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The Graduate School
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ANALYTICAL AND SIMULATION MODELS FOR EVALUATING CASH-FLOW BULLWHIP IN SUPPLY CHAINS

A Thesis in
Industrial Engineering
by
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ABSTRACT

Inventory bullwhip in supply chain can have numerous adverse consequences and has been widely investigated over the last several years. Recent research has begun to investigate the plausibility bullwhip in cash-flow and has focused on developing theoretical models using analytical and simulation tools. This thesis extends these theoretical investigations by studying the effect of cash collection size and the variance in cash collection. In particular, a series of designed experiments are conducted using computer simulation of a four-tier supply chain in which cash collection is treated as a random variable. From the observation of these experiments, the size of cash collection is either irrelevant or marginally relevant to the cash flow bullwhip effect. Moreover, analytical models are developed to characterize the effect of variance in cash collection when it is has a normal or exponential distribution. No matter which distribution is assumed for the cash collection, the models tend to overestimate the cash flow bullwhip. When the cash collection follows normal distribution, the model over estimates cash flow bullwhip by approximately 30%. Whereas the model over estimates cash flow bullwhip by approximately 150% when the cash collection follows exponential distribution.
# TABLE OF CONTENTS

List of Figures ........................................................................................................ v

List of Tables ........................................................................................................ vi

Acknowledgements ............................................................................................... vii

Chapter 1 Introduction ......................................................................................... 1

Chapter 2 Literature Review .................................................................................. 4
   2.1. Supply Chain Management ........................................................................ 4
   2.2 Bullwhip Effect in Supply Chain ................................................................. 4
   2.3. Cash Flow Bullwhip Effect in Supply Chain .............................................. 6
   2.4. Extended Beer Game Model ...................................................................... 6

Chapter 3 An Effect of Cash Collection Size on Supply Chain Cash Flow ............. 9
   3.1. Motivation and Problem ............................................................................ 9
   3.2. Methodology ............................................................................................. 10
   3.3. Result ......................................................................................................... 12
      3.3.1. CV(PaymentRetailer) ........................................................................ 13
      3.3.2. CV(PaymentDistributor) .................................................................. 14
      3.3.3. CV(Paymentmanufacturer) ............................................................... 15
      3.3.4. CV(PaymentSupplier) ...................................................................... 16
   3.4. Conclusive Remarks .................................................................................. 17

Chapter 4 An effect of Cash Collection Ratio Variation on Supply Chain Cash Flow ... 20
   4.1. Motivation and Problem ............................................................................ 20
   4.2. Methodology ............................................................................................. 20
   4.3. Derivation and Result ................................................................................ 21
      4.3.1. Normal Distribution and Exponential Distribution .......................... 24
      4.3.3. Family of Normal Distribution ....................................................... 29
   4.4. Conclusive Remarks .................................................................................. 31

Chapter 5 Summary and Future Research ............................................................ 32
   5.1. Summary ................................................................................................... 32
   5.2. Future Research ....................................................................................... 33

References ............................................................................................................ 35

Appendix  Matlab Code for Extended Beer Distribution Game Model ................. 37
LIST OF FIGURES

Figure 2 - 1 Supply Chain.................................................................................................................4
Figure 2 - 2 Bullwhip Effect ................................................................................................................5
Figure 2 - 3 Supply Chain Model with Cash Flow .............................................................................7
Figure 2 - 4 Normalized Payments for Supply Chain Member vs Time .......................................8
Figure 3 - 1 Supply Chain ....................................................................................................................9
Figure 4 - 1 Comparison between Simulation and Analytic (Normal) Result ............................27
Figure 4 - 2 Comparison between Simulation and Analytic (Exponential) Result .....................28
Figure 4 - 3 Comparison of C.V. .........................................................................................................28
Figure 4 - 4 Family of Normal Distribution .....................................................................................29
Figure 4 - 5 Comparison between Family of Normal Dis. and Exponential Dis. ......................30
LIST OF TABLES

Table 3 - 1 How cash flow bullwhip effect can be compared to bullwhip effect .................. 10
Table 3 - 2 Decision Variables for the DOE ................................................................. 11
Table 3 - 3 Design of Experiment ................................................................................ 12
Table 3 - 4 Independent Variables included in the Regression Model for Retailer .......... 13
Table 3 - 5 Independent Variables included in the Regression Model for Distributor ...... 14
Table 3 - 6 Independent Variables included in the Regression Model for Manufacturer .... 15
Table 3 - 7 Independent Variables included in the Regression Model for Supplier ........ 16
Table 3 - 8 Comparison between Liu’s and Shin’s DOE Model ....................................... 18
Table 3 - 9 Variables included in the Regression Models ............................................... 18
Table 4 - 1 Coefficient of variation of O_j .................................................................... 26
Table 4 - 2 Family of Normal Distribution with Mean of 0.45 ........................................ 30
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Chapter 1

Introduction

Many supply chain management textbooks teach that there are three components in supply chain management: goods, information and cash (Handfield, 1999, Fawcett, 2006 and Bernd 2002). However, for many years, not only practitioners but also many researchers have only focused on the flow of physical goods and information in supply chain. The study of cash flow is neglected. Consequently, average inventory level of a firm was reduced by 35% while account receivables were reduced by only 16% for the last 20 years, according to the report of the TradeBeam by Hausman in 2009.

However, the studied trend has slowly changed from the flow of physical goods and information to the flow of cash in supply chain, as the world recently faced its worst financial crisis in history (Kirkup, 2011). During the recent financial crisis, financial institutions were reluctant to lend money and companies were afraid of running their business with credit because financial asset’s nominal value was dropped abruptly (“Worldbank.org”, 2013). As a result, companies, especially small and medium sized enterprises, which do not hold adequate cash to pay what they owed, tried to extend their account payable term and shorten their account receivable term at the same time in order to stay in business. Unfortunately all of these attempts did not prevent them from going bankrupt during this time (“Worldbank.org”, 2013). Therefore, survived companies and researchers realize the importance of holding sufficient cash on hand and cash flow during and after unstable economic condition.

While practitioners realize the importance of the cash due to the financial crisis, the field of supply chain management also faced new paradigm due to the progress of information
technology. In the past, the role of supply chain management was limited. The researchers in the field of supply chain management focused on production or logistic aspects of business as it is mentioned above. However, as the information technology progresses, the role of supply chain management is expanded to all aspects of business such as marketing and finance (Camerinelli, 2009). As a result, there are few papers that suggest integration between supply chain management and cooperate cash management. For example, in 2006 Hofmann conceptually suggested a way to manage working capital with collaboration of players in supply chain.

Not until recently, the study of cash flow in supply chain is proposed. In 2010, Sasisekaran reported the prominence of the demand forecasting on the cash flow in supply chain. Sasisekaran concluded based on the VBA simulation that, when the forecasting model is correctly implemented, there is a possibility to free up large amounts of capitals which could go into new investment (Saisekaran, 2010). Moreover, in 2013, He was focused on studying cash flow forecasting in different economic conditions. He tried to find out how the customer payment time which probability is represented by Weibull distribution, affects the proper value of working capital requirement of supply chain members (He, 2013). In addition, Tangsucheeva proposed a new cash collection prediction model which was built by combining Markov Chain and exponential smoothing forecasting technique. This new attempt tries to help SMEs’ cash flow by improving the accuracy of cash collection prediction rate (Tangsucheeva, 2013).

Meanwhile, others focused on the cash flow bullwhip effect in supply chain. In 2011, Liu proposed a simple model which extended the beer distribution game model by introducing a cash flow component in the model (Liu, 2011). In his dissertation, Liu proposed the existence of cash flow bullwhip effect using the model he developed. In addition, he proposed another model which illustrates the effect of payment decision rule on cash flow in supply chain (Liu, 2011). While Liu was focused on studying existence of cash flow bullwhip effect using the beer distribution game model, Tangsucheeva proposed cash flow bullwhip effect in a different perspective.
(Tangsucheeva, 2013). Instead of using the beer distribution game, Tangsucheeva proposed the existence of cash flow bullwhip effect using an analytical model which is derived from the Cash Conversion Cycle. In addition, he proposed that 20% of cash flow bullwhip effect is contributed by the bullwhip effect (Tangsucheeva, 2013).

