EFFECT OF GRID BLOCK SIZE AND HETEROGENEITY ON MISCIBLE RESIDUAL OIL SATURATION

A Thesis in
Energy and Mineral Engineering

by

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Submitted in Partial Fulfillment of the Requirements for the Degree of

Master of Science

December 2013
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ABSTRACT

Compositional simulators almost always predict excessive vaporization of oil near the injector (nearly 100% recovery). Such high recovery may be possible for multi-contact PVT experiments or in a slim-tube displacement but not at the scale of a simulator grid block, where oil is bypassed by injected gas. Bypassed oil will not be vaporized and the amount of oil bypassed will be affected by the heterogeneity of the reservoir as gas will tend to flow through high permeability channels. This bypassed oil is one part of the oil that is quantified as miscible residual oil saturation or $S_{orm}$. The other part of $S_{orm}$ is heavy hydrocarbon components that are not vaporized even after contacted by a significant volume of gas.

Accurate prediction of $S_{orm}$ is important to forecast recovery from a reservoir. Large grid blocks are often used in gas flooding simulations. Numbers of hydrocarbon components are also upscaled to very few components in these simulations. All these assumptions lead to inaccurate recovery predictions considering that $S_{orm}$ can be as large as 10% of oil saturation depending on the oil gravity. If small grid blocks are used in compositional simulation with many hydrocarbon components representing the fluid then $S_{orm}$ would be very small or negligible.
In order to avoid excessive vaporization of the oil, simulators can specify residual oil in grid blocks during gas injection processes. There are several methods available for predicting $S_{orm}$. Most of these methods do not consider spatial and temporal variation of $S_{orm}$. This thesis focuses on studying how $S_{orm}$ changes with distance from the injection well based on upscaling the reservoir to larger grid blocks. The effect of different scaling groups based on fluid and reservoir properties on $S_{orm}$ has also been studied.

Upscaling is often used in the simulation of gas injection processes to save computational time. Permeability upscaling causes a change in the heterogeneity of the reservoir. The amount of mixing between solvent and reservoir oil changes with upscaling. However, mixing is ignored while performing upscaling in compositional simulators even though mixing has a significant effect on oil recovery. In this research we discuss the importance of mixing in oil recovery predictions and on miscible residual oil saturation.

We present an efficient way of calculating $S_{orm}$, which is based on permeability upscaling. We simulate first contact miscible flooding and concentrations in grid-blocks have been averaged locally and globally in order to calculate $S_{orm}$. Scaling groups are generated from inspectional analysis based on reservoir and fluid properties and reservoir heterogeneity. The effect of each scaling group on $S_{orm}$ has been shown in this research.
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NOMENCLATURE

A  Mobile phase
B  Immobile phase
C  Concentration of components
F  Flowing hydrocarbon mol fraction
H  Distance in transverse direction, ft
I  Gas injection rate (mol/day/unit volume)
L  Distance in longitudinal direction, ft
P  Pressure, psia
Pwf  Flowing bottom hole pressure, psia
V  Vapour (gas) mole fraction
Q  Production rates (mol/day/unit volume)
S  Hydrocarbon saturation
Krow  Oil relative permeability to water
Krog  Oil relative permeability to gas
Sorm  Residual oil saturation to miscible gas
Sorw  Residual oil saturation to water, fraction
Sorg  Residual oil saturation to gas, fraction
KL  Dispersion coefficient in longitudinal direction
KT  Dispersion coefficient in transverse direction
\( N_{Pe} \)  \hspace{1cm} \text{Peclet number}

\( R_L \)  \hspace{1cm} \text{Effective aspect ratio}

\( N_D \)  \hspace{1cm} \text{Dispersion number}

\( M \)  \hspace{1cm} \text{Mobility ratio}

\( V_{DP} \)  \hspace{1cm} \text{Dykstra Parsons coefficient}

\( \varphi \)  \hspace{1cm} \text{Porosity, fraction}

\( \mu \)  \hspace{1cm} \text{Viscosity, cp}

\( \mu_m \)  \hspace{1cm} \text{Viscosity of oil and gas mixture, cp}

\( z \)  \hspace{1cm} \text{Overall mole fraction of component in hydrocarbon present}

\( u \)  \hspace{1cm} \text{Darcy velocity (m/day)}

\( \alpha \)  \hspace{1cm} \text{Alpha-factor}

\( x \)  \hspace{1cm} \text{Mole fraction of component in oil phase}

\( y \)  \hspace{1cm} \text{Mole fraction of component in gas phase}

\( q \)  \hspace{1cm} \text{Injection/ production rate (mols/m}^3\text{/day)}

\( \rho \)  \hspace{1cm} \text{Molar density (mol/m}^3\text{)}

\( \delta \)  \hspace{1cm} \text{Solubility parameter, m}^{0.5}\text{/L}^{0.5}\text{, (cal/cm}^3\text{)}^{0.5}

\( \Delta u_{vap} \)  \hspace{1cm} \text{Molar energy of vaporization, mL}^2\text{/t}^2\text{, cal/mol}

\( k_x \)  \hspace{1cm} \text{Permeability in the x-direction, mD}

\( k_z \)  \hspace{1cm} \text{Permeability in the z-direction, mD}

\( \alpha_{Li} \)  \hspace{1cm} \text{Dispersivity of the component i in longitudinal direction}

\( \alpha_{Ti} \)  \hspace{1cm} \text{Dispersivity of the component i in transverse direction}
$\lambda_x$  Longitudinal correlation length in $x$-direction, ft

$\lambda_z$  Transverse correlation length in $z$-direction, ft

$x$-  $x$ direction

$z$-  $z$ direction

$\beta_o$  Intercept coefficient in response function

$\beta_i$  Linear coefficients in response function

$\beta_{ii}$ and $\beta_{ij}$  Quadratic coefficients in response function

$H_x$  Upper level value of scaling group

$L_x$  Lower level value of scaling group

$k$  Average permeability, mD

$\bar{X}$  Normalized value of the intermediate level
Subscripts

1* Multiplicative term in inspectional analysis
2* Additive term in inspectional analysis
c Connate (irreducible)
D Dimensionless parameter
f Flowing
g Gas
i Relative to component i
j Relative to component j
s Solvent
o Oil
r Residual
w Water

Superscripts

inj Injected
res Reservoir
ACKNOWLEDGEMENTS

I would like to thank my supervisor Dr. Russell T. Johns for his support, guidance and encouragement throughout my research. I would like to thank him for continuing to believe in me. I have learned a lot from his vast knowledge and enthusiastic observations. He always encouraged me to develop independent thinking.

I would like to extend my appreciation to my committee members, Dr. Turgay Ertekin and Dr. John Yilin Wang for their valuable time to review my thesis. Their knowledge and insights was extremely valuable and greatly enriched my research.

I would like to acknowledge the faculty in the petroleum and natural gas engineering department: Dr. Luis F. Ayala, Dr. Zuleima T. Karpyn, Dr. Li Li and Dr. Derek Elsworth. I have learned a lot from them during my coursework and from various seminars. I would also like to thank the administrative staff of the Energy and Mineral engineering department: Robert Byers, Carole Donald, Crystal Renaud, Sara Peterson and Tatyana Kalinchuk.

I also extend my appreciation to the members of our JIP gas flooding research group, Michael Connolly, Amey Khanzode, Emre Gorucu and Ashwin Venkatraman for valuable discussions that we have had. I would like to thank the
industrial members of JIP gas flooding group: Maersk oil, OMV, Shell, Chevron and BP for their financial support.

It has been a great pleasure working with several colleagues. I would like to thank my friends and colleagues in the department of Energy and Mineral engineering, Sachin Rana, Tushar Vatsa, Phani Kiran, Phani Bhushan, Kelvin Abba, Tunde Osholake, Yogesh Bansal, Talal Almousa, Taha Hussain, Debanjan Das and Shashank Sharma.

Finally, I would like to thank my parents, uncles, aunts, brother, sister and cousins for their love and affection. Without their support it would not have been possible.
CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Primary recovery processes contribute to about 10-15% recovery of the original oil in place (OOIP). Once the reservoir pressure declines, secondary and tertiary enhanced oil recovery (EOR) stages of the operation are conducted, which consists of water flooding, gas injection and pressure maintenance. The definition of EOR is ‘the oil recovery by the injection of materials not normally present in the reservoir’ (Lake, 1989). Miscible gas flooding is one of the most reliable EOR methods. In the gas flooding process, gas is injected into the reservoir, which mixes with the oil and causes a change in the viscosity and density of the oil.

Natural reservoir energy is utilized for the initial production of hydrocarbons from the underground reservoir and this production is termed as primary production. When this natural energy of the reservoir is depleted, it becomes necessary to use external sources in order to augment the reservoir energy, which is accomplished by injection of fluids like gas, solvent or water. When water is injected in the reservoir, the process is termed water flooding. The purpose of water flooding, which is typically done as a secondary recovery process, is to pressurize the reservoir and mechanically displace oil to the
production wells. The recovery efficiency of a water flood is a function of sweep efficiency and the viscosities of oil and water.

Sweep efficiency is a measure of how well water or gas has contacted the available pore space in the oil bearing zone. Presence of fractures, faults and high permeability zones in the reservoir can lead to low sweep efficiency. If the injected water is much less viscous than the oil present in the reservoir, water could start to channel through the reservoir. Channeling causes bypassing of oil and causes lower flooding efficiencies. Bypassing is a very important issue in applying enhanced oil recovery techniques.

When the secondary recovery processes become uneconomic, tertiary recovery processes can be implemented for incremental oil recovery. In general, the term enhanced oil recovery is used in referring to any recovery process that enhances the recovery of oil beyond what primary and secondary production would normally be expected to yield.

Enhanced oil recovery processes can be classified into four categories:

1. Miscible flooding processes
2. Chemical flooding processes
3. Thermal flooding processes
4. Microbial flooding processes

Miscible flooding includes single-contact and multiple-contact miscible processes. Injection of polymers, micelle-polymer and alkaline flooding falls
under chemical flooding. Thermal processes include steam injection, steam drive
and in-situ combustion. Microbial processes use microorganisms to assist in oil
recovery.

The National Petroleum Council (NPC) defines enhanced oil recovery as
“incremental oil that can be economically produced over that which can be
economically recoverable by conventional primary and secondary processes”. In
chemical flooding, the main goals are to provide favorable mobility ratios \((M<1.0)\)
and increasing the capillary number. In gas flooding processes, high capillary
number is not needed because gas flooding causes vaporization of oil.

For a first contact miscible (FCM) flood, the mobility ratio is defined as the
ratio of mobility of the displacing fluid to that of the displaced fluid.

\[
M = \frac{\mu_o}{\mu_s}
\]

The capillary number is defined as the ratio of viscous forces to capillary
forces,

\[
N_{ca} = \frac{\text{Viscous Forces}}{\text{Capillary Forces}} = \frac{\nu \mu}{\sigma \cos \theta}
\]

where \(\sigma\) is the oil-water interfacial tension and \(\theta\) is the angle between the oil-water
interface and rock surface.
1.2 MISCIBLE GAS FLOODING

In this research we focused on miscible flooding. The overall efficiency of an EOR process depends on both the microscopic and macroscopic sweep efficiencies. Macroscopic efficiency is affected by the difference in fluid densities and rock heterogeneity. Microscopic efficiency is affected by the interfacial interactions involving interfacial tension and dynamic contact angles.

Miscibility is defined as the development of a zero interfacial tension between oil and gas (Thomas et al. 1995). Miscibility does not have to occur in order to have the optimal gas injection or optimal recovery. For example in the case of a water-wet reservoir where water adheres more closely to the smaller pore throats, we do not need to enter into the smallest pore throats to recover all the oil. But for oil-wet systems where significant amount of oil is associated with the smallest pore throats, zero IFT must be obtained in order to make the smallest pore throats accessible to the injection phase (Thomas et al. 1995).

One of the first successful miscible gas floods using carbon dioxide was tested in North Dakota by Gulf Oil Company (Gulf) in 1981 at Little Knife North Field (Thakur et al. 1984; Desch et al. 1984). A CO₂ miscible oil recovery of 13% of the original oil in place (OOIP) was achieved by Gulf. There is a vast amount of literature available on how miscibility works in gas flooding to improve
hydrocarbon recovery (Holm et al. 1974; Yellig et al. 1980; Novosad et al. 1991; 

1.3 MISCIBLE RESIDUAL OIL SATURATION ($S_{ORM}$)

Theoretical considerations (Perrinee, 1961), laboratory experiments (Habermann, 1960) and pilot tests (Relph, 1958) have shown that miscible displacement in the field is unstable. When these miscible processes are simulated using compositional simulators, oil saturation at the end of the simulation is often zero because oil recovery from grid blocks swept by the gas is often complete. However, field observations and lab experiments indicate that oil recovery is not 100%. In a heterogeneous reservoir, solvent fingers through the reservoir, which leads to early breakthrough of the solvent (Koval, 1963).

Compositional simulators predict excessive vaporization of the oil because in the conventional compositional simulators there is no facility to define residual oil or non-vaporizing oil. This phenomenon is observed very often in the vicinity of the injector, where oil saturation of zero is sometimes predicted (Hiraiwa et al. 2004). But such lower residual oil saturations are rarely seen in laboratory data such as core flood experiments, slimtube tests or in cores taken near injection wells. This causes optimistic results in which all the oil in the grid blocks can be stripped by the injected gas.
The oil trapped in dead end pores and in lower permeability regions do not contact the injected gas. Trapped oil is not considered in compositional simulation when grid blocks are made large, which causes the difference in oil saturation values of compositional simulation and laboratory results. Bypassing of oil occurs even when the solvent is above the minimum miscibility pressure (MMP) (Stalkup F., 1983), which implies even under the miscible flooding conditions, oil saturation will not reach zero percent in heterogeneous reservoirs.

Figure 1-1 shows the typical permeability map of a heterogeneous reservoir. A real heterogeneous reservoir has regions of varying permeabilities especially in the vertical direction. Injected gas channels through these high permeability regions and breakthroughs early causing bypassing of the oil in the lower permeability zones as shown in Fig. 1-2. This bypassed oil has not been fully contacted by the solvent so it does not vaporize it. In compositional simulators this oil may or may not be considered while laboratory tests consider the other type of $S_{orm}$ caused by slow evaporation of heavy components. In compositional simulation the saturation of this bypassed oil is quantified as miscible residual oil saturation ($S_{orm}$) (Spence et al. 1980).

Incorporation of true residual oil saturation based on laboratory results into compositional simulators has been a critical problem in the oil industry particularly in gas flooding processes. In gas injection processes, the proposed mechanism of oil recovery includes low IFT displacement, extraction and oil
swelling (Stern, 1991). Common parameters used in the design of miscible flooding processes are minimum miscibility pressure (MMP) or minimum miscibility enrichment (MME).

Miscible recovery processes have been studied extensively and various schemes have been developed to predict MMP (Yellig et al. 1980; Holm et al. 1982; Sebastian et al. 1985; Orr, 1987). But only a few methods have been proposed to predict $S_{orm}$ to date. We discuss some of these methods in the literature review. These methods have been used in the industry to predict miscible residual oil saturation and to include $S_{orm}$ in compositional simulators.

### 1.4 OBJECTIVES

The main aim of this research is to show that miscible residual oil saturation changes with distance (from the injection well) and with the size of the grid blocks used in the simulation.

Other objectives of this research are to:

- Examine the methods that are used in compositional simulators in the petroleum industry to account for $S_{orm}$
- Define miscible residual oil saturation based on permeability upscaling.
- Determine the effect of upscaling (grid block size) on $S_{orm}$.
- Determine the effect of pore volumes of solvent injected (PVI) on $S_{orm}$.
• Perform a sensitivity analysis of $S_{orm}$ for different reservoir and fluid scaling groups.

• Suggest a method to predict $S_{orm}$ based on the scaling group values.
Figure 1-1: Typical permeability map of a heterogeneous reservoir (Source: Coll et al. 1998)

Figure 1-2: Visualization of channeling of injected solvent into a heterogeneous reservoir (Source: Jha et al. 2010)
CHAPTER 2

LITERATURE REVIEW

2.1 OVERVIEW

The main objective of this research is to understand the concept of miscible residual oil saturation ($S_{orm}$) and to examine the effect of grid block size on $S_{orm}$. We also study the effects of various reservoir and fluid parameters on $S_{orm}$. Despite being an important factor in gas flooding, not much research has been done to simulate miscible residual oil saturation effectively. In this section we present a comprehensive review of the literature on $S_{orm}$ and discuss the methods and correlations available to effectively simulate $S_{orm}$.

