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STOCHASTIC FINANCIAL ANALYTICS FOR CASH-FLOW BULLWHIP, CASH-FLOW FORECAST, AND WORKING CAPITAL OPTIMIZATION

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Rattachut Tangsucheeva

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The dissertation of Rattachut Tangsucheeva was reviewed and approved* by the following:

Vittaldas V. Prabhu
Professor of Industrial and Manufacturing Engineering
Dissertation Advisor
Chair of Committee

A. Ravi Ravindran
Professor of Industrial Engineering, Chair-Enterprise Integration Consortium

Tao Yao
Associate Professor of Industrial and Manufacturing Engineering

Douglas Thomas
Professor of Supply Chain and Information Systems, MBA Faulty Director

Paul Griffin
Peter and Angela Dal Pezzo Department Head Chair
Head of the Department of Industrial and Manufacturing Engineering

*Signatures are on file in the Graduate School

ABSTRACT

Managing modern supply chains involves dealing with complex dynamics of materials, information, and cash flows around the globe, and is a key determinant of business success today. One of the well-recognized challenges in supply chain management is the inventory bullwhip effect in which demand forecast errors are amplified as it propagates upstream from a retailer, in part because of lags and errors in information flows. Adverse effects of such bullwhip effect include excessive inventory, stock-outs, backorders, and wasteful swings in manufacturing production. In this dissertation we theorize that inventory bullwhip also leads to cash-flow bullwhip (*CFB*). Specifically, this research focuses on studying *CFB* by developing mathematical and simulation models to analyze the relationship between inventory and cash-flow bullwhip by using Cash Conversion Cycle (*CCC*) as a metric. *CFB* predicted by the proposed mathematical models approximately differ 14% from detailed simulation models. We find that increasing variability increases inventory and cash-flow bullwhip along with lead time, whereas increasing the demand observation period has the opposite effect. The average marginal impact of the bullwhip effect on the *CFB* is approximately 20%. Additionally, the *CFB* is also an increasing function of an expected value of inventory and a decreasing function of an expected value of demand.

Next, we develop stochastic financial analytics for cash flow forecasting for firms by integrating two models: (1) Markov chain model of the aggregate payment behavior across all customers of the firm using accounts receivable aging and; (2) Bayesian model of individual customer payment behavior at the individual invoice level. As the stochastic dynamics of cash flow evolves every day, the forecast can be updated every time an invoice is paid. The proposed model is back-tested using empirical data from a small manufacturing firm and found to differ 3%-6% from actual monthly cash flow, and differs approximately 2%-4% compared to actual

annual cash flow. The forecast accuracy of the proposed stochastic financial analytics model is found to be considerably superior to other techniques commonly used. Furthermore, in computer simulation experiments, the proposed model is found to be largely robust to supply chain dynamics, including when subjected to severe bullwhip effect. The proposed model has been implemented in Excel, which allows it to be easily integrated with the accounts receivable aging data, making it practicable for small and large firms.

Lastly, we identify a potential strategy to engineer a solution for dynamic financial decisions. This part focuses on maximizing profit of two types of manufacturing firms: firms with non-recurring customers and firms with recurring customers. The proposed model determines the optimal pricing of products sold to different customers using an integer programming model in which Friedman's model is used to estimate bid winning probability and the model is constrained by several operational factors including working capital and customer credit risk. Customer credit risk is modeled as the probability of payment delay or default, which is used to add a risk premium into the bid price. The model can be used for decision-support in business development to select an optimal portfolio of customer projects or bids to pursue. Detailed industrial case studies used to test the efficacy of the proposed model show that the Price/Cost ratio has an inversely proportional relationship to the winning probability, and the winning probability has the inversely proportional relationship to the risk premium. However, at a high winning probability, the firm may not be able to make a profit due to the unrealistically low bidding price. The results also show the bidding price at which the firm is expected to maximize its profit.

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Chapter 1

INTRODUCTION

1.1 Background

Supply chain management is currently a fast growing area in most businesses and is considered as key to the success of most leading companies. Supply chain is a dynamic system involving coordination of three major flows, which are product flow, financial flow, and information flow, between different supply chain stages such as supplier, manufacturer, wholesaler, distributor, and retailer (Chopra and Meindl, 2007). Typically, a stage of a supply chain can be one of these following processes: sourcing, manufacturing, distributing, transporting, and retailing. However, it is not necessary for a supply chain to contain all the processes above. For example, the manufacturer may produce and ship products directly to retailers. Therefore, in this case, the supply chain does not contain a distributor.

Figure 1-1 shows product flow, financial flow, and information flow, between a supplier and a manufacturer at one stage of a multi-stage supply chain. These flows continue in all stages across the supply chain.

Product flow starts from a supplier providing raw materials or parts to a manufacturer. Then a manufacturer begins manufacturing processes and then ships finished products to a distributor who transfers the replenishment orders to retailers. Eventually, retailers fill orders to its customers. Understanding these fundamental product flows provide us a big picture of how products move from place to place and real understanding in order to improve the cash flow of a supply chain. Financial flow usually transfers backward which is different from that of the product flow.

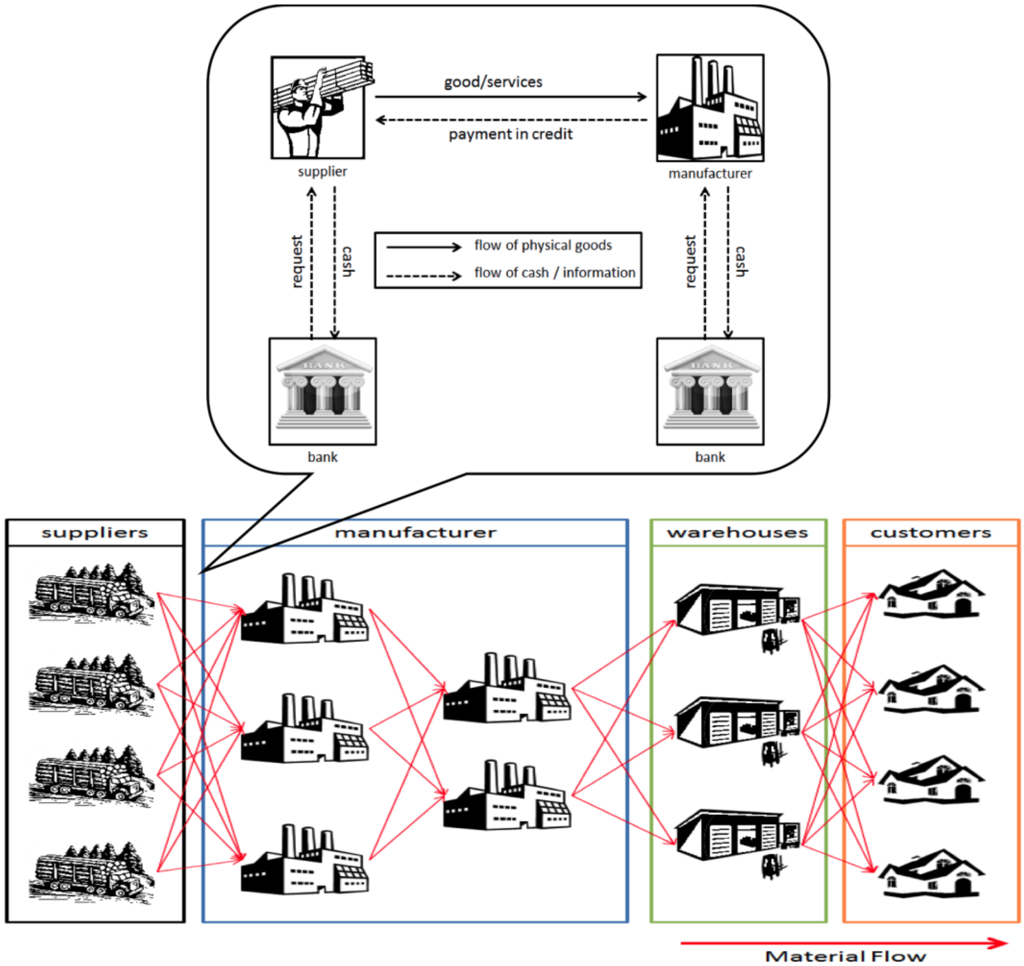


Figure 1-1. Flows in supply chain.

Financial flow starts from customers making payments to retailers. Then, the retailers pay the bills to their distributors and the distributors pay to their manufacturers, and manufacturers pay to their suppliers.

Information flow plays an important role to a supply chain management because it analogously functions as a bridge to link among other supply chain drivers which are facilities, inventory, transportation, information, sourcing, and pricing. A goal of good information flow is to integrate and coordinate factors in a supply chain for a decision maker to execute transactions.

Information makes a supply chain visibility. Without it, a manager is blind and unable to know what the demand of products is, how much products left in stock, how much more to produce and ship these products (Chopra and Meindl, 2007). A lack of supply chain coordination may cause negative effects to the performance of the supply chain such as the Bullwhip effect. Therefore, accurate information is crucial to the decision making and the performance of a supply chain.

In a real business world, a manufacturer may have several sources of raw material from many suppliers from many locations shipped to assemble and then distribute the finished products to many distributors and retailers. For a larger and more complex supply chain containing more than one suppliers, manufacturers, distributors, and retailers, it is probably more accurate to call a *supply network* rather than a supply chain (Chopra and Meindl, 2007). Moreover, in this globalization era in which modern communications and transportation technologies have been developed tremendously, a supply network may have suppliers and retailers located in different continent where it can have competitive advantages. A supply network, which has sources of raw material, manufactures products, and sells to its customers in more than one country is considered be a *global supply network*. The growing internationalization of American business increased sharply in year 1998 to 2000 and the cumulative foreign direct investment in the U.S. from 2008 to 2012 reached $2.65 trillion (Shapiro, 2006; investment, 2013). The internationalization of commerce provides firms to seek for raw materials, markets, and cost minimization. For example, an mp3 player from Apple Inc. called iPod, it was engineered in India, designed in America, manufactured in China and sold around the world (Ravindran, 2008).

Even though the structure and size of a supply chain is varied, it focuses on the same direction where its objective is the same as that of commercial firms, maximize overall profit. The higher the profit, the more successful the supply chain is. The source of revenues comes from customers who pay positive cash flow; whereas, other activities in product, information and financial flows incur cost into the supply chain (Chopra and Meindl, 2007). Therefore, a good supply chain management plays an important role to a successive firm in this competitive business world.

Generally, financial management is categorized into two basic functions: (1) the acquisition of funds and (2) the investment of those funds. The first function, also known as the financing decision, involves the processes to acquire funds from both internal and external sources to the firm at the lowest cost possible. This also includes how to transfer funds from place to place when buying raw materials and selling products. The second function, the investment decision, involves allocation of funds to earn maximum profit.

Other financial management, for example, internal financial flow of a corporate such as loan repayments usually is achieved by accessing to sources of funds that are already exist. Other financial flows, such as dividend payments, may be managed to provide to shareholders in order to reduce tax burdens or currency risk. Capital structure and other financing decisions are usually undertaken to reduce investment risks and financing costs. These actions frequently are motivated by firm's financial situation and world's economic status at that time.

However, in the current economic downturn and credit squeeze, sales are dropping, cash reserves are decreasing, and banks are very careful to lend money. Only a good supply chain management may not adequate for a firm to survive in this critical situation. A firm has to pay more attention to the importance of cash flow and working capital management. Particularly, a multinational corporate should focus on real-time visibility of cash balance among corporations worldwide. Cash flow is crucial of survivability of a firm. Without cash a firm fails to meet basic financial obligations such as payroll, taxes, and payment to suppliers (Pate-Cornell et al., 1990). All these factors eventually may risk a firm with a poor cash flow management bankruptcy.

Cash flow forecasting is one of the keys for efficient cash flow management and business operation. Nonetheless, each company may have different purposes for its cash flow forecast. A company, such as a start-up company, may just want to forecast its cash flow in order to know how fast it uses up cash and whether it needs to prepare for additional funds whereas a big company may want to perform the forecast to maximize its return on investment. More generally, most companies may simply want to manage their obligations such as debt repayment, wages, and so forth. Therefore, the use of the cash position forecast for treasury roles can be summarized as follows (WWCP, 2012):

1. To meet external obligations
2. To minimize external borrowing costs
3. To maximize investment outcomes
4. To manage currency exposure

In addition to the usage of the core treasury roles, the company may also use the cash position forecast in order to exercise control over group companies, to perform a general management role, and to perform a strategic role. Despite great benefits of the forecast, most SMEs fail to do it because they may not have sufficient and effective resources to do so. Some of the difficulties in cash position forecasting are as follows (WWCP, 2012).

1. Different countries and time zones
2. Complexity of data integration from other external sources
3. Multicurrency forecasts
4. Status and quality of underlying data

Therefore, effective cash and liquidity management are ultimately important at all times, especially during this economic downturn time. A firm must enhance its treasury functions throughout its branches and subsidiaries to achieve accurate and real-time visibility of its cash balances. Effective financial manager should be able to acquire cash whenever it is required and minimize the need for unnecessary borrowing. The challenging problem is to formulate relationships between supply chain management and cash flow management to improve cash flow in a supply chain by integrating these two concepts together.

1.2 Motivation

Among the product flow, information flow, and financial flow of a supply chain, previous supply chain studies focused more on the first two flows. However, the relationship between the product flow and financial flow, in particular, cash flow, is less explored (Comelli et al., 2008; Tsai, 2008). Cash is not only an essential resource needed to support almost all activities in a firm; it also provides liquidity for companies during financial difficult time and allows them to take advantage of expansion during the good time. Particularly, in a growth firm with few customers or a firm paying high interest rate, good cash flow management is very essential (Milling, 1983). A firm with high growth rate requires a large amount of capital to expand its factory, hire more personnel, and increase inventory level in anticipation of high demand. Thus, a high growth firm generally has large financial burden to pay in a near term whereas the source of revenue comes from a small group of customers, which leads to increasing in uncertainty of revenue. Finally, a high interest rate can incur a high cost of borrowing and a high opportunity cost to maintain high cash balances.

For any supply chain, since revenues only come from customers who pay for products or services, other activities in supply chain flows (product, information, and financial flows) generate cost to the supply chain (Chopra and Meindl, 2007). Thus, most previous researches focused on improving product flow, which is one way to reduce the cost of supply chain flows. Many existing studies have done reducing the cost of product flow such as the just-in-time system (JIT) and economic order quantity (EOQ) model. The goal of JIT is simply that the required parts from upstream workstations are received precisely as needed (just in time) which could be done by eliminating waste or having zero inventories in order to reduce cost (Hopp and Spearman, 2001). The EOQ model aims to optimize the order quantity to minimize the total cost, which contains fixed ordering cost, holding cost, and unit cost.

The problem of cash flow management, especially during the economic downturn, and the lack of researches that integrate between product flow and financial flow of a supply chain, therefore, inspires this research to focus on the relationship between these two flows of a firm in supply chain in order to optimize the maximize profit of a firm within a limited cash flow. So, a financial manager can maintain minimum cash balance to sufficiently run a business (because cash sitting in the account does not generate income) and prepare extra cash when it is needed to prevent from cash shortage.

1.3 Research and objectives

Managing modern supply chains involves dealing with complex dynamics of materials, information, and cash flows around the globe, and is a key determinant of business success today. One of the well-recognized challenges in supply chain management is the inventory bullwhip effect in which demand forecast errors are amplified as it propagates upstream from a retailer, in part because of lags and errors in information flows. Adverse effects of such bullwhip effect include excessive inventory and wasteful swings in manufacturing production. In this dissertation we postulate that there can be a corresponding bullwhip in the cash flow across the supply chain and it can be one of many reasons why firms run out of cash. Furthermore, we explore ways to predict cash flow using stochastic financial analytics model to develop the forecast model and finally identify a potential strategy to engineer a solution for maximizing profit within a limited working capital by using mixed integer linear programming.

The ultimate goal of this research is to develop an integrated model that combines the supply chain management concept and financial management concept together to maximize profit of a firm. The integrated model is the dynamic financial decision model in order to assist a manager to optimize the decision on cash flow in supply chain. The cost of shortage of cash flow and the cost of credit risk are involved while maintaining adequate cash balance to run a business. In developing an analytical model to express relationships between the product flow and cash flow, there are some involvements of uncertainties from demand forecast and payment delay, which need stochastic process to assist in formulating this model. While a rich historical research and body of literature on supply chain and cash management exists separately, there is a gap between the literature and this research to combine the concept of supply chain management and cash flow management together.

1.4 Research Contributions

Some of the key concepts and the potential research contributions of this research are summarized below:

1. The analytical models of the inventory bullwhip and the cash flow bullwhip in a supply chain enable firms to better understand the phenomenon and the relationship among inventory, bullwhip effect, and the cash flow bullwhip. These models lead to the estimation of the cash flow bullwhip in a supply chain and how to reduce its adverse impacts. The cash flow bullwhip can also be potentially set as a benchmark to measure performance among supply chains.
2. The stochastic financial analytics model for cash flow forecasting is back-tested using empirical data from a small manufacturing firm and found to be considerably superior to other techniques commonly used. Furthermore, in computer simulation experiments, the

proposed model is found to be largely robust to supply chain dynamics, including when subjected to severe bullwhip effect. The proposed model has been implemented in Excel, which allows it to be easily integrated with the accounts receivable aging data, making it practicable for small and large firms.

3. The optimal pricing model is able to provide a decision support to allocate proper amount of investment in projects within limited working capital and selecting the projects that maximize profit and minimize credit risk of a firm. The analysis also provides a better understanding among price, cost, winning probability, profit, and credit risk. This leads to some managerial insights and strategic planning to maximize profit.

1.5 Outline of the Dissertation

The remainder of this dissertation is organized into five chapters. First, Chapter 2 provides the comprehensive literature review related to cash flow. The review also includes cash flow management in various situations and different objectives, cash flow forecast, cash flow risk, credit risk, and cash flow decision. Lastly, this chapter provides the review of the applications and integrations of cash flow management concept and supply chain management concept that improve supply chain. Chapter 3 presents how to develop the analytical model of the inventory bullwhip and the cash flow bullwhip. This model is extended from the quantitative model of the bullwhip effect. Then the models are compared with the simulation model. In Chapter 4, the stochastic financial analytics model for cash flow forecast is developed. The model integrates the aggregate payment behavior across all customers of the firm using accounts receivable aging and individual customer payment behavior at the individual invoice level to perform the cash flow forecast. The proposed model is back-tested using empirical data from a small manufacturing firm and found to be significantly improved. Chapter 5 presents the optimal pricing with constraints on working capital and payment delay risk. The model is developed to assist management to allocate capital investment in projects within limited working capital and selecting the projects that maximize profit and minimize credit risk of a firm. Finally, conclusions of this dissertation and the direction for the future research are addressed in Chapter 6.

Chapter 2

LITERATURE REVIEW

This chapter provides a review of existing literature related to cash flow management in general and cash flow in supply chain. The review also discusses several techniques to manage cash flow in various dimensions such as cash balance optimization and financial planning. Then, the importance of cash flow forecast and several approaches to predict the cash flow are addressed. Next we review how the cash flow risk and credit risk are measured, how cash flow is impacted by these risks, and how these risks are mitigated. Then, the dynamic financial decisions and project selection are reviewed. This review includes allocate funds to projects and how to select projects to achieve firm's objective such as maximizing profit or minimizing risk. Finally, previous works relating to an integration of cash flow and supply chain are summarized.

2.1 Cash Flow Management

Cash flow management is one of the top concerns of many small and medium enterprises (SMEs). Especially, when the enterprise progresses through various life-cycle stages, they face more problematic situations in managing their financial assets (Mcmahon, 2001). Furthermore, SMEs usually incur high interest rate for any types of financial services due to their higher credit risk, and resource constraints they face when they use financial services (Baas and Schrooten, 2006). The need for more careful and effective cash flow management is of critical importance to researchers and managers of SME's.

Cash flow of a supply chain refers to the movement of revenue (cash inflows) or expense (cash outflows) stream through its business during a specific period. Financial flow can be assessed from the financial statements, which are balance sheet, income statement, and cash flow statement. In this research, we focus on the relationship between product flow and financial flow, especially cash flow.

A substantial body of research has focused on improving supply chain product flows such as work in process and inventory. For example, the work of Hopp and Spearman, 2001 focuses on determining the relationship among work in process, cycle time, and throughput and Chauhan et al., 2007proposed the scheduling technique to minimize work in process. Other works are focused on optimizing inventory level (Tempelmeier, 2006; Wu and Hwang, 2011) while only limited research focuses on improving cash flow.

Cash flow is one of the most important financial statistics. It can be used to measure rate of return and liquidity of a firm. Cash flow determines business's solvency. It is crucial for business to survive. Without cash, a firm fails to meet basic financial obligation to its creditors, employees, and the others. A firm with insufficient cash is likely to bankrupt if the insolvency continue. Therefore, economists try to forecast and improve cash flow. In this literature reviews show many researches attempted to forecast and improve cash flow.

The literature on cash flow management applying the inventory management concept has developed more than a century. An early attempt of some economists and mathematicians to manage a cash balance using the techniques in inventory management was provided by Baumol, 1952. Some similarities between managing cash balance and inventory balance was recognized. Baumol, 1952 proposed a deterministic model where parameters such as cash outflow was predetermined; interest rate and transfer fee were constant. This work implemented the classical "lot size" model of the inventory management and aimed to optimize the amount of transferred cash in order to minimize transfer cost and holding cost while maintaining sufficient cash balance to pay the bills. In Baumol's model, the cash inflows and cash outflows were assumed to be predetermined. The cash inflows periodically flow into the noninterest cash account for operating the firm while the cash outflows are constant rate expenditures. The cash balance corresponding to this model is in the simple saw tooth pattern, which is shown in Figure 2-1.

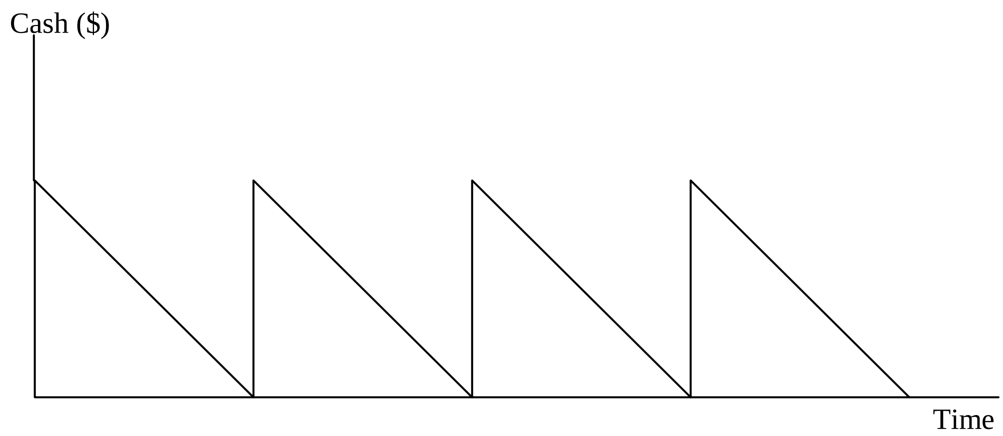


Figure 2-1. Saw tooth cash balance (Miller and Orr, 1966).

However, in reality, the cash balance of the firm is not as simple as shown in Figure 2-1. On the contrary, the cash balance normally fluctuates depending on the amount and timing of cash inflows and cash outflows. It is more complex and fluctuates over time in both positive and negative directions (Miller and Orr, 1966) as shown in Figure 2-2. The cash balance becomes more complex for multinational corporations and global supply networks which internal and external economical aspects such as exchange rates are involved. Miller and Orr (1966) assumed cash balance to be a stochastic model rather than assuming that cash flow occurs constantly. The net cash flows fluctuate over time. There were probabilities of cash balance to increase and decrease. The objective function was to obtain the optimal level of cash balance while minimizing the long-run average cost (Miller and Orr, 1966).

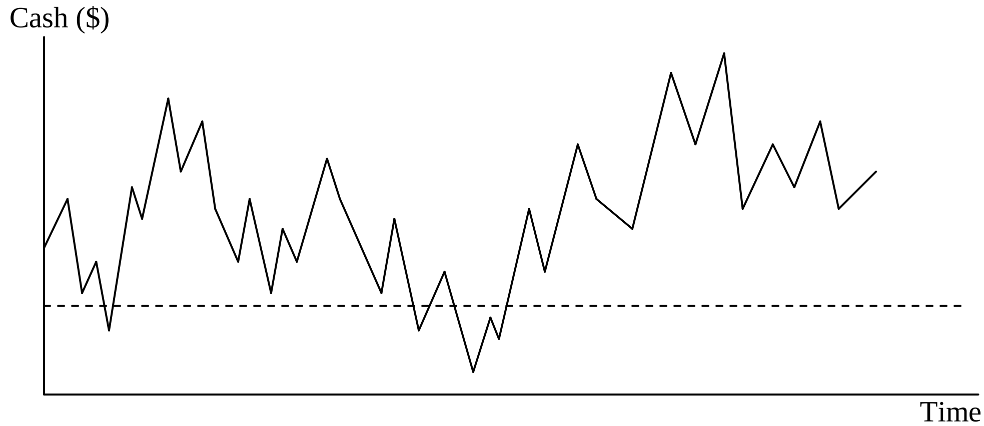


Figure 2-2. Real cash balance (Miller and Orr, 1966).

From Figure 2-2 we can see that in some point in time the firm may have very low cash balance or even negative cash balance may be possible. This situation can damage the firm in many ways such as reduce credibility and reliability to its suppliers and customers. Both studies by Baumol, 1952 and Miller and Orr, 1966 focus on minimizing transfer cost between two financial accounts to maintain cash balance to an adequate working level. These two models introduce ideas and provide good foundation of how to manage cash balance by applying supply chain concept. Meanwhile, these two models leave some opportunity for further improvement in this area to be more practical in the real business world. Some assumptions can be relaxed. For example, transfer cost is assumed to be a constant independent of the amount transferred which is not practical. Clearly, there is an opportunity to further study in this area. There are many dimensions that need to be addressed. By understanding the fundamental behavior of the supply chain, the manufacturing system, and the cash flow forms the basis of this research.

Charnes et al., 1963 suggested that the financial planning and operation planning should be considered together to improve the revenues of a firm. Robichek et al., 1965 recognized the simultaneous approach of the short term and long term capital budget instead of compute it sequentially. However, the linear programming and financial management were not able to simply capture the overall financial solution because the whole financial problem was too complex to be analyzed. Therefore, this work isolates the problem into sub-problems and then determines the solutions (Robichek et al., 1965). Girgis, 1968 applied the inventory management concept to the cash management by developing optimal policies for maintaining cash balance in anticipation of future net expenses. The cash flow was assumed to be independent and identically distributed as well as the holding cost and the shortage cost were assumed to be convex (Girgis, 1968). Gormley and Meade, 2007 proposed a dynamic simple policy (DSP) to minimize transaction cost considering the cash balance as a stochastic problem and the cash flow were not independent or identically distributed.

Some financial techniques have been developed to mitigate the currency exchange risk such as options, forwards, and swaps. Kazaz et al., 2005 formulated the optimal policy, which involves the impact of currency exchange rate uncertainty, for the production planning of a multinational corporation. The two-stage recourse program, production hedging and allocation hedging, was formulated to maximize expected profit. This two-stage stochastic program was incorporated with exchange rate realizations. The first stage determines the production planning of how much to produce and then the second stage allocates the production to markets (Kazaz et al., 2005).

Managing a firm's working capital is very important for its operational and financial success. The objective is to maintain cash flow to sufficiently cover its short-term debt obligations and day-to-day operating expenses meanwhile keeping excess cash as low as possible since the firm does not earn profit from cash sitting in the account. In other words, the firm loses the opportunity to make a profit from investing this excess cash in other assets such as security assets. Sustainable working capital allows a firm to be more flexible to expand business, improve liquidity, and enhance responsiveness to economic situation.

Improving working capital has become crucial to the growth and profitability of most firms since working capital is very sensitive to cash flow fluctuations (Fazzari and Peterson, 1993). Most of the previous works relating to working capital focus on application of procurement policy, production capacity, production planning, and scheduling problem under working capital constraints (Chung and Lin, 1998; Guillen et al., 2006; Guillen et al., 2007; Comelli et al., 2008; Zeballos and Seifert, 2013). Ke and Ai, 2008 determined superior ordering point while achieving working capital target. Protopappa-Sieke and Seifert, 2010 proposed a model to determine optimal purchasing order quantity under working capital restrictions and payment delays.

A firm hold a large amount of cash in order to operate its daily activities smoothly. Hence, efficient cash flow management and working capital management are highly beneficial to firm's financial status. Orgler (1970) documented that a firm should have sufficient amount of cash to operate its normal business and also to prepare for unexpected situations since it is very difficult to forecast the cash flow accurately and to have cash inflows and outflows synchronize perfectly.

A perfect cash flow forecast is impossible because of the uncertainty of cash inflows. Uncertainty may come from customer payment behavior, liquidity problems of customers, and economic situation. Therefore, a firm creates a cash buffer by determining a lower and upper bound of cash flow. There are two types of metrics that are widely used to optimize cash flow, cash position and cash flow. Cash position tells the level of cash available at the end of the period while cash flow tells the amount of cash generated during the period. Several techniques to compute the optimal cash level were developed (Orgler, 1969; Miller and Orr, 1966). However, Orgler (1969) assumed that the cash outflows are controllable and not stochastic; therefore, the forecast of the cash outflows was developed by a deterministic model. On the other hand, the cash inflows contain uncertainty, but they are predictable. In this case, a linear programming model can be applied to optimize the cash flow level. These mathematical programming techniques are very useful and practically efficient in a real world (Graham and Harvey, 2001).

Yet cash flow is usually restrained in the form of accounts receivable and is over looked when a firm tries to optimize its working capital. A number of literatures in finance reveals information about the fact of how much cash is locked up in the accounts receivable, inventories, accounts payable, and working capital. Classic financial models, which relate to optimization of current assets such as accounts receivable, show that the extension of the payment terms on the accounts receivable is a tradeoff between controlling the risks of payment delay from customers and gaining new customers (Michalski, 2007).

2.2 Cash Flow Forecast

Accurate cash flow forecasting models that are easy to use are becoming important for businesses to manage their finances efficiently. Such models will be especially critical for SMEs when liquidity and credit decrease in the economy. Cash flow forecast can be performed in several ways depending on the nature of businesses and the purpose of the forecast. Basically, it can be categorized into two core techniques: the receipts and disbursements forecast, and statistical modeling such as moving average, exponential smoothing, regression analysis, and distribution model (WWCP, 2012). There is no perfect technique for forecasting because each company has its own unique financial activity and characteristics.

Groeye and Mellyn, 2000 showed that for the past 20 years, days of inventory reduced by 35% (to 48 days) whereas days of receivables reduced by only 16% (to 57 days). A significant portion of a firm's asset can be tied up in the accounts receivable rather than in the inventory (Cyert et al., 1962). One of the reasons why a large number of assets are tied up in the accounts receivable is that the focus of efforts for past years was on reducing the inventory in a system while little effort was put on reducing days of outstanding accounts receivable. As a result, payment delay and bad debt can lead to major problems that can jeopardize a company, especially in the case of an SME. Therefore, an accurate cash flow forecast is essential for a company to be able to prepare itself for an unexpected and undesirable financial situation.

Pate-Cornell et al., 1990 formulated a stochastic model for monitoring cash flow and making a short term decision using a signal-response model (Pate-Cornell, 1986; Pate-Cornell et al., 1990). Elton and Gruber, 1974 generally formulated dynamic programming models for the cash management policies under different assumptions of transaction costs and demand for cash Elton and Gruber, 1974.

Kaka, 1996 proposed a computer-based cash flow forecasting model applying cumulative curves of corresponding parameters such as cash in and cash out. In addition, this model included some of the risk associated with a firm since construction projects were known to encounter high level of risk (Kaka, 1996). Navon, 1996 developed a cash flow management model for a company-level by gathering as much as from the model's database (Navon, 1996; Kaka and Lewis, 2003) developed a computer-based model to forecast a company-level cash flow. The model basically involved with many uncertainties and it was needed to update new data frequently; hence, a fully stochastic simulation model and dynamic data updating were applied (Kaka and Lewis, 2003).

While cash is very essential for running a business as a current asset, accounts receivable, another current asset which can be converted into cash very quickly become more imperative in business transactions because most of them are credit transactions where sellers offer their customers payment terms rather than cash on delivery. However, if the receivables cannot be collected on the due dates, sooner or later the company will be insolvent. In order to avoid this crisis, a company must have good cash flow management, especially during the economic recession where cash is the main bloodline for surviving in a business.

Accounts receivable aging is a technique to evaluate the financial health of a company by identifying whether irregularities exist. It shows a company's accounts receivable according to the length of time the amounts have been outstanding. The typical aging time period is 30 days, 60 days, 90 days, and over 120 days. In 1962, Cyert, Davidson, and Thompson successfully developed a model to estimate the allowance for doubtful accounts using a Markov chain approach. This CDT model was made more practicable by Corcoran, 1978 by using exponential smoothing to the transition matrix and changed the method of accounts receivable aging from the oldest balance method to the partial balance method, which is more commonly the standard practice in most businesses. In addition, this study focuses more on the transient state rather than the steady state as in the CDT model (Corcoran, 1978). Kuelen et al., 1981 reexamined the CDT model and found that the use of the total balance aging method in the model did not reflect the real age of the dollars in the accounts. The model was modified in order to perform more accurately (Kuelen et al., 1981).

2.3 Cash Flow Risk

Tsai, 2008 proposed a model to forecast a cash flow risk measured by its standard deviation. The main purpose was to gain insight cash information in order to practically improve the cash conversion cycle (CCC). CCC, known as cash-to-cash cycle or cash cycle, is one of the widely used performance measures on cash flow (Tsai, 2008). Since cash itself is a non-productive asset, excess cash apart from covering day-to-day operating expenses is maintained as little as possible (Bertel et al., 2008). A firm has to utilize cash efficiently.

In order to reduce CCC, a firm can reduce days-in-inventory, shorten days-in-receivables, and prolong days-in-payables. However, the study found that an early payment discount to reduce days-in-receivables, which reduced CCC, increased cash inflow risks. The additional risk in the cash inflow came from uncertain early collection pattern. This risk could be reduced in case that early collection ratio was a constant. In other words, the more the early collection pattern was known, the lower the risk. Since the cash inflow risks dominated the net cash flow risks, therefore, reducing the cash inflow risks could reduce the overall cash flow risks (Tsai, 2008).

The firm with flexible credit lines from its lender could take advantage to tolerate more risks such as shortening the credit period and offering early payment discount; whereas, the firm with tight credit could also tolerate the cash inflow risks by having longer credit terms for better future cash flow prediction. However, the latter case led to worse CCC. The author also showed that the Asset Based Securities (ABS) was the best policy to finance account receivables in order to reduce CCC and cash inflow risk (Tsai, 2008).

2.4 Credit Risk

The concept of credit risk assessment becomes more intriguing to many financial analysts and practitioners in financial area. Over the last three decades, a considerable number of efforts have focused on developing credit risk assessment in both theoretical and practical models. Financial characteristics of a firm such as financial ratios and financial performance measures are studied to determine their relationships with the credit risk. As a result, these relationships are identified and included in the decision support models for a firm to improve accuracy of the credit risk and creditworthiness assessment as much as possible. A comprehensive review of credit risk assessment over the last two decades is presented by Altman and Saunders (1998).

To determine the optimal credit-granting decision which a firm has to tradeoff between the risk of payment loss and the chance of earning more profit from granting credit, the credit risk assessment must take both financial and non-financial aspects into consideration (Srinivasan and

Kim, 1987; Srinivasan and Ruparel, 1990). The information of creditworthy and insolvent firms can be obtained when a firm seeks for line of credit from banks or financial institutions.

A large number of factors usually required to evaluate the credit risk is considered as one of major obstacles for the credit risk assessment process. These factors include financial characteristics of firms, qualitative and quantitative performance measures of firms, macroeconomic factors such as inflation and interest rate. To evaluate the credit risk, credit analysts have to look into all these factors, screen out the least relevant factors, and then focus their analysis on the rest of the relevant factors. The other major obstacle is the aggregation of these factors from the previous phase. The complexity of these factors and analysis make it more difficult and time consuming to make a final credit risk assessment. This obstacle can lead to conflicting results and decisions. During the final process of credit risk assessment, the credit and financial analysts try to balance these conflicting criteria corresponding to their preference system. Consequently, the optimal outcome can be concluded from an appropriate aggregation of evaluation criteria [Bergeron et al., 1996].

