The Pennsylvania State University
The Graduate School
Industrial and Manufacturing Engineering

KEEPING RETAILER’S CYCLE SERVICE LEVEL UNDER VMI STRATEGY
A REGRESSION MODEL

A Thesis in
Industrial Engineering
by
Yu Zhang

© 2012 Yu Zhang

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

December 2012
The thesis of Yu Zhang was reviewed and approved* by the following:

Douglas Thomas  
Associate Professor of Supply Chain and Information System  
Thesis Co-Advisor

Tao Yao  
Associate Professor of Industrial Engineering

Paul Griffin  
Professor of Industrial Engineering  
Thesis Co-Advisor  
Head of the Department of Industrial Engineering

*Signatures are on file in the Graduate School
ABSTRACT

As a product or service moving from supplier to the customer, it needs to go through many intermediate links, for example distributor and wholesaler. Each partner in the supply chain has their own strategies, such as different cycle service level or lead time, which led to the distortion of demand information from downstream to upstream of the supply chain. This is the well-known bullwhip effect. An initiative called vendor-managed inventory (VMI) strategy appears to improve the efficiency of the supply chain. Its core idea is to share information between each levels of the supply chain, trying to reduce the variability of demand information. J&J is one of companies that earliest implement VMI and J&J is in charge of managing the inventories of many of its customers. One of its customers, Dollar General, is asking J&J to keep the cycle service level while at the same time reduce the stock level. In this article, we will analyze the inner connections between days of supply and cycle service level. We will use statistic method to analyze two years demand data of DG. Finally, we use a regression model to represent this relationship.
# TABLE OF CONTENTS

List of Figures .................................................................................................................. vi

List of Tables .................................................................................................................... viii

Chapter 1 Introduction ....................................................................................................... 1

Bullwhip Effect .................................................................................................................. 2
Vendor-Managed Inventory ............................................................................................... 3
Electronic Data Interchange ............................................................................................. 5

Chapter 2 Problem Formation ......................................................................................... 6

Replenishment Policy ........................................................................................................ 7

Chapter 3 Relationship between CSL and DOS ............................................................ 10

Days of Supply (DOS) ..................................................................................................... 10
    Simple EOQ model ....................................................................................................... 10
    When demand curve is convex ................................................................................ 11
    When demand is stair stepping .............................................................................. 12
    Concave stair demand ............................................................................................. 13
    Convex stair demand ............................................................................................... 15
    Using weeks as time unit .......................................................................................... 16
Cycle Service Level (CSL) ............................................................................................... 17
    Periodic review policy ............................................................................................. 18
    Real Application ........................................................................................................ 19

Chapter 4 Data Analysis .................................................................................................. 22

Data Statistics .................................................................................................................. 22
Regression Analysis ........................................................................................................ 23
    Choose potential predictors .................................................................................... 24
Choosing candidate models ............................................................................................ 26
    Out of stock rate = $F(\text{Demand CV})$ ................................................................ 27
    Out of stock rate = $F(\text{Demand Mean})$ ................................................................. 28
    Out of stock rate = $F(\text{Demand SD})$ ................................................................... 30
    Out of stock rate = $F(\text{On hand Mean})$ ............................................................... 31
    Out of stock rate = $F(\text{Demand CV, On hand mean})$ ........................................ 33
    Out of stock rate = $F(\text{Demand mean, On hand mean})$ .................................... 34
    Out of stock rate = $F(\text{Demand SD, On hand Mean})$ .......................................... 35
    Out of stock rate = $F(\text{Demand CV, On Hand Mean/Demand Mean})$ ............ 36
    Out of stock rate = $F(\text{Demand CV, On hand Mean/Demand SD})$ ..................... 38
    Out of stock rate = $F(\text{Demand mean, Demand SD, On hand mean})$ ............... 39
Out of stock rate = F (Demand Mean, On hand mean, On hand CV) ........ 41

Chapter 5 A Regression Model ........................................................................................................ 43
  Regression Tests ...................................................................................................................... 43
  Transformation .......................................................................................................................... 47

Chapter 6 Conclusion .................................................................................................................. 49
  Reference ...................................................................................................................................... 50
LIST OF FIGURES

Figure 1-1. The distortion of order information (Lee et al., 2004) ........................................3
Figure 2-1. Simple EOQ model ..............................................................................................7
Figure 3-1. Simple EOQ model ..............................................................................................11
Figure 3-2. Demand curve is convex ....................................................................................12
Figure 3-3. Stair stepping demand .......................................................................................13
Figure 3-4. Concave stair demand .......................................................................................15
Figure 3-5. Convex stair demand .........................................................................................16
Figure 3-6. Using weeks as time units ..................................................................................17
Figure 3-7. Simple model with safety stock .........................................................................18
Figure 3-8. When demand is not constant ..........................................................................20
Figure 4-1. Scatter plot of variables ....................................................................................25
Figure 4-2. Predictor Demand CV .......................................................................................28
Figure 4-3. Predictor Demand Mean ...................................................................................29
Figure 4-4. Predictor Demand SD .......................................................................................31
Figure 4-5. Predictor On Hand Mean ..................................................................................32
Figure 4-6. Predictor Demand CV, On Hand Mean .............................................................34
Figure 4-7. Predictor Demand Mean, On Hand Mean ..........................................................35
Figure 4-8. Predictor Demand SD, On Hand Mean ..............................................................36
Figure 4-9. Predictor Demand CV, On Hand Mean/Demand Mean .....................................37
Figure 4-10. Predictor Demand CV, On Hand Mean/Demand SD .......................................39
Figure 4-11. Predictor Demand Mean, Demand SD, On hand Mean ....................................40
Figure 4-12. Predictor Demand Mean, On hand Mean, On hand CV ....................................42
Figure 5-1. Residual Plots ....................................................................................................45
Figure 5-2. Normality Plot.................................................................46

Figure 5-3. Residual Plots after transformation............................................48
LIST OF TABLES

Table 2-1. Data provide by DG every day.................................................................6
Table 2-2. Total demand information transferred..................................................8
Table 4-1. Two years data of DG...........................................................................22
Table 4-2. The statistic data......................................................................................23
Table 4-3. Descriptive Data......................................................................................24
Table 4-4. Variables Correlation.............................................................................25
Table 4-5. VIF test for all variables..........................................................................26
Table 4-6. VIF test without on hand SD.................................................................26
Table 4-7. Predictor Demand CV ...........................................................................27
Table 4-8. Predictor Demand Mean..........................................................................29
Table 4-9. Predictor Demand SD.............................................................................30
Table 4-10. Predictor On Hand Mean......................................................................32
Table 4-11. Predictor Demand CV, On Hand Mean.................................................33
Table 4-12. Predictor Demand Mean, On hand Mean..............................................34
Table 4-13. Predictor Demand SD, On hand Mean..................................................35
Table 4-14. Predictor Demand CV, On hand Mean/Demand Mean.........................37
Table 4-15. Predictor Demand CV, On hand Mean/Demand SD............................38
Table 4-16. Predictor Demand Mean, Demand SD, On hand Mean .......................40
Table 4-17. Predictor Demand Mean, On hand Mean, On hand CV......................41
Table 5-1. ANOVA Result.......................................................................................44
Table 5-2. ANOVA after transformation.................................................................47
Chapter 1

Introduction

Supply chain refers to the system that integrated all the management, information and resources as a product or servicing moving from the supplier to the customer. It involves the planning and management of sourcing, procurement, conversion and logistics. When the raw materials and components finally turned into the products in the customers’ hands, they go through many intermediate partners in the supply chain, such as wholesalers and distributors.

