WINGLET DESIGN FOR THE PIPISTREL TAURUS ELECTRO G2
AND TWIN-TAURUS G4 WITH PERFORMANCE COMPARISON
MADE USING FLIGHT-PATH OPTIMIZATION

A Thesis in
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by
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Abstract

Winglets have been accepted widely by the aviation community as a means of improving performance. In particular, the manufacturers of racing sailplanes have employed winglets as a means of increasing overall cross-country performance. Many companies have incorporated these features into their modern designs so much so that a racing sailplane seldom leaves the factory without winglets.

A winglet design for the Pipistrel Taurus Electro G2 self-launching sailplane that improves cross-country performance is presented. The design was made by analyzing the airplane using a component drag build-up that considers the airplane flight regime, flap settings, and trim drag penalties. Induced drag is predicted using a multiple lifting-line analysis method. This method has been used to evaluate the impact winglets have on the Schempp-Hirth Discus and the Schleicher ASW-20. The predicted performance of the unaltered airplanes is shown to be comparable to flight testing results. The Taurus G2 is simulated to experience optimal conditions (zero wind field) in order to predict the energy required to climb to cruising altitude and maximum range. The predicted performance of the Pipistrel Taurus Electro G2 with winglets shows gains that are consistent with the improvements made to the Discus and ASW-20.

A winglet design for the CAFE Foundation Green Flight Challenge winning airplane, the Pipistrel Twin-Taurus G4, is also presented. The Twin-Taurus G4 was built using two, reacquired Taurus G2 fuselages and wings using a new center section to join the fuselages together. Outboard wingspans are shared with the Taurus G2. Winglets were designed in the same manner as for the G2 but tailored to improve performance of the airplane specific to the requirements of the competition. The performance of the airplane is evaluated using a MATLAB gradient-based minimizer that finds the most energy-efficient flight path given an airplane model, wind speed model, and course turnpoint coordinates. The flight performance with winglets is compared to that without winglets. A significant improvement in performance is gained for both airplanes (on the order of 2%-4%).
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\begin{itemize}
  \item $C_D = \text{airplane drag coefficient}$
  \item $C_L = \text{airplane lift coefficient}$
  \item $C_P = \text{power coefficient}$
  \item $C_T = \text{thrust coefficient}$
  \item $D = \text{magnitude of drag force}$
  \item $D_{\text{prop}} = \text{propeller diameter}$
  \item $J = \text{advance ratio, } \frac{V}{nD_{\text{prop}}}$
  \item $L = \text{magnitude of lift force}$
  \item $L/D = \text{lift-to-drag ratio, range efficiency}$
  \item $P = \text{power}$
  \item $S = \text{wing area}$
  \item $T = \text{magnitude of thrust force}$
  \item $V = \text{airspeed, } v_a + |w| = V$
  \item $a_i = \text{polynomial coefficients}$
  \item $c_d = \text{airfoil drag coefficient}$
  \item $c_l = \text{airfoil lift coefficient}$
  \item $g = \text{acceleration due to gravity, } 9.81 \text{ m/s or 32}$
\end{itemize}
$m$ = mass

$\alpha$ = angle of attack

$\rho$ = air density

Subscripts

$t$ = in-track component

$c$ = cross-track component

$z$ = vertical component
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1.1 Benefits of Winglets

Winglets are an established way of improving the performance of an airplane without the need of drastically modifying the design of the airplane. Studies of winglet performance for sailplanes began in the early 1980s. This research led to the first telling use of winglets in competition at the 1991 World Soaring competition in Udvalde, Texas. Ten sailplanes flew with winglets in various classes. Five of the 15 meter class ships that flew with winglets placed in the top five places on the fastest day of competition [1]. Soon after this debut of technology, winglet use on sailplanes became more and more relevant. It is now rare for a high-performance sailplane to leave the factory without winglets.

Properly designed winglets improve several aerodynamic aspects of the sailplane they are mounted on. Winglets influence the airflow near the wingtip such that flow can stay attached in this region at higher angles of attack. This can increase the maximum lift coefficient that the wing is capable of generating and allows the sailplane with winglets to climb in thermals that would otherwise be too weak. A sailplane could also bank at a steeper angle within a thermal, which allows the sailplane to be closer to the core of thermal thereby allowing it to climb faster. Winglets can also improve the span efficiency of a sailplane, which reduces induced drag.
1.2 Pipistrel Taurus G2 Electro

The Pipistrel Taurus G2 Electro is an electric, self-launching 15 meter sailplane designed and built in Slovenia. Its side-by-side cockpit layout and self-launching capabilities make it very suitable for training flights as well as recreation. It comes from the factory without winglets, currently. On airplanes of similar span, such as the ASW-20, winglets have shown to improve the span efficiency, turning performance, climb performance, and sink rate [1]. It can safely be assumed that well-designed winglets could improve the same characteristics of the G2. The amount of energy required to reach a desired altitude is crucial for an airplane, especially for an electric airplane. With winglets, a Taurus G2 could use less energy while using the self-launching system to reach the same altitude a stock G2 would use. Winglets could also allow the G2 to climb in thermals that would otherwise be too weak, resulting in more training opportunities for the student pilot.

1.3 The Green Flight Challenge

The Green Flight Challenge (GFC) was named one of NASA’s Centennial Challenges at Oshkosh AirVenture 2009. The motivation behind the competition was to draw excitement and innovation into the design of a general aviation (GA) airplane for energy-efficient flight by having a set of challenging requirements. In essence, the rules for the competition laid the ground work for designing a “flying Prius.” The rules for the competition did not prohibit any energy source (except for nuclear, of course) or combination of energy sources, such as gas-electric hybrids. The rules are summarized in Table 1.1.
Range | 200 statute miles with 30 min. reserve, day VFR at ≥ 4000′ MSL over coastal terrain
---|---
Efficiency | ≥ 200 Passenger-MPGe energy equivalency
Speed | ≥ 100 mph average ground speed
Minimum speed | ≤ 52 mph in level flight, power and flaps allowed
Take-off distance | ≤ 2000′ from brake release to clear 50′ obstacle
Noise level | ≤ 78 dBA at full power take-off, measured from 250′ sideways to take-off brake release

Table 1.1: Green Flight Challenge requirements, [21]

Restrictions were made on vehicle weight, wingspan, height (for hangar clearance), and seating arrangement. There were further restrictions on the course. The airplane had to follow a set of GPS turnpoints and not stray more than 300′ off the centerline of the course trajectory. The airplane also had to fly above 4000′ MSL and below 6500′ MSL once the airplane reached 4000′ MSL. These constrain the ability to possibly utilize the best winds.

The Taurus G2 was used as the foundation of the Pipistrel’s entry into the Green Flight Challenge because of the team’s limited time and monetary budget before the competition. The mission for the G4 was very specific, as described by the competition rules. The focus of the winglet design is for performance around 100 mph without hindering performance in climb after take-off or in high speed flight to the finish line. Ultimately, the goal of the winglet design is to improve the energy efficiency of the G4 by requiring less energy to fly the course.
Energy savings can be furthered through flight-path planning. The wind can be a significant hinderance and/or benefit to the energy consumption of an airplane, with much variation involved in a closed circuit flight. A method of analyzing the course by the energy consumed is used to determine the best path to fly, while meeting the requirements of the competition. This baseline performance is compared to performance predicted with various winglet designs.