Due to a large number of relevant factors and the complexity of the process to evaluate the credit risk, the systematic credit risk assessment models are developed based on the sorting approach. These models facilitate credit and financial analysts as evaluation systems to evaluate credit risk of new firms which seek for financing and as screening tools for bank and financial institution to evaluate existing borrowers (Lane, 1972; Grablowsky and Talley, 1981; Altman et al., 1983; Srinivasan and Kim, 1987; Srinivasan and Ruparel, 1990).

Many articles and academic articles studied how optimal working capital can improve financial status of companies, but these studies do not usually include credit risk into the project selection problem with a working capital constraint. The question discussed in this dissertation concerns the possibility of using integer programming in making decisions about selecting which projects or customers should be invested in. This is certainly one of the most common objectives.

Therefore, this dissertation addresses the implementation of bidding strategies with the project selection. The target is to obtain trade-off solutions during the routine practice preserving at most the profit and liquidity while compensating risk. The main purpose of this dissertation is to determine the optimal project selection that maximize profit and minimize credit risk. The proposed formulation combines a project selection problem and a credit risk problem with a working capital constraint using an integer programming approach.

2.5 Dynamic Financial Decisions

Dynamic financial decisions are decisions that involve: (1) determining the proper amount of funds to employ in a firm; (2) selecting projects and capital expenditure analysis; (3) raising funds on the most favorable terms possible; and (4) managing working capital such as inventory and accounts receivable, while an environment changes over time leading to the change in financial decision-making. This dissertation focuses on a combination of allocating proper amount of investment in projects within limited working capital and selecting projects that maximize profit and minimize credit risk of a firm.

The art and science of project selection plays a critical role in many firms. Firms in each industry develop their own highly sophisticated methods to screen and select projects based on their specific industrial characteristics. These methods are created to ensure that the selected projects provide the highest benefits and a promising success. MTO firms take it very seriously to select projects they want to bid since selection of the right projects for future investment is crucial for the firms meanwhile selection of the wrong projects may risk the firms to loss. Generally, firms have numerous opportunities to invest in several projects. However, with limited working capital and other resources, the firms cannot pursue all opportunities that present themselves. The best choice must be made to secure the most viable projects. Several priority systems and project selection guidelines are developed in order to utilize firms' resources effectively and balance between opportunities and costs entailed by each alternative.

Project selection methods vary depending on the firms, the customers, the criteria, and the characteristics of the projects. Some projects should be evaluated by qualitative approaches such as SWOT analysis while the others should be evaluated by quantitative approaches such as scoring models, economic model, cost-benefit analysis, etc. A number of decision support models to assist firms to screen and select potential project candidates are developed in many areas. Followings are examples of project selecting decision support models: dynamic selection of risky capital investments (Cord, 1964; Magee, 1964; Hespos and Strassman, 1965; Prastacos, 1983), allocation of strategic resources such as production capacities (Naylor, 1984), and dynamic selection and trimming of R&D investments (Hess, 1962; Rosen and Souder, 1965; Atkinson and Bobis, 1969; Flinn and Turban, 1970; Bobis et al., 1971; Aldrich and Morton, 1975; Hopp, 1987; Popp, 1987; Gupta and Mandakovic, 1992) (Heidenberger, 1996).

Most project selection methods take profit and loss, cost and benefit, and working capital into account. Therefore, asset management techniques are also applied in many project selections with regard to financial aspects. Several business decisions such as manufacturing redesign and capital budgeting are examples of where asset management can be applied. The former example deals with managing locations, scheduling, and capacity changes in physical assets such as production facility, warehouse, and distribution facility. The latter example involves the allocation of financial assets such as allocating of capital to procure materials, produce products, and invest in new facilities (Naraharisetti et al., 2008).

Financial aspects are seemed to be the most common basis for project appraisal, however, there are other considerations that can be taken into account. Meredith and S.J., 1995 reported other areas of considerations that affect the project selection. Such considerations are production considerations, marketing considerations, personnel considerations, and administrative considerations in addition to financial considerations. In addition to these considerations, risk of a project uncertainty and risk of incomplete project information are factors that make the project evaluation more complex and difficult. In most cases, decisions are efficiently successful when they are made with accurate information and in a timely manner. Pascale et al. (1997) researched ways to balance competing demands of time and benefits (Pascale et al., 1997).

2.6 Integration of Cash Flow and Supply Chain

Some previous works study on integration between supply chain concept and cash flow concept together in order to improve the cash flow of a firm. A supply chain typically contains many physical facilities such as manufacturing factories, distribution centers, and warehouses. The supply chain usually involves three types of flows, which are forward physical flow, backward financial flow, and backward information flow (Comelli et al., 2008). Many areas indicate that supply chain can combine with cash flow management to improve cash flow of a supply chain.

The following papers studied on the product flow and the financial flow of supply chain in order to improve supply chain's efficiency. Traditionally, these two flows are considered separately; however, these two papers integrated product flow and financial flow together. The information flow was out of their scopes. These two papers focused on supply chain planning; however, one focused on the tactical planning, and the other one focused on operational planning.

Comelli et al., 2008 proposed an approach to evaluate a *tactical production planning* in supply chains by integrating budget constraints into account. This paper aimed to maximize supply chain evaluation function to achieve job schedules. The evaluation of supply chain performance was usually based on quantitative parameters such as stock level and demand satisfaction. The Activity Based Costing (ABC), cost drivers, and payment terms were implemented to evaluate financial flow which was generated from the tactical production planning; however, due to the difficulty in implementing ABC directly to supply chain activities, many activities in supply chain such as sourcing and manufacturing were required to model with the Supply-Chain Operations Reference (SCOR) process before implementing ABC model.

A computer model called PRocess EVAluation or PREVA to evaluate the planning was proposed. The PREVA contained two main steps; (1) physical process evaluation and (2) financial flow evaluation. The physical process evaluation created a physical flow planning. Then this planning became an input data for the second step, the financial flow evaluation. This latter evaluation contained mathematical model to compute cost of each process and integrate cash flow into the physical flow. During the financial evaluation, it determined differences between activity-based cost and transferred price (profit) of each process. These differences provided a value creation assessment in every business unit. The evaluation function selected the planning with the highest value (Comelli et al., 2008).

Rather than integrate financial aspects to the tactical planning like Comelli et al., 2008, Bertel et al., 2008 proposed a method to integrate financial aspects into *operational production planning* in order to obtain optimal solutions, particularly the stock level, for a supply chain manager. This supply chain was modeled as a flowshop. In addition, since this supply chain was a complex manufacturing system, the hybrid flowshop was implemented to this problem. The hybrid flowshop or multiprocessor flowshop was a generalization of flowshop problems, which at least one stage had several resources. Performance measures for this type of problems were demand satisfaction, payback time, stocking cost, cash position, production cost and cash flow (Bertel et al., 2008).

The method, which the authors proposed contained two steps: (1) multiprocessor flowshop or hybrid flowshop generation and (2) cash management. The hybrid flowshop generated sequences of jobs for the tactical planning. Next, the cash management adjusted the balance between non-invested cash and security-invested cash in order to have sufficient cash to cover the day-to-day operating expenses with minimum excess cash. In the cash management, there was a mathematical model applying mixed integer linear programming (MILP) to determine an optimal average cash position. The mathematical model was run by a mathematical programming language (AMPL), and tested by CPLEX solver. Another method to schedule job process was a heuristic model, which contained two algorithms. An algorithm1 applied greedy search algorithm to schedule plant while an algorithm2 applied the BFRT algorithm to provide a list of jobs to be processed.

Another study which integrated financial cross functions into production scheduling and planning in chemical process industries, Badell et al., 2007 applied a Symmetric/Asymmetric Travelling Salesman Problem (TSP/ATSP) that took the overlap times between batches with economic weight to simplify the scheduling task. In addition, the financial and operative scheduling tasks were adapted to work with the advanced planning and scheduling (APS) systems. Badell et al., 2007 developed two models, which were an operative model and a budgeting model. The former model contains the profit function and process time function using mixed integer linear programming (MILP), which is solved by GAMS-CPLEX. The latter model take cash inflows, outflows, assets, and liabilities into account in the objective function which maximizes the dividends to shareholder in month 4, 8, and 12. Even though these strategies could not provide the optimal solution, they improved revenues, profit, and computational time. The results showed that the integrated model that the authors proposed provided less debt and smaller inventory stock (Badell et al., 2007).

Another review of previous integration between financial and supply chain operations at plant level in a short term planning was proposed by Badell et al., 2005. This work aimed to use the advanced planning and schedule (APS) and MILP formulation to assist a manager in making a decision (Badell et al., 2005). Another application of MILP model based on simultaneous optimization of financial flow and planning model was proposed by Guillen et al., 2006. This approach integrated planning and scheduling of chemical supply chains with multi-product, multi-echelon distribution networks, and financial aspects together; whereas, a traditional method solves for the planning first and then fits the financial aspects afterwards (Guillen et al., 2006). Instead of maximizing the profit or minimizing cost for a firm, this model maximizes the profit of shareholders. Another application of this concept was applied to multi-product batch chemical supply chain in Europe (Guillen et al., 2007).

In the past (before the cash flow management is combined with the supply chain management), the cash flow concept was mostly used to evaluate a supply chain elements, especially inventory policies for both deterministic (Kim et al., 1986) and stochastic models (Copeland and Weston, 1988).

Inderfurth and Schefer, 1996 applied the capital asset pricing model (CAPM) to evaluate the inventory policy, the order-up-to-S policy, in a multi period framework. Kim and Chung, 1990 proposed an integrated cash flow model to evaluate inventory and account receivables using the net present value (NPV) maximization framework. Chung and Lin, 1998 later refuted some conclusions of Kim and Chung, 1990 and proposed the exact solution of cash flow for an integrated evaluation of investment in inventory and credit.

Yi and Reklaitis, 2007 proposed an integrated work between the supply chain and the financial decisions of a global supply network. The analytical model was constructed to quantify the effect of exchange rate and taxes to the production lot and storage sizes of a multinational corporation. The model applied the periodic square wave (PSW) method and multistage batch-storage network (BSN) in order to determine optimal design of a parallel batch-storage system and to represent the global production plants, respectively. This study aimed to “minimize the opportunity costs of annualized capital investment and currency/material inventory minus the benefit to stockholders in the numeraire currency” (Yi and Reklaitis, 2007).

Research on other areas of supply chain integrating product flow and cash flow together is inventory management. Many prior studies have improved financial status of a firm by improving inventory management (Brown and Haegler, 2004; Buzacott and Zhang, 2004; Chao et al., 2008; Chen, 2008; Yang et al., 2008; Yang et al., 2008).

2.7 Bullwhip Effect and Cash Flow

When a supply chain management has become a business essential philosophy, information flow plays an important role to coordinate between product flow and financial flow of each supply chain stage. Information is linked among suppliers, manufacturers, distributors, retailers, and customers in order to operate smoothly. As shown in Figure 2-3, the bidirectional arrows represent information flow conveying information in both ways back and forth among supply chain stages.

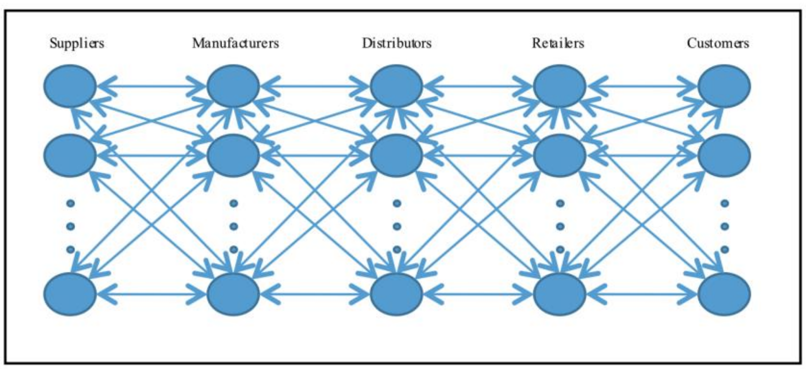


Figure 2-3. Supply network (Fayazbakhsh and Razzazi, 2008).

Effective information flow allows a supply chain to produce products at the right quantities, at the right time, and at the right place. Inefficient information sharing can cause negative effects on a supply chain such as the bullwhip[1] effect, lead time increase, operational cost (production cost, labor cost, transportation cost, and inventory cost) increase, and customer service level drop (Fayazbakhsh and Razzazi, 2008).

The bullwhip effect or whiplash or whipsaw effect is the effect of demand uncertainty, demand amplification, and information distortion from their immediate downstream order placement. (Lee et al., 1997; Mason-Jones and Towill, 2000). The explanation of the bullwhip effect that is universally accepted is described by Lee et al., 1997. Such phenomenon arises when a downstream member in the supply chain place orders containing large variance compared to its actual sales (demand distortion), and this demand distortion propagates to its upstream member causing the demand amplification (Kahn, 1987; Metters, 1997; Cohen, 1998; Lee et al., 2004; Lee et al., 2004). In this study we postulate that the bullwhip effect may also impact the cash flow in the same way as it does to the product flow and lead to the cash flow bullwhip (CFB).

[1] Fluctuation and amplification of demand from downstream to upstream stage of a supply chain

Chapter 3

MODELING AND ANALYSIS OF CASH-FLOW BULLWHIP IN SUPPLY CHAIN

3.1 Introduction

Most supply chains suffer from the effects of demand uncertainty, demand amplification, and information distortion from their immediate downstream order placement known as the "Bullwhip Effect" or "Whiplash" or "Whipsaw" effect. (Lee et al., 1997; Mason-Jones and Towill, 2000). The bullwhip effect has been recognized in many companies. For example, Procter & Gamble and 3M found that the orders placed by the distributors had large fluctuation and the phenomenon was more severe in the upstream members while the customer demand was quite stable. The explanation of the bullwhip effect that is universally accepted is described by Lee et al., 1997. Such phenomenon arises when a downstream member in the supply chain place orders containing large variance compared to its actual sales (demand distortion), and this demand distortion propagates to its upstream member causing the demand amplification (Kahn, 1987; Metters, 1997; Baganha and Cohen, 1998; Lee et al., 2004). The bullwhip effect is illustrated in Figure 3-1.

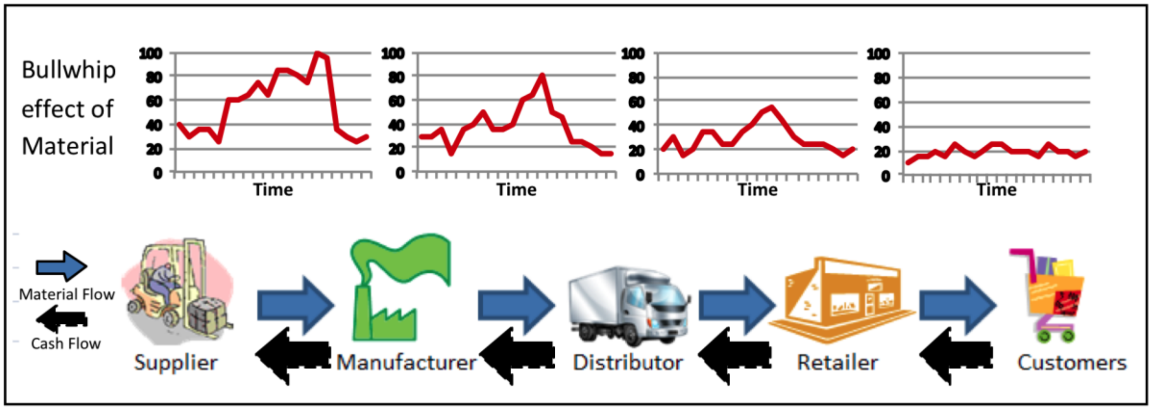


Figure 3-1. Bullwhip effect of material flow in the supply chain.

The graphs in Figure 3-1 show the order quantity of each supply chain member over time. Customer demand (the rightmost graph) has little variation of the order quantity and then it becomes larger and larger when demand distortion propagates to the upstream member (the leftmost graph). The further upstream member in the supply chain the company is, the worse the bullwhip effect will be. We postulate that the bullwhip phenomenon in material flow may similarly happen to the cash flow across supply chain. The term "Cash Flow Bullwhip (*CFB*)" is introduced here in order to capture the bullwhip effect of the cash flow. The *CFB* is a similar phenomenon to the bullwhip effect of material except that it happens to the cash flow. Our motivation stemmed from the importance of cash as a crucial asset for operating a business, especially during the economic recession.

In this study, we develop the *CFB* from the Cash Conversion Cycle (*CCC*), which can be explained as follows.

$$CCC = \frac{Average\ Inventory}{COGS/365} + \frac{Average\ Account\ Receivable}{Revenue/365} - \frac{Average\ Account\ Payable}{COGS/365} \tag{3.1}$$

where *COGS* is a cost of goods sold.

The Cash Conversion Cycle (*CCC*) is the average days required to convert a dollar invested in raw material into a dollar collected from a customer (Stewart, 1995). It is one of the critical factors for a company to be successful in running business by representing how well the company manages its liquidity. A low *CCC* indicates that the company has lower financial cost to fund its business operation. A good example is Dell Computer Corporation, which manages its *CCC* to be negative. In other words, Dell uses other people's money to operate its business (Farris II and Hutchison, 2002). The *CCC* can be lowered by one or more of the followings; lower days-in-inventory outstanding, lower account receivable days, and higher accounts payable days. The smaller number of days-in-inventory outstanding, the lower the *CCC*. However, Tsai, 2008, who investigated the cash flow risks of a simple supply chain using an auto regression model, showed that some common practices used to lower the *CCC* can lead to higher cash flow risks. Some other existing literatures relating the *CCC* to the supply chain, such as the work of Banomyong, 2005, measured the *CCC* of the international supply chain. On the other hand, high *CCC* can lead to an opposite scenario. As shown in Eq. (3.1), the *CCC* can be increased by an increment of an average inventory, assuming that the other terms do not change. Disney and Towill, 2003 found that the inventory variance increases when the production lead-time increases. Associated with the work of Chen et al., 2000, the increase of the lead-time results in the increase in the bullwhip effect.

Most of the existing literatures regarding the bullwhip effect emphasize the existence of the bullwhip effect, the reasons of its occurrence, and possible ways to lower it. For example, Sterman, 1989 provided evidence of the bullwhip effect via the study of the 'Beer distribution game'. Similarly, Burbidge, 1989 studied the bullwhip effect, prescribed reasons for its existence, and then concluded that demand amplification occurring across supply chains is a system induced phenomena influenced by information and material delays in the supply chain. Later on, four major causes of the bullwhip effect, which are (1) demand forecast updating, (2) order batching, (3) price fluctuation, and (4) rationing and shortage gaming, were identified (Lee et al., 1997). Mason-Jones and Towill, 2000 studied the influence of the bullwhip effect to supply chain uncertainties known as the Uncertainty Circle, which is (1) supply side, (2) manufacturing process, (3) process controls, and (4) demand side. Additionally, they found that forecasting error can lead to shortage of supply which not only results in a loss of sale but also a loss of consumer confidence, which may impact future sales (Mason-Jones and Towill, 2000). The bullwhip effect or the demand amplification may also result in numerous negative effects: excessive inventory level, stock-outs and backorders, expensive production capacity swings, uncertain production planning, ineffective transportation, expensive cost for correction, distorted demand forecasting, and so forth (Lee et al., 1997; Chen et al., 1998). Lee et al., 2004 studied the flow of demand information across the supply chain and made observations regarding the distortion in demand information as it propagates up the supply chain as orders. A number of studies dedicated to quantify the bullwhip effect as follows. Chen et al., 2000 formulated a model to quantify the bullwhip effect for a simple supply chain. Later on, Kim et al., 2006 developed a model for a stochastic lead time as well as Fioriolli and Fogliatto, 2008 developed a model for a stochastic demand and lead time.

All the works surveyed in this literature review, however, mainly focus on analyzing and mitigating adverse effects of the bullwhip effect, or studying the effect of this phenomenon on inventory and ordering policies. On the other hand, the focus of this dissertation lies in modeling and analyzing the *CFB* as well as understanding its causes and managerial implications. The important contribution that our work seeks to make in comparison to previous research is to analyze its impact on the cash flow, particularly, the Cash Conversion Cycle (*CCC*). We postulate that the bullwhip effect may also impact the cash flow in the same way as it does to the material flow. Consequently, when the bullwhip effect of the material occurs, the *CFB* is anticipated to take place in a supply chain. We will explain and present how to model the *CFB* in Section 3.2.

The rest of this chapter is organized as follows. The next section provides the analytical model to determine the *CFB*, which is derived from the variability of inventory and the *CCC* for a simple supply chain and multi-stages supply chain. Section 3.3 gives an overview of the simulation model used for experimentation. Section 3.4 presents the results and discusses the impact of the bullwhip effect on the variability of inventory and *CFB*. Lastly, in Section 3.5 the conclusion is discussed.

3.2 Analytical Model for CFB

In this section, we develop the analytical models for inventory bullwhip effect in a simple supply chain, and then extends the model for a multi-stages supply chain. The model shows how the bullwhip effect impacts the inventory. Then, we extend the inventory bullwhip effect model to the *CFB* model, which is derived from the Cash Conversion Cycle (*CCC*).

3.2.1 Impact on Inventory in Simple Supply Chain

Consider a simple supply chain which contains a single retailer and a single manufacturer. The retailer observes his inventory level at time t, I_t. By the end of period t, the retailer places an order q_t to the manufacturer. Assume the lead time L is fixed, thus, the order will be received at the start of period $t+L$. After the order is received, the retailer fills the customer demand D_t and backlogs any excessive demands. Kahn, 1987 provides the demand model that the retailer faces in the form of

$$D_t = d + \rho D_{t-1} + \mu_t \tag{3.2}$$

where d is a nonnegative constant, ρ is a correlation parameter satisfying $|\rho|<1$, and μ_t is an independent and identically normally distributed random variable with zero mean and variance σ^2. The demand model of this form has been used by many authors to analyze the bullwhip effect (Chen et al., 2000).

The approach in Chen et al., 2000 is used in this research as a starting point in the development of the proposed Cash Flow Bullwhip model. The order quantity (q_t) can be written relative to the customer demand (D_t) as

$$q_t = y_t - y_{t-1} + D_{t-1} \tag{3.3}$$

where y_t is the order-up-to point which is estimated from the observed demand as

$$y_t = \widehat{D}_t^L + z\hat{\sigma}_{e,t}^L \tag{3.4}$$

where $\widehat{D}_t^L$ is an estimate of the mean lead time demand using a simple moving average, z is a constant to meet a desired service level, and $\hat{\sigma}_{e,t}^L$ is an estimate of the standard deviation of the L period forecast error which are shown below.

$$\widehat{D}_t^L = L\left(\frac{\sum_{i=1}^{p} D_{t-i}}{p}\right) \tag{3.5}$$

$$\hat{\sigma}_{et}^L = C_{L,\rho}\sqrt{\frac{\sum_{i=1}^{p}(e_{t-i})^2}{p}} \tag{3.6}$$

where p is the period in days of a simple moving average to estimate the customer demand $\widehat{D}_t^L$, $C_{L,\rho}$ is a constant function of L, p and ρ, and e_t is the one-period forecast error.

In order to show the impact of the bullwhip effect to the inventory level and *CCC*, we determine the variance of the inventory level at time t as I_t corresponding to the bullwhip effect ($Var(q)/Var(D)$). Therefore, we write I_t as

$$I_t = I_{int} - \sum_{i=1}^{t} D_i + \sum_{i=1}^{t} q_{i-L}$$

where I_{int} is an initial inventory level.

From $q_t = y_t - y_{t-1} + D_{t-1}$, hence,

$$\sum_{i=1}^{t} q_{i-L} = y_{t-L} + \sum_{i=1}^{t-1-L} D_i\,;\, y_0 \, and \; D_0 = 0$$

Thus, I_t becomes

$$I_t = I_{int} + y_{t-L} - \sum_{i=t-L}^{t} D_i$$

Since we want to demonstrate variance of inventory in the form of the bullwhip effect, we need the term q.

From Eq. (3.3), we rearrange the equation to write y_{t-L} in the form of q_{t-L}

$$y_{t-L} = q_{t-L} + y_{t-L-1} - D_{t-L-1}$$

we obtain

$$I_t = I_{int} + q_{t-L} + y_{t-L-1} - D_{t-L-1} - \sum_{i=t-L}^{t} D_i$$

$$= I_{int} + q_{t-L} + y_{t-L-1} - \sum_{i=t-L-1}^{t} D_i$$

$$= I_{int} + q_{t-L} + y_{t-L-1} - D_t - \sum_{i=1}^{L+1} D_{t-i} \qquad (3.7)$$

Then we determine the variance of I_t.

$$Var(I_t) = Var\left(q_{t-L} + y_{t-L-1} - D_t - \sum_{i=1}^{L+1} D_{t-i}\right); Var(I_{int}) = 0$$

$$Var(I_t) = Var(q_{t-L}) + Var(y_{t-L-1}) + Var(D_t) + Var\left(\sum_{i=1}^{L+1} D_{t-i}\right) + 2Cov(q_{t-L}, y_{t-L-1})$$

$$- 2Cov(q_{t-L}, D_t) - 2Cov(q_{t-L}, \sum_{i=1}^{L+1} D_{t-i}) - 2Cov(y_{t-L-1}, D_t)$$

$$- 2Cov\left(y_{t-L-1}, \sum_{i=1}^{L+1} D_{t-i}\right) + 2Cov(D_t, \sum_{i=1}^{L+1} D_{t-i})$$

We need to determine $Var(y_{t-L-1})$, $Var\left(\sum_{i=1}^{L+1} D_{t-i}\right)$ and all the covariance in the equation. (We do not touch $Var(q)$ since we want the term $Var(q)/Var(D)$ to remain in the equation.) If the retailer uses a simple moving average forecasting method with p demand observations and under the condition of Lemma 2.1 of Chen et al., 2000, then we obtain the variance of I_t as follows:

$$\frac{Var(I)}{Var(D)} = \frac{Var(q)}{Var(D)} + \left(\frac{L}{p}\right)^2 \left[p + 2p\left(\frac{\rho - \rho^p}{1-\rho}\right) - 2\rho\left(\frac{(p-1)\rho^p - p\rho^{p-1} + 1}{(1-\rho)^2}\right)\right] \tag{3.8}$$
$$+ 1$$
$$+ \left[(L+1) + 2L\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right)\right.$$
$$\left. - 2\rho\left(\frac{L\rho^{L+1} - (L+1)\rho^L + 1}{(1-\rho)^2}\right)\right]$$
$$+ 2\left(\left(1 + \frac{L}{p}\right)\left(\frac{L}{p}\right)\left(\frac{\rho - \rho^{p+1}}{1-\rho}\right) - \left(\frac{L}{p}\right)^2\left(\frac{1-\rho^p}{1-\rho}\right)\right)$$
$$- 2\left(\left(1 + \frac{L}{p}\right)\rho^{L+1} - \left(\frac{L}{p}\right)\rho^{L+p+1}\right)$$
$$- 2\left(\left(1 + \frac{L}{p}\right)\left(\frac{1-\rho^{L+1}}{1-\rho}\right) - \left(\frac{L}{p}\right)\left(\frac{\rho^p - \rho^{L+p+1}}{1-\rho}\right)\right)$$
$$- 2\left(\frac{L}{p}\right)\rho^{L+1}\left(\frac{\rho - \rho^{p+1}}{1-\rho}\right)$$
$$- 2\left(\frac{L}{p}\right)\rho^{L+1}\left(\frac{\rho - \rho^{p+1}}{1-\rho}\right)\left(\frac{1-\rho^{-(L+1)}}{\rho - 1}\right) + 2\left(\frac{\rho - \rho^{L+2}}{1-\rho}\right)$$
$$+ \frac{z^2 Cov\left(\hat{\sigma}^L_{e,t-L}, \hat{\sigma}^L_{e,t-L-1}\right)}{Var(D)}$$

Then, we divide *Var(I)* by *Var(D)* and rearrange the equation, we obtain

$$\frac{Var(I)}{Var(D)} = \frac{Var(q)}{Var(D)} + f(L, p, \rho) + g(z, \hat{\sigma}^L_{e,t}, D)$$

where

$$f(L,p,\rho) \;=\; \frac{1}{(1-\rho)^2}\Bigg[\left(\frac{L}{p}\right)^2 [2\rho^{p+2} - 2\rho^{p+1} + 2\rho^{p} - (p+2)\rho^2 + 2\rho + p - 2]$$

$$- 2\left(\frac{L}{p}\right)\left[\rho^{L+p+2} - \rho^{(L+2)} - \rho^{(p+2)} + \rho^{p+1} - \rho^{p} + \rho^2 - \rho + 1\right] + (2\rho^{L+2}$$

$$- (L+2)\rho^2 + L)\Bigg]$$

$$g\left(z, \hat{\sigma}^{L}_{e,t}, D\right) = \frac{z^2 Cov\left(\hat{\sigma}^{L}_{e,t-L}, \hat{\sigma}^{L}_{e,t-L-1}\right)}{Var(D)}$$

and we obtain the lower bound of its variance as

$$\frac{Var(I)}{Var(D)} \geq \frac{Var(q)}{Var(D)} + f(L,p,\rho) \tag{3.9}$$

The bound is tight when z = 0. See Appendix A for a proof of Eq. (3.8).

Eq. (3.9) mathematically shows us that the variability of the inventory depends on the bullwhip effect *Var(q)/Var(D)*, and the function of *L*, *p*, and *ρ*. The relationship between the variability of the inventory and the bullwhip effect seems to be a linear function, which will be compared to the results from the simulation in Section 3.4 later. In order to see the impact of the rest of the factors, we firstly plot the graphs of *Var(I)/Var(D)* versus *L* with various *ρ* for $\rho > 0$, which is more reasonable in practice. We find that the increase in the variability of inventory is an increasing function of *L* and *ρ* as shown in Figure 3-2. Unlike the bullwhip effect, the lead time *L* has a non-linear relationship with the variability of the inventory.

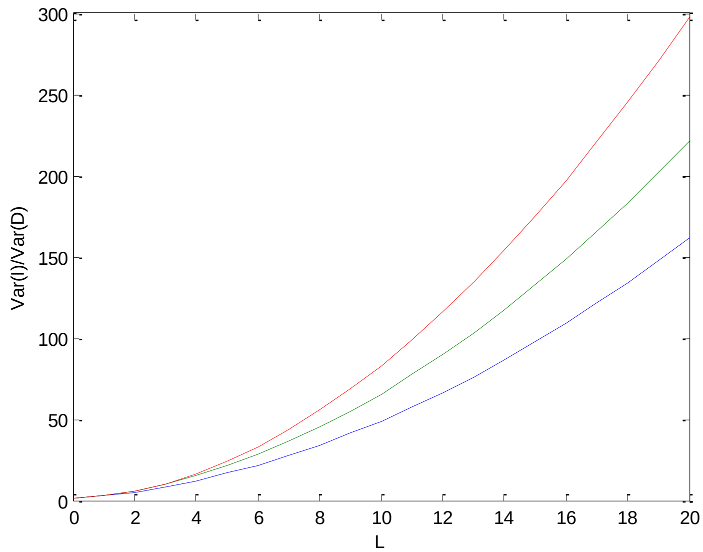


Figure 3-2. Effect of lead time L on inventory bullwhip.

Figure 3-3 below shows that the increase in the variability of inventory is a decreasing function of the demand observation period *p*. As *p* is increased, the variability of inventory is reduced significantly. These results are similar to the findings of Chen et al., 2000 where the increase in the variability of the order quantity results from the increase of *L* and the decrease of *p*. From the results here lead us to a conclusion that if we can reduce the lead time *L* and increase the demand observation period *p* (moving average) as much as we can, we can reduce the variability of the inventory and its adverse impacts. On the other hand, since the bullwhip effect cannot be completely removed (Chen et al., 2000), the variability of the inventory still remains. In addition, when we take a closer look at Eq. (3.9), we see that the variability of inventory is greater than or equal to the bullwhip effect itself because the right hand side of Eq. (3.9) contains the summation of the bullwhip effect and the function of *L*, *p*, and *ρ*. With that said, as long as the lead time *L* is not zero and the demand forecast is applied, the variability of the inventory is always more severe than the bullwhip effect.

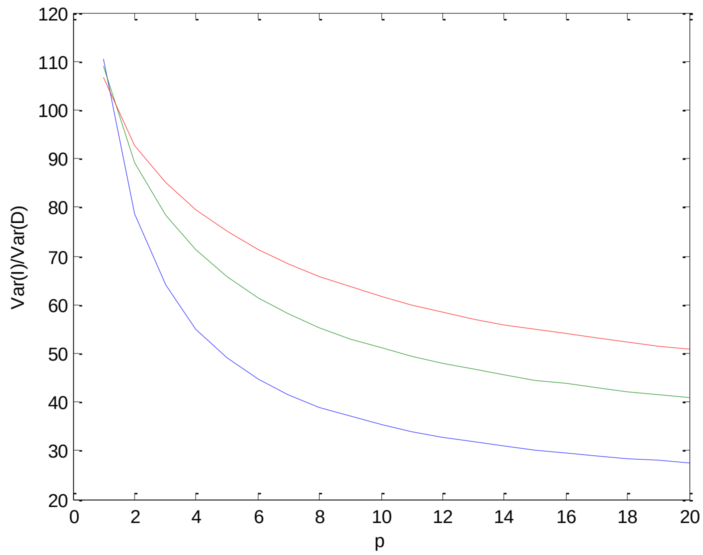


Figure 3-3. Effect of demand observation period *p* on inventory bullwhip

In addition, the marginal increase of lead time *L* worsens the variability of inventory more than the marginal decrease of *p*. In other words, the increase in one unit of lead time can hurt the fluctuation of inventory more than the increase in one unit of the number of observations used in demand forecasting can relieve the fluctuation. Thus, large lead time *L* causes more fluctuation in inventory, whereas, large value of *p* smoothes the demand forecasting data and hence reduces this fluctuation. One more observation from Eq. (3.9), with unchanged average account receivable and average account payable when the bullwhip effect occurs, the increase in the bullwhip effect, *Var(q)/Var(D)*, on the right hand side of Eq. (3.9) makes the term $Var(I_t)/Var(D)$ increase which leads to the increase in the *CCC*. Even though, the bullwhip effect is not the reason to increase *CCC* directly in Eq. (3.9), it increases the excessive inventory, which sits in the warehouse longer. This situation makes a company takes a longer time to convert the inventory into cash and hence makes the cash conversion cycle larger. Section 3.2.3 shows how the bullwhip effect really impacts the *CCC*.

3.2.2 Impact on Inventory in Multi-Stages Supply Chain with Centralized Demand Information

In this section, a multi-stages supply chain with all demand information shared from a retailer is considered. Assume that a moving average forecasting method and the order-up-to inventory policy are applied through all stages in the supply chain. Therefore, an estimation of the mean demand per period for each stage is $\widehat{D}_t = \sum_{i=1}^{p} D_{t-i} / p$, and the order-up-to point for each stage k is

$$y_t^k = L_k \widehat{D}_t^L + z_k \hat{\sigma}_{e,t}^{Lk}$$

where L_k is the lead time between stages k and $k+1$, $\widehat{D}_t$ is the estimate of the mean demand per period, z_k is a constant, and $\hat{\sigma}_{et}^{Lk} = C_{L,\rho}\sqrt{\sum_{i=1}^{p}(e_{t-i})^2 / p}$ (Chen et al., 2000).

