INTERACTIONS IN A DILUTE FERMI GAS

A Dissertation in
Physics
by
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Abstract

This dissertation investigates the role of interactions in a dilute Fermi gas of $^6$Li atoms. Interactions between atoms play an important role in both strongly and weakly interacting systems. In strongly interacting systems the interactions create correlations and pairing that give rise to superfluidity and other emergent phenomena. On the other hand, weak interactions cause unwanted frequency shifts in atomic clocks. I will discuss two experiments that we have conducted that look into interactions in these two regimes. For the strongly interacting case I will discuss our investigation into a narrow Feshbach resonance. I will show that the atom loss near this resonance and the interactions are both energy dependent. Due to this energy dependence the narrow Feshbach resonance must be described by a finite effective range. Fermi gases with energy dependent interactions may yield novel forms of superfluidity and can be used to simulate the equation of state for neutron star matter.

Identical fermions do not interact at ultracold temperatures since only $s$-wave collisions are allowed. Therefore, identical fermions in an atomic clock do not interact provided that each atom feels the same interrogation field. If an inhomogenous interrogation field is used, then different atoms feel different interrogation fields and the atoms are made distinguishable resulting in an $s$-wave collisional frequency shift. Interestingly, to observe the shift, it is essential the clock field itself produce the distinguishability. No clock shifts are observed for distinguishable two-state fermions if they are interrogated by a homogeneous field. We characterize the clock shift for an inhomogeneous field using Ramsey spectroscopy. We observe that the collisional shift has a weak dependence on the first Ramsey pulse area and depends sensitively on the duration of the second Ramsey pulse. We demonstrate linear scaling of the shift with the $s$-wave scattering length showing that it is indeed $s$-wave in nature. The results of these findings will help provide a blue print for how to eliminate uncertainties due to density dependent shifts in atomic clocks.
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Dedication

For those whose dreams were crushed, only to have them come back stronger.
Chapter 1

Introduction

Everyday we experience some sort of interaction. These interactions affect how we live our lives. The same is true for atoms. Atoms interact with the world and each other. The investigation and understanding of the how the universe operates via interactions is at the core all physics research. Because, where is the fun in a universe without interactions? The interactions that I will cover in this dissertation cover many orders of magnitude. I will investigate how small interactions can affect the accuracy of atomic clocks. I will also investigate resonant interactions that must be described with a finite effective range that is 4 orders of magnitude larger than the strength of the weak clock interactions observed in the atomic clocks. The ability to study interactions both weak and strong in one experimental system is one of the great advancements of atomic physics. The work done in this dissertation assists in the advancement of this field.

But, let us first start with how we got where we are in the field of atomic physics. Over the past 20 years the field of ultra-cold atoms has boomed. This was mainly due in part to the ability to laser cool and trap atoms pioneered by Steven Chu, William Phillips, and Claude Cohen-Tannoudji and others. These three named here were given the Nobel Prize for this work. This has led to the first observation of a Bose-Einstein condensate by the groups of Cornell and Wieman, Ketterle, and Hulet of which the first three were awarded the Nobel Prize.

Cold atoms haven’t just given us colder temperatures and new phase transitions, but have opened up a whole new playground to test the theories of quantum mechanics and model systems in other fields. This includes creating optical lattices
where laser beams create a crystal of light where atoms are attracted to regions of high or low intensity depending on the frequency of the light with respect to the atomic transition. This creates periodic structure that can be tuned by the orientation of the laser beams allowing for testing of condensed matter systems where the extra knob of controlling the lattice geometry is allowed. Also, Feshbach resonances enable us to tune interactions to study gases in the strongly interacting regime where calculations become difficult. It is thought that this is where the mechanisms behind high temperature superconductivity and neutron stars is thought to lie. Also the ability to control laser fields with great precision has allowed for ultra precise measurements on cold atoms which will help create more accurate atomic clocks and measurements of fundamental constants. For a review of the field see [11, 12]

1.1 Dilute Ultracold Gases

When we talk about gases we immediately think about the air around us. This is the air that we hardly notice is around us. We know from the ideal gas law that this air has a density around $2 \times 10^{21}$ atoms per cubic centimeter. Now let us think about what we think of as dilute. A very, very low amount of Radon in air that we breath is at the 1 part per billion level. If we consider this a very dilute amount of Radon in the air we breath there could be $\sim 10^{19}$ Radon atoms per cubic centimeter in the air. The gas of $^6$Li atoms that we want to investigate are at densities still many orders of magnitude lower than that. The typical densities that we work with are below $10^{14}$ atoms per cm$^3$. As you can tell from the eight orders of magnitude difference between these two systems, we weren’t joking around with our use of the term dilute. Working with a dilute gas also has an advantage that the dynamics are dominated by two-body processes, which simplify our investigations greatly.

For the experiments that I will discuss the gas is at temperatures between 860 nK and 50 $\mu$K. These cold tempertures allow for large de Broglie wavelengths. At these temperatures we effectively freeze out higher angular momentum scattering. Because the de Broglie wavelength is much longer than the Van der Waals length we can model the interactions with a contact potential. This means that the s-wave scattering length will characterize most of the interactions that we can investigate.
1.2 Interactions with Finite Range

One of the goals of current atomic physics is the idea of quantum simulation. This is the idea that we can only efficiently simulate a quantum system with another quantum system. Thus if we are to reach this goal we need an atomic system with a lot of different control knobs that we can adjust to simulate quantum systems that would otherwise be experimentally difficult to investigate.

Most interactions in cold atom gas experiments can be classified by contact interactions. This means that atoms interact via a delta function potential. This gives rise to a characteristic s-wave scattering length discussed in your favorite quantum mechanics course [13]. Now in our cold gases we are fortunate that we can tune this interaction via a Feshbach resonance [7]. This has led to observations of BCS superfluids and the association of molecules and subsequent observation of a BEC of these molecules. All of BCS-BEC crossover experiments performed so far have been in the context of this contact potential.

Now if we want to expand the reach of the simulations to systems that do not interact via a contact potential then it seems like cold atoms are not a good simulator unless we find a way to experimentally create a system with finite range interactions. This is exactly what we have done. We have realized a Fermi system where two components of this Fermi system do indeed interact via a finite effective range. This is done by tuning near a narrow Feshbach resonance that is 0.1 G wide at 543.286(3)G. By creating a stable magnetic field, as describe in Appendix-A, we are able to investigate the properties of this narrow resonance. In Chapters 3 and 4 I will describe both the theory behind creating a Fermi gas with a finite effective interaction range and the experiments we have conducted confirming this realization. I will also describe the potential new physics that could be explored with such a system.

1.3 Interactions and Clocks

Einstein taught us that time is relative. If we move at speeds close to the speed of light time moves more slowly compared to when we are at rest. Time also moves more slowly near massive objects. To measure this temporal relativity he
postulated thought experiments that involved a system of synchronized clocks. These clocks would have arbitrary precision such that slight time differences due to relativity would be measurable. While Einstein postulated this as a thought experiment we have made clocks that can measure these relativistic effects to great accuracy and precision.

The clocks I am talking about are of course atomic clocks. Nature has provided us with a set of identical atoms. This means that all the atoms of a specific element are identical no matter where they occur in the universe. A $^{6}$Li atom here on earth is the same as a $^{6}$Li atom in space or near a quasar. Thus the energy levels of an atom are the same regardless of location. If we can precisely measure the frequency between two energy levels in an atom we can have a local measurement of time. This is how an atomic clock works. We have these lovely energy levels that are the same for each atom everywhere, so we just need to resolve the energy difference in the atom and we will have a local measurement of time. Just because there are identical atoms everywhere doesn’t mean that the environment they are in is the same. The environment the atom is in will affect the energy levels used in the clock transition. These environmental effects may or may not be something we are interested in measuring. We must take care to eliminate systematic uncertainties that we introduce in our construction of the clock. These effects may include light shifts, black body radiation, and density dependent effects to name a few. By understanding these effects we can compensate for them and only isolate the interactions that we wish to measure. As of today the two biggest sources of uncertainty due to environmental conditions are coupling to black-body radiation and the density-dependent interaction shifts of the gas [14]. We will address the latter in our experiments.

The density is an important factor because collisions can cause frequency shifts. If we can eliminate or understand the nature of these shifts we can create more accurate clocks. Identical fermions are forbidden from interacting with each other due to the Pauli exclusion principle. If fermionic isotopes are used there should be no interactions between the atoms and thus no collisional frequency shifts. This is indeed the case when there is a driving field that is interrogating the two energy levels is the same for every atom that is being interrogated. If the atoms feel different driving field then a degree of distinguishability is introduced. What can
happen when fermions become distinguishable? Well, they can interact and cause a collisional frequency shift of the clock transition. We will show that when atoms feel different strengths of driving fields a collisional frequency shift does exist. We will characterize its dependence on Ramsey pulse sequence, scattering length, and degree of inhomogeneity in the driving field. This measurement is the first direct measurement of a collisional frequency clock shift in a Fermi gas and these effects must be considered when designing new atomic clocks with fermionic isotopes.

1.4 Outline of Dissertation

This dissertation is divided into three different parts. In Chapter 2 I will describe the background physics of Fermi gases and the experimental procedures that we use to manipulate and probe these atoms. These procedures will set the foundation for the two experiments that we conducted investigating interactions in a dilute Fermi gas. In Chapters 3 I will cover the physics of Feshbach resonances and how a narrow Feshbach resonance will lead to new physics that can be simulated in ultra-cold gases. Chapter 4 will discuss our experimental realization of a Fermi gas with finite range interactions by exploring the physics near the narrow Feshbach resonance.

From this experiment we will continue on the experiment observing a collisional frequency shift in a Fermi gas. In Chapter 5 I will discuss the experiments that motivated our investigations as well as the theory behind this shift. In Chapter 6 I will discuss all of the experimental details on how we have realized and characterized the nature of the collisional frequency shift for Fermions.

- Chapter 2: Background quantum and statistical mechanics of Fermi gases as well as the atomic physics of $^6$Li. This chapter also discusses the experimental detail on how we trap, manipulate, and probe our Fermi gases.
- Chapter 3: Theory of Feshbach resonances in ultracold gases and how we can create a Fermi gas with a finite effective range.
- Chapter 4: Experimental investigation of a narrow feshbach resonance that has energy dependent interactions and must be characterized by a finite effective range.
• Chapter 5: Background theory and experiments that have inspired our investigations of an s-wave collisional frequency shift in a Fermi gas.

• Chapter 6: The experimental realization of an s-wave collisional frequency shift in an ultra cold Fermi gas.
Chapter 2

Understanding, Creating, and Probing Ultracold Gases

2.1 Understanding Fermi Gases

In this chapter I will discuss the background physics and experimental methods used for our investigations of interactions in a Fermi Gas. I will mainly focus on $^{6}\text{Li}$ fermionic isotope that we study. Although I will use specific details about $^{6}\text{Li}$ in this dissertation, a similar procedure can be used for other atoms assuming that their electronic structure is known. Since we work with $^{6}\text{Li}$ in our experiments I will mainly focus on its properties. Lithium being the lightest of the alkali atoms has two stable isotopes $^{6}\text{Li}$ and $^{7}\text{Li}$. Both of these isotopes have one valance electron which makes the total electron spin $1/2$. The $^{7}\text{Li}$ isotope has a nuclear spin if I=$3/2$. The $^{6}\text{Li}$ atom has a nuclear spin of I=$1$. If we count up the number of protons, neutrons, and electrons in each isotope we see that $^{7}\text{Li}$ has an even number. Thus its total spin must be integer making it bosonic. For $^{6}\text{Li}$ the total number is odd thus $^{6}\text{Li}$ is a fermion.

For laser cooling and trapping of fermionic alkali atoms there are two stable isotopes to choose from, $^{6}\text{Li}$ and $^{40}\text{K}$. While both can and have been used with much success, the decision to investigate interactions in $^{6}\text{Li}$ was based on several factors. First since I am not the PI of the project I didn’t really have a choice on what fermion atom to use, because I did not have the funds to start my own research.
I only had to choose a lab that was studying fermionic atoms. Ken’s lab was the only ultra cold lab studying lithium so I studied lithium. Other, more objective, reasons to use $^6$Li are the following. For one $^6$Li is more abundant in nature thus easier to obtain than $^4$He. Inelastic loss and heating mechanisms occur in $^4$He due to $p$-wave collisions that are not problematic in $^6$Li, allowing lower reduced temperatures to be achieved with $^6$Li atoms. And, the most important reason is that for the Feshbach resonances, which allow for control over interactions, are very broad and easily accessible. In an applied magnetic field, different mixtures of the three lowest Zeeman sub-levels can be made by applying radio-frequency fields. For fields above 100 G these three levels are electronic-spin polarized and have no energetically allowed spin exchange collisions for temperatures less than 10 mK. I will first discuss the thermodynamics and quantum mechanics involved in investigations of Fermi gases. Then I will go into more detail on the experimental details of the system that we use. This will give a good background into how we use this system to investigate interactions in a dilute Fermi gas.

2.1.1 Thermodynamics and Quantum Mechanics of Fermi Gases

Symmetry is a beautiful thing and is at the heart of many if not all physical systems. We know from Fourier analysis that we can comprise any function by adding up sines and cosines. The sine being an odd, or anti-symmetric, function about 0 and the cosine being an even, or symmetric, function about 0. We use this Fourier analysis to mathematically describe the world and there is no surprise that all the particles that comprise the universe have a similar symmetry. If we look at a system of particles we notice that if we exchange particles the wave function is either symmetric or anti-symmetric under particle exchange depending on the spin of the constituent particles. If the particles have integer spin then their wave function is symmetric upon exchange and the particles are bosons. If the particles have half-integer spin then their wave function is anti-symmetric upon exchange and the particles are fermions.

We know from quantum mechanics that all particles of a specific type are identical. That symmetry constraint on the bosonic wave function means that
multiple identical bosons can occupy the same quantum state. Examples of this are Bose-Einstein condensation, lasers and $^4\text{He}$ superfluidity. All of those phenomena arise from the fact that boson tend to bunch together. Bosons prefer to occupy the same states. The opposite is true of fermions. The manifestation of this is the Pauli exclusion principle which states that identical fermions cannot occupy the same quantum state. This gives us the different electronic states in atoms and Fermi surfaces and energies in materials. Since $^6\text{Li}$ is a fermion we need a brief discussion on the thermodynamics and quantum mechanics of our favorite fermion: $^6\text{Li}$.

2.1.1.1 Thermodynamics

All the thermodynamic properties of a non-interacting ensemble of particles can be realized by counting, specifically by the statistics of counting those particles and imposing the constraints described above. The treatment can be found in your favorite statistical mechanics book [15] and I will give a brief treatment here.

Let us start with a system of $N$ indistinguishable particles with a total energy of $E$ in a volume $V$. All we know about these particles is that they are fermions and are thus governed by Fermi-Dirac statistics. In the system we are looking at we have energy levels $\varepsilon_i$ of which there are a number $g_i$ accessible states for the $i^{th}$ level of the system. Now our job as laid out in statistical mechanics is to calculate the number of microstates available to the system of particles. So if we say that the probability that a particle occupies a certain state is $n_i$ the we must have that,

$$\sum_i n_i = N$$

$$\sum_i \varepsilon_i n_i = E$$

(2.1)

By counting up all of the all the accessible states in the system, assuming large $n$, employing Stirling’s approximation, and using Legrange multipliers we arrive at our Fermi-Dirac distribution

$$f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon}{k_B T}} - e^{\frac{\varepsilon_F}{k_B T}} + 1}$$

(2.2)
This is of course assuming that there is a degeneracy \( g_i \) of 1. Here \( k_B \) is Boltzmann’s constant and \( T \) is the temperature. \( E_F \) is the Fermi energy of the system. This is the energy of the highest level occupied for a zero temperature Fermi gas. There will be more discussion on this later.

Using 2.2 we can perform all the calculations of statistical mechanics, assuming we know the energy levels \( \varepsilon_i \) and their degeneracy\[15\]. Once again this is for an ideal Fermi gas. When interactions are involved things become much more complicated.

### 2.1.2 Quantum Mechanics for Fermions

Here I will discuss the quantum mechanical implementations of the Pauli exclusion principle as it pertains to our research. This will include derivation of the Fermi energy and temperature for a harmonic trap, and basic scattering of fermions.

As already mentioned, due to the Pauli exclusion principle fermions must have an antisymmetric wave function under exchange. So, for example, if we consider the two body wave functions below for the states \( \phi_1 \) and \( \phi_2 \) and particles at positions \( x_1 \) and \( x_2 \).

\[
\Psi(x_1, x_2) = \phi_1(x_1)\phi_2(x_2) + \phi_1(x_2)\phi_2(x_1) \quad (2.3)
\]

\[
\Psi(x_1, x_2) = \phi_1(x_1)\phi_2(x_2) - \phi_1(x_2)\phi_2(x_1) \quad (2.4)
\]

They look similar, but the + sign in 2.3 makes the wave function symmetric under exchange and thus a bosonic wave function. Thus 2.4 is anti-symmetric under exchange and is a fermionic wave function. In general you can use a Slater determinant to construct an antisymmetric wave function for your system of fermions \[13\].

The Pauli exclusion principle states that no two identical fermions can occupy the same state. This can be related to the anti-symmetry of the wave function in 2.4 by setting \( \phi_1 = \phi_2 = \phi \). Such a two-body wave function is identically zero and not allowed. On the other hand, even if two fermions occupy different states, they cannot be found at the same position. This is seen from 2.4 when \( x_1 = x_2 \). Then the two terms cancel out and there is zero probability of finding the two particles.
The fact that two fermions cannot occupy the same quantum state has important implications for the statistical mechanics of fermions. If we have a Fermi gas with $N$ particles at zero temperature the lowest $N$ energy levels will be filled. The $N^{th}$ energy level is called the Fermi energy. This is a ubiquitous concept in condensed matter physics, but in condensed matter physics the Fermi temperatures are 1000K and thus the ratio $T/T_F$ is $\ll 1$. The Fermi temperature is defined by, $T_F = k_B E_F$.

In our experiments we use a harmonic trap in three dimensions. For such a system the Hamiltonian is given by

$$ H(r, p) = \frac{\hbar}{2m}(p_x^2 + p_y^2 + p_z^2) + \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) $$

(2.5)

For temperatures much greater than the harmonic oscillator spacing, which is normal for atomic experiments, the density of states can be written as [16]

$$ g(\epsilon) = \frac{\epsilon}{2(h\varpi)^3} $$

(2.6)

where $\varpi = (\omega_x\omega_y\omega_z)^{1/3}$. We know that we have $N$ particles in the gas. So to compute the Fermi energy we simply compute the integral

$$ N = \int d\epsilon f(\epsilon)g(\epsilon) = \int d\epsilon \frac{\epsilon^2}{2(h\varpi)^3} * \left(\frac{\epsilon}{h\hbar T} - \frac{E_F}{k_B T} + 1\right) $$

(2.7)

which when solving for $E_F$ we obtain:

$$ E_F = (6N)^{1/3}\hbar\varpi $$

(2.8)

Which is the form for the Fermi energy that we use for our experiments. Typical values for the Fermi energy in our cold gas experiments range between 10 nK and 50 $\mu$K. Most of the Fermi gas experiments described in this dissertation operate at temperatures such that $T/T_F$ is of order 1. The lowest reduced Fermi temperatures reached are on the order of $T/T_F = 0.05$ [12].
2.1.2.1 Quantum Scattering

Now that we have seen the results of the Pauli exclusion principle on the distribution of fermions among energy levels we still need a picture of its effect on the scattering fermions. To do this we will go through the quantum mechanics of scattering of identical particles which can be found in your favorite quantum mechanics text [13].

Let us start by describing a system of two atoms colliding. For the case of two alkali atoms scattering, the effective potential between the two of them can be described in center of mass coordinates by,

$$V(R) = -V_{vdW}(R) + \frac{\hbar^2 l(l + 1)}{2\mu R^2}$$  \hspace{1cm} (2.9)

where the first term, $V_{vdW}$, is the van der Waals potential and the second term is the so called centrifugal term that involves the angular momentum of the two particles with a reduced mass $\mu$ and separation $R$. We will go into a more in depth treatment for the van der Waals potential later, but for now we will just consider a general isotropic central potential. In this case we can describe the incoming and outgoing waves as a decomposition of partial waves written as

$$\psi(k, r) = \sum_{l=0}^{l=\infty} (2l + 1) i^l j_l(kr) P_l(\cos\theta)$$ \hspace{1cm} (2.10)

Where $j_l(kr)$ are spherical Bessel functions, and $P_l(\cos\theta)$ is a Legendre polynomial of $\cos(\theta) = \hat{k} \cdot \hat{r}$. If we are in the far field where $kr \gg 1$ then we write our scattered wave function as

$$\psi(k, r) = e^{ikr} + f e^{ikr}$$ \hspace{1cm} (2.11)

where $f$ is defined as the scattering amplitude by

$$f(k, \theta) = \frac{1}{k} \sum_l (2l + 1) e^{i\delta_l(k)} \sin \delta_l(k) P_l(\cos\theta) = \sum_l (2l + 1) f_l(k) P_l(\cos\theta).$$  \hspace{1cm} (2.12)

Here, $f_l(k)$ is the partial scattering amplitude. The scattering amplitude is related
to the scattering T matrix by

\[ T(k, \theta) = \frac{2\pi \hbar^2}{m} f(k, \theta). \] (2.13)

The consequence of all of this is that to describe the scattering we need to determine the phase shift \( \delta_l(k) \) for each partial wave which can be done by solving the Hamiltonian of the system and then invoking the boundary conditions or asymptotic boundary conditions [17].

Now that we have our phase shifts calculated, we are ready to calculate the differential cross sections which is just the modulus squared of the scattering amplitude,

\[ \frac{d\sigma}{d\Omega} = |f|^2 \] (2.14)

Then by placing our expression for the scattering amplitude into 2.14 and carrying out the integration over all solid angles we obtain the expression for our total cross section.

\[ \sigma(k) = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l + 1) \sin^2(\delta_l(k)) \] (2.15)

Thus as stated before to find the cross section all one has to do is calculate the phase shift associated with each scattering process. The total cross section can also be calculated by invoking the optical theorem which relates the total cross section to the imaginary part of the scattering amplitude.

\[ \sigma(k) = \frac{4\pi}{k} \Im[f(k, 0)] \] (2.16)

We can write the partial scattering amplitude as [18]

\[ f_l(k) = \frac{1}{kcot\delta_l - ik}. \] (2.17)

We can define a length scale \( a_l \) associated with the scattering phase shift such that

\[ \frac{1}{a_l} = k\cot\delta_l \] (2.18)

\[ a_l = -\Re[f_l(k)] \] (2.19)
For contact interactions, we only have s-wave ($l = 0$) scattering and $a_0 = a$ is the s-wave scattering length. For s-wave scattering by finite-range potentials, as will be discussed for the study of the narrow Feshbach resonance, it is important to take the effective range expansion of 2.18 given by.

$$k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} r_0 k^2$$  \hspace{1cm} (2.20)

where $r_0$ is the effective range and $a$ the scattering length. These expressions will play an important role in later discussions so take note of them here. There will be a quiz later.

The treatment above is for distinguishable particles. Quantum statistics must be taken into account for identical particles. Since the particles are indistinguishable we cannot discern whether or not the particles backscatter (at an angle $\pi$ away from the scattering angle $\theta$) or not thus we must take into account both scattering amplitudes and the equation for the differential cross section becomes

$$\frac{d\sigma}{d\Omega} = |f(k, \theta) \pm f(k, \pi - \theta)|^2$$  \hspace{1cm} (2.21)

Here the plus sign corresponds to the scattering of bosons and the minus sign corresponds to the scattering of fermions. The consequence is that for identical fermions the differential cross section will only be finite for odd $l$ values and for bosons it is only finite for even $l$ values and $l = 0$. This brings us to the discussion of how many partial waves do we need to consider when calculating scattering amplitudes. This of course will take us to the Hamiltonian of the system and the exact form of the potential would be needed to exactly calculate the different partial wave cutoffs. With good accuracy we can use a van der Waals potential given by

$$V_l(R) = -\frac{C_6}{R^6} + \frac{\hbar^2}{2\mu} \frac{l(l+1)}{R^2}$$  \hspace{1cm} (2.22)

Where the $C_6 = 1393.39$[Hartree $a_0^6$] for $^6$Li [7]. The second term in 2.22 is the centrifugal term. All that is needed is to do is to calculate the energy of this term for different partial waves and we can determine which partial waves to keep for various temperatures. For $l = 1$ in lithium we must have a energy of
$E = k_B 6.8 mK$ to break the p-wave barrier. In our experiments we are well below 1mK, thus we only need to take into account s-wave interactions. Keeping only the $l = 0$ partial waves means that we must have at least two species of fermions to observe interactions. It turns out that we indeed do have easy access to three different species which we can populate with RF or Raman transitions. But, of course the situation is more complicated than just having two species. We must have an incoherent mixture of these two species, which will be discussed in much more detail later on.