As discussed earlier, miscible displacements are highly unstable. Many authors have presented their research on the prediction of unstable miscible displacements in heterogeneous reservoirs (Koval, 1963; Todd, 1972). Koval presented a method analogous to the Buckley-Leverett theory for calculating oil recovery as a fraction of fingering (oil-solvent mobility ratio) and channeling (level of heterogeneity) in porous media. Todd & Longstaff (Todd et al. 1972) developed a two-component numerical simulator for prediction of miscible flood performance in a modified black oil simulator.
In this research we concentrate on first contact miscible (FCM) flooding, which means that all component fronts move at a piston-like speed. Miscible displacements are affected significantly by dispersion. Very often numerical dispersion is ignored when determining miscible residual oil saturation. In the next few sections we discuss the impact of mixing on oil recovery.

2.2 IMPORTANCE OF MIXING

Reservoir mixing or dispersion in porous media is caused by molecular diffusion and mechanical spreading. Molecular diffusion occurs because of the mass transfer of a component in the direction of decreasing concentration. Dispersion in porous media has been studied extensively for decades and there is a vast amount of literature available on dispersion. Garmeh (2009) and Adepoju (2012) have explained different types of dispersion and how dispersion is different from convective spreading.

Oil recovery from miscible gas floods is highly dependent on the amount of mixing. Mixing of the reservoir fluid with the injected fluid can decrease the strength of the injected fluid and cause early breakthrough. All multi-contact (MCM) floods develop two-phase flow because mixing causes the composition route to move deeper into the two-phase region, decreasing the local displacement
efficiency significantly (Baker et al. 1977; Walsh et al. 1990; Fayers et al. 1994; Johns et al. 1994). Figure 2-1 shows the effect of mixing on oil recovery in a 1-D model of MCM displacement after 2.0 hydrocarbon pore volumes injected (Garmeh, 2009). Recovery in 1–D is decreased by increased mixing.

In the case of immiscible floods, dispersion has no significant effect on recovery as fluid compositions are relatively constant with negligible mass transfer between phases. The component shock fronts tend to be more self-sharpening in an immiscible flood. Mixing plays a very important role for MCM and FCM displacements. In the case of large reservoir mixing it is desirable to operate at pressures above minimum miscibility pressure in order to have good recovery efficiency (Johns et al. 2002; Solano et al. 2001; Stalkup et al. 1998). In light of the significant effects of mixing on oil recovery it is important to consider reservoir mixing while handling miscible residual oil saturation.

2.3 SCALE DEPENDENCY OF MIXING

In the previous section we saw that oil recovery in miscible floods is affected by dispersion. One of the objectives of this research is to show how \( S_{orm} \) changes with distance from the injection well. Since \( S_{orm} \) is directly related to the
oil recovery it is important to know the scale dependency of mixing. In this section we review the literature, which shows how mixing changes spatially.

The primary cause of mixing in a reservoir is molecular diffusion of gas from one streamline to the next within the pores. The effect of diffusion increases as the contact area between gas and oil increases. Reservoir mixing can be enhanced by any mechanism that increases the area of contact between the injected gas and the reservoir oil. That is what makes dispersion scale dependent. The longer the distance travelled by the gas the greater the area exposed to diffusion.

The level of mixing is quantified by measuring the dispersivity or dispersion coefficient. Large values of dispersion mean more mixing. There is a vast amount of literature available, which examines the scale dependency of dispersion (Arya et al. 1988; Mahadevan et al. 2003; Pickens et al. 1981; Gelhar et al. 1992). Figure 2-2 shows field and laboratory data for dispersivity confirming scale dependency of mixing.

In this research to estimate $S_{orm}$ we use a method based on permeability upscaling. Upscaling of miscible floods has a significant effect on the amount of mixing in a grid block. Geological models are often converted into simulation models using permeability upscaling. Large grid block sizes are typically used in simulation to save computational time. These large grid block sizes create artificial
mixing and result in significant errors in recoveries (Stalkup, 1998; Haazijadeh et al. 1999; Solano et al. 2001; Moulds et al. 2005). While upscaling geological models little attention is paid to the reservoir mixing. Permeability upscaling of reservoirs results in smaller dispersivities owing to more homogeneous structure of upscaled models.

A number of authors have investigated the impact of upscaling on dispersion (Degan, 1994; Kitanidis, 1990). Garmeh (2009) suggested a procedure to upscale miscible floods considering the appropriate level of reservoir mixing based on a response surface for both FCM and MCM displacements (Garmeh, 2009).

Large reservoir mixing causes enhanced sweep but might result in lower displacement efficiency. Depending on the amount of mixing it could lead to smaller miscible residual oil saturations ($S_{orm}$). In such cases it becomes important to estimate the optimum grid block size of the upscaled model to avoid artificial mixing.

As stated in the introduction chapter, there are methods and correlations developed to predict miscible residual oil saturation. The next few sections discuss these methods in brief.
2.4 METHODS AVAILABLE TO ESTIMATE $S_{orm}$

$S_{orm}$ is a key property for simulation and screening studies of the gas injection EOR processes. The miscible residual oil saturation represents the local displacement efficiency of oil by the injected gas. Although this area of miscible flooding has not been explored in detail yet there are some methods and correlations suggested by various authors to predict $S_{orm}$. In this section we explore the methods available to estimate $S_{orm}$ and discuss the advantages and disadvantages of each of the methods.

2.4.1 ALPHA FACTORS

A method of imposing miscible residual oil saturation based on use of alpha factors was suggested by Barker et al. (2005). These alpha factors can be found by analytic calculations.

Alpha factors were first introduced by Barker and Fayers (Barker et al. 1994) as a method for modeling sub-grid scale heterogeneity, viscous fingering and gravity over-ride in compositional simulation of gas injection processes. Alpha factors work as transport coefficients applied to the components, which modify the composition of oil or gas flowing out of a grid block. The mass balance
equations for components in the presence of alpha factors become (Barker et al. 2005)

\[
\frac{\partial}{\partial t} (\phi \rho \alpha_i) + \nabla \cdot (u_o \alpha_o \rho_o x_i + u_g \alpha_g \rho_g y_i) = q_i \nonumber 
\]

We can see from equation 2.1 that the mole fraction of component \( i \) in oil (gas) phase is now \( \alpha_{oi} x_i \) (\( \alpha_{gi} y_i \)) instead of \( x_i \) (\( y_i \)). Alpha factors alter the flow of heavy hydrocarbon components and allow the flow of lighter hydrocarbon components.

In FCM gas flooding, a mixture of injected gas and reservoir oil always form a single-phase composition, which can be expressed as a linear combination of injected gas composition and reservoir oil composition.

\[
z_i = V y_i^{inj} + (1 - V) x_i^{res} \nonumber 
\]

In order to impose \( S_{orm} \), the hydrocarbon phase is split into two parts, one is the flowing phase and the other is the non-flowing phase. The non-flowing phase is used for the miscible residual oil saturation:

\[
z_i = F z_i^{f} + (1 - F) x_i^{res} \nonumber 
\]

In case of ideal mixing between injected gas and reservoir oil equation 2.3 can also be written as:

\[
F = 1 - \frac{\rho_{o^{res}} S_{orm}}{(V \rho_{o^{inj}} + (1 - V) \rho_{o^{res}})(1 - S_{we})} \nonumber 
\]
where $S_{wc}$ is the fixed irreducible water saturation and $S_{orm}$ is the desired residual oil saturation.

For a given value of $V$, alpha factors can be found using equations 2.2 to 2.4 as,

$$\alpha_{oi} = \alpha_{gi} = \frac{z_i^f}{z_i} \ldots$$

(2.5)

We can see that the alpha factor approach is purely numerical. Alpha factors can be thought as analogous to pseudo relative permeabilities. Pseudo relative permeabilities are applied to phases while alpha factors are applied to components. In theory, alpha factors should be used with pseudo relative permeabilities. During upscaling, pseudo relative permeabilities are used to correct the flow of each phase while alpha factors are used to correct the flow of each component. Because of their similarity with pseudos, alpha factors suffer the same drawbacks as pseudo relative permeabilities. Some of these drawbacks include dependence on well rates and positions, different tables of values for each grid block, the need of a fine grid simulation to generate them (Barker J. S., 2005).

These drawbacks and need of handling pseudos while using alpha factors make the process very complex. The alpha-factor theory was developed for single-phase fully miscible flow. Many researchers have shown its application to two-
phase flow as well, but for WAG schemes and three-phase flow additional research needs to be performed (Bourgeois, 2012).

The alpha factor approach was also presented by Hirawa and Suzuki, where they combined this approach with relative permeability modifications (Hirawa, 2007). In their approach, they refined the fluid system so that each component has a mobile portion and an immobile portion as shown in Fig. 2-3. The $S_{orm}$ is referred to as the immobile oil in both the miscible and immiscible conditions.

The mass balance equations are written in the same way as used by Barker (Barker J. S., 2005).

$$\frac{\partial}{\partial t}(\phi F_{zi}) + \nabla \cdot (\alpha_i u_x \rho_x x_i + \alpha_g u_g \rho_g y_i) - Q_{zi} - Q_{yi} = 0 \quad \text{................................. (2.6)}$$

where $F$ indicates total hydrocarbon in a grid block of interest.

Hirawa and Suzuki (2007) used a different approach that utilized these transport coefficients or alpha factors to immobilize particular hydrocarbon components. They assigned zero value of alpha factors to the immobile portion of certain components in order to prevent them from moving out of the grid block. Alpha factor of unity does not make any change in the flow velocity or mobility of the component. By using this method, any mole fraction of any component can be immobilized in the grid block.

$$\alpha_i = \begin{cases} 
0 & \text{Immobile Component} \\
1 & \text{Mobile Component} 
\end{cases} \quad \text{................................. (2.7)}$$
Alpha factors change the velocity of each component of the oil/gas phase, which causes a change in the total phase velocity as well. To address this problem, Hiraiwa (2004) introduced a correction factor ‘C’ which can be written as:

\[
C = \left( \frac{z_{i_A}}{z_{i_B}} \right)^{-1} = \frac{z_{i_B}}{z_{i_A}} = \frac{S_o}{S_o - S_{orm}}
\]

Equation 2.8

The ‘C’ factor is used to adjust relative permeabilities.

This method provides better results than the one presented by Barker and Fayers. Hirawa and Suzuki assumed a single value of $S_{orm}$ through their model. This approach is still complicated to apply in real reservoirs due to heterogeneity.

2.4.2 EXCLUSION OF CERTAIN OIL VOLUME FROM FLASH CALCULATIONS

Another way of avoiding excessive vaporization of oil in compositional simulation is to exclude a certain volume of oil from flash calculations. This approach has been used by Total oil & gas, implemented as CSORM in VIP (VIP is a trademark of Landmark Graphics Corporation) for simulation of miscible gas injections in a North Sea reservoir (Barker J. S., 2005).

Total used this approach to study whether miscible gas injection is economically viable in reservoirs that have already been extensively water flooded and ultimately depressurized. They compared two simulations: one in which
miscible gas was injected after a water flood and before the blow down and one in which blow down took place immediately after the water flood. The approach where a certain amount of oil was excluded from flash calculations would not simulate blow down correctly because excluded oil cannot release its solution gas. Therefore, this approach could not be used.

Eclipse (trademark of Schlumbeger) also uses the excluded volume approach in order to avoid 100% recovery in compositional simulation. ECLIPSE also has the alpha factor approach implemented in Eclipse-300. Exclusion of a fixed amount of oil is probably the easiest way to prevent simulators from predicting 100% recovery. But this is not the correct way of handling \( S_{orm} \) as \( S_{orm} \) should change with time and space and the composition of the oil should also change.

2.4.3 DUAL POROSITY APPROACH

In 2007 another approach was developed by Coats et al. (2004) to account for miscible residual oil saturation. The dual porosity approach treats the residual oil as part of the immobile rock thereby assigning different porosities to the reservoir. Since the residual oil presumably resides in the poor quality rocks and isolated pores, the dual porosity approach approximates the true physical situation.
The reservoir under study is partitioned into two porosities, one equal to the specified amount of bypassed oil (Coats, 2004),

\[ \phi S_{orm} = \frac{\phi \times S_{orm}}{1 - S_{wi}} \]  

(2.9)

where \( S_{wi} \) is the initial water saturation. The other porosity is equal to total porosity minus the bypassed oil porosity,

\[ \bar{\phi} = \phi - \phi_{orm} \]  

(2.10)

In single-porosity mode, the reduction in oil saturation is limited to the input value of \( S_{orm} \) as we have seen before,

\[ S_o \geq S_{orm} \]  

(2.11)

where

\[ S_{orm} \leq S_{orw} \text{ and } S_{org} \]  

(2.12)

The dual porosity approach considers the residual oil as a secondary medium contributing very little to flow but significantly to volume.

### 2.4.4 METHODS TO PREDICT \( S_{ORM} \)

In 1998, Lange developed a correlation to predict miscible or near miscible residual oil saturation based on physical and chemical properties of the crude oil and injected gas (Lange, 1998). His method is based on a correlation of laboratory
residual oil saturation data as measured in core flood tests, for a wide range of injected gas compositions, reservoir oil, temperature and pressure using Hildebrand solubility parameters to represent the properties of reservoir oil and injected gas. Thus, the residual oil in this case is due to the portion of the oil contacted by the gas that is not effectively vaporized.

The solubility parameter for a pure compound is defined as,

$$\delta = \left( \frac{\Delta u_{\text{vap}}}{V} \right)^{0.5}$$

(2.13)

where $\Delta u_{\text{vap}}$ is the molar energy of vaporization and $V$ is the molar volume. The solubility parameter for a high pressure gas can be estimated as follows:

$$\delta_g = 0.326 \left( P_c \right)^{0.5} \frac{\rho_r}{\rho_r(\text{liq})}$$

(2.14)

where $P_c$ is critical pressure in psia, $\rho_r$ is the reduced density and $\rho_r(\text{liq})$ is the density of gas compressed to a liquid state.

Different hydrocarbon components have different solubility with injected gas. Based on the difference in solubility parameters between injected gas and crude oil, Lange developed a correlation to predict miscible residual oil saturation (Lange, 1998) as,

$$\frac{S_{\text{orm}}}{S_{\text{orw}}} = 0.12 \left( |\delta_{\text{oil}} - \delta_g| \right) - 0.11$$

(2.15)
The solubility parameter correlation (Lange, 1998) was used by ExxonMobil for estimating the conditions under which gas will be miscible with oil (Teletzke, 2005).

There are some other correlations available to predict $S_{orm}$. Shyeh-yung developed correlations based on capillary number and reduced density of injected gas for one crude oil with several hydrocarbon gases (Shyeh-Yung, 1991; Shyeh-Yung, 1995).

### 2.5 SUMMARY AND CONCLUSION

In this section we discussed in brief the advantages and disadvantages of the methods to determine $S_{orm}$ and model it in a simulator. The alpha factors approach is used more frequently than the other methods available in compositional simulation. But as discussed earlier it suffers the same disadvantages as the pseudo relative permeability approach. Generation of alpha factors in case of non-FCM floods require fine grid simulation, which makes this technique complicated.

Dual porosity approach works well with gas injection processes for both FCM and non-FCM cases. But in case of water alternating gas or processes which require injection of water and gas, this approach is not demonstrated to work.
Though dual porosity manages to exclude the residual oil from being flashed with the rest of the content of a grid cell it can’t prevent it from responding to changes in pressure. If pressure falls below bubble point this approach can’t prevent excluded oil from releasing its solution gas. Also Coats et al. (2004) did not provide any information about what permeability should be assigned to the second medium and whether or not it should have connate water saturation.

In the case of a water flood before gas injection, the alpha factor approach and exclusion of oil from a flash works well as they don’t have any effect on the water flood simulation. But in the case of water flooding after gas injection, the alpha factor approach results in recovery of some of the oil by gas, but the user has no control over how much is recovered.

From the above discussion we conclude that use of these approaches is case dependent and depending on the working reservoir conditions one approach might be better than the other. None of these approaches consider dispersion or reservoir mixing in their models. These approaches do not consider the effect of grid block size and the scale dependency of $S_{orm}$. Though the alpha factor approach and dual porosity approach can be implemented on the local scale (for each grid block), the implementation and calculations will be complicated. Lange’s correlation, which is based on solubility parameters of injected gas and oil components, also fails to provide information about local values of miscible residual oil saturation.
Mixing has shown to have significant impact on oil recovery in miscible and near-miscible gas floods. The concept of reservoir mixing has been ignored in all the approaches we have discussed so far. Heterogeneity in the reservoir is the origin of bypassed oil and mixing, and all these approaches become very complicated when the reservoir heterogeneity is considered in the compositional simulation.

