INVESTIGATION OF NEEDLE-TISSUE FRICTION FORCE AND GEOMETRIC MODELING OF HYPODERMIC NEEDLE

A Thesis in
Mechanical Engineering

by

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ABSTRACT

This thesis investigates how surface roughness and insertion speed affects the frictional force at the needle-tissue interface and forms an analytical model for defining the tip geometry of a three bevel hypodermic needle.

High frictional forces between the needle surface and tissue create high insertion forces that reduce needle positioning accuracy in procedures such as brachytherapy radiation treatment and needle biopsy. Failure to accurately place needles inside the body reduces the efficacy of the procedure and can lower the patient’s quality of life. The first part of this thesis experimentally investigates the frictional force and needle positioning accuracy at the needle-tissue interface. It was found that, rougher surface texturing of the needles generally reduced the friction forces and higher insertion speeds resulted in increased frictional forces. Texturing was also shown to improve the targeting accuracy of the needles across all the speeds.

Analytical models for defining needle tip geometry can play important roles in needle performance evaluation and comparison by providing insight about different needle tip features. These models can also be applied to calculate the needle tip volume for a given insertion depth. Knowledge about how the needle tip volume changes with increasing insertion depth is important as needles with thinner profile near the tip have been reported to reduce patients’ pain. Minimizing needle insertion pain can improve patient adherence to diabetes treatments. The second part of this thesis formulates an analytical model for defining the geometry of a three-bevel hypodermic needle and uses the model to calculate the needle volume at varying insertion depths.
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NOMENCLATURE

$F_S$ = Tissue stiffness force.
$N$ = Normal force.
$F_T$ = Total needle force.
$F_F$ = Frictional force.
$d$ = Needle insertion depth.
$F_r$ = Residual tissue stiffness force.
$\mu$ = Friction coefficient.
$S$ = Needle insertion speed.
$R_{RMS}$ = Root mean square value of needle surface roughness.
$\eta$ = Tissue fluid viscosity.
$P_{av}$ = Average pressure compressing the liver.
$K$ = Tissue mechanical stiffness.
$A$ = Needle surface area.
$R_s$ = Arithmetic average of the absolute surface asperity heights.
$R_p$ = Maximum peak height measured from the surface mean line.
$R_z$ = Average distance between the highest peak and the lowest valley.
$r_i$ = Inner tube radius.
$r_o$ = Outer tube radius.
$\gamma$ = Radial angle of a needle point with the X-axis.
$\xi$ = Primary bevel angle.
$\varphi$ = Secondary bevel angle.
$\beta$ = Rotation angle.
$q$ = Offset distance.
$\gamma_d$ = Inner radial angle at the transition of primary and secondary bevels.
$\gamma_c$ = Outer radial angle at the transition of primary and secondary bevels.
$m$ = Slope of the line at the transition of primary and secondary bevels.
$t$ = Parameter for defining the cutting edges.
$z_d$ = Needle insertion depth from the tip.
$\delta$ = Angle between tip cutting edge and the YZ plane.
\( \gamma_{ao} \) = Outer radial angle on the needle tip at a given insertion depth.

\( \gamma_{ai} \) = Inner radial angle on the needle tip at a given insertion depth.

\( \gamma_{am} \) = Intermediate radial angle on the needle tip at a given insertion depth.

\( r_m \) = Intermediate radius on the needle tip at a given insertion depth.
Chapter 1

Introduction

This thesis contains four chapters. Chapter 1, Introduction, discusses research motivation, needle placement inside the human body, needle tip geometry modeling, previous relevant works and thesis objectives. Chapter 2, Needle-Tissue Friction, investigates the effects of needle surface roughness and insertion speed on the frictional force and positioning accuracy at the needle tissue interface. Chapter 3, Analytical Modeling of Hypodermic Needles, develops an analytical model to define the tip geometry of three bevel hypodermic needles and uses that model to calculate needle tip volume at given insertion depths. Chapter 4, Conclusion, lists summary, contribution of this thesis and future works.

1.1 Research Motivation

This thesis addresses two issues pertinent to invasive medical procedures. The first one is precise needle positioning inside the human body and the second one is defining hypodermic needle geometry to allow for future geometry optimization. This section discusses the motivation for pursuing research in the mentioned directions.

Accurate needle placement inside the human body is crucial for many medical procedures including biopsy, intravenous drug delivery, percutaneous local ablative therapies (ablation through ethanol injection or by radio frequency) and brachytherapy, where radioactive seeds are precisely placed in and around the cancerous tissue [1]. Friction between the needle and tissue hinders precise needle positioning which reduces the efficacy of the procedure and the patient’s quality of life. Brachytherapy is widely used to treat cancerous growths of the cervix [2], esophagus [3], lung [4], tongue [5], skin [6], prostate [7], brain [8], eye [9], bladder [10] and breast [11]. For interstitial brachytherapy, radioactive seeds are placed directly inside the tissue [12]. Accurate needle positioning is crucial for interstitial prostate brachytherapy. In this procedure, the
placement locations of 35-140 radioactive seeds [13] based on the size of the prostate gland and cancer are determined and needles are then accurately placed into the tissue to permanently deliver the seeds to the planned location. The efficacy of prostate brachytherapy depends on the accuracy of seed placement. Radioactive seed misplacement inside the body can cause the treatment to be ineffective (radiation missing desired location) and can lower the patient’s quality of life (radiation in wrong location) by causing side effects such as bowel and urinary incontinence, rectal bleeding, erectile dysfunction, and substantial tissue damage [14-16]. Elevated forces at the needle-tissue interface cause the prostate to splay and roll as shown in Fig. 1, which can result in needle deflection that ultimately leads to seed misplacement. Frictional forces correspond to a significant portion of the total forces at the needle-tissue interface [17]. Detailed studies investigating surface texture and insertion speed effects on needle-tissue frictional force and positioning accuracy are scarce.

Figure 1. Needle and seed misplacement due to prostate motion
Hypodermic needle is one of the most common medical devices as that is frequently used for the infusion of substances during Intravenous therapies and extraction of samples from the patients’ body [18]. A typical hypodermic needle consists of a cannula, a pointed tip and three bevel edges as shown in Fig. 2. For characterization of the performance, accurate comparison and improved design of the hypodermic needles, analytical modeling of the tip geometry is necessary. These models can be applied to determine how needle tip design affects patients’ pain. Pain associated with the use of hypodermic needles is a major concern. Diabetic patients need to use insulin injections on a daily basis for keeping their blood sugar levels under control. It has been reported that, a substantial majority of patients would like to reduce the number of injections they take each day, and almost half said that they would be more likely to take their insulin
injections regularly if a product was available to ease the pain [19]. As reduced puncture
forces during needle insertion have been reported to be related to reduced patients’ pain
[20], majority of the earlier literature focused on the reduction of insertion forces. However, higher insertion depth of the hypodermic needles can also play a significant role in the feeling of pain and discomfort [21]. Higher insertion depth will result in elevated volume of the needle tip inside the patients’ skin and consequently more pain to the patient. Analytical models defining the tip geometry can be utilized for calculating the hypodermic needle tip volume inside the patients’ body at a given insertion depth.

1.2 Accurate Needle Placement

Needle misplacement is due to a combination of edema, needle deflection and bending, poor image quality and target deflection [22]. Thin needles that are typically used to implant radioactive sources can deflect from the planned trajectory [23] and bend [24]. Ultrasound imaging facilitates real time confirmation of needle placement [25] during brachytherapy and hence poor image quality may hamper needle positioning accuracy. Needle templates used for manual brachytherapy imposes restrictions on needle placement and spacing [26]. Target deflections resulting from tissue movement and deformation are directly related to the forces imparted onto the tissue by the needle [27]. For a symmetric needle experiencing no imbalance in lateral forces, the only acting forces are axial, which include the cutting and stiffness forces at the leading cutting edge of the needle and the frictional force (force resisting the relative motion between the needle and tissue) that act along the side surface of the needle. Different forces acting on the needle are shown in Fig. 3.
Figure 3. Forces acting on the needle during needle insertion

Tissue deformation moves the target away from the initial position as illustrated in Fig. 4. Before insertion, a target position is determined inside the tissue, Fig. 4(a). Upon insertion, the tissue friction and cutting forces cause the tissue to deform, thereby moving the target, Fig. 4(b). The amount of tissue deformation can be reduced by minimizing the friction between the needle and tissue [28, 29] which can ultimately improve needle positioning accuracy.

Figure 4. Target location in soft tissue (a) before needle insertion and (b) after needle insertion
1.3 Hypodermic Needle Tip Modeling

Defining hypodermic needle geometry in terms of analytical models helps to compare the performances of different needles, understand insertion force behavior and identify important needle designing parameters.

Defining the geometry of a hypodermic needle can lead to improved needle designs for reducing patients’ pain. It has been reported that patients’ pain depends linearly on the needle insertion depth [30]. This happens because increased insertion depth will result in increased volume of the needle inside the body and consequently more pain to the patient. However, needles of different tip profiles will have different amounts of tip volume inside the patients’ body at a given insertion depth. For example, two needles are shown in Fig. 5. All the grinding parameters of these two needles are same except the secondary bevel angle which determines the lengths of the secondary bevels. Due to the differences in profile at the needle tips, needle (b) will have more volume than needle (a) for a given insertion depth and will result in more pain to the patient. Knowledge about how the needle tip volume depends on different needle grinding parameters will be beneficial in reducing patients’ pain and discomfort. This valuable information can be obtained by the analytical model.
Figure 5. Hypodermic needles with a (a) lower secondary bevel angle and (b) higher secondary bevel angle

1.4 Literature Review

This section presents the related works on needle-tissue interaction, surface texturing of components, geometry based needle modeling and needle designing with a view to reduce patient pain and discomfort.
1.4.1 Needle-Tissue interaction

Review on this topic has been divided into needle-tissue interaction forces, tissue deformation, needle deflection and robotic needle steering.