When managers are talking about supply chain management, especially big retail businesses, the most annoying topic must be Bullwhip Effect, which refers to the distortion of demand information transferring from lower suppliers to the upper buyers in the supply chain (Arntzen et al., 1995). Due to the uncertainty of demand, each member in a supply chain has to keep some safety stocks. The uncertainty would be amplified by each layer just like a bullwhip (Lee et al., 2004).

The VMI is initiated under this circumstance where the vendor (supplier) is authorized to manage the inventories of its upper-stream buyers. In this way, the vendors are trying to reduce their safety stock then further reduce their inventory cost.

Let us first review the details of bullwhip effect and vendor-managed inventory. We will further discuss the functions of electronic data interchange in the implementations of the vendor-managed inventory.
Bullwhip Effect

In a “push” supply chain system, like the large grocery stores, since customer demand can never be perfectly predicted, vendors need to do forecast of customer demand to replenish the inventory. Because there are always some variances between the forecast and real demand, vendors need to keep more inventory as a buffer to unexpected coming demand, which is known as safety stock. Each members of the supply chain is actually the customers of its downstream members in the supply chain. If each members in the supply chain are having their own safety stock and do not communicate with each other, the demand information will inevitably be distorted from the end-consumer to the upstream suppliers, which is the formation of bullwhip effect.

The bullwhip effect has been found in many different fields. P&G found that the consumer demand fluctuation alone cannot count for all the variance of the distortion of the total amount ordered by the distributors (Lee et al., 2004). They found out that the variances of demand will inevitably lead to excessive inventory as demand variance is amplified from the buyer to the suppliers. The distortion of demand information indicates that the downstream suppliers who observe the order from its nearest customer will be misled by the order information.
We can see from the figure above that the retailer’s orders do not coincide with the actual retail sales (Lee et al., 1997). This can lead to serious cost implications. For example, manufacturers are spending more money on the purchases of raw materials. Distributors are expending excess expense of transportation due to the inefficient planning and shipping. Trade estimates suggest that these activities can result in excess costs in the range between 12.5% to 25% (Kurt Salmon Associates 1993).

**Vendor-Managed Inventory**

Under the circumstances that demand order distortion can increase the unnecessary expense of all the members in the supply chain, information sharing becomes an inevitably solution. Lots of methods emerged such as Kanban, Just In Time
(JIT) or Vendor Managed Inventory (VMI). The core idea of all these methods is to reduce the information distortion through the supply chain. However, Kanban and JIT methods are mostly used in the “pull” supply chain system, in which the actual demand will trigger the replenishment order (Yano et al., 1989). VMI is still using in the “push” supply chain system, meaning that the vendors still need to do the forecast. But it focuses on the communication between different members in the supply chain to reduce the bullwhip effect.

The popularity of VMI is largely due to the successful implementation of Wal-Mart, the biggest retail business in the world. In the VMI system, the vendor is responsible for managing its customers’ inventory. The information of inventory level is exchanged between the customer and vendor normally through some electronic data interchange system (Aviv and Federgruen, 1998, Parker, 1996).

There are generally two kinds of VMI implantations. The first one is that the entire inventories in the retailers’ shelves are belonging to the retailers. The vendors are only in charge of the replenishment policy. Most of the time the retailers will set the threshold Cycle Service Level or Days of Supply (Yu et al., 2009a). The second one is that the retailers are actually renting a sell space for the vendors. All the products on the retailers’ shelves are belonging to the vendors (Birendra and Srinivasan, 2004). J&J’s VMI implementation pertains to the first category.
Electronic Data Interchange

Electronic data interchange (EDI) refers to the transmission of information between different members through electronic medium, mostly the Internet. Many instances are actually belonging to the EDI scope, for example, the web storage. Some parts of the ERP software, like SAP, are also based on the ideas of EDI.

Even though the start of VMI system is not mandatorily relied on the electronic data interchange, like we mentioned before that the core idea of VMI is the communication between two members, EDI do play the most important part of the prevailing of VMI (Funda et al., 2002). The idea of VMI is not fresh. It was just limited by the age. Before the age of internet, vendors and retailers are using telephone or letters to communicate. It is efficient enough to deal with small amount of data. But under the current circumstance, considering the variability and customization of goods, it is hard to image using telephone or letter to exchange information. Without sophisticated computer platforms and communication technology, it is hard to efficiently and successfully start the VMI when the number of SKUs is huge, like the retail business.
J&J is one of the largest medical, pharmaceutical and consumer packaged goods manufacturers in America. Its products include some of the famous bands in market such as Clear&Clean and Johnson’s Baby.

It is also one of companies earliest adopted VMI in businesses. J&J have VMI contract with one of its customers, Dollar General (DG), a grocery retailer across the America. So it has the obligation to manage the inventories of its customer. Specifically, J&J is in charge of 9 Distribution Centers of DG.

Dollar General sends the required information (in spreadsheet) to J&J on a daily basis through their EDI system. This spreadsheet contains the on hand quantity, on order quantity, demand, sales etc. of every day. J&J will analyze the data and place the replenishment order to manage the inventory.