In this research we present an effective way of calculating $S_{orm}$ to account for oil bypassing that will not only consider reservoir heterogeneity, but also provide information about the effect of grid block size on $S_{orm}$. 
Figure 2-1: Effect of reservoir mixing on ultimate oil recovery from a 1-D MCM displacement after 2.0 hydrocarbon PVI flood (Garmeh, 2009).
Figure 2-2: Field and laboratory measured dispersivities (Arya, 1988; Lake, 1989).
Figure 2-3: Refinement of fluid system to define components as mobile and immobile (Hirawa, 2007).
CHAPTER 3

METHODOLOGY AND SIMULATION MODEL

This chapter presents the methodology used in this research to estimate $S_{orm}$. We also present the upscaling and the simulation model used in this research. Dimensionless groups were derived from an inspectional analysis for a first contact miscible (FCM) gas flood in a 2-D heterogeneous reservoir.

3.1 OVERVIEW

$S_{orm}$ represents the amount of bypassed oil in gas flooding processes. There are many papers that discuss various techniques and correlations for prediction of $S_{orm}$. Upscaling is widely used in oil and gas simulators to save computational time. When the permeability field is upscaled the new recovery predictions are different from what they would be for the fine scale model because the reservoir has been made more homogeneous. We have used the same concept of permeability upscaling to calculate $S_{orm}$ and eventually to find out how $S_{orm}$ will change with fluid and reservoir properties.

Generally in order to handle the problem of excessive vaporization of oil, a constant value of $S_{orm}$ is introduced in the corresponding upscaled model, which is
treated as immobile oil saturation by the simulator. A constant value of $S_{orm}$ for the entire reservoir normally does not solve the problem as vaporization of oil changes with time and space in the reservoir and these spatial and temporal variations are ignored while dealing with this problem. The alpha factors approach (Barker et al. 2005) is extended to consider these variations by assigning different alpha factors to each grid block but the calculations are very tedious and time consuming. This chapter gives an overview of how $S_{orm}$ changes with time and space. The effect of grid block size will also be discussed.

### 3.1.1 ASSUMPTIONS

The main assumptions made in the simulation model are single-phase flow, 2-D geometry, uniform and constant porosity, no chemical reactions and no gravity. The top and bottom of the reservoir are no flow boundaries. The injection well is constant rate while the producer well is at constant pressure. We have used a simple mixing rule in order to calculate the viscosity of the multi-component single-phase mixture. The density of the fluid changes with space and time.
3.1.2 GOVERNING EQUATIONS FOR FCM

The governing equations and auxiliary relations for first contact miscible flow are the component material balance equations, the continuity equation, Darcy’s law, dispersion tensor and the viscosity mixing rule. We start with the overall compositional balance equation written over all phases for each component and then reduce that equation as per our assumptions.

The overall material balance equation for all phases and for each component is given by,

\[
\frac{\partial}{\partial t} \left( \phi \sum_{j=1}^{N_p} \rho_j S_j w_{ij} + (1-\phi) \rho_s w_s \right) + \nabla \cdot \left( \sum_{j=1}^{N_p} \left( \rho_j w_j \bar{u}_j - \phi S_j \bar{K}_y \nabla (\rho_j w_j) \right) \right) = 0 \quad \text{...........(3.1)}
\]

where subscript \(i\) represents each component, subscript \(j\) represents each phase, subscript \(s\) represents the solid phase, \(N_p\) is the number of phases present, \(S_j\) is the saturation of phase \(j\), \(\rho_j\) is the density of phase \(j\), \(w_{ij}\) is the mass fraction of component \(i\) in phase \(j\), \(u\) is the Darcy velocity, \(\phi\) is porosity and \(K_{ij}\) is the dispersion tensor. The dispersion tensor is given by,

\[
\bar{K} = \begin{bmatrix}
K_{xx} & K_{xz} \\
K_{zx} & K_{zz}
\end{bmatrix}
\]

Dispersion coefficients in longitudinal and transverse directions are given by,

\[
K_{Li} = \frac{D_l}{\phi F} + \frac{\alpha_l u_m}{\phi} \quad \text{.................................................. .................................................. (3.2)}
\]
\[ K_{\tau_i} = \frac{D_i}{\phi F_i} + \frac{\alpha_{\tau_i} \mu_{\tau_i}}{\phi} \] ................................................................. ......................................................... (3.3)

In equations 3.1, the first term on the left hand side is the accumulation term and the second term is the flux term (advective and dispersive transport). The right hand side in Eqs. 3.1 is zero because we have assumed no chemical reactions and no source terms in our model.

Assuming a single phase, no adsorption and using the relation \( \rho_j w_{ij} = C_i \) (where \( C_i \) is the mass concentration (mass/volume) of component \( i \)), Eqs. 3.1 can be reduced to

\[ \frac{\partial}{\partial t} (\phi C_i) + \nabla \left( (C_i \mu_i) - \phi \vec{K}_i \cdot \nabla (C_i) \right) = 0 \] ................................................................. ......................................................... (3.4)

We use two components in our model to represent oil and gas compositions. Equations 3.4 can be written for both the components. Concentrations of both components when added give the density of the mixture. That is,

\[ C_o + C_s = \sum_{i=1}^{2} w_{ij} \rho_j = \rho_j \] ................................................................. ......................................................... (3.5)

Addition of the two component equations given by Eqs. 3.4, gives,

\[ \frac{\partial}{\partial t} (\phi \rho) + \nabla (\rho \mu) = 0 \] ................................................................. ......................................................... (3.6)
In the derivation of equation 3.6, the dispersion term will vanish when we add the equations for the two components. For uniform porosity and 2-D flow, equation 3.6 can be written as,

\[
\frac{\partial}{\partial t}(\phi \rho) + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial z}(\rho u_z) = 0 \quad .......................................................... (3.7)
\]

For an isothermal system, density is a function of pressure and composition. In our simulations we take the properties of both components in the cubic EOS to be the same, i.e. $T_C$, $P_C$ etc. Thus, density does not vary in our simulations owing to composition changes. Density can only vary if pressure changes as a result of fluid compressibility.

The velocities in equation 3.7 can be calculated using Darcy’s law, which is given by

\[
u = \frac{k}{\mu_m} (\nabla P - \rho g \Delta Z) \quad .......................................................... (3.8)
\]

where $\mu_m$ is the viscosity of mixture of the components and $Z$ is the elevation. The mixture viscosity is a function of composition changes only in our model and is calculated using the fourth power mixing rule as follows:

\[
\mu_m = \left[ \mu_o^{-1/4} + \beta (\mu_s^{-1/4} - \mu_o^{-1/4}) \right]^{-4} \quad .......................................................... (3.9)
\]

where
\[ \beta = \frac{C_i - C_i'}{C_i' - C_i''}. \]

The mixture viscosity therefore varies with time and space causing the flow rate changes spatially and with time. There are many other methods available for calculation of mixture viscosity (Maria A. Barrufet, 2003). Substitution of equation 3.8 into equation 3.7 with constant porosity gives the pressure equation for our model as,

\[ \phi \frac{\partial P}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\rho k_x}{\mu_m} \nabla P \right) + \frac{\partial}{\partial z} \left( \frac{\rho k_z}{\mu_m} \nabla P \right) = 0 \]

For incompressible single-phase flow, Eq. 3.10 reduces to

\[ \nabla \cdot (\vec{u}) = 0 \]

or,

\[ \frac{\partial}{\partial x} \left( \frac{k_x}{\mu_m} \nabla P \right) + \frac{\partial}{\partial z} \left( \frac{k_z}{\mu_m} \nabla P \right) = 0 \]

\[ \textbf{3.2 DIMENSIONLESS SCALING GROUPS} \]

From inspectional analysis we found seven scaling groups that govern recovery for a FCM model. Four of the scaling groups are obtained from the governing equations for FCM flow while the remainders are used to describe the
reservoir heterogeneity. Inspectional analysis for FCM flow is presented in appendix A.

3.2.1 SCALING GROUPS FROM FCM FLOW

Four independent dimensionless groups, which have been obtained from inspectional analysis of the governing equations of the FCM flow are given as described below

LONGITUDINAL PECLET NUMBER

The Peclet number is given by,

\[ N_{Pe} = \frac{v_{inj} L}{K_i} \]

where \( L \) is the length across the wells in the primary flow direction, \( v_{inj} \) is the injection pore velocity and \( K_i \) is the longitudinal dispersion coefficient.

The Peclet number is the ratio of characteristic times for dispersive transport of a particle in the direction of flow to convective transport of the particle in the same direction. If Peclet number is zero, dispersive transport completely dominates convective transport. Taking the numerical dispersion into account, the Peclet number can be approximated (Garmeh et al. 2009) as,
\[ N_{Pe} \approx \frac{2L}{2\alpha_L + \Delta x} \]  \[ \text{...............................................} \]  \[ \text{...............................................} \]  \[ (3.14) \]

where we have assumed \( v_{inj} \approx |v| \approx v_x, \alpha_T v_z^2 / |v| \ll \alpha_L v_x^2 / |v| \) and time step sizes are small. We have assumed that the diffusion term in equation 3.15 is negligible compared to other terms.

**DISPERSION NUMBER**

The dispersion number is defined as the square root of the ratio of the time scales for a particle to be transported a distance \( H \) by transverse dispersion to that transported a distance \( L \) by longitudinal dispersion (Garmeh 2009). It is given by,

\[ N_D = \frac{L}{H} \sqrt{\frac{K_T}{K_L}} \]  \[ \text{...............................................} \]  \[ \text{...............................................} \]  \[ (3.15) \]

A dispersion number of zero means there is no transverse mixing and there will be no scaling effects on dispersivity. We can use the above equation to account for numerical dispersion in order to obtain,

\[ N_D = \frac{L}{H} \sqrt{\frac{2\alpha_L |v_z / v_x|^2 + 2\alpha_T + |v_z / v_x| \Delta z}{2\alpha_L + \Delta x}} \]  \[ \text{...............................................} \]  \[ (3.16) \]

From Equation 3.18, the dispersion number is dependent on dispersivities and grid block sizes in both directions and also on the ratio of transverse to longitudinal velocities.
EFFECTIVE ASPECT RATIO

Effective aspect ratio is given by,

\[ R_e = \frac{L}{H} \sqrt{\frac{k_z}{k_x}} \] .......................... ........................................(3.17)

Effective aspect ratio is the ratio of time required for fluid to cross the reservoir in the longitudinal direction to that in the transverse direction. Large values of aspect ratio mean that fluid crosses the reservoir in the transverse direction faster than in the longitudinal direction. Thus, the fluid fluctuations in the transverse direction decay faster than the longitudinal direction. If aspect ratio is zero there is no cross flow in the reservoir.

MOBILITY RATIO

Mobility ratio of the reservoir oil and injected gas has significant impact on the amount of bypassed oil. If the viscosity is constant and flow is first contact as in our case, the mobility ratio is given by,

\[ M = \frac{\mu_o}{\mu_s} \] .......................... ........................................(3.18)

Larger mobility ratios mean lower oil recovery owing to increased channeling. Increased channeling will increase \( S_{orm} \).
3.2.2 SCALING GROUPS FROM RESERVOIR HETEROGENEITY

To capture the effect of reservoir heterogeneity on miscible residual oil saturation, we have included the correlation length in the longitudinal and transverse directions. Dykstra Parson’s coefficient is also used in our scaling groups to allow for the various permeability variations.

CORRELATION LENGTH IN LONGITUDINAL DIRECTION

The correlation length is the maximum length of autocorrelation between variables of the same type taken at different locations (G. Garmeh, 2007). The structure of the autocorrelation has been modeled using semivariograms. A semivariogram is defined as,

\[
\gamma(h) = \frac{1}{2n} \sum_{i=1}^{n} \left[ k(x_i) - k(x_i + h) \right]^2
\]

Equation 3.21 is used to calculate the mean of the squared difference between the pairs of data points in a direction separated by a given distance (Deutsch, 2002). In equation 3.21 \( \gamma \) is the semivariance of the permeability data, \( k(x_i) \) is the permeability value at \( x_i \), \( k(x_i+h) \) is the permeability value at \( (x_i+h) \), \( \bar{h} \) is the lag distance vector in a specified direction and \( n \) is the number of data pairs used to estimate the semivariance.
There are other components of the semivariogram model called the sill, nugget and range. The maximum variability of the reservoir or maximum value of the semivariogram model at a large distance is called the sill. The nugget is defined as the value of the semivariogram model extrapolated to zero lag distance. The range is defined as the lag distance that a semivariogram approaches the sill and is normalized by total length $L$ in the longitudinal direction and by total width $H$ in the transverse direction. The semivariogram model which has been used in this research and the user manual for using FFTSim (Jennings, 2000) to generate permeability fields can be found in Garmeh’s dissertation (Garmeh, 2009).

**CORRELATION LENGTH IN TRANSVERSE DIRECTION**

The correlation length in the transverse direction has the same properties as the correlation length in the longitudinal direction except that it is related to the permeability distribution in the transverse direction. As longitudinal correlation length is normalized by $L$, transverse correlation length is normalized by $H$.

**DYKSTRA-PARSONS COEFFICIENT**

Dykstra-Parsons coefficient is a measure of the reservoir heterogeneity. Larger Dykstra Parsons coefficients imply large reservoir heterogeneity. It is a
dimensionless number that relates permeability distribution to the standard deviation of its mean (Dykstra and Parson 1950).

\[
V_{DP} = \frac{\bar{k} - k_\sigma}{\bar{k}}
\].............(3.20)

where \( \bar{k} \) is the median permeability and \( k_\sigma \) is the permeability at one standard deviation below median on a log-probability plot. Jensen (2000) provided an alternative formula for the estimation of Dykstra Parsons coefficient which is given by,

\[
V_{DP} = 1 - \exp(-\sigma_{\ln k})
\].............(3.21)

where \( \sigma_{\ln k} \) is the sample standard deviation of \( \ln k \). Values of Dykstra Parsons coefficient lies between zero and one. A zero value of \( V_{DP} \) means the formation is completely homogeneous and if \( V_{DP} \) is 1.0 the formation is completely heterogeneous. Typical values of the Dykstra Parsons coefficients are between 0.4 and 0.9.

3.3 METHODOLOGY USED FOR ESTIMATION OF \( S_{ORM} \)

This section explains the methodology used in this research to estimate the miscible residual oil saturation from the simulation.
We have used the concept of permeability upscaling to define miscible residual oil saturation. Generally the amount of oil left behind in the fine grid model is more than the amount of oil left behind in the corresponding upscaled or coarse scale grid due to the more homogeneous nature of the coarse scale grid as compared to the fine scale model. When the gas flooding simulations are run on a coarse scale model the simulators predict excessive recovery, and thus, to avoid these excessive recovery predictions some oil shall be forced to remain at the end of the gas flooding simulation otherwise oil recovery will be too large. The oil saturation which is used in simulators to avoid excessive recovery predictions is called miscible residual oil saturation or $S_{orm}$. There are several methods available to predict $S_{orm}$ as discussed in the literature review section. This research provides an idea of how $S_{orm}$ changes based on the reservoir and fluid scaling groups and numerical parameters.

To obtain the value of the miscible residual oil saturation, we subtract the oil saturation for the coarse scale model from the oil saturation for the fine scale model. There will be different values of $S_{orm}$ for different levels of upscaling. The coarser the grid model more oil will be recovered and the greater should be the $S_{orm}$ in most cases. In this research we have used two levels of upscaling.
The level of upscaling has been defined as the ratio of the fine scale grid block size in \(x\)- or \(z\)-directions to the coarse scale grid block size in the same direction.

\[
\delta(\Delta x) = \frac{\Delta x \text{ (fine scale grid)}}{\Delta x \text{ (coarse scale grid)}}
\]

\[
\delta(\Delta z) = \frac{\Delta z \text{ (fine scale grid)}}{\Delta z \text{ (coarse scale grid)}}
\]

where \(\delta(\Delta x)\) and \(\delta(\Delta z)\) are the levels of upscaling (called ‘upscaling factor’ in this research) in the \(x\)- and \(z\)-directions respectively and \(\Delta x\) and \(\Delta z\) are the grid block sizes in the \(x\)- and \(z\)-directions respectively.

We determine the oil saturation for fine and coarse scale models after injection of various amounts of gas. The difference between the two oil saturations are then calculated at different pore volumes injected.

### 3.3.1 UPSCALING METHOD

The term upscaling is often used in reservoir engineering. Upscaling is performed to save computational time. Permeability upscaling finds a homogeneous effective permeability that has the same flow characteristics as the fine scale.