1.4 Summary and Outline

Chapter 2 summarizes a history of wingtip/winglet design and the transition of winglets from theoretical to practical design features. Previous work done on path planning is also discussed in Chapter 2, summarizing work done in the late 19th century up to complex computer programs developed recently for unmanned aerial vehicles. The wind model generation is acknowledged, along with a discussion of the prediction software. The physics of winglets as well as the design and analysis method of winglets is discussed in Chapter 3. Chapter 4 summarizes the method used to predict the energy performance of the Twin-Taurus G4 flying the GFC course. Chapter 5 presents and discusses the winglet designs and the effect on performance of both airplane. Finally, Chapter 6 suggests future work for both winglets and path planning.
Chapter 2

Previous and Related Research

2.1 Brief History of Wingtips

The concept of adding a device to the tip of a wing in order to influence the flow around the tip began around the early 20th century. Frederick W. Lancaster, a British engineer, acquired the first patent for endplates on wingtips in 1897. Through his theory of lift via circulation, he noted that the high pressure under the wing would “spill” to the upper surface around the wingtips [2]. This “spillage” would generate vortices that would cause the lift-force vector to tilt backwards in the freestream direction, creating an “induced drag.” Thus, he developed endplates to minimize this “spillage” [3]. Later, in the United States, Scottish-born engineer William E. Somerville filed a patent for a biplane with upturned wingtips on the upper wing. While they bore a resemblance to winglets, the tips were intended to improve roll stability through dihedral rather than reduce induced drag [4]. In 1930, Vincent J. Burnelli patented the use of wing end plates as a means of increasing the effectiveness of ailerons whose span extends to the wingtip. Because of the tip vortex, there is a pressure gradient loss near the wing tip, and the effective aileron span diminishes. Burnelli proved that a flat plate mounted on the wing tip would...
reduce this pressure gradient loss [5].

The German mathematician W. Mangler in 1937 showed the effect of an endplate on the lift distribution and induced drag of an elliptically-loaded wing. He noted that if the wing is operating at high lift coefficients, then the presence of an endplate provides a means of reducing the induced drag of the wing [6]. The first rigorous experimental research of wingtip design to reduce drag was conducted by Sighard Hoerner at Wright-Patterson Air Force base in 1952. These experiments investigated the mechanism of tip vortices as well as the effect of various wingtip shapes on lift-to-drag ratio [7]. Hoerner wingtips became common design features on numerous general aviation airplane of that time, because the ease of construction of these tips did not increase airplane manufacturing cost.

Richard T. Whitcomb et al. began the first research on what are now known as “winglets” in the 1970s. Essentially, the difference between an endplate and a winglet is that a winglet acts as a small wing at the tip that is generating a lift force directed inboard that affects the flow on the upper surface of the main wing. The flow on the upper surface is altered such that “the reductions in induced drag are associated with a spreading of the vortex crossflows behind the wing tip” [8]. This flow is illustrated in Figure 2.3.

Figure 2.2: Photo of a Hoerner Wingtip, MetCoAire [33]

Figure 2.3: Comparison of flow field near the wingtip, NASA TN D-8264 [8]
Whitcomb also described how not only is the aerodynamic benefit of winglets important, but also the effect of winglets on root-bending moment of the main wing and wing weight. This analysis was confirmed and elaborated in a parametric study conducted at NASA by Harry H. Heyson et al. in 1977 [9]. This report explains that although an increase in empty weight would occur due to the addition of a winglet, there would be an overall decrease in gross take-off weight due to fuel savings.

Peter Masak began notable research in winglet use on sailplanes. He was among the first to apply winglets to a competition sailplane successfully. In this case, success was measured by how well the winglets helped the overall cross-country performance of the sailplane. Sailplane operation involves low speed, high lift coefficient flight in thermals and high speed, low lift coefficient flight between thermals. Masak enlisted the help of Maughmer and Selig to design a winglet for a racing sailplane. The design indeed produced an improvement, as can be seen in the contest results of the 1991 World Soaring contest [1]. Through the design work, Maughmer et al. realized the necessity of having an airfoil section designed specifically for air flow at the wingtip. This obviously led to the need for an airfoil section that could perform at those low Reynolds numbers. One of the main challenges in designing this airfoil was the ever-present laminar separation bubble prevalent in low Reynolds number flows. Careful design of the surface pressure gradients lead to a successful airfoil design that would be able to operate in the necessary flight regime, the PSU 90-125 [1]. Winglets have now become the standard tip design of most high-performance sailplanes.

Research has continued in wingtip design with various other non-planar designs, such as spiroid tips and C-wings [22][30]. C-wings have proved to be most beneficial
for tail-less airplane because the top of the C-wing can provide a stable positive-pitching moment about the center of gravity while not reducing the efficiency of the main wing.

![Figure 2.5: A conceptual airplane designed with a C-wing, [30].](image)

### 2.2 Flight-Path Optimization

Flight-path planning is an integral part in the operation of any airplane. Careful attention must be paid to weather conditions, possible emergency landing locations, and fuel-consumption calculations before any flight of significant length. This is very crucial for small, general aviation airplane and even smaller uninhabited aerial vehicles (UAVs), because of the relatively small amount of fuel or energy available on-board. The weather can play an even more crucial role in the operation of sailplanes, since most have a very limited energy supply if any. The effect of the wind on a sailplane is an integral part of the ship’s performance.

Much work has been done with focus on soaring flight. Lord Rayleigh described the flight of birds that maintain altitude without flapping. He deduced that the bird must gain energy from the wind, be it a non-horizontal wind (thermals, slope soaring) or velocity gradients (dynamic soaring) [10].
A lack of a substantial propulsion system on sailplanes leads to many problems that must be overcome. Paul MacCready found the optimal speed to fly between thermals if the strength of the next thermal is known or assumed [11]. This analysis lead to the development of the MacCready speed ring, a simple instrument that pairs the airspeed indicator with the variometer (vertical velocity), and allows the pilot to know the optimal speed to fly to the next thermal.

Langelaan has done much research in using atmospheric lift as a means of greatly increasing the endurance and range of small UAVs, including simulated flight using a polynomial model of wind and a real-time simulation of wind [12]. Along with this, Chakrabarty applied graph-based planning methods, the Energy Map and A*, to autonomous soaring flight in realistic wind fields at constant altitude [13]. The Energy Map provides a means of finding the path of minimal energy expended starting at a given location and ending at the goal. The A* method can constrain the flight path such that there is a weight function relating energy required to the goal and, say, time to reach goal. Fig. 2.6 shows energy efficient paths to a goal located at (45km, 40km).

![Figure 2.6: Example energy map plotted for winds at altitude of 1km over central Pennsylvania. Red regions are areas of upward wind and blue is downward wind, [13].](image-url)
Genetic algorithms have been implemented in path planning by Torroella [14]. This work was done to find an optimal path in four dimensions (three spatial, one temporal) around hazardous weather systems rather than harvesting energy, although utilizing horizontal winds was investigated. This method also allows the plan to be modified dynamically, i.e. in-flight rather than pre-flight.

2.3 Wind Modeling

This section pertains to a portion of the path-planning program only. The simulated wind field was generated using the Weather Research and Forecasting system. This system is community-supported and is used for flexible, state-of-the-art atmospheric simulations capable of being run on a desktop computers and parallel clusters alike. It is capable of simulating weather system sizes that range from synoptic scales of over 100 km to large eddies on the scale of 100 m [15].