### 2.2 $^6$Li properties

Here I will discuss in more detail all of the properties needed to laser cool and trap my favorite fermion: $^6$Li. This will mainly consist of discussions about its electronic structure and how the properties are altered in the presence of a magnetic or electric field. For a more thorough discussion I suggest you visit your favorite graduate thesis, graduate quantum text, or atomic physics books [1, 19, 20, 13]. Our laser cooling and trapping of $^6$Li requires transitions between the $1s^2 2s^1$ ground state and the first excited energy level $1s^2 2p^1$, known as the D line. The D lines split up into a pair of lines aptly named the D1 and D2 lines. This fine structure splitting arises from the spin-orbit coupling of the spin of the valence electron and the orbital angular momentum, relativistic correction, a Darwin term (for $L = 0$) and the Lamb shift 1. It then becomes natural to express the eigenstates by quantum number $J$ which is the sum of the orbital angular momentum $L$ and the spin of the electron $S$. Thus the total angular momentum is expressed by $J = L + S$. The possible values of $J$ become integer values between $|L - S| \leq J \leq L + S$.

For the ground state of $^6$Li $^2S_{1/2}$ there is only an offset from the unperturbed energy level and no splitting and the $J=1/2$ is the only possible value because $L=0$ and $S=1/2$. Now for the first excited state in $^6$Li we have that $L=1$ and $S=1/2$. Thus we can have $J=1/2$ and $J=3/2$. This breaks the D lines into two

---

1The fine structure constant is the energy perturbations of the Hamiltonian that are first order in the fine structure constant $\alpha$. Likewise the hyperfine structure arises from the terms that are second order in $\alpha$.
Figure 2.1. Energy level diagram for the ground state $2^2S$ and the first excited state $2^2P$ of $^6$Li with the fine structure and hyperfine structure included. Note that the energy levels are not drawn to scale.

Separate lines $D_1$ and $D_2$ and the fine structure splitting between these two lines is $\Delta_{fs} = 10.053$ GHz [21]. These lines have a linewidth of 5.9 MHz which will have ramifications on laser cooling that will be discussed later.

Since lasers with narrow linewidths are readily available the resolution of the hyperfine structure is easily achieved for the alkali atoms. The hyperfine corrections arise from the coupling of the total angular momentum of the electron coupling to the spin of the nucleus. For $^6$Li the nuclear spin is $I = 1$. Thus in the same manner as the spin-orbit coupling that gave us the $J$ quantum number we introduce the quantum number $F$. $F$ can take the integers values that satisfy $|J - I| \leq F \leq J + I$. Thus this splits ground electronic state into a $F=1/2$ and $F=3/2$ level. The $2p^2P_{1/2}$ state has $J=1/2$ and thus also splits into $F=1/2$ and $F=3/2$ levels. The $2p^2P_{3/2}$ state has $J=3/2$ thus it splits into three separate lines $F=1/2$, $F=3/2$, and
F=5/2.

The fine structure and the hyperfine structure of $^6$Li is shown in Figure 2.1. To calculated the energy shifts for each of the hyperfine states we need to look at the hyperfine Hamiltonian which is composed of a magnetic dipole term and an electric quadrupole term. The hyperfine Hamiltonian is given by:

$$ H_{Hf} = A_J I \cdot J + \frac{B_J [3(I \cdot J)^2 + 3/2I \cdot J - I(I + 1)J(J + 1)]}{2I(2I - 1)J(2J - 1)} $$

(2.23)

Here the $A_J$ and $B_J$ are the magnetic dipole and electric quadrupole constants for the specific hyperfine state of the atom. Values are given in Table 2.1. From these values we have the foundation of the knowledge of all of the electronic levels needed to trap and cool $^6$Li. Since magnetic fields are necessary to manipulate $^6$Li and make it do our bidding we shall look at that next.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic Dipole Constant</td>
<td>$A_{2^2S_{1/2}}$</td>
<td>152.1368407</td>
</tr>
<tr>
<td></td>
<td>$A_{2^2P_{1/2}}$</td>
<td>17.375</td>
</tr>
<tr>
<td></td>
<td>$A_{2^2P_{3/2}}$</td>
<td>-1.155</td>
</tr>
<tr>
<td>Electric Quadrupole Constant</td>
<td>$B_{2^2P_{3/2}}$</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

### 2.2.1 $^6$Li in a Magnetic Field

The use of magnetic fields are essential for studying ultracold atoms. For example if you have a MOT without a magnetic field you would just have an OT and it is very difficult to load atoms directly into an OT. Since the effects of magnetic field are integral not only in trapping and cooling, but also in adjusting interactions via a feshbach resonance, we need to spend a few minutes discussing them.

So, how exactly does the magnetic field come into play. Looking in your favorite quantum textbook [13] one will find that the application of a magnetic field will produce a Zeeman shift with a perturbation of the form:

$$ H_Z = -\mu_B (g_J J + g_I I) \cdot B. $$

(2.24)
Here we are working under the assumption that the Zeeman energy is comparable to the hyperfine splitting. At zero magnetic field we operate in the F basis where F and m_F are the good quantum numbers and states are label by |F, m_F⟩. Once a magnetic field is applied it lifts the degeneracy of the |F, m_F⟩ states into different Zeeman sublevels. Once the Zeeman energy becomes comparable to the hyperfine splitting of 228.2 MHz, which we quickly reach, we cannot treat the Zeeman term as a perturbation on the hyperfine Hamiltonian. We must diagonalize the full Hamiltonian:

\[ H = H_Z + H_{hf} \]
\[
H = -\mu_B (g_J J + g_I I) \cdot B + A_J J \cdot I + B_J [3(I \cdot J)^2 + 3/2 I \cdot J - I(I + 1)J(J + 1) / 2I(2I - 1)J(2J - 1)] 
\]

(2.25)

By diagonalizing this hamiltonian for the ground state 2^2S we get that there are six Zeeman sublevels that are labeled |1⟩ to |6⟩ in order from lowest to highest energy and are expressed in the |m_S, m_I⟩ basis [22]. For the ground state, and all states where L=0, the electric quadrupole constant ,B_j, is zero due to spherical symmetry.

\[
|1⟩ = \sin\Theta_+ |1/2, 0⟩ - \cos\Theta_+ | -1/2, 1⟩ \\
|2⟩ = \sin\Theta_- |1/2, -1⟩ - \cos\Theta_- | -1/2, 0⟩ \\
|3⟩ = | -1/2, -1⟩ \\
|4⟩ = \cos\Theta_- |1/2, -1⟩ + \sin\Theta_- | -1/2, 0⟩ \\
|5⟩ = \cos\Theta_+ |1/2, 0⟩ + \sin\Theta_+ | -1/2, 1⟩ \\
|6⟩ = |1/2, 1⟩
\]

(2.26)
Figure 2.2. The dependence on magnetic field of the lowest six Zeeman sublevels. Note how quickly the states enter the Pashen-Bach regime where they tune linearly with magnetic field. States $|1\rangle$ and $|3\rangle$ are highfield seeking states and are essentially spin-polarized up with the applied magnetic field and are high field seeking states. The other three states are spin-polarized down and are low field seeking states.

Where the cosine and sine terms are defined as:

$$\sin\Theta_\pm = \frac{1}{\sqrt{1 + (Z^\pm + R^\pm)^2/2}}$$

$$\cos\Theta_\pm = \sqrt{1 - (\sin\Theta_\pm)^2}$$

$$Z^\pm = \frac{\mu_B B}{A^2 S_{1/2}} (-g_J J_{2S_{1/2}} + g_I I_i) \pm \frac{1}{2}$$

$$R^\pm = \sqrt{(Z^\pm)^2 + 2}$$

By inserting these eigenstates into Hamiltonian 2.25 we can extract the energy levels of the states via the Briet-Rabi formula [23]. The results of this calculation are shown in figure 2.2. It is useful to note that at high fields the cos term approaches 1 and the sin term approaches 0.

From equation 2.27 we see that the $\cos\Theta_\pm$ terms approach unity at high fields. At these high fields the first three energy levels the states are mostly $m_S = -1/2$ or electron spin polarized down and vice versa for the other three states. As we reach these high magnetic fields the Zeeman energy becomes the dominant term in the Hamiltonian. We can then treat the hyperfine Hamiltonian as a perturbation.
This is known as the Pashen-Bach regime. Here the lines split into spectroscopic triplets where |1⟩ - |3⟩ are highfield seeking states and |4⟩ - |6⟩ are low field seeking states. Also equation 2.25 conserves the quantity \( m_F = m_S + m_I \) thus we can label the states by their \( m_F \) values: \((1/2, -1/2, -3/2, 1/2, 1/2, +3/2)\) for \(|1⟩ , |2⟩ , |3⟩ , |4⟩ , |5⟩ , |6⟩ \) respectively.

The same results can be applied to the excited states also. This gives us all knowledge of the energy levels of all the different ground and excited state. Using this knowledge we can make transitions between levels either optically as done when imaging or when making Raman transitions, or with RF fields for state manipulation. The consequence of \(^6\)Li ground state splitting into these triplets is that we can spectroscopically image these states individually without repumping. Magnetic fields also play a big role in scattering phenomena such as Feshbach resonances and will be discussed in more detail later.

### 2.2.2 Light Shifts: \(^6\)Li in an Electric Field

We use laser light to cool and trap our lithium atoms. Thus we must take into account how laser fields interact with the atoms. The effect of the laser is to apply a time-varying electric field upon the atom. This shift is called the light shift or the AC Stark shift. Our job is to then calculate how this field shifts the levels of the atom. To calculate this we use time-dependent perturbation theory to calculate the shifts. We will use the case of a two level atom and then calculate the shift. This treatment can be found in your favorite quantum or atomic physics book so I will highlight the results here \([19, 20, 13]\).

To start we have a perturbation to the atomic Hamiltonian caused by an electric field which is

\[
H_E = -e \vec{E}(\vec{r}, t) \cdot \vec{r}
\]  
\[
\vec{E}(\vec{r}, t) = E_0 \hat{\epsilon} \cos(kz - \omega_l t)
\]

Where \( e \) is the charge of the electron, \( \hat{\epsilon} \) is the polarization of the laser field with wave number \( k \) and frequency \( \omega_l \). This is the dipole term of the interaction. We are making the assumption here that the spatial variation of the electric field is large in comparison to the atomic wave functions. This is called the dipole approximation.
Larger multipole contributions also contribute, but they are negligible for our considerations. Also, to calculate the dynamics and energy levels of the system we use the rotating wave approximation which allows us to ignore terms that are proportional to $1/\omega_l$ compared to the terms $1/\delta$. Here $\delta$ is the detuning of the light from the atomic transition frequency, $\delta = \omega_{Atom} - \omega_{Laser}$.

In time dependent perturbation theory we write down the total wave function of our system as a sum of the wave function for each level of the atom, which form a complete set. We then solve the set of Schrödinger equations to extract the time varying coefficients for each level. Here we are considering a two level system so we have $c_g(t)$ and $c_e(t)$ for the ground and excited states of the transition. Like with most quantum mechanics the modulus squared of each coefficient gives us the probability finding the atom in that state. The sum of all of the probabilities is unity.

After running through the calculation we find that the probability of finding an atom in the excited state is

$$|c_e(t)|^2 = \frac{\Omega'^2}{\Omega'^2 + \delta^2} \sin^2 \left(\Omega' t \frac{\pi}{2}\right)$$

(2.30)

Where $\Omega$ is the Rabi frequency and $\Omega'$ is the effective Rabi Frequency given by.

$$\Omega = -\frac{eE_0}{\hbar} \langle e| r |g \rangle$$

(2.31)

$$\Omega'^2 = \Omega^2 + \delta^2$$

(2.32)

Where $\langle e| r |g \rangle$ is the dipole matrix element between ground and the excited state. For this discussion it is the electric dipole matrix element, but a similar treatment can be done for a magnetic dipole. The only thing that changes is what field is driving the transition, the specific selection rules for that Hamiltonian, and what matrix element to use when calculating the Rabi frequency. In practice one rarely calculates the Rabi frequency from the matrix elements, but does the experiment and then measures the Rabi frequency. It is worthwhile to note that $\Omega$ depends linearly on the amplitude of the field ($E_0$), and thus varies as the square root of the intensity.

Equation 2.30 also introduces us to the concept of a $\pi$-pulse. A $\pi$-pulse is the
pulse of the driving field such that all of the population is transferred from the initial to the final state. It is called such because the product of the amplitude and duration of the pulse is chosen such that $\Omega t = \pi$

Now that we have the coefficients for the states, we can then proceed to insert them back into the Schrödinger equation and calculate the energy shift associated with the application of the light. This involves diagonalizing the Hamiltonian which gives the light shift to be,

$$\Delta E_g = -\frac{\hbar \Omega^2}{4\delta}$$  \hspace{1cm} (2.33)

$$\Delta E_e = \frac{\hbar \Omega^2}{4\delta}$$  \hspace{1cm} (2.34)

$$\Delta E_e^\prime = \frac{\hbar \Omega^2}{4\delta}$$  \hspace{1cm} (2.35)

This is the case when $\delta \gg \Omega$. The more general expression is given by

$$E_{g,e} = \frac{\hbar}{2} (\delta \pm \Omega^\prime)$$  \hspace{1cm} (2.36)

We can see from these equations that when the laser frequency is red detuned $\delta > 0$ and the ground state is shifted to a lower energy. If we have a gradient of this light shift then we can have a force on the atoms. This is the dipole force that is used to trap the atoms. We can derive the light shift from the dipole force.

Starting from the time averaged force seen by the atoms [20] we can write down the dipole force as

$$F_z = -\frac{e \langle \epsilon | r | g \rangle}{2} \left\{ u \frac{\partial E_0}{\partial z} - v E_0 k \right\}$$  \hspace{1cm} (2.37)

Where $u$ and $v$ are the in-phase a quadrature components of the dipole in the frame rotating at the transition frequency $\omega$. They also define two axes of the Bloch sphere with the third axis being $w$, defined below. The three axes are defined from components of the density matrix $|\Psi\rangle\langle\Psi|$ where $\rho_{i,j} = c_i c_j^\dagger$ where the subscripts correspond to the ground or excited state. These components $u,v,$ and $w$ are defined as
\begin{align*}
u &= \rho_{12} + \rho_{21} & (2.38) \\
v &= i(\rho_{12} - \rho_{21}) & (2.39) \\
w &= \rho_{22} - \rho_{11} & (2.40)
\end{align*}

In 2.37 we see there are two components to the force on the atoms. The first is the dipole force and the second it the scattering force. And they are given by

\begin{align*}
F_{\text{dipole}} &= -\frac{\hbar \delta}{2} \frac{\Omega}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \frac{\partial \Omega}{\partial x} & (2.41) \\
F_{\text{scattering}} &= \hbar k \frac{\Gamma}{2} \frac{\Omega}{\delta^2 + \Omega^2/2 + \Gamma^2/4} & (2.42)
\end{align*}

Then for detuning far away from resonance \(|\delta \gg \Gamma|\) the force term can be written as

\begin{align*}
F_{\text{dipole}} &= -\hbar \Omega \frac{\partial \Omega}{\partial x} = -\frac{\partial}{\partial x} \left( \frac{\hbar \Omega^2}{4\delta} \right) = -\nabla U & (2.43)
\end{align*}

Which is the same expression for the light shift as 2.35. In this limit the dipole force dominates, but there is still some scattering of the light. The rate of scattering is

\begin{align*}
R_{\text{scattering}} \simeq \frac{\Gamma \Gamma^2}{8 \delta^2} \frac{I}{I_{\text{sat}}}. & (2.44)
\end{align*}

Here \(I_{\text{sat}}\) is the saturation intensity of the transition. Because of spontaneous emission as we increase the intensity of the light that is applied, the transition will broaden and the transition will start to saturate. The saturation intensity is given by

\begin{align*}
I_{\text{sat}} &= \frac{\pi \hbar c}{3 \lambda^3 \tau}. & (2.45)
\end{align*}

The saturation intensity for the \(D_2\) line is calculated to be 2.54 mW/cm\(^2\).
2.3 Transitions between different $^6\text{Li}$ Sublevels

$^6\text{Li}$

Having control over the different levels in a system is one of the luxuries of atomic physics. For our lithium system we can use driving RF or optical fields to populate different Zeeman sub-levels and create interesting atomic mixtures. Since $^6\text{Li}$ is a fermion, the only way to get interactions in our system at the temperatures we operate at is to populate different internal states. In our experiments we mainly use the three lowest Zeeman sublevels $|1\rangle$, $|2\rangle$, and $|3\rangle$. To transfer atomic population between these three states we use either a RF transition which is a magnetic dipole transition, or a Raman transition which is a two-photon electric dipole transition.

As is the case with most transitions there are certain selection rules that allow for certain transitions. The transitions we are concerned with are all dipole transitions. Since we have both electric and magnetic dipole transitions we shall look at the different selection rules individually.

For electric dipole transitions, the light is interacting with the electron’s orbital angular momentum and it does not directly couple to the spin. Thus we have the first selection rule that $\Delta S = 0$. Also since we have that the dipole operator is of odd parity, the transition must change the parity of the state. Thus we can’t make any nuclear transitions so $\Delta I = 0$ and $\Delta M_I = 0$. Since we are adding a photon with an angular momentum of 1 we can only make $\Delta L = \pm 1$ transitions. Depending on the projection of the angular momentum we can make $\Delta M_l = 0, \pm 1$ transitions for linear and $\sigma_\pm$ polarized light respectively. These are just statements of the angular momentum conservation. For $J$ we have the selection rules that $\Delta J = 0, \pm 1$. We can have the $\Delta J = 0$ transition because the spin is coupled to the angular momentum. If we go to high magnetic field $J$ and $m_J$ stop becoming good quantum numbers and we have to go back to the $L$ and $M_L$ selection rules. The same argument is applied for the hyperfine structure where we can have $\Delta F = 0, \pm 1$. If the initial and final state both have $J = 0$ or $F = 0$ then the transition is dipole forbidden because that would imply that $\Delta L = 0$ which is not allowed.

For the magnetic dipole transition we are only changing the angular momentum of the spin of the electron. In principle we can flip the spin of the nucleus. Since
Figure 2.3. The transition frequencies between the lowest three Zeeman levels in $^6$Li. The direct transition between $|1\rangle$ and $|3\rangle$ is not allowed for a one-photon transition.

2.3.1 Radio Frequency Transitions

The most common way to transfer atoms between different Zeeman states is to use radio frequency (RF) waves. As described above we can calculate from the Briet-Rabi formula the transition frequency between the different Zeeman levels. For the experiments I describe we only drive transitions between the lowest three Zeeman sublevels, $|1\rangle$ to $|2\rangle$ and $|2\rangle$ to $|3\rangle$. The frequency of these transitions are between 70 MHz and 90 MHz and are shown in figure 2.3.
As shown in the figure both frequencies asymptote to near 80 MHz at very high field. RF is much easier to work with than microwaves, thus we can just use a dipole antenna to broadcast into the chamber. Our system has the advantage that we have the RF coils inside our vacuum. This allows us to deliver a lot of power to the atoms. By using a 100W amplifier from Minicircuits we can obtain $\pi$-pulses on the order of 10 uS at fields of 500G. We create these field either by using Agilent frequency generators or by home-made amplifiers built following [24].

The reason the field was stated when quoting the $\pi$-pulse duration is that the transition strength changes with field. As stated we are making a magnetic dipole transition that changes $M_S$. As we can we see from 2.26 that in order make the $|1\rangle - |2\rangle$ and $|2\rangle - |3\rangle$ we are counting on $|1\rangle$ and $|2\rangle$ to be a mixture of states. At high fields the cos terms approach 1 and the sin terms approach 0. The transition between the lowest three states contains one sin term thus as we increase field the transition strength decreases.

In addition to transferring population between the states with resonant RF we also employ the use of adiabatic rapid passage ARP[19, 12]. In ARP a far-detuned-below-resonance RF field is initially applied to the atoms in the initial state. Then the RF frequency is swept through the transition to a final detuning above the resonance frequency. This sweeping of the field will transfer the atoms for the initial to the final state with great efficiency with the amount of population transferred given by

$$P_{|i\rangle - |f\rangle} = 1 - \exp[-2\pi \frac{\Omega^2}{\dot{\delta}}]$$

(2.46)

where $\dot{\delta}$ is rate of change of the detuning. To increase the efficiency of the transition it is important to ramp on the RF magnetic field to reduce its spectral weight. For our experiments we usually have $\Omega = 2\pi 1500$ Hz, so we turn on the RF field in a linear manner over 2ms and then sweep from $\delta = -4.5$ MHz to $\delta = 4.5$MHz in 10ms and ramp the field off in 2ms. This gives us a near 100% transfer (we do not observe any atoms left in the initial state.)

2.3.2 Raman transitions

The other way to drive transitions is to use two lasers that drive a two-photon Raman process. In a Raman process we have that each of the beams is detuned
Figure 2.4. A two photon Raman process to transfer atoms from $|1\rangle$ to $|2\rangle$. There is an overall detuning $\Delta$ from the intermediate state which can be either positive or negative. In our experiments we are detuned $\sim 4$GHz from the $M_J=3/2$ manifold of $D_2$ line for fields near 525 G. This is to minimize the differential light shift between $|1\rangle$ and $|2\rangle$.

from an intermediate excited level as shown in figure 2.4.

In transferring population between different states we end up with the same equation 2.30 except that we now have a different expression for $\Omega$. The two-photon transition is a transition from an initial state to a final state via intermediate states. The two lasers that drive the transition operate at frequencies $\omega_{L1}$ and $\omega_{L2}$. We define the detuning from each intermediate state $|i\rangle$ as $\Delta = \omega_1 + \omega_{L1} - \omega_i$. The thing to note is that the atom never spends any time in the intermediate state. Thus when calculating the Rabi frequency we take this into account. Then our Raman Rabi frequency between $|1\rangle$ and $|2\rangle$ becomes

$$\Omega_{\text{Raman}} = E_0 \sum_i \frac{\langle 2|er|i\rangle \langle i|er|1\rangle}{\hbar^2(\omega_i - \omega_1 - \omega_{L1})} + \frac{\langle 2|cr|i\rangle \langle i|cr|1\rangle}{\hbar^2(\omega_i - \omega_1 - \omega_{L2})}$$

(2.47)

where only one of the terms of the sum will be on resonance with the transition. Then if we use the expression for the Rabi frequency from above 2.31 we find that our Raman Rabi frequency is given by

$$\Omega = \frac{\Omega_{2,i} \Omega_{1,i}}{2\Delta_i}$$

(2.48)

where the $\Omega_{1,2,i}$ involves the matrix elements for the dipole transition to all possible intermediate states $|i\rangle$. Then all we have to do is keep track of all the possible transitions and the detuning from these transitions and we can find our Raman Rabi frequency. For the transition shown in figure 2.4 we of course have to conserve
angular momentum. Thus one of the beams must be $\pi$ polarized while the other is $\sigma^-$.  

### 2.4 Creating an Ultracold Gas

There are many ways to skin a cat. None of them involve lasers. Lasers are used more often to cool and trap atoms than cats. There are many different schemes, methods, or ways to do this and a more detailed account of these ways can be found in the literature [11, 19, 20]. I will describe the process that we employ to create an ultracold gas of $^6$Li starting from a hot beam of atoms in an oven. Then trapping those atoms in a magneto-optical trap (MOT). Loading the atoms from MOT to a dipole trap and then cooling them further by forced evaporative cooling. The main experimental system is shown below 2.5. 

The whole experimental chamber is under vacuum. We use standard vacuum techniques to create this vacuum. This starts by cleaning all components that will be in vacuum with acetone and then methanol. After the system is assembled a
rough mechanical pump is used to get down to $\sim 10^{-2}$ Torr where the turbo pump can be started. The turbo pump is used to get the system down to $\sim 10^{-7}$ Torr level where the ion pumps can take over. But before we run the ion pumps it is important to bake out the system. The baking out of the system involves heating the system up to a high temperature ($150^\circ - 250^\circ$ C). This heating up of the system increases the rate at which gases trapped in the surface can outgas and be pumped out of the system. If you have infinite time this step would not be necessary, but since we don’t have that luxury we must bake the system.

After the bake we then turn on the ion pumps, which continuously run, that take us down to the $10^{-10}$ level. We also have non-evaporable getters that absorb atoms to help create the vacuum. Also we use a titanium sublimation (Ti-sub) pump. The Ti-sub pump is only run occasionally to replenish a monolayer film of Ti atoms on the chamber walls which adsorb atoms/molecules continuously. The Ti-sub pump is run by applying 48 Amps of current through Ti filaments that are inside the vacuum system. At these high currents Ti atoms are emitted by the filament and coat the vacuum system. The Ti acts as a getter, absorbing atoms from the chamber thus assisting in obtaining a vacuum.

There are two main regions in the system, the MOT region and the oven region. Since we have to break vacuum to load more lithium we have installed a gate valve that can separate the two regions and change the oven without having to pump down the whole system. I direct you to Jason’s thesis for a more detailed description of the pumps in our systems [1]. The MOT region kept at a pressure of $\sim 10^{-10}$ Torr and the oven region is in the low $10^{-8}$ Torr when the ovens are at full temperature.

### 2.4.1 Lasers involved in the system

Of course we have lasers in our lab. We essentially have two different types of lasers in the lab. Lasers that operate near 671 nm which are used for the MOT, slower, and probe beams. The other set of lasers are far off resonance at 1070 nm and 1064 nm. These lasers are used as our dipole traps.
2.4.1.1 Near resonant lasers

These lasers are all near the $^6$Li resonance. All of these lasers are some subset of the Topitca DL100 laser which is a grating stabilized diode laser. There are many options that come with these lasers, but I will describe only two versions that we use. Both options allow for the piezo feedback to stabilize the frequency of the laser. The main difference between the two types of DL100s that we have is that one type has a FET that we can provide current feedback, in addition to piezo feedback, and the other does not. The current feedback to the FET allows for a more versatile and robust lock than just the piezo feedback.

