AN OPTION-BASED METHOD FOR REVENUE MANAGEMENT IN THE AIRLINE INDUSTRY WITH TWO CLASSES

A Thesis in
Industrial Engineering

by

Chang Liu

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The thesis of Chang Liu was reviewed and approved* by the following:

Jeya M. Chandra  
Professor, Department of Industrial and Manufacturing Engineering  
Thesis Advisor

Saurabh Bansal  
Assistant Professor, Department of Supply Chain and Information Systems

Paul Griffin  
Professor, Department of Industrial and Manufacturing Engineering  
Head of the Department of Industrial and Manufacturing Engineering

*Signatures are on file in the Graduate School
ABSTRACT

In the context of airline industry, the revenue management is the practice of managing the ticket sales in an airline booking process with an important target of maximizing the sales revenues. The topic has been studied since the early 1970s. In recent years, an application model, which uses the financial option theory in revenue management for single-fare single-leg problem in airline industry, has been designed.

The purpose of this thesis is to advance the model to make it more reasonable to apply. An airline may be considered as the holder of both call option tickets for customers and put option tickets for travel agents. Based on different demand situations, the airline could offer two levels of tickets to its customers at any period for a single leg. Also, at the last minute before departure, the overbooked passengers may choose to travel in a different class or not to travel, rather than waiting for the next flight. The potential revenue loss caused by customer dissatisfaction, which results from denied boarding, is also taken into consideration. These factors determine the optimal booking limits for call and put option tickets, and the optimal prices for the two options. A general description of the whole booking process is presented. At last, a numerical example is given to illustrate the application of the model.
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Chapter 1

Introduction

1.1 Introduction on Revenue Management and Financial Option Theory

1.1.1 Revenue Management in the Airline Industrial

Revenue management has got much attention as a fruitful practice in the airline industry, since the deregulation of the fares for airlines in 1978 [1]. It is precisely because of deregulation that airlines start to sell the same seats in the cabin of an aircraft to different customers at different prices during the booking period. For airline industry, the revenue management is defined as the applications of managing the booking requests with a goal of maximizing the sales profit. The basic theory uses the revenue management methods, which are demand forecasting, capacity control, overbooking and dynamic pricing, to decide the fares and the number of reserved seats for each customer segment. In addition to the airline industry, revenue management makes significant contribution to the performance of other areas, such as hotel management and car rental.

Ever since the mid 1980s, major US airlines began to refer to the revenue management to combat the influx of low-cost carrier airlines and earn enormous profits. American Airlines estimated that RM techniques improved their revenues by $500m annually from 1989 to 1992. They took advantage of the demand forecast data to determine the right number of seats that should be reserved at the right discount rate to maximize the profits of each flight [2]. Other companies such as United Airlines made similar records on their budget report for the revenue improvement by implementing revenue management in their business.
However, the implementation of revenue management also has its shortcomings. One of the shortcomings is the latent undesirable impact on customer relations that the methodology generates. Being denied boarding caused by overbooking or price fluctuations which result from seat inventory control may lead to the generation of dissatisfied customers. Consistent with the theory of service-profit-chain [3], negative views of service performance may result in diminished customer satisfaction and therefore in the fading of the company’s economic success.

1.1.2 Financial Option Theory

In general, financial options are used as a tool for moving the firm’s risk disclosure. They are widely used in many industries, such as real state, metals, etc. to cope with the risk and inventory. In finance, an option is a contract which gives the owner the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified strike price on or before a specified date (from Wikipedia). Hence, the options can be categorized into two groups. One option, which takes the right to purchase something at a specific price, is called a call option; the other, which conveys the right to sell something at a specific price, is called a put option. There are several types of options in the world. Among them, two types, which are American option and European option, are widely used. The only difference between these two types is the execution time. A European option allows for execution of the option only on the specified date. However, an American option allows executing the option on or before the date.

First call option is considered. For a certain asset, a buyer could first pay a minor amount of capital, which is called a Premium, to obtain a right. The right means the buyer could decide whether to execute the right on or before an Expiration date or not. If the buyer decides to execute
the right, he/she just needs to pay the Striking Price to buy the asset. The two counterparties would have a contract with the terms of the option specified in a term sheet. The Premium, the Expiration date and the Striking Price are all pre-decided on the contract. There is a time-difference between the time of buying the call option and the date on which the option could be executed. The market price would fluctuate during the period. If on the Expiration date, the market price is higher than the accumulation of the premium and the striking price, executing the option at the striking price is a more cost-effective choice. The difference between the market price and the accumulation of the two parts is how much the buyer would save. On the contrary, the buyer would prefer not to execute, and the loss is only the premium. So the call option helps the buyer to cushion the risk as well as benefits the buyer.

The airline tickets can also be seen as an asset, because it is difficult to forecast the market price. After all, many factors affect the market price. In this way, an airline could pay certain premium to buy the right that it can call back the tickets, which have been sold to customers previously, at a striking price when the market price was higher than the aggregation of the premium and the striking price. Here, customers could be either individual customers or travel agents.

The above just simply justifies that the financial options theory is applicable to the revenue management in the airline industry. In the subsequent part of the thesis, a specified and thorough rationalization will be given.

1.2 Literature Review

Revenue Management was born with the deregulation of airline industries in the 1970s. It
is a method using which an airline earn maximize its revenue or profit by selling the right seat to the right type of customer, at the right time and for the right price, according to American Airlines mentioned in Weatherford and Bodily [4]. To obtain this goal, several ideas and techniques have been widely proposed in literature. To better understand the problem, this review presents an overview of these studies. It can be categorized into three groups: dynamic pricing, overbooking and techniques used in capacity allocation process.

1.2.1 Dynamic Pricing

The airline faces the challenge of selling a fixed amount of seats within a specific time horizon. Furthermore, each airline always offers different kinds of ticket to its passenger, which will compete with each other. The only thing the airline could do is to set prices at each stage of the given selling horizon in order to maximize revenues while facing an uncertain demand.

The revenue management literature generally assumes a fixed range of pre-determined prices and attempts to allocate capacity for different price levels. This assumption can be tracked back to Littlewood, when he first tackles the network capacity control problem [5]. Talluri and Van Ryzin further propose the concept of bid-price controls [1]. On the other hand, the dynamic pricing literature assumes that demand follows a certain probability distribution. Elmaghraby and Keskinocak (2003) present a good overview of the literature and current practices in dynamic pricing [6]. Gallego and van Ryzin (1997) and Paschalidis and Tsitsiklis (2000) study dynamic pricing models [7]. However, a disadvantage of dynamic pricing is that the accurate demand distribution is difficult to determine. To avoid this problem, using historical data is proposed. By using past historical data, Van Ryzin and McGill first introduces an adaptive algorithm for booking limits [8]. Another typical form of data-drive approach is known as Sample Average
Approximation (SAA) according to Lobel [9]. Kleywegt et al. show that the SAA solution is optimum for certain scenario [10].

1.2.2 Overbooking

Overbooking is the practice of selling seats beyond the capacity of an aircraft in attempt to maximize seat utilization, avoid empty seats and minimize revenue losses. The characteristics of an airline seat are perishability, advanced bookings and high fixed cost vs. low variable cost, which encourage overbooking to be a necessary opportunity to improve revenue. Smith et al. from American Airlines estimate that 15% of seats on sold-out flight would be lost without overbooking and that the benefit of overbooking in America in 1990 exceeded $225 million [2].

There are a lot of researches concerning the overbooking problem. By using different techniques and algorithms to forecast the no-show rate and control the overbooking level, researchers in this field proposed different models to gain the optimal profit. Littlewood is the first to apply the marginal seat revenue principle to optimize seat overbooking levels for two classes and single-leg flights [5]. Chatwin (1999) analyzes a model of airline overbooking in which customer cancellations and no-shows are explicitly considered. He models the reservations process as a continuous-time birth-and-death process with rewards representing the fares received and refunds paid representing the penalty [11]. Gallego (1996), Lee and Hersh (1993) and Rothstein (1985) study the overbooking policy and bid-price control [12]. However, the implementation of revenue management and overbooking also has its drawbacks. The researches of McMahon-Beattie, Yeoman, Palmer, and Mudie (2002) and McMahon-Beattie (2006) are primarily concerned with the construct of customer trust. Managerial recommendations for the reduction of customer dissatisfaction can be found in Noone, Kimes, and Renaghan (2003) and
Wirtz et al. (2003). Lindenmeier and Tscheulin (2008) analyze negative effect of denied boarding due to overbooking or price fluctuations caused by seat inventory control on customer satisfaction and found that denied boarding leads to distinct dissatisfaction reactions [13].

1.2.3 Techniques used in Capacity Allocation Process

The capacity allocation problem in airline revenue management concerns the allocation of a finite seat inventory to the demand that occurs over time for both the single-leg flights. Lack of an accurate demand forecast and the difficulty in solving large-scale dynamic programming problems are the two major challenges confronted by the airlines.

For single-leg capacity control problem, there are two categories of the solution methods: static and dynamic. In static problem, it is assumed that demand for different fare classes arrives sequentially, which means that the booking requests for the lowest class come first. However, for the models, which use dynamic method, the booking limits are updated through the airline booking process, according to the actual status of booking.

Littlewood (1972) was the pioneer who developed an Expected Marginal Seat Revenue (EMSR) approach to get an approximated booking limit for the single-leg, two-class problem. The booking limitation he proposed is that given the average high fare($f_1$), discounted fare($f_2$), the high fare demand($d_1$), and the high fare protection level($p_1$), the booking limitation should satisfy the following condition:

$$f_2 \geq f_1 \Pr(d_1 > p_1),$$

It means the airline sells discounted fare seats as long as the discounted revenue equals or exceeds the expected marginal return from a full fare booking of the last remaining seat [5]. Then, Belobaba (1987, 1989) extend the results by developing an EMSR approach for a multiple
classes. Because of the lighten computation, his EMSR heuristic provides a natural alternative to
the optimal policy [14].

Van Ryzin and McGill (2000) investigate a simple adaptive approach to optimize seat
protection levels, which uses historical observations to guide adjustments of the protection levels.
The distinctive advantage of this approach is that it is not based on demand forecasting and
therefore it is a way to get around all difficulties related to forecasting. However, in order to set a
good protection level, the updating process needs a sufficiently large sequence of flights to
converge.

Lee and Hersh (1993), Chatwin (1996, 1998), Liang (1999), and Subramanian et al.
(1999) study this dynamic type of seat inventory control. Gosavi et al. (2002) accommodate all
the realistic factors and model the single leg problem as a Semi-Markov decision problem
(SMDP). These factors include multiple fare classes, overbooking of the flight, concurrent
demand arrivals of passengers from the different fare classes, and class-dependent, random
cancellations. They use a stochastic optimization technique, which is called reinforcement
learning to solve their model [15]. In recent years, new types of revenue management
optimization models have been proposed, which effectively handle demand dependencies across
fare classes. These works include displacement adjusted virtual nesting for dependent demands
(Fig et al, 2010) and choice-based Expected Marginal Seat Revenue (EMSR) (Gallego et al,
2009) [16].