The user supplies initial boundary conditions to the preprocessor by using “ideal” data examples or “real” data. The user also determines the grid size of each nested grid within the simulation such that the effect of large weather systems on smaller systems is resolved accurately.

2.4 Summary

This chapter discussed the research history of wingtip devices, flight-path planning, and mentioned the weather prediction software used to simulate a realistic wind field that would be experienced at the Green Flight Challenge. Wingtip design research began around the early 20th century but did not become prevalent until the 1970s when the technology was available to accurately analyze the designs. Studies in soaring flight began around the same time as wingtip design. The interest in soaring has since lead to research in optimal flight paths that are influenced by the three dimensional wind field for both unpowered and powered airplanes.
Chapter 3

Winglet and Analysis

This section follows the work described by Maughmer et al., Refs. [16], [17] and is summarized here. Winglets have become an ever-present design feature on modern, high-performance sailplanes. Only until recently have methods of design and analysis become accurate and dependable enough for companies to justify the complexity in manufacturing a winglet to the gains in performance the winglet provides.

3.1 Crossover Point

The driving point in the design of a winglet is the point at which the induced drag benefit is outweighed by the parasite drag penalty. This point is called the crossover point and is represented as

$$\frac{1}{2} \rho V^2 (S_{WL} C_{DP_W} - S_W C_{DP_W}) = \left( \frac{2W^2}{\pi \rho V^2} \right) \left( \frac{K_1}{b_1^2} - \frac{K_2}{b_2^2} \right)$$

(3.1)

where the subscripts $WL$ and 1 belong to the winglet and $W$ and 2 belong to the wing. In the parentheses on the left side of the equality is the difference in equivalent flat-plate area of the winglet and the flat-plate area removed from the main wing after the addition of a winglet. The right side of the equality is the change in span efficiency of a wing with a winglet and one without, respectively. This relationship shows that the winglet design must maximize the right side of the equation while minimizing the left side. The factor $K$ is the induced drag
factor, with $K = 1$ equivalent to an elliptically loaded wing.

Winglets show their best aerodynamic benefit when the airplane is operating at low speeds and high lift coefficient. As the airplane speeds increase and the lift coefficient decreases, the profile drag of the winglet increases and benefit is reduced; therefore, the design of the winglet must take into account the flight regime of the airplane in order to not greatly penalize the performance of the airplane over parts of the flight envelope where it would be lease desired.

### 3.2 Physics of Winglets

Lift is the useful force generated from a pressure differential between the upper surface and lower surface of a wing. Basic fluid mechanics describes fluid flow as particles moving from an area of high pressure to an area of lower pressure, as shown in Fig. 3.1. So a consequence of generating lift on a finite-span wing is spanwise flow.

![Figure 3.1: Spanwise flow on a wing, [20].](image)

As the spanwise flow is outboard on the lower surface and inboard on the upper surface, a shearing effect occurs that is focused in the area of the wingtip, creating the wingtip vortex as seen in Fig. 3.2.

The purpose of the winglet is to reduce this spanwise flow by generating a lift force directed inboard on the upper surface, as described in Section 2.1. The flow on the upper surface near the wingtip with a winglet looks like flow that is found more inboard of the tip as if the wing had a greater span. The flowfield created near and around the winglet causes the tip vortex to diffuse. The spreading
of the vortex doesn’t necessarily reduce the induced drag, but it is the overall effect the winglet has on the flowfield that provides the benefit in aerodynamic performance. It should be noted that a downward-oriented winglet does not have the same performance benefit as an upward-oriented winglet. A downward winglet causes a contraction in the wake, and the benefit is not as great. The trade-off between a winglet and a span extension should be weighed on a mission basis, as the results and effects of both wing designs affect the mission quality of the airplane. In general, a winglet can provide the same benefit to induced drag as a span extension but with less parasite drag if the winglet wetted area can be made less.

3.3 Winglet Geometry

Since the design of the winglet is greatly influenced by the speeds at which it operates, the geometry of the winglet must be iterated until the performance is as desired. This becomes a very complicated problem since every aspect of the geometry influences most other aspects. Therefore, the design needs to consider the height, chord distribution, twist, sweep, toe angle, and airfoil section seen in Fig. 3.3.
3.3.1 Height and Chord Distribution

Winglet height is mainly determined by the trade-off of induced drag benefit to the parasite drag penalty, but structural weight can become an issue as well.

Determining the chord distribution becomes a much more difficult problem. Foremost, the winglet must produce the loading, $cc_l$, that is required to have the desired effect on the flowfield. At low speeds, if the chord length is too small, then the design could require the airfoil section to generate a lift coefficient beyond what it can produce at such low Reynolds numbers. Likewise, if chord lengths are too large, then the high loading of the winglet could alter the flow at the tip such that the span efficiency is penalized. So, depending on the airfoil section for the winglet, the chord distribution becomes bounded by low-Reynolds effects at the low operating speeds and parasite drag at higher operating speeds.

3.3.2 Twist, Sweep, and Toe Angle

Once the height and chord distribution is set, the geometry can be further manipulated and tailored by changing the sweep, twist, and toe angle.

Figure 3.3: Geometry aspects of a winglet, [20].

Changing sweep has similar effect as twist: increasing sweep affects the winglet
load distribution the same as does wash-in; therefore, one element can be set while the other is tailored for best performance.

Toe angle is the another parameter that must be considered in the design. The toe angle can shift the effect of the winglet to the desired flight condition once all other parameters are fixed. The toe angle of a winglet is generally unique to that design, so this angle can perhaps be considered the most important aspect of the geometry.

3.3.3 Airfoil Section

For the winglet to not inhibit performance at all flight conditions, the airfoil section is perhaps the most critical aspect. Like most other airfoil designs, the goal is to be able to operate at the necessary lift coefficients with the lowest drag possible. This can be difficult because of the Reynolds numbers at which wingtips operate. The low drag region of the winglet should match the low drag region of the main wing. Because of this, the lift coefficient range of the winglet is specific for that of the main wing it is mated with. Therefore, each new winglet should have a specific airfoil in order to perform optimally. In general, the gain in performance found with a unique winglet airfoil is comparable to the minute change in the airfoil design for similar aircraft, and the effort is not worth the diminishing returns.

The trade-off of reducing winglet chord in order to decrease wetted area and the cost of high drag coefficients at low Reynolds numbers becomes significant. At these low Reynolds numbers, the ever-present laminar separation bubble along with other effects influencing the profile drag become more important. This is mitigated by the fact the winglet airfoil will have a smaller range of lift coefficients over which it operates in compared to an airfoil section of the main wing. This is illustrated in Fig. 3.4
3.4 Effect of Winglets on Performance

The ultimate goal of a winglet design is to improve overall mission performance. For racing sailplanes, the mission performance is measured by overall cross-country speed. Racing sailplanes need to be able to climb effectively in thermals of varying strengths at low speeds while also being able to cruise efficiently at high speeds, known as the MacCready speed-to-fly. Winglets generally help very well at low speeds, where induced drag is high, and have less effect at high speeds (perhaps even hurting performance). Fig 3.5 and Fig. 3.6 show the behavior of well-designed winglets on a racing sailplane’s performance.

Figure 3.5: Percent change of L/D with winglets for the Discus 2 ballasted and unballasted, [20].
Figure 3.6: Percent change in average cross-country speed for the Discus 2 with winglets, [20].

Figs. 3.5, 3.6 show the effect of properly designed winglets on percent change of L/D and average cross-country speed for the Discus 2, respectively. The greatest improvement on performance is at low speeds, while a marginal, albeit positive, gain is possible at the higher speeds.