Needle to tissue interaction forces have been a topic of significant research interest for long time. Okamura et. al. [31] created and verified a model of different needle-tissue interaction forces for liver tissue. A general force model for needle insertion through the skin, fat and muscle to reach the liver [32] and force models based on nonlinear dynamics [33] have been developed. Effects of friction and needle geometry during robotic needle insertion into soft tissues have also been investigated [34]. An extensive friction force model was developed by Kobayashi et al. [35] who, examined the relationship between slow needle insertion speeds (from 0.01 to 10 mm/s) and frictional forces. As needle insertion simulation facilitates training and planning for medical procedures [29], considerable research effort has been given in this field. Simulation of resistance forces on needles [36], radioactive seed implantation for prostate brachytherapy [37] and effect of tissue deformation on needle placement accuracy have been reported [38]. The effects of surface coatings on frictional forces have been briefly investigated [28].

Due to the inhomogeneous, nonlinear, anisotropic, elastic and viscous behavior of soft tissue, real-time and accurate calculation of tissue deformation is complex [1]. Acoustic remote palpation imaging method for investigating tissue stiffness [39], real-time ultrasound indentation system for measuring soft tissue mechanical properties in vivo [40], and devices for measuring for tissue extension, tissue indentation and instrument-tissue interaction forces [41] have been developed. Mass-spring [42] and finite element based [43] models have also been reported for modeling tissue deformation.

Needle deflection is one of the reasons for inaccuracy in the needle insertion procedures [1]. It is known that needles with lower diameter and beveled tips bend more than higher diameter and triangular tipped needles [31]. As needles with bevel tips are asymmetric, they receive higher forces on one side that leads to their pronounced bending. However, triangular tipped needles also experience some bending due to local
density variation inside the tissue. Needle deflection models based on mechanics [44], beam deflection [45] and dynamics [46], needle deflection experiments on biological tissue [47] and soft tissue stimulant [48], simulation of deflection during needle insertion procedures [49] and methods for needle deflection estimation [50] have been reported.

Robotic needle insertion has the potential to overcome some shortcomings of the traditional therapies and diagnosis [51]. Motion planning techniques based on potential-field approach [52] and sensor less planning algorithm [53], kinematic model of needle-tissue interaction [54] and image-based controlling methods [55] for robotic needle steering have been investigated. Other aspects of robotic needle insertion such as needle designs [56-59], steering techniques [60, 61] and robotic systems [62-64] have also been reported.

1.4.2 Surface texturing

Surface texturing is a proven method for reducing friction between two lubricated surfaces. Research has explored laser surface texturing [65], electric discharge texturing [66] and ion beam etching [67] to improve the tribological properties for reduction of friction and wear. The effect of surface texture on the friction of metal-metal contacts such as bearings [68], mechanical seals [68-70], and piston rings [71-74] have been well investigated. Friction between soft contacts was studied by Huang et al. [75], who reported the coupled influence of surface texture and surface wettability on the lubrication between two elastomers in contact by experimenting with contacts (lubricated by water and glycerol) between textured PDMS (Polydimethylsiloxane) disks and a PDMS pin.

None of the works mentioned above have explored how friction at the needle-tissue interface is affected by needle surface texture and high insertion speeds (>100 mm/s). Investigations at high insertion speeds are necessary as hand placed needle insertion speeds during brachytherapy vary from near 0 mm/s to 900 mm/s [76].
1.4.3 Needle Geometry Based Modeling and Needle Design for Minimal Pain

Much work has been done in the field of needle modeling based on their geometry. Insertion force model for hollow needle [77] and Cutting edge geometry modeling of plane [78], curved [79] and novel enhanced cutting edge needles [80] have been reported. Wang et. al. formed a tissue insertion force model for three bevel hypodermic needles [81]. All these studies focused on cutting edge inclination and rake angles as they play important roles on the insertion force required by the needles.

Exploration of improved needle designs for reduced pain has also been centered on insertion forces. Comparative studies of five bevel needles revealed that they require lesser insertion forces than their three bevel conventional counterparts and also less painful as they have thinner profiles and less inserted volume at a given insertion depth [20, 82]. For overcoming problems associated with hypodermic needles and improving patient compliance, micro needles have been proposed for different medical procedures [83]. They have been shown to be less painful than regular 25 [84] and 26 [85] gauge hypodermic needles. Design and detailed characterization of micro needles [86], effects of vibration on micro needle insertion force [87] and effects of micro needle length on pain [88] have also been reported.

Although analytical models for defining the needle geometry are found in the literature, models for calculating the needle tip volume are currently lacking. These types of models can provide insight on solving problems pertinent to different medical procedures including patients’ pain reduction.

1.5 Objectives

The goal of the first part of this thesis is to investigate how the friction force and needle placement accuracy at the needle tissue interface is affected by high needle insertion speeds (>100 mm/s) and surface texture. A friction testing device was constructed and two sets of needle insertion experiments with needles of varying surface texture were performed on bovine liver samples to form and verify a frictional force model and determine targeting accuracy at the needle tissue interface.
The goal of the second part of this thesis is to formulate an analytical model for defining the geometry of a three bevel hypodermic needle. The model is formed by using the parametric equations of the different edges. This model was used to calculate the volume of the needle inside patients’ body at a given insertion depth. Results of how the needle tip volume depends on different grinding parameters have been reported.
Chapter 2

Needle-Tissue Friction

This chapter will discuss about the effects of needle surface roughness and insertion speed on needle-tissue friction. For clarity, this chapter has been divided into sections that discusses friction force model, methods, results and conclusions.

2.1 Friction Force Model

For accurate modeling of forces resulting from needle insertion, it is necessary to separate total insertion force ($F_T$), into the two main components namely tissue stiffness ($F_S$), caused by the deflection of tissue, and needle to tissue frictional force ($F_F$), as illustrated and Eq. (1).

$$F_T = F_S + F_F$$ (1)

A typical total insertion force versus insertion depth curve of an 18 gauge needle going into soft tissue is shown in Fig. 6 that includes both $F_S$ and $F_F$ forces. Two distinct phases as of cutting tissue reported by Heverly et al. [89] are, Phase one tissue deflection and Phase 2 tissue cutting. These two phases continue to occur one after another until the needle reaches its desired target as illustrated in Fig. 6. This buildup and release of forces is results of the changing tissue stiffness force as the needle deflects the tissue in phase one and then breaks through in phase two.
Figure 6. Example plot of total needle insertion force for an 18 gauge needle into soft tissue.

Figure 7(a) illustrates how the tissue stiffness force changes based on insertion depth. Points C1, C2, and C3 in Fig. 7(a) mark the areas where the residual tissue stiffness force ($F_{r1}$, $F_{r2}$, and $F_{r3}$) equals the total tissue stiffness force. In these locations, the needle has just punctured through the tissue and a low value of residual stiffness remains. The residual tissue stiffness will vary from C1 to C2 due to inhomogeneous tissue that creates variations in the tissue stiffness based on the location inside the tissue. However, for simplicity it is assumed that $F_{r1}$ equals $F_{r2}$ and $F_{r3}$ for a given trial and this residual tissue stiffness is denoted as $F_r$. Figure 7(b) illustrates how the frictional force increases with respect to insertion depth, after the point of needle insertion into tissue, point C1, which occurs after the initial phase one.
Figure 7. Illustration of how (a) Tissue stiffness and (b) Frictional force vary at different needle insertion depths

Upon needle insertion both dry and viscous friction will occur, as shown in Fig. 8. Dry friction occurs where there is direct contact between the needle and tissue while viscous friction occurs where there is fluid film separating the needle and tissue. Due to the presence of surface irregularities, fluid from the tissue is deposited into the valleys between the needle peaks. Therefore the ratio of dry to viscous friction is directly correlated to the surface texture of the needle. This phenomenon closely resembles lubricated journal bearings, where lubricants are interposed between the journal (portion of shaft in contact with the bearing) and bearing surfaces for reducing wear by decreasing the metal to metal contact.
Journal bearing lubrication can be characterized by the Strubeck curve [90] which illustrates how the frictional coefficient changes depending on the boundary condition established by the amount of viscous and dry friction. According to the Strubeck curve, the frictional coefficient increases with increasing speed between the moving parts at the higher speed ranges due to the dominance of viscous friction over dry friction. Research has also shown that, modification of the surface texture affects the Strubeck curve and can decrease the level of friction [91]. Hence, by combining Coulomb’s law and the factors that have been shown to influence journal bearing friction, the friction at the needle tissue interface for a given needle can be written as:

\[
F_F = \mu(S, R_{RMS}) N = \mu(S, R_{RMS}, \eta, P_{av}, K) P_{av} A(d)
\]  \hspace{1cm} (2)

Here, \( \mu \) is the frictional coefficient and it is a function of needle insertion speed \( (S) \), needle surface roughness \( (R_{RMS}) \), viscosity of the tissue fluid \( (\eta) \), average pressure compressing the liver \( (P_{av}) \) and the mechanical stiffness of the tissue \( (K) \). The tissue’s mechanical stiffness, \( K \), is necessary in Eq. (2) because unlike journal bearings tissue is a soft material that easily deforms and this would affect the contacts occurred between the needle and the tissue. The normal force, \( N \), on the needle is found by the product of the average pressure compressing the liver, \( P_{av} \), and the needle surface area, \( A \), in

Figure 8. Dry and viscous friction regimes on the needle surface are dependent on needle and tissue texture.
contact with the tissue. The $A$ will increase linearly as insertion depth, $d$ increases. The first set of experiments are conducted to solve for $\mu(S, R_{RMS}, \eta, P_{av}, K)$ using Eq. (2).