Lately, Akgunduz et al. (2007) propose that financial option theory could be used to
maximize the airline revenue. Call options are used to recall the tickets already sold when the
total forecast exceeds the total capacity. Otherwise, put options are exercised to sell low-fare
tickets in the last booking period to avoid empty seats [17]. Then, Ravelojaona (2008) works on a
similar topic, and advances the model from Akgunduz et al. (2007). She assumes that ticket price
follows a random walk, which is more realistic than a constant market price [18]. However, the
option she uses is the European option, which means that the options could be executed only at some specific dates. The inflexible execution may not result in the optimal solution all the times. Zhang (2009) divides the whole booking process into several exclusive stages and uses both call and put options to improve expected revenue. She also adopts the American option to give more flexibility to the airline to manage the whole booking process [19]. The work of Hui et al. (2008) is also based on the American option, but the price is modeled as a random process [20]. The works of Zhang (2009) and Hui et al. (2008) have a similar limitation, which is that only one fare class is considered. Also, even though both of them take no-shows into consideration, they ignore the potential revenue loss caused by overbooking and denied boarding.

### 1.3 Contribution of the Thesis

The objective of this thesis is to develop a model with call and put options that can be used by the airlines in attempt to maximize their revenues. The thesis is inspired by Zhang [19] and it will use similar booking stages. Besides this, it is different in the following:

- **a.** Two-fare classes are studied.
- **b.** The overbooked passengers may choose to travel in a different class or not to travel, rather than waiting for the next flight.
- **c.** The potential revenue loss caused by customer dissatisfaction, which results from denied boarding, is also included.

Based on the literature review, there are no papers addressing these problems. Also, the model enables airline companies to improve revenue according to their own scenario.

The research presented in this thesis is categorized according to the following:

- Chapter 2 provides a description of the problem and model formulation.
• Chapter 3 gives a numerical example of the application of the proposed model.

• Chapter 4 includes the conclusion of the thesis and some ideas for future research.
Chapter 2

Introduction

2.1 Problem Description

This thesis studies a two-fare single-leg problem in the airline tickets booking process. “Two fares single-leg” indicates there are two classes, which are first class and economic class, on the plane, and the plane is used to offer trips from one origin to one destination only. It is assumed that the first class passengers could get more convenience service than the passengers in the economy class. In the real world, the booking period is continuous. However, in the model, it is assumed that the selling period is discrete and the total number of periods is a constant number determined by the airline. Also, the selling periods and the total number of intervals are the same for the two classes.

The airline offers three types of tickets for each class passengers: call option tickets, put option tickets and standard tickets. For each customer, he/she only orders one ticket at a time to simplify the model. The put option tickets are provided only to travel agents, while the other two types of tickets are offered to customers. For standard tickets, they are just normal tickets, which are sold at market price. For the other two tickets, as explained in Chapter 1, call option buyers (customers) have the right to buy the tickets at a discounted price, while the put option buyers (travel agents) have the right to sell tickets. To better understand the model, the two options are explained in detail first.

Call option

Customers buy the call option tickets, but the airline retains the right to recall these tickets back at any time. In order to get this right, the airline needs to pay a premium to the customer. That means a customer would save an amount of money, which is equal to the
premium and this could be seen as a discounted ticket. If the airline decides to execute the right and recall these tickets, it has to pay a compensation, which is the striking price, to them. Call option tickets are used to not only attract price-sensitive customers, who may not have any urgent requirements for travelling, but also to create the opportunity to resell these tickets at higher price to gain extra revenue.

**Put Option**

The airline meets the risk that the actual total demand is less than the total capacity. If it happens, some of the seats will be empty, and it causes a negative effect on the revenue of the airline. In order to buffer the risk, the airline pays the put option premium to the travel agents to buy the right. This forces travel agents to buy the put option tickets at the put option striking price, if there are unsold tickets at the end of the selling period. To avoid the unnecessary procedures, travel agents could not physically get the tickets until the time at which the airline decides to execute the right. However, if the actual demand is larger than the total capacity, the airline would not execute the right to gain more revenue.

**Four Stages Model**

In a similar problem studied by Nan Zhang [19] with four stages, only one class has studied. In this research, a similar four stages model is studied with two classes. The four stages selling process is the same for the two classes, so the first class is considered as an illustration.

**Stage 1**

The call option tickets are sold at the beginning of each interval at discounted price to the customers who agree to offer the airline the right that these tickets can be called back anytime in the future at the call option striking price, which has been pre-settled on the contract which both customers and the airline agreed. The penalty to the airline for getting the call back right is that it needs to pay the call option premium as the buying price.
Meanwhile, the airline buys the put option right from travel agents at the price of put option premium. In this way, travel agents can sell at the put option striking price in the future. However, the airline keeps these tickets first rather than giving them to travel agents. There exists the probability that not all the promised tickets could be sold to the travel agents.

**Stage 2**

Stage 2 will start when all the assigned call option tickets are sold. In stage 2, the airline starts to sell standard tickets to customers at the market price. It is assumed that the fluctuation of market price follows a binomial distribution. Different from call option tickets, these tickets cannot be called back any more. The stage ends when all the assigned standard tickets are sold out.

**Stage 3**

Stage 3 will begin if all the assigned call option and standard tickets are sold out and the demand for tickets continues. Also, for customers, the price is still the market price. In this stage, the airline decides whether to sell the remaining tickets to travel agents or customers based on the relationship between striking price and market price. If the market price was greater than the striking price, to improve revenue, the airline would prefer to sell the tickets to customers directly. Otherwise, the airline gives the tickets to travel agents.

Stage 3 will continue until all the allocated put option tickets are sold out. For the situation in which there are still some put option tickets left until the departure of the flight, they will be dumped to the travel agents.

**Stage 4**

If all the tickets are sold out and there still exist customers who need tickets, the airline begins to recall the call option tickets back from those customers at the striking price and then sells them to new customers at the market price. The airline will only recall when the current
market price is higher than the striking price to make sure it can earn more revenue through the process.

Actually, for a certain flight, not all the stages could be covered. The stage at which the booking process may stop is based on the demand for that flight. For instance, if the summation of call option tickets and standard tickets can fulfill the total demand for that flight, stage 2 will be the last stage and stage 3 and stage 4 will not occur.

**Overbooking and Denied Boarding Problems**

It is likely that some customers will not show up at the time of boarding. The airline overbooks its aircraft in an attempt to reduce the revenue loss associated with passenger no-shows. For each class, the airline uses the forecasted no-show rate to decide its total booking limit. For example, if the forecasted no-show rate is higher than the actual value (overestimated forecast), the first class cannot accommodate all the coming passengers who bought the first class tickets and hence some are denied boarding. However, if the forecasted no-show rate is lower than the actual value (underestimated forecast), which means some seats are available for more first class passengers. For the case in which the forecasted no-show rate is exactly equals to the actual no-show rate, no passenger is denied boarding. Due to the existence of two classes, there exists a situation that one of the two classes is underestimated and the other is overestimated. When this situation occurs, the airline offers choices to the denied boarding passengers: change the class or do not fly. Now, it is necessary to illustrate the possible events in detail and define the passenger groups accordingly.

**Drop class passenger:** a first class passenger who decides to fly in the economic class, when overbooking occurs in first class only.

**Up class passenger:** an economic class passenger who decides to fly in the first class, when overbooking occurs in economic class only.
**Not flying passenger:** There are two scenarios, in which a passenger will be labeled as a “not flying passenger”. In the first scenario, when overbookings occur in both the first and economic classes, the passenger who would be denied boarding is called a “not flying passenger”. In the second scenario, when overbooking occurs just in one of the two classes, and the denied boarding passenger chooses to not fly at all, the passenger is also called a “not flying passenger”.

Both the drop class passengers and not flying passengers would get certain amount of compensation. Meanwhile, the compensation for not flying passengers should be more than that for drop class passengers. However, up class passengers, need to pay extra up class fee, which should be less than the ticket price difference between the first and the economic classes.

Another problem is the negative impact caused by overbooking. Drop class passengers and not flying passengers may be dissatisfied with the service, even though they have already obtained certain amount of compensations. Similarly, up class passengers also may be displeased, because they need to pay extra money to get the service, which they might not need. Those dissatisfactions might lead to the loss of customers and eventually result in revenue loss. The thesis adds this negative part as a component of the expected revenue.

The objective of the problem is to maximize the revenue for the airline.

This thesis will mainly discuss two problems:

1) How to determine the optimum number of call and put option tickets?

2) How to determine the striking prices for both call and put option tickets?
2.2 Model Formulation

2.2.1 Variables

\(T\): Total number of periods

\(A_i\): Booking limitation of call option tickets for class \(i\), \(i = 1, 2\)

\(B_i\): Booking limitation of put option tickets for class \(i\), \(i = 1, 2\)

\(C_i\): Total capacity of class \(i\) of a single plane.

\(c_{ci}\): Premium for call option for class \(i\), \(i = 1, 2\)

\(c_{pi}\): Premium for put option for class \(i\), \(i = 1, 2\)

\(D_i\): Total demand of the tickets for class \(i\), \(i = 1, 2\)

\(d_{ij}\): Demand of the tickets in period \(j\) for class \(i\), \(j = 0, 1 \ldots T, i = 1, 2\)

\(l_{ij}\): Upper limit for the number of tickets sold at period \(j\) for class \(i\), \(j = 0, 1 \ldots T, i = 1, 2\)

\(n_{ij}\): Number of the tickets sold at period \(j\) for class \(i\), \(i = 1, 2, j = 0, 1 \ldots T\)

\(n_{ij}^c\): Number of tickets sold as call option tickets in period \(j\) for class \(i\), \(j = 0, 1 \ldots T, i = 1, 2\)

\(n_{ij}^p\): Number of tickets sold as put option tickets in period \(j\) for class \(i\), \(j = 0, 1 \ldots T, i = 1, 2\)

\(n_{ij}^r\): Number of tickets recalled in period \(j\) for class \(i\), \(j = 0, 1 \ldots T, i = 1, 2\)

\(n_{ij}^s\): Number of standard tickets sold in period \(j\) for class \(i\), \(j = 0, 1 \ldots T, i = 1, 2\)

\(P_{ij}\): Price of the standard tickets in time period \(j\) for class \(i\), \(j = 0, 1 \ldots T, i = 1, 2\)

\(q_i\): Estimated no-show rate for class \(i\), \(i = 1, 2\)

\(q_i^a\): Actual no-show rate for class \(i\), \(i = 1, 2\)

\(p_{oi}\): Probability of overestimating the no-show rate for class \(i\), \(i = 1, 2\)

\(M_i\): Compensation paid to a “not flying passenger” in class \(i\), \(i = 1, 2\)

\(M_{12}\): Compensation paid to a “drop class passenger”
$M_{21}$: Extra charge for an “up class passenger”

$p_{12}$: Probability that a first class passenger chooses to be a “drop class passenger”

$p_{21}$: Probability that an economic class passenger chooses to be an “up class passenger”

$S_{ci}$ : Striking prices of call option tickets for class $i, i = 1,2$

$S_{pi}$ : Striking prices of put option tickets for class $i, i = 1,2$

### 2.2.2 Assumption

The following assumptions are made to build the model:

1) The selling period is discrete.

