STUDY OF THE RELATIONSHIP BETWEEN MACHINE COMPLIANCE AND GRINDING FORCES IN CYLINDRICAL GRINDING

A Dissertation in
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by
Theodore R. S. Deakyne

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The dissertation of Theodore R. S. Deakyne was reviewed and approved* by the following:

Eric R. Marsh  
Professor of Mechanical Engineering  
Dissertation Advisor, Chair of Committee

H. Joseph Sommer III  
Professor of Mechanical Engineering

Panagiotis Michaleris  
Associate Professor of Mechanical Engineering

Sanjay Joshi  
Professor of Industrial and Manufacturing Engineering

Karen A. Thole  
Professor of Mechanical Engineering  
Head of the Department of Mechanical and Nuclear Engineering

*Signatures are on file in the Graduate School.
Abstract

This work demonstrates a novel strategy for improving size and form control when grinding with abrasive wheels. A comprehensive math model is proposed to capture the relevant physics and is used in conjunction with in-process measurements to provide better workpiece quality. Key issues to the success of this approach include rigorous accounting of the many (nonlinear) sources of structural loop compliance and their interaction with the grinding force.

This work is particularly valuable in plunge and traverse grinding because any motion of the abrasive wheel translates directly into workpiece form errors. During a typical plunge grinding operation, the rapidly spinning abrasive wheel is advanced at constant rate into the slower workpiece. The literature documents many modeling attempts to predict the material removal as a function of these feeds and speeds. However, our novel experimental apparatus provides additional in-process information, most notably the grinding force, and clearly shows the inability of the traditional linear models to accurately predict workpiece size.

As will be shown, there are additional physics to consider as evidenced by the lag between grinding force and material removal. This lag is not fully predicted by existing models and is a key result of this research. The model presented here correctly predicts this apparent lag by including nonlinear phenomena such as the depth-dependent energy of material removal as well as nonlinear contact stiffness between workpiece and wheel.

The key contributions of this work are: 1) recognition of the errors of classical grinding efforts based on our results from a one-of-a-kind apparatus not available to other researchers; 2) inclusion of a physics-based description of the nonlinear relationship in material removal as a function of grinding parameters; 3) the prediction of workpiece size without off-line inspection; and 4) accurate simulation in the presence of ever-changing wheel condition, workpiece type, and grinding parameters.
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List of Symbols

\( k_s \)  The machine structure linear spring stiffness support of the wheel in Newtons per microns, p. 10

\( k_w \)  The machine structure linear spring stiffness support of the work in Newtons per microns, p. 10

\( k_e \)  The machine structural equivalent stiffness in Newtons per microns, p. 10

\( v_r(t) = r'(t) \)  Actual material removal rate from workpiece in microns per second, p. 47, 10

\( v_{wear} \)  Actual wheel wear rate in microns per second, p. 10

\( x_c(t) \)  The commanded position of the grinding machine in microns, p. 10, 59

\( v_f \)  Commanded infeed rate, p. 10

\( x(t) \)  The actual position of the grinding wheel in microns, p. 10, 59

\( v(t) \)  The actual wheel in-feed velocity in microns per second, p. 10

\( r(t) \)  Actual material removed from workpiece in microns, p. 10, 47,

\( \delta_m'(t) \)  Rate of change of the radial elastic deflection of the grinding machine in microns per second, p. 10

\( \delta_m \)  Machine deflection in microns, p. 12

\( a \)  Wheel depth of cut, p. 12
The grinding force coefficient which relates the component of the grinding force acting normal to the cut surface to the depth of cut in Newtons per micron, p. 59, 66, 71, 73

\( \eta_w \) The rotational speed of the workpiece in revolutions per second, p. 12

\( \tau_w \) The time it takes the workpiece to make one rotation in seconds, p. 12

\( \tau \) The system time constant in seconds, p. 60

\( F_n \) The normal component of the grinding force in Newtons, p. 66

\( F_t \) The tangential component of the grinding force in Newtons, p. 70, 71

\( w(t) \) Radial wheel wear in microns, p. 13

\( G \) Grinding ratio, no units, p. 13

\( b_s \) Width of the grinding wheel in millimeters, p. 13

\( d_s \) Diameter of the grinding wheel in millimeters, p. 13

\( \Delta r_s \) Depth in wheel radius wear in microns, p. 14

\( V_s \) Volumetric radial wheel wear for plunge grinding given in millimeters cubed, p. 14

\( V_w \) Volumetric radial removed from the work for plunge grinding given in millimeters cubed, p. 14

\( G \) Grinding ratio, no units, p. 13

\( \delta_k \) Deformation of the grinding contact area given in microns, p. 17

\( K_m \) Static stiffness of the machine’s structural loop in Newtons per micron, p. 37

\( K_{eq} \) Equivalent static stiffness of the machine’s x-axis slide and grinding spindle in Newtons per micron, p. 37

\( K_{spindle} \) Static stiffness of the machine’s work spindle in Newtons per micron, p. 37
$K_c$  The grinding constant found from actual measured data in Newtons, p. 73

$\tau_w$  Period of the workpiece rotation in seconds, p. 66

$K_{instr}$  Instrumented spindle's force coefficient in Newtons per micron, p. 26

$K_1$  Inverse compliance of the x-axis slide in the grinding-infeed direction in Newtons per micron, p. 37

$K_2$  Inverse compliance of the wheel spindle and overhang in the grinding-infeed direction in Newtons per micron, p. 37

$K_4$  Inverse compliance of the work spindle and overhang in the grinding-infeed direction in Newtons per micron, p. 37

$K_5$  Inverse compliance of the z-axis slide in the grinding-infeed direction in Newtons per micron, p. 37

$y$  Deflection of spindle and workpiece in micron, p. 49

$\mu$  Grinding force ratio, indirect information about the efficiency of grinding no units, p. 71

$e_t$  Total grinding energy in joules per millimeter cubed, p. 69, 71

$Q_s$  Specific volumetric removal rate in millimeters squared per second, p. 73

$Q$  Volumetric removal rate in millimeters cubed per second, p. 69

$h_{eq}$  Equivalent chip thickness in nanometers, p. 73
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Overview

1.1 Introduction

1.1.1 Problem statement

This dissertation seeks to solve the challenging problem of representing the physics of precision grinding with an efficient and accurate model. This work is motivated by the particular importance of abrasive grinding processes to impart size and form accuracy while maintaining good surface roughness, particularly in hardened steels, ceramics, and optical glasses, which cannot be economically machined by other processes. The grinding process is complex and fickle, but is ultimately dictated by the interface between the abrasive wheel and workpiece. As will be seen, deflection in a grinding machine’s structural loop, both steady-state and time-varying, complicates the goal of achieving suitable control of the wheel/work interface and therefore workpiece quality.

Decades of research identify the numerous parameters involved in the grinding process and the sometimes subtle interactions taking place between them. Given these many contributing parameters, it can be extremely difficult to distinguish between cause and effect in order to explain observed behavior [2]. There is universal agreement in the grinding community that process parameters (i.e., peripheral speeds, feeds, depths of cut, along with proper wheel conditioning) need to be represented in a comprehensive and accurate model [2, 3, 4, 5]. However, this task is difficult and is generally not considered to be close to completion.
This work differs from previous efforts in its use of one-of-a-kind instrumentation that allows more direct access to the key physics of a particular grinding process. This instrumentation reflects over ten years of refinement, beginning with work by Knapp and Couey [6, 7]. This dissertation describes that application of this instrumentation to the problem of measuring, quantifying, and modeling precision grinding by focusing on the interaction between the forces and resulting deflection generated at the workpiece and abrasive wheel interface.

1.1.2 Research aims

The goal of this research is to capture the physics of grinding using the unique hardware at our disposal in a robust model. The ultimate objective is to develop practical means for improving workpiece quality. Here a model is developed that relies upon previous work as well as insight gained from our unique hardware to improve the predicted material removal from the workpiece. This work includes thoughtful comparison of actual grinding data to results predicted by the model.

This research recognizes that relative displacement between the grinding wheel and workpiece is the primary issue and leads to size and form errors as well as objectionable visible patterns on ground workpieces. In the first part of this work, the relationship between a machine’s structural loop compliance and workpiece quality are reviewed. The second, main part of this work develops and experimentally verifies a model of the grinding system based upon knowledge about the relationship between grinding forces and machine compliance during precision grinding.

1.2 Effects of machine compliance

Precision grinding is carried out in a number of machine configurations including cylindrical, internal, and surface grinding. The kinematics of each configuration influences the characteristic surfaces resulting from the inevitable machine compliance. In each case, accurate size, shape, and surface texture can be achieve with tolerances specified in microns (µm) if the compliance is understood or if corrected by ad hoc methods. For high precision applications and/or small parts including contact lenses, optics, electronic components, silicon wafers, etc. tolerances can be
extended to sub-micron levels. Surface roughnesses for a ground part can be reduced down to mirror finishes and optical quality of flatness [8]. For lack of suitable models, a certain amount of iteration is required to achieve the desired results.

Figure 1.1. Examples of the interaction between the machine tool structure and those of the abrasive grinding process: (a) surface grinding mild steel, (b) rotary grinding 416 stainless steel, (c) surface grinding, (d) OD grinding stainless steel.

Machine compliance leads to challenges in achieving adequate process control and is manifested as a reduction in form accuracy as well as increases in surface roughness. Furthermore, time-varying deflection (i.e., vibration), shorten the life of the grinding wheel and prevent a machine tool from operating at its full capacity. Often these problems are difficult to eliminate even if the sources are known [9, 10].

The best known example of structural loop compliance is an instability known as regenerative chatter and the attendant chatter marks. They are visible to the naked eye and show repeatable structure as seen by the examples in Figure 1.1.
This visible deterioration in surface quality is clearly objectionable to the human eye, but can be difficult to quantify using modern visible light inspection techniques because of its characteristic wavelengths. While the amplitude of grinding chatter waves is generally small, it can be detected with contact stylus instruments.

![Figure 1.2. Chatter marks on a tungsten carbide workpiece.](image)

Surfaces of ground workpieces can also show other types of visible patterns such as diagonal grids or parallel lines resulting from the phasing of vibrations as shown in Figure 1.2. These patterns impair the aesthetic of the parts causing customer rejection, but in some cases, such as in molds or dies, may also cause defects in finished products when these patterns are reproduced, such as molded transparent plastic parts [11].

1.3 The effect of structural loop compliance on workpiece quality

This section addresses the well-known and most completely documented result of structural loop compliance to further motivate the importance of this dissertation. The results shown here turned out to be particularly relevant to this dissertation.
as some of these very issues were encountered and hampered early progress in the modeling effort.

Figure 1.3. Pattern changes resulting from a work speed change. The pattern would change if the vibration was a forced vibration (i.e., not chatter).

Excessive structural loop compliance may be recognized by a visible, facet-like structure on a ground surface. If the pattern’s wavelength corresponds to the wheel speed, the driving energy source is one of several possibilities including: 1) wheel or spindle imbalance, 2) geometrical runout of the (untrued) wheel, 3) local high spots and cutting variations in the grinding wheel, and 4) periodic excitation from sources such as bearing-induced vibration, floor vibrations, and hydraulic oil pressure pulsations, etc [9, 12]. Surface patterns that are not explained by these three phenomena are most likely due to self-excited vibrations.
The importance role of compliance in a grinding machine was encountered unexpectedly in the course of this work. Both unbalanced wheels and out-of-true abrasive wheels were found to be causing visibly poor workpiece surface finishes. Usually these two conditions are coupled and can be difficult to remove. An example is shown in Figure 1.2, where 26 complete waves were counted around the part’s circumference. In the example shown in the figure, rotational speed of the wheel was 3120 rpm and the work was 120 rpm. Changing the ratio of wheel rotational velocity to workpiece rotational velocity to non-integer ratios reduced the patterns’ severity [13]. Figure 1.3 shows how the visible pattern is affected by a wheel speed change of a few rpms during a traverse grinding operation.

In preparation for our work the solution was to balance the wheel/hub assembly using set screws in the hub assembly as shown in Figure 1.4. The weight of the set screws were varied down to 0.05 grams as shown in Figure 1.5. A certain amount of iteration between balancing and truing was necessary to provide a truly round and balanced wheel. Once balanced a final truing pass with a silicon carbide wheel was taken. This was a second truing operation to remove the geometric run out caused by truing an unbalanced wheel. Finally, the balance was checked one final time with the feedback error position of the machine’s x-axis while the grinding spindle was rotating at speed.

The observance of forced vibration in our early experiments provides a good
Original screw 0.52 g
Filed screw 0.57 g
Reduced error amplitude of x-axis by 70 percent at wheel speed 3600RPM (60Hz)

Figure 1.5. Screen shot of the controller interface showing the FFT of the x-axis error position. Set screws were used to adjust the balance of the wheel. The error position is given in counts where 1 ctn = 0.927 nm.

opportunity to explore the sensitivity of the in-process measurement capability of our test apparatus. The output of the instrumented spindle, shown in Figure 1.6, shows the measured force for different speed ratios. In this figure, the plots are made by unwrapping the measured force signal from around the circumference of the work. The y-axis shows increasing time and the z-axis corresponds to the measured grinding force. In the initial part of each grinding cycle, the wheel is not in contact with the work as shown by the blue section of the plots starting at the
first revolution. As the wheel engages the work, forces develop as shown in the multi-colored section. Eventually, the wheel traverses off the back of the work and the forces cease, shown in blue. In each trial of the three grinding tests the wheel speed is 2710 rpm and the traverse speed is constant. The work speed of 387 rpm creates an integer speed ratio of 7 shown in the middle plot with 7 distinct lines. The plot above and below are non-integer speed ratios resulting in the 7 line pattern being mostly cancelled. While speed ratios can reduce the workpiece waviness, they can also add frustration in identifying the class of forced vibration. The correct solution is to have a perfect wheel both trued and balanced [14, 15, 16]. The calibration, capabilities, and use of the instrumented spindle and data collection will be discussed in Chapter 2.

1.4 Machine tool structural compliance

During grinding, the grinding forces cause deformation and deflection of the machine, the grinding wheel, and the workpiece. In some cases, the deflection between the wheel and workpiece may exceed, possibly significantly, the depth of cut of the spinning abrasive wheel. A spark-out, or dwell cycle, is commonly used to minimize some of these inaccuracies. In theory, additional asynchronous revolutions of the work and wheel with no further infeed should recover some of the elastic deflection at the expense of total cycle time [17, 18, 19, 20, 21]. As will be shown, this is not generally as effective as may be believed. However, it is important to take note of previous work, so this section shows the classic linear analyses that is used in an attempt to account for the differences between the machine’s infeed and actual stock removal for continuous infeed operations. As will be seen, the math that follows is not particularly good at predicting the observed behavior of real grinding.

1.4.1 Grinding deflections

The classic cylindrical plunge grinding model assumes continuous infeed and attempts to account for the difference between a grinding machine’s commanded (i.e., CNC programmed) infeed and the actual material removed from a workpiece.
Figure 1.6. Normal grinding force measured at a wheel speed of 2710 rpm, a depth of cut of 1.2 microns/rev, and traverse speed of 55 mm/min. The work speed was (a) 371 rpm, (b) 387 rpm, and (c) 403 rpm.

Some of the assumptions and calculated parameters will carry over to the new force model proposed in Chapter 3.