The main contribution of both Liu’s thesis and Tangsucheeva’s paper is that they both identified the cash flow bullwhip effect in supply chain. Thus, it is natural to pay attention to the drivers of cash flow bullwhip effect like decades ago, researchers had focused on studying the drivers of the bullwhip effect and how to mitigate this phenomenon, since bullwhip effect has been identified.

In Liu’s thesis, he designed experiments to identify factors that affect the cash flow bullwhip effect. More specifically speaking, he focused on the finding impact of moving average term which is used in demand forecast and cash collection forecast on cash-flow bullwhip effect. However, he did not put much attention on impact of cash collection ratio itself on cash flow bullwhip effect, even though cash collection is the only financial component used in the model. Thus, in this thesis, the effect of cash collection ratio is studied in depth based on Liu’s model: (1) how the size of cash collection affects the cash flow bullwhip effect, and (2) how the variation of cash collection ratio affects the cash flow bullwhip effect.

And the rest of the thesis is organized as follows: Chapter 2 provides summary of bullwhip effect by introducing some important papers since the existence of bullwhip effect was detected. At the same time the concept of cash flow bullwhip effect is introduced. Furthermore, overview of Liu’s model is presented in detail. In Chapter 3, a problem, “how the size of cash collection ratio affects the cash flow bullwhip effect is studied via Design of Experiment. The details of the methodology are explained in Chapter 3. Chapter 4 proposes an analytic model to determine the variation of cash collection ratio effect on the cash flow bullwhip effect. Lastly, in chapter 5, the conclusive remarks and direction for future works are discussed.
Chapter 2

Literature Review

2.1. Supply Chain Management

Since late 1990s, a term called Supply Chain Management became popular (Cooper, 1997). The number of conference sessions which titles contains “supply chain” increase from 13.5% to 22.4% in two years at the annual conference of the Council of Logistics Management (Cooper, 1997). However, the definition of supply chain management was not completely determined by researchers. During late 1990s, the concept of supply chain management is something that is similar to logistic management (Cooper, 1997).

Figure 2-1 Supply Chain

Therefore, the more comprehensive definition was needed. In 2001, Mentzer defines supply chain as “a set of three or more entities (organizations or individuals) directly involved in the upstream and downstream flows of products, services, finances, and/or information from a source to a customer.”

2.2 Bullwhip Effect in Supply Chain

The phenomenon in which product order size is amplified as it goes to the upper player in the supply chain is called “Bullwhip Effect” (Bernd, 2002) as it is shown in the Figure 2 – 2.
Since this phenomenon was identified first in 1989, researchers focused on studying causes or
drivers of the bullwhip effect and how to mitigate this phenomenon.

![Figure 2 - 2 Bullwhip Effect](image)

In 1989, Sterman successfully introduced and identified the existence of bullwhip effect using Beer Distribution Game. In the same year, Burbidge also introduced the bullwhip effect by describing cause or reason of existence. He proposed that bullwhip effect exists in the supply chain because of following reasons: deluded information and delays of materials.

After Burbidge’s attempt to explain the cause of the bullwhip effect, many of other researchers also focused on explaining causes of the bullwhip effect. For example, Lee et al in 1997 listed four reasons in his paper: demand forecast update, batch order, fluctuation in price and rationing game.

While some researchers had focused on identifying causes of the bullwhip effect, some had focused on influences of bullwhip effect. For example, researchers reported that bullwhip effect brings negative influences to the supply chain such as excessive inventory level, backorders, uncertainty in production planning, and stock out (Chen, 1998, Jones, 2000 and Lee et al, 2004).

After identifying the causes of the phenomenon, researchers had put their efforts on quantifying the bullwhip effect. In 2000 Chen was able to quantify the bullwhip effect for simple supply chain. After Chen, much more sophisticated supply chain was quantified by other researchers such as Kim in 2006 and Fioriolli in 2008.
2.3. Cash Flow Bullwhip Effect in Supply Chain

Not many papers about the cash flow bullwhip effect exist because the term, cash flow bullwhip effect is newly introduced by Tangsucheeva and Prabhu in 2013. Prior to Tangsucheeva and Prabhu, a similar work can be found in 2011.

In 2011, Liu introduced a concept which states that a similar effect to bullwhip effect exists in supply chain cash flow. In Liu’s paper, he extended Chatfield’s beer game by introducing a cash flow component to the model. In addition, Liu also suggested the metric which can capture the cash flow bullwhip effect in supply chain, which is Coefficient of Variation of Payment.

While Liu’s thesis used coefficient of variation of payment to illustrate cash flow bullwhip effect in the supply chain, Tangsucheeva and Prabhu used different approaches to explain the cash flow bullwhip effect in the supply chain. Tangsucheeva and Prabhu proposed that inventory bullwhip effect leads to cash flow bullwhip effect and it can be explained by using Cash Conversion Cycle as the metric. According to Tangsucheeva and Prabhu, approximately 20% of cash flow bullwhip effect is due to the inventory bullwhip effect (Tangsucheeva and Prabhu, 2013).

2.4. Extended Beer Game Model

In order to see whether the financial factors, such as payment and collection of cash uncertainty, makes impact on the cash flow bullwhip effect, Liu extended the popular model, beer distribution game by introducing a financial component to the model.
In this model, there are five players: Supplier, Manufacturer, Distributor, Retailer and Customer. The orders and cash flows upstream the supply chain while the goods flow downstream the supply chain as it is shown in Figure 2 - 3.

**Figure 2 - 3 Supply Chain Model with Cash Flow**

In Liu’s extended model, the inventory replenishment policy is set to be order-up-to policy and expressed as following:

$$O^t = L\bar{X} + k\sigma - (\text{inventory on hand at } t + \text{inventory on order at } t - \text{backorder at } t)$$

where, $L$ is lead time and $\bar{X}$ is demand mean over the lead time which is updated by moving average forecasting method.

Liu introduced a simple rule for payment decision under which a player expects to pay more if a players expect to collect more from its customer (Liu, 2011). Specifically in each time, each member calculates how much they are going to collect from its downstream player. This is expressed in the following equation:

$$\beta^t = \begin{cases} \frac{\text{collection for sale at } (t-2)}{\text{sales at } (t-2)}, & \text{if } \text{sales at } (t-2) \neq 0 \\ 0, & \text{if } \text{sales at } (t-2) = 0 \end{cases}$$

In addition, the payment decision of each player is as follows:

$$PAY^{t+1} = \hat{\beta}^{t+1} \ast O^{t-1}$$

where, forecasting of collection ratio is expressed by

$$\hat{\beta}^{t+1} = \alpha \beta^t + (1 - \alpha) \hat{\beta}^t$$

Lastly, in Liu’s model, the cash flow bullwhip effect phenomenon is captured by comparing Coefficient of Variation of Payment of each player in supply chain.
\[ cv(P_{AY_j}) = \frac{std(P_{AY_j})}{mean(P_{AY_j})} \]

In Liu’s paper, the simulation model is built using the above equations. Then \( cv(P_{ay}) \) was calculated for each player. When CVs were compared, the CV aggrandized as it moves to upstream player of the supply chain. This is shown in Figure 2 – 4. In other words, the cash flow bullwhip effect is identified.

Figure 2 - 4 Normalized Payments for Supply Chain Member vs Time
Chapter 3

An Effect of Cash Collection Size on Supply Chain Cash Flow

The literature review shows that the existence of bullwhip effect in the supply chain physical goods flow is not questionable. As the importance of managing cash is embossed due to the current economic condition, the bullwhip effect in the supply chain cash flow is also recently studied and proven to exist.

3.1. Motivation and Problem

Having a question that “how are they different?” or “how do they compare?” is very natural for those who know present of both cash flow bullwhip effect and bullwhip effect in supply chain.

Figure 3-1 Supply Chain

Since the logic for two flows are very similar in supply chain as it is shown in Figure 3-1, finding similarity and difference between cash flow bullwhip effect and the bullwhip effect is very important because it may help to understand cash flow bullwhip effect better. If similarity
can be found, then the cash flow bullwhip effect can be removed or mitigated by way to remove or mitigate bullwhip effect. The following questions are some possible questions that can be asked: (1) does impact of lead time on inventory bullwhip effect compare to impact of cash collection term on cash flow bullwhip effect? (2) does impact of order size on inventory bullwhip effect compare to impact of cash collection size on cash flow bullwhip effect?