2) Demand in each period is known at the beginning of the period and independent.

3) Although in reality, customers have the right to choose between discounted and standard tickets, in order to make the model numerically easy to solve, it is assumed that all customers prefer the discounted tickets. After such tickets are sold out, the customers have no choice but to buy the standard tickets.

4) Tickets sold to customers as call option could be called back at any period.

5) The market price of the tickets varies according to a binomial distribution.

6) Striking prices are constant for both call and put option tickets.

7) When first class is overbooked and economic class still have some seats left, first class passengers can make their own decisions from two choices. They are to be “drop class passenger” or to be “not flying passenger”.
8) When economic class is overbooked and first class still have some seats left, economy class passengers can make their own decisions from two choices. They are to be “up class passenger” or to be “not flying passenger”.

9) Compensations are constant for every not flying passenger. However, for first and economy classes, the two constant compensations are different.

10) Compensation is constant for every drop class passenger.

11) Extra charge is constant for every up class passenger.

12) The upper bound of call option tickets for each class is 25 percent of its capacity.

13) The upper bound of put option tickets for each class is 30 percent of its capacity.

2.2.3 General Model Objective Function

In this model, the selling period is split into 4 stages. Stage 1 is the period for selling call option tickets; stage 2 is the period when the standard tickets are sold; stage 3 sells the put option tickets; and stage 4 is the period for recalling call option tickets. When the selling period finishes and the boarding time arrives, the airline needs to deal with the denied boarding or empty flying situation. Now, the 4 stages and the boarding time by turns will be discussed.

Before the selling period begins

Here, using put options, the airline just makes promises to the travel agents. The number of tickets hedged with put option is \( B_i \). The premium per ticket is \( c_{pi} \). So the current cash flow is negative, which could be expressed as

\[
R_0 = - \sum_{i=1}^{2} B_i \cdot c_{pi} \quad [2.1]
\]
Stage 1

The first period is call option tickets selling period. In period \( j \), the number of call option tickets sold is \( n_{ij}^c \) for class \( i \). The premium for the airline buying such tickets is \( c_{ci} \) per ticket for class \( i \). For each period, the current market price of a ticket is \( P_{ij} \) . So the revenue for this period is

\[
R_1 = \sum_{i=1}^{2} \sum_{j=1}^{T} n_{ij}^c * (P_{ij} - c_{ci}) \tag{2.2}
\]

For each class \( i \), because call option tickets are sold first, the variable number of call option tickets should either be the demand for period \( j \) or the balance of the call option tickets remaining, which is equal to the booking limit of call option tickets subtracts the number of tickets that have already being sold. So it has the following expression:

\[
n_{ij}^c = \min\left( d_{ij}, \max\left[ 0, A_i - \sum_{k=1}^{j-1} n_{kj}^c \right] \right) \tag{2.3}
\]

Then the revenue is

\[
R_1 = \sum_{i=1}^{2} \sum_{j=1}^{T} \min\left( d_{ij}, \max\left[ 0, A_i - \sum_{k=1}^{j-1} n_{kj}^c \right] \right) * (P_{ij} - c_{ci}) \tag{2.4}
\]

Stage 2

This is the stage of the standard tickets. The number of tickets sold at standard price \( P_{ij} \), for period \( j \) is \( n_{ij}^s \). So the revenue for this stage is

\[
R_2 = \sum_{i=1}^{2} \sum_{j=1}^{T} n_{ij}^s * P_{ij} \tag{2.5}
\]

For each class \( i \), the variable number of call option tickets sold \( n_{ij}^s \) is \( \geq 0 \), and the value should either the demand left for period \( j \) after selling call options tickets or the balance of the standard tickets remaining. Here, no-show is considered in the model, so the total booking limit
for both discounted tickets and the standard tickets is \( \frac{c_i}{1-q_i} \) for class \( i \), and \( \left( \frac{c_i}{1-q_i} - B_i \right) \) is the total of call option tickets and standard tickets for class \( i, i=1,2 \). \( n_{ik} \) is the number of tickets sold at period \( k \) for class \( i \) by the airline regardless of the type of ticket. The number of tickets sold is subtracted from \( \frac{c_i}{1-q_i} - B_i \) to get the remaining standard tickets, if any. Hence, the number of tickets sold for class \( i \) in period \( j \) and the revenue for this stage is respectively

\[
n^s_{ij} = \min \left( d_{ij} - n^c_{ij}, \max \left[ 0, \frac{c_i}{1-q_i} - B_i - \sum_{k=1}^{j-1} n_{ik} - n^c_{ij} \right] \right)
\]

[2.6]

And

\[
R_2 = \sum_{i=1}^{2} \sum_{j=1}^{T} \min \left( d_{ij} - n^c_{ij}, \max \left[ 0, \frac{c_i}{1-q_i} - B_i - \sum_{k=1}^{j-1} n_{ik} - n^c_{ij} \right] \right) P_{ij}
\]

[2.7]

**Stage 3**

In this stage, the airline sells the put option tickets to travel agents at striking price or to customers at market price. If the striking price is higher than the market price, the airline will sell the tickets to travel agent, otherwise, it will sell them to customers. The number of tickets sold for period \( j \) is \( n^p_{ij} \). The revenue could be expressed as

\[
R_3 = \sum_{i=1}^{2} \sum_{j=1}^{T} n^p_{ij} \max (P_{ij}, S_{pi})
\]

[2.8]

For each class \( i \), the number of put option tickets sold \( n^p_{ij} \) should either be the rest of demand for period \( j \) or the rest of the put option tickets. Here, \( \frac{c_i}{1-q_i} \) is the total number of tickets that can be sold. The total number of tickets sold is subtracted from \( \frac{c_i}{1-q_i} \) to obtain the remaining put option tickets, if any. Hence, the number of tickets sold for class \( i \) in period \( j \) and the revenue for this stage is respectively
\( n_{ij}^p = \min \left( d_{ij} - n_{ij}^c - n_{ij}^s, \max \left[ 0, \frac{C_i}{1-q_i} - \sum_{k=1}^{j-1} n_{ik} - n_{ij}^c - n_{ij}^s \right] \right) \) \[2.9\]

And

\[ R_3 = \sum_{i=1}^{2} \sum_{j=1}^{T} \min \left( d_{ij} - n_{ij}^c - n_{ij}^s, \max \left[ 0, \frac{C_i}{1-q_i} - \sum_{k=1}^{j-1} n_{ik} - n_{ij}^c - n_{ij}^s \right] \right) \times \max (P_{ij} - S_{ci}, 0) \] \[2.10\]

**Stage 4**

In this stage, because the excessive demand for the tickets is still coming, the airline could begin to recall the tickets sold as call options and sells them at the market price \( P_{ij} \). However, for each ticket called back, the airline has to pay the customer compensation, which is \( S_{ci} \). To gain more revenue, the airline would recall call option tickets when the market price is greater than the compensation. So the revenue for this stage is

\[ R_4 = \sum_{i=1}^{2} \sum_{j=1}^{T} n_{ij}^r \times \max (P_{ij} - S_{ci}, 0) \] \[2.11\]

Since call option tickets can be recalled, the actual number of tickets that could be sold is \( \left( \frac{C_i}{1-q_i} + A_i \right) \) for class \( i \). Subtracting the number of tickets sold already could result in the number of call option tickets that have not been recalled yet. It could be expressed as

\[ n_{ij}^r = \min \left( d_{ij} - n_{ij}^c - n_{ij}^s - n_{ij}^p, \max \left[ 0, \frac{C_i}{1-q_i} + A_i - \sum_{k=1}^{j-1} n_{ik} - n_{ij}^c - n_{ij}^s - n_{ij}^p \right] \right) \] \[2.12\]

Then, the revenue could be expressed as
\( R_4 = \sum_{i=1}^{S} \sum_{j=1}^{T} \min \left( d_{ij} - n_{ij}^c - n_{ij}^s - n_{ij}^p, \max \left[ 0, \frac{c_i}{1-q_i} + A_i - \sum_{k=1}^{j-1} n_{ik} - n_{ij}^c - n_{ij}^s - n_{ij}^p \right] \right) \times \max (P_{ij} - S_{ei}, 0) \)  

[2.13]

After all or some of the stages come to an end, the whole selling season may stop. At the end, the airline also has to deal with the rest of put option tickets. Because some put option tickets have been sold already, the balance of the tickets, if any, would be sold to travel agents. The selling price is the striking price, which is \( S_{pi} \). Then the revenue is

\[ R_5 = \max \left[ 0, B_i - \sum_{j=1}^{T} n_{ij}^p \right] \times S_{pi} \]  

[2.14]

\[ R_5 = \max \left[ 0, B_i - \sum_{j=1}^{T} \min \left( d_{ij} - n_{ij}^c - n_{ij}^s - n_{ij}^p, \max \left[ 0, \frac{c_i}{1-q_i} + A_i - \sum_{k=1}^{j-1} n_{ik} - n_{ij}^c - n_{ij}^s - n_{ij}^p \right] \right) \right] \times S_{pi} \]  

[2.15]

**Overbooking and denied boarding problems**

For overbooking and denied boarding problems, four scenarios might happen in this model:

a) If \( q_i > \bar{q}_i, i = 1, 2 \), both first class and economic class are overbooked.

b) If \( q_i \leq \bar{q}_i, i = 1, 2 \), all the coming customers can be seated in the flight.

c) If \( q_1 > \bar{q}_1, q_2 \leq \bar{q}_2 \), first class is overbooked and economic class still has some seats left.

d) If \( q_1 \leq \bar{q}_1, q_2 > \bar{q}_2 \), first class still has some seats left and economic class is overbooked.