In its simplest form, the structural loop of a cylindrical plunge grinding machine may be modeled as three springs in series as shown in Figure 1.7 [17, 20, 22]. The equivalent system stiffness $k_e$ at the point of contact is given by
\( k_w = \text{Machine structure support stiffness of the work} \)
\( k_s = \text{Machine structure support stiffness of the wheel} \)
\( k_a = \text{Material + flexible contact stiffness} \)
\( x_c(t) = \text{Commanded infeed} \)
\( r(t) = \text{Material removed} \)

During grinding with a controlled infeed \( x_c(t) \), a time-dependent radial infeed velocity \( v_f \) such that \( x_c(t) = v_f t \) is input to the process. The actual velocity corresponding to the radial size reduction rate of the workpiece \( v_r(t) \) lags the radial infeed velocity \( v_f \) due to structural loop compliance in the machine. Continuity requires that the difference between the controlled rate \( v_f \) and the actual material removal rate \( v_r(t) \) be equal to the time rate of change of the radial elastic deflection \( \delta_m(t) \) of the grinding system plus the wheel wear rate \( v_{wear}(t) \) [17, 18, 19, 20, 21].

\[
\dot{v}_f - v_r(t) = \dot{\delta}_m(t) + v_{wear}(t) \tag{1.2}
\]
Some authors do not make the distinction between the actual material removed \( r(t) \) and the actual wheel infeed \( x(t) \) and the actual material removal rate \( v_r(t) \) and the actual wheel infeed velocity \( v(t) \). We found this to be a significant error as the wheel and work can make contact but no material is removed from the work until the attendant force grow larger than the plowing and rubbing threshold. Eventually the infeed rate and the material removal rate converge to the same value \( v_f \) at steady-state but not initially. This error in the classic analysis can lead to a time discrepancy and confusion as shown in Figure 1.8. The important note here is, if the equivalent stiffness of the machine is used, than the equation can only be solved for the actual material removed, not the actual infeed and will not account for plowing and friction with no measurable material removal. This will
be shown later in Chapter 3.

For the idealized model, the deflection is given by

\[ \delta_m = \frac{F_n}{k_e} \]  

where \( F_n \) is the normal grinding force component.

A linear cutting force model is used such that the normal grinding force is equal to the product of an empirical grinding coefficient \( k_c \) and the wheel depth of cut \( a \). This model has been adopted by nearly all metal cutting researchers [4, 5, 8, 13, 17, 20, 1, 23, 24, 25, 26, 27].

\[ F_n = k_c a \]  

This model performs well, but only after the grinding has progressed beyond the initial contact of the wheel with the workpiece [21]. The instantaneous depth of cut \( a \) is equal to the actual material removal rate \( \dot{v}_r \) divided by the rotational speed \( \eta_w \) of the work in revolutions per second such that

\[ a = \frac{\dot{v}_r(t)}{\eta_w} = \dot{v}_r(t)\tau_w \]  

where \( \tau_w \) is the time taken by the work to make one rotation. Taking Equations 1.3, 1.4, 1.5, and substituting them into Equation 1.2 leads to:

\[ \frac{k_c\tau_w}{k_e}\dot{v}_r(t) + \dot{v}_r(t) = v_f - v_{wear}(t) \]  

where

\[ \frac{k_c\tau_w}{k_e} = \tau \]  

\( \tau \) is a grinding characteristic time constant. Typical time constants fall between 0.5-1 seconds for external grinding and between 1-10 seconds for internal grinding [17]. Longer time constants are usually obtained for internal grinding, owing mainly to a much lower wheel support stiffness. The wheel in internal grinding is usually mounted on the free end of a long and narrow shaft to grind internal features such as holes [17].

The radial wheel wear rate \( v_{wear}(t) \) is assumed to be characterized by a grind-
ing ratio $G$, which is defined as the ratio of volumetric stock removal to wheel consumption (due to wheel wear). For cylindrical plunge grinding

$$G = \frac{\pi d_w b_s v_r(t)}{\pi d_s b_s v_{wear}(t)} = \frac{d_w v_r(t)}{d_s v_{wear}(t)}$$  \hspace{1cm} (1.8)

where $b_s$ is the width of the grinding wheel and $d_s$ is the diameter of the grinding wheel. Combining Equations 1.7 and 1.8 into Equation 1.6 yields the first order system

$$\tau' v_r(t) + v_r(t) = v'_f$$  \hspace{1cm} (1.9)

where

$$\tau' = \frac{\tau}{1 + \frac{d_w}{d_s G}}$$  \hspace{1cm} (1.10)

and

$$v'_f = \frac{v_f}{1 + \frac{d_w}{d_s G}}$$  \hspace{1cm} (1.11)

The standard form of the ordinary differential equation with constant infeed $u'$ is given as

$$\dot{v}_r(t) + \frac{1}{\tau'} v_r(t) = \frac{1}{\tau'} v'_f$$  \hspace{1cm} (1.12)

Multiplying the standard form of the ODE by the integrating factor and integrating, solving for $v_r(t)$ gives the solution of

$$v_r(t) = v'_f + C e^{-t/\tau'}$$  \hspace{1cm} (1.13)

where $C$ is a constant of integration. At this point the rate of workpiece radius reduction (material removal rate) is usually said to be equal to the actual infeed velocity if the wear of the wheel is ignored [17, 22]. This assumption, made throughout the literature of grinding modeling, is simply not true because of the inherent nonlinearity between the specific grinding energy. This nonlinearity, which we confirm experimentally, invalidates Eq. 1.4. This means that the previously customary statement is false so that:
\[ \dot{r} = v_r(t) \neq v(t) \] (1.14)

Neglecting this error, we complete the analysis by solving for \( r(t) \) the accumulated reduction of the workpiece radius. For controlled infeed velocity \( v'_f \) input to the machine, the size-reduction process is described by Equation 1.12 and 1.14 together with the initial conditions

\[ v_r(0) = v_0 \] (1.15)

and

\[ r(0) = r_0 \] (1.16)

such that

\[ \dot{r}(t) = v'_f(1 - e^{-t/\tau'}) + v_0e^{-t/\tau'} \] (1.17)

Integrating and substituting the initial conditions for the material removed and rate gives an equation for the total material removed from the work:

\[ r(t) = v'_f(t - \tau' + \tau' e^{-t/\tau'}) + v_0(\tau' - \tau' e^{-t/\tau'}) + r_0 \] (1.18)

### 1.4.2 Grinding wheel and work wear

The wear of a grinding wheel is usually expressed as a volumetric loss of material. For plunge grinding, the volume of radial wheel wear is simply

\[ V_s = \pi d_s b_s \Delta r_s \] (1.19)

where \( \Delta r_s \) is the measured decrease in wheel radius, \( d_s \) is the mean of the wheel diameter before and after wear has occurred, and \( b_s \) is the grinding wheel width.

The volumetric material removed from the work during a plunge grinding operation is expressed as

\[ V_w = \pi d_w b_s \Delta r \] (1.20)
The volumetric material removed from the work is compared to the wear of the wheel by the grinding ratio $G$ such that

$$G = \frac{V_w}{V_s}$$

(1.21)

The so-called $G$ ratio is commonly used as an index to characterize wheel performance. $G$-ratios cover an extremely wide range of values. On vanadium-rich high speed steels, $G$-ratios less than unity may be obtained in which case the work appears to be grinding the wheel. At the other extreme, $G$-ratios of above 60,000 have been reported for internal grinding of bearing races using superabrasive wheels such as CBN [17].

The important consequence to take away from the grinding ratio is that the actual stock removal rate will be less than the commanded infeed velocity since part of the infeed motion corresponds to the shrinking wheel diameter as it wears. This discrepancy becomes significant with small grinding ratios as given in the grinding infeed and time constant from Equations 1.10 and 1.11.

Wheel wear is difficult to measure and the current setup does not address this shortcoming. This is likely an acceptable compromise for the conditions used in the work that follows because of the high hardness of the superabrasive (diamond) wheels and free-grinding workpiece material (tungsten carbide). Furthermore, this work is based entirely on short experimental grinding cycles (at most one minute long).

One interesting difference between production and this experimental work is our more frequent dressing of the wheel’s cutting surface so that the grinding conditions may be accurately known and controlled. In production, the redress/retrue rate is minimized for efficiency (many grind cycles between wheel conditioning steps) whereas in this work the wheel is refreshed between each cycle.

### 1.4.3 Grinding vibrations

The literature makes extensive use of a particular model. This model is linear, but does usually include a delay term to account for the difference in material removed between two consecutive revolutions of the workpiece. In some cases, this leads to variations in the local depth of cut during successive passes of the wheel, thereby
causing regenerative undulations or lobes on the workpiece. So while the classic model is good for predicting regenerative chatter it is inadequate for maximizing workpiece quality in stable processes [15, 16, 17, 28].

\[ r(t) + r_s(t) = x_c - \delta_k - \delta_m \]

\[ r(t) + r_s(t) = x_c - \delta_k - \delta_m \]

**Figure 1.9.** Closed loop representation of plunge grinding operation.

The classic linear grinding chatter model from Figure 1.7 is redrawn in a block diagram form, shown in Figure 1.9 and appears in papers dating back to 1969 [1]. Included in the model is a continuity condition analogous to Equation 1.2 and dictates that the material removed from the workpiece \( r(t) \) plus the wear of the grinding wheel \( r_s(t) \) must, at all times, be equal to the commanded infeed \( x_c(t) \) minus both the deformation of the area of contact \( \delta_k(t) \), and the machine deformation \( \delta_m(t) \).
\[ r(t) + r_s(t) = x_c(t) - \delta_k(t) - \delta_m(t) \quad (1.22) \]

Also included in the model is the customary proportional relationship between normal force and actual depth of cut analogous to Equation 1.4. The compliance of the machine is the inverse of its stiffness and in the previous section was defined as \( k_e \) but this was related to a zero excitation frequency or static stiffness. In this model the dynamic response of the machine structure is introduced in terms of its directional frequency response, which can be considered as the dynamic deflection due to a frequency-dependent unit excitation force between the wheel and the workpiece. This response has to be determined experimentally, usually by experimental modal analysis in which several modes are detected. However, good results are obtained by retaining just the main mode and including the effect of higher order modes on lower frequencies using a residual flexibility \[23\].

The relationships between vibrations, depth of cut, grinding force, machine system stiffness, and workpiece shape is represented by the block diagram shown in Figure 1.9 as demonstrated by Snoeys et al \[1\].

Using the dynamic grinding model proposed by Snoeys et al. the effect of machine stiffness can be simulated and grinding stability analyzed. The threshold condition between stable and unstable behavior of the grinding machine can be derived by applying classical feedback techniques to the mathematical model. The limiting stability condition which is obtained can be written as

\[ \left| Re_m \right| \leq \frac{1}{2k_c} \left(1 + \frac{v_w}{v_s}G\right) + \frac{1}{k_a} \quad (1.23) \]

Where \( Re_m \) is the negative real part of the machine response and \( v_w \) and \( v_s \) are the workpiece and wheel surface speed velocities. The left side of this equation is considered to represent the dynamic compliance of the machine in the direction of the normal force. On the right hand side, the first term equals half the combined cutting and wheel-wear compliances and the second is the contact compliance.
1.5 Superabrasives grinding wheels

Grinding depends on the cumulative action of indeterminate cutting edges provided by abrasive grits to remove material at high speed and improve or modify the shape, dimensions, and/or the surface quality of the workpiece. For this work, resin-bonded diamond abrasives are used for their excellent hardness as shown in Figure 1.10.

The term superabrasive stems from the General Electric Specialty Materials Department to differentiate the hardest abrasives known, i.e., diamond and cubic boron nitride, from conventional abrasives such as aluminum oxide and silicon carbide. Superabrasive wheels are used for extremely difficult to grind materials with high hardness, toughness, and resistance to abrasive wear like tungsten carbide, high speed steel, glass, and ceramics [2, 29].

![Hardness of Abrasives Including Selected Materials](image)

**Figure 1.10.** Indentation hardness of abrasives and some selected materials.

Wheel preparation is critical in avoiding grinding problems as shown previously in this chapter. Preparing a wheel prior to grinding is much more complicated for
superabrasives than conventional wheels. Preparations for superabrasives require two distinct operations, truing and dressing. Unlike conventional wheels used in production, which can be dressed and trued at grinding speed in a few quick passes using a single point diamond tool at about 5-25 microns per pass, the superabrasive wheel truing operations requires a second spindle with a silicon carbide wheel spinning at 3000rpm with respect to a slowly rotating diamond wheel at 100 rpm at 2 microns per pass. Once the wheel’s geometry is such that the rim, in its entirety will contact the workpiece during each wheel revolution, only then the wheel is said to be trued.

Dressing, as distinct from truing, is a separate surface topography conditioning of the grits to ensure the grits’ cutting edges are protruding above the bond. This was accomplished using a Norton vitrified soft aluminum oxide abrasive 400 grit dressing stick [NMVC400-J5VCA] which was applied by hand pressure to the diamond wheel at grinding speed. The dressing stick is applied by hand until the stick rapidly wore away indicating the grits are protruding sufficiently to guarantee proper cutting action.
Development of the Test Apparatus

2.1 Introduction

Compliance is widely considered one of the most important considerations for ultraprecision machines [30, 31, 32]. The material removed from the work, grinding force, surface finish, and workpiece form are all affected by the compliance of the machine structure. To understand the effects of input parameters on the process outputs the compliance of the machine, static and dynamic stiffness of the structural loop, is investigated.

In this chapter, the grinding setup is introduced and the wheel-work-spindle-machine structural dynamics at the grinding interface are determined. The dynamics of the process will be formulated using modal fitting from the measured spindle-machine receptances at the grinding interface. The measured parameters of the grinding process, grinding forces, and the radial material removed from the work will be discussed.

As shown by Knapp [33], the approach taken here is required considering common grinding configurations, such as inner diameter or outer diameter grinding, are difficult to instrument because of the duel rotating bodies of both the workpiece and abrasive wheel. Therefore, the motivation for developing and building custom instrumented spindles is found to meet many of the challenges for precision grinding.

The instrumentation in this work addresses the need for precision grinding feedback through the use of a spindle with embedded non-contact displacement
sensors that measure the relative tilt motion between the rotor and stator. Additional, non-contact/contact displacement sensors measure the commanded table position and the workpiece’s shrinking radius.

2.2 Setup

2.2.1 General

Figure 2.1. The grinding testbed with the instrumented spindle.
The custom machine tool for grinding experimentation is shown in Figure 2.1. The machine base is an ultra precision lathe [Nanotechnology Systems]. The axes are supported by fully constrained oil hydrostatic, box way slides and are controlled by linear motor drives with sinusoidal drive amplifiers. The system is suitable for both single point diamond turning and deterministic micro-grinding of optical components and has successfully diamond-turned a mirror with a surface roughness below 1.6 nanometers Ra (arithmetic average of absolute amplitudes) as measured by an optical white light interferometer, Zygo NewView 5000.

The grinding spindle is a Professional Instruments Model 4R Twin-Mount BLOCK-HEAD with a frameless, brushless DC motor chosen for its high stiffness and very low error motions (5 to 15 nm).

2.2.2 Instrumented spindle

The instrumented spindle in Figure 2.2 has previously demonstrated the force-sensing capabilities for workpiece contact detection, sensitivity in detecting workpiece defects, and detecting degradation of abrasive wheels [6, 33].

The spindle has been successfully used to detect workpiece touch-off force as low as 60 mN, and the measured forces have been shown to correlate well with radial depths of cut below 1 µm. It provides stable feedback sufficient to observe workpiece defects and detect quasi-static increases in force that result from long term loading and wear of the abrasive wheels [6, 33]. More recently, the instrumented spindle has been used for monitoring the real-time force feedback that allowed repeatable processing of cylindrical ground parts with tighter tolerances and control of workpiece diameters to 0.25 microns [21].

2.2.2.1 Calibration dependencies on the grinding process

The instrumented spindle’s grinding force calibration coefficient $K_{instr}$ is dependent on the active grinding surface and kinematics for the particular grinding process. In surface, external, and centerless grinding, the abrasive is on the periphery surface of the grinding wheel and the workpiece material is cut with this surface usually with a traverse feed. In the case of face grinding with axial feed, the abrasive is on the circular area of the wheel face, and a circular cross section is removed from
The displacement measurements of the rotor with respect to the stator are dependent on the location and orientation of the embedded capacitance sensors. The 1 V/µm Lion Precision capacitance probes (Model C2C), shown in Figure 2.3, can measure the axial and tilt displacements of the rotor but not radial. Also the direction and magnitude of the input load depends on the type of grinding process used and affects how the rotor will deflect relative to the capacitance sensors in the stator.

For cylindrical plunge grinding the wheel makes contact with the OD of the workpiece resulting in a normal and tangential grinding force as shown in Figure 2.3. The spindle behaves as a torsional spring with the sensors capturing the tilt deflections of the rotor. In a different operation, such as face plunge grinding, the wheel abrasives make contact with the front face of the work creating normal and tangential forces, but now the spindle behaves as a compression spring combined with a torsional spring [33]. In this case the sensors measure the combined effects of axial and tilt deflections of the rotor. Depending on the type of process, different coefficients are determined and multiplied to the measured displacements in order to monitor the grinding forces.
Figure 2.3. View of the embedded probes that measure the time varying displacement between the stator and the rotor during grinding. The grinding force is calculated using the measured displacement and the known stiffness of the spindle.