3.5 Polar Generator (PGEN)

PGEN is a program that was developed by Kunz for assessing the performance of sailplanes [31]. It calculates the lift-to-drag ratio and sink rate performance for varying airspeeds. This program estimates the effects of trim drag, fuselage drag, airfoil selection, flap deflection schedule, flap geometry, and static margin on the airplane performance. Of course, the most important aspect of this analysis is the calculation of the wing planform lift and drag characteristics. This tool allows a new design to be readily be analyzed and compared to a baseline design, with modifications applied quickly to tailor the new design.

The wing planform analysis is done by combining a non-planar, multiple lifting-line method, which accounts for induced drag, with an interpolation method for airfoil data, which estimates the profile drag. Induced drag is calculated by determining the wing and tail planform efficiencies as well as the effect each lifting surface has on the other. The nonplanar multiple-lifting-line method developed by Horstmann is used [18]. The method can determine the lift distribution of the
planform by assuming a continuously distributed vorticity distribution across each wing section, or panel, with forced continuity of vorticity between panels. This method has proved to be accurate for high aspect ratio wings when compared to a full lifting-surface panel code [19]. For the importance of computational speed rather than accuracy in the initial stages of winglet design the chordwise distribution of vorticity is not taken into account. This method is also used on the horizontal tail. The total effect of planform efficiency and lifting surface separation distance is determined using a combination of Prandtl’s Biplane Equation and Munk’s Stagger Theorem, which assumes a fixed wake geometry

\[
C_{D_i} = \frac{C_{L_w}^2}{\pi R_w e_w} + 2\sigma \frac{S_{tail} C_{L_w} C_{L_{tail}}}{b_w b_{tail}} + \frac{S_{tail}}{S_w} \frac{C_{L_{tail}}^2}{\pi R_{tail} e_{tail}}
\]  

(3.2)

where \(\sigma\) is the biplane factor, \(R\) is the aspect ratio of the appropriate surface, and \(b\) is the span of the appropriate surface.

Airfoil data \((C_l, C_d, \text{ and } C_m)\) are tabulated in a specific format with respect to a Reynolds number range that spans slightly beyond that the wing experiences. Flap deflection is taken into account in the modified lifting-line method by a simple modification of the zero-lift angle, essentially creating twist on the wing at the respective section. Once the lift of the wing is determined by interpolating the airfoil section data, the necessary lift force of the tail is calculated by a simple moment summation about the airplane center of gravity. Trim drag can then be calculated and iterated based on changes in the planform induced and profile drag.

Fuselage drag is an input for the program, in other words determined by the user outside of the software. Fuselage skin friction drag is given an equivalent flat plate area. This flat plate area can also be used as a means of tuning or calibrating the total airplane drag for anything drag not accounted for, such as interference drag.

The output of the PGEN software is a L/D curve of the airplane with respect to the airspeed range at a fixed altitude as well as sink rate with respect to airspeed. The results of this program have been compared to flight test data of the
Schempp-Hirth Discus 2, Table 3.1, and a Schleicher ASW-20, Table 3.2.

<table>
<thead>
<tr>
<th>S-H Discus</th>
<th>Stall Speed</th>
<th>Maximum L/D</th>
<th>Minimum Sink Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V$ [km/h]</td>
<td>$V_{sink}$ [m/s]</td>
<td>$L/D$</td>
</tr>
<tr>
<td>Test Data</td>
<td>74</td>
<td>0.61</td>
<td>41</td>
</tr>
<tr>
<td>PGEN</td>
<td>68.4</td>
<td>0.78</td>
<td>39.9</td>
</tr>
</tbody>
</table>

Table 3.1: Performance comparison of a Schempp-Hirth Discus without winglets, weighted to 320 kg. or 700 lbs., [31].

<table>
<thead>
<tr>
<th>ASW-20</th>
<th>Stall Speed</th>
<th>Maximum L/D</th>
<th>Minimum Sink Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V$ [km/h]</td>
<td>$V_{sink}$ [m/s]</td>
<td>$L/D$</td>
</tr>
<tr>
<td>Test Data</td>
<td>69</td>
<td>0.75</td>
<td>42</td>
</tr>
<tr>
<td>PGEN</td>
<td>68.4</td>
<td>0.63</td>
<td>42.5</td>
</tr>
</tbody>
</table>

Table 3.2: Performance comparison of a ASW-20 without winglets, weighted to 350 kg. or 770 lbs., [31].

### 3.6 Summary

This chapter discussed the philosophy of winglet design. The crossover point is the velocity above which the winglet is no longer a benefit to the original airplane. This means the added profile drag of adding a winglet exceeds the induced drag reduction the winglet creates by its design. The profile drag penalty is greatly influenced by the height of the winglet, chord distribution, toe and twist angles, and airfoil choice. The induced drag benefit is affected by the load distribution of the winglet, *i.e.* the twist distribution and toe angle of the winglet. Using the PGEN software, the L/D performance of the modified airplane can be compared to that of the original airplane.
Flight-Path Planning

4.1 Airplane Kinematic Model

The airplane kinematics model follows the method described in [12] and is summarized here. A point mass model rather than an inertial model is used because it is assumed the response time to control inputs is significantly shorter than the duration of the input. The kinematic equations of the airplane are given by

\[
\begin{align*}
\dot{x} &= v_a \cos \gamma \cos \psi + w_x \\
\dot{y} &= v_a \cos \gamma \sin \psi + w_y \\
\dot{z} &= v_a \sin \gamma + w_z
\end{align*}
\]  

(4.1) (4.2) (4.3)

where \( \dot{x}, \dot{y}, \) and \( \dot{z} \) are translational velocities and vertical velocity, \( \gamma \) is the flight-path angle, and \( \psi \) is heading. Using a small angle approximation and resolving the relevant forces as shown in Fig.4.1 relative to the flight path angle, flight-path perpendicular and parallel forces become

\[
\begin{align*}
mg \cos \gamma &= L + T \sin \alpha \\
mg \sin \gamma &= D - T \cos \alpha
\end{align*}
\]  

(4.4) (4.5)

The small angle approximation assumes the thrust vector is aligned with the body x-axis of the vehicle. Using the relations for force coefficients, Eq.(4.4) and (4.5)
Figure 4.1: Point Mass Model, [12]

become

\[
\cos \gamma = \frac{qS}{mg} (C_L + C_T \sin \alpha) \tag{4.6}
\]

\[
\sin \gamma = \frac{qS}{mg} (C_D - C_T \cos \alpha) \tag{4.7}
\]

where \( q \) is the dynamic pressure given as \( q = \frac{1}{2} \rho v_a^2 \).

The airplane drag polar can be approximated using a \( k^{th} \)-order polynomial.

\[
C_D = \sum_{i=0}^{k} a_i C_L^i \tag{4.8}
\]

Fourth-order polynomials are typically sufficient for this to capture the behavior of the low-drag region and to reduce computation time. Alternatively, a linear interpolation is used to calculate data using the output from \texttt{PGEN}, [24]. Substituting this into Eq.(4.7) and assuming angle of attack is of small magnitude at cruise, the flight path angle can be found using

\[
\sin \gamma = \frac{qS}{mg} \left( \sum_{i=0}^{k} a_i C_L^i - C_T \right) \tag{4.9}
\]

Combining this with the equation for lift coefficient,

\[
C_L = \frac{mg}{qS} = \frac{2mg}{\rho v_a^2 S} \tag{4.10}
\]
allows the flight path angle to be defined given the inputs $u = [v_a C_T]^T$ and the wind vector $w = [w_x w_y w_z]$. This point-mass model is sufficient given that the duration of each segment is longer than the airplane’s response time to a step input.