These are illustrated in proper order as follows.
a) In the first scenario, both classes are overbooked. The number of overbooked customers in class $i$ is $c_i = \frac{c_i}{1-q_i} \cdot (1 - q_i) - C_i = C_i \cdot \frac{q_i - q_1}{1-q_i}$. Hence, the total unseated number is $\sum_{i=1}^{2} C_i \cdot \frac{q_i - q_i}{1-q_i}$. Assuming that the compensation of denied boarding for every customer in class $i$ is $M_i$.

$$R_{D1} = - \sum_{i=1}^{2} [M_i \cdot C_i \cdot \frac{q_i - q_i}{1-q_i}] \quad [2.33]$$

b) Since overbooking does not happen, the compensation fee is zero.

$$R_{D2} = 0 \quad [2.34]$$

c) First class passengers could make their own decisions from two options, which are flying in economic class or not booking this flight. The number of vacant seats in economic class is $C_2 - \frac{c_2}{1-q_2} \cdot (1 - q_2) = C_2 \cdot \frac{q_2 - q_2}{1-q_2}$. The number of overbooked customers in first class is $\frac{c_1}{1-q_1} \cdot (1 - q_1) - C_1 = C_1 \cdot \frac{q_1 - q_1}{1-q_1}$. Assuming that if a customer decides to fly in economic class, the compensation he/she would get is $M_{12}$, else the compensation would be $M_1$. Also, assuming $p_{12}$ is the probability that customers accept the rearrange plan, the revenue is

$$R_{D3} = - \min \left(C_2 \cdot \frac{q_2 - q_2}{1-q_2}, C_1 \cdot \frac{q_1 - q_1}{1-q_1} \right) \cdot \left[p_{12} \cdot M_{12} + (1-p_{12}) \cdot M_1\right] \quad [2.35]$$

d) Economic class passengers could make their own decisions from two options, flying in first class or being denied boarding. The number of vacant seats in first class is $C_1 \cdot \frac{q_1 - q_1}{1-q_1}$, and the number of overbooked customers in economic class is $C_2 \cdot \frac{q_2 - q_2}{1-q_2}$. Assuming that if a customer decides to fly in first class, he/she needs to pay $M_{21}$, otherwise he/she could get $M_2$ as compensation. Also, assuming $p_{21}$ customers accept the rearrange plan, the revenue is

$$R_{D4} = \min \left(C_1 \cdot \frac{q_1 - q_1}{1-q_1}, C_2 \cdot \frac{q_2 - q_2}{1-q_2} \right) \cdot \left[p_{21} \cdot M_{21} - (1-p_{21}) \cdot M_2\right] \quad [2.36]$$

It is assumed that $p_{oi}$ is the probability of overestimating the no-show rate for class $i$, $i = 1, 2$, and hence $(1 - p_{oi})$ is the probability of underestimating or accurately estimating the no-show
rate for class $i, i = 1,2$. Hence, the expected revenue caused by denied boarding and changing class is

$$R_D = R_{D1} * p_{o1} * p_{o2} + R_{D3} * p_{o1} * (1 - p_{o2}) + R_{D4} * (1 - p_{o1}) * p_{o2} \tag{2.37}$$

$$= - \sum_{i=1}^{2} [M_{1i} * C_i * \left(\frac{q_i - q_i}{1 - q_i}\right)] * p_{o1} * p_{o2} - \min \left(C_2 * \left(\frac{q_2 - q_2}{1 - q_2}\right), C_1 * \left(\frac{q_1 - q_1}{1 - q_1}\right)\right) * [p_{12} * M_{12} + (1 - p_{12}) * M_{1}] * p_{o1} * (1 - p_{o2}) + \min \left(C_1 * \left(\frac{q_1 - q_1}{1 - q_1}\right), C_2 * \left(\frac{q_2 - q_2}{1 - q_2}\right)\right) * [p_{21} * M_{21} - (1 - p_{21}) * M_{2}] * (1 - p_{o1}) * p_{o2}$$

The whole denied boarding and changing class processes are summarized in Figure 2-1.
Although denied boarding passengers get certain amount of compensation, some of them may still feel dissatisfied about the service.
It is assumed that the dissatisfied rate for each not flying passenger in class $i$ is $u_i$ and the potential revenue loss for each of them is $r_i, i = 1, 2$. Hence, the total potential revenue loss of this category is

$$R_{L1} = - \left\{ \sum_{i=1}^{2} [u_i \cdot C_i \cdot \left( \frac{q_i - \tilde{q}_i}{1 - q_i} \right) \cdot r_i] + p_{o1} \cdot (1 - p_{o2}) \cdot (1 - p_{12}) \cdot \min \left( \frac{q_1 - \tilde{q}_1}{1 - q_1}, \frac{q_2 - \tilde{q}_2}{1 - q_2} \right) \cdot u_1 \cdot r_1 + (1 - p_{o1}) \cdot p_{o2} \cdot (1 - p_{21}) \cdot \min \left( \frac{q_1 - \tilde{q}_1}{1 - q_1}, \frac{q_2 - \tilde{q}_2}{1 - q_2} \right) \cdot u_2 \cdot r_2 \right\}$$

[2.38]

Also, the passengers who choose to change their initial classes may also feel dissatisfied about this rearrangement.

First, for up class passenger, it is assumed that the dissatisfied rate is $u_{12}$ and the potential revenue loss for each of them is $r_{12}$. Hence, the total potential revenue loss of this category is

$$R_{L2} = -p_{o1} \cdot (1 - p_{o2}) \cdot p_{12} \cdot \min \left( \frac{q_2 - \tilde{q}_2}{1 - q_2}, \frac{q_1 - \tilde{q}_1}{1 - q_1} \right) \cdot u_{12} \cdot r_{12}$$

[2.39]

Second, it is assumed that the dissatisfied rate for drop class passenger is $u_{21}$ and the potential revenue loss for each of them is $r_{21}$. Hence, the total potential revenue loss of this category is

$$R_{L3} = -(1 - p_{o1}) \cdot p_{o2} \cdot p_{21} \cdot \min \left( \frac{q_1 - \tilde{q}_1}{1 - q_1}, \frac{q_2 - \tilde{q}_2}{1 - q_2} \right) \cdot u_{21} \cdot r_{21}$$

[2.40]

Hence, the total potential revenue loss is

$$R_L = R_{L1} + R_{L2} + R_{L3}$$

[2.41]
\[ R = R_1 + R_2 + R_3 + R_4 + R_5 + R_D + R_b \]
\[ - \sum_{i=1}^{2} B_i \cdot c_{pi} + \sum_{i=1}^{2} \sum_{j=1}^{T} \min \left( d_{ij} - n_{ij}^c, \max \left[ 0, \frac{c_{i}}{1 - q_i} - B_i - \sum_{k=1}^{j-1} n_{ik} - n_{ij}^c \right] \right) \cdot \left( p_{ij} - c_{ci} \right) + \]
\[ \sum_{i=1}^{2} \sum_{j=1}^{T} \min \left( d_{ij} - n_{ij}^c, \max \left[ 0, \frac{c_{i}}{1 - q_i} - B_i - \sum_{k=1}^{j-1} n_{ik} - n_{ij}^c \right] \right) \cdot \left( P_{ij} \cdot S_{pi} \right) + \sum_{i=1}^{2} \sum_{j=1}^{T} \min \left( d_{ij} - n_{ij}^c - n_{ij}^p, \max \left[ 0, \frac{c_{i}}{1 - q_i} - A_i - \sum_{k=1}^{j-1} n_{ik} - n_{ij}^c - n_{ij}^p \right] \right) \cdot \max \left( P_{ij} - S_{ci}, 0 \right) + \]
\[ \sum_{i=1}^{2} \left[ M_{1i} \cdot C_i \cdot \left( \frac{q_{i} - q_{1i}}{1 - q_{i}} \right) \right] \cdot p_{o1} \cdot p_{o2} - \min \left( C_2 \cdot \left( \frac{q_2 - q_2}{1 - q_2} \right), C_1 \cdot \left( \frac{q_1 - q_1}{1 - q_1} \right) \right) \cdot \left[ p_{12} \cdot M_{12} + \right] \]
\[ (1 - p_{12}) \cdot M_1 \cdot p_{o1} \cdot (1 - p_{o2}) + \min \left( C_1 \cdot \left( \frac{q_1 - q_1}{1 - q_1} \right), C_2 \cdot \left( \frac{q_2 - q_2}{1 - q_2} \right) \right) \cdot \left[ p_{21} \cdot M_{21} - \right] \]
\[ (1 - p_{21}) \cdot M_2 \cdot p_{o1} \cdot (1 - p_{o2}) - \left\{ \sum_{i=1}^{2} u_i \cdot C_i \cdot \left( \frac{q_i - q_1}{1 - q_1} \right) \cdot r_i \right\} + \left( 1 - p_{o1} \right) \cdot p_{o2} \cdot \left( 1 - p_{o2} \right) \cdot \left[ p_{12} \cdot \min \left( C_2 \cdot \left( \frac{q_2 - q_2}{1 - q_2} \right), C_1 \cdot \left( \frac{q_1 - q_1}{1 - q_1} \right) \right) \cdot u_1 \cdot r_1 + \left( 1 - p_{o1} \right) \cdot p_{o2} \cdot \left( 1 - p_{o2} \right) \cdot \left[ \min \left( C_1 \cdot \left( \frac{q_1 - q_1}{1 - q_1} \right), C_2 \cdot \left( \frac{q_2 - q_2}{1 - q_2} \right) \right) \cdot u_2 \cdot \min \left( C_1 \cdot \left( \frac{q_1 - q_1}{1 - q_1} \right), C_2 \cdot \left( \frac{q_2 - q_2}{1 - q_2} \right) \right) \right] \cdot u_{21} \].

