear stress was localized at the root and tip of the piezo. The 1-2 shear stress is the stress that attempts to lift the piezo off of the beam. It was predicted at 1.1 MPa.
while the bonding epoxy (EP-21) has a shear strength over 20 MPa. Therefore, the bond layer was not in danger of failing and this, and even higher amplitudes.

Normal strain in the 1-1 direction on the top surface of the lab asset piezo was predicted and is displayed in Figure 4-18. The maximum strain was \( \sim 81 \, \mu \varepsilon \) and the mean strain over the entire surface was \( \sim 55 \, \mu \varepsilon \). The strain profile was nearly constant on the top surface due to the tapered cantilever design. With a maximum of only \( 81 \, \mu \varepsilon \), higher strain systems can be designed, which was taken into account during the design of the compact harvester. The reduction of strain near the edges of the patch was due to the unconstrained nature of the patch itself. This allowed for expansion of the top surface which led to very little strain production.

![Figure 4-18. Strain at 1.0 g on the top surface of the piezoelectric patch on the lab asset. Note the near-constant profile due to the tapered cantilever design.](image-url)
Overall, relying on contact between the piezoelectric and the beam surface is not good design methodology. It allows for unpredictable failures in electrical connections yielding unacceptable voltage losses even at vibration levels around 1.0 g. However, the shear stress and strain on the piezo are much lower than failure limits for the bonding agent or the material itself. Therefore, high strain systems may be designed to increase performance.

4.2.2 Lab Asset Results – Design 2 – Electrical Connection using Wire Underneath Piezo

The second design used a small-diameter wire attached to the underside of the piezo. This design proved to give a reliable electrical connection, however, it lowered the overall output voltage (from $20 \, V_{\text{RMS}}$ to $8 \, V_{\text{RMS}}$) and resonance frequency (from 720 Hz to 695 Hz). Figure 4-19 illustrates the resonance and Figure 4-20 displays the open-circuit voltage. From the initial data point, resonance never changed more than 1%. Voltage, however, exhibited a larger percent change over time. In the first few hours (during starting transient) voltage changed up to 35% of the original value. After a longer transient time (approximately 20 hours for this design), the

![Figure 4-19. Resonance frequency of the endurance test on lab asset design two at 1.0 g acceleration. Resonance is very stable changing less than 1%.](image-url)
voltage stabilized and only changed a maximum of 13%. There were no abrupt losses in voltage or the static capacitance (Figure 4-21) indicating a reliable electrical connection to the underside of the piezoelectric material.

Figure 4-20. Comparison of output voltages from lab asset design one and design two. Note the lack of abrupt voltage losses in design two indicating a reliable electrical connection and the lower overall output.

Figure 4-21. Static capacitance of the endurance test on lab asset design two at 1.0 g acceleration. The stable nature indicates no electrical failures were present during the tests.
The overall lower voltage could be attributed to the change in strain profile with the “sandwiched” wire. The wire caused an area of lower strain (Figure 4-22) on the piezo surface which lowered the overall average strain, and thus the open-circuit voltage. Here, the maximum strain was 71 με but the average strain reduced to 35 με at 1.0 g acceleration.

![Figure 4-22. Strain profile with wire underneath piezoelectric patch on lab asset design two at 1.0 g acceleration. The profile is no longer constant and the mean strain decreased from 55 to 35 microstrain.](image)

Although attaching a wire underneath the piezo proved to have a reliable electrical connection, the decrease in output voltage is undesirable. The first design showed higher output voltages (2x the second design) are possible. It was beneficial to try to keep the high level of
voltage and introduce a high level of reliability in the electrical connection. Therefore a third design was investigated using a thin piece of copper under the piezo.

4.2.3 Lab Asset Results – Design 3 – Electrical Connection Using Copper Tab

The final design iteration of the lab asset utilized a small, thin copper tab epoxied with conductive silver epoxy to the underside of the piezo which then sticks out from underneath. The lead wire was soldered to the copper outside of the piezo. This design proved the best as it exhibited both stable and high output voltage.

Resonance frequency (Figure 4-23) was stable around 700 Hz changing only ~1% from the initial data point. After the thermal transient ended around four hours, the percent change was less than 0.5%. The open-circuit voltage was also very stable around 25 V_{RMS} as shown in Figure 4-24. The voltage magnitude changed a maximum of 22% from the initial data point. After the transient died out (around five hours) the percent change was less than 13% throughout the remainder of the test time.

![Figure 4-23. Resonance frequency of the endurance test on lab asset design three at 1.0 g acceleration. Resonance is very stable changing approximately 1% from the initial data point.](image-url)
Figure 4-24. Open-circuit voltage of the endurance test on lab asset design three at 1.0 g acceleration. The voltage is higher than design two and without any abrupt losses, showing the third design is best.

It is important to note how the voltage data is presented. Due to the compliance of the clamping fixture, the voltages for the endurance test and the temperature test were transmissibility normalized to the performance testing data. The performance test setup on the floor shaker allowed torqueing the clamp bolts to 35 ft-lbs, lessening the effects of clamp compliance. The test setups utilizing the F4 shaker did not allow for the same level of torque. Therefore, the performance test was used for a comparison baseline. The transmissibility between the tip acceleration and the base acceleration was calculated for each test. To normalize the values, the raw voltage data was multiplied by the ratio of the performance test transmissibility to the test of interest’s transmissibility shown by

\[ Voltage_{\text{normalized}} = Voltage_{\text{Raw}} \frac{T_{\text{Performance}}}{T_{\text{Interest}}}. \]  

(20)

This allowed the data from all three tests to be presented assuming the same boundary conditions.
The static capacitance was measured and exhibited a constant behavior illustrated in Figure 4-25. Similar to the second design iteration, the stable voltage and capacitance lead to the assumption of a reliable electrical connection.

![Figure 4-25. Static capacitance of the endurance test on lab asset design three at 1.0 g acceleration. The stable behavior eludes to the electrical connection being reliable.](image)

Along with the resonance, voltage, and capacitance, the system’s electromechanical coupling factor (k_{31}) was calculated for this endurance test and is presented in Figure 4-26. The factor remained very stable throughout the entire testing period indicating no degradation or

![Figure 4-26. Electromechanical coupling factor for the third lab asset design. The stable nature indicates there is no change in the piezoelectric material for the test period.](image)
failures of the piezoelectric material. The coupling factor was calculated by means of Equation 7. Open circuit resonance was measured as previously explained. The short circuit resonance was measured utilizing the same method except a small wire was attached from the bottom surface to the top surface of the piezo. This short-circuited the piezo during vibrations and allowed for measurements of short-circuit resonance frequency.

The third design iteration proved the optimal option for energy harvester design. It provided the highest voltage magnitude while maintaining high electrical reliability. When designing future harvesters, a smaller, thinner footprint of the underside electrical lead is the best option for harvester design.

4.3 Temperature Testing Results

Temperature dependence of the energy harvester performance was investigated for a temperature range of 0 to 70 degrees Celsius at 0.25 g and 1.0 g base accelerations. These tests provided data regarding resonance frequency, open-circuit voltage, and strain on the piezo with respect to temperature. Two different harvesters were tested, the first using a 32% composition of PMN-PT (PMN-.32PT) and the second using 28% PT (PMN-.28PT). The different compositions provided different piezoelectric properties which correlate to a difference in harvester performance. Table 4-3 summarizes the piezoelectric properties of the two different compositions. Temperature dependence of energy harvesters is important for designers since rotorcraft see very hot (~70 °C) and very cold (below 0 °C) environments. The behavior of harvesters as a function of ambient temperature is paramount to harvester designers.
### 4.3.1 Lab Asset Results – PMN-.32PT Sample

First, temperature tests were run on the lab asset using PMN-.32PT as the active material. Before strain measurements were made, tests without an attached strain gage were conducted to reduce any effect the gage and wiring had on the system. While the temperature was increased, resonance frequency decreased ~6% from 710 Hz to 665 Hz as shown in Figure 4-27. Also, the agreement of measured resonances between the increasing temperature data and the decreasing temperature data (meaning from 0 to 70 °C then from 70 to 0 °C) was very good. There was no hysteresis effect present presumably because the phase transition temperature was not exceeded.

#### Table 4-3. Comparison of piezoelectric properties for PMN-.32PT and PMN-.28PT.

<table>
<thead>
<tr>
<th></th>
<th>PMN-.28PT</th>
<th>PMN-.32PT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{31}$ (pC/N)</td>
<td>-1000</td>
<td>-920</td>
</tr>
<tr>
<td>$K_{33}^T$</td>
<td>4000 to 5500</td>
<td>4950</td>
</tr>
<tr>
<td>$T_{Phase}$ (°C)</td>
<td>95</td>
<td>75</td>
</tr>
</tbody>
</table>

*Figure 4-27. Resonance frequency versus temperature for the PMN.32PT lab asset harvester. Note the good correlation between increasing and decreasing temperature data points.*
Bare beam resonance tests were conducted to establish the cause of the resonance change. The resonance of an aluminum tapered cantilever beam did not change with temperature (~1%). Strain on the piezo patch’s surface was measured and remained relatively constant meaning the bond layer was not degrading with temperature. Therefore, the resonance shift in the energy harvester was due mostly to changes in the piezoelectric material.

Tests were run at 0.25 g first since previous research used that same base acceleration. Figure 4-28 illustrates the non-transmissibility-normalized output voltage. The values were not normalized as discussed in section 4.2.3 since performance tests were not run at 0.25 g. Therefore, the transmissibility value was unknown for this acceleration level and the normalization was not possible. This led to the large scattering effect shown in Figure 4-28. As temperature was increased, the voltage decreased substantially. From 0 to 70 °C it reduced ~92% of its original value.

![Figure 4-28. Open-circuit voltage at 0.25 g for the 32% PT lab asset harvester. Voltage decreases 92% from 0 to 70 °C. Scattering effect is due to lack of voltage normalization.](image)
At 1.0 g, the voltage was normalized as described in section 4.2.3. This allowed the effects of the clamping fixture to be eliminated and the data presented to be an accurate representation of harvester performance. Figure 4-29 displays the normalized voltage output as a function of temperature. Similar to 0.25 g acceleration, the percent reduction over the temperature range was ~92%. Even a 30 degree change (from room temperature to 50 °C) showed a decrease of 60%.

![Figure 4-29. Open-circuit voltage at 1.0 g for the 32% PT lab asset harvester. Voltage decreases 92% from 0 to 70 °C and exhibits very high correlation during increasing/decreasing temperature.](image)

The large reduction in voltage at high temperatures is attributed to the relatively low phase transition temperature of PMN-.32PT which is ~75 °C. This voltage loss is problematic for harvester performance at elevated temperatures. A different composition (PMN-.28PT) was tested given its higher phase transition temperature (~95 °C).

The static capacitance of the energy harvester was measured with respect to temperature. Capacitance tended to increase with increasing temperature and offered good correlation between data taken while increasing and decreasing temperature (Figure 4-29). Open-circuit
voltage is inversely related to the permittivity (Equation 13) and permittivity is directly proportional to capacitance (Equation 8). Therefore, if capacitance increases, so does permittivity, and thus open-circuit voltage drops. The increase in capacitance illustrated in Figure 4-30 correlates to the decrease in voltage shown in Figures 4-28 and 4-29.

Finally, strain on the top surface of the piezo was measured on the 32% PT lab asset using a foil strain gage. This test was conducted to determine if the reduction in voltage was caused by changes in the piezoelectric material, or degradation of the bond line at elevated temperature. Figure 4-31 shows the strain as a function of temperature. Overall, the strain remained relatively constant (changing only ~15%) over the temperature range and did not have any decreasing trend. Given the relationship between capacitance, permittivity, and voltage (Equations 5 and 13) as well as the non-decreasing trend of surface strain, it is shown that voltage reduction at elevated temperatures is attributed to changes in the piezoelectric properties of the material and not mechanical degradation. Substituting Equation 4 into Equation 13, further proves how the piezoelectric properties cause the voltage to decrease.
As discussed in Chapter 2, the electromechanical coupling factor \( k_{31} \) remains constant with temperature when the phase transition temperature is not exceeded. The piezo thickness \( t \) is a constant and the strain \( \delta \) does not decrease. Equation 21 then shows the charge coefficient \( d_{31} \) must increase for voltage to decrease.

\[
V_{OC} = \frac{k_{31}^2}{d_{31}} t\delta
\]  

Equation 21

In general, voltage substantially decreases (~92%) as ambient temperature increases. The 32% PT composition shows very poor performance at high temperatures which is problematic in energy harvesting applications. The voltage reduction is due mainly to changes in the piezoelectric properties of the single crystal, not mechanical degradation.