There are two main laser systems anchored by the FET option DL100 in our lab. To produce enough power we send each of these beams through a tapered amplifier based off a TA0670 crystal purchased from Toptica. After isolation of $\sim 60$ dB we get between 375 mW and 400 mW. Some of the light deflected from the isolators are used to create a beat note signal that we can use to lock other lasers to these two systems: the Slower laser system and the MOT laser system. I like to think of these as our Master and subMaster laser systems. The slower is the master system because it is locked to the iodine cell that we use as our reference. We lock to a line in iodine which is 931 MHz to the red of the $^6$Li D$_2$ line. This allows us to produce laser light at 814 MHz red-detuned from the D$_2$ line which is used as our slower light. I direct you to Jason William’s thesis [1] for a more detailed description of the laser system. We use saturated absorption spectroscopy to lock to this iodine line. This involved modulating an AO and then using a lock-in amplifier to produce the line to lock to. We then have our own feedback electronics that uses a PID controller to lock to this signal $^2$.

Now that we have one laser locked to a reference we can lock other lasers to it. The way we do this is to interfere light from the slower laser with light from the MOT laser. Then we can detect the beat signal that is produced and by comparing that beat signal with an RF signal that we provide we can lock the MOT laser to Slower laser frequency offset by a multiple of 8, 16, 32, or 64 times the reference RF signal we provide. By changing this reference RF signal during the experiment we can tune the laser over a GHz range. We use an Analog Devices ADF4007

$^2$Appendix B contains the schematics of all of our home built electronics.
evaluation board and more locking electronics to create an error signal with which we can lock the laser with feedback both to the piezo and the FET that controls the current. All of the DL100’s that we use are locked to either the slower or MOT laser system using this procedure. This procedure that we use is outlined in [25]. This type of locking can phase lock the lasers when the system is fully peak up, but this phase locking is not currently necessary for the experiments described here. As described above, the energy levels of lithium greatly tune in a magnetic field. Since we use the MOT laser for not only the MOT, but also the imaging we must be tune over this long range. To speed this locking up we also apply a feed forward voltage to the piezo. The feed forward puts the voltage to the piezo near where it needs to be to produce the proper frequency and then the lock takes over from there.

2.4.1.2 Far off Resonance Lasers

The far off resonance lasers that we have are a 1064nm 110W IPG Photonics multimode laser and a 1070nm 50W laser. These lasers are free running in the sense we don’t lock them. The 1064nm laser is the workhorse of the lab. If I learned one thing about IPG it is that these multimode lasers are robust. The single frequency lasers on the other hand are very touchy and very susceptible to back reflections, even when isolated properly. We actually had multiple of them break on us.

2.4.2 Oven to MOT: 750 K to T < mK

The source of our $^6$Li is our oven. The oven consists of a bottom part that we keep at 435°C when operating and a top part which is kept at 465°C during operation. This temperature difference helps keep lithium in the bottom part of the oven. Each part is heated via band heaters and controlled via Omega temperature controllers. When the system is off we set the top part of the oven to 225°C and turn the bottom part of the oven off.

The bottom part is where $^6$Li is loaded into our vacuum chamber. It is a sealed off mini-conflat nipple. We machine an aluminum housing that fits around this oven and to which a band heater is attached. The top part of the oven consists
of a mini-conflat elbow which is connected to a blank mini-conflat flange with a 0.25 inch hole drilled in the center and then also connected to a mini-conflat nipple whose outside diameter is machined down 0.1 inch which acts as a thermal break. This break lets us keep the oven at 465° without heating up the rest of the oven region of the vacuum chamber. An aluminum housing is also machined for the elbow and drilled blank flange in order to attach the band heaters to this part of the oven.

In the bottom region of the oven we have liquid lithium and at the top we have lithium vapor that is around a pressure of $3 \times 10^{-3}$ Torr at 465°. This will lead to an effusive flux of around $8 \times 10^{10}$ atoms/s coming out of the 0.25 diameter hole at a mean speed of 1610 m/s. Of course our goal is to obtain a gas at temperatures of a few mK (1 mK is 1.8 m/s) but the fact that we have this fast atomic beam is actually a good start.

So, we have this beam of really fast atoms heading to our MOT chamber. How do we stop them? Use lasers of course, with the help of a magnetic field. As they travel down the chamber the atoms absorb and re-emit photons from the slower laser, which transfers momentum to the atom. The momentum from the absorbed photon is anti-parallel to the velocity of the atoms, but the emitted photon can go in any direction. This has been equated to slowing a semi down by means of using many ping pong balls. Although I have yet to see this occur in the truck case we can see it happen with the atoms on a daily basis. Since the atoms are flying down the vacuum system at a non-zero velocity their energy levels are doppler shifted. We use a laser that operates at a single frequency which means that we can only slow down atoms that are on resonance with this light in the moving frame of the atoms. Thus we must create a way for the atoms to continuously absorb photons as they travel down to where we trap them in the MOT. This can be done by either varying the laser frequency or shifting the energy levels of the atoms with a magnetic field. We choose the later for our slower. By applying a spatially varying magnetic field which will Zeeman shift the energy levels of the atoms into resonance as they traverse the chamber. Thus we have a Zeeman slower.

The Zeeman slower consists of a laser beam that is -814 MHz detuned from the D$_2$ line of $^6$Li. We put as much power into this beam as possible which is
Figure 2.6. Side view of the experimental chamber. Also shown is the magnetic field used for the Zeeman slowing of the atoms. Figure created by Jason Williams [1]

about 100mW of power. This laser light also has sidebands at 228.2 MHz, the hyperfine splitting of the $^2S$ state for repumping, and it is $\sigma^+$ polarized. The main slowing transition is between the $F=3/2$ to $F'=5/2$ states which is closed for fields in the Paschen-Bach regime. At lower magnetic fields the transition is not closed and the atoms can fall back into the $F=1/2$ level. This is why we have repumper light applied via that Electric-Optic Modulator to optically pump them back into the $F=3/2$ state. The magnetic field is created by running 130 amps of current through water cooled copper wire wrapped at different densities gives us the magnetic field profile shown in figure 2.6. Putting it all together, $^6$Li atoms continuously absorb photons and slow down to speeds of the order of 30 m/s where we can trap them in our magneto-optical trap.

---

3Atoms that are not trapped hit the window that allows the slower laser to enter the chamber. Over time this gets coated with lithium. This obviously decreases the amount of slower light (or any other light that passes through this window) that enters the chamber. We found that by shining blue light, we use 405 nm LEDs, we can desorb the atoms and clean the window. It does not entirely clean it, but noticeable improvement is easily seen.
2.4.3 From MOT to Trapped

Now that we have this cold gas we are ready to load our magneto-optical trap. The magneto-optical trap marked a very big advance for the field. It starts with an optical molasses setup [?, 26] where there are three sets of counter propagating laser beams that are orthogonal to each other and that are red-detuned from the transition. An atom moving with a non-zero momentum will be Doppler shifted into resonance with the laser beam that will give it a momentum kick opposite its direction of motion. The force applied to the atoms is the scattering force which is given in equation 2.42, but written here in terms of the saturation intensity $I_{\text{sat}}$ and taking the assumption that we are at low velocities and including the Doppler shift in the detuning [?]. The total force experienced by the atom due to the two counter propagating beams at a detuning $\delta$ from the transition is

$$F = 4\hbar k^2 \frac{I}{I_{\text{sat}}} \frac{2\delta/\Gamma}{[1 + (2\delta/\Gamma)^2]^2} \cdot v.$$ (2.49)

This is the same as a viscous damping force $F = -\alpha v$ which is why the moniker "optical molasses" makes sense. Since there is heating due to scattering and cooling due to the viscous force we see we can balance these two rate to get the temperature of the gas.

$$T = \frac{\hbar \Gamma}{4k_B} \frac{1 + (2\delta/\Gamma)^2}{2\delta/\Gamma}$$ (2.50)

Optimal cooling is done when we set the detuning of the laser field to

$$\delta = -\frac{\Gamma}{2}.$$ (2.51)

Which for the $D_2$ line of $^6$Li we get a cooling limit of $T=142\mu$K. We don’t quite get down to that level, but we do get on the order of 300 uK temperatures.

Of course one of the key components of a MOT is the magnetic field. By using anti-Helmholtz coils we can create linear gradients in all directions about the center of the MOT. Then, due to the Zeeman effect, the energy levels of the different sublevels will be shifted. With the red detuned beams when an atom wanders too far from the center of the trap it becomes on resonance only with the correct laser field that will push it back. This requires that the polarization of the
different beams be correctly chosen [19, 20].

We cool the atoms on the F=3/2 to the $D_2$ line. As stated before the $D_2$ hyperfine lines are not resolved for low fields which is the case in our MOT. Thus we must add a repumper beam that will transfer the atoms that fall to the F=1/2 level back up to be cooled. At the end of our MOT cooling stage we actually turn off the repump beam to optically pump all to the atoms into that F=1/2 level which when we apply a magnetic field we will have atoms in $|1\rangle$ and $|2\rangle$.

For our MOT procedure we start with a large detuning on the order of $5\Gamma$. This allows for a large capture volume, but a high temperature. Then right before we turn off the MOT to load the dipole trap we tune the frequency to a detuning near $\Gamma/2$ to cool the atoms even more. Then we ramp up a crossed dipole trap formed by two 1064 nm beam that are crossed at an angle of 12° from co-propagating. We discussed the dipole force above in section 2.2.2. The intensity profile of the a gaussian beam is given by the equation,

$$I(r, z) = \frac{I_0}{1 + (z/z_0)^2} \exp\left(\frac{2x^2}{w_0^2(1 + (z/z_0)^2)}\right). \tag{2.52}$$

Where the beam is characterized by the position of the focus $z = 0$ and the waist $w_0$ is the $1/e^2$ radius of the beam at $z = 0$. The parameter $z_0$ is the Rayleigh range which is the $z$ distance that it take for the intensity to double in size and is given by $z_0 = \frac{\pi w_0}{\lambda}$. From the knowledge of the focal position, wavelength, and $w_0$ we can determine the shape of the beam at any position $z$ and can propagate it through lenses and other optical elements [27]. The term $I_0$ is the intensity of the beam at $z = 0$ and is given by $I_0 = 2P/(\pi w_0^2)$, where $P$ is the power of the beam.

Since the light shift is proportional to the intensity of the light we can Taylor expand the intensity of the laser light and get the potential that the atoms see to be,

$$U(r, z) = U_0\left(1 - \frac{z^2}{z_0} - 2\frac{r^2}{w_0^2} + \ldots\right). \tag{2.53}$$

where $U_0$ is the trap depth. Thus we have a harmonic potential that the atoms see in the radial and axial directions. Using this potential we can see that the harmonic oscillator frequencies in the radial and axial directions are given by
\[ \omega_r = \sqrt{\frac{2U_0}{mw_0^2}} \]  
\[ \omega_z = \sqrt{\frac{2U_0}{mz_0^2}}. \]  

If there are multiple beams creating the dipole trap then the trap frequencies are given by \( \omega_{total}^2 = \omega_1^2 + \omega_2^2 + \ldots \).

So after loading the gas into the dipole trap we still need to make the gas colder. To do this we perform forced evaporative cooling on the gas. This works essentially in the same manner as your hot coffee becomes cold. From statistical mechanics we know that we have distribution of energies in our gas. If we remove the hottest atoms and let the atoms re-thermalize then the final temperature becomes lower. This requires the loss of atoms, but also their re-thermalization. Since we have fermions we must have two different sublevels populated in order to have collisions which re-thermalizes the gas. Also the stronger the interactions are, the faster the rethermalization. We use a Feshbach resonance to tune the interactions and perform evaporative cooling near 315 G or 840 G where the interactions are the strongest.

Now we have a cold gas at the temperatures that we want for the experiment. From there all that we need to do is to adjust our experimental parameters for the experiment that we want to perform and image the gas.

### 2.4.4 Imaging the Gas

So far, we have covered how to create our gas but not how to take data. The way that we do it is to take an absorption image with a CCD camera. For our experiments we image the gas at high field where, as discussed earlier, the \( m_J \) basis is the best way to describe the atoms. We only image \( |1\rangle \), \( |2\rangle \), or \( |3\rangle \) thus for these states we have \( m_J = -1/2 \). Then we use \( \sigma_- \) light tuned to the \( D_2 \) line. This means that we only couple the \( m_J = -3/2 \) level. This transition is a closed transition because the excited state can only decay back down to the original state it started in. Thus there is no need for optical pumping. If we do not saturate the
transition we get an image which is darker depending on optical thickness caused
by the atoms absorbing the light. Thus we operate at \( I/I_{\text{sat}} \simeq 0.1 \). The amount of
light reaching the camera is

\[
I(x, y) = I_0(x, y)e^{-D(x,y)}. \tag{2.55}
\]

\( D(x,y) \) is optical density which is related to the optical cross-section \( (\sigma_0) \) and the
number of atoms the laser passes through, or the column density. This gives us
that \( D(x, y) = \sigma_0 n_z(x, y) \). Thus the column density \( n_z(x, y) \) is the number of
atoms in the light passes through in the z-direction at position \( (x,y) \). This gives
us the profile of the cloud from which we can extract the temperature \cite{1}.

\[
T \simeq \frac{m\sigma_0^2}{2k_BT^2} \tag{2.56}
\]

where \( \sigma \) is the fitted width of the imaged cloud. To use the expression for the
temperature we must allow the gas to ballistically expand for a time that is much
longer than the time scale associated with the harmonic oscillator time scale of the
trap.

In practice we do not use one image, but multiple images to construct the
density profile. In fact we take four intensity pictures, an absorption image \( I_{\text{abs}} \) in
which we image the atoms, a reference image which is the same as the absorption
image, but with no atoms \( I_{\text{ref}} \), and two background images with no probe light
\( I_{\text{bkgnd}} \). Then we compose these images and construct the picture that we analyze.
These four images are composed into one by the equation

\[
T(x, y) = \frac{I_{\text{abs}} - I_{\text{bkgnd}}}{I_{\text{ref}} - I_{\text{bkgnd}}} \tag{2.57}
\]

Then the column density that contains atom number information is

\[
n_z(x, y) = -\frac{1}{\sigma_0 M} \ln(T(x, y_0)) \tag{2.58}
\]

where \( M \) is the magnification of our system.

This gives us all the background tools needed to discuss the experiments per-
formed. Of course each experiment requires different parameters to create the
desired system. These are just the experimental details of each experiment, but
they all stem from the topics covered in this chapter, so I hope good notes were taken.
Chapter 3

Theory of Feshbach Resonances, Broad and Narrow

3.1 Feshbach Resonances

With relative ease we can adjust the laser beams to create different potentials for the atoms such as optical lattices in many different configurations. While this gives us a high degree of flexibility we demand more. We want to not only control in potential, but also the interactions between atoms. Luckily we have a Feshbach resonances that allow us to do just that.

The basics physics behind a Feshbach resonance can be seen by looking at the different scattering channels and the potentials associated with them. We define a scattering channel as either being open or closed. The open channel or entrance channel is the channel in which the atoms have enough energy in which to scatter. The atoms are allowed to energetically enter and leave this state. The closed channel is a scattering channel in which the two atoms involved cannot return to the open channel without additional energy being added to the system. A Feshbach resonance occurs when atoms colliding with energy $E$ in the entrance channel becomes becomes energetically on resonance with a bound state of the closed channel potential as pictorially shown in figure 3.1. Since the open and closed channel have different magnetic moments they tune differently with magnetic field. This allows us to tune the magnetic field to access the resonance. A scattering resonance, such
Figure 3.1. Cartoon depiction of a Feshbach resonance. Two atoms, with energy $E$, collide via the entrance channel $V_{\text{open}}(r)$. A Feshbach resonance occurs when their energy $E$ becomes on resonance with the bound state of the closed channel potential $V_{\text{closed}}$.

as Feshbach resonance, is characterized when the scattering phase shift changes by $\sim \pi$. As described in Chapter 2, the scattering length depends on the phase shift caused by the scattering event. Since the scattering length is also a measure of the interaction strength, the Feshbach resonance also allows us to tune the interactions in the gas. Now that we have a qualitative picture let's get down to the some of the nitty-gritty details. The physics of the Feshbach resonances contains many detail and of which I will give a description of main results. For a more thorough description of all the details see the review article by Chin, Grimm, Julienne, and Tiesinga [7].

Above we described the concept of open and closed channel discussed briefly how they bring about a Feshbach resonance. As discussed earlier the $2^3S$ ground state splits into six different Zeeman levels 2.26. Since these states are admixture of different $|m_S, m_I\rangle$ projections the collisions between them will require the projection onto both singlet and triplet scattering potentials [17]. These two potentials arise from the possible electronic configuration of the two atoms involved.
in the scattering event. For most cold atom experiments the two atoms involved in a collision are in the electronic ground state with angular momentum $L = 0$ and one valance electron which has a spin of $S = 1/2$. Thus the potentials can be of the singlet nature where $S = S_1 + S_2 = 0$ or of the triplet potential $S = S_1 + S_2 = 1$. For interactions in a central potential angular momentum is conserved, also we will look at collisions at a finite magnetic field the magnetic quantum number $M = m_{f1} + m_{f2}$ must also be conserved in the collision. Now let us look into a scattering event between a $|1\rangle$ atom and $|2\rangle$ atom. This has a magnetic quantum number of $M=0$. Thus the possible scattering channels with $M=0$ are $\{|1\rangle, |2\rangle\}, \{|1\rangle, |4\rangle\}, \{|2\rangle, |5\rangle\}, \{|3\rangle, |6\rangle\}$, and $\{|4\rangle, |5\rangle\}$. For the temperatures that we are concerned with $T<1$ mK atoms entering the scattering event on the $\{|1\rangle, |2\rangle\}$ channel does not possess enough energy to couple to the other ”open” channels.

To investigate the Feshbach resonance between these levels we look at the last vibrational bound state $v=38$ of the singlet potential $^1\Sigma_g^+$. This state has a possible nuclear spin of $I = 0$ or $I = 2$ both of which have a state with $M_I = 0$. These two states are associated with the singlet scattering length of $a=45.17a_0$ [28, 10]. If we
look at the last bound state of the triplet potential \( ^3\Sigma_u^+ \) we find that there are three \( M=0 \) states of the v=9 vibrational level that bring rise to large triplet scattering length of \(-2140a_0 \) [28, 10].

Now that we have our open channel \( \left\{ |1\rangle, |2\rangle \right\} \) and the closed channel (the \( M=0 \) levels of the v=38 vibrational level of \( ^1\Sigma_g^+ \)) we are ready to discuss the consequences on these potentials on the scattering lengths. As discussed in Chapter 2 the scattering properties we need to calculate the phase shifts. We can describe the phase shift associated with this resonance as a phase shift with a contribution from the background in the open channels (from the singlet and triplet potentials) and a term due to the Feshbach resonance.

\[
\delta(E) = \delta_{\text{bkgnd}}(E) + \delta_{\text{res}}(E) \tag{3.1}
\]

The phase shift \( \delta_{\text{res}}(E) \) is given by the Briet-Wigner form [7, 29]

\[
\delta_{\text{res}}(E) = \tan^{-1}\left(\frac{\frac{1}{2}\Gamma(E)}{E - E_c - \delta E(E)}\right), \tag{3.2}
\]

where \( \Gamma(E_c) \) is the energy dependent width, \( E_c \) is the energy of the closed channel bound state and \( \delta E(E) \) is the how the bound state tunes with energy. For low energy scattering where \( k \to 0 \) we can express the width and energy as energy independent parameters: \( \frac{1}{2}\Gamma(E) \to (ka_{\text{bkgnd}})\Gamma_0 \) and \( E_c + \delta E(E) \to E_0 \). If we also add a decay rate (which occurs in optical feshbach resonances, but is neglected in magnetically tuned resonances) \( \gamma/2 \) we can write the scattering length as

\[
a(E) = a_{\text{bkgnd}} \left(1 + \frac{\Gamma_0}{-E_0 + i\gamma/2}\right) \tag{3.3}
\]

Then what we have to do is take into account how the resonance tunes. For a magnetically tunable resonance the energy of the bound state tune in respect to the energy of the entrance channel by the difference in magnetic moments between the two

\[
E_c = \delta \mu(B - B_c) \tag{3.4}
\]

Where \( B_c \) is the magnetic field where the bound state crosses threshold. Then
Figure 3.3. Scattering length for the pair-wise interactions of the three lowest Zeeman levels in $^6$Li showing the location of the Feshbach resonances [2]. The locations of the resonances are near 834G, 811G, and 690G for $a_{12}, a_{23}$, and $a_{13}$ respectively. The inset shows a zoomed in view of the narrow Feshbach resonance at 543.286G[3]

using 3.3 we arrive at the well known expression for the scattering length,

$$a(B) = a_{bkgnd} \left(1 - \frac{\Delta}{B - B_0}\right)$$  \hspace{1cm} (3.5)

where $\Delta = \Gamma/\delta \mu$ and $B_0 = B_c + \delta B$. For the case of $^6$Li $\delta \mu = 2\mu_B$. The same treatment can be done for the other pairwise interacting between $|1\rangle$ and $|3\rangle$ and the interactions between $|2\rangle$ and $|3\rangle$. Since there are only slight differences between these states the location and widths of these Feshbach resonances are similar, except that the $|1\rangle - |2\rangle$ resonance is the only one that displays the narrow resonance. The three overlapping Feshbach resonances for the lowest three Zeeman levels of $^6$Li are shown in figure 3.3.

This gives us insight how we can use a Feshbach resonance tune the scattering length and the interactions. The relation between the interaction energy and the
scattering length is [30]
\[ U = \frac{4\pi \hbar^2 a}{m}. \] (3.6)

The scattering length can be positive or negative for repulsive or attractive interactions respectively. The consequences of this have been a boon for the field. It has opened up the ability to study strong interactions, superfluidity, molecular formation and Bose-Einstein condensation of those molecules, BEC-BCS crossover physics, universal physics at unitarity, and the list goes on [31, 32, 33, 34, 35, 7, 11, 12, 36, 37, 38, 39, 40] Some of this phenomena will be discussed later on in this chapter.

Let us return the narrow and broad Feshbach resonances associated with the \( M_I = 0 \) bound states of the singlet potential with \( I = 0, 2 \). The question arises is, why does one cause a narrow resonance and the other causes a broad resonance. For this it is helpful to define a few length and energy scales [41] based off the van der Waals length \( R_{vdW} = 31.26a_0 \) for \(^6\)Li. If we define a mean scattering length for \( R_{vdW} \) and the energy associated with it as

\[
\bar{a} = \left( \frac{4\pi}{\Gamma(1/4)} \right) \approx 0.956 R_{vdW} \\
\bar{E} = \frac{\hbar^2}{2\mu \bar{a}^2}
\] (3.7)

where \( \Gamma(x) \) is the gamma function of \( x \). Following the formalism of [7] we can define a strength parameter

\[ s_{res} = \frac{a_{bkgnd} \delta \mu \Gamma_0}{\bar{E}}. \] (3.8)

The values of \( s_{res} \) are given in the table 3.1. For the broad resonance we see that we have \( s_{res} = 59 \) and for the narrow \( s_{res} = 0.001 \). From this it is clear that broad resonances are classified as those which \( s_{res} \gg 1 \) and narrow resonances are those which \( s_{res} \ll 1 \). Which it is usually the case that a narrow width causes

\[ ^1 \text{There are numerous theoretical and experimental works on this subject. It would be a thesis in it self to describe them all here. As of this thesis being written I direct you to the works from the labs of Grimm, Ketterle, Jin, Zwierlein, Thomas, O’Hara, Hulet, Salomon, Esslinger, Bloch, and Jochim for some good insight into these works.} \]
Table 3.1. Properties of the Feshbach resonances in $^6\text{Li}$. These values were collected from [7, 8, 9, 3, 10]

<table>
<thead>
<tr>
<th>Scattering Channel</th>
<th>Partial Wave</th>
<th>$B_0 (\text{G})$</th>
<th>$\Delta$</th>
<th>$s_{\text{res}}$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>1\rangle\langle2</td>
<td>$</td>
<td>s</td>
<td>834.1</td>
<td>-300</td>
</tr>
<tr>
<td>$</td>
<td>1\rangle\langle2</td>
<td>$</td>
<td>s</td>
<td>543.286(3)</td>
<td>0.1</td>
</tr>
<tr>
<td>$</td>
<td>2\rangle\langle3</td>
<td>$</td>
<td>s</td>
<td>811.2</td>
<td>-222.3</td>
</tr>
<tr>
<td>$</td>
<td>1\rangle\langle3</td>
<td>$</td>
<td>s</td>
<td>690.4</td>
<td>-122.3</td>
</tr>
<tr>
<td>$</td>
<td>1\rangle\langle1</td>
<td>$</td>
<td>p</td>
<td>159.14</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>1\rangle\langle2</td>
<td>$</td>
<td>p</td>
<td>185.09</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>2\rangle\langle2</td>
<td>$</td>
<td>p</td>
<td>214.94</td>
<td></td>
</tr>
</tbody>
</table>

corresponds to a narrow resonance that is not always true, but using the $s_{\text{res}}$ criterion we can better classify the these two different types of resonances.

