ACOUSTIC INTENSITY METHODS IN CLASSICAL SCATTERING

A Thesis in
Acoustics
by
Brian Richard Rapids

© 2004 Brian Richard Rapids

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

December 2004
The thesis of Brian Richard Rapids was reviewed and approved* by the following:

Gerald C. Lauchle  
Professor of Acoustics  
Thesis Advisor  
Chair of Committee

Thomas B. Gabrielson  
Senior Research Associate and  
Associate Professor of Acoustics

Russell C. Burkhardt  
Research Associate and  
Assistant Professor of Acoustics

Martin W. Trethewey  
Professor of Mechanical Engineering

Anthony A. Atchley  
Professor of Acoustics  
Head of the Graduate Program in Acoustics

*Signatures are on file in the Graduate School
Measurements made with scalar pressure sensors are only able to provide an estimate of the magnitude of the total intensity associated with an equivalent plane wave. This equivalence is an assumption that the relative phase between pressure and velocity is identically zero. True intensity sensors simultaneously measure the acoustic pressure and components of particle velocity (or related quantity such as acceleration, displacement, or pressure gradient) at a single coordinate in space. Numeric computations regarding the scattering of a steady-state acoustic field by a rigid spheroid indicate that the equivalent plane wave intensity field only varies by ±0.5dB rel. incident acoustic intensity many object lengths away in the forward direction at high frequencies. These computations also predict a phase difference of up to 5° between pressure and particle velocity at ranges exceeding 10 objects lengths away from the scattering body. The reactive intensity estimate, which is identically zero in the presence of a plane wave, also took on values as high as -25dB rel. incident acoustic intensity. The theoretical investigations concluded that the presence of a spheroid in a steady-state harmonic field would perturb the acoustic intensity field more significantly than the scalar pressure field. Experimental measurements involving a biaxial intensity sensor and a model prolate spheroid were conducted to determine if perturbations in the acoustic field due to a scattering body would be more readily observable with an intensity sensor than with a traditional pressure sensor. The experimental data also indicates that measurements of acoustic intensity may be more sensitive to the presence of the spheroid than measurements made solely with the pressure sensor.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>List of Figures</th>
<th>vii</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Tables</td>
<td>xxiv</td>
</tr>
<tr>
<td>List of Symbols</td>
<td>xxv</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>xxxi</td>
</tr>
</tbody>
</table>

Chapter 1. Introduction ................................................................. 1
  1.1. Overview .............................................................................. 1
  1.2. Overview of Acoustic Intensity ............................................. 2
  1.3. Overview of Scattering from Prolate Spheroids ......................... 9
  1.4. Problem Statement and Objective .......................................... 12
  1.5. Scope of Thesis Work ......................................................... 12
  1.6. Outline of Thesis .............................................................. 13

Chapter 2. Concepts in Acoustic Intensity .......................................... 15
  2.1. Derivation of Intensity ....................................................... 15
  2.2. Time-Domain Representation of Acoustic Intensity ..................... 17
  2.3. Frequency Domain Representation of Intensity .......................... 23
  2.4. Summary ............................................................................ 25

Chapter 3. Scattering from a Rigid Prolate Spheroid ............................ 27
  3.1. Introduction ........................................................................ 27
  3.2. Overview of Prolate Spheroidal Coordinate System .................... 28
3.3. Solving for the Acoustic Field Scattered from a Rigid Prolate Spheroid…33

3.4. Total Pressure Field Caused by the Scatter of an Incident Plane Wave by a
Rigid Prolate Spheroid……………………………………………………43

3.5. Total Acoustic Intensity Field Caused by the Scatter of an Incident Plane
Wave by a Rigid Prolate Spheroid……………………………………….46

3.6. Summary………………………………………………………………60

Chapter 4. Frequency Domain Estimates of Acoustic Intensity……………………...62

4.1. Introduction…………………………………………………………………62

4.2. Frequency Domain Estimators of Acoustic Intensity………………………63

4.3. Frequency Domain Estimators of Acoustic Intensity Employing Pressure
Transducers and Accelerometers (p-a probes)………………………………86

4.4. Analysis of Measurements made with an Underwater p-a Probe in the Near
Field of an Acoustic Source………………………………………………94

4.5. Summary………………………………………………………………….123

Chapter 5. Experimental Measurement of Acoustic Intensity in the Forward Scatter
Direction……………………………………………………………………125

5.1. Introduction………………………………………………………………125

5.2. Fabrication of 10:1 Prolate Spheroid……………………………………..126

5.3. Initial Testing at Flooded Quarry in Jacksonville, PA…………………...130

5.4. Testing at Seneca Lake Sonar Test Facility in Dresden, NY……………..135

5.5. Summary …………………………………………………………………187
Chapter 6. Concluding Remarks

6.1 Summary

6.2 Future Work

BIBLIOGRAPHY

APPENDIX A: Details Regarding Scattering Computations

APPENDIX B: Histograms of Acoustic Intensity from Seneca Lake Experiment
LIST OF FIGURES

Figure 1-1: Impact of $\phi_{pu}$ on radial acoustic intensity for spherical waves in water having unit amplitude pressure. (Left) blue: Magnitude of peak instantaneous active intensity; green: Magnitude of peak instantaneous reactive intensity; red: Magnitude of time-averaged intensity. (Right) Power factor angle versus $r/\lambda$ ................................................................. 8

Figure 1-2: Bias in time-averaged intensity magnitude when the power factor angle is ignored..................................................................................................... 9

Figure 3-1: Two dimensional elliptical coordinate system. Figure was reproduced from Flammer7 ...................................................................................................... 29

Figure 3-2: Three-dimensional prolate spheroidal coordinate system. Figure was reproduced from Kollars27 ................................................................................... 30

Figure 3-3: Farfield scattering patterns for rigid prolate spheroid ensonified by a monochromatic plane wave ($\theta_{inc} = 60^\circ, \phi_{inc} = 0^\circ$). Patterns present the magnitude of scattered pressure over $\theta = \cos^{-1} \eta$ normalized to the respective maximum scattered pressures. The arrow represents the wavenumber vector $\vec{k}$ of the incident plane wave. ............................................................................... 37

Figure 3-4: Farfield scattering patterns for rigid sphere ensonified by a monochromatic plane wave ($\theta_{inc} = 0^\circ, \phi_{inc} = 0^\circ$). Patterns present the magnitude of scattered pressure over $\theta$ normalized to the respective maximum scattered pressure. The arrow represents the wavenumber vector $\vec{k}$ of the incident plane wave. ............................................................................... 39

Figure 3-5: Pressure field scattered by a 10:1 fineness ratio rigid prolate spheroid from a monochromatic plane wave ($\theta_{inc} = 60^\circ, \phi_{inc} = 0^\circ$). The spheroid can be seen at the center of each field plot where the field in the immediate vicinity of the spheroid has not been computed. Axis dimensions are number of object lengths ($L$) ........................................................................................................ 42

Figure 3-6: Total pressure field established by the scattering of a monochromatic plane wave ($\theta_{inc} = 60^\circ, \phi_{inc} = 0^\circ$) by a 10:1 fineness ratio rigid prolate spheroid. The spheroid can be seen at the center of each field plot where the field in the immediate vicinity of the spheroid has not been computed. .............. 44
Figure 3-7: Total pressure field established by the scattering of a monochromatic plane wave by a rigid sphere. Arrow denotes the incident plane wave. Axis dimensions are in number of radii. .................................................................45

Figure 3-8: $\hat{\xi}$ and $\hat{\eta}$ unit vectors on the z-x plane. An enlarged image of the spheroid has been added for reference. The $\hat{\phi}$ unit vector points out of the page and is orthogonal to the z-x plane.................................................................48

Figure 3-9: Phase of the $\hat{\xi}$ complex intensity field when a monochromatic plane wave originating from $\cos^{-1} \eta_{inc} = 60^\circ$ is perturbed by the presence of a rigid prolate spheroid having a fineness ratio of 10:1. Axis dimensions are number of object lengths ($L$). ..............................................................................50

Figure 3-10: Phase of the $\hat{\eta}$ complex intensity field when a monochromatic plane wave originating from $\cos^{-1} \eta_{inc} = 60^\circ$ is perturbed by the presence of a rigid prolate spheroid having a fineness ratio of 10:1. Axis dimensions are number of object lengths ($L$). ...........................................................................................51

Figure 3-11: Phase of the $\hat{\phi}$ complex intensity field when a monochromatic plane wave originating from $\cos^{-1} \eta_{inc} = 60^\circ$ is perturbed by the presence of a rigid prolate spheroid having a fineness ratio of 10:1........................................52

Figure 3-12: Total active intensity fields in the $\hat{\xi}$, $\hat{\eta}$, and $\hat{\phi}$ directions resulting from the scattering of a monochromatic plane wave originating from $\cos^{-1} \eta_{inc} = 60^\circ$ by a rigid prolate spheroid having a fineness ratio of 10:1. Axis dimensions are number of object lengths ($L$). Patterns depict the cosine response of the vector sensors and represent the gross features of the field for $h=1$ to 40 due to the coarse color scale. Finer details of the total fields can be observed in Figure 3-13 .................................................................................................55

Figure 3-13: Total active intensity fields in the $\hat{\xi}$ direction resulting from the scattering of a monochromatic plane wave originating from $\cos^{-1} \eta_{inc} = 60^\circ$ by a rigid prolate spheroid having a fineness ratio of 10:1. Fields corresponding to four values of $h$ are presented. Axis dimensions are number of object lengths ($L$). The color scale emphasizes the details of the field in the forward scatter direction. The field magnitude at other locations in the diagram do not fall on the narrow color scale, but can be seen with the coarse color scale in Figure 3-12 .................................................................................................56

Figure 3-14: Total reactive intensity fields in the $\hat{\xi}$ direction for the field resulting from the presence of a rigid prolate spheroid having a fineness ratio
of 10:1 in a monochromatic plane wave \((\theta_{inc} = 60^\circ, \varphi_{inc} = 0^\circ)\). Fields corresponding to values of \(h\) are presented. Axis dimensions are number of object lengths (\(L\)).

Figure 3-15: Total reactive intensity fields in the \(\hat{\eta}\) direction for the field resulting from the presence of a rigid prolate spheroid having a fineness ratio of 10:1 in a monochromatic plane wave \((\theta_{inc} = 60^\circ, \varphi_{inc} = 0^\circ)\). Fields corresponding to values of \(h\) are presented. Axis dimensions are number of object lengths (\(L\)).

Figure 4-1: Linear model of complex intensity measurement.

Figure 4-2: Auto-correlations and cross-correlations of outputs from a pressure sensor and three orthogonal velocity sensors in spherically isotropic noise.

Figure 4-3: Simplified linear measurement model including the construction of the acoustic intensity estimate via the cross-spectrum.

Figure 4-4: Relationship between \(\kappa_{pu}^2\) and SNR of pressure and velocity.

Figure 4-5: Normalized random error for the autospectral estimators \(\hat{G}_{pp}\) or \(\hat{G}_{uu}\).

Figure 4-6: Normalized random error for the magnitude of the cross-spectral estimator \(\hat{G}_{pu}\) as a function of the number of independent samples contributing to the ensemble average. The relationship is plotted for several values of \(\kappa_{pu}^2\) which annotate their respective lines.

Figure 4-7: Normalized random error for the magnitude of the cross-spectral estimator \(\hat{G}_{pu}\) as a function of the SNR of the pressure and velocity signals. It is assumed here that the two signals have identical values of SNR. The relationship is plotted for several values of \(n_d\) which annotate their respective lines.

Figure 4-8: Normalized random error for the magnitude of the cross-spectral estimator \(\hat{G}_{pu}\) as a function of the SNR of the pressure or the velocity signal. The relationship is plotted at four different scenarios in which the complementary signal has an SNR lower than that indicated on the abscissa. The lines are annotated with their respective amount of SNR differential (in dB) between the pressure and velocity. It is assumed that \(n_d = 1\).
Figure 4-9: Standard deviation of the phase estimated from the cross-spectral estimator $\hat{G}_{pu}$ as a function of the number of independent samples contributing to the ensemble average. The relationship is plotted for several values of $\kappa_{pu}^2$ which annotate their respective lines. ............................................82

Figure 4-10: Standard deviation of the phase estimate from the cross-spectral estimate $\hat{G}_{pu}$ as a function of the SNR of the pressure and velocity signals. It is assumed here that the two signals have identical values of SNR. The relationship is plotted for several values of $n_d$ which annotate their respective lines.................................................................83

Figure 4-11: Standard deviation of the phase estimate from the cross-spectral estimate $\hat{G}_{pu}$ as a function of the SNR of the pressure or the velocity signal. The relationship is plotted at four different scenarios in which the complementary signal has an SNR lower than that indicated on the abscissa. The lines are annotated with their respective amount of SNR differential between the pressure and velocity. It is assumed that $n_d = 1$........................................84

Figure 4-12: Normalized random error of the coherence function estimated from the cross-spectral estimates $\hat{G}_{pp}, \hat{G}_{uu},$ and $\hat{G}_{pu}$. The function is plotted against the number of independent samples contributing to the ensemble average. The relationship is plotted for several values of the true coherence function which annotate their respective lines........................................85

Figure 4-13: Dual axis p-a probe designed and constructed by Acoustech ...............89

Figure 4-14: Directivity of the pressure sensor used in the p-a probe.......................89

Figure 4-15: Sensitivity of pressure sensor with preamplifier providing +27dB of gain .................................................................................................................................90

Figure 4-16: Directivity of x-accelerometer in sensor package ..................................90

Figure 4-17: Sensitivity of x-accelerometers in sensor package (PCB Model 480E09 ICP signal conditioner S/N 23076) .................................................................91

Figure 4-18: Directivity of y-accelerometers in sensor package. ..............................91

Figure 4-19: Sensitivity of y-accelerometers in sensor package. (PCB Model 480E09 ICP signal conditioner S/N 23077) .................................................................92

Figure 4-20: Block diagram of measurement and estimation process with dual axis p-a probes.................................................................................................93
Figure 4-21: Geometry of Acoustic Test Facility at the Applied Research Laboratory ............................................................................................................95

Figure 4-22: Estimation of TVR by means of the Laser Doppler Vibrometer and by calibration with the USRD F33 reference hydrophone. ..............................................98

Figure 4-23: Measured FFVS for the hydrophone of the dual axis \(p-a\) probe by means of ITC-1032 transducer. No preamplifier was employed for the hydrophone. ..........................................................................................................99

Figure 4-24: Measured FFVS for the \(x\)-channel accelerometer of the dual axis \(p-a\) probe by means of ITC-1032 transducer. The accelerometer was connected to PCB Model 480E09 ICP signal conditioner (S/N 5633). .................................................100

Figure 4-25: Measured phase difference between hydrophone and \(a_{x}\) accelerometer. No preamplifier was employed for the hydrophone. The accelerometer was connected to PCB Model 480E09 ICP signal conditioner (S/N 5633). ...........................................................................................................100

Figure 4-26: Sample time series at different source-receiver separations: 26cm (top), 102cm (middle), 316cm (bottom). Esonification frequency was 3kHz.............................................................103

Figure 4-27: Sample time series at different source-receiver separations: 26cm (top), 102cm (middle), 316cm (bottom). Esonification frequency was 5kHz......104

Figure 4-28: Application of gate to data record. A data segment of 96pts for the 5kHz pulse was windowed and operated upon by a 96pt FFT. (top) Voltage signal from the hydrophone. (bottom) Voltage signal from the accelerometer...105

Figure 4-29: Autospectra of the pressure transducer (top) and accelerometer (bottom) during 3kHz pulse and prior to pulse arrival when sensor was 316cm from ITC-1032. Spectra have been generated using a single data record. (red): Autospectra of 96pt windowed signal with 96pt DFT; (lt blue): Autospectra of 96pt windowed noise record with 96pt DFT; (dk blue): Autospectra of 1024pt windowed noise record with 1024pt DFT. ......................107

Figure 4-30: Autospectra of the pressure transducer (top) and accelerometer (bottom) during 5kHz pulse and prior to pulse arrival when sensor was 316cm from ITC-1032. Spectra have been generated using a single data record. (red): Autospectra of 96pt windowed signal with 96pt DFT; (lt blue): Autospectra of 96pt windowed noise record with 96pt DFT; (dk blue): Autospectra of 1024pt windowed noise record with 1024pt DFT. ......................108

Figure 4-31: Scatterplot of active acoustic intensity at 1m from the face of the of ITC-1032 generated by removing the propagation loss associated with the
active acoustic intensity estimate made at the $p-a$ probe. The ITC-1032 was driven with a 1Vrms CW pulse which was amplified by +40dB. The scatterplot has been plotted on top of the output for the ITC-1032 (TVR+40dB).