4.3.2 Lab Asset Results – PMN-.28PT Sample

To look into better high temperature performance, a new harvester was fabricated using PMN-.28PT as the active material. This composition has a phase transition temperature of 95 °C, which is 20 degrees higher than the 32% PT sample. The higher phase transition may abate
some of the large voltage reductions at high temperature shown with the 32% PT harvester. The 28% PT was chosen because of the higher phase transition, but also because of the piezoelectric properties. Use of Equation 13 with typical properties of PMN-.28PT showed PMN-.28PT outperformed PMN-.32PT at 56 V to 40 V at room temperature. Better performance at elevated temperatures yields a more desirable harvesting device.

Resonance frequency of the 28% PT harvester was vastly different than then 32%. Between 30 and 40 °C there was a discontinuity that increased the frequency almost 100 Hz that was present in two tests and not with the 32% samples. In general, the frequency exhibited decreasing behavior with an increase in temperature shown in Figure 4-32. From 0 to 30 °C resonance decreased ~8% from 710 Hz to 650 Hz. There was then a large jump back up to ~710 Hz and another decrease of ~6% to 670 Hz. The correlation between increasing and decreasing temperature data points was good and repeatable, indicating this trend is not an anomaly. Tests were run on two different PMN-.28PT samples to eliminate the possibility of a bad piezo patch. The reason for the frequency discontinuity is unknown, and is an area for future research.

![Figure 4-32. Resonance frequency of the 28% PT lab asset energy harvester. The reason for the discontinuity at 30 degrees Celsius is unknown at this time. Overall, the resonance still decreases with increasing temperature.](image)
Although the resonance displayed a discontinuity, the voltage was unaffected and showed a continuous decrease over the temperature range. At 1.0 g, the normalized voltage still exhibited the same decreasing trend as the 32% PT sample, however, the magnitude was larger overall as evidenced in Figure 4-33. Voltage decreased ~80% from ~42 $V_{\text{RMS}}$ to ~ 9 $V_{\text{RMS}}$ which is 12% less of a reduction than shown with the PMN-.32PT harvester. AT 70 °C, voltage output was nearly three times that of the 32% harvester.

![Figure 4-33. Temperature performance comparison of output voltage between 32% and 28% PT lab asset energy harvesters. The 28% composition offers better overall performance including almost three times the voltage at 70 °C](image)

Overall, the performance of single crystal energy harvesters degraded with increasing temperature. At high temperatures, output voltage of a 32% PT composition may be inadequate for proper energy requirements. However, the losses at elevated temperatures was abated by using a different composition of single crystal material that possesses a higher phase transition temperature. This increased voltage output over the entire temperature range including an almost factor of three increase at 70 degrees Celsius.
4.4 Performance Testing Results

Performance tests were conducted initially on the lab asset to characterize its performance at varying acceleration levels. The compact harvester was then used to investigate not only amplitude dependence, but also to help validate the FE model and to examine output power between 0.5 and 5.0 g acceleration. FEM validation was accomplished by experimentally measuring damping and using that as input for the FE model. Mean strain was then predicted and used in Equation 13 to predict the open-circuit voltage. The tests utilized two samples of PMN-.32PT and some common conventional PZT-5A ceramic for comparison of performances. It is important to characterize behavior of energy harvester performance at varying base accelerations. Currently, most energy harvesters are designed for a specific application. Knowing the behavior at a variety of vibration amplitudes will help designers create better devices for numerous applications.

4.4.1 Lab Asset Results

The initial 32% PT lab asset tests provided insight into behavior of the energy harvester with respect to base amplitude. In general, open-circuit voltage followed a linear line, however, was non-linear in the sense that the voltage at 2.0 g was not twice that of 1.0 g. Figure 4-34 illustrates the open-circuit voltage and the FEM voltage prediction for the lab asset. The compact device produced $\sim20 \text{ V}_{\text{RMS}}$ at 0.5 g which increased to over $100 \text{ V}_{\text{RMS}}$ at 5.0 g. It is worth mentioning that the true damping ratio was not used in the predictions and that the piezoelectric properties carry a $\pm20\%$ error, which will result in error in voltage predictions.
4.4.2 Compact Harvester Results

Tests for the PMN-.32PT compact harvester began with open-circuit voltage. They also included optimal resistance for the system, damping measurements, and output power. Voltage and power for a commercially-inspired representative harvester are paramount performance parameters for energy harvester use and design.

The compact harvester exhibited more open-circuit voltage roll-off than the lab asset. At higher acceleration levels, the voltage tended to level off more rapidly. Figure 4-35 shows the measured open-circuit voltage and the FE predicted values. True damping ratio was used for FE predictions, which correlated to measured data well. At 0.5 g, the harvester produced ~12 V_{RMS} while at 5.0 g, it produced ~72 V_{RMS}. The compact harvester generated ~30 V_{RMS} at 1.0 g base acceleration.

![Figure 4-34. Voltage versus base acceleration amplitude for the 32% PT lab asset harvester. A wide range of voltages are achievable up to 5.0 g. The FE model predictions match well with experimental measurements.](image)
Optimal resistance for the harvester system was needed in order to measure usable power. This resistance was determined experimentally by measuring the output power at 1.0g using various load resistances in conjunction with the harvesting circuitry. Figure 4-36 displays the power versus resistance curve for the compact harvester. The optimal resistance was found to be 10 kΩ for the compact harvester using a 0.5 mm thick PMN-.32PT piezo and calculated to be
~9.5 kΩ using Equation 19. An interesting feature to note is the relatively flat portion of the power curve. Typically power versus resistance curves resemble a perfect quadratic relation closely. The “flattened” power peak may lead to usable power generated over a wide range of load resistances.

Power measurements were taken using the harvesting circuitry and the optimal load resistance. First, voltage with circuitry and load resistor was measured and is plotted in Figure 4-37. With the full circuitry and load resistance, the non-linear behavior at high acceleration levels disappeared. Also, the voltage levels are substantially reduced. For example, at 1.0 g the open-circuit condition produced 30 V_{RMS} while the full system produced only 5 V_{RMS}. This was due to the increased electrical damping introduced with the circuitry and load resistance. The mechanical damping for the monoblock design is inherently very low, therefore electrical damping tends to dominate the overall system damping.

![Figure 4-37](image)

Figure 4-37. Voltage across the load resistor as a function of base acceleration for the compact harvester using a 0.5 mm thick PMN-.32PT piezo. Due to the different system damping, the voltage shows more linearity than the open-circuit condition.
Power was calculated by taking the square of the output voltage and dividing it by the resistance for this simple harvesting study. Since the voltage showed a relatively linear behavior, the power showed a fairly quadratic nature. At low acceleration levels, the harvester produced low power (about 0.7 mW\textsubscript{RMS} at 0.5 g), however, output increased quadratically to 45 mW\textsubscript{RMS} at 5.0 g (Figure 4-38). The compact harvester with a 10 k\Omega resistance and a 0.5 mm thick 32\% PT crystal at 1.0 g base acceleration produced ~2.7 mW\textsubscript{RMS} of usable power.

Another power test was run to compare the performance of the PMN-.32PT single crystal to that of a common PZT-5A conventional ceramic. The same harvesting circuitry was used but the optimal resistance was 82 k\Omega for the PZT system. Figure 4-39 compares the power output of the single crystal device to the PZT device. At all acceleration levels, the single crystal produced over twice the power of the PZT-based device.
One final test was conducted in an attempt to increase output power. This last test used a PMN-.32PT sample that was 1.0mm thick (twice that of the initial design). As Equation 13 shows, the open-circuit voltage is directly proportional to the material thickness. Therefore, increasing the piezo thickness should theoretically increase the output voltage. However, testing proved that increasing the thickness of the piezo, also increased the system damping ratio (Figure 4-40). The increased damping led to less voltage output and power as evidenced in Figures 4-41 and 4-42.
In order to understand the effect of thicker-chip damping, tests were run using aluminum patches rather than the piezoelectric patches. The patches remained 10x10x0.5 mm and 10x10x1.0 mm to emulate the same thicknesses and relative thickness of the piezo. The damping ratio of the all-aluminum assembly was low, on the order of $\zeta = 0.0015$. Unlike the damping with the piezoelectric material, damping with aluminum chips remained constant with increasing amplitude. There was also no increase in damping with an increase in aluminum chip
thickness (Figure 4-43). Since damping did not increase with the increase in chip thickness of an aluminum chip, all damping changes were related to the piezoelectric material.

![Graph showing damping ratio comparison between a 0.5 mm and 1.0 mm aluminum chip used in place of the piezo. This test shows the increase in system damping is due to an increase in the piezoelectric chip thickness.](image)

Overall, performance of energy harvesters are highly dependent on base vibration amplitude. Approximately 3 mW$_{\text{RMS}}$ of power was produced through the fully-assembled harvester using 0.5 mm PMN-32PT single crystal. The harvester produces less than half the output power when the single crystal was replaced with conventional PZT ceramic. This shows that single crystal harvesters can be designed which produce the same amount of power as a PZT device, yet with a smaller footprint and less mass. Or, a device that is the same size can produce more power, as evidenced in this work. Finally, thickness of the piezoelectric material does play an important role in harvester performance. A thicker piezoelectric patch should yield more power due to the increased thickness; however, it introduces more mechanical damping that eliminates any performance gain. Therefore, determination of piezoelectric chip thickness is important in designing new harvesting devices.
4.5 Damping Considerations of Energy Harvesters

System damping ratio proved to be an important parameter in harvester performance. In energy harvesting applications, it is desirable to drive the system damping ratio as low as possible in order to get the maximum amount of output power. Understanding the effect damping has on the energy harvester will aid future designers in developing new energy harvesters more suited for commercial use. True damping ratio measurements were taken on the lab asset and compact harvester in a variety of different configurations. These include damping on one lab asset that has no piezoelectric material, one that uses the 32% PT piezo, and one that uses the 28% PT piezo. The compact harvester had more configurations available for damping measurements. These include the harvester with no piezoelectric material, with a 0.5 mm thick 32% PT crystal in open-circuit configuration, the 0.5 mm thick 32% material in shunted configuration (no circuitry but with load resistor), and the 0.5 mm thick 32% composition in full assembly. Data was also taken for the compact harvester with 1.0 mm thick 32% PT piezo in open-circuit and full assembly configurations, with a 0.5 mm thick PZT-5A piezo in fully assembled configuration, and the 0.5 mm and 1.0 mm thick aluminum chips in place of the piezo material. Damping measurements for the lab asset and the compact harvester with a bare beam and no tip mass were not able to be obtained due to the very high resonance frequency of the systems. Table 4-4 summarizes the configurations and available damping data.
Table 4-4. Summary of harvester configurations and available damping data for each.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Piezo Type</th>
<th>Tip Transmissibility Data?</th>
<th>Damping Ratio Data?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lab Asset Energy Harvester</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bare Beam Only</td>
<td>-</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Bare Beam + Tip Mass</td>
<td>-</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Beam + Mass + Piezo</td>
<td>PMN-.32PT</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Beam + Mass + Piezo</td>
<td>PMN-.28PT</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Compact Energy Harvester</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Bare Beam Only</td>
<td>-</td>
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<td>No</td>
</tr>
<tr>
<td>Bare Beam + Tip Mass</td>
<td>-</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Beam + Mass + Piezo Open</td>
<td>0.5mm PMN-.32PT</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Beam + Mass + Piezo Shunt</td>
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</tr>
<tr>
<td>Beam + Mass + Piezo Full</td>
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<td>Yes</td>
</tr>
<tr>
<td>Beam + Mass + Piezo Open</td>
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<td>Yes</td>
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<td>Yes</td>
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<tr>
<td>Beam + Mass + 1.0mm Al</td>
<td>-</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
The lab asset bare beam damping ratio was very low due to the simplistic system. No bond line or piezo were present to introduce damping. The bare beam showed a damping ratio of approximately 0.0015 at 1.0 g base acceleration. The 28% PT material showed more material damping than the 32% PT sample at 1.0 g vibration and 0.5 mm piezo patches. The system damping increased to ~0.0025 with the PMN-.32PT and up to ~0.0035 for the PMN-.28PT sample. Figure 4-44 illustrates the damping ratios for the lab asset harvester. Measurements with the piezo are at the open-circuit condition.