### 4.2 Flight-Path Determination

To consider the effects of the wind on the flight, the wind must be decomposed into in-track and cross-track components as seen in Fig. 4.2. Vertical components are congruent and can be added directly.

![Figure 4.2: Track reference frame coordinates (left) and wind and airspeed decomposition (right), [12].](image)

Airspeed is also decomposed into in-track, $v_t$, and cross-track, $v_c$, components. The desired heading is thus found by satisfying the constraint of not straying from the trajectory from node to node, i.e. $v_c = 0$. The coordinate frame transformation leads to

$$v_t = \sqrt{v_a^2 \cos^2 \gamma - w_c^2} \quad (4.11)$$
$$v_g = v_t + w_t \quad (4.12)$$
$$v_a \cos \gamma \sin \beta = w_c \quad (4.13)$$

Since the flight path angle is assumed to be small, the ground speed, $v_g$, is closely
related to

\[ v_g \approx \sqrt{v_a^2 - w_c^2} + w_t \]  (4.14)

By examining the track reference frame coordinates, the desired track heading is

\[ \psi = \psi_t - \beta \]  (4.15)
\[ \psi = \psi_t - \sin^{-1} \frac{w_c}{v_a} \]  (4.16)

As heading is shown to be dependent on airspeed, the problem has been resolved to be one of determining the airspeed and thrust setting for minimum energy required to fly between nodes.

### 4.3 Energy Required Between Nodes

To find the minimum energy required to fly between nodes, total energy must be determined using steady-state airspeed and thrust setting. Total energy is given by

\[ E_{\text{total}} = mgh + \frac{1}{2}mv_a^2 + E_{\text{OBS}} \]  (4.17)

where \( E_{\text{OBS}} \) is the onboard stored energy, such as battery packs or fuel. Specific total energy is given by

\[ e_{\text{total}} = \frac{E_{\text{total}}}{mg} \]  (4.18)

So Eq.(4.18) becomes

\[ e_{\text{total}} = h + \frac{v_a^2}{2g} + \frac{E_{\text{OBS}}}{mg} = h + \frac{v_a^2}{2g} + e_{\text{obs}} \]  (4.19)

Minimal energy loss over each segment is the same as maximizing the change in total energy over a segment, \( \frac{\Delta e_{\text{total}}}{\Delta s} \). This involves an increase in altitude, increase in speed, and/or an increase in the onboard stored energy. In a zero-strength wind field, this would not be possible. For steady-state flight, this objective becomes one of maximizing \( \frac{\dot{e}_v}{v_g} \). With this in mind, Eq.(4.19) becomes

\[ \dot{e}_{\text{total}} = \dot{h} + \frac{v_a \dot{v}_a}{g} + \dot{e}_s \]  (4.20)
Steady-state flight implies no acceleration; therefore

\[ \dot{e}_{\text{total}} = \dot{h} + \dot{e}_s \]  

(4.21)

As altitude is congruent with the body z-axis and positive down, \( \dot{h} \) is equivalent to \( -\dot{z} \). The time rate of change of the onboard stored energy is dependent on the energy-to-power conversion of the propulsion system and thrust setting.

\[ \dot{e}_{\text{obs}} = -\frac{T v_a}{m g \eta_{ec} \eta_p} = -\frac{q S C T v_a}{m g \eta_{ec} \eta_p} \]  

(4.22)

Since a force multiplied by a velocity gives units of power, then \( T v_a = P \), so

\[ \dot{e}_{\text{obs}} = -\frac{P}{m g \eta_{ec} \eta_p} = -\frac{q S C_P}{m g \eta_{ec} \eta_p} \]  

(4.23)

Combining this with Eq.(4.3) results in the time rate of change of specific energy in steady-state flight,

\[ \dot{e}_{\text{total}} = -(v_a \sin \gamma + w_z) - \frac{q S C_P}{m g \eta_{ec} \eta_p} \]  

(4.24)

As given earlier the objective is to maximize \( \frac{\dot{e}_{\text{obs}}}{v_g} \), such that Eq.(4.24) can be used to obtain

\[ \frac{\dot{e}_{\text{total}}}{v_g} = -\frac{v_a \sin \gamma + w_z}{\sqrt{v_a^2 - w_c^2 + w_t}} - \frac{q S C_P}{m g \eta_{ec} \eta_p(\sqrt{v_a^2 - w_c^2 + w_t})} \]  

(4.25)

The point-mass model and calculations for energy required between nodes are both used in simulations of the Taurus G2 and Twin-Taurus G4.

4.4 Flight-Path Optimization

It would be very difficult to gain energy while flying the predetermined course given the constraints on the flight path of the airplane for the Green Flight Challenge competition. The problem becomes not one of maximizing, but minimizing. The described method is implemented in a two-step optimization routine to find a path for the airplane to fly that consumes the least energy. The first step is an altitude sweep that found the best altitude that conforms to the constraints of
the course that utilizes the winds best. The second step is to optimize the power setting and airspeed for each segment of the course such that the energy consumed over the whole course is minimized while constrained to follow the altitudes determined by the first step. A generic, gradient-based minimizer, MATLAB’s script \texttt{fmincon}, is used to do this. This script searches for the minimum of a multivariable, scalar function subject to linear and/or nonlinear constraints [23]. Starting at an initial guess, the script uses a Hessian matrix to find the direction of greatest value decline. The Hessian matrix describes the local curvature of the scalar function, so the script follows the direction of steepest descent.

The Green Flight Challenge presented an obvious constrained problem suitable for the script: minimize the energy consumed while flying a course with ground speed and altitude constraints. The airspeed and energy consumption are directly related to pilot power setting, so based on the airplane control inputs described in Section 4.1, the problem becomes

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{N} \frac{P_i}{\eta_{ee} v_{gi}} \\
\text{subject to} & \quad C_{Pi} = f^{(4)}(J_i) \quad (4.27) \\
& \quad C_{Ti} = f^{(4)}(J_i) \quad (4.28) \\
& \quad J_{min} \leq J_i \leq J_{max} \quad (4.29) \\
& \quad h_{min} \leq h_i \leq h_{max} \quad (4.30) \\
& \quad v_{g,min} \leq v_{gi} \leq v_{g_i} \quad (4.31) \\
& \quad \gamma_{req} = \gamma_i \quad (4.32)
\end{align*}
\]

where the subscript \( i \) signifies each individual segment. Power and thrust coefficients are given as fourth-order polynomials of the advance ratio, \( J \). The advance ratio is constrained such that \( C_P \) and \( C_T \) cannot have a negative value, which would signify power regeneration and reverse thrust, respectively. Although the airplane was designed to be capable of power regeneration, it was not implemented during the competition. Reverse thrust was not used in order to conserve energy, but could be used to assist in landing.
A similar method of optimization is used for the Taurus G2 for the climb to altitude. Unlike the G4, ground speed was not constrained for the G2. Power setting was constrained to the maximum setting to be compared with an unconstrained flight.