Table 3 - 1 How cash flow bullwhip effect can be compared to bullwhip effect

<table>
<thead>
<tr>
<th>Inventory</th>
<th>Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bullwhip effect on Inventory</td>
<td>Bullwhip Effect on Payment?</td>
</tr>
<tr>
<td>Lead Time</td>
<td>Cash Collection Term?</td>
</tr>
<tr>
<td>Batch Order Size</td>
<td>Cash Collection Size?</td>
</tr>
</tbody>
</table>

Of the questions that are stated in Table 3 – 1, the question about whether the cash collection size gives similar impact on cash flow bullwhip effect as the batch order size does to inventory bullwhip effect, is going to be examined in this chapter.

3.2. Methodology

In order to find out that whether the cash collection size gives similar impact on the cash flow bullwhip effect as the batch order size, the regression model, whose response variable is coefficient of variation of payment is built through two full factorial design. Unless changes or modification are explicitly stated in the thesis, all of assumptions and details are same as theoretical model introduced in 2011. The decision variables used to build this regression models are time, cash collection size, lead time, and order size.

Since the experiment is designed to have two levels (Low and High), two levels for each independent variables are need to be pre-defined: The Low is 60, and the high is 120 for Time.
The low is 0.2 and the high is 0.8 for payment ratio of the customer. The low is 2 and the high is 8 for lead time. The low is 40 and high is 160 for order size of the customer as shown Table 3 - 2.

<table>
<thead>
<tr>
<th>Name</th>
<th>Range</th>
<th>low</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>1 to ∞</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>Collection Ratio</td>
<td>0 to 1</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>Lead Time</td>
<td>0 to 10</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Order Size</td>
<td>0 to 200</td>
<td>40</td>
<td>160</td>
</tr>
</tbody>
</table>

When the variable is a continuous number like this case, it is very difficult to define low and high value. However, low and high value must be predefined in a way that they are distinct from each other, yet reasonable. Thus, in order to be reasonable, each variables low and high value must be chosen within the range of each variable. For variables which are drivers of bullwhip effect, low is set to be a small number that cannot obviously observe bullwhip effect, and high is set to be a large number that can obviously observe bullwhip effect. For collection ratio, 0 for low and 1 for high is too extreme in this case. Therefore, low and high is set to be a point where they are 0.2 away from minimum or maximum.

In this study, three regression models are going to be built by keeping all variables the same, but differentiating only the response variables: CV of PAY_{Retailer}, CV of PAY_{Distributor}, and CV of PAY_{Manufacturer}

Since each variable has two levels, only a total of $2^4$ or 16 runs is required when the experiment is built using full factorial design. Furthermore, randomizing the order of the experiment is unnecessary since the experiment is going to be conducted via computer simulation, but the order of experiments is randomized anyway. The experiment order and other details of the design of experiment is summarized in the following Table 3 – 3.
Table 3 - Design of Experiment

<table>
<thead>
<tr>
<th>StdOrder</th>
<th>RunOrder</th>
<th>CenterPt</th>
<th>Blocks</th>
<th>Time</th>
<th>Payment Ratio</th>
<th>Lead Time</th>
<th>Order Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>60</td>
<td>0.2</td>
<td>2</td>
<td>160</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>1</td>
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<td>40</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>120</td>
<td>0.8</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>120</td>
<td>0.8</td>
<td>8</td>
<td>160</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>6</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>120</td>
<td>0.2</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>120</td>
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<td>2</td>
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<td>14</td>
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<td>0.2</td>
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<td>160</td>
</tr>
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<td>9</td>
<td>1</td>
<td>1</td>
<td>60</td>
<td>0.8</td>
<td>8</td>
<td>160</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>60</td>
<td>0.8</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>13</td>
<td>11</td>
<td>1</td>
<td>1</td>
<td>60</td>
<td>0.2</td>
<td>8</td>
<td>160</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>1</td>
<td>1</td>
<td>120</td>
<td>0.8</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>1</td>
<td>1</td>
<td>120</td>
<td>0.8</td>
<td>2</td>
<td>160</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>1</td>
<td>1</td>
<td>60</td>
<td>0.8</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
<td>1</td>
<td>1</td>
<td>60</td>
<td>0.8</td>
<td>2</td>
<td>160</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>1</td>
<td>1</td>
<td>120</td>
<td>0.2</td>
<td>2</td>
<td>160</td>
</tr>
</tbody>
</table>

3.3. Result

Only variables which p-value is less than 0.05 are included in the final model. Furthermore, variables only up to 2-way interactions are considered as candidate. In other words, any variables 3-way interactions or above are automatically removed from the final model.
### 3.3.1. CV\(\text{Retailer}\)

The regression model for the retailer which plays at one level above the customer, is composed of following six variables: Time, Lead Time, Order Size, Time*Lead Time, Time*Order Size and Lead Time*Order Size. The regression model’s adjusted coefficient determination is 100%.

**Table 3 - 4 Independent Variables included in the Regression Model for Retailer**

<table>
<thead>
<tr>
<th>C.V</th>
<th>P</th>
<th>PR</th>
<th>LT</th>
<th>OS</th>
<th>T*PR</th>
<th>T*LT</th>
<th>T*OS</th>
<th>PR*LT</th>
<th>PR*OS</th>
<th>LT*OS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>

Variables included in the final regression model are marked with O

C.V.: Coefficient Variance, T: Time, PR: Payment Ratio, LT: Lead Time, OS: Order Size

<table>
<thead>
<tr>
<th>Term</th>
<th>Effect</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
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<td>0.000217</td>
<td>3126.46</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>-0.3255</td>
<td>-0.1628</td>
<td>0.000217</td>
<td>-750.95</td>
<td>0.000</td>
</tr>
<tr>
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<td>0.0000</td>
<td>0.000217</td>
<td>0.00</td>
<td>1.000</td>
</tr>
<tr>
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<td>-0.0121</td>
<td>0.000217</td>
<td>-55.77</td>
<td>0.000</td>
</tr>
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<td>-0.0055</td>
<td>0.000217</td>
<td>-25.19</td>
<td>0.000</td>
</tr>
<tr>
<td>Time*Payment_Ratio</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.000217</td>
<td>0.00</td>
<td>1.000</td>
</tr>
<tr>
<td>Time*Lead_Time</td>
<td>0.0070</td>
<td>0.0035</td>
<td>0.000217</td>
<td>16.04</td>
<td>0.000</td>
</tr>
<tr>
<td>Time*Order_Size</td>
<td>0.0045</td>
<td>0.0023</td>
<td>0.000217</td>
<td>10.42</td>
<td>0.000</td>
</tr>
<tr>
<td>Payment_Ratio*Lead_Time</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.000217</td>
<td>0.00</td>
<td>1.000</td>
</tr>
<tr>
<td>Payment_Ratio*Order_Size</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.000217</td>
<td>0.00</td>
<td>1.000</td>
</tr>
<tr>
<td>Lead_Time*Order_Size</td>
<td>0.0020</td>
<td>0.0010</td>
<td>0.000217</td>
<td>4.60</td>
<td>0.006</td>
</tr>
</tbody>
</table>

\(S = 0.000867030\) \(\text{PRESS} = 0.0000384892\)

\(R^2 = 100.00\%\) \(R^2(\text{pred}) = 99.99\%\) \(R^2(\text{adj}) = 100.00\%\)

**Analysis of Variance for CV\_Retailer**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Effects</td>
<td>4</td>
<td>0.426738</td>
<td>0.426738</td>
<td>0.106684</td>
<td>141916.25</td>
<td>0.000</td>
</tr>
<tr>
<td>Time</td>
<td>1</td>
<td>0.423922</td>
<td>0.423922</td>
<td>0.423922</td>
<td>563919.87</td>
<td>0.000</td>
</tr>
<tr>
<td>Payment_Ratio</td>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Lead_Time</td>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Order_Size</td>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>2-Way Interactions</td>
<td>6</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>64.53</td>
<td>0.000</td>
</tr>
<tr>
<td>Time*Payment_Ratio</td>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Time*Lead_Time</td>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Time*Order_Size</td>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Payment_Ratio*Lead_Time</td>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Payment_Ratio*Order_Size</td>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>
3.3.2. CV(Payment\textsubscript{Distributor})

The regression model for the distributor which plays at two levels above the customer, is composed of following five variables: Time, Lead Time, Order Size, Time*Lead Time, and Lead Time*Order Size. The regression model’s adjusted coefficient determination is 99.94%.