![Figure 4-44. Comparison of damping ratios for the lab asset harvester at 1.0 g base acceleration. The PMN-.28PT material has more material damping than the PMN-.32PT sample.](image)

Various damping values for the compact harvester were measured. The data compares damping from a bare beam system through a system that was fully assembled with full circuitry. The bare beam showed a damping ratio around 0.00075 at 1.0 g base amplitude. Figure 4-45 illustrates the comparison of damping ratios for various assembly configurations that used 0.5 mm thick PMN-.32PT piezo patches. As more components were added, the damping ratio
increased. The open-circuit damping ratio was approximately 0.0025 at 1.0 g acceleration. Adding the load resistance only (shunted configuration) increased damping to 0.0035, and finally, the full circuitry with load resistance was 0.004. The electrical damping contributed by the circuitry and resistor was ~40% of the total system damping. Better matching impedances of the piezo and the circuitry and optimizing the circuitry will decrease the percentage of electrical damping and increase energy harvester performance.

![Figure 4-45. Damping ratio comparisons of various compact harvester configurations at 1.0 g base acceleration and 0.5mm piezoelectric patches. Electrical damping is ~40% of the total system damping.](image)

Figure 4-46 compares all compact harvester configurations. They are grouped by configuration type: bare beam, open circuit, shunted circuit, full circuitry, and finally, the aluminum chips. The data compares bare beam to harvesters with 0.5 mm and 1.0 mm PMN-.32PT and aluminum chips, as well as to a 0.5 mm PZT chip. The 1.0 mm chip showed more damping than the 0.5 mm chip. However, electrical damping for the thicker chip was essentially
the same between the open circuit and the full circuit configurations. It is possible that the impedance of the 1.0 mm chip better matches the impedance of the circuitry than the 0.5 mm chip.

![Figure 4-46. Summary of damping ratios of all compact harvester configurations at 1.0 g base acceleration.]

Overall, damping plays an important role in energy harvester performance. Although a PMN-.28PT material provided higher output voltage at 1.0 g than the PMN-.32PT material, the damping for the 28% PT material is larger. Higher damping yields lower output and would therefore be of high concern to harvester designers. A 0.5 mm PMN-.32PT piezo showed large increases in damping from an open-circuit condition to the full circuitry and load resistance configuration. The total system damping consists of over 40% electrical damping with the 0.5 mm chip. If the impedances or the circuitry and piezo better match, the electrical damping would have a lower percentage of total damping; however, the total damping will increase due to the increase in output power. The 1.0 mm piezo proved to have no change in damping from open-circuit to full circuit conditions. This may mean the impedances of piezo and chip better
match. Damping within the materials themselves and the overall system must be taken into account in future harvester designs.
Chapter 5 – Scalability Study on Compact Harvester

Scalability studies were conducted to determine the size/weight changes required of the compact harvester in order to keep the output power the same while tuning the fundamental resonance frequency to different design frequencies. The study was completed using the FE software applying the validated models and simulation techniques developed in this work. Figure 2-3 shows the flowchart used during scalability designs. It was assumed that an equal strain on the piezo surface will yield the same output voltage and power. A mean strain of ~240 με was used to correspond to the 1.0 g acceleration of the compact harvester at 800 Hz. It also assumed that damping remained the same as the 800 Hz harvester driven at 1.0 g. Two designs are presented, one at 500 Hz and one at 1500 Hz.

5.1 Study on 500 Hz Design

The harvester tuned to 500 Hz (Figure 5-1) consisted of a longer and thinner tapered cantilever beam, a slightly larger frame, and a lower tip mass than the 800 Hz design. The beam length was 35 mm and the thickness decreased to 2 mm at the root with 0.8 mm at the tip. The frame increased to 50 mm in length but remained the same in height and width and the tip mass decreased to 1.42 grams. Overall, these changes did not affect the overall harvester mass (staying at 36 grams) but did increase the volume to 19 cubic centimeters. Finite element software predicted the average strain on the surface of the piezoelectric material at 235 με, which was only 2% off of the 240 με requirement.
5.2 Study on 1500 Hz Design

Tuning to 1500 Hz proved more difficult to design. A slight design change was required that made the beam a double tapered cantilever as shown in Figure 5-2. The beam length was reduced to 17 mm and its thickness to 2 mm at the root ending at 0.7 mm at the tip. The shorter beam allowed the frame length to shrink to 30 mm. Obtaining the same strain output was the
reason for the double taper to decrease the strain required on the piezoelectric patches. A piezoelectric patch was bonded to both the upper and lower surfaces of the beam. Using two pieces of material made the required strain on each piezo half of the original value, which was easier to obtain at higher frequencies. The tip mass decreased to 2.5 grams since the required strain was split between the two piezos. Due to the design changes, the overall mass decreased to 27 grams and the volume to 9 cubic centimeters, however, using two piezos will increase cost.

5.3 Comparison of the Three Designs

Overall, the energy harvester was scalable to a wide range of resonance frequencies with very minor design changes. This approach is valuable to future designers to assess new energy harvesting applications. Having a validated FE model allows designers to create numerous designs and predict their output without the time and resources required to fabricate and test harvester designs. In general, the length of the beam and the frame, and overall volume all decrease as the desired resonance frequency is increased. Table 5-1 compares dimensions, masses, and other design parameters for the three designs.

| Table 5-1. Comparison of important design parameters for 500 and 1500 Hz scalability study keeping strain level at 240 με and base acceleration at 1.0 g. |
|-----------------|-----------------|-----------------|
| **Beam length (mm)** | 35 | 20 | 17 |
| **Beam Thickness (mm)** | 2 to 0.8 | 3.3 to 1.2 | 2 to 0.7 (Double Taper) |
| **Frame length (mm)** | 50 | 40 | 30 |
| **Tip Mass (g)** | 1.42 | 6.5 | 2.5 |
| **Mass (g)** | 36.3 | 36.7 | 27 |
| **Volume (cm³)** | 18.75 | 15 | 9 |
| **Frequency (Hz)** | 502.9 | 800 | 1499 |
| **Mean Strain (με)** | 235 | 240 | 236 (118 x 2) |
Changing the resonance of the first bending mode also shifted the modes and frequencies of subsequent modes. The mode shapes of the 800 Hz and 1500 Hz harvesters remained the same, albeit at different frequencies. For example, the 1st plate bending mode of the front cover occurs at 1717 Hz for the 800 Hz design and 2010 Hz for the 1500 Hz design. The 1st plate bending mode of the back covers shifted from 1759 Hz to 2068 Hz. However, mode shapes did not line up between the 500 Hz and the 800 Hz design. For example, the third mode of the 800 Hz design is the 1st plate bending mode of the back cover, while the third mode of the 500 Hz design is the 1st plate bending mode of the top cover. Table 5-2 summarizes the first few modes of the three harvester designs.

### Table 5-2. Summary of first five mode shapes and resonance frequencies for the three scaled harvester designs.

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Description 500 Hz</th>
<th>500 Hz Design</th>
<th>Description 800 Hz</th>
<th>800 Hz Design</th>
<th>Description 1500 Hz</th>
<th>1500 Hz Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1st Beam Bending</td>
<td>502.9</td>
<td>1st Beam Bending</td>
<td>860.1</td>
<td>1st Beam Bending</td>
<td>1499</td>
</tr>
<tr>
<td>2</td>
<td>1st Plate Bending Front Cover</td>
<td>978.8</td>
<td>1st Plate Bending Front Cover</td>
<td>1717.1</td>
<td>1st Plate Bending Front Cover</td>
<td>2010</td>
</tr>
<tr>
<td>3</td>
<td>1st Plate Bending Top Cover</td>
<td>1065.2</td>
<td>1st Plate Bending Back Cover</td>
<td>1759.4</td>
<td>1st Plate Bending Back Cover</td>
<td>2068</td>
</tr>
<tr>
<td>4</td>
<td>2nd Plate Bending Back Cover</td>
<td>1111.3</td>
<td>2nd Plate Bending Front Cover</td>
<td>1834.7</td>
<td>2nd Plate Bending Front Cover</td>
<td>2267</td>
</tr>
<tr>
<td>5</td>
<td>2nd Plate Bending Front Cover</td>
<td>1497.8</td>
<td>2nd Plate Bending Back Cover</td>
<td>1868</td>
<td>2nd Plate Bending Back Cover</td>
<td>2325</td>
</tr>
</tbody>
</table>

Overall, the compact harvester was easily scaled between 500 Hz and 1500 Hz with few design changes. Lower frequencies were easier to design when keeping output power the same. It was more difficult to design the high frequency device due to the increased stiffness and
reduced inertial mass to obtain high frequencies. Low frequency devices have an advantage in energy harvesting systems. Lower design and drive frequencies typically will feature larger masses and skinnier/longer beams. These all equate to larger strains seen in the beam and subsequently, on the piezo patch. Therefore, lower frequency devices can produce higher power more easily. Higher frequency devices would typically produce less strain and less power due to the increased stiffness and reduced inertial mass. As seen previously, system damping ratio tends to increase with thicker piezoelectric patches. This is a limitation to the design space that needs to be considered when designing future harvesters.
Chapter 6 – Thermal Energy Harvesters

Other technologies are available for energy harvesting applications. One such technology is thermal harvesting. Thermal energy harvesting, like vibrational harvesting, is another promising area for lightweight and compact power conversion. This chapter aims to quickly describe thermal energy harvesting and to compare performance between thermal and piezoelectric energy harvesters.

6.1 Background on Thermal Harvesting

One other prominent harvesting technique is thermal energy harvesting. Natural and man-made thermal gradients and heat flows are used to harvest electrical potential and power [41, 58, 59]. According to Synder [58], the temperature gradient yields a voltage and the flow is directly related to the converted power and current. Thermoelectric harvesters are of interest due their high reliability and low maintenance (no moving parts under stationary operation), very low noise output, and design scalability [58, 59].

When a gradient is applied to the thermoelectric harvester, charged particles are collected at the cold ends of the material [58]. This results in an electrostatic voltage between the hot and cold ends of the thermoelectric materials. This voltage is calculated as

\[ V = \alpha \Delta T, \]  

(22)

where \( \alpha \) is the Seebeck Coefficient (a material-dependent property measured in Volts per Kelvin) and \( \Delta T \) is the temperature difference in Kelvin. The heat flux, \( q' \), determines the current and power [58]. The heat flux (Watts per Meter squared) is
\[ q' = \kappa \frac{\Delta T}{L}. \]  

where \( \kappa \) is the material’s thermal conductivity (Watts per Meter-Kelvin), and \( L \) is the length of the thermoelectric material between the hot and cold ends of the device in meters [41].

Before output power is discussed and presented, the efficiency of a thermoelectric harvester must be discussed. An important parameter for efficiency is the *figure of merit* of a thermoelectric device. The figure of merit \((zT)\) is directly related to the material properties and the absolute temperature. It is given as

\[ zT = \frac{\alpha^2 T}{\rho \kappa}. \]  

The variable \( T \) is the absolute temperature in Kelvin and \( \rho \) is the electrical resistivity in Ohm-Meter [58]. The figure of merit is multiplied to the Carnot efficiency to give a more accurate value of true thermal harvester efficiency. The Carnot efficiency is the theoretical maximum efficiency a heat exchanger or thermal device can achieve.

Output power per unit area is given by

\[ \frac{P}{A} = \eta q', \]  

where \( q' \) is the heat flux defined earlier and \( \eta \) is the thermoelectric efficiency defined as

\[ \eta = \eta_{Carnot} \frac{\sqrt{1+zT}-1}{\sqrt{1+zT}+T_e/T_h}. \]  

The variables \( T_e \) and \( T_h \) are the cold and hot temperatures respectively [58]. The overall efficiency of a thermal harvester is typically very low, less than 5% [60].
6.2 Examples of Thermal Harvesters

Few examples of thermal energy harvesters exist and are mostly research-related. There is one commercially available device, however. The Seiko Thermic Watch converts body heat into electrical energy that powers the watch [58]. It produces 22 μW of power with 300 mV at a 1.5 °C temperature difference. The harvester is 0.0065 cm³ and has an efficiency of 0.1%.

KCF Technologies [31] also reports data for a thermal harvester. It is a full assembly including harvester, sensor, and wireless transmitter all enclosed in a 20.5 cubic centimeter package weighing 75 grams. It produces 0.35 mW of power with a 25 °C temperature gradient using natural convection for heat removal. With forced convection at 0.25 m/s air velocity and the same 25 °C temperature difference, the power increases to 0.9 mW.