Now that we have a parameter to classify the resonances it still doesn’t really solve the question of what makes a resonance narrow? To answer this let us think about the coupling between the closed channel and the open channel. In order to have a resonance at all these two channels must have the same spin characteristics or else we would violate conservation of angular momentum. The strength of this coupling will which is essentially what $s_{\text{res}}$ gives us. This is why it is a good way to characterize the resonance. Another way to think of these resonances is to determine over what range do the molecular states have the spin characteristic of the open channel. The greater the range the broader the resonance. Conversely if the closed channel fraction is high over a wide range then the resonance is narrow. Near the resonance, where $a \to \infty$ we can express the closed channel fraction as

$$Z = \frac{1}{\zeta} \left| \frac{B - B_0}{\Delta} \right| \quad (3.9)$$

$$\zeta = \frac{1}{2} s_{\text{res}}|r_{\text{bg}}| \quad (3.10)$$

Where values of $\zeta$ are given in table 3.1. Thus for the narrow resonance the closed channel fraction is only small near the resonance and for the broad resonances the closed channel fraction is low over a broad range.

As discussed above the M=0 states of $I = 2$ and $I = 0$ states cause the narrow and the broad resonance. The narrow resonance is small because the $I = 2$ state
retains its closed channel character as we cross the resonance near 543G where in only weakly couples to the entrance channel. But, for the broad resonance the \( I = 0 \) state couples strongly to the entrance channel. This coupling is strong enough that it changes its spin character to that of a universal halo-dimer that is of entrance channel character and couples strongly with the entrance channel and crosses threshold at the resonance location at 834 G. This state allows for molecular formation which will be discussed below.

### 3.1.1 Physics near Broad Feshbach Resonances

The broad Feshbach resonances are important for they allow for the tuning of the scattering length without the need for stable magnetic fields. All that you need is a good enough power supply to provide the currents to create those fields. For our experiment we need about 700 Amps through our coils to produce a field of 1000G.

As discussed before we need interactions to in order for the gas thermalize during evaporative cooling. It is better to have attractive interactions, because for large repulsive interactions the atoms undergo three-body recombination that causes large loss rates in the atoms [42]. Thus for our experiments we tend perform evaporative cooling at fields near 315 G or 900G.

This broad resonance creates a wide field range where the scattering length is large. When the scattering length becomes this large the details of the scattering potential drop out of the problem and only the scattering length becomes important. This is called the unitarity regime. This has the important consequence that the physics here becomes universal in the since that it doesn’t matter what species of atom we use, we will extract the same physics from the system. Around this resonance we see that we have both positive and negative scattering lengths that are large thus we have repulsive and attractive interactions. The pairing mechanism for the positive scattering length side is to create molecules, which are composite bosons (fermion+fermion=boson), and then the ground state at zero temperature is of the Bose-Einstein condensation (BEC) nature. For negative scattering length the there are no bound molecular states below threshold. Here the pairing between two different species is governed by the physics of Cooper pairing which is the
same physics as conventional Bardeen-Cooper-Schrieffer (BCS) superconductors. The broad Feshbach resonance then allows us to turn between the two regimes. This is called the BEC-BCS crossover and is the subject of many experiment and theoretical pursuits [11, 12, 43, 34].

For the positive scattering length there exists a bound molecular state that is a weakly bound "halo" molecule with the binding energy $E = \hbar^2/(2ma^2)$. These are halo molecules because the separation between the two atom species that comprise the molecule have a characteristic length scale on the order of the scattering length is quite large. These molecules are the ground state of the system and can be created via two different ways. One method is to cool the atomic gas down into the molecular state. Here the molecules are created by three-body combination where two atoms that couple to the molecular state collide with a third free atom that takes the binding energy away [44]. The other way to create the molecules is to start at a magnetic field above the Feshbach resonance and adiabatic sweep the field across the Feshbach resonance. This way the open scattering channel will be adiabatically couple to the molecular closed channel as the gas is swept across resonance. Since the sweep is done adiabatically the gas will follow the ground state of the system since the sweep is done slow enough, the coupling to the molecular state is strong, and the binding energy of the molecular state approaches zero at the location of the Feshbach resonance we can associate molecules in this manner [12].

In our lab we have created molecules via both mechanisms. A series of false color images that show the creation of a molecular BEC is shown in figure 3.4. In this experiment we start with a $|1\rangle - |2\rangle$ mixture at a field of 850G. We then adiabatically cross the Feshbach resonance and sweep to a field near 800G on the BCS side of the resonance. This sweep associated molecules. After the sweep we perform additional evaporation on the molecular gas to varying degrees. Once we reach the critical temperature for BEC we start to see a bimodal distribution. With continued evaporation we see that a we can create an almost pure BEC.

On the BCS side of the resonance the physics is just as rich. Of course the BCS mechanism is responsible for superconductivity in type-1 superconductors. What happens when you have BCS pairing in a gas that is not charged? Instead of superconductivity you get superfluidity. This is the case for $^6$Li gas with negative
Figure 3.4. The creation of molecular BEC in $^6$Li. These are false color absorption images of $|1\rangle$ at a magnetic field near 800 G. Evaporative cooling was performed to various degrees with the top right image being the hottest cloud and the bottom right image being the coldest. In the sixth image a bimodal distribution can be seen to start to form. The dense part of the cloud corresponds to the creation of a molecular BEC. In the final image we can see an almost pure BEC created. Since the molecules are of entrance channel character we can image them via the same procedure as that for free atoms.

scattering lengths on the BCS side of the resonance. Evidence of superfluidity has been shown by observing the collective modes of the gas [38], observation of the BCS pairing gap [37], and observation of vortices [39].

### 3.1.2 Physics near Narrow Feshbach Resonances

Above I discussed the basics of scattering near a Feshbach resonance in a general manner. Here we will go into more depth into some of the physics near a narrow Feshbach resonance following [45, 3]. First let us define some notation. Here I will define $\mu_r$ as the difference between the magnetic moment of the open and closed
channels which for our case is $\mu_r = 2\mu_B$. We can also introduce the width of the
resonance in a different manner where $\Gamma(B) = \mu_r(B_0 - B_\infty)$ which defines $B_0$ and
then we define the location of the resonance $B_\infty$ by the relation

$$a(B) = a_{bkgnd} \left( \frac{\mu_r(B - B_0)}{\mu_r(B - B_\infty)} \right).$$

Then we go back to our Breit-Wigner form of the phase shift from equation 3.2 to calculate the phase associated with the resonance. Figure 3.5 shows the $\sin^2(\delta)$ for both the broad and narrow Feshbach resonances in $^6$Li for comparison. One can easily see the main difference between the two resonances. For the broad resonance the only real way to cross the resonance is by sweeping the magnetic field. Although it does have some energy dependance then wide nature of it smears it out and makes it experimentally very difficult to create a gas that can cross the resonance energetically. On the other hand because of the small width of the narrow resonance we can see the resonance can be crossed not only by means of a magnetic field sweep, but by also tuning the energy of the gas. Thus if a gas was prepared with a temperature of 35$\mu$K then as the magnetic field is swept the gas would cross the resonance near 543.5 G instead of 543.286 G for a 0K gas.

As discussed above since the closed channel bound state couples very weakly with the entrance channel, for the narrow resonance, the bound state retains its spin character across the resonance. After crossing threshold this state becomes imbedded in the continuum and still be coupled to if the energy of the collision is on resonance. This is why the narrow resonance is very energy dependent. Since, we have this molecular state that can be coupled this opens up the possibility of three-body recombination loss due to coupling to this state. In contrast to the broad resonance this loss would occur on the BCS or atomic side of the resonance where we have this molecular state in the continuum. The observation of such features will be discussed in the next chapter describing the experiment.

The phase shift in figure 3.5 is derived from looking at the energy dependance of the scattering amplitude from equations 2.17 and 2.18 we have that $1/a(k) = k \cot(\delta(k))$. Using our the expression for the phase given in equations 3.1 and 3.2 and using the energy dependent scattering length given by [45],
Figure 3.5. Plots of $\sin^2(\delta)$ for the Feshbach resonances in $^6$Li. A) shows $\sin^2(\delta)$ for the broad resonance located at 834G. B) shows $\sin^2(\delta)$ for the narrow resonance at 542.286G. Both plots have the same color scale. It is clear that for the narrow resonance we have a pronounced energy dependence due to the molecular state being imbedded in the continuum.

\[ a(k, B) = a_{bgnd} \left( \frac{\hbar^2 k^2}{m} - \mu_r(B - B_0) \right) \left( \frac{\hbar^2 k^2}{m} - \mu_r(B - B_\infty) \right)^{-1} \]  \hspace{1cm} (3.12)

we can express the scattering phase shift relation by

\[ k \cot(\delta) = -\frac{1}{a_{bgnd}} \left( \frac{\hbar^2 k^2}{m} - \mu_r(B - B_\infty) \right) \left( \frac{\hbar^2 k^2}{m} - \mu_r(B - B_\infty) + \mu_r \Delta \right)^{-1}. \]  \hspace{1cm} (3.13)

This expression is obtained by adding and subtracting $\mu_R B_\infty$ from the numerator of equation 3.12. If we look at the expression for the scattering length as $k \to 0$ and near the resonance location where $\mu_r \Delta \gg \frac{\hbar^2 k^2}{m}$ and $\mu_r(B - B_\infty)$ we can then rewrite equation 3.13 as

\[ k \cot(\delta) = -\frac{(B_\infty - B)}{a_{bgnd} \mu_r \Delta} - \frac{\hbar^2}{m a_{bgnd} \mu_r \Delta} \frac{1}{k^2}. \]  \hspace{1cm} (3.14)

We can readily see that the first term corresponds to our expression for the scattering length of the system near the feshbach resonance where $a = a_{bgnd} \mu_r \Delta/(B - B_\infty)$ and then we can interpret the second term as a contribution due to a finite range correction from equation 2.20. In this finite range expansion we see that the
effective range our the interaction is

\[ r_0 = -2 \frac{\hbar^2}{m_{\text{bkgnd}}} \frac{1}{\mu_r \Delta}. \]  

By using the theoretical parameters for narrow Feshbach resonance from [7] we see that \( a_{\text{bkgnd}} = 60a_0 \), \( \Delta = .1 \), and \( \mu_r = 2\mu_B \) we obtain that we get an effective range of \( r_{\text{eff}} \simeq -7 \times 10^4a_0 \) which is orders of magnitude greater than the effective range for the broad resonance which is \( \sim 1a_0 \).

The goal of our experiment will be to investigate the consequences of a finite range and energy dependent interactions. There has been one previous experiment in which this resonance has been investigated and that was in at Rice [46]. This experiment looked into loss and molecular formation near this resonance. At the time they did not have the magnetic field stability to fully investigate the interaction and precisely determine the location of the resonance. Our experiment will pick up from here and characterized this narrow resonance and discuss the implications of the finite range system.

### 3.2 Physics with Finite Range Interactions

#### 3.2.1 Neutron Matter

Here I will talk about some of the physics that can be studied with a system with finite range interactions and the narrow Feshbach resonance. As previously stated cold atoms have the advantage of being a system with a lot of knob that we can turn. Physics with finite range interactions gives us another knob that we can hopefully tune into some good stuff.

One system we can dial up is the physics of neutron matter [47]. The scattering length for neutrons it anomalously large at a value of \( a = -18.5 \pm 0.3 \text{ fm} \). This is unexpected because the range of the nuclear interactions via pion exchange is of \( \sim 1.4\text{fm} \). The effective range of the neutron-proton system is measured to be \( 2.7 \text{ fm} \) and the it effective range for the neutron system is expected to be charge independent and thus of the same value. From universal scaling we find that for a
resonantly interacting Fermi gas we have a relation that

\[ \frac{E}{N} = \xi \frac{3k_F^2}{10m} \]  

(3.16)

Where \( E \) is the energy of the system of \( N \) particles of mass \( m \) with a Fermi wave number of \( k_F \). The universal constant \( \xi \) has been measured to be \( \xi = 0.52 \pm 0.04 \) [48], \( \xi \simeq 0.7 \) [49], and \( \xi = 0.44^{+0.12}_{-0.09} \) [43]. While numerical calculations put the value near 0.55 [50] The universal function is only valid when the density of the gas is such that \( k_F r_e \ll 1 \) where \( r_e \) is the effective range of the interactions. Using the range of interaction for neutrons, this yields that we would have to produce a neutron gas with densities less than \( 10^{-4} \) fm\(^{-3} \). In neutrons there is a parameter called the drip density. Above this density the neutrons can be unbound. The neutron drip density is \( 2.3 \times 10^{-4} \) fm\(^{-3} \) which would cause the \( k_F r_e \sim 1 \). This means that we cannot use the universal relation above to characterize the equation of state for neutron matter. Since the inequality requirement above is related to the Fermi wave number and the effective range of the interactions cannot be satisfied we can expect for a non-universal scaling for a neutron gas. We can expect that the energy per particle will vary as

\[ \frac{E}{N} = \xi(k_F r_e) \frac{3k_F^2}{10m}. \]  

(3.17)

Since \( k_F = 1/a_F \) all we must due is find a system where the Fermi wave number is comparable to the effective range of the interactions.

For our narrow resonance we have a range of interactions to be \( r_{eff} \simeq -7 \times 10^4 a_0 \) which would mean that if we can reach a degenerate Fermi gas with a fermi temperature of 1\( \mu \)K our \( k_F r_e \sim 17 \) which would put us in the right regime. Then by changing the Fermi temperature of this gas we could map out the form of \( \xi(k_F r_e) \).

### 3.2.2 Breached Pair

Another possible research direction is to look into the possibility of a breached pair superfluid [51]. In an normal superfluid a the BCS pairing mechanism happens at the Fermi surface. In a breached pair the pairing occurs inside the Fermi surface. The energy difference would then be made up by different masses of the paired
particles, for example a heavy and light fermion. For contact interactions it is unclear if this state is stable. But, it was shown that if there are finite range interactions stability can be obtained [52].

The breached pair phase would occur for pairing between particles of unequal mass. This would mean that we would need to different fermion species that have can interact via narrow feshbach resonance. Another way to obtain this would be to create a state dependent lattice for $^6$Li. Since the mass of a particle is proportional to the curvature of the band structure if the $|1\rangle$ and $|2\rangle$ experience difference lattices they will see different curvatures and thus have different effective masses.
In this chapter I will discuss our investigation into the narrow Feshbach resonance in $^6\text{Li}$. This experiment extends the work done by Randy Hulet’s lab at Rice [46]. In that work they looked into molecular formation using the narrow Feshbach resonance. They also looked into loss in the resonance by sweeping the magnetic field at different sweep rates. They observed loss, but did not investigate the energy dependence of this loss. Also interactions were not measured near this resonance. Also, the magnetic field stability of their system was not yet at the level to fully characterize this 0.1G wide resonance.

This brings us to the situation where we have this Feshbach that has some interesting physics associated with it and it is left uncharacterized. This is where our experiment comes into play. We determined the three-body loss rate associated with the molecular state that is embedded in the continuum. We also utilize RF-spectroscopy to determine the interactions caused by resonant scattering. This gives us a measure of the scattering length of the $|1\rangle - |2\rangle$ system near this resonance. By adjusting the absolute temperature of the gas we observe the energy dependence of this resonance and show that we have to take into account the finite-range of the interactions when fitting the data.
4.1 A Narrow Feshbach Resonance

Science can work in interesting ways. When tackling one problem, other interesting and unexpected experiments present themselves to you. When you happen to stumble on this unexpected observation it is often useful to investigate it. It might even result in some interesting physics and a quick publication. One has to be aware to capitalize on this being in the right place at the right time type of opportunity. Luckily we were able to take advantage of exactly this situation.

We were working on the clock experiment that will be described in later chapters, when I noticed that a large number of atoms were being lost from our trap. This occurred when we were ramping the field from around 520G to the location of the $|1\rangle - |3\rangle$ zero scattering length crossing at 568.81(2)G. I knew that the narrow resonance was between this range, but wasn’t really convinced this loss was due to the resonance. So, to investigate this I decreased the field that we ramped to from 568 G to lower values\(^1\). At first I thought that the system was not running effectively, so I checked my atom number at 520G and saw that I was getting the normal atom number here so this got me excited that something was happening during the magnetic field sweep. I started to decrease the final field value that I ramped to. The loss was still there until I reached a field value near 540G, which is close to the location of the narrow Feshbach resonance. It was at this point I was convinced that the loss was due to the narrow resonance. So, I went up to my advisor Ken to see if we could get the green light to take a break from the clock research to investigate this loss more quantitatively. He gave the green light to go ahead and that is what we did.

The first thing that I did was to look closely at the resonance and sweep to different field values above and below the resonance. To do this we prepared a $|1\rangle - |2\rangle$ mixture at 7 $\mu$K at a field 535G and 545G. To create the $|1\rangle - |2\rangle$ mixture at 545G we did an adiabatic rapid passage (ARP) from $|2\rangle$ to $|3\rangle$ and then swept to 545 G and performed another ARP to bring the atoms in $|3\rangle$ back to $|2\rangle$. We

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\(^1\)Things weren’t really going well with the clock shift experiment, so I wanted to clear my head and play around with the experiment to boost my morale and maybe stumble onto something good, which it turned out worked out. That is not to say every time one takes a break from daily tasks they will be rewarded, but I think that it is good to take a break from what you are doing and have a little fun every once in a while. Not only did it refocus my mind, but also led to a PRL publication!
Figure 4.1. Fraction of atoms lost near the narrow Feshbach resonance. The fraction lost is due to a 100 ms dwell time at the field of interest. The blue circles indicate that the gas was initially prepared at 535G and the field was then swept up to the field of interest. The red triangles indicate the gas was prepared at 545G and the field was then swept down to the field of interest. The vertical line is the location of the resonance is at 543.286 G. The loss due to the resonance is asymmetric to the high field side due to coupling to a bound state embedded in the continuum.

then recorded the initial atom number that we prepared at these two fields and ramped to the field of interest in 10 ms and held for 100 ms at each field. Then we would image the amount of atoms lost at each field. The results are shown in figure 4.1. From this we can see that the gas is asymmetric on the high field side of the resonance. If we attribute the loss that we see due to three-body recombination with the molecular state associated with the resonance we can understand why the loss is asymmetric. Looking back to figure 3.5 we can see that the coupling to the molecular state is very energy dependent on the high field side of the resonance. Then when two atoms couple to the molecular state a third atom can interact with this molecular state resulting in loss from the trap.
4.1.1 Atom Loss

In the previous section we discussed how we came about first observing the resonance. Now I will talk about how we made qualitative measurements of the gas. Since we observed substantial loss at 100ms we can extract the loss rate by measuring the loss as a function of time. To do this we prepared an even mixture of $|1\rangle$ and $|2\rangle$ atoms at the desired temperature at a field of 545G. Then we swept to the field of interest in 10 ms and held the gas there for a variable amount of time. Then we swept the field back to 545G in 10ms and imaged the gas. Since we will be looking at the loss rate at various fields, the loss that occurs during the ramp will just adjust our initial atom number and the temperature of the gas. The loss will just give us an offset to the data. The temperature will cause the density of our gas to change which can be taken into account. Figure 4.2 shows the final atom number as a function of field for this experiment.

The first thing that jumped out at us in figure 4.2 is that the loss feature is much broader for the 10 $\mu$K data as compared to the 3$\mu$K data. To analyze this
data we model the loss rate with a model that the molecular state has a finite lifetime of $\frac{\hbar}{\Gamma_0}$ and a total linewidth of $\Gamma_{tot} = \Gamma_0 + \Gamma(E)$. Following [53], we can express the loss as a two-body loss rate given by

$$L_2 = \frac{\hbar^2}{m k E - 2\mu_B(B - B_\infty) + \Gamma_{tot}^2/4}. \quad (4.1)$$

From the previous chapter we have that $\Gamma(E) = 4\mu_B a_{bkgnd} \Delta \sqrt{m E/\hbar^2}$. Then from statistical mechanics we have the probability of two atoms having a relative kinetic energy $E$ at a temperature $T$ is given by

$$P(E) = \frac{2}{\sqrt{\pi}} \frac{1}{(k_B T)^{3/2}} \sqrt{E} \exp[-E/(k_B T)] \quad (4.2)$$

Then we can use 4.1 and 4.2 to obtain the thermal average of the two body loss rate for atoms that have energy on resonance with the molecular state.

$$\langle L_2 \rangle = \frac{1}{\hbar^2} \frac{(2\pi\hbar)^3}{(\pi m k_B T)^{3/2}} \frac{\Gamma(2\mu_B(B - B_\infty))\Gamma_0}{\Gamma_0 + \Gamma(2\mu_B(B - B_\infty))} \exp[-2\mu_B(B - B_\infty)/(k_B T)] \quad (4.3)$$

Then if we take the assumption that we are at a temperature that is sufficiently high such that $\Gamma_0 \ll \Gamma(E)$ we and thus express the above expression as.

$$\langle L_2 \rangle = \frac{1}{\hbar^2} \frac{(2\pi \hbar)^3}{(\pi m k_B T)^{3/2}} \Gamma_0 \exp[-2\mu_B(B - B_\infty)/(k_B T)] \quad (4.4)$$

If we assume that $\Gamma_0$ is a measure of the atom-dimer loss rate then we can write it as $\Gamma_0 = \hbar K_{AD}(n_1 + n_2)$. Here $n_i$ is the density of atoms in state $i$ and $K_{ad}$ is the atom dimer loss rate. Assuming that $n_1 = n_2$ and the $L_2 \propto K_{AD}n$ thus the loss rate is actually a three-body process $\dot{n} = -L_3 n^3$ where are loss coefficient $L_3$ is proportional to the atom-dimer loss rate $K_{AD}$.

$$L_3 = \frac{2\hbar^3 K_{AD}}{(\pi m k_B T)^{3/2}} \exp[-2\mu_B(B - B_\infty)/(k_B T)] \quad (4.5)$$

Thus if we take the data from figure 4.2 and fit it to three body loss function as $\dot{n} = -L_3 n^3$. Where $n$ is the density which is calculated from the atom number and the temperature. The volume of the gas can be found using the following
expression.

\[ V = \pi^{3/2} \sigma_x \sigma_y \sigma_z \]  \hspace{1cm} (4.6)

where

\[ \sigma_i = \sqrt{\frac{k_B T}{m \omega_i}} \]  \hspace{1cm} (4.7)

and \( \omega_i \) is the trap frequency in the \( i \) direction.

This means that we can fit the \( L_3 \) data as a function of field with \( K_{AD} \) as our only free parameter. The value for location of the resonance \( B_\infty = 543.286 \) is obtained from the interaction data which will be discussed in the following section. Figure 4.3 shows our data for the 3.2 \( \mu \)K data and the 10 \( \mu \)K data. The fits agree very well with the data. From these two fits we extract \( K_{AD} = 3(1) \times 10^{-10} \) cm\(^3\)/s and \( K_{AD} = 2(1) \times 10^{-10} \) cm\(^3\)/s for the 3.2 \( \mu \)K data and the 10 \( \mu \)K data respectively. It is also interesting to look at the final temperature of the gas as a result of these loss rates. Since the loss is due to the resonant coupling to the molecular state, which is energy dependent, the loss is also energy dependent. An energy dependent loss feature could potentially be used as a method for cooling the gas. If we could selectively lose the high energy atoms and leave the low energy atoms, the gas will re-thermalize to a lower temperature. In this manner the energy dependent loss could be used as a knife to perform forced evaporative cooling. Figure 4.4 shows the final temperature of the gas after 80ms of dwell time at the field of interest. Slight cooling is observed at fields near 543.4G in the 3.2 \( \mu \)K initial temperature data.

### 4.2 Measuring Interactions

Now that we have characterized the loss due to the narrow resonance we will measure the interactions. The way that we will do this is through RF spectroscopy [54]. The interactions that we want to measure are between atoms in \( |1\rangle \) and atoms in \( |2\rangle \). When atoms interact the interaction energy causes their energy levels to change based on the mean field energy they experience. Figure 4.5 shows qualitatively how interactions between atoms cause a shift in the transition frequency as well as the experimental realization. In our experiment we use the lowest three
Figure 4.3. Three-body recombination loss rates for a $|1\rangle - |2\rangle$ gas near the narrow Feshbach resonance. a) shows data for 3.2 µK and b) shows data for 10 µK. Both solid lines are fits to equation 4.5, which show very good agreement with the data.

Zeeman levels of $^6$Li to perform the RF spectroscopy\(^2\). If we only have atoms in $|2\rangle$ the frequency needed to resonantly transfer atoms from $|2\rangle$ to $|3\rangle$ will be given by the bare transition frequency $\nu_0$. If there are atoms in $|1\rangle$ that are incoherent with the atoms in $|2\rangle$ then the energy level of $|2\rangle$ and $|3\rangle$ will feel a mean field shift due to these atoms in $|1\rangle$. The atoms must be decoherent with the atoms in state two because Pauli exclusion principle prohibits fermions in a coherent superposition from interacting [54, 55]. The amount the frequency of the transition is shifted is thus given by

$$\Delta \nu = \frac{2\hbar}{m} n_1 (a_{13} - a_{12})$$  \hspace{1cm} (4.8)

\(^2\)Since we are using fermions we have to use a third state to read out interactions with RF spectroscopy. This is due to the fact that the RF field is coherent and there is no s-wave shift that is measurable with a homogenous field [54, 55, 56].
Figure 4.4. Temperature of the gas after 80ms of hold time at the field of interest for an initial temperature of A) 3.2 µK and B) 10 µK. Slight cooling is observed near 543.4G for the 3.2 µK data and near 543.7 G for the 10 µK data. The lines are smoothing splines that were used to extract the temperature for the extraction of $L_3$.

where $a_{12}$ is the scattering length between $|1\rangle$ and $|2\rangle$ and $a_{13}$ is the scattering length between the $|1\rangle$ and $|3\rangle$. The density of the atoms in $|1\rangle$ is given by $n_1$. The same result holds if we start with atoms in $|3\rangle$ and transfer atoms to $|2\rangle$ except that the sign in front of each scattering length changes.