Figure 4-32: Scatterplot of active acoustic intensity at face of sensor. Black lines are results of a least-square fit of the data points, the slope of the resulting lines are indicated in the legend.

Figure 4-33: Histogram of the normalized error associated with the active intensity estimates made from approximately 200 records at each frequency and range. The $y$-axes of the histograms have been normalized by the number of samples.

Figure 4-34: Scatterplot of the reactive intensity at the face of the $p-a$ probe across frequency and source-receiver separation. Black lines are results of a least-square fit of the data points, the slope of the resulting lines are indicated in the legend.

Figure 4-35: Histogram of the normalized error associated with the reactive intensity estimates made from approximately 200 records at each frequency and range. The $y$-axes of the histograms have been normalized by the number of samples.

Figure 4-36: Scatterplot of the magnitude of complex acoustic intensity at the face of sensor. Black lines are results of a least-square fit of the data points, the slope of the resulting lines are indicated in the legend.

Figure 4-37: Histogram of the normalized error associated with the magnitude of the complex intensity estimates made from approximately 200 records at each frequency and range. The $y$-axes of the histograms have been normalized by the number of samples.

Figure 4-38: Scatterplot of the phase of the complex intensity estimate made at the face of the $p-a$ probe across frequency and source-receiver separation. The theoretical value for the relative phase is plotted in black for reference.

Figure 4-39: Histogram of the normalized error associated with the phase of the complex intensity estimates made from approximately 200 records at each frequency and range. The $y$-axes of the histograms have been normalized by the number of samples.

Figure 5-1: CAD drawings of 10:1 prolate spheroid.

Figure 5-2: Construction of wooden 10:1 prolate spheroid.
Figure 5-3: Prolate spheroid employed for scattering studies: 2m long with 10:1 aspect ratio. .......................................................................................................................... 130

Figure 5-4: Flooded quarry in Jacksonville, PA which was the site of initial dual-axis intensity sensor testing. ........................................................................................................ 132

Figure 5-5: 10:1 Prolate spheroid scattering body suspended in flooded quarry. .... 132

Figure 5-6: Deployment of dual-axis p-a probe and transducer. ........................ 133

Figure 5-7: Block diagram of equipment setup for quarry test. ........................... 134

Figure 5-8: Planned experimental geometry of scattering study aboard SMP at Seneca Lake Sonar Test Facility. Predicted arrival times for the direct path and surface reflections are indicated................................................................. 136

Figure 5-9: Deployment of spheroid rigidly mounted to pipe stringers. ............... 137

Figure 5-10: Theoretical prediction of total active acoustic intensity, \( I \), in the \( \mathbf{\hat{\xi}} \) direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of \( \left( \theta_{\text{inc}} = 90^\circ, \varphi_{\text{inc}} = 0^\circ \right) \) ....................................................... 138

Figure 5-11: Theoretical prediction of total reactive acoustic intensity, \( Q \), in the \( \mathbf{\hat{\xi}} \) direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of \( \left( \theta_{\text{inc}} = 90^\circ, \varphi_{\text{inc}} = 0^\circ \right) \) ....................................................... 139

Figure 5-12: Theoretical prediction of magnitude of the power factor angle, \( \varphi_{pu} \) in the \( \mathbf{\hat{\xi}} \) direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of \( \left( \theta_{\text{inc}} = 90^\circ, \varphi_{\text{inc}} = 0^\circ \right) \) ....................................................... 140

Figure 5-13: Theoretical prediction of total active acoustic intensity, \( I \), in the \( \mathbf{\hat{\eta}} \) direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of \( \left( \theta_{\text{inc}} = 90^\circ, \varphi_{\text{inc}} = 0^\circ \right) \) ....................................................... 141

Figure 5-14: Theoretical prediction of total reactive acoustic intensity, \( Q \), in the \( \mathbf{\hat{\eta}} \) direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of \( \left( \theta_{\text{inc}} = 90^\circ, \varphi_{\text{inc}} = 0^\circ \right) \) ....................................................... 142
Figure 5-15: Theoretical prediction of magnitude of the power factor angle, $\varphi_{pu}$ in the $\hat{n}$ direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of $\left(\theta_{inc} = 90^\circ, \varphi_{inc} = 0^\circ\right)$. .............................................................. 143

Figure 5-16: Sound speed profiles estimated from CTD measurements during week of 15 September 2003. ........................................................................................................ 144

Figure 5-17: Nominal transmit voltage response of ITC-1007 spherical transducer. Data provided by International Transducer Corporation, Santa Barbara, CA......147

Figure 5-18: Vertical array of dual axis $p-a$ probes deployed for scattering tests. Upper sensor was S/N 2002-4 and lower sensor was S/N 2002-5. .................148

Figure 5-19: Block diagram of instrumentation for Seneca Lake scattering experiment. ........................................................................................................... 149

Figure 5-20: (top) Data record from pressure sensor with 1ms 8kHz pulse transmitted from ITC-1007. (bottom) Mean square voltage level from pressure sensor over 20 records. The source to receiver separation is 38m (125ft)........................................................................................................................................ 150

Figure 5-21: (top) Data record from pressure sensor with 35ms 8kHz pulse transmitted from ITC-1007. (bottom) Mean square voltage level from pressure sensor over 20 records. The source to receiver separation is 38m (125ft)........................................................................................................................................ 150

Figure 5-22: Time series for the three channels of the $p-a$ probe for a 35msec 10kHz pulse for which the 1200pt processing window is highlighted in red. (top) Voltage signal from the hydrophone. (middle) Voltage signal from the $x$-axis accelerometer. (bottom) Voltage signal from the $y$-axis accelerometer. The source to receiver separation is 53m (175ft). ........................................................................................................... 153

Figure 5-23: Autospectra from a single data record for the pressure transducer channel for the 35ms 10kHz CW pulse with src/rcv separation of 53m (175ft). Spectra corresponding to the background noise recorded on 16 Sept (cyan) and 18 Sept (blue) are also presented in the same analysis bandwidth for comparison................................................................. 154

Figure 5-24: Autospectra from a single data record for the $x$-axis accelerometer channel for the 35ms 10kHz CW pulse with src/rcv separation of 53m (175ft). Spectra corresponding to the background noise recorded on 16 Sept (cyan) and 18 Sept (blue) are also presented in the same analysis bandwidth for comparison................................................................. 155
Figure 5-25: Autospectra from a single data record for the y-axis accelerometer channel for the 35ms 10kHz CW pulse with src/rcv separation of 53m (ft). Spectra corresponding to the background noise recorded on 16 Sept (cyan) and 18 Sept (blue) are also presented in the same analysis bandwidth for comparison...

Figure 5-26: Frequency domain estimates of $I_{\text{equiv}}$, $I_t$, $Q_t$, $\theta_{xy}$, and $\theta_{Qy}$ across 1200pt data records for the 10kHz steady state signal corresponding to a src/rcv separation of 38m (125ft) relative to the $xy$ coordinate frame. The time on the abscissa represents the time of the data record from which the estimate was made. The spheroid was present during data recording.

Figure 5-27: Frequency domain estimates of $I_{\text{equiv}}$, $I_t$, $Q_t$, $\theta_{xy}$, and $\theta_{Qy}$ across 1200pt data records for the 10kHz pulsed CW signal corresponding to a src/rcv separation of 38m (125ft) relative to the $xy$ coordinate frame. The time on the abscissa represents the time of the data record from which the estimate was made. The spheroid was present during data recording.

Figure 5-28: Geometry of the rotated intensity vectors in the $ab$ frame relative to the original $xy$ frame and the incident signal.

Figure 5-29: Frequency domain estimates of $I_{\text{equiv}}$, $I_t$, $Q_t$, $\theta_{ab}$, and $\theta_{Qab}$ across 1200pt data records for the 10kHz steady state signal corresponding to a src/rcv separation of 38m (125ft) relative to the $ab$ coordinate frame. The time on the abscissa represents the time of the data record from which the estimate was made. The spheroid was present during data recording.

Figure 5-30: Frequency domain estimates of $I_{\text{equiv}}$, $I_t$, $Q_t$, $\theta_{ab}$, and $\theta_{Qab}$ across 1200pt data records for the 10kHz pulsed CW signal corresponding to a src/rcv separation of 38m (125ft) relative to the $ab$ coordinate frame. The time on the abscissa represents the time of the data record from which the estimate was made. The spheroid was present during data recording.

Figure 5-31: Histograms of $\hat{G}_{pp}$ for src/rcv separation for 10kHz steady state CW at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure 5-32: Histograms of $\hat{G}_{pp}$ for a 10kHz CW pulse at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure 5-33: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz steady state CW at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.............................172

Figure 5-34: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz CW pulse at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................................173

Figure 5-35: Total active intensity in the $\hat{\xi}$ direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of $\left(\theta_{inc} = 90^\circ, \phi_{inc} = 0^\circ\right)$. The computational plane lies in the yz (vertical) plane perpendicular to the incident wavenumber vector which resides in the horizontal plane. The scale of the plot is in dB relative to the incident plane wave intensity.......................................................................................................174

Figure 5-36: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz steady state CW at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.............................................177

Figure 5-37: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz CW pulse at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.............................................178

Figure 5-38: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz steady state CW at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase
angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent. .................. 180

Figure 5-39: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz CW pulse at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent. .................. 181

Figure 5-40: Histograms of normalized co-spectra for 10kHz steady state CW at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent. .................. 183

Figure 5-41: Histograms of normalized co-spectra for 10kHz CW pulse at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent. .................. 184

Figure 5-42: Histograms of normalized quad-spectra estimates for 10kHz steady state CW at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent. .................. 185

Figure 5-43: Histograms of normalized quad-spectra estimates for 10kHz CW pulse at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent. .................. 186

Figure B-1: Histograms of $\tilde{G}_{pp}$ for 8kHz steady state CW at a src/rcv separation of 38m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent. ................................. 202

Figure B-2: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 8kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent. ................................. 203

Figure B-3: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 8kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum.
sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..........................................................204

Figure B-4: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 8kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent........................................205

Figure B-5: Histograms of $\tilde{G}_{pp}$ for 12kHz steady state CW at a src/rcv separation of 38m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..........................................................206

Figure B-6: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent...............................................207

Figure B-7: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent...............................................208

Figure B-8: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent........................................209

Figure B-9: Histograms of $\tilde{G}_{pp}$ for 8kHz CW pulse at a src/rcv separation of 38m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................................210

Figure B-10: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 8kHz CW pulse at a src/rcv separation of 38m. From top to
bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................................211

Figure B-11: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 8kHz CW pulse at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.................................212

Figure B-12: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 8kHz CW pulse at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.................................213

Figure B-13: Histograms of $\tilde{G}_{pp}$ for 12kHz CW pulse at a src/rcv separation of 38m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.....................................................................214

Figure B-14: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 12kHz CW pulse at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................................215

Figure B-15: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz CW pulse at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................................216

Figure B-16: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz CW pulse at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are...
presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-17: Histograms of $\tilde{G}_{pp}$ for 8kHz steady state CW at a src/rcv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-18: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 8kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-19: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 8kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-20: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 8kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-21: Histograms of $\tilde{G}_{pp}$ for 10kHz steady state CW at a src/rcv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-22: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-23: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 10kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................................224

Figure B-24: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 10kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................225

Figure B-25: Histograms of $\tilde{G}_{pp}$ for 12kHz steady state CW at a src/rcv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................................226

Figure B-26: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 12kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................................227

Figure B-27: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................................228

Figure B-28: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................229
Figure B-29: Histograms of $\tilde{G}_{pp}$ for 8kHz CW pulse at a src/rcv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.................................230

Figure B-30: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 8kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..................................................231

Figure B-31: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 8kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent ..........................................................232

Figure B-32: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 8kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.................................233

Figure B-33: Histograms of $\tilde{G}_{pp}$ for 10kHz CW pulse at a src/rcv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent..........................................................234

Figure B-34: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.................................235

Figure B-35: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 10kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in
degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-36: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 10kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-37: Histograms of $\tilde{G}_{pp}$ for 12kHz CW pulse at a src/rcv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-38: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 12kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-39: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure B-40: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
LIST OF TABLES

Table 1-1: Acoustic Quantities for Plane Waves and Radial Component of Spherical Waves ........................................................................................................6

Table 1-2: Acoustic Intensities for Plane Waves and Radial Component of Spherical Waves ........................................................................................................6

Table 4-1: Ensemble means (top) and standard deviations (bottom) for the active intensity estimates presented in Figure 4-32 and Figure 4-33. Units of values are Watts/m² .........................................................................................................114

Table 4-2: Ensemble means (top) and standard deviations (bottom) for the reactive intensity estimates presented in Figure 4-34 and Figure 4-35. Units of values are Watts/m². .................................................................................................117

Table 4-3: Ensemble means (top) and standard deviations (bottom) for the magnitude of the complex intensity estimates presented in Figure 4-34 and Figure 4-35. Units of values are Watts/m² .................................................................120

Table 4-4: Ensemble means (top) and standard deviations (bottom) for the phase of the complex intensity estimates presented in Figure 4-34 and Figure 4-35. Units of values are degrees. .................................................................123

Table 5-1: Number of 1200pt data records for each Seneca Lake Experiment permutation and the actual source to receiver separations. ...........................................156

Table 5-2: Acoustic field indicators and metrics employed to quantify disturbances to the acoustic field due to the presence of a prolate spheroid. ...............166
LIST OF SYMBOLS

\( p \) Complex acoustic pressure

\( P_{\text{rms}} \) RMS magnitude of complex pressure

\( P \) Magnitude of complex acoustic pressure

\( \varphi_p \) Phase of complex acoustic pressure

\( \ddot{u} \) Complex acoustic particle velocity vector

\( U_{\text{rms}} \) RMS magnitude of complex velocity

\( U \) Magnitude of complex acoustic particle velocity vector

\( \varphi_u \) Phase of complex acoustic particle velocity

\[
\begin{bmatrix}
  u_i \\
  u_j \\
  u_k
\end{bmatrix}
\] Components of acoustic particle velocity vector

\[
\begin{bmatrix}
  U_i \\
  U_j \\
  U_k
\end{bmatrix}
\] Magnitude of acoustic particle velocity vector components

\[
\begin{bmatrix}
  \varphi_{u_i} \\
  \varphi_{u_j} \\
  \varphi_{u_k}
\end{bmatrix}
\] Phase of acoustic particle velocity vector components