The sequence of these supply chain events follows the Beer Game Distribution of Sterman, 1989. The sequence is briefly described as follows. The retailer (k = 1) observes its customer demand by the end of period t-1 and then determines its order quantity q_t^1 to fill its inventory level to y_t^1. The manufacturer (k = 2) receives the order q_t^1 with the demand information D_{t-1} by the end of period t-1, assuming that there is no information lead time. Then its order quantity q_t^2 is calculated in order to raise its inventory level to y_t^2. Then the process continues in the same way for further stages (k = 3, 4, 5, ...). To determine $Var(I_t)/Var(D)$ of the multi-stages supply chain, first, we consider the inventory level at stage 1 (k = 1),

$$I_t^1 = I_{int} + q_{t-L} + y_{t-L-1} - D_t - \sum_{i=1}^{L+1} D_{t-i}$$

and at all other stages (for $k \geq 2$),

$$I_t^k = I_{int}^k + q_{t-L}^k + \frac{L_k}{p}\sum_{i=1}^{p} D_{t-L-1-i} + \frac{\sum_{i=1}^{k-1} L_i}{p} D_{t-p} - \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right) D_t$$

$$- \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\sum_{i=1}^{L+1} D_{t-i} - \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\sum_{i=1}^{L+1} D_{t-p-i}$$

and its variance is

$$Var(I_t^k) = Var(q_{t-L}^k) + \left(\frac{L_k}{p}\right)^2 Var\left(\sum_{i=1}^{p} D_{t-L-1-i}\right) + \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var(D) + \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var(D)$$

$$+ \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var\left(\sum_{i=1}^{L+1} D_{t-i}\right) + \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var\left(\sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$+ 2\frac{L_k}{p} Cov\left(q_{t-L}^k, \sum_{i=1}^{p} D_{t-L-1-i}\right) + 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(q_{t-L}^k, D_{t-p}\right)$$

$$- 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov(q_{t-L}^k, D_t) - 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(q_{t-L}^k, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$- 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(q_{t-L}^k, \sum_{i=1}^{L+1} D_{t-p-i}\right) + 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, D_{t-p}\right)$$

$$- 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, D_t\right)$$

$$- 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$- 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, \sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$- 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(D_{t-p}, D_t\right)$$

$$- 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(D_{t-p}, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$- 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Cov\left(D_{t-p}, \sum_{i=1}^{L+1} D_{t-p-i}\right) + 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Cov\left(D_t, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(D_t, \sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(\sum_{i=1}^{L+1} D_{t-i}, \sum_{i=1}^{L+1} D_{t-p-i}\right); Var(I_{int}^k) = 0$$

$$Var(I_t^k) = Var(q_{t-L}^k) \tag{3.10}$$

$$+ \left(\frac{L_k}{p}\right)^2 \left[p + 2p\left(\frac{\rho - \rho^p}{1-\rho}\right)\right.$$

$$\left. - 2\rho\left(\frac{(p-1)\rho^p - p\rho^{p-1} + 1}{(1-\rho)^2}\right)\right] Var(D) + \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var(D)$$

$$+ \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var(D)$$

$$+ \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \left[(L+1) + 2L\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right)\right.$$

$$\left. - 2\rho\left(\frac{L\rho^{L+1} - (L+1)\rho^L + 1}{(1-\rho)^2}\right)\right] Var(D)$$

$$+ \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \left[(L+1) + 2L\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right)\right.$$

$$\left. - 2\rho\left(\frac{L\rho^{L+1} - (L+1)\rho^L + 1}{(1-\rho)^2}\right)\right] Var(D)$$

$$+ 2\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{L_k}{p}\right)\left(\frac{\rho - \rho^{p+1}}{1-\rho}\right)\right.$$

$$\left. - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{L_k}{p}\right)\left(\frac{1-\rho^p}{1-\rho}\right)\right) Var(D)$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{|L+1-p|}\right.$$

$$\left. - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+1}\right) Var(D)$$

$$- 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+1}\right.$$

$$\left. - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+p+1}\right) Var(D)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\left(1+\frac{\sum_{i=1}^{k}L_i}{p}\right)\left(\frac{1-\rho^{L+1}}{1-\rho}\right)-\left(\frac{\sum_{i=1}^{k}L_i}{p}\right)\left(\frac{\rho^{p}-\rho^{L+p+1}}{1-\rho}\right)\right)Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\left(1+\frac{\sum_{i=1}^{k}L_i}{p}\right)\sum_{i=1}^{L+1}\rho^{|L-p+1-i|}-\left(\frac{\sum_{i=1}^{k}L_i}{p}\right)\left(\frac{1-\rho^{L+1}}{1-\rho}\right)\right)Var(D)$$

$$+2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=L-p+2}^{L+1}\rho^{|i|}\,Var(D)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\frac{L_k}{p}\right)\rho^{L+1}\left(\frac{\rho-\rho^{p+1}}{1-\rho}\right)Var(D)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\frac{L_k}{p}\right)\rho^{L+1}\left(\frac{\rho-\rho^{p+1}}{1-\rho}\right)\left(\frac{1-\rho^{-(L+1)}}{\rho-1}\right)Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1}\rho^{|L-p+1+i-j|}\,Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\rho^{p}Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\sum_{i=1}^{L+1}\rho^{|p-i|}\,Var(D)-2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)^2\left(\frac{\rho-\rho^{L+2}}{1-\rho}\right)Var(D)$$

$$+2\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)^2\left(\frac{\rho-\rho^{L+2}}{1-\rho}\right)Var(D)$$

$$+2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\frac{\rho^{p+1}-\rho^{L+p+2}}{1-\rho}\right)Var(D)$$

$$+2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\sum_{i=1}^{L+1}\sum_{j=1}^{L+1}\rho^{|p+j-i|}\,Var(D)\,;Var\left(I_{int}^{k}\right)=0$$

Therefore, $Var(I_t^k)/Var(D)$ can be written as

$$\frac{Var(I^k)}{Var(D)}\geq\frac{Var\left(q^k\right)}{Var(D)}+h(L,p,\rho)$$

where $h(L,p,\rho)$ is

$$h(L,p,\rho) = \frac{1}{(1-\rho)^2}\left(\frac{L_k}{p}\right)^2 [2\rho^{p+1} - p\rho^2 - 2\rho + p]$$

$$+ 2\frac{1}{(1-\rho)^2}\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \Bigg[\rho^{L+p+3} - \rho^{L+p+2} + 2\rho^{L+2} - 2\rho^{p+2} + 3\rho^{p+1} - \rho^p$$

$$- L\rho^2 - 4\rho + L + 2 - 2(1-\rho)^2 \sum_{i=1}^{L+1} \rho^{|p-i|} + 2(1-\rho)^2 \sum_{i=1}^{L+1}\sum_{j=1}^{L+1} \rho^{|p+j-i|}\Bigg]$$

$$- 2\frac{1}{(1-\rho)^2}\left(\frac{L_k}{p}\right)[\rho^{L+p+3} - \rho^{L+3} - \rho^{p+2} + \rho^2]$$

$$+ 2\frac{1}{1-\rho}\left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{L_k}{p}\right)[\rho^{p+1} - \rho^p - \rho + 1]$$

$$+ 2\frac{1}{(1-\rho)^2}\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\Bigg[(1-\rho)^2\rho^{|L+1-p|} + \rho^{L+p+3} - \rho^{L+p+2} + \rho^{L+3} + \rho^{L+2}$$

$$- 2\rho^{p+2} + 3\rho^{p+1} - \rho^p - (L+2)\rho^2 - \rho + L + 1$$

$$+ (1-\rho)^2\left(\sum_{i=1}^{L+1}\sum_{j=1}^{L+1}\rho^{|p+j-i|} - \sum_{i=1}^{L+1}\rho^{|L-p+1-i|} - \sum_{i=1}^{L+1}\rho^{|p-i|}\right)\Bigg]$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\Bigg[\rho^{|L+1-p|} + \rho^{L+p+1} - \sum_{i=1}^{L+1}\rho^{|L-p+1-i|} - 2\rho^{L+1}$$

$$+ \left(\frac{\rho^p - \rho^{L+p+1}}{1-\rho}\right)\Bigg]$$

$$+ 2\left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left[\rho^{L+p+1} - \rho^{L+1} - \left(\frac{\rho^{L+p+1} - \rho^{L+1} - \rho^p + 1}{1-\rho}\right)\right]$$

$$- 2\frac{1}{(1-\rho)^2}\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\Bigg[\rho^{L+p+3} - \rho^{L+3} - \rho^{p+1} - \rho$$

$$+ (1-\rho)^2\left(\sum_{i=1}^{p}\sum_{j=1}^{L+1}\rho^{|L-p+1+i-j|} - \sum_{i=L-p+2}^{L+1}\rho^{|i|}\right)\Bigg]$$

$$+\frac{1}{(1-\rho)^2}\left[2\rho^{L+2}-(L+2)\rho^2+L\right]$$

This bound is tight when $z_i = 0$ for $i = 1, 2, 3, \ldots, k$, when $k \geq 2$.

See Appendix B for a proof of Eq. (3.10).

Notice that $Var(I_t)$ in Eq. (3.10) are similar to that of Eq. (3.8) plus some additional terms which are caused by the additional stages in the supply chain. The increase in the variability of the inventory in multi-stages supply chain is obviously larger than that of the simple supply chain. Even though the demand information is centralized and the demand forecasting technique as well as the inventory policy is the same, the bullwhip effect still exists and there is still an increase in variability of the inventory in the supply chain.

3.2.3 Impact on CCC in Simple Supply Chain

As a deduction from the previous section that the increase in the bullwhip effect amplifies the variability of the inventory level resulting in the increase in the *CCC*, in this section, we demonstrate how the bullwhip effect impacts the *CCC*. For the sake of simplicity, assume that all sale and purchase transactions are credit sales and all revenues come from account receivable sales only, no other income such as income from other investment or interest. These assumptions can be relaxed later by setting the proportion of sales made by cash and credit, and the proportion of revenues from sales and other incomes. For example, 30% of sales is cash transaction and 70% of it is credit transaction. No discount rate or penalty fee for early or late payment. In addition, changes in credit and collection policies effect the accounts receivable in different ways (Richards and Laughlin, 1980), and hence play important roles in changing *CCC*. In this study, the credit and collection policies are assumed to be fixed and same policies are applied throughout the supply chain.

Let us consider the components of the *CCC* in Eq. (3.1), which are days inventory outstanding, days sales outstanding, and days payable outstanding, respectively. The Days Inventory Outstanding (*DIO*) can be expressed as

$$DIO = \frac{Average\ Inventory}{COGS/365}$$

$$= \frac{sI}{cD/365} = 365\left(\frac{s}{c}\right)\left(\frac{I}{D}\right)$$

where s is the sales price per unit, *I* is the average inventory level, *c* is the unit cost, and *D* is the average demand.

Then, consider Days Sales Outstanding (*DSO*) or Days Receivables which can be written as

$$DSO = \frac{Average\ Account\ Receivable}{Revenue/365}$$

where the account receivable (*AR*) can be expressed in terms of demand and inventory level as follows:

$$AR = m\,min\,(sD, sI) \tag{3.11}$$

where *m* denotes the collection policy of the firm; $0 \le m \le 1$, however, in this dissertation, we assume $m = 1$ for all credit sales. Replace Eq.(3.11) in *DSO*, obtain

Case I: $D \leq I$

$$DSO = m\left(\frac{sD}{sD/365}\right) = 365$$

Case II: $D > I$

$$DSO = m\left(\frac{sI}{sD/365}\right) = 365\left(\frac{I}{D}\right)$$

Lastly, consider Days Payable Outstanding (*DPO*) which can be written as

$$DPO = \frac{Average\ Account\ Payable}{COGS/365}$$

where the account payable (*AP*) can be written as

$$AP = ncq \tag{3.12}$$

where $0 \leq n \leq 1$, is the payment type of the firm and is assumed to be equal to 1 for all credit purchases; q is the order quantity. Replace Eq. (3.12) in *DPO*, we get

$$DPO = n\left(\frac{cq}{cD/365}\right) = 365\left(\frac{q}{D}\right)$$

Hence, from Eq. (3.1), we obtain *Var(CCC)/Var(D)* for case I as follows:

Case I: $D \leq I$

$$\frac{Var(CCC)}{Var(D)} = \frac{(365)^2}{Var(D)}\left(\frac{s}{c}\right)^2 Var\left(\frac{I}{D}\right) + 0 + \frac{(365)^2}{Var(D)} Var\left(\frac{q}{D}\right)$$

Typically, there are no exact formula to determine the variance of the quotient of two random variables. However, Mood et al., 1974 developed the approximate model for two independent random variables as follows:

$$Var\left(\frac{x}{y}\right) \cong \left(\frac{E(x)}{E(y)}\right)^2 \left[\frac{Var(x)}{E(x)^2} + \frac{Var(y)}{E(y)^2}\right]$$

where *E(x)* and *E(y)* are the expected value of *x* and *y*, respectively. Hence, case I can be written as

$$\frac{Var(CCC)}{Var(D)} \cong \frac{(365)^2}{E(D)^2}\left(\frac{s}{c}\right)^2 \left[\frac{Var(I)}{Var(D)} + \frac{E(I)^2}{E(D)^2}\right] + \frac{(365)^2}{E(D)^2}\left[\frac{Var(q)}{Var(D)} + \frac{E(q)^2}{E(D)^2}\right]$$

Then, replace *Var(I)/Var(D)* by Eq. (3.9), we obtain

$$\frac{Var(CCC)}{Var(D)} \cong \frac{(365)^2}{E(D)^2}\left[\left(\frac{s}{c}\right)^2 \left(\frac{Var(q)}{Var(D)} + f(L,p,\rho) + \frac{E(I)^2}{E(D)^2}\right) + \left(\frac{Var(q)}{Var(D)} + \frac{E(q)^2}{E(D)^2}\right)\right] \tag{3.13}$$

Similarly, Case II can be written as

Case II: *D* > *I*

$$\frac{Var(CCC)}{Var(D)} = \frac{(365)^2}{Var(D)}\left(\frac{s}{c}\right)^2 Var\left(\frac{I}{D}\right) + \frac{(365)^2}{Var(D)} Var\left(\frac{I}{D}\right) + \frac{(365)^2}{Var(D)} Var\left(\frac{q}{D}\right)$$

and then, in the same way, apply the approximation of Mood et al., 1974 and replace *Var(I)/Var(D)* by Eq. (3.9), obtain

$$\frac{Var(CCC)}{Var(D)} \cong \frac{(365)^2}{E(D)^2}\left[\left(\frac{s}{c}\right)^2\left(\frac{Var(q)}{Var(D)} + f(L,p,\rho) + \frac{E(I)^2}{E(D)^2}\right) + \left(\frac{Var(q)}{Var(D)} + \frac{E(q)^2}{E(D)^2}\right)\right] \tag{3.14}$$

Eq. (3.13) and (3.14) represent the approximation of the variance of *CCC* over the variance of demand, *Var(CCC)/Var(D)*. Let us call this variability the Cash Flow Bullwhip effect (*CFB*). Since fluctuation of the order quantity (*q*) represents the demand amplification in the supply chain, Chen et al., 2000 determines *Var(q)/Var(D*) as a quantification of the bullwhip effect. Similarly, for cash flow, fluctuation of the payment or cash collection can represent the bullwhip effect in cash flow. However, instead of using these two variables, we use the *CCC* which is a widely used measure of liquidity and cash flow. *CCC*, by integrating the time pattern of cash inflow from current asset investments and cash outflow from current liabilities, expresses the period of time required to convert a dollar of cash invested in raw material into a dollar of cash collected from customers (Richards and Laughlin, 1980; Stewart, 1995). It should also be empahsized that our choice of *CCC* was also motivated by the fact that it is an effectively links cash flow and inventory which would be useful for studying relationship between bullwhip effect and *CFB*.

Unlike $Var(I_t)/Var(D)$, *CFB* is not only the function of *Var(q)/Var(D)*, *L*, *p*, and *ρ*, but also the function of *E(I)*, *E(q)*, *E(D)*, *s*, and *c*. Figure 3-4 below shows that the *CFB* is also an increasing function of the lead time *L*. The increase in lead time *L* leads to the increase in *CFB*. In other words, when the whole processing time from the order placement to the finished goods shipment extends, it is likely that the firm will have high *CFB*, which leads to more fluctuation in cash flow. Therefore, one way to reduce the *CFB* is to shorten the lead time *L*.

Figure 3-5 shows a surface of *CFB* corresponding to the bullwhip effect and *p* values. The *CFB* is a decreasing function of *p*. Therefore, another way to reduce the *CFB* is to extend the moving average period *p* to smooth the demand forecasting. Thus, when *p* is increased, the bullwhip effect and the *CFB* decreases. As shown in the figure, the *CFB* is an increasing function of the bullwhip effect. This figure shows how the bullwhip effect impacts the *CFB* at different values of *p*. It implies that the firm exposes more fluctuation of converting the input resources into cash when the bullwhip effect becomes more severe. The increase in the bullwhip effect causes the increase in the *CFB* and the average marginal impact of the bullwhip effect on the increase of the *CFB* is approximately 20%. High *CFB* implies that the firm may find it is more difficult to manage its cash level and may incur higher working capital cost to operate its business. Therefore, the low *CFB* is perferable. From this figure, we can see that in order to minimize the *CFB*, the firm needs to increase the moving average period *p* and reduce the bullwhip effect.

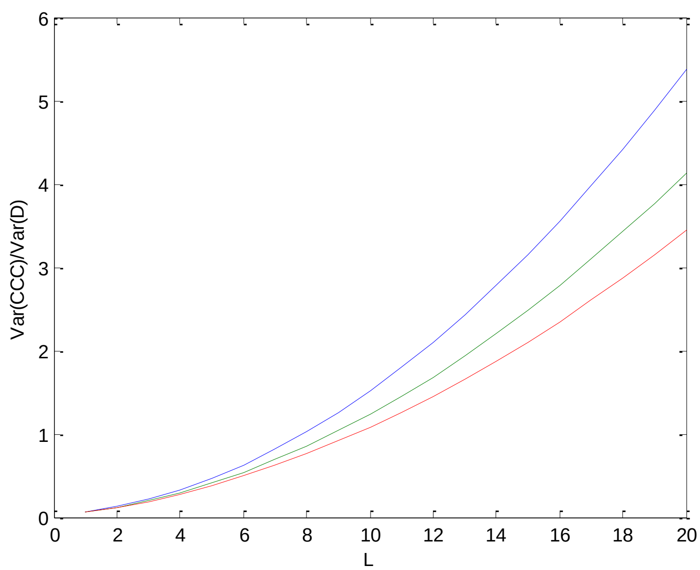


Figure 3-4. Effect of lead time *L* on retailer CFB.

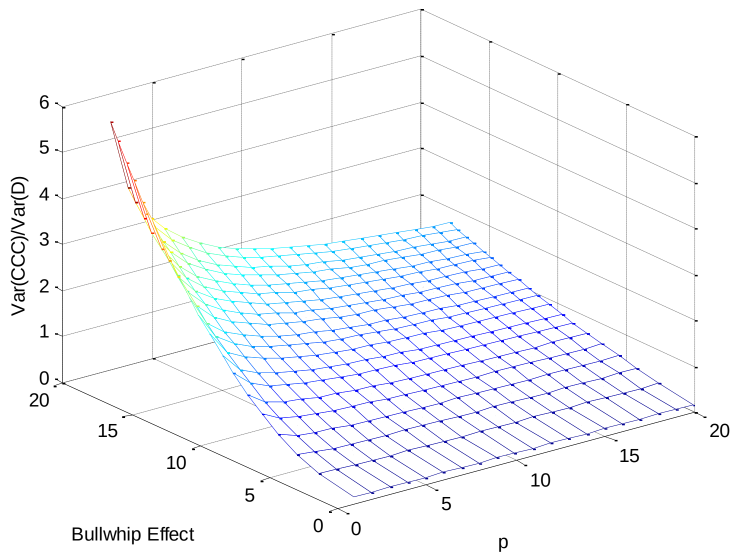


Figure 3-5. *CFB* corresponding to the bullwhip effect and *p* value.

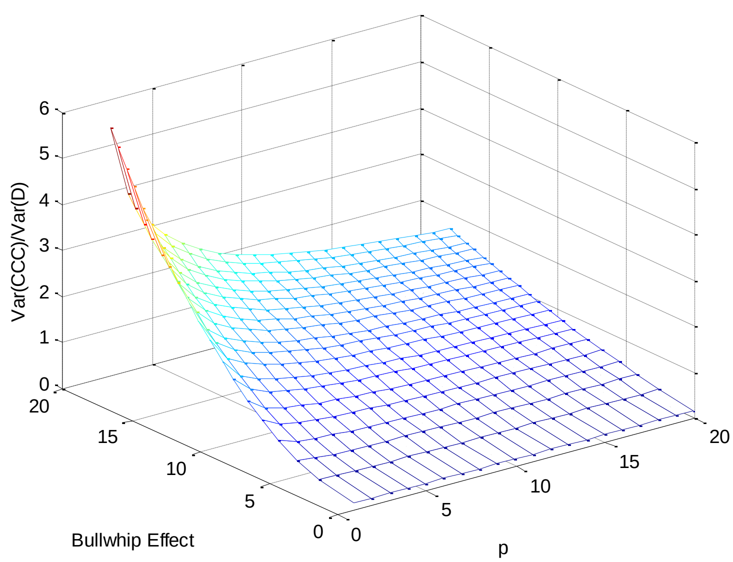


Figure 3-6. *CFB* corresponding to the bullwhip effect and ρ value.

Figure 3-6 depicts the relationship among the *CFB*, the bullwhip effect, and demand correlation ρ. This figure shows that the *CFB* is an increasing function of ρ when ρ is larger than 0 but less than 0.85. For the value of ρ greater than 0.85 but less than 1.0, the *CFB* start decreasing from its peak value. Figure 3-7 shows the *CFB* corresponding to the expected value of inventory *E(I)* and the expected value of demand *E(D)* from Eq. (3.13) and Eq. (3.14). As the value of *E(D)* increases, the *CFB* tends to decrease. On the other hand, it seems that either the expected value of inventory *E(I)* or the expected value of order quantity *E(q)* (graph not shown) does not have significant impact on the *CFB*. Last, the mark up price ratio (*s/c*) can be considered as the coefficient of the equations and the *CFB* is a direct proportional to it.

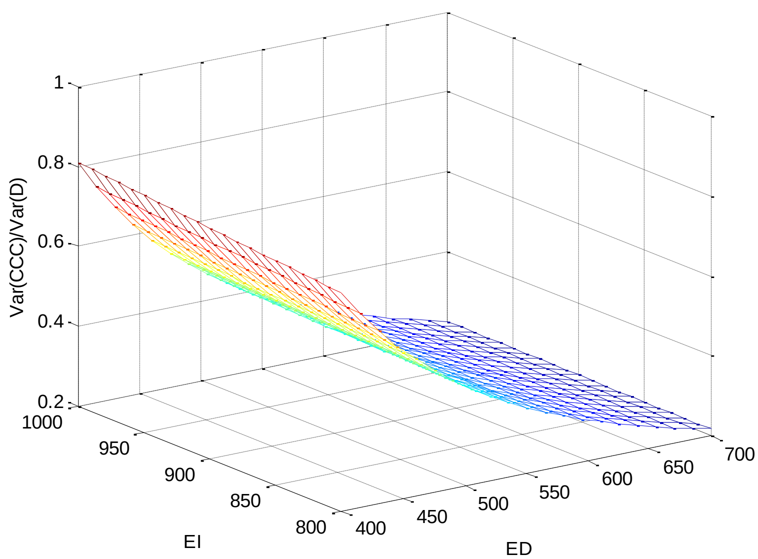


Figure 3-7. *CFB* corresponding to the *E(I)* and *E(D)*.

3.3 Simulation Model

Simulation is an effective tool to analyze the supply chain with uncertainty. The use of simulation in this dissertation mainly focuses on comparing the results from the analytical model and the simulated model. The simulation is used to mimic the activities in a supply chain, which contains a manufacturer, a distributor and a retailer. The model is composed of two parts, a demand forecasting part and a supply chain activity part.

The first part is done by the Visual Basic for Applications (VBA). For the experiment in this dissertation, the moving average forecasting method is applied in the simulation in order to generate the customer demand from the historical data. In the simulation, we assume that all supply chain members apply the same forecasting method throughout each simulation run. Below is the formula of the moving average forecasting method used in the simulation.

$$F_{t+1} = \frac{1}{p} \sum_{i=t-p+1}^{t} Y_i$$

where F_{t+1} is the forecast for period $t + 1$, p is the moving average period, and Y_t is the observed value of sales in period t.

The second part is simulated by the AutoMod (version 12.3 student version). This software runs a discrete event simulation that allows easy configuration. The AutoMod simulates the operational level of the supply chain from the ordering and shipping, to the billing process among supply chain stages.

In the real supply chain, the bullwhip effect may result from several factors. Therefore, while performing the simulation of the supply chain, these factors are varied to gauge how these changes affect the cash flow bullwhip (*CFB*) effect and in turn the cash flow of the member in supply chain. In this simulation, the bullwhip effect is caused by the following factors: mean lead time and its variance, demand forecasting technique and its input parameters, excessively reactive ordering in response to spikes in sales, wide variability in order quantity, ordering policy, and ordering frequency.

For the supply chain activity part, we assume purchasing cost (fixed cost and variable cost), inventory holding cost, and backorder cost are stationary over time. Periodic review and lead time can be changed to see their effects to the cash flow. Storage capacity is assumed to be unlimited. The model logic is shown in Figure 3-8 below. Each supply chain stage, for example, a manufacturer, simply has a single customer, which is a retailer in this example, buying products whenever replenishment is needed. When the order is received, the manufacturer checks its inventory level and then ships the finished goods to the retailer in case of sufficient inventory level, otherwise backorder occurs.

The decision variables, such as the parameters of the forecasting methods and periodic review, are varied depending on the cases. The output data from a simulation is focused on how input parameters such as lead time and the moving average period, which result in the bullwhip effect, impact the variance of the inventory level of each supply chain stage. These output data, which is the inventory level, the order quantity, and the *CFB* of each supply chain stages is monitored. The results of how the input parameters affect the inventory and the *CFB* are discussed in the next section.

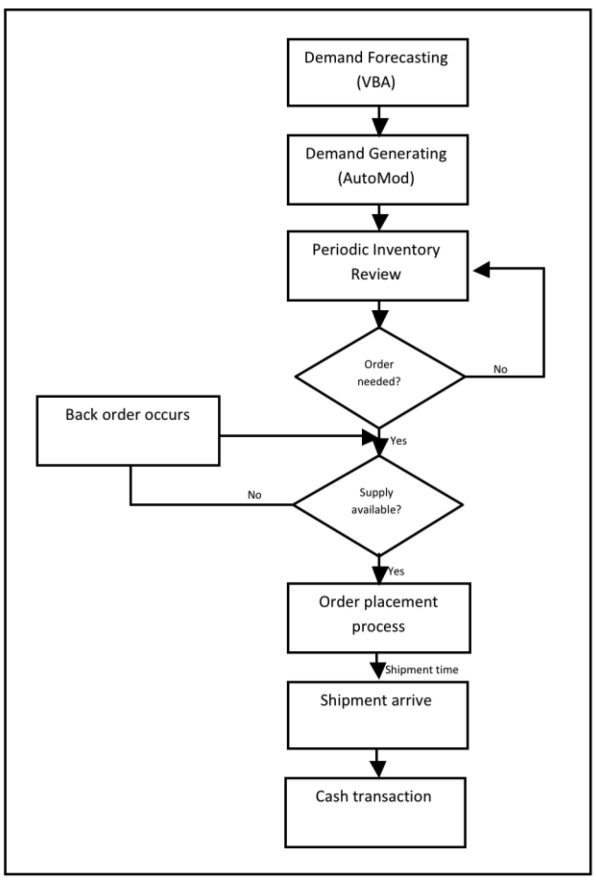


Figure 3-8. Model logic for each supply chain stage.

3.4. Results and Discussion

This section presents the results and interpretations of the analytical model and the simulation from the previous sections. It seeks to analyze the impacts of the bullwhip effect on the inventory and the *CFB* of the supply chain. Note that practical bullwhip measurement based on standard assumptions such as normality and independence may be misleading unless the true sample size is very large. However, the bullwhip effect values in this dissertation are irrespective of the sample size due to the use of the order quantity and the order-up-to policy of the form given in Eq. (3.3) and (3.4) (Hejn Nielsen, 2012).

3.4.1 Impact on the Inventory Variance

Figure 3-9 shows the variance of inventory over the variance of demand ($Var(I_t)/Var(D)$) from the analytical model (Eq. (3.9)) compared with the results from the simulation in Section 3.3 on the increase of the bullwhip effect ($Var(q)/Var(D)$) for z = 0, p = 5 and ρ = 0.5 . Both graphs show that the increase in the bullwhip effect leads to the increase in the variability of inventory.

Some observations from the graph regarding to the increase of the inventory variance can be made from Eq. (3.9) and the simulation. The variability of the inventory increases when the bullwhip effect increases. (Such increases eventually result in the increase in the *CFB* assuming that the other factors in Eq. (3.1) are not changed. We will elaborate it in the next sub section.) The observed variability of retailer's inventory is an increasing function of the bullwhip effect; more specifically, it is approximately a linear fashion as shown in Eq. (3.9) and Figure 3-9.

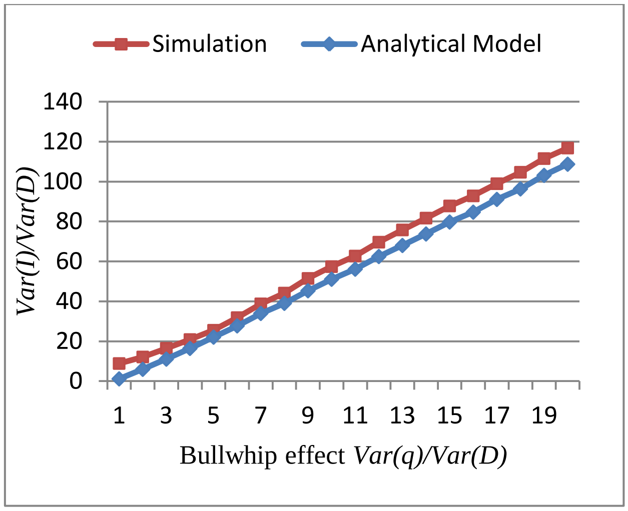


Figure 3-9. Variance of inventory versus bullwhip effect.

Although there are some gaps, approximately 10%, between the analytical model and the simulation, both models still show the same increasing trend. These gaps come from a different assumption that the analytical model is based on the excess inventory can be returned without cost whereas the simulation model is not. This gap becomes large when the bullwhip effect and the lead time L increase. Additionally, the variability of inventory obtained from Eq. (3.9) is calculated directly from the bullwhip effect $Var(q)/Var(D)$, L, p, and ρ, as numerical values plugging into the equation without taking the variance of demand into account. Meanwhile, the variability of inventory obtained from the simulation is determined by individually collecting all actual data such as inventory level, customer demand, and order quantity, which is affected by the demand forecasting error, the moving average period p, the demand correlation ρ, the real condition of the order-up-to policy, and the fixed lead time L during the simulation. These conditions cause the fluctuation in the results.

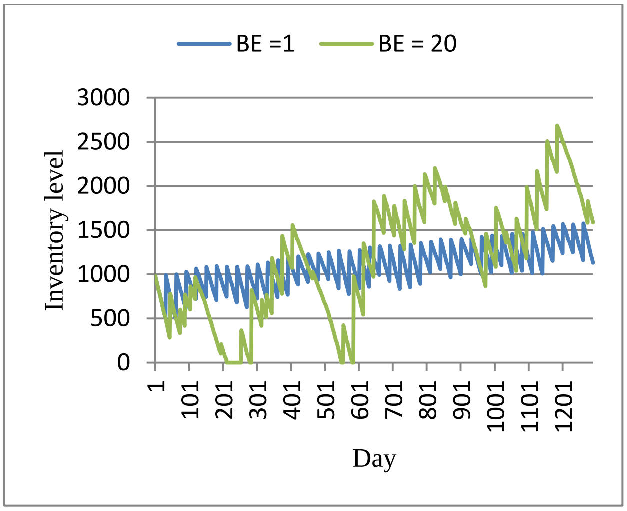


Figure 3-10. Inventory level over time.

The results from the simulation, shown in Figure 3-10, are daily inventory level of the retailer over time at different bullwhip effect (*BE*) values. As seen from the figure, the inventory level of the *BE* = 20 has greater variance than that of the *BE* = 1. Therefore, the more severe the bullwhip effect, the greater the variability of the inventory. This situation can lead to several undesired impacts such as excessive inventory causing higher inventory holding cost and excessive capacity causing swing production plan and higher operational cost.

3.4.2 Impact on the CFB

As shown in Eq. (3.13) and Eq. (3.14), the *CFB* model is a function of nine parameters, which are *Var(q)/Var(D), L, p, ρ, E(I), E(q), E(D), s,* and *c*. First, we would like to show the relationship between the *CFB* and the Bullwhip effect (*Var(q)/Var(D))*. below depicts the impact of the bullwhip effect, for the case which $z = 0$, $p = 5$ and $\rho = 0.5$, on the Cash Flow Bullwhip (*CFB*) or the variance of the cash conversion cycle (*CCC*) over the variance of demand, (*Var(CCC)/Var(D)*).

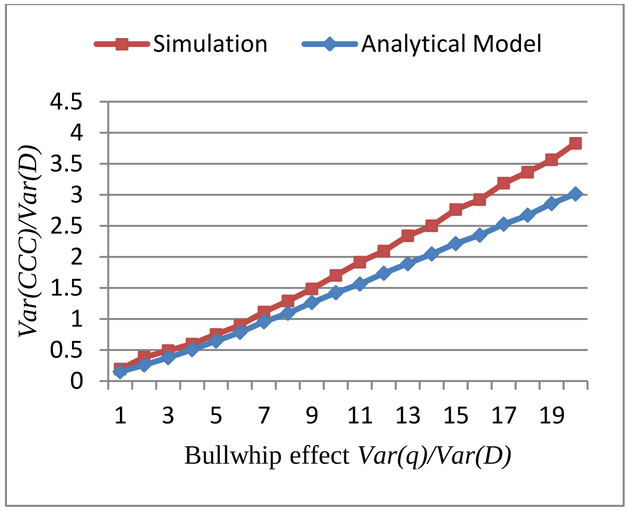


Figure 3-11. Cash Flow Bullwhip (*Var* (*CCC*)/*Var*(*D*)) vs Bullwhip Effect (*Var*(*q*)/*Var*(*D*)).

This figure shows the increase of the *CFB* when the bullwhip effect, *Var(q)/Var(D)*, increases. The results from the analytical model and the simulation represent the lower bound and what really happens in the supply chain, respectively. The graph from both models seem to increase linearly. Even though there is some difference between these two results, they carry the same increasing trend. The difference between these two results comes from the diferent assumption that we mentioned in the previous section and also from the followings. The result from the analytical model is the approximation of the lower bound, therefore, the result from the simulation is larger than that of the analytical model. In addition, there is some error from the approximation of the variance of quotience from the model of Mood et al., 1974. The estimated model of the variance of quotience is underestimated. That is why the results from the analytical model is smaller than that of the simulation model. The average difference between these two models is approximately 14% as shown in Table1. Theorectically, two results can be matched when we can determine the exact value of the bullwhip effect and the results from the following equations are equal.

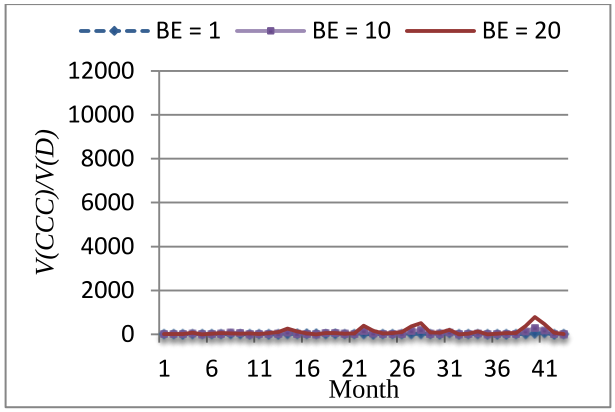
$$\frac{Var\left(\frac{I}{D}\right)}{Var(D)} = \frac{1}{E(D)^2}\left(\frac{V(I)}{V(D)} + \frac{E(I)^2}{E(D)^2}\right) \text{ and } \frac{Var\left(\frac{q}{D}\right)}{Var(D)} = \frac{1}{E(D)^2}\left(\frac{V(q)}{V(D)} + \frac{E(q)^2}{E(D)^2}\right)$$

In case the right hand side of the equations are larger than that of the left hand side, the analytical model is overestimated and vice versa is underestimated.

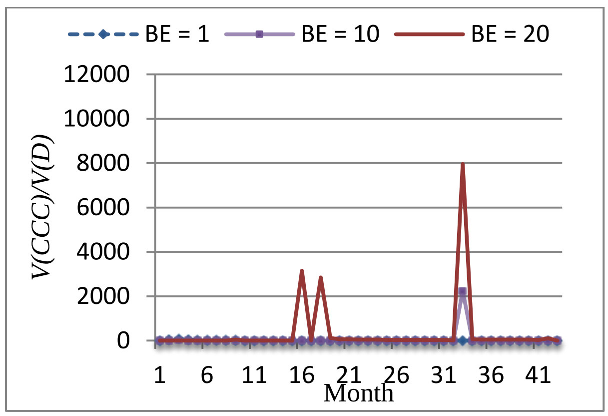
Table 3-1. Comparison of average results of the analytical model with the simulation.