Combining Eqs. (1) and (2), the generic total force equation can be written as:

$$F_T = F_S + \mu(S, R_{RMS}, \eta, P_{av}, K) P_{av} A(d)$$  \hfill (3)

This thesis only investigated the effects surface roughness and insertion speed on the frictional force. Hence, $\mu(S, R_{RMS}, \eta, P_{av}, K)$ has been written as $\mu(S, R_{RMS})$ in the rest of the thesis. At the transition between Phase two and Phase one, as illustrated in Fig. 7(a), at points $C_1$, $C_2$, and $C_3$, the $F_T$ can be expressed as:

$$F_T = F_r + F_F \quad \text{(At transition of Phase 2 and Phase 1)}$$  \hfill (4)

For $C_1$, the frictional force component, $F_F$ will be zero as the needle has just punctured through the tissue. Hence at this point, $F_T$ will be equal to $F_r$. This $F_r$ is later used for verifying friction model. Equation 4 for the second set of phases can be rewritten as:

$$F_F = F_T - F_r \quad \text{(At transition of second Phase 2 and Phase 1)}$$  \hfill (5)

Equation 5 is used in the second set of experiments to verify the friction force model, Eq. (2), at the specific point of $C_2$. 
2.2 Method

Two sets of experiments were carried out to investigate the effects of needle surface texture on needle friction. In the first set of experiments, the $F_F$ was measured on textured 11 gauge needles sliding through bovine liver tissue with no rotation to determine $\mu(S, R_{RMS})$ for the development of the frictional force model. In the second set of experiments, 18 gauge needles were inserted into bovine liver tissue with no rotation and the $F_T$ is measured along with position accuracy to verify the friction force model and quantify the effect of texture and speed on needle deflection.

2.2.1 Overview of Experimental Setup

The experimental setups for the first and second set of experiments are shown in Fig. 9. For both the experiments, the setup consists of a linear motor (Dunkermotoren, Germany), a 6 DOF force sensor (Nano17 from ATI Industrial Automation, NC), and a tissue box that holds the bovine liver. For all of the experiments fresh bovine liver was used and stored saturated in its fluid while not in use. Little fluid loss was observed during the trials from the tissue box. The tissue box held the tissue under a fixed compression of 11.65 kPa allowing for consistent boundary conditions on the needle. Consistent fixturing of soft materials is crucial for repeatable machining results as illustrated by Shih et al. in machining elastomers [92].

For the first set of experiments, Fig. 9(a), the bovine liver box allowed for a constant 90 mm of the needle to be in contact with the liver. The needle was never fully retracted from the bovine liver; therefore, only $F_F$ was measured.

For the second set of experiments, 18 gauge needle deflections were quantified with a six DOF electromagnetic based position sensor (3D Guidance trackSTAR from Ascension Technology Corporation, VT) as shown in Fig. 9(b). The 18 gauge needles were inserted 120 mm into the liver through a brachytherapy template and the $F_T$ consisting of tissue stiffness and friction was recorded. After each insertion, the needles were freed from the force sensor and the magnetic sensor cable was inserted into the
needle at a depth of 100 mm to obtain the x, y and z position of the needle inside the liver.

Figure 9. Overview of the experimental setup for (a) first set of experiments and (b) second set of experiments.
2.2.2 Needles

Four 11 gauge (3.048 mm diameter) stainless steel needles of varying surface roughness denoted N1, N2, N3, and N4, were used in the first set of experiments and are shown in Fig. 10. In the second set of experiments three 18 gauge brachytherapy needles (1.27 mm diameter) (Mick Radio-Nuclear Instruments, Inc.), denoted N5, N6, and N7, were used and needles N6 and N7 were modified with higher surface roughness. The varying surface roughness was created by turning the needles against an abrasive cloth of varying grit sizes listed in Table 1. The surface roughness parameters were measured using the non-contact method of optical profilometry illustrated for the 11 gauge needles shown in Fig. 10. Table 1 lists parameters measured where; $R_a$ is the arithmetic average of absolute heights measured from the mean line of the surface, $R_{RMS}$ is the root mean square value of the roughness profile, $R_p$ is the maximum peak height measured from the mean line, and $R_z$ is the average distance between the highest peak and the lowest valley in each sampling length. Among the four 11 gauge needles, needle N1 is the smoothest and N4 is the roughest. Among the three 18 gauge needles, needle N5 is the smoothest and N7 is the roughest.

Frictional forces on 11 gauge needles are more pronounced than that on the 18 gauge needles due to the higher surface area per insertion depth. This made the 11 gauge needles more ideal for accurately determining $\mu(S, R_{RMS})$ for the friction model. The 18 gauge needles, currently used in brachytherapy, allow for accurate verification of the model and position accuracy measurement.
Figure 10. Pictures and optical profilometry of needles (a) N₁, (b) N₂, (c) N₃, and (d) N₄. The field of view for profilometry images was 0.28 mm × 0.22 mm.

Table 1. Surface roughness values of the needles used in the experiment

<table>
<thead>
<tr>
<th>Needle type</th>
<th>Ra (μm)</th>
<th>R_RMS (μm)</th>
<th>R_P (μm)</th>
<th>R_Z (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₁ (11 gauge, no blasting)</td>
<td>0.23 ± 0.03</td>
<td>0.35 ± 0.05</td>
<td>7.10 ± 0.96</td>
<td>5.51 ± 1.05</td>
</tr>
<tr>
<td>N₂ (11 gauge, 1200 grit)</td>
<td>0.41 ± 0.07</td>
<td>0.55 ± 0.10</td>
<td>10.68 ± 1.08</td>
<td>9.21 ± 0.74</td>
</tr>
<tr>
<td>N₃ (11 gauge, 600 grit)</td>
<td>0.49 ± 0.08</td>
<td>0.65 ± 0.11</td>
<td>11.09 ± 1.39</td>
<td>9.73 ± 0.93</td>
</tr>
<tr>
<td>N₄ (11 gauge, 240 grit)</td>
<td>0.64 ± 0.08</td>
<td>0.83 ± 0.11</td>
<td>13.18 ± 1.75</td>
<td>11.50 ± 1.05</td>
</tr>
<tr>
<td>N₅ (18 gauge, no blasting)</td>
<td>0.19 ± 0.02</td>
<td>0.25 ± 0.03</td>
<td>5.88 ± 2.38</td>
<td>4.84 ± 1.72</td>
</tr>
<tr>
<td>N₆ (18 gauge, 600 grit)</td>
<td>0.47 ± 0.10</td>
<td>0.62 ± 0.13</td>
<td>9.75 ± 1.30</td>
<td>8.04 ± 1.14</td>
</tr>
<tr>
<td>N₇ (18 gauge, 240 grit)</td>
<td>0.65 ± 0.07</td>
<td>0.86 ± 0.10</td>
<td>12.32 ± 1.58</td>
<td>10.45 ± 0.96</td>
</tr>
</tbody>
</table>
2.2.3. Experimental Parameters

Each of the four 11 gauge needles was tested at speeds of 25, 50, 100, 150, 200, 400, 600, 800 and 900 mm/s for a total of 36 trials. Each of the trials was carried out at different locations. In each trial the needle was advanced 200 mm and retracted 200 mm for 10 cycles producing a steady state insertion and retraction force for each cycle. The steady state force was defined as the average force during the final 15 mm of insertion and retraction. One cycle is shown in Fig. 11 that illustrates where steady state insertion and retraction force was determined. Friction acts on the needle opposite to the direction of needle motion. The force recorded from the force sensor is negative for compression and positive for tension. As a result, the friction during insertion is recorded as negative forces and friction during retraction is recorded as positive forces. The friction force varied greatly in the first few cycles. This can be attributed to the dynamic interaction of the tissue with the boundary conditions of the box where there is a dynamic movement of the tissue that affects the insertion force in the first few cycles before the tissue settles. The last 5 cycles produced fairly consistent results as shown in Fig. 12. These cycles were averaged together to determine $F_F$ for the given parameters of the experiment.

The three 18 gauge needles were each tested four times at each of the speeds of 25, 200, 400 and 900 mm/s for a total of 48 trials. The $F_F$ was measured during the insertion into tissue. The $x$, $y$ and $z$ position of the needle just before the insertion and at an insertion depth of 100 mm were determined with the position sensor and the corresponding needle deflection was calculated from these values. The much larger tissue box helped prevent dynamic movement of the tissue during insertion from affecting the insertion force readings.
Figure 11. Friction force for one cycle of insertion and retraction

Figure 12. Example plot showing the frictional force values obtained at different cycles
2.3 Results and Analysis

This section discusses the results of the frictional force investigation. Frictional force, calculation of $\mu(S, R_{RMS})$, total needle force, needle deflection results and model verification have been presented in different sub-sections.

2.3.1 Speed and Roughness Effect on Friction

The absolute values of the average friction force, $F_F$, during the first set of experiments were determined for insertion, Fig. 13, and retraction, Fig. 14, across the nine tested speeds. It was found that, on average increasing insertion speed increases the frictional force for needles N1-N4. Of the four needles tested, needle N3 ($R_{RMS}$ of 0.655 ± 0.11 µm) had on average the lowest frictional forces, outperforming the smooth surface textured needle, needle N1. During insertion, needle N3 experienced 20, 30, 15, 9, 24, 28, 36, 11 and 13% less force and during retraction, it experienced 36, 16, 42, 34, 24, 36, 41, 11 and 27% less force than needle N1 for the speeds of 25, 50, 100, 150, 200, 400, 600, 800 and 900 mm/sec respectively. This indicates that surface texture can help to improve needle to tissue friction. The increase in friction force with increasing speeds can be justified from the Stipeck curve as in the hydrodynamic lubrication regime, the friction coefficient increases with increase in speed due to viscous drag.
Figure 13. Absolute value of insertion friction force for 11 gauge needle experiments.

Figure 14. Absolute value of retraction friction force for 11 gauge needle experiments.
In general, the roughened 11 gauge needles, N₂ – N₄ experienced lower friction forces at the needle tissue interface. This can be explained by the presence of more irregularities on the surface of the rougher needles. Surface asperities replace dry friction by viscous friction by reducing the contact area at the needle tissue interface and collecting tissue fluids in the depressions. Viscous friction is much lower than dry friction in magnitude and therefore the total friction force is less for rougher needles. The concept of replacing dry friction by viscous friction is shown in Fig. 15.