\( \ddot{a} \) Complex acoustic particle acceleration vector

\[
\begin{bmatrix}
  a_i \\
  a_j \\
  a_k
\end{bmatrix}
\] Components of acoustic particle velocity vector

\[
\begin{bmatrix}
  \varphi_{a_i} \\
  \varphi_{a_j} \\
  \varphi_{a_k}
\end{bmatrix}
\] Phase of acoustic particle acceleration vector components

\( \ddot{r}, \ddot{d} \) Displacement vectors

\( t \) Time

\( \vec{k} \) Acoustic wavenumber vector

\( \lambda \) Acoustic wavelength

\( \omega \) Frequency (radians)
$f$ Frequency (Hz)

$\bar{I}_i$ Instantaneous intensity vector

$\varphi_{pu_n}$ Phase difference between $p$ and $u$ in the $n$ direction (power factor angle)

$\bar{I}$ Time averaged acoustic intensity or active acoustic intensity vector

$I_n$ Active acoustic intensity in the n-direction

$\bar{Q}$ Reactive acoustic intensity vector

$Q_n$ Reactive acoustic intensity in the $n$ direction

$\bar{I}_c$ Complex acoustic intensity vector

$I_{c,n}$ Complex acoustic intensity in the $n$ direction

$\rho$ Bulk density of medium

$c$ Bulk sound speed in medium

$e_{tot}$ Total mechanical energy density

$e_{kinetic}$ Kinetic energy density

$e_{potential}$ Potential energy density

$E$ Total energy

$\gamma = 2\varphi_p - 2\omega t$

$R_{pu}$ Cross-correlation function between $p$ and $u$

$t$ Time lag

$T$ Observation time interval

$S_{pu}$ Two-sided cross-spectral density function between $p$ and $u$
Single-sided cross-spectral density function between $p$ and $u$

Co-spectra between $p$ and $u$

Quad-spectra between $p$ and $u$

Single-sided auto-spectral density function for $p$ and $u$

Coordinates in Cartesian coordinate system

Coordinates in prolate spheroidal coordinate system

Constant value defining a prolate spheroidal surface

Focal points for spheroidal surface defined by $\xi_0$

Major axis of spheroid defined by $\xi_0$

Minor axis of spheroid defined by $\xi_0$

Interfocal distance between focal points

Fineness ratio

Eccentricity

Elements of the metric tensor in prolate spheroidal coordinates

Arbitrary vector field

Arbitrary scalar field

Magnitude of plane wave incident upon spheroid

Incident angle of plane wave upon spheroid

Reduced frequency

Prolate spheroidal angle function of the 1st kind
\( R_{m,n}^{(j)}(h, \xi) \) Prolate spheroidal radial function of the \( j \)th kind

\( P_{m,n}(\xi, \eta, \varphi) \) Eigenfunction in prolate spheroidal coordinates

\( P_{n}^{m}(\eta) \) Associated Legendre function

\( \varepsilon_m \) Neumann factor

\( N_{m,n} \) Normalization factor

\( r_0 \) Radius of sphere

\( p_{tot} \) Total complex acoustic pressure (incident + scattered)

\( u_{tot} \) Total complex acoustic particle velocity (incident + scattered)

\( p_k \) Discrete Fourier Transform of the \( k \)th pressure data record

\( u_k \) Discrete Fourier Transform of the \( k \)th velocity data record

\( n_d \) Number of independent samples in ensemble

\( \hat{I} \) Estimate of active acoustic intensity

\( \hat{Q} \) Estimate of reactive acoustic intensity

\( \hat{I}_c \) Estimate of complex acoustic intensity

\( \hat{\phi}_{pu} \) Estimate of power factor angle

\( z_{acs} \) Specific acoustic impedance

\( H_z \) Transfer function due to \( z_{acs} \)

\( H_p \) Transfer function due to acoustic pressure sensor

\( H_u \) Transfer function due to acoustic particle velocity sensor
Transfer function due to acoustic particle acceleration sensor

Acoustic pressure signal of interest

Acoustic particle velocity signal of interest

Acoustic particle acceleration signal of interest

Voltage output of acoustic pressure sensor

Voltage output of acoustic particle velocity sensor

Pressure associated with individual interfering plane wave

Particle velocity associated with individual interfering plane wave

Spatially integrated acoustic pressure noise at face of sensor

Spatially integrated acoustic particle velocity noise at face of sensor

Pressure sensor thermal noise

Particle velocity sensor thermal noise

Noise due pressure sensor signal conditioning electronics

Noise due particle velocity sensor signal conditioning electronics

Total noise in acoustic pressure sensor channel

Total noise in acoustic particle velocity sensor channel

Ratio of signal power to noise power in pressure sensor channel

Ratio of signal power to noise power in particle velocity sensor channel

Signal-to-noise ratio in pressure sensor channel (decibels)
\( SNR_{\text{rel}} \) Signal-to-noise ratio in particle velocity sensor channel (decibels)

\( \kappa_{pu}^2 \) Ordinary coherence function between pressure and velocity

\( \varepsilon_x \) Normalized random error associated with ensemble of estimates

\( \bar{x} \) Mean value associated with ensemble of estimates

\( \sigma[\hat{x}] \) Standard deviation associated with ensemble of estimates

\( \sigma^2[\hat{x}] \) Variance associated with ensemble of estimates

\( I_{\text{equiv}} \) Equivalent plane wave intensity

\[ I_{\text{ref}} = 6.67 \times 10^{-14} \frac{W}{m^2} \] Reference intensity

\( \theta_{xy} \) Bearing of active intensity vector in \( xy \) coordinates

\( \theta_{ab} \) Bearing of active intensity vector in \( ab \) coordinates

\( \theta_{Qxy} \) Bearing of reactive intensity vector in \( xy \) coordinates

\( \theta_{Qab} \) Bearing of reactive intensity vector in \( ab \) coordinates

\( I_t \) Total active intensity magnitude

\( Q_t \) Total reactive intensity magnitude

\( I_{\text{norm},n} \) Normalized active intensity component in the \( n \) direction

\( Q_{\text{norm},n} \) Normalized reactive intensity component in the \( n \) direction

\( I_{\text{norm}} \) Normalized magnitude of active intensity vector

\( Q_{\text{norm}} \) Normalized magnitude of reactive intensity vector
ACKNOWLEDGEMENTS

The author wishes first and foremost to thank his wife Lori Rapids for being his sun and moon. Without her he would not have had the strength nor the wherewithal to perform this work. Without his children, Andrew, David, and Christopher, he would not have had a purpose or a reason to pursue the degree. To all of them he is deeply indebted.

The guidance and friendship of Dr. Gerald Lauchle will also not be forgotten. It is because of his support that the author was able to finish school before his own children started.

Lastly the support of the entire staff on the Torpedo Defense Program at the Applied Research Laboratory was greatly appreciated. In particular, special thanks need to be extended to Leo Schneider and Dr. Russell Burkhardt for their support and encouragement.

The author would also like to acknowledge that this work was supported by the Office of Naval Research under grant number N00014-01-1-0108.
Chapter 1

Introduction

1.1 Overview

The scattering of a scalar wave by an obstacle (having ideal boundary conditions) is a classic problem in theoretical physics. Past investigations regarding perturbations in the total scalar wave field due to the presence of an obstacle have emphasized the significance of forward scattering by the obstacle at high frequencies. Forward scattering at high frequencies has been a subject of many investigations in electromagnetics, acoustics, and optics due to its application in imaging, radar, and sonar systems. For example, when light is obstructed by a circular aperture, it is in the forward scatter region in which the Poisson spot can be observed and the classic Airy diffraction pattern manifests. In atmospheric optics, forward scattering from small water droplets causes the colored rings around the moon (lunar corona). It is also the forward scatter from a sphere in the high frequency limit that gives rise to the extinction paradox in which the total scattering cross section of a sphere is twice the geometrical cross section of the sphere. Investigations into the scattering of scalar waves often emphasize calculations or measurements of the scalar wave magnitude in the forward scatter region. In acoustics, this amounts to computing or measuring the pressure field in the
Fraunhofer region of the scattering body which is a quantity that is routinely measured in underwater acoustics.

The research presented here expands upon these areas by theoretically and experimentally investigating the perturbation in the total acoustic intensity field due to a scattering body. The theoretical analysis involves analytic computations of the scalar acoustic pressure and particle velocity vector in order to construct the total acoustic intensity vector field. The experimental measurements are designed to determine if novel pressure-accelerometer type underwater acoustic intensity sensors are able to measure a statistical difference in the estimated acoustical quantities that indicate an observable perturbation in the acoustic intensity field due to the presence of the obstacle.

1.2 Overview of Acoustic Intensity

In wave theory, a common way to describe the intensity of a sinusoidal wave is to define it as the power transported across a unit area normal to the wave’s direction of propagation. This description relies upon the definition of power as energy per unit time to provide a time scale for the energy transport under consideration. However this definition can be misleading. Intensity may either be instantaneous or time-averaged over one period of the sinusoid. The instantaneous intensity of an acoustic wave is the product of its instantaneous pressure and particle velocity and is therefore a vector quantity. This instantaneous vector points in the direction of the particle velocity vector as expressed for the one-dimensional case in Eq. 1.1 and Eq. 1.2 for which $\hat{u}$ has a single component. In general, $\hat{u}$ has three components $\hat{u} = u_i \hat{i} + u_j \hat{j} + u_k \hat{k}$, which each have a
corresponding phase term $\begin{bmatrix} \varphi_{u_i} & \varphi_{u_j} & \varphi_{u_k} \end{bmatrix}$. While not explicitly stated, it should be understood that $\varphi_p = \varphi_p (\vec{r})$ and $\varphi_u = \varphi_u (\vec{r})$.

$$p(\vec{r},t) = P(\vec{r}) \cos(\varphi_p - \omega t) \quad 1.1$$
$$\vec{u}(\vec{r},t) = \vec{U}(\vec{r}) \cos(\varphi_u - \omega t)$$

$$\overline{I}_r = p\vec{u} \quad 1.2$$

Mathematically, the product of the two sinusoids causes a heterodyne effect resulting in the sum of a DC component and a sinusoid oscillating at twice the frequency of either the pressure or particle velocity as given by Eq. 1.3. The vector instantaneous flow of energy is not independent of time, but instead is a sinusoidal quantity having a DC offset that may even take on negative values under certain circumstances. The instantaneous intensity may be regarded as having both a radiative (convective) component and an oscillatory (non-convective) component as shown in Eq. 1.4 and Eq. 1.5. In these equations, the variable $\varphi_{pu} = \varphi_p - \varphi_u$ has been introduced. The non-convective energy flow has no DC component and is purely sinusoidal in nature and will be shown to result from the spatial gradient of the pressure field in Chapter 2. The instantaneous intensity can be integrated over time to generate the time-averaged intensity in Eq. 1.6 which is the DC component of the convective instantaneous intensity given in Eq. 1.5. This time averaged quantity quantifies the real power transport of the wave field across the reference plane and points in the direction of the wavenumber vector $\vec{k}$ which may not coincide with the direction of the instantaneous particle velocity $\vec{u}$ that governs the
direction of the instantaneous intensity. For this reason, the time-averaged intensity vector is considered to be analogous to the Poynting vector in electromagnetic theory.

\[ \bar{I}_i = \frac{P\bar{U}}{2} \left[ \cos(\phi_p - \phi_u) + \cos(\phi_p + \phi_u - 2\omega t) \right] \]  \hspace{1cm} 1.3

\[ \bar{I}_i = \frac{P\bar{U}}{2} \left[ \cos \phi_{pu} \left( 1 + \cos(2\omega t) \right) + \sin \phi_{pu} \sin(2\omega t) \right] \]  \hspace{1cm} 1.4

\[ \bar{I}_i = \left\{ \frac{P\bar{U}}{2} \cos \phi_{pu} \right\} + \left\{ \frac{P\bar{U}}{2} \left[ \cos \phi_{pu} \cos(2\omega t) + \sin \phi_{pu} \sin(2\omega t) \right] \right\} \]  \hspace{1cm} 1.5

\[ \bar{I} = \langle \bar{I}_i \rangle_{\text{time}} = \frac{P\bar{U}}{2} \cos \phi_{pu} \]  \hspace{1cm} 1.6

Thus the time-averaged intensity is equal to the product of the RMS magnitudes of the pressure, \( P_{\text{rms}} = \frac{P}{\sqrt{2}} \), and particle velocity, \( \bar{U}_{\text{rms}} = \frac{\bar{U}}{\sqrt{2}} \), oscillations weighted by the cosine of the phase difference between pressure and velocity \( \phi_{pu} \). This phase difference, \( \phi_{pu} = \phi_p - \phi_u \), shall be referred to as the power factor angle. If the pressure and velocity oscillations are in-phase, then the power factor angle is zero and the time-averaged intensity is maximized (e.g. the case of a plane wave). Conversely if the two oscillations are in quadrature, then the power factor angle is 90° and the time-averaged intensity is identically zero (e.g. the case of a standing wave). The reference of power factor angle has been adopted from the variable of the same name used for power calculations associated with steady-state voltages and currents of circuits driven by
sinusoidal sources. In that field of power transfer, the power factor angle represents the phase difference between voltage and current. The power dissipated by resistive loads is referred to as the real (or active) power and has both an instantaneous and time-averaged value. When the power factor angle is zero, the average power dissipated by the resistive components is maximized and is always positive. When circuits contain capacitive or inductive components, the power factor angle can become non-zero and the power dissipated by the circuit may be reduced. If the circuit contains only inductive or only capacitive elements, then the power factor angle is ±90°, respectively, and there is no average power dissipation since these components do not provide a real resistive load.

Power that is associated with these reactive components (i.e. inductors and capacitors) is referred to as reactive power which has an instantaneous value proportional to the product of the RMS magnitudes of the voltage and current oscillations weighted by the sine of the power factor angle. While this analogy is useful, the pressure and particle velocity of an acoustic wave are not directly analogous to electrical voltage and current, the correct acoustic quantities for this analogy are pressure and volume velocity.

However the analogies drawn between the real electrical power and convective acoustic intensity and between the reactive electrical power and the non-convective acoustic intensity are still useful. Because of their electrical counterparts, the acoustical quantities are typically referred to as active and reactive acoustic intensity.

Both types of acoustic intensity provide information regarding the nature of the acoustic field. For example, consider a simple acoustic source residing in an unbounded, lossless medium generating spherical waves. Common rules of thumb typically delineate two regions of the field, the near field and the far field, by a range value of 10
wavelengths or equivalently a value of \( kr = 20\pi \). This delineation is intended to separate those observation points for which the acoustic wave can locally be approximated as planar from those points where the full spherical nature of the wavefield should be recognized. Employing the same real notation for the wavefields as in Eq. 1.1, then the pressure and particle velocity for spherical and plane waves follow the expressions given in Table 1-1 and the corresponding intensity related quantities are summarized in Table 1-2.