Table 3 - 5 Independent Variables included in the Regression Model for Distributor

<table>
<thead>
<tr>
<th>C.V</th>
<th>P</th>
<th>PR</th>
<th>LT</th>
<th>OS</th>
<th>T*PR</th>
<th>T*LT</th>
<th>T*OS</th>
<th>PR*LT</th>
<th>PR*OS</th>
<th>LT*OS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributor</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>

Variables included in the final regression model are marked with O

C.V.: Coefficient Variance, T: Time, PR: Payment Ratio
LT: Lead Time, OS: Order Size

| Estimated Effects and Coefficients for CV\textsubscript{Distributor} (coded units) |
|---------------------|-------|-------|-------|-------|-------|
| Term                | Effect | Coef  | SE Coef | T     | P     |
| Constant            | 0.8451 | 0.001418 | 595.97 | 0.000 |
| Time                | -0.4417 | -0.2208 | 0.001418 | -155.73 | 0.000 |
| Payment\_Ratio      | 0.0000 | 0.0000 | 0.001418 | 0.00  | 1.000 |
| Lead\_Time          | 0.0460 | 0.0230 | 0.001418 | 16.22 | 0.000 |
| Order\_Size         | -0.0037 | -0.0018 | 0.001418 | -1.30 | 0.251 |
| Time*Payment\_Ratio | 0.0000 | 0.0000 | 0.001418 | 0.00  | 1.000 |
| Time*Lead\_Time     | -0.0280 | -0.0140 | 0.001418 | -9.88 | 0.000 |
| Time*Order\_Size    | 0.0020 | 0.0010 | 0.001418 | 0.71  | 0.507 |
| Payment\_Ratio*Lead\_Time | -0.0000 | -0.0000 | 0.001418 | -0.00 | 1.000 |
| Payment\_Ratio*Order\_Size | 0.0000 | 0.0000 | 0.001418 | 0.00  | 1.000 |
| Lead\_Time*Order\_Size | 0.0121 | 0.0061 | 0.001418 | 4.28  | 0.008 |

S = 0.00567205  PRESS = 0.00164721
R-Sq = 99.98%  R-Sq(pred) = 99.79%  R-Sq(adj) = 99.94%

Analysis of Variance for CV\textsubscript{Distributor} (coded units)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Effects</td>
<td>4</td>
<td>0.788782</td>
<td>0.788782</td>
<td>0.197195</td>
<td>6129.39</td>
<td>0.000</td>
</tr>
<tr>
<td>Time</td>
<td>1</td>
<td>0.780258</td>
<td>0.780258</td>
<td>0.780258</td>
<td>24252.64</td>
<td>0.000</td>
</tr>
<tr>
<td>Payment_Ratio</td>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Lead_Time</td>
<td>1</td>
<td>0.008469</td>
<td>0.008469</td>
<td>0.008469</td>
<td>263.25</td>
<td>0.000</td>
</tr>
</tbody>
</table>
3.3.3. CV(Payment\textsubscript{manufacturer})

The regression model for the manufacturer which plays at three levels above the customer, is composed of following five variables: Time, Lead Time, Order Size, Time*Lead Time, and Lead Time*Order Size. The adjusted coefficient determination is 99.04%.

Table 3 - 6 Independent Variables included in the Regression Model for Manufacturer

<table>
<thead>
<tr>
<th>C.V</th>
<th>P</th>
<th>PR</th>
<th>LT</th>
<th>OS</th>
<th>T*PR</th>
<th>T*LT</th>
<th>T*OS</th>
<th>PR*LT</th>
<th>PR*OS</th>
<th>LT*OS</th>
</tr>
</thead>
<tbody>
<tr>
<td>MFG</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>

Variables included in the final regression model are marked with O

C.V.: Coefficient Variance, T: Time, PR: Payment Ratio
LT: Lead Time, OS: Order Size

Estimated Effects and Coefficients for CV\textsubscript{MFG} (coded units)

<table>
<thead>
<tr>
<th>Term</th>
<th>Effect</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.3919</td>
<td>0.01912</td>
<td>72.79</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>-1.1489</td>
<td>-0.5744</td>
<td>0.01912</td>
<td>-30.04</td>
<td>0.000</td>
</tr>
<tr>
<td>Payment_Ratio</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.01912</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Lead_Time</td>
<td>0.8147</td>
<td>0.4074</td>
<td>0.01912</td>
<td>21.30</td>
<td>0.000</td>
</tr>
<tr>
<td>Order_Size</td>
<td>0.0912</td>
<td>0.0456</td>
<td>0.01912</td>
<td>2.39</td>
<td>0.063</td>
</tr>
<tr>
<td>Time*Payment_Ratio</td>
<td>-0.0000</td>
<td>-0.0000</td>
<td>0.01912</td>
<td>-0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Time*Lead_Time</td>
<td>-0.6236</td>
<td>-0.3118</td>
<td>0.01912</td>
<td>-16.31</td>
<td>0.000</td>
</tr>
<tr>
<td>Time*Order_Size</td>
<td>-0.0733</td>
<td>-0.0396</td>
<td>0.01912</td>
<td>-2.07</td>
<td>0.093</td>
</tr>
<tr>
<td>Payment_Ratio*Lead_Time</td>
<td>-0.0000</td>
<td>-0.0000</td>
<td>0.01912</td>
<td>-0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Payment_Ratio*Order_Size</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.01912</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Lead_Time*Order_Size</td>
<td>0.1021</td>
<td>0.0511</td>
<td>0.01912</td>
<td>2.67</td>
<td>0.044</td>
</tr>
</tbody>
</table>

S = 0.0764875  PRESS = 0.299538
R-Sq = 99.70%  R-Sq(pred) = 96.89%  R-Sq(adj) = 99.09%
## Analysis of Variance for CV_MFG (coded units)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Effects</td>
<td>4</td>
<td>7.96835</td>
<td>7.96835</td>
<td>1.99209</td>
<td>340.51</td>
<td>0.000</td>
</tr>
<tr>
<td>Time</td>
<td>1</td>
<td>5.27981</td>
<td>5.27981</td>
<td>5.27981</td>
<td>902.48</td>
<td>0.000</td>
</tr>
<tr>
<td>Payment_Ratio</td>
<td>1</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead_Time</td>
<td>1</td>
<td>2.65523</td>
<td>2.65523</td>
<td>2.65523</td>
<td>453.86</td>
<td>0.000</td>
</tr>
<tr>
<td>Order_Size</td>
<td>1</td>
<td>0.03330</td>
<td>0.03330</td>
<td>0.03330</td>
<td>5.69</td>
<td>0.063</td>
</tr>
<tr>
<td>2-Way Interactions</td>
<td>6</td>
<td>1.62257</td>
<td>1.62257</td>
<td>0.27043</td>
<td>46.22</td>
<td>0.000</td>
</tr>
<tr>
<td>Time*Payment_Ratio</td>
<td>1</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time*Lead_Time</td>
<td>1</td>
<td>1.55571</td>
<td>1.55571</td>
<td>1.55571</td>
<td>265.92</td>
<td>0.000</td>
</tr>
<tr>
<td>Time*Order_Size</td>
<td>1</td>
<td>0.02515</td>
<td>0.02515</td>
<td>0.02515</td>
<td>4.30</td>
<td>0.093</td>
</tr>
<tr>
<td>Payment_Ratio*Lead_Time</td>
<td>1</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Payment_Ratio*Order_Size</td>
<td>1</td>
<td>0.04171</td>
<td>0.04171</td>
<td>0.04171</td>
<td>7.13</td>
<td>0.044</td>
</tr>
<tr>
<td>Residual Error</td>
<td>5</td>
<td>0.02925</td>
<td>0.02925</td>
<td>0.00585</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CV(PAY_{mfg}) = 1.3919 - 0.5744 Time + 0.4074 Lead Time + 0.0456 Order Size - 0.3118 Time * Lead Time + 0.0511 Lead Time * Order Size

### 3.3.4. CV(Payment_{Supplier})

The regression model for the supplier which plays at four levels above the customer, is composed of following five variables: Time, Lead Time, Order Size, Time*Lead Time, and Lead Time*Order Size. The regression model’s adjusted coefficient determination is 99.98%.