To perform RF spectroscopy on our gas we need to find the Rabi frequency for a bare transition of atoms from $|2\rangle$ to $|3\rangle$. Then we apply a $\pi$ pulse which transfers all of the atoms from $|2\rangle$ to $|3\rangle$ with no atoms present. We then change the frequency of our $\pi$ pulse and record the amount of atoms transferred. When we are off resonance we do not transfer the full population. In this manner we can fine the resonant frequency for the transition. This method is called Rabi spectroscopy. Applying this Rabi spectroscopy on the transition between $|2\rangle$ and $|3\rangle$ with and without $|1\rangle$ present we can discern the mean field shift given by equation 4.8.

If we look at the RF spectrum in figure 4.5 we can see that we observe sidebands where there is a local maximum amount of atoms transferred. This can be understood by looking at the applied pulse that we use. Since for this experiment we use a square window function for our RF the spectral shape of the frequency
of the field applied to the atoms is proportional to the Sinc functions which is

\[
\text{sinc}(\omega \frac{T}{2}) = \frac{\sin(\omega \frac{T}{2})}{\omega \frac{T}{2}}. \tag{4.9}
\]

Where the pulse duration is given by T. We can also see that the first node of the sinc function occurs for a frequency of \( f = \frac{1}{T} \) away from the carrier frequency, because \( \text{sinc}(\pi) = 0 \). Thus we can see that for longer pulse duration we have a smaller spectral width. Thus the sidebands shown in figure 4.5 are due to the antinodes of the spectral width of the square pulse of near 90 µS which is what was done in the experiment.

We can take this a bit further. From equation 2.30 we know the expression for the amount of atom transfer at a certain detuning \( \delta \). If we then compute the convolution of this probability of atom transfer with the spectral shape of the intensity of the RF pulse that we apply, which is given above in 4.9 we will get an expression for atom transfer for an applied square pulse of duration \( \tau \) and detuning
δ. After convolution we arrive at the form for atom transfer to be

\[ P_{Rabi}(\nu) = \frac{\tau^2 \Omega^2}{4} \text{sinc}^2 \left( \frac{\tau \Omega}{2} \sqrt{1 + \left(\nu_{res} - \delta\right)^2/\Omega^2} \right) \] (4.10)

Then we can apply this to the RF spectrum we observe and obtain the fits shown in figure 4.6. From the figure we notice that we have excellent agreement between the derived line-shape and the experimental results. Now that we have a handle on the effects of our pulse shape on the amount of atoms transferred between two states we need to look into the effect of the trapping potential on the density of the gas via a local density approximation.

### 4.2.1 Local Density Approximation Lineshape

Looking at equation 4.8 we notice that the shift is dependent on the density of the gas. From chapter 2 we know that the trapping potential for our gas is gaussian in shape and yields a harmonic potential for the atoms. The result of this is that each atom doesn’t experience the same density, because the cold atoms tend to congregate near the center of the trap and the hotter atoms can spread out into the wings. This creates a thermal density distribution of the form

\[ P_{Rabi}(\nu) = \frac{\tau^2 \Omega^2}{4} \text{sinc}^2 \left( \frac{\tau \Omega}{2} \sqrt{1 + \left(\nu_{res} - \delta\right)^2/\Omega^2} \right) \] (4.10)
\[ n(r) = n_0 \exp \left( -\frac{r^2}{\sigma_r^2} - \frac{z^2}{\sigma_z^2} \right) \]  \hspace{1cm} (4.11)

where \( \sigma \) is given in equation 4.7 and we define the density at the center of the trap as \( n_0 \). We are also assuming that our trap is cylindrically symmetric and can be parameterized by a radial parameter \( r \) and an axial parameter \( z \). We can then write down the expression for \( N \) atoms making the transition from \( |2\rangle \) to \( |3\rangle \) as

\[ NP(\nu) = \int \delta(\nu - \nu(r)) n_2(r) d^3r \]  \hspace{1cm} (4.12)

Then we can express our shift \( \nu(r) \) as

\[ \nu(r) = \frac{2\hbar}{m} n_1(r)(a_{13} - a_{12}). \]  \hspace{1cm} (4.13)

For our experiment we make the assumption that \( n_1(r) = n_2(r) = n(r) \), and each have the density distribution given in 4.11. Using these relations we obtain the probability density for the \( |2\rangle \) - \( |3\rangle \) transition to be

\[ P_{\text{int}}(\nu) = \frac{2\pi n_0}{N} \int_{-\infty}^{\infty} \int_{0}^{\infty} \delta(\nu - \Delta\nu_0) \exp \left[ -\frac{r^2}{\sigma_r^2} \right] \exp \left[ -\frac{z^2}{\sigma_z^2} \right] \times \]
\[ \exp \left[ -\frac{r^2}{\sigma_r^2} \right] \exp \left[ -\frac{z^2}{\sigma_z^2} \right] r \, dr \, dz \]  \hspace{1cm} (4.14)

Then after performing the integral we obtain the probability density of the transition to be

\[ P_{\text{int}}(\nu) = \frac{2}{\sqrt{\pi} \Delta\nu_0} \left[ \sqrt{\ln \left( \frac{\Delta\nu_0}{\nu} \right)} \right] \]  \hspace{1cm} (4.15)

Where \( \Delta\nu_0 \) is the max shift felt by the atoms at the center of the trap. We can then straight forwardly calculate the rms shift, which is what we want, by using this probability density and weighting it by \( \nu \) and \( \nu^2 \) and we find the \( \nu_{\text{rms}} \approx 0.259\nu_0 \). Of course now that we have this probability density we will have to convolute it with the line-shape obtained in 4.10 to obtain the experimental line-shape that we will observe.
\[ P_{\text{obs}} = \int_{-\infty}^{\infty} P_{\text{Rabi}}(v - v') P_{\text{int}}(v') dv' \]

\[ P_{\text{obs}} = A \int_0^{\Delta \nu_0} \text{sinc}^2 \left( \frac{\tau \Omega}{2} \sqrt{1 + 4\pi^2 (\nu - \nu')^2 / \Omega^2} \right) \sqrt{\ln \left( \frac{\Delta \nu_0}{\nu} \right)} dv'. \] (4.16)

The result of the integral must be calculated numerically. An amplitude A is included in order to fit the experimental data to an atom number because that is how we take data. To fit the data we evaluate this integral numerically to extract the value of \( \nu_0 \). To make results of the numerical calculation easier to calculate we can see that the integral is a function of \( \nu \) and \( \Omega \). Thus, we can parameterize the integral as a function of \( f = \frac{\nu}{\nu_0} \) and \( \bar{\Omega} = \frac{\Omega}{2\pi \nu_0} \). Since we know our Rabi frequency \( \Omega \) and our pulse duration \( \tau \), all we need to do is fit our experimental data to the line-shape given by this integral of the above equation and extract \( \Delta \nu \).

If we look back to figure 4.5b we see that the line-shape with interactions doesn’t seem to be that much different than the bare transition that is not effected by interactions. This is okay, because the data in this figure was taken with a gas in a shallow trap where the spectral linewidth of the Rabi pulse is not fine enough to resolve the lineshape. If we look at a spectrum with a higher density where we have enough spectral width with a 95\( \mu \)s RF pulse to resolved the line shape we obtain a result as shown in figure 4.7. The results of equation 4.16 are in good agreement with the observed lineshape with and without allowing the Rabi frequency to be a fit parameter. To obtain the value of the interaction shift, from which we can extract scattering length, we use the fits that allow \( \Omega \) as a fit parameter.

Now we have the machinery needed to extract the information about the interactions near this narrow resonance. To observe this we need to take this kind of data at fields around the resonance and see how the interactions change. Since we have already discussed the fact that the resonance is energy dependent we will have to do this for at different absolute temperatures to change the energy of our gas.
Figure 4.7. RF Rabi spectrum for atoms transferred from $|2\rangle$ to $|3\rangle$ for an applied RF $\pi$-pulse of 95 $\mu$s at a field of 543.17G. The red triangles are the bare RF transition and the dash-dotted line is a fit using the lineshape function given in equation 4.6. The blue squares show the experimental lineshape due to interactions. It is broaden due to the fact that the density is not the same for all atoms. The grey dotted line is a fit using the result of equation 4.16 with the $\Omega$ fixed for the 95$\mu$s $\pi$-pulse. The solid blue line is also a fit using the result of 4.16, but allowing $\Omega$ to be another fit parameter.

4.2.2 Experimental realization

In this section I will describe the experimental procedure and results of our investigation into the narrow Feshbach resonance. I will first start out with a more detailed description of the experimental procedure and dipole traps used to create the different temperatures used.

We first start our experiment by loading atoms into either a crossed-dipole or single beam trap formed by our 1064 nm laser. By lowering the trap depth we perform forced evaporative cooling at 315G to reach the desired temperature. After evaporation we raise the trap depth to halt evaporation[57]. To ensure that we start with an even $|1\rangle - |2\rangle$ mixture we apply 20ms of an RF field to balance the population [58]. The RF is frequency modulated with a depth of 10 kHz at a rate of 100kHz.

We also have the option to add a 740 nm beam intersecting the trap at an angle of 34° from vertical to get the trap parameters that we want. A description of the trap parameters used is given in table 4.1 and a cartoon showing the laser
setup is shown in figure 4.8. Once we have the gas at the temperature that we want we perform an adiabatic rapid passage to transfer the atoms from $|2\rangle$ to $|3\rangle$. Then we ramp the magnetic field to the field of interest where we perform the RF spectroscopy to probe the interactions. In order to prevent loss due to three-body recombination [57, 58] we immediately release the atoms for a time of flight of 600 $\mu$s and image the cloud at the field of interest. In order to get a precise value of the magnetic field we interleave the RF spectroscopy with and without atoms in $|1\rangle$. This gives us the bare transition frequency which can be used with the Breit-Rabi formula to find the exact magnetic field we are operating at. The scattering length between $|1\rangle$ and $|2\rangle$ atoms is $a_{13} \simeq -267a_0$ and the background scattering length between the $|1\rangle$ and $|2\rangle$ atoms is $a_{12} \simeq 63a_0$. Thus from 4.8 we expect that the transition frequencies will be shifted to frequencies below the bare transition frequency. The range of RF values that we use is $\sim 40$kHz below to $\sim 20$kHz above the bare transition frequency.

Once again, the spectrum that we observe is shown in figure 4.6. From this
Table 4.1. Trap parameters for the different temperatures used to investigate the energy dependence of the narrow Feshbach resonance. The laser setup is given in figure 4.8.

<table>
<thead>
<tr>
<th>Trap Type</th>
<th>Trap frequencies (x,y,z)</th>
<th>Temperature</th>
<th>$T_{Fermi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single 1064nm beam</td>
<td>(809,255,6.4)Hz</td>
<td>860 nK</td>
<td>360 nK</td>
</tr>
<tr>
<td>Crossed 1064nm</td>
<td>(2290,4780,150)Hz</td>
<td>3.8 µK</td>
<td>6.2 µK</td>
</tr>
<tr>
<td>Crossed 1064nm + 740 nm</td>
<td>(2140,6340,4650)Hz</td>
<td>10.4 µK</td>
<td>21 µK</td>
</tr>
<tr>
<td>Crossed 1064nm</td>
<td>(3647,7611,181)Hz</td>
<td>23.8 µK</td>
<td>11.3 µK</td>
</tr>
</tbody>
</table>

We can see from these plots that the interactions change greatly as the energy of the gas is increased. To analyze this data we first rewrite equation 4.8 in terms of the scattering amplitude.

$$\Delta \nu = \frac{2\hbar}{m} (-\text{Re}[f_{13}] + \text{Re}[f_{12}])$$  \hspace{1cm} (4.18)$$

From Chapter two we know that $f = 1/(k \cot(\delta) - ik)$ and we have our expression for $k \cot(\delta)$ including an effective finite range in equation 3.14. Then we can express the real part of the scattering amplitude as

$$\text{Re}[f] = -a_{\text{bkgnd}} \frac{(E_r - 2\mu_B(\delta - \Delta))(E_r - 2\mu_B \delta)}{(E_r - 2\mu_B \delta)^2 + E_{\text{bkgnd}}^2}.$$  \hspace{1cm} (4.19)$$

where we define $\delta \equiv (B - B_\infty)$, $\Delta \equiv (B_0 - B_{\text{int}})$, $E_r = (\hbar^2 k^2)/m$, and $E_{\text{bkgnd}} = h^2/(ma_{\text{bkgnd}}^2)$. From table 4.1 we see that we are above 0.5 T$_F$ so we can model our gas via a Boltzmann distribution [57] and this gives the probability of two atoms interacting with a relative kinetic energy of $E_r$ given in equation 4.2. We can then
Figure 4.9. Plot of effective scattering length difference $a_\ast$ from equation 4.17 vs. magnetic field for various temperatures a) 860 nK, b) 3.8 $\mu$K, c) 10.5 $\mu$K and 23.8 $\mu$K. The trap parameters for these different plots are given in table 4.1. The solid line is the theoretical prediction for a Feshbach resonance acting via a contact potential with a finite range contribution.

calculate the thermal average of $\langle \text{Re}[f] \rangle$ given by

$$
\langle \text{Re}[f] \rangle = \int_0^\infty -a_{bkgnd} \frac{(E_r - 2\mu_B(\delta - \Delta))(E_r - 2\mu_B\delta)}{(E_r - 2\mu_B\delta)^2 + \frac{E_r}{E_{bkgnd}}(E_r - 2\mu_B(\delta - \Delta))^2} \times \frac{2}{\sqrt{\pi}} \frac{1}{(k_BT)^{3/2}} \sqrt{E_r} \exp[-E_r/(k_BT)]dE_r
$$

(4.20)

By using the knowledge of the background scattering lengths we can express $-\text{Re}[f_{13}] = -267a_0$ and $a_{bkgnd} = 63a_0$ and include their slight variation as a function of field $[2]$ we can fix these values. We also know the temperature of the gas, which leaves the only fit parameters to be the width $\Delta$ and the location of the resonance $B_\infty$. From our expression for the effective scattering length the
difference is given by

\[
a_\ast(a_{13}, a_{bkgrd}, \Delta, B_\infty, B, T) = -\frac{m}{2\hbar n}(\langle -\text{Re}[f_{12}(a_{bkgrd}, \Delta, B_\infty, B, T)] \rangle - a_{13}) \quad (4.21)
\]

We use the 860 nK data in figure 4.9a) to \( A \times a_\ast(a_{13}, a_{bkgrd}, \Delta, B_\infty, B, T) \) in order to extract the value for \( B_\infty \) and \( \Delta \). The amplitude \( A \) is included to compensate for the systematic uncertainties in atom number and trap frequency. The result of this fit yields the location of the narrow resonance to be at \( B_\infty = 543.286(3) \) G with a width of \( \Delta = 0.10(1) \) G. Where the uncertainties here include the uncertainty in the field. The fit to the data is shown in figure 4.10. Now that we have the location and the width of the resonance we can fit the rest of the data fixing these parameters leaving only the amplitude \( A \) as a fit parameter. We can see from the figure that we get very good agreement between experimental data and the fit equation 4.21 indicating that we have to use the effective range expansion in order to describe our data. The dashed line in figure 4.10 is the predicted \( a_\ast \) for a contact potential where \( -\text{Re}[f_{12}] = a_{bkgrd}(1 - \Delta/(B - B_\infty)) \).

The interaction data also agrees with the idea that there is coupling to molecular state embedded in the continuum on the high field side of the resonance. This causes interactions to have an asymmetry to this side of the resonance as is shown in the data. Also for higher temperature the interactions get washed out due to the fact the there is a wider distribution of energies, which means that the gas doesn’t traverse the resonance at the same field. For the 860 nK data we have a narrow distribution of energies and the shape of the resonance looks more like that of a broad Feshbach resonance. This is because of the low energy of these atoms which results in the cloud traversing the resonance almost at the same field value.

### 4.2.3 Future Directions

Above I discussed our investigation into the energy dependent loss near a narrow Feshbach resonance. We have shown that coupling to the molecular state embedded in the continuum causes three-body recombination that is highly energy dependent. From figure 4.4 we see that we see some cooling in our gas. This is one of the future directions that we can take this experiment. If we can use this resonance as a knife to cut off the high energy tail we might be able to perform efficient evaporation
Figure 4.10. Plot of effective scattering length difference \( a_e \) from equation 4.17 vs. magnetic field for various temperatures a)860 nK, b)3.8 \( \mu \)K, c)10.5 \( \mu \)K and 23.8 \( \mu \)K. The trap parameters for these different plots are given in table 4.1. The dashed line shows the prediction for a resonance interacting via a contact potential. For a) the solid blue line is a fit to the data using equation 4.21 in order to extract \( B_\infty = 543.286(3) \) G and \( \Delta = 0.10(1) \) G. For b), c), and d) the solid blue line is a fit with \( B_\infty \) and \( \Delta \) fixed to the values found from fitting a) with only the amplitude \( A \) as a fitting parameter.

which was proposed for an optical Feshbach resonance [53]. This proposal used an optical Feshbach resonance that has the downside of having to use high power to
create the resonance and the side effect of light scattering that could possibly heat the gas. If we were to use the narrow resonance, we would start with a $|1\rangle - |2\rangle$ mixture at a field above the resonance and slowly ramp the field down. This would allow the high energy atoms to be lost from the trap and, after rethermalization, the gas would be colder.

Another possible way to use the resonance as a knife is to use Raman beams to couple to the molecular state that causes the loss. To do this we would apply Raman beams to couple the energetic atoms to the molecular state. This would enable us to selectively transfer atoms with lasers instead of magnetic field sweeps. This would allow for us to stay at fixed field and couple the atom to the state only when the light is applied. Then we could easily allow for enough time for the atoms to re-thermalize.

In my opinion I believe that this is the first direction to investigate with the narrow Feshbach resonance. The other main directions were discussed at the end of chapter 3. They include looking into the equation of state of the gas. Now this would give great insight into the physics of neutron matter and interactions with finite range effects, but would of course require much more work. As of this publication the easiest feasible way to measure the equation of state would be to follow the blueprint put forth by Salomon’s group and Zwierlein’s group. They used in-situ measurements of the gas to extract the compressibility and then used that information to determine the equation of state [59, 60]. This could be a long term goal, but there are other interesting goals in the lab that I think are of higher priority.
Chapter 5

S-Wave Clock Shift for Fermions

In this chapter I will discuss the background motivation and theory behind observing an s-wave collisional shift in an ultracold Fermi gas. I will start by briefly discussing how atomic clocks work and why we need to even consider an s-wave clock shift for fermions. Then I will proceed into how we can observe this shift and a discussion of the current state of investigations into this shift. Motivation for experiments has to start from somewhere, and ours was from some precision work done in Boulder and also some convincing from our colleague from down the hall, Kurt Gibble.

5.1 Experimental Motivation

Time still remains the most important dimension that we cannot see. In an effort to see time and keep track of everything we observe, in a timely manner, we have developed atomic clocks. As stated in the introduction an atomic clock works by resolving the transition frequency between two energy levels in an atom. With the advances in atomic physics we can now use ultra cold atoms as atomic clocks [61]. As long as we can correctly understand how the energy levels of the clock move due to experimental and environmental factors there really is no fundamental limit on the precision and accuracy of these atomic clocks. Our investigation into an s-wave clock shift in fermions will help with the understanding of frequency shifts that limit our precision and accuracy, and will hopefully help to usher in a new class of optical clocks based on fermionic atoms.
The biggest two sources of uncertainty in fermionic optical clocks are due to black-body radiation and the density of the atomic gas [14]. The density dependent shift is caused by collisional frequency shifts. The higher the density the higher the rate of collisions and the higher the collisional frequency shift. It would be nice to be in a situation where we do not have any collisions at all and thus no shift. This is the appeal of making an atomic clock with a fermionic atom. If we could cool the gas of atoms down such that the only allowed collisions are s-wave in nature then, due to the Pauli exclusion principle, we could eliminate collisional shifts from our error budget [62] for fermionic atoms. This was indeed the case, as shown by experiments conducted at MIT [55, 54].

These experiments were conducted with $^6$Li in the lowest two Zeeman states. In these experiments they looked for a shift in transition frequency between by driving the atoms between $|1\rangle$ and $|2\rangle$. They found that no matter what the original coherence of the system they do not see a shift due to the interactions between the two states. This can be interpreted as the RF reintroducing coherence in the system. If the system of fermions is in a coherent superposition of states then no two fermions can occupy the same state and thus they cannot interact via s-wave collisions, and thus no frequency shift. It turns out that what is needed to reintroduce coherence is not just an RF transition, but a field which all the atoms experience the same Rabi frequency[56]. This will be discussed in more detail in the next section.

These experiments confirmed the prediction that an atomic clock made from a fermionic atom will result the clock transition being insensitive to density-dependent collisional frequency shifts. Thus people started building clocks out of fermionic isotopes. For a clock you want an atom with a narrow linewidth and a large transition frequency so that the fractional uncertainty is small. This meant that the alkali fermions are out of the question. The first two atoms that were chosen as candidates were $^{87}$Sr and $^{171}$Yb.

The researchers in Jun Ye’s lab at JILA first created an optical clock with Sr and showed fractional uncertainty at a level of $10^{-16}$ [14]. Further investigation into the uncertainty of this fermionic clock led to an observation of a density-dependent collisional frequency shift [63]. They observed this shift using Rabi spectroscopy on the gas, even for what was claimed to be a purely s-wave gas.
The resulting explanation they gave for the shift was that their probe beam was misaligned and thus different atoms experienced a different Rabi frequency and this introduced distinguishability. Then a mean-field shift could arise if there is different partial densities in the ground and excited states of the clock. While this explanation seemed to fit the data, it was not a complete picture. This sparked a lot of theoretical work into this subject [56, 64, 65]. While all three of these results are mathematically equivalent in their treatment of Rabi spectroscopy, the theory presented by Kurt Gibble provides the clearest picture of what is happening, and in my opinion that carries a lot of weight in physics. Kurt Gibbles treatment considers Ramsey spectroscopy in addition to Rabi spectroscopy which elucidates the origin of the s-wave collisional frequency shift of a Fermi gas.

The JILA group continued to investigate this phenomena in their Sr gas and conducted other experiments [66, 67] that supported the three theories, but did not investigate with Ramsey spectroscopy which would be definitive proof of correlation with Kurt’s theory. The interesting thing is that the results for Rabi spectroscopy could not resolve weather or not the shifts observed in the gas were due to s-wave physics and a s-wave collisional shift or if they were p-wave in nature.

Then in 2011 Chris Oates group working with Yb found that the density-dependent shifts that they were observing in their Yb gas was due almost entirely to p-wave interactions [68]. This led to a reanalysis of the experiments that were done on Sr [63, 66, 67] and it was found the results were consistent with p-wave physics and that the p-wave interactions were the main contribution of these density dependent frequency shifts 1.

5.2 Origin of S-wave Clock Shift for Fermions

In this section I will discuss how there can be a density-dependent collisional frequency shift for fermions that only interact with s-wave interactions. As stated above much theory has been explored investigating this effect [56, 64, 65] and it seems that all three theories are essentially different ways of saying the same thing I will follow the formulation presented in [56]. I do this because for the experiment

\[ \text{I received this information from conversations with Gretchen Campbell (who was first author on [63]) and from a conversation Kurt Gibble had with Chris Oates and Jun Ye.} \]
we use Ramsey spectroscopy in our experiment and this formalism is most suited
for this type of spectroscopy. Also I believe that this formalism provides the most
physical insight into the situation. I will first start with a description of Ramsey
Spectroscopy. This will lead into describing the background theory from [56]. I will
then end this chapter by stating the current status of experimental investigation
into observing a collisional frequency shift for fermions. This will give a good setup
for our experimental investigation into CFS.

5.2.1 Ramsey Spectroscopy

The method that we will use to investigate the s-wave Collisional frequency shift
(CFS) will be Ramsey spectroscopy. This method was originally developed by
Norman Ramsey building off the work of his advisor, Isidor Rabi [69]. In the
previous chapters we discussed the method of RF-spectroscopy which is actually
a form of Rabi spectroscopy. Rabi spectroscopy uses a pulsed driving field of
a π-pulse duration. Then the frequency of the driving field is adjusted and the
transferred population is recorded. The resonance frequency is determined by
location peak atom transfer.

Ramsey spectroscopy does essentially the same thing as Rabi spectroscopy
except that instead of using one π-pulse, two π/2 pulses separated by interrogation
time T are used. During the interrogation time T the driving field is turned off
and the atom is free to evolve. To look into Ramsey spectroscopy further I will
follow the treatment given in [20].