**Table 1-1: Acoustic Quantities for Plane Waves and Radial Component of Spherical Waves**

<table>
<thead>
<tr>
<th></th>
<th>Plane Wave</th>
<th>Spherical Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>( p = P \cos(kx - \omega t) )</td>
<td>( p = \frac{P}{r} \cos(kr - \omega t) )</td>
</tr>
<tr>
<td>Particle Velocity</td>
<td>( u_x = \frac{P}{\rho c} \cos(kx - \omega t) )</td>
<td>( u_r = \frac{P}{\rho c} \left[ \frac{\cos(2kr - 2\omega t)}{r} - \frac{\sin(2kr - 2\omega t)}{kr^2} \right] )</td>
</tr>
</tbody>
</table>

**Table 1-2: Acoustic Intensities for Plane Waves and Radial Component of Spherical Waves**

<table>
<thead>
<tr>
<th></th>
<th>Plane Wave</th>
<th>Spherical Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instantaneous Active Intensity</td>
<td>( \frac{P^2}{2\rho c} \left[ 1 + \cos(2kx - 2\omega t) \right] )</td>
<td>( \frac{P^2}{2\rho c r^2} \left[ 1 + \cos(2kr - 2\omega t) \right] )</td>
</tr>
<tr>
<td>Instantaneous Reactive Intensity</td>
<td>0</td>
<td>( \frac{P^2 \sin(2kr - 2\omega t)}{2k \rho c r^3} )</td>
</tr>
<tr>
<td>Power Factor Angle (( \varphi_{pu} ))</td>
<td>0</td>
<td>( \varphi_{pu} = \tan^{-1} \left( \frac{1}{kr} \right) )</td>
</tr>
<tr>
<td>Time-Averaged Intensity</td>
<td>( \frac{P^2}{2\rho c} )</td>
<td>( \frac{P^2}{2\rho c r^2} \cos \varphi_{pu} )</td>
</tr>
</tbody>
</table>
Figure 1-1 depicts the intensity relationships given in Table 1-2 for a unit amplitude pressure wave in water and illustrates that the aforementioned rule of thumb allows for a power factor angle on the order of 1° before the acoustic wave can be approximated by a plane wave having a power factor angle of 0°. Figure 1-2 shows that when this approximation is made, the 1° of phase difference between pressure and velocity at a range of \( r = 10 \lambda \) from the simple source amounts to a 

\[
10 \log_{10} \left( \cos 1^\circ \right) = -6.6 \times 10^{-4} \text{ dB}
\]

error in the calculation of average intensity. This example serves to provide an explanation for the rule of thumb in terms of power transfer rather than an alternative explanation involving local wavefront curvature. However it is interesting to point out that in the comparison of spherical and plane waves in a lossless, unbounded media, it is the wavefront curvature associated with the spherical wavefield that generates a non-zero power factor angle due to the presence of reactive intensity necessary to support the spatial gradient of pressure. This also serves to illustrate that if the phase difference between pressure and particle velocity (or alternatively the magnitude of the active and reactive acoustic intensity components) can be measured at an observation point, then an investigator can quantify the degree to which the observed wavefield locally deviates from ideal plane wave conditions. Furthermore, this can be done without *apriori* knowledge of the source dimensions or range from the observation point. Metrics such as this have been developed under the nomenclature of sound field indicators to assist with analysis of acoustic intensity measurements\(^{19}\).
Figure 1-1: Impact of $\phi_{pm}$ on radial acoustic intensity for spherical waves in water having unit amplitude pressure. (Left) blue: Magnitude of peak instantaneous active intensity; green: Magnitude of peak instantaneous reactive intensity; red: Magnitude of time-averaged intensity. (Right) Power factor angle versus $r/\lambda$. 
1.3 Overview of Scattering from Prolate Spheroids

Scattering from prolate spheroids is an ongoing research area that employs both analytical methods (e.g. normal modes) and numerical techniques (e.g. T-matrix, finite element methods, boundary element methods) to solve for the scattered acoustic field. The prolate spheroid is an ellipsoid of revolution whose axis of symmetry coincides with the major axis of the underlying ellipse. The eccentricity (e) of the ellipsoid allows the scattering body to degenerate to a sphere (e→0), or to an infinitely thin line segment whose length is equal to the interfocal distance (e→1). The importance of the prolate spheroid comes from the variety of scattering shapes attainable by the spheroid, coupled with the fact that the prolate spheroidal coordinate system is one of eleven that provide a separable solution to the wave equation. The analytical method for generating an exact
solution to the scattering problem pivots on the partial wave expansion of the incident wave in terms of basis functions and the adoption of classical boundary conditions (i.e. Neumann, Dirchlet, or Robin). The basis functions for the azimuthal angle and radial portions of the separable solution both employ spheroidal harmonics. Computational difficulty arises when attempting to evaluate these basis functions. As a result, analytical investigations have been limited to expanding the spheroidal harmonics in terms of more familiar spherical harmonics and constraining the analyses to a high or low frequency limit and observation points in the far-field of the scattering body. In order to mitigate these issues, various numerical techniques have been applied to the problem and have allowed investigators to study the scattering from elastic spheroidal solids and shells and study phenomena associated with resonances and creeping waves. Van Buren has developed techniques to evaluate the spheroidal wave functions over a wide range of parameters with sufficient precision and accuracy for rigorous scattering studies. This has enabled the evaluation of the three-dimensional field scattered from a prolate spheroid in both the near and far field of the scatterer over a wide range of incident frequencies and incident angles. In light of these new techniques, the analytical solution to the scattering from a rigid spheroid has been selected as the foundation for this research. This enables the formulation of a classic solution to boundary value (i.e. scattering from a rigid surface) problem by employing the method of separation of variables. The resulting analytical solutions involving the prolate spheroidal wave functions can be evaluated (within a user-specified precision and accuracy) over a wide range of input parameters using the recently developed numerical techniques.
Based upon the principle of superposition in linear acoustic, the total acoustic field observed when a scattering body is placed in an incident wavefield is equal to the sum of the incident and the scattered fields. Superposition applies to linear acoustic quantities (e.g. pressure and particle velocity) and cannot be applied to intensities or energies. Computation of the total pressure and particle velocity fields enables evaluation of both the active (convective) and reactive (non-convective) components of the instantaneous intensity at all points around the scattering body. This permits investigations regarding how the presence of a spheroid perturbs the intensity field of a harmonic wave in an unbounded, lossless media. Such an investigation may examine how the perturbations are affected by combinations of incident wave frequency and spheroid dimensions which span the scattering regimes of Rayleigh, resonant, and geometrical acoustics. Of particular interest are perturbations when the scattering parameters place the problem in the geometrical acoustics regime (high-frequency limit). In this scenario, an acoustic shadow is formed on the side of the scattering body opposite of the direction of ensonification\textsuperscript{34}. The shadow region only exists in the very-near field of the scattering body while an interference pattern becomes established in the far field. From the principle of superposition, it can be concluded that the acoustic field scattered by the spheroid must have an appropriate magnitude and phase in order to cause sufficient destructive interference to support the formation of an acoustic shadow. In fact, this strong forward scattering by the spheroid in the high-frequency limit of geometrical acoustics is what gives rise to the classic extinction paradox for the degenerate case of a sphere (i.e. the total scattering cross section is twice the geometrical cross section of the sphere). It is in regions such as this where strong gradients in the
pressure field due to interference reduces the active (convective) intensity but should increase the reactive acoustic intensity according to the theory presented in Chapter 2.

1.4 Problem Statement and Objective

The problem considered by this body of work is the application of underwater acoustic intensity concepts to the area of underwater scattering measurements. The recent development of novel underwater acoustic intensity sensors permits investigators to measure the true acoustic intensity field in the presence of a scattering body. To the best knowledge of the author, this type of investigation has not yet been performed either theoretically or experimentally. The objective of this research is therefore to learn if the measurement of the acoustic intensity field provides new and more reliable information regarding the presence of a scattering body over that obtained with scalar pressure sensors alone.

1.5 Scope of Thesis Work

This research fundamentally requires the analytical calculation and experimental measurement of acoustic pressure and acoustic particle velocity in order to investigate the problem. A physical understanding of the problem will be developed through computations of harmonic plane wave scattering by a rigid prolate spheroid in an unbounded and lossless medium. A limited investigation into the random errors associated with the estimation of acoustic intensity by cross-spectral methods will also be
performed. As a precursor to the field experiment, a pressure-accelerometer sensor (i.e. $p-a$ probe) will be calibrated in the near field of a spherical source to verify the integrity of the sensor and sensor processing stream. The field experiment, conducted at the Seneca Lake Sonar Test Facility, will explore the feasibility of measuring the perturbation of the acoustic intensity field due to a scattering body. The underwater scattering experiment will be designed to compare the values of active acoustic intensity, reactive acoustic intensity, and power factor angle that are estimated when the scattering body is present to those values that are estimated when the scattering body is not present. The comparison of high frequency underwater acoustic intensity measurements in the presence and absence of a model prolate spheroid is the fundamental goal of the experimental work.

### 1.6 Outline of Thesis

The thesis is organized such that Chapter 1 and Chapter 2 will respectively describe the underlying theory associated with acoustic intensity and the techniques employed to estimate the metrics experimentally.

Scattering from a rigid prolate spheroid is discussed in Chapter 3. This will serve to introduce the reader to the prolate spheroidal coordinate system and the exact solutions for the scattered scalar field and the intensity in the total field. While the analytical material for the scalar pressure or velocity potential is well documented elsewhere, the solutions given in this thesis for the scattered and total acoustic particle velocity vector and corresponding vector intensity are new. These computational results have not been
documented before. Their inclusion in this background material will assist with the interpretation of the subsequent results.

Chapter 4 discusses the estimators that are used to construct the analytical quantities defined in Chapter 2. The estimators themselves are important because the operations which generate the estimates determine how random errors will affect the ability to approximate the true value of an observable. The random error associated with those estimators will then be compared against the predicted values of the phenomena presented in Chapter 3 to determine the feasibility of measuring them using the defined frequency domain techniques. The estimators are then applied to data collected with a single-axis intensity probe in the near-field of a spherical radiator. The subsequent data analysis demonstrates the analysis techniques against the theoretical values. Chapter 5 contains a description of underwater scattering experiments involving the use of a recently developed pressure-particle acceleration intensity probe (p-a probe). The experiments were designed to measure the acoustic intensity in the forward-scattered direction in order to verify the computational results from Chapter 3. Finally, Chapter 6 will summarize the findings of the investigation and make recommendations for future work. A concise discussion of the mechanics associated with the computation of the acoustic intensity fields presented in Chapter 3 is provided in Appendix A. Also, since a limited set of experimental results is provided in the Chapter 5 discussion, the balance of the experimental data is archived in Appendix B for completeness.
Chapter 2

Concepts in Acoustic Intensity

2.1 Derivation of Intensity

There are several ways to define acoustic intensity, rigorous approaches define the concept using the fundamental relationship of conservation of energy. This approach begins with the total mechanical energy density associated with linear acoustics in a stationary medium \( e \) which is composed of the kinetic and potential energy per unit volume as given in Eq. 2.1 and Eq. 2.2. The derivation of these quantities is not included here since they can be obtained in standard acoustics textbooks.

\[
e_{\text{tot}} = e_{\text{kinetic}} + e_{\text{potential}} \tag{2.1}
\]

\[
e = \frac{\rho \ddot{u} \cdot \ddot{u}}{2} + \frac{p^2}{2 \rho c} \tag{2.2}
\]

The total energy density of the acoustic process can be integrated over an arbitrary volume to obtain the total energy as is done in Eq. 2.3. The rate of change of total energy is given by Eq. 2.4.

\[
E = \iiint \left( \frac{\rho \ddot{u} \cdot \ddot{u}}{2} + \frac{p^2}{2 \rho c} \right) dV \tag{2.3}
\]
The linearized equations governing momentum and continuity can be applied to the integrand in Eq. 2.4 to yield Eq. 2.5 which is a corollary to the conservation of energy and shows that if the total energy in the volume does not change over time, then the divergence of the product of pressure and velocity must also be zero. Application of the divergence theorem to Eq. 2.5 completes the definition of instantaneous intensity with Eq. 2.6 and Eq. 2.7 for a particle velocity vector \( \hat{u} \) pointing out of the volume which the surface \( S \) encapsulates into the \( \hat{n} \) direction. Thus we find that the rate of change of total energy in a volume is equal to the acoustic power flowing across the surface of the volume. Acoustic power per unit area is the instantaneous intensity and is defined as the product of pressure and particle velocity.

\[
\frac{dE}{dt} = \iiint_V \left( \rho \hat{u} \cdot \frac{\partial \hat{u}}{\partial t} + \frac{p}{\rho c} \frac{\partial \hat{p}}{\partial t} \right) dV \quad 2.4
\]

\[
\frac{dE}{dt} = \iiint_V \nabla \cdot (p \hat{u}) \ dV \quad 2.5
\]

\[
\frac{dE}{dt} = \iint_S p \hat{u}_n \ dS \quad 2.6
\]

\[
\frac{dE}{dt} = \iint_S \hat{T} \cdot \hat{n} \ dS \quad 2.7
\]

If the time average of Eq. 2.4 is taken and there are no sources present in the volume, then it becomes apparent that the divergence of the time-averaged intensity is zero since the average total energy in the volume does not change. This defines the sound power radiated by a source as the integration of the time-averaged intensity over an
arbitrary surface which encapsulates the source in question but no other source or sink of energy.

2.2 Time-Domain Representations of Acoustic Intensity

Employing complex notation and an $e^{-j\omega t}$ time convention, the example given in Chapter 1.1 can be explored in greater detail. First the expressions for components of acoustic intensity are derived using an arbitrary pressure and particle velocity to show that the power factor angle is the phase difference between those two quantities. Second, the same intensity expressions are developed using an arbitrary pressure field and the linearized momentum equation to show how the active and reactive intensities are related to the mean squared pressure field and the gradient of the mean square pressure field respectively. Lastly, the concept of complex acoustic intensity is developed and used to relate the power factor angle to the active and reactive intensities.