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>ON HAND</th>
<th>ON ORDER</th>
<th>SALES</th>
<th>1 Week Sales</th>
<th>DEMAND</th>
<th>1 Week Demand</th>
<th>OUT OF STOCK</th>
<th>AVG ON HAND</th>
<th>DAY</th>
<th>WOS</th>
<th>DOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOLL-96000</td>
<td>1,304.00</td>
<td>580</td>
<td>157</td>
<td>549.5</td>
<td>32</td>
<td>182</td>
<td>0</td>
<td>2,108.00</td>
<td>2</td>
<td>11.2307932</td>
<td>78.61538462</td>
</tr>
<tr>
<td>DOLL-96300</td>
<td>1,688.60</td>
<td>684</td>
<td>376</td>
<td>1316</td>
<td>116</td>
<td>406</td>
<td>0</td>
<td>2,648.00</td>
<td>2</td>
<td>3.842364332</td>
<td>40.8955172</td>
</tr>
<tr>
<td>DOLL-96500</td>
<td>1,388.60</td>
<td>468</td>
<td>221</td>
<td>773.5</td>
<td>52</td>
<td>182</td>
<td>0</td>
<td>2,104.00</td>
<td>2</td>
<td>10.1978022</td>
<td>71.39461538</td>
</tr>
<tr>
<td>DOLL-96600</td>
<td>1,552.60</td>
<td>668</td>
<td>282</td>
<td>987.5</td>
<td>80</td>
<td>280</td>
<td>80</td>
<td>379.00</td>
<td>2</td>
<td>3.785714286</td>
<td>40.5</td>
</tr>
<tr>
<td>DOLL-96700</td>
<td>1,888.00</td>
<td>988</td>
<td>215</td>
<td>822.5</td>
<td>52</td>
<td>182</td>
<td>0</td>
<td>2,484.00</td>
<td>2</td>
<td>10.93405939</td>
<td>97.5384612</td>
</tr>
<tr>
<td>DOLL-96800</td>
<td>1,020.60</td>
<td>396</td>
<td>213</td>
<td>745.5</td>
<td>60</td>
<td>210</td>
<td>0</td>
<td>1,390.00</td>
<td>2</td>
<td>6.742857143</td>
<td>47.2</td>
</tr>
<tr>
<td>DOLL-96900</td>
<td>1,712.60</td>
<td>127</td>
<td>207</td>
<td>724.5</td>
<td>64</td>
<td>224</td>
<td>0</td>
<td>2,632.00</td>
<td>2</td>
<td>7.662222222</td>
<td>53.5</td>
</tr>
<tr>
<td>DOLL-96910</td>
<td>20</td>
<td>1,836.00</td>
<td>241</td>
<td>843.3</td>
<td>78</td>
<td>256</td>
<td>0</td>
<td>106</td>
<td>2</td>
<td>6.977443369</td>
<td>48.8610526</td>
</tr>
<tr>
<td>DOLL-96920</td>
<td>679</td>
<td>396</td>
<td>169</td>
<td>591.5</td>
<td>48</td>
<td>168</td>
<td>0</td>
<td>1,062.00</td>
<td>2</td>
<td>6.380952381</td>
<td>44.06666667</td>
</tr>
</tbody>
</table>

Table 2-1. Data provide by DG every day.
Keeping a higher inventory level can help you achieve a higher cycle service level. High inventory level has the same meaning of high days of supply, which would lead to high inventory cost. DG believes that they are keeping a high inventory level. They ask J&J to reduce the days of supply but at the same time keep the cycle service level above their targets.

**Replenishment Policy**

One day of the week has been fixed across all DCs when replenishment quantities are determined. In this case it is the Tuesday of every week. So every Tuesday, the system determines the quantity to be sent to the DC and it is delivered on Wednesday of the following week. J&J uses a software solution (JDA) to determine the quantity to be delivered. The JDA software looks at the inventory on hand, inventory on order and the demand provided by Dollar General to determine the order quantity.

![Figure 2-1. Simple EOQ model](image)

Dollar General sends the required information to J&J on a daily basis, which including the following terms:
Demand – This is the demand that the Dollar General DC has received from its retail stores. If the information is transmitted on Friday, the DC has already observed the demand of the stores from Saturday to Thursday which will say Day 6. The demand listed in the spreadsheet is the sum of the store demands. If the information is transmitted on Wednesday, it will include the demand from Saturday to Tuesday which will say Day 4. This assumes that the retail stores place orders with the DC daily and the DC to retail store lead-time is one day.

<table>
<thead>
<tr>
<th>Days</th>
<th>Sat</th>
<th>Sun</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>7</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

Table 2-2. Total demand information transferred.

On hand – This is the quantity on hand on the date when the spreadsheet is generated. That is, the spreadsheet sent on the first day indicates the quantity on hand on the first day and the spreadsheet sent on the second day indicates the quantity on hand on the second day.

On order – This is the quantity that is currently on order and will be received by the coming Wednesday. The quantity on order mentioned in Monday’s transmission (which contains demand information for Sunday – Tuesday) indicates the quantity that is expected to arrive two days later.

Sales – This is the actual sales of the retail stores that DC supplies.
Out of stock – The out of stock value indicates the total week to date quantity that the DC could not fulfill for orders placed by the stores. For example, suppose that there are 2 stores (A and B) that place orders of 100 units each every day from Sunday to Wednesday. If the DC is not able to meet stores A’s demand on Monday and sends it only 50 units and similarly is able to send only 70 units to store B on Tuesday, then the out of stock number transmitted on Wednesday will be 80 (50 + 30).

Average on hand – This is the average quantity on hand for the week.
Chapter 3

Relationship between CSL and DOS

We need to explore some of the concepts that will be used later, including the days of supply and cycle service level. Then we will see what is the relationship between these two terms.

Days of Supply (DOS)

The definition of DOS is quite simple: the DOS equals the inventory divided by demand. If we use a formula to represent it:

\[
\text{DOS} = \frac{\text{Inventory}}{\text{Demand}}
\]

It is also easy to explain it in a plain language, DOS stands for how many days you can supply your customers. However, in practice, different time units we chose may lead to different DOS results, which will be showed in the following. Similarly, weeks of supply (WOS) refers to the weeks that you can supply your customers.

Simple EOQ model

We will start with a simple EOQ model. In this model, the demand is constant through a period. We can easily know that the DOS is 3.5, which is equally 1/2 weeks of supply.
When demand curve is convex

When the demand is not constant through the week, but like the convex curve below, what are the DOS and WOS? We need to use calculus to do the math. We can make sure that the average inventory is higher than the simple EOQ model. But we cannot give simple conclusions about what the DOS would change comparing to the EOQ model. We will use discrete methods to illustrate the relationship in the following examples.
When demand is stair stepping

The above two examples are just theoretical models. In reality, the inventory curve will always have steps like the following. Now how can we calculate the days of supply?

\[
\begin{align*}
\text{DOS}_1 &= 70/10 = 7 \\
\text{DOS}_2 &= 60/10 = 6 \\
\text{DOS}_3 &= 50/10 = 5 \\
\text{DOS}_4 &= 40/10 = 4 \\
\text{DOS}_5 &= 30/10 = 3 \\
\text{DOS}_6 &= 20/10 = 2 \\
\text{DOS}_7 &= 10/10 = 1 
\end{align*}
\]
So if we use days as our basic time units, we can say that the average days of supply is 4 and the average weeks of supply is $4/7=0.57$.

If we use weeks as our basic units, we can calculate like the following:

Demand of the week: $10*7=70$.

Average inventory level: $(70+60+50+40+30+20+10)/7=40$.

So the average weeks of supply is $40/70=4/7$ and the days of supply is $4/7*7=4$.

**Figure 3-3. Stair stepping demand**

**Concave stair demand**

Now let us see what if the demand is not constant any more. Let us see the following curve.

If we use the days as basic time units, we can do the following calculation:

$DOS_1=70/35=2$

$DOS_2=35/10=3.5$
\[ \text{DOS}_3 = \frac{25}{5} = 5 \]
\[ \text{DOS}_4 = \frac{20}{5} = 4 \]
\[ \text{DOS}_5 = \frac{15}{5} = 3 \]
\[ \text{DOS}_6 = \frac{10}{5} = 2 \]
\[ \text{DOS}_7 = \frac{5}{5} = 1 \]

So the average days of supply is \( \frac{2 + 3.5 + 5 + 4 + 3 + 2 + 1}{7} = 2.93 \)

Average week of supply is \( \frac{2.93}{7} = 0.42 \)

Then let us use weeks as basic time units.