2.2.2.2 Response of the measuring force spindle

It is important to verify that the output information truly represents the input grinding forces seen by the work from the wheel. Therefore the ability to precisely sense, transmit, and present all the pertinent force information has been evaluated by the response characteristics of the instrumented spindle [34].

The frequency response of the instrumented spindle is one way to characterize the performance of the system. The compliance of the instrumented spindle is readily estimated using the capacitance probes in the stator. For the measurement shown in Figure 2.4, the spindle was completely assembled so that the dynamics include the effects of the mass of the motor and mass of the workpiece. A 500 N Kistler impact hammer (Model #9722A500) is used to excite the instrumented spindle while the response is measured with the embedded Lion Precision capacitance probes.

The region of the compliance for which the displacements are linear with the applied force defines the usable bandwidth of the force measurement. For a maximum acceptable deviation from the static compliance of 10 percent, the bandwidth of the force measurement is 200 Hz.
2.2.2.3 Verification of static calibration factor

The impulse force from the impact hammer was applied to the instrumented spindle at the same location where the wheel would make contact. The 0.019 μm/N (52.6 N/μm) compliance at 0 Hz is a result of the rotor tilt stiffness of the spindle. This value is the grinding force calibration coefficient $K_{instr}$ used for an external cylindrical grinding process.

The value of the calibration coefficient is explained using statics on the rotor of the air bearing spindle. For an external cylindrical plunge grinding configuration, the capacitance sensor measures a deflection $\delta_{rotor}$ when a force $F$ is applied to the rotor as shown in Figure 2.5. The applied force is countered by the torsional stiffness $k_t$ created by the air film between the rotor and stator as the rotor is tilted through angle $\theta$ such that
Figure 2.5. Cross section from the right side of the instrumented 4R airbearing work spindle. The I-shaped cross section (in yellow) is the spindle rotor. The capacitance sensors measure the back of the rotor as shown and are embedded in the stator (colored pink).

\[ F = K_{\text{instr}} \delta_{\text{rotor}} \]  \hspace{1cm} (2.1)

\[ FL - k_t \theta = 0 \]  \hspace{1cm} (2.2)

where \( K_{\text{instr}} \) is the inverse of the compliance measured in Figure 2.4 at 0 Hz. \( L \) is the axial distance from the capacitance probe target to the location of the input force. Using the geometry of the 4R spindle shown in Figure 2.5 the deflection measured by the capacitance sensors is related to the tilt angle of the rotor

\[ \delta_{\text{rotor}} = l \cos \theta + x \sin \theta - l \]  \hspace{1cm} (2.3)
where the length $l$ is half the rotor shaft, from the centerline to the thrust plate and $x$ is the radial length to the capacitance probe. Rearranging Equation 2.3 to solve for the rotor tilt angle in terms of deflection measured at the capacitance probe is complicated. Therefore, a simplification based on small angle approximation is made, $\delta_{\text{rotor}} = x\theta$. This is justified in the small deflections of the rotor measured by the sensors. Therefore the effective stiffness is equal to

$$ K_{\text{instr}} = \frac{k_t}{Lx} \ (2.4) $$

In this equation the length to the load $L$ and $x$ are known. The spindle tilt stiffness $k_t$ is 0.45Nm/µrads at 1035 kPa.

The final and positive proof of the instrumented spindle’s performance is direct measurement of the actual spindle’s response to a completely defined and known
input force from a force transducer as shown in Figure 2.6.

2.3 Machine structural loop

In this section the influence of the grinding machine’s stiffness and damping on the work accuracy and material removal rates will be considered. In particular the machine’s motion between the wheel and work in the direction of the normal grinding force component.

**Figure 2.7.** Structural loop of grinding machine set up for cylindrical plunge grinding.

An illustrated simplified structural loop of the grinding machine is shown in Figure 2.7. The infeed direction (x-direction) is supported by the x-slide with stiffness $K_1$ and the spindle stiffness $K_2$. The work is supported by the z-slide in the x-direction by stiffness $K_5$ and the spindle $K_4$. Between the work and the grinding wheel there is another stiffness $K_3$ which is a combination of the grinding wheel stiffness and abrasive grain penetration into the work. Through the following sections, the dynamics and compliance of the grinding machine’s simplified structural
loop will be examined.

2.3.1 Machine dynamic stiffness and damping

![Diagram of machine with axes and labels](image)

**Figure 2.8.** Measured vibrational response of the machine’s x-axis output

For the work here, modes of vibration that have relatively small influences on the accuracy of the process are not considered. The relative vibration between the two spindles is the critical point of interest for workpiece form. Therefore, the main elements of the cylindrical grinding machine force loop include the machine base,
axis tables, the grinding wheel spindle and components, work spindle and components, and the drivers used to control the system. The structural loop is simplified so that the x-axis and z-axis are each treated separately connected to ground (the machine’s granite base). The estimate of the system response is calculated from the direct and cross responses from two points on the machine. One point on the wheel attached to the x-axis and the second point from the work on the z-axis.

The frequency responses are measured in the XYZ coordinate system, but only the direction of the grinding infeed is of significance. For each axis a force-instrumented hammer [500 N Kistler impact hammer] is used to excite the axes and an accelerometer [5 G Kistler piezo triaxial accelerometer] is used to capture the response.

The accelerometer is glued to the center of the wheel hub on the x-axis. The input from the hammer impulse is located on the outer perimeter of the grinding wheel, where the work would make contact with the wheel. This process is then repeated for the z-axis on the work. The x-direction, as labeled in Figures 2.8 and 2.10, is the sensitive direction that will give insight to the behavior of the grinding force and deflection acting normal to the cut surface to the depth of cut.

The response of the machine’s x-axis from the grinding wheel hub is shown in Figure 2.8. This axis is responsible for setting the commanded depth of cut into the workpiece $x_c$. The hydrostatic ways support the x-axis in the vertical, y-direction, and perpendicular to the depth of cut, z-direction, while allowing the table to move freely in the x-direction. The x-direction motion is controlled by the linear motor and a PID controller using feedback from a Sony laser scale BS75A and BD15 detector. The scale has a 410 mm measuring length and the output position resolution of the x-axis table is 1 nm. This may lead one to believe that the part tolerances can be held quite easily down to the nanometer level with the CNC program and machine controller. However, it is important to make the distinction between where the table position is measured and where the actual grinding interface takes place as shown in Figure 2.9. The table position feedback is measured 508mm behind and 356mm below the grinding interface and does not account for the x-axis table twist or the deflection in the grinding or work spindles. Also the z-axis table position feedback only measures perpendicular to the x-direction so that the z-axis deflections in x are not accounted for by the machine’s
control. These displacements can lead to several microns difference between the commanded depth of cut and the actual amount of material removed, not including the time varying effects of wheel wear and temperature changes during the grind also not accounted for in the grinding controls.

The response of the machine’s x-axis is important for self excited vibration identification. The x-axis is commanded in the sensitive direction, normal to the depth of cut and the table’s feedback may not even detect the chatter occurring.

In addition to the two direct measured responses there are two cross measured responses. The cross response is where the input hammer is on one axis and the accelerometer output is measured on the other axis. Together these four measured responses can be used to calculate an equivalent machine response of the grinding interface. Comparing the cross to the direct responses, it shows that the cross components can be safely neglected as shown in Figure 2.11. In the legend the
Figure 2.10. Measured vibrational response of magnitude and phase for the machine’s x-axis output as a function of frequency in comparison to the input from the force instrumented hammer (µm/N).

The first letter corresponds to the axis with the accelerometer measuring the output while the second letter is the axis with the measured input from the hammer. Both the dark blue line (XZ cross) and dark green line (ZX cross) have an insignificant magnitude through most of the frequency range when compared to the directed measurements. The lower frequency peaks are believed to be the machine’s bed rocking (i.e., rigid body motion).
Figure 2.11. Total machine response

The resulting machine dynamics of the relative motion between the wheel and work are calculated by adding the real and imaginary components of the direct FRFs for the x and z-axis as shown in the bottom graph of Figure 2.11.

2.3.2 System identification

The experimental determined machine’s dynamic can be used to characterize the behavior of the dynamic deflection at the wheel and workpiece due to the grinding
force. As seen in Figure 2.11 several modes and modal frequencies were detected and all of them influence the dynamics of the machine. However, good results can be obtained with a simpler representation by retaining the main modes and including effects of higher order modes on lower frequencies using a residual flexibility, $1/k_r$.

The behavior of the machine flexibility can then be represented by the transfer function

$$H(s) = \sum_{i=1}^{N} \frac{\omega_i^2}{A_i(s^2 + 2\zeta_i\omega_i + \omega_i^2)} + \frac{1}{k_r} \quad (2.5)$$

The mass, damping, and stiffness of the system are unknown. While approximations can be made for these parameters of the grinding assembly, it is easier to use the measured response and estimate the modal frequency $\omega_i$, damping $\zeta_i$, and amplitude $A_i$ in equation 2.5 performing a modal fit to the measured responses. This is known as a peak picking method.

The peak picking method was used on the real and imaginary parts of the measured response of the system to identify the modal parameters and build the transfer function used in the Simulink model. This technique works best when the system modes are not in close proximity to each other, but even if two of the modes are relatively close, a reasonable modal fit can be obtained. The transfer function built with Equation 2.5 is shown in Figure 2.12.

### 2.3.3 Static stiffness

Most machine’s control systems are based on the concept of a rigid machine tool and workpiece. This methodology usually assumes that the machine tool-workpiece system has little or no compliance and that the programmed feed rates and geometric positions will be achieved. However, as shown in the previous sections, the feedback of the table position cannot capture the true position of the machining interface and the machine does not have infinite stiffness. Deflections occurring from the cutting forces and the machine’s, lack of ability to compensate for the combined deflections, leads to size errors of the part [19, 18].

In this section the machine’s static stiffness at the grinding interface is examined. A capacitance sensor is looped from the grinding spindle’s wheel hub and
aimed at the tungsten carbide part on the work spindle as shown in Figure 2.13. A second capacitance sensor is aimed at the x-table in the same location that the Sony detector feedback is located to measure the position. Between the wheel and work is a strain gauge calibrated to measure force. The grinding wheel on the x-axis is stepped at 1 \( \mu \text{m} \) increments into the part on the z-axis. Eventually, the strain gauge makes contact between the wheel and the work. As the table continues to step in the strain gauge is compressed and the part begins to lag the table position. The results from this test just when the strain gauge starts to make contact is shown in Figure 2.14. The differences between the commanded table position, at the sony detector location, and the actual position, work spindle (interface), is the deflection. From the deflection and the measured force during this test an estimated machine stiffness of about 5 N/\( \mu \text{m} \) was calculated.

A second static stiffness experiment was conducted without using the motor of the x-table to sandwich the wheel and work against the strain gauge. For this
second test an intertube was placed between the wheel and the strain gauge. The intertube was then filled with air, expanding between the wheel and work, pushing the two slides apart. The position of the work with respect to the wheel was measured along with the force as shown in Figure 2.15. The resulting static stiffness is the same order of magnitude as before and estimated at about 4 N/µm.

From these tests an idealized model of the grinding machine is built and shown in Figure 2.16. The left hand side represents the wheel side with three springs in series which are the slide stiffness, spindle stiffness, and the wheel contact stiffness including grit penetration of the abrasive grains into the aluminum of the strain gauges. This penetration stiffness, $K_3$, will be higher in actuality since the grains will be penetrating tungsten carbide instead of aluminum. The right hand side represents the work side. The z-slide stiffness in the infeed direction (x-direction) can be considered infinite when compared to the other spring structures in the loop stiffness. Therefore, only the spindle stiffness, $K_4 = K_{spindle}$, between the work and
Figure 2.14. Measurements of the force and deflection at the interface of the grinding machine.

z-slide is on the right hand side.

The impact hammer is used to find the compliance of the work overhang from the z-slide. Figure 2.17 is the response measured from the part’s probe with the hammer input at the same location where the wheel would make contact. The inverse of the given compliance is the stiffness value, $K_4 = 11 \text{ N/µm}$.

Free body diagrams of the model in Figure 2.16 helped create equations for the five unknowns $q_1, q_2, q_3, q_4,$ and $F$. The $q$s are displacements and $F$ is the force exerted between the wheel and work. On the wheel side the three equations are

\[
F = -K_1 q_1 \quad (2.6)
\]
\[
F = K_2 (q_1 - q_2) \quad (2.7)
\]
\[
F = K_3 (q_2 - q_3) \quad (2.8)
\]
These three equations can be combined to get one equation in terms of the equivalent stiffness of the wheel side $K_{eq}$ and $q_3$ by eliminating $q_1$ and $q_2$.

$$q_3 = -F \left( \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} \right)$$  
(2.9)

$$q_3 = -\frac{F}{K_{eq}}$$  
(2.10)

For the work side, the equation is

$$q_4 = \frac{F}{K_4} = \frac{1}{K_{spin}}$$  
(2.11)

The final equation is between the work and wheel interface
Figure 2.16. Idealized model for static stiffness measurement of the grinding machine

\[ \Delta q = q_4 - q_3 \]  

(2.12)

The final equation is made by combining Equation 2.11 and 2.10 into Equation 2.12 such that

\[ \frac{\Delta q}{F} = \frac{1}{K_{spin}} + \frac{1}{K_{eq}} \]  

(2.13)

Using equation 2.13 the equivalent stiffness, \( K_{eq} \) of the three springs in series on the wheel side was found to be equal to about 6.29 N/\( \mu \)m.

### 2.4 Data acquisition and analysis

A total of six non-contact cap sensors capable of high-resolution measurement of the position and/or change of position of any conductive target were used in this work. Four of these are permanently embedded in the stator of the instrumented spindle. A fifth measures the position of the x-slide and the sixth probe is coupled with an air piston to become a touch probe for the work material removal measurement. In typical capacitive sensing applications, the probe or sensor is one of the conductive objects, and the target object is the other. The capacitive sensors are
Figure 2.17. Compliance of the work spindle, $1/K_4$, between the z-slide and the overhang of the workpiece measured from the part probe.

able to measure changes in capacitance and translate these changes into distance measurements.

$$\text{Capacitance } \propto \frac{1}{\text{distance}}$$  \hspace{1cm} (2.14)

These capacitive sensors use an alternating voltage which causes the charges to continually reverse their positions. The moving of the charges creates an alternating electric current which is detected by the sensor. Capacitance is determined by the area and proximity of the conductive objects which effects the current flow measured.

The Lion Precision amplifier that comes with the sensors is calibrated to generate specific output voltage changes for corresponding changes in capacitance. These voltages are calibrated to represent specific changes in distance.
The continuous output voltage signals from the amplifiers are sampled using a computer on board National Instruments PCI-6032E analog to digital converter. The sampling of the continuous signal into a sequence of samples is triggered by the encoder of the work spindle. Each revolution of the work provides 1024 samples. The digital form of the sensors’ voltages signals are manipulated in MATLAB code.

The following sections discuss some of the data manipulations used to provide quantitative information on the actual grinding process.

2.4.1 Removal of rotor form error

The instrumented spindle used in this work has been rebuilt from the previous generation used in earlier work [33, 35, 36]. The main parts of the rebuild included replacement of the thrust plates for the rotor and installing a new Motion Control Systems motorization, model B13 similar to the Nanotech 5.5 ISO spindles, which is the premier work spindle used for diamond turning and grinding in the machining industry today.

As mentioned, the capacitance sensors embedded in the stator measure the back side of the thrust plates, component of the rotor, as shown in Figure 2.18 (a). As the rotor spins the probes measure an axially change in displacement represented by the circular plot shown in 2.18 (b). The measured displacements include the surface form of the rotor in addition to any displacement of the thrust plate due to the grinding forces. In order to determine the force applied to the instrumented spindle, the form of the rotor must be removed from the capacitance sensor output. The axial synchronous form error for one revolution of the rotor is shown in Figure 2.18(c). Deviations from this form during grinding are attributed to loads applied to the instrumented spindle in the radial direction. Algorithms written in MATLAB remove this form error. Figure 2.18(e) shows the remaining information after the form error is removed. Other spindles which use rolling element bearings would not readily allow this technique due to the higher asynchronous error motion involved with those types of bearings.