2.2.1 Instantaneous Intensity in Terms of an Arbitrary Pressure and Arbitrary Particle Velocity

The analysis begins with the complex field variables in Eq. 2.8 and the corresponding definition of instantaneous intensity in Eq. 2.9 as the product of the real parts of the complex pressure and velocity.
The real part of a complex quantity can be represented as one-half the sum of the quantity and its complex conjugate which is denoted by \{\}^*\. This algebra yields Eq. 2.10 and Eq. 2.11.

\[
\bar{I}_i(\bar{r}, t) = \Re\{p(\bar{r}, t)\} - \Re\{\bar{u}(\bar{r}, t)\}
\]

Substitution of the variable expressions of Eq. 2.8 into Eq. 2.11 results in Eq. 2.12. The product of the two oscillations produces a heterodyne effect in which some of the terms have been shifted to DC and the cross terms have been shifted up in frequency and oscillate at 2\omega.

\[
\bar{I}_i = \frac{P}{4} U_i \left[ e^{i(\phi_r + \phi_{r_0} - 2\omega t)} + e^{-i(\phi_r + \phi_{r_0} - 2\omega t)} + e^{i(\phi_r + \phi_{r_0})} + e^{-i(\phi_r + \phi_{r_0})} \right] i
\]

\[
\bar{I}_j = \frac{P}{4} U_j \left[ e^{i(\phi_r + \phi_{r_j} - 2\omega t)} + e^{-i(\phi_r + \phi_{r_j} - 2\omega t)} + e^{i(\phi_{r_j} + \phi_{r_0})} + e^{-i(\phi_{r_j} + \phi_{r_0})} \right] j
\]

\[
\bar{I}_k = \frac{P}{4} U_k \left[ e^{i(\phi_r + \phi_{r_k} - 2\omega t)} + e^{-i(\phi_r + \phi_{r_k} - 2\omega t)} + e^{i(\phi_r + \phi_{r_k})} + e^{-i(\phi_r + \phi_{r_k})} \right] k
\]
The sums of complex exponentials with complementary arguments have been reduced to simple trigonometric functions and re-expressed in Eq. 2.13 through Eq. 2.15 which are respectively identical to Eq. 1.3 and Eq. 1.4. Four new variables are defined for an arbitrary unit vector $\hat{n}$ here, $I\hat{n} = \frac{PU}{4} \cos \varphi_{pu} \hat{n}$, $Q\hat{n} = \frac{PU}{4} \sin \varphi_{pu} \hat{n}$,

$$\gamma = 2\varphi_p - 2\omega t$$, and $\varphi_{pu} = \varphi_p - \varphi_{u_i}$. These allow Eq. 2.14 to be re-written as Eq. 2.15 and yield compact expressions for the instantaneous active and reactive intensities.

$$\bar{I}_i = \frac{P}{2} \left[ U_i \left[ \cos \left( \varphi_p - \varphi_{u_i} \right) + \cos \left( \varphi_p - \varphi_{u_i} - 2\omega t \right) \right] \hat{i} \right]$$

$$\bar{I}_j = \frac{P}{2} \left[ U_j \left[ \cos \left( \varphi_p - \varphi_{u_i} \right) + \cos \left( \varphi_p - \varphi_{u_i} - 2\omega t \right) \right] \hat{j} \right]$$

$$\bar{I}_k = \frac{P}{2} \left[ U_k \left[ \cos \left( \varphi_p - \varphi_{u_i} \right) + \cos \left( \varphi_p - \varphi_{u_i} - 2\omega t \right) \right] \hat{k} \right]$$

$$\bar{I}_i = \bar{I} \left( 1 + \cos \gamma \right) + \bar{Q} \sin \gamma$$

2.2.2 Alternative Form of the Instantaneous Intensity in Terms of an Arbitrary Pressure

If Eq. 2.8 is combined with the linearized Euler equation given in Eq. 2.16, an alternative form of the instantaneous intensity $\bar{I}_i$ can be derived.
\[
\frac{\partial \hat{u}(\hat{r}, t)}{\partial t} = -\frac{1}{\rho} \hat{\nabla} p \tag{2.16}
\]

Since the investigation is restricted to harmonic signals, the \(e^{-i\omega t}\) time dependence permits integration of both sides over time to obtain Eq. 2.17. It is worth pointing out that the direction of the instantaneous velocity vector is governed by two gradient operations and is affected by the local spatial variations of the phase and amplitude of the pressure wave.

\[
\hat{u}(\hat{r}, t) = \frac{-i}{\omega \rho} \left[ P \hat{\nabla} \varphi_p - i \hat{\nabla} P \right] e^{i(\varphi_p-\omega t)} \tag{2.17}
\]

An expression for the instantaneous intensity is obtained by substituting the pressure given in Eq. 2.8 and the particle velocity given in Eq. 2.17 into Eq. 2.11 to obtain Eq. 2.18. Further algebraic manipulation leads to Eq. 2.19 and Eq. 2.20.

\[
\tilde{I}_i = \frac{1}{4\omega \rho} \left[ \left( P^2 \hat{\nabla} \varphi_p - iP \hat{\nabla} P \right) e^{i\gamma} + \left( P^2 \hat{\nabla} \varphi_p - iP \hat{\nabla} P \right) \ldots 
+ \left( P^2 \hat{\nabla} \varphi_p - iP \hat{\nabla} P \right) + \left( P^2 \hat{\nabla} \varphi_p + iP \hat{\nabla} P \right) e^{-i\gamma} \right] \tag{2.18}
\]

\[
\tilde{I}_i = \frac{1}{4\omega \rho} \left[ 2P^2 \hat{\nabla} \varphi_p + P^2 \hat{\nabla} \varphi_p \left( e^{i\gamma} + e^{-i\gamma} \right) + iP \hat{\nabla} P \left( e^{-i\gamma} - e^{i\gamma} \right) \right] \tag{2.19}
\]

\[
\tilde{I}_i = \frac{P^2 \hat{\nabla} \varphi_p}{2\omega \rho} (1 + \cos \gamma) + \frac{P \hat{\nabla} P}{2\omega \rho} \sin \gamma \tag{2.20}
\]

Comparison of Eq. 2.20 with Eq. 2.15 indicates that the variables \(\tilde{I}\) and \(\tilde{Q}\) which contain the spatial dependencies of the instantaneous active and reactive intensities can be defined more rigorously in Eq. 2.21 through Eq. 2.23. Due the fact that both \(\tilde{I}\) and \(\tilde{Q}\) contain all the spatially related information (no temporal dependencies) of the
instantaneous active and reactive intensities, they are commonly referred to as the active and reactive intensities. It can be seen that the direction (and to some degree the magnitude) of the active intensity is governed by the gradient of the pressure phase which is parallel to the wavenumber vector, $\vec{k}$, of the harmonic wave component. Conversely, the reactive intensity vector is governed by the gradient of the squared pressure amplitude.

$$\vec{I}(\vec{r}) = \frac{P^2 \vec{\nabla} \phi_p}{2\omega \rho}$$ \hspace{1cm} 2.21$$

$$\vec{Q}(\vec{r}) = \frac{P^2 \nabla \nabla P}{2\omega \rho}$$ \hspace{1cm} 2.22$$

The vector relationship of $P\vec{\nabla}P = \frac{1}{2} \vec{\nabla}P^2$ allows Eq. 2.22 to be rewritten in terms of the gradient of the square of the pressure amplitude in Eq. 2.23.

$$\vec{Q}(\vec{r}) = \frac{\vec{\nabla}P^2}{4\omega \rho}$$ \hspace{1cm} 2.23$$

### 2.2.3 Alternative Form of Instantaneous Intensity in Terms of a Complex Intensity

In Section 2.2.2, the instantaneous intensity in the $\hat{n}$ direction has been expressed in terms of an arbitrary phase difference between the acoustic pressure and particle velocity in the $\hat{n}$ direction $\phi_{pu}$. This resulted from expressions for an arbitrary acoustic pressure field. In this section, Eq. 2.15 is expressed in terms of a complex intensity $\vec{I}_c$
and related to the active and reactive intensities $\bar{I}$ and $\bar{Q}$. This process begins by re-
writing Eq. 2.15 as Eq. 2.24 which separates the time and space dependent quantities.

$$\bar{I}_i = \text{Re}\left\{ (\bar{I} + i\bar{Q})(1 + \cos \gamma - i \sin \gamma) \right\} \quad 2.24$$

The instantaneous intensity field is then expressed as Eq. 2.25 with the substitution of $I_c = I + iQ$. As before, this only represents the spatial information regarding the instantaneous intensity field.

$$\bar{I}_i = \text{Re}\left\{ \bar{I}_i \left(1 + e^{-i\gamma}\right) \right\} \quad 2.25$$

The novelty of this new variable is that the complex intensity can be related to the expressions for $\bar{I}$ and $\bar{Q}$ that were derived in sections 2.2.1 and 2.2.2 via Eq. 2.26 and Eq. 2.27. Then it can be seen that when complex notation is employed for the acoustic variables, the complex intensity becomes the product of the acoustic pressure and the complex conjugate of particle velocity as shown in Eq. 2.28. Here, $\bar{I}$ and $\bar{Q}$ are respectively the real and imaginary parts. Since the complex intensity is a complex valued function, any of its components in the $\hat{n}$ direction may be represented as a phasor having a magnitude equal to the envelope of $I_{c,n} \hat{n}$ and a phase angle determined by $I_n$ and $Q_n$ according to Eq. 2.29.

$$\bar{I}_c = \bar{I} + i\bar{Q} = \frac{P}{2} \begin{bmatrix} U_i e^{i\phi_{nu}} \hat{i} \\ U_j e^{i\phi_{nj}} \hat{j} \\ U_k e^{i\phi_{nk}} \hat{k} \end{bmatrix} \quad 2.26$$
It is important to note that the time-averaged intensity given in Eq. 1.6 is equal to the active intensity, \( \bar{I} \), introduced in Eq. 2.15 and Eq. 2.21 despite the fact that one was derived using real trigonometric functions while the other two were derived using complex variables.

\[
\bar{I}_c = \bar{I} + i\bar{Q} = \frac{p^2}{2\omega \rho} \nabla \phi_p + i \frac{\nabla P^2}{4\omega \rho} \tag{2.27}
\]

\[
\bar{I}_c = \frac{pu^*}{2} \tag{2.28}
\]

\[
I_{c,n} = \sqrt{|I_n|^2 + |Q_n|^2} e^{i\phi_{pu}} \quad \text{where} \quad \phi_{pu} = \tan^{-1}\left(\frac{Q_n}{I_n}\right) \tag{2.29}
\]

2.3 Frequency Domain Representations of Intensity

This section serves to define the spectrum of acoustic intensity and demonstrate that the time-averaged acoustic intensity (i.e. active intensity) is equal to the co-spectrum of the single sided cross-spectrum between pressure and particle velocity. Similarly, the reactive intensity spectrum is shown to be the quad-spectrum of the single sided cross-spectrum between pressure and particle velocity.
2.3.1 Frequency Domain Representation of Active Intensity

The frequency domain representation of active intensity assumes that the pressure and particle velocity are both stationary and ergodic random processes. The time averaged acoustic intensity in Eq. 1.6 was obtained with the use of the operation in Eq. 2.30.

\[ \bar{I}(\bar{r}) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} p(\bar{r},t) \cdot u(\bar{r},t) dt \] \hspace{1cm} 2.30

Recognizing the resemblance between Eq. 2.30 and the cross-correlation function given in Eq. 2.31, Eq. 2.32 arises directly.

\[ R_{pu}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} p(\bar{r},t) \cdot u(\bar{r},t + \tau) dt \] \hspace{1cm} 2.31

\[ R_{pu}(0) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} p(\bar{r},t) \cdot u(\bar{r},t) dt \] \hspace{1cm} 2.32

Therefore the active intensity, or the time-averaged intensity, can be constructed by evaluating the cross-correlation at a lag of zero, \( \bar{I}(\bar{r}) = R_{pu}(0) \). The cross correlation function and the two-sided cross spectral density functions are a Fourier transform pair following Eq. 2.33.

\[ R_{pu}(\tau) = \int_{-\infty}^{+\infty} S_{pu}(f) e^{-i2\pi f \tau} df \] \hspace{1cm} 2.33

Since \( p \) and \( \bar{u} \) are real functions (rather than analytic), the real part of \( S_{pu} \) is an even function while the imaginary part is an odd function. Evaluation of Eq. 2.33 at \( \tau = 0 \) yields Eq. 2.34 which in turn can be written in terms of the co-spectrum \( C_{pu} \) and
quad-spectrum \( Q_{pu} \) components of the single sided cross spectrum

\[ G_{pu}(f) = C_{pu}(f) + iQ_{pu}(f). \]

Without any loss of generality, this results in Eq. 2.35.

\[ R_{pu}(0) = \int_{-\infty}^{+\infty} S_{pu}(f) df \]

\[ R_{pu}(0) = \int_{0}^{+\infty} C_{pu} df \]

Combining these equations together, the time-averaged acoustic intensity can be expressed in terms of the integrated co-spectrum between pressure and velocity by Eq. 2.36.

\[ \bar{I}(\vec{r}) = \int_{0}^{+\infty} C_{pu} df \]

Therefore the \( C_{pu}(f) \) can be interpreted as the active intensity spectrum \( \bar{I}(\vec{r}, f) \).

### 2.4 Summary

This chapter developed the formal concepts of acoustic intensity for steady-state harmonic acoustic fields. The quantities of active, reactive, and complex acoustic intensity were introduced. The phase of the complex intensity was identified as the phase difference between the acoustic pressure and particle velocity, \( \varphi_{pu} \). It was also established that the average flow of acoustic energy per unit area (active acoustic
intensity) is equal to the product of the magnitudes of the pressure and particle velocities weighted by the cosine of $\varphi_{pu}$. Since this phase difference cannot be measured with a single pressure sensor, it can be concluded that an accurate measurement of the flow of acoustic energy cannot be made by a single scalar pressure sensor unless it can be correctly assumed that $\varphi_{pu} = 0$.

It can also be concluded that if $\varphi_{pu}$ or the reactive intensity could be reliably measured, then an inference could be made regarding whether the mean square pressure field exhibits spatial gradients in the vicinity of the measurement. It was established in this chapter that a plane wave field contains no reactive intensity and $\varphi_{pu} = 0$ everywhere in the field. Therefore, if a scattering body is able to perturb the plane wave field, then neither the reactive intensity nor $\varphi_{pu}$ should be identically zero. Chapter 3 develops these ideas further by investigating the spatial gradients in the acoustic pressure field established by the scattering of a plane wave by a rigid prolate spheroid and computing various acoustic intensity quantities.
Chapter 3
Scattering from a Rigid Prolate Spheroid

3.1 Introduction

A significant amount of work has previously been done in the area of scattering from prolate spheroids. Analytical expressions for the partial wave expansion of the scattered field are obtained when classical boundary conditions are considered (i.e. Dirichlet, Neumann, or Robin). However, the evaluation of the partial wave expansion is difficult due to spheroidal wave functions that arise from solving the Helmholtz equation in prolate spheroidal coordinates. For this reason, some efforts have focused upon asymptotic expansions of the spheroidal wave functions, while others have focused upon numerical solutions to the scattering problem. The approach taken here is to employ recently developed algorithms to directly evaluate the radial and angular spheroidal wavefunctions in a modal solution to the scattering problem. These techniques allow the scattered field to be evaluated in both the nearfield and farfield as part of investigations regarding the application of acoustic intensity techniques to underwater detection. Modern computers make this a viable approach despite the fact that at high frequencies, the number of eigenfunctions required for an accurate solution becomes large in accordance with Debeye’s theorem. The particle velocity associated with the scattered
field may then be evaluated by applying the linearized Euler equation to the analytical expression for scattered pressure. The scattered fields are then combined with expressions for the incident field by superposition to obtain expressions for the total pressure and particle velocity fields. Expressions presented in Chapter 2 may then be employed to produce values of the various parameters related to the acoustic intensity of the total monochromatic, steady-state sinusoidal field.