4.5 Summary

This chapter discusses the path-planning method used to optimize the flight of the Green Flight Challenge airplane, the Twin-Taurus G4. The method uses a point-mass model for the airplane because the duration of a control input is significantly shorter than the duration of travelling along a course segment. The energy required to traverse each segment is calculated given a three-dimensional wind field and performance characteristics of the airplane. The energy required to fly the course is then minimized using the MATLAB minimizer \texttt{fmincon} while subject to the constraints dictated by the GFC requirements. The point-mass model and energy required to fly a segment is slightly modified to be used as the method of calculating the maximum range for the Taurus G2 Electro.
Results

This chapter presents a method of determining a winglet design and the subsequent changes the winglets cause on the performance of the Pipistrel Taurus G2 and Twin-Taurus G4. Comparisons of airplane performance are presented between the nominal simulation and the simulation with winglets. The performance curve using each winglet design was implemented into the airplane model described in Sec. 4.1. Performance of the Twin-Taurus G4 is evaluated using the flight path-planning method described in Sec. 4.2. The Taurus G2 Electro is evaluated by calculating energy required to climb and maximum range. Each winglet design is compared to the nominal performance of each airplane to show the benefits of winglets in each case. Results show that winglets improve the performance of both airplane.

5.1 Design of Winglets

Winglet design is an iterative process. The manipulation of one parameter will change the performance across the entire operating airspeed range. A change in one parameter can also balance or intensify the effect of another parameter. This can certainly complicate the design process. A height is first chosen as an initial design point. Given a height, chord distribution is selected such that the winglet area is reduced. The sweep of the winglet is then altered such that the leading edge angle is not greater than 30°, to avoid spanwise flow effects. The
twist angle is then determined, given a linear twist distribution, in order to give the winglet an elliptical load distribution. An initial toe angle is found to tune the cross-over point. Chord distribution is then iterated with toe angle in order to gain the desired performance across the airspeed range. This thesis shows another method of design. After chord distribution and sweep angle were selected, a simple parameter sweep was done to see the effect of height, toe angle, and twist on a winglet’s performance across the operating airspeed range. The PSU94-097 was the airfoil section used for each winglet.

Height was varied, for example for the G4 winglet, from 0.2 meter to 2 meter in 0.2 meter increments. For every height, the toe angle was varied from 0 degrees to 5 degrees in 0.5 degree increments. Then for every toe angle, the twist was varied from -2 degrees to 5 degrees in 0.5 degree increments. The twist distribution is linear between the root and tip. The following plots are examples to illustrate the effect of each design parameter on the airplane performance. The L/D was calculated with the optimal flap schedule and compared to the nominal case by a percent change.

\[
\%\Delta \frac{L}{D} = \frac{\frac{L}{D}_{WL} - \frac{L}{D}_{NOM}}{\frac{L}{D}_{NOM}} \times 100 \tag{5.1}
\]

Figure 5.1: The effect of height on L/D with zero toe angle and zero twist.

In Fig. 5.1, the effect of adding height to a winglet with zero toe angle and zero
twist is illustrated. Height is increased with no change in the chord distribution; therefore, winglet area is also increased. Not tailoring the chord distribution with the change in height clearly shows a relationship with Reynolds number and profile drag at the high speeds, where the winglet actually detracts from the airplane’s performance. At lower speeds, as height is increased, the maximum gain in L/D is increased until a height at which the inefficiently loaded winglet does not perform well and the gain decreases. This effect can be mitigated by altering the toe angle and twist of the winglet.

![Figure 5.2: The effect of toe angle on L/D on a 1 meter tall winglet and zero twist.](image)

Fig. 5.2 shows the effect of increasing toe angle on an untwisted winglet. The height of the winglet is 1 meter, which is the height with the best peak performance determined from Fig. 5.1. As toe angle is increased, performance is generally improved for all speeds. There is a point where too much toe angle can cause the winglet to begin to detract from the performance of the airplane, which can be seen with toe angles greater than 2 or 3 degrees. Adjusting the twist of this winglet can improve the performance at the higher speeds, so a toe angle of 5 degrees is chosen because of the high peak performance and twist is added to improve the performance at higher speeds.

Fig. 5.3 shows the effect of twist distribution on a winglet with set height and
Figure 5.3: The effect of twist on L/D on a 1 meter tall winglet with 5 degree toe angle.

To angle. The twist distribution is designed to be linear, so the twist variation from winglet root to tip is determined by the difference between toe angle and tip angle. For this winglet design, the best tip angle is shown to be 3 degrees, with the crossover point moving to slower speeds with decreasing twist. -2 degrees of twist along the span of the winglet appears to have a significant benefit at low speeds and marginal (if any) penalty at high speeds. Fig. 5.3 shows that twist can have a small effect at low speeds and a significant effect at high speeds; therefore, twist can be used as a tuner to adjust the crossover point of the winglet.

From the previous plots, a winglet was designed which is 1 meter tall with a 5 degree toe angle and -2 degree linear twist. The toe angle and twist improved the performance from a 1.125% maximum improvement at low speeds with significant performance penalty above 90 mph to a maximum benefit of 1.86% and no penalty between 90 and 135 mph. This example is a simple exercise in determining the design of a winglet. This method does not account for the possibility of more performance gain from a different configuration or a better distribution of performance across the speed range. A higher gain is found using a lower toe angle of 4.5 degrees, for instance, with the same height and twist.
5.1.1 Taurus G2 Electro

Unlike a racing sailplane, the Taurus G2 Electro does not have an obviously specific mission. If the G2 were a racing sailplane, then the winglet design parameters and constraints would be straight-forward: gain the greatest improvement in average cross-country speed over a wide range of thermal strengths. Since the G2 is mainly a recreational airplane or even a trainer, the design of the winglet was focused on giving the best improvement in energy usage and maximum range to improve time aloft.

The owner and operator manual provides much information into the operation of a Taurus G2. The L/D curve and sink rate polar found there is shown in Fig. 5.4. Also listed is the total energy stored on-board in batteries. The G2 can store up to 4.75 kWh with the standard battery pack, although the nature of Lithium-Ion Polymer batteries allows only 3.8 kWh to be used.

![Figure 5.4: Performance curves of the Pipistrel Taurus G2, [29].](image)

Also published is information about climbing to altitude after take-off. Zeroed flaps and maximum power are suggested until the airplane has reached the desired altitude. The manual also lists the $V_{NE}$ as 225 km/h or roughly 140 mph. Given
the assumed mission of the G2 (recreational flying or trainer), the constraint on the winglet for no performance deficit approaching $V_{NE}$ was relaxed.

Accordingly, the G2 model is analyzed with zeroed flaps, and the climb to altitude is simulated to use maximum power while minimizing energy consumption (so the airplane flies at optimal airspeed and advance ratio). Fig. 5.5 compares the published performance curve of the G2 to simulation results of the G2 model with best flap schedule and zeroed flaps without winglets.

![Figure 5.5: Performance comparison of the Taurus G2 operator manual data and PGEN simulations, airplane weight 472 kg.](image)

The constraints specific to the mission govern the winglet design. The G2 operator manual describes the climb to altitude should be at constant full power. This causes the airplane to fly at high airspeed in order for the propeller to be operating at its highest efficiency. Once at cruising altitude the airplane will fly at maximum L/D, which is shown to be at much slower speeds. The winglet must then provide significant benefit at slow speeds while not detracting from the airplane performance at high speeds.
The winglets were designed using a technique similar to that as described in Sec. 5.1. Varying winglet height was evaluated to find which height no longer provided significant benefit to the L/D. This height was then used to evaluate the effect of toe angle on L/D. Once a toe angle was selected, a twist was found for the best performance. These parameters were iterated to find an appropriate design. Several winglet designs are compared to find the best climb and cruise performance. Each winglet has an elliptical chord distribution and linear twist.