Average inventory level = \( \frac{70 + 20 + 20 + 20 + 10 + 10 + 0}{7} = \frac{150}{7} = 21.4 \)

Weekly demand = 70

The average weeks of supply is \( \frac{150}{490} = 0.31 \)

The average days of supply is \( \frac{150}{490} \times 7 = 2.14 \)

When the demand is not constant throughout the week, within the same example, using different time unit to calculate the DOS and WOS may give us different results.

So we should always use the length of the replenishment cycle as our time unit to do the calculation.
In this example, we will only use weeks as the basic time unit.

Average inventory level = \( \frac{70+65+60+55+45+40+5}{7} = 48.6 \)

Weekly demand = 70

Week of supply = \( \frac{48.6}{70} = 0.69 \)

Days of supply = \( 0.69 \times 7 = 4.86 \)

Comparing this result with Example 3 and 4, in these three examples, the weekly demand in one week are the same (70). But the results we got are different. We can easily find out that different demand distribution will cause different average inventory levels, which further lead to different WOS.

**Convex stair demand**

In this example, we will only use weeks as the basic time unit.

Average inventory level = \( \frac{70+65+60+55+45+40+5}{7} = 48.6 \)

Weekly demand = 70

Week of supply = \( \frac{48.6}{70} = 0.69 \)

Days of supply = \( 0.69 \times 7 = 4.86 \)

Comparing this result with Example 3 and 4, in these three examples, the weekly demand in one week are the same (70). But the results we got are different. We can easily find out that different demand distribution will cause different average inventory levels, which further lead to different WOS.
Using weeks as time unit

Recall that J&J’s replenishment cycle is 7 days, which means that even if J&J can exactly predict the further demand, the WOS may differ each week. So I prefer to use another way to stand for the WOS, which is take each week as one unit and then ignore the daily demand. Then the inventory curve would be like the red line.
Next, let us go into the details about the cycle service level (CSL). In reality, the situation is more complicated than the model we talked above. Sometimes the actual demand can surpass your predicted demand. That is why we need to keep the safety stock. The curve below shows the simplest EOQ model with safety stock.
Periodic review policy

When we use the periodic review policy, the analytical model would be like the following:

Definition of variables:

L = lead time
R = reorder interval
D = average demand per unit time
σ = standard deviation of demand per unit time
σ_{L+R} = standard deviation of demand during L+R period
F(z) = Cycle service level
z = safety stock factor
ss = safety stock
S = order up to quantity
AI = average inventory

Formula:

\[ \sigma_{L+R} = \sigma\sqrt{L + R} \]

\[ ss = z^* \sigma_{L+R} \]

\[ S = D*(R+L) + ss = D*(R+L) + z \sigma\sqrt{L + R} \]

\[ AI = D*R/2 + ss = D*R/2 + z \sigma\sqrt{L + R} \]

Based on the definition and formulas above, we noticed that there are indeed some kind of connections between the CSL and Week of supply.

\[ AI = D*R/2 + z \sigma\sqrt{L + R} \text{ and } CSL = F(z) \]

\[ \rightarrow AI = D*R/2 + F^{-1}(CSL) \sigma\sqrt{L + R} \]

\[ \rightarrow WOS = \frac{AI}{D} = \frac{D*R/2 + F^{-1}(CSL) \sigma\sqrt{L + R}}{D} \]

Of course, the premise of the above equation is the assumption that the demands of each day in the cycles have the same distribution. In another word, we can use half of D*R to represent the average inventory, which is not always true in the real life.

**Real Application**

In reality, the demand may not be constant through each replenishment cycle. In the J&J’s problem, we know the lead time is 7 days. Reorder interval is also 7 days. Since we cannot get the daily data, we will use 7 days (one week) as the time units. We will
define these two weeks as Week1 and Week2. And the demands in the following two weeks are normally distributed with mean $D1$ and $D2$ with standard deviation $\sigma_1$ and $\sigma_2$. $D_{1+2}$ and $\sigma_{1+2}$ will be the mean and standard deviation in the two weeks. $D_{1+2} = D1 + D2$. $\sigma_{1+2} = \sqrt{\sigma_1^2 + \sigma_2^2}$.

![inventory](image)

Figure 3-8. When demand is not constant

In the real world the inventory curve is more like the following pattern. The demand will always change due to different factors, promotion, seasonality, or price. So each week we will make a new predict of the next two weeks.

The order up to quantity $S = D1 + D2 + F^{-1}(SCL) \sqrt{\sigma_1^2 + \sigma_2^2}$.

The initial inventory level you have when you making the order is $IL_{initial}$. If we assume that the demand is linear in the week, then we can have the equation:

$$WOS_2 = \frac{A1}{D} = \frac{S - D1 - D2/2}{D2}$$
If we don’t consider how the demand is distributed in each week like in the previous examples:

\[ WOS_2 = AI/D = \frac{S-D_1}{D^2} \]
Chapter 4

Data Analysis

J&J don’t keep record of the daily data which transferred from DG. Every week they will calculate the weekly demand, on hand and out of stock quantities and record them. What we have is the record of 1008 different SKUs’ weekly data for two years which started on January 2010.

Table 4-1. Two years data of DG

<table>
<thead>
<tr>
<th>Demand Qty</th>
<th>Avg On Hand Qty</th>
<th>Weeks Of Supply</th>
<th>Out Of Stock Qty</th>
<th>Avg On Hand Qty</th>
<th>Weeks Of Supply</th>
<th>Out Of Stock Qty</th>
</tr>
</thead>
<tbody>
<tr>
<td>100273800</td>
<td>291</td>
<td>87.86</td>
<td>1.33</td>
<td>315</td>
<td>654.43</td>
<td>2.08</td>
</tr>
<tr>
<td>100223800</td>
<td>218</td>
<td>200.14</td>
<td>0.95</td>
<td>201</td>
<td>422.14</td>
<td>2.1</td>
</tr>
<tr>
<td>100232300</td>
<td>204</td>
<td>310.29</td>
<td>1.2</td>
<td>357</td>
<td>653.80</td>
<td>1.86</td>
</tr>
<tr>
<td>100233300</td>
<td>303</td>
<td>374.14</td>
<td>1.23</td>
<td>303</td>
<td>660</td>
<td>1.98</td>
</tr>
<tr>
<td>100234400</td>
<td>300</td>
<td>592.29</td>
<td>1.97</td>
<td>303</td>
<td>754.29</td>
<td>2.49</td>
</tr>
<tr>
<td>100236700</td>
<td>255</td>
<td>290.14</td>
<td>1.14</td>
<td>216</td>
<td>541.71</td>
<td>2.51</td>
</tr>
<tr>
<td>100245700</td>
<td>528</td>
<td>765.71</td>
<td>1.46</td>
<td>464</td>
<td>1,257.86</td>
<td>2.71</td>
</tr>
<tr>
<td>100271400</td>
<td>389</td>
<td>252</td>
<td>0.99</td>
<td>347</td>
<td>721.57</td>
<td>2.11</td>
</tr>
<tr>
<td>100281500</td>
<td>389</td>
<td>174</td>
<td>0.3</td>
<td>499</td>
<td>1,304.71</td>
<td>2.61</td>
</tr>
</tbody>
</table>

Data Statistics

We need to do further statistics to analysis the data. We can see from table 4-1 that some of the grids are empty. Some of these data are missing because these SKUs are not in the market at that time. Some are missing due to recording issues. We need to filter out these empty blanks before we can do the statistical analysis.
Table 4-2. The statistic data

The organized statistic data are in table 4-2. There are several terms in the table that need to be explained:

Demand Mean – The average of weekly demand
Demand SD – The standard deviation of weekly demand
On Hand Mean – The average of weekly inventory
On hand SD – The standard deviation of weekly inventory
Rate of out of stock – rate of number of weeks that are out of stock

We count 1008 different SKUs to make sure that the examples are large enough, 7 of them are showed in table 4-2. We use serial number to represent each SKU.