3.2 Overview of Prolate Spheroidal Coordinate System

By rotating the two-dimensional elliptical coordinate system shown in Figure 3-1 about the major axis of the ellipse, the prolate spheroidal coordinate system shown in Figure 3-2 can be obtained. The natural coordinates in this system are \((\xi, \eta, \varphi)\). The \(\xi\) coordinate acts as a radial coordinate and specifies the ellipsoidal surface and resides on the interval \([1, \infty]\). A surface of constant \(\xi\) is defined as the locus of points for which the sum of distances to two focal points, \(f_1\) and \(f_2\), is a constant. Furthermore, this constant is equal to the product of the radial coordinate \(\xi\) and the interfocal distance, \(a\), as given in Eq. 3.1.
Figure 3-1: Two dimensional elliptical coordinate system. Figure was reproduced from Flammer.”
Figure 3-2: Three-dimensional prolate spheroidal coordinate system. Figure was reproduced from Kollars.\textsuperscript{27}
The coordinate system also contains two hyperboloid sheets which are defined by the locus of points for which the difference of the distances to the two focal points is a constant equal to the product of the hyperbolic coordinate $\eta$ and the interfocal distance as given in Eq. 3.2. The hyperbolic coordinate $\eta$ resides on the interval $[1, -1]$. The hyperboloid sheets are asymptotic to a conical surface which pass through the origin and make an angle of $\cos^{-1} \eta$ with the z-axis. Lastly, the $\phi$ coordinate is the rotational coordinate residing on the interval $[0, 2\pi]$ and specifies a unique plane stemming from the z-axis.

\begin{align*}
    r_1 + r_2 &= \xi a \quad \text{(3.1)} \\
    r_1 - r_2 &= \eta a \quad \text{(3.2)}
\end{align*}

The confocal ellipsoids specified by $\xi$, the hyperboloids defined by $\eta$, and the plane defined by $\phi$ are all orthogonal to one another and their intersection specifies a single unique point in space. Any point specified by $(\xi, \eta, \phi)$ can be projected onto a Cartesian coordinate system for the same unique point in $(x, y, z)$ space according to the conversions given in Eq. 3.2, Eq. 3.3, and Eq. 3.4.

\begin{align*}
    x &= \frac{a}{2} \left[ \left(1 - \eta^2\right) (\xi^2 - 1) \right]^{1/2} \cos \phi \quad \text{(3.3)} \\
    y &= \frac{a}{2} \left[ \left(1 - \eta^2\right) (\xi^2 - 1) \right]^{1/2} \sin \phi \quad \text{(3.4)}
\end{align*}
A prolate spheroid is constructed by restricting a locus of points to a constant value of $\xi = \xi_0$. This value alone is sufficient to specify the fineness ratio, $\varepsilon$, of the prolate spheroid which is defined as the ratio of major to minor axis of the ellipsoid and is given in Eq. 3.6.

$$
\varepsilon = \frac{L}{2R} = \frac{\xi_0}{\left(\frac{\xi^2}{\xi_0^2} - 1\right)^{\frac{1}{2}}}
$$

3.6

The eccentricity of the ellipsoid is also completely specified by $\xi_0$ according to Eq. 3.7.

$$
e = \frac{a}{L} = \frac{1}{\xi_0}
$$

3.7

The prolate spheroidal coordinate system, as any orthogonal curvilinear coordinate system, allows various vector operations (e.g. divergence, gradient, and Laplacian) to be expressed in terms of the elements of the metric tensor (scale factors) $[h_\xi \ h_\eta \ h_\phi]$ which are given in Eq. 3.8, Eq. 3.9, Eq. 3.10.

$$
h_\xi = \frac{a}{2}\left[(1-\eta^2)(\xi^2 - 1)\right]^\frac{1}{2}
$$

3.8

$$
h_\eta = \frac{a}{2}\left[(1-\eta^2)(\xi^2 - 1)\right]^\frac{1}{2}
$$

3.9

$$
h_\phi = \frac{a}{2}\left[(1-\eta^2)(\xi^2 - 1)\right]^\frac{1}{2}
$$

3.10
These scale factors enable the gradient of a scalar function $F$ in prolate spheroidal coordinates to be expressed by Eq. 3.11; the divergence of a vector $\mathbf{G}$ by Eq. 3.12, and the Laplacian of a scalar function $F$ by Eq. 3.13.

$$\nabla F = \frac{1}{h_\xi} \frac{\partial F}{\partial \xi} \hat{\xi} + \frac{1}{h_\eta} \frac{\partial F}{\partial \eta} \hat{\eta} + \frac{1}{h_\phi} \frac{\partial F}{\partial \phi} \hat{\phi}$$

3.11

$$\nabla \cdot \mathbf{G} = \frac{1}{h_\xi h_\eta h_\phi} \left[ \frac{\partial}{\partial \xi} (h_\eta h_\phi G_\xi) + \frac{\partial}{\partial \eta} (h_\xi h_\phi G_\eta) + \frac{\partial}{\partial \phi} (h_\xi h_\eta G_\phi) \right]$$

3.12

$$\nabla^2 F = \frac{1}{a^2 (\xi^2 - \eta^2)} \left[ \left( \frac{\partial}{\partial \xi} (\xi^2 + 1) \frac{\partial}{\partial \xi} \right) + \left( \frac{\partial}{\partial \eta} (1 - \eta^2) \frac{\partial}{\partial \eta} \right) + \left( \frac{\partial^2}{\partial \phi^2} \right) \right] F$$

3.13

3.3 Solving for the Acoustic Field Scattered from a Rigid Prolate Spheroid

The scattering problem begins with a statement that a monochromatic plane wave in a lossless, unbounded medium is $p(\mathbf{r}, t) = P_0 e^{i(k \cdot \mathbf{r} - \omega t)} = P_0 e^{-i\omega t}$. It is incident upon the prolate spheroid from an arbitrary angle defined by $(\cos^{-1} \eta_{inc}, \phi_{inc})$. The medium has a bulk sound speed $c$ and a bulk density $\rho$. The acoustic wave has a wavenumber vector $\mathbf{k}$ and a frequency $\omega$ which are related to the sound speed by $|\mathbf{k}| = \frac{\omega}{c}$. Substitution of this pressure field into the wave equation, $\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$, results in a time-
independent version of the wave equation better known as the Helmholtz equation in Eq. 3.14 for which $k = \sqrt{k \cdot k}$.

$$\nabla^2 P + k^2 P = 0$$  \hspace{1cm} 3.14

The prolate spheroidal coordinate system is one of eleven in which the Helmholtz equation can be solved via separation of variables\(^{33}\). This technique assumes a solution can be written in terms of the eigenfunction $P_{mn} = R_{mn}^{(j)}(h, \xi) S_{mn}(h, \eta) \left[ \cos m\phi \sin m\phi \right]$ for which the substitution of $h = \frac{k\alpha}{2}$ has been made. The function $S_{mn}(h, \eta)$ is known as the prolate spheroidal angle function of the first kind of order $m$ and degree $n$. The function $R_{mn}^{(j)}(h, \xi)$ is known as the prolate spheroidal radial function of the $j$th kind of order $m$ and degree $n$. The radial functions may be tailored to represent standing or traveling waves. For the problem under consideration, traveling waves are more appropriate and the radial functions of the third kind, Eq. 3.15, and the fourth kind, Eq. 3.16, are employed. These wavefunctions are derived as solutions to the ordinary differential equations that are generated when solving the Helmhotz equation via separation of variables with the given ansatz. When an $e^{-i\omega t}$ time dependence is employed, then $R_{mn}^{(3)}$ represents a diverging wave traveling away from the origin while $R_{mn}^{(4)}$ represents an inward traveling wave that converges at the origin. Since a scattered field represents a traveling wave that is diverging from the scattering body, only the $R_{mn}^{(3)}$ will be employed in the solution to the scattered field.

$$R_{mn}^{(3)}(h, \xi) = R_{mn}^{(1)}(h, \xi) + iR_{mn}^{(2)}(h, \xi)$$  \hspace{1cm} 3.15
The incident plane wave can be expanded in terms of these eigenfunctions as given by Eq. 3.17.

\[ P_{\text{inc}}(\xi, \eta, \phi) = 2P_0 \sum_{m=0}^{\infty} \sum_{n=-m}^{m} i^n \frac{\mathcal{E}_m}{N_{mn}} S_{mn}(h, \eta_{\text{inc}}) S_{mn}(h, \eta) R_{mn}^{(1)}(h, \xi) \cos m(\phi - \phi_{\text{inc}}) \]  

A Neumann boundary condition is imposed at the surface of the rigid prolate spheroid, \( \xi = \xi_0 \), requiring that \( \left\{ \hat{n} \cdot \nabla P_{\text{sc}} + \hat{n} \cdot \nabla P_{\text{inc}} \right\}_{\xi=\xi_0} = 0 \). Substitution of Eq. 3.17 into this boundary condition allows the scattered field to be expressed analytically in Eq. 3.18.

\[ P_{\text{sc}}(\xi, \eta, \phi) = -2P_0 \sum_{m=0}^{\infty} \sum_{n=-m}^{m} i^n \frac{\mathcal{E}_m}{N_{mn}} \frac{R_{mn}^{(1)}(h, \xi_0)}{R_{mn}^{(3)}(h, \xi_0)} S_{mn}(h, \eta_{\text{inc}}) \cdot S_{mn}(h, \eta) R_{mn}^{(3)}(h, \xi) \cos m(\phi - \phi_{\text{inc}}) \]  

In the analytical solution to the scattered field, the eigenfunction in the rotational coordinate, \( \begin{bmatrix} \cos m\phi \\ \sin m\phi \end{bmatrix} \), has been reduced to \( \cos m\phi \) with \( m \) being an integer greater than or equal to zero in order to satisfy a required periodicity over \( 2\pi \). Two new variables have been introduced by Eq. 3.18. The Neumann factor, \( \mathcal{E}_m = \begin{cases} 1 & \text{for } m = 0 \\ 2 & \text{for } m \geq 1 \end{cases} \), and a normalization factor \( N_{mn} \) which has been selected in Eq. 3.19 which requires the prolate spheroidal angular functions to have the same normalization as the associated Legendre function \( P_n^m(\eta) \). Computational details regarding these calculations are contained in Appendix A.
The farfield scattering pattern in the z-x plane generated by a plane wave incident from the spherical coordinates of \((\theta_{\text{inc}} = 60^\circ, \phi_{\text{inc}} = 0^\circ)\) have been computed for \(h = 0.5\) to \(h = 40\) from a scattering body having a 10:1 fineness ratio. The resulting scattering patterns are shown in Figure 3-3. For comparison, analogous scattering patterns for the pressure scattered from a rigid sphere ensonified with a plane wave are presented in Figure 3-4. At the lowest value of \(h\), the scattering is predominantly in the backscattered direction. The low value of \(h\) indicates that the wavelength associated with the incident plane wave is much larger than the characteristic dimension of the the prolate spheroid which is traditionally taken as \(a/2\). When \(ka\ll1\) the spheroidal scattering is analagous to Rayleigh scattering from a sphere having radius \(r_0 = a/2\). If the scatterer was a sphere or the plane wave was impinging the spheroid from a broadside direction \((\theta_{\text{inc}} = 90^\circ)\) then the direction of maximum backscatter coincides with the wavenumber vector of the incident plane wave rotated 180°. The results for \(h \ll 1\) in Figure 3-3 depict a peak in the forward-scattered hemisphere that is not in the direction of the incident wavefield. This occurs because the local surface curvature at the point where the incident wavenumber vector intersects the surface of the spheroid is comparable to the acoustic wavelength.

\[
N_{mn} = \frac{1}{2\pi} \int_{\mathbb{R}} \left| S_{mn}(h, \eta) \right|^2 d\eta = \frac{1}{(2n+1)(n-m)!} \int_{\mathbb{R}} \left| P_n^m(\eta) \right|^2 d\eta = \frac{2(m+n)!}{(2n+1)(n-m)!}
\] 3.19
Figure 3-3: Farfield scattering patterns for rigid prolate spheroid ensonified by a monochromatic plane wave ($\theta_{inc} = 60^\circ, \varphi_{inc} = 0^\circ$). Patterns present the magnitude of scattered pressure over $\theta = \cos^{-1} \eta$ normalized to the respective maximum scattered pressures. The arrow represents the wavenumber vector $\vec{k}$ of the incident plane wave.
As the frequency of the incident plane wave increase to $h \in \{2, 4\}$, the dominant backscatter lobe narrows and rotates from a backscatter direction towards the direction of specular reflection. At $h = 4$, the specularly scattered lobe falls short of the true direction in which specular reflection would reside. This is because the local curvature of the spheroid is still significant compared to the incident wavelength. Values of $h$ of order unity are analogous to resonance scattering (i.e. Mie scattering) involving spheres in which $kr_0 \sim 1 \rightarrow 10$. 
Figure 3-4: Farfield scattering patterns for rigid sphere ensonified by a monochromatic plane wave \((\theta_{inc} = 0^\circ, \phi_{inc} = 0^\circ)\). Patterns present the magnitude of scattered pressure over \(\theta\) normalized to the respective maximum scattered pressure. The arrow represents the wavenumber vector \(\vec{k}\) of the incident plane wave.
As the frequency of the incident plane wave increases such that the wavelength is much smaller than the characteristic dimension of the spheroid \((h > 10)\), the scattering problem approaches the asymptotic condition of geometrical acoustics. The specular reflection asymptotically approaches the true specular direction because the local curvature at the surface of the spheroid becomes larger than the wavelength of the incident plane wave. Likewise, the pressure scattered in the forward direction rapidly acquires a more significant magnitude. This forward scattering lobe grows in magnitude and narrows in angular extent as \(h \gg 10\). The magnitude of the forward scattered signal must asymptotically approach that of the incident signal. This happens because according to geometrical acoustics and the rigid boundary condition, a shadow is generated in the near field immediately behind the scattering body. According to superposition in linear acoustics, the total pressure behind the scattering body is the sum of the incident and scattered pressures. In order for the shadow to exist in the high-frequency limit, the scattering body must scatter an acoustic wave in the forward direction that is equal in magnitude but opposite in sign from the incident wave. Destructive interference between the incident and scattered signal provide for the acoustic shadow immediately behind the scatterer. In the far-field forward scatter direction, the scattered wave and incident waves constructively interfere to form the Poisson cone as discussed in Section 3.4.

The far-field scattering patterns of the spheroid shown in Figure 3-3 depict the dependence of the scattered field in the \(\eta = \cos^{-1} \theta\) directions since \(\varphi \in (0, \pi)\) defines
two planes which intersect on the z-axis. Alternatively, the scattered pressure can be
evaluated over both $\xi$ and $\eta = \cos^{-1} \theta$ for the same $\phi \in (0, \pi)$ and presented such that
the value of scattered pressure corresponds to a color as done in Figure 3-5. The planes
specified by $\phi \in (0, \pi)$ correspond to the x-z plane which contains the wavenumber
vector of the incident plane wave $\left( \theta_{inc} = 60^\circ, \phi_{inc} = 0^\circ \right)$. These figures show that the
overall scattering levels are very small when $h \ll 1$ and become appreciable only when
$h \gg 1$. At very high values of $h \in \{20, 40\}$, the magnitude of the scattered pressure can
be seen to approach that of the incident wave.
Figure 3-5: Pressure field scattered by a 10:1 fineness ratio rigid prolate spheroid from a monochromatic plane wave \((\theta_{inc} = 60^\circ, \phi_{inc} = 0^\circ)\). The spheroid can be seen at the center of each field plot where the field in the immediate vicinity of the spheroid has not been computed. Axis dimensions are number of object lengths \((L)\).
3.4 Total Pressure Field Caused by the Scatter of an Incident Plane Wave by a Rigid Prolate Spheroid

The principle of superposition in linear acoustics states that the total acoustic field at any point and time is the sum of the incident and scattered fields evaluated at the same point and time. The scattered pressure fields shown in Figure 3-5 can be combined with that of the incident plane wave to depict the total pressure field presented in Figure 3-6. The total pressure field illustrates the interference between the incident and scattered fields which is only apparent when the scattered field has an appreciable magnitude, \( i.e., \) when \( h \gg 1 \). In the case of high frequency scattering from a rigid sphere having radius \( r_0 \) shown in Figure 3-7, the total pressure field contains several classic features that can also be observed in the total field of the spheroid. The bright spot in the center of the resulting shadow known as the Poisson spot is a cross section of the Poisson cone which manifests at ranges on the order of \( (kr_0)^{1/2} r_0 \) with an angular extent on the order of \( 1/kr_0 \) radians\(^{34}\). The Poisson cone merges with other features of the diffracted field in the Fraunhaufer region to construct interference fringes whose “intensity” is represented by the classic Airy diffraction. The farfield forward diffraction peak is the mainlobe of the Airy diffraction pattern which contains a significant portion of the total diffracted intensity and has an angular extent of approximately \( 0.61 \lambda/r_0 \) radians\(^{34}\).
Figure 3-6: Total pressure field established by the scattering of a monochromatic plane wave ($\theta_{inc} = 60^\circ, \varphi_{inc} = 0^\circ$) by a 10:1 fineness ratio rigid prolate spheroid. The spheroid can be seen at the center of each field plot where the field in the immediate vicinity of the spheroid has not been computed.
Figure 3-7: Total pressure field established by the scattering of a monochromatic plane wave by a rigid sphere. Arrow denotes the incident plane wave. Axis dimensions are in number of radii.
While the scattered pressure field significantly interferes with the incident field at high values of $h$, the constructive and destructive interference generating the farfield diffraction pattern does not cause the overall pressure levels to deviate significantly from the incident pressure level. It can therefore be said that the presence of the scattering body does not appreciably perturb the total pressure field.