2.2.4 Binomial Option Pricing Formula (BOPF)

As suggested by Cox et al. (1979), it is assumed that the ticket price follows a multiplicative binomial process during the selling periods [21]. At the beginning of the selling time, the market price for class i is \( P_{i0} \). Then at the next period, the price might increase to
\( \alpha_i \cdot P_{10} \) (\( \alpha_i > 1 \)) with possibility \( p_1 \), or decrease to \( \beta_i \cdot P_{10} \) (\( \beta_i < 1 \)) with probability \( 1 - p_1 \) for class \( i \). Taking first class as an example, which means \( i = 1 \). The following Figure 2-2 depicts this process:

\[
\begin{align*}
P_{11} &= \alpha_1 \cdot P_{10} \quad \text{With probability } p_1 \\
P_{11} &= \beta_1 \cdot P_{10} \quad \text{With probability } 1 - p_1
\end{align*}
\]

Figure 2-2 The Ticket Market Price for the First Class in the 1st Period

So \( P_{11} \) is calculated as follows:

\[
E[ P_{11} ] = p_1 \cdot \alpha_1 \cdot P_{10} + (1 - p_1) \cdot \beta_1 \cdot P_{10} \quad [2.43]
\]

\[
= \sum_{m=0}^{1} \left( \frac{1}{m} \right) p_1^m (1 - p_1)^{1-m} \cdot \alpha_1^m \cdot \beta_1^{1-m} \cdot P_{10}
\]

Similarly, \( P_{12} \) can be calculated based on \( P_{10} \), using Figure 2-3

\[
\begin{align*}
P_{12} &= \alpha_1 \cdot \alpha_1 \cdot P_{10} \quad \text{With probability } p_1^2 \\
P_{12} &= \alpha_1 \cdot \beta_1 \cdot P_{10} \quad \text{With probability } 2 \cdot (1 - p_1) \cdot p_1 \\
P_{12} &= \beta_1 \cdot \beta_1 \cdot P_{10} \quad \text{With probability } (1 - p_1)^2
\end{align*}
\]

Figure 2-3 The Ticket Market Price for the First Class in 2nd Period

\[
E[ P_{12} ] = [ p_1^2 \cdot \alpha_1^2 + 2 \cdot (1 - p_1) \cdot p_1 \cdot \alpha_1 \cdot \beta_1 + (1 - p_1)^2 \cdot \beta_1^2 ] \cdot P_{10} \quad [2.44]
\]

\[
= \sum_{m=0}^{2} \left( \frac{2}{m} \right) p_1^m (1 - p_1)^{2-m} \cdot \alpha_1^m \cdot \beta_1^{2-m} \cdot P_{10}
\]

From the example above, \( P_{1j} \), for \( j = 1, 2 \ldots T \), is obtained as
\[ E[P_{ij}] = \sum_{m=0}^{j} \binom{j}{m} p_1^m (1 - p_1)^{j-m} * \alpha_1^m * \beta_1^{j-m} * P_{10} \]  

After applying the same principles to economic class, the expected market price for class \( i, i = 1,2 \) at period \( j, j = 1,2 \ldots T \) can be expressed as

\[ E[P_{ij}] = \sum_{m=0}^{j} \binom{j}{m} p_i^m (1 - p_i)^{j-m} * \alpha_i^m * \beta_i^{j-m} * P_{i0} \]  

Once the price evolution is specified, the premiums are set for the tickets according to Cox et al. (1979). The premiums for a call option or a put option for class \( i = 1,2 \) at period \( j, j = 1,2 \ldots T \) are as follows:

Put option:

\[ c_{pij} = \max\{0, S_{pi} - E[P_{ij}]\} \]

\[ = \max\left\{0, S_{pi} - \sum_{m=0}^{j} \binom{j}{m} p_i^m (1 - p_i)^{j-m} * \alpha_i^m * \beta_i^{j-m} * P_{i0}\right\} \]

Call option:

\[ c_{ci} = \max\{0, E[P_{ij}] - S_{ci}\} \]

\[ = \max\left\{0, \sum_{m=0}^{j} \binom{j}{m} p_i^m (1 - p_i)^{j-m} * \alpha_i^m * \beta_i^{j-m} * P_{i0} - S_{ci}\right\} \]

### 2.2.5 Option pricing

As suggested by Nan Zhang [19], the exact values of \( c_{ci} \) for call option in class \( i, i = 1,2 \) are obtained using a proper linear combination of \( c_{ci} \). Same principles would be applied to get the exact values of \( c_{pi} \) for put option in class \( i, i = 1,2 \).
Here, for easy calculation, it is assumed that the weights for both call and put options are the same, and the weight sequence for different periods in class $i$ should be $W = \{w_{i1}, w_{i2} \ldots w_{ij}, w_{iT}\}$. So the final premiums for put option in class $i$ should be

$$c_{pi} = \sum_{j=1}^{T} c_{pij} \cdot w_{ij}$$

$$= \sum_{j=1}^{T} \max \left\{ 0, S_{pi} - \sum_{m=0}^{j} \left(^j_m\right) p_{i}^{m}(1 - p_{i})^{j-m} \cdot \alpha_i^m \cdot \beta_i^{j-m} \cdot p_{i0} \right\} \cdot w_{ij}$$

$$c_{ci} = \sum_{j=1}^{T} c_{cij} \cdot w_{ij}$$

$$= \sum_{j=1}^{T} \max \left\{ 0, \sum_{m=0}^{j} \left(^j_m\right) p_{i}^{m}(1 - p_{i})^{j-m} \cdot \alpha_i^m \cdot \beta_i^{j-m} \cdot p_{i0} - S_{ci} \right\} \cdot w_{ij}$$

### 2.3 Model Objective Function

As the result of the earlier discussion, the final objective function is as follows:

**Objective:** $\max(R)$

$$R = -\left( \sum_{i=1}^{2} \sum_{j=1}^{T} \max\{0, S_{ci} - \sum_{k=0}^{j-1} \left(^j_k\right) p_{i}^{k}(1 - p_{i})^{j-k} \cdot \alpha_i^k \cdot \beta_i^{j-k} \cdot P_{i0} \} \cdot w_{ij} \right) \cdot B_{i} +$$

$$\sum_{i=1}^{2} \sum_{j=1}^{T} \min \{d_{ij}, \max\{0, A_{i} - \sum_{k=0}^{j-1} n_{ik}\} \} \cdot (P_{ij} - \sum_{j=1}^{T} \max\{0, \sum_{k=0}^{j} \left(^j_k\right) p_{i}^{k}(1 - p_{i})^{j-k} \cdot \alpha_i^k \cdot \beta_i^{j-k} \cdot P_{i0} \} \cdot w_{ij} \} +$$

$$\sum_{i=1}^{2} \sum_{j=1}^{T} \min \{\max\{0, d_{ij} - n_{ij}^{e} - n_{ij}^{x}\}, \max\{0, \frac{c_{i}}{1 - q_{i}} - B_{i} - \sum_{k=0}^{j-1} n_{ik}\} \} \cdot \max(P_{ij}, S_{pi}) +$$

$$\sum_{i=1}^{2} \sum_{j=1}^{T} \min \{\max\{0, d_{ij} - n_{ij}^{e} - n_{ij}^{x} - n_{ij}^{p}\}, \max\{0, \frac{c_{i}}{1 - q_{i}} + A_{i} - \sum_{k=0}^{j-1} n_{ik}\} \} \cdot \max(P_{ij} -$$
\[ S_{ci} , 0 \] \[ + \max \left[ 0, B_i - \sum_{j=1}^{T} \min \left( \max \left[ 0, d_{ij} - n_{ij}^c - n_{ij}^s \right], \max \left[ 0, \frac{c_i}{1-q_i} - \sum_{k=1}^{i-1} n_{ik} \right] \right) \right] \] \[ \times S_{pi} - \]

\[ \sum_{i=1}^{2}[M_{1i} \times C_i \times \left( \frac{q_i - q_1}{1-q_i} \right)] \times P_{o1} \times P_{o2} - \min \left( C_2 \times \left( \frac{q_2 - q_1}{1-q_1} \right), C_1 \times \left( \frac{q_1 - q_1}{1-q_1} \right) \right) \] \[ \times \left[ P_{12} \times M_{12} + \right. \]

\[ (1 - P_{12}) \times M_{1} \] \[ \times P_{o1} \times (1 - P_{o2}) + \min \left( C_1 \times \left( \frac{q_1 - q_1}{1-q_1} \right), C_2 \times \left( \frac{q_2 - q_1}{1-q_1} \right) \right) \] \[ \left. \times \left[ P_{21} \times M_{21} - \right. \right. \]

\[ (1 - P_{21}) \times M_{2} \] \[ \times (1 - P_{o1}) \times P_{o2} - \{ \sum_{i=1}^{2}[u_i \times C_i \times \left( \frac{q_i - q_1}{1-q_i} \right) \times r_i] + P_{o1} \times (1 - P_{o2}) \times \]

\[ (1 - P_{12}) \times \min \left( C_2 \times \left( \frac{q_2 - q_1}{1-q_2} \right), C_1 \times \left( \frac{q_1 - q_1}{1-q_1} \right) \right) \times u_1 \times r_1 + (1 - P_{o1}) \times P_{o2} \times (1 - P_{21}) \times \]

\[ \min \left( C_1 \times \left( \frac{q_1 - q_1}{1-q_1} \right), C_2 \times \left( \frac{q_2 - q_1}{1-q_1} \right) \right) \times u_2 \times r_2 \} - P_{o1} \times (1 - P_{o2}) \times P_{12} \times \min \left( C_2 \times \left( \frac{q_2 - q_2}{1-q_2} \right), C_1 \times \right. \]

\[ \left. \left( \frac{q_1 - q_1}{1-q_1} \right) \right) \times u_{12} \times r_{12} - (1 - P_{o1}) \times P_{o2} \times P_{21} \times \min \left( C_1 \times \left( \frac{q_1 - q_1}{1-q_1} \right), C_2 \times \left( \frac{q_2 - q_2}{1-q_2} \right) \right) \] \[ \times u_{21} \times r_{21} \]

\[ \[2.51] \]
Chapter 3

Numerical Example

3.1 Model Application

Based on the model proposed in Chapter 2, a numerical example will be given in this chapter to show how the model can be applied in practice. The method used is numerical search, which may only result in a local optimum. It is recommended that using several different initial values for the variables, will help the user, find a solution as close to the global optimal solution as possible. Then demand path analysis will be given.

3.1.1 Input Parameters

- **Demand Parameters**

  Demand is a significant parameter in the model and it is one of the factors that motivate the whole process. In reality, demand is a discrete random variable. As illustrated in chapter 2, it is assumed that the demand in each period is known at the beginning of the period. In the numerical example, the demand in period \( j \) for class \( i \) is \( d_{ij} \) is given in Table 3-1.

<table>
<thead>
<tr>
<th>Period ( j )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Class ( d_{1j} )</td>
<td>30</td>
<td>35</td>
<td>25</td>
<td>25</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>Economy Class ( d_{2j} )</td>
<td>10</td>
<td>30</td>
<td>15</td>
<td>25</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>
• **Price Parameters**

It is assumed that the market prices for the two classes follow the binomial distribution. As illustrated before, the ticket price in period $j$ for class $i$ is $P_{ij}$ and it could be calculated as follows:

$$P_{i,j+1} = \alpha_i \cdot P_{i,j} \quad \text{With probability } p_i$$

$$P_{i,j+1} = \beta_i \cdot P_{i,j} \quad \text{With probability } 1 - p_i$$

Here, the parameters are set as illustrated in the following table:

<table>
<thead>
<tr>
<th>Class $(i)$</th>
<th>First Class</th>
<th>Economy Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of increase of ticket price ($\alpha_i$)</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>Proportion of decrease of ticket price ($\beta_i$)</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>$p_i$</td>
<td>0.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

For this numerical example, the price paths for the two classes are listed in Table 3-3.