**Regression Analysis**

The sample size is large enough to do the regression analysis. The goal of regression analysis is trying to find out that whether there is a relationship between the out of stock rate and the replenishment strategies. The response variable is the out of stock rate. There are many candidate predictor variables, for example, the demand mean and demand standard deviation. All the predictor variables are quantitative.
**Choose potential predictors**

Since all the variables are quantitative, we don’t need to do the coding work. We will try to examine the data using the descriptive method to give us a whole picture of the data. We can see from the table that for most of the variables, their mean value is close to the median value, this shows that the values are evenly separately. Several variables’ median value has a larger deviation from mean value. All the variables’ median value is smaller than the mean value, which indicates that there are fewer samples which have a larger value.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>N*</th>
<th>Mean</th>
<th>SE Mean</th>
<th>StDev</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Mean</td>
<td>1008</td>
<td>0</td>
<td>1075.8</td>
<td>28.3</td>
<td>697.1</td>
<td>134.0</td>
<td>470.7</td>
<td>745.3</td>
<td>1425.4</td>
<td>5413.5</td>
</tr>
<tr>
<td>Demand SD</td>
<td>1008</td>
<td>0</td>
<td>930.9</td>
<td>33.5</td>
<td>1084.0</td>
<td>34.8</td>
<td>222.3</td>
<td>575.5</td>
<td>1107.3</td>
<td>7414.1</td>
</tr>
<tr>
<td>On hand Mean</td>
<td>1008</td>
<td>0</td>
<td>2232.1</td>
<td>58.7</td>
<td>1864.6</td>
<td>155.8</td>
<td>1072.4</td>
<td>1664.2</td>
<td>2017.7</td>
<td>19892.2</td>
</tr>
<tr>
<td>On hand SD</td>
<td>1008</td>
<td>0</td>
<td>1687.0</td>
<td>59.8</td>
<td>1897.8</td>
<td>88.7</td>
<td>595.3</td>
<td>1145.6</td>
<td>2196.9</td>
<td>20717.0</td>
</tr>
<tr>
<td>Rate %</td>
<td>1008</td>
<td>0</td>
<td>19.457</td>
<td>0.564</td>
<td>16.527</td>
<td>0.000</td>
<td>6.931</td>
<td>12.646</td>
<td>25.790</td>
<td>100.000</td>
</tr>
<tr>
<td>Demand CV</td>
<td>1008</td>
<td>0</td>
<td>0.8521</td>
<td>0.0190</td>
<td>0.6233</td>
<td>0.1129</td>
<td>0.4046</td>
<td>0.6887</td>
<td>1.1117</td>
<td>3.0042</td>
</tr>
<tr>
<td>On hand CV</td>
<td>1008</td>
<td>0</td>
<td>0.71243</td>
<td>0.00977</td>
<td>0.31011</td>
<td>0.22383</td>
<td>0.40185</td>
<td>0.53738</td>
<td>0.66504</td>
<td>2.28210</td>
</tr>
<tr>
<td>On hand / Demand</td>
<td>1008</td>
<td>0</td>
<td>2.3999</td>
<td>0.0377</td>
<td>1.1977</td>
<td>0.0410</td>
<td>1.0335</td>
<td>2.1700</td>
<td>2.6572</td>
<td>11.9041</td>
</tr>
<tr>
<td>On hand/Demand SD</td>
<td>1008</td>
<td>0</td>
<td>4.412</td>
<td>0.120</td>
<td>3.805</td>
<td>0.139</td>
<td>1.841</td>
<td>3.115</td>
<td>6.051</td>
<td>38.722</td>
</tr>
<tr>
<td>On hand CV/Demand CV</td>
<td>1008</td>
<td>0</td>
<td>1.2199</td>
<td>0.0261</td>
<td>0.6936</td>
<td>0.1252</td>
<td>0.5928</td>
<td>1.1000</td>
<td>1.5396</td>
<td>7.6270</td>
</tr>
</tbody>
</table>

**Table 4-3. Descriptive Data**

Then we wanted to examine the scatter plot of the variables. We know that multicollinearity may cause unnecessary inflation of MSE in our model. So we wanted to avoid the multicollinearity as much as possible. We have sensed from the plot that there is no strong correlation with each other. Only the on hand mean and on hand SD are showing a strong multicollinearity. So we need to further examine the correlations between them. Following is the correlation test between the two variables. We can see that on hand mean and on hand SD are showing extremely strong correlations with each other (higher than 0.8).
We can rely on the variance inflation factors (VIF) to help detect multicollinearity. VIF quantifies how much the variance is inflated; it is a measure of how much the variance of the estimated regression coefficient is "inflated" by the existence of correlation among the predictor variables in the model. If we run the regression with
these predictors, we can also see from the VIF test that the on hand mean and on hand SD have very high VIF values.

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>7.421</td>
<td>1.694</td>
<td>4.36</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Demand Mean</td>
<td>0.00646</td>
<td>0.001065</td>
<td>10.00</td>
<td>0.000</td>
<td>9.828</td>
</tr>
<tr>
<td>Demand SD</td>
<td>-0.0034765</td>
<td>0.0008159</td>
<td>-4.26</td>
<td>0.000</td>
<td>6.120</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>-0.0031084</td>
<td>0.0005818</td>
<td>-4.65</td>
<td>0.000</td>
<td>17.413</td>
</tr>
<tr>
<td>On hand SD</td>
<td>0.0028535</td>
<td>0.0005468</td>
<td>5.22</td>
<td>0.000</td>
<td>11.601</td>
</tr>
<tr>
<td>Demand CV</td>
<td>-3.821</td>
<td>1.290</td>
<td>-2.99</td>
<td>0.003</td>
<td>8.424</td>
</tr>
<tr>
<td>On Hand CV</td>
<td>21.237</td>
<td>2.312</td>
<td>9.21</td>
<td>0.000</td>
<td>5.540</td>
</tr>
<tr>
<td>On hand / Demand</td>
<td>-0.2370</td>
<td>0.5632</td>
<td>-0.42</td>
<td>0.674</td>
<td>4.901</td>
</tr>
<tr>
<td>On hand/Demand S3</td>
<td>-1.6438</td>
<td>0.2115</td>
<td>-7.77</td>
<td>0.000</td>
<td>6.979</td>
</tr>
<tr>
<td>On hand CV/Demand CV</td>
<td>1.6235</td>
<td>0.9188</td>
<td>1.77</td>
<td>0.078</td>
<td>7.263</td>
</tr>
</tbody>
</table>

Table 4-5. VIF test for all variables

After we take away the on hand SD, the VIF is much smaller right now.