#### Table 3 - 7 Independent Variables included in the Regression Model for Supplier

<table>
<thead>
<tr>
<th>C.V</th>
<th>P</th>
<th>PR</th>
<th>LT</th>
<th>OS</th>
<th>T*PR</th>
<th>T*LT</th>
<th>T*OS</th>
<th>PR*LT</th>
<th>PR*OS</th>
<th>LT*OS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>

Variables included in the final regression model are marked with O

Estimated Effects and Coefficients for CV_Supplier (coded units)

<table>
<thead>
<tr>
<th>Term</th>
<th>Effect</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.9236</td>
<td>0.003773</td>
<td>509.79</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Time</td>
<td>-0.9867</td>
<td>-0.4934</td>
<td>0.003773</td>
<td>-130.75</td>
<td>0.000</td>
</tr>
<tr>
<td>Payment_Ratio</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.003773</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Lead_Time</td>
<td>1.5201</td>
<td>0.7601</td>
<td>0.003773</td>
<td>201.43</td>
<td>0.000</td>
</tr>
<tr>
<td>Order_Size</td>
<td>0.2206</td>
<td>0.1621</td>
<td>0.003773</td>
<td>42.97</td>
<td>0.000</td>
</tr>
<tr>
<td>Time*Payment_Ratio</td>
<td>-0.0000</td>
<td>-0.0000</td>
<td>0.003773</td>
<td>-0.0000</td>
<td>1.000</td>
</tr>
<tr>
<td>Time*Lead_Time</td>
<td>-0.3243</td>
<td>-0.1621</td>
<td>0.003773</td>
<td>-42.97</td>
<td>0.000</td>
</tr>
<tr>
<td>Time*Order_Size</td>
<td>-0.0121</td>
<td>-0.0060</td>
<td>0.003773</td>
<td>-1.60</td>
<td>0.170</td>
</tr>
<tr>
<td>Payment_Ratio*Lead_Time</td>
<td>-0.0000</td>
<td>-0.0000</td>
<td>0.003773</td>
<td>-0.0000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
### 3.4. Conclusive Remarks

Prior to discussing the observation of this experiment, it is important to mention the difference between the Liu’s observation and this thesis’ observation. Liu’s experiment and this experiment differ in the focus of experiment; In 2011, Liu also built a regression using DOE. However, Liu’s model is more focused on identifying the effect of the moving term used in demand forecast and payment forecast, while the object of this experiment is more focused on identifying the effect of the cash collection size. As a result, the variables used to build the model is also different. In Liu’s model, the independent variable candidates are: Payment Forecast, Credit Term, Demand Forecast and Lead Time. While this model’s independent variable candidates are: Time, Cash Collection Ratio, Lead Time and Order Size.
Table 3 - 8 Comparison between Liu’s and Shin’s DOE Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>Liu’s DOE</th>
<th>Shin’s DOE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Payment Forecast</td>
<td>Time</td>
</tr>
<tr>
<td></td>
<td>Credit Term</td>
<td>Cash Collection Ratio</td>
</tr>
<tr>
<td></td>
<td>Demand Forecast</td>
<td>Order size</td>
</tr>
<tr>
<td></td>
<td>Lead Time</td>
<td>Lead Time</td>
</tr>
<tr>
<td>Objective</td>
<td>Effect of moving term in used forecasting</td>
<td>Effect of cash collection size</td>
</tr>
</tbody>
</table>

The four regression models built via design of experiment reveal some interesting observations: First of all, generally speaking the coefficient determination drops as it move to the upstream player in the supply chain, although the same independent variables are used to build a model. This indicates that, as it moves upstream players, there are more factors that cannot be explained by independent variables that are used to build a regression model. Secondly, all four regression models did not include the cash collection ratio, which decides how much a player is going to pay for its customer. Furthermore, none of the models include any 2-way interaction variables with the payment ratio as it is shown in Table 3.

Table 3 - 9 Variables included in the Regression Models

<table>
<thead>
<tr>
<th>C.V</th>
<th>P</th>
<th>PR</th>
<th>LT</th>
<th>OS</th>
<th>T*PR</th>
<th>T*LT</th>
<th>T*OS</th>
<th>PR*LT</th>
<th>PR*OS</th>
<th>LT*OS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Distributor</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Supplier</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>

Variables included in the final regression model are marked with O

C.V.: Coefficient Variance, T: Time, PR: Payment Ratio
LT: Lead Time, OS: Order Size
This observation indicates that the $cv(\text{Payment})$ is not depended on the size of cash collection. In other words, the size of cash collection either doesn’t give any effect or gives to insignificant effect to the cash flow bullwhip effect.

In sum, from the observations of this experiment, the following points can be concluded: first of all, variables, such as: lead time, order size and time are also identified as variables that have some impact on cash flow bullwhip effect. According to the regression models, the average of approximately 98% of $cv(\text{payment})$ variation can be explained with variables mentioned above. Secondly, the size of cash collection is not included in a regression model. In other words, whether the customer pays in full or a small fraction of what he or she owes, it really doesn’t matter to the cash flow bullwhip effect. The cash flow bullwhip effect either is not affected or is marginally affected by the size of cash collection.
Chapter 4

An effect of Cash Collection Ratio Variation on Supply Chain Cash Flow

4.1. Motivation and Problem

From the result of the previous chapter, the size of cash collection does not affect the cash flow bullwhip effect. However, as it is mentioned in the introduction, many of the SMEs suffered because their customers did not pay what they owed in times of financial crisis as the economic conditions change. In other words, this fact illustrates that cash collection ratio is somehow related to cash flow in supply chain.

If the size of cash collection ratio is irrelevant to the cash flow bullwhip effect, than there must be a hidden relationship between them. In the end, this leads to another problem that needs to be answered. As it is shown in the chapter 3, if the size of cash collection is irrelevant to the cash flow bullwhip effect, how or in what way does cash collection ratio affect the cash flow bullwhip?

4.2. Methodology

In this chapter, in order to find the hidden relationship between cash flow bullwhip and the cash collection ratio, the components which impact the cash flow bullwhip effect are analytically derived from cv(payment). As the next step, the way in which the cash collection ratio affects cv(Payment) is analytically identified. After that, the cv(Payment) is calculated for every players in the supply chain using equation derived analytically. In the end, the analytic model’s result is compared to the result of the simulation model for the validation purpose.
4.3. Derivation and Result

The cash flow bullwhip effect is measured by $\text{cv}(\text{Payment})$. The coefficient of variation is the ratio of standard deviation over the mean, also known as normalized standard deviation. The reason normalized standard deviation is used in the model is because the price for different members are different. Thus, in order to compare variance of each player, the normalized standard deviation is required. The Payment is defined as following:

$$PAY_{t+1} = \hat{\beta}_{t+1} \cdot O_{t-1}$$

Therefore, payment can also be expressed as following:

$$PAY_t = \hat{\beta}_{t} \cdot O_{t-2} \quad (1)$$

The Payment is composed of forecast of collection ratio and the order made. In addition, the forecasting is made based on exponential smoothing. Therefore, forecast of collection ratio can be expressed as following:

$$\hat{\beta}_{t+1} = \alpha \beta_t + (1 - \alpha)\hat{\beta}_t$$
$$\hat{\beta}_t = \alpha \beta_{t-1} + (1 - \alpha)\hat{\beta}_{t-1} \quad (2)$$

in which, $\beta_t$ is collection ratio at time $t$ which is expressed as shown below:

$$\beta_t = \begin{cases} 
\frac{\text{collection for sale at } (t-2)}{\text{sales at } (t-2)}, & \text{if } \text{sales at } (t-2) \neq 0 \\
0, & \text{if } \text{sales at } (t-2) = 0 
\end{cases} \quad (3)$$

Using equation (1) and (3), the relationship between the collection ratio of player $j$ and the forecast collection ratio of player $j-1$ can be derived as following:

$$\beta_j^t = \frac{\text{collection for sale at } (t-2)}{\text{sales at } (t-2)} = \frac{PAY_{j-1}^t}{O_{j-1}^{t-2}} = \frac{\hat{\beta}_{j-1}^t \cdot O_{j-1}^{t-2}}{O_{j-1}^{t-2}} = \hat{\beta}_{j-1}^t$$

This derivation shows that the collection ratio of player $j$ is dependent on the previous player’s forecast collection ratio. In sum, every players’ collection ratio in the supply chain is
dependent on the payment of the last player of the supply chain, which is also known as the
customer of the supply chain.