Metric	Simulation	Analytical model	Difference
Inventory Bullwhip	60.50	53.88	10.95%
Cash Flow Bullwhip (*CFB*)	1.87	1.62	13.40%

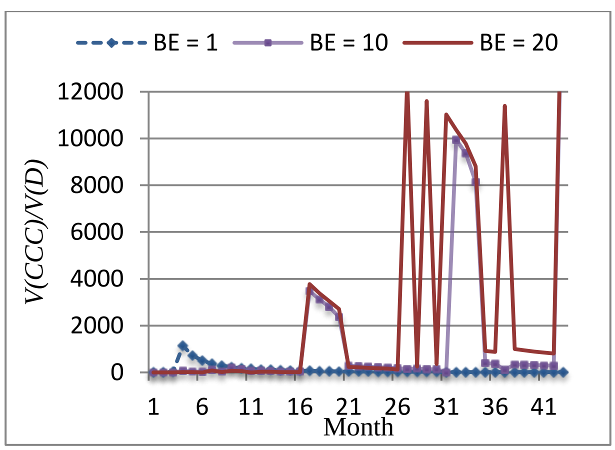
Figure 3-12 shows the simulation results of the *CFB* of the retailer, the distributor, and the manufacturer over time at different bullwhip effect values (*BE*). As seen, the more severe the bullwhip effect, the more *CFB* of the supply chain members. For example, in Figure 3-12 (a), around month 13 and month 22, there is small fluctuation in *CFB*, when the *BE* becomes more severe (*BE* = 10 and *BE* = 20), the effect becomes larger and remains longer than that of the small bullwhip effect (*BE* = 1). The effect also propagates to the upstream members with some time lags due to the lead time and becomes larger than its downstream member as shown in Figure 3-12 (b) and (c).



(a)

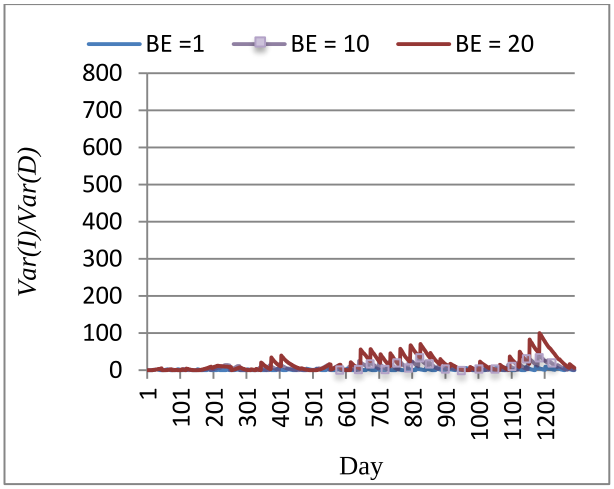


(b)

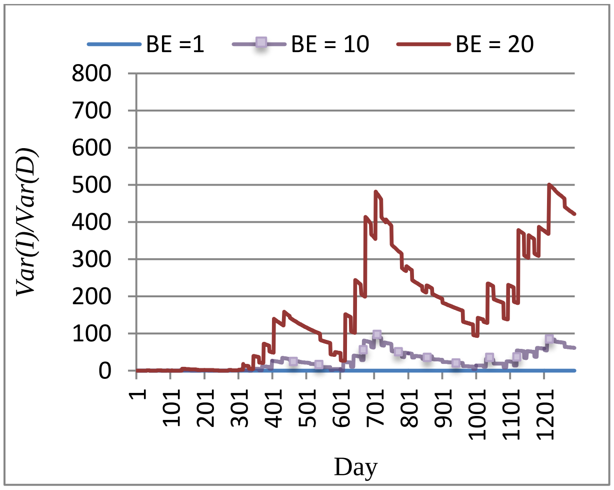


(c)

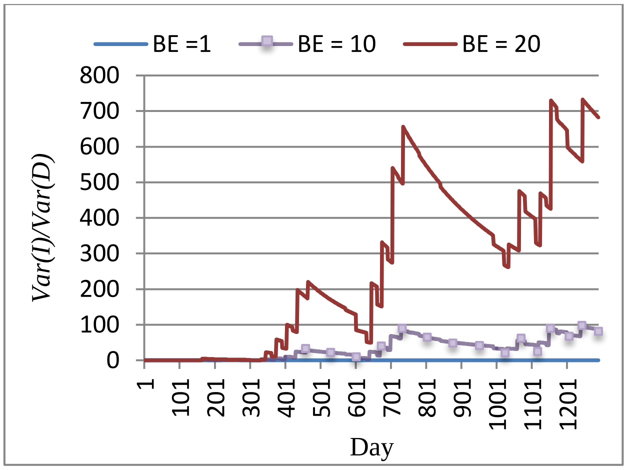
Figure 3-12. *CFB* over time, (a) Retailer, (b) Distributor, and (c) Manufacturer.



(a)



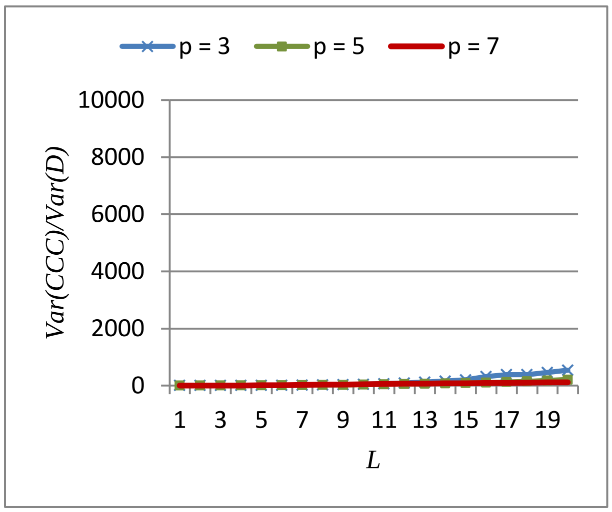
(b)



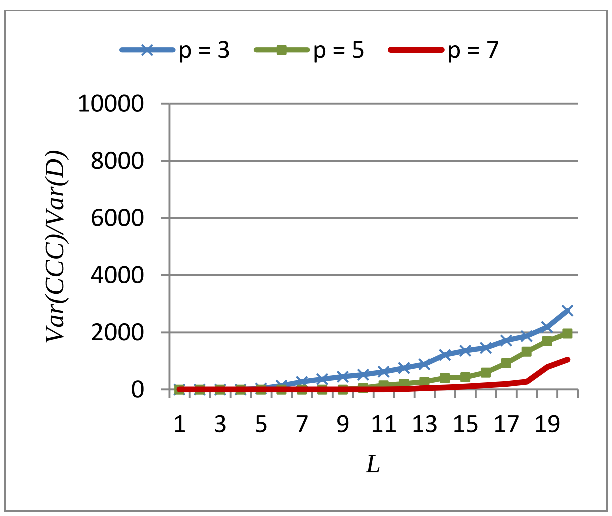
(c)

Figure 3-13. *Var(I)/Var(D)* over time, (a) Retailer, (b) Distributor, and (c) Manufacturer.

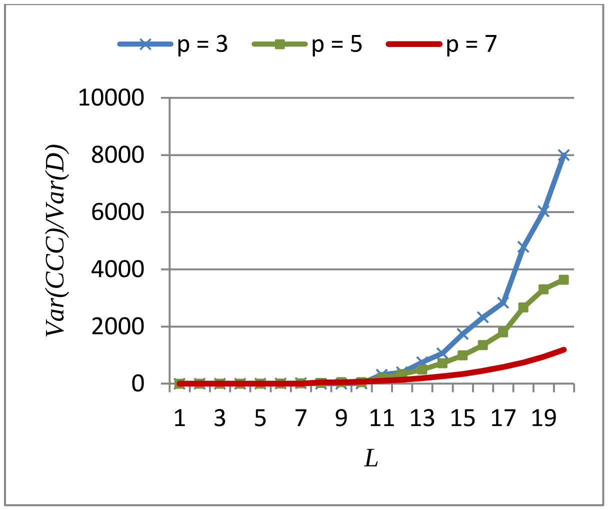
Figure 3-13 shows similar results of *Var(I)/Var(D)* over time when the bullwhip effect becomes more severe for all three supply chain members. Severe bullwhip effect (*BE* = 20) causes larger variance than that of the mild bullwhip effects and this impact is greater in upstream members, the distributor and the manufacturer, respectively. As shown in Eq. (3.13) and Eq. (3.14), the *CFB* contains the term *Var(q)/Var(D) + f(L,p,ρ)*, which is equal to *Var(I)/Var(D)*. Hence, when there is a surge of *Var(I)/Var(D)* , the *CFB* also spikes up around the same time.



(a)



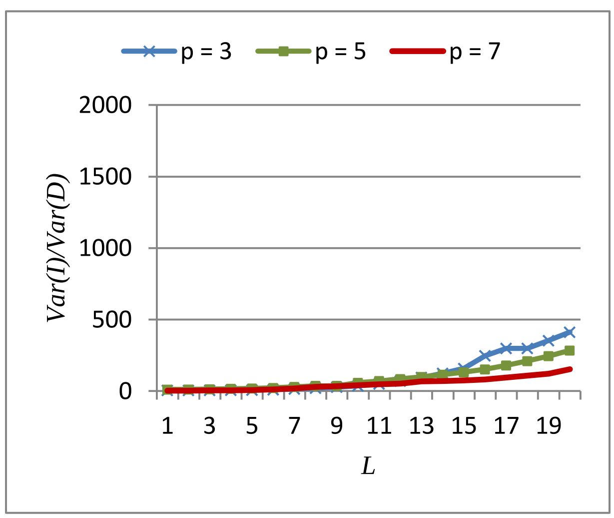
(b)



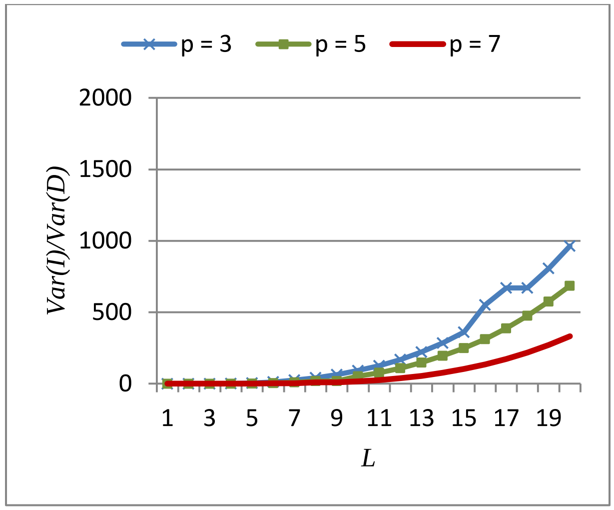
(c)

Figure 3-14. *CFB* over lead time *L*, (a) Retailer, (b) Distributor, and (c) Manufacturer.

Figure 3-14 shows the results from the simulation of the *CFB* over lead time *L* for each supply chain member. The results from the simulation are not as smooth as the result from the analytical model shown in Figure 3-4. However, they increase in a similar way to the analytical model when the lead time *L* increases.



(a)



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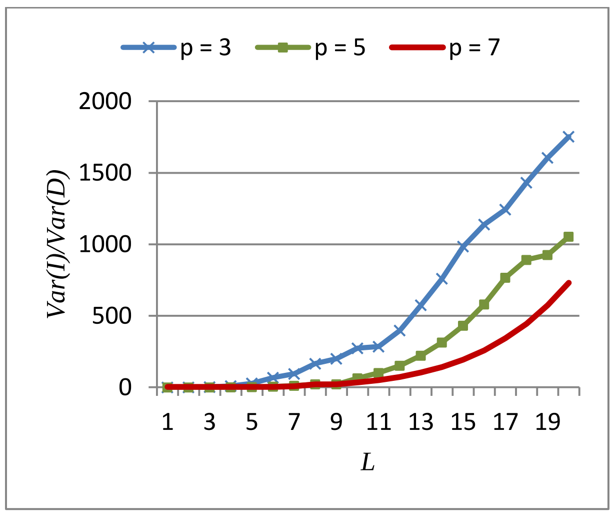


(c)

Figure 3-15. *Var(I)/Var(D)* over lead time *L*, (a) Retailer, (b) Distributor, and (c) Manufacturer.

Figure 3-15 shows the simulation results of *Var(I)/Var(D)* when the lead time *L* increases over time. The graphs do not increase as smooth as that shown in Figure 3-2, nor look much similar to the increasing trend. This fluctuation may come from the random variables such as demand and uncertainty in the supply chain process in the simulation.

As we know from the bullwhip effect, the further upstream the member of the supply chain is, the more severe the bullwhip effect. Therefore, in this dissertation we show that when the bullwhip effect becomes more severe, the *CFB* also increases. Then, in a similar way, the further upstream the member of supply chain is, the more severe the *CFB*, as shown in the simulation results. High *CFB* impacts supply chains in several negative ways such as high working capital and high financial cost. Therefore, in order to mitigate the *CFB,* the firm needs to reduce the bullwhip effect and the lead time L as well as increase the moving average period p.

3.5 Conclusions

In summary, the bullwhip effect is one of the most common problems found in supply chain management. It not only has tremendous impact on supply chain efficiencies, but it also has an impact on the cash flow in many ways. In this study, we particularly focus on modeling and analyzing the *CFB,* which is developed from the cash conversion cycle (*CCC*). Low *CCC* indicates lower financial cost it has to fund its business operation. In order to reduce the *CCC*, one way is to reduce the days-in-inventory. In this study, we mathematically show that the increase in the bullwhip effect results in the increase in the variance of inventory and the *CCC*, leading to the increase in the *CFB* eventually. In addition, the impact of the bullwhip effect becomes more severe for the upstream member of the supply chain. Such phenomenon eventually leads to worse *CFB* in upstream members.

We found that the *CFB* is a function of the following parameters: (1) the bullwhip effect $Var(q)/Var(D)$, (2) the number of observations used in the moving average p, (3) the lead time of the order placement and the arrival of goods L, (4) the demand correlation ρ, (5) the expected value of inventory $E(I)$, (6) the expected value of demand $E(D)$, (7) the expected value of order quantity $E(q)$, and (8) the mark up price ratio s/c. The *CFB* is an increasing function of the bullwhip effect $Var(q)/Var(D)$ and the lead time L, but it is a decreasing function of the moving average period p. This result supports the idea that the large lead time L and small number of moving average period p may produce the bullwhip effect. From the experiment, the bullwhip effect and the lead time L are the most significant factors which impact the *CFB*. Once the bullwhip effect occurs, it causes the increase in the variance of inventory and the *CFB*. In other words, since the amplified order does not reflect the actual demand, which is lower than the order, therefore, the products will sit in the inventory and this will cause the company not only incur the high inventory holding cost but also incur high opportunity cost, financial cost and working capital. In addition, for multi-stages supply chain with centralized demand information, the increase in the variability of the inventory and the *CFB* still exists caused by the bullwhip effect, L, p, and ρ.

Chapter 4

STOCHASTIC FINANCIAL ANALYTICS FOR CASH FLOW FORECASTING

4.1 Introduction

In order to keep businesses running with sufficient working capital and to manage cash flow efficiently, an accurate cash flow forecast is critical. Smaller firms usually incurs higher financing cost due to its credit risk and resource constraints (Baas and Schrooten, 2006). On the other hand, larger firms may find it is difficult to manage its financial assets when it moves to another stage of business life cycle (Mcmahon, 2001). Essentially, cash is the main bloodline of all firms. A firm without profit may be able to survive for a while, but without cash a firm can become insolvent, and risks bankruptcy. Cash flow forecast can serve different purposes, as summarized below.

- ***Treasury Management*** is the process of administering a firm's financial assets and holdings with the goal to ensure its operations are fully funded, make sound financial investments, and reduce financial risks. Effective cash flow forecasting can help optimize liquidity, maximize investment outcomes, and manage currency exposure. For example, a firm may use the forecast to estimate how long its surplus cash is available in order for the treasury to maximize return on investment from this surplus cash.
- ***Working Capital Financing*** can be referred to activities to control and manage short-term assets and short-term liabilities with a goal to ensure that a firm is in the position to meet its immediate obligations and continues to run its operations. Without an accurate forecast, the firm may have unnecessary high liquidity buffer or end up with cash shortage and unable to meet short-term obligations.

By improving cash forecasting, the firm can reduce financing cost and increase efficiency of working capital.

- ***Business Valuation*** is a process to estimate the economic value of a firm, which is critical to establish fair value of a firm. One of the key steps in business valuation is future cash flow forecast, which is then discounted to determine the present value.

Besides the aforementioned purposes, the cash flow forecast may be used for various strategic purposes such as controlling a group of subsidiary companies, and for general management (WWCP, 2012). For example, a firm may use variance between the actual cash flow and forecasted cash flow to diagnose underlying problems and respond in a timely manner.

Two widely used techniques for a short-term cash flow forecast are the receipts and disbursements forecast technique, and a statistical technique. The former technique is to determine all expected cash inflows and cash outflows over the forecast period whereas the latter technique can be a bit more sophisticated in that it considers historical trends. Such statistical techniques include, for example, simple moving average, exponential smoothing, regression analysis, and distribution model (WWCP, 2012). Many commercial software packages for enterprise resource planning (ERP) and treasury management system (TMS) use these statistical techniques for cash flow forecast.

Several research studies have focused on developing and improving cash flow forecasting techniques. In 1950s, the idea of treating cash as products in inventory management was used to forecast and optimize the cash position (Baumol, 1952; Whitin, 1953). However, the assumption of predetermined cash flow in this body of work may not be realistic in many practical situations. Hence, another model was developed to maintain the cash position and minimize transaction fee (Miller and Orr, 1966). Several techniques have also been developed with a specific focus on construction industry (Bromilow and Henderson, 1977; Hudson, 1978; Singh and Woon, 1984;

Kenley and Wilson, 1986; Miskawi, 1989; Khosrowshahi, 1991; Skitmore, 1992; Evans and Kaka, 1998; Skitmore, 1998).

Accounts receivable (AR) aging is a report classifying the length of time since invoices have been sent to various customers. This report is a part of an accounting analytics routinely used by many companies to identify irregular payments and closely monitor overdue accounts. A typical AR aging report consists of many customer accounts in the report and each different customer may have different payment behavior. AR aging and related data can be used in many ways for cash flow forecast. The pioneering efforts of Cyert et al., 1962 used Markov Chain for estimating the allowance for doubtful accounts. This was further improved by incorporating exponential smoothing to AR aging for forecasting cash flow (Cyert et al., 1962; Corcoran, 1978). Later, Kuelen et al., 1981 modified the model and improved the accuracy of the forecast by changing the total balance aging to determine probabilities of the next payments by using the accounts receivable aging. These two techniques outperform other common practices such as moving average and exponential smoothing techniques which are still widely used in practice (Beattie, 2011; WWCP, 2012). Another key development in this area is to model cash flow as a stochastic process to predict cash on-hand for short-term financial planning (Pate-Cornell, 1986; Pate-Cornell et al., 1990).

In general, cash flow can be viewed as a stochastic process which is sequence of random variables that depend upon a number of factors including macro-economic conditions that influence liquidity in the economy, customer payment behavior that can vary from time to time as well by the industry, and dynamics of the particular supply chain itself. For example, one of the prominent and widely studied dynamics of supply chain is the bullwhip effect in inventory. Bullwhip effect in inventory is an undesirable phenomenon in forecast-driven distribution channels where the variance of orders from downstream supply chain gets amplified as it propagates upstream as shown in Figure 4-1 (Kahn, 1987; Lee et al., 1997; Metters, 1997; Cohen, 1998; Lee et al., 2004). Adverse impacts of the bullwhip effect can result in excessive inventory, stock-outs, backorders, production swing, and low utilization of distribution channels. The majority of past research of the bullwhip effect concentrated on key factors, which cause such phenomenon and explanation of its existence (Burbidge, 1989; Sterman, 1989; Lee et al., 1997; Mason-Jones and Towill, 2000). Such key factors are disorganization, lack of communication, order batching, and price variations. Interest in this area was shifted to its impacts and techniques to reduce it (Lee et al., 1997; Chen et al., 1998). Several approaches were developed to mathematically quantify the bullwhip effect (Chen et al., 2000; Kim et al., 2006; Fioriolli and Fogliatto, 2008). Tangsucheeva and Prabhu, 2013 studied the impact of the inventory bullwhip effect on the corresponding cash flow bullwhip (CFB) in a supply chain as shown in the lower graph of Figure 4-1. It needs to be emphasized that the cash flow of a firm not only depends on its immediate customers but potentially also on the system dynamics of its supply chain.

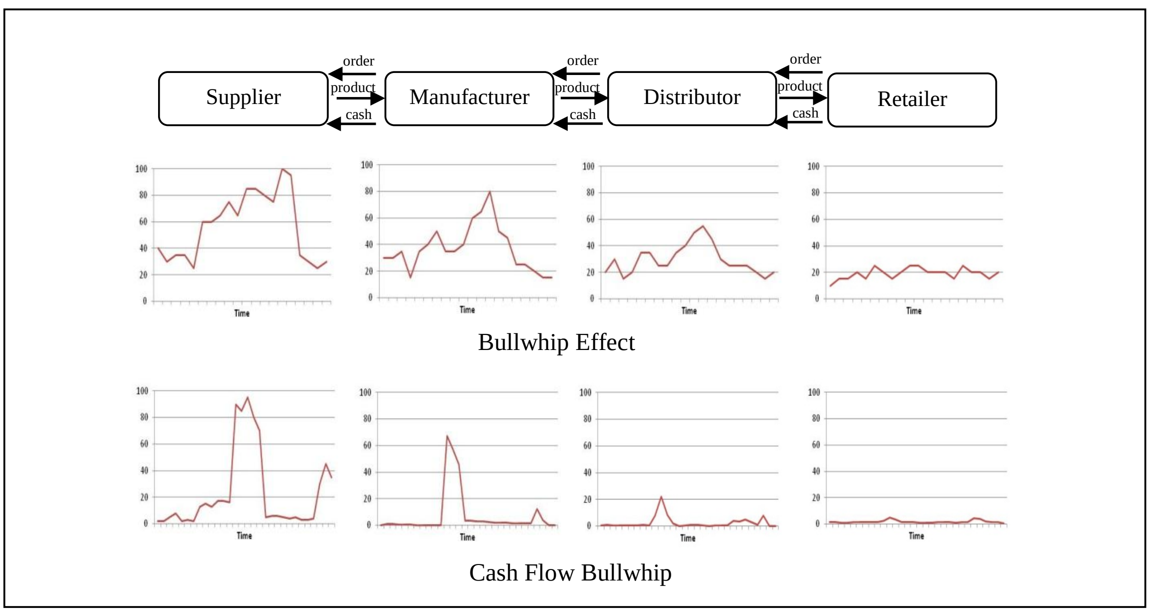


Figure 4-1. Bullwhip Effect and Cash Flow Bullwhip in the supply chain.

There is a need for cash flow models and analytics that more fully utilize the available financial data to improve the accuracy of cash flow forecasts. In this dissertation, we focus on improving the accuracy of cash flow forecasting technique by modeling individual customer payment behavior for determining payment probability of individual invoices by using historic AR aging data. This individual customer level stochastic model is implemented in Excel, which is the tool used by about 70% of companies for accounting and cash flow forecasts (Fuchs, 2011). This development potentially provides users a practical and convenient forecasting tool without having to dwell on the intricacies cash flow forecasting techniques.

The rest of the chapter is organized as follows. The next section presents the stochastic financial analytics model for cash flow forecasting. In Section 4.3, the proposed model is applied and back-tested for a small manufacturing firm. In Section 4.4, the proposed model is applied and tested for a 4-tier supply chain using computer simulation models. Additionally, Section 4.4 also investigates the impact of the bullwhip effect on the cash flow forecasting models. Lastly, in Section 4.5 provides conclusions and directions for future research.

4.2 Stochastic Financial Analytics Model

In this section we propose a stochastic analytics model by building on the work done by Cyert et al., Corcoran, and Pate-Cornell et al. Cyert et al., 1962; Corcoran, 1978; Pate-Cornell et al., 1990. Based on this model we then suggest a computational algorithm for cash flow forecast.

Since the accounts receivable aging is one of the accounting routines, it is quite convenient to use the data from the accounts receivable aging to perform the cash flow forecast. However, the forecasting accuracy of the model could be improved by explicitly modeling the individual customer payment behavior as the concept of the proposed model shown in Figure 4-2 (a). Figure 4-2 (b) illustrates the process of the proposed cash flow forecasting model. It consists of the following six steps: (1) Determine payment probabilities for the next period from AR aging, $\boldsymbol{P}'_{\boldsymbol{P}}$, by estimating the transition probabilities of the Markov chain, (2) Determine payment probabilities from customer specific payment behavior, $\boldsymbol{P''_P}$, using a Bayesian model , (3) Create a transition matrix, (4) Apply exponential smoothing, (5) Perform the cash flow forecast, and (6) Update the new actual data.

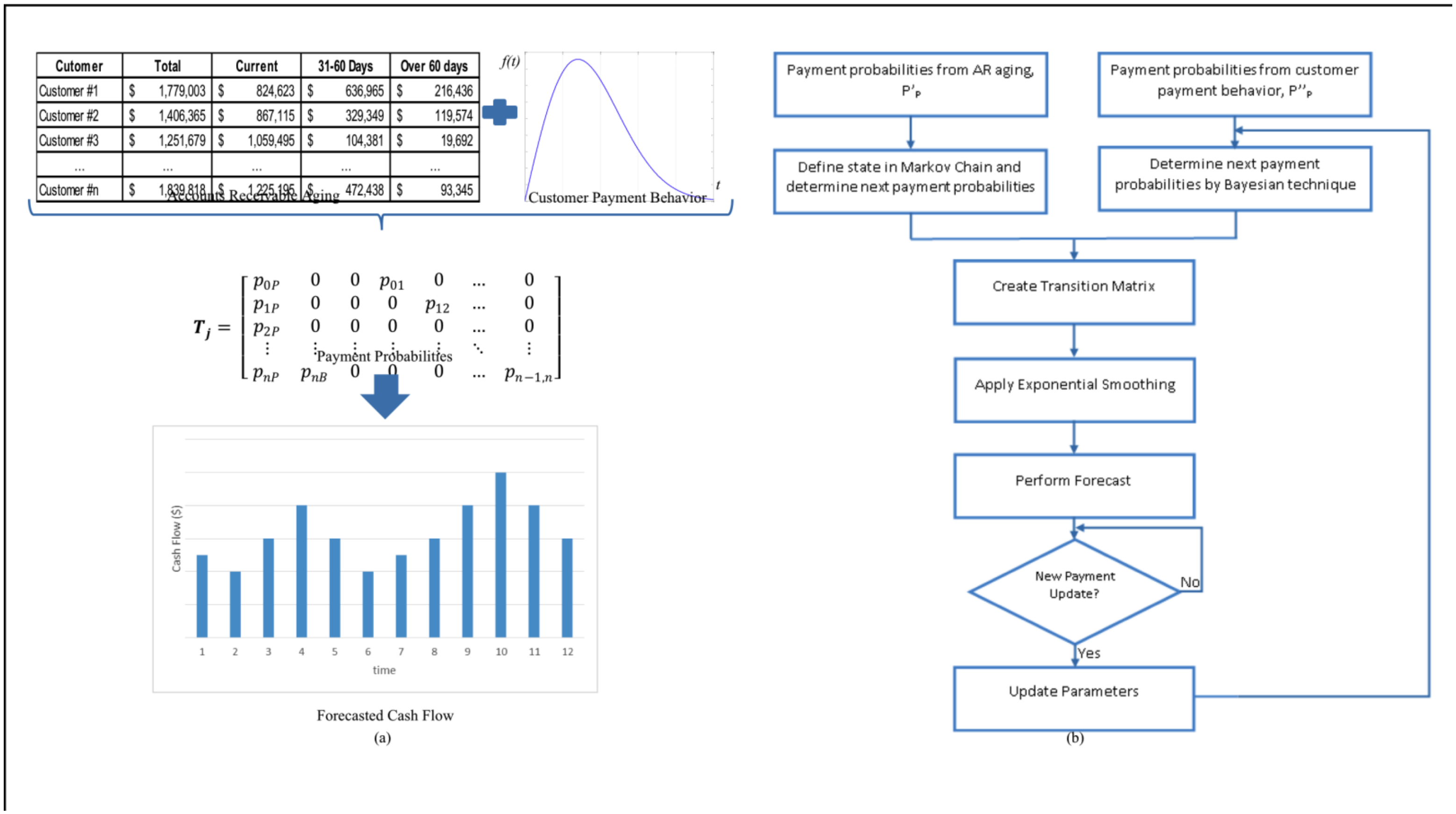


Figure 4-2. (a) Model concept and (b) process of the proposed cash flow forecast.

The first step is to define the Markov chain states in order to prepare a transition matrix to calculate the cash flow forecast. To formulate the Markov Chain, the transient states and the absorbing states are defined from the accounts receivable aging. Let $S_t = [0, 1, 2, \ldots, n]$ be a set of transient states where *0*, *1*, *2*, and *n* represent the AR aging states in the Markov Chain. Typically in practice there are 4 states corresponding to 0-30 days, 31-60 days, 61-90 days, and over 120 days, as summarized in Table 4-1 below.

Table 4-1. State definition.

AR aging	State
0-30 day	0
31-60 days	1
61-90 days	2
…	…
Last AR aging range	n

Additionally, two absorbing states are defined as $S_a = [P, B]$ where *P* and *B* represent *Paid* and *Bad Debt* states. An absorbing state is a Markov chain state, which can be reached from any state. However, once it enters the absorbing state, it cannot leave this state. Table 4-2 shows an example of the accounts receivable aging and defined states of the Markov chain in parenthesis.

Table 4-2. Accounts Receivable Aging and Bad Debt.

Month (period *j*)	Total	Account Receivable Aging (state *i*)					
		Current *(0)*	30days *(1)*	60days *(2)*	90days *(3)*	Over120days *(4)*	Bad debt *(B)*
November (11)	1,609,405	702,560	623,810	101,470	36,480	127,085	16,270
December (12)	2,655,895	1,418,250	530,080	528,400	75,295	103,870	12,328
January (1)	2,287,070	896,140	505,465	669,020	76,330	140,115	4,556
February (2)	2,109,595	829,995	586,160	450,745	66,365	107,700	13,590

The accounts receivable aging in Table 4-2 can be converted into the accounts receivable aging matrix (***R***).

$$\boldsymbol{R} = \begin{bmatrix} r_{10} & r_{11} & r_{12} & \cdots & r_{1n} & r_{1B} \\ r_{20} & r_{21} & r_{22} & \cdots & r_{2n} & r_{2B} \\ r_{30} & r_{31} & r_{32} & \cdots & r_{3n} & r_{3B} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{j0} & r_{j1} & r_{j2} & \cdots & r_{jn} & r_{jB} \end{bmatrix} \tag{4.1}$$

where $r_{j,i}$ is the amount of the accounts receivable aging in period *j* at *State i* and $r_{j,B}$ is the amount of bad debt in period *j*.

Figure 4-3 shows the state transition diagram and the notation for corresponding transition probabilities. The process starts from *State 0*, indicating the current charges (0-30 days) in the AR aging, and transitions to *State P* if the invoice is *Paid* while in this state; or transitions to *State 1* (31-60 days) if the invoice is not paid. The probability of an invoice getting paid in State 0 is modeled as the transition probability from *State 0* to *State P* and denoted by p_{0P}. Therefore probability from *State 0* to *State 1* can be modeled as $p_{01} = 1 - p_{0P}$. The process continues to the last state *n* where it can move to either *State P* (*Paid*) or *B* (*Bad Debt*). Since *P* and *B* are the absorbing states, $p_{PP} = 1$ and $p_{BB} = 1$ in this model.

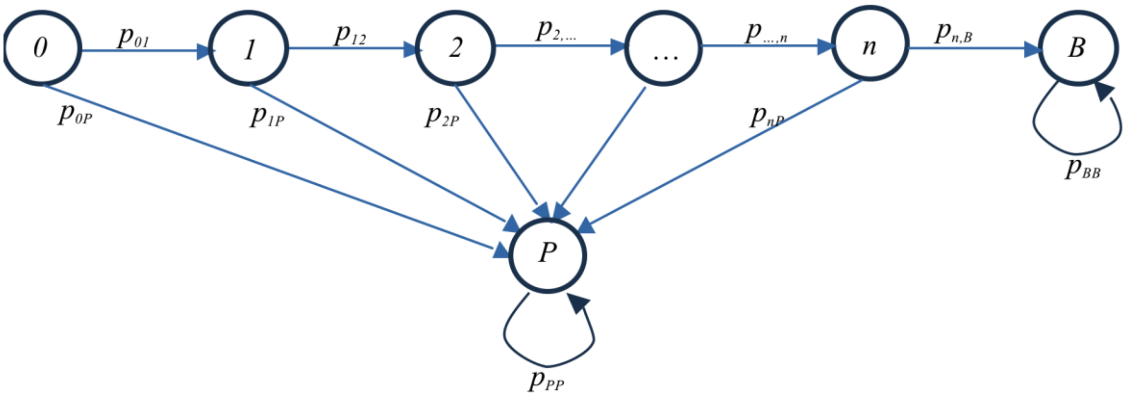


Figure 4-3. Markov chain state diagram.

Then determine the remaining 2(*n*+*1*) payment probabilities. Let $\boldsymbol{P_P}$ be the *(n+1)×2* matrix of the payment probability, which is the transition probability from any transient state *n* to *State P*.

$$\boldsymbol{P_P} = \beta \boldsymbol{P'_P} + (1-\beta)\boldsymbol{P''_P} \tag{4.2}$$

where β is a weighting parameter, $\boldsymbol{P'_P}$ is the payment probability matrix from the AR aging and $\boldsymbol{P''_P}$ is the payment probability matrix from the customer payment behavior. Here the payment probability matrix $\boldsymbol{P_P}$ is modeled to consist of two parts: the first part, $\boldsymbol{P'_P}$ models the aggregate payment behavior across all customers of the firm in the recent past. This can be expected to model any macro-economic trend that has influenced payment behavior across customers and other trends across the industry. The second part, $\boldsymbol{P''_P}$ models payment behavior of a specific customer at the individual invoice level based on all know payment history of the customer.

The weighting parameter β can be obtained from back testing using historic data of individual customers. The value of β is selected to provide the most accurate forecasting result. Furthermore, β may vary from customer to customer, and over time. For a new customer, when there is no payment history, β would be set to unity thereby treating the customer as a "typical" customer. However, for a longstanding customer, β could be smaller thereby increasing the weight for customer-specific payment behavior. The proposed model in Eq. (4.2) can be viewed as a convex combination of Corcoran's model and Pate-Cornell et al.'s model. If β = 1, Eq. (4.2) becomes Corcoran's model and if β = 0, it becomes Pate-Cornell et al.'s model.

$\boldsymbol{P'_P}$ can be determined from the changes in the accounts receivable aging from the previous period to the current period Corcoran, 1978.

$$\boldsymbol{P'_P} = \begin{bmatrix} p'_{0P} & 0 \\ p'_{1P} & 0 \\ \vdots & \vdots \\ p'_{nP} & p'_{nB} \end{bmatrix} \tag{4.3}$$

where

$$p'_{iP} = \left(r_{j,i} - r_{j+1,i+1}\right)/r_{j,i} \tag{4.4}$$

$$p'_{nB} = r_{j,B}/r_{j-1,n} \tag{4.5}$$

where p'_{iP} is the payment probability from *State i* to *State P (Paid)*, p'_{nB} is the transition probability from *State n* to *State B (Bad Debt)*, $r_{j,i}$, $r_{j+1,i+1}$, $r_{j,B}$, and $r_{j-1,n}$ are the elements from the accounts receivable aging matrix ($\boldsymbol{R}$) in Eq. (4.1).