The previous form error was measured below 0.4 microns where the new spindle has increased over 0.16 percent. Measuring the axial runout over the front face of the thrust plate shown in Figure 2.18 (a) results in a total runout reading below a
Figure 2.18. (a) Instrumented spindle with exaggerated gap between stator and thrust plate. Probe 1 targets the hidden face. (b) Measured axial form error of the hidden face. (c) Unwrapped polar plot. (d) Multiple revolutions for a grinding operation. (e) Removed form error, deviations are attributed to loads.

A typical air gap between the stator and rotor is about a half micron. A form error of 2.6 µm peak to valley would cause major problems in an air bearing spindle, and depending on the high spots, could even cause a crash due to contact between the rotor and stator. However, this is not the case since the rotor spins freely with low
error motion. To explain this discrepancy the manufacturing of a thrust plate was examined.

The components of a standard 4R air bearing spindle are shown in Figure 2.19. The rotor is composed of two thrust plates connected by a shaft. The thrust plates are initially rough ground and a coating of Teflon is applied to the rough ground surface. The coating is only applied on the back of the thrust plate that faces the stator. A fine grinding process is then applied to this Teflon coated face. It is believed that the capacitance sensors embedded in the stator of the instrumented spindle are looking through the Teflon coating. The fine ground Teflon coating is a non-conductive material. The rough ground form error of the thrust plate is therefore the true target. The earlier version of this spindle was finely ground and lapped on both surfaces. Even with the larger synchronous form error it was still possible to isolate the displacements of the thrust plate due to the grinding forces.
2.4.2 Grinding force processing

![Diagram of force and revolutions](image)

**Figure 2.20.** Channel 1 measurement with synchronous error removed during a grinding cycle

A description and illustration of the post-collection data processing follows. The measured grinding forces attributed to the applied loads from the wheel on the work are found when the synchronous error (form error) is removed. Figure 2.20 shows the remaining signal after the synchronous error is removed. The angle is over the circumference of the workpiece, division is \(2\pi/1024\) (radians per sample). The revolutions are collected through time and the z-axis is the force in Newtons. The once-per-revolution average is shown in black for the entire grinding cycle. The dominant effect of the grinding force for this channel is low frequency, once-per-rev, and will effect the size of the ground part. The higher frequencies will effect the form and finish of the ground part.

During a grinding cycle all four channels, spaced 90 degrees apart, are used to
capture the process, as shown in Figure 2.21. A low-frequency drift was observed in the individual channels that is thought to be attributable to thermal growth effects of the rotor. Previously, this drift was removed using a second-order curve fit based on manually selected points. This method is susceptible to user error as it is difficult to select the appropriate points to build the curve. In previous work, the data acquisition card only had four channels, two channels occupied by the instrumented spindle and the other two were taken by the commanded infeed and actual material removed from the part. There was also limitations of the board due to real time control of the grinding process.

The four channels make two pairs. The probes 180 degree apart measure, roughly, the same event but in an opposite orientation as seen in 2.21. For example, channel 1 and channel 3 are a pair 180 degrees apart that measure the grinding force equal and opposite, while the thermal effect is the same in both
channels. This observation will allow a separation between the force and some of the undesired effects. Using the equations

\[ F_{\text{norm}} = \frac{-(Ch_1 + Ch_2) + (Ch_3 + Ch_4)}{4} \]  
(2.15)

\[ F_{\text{tan}} = \frac{(Ch_2 + Ch_3) - (Ch_1 + Ch_4)}{4} \]  
(2.16)

**Figure 2.22.** Processed once per revolution tangential and normal forces for a grinding cycle

the normal and tangential grinding forces can be calculated from the grinding force data. Figure 2.22 shows the tangential and normal grinding forces acquired from the four channels of the instrumented spindle.
2.4.3 Measuring the material removal

It is necessary to measure the workpiece radius in-process to determine the actual material removed. The actual depth of cut $\Delta r$ can be as low as a quarter of the programmed depth of cut $\Delta x_c$ depending on the workpiece hardness. The fraction also depends on grinding wheel sharpness, machine tool stiffness, grinding wheel stiffness, contact width, work speed, and wheel speed. All of these can affect grinding forces substantially resulting in deflection of a system [8].

\[
K_{spin} = K_4 \\
gap = y - r \\
\}
\text{probe} \\
\text{work} \\
F_r \\
\omega_w \\
d_w \\
y
\]

\textbf{Figure 2.23.} Measuring the actual material removed from the workpiece.

In this work the material removal is measured with a capacitance sensor coupled to an air piston with a diamond stylus used to touch the part during a grinding cycle. The probe measures the actual material removed from the part $r(t)$ as shown in Figure 2.23. In the figure the the probe is shown above the work’s center. In actuality the probe is align to the work’s center, and is moved above in the figure so it was not obstructed by the spring $K_{spin}$. 
Due to the compliance of the grinding system and the orientation of the probe, the material removed from the workpiece was not the only displacement measured. Instead, the measurement from this probe was called gap, to account for other displacements in the signal, as in the gap between the work and probe. The blue line in Figure 2.24 is the actual measured signal from the part probe. The raw signal from the probe measures a positive voltage as an object gets closer and a negative voltage as an object gets further away. Here, the signal has been inverted and a sensitivity factor applied to convert to microns. Initially, disturbing was the negative material removed for the first 7 to 10 seconds of the grinding cycle, which meant the part was increasing in size as the wheel was trying to remove material.

A initial increase in size, this large, has not been seen in the previous work with this spindle [21, 36, 37, 38] and was believe to be a result of the flood coolant used in this setup. For the previous work a mist was used instead. However, similar
experiments in Couey et al [6] disproved this theory.

The second hint came at the end of the grinding cycle where a large jump, part get instantaneously smaller, of about 1.5 microns as the wheel is quickly retracted as shown in Figure 2.24. Clearly from these pieces of information the probe has measured a combination of the material removed from the workpiece and the compliance of the work spindle/work overhang (rotor itself cannot deflect more than 0.5 µm without crashing).

In Figure 2.17 the compliance of the work spindle/overhang was found, inverting this compliance, the stiffness for the spindle and work overhang was 11 N/µm as measured from the part probe. In addition to compliance the grinding force experienced by the workpiece was measured by the instrumented spindle. The total deflection of the work spindle resulting from the grinding action is

\[ y = \frac{F_n}{K_{spin}} \]  \hspace{1cm} (2.17)

This spindle/overhang deflection can now be separated from the measured gap to provide the material removed from the work piece as

\[ r = gap - y \]  \hspace{1cm} (2.18)

The reason for these larger spindle deflections in this work compared to previous work was the material. The previous work used a mild steel workpiece with a soft, E-grade, aluminum oxide wheel. In this work the material is tungsten carbide ground with a diamond superabrasive wheel.

2.4.4 Start of material removal

Determining the point at which material removal begins is somewhat difficult. The raw data has an arbitrary DC offset determined by the capacitance probe amplifier, rendering direct comparison of unprocessed data meaningless as can be seen in Figure 2.25. The data is synchronized by the encoder of the work spindle and is aligned in time, but the Y-alignment requires additional consideration. One way to align the dataset in the Y-axis is to use the point at which the grinding wheel makes contact with the workpiece.
Instead of selecting the location on the measured gap data, the point at which the normal force begins to increase will be the point of contact. Razavi and Kurfess use a similar technique using a piezoelectric force transducer in their work to improve grinding automation [39]. Figure 2.26 shows the processed commanded infeed position $x_c(t)$ aligned with the material removed from the workpiece $r(t)$ using the grinding force as the point of contact the the force starts to increase.

In addition to the point of contact a pre-grind was used in every dataset to increase repeatability. Due to the system compliance the workpiece does not shrink as much as it is commanded to do so as shown in Figure 2.26. In this dataset there is a difference of 5.5 microns between what the operator commanded the workpiece size to be and what it actually is. Depending on the grinding parameters

**Figure 2.25.** Raw voltage signal from the capacitance probe amplifier
Figure 2.26. Using the grinding force the commanded infeed and the material removed is aligned in the Y-axis.

a discrepancy as high as 10 microns was measured. As a result, the next grinding experiment will start earlier than expected due to this size difference, i.e. the surface of the part is believed to be 5.5 microns further from the wheel than it truly is. The pre-grind is used to eliminate the guess work of grinding wheel touch off.

Initially, the wheel is fed in slowly to avoid crashing into the work, a 10 micron continuous infeed is used to remove the previous surface on the work, afterwards a long dwell period is used. Finally, the wheel is retracted and then the actual grinding experiment will begin as shown in Figure 2.27.

This method works quite well, as shown in Figure 2.27 for three different grinding cycles with the same grinding parameters. Initially, the measurements in the pre-grind are slightly different due to the uncertainty of the true location for the
Figure 2.27. Comparing the commanded/material removal alignment of three consecutive tests. Initial, during the first touch off, the grinding force varies as a result from the uncertainty of the work surface, i.e. part is slightly over sized from previous experiment work surface. After the pre-grind, the experimental data used in this work is very repeatability in that the table motion, force, and material removed from the work-piece measurements line up, which they quite clearly do.
Grinding model

3.1 Introduction

Grinding a batch of parts with the same CNC code and setup does not guarantee the parts will be identical down to the nearest micron. In fact, the parts will likely show a significant variation in size at the 2 to 5 micron level. These variations reflect the condition of the grinding wheel and its wear, variations in the initial size of the workpieces, grinding conditions, etc. Each of these fluctuating effects are mirrored in the grinding force and system deflections. In order to compensate for these errors, engineers have adopted control strategies based on process modeling (with limited success) along with in-process monitoring for adaptive size control.

As mentioned, this work focuses on the use of force as the parameter for quantifying the grinding process. We proceed by building upon work found in the literature and then provide verification using the unique hardware described in Chapter Two. After a brief review of previous models we compare their predictive capability with actual grinding process data.

3.2 Mathematical modeling

3.2.1 Conventional plunge grinding cycle

The simple grinding cycle shown in Figure 3.1, consisting of a constant velocity plunge followed by a spark-out dwell, is used throughout this work as a represen-
Figure 3.1. Conventional plunge grinding cycle measured on the current setup with a 177.8 mm diameter by 6.6 mm width 320 grit resin-bonded diamond wheel and 50.8 mm diameter tungsten carbide workpiece.

The grinding community has long used the equivalent chip thickness $h_{eq}$ to reduce the number of parameters that need to be tested in grinding trials. This quantity corresponds to the thickness of a continuous layer of material being removed at a volumetric rate $Q_s$ per unit width at a cutting velocity $v_s$. The equivalent chip thickness correlates with performance characters including the surface roughness as

$$R_a = C_1 \left( \frac{Q'}{v_s} \right)^{C_2}$$  \hspace{1cm} (3.1)$$

where $C_1$ and $C_2$ are experimentally determined constants. The exponent $C_2$ typically falls in the range of 0.15 to 0.6 [17]. Another critically important parameter effecting workpiece quality is the surface topography of the wheel, including the dressing conditions as well as the abrasive grit size and condition. For this work the
dressing and grit size is constant and the wheel is redressed at frequent intervals throughout all testing. The purpose of this nearly continuous wheel conditioning is to remove at least one variable from the list of contributing factors of workpiece variation.

As will be shown, and somewhat contrary to conventional wisdom, there is a limit to how much improvement in size accuracy can be achieved by using a lengthy dwell (spark-out) cycle. Also the spark-out significantly increases the process time reducing efficiency. These issues are among the biggest motivations for our development of a mathematical model capable of capturing these effects to predict and compensate for inaccuracies. These models also increase comprehension of the process to further increase product quality.

### 3.2.2 Lumped parameter machine model

A lumped parameter model of the grinding machine setup is shown in Figure 3.2. The geometry of this setup is representative of a broad class of grinding machines. The machine’s stiffness values that were measured in Chapter 2 are used extensively here as necessary values for the modeling effort.

Figure 3.2 shows six equations with six unknowns that may be found using a free body diagram analysis of the lumped parameter representation. These equations will be rearranged and built into a block diagram in MATLAB/simulink following some preliminary simplifications.

The displacement of the grinding wheel hub center $q$ is eliminated by rearranging Equation (2) in terms of $q$ and substituting into Equation (1) as

$$
q = \frac{F}{K} + x
$$

$$
\frac{F}{K_{eq}} = x_c - \frac{F}{K} - x
$$

Solving for the actual infeed $x$

$$
x = x_c - \frac{F}{K_{eq}} - \frac{F}{K}
$$

The material removed from the radius of the workpiece $r$ is found by elimi-
Work spindle with embedded probes to measure grinding force

\[ K_{eq} = \frac{K_{qy} K_q}{K_{qy} - K_m} \]

Equations

1. \( F = K_{eq} (x_c - q) \)
2. \( F = K (q - x) \)
3. \( F = K_{qy} (y) \)
4. \( y = x - r \)
5. \( \text{gap} = y - r \)
6. \( F = f(\Delta r) \)

Unknowns

\( F, q, x, y, r, \text{gap} \)

Figure 3.2. Unique model for a Moore/Nanotech Systems UPL350 lathe used in these experiments.

Rounded the displacement of the workpiece center \( y \) by substituting Equation 4 into Equation 3 such that

\[ r = x - \frac{F}{K_{spin}} \]  

At this point Equation 3.5 can be rearranged to solve for the actual infeed and substituted into Equation 3.4 to have an equation for the material removed, \( r \), and corresponding nonlinear process force \( F \). However, this is not done for this work to make the distinction between the actual infeed of the grinding wheel and the actual material removed from the workpiece which was previously shown in this work to not be equal.
Equations 3.4 and 3.5 can be re-written in a block diagram form as shown in Figure 3.3. The workpiece material removed in time is equal to the removed material one revolution before \( r(t - \tau_w) \), plus the instantaneous depth \( \Delta r \) where \( \tau_w \) is the time required for one revolution of the workpiece.

\[
r(t) = r(t - \tau_w) + \Delta r
\]  

(3.6)

Since the current and previous material removed from the workpiece will be calculated in simulink, Equation 3.6 can be rearranged to solve for the instantaneous depth of cut \( \Delta r \). It is usually assumed that this instantaneous depth of cut is proportional to the cutting force \( F \). This simplification is similar to the one made when using a single point tool in cutting operations [1].
Constitutive Equation (6) is the main interest for this work and is described in detail in the following section.

3.2.3 Modeling the grinding force

Grinding models have always made the assumption that the component of the grinding force acting normal to the cut surface is generated by the instantaneous depth of cut and depends on a grinding coefficient referred to as the specific cutting energy or cutting stiffness. While this coefficient does have units of N/m, it is misleading to consider it a stiffness, as it has no physical meaning as such; it is a characteristic of the wheel-workpiece interaction [18, 19, 20, 24, 40, 41]. For this work, this coefficient is extended in order to more accurately capture its strong dependence on the equivalent chip thickness. The instrumented spindle measures the time varying deflection during the grinding cycle.

3.2.4 Continuous infeed

The classical grinding model represents the relationship between the programmed infeed rate and the normal grinding force as a first-order system and has been shown that the actual infeed position can be found from the solution of the Equation [18, 19]

\[ \ddot{x} = \frac{1}{\tau} (x_c - \dot{x}) \]  \hfill (3.7)

where \( \tau \) is the system’s time constant. For a constant programmed infeed rate of \( \dot{x}_c = v_f \) so that \( x_c = v_f t \) the solution for the actual material removed \( r(t) \) is given as

\[ r(t) = v_f (t - \tau + \tau e^{-t/\tau}) \quad t(0) < t < t(1) \]  \hfill (3.8)

where \( t(0) \) is the start of the constant infeed and \( t(1) \) the end. The normal force as

\[ F_n = \frac{k_c v_f}{\eta_w} (1 - e^{-t/\tau}) \quad t(0) < t < t(1) \]  \hfill (3.9)
The system’s deflection is given by

\[ \delta = x_c - x \]  
(3.10)

so that

\[ \delta = v_f \tau (1 - e^{-t/\tau}) \quad t(0) < t < t(1) \]  
(3.11)

At the start of the dwell cycle the depth of cut is a step input which will be equal to the deflection of the system

\[ \delta_{t=t1} = v_f \tau (1 - e^{-t1/\tau}) \]  
(3.12)

and the actual infeed position during the dwell cycle is given by

\[ x(t) = x(t = t1) + \delta_{t=t1}(1 - e^{-(t-t1)/\tau}) \quad t1 < t < t2 \]  
(3.13)

Using these equations with the grinding parameters of Figure 3.1 leads to the well-known result shown in Figure 3.4. An estimated system time constant \( \tau \) of 4 seconds is used for the model.