For incompressible and single-phase flow, the flow equation is given by the continuity equation and Darcy’s law
\[ \nabla \cdot (u) = 0 \]  ...........................................................(3.23)

\[ u = -\frac{k}{\mu} \nabla P \]  ...........................................................(3.24)

where \( u \) is the local velocity vector. We can derive the pressure equation by substitution of Darcy’s law into the continuity equation as given by,

\[ \nabla \cdot \left( -\frac{k}{\mu} \nabla P \right) = 0 \]  ...........................................................(3.25)

Upscaling can be represented as in Fig. 3-1. To upscale the permeability field we have taken an arithmetic average of the fine scale permeabilities in the direction of flow and a harmonic average in the direction perpendicular to the flow, as shown in Fig. 3-2.

Figure 3-3 shows the permeability distributions for a fine and upscaled reservoir. From Fig. 3-3 we can see that upscaling doesn’t change the permeability structure in the reservoir but permeability variations in the reservoir are made more homogeneous. We have also plotted concentration profiles for the same reservoir models in Fig. 3-3.

Figure 3-4 shows the solvent concentrations after 1.0 pore volumes of injection in the fine and upscaled models. The concentrations in Fig. 3-4 show that the upscaled models accurately reflect the displacement for their fine scale model.
Permeability fields, which result in the alteration of the permeability structure after upscaling, are not supposed to be upscaled.

The permeability variations in the reservoir become more homogeneous as upscaling proceeds thereby changing the values of correlation lengths and Dykstra Parsons coefficient. Dispersion number is also affected by grid block sizes in both longitudinal and transverse directions (Garmeh, 2009). This alteration of scaling groups changes the level of mixing in the reservoir. The amount of mixing in the upscaled reservoir is smaller than the fine scale model owing to the alteration of scaling groups. But the level of numerical dispersion is increased during upscaling. Thus, it is possible to achieve the same level of mixing as the original fine-scale model. Therefore, it is needed to find the maximum grid block size, which can be used to match the level of mixing at the fine and coarse scales (Garmeh et al. 2009; Adepoju et al. 2012).

The concept of mixing is often ignored in compositional simulators. Therefore, upscaling without matching the level of mixing in fine and coarse scale models can result in larger recovery predictions, which requires miscible residual oil saturation in simulators to avoid excessive recovery.
3.4 SIMULATION MODEL

In this research we have used computer modeling group’s GEM simulator (User’s Manual 2010) combined with MATLAB in order to find the effect of each scaling group on miscible residual oil saturation by changing the parameters that go into each scaling group. This method has been used by Garmeh (Garmeh et al. 2009) in his research of scale dependency of dispersivity.

We have previously defined the assumptions used in the model. The reservoir model is a 2-D heterogeneous reservoir with constant and uniform porosity. The base case model is 512 ft by 64 ft with a Cartesian grid of 256 x 64 blocks in the x- and z- directions respectively. All the grid blocks have equal size and the thickness of the reservoir is 10 ft. We are injecting solvent at a constant rate and the producer is at constant pressure.

We have used the same properties for the injection component and the component which is present in the reservoir except for viscosity. In case of a unit mobility ratio, viscosity will remain constant. Viscosity and density are functions of $T$, $P$ and composition. As the reservoir temperature is fixed, viscosity and density will only be affected by pressure and composition. Since we are using similar properties for both the components including critical pressure and critical temperature, composition will not have any effect on density changes. For non-unit mobility ratios, viscosity of the mixture changes as gas is injected.
In order to generate permeability fields for the reservoir we have used FFTSim (Jennings, 2000). The parameters that are assigned for generating permeability fields are correlation length in the longitudinal and transverse directions, Dykstra Parsons coefficient and mean permeability. The mean permeability in all simulations is 200 md.

We have injected 5.0 pore volumes of solvent in all simulations to ensure that the solvent breakthroughs to the producer well and also to ensure a sufficient increase in the concentrations in each grid block. Typically 0.5 to 1.5 pore volumes of solvent are injected in reservoirs. We inject continuously but water alternating gas processes (WAG) could be done in the future.

After each GEM simulation we export the concentration histories to MATLAB and save it in a database. To find the impact of each scaling group on the oil saturation we change the parameter that goes into each scaling group. The procedure is repeated for the upscaled model and data for the upscaled model is also stored in the same database for analysis. The concentrations from each grid block are then averaged over the entire reservoir in order to find the total oil saturation left in the reservoir after the gas flood. Component concentrations are then plotted against the pore volume injected in order to capture the temporal variation of $S_{orm}$. Depiction of spatial variation of $S_{orm}$ is difficult because of the upscaling factor. To capture the spatial variation of $S_{orm}$ we have averaged the
concentrations of fine scale grid blocks to match the grid block size of the coarse
scale model. After averaging the fine scale concentrations we calculate $S_{orm}$ at
different locations in the reservoir.

As stated earlier, we have used FFTSim in order to generate the
permeability field. Based on the seed provided to the input file FFTSim generates
different realizations of permeability. For example in Fig. 3-5 three different
permeability fields are shown, which are generated for the same values of the
scaling groups and parameters.

The difference between permeability fields in Fig. 3.5 is the seed number
provided in the input file for FFTSim to generate them. Depending on the
permeability field, the injected gas will have different breakthrough times.
Different realizations of permeability will generate different oil saturations and
hence different $S_{orm}$ values even for the same scaling groups. We have used 10
different realizations of permeability for one set of scaling groups to obtain a
better average. We have averaged the concentrations for those 10 different cases to
obtain concentrations for that particular set of scaling groups.

We have run an example case in which we have used 50 different seed
numbers to generate 50 different permeability fields. This was done to see how
many realizations of permeability are required to obtain a constant averaged
concentration history. Figure 3-6 shows $S_{orm}$ values for 50 realizations of permeability for a particular set of scaling groups.

Figure 3-6 shows that $S_{orm}$ changes with the seed number. We have averaged the $S_{orm}$ values for different realizations. We can see from the plot that after 20 realizations, the oscillations in $S_{orm}$ with PVI are minimized. We have used 256x32 grid blocks in the fine grid case and the dimensions of each grid block are 1ft x 1ft. The upscaling groups are given in Fig. 3-6. Sorm decreases with PVI.

We ran another set of simulations with different scaling groups and parameters to study the effect of permeability realizations on $S_{orm}$. $S_{orm}$ in Fig. 3-7 has been plotted for the similar scaling group parameters used as base case in sensitivity analysis except for mobility ratio of 125. The trends in Fig. 3-7 for different PVI are explained in the discussion of mobility ratio on $S_{orm}$ in Chapter 4.

In this research we have used 10 different realizations of permeability for each set of scaling groups to keep the computational time reasonable. The results could be improved if concentrations are averaged over 30 to 50 realizations of permeability.
3.5 TEMPORAL AND SPATIAL VARIATIONS OF $S_{ORM}$

In this section we discuss how $S_{orm}$ changes with time and space. We have used continuous gas injection in all of our simulations.

3.5.1 SPATIAL VARIATION OF $S_{ORM}$

This section presents how $S_{orm}$ changes with the distance from the injection well. The scaling groups and other parameters for the base case model, which has also been used in sensitivity analysis, are given in table 3-1.

We have run simulations for 5.0 pore volumes of injection to make sure sufficient concentration of the solvent passes through the reservoir. Depending on the permeability field, the solvent will breakthrough into the reservoir. Solvent will travel faster in high permeability regions than in low permeability regions.

To calculate $S_{orm}$, we have taken the difference of oil saturation for the fine scale grid and coarse scale grid. We have mapped the coarse scale grid model over the fine scale grid model to obtain $S_{orm}$ at different locations from the injector well. This method can be depicted as in Fig. 3-8.

In Fig. 3-8, fine scale grids have been mapped to match the size of upscaled grids at the corresponding locations. We take the arithmetic mean of the oil saturations in fine grid blocks which are mapped to the coarse grid blocks and then
subtract the oil saturation of the grid block in the coarse scale model from the fine scale model. It gives us the value of $S_{orm}$ at that location and similarly we find the $S_{orm}$ at different locations and distances from the injector well. We take the average of the $S_{orm}$ values which are at the same distance from the injection well and plot them against the normalized distance in order to obtain the spatial variation of $S_{orm}$. Figure 3-9 shows the variations of the oil saturations for the fine and coarse scale models with the normalized distance from the injection well.

As mentioned earlier, $\delta \Delta x$ represents the level of upscaling and is the ratio of grid block dimension for the fine scale model to the coarse scale model. We use two different levels of upscaling in this research. Table 3-2 shows the parameters that have been used in upscaled cases.

Normalized length travelled is the dimensionless distance from the injector well and is computed as $x/L$, where $x$ is the distance from the injector well and $L$ is the distance between the wells or the length of the reservoir.

The oil saturation ($S_o$) increases as we go away from the injector well since the grid blocks closer to the injection well are subjected to greater pore volumes of injection than the grid blocks farther away. We can also see from Fig. 3-9 that $S_o$ for coarse scale model is almost always lower than the $S_o$ for the fine scale model due to the fact that upscaling induces more homogeneity into the permeability structure and causes more recovery and hence less oil saturation after gas flooding.
Figure 3-10 shows the spatial variation of miscible residual oil saturation. Figure 3-10 shows that $S_{orm}$ changes from a minimum value of zero percent to maximum of almost 6% with distance. This implies that providing the same value of $S_{orm}$ at different locations in gas flooding simulations is not the correct way of accounting for excessive vaporization of oil. Next we have shown variation of $S_o$ for fine scale model for different pore volumes injected in Fig. 3-11.

We have run simulations for continuous gas injection. In this research we are not considering slug or WAG injection processes. As more pore volumes of solvent are injected into the reservoir more oil is vaporized and recovered and hence smaller oil saturation is achieved. In Fig. 3-12, we have shown the corresponding $S_{orm}$ trends for an upscaling factor of 0.125.

3.5.2 TEMPORAL VARIATION OF $S_{ORM}$

Miscible residual oil saturation changes with the distance as we have seen in the previous section. In this section we see that it changes with time as well. We use the same base case model as was used in the previous section. The parameters and scaling groups are given in table 3-1.

As the gas is injected into the reservoir it causes the vaporization of oil. The amount of vaporization or sweep depends upon the amount of gas that contacts the
oil. Continuous injection of a large amount of gas will cause more vaporization of oil as there will be enough gas to contact more and more oil.

Figure 3-13 shows the variation of oil saturation for fine and coarse scaled models with pore volumes injected. Oil saturation decreases as we inject more and more pore volumes of gas. Pore volumes of gas injected is one measure of time since we are using continuous gas injection. Figure 3-13 shows the global oil saturation at different PVI. Global oil saturation means that we have taken the average of oil saturations over the entire reservoir. Figure 3-14 presents the variations of $S_{orm}$ with pore volumes injected.

We can also calculate local $S_{orm}$ ($S_{orm}$ at a particular distance from the injection well or $S_{orm}$ in a particular cell) in the similar fashion as in the section 3.5.1. We have shown an example case of how local $S_{orm}$ will change with pore volumes of gas injected. Figure 3-15 shows the variation of local $S_o$ with PVI at $x_D=0.625$ and Fig. 3-16 shows the variation of local $S_{orm}$ with time at the same distance from the injection well.

The local $S_{orm}$ changes significantly with pore volumes injected. Using $S_{orm}$ in compositional simulators for each grid block may be more accurate than using a constant $S_{orm}$ for the entire reservoir. Different grid blocks are subjected to different pore volumes injected, which results in different $S_{orm}$ values in different grid blocks. Grid blocks near the injection well are subjected to far greater PVI
than the grid blocks farther away from the injection well. Thus, grid blocks near the injection well will have lesser $S_{orm}$ than the blocks farther away.

### 3.5.3 EFFECT OF UPSCALING FACTOR ON $S_{ORM}$

Before we move to the sensitivity analysis, we study the effect of the upscaling factor on $S_{orm}$. The upscaling factor is the ratio of grid dimension for the fine scale grid model to that of the coarse scale grid model. Figure 3-17 shows the global oil saturation for different levels of upscaling. The case when the upscaling factor is equal to one is the fine scale model with no upscaling. In this research, we are using $\delta \Delta x$ (upscaling factor in $x$-direction) = $\delta \Delta z$ (upscaling factor in $z$-direction). We can upscale for different values of $\delta \Delta x$ and $\delta \Delta z$ as well. $S_{orm}$ is predicted for a particular upscaling factor.

We can see from Fig. 3-17 that recovery increases with upscaling as discussed in earlier sections. Figure 3-18 shows that $S_{orm}$ increases with upscaling factor. Larger upscaling causes a more homogeneous structure, which results in increased recovery in all grid blocks and hence increased $S_{orm}$. At large amounts of PVI, $S_{orm}$ approaches zero as we have injected more than enough pore volumes of solvent to sweep through the reservoir for both fine and coarse scale models. If we change the scaling group values, the difference in $S_{orm}$ between different upscaling
levels doesn’t approach zero even for larger PVI as shown in Fig. 3-19, where a mobility ratio of 25 has been used. While running gas flooding simulations and using a constant value of $S_{orm}$ for avoiding excessive vaporization, upscaling factor plays an important role in recovery predictions.

3.6 SUMMARY AND CONCLUSION

Miscible residual oil saturation varies with time and space in the reservoir. Using a constant value of $S_{orm}$ throughout the reservoir to account for excessive recovery may lead to wrong recovery predictions. Effective handling of permeability fields is also very important while running simulations since the same reservoir parameters generate different permeability fields and hence cause different recovery predictions. To neglect the oscillations caused by multiple permeability fields generated related to a single set of reservoir properties, we averaged concentrations for a number of permeability realizations.

We presented an effective method for prediction of miscible residual oil saturation ($S_{orm}$) based on permeability upscaling. $S_{orm}$ calculations based on this scheme not only consider reservoir heterogeneity but also include the impact of reservoir mixing on oil recovery. Different levels of upscaling result in different values of miscible residual oil saturation. The greater the upscaling factor, the
larger is $S_{orm}$. In this research we are not generating any function or correlation which can predict $S_{orm}$ based on grid block size but this can be done easily using the methodology presented in chapter 5. Our $S_{orm}$ calculations are based on the fine scale model. If the grid block size in fine scale model is made even smaller, $S_{orm}$ will increase.
<table>
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</tr>
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</table>

*Table 3-1: Parameters and scaling groups used in base case simulation model.*
 Fine Scale | Coarse Scale \[\delta x = \frac{\Delta x(\text{Fine Scale})}{\Delta x(\text{Coarse Scale})}\] | Coarse Scale \[\delta x = \frac{\Delta x(\text{Fine Scale})}{\Delta x(\text{Coarse Scale})}\]
--- | --- | --- | ---
\(N_x\) | 256 | 64 | 32
\(N_z\) | 64 | 16 | 8
\(\Delta x\) | 2 | 8 | 16
\(\Delta z\) | 1 | 4 | 8
\(L/ H\) | 8 | 8 | 8

*Table 3-2: Reservoir dimensions used in upscaled models.*
Figure 3-1: Schematic representation of local flow upscaling scheme. Upscaling represents a heterogeneous porous medium by its equally effective homogeneous medium.
Figure 3-2: Effective permeability calculation methodology based on flow direction.
Figure 3-3: Permeability distribution for fine scaled and upscaled models. a) Fine scale. b) For upscaling factor of 0.250. c) For upscaling factor of 0.125.
Figure 3-4: Solvent concentrations after 1.0 pore volumes of injection in fine and upcaled models. a) Fine scale model. b) Upscaling factor of 0.250. c) Upscaling factor of 0.125.
Figure 3-5: Permeability fields generated for different seed numbers using FFTSim. Other parameters are $N_x=256$, $N_z=64$, $\Delta x=2\,\text{ft}$, $\Delta z=1\,\text{ft}$, $\lambda_x D=1.0$, $\lambda_z D=0.25$ and $V_{DP}=0.6$. a) Permeability field for seed one. b) Permeability field for seed two. c) Permeability field for seed three.
Figure 3-6: $S_{orm}$ for different realizations of permeability. $\delta \Delta x=0.125$. Scaling groups are $\lambda_D=2$, $\lambda_Z=0.25$, $R_L=8$, $M=30$, $N_D=1.38$, $N_{Pe}=170$, $V_{DP}=0.4$. 
Figure 3-7: $S_{orm}$ for different realizations of permeability. $\delta \Delta x=0.125$. Scaling groups are $\lambda_{xD}=1$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=125$, $N_D=2.93$, $N_{Pe}=128$, $V_{DP}=0.6$. 
Figure 3-8: Mapping of fine scaled model over coarse scaled model.
Figure 3-9: Variation of oil saturation with distance after 2.5 PVI. Concentrations are averaged for 10 permeability realizations. Scaling groups are $\lambda_{xD}=1$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{pe}=128$, $V_{DP}=0.6$. 
Figure 3-10: Spatial variation of $S_{orm}$ after 2.5 PVI. Scaling groups are $\lambda_{xD}=1$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{Pe}=128$, $V_{Dp}=0.6$. 
Figure 3-11: $S_o$ at different pore volumes of injection for fine scale model. Scaling groups are $\lambda_{xD}=1$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{pe} = 128$, $V_{DP}=0.6$. 
Figure 3-12: $S_{orm}$ for different PVIs. Scaling groups are $\lambda_{xd}=1$, $\lambda_{zd}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{pe} = 128$, $V_{dp}=0.6$. 
Figure 3-13: Variation of $S_o$ with pore volumes injected. Scaling groups are $\lambda_{xD}=1$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{Pe} = 128$, $V_{DP}=0.6$. 