In general, exponential smoothing forecast can be defined as following:

\[ S_t = \alpha X_{t-1} + (1 - \alpha)S_{t-1} \]

And if we expand the above equation. We get:

\[ S_t = \alpha[X_{t-1} + (1 - \alpha)X_{t-2} + (1 - \alpha)^2X_{t-3} + \cdots + (1 - \alpha)^{t-1}X_0] + (1 - \alpha)^tS_0 \]

\[ S_t = \sum_{i=0}^{t-1} \alpha(1 - \alpha)^iX_{t-i-1} \tag{4} \]

Furthermore, writing equation (4) in terms of collection ratio (\( \beta_t^f \)) and forecast collection ratio (\( \hat{\beta}^f_t \)), can be express as following:

\[ \hat{\beta}^t = \sum_{i=0}^{t-1} \alpha(1 - \alpha)^i\beta_{t-i-1} \tag{5} \]

By the definition of cv(Payment),

\[ cv(PAY_j) = \frac{std(PAY_j)}{mean(PAY_j)} \tag{6} \]

cv(\( PAY_j \)) can be also expressed as following:
\[ cv(PAY_j) = \frac{\text{std}(PAY_j)}{\text{mean}(PAY_j)} = \frac{\text{std}(\beta_j \times Pr_{j+1} \times O_j)}{E(\beta_j \times Pr_{j+1} \times O_j)} = \frac{Pr_{j+1} \times \text{std}(\beta_j \times O_j)}{Pr_{j+1} \times E(\beta_j)E(O_j)} \]

\[ = \frac{\text{Var}(\beta_j \times O_j)}{\sqrt{(E(\beta_j)E(O_j))^2}} \]

\[ = \frac{E(\beta_j)^2 \text{Var}(O_j) + E(O_j)^2 \text{Var}(\beta_j) + \text{Var}(\beta_j) \text{Var}(O_j)}{\sqrt{(E(\beta_j)E(O_j))^2}} \]

\[ = \frac{\text{Var}(O_j) + \frac{\text{Var}(\beta_j)}{E(\beta_j)^2} + \frac{\text{Var}(\beta_j)}{E(O_j)^2} \cdot \text{Var}(O_j)}{\sqrt{E(O_j)^2 + \frac{\text{Var}(\beta_j)}{E(\beta_j)^2} + \frac{\text{Var}(\beta_j)}{E(O_j)^2} \cdot \text{Var}(O_j)}} \]

\[ = cv(O_j) + cv(\beta_j) + cv(\beta_j) \cdot cv(O_j) \]

\[ = cv(O_j) + (1 + cv(O_j)) \cdot cv(\beta_j) \]

(7)

Equation 7, derived from the definition of cv(Payment), indicates that cv(Payment) is composed of cv(Oj), cv(\hat{\beta}_j) and interaction of these two. Furthermore, since the \hat{\beta}_j is derived from the exponential smoothing method, cv(Payment) can also be expressed as following:

\[ cv(PAY_j) = cv(O_j) + (1 + cv(O_j)) \cdot cv(\hat{\beta}_j) = cv(O_j) + (1 + cv(O_j)) \cdot \frac{\text{Var}(\hat{\beta}_j)}{E(\hat{\beta}_j)^2} \]

\[ = cv(O_j) + (1 + cv(O_j)) \cdot \frac{\text{Var}(\sum_{i=0}^{t-1} \alpha(1-\alpha)^i \beta_{t-i-1})}{E(\hat{\beta}_j)^2} \]

(8)

As it is mentioned above, the equation (7) suggests that cv(Payment) is subjected by two coefficients of variation: order made and collection ratio. Since this thesis is focused on the effect of cash collection, instead of identifying the impact of all components of cv(Payment), the rest of this section is focused on studying the effect of C.V(\hat{\beta}_j) to the cash flow bullwhip effect.
4.3.1. Normal Distribution and Exponential Distribution

Let’s assume that collection ratio follows normal distribution \([\beta \sim N(\mu, \sigma^2)]\) and it is independent. Then the equation (8) can be expressed as following:

\[
cv(PAY_j) = cv(O_j) + (1 + cv(O_j)) \cdot \frac{\sqrt{\text{Var}(\sum_{i=0}^{t-1} \alpha(1 - \alpha)^i \beta_{t-i-1})}}{\sqrt{\text{E}(\sum_{i=0}^{t-1} \alpha(1 - \alpha)^i \mu_{\beta_{t-i-1}})^2}}
\]

\[
= cv(O_j) + (1 + cv(O_j)) \cdot \frac{\sqrt{\sum_{i=0}^{t-1} (\alpha(1 - \alpha)^i)^2 \text{Var}(\beta_{t-i-1})}}{\sqrt{\left(\sum_{i=0}^{t-1} \alpha(1 - \alpha)^i \text{E}(\beta_{t-i-1})\right)^2}}
\]

\[
= cv(O_j) + (1 + cv(O_j)) \cdot \frac{\sum_{i=0}^{t-1} (\alpha(1 - \alpha)^i)^2 \sigma_{\beta_{t-i-1}}^2}{\sqrt{\left(\sum_{i=0}^{t-1} \alpha(1 - \alpha)^i \mu_{\beta_{t-i-1}}\right)^2}}
\]

\[
= cv(O_j) + (1 + cv(O_j)) \cdot \frac{\sum_{i=0}^{t-1} (\alpha(1 - \alpha)^i)^2 \sigma_{\beta_{t-i-1}}}{\sum_{i=0}^{t-1} (\alpha(1 - \alpha)^i) \mu_{\beta_{t-i-1}}}
\]

\[
= cv(O_j) + (1 + cv(O_j)) \cdot \frac{\sum_{i=0}^{t-1} (\alpha(1 - \alpha)^i)^2 \sigma_{\beta_{t-i-1}}}{\mu_{\beta}}
\]

Instead of assuming normal distribution, let’s assume that collection ratio follows exponential distribution \([\beta \sim \exp\left(\frac{1}{\lambda}\right)]\) and is independent. Then, the equation (8) can be expressed as following:
The equation (9) and equation (10) is similar in which both equations start with \( cv(O_j) + (1 + cv(O_j)) \). However, two equations differ in the last component of the equations because different distribution is assumed. Thus, in order to see the impact of the difference in the distribution of the cash collection ratio, the cv(Payment) is calculated when both distributions have same mean and variance but when they have different distributions. The mean is set to be 0.45 because the replications of simulation model’s cash collection ratio is approximately 0.45 and variance is set to be 0.2025 for both distribution because the variance of exponential distribution is pre-defined as \( E^2 \) by the definition. Given that \( \beta \sim N(0.45, 0.2025) \) or \( \beta \sim exp \left( \frac{1}{\lambda} = 0.45 \right) \) and \( cv(O_j) \) is as shown in Table 4 – 1, cv(Payment) for each player can be calculated as following:
Table 4 - 1 Coefficient of variation of $O_j$

<table>
<thead>
<tr>
<th></th>
<th>Coefficient of variation of $O_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer</td>
<td>0.4853</td>
</tr>
<tr>
<td>Retailer</td>
<td>0.5929</td>
</tr>
<tr>
<td>Distributor</td>
<td>0.8366</td>
</tr>
<tr>
<td>MFG</td>
<td>1.2917</td>
</tr>
<tr>
<td>Supplier</td>
<td>1.9112</td>
</tr>
</tbody>
</table>