Research performed by Knight demonstrates a thermal energy harvester used at an air-water interface [59]. They tested three sizes of thermoelectric material discs 0.020m², 0.031 m², and 0.045 m² in area. At ~21 °C temperature difference the small device produces 22 mW, the medium device yields 31 mW and the large device gives 48 mW of usable power. However, when normalized to the area, all three devices produce a power density of ~1 mW/m² at an efficiency of <0.2%.

Lu, reference 60 also presents data on a thermal harvester for use in home radiator applications. Using a 3.3 cm³ device and a 30 °C temperature differential, the device produces ~28 mW of power with 0.29 Volts and 97 mA of current. Table 6-1 summarizes these available thermal harvesters and their performance characteristics.
Table 6-1. Summary of performance characteristics of various thermal harvesting devices

<table>
<thead>
<tr>
<th></th>
<th>Temperature Gradient (°C)</th>
<th>Power (mW)</th>
<th>Volume (cm³)</th>
<th>Power Density (mW/cm³)</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>KCF [31]</td>
<td>25</td>
<td>0.35</td>
<td>20.5</td>
<td>0.017</td>
<td>-</td>
</tr>
<tr>
<td>Seiko [58]</td>
<td>1.5</td>
<td>0.022</td>
<td>0.0065</td>
<td>3.38</td>
<td>0.1%</td>
</tr>
<tr>
<td>Knight [59]</td>
<td>21</td>
<td>22</td>
<td>200*</td>
<td>0.11*</td>
<td>&lt;0.2%</td>
</tr>
<tr>
<td>Lu [60]</td>
<td>30</td>
<td>28</td>
<td>3.3</td>
<td>8.5</td>
<td>-</td>
</tr>
</tbody>
</table>

*Given in square centimeters.

6.3 Comparison of Thermal Harvesters to Vibration Harvesters

Similar to vibrational energy harvesters, the output power of a thermal harvester is highly dependent on the environment in which it is used. Higher vibration levels produce higher power; likewise larger thermal gradients produce larger voltages and power. Roundy [41] presents comparison data on various power sources. The data show one specific thermal harvester has a power density of ~0.04 mW/cm² at 5 °C temperature difference and a vibrational harvester has 0.3 mW/cm³. Becker [61] also presents data on power to mass ratio for various harvesters. With a 10 °C temperature difference, a thermal harvester can produce 800 mW/kg while a vibrational harvester at 1.0g and 60 Hz produces 37 mW/kg.

The harvester presented in this work produces ~3 mW of power in a 40 gram and 15 cm³ package. This corresponds to a power density of 75 mW/kg and 0.2 mW/cm³. Thermal harvesters were not investigated on an experimental level in this work. However, data provided by industry [62] shows typical thermal gradients of ~16 °C per inch with a hot temperature of ~250 °C. If it is assumed a thermal harvesting device has a figure of merit of 1, a thermal conductivity of 1.20 W/m-K, and a length of .1 meter, the power density will be 0.216 mW/cm². Table 6-2 presents comparisons of thermal harvesters to vibrational harvesters.
In general, vibrational energy harvesters tend to have better power per unit volume performance than thermal harvesters. However, power per unit mass does not show favor toward one or the other. Larger thermal gradients will produce inflated power to mass densities for thermal harvesters and high vibrations will produce higher values for vibrational harvesters. Performance of any energy harvester device is highly dependent on the environment in which the harvester is operating.

Disadvantages of thermal harvesters include the required thermal gradients and electronics. Small thermal gradients are present on rotorcraft. These will yield power; however, it may be low. Adding heat pipes could increase the gradient and thus increase power, but they will add overall mass. Electronics also pose a problem. Harvesting circuitry is still required and would be placed close to the heat source in a compact device. Electronics lose efficiency and reliability in high heat environments and would not be best placed near the hot surfaces for thermal harvesting.
Chapter 7 – Summary and Conclusions

This work identified performance and critical design parameters and presented results for a single crystal energy harvester in long duration, high temperature, and high amplitude environments. The single crystal harvesters saw 120 hours of continuous use at 1.0 g base acceleration, temperatures between 0 and 70 °C at 1.0 g base acceleration, and vibration amplitudes between 0.5 g and 5.0 g. Performance comparisons between PMN-.32PT and PZT-5A were reported for varying amplitudes, as well as between PMN-.32PT and PMN-.28PT for temperature variations.

In general, the single crystal energy harvester proved stable for long periods of continuous use and showed a non-linear behavior with base acceleration. Piezoelectric theory predicts non-linear behavior which was shown in this work. Performance degraded with increasing temperature, however, the 28% PT composition provided better overall performance than the 32% PT sample. The 32% compact harvester showed over twice the output power than a conventional ceramic-based PZT harvester at all base amplitudes.

Validated FE models and simulation techniques were developed to aid designers in harvester design. The model predictions matched very well with experimental measurements given appropriate material properties and system damping ratios. These models are valuable design tools for future application assessments.

7.1 Endurance Testing Conclusions

Vibration testing for 120 hours at 1.0 g base acceleration showed the energy harvester remained mechanically and electrically stable. The tapered cantilever beam design produced a
near-constant strain profile on the piezoelectric material’s surface and showed no mechanical failures. A constant electromechanical coupling factor and stable output voltage indicate there was no piezoelectric degradation of the material. At approximately 700 Hz, the harvester experienced over 300 million cycles at 1.0 g and a maximum strain in the piezo around 80 microstrain with no reduction in performance. True rotorcraft missions will never see 120 hours of continuous use, rather, they are often short in duration. Therefore the harvester may offer reliable long-term power solutions.

The electrical connection to the underside of the piezoelectric material was more important in this study than the connection to the top. Three methods were tested including using the inconsistencies in the bond layer to create contact between the beam and the piezo surface, attaching a small-diameter wire to the bottom of the piezo, and finally, using a thin copper tab epoxied to the bottom of the piezo. Using the beam-piezo contact method proved unreliable and introduced unpredictable loss of voltages. This method was not acceptable for this study’s application. Placing a small wire underneath the piezo in the bond line itself showed high reliability, albeit at the cost of lower output voltages. The wire effectively created an area approximately 1/3 of the piezo bottom surface that was not bonded to the beam. This caused a reduction in average strain, and thus the output voltage. The best bonding method was using a very thin copper tab epoxied to the underside of the piezo. This provided a reliable electrical connection while maintaining a high level of output voltage.
7.2 Temperature Testing Conclusions

Temperature dependence of harvester performance was investigated for two different single crystal material compositions. Both materials (PMN-.32PT and PMN-.28PT) showed voltage reductions at elevated temperature. From 0 to 70 °C, the 32% PT sample lost 92% of the voltage and the 28% PT sample lost nearly 80% of its open circuit output voltage at 1.0 g base acceleration. The PMN-.32PT was also tested at 0.25 g, where it also lost 92% of the output voltage over the temperature range. In general, the 28% PT composition showed better overall performance offering over twice the output voltage at temperatures above 40 °C.

The reduction in voltage was attributed to a change in the piezoelectric properties with temperature. Strain measurements showed crystal surface strain is relatively constant, with a single drop of ~15% near 30 °C. The strain measurements do not decrease with increasing temperature, which explained the piezoelectric properties must change for output voltage to decrease. The charge coefficient ($d_{31}$) and permittivity ($\varepsilon_{33}$) both increase, however, permittivity increases faster which led to a reduction of voltage (shown in Equation 11). Static capacitance was shown to increase with temperature (Figure 4-29) and is directly related to permittivity through Equation 6. As capacitance increases the permittivity increases, further proving the piezoelectric properties change with temperature.

Resonance frequency for the 32% PT sample was relatively stable only reducing 6% from 0 to 70 °C. Therefore, the harvester was mechanically stable at least between 0 and 70 °C. In contrast, the 28% PT sample had a large discontinuity in resonance between 30 and 40 °C where resonance changes by nearly 100 Hz. Reasons for this discontinuity are not fully understood and are area for future investigation.
7.3 Performance Testing Conclusions

With respect to amplitude dependence, the single crystal energy harvesters showed non-linear behavior. The lab asset had less non-linearity than the compact harvester continuing with a straight-line trend up to 5.0 g while the compact harvester output started to level off at high accelerations. This non-linearity of the compact harvester may mean at very high acceleration levels (over 10 g) the voltage may not show appreciable increases with any further increase in base acceleration. High amplitude testing is required to assess the behavior at these levels.

Simple rectifying circuitry and optimal load resistances were required for power measurements. The single crystal harvester had an optimal resistance of 10 kΩ and the PZT harvester was 82 kΩ. The power versus load resistance curve (Figure 4-35) showed a relatively flat area of power production from ~10 kΩ to 1 MΩ. This could allow the harvester to produce usable power over a large range of load resistances.

Power measurements were taken for the compact harvester using PMN-.32PT and PZT-5A to compare newer single crystal performance to the conventional PZT-based ceramics. The single crystal harvester produced over twice the output power of the PZT harvester at all vibration amplitudes. The difference was especially noticeable at high acceleration levels i.e. at 5.0 g where the single crystal harvester produced 45 mW of power and the PZT harvester only produced 15 mW.

Measurements were also taken to compare a 1.0 mm thick PMN-.32PT piezo to the original 0.5 mm thick piezo. The thicker material had no better performance than the 0.5 mm chip. In fact, it yielded lower voltage and power output due to increased system damping
introduced by the thicker material. Even though output voltage is directly proportional to piezo thickness, the overall system damping changed causing drastic changes in performance.

Thermal energy harvesters may be simple in design, but they do have some disadvantages for harvesting applications. Thermal gradients and heat flow may not be present in the area where sensors are desired. This would lead to adding wires to the system in order to power the sensors. It is difficult to compare and contrast thermal and vibrational energy harvesters. Each harvesting technology is heavily dependent on the environment in which they operate. Thermal devices will produce more power under larger temperature gradients just as vibrational harvesters will produce more power under higher base accelerations.

7.4 Damping Conclusions

Damping is an important factor to consider when designing energy harvesters. Different materials possess different internal damping. For example, PMN-.28PT showed higher damping than PMN-.32PT material. When designing energy harvesters, damping should be minimized to maximize output power. This may be achieved through structural design coupled with selection of appropriate materials.

Single crystal materials exhibited non-linear damping with respect to vibration amplitude as evidenced by the non-linear behavior of the compact harvester. Even slight increases in damping had large effects on harvester performance. This is the reason open-circuit voltage tended to level off at higher vibration amplitudes. Also, increasing thicknesses of the active material increased system damping. A 1.0 mm think chip had nearly twice the damping ratio of a 0.5 mm piezo in the open-circuit condition.
In general, adding more components increased damping. The compact harvester utilizing a 0.5 mm PMN-.32PT chip increased damping nearly 50% from an open-circuit condition to the full circuit and load resistance configuration. However, damping remained the same between open-circuit and full circuit conditions with a 1.0 mm PMN-.32PT chip.

With non-optimized harvesting circuitry, the impedances of the crystal and electrical components do not match well. This caused high levels of electrical damping, which was approximately 40% of total system damping. If the circuitry was optimized to reduce the percentage of electrical damping in the overall system damping ratio, harvester performance and efficiency would increase. The validated FE models could be used for design and electrical optimization in future studies.

7.5 Future Work

Reasons for the large jump discontinuity seen in the 28% PT piezo sample are unclear and should be investigated further. Electrical impedance of the piezoelectric material may offer insight into this phenomenon. It would be beneficial to test a bare piece of material with electrical leads to measure impedance. Tests would still be conducted between 0 and 70 °C with data taken every 10 degrees. Between 30 and 40 degrees, data should be taken in at least two degree increments. Piezoelectric properties do not change with the resonance shift since the voltage showed no discontinuity. Mechanical tests at various temperatures would aid in characterizing the composition’s strength properties. Simple tensile tests would help in this endeavor. Also, it may be possible crystalline structure is changing significantly at this temperature. Literature searches returned no definitive answer on phase transitions in that
temperature region for PMN-.28PT. Therefore, investigating crystal structure will add to the existing knowledge base and confirm or disprove the existence of a phase transition.