<table>
<thead>
<tr>
<th>Period $j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Class ($P_{1j}$)</td>
<td>360</td>
<td>288</td>
<td>345.6</td>
<td>414.72</td>
<td>497.664</td>
<td>398.131</td>
</tr>
<tr>
<td>Economy Class ($P_{2j}$)</td>
<td>180</td>
<td>162</td>
<td>145.8</td>
<td>160.38</td>
<td>144.342</td>
<td>158.776</td>
</tr>
</tbody>
</table>

• **No-show and Denied Boarding Parameters**

In this model, no-show is taken into consideration, which might cause denied boarding or empty seats left before departure. To deal with the problem, the airline could either pay compensation or persuade passengers to change class in different circumstances. Also, under each
circumstance, some passengers might feel unsatisfied and potential revenue loss might occur accordingly. These parameters are listed in the table as follows:

Table 3- 4 No-show and Denied Boarding Parameters

<table>
<thead>
<tr>
<th>Class (i)</th>
<th>First Class</th>
<th>Economy Class</th>
<th>Drop Class</th>
<th>Up Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of Overestimating</td>
<td>0.4</td>
<td>0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of Change</td>
<td></td>
<td></td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Compensation</td>
<td>300</td>
<td>200</td>
<td>50</td>
<td>-50</td>
</tr>
<tr>
<td>Probability of Dissatisfied</td>
<td>0.4</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Revenue Loss</td>
<td>2,000</td>
<td>1,000</td>
<td>800</td>
<td>400</td>
</tr>
</tbody>
</table>

- **Other Parameters**

Because a numerical search method is used in this model, some general parameters should also be assumed. These are set in the following table:

Table 3- 5 General Parameters

<table>
<thead>
<tr>
<th>Class (i)</th>
<th>First Class</th>
<th>Economy Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of Periods (T)</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Estimated no-show Rate of customers (Q_i)</td>
<td>0.15</td>
<td>0.1</td>
</tr>
<tr>
<td>Total Capacity of Class i (C_i)</td>
<td>80</td>
<td>140</td>
</tr>
<tr>
<td>Initial Ticket Price (P0_i)</td>
<td>300</td>
<td>200</td>
</tr>
</tbody>
</table>

3.1.2 Initial Values for Variables

As illustrated before, the numerical search method may result in a local optimum. It is assumed that several different initial values for the decision variables are used to find a solution as close to the global optimal solution as possible.
3.1.3 Optimal Results

In the model, based on financial option, four stages theory is used to sell tickets. The model also takes potential revenue loss caused by no-show and denied boarding into consideration to get more accurate and practical results. The table compares the expected revenues yields of the option based theory and traditional selling method with/without no-shows, to illustrate the benefit of the model.

Table 3-6 Results

<table>
<thead>
<tr>
<th></th>
<th>Expected Revenue</th>
<th>Optimal Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional selling method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without no-show</td>
<td>$44801.64</td>
<td></td>
</tr>
<tr>
<td>With no-show</td>
<td>$49049.89</td>
<td></td>
</tr>
<tr>
<td>Option based method</td>
<td>$55108.87</td>
<td></td>
</tr>
</tbody>
</table>

As the results shown in Table 3-6, using financial option theory, the airline could improve its revenue by 12.35%. Hence, it is suggested that the airline may adopt financial option theory in practice to improve its revenue.

3.2 Demand path Analysis

In the example given above, the numerical search method is used to calculate the expected revenue for airline. The demand path is generated randomly. However, it is possible that
the demand path may have an impact on the results. So in this section, several representative demand paths will be chosen to demonstrate this problem.

In the example, according to the total capacity ($C_i$) and estimated no-show rate ($Q_i$) for each class, its corresponding booking limit can be calculated, which is called supply and denoted by $S_i$. Also, the total demand for each class can be obtained from the demand path, which is denoted by $D_i$. The calculation results for $S_i$ and $D_i$ are shown in Table 3-7.

Table 3- 7 Booking Limit and Total Demand

<table>
<thead>
<tr>
<th>Class ($i$)</th>
<th>First Class ($i = 1$)</th>
<th>Economy Class ($i = 2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total capacity ($C_i$)</td>
<td>80</td>
<td>140</td>
</tr>
<tr>
<td>Estimated no-show Rate ($Q_i$)</td>
<td>0.15</td>
<td>0.1</td>
</tr>
<tr>
<td>Booking Limit ($S_i = \frac{C_i}{1 - Q_i}$)</td>
<td>94</td>
<td>155</td>
</tr>
<tr>
<td>Total Demand ($D_i = \sum_{j=1}^{\tau} d_{ij}$)</td>
<td>160</td>
<td>120</td>
</tr>
</tbody>
</table>

The relationships between $S_i$ and $D_i$ are shown in Table 3-8, which gives the four possible scenarios. The numerical example above is for the first scenario and the results shown in Table 3-9 to Table 3-11 are for the other three scenarios.

Table 3- 8 Relationship Between $S_i$ and $D_i$

| | 1. $S_1 < D_1, S_2 > D_2$ | 2. $S_1 > D_1, S_2 > D_2$ |
| | 3. $S_1 < D_1, S_2 < D_2$ | 4. $S_1 > D_1, S_2 < D_2$ |

Table 3- 9 The Result for the 2nd Scenario

<table>
<thead>
<tr>
<th></th>
<th>$D_i$</th>
<th>$S_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period $j$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>First Class ($d_{1j}$)</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Economy Class ($d_{2j}$)</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Traditional selling method</td>
<td>Expected Revenue</td>
<td></td>
</tr>
<tr>
<td>----------------------------</td>
<td>------------------</td>
<td></td>
</tr>
<tr>
<td>Without no-show</td>
<td>$44183.67</td>
<td></td>
</tr>
<tr>
<td>Traditional selling method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With no-show</td>
<td>$50275.70</td>
<td></td>
</tr>
<tr>
<td>Option based method</td>
<td>$57433.23</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>First Class</th>
<th>Economy Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_i$</td>
<td>0</td>
</tr>
<tr>
<td>$B_i$</td>
<td>18</td>
</tr>
</tbody>
</table>

**Table 3-10 The Result for the 3rd Scenario**

<table>
<thead>
<tr>
<th>Period $j$</th>
<th>$D_i$</th>
<th>$S_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Class ($d_{1j}$)</td>
<td>125</td>
<td>94</td>
</tr>
<tr>
<td>Economy Class ($d_{2j}$)</td>
<td>160</td>
<td>155</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional selling method</td>
</tr>
<tr>
<td>Without no-show</td>
</tr>
<tr>
<td>Traditional selling method</td>
</tr>
<tr>
<td>With no-show</td>
</tr>
<tr>
<td>Option based method</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>First Class</th>
<th>Economy Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_i$</td>
<td>20</td>
</tr>
<tr>
<td>$B_i$</td>
<td>0</td>
</tr>
</tbody>
</table>

Optimal Values
Table 3-11 The Result for the 4th Scenario

<table>
<thead>
<tr>
<th></th>
<th>Period $j$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>First Class ($d_{1j}$)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>30</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Economy Class ($d_{2j}$)</td>
<td>30</td>
<td>15</td>
<td>30</td>
<td>35</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional selling method Without no-show</td>
</tr>
<tr>
<td>Traditional selling method With no-show</td>
</tr>
<tr>
<td>Option based method</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$A_i$</th>
<th>$B_i$</th>
<th>First Class</th>
<th>Economy Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_i$</td>
<td>0</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_i$</td>
<td>10</td>
<td>36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3-9 to Table 3-11 show that the demand path actually has an impact on the optimal values of call and put option tickets. Hence accurate demand forecast at the beginning of the tickets selling process is very important. However, regardless of the demand path, the option-based method could help the airline earn more revenue than the traditional selling method.
Chapter 4

Conclusion and Future Work

4.1 Conclusions

Based on the financial option theory, the thesis builds a practical model to optimize the revenue for airline selling processes. The call and put option concepts are applied to the airline tickets and airlines could buy such options from customers and travel agents. Before the selling process, the airline will set the upper bounds for both call and put option tickets according to its own rule, like the bound of 30 percentage of the capacity for each class. During the tickets selling period, the airline will make decisions, such as to sell tickets to the travel agents or customers and to recall the tickets already sold or not. To gain the call and put options, the airline has to pay the customers and the travel agents certain premiums as the buying prices. The limitations and premiums of the call and put option tickets are significant variables in the model.

The airline could either recall the call option tickets back from the customers to gain more revenue or it would give the excess tickets to the travel agents as put option tickets to minimize the risk when the demand is less than the supply. Also, since no-show rate is being considered in this model, denied boarding or extra seats left might occur as a result of the inaccurate forecast. The airline offers two kinds of tickets to its customers, and hence the model adopts changing classes as a method to deal with the problem. Also, it uses potential revenue loss as a measurement to weigh customer dissatisfaction caused by denied boarding or class changing. Those considerations would result in more accurate expected revenue to the airline.

In addition, the market price is following a binomial distribution in this model. Based on the expected market price for each period, striking call and put option prices would be obtained. However, for every selling process, the price and demand will just follow one path and the airline
will forecast them in advance. Hence, in this model the airline could set the price and demand path according to its forecast, which makes the model easier to practice, compared to stochastic dynamic programming models.

4.2 Future Work

A number of areas exist in which future research could extend the effectiveness and utilization of this model. First, it is assumed that the demand and the price are independent. However, it is very reasonable that the price of the tickets has a huge effect on the demand. Building a relationship between them could be an interesting future research. Second, as mentioned before in the numerical example, the search method is usually result in a local optimum rather than a global one. Hence, departing from the initial value of the variables to obtain the global optimum solution is suggested for future work. Last but not least, the method used in this model to solve no-show and overbooking problem needs several forecasted values, which is difficult to deal with. A more practical and direct method should be considered in the future research.
REFERENCES


T=6;%initial the period
demand=zeros(2,T);%initial demand for each class
a=[0.35,0.7,0.8,0.9,1;0.2,0.45,0.65,0.8,1];%initial probability of demand distribution for each class
d=[10,15,20,25,30;15,20,25,30,35];%demand for each class
u=zeros(1,6);%initial random number generator
for i=1:2
    for j=1:T
        u(i,j)=rand(1);
        if u(i,j)<a(i,1)
            demand(i,j)=d(i,1);
        elseif u(i,j)<a(i,2)
            demand(i,j)=d(i,2);
        elseif u(i,j)<a(i,3)
            demand(i,j)=d(i,3);
        elseif u(i,j)<a(i,4)
            demand(i,j)=d(i,4);
        elseif u(i,j)<a(i,5)
            demand(i,j)=d(i,5);
        end
    end
end
display(demand);

% u=zeros(2,6);%initial random number generator
u=[0.5,0.7,0.5,0.5,0.5,0.7;0.7,0.7,0.7,0.5,0.7,0.5];
w=[1.2,0.68;1.1,0.9];%initial price change percentage for each class
P0=[300;200];%initial original price for each class
p_up=[0.6;0.6];%initial the probability that the price goes up for each class
Price=zeros(2,7);%initial the market price for each period in each class

for h=1:2
    Price(h,1)=P0(h,1);
    for i=1:T
        u(h,i)=rand(1);
        if u(h,i)<p_up(h,1)
            Price(h,i+1)=w(h,1)*Price(h,i);
        end
    end
end
else 
    Price(h,i+1)=w(h,2)*Price(h,i);
end
end
end
display(Price);