For a two-level system that is being driven by an oscillating field we know from
time-dependent perturbation theory that the coefficients for the two states obey
the equations of motion

\[ i \dot{c}_1 = \Omega \cos(\omega t) e^{-i\omega_0 t} c_2 \]  
\[ i \dot{c}_2 = \Omega^* \cos(\omega t) e^{i\omega_0 t} c_1 \]  

Here Ω is the Rabi frequency, \( \omega_0 \) is the resonant frequency for the transition,
and \( \omega \) is the frequency of the driving field. With an initial condition we can solve
these equations of motion for our Ramsey sequence. The two-levels that we will
Figure 5.1. Atom Transfer as a function of A) detuning and B) phase $\delta = 0$. The peak of the central fringe in A) is the location of the resonance. In our experiments we adjust the phase while keeping the frequency near resonance. Both methods can be used to find the resonance location and contain the same information.

talk about for out $^6$Li system are $|1\rangle$ and $|2\rangle$. We will start out with the atoms initially populating only $|1\rangle$. This makes our initial condition be that $c_1 = 1$ and $c_2 = 0$. Then we apply a pulse of length $\tau_1$ at a frequency $\omega$. After this pulse we let the gas freely evolve for an interrogation time $T$. Then after this interrogation time we apply a second pulse of duration $\tau_2$. We also have the ability to change the phase of the second pulse by a phase factor of $\phi$. When solving these equations of motion we obtain the probability of finding the atom in $|2\rangle$ for two equal pulses to be,

$$|c_{2}|^2 = \left| \frac{\Omega \tau}{2} \sin(\frac{\delta \tau}{2})^2 \cos(\frac{\delta T}{2} + \phi)^2 \right|^2$$

(5.3)

By looking at this equation we can tell many things. First we see that we have an overall envelope that is governed by the sinc function which is proportional to the length of the pulse $\tau$. Underneath this window is a cosine function whose periodicity is governed by the interrogation time. Figure 5.1A shows a theoretical Ramsey fringe. To use this for spectroscopy we know that the central fringe has the peak number of atoms transferred. If we can locate this then we can obtain the resonant frequency of the transition. The advantage of this method is the width of this central fringe is given by.
\[ \Delta \nu = \frac{1}{2T} \] (5.4)

Which means that for the same pulse setup, all that is needed to change the resolution of your experiment is to make the interrogation time longer. Then you either sweep the frequency of the pulses or adjust the phase difference between the two pulses. Figure 5.1B shows the result of the Ramsey fringe found by adjusting the phase between the two pulses instead of the frequency detuning. Since both methods give us the same information we can operate either way. To convert a phase to frequency we use the relation,

\[ \Delta \nu = \frac{\Delta \phi}{2\pi T} \] (5.5)

Each of these methods give us the same information. We choose to use the phase method, because the effects of light shifts could be more detrimental when using the frequency method.

## 5.3 Origin of the Collisional Frequency Shift

In this section I will go into detail on how an s-wave collisional frequency shift (CFS) for fermions can manifest. As stated above I will follow the formalism of [56]. First of all let’s remind ourselves of the ever present Pauli exclusion principle. This is essentially a statement that the wave-function for a system of fermions must be antisymmetric under particle exchange (as was discussed in chapter 2). In our case where we have fermions that can occupy different spatial and internal states the total wave function must be anti-symmetric. Then for this system we can write down the Hamiltonian as

\[ H = H_0 + \frac{1}{2} \sum_{i=1...N} \sum_{\eta=a,b,c} (\Omega_\eta |S\psi_\eta(\vec{r}_i)\rangle \langle P\psi_\eta(\vec{r}_i)| + h.c) + \sum_{i<j} V(\vec{r}_i - \vec{r}_j) \] (5.6)

where h.c. is the Hermetian conjugate of the previous term. \( H_0 \) is the Hamiltonian for the trap which we take to be internal state independent. S and P are the internal state wave functions, for our case this would correspond to the Zeeman
levels $|1\rangle$ and $|2\rangle$. The spatial wave functions are given by $\psi_\eta(\vec{r}_i)$. We label the particles by the index $i = 1\ldots N$, where $N$ is the total number of particles in the trap. We denote the different trap states by the indices $\eta = a, b, c, \ldots$. The coupling between the different internal states is done via a driving field $\Omega_\eta$. We use the trap state index to allow for a driving field that can be different for different trap states. As is the usual case we operate in a frame rotating at the resonant frequency of the transition. Then our driving field becomes

$$\Omega_\eta = \Omega_{0\eta} e^{i\delta t + i\phi}$$ \hspace{1cm} (5.7)

where $\phi$ is the phase of the pulse. $V(\vec{r}_i - \vec{r}_j)$ is the interaction potential that we take to be a contact potential

$$V(\vec{r}_i - \vec{r}_j) = \frac{4\pi \hbar^2 a}{m} \delta(r_1 - r_2).$$ \hspace{1cm} (5.8)

And we define the interaction strength, $g$, as

$$g \equiv \frac{1}{2\hbar} \langle \Psi | V | \Psi \rangle$$ \hspace{1cm} (5.9)

Now we have the setup for our experiment. To review we have a gas of fermions that are all in the same internal state $g$. Then we apply a driving field to move the atoms from $g$ to $e$. This driving field is different for atoms in different trap states. The question is, does this cause a density dependent frequency shift?

To answer this question it is instructive to treat the case for two atoms and then extend this to the full gas. For this two atom picture begin with two atoms in different trap states which we call $a$ and $b$. The trap sidebands are spectroscopically resolved such that the Ramsey pulses do not change the trap state. We will use the particle labels 1 and 2 for the two atoms. We can write the wave function for these two particles as

$$\Psi_{ab} = \chi_{12} \psi_{ab}(\vec{r}_1, \vec{r}_2)$$ \hspace{1cm} (5.10)

Where I designate the internal state wave function ($g$ and $e$) by the function $\chi_{12}$ and the spatial wave function as $\psi_{ab}(\vec{r}_1, \vec{r}_2)$. We know that the total wave function given in 5.10 must be anti-symmetric under exchange because we are treating
fermions. That means that if the spatial wave function is symmetric the internal wave function must be anti-symmetric and vice-versa. Thus the two-particle wave functions can be written as

\[ \psi_{ab}^\pm(r_1, r_2) = \frac{1}{\sqrt{2}} [\psi_a(r_1)\psi_b(r_2) \pm \psi_a(r_2)\psi_b(r_1)] \] (5.11)

Where the +(-) sign refers to the symmetric (anti-symmetric) wave function.

Then we can express the states in a singlet-triplet basis where the three triplet states are denoted by \( |u\rangle, |t\rangle, \) and \( |d\rangle \) and the singlet state is given by \( |s\rangle \). The triplet states all have anti-symmetric spatial wave functions while the singlet state has a symmetric spatial wave function and vice-versa for the internal state wave function. The states are given in equation 5.12.

\[|u\rangle = |e_1 e_2\rangle \psi_{ab}^-(r_1, r_2)\]
\[|t\rangle = \frac{1}{\sqrt{2}} (|e_1 g_2 + g_1 e_2\rangle \psi_{ab}^-(r_1, r_2)\]
\[|s\rangle = \frac{1}{\sqrt{2}} (|e_1 g_2 - g_1 e_2\rangle \psi_{ab}^-(r_1, r_2)\]
\[|d\rangle = |g_1 g_2\rangle \psi_{ab}^-(r_1, r_2)\]

In the basis of these states we can write down a general wave function for these two particles as \( \psi = |d\rangle|d\rangle + |s\rangle|s\rangle + |t\rangle|t\rangle + |u\rangle|u\rangle \). Then we can proceed to calculate the evolution of these states via time dependent perturbation theory. The equations of motion for this system can be written as

\[i \dot{d} = \frac{\Omega}{\sqrt{2}} t - \frac{\Delta \Omega}{\sqrt{2}} s\]
\[i \dot{t} = -\frac{\Omega}{\sqrt{2}} u + \frac{\bar{\Omega}^*}{\sqrt{2}} d\]
\[i \dot{u} = \frac{\bar{\Omega}^*}{\sqrt{2}} t + \frac{\Delta \Omega^*}{\sqrt{2}} s\]
\[i \dot{s} = \frac{\Delta \Omega}{\sqrt{2}} u - \frac{\Delta \Omega^*}{\sqrt{2}} d + 2g_{eg}s\]

where we define \( \Delta \Omega \equiv (\Omega_a - \Omega_b)/2 \) and \( \bar{\Omega} \equiv (\Omega_a + \Omega_b)/2 \). Without solving these
equations we can understand several things that are going on. First is that the only way to couple to the singlet state $|s\rangle$ is to have the Rabi frequencies for the two trap states be different. Another way to look at this situation is that in order to couple the two different spatial wave function there must be a spatial perturbation. For the homogenous case we cannot couple to the singlet state, but if we do have a spatial inhomogeneity then coupling to the singlet state is possible. Once the atoms are in the singlet state $|s\rangle$ interactions can take place. This is because this state has a symmetric spatial wave function that will allow for the two states to interact.

We know from previous chapters that fermions in different internal states can interact if they are incoherent or distinguishable. Thus the different trap states that have different Rabi frequencies means that after they are driven between the different states there is a certain amount of distinguishability between the two atoms. For an extreme example if we could set $\Omega_a \gg \Omega_b$ then after a short amount of applied driving field we know that the probability of the atom making the transition from $|e\rangle$ to $|g\rangle$ must be greater for atom in trap state $a$. And if we can create this distinguishability interaction will occur and thus scramble the coherence of the system.

The other thing that we can take from these equations is that if we have that $\Omega_a = \Omega_b$ and $\Delta \Omega = 0$ then we have no possibility of coupling to $|s\rangle$. This means we have no way to create a decoherent gas which would have feel an interaction shift, since the interactions only occur when there is coupling to $|s\rangle$ and a homogenous field cannot couple to $|s\rangle$. This explains why the MIT group did not see a clock shift when applying a homogenous driving field on their gas [54, 55].

We are now ready to calculate the frequency shift for a Ramsey pulse sequence. To do this we will solve the equations of motion in the short pulse approximation. Essentially what the short pulse approximation means is that we have no phase evolution due to interactions during the pulse sequence. So for our conditions we will start with the initial condition that the whole population is state $|d\rangle$, which means that $d = 1$. Then we will apply a pulse Ramsey pulse length $\tau_1$ keeping $g = 0$ using equation 5.13. After this pulse we let the state freely evolve in the presence of interactions. During this time the driving field acquires a phase of $e^{i\delta T}$ where $\delta$ is the detuning of the applied field from the resonance frequency. There is
no effect due to the interactions on the triplet states because the wave function is spatially anti-symmetric. During this time the singlet state also acquires a phase shift due to interactions of $e^{i2g_{eg}T}$. We then can add an additional phase $\phi$ to the driving field in the same manner as described in the previous subsection.

After this evolution we apply a second Ramsey pulse of duration $\tau_2$ again in the short pulse approximation. In our Ramsey measurement we measure the excited state population $2|u|^2 + |t|^2 + |s|^2$. By setting $\delta = 0$ and $\phi = 0$ we can calculate the difference in phase between the $g_{eg} = 0$ case and the case where $g_{eg} \neq 0$ and convert this to frequency by dividing by $2\pi T$. This will tell us the size of the frequency shift. We define our tipping angles $\bar{\theta}_j \equiv \bar{\Omega}_j$ and $\Delta \theta_j \equiv \Delta \theta_j$. This allows us to write the total frequency shift as

$$\Delta \nu = -\frac{1}{2\pi T} \times \tan^{-1}\left(\frac{\sin 2\Delta \theta_1 \sin \Delta \theta_2 \cos \theta_2 \sin 2g_{eg}T}{2 (\cos \theta_1 \cos \theta_2 \sin \Delta \theta_1 \sin \Delta \theta_2 \cos 2g_{eg}T + \sin \theta_1 \sin \theta_2 \cos \Delta \theta_1 \cos \Delta \theta_2)}\right)$$

There is a lot going on here and if you don’t see the subtleties don’t worry you are not alone. Like most things in physics the goal is not to solve the equation, but to gain physical insight into the problem, and the exact solution will come in the details. In that spirit we will make a couple of approximations. The first is that the spread of the Rabi frequency is small compared to the Rabi frequency which means that $\Delta \Omega/\Omega$ is small. Then we will also consider the case of small interactions such that $2g_{eg}T$ is small enough that we can take the small angle approximation of the sines and cosines. After taking these simplifications into account we arrive at a more interpretable form of the frequency shift [56]

$$\Delta \nu = \frac{g_{eg}}{A} \sin(2\Delta \theta_1) \sin \Delta \theta_2 \cos(\bar{\theta}_2),$$

where $A$ is the amplitude of the Ramsey fringe $A = \sum_m \sin(\theta_{1m}) \sin(\theta_{2m})$ which is just the contrast of the for each atoms which for large atom numbers becomes $\approx N \sin(\bar{\theta}_1) \sin(\bar{\theta}_2)$. In our case where we have large atom numbers we must find the shift as a function of density in order to extract $g_{eg}$ to compare the experimental

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2When asking Dave Weiss about a homework problem in his AMO class he said "You don’t need to carry all these terms around. You’re a grown up physicist now, you can make approximations."
Figure 5.2. Theoretical shift as a function of first $\theta_1$ and second $\theta_2$ pulse area. The amount inhomogeneity is $\Delta \Omega/\Omega = 0.1$. This shows that strong dependence of the shift on $\theta_2$ and the weak dependence on $\theta_1$. The solid lines are for fixed $\theta_1 = \pi/2$ or $\theta_2 = \pi/2$. When $\theta_2$ is set to $\pi/2$ then we can see the shift is eliminated.

results to theory.

Figure 5.2 shows how the CFS changes as a function first and second pulse area. As one can see from the surface the shift has a strong dependence on second pulse area $\theta_2$. Also, the CFS shows a weak dependence on first pulse area $\theta_1$. The solid lines are for fixed first and second pulse area at $\pi/2$. When the second pulse area is $\pi/2$ we see that the CFS shift goes to zero regardless of first pulse area. This is contrary to the idea that the shift is proportional to the partial density that is created in the first pulse [63]. The idea is that Bloch vector for the two atoms experience different phase shifts during the first pulse. If $\theta_2 = \pi/2$ the second Ramsey pulse is insensitive to this phase acquisition and it thus reads out the atom transfer at the same sensitivity thus resulting in no shift of the Ramsey Fringe and no CFS. Another way of stating this is that if $\theta_2 = \pi/2$ then the measured excited state population is the same regardless of the first pulse ares. This is for the Bloch vector picture of the system.

If we look to the singlet triplet we picture we that when the $\theta_2 = \pi/2$ the resulting Ramsey fringe for atom in trap state a has the exactly opposite frequency
shift as that in trap state b. Since what we measure is the shift for both atoms we see is no effect of the collisional frequency shift we would measure no collisional shift for gas. All the equations of motion still take place, but the net effect is that they all cancel out.

From equation 5.15 we can also see that we need to have inhomogeneity in both pulses. If the second Ramsey pulse is homogenous then there is no coupling to the singlet state and thus we are insensitive to any interaction shift. This is also consistent with the previous experiments done investigating the possibility of a clock shift with a fully decoherent gas [54, 55].

5.4 Current Status of Investigations

Before I describe our experimental investigation into the s-wave CFS there has not been a direct measurement of a CFS for fermions, but a shift has been seen in an almost equivalent bosonic system. It is most useful to think of the system in the singlet-triplet basis. Figure 5.3 shows the energy levels for the singlet and triplets states for a fermionic system. For this fermionic case we have a collisional shift due to coupling to the singlet state on the right hand side of this figure. As we adjust the coupling strength $g$ then the singlet state is shifted in energy with respect to the triplet state. For the bosonic case the total wave function must be symmetric thus in figure ?? the $\psi_A$'s become $\psi_s$ and vice versa. Thus the triplet states have a spatially symmetric wave functions and thus they feel the effects of interactions. If all three triplet states shift the same amount it is equivalent to the singlet state shifting. In this manner a fermionic CFS could be simulated. This is just what has been done [4].

Their investigation concentrated mostly on spinwave propagation and how that is associated with the CFS. They investigated the dependence of the CFS on first and second pulse areas. While they observed the expected dependence on second pulse area, they also observed an unexpected and unexplained dependence on first pulse area. This unexplained dependence on first pulse area may have been a result of the fact that they did not work in the resolved sideband limit and drove transitions between vibrational states whenever they drove Rabi oscillations. In the next chapter I will describe our experiment investigating the CFS for a pure
Figure 5.3. Energy levels for the singlet and triplet states. For fermions the singlet state is shifted in energy due to the CFS. For bosons the triplet states are shifted. In [4] they use $^{87}\text{Rb}$ such that each triplet state shifts the same (within 5%) due to interactions. Thus making it equivalent to having a fermionic system.

fermionic system and show how the CFS depends on Ramsey pulse areas, scattering length, and inhomogeneity. Our work was done in the resolved sideband limit.
Chapter 6

S-Wave Clock Shift for Fermions

In chapter I will discuss the experimental results of our investigation into the collisional frequency shift motivated by the theoretical and experimental interests discussed in the previous chapter. We will start with a discussion our experimental set-up and how we create an inhomogeneous Raman field. Then I will discuss the light shifts that we observe and how we use as other lasers to clear out atom population in $|5\rangle$ and $|6\rangle$ before getting into our results.

For an atomic clock you essentially are measuring the frequency splitting between two different energy levels. A desirable clock transition has a narrow linewidth and a high transition frequency, the higher the frequency of the transition the better. For cesium the clock transition frequency is 9,192,631,770 Hz and for our transition we are around 76 MHz, thus cesium wins this round. Also for a clock you want the transition to be insensitive to magnetic fields, another win for cesium. With cesium seeming like a better option why should we use $^6$Li. Well Donald Rumsfeld once said ”You go to war with the army you have, not the army you want.” While objectively this statement is true, but we will not discuss the politics of war here. Applied to us we study clock shifts with the fermionic isotope we have, not the isotope we want. And the isotope we have is of course $^6$Li. Also we want the wide tunability that $^6$Li has to offer. As discussed earlier our the Feshbach resonances in $^6$Li provide us with a wide range of interaction strengths. This is not good for clocks because interactions could shift the energy levels around, but this tunability is good for studying how interaction shifts manifest themselves.
6.1 Experimental Details

The first thing that we need for investigating the collision frequency shift (CFS) is an inhomogenous driving field. We are going to use the $|1\rangle - |2\rangle$ transition as our clock transition. As discussed in chapter 2 we have two different ways of transferring atoms between these states. For our set-up we have designed the RF coils in our chamber to apply a uniform RF field to the atoms. Using RF as our drive field would not produce an inhomogenous driving field for the transition. Thus we are left with Raman transitions. To perform Raman transitions we apply laser light such that the frequency difference between the two-beams is tuned to be the transition frequency between the two states. If we can focus the Raman light down such that the waist of each beam is comparable or smaller than the width of the trapped cloud of atoms we can create the inhomogenous field we desire. Also by using laser beams, we can easily change the shape of the beams and have direct control over the inhomogeneity.

6.1.1 Minimizing Light Shifts

From chapter two we know that when we apply light to the $^6$Li atoms we experience a light shift. To calculate the light shift we must take into account all of the different levels the $|1\rangle$ and $|2\rangle$ atoms can couple to. These are the different levels of the D$_1$ and D$_2$ lines. We also have to take into account the polarizations. For our Raman beam configuration we have one beam linearly polarized along the quantization axis $z$ defined by our applied magnetic field. The other beam is perpendicular to the first resulting in a superposition of $\sigma_+$ and $\sigma_-$ light. This allows us to make $\Delta m_s = 1$ transitions, which is the transition we want. Because of the splitting between the D$_1$ and D$_2$ lines we cannot fully cancel out the light shifts felt by these states without increasing our scattering rate to undesirable levels. Thus we do the best that we can.

Both Raman beams originate from the same laser which is beatnote locked to the slower laser system. By measuring the light shift as a function of the applied beatnote frequency we can find the minimum light shift which is where we will choose to operate. Figure 6.1 shows the light shift as a function of beatnote frequency applied to the lock [25]. The minimum in the light shift is when a
118 MHz reference is applied to the beatnote lock. For this lock we have set the reference multiplier to a factor of 32. Thus the Raman laser is locked to 3.776 GHz to the blue of the Raman laser.

### 6.1.2 Removing population in higher Zeeman states

Whenever we have light applied to the atoms we have the possibility of scattering due to spontaneous emission. Because of angular selection rules and the Raman polarization application of the Raman light on or off-resonance could result in population transfer from $|1\rangle$ to $|6\rangle$ and from $|2\rangle$ to $|5\rangle$. If there are atoms in these states they could interact with the atoms in $|1\rangle$ and $|2\rangle$ causing mean-field shifts that would be indistinguishable from the signal that we want to measure.

The solution to this is problem is to either clear the atoms away with the imaging laser or to optically pump $|5\rangle$ atoms back to $|2\rangle$. The clearing pulse is $\sigma_+^\text{light}$ and contains light resonant with the cycling transition to the $D_2$ line for $|5\rangle$ and $|6\rangle$. We only optically pump the atoms from $|5\rangle$ to $|2\rangle$ because $|5\rangle$ atoms...
Figure 6.2. RF spectroscopy between the higher Zeeman levels and $|1\rangle$ and $|2\rangle$. The legend indicates laser beams used to clear the higher states. If no clearing beam is used we can see a large shift in the spectrum due to interactions with the higher states. This shift is eliminated with the use of optical pumping and clearing pulse.

To ensure that we are not seeing frequency shifts due to population in these higher states we perform RF spectroscopy on the $|2\rangle - |1\rangle$ transition. As stated before, there will be no clock shift for these states when using RF to drive the transition and this is indeed what we see. We prepare a gas near 528G and start with all of our population in $|2\rangle$. To see the effects of $|5\rangle$ and $|6\rangle$ on a $|1\rangle - |2\rangle$ mixture we apply Raman a 1 ms long pulse at double the intensity that we normally operate the Raman light. This Raman light is detuned from the $|1\rangle - |2\rangle$ transition by -50 kHz in order to isolate the effects of spontaneous emission. Then we have the option of additionally applying the $|5\rangle |6\rangle$ clearing laser and the optical pumping laser.

After this pulse we perform RF spectroscopy on the gas and observe the resulting frequency shifts. The results are shown in figure 6.2. From this figure we can see if we do not address the problem of population in states $|5\rangle$ and $|6\rangle$ we observe a shift in the RF spectrum. If we only apply a clearing pulse this reduces this...
effect, but does not eliminate it. With the application of both the optical pumping light and clearing light we can see the shift is eliminated. Also we can see that with application of the optical pumping light alone this shift can be eliminated, but for the experiment we choose to keep the clearing light on just to be sure no population stays in |5⟩ and |6⟩.

6.1.3 Varying Density without Heating

Measurement of the CFS will require us to measure the shift as a function of density. Thus in order to accurately do this we must vary the density without heating the gas. To do this we will need to operate at a field where interactions are eliminated or where the scattering length is zero. For this experiment we adjusted the density where \( a_{13} = 0 \). The first thing that needed to be done was to find this location. From theory [2] we knew that it was close to 570 G. But, we wanted to know exactly the location. We determine the location in a manner similar to [70].

To find this we looked at rates of thermalization of a gas comprised of atoms in |1⟩ and |3⟩. Like always we start with a |1⟩ -|2⟩ mixture. Then we ramp to a field of 575 G and perform an ARP to transfer atoms from |2⟩ to |3⟩. We then

---

**Figure 6.3.** x-width of |3⟩ as a function of magnetic field after a 500 µs laser pulse is used to heat |3⟩ and a 115 ms hold time allows for possible thermalization with |1⟩. We interpret the peak to correspond to the location where \( a_{13} = 0 \). The solid line is a Lorentzian fit to the data to find the field value of the peak to be 568.81(2)G.
Figure 6.4. x-width of $|2\rangle$ atoms as function of density. A density of 1 corresponds to the peak density of our gas for an experimental run.

apply imaging light that is near resonance to the imaging frequency for $|3\rangle$ for 500 µs. This heats up the atoms in $|3\rangle$. After this pulse we allow for a 115 ms hold time for the atoms in the trap to thermalize. We then image $|3\rangle$ and record the width of the gas which is proportional to temperature. The idea is that in order for thermalization to occur there has to be interactions between the two species of atoms. We heat up state three atoms much more than state one atoms because the light that we apply is 16 MHz detuned from the $|3\rangle$ image frequency and around 175 MHz from the $|1\rangle$ image frequency. If the gas thermalizes we should see a lower width for the state three gas. If no or incomplete thermalization occurs then the width of the $|3\rangle$ gas will be higher because less energy was transferred via interactions. The field where the width is the highest will correspond to the location where the scattering length is zero. The results are shown in figure 6.3. By fitting this data to a Lorentzian we find that the $|1\rangle$-$|3\rangle$ zero crossing to occur at a field 568.81(2) G. To our knowledge this is the most precise measurement of this value.

Now that we have determined a location where we can eliminate the interactions between $|1\rangle$ and $|3\rangle$ we need a procedure to adjust the density of the gas. Starting from our $|1\rangle$-$|2\rangle$ mixture after evaporation we ramp the field to 568.81 G. Here we
do an ARP to transfer the $|2\rangle$ atoms to $|3\rangle$ and then clear the $|3\rangle$ atoms. We use two 100 $\mu$s clearing pulses separated by 1 ms to ensure all of the atoms are cleared from $|3\rangle$. At this point we apply a RF pulse of the desired fraction of a $\pi$-pulse to transfer atoms from $|1\rangle$ to $|2\rangle$. The amount of atoms left in $|1\rangle$ will be the density we work with. To clear the atoms from $|2\rangle$ we perform the same sequence as our initial clearing of $|2\rangle$ atoms. We ARP the $|2\rangle$ atoms to $|3\rangle$ and clear with the same two pulses. After this we also perform an ARP to bring any $|3\rangle$ atoms left back to $|2\rangle$. Since the difference between $a_{23}$ and $a_{13}$ is about 660 $a_0$ which is two orders of magnitude greater than the $a_{12} \sim 8$ that we are trying to measure, any atoms left in $|3\rangle$ will really mess up our measurement. In this manner we get a variable density gas that is essentially at the same temperature. We can see from 6.4 that the gas is essentially at the same width for each density, indicating that we are not inducing heating as we change the density of the gas.