For this model, the response of the system (force and actual infeed) and the dwell time required to produce components within the size tolerance are dependent on the commanded infeed rate \( v_f \) and the system time constant \( \tau \). The time constant is defined as

\[ \tau = \frac{k_c \tau_w}{k_c} \]  
(3.14)

It should be noted that it is not practical to calculate the time constant from Equation 3.14 because the grinding force coefficient \( k_c \) is constantly varying. The grinding force coefficient varies with the type of wheel and also changes with the wheel sharpness. Therefore, it is necessary to measure the system time constant in real-time from readily available process data.
Figure 3.4. Representing the relationship between the programmed infeed rate and the normal grinding force as a first order system

3.2.5 Measurement of the system time constant $\tau$

Adaptive grinding cycles have been previously implemented using measured system time constant to control the target axis position and the dwell time. In this way the control system adapts the machining cycle for the wheel condition at the time of grinding, and can cope with the large variations in deflection that occur when using high feed rates. While difficult to do in practice, the notion of the system time constant is helpful and is documented in this section with comparison to actual grinding data.

The predictive equation for the time constant $\tau$ including the wear of the wheel is given by the equation

$$\tau = \frac{d_w \pi k_c}{v_w k_c (1 + \frac{d_w}{d_{CG}})}$$  \hspace{1cm} (3.15)
As mentioned previous, the time constant is related to factors such as grinding system stiffness, the grinding wheel, and the workpiece material. It reflects the response speed of the grinding control system and in control engineering terms, the value of the time constant equals the time lag or delay during the steady state period of a ramp type input signal [42, 43]. Some of these effects appear explicitly in Equation 3.15 and others are hidden within stiffness parameters.

Large values of $\tau$ may indicate that the grain cutting edges are not sharp enough, the workpiece material is difficult to grind, and/or the static stiffness of the grinding machine is not high enough. As the sharpness of the grain cutting edges reduces, the time constant increases and as such it can be considered as a useful indication of tool life of the grinding wheel [44, 42].

For these reasons the time constant is usually experimentally estimated using measured grinding data. Since the time constant will be obtained with measured data these dependency will be automatically built into the value.

### 3.2.5.1 Steady state system lag

The most common way that researchers have experimentally estimated the time constant is to measure the steady state lag in the roughing stage by in-process gaging of the part diameter and dividing it by the commanded infeed rate. After an initial transient the lag of the actual infeed behind the accumulated commanded infeed approaches a steady state value as shown in Figure 3.5 [17, 18, 20]. Using this method the time delay between the commanded and actual infeed for the given input parameters was about 11.8 seconds. This delay can also be measured by the difference in time to reach a required diameter. The time delay between the commanded and actual infeed for the given input parameters to remove 14.36 microns from the work was 11.7 seconds. The two measured values have a percent difference less than one. This time constant is more than doubled the previously estimated 4 seconds.

### 3.2.5.2 Dwell phase

Another method for calculating the system time constant uses the force data collected during spark out. After the initial roughing a spark out phase with zero
controlled infeed velocity is used. The material removal continues at a decreasing rate in an exponential manner [18, 19, 20, 45, 46]. Assuming a first order system response and normalizing the measured normal force $F_n$ the equation during spark out becomes

$$F_n = e^{-t/\tau} \quad (3.16)$$

Taking the log of Equation 3.16 the time constant is then described by the negative inverse of the slope. A line fitted using this slope is shown in Figure 3.6. For this particular data set the time constant is calculated to be 6.7 seconds, which is between the first two values used.
3.2.5.3 Infeed phase

A third method for estimating the system time constant uses the roughing stage of the grind. Normalizing the measured force resembles a unit step response of a first order system as described by Equation 3.9. One important characteristic of the normalized exponential response curve is that at $t = \tau$ the response has reached 63.2% of its total change [5, 43]. However, due to the additional delay the time constant appears larger if it is assumed the response has reached 63.2% in one time constant. Therefore, the value of the time constant is taken between the time it takes the response to reach 86.5% from 63.2% of the final value. This calculation is carried out in MATLAB using the normalized measured force response as shown in Figure 3.7.

Another important characteristic of the exponential response curve is that the slope of the tangent line is $1/\tau$. By normalizing and then differentiating Equation 3.9 such that
Figure 3.7. Modeling the measured normalized force as a unit step response for a first order system to find the time constant.

\[ F_n = (1 - e^{-\frac{t}{\tau'}}) \tag{3.17} \]

and resetting the time to start at the beginning of the model so that

\[ \frac{dF'_n}{dt} = \frac{1}{\tau} e^{-\frac{t}{\tau}} = \frac{1}{\tau} \tag{3.18} \]

Researchers prefer this alternative method for adaptive control of grinding systems over the dwell calculation. Using the dwell period method means optimizing the feed cycle using the value of time constant computed from previous grinding operations rather than from the operation which is currently being undertaken. However, most other papers deal with the much less accurate information of the spindle power consumption instead of the normal force for the adaptive control of
grinding [18, 44].

3.2.5.4 Time constant discrepancies

From the three cases above, three different numbers for the time constant were found which is a bit confusing. Theoretically, the system time constant should not change unless different events are taking place.

![Graph comparing measured actual infeed during a plunge grinding cycle to modeled response with different time constants.](image)

**Figure 3.8.** Comparing the measured actual infeed during a plunge grinding cycle to modeled response with different time constants.

In the case of the steady-state system lag, a distinction is made for this work between the actual infeed \( x(t) \) and the material removed from the workpiece. In Figure 3.5 the probe targeted the OD of the workpiece as shown in Chapter 2 and measured the actual material removed, \( r(t) \) from the workpiece. This is significantly different from the actual infeed referred to by other researchers. In this work, it
is shown that there can be an actual infeed \( x(t) \neq 0 \) but no material removed \( r(t) = 0 \) leading the longer delay shown. This will be discussed at length in a later section for the actual model used.

Between the infeed and dwell stages it is noted by other authors that the time constant may differ [8] and was witnessed first hand in this work. Figure 3.7 compares the two time constants found from the two different stages, the 4 second time constant calculated from the infeed and 7 second calculated from the dwell. It is stated that for high accuracy it is best to determine the time constant during the dwell [8]. However, we will show that behavior during infeed and the dwell stage differ due to the initial contact between the wheel and work and the transition from no material removed to steady state removal.

### 3.3 Grinding force modeling

The relationship between grinding force and the equivalent chip thickness is the primary challenge in grinding force modeling. As shown previously, the literature assumes the grinding force is proportional to the wheel depth of cut:

\[
F_n = k c a
\]  
(3.19)

Where \( k_c \) is the grinding force coefficient and \( a \) is the real depth of cut per revolution of the workpiece, also known as the instantaneous depth of cut. For the first order model the wheel depth of cut \( a \) can be calculated by the material removal rate \( \dot{r} = \Delta r / \tau_w = v_r \) multiplied by the time it takes the work to make one revolution \( \tau_w \).

\[
a = \frac{v_r}{\eta_w} = v_r \tau_w
\]  
(3.20)

Usually the material removed is estimated from the monitored force by a calculated time constant as shown earlier. The material removed is derived from Equation 3.7 and the assumed force is proportional to the wheel depth of cut as Equation 3.9. However, in this work the material removed from the work is measured so that the rate may be calculated as seen in Figure 3.9. At this point it would be interesting to compare the equation for the modeled force to the measured force.
Figure 3.9. Measured material removed from the work top and the calculated rate of material removed bottom.

from the instrumented spindle. The material removal rate (in microns per second) calculated in the lower plot of Figure 3.9 is multiplied by a grinding coefficient of 255 N/µm, found from the grinding force data, and the work revolution period of 0.1154 sec and compared to the measured force shown in Figure 3.10.

There is a discrepancy between the force model prediction and the measured data as shown in the figure. This difference is due to the compliance in the machine and the rubbing forces that are significant at very low depths of cut. This is supported by data in Figure 3.9 and the results shown in Chapter 4 which show material removal to be nearly zero during the first couple of seconds, during which time the force is starting to increase rapidly. As the grinding continues, eventually, material is removed at the same rate as the commanded plunge rate as shown.
Figure 3.10. For a given workpiece period and grinding width the normal grinding force component is proportional to the wheel depth of cut.

This discrepancy is not addressed in much of the literature. In the grinding force plots of Hashimoto [5] the forces appear to follow the estimated first order response, unlike the plots here which initially appear to have an exponential growth in the beginning of the grind. One significant difference is in the equivalent chip thickness and the final steady-state forces reached. In Hashimoto, the steady state equivalent chip thickness is calculated at 31.6 nm, where the equivalent chip thickness for this data of 6.52 is only 0.20 percent of this value. The steady state force measured in this work reach around 20 Newtons, which is only 0.22 to 0.50 percent of the forces reached in Hashimoto’s work of 40 to 90 Newtons. Another difference is the material being ground. The material used in Hashimoto et al was hardened Cr-Mo high strength steel, which is three times less stiff when compared to the tungsten carbide used in this work. Also it is not comparable with corundum or sapphire in hardness like the tungsten carbide used in this work.
Material removal occurs by chip formation, however it would seem that much of the grinding energy must be expended by mechanisms other than chip formation, such as friction and grain penetration. This is especially more pronounced at lower removal rates. In order to account for these differences a more in depth model for the grinding forces is needed. This will begin with the understanding of the grinding force coefficient.

### 3.4 The grinding force coefficient

Of the many factors affecting a grinding process, the largest contributors to workpiece size are the machine’s compliance and the specific grinding energy. A long list of additional, and often time-varying, factors such as lubrication and abrasive wheel condition clearly affect the grinding force as well. The machine’s compliance is accurately described by linear lumped parameter elements and quantified with straightforward, if tedious, measurements.

The grinding coefficient $k_c$ relates the component of the grinding force acting normal to the cut surface to the depth of cut. It can be thought of as the normal force required to take a unit depth of cut and is one of the fundamental parameters in the regenerative chatter phenomena when compared to the system stiffness $k_m$ as seen in Chapter 1 [45, 47].

This coefficient has units of force per length and is also referred to as a work wear or cutting stiffness. It is a function of the workpiece material, abrasive wheel design, machine parameters, wheel width, lubrication, wheel dressing, and many other variables. It is usually normalized by the grinding width reported in N/m².

The value of this coefficient can be derived from the total specific grinding energy. The specific energy is typically between 15 and 700 J/mm³, which was half the value of the results found in this work and particularly depends on the workpiece hardness and wheel sharpness [8]. The total specific energy of the cutting action $e_t$ is the power $P$ per unit volumetric removal rate $Q$ as

\[ e_t = \frac{P}{Q} \]  \hspace{1cm} (3.21)

The power required by the grinding spindle during the cutting action has of-
ten been used in the literature for its ease of instrumentation and reliability [18]. Usually the voltage and current inputs to the grinding spindle have been measured. This grinding power is associated with the grinding force components, and for this work the grinding power can be identified from the instrumented spindle’s measured grinding forces. For plunge grinding the power is resolved into two components from the tangential and normal grinding force as

\[ P = F_t (v_s + v_w) + F_n v_f \]  

(3.22)

![Figure 3.11. Grinding power calculated from the instrumented spindle’s measured grinding forces](image)

In production, taking account of the workpiece speed has a small effect as \( v_s \) typically is 60-200 times larger than \( v_w \) [8]. For this work, the wheel speed, \( v_s \) was
11-22 times larger than the work \(v_w\). The infeed velocity \(v_f\) was much smaller than the wheel speed so that it can be ignored in Equation 3.22. The grinding power, ignoring the work and infeed velocity, is given as

\[
P = F_t v_s
\]  

(3.23)

Figure 3.11 compares the power calculated from Equations 3.22 and 3.23. For this data set the wheel surface speed was 11 times larger than that of the work. Even at the lower end of the speed range there is only an 0.08 percent difference between these two equations. This simplification to the grinding power will ease the formulation of the Simulink model as shown later. Plugging the power equation into Equation 3.21 and solving for the tangential force gives

\[
F_t = e_t \frac{Q}{v_s}
\]  

(3.24)

The numerator or volumetric removal rate is usually given in terms of the grinding parameters as \(Q = \pi d_w b_s v_f\) where \(v_f\) is the constant infeed rate. However, this is not true. The volumetric removal rate is not constant and will eventually approach this value as the wheel penetrates deeper into the work. The material removal rate \(v_r\) approaches the infeed rate \(v_f\) as shown in Figure 3.9. Therefore in this work the volumetric removal rate is given as \(Q = \pi d_w b_s v_r\). The wheel surface speed velocity, \(v_s\) is equal to \(\pi d_s / \tau_s\). The final equation of the tangential force is equal to

\[
F_t = e_t \frac{d_w}{d_s} b_s \tau_s v_r
\]  

(3.25)

The normal force is related to the tangential cutting force by the grinding force ratio \(\mu\) sometimes referred to as the contact angle coefficient as \(F_t / F_n = \mu\). The grinding force ratio is similar to a friction coefficient and employs the same symbol. This ratio is another useful parameter and will be discussed in depth later. Comparing Equation 3.25 to the Equations 3.19 and 3.20 the grinding coefficient is equal to

\[
k_c = \frac{1}{\mu} e_t \frac{d_w}{d_s} \tau_s b_s
\]  

(3.26)
From Equation 3.26 the grinding coefficient is a function of the workpiece material properties \( e_t \) the grinding coefficient, \( \mu \), and the grinding width \( b_s \) as well as the grinding wheel and workpiece formulation, coolant, etc [8, 23, 45]. We now know that this coefficient is not constant and varies nonlinearly with chip thickness, exhibiting a so-called size effect when the chip thickness is very small [8, 48].

The challenge in grinding modeling is striking the appropriate balance between model sophistication and the ability to obtain suitable numerical values for the key parameters so that the model can be applied. The experimental determination of a constant cutting stiffness does not present any major difficulty as it can easily be worked out from the measurements of the cutting forces during the grinding
processes such that

$$F_n(t \to \infty) = k_c v_f \tau_w = K_c \quad (3.27)$$

Where $K_c$ is the steady state grinding force shown in Figure 3.12 with units of force. This linear cutting force model relies on the collected grinding force to refresh the empirical values in order to maintain an accurate material removal prediction in Equation 3.8. In general this model shows a similar trend to the measured data for both the force and material removed from the workpiece, however in previous work [21, 37] the effects of rubbing and grit penetration were unknown. As a result the estimated data has unknown time delays. The model also exhibits a different initial shape for the force as shown in Figure 3.7. These anomalies were removed from the estimated data and the remaining estimate aligned to the measured data.

Using the linear model of force $F = k_c \Delta r$ for Equation (6) in Figure 3.3 of the machine’s model produced an estimated force lower than the measured. The specific grinding energy was calculated from the estimated results of the force and material removed as $e_t = (F_n \mu d_s \tau_w)/(d_w b_s \Delta r \tau_s)$ and is plotted in Figure 3.13. A better approximation can be made of the grinding force if the behavior of the specific energy can be modeled.