%Traditional selling method without no-show
clc;
T=6;
demand=[35,5,20,25,15,20;45,15,40,30,35,55];%first class 160,120
display(demand);
Price=[360,288,345.6,414.72,497.664,398.1312;
     180,162,145.8,160.38,144.342,158.7762];
display(Price);

C=[68;126];
R=zeros(2,T);%initial revenue for each period
R_total=zeros(2,1);%initial total revenue gained for each class
n=zeros(2,6);%
initial the tickets sold for each period for each class
n_total=[0;0]%initial the total tickets sold for each class

for h=1:2
    for i=1:T
        n(h,i)=min(demand(h,i),max(0,C(h,1)-n_total(h,1)));
        R(h,i)=n(h,i)*Price(h,i);
        n_total(h,1)=n_total(h,1)+n(h,i);
        R_total(h,1)=R(h,i)+R_total(h,1);
    end
end
display(n);
display(R);
display(n_total);
display(R_total);
Revenue_final=sum(R_total(:,1));
fprintf('final revenue without no-show is %f
',Revenue_final);

%Traditional selling method with no-show
clc;
T=6;
demand=[30,35,25,25,15,30;10,30,15,25,20,20];%first class 160,120
display(demand);

L=500000;
rd=zeros(2,L);
C=[80;140];
Q=[0.15;0.1];%initial the estimated no-show rate for each class
M=[300;200;50;50];%initial the compensation for the first class, economy class, drop class and up class passenger
RD=zeros(4,L);%initial the revenue caused by overbooking and denied boarding
RL=zeros(4,L); % initial the revenue caused by customer dissatisfied
Revenue_Denied=zeros(1,L); % initial the total denied boarding revenue
Revenue_Dissatisfied=zeros(1,L); % initial the revenue loss caused by customer dissatisfied
p_overestimating=[0.4;0.45]; % initial the probability of overestimating the no-show rate for each class
p_change=[0.3;0.4]; % initial the probability of drop class/up class passenger
p_dissatisfied=[0.4;0.2;0.1;0.2]; % initial the dissatisfied probability for first class, economy class, drop class and up class
loss=[2000;1000;800;400]; % initial the potential loss for first class, economy class, drop class and up class
for i=1:L
    for h=1:2
        rd(h,1)=rand(1);
    end
    if rd(1,1)<p_overestimating(1,1)&&rd(2,1)<p_overestimating(2,1) % both classes are overbooked
        q(1,1)=Q(1,1)*rand(1);
        q(2,1)=Q(2,1)*rand(1);
        RD(1,i)=-M(1,1)*C(1,1)*(Q(1,1)-q(1,1))/(1-Q(1,1))-M(2,1)*C(2,1)*(Q(2,1)-q(2,1))/(1-Q(2,1));
        RL(1,i)=-C(1,1)*(Q(1,1)-q(1,1))/(1-Q(1,1))*p_dissatisfied(1,1)*loss(1,1)-C(2,1)*(Q(2,1)-q(2,1))/(1-Q(2,1))*p_dissatisfied(2,1)*loss(2,1);
    else if rd(1,1)>p_overestimating(1,1)&&rd(2,1)>p_overestimating(2,1) % both classes have empty seats
        RD(2,i)=0;
        RL(2,i)=0;
    else if rd(1,1)<p_overestimating(1,1)&&rd(2,1)>p_overestimating(2,1) % the first class is overbooked and economy class has seats left
        q(1,1)=Q(1,1)*rand(1)+0.1;
        q(2,1)=Q(2,1)*rand(1);
        RD(3,i)=-min(C(2,1)*(Q(2,1)-q(2,1))/(1-Q(2,1)),C(1,1)*q(1,1)-(1-Q(1,1)))/(1-Q(1,1))*p_change(1,1)*M(3,1)+(1-p_change(1,1))*M(1,1);
        RL(3,i)=-min(C(2,1)*q(2,1)-(1-Q(2,1)),C(1,1)*q(1,1)-(1-Q(1,1)))/(1-Q(1,1))*p_change(1,1)*M(3,1)+(1-p_change(1,1))*p_dissatisfied(1,1)*loss(1,1)+p_change(1,1)*p_dissatisfied(3,1)*loss(3,1);
    end
    else if rd(1,1)>p_overestimating(1,1)&&rd(2,1)<p_overestimating(2,1) % first class has seats left and economy class is overbooked
        q(1,1)=Q(1,1)*rand(1)+0.15;
        q(2,1)=Q(2,1)*rand(1);
        RD(4,i)=-min(C(1,1)*(Q(1,1)-q(1,1))/(1-Q(1,1)),C(2,1)*q(2,1)-(1-Q(2,1)))/(1-Q(2,1))*p_change(2,1)*M(4,1)+(1-p_change(2,1))*M(2,1);
        RL(4,i)=-min(C(1,1)*q(1,1)-(1-Q(1,1)),C(2,1)*q(2,1)-(1-Q(2,1)))/(1-Q(2,1))*p_change(2,1)*M(4,1)+(1-p_change(2,1))*p_dissatisfied(2,1)*loss(2,1)+p_change(2,1)*p_dissatisfied(4,1)*loss(4,1);
    end
end
end

Revenue_Denied(1,i)=p_overestimating(1,1)*p_overestimating(2,1)*RD(1,i)+p_overestimating(1,1)*(1-p_overestimating(2,1))*RD(3,i)+(1-
\[ p_{\text{overestimating}}(1,1) \times p_{\text{overestimating}}(2,1) \times RD(4,i); \]

\[ \text{Revenue}_{\text{Dissatisfied}}(1,i) = p_{\text{dissatisfied}}(1,1) \times RL(1,i) + p_{\text{dissatisfied}}(3,1) \times RL(3,i) + p_{\text{dissatisfied}}(4,1) \times RL(4,i); \]

\[ \text{end} \]

\[ RD_{\text{total}} = \text{sum}(\text{Revenue}_{\text{Denied}}(1,:))/L; \]
\[ \text{display}(RD_{\text{total}}); \]

\[ RL_{\text{total}} = \text{sum}(\text{Revenue}_{\text{Dissatisfied}}(1,:))/L; \]
\[ \text{display}(RL_{\text{total}}); \]

\% price

\[ \text{Price} = [360, 288, 345.6, 414.72, 497.664, 398.1312; 180, 162, 145.8, 160.38, 144.342, 158.7762]; \]

\[ R = \text{zeros}(2,T); \] \% initial revenue for each period

\[ R_{\text{total}} = \text{zeros}(2,1); \] \% initial total revenue gained for each class

\[ n = \text{zeros}(2,6); \] \% initial the tickets sold for each period for each class

\[ n_{\text{total}} = [0; 0]; \] \% initial the tickets sold for each class

\[ \text{for } h = 1:2 \]
\[ \text{for } i = 1:T \]
\[ n(h,i) = \text{min}(\text{demand}(h,i), \text{max}(0, C(h,1)/(1-Q(h,1))-n_{\text{total}}(h,1))); \]
\[ R(h,i) = n(h,i) \times \text{Price}(h,i); \]
\[ n_{\text{total}}(h,1) = n_{\text{total}}(h,1) + n(h,i); \]
\[ R_{\text{total}}(h,1) = R(h,i) + R_{\text{total}}(h,1); \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{display}(\text{Price}); \]
\[ \text{display}(n); \]
\[ \text{display}(R); \]
\[ \text{display}(n_{\text{total}}); \]
\[ \text{display}(R_{\text{total}}); \]
\[ \text{Revenue}_{\text{final}} = \text{sum}(R_{\text{total}}(:,1)) + RD_{\text{total}} + RL_{\text{total}}; \]
\[ \text{fprintf(''final revenue='', Revenue}_{\text{final});} \]

\% Option-based method

\[ \text{clc;} \]
\[ A0 = [20; 30]; \]
\[ B0 = [0; 36]; \]
\[ S0 = [A0, B0]; \] \% Starting guess
\[ \text{options} = \text{optimset('LargeScale','on');} \]
\[ [S, fval] = \text{fmincon('total', S0, [], [], [], [], [0, 0, 0, 0], [20, 30, 24, 36]);} \]