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>3.593</td>
<td>1.546</td>
<td>2.32</td>
<td>0.020</td>
<td></td>
</tr>
<tr>
<td>Demand Mean</td>
<td>0.008749</td>
<td>0.001014</td>
<td>8.63</td>
<td>0.000</td>
<td>8.682</td>
</tr>
<tr>
<td>Demand SD</td>
<td>-0.0032786</td>
<td>0.0008257</td>
<td>-3.97</td>
<td>0.000</td>
<td>8.103</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>-0.0001991</td>
<td>0.0003806</td>
<td>-0.52</td>
<td>0.601</td>
<td>5.298</td>
</tr>
<tr>
<td>Demand CV</td>
<td>-4.174</td>
<td>1.295</td>
<td>-3.22</td>
<td>0.001</td>
<td>6.406</td>
</tr>
<tr>
<td>On Hand CV</td>
<td>28.615</td>
<td>1.963</td>
<td>15.36</td>
<td>0.000</td>
<td>3.503</td>
</tr>
<tr>
<td>On hand / Demand</td>
<td>-0.7255</td>
<td>0.5826</td>
<td>-1.29</td>
<td>0.197</td>
<td>4.766</td>
</tr>
<tr>
<td>On hand/Demand SD</td>
<td>-1.8125</td>
<td>0.2142</td>
<td>-7.53</td>
<td>0.000</td>
<td>6.973</td>
</tr>
<tr>
<td>On hand CV/Demand CV</td>
<td>1.6174</td>
<td>0.9307</td>
<td>1.74</td>
<td>0.083</td>
<td>7.263</td>
</tr>
</tbody>
</table>

Table 4-6. VIF test without on hand SD

**Choosing candidate models**

There are many candidate models. We will try to start with the simple and intuitive one to see whether it can represent the data. Then we will explore the more complicated ones.
Out of stock rate = F (Demand CV)

The most intuitive function is using the demand CV as arguments. The demand CV stands for the mean of Demand divided by the standard deviation of Demand. It represents the fluctuation of the demand information. We expect lower the demand CV; lower the out of stock rate, so the coefficient of demand CV should be positive.

We can see from the Minitab output that the coefficient of demand CV is negative, which is not what we expected. The R2 value is only 0.4%, which means that only 0.4% of the variance in the MSE can be explained by our model.

When we further look into the residuals plots, the normality assumption is seriously violated. And the fitted value plot shows an obvious pattern of increasing trend with the increase of fitted value. This is the sign of non-constant variance.

All of these indicate that this is not an adequate model to represent this problem. We have to abandon this model.

The regression equation is
Rate % = 21.2 - 2.08 Demand CV

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>21.21</td>
<td>1.008</td>
<td>21.03</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand CV</td>
<td>-2.0581</td>
<td>0.9661</td>
<td>-2.13</td>
<td>0.033</td>
</tr>
</tbody>
</table>

S = 16.4944  R-Sq = 0.4%  R-Sq(adj) = 0.4%

Table 4-7. Predictor Demand CV
Out of stock rate = F (Demand Mean)

Next we would like to use the demand Mean as arguments. It represents the level of the demand information. Because the demand mean has no intuitive impact on the out of stock rate. Its coefficient can be either positive or negative.

We can see from the Minitab output that the coefficient of demand mean is positive, which means that higher demand level may cause higher out of stock rate. The R2 value is only 33.7%, which means that only 33.7% of the variance in the MSE can be explained by our model.
When we further look into the residuals plots, the normality assumption is seriously violated. And the fitted value plot shows an obvious pattern of increasing trend with the increase of fitted value. This is the sign of non-constant variance.

All of these indicate that this is not an adequate model to represent this problem. We have to abandon this model.

The regression equation is

\[ \text{Rate } % = 6.55 + 0.0120 \times \text{Demand Mean} \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>6.5601</td>
<td>0.7425</td>
<td>8.83</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand Mean</td>
<td>0.0104882</td>
<td>0.0005302</td>
<td>22.61</td>
<td>0.000</td>
</tr>
</tbody>
</table>

\[ S = 15.0935 \quad \text{R-Sq} = 33.7\% \quad \text{R-Sq(adj)} = 33.6\% \]

Table 4-8. Predictor Demand Mean

Residual Plots for Rate %

![Residual plots](image)

Figure 4-3. Predictor Demand Mean
Out of stock rate = F (Demand SD)

Then let us try to use Demand SD as arguments. The demand SD also represents the spread of the demand level. We expect lower the demand SD; lower the out of stock rate, so the coefficient of demand SD should be positive.

The Minitab output shows that the coefficient of demand CV is positive, which is what we expected. But the R2 value is only 5.5%, which means that only 5.5% of the variance in the MSE can be explained by our model.

The residuals plots show that the normality assumption is seriously violated. And the fitted value plot shows an obvious pattern of increasing trend with the increase of fitted value. This is the sign of non-constant variance.

All of these showed that it is not a sufficient model.

The regression equation is
Rate % = 15.7 + 0.00409 Demand SD

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>15.8540</td>
<td>0.7549</td>
<td>20.75</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand SD</td>
<td>0.0040869</td>
<td>0.0005536</td>
<td>7.65</td>
<td>0.000</td>
</tr>
</tbody>
</table>

S = 18.0185   R-Sq = 5.5%   R-Sq(adj) = 5.4%

Table 4-9. Predictor Demand SD
Out of stock rate = F (On hand Mean)

Next let us see whether the on hand demand information has any relationship with the out of stock rate. The on hand mean represents inventory level. The on hand mean alone has no significant relationship with the increase of out of stock rate, so the coefficient may be positive or negative.

The Minitab output shows that the coefficient of on hand mean is positive, which means that higher inventory level may lead to higher out of stock rate. The R2 value is only 2.6%, which means that only 2.6% of the variance in the MSE can be explained by our model.
The residuals plots show that the normality assumption is seriously violated. And the fitted value plot shows an obvious pattern of increasing trend with the increase of fitted value. This is the sign of non-constant variance.

We need to add more variables into our model.

The regression equation is

\[ \text{Rate \%} = 15.9 + 0.00160 \text{ On Hand Mean} \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>15.8855</td>
<td>0.8990</td>
<td>17.67</td>
<td>0.000</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>0.0016003</td>
<td>0.0003092</td>
<td>5.18</td>
<td>0.000</td>
</tr>
</tbody>
</table>

\[ S = 18.2941 \quad R-Sq = 2.6\% \quad R-Sq(\text{adj}) = 2.5\% \]

Table 4-10. Predictor On Hand Mean

![Residual Plots for Rate %](image)

Figure 4-5. Predictor On Hand Mean
Out of stock rate = F (Demand CV, On hand mean)

We try to combine the demand CV and on hand mean as the arguments. And we expected the coefficients of these two variables to be positive. We can see from the output that the coefficient of demand CV is negative, which is not what we want. The R2 value is only 3.1%, which is very low. And the residuals plots show that the normality and constant variance assumptions are not met too. This model is not good enough for us.