A large area for future research is circuitry optimization and energy storage. Optimizing the AC-DC rectifier and smoothing capacitor to better match their impedances to the piezoelectric crystal will yield better performance and efficiency of energy conversion. Energy storage and power distribution circuitry is another important feature. The energy harvester requires some sort of energy storage in order to properly power sensors when vibrations are low or not present. A storage capacitor or battery is required. Charge and discharge times along with electrical impedances are important parameters that must be taken into account when choosing an energy storage device. Also, most sensors have minimum and maximum levels or voltage and/or current for operation. The energy harvester produces high voltage and low current which would have to be transformed to the appropriate levels. A voltage regulator is required to drop the voltage to a more usable level.

Other tests could be completed that compare various PMN-PT compositions among themselves as well as to other single crystal materials. This will help gather pertinent performance data for single crystal harvesters and help open new doors for harvester design and usage. Materials such as PMN-PT with PT percentages between 20% and 40% and other single crystals such as PZN-PT should be investigated. This will allow comparisons of various piezoelectric properties and show trade-offs in terms of material price, strength, temperature dependence, and output performance. All tests utilized in this work should be used in material characterization. Endurance testing helps characterize fatigue life, thermal tests will show harvester performance as a function of temperature. Finally, performance testing will provide
data on how high and low amplitude vibrations affect the harvester performance and the piezoelectric material.

Testing to complete a power versus frequency can also be conducted. This work presented power versus base amplitude with the drive frequency changed to match the resonance at each acceleration level. However, the driving frequency in operational use will not change with amplitude. Therefore, creating a profile of power versus frequency at a specified excitation level is important for further understanding energy harvester applications. The tests can be expanded to create a two dimensional plot of power versus frequency and amplitude.

The FE models are another area for future work. The models presented in this work were only mechanical models. They predicted resonances, stresses, strains, and other dynamic quantities. Prediction of electrical quantities were calculated using the mechanical output variables. Further modeling may be completed that expands the model to an electromechanical model. By using mechanical and piezoelectric properties of the piezoelectric material and coupling the mechanical and electrical portions of the model, accurate performance predictions could be obtained directly from the FE model. The model may also be improved by either creating the hardware that holds the inertial mass to the beam and the covers to the frame and using appropriate interaction parameters to capture effects of bolts and screws. The model can be used to explore a double taper design as seen in the scalability study. If lower frequencies were designed with a double taper, it may be possible create a smaller, lighter device that produces the same output power.

This work presented data dependent on base excitation amplitude and time, however, did not look at shock properties of the harvester or material. Tests that investigate how the harvester
performs and withstands shock should be conducted. This may include introducing an impulse to the system while running that investigates how the output behaves. Simple tests that use the material alone may be conducted to evaluate strength properties of the crystal. Varying levels of impulses or other shock methods could be used, then the material would be tested for strength and electromechanical properties.

Finally, the energy harvester itself may still be optimized further to produce higher strain levels and less weight. Larger strain magnitudes, less weight, and more optimized circuitry corresponds to more output power in a more desirable package for rotorcraft applications. Structural changes to the frame can reduce weight by removing unnecessary material in the frame walls. Optimization software exists that may be utilized to better design the harvesting beam. Constrains in beam dimensions, overall mass, frequency, etc. can be input to the program and various optimal designs would be output that minimizes mass while maintaining the desired size, frequency, and other harvester details. There is still a large margin to design higher strain systems. At 1.0 g, the compact harvester shows a maximum strain around 300 microstrain. This is still well below the 1500 microstrain limit allowing for higher strain systems to be designed.
Appendix A – Matlab Codes for Beam Design

The Matlab code presented below was used for preliminary design of the lab asset and compact harvesters. The output of this code was the beam’s natural frequency, strain on the beam surface and piezo surface, and the percentage loss of strain from the root for the beam and piezo surfaces. Dimensions of the beam were systematically changed to find an adequate design. These values were then used as a starting point in the FEM software for detailed design.

Main Code:

```matlab
L = .020; % Beam Length (m)
Lc = .010; % Piezoelectric Length (m)
b = .010; % Beam Width (m)
tc = .0005; % Piezoelectric Thickness (m)
t1 = .0033; % Root Thickness (m)
t2 = .0006; % Thickness at End of Taper (m)
m = .00665; % Tip Mass on Beam (grams)

%% BEAM Properties
%%%%%%%%%%%%%%%%%%
Eb = 70e9; % Youngs Modulus of aluminum in Pa
rho = 2700; % Density in kg/m^3
% L = .030; % Length in meters
% b = .010; % Width in meters

%% CERAMIC PROPS
%%%%%%%%%%%%%%%%%
%Lc = .010; % Length of piezo in meters
bc = b; % Width of piezo in meters
Ec = 20e9; % Youngs Modulus of Piezo in Pa (25 GPa for initial design)
tc = .0005; % Thickness of Piezo (m)
rhoc = 8200; % Density of piezo in kg/m^3

%% Misc Props
%%%%%%%%%%%%%%%%%
n_points = 1000; % Number of Nodes for FE Analysis
ratio = L/Lc; % Ratio of Beam Length to Piezo Length for Indexing Thickness Array
e_des = 800e-6; % Original Strain Design

%% Tip Mass
%%%%%%%%%%%%%%%%
%m = 0.0051; % Mass of tip mass in kg (.450 ~ 1 lb)
MassCG = .0025; % Location of COM for Tip Mass from Beam Tip
Mtip = m; % Tip Mass

%% Initial and Final Taper Thicknesses
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t_beam_1 = t1; % Begin Beam Thickness Arrays with Root Thickness
t2 = .0010; % Ending Thickness

%% FORCE PROPERTIES
```

127
% Assumed System Quality Factor
Q = 50;

% Acceleration due to gravity in m/s^2
g = 9.8;

% Base input acceleration
base = 1*g;

% Acceleration of tip due to base input acceleration
w_dd = base*Q;

% Force at the tip of the Beam
F = m*w_dd;

%% CREATE THICKNESS ARRAY

% Create Tapered Profile

% Find Index of Length for End of Piezo
piezo_end = ceil(n_points/ratio);

% Set Taper Ending Index
%taper_end = ceil((3/4)*n_points);
% Set Taper End at 3/4 the total Beam Length
% Make sure Taper ends at Ending Thickness
% Set rest of beam to Ending Thickness

%% CENTROID CALCULATION

% Cross-sectional Area of Beam (m^2)
Area_beam = t_beam.*b;

% Cross-sectional Area of Ceramic (m^2)
Area_ceramic = tc*b;

% Distance from Neutral Axis to Surface of Beam
y_beam = t_beam./2;

% Distance from N.A. to Ceramic Surface
y_ceramic = t_beam + (tc/2);

% Calculation of Centroid of System

% Cross-sectional Area of Beam (m^2)

% Cross-sectional Area of Ceramic (m^2)

% Distance from Neutral Axis to Surface of Beam

% Distance from N.A. to Ceramic Surface

% Calculation of Centroid of System

%% INERTIA CALCULATION

% Combined E*I for System with Piezo
EI_tot1 = (Eb*I_beam) + (Ec*I_ceramic);

% Combined E*I for System after Piezo
EI_tot2 = Eb*I_beam;

% Create EI Array for Whole System and Full Length
EI_tot(1:((piezo_end)-1)) = EI_tot1(1:((piezo_end)-1));
EI_tot(piezo_end:n_points) = EI_tot2(piezo_end:n_points);
EI_tot = EI_tot(1:n_points);

%% CREATE LENGTH ARRAYS FOR DEFLECTION, THICKNESS PROFILE, STRAIN

% X Array for Piezo Top
x = linspace(0, L, n_points);

% X Array for Piezo Top
x_top = linspace(0, Lc, piezo_end-1);

% X Array for Strain Loss
x_loss = linspace(0, bc, (piezo_end-1));

% Variable for Graphing

t_top = 0;

% Variable for Graphing
t_piezo(1:piezo_end-1) = -tc;

%% Z-VALUES FOR STRAIN

% Distance from N.A. for Beam Strain Calc
z_des = t_beam - centroid;

% Distance from N.A. for Piezo Strain Calc
z_des_top = z_des + tc;

%% DEFORMATION AND STRAIN CALCULATION

% Deflection
W_des = (((1/3)*F.*(x.^3) - (1/2)*F.*L.*x.^2)./EI_tot;

% Beam Strain
strain = (-z_des*F.*(x - (L + MassCG)))./EI_tot;

% Strain on Piezo Surface
strain_top = -z_des_top*F.*(x - (L + MassCG))./EI_tot;

% Strain Loss Calculation

for i = 1:(piezo_end-1);
    loss(i) = (1 - (strain(i)/strain_top(i)))*100;
    top_loss(i) = (1 - (strain_top(i)/strain_top(i)))*100;
end

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%% PLOTS

figure(1)
subplot(3,1,1)
plot(x*1e3,W_des)
title('Displacement versus distance on Beam Linear Taper')
xlabel('x (millimeters)')
ylabel('W (meters)')
subplot(3,1,2)
plot(x*1e3,t_beam*1e3, 'b', x*1e3,t_top, 'b', x_top*1e3,t_piezo*1e3, 'r')
title('Thickness versus Distance on Beam Linear Taper')
xlabel('x (millimeters)')
ylabel('t (millimeters)')
set(gca, 'ydir', 'reverse')
axis([0,L*1e3,-2,6])
subplot(3,1,3)
plot(x*1e3,strain*1e6, 'b', x_top*1e3,strain_top*1e6, 'r')
title('Strain versus distance on Beam Linear Taper')
xlabel('x (millimeters)')
ylabel('\epsilon (microstrain)')

figure(2)
plot(x_loss*1e3,loss, 'b', x_loss*1e3, top_loss, 'r')
title(['Percentage Loss of Root Strain (', num2str(e_des*1e6), ') microstrain'])
xlabel('x (millimeters)')
ylabel('Percent Loss')

%% FREQUENCY CALCULATION

freq = FEMBeamFunction(EI_tot, EI_tot2, t_beam, tc, n_points, Mtip, rho, rhoc, b, bc, L, MassCG)

Avg_Strain = mean(strain_top)*(1e6)
function [freq] = FEMBeamFunction(EI_tot, EI_tot2, t_beam, tc, n_points, Mtip, rho, rhoc, b, bc, L, l_end)

format long;

Nel = (n_points/2); % Number of Elements
l = L/Nel; % Element Length
piezo_end = ceil(bc/l); % Index of End of Piezo

Nu = 2*(Nel+1); % total number of coordinates in unconstrained system
Nu = Nu + 2; % Number of Nodes
K = zeros(Nu); % Initialize Global Stiffness Matrix
M = K; % Initialize Global Mass Matrix

% Create Thickness, Inertia, and Mass Terms in FE Matrices
for i=1:Nel
    j = floor((n_points/(2*Nel))) + (i-1).*floor((n_points./Nel)); %EI
    k2 = floor((n_points./Nel)) + i.*floor((n_points./Nel)) - floor(n_points./Nel);
    %second thickness
    k1 = floor((n_points./Nel)) + (i-1).*floor((n_points./Nel)) - floor(n_points./Nel);
    %first thickness
    t(i) = (t_beam(k2) + t_beam(k1 + 1))./2;
    A(i) = b*t(i);
    Ac = bc*tc;
    EI1(i) = EI_tot(j);
    EI2(i) = EI_tot2(j);
    m1(i) = (rho*A(i)*l) + (rhoc*Ac*l);
    m2(i) = rho*A(i)*l;
end

% Create Inertia Array
EI(1:piezo_end) = EI1(1:piezo_end);
EI((piezo_end+1):Nel) = EI2((piezo_end+1):Nel);

% Create Density*Area Array
rhoAl(1:piezo_end) = m1(1:piezo_end);
rhoAl((piezo_end+1):Nel) = m2((piezo_end+1):Nel);

% Create Full Arrays for Constants on Element Matrices
EI = EI(1:Nel);
rhoAl = rhoAl(1:Nel);

for i = 1:Nel
    % Create Element Stiffness and Mass Matrices
    k = (EI(i)./l^3).* [12 6*l -12 6*l; 6*l -12 6*l; 6*l 2*l^2 -6*l; -6*l 12 6*l; 6*l 2*l^2 -6*l; -6*l -12 6*l; 6*l 2*l^2 -6*l; -6*l 12 6*l; 6*l 2*l^2 -6*l];
    m = rhoAl(i)./420 .* [156 22*l 54 -13*l; 22*l 4*l^2 13*l -3*l^2; 54 13*l 156 -22*l; -13*l -3*l^2 -22*l 4*l^2];
    % Assemble Global Matrices from Local Matrices
    %clc;
    n0 = 2*(i-1)+1; % Start Index
    n1 = 2*(i-1)+4; % End Index
    K(n0:n1, n0:n1) = K(n0:n1, n0:n1) + k; % Append Stiffness Matrix
    M(n0:n1, n0:n1) = M(n0:n1, n0:n1) + m; % Append Mass Matrix
end;

l_end = L; %l_end = .0075;
k_end = ((EI(Nel)*10e6)/l_end^3).* [12 6*l_end -12 6*l_end; 6*l_end 4*l_end^2 -6*l_end 2*l_end^2; 12 6*l_end -12 6*l_end; 6*l_end 2*l_end^2 -6*l_end; 6*l_end 2*l_end^2 -6*l_end; -12 6*l_end -12 6*l_end; 6*l_end 2*l_end^2 -6*l_end; -12 6*l_end -12 6*l_end; 6*l_end 2*l_end^2 -6*l_end; -12 6*l_end -12 6*l_end; 6*l_end 2*l_end^2 -6*l_end; 6*l_end 2*l_end^2 -6*l_end];