$\boldsymbol{P'_P}$ models aggregate behavior across all customers of the firm, however, it does not include customer specific payment behavior. Therefore, the second step, taking these factors from distinct customers into account by modeling $\boldsymbol{P''_P}$ can be expected to improve the forecasting accuracy. Assume that information of the customer payment behavior such as the number of days that the invoices have been sent out to customers and the minimum number of days that individual customer makes a payment are available.

$$\boldsymbol{P''_P} = \begin{bmatrix} p''_{0P} & 0 \\ p''_{1P} & 0 \\ \vdots & \vdots \\ p''_{nP} & 0 \end{bmatrix} \tag{4.6}$$

To compute the probability of a given invoice that will be paid between time t_0 and $t_0 + \Delta t$, the distribution of the payment time of each customer can be modeled conveniently as Weibull distribution (Pate-Cornell, 1986; Pate-Cornell et al., 1990).

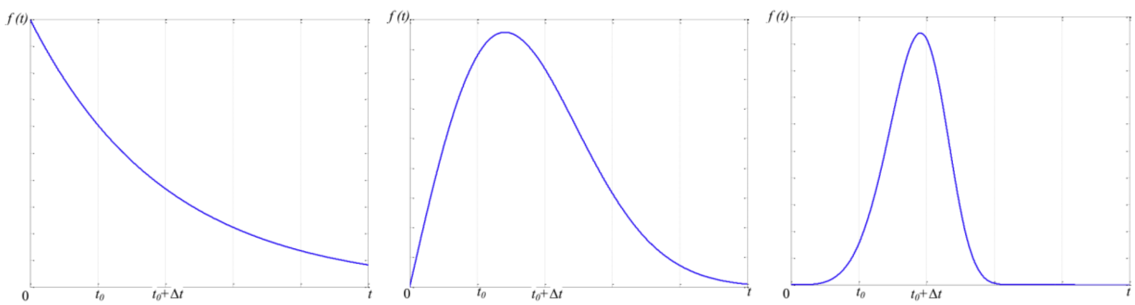


Figure 4-4. Weibull distribution with the shape parameter k = 1, 2, and 5, respectively.

Figure 4-4 shows Weibull distribution with the scale parameter λ = 1 and the shape parameter k = 1, 2, and 5, respectively. Notice that for the shape parameter k = 2, this particular distribution provides a linearly increasing rate, which rises to a peak quickly and then decreases over time. This model of individual customer payment behavior can be used to characterize payment lead-time that is used to set payment terms, probability of payment delay beyond the expected lead-time, and probability of bad debt. In this dissertation, individual customer is modeled using a Weibull distribution with the shape parameter k = 2

$$f(x;\lambda,k) = \begin{cases} \frac{2x}{\lambda^2} e^{-\left(\frac{x}{\lambda}\right)^2}, & x \geq 0 \\ 0 & , x < 0 \end{cases} \tag{4.7}$$

where $\lambda > 0$ is the scale parameter.

The probability that an invoice will be paid during time $t_0 + \Delta t$ can be written as

$$\begin{aligned} p''_{iP} &= p[t \leq t_0 + \Delta t | t \geq t_0] \\ &= \frac{p[t_0 \leq t \leq t_0 + \Delta t]}{p[t \geq t_0]} \\ &= \frac{[F_j(t_0 + \Delta t - t_b) - F_j(t_0 - t_b)]}{[1 - F_j(t_0 - t_b)]} \end{aligned}$$

where p''_{iP} is the payment probability from the customer payment behavior of *State i*, t_b is the time a given invoice is billed to the customer, and $F_j(.)$ is the cumulative distribution for the payment time of the accounts receivable. From the cumulative distribution function, we can determine three possible cases of the payment probability.

Case I: $t_0 - t_b \geq \gamma$

$$p''_{iP} = 1 - \exp\left\{-\frac{[2(t_0 - t_b - \gamma)(\Delta t) + (\Delta t)^2]}{\hat{\lambda}_j^2}\right\} \tag{4.8}$$

Case II: $t_0 - t_b \leq \gamma$ and $t_0 - t_b + \Delta t \geq \gamma$

$$p''_{iP} = 1 - \exp\left\{-\frac{[(t_0 - t_b + \Delta t - \gamma)^2]}{\hat{\lambda}_j^2}\right\} \tag{4.9}$$

Case III: $t_0 - t_b + \Delta t \leq \gamma$

$$p''_{iP} \ = p[t \leq t_0 + \Delta t | t \geq t_0, \lambda] = 0 \tag{4.10}$$

where γ is the minimum payment time and $\widehat{\lambda_j}$ is the estimate scale parameter which characterizes customer's payment lead time (Pate-Cornell et al., 1990). These model parameters can be estimated periodically or even after invoice payment.

Once the payment probabilities $\boldsymbol{P'_P}$ and $\boldsymbol{P''_P}$ are obtained, $\boldsymbol{P_P}$ can be determined by Eq. (4.2) and then $\boldsymbol{P_D}$, the *(n+1)* square matrix of the delay payment probability, can be determined by

$$\boldsymbol{P_D} = \begin{bmatrix} 0 & p_{01} & 0 & \dots & 0 \\ 0 & 0 & p_{12} & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & p_{n-1,n} \end{bmatrix}$$

where

$$p_{i,i+1} = 1 - p_{iP} \tag{4.11}$$

$p_{i,i+1}$ is the probability of the amount that will age from *State i* to *State i+1* and p_{iP} is the payment probability from matrix $\boldsymbol{P_P}$. Now it is ready to construct a transition matrix for period *j*, $\boldsymbol{T_j}$**,** from the probabilities in step 1 and step 2 as shown in Eq. (4.12).

$$\boldsymbol{T_j} = \ [\boldsymbol{P_P} \quad \boldsymbol{P_D}]$$

$$\boldsymbol{T_j} = \begin{bmatrix} p_{0P} & 0 & 0 & p_{01} & 0 & \dots & 0 \\ p_{1P} & 0 & 0 & 0 & p_{12} & \dots & 0 \\ p_{2P} & 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{nP} & p_{nB} & 0 & 0 & 0 & \dots & p_{n-1,n} \end{bmatrix} \tag{4.12}$$

Then, we use an exponential smoothing technique to smooth the data and then perform the forecast of the next period cash flow. The formula of the exponential smoothing technique can be written as

$$\overline{\boldsymbol{A_j}} = \alpha \boldsymbol{T_j} + (1-\alpha)\overline{\boldsymbol{A_{j-1}}} \tag{4.13}$$

where

$\overline{\boldsymbol{A_j}}$ is the estimated transition matrix or exponentially smoothed matrix for period *j*

α is the smoothing factor

$\boldsymbol{T_j}$ is the transition matrix for period *j*

The smoothing factor α can be identified by back-testing historic data of a firm to provide the most accurate forecast.

Then, the cash inflow forecast and bad debt can be obtained by Corcoran, 1978; Tangsucheeva et al., 2013

$$F_{j+1} = R_j\overline{\boldsymbol{A_j}}$$

$$F_{j+1} = [r_{j0} \quad r_{j1} \quad r_{j2} \quad \cdots \quad r_{jn}] \begin{bmatrix} p_{0P} & 0 & 0 & p_{01} & 0 & \cdots & 0 \\ p_{1P} & 0 & 0 & 0 & p_{12} & \cdots & 0 \\ p_{2P} & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{nP} & p_{nB} & 0 & 0 & 0 & \cdots & p_{n-1,n} \end{bmatrix} \tag{4.14}$$

where

F_{j+1} is the forecast vector of dollar amount in each state for period $j+1$

R_j is the vector of actual accounts receivable aging report in period *j* from matrix ***R***

The final step checks the updated actual payment in order to update the payment probabilities and then repeat Step 2 through 6 to dynamically update the cash flow forecast. This individual customer payment forecast is repeated for all customers with outstanding invoices to compute the firm level forecast.

4.3 Enterprise Level Application

To illustrate the application of the proposed model, we use AR aging data from a small manufacturing firm in Pennsylvania, which is shown in Table 4-2 along with states of the Markov chain in parenthesis. For the sake of simplicity, assume that the firm needs a close attention to a particular customer. Based on the past payment history in 5 months of this customer, the average payment time is 25 days and the minimum payment time that this customer pays his invoice is 20 days. This individual customer is modeled by fitting the corresponding AR aging data to identify the Weibull distribution parameters. By using Minitab, given the lowest AD (Anderson-Darling Statistic) = 0.304 and the significant LRT P value = 0.000, Weibull distribution with a shape parameter k = 2 is selected.

We follow the process presented in Section 4.2. First, define the Markov chain state from the accounts receivable aging as shown in Table 4-2 and then convert it to a matrix $\boldsymbol{R}$ as shown below.

$$\boldsymbol{R} = \begin{bmatrix} 702{,}560 & 623{,}810 & 101{,}470 & 36{,}480 & 127{,}085 & 16{,}270 \\ 1{,}418{,}250 & 530{,}080 & 528{,}400 & 75{,}295 & 103{,}870 & 12{,}328 \\ 896{,}140 & 505{,}465 & 669{,}020 & 76{,}330 & 140{,}115 & 4{,}556 \\ 829{,}995 & 586{,}160 & 450{,}745 & 66{,}365 & 107{,}700 & 13{,}590 \end{bmatrix}$$

Then, determine the first part of the probabilities of the next payment, $\boldsymbol{P'_P}$**,** using Eq. (4.3) and Eq. (4.4).

$$p'_{iP} = (r_{j,i} - r_{j+1,i+1})/r_{j,i}$$

$$p'_{0P} = (r_{1,0} - r_{2,1})/r_{1,0} = (896{,}140-586{,}160)/896{,}140 = 0.3459$$

Continue in the same manner to determine the rest of the payment probabilities, the transition probabilities of *State i* to *State P (Paid)*, for i = 0, 1, 2, 3, 4. Use Eq. (4.5) to determine the probability of bad debt.

$$p'_{nB} = r_{j,B}/r_{j-1,n}$$

$$p'_{4B} = r_{2,B}/r_{1,4} = 13{,}590/140{,}115 = 0.0970$$

To determine the second part of Eq. (4.2), $\boldsymbol{P''_P}$, the scale parameter λ of the Weibull distribution must be determined first. Let X be the random variable of the payment time in the Weibull distribution. The mean of this distribution for the shape parameter k = 2 can be determined by

$$\bar{X}(n) = \gamma + \lambda\left(\frac{\sqrt{\pi}}{2}\right)$$

Hence, an estimate of scale parameter λ can be obtained as follows:

$$\hat{\lambda}(n) = \frac{2(\bar{X}(n) - \gamma)}{\sqrt{\pi}}$$

where $\hat{\lambda}(n)$ is an estimate of scale parameter λ, $\bar{X}(n)$ is an average of the n past observations of the payment time of the customer, and γ is the minimum payment time. Based on the given information, the scale parameter of the Weibull distribution is

$$\hat{\lambda}(n) = \frac{2(25-20)}{\sqrt{\pi}} = 5.6419$$

Given that the invoice was sent out to the customer 27 days ago and Δt = 3 days, $t_0 - t_b =$ $27\ days \geq \gamma$ which is 20 days, the probability of receiving the payment from this customer is determined by Eq. (4.8) as follows:

$$p''_{iP} = 1 - \exp\left\{-\frac{[2(t_0 - t_b - \gamma)(\Delta t) + (\Delta t)^2]}{\hat{\lambda}_j^2}\right\}; t_0 - t_b \geq \gamma$$

$$p''_{0P} = 1 - \exp\left\{-\frac{[2(27-20)(3) + (3)^2]}{5.6419^2}\right\} = 0.7986$$

Hence, the payment probability from *State 0* to *State P* and the probability of the invoice will be postponed to the next aging can be obtained from Eq. (4.2) and Eq. (4.11), respectively.

$$p_{iP} = \beta p'_{iP} + (1-\beta) p''_{iP}$$

$$p_{0P} = 0.2 \times 0.3459 + 0.8 \times 0.7986 = 0.7081$$

$$p_{i,i+1} = 1 - p_{iP}$$

$$p_{01} = 1 - p_{0P} = 1 - 0.7081 = 0.2919$$

The weighting parameter β = 0.2 is obtained from back testing using historic data of a firm. Once p_{0P} and p_{01} are obtained, next, repeat the processes to determine probabilities for

State 1, 2, 3, and *4*. Once we got all probabilities from all accounts receivable aging of this customer, we can construct the transition matrix $\boldsymbol{T}$. Figure 4-5 illustrates an example of transition matrix $\boldsymbol{T_2}$ (for month 2, February), which contains probabilities of cash inflows at each state transition.

$$\boldsymbol{T_2} = \begin{bmatrix} 0.7081 & 0 & 0 & 0.2919 & 0 & 0 & 0 \\ 0.8211 & 0 & 0 & 0 & 0.1789 & 0 & 0 \\ 0.9801 & 0 & 0 & 0 & 0 & 0.0199 & 0 \\ 0.8800 & 0 & 0 & 0 & 0 & 0 & 0.1200 \\ 0.8922 & 0.0970 & 0 & 0 & 0 & 0 & 0.0108 \end{bmatrix}$$

Figure 4-5. Transition Matrix $\boldsymbol{T_2}$.

Then, the exponential smoothing technique is applied to smooth the data. Assume the exponentially smoothed matrix for January $\overline{\boldsymbol{A_1}}$ is known, the exponentially smoothed matrix for February $\overline{\boldsymbol{A_2}}$ can be determined by Eq. (4.13). The alpha value of 0.8 is back tested from historic data in order to calculate $\overline{\boldsymbol{A_2}} = 0.8\boldsymbol{T_2} + (0.2)\overline{\boldsymbol{A_1}}$. Once, the matrix $\overline{\boldsymbol{A_2}}$ is obtained, the estimated vector of dollar amount for March can be determined by $F_3 = R_2\overline{\boldsymbol{A_2}}$, using Eq. (4.14).

$$\overline{\boldsymbol{A_1}} = \begin{bmatrix} 0.6010 & 0 & 0 & 0.3984 & 0 & 0 & 0 \\ 0.6638 & 0 & 0 & 0 & 0.3362 & 0 & 0 \\ 0.5892 & 0 & 0 & 0 & 0 & 0.4107 & 0 \\ 0.4400 & 0 & 0 & 0 & 0 & 0 & 0.5600 \\ 0.5439 & 0.0000 & 0 & 0 & 0 & 0 & 0.4561 \end{bmatrix}$$

$$\overline{\boldsymbol{A_2}} = \begin{bmatrix} 0.6867 & 0 & 0 & 0.3133 & 0 & 0 & 0 \\ 0.7896 & 0 & 0 & 0 & 0.2104 & 0 & 0 \\ 0.9019 & 0 & 0 & 0 & 0 & 0.0980 & 0 \\ 0.7920 & 0 & 0 & 0 & 0 & 0 & 0.2080 \\ 0.8225 & 0.0776 & 0 & 0 & 0 & 0 & 0.0998 \end{bmatrix}$$

$$F_3 = [1{,}580{,}481 \quad 8{,}358 \quad 0 \quad 260{,}054 \quad 123{,}305 \quad 44{,}209 \quad 24{,}559]$$

Figure 4-6. Exponentially Smoothed Matrix $\overline{\boldsymbol{A_1}}$ and $\overline{\boldsymbol{A_2}}$, and Forecasted vector F_3.

The estimated vector F_3 forecasts that the firm will have \$1,580,481 collected cash from the accounts receivable and \$8,358 in bad debt whereas the actual cash collection and bad debt are \$1,540,900 and \$6,275, respectively. Figure 4-6 illustrates Matrix $\overline{\boldsymbol{A_1}}$, Matrix $\overline{\boldsymbol{A_2}}$, and the forecasted cash collection as well as bad debt. The difference of the cash collection between the forecast and the actual value of the whole year is 2.31% whereas the difference of the bad debt is 14.79%. (We do not have information of detailed bad debt, so, we will not focus on the bad debt.)

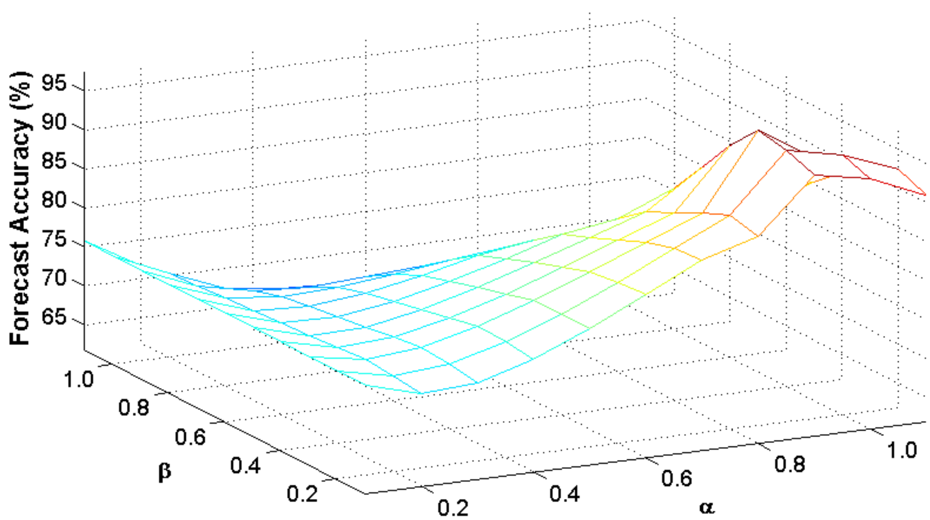
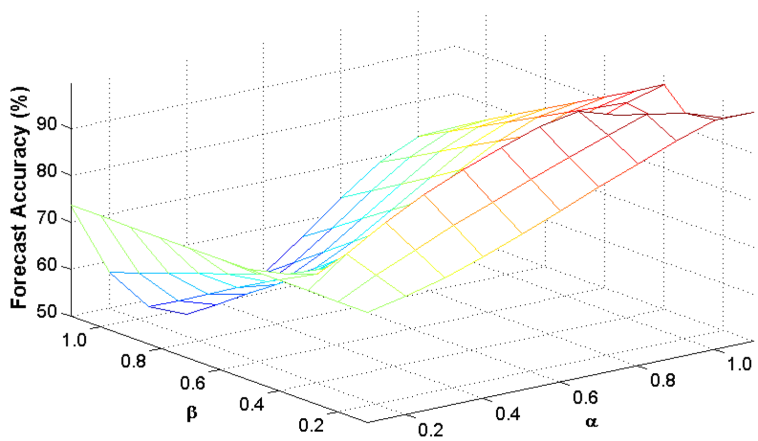
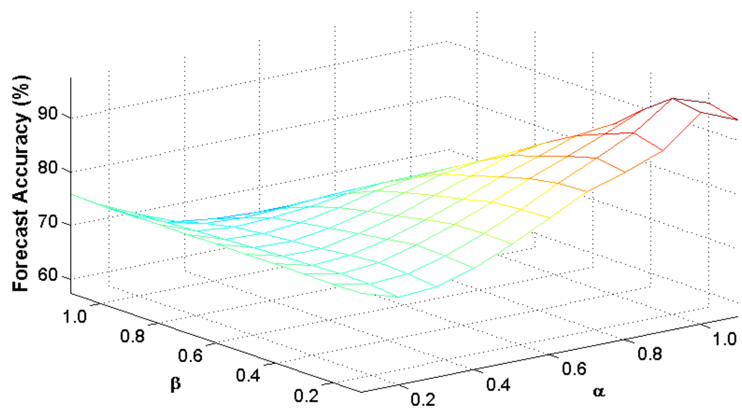


Figure 4-7. Forecasting accuracy and $\beta$and α value.

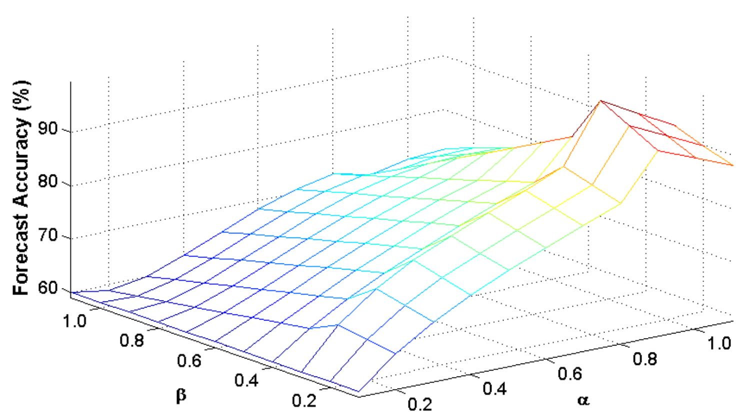
Figure 4-7 shows relationship among forecasting accuracy, the smoothing factor, and the weighting parameter β. α and β are back tested from historic data: AR aging and customer specific payment behavior. The selected values are chosen from the ones that provide the most accurate average forecasting accuracy, which in this example, α = 0.8 and β = 0.2.


Forecast Accuracy (%)
50
60
70
80
90
1.0
0.8
0.6
0.4
0.2
β
0.2
0.4
0.6
0.8
1.0
α

(1)


Forecast Accuracy (%)
60
70
80
90
1.0
0.8
0.6
0.4
0.2
β
0.2
0.4
0.6
0.8
1.0
α

(2)


Forecast Accuracy (%)
60
70
80
90
1.0
0.8
0.6
0.4
0.2
β
0.2
0.4
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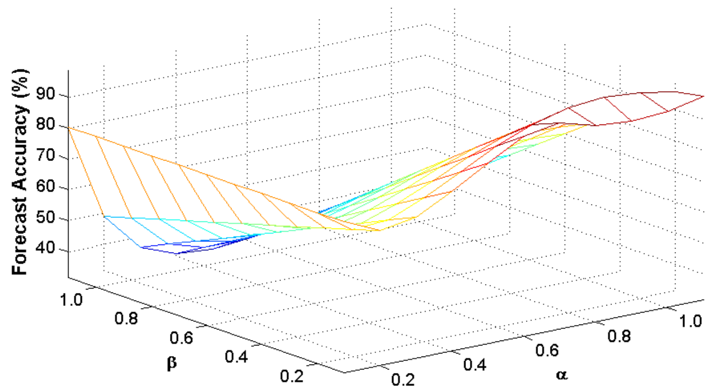
0.8
1.0
α

(3)



(4)

Figure 4-8. Sensitivity analysis of α and β for quarter 1, 2, 3, and 4.

Figure 4-8 shows the sensitivity analysis of how the forecasting accuracy changes over α and β values for this specific customer. First observation is the highest forecasting accuracy of each quarter is in the range of α between 0.7 and 0.9 and in the range of β between 0.1 and 0.3. Since these parameters are quite sensitive to customer payment behavior, for the best of α and β value, they can be varied from industry to industry, customer to customer, and time to time. If the past data shows that α and β do not change with time then these parameters can be estimated infrequently. Second, α value is the exponential smoothing factor of the forecasting. The larger of the α is, the more weight on the most recent data is used to calculate the forecast. When $\alpha \to 1$ provides high forecasting accuracy, it implies that the payment behavior of this customer changes significantly from its past payment. Therefore, using the outdated data or the further past data may decrease the forecasting accuracy. Third, for this customer, the forecasting accuracy is higher when β value decreases. This implies that the payment behavior of this customer is quite different from those of other customers. In other words, when $\beta \to 0$, the payment behavior of this customer is distinct from prevailing industry practice. Since the proposed forecasting model is a convex combination of Corcoran's model and Pate-Cornell et al.'s model, when $\beta \to 0$, the proposed forecasting model relies more on Pate-Cornell et al.'s model which incorporates customer specific payment behavior. On the other hand, when $\beta \rightarrow 1$, the proposed model relies more on Corcoran's model, which represents the aggregate customer behavior.

Let us consider another customer, customer #1, to see how the proposed model performs the forecasting. This customer makes approximately 31 transactions on average per month. The average payment time of this customer is 73 days and the minimum payment time is 48 days. Table 4-3 shows the actual cash flow compared with the forecasted cash flow, and the percent difference between the actual and the forecasted cash flow determined by five models: (1) the proposed model, (2) Corcoran's model to forecast cash flow from the accounts receivable aging, (3) Pate-Cornell's model, (4) the moving average technique, and (5) the exponential smoothing technique. For the proposed model, we extracted the accounts receivable aging of this customer and followed the process mentioned in Section 4.2 to forecast the cash flow corresponding to this customer. For Corcoran's model and Pate's model, please see detailed explanation in Corcoran, 1978, and Pate-Cornell et al., 1990, respectively. For the moving average technique, we use 3 periods moving average to estimate the cash flow. Finally, for the exponential smoothing technique, the smoothing factor is assigned to 0.8.

Table 4-3. Cash Flow and Forecasting Difference of Customer #1.

Month	Actual Cash Flow ($)	Proposed Model		Corcoran's Model		Pate-Cornell's Model		Moving Average		Exponential Smooth	
		Forecast ($)	Difference (%)	Forecast ($)	Difference (%)	Forecast ($)	Difference (%)	Forecast ($)	Difference (%)	Forecast ($)	Difference (%)
1	21,547	21,922	1.74	22,920	6.37	24,890	15.52	31,604	46.67	31,604	46.67
2	21,145	21,045	0.47	25,244	19.39	21,679	2.53	26,576	25.68	23,558	11.41
3	50,890	50,616	0.54	36,974	27.35	45,018	11.54	24,765	51.34	21,627	57.50
4	56,767	59,468	4.76	60,752	7.02	67,564	19.02	31,194	45.05	45,037	20.66
5	43,881	43,506	0.85	50,029	14.01	54,883	25.07	42,934	2.16	54,421	24.02
6	44,776	43,634	2.55	48,354	7.99	42,664	4.72	50,512	12.81	45,989	2.71

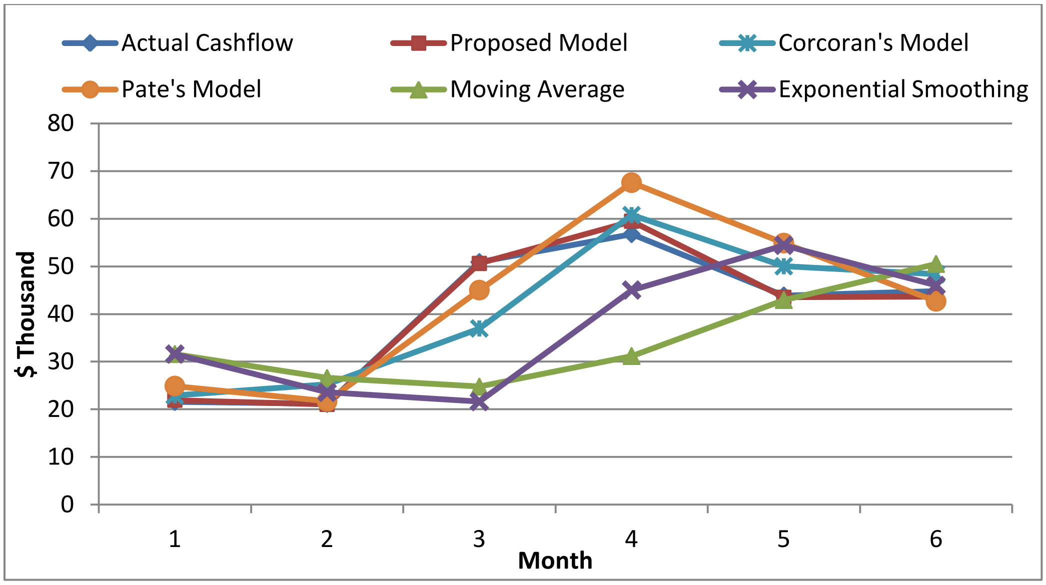


Figure 4-9. Cash flow from actual cash flow and various forecasting models.

Figure 4-9 illustrates how a customer's actual cash flow changes during month 1 through month 6 compared to 5 other cash flow forecasting models. This figure shows how well the forecasting models can keep track of the actual cash flow. While the cash flows forecasted by the moving average technique and the exponential smoothing technique have some time lag to follow the change of the actual cash flow, Corcoran's model and Pate's model do not have this issue. Thus, they can keep track the actual cash flow better than the former two models. However, they still cannot keep track of the sudden change since the model determines the payment probability based on only previous payment regardless of customer payment behavior. Consequently, when the next payment drastically changes, the estimation from Corcoran's model and Pate's model have big gaps. On the other hand, the proposed model takes the customer payment behavior into account. It considers the average payment time and the minimum payment time of the customer to calculate the payment probability. Hence, the proposed model is able to keep track of the actual cash flow as if they are the same line.

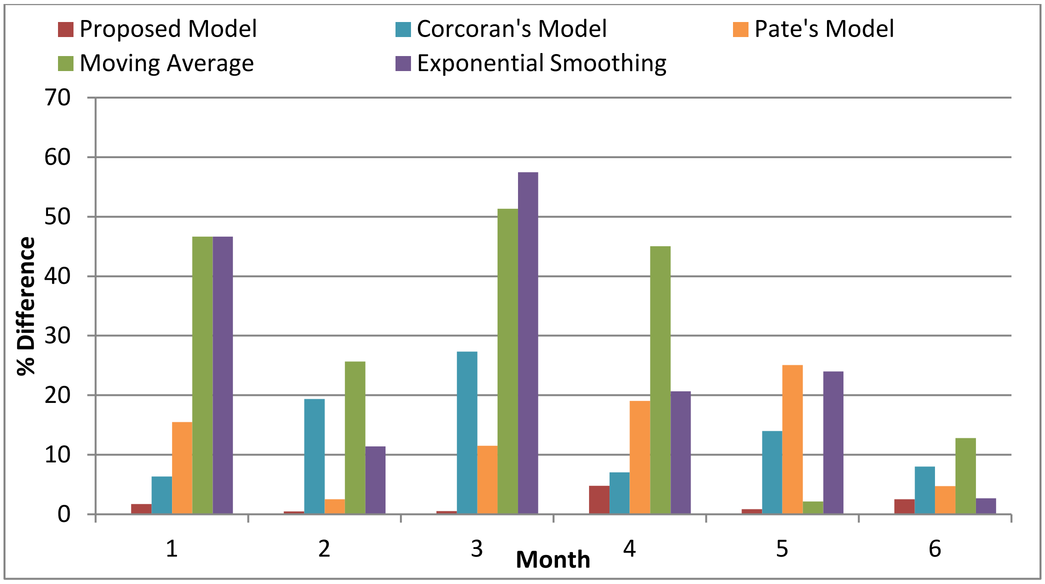


Figure 4-10. Percent difference of forecasting models.

Figure 4-10 depicts percent difference between the actual cash flow and the forecasting models. As shown in the figure, the propose model has the smallest difference among four models followed by Corcoran's model.

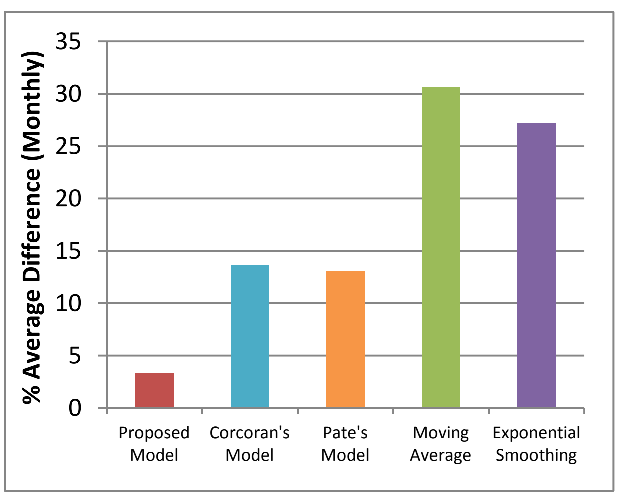


Figure 4-11. Average percent monthly difference.

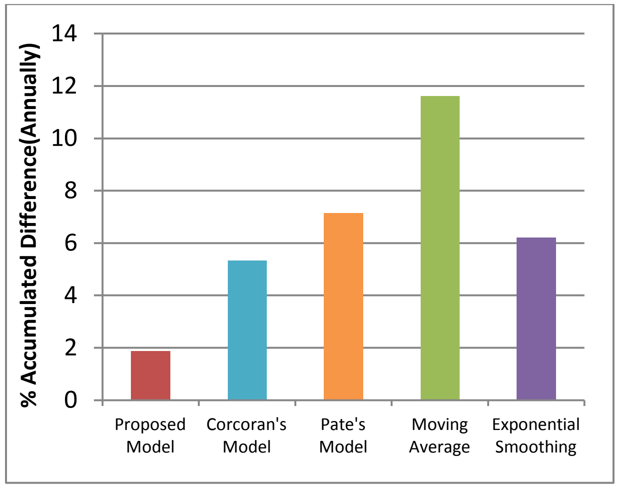


Figure 4-12. Accumulated percent annually difference.

Figure 4-11 and Figure 4-12 show forecasting performance of each forecasting model on an average monthly basis and average annually basis, respectively. In Figure 4-11, the proposed model performs the best with approximately 3% difference on average while the difference from Corcoran's model and Pate's model are almost 15% on average, and the differences from the moving average and the exponential smoothing technique are approximately 30% on average. The percent difference of the last two models is relatively a lot larger than that of the proposed model since these two models use only the average values from the historical data to smooth out fluctuation. This is why these two models cannot capture the change of the actual cash flow in a timely manner.

The accumulated differences shown in Figure 4-12 are, of course, smaller than those in Figure 4-11 since the overestimate and underestimate are offset. Still the results shown in Figure 4-12 are consistent to those shown in Figure 4-11, the proposed model still performs the best among these five methods.

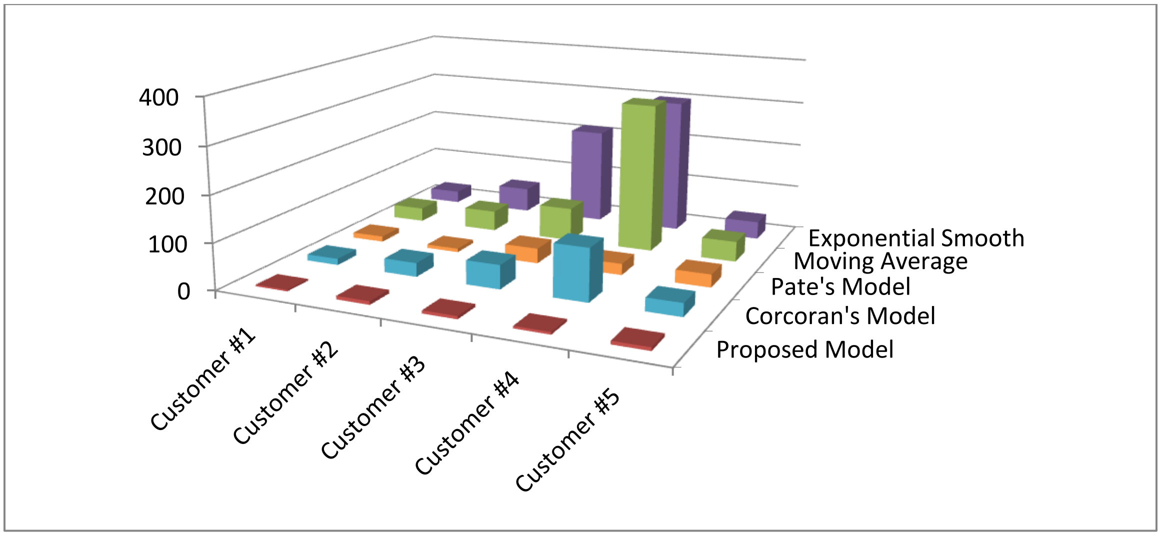


Figure 4-13. Forecasting differences (monthly).

Next, the experiment was conducted to five sample customers over 12 months, who have different payment behavior, to see how the forecasting models perform in different scenarios. In this case, the customers have different number of days in accounts receivable, average payment time, and minimum payment time. Customer #1 represents customers who have an average payment time longer than 60 days with α = 0.8 and β = 0.2, customer #2 and customer #3 represent customers who have an average payment time approximately between 45-60 days with α = 0.6 and β = 0.3, customer #4 and customer #5 represent customers who have an average payment time approximately 31-50 days and less than 30 days with α = 0.9 and β = 0.1, respectively.

Figure 4-13 shows the differences of the five forecasting models compared to the actual cash flow of each customer. The proposed model still performs the best by providing the smallest difference among five forecasting models. The percent difference of the proposed model on average from five customers is approximately 6% while the percent differences of the other models are much greater as shown in Figure 4-14. The proposed model is able to adjust the parameters relating to the payment behavior so that it can manage to keep the gap of the difference small compared to other models, which do not have this parameters involve in the forecast.