Coarse Scale Model ($\delta\Delta x=0.125$)

Fine Scale Model

Pore Volumes Injected
Figure 3-14: Variation of $S_{orm}$ with pore volumes injected. Scaling groups are $\lambda_{xD}=1$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{Pe} = 128$, $V_{DP}=0.6$. 
Figure 3-15: Variation of $S_o$ with time at $x_D=0.625$. Scaling groups are $\lambda_{xD}=1$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{Pe} = 128$, $V_{DP}=0.6$. 
Figure 3-16: Variation of $S_{orr}$ with pore volumes injected at $x_D=0.625$. Scaling groups are $\lambda_{xD}=1$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{Pe} = 128$, $V_{DP}=0.6$. 
Figure 3-17: Effect of upscaling on oil saturation. Scaling groups are $\lambda_{x_D}=1$, $\lambda_{z_D}=0.25$, $R_L=2.5$, $M=5$, $N_D=2.93$, $N_{Pe}=128$, $V_{DP}=0.6$. 
Figure 3-18: Effect of upscaling on $S_{orm}$. Scaling groups are $\lambda_{xD}=1$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=5$, $N_D=2.93$, $N_{Pe} = 128$, $V_{DP}=0.6$. 

\[ \delta \Delta x = 0.125 \]

\[ \delta \Delta x = 0.250 \]
Figure 3-19: Effect of upscaling on $S_{orm}$. Scaling groups are $\lambda_{xD}=1$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{Pe} = 128$, $V_{DP}=0.6$. 
CHAPTER 4

SENSITIVITY ANALYSIS FOR MISCIBLE RESIDUAL OIL SATURATION ($S_{orm}$)

This chapter presents a sensitivity analysis of $S_{orm}$. We have seven scaling groups based on fluid and reservoir properties derived from inspectional analysis. We examine the effect of these scaling groups on $S_{orm}$ by changing the parameters that go into each group.

4.1 OVERVIEW

In inspectional analysis (Shook et al. 1992; Gharbi et al. 1998), scaling factors are introduced in the governing equations and boundary conditions. These factors are then combined into dimensionless groups in order to retain the original form of the equations. The governing equations for 2-D FCM flow and inspectional analysis are in Appendix A.
4.2 SENSITIVITY ANALYSIS

Sensitivity analysis has been performed to capture the effect of each scaling group on $S_{orm}$. For sensitivity analysis scaling groups have been varied one at a time.

We have considered four different cases for each scaling group. Though some of the cases are not physically possible, for example zero value of effective aspect ratio, zero longitudinal correlation length etc. but including them give us a better understanding of how $S_{orm}$ behaves with change in these scaling group values.

For each case we have run simulations for 10 different realizations of permeability as discussed in earlier sections. Concentrations from these 10 different realizations are then averaged to obtain oil saturation for that particular case. We have run a total of 22 different cases for sensitivity analysis and each case has been run for 10 different realizations of permeability, which gives 220 total simulations. We have also run 32 other cases (320 simulations), where scaling groups have been varied two at a time to generate a dataset of $S_{orm}$ values. The effect of each scaling group on $S_{orm}$ has been discussed in the following sections.
4.2.1 CORRELATION LENGTH IN LONGITUDINAL DIRECTION

As stated earlier, correlation length is the maximum length of autocorrelation between variables of the same type taken at different locations. Figure 4-1 shows that miscible residual oil saturation increases with an increase in longitudinal correlation length.

We have shown global $S_{orm}$, which is calculated over the entire reservoir. A larger correlation length means an early breakthrough of the solvent because of increased channeling through the more continuous layers as shown in Fig. 4-2. It is clear from Fig. 4-2 that recovery is more for smaller longitudinal correlation length owing to little channeling. This increase in channeling for larger longitudinal correlation length causes larger average oil saturations at the end of the simulation as shown in Fig. 4-3. An increase in $S_{orm}$ means that the difference of oil saturations between fine scale and coarse scale is larger.

As shown earlier, oil saturations are lower for upscaled models compared to their corresponding fine scale models. Figure 4-1 shows the effect of longitudinal correlation length on miscible residual oil saturation for an upscaling factor of 0.25. For smaller upscaling factor $S_{orm}$ will be larger as shown in Fig. 4-4. For zero longitudinal correlation length, oil saturation $(S_o)$ and $S_{orm}$ decreases to zero after 2.0 pore volumes injected owing to little channeling. For zero longitudinal
correlation length at low PVI, $S_{orm}$ is higher compared to other cases because the gas has not reached the breakthrough yet in other cases.

4.2.2 DYKSTRA PARSONS COEFFICIENT ($V_{DP}$)

Dykstra Parsons coefficient is a measure of reservoir heterogeneity. Figure 4-5 shows the effect of Dykstra Parsons coefficient on oil saturation for the fine scale model. Larger values of $V_{DP}$ imply more heterogeneous structure and therefore more channeling, leaving behind more oil. Figure 4-6 shows the effect of Dykstra Parsons coefficient on $S_{orm}$.

An increase in the value of Dykstra Parsons Coefficient causes higher values of $S_{orm}$. In some cases $S_{orm}$ curves overlap or intersect each other. This is because we are using only 10 different realizations of permeability and upscaling of permeability might not be representative of the average permeability structure. If we use many more realizations, the oscillations in $S_{orm}$ should vanish.

4.2.3 MOBILITY RATIO (M)

The mobility ratio is defined as the ratio of viscosity of the oil to the viscosity of the injected solvent. Values of mobility ratio can vary from 20 to 100 or even more based on the type of solvent used. We have covered a valid range of
mobility ratios from 1 to 125 to capture its effect on $S_{orm}$. Figure 4-7 shows the effect of mobility ratio on oil saturation for the fine scale model.

Larger values of mobility ratio are caused by either highly viscous reservoir oil or a solvent having very low viscosity. Both of these conditions cause less recovery because of excessive channeling and hence more oil saturation left behind in the reservoir. The solvent concentration in high permeability zones will increase for high mobility ratio until gas breaks through as seen in Fig. 4-8-a. In case of high mobility ratio and for small volumes of gas injection $S_{orm}$ increases with PVI, but once gas breaks through in the reservoir $S_{orm}$ becomes nearly constant with PVI. Even after 5.0 PVI gas do not contact all the oil in the reservoir owing to channeling. As the oil saturation increases with an increase in mobility ratio, we expect large $S_{orm}$ values for higher mobility ratio. Figure 4-9 shows the effect of mobility ratio on $S_{orm}$.

As expected $S_{orm}$ increases with an increase in mobility ratio. Residual oil saturation decreases with decreasing mobility ratio and with pore volumes injected. $S_{orm}$ should approach zero as mobility ratio decreases.
4.2.4 DISPERSION NUMBER (Nd)

Figure 4-10 shows the effect of dispersion number on oil saturation. Dispersion number is a measure of mixing in the reservoir between reservoir oil and solvent injected. Very low values of dispersion number results in low recovery factor and cause high oil saturation left behind in the reservoir. With an increase in dispersion number transverse dispersion becomes more dominating over the heterogeneities. In that case we have large mixing in the porous media, which causes large recovery and hence lower values of oil saturation as shown in Fig. 4-10. Figure 4-11 shows the variation of $S_{orm}$ with dispersion number.

Miscible residual oil saturation increases with a decrease in dispersion number. Transverse dispersion becomes less dominate with decrease in dispersion number causing less recovery and hence more $S_{orm}$.

4.2.5 PECLET NUMBER (NPE)

The Peclet number is a measure of dispersive and convective transport of particles in the flow direction. Lower values of Peclet number means the dispersive transport dominates over the convective transport and higher values of Peclet number favors convective transport. Figure 4-12 shows the effect of Peclet number on oil saturation.
We can see from Fig. 4-12 that oil saturation increases with increasing Peclet number. Increasing Peclet number favors convective transport of particles causing lower mixing. Lower mixing leads to low recovery and causes more oil saturation at the end of the simulation. As stated earlier, the Peclet number is dependent on grid block size and physical dispersivity in longitudinal direction. Table 4-1 shows the reservoir dimensions and parameters which have been used in the sensitivity test of Peclet numbers. All other scaling groups are kept the same.

Figures 4-13 and 4-14 show the effect of Peclet number on miscible residual oil saturation ($S_{orm}$). $S_{orm}$ increases with an increase in Peclet number owing to a decrease in the amount of mixing.

### 4.2.6 EFFECTIVE ASPECT RATIO (R$_L$)

Effective aspect ratio is defined as

$$R_L = \frac{L}{H} \sqrt{\frac{k_z}{k_x}}$$

To examine the sensitivity of $S_{orm}$ on $R_L$, we have changed the permeability ratio to obtain different values of $R_L$. We have used permeability ratios of 0, 0.1, 1, 10 and 25 to obtain effective aspect ratios of 0, 2.5, 8, 25 and 40 respectively.
Parameters for the simulation models for sensitivity of $S_{orm}$ on aspect ratio are given in Table 4-2. All other scaling groups are kept constant.

Zero value of effective aspect ratio means there is no cross flow in the reservoir. We have used $k_c=0$, which implies that the velocities in transverse direction are zero and the fluid will travel in the longitudinal direction only. This causes channeling and early breakthrough of the solvent resulting in low recovery. Figure 4-15 shows the breakthrough of the solvent in the reservoir after 0.7 pore volumes of solvent injected. This early breakthrough causes low production and hence larger values of residual oil. Figure 4-16 shows the solvent concentrations after the end of simulation (5.0 PVI).

Figure 4-17 shows large values of oil saturation at zero $R_L$. Larger values of aspect ratio increases reservoir mixing (Garmeh, 2009). Mixing is increased over the cross-sectional area as the aspect ratio increases. Because of increase in mixing, recovery increases causing lower oil saturations as shown in Fig. 4-17. With increase in aspect ratio, concentrations reach transverse boundaries quicker but once they reach the entire reservoir dispersivities approach an asymptotic limit and mixing doesn’t increase anymore. After that, oil saturation increase with increase in aspect ratio.
Miscible residual oil saturation decrease with increase in aspect ratio owing to greater mixing, but once dispersivity approach an asymptotic value, reservoir mixing doesn’t increase anymore causing an increase in $S_{orm}$. Figure 4-18 to 4-21 show the effect of aspect ratio on $S_{orm}$ for different upscaling factors.

4.2.7 CORRELATION LENGTH IN TRANSVERSE DIRECTION ($\lambda_{zD}$)

We have run simulations for four different values of transverse correlation length. A zero value transverse correlation length means that permeabilities in that direction are highly uncorrelated and fluid tends to flow in the longitudinal direction and cause lower residual oil. Figure 4-22 shows solvent concentrations at $\lambda_{zD}=0$ in the grid blocks after 0.8 PVI. Figure 4-22 shows that solvent hasn’t reached breakthrough at all and as we inject more it sweeps through the entire reservoir leaving behind essentially zero residual oil saturations.

Figure 4-23 shows solvent concentrations at $\lambda_{zD}=0.25$ after 0.8 PVIs. Because of early breakthrough of solvent even after 5.0 PVI there is some oil left unswept by the solvent causing higher residual oil saturations as shown in Fig. 4-25. Figure 4.24 presents solvent concentrations at $\lambda_{zD}=1.0$ after 0.8 PVI. Breakthrough time in this case is more than in previous case which causes slightly lower residual oil saturations.
With an increase in transverse correlation length fluid tends to go in the transverse direction which avoids early breakthrough of solvent and cause larger recovery. Figures 4.25-4.27 show the effect of transverse correlation length on oil saturation and miscible residual oil saturation.

### 4.3 SUMMARY AND DISCUSSION

The spatial and temporal variations of $S_{orm}$ were presented in chapter 3. In this chapter we observed the effects of various reservoir and fluid scaling groups on miscible residual oil saturation ($S_{orm}$).

In this research we present a method of calculating $S_{orm}$ based on upscaling and this method can be used to model $S_{orm}$ effectively combined with the scaling groups. Though in this research we have shown global $S_{orm}$ ($S_{orm}$ for the entire reservoir) only but this method can be used for prediction of local $S_{orm}$ ($S_{orm}$ for each grid block) as well. A surface response function based on Box-Behnken analysis (discussed in future research and recommendations) can be generated, which should be able to predict global miscible residual oil saturation based on the scaling groups. Global $S_{orm}$ predictions can be very useful in the simulations where we use a constant value of $S_{orm}$ to force the simulators from predicting 100% recovery.
Additional conclusions are:

- Miscible residual oil saturation can be predicted based on fluid and reservoir scaling groups.
- $S_{orm}$ increases with heterogeneity and correlation length in the longitudinal direction.
- Increase in dispersion number causes decreasing values of $S_{orm}$ owing to domination of transverse dispersion over heterogeneities.
- $S_{orm}$ increases with mobility ratio owing to lower recovery at higher mobility ratios.
- $S_{orm}$ increases with increase in Peclet number.
- At first, $S_{orm}$ decreases with increase in effective aspect ratio but when dispersivities reach an asymptotic limit, $S_{orm}$ increases with increase in aspect ratio.
- $S_{orm}$ decreases with an increase in transverse correlation length.
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*Table 4-1: Reservoir dimensions and parameters used for sensitivity analysis of Peclet number. Other scaling groups are \(\lambda_{x_D}=1.0, \lambda_{z_D}=0.25, R_L=2.5, M=25, N_D=2.93, V_{DP}=0.6.\)*
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*Table 4-2: Reservoir dimensions and parameters used for sensitivity analysis of effective aspect ratio. Other scaling groups are $\lambda_{x,D}=1.0$, $\lambda_{z,D}=0.25$, $M=25$, $N_D=2.93$, $N_{Pe}=128$ $V_{DP}=0.6$.  


Figure 4-1: Effect of longitudinal correlation length on $S_{orm}$ for $\delta x=0.250$. Other scaling groups are $\lambda_\zeta D=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{Pe}=128$, $V_{DP}=0.6$. 
Figure 4-2: Solvent concentration after 0.7 PVI in fine scale models. Other scaling groups are $\lambda_{x}D=0.25$, $R_{L}=2.5$, $M=25$, $N_{D}=2.93$, $N_{Pe}=128$, $V_{DP}=0.6$. a) Solvent concentration after 0.7 PVI in fine scale model for $\lambda_{x}D=4.0$. b) Solvent concentration after 0.7 PVI in fine scale model for $\lambda_{x}D=0.25$. 
Figure 4-3: Effect of longitudinal correlation length on oil saturation for fine scale model. $R_L=2.5$, $M=25$, $N_D=2.93$, $N_{Pe}=128$, $V_{DP}=0.6$. 
Figure 4-4: Effect of longitudinal correlation length on $S_{orm}$ for $\delta \Delta x = 0.125$. Other scaling groups are $\lambda_{zD} = 0.25$, $R_L = 2.5$, $M = 25$, $N_D = 2.93$, $N_{Pe} = 128$, $V_{DP} = 0.6$. 
Figure 4-5: Effect of Dykstra Parsons coefficient on $S_o$ for fine scale model. Other scaling groups are $\lambda_{xD}=1.0$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$ and $N_{Pe} = 128$. 
Figure 4-6: Effect of Dykstra Parsons coefficient on $S_{orm}$ for $\delta A_x=0.250$. Other scaling groups are $\lambda_{x_D}=1.0$, $\lambda_{z_D}=0.25$, $R_L=2.5$, $M=25$, $N_D=2.93$ and $N_{Pe} = 128$. 
Figure 4-7: Effect of mobility ratio on $S_o$ for fine scale model. Other scaling groups are $\lambda_{xD}=1.0$, $\lambda_{zD}=0.25$, $R_L=2.5$, $V_{DP}=0.6$, $N_D=2.93$ and $N_{Pe}=128$. 
Figure 4-8: Solvent concentration at different PVI in fine scale model. Other scaling groups are $\lambda_{x,D}=1.0$, $\lambda_{z,D}=0.25$, $R_L=2.5$, $N_D=2.93$, $N_{Pe}=128$, $V_{DP}=0.6$. a) Solvent concentration after 1.0 PVI in fine scale model for $M=125$. b) Solvent concentration after 5.0 PVI in fine scale model for $M=125$. 
Figure 4-9: Effect of mobility ratio on $S_{orm}$ for $\delta \Delta x=0.250$. Other scaling groups are $\lambda_x D=1.0$, $\lambda_z D=0.25$, $R_L=2.5$, $V_{DP}=0.6$, $N_D=2.93$ and $N_{pe} = 128$. 

\[\Delta S_{orm}\] vs Pore Volumes Injected

M=125, M=5, M=25, M=1
Figure 4-10: Effect of dispersion number on $S_o$ for fine scale model. Other scaling groups are $\lambda_{xD}=1.0$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $V_{DP}=0.6$ and $N_{Pe}=128$. 
Figure 4-11: Effect of dispersion number on $S_{orm}$ for $\delta \Delta x=0.250$. Other scaling groups are $\lambda_{xD}=1.0$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $V_{DP}=0.6=2.93$ and $N_{Pe} = 128$. 
Figure 4-12: Effect of Peclet number on $S_o$ for fine scale model. Other scaling groups are $\lambda_D=1.0$, $\lambda_z=0.25$, $R_l=2.5$, $M=25$, $V_{DP}=0.6=2.93$ and $N_D = 2.93$. 
Figure 4.13: Effect of Peclet number on $S_{orm}$ for $\delta \Delta x=0.250$. Other scaling groups are $\lambda_{xD}=1.0$, $\lambda_{zD}=0.25$, $R_L=2.5$, $M=25$, $V_{DP}=0.6=2.93$ and $N_D = 2.93$. 
Figure 4-14: Effect of Peclet number on $S_{orm}$ for $\delta \Delta x = 0.125$. Other scaling groups are $\lambda_{\alpha D} = 1.0$, $\lambda_{z D} = 0.25$, $R_L = 2.5$, $M = 25$, $V_{dp} = 0.6 = 2.93$ and $N_D = 2.93$. 
Figure 4-15: Solvent concentrations after 0.7 PVI for aspect ratio=0.