### 3.5 New force model

Snoey et al [1] showed for cylindrical grinding that the tangential and normal force components can be approximated by the power function relationships

$$F_t = F_1 \left( \frac{Q_s}{v_s} \right)^f = F_1(h_{eq})^f \quad (3.28)$$

and

$$F_n = F_2 \left( \frac{Q_s}{v_s} \right)^f = F_2(h_{eq})^f \quad (3.29)$$

where $F_1$, $F_2$, and $f$ are constants with the exponent typically in the range of 0.4-0.9, and $h_{eq}$ is the equivalent chip thickness [17]. The corresponding specific energy is
**Figure 3.13.** Comparing specific grinding energy versus volumetric removal rate per unit width for the measured data and linear force modeled data

\[ e_t = F_1(h_{eq})^{f-1} \]  \hspace{1cm} (3.30)

Usually these and other empirical relationships are regarded as having limited practical use for predicting grinding performance because the constants \( F_1, F_2, \) and \( f \) depend on a particular wheel, workpiece, grinding fluid, and dressing conditions, as well as on the accumulated stock removal.

Obtaining these parameters using a purely empirical approach will lead to accurate results for conditions close to the training data, but will not constitute a robust physical description of the grinding process. Here, the grinding force model will be a mix between the two. This work shows how force measurement can be used
to characterize the specific grinding energy for the grinding operation. In practice, this step will always be necessary because it is all but impossible to parameterize the myriad variables effecting workpiece quality.

This work benefits from a much better experimental apparatus than those reported in the literature. In most cases accurate determination of the grinding energies $e_t$ even over a narrow range of properties (tool/workpiece material combinations are constant) is tedious and time consuming. Our test rig allows the measurement of huge amounts of data in relatively little time.

![Figure 3.14.](image)

Figure 3.14. Top plot is the material removed from the workpiece. Lower plot is the first numerical differentiation of the measured material removed or material removal rate

The material removal rate is calculated using the first numerical differentiation of the measured material removed from the workpiece. The rate of change of $r(t)$ with respect to time, that is $\Delta r/\Delta t$, which is interpreted as the slope of the
tangent to the signal at each point. The time-interval between adjacent points is constant, and is equal to the period of work rotation \( \tau_w \) due to the synchronization of the instrumented spindle's encoder and the reliability for the spindle to hold its rotational speed. The first derivative is taken by the difference between adjacent points of the material removed divided by the difference in adjacent points in time as shown in the lower plot of Figure 3.14. As is commonly observed, the differentiation degrades signal-to-noise ratio. In order to control the signal-to-noise degradation a curve was fitted to the data.

**Figure 3.15.** Power calculated from the tangential grinding force. Material starts to be removed after 3.4 seconds.

The power is calculated from the measured tangential force as stated in Equation 3.23 and is shown in Figure 3.15. The calculated power contains one data point per revolution. Each of the data points are averaged twice, once from the
previous sampled point and the second from the post sample points. This averaging generates the a smoother function as shown in red in Figure 3.15.

\[e_t = \frac{d_s}{d_w} \frac{\tau_w}{\tau_s} \frac{F_t}{\Delta rb_s}\]  

(3.31)

where \(d_s\), \(\tau_w\), \(b_s\), and \(\tau_s\) are constant input parameters per experiment. The diameter of the work \(d_w\) decreases about 20 microns per experiment and the tangential force \(F_t\) and material removed \(\Delta r\) were measured throughout the experiment. The total specific energy was calculated through Equation 3.31 and plotted over the
grinding cycle time as shown in Figure 3.16. As stated earlier the specific energy is typically between 15 and 700 J/mm$^3$. Higher values are usually caused by difficult-to-grind materials such as tungsten carbide [8].

Obtained results such as those in Figure 3.16 are usually not possible for most experimental setups as mentioned before. It is difficult to continuously measure both the change in the tangential force $F_t$ and depth of cut $\Delta r$ as the wheel is fed into the part, as in this research. In the literature it is common that the measured steady state values are used to calculate the specific energy. In Equation 3.31 the depth of cut $\Delta r$ will eventually reach a steady state depth of cut per revolution of the workpiece $a$. This steady state depth of cut per revolution $a$ divided by the period of rotation for the workpiece in seconds $\tau_w$ is equal to the commanded infeed velocity $v_f$, as seen in the lower plot of Figure 3.14. Therefore at steady state, Equation 3.31 becomes

\[
e_t = \frac{d_s}{d_w} \frac{1}{\tau_s v_f b_s} F_t
\]

(3.32)

where $d_s$, $d_w$, $\tau_w$, $b$, $v_f$ and $\tau_s$ are treated as constants and the tangential force $F_t$ is carefully measured at the end of the grind which provides a single data point. One of the six constants are then varied and the tangential force remeasured to acquire other data points [49, 50].

In Figure 3.16 the total specific grinding energies are plotted versus time for four datasets out of the 54 sets. One dataset was chosen at the beginning of experimentation, one at the end and two in the middle. These datasets span a three month time period. These results for the material combination of a tungsten carbide workpiece with a 320 grit diamond wheel show the same type of inverse behavior. This behavior is related to the equivalent chip thickness $h_{eq}$ which is increasing to a steady state value with the depth of cut per revolution $\Delta r$.

Initially, at time equal to zero the wheel is in contact with the work but the abrasives have not yet penetrated the work. At this zero depth of cut a finite force is required due to friction between the wheel and workpiece even though no material is removed. The result is that specific energy is infinite. Only elastic and plastic deformations take place as evidenced by polishing of the surface [8]. This first phase is known as rubbing and is proportional to the grain wear flat area,
dulled flatted tips of the abrasive grain.

Afterwards from 0 to 3.4 seconds ploughing begins. Scratches appear and ridges are formed at the sides of these scratches. Plastic deformation increases but the material removal is still negligible. There is still rubbing as well within this interval.

Ultimately, after 3.4 seconds with further increases in force, material removal begins with a rate that approaches the commanded infeed velocity. The rubbing and ploughing energy becomes a smaller proportion for increasing material removal rates. The cutting energy remains constant with increasing removal rates and can be identified as the minimum asymptote lines shown in Figure 3.16. As shown in the figure, the energy for the same combo of workpiece and wheel approach a narrow range of cutting energy.

**Figure 3.17.** Concept of the equivalent chip thickness by mass continuity after Snoeys et al [1]
This phenomenon, where the specific grinding energy reduces as the depth of cut and removal rate increases, is known as the size effect. This effect is a directly related to the equivalent chip thickness from Equation 3.30.

The equivalent chip thickness \( h_{eq} \) has been proposed as the grinding parameter that may best describe the output of the process. The physical interpretation of the equivalent chip thickness \( h_{eq} \), is described as the mass continuity of material into and out of the grinding zone as shown in Figure 3.17. The depth of material removed \( \Delta r \) entering the grinding zone at the work speed \( v_w \) is very much larger than the thickness of the layer emerging from the grinding zone at wheel speed \( v_s \), and is expressed by the following equation

\[
v_s h_{eq} = v_w \Delta r
\]  

(3.33)

The material is speed up from work speed to wheel speed, and if the material emerged as a solid extruded sheet it would have a thickness correspondingly reduced to the value known as the equivalent chip thickness \( h_{eq} \). Obviously, the material does not emerge as a solid sheet. It is cut into many smaller chips by randomly spaced grains. The thickness of the chips must greatly exceed the equivalent chip thickness to account for the discrete nature of material emerging [8, 17].

Figure 3.18 shows the inverse behavior of the specific energy for four different datasets decreasing with \( h_{eq} \) at a diminishing rate toward a minimum value. The particular significance of the specific energy is the fact that any plausible mechanism of abrasive-metal interaction must be able to account for its magnitude and its dependence on process parameters.

For grinding metals, it has been assumed that material removal occurs by a shearing process of chip formation, similar to that found with other machining methods such as turning and milling. In grinding, the debris from the process is called swarf. Examining this swarf with scanning electron microscope has shown it is composed mostly of curled chips very much like those found in turning and milling [17].

In the early 1950s careful measurements of the grinding forces and specific energies have showed that the specific grinding energies are much higher than for other metal cutting operations. Also larger specific grinding energies were found when the process parameters were adjusted to decrease the equivalent chip thickness.
Figure 3.18. Specific grinding energy’s dependence on the equivalent chip thickness

Perhaps the most disturbing factor is in the magnitude of the specific grinding energy. Under adiabatic conditions, such as in grinding with extremely rapid chip formation with no time for heat to be conducted away, the plastic energy input per unit volume should be limited by the energy it takes to bring a unit volume of material from its ambient condition to its molten state [17].

For example the melting energy per unit volume of iron is 10.5 J/mm$^3$. In production grinding of steels the specific energies are typically ranging from 20 to 60 J/mm$^3$, and higher values have been found in fine grinding [17].

From these results it would seem that much of the grinding energy must be expended by mechanisms other than chip formation. Even though metal removal occurs mostly by chip formation, as seen from the swarf.
This behavior has been described by Hahn [51] who proposed three aspects of the grinding wheel abrasive interaction with the work which are rubbing, ploughing and cutting described earlier. In Figure 3.18 the specific energies appear to be converging on a narrow range as the equivalent chip thickness is increased which is theorized to be the chip formation energy. However, at small equivalent chip thicknesses sliding and ploughing dominate the process and the specific energy goes off to infinity. The energy seen in this figure will capture the three abrasive mechanisms and be used to calculate the force as

$$F_n = \frac{1}{\mu} \varepsilon_t \left( \frac{d_w}{d_s} \frac{\tau_s}{\tau_w} \Delta r b_s \right) = \frac{1}{\mu} \varepsilon_t h_{eq} b_s$$  \hspace{1cm} (3.34)

For this work the grinding force ratio $\mu$ is a wheel sharpness property and was found to be between 0.35-0.48, the grinding width $b_s$ was constant for all the tests, and the equivalent chip thickness is built into Simulink, as the input to the force model, through the material that enters the grinding zone per revolution of the workpiece $\Delta r$. The final piece is to characterize the specific grinding energy shown in Figure 3.18 in terms of the equivalent chip thickness.

The dependence of the specific energies on the equivalent chip thickness is plainly seen on a log-log plot as shown in Figure 3.19. Comparing these four different dataset a clear logarithmic trend is shown for the specific grinding energies versus the equivalent chip thickness. The energy model for this figure is given as

$$e_t = E(h_{eq})^{-0.5}$$  \hspace{1cm} (3.35)

The power of -0.5 represents the slope which all of the datasets appear to follow. The coefficient $E$ was found to have a very narrow band for this wheel and workpiece combination between 0.3 - 0.5, with a core group of the datasets using a 0.31 coefficient which will be shown in Chapter 4. In order to strike an appropriate balance between model sophistication and our ability to obtain suitable numerical values for the key parameters a power of $n = -0.5$ and a coefficient of $E = 0.31$ was used in the Simulink model and will be shown to predict the grinding force and material removed accurately in Chapter 4.

This representation for the specific energy is the same as given in Equation 3.30, where the coefficients are from the material properties of the workpiece and wheel.
Figure 3.19. Log-Log plot of specific grinding energy and model for specific energy vs the equivalent chip thickness

For different wheel/workpiece combinations these values would have to be recalibrate with a simple grinding test.

The conclusion here is that increasing chip thickness reduces specific grinding energy. This conclusion is invaluable for understanding the implications of changing process parameters and empirically deriving a new grinding force coefficient for a greater prediction accuracy in the model.
3.6 Calculation of the contact stiffness and grain penetration

The contact stiffness relates the cutting force with the deformation of the contact area between the grinding wheel and workpiece. This elastic deformation at the area of contact is an additional flexibility, and needs to be modeled as it induces another error in the stock removal of the workpiece and affects the grinding force [45, 52].

Considering conventional grinding the contact stiffness $K$ between the work and grinding wheel is the same order of magnitude as the grinding coefficient $k_c$. However, in tests conducted with superabrasive wheels the obtained contact stiffness was an order of magnitude superior [13]. Consequently, the contact stiffness between the workpiece and superabrasive wheel is ignore in the literature’s simulations. The reasoning, that the contact is produced at one single point and that the flexibility at the contact point can be neglected for high stiffness. This phenomena cannot be ignored in light depths of cut with superabrasive wheels and believed to be the cause of the initial increasing exponential shape of the grinding force.

It is difficult to get a reliable representation of the contact stiffness $K$ which tends to be non-linear. The empirical determination of the contact area stiffness is complicated because of the large number of factors affecting it and the difficulty in setting up the experiments [15, 23, 45, 52]. One of the main factors for the contact stiffness is the number of the abrasive grains in contact with the workpiece and the support stiffnesses’ of those abrasive grains. These grains have irregular geometry, sometimes multiple cutting points and are randomly distributed over the surface of the wheel. They are hard enough to see with the naked eye let alone acquire a quantitative estimate. Also the number of grains in contact with the work will increase as the contact area increases due to the deformation of the wheel. However, if the number of the abrasive grains in contact with the workpiece can be estimated statically, and the support stiffness of a single abrasive grain can be obtained, the theoretical contact stiffness can be estimated [53].

Snoeys and Wang [1] developed and describe a system for the static determination of the contact stiffness. The tests are performed in a stationary state, without any relative movement between the grinding wheel and the workpiece in the tan-
Figure 3.20. Measured process infeeds, material removed, and deflections

gential direction. A steel shoe is pushed towards the wheel with a preload and a
dynamic exciting force superimposed. Both the static and the dynamic forces are
measured by means of a load cell and the deformation of the contact zone by a
non-contacting inductive probe. The experimental results show an exponential re-
lation between the contact stiffness and the radial force. However, they deduce
a theoretical expression of the contact stiffness, assuming that each grain of the
surface of the wheel is supported by a single spring. From the obtained expression
it can be said that the contact stiffness depends on the wheel composition, the
dressing condition, the wheel and workpiece diameters and the normal force.

Yamada [54] empirically measured the contact stiffness under grinding opera-
tions stating different values than those calculated from the stationary state. For
this work the contact stiffness is measured similar to Yamada [54] to quantify the value in the grinding operation for various parameters.

![Simplified model with no material removal](image)

**Figure 3.21.** Simplified model with no material removal

The measurement of the contact deflection was calculated as follows. The difference between the measured commanded infeed of the wheel and the measured material removed from the work is equal to the total deflection of the process. The normal component of grinding force is also measured for this process and the measured machine stiffness was calculated in Chapter 2. The deflection of the machine is calculated by the force divided by the machine stiffness. Subtracting the total process deflection from the machine deflection is the contact deflection of the wheel and work.

For this work the initial period before material removal was used to calculate the contact stiffness. Figure 3.20 shows this initial period where the red line holding zero represents the measured material removed from the work. This corresponds to all of the infeed going into the system deflection $x_c(t) = \delta_{sys}(t)$. The purple line represents the estimated machine deflection given the measured process force and
machine stiffness. The black line is the contact deflection for this process.

Figure 3.22. Comparing the measured and estimated normal component of the grinding force

The unique model is modified into the Simulink model shown in Figure 3.21 for the special case of $r(t) = 0$. For this case the normal grinding force component will be equal to the actual infeed multiplied by the work spindle stiffness from Equation 6 and 3 with $r(t) = 0$. Using this simplified model an empirical contact stiffness was found to be a power function of the grinding force such that

$$\delta_c = 0.6(F)^{0.2} \quad (3.36)$$

Using this function the estimated contact stiffness is plotted as the cyan colored line in Figure 3.20. The coefficient and power were found to work well throughout
the grinding and for every test, even with varying parameters. The final check is to compare the measured force to the estimated as seen in Figure 3.22.

### 3.7 Simulink model of the grinding process

\[ x = x_c - \frac{F_n}{K_{eq}} - \frac{F_n}{K} \]

**Figure 3.23.** Block diagram representation of the actual infeed of the wheel

This section presents the complete time domain model for the external plunge grinder with a diamond grinding wheel and tungsten carbide workpiece. The modeling employs the simulation tools of MATLAB and Simulink where diagram blocks can be visualized and modified directly. This work addresses the industry’s need for a realistic but simple model which can predict the grinding process results, with the intention of optimizing the process of a particular machine, by testing different input parameters.

Equation 3.4 for the actual infeed of the grinding wheel can be used to create the block diagram shown in Figure 3.23. Where the two gains \(1/K_{eq}\) and \(1/K\) are the measured inverse equivalent stiffness of the wheel side and the contact stiffness.
at the point of contact found from the procedures in the previous section.

\[ r = x - \frac{F_n}{K_{spin}} \]

- gap = \( y - r \)
- \( F_n = F_i(\Delta r) \)

**Figure 3.24.** Block diagram representation of the material removed

The actual infeed \( x \) is fed into Equation 3.5 represented by the block diagram shown in Figure 3.24. The gain \( 1/K_{spin} \) is the inverse equivalent stiffness of the work spindle. A transport delay block is used to delay the material removed \( r \) by the work’s period \( \tau_w \) simulating the material removed from the previous revolution of the workpiece \( r(t - \tau_w) \). This allows the calculation of the depth of cut \( \Delta r \) used in the force model for Equation (6).