Three designs are compared:

1. 0.4m Height, 2° Toe angle, 2° Twist
2. 0.6m Height, 1.5° Toe angle, 1.5° Twist
3. 0.4m Height, 2° Toe angle, −1° Twist

![Figure 5.6: A comparison of three winglet designs for the Taurus G2.](image)

The winglets shown behave as expected: they have a significant effect at slow speeds and almost negligible effect at high speeds. Winglet 3 shows a slightly lower gain at lower speeds compared to Winglet 2. This may be balanced by the better performance at higher speeds.
5.1.2 Twin-Taurus G4

The mission of the Twin-Taurus G4 was very specific, the purpose being that of winning the CAFE Foundation’s Green Flight Challenge in the Fall of 2011. The main constraints of the mission were to fly 200 miles around a closed circuit at an average ground speed of at least 100 mph. The entrants to the competition could use any fuel/energy source desired as long as the airplane could demonstrate an equivalent passenger-miles-per-gallon fuel economy of at least 200 pMPGe. The design of the G4 was such that it needed to be as efficient at or above 100 mph as possible. This constraint was the focus of the winglet design such that it needed to provide as significant a benefit at this velocity as possible. The optimal winglet design is not necessarily the one that best fulfills this requirement, but the one that allows the G4 to fly the GFC course using the least energy.

Several winglet configurations were compared:

1. The example winglet chosen from Figs. 5.1-5.3 \Rightarrow [1 \text{ m.}, 5^\circ \text{ toe}, -2^\circ \text{ twist}]
2. Best maximum \(\% \Delta \frac{L}{D}\) \Rightarrow [1.8 \text{ m.}, 5^\circ \text{ toe}, 1.5^\circ \text{ twist}]
3. Best maximum \(\% \Delta \frac{L}{D}\) without performance deficit at any speed \Rightarrow [1.2 \text{ m.}, 4.5^\circ \text{ toe}, -2^\circ \text{ twist}]
4. Winglet 1 with a reduced area \Rightarrow [1 \text{ m.}, 5^\circ \text{ toe}, -2^\circ \text{ twist}]

Winglet 1, 2, & 3 share common root and tip chords, root chord of 11.4 in. and tip chord of 4.75 in., with an elliptical distribution in between. Winglet 4 has a root chord 9.2 in., tip chord of 3.75 in., and is otherwise the same as Winglet 1.

Fig. 5.7 clearly shows that, given the constraint of no performance deficit at any speed, the example winglet design from Sec. 5.1 is not the best winglet design to satisfy this constraint. A slight decrease in toe angle provides a greater performance gain. This winglet design may not be the best for the Twin-Taurus G4 though. Other factors are involved with the G4 mission rather than cruise, such as climb to altitude and high-speed descent. The climb to altitude requires a high power setting compared to the cruise power setting. Also, the high-speed descent
does not require a high power setting; the propeller essentially needs enough power to generate thrust to balance its drag force while the weight of the vehicle can provide ample power to descend at speed. The winglet design that provides the best maximum $\%\Delta \frac{L}{D}$ is compared to the other designs because of the considerations beyond the point design of 100 mph. A modification to Winglet 1, where the area is reduced, is also compared.

**PGEN** cannot predict the effects of a fuselage within the semi-span of a wing, such is the case as for the Twin-Taurus G4. A continuous wing with a separate flap schedule at the junction where the fuselage would be was modeled, as seen in Fig. A.6. To better approximate the behavior of a fuselage in the semi-span, a flap schedule at this point is necessary because a fuselage can carry the load distribution more smoothly than a non-flapped wing section.
5.2 Winglet Design Evaluation

The airplane model of the Taurus G2 and the Twin-Taurus G4 was modified to include the geometry of each winglet design for the appropriate airplane. The software PGEN uses this information to calculate the drag polar for the allotted airspeed range. Each respective L/D curve was implemented in the appropriate simulation for evaluation.

5.2.1 Taurus G2 Electro

The winglet designs for the Taurus G2 Electro were evaluated using a simulation that calculates the minimum energy required to climb to the cruising altitude of 3000 feet and 6000 feet individually. This energy required is subtracted from the total usable on-board energy, listed in the operator manual as 3.8 kWh, to determine the energy available to maintain constant altitude. Once the energy is depleted, the airplane is then simulated to descend. During the cruise and descent stages, the airplane is forced to fly at the airspeed for best L/D since this is known to be the airspeed for maximum range. Results of simulations without a power setting during climb constraint are also presented.

Results of the simulations show the airplane flies at a high airspeed while at high power in order to have the propeller operate at best efficiency. Airspeeds are compared in Fig. 5.8 and Fig 5.9. The energy to climb and best cruise performance of each winglet is compared to the baseline performance in Table 5.1 and Table 5.2 for 3000 feet and 6000 feet, respectively. By examining the tables, each individual design has negligible effect on energy required to climb. The reason for this can be seen in Fig. 5.8 and Fig. 5.9. Each winglet has a very small effect on L/D at those high airspeeds. Once the airplane gets to altitude, the effect of the winglet on best L/D becomes more apparent. Range is appreciably increased above the nominal performance.
Figure 5.8: A comparison of airspeed and L/D during climb to 3000 ft. for the Taurus G2 winglet design.

<table>
<thead>
<tr>
<th></th>
<th>Nominal</th>
<th>Winglet 1</th>
<th>Winglet 2</th>
<th>Winglet 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{climb}$ [kWh]</td>
<td>1.8696</td>
<td>1.8696</td>
<td>1.8700</td>
<td>1.8700</td>
</tr>
<tr>
<td>Range [mi]</td>
<td>86.97</td>
<td>87.49</td>
<td>88.83</td>
<td>89.50</td>
</tr>
<tr>
<td>% Change in Range</td>
<td>0.60</td>
<td>2.10</td>
<td>2.83</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Comparison of energy required to climb and maximum range at 3000 ft. cruise altitude.
Figure 5.9: A comparison of airspeed and L/D during climb to 6000 ft. for the Taurus G2 winglet design.

<table>
<thead>
<tr>
<th></th>
<th>Nominal</th>
<th>Winglet 1</th>
<th>Winglet 2</th>
<th>Winglet 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range [mi]</td>
<td>57.07</td>
<td>57.82</td>
<td>57.95</td>
<td>57.78</td>
</tr>
<tr>
<td>% Change in Range</td>
<td>1.30</td>
<td>1.52</td>
<td>1.23</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Comparison of energy required to climb and maximum range at 6000 ft. cruise altitude.
<table>
<thead>
<tr>
<th></th>
<th>Nominal</th>
<th>Winglet 1</th>
<th>Winglet 2</th>
<th>Winglet 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{climb}$ [kWh]</td>
<td>1.859</td>
<td>1.832</td>
<td>1.832</td>
<td>1.833</td>
</tr>
<tr>
<td>Range [mi]</td>
<td>87.29</td>
<td>88.64</td>
<td>90.03</td>
<td>90.60</td>
</tr>
<tr>
<td>% Change in Range</td>
<td>1.52</td>
<td>3.04</td>
<td>3.65</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3: Comparison of energy required to climb and maximum range at 3000 ft. cruise altitude without using maximum power.