Figure 4-16: Solvent concentrations after 5.0 PVI for aspect ratio=0. Solvent reaches breakthrough early in the simulation, causing more unswept oil.
Figure 4-17: Effect of effective aspect ratio on oil saturation. Other scaling groups are $\lambda_{xD}=1.0$, $\lambda_{zD}=0.25$, $M=25$, $V_{DP}=0.6$, $N_{Pe}=128$ and $N_{D} = 2.93$. 
Figure 4-18: Effect of effective aspect ratio on $S_{orm}$ for $\deltaAx=0.250$. Other scaling groups are $\lambdaxD=1.0$, $\lambdazD=0.25$, $M=25$, $VDP=0.6$, $NPe=128$ and $ND = 2.93$. 
Figure 4-19: Effect of effective aspect ratio on $S_{orm}$ for $\delta Ax=0.125$. Other scaling groups are $\lambda_{xD}=1.0$, $\lambda_{zD}=0.25$, $M=25$, $V_{DP}=0.6$, $N_{Pe}=128$ and $N_{D} = 2.93$. 
Figure 4-20: Effect of effective aspect ratio on $S_{orm}$ for $\delta \Delta x=0.250$. Other scaling groups are $\lambda_{xD}=1.0$, $\lambda_{zD}=0.25$, $M=25$, $V_{DP}=0.6$, $N_{Pe}=128$ and $N_D = 2.93$. 
Figure 4-21: Effect of effective aspect ratio on $S_{orm}$ for $\delta x=0.125$. Other scaling groups are $\lambda_{xD}=1.0$, $\lambda_{zD}=0.25$, $M=25$, $V_{DP}=0.6$, $N_{Pe}=128$ and $N_{D} = 2.93$. 
Figure 4-22: Solvent concentrations at different PVI for \( \lambda_{D}=0 \). Other scaling groups are \( \lambda_{D}=1.0, M=25, V_{DP}=0.6, N_{D}=2.93, N_{Pe}=128 \) and \( N_{D} = 2.93 \). a) solvent concentrations after 0.8 PVI for \( \lambda_{D}=0 \). b) solvent concentrations after 5.0 PVI for \( \lambda_{D}=0 \). Solvent has swept the entire reservoir leaving behind nearly zero residual oil saturation.
Figure 4-23: Solvent concentrations at different PVI for $\lambda_{zD}=0.25$. Other scaling groups are $\lambda_{xD}=1.0$, $M=25$, $V_{DP}=0.6$, $N_{D}=2.93$, $N_{Pe}=128$ and $N_{D} = 2.93$. a) Solvent concentrations after 0.8 PVI for $\lambda_{zD}=0.25$. Solvent breaks through in the reservoir early leaving behind unswept oil. b) Solvent concentrations after 5.0 PVI for $\lambda_{zD}=0.25$. 

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Figure 4-24: Solvent concentrations at different PVI for $\lambda_D=1.0$. Other scaling groups are $\lambda_D=1.0$, $M=25$, $V_{DP}=0.6$, $N_D=2.93$, $N_{Pe}=128$ and $N_D = 2.93$. a) Solvent concentrations after 0.8 PVI for $\lambda_D=1.0$. Compared to Fig. 4-23-a solvent still has not reached breakthrough yet. b) Solvent concentrations after 5.0 PVI at $\lambda_D=1.0$. 
Figure 4-25: Effect of transverse correlation length on $S_\nu$. Other scaling groups are $\lambda_{\alpha D}=1.0$, $M=25$, $V_{DP}=0.6$, $N_{Pe}=128$, $R_L=0.25$ and $N_D=2.93$. 

$\lambda_{zD}=0.25$, $\lambda_{zD}=0.75$, $\lambda_{zD}=1.0$.
Figure 4-26: Effect of transverse correlation length on $S_{orm}$ for $\delta \Delta x=0.250$. Other scaling groups are $\lambda_{xD}=1.0$, $M=25$, $V_{DP}=0.6$, $N_{Pe}=128$, $R_{L}=0.25$ and $N_D = 2.93$. 
Figure 4-27: Effect of transverse correlation length on $S_{orm}$ for $\delta \Delta x = 0.125$. Other scaling groups are $\lambda_{xD} = 1.0$, $M = 25$, $V_{DP} = 0.6$, $N_{Pe} = 128$, $R_L = 0.25$ and $N_D = 2.93$. 
CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 SUMMARY AND CONCLUSIONS

Accurate modeling of miscible residual oil saturation in compositional simulators is very important for correct predictions of oil recovery. Channeling and dead end pores in miscible floods lead to bypassed oil, which is ignored by the conventional simulators causing optimistic recovery predictions.

There are a number of methods available suggested by many authors to model $S_{orm}$. Though these methods provide a good insight of the problem, many of them are not accurate and others are too complicated to apply. The alpha factor approach to predict $S_{orm}$ is likely the best approach, but calculations become very tedious and time consuming when applied at the local scale (grid block scale). Exclusion of oil volume from flash calculation and dual porosity methods are easy to implement but they often lead to wrong recovery predictions because a constant value of $S_{orm}$ is used to prevent the simulators from predicting 100% recovery.

Oil recovery from miscible gas floods is highly dependent on mixing. Mixing of oil and gas decreases the strength of the injected fluid, and causes early breakthrough. The level of mixing is often ignored in reservoir simulations where
very large grid blocks are used. This can cause errors in predicting \( S_{orm} \) and oil recovery. Almost all the methods available for modeling \( S_{orm} \) do not consider the amount of mixing while simulating miscible floods.

We presented an effective way to estimate \( S_{orm} \) based on permeability upscaling and bypassing of oil. We have shown that different permeability realizations for the same input values lead to different \( S_{orm} \) predictions. It is important to run a number of permeability realizations to dampen the effect of different permeability structures on \( S_{orm} \) for the same set of scaling groups. We have also shown that \( S_{orm} \) varies with the distance from the injection well, a very important observation, which is ignored by the methods that are used in simulators to incorporate \( S_{orm} \).

The effect of grid block size on \( S_{orm} \) has been studied in this research. We have shown that the \( S_{orm} \) increases with the increasing size of the grid block in the upscaled model. This observation is important given the fact that geological models are often converted into simulation models using upscaling. In those cases, the value of \( S_{orm} \) will change with the amount of upscaling. Larger grid block size will lead to greater \( S_{orm} \).

We derived scaling groups from an inspectional analysis for a first contact miscible gas flood in a 2-D heterogeneous reservoir. Over 250 2-D numerical simulations were carried out to find the effects of different dimensionless groups.
on $S_{orm}$. These scaling groups are representative of fluid and reservoir properties and amount of mixing in the reservoir between injected gas and reservoir oil. These scaling groups can be used to predict $S_{orm}$ in a reservoir for a given level of upscaling using response surface function.

We performed a sensitivity analysis of $S_{orm}$ for scaling groups generated from inspectional analysis and reservoir heterogeneity. Miscible residual oil saturation increases with heterogeneity and with correlation length in the longitudinal direction. $S_{orm}$ decreases with increasing dispersion number because transverse dispersion dominates over channeling. $S_{orm}$ increases with increasing mobility ratio and increasing Peclet number. $S_{orm}$ decreases with increasing effective aspect ratio until dispersivities reach an asymptotic limit, after that $S_{orm}$ increases with increasing effective aspect ratio. Increasing transverse correlation length causes a decrease in $S_{orm}$.

### 5.2 Future Research and Recommendations

In this section we present recommendations for future research to model miscible residual oil saturation in a more effective way. We also discuss Box Behnken analysis, which can be used to generate a response function and should
be able to predict $S_{orm}$ for a given reservoir based on scaling groups and for a given upscaling factor.

We have ignored gravity, reaction and adsorption terms in our inspectional analysis; therefore, derived scaling groups can be extended in order to achieve much better results. We have considered single phase and incompressible simulations in this research. Single phase and incompressible assumptions can be extended to multi-phase models by updating relative permeabilities and capillary pressure.

In the upscaling process grid block sizes and porosities are assumed uniform spatially. The upscaling procedure can be extended for spatially variable properties. In this research we have used similar upscaling factors in both $x$- and $z$-directions. We can extend upscaling for different upscaling factors in $x$- and $z$-directions.

FFTSim simulator, which we used to generate heterogeneous permeability fields, gives different permeability fields for the same correlation lengths and heterogeneity factor but for a different input seed. In this research we have considered 10 realizations of permeability, but it is recommended that a larger number of realizations of permeability should be used to produce better predictions. We have used continuous gas injection in this research, but it can be
extended for water alternating gas injection processes (WAG) with more realistic relative permeability considerations including gas trapping.

An experimental design can be performed to predict miscible residual oil saturations based on three-factorial Box-Behnken design for seven scaling factors. A response surface using a quadratic model can be fit to the global miscible residual oil saturation for a given pore volume and upscaling factor (Appendix D).
APPENDIX A

SCALING GROUPS BY INSPECTIONAL ANALYSIS

Dimensionless scaling groups for first contact miscibility have been derived by inspectional analysis (Gharbi, 1998; Shook, 1992). Governing equations for FCM flow based on the assumptions stated in chapter 3 (see Eq. 3.4 with constant porosity) are,

\[
\frac{\partial C}{\partial t} + \frac{u_x}{\phi} \frac{\partial C}{\partial x} + \frac{u_z}{\phi} \frac{\partial C}{\partial z} - K_L \frac{\partial^2 C}{\partial x^2} - K_T \frac{\partial^2 C}{\partial z^2} = 0
\]

(A.1)

where

\[
K_L = \frac{D}{\tau} + \frac{\alpha_L |u|}{\phi}
\]

(A.2)

\[
K_T = \frac{D}{\tau} + \frac{\alpha_T |u|}{\phi}
\]

(A.3)

and \(\tau = \phi F\). where \(F\) is formation resistivity factor, \(K_L\) and \(K_T\) are dispersion coefficients in longitudinal and transverse direction and \(\alpha\) is dispersivity. We have assumed that dispersion coefficients are constant. Velocity in our model changes spatially owing to changes in mixture viscosity. For a mobility ratio of unity, velocity will remain constant. We derive our scaling groups based on constant velocity, but in the case of spatially changing velocity; additional scaling groups will be required.
The continuity equation under the same assumptions is given as,

\[
\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} = 0 \tag{A.4}
\]

Velocities are calculated using Darcy’s Law,

\[
u_x = -\frac{k_x}{\mu_m} \left( \frac{\partial P}{\partial x} \right) \tag{A.5}
\]

\[
u_z = -\frac{k_z}{\mu_m} \left( \frac{\partial P}{\partial z} \right) \tag{A.6}
\]

We use the fourth power mixing rule to calculate mixture viscosity,

\[
\mu_m = \left[ \mu_o^{-1/4} + \beta \left( \mu_i^{-1/4} - \mu_o^{-1/4} \right) \right]^{-4} \tag{A.7}
\]

\[
\beta = \frac{C - C_i}{C_i' - C_i} \tag{A.8}
\]

and \( C_i \) is the concentration of the injected component.

The independent parameters in the system of equations for FCM under these assumptions are \( x, z, t \), while the dependent variables are \( C \) and \( P \). Oil recovery is a function of reservoir parameters, independent variables (space and time) and dependent variables (concentration and pressure). Darcy’s law is used to calculate velocities in \( x \)- and \( z \)- directions.
Introducing Arbitrary Scaling Factors

Now we introduce two arbitrary scaling factors for each variable assuming a linear relationship which are substituted in the equations to obtain the dimensionless form of the equations.

\[ C = C_1 C_{iD} + C_{2s} \]

\[ x = x_{1s} x_D + x_{2s} \]

\[ z = z_{1s} z_D + z_{2s} \]

\[ t = t_{1s} t_D + t_{2s} \]

\[ P = P_{1s} P_D + P_{2s} \]

\[ u_z = u_{zs} u_D + u_{zs} \]

Next we substitute these dimensionless variables in the governing system of equations for FCM.

The mass balance equation becomes:

\[
\frac{\partial (C_1 C_D + C_{2s})}{\partial (t_{1s} t_D + t_{2s})} + \left( \frac{u_{zs} u_D + u_{zs}}{\phi} \right) \frac{\partial (C_1 C_D + C_{2s})}{\partial (x_{1s} x_D + x_{2s})} + \\
\left( \frac{u_{zs} u_D + u_{zs}}{\phi} \right) \frac{\partial (C_1 C_D + C_{2s})}{\partial (z_{1s} z_D + z_{2s})} - K_L \frac{\partial^2 (C_1 C_D + C_{2s})}{\partial (x_{1s} x_D + x_{2s})^2} = 0 
\]
The continuity equation for constant porosity takes the form,

$$\frac{\partial}{\partial \chi}(u_{t,1} + u_{x,D} + u_{z,2*}) + \frac{\partial}{\partial z^*}(u_{t,1} + u_{z,D} + u_{z,2*}) = 0$$

(A.10)

Here we have assumed constant density and porosity so that number of scaling groups is reduced. This is a reasonable assumption since mean permeability is fairly large. However, for non-unity mobility ratio, viscosity is still varying spatially in Eq. A.10. Using dimensionless variables in Darcy’s law

$$u_{t,1} + u_{x,D} + u_{z,2*} = \frac{k_x}{\mu_m} \left( \frac{\partial (P_v P_D + P_{z,2})}{\partial (x_1, x_D + x_{2*})} \right)$$

(A.11)

$$u_{z,1} + u_{z,D} + u_{z,2*} = -\frac{k_z}{\mu_m} \left( \frac{\partial (P_v P_D + P_{z,2})}{\partial (z_1, z_D + z_{2*})} \right)$$

(A.12)

We use the chain rule for first and second order derivatives to simplify the system of equations. With the help of chain rule, dimensional derivatives can be converted into their dimensionless forms.