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m_end = [156 22*l_end 54 -13*l_end; 22*l_end 4*l_end^2 13*l_end -3*l_end^2; 54 13*l_end 156 -22*l_end -13*l_end -3*l_end^2 -22*l_end 4*l_end^2];

n0 = 2*((Nel+1)-1)+1; % Start index
n1 = 2*((Nel+1)-1)+4; % End index
K(n0:n1, n0:n1) = K(n0:n1, n0:n1) + k_end;
M(n0:n1, n0:n1) = M(n0:n1, n0:n1) + m_end;

% Add Tip Mass
M(Nu-1,Nu-1) = M(Nu-1,Nu-1) + Mtip;

% Enforce Cantilevered Boundary Condition (w(0) = 0, theta(1) = 0)
Ng = Nu - 2; % Eliminate the first two rows and columns from the global matrices
Kg = K(3:Nu, 3:Nu); % Constrained Global Stiffness Matrix
Mg = M(3:Nu, 3:Nu); % Constrained Global Mass Matrix

% Eigenvalue Calculation
[phi, omg2] = eig(Kg, Mg);

% Mode Sorting and Mode Shape (eigenvector) Normalization
c = phi'*Mg*phi; % Initial Values of Modal Mass
phi1 = phi;
for n=1:Ng, phi1(:,n)=phi1(:,n)./sqrt(c(n,n)); end % Scale to Unit Modal Mass

[omg, index] = sort(sqrt(diag(omg2))); % Sort Natural Frequencies

freq = omg(1)/(2*pi); % Prints Value of First Natural Frequency
Appendix B – Data Acquisition Setup and Recording Parameters

Data acquisition was accomplished through two different software programs a function generator (FGen) and a data recording suite (Modal Impact). The programs were used together to create the output signals to the shakers and to record the data given by the various sensors.

B.1 Function Generator Software

In order to produce driving forces, a function generator was utilized. This program was written by ARL and is called FGen (for Function Generator). The software had options to output sine wave drive, Gaussian white noise drive, sine sweep drive, and a location to import arbitrary signals. These are shown in Figure B-1. Arbitrary signals and sine sweeps were not used in this work.

Figure B-1. Screenshot of the Function Generator program while running a sine drive test. Notice the Sine Parameters channel is turned on and the others are turned off.
B.1.1 Resonance Frequency Test Parameters

Two types of tests were run that utilize the Gaussian white noise parameters and the sine drive parameters. Each test (endurance, temperature, and performance) began with determining the resonance frequency. In order to do this, the Gaussian white noise parameters were set as follows. The RMS field was set to 0.5 \( V_{\text{RMS}} \), the window and window parameter fields were left as the default “Rectangle” and 0, and the standard deviation was set to 3. The radio button for bandpass filter was selected to be on. For resonance tests, the filter was set between 10 and 5000 Hz.

B.1.2 Sinusoidal Drive Test Parameters

Running tests at the resonance frequency used the sine parameters section of FGen. The sine parameters changed with each specific test. The frequency field took on the value of the resonance frequency in Hertz and the amplitude was the output voltage of the FGen program. This value could not exceed 3.5, as this is the overload limit of FGen. The amplitude can be changed to produce the desired base acceleration. All other fields (Phase, Window, Window Parameter, and Reset Phase) were left as the default zero, Rectangle, zero, and unchecked, respectively.

B.1.3 Common Function Generator Parameters

Regardless of which test is operating, a few parameters remained the same. The Output Channel must be changed to use Dev1/ao0. The update rate must be one of many specific values that the generator will accept. Also, this rate must match the sample rate assigned in the data acquisition file. For this work the value was 12800. The field labeled DAC Type must read
USB-4431, which corresponds to the National Instruments USB-4431 Data Acquisition Card used in data collection. All other fields were the default values.

The right hand side of the generator displays the output signals. The top graph shows the autospectra of the output signal. In Figure B-1 it shows a single driving frequency of 696.9 Hz. The middle and lower graphs illustrate the amplitude output of the signal. The lower graph is the most useful as it shows amplitude as a function of time. For a sinusoidal signal, the amplitude should take on the value prescribed in the Amplitude field of the sine wave parameters.

**B.2 Data Collection Software**

The software used to acquire the measured data was also written by ARL and is called Modal Impact. The program has four tabs, Setup, Plot Setup, Ave Plots, and This Hit. Each tab has important aspects for acquiring data.

**B.2.1 Modal Impact Setup Tab**

The setup tab, Figure B-2, shows the parameters used for data collection such as sensor sensitivities, sample rates, time per data point, frequency step, etc. ICP current must be set at 2mA for the program to work correctly. All tabular fields were filled in by selecting an input file using the Load Setup File button. When ready to acquire data, the Arm button was pressed.

The program was setup to read in an Excel file that describes all of the parameters needed. Table B-1 describes the Channel Setup tab, seen in Figure B-3 and Table B-2 describes the Acquisition Setup tab seen in Figure B-4.
Figure B-2. Modal Impact software Setup tab that reads in Excel setup files.

Figure B-3. Channel Setup tab in the Microsoft Excel setup file used for data acquisition in Modal Impact software.
Table B-1. Description of the Channel Setup parameters in the setup file.

<table>
<thead>
<tr>
<th>Column Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel Number</td>
<td>Tells the channel number of the sensor</td>
</tr>
<tr>
<td>On/Off/Out</td>
<td>Tells whether the input channel is active or not</td>
</tr>
<tr>
<td>Device Name</td>
<td>Name of the data acquisition card in use</td>
</tr>
<tr>
<td>Analog I/O #</td>
<td>Channel number of the sensor relative to the data acquisition card</td>
</tr>
<tr>
<td>Range (V)</td>
<td>+/- range of the data acquisition card for input channels</td>
</tr>
<tr>
<td>Sensitivity (mV/EU)</td>
<td>Sensitivity of the sensor in mV per engineering unit</td>
</tr>
<tr>
<td>Units</td>
<td>The engineering unit desired (i.e. m/s for velocity)</td>
</tr>
<tr>
<td>Range (EU)</td>
<td>+/- limit of measurement from sensor</td>
</tr>
<tr>
<td>ICP Power</td>
<td>Determines if sensor needs 2 mA of power to operate</td>
</tr>
<tr>
<td>Coupling</td>
<td>Coupling of ICP power to sensor</td>
</tr>
<tr>
<td>SN</td>
<td>Serial Number of sensor if desired</td>
</tr>
<tr>
<td>Description</td>
<td>Short description of the sensor if desired</td>
</tr>
<tr>
<td>Point Number</td>
<td>Unused</td>
</tr>
<tr>
<td>Increment</td>
<td>Unused</td>
</tr>
</tbody>
</table>
Table B-2. Description of the Acquisition Setup parameters in the setup file.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>Frequency range of data acquisition</td>
</tr>
<tr>
<td>Blocksize</td>
<td>Number of samples in data set</td>
</tr>
<tr>
<td>Fs (Hz)</td>
<td>Sampling Frequency in Hertz (Calculated)</td>
</tr>
<tr>
<td>T (s)</td>
<td>Time to run one data set (Calculated)</td>
</tr>
<tr>
<td>df (Hz)</td>
<td>Frequency step size (Calculated)</td>
</tr>
<tr>
<td>dt (s)</td>
<td>Time step size (Calculated)</td>
</tr>
<tr>
<td>Analog Trigger – Pretrigger</td>
<td>Setting for acquisition triggers, unused</td>
</tr>
<tr>
<td>Window</td>
<td>Acquisition window selection</td>
</tr>
<tr>
<td>Averaging Mode</td>
<td>Mode of averaging data</td>
</tr>
<tr>
<td>Weighting</td>
<td></td>
</tr>
<tr>
<td>Number of Averages</td>
<td>Total number of samples run for a full data set</td>
</tr>
<tr>
<td>Total # AI</td>
<td>Number of input channels on acquisition card</td>
</tr>
<tr>
<td>Total # AO</td>
<td>Number of output channels on acquisition card</td>
</tr>
<tr>
<td>Reference Channel</td>
<td>Reference Channel for FRF based on Channel Setup tab</td>
</tr>
<tr>
<td>Save Mode</td>
<td>Option to save time histories or not</td>
</tr>
</tbody>
</table>

Figure B-4. Acquisition Setup tab in the Microsoft Excel setup file used for data acquisition in Modal Impact software.
B.2.2 Modal Impact Plot Setup and Ave Plots Tabs

The software gave the option to view up to six plots of average data. These plots were chosen in the Plot Setup tab (Figure B-5) and included time, coherence, FRF phase, FRF magnitude, cross spectra phase, cross spectra magnitude, and the autospectra. As data was collected and analyzed, the average graphs were displayed in the Ave Plot tab seen in Figure B-6.

Figure B-5. Modal Impact software Plot Setup tab. This interface has the option chose up to 6 plots showing various average data. The available data options are: Coherence, FRF Phase, FRF Magnitude, Cross Spectra Phase, Cross Spectra Magnitude, Autospectra, and Time.
Figure B-6. The Average Plot tab displays the averages for the data selection in the Plot Setup tab. Shown here is an example of FRF Phase in the top left corner, FRF magnitude in the top right, Autospectra for an accelerometer and laser vibrometer in the bottom left, and coherence in the bottom right.

B.2.3 Modal Impact This Hit Tab

This tab monitored the instantaneous data collection. Figure B-7 illustrates the This Hit interface. There were two graphical options available, time domain traces, and frequency domain traces. Each graph had the option to choose whichever sensor data is of interest. Status messages were shown to relay whether or not overloads in the acquisition setup were detected. The buttons in the middle of the interface allowed choosing whether to accept or reject the last data point, automatically accept all data points, automatically reject all overloaded points, and whether or not to restart the data averages.
Figure B-7. Modal Impact software This Hit tab that shows instantaneous time domain and frequency domain data points.
Appendix C – Matlab Codes for Data Extraction from FE Software

These Matlab codes were used to extract transmissibility, stress, and strain data from the FE Software. The first code extracted transmissibility and created a predicted FRF graph for the energy harvester. For the code to operate correctly, a single node on the tip of the mass had to be chosen. This node was the global node on the FE mesh that most closely corresponded to the point at which the laser vibrometer was measuring. Also, the base acceleration of the analysis was required. The code then extracted the acceleration gain and phase data from the data file and calculated, then plotted, the FRF for the system.

The stress/strain extraction code required input of the name of the part instance in the assembly, and of the node/element set being analyzed. The code then systematically checked the data file for predetermined keywords to find the data asked for. It extracted data at each frequency increment in the analysis and stored it in various matrices. The code also needed a frequency increment number (that usually corresponded to the resonance frequency) to produce a plot of stress or strain at a particular frequency. This was input near the end of the code.