![Figure 15. Illustration of how needle surface roughness affects ratio of dry to viscous friction for a (a) smooth needle, and (b) rough needle](image)
The frictional force results also conform to the surface wettability studies. Stainless steel is hydrophilic to blood meaning, blood will readily get collected into the surface asperities after its exposure. Surface roughness has been reported to enhance the surface wettability [93]. Needles with higher surface roughness values undergone increased wetting as more blood got deposited into the surface valleys. This ultimately resulted in the dominance of viscous friction over dry friction which caused reduced overall frictional forces for rougher needles.

### 2.3.2 Calculation of $\mu(S, R_{\text{RMS}})$

The friction coefficient, $\mu(S, R_{\text{RMS}})$, relationship to $S$ and $R_{\text{RMS}}$ was determined by using Eq. (2) to calculate $\mu$ for the varying $S$ and $R_{\text{RMS}}$ parameters tested during needle insertion, as shown by the points in Fig. 16. The surface in Fig. 16, corresponds to a second order multivariable ($R_{\text{RMS}}$ and $S$) best fit polynomial of the calculated $\mu$ values. This surface is expressed in Eq. (6) where $R_{\text{RMS}}$ is in units of $\mu$m and $S$ is in units of mm/s.

$$\mu(R_{\text{RMS}}, S) = 0.201 - 0.256R_{\text{RMS}} + 0.000855S + 0.181R_{\text{RMS}}^2$$
$$\quad - 0.000298R_{\text{RMS}}S - 3.158e-007S^2$$  \hspace{1cm} (6)

The $R^2$ value of the $\mu(R_{\text{RMS}}, S)$ model is 0.91, which shows reasonable correlation to the experimental results. This model indicates that friction coefficient decreases as the needle surface roughness increases up to an $R_{\text{RMS}}$ of 0.83 $\mu$m and the reduction in the friction coefficient is more pronounced at higher speeds. Around a surface roughness of $R_{\text{RMS}}$ of 0.83 $\mu$m the improvement in frictional force becomes less pronounced and it is expected that a high enough increase in surface roughness above 0.8 $\mu$m would cause an increase in friction.
2.3.3 Effect of Surface Area and Coefficient of Friction on Total Needle Force

The friction coefficients for the three 18 gauge needles at different speeds were calculated using Eq. 6. The area of the needle in contact with the tissue, $A$ was calculated and the total force on the needle, $F_T$ after the second transition from phase one to phase two (point $C_2$ in Fig. 6) was separated from the force vs. insertion depth
plot to produce the points illustrated in Fig. 17. The plane in Fig. 17 is a first order multivariable best fit plane using least squares method with an $R^2$ value of 0.90 that shows the general trend. It is shown that the total force increases with increase in $A$ and $\mu$ as predicted by the model given in Eq. (3).

Figure 17. Total force just after the second puncture vs. calculated friction coefficient and needle surface area
2.3.4 Effect of Surface Texture and Insertion Speed on Needle Deflection

The rougher needles, $N_6$ and $N_7$, experienced lower needle deflection than the standard smooth 18 gauge brachytherapy needle across their 100 mm of insertion length as illustrated in Fig. 18. $N_7$ experienced 19, 41, 49 and 45\% less deflection than the smoothest 18 gauge needle $N_5$ at the speeds of 25, 200, 400 and 900 mm/s. This can be explained by the fact that, the friction force was lowest on the interface of $N_7$ and therefore there is less tissue deflection and needle bending resulting in lower deflection.

The highest insertion speed of 900 mm/sec was shown to improve needle position accuracy for all three needles tested compared to the slow insertion speed of 25 mm/s. This phenomenon can be explained because high cutting speed creates high cutting force at the needle tip that makes for a shorter Phase one of tissue deflection. This means the tissue does not deflect as much and therefore the position accuracy is better.

![Figure 18. Needle deflection vs. Insertion speed plot for the three 18 gauge needles](image-url)
2.3.5 Verification of Model

The predicted friction force is found using Eq. (2) and inserting Eq. (6). Equation (5) is used to calculate $F_F$ for the 18 gauge needles. The residual stiffness is determined by the value of $F_T$ at point C$_1$ and then this value is subtracted from the value of $F_T$ at C$_2$. The model and force results from the 18 gauge needle experiments can then be compared as in Fig. 19, where the line represents an ideal correlation between the model and actual values.

The model is not ideally in line with the experimental results; however, a general trend of correlation can be seen in Fig. 19 where it is illustrated that 87.5% (42 of 48) of the 18 gauge needle results fall within 1.1 N of the model. The inaccuracy of the model can be explained by the inhomogeneity of the tissue and the assumption that residual stiffness at point C$_1$ equals the residual stiffness at point C$_2$. The residual stiffness will vary based on needle depth and it is very difficult to separate this value from $F_T$.

Figure 19. Friction force calculated from the model compared to measured force from the 18 gauge needles experiments
2.4 Conclusions

Surface roughness and insertion speed effects on needle-tissue friction have been investigated. A friction force model was developed and correlated to 18 gauge needle insertion results. It was found that a decrease in insertion speed and an increase in needle roughness, up to 0.83 µm, generally reduced needle friction. An increase in surface roughness was also shown to improve needle position accuracy over that of traditional smooth 18 gauge needles.
Chapter 3

Analytical Modeling of Hypodermic Needles

This chapter develops an analytical model to define the geometry of three bevel hypodermic needles and then uses the model to determine the needle tip volume at given insertion depths. For convenience, this chapter has been divided into sections that describe needle definition, needle modeling, needle tip volume estimation from the model, results of tip volume calculation and conclusion.

3.1 Needle Definition

A typical three bevel hypodermic needle is shown in Fig. 20. Here \( r_i \) is the inner tube radius and \( r_o \) is the outer tube radius. The X, Y and Z axis shown in the figure is adopted as the convention for coordinates and is followed for subsequent mathematics. The X-axis passes through the lowest point of the needle tip and the Z-axis coincides with the needle axis. \( A \) is any point on the needle cutting surface and \( \gamma \) is the radial angle of \( A \) measured from the X-axis. Points \( P \) and \( Q \) are the points of intersection of the two secondary bevels on the inner and outer tubes, respectively, as shown in Fig. 20.
The hypodermic needle is defined by the primary bevel angle (\( \zeta \)), secondary bevel angle (\( \varphi \)), rotation angle (\( \beta \)), inner tube radius (\( r_i \)) and the outer tube radius (\( r_o \)). The two radii are shown in Fig. 20 and the three angles are shown in Fig. 21. The offset distance, \( q \) ensures the existence of point P in the ground needle and it can be expressed in terms of the two radii and the three angles.
3.2 Needle Modeling

This model defines the needle geometry by expressing the needle cutting edges in terms of parametric equations. The needle with different cutting edges marked is shown in Fig. 22.
Points D and E denote the transition between the primary and secondary bevels on the right side of the needle. The location of these two points with respect to the origin are expressed in terms of angles $\gamma_d$ and $\gamma_e$. These have been shown in Fig. 23.
Wang et. al. [81] expressed $\gamma_d$ and $\gamma_e$ in terms of a slope $m$. $m$ is the slope of the right cutting edge in the XY plane. The equations for $m$, $\gamma_d$, $\gamma_e$ and $q$ are written as:

\[
m = \frac{\cos \beta - \cot \xi \tan \varphi}{\sin \beta}
\]

\[
\gamma_d = \arccos \left( \frac{1 - m^2}{1 + m^2} \right)
\]

\[
\gamma_e = \arccos \left( -m^2 r_i + \sqrt{r_o^2 + m^2 (r_o^2 - r_i^2)} \right)
\]

\[
q = r_i (1 - \cos \beta) \cos \varphi - \frac{\sin(\varphi - \xi)(r_o - r_i)}{\sin \xi}
\]

(7)

The equations for the tip cutting edge can be expressed in terms of parameter $t$ as:

\[
x = -r_i + t(r_i - r_o)
\]

\[
y = 0
\]

\[
z = (r_o + r_i) \cot \xi + t(r_o - r_i) \cot \delta
\]

\[0 < t < 1\]

(8)

The equations for the right and left cutting edges can also be expressed in terms of $t$ as:

\[
x = r_i \cos \gamma_d + t(r_o \cos \gamma_e - r_i \cos \gamma_d)
\]

\[
y_{\text{right}} = r_i \sin \gamma_d + t(r_o \sin \gamma_e - r_i \sin \gamma_d)
\]

\[
y_{\text{left}} = -r_i \sin \gamma_d - t(r_o \sin \gamma_e - r_i \sin \gamma_d)
\]

\[
z = r_i (1 - \cos \gamma_d) \cot \xi + (r_o - r_i) \cot \xi + t(r_i \cos \gamma_d \cot \xi - r_o \cos \gamma_e \cot \xi)
\]

\[0 < t < 1\]

(9)
The equations for the inner cutting edge are:

\[
\begin{align*}
x &= r_i \cos \gamma \\
y &= r_i \sin \gamma \\
z &= \begin{cases} 
(r_o - r_i \cos \gamma) \cot \xi, & 0 \leq \gamma \leq \gamma_d \\
-(r_o - r_i \cos(\gamma - \beta)) \cot \varphi + 2r_o \cot \xi + \frac{q}{\sin \varphi}, & \gamma_d \leq \gamma \leq \pi \\
-(r_o - r_i \cos(\gamma + \beta)) \cot \varphi + 2r_o \cot \xi + \frac{q}{\sin \varphi}, & \pi \leq \gamma \leq 2\pi - \gamma_d \\
(r_o - r_i \cos \gamma) \cot \xi, & 2\pi - \gamma_d \leq \gamma \leq 2\pi
\end{cases}
\end{align*}
\]  