Applying chain rule on the time derivative of concentration,

$$\frac{\partial C}{\partial t} = \left( \frac{\partial C}{\partial C_D} \right) \left( \frac{\partial C_D}{\partial t_D} \right) = C_D$$

(A.13)

Now, we can use the dimensionless form of the variables to find the first and second terms on right side of Eq. A.13 as below,

$$C = C_{1*}C_D + C_{2*} \rightarrow \frac{\partial C}{\partial C_D} = \frac{\partial}{\partial C_D} = C_{1*}$$
Using above equations in Eq. A.13 to obtain the time derivative of concentration

$$\frac{\partial C}{\partial t} = \frac{C_{1v}}{t_v} \left( \frac{\partial C_D}{\partial t_D} \right)$$

Similarly, we can obtain spatial derivatives of concentration in both $x$- and $z$- direction in dimensionless forms as below,

$$\frac{\partial C}{\partial x} = \frac{C_{1v}}{x_v} \left( \frac{\partial C_D}{\partial x_D} \right)$$

$$\frac{\partial C}{\partial z} = \frac{C_{1v}}{z_v} \left( \frac{\partial C_D}{\partial z_D} \right)$$

Second order derivatives can also be converted into their dimensionless forms using chain rule.

$$\frac{\partial^2 C}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial C}{\partial x} \right) = \frac{\partial}{\partial x} \left( \frac{C_{1v}}{x_v} \left( \frac{\partial C_D}{\partial x_D} \right) \right) = \frac{C_{1v}}{x_v} \left( \frac{\partial^2 C_D}{\partial x_D^2} \right)$$

$$\frac{\partial^2 C}{\partial z^2} = \frac{\partial}{\partial z} \left( \frac{\partial C}{\partial z} \right) = \frac{\partial}{\partial z} \left( \frac{C_{1v}}{z_v} \left( \frac{\partial C_D}{\partial z_D} \right) \right) = \frac{C_{1v}}{z_v} \left( \frac{\partial^2 C_D}{\partial z_D^2} \right)$$

We can use the dimensionless derivatives given by equations A.14 to A.18 in the mass balance equation given by equation A.9 to obtain
\[
\frac{C_{v}'}{t_{v}'} \left( \frac{\partial C_{D}}{\partial t_{D}} \right) + \left( \frac{u_{x_1} u_{z_{D}} + u_{z_2}}{\phi} \right) \frac{C_{v}'}{x_{1}'} \left( \frac{\partial C_{D}}{\partial x_{D}} \right) + \frac{u_{x_2} u_{z_{D}} + u_{z_2}}{\phi} \frac{C_{v}'}{z_{1}'} \left( \frac{\partial C_{D}}{\partial z_{D}} \right) - k_{t} \frac{C_{v}'}{x_{1}'} \frac{\partial^{2} C_{D}}{\partial x_{D}^{2}} - k_{t} \frac{C_{v}'}{z_{1}'} \frac{\partial^{2} C_{D}}{\partial z_{D}^{2}} = 0
\]

(A.19)

The convection terms in equations A.19 can be expanded to obtain

\[
\frac{C_{v}'}{t_{v}'} \left( \frac{\partial C_{D}}{\partial t_{D}} \right) + \frac{u_{x_1} C_{v}'}{\phi x_{1}'} \left( \frac{\partial}{\partial x_{D}} (u_{x_{D}} C_{D}) \right) + \frac{u_{z_2} C_{v}'}{\phi z_{1}'} \left( \frac{\partial}{\partial z_{D}} (u_{z_{D}} C_{D}) \right) - \frac{k_{t} C_{v}'}{x_{1}'} \frac{\partial^{2} C_{D}}{\partial x_{D}^{2}} - \frac{k_{t} C_{v}'}{z_{1}'} \frac{\partial^{2} C_{D}}{\partial z_{D}^{2}} = 0
\]

(A.20)

The continuity equation can also be converted into dimensionless form using chain rule for derivatives as below,

\[
\frac{\partial u_{_{x}}}{\partial x} + \frac{\partial u_{_{z}}}{\partial z} = 0
\]

Applying chain rule to the continuity equation,

\[
\left( \frac{\partial u_{_{x}}}{\partial x} \right) \left( \frac{\partial u_{_{x}}}{\partial u_{_{x_{D}}}} \right) \left( \frac{\partial x_{D}}{\partial x} \right) + \left( \frac{\partial u_{_{x}}}{\partial z} \right) \left( \frac{\partial u_{_{x}}}{\partial u_{_{z_{D}}}} \right) \left( \frac{\partial z_{D}}{\partial z} \right) = 0 \quad \text{(A.21)}
\]

To obtain the dimensionless form of Eq. A.21, we use the definition of dimensionless variables as given below,

\[
u_{_{x}} = u_{_{x_1}} u_{_{x_{D}}} + u_{_{z_2}} \rightarrow \frac{\partial u_{_{x}}}{\partial u_{_{x_{D}}}} = u_{_{x_1}} \quad \text{(A.22)}
\]

\[
u_{_{z}} = u_{_{z_1}} u_{_{z_{D}}} + u_{_{z_2}} \rightarrow \frac{\partial u_{_{z}}}{\partial u_{_{z_{D}}}} = u_{_{z_1}} \quad \text{(A.23)}
\]

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Using equation A.22 and A.23 in equation A.21 to obtain the dimensionless form of the continuity equation

\[
\frac{u_{x^*}}{x_{i^*}} \left( \frac{\partial u_{x^*}}{\partial x_D} \right) + \frac{u_{z^*}}{z_{i^*}} \left( \frac{\partial u_{z^*}}{\partial z_D} \right) = 0 \quad \text{.................................(A.24)}
\]

Equation A.24 can be simplified to obtain

\[
\left( \frac{\partial u_{x^*}}{\partial x_D} \right) + \frac{x_i u_{z^*}}{u_{z^*} z_{i^*}} \left( \frac{\partial u_{z^*}}{\partial z_D} \right) = 0 \quad \text{.................................(A.25)}
\]

The chain rule is applied to Darcy’s law and with the use of dimensionless variables we can obtain the equations A.26 and A.27

\[
(u_{x^*} u_{x^*} + u_{z^*} u_{z^*}) = -\frac{k_i}{\left[ \mu_o^{-1/4} + \beta \left( \mu_s^{-1/4} - \mu_o^{-1/4} \right) \right]^{-4}} \left( \frac{P_{p^*}}{x_{i^*}} \right) \frac{\partial P_{D}}{\partial x_D} \quad \text{.................................(A.26)}
\]

\[
(u_{z^*} u_{x^*} + u_{z^*} u_{z^*}) = -\frac{k_i}{\left[ \mu_o^{-1/4} + \beta \left( \mu_s^{-1/4} - \mu_o^{-1/4} \right) \right]^{-4}} \left( \frac{P_{p^*}}{z_{i^*}} \right) \frac{\partial P_{D}}{\partial z_D} \quad \text{.................................(A.27)}
\]

Further simplification of equations A.26 and A.27 leads to

\[
u_{x^*} = -\frac{1}{\left[ \frac{u_{x^*} x_{i^*} [\beta^{-4}] \mu_s}{k_x P_p} \right]^{1/4} - \left[ \frac{u_{x^*} x_{i^*} [\beta^{-4}] \mu_o}{k_x P_p} \right]^{1/4} + \left[ \frac{u_{x^*} x_{i^*} [\beta^{-4}] \mu_o}{k_x P_p} \right]^{1/4} - \frac{\partial P_{D}}{\partial x_D} \quad \text{.................................(A.28)}
\]

\[
u_{z^*} = -\frac{1}{\left[ \frac{u_{z^*} z_{i^*} [\beta^{-4}] \mu_s}{k_z P_p} \right]^{1/4} - \left[ \frac{u_{z^*} z_{i^*} [\beta^{-4}] \mu_o}{k_z P_p} \right]^{1/4} + \left[ \frac{u_{z^*} z_{i^*} [\beta^{-4}] \mu_o}{k_z P_p} \right]^{1/4} - \frac{\partial P_{D}}{\partial z_D} \quad \text{.................................(A.29)}
\]
Initial and Boundary Conditions

Now we introduce initial and boundary conditions and obtain their dimensionless forms:

\[ C_{i1}C_1 + C_{2*} = C_i' \] at \( t_{1i}t_D + t_{2*} = t_o \)

\[ P_{1p}P_D + P_{2*} = P_o \] at \( t_{1p}t_D + t_{2*} = 0 \)

\[ C_{s1}C_{sD} + C_{s2*} = C_s' \] at \( x_{1s}x_D + x_{2*} = x_o \)

\[ u_{x1}u_{xD} + u_{x2*} = u_{inf} \] at \( x_{1s}x_D + x_{2*} = x_o \)

\[ P_{1p}P_D + P_{2*} = P_{wf} \] at \( x_{1s}x_D + x_{2*} = L \)

\[ u_{z1}u_{zD} + u_{z2*} = 0 \] at \( z_{1s}z_D + z_{2*} = z_o \)

\[ u_{z1}u_{zD} + u_{z2*} = 0 \] at \( z_{1s}z_D + z_{2*} = H \)

In dimensionless forms initial and boundary conditions are set to one or zero, therefore, the dimensionless forms of the variables are given by:

\[ C_D = \frac{C_i' - C_{2*}}{C_{i1}} \] at \( t_D = \frac{t_o - t_{2*}}{t_{1*}} \)

\[ P_D = \frac{P_o - P_{2*}}{P_{1*}} \] at \( t_D = -\frac{t_{2*}}{t_{1*}} \)

\[ C_{sD} = \frac{C_s' - C_{s2*}}{C_{s1*}} \] at \( x_D = \frac{x_o - x_{2*}}{x_{1*}} \)
We set boundary and initial conditions equal to zero or one wherever appropriate.

$$u_{xD} = \frac{u^{ai}}{u_{x1^*}} \text{ at } x_D = \frac{x_o - x_{2^*}}{x_{1^*}}$$

$$P_D = \frac{P_{w_f} - P_{2^*}}{P_{1^*}} \text{ at } x_D = \frac{L - x_{2^*}}{x_{1^*}}$$

$$u_{zD} = 0 \text{ at } z_D = \frac{z_o - z_{2^*}}{z_{1^*}}$$

$$u_{zD} = 0 \text{ at } z_D = \frac{H - z_{2^*}}{z_{1^*}}$$

These initial and boundary conditions can be used to obtain the scaling factors as below,
Dimensionless Forms of the Equations

Dimensionless forms of the system of equations for FCM can be obtained using scaling factors and initial and boundary conditions. Dimensionless form of the conservation equation is given by,
\[
\frac{\partial C_D}{\partial t_D} + \left( \frac{u_{s,i}I_v}{\phi x_{i,v}} \right) \frac{\partial}{\partial x_D} (u_{sD} C_D) + \left( \frac{u_{s,i}I_v}{\phi z_{i,v}} \right) \frac{\partial}{\partial z_D} (u_{zD} C_D)
\]
\[
= \left( \frac{K_{l,i}I_v}{x_{i,v}^2} \right) \frac{\partial^2 C_D}{\partial x_D^2} + \left( \frac{K_{l,i}I_v}{z_{i,v}^2} \right) \frac{\partial^2 C_D}{\partial z_D^2}
\]

The continuity equation is given by,

\[
\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} = \frac{\partial u_{sD}}{\partial x_D} + \left( \frac{x_{i,v}u_{s,i}}{u_{s,i}z_{i,v}} \right) \frac{\partial u_{zD}}{\partial z_D}
\]

Velocities in both directions are given by,

\[
u_{sD} = -\left( \frac{u_{s,i}x_{i,v}}{k_x P_{i,v}} \right) \left[ \beta \right]^{-1/4} \mu_x^{-1/4} - \left( \frac{u_{s,i}x_{i,v}}{k_x P_{i,v}} \right) \left[ \beta \right]^{-1/4} \mu_x^{-1/4} + \left( \frac{u_{s,i}x_{i,v}}{k_x P_{i,v}} \right) \left[ \beta \right]^{-1/4} \mu_x^{-1/4}
\]

\[
u_{zD} = -\left( \frac{u_{z,i}z_{i,v}}{k_z P_{i,v}} \right) \left[ \beta \right]^{-1/4} \mu_z^{-1/4} - \left( \frac{u_{z,i}z_{i,v}}{k_z P_{i,v}} \right) \left[ \beta \right]^{-1/4} \mu_z^{-1/4} + \left( \frac{u_{z,i}z_{i,v}}{k_z P_{i,v}} \right) \left[ \beta \right]^{-1/4} \mu_z^{-1/4}
\]
In equation A.30, set the first and second group as 1.0, therefore, group three in the equation A.26 becomes

\[
\frac{\phi K_k}{L u_{\text{inj}}} = \frac{1}{N_{Pe_x}}
\] .......................................................... .......................................................... (A.34)

Group 4 in equation A.30 is given by,

\[
\frac{\phi K_T}{H u_{\text{inj}}} \frac{L}{H} = \frac{1}{N_{Pe_z}}
\] .......................................................... .......................................................... (A.35)

Group 5 will be reduced to 1.0 using groups one and two. Darcy’s equation simplifies to the form,

\[
\frac{1}{1 + \left( \frac{1}{\beta^+} \right)^{-1} \left( \frac{\mu_o}{\mu_s} \right)^{-1/4}} \frac{\partial P_D}{\partial x_D}
\] .......................................................... .......................................................... (A.36)

\[
\frac{1}{1 + \left( \frac{1}{\beta^+} \right)^{-1} \left( \frac{\mu_o}{\mu_s} \right)^{-1/4}} \frac{\partial P_D}{\partial z_D}
\] (A.37)

The first group in Darcy’s equation is set to 1.0 and the second group is mobility ratio

\[
M = \frac{\mu_o}{\mu_s}
\] .......................................................... .......................................................... (A.38)

As \(u_{x1} = u_{\text{inj}}\), so Darcy’s equation in \(z\)- direction reduces to

\[
\frac{k_z}{k_i} \frac{L^2}{H^2} \left[ \frac{1}{\mu_{m_D}} \frac{\partial P_D}{\partial z_D} \right]
\] .......................................................... .......................................................... (A.39)
Therefore, the last group is defined as

\[ R_L = \sqrt{\frac{k_L L^2}{k_x H^2}} \] .................................................................(A.40)

**Final Dimensionless Scaling Groups**

Final form of the dimensionless scaling groups can be represented as:

\[ G_1 = \frac{\mu_{ij} L}{\phi K_L} \] .................................................................(A.41)

\[ G_2 = \frac{u_{ij} H^2}{\phi K_T L} \] .................................................................(A.42)

\[ G_3 = \frac{L^2 k_x}{H^2 k_x} \] .................................................................(A.43)

\[ G_4 = \frac{\mu_o}{\mu_s} \] .................................................................(A.44)

These scaling groups can be minimized using the matrix rank method (Shook, 1992). After minimization, scaling groups for first contact miscible displacement can be defined as:

\[ G_1 = N_{Pe} = \frac{u_{ij} L}{\phi K_L} \] .................................................................(A.45)

\[ G_2 = N_D = \frac{L}{H \sqrt{K_T K_L}} \] .................................................................(A.46)
\[ G_3 = R_L = \left[ \frac{L}{H} \sqrt{\frac{k_x}{k_z}} \right] \] (A.47)

\[ G_4 = M = \left[ \frac{\mu_0}{\mu_s} \right] \] (A.48)
APPENDIX B

MATLAB INPUT FILES

The following MATLAB files are sample input files used to generate permeability fields, input files for GEM and to generate sensitivity plots from concentration data imported from GEM.

MATLAB file below generates permeability fields from the output files generated using FFTSim

```matlab
clc
clear all
x=256;y=64;
mean=5.3;
sdev=0.92;
for i=1:10
    run2= num2str(i);
data = ['out' run2 '.txt '];
[a1 ] = textread(data,'%f ','headerlines',3);
random(:,i) = a1;
end
for h=1:10
    eval(['sorm_conc' num2str(h),'(x,k)=SORM', num2str(h),'(i,j);']);
    perm = exp(mean+sdev*random(:,h));
end
```
for h=1:10
    h
    z=1;
for i=1:y
    for j=1:x
        eval(['permx' num2str(h),'(i,j)=perm', num2str(h),'(z);'])
        z=z+1;
    end
end
end
end
for h=1:10
    for i=1:256
        for j=1:64
            eval(['permy' num2str(h),'=permx' num2str(h);])
            eval(['permy' num2str(h),'=0.1*permx' num2str(h);])
        end
    end
end

File. 1: MATLAB input file for generating permeability fields from the output of FFTSim.

Next MATLAB file is the input file for generating input file for GEM for FCM displacement. A batch file generated from this code can be run using GEM for different permeability realizations.

clear all
clc
for i=1:10
    RF(i,1)=i;
end

nam2 = 'fcm_componet.bat';
fidbatch = fopen(nam2,'w');
for i=1:10
    nam = ['fcm_componet' num2str(i) '.dat'];
    fid = fopen(nam,'w');
    fprintf(fid,'RESULTS SIMULATOR GEM 200900
    
    **$ ****************************************
    
    **FILENAMES   *INDEX-IN      file name
    
    *INUNIT *Field**units are used for both input and output.
    