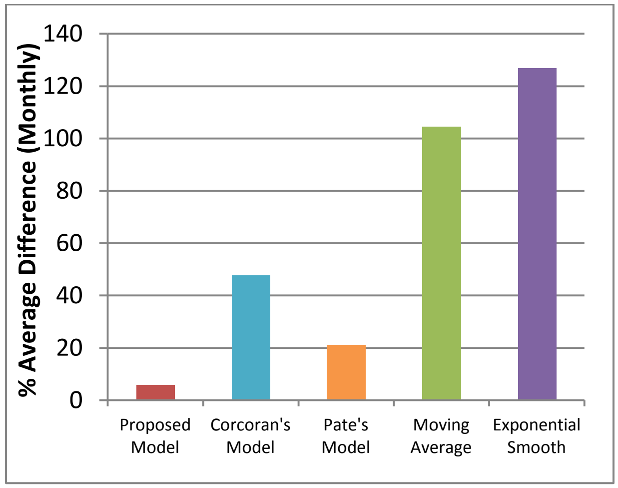


Figure 4-14. Average forecasting difference (monthly).

Notice that the difference of the moving average and the exponential smoothing techniques of customer #3 and customer #4 are large because there is so large fluctuation at some points in time that these two models create big gaps between the actual cash flow and the forecasted cash flow.

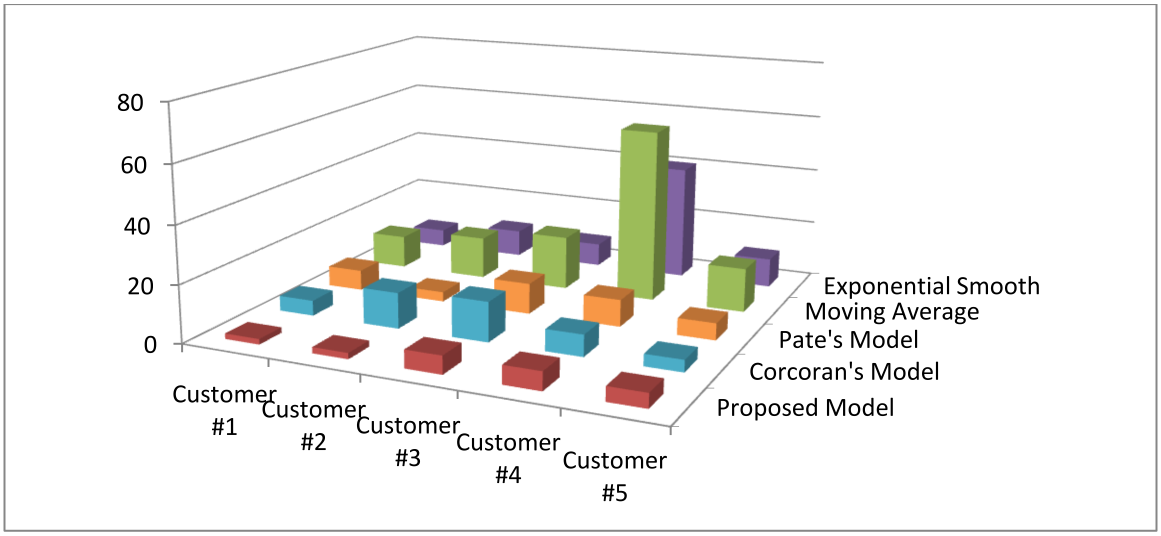


Figure 4-15. Forecasting difference (annually).

Figure 4-15 and Figure 4-16 show the performance of the five forecasting techniques in aggregated level (annually). The results here are still consistent with the results of the monthly basis shown in Figure 4-13 and Figure 4-14. The forecasting models perform better because of the aggregated amount offset. The proposed model performs the best among five methods with an average of approximately 4% different from the actual cash flow while Corcoran’s model has a difference of approximately 9%, Pate's model has a difference approximately 7%, the moving average technique has a difference of approximately 25%, and the exponential smoothing has a difference of approximately 15%.

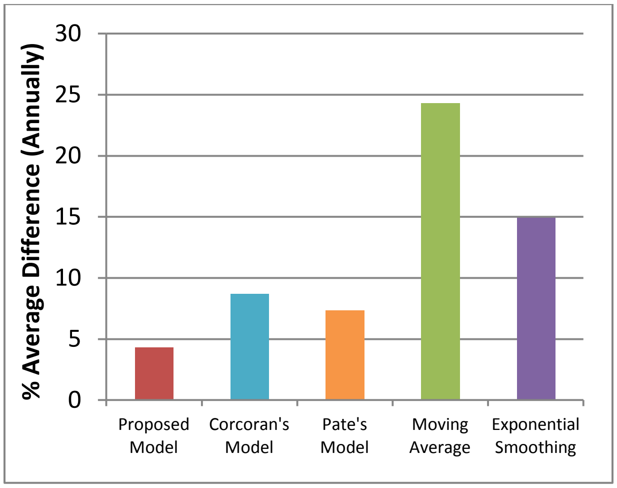


Figure 4-16. Average forecasting difference (annually).

4.4 Supply Chain Level Application

This section discusses how the proposed model performs in the supply chain and how it is impacted by one of the most common problems in supply chain, the bullwhip effect. The following section defines the bullwhip effect (BWE), the inventory bullwhip (IBW), and the cash flow bullwhip (CFB), respectively.

4.4.1 Bullwhip Effect (BWE)

One of the bullwhip effect models that is widely accepted was developed by Chen et al., 2000. The model starts with a simple supply chain, which contains a single manufacturer and a single retailer. The customer demand is in the form of Kahn, 1987

$$D_t = d + \rho D_{t-1} + \mu_t \tag{4.15}$$

where d is a nonnegative constant, ρ is a correlation parameter satisfying $|\rho|<1$, and μ_t is an independent and identically normally distributed random variable with zero mean and variance σ^2. Assume the lead time L is fixed and the order-up-to policy is applied; hence, the order quantity (q_t) can be written as

$$q_t = y_t - y_{t-1} + D_{t-1} \tag{4.16}$$

where y_t is the order-up-to point which is estimated from the observed demand as

$$y_t = \hat{D}_t^L + z\hat{\sigma}_{e,t}^L \tag{4.17}$$

where $\hat{D}_t^L$ is an estimate of the mean lead time demand using a simple moving average, z is a constant to meet a desired service level, and $\hat{\sigma}_{e,t}^L$ is an estimate of the standard deviation of the L period forecast error. The bullwhip effect of a simple supply chain can be obtained from the following (Chen et al., 2000):

$$\frac{Var(q)}{Var(D)} \geq 1 + \left(\frac{2L}{p} + \frac{2L^2}{p^2}\right)(1 - \rho^p) \tag{4.18}$$

The bound is tight when $z = 0$.

More detail of this model is discussed in Chen et al., 2000.

4.4.2 Inventory Bullwhip (IBW)

From the previous section, the inventory bullwhip model is extended to see how the bullwhip effect impacts the inventory. For a simple supply chain containing a single retailer and a single manufacturer, the lower bound of the inventory bullwhip can be written as Tangsucheeva and Prabhu, 2013

$$\frac{Var(I)}{Var(D)} \geq \frac{Var(q)}{Var(D)} + f(L, p, \rho) \tag{4.19}$$

where I is inventory level. The bound is tight when $z = 0$.

$$\begin{aligned} f(L, p, \rho) = \frac{1}{(1-\rho)^2} \Bigg[& \left(\frac{L}{p}\right)^2 [2\rho^{p+2} - 2\rho^{p+1} + 2\rho^p - (p+2)\rho^2 + 2\rho + p - 2] \\ & - 2\left(\frac{L}{p}\right) [\rho^{L+p+2} - \rho^{(L+2)} - \rho^{(p+2)} + \rho^{p+1} - \rho^p + \rho^2 - \rho + 1] \\ & + (2\rho^{L+2} - (L+2)\rho^2 + L) \Bigg] \end{aligned} \tag{4.20}$$

More detail of this model has been discussed elsewhere in Tangsucheeva and Prabhu, 2013.

4.4.3 Cash Flow Bullwhip (CFB)

The model above is further extended for the cash flow bullwhip in order to determine the effect of the bullwhip effect and the inventory bullwhip to the cash flow (Tangsucheeva and Prabhu, 2013). CFB is the variance of the cash conversion cycle (CCC) over the variance of demand. CCC is the most widely used measurement for cash flow. It contains inventory (days inventory outstanding), cash inflow (days sales outstanding), and cash outflow (days payable outstanding) and this is the reason why CCC is used to represent the cash flow.

CFB for a simple supply chain can be obtained from the following formulae (Tangsucheeva and Prabhu, 2013):

Case I: $D \leq I$

$$\frac{Var(CCC)}{Var(D)} \cong \frac{(365)^2}{E(D)^2}\left[\left(\frac{s}{c}\right)^2\left(\frac{Var(q)}{Var(D)} + f(L,p,\rho) + \frac{E(I)^2}{E(D)^2}\right) + \left(\frac{Var(q)}{Var(D)} + \frac{E(q)^2}{E(D)^2}\right)\right] \tag{4.21}$$

Case II: $D > I$

$$\frac{Var(CCC)}{Var(D)} \cong \frac{(365)^2}{E(D)^2}\left[\left(\left(\frac{s}{c}\right)^2 + 1\right)\left(\frac{Var(q)}{Var(D)} + f(L,p,\rho) + \frac{E(I)^2}{E(D)^2}\right) + \left(\frac{Var(q)}{Var(D)} + \frac{E(q)^2}{E(D)^2}\right)\right] \tag{4.22}$$

where *s* is the sales price per unit, *c* is the unit cost, *E(D)* is the expected value of demand, and *E(I)* is the expected value of inventory.

Further information regarding to this model is presented in Tangsucheeva and Prabhu, 2013.

4.4.4 Simulation Model

A supply chain simulation is one of a powerful and widely used tool for supply chain analysis. Particularly, in case the analytical model cannot provide a clear analysis or stochastic nature exists in the supply chain. Therefore, in this study, simulation experiments are developed for the purpose of analyzing the cash flow forecasting accuracy at different stages of supply chain and at different intensity of bullwhip effect.

4.4.4.1 Simulation Structure and Process

A simulation model of a multi-stage supply chain contains a single supplier, three manufacturers, three distributors, and three retailers. Its structure and product flow of the supply chain are shown in Figure 4-17 while its cash flow is in the reverse direction of the product flow. Originally, the simulation model was created for a simple supply chain, which contains only a single member in each stage and then more members were added into the model for more practical illustration. The supply chain members in each stage can be adjusted later for more proper use in each supply chain.

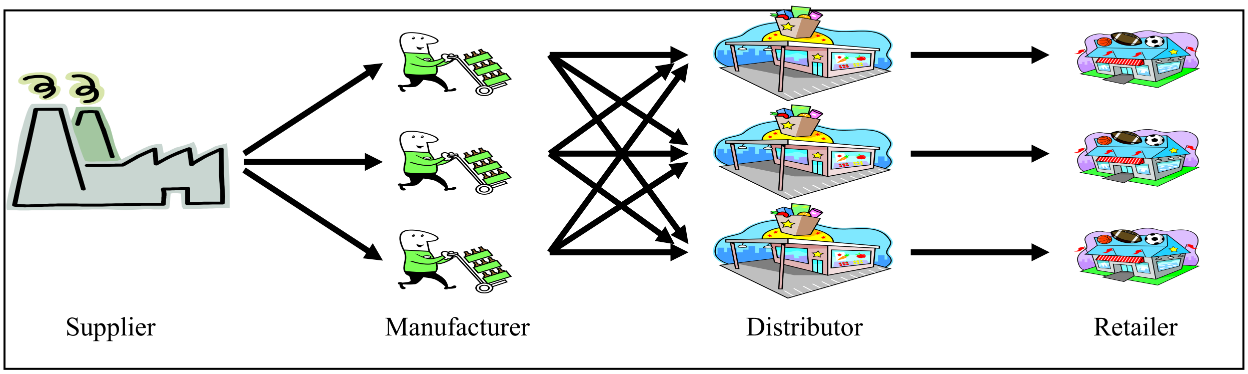


Figure 4-17. Structure of a supply chain in this simulation.

A simulation starts from a customer demand obtained from Eq. (4.15) with a 0.5 demand correlation. The customer demand is produced for 72 months including a 12 months warm up period for each input variable and it is generated separately for each retailer in order to represent three retail stores that have different customer demands. Assume that the initial inventory is twice as much of the customer demand. Each retailer fills the customer demand and periodically observes its inventory level. Excessive demand is backordered and will be filled first when the inventory is replenished. By the end of the observation period *t*, it places an order to the distributor to replenish its inventory level. The order quantity follows Eq. (4.16). A fixed lead time L is applied, thus, the order will be received at the start of period $t+L$ and then the process repeats. The supply chain process of the four members—retailers, distributors, manufacturers, and a supplier—are treated in a similar way except that each manufacturer receives orders from three distributors. The process continues like this through the upstream members. The simulation runs for 2,160 days (72 months including the warm up period) for each different bullwhip effect value (from BWE = 1 through BWE = 20) to observe the inventory bullwhip, the cash flow bullwhip, and the cash flow forecast error at all different stages in supply chain. The simulation model focuses on the material flow and the cash flow among supply chain members.

In order to obtain preliminary insights, some known sources of variability are reduced. Therefore, in this simulation, all supply chain members apply the same moving average forecasting method and the same order-up-to inventory policy as well as the same lead time throughout the supply chain. Additional assumption is that the accounts payable and the accounts receivable are 80% of total purchases and sales each period with a 30 days payment term, respectively. However, not all accounts receivable is collected within the pre-determine payment term. The uncollectible amount over 120 days in the accounts receivable aging is written off as a bad debt.

4.4.4.2 Simulation Tool

This simulation model is developed by Visual Basic for Applications (VBA) in Microsoft Excel 2007. The three main reasons why we use the VBA in Excel is that, firstly, the data regarding to the inventory and the cash flow is mostly in the tabular form or spreadsheet. Therefore, input and output variables in this simulation are easy to manage in Excel. Secondly, Excel provides adequate capabilities for the required computations, statistical analysis, and chart generation. Lastly, Excel is a very common program used in many companies. Therefore, it is very practical to develop the simulation in Excel. Our simulation model has a friendly user interface, which is easy to use and configure.

Tangsucheeva and Prabhu, 2013 shows that an increase in the bullwhip effect leads to an increase in the inventory bullwhip (IBW) and the cash flow bullwhip (CFB). Adverse impacts from these effects include inventory, cost, and cash flow management difficulty. These effects also propagate upstream in supply chain the same way as the bullwhip effect. In other words, the impact of these phenomenon is worse in upstream members, particularly the supplier (Tangsucheeva and Prabhu, 2013).

In this section, five cash flow forecasting models are compared among others on the increase of the bullwhip effect to see how they are impacted as shown in Figure 4-18. This experiment was conducted in the simulation model, which contains a supplier, three manufacturers, three distributors, and three retailers as mentioned in Section 4.4.4.1.

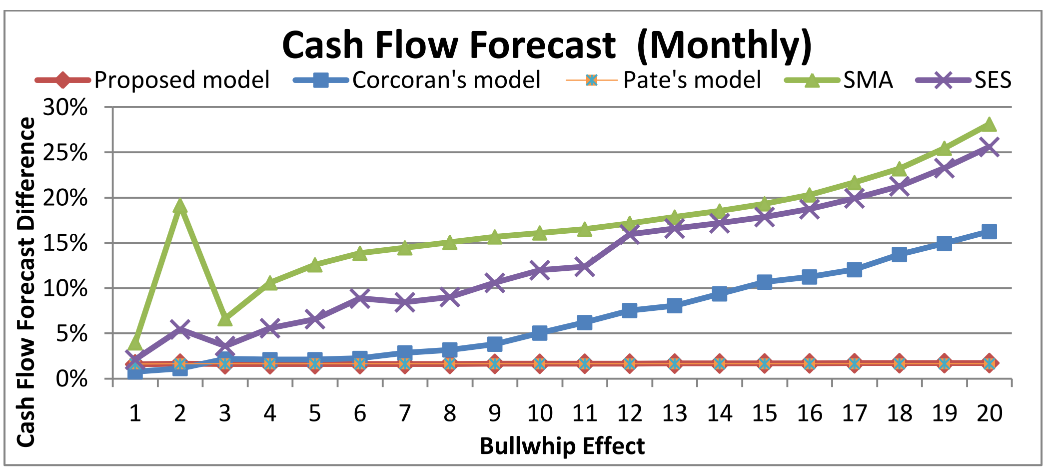


Figure 4-18. Comparison among five techniques at a manufacturer stage.

As the bullwhip effect increases, gaps of the differences between the actual cash flow and the forecasted cash flow also increases leading to worse accuracy, except the proposed model. Figure 4-18 shows that the result from the proposed model with α = 0.8 and β = 0.2 and the

Pate's model are largely independent of the bullwhip effect. The gap of the difference between the actual cash flow and the forecasted cash flow remains approximately the same. On the other hand, the other three models perform the cash flow forecast based heavily on the historical data only. Therefore, when the data contains large fluctuation, especially when the bullwhip effect is large, it significantly impairs the forecasting accuracy. This is the reason why Corcoran's model, the simple moving average technique (SMA), and the exponential smoothing technique, have their cash flow forecasting differences increase.

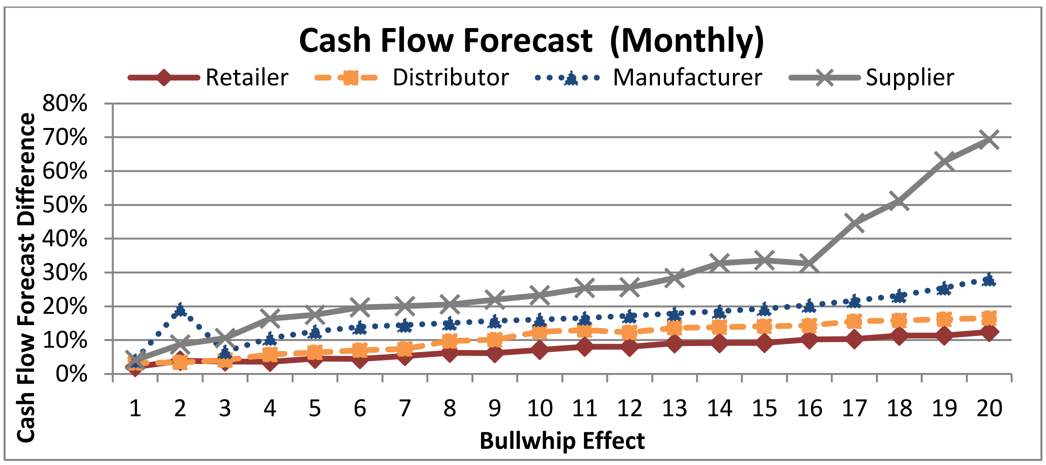


Figure 4-19. Cash flow forecast difference at different stages of supply chain (Moving average).

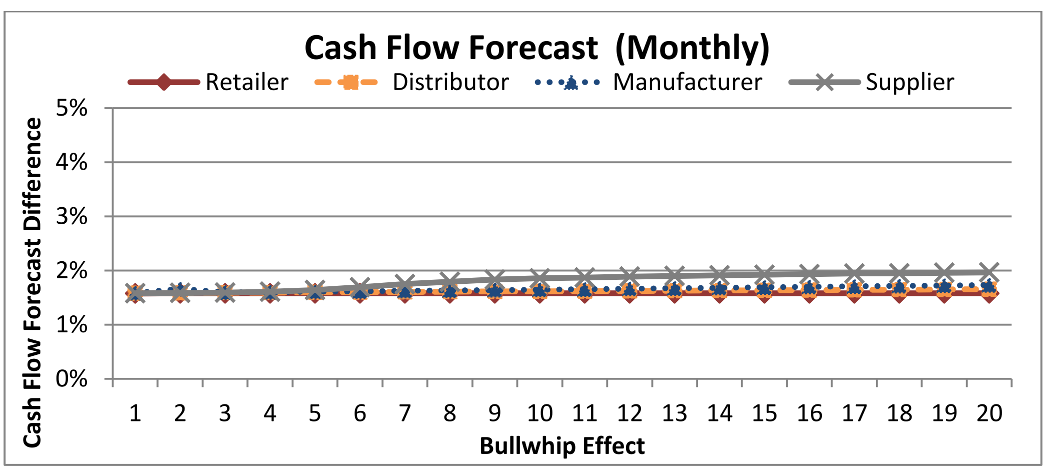


Figure 4-20. Cash flow forecast difference at different stages of supply chain (Proposed model).

Figure 4-19 and Figure 4-20 break down the impact of the bullwhip effect on the cash flow forecast to each supply chain level and compare the performance of the forecasting techniques between the two models, the moving average and the proposed model with α = 0.8 and β= 0.2. To be easily distinguishable, the moving average technique is selected to compare with the proposed model since it performs the worst among four models. As shown in Figure 4-19, the gaps of the forecasting differences from the moving average technique increase when the bullwhip increases and this inaccuracy continues to amplify when moving upstream toward the supplier. In the worst case scenario, when the bullwhip effect is very high (BW = 20), the cash flow forecast of the supplier may approximately result in 70% deviate from the actual cash flow. This effect can cause the supplier a huge amount of interest expense, working capital, opportunity cost, and so forth, meanwhile the forecasted cash flow from the proposed model remains about the same, under 2% for all supply chain levels.

4.5 Conclusions

The proposed cash flow forecasting model was developed to assist a firm to have accurate results and be convenient to use since the model obtains input data from the accounts receivable aging, which is one of the accounting routine the firm already has. The model is based on the combination of the work done by Cyert et al., 1962, Corcoran, 1978, and Pate-Cornell et al Pate-Cornell, 1986. The proposed cash flow forecasting for firms is developed by integrating two models: (1) Markov chain model of the aggregate payment behavior across all customers of the firm using accounts receivable aging and; (2) Bayesian model of individual customer specific payment behavior at the individual invoice level.

Actual data from a small manufacturing firm in Pennsylvania was used as the empirical study to evaluate the performance of the forecasting models. The experimental results were generated by the calculation from the analytical models and from a series of simulations. The overall performances of the proposed model were compared with the Corcoran's model, Pate's model, and other two common practice models, the moving average and the exponential smoothing techniques, which are widely used in small and medium companies.

In summary, the proposed cash flow forecasting model was demonstrated to be a simple and most accurate forecasting technique for the firm in comparison with other models. The proposed model performs the best with approximately 3%-6% different from the actual cash flow and approximately 2%-4% difference on an aggregate level. For the supply chain level application, the proposed model was shown to be independent of the bullwhip effect. The forecasting accuracy of the proposed model is high and seems to be largely robust to supply chain dynamics, including when subjected to severe bullwhip effect whereas those of other models become worse when the bullwhip effect is severe. Potential key impact of accurate forecast is efficient management of working capital. By reducing the forecast error from 20% (using simple moving average) to 2% we can reduce the cost of running a business, especially for suppliers who are likely to be SME. In addition, the proposed model was developed in an Excel spreadsheet format which links to the accounts receivable aging. This development provides users a practical and convenient forecasting tool to use even for a person who is not familiar with cash flow forecast.

Regarding the exponential smoothing factor α and the weighting parameter β, these two parameters are back tested from the payment historic data and selected from best values, which provide the highest forecasting accuracy. We found that these parameters are quite sensitive and can vary from industry to industry, customer to customer, and time to time. Since the proposed forecasting model is a convex combination of Corcoran's model and Pate-Cornell et al.'s model, when $\beta \to 0$, the proposed forecasting model relies more on Pate-Cornell et al.'s model which incorporates customer specific payment behavior. This implies that the payment behavior of this specific customer is different from the others. A firm needs to pay close attention to any customer that has $\beta \to 0$ because of its distinct from prevailing industry practice, especially when it is an important customer. On the other hand, when $\beta \to 1$, the proposed model relies more on Corcoran's model, which represents more aggregate customer behavior. The future direction of this study could be dedicated to the testing of the depth of historic data, how far back of the data we should go, exploit all data or just look one season back in order to determine the weighting parameter β.

Chapter 5

OPTIMAL PRICING WITH CONSTRAINTS ON WORKING CAPITAL AND PAYMENT DELAY RISK

5.1 Introduction

Dynamic financial decisions are decisions concerning financial activities while a situation and environment change over time leading to the change in financial decision-making. Such decisions include allocating amount of funds to operate a firm, selecting projects and managing their capital expenditure, financing both long-term and short-term, and managing working capital. This dissertation focuses on a combination of allocating proper amount of investment in projects within limited working capital and selecting projects that maximize profit and minimize credit risk of a firm.

Allocating proper amount of investment in projects is similar to asset allocation problem in financial investment strategy which diversifies investments in various categories of assets such as stocks and bonds in order to balance risk and return. Similarly, a firm seeks to invest in projects from several potential candidates, which have different profit and risk in order to balance its risk and return corresponding to its objectives.

The problem of project selection has caught fully attention to many researchers in project management and engineering management. Due to a complex decision-making process and various types of problems, many quantitative and qualitative techniques for selecting projects have been proposed over the last 50 years. Project selection problem is involved by several critical factors such as the market conditions, raw materials availability, probability of technical success, government regulations, etc. (Bard et al., 1988). Make-to-order (MTO) firms are a type of firms that may involve in project selection frequently. MTO firms must select the projects they decide to pursue from among numerous opportunities. Apparently, this is not a simple problem to solve. Poor decision-making may incur the firms to enormously expensive cost, which consequently affects profit and loss. Particularly, an MTO firm participating in bidding has to deal with several decision-making for project selection and bidding competition.

The whole bidding process can be divided into four phases, which are acquisition of the project information, pre-evaluation of the project (which project to bid), mid-evaluation (how to bid), and implementation of the project as shown in Figure 5-1 (Wang et al., 2009). This dissertation focuses on which projects to bid in the second phase, the pre-evaluation of the project, of Figure 5-1.

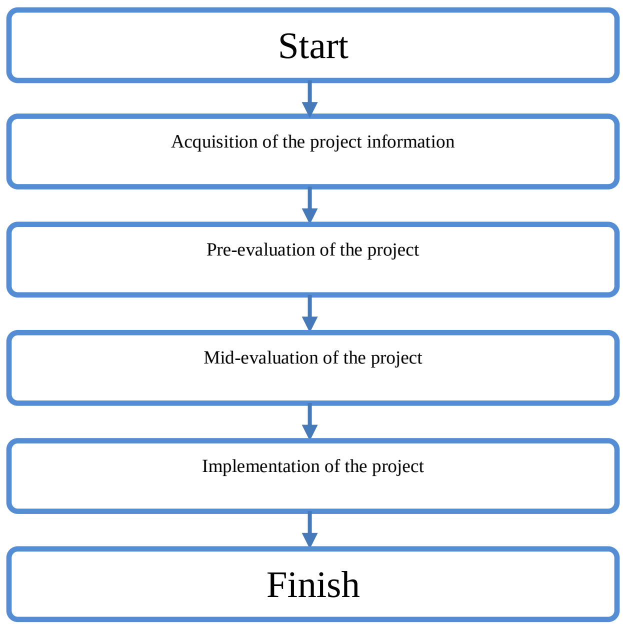


Figure 5-1. Phase classification of design project bidding decision (Wang et al., 2009).

The process of project bidding decision begins with a need for a custom product from a prospective customer. Then the customer identifies a number of qualified manufacturers, and solicit proposals, or bids, to satisfy its requirements. On the bidder side, an MTO firm starts with a detailed investigation of the project and then extensively collects and carefully reviews the information of the tender project. This information includes specifications of the products and services corresponding to customer's requirements (King and Mercer, 1988). MTO firms must estimate all project requirements such as raw material, labor, time frame and completion time, costs and working capital of the project and then tender their terms to meet the customer's requirements. Once all information is complete, the MTO firm is able to decide whether to bid or not, which project to bid and how to bid, in order to develop a strategy to make an optimal bid for projects.

On the other hand, the prospective customer also tries to have many options to select the best one. Therefore, the customer usually solicits proposals to several different suppliers or manufacturers and compares their terms. Some of the competing MTO firms may be invited to submit bids for other jobs as shown in Figure 5-2. Therefore, not every bid submitted by an MTO firm is likely to be successful. Finally, the project is awarded to a single bidder. The risk of the MTO firm is that it may lose the biding competition to another competitor. According to the survey of Tobin et al., 1988 about the bidding competition in UK, it shows that the proportion of successful tendered bids that become orders of the firms is very varied between 3% and 100%. The success of a bidding competition may depend on many different aspects, but bidding price is frequently an important key to win the bid. Kingsman et al., 1989 identifies that bidding price plays an important and critical role in winning or losing bid. Dempsey's survey reveals that price is among the top three factors with delivery capability and quality, which influence industrial purchase decisions (Dempsey, 1978). Similarly, Dickson, 1966 findings show that there are 22 other important factors which customers evaluate during the selection process. These factors include good quality and reliable delivery. Understand how these factors interact and affect each other allows firms to construct the bidding strategies to win the bid. Once the customer decides which bidder is a winner, occasionally, it may initiate negotiations with the bidder for more favorable terms. Each stage of the bidding process may involve a considerable time lag (Slatter, 1990).

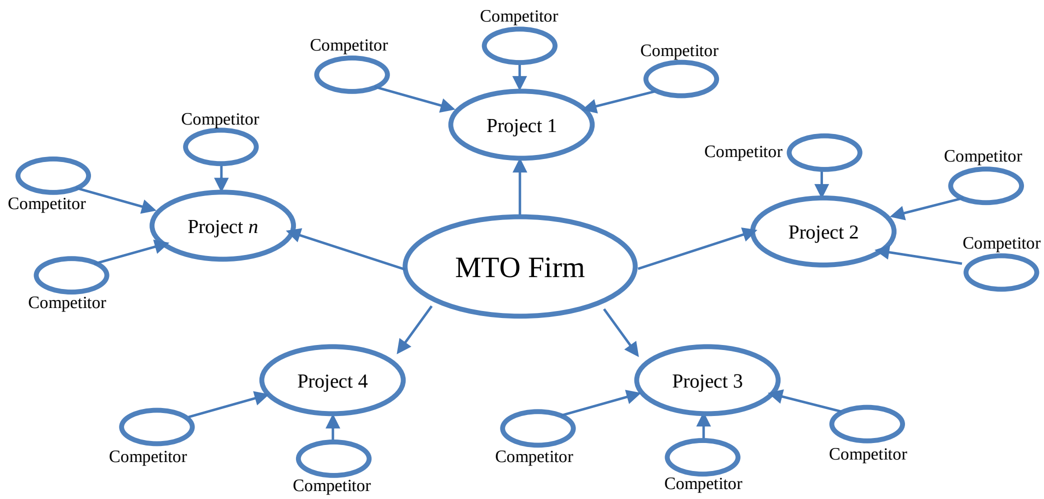


Figure 5-2. Scenario diagram of the MTO firm bidding.

The ultimate goal of MTO firms to participate in the bidding competition is not only to win the bid, but also to earn profit for themselves. An MTO firm may win a bidding competition with an unrealistically low price or delivery terms; as a result, this may risk the firm to earn so low revenue that it cannot cover overall costs and eventually loss a profit – a phenomenon sometimes called "the winner's curse". Friedman, 1956 and Gates, 1967 proposed bidding strategies that manage bidding prices to maximize expected profits. Berkson's binary choice logit model was among the pioneers to apply an S-shape multi attribute model for choice probabilities (Berkson, 1953). Later on, Easton and Moodie, 1999 included the lead time to deliver the project to the S-shape logistical response function (logit model) to provide the winning probability of the bidding competition. To optimize the expected benefits from a bidding competition, MTO firms need to understand the relationship among prices, revenues, expenses, winning likelihood, and return and risk.

Default risk, a possibility of a default payment, is one of many problems MTO firms encounter after deliver complete projects. Therefore, credit risk assessment and risk compensation are also necessary for MTO firms to evaluate their customers. Credit risk refers to the risk that a customer will default on its payments. The risk includes lost in principal and interest, disruption to cash flows, and increased collection costs. The loss may be complete or partial and can arise in a number of many possible circumstances. Credit risk also leads to cash flow and working capital problem. To compensate this risk, additional amount called default premium or risk premium can be added into bidding price.

Typically, the process of setting a bidding price of MTO firms is to evaluate all associated costs and using a standard mark-up approach (Kingsman et al., 1993; Wisner and Siferd, 1995). Mark-up is the additional amount that a firm adds to its estimated total costs for the project to cover overhead, unplanned expenses, and desired profit (Ahmad and Minkarah, 1987). Many literature in the competitive bidding focuses on these key decision variables (King and Mercer, 1988). To construct an optimal bidding strategy, it is important that a firm realizes and understands its objectives clearly since there are many possible objectives in a real business world such as maximizing profit, minimizing loss, minimize profits of competitors, and just obtaining the contract to keep production going even at a loss (Friedman, 1956). Different strategies are required based on different objectives of the firms.

In what follows we first explains the analytical models of how the integer programming is formulated in order to determine the combination of the project investment that maximize profit and minimize credit risk in Section 5.2. Then, Section 5.3 describes the setup of the problems and illustrates industrial case numerical examples. This section shows how the model can be applied to the project selection problem. Then, the results and discussion are addressed in Section 5.4. Lastly, in Section 5.5 the conclusion and direction for future work are discussed.

5.2 Pricing Model

This section we identify a potential strategy to engineer a solution for maximizing profit and minimizing credit risk by using integer programming model. In practice, financial problems involve multiple objectives that are dependent on the complex financial and nonfinancial relationships that define the problem. The objective of this section is to apply an integer programming model to this project selection problem.

To formulate this integrated project selection problem, credit risk minimization, and working capital maximization, an integer programming approach is applied to determine an optimal investment strategy. We will start with the objective functions. The first objective is to maximize the expected profit.

5.2.1 Objective Functions

Expected Profit

In our problem, the uncertainty associated with the bidding is represented by a chance of winning a bid. Let $P(b_{ij})$ represent the probability that a bid of b_{ij} will be the lowest and win the project, p_{ij} represent the amount of actual profit obtained from each selected project i in period j. Then the expected profit, $E(b_{ij})$, if a bid of b_{ij} wins, can be written as

$$E(b_{ij}) = P(b_{ij})p_{ij} \tag{5.1}$$

where the probability of winning bid $P(b_{ij})$ can be obtained from Friedman (1965). Presumably the results of previous bidding on contracts are always announced, and from these announced bids the 'bidding patterns' of potential competitors may be studied. Additionally, each competitor is likely to bid as he has done in the past, which is the best assumption in the absence of additional information

$$P(b_{ij}) = \exp\left[-\pi\left(1 - \sum_{i=0}^{d} \frac{1}{i!}\left\{\frac{ab_{ij}}{c_{ij}}\right\}^{i} e^{-\frac{ab_{ij}}{c_{ij}}}\right)\right] \tag{5.2}$$

where π is the estimated number of bidders, c_{ij} is the cost of the project *i* in period *j*, *a* and *d* are constants obtained from curve fitting frequency data to the gamma distribution. A gamma distribution is a good fit to this sort of data, the probability density function of the ratio of the average bidder's bid to the firm cost estimate. The actual profit p_{ij} can be obtained from

$$p_{ij} = b_{ij} - c_{ij} \tag{5.3}$$

where b_{ij} is the bidding price and c_{ij} is the cost of the project *i* in period *j*. Note that the cost c_{ij} already includes both fixed cost (i.e. production set up cost and overhead) and variable cost (i.e. unit cost and holding cost) of project *i*. The bidding price can be calculated by

$$b_{ij} = c_{ij}(1 + \delta_{ij} + \phi_{ij}) \tag{5.4}$$

where δ_{ij} is the required percent markup of a firm and ϕ_{ij} is the percent of the risk premium of the project *i* in period *j*. Therefore, the actual profit p_{ij} can be rewritten as

$$p_{ij} = c_{ij}(\delta_{ij} + \phi_{ij}) \tag{5.5}$$

Finally, the first objective function, maximizing the expected profit, can be written as

$$Max\ Z_1 = \sum_{i=1}^{n}\sum_{j=1}^{m}\left((\delta_{ij}+\phi_{ij})c_{ij}x_{ij}\right)\exp\left[-\pi\left(1 - \sum_{i=0}^{d}\frac{1}{i!}\left\{\frac{ab_{ij}}{c_{ij}}\right\}^{i} e^{-\frac{ab_{ij}}{c_{ij}}}\right)\right] \tag{5.6}$$

where x_{ij} is the binary 0-1 decision variables of the project *i* in period *j* where x_{ij} = 1 if the project is selected and x_{ij} = 0 otherwise.