6.1.4 Creating an Inhomogenous Driving Field

6.1.4.1 Creating the Raman Field

The pieces are starting to fall into place and the next piece of the puzzle is the creation of the inhomogenous Raman driving field. First let us start with the creation of the Raman field via one laser. To do this we use the setup as shown in figure 6.6. The laser light originates from a Toptica DL-100. It is then passed through an optical isolator to prevent back scattering from damaging the laser. There is a beam sampler used to pick-off $\sim 500 \mu W$ of power to be used to lock the frequency of the laser to the slower system as describe above. The light is then sent through an Acoustic-Optical Modulator (AOM) used to control the intensity of the beam, I will call this the switching AOM.

As discussed in chapter 4 if we have a square-pulse window we get frequencies governed by a sinc function. The most ideal way to apply a pulse is to use a Gaussian pulse shape. Unfortunately, a Gaussian starts at a time of $t = -\infty$ and is turned off at a time $t = \infty$. Since we don’t have that kind of time we turn on and off the light using a Blackman window. The intensity of the light is ramped up over a time equal to $1/12^{th}$ of the total pulse duration. The field is also ramped off using a Blackman profile, with the same time of $1/12^{th}$ of the total pulse duration.
For the $5/6^{\text{th}}$ of the time between the turning on and off of the light we keep the light at a constant intensity. This is done, because the experiment is repeated for different pulse durations and we want to keep the phase evolution of the system due to light shifts as constant between shots as possible without adding spectral width to our pulses. The intensity is varied by using a voltage-variable attenuator that is part of the RF amplifier that we use. The transfer function was found in order to correctly produce the correct intensity profile that is shown in figure 6.5.

We use the $-1^{\text{st}}$ order from the 80MHz AOM to produce our Raman beams. We need Raman beams that are linearly polarized orthogonal to each other. To do this we use another AOM that is operating at the resonance frequency between $|1\rangle$ and $|2\rangle$, I will call this the Raman AOM. We use the $0^{\text{th}}$ and $-1^{\text{st}}$ order beams from this AOM. To generate the frequencies we use two Agilent frequency generators that are synced to the same stable GPS signal. These two Agilents are sent into a RF switch where we can decide which frequency to use. The reason we use two frequencies is that we take Ramsey fringes by adjusting the phase difference between the two Ramsey pulses. The Agilents allow us to program the phase of the applied frequency. For the Ramsey pulse sequence we use one Agilent to run the Raman AOM and during the Ramsey interrogation time we switch the operation
of the Raman to the other Agilent, of which we can control the phase. Wave plates are used to adjust the polarization of each beam such that they are crossed polarized. After this they are combined via a polarizing beam splitter where one of the beams is transmitted and the other reflected. The two beams are then sent into a polarization maintaining fiber and sent to the experimental table where they will be shaped to their final size. The $0^{th}$ order beam also has an electro-optic modulator (EOM) in its path. What this EOM does is put side bands on the laser light passing through at the resonant frequency the EOM operates, which for us is 20 MHz. The application of these sidebands results in a lower amplitude for the carrier light. Then in this manner we can adjust the amount of resonant light the atoms sees while keeping the total light intensity constant. This allows us to apply pulses of small pulse area at a duration that keeps us sideband resolved.

6.1.4.2 Shaping of the Raman lasers and 1070nm lasers

Now we have our Raman beams at the correct absolute and relative frequencies we need to shape them. To create our inhomogenous driving field we need shape the beam such that it is smaller than the trapping beam. As stated above we want to have one of the Raman beams be polarized along the direction of the applied magnetic field. We define the direction of our magnetic field as the z-axis. That means in order to get z-polarized light we have to insert the Raman beams along the x-y plane. For technical reasons we insert the beam at about a 9° angle relative to the slower beam which is parallel to horizontal in the lab frame. Figure
Figure 6.7. Schematic of the laser setup used for the clock experiment. The red beam is the Raman beam and the grey co-propagating beam is the 1070 nm beam used for additional trapping. The two vertical beams comprise the 1064 nm crossed-dipole trap. The two concentric annuli are the Helmholtz coils used to create the magnetic field that is in the z-direction as defined by the axis between the center of these two coils. The atoms are at the point where the three infrared beams intersect. The RF antennas are also shown above and below the atom cloud. Also shown are the false color absorption images for (from left to right) only the crossed 1064nm beams, only the 1070nm beams, and the cloud with both beams. All images have zero time-of-flight.

6.7 shows the alignment of the dipole lasers and the Raman laser as used in the experiment. As stated in chapter 5 we want to be sideband resolved so we can define atoms in distinct harmonic oscillator states. This constraint is only for the directions that we have created inhomogeneity in. For our setup the inhomogeneity is in the x-z plane. From the narrow Feshbach resonance experiment we showed that we can get a trap such that in this frame the trap frequencies for the crossed dipole trap are around (181,3400, 7611) Hz in the (x,y,z) directions. Since we have weak trapping in the x-direction we add an additional 1070nm beam co-propagating with the Raman beam for additional trapping in order to be sideband resolved in this direction.

Since we want the Raman and the 1070 nm beam to be co-propagating we
Figure 6.8. Optical setup for the Raman and 1070nm lasers. Also an image of the different sizes of the beams at the focus of the system. This location is also where the atoms are.

designed an optical setup as shown in 6.8. This set-up is based on both the Raman and 1070 nm laser being fiber launched. This allowed us to set up the system on a separate bread-board and once it was aligned we mounted it vertically to the experimental table and used only one additional gold mirror (not-shown) to steer the beams onto the atoms. We then used a camera to image the beam at the location of the focus. This image is also shown in figure 6.8. From these images we get a mean waist of the Raman beam to be \( w_{\text{Raman}} \approx 15\mu\text{m} \) and for the 1070 nm beam we get a waist of \( w_{1070} \approx 29\mu\text{m} \). We find that this results in a trap with frequencies \( (7.2, 3.4, 11.2) \) kHz. This constitutes the laser setup for our system.

The theory formulated in chapter 5 assumes that we have atoms in well defined harmonic oscillator states and we only make transitions between internal states not trap states. This means that the spectral width of our pulse should carry no weight at the trap frequencies. Another way to state this is that our trap is sideband resolved. In order to investigate this we perform Rabi spectroscopy on the gas. We start with atoms in \(|1\rangle\) set the pulse duration to give us a \( \pi \)-pulse, \( \sim 900 \mu\text{s} \). We then sweep the Raman frequency from -25kHz to 25 kHz. The resulting spectrum is shown in figure 6.9A. From this we can see that we are sideband resolved. If we look back at our quantum mechanics book [13] we see
that the harmonic oscillator wave functions are,

\[ \psi_n(x) = \sqrt{\frac{1}{2^n n! \sqrt{\pi}}} \frac{1}{\sqrt{a_{ho}}} \exp \left[ -\frac{x^2}{2 a_{ho}^2} \right] H_n \left( \frac{x}{a_{ho}} \right). \]  

(6.1)

Where \( H_N \left( \frac{x}{a_{ho}} \right) \) are Hermite polynomials which have even or odd parity depending for even and odd \( n \), and \( a_{ho} = \sqrt{\hbar/(m\omega)} \) is the harmonic oscillator length.

We can see from this figure 6.9 that we are indeed sideband resolved. We can tell that the different peaks correspond to trap frequencies in the different directions. Our Raman beams have a Gaussian profile which is an even function. This means that we cannot change the parity of the atoms when we drive Raman transitions. But, this is only the case when we are aligned with the trap. If we are misaligned this introduces an odd parity component to the transition. This is what allows us to make the \( \Delta n = \pm 1 \) transitions. This also gives us a great feedback signal to determine whether or not the Raman beams are aligned with the trap. If we park the frequency on the 7.2 kHz or 11.2 kHz sidebands we can align the beam such that we see no population transfer to the \( \Delta n = \pm 1 \) transitions. After this alignment if we take the Rabi spectrum again we will only see transitions between the \( \Delta n = \pm 2 \) transitions. This is exactly what we see in figure 6.9B.
Now that we have all of the technical details accounted for we can start investigating the effects of inhomogeneity and if it creates an s-wave collisional frequency shift. I will first discuss the shape of our observed Rabi oscillation and the model that we use to fit this Rabi data. This will lead into our investigation into measuring the interactions via RF spectroscopy on the $|2\rangle - |3\rangle$ transition. This gives us proof that we are observing decoherence due to coupling to the singlet state. Then I will discuss our observation of spinwaves propagating which is a result of coupling to the singlet state. From these experiments we will have all the information needed to characterize the collisional frequency shift.

6.2 Rabi Oscillations with a Inhomogeneous driving field

Normally the whole atomic cloud would see a driving field of uniform density. Then the formulation for Rabi flopping given in equation 2.30 would hold, where the coefficients would represent the components of the density matrix. For the case where we have an inhomogenous driving field such as the case described above our Rabi oscillation becomes a bit more complicated. For the two-photon transition the driving field varies as the gaussian profile of the laser beams that we use. As we can see from figure 6.7 our beam is inhomogenous in the x and z directions. Since the Rayleigh length for the 15 $\mu m$ beam is on $\sim 1$mm for the y direction the driving field is homogenous in this direction. Thus we can write our Rabi frequency as a function of position in the x and z directions as

$$\Omega(x, y) = \Omega_0 e^{\frac{2x^2}{w^2}} e^{\frac{2z^2}{w^2}}$$

(6.2)

Where $\Omega_0$ is the Rabi frequency at the center of the trap. Then to calculate the Rabi frequency for each harmonic oscillator level, with quantum numbers n, and m, we write down the Rabi frequency for this atom as

$$\Omega_{n,m} = \langle \psi_{n,m} | \Omega(x, y) | \psi_{n,m} \rangle.$$  

(6.3)

Let us scale all of our length scales by the harmonic oscillator length scale $a_{ho}$
such that \( \bar{x} = x/a_{ho} \), \( \bar{z} = z/a_{ho} \) and \( \bar{w} = w/a_{ho} \). After inserting this into the above equation we get the result that

\[
\Omega_{m,n} = \Omega_0 \frac{1}{2^m 2^n m! n!} \frac{1}{\pi} \int_{-\infty}^{\infty} H_m^2(\bar{x}) \exp \left[ -\left( \frac{2}{\bar{w}^2} + 1 \right) \bar{x}^2 \right] d\bar{x} \times \int_{-\infty}^{\infty} H_n^2(\bar{z}) \exp \left[ -\left( \frac{2}{\bar{w}^2} + 1 \right) \bar{z}^2 \right] d\bar{z}. \quad (6.4)
\]

For waists much much larger than the harmonic oscillator length the integrals approach unity which is the case for a uniform field. If we take the parameterization a step further let us define a parameter \( \alpha = (2/\bar{w} + 1) \). Then we can write the Rabi frequency as

\[
\Omega_{n,m} = \Omega_0 \mathcal{K}_m(\alpha) \mathcal{K}_n(\alpha) \quad (6.5)
\]

\[
\mathcal{K}_n(\alpha) = \frac{1}{2^m m!} \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} H_m^2(\bar{x}) \exp \left[ -\alpha \bar{x}^2 \right] d\bar{x} \quad (6.6)
\]

The calculation of this integral will allow us to calculate the Rabi frequency for a particular atom with harmonic oscillator quantum numbers \( n \) and \( m \). Of course we don’t have a single atom, but a thermal gas of atoms. Since we operate at a temperature near 49 \( \mu \)K and our Fermi temperature is \( \sim 25 \mu \)K we can say that the atoms populate the 3D-harmonic oscillator state via a Boltzmann distribution. For a given number of atoms we can assign a different quantum harmonic oscillator quantum number \( (n,p,m) \) using the trap frequencies in the \( (x,y,z) \) direction. To simulate our experiment we thermally distribute the atoms \[71\] such that the energies of the atoms reproduce the proper Boltzmann distribution as covered in chapter 2. Even though we don’t have any inhomogeneity in the \( y \)-direction we must include it in the Boltzmann distribution in order to properly populate the states.

Now that we have the distribution of energy levels we can thus calculate the overlap of the atom in state \( n,m \) with the Raman light as given in equation 6.5. Calculating this for each atom in our simulation\(^1\) we obtain a value for \( \Omega \) for a specific input \( \alpha \) and \( \Omega \). We can then calculate the mean of this distribution to

\(^1\)For simulation of this on my laptop I use run a simulation for 20,000 atoms which gives a good simulation of the \( \sim 100000 \) atoms we use in the experiment.
Figure 6.10. Atom transfer from $|1\rangle$ to $|2\rangle$ as a function of pulse duration showing Rabi oscillation data for the three different beam sizes used in the experiment. A) is the largest beam size thus the least amount of inhomogeneity. B) is a smaller beam than A) and one can see that the oscillations damp our more quickly. C) is the smallest beam imparting the most inhomogeneity. All of the fits are done with a 20,000 atom model and equation ???. The results of the fits for each beam is shown in table 6.1

\[ \bar{\Omega} \]

\[ \text{and then from the standard deviation of this distribution we obtain a value for } \Delta \Omega. \]

We still have this little pesky issue of light shifts to deal with. Remember, there is a reason people don’t build clocks out of $^6\text{Li}$. But, luckily we have the same machinery to deal with the light shift. Looking back to chapter 2 we know we can write the light shift as a function of Rabi frequency, thus we can write the light shift for atom in state $n,m$ as

\[ \Delta E_{n,m} = \frac{\hbar \Omega_{n,m}^2}{4\delta} \]  

(6.7)

where $\delta$ is the detuning from the different levels. When we run our experiment we adjust the detuning such that we are the mean Rabi frequency is on resonance.
Then we can express the detuning from this mean Rabi frequency as

$$\delta_{nm}(\alpha) = \bar{\delta}((\mathcal{K}_m(\alpha)\mathcal{K}_n(\alpha))^2 - (\mathcal{K}_m(\alpha)\mathcal{K}_n(\alpha))^2), \quad (6.8)$$

Where $\bar{\delta}$ is the mean light shift for atoms that experience $\bar{\Omega}$. Using this expression for the light shift we find the probability of transferring an atom to from $|1\rangle$ to $|2\rangle$ to be

$$|2\rangle = \frac{(\Omega_0\mathcal{K}_m(\alpha)\mathcal{K}_n(\alpha))^2}{(\Omega_0\mathcal{K}_m(\alpha)\mathcal{K}_n(\alpha))^2 + (\bar{\delta}(\mathcal{K}_m(\alpha)\mathcal{K}_n(\alpha)) - (\mathcal{K}_m(\alpha)\mathcal{K}_n(\alpha))^2)^2} \times \sin^2 \left( \frac{t\sqrt{(\Omega_0\mathcal{K}_m(\alpha)\mathcal{K}_n(\alpha))^2 + (\bar{\delta}(\mathcal{K}_m(\alpha)\mathcal{K}_n(\alpha)) - (\mathcal{K}_m(\alpha)\mathcal{K}_n(\alpha))^2)^2}}{2} \right)$$

$$\quad (6.9)$$

This is the expression for one atom. To get the result for all the atoms we take the sum this term for each atom and divide by the number of particles $N$. This will give us the amount transferred. This model is then fit to the experimental data to this equation to extract $\Omega_0$, $\bar{\delta}$ and $\alpha$. Since we have parameterized our inhomogeneity by $\alpha$ we can then use this $\alpha$ to create a distribution of Rabi frequencies from which we can determine $\bar{\Omega}$ and $\Delta\Omega$. For a field of 525.5G figure 6.10 shows the three different beam sizes that we used in the taking of the data. Table 6.1 shows the calculated $\Delta\Omega/\Omega$ for the various fields and beam sizes that we have used. For this calculation I assumed that the Raman beam was cylindrically symmetric.

This gives us a handle on the amount of inhomogeneity we have in our system for the different beam sizes. What this doesn’t tell us is if there we are actually coupling to the singlet state. We need a way of directly probing the interactions to determine that.
Table 6.1. Results of fitting the Rabi Oscillation Data for each field and beam size that we use.

<table>
<thead>
<tr>
<th>Field</th>
<th>Beam Size</th>
<th>$\Delta \Omega / \Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>525.5G</td>
<td>Smallest</td>
<td>$0.346 \pm 0.008$</td>
</tr>
<tr>
<td>525.5G</td>
<td>Smallest</td>
<td>$0.314 \pm 0.007$</td>
</tr>
<tr>
<td>525.5G</td>
<td>Smallest</td>
<td>$0.359 \pm 0.003$</td>
</tr>
<tr>
<td>527.5G</td>
<td>Smallest</td>
<td>$0.324 \pm 0.006$</td>
</tr>
<tr>
<td>527.9G</td>
<td>Smallest</td>
<td>$0.358 \pm 0.008$</td>
</tr>
<tr>
<td>528.7G</td>
<td>Smallest</td>
<td>$0.378 \pm 0.016$</td>
</tr>
<tr>
<td>Average</td>
<td>Smallest</td>
<td>$0.347 \pm 0.010$</td>
</tr>
<tr>
<td>525.5G</td>
<td>Medium</td>
<td>$0.146 \pm 0.008$</td>
</tr>
<tr>
<td>525.5G</td>
<td>Largest</td>
<td>$0.087 \pm 0.004$</td>
</tr>
</tbody>
</table>

6.3 Measuring Interactions with RF spectroscopy

The way we directly probe whether or not we have transferred population to the singlet state is to perform RF spectroscopy on the sample. As previously stated if we use a homogenous field to drive the transition we will not couple to the singlet state. Then if we perform RF spectroscopy on the transferred atoms we will not observe a frequency shift because there was no coupling to the symmetric spatial singlet state where atoms can interact. Now if we have an inhomogenous driving field we can couple to this singlet state. This would result in a frequency shift that can be measured via RF spectroscopy. The amount of shift that we observe would be proportional to the amount population transferred to the singlet state.

The first step is that we prepare the gas of atoms in $|1\rangle$ in the same manner as described above. Then we create a $|1\rangle - |2\rangle$ mixture in one of three different ways. To prepare a coherent mixture we use a homogenous RF $\pi/2$-pulse, this does not couple to the singlet state at all. To create a fully decoherent mixture we use the optical pumping step used as the last step of the MOT preparation. Then we also prepare the gas via a Raman $\pi/2$ or $3\pi/2$-pulse. The Raman pulse which is inhomogenous couples to the singlet state which can have interactions. If we are coupling to the singlet state then we should see a shift proportional to the
Figure 6.11. RF spectroscopy on the $|2\rangle \rightarrow |3\rangle$ transition showing that we do indeed see coupling to the singlet state. The top graph shows the data for the smallest beam and largest $\Delta \Omega/\Omega$. As the beam size is increased we see a decrease in size of the interactions for the Raman $\pi/2$ and $3\pi/2$ pulses indicating that there is less coupling to the singlet state. The inset of each graph shows the Raman-Rabi oscillation for each beam size.

amount of atoms transferred to the singlet state. The results for the three different beam-sizes are shown in figure 6.11.

Seeing this shift confirms that we are indeed coupling to the singlet state. This also gives us a scale of the interaction strength $g$ between two atoms interacting at this density. This interaction strength will determine how fast the phase winds during the interrogation time of the Ramsey sequence. In order to compare the results to theory we want to be in the regime where we are linear in $g$. I will talk more about this in the next section.

Now that we have our interaction shift for Raman $\pi/2$ and $3\pi/2$ pulses we can use this as an independent measure of $\Delta \Omega/\Omega$. To do this we calculate how
much coupling to the singlet state is needed to create the amount of incoherence that would cause the frequency shift that we measure. For a two atoms in trap states $a$ and $b$ the expectant shift observed doing RF spectroscopy on the $|2\rangle$ to $|3\rangle$ transition is

$$
\Delta \nu = \frac{\nu_{12} - \nu_{23}}{\pi} \frac{\sin^2(\Delta \theta_{ab})}{2(1 - \cos(\Delta \theta_{ab}) \cos(\hat{\theta}_{ab}))}
$$

Comparing the values for $\Delta \Omega/\Omega$ obtained in this manner compared to the values extracted from the Rabi oscillation data we see that they differ. The discrepancy could be due to the fact that the trapping potential is not truly harmonic but
really is a Gaussian potential. The value of $\Delta \Omega/\Omega$ that we will use to ultimately
fit our data will be an average of these two values.

Before we move on to Ramsey spectroscopy there is one other sanity check that
can be done. This check is observing the shift in the Rabi spectrum due to coupling
to the singlet state is linear as a function of density. To perform this experiment we
interleave a Raman $\pi/2$ on clouds of varying density with an RF-pulse to create a
$|1\rangle - |2\rangle$ mixture. Then by measuring the frequency difference between the Raman
and RF spectra we can see if our shift is linear. Figure 6.13 shows the results of
this run. The offset is fixed at 0 Hz. A linear fit yields the best $\chi^2$ of 1.15. This
means that all of our checks are marked off and that we can start to explore this
shift via Ramsey Spectroscopy.

### 6.4 Measuring Spinwaves

Since we are driving population between the different internal states for different
trap states it only makes sense that we imprint a spin profile onto our gas. The
propagation of this spin imbalance can thus propagate in the gas and would create
spin waves. These spin waves have been explored for the boson in [72, 73, 74, 75, 4],
and for fermions by John Thomas’s group [76, 77].

The mechanism behind these spin-waves is a mechanism that is called identical
spin rotation (ISR). When two atoms of different spin projections interact they
rotate about their total spin projections at a rate of $\omega_e$ [78, 4]. This exchange
rate is the same as twice the interaction rate ($2g_{eg}$) described in chapter 5. After the
atoms collide for a time equal to $\pi/\omega_e$, the atoms exchange their spin polarizations.
This will result in a propagation of the spin polarization in the trap. This can be
seen by varying the interrogation time in the Ramsey sequence.

The first pulse in the Ramsey sequence imprints a spin-profile on the gas. We
use the inhomogenous Raman beam for the Ramsey spectroscopy because in order
to see the interaction shift and the spinwaves we must couple to the singlet state.
Then during the interrogation time phase of the gas in the frame oscillating at
the resonance frequency evolves at $e^{-i2gT}$. Then the phase is read out with an
additional Raman pulse. It is important to note that we are sideband resolved in
the trap. This means that the Ramsey pulses do not change the trap states of the
atoms. Also it means that in order for the spin wave to propagate it must be ISR.

Taking the full form of the frequency shift calculated in chapter 4 we obtain
that the observed frequency shift for a $\pi/2-3\pi/4$ Ramsey sequence to be for small
$\Delta\theta$.

$$\Delta \nu \propto -\frac{\sin 2\Delta\theta_1 \sin \Delta\theta_2}{2\pi T} \sin 2g_{eg}T$$

(6.11)

We can see from this equation that we are looking for oscillatory behavior in
interrogation time $T$. We should also see oscillatory behavior at a fixed time and
by varying the density of the gas. This is due to the fact that $g$ is a measure of
the interactions which are density dependent.

Figure 6.14 shows our results for spinwaves. For this data we performed Ramsey
spectroscopy with a $\pi/2 - 3\pi/4$ pulse sequence which created a large collisional
frequency shift. We then changed the interrogation as noted on the bottom axis.
The plotted points are the difference between the frequency shift measured for
highest density gas and the frequency shift where we believe zero frequency shift
occurs. To find our zero frequency we fit a line to our lowest three density points to
determine zero. We ran this experiment for both positive and negative scattering
Figure 6.14. Frequency difference between the highest density and the fitted zero density frequency. This data is in agreement with spinwaves propagating in our gas. The insert also shows the measured frequency shift as a function of density. It is non-linear in density for the ∼4ms data, also as predicted for the propagation of spinwaves.

length. The red squares are at a positive scattering length ∼8\(a_0\) and the blue circles are at a negative scattering length ∼−10\(a_0\). The inset to the figure shows the frequency measured for the 6 different densities or the boxed in data points. It is clear that the frequency shifts show a clear non-linearity. The fits in the inset are fits to a sine function. This shows good agreement that we are indeed seeing a signature of spinwaves in the sideband resolved limit.

Now that we have seen spinwaves, which occur whenever we have a fermionic collisional shift [4], we are now ready to take data on and characterize the CFS dependence on Ramsey pulse area and inhomogeneity. To take this data we want to be in a regime where \(2gT < \pi/4\). Thus we look at the spin-wave data and see that at 2.7 ms we should be safe from non-linearities and we will take our data here.

The spinwaves observed by the Paris group were not in the sideband regime. They predicted that the spin waves would still propagate in this regime and we
Figure 6.15. Frequency shift as a function of density for a $\sim \pi/2 \sim 3\pi/4$ Ramsey pulse sequence. The fits to all the data are the solid lines where only one scattering length dependent frequency $\omega_{ex}$ is used for all three fits.

We have confirmed this above. We also observed the effects of spinwaves at constant interrogation time and by adjusting the density for different scattering lengths. This is the same case as above except that instead of changing the interrogation time $T$ we change the value of $g_{eg}$. This experiment was done with a $\Delta \Omega/\Omega = 0.33$ at fields of 524.5G, 525.5G, and 526.5G where the scattering length is near $-9a_0$, $-6a_0$, and $-3a_0$ respectively. The Ramsey pulse sequence was fixed at $\sim \pi/2$ and $\sim 3\pi/4$.