3.5 Total Acoustic Intensity Field Caused by the Scatter of an Incident Plane Wave by a Rigid Prolate Spheroid

An expression for pressure waves scattered from a rigid prolate spheroid was presented in Eq. 3.18 and later combined with the incident pressure field given by 3.17 to construct the total pressure field. The scattered particle velocity

$$ u_{\text{scat}} = u_{\text{scat}}^{(\xi)} \hat{\xi} + u_{\text{scat}}^{(\eta)} \hat{\eta} + u_{\text{scat}}^{(\phi)} \hat{\phi} $$

can be derived with assistance from the linearized Euler equation in Eq. 3.20. This relationship is fundamental in linear acoustics and relates the gradient of the pressure field (force) to the product of the ambient fluid density (mass) and the time derivative of particle velocity (acceleration). When an acoustic wave with harmonic time dependence of $e^{-i\omega t}$ is employed, the relationship can be rewritten to solve for $\vec{u}(\vec{r},t)$ in terms of a gradient operator acting on the scattered pressure in Eq. 3.21.

$$ \nabla p(\vec{r},t) = \rho \frac{\partial}{\partial t} \vec{u}(\vec{r},t) \tag{3.20} $$

$$ \vec{u}_{\text{scat}}(\vec{r},t) = \frac{i}{\omega \rho} \nabla p_{\text{scat}}(\vec{r},t) \tag{3.21} $$
Application of the gradient operator from Eq. 3.11 to the analytical solution of the scattered pressure field in Eq. 3.18, yields an expression for the scattered particle velocity vector in Eq. 3.22. The prolate spheroidal unit vectors \((\hat{\xi}, \hat{\eta}, \hat{\phi})\) are depicted on the z-x plane from Figure 3-8 in relation to the scattering body.

\[
\vec{u}_{\text{sc}} = \frac{-i}{\omega p} \left( \frac{2}{a} \sqrt{\frac{\xi^2 - 1}{\xi^2 - \eta^2}} \frac{\partial P_{\text{sc}}}{\partial \xi} \hat{\xi} + \frac{2}{a} \sqrt{\frac{1 - \eta^2}{\xi^2 - \eta^2}} \frac{\partial P_{\text{sc}}}{\partial \eta} \hat{\eta} + \frac{2}{a \sqrt{(\xi^2 - 1)(1 - \eta^2)}} \frac{\partial P_{\text{sc}}}{\partial \phi} \hat{\phi} \right) \tag{3.22}
\]
The scattered particle velocity can be combined with the particle velocity due to the incident plane wave by superposition to yield the total particle velocity, $\mathbf{u}_{\text{tot}}(\mathbf{r},t)$. Supressing the $e^{-i\omega t}$ time dependence, expressions for the total pressure $p_{\text{tot}}(\xi,\eta,\phi)$ and the total particle velocity $\mathbf{u}_{\text{tot}}(\xi,\eta,\phi)$ are used to get the complex acoustic intensity from Eq. 2.28. This enables the phase difference between the total pressure and vector

Figure 3-8: $\hat{\xi}$ and $\hat{\eta}$ unit vectors on the z-x plane. An enlarged image of the spheroid has been added for reference. The $\hat{\phi}$ unit vector points out of the page and is orthogonal to the z-x plane.
components of particle velocity to be calculated from the argument of the complex intensity components. The phase from the complex intensities in the $\hat{\xi}$, $\hat{\eta}$, and $\hat{\phi}$ directions are presented in Figures 3-9 through 3-11, respectively. The power factor angle for the complex intensity in the $\hat{\phi}$ direction is presented even though the magnitude of $I_c$ is zero since $\bar{k} \cdot \hat{\phi} = 0$. 
Figure 3-9: Phase of the $\hat{\xi}$ complex intensity field when a monochromatic plane wave originating from $\cos^{-1} \eta_{inc} = 60^\circ$ is perturbed by the presence of a rigid prolate spheroid having a fineness ratio of 10:1. Axis dimensions are number of object lengths ($L$).
Figure 3-10: Phase of the $\tilde{\eta}$ complex intensity field when a monochromatic plane wave originating from $\cos^{-1} \eta_{inc} = 60^\circ$ is perturbed by the presence of a rigid prolate spheroid having a fineness ratio of 10:1. Axis dimensions are number of object lengths ($L$).
Figure 3-11: Phase of the $\hat{\phi}$ complex intensity field when a monochromatic plane wave originating from $\cos^{-1} \eta_{inc} = 60^\circ$ is perturbed by the presence of a rigid prolate spheroid having a fineness ratio of 10:1.
The spatial distribution of power factor angle indicates the development of significant and localized disturbances in the phase of the complex intensity that are strongly dependent upon $h$. The phase of the $\hat{\xi}$ complex intensity field shows the development of a phase difference between pressure and the $\hat{\xi}$ component of particle velocity that exceeds 5° in the specularly scattered direction and at distances that exceed $20L$. Conversely the phase of the $\hat{\eta}$ complex intensity field exhibits an anomaly that is localized to the forward-scattered region and only slightly in the region corresponding to specular reflection. The cause for the phase difference and its manifestation in the $\hat{\xi}$ and $\hat{\eta}$ intensity components can be explained by examining the real and imaginary components of the complex intensity which correspond to the active and reactive intensity.

The active intensity is the real part of the complex intensity. The active intensity for the $\hat{\xi}$, $\hat{\eta}$, and $\hat{\phi}$ directions are presented in Figure 3-12. The fields presented in Figure 3-12 represent vector quantities and therefore contain a spatial response corresponding to the dot product of the local acoustic wavenumber vector and the unit vectors of the coordinate system. This dot product results in a cosine shading (a dipole spatial response). The radial component$^1$ of active intensity, $I_\xi$, quickly drops to zero at those locations where the radial unit vector is orthogonal to the incident wavenumber vector (i.e. $\hat{k} \cdot \hat{\xi} = 0$). Since $\hat{\xi}$ and $\hat{\eta}$ are orthogonal to one another, the tangential

$^1$ Note that the $\hat{\xi}$ component is “radial” only for $\xi >> 1$. The term is used here to offer a more familiar term borrowed from scattering in spherical coordinates.
component of active intensity is at a maximum when the radial component of active intensity is at a minimum. The \( \hat{\phi} \) component of active intensity is at a minimum for all the field points evaluated on the \( z-x \) plane because the \( \hat{\phi} \) unit vector is orthogonal to the \( z-x \) plane which contains \( \vec{k}_{\text{inc}} \).

In Chapter 2, it was shown that the active intensity was proportional to the square of the pressure field. Therefore the features that were observed in Figure 3-6 are present in the active intensity and displayed in Figure 3-13. The \( \hat{\eta} \) component of active intensity does not exhibit those anomalies. The only difference between Figure 3-6 and Figure 3-13 is the fact that the \( \hat{\xi} \) component of active intensity becomes small as the unit vector becomes orthogonal to the incident wavenumber vector while the pressure field is a scalar quantity and contains the information contained in both the \( \hat{\xi} \) and \( \hat{\eta} \) active intensity fields.
Figure 3-12: Total active intensity fields in the $\hat{\xi}$, $\hat{\eta}$, and $\hat{\phi}$ directions resulting from the scattering of a monochromatic plane wave originating from $\cos^{-1} \eta_{inc} = 60^\circ$ by a rigid prolate spheroid having a fineness ratio of 10:1. Axis dimensions are number of object lengths ($L$). Patterns depict the cosine response of the vector sensors and represent the gross features of the field for $h=1$ to 40 due to the coarse color scale. Finer details of the total fields can be observed in Figure 3-13.
Figure 3-13: Total active intensity fields in the $\hat{\xi}$ direction resulting from the scattering of a monochromatic plane wave originating from $\cos^{-1} \eta_{inc} = 60^\circ$ by a rigid prolate spheroid having a fineness ratio of 10:1. Fields corresponding to four values of $h$ are presented. Axis dimensions are number of object lengths ($L$). The color scale emphasizes the details of the field in the forward scatter direction. The field magnitude at other locations in the diagram do not fall on the narrow color scale, but can be seen with the coarse color scale in Figure 3-12.
The reactive intensity is the imaginary part of the complex intensity and corresponds to the gradient of the mean squared pressure field. The reactive intensity for the $\hat{\xi}$ and $\hat{\eta}$ directions are presented in Figure 3-14 and Figure 3-15 respectively. The results in the $\hat{\phi}$ direction are absent since $\vec{k} \cdot \hat{\phi} = 0$. The $\hat{\xi}$ reactive intensity in Figure 3-14 at values of $h < 1$ indicates the development of very small levels of reactive intensity in the backscattered direction for values. Only when $h >> 1$ does the $\hat{\xi}$ reactive intensity become a feature that is localized in the direction of specular reflection. The justification for this is that Eq. 2.23 defines the $\hat{\xi}$ component of reactive intensity as having significant magnitude when the radial portion of the gradient of the mean squared pressure is significant. In the steady-state scattering problem, the incident plane wave is always interfering with the specularly scattered wave in a fashion that causes the pressure gradient to take on significant magnitude. This development can also be observed in Figure 3-9 which shows that the power factor angle for the $\hat{\xi}$ direction is also localized to the same region at large values of $h$.

A similar explanation is used to explain the development of reactive intensity in the $\hat{\eta}$ direction presented in Figure 3-15. The reactive intensity in the specular direction at high reduced frequencies exists because of the interference between the incident and scattered waves. However reactive intensity has also developed in the forward direction at large values of $h$ which mimics the development of appreciable power factor angles in the forward scattered direction in Figure 3-10. This can be explained by referring to Figure 3-13 and observing that the gradient of the mean square pressure in the $\hat{\eta}$ direction can become significant due to the fringes of the interference pattern.
Figure 3-14: Total reactive intensity fields in the $\hat{\xi}$ direction for the field resulting from the presence of a rigid prolate spheroid having a fineness ratio of 10:1 in a monochromatic plane wave ($\theta_{inc} = 60^\circ, \phi_{inc} = 0^\circ$). Fields corresponding to values of $h$ are presented. Axis dimensions are number of object lengths ($L$).
Figure 3-15: Total reactive intensity fields in the $\hat{n}$ direction for the field resulting from the presence of a rigid prolate spheroid having a fineness ratio of 10:1 in a monochromatic plane wave ($\theta_{inc} = 60^\circ$, $\varphi_{inc} = 0^\circ$). Fields corresponding to values of $h$ are presented. Axis dimensions are number of object lengths ($L$).
3.6 Summary

This chapter has analytically calculated the acoustic intensity field associated with the scattering of a steady-state harmonic plane wave by a rigid prolate spheroid in an unbounded, lossless medium. The calculations indicate that there are areas in the total acoustic field in which the presences of the scattering body perturbs the active and reactive intensity field as well as the power factor angle. The areas in which the perturbation is most apparent is the forward scattering and specular scatter directions. Perturbation in the backscatter direction would be a special case that is degenerative at broadside incident angles in which the specular scatter direction coincides with the backscatter direction.

The $\hat{\xi}$ and $\hat{\eta}$ reactive intensities were found to reach values greater than 20dB below the incident acoustic intensity in the specular direction. While this may not appear significant, one must consider that if the spheroid was not present, the reactive intensity would be identically zero or $-\infty$ dB below the incident acoustic intensity. Even though the reactive intensity is appreciable in the specular directions, the power factor angle is $<1^\circ$ for the $\hat{\eta}$ complex intensity and near $3^\circ$ for the $\hat{\xi}$ complex intensity at 10 object lengths away in the specular direction $h=40$. In the forward scatter direction, the $\hat{\eta}$ reactive intensity was calculated to be 25-30dB below the incident acoustic intensity while having a corresponding power factor angle of $3^\circ$-$5^\circ$ at distances of 10 object length and reduced frequencies of $h=40$. 
These theoretical results have established that the scattering problem generates an acoustic field that contains information that can only be extracted through the combined measurement of acoustic pressure and particle velocity. Furthermore, the ability to reliably measure power factor angles of $3^\circ-5^\circ$ and differentiate then from a scenario in which the power factor angle is $0^\circ$ could allow an investigator to infer that a scattering body is present when the measurement is made at high reduced frequencies in the forward scatter direction.
Chapter 4

Frequency Domain Estimates of Acoustic Intensity

4.1 Introduction

Expressions for acoustic intensity derived in Chapter 2 were analytic in nature and therefore cannot be used directly in experimental investigations employing discrete observations. Experiments typically employ sensors to convert physical quantities into electrical signals, electronics to condition the electrical signals, and analog-to-digital converters to discretely sample the signal. Therefore the analytic Fourier transforms, continuous integrals, and the asymptotic integration limits are not directly applicable to discrete experimental data having finite observation lengths. This chapter constructs expressions for the estimates of acoustic intensity constructed from finite amounts of digitized sensor outputs. There can be many ways to construct an estimate of a quantity due to the variety of sensors that can be employed and the subsequent digital time series analysis that follows. A variety of estimators can be employed to generate an estimate of a particular quantity. The performance of one estimator is not necessarily indicative of the performance of another. These techniques will be used to process the experimental data discussed in Section 4.4 and Chapter 5.
4.2 Frequency Domain Estimators of Acoustic Intensity

In section 2.3.1, it was shown that the frequency domain version of the complex intensity, $I_c(f)$, can be constructed from the cross-spectrum of acoustic pressure and particle velocity. This complex acoustic intensity may then be interpreted in terms of a co-spectrum which represents the active acoustic intensity and the quad-spectrum which represents reactive intensity. Alternatively, the same complex intensity may be interpreted in terms of a magnitude and phase angle.