5.2.2 Constraints

Credit Risk

Tangsucheeva and Prabhu, 2014 proposes a stochastic analytics model to forecast cash flow by building on the work done by Cyert et al., 1962; Corcoran, 1978; Pate-Cornell et al., 1990. A part of this proposed model applies Bayesian technique to take customer specific payment behavior into account to calculate the payment probability. By modeling the distribution of the payment time of each customer as Weibull distribution with scale parameter λ = 1 and the shape parameter k = 2, the probability of a given invoice that will be paid between time t_0 and $t_0 + \Delta t$ can be computed as follows:

Case I: $t_0 - t_b \geq \gamma$

$$p''_{nP} = 1 - \exp\left\{-\frac{[2(t_0 - t_b - \gamma)(\Delta t) + (\Delta t)^2]}{\hat{\lambda}_j^2}\right\} \tag{5.7}$$

Case II: $t_0 - t_b \leq \gamma$ and $t_0 - t_b + \Delta t \geq \gamma$

$$p''_{nP} = 1 - \exp\left\{-\frac{[(t_0 - t_b + \Delta t - \gamma)^2]}{\hat{\lambda}_j^2}\right\} \tag{5.8}$$

Case III: $t_0 - t_b + \Delta t \leq \gamma$

$$p''_{nP} = p[t \leq t_0 + \Delta t | t \geq t_0, \lambda] = 0 \tag{5.9}$$

where p''_{nP} is the payment probability from the customer payment behavior (the transition probability from the last state, *State n*, to *State P (Paid)* in the accounts receivable aging), t_b is the time a given invoice is billed to the customer, γ is the minimum payment time and $\hat{\lambda}_j$ is the estimate scale parameter which characterizes customer's payment lead time (Pate-Cornell et al., 1990). These model parameters can be estimated periodically or even after invoice payment.

Generally, if the invoice ages over the last *State n* in the accounts receivable aging, for instance, 120 days, this invoice is usually written off as a bad debt in accounting. Hence, the complement of p''_{nP} , which is $p''_{nB} = 1 - p''_{nP}$, is the transition probability of *State n* to *State (B) Bad Debt* within Δt days. In other words, it is the probability of default payment. Therefore, the probability of default becomes

Case I: $t_0 - t_b \geq \gamma$

$$p''_{nB} = 1 - p''_{nP} = \exp\left\{-\frac{[2(t_0 - t_b - \gamma)(\Delta t) + (\Delta t)^2]}{\hat{\lambda}_j^2}\right\} \tag{5.10}$$

Case II: $t_0 - t_b \leq \gamma$ and $t_0 - t_b + \Delta t \geq \gamma$

$$p''_{nB} = \exp\left\{-\frac{[(t_0 - t_b + \Delta t - \gamma)^2]}{\widehat{\lambda}_j^2}\right\} \tag{5.11}$$

Case III: $t_0 - t_b + \Delta t \leq \gamma$

$$p''_{nB} = 1 \tag{5.12}$$

To compensate these credit risk or default risk, we need to determine risk premium. Typically, risk premium in the financial investment can be calculated by

$$Risk\ premium = r_a - r_f$$

where r_a is the return of a risky asset such as stock and r_f is the return of the risk-free asset such as T-bill.

However, in the trade credit, the concept of the risk and return is different from that of the financial investment. Risk premium in the trade credit can be determined by

$$Risk\ premium = r_c - E(r_t)$$

where r_c is the return from cash trade, which represents the return from the risk-free asset, and $E(r_t)$ is the expected return from the trade credit and can be determined by

$$E(r_t) = p''_{nP} r_c$$

where p''_{nP} is the average payment probability of a customer within the payment term (depend on the payment term i.e. 45 days, 60 days, 90 days, etc.).

Generally, in financial investment, the higher risk you invest in assets, the higher return you get from them. However, this is not the case for the trade credit. Trade credit may expose to credit risk or risk from delay payment whereas the cash trade does not have these kinds of risk since the latter case a firm receive payment before or on delivery. Nonetheless, trade credit is essential for business to business (B2B) sellers since it is one of several tools to attract buyers in a competitive market.

However, the previous equation does not include the impact from the payment delay. Therefore, taking time value of money into account, the risk premium for trade credit can be rewritten as

$$Risk\ premium = \left(r_c - E(r_t)\right)(1+i)^n$$

where *i* is the required rate of return of a firm and *n* is the period of the delay payment. Finally, the percent risk premium ϕ_{ij} can be obtained from

$$\phi_{ij} = (1 - p''_{nP})(1+i)^n = p''_{nB}(1+i)^n \tag{5.13}$$

The financial risk associated with this project bidding under uncertainty of the trade credit is credit risk. It is defined as the risk premium which compensates the default risk.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\phi_{ij} x_{ij}\right) \le \Phi \tag{5.14}$$

where ϕ_{ij} is the risk premium of the project *i* in period *j* and Φ represents the maximum risk the firm can expose to.

Machine Utilization

The sum of machine utilization of all selected projects must not exceed the total machine utilization.

$$\sum_{i=1}^{n}\sum_{j=1}^{m} E(u_{ij})x_{ij} \leq U$$

where $E(u_{ij})$ is the expected machine gross utilization available based on all committed project *i* in period *j* and *U* is the total machine utilization of the manufacturer.

Raw Materials

The amount of raw materials supplied by a supplier to various projects (to produce for customers) at production facility, must be sufficient.

$$\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}x_{ij} \leq W$$

where w_{ij} is the total required amount of raw materials for the selected projects.

Labor

Labor or man-hour required for the selected projects must not exceed the total number of maximum man-hours *L*.

$$\sum_{i=1}^{n}\sum_{j=1}^{m} l_{ij}x_{ij} \leq L$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} o_{ij}x_{ij} \leq O$$

where l_{ij} is the required number of hours for the selected projects and o_{ij} is the overtime hour for the selected project must not exceed the maximum overtime *O*.

Storage Space

As occurs with the plants, it is defined a continuous variable in order to represent the capacity of the warehouse or storage space. Therefore, the total inventory of s_{ij} kept at warehouse during period *j* must be lower than the capacity of the warehouse. Warehouse space must be sufficient.

$$\sum_{i=1}^{n}\sum_{j=1}^{m} s_{ij}x_{ij} \leq S$$

where s_{ij} is the space required to store in the warehouse for the project *i* in period *j*.

Working Capital

Cash inflows and outflows are normally not synchronized, so that a positive cash balance is required to operate the firm (Robichek et al., 1965). Lutz and Lutz, 1951 point out that the synchronization of inflows and outflows might be improved, but at a cost. Capital budgeting, Capital investment planning (limited capital and credit line). Each project requires the amount of capital investment c_{ij} and the total capital investment must not exceed the available capital C.

Assume that C is the maximum working capital the firm can invest in selected projects. This maximum working capital C is available for raw material cost, ordering cost, labor cost, production cost, facilities cost (i.e. EE, gas, fuel, etc.), inventory holding cost, material handling cost, G&A cost, overhead cost, and all other requirements for completing the selected projects.

$$\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} \leq C$$

These constraints are applicable to this empirical study. In practice, constraints can be added or removed based on specific constraints of firms.

5.3 Industrial Case Numerical Examples

Two industrial cases, a firm with non recurring customers and a firm with recurring customers, are presented in this paper. A small manufacturing make-to-order (MTO) firm in Pennsylvania presented as a firm with non recurring customers has few standard products and volatile. The second case is another manufacturing firm with recurring customers. For the MTO firm, a production order usually stems from a successful bid. To participate in a bidding competition, the MTO firm must estimate the resource requirements, the completion time, direct costs of the project, and then tender its term to meet the customer’s requirements. The customer may decide to accept, reject, or modify these terms. However, in this study, we assume that there is no modification of these terms. Many of the important operational decisions that associate with the MTO firm do not begin until a customer places an order. This scenario is not a mutually exclusive, the company can select more than one project as long as all constraints are not violated and all projects are independent. Periodically, the MTO firm has several projects to bid with several different customers, but with a limited working capital. Each project provides different profit and chance to win the bid, exposes to different level of credit risk, requires different amount of working capital and capacity constraints.

The firm has a 30 days fixed payment term to its supplier in order to have an edge of low raw material cost over its competitors. On the other hand, the payment term from its accounts receivable may vary from 60-90 days assuming all projects are trade credit. Some customers pay their invoices on time or before time while the others pay late. The accounts receivable that age over 120 days are considered as a bad debt. These delay payments and default risk effect the firm's cash flow and working capital. The firm's cash flow is forecasted by the stochastic financial analytics model (Tangsucheeva & Prabhu, 2014). Thus, the firm knows its cash flow and want to manage it efficiently. The firm can have a large amount of working capital sitting in its account, but doing so, it may lose an opportunity to profit from excess cash. Meanwhile, too little of working capital may risk the company to be insolvent and lose opportunity to profit from investing in other projects.

Therefore, the challenge here for the firm to be decided is to select the projects which maximize profits, minimize risk, and maximize working capital within a limited resources. The firm may have to trade-off profit with risk and working capital with opportunity cost (excess cash) and liquidity problem (cash shortage). These types of decisions repeat every period.

5.3.1 Industrial Case - Firm with Non Recurring Customers

To illustrate the model, we use actual cash flow data from a small manufacturing firm where we know cash inflow and cash outflow. Thus, the next period working capital can be forecasted. The problem consists of determining the optimal strategy to allocate working capital to various projects in order to maximize profit and minimize credit risk.

In this illustration, we consider five potential project candidates. We consider a planning horizon of 6 months. The process starts by estimating credit risk of customers. Most companies typically write off the remaining balance (delinquent amount) after 120 days of accounts receivable aging as a bad debt. p''_{nP} is the payment probability of the outstanding balance that will be paid within 91-120 days, hence, $1 - p''_{nP}$ is the probability that this project will not get paid within 120 days. In other words, it is the probability of the bad debt.

For customer #1, since the outstanding balance age, $t_0 - t_b$, is 91 days whereas the average payment time is 95 days and the minimum payment time γ is 60 days, therefore, Eq. (5.10) is used to calculate $1 - p''_{nP}$ for Case I.

$$p''_{nB} = \exp\left\{-\frac{[2(t_0 - t_b - \gamma)(\Delta t) + (\Delta t)^2]}{\hat{\lambda}_j^2}\right\}; t_0 - t_b \geq \gamma$$

$$p''_{nB} = \exp\left\{-\frac{[2(91-60)(30)+(30)^2]}{39.4933^2}\right\}=0.1704$$

Then we are able to calculate the risk premium of this customer using Eq. (5.13) where the required rate of return of a firm *i* is 20%.

$$\phi_{11} = p''_{nB}(1+i)^n = 0.1704(1.0167) = 0.1732 = 17.32\%$$

Once we know the risk premium, we can determine the required minimum bidding price b_{11} and the actual profit p_{11} if winning the bid, using Eq. (5.4) and (5.5), respectively, assuming the required working capital c_{11} investing in this project is $1M and the required percent markup δ_{ij} is 30%.

$$b_{11} = c_{11}(1 + \delta_{11} + \phi_{11}) = \$1M(1 + 0.30 + 0.1732) = \$1{,}473{,}200$$

$$p_{11} = c_{11}(\delta_{11} + \phi_{11}) = \$473{,}200$$

Next, to determine the expected profit $E(b_{11})$, if a bid b_{11} wins, apply Eq. (5.1) and Eq. (5.2) assuming there are 10 bidders in this project, *a* and *d* are 2 and 10, respectively.

$$E(b_{11}) = P(b_{11})p_{11} = \$472{,}011$$

Repeat these steps to determine risk and return of other projects. Table 5-1 summarizes parameters for determining risk and return of 5 projects to be considered in period 1.

Table 5-1. Objective function parameters.

Project	Credit Risk	Risk Premium	Percent Markup	Working Capital	Minimum Bidding Price	Winning Probability $P(b_{ij})$	Actual Profit
1	0.1704	17.32%	30%	$1,000,000	$1,473,200	99.75%	$473,200
2	0.3210	32.64%	25%	$700,000	$1,103,480	99.65%	$403,480
3	0.5367	54.57%	30%	$400,000	$738,280	99.23%	$338,280
4	0.7295	74.17%	45%	$1,500,000	$3,287,550	98.35%	$1,787,550
5	0.8841	89.89%	30%	$800,000	$1,759,120	93.42%	$959,120

However, the purpose of a firm is not just to win the bid, but to make profit too. The required minimum bidding price of a firm may not provide the highest expected profit. Thus, we construct to vary the bidding price of Project 1 to see how the expected profit changes.

Table 5-2. Various bidding price vs expected profit.

Bidding Price ($)	Expected Profit ($)
1,473,200	472,011
1,600,000	597,024
1,800,000	789,894
2,000,000	972,002
2,200,000	1,133,646
2,400,000	1,261,503
2,600,000	1,340,462
2,800,000	1,357,392
3,000,000	1,305,959

From Table 5-2, the bidding price at $2.8M provides the highest expected profit of $1.357M with the winning probability of 75.41%. Table 5-3 shows the bidding prices which provide the highest expected profit of 5 projects.

Table 5-3. Bidding prices and expected profits

Project	Minimum Bidding Price	Number of Bidders	Bidding Price providing Highest Expected Profit	Winning Probability $P(b_{ij})$	Expected Profit $E(b_{ij})$
1	$1,473,200	10	$2,800,000	75.41%	$1,357,392
2	$1,103,480	8	$2,030,000	75.64%	$1,500,986
3	$738,280	5	$1,240,000	77.32%	$649,496
4	$3,287,550	3	$5,250,000	74.41%	$2,790,424
5	$1,759,120	12	$2,160,000	76.32%	$1,038,025

Table 5-4. Constraint parameters.

Project	Machine Utilization	Raw Materials (lbs)	Labor (hrs.)	OT (hrs.)	Storage Space (ft^2)	Working Capital
1	0.2	3,800	3,200	960	3,600	$1,000,000
2	0.17	2,500	2,800	840	3,000	$700,000
3	0.10	2,000	2,400	800	2,500	$400,000
4	0.42	4,000	3,800	740	9,400	$1,500,000
5	0.23	3,200	3,000	700	4,300	$800,000

Table 5-5. Upper and Lower bound.

Machine Utilization	Raw Materials (lbs)	Labor (hrs.)	OT (lbs.)	Storage Space (ft^2)	Working Capital
0.95	10,000	10,000	2,500	20,000 ft^2	$4,000,000

Table 5-4 and Table 5-5 summarize the constraints and upper and lower bound for integer programming formulation. Once we know the optimal bidding price and maximum expected profit for each pricing, then, from the data in Table 5-1 through Table 5-5, we can formulate the integer programming model to determine which project combination provides maximum profit and minimum risk.

5.3.2 Industrial Case - Firm with Recurring Customers

We also test the model with another small manufacturing firm which has its customers constantly coming back for reordering. Even though there is no bidding competition in this scenario, it is similar to the first scenario since the firm has to compete with its competitors through pricing, assuming that price is the decision variable to convert the purchasing quotation to the purchasing order. The objective of the firm is the same as that of the first scenario which is to maximize profit and minimize credit risk. Therefore, pricing strategy plays an important role in getting the order. The differences between these two scenarios are that the firm with recurring customers is more familiar with its customers and customers' payment behavior, therefore, it can assess customers' credit risk and hence forecast its cash flow more accurately.

We use the model to run the experiment to see how it works with this scenario. In this illustration, five customers request the quotations from the firm and competitors of the firm. We consider a planning horizon of 6 months. To formulate the integer programming, we follow the same procedure as the previous section by starting to determine the price, calculating credit risk and risk premium of these five customers, and then setting up the required percent markup as shown in Table 5-6. Next we use Eq. (5.1) and Eq. (5.2) to determine the price which provides the highest expected profit as shown in Table 5-7.

Table 5-6. Objective function parameters.

Customer	Credit Risk	Risk Premium	Percent Markup	Working Capital	Minimum Quote Price	Winning Probability $P(b_{ij})$	Actual Profit
1	0.2920	29.80%	25%	\$12,000	\$18,577	99.88%	\$6,577
2	0.1536	15.68%	25%	\$15,000	\$21,102	99.95%	\$6,102
3	0.3404	34.75%	25%	\$23,000	\$36,743	99.85%	\$13,743
4	0.1867	19.06%	25%	\$18,000	\$25,931	99.94%	\$7,931
5	0.3495	35.68%	25%	\$26,000	\$41,777	99.85%	\$15,777

Table 5-7. Bidding prices and expected profits

Customer	Minimum Quote Price	Number of Competitors	Quote Price Providing Highest Expected Profit	Winning Probability $P(b_{ij})$	Expected Profit $E(b_{ij})$
1	\$18,577	1	\$98,800	74.60%	\$54,312
2	\$21,102	3	\$48,000	78.22%	\$25,812
3	\$36,743	2	\$80,500	74.41%	\$42,787
4	\$25,931	4	\$55,800	77.32%	\$29,227
5	\$41,777	5	\$78,000	77.43%	\$40,266

Then we list the constraints of 5 customers and upper and lower bound of the firm, respectively, as shown in Table 5-8 and Table 5-9.

Table 5-8. Constraint parameters.

Customer	Machine Utilization	Raw Materials (lbs)	Labor (hrs.)	OT (hrs.)	Storage Space (ft^2)	Working Capital
1	0.12	1,700	1,600	300	2,500	\$100,000
2	0.14	2,200	1,800	300	2,800	\$100,000
3	0.17	3,500	2,100	900	4,800	\$100,000
4	0.15	2,800	1,900	600	6,000	\$100,000
5	0.20	4,300	3,000	900	7,000	\$100,000

Table 5-9. Upper and Lower bound.

Machine Utilization	Raw Materials (lbs)	Labor (hrs.)	OT (lbs.)	Storage Space (ft^2)	Working Capital
0.80	13,000	10,000	2,500	20,000 ft^2	\$100,000

Then we are ready to formulate the integer programming model where x_{ij} is the binary 0-1 decision variables of the customer i in period j where x_{ij} = 1 if the customer is selected and x_{ij} = 0 otherwise.

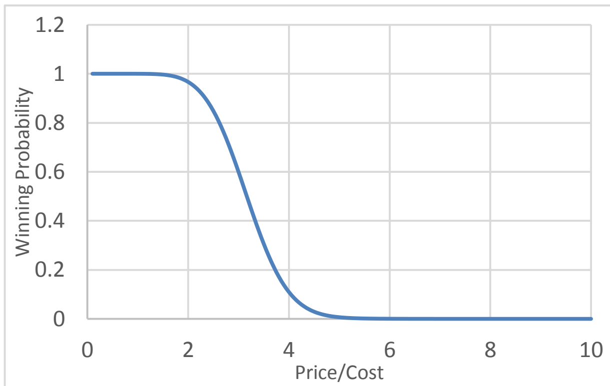
5.4 Results and Discussion

We consider 5 projects for the project bidding competition scenario and 5 customers for the purchasing quotation of recurring customer scenario in 6 periods time horizon. The model was solved by LINDO on a Window 8 based Dell workstation with Intel Xeon 3.6GHz dual processor. The model was terminated after 4 second of CPU time. Table 5-10 shows that for the first scenario Project 2, 4, and 5 are selected to invest in this period with the expected profit of \$5.3M and credit risk of 1.967 within a \$3M working capital. For the second scenario, Customer 1, 2, 3, and 5 are selected in this period with the expected profit of \$163k.

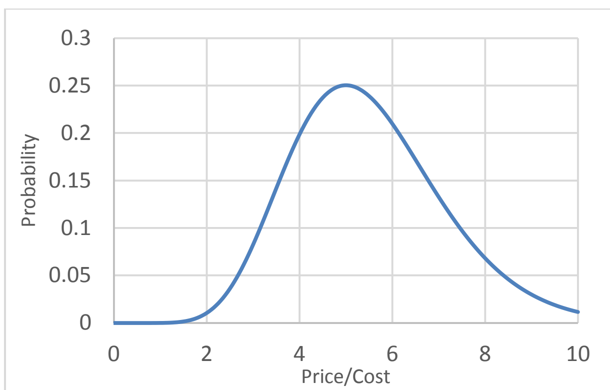
Table 5-10. Results.

	Results	
	Bidding competition	Purchasing quotation
Selected Projects/Customers	2, 4, and 5	1, 2, 3, and 5
Expected Profit	\$5,329,435	\$163,177
Credit Risk	1.967	1.136
Working Capital	\$3,000,000	\$76,000

For the analysis part, since two scenarios provide similar results, we present only the bidding competition scenario. Figure 5-3 (a) and Figure 5-3 (b) show the winning probability among 10 bidders on Project 1 and the bidding pattern of 10 bidders in this bidding competition, respectively. The winning probability is very high and is almost 100% chance of winning the bid at the low price-cost ratio (PC ratio) range of 0.1 to 1.7, and then it drops sharply when PC ratio increases over 2.0. Eventually, the winning probability drops to almost zero chance at the PC ratio over 5.0.



(a)



(b)

Figure 5-3. Winning probability (a) and Bidding pattern of average bidder (b).

Figure 5-4 shows the expected profit of Project 1 over the PC ratio. At the beginning, the expected profit is negative (loss) until the PC ratio increases over 1.0, then the firm starts to make a profit. The expected profit keeps increasing quite linearly and reaches its peak of $1.36M at the PC ratio of 2.8, and then it starts to decrease.

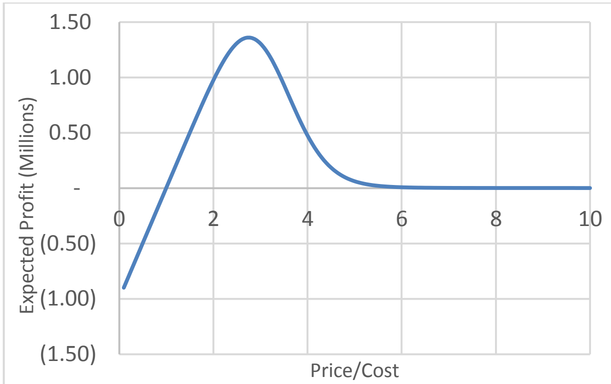


Figure 5-4. Expected profit over Price-Cost ratio (PC ratio).

Figure 5-3 (a) and Figure 5-4 show that the firm does not have much profit or even loss when the firm try to bid at a very low price (low PC ratio) in order to just to win the bid (low price, high winning chance). Since the objective of the firm is not only to win the bid, but also to maximize its profit, thus, bidding at the lowest price or at a low profit may not be the best strategy in this case.

Figure 5-5 shows the relationship between the expected profit and credit risk of Project 1. The expected profit increases at the beginning when the credit risk is low and it keeps increasing until the credit risk is 0.3636. Then, the expected profit starts to fall. The expected profit increases while the credit risk increases because the firm compensates this risk by adding the risk premium into the bidding price. Hence, when credit risk increases, the risk premium increases and the expected profit still increases. However, at one point, the increase in the risk premium is not sufficient to compensate the rising of credit risk. Thus, at this point, the expected profit starts to decrease. At the point where the risk premium is not sufficient to compensate the credit risk, the firm may need to re-evaluate its risk premium to compensate the high credit risk.

From Figure 5-5 implication, consider two projects that have the same characteristics such as profit and required production capacity, but the credit risks are different. One project has the credit risk of 0.3636, which provides the highest expected profit due to the additional risk premium, and the other project has the credit risk of 0.25. The firm should select the former project if the objective of the firm is to maximize its profit. Otherwise, the firm may choose the latter project for lower credit risk.

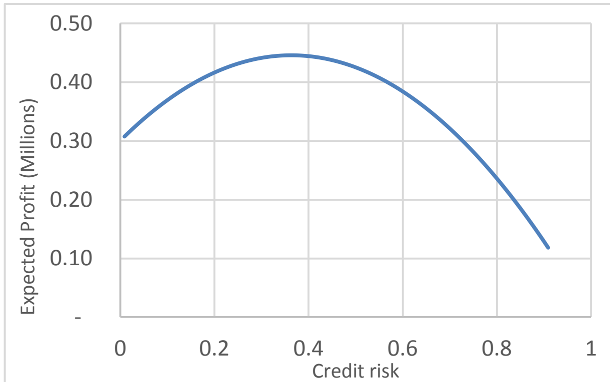


Figure 5-5. Expected profit vs Credit risk.

Figure 5-6 illustrates the relationship between the winning probability and the risk premium. As the risk premium increases, it reduces the chance of winning the bid since the risk premium is added to the bidding price to protect from the credit risk.

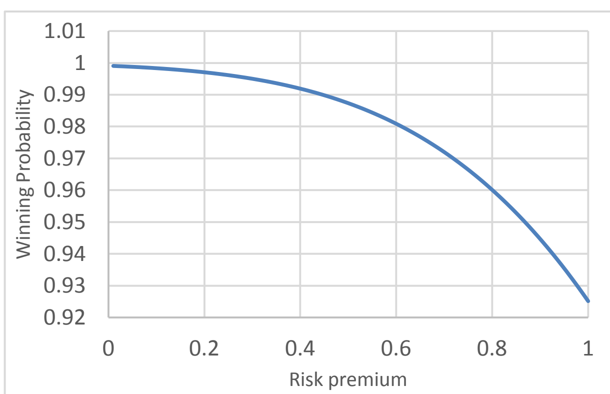


Figure 5-6. Winning probability vs Credit risk.

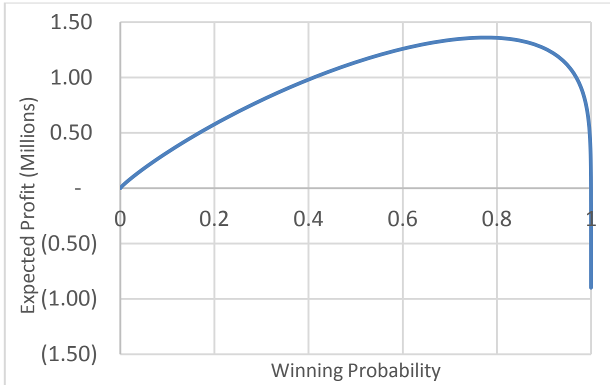


Figure 5-7. Expected profit vs Winning probability.

Figure 5-7 depicts the relationship between the expected profit and the winning probability. This figure shows the phenomenon called winner's curse where the increase in the winning probability may lead to the loss of the firm. The winner of the bidding competition may not be able to make a profit from the project since the bidding price may be too low to offset the cost.

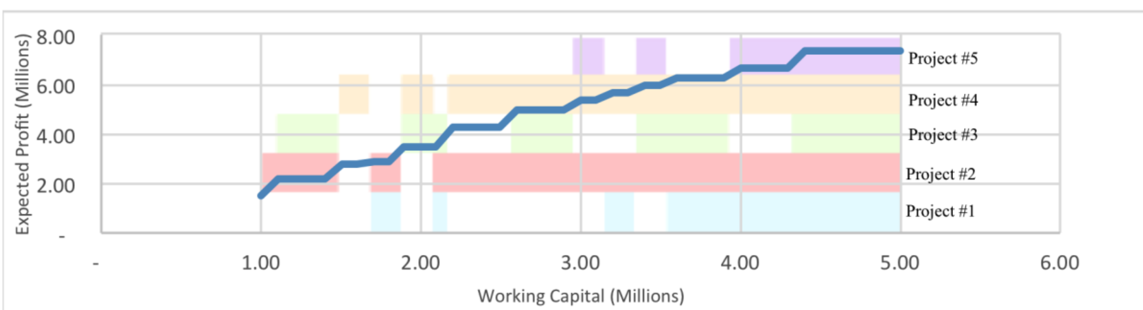


Figure 5-8. Expected profit vs Working capital.

Figure 5-8 shows the relationship among the expected profit, working capital, and the selected projects. We relax other constraints to see how the expected profit increases when the firm has more working capital. The solid line shows the increase in the expected profit when the working capital is increased. The horizontal color bar chart in the background represents the selected projects (Project #1 through Project #5) over the range of working capital. For example, if the firm has $2M working capital, investing in Project #3 and Project #4 provides it the maximum expected profit of $3.44M. If the firm obtains more working capital from its line of credit to $4M, then investing in Project #1, Project #2, Project #4, and Project #5 gives it maximum expected profit of $6.69M. This graph helps the management team to make a decision conveniently and effectively.

Table 5-11. Results for 6 periods.

Period	Working Capital	Credit Risk	Expected Profit	Selected Projects
1	$3,000,000	1.9670	$5,329,435	2, 4, and 5
2	$3,000,000	1.5783	$5,473,020	1, 3, and 4
3	$4,000,000	1.5821	$6,120,323	1, 2, and 4
4	$4,500,000	0.9647	$6,783,705	2, 3, 4, and 5
5	$6,000,000	1.4581	$8,539,460	1, 2, 4, and 5
6	$6,000,000	1.8937	$8,152,905	1, 2, 3, and 5

Once the decisions are made, the firm follows the strategy from the investment model in selected projects. Before the end of the period, the firm has to prepare for the bidding competition of the next period using the cash flow forecast to estimate its working capital for the next period. The objective functions remain the same, which are maximized profit and minimize credit risk. However, the constraints may change over time. The process repeats like this each period. The results for 6 periods are shown in Table 5-11.

5.5 Conclusions

The problem of project selection has been given fully attention in engineering management. How to select the best project or portfolio for more benefits. Especially for a firm with non recurring customers, an MTO firm, participating in a bidding has to deal with project selection and bidding competition. The purpose of bidding is to earn a profit, not simply to win orders. An MTO firm might win a competition with an unrealistically low price, hence, the revenue does not cover its costs and eventually results in loss of the firm.

We have applied an integer programming model to the firms to select which projects it should invest in for maximizing profit by applying a competitive bidding strategy of Friedman (1956) to determine the appropriate bidding price. In addition, we also take the credit risk into account in the objective functions in order to control the loss from payment delay and default risk at the acceptable level of the firm. The probability of default payment is calculated to determine the risk premium which is added to compensate the risk. The model treats diverse manufacturing and financial constraints such as production capacity and working capital. Other constraints can be added or removed depending on the project requirements.

The solution of a model is a combination of selected projects which tradeoff between risk and return and could be a basis for strategies and options for management to maximize their profit, minimize risk, and avoid liquidity problem. A detailed numerical case study also shows that the firm should invest in which projects corresponding to the available working capital in order to maximize the profit and minimize the credit risk. In addition, the results show that the PC ratio has an inversely proportional relationship to the winning probability and the winning probability has the inversely proportional relationship to the risk premium. That is the winning probability of the bidding is very high at a low PC ratio and the winning probability starts to decrease when the PC ratio increases. Therefore, at a very low bidding price, the firm has a very high chance to win the bid.

Similar to the risk premium, the increase in risk premium leads to the decrease in winning probability. Since the risk premium is added to the bidding price to compensate the credit risk, thus, the price is increased and the winning probability is decreased. However, at a high winning probability, the firm may not be able to make a profit due to the unrealistically low bidding price. The results also show that at a specific bidding price, the firm obtains the highest expected profit. As a result, the firm should bid at this price in case its purpose is to maximize the profit.

Lastly, the relationship between the expected profit and the credit risk is concave function where the expected profit increases at the beginning, reaches its peak, and then starts to fall when the credit risk keeps increasing. In case there are two projects that have the same characteristics such as profit and required production capacity, but the credit risks are different. From the expected profit and credit risk relationship, the firm can see which project provides how much profit at a specific credit risk and select the project based on its need to maximize profit or minimize risk.

Chapter 6

CONCLUSIONS AND FUTURE DIRECTIONS

This chapter concludes all of the work done in this dissertation and summarizes future directions of this research.

6.1 Conclusions

In Chapter 3, we postulate that there can be a corresponding bullwhip in the cash flow across the supply chain. The bullwhip effect is one of the most common problems found in supply chain management. It not only has tremendous impact on supply chain efficiencies, but it also has an impact on the cash flow in many ways. We successfully develop and analyze the cash flow bullwhip model and find that the increase in the bullwhip effect results in the increase in the variance of inventory and the cash conversion cycle, leading to the increase in the cash flow bullwhip eventually. In addition, severe cash flow bullwhip takes place in upstream members such as suppliers and manufacturers once the bullwhip effect propagates upstream to these members.

We found that the cash flow bullwhip is an increasing function of the bullwhip effect and the lead time, but it is a decreasing function of the demand observation period. This result supports the idea that the large lead time and small number of moving average observation period may produce the bullwhip effect. Once the bullwhip effect occurs, it causes the increase in the variance of inventory and the cash flow bullwhip. In other words, since the amplified order does not reflect the actual demand, which is lower than the order, therefore, the products will sit in the inventory and this will cause the company not only incur the high inventory holding cost but also incur high opportunity cost, financial cost and working capital.

In Chapter 4, we develop the stochastic financial analytics model for cash flow forecast. The proposed model integrates Markov chain model of the aggregate payment behavior across all customers of the firm using accounts receivable aging and Bayesian model of individual customer specific payment behavior at the individual invoice level.

Actual data from a small manufacturing firm in Pennsylvania is used as the empirical study to evaluate the performance of the forecasting models. The experimental results are generated by the calculation from the analytical models and from a series of simulations. In summary, the proposed cash flow forecasting model is demonstrated to be a simple and most accurate forecasting technique for the firm in comparison with Corcoran's model and other two common practice models, the moving average and the exponential smoothing techniques, which are widely used in small and medium companies. For the supply chain level application, the proposed model was shown to be independent of the bullwhip effect. The forecasting accuracy of the proposed model is high and seems to be largely robust to supply chain dynamics, including when subjected to severe bullwhip effect whereas those of other models become worse when the bullwhip effect is severe. Potential key impact of accurate forecast is efficient management of working capital. By reducing the forecast error from 20% (using simple moving average) to 2% we can reduce the cost of running a business, especially for suppliers who are likely to be SME. In addition, the proposed model is developed in an Excel spreadsheet format which links to the accounts receivable aging. This development provides users a practical and convenient forecasting tool to use even for a person who is not familiar with cash flow forecast.

Regarding the exponential smoothing factor α and the weighting parameter β, these two parameters are back tested from the payment historic data and selected from best values, which provide the highest forecasting accuracy. We found that these parameters are quite sensitive and can vary from industry to industry, customer to customer, and time to time. Since the proposed forecasting model is a convex combination of Corcoran's model and Pate-Cornell et al.'s model, when $\beta \to 0$, the proposed forecasting model relies more on Pate-Cornell et al.'s model which incorporates customer specific payment behavior. This implies that the payment behavior of this specific customer is different from the others. A firm needs to pay close attention to any customer that has $\beta \to 0$ because of its distinct from prevailing industry practice, especially when it is an important customer. On the other hand, when $\beta \to 1$, the proposed model relies more on Corcoran's model, which represents more aggregate customer behavior.

In Chapter 5, we discuss the problem of how to select the projects or portfolios for optimal benefits. Especially for an MTO firm participating in bidding has to deal with project selection and bidding competition. The purpose of bidding is to earn a profit, not simply to win orders. An MTO firm might win a competition with an unrealistically low price, hence, the revenue does not cover its costs and eventually results in loss of the firm.

We present an integer programming model for an MTO firm to select which projects it should invest in for maximizing profit by applying a competitive bidding strategy of Friedman (1956) to determine the appropriate bidding price. In addition, we also take the credit risk into account in the objective functions in order to control the loss from payment delay and default risk at the acceptable level of the firm. The probability of default payment is calculated to determine the risk premium, which is added to compensate the risk. The model treats diverse manufacturing and financial constraints such as production capacity and working capital. Other constraints can be added or removed depending on the project requirements.

The solution of a model is a combination of selected projects which tradeoff between risk and return and could be a basis for strategies and options for management to maximize their profit, minimize risk, and avoid liquidity problem. A detailed numerical case study also shows that the firm should invest in which projects corresponding to the available working capital in order to maximize the profit and minimize the credit risk. In addition, the results show that the PC ratio has an inversely proportional relationship to the winning probability and the winning probability has the inversely proportional relationship to the risk premium. That is the winning probability of the bidding is very high at a low PC ratio and the winning probability starts to decrease when the PC ratio increases. Therefore, at a very low bidding price, the firm has a very high chance to win the bid.

Similar to the risk premium, the increase in risk premium leads to the decrease in winning probability. Since the risk premium is added to the bidding price to compensate the credit risk, thus, the price is increased and the winning probability is decreased. However, at a high winning probability, the firm may not be able to make a profit due to the unrealistically low bidding price. The results also show that at a specific bidding price, the firm obtains the highest expected profit. As a result, the firm should bid at this price in case its purpose is to maximize the profit.

Lastly, the relationship between the expected profit and the credit risk is concave function where the expected profit increases at the beginning, reaches its peak, and then starts to fall when the credit risk keeps increasing. In case there are two projects that have the same characteristics such as profit and required production capacity, but the credit risks are different. From the expected profit and credit risk relationship, the firm can see which project provides how much profit at a specific credit risk and select the project based on its need to maximize profit or minimize risk.