\[ \text{function } f = \text{total}(S) \]
\[ A = S(:,1); B = S(:,2); \]
\[ \text{clc;} \]
\[ T = 6; \]
\[ \text{demand} = [31, 35, 25, 15, 30; 10, 30, 15, 25, 20, 20]; \] \% first class 160, 120
display(demand);
%no_show
L=500000;
rd=zeros(2,L);
C=[80;140];
Q=[0.15;0.1];%initial the estimated no-show rate for each class
M=[300;200;50;50];%initial the compensation for the first class, economy class, drop class and up class passenger
RD=zeros(4,L);%initial the revenue caused by overbooking and denied boarding
RL=zeros(4,L);%initial the revenue caused by customer dissatisfied
Revenue_Denied=zeros(1,L);%initial the total denied boarding revenue
Revenue_Dissatisfied=zeros(1,L);%initial the revenue loss caused by customer dissatisfied
p_overestimating=[0.4;0.45];%initial the probability of overestimating the no-show rate for each class
p_change=[0.3;0.4];%initial the probability of drop class/up class passenger
p_dissatisfied=[0.4;0.2;0.1;0.2];%initial the dissatisfied probability for first class, economy class, drop class and up class
loss=[2000;1000;800;400];%initial the potential loss for first class, economy class, drop class and up class
for i=1:L
    for h=1:2
        rd(h,1)=rand(1);
    end
    if rd(1,1)<p_overestimating(1,1)&&rd(2,1)<p_overestimating(2,1)%both classes are overbooked
        q(1,1)=Q(1,1)*rand(1);
        q(2,1)=Q(2,1)*rand(1);
        RD(1,i)=-M(1,1)*C(1,1)*(Q(1,1)-q(1,1))/(1-Q(1,1))-M(2,1)*C(2,1)*(Q(2,1)-q(2,1))/(1-Q(2,1));
        RL(1,i)=-C(1,1)*(Q(1,1)-q(1,1))/(1-Q(1,1))*p_dissatisfied(1,1)*loss(1,1)-C(2,1)*(Q(2,1)-q(2,1))/(1-Q(2,1))*p_dissatisfied(2,1)*loss(2,1);
    else if rd(1,1)>p_overestimating(1,1)&&rd(2,1)>p_overestimating(2,1)%both classes have empty seats
        RD(2,i)=0;
        RL(2,i)=0;
    else if rd(1,1)<p_overestimating(1,1)&&rd(2,1)>p_overestimating(2,1)%the first class is overbooked and economy class has seats left
        q(1,1)=Q(1,1)*rand(1)+0.1;
        q(2,1)=Q(2,1)*rand(1);
        RD(3,i)=min(C(2,1)*(Q(2,1)-q(2,1))/(1-Q(2,1)),C(1,1)*(q(1,1)-Q(1,1))/(1-Q(1,1)))*(p_change(1,1)*M(3,1)+(1-p_change(1,1))*M(1,1));
        RL(3,i)=-min(C(2,1)*(q(2,1)-Q(2,1))/(1-Q(2,1)),C(1,1)*(Q(1,1)-q(1,1))/(1-Q(1,1)))*(1-p_change(1,1))*p_dissatisfied(1,1)*loss(1,1)+p_change(1,1)*p_dissatisfied(3,1)*loss(3,1));
    else if rd(1,1)>p_overestimating(1,1)&&rd(2,1)<p_overestimating(2,1)%the first class has seats left and economy class is overbooked
        q(1,1)=Q(1,1)*rand(1)+0.15;
        q(2,1)=Q(2,1)*rand(1);
        RD(4,i)=min(C(1,1)*(Q(1,1)-q(1,1))/(1-Q(1,1)),C(2,1)*(q(2,1)-Q(2,1))/(1-Q(2,1)))*(p_change(2,1)*M(4,1)+(1-p_change(2,1))*M(2,1));
        RL(4,i)=-min(C(1,1)*(q(1,1)-Q(1,1))/(1-Q(1,1)),C(2,1)*(Q(2,1)-q(2,1))/(1-Q(2,1)))*p_change(2,1)*M(4,1)-C(1,1)*(q(1,1)-Q(1,1))/(1-Q(1,1))*p_dissatisfied(1,1)*loss(1,1)-C(2,1)*(Q(2,1)-q(2,1))/(1-Q(2,1))*p_dissatisfied(2,1)*loss(2,1);
    else if rd(1,1)<p_overestimating(1,1)||rd(2,1)<p_overestimating(2,1)%the first class is not overbooked and economy class is not empty
        q(1,1)=Q(1,1)*rand(1);
        q(2,1)=Q(2,1)*rand(1);
        RD(5,i)=-M(1,1)*C(1,1)*(Q(1,1)-q(1,1))/(1-Q(1,1))-M(2,1)*C(2,1)*(Q(2,1)-q(2,1))/(1-Q(2,1));
        RL(5,i)=-C(1,1)*(Q(1,1)-q(1,1))/(1-Q(1,1))*p_dissatisfied(1,1)*loss(1,1)-C(2,1)*(Q(2,1)-q(2,1))/(1-Q(2,1))*p_dissatisfied(2,1)*loss(2,1);
\[
Q(2,1)) \times (1 - p_{\text{change}(2,1)}) \times p_{\text{dissatisfied}(2,1)} \times \text{loss}(2,1) + p_{\text{change}(2,1)} \times p_{\text{dissatisfied}(4,1)} \times \text{loss}(4,1));
\]

\[
\text{Revenue\_Denied}(1,i) = p_{\text{overestimating}(1,1)} \times p_{\text{overestimating}(2,1)} + \text{RD}(1,i) + p_{\text{overestimating}(1,1)} \times (1 - p_{\text{overestimating}(2,1)}) \times \text{RD}(3,i) + (1 - p_{\text{overestimating}(1,1)}) \times p_{\text{overestimating}(2,1)} \times \text{RD}(4,i);
\]

\[
\text{Revenue\_Dissatisfied}(1,i) = p_{\text{dissatisfied}(1,1)} \times \text{RL}(1,i) + p_{\text{dissatisfied}(3,1)} \times \text{RL}(3,i) + p_{\text{dissatisfied}(4,1)} \times \text{RL}(4,i);
\]

\[
\text{RD\_total} = \frac{\text{sum}(\text{Revenue\_Denied}(1,:))}{L};
\]

\[
\text{display}(\text{RD\_total});
\]

\[
\text{RL\_total} = \frac{\text{sum}(\text{Revenue\_Dissatisfied}(1,:))}{L};
\]

\[
\text{display}(\text{RL\_total});
\]

\[
\% \text{expected market price and striking price}
\]

\[
w = [1.2, 0.8; 1.1, 0.9]; \% \text{initial price change percentage for each class}
\]

\[
P0 = [300; 200]; \% \text{initial original price for each class}
\]

\[
p_{\text{up}} = [0.6; 0.6]; \% \text{initial the probability that the price goes up for each class}
\]

\[
\text{CC} = \text{zeros}(2, 6, 6);
\]

\[
\text{P} = \text{zeros}(2, 6); \% \text{initial the expected market price for each period in each class}
\]

\[
\text{S} = \text{zeros}(2, 1);
\]

\[
\text{for } h = 1:2
\]

\[
\text{for } i = 1:T
\]

\[
\text{CC}(h,i,1) = (1 - p_{\text{up}(h,1)})^i \times w(h,2)^i \times P0(h,1);
\]

\[
c = 0;
\]

\[
\text{for } j = 1:i
\]

\[
\text{CC}(h,i,j+1) = \text{nchoosek}(i,j) \times p_{\text{up}(h,1)}^j \times (1 - p_{\text{up}(h,1)})^{i-j} \times w(h,1)^j \times w(h,2)^{(i-j)} \times P0(h,1);
\]

\[
c = c + \text{CC}(h,i,j+1);
\]

\[
\text{end}
\]

\[
\text{P}(h,i) = \text{CC}(h,i,1) + c;
\]

\[
\text{end}
\]

\[
\text{S}(h,1) = \text{sum}(\text{P}(h,:))/6;
\]

\[
\text{end}
\]

\[
\text{display('the expected price')};
\]

\[
\text{display(P)};
\]

\[
\text{display('Striking price')};
\]

\[
\text{display(S)};
\]

\[
\% \text{actual market price}
\]

\[
\% 1.2
\]

\[
\text{Price} = [360, 288, 345.6, 414.72, 497.664, 398.1312, 180, 162, 145.8, 160.38, 144.342, 158.7762];
\]
display(Price);

call option and put option tickets premium
C_call=zeros(2,1);C_put=zeros(2,1);
%stage 0
c_put=zeros(2,T);% premium for put option tickets for each class
%stage 1
c_call=zeros(2,T);% premium for call option tickets for each class
n_call=zeros(2,T);% initial call option tickets for each period
R_call=zeros(2,T);% initial revenue for call option tickets in each period
%stage 2
n_standard=zeros(2,T);% initial standard tickets for each period
R_standard=zeros(2,T);% initial revenue for standard tickets in each period
%stage 3
n_put=zeros(2,T);% initial put option tickets for each period
R_put=zeros(2,T);% initial revenue for put option tickets in each period
%stage 4
n_recall=zeros(2,T);% initial recall tickets for each period
R_recall=zeros(2,T);% initial revenue for recall tickets in each period

for h=1:2
    for i=1:T
        c_put(h,i)=max(0,S(h,1)-Price(h,i));
        c_call(h,i)=max(0,Price(h,i)-S(h,1));
    end
    C_put(h,1)=sum(c_put(h,:))/T;
    C_call(h,1)=sum(c_call(h,:))/T;
end
display(c_put);
display(c_call);
display('premium for put option ticket');
display(C_put);
display('premium for call option ticket');
display(C_call);

R=zeros(2,T);% initial revenue for each period
R_before=zeros(2,1);% initial revenue before the four stages
R_stage=zeros(2,1);
R_end=zeros(2,1);
R_total=zeros(2,1);
n_total=zeros(2,1);% initial sold tickets
for h=1:2
    %stage 0
    R_before(h,1)=-B(h,1)*C_put(h,1);
    for i=1:T
        %stage 1
        n_call(h,i)=min(demand(h,i),max(0,A(h,1)-n_total(h,1)));
        R_call(h,i)=min(demand(h,i),max(0,A(h,1)-n_total(h,1)))*(Price(h,i)-C_call(h,1));
        n_total(h,1)=n_total(h,1)+n_call(h,i);
    end
end

for h=1:2
    %stage 3
    for i=1:T
        n_put(h,i)=min(demand(h,i),max(0,A(h,1)-n_total(h,1)));
        R_put(h,i)=min(demand(h,i),max(0,A(h,1)-n_total(h,1)))*(Price(h,i)-C_put(h,1));
        n_total(h,1)=n_total(h,1)+n_put(h,i);
    end
end

R=ones(2,T);% initial revenue for each period
R_before=ones(2,1);% initial revenue before the four stages
R_stage=ones(2,1);
R_end=ones(2,1);
R_total=ones(2,1);
n_total=ones(2,1);% initial sold tickets
n_standard(h,i)=min(demand(h,i)-n_call(h,i),max(0,C(h,1)/(1-Q(h,1))-B(h,1)-n_total(h,1)))*Price(h,i);
R_standard(h,i)=min(demand(h,i)-n_call(h,i),max(0,C(h,1)/(1-Q(h,1))-B(h,1)-n_total(h,1)))*Price(h,i);
n_total(h,1)=n_total(h,1)+n_standard(h,i);

n_put(h,i)=min(demand(h,i)-n_call(h,i)-n_standard(h,i),max(0,C(h,1)/(1-Q(h,1))-n_total(h,1)))*max(Price(h,i),S(h,1));
R_put(h,i)=min(demand(h,i)-n_call(h,i)-n_standard(h,i),max(0,C(h,1)/(1-Q(h,1))-n_total(h,1)))*max(Price(h,i),S(h,1));
n_total(h,1)=n_total(h,1)+n_put(h,i);

n_recall(h,i)=min(demand(h,i)-n_call(h,i)-n_standard(h,i)-n_put(h,i),max(0,C(h,1)/(1-Q(h,1))+A(h,1)-n_total(h,1)))*max(0,Price(h,i)-S(h,1));
R_recall(h,i)=min(demand(h,i)-n_call(h,i)-n_standard(h,i)-n_put(h,i),max(0,C(h,1)/(1-Q(h,1))+A(h,1)-n_total(h,1)))*max(0,Price(h,i)-S(h,1));
n_total(h,1)=n_total(h,1)+n_recall(h,i);
R(h,i)=R_call(h,i)+R_standard(h,i)+R_put(h,i)+R_recall(h,i);
end
R_stage(h,1)=sum(R(h,:));
R_end(h,1)=max(0,B(h,1)-sum(n_put(h,:)))*S(h,1);
R_total(h,1)=R_before(h,1)+R_stage(h,1)+R_end(h,1);
end
display(n_call); display(n_standard); display(n_put); display(n_recall); display(n_total);
display(R); display(R_before); display(R_stage); display(R_end); display(R_total);
Revenue_final=sum(R_total(:,1))+RD_total+RL_total;
fprintf('final revenue=%f
',Revenue_final);
f=Revenue_final;