The cross-spectral density estimate of acoustic intensity can be directly constructed with the discrete Fourier transforms of sampled pressure and velocity time series. Alternatively the times series may be of two quantities that can be linearly related to pressure and velocity. Having employed a time dependence of $e^{-i\omega t}$, the corresponding discrete forward Fourier transforms for a data record $k$ having finite length is given by Eq. 4.1 for which $f = \frac{m}{N\Delta t}$ and $m = 0...N$. The vector notation for particle velocity will be suppressed in order to keep the notation from becoming cluttered. The dependence of all physical quantities on $\bar{r}$ is also understood and not explicitly stated.

$$p_k(f,T) = \Delta t \sum_{n=0}^{N-1} P_k[n\Delta t] e^{i2\pi mn/N}$$

$$u_k(f,T) = \Delta t \sum_{n=0}^{N-1} U_k[n\Delta t] e^{i2\pi mn/N}$$

4.1
The true cross spectrum is constructed by taking an infinitely large observation time, $T$, as given by Eq. 4.2 which is only achievable in the asymptotic limit of $T \to \infty$ and $k \to \infty$.

$$S_{pu}(f, T) = \lim_{T \to \infty} \frac{1}{T} E \left[ p_k^*(f, T) u_k(f, T) \right]$$  \hspace{1cm} 4.2

Evaluating this expression is not practical because of its asymptotic nature. By bounding the value of $T$ and $k$, the cross-spectrum is estimated by Eq. 4.3 which employs an ensemble average over $n_d$ independent finite cross-spectra in place of the expectation operator.

$$\hat{S}_{pu}(f) = \frac{1}{n_d T} \sum_{k=1}^{n_d} p_k^*(f, T) u_k(f, T)$$  \hspace{1cm} 4.3

From this, estimates corresponding to the various interpretations of the complex acoustic intensity can be constructed by the estimators given in Eq. 4.4, Eq. 4.5, Eq. 4.6, and Eq. 4.7.

$$\hat{I}(f) = \text{Re} \left[ \hat{I}_c(f) \right]$$  \hspace{1cm} 4.4

$$\hat{Q}(f) = \text{Im} \left[ \hat{I}_c(f) \right]$$  \hspace{1cm} 4.5

$$|\hat{I}| = \sqrt{\hat{I}^2 + \hat{Q}^2}$$  \hspace{1cm} 4.6

$$\hat{\phi}_{pu} = \tan^{-1} \frac{\hat{Q}}{\hat{I}}$$  \hspace{1cm} 4.7
In order to examine the performance of the frequency domain estimators, the linear system in Figure 4-1 is employed to model the measurement. This model employs two sensors which respond to the acoustic pressure and particle velocity. The components of the pressure and particle velocity due to the signal $S_p$ are related by the linearized Euler equation in Eq. 2.16. Several sources of noise exist in the measurement model and corrupt the signal. The ambient acoustic field can be modeled as a superposition of interfering plane waves arriving from all directions $(\theta, \phi)$. Under this model, the pressure and particle velocity due to each of the interfering plane waves is also related by the linearized Euler equation as shown in Figure 4-1 in which a transfer function $H_z$ represents the specific acoustic impedance $z_{acs} = p/u$. The total noise contribution is computed by integrating the interfering pressure and particle velocity fields over the solid angle defined by a sphere centered on the two co-located sensors. The two integrations yield the acoustic noise and add to the pressure and particle velocity signals at the face of the two sensors. This ambient acoustic noise is assumed to be a zero-mean Gaussian process whose phase angle of the noise is uniformly distributed between 0 and $2\pi$. While it is true that the pressure and particle velocity, $N_p(\theta, \phi)$ and $N_u(\theta, \phi)$, due to any individual interfering plane wave is correlated, the integrated noise fields $N_p$ and $N_u$ are not. Hawkes$^{18}$ and Jacobsen$^{20}$ have found the spatial auto- and cross-correlation functions for acoustic pressure and the three components of particle velocity to be given by Eq. 4.8 through Eq. 4.11 when the sensitivity axis of the velocity sensor, $\hat{s}_u$, and displacement vector $\vec{d}$ subtend the angle $\alpha$. The cross-correlations
between a pair of velocity sensors involves two angles, \( \alpha_1 \) and \( \alpha_2 \), which denote the angles between the displacement vector \( \vec{d} \) and the sensitivity axis for each sensor, \( \hat{s}_{u_1} \) and \( \hat{s}_{u_2} \).

\[
R_{pp}(d) = j_0(|kd|) \tag{4.8}
\]

\[
R_{pu}(d) = R_{pu_1}(d) = R_{pu_2}(d) = \sqrt{3} j_1(kd) \cos \alpha \tag{4.9}
\]

\[
R_{u_iu_j}(d) = R_{u_iu_j}(d) = R_{u_iu_j}(d) = -3 j_2(kd) \cos \alpha_1 \cos \alpha_2 \tag{4.10}
\]

\[
R_{u_iu_j}(d) = R_{u_iu_j}(d) = R_{u_iu_j}(d) = 3 \left( \frac{j_1(kd)}{kd} - j_2(kd) \cos^2 \alpha \right) \tag{4.11}
\]
Figure 4-1: Linear model of complex intensity measurement.
These correlation functions are shown in Figure 4-2 for five special cases and have been normalized to conform to those presented by Jacobsen\textsuperscript{20}. The first case involves the cross-correlation between two omnidirectional pressure sensors in Eq. 4.8 which is only a function of the acoustic wavenumber and the magnitude of the sensor separation. When the sensors are co-located and placed in a spherically isotropic noise field, their outputs are perfectly correlated. This makes physical sense because the sensors are responding to the same stimulus. When the two pressure sensors are separated by a distance of $\lambda/2$, the output of the two sensors are perfectly uncorrelated with an infinitely long observation time. The second case involves the cross-correlation between an omnidirectional pressure sensor and a velocity sensor with a sensitivity axis oriented in the $\hat{s}_u$ direction. When the displacement vector between these two sensors, $\vec{d}$, is oriented in the same direction as $\hat{s}_u$ (i.e $\vec{d} \cdot \hat{s}_u = 0$), then $\alpha = 0$ and the cross-correlation function in Eq. 4.9 is again only a function of the acoustic wavenumber and the magnitude of the displacement vector. This case is different from the first case since the cross-correlation of the two sensors is now zero when they are co-located in a spherically isotropic noise field and have a maximum correlation at a spacing of $\lambda/2$. This is significant because the intensity sensors under consideration are composed of a co-located pressure sensor and velocity sensor indicating that their outputs are perfectly uncorrelated in this type of noise field. A third case involves two velocity sensors with their sensitivities oriented in the $\hat{s}_{u_1}$ and $\hat{s}_{u_2}$ directions allowing the angles $\alpha_1$ and $\alpha_2$ to be defined for Eq. 4.10. When these sensors are oriented orthogonally such that $\hat{s}_{u_1} \cdot \hat{s}_{u_2} = 0$ and displaced in such a way that $\vec{d} \cdot \hat{s}_{u_1} = 0$ and $\vec{d} \cdot \hat{s}_{u_2} = 0$, then the cross-
correlation is identically zero for all magnitudes of acoustic wavenumber and
displacement vector. The fourth and fifth cases arise with Eq. 4.11 for which the two
velocity sensors are pointed in the same direction implying that \( \mathbf{s}_{u1} \cdot \mathbf{s}_{u2} = 1 \) and allowing
for \( \alpha_1 = \alpha_2 = 0 \). When the sensors are co-located, the outputs of the two sensors are
perfectly correlated since they are responding to the same stimuli as did the two pressure
sensors. The correlations then decrease at differing rates for the two cases in which the
displacement vector is perpendicular to the sensitivity axis (case #4: \( \alpha = \frac{\pi}{2} \)) and when
the displacement vector is parallel to the sensitivity axis (case #5: \( \alpha = 0 \)).
Figure 4-2: Auto-correlations and cross-correlations of outputs from a pressure sensor and three orthogonal velocity sensors in spherically isotropic noise.
There are two other noise sources on each output line that are identified in the model. The sensor noise is noise generated by the sensor itself when there is no physical stimulation (random ambient noise or deterministic signal) at the input to the sensor. This type of noise is governed by the thermodynamics and statistical mechanics associated with the particular type of transduction in the sensor. Detailed analyses of these noise sources are beyond the scope of this investigation. The reader is referred to Gabrielson$^9$ for details. The sensor noise shall be modeled as an additive zero-mean random process whose amplitude follows a Gaussian distribution and phase that is uniformly distributed between 0 and $2\pi$. The signal conditioning electronics which follow the sensor include amplifiers and filters. These components all contribute electrical noise to the measured output and are collectively modeled as an additive zero-mean random process whose amplitude follows a Gaussian distribution and phase that is again uniformly distributed between 0 and $2\pi$. It is assumed that the signal, $S_p$, is uncorrelated with any noise term. Since any linear combination of Gaussian processes results in a Gaussian process, the various noise terms are combined to simplify the measurement model as shown in Figure 4-3. Two transfer functions, $H_p$ and $H_u$, have been included to represent the analog processing and signal conditioning of the pressure and velocity signals respectively.
Figure 4-3: Simplified linear measurement model including the construction of the acoustic intensity estimate via the cross-spectrum
The ordinary coherence function between pressure and velocity for this measurement model is given by Eq. 4.12 and re-written in terms of the two signal-to-noise ratios, \( snr_{\text{pres}} = \frac{S_{v,p} S_{v,p}}{S_{n,p}} \) and \( snr_{\text{vel}} = \frac{S_{v,u} S_{v,u}}{S_{n,u}} \), in Eq. 4.13. This relationship between the coherence function and the \( snr \) of the two outputs is shown in Figure 4-4 as a contour plot for which \( SNR = 10 \cdot \log_{10}(snr) \). The sensitivity of the coherence function to variations in the output \( SNR \) of particle velocity or pressure can be observed with the conclusion that the coherence function is principally dominated by the lower of the two \( SNRs \).

\[
\kappa_{pu}^2 = \frac{S_{pu} S_{pu}^*}{S_{pp} S_{uu}} = \left( \frac{S_{v,p} v_{p}}{S_{pp}} \right) \left( \frac{S_{v,v}}{S_{uu}} \right)
\]

\[
\kappa_{pu}^2 = \frac{1}{1 + \frac{S_{n,p}}{S_{v,p} v_{p}}} \left( 1 + \frac{S_{n,u}}{S_{v,u} v_{u}} \right) \left( 1 + \frac{1}{snr_{\text{pres}}} \right) \left( 1 + \frac{1}{snr_{\text{vel}}} \right)
\]
One figure of merit which can be used to quantify the error of an estimate is the normalized random error. For an estimate of a quantity $x \neq 0$, the normalized random error is given by $\varepsilon_x = \frac{\sigma[\hat{x}]}{x}$ where $\sigma^2[\hat{x}]$ is the variance of the estimate. When the true value of $x$ is not known, the mean value, $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} \hat{x}_i$, of the ensemble is used to estimate the standard deviation $\sigma[\hat{x}]$. The normalized random error for a variety of auto- and cross-spectral parameter estimators have been investigated elsewhere$^{5,6,21}$, and will not be derived here. The autospectrum $\hat{G}_{pp}$ or $\hat{G}_{uu}$ of a Gaussian random process results

Figure 4-4: Relationship between $\chi^2_{pu}$ and SNR of pressure and velocity.
in a random process whose amplitude is distributed according to a Chi-square distribution with two degrees of freedom. The significance is that the variance of this distribution is twice its expected value. When the square root of the variance of a random process (i.e. standard deviation) is of comparable magnitude to the expected value of that process, it is not plausible to accurately estimate the mean value of that process without a large sample population. If an ensemble average is used (i.e. averaged periodogram) as was done in Eq. 4.3, this variance is reduced by the number of independent samples, \( n_d \), used in the ensemble average. Thus the normalized random error of the autospectral estimators is given by Eq. 4.14 and plotted against the number of independent samples in Figure 4-5 to emphasize the significance of this quantity. If a single record is used to calculate the magnitude of the autospectrum, the standard deviation of the random error associated with that estimate is the value of the estimate itself since the normalized random error is unity.

\[
\varepsilon \left[ \hat{G}_{pp} \right] = \varepsilon \left[ \hat{G}_{nn} \right] = \frac{1}{\sqrt{n_d}} \quad 4.14
\]
If a deterministic process, such as a steady-state scattered signal, is added to the random process then the ability to estimate the magnitude of that deterministic process or even determine that an underlying deterministic process is present can be affected by this type of random error. The autospectrum of a sinusoidal process is deterministic and is equal to the mean square value of that sinusoid. Therefore the presence of a sinusoidal signal in a random noise field will cause the expected value of the chi-square distribution describing the random process to become biased. The amount

Figure 4-5: Normalized random error for the autospectral estimators $\hat{G}_{pp}$ or $\hat{G}_{uu}$. 
of this bias is equal to the mean square value of the sinusoid. Therefore if the normalized random error is large, then it may be difficult to infer the presence of the deterministic process.

The normalized error for the magnitude of the cross-spectrum is found to be similar to that of the autopectral estimates except for a dependency upon the magnitude squared coherence between the two spectral quantities as given by Eq. 4.15 and shown in Figure 4-6. As the coherence between the pressure and particle velocity signals decreases, the normalized error will increases if additional records are not employed in the ensemble average. The coherence between the pressure and particle velocity signals has previously been found to be dependent upon the SNR of those two channels. Therefore after substituting Eq. 4.13 into Eq. 4.15, the normalized error can be expressed in terms of the SNR of the pressure and velocity. This relationship is shown in Figure 4-7 for the pressure and velocity having the same SNR, but with different values of $n_d$. By keeping the value of $n_d$ constant at unity to represent no ensemble averaging, the sensitivity of the error to a difference in SNR between pressure and velocity signals can be seen in Figure 4-8. The importance of SNR in the pressure and particle velocity channels cannot be under-emphasized since the normalized error grows very quickly as SNR decreases. When SNR $<< 0$dB, the theoretical normalized error can exceed 10 indicating that the standard deviation of the random process is 10 times that of the mean value being estimated. A poorly designed acoustic intensity sensor in which the SNR of the pressure and particle velocity channels are significantly different can cause the random error to become unmanageable as well.
Figure 4-6: Normalized random error for the magnitude of the cross-spectral estimator $\hat{G}_{pu}$ as a function of the number of independent samples contributing to the ensemble average. The relationship is plotted for several values of $\kappa_{pu}$ which annotate their respective lines.
Figure 4-7: Normalized random error for the magnitude of the cross-spectral estimator $\hat{G}_{pu}$ as a function of the SNR of the pressure and velocity signals. It is assumed here that the two signals have identical values of SNR. The relationship is plotted for several values of $n_y$ which annotate their respective lines.
The phase of the cross-spectral estimate is also subject to random error but cannot be expressed as a normalized random error due to the fact that zero is an acceptable value of the true phase angle and would therefore drive the normalized error to infinity. Accordingly, the standard deviation of the phase angle estimate is given in Eq. 4.16 and plotted versus $n_d$ for several values of the coherence function in Figure 4-9. As was done with the magnitude of the cross-spectrum, the standard deviation of the phase angle

Figure 4-8  Normalized random error for the magnitude of the cross-spectral estimator $\hat{G}_{pu}$ as a function of the SNR of the pressure or the velocity signal. The relationship is plotted at four different scenarios in which the complementary signal has an SNR lower than that indicated on the abscissa. The lines are annotated with their respective amount of SNR differential (in dB) between the pressure and velocity. It is assumed that $n_d = 1$. 

![Normalized random error for the magnitude of the cross-spectral estimator](image)
is plotted as a function of the signal SNR in Figure 4-10 and Figure 4-11. In Chapter 3, it was determined that the power factor angle in the forward scatter direction could reach values as high as 5° or 0.087 radians at a distance