(10)

The equations for the outer cutting edge are:

\[
\begin{align*}
x &= r_o \cos \gamma \\
y &= r_o \sin \gamma \\
z &= \begin{cases} 
r_o(1 - \cos \gamma) \cot \xi, & 0 \leq \gamma \leq \gamma_e \\
-(r_o - r_o \cos(\gamma - \beta)) \cot \varphi + 2r_o \cot \xi + \frac{q}{\sin \varphi}, & \gamma_e \leq \gamma \leq \pi \\
-(r_o - r_o \cos(\gamma + \beta)) \cot \varphi + 2r_o \cot \xi + \frac{q}{\sin \varphi}, & \pi \leq \gamma \leq 2\pi - \gamma_e \\
r_o(1 - \cos \gamma) \cot \xi, & 2\pi - \gamma_e \leq \gamma \leq 2\pi
\end{cases}
\end{align*}
\]  

(11)

3.3 Needle Tip Volume Calculation from the Model

For finding the needle tip volume at an insertion depth, first the tip area of the needle at that particular insertion depth is found. After that, the area expression is integrated up to the point Q to find the inserted needle tip volume. Presence of multiple cutting edges results in a number of transitions on the needle. These transitions are shown in Fig. 24.
Here, $z_d$ is the insertion depth and is measured from the tip point, $Q$. Beyond transition 5, the needle tip becomes a regular tube having needle outer and inner radii as its outer ($r_o$) and inner radii ($r_i$).

For the convenience of defining different transition lengths and find corresponding tip areas, the angle between the tip cutting edge and the YZ plane has been defined as $\delta$. $\delta$ is shown in Fig. 25 and it can be written as:

$$\delta = \arcsin \frac{\sin \phi}{\sqrt{\sin^2 \phi + \cos^2 \beta \cos^2 \phi}}$$

(12)
Lengths for different transitions are shown in Fig. 26. These lengths can be obtained by using Eq. 8 - 12.

Equations for obtaining the needle tip volumes at different transitions have been obtained from Eq. 8 – 12. The volume equations were compiled into a MATLAB code shown in Appendix.
3.4 Results of Needle Tip Volume Calculation

This section reports about how the three bevel needle volume changes from one transition to another and how it can be modified by varying different needle grinding parameters.

3.4.1 Volume of a Standard 16 Gauge Needle

A standard 16 gauge needle has an inner radius ($r_i$) of 0.60 mm, outer radius ($r_o$) of 0.83 mm, primary bevel angle ($\xi$) of 11.3°, secondary bevel angle ($\varphi$) of 18.18° and rotation angle ($\beta$) of 56.74°. Figure 27 shows how its volume increases with increasing insertion depth across and beyond the 5 transitions.

Figure 27. Needle volume vs. insertion depth for a standard 16 gauge needle ($r_i$=0.60 mm, $r_o$=0.83 mm, $\xi$ = 11.3°, $\varphi$ =18.18° and $\beta$ = 56.74°).
Fig. 27 shows that, the volume does not increase that much in transition 1. That happens because a very small portion of the needle gets inserted into the body with increasing insertion depth during transition 1. Volume changes are much more significant for the subsequent transitions. The curves illustrating different transitions have different curvature and lengths owing to differences in the geometry and length of the transitions. Beyond transition 5, the primary bevel ends and the needle becomes a hollow tube whose volume depends only on the needle inner and outer radii that are constant. Hence, the volume increases linearly after transition 5.

3.4.2 Effects of Primary Bevel Angle

For evaluating the effects of the primary bevel angle, $\zeta$, tip volume of a number of needles with varying $\zeta$ were evaluated keeping all the remaining grinding parameters constant. The results are shown in Fig. 28.

![Figure 28. Effects of $\zeta$ on needle volume for a needle of $r_i = 0.60$ mm, $r_o = 0.83$ mm, $\varphi = 18.18^\circ$ and $\beta = 56.74^\circ$.](image)
Figure 28 was generated considering the whole needle tip lengths for the needles. \( \xi = 11.3^\circ \) is the standard needle and \( \xi = 18.18^\circ \) represents the configuration of maximum \( \xi \) with this combination. This happens because for grinding three bevel hypodermic needles, \( \varphi \) has to be greater than or equal to \( \xi \). Lower values of \( \xi \) resulted in higher needle tip length, lower volume change rate and lower inserted volume at a given insertion depth. Another significant finding is that, the needle tip volume is independent of \( \xi \) for the first three transitions. It occurs as the geometry for the first three transition is determined by \( \varphi \) and \( \beta \). To get an idea of how needles with very high and low values of \( \xi \) would look like, two needles are shown in Fig. 29.

As shown in Fig. 29, lower \( \xi \) makes the profile thinner near the tip which ultimately causes lower volumes at a given insertion depth. Lower \( \xi \) also results in higher primary bevel length to make the needle tip length greater than that having a higher \( \xi \).
3.4.3 Effects of Secondary Bevel Angle

Figure 30 shows the effects of the secondary bevel angle, $\varphi$ on the tip volume of four needles with varying $\varphi$ while keeping the remaining grinding parameters constant. Figure 30 considered the total needle tip length for the needles.

![Graph showing effects of $\varphi$ on needle volume](image)

**Figure 30.** Effects of $\varphi$ on needle volume for a needle of $r_i=0.60$ mm, $r_o=0.83$ mm, $\xi=11.3^{\circ}$ and $\beta=56.74^{\circ}$.

In Fig. 30, $\varphi=18.18^{\circ}$ is the standard needle and $\varphi=11.3^{\circ}$ represents the minimum $\varphi$ with this configuration as for grinding a three bevel hypodermic needle, $\varphi$ has to be greater than or equal to $\xi$. Increasing the secondary bevel angle resulted in lower needle tip length and higher volume change rate. Increased $\varphi$ also resulted in higher inserted volume at a given insertion depth as higher values of $\varphi$ led to less grinding of the needle tip which ultimately caused the profile to be thicker. Figure 31 shows how the needle volume changes during the first transition for various $\varphi$. 
Figure 31. Effects of $\phi$ on needle volume for a needle of $r_i = 0.60$ mm, $r_o = 0.83$ mm, $\xi = 11.3^\circ$ and $\beta = 56.74^\circ$ during the first transition.

Figure 31 shows that, higher values of $\phi$ resulted in earlier termination of the first transition and also increased volume change rate for the needles during the first transition. To get an idea of how needles with very high and low values of $\phi$ would look like, two needles are shown in Fig. 32.
Figure 32. Needle profiles for very low and high values of $\varphi$.

It can be seen from Fig. 32 that, a low value of $\varphi$ reduces the profile breadth at the needle tip and also increases the first transition and needle tip length. These observations support the trends seen at Fig.(s) 30 and 31.

3.4.4 Effects of Rotation Angle

For evaluating the effects of the rotation angle, $\beta$, tip volume of four needles with varying $\beta$ were evaluated keeping all the remaining grinding parameters constant. The results are shown in Fig. 33. Figure 33 was generated considering the needle tip lengths (from the tip up to transition 5) for the needles.
Figure 33. Effects of $\beta$ on needle volume for a needle of $r_t=0.60$ mm, $r_o=0.83$ mm, $\xi=11.3^\circ$ and $\varphi=18.18^\circ$.

In Fig. 33, $\beta = 56.74^\circ$ is the standard needle and other values of $\beta$ represent different configurations. Modifying the $\beta$ changes the total tip lengths for the needles. Apart from $\beta = 85^\circ$, all the other needles resulted in similar volumes near the end of the tip lengths. For the first two transitions, it is logical that, needles with very low rotation angles should result in higher volumes at a given insertion depth owing to lesser grinding. But in Fig. 33, the needle with very high rotation angle ($85^\circ$) also resulted in higher volumes for the first 2 transitions. This phenomena is shown more clearly in Fig. 34 for the first 2 transitions of all the needles.
Figure 34. Effects of $\beta$ on needle volume for a needle of $r_i=0.60$ mm, $r_o=0.83$ mm, $\zeta=11.3^\circ$ and $\phi=18.18^\circ$ during the first two transitions.

As shown in Fig. 34, the volume for $\beta=85^\circ$ was initially higher than that of $\beta=15^\circ$ but eventually the needle with $\beta=15^\circ$ crosses the one with $\beta=85^\circ$. The volumes are lower and the transition lengths are higher for the needles with intermediate rotation angles ($35^\circ$ and $56.74^\circ$). To get an idea of how needles with high, intermediate and low values of $\beta$ would look like, three needles are shown in Fig. 35.
As shown in Fig. 35, a low value of $\beta$ shortens the first two transitions and makes the needle profile broader that ultimately results in higher volume. Needles with very high $\beta$ caused the profile to be thicker as shown in Fig. 35 and that resulted in higher volumes. It may seem counterintuitive as why needles with very high rotation angles still end up with higher volumes because higher rotation angles open the way for more needle grinding near the tip. The reason behind this is, apart from $\beta$, offset distance, $q$
also determines the amount of grinding undergone by a particular needle. Increasing the \( \beta \) too much eventually increases the offset distance \( q \) so much that it reduces any potential increase in the grinding amount and leave the needle in conditions similar to the one having a low \( \beta \). For this reason, although the profiles of needles with very high and low \( \beta \) are much different, both of them have higher volumes than those having intermediate values of \( \beta \). This indicates that, there are some optimal values of the angle \( \beta \) in between very high and low values that can yield suitable values of \( q \) to result in minimal needle volume.