The force model was developed in the previous sections and is represented by the block diagram shown in Figure 3.25. The depth of cut \( \Delta r \) is fed into the Simulink force model and used to calculate the equivalent chip thickness by Equation 3.33, \( h_{eq} = \left( \frac{v_w}{v_s} \right) \Delta r \) or \( h_{eq} = \left( \frac{d_w}{d_s} \right) \left( \frac{\tau_s}{\tau_w} \right) \Delta r \).

The grinding energy is a material property of the wheel and workpiece and found from a grinding pretest. Equation 3.35 is the energy model used in the function block of the Simulink model. A coefficient \( E \) of 0.31 was found to work well with most of the datasets as will be shown in the following chapter.
With the specific grinding energy and equivalent chip thickness calculated the normal grinding force is found through Equation 3.34. In most cases for this work the grinding ratio $\mu$ was found to be around 0.35.

A saturation block was added to the force model as shown in Figure 3.25. The saturation block was used to prevent the initial infinite specific grinding energy generated in the model and caused difficulty in the simulation. An upper limit of 10,000 J/mm$^3$ is used.

These three different pieces are combined to form the process model shown in Figure 3.26. This model uses basic input parameters shown in purple combined with the material properties of the workpiece and wheel combination and this machine’s physical properties shown in orange to produce predicted results of the process including grinding force, the material removed from the workpiece in addition to the deflections of the machine and spindles shown in green. The model is accompanied by a definition index file, m-file, which provides for the purple parameter variables specifying the process in particular before executing the simulation of the model. In the configuration parameters for Simulink a fixed-step type is chosen for the solver. The fixed-step size is a fraction of the work period $\tau_w$. The integration solver for the grinding process uses the Runga-Kutta integrators of the fifth order ode5.

This model enables quantitative results to be obtained with relative speed and allows optimization of the grinding process for this particular machine by observing how the influence of the systematic variation of the parameter space affects the final results of the workpiece simulation. The final model produced in Simulink by
this chapter is shown in Figure A.1.
$F_n$ Normal force
$F_t$ Tangential force
$x$ Actual infeed
$r$ Material removed
$\delta$ De/f_lections

$\tau_w$ Work speed
$\tau_s$ Wheel speed
$ds$ Wheel diameter
$dw$ Work diameter
$bs$ Grinding width

$K_{spin}$ Spindle stiffness
$K_{eq}$ Wheel side stiffness
$K_{con}$ Contact stiffness

Figure 3.26. Process model
4 Discussion of results

4.1 Introduction

Closed-form solutions to the nonlinear grinding force model are unlikely, but numerical simulation is particularly well-suited to grinding processes because of its ability to handle complexity and broad range of operating parameters. Numerical simulation through function calls from MATLAB m-code allows rapid evaluation of the effects of changing parameters on workpiece quality. Furthermore, numerical simulation can be used to explore the parameter space to greatly reduce the number of grinding trials by facilitating selection of the most important grinding parameters.

In this chapter the predicted forces and size errors of the grinding process are discussed for various operating conditions (i.e., feeds and speeds). The effects of wheel and workpiece material properties are also incorporated in the simulation by the nonlinear specific grinding energy developed in Chapter 3. The performance of the simulation is tested by comparing the predicted estimates to actual grinding results. These results demonstrate the success of this work’s improved grinding simulation over previous research.
4.2 Contrasts and alignment of the classic model

This section emphasizes the differences between the new simulation and the classic cylindrical plunge grinding model for light depths of cut in addition to presenting the tedious alignment procedure used to match the classic model to the measured data.

Generally, in the literatures’ simulations, the contact stiffness between the workpiece and the grinding wheel have been ignored along with the sliding and plowing forces. As a result of this neglect, the rate of the workpiece’s radial reduction is equal to the actual infeed velocity of the wheel. Therefore, the response of the first order cylindrical plunge grinding model is better represented by Figure 4.1 instead of the previous model shown in Chapter 1, Figure 1.7.

\[
x_c = v_f t
\]

\[
F_n = k_c \tau w v \quad \text{(no plowing, sliding, or contact deformation)}
\]

Figure 4.1. Model neglecting plowing, sliding, and the wheel contact deformation

This model is represented by the differential equation, Eqn 3.7, from Chapter 3. The solution to the differential equation, using a zero velocity initial condition, is the actual velocity \( v(t) = v_f (1 - e^{-t/\tau}) \). Substituting this result into Equation 3
gives the response of the normal grinding force component, Eqn 3.9.

\[ F_n = k_c \tau_w v_f (1 - e^{-t/\tau}) \]

Figure 4.2. Contrast between forces

The behavior of the force response from Equation 3.9 is plotted in Figure 4.2 together with the new simulation and actual measured results versus time. After an initial transient, the grinding force approaches a steady state magnitude. This magnitude is dependent on the grinding parameters \( v_f, \tau_w \) and the shearing energy for chip formation.

As seen in Chapter 3, this shearing energy for chip formation is built into the grinding coefficient \( k_c \). The speed of the transient, how fast the grinding force reaches its steady state magnitude, is dependent on the time constant \( \tau \).

The time constant can be estimated by Equation 3.14 as the ratio of the grinding coefficient and work speed versus the machine’s effective stiffness. In Figure 4.2 the time constant has been estimated at 6 seconds.

This result shows the effects of not including the nonlinearities of the contact
stiffness and the sliding and plowing energies. The predicted first order response clearly does not have the same initial shape as the Simulink and measured forces. Also it appears to increase much earlier, yet reaches steady state later than the other two.

![Graph](image)

**Figure 4.3.** Contrast between the material removed

Equation 3.8 from Chapter 3 represents the actual position, which is equal to the material removed as shown in the model, and can be found by integrating the actual velocity $v(t)$ with respect to time and including an initial condition of zero material removed. The behavior of the actual infeed from Equation 3.8 is plotted in Figure 4.3.

The rate, corresponding to the slope of the predicted curve, is the same as both the measured and Simulink response. However, it approaches this rate much faster than the other two results. This faster response leads to an increased predicted amount of material removed by the first order case. In Figure 4.3 it is shown
that the first order response would predict a workpiece diameter about 11 microns smaller than measured or predicted by the Simulink simulation.

Figure 4.4. Introduce offset in first order response to neglect the nonlinear contact stiffness

To adjust the solution for the nonlinearities neglected at the beginning of the force development, the first order response uses an offset or time delay as shown in Figure 4.4. This offset was found in MATLAB using prior knowledge of the system from the measured data.

As seen in Figure 4.4 the response time of the estimated force is slower than both the measured and Simulink result. This is theorized to result from the neglect of the sliding and plowing forces.

Figure 4.5 applies the 3 second time delay, used in the grinding force response, to the first order predicted response of the material removed. There is still a smaller predicted work diameter of 3 microns than the actual measured and Simulink simulated results, but with the additional 3 second time delay the result of the first
Figure 4.5. Introduce offset in the first order response for the material removed

order response corresponds very well to the measured actual infeed of the grinding
gwheel. This makes sense since the actual material removed lags the actual infeed
of the grinding wheel due to the sliding and plowing effects not yet considered.

Reexamining the predicted time constant, it was previously mentioned in Chap-
3, it is not practical to calculate the time constant from Equation 3.27. The
grinding force coefficient $k_c$ varies with the type of wheel and changes with the
wheel sharpness. The stiffness of the machining system $k_m$ has many components,
the stiffness of x-slide, the stiffness of the wheel spindle and overhang, the stiffness
of the z-slide and the stiffness of the work spindle and overhang. Another method
is available for measuring the system time constant from the readily available pro-
cessed force data.

Mentioned in Chapter 3, an important characteristic of the exponential re-
sponse curve described by Equation 3.9 is that at $t = \tau$ the value of the force
Figure 4.6. Force response using a faster time constant and longer time delay has reached 0.632 percent of its total change and at $t = 2\tau$ the value will have reached 0.865 percent of its total change. For this dataset, 20P52016430670D, the steady state force was 15 Newtons. Approximately, 13 Newtons is 0.86 percent and 9.5 Newtons is 63 percent of the steady state magnitude. The force response took 3.2 seconds to climb between these values. This new time constant is half the value calculated by Equation 3.14.

Using the two points of (8.3 sec, 9.5 N) and (11.5 sec, 13 N) as references for the classic predicted force, a new alignment with an offset of 4.9 seconds is found. The result for this dataset with the new time constant and time delay is shown as the orange line in Figure 4.6.

A grinding coefficient $k_c$ of 192 N/µm is found from the measured force and Equation 3.27. This is the same coefficient used in Equation 3.14, $\tau = (k_c \tau_w)/k_m$, to find the previous 6 second grinding system time constant. Neither the work speed
\[ r = v_f (t - \tau - t e^{-\xi t}) \]

Figure 4.7. predicted material removal using a faster time constant and longer time delay

\( \tau_w \) nor the machine’s equivalent stiffness \( k_m \) has changed. Rearranging Eqn 3.14 to solve for the grinding coefficient with the new 3.2 second time constant results in a lower coefficient of 102 N/µm, which when substituted into Eqn 3.27 a new steady state grinding force of 8 Newtons is found. These results further show the breakdown of the classic model.

Interestingly, the performance of predicted material removal equation using the 3.2 second time constant along with the longer 4.9 second time delay, that worked so well with the predicted force equation, was worse. Comparing the results from Figure 4.7 to Figure 4.5 shows an increase in error. The slope of the predicted material removal is too fast, but the longer delay appears to match the starting point of the material removal better.

The smaller time constant results in a faster system response. This faster re-
Figure 4.8. Predicted material removed from first order response using a longer time delay and 6 second time constant

Figure 4.8. Predicted material removed from first order response using a longer time delay and 6 second time constant works well with the predicted force but not with the predicted material removed as shown in the previous results. This smaller time constant is theorized to include some of the plowing and sliding components which cause the normal grinding force to climb faster, but the plowing and sliding only delay the material removed and not the removal rate. Using the longer time delay of 4.9 seconds with the 6 second time constant results in a more accurate material removal prediction as shown in Figure 4.8.

A second interesting result is shown in Figure 4.9 plotting the material removed, equivalent chip thickness, and total specific energy versus time. As seen in the measured results the region where sliding and plowing dominate and the transition
Figure 4.9. Material removed, equivalent chip thickness, and the total specific grinding energy versus time

from sliding and plowing to cutting are ignored by the 4.9 second delay used in the first order predictions.

The results in this section demonstrate many of the inadequacies of the classical modeling effort. Only after careful and tedious examination of the actual measured data could the predicted force and material removal perform accurately. It also appears from these results that two system time constants exist, one for the force prediction and a second for material removal prediction. Without this tedious alignment procedure the first order response predicts grossly larger numbers for the material removed than was actually removed. This tedious alignment requires knowledge from the process that can only be acquired by monitoring the grinding force. This monitoring was simplified by the unique hardware at our disposal not available to other researchers.

In the following sections the results of the Simulink simulation using the power energy model $e_t = 0.31(h_{eq})^{-0.5}$ will be presented and compared to the measured
data of the process along with the first order predicted results pre-aligned by the tedious methods presented in this section. The benefits of the new simulation are apparent in the results of the predicted force and material removed which do not require this alignment procedure. Only one pre-grinding test is required using this unique hardware and then other grinding cycles on the machine may be predicted by varying the grinding parameters.

4.3 Varying commanded infeed velocity

![Graph showing comparison of normal grinding force of first order model to the actual measured normal grinding force with varying infeed velocity.](image)

**Figure 4.10.** Comparison of normal grinding force of first order model to the actual measured normal grinding force with varying infeed velocity.

The first variable to be explored was the commanded infeed velocity, which
was increased in increments of 0.33 \( \mu \text{m/sec} \) while the wheel and work speeds were held constant. The first order model from Chapter 1 is used as a baseline for these preliminary simulations.

Table 4.3 shows five grinding cases with increasing infeed. The case number identifies the grinding test conditions using by the scheme of: test number, followed by the letter P for processed data, then the work speed in rpm, wheel speed in rpm, infeed velocity in microns per second, and ending in a 0 for no dwell at the end of the grind cycle.

In testing, every cycle must be monitored to acquire the steady state grinding force \( K_c \) and the system time constant \( \tau \) (force response speed). The system time constant is calculated by the monitored force’s time difference between 63.2\% to 86.5\% of its final steady state value as mentioned in the previous section. The time constant used for the predicted material removal was calculated using Equation 3.14. These results are shown in Table 4.3.

<table>
<thead>
<tr>
<th>Case File</th>
<th>( v_s ) (m/s)</th>
<th>( v_w ) (m/s)</th>
<th>( v_f ) (( \mu \text{m/s} ))</th>
<th>( \tau ) (s)</th>
<th>( K_c ) (N)</th>
<th>( \mu )</th>
<th>Delay (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20P52016430670D</td>
<td>15.3</td>
<td>54.5</td>
<td>0.67</td>
<td>3.20</td>
<td>15.0</td>
<td>0.35</td>
<td>4.89</td>
</tr>
<tr>
<td>21P52016431000D</td>
<td>-</td>
<td>-</td>
<td>1.00</td>
<td>2.78</td>
<td>18.6</td>
<td>0.36</td>
<td>3.41</td>
</tr>
<tr>
<td>22P52016431330D</td>
<td>-</td>
<td>-</td>
<td>1.33</td>
<td>2.13</td>
<td>20.1</td>
<td>0.36</td>
<td>2.82</td>
</tr>
<tr>
<td>24P52016431670D</td>
<td>-</td>
<td>-</td>
<td>1.67</td>
<td>1.94</td>
<td>23.4</td>
<td>0.35</td>
<td>2.40</td>
</tr>
<tr>
<td>27P52016432000D</td>
<td>-</td>
<td>-</td>
<td>2.00</td>
<td>1.61</td>
<td>25.7</td>
<td>0.36</td>
<td>2.13</td>
</tr>
</tbody>
</table>

Figure 4.10 compares the force results for the predicted baseline to the actual measured datasets. The alignment for the predicted force response uses the inverse slope of the calculated time constant as seen in the previous section. The starting point is where the inverse slope, tangent to the measured force crosses the horizontal axis as shown by the black line. The starting point of the first order model always occurs behind the initial increase of force measured. This starting point neglects the contact and energy nonlinearities and results in a time offset for the model. This offset is recorded as a delay in Table 4.3.

The material removed is predicted with Equation 3.8 from Chapter 3 and uses a different system time constant than acquired from the force, but uses the same
Figure 4.11. Comparing size errors for the part of a first order model to the actual measured material removed.

time delay. The results for the predicted material removed are compared the the measured dataset in Figure 4.11.

In the figure the material removed by the first order predictions performs well after the tedious alignment used in the previous section. The slopes in Figure 4.11 show that the commanded and material removal rates are ultimately equal as the process reaches steady state.

Over the course of this research the representative time constants were collected from each grinding test force. A trend was found to exist between the time constant and the commanded infeed velocity. Figure 4.12 plots 42 dataset’s time constant as a function of the commanded infeed velocity. The red plus symbols in the figure designate the five time constants in Table 4.3. This figure shows that the system
time constant decrease logarithmically to the increasing commanded infeed velocity by the function

\[ \tau = 2.72(v_f)^{-0.7} \]  

(4.1)

This result shows an asymptote to the rise time of steady state normal force. After a 2.2 \( \mu \text{m/sec} \) commanded infeed there is no effect on the grinding forces rise time to steady state.

In addition to the collected time constant, there are also 42 data points collected for the steady state normal grinding force \( K_c \). The steady state normal grinding force also appeared to be linked to the commanded infeed velocity as seen in Figure 4.13. The five cases in Table 4.3 are highlighted by the red plus

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**Figure 4.12.** System time constant vs the commanded in-feed velocity.