    **TIME   **write well result every time step
    
    WPRN GRID 0
    
    **ALL
    
    OUTPRN WELL BRIEF
    ');
    fclose(fid);
end
fprintf(fid,'**ALL\n');
fprintf(fid,'OUTPRN RES ALL\n');
fprintf(fid,'WSRF GRID TIME\n');
fprintf(fid,'WSRF WELL TIME\n');
fprintf(fid,'**RESTART      **write restart file WITH TIME STEP\n');
fprintf(fid,'%s\n','    OUTSRF GRID DENO PRES SO Z ''SOL ''');
fprintf(fid,'%s\n','    OUTSRF GRID DENG PRES SO VELOCRC VISO X "OIL ");
fprintf(fid,'OUTSRF RES ALL\n');
fprintf(fid,'SUMMARY \n');
fprintf(fid,'**MAXIMUM NUMBER OF NON-ZERO VALUES ENTERED FOR L & U IN AIMSOL \n');
fprintf(fid,'DIM MDICLU 200000 \n');
fprintf(fid,'**FRACTION OF GRID BLOCK IN FULLY IMPICIT MANER \n');
fprintf(fid,'DIM MDIMPL 100 \n');
fprintf(fid,'**$ Distance units: ft \n');
fprintf(fid,'RESULTS XOFFSET 0.0000 \n');
fprintf(fid,'RESULTS YOFFSET 0.0000 \n');
fprintf(fid,'RESULTS ROTATION 0.0000  **$ (DEGREES) \n');
fprintf(fid,'**$ Definition of fundamental cartesian grid\n');
fprintf(fid,'**$  **************************************************
**$        ****************************************************
************************

% Reservoir Dimensions

fprintf(fid,'GRID VARI 256 64 1\n');
fprintf(fid,'KDIR DOWN\n');
fprintf(fid,'DI IVAR \n');
fprintf(fid, 256*2\n');%% No. of grid blocks and delta X
fprintf(fid,'DJ JV\n');
fprintf(fid,' 64*1\n');%% No. of grid blocks and delta Y
fprintf(fid,'DK ALL\n');
fprintf(fid,' 16384*10\n');%% Thickness
fprintf(fid,'DTOP\n');
fprintf(fid,' 16384*5000\n');%% DTOP

% Property: NULL Blocks Max: 1 Min: 1
fprintf(fid,'**$ Property: NULL Blocks Max: 1 Min: 1\n');
fprintf(fid,'**$  0 = null block, 1 = active block\n');
fprintf(fid,'NULL CON 1\n');
fprintf(fid,'**$ Property: Porosity Max: 0.25 Min: 0.25\n');
fprintf(fid,'POR CON 0.25\n');
fprintf(fid,'*INCLUDE permx%d.inc\n',RF(i,1));
fprintf(fid, '*INCLUDE permy%d.inc\n',RF(i,1));
fprintf(fid,'**$ Property: Permeability K (md)  Max: 0.005  Min: 0.005\n');
fprintf(fid,'PERMK CON 1\n');
fprintf(fid,'**$ Property: Pinchout Array  Max: 1  Min: 1 \n');
fprintf(fid,'**$  0 = pinched block, 1 = active block \n');
fprintf(fid,'PINCHOUTARRAY CON 1\n');
fprintf(fid,'CPOR MATRIX 1.0E-5\n');
fprintf(fid,'** USER COMPONENT IS NOT A HYDROCARBON COMP.\n');
fprintf(fid,'HCFLAG\n');
fprintf(fid,'0 0\n');
fprintf(fid,'** FLUID PROPERTIES \n');
fprintf(fid,'**$ Model and number of components\n');
fprintf(fid,'MODEL PR\n');
fprintf(fid,'NC 2 2\n');
fprintf(fid,'%s\n','    COMPNAME  ''OIL''   ''SOL''');
fprintf(fid,'** USER COMPONENT IS NOT A HYDROCARBON COMP.\n');
fprintf(fid,'HCFLAG\n');
fprintf(fid,'0 0 \n');
fprintf(fid,'PVC3 0.05
');
fprintf(fid,'MW
');
fprintf(fid,'18.01 18.01 \n');
fprintf(fid,'**PCHOR           41.00 41.00\n');
fprintf(fid,'AC\n');
fprintf(fid,'0.04 0.04 \n');
fprintf(fid,'**atm\n');
fprintf(fid,'PCRIT\n');
fprintf(fid,'220.0 220.0 \n');
fprintf(fid,'**m3/kmole\n');
fprintf(fid,'VCRIT\n');
fprintf(fid,'0.055 0.055 \n');
fprintf(fid,'**k\n');
fprintf(fid,'TCRIT\n');
fprintf(fid,'650.0 650.0 \n');
fprintf(fid,'SG\n');
fprintf(fid,'1.0 1.0 \n');
fprintf(fid,'**deeg F\n');
fprintf(fid,'TB\n');
fprintf(fid,'212.0 212.0 \n');
fprintf(fid,'OMEGB\n');
fprintf(fid,'0.077 0.077 \n');
fprintf(fid,'VISCOSITY\n');
fprintf(fid,'25.0 1.0 \n');
fprintf(fid,'BIN\n');
fprintf(fid,'0.001 \n');
fprintf(fid,'**deg F\n');
fprintf(fid,'TRES 60.0 \n');
fprintf(fid,'** ALL SINGLE PHASE GRID BLOCK IDENTIFY BY OIL \n');
fprintf(fid,'PHASEID OIL\n');
fprintf(fid,'**Psi \n');
fprintf(fid,'PSAT -1\n');
fprintf(fid,'** Lbm/ft3 \n');
fprintf(fid,'DENW 62.4\n');
fprintf(fid,'**WATER PROPERTY \n');
fprintf(fid,'** /kpa -1\n');
fprintf(fid,'CW 0.0\n');
fprintf(fid,'** Psi\n');
fprintf(fid,'REFPW 14.7\n');
fprintf(fid,'** cp\n');
fprintf(fid,'VISW 1.00\n');
fprintf(fid,'ROCKFLUID\n');
fprintf(fid,'****************************************************\n');
fprintf(fid,'** Rock-Fluid Property Section\n');
fprintf(fid,'****************************************************\n');
fprintf(fid,'**SIGMA        \n');
fprintf(fid,'**NTENA\n');
fprintf(fid,'RPT 1\n');
fprintf(fid,'**SNORM KRW NKRWO NPCWOD\n');
fprintf(fid,'SWT\n');
fprintf(fid,'**$        Sw       krw      krow      Pcow\n');
fprintf(fid,' 0.00      0.00      1.00       0.0\n');
fprintf(fid,' 1.00      1.00     0.000       0.0\n');
fprintf(fid,'**SL KRG   KROG PCGOD\n');
fprintf(fid,'SGT\n');
fprintf(fid,'**$        Sg       krg      krog      Pcog\n');
fprintf(fid,'0.00      0.00      1.00       0.0\n');
fprintf(fid,'1.00      1.00      0.00       0.0\n');
fprintf(fid,'**$ Property: Rel Perm Set Num  Max: 1  Min: 1\n');
fprintf(fid,'RTYPE CON            1\n');
fprintf(fid,'**ft\n');
fprintf(fid,'**$ Property: Longitudinal Dispersivity (ft)   Max: 8  Min: 8\n');
fprintf(fid,'DISPERARRAY-LNG CON  3\n');

fprintf(fid,'**$ Property: Transverse Dispersivity (ft)  Max: 1.27  Min: 1.27\n');

fprintf(fid,'DISPERARRAY-TRN CON  0.51\n');

fprintf(fid,'**DISPERARRAY-TNG *CON 0.0 \n');

fprintf(fid,'**DISPERARRAY-ISO *CON 0.0\n');

fprintf(fid,'**INITIAL\n');

fprintf(fid,'********************************************************
');

fprintf(fid,'** Initial Conditions Section\n');

fprintf(fid,'********************************************************
');

fprintf(fid,'**USER_INPUT\n');

fprintf(fid,'**off\n');

fprintf(fid,'USER_INPUT\n');

fprintf(fid,'**$ Property: Pressure (psi)  Max: 4500  Min: 4500\n');

fprintf(fid,'PRES CON  4500\n');

fprintf(fid,'**$ Property: Water Saturation  Max: 0  Min: 0\n');

fprintf(fid,'SW CON  0\n');

fprintf(fid,'**$ Property: Global Composition(OIL)  Max: 1  Min: 1\n');

fprintf(fid,'%s\n','    ZGLOBALC "OIL" CON  1');

fprintf(fid,'**$ Property: Global Composition(SOL)  Max: 0  Min: 0\n');
fprintf(fid,'%s','    ZGLOBALC "SOL" CON 0');

fprintf(fid,'******************************************************
');

fprintf(fid,'** Numerical Control Section
');

fprintf(fid,'******************************************************
');

fprintf(fid,'*NUERICAL
');

fprintf(fid,'NORM PRESS 1000
');

fprintf(fid,'NORM SATUR 0.25
');

fprintf(fid,'NORM GMOLAR 0.25
');

fprintf(fid,'**PRECC 0.0005    ** Convergence tolerance for linear solver default=0.0005
');

fprintf(fid,'** CONVERGE ****PRESS, HC, WATER, MAXRES
');

fprintf(fid,'** LOOSER, LOOSE, NORMAL, TIGHT, TIGHTER
');

fprintf(fid,'CONVERGE MAXRES NORMAL
');

fprintf(fid,'**CONVERGE *PRESS 0.25
');

fprintf(fid,'**CONVERGE *WATER 0.01
');

fprintf(fid,'**CONVERGE *HC 0.01
');

fprintf(fid,'***Default value is 10
');

fprintf(fid,'NEWTONCYC 30
');

fprintf(fid,'SDEGREE 3
');

fprintf(fid,'SORDER NATURAL
');

145
fprintf(fid,'**PIVOT *OFF ***Control the diagonal submatrix pivot stabilization, default is OFF
');

fprintf(fid,'**SOLVER **PARASOL **/AIMSOL
');

fprintf(fid,'******************************************************
');

fprintf(fid,'** Well and Recurrent Data Section
');

fprintf(fid,'******************************************************
');

fprintf(fid,'*RUN
');

fprintf(fid,'*DATE 2000 1 1
');

fprintf(fid,'DTWELL 1e-005
');

fprintf(fid,'DTMAX 1.0E-1
');

fprintf(fid,'DTMIN 1E-5
');

fprintf(fid,'**$
');

fprintf(fid,'%s', '    WELL  ''Well-1''
');

fprintf(fid,'%s', '    INJECTOR ''Well-1''
');

fprintf(fid,'INCOMP SOLVENT 0. 1.
');

fprintf(fid,'OPERATE MAX STG 2048. CONT
');

fprintf(fid,'***OPERATE MAX BHP 5200. CONT
');

fprintf(fid,'** if jf kf wi
');

fprintf(fid,'**$ rad geofac wfrac skin
');

fprintf(fid,'GEOMETRY J 0.1 0.35 1. 0.
');
fprintf(fid,'%s\n', 'PERF GEO "Well-1"');
fprintf(fid,'**$ UBA ff Status Connection \n');
fprintf(fid,'%s\n', '1 1 1. OPEN FLOW-FROM "SURFACE" "REFLAYER"');
fprintf(fid,'** if jf kf wi\n');
fprintf(fid,' 1:1 1:64 1:1 1.0 *OPEN\n');
fprintf(fid,'**$\n');
fprintf(fid,'%s\n', 'WELL "Well-2"');
fprintf(fid,'%s\n', 'PRODUCER "Well-2"');
fprintf(fid,'OPERATE MIN BHP 4500. CONT\n');
fprintf(fid,'*****OPERATE MAX STL 2000. CONT\n');
fprintf(fid,'** if jf kf wi\n');
fprintf(fid,'**$ rad geofac wfrac skin\n');
fprintf(fid,'GEOMETRY J 0.15 0.35 1. 0.\n');
fprintf(fid,'%s\n', 'PERF GEO "Well-2"');
fprintf(fid,'**$ UBA ff Status Connection \n');
fprintf(fid,'%s\n', '1 1 1. OPEN FLOW-FROM "SURFACE" "REFLAYER"');
fprintf(fid,'** if jf kf wi\n');
fprintf(fid,'256:256 1:64 1:1 1.0 *OPEN\n');
fprintf(fid,'** 0=explicit 1=implicit 2=explicity always 3=impicit always\n');
fprintf(fid,' AIMSET  *CON 3\n');

fprintf(fid,'DATE 2000 1  6.00000\n');

fprintf(fid,'DATE 2000 1  11.00000\n');

fprintf(fid,'DATE 2000 1  16.00000\n');

fprintf(fid,'DATE 2000 1  21.00000\n');

fprintf(fid,'DATE 2000 1  26.00000\n');

fprintf(fid,'DATE 2000 1  31.00000\n');

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fprintf(fid,'DATE 2000 2  10.00000\n');

fprintf(fid,'DATE 2000 2  15.00000\n');

fprintf(fid,'DATE 2000 2  20.00000\n');

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fprintf(fid,'DATE 2000 3  31.00000\n');

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fprintf(fid,'DATE 2000 4 20.00000\n');
fprintf(fid,'DATE 2000 4 25.00000\n');
fprintf(fid,'DATE 2000 4 30.00000\n');
fprintf(fid,'DATE 2000 5  5.00000\n');
fprintf(fid,'DATE 2000 5 10.00000\n');
fprintf(fid,'DATE 2000 5 15.00000\n');
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fprintf(fid,'DATE 2000 5 25.00000\n');
fprintf(fid,'DATE 2000 5 30.00000\n');
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fprintf(fid,'DATE 2000 6 14.00000\n');
fprintf(fid,'DATE 2000 6 19.00000\n');
fprintf(fid,'DATE 2000 6 24.00000\n');
fprintf(fid,'DATE 2000 6 29.00000\n');
fprintf(fid,'DATE 2000 7  4.00000\n');
fprintf(fid,'DATE 2000 7  9.00000\n');
fprintf(fid,'DATE 2000 7 14.00000\n');
fprintf(fid,'DATE 2000 7 19.00000\n');
fprintf(fid,'RESULTS PINCHOUT-VAL 0.0002\n');
fclose(fid);

fprintf(fidbatch,'"%s","C:\Program Files (x86)\CMG\GEM\2009.13\Win_x64\EXE\gm200913.exe" -f '); %office link

fprintf(fidbatch,'%s\n',nam);

end

close(fidbatch);

File. 2: MATLAB file to generate input files for GEM for FCM displacement.
APPENDIX C

SCALING GROUPS AND $S_{ORM}$

In this appendix the values of scaling groups which are used in the sensitivity analysis are presented. $S_{orm}$ values for corresponding simulations are also been presented for 2.0, 3.0 and 4.0 pore volumes of injection.

<table>
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<tr>
<th>Case</th>
<th>$\lambda x_D$</th>
<th>$\lambda y_D$</th>
<th>$V_{DP}$</th>
<th>M</th>
<th>$R_L$</th>
<th>$N_D$</th>
<th>$N_{Pe}$</th>
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*Table C-1: Scaling groups for sensitivity analysis of $S_{orm}$*
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**Table C-2:** $S_{orm}$ values at different pore volume injections for $\delta\Delta x=0.250$ for the cases mentioned in table C-1
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*Table C-3: $S_{orm}$ values at different pore volume injections for $\delta A x=0.125$ for the cases mentioned in table C-1*
APPENDIX D

RESPONSE FUNCTION FOR $S_{orm}$ PREDICTION

In this section we present a method for predicting $S_{orm}$ based on scaling groups found by inspectional analysis. A Box-Behnken experimental design (Box & Behnken, 1960) with seven scaling groups found by inspectional analysis can be used to run the simulations. A Box-Behnken experimental design for an independent quadratic design for an independent quadratic design require three levels (high, low and intermediate) for each scaling factor to capture quadratic effect.

There are seven scaling groups to fit the response function and therefore 35 coefficients in the response function must be determined. A second order polynomial can be used as the response surface function as given below,

$$ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots \ldots \beta_n x_n + \sum_{i<j}^{n} \beta_{ij} x_i x_j + \sum_{i=1}^{n} \beta_{ii} x_i^2 \ldots \ldots \ldots \ldots \ldots (D.1) $$

where $n$ is the number of scaling groups (seven in our case), $y$ is the objective function ($S_{orm}$), $\beta_0$ is the intercept constant, $\beta_i$ are linear coefficients, $\beta_{ij}$ and $\beta_{ii}$ are quadratic coefficients and $x_i$ are the normalized scaling group values.
The scaling groups are normalized between -1 and +1 prior to determining the coefficients of response function. The intermediate value can be normalized linearly into a value between upper and lower limit as in equation D.2.

\[
\overline{X} = 2 \left[ \frac{X - L_x}{H_x - L_x} \right] - 1 \]

.......................................................... .......................................................... (D.2)

where \( \overline{X} \) is the normalized value of the intermediate level, \( X \) is the dimensionless group’s intermediate value, \( H_x \) is the upper level and \( L_x \) is the lower level. The importance of each scaling group can be identified by using normalization.

Using above methodology we can generate a response function, which will be based on scaling group and will be able to predict \( S_{orm} \) for different upscaling factors and at different pore volumes of solvent injected. By performing t-tests importance of each scaling group can be determined.
REFERENCES


Spence, A. a. (Sep, 1980). The Effect of Microscopic Core Heterogeneity on Miscible Flood Residual Oil Saturation. *SPE 55229*.