The regression equation is
Rate % = 17.7 - 2.23 Demand CV + 0.00183 On Hand Mean

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>17.73</td>
<td>1.195</td>
<td>14.84</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand CV</td>
<td>-2.23</td>
<td>0.9540</td>
<td>-2.34</td>
<td>0.020</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>0.0018252</td>
<td>0.0003037</td>
<td>5.23</td>
<td>0.000</td>
</tr>
</tbody>
</table>

S = 18.2536  R-Sq = 3.1%  R-Sq(adj) = 2.9%

Table 4-11. Predictor Demand CV, On Hand Mean
Out of stock rate = F (Demand mean, On hand mean)

In this model, we can see from the output that the R2 value is only 39.1%. And the constant variance assumption is not met.

The regression equation is:
Rate % = 9.14 + 0.0156 Demand Mean - 0.00290 On Hand Mean

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>9.1443</td>
<td>0.7621</td>
<td>12.00</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand Mean</td>
<td>0.0155980</td>
<td>0.0006349</td>
<td>24.57</td>
<td>0.000</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>-0.0028976</td>
<td>0.0003054</td>
<td>-9.49</td>
<td>0.000</td>
</tr>
</tbody>
</table>

S = 14.4672  R-Sq = 39.1%  R-Sq(adj) = 39.0%

Table 4-12. Predictor Demand Mean, On hand Mean
Out of stock rate = F (Demand SD, On hand Mean)

In this model, we expect the coefficient of demand SD to be positive. We can see from the output that the R2 value is 5.9%, which is fairly low. And the normality and constant variance assumption are seriously violated.

The regression equation is
\[
\text{Rate \%} = 14.6 + 0.00354 \text{ Demand SD} + 0.000720 \text{ On Hand Mean}
\]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>14.5587</td>
<td>0.9114</td>
<td>15.97</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand SD</td>
<td>0.003535</td>
<td>0.0005920</td>
<td>5.97</td>
<td>0.000</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>0.0007203</td>
<td>0.0003378</td>
<td>2.13</td>
<td>0.033</td>
</tr>
</tbody>
</table>

S = 17.9069     \quad R-Sq = 5.9\%     \quad R-Sq(adj) = 5.7\%

Table 4-13. Predictor Demand SD, On hand Mean
Out of stock rate = \( F(\text{Demand CV, On Hand Mean/Demand Mean}) \)

In this model, we expect the coefficient of Demand CV to be positive and the coefficient of the second variable to negative. The output shows that the coefficients did not meet our expectation.

R2 value is fairly low. The residual plots show that the normality assumption is not met. And the linearity assumption is not met too.
The regression equation is
Rate % = 33.3 - 0.964 Demand CV - 5.43 On hand / Demand

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>33.296</td>
<td>1.391</td>
<td>23.93</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand CV</td>
<td>-0.9639</td>
<td>0.9102</td>
<td>-1.06</td>
<td>0.290</td>
</tr>
<tr>
<td>On hand / Demand</td>
<td>-5.4265</td>
<td>0.4585</td>
<td>-11.84</td>
<td>0.000</td>
</tr>
</tbody>
</table>

S = 17.3348  R-Sq = 12.6%  R-Sq(adj) = 12.5%

Table 4-14. Predictor Demand CV, On hand Mean/Demand Mean

Figure 4-9. Predictor Demand CV, On Hand Mean/Demand Mean
Out of stock rate = F (Demand CV, On hand Mean/Demand SD)

In this model, we expect the coefficient of Demand CV to be positive and the coefficient of the second variable to negative. The output shows that the coefficients did not meet our expectation.

R2 value is fairly low. The residual plots show that the normality assumption is not met. And the linearity assumption is not met too.

The regression equation is
\[ \text{Rate} = 44.6 - 13.5 \text{ Demand CV} - 3.08 \text{ On hand/Demand SD} \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE</th>
<th>Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>44.613</td>
<td>1.517</td>
<td>29.42</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Demand CV</td>
<td>-13.579</td>
<td>1.032</td>
<td>-13.15</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>On hand/Demand SD</td>
<td>-3.0793</td>
<td>0.1637</td>
<td>-18.01</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

\[ S = 15.9125 \quad \text{R-Sq} = 26.4\% \quad \text{R-Sq(adj)} = 26.2\% \]

Table 4-15. Predictor Demand CV, On hand Mean/Demand SD
Out of stock rate = F (Demand mean, Demand SD, On hand mean)

Now we try to use three variables to represent the model. When we include both the on hand mean and demand mean in the model. We expected the demand mean to be positive, on hand mean to be negative and the demand SD to be positive.

However, the output shows that the coefficient did not meet our expectations. R2 value is 43.2%, which is not high enough. And the residual plots showed that the constant variances assumption is not met.
The regression equation is

\[ \text{Rate } \% = 9.35 + 0.0193 \text{ Demand Mean} - 0.00477 \text{ Demand SD} - 0.00276 \text{ On Hand Mean} \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>9.3536</td>
<td>0.7370</td>
<td>12.69</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand Mean</td>
<td>0.0192525</td>
<td>0.0007500</td>
<td>25.67</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand SD</td>
<td>-0.0047669</td>
<td>0.0005625</td>
<td>-8.47</td>
<td>0.000</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>-0.0027649</td>
<td>0.0002356</td>
<td>-9.35</td>
<td>0.000</td>
</tr>
</tbody>
</table>

\[ S = 13.9825 \quad R^2 = 43.2\% \quad R^2(\text{adj}) = 43.0\% \]

Table 4-16. Predictor Demand Mean, Demand SD, On hand Mean

Figure 4-11. Predictor Demand Mean, Demand SD, On hand Mean
Out of stock rate = F (Demand Mean, On hand mean, On hand CV)

In this model, we expected the demand mean to be positive, on hand mean to be negative and on hand CV to be positive. We can see from the output that the coefficient meets our need.

The R2 value is 66.1%, which means that 66.1% of the variance in MSE can be explained by our model. This is a fairly high value.

Further examination of the residual plots showed that the normality assumption is violated, but not seriously. There is a trend of increased residual with the increase of fitted value.

But since the R2 value is high enough in this model. We decided to choose this model as candidate. We will further do transformation to solve the residual issues.

The regression equation is

\[ \text{Rate} % = -10.6 + 0.0104 \text{ Demand Mean} - 0.00234 \text{ On Hand Mean} + 33.8 \text{ On Hand CV} \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-10.5601</td>
<td>0.8998</td>
<td>-11.74</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand Mean</td>
<td>0.0104008</td>
<td>0.0002500</td>
<td>20.40</td>
<td>0.000</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>-0.0023415</td>
<td>0.0002283</td>
<td>-10.23</td>
<td>0.000</td>
</tr>
<tr>
<td>On Hand CV</td>
<td>33.751</td>
<td>1.194</td>
<td>23.27</td>
<td>0.000</td>
</tr>
</tbody>
</table>

\[ S = 10.7995 \quad R-Sq = 66.1\% \quad R-Sq(adj) = 66.0\% \]

Table 4-17. Predictor Demand Mean, On hand Mean, On hand CV
Figure 4-12. Predictor Demand Mean, On hand Mean, On hand CV
Chapter 5

A Regression Model

We found out that the model with Demand Mean, On hand mean, On hand CV could be an eligible candidate. After we have selected our potential model, we need to do some further analysis to see whether the model needs to be transformed.