### 3.4.5 Effects of Needle Wall Thickness

For evaluating the effects of the needle wall thickness, tip volume of four needles with varying needle inner radius, \( r_i \) were evaluated keeping all the remaining grinding parameters constant. The results are shown in Fig. 36. Figure 36 was generated considering the whole needle tip lengths (from the tip up to transition 5) for the needles.
In Fig. 36, \( r_i = 0.60 \text{ mm} \) represents the standard 16 gauge needle and other values of \( r_i \) are investigated to see the effects of the needle wall thickness. Increased \( r_i \) resulted in higher needle tip length, lower volume change rate and lower volume at a given insertion depth. The volume is independent of \( r_i \) for the first transition. This is due to the more pronounced effects of the secondary bevel angle, \( \varphi \) and rotation angle, \( \beta \) on that transition. To get an idea of how needles with very high and low values of \( r_i \) would look like, two needles are shown in Fig. 37.
As shown in Fig. 37, Needles with higher $r_i$ have thinner walls and consequently lesser volume at a given insertion depth. Higher $r_i$ also resulted in increased needle tip length.

### 3.4 Conclusions

An analytical model for defining the geometry of a three bevel hypodermic needle has been created and applied to find the needle volume at given insertion depths. Effects of various needle grinding parameters on needle volume have been investigated. It was found that, the inner needle radius, $r_i$ and the primary bevel angle, $\xi$ mainly affects the needle volume for the last two transitions while, the secondary bevel angle, $\varphi$ and the rotation angle, $\beta$ primarily affects the needle volume for the first two transitions. Needles with very high or low values of $\beta$ have higher volumes than those having intermediate values.
Chapter 4
Conclusions, Contributions and Future works

This thesis investigated the effects of needle surface roughness and insertion speed on needle-tissue friction, developed an analytical model for describing the geometry of a three bevel hypodermic needle and applied the developed model for calculating needle tip volume at given insertion depths. The previous chapters discussed about research motivation, related literature, friction force modeling, results of the frictional investigation, modeling of the hypodermic needle and results of the needle tip volume calculation. This chapter will list the conclusions drawn from this thesis, major contributions and future works.

4.1 Conclusions

This section is divided into two parts. The first part discusses about the conclusions of the needle-tissue friction investigation and the second part reports about the findings of the hypodermic needle tip modeling work.

4.1.1 Needle-Tissue Friction

For this work, the problem of accurate needle positioning inside the human body and related literature on needle-tissue interaction forces, tissue deformation, needle deflection, robotic needle steering and surface texturing of different components were reported first. After that, a friction force model was created and the experimental procedures were discussed. Finally, results of the frictional forces and position accuracy were presented.

Surface texturing was shown to improve the frictional characteristics of the needles. All the needles experienced higher frictional forces at higher speeds. In general, the rougher 11 gauge needles experienced lower frictional forces than the smoother needles.
across all the speeds. Texturing also improved the positioning accuracy of the 18 gauge needles. Discrepancies between the frictional force results and the model can be attributed to the local variation of the tissue properties.

4.1.1 Hypodermic Needle Modeling

Importance of analytic modeling of hypodermic needles and literature review on needle geometry based modeling and needle design for minimal pain have been reported first. After that, a three bevel hypodermic needle was defined, analytic model defining needle geometry was developed and application of the developed model for calculating needle tip volume was discussed. Finally, results of the needle tip volume calculation were reported.

Results of the needle tip volume calculation showed dependence on the needle grinding parameters. The secondary bevel angle and the rotation angle affected the needle volume for the first two transitions, the primary bevel angle and the needle inner radius affected the volume for the last 2 transitions and increasing/decreasing the rotation angle too much resulted in needles with higher volumes.

4.2 Contributions

This section discusses the contributions made by this thesis in the studies of needle-tissue friction and hypodermic needle modeling.

Although needle-tissue interaction has been a topic of much research interest, very few studies concentrated on the friction at the needle-tissue interface. Application of surface texture and insertion speed with a view to reduce friction and improve needle positioning accuracy is a novel approach. The first part of the thesis developed a frictional force model considering all the components of the needle insertion force. Frictional force and positioning accuracy measurements were done up to 900mm/s compared to slow speed investigations reported in earlier literature. Frictional forces for textured 18 gauge needles were extracted from the total insertion force and compared with the frictional force model. Findings from this work can be applied to needle insertion
procedures for reducing insertion force and ensuring better needle placement inside the patients’ body.

Modeling of surgical needles based on needle geometry has been the topic of research for quite a bit of time. While most of the earlier works focused on cutting force modeling for different types of needles based on their geometry, analytical models of needles for calculating the needle tip volume at an insertion depth are currently lacking. The second part of this thesis developed an analytical model of a three bevel hypodermic needle and applied that model for calculating needle tip volume at various insertion depths. Needle volume dependence on various needle grinding parameters has also been reported. Findings from this work can be applied to determine critical needle designing parameters for minimal pain during needle insertion and characterize the needle tip area for blunt needle studies.

4.3 Future Works

This section has been divided into two parts. The first part discusses the future works for needle-tissue friction and the second part discusses that for the analytical modeling of hypodermic needles.

4.3.1 Needle-Tissue Friction

This work can be expanded in the following directions:

♦ For modeling the friction force at the needle-tissue interface, steady state frictional force was considered. Dynamic modeling of the needle insertion force could be done as during medical procedures needle is inserted only once into the tissue.

♦ Needles of varying surface texture were obtained by using abrasive cloths. Alternative surface texturing techniques, different coatings and an in-depth study of surface texture characteristics can be done to investigate their effects on needle friction.

♦ Of the five factors: needle surface roughness ($R_{\text{RMS}}$), tissue fluid viscosity ($\eta$), tissue mechanical stiffness ($K$), average pressure ($P_{\text{av}}$) and needle insertion
speed (S), only the effects of needle surface roughness ($R_{RMS}$) and insertion speed (S) were investigated in this thesis. However, tissue mechanical stiffness, tissue fluid viscosity and average pressure may also affect the frictional force and studies focusing to characterize their effects can be done.

- Clinicians often rotate the needles during insertion for medical procedures. For this thesis, needles were inserted without any rotation. Investigations can be done to see whether rotating the textured needles during insertion affects the frictional force.

4.3.2 Analytical Modeling of Hypodermic Needles

Possible research directions for this work would be:

- This thesis formed an analytical model for defining the geometry of three bevel hypodermic needles. Five bevel needles are preferred over their three bevel counterparts as they require lesser insertion force and cause reduced pain owing to their thinner profiles. Modeling of five bevel needles can be explored.

- The developed model was used to calculate the needle volume at given insertion depths and effects of various needle grinding parameters on needle volume were reported. Findings from this study are beneficial for reducing patients’ pain. Clinical studies with different needles can be performed to confirm the findings of this study.

- Apart from the calculation of needle volume at a given insertion depth, this model can also be applied to characterize tip area for blunt needles and provide insight about cutting performances of regular hypodermic needles. Those directions can be pursued in the future.

- This thesis reported about the needle volume dependence on different needle grinding parameters. It is possible to formulate an optimization problem for minimizing the needle volume at a given insertion depth/ tip area for blunt needle at a particular blunting length. This type of problem can be solved for optimizing needle performance.
Appendix: Volume Equations

Volume equations for different transitions are derived below using the needle geometry model.

Transition 1: This occurs when insertion depth, \( z_d \) is between 0 and \((r_o-r_i)\cot \delta\). The tip area at that instance is shown below:

![Diagram](image)

Figure 38. Tip area for transition 1.

Points \( P', T \) and \( T' \) are the needle tip points at an insertion depth of \( z_d \). \( CF \) is the perpendicular on the needle outer diameter from the center \( C \). \( \gamma_{ao} \) is the angle \( \angle SCX \).
This angle can be found by using Eq. 11 for $\gamma_e \leq \gamma \leq \pi$. Putting $\gamma = 180^\circ$ and $\gamma_{ao}$ into Eq. 11 and then subtracting the later from the earlier one gives the value of $\gamma_{ao}$:

$$
\gamma_{ao} = \arccos \left( \frac{z_d}{r_o \cot \phi} - \cos \beta \right) + \beta
$$

The area $\text{PTSP}'$ can be found by the following manner:

Area of $\text{PTSP}' = 2 \times (\text{area of } \text{P'FSP}')$

$$
= 2 \times (\text{area of CFSC} - \text{area of CP'SC})
= 2 \times \left\{ \left( \frac{\pi - \gamma_{ao}}{2\pi} \right) r_o^2 - \frac{r_o \sin(\gamma_{ao}) (r_o - z_d \tan \delta)}{2} \right\}
= (\pi - \gamma_{ao}) r_o^2 - r_o \sin \gamma_{ao} (r_o - z_d \tan \delta)
$$

The volume can be got by integrating the area from $0$ to $z_d$.

Volume from tip to $z_d = \int_0^{z_d} \text{Area of } \text{PTSP}'$

$$
= \int_0^{z_d} \left\{ (\pi - \gamma_{ao}) r_o^2 - r_o \sin \gamma_{ao} (r_o - z_d \tan \delta) \right\} dz_d
$$
Here,

\[ c_1 = r_o \cot \varphi \]
\[ c_2 = \cos \beta \]
\[ c_3 = \tan \delta \]

When \( z_d = (r_o - r_i) \cot \delta \), \( P' \) will coincide with \( P \) and \( \int_0^{z_d} \) Area of \( P'TSP' \) will give the volume from the tip point (Q) to point P, where case 2 begins. The volume from Q to P will be referred as case 1 volume for the rest of this thesis.

Transition 2: This occurs when \( z_d \) is between \( (r_o - r_i) \cot \delta \) and

\[ (r_i - r_o) \cot \varphi + (r_o - r_i) \cot \delta + (r_o + r_i \cos (\gamma_d - \beta)) \cot \phi - \frac{q}{\sin \phi} \]. The tip area at that instance will look like:
Figure 39. Tip area for transition 2.