The results are shown in figure 6.15. The solid lines plotted are the theoretical dependence of the shift as a function of density which is a sine wave of frequency $\omega_{ex} = 4\pi hna_{12}/m$ [4]. Where $\omega_{ex} = 2g$ and $n$ is the density. From this expression for $\omega_{ex}$ we see that it is linearly dependent on scattering length and from theory [2] we know the relative scattering lengths for these field, we can fit all the data to one value of $\omega_{ex}$ which are the solid lines in figure 6.15. It is clear this model is in good agreement with the data showing the ISR model and spin wave propagation does indeed occur in the sideband resolved regime.
6.5 Measuring the S-Wave collisional frequency shift

We are now ready to start taking data measuring the collisional frequency shift (CFS) in our system. For review we have developed a way to change the density of our gas without causing heating, found a way to clear the population in the higher Zeeman levels, and observed that we are actually coupling to the singlet state by measuring interactions. Now we are ready to take some data!

The procedure for taking this data is to perform Ramsey spectroscopy on gases of varying density by adjusting the phase difference between the two Ramsey pulses. This was described in Chapter 5 if you wish to refresh your memory. From these Ramsey fringes we will be able to calculate the frequency shift for each density. Since we have these light shifts that cause a systematic phase evolution we are not that sensitive to absolute frequency measurement, but that is fine. We want the shift as a function of density for different Ramsey pulse areas so all that matters is phase difference between Ramsey fringes of different density.

From this phase difference we can calculate the frequency shift, if we know the interrogation time. In the previous section we described how observation of spinwaves led us to use an interrogation time of 2.7 ms. What this is is actually the effective dark time. If there is any phase evolution during the Ramsey pulses then the interrogation time that we see by measuring the fringe frequency will be adjusted. The way I think about it is that as soon as the phase starts to evolve that is when your interrogation time begins. In order find our effective interrogation time we measured the frequency of the Ramsey fringe as a function of first and second pulse area and intensity. With this knowledge we adjusted the programmed interrogation time such that for each Ramsey measurement we have the same fringe frequency and thus the same effective interrogation time.

To map out the surface of figure 5.2 we need to find the slope as a function of density for each pulse area. For each set of pulse areas we take a Ramsey fringe by adjusting the phase from $-\pi$ to $\pi$ in steps of $\pi/9$. To minimize the systematic shifts that could arise we interleave the fringes for the five different densities. We also sample the fringe four times to build up statistics. The experimental run consists of sweeping the phase from $-\pi$ to $\pi$ and then back to $-\pi$ twice taking an
Figure 6.16. Transferred atom fraction as a function of phase for three of the five different densities used to extract the shift as a function density. To extract the frequency we extract the phase of each fringe and divide by $2\pi T$ where $T = 2.7\text{ms}$.

We first performed these experiments at the largest value of $\Delta\Omega/\Omega = 0.34$. We took data at four different fields: 525.5G, 527.5G, 527.9G, and 528.7G. The predicted scattering lengths for these fields are: $\sim -4a_0$, $0a_0$, $2a_0$, and $4a_0$ respectively. The data is shown in figure 6.17. The surfaces shown are fits by taking the thermal average of equation 5.15 for all of the pairs of atoms as was similarly done for the Rabi oscillation and interaction data. All fits were done with the strength of the interactions $g_{eg}$ as the only fit parameter.

From these plots we can already tell that we have confirmed a variety of the
Figure 6.17. Collisional frequency shift per density as a function of first pulse area $\bar{\theta}_1$ and second pulse area $\bar{\theta}_2$ of the Ramsey sequence. All four graphs are plotted on the same scale. The surfaces are a fit to the data using an equation 5.15 that is thermally averaged for all pairs in the trap. The lines emphasize the cuts along the surface where the data was taken. Green is $\bar{\theta}_2$ fixed at 0.445$\pi$, Red is $\bar{\theta}_1$ fixed at 0.445$\pi$, blue is $\bar{\theta}_1 = \bar{\theta}_2$, and purple is $\bar{\theta}_1 = \pi - \bar{\theta}_2$

predictions of the theory. First we can see that the shift is highly dependent on second pulse area and weakly dependent on second pulse area. For all of the fields we see that we have almost zero dependence on first pulse area when our second pulse area is fixed at 0.445 $\pi$. This clearly refutes the postulate that the CFS is due to different partial densities created during the first Ramsey pulse.

From these graphs we can also see that the CFS is dependent on the sign of
the scattering length. This tells us that the CFS is proportional to the s-wave scattering length. Thus, this is the first direct observance of an s-wave collisional frequency shift in a Fermi gas.

Let us take a closer look at the data in a format that is easier to see. While the 3D plots look great in showing qualitatively looking directly at the data as we vary the different pulse areas will corroborate the claims that we just made above. Figure 6.18 shows the CFS data we took for this field as a function of pulse area as described in for each graph. From A) of figure 6.18 we can see that when we fix $\bar{\theta}_2 = 0.455\pi$ and vary $\bar{\theta}_1$ we see almost no dependence on first pulse area for the thermal fit. The numerical simulation taking into account all light shifts reproduces the slight upward trend for large first pulse area.
**Figure 6.19.** Collisional frequency shift per density as a function of Ramsey pulse area. Top row graph is for 527.5 G. Middle row two graphs are for 527.9G. Bottom row graph is for 528.7 G. For all the graphs A) is keeping $\bar{\theta}_2 = 0.445\pi$ and varying first pulse area. B) is keeping $\bar{\theta}_1 = 0.455\pi$ and varying second pulse area. And C) is for $\bar{\theta}_1 = \bar{\theta}_2$. For each plot the solid line represents a fit using a thermal average of equation 5.15 for all possible pairs in the trap. The dotted line is a fit to the two-atom model. The dashed line is a fit to a numerical simulation of the equations of motion given in 5.13 taking into account the light shifts and pulse shapes.

This is quite a remarkable effect. It shows that if the second pulse area of the Ramsey sequence is $\pi/2$ then we become insensitive to the CFS. Thus even if we do not have a homogenous driving field, for instance in a cavity, then the CFS that could accompany the inhomogeneity doesn’t matter because the second pulse would be read out the atoms with the same sensitivity and no collisional shift would be measured. This thus provides proof that the origin of the shift is NOT due to partial densities created during the first pulse.

From plot B) of 6.18 we can see that the readout pulse $\bar{\theta}_2$ is an important part of actually measuring the CFS. For $\bar{\theta}_2 < \pi/2$ then the CFS is negative and crosses 0 near $\pi/2$ and then varies strongly creating a large shift for pulse areas
Figure 6.20. Extracted scattering lengths as a function of magnetic field. From this fit we see that the slope is $3.1 a_0$ per G and the zero crossing location is 527.51(11)G.

approaching $\pi$. I think of the process as you transfer atoms to the singlet state during the first pulse and the second pulse reads out the the phase difference between atoms evolving in the singlet and triplet states. If the second Ramsey pulse is homogenous then there no way to couple to the singlet state which is the only way the CFS can manifest.

Figure 6.18 also shows the shift when the first and second pulse areas are equal as is normally done in a Ramsey sequence in part C) and then the orthogonal sequence where $\bar{\theta}_1 = \pi - \bar{\theta}_2$ in D). We also see good agreement with the theoretical predictions supporting the validity of the theory. Thus if one were to design a clock that must use an inhomogenous driving field then it would be wise to take a lesson from these CFS results. That lesson is of course to first of all do whatever you can have a homogenous field and the other lesson, if you can't follow the first lesson, is have your second pulse area be $\pi/2$ in order to be insensitive to the CFS.

In looking at Figure 6.17 take note that the sign of the shift is proportional to the sign of the scattering length. This means that from the fits to this data we have a direct measurement of scattering lengths. The explicit CFS for a pair of atoms is given in equation 5.15. From this we can then calculate the scattering length for each field using the equation for $g_{eg}$ for atoms in trap states a and b given by
Figure 6.21. Collisional frequency shift as a function of second pulse area for varying values of $\Delta \Omega / \Omega$. A) shows the CFS shift as a function of second Ramsey pulse area. B) shows the shift per density extracted from A) as a function of $\Delta \Omega / \Omega$. The solid line in B) is a fit to theory with one free parameter.

$$g_{eg,ab} = \frac{4 \pi \hbar a}{m} \int \psi_a^2(r) \psi_b^2(r) dV. \quad (6.12)$$

Thus in this manner we can extract the scattering length for each field. Now we know that we have some uncertainty in the absolute number of atoms that we have. This would only affect the magnitude of the shift that we observe. Since we know from [2] the scattering lengths change linearly in the field region we are looking at we can use this to determine the location of the zero crossing of $|1\rangle - |2\rangle$ scattering length.

The resultant scattering lengths extracted from the fits are shown in figure 6.20. This fit gives a slope of $3.1(2) a_0/G$ which is close to the predicted $3.5 a_0/G$ slope from the theory [2]. This discrepancy is most likely due to the systematic error in atom number. The extracted zero crossing location is at $527.51(11)G$. This is in good agreement with value $525.5(2)G$ as determined in [76]. The fact that these numbers are all in agreement confirms that we are seeing effects due to s-wave interactions.

All that is left is to see how our shift scales as we vary inhomogeneity. From table 6.1 we see that we have two other beam sizes that give us less inhomogeneity. From equation 5.15 we see that we the frequency shift should be proportional to $\sin(2\Delta \theta_1) \sin(\Delta \theta_1)$. Figure 6.21 A shows the dependence on the CFS as a function
of second pulse area for various values of $\Delta \Omega / \Omega$. The shift decreases as the amount of inhomogeneity decreases. Figure B of 6.21 explicitly shows this dependence. This shows that for our data the CFS scales as expected from the theory.

### 6.5.1 Conclusion of Results

From all of these results we can conclude that even for fermion systems a CFS can still exist. The theory behind this shift as described in Chapter 5 shows that it is the coupling to a singlet state that has a spatially symmetric wave function which will allow for interactions. The consequence of this is that the CFS is strongly dependent on second Ramsey pulse area and varies weakly with first pulse area. This is confirmed in our experiments. We also show that this shift is due to s-wave interactions because it scales directly with the s-wave scattering effects. This allows us to determine the location of the zero of the s-wave scattering length to be at $527.51(1)$G. We also see that as we decrease the inhomogeneity the shift scales in accordance with the theory. Also we observe spin waves which occurs simultaneously when a CFS exists.

From this information we can then compensate for these effects in an atomic clock made with fermionic isotopes. If for technical reasons a homogenous driving field cannot be produced then operating with a second pulse area of $\pi / 2$ will produce an insensitivity to the CFS.

### 6.6 Future Directions

For experiments that we performed above the singlet state is always within the bandwidth of the Ramsey pulse because the interactions are small. It is possible that we could increase the scattering length such that the driving pulse cannot spectroscopically couple to the singlet state. If we do not couple to this state we cannot have a CFS. This would be another way to create a clock that is immune to CFS. We could see this by spectroscopically coupling to the singlet and triplet state via Rabi spectroscopy and see peaks in the spectrum with a gap of $2g_{eg}$ between the two peaks. Another way to see evidence of this would be to look at Rabi oscillations. If the singlet state is spectroscopically resolved, then we only
Figure 6.22. Rabi oscillation data for different scattering lengths. As the scattering length increases the coherence time increases.

have coupling between the three triplet states and observe Rabi oscillation at the mean Rabi frequency. This would result in a longer coherence time and all the atoms oscillating between the two levels at the same Rabi frequency. We have already seen evidence of this effect as shown in Figure 6.22. This figure shows Rabi oscillation data for roughly the same Rabi frequency at different scattering lengths. We can see that for higher scattering lengths the coherence time increases. This is consistent with the idea that we are only coupling to the triplet state.
Appendix A

Magnet Control System

A.1 Introduction

Discussed here is the magnet control system that we use to control our Chandlee and Corridor coils that produce our Feshbach magnetic fields. The system consists of a computer controlled fine and coarse analog setpoint that is converted to a digital signal. This single is ran over to the Setpoint Box where is converted to an analog voltage. This setpoint is stabilized by a digital feedback loop. This analog setpoint is then sent to the Control Box where is use as a reference for out main current control. The Control Box also houses the burden resistors and analog control circuit that creates our process variable. Each process variable is then sent to a set of 10 MOSFETS which control our coils.

A.2 Overview

The magnet system that we use for creating bias magnetic fields consists of three parts; the power supply, the coils, and the control electronics. The power supplies are the Sorenson DHP series supplies. They are capable of generating 660 A. We run them at 15V which gives us enough overhead to drive the current. There are two coils configured in Helmholtz configuration. Each coil consists of 16 turns of 3/16" square wire with 1/16" hollow core that is used for water cooling purposes. For convenience we named these coils Chandlee and Corridor. The coil that is closest to the corridor in the lab is called Corridor. The other is called Chandlee
Figure A.1. Top view of the magnetic coils used to produce our bias magnetic field. The Primary coils are the Chandlee and Corridor magnets that use the control system described in this write up. The Booster Coils are now our MOT coils.

The current to these coils is controlled by tuning the gates of an array of MOSFETs. The gate voltage is controlled by a current control feedback system. A description of this control system is the main focus of the report. For a more in depth look at the magnetic coils consult Jason Williams thesis. In Fig. 2 I present a flowchart of the whole magnet system. Skinny blue wires indicate analog lines and the fat lines are digital signals. This chart is only for one of the coils and an identical system is in place for the other coil.

A.3 Magnets

Here I will present some of the technical details of our magnets. Using the dimensions above the and permeability of free space the calculated inductance of each coil is .55 mH. By feeding modulating the gate voltage to the MOSFETs we are able to determine the transfer function of the Corridor magnet (It is assumed that Chandlee has a similar transfer function and all tests have indicated that this is...
Figure A.2. Flowchart of the magnet control system for one of the coils. A coarse and fine analog signal enter the FPGA. The FPGA converts the coarse signal to a 10 bit digital signal and the fine signal to a 13 bit signal. This signal is then sent across the room to another FPGA. Here the FPGA runs a 24bit Digital-to-Analog converter (DAC). The output signal is also set back to the FPGA through a temperature stabilized analog-to-digital converter (ADC). The FPGA uses this as a feedback for the digital control filter that the FPGA runs. This control filter is used to ensure that the output of the DAC does not drift. The stable setpoint is then sent to the analog controller. The analog controller is a feedback circuit that is the main control of our current. A Danfisyk current sensor outputs a current proportional to the current through our coils. This current is converted into a voltage via temperature controlled ultra-stable burden resistors on the analog control board. This voltage is compared to the setpoint and an error signal is produced and processed to create a voltage to drive the gates of the MOSFET array.

Below are Bode plots of the gain and a phase plot of the response.
A.4 Current Current Control System

The current control system consists of many different parts. Figure two is a flowchart of the main points of the system. This system will be broken down into the following parts:

- Creation of a digital signal
- Creation of a stable analog set point
- The Feedback control of the current

The goal of this whole system is to get a precision of 1 part in $10^6$ or to miliGauss or at least 20-bit resolution. With the system described here we only get about 70 mG stability. I will also discuss future planned improvement to get us to 1mG stability.

A.5 Digital Setpoint from two Analog Signals

From discussions with colleagues here at Penn State and while attending DAMOP we found that a key part of producing a stable magnetic field is to ensure that the setpoint that we feed to the feedback control circuit is stable. Our previous
version had just one analog output for each coil. This output was also feed across the room potentially picking up noise. The solution to this problem was to use an FPGA (NI sbRIO-9631) to create a digital signal that could be sent across the room without any loss. This FPGA board has analog inputs that can function as a 16 bit analog to digital converter. We would need at least 20-bit resolution (if we have a perfect controller) we have to use multiple analog signals witch is equivalent to a fine and coarse analog line for each coil. The way this is done is that the desired field, 0-1024 Gauss, is entered. The LabView program we use to run the system (there will be another writeup on this subject) then converts this number into a course and fine analog output. Essentially the course analog output divides the 0-10V output into $2^{10}$ (1024) steps. Then the remainder is put sent to the fine control analog output which is also a 0-10V signal.

As stated above the FPGA has a digital resolution of 16-bit, but unfortunately we don’t get that resolution from our National instuments analog output. To get our 20+ bit resolution we feed a fine and coarse line each into a separate analog input in the FPGA. Because we split the coarse signal into steps 1024 the most resolution we can attain is $2^{10}$. In a perfect world we could then get 16-bit resolution from the fine control and that would give 26-bit resolution (or one part in 67,108,864). Since we are building this system we are not in a perfect world, so

![Figure A.4. Phase of the response of the magnet system as a function of driven frequency.](image-url)
adjustments need to be made. For the fine control we observe that we are stable at 13-bit resolution. Even though the ADC has 16-bit resolution our source is only stable to the 13-bit level.

The FPGA program then combines these signals to create one 23-bit \((8,388608)\) signal for each magnet. This signal is then sent out of the FPGA via digital outputs into a 64-pin cable that then takes it to the Digital to Analog conversion box that will create the stable setpoint to send the control box. The main advantage here is that we have created a stable digital setpoint that can be sent across the room without noise.

A.6 Digital to Analog Conversion

We hit a couple of snags creating the Analog signal from the digital signal. First we wanted to do all digital control of the magnets. This involved feeding our Danfisik hall probes into the a FPGA board via an ADC converter. Then all of our servo control would be done on the FPGA and we would output an super precise process variable. This would have worked except for the fact that the ADC board had a delay on the line. This limited our bandwidth to only 500 Hz more importantly it sent our phase margin threw 180 and that caused us to lose control of the magnets. Like most things in the lab we found a way around this problem.

The solution was to create a hybrid Digital and Analog control servo. The digital part creates a stable setpoint at an ideal 24 bit resolution that is sent to the Analog control. This setpoint is also fed back into the FPGA via the ADC board that we wanted to use earlier. As stated the delay limited our bandwidth to 500Hz, but the only drifts that we expect in the setpoint should be within this limit (slow thermal drifts of the chip) thus it should create our stable setpoint.

A.6.1 Isolation

The first step in our creation of the signal is to isolate it from the other FPGA that creates the digital setpoint. This is done by the use of HCPL-0723 optocouplers. There is one for each line. Each side of this chip is controlled by different voltage sources and grounds to prevent ground loops and noise on being transferred across
the room. After the isolators the signal is sent to the FPGA.

### A.6.2 DAC

After the isolators the signal is sent directly into the FPGA. The FPGA takes these digital lines and then converts them into an analog signal. To perform the Digital to analog conversion we take advantage of a PCM1794A chip from Texas Instruments. This is a 24-BIT digital to analog converting chip that is programmed via the I²S method. This chip is actually a similar chip used in Ipod and other digital music players. It has two different channels that it can program, the left and the right speaker. In our case we of course do not have two different speaker, but two different coils. The schematic for this process is shown in Figure A.5.

The output of each channel goes to the analog board for control of the current. It is also fed back to the FPGA via a NI 9239E Analog to Digital converter. This converter has a delay that limits our bandwidth at only 500 Hz. This prevents us from only operating with a digital feedback loop. But, this feedback loop can be used to prevent slow drifts of the setpoint due to temperature effects. The temperature of the 9239E is stabilized via a TECs and an Thor Labs temperature controller.

### A.7 Analog control

The analog control for each of the coils is given by the following circuit in A.6. The purpose of this board is to take the a measurement of the current via inputs J2 and J3, and compare it to the input J1 which is the stable digital setpoint described above. To sense the current in each coil we use a Danfisyk closed loop Hall sensor. The output of this device is a current proportional to the current you wish to measure. The output current is between 0-400mA. Then we pass this current through four 0.001% 10 Ohm resistors that allow us to convert the current to a voltage between 0-1 V. The rest of the components are OP27’s that are fast and precise. Where possible we used the highest precision resistors. The two inputs are compared and processed using an OP27 and a PI feedback loop. The output is then fed to the gates of the MOSFETs. There is an additional output that is
**Figure A.5.** Schematic of the Digital to analog board that is used to create an stable setpoint. The to BNC outputs are sent to the analog control board and also feedback to the FPGA via an ADC. The FPGA then uses a digital feedback loop to stabilize this setpoint.

used for error monitoring will be used for a faster tweeter system in the future.
A.7.1 MOSFETS

They say that your strongest memories are attached to smell. I believe this to be true, because I cringe at just the thought of smelling burnt MOSFETs that have exploded. But, I believe that future students will not have this stench ingrained into their brain due to the improvements that have been done.

The type of we use is the IXFN 180N10 HiPerFET made by IXYS. These work well, but get mad when they overheat. By, get mad I mean explode. Thus to keep them cool we mount them a water cooled aluminium heat sink and use thermal paste when mounting them to insure good thermal contact. It is important that the water comes directly from the wall to the heat sink. If the water cools another heat sink first the 1-2 degree change in temperature is enough to cause the MOSFETs to fail. This is why I know what burnt MOSFETs smell like. Once we got a separate water line for each set of MOSFETs we have not experienced a failure (as of June 2012).

A.8 Results of the system

The easiest way to test the results of our system is to use the atoms. If by using Ramsey spectroscopy and sitting on the edge of a Ramsey fringe we can test the stability of our system. The fluctuation of the magnetic field will cause fluctuations in atom number. By knowing the interrogation time we can set an upper limit to the field stability by the relation

\[
\Delta B = \frac{\Delta N}{\pi T} \frac{1}{\partial B} \frac{\partial \nu}{\partial B} (A.1)
\]

Where \(\Delta B\) is our field instability, \(\Delta N\) is the measured standard deviation of the atom number, \(T\) is the interrogations time and \(\frac{\partial \nu}{\partial B}\) is the sensitivity of the transition in \(\frac{Hz}{G}\). Then for a 25ms interrogation time we obtain a Ramsey fringe as shown in figure A.7. Taking data at the side of the fringe where the slope of the fringe is greatest we get our value for \(\Delta N=0.0756\) which translates into a field stability of at most 240 \(\mu G\).

This stability is great, but how fast can we switch field. By imaging the atoms after a variable time magnetic field sweep we find that we need about \(\sim 25\) ms to
Figure A.6. Schematic of the analog feedback control board used to control the current of the magnets.

ensure that the field has settled down after a $\sim$100G sweep. Which is OK and could be improved by adding a tweeter to the system.
Figure A.7. Ramsey fringe with RF at an interrogation time of 25 ms. The fluctuations at the side have a standard deviation of 0.0756 which translates into a field stability of at least 240 $\mu$G.
Other Experimental Details

For the sake passing my knowledge of the experimental details after I move on from Penn State. This appendix will provide insight into these experimental details in case they were to break. Which unfortunately might happen one day.

B.1 Sync to 60 Hz

Currently in our lab we use a PulseBlaster from SpinCore technologies to control our digital signals. This device handles all of our digital timing as well as triggers our analog control devices which are NI-6733 boards hooked up to the BNC-2110 module. NI board is triggered via RTSI clock that is actually a trigger from the pulse blaster. Thus all of our timing is controlled by the PulseBlaster.

Since all of our instruments are run off a 60 Hz line voltage there is a potential of having drift of magnetic field depending on what phase we started the experiment running. An example is if a small magnetic field is oscillating at the 60 Hz, caused by the line voltage, our interrogation of the atoms (via imaging, RF transitions...) will depend on the phase of this 60 Hz voltage. Thus if the experiment does not start at the same phase it is possible to interrogate the atoms in a non-systematic way. This could lead to a systematic error in our signal. To help eliminate this error we feedback on the current using a digital control circuit (discussed elsewhere). We will also trigger off the same phase 60 Hz line that runs all of our equipment.

To trigger the PulseBlaster it requires a low logic state (in this case 0 V) on the hardware trigger. Normally it should trigger off a falling edge, but for some reason
we found that a constant 0 V will trigger it. This leads us to desire a trigger that is a square pulse (between 0 and 5 V) with a very high duty cycle. The circuit below does just this. It takes in a 60 Hz 115 V\textsubscript{RMS} single and transforms it to $\sim 6$V\textsubscript{RMS}. After this we have a voltage divider. This signal is fed into a 74HC14 inverting Schmitt trigger. This was the original design of the circuit. This produce the desired square wave with a duty cycle of $\sim 65\%$. This means that signal would be at 0v for 7.5 ms. Thus the start of the experiment could drift by this amount from shot to shot. To correct this I fed this signal in to into an LT1097 opamp configured as a differentiator. Since I only have +5 V connected as the power we only get two sharp signals instead of four as the output. These are then fed into the Schmitt trigger. This produces a square wave a 1.5 \% duty cycle. Sending this trough the Schmitt trigger again inverts the signal and produces our desired square wave between 0V and 5V with a 98.5 \% duty cycle. This was put into the hardware signal and it was observed that the experiment becomes synchronized to line voltage within $0.03\pi$ radians.

### B.2 Important Circuits

We have had a bunch of homebuilt circuits in our lab. Sometimes it is not straight forward to reverse engineer these components. The two most important homemade circuits in our lab are for locking the lasers and controlling the magnets. Since
Figure B.2. The schematic of the circuit used to beatnote lock the lasers.

the magnets were already covered in Appendix A, I will not include those circuits here.

B.2.1 Beat-Note Lock

As described in Chapter 2 we lock our laser systems by use of a beat-lock lock system based off of [25]. With this lock we feedback not only to the piezo that controls, but the current of the laser diode. To do this we employ the use of the circuit in figure B.2.


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