Regression Tests

There are several things that need to be examined in the output. First, we can see the regression equation is Rate % = - 10.6 + 0.0104 Demand Mean - 0.00234 On Hand Mean + 33.8 On Hand CV. This equation reveals the relationship between the out of stock rate and the predictors. The coefficient of Demand Mean is positive, which means that if the demand is higher, we may get a higher of out of stock rate. The coefficient of On Hand Mean is negative, which indicates that a higher inventory will reduce the chance of out of stock. The coefficient of On Hand CV is positive, which shows that if the fluctuation of your inventory level is high, you will get a higher chance of out of stock. These all fit the logic of the supply chain.
The regression equation is
\[ \text{Rate} \% = -10.6 + 0.0104 \times \text{Demand Mean} - 0.00234 \times \text{On Hand Mean} + 33.8 \times \text{On Hand CV} \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-10.6801</td>
<td>0.8996</td>
<td>-11.74</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand Mean</td>
<td>0.0104089</td>
<td>0.0005082</td>
<td>20.48</td>
<td>0.000</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>-0.0023415</td>
<td>0.0002289</td>
<td>-10.23</td>
<td>0.000</td>
</tr>
<tr>
<td>On Hand CV</td>
<td>33.751</td>
<td>1.194</td>
<td>28.27</td>
<td>0.000</td>
</tr>
</tbody>
</table>

\[ S = 10.7999 \quad \text{R}-\text{Sq} = 66.1\% \quad \text{R}-\text{Sq(adj)} = 66.0\% \]

### Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>3</td>
<td>228645</td>
<td>76182</td>
<td>653.15</td>
<td>0.000</td>
</tr>
<tr>
<td>Residual Error</td>
<td>1004</td>
<td>117104</td>
<td>117</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1007</td>
<td>345849</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Mean</td>
<td>1</td>
<td>116469</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>1</td>
<td>18635</td>
</tr>
<tr>
<td>On Hand CV</td>
<td>1</td>
<td>93241</td>
</tr>
</tbody>
</table>

Table 5-1. ANOVA Result

The ANOVA table summarizes the relationship between the amounts of variability the regression model accounts for sales as compared to the amount of variability not accounted for by the model. In the ANOVA table, we can see that the p value of the F test is near zero (less than 0.05) which means that the amount of variability accounted for by the predictors is significantly greater than the amount of variability that is left over after the model is applied.

Finally, the R2 value in the output is 66.1%, which means that 66.1% of the variances in the sales can be explained by the density. This value is fairly high since there is no statistical test for significance.
Then let us examine the residual plots. The left two plots show the normality of the residuals. We can see that the residuals didn’t fit the straight line quit well. However, considering the sample size we have, this result is OK. Both of these plots show that the normality assumption is met. The right top plot shows a significant pattern of residuals against the fitted value, we can see that with fitted value increasing, the residuals are showing an increase pattern. This shows that the constant variances assumption is not met. However, there is no significant evidence that there is none linearity. The final plot indicts that there does not appear to be any time trend so it is possible to conclude the residuals are independent.
In addition to visually assessing the approximate linearity of the points plotted in the plot. The Kolmogorov-Smirnov test can be used for examining the normality of the error terms. This test compares the empirical cumulative distribution function of the residuals with the distribution expected if the data were normal. If this observed difference is sufficiently large, we will reject the null hypothesis and conclude that the normality assumption is not met. If the p-value of this test is less than the chosen a-level, we can reject the null hypothesis and conclude that the residual is non-normal. From the table beside the plot, we can see that the p value of the KS test is less than 0.01. So we can conclude that the normality assumption is not met.
Transformation

Because some of the basic assumptions are not met, we would like to do some transformation. After we take the square root of out of stock rate, we run the regression again.

In the following output, we can see that the coefficients of the variables are what we expected. The positive coefficient of demand mean indicates that higher demand level may lead to higher out of stock rate. The negative on hand mean shows that a higher inventory level can prevent the out of stock rate. And the on hand CV indicates that the fluctuation of the inventory level will lead to higher out of stock rate.

The R2 value is 58.6%, which means that 58.6% of the variances in MSE can be explained by our model. This value decreased from the model before transformation, but is still higher enough.

The regression equation is

\[ \sqrt{Y} = 0.900 + 0.000857 \text{Demand Mean} - 0.000161 \text{On Hand Mean} + 3.53 \text{On Hand CV} \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.8995</td>
<td>0.1026</td>
<td>8.77</td>
<td>0.000</td>
</tr>
<tr>
<td>Demand Mean</td>
<td>0.00085685</td>
<td>0.00005795</td>
<td>14.78</td>
<td>0.000</td>
</tr>
<tr>
<td>On Hand Mean</td>
<td>-0.00016080</td>
<td>0.00002610</td>
<td>-6.16</td>
<td>0.000</td>
</tr>
<tr>
<td>On Hand CV</td>
<td>3.5332</td>
<td>0.1561</td>
<td>25.92</td>
<td>0.000</td>
</tr>
</tbody>
</table>

\[ S = 1.23152 \quad R^2 = 58.8\% \quad R^2(adj) = 58.5\% \]

<table>
<thead>
<tr>
<th>Analysis of Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
</tr>
<tr>
<td>Regression</td>
</tr>
<tr>
<td>Residual Error</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

Table 5-2. ANOVA after transformation
Then left top residual plot shows that the normality assumption is met. The right top plot indicates that the constant variance assumption is met. The right bottom plot shows that the independence assumption is met.

![Residual Plots after transformation](image)

Figure 5-3. Residual Plots after transformation

After the above analysis, we finally define the model to be:

$$\sqrt{\text{Out of Stock rate}} = 0.9 + 0.000857 \times \text{Demand Mean} - 0.000161 \times \text{On Hand Mean} + 3.53 \times \text{On hand CV}$$

We see that the fluctuation of their inventory level is the main source of the out of stock rate. So J&J should try to reduce their on hand CV to achieve better cycle service level.
Chapter 6

Conclusion

Bullwhip effect is one of the main problems in the big retail supply chain strategies. Lots of methods have emerged trying to decrease the bullwhip effect, such as JIT or VMI. In VMI, the vendor is charge of the replenishment of its customers’ inventory. And demand information is exchanged between each levels of the supply chain, which further lead to the distortion of demand order.

In this article, we first examine the strategy adopted by J&J, one of the largest medical, pharmaceutical and consumer packaged goods manufacturers in America. Then we further get into the relationship between the days of supply and cycle service level. We can see the disadvantage of measuring inventory level using the days of supply.

Then we try to analyze two years demand data of J&J and using a regression model to represent the relationship between the out of stock rate and other variables. The result shows that the fluctuation of the on hand inventory is the most important case of the out of stock. Increasing the on hand inventory level can also improve the performance.
Reference


(June) 2A–14A.