Points T, T', S and S' are on the needle tip at an insertion depth of \( z_d \). CF'F is the perpendicular drawn from the Center C on the needle tip. G is a point on the X axis got by extending the line SS'. \( \gamma_{ao} \) is the angle \( \angle SCX \) and \( \gamma_{ai} \) is the angle \( \angle S'CX \). \( \gamma_{ao} \) can be found by using Eq. 11 for \( \gamma_d \leq \gamma \leq \pi \). Replacing \( z \) by \((r_i + r_o)\cot \xi + (r_o - r_i)\cot \delta - z_d\) and \( \gamma \) by \( \gamma_{ao} \):

\[
\gamma_{ao} = \arccos \left( \frac{(r_o - r_i)\cot \xi - (r_o - r_i)\cot \delta}{r_o \cot \varphi} + \frac{z_d}{r_o \cot \varphi} - 1 + \frac{q}{r_o \sin \varphi \cot \varphi} \right) + \beta
\]

\( \gamma_{ai} \) can be found by using Eq. 10 for \( \gamma_d \leq \gamma \leq \pi \). Replacing \( z \) by \((r_i + r_o)\cot \xi + (r_o - r_i)\cot \delta - z_d\) and \( \gamma \) by \( \gamma_{ai} \):
\[ \gamma_{ai} = \arccos \left( \frac{(r_o - r_i) \cot \xi}{r_i \cot \varphi} - \frac{(r_o - r_i) \cot \delta}{r_j \cot \varphi} + \frac{z_d}{r_i} - \frac{r_o}{r_j} + \frac{q}{r_j \sin \varphi \cot \varphi} \right) + \beta \]

The area T'TSS'T' can be found by the following manner:

Area of T'TSS'T' = 2*( area of F'FSS'F')

= 2*( area of CFSC + area of CSGC – area of CS'F'C – area of CS'GC)

= \[2 \left( \frac{(\pi - \gamma_{ao}) r_o^2}{2\pi} + r_o \sin \gamma_{ao} \left( z_d \tan \delta - r_o \right) \right) - \frac{(\pi - \gamma_{ai}) r_i^2}{2\pi} - r_i \sin \gamma_{ai} \left( z_d \tan \delta - r_o \right) \]

Volume from tip to \(z_d\) = case 1 volume + \[ \int_{(r_i - r_j)}^{z_d} \text{Area of T'TSS'T'} d\zeta \]

= case 1 volume + \[ \int_{(r_i - r_j)}^{z_d} \left( \frac{(\pi - \gamma_{ao}) r_o^2}{2\pi} + r_o \sin \gamma_{ao} \left( z_d \tan \delta - r_o \right) \right) d\zeta \]}
Here,

\[ d_1 = \frac{1}{r_o \cot \phi} \]
\[ d_2 = \frac{(r_o - r_i) \cot \xi - (r_o - r_i) \cot \delta}{r_o \cot \phi} + \frac{q}{r_o \sin \phi \cot \phi} - 1 \]
\[ d_3 = \frac{1}{r_i \cot \phi} \]
\[ d_4 = \frac{(r_o - r_i) \cot \xi - (r_o - r_i) \cot \delta}{r_i \cot \phi} + \frac{q}{r_i \sin \phi \cot \phi} - \frac{r_o}{r_i} \]
\[ d_5 = \tan \delta \]
When \( z_d = (r_i - r_o) \cot \xi + (r_o - r_i) \cot \delta + \left( r_o + r_i \cos(\gamma_d - \beta) \right) \cot \phi - \frac{q}{\sin \phi} \), \( S' \) will coincide with \( D \) and \( \int_{(r_o - r_i) \cot \delta}^{z_d} \) Area of \( T'TSS'T' \) will give the volume from the point \( P \) to point \( D \), where transition 3 begins. The volume from \( P \) to \( D \) will be referred as case 2 volume for the rest of this thesis.

Transition 3: This occurs when \( z_d \) is between

\[
(r_i - r_o) \cot \xi + (r_o - r_i) \cot \delta + \left( r_o + r_i \cos(\gamma_d - \beta) \right) \cot \phi - \frac{q}{\sin \phi} \quad \text{and} \quad (r_i - r_o) \cot \xi + (r_o - r_i) \cot \delta + \left( r_o + r_i \cos(\gamma_e - \beta) \right) \cot \phi - \frac{q}{\sin \phi}
\]

The tip area at that instance will look like:
Figure 40. Tip area for transition 3.

Here, point T', M, T, S, N and S' are on the needle tip at an insertion depth of $z_d$. CF'F is a perpendicular drawn from the centre on the needle tip. G is a point on the X axis got by extending the line SN. $\gamma_{am}$ is the angle $\angle NCX$, $\gamma_{ai}$ is the angle $\angle S'CX$ and $\gamma_{ao}$ is the angle $\angle SCX$. $\gamma_{ai}$ is found by using Eq. 10 for $0 \leq \gamma \leq \gamma_d$. Replacing $z$ by $(r_i + r_o)\cot \xi + (r_o - r_i)\cot \delta - z_d$ and $\gamma$ by $\gamma_{ai}$:

$$
\gamma_{ai} = \arccos \left\{ \frac{z_d}{r_i \cot \xi} - \frac{(r_o - r_i)\cot \delta}{r_i \cot \xi} - 1 \right\}
$$
\( \gamma_{ao} \) is found by using Eq. 11 for \( \gamma_c \leq \gamma \leq \pi \). Replacing \( z \) by 
\[(r_i + r_o) \cot \xi + (r_o - r_i) \cot \delta - z_d \] and \( \gamma \) by \( \gamma_{ao} \):

\[
\gamma_{ao} = \arccos \left\{ \frac{z_d}{r_o \cot \varphi} + \frac{q}{r_o \sin \varphi \cot \varphi} - 1 - \frac{r_i \cot \xi}{r_o \cot \varphi} + \frac{\cot \xi}{r_o \cot \varphi} - \frac{(r_o - r_i) \cot \delta}{r_o \cot \varphi} - 1 \right\}
\]

Here, \( r_m \) and \( \gamma_{am} \) can be written as:

\[
r_m = \frac{r_i \cos \gamma_{ai}}{\cos \gamma_{am}}
\]

\[
\gamma_{am} = \arctan \left( \frac{r_o \cos (\gamma_{ao} - \beta)}{r_i \sin \beta \cos \gamma_{ai} - \cot \beta} \right)
\]

The area \( T'MTFSNS'F'T' \) can be found in the following manner:

Area of \( T'MTFSNS'F'T' = 2 \times (\text{area of } F'FSNS'F') \)

\[
(\pi - \gamma_{ao}) r_i^2 + r_o \sin \gamma_{ao} (z_d \tan \delta - r_o) = - (\pi - \gamma_{ai}) r_i^2 - r_i \cos \gamma_{ai} \tan \gamma_{am} (z_d \tan \delta - r_o) + r_i^2 \cos^2 \gamma_{ai} \tan \gamma_{am} - r_i^2 \cos \gamma_{ai} \sin \gamma_{ai}
\]

Volume from tip to \( z_d = \text{case 1 volume} + \text{case 2 volume} + \)

\[
\int_{(r_i - r_o) \cot \xi + (r_o - r_i) \cot \delta + [r_o \cos (\gamma_{ao} - \beta)] \cot \varphi - \frac{q}{\sin \varphi}}^{z_d} \text{Area of } T'MTFSNS'F'T
\]

\[
= \text{case 1 volume} + \text{case 2 volume} +
\]
\[
\begin{align*}
\mathbf{r}_o^2 & = \left( -\beta z_d + \pi z_d - z_d \arccos(e_2 + e_1 z_d) \right) \\
& + \sqrt{1 - e_2^2 - 2e_2z_d - e_1^2 z_d^2 + e_2 \arcsin(e_2 + e_1 z_d)} \\
& + \frac{3e_1 r_o (e_2 + e_1 z_d)}{e_1} \left( 2 + e_2^2 - e_1 e_2 z_d - 2e_1^2 z_d^2 \right) \\
& + 3 \cos \beta (e_2 e_4 + e_4 r_o) \arcsin(e_2 + e_1 z_d) \\
& + e_1 \sin \beta \left( 3e_2 r_o + e_1^2 z_d^2 \left( 3r_o - 2e_2 z_d \right) - 3e_1 e_2 z_d \left( -2r_o + e_2 z_d \right) \right) \\
& \left( (r_o - r_c) \cot \xi + (r_o - r_c) \cot \delta \right) \\
& \left( r_o + r_c \cos(\gamma - \beta) \right) \cot \phi \cdot \frac{q}{\sin \phi} \\
\mathbf{r}_o^2 & = \left( -\pi r_i^2 z_d - r_i^2 z_d \arccos(e_4 + e_3 z_d) \right) \\
& + \frac{r_i^2 \left( 1 - e_4^2 - 2e_4 z_d - e_3^2 z_d^2 + e_4 \arcsin(e_4 + e_3 z_d) \right)}{e_3} \\
& \left( (r_o - r_c) \cot \xi + (r_o - r_c) \cot \delta \right) \\
& \left( r_o + r_c \cos(\gamma - \beta) \right) \cot \phi \cdot \frac{q}{\sin \phi} \\
\mathbf{r}_o^2 & = \frac{e_4 e_7 (6r_o - 3e_5 z_d) + z_d^2 (e_4 e_6 - e_4 e_7) (-3r_o + 2e_2 z_d)}{6} \\
& + e_2 e_6 \left( - 6r_o + 3e_5 z_d \right) \\
& \left( (r_o - r_c) \cot \xi + (r_o - r_c) \cot \delta \right) \\
& \left( r_o + r_c \cos(\gamma - \beta) \right) \cot \phi \cdot \frac{q}{\sin \phi} \\
\mathbf{r}_o^2 & = \frac{- e_4 z_d (- e_2 e_6 + e_4 e_7) + z_d^2 (e_2 e_5 e_6 + e_1 e_4 e_6 - 2e_1 e_4 e_7)}{2} \\
& - e_3 z_d^3 (- e_2 e_6 + e_4 e_7) \\
& \frac{3}{3} \\
& \left( (r_o - r_c) \cot \xi + (r_o - r_c) \cot \delta \right) \\
& \left( r_o + r_c \cos(\gamma - \beta) \right) \cot \phi \cdot \frac{q}{\sin \phi} \\
\mathbf{r}_o^2 & = \frac{1 - (e_4 + e_5 z_d)^2}{3} \left( (r_o - r_c) \cot \xi + (r_o - r_c) \cot \delta \right) \\
& \left( r_o + r_c \cos(\gamma - \beta) \right) \cot \phi \cdot \frac{q}{\sin \phi}
\end{align*}
\]
Here,

\[ e_1 = \frac{1}{r_o \cot \varphi} \]
\[ e_2 = \frac{(r_o - r_i) \cot \xi - (r_o - r_i) \cot \delta}{r_o \cot \varphi} + \frac{q}{r_o \sin \varphi \cot \varphi} - 1 \]
\[ e_3 = \frac{1}{r_i \cot \xi} \]
\[ e_4 = \frac{(r_o - r_i) \cot \xi}{r_i \cot \xi} - \frac{(r_o - r_i) \cot \delta - r_o}{r_i} \]
\[ e_5 = \tan \delta \]
\[ e_6 = \frac{r_o}{r_i \sin \beta} \]
\[ e_7 = \cot \beta \]