Transmissibility Code

```matlab
clear
clc

%%% In Abaqus, PU gives phase and magnitude of displacement
%%% Node 6776 is the GLOBAL Node of node 1083 on the top mass

tic % Start timer

%%% BASE ACCELERATION
Base_Input = 3;
Base_Accel = Base_Input*9.80665; % Magnitude of the Base Acceleration in Abaqus

%%% DATA FILE SEARCH PARAMETERS
Name_begin = 8; % Where node number begins in PHYSICAL columns (changed for when nodes change order of magnitude)
Num_begin = 8;
Column_begin = 17; % Beginning PHYSICAL column number for the first data of interest
Column_end = 29; % Last PHYSICAL column number in the tabular data
real_imag_col = 1; % Which TABULAR column is the real and imaginary data in?
mag_phase_col = 1; % which TABULAR column is the magnitude and phase data in?
```

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fid = fopen('Beam_Analysis.dat','r');
i=0;

%% MAIN DATA EXTRACTION

while (feof(fid)~=1)                     % Physical Column to look for Frequency
    tmp = fgets(fid);
    if(regexp(tmp,'FREQUENCY')~=31)    % Physical Column to look for Frequency
        i=i+1;
        tmp=str2num(tmp(42:55));
        freq(i) = tmp;
    elseif(regexp(tmp,'MINIMUM')~=2)  % Physical Column to look for Frequency
        p(i) = tmp(2);
    elseif(regexp(tmp,'NODE')=='Name_begin')   % Physical Column to look for Frequency
        while(isempty(regexp(tmp,'ITERATION', 'once')))
            tmp = fgets(fid);                     % Name Begin is the column the node
            if(regexp(tmp,'6766', 'once')=='Num_begin') % 6766 is the GLOBAL node of
                tmp=str2num(tmp(Column_begin:Column_end)); % Real Portion Number in tmp() corresponds
                real(i)=tmp(real_imag_col);                 % TABULAR column of data
                %U_mag(i)=tmp(mag_phase_col);             % Magnitude Number in tmp() corresponds to
                %U_phase(i)=tmp(mag_phase_col);                 % TABULAR column of data
                break;
            end
        end
    end
end
fclose(fid);
clear i;
disp('Done!!!!')

%% FRF CALCULATION

Magnitude_WRONG = sqrt(real.^2 + imaginary.^2);
FRF_Mag_WRONG = (Magnitude_WRONG + Base_Accel)./Base_Accel;

Acceleration = real + i.*imaginary;
Total_Accel = Acceleration + Base_Accel;
Accel_Mag_FRF = Total_Accel./Base_Accel;
FRF_Mag = abs(Accel_Mag_FRF);

%% PLOTTING FRF

deployed(freq,FRF_Mag)
title('Abaqus Acceleration Transmissibility')
xlabel('Frequency (Hz)')
ylabel('Magnitude ((m/s^2)/(m/s^2)')

time = toc;       % Stop timer
Seconds_to_Run = time
clear all
close all
clc
datfilename='Strain_Analysis';
inpfilename='Strain_Analysis';
var1=0; %U
var2=1; %S12;
var3=1; %S23;
var4=0; %Charge

Uset=[];
Upart=[];
Sset='PIEZO_TOP';  % Set of Interest IN ALL CAPS
Spart='PIEZ0';  %instance name IN ALL CAPS
V=50;  % Used with Impedance
Zmin_option=0;  %Impedance (0 for no, 1 for yes)
dimension=3;
elem_nodes=8;  % Number of nodes per element (8-noded brick)
num_act=1;
rectmesh_option = 0;  % set to '1' if rectangular mesh is used for specified set, and output sorted by x-y location is desired

%% Data Line Indicators and Formats
% Specific strings used to locate values and parameters in .inp and .dat
% files, as well specific data formats used to extract values from these % files.

flagline1 = '  INCREMENT NUMBER';
flagline2 = ' NOTE ';
flagline3 = ['  Sset ':'];
flagline4 = '  number of elements:';
flagline5 = '  FREQUENCY RANGE DEFINITIONS (CYCLES/TIME)';
flagline6 = ['  Instance, name=' Spart];
flagline7 = '  Element, type=';
flagline8 = '  GLOBAL TO LOCAL NODE AND ELEMENT MAPS';
flagline9 = ['  Nset, nset=' Sset ', instance=' Spart];
flagline10 = ['  Elset, elset=' Sset ', instance=' Spart];% ', generate'];%
flagline11 = ['  Nset, nset=' Uset ', instance=' Upart];
flagline12 = ['  Elset, elset=' Uset ', instance=' Upart];%]
flagline13 = '  ALL VALUES IN THIS TABLE ARE ZERO';
flagline14 = ['  Uset '];
flagline15 = ['  Part, name=' Upart];
flagline16 = '  number of nodes:';
flagline17 = '  NODE FOOT- PHCHG  ';
flagline18 = '  NODE FOOT- PU1   PU2';
flagline19 = '  ELEMENT ND FOOT- PHS';
flagline20 = '  element sets:';

format1 = ['    FREQUENCY =  %f'];
format2 = ['        %d %d %e %f' sprintf('
')... , %d %d SSD %f %f' sprintf('
')];
format3 = ['        number of elements: %d' sprintf('
')];
format4 = ['        %e %e %d %f %f Linear', sprintf('
')];
if dimension == 3
format5 = ['        %d, %f, %f, %f' sprintf('
')];
elseif dimension == 2
format5 = ['        %d, %f, %f, %f' sprintf('
')];
end
format6 = ['        %d %d %s'];
format7 = ['        %d %d %d %d %d %d %d %d %d %d %d %d', sprintf('
')];
format8 = ['        THE FOLLOWING TABLE IS PRINTED FOR NODES BELONGING TO NODE SET ASSEMBLY', sprintf('
')];
format9 = ['        %d %e' sprintf('
')...
'     %d SSD  %f' sprintf('n');

format10 = ['     %d      %d      %f', '
     %d      %d SSD     %f' sprintf('n')];

if dimension==3
    format11 = ['     %d      %d      %e  %e  %e', '
     %d SSD  %f      %f      %f' sprintf('n')];
else
    format11 = ['     %d      %d      %e', '
     %d SSD  %f      %f' sprintf('n')];
end

format12 = ['      number of nodes: %d', sprintf('n')];

if dimension==3
    skip1 = 8;  
    % values for data formatting, for displacement output
    skip2 = 8;  
    % values for data formatting, for both S13 and S23 output
else
    skip1 = 6;  
    % values for data formatting, for displacement output
    skip2 = 6;  
    % values for data formatting, for both S13 and S23 output
end

skip3 = 6;  
% values for data formatting, for single S13 or S23 output
skip4 = 4;  
% values for data formatting, for charge output

% % Open .dat and .inp Files
% Open .dat and .inp files and assign them file identifiers to access them
% later.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

OldFolder=cd;

% Assign file identifiers for .dat and .inp files
% OldFolder=cd;
% datfolder=OldFolder;
cd(datfolder);

% Open .dat and .inp files and assign them file identifiers to access them
% later.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% The following will execute if 'PHS13' or 'PHS23' shear stress output was requested
if var2==1 || var3==1
    fseek(fid_inp,0,'bof');
    eofflag = 0;
    nodeinx=1;
    wbh=waitbar(0,'Determining Nodal Coordinates for Part "' Spart '"');
    while eofflag == 0
        currentline = fgetl(fid_inp);
        if currentline == -1
            eofflag = 1;
        elseif strncmp(currentline,flagline6,length(flagline6))==1
            for i=1:1
                currentline = fgetl(fid_inp);
                if currentline == -1
                    eofflag = 1;
                end
            end
            currentline = fgetl(fid_inp);
            fseek(fid_inp,prevline,'bof');
            currentline = fgetl(fid_inp);
            fseek(fid_inp,prevline,'bof');
        end
    end
    close(wbh)
end
%% Determine Number of Nodes in Set(s)
% Determine the total number of elements in the specified set(s),
% from the .dat file.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%\n\n% The following will execute if 'PHS13' or 'PHS23' shear stress output was requested
if var2==1 || var3==1
    fseek(fid,0,'bof');
    eofflag = 0;
    wbh=waitbar(0,['Determining Number of Nodes in Set "' Sset '"]);
    while eofflag == 0
        currentline = fgetl(fid);
        if currentline == -1
            eofflag = 1;
        elseif strncmp(currentline,flagline3,length(flagline3))==1
            % IN .DAT: number of elements: 6 (Line 299)
            location = ftell(fid);
            % 11994 bytes at the beginning of the current
            % line above (Line 299)
            if strncmp(currentline,flagline16,length(flagline16)-2)==1
                fseek(fid,prevline,'bof');
                % Resets cursor to 1196 bytes after the
                % beginning of file (Line 298)
                ftell(fid);
                Snum_nodes=fscanf(fid,format12,1);
                eofflag = 1;
            end
        end
    end
end
% close(wbh)

fseek(fid,0,'bof');
%%% RESET CURRENT LINE TO FIRST LINE

%% THIS IS CORRECT %%%%\n\ndisp('Number of Nodes in Set(s) Determined!')

%% Compile Node List for Set(s)
% Read the list of nodes that comprise the specified set(s), from the
% .inp file.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%\n\n% The following will execute if 'PHS13' or 'PHS23' shear stress output was requested
if var2==1 || var3==1
    fseek(fid,0,'bof');
    eofflag = 0;
    SSetNodes=zeros(Snum_nodes,1);
    wbh=waitbar(0,['Determining Nodes in Set "' Sset '"]);
    while eofflag == 0
        currentline = fgetl(fid);
        if currentline == -1
            eofflag = 1;
        elseif strncmp(currentline,flagline3,length(flagline3))==1
            % IN .DAT: number of elements: 6 (Line 299)
            location = ftell(fid);
            % 11994 bytes at the beginning of the current
            % line above (Line 299)
            if strncmp(currentline,flagline16,length(flagline16)-2)==1
                fseek(fid,prevline,'bof');
                % Resets cursor to 1196 bytes after the
                % beginning of file (Line 298)
                ftell(fid);
                for count=1:3
                    currentline = fgetl(fid);
                end
                tempread = fscanf(fid,format7);
                setinx1 = 1;
                for count=1:3
                    currentline = fgetl(fid);
                end
                setinx2 = setinx1 + length(tempread)-1;
                SSetNodes(setinx1:setinx2) = tempread;
                prevline=f.tell(fid);
                currentline = fgetl(fid);
            end
        end
    end
end

fseek(fid,0,'bof');
%%% RESET CURRENT LINE TO FIRST LINE

%% THIS IS CORRECT %%%%\n\ndisp('Nodal Coordinates for Part(s) Determined!')

%% THIS IS CORRECT %%%%
fseek(fid,prevline,'bof');
setinx1 = setinx2 + 1;
end
eofflag = 1;
clear tempread
end
end
eofflag = 0;
tempread=fscanf(fid,format6,3);
partname=char(tempread(3:end));
wbh=waitbar(0,['Global to Local Node Mapping for Part "',Spart,'"']);
while eofflag == 0
  currentline = fgetl(fid);
  if currentline == -1
    eofflag = 1;
  elseif strncmp(currentline,flagline8,length(flagline8))==1
    for ii=1:7
      currentline=fgetl(fid);
    end
  end
end
close(wbh)
end
%%% THIS IS CORRECT %%%
disp('Number of Elements in Set(s) Determined!')

% Determine Global to Local Node Mapping
% Determine the relationship between local and global node
% numbering from the list in the .dat file in order to directly compare
% nodal coordinates (using local node numbering) and output variable
% values (using global node numbering).

%%% THIS IS CORRECT %%%
disp('Node List for Sets Compiled!')

% Determine Number of Elements in Set
% Determine the total number of elements in the specified set for stresses,
% from the .dat file.

if var2==1 || var3==1
  eofflag = 0;
  wbh=waitbar(0,['Determining Number of Elements in Set "',Sset,'"']);
  while eofflag == 0
    currentline = fgetl(fid);
    if currentline == -1
      eofflag = 1;
    elseif strncmp(currentline,flagline3,length(flagline3))==1
      fgetl(fid);
      prevline=f.tell(fid);
      currentline = fgetl(fid);
      if strncmp(currentline,flagline4,length(flagline4))==1
        fseek(fid,prevline,'bof');
        num_elem=fscanf(fid,format3,2);
        eofflag = 1;
      end
    end
  end
end
close(wbh)
end

% The following will execute if 'PHS13' or 'PHS23' shear stress output was requested
if var2==1 || var3==1
  fseek(fid,0,'bof');
  eofflag = 0;
  tempread=fscanf(fid,format6,3);
  partname=char(tempread(3:end));
  wbh=waitbar(0,['Global to Local Node Mapping for Part "',Spart,'"']);
  while eofflag == 0
    currentline = fgetl(fid);
    if currentline == -1
      eofflag = 1;
    elseif strncmp(currentline,flagline8,length(flagline8))==1
      for ii=1:7
        currentline=fgetl(fid);
      end
    end
  while strncmp(partname,Spart,length(Spart))==0

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tempread=fscanf(fid,format6,3);
partname=char(tempread(3:end));
end
Sglobal_num = tempread(1)-1;
eofflag = 1;
end
close(wbh)
end

%%% THIS IS CORRECT %%%%
disp('Global to Local Node Map Created')

%% Determine Number of Frequency Increments in Step
% Determine the total number of frequency increments in the .dat file.
%******************************************************************************
listflag = 0;
wbh=waitbar(0,'Determining Number of Frequency Increments in Step');
while listflag == 0
    currentline = fgetl(fid);
    if strncmp(currentline,flagline5,length(flagline5))==1
        for ii=1:3
            fgetl(fid);
        end
        tempread = fscanf(fid,format4,5);
        numinc = tempread(3); % Number of Frequency Steps
        clear tempread
        listflag = 1;
    end
end
close(wbh)

%%% THIS IS CORRECT %%%
disp('Frequency Increment Determined!')