Result of the Normal Distribution

\[ cv(\text{PAY}_{j=1}) = cv(O_{j=1}) + \left(1 + cv(O_{j=1})\right) \times \frac{\sum_{i=0}^{t-1} \alpha(1 - \alpha)^i \sigma_{\beta_{t-i-1}}}{\mu_{\beta}} \]

\[ = 0.49 + (1 + 0.49) \times \frac{0.1034}{0.45} = 0.832369 \]

\[ cv(\text{PAY}_{j=2}) = cv(O_{j=2}) + \left(1 + cv(O_{j=2})\right) \times \frac{\sum_{i=0}^{t-1} \alpha(1 - \alpha)^i \sigma_{\beta_{t-i-1}}}{\mu_{\beta}} \]

\[ = 0.5929 + (1 + 0.5929) \times \frac{0.1034}{0.45} = 0.958913 \]

\[ cv(\text{PAY}_{j=3}) = cv(O_{j=3}) + \left(1 + cv(O_{j=3})\right) \times \frac{\sum_{i=0}^{t-1} \alpha(1 - \alpha)^i \sigma_{\beta_{t-i-1}}}{\mu_{\beta}} \]

\[ = 0.8366 + (1 + 0.8366) \times \frac{0.1034}{0.45} = 1.25861 \]

\[ cv(\text{PAY}_{j=4}) = cv(O_{j=4}) + \left(1 + cv(O_{j=4})\right) \times \frac{\sum_{i=0}^{t-1} \alpha(1 - \alpha)^i \sigma_{\beta_{t-i-1}}}{\mu_{\beta}} \]

\[ = 1.2917 + (1 + 1.2917) \times \frac{0.1034}{0.45} = 1.818282 \]

\[ cv(\text{PAY}_{j=5}) = cv(O_{j=5}) + \left(1 + cv(O_{j=5})\right) \times \frac{\sum_{i=0}^{t-1} \alpha(1 - \alpha)^i \sigma_{\beta_{t-i-1}}}{\mu_{\beta}} \]

\[ = 1.9112 + (1 + 1.9112) \times \frac{0.1034}{0.45} = 2.580129 \]
Result of the Exponential Distribution

\[ \text{cv}(PAY_{j=1}) = \text{cv}(O_{j=1}) + \left(1 + \text{cv}(O_{j=1})\right) \times 1 = 0.4853 + (1 + 0.4853) \times 1 = 1.9706 \]

\[ \text{cv}(PAY_{j=2}) = \text{cv}(O_{j=2}) + \left(1 + \text{cv}(O_{j=2})\right) \times 1 = 0.5929 + (1 + 0.5929) \times 1 = 2.1858 \]

\[ \text{cv}(PAY_{j=3}) = \text{cv}(O_{j=3}) + \left(1 + \text{cv}(O_{j=3})\right) \times 1 = 0.8366 + (1 + 0.8366) \times 1 = 2.6732 \]

\[ \text{cv}(PAY_{j=4}) = \text{cv}(O_{j=4}) + \left(1 + \text{cv}(O_{j=4})\right) \times 1 = 1.2917 + (1 + 1.2917) \times 1 = 3.5834 \]

\[ \text{cv}(PAY_{j=5}) = \text{cv}(O_{j=5}) + \left(1 + \text{cv}(O_{j=5})\right) \times 1 = 1.9112 + (1 + 1.9112) \times 1 = 4.8224 \]
Although all other components of \( cv(\text{Payment}) \) remain the same, the result of the two analytic models is significantly different as this is shown in Figure 4 – 3. This difference comes from the difference in the distribution of the cash collection ratio.
4.3.3. Family of Normal Distribution

As it shown in the Figure 4-4, there are thousands of normal distributions with the same mean. In the previous section, the result of the normal model is similar to the result of the simulation model, but significantly different from the result of the exponential model although assumed distribution of the cash collection ratio is different.

One of possible reasons is due to the difference in variance. In other words, the result of the normal model may differ depending on the variance of the distribution.

![Figure 4 - 4 Family of Normal Distribution](image)

Thus in this section, the changes of C.V. Payment are observed when the variance of normal distribution changes. By doing so, the effect of different variance size can be identified. A family of normal distribution with mean of 0.45 is tested: \( N(0.45, 0.1), N(0.45, 0.2), N(0.45, 0.5), N(0.45, 1), N(0.45, 3), N(0.45, 5), N(0.45, 7), N(0.45, 9), \) and \( N(0.45, 10) \).

The C.V. Payment for each player in the supply chain is calculated using equation (9) and the result of calculation is summarized. The summary is shown in Table 4 – 2.
Table 4 - 2 Family of Normal Distribution with Mean of 0.45

<table>
<thead>
<tr>
<th></th>
<th>v(0.1)</th>
<th>v(0.2)</th>
<th>v(0.5)</th>
<th>v(1)</th>
<th>v(3)</th>
<th>v(5)</th>
<th>v(7)</th>
<th>v(9)</th>
<th>v(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer</td>
<td>0.5</td>
<td>0.52</td>
<td>0.57</td>
<td>0.66</td>
<td>1.01</td>
<td>1.35</td>
<td>1.7</td>
<td>2.05</td>
<td>2.22</td>
</tr>
<tr>
<td>Retailer</td>
<td>0.61</td>
<td>0.63</td>
<td>0.68</td>
<td>0.78</td>
<td>1.15</td>
<td>1.52</td>
<td>1.9</td>
<td>2.27</td>
<td>2.45</td>
</tr>
<tr>
<td>Distributor</td>
<td>0.86</td>
<td>0.88</td>
<td>0.94</td>
<td>1.05</td>
<td>1.48</td>
<td>1.91</td>
<td>2.34</td>
<td>2.77</td>
<td>2.98</td>
</tr>
<tr>
<td>MFG</td>
<td>1.32</td>
<td>1.35</td>
<td>1.42</td>
<td>1.56</td>
<td>2.1</td>
<td>2.63</td>
<td>3.17</td>
<td>3.71</td>
<td>3.97</td>
</tr>
<tr>
<td>Supplier</td>
<td>1.94</td>
<td>1.98</td>
<td>2.08</td>
<td>2.25</td>
<td>2.93</td>
<td>3.61</td>
<td>4.29</td>
<td>4.98</td>
<td>5.31</td>
</tr>
</tbody>
</table>

As Table 4 – 2 and Figure 4 – 5 show, the C.V. Payment of each player in the supply chain increases as the variance of the normal distribution increases. In addition, when the size of variance is big enough, $cv(Payment)$ of each player, which cash collection follows normal distribution, is similar to $cv(Payment)$ which cash collection follows exponential distribution. This is shown in the Figure 4 – 5.

![Family of Normal Distribution (μ=0.45) vs Exp](image)

Figure 4 - 5 Comparison between Family of Normal Dis. and Exponential Dis.

In addition to size of variance, in case of the normal distribution, the smoothing parameter of the forecast method cannot be ignored. On the other hand, in case of the exponential distribution, the smoothing parameter of the forecast method can be ignored; it is because as it is
shown in the equation (10), in the case of exponential distribution, the smoothing parameter is canceled out in the final equation.

4.4. Conclusive Remarks

In this chapter, the effect of cash collection ratio’s variance on Cash bullwhip is studied. By analytically deriving the components of $cv(Payment)$, the measurement of the cash flow bullwhip effect, $cv(\hat{\beta}_j)$, is identified as one of the components of $cv(Payment)$. Thus, even though the size of cash collection does not have effect on supply chain cash flow, this finding illustrates that variation of cash collection is related to the cash flow bullwhip effect.

In addition to deriving the analytical model, $cv(Payment)$ is calculated assuming that the cash collection ratio follows some types of distributions such as normal and exponential. From this, both models’ result overly estimates $cv(Payment)$ compared to the result of the simulation model. Moreover, not only the difference in the type of distributions assumed, but also the variance size of the cash collection ratio gives effect to the cash flow bullwhip. As the variance of cash collection increases by 0.1, the $cv(Payment)$ of customer, retailer, distributor, MFG, and supplier also increases by 0.10, 0.11, 0.12, 0.15 and 0.19 respectively.
Chapter 5

Summary and Future Research

5.1. Summary

The bullwhip effect brings adverse influence to the supply chain. Because of the bullwhip effect, players of the supply chain suffer with excessive inventory level, backorders, uncertainty in production planning and stock out. For this reasons, numerous researchers and practitioners have focused on removing the bullwhip effect in supply chain.