6.2 Future Directions

The future work for modeling and analysis of cash flow bullwhip will take various order policies such as *(Q, r)*, *(s, S)*, and so forth into account to see the impact of the bullwhip effect on the *CFB*. We also want to research these effects' impact the supply chain finance (*SCF*) and financial supply chain (*FSC*). The future direction of the stochastic financial analytics for cash flow forecast could be dedicated to the testing of the depth of historic data, how far back of the data we should go, exploit all data or just look one season back in order to determine the weighting parameter β.

Currently, the optimal pricing model in Chapter 5 considers only the stochastic nature of the successful bid which effects the expected profit and the working capital. If a bid is successful then the firm will need appropriate working capital which will be freed up when the customer makes payments. If the customer does not make payments in a timely manner then there is a risk of working capital shortage for the ongoing project(s). Only bidding for a project does not imply that the firm needs to set aside the working capital required for the project. This is because the bid may not be successful and not all the working capital would be required at the same time, for instance, it will depend on the payment terms because long duration projects may involve partial payments based on project milestones or schedule. This element of dynamics of cash flow can be a possible future research extension that the current model does not explicitly consider.

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Appendix A

PROOF OF EQUATION (3.8)

From Eq. (7), we have the inventory at time t (I_t) in the form of order quantity and customer demand as

$$I_t = I_{int} + q_{t-L} + y_{t-L-1} - D_t - \sum_{i=1}^{L+1} D_{t-i}$$

Then, we determine its variance as follows.

$$Var(I_t) = Var(q_{t-L}) + Var(y_{t-L-1}) + Var(D_t) + Var\left(\sum_{i=1}^{L+1} D_{t-i}\right) \tag{A.1}$$

$$+ 2Cov(q_{t-L}, y_{t-L-1}) - 2Cov(q_{t-L}, D_t)$$

$$- 2Cov(q_{t-L}, \sum_{i=1}^{L+1} D_{t-i}) - 2Cov(y_{t-L-1}, D_t)$$

$$- 2Cov\left(y_{t-L-1}, \sum_{i=1}^{L+1} D_{t-i}\right) + 2Cov(D_t, \sum_{i=1}^{L+1} D_{t-i})$$

We need to determine $Var(y_{t-L-1})$, $Var\left(\sum_{i=1}^{L+1} D_{t-i}\right)$ and all the covariance as follows. (We do not touch $Var(q)$ since we want the term $Var(q)/Var(D)$ to remain in the equation.) From Eq. (4) and (5), we rearrange

$$Var(y_{t-L-1}) = Var\left(L\left(\frac{\sum_{i=1}^{p} D_{t-L-1-i}}{p}\right) + z\hat{\sigma}_{e,t-L-1}^{L}\right)$$

$$= \left(\frac{L}{p}\right)^2 Var\left(\sum_{i=1}^{p} D_{t-L-1-i}\right) + z^2 Var(\sigma) + 2\left(\frac{L}{p}\right)(z) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}\ ,\hat{\sigma}_{e,t-L-1}^{L}\right)$$

From Lemma 2.1 of Chen et al (2000), $Cov\left(D_{t-i}, \hat{\sigma}_{e,t}^{L}\right) = 0$ for all $i = 1,2, \ldots, p$, hence

$$Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}\ ,\hat{\sigma}_{e,t-L-1}^{L}\right) = 0; for\ all\ i = 1,2, \ldots, p$$

$$Var(y_{t-L-1}) = \left(\frac{L}{p}\right)^2 Var\left(\sum_{i=1}^{p} D_{t-L-1-i}\right) + z^2 Var(\sigma)$$

$$Var(y_{t-L-1}) = \left(\frac{L}{p}\right)^2 \left[p + 2p\left(\frac{\rho-\rho^p}{1-\rho}\right) - 2\rho\left(\frac{(p-1)\rho^p - p\rho^{p-1} + 1}{(1-\rho)^2}\right)\right] Var(D) + z^2 Var(\sigma) \tag{A.2}$$

See Chen et al (2000) for the proof of $Var\left(\sum_{i=1}^{p} D_{t-L-1-i}\right)$

Next, we determine

$$Var\left(\sum_{i=1}^{L+1} D_{t-i}\right) = \sum_{i=1}^{L+1} Var(D_{t-i}) + 2\sum_{i<j} Cov(D_{t-i}, D_{t-j})$$

$$= (L+1)Var(D) + 2\sum_{i<j} \rho^{j-i}\, Var(D)$$

$$= (L+1)Var(D) + 2\sum_{i=1}^{L} (L+1-i)\rho^{i}\, Var(D)$$

$$Var\left(\sum_{i=1}^{L+1} D_{t-i}\right) = \left[(L+1) + 2L\left(\frac{\rho-\rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho-\rho^{L+1}}{1-\rho}\right) - 2\rho\left(\frac{L\rho^{L+1}-(L+1)\rho^{L}+1}{(1-\rho)^2}\right)\right]Var(D) \tag{A.3}$$

Then we determine all the covariances in Eq. (A.1) under the condition of Lemma 2.1 of Chen et al (2000) as follows:

$$\begin{aligned} 2Cov(q_{t-L}, y_{t-L-1}) &= 2Cov(y_{t-L} - y_{t-L-1} + D_{t-L-1}, y_{t-L-1}) \\ &= 2Cov(\widehat{D}_{t-L}^{L} + z\hat{\sigma}_{e,t-L}^{L} - \widehat{D}_{t-L-1}^{L} - z\hat{\sigma}_{e,t-L-1}^{L} + D_{t-L-1}, \widehat{D}_{t-L-1}^{L} + z\hat{\sigma}_{e,t-L-1}^{L}) \\ &= 2Cov\left(\left(1+\frac{L}{p}\right)D_{t-L-1} - \left(\frac{L}{p}\right)D_{t-L-p-1} + z\left(\hat{\sigma}_{e,t-L}^{L} - \hat{\sigma}_{e,t-L-1}^{L}\right), \left(\frac{L}{p}\right)\sum_{i=1}^{p} D_{t-L-1-i} + z\hat{\sigma}_{e,t-L-1}^{L}\right) \\ &= 2\left(\left(1+\frac{L}{p}\right)\left(\frac{L}{p}\right)\sum_{i=1}^{p}\rho^{i} - \left(\frac{L}{p}\right)^{2}\sum_{i=0}^{p-1}\rho^{i}\right)Var(D) + z^2 Cov\left(\hat{\sigma}_{e,t-L}^{L}, \hat{\sigma}_{e,t-L-1}^{L}\right) - z^2 Var(\sigma) \end{aligned} \tag{A.4}$$

$$\begin{aligned} 2Cov(q_{t-L}, D_t) &= 2Cov(y_{t-L} - y_{t-L-1} + D_{t-L-1}, D_t) \\ &= 2Cov\left(\left(1+\frac{L}{p}\right)D_{t-L-1} - \left(\frac{L}{p}\right)D_{t-L-p-1} + z\left(\hat{\sigma}_{e,t-L}^{L} - \hat{\sigma}_{e,t-L-1}^{L}\right), D_t\right) \\ &= 2\left(\left(1+\frac{L}{p}\right)\rho^{L+1} - \left(\frac{L}{p}\right)\rho^{L+p+1}\right)Var(D) \end{aligned} \tag{A.5}$$

$$2Cov\left(q_{t-L}, \sum_{i=1}^{L+1} D_{t-i}\right) = 2Cov\left(y_{t-L} - y_{t-L-1} + D_{t-L-1}, \sum_{i=1}^{L+1} D_{t-i}\right) \tag{A.6}$$

$$= 2Cov\left(\left(1+\frac{L}{p}\right)D_{t-L-1} - \left(\frac{L}{p}\right)D_{t-L-p-1} + z\left(\hat{\sigma}_{e,t-L}^{L} - \hat{\sigma}_{e,t-L-1}^{L}\right), \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$= 2\left(\left(1+\frac{L}{p}\right)\sum_{i=0}^{L}\rho^{i} - \left(\frac{L}{p}\right)\sum_{i=p}^{p+L}\rho^{i}\right)Var(D)$$

$$2Cov(y_{t-L-1}, D_t) = 2Cov\left(\left(\frac{L}{p}\right)\sum_{i=1}^{p} D_{t-L-1-i} + z\hat{\sigma}_{e,t-L-1}^{L}, D_t\right) \tag{A.7}$$

$$= 2\left(\frac{L}{p}\right)\sum_{i=1}^{p} Cov\left(D_{t-L-1-i}, D_t\right)$$

$$= 2\left(\frac{L}{p}\right)\sum_{i=1}^{p} \rho^{L+1+i}\, Var(D)$$

$$2Cov\left(y_{t-L-1}, \sum_{i=1}^{L+1} D_{t-i}\right) \tag{A.8}$$

$$= 2Cov\left(\left(\frac{L}{p}\right)\sum_{i=1}^{p} D_{t-L-1-i} + z\hat{\sigma}_{e,t-L-1}^{L}, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$= 2\left(\frac{L}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1} Cov\left(D_{t-L-1-i}, D_{t-j}\right)$$

$$= 2\left(\frac{L}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1} \rho^{L+1+i-j}\, Var(D)$$

$$2Cov\left(D_t, \sum_{i=1}^{L+1} D_{t-i}\right) = 2\sum_{i=1}^{L+1} Cov\,(D_t, D_{t-i}) \tag{A.9}$$

$$= 2\sum_{i=1}^{L+1} \rho^i\, Var(D)$$

If the retailer uses a simple moving average forecasting method with p demand observations and under the condition of Lemma 2.1 of Chen et al (2000), then we obtain the variance of I_t by plugging Eq. (A.2) to Eq. (A.9) in Eq. (A.1) as follows:

$$Var(I_t) = Var(q_{t-L}) + Var(y_{t-L-1}) + Var(D_t) + Var\left(\sum_{i=1}^{L+1} D_{t-i}\right) + 2Cov(q_{t-L}, y_{t-L-1})$$

$$- 2Cov(q_{t-L}, D_t) - 2Cov(q_{t-L}, \sum_{i=1}^{L+1} D_{t-i}) - 2Cov(y_{t-L-1}, D_t)$$

$$- 2Cov\left(y_{t-L-1}, \sum_{i=1}^{L+1} D_{t-i}\right) + 2Cov(D_t, \sum_{i=1}^{L+1} D_{t-i})$$

$$Var(I) = Var(q) + \left(\frac{L}{p}\right)^2 \left[p + 2p\left(\frac{\rho - \rho^p}{1-\rho}\right) - 2\rho\left(\frac{(p-1)\rho^p - p\rho^{p-1} + 1}{(1-\rho)^2}\right)\right] Var(D)$$

$$+ z^2 Var(\sigma) + Var(D)$$

$$+ \left[(L+1) + 2L\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right)\right.$$

$$\left. - 2\rho\left(\frac{L\rho^{L+1} - (L+1)\rho^L + 1}{(1-\rho)^2}\right)\right] Var(D)$$

$$+ 2\left(\left(1 + \frac{L}{p}\right)\left(\frac{L}{p}\right)\sum_{i=1}^{p}\rho^i - \left(\frac{L}{p}\right)^2\sum_{i=0}^{p-1}\rho^i\right) Var(D) + z^2 Cov\left(\hat{\sigma}_{e,t-L}^L, \hat{\sigma}_{e,t-L-1}^L\right)$$

$$- z^2 Var(\sigma) - 2\left(\left(1 + \frac{L}{p}\right)\rho^{L+1} - \left(\frac{L}{p}\right)\rho^{L+p+1}\right) Var(D)$$

$$- 2\left(\left(1 + \frac{L}{p}\right)\sum_{i=0}^{L}\rho^i - \left(\frac{L}{p}\right)\sum_{i=p}^{p+L}\rho^i\right) Var(D) - 2\left(\frac{L}{p}\right)\sum_{i=1}^{p}\rho^{L+1+i}\, Var(D)$$

$$- 2\left(\frac{L}{p}\right)\sum_{i=1}^{p}\ \sum_{j=1}^{L+1}\rho^{L+1+i-j}\, Var(D) + 2\sum_{i=1}^{L+1}\rho^i\, Var(D)$$

Divided by *Var(D)*, we obtain

$$\frac{Var(I)}{Var(D)} = \frac{Var(q)}{Var(D)} + \left(\frac{L}{p}\right)^2 \left[p + 2p\left(\frac{\rho - \rho^p}{1-\rho}\right) - 2\rho\left(\frac{(p-1)\rho^p - p\rho^{p-1} + 1}{(1-\rho)^2}\right)\right] + 1$$

$$+ \left[(L+1) + 2L\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) - 2\rho\left(\frac{L\rho^{L+1} - (L+1)\rho^L + 1}{(1-\rho)^2}\right)\right]$$

$$+ 2\left(\left(1 + \frac{L}{p}\right)\left(\frac{L}{p}\right)\sum_{i=1}^{p}\rho^i - \left(\frac{L}{p}\right)^2\sum_{i=0}^{p-1}\rho^i\right) - 2\left(\left(1+\frac{L}{p}\right)\rho^{L+1} - \left(\frac{L}{p}\right)\rho^{L+p+1}\right)$$

$$- 2\left(\left(1+\frac{L}{p}\right)\sum_{i=0}^{L}\rho^i - \left(\frac{L}{p}\right)\sum_{i=p}^{p+L}\rho^i\right) - 2\left(\frac{L}{p}\right)\sum_{i=1}^{p}\rho^{L+1+i}$$

$$- 2\left(\frac{L}{p}\right)\sum_{i=1}^{p}\ \sum_{j=1}^{L+1}\rho^{L+1+i-j} + 2\sum_{i=1}^{L+1}\rho^i + \frac{z^2 Cov\left(\hat{\sigma}^L_{e,t-L}, \hat{\sigma}^L_{e,t-L-1}\right)}{Var(D)}$$

$$\frac{Var(I)}{Var(D)} = \frac{Var(q)}{Var(D)} + \left(\frac{L}{p}\right)^2 \left[p + 2p\left(\frac{\rho - \rho^p}{1-\rho}\right) - 2\rho\left(\frac{(p-1)\rho^p - p\rho^{p-1} + 1}{(1-\rho)^2}\right)\right] + 1$$

$$+ \left[(L+1) + 2L\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) - 2\rho\left(\frac{L\rho^{L+1} - (L+1)\rho^L + 1}{(1-\rho)^2}\right)\right]$$

$$+ 2\left(\left(1+\frac{L}{p}\right)\left(\frac{L}{p}\right)\left(\frac{\rho - \rho^{p+1}}{1-\rho}\right) - \left(\frac{L}{p}\right)^2\left(\frac{1-\rho^p}{1-\rho}\right)\right)$$

$$- 2\left(\left(1+\frac{L}{p}\right)\rho^{L+1} - \left(\frac{L}{p}\right)\rho^{L+p+1}\right)$$

$$- 2\left(\left(1+\frac{L}{p}\right)\left(\frac{1-\rho^{L+1}}{1-\rho}\right) - \left(\frac{L}{p}\right)\left(\frac{\rho^p - \rho^{L+p+1}}{1-\rho}\right)\right) - 2\left(\frac{L}{p}\right)\rho^{L+1}\left(\frac{\rho - \rho^{p+1}}{1-\rho}\right)$$

$$- 2\left(\frac{L}{p}\right)\rho^{L+1}\left(\frac{\rho - \rho^{p+1}}{1-\rho}\right)\left(\frac{1-\rho^{-(L+1)}}{\rho - 1}\right) + 2\left(\frac{\rho - \rho^{L+2}}{1-\rho}\right)$$

$$+ \frac{z^2 Cov\left(\hat{\sigma}^L_{e,t-L}, \hat{\sigma}^L_{e,t-L-1}\right)}{Var(D)}$$

$\blacksquare$

Appendix B

PROOF OF EQUATION (3.10)

To determine $Var(I_t)/Var(D)$ of the multi-stages supply chain, first, we consider the following equation.

$$
\begin{aligned}
Var(I_t^k) = {} & Var(q_{t-L}^k) + \left(\frac{L_k}{p}\right)^2 Var\left(\sum_{i=1}^{p} D_{t-L-1-i}\right) + \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var(D) \\
& + \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var(D) \\
& + \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var\left(\sum_{i=1}^{L+1} D_{t-i}\right) \\
& + \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var\left(\sum_{i=1}^{L+1} D_{t-p-i}\right) \\
& + 2\frac{L_k}{p} Cov\left(q_{t-L}^k, \sum_{i=1}^{p} D_{t-L-1-i}\right) \\
& + 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(q_{t-L}^k, D_{t-p}\right) \\
& - 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(q_{t-L}^k, D_t\right) \\
& - 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(q_{t-L}^k, \sum_{i=1}^{L+1} D_{t-i}\right) \\
& - 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(q_{t-L}^k, \sum_{i=1}^{L+1} D_{t-p-i}\right)
\end{aligned}
\tag{B.1}
$$

$$+2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, D_{t-p}\right)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, D_t\right)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$-2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, \sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$-2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(D_{t-p}, D_t\right)$$

$$-2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(D_{t-p}, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$-2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Cov\left(D_{t-p}, \sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$+2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Cov\left(D_t, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$+2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(D_t, \sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$+2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(\sum_{i=1}^{L+1} D_{t-i}, \sum_{i=1}^{L+1} D_{t-p-i}\right); Var\left(I_{int}^k\right) = 0$$

Then, we determine each variance and covariance as follows:

$$Var\left(\sum_{i=1}^{L+1} D_{t-p-i}\right) = \sum_{i=1}^{L+1} Var\big(D_{t-p-i}\big) + 2\sum_{i<j} Cov(D_{t-p-i}, D_{t-p-j}) \tag{B.2}$$

$$= (L+1)Var(D) + 2\sum_{i<j} \rho^{j-i}\, Var(D)$$

$$= \left[(L+1) + 2L\left(\frac{\rho-\rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho-\rho^{L+1}}{1-\rho}\right)\right.$$

$$\left. - 2\rho\left(\frac{L\rho^{L+1}-(L+1)\rho^{L}+1}{(1-\rho)^2}\right)\right] Var(D)$$

$$\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var\left(\sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$= \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \left[(L+1) + 2L\left(\frac{\rho-\rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho-\rho^{L+1}}{1-\rho}\right)\right.$$

$$\left. - 2\rho\left(\frac{L\rho^{L+1}-(L+1)\rho^{L}+1}{(1-\rho)^2}\right)\right] Var(D)$$

From Eq. (3-21), the covariance can be rewritten to

$$2\frac{L_k}{p}Cov\left(q_{t-L}^k, \sum_{i=1}^{p} D_{t-L-1-i}\right) \tag{B.3}$$

$$= 2\frac{L_k}{p}Cov\left(\left(1+\frac{\sum_{i=1}^{k} L_i}{p}\right)D_{t-L-1}\right.$$

$$\left. - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)D_{t-L-p-1}, \sum_{i=1}^{p} D_{t-L-1-i}\right)$$

$$= 2\left(\left(1+\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\rho^i - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=0}^{p-1}\rho^i\right)Var(D)$$

$$2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)Cov\left(q_{t-L}^k, D_{t-p}\right) \tag{B.4}$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)Cov\left(\left(1+\frac{\sum_{i=1}^{k} L_i}{p}\right)D_{t-L-1}\right.$$

$$\left. - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)D_{t-L-p-1}, D_{t-p}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1+\frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{|L+1-p|} - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+1}\right)Var(D)$$

$$2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(q_{t-L}^k, D_t\right) \tag{B.5}$$

$$= 2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(\left(1+\frac{\sum_{i=1}^{k} L_i}{p}\right) D_{t-L-1}\right.$$

$$\left. - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right) D_{t-L-p-1}, D_t\right)$$

$$= 2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1+\frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+1} - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+p+1}\right) Var(D)$$

$$2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(q_{t-L}^k, \sum_{i=1}^{L+1} D_{t-i}\right) \tag{B.6}$$

$$= 2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(\left(1+\frac{\sum_{i=1}^{k} L_i}{p}\right) D_{t-L-1}\right.$$

$$\left. - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right) D_{t-L-p-1}, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$= 2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1+\frac{\sum_{i=1}^{k} L_i}{p}\right)\sum_{i=0}^{L}\rho^{i} - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\sum_{i=p}^{p+L}\rho^{i}\right) Var(D)$$

$$2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(q_{t-L}^k, \sum_{i=1}^{L+1} D_{t-p-i}\right) \tag{B.7}$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(\left(1+\frac{\sum_{i=1}^{k} L_i}{p}\right) D_{t-L-1}\right.$$

$$\left. - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right) D_{t-L-p-1}, \sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\sum_{i=1}^{L+1} \rho^{|L-p+1-i|} - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\sum_{i=0}^{L} \rho^{i}\right) Var(D)$$

(B.8)

$$2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, D_{t-p}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, D_{t-p}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p} Cov\left(D_{t-L-1-i}, D_{t-p}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=L-p+2}^{L+1} \rho^{|i|}\, Var(D)$$

(B.9)

$$2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, D_t\right)$$

$$= 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p} Cov\left(D_{t-L-1-i}, D_t\right)$$

$$= 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p} \rho^{L+1+i}\, Var(D)$$

(B.10)

$$2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right) Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$= 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1} Cov\left(D_{t-L-1-i}, D_{t-j}\right)$$

$$= 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1}\rho^{L+1+i-j}\,Var(D)$$

(B.11)

$$2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)Cov\left(\sum_{i=1}^{p} D_{t-L-1-i}, \sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1} Cov\left(D_{t-L-1-i}, D_{t-p-j}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1}\rho^{|L-p+1+i-j|}\,Var(D)$$

(B.12)

$$2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)Cov\left(D_{t-p}, D_t\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\rho^{p} Var(D)$$

(B.13)

$$2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)Cov\left(D_{t-p}, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\sum_{i=1}^{L+1} Cov\left(D_{t-p}, D_{t-i}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\sum_{i=1}^{L+1}\rho^{|p-i|}\,Var(D)$$

(B.14)

$$2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Cov\left(D_{t-p}, \sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \sum_{i=1}^{L+1} Cov\left(D_{t-p}, D_{t-p-i}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \sum_{i=1}^{L+1} \rho^i \, Var(D)$$

$$2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Cov\left(D_t, \sum_{i=1}^{L+1} D_{t-i}\right)$$

$$= 2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \sum_{i=1}^{L+1} Cov\left(D_t, D_{t-i}\right) \tag{B.15}$$

$$= 2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \sum_{i=1}^{L+1} \rho^i \, Var(D)$$

(B.16)

$$2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) Cov\left(D_t, \sum_{i=1}^{L+1} D_{t-p-i}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) \sum_{i=1}^{L+1} Cov\left(D_t, D_{t-p-i}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) \sum_{i=p+1}^{p+L+1} \rho^i \, Var(D)$$

$$2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)Cov\left(\sum_{i=1}^{L+1} D_{t-i}\,, \sum_{i=1}^{L+1} D_{t-p-i}\right) \tag{B.17}$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\sum_{i=1}^{L+1}\sum_{j=1}^{L+1} Cov\left(D_{t-i}, D_{t-p-j}\right)$$

$$= 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\sum_{i=1}^{L+1}\sum_{j=1}^{L+1} \rho^{|p+j-i|}\, Var(D)$$

Then, substitute Eq. (B.2) to Eq. (B.17) into Eq. (B.1), we obtain Eq. (B.18) as follows:

$$Var(I_t^k) = Var(q_{t-L}^k) \tag{B.18}$$

$$+\left(\frac{L_k}{p}\right)^2\left[p + 2p\left(\frac{\rho-\rho^p}{1-\rho}\right)\right.$$

$$\left.- 2\rho\left(\frac{(p-1)\rho^p - p\rho^{p-1}+1}{(1-\rho)^2}\right)\right]Var(D)$$

$$+\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)^2 Var(D) + \left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)^2 Var(D)$$

$$+\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)^2\left[(L+1) + 2L\left(\frac{\rho-\rho^{L+1}}{1-\rho}\right)\right.$$

$$\left.+ 2\left(\frac{\rho-\rho^{L+1}}{1-\rho}\right) - 2\rho\left(\frac{L\rho^{L+1}-(L+1)\rho^L+1}{(1-\rho)^2}\right)\right]Var(D)$$

$$+\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)^2\left[(L+1) + 2L\left(\frac{\rho-\rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho-\rho^{L+1}}{1-\rho}\right)\right.$$

$$\left.- 2\rho\left(\frac{L\rho^{L+1}-(L+1)\rho^L+1}{(1-\rho)^2}\right)\right]Var(D)$$

$$+ 2\left(\left(1+\frac{\sum_{i=1}^{k}L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\rho^i\right.$$

$$\left.-\left(\frac{\sum_{i=1}^{k}L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=0}^{p-1}\rho^i\right)Var(D)$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\left(1+\frac{\sum_{i=1}^{k}L_i}{p}\right)\rho^{|L+1-p|}\right.$$

$$\left.-\left(\frac{\sum_{i=1}^{k}L_i}{p}\right)\rho^{L+1}\right) Var(D)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\left(1+\frac{\sum_{i=1}^{k}L_i}{p}\right)\rho^{L+1}-\left(\frac{\sum_{i=1}^{k}L_i}{p}\right)\rho^{L+p+1}\right)Var(D)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\left(1+\frac{\sum_{i=1}^{k}L_i}{p}\right)\sum_{i=0}^{L}\rho^{i}\right.$$

$$\left.-\left(\frac{\sum_{i=1}^{k}L_i}{p}\right)\sum_{i=p}^{p+L}\rho^{i}\right)Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\left(1+\frac{\sum_{i=1}^{k}L_i}{p}\right)\sum_{i=1}^{L+1}\rho^{|L-p+1-i|}\right.$$

$$\left.-\left(\frac{\sum_{i=1}^{k}L_i}{p}\right)\sum_{i=0}^{L}\rho^{i}\right)Var(D)$$

$$+2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=L-p+2}^{L+1}\rho^{|i|}\,Var(D)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\rho^{L+1+i}\,Var(D)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1}\rho^{L+1+i-j}\,Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1}\rho^{|L-p+1+i-j|}\,Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\rho^{p}Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1}L_i}{p}\right)\sum_{i=1}^{L+1}\rho^{|p-i|}\,Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1}L_i}{p}\right)^{2}\sum_{i=1}^{L+1}\rho^{i}\,Var(D)$$

$$+2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \sum_{i=1}^{L+1} \rho^i \, Var(D) + 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) \sum_{i=p+1}^{p+L+1} \rho^i \, Var(D)$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right) \sum_{i=1}^{L+1} \sum_{j=1}^{L+1} \rho^{|p+j-i|} \, Var(D) \, ; Var\left(I_{int}^{k}\right) = 0$$

$$Var(I_t^k) = Var(q_{t-L}^k) + \left(\frac{L_k}{p}\right)^2 \left[p + 2p\left(\frac{\rho - \rho^p}{1-\rho}\right) - 2\rho\left(\frac{(p-1)\rho^p - p\rho^{p-1} + 1}{(1-\rho)^2}\right)\right] Var(D)$$

$$+ \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var(D) + \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 Var(D)$$

$$+ \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \left[(L+1) + 2L\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right)\right.$$

$$\left. - 2\rho\left(\frac{L\rho^{L+1} - (L+1)\rho^L + 1}{(1-\rho)^2}\right)\right] Var(D)$$

$$+ \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \left[(L+1) + 2L\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right)\right.$$

$$\left. - 2\rho\left(\frac{L\rho^{L+1} - (L+1)\rho^L + 1}{(1-\rho)^2}\right)\right] Var(D)$$

$$+ 2\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{L_k}{p}\right)\left(\frac{\rho - \rho^{p+1}}{1-\rho}\right) - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{L_k}{p}\right)\left(\frac{1-\rho^p}{1-\rho}\right)\right) Var(D)$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{|L+1-p|} - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+1}\right) Var(D)$$

$$- 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+1} - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+p+1}\right) Var(D)$$

$$- 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{1-\rho^{L+1}}{1-\rho}\right)\right.$$

$$\left. - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{\rho^p - \rho^{L+p+1}}{1-\rho}\right)\right) Var(D)$$

$$- 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\sum_{i=1}^{L+1} \rho^{|L-p+1-i|}\right.$$

$$-\left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{1-\rho^{L+1}}{1-\rho}\right)\Bigg)Var(D)+2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=L-p+2}^{L+1}\rho^{|i|}\,Var(D)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\rho^{L+1}\left(\frac{\rho-\rho^{p+1}}{1-\rho}\right)Var(D)$$

$$-2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\rho^{L+1}\left(\frac{\rho-\rho^{p+1}}{1-\rho}\right)\left(\frac{1-\rho^{-(L+1)}}{\rho-1}\right)Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1}\rho^{|L-p+1+i-j|}\,Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\rho^{p}Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\sum_{i=1}^{L+1}\rho^{|p-i|}\,Var(D)$$

$$-2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2\left(\frac{\rho-\rho^{L+2}}{1-\rho}\right)Var(D)+2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2\left(\frac{\rho-\rho^{L+2}}{1-\rho}\right)Var(D)$$

$$+2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{\rho^{p+1}-\rho^{L+p+2}}{1-\rho}\right)Var(D)$$

$$+2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\sum_{i=1}^{L+1}\sum_{j=1}^{L+1}\rho^{|p+j-i|}\,Var(D)\,;Var\left(I_{int}^{k}\right)=0$$

■

Therefore, $Var(I^k)/Var(D)$ can be written as

$$\frac{Var(I^k)}{Var(D)}\geq\frac{Var\left(q^k\right)}{Var(D)}+h(L,p,\rho)$$

where $h(L,p,\rho)$ is

$$h(L,p,\rho) = \left(\frac{L_k}{p}\right)^2 \left[p + 2p\left(\frac{\rho - \rho^p}{1-\rho}\right) - 2\rho\left(\frac{(p-1)\rho^p - p\rho^{p-1} + 1}{(1-\rho)^2}\right)\right] + \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2$$

$$+ \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2$$

$$+ \left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \left[(L+1) + 2L\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right)\right.$$

$$\left. - 2\rho\left(\frac{L\rho^{L+1} - (L+1)\rho^L + 1}{(1-\rho)^2}\right)\right]$$

$$+ \left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \left[(L+1) + 2L\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right) + 2\left(\frac{\rho - \rho^{L+1}}{1-\rho}\right)\right.$$

$$\left. - 2\rho\left(\frac{L\rho^{L+1} - (L+1)\rho^L + 1}{(1-\rho)^2}\right)\right]$$

$$+ 2\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{L_k}{p}\right)\left(\frac{\rho - \rho^{p+1}}{1-\rho}\right) - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{L_k}{p}\right)\left(\frac{1 - \rho^p}{1-\rho}\right)\right)$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{|L+1-p|} - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+1}\right)$$

$$- 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+1} - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\rho^{L+p+1}\right)$$

$$- 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{1 - \rho^{L+1}}{1-\rho}\right) - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{\rho^p - \rho^{L+p+1}}{1-\rho}\right)\right)$$

$$- 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\left(1 + \frac{\sum_{i=1}^{k} L_i}{p}\right)\sum_{i=1}^{L+1} \rho^{|L-p+1-i|} - \left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{1 - \rho^{L+1}}{1-\rho}\right)\right)$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=L-p+2}^{L+1} \rho^{|i|} - 2\left(1 + \frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\rho^{L+1}\left(\frac{\rho - \rho^{p+1}}{1-\rho}\right)$$

$$- 2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\rho^{L+1}\left(\frac{\rho-\rho^{p+1}}{1-\rho}\right)\left(\frac{1-\rho^{-(L+1)}}{\rho-1}\right)$$

$$- 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\sum_{i=1}^{p}\sum_{j=1}^{L+1}\rho^{|L-p+1+i-j|} - 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\rho^{p}$$

$$- 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\sum_{i=1}^{L+1}\rho^{|p-i|} - 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2\left(\frac{\rho-\rho^{L+2}}{1-\rho}\right)$$

$$+ 2\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2\left(\frac{\rho-\rho^{L+2}}{1-\rho}\right)$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{\rho^{p+1}-\rho^{L+p+2}}{1-\rho}\right)$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(1+\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\sum_{i=1}^{L+1}\sum_{j=1}^{L+1}\rho^{|p+j-i|}$$

$$h(L,p,\rho) = \frac{1}{(1-\rho)^2}\left(\frac{L_k}{p}\right)^2 [2\rho^{p+1} - p\rho^2 - 2\rho + p]$$

$$+ 2\frac{1}{(1-\rho)^2}\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)^2 \Bigg[\rho^{L+p+3} - \rho^{L+p+2} + 2\rho^{L+2} - 2\rho^{p+2} + 3\rho^{p+1} - \rho^p$$

$$- L\rho^2 - 4\rho + L + 2 - 2(1-\rho)^2 \sum_{i=1}^{L+1} \rho^{|p-i|} + 2(1-\rho)^2 \sum_{i=1}^{L+1}\sum_{j=1}^{L+1} \rho^{|p+j-i|}\Bigg]$$

$$- 2\frac{1}{(1-\rho)^2}\left(\frac{L_k}{p}\right)[\rho^{L+p+3} - \rho^{L+3} - \rho^{p+2} + \rho^2]$$

$$+ 2\frac{1}{1-\rho}\left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left(\frac{L_k}{p}\right)[\rho^{p+1} - \rho^p - \rho + 1]$$

$$+ 2\frac{1}{(1-\rho)^2}\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\Bigg[(1-\rho)^2\rho^{|L+1-p|} + \rho^{L+p+3} - \rho^{L+p+2} + \rho^{L+3} + \rho^{L+2}$$

$$- 2\rho^{p+2} + 3\rho^{p+1} - \rho^p - (L+2)\rho^2 - \rho + L + 1$$

$$+ (1-\rho)^2\left(\sum_{i=1}^{L+1}\sum_{j=1}^{L+1}\rho^{|p+j-i|} - \sum_{i=1}^{L+1}\rho^{|L-p+1-i|} - \sum_{i=1}^{L+1}\rho^{|p-i|}\right)\Bigg]$$

$$+ 2\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\Bigg[\rho^{|L+1-p|} + \rho^{L+p+1} - \sum_{i=1}^{L+1}\rho^{|L-p+1-i|} - 2\rho^{L+1}$$

$$+ \left(\frac{\rho^p - \rho^{L+p+1}}{1-\rho}\right)\Bigg]$$

$$+ 2\left(\frac{\sum_{i=1}^{k} L_i}{p}\right)\left[\rho^{L+p+1} - \rho^{L+1} - \left(\frac{\rho^{L+p+1} - \rho^{L+1} - \rho^p + 1}{1-\rho}\right)\right]$$

$$- 2\frac{1}{(1-\rho)^2}\left(\frac{\sum_{i=1}^{k-1} L_i}{p}\right)\left(\frac{L_k}{p}\right)\Bigg[\rho^{L+p+3} - \rho^{L+3} - \rho^{p+1} - \rho$$

$$+ (1-\rho)^2\left(\sum_{i=1}^{p}\sum_{j=1}^{L+1}\rho^{|L-p+1+i-j|} - \sum_{i=L-p+2}^{L+1}\rho^{|i|}\right)\Bigg]$$

$$+\frac{1}{(1-\rho)^2}\left[2\rho^{L+2}-(L+2)\rho^2+L\right]$$

■

This bound is tight when $z_i = 0$ for $i = 1, 2, 3, \ldots, k$, when $k \geq 2$.

VITA

Rattachut Tangsucheeva

Rattachut Tangsucheeva graduated from Chiang Mai University in Thailand with a Bachelor degree of Electrical and Telecommunication Engineering in 1999. After his graduation, he worked as a Production Engineer at Daikin Industry Co., Ltd. and a Technical Support Engineer at SKF Co., Ltd. before he came to United States of America to pursue his Master's degree.

He graduated from University of Southern California in Los Angeles, California, with an Outstanding Academic Achievement Honor of Master of Science in Engineering Management in 2005. He joined the DISCRETE laboratory at the Pennsylvania State University in 2009 to research supply chain optimization, cash flow forecast, cash flow analysis, and inventory management. Throughout his study at Penn State, he is also a teacher assistant of Financial Services Engineering (IE 597A) and Retail Services Engineering (IE 478) in several semesters. He won the best paper award in College of Engineering Research Symposium (CERS) from his paper, "Why firm run of out cash? A dynamic perspective" in 2013. Additionally, he was selected to present his dissertation in the First Annual Amazon PhD Symposium in Seattle, Washington in 2013.