%% Extract Data at All Frequencies
% Extract the requested output variable data from the tables in the
% .dat file for all frequencies.
%******************************************************************************
if Zmin_option ~= 1
    wbh=waitbar(0,'Extracting Output Data from .dat File...');
    finx=1;
    prevline=0;
    eofflag = 0;
    FData=zeros(numinc,1);
    if var1==1
        % Creates a 48 x 5 x # frequency steps matrix
        RawUData=zeros(Unum_nodes,7,numinc);
    elseif var2==1 || var3==1
        RawSData=zeros(num_elem*elem_nodes,3,numinc);
    end
    while eofflag == 0
        currentline = fgetl(fid);
        if currentline == -1
            eofflag = 1;
        elseif strncmp(currentline,flagline1,length(flagline1))==1
            prevline;
            currentline;
            % Current line first time through is INCREMENT NUMBER 1
else

fseek(fid, prevline, 'bof'); % Cursor now at beginning of INCREMENT NUMBER line
currentline = fgetl(fid); % Cursor now at beginning of INCREMENT NUMBER line
tempread = fscanf(fid, format1, 1);
FData(finx) = tempread(1);

%%% CORRECT AFTER FREQUENCY ARRAY %%%

for ii = 1:16
    currentline = fgetl(fid);
end

clear tempread

% Current Line - NOTE (footnote portion in data table)

% The following will execute if 'PHS13' and/or 'PHS23' shear stress
% output was requested
if var2 == 1 || var3 == 1
    nodeinx = 1;
    if var2 == 1 && var3 == 1
        DataTemp = zeros(num_elem*elem_nodes, 5);
    else
        DataTemp = zeros(num_elem*elem_nodes, 3);
    end
    while nodeinx <= num_elem*elem_nodes
        if var2 == 1 && var3 == 1
            tempread = fscanf(fid, format2, skip2);
            DataTemp(nodeinx, :) = [tempread(2) tempread(3) tempread(7) tempread(4) tempread(8)];
        else
            tempread = fscanf(fid, format10, skip3);
            DataTemp(nodeinx, :) = [tempread(2) tempread(3) tempread(6) tempread(7) tempread(8)];
        end
        nodeinx = nodeinx + 1;
    end
    RawSData(:, :, finx) = sortrows(DataTemp);
    clear tempread

    currentline = fgetl(fid); % MAXIMUM (Line 131993)
    for ii = 1:6
        currentline = fgetl(fid);
    end
end

waitbar(finx/numinc, wbh)

finx = finx + 1;

end

prevline = ftell(fid);

close(wbh)
end

%%% THIS IS CORRECT %%%

disp('Data Extraction Complete!')

% Calculate Impedance Values from Charge Data

clear i;
if var2 == 1 || var3 == 1
    else
        RawSDataComp(:, 1, :) = RawSData(:, 1, :); % Node Number

    end

clear tempread

% Current Line - NOTE (footnote portion in data table)

% The following will execute if 'PHS13' and/or 'PHS23' shear stress
% output was requested
if var2 == 1 || var3 == 1
    nodeinx = 1;
    if var2 == 1 && var3 == 1
        DataTemp = zeros(num_elem*elem_nodes, 5);
    else
        DataTemp = zeros(num_elem*elem_nodes, 3);
    end
    while nodeinx <= num_elem*elem_nodes
        if var2 == 1 && var3 == 1
            tempread = fscanf(fid, format2, skip2);
            DataTemp(nodeinx, :) = [tempread(2) tempread(3) tempread(7) tempread(4) tempread(8)];
        else
            tempread = fscanf(fid, format10, skip3);
            DataTemp(nodeinx, :) = [tempread(2) tempread(3) tempread(6) tempread(7) tempread(8)];
        end
        nodeinx = nodeinx + 1;
    end
    RawSData(:, :, finx) = sortrows(DataTemp);
    clear tempread

    currentline = fgetl(fid); % MAXIMUM (Line 131993)
    for ii = 1:6
        currentline = fgetl(fid);
    end
end

waitbar(finx/numinc, wbh)

finx = finx + 1;

end

prevline = ftell(fid);

close(wbh)
end

%%% THIS IS CORRECT %%%

disp('Data Extraction Complete!')

% Calculate Impedance Values from Charge Data

clear i;
if var2 == 1 || var3 == 1
    else
        RawSDataComp(:, 1, :) = RawSData(:, 1, :); % Node Number

    end

RawSDataComp(:,2,:) = RawSData(:,2,:).*exp(1i*RawSData(:,3,:)/180*pi); % Real and Imaginary of S12
if var2==1 && var3==1
    RawSDataComp(:,3,:) = RawSData(:,4,:).*exp(1i*RawSData(:,5,:)/180*pi); % Real and Imaginary of S23
    RawSDataComp(:,4,:) = sqrt(RawSDataComp(:,2,:).^2 + RawSDataComp(:,3,:).^2); %"Magnitude" of Columns 2 and 3
end
end
end

disp('Data Rewritten in Complex Terms!')

%% Average at Unique Nodes
% Average complex output variable values at each unique nodes. Since nodes are shared by up to 4 adjacent elements, multiple output values will exist for various nodes. These values may be different from one another due to inaccuracies in extrapolation from the integration point(s).

if var2==1 || var3==1
    if Zmin_option == 1
        %
        %
    else
        RawSDataComp(:,1,:)=RawSDataComp(:,1,:)-Sglobal_num;
caca=unique(RawSDataComp(:,1,:));
x=1;
for kap=1:length(caca)
    if caca(kap)>0
        uniq_nodes(x)=caca(kap);
        x=x+1;
    end
end
SortedSData=zeros(length(uniq_nodes),dimension+1,numinc);
wbh=waitbar(0,['Averaging at Unique Nodes in Set "' Sset '"']);
for uni=1:length(uniq_nodes)
    node_inx=find(squeeze(RawSDataComp(:,1,:))==uniq_nodes(uni));
    if length(node_inx) == 1
        SortedSData(uni,:,:)=RawSDataComp(node_inx,:,:);
    else
        SortedSData(uni,:,:)=mean(squeeze(RawSDataComp(node_inx,:,:)),1);
    end
end
close(wbh)
end
%clear RawSDataComp
end

disp('Complex Data Averaged at Unique Nodes!')

%% Remove Nodes not in Specified Node Set
% Remove any nodal values that are not part of the specified node set. Since stress is an element value, nodal output for nodes contained in the element set but not the node set exist in the extracted stress data.

if var2==1 || var3==1
    if Zmin_option == 1
        %
        %
    else
        [aa, ia, cc] = intersect(squeeze(SortedSData(:,1,:)), (SSetNodes - Sglobal_num));
        StressData = SortedSData(ia,:,:);
        clear aa cc
end
end

[a, ia, cc] = intersect(SPartNodes(:,1), (SSetNodes - Sglobal_num));
SSetNodeCoords(:,1) = SSetNodes - Sglobal_num;
SSetNodeCoords(:,2:dimension+1) = SPartNodes(ia,2:dimension+1);

%clear SortedSData
%clear aa cc
end

disp('Removed Nodes not in the Node Set!')

%% Average Shear Stress Values over Set when at Local Maximum
% Calculate average shear stress values over the interface, for each node
% when the stress field is at a maximum locally.
%******************************************************************************
if var2==1 || var3==1
  else
    if dimension==3
      S13_avg = mean(squeeze(abs(StressData(:,2,:))),1);
      S23_avg = mean(squeeze(abs(StressData(:,3,:))),1);
      Smag_avg = mean(squeeze(abs(StressData(:,4,:))),1);
    elseif dimension ==2
      S12_avg = mean(squeeze(abs(StressData(:,2,:))),1);
    end
  end
end

disp('Average Shear Stress Over Set when at Local Maximum Calculated!')

%% Save Data to .mat Files
% Save extracted and sorted data to .mat file named after .dat file name.
%******************************************************************************
wbh=waitbar(0,'Saving Data to File');
savefolder=OldFolder;
cd(savefolder)
save(['FOutputData_'.datfilename '.mat'], 'FData');
if var2==1 && var3==1
  SNodeData=struct('Headings',{ 'Node Numbers', 'x', 'y', 'z' }, 'Data', SSetNodeCoords);
  if rectmesh_option == 1
    SData=struct('Headings', { 'Node Numbers', 'S13', 'S23', 'Smag' }, 'Data', StressData, ...'
  else
    SData=struct('Headings', { 'Node Numbers', 'S13', 'S23', 'Smag' }, 'Data', StressData);
  end
  SavgData=struct('Headings', { 'S13 avg', 'S23 avg', 'Smag avg' }, 'Data', [S13_avg S23_avg Smag_avg]);
  save(['SOutputData_'.datfilename '.mat'], 'SNodeData', 'SData', 'SavgData');
end
disp('Data Saved to .mat File!')

% close(wbh)
clear x
cd(OldFolder)

x = unique(SSetNodeCoords(:,2));
y = unique(SSetNodeCoords(:,4));
Increment = 252;  % Frequency Increment of Resonance
disp('Done with Data Processing!')

start = Snum_nodes - sqrt(num_elem);
for counter = 1:((sqrt(num_elem)) + 1)
    StressMatrix13Comp(:,counter) = StressData(start:start + sqrt(num_elem),2,Increment);
    StressMatrix23Comp(:,counter) = StressData(start:start + sqrt(num_elem),3,Increment);
    start = start - (sqrt(num_elem) + 1);
end

StressMatrix13 = abs(flipud(StressMatrix13Comp));
StressMatrix23 = abs(flipud(StressMatrix23Comp));
Micro = 1e6;

inter = mean(StressMatrix13);
Mean_Strain = mean(inter)*Micro

interm = max(StressMatrix13);
Max_Strain = max(interm)*Micro

figure()
contourf(x*1e3,y*1e3,StressMatrix13*Micro)
gtext
xlabel('x-coord (mm)')
ylabel('y-coord (mm)')
set(gca,'YDir','reverse')
title('E11 Strain on Top of Piezo Patch in Microstrain')
colorbar

figure()
contourf(x*1e3,y*1e3,StressMatrix23*Micro)
gtext
xlabel('x-coord (mm)')
ylabel('y-coord (mm)')
set(gca,'YDir','reverse')
title('E22 Strain on Top of Piezo Patch in Microstrain')
colorbar
**Appendix D – Methods of Bonding Piezoelectric Material to Beam**

Three different methods of obtaining an electrical connection to the bottom of the piezoelectric patch were investigated and tested. These methods were:

1) Using contact between the piezoelectric patch’s bottom surface and the beam surface through the bond layer,

2) Attaching a thin wire to the piezo bottom in and bonding the piezo to the beam, and

3) Epoxying a very thin copper tab to the bottom of the piezo that sticks out for a lead wire to be soldered.

The first method relied solely on the roughness of the metal beam and piezoelectric patch to create an electrical connection based on inconsistencies in the bond layer. Figure D-1 illustrates the theory behind this connection method.

![Figure D-1. First method of obtaining electrical connection to underside of piezoelectric patch on the lab asset.](image)

This method was unreliable as the contact points were shown to get disconnected under vibration, leading to unpredictable losses in voltage.
The second method used a thin wire attached directly to the piezoelectric surface, and then the whole wire/piezo assembly was bonded to the beam. Figure D-2 illustrates this bonding method.

This method proved to be electrically reliable, however, the wire created areas of lower strain on the piezo, which reduced overall output voltage and deeming this method unfit for energy harvester use.

The final method utilized a very thin copper tab epoxied to the piezo using conductive epoxy to establish good electrical connection. The thin material minimized the area taken from the piezo to beam epoxy. Figure D-3 illustrates the final method which was used in the final lab asset tests and all compact harvester designs.
References


50. Course notes from Aersp/E Mch/M E 571, Penn State Graduate Program in Aerospace Engineering, 2011.