---
symbols. This result shows that the steady state force increases linearity with the commanded infeed velocity. The three low infeed cases deviated from the modeled line, presumably because of the disproportionately large influence of the plowing and rubbing forces (which do not cause material removal).

These results can be used to make estimates of the time constant and steady state grinding force for various grinding parameters to be used in the first order model without additional testing.

With the baseline complete, results for the model developed in Chapter 3 can be compared to the actual grinding data in Table 4.3. The inputs given to the simulation are the first three columns in Table 4.3; $v_s$, $v_w$, and $v_f$. The specific
grinding energy used in this simulation was acquired from an initial grinding experiment and is the same as equation 3.35 from Chapter 3. The contact stiffness derived in Chapter 3 is also included in the simulation.

![Diagram showing force over time for measured and modeled forces.](image)

**Figure 4.14.** Output force of simulink model given three input parameters, and compared to the measured normal and tangential grinding forces

Figure 4.14 shows the results of the simulation’s output force compared to the actual grinding data. The simulation estimates both the tangential and normal grinding forces. The initial nonlinearity is clearly accounted for in the force results and avoids the alignment procedure used for the baseline. The estimated and measured tangential grinding force are in good agreement. For the normal force comparison, initially the results are in agreement but tend to deviate from steady state.
Figure 4.15. Calculated error between the measured and simulated force

A more quantitative comparison is given in Figure 4.15 for the first dataset. In the top plot the two components of forces are compared and appear to be in agreement. The lower plot is the calculated difference between the compared forces. The straight line, in the lower plot, corresponds to the average difference in force which is quite small considering the the steady state forces’ magnitude. The error is computed between the simulation and measured dat using the measured steady state forces in the top plot divided by the average difference in the lower plot. For this dataset a result of 2.2 percent error for the normal force and 2.7 percent for the tangential force was calculated. These values are recorded in Table 4.3 along with the calculated values for the remaining four datasets.

The results of the simulation’s material removal and actual infeed compared to
Table 4.2. Simulation errors with varying infeed velocity

<table>
<thead>
<tr>
<th>Case</th>
<th>$F_n$</th>
<th>$F_t$</th>
<th>$\text{gap}$</th>
<th>$\text{gap}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>File</td>
<td>% error</td>
<td>% error</td>
<td>Average error ($\mu$m)</td>
<td>max error ($\mu$m)</td>
</tr>
<tr>
<td>20P52016430670D</td>
<td>2.2</td>
<td>2.7</td>
<td>0.12</td>
<td>0.5</td>
</tr>
<tr>
<td>21P52016431000D</td>
<td>3.2</td>
<td>2.8</td>
<td>0.07</td>
<td>0.2</td>
</tr>
<tr>
<td>22P52016431330D</td>
<td>2.0</td>
<td>1.6</td>
<td>0.07</td>
<td>0.2</td>
</tr>
<tr>
<td>24P52016431670D</td>
<td>2.4</td>
<td>1.5</td>
<td>0.13</td>
<td>0.2</td>
</tr>
<tr>
<td>27P52016432000D</td>
<td>4.2</td>
<td>4.0</td>
<td>0.13</td>
<td>0.2</td>
</tr>
</tbody>
</table>

the actual grinding data is seen in Figure 4.16. The outputs are closely aligned to the actual measured data.

Since the gap between the work and probe was the actual measurement, a qualitative assessment of the simulation’s output will be made using this measurement as a comparison. Using Equation 5 from Chapter 3 with the simulation’s estimates of material removed and deflection of the work spindle a gap can be estimated.

The comparison between the estimated and measured gap is shown in the top plot of Figure 4.17 for the first dataset. The lower plot is the calculated difference between the two values. The average deviation between the measured and the simulation is about 0.12 microns and the maximum error is about a half micron. These results are given in Table 4.3.

4.4 Varying commanded spindle work speed

In this next section the simulation’s output with a varying work spindle speed is investigated. Table 4.4 shows six cases used for this section. The commanded infeed velocity is constant along with the wheel speed. As before the first order model response will be used as a benchmark.

The benchmark force is compared to the measured data for varying work spindle speed in Figure 4.18. These results are similar to the previous with one exception, the work spindle speed did not affect the force as the commanded infeed velocity parameter. The normal and tangential force are constant with varying work spindle speed.

The benchmark material removed is compared to the measured data for varying
Figure 4.16. Output material removed and actual infeed of simulink model

work spindle speed in Figure 4.19. Changing work spindle speed also has no affect on the material removed. As before the blue line in Figure 4.19 performs well after the alignment procedure.

Figure 4.20 shows the results of the simulated force compared to the measured data set for varying work spindle speed. A quantitative measurement of these results is provided in Table 4.4.

Figure 4.21 shows the results of the simulated material removed and actual infeed compared to the measured data set for varying work spindle speed. A quantitative measurement of these results is provided in Table 4.4.

The contribution of these results conclusively demonstrates the inadequacies of classical modeling efforts and the improvements made by the newly develop
Figure 4.17. Comparison of the modeled and simulated gap.

simulation. One of the main issues with the classical modeling effort is its inability to differentiate the material removed from the workpiece and the actual wheel infeed, which are usually stated as being equivalent. Confusing the actual infeed with the material removed will lead to grossly over predicted sizes of several microns as shown in these results. Other deficiencies of the classical model include the neglect of the nonlinear contact stiffness resulting in a time delay between the start of the model and measured data producing further errors in predicted values.

Overall the new simulation is work performed excellent, with good agreement of predicted values and shape over the entire dataset. Discrepancy in the simulation are due to the nonlinear specific grinding energy and grinding efficiency expressed in the grinding force ratio $\mu$. These two elements are discussed in the following
Table 4.3. Model results with varying work spindle speed

<table>
<thead>
<tr>
<th>Case File</th>
<th>$v_s$ (m/s)</th>
<th>$v_w$ (m/s)</th>
<th>$v_f$ ($\mu$m/s)</th>
<th>$\tau$ (s)</th>
<th>$K_e$ (N)</th>
<th>$\mu$</th>
<th>Delay (s)</th>
</tr>
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<tr>
<td>37P52022641331D</td>
<td>21.1</td>
<td>54.5</td>
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<td>1.99</td>
<td>10.3</td>
<td>0.33</td>
<td>2.45</td>
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<td>75.5</td>
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<td>-</td>
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</tr>
<tr>
<td>41P75522641331D</td>
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<td>79.1</td>
<td>-</td>
<td>1.93</td>
<td>10.5</td>
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<td>2.65</td>
</tr>
<tr>
<td>42P37822641331D</td>
<td>-</td>
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<td>-</td>
<td>2.04</td>
<td>10.5</td>
<td>0.34</td>
<td>2.50</td>
</tr>
<tr>
<td>43P28322641331D</td>
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<td>29.6</td>
<td>-</td>
<td>1.90</td>
<td>10.3</td>
<td>0.34</td>
<td>2.84</td>
</tr>
</tbody>
</table>

Table 4.4. Simulation errors with varying work spindle speed velocity

<table>
<thead>
<tr>
<th>Case File</th>
<th>$F_n$ % error</th>
<th>$F_t$ % error</th>
<th>gap (Average error) (µm)</th>
<th>max error (µm)</th>
</tr>
</thead>
<tbody>
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<td>37P52022641331D</td>
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<td>1.9</td>
<td>0.10</td>
<td>0.3</td>
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<tr>
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<td>0.26</td>
<td>0.7</td>
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<tr>
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<td>0.16</td>
<td>0.3</td>
</tr>
<tr>
<td>43P28322641331D</td>
<td>2.6</td>
<td>2.8</td>
<td>0.33</td>
<td>0.6</td>
</tr>
</tbody>
</table>

4.5 Irregularities in the simulation

One of the main contributions of this work is the incorporation of the equivalent chip thickness-dependency of the specific grinding energy model. In this section the results of the specific grinding energy are introduced through the 42 measured datasets out of 54.

Figure 4.22 shows the nonlinear (logarithmic) relationship between the equivalent chip thickness and the specific grinding energy for this particular wheel/work combination under changing grinding parameters. In the figure there are two different symbols to differentiate the wheel speeds used in the experiments. The plus symbol corresponds to wheel speed of 2264 rpm and the o symbol is for 1643 rpm. The colors represent the different infeeds of the datasets with the measured nu-
Figure 4.18. Comparing normal grinding force of first order model to the actual measured normal grinding force with varying work spindle speed.

The experimentally-determined specific grinding energies fall within a narrow range. This range appears to converge as the equivalent chip thickness increases. At about 8 nm for the equivalent chip thickness most of the specific grinding energies lie on top of one another.

As the equivalent chip thickness increases above the threshold for material removal, rubbing and ploughing energy becomes relatively smaller in comparison with cutting energy. The chip formation energy remains constant with increasing removal rate and can be identified as the proportion where the dataset’s range...
Figure 4.19. Comparing size errors for the part of a first order model to the actual measured material removed

converges. From the figure the cutting energy appears to be related only to the equivalent chip thickness and the other two energies dependent on the grinding parameters.

The spread of the specific grinding energies’ range is easier to see on a log-log plot as shown in Figure 4.23. The model for specific grinding energy developed in Chapter 3 is represent by the black line. The core of the dataset, from about 300 to 75 J/mm³, agree with the energy model and is hard to differentiate the colors to the various datasets. The two exception from this model is the very low infeed grinding rate shown as the cyan color, with rates 90 percent lower than the average rate used in these experiments, and the red dataset with the second lowest infeed rate.
The datasets marked in red actually still follow the general trend of the energy model and the simulation’s results for this set are in agreement with the measured tests as seen by dataset 20P52016430670D in Table 4.3.

The datasets marked in cyan look to follow the general trend as seen in Figure 4.22 but at a lower cutting energy. In Figure 4.23 the dataset looks to have an entirely different slope and has a large range. Due to the current input parameters the equivalent chip thickness will not increase above 1.2 nm for this dataset, the work spindle would have to be increased or the wheel speed decrease to see if the dataset would eventually converge on the model.

Other irregularities in the simulation could be cause by the grinding force ratio.
Figure 4.21. Comparing simulated material moved and infeed to the actual measured material removed and actual in-feed for varying work spindle speed.

This ratio has been used to characterize the friction conditions in the contact zone between the cutting edges and the workpiece. It is a quantitative measurement for the cutting ability of the grinding wheel and the effectiveness of material removal. Sharp wheels tend to have higher grinding force ratios as the normal force is low when compared to the tangential force. Conversely, when grinding with blunt wheels, the grinding force ratio is low.

Figure 4.24 plots the quotient of the tangential force and the normal force during a grinding cycle. The slope of this line is the grinding force ratio.

Figure 4.25 shows the calculated grinding force ratio for all 42 datasets. The wheel was maintained throughout this research by redressing the surface after each
Figure 4.22. Specific grinding energy versus the equivalent chip thickness for the 42 measured datasets.

The grinding force ratio average is about 0.35 as shown in the figure. However, there are deviations of the ratio up to 0.13.

The simulation estimates the cutting, or tangential force. The normal force is calculated through the tangential force and this grinding force ratio in the simulation. There is no model of the grinding force ratio in the simulation and these deviations can increase or lower the normal grinding force which affects the rest of the simulation.
Figure 4.23. Log-Log plot of specific grinding energy compared to logarithmic energy model for the 42 measured datasets

These results show an overall improvement in the simulation of the grinding force and material removed from the work. The predictions of the simulation were consistently in agreement with the actual grinding experiments as seen here. This work conclusive demonstrates the inadequacies of classical modeling efforts and takes into account the nonlinearities in the grinding process.
Figure 4.24. Top plot is tangential and normal force versus time. Lower plot is the quotient of the tangential force and the normal force.
Figure 4.25. Grinding force ratio for 42 datasets
Conclusions

5.1 Summary

The physical description of the machine used six equations derived from the free body diagrams of the structural elements and corresponded to six unknowns for deflection and force. These equations and measured stiffnesses, exclusive to this machine, was outline in the procedures given in Chapters 2 and 3.

The normal component of the grinding force was modeled in Chapter 3 by the nonlinear, equivalent chip thickness dependent, energy of material removal. The specific grinding energy captures three components corresponding to the abrasive mechanisms; rubbing, plowing, and cutting. This energy is treated as a material property of the work/wheel combination and found through a simple pre-grinding test using the current setup. The additional nonlinearity included in the model was the grinding wheel contact stiffness developed at the end of Chapter 3 with measured data.

The procedures described in this work allow an individual with a minimum grinding experience to set up a system of equations for a unique cylindrical grinder and estimate the grinding process relationship between the machine compliance, depth of cut, and grinding force so that the size errors of a workpiece can be simulated with minimal testing.
5.2 Contributions of work

The first contribution of this work is conclusive demonstration of the inadequacies of classical modeling efforts. It is well known that the material removed from the workpiece lags the commanded infeed due to the compliance of the machine. However, there is some confusion to the predicted material removed. In this work it was found that the actual infeed of the wheel does not equal the material removed as numerously cited in the literature. Initially, rubbing and plowing dominate as the wheel contact with the workpiece commences. Machine compliance causes the actual infeed of the wheel to lag the commanded infeed. During this initial contact the forces rise due to the plowing and rubbing, but the material removed from the work is negligible. As the actual infeed increases a threshold is reached and material removal begins. Confusing the actual infeed with the material removed will lead to grossly over predicted sizes of several microns.

It was previously known that different parameters affect grinding, and that most of them are captured by an equivalent chip thickness. This work made use of the nonlinear (logarithmic) relationship between the equivalent chip thickness and the specific grinding energy. Our model reflects the specific grinding energy as a material property of the work and wheel, and is found through an initial grinding test. In this work, the experimentally-determined specific grinding energies typically fell within a narrow range because of the use of a single workpiece and abrasive wheel. The second contribution of this work is the incorporation of the equivalent chip thickness-dependency of the specific grinding energy model. This improvement led to predictions that were consistently in agreement with the actual grinding experiments. This verified predictive ability is invaluable for exploring the implications of different process parameters on the grinding force and material removed from the workpiece.

The final contribution of this work is recognition that the contact stiffness between the wheel and work is the second nonlinear phenomena that cannot be ignored, particularly in grinding with light depths of cut with superabrasive wheels. The response of the grinding force at the beginning of a grinding cycle increases exponentially until steady-state is reached (the point at which the material removal rate matches the commanded infeed velocity). The contact stiffness was extracted
from the measurement of the total system deflection and the calculated grinding machine deflection under the grinding load. The contact deflection was found to increase with the normal grinding force in an exponential manner eventually reaching an asymptotic steady state value.
Simulink Model
Processes' parameter inputs (controller)
Machine's physics (unique to machine)
Material's physics (unique to wheel/workpiece combo)
Predicted outputs

\[ x_c(t) \]
\[ y(t) \]
\[ \delta_k(t) \]
\[ \text{gap}(t) \]
\[ r(t) \]
\[ \Delta r(t) \]
\[ 1/\tau_w \text{ delay} \]
\[ \tau_w dw/ds \]
\[ \tau_s \]
\[ v_r(t) \]
\[ \text{bs} \]
\[ \text{heq}(t) \]
\[ \text{eq}(0) \]
\[ \text{et}(t) \]
\[ \text{etheqbs} \]
\[ F_t(t) \]
\[ 1/K_s \]
\[ 1/Keq \]
\[ 1/0.2 \]
\[ (F_n)_{\text{finite}} \]
\[ \text{value} \]

Contact deflection
X-side deflection
Z-side deflection
Grinding width
Grinding ratio

Figure A.1. Simulink model of the grinding process
Bibliography


Vita
Theodore R. S. Deakyne

Theodore R.S. Deakyne was born October 19, 1982. He obtained his Bachelors in Mechanical Engineering from the College of Engineering, Pennsylvania, The Pennsylvania State University, in the year 2005. He joined the doctoral program in the Department of Mechanical Engineering at The Pennsylvania State University in 2007. During his doctoral program he published in reputed conferences in his research area, ASPE, and coauthored with other members in his lab. He spent two summers working at Confero solutions, a branch of Heidenhain, a summer working at the Professional Instruments Company, and a year working as a consulting engineer in the design and development for companies such as ASML. He also spent two semesters teaching junior level engineering classes at The Pennsylvania State University.