Figure 5.10: A comparison of airspeed and L/D during climb to 3000 ft. for the Taurus G2 winglet design with no power requirement.
Table 5.4: Comparison of energy required to climb and maximum range at 6000 ft. cruise altitude without using maximum power.

<table>
<thead>
<tr>
<th></th>
<th>Nominal</th>
<th>Winglet 1</th>
<th>Winglet 2</th>
<th>Winglet 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{climb}$ [kWh]</td>
<td>3.680</td>
<td>3.683</td>
<td>3.679</td>
<td>3.679</td>
</tr>
<tr>
<td>Range [mi]</td>
<td>59.48</td>
<td>60.15</td>
<td>60.43</td>
<td>60.30</td>
</tr>
<tr>
<td>% Change in Range</td>
<td>1.12</td>
<td>1.59</td>
<td>1.34</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.11: A comparison of airspeed and L/D during climb to 6000 ft. for the Taurus G2 winglet design with no power requirement.
Figure 5.12: A comparison of flight path during climb to 3000 ft. for the Taurus G2 winglet design with no power requirement.
If the power setting constraint is relaxed, then the benefit of the winglet is much more pronounced. The simulation was evaluated for climb to 3000 feet again without the power setting constraint. The flight path of the climb for each simulation is considerably different as shown more clearly in Fig 5.12. The airplane also flies much slower in all cases, which allows for flight closer to optimal energy-consumption performance.

Winglet 2 creates significant performance gains for climb to and cruise at 3000 feet and 6000 feet compared to the other designs; however, Winglet 3 provides very similar results as Winglet 2 and provides these results with less height. Less height means that winglet would need less structure to support the weight added and forces generated at the wing tip. By taking into account the entire system, Winglet 3 would be the better design.
5.2.2 Twin-Taurus G4

The winglet designs shown in Sec. 5.1.2 for the Twin-Taurus G4 were evaluated using the same path-planning simulation that was developed for use at the Green Flight Challenge. The flight path for the G4 involved a moderate climb to the minimum allowable altitude, a cruise at slightly varying altitude, and a high-speed descent. The best winglet must provide the greatest increase in performance, specifically energy consumption, across this flight plan. Results are listed in Table 5.5 for varying average ground speed.

<table>
<thead>
<tr>
<th></th>
<th>Nominal</th>
<th>Winglet 1</th>
<th>Winglet 2</th>
<th>Winglet 3</th>
<th>Winglet 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 mph</td>
<td>57.662</td>
<td>57.404</td>
<td>57.428</td>
<td>57.411</td>
<td>55.011</td>
</tr>
<tr>
<td>105 mph</td>
<td>59.836</td>
<td>59.604</td>
<td>59.596</td>
<td>59.603</td>
<td>57.127</td>
</tr>
<tr>
<td>110 mph</td>
<td>62.704</td>
<td>62.490</td>
<td>62.678</td>
<td>62.539</td>
<td>59.983</td>
</tr>
<tr>
<td>Avg. %Δ</td>
<td>0.393</td>
<td>0.270</td>
<td>0.363</td>
<td>4.40</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.5: Energy consumption [kWh] comparison of each winglet design at varying average ground speed.

As shown in Table 5.5, each winglet provides some benefit in energy consumption. The parts of the flight where the most performance is gained can be seen in comparing energy consumption per segment. The most dramatic gain in performance is with Winglet 4. Fig. 5.13 shows a comparison of energy consumption per segment between the nominal case and Winglet 4. The nominal energy consumption results in Table 5.5 differ from published GFC simulation results for several reasons. **PGEN** was not used for generating L/D vs. airspeed for the Green Flight Challenge flight-path planning. A method that was calibrated using a narrow flight-test data set was used for the GFC. Also, the results shown in Table 5.5 were generated in part by using a linear interpolation of **PGEN** data rather than the polynomial curve fit in Equ. 4.8.
Figure 5.13: Energy consumption per segment comparison of the nominal case and Winglet Design 4.
Fig. 5.14 shows the operating L/D of the G4 with and without Winglet Design 4. Although the airplane without winglets operates at airspeeds generally lower than with winglets, the L/D of the airplane with winglets is better. The airplane could fly at higher speeds for less energy because of this, as seen in Table 5.5.
Chapter 6

Conclusions and Future Work

A winglet design for the Pipistrel Taurus Electro G2 and the Twin-Taurus G4 was presented. Winglet design parameters, such as height, toe angle, and twist were iterated until appropriate designs were found. Each design was compared with similar and extreme designs to show the effect of properly assumed mission constraints on winglet design and performance. The effect of winglets on the L/D of each airplane was calculated using a detailed drag build-up program called PGEN.

The new performance curve of the Taurus G2 was implemented with a point-mass model and propulsion performance curves into a simulation that calculated energy to climb and maximum range. This was done for climb to 3000 feet and 6000 feet cruise altitude in a zero-wind field. A winglet design using a root chord of 6.92 inches, a tip chord of 3.92 inches, a 15.75 inch (or 0.4 meter) height, a $2^\circ$ toe angle, and a $-1^\circ$ linear twist. Performance gains in L/D max are on the same order as gains in range. L/D max was improved by 1.5%. If the climb to 3000 feet is constrained with power setting, then the range was increased by about 2.8%. If the climb was unconstrained, then range was increased by 3.65%. The gains in range for the flights with constrained and unconstrained climbs to 6000 feet were 1.23% and 1.34%, respectively. This winglet design was chosen because although similar results were achievable with a taller winglet, the shorter winglet would have a lower weight; thus, this design has a better system efficiency.
The new performance curve of the Twin-Taurus G4 was implemented, along with a point-mass model and propulsion performance curves, into a flight-path planning simulation developed for the 2011 Green Flight Challenge. This simulation used up-to-date wind data created by a state-of-the-art weather software package. The flight-path simulation optimized the energy required to fly the GFC course while maintaining a minimum average groundspeed. Given these constraints, the performance gains are comparable to results from simulations done of high-performance sailplanes at the same airspeeds. The winglet design chosen for the Twin-Taurus G4 has a root chord of 9.2 inches, a tip chord of 3.75 inches, a height of 39.4 inches (or 1 meter), toe angle of $5^\circ$, and a linear twist of $-2^\circ$.

A more accurate means of optimizing minimum-energy required would be a benefit to this endeavour. The use of MATLAB's \texttt{fmincon} can prove to be a challenge when trying to match performance to a significant degree, such as on the order of $10^{-3}$. An optimizer tuned for the specific task could prove to be better.

Improvements can be made in this design and analysis of winglets. A wider parameter sweep can be done to ensure a global optimal for the design; although much of the parameter values can be neglected with some foresight and intuition. Ultimately, the design of winglets could be an extraordinary case to explore for an evolutionary algorithm. A more stable algorithm for the drag build-up would have to be implemented for the EA to be successful.
Figure A.1: A 3-view of the Pipistrel Taurus Electro G2
Figure A.2: An isometric view of the semi-span of the Taurus G2 with the designed winglet.

Figure A.3: A rear view of the semi-span of the Taurus G2 with the designed winglet.
Figure A.4: A side view of the Taurus G2 semi-span with the designed winglet.
Figure A.5: A 3-view of the Pipistrel Twin-Taurus G4.
Figure A.6: An isometric view of the semi-span of the Twin-Taurus G4 with the designed winglet.

Figure A.7: A rear view of the semi-span of the Twin-Taurus G4 with the designed winglet.
Figure A.8: A side view of the Twin-Taurus G4 semi-span with the designed winglet.
Bibliography