Recently, a similar phenomenon has been detected from cash flow in supply chain. However, because it is newly discovered, many questions are still out there to be answered. Therefore, this thesis extended former discovers and studies by studying the effect of cash collection on the cash flow bullwhip. First, a series of designed experiments are conducted to study the effect of the cash collection’s size. Then, the analytical models are developed to illustrate the effect of variance in cash collection the cash collection’s variance when it has a normal or exponential distribution.

The purpose of conducting a series of designed experiments is to see whether the size of the cash collection affect the cash flow bullwhip as the batch order size does to the inventory bullwhip. Although it seems the size of the cash collection is similar to the batch order size, the cash collection size either does not affect or marginally affect the cash flow bullwhip effect. The experiment conducted in Chapter 3 leads to the conclusion that the size of cash collection is irrelevant to the cash flow bullwhip effect.

The purpose of developing analytical models is to find out the relationship between the cash flow bullwhip and the cash collection ratio. If the size of cash collection is irrelevant, there must be other ways to connect between the cash collection and the cash flow bullwhip effect.
Chapter 4’s result leads to the conclusion that the uncertainty of cash collection increases the cash flow bullwhip in the supply chain. As the variance of cash collection increases by 0.1, the \( cv(\text{Payment}) \) of customer, retailer, distributor, MFG, and supplier also aggrandizes by 0.10, 0.11, 0.12, 0.15 and 0.19 respectively. In other words, cash flow bullwhip effect also aggrandized.

Moreover, the result of Chapter 4 reveals that the cash flow bullwhip can be estimated differently depending on the distribution of cash collection ratio. Whether the cash collection ratio follows normal or exponential distribution, the analytical models overestimate cash bullwhip. However, when the cash collection ratio follows normal distribution, the model overestimates cash flow bullwhip approximately by 30%. Whereas, when the cash collection ratio follows the exponential distribution, the model overestimates cash flow bullwhip approximately by 177%. Furthermore, the normal distribution model over estimates the cash bullwhip approximately by 15% when the mean and variance is same as the simulation model. This difference, due to the difference in the distribution of the cash collection ratio, reinforces the finding of this thesis that the uncertainty of cash collection increases the cash flow bullwhip in supply chain because inaccurately assumed distribution of the cash collection ratio aggrandizes the cash flow bullwhip in supply chain.

### 5.2. Future Research

The findings summarized above help to explain the cash flow bullwhip effect in the supply chain. However, there are still numerous problems that need to be addressed by researchers. An in-depth study of the drivers of cash flow bullwhip effect other than cash collection ratio, is a great example for a future study. Furthermore, it would be great if future studies can address how to mitigate the cash flow bullwhip effect or how to remove the cash flow bullwhip effect from the supply chain. Also, it would be better if future studies could address how
much of the cash flow bullwhip effect is purely due to the financial component of the supply chain.
References


Fawcett, S., 2006. Supply Chain Management (p. 600).

"Financial Crisis - The World Bank." Financial Crisis - The World Bank. The World Bank,


Appendix

Matlab Code for Extended Beer Distribution Game Model

```matlab
%-----------------------Initialization-----------------------
% Number of total time periods in the simulation
for k=1:1000
N=121;

clear inv d o onorder bo f
clear nn index rand1
clear rec recratio fratio cv
% Assigning initial values
% inv: inventory level
% bo: backorder amount
% onorder: inventory on order
% d: demand to a SC member from its downstream member
% o: order placed by a SC member to its upstream member
% f: order-up-to level
% rec: collection of sales
% recratio: collection ratio
% fratio: forecasted collection ratio
for j=1:5
for n=1:20
inv(n,j)=100;
bo(n,j)=0;
onorder(n,j)=0;
d(n,j)=0;
f(n,j)=0;
o(n,j)=0;
rec(n,j)=0;
reccratio(n,j)=0;
fratio(n,j)=0;
end
end
% Prices
p(1)=2;
p(2)=1.75;
p(3)=1.5;
p(4)=1.25;
p(5)=1;

% Lead time
for j=1:5
t(j)=5;
end
```
% Generate normally distributed customer demand
a = 0; b = 1000; c =100; nn = N*2;mm = 50;
x = randn(1,nn);
x = x/std(x)*sqrt(c);
x = x-mean(x)+mm;
index = find(x>=a & x<b);
rand1= fix(x(index));

%------------------------------- Simulation-------------------------------
while n<N
  for j=1:4
    % Receive products and update inventory level
    inv(n,j)=inv(n-1,j)-bo(n-1,j)+o(n-t(j+1),j)-d(n-1,j);
    if inv(n,j)<0
      bo(n,j)=0-inv(n,j);
      inv(n,j)=0;
    else
      bo(n,j)=0;
    end
  end
  % Forecast demand by moving average
  for j=1:4
    s(n,j)=0;
    for i=1:9
      s(n,j)=s(n,j)+d(n-i,j);
    end
    f(n,j)=s(n,j)/9;
  end
  % Calculate inventory on order
  for j=1:4
    onorder(n,j)=0;
    for i=1:t(j+1)-1
      onorder(n,j)=onorder(n,j)+o(n-i,j);
    end
  end
  % Decide order size by order-up-to policy
  for j=1:4
    if inv(n-1,j)-d(n-1,j)-bo(n-1,j)+o(n-
      t(j+1),j)+onorder(n,j)<(inv(n-1,j)-d(n-1,j)-bo(n-1,j)+o(n-
      t(j+1),j)+onorder(n,j))
      o(n,j)=t(j+1)*f(n,j)-(inv(n-1,j)-d(n-1,j)-bo(n-1,j)+o(n-
      t(j+1),j)+onorder(n,j));
    else
      o(n,j)=0;
    end
  end
  % Generate place orders to upstream member
  % Demand to the Retailer
  d(n,1)=rand1(n);
  % Demand to each member is exactly the orders placed by
  % its downstream member
  for j=2:5
    d(n,j)=o(n,j-1);
  end
  % Calculate the collection ratio at current time unit
  for j=1:5
if d(n-2,j)==0
reccratio(n,j)=0;
else
reccratio(n,j)=rec(n,j)/(d(n-2,j)*p(j));
end
end

% Payment ratio to Retailer by customer follow U(0,1)
rec_c(n+1,1)=rand(1,1);
rec(n+1,1)=rec_c(n+1,1)*d(n-1,1)*p(1);

% Smoothing parameter for exponential smoothing in
% collection ratio forecast
al=0.1;
% Forecast collection ratio in the next time unit,
% and decide payment sizes
for j=1:4
fratio(n,j)=0;
fratio(n,j)=al*reccratio(n,j)+(1-al)*fratio(n-1,j);
rec(n+1,j+1)=fratio(n,j)*o(n-1,j)*p(j+1);
end
n=n+1;
end

%---------------------- Output Results ----------------------
% Calculate coefficient of variance
stdrec=std(rec);
meanrec=mean(rec);
stdo=std(o);
meano=mean(o);

cvo(1,1) = std(d(:,1))/mean(d(:,1));
for j = 2:5
    cvo(j)=stdo(j-1)/meano(j-1);
end
for j=1:5
cvpay(j)=stdrec(j)/meanrec(j);
end

comparison_c(k,1) = cvpay(1);
comparison_c(k,2) = cvo(1)+(1+cvo(1))*0.033/0.45;
comparison_r(k,1) = cvpay(2);
comparison_r(k,2) = cvo(2)+(1+cvo(2))*0.033/0.45;
comparison_d(k,1) = cvpay(3);
comparison_d(k,2) = cvo(3)+(1+cvo(3))*0.033/0.45;
comparison_m(k,1) = cvpay(4);
comparison_m(k,2) = cvo(4)+(1+cvo(4))*0.033/0.45;
comparison_s(k,1) = cvpay(5);
comparison_s(k,2) = cvo(5)+(1+cvo(5))*0.033/0.45;
end