When \( z_d = (r_i - r_o) \cot \xi + (r_o - r_i) \cot \delta + \{r_o + r_o \cos(\gamma_c - \beta)\} \cot \phi - \frac{q}{\sin \phi} \), S and N will coincide with E and

\[ \int_{(r_i - r_o) \cot \xi + (r_o - r_i) \cot \delta + \{r_o + r_o \cos(\gamma_c - \beta)\} \cot \phi - \frac{q}{\sin \phi}} \]

will give the volume from the point D to point E, where transition 4 begins. The volume from D to E will be referred as case 3 volume for the rest of this thesis.

Transition 4: This occurs when \( z_d \) is between

\( (r_i - r_o) \cot \xi + (r_o - r_i) \cot \delta + \{r_o + r_o \cos(\gamma_c - \beta)\} \cot \phi - \frac{q}{\sin \phi} \) and

\( 2r_i \cot \xi + (r_o - r_i) \cot \delta \). The tip area at that instance will look like:
Here, point T', T, S and S' are on the needle tip at an insertion depth of \( z_d \). CF'F is a perpendicular drawn from the centre on the needle tip. G is a point on the X axis got by extending the line SS'. \( \gamma_{ai} \) is the angle \( \angle S'CX \) and \( \gamma_{ao} \) is the angle \( \angle SCX \). \( \gamma_{ai} \) and \( \gamma_{ao} \) can be written as:

\[
\gamma_{ai} = \frac{(r_i - r_o) \cot \delta}{r_i \cot \xi} - 1 + \frac{z_d}{r_i \cot \xi}
\]

\[
\gamma_{ao} = \frac{(r_i - r_o) \cot \delta - r_i \cot \xi}{r_o \cot \xi} + \frac{z_d}{r_o \cot \xi}
\]

The area T'TFSS'F'T' can be found in the following manner:
Area of T'TFSS'F'T' = \( 2 \times (\text{area of } F'FSS'F') \)
Volume from tip to \( z_d \) = case 1 volume + case 2 volume + case 3 volume +

\[
\int_{z_d}^{z_f} \left[ \frac{\pi r_o^2 z_d - r_o^2 z_d}{3 f_1} - \frac{\pi r_i^2 z_d}{3 f_3} \right] \ \text{Area of T'TFSS'F'T'}
\]

= case 1 volume + case 2 volume + case 3 volume +

\[
\left[ \frac{\pi r_o^2 z_d - r_o^2 z_d \arccos \left( f_2 + f_1 z_d \right) - r_o^2 \sqrt{1 - f_2^2 - 2 f_1 f_2 z_d - f_1^2 z_d^2} + f_2 \arcsin \left( f_2 + f_1 z_d \right)}{f_1} \right]_{z_d}^{z_f}
\]

\[
\left[ \frac{\pi r_i^2 z_d - r_i^2 z_d \arccos \left( f_4 + f_3 z_d \right) - r_i^2 \sqrt{1 - f_4^2 - 2 f_3 f_4 z_d - f_3^2 z_d^2} + f_4 \arcsin \left( f_4 + f_3 z_d \right)}{f_3} \right]_{z_d}^{z_f}
\]

\[
+ \left[ \frac{-r_o^2 \left[ 1 - (f_2 + f_1 z_d)^2 \right]^{3/2}}{3 f_1} \right]_{z_d}^{z_f}
\]

\[
- \left[ \frac{-r_i^2 \left[ 1 - (f_4 + f_3 z_d)^2 \right]^{3/2}}{3 f_3} \right]_{z_d}^{z_f}
\]

Here,

\[
f_1 = \frac{1}{r_o \cot \xi}
\]

\[
f_2 = \frac{r_i}{r_o} \left( r_o - r_i \right) \cot \delta
\]

\[
f_3 = \frac{1}{r_i \cot \xi}
\]

\[
f_4 = \frac{r_o}{r_i} \left( \cot \delta - \cot \xi \right)
\]

\[
\frac{\pi}{2} \left( r_o - r_i \right) \cot \delta \]
When \( z_d = 2r_i \cot \xi + (r_o - r_i) \cot \delta \), \( S' \) and \( T' \) will coincide with \( B_i \) and

\[
\int_{(r_i-r_o)\cot \xi + (r_o-r_i)\cot \delta}^{(r_i+r_o)\cot \xi + (r_o-r_i)\cot \delta} \frac{q}{\sin \phi} \, d\phi
\]

Area of \( T'FSS'FT' \) will give the volume from the point \( E \) to point \( B_i \), where transition 5 begins. The volume from \( E \) to \( B_i \) will be referred as case 4 volume for the rest of this thesis.

Transition 5: This occurs when \( z_d \) is between \( 2r_i \cot \xi + (r_o - r_i) \cot \delta \) and \( (r_i + r_o) \cot \xi + (r_o - r_i) \cot \delta \). The tip area at that instance will look like:

![Figure 42. Tip area for transition 5.](image-url)
Here, point T, G and S are on the needle tip at an insertion depth of $z_d$. CF'F and CG'G are perpendiculars drawn from the centre on the needle tip. $\gamma_{ao}$ is the angle $\angle SCX$. $\gamma_{ao}$ can be written as:

$$\gamma_{ao} = \frac{z_d}{r_o \cot \xi} - \frac{r_i}{r_o} \frac{(r_o - r_i) \cot \delta}{r_o \cot \xi}$$

The area $F'G'GTFSGG'F'$ can be found in the following manner:

Area of $F'G'GTFSGG'F' = 2^* \text{(area of F'FSGG'F')}$

$$= \left( \pi - \gamma_{ao} \right) r_o^2 + r_i^2 \cos \gamma_{ao} \sin \gamma_{ao} - \pi r_i^2$$

Volume from tip to $z_d = \text{case 1 volume} + \text{case 2 volume} + \text{case 3 volume} + \text{case 4 volume} + \int_{2r_i \cot \xi + (r_i - r_f) \cot \delta}^{z_d} \text{Area of F'G'GTFSGG'F'} \, dz$$

$$= \text{case 1 volume} + \text{case 2 volume} + \text{case 3 volume} + \text{case 4 volume} +$$

$$\left[ \frac{\pi r_o^2 z_d - r_i^2 z_d \arccos \left( g_2 + g_1 z_d \right)}{g_1} \right]_{2r_i \cot \xi + (r_i - r_f) \cot \delta}^{z_d}$$

$$+ \left[ -r_i^2 \left( g_2 + g_1 z_d \right)^2 \right]_{3g_1}^{z_d} - \left( g_3 z_d \right)_{2r_i \cot \xi + (r_i - r_f) \cot \delta}^{z_d}$$

Here,

$$g_1 = \frac{1}{r_o \cot \xi}$$

$$g_2 = \frac{r_i}{r_o} \frac{(r_o - r_i) \cot \delta}{r_o \cot \xi}$$

$$g_3 = r_i^2$$
When \( z_d = (r_i + r_o) \cot \xi + (r_o - r_i) \cot \delta \), \( S \) and \( T \) will coincide with \( B_o \) and

\[
\int_{2r_i \cot \xi + (r_o - r_i) \cot \delta}^{z_i} \text{Area of } F'G'GTSGG'F' \text{ will give the volume from the point } B_i \text{ to point } B_o,
\]

where transition 5 begins. The volume from \( B_i \) to \( B_o \) will be referred as case 5 volume for the rest of this thesis.

**Beyond transition 5:** This occurs when \( z_d \) is greater than \((r_i + r_o) \cot \xi + (r_o - r_i) \cot \delta\).

The tip area at that instance will look like:

As shown in Fig. 43, the tip assumes a ring shape beyond transition 5. The area is simple to calculate and can be written as:

\[
\text{Tip area beyond transition 5} = \pi \left( r_o^2 - r_i^2 \right)
\]
Volume from tip to \( z_d \) = case 1 volume + case 2 volume + case 3 volume

\[ + \text{ case 4 volume} + \int_{(r_e + r_i)\cot \xi + (r_e - r_i)\cot \delta}^{z_d} \text{Area of F'G'GTFSGG'F'} \]

= case 1 volume + case 2 volume + case 3 volume

\[ + \text{ case 4 volume} + \left[ \pi \left( r_o^2 - r_i^2 \right) \right]_{(r_e + r_i)\cot \xi + (r_e - r_i)\cot \delta}^{z_d} \]

= case 1 volume + case 2 volume + case 3 volume

\[ + \text{ case 4 volume} + \left[ \pi \left( r_o^2 - r_i^2 \right) \right] \left( z_d - (r_e + r_o)\cot \xi \right) \left( - (r_o - r_i)\cot \delta \right) \]
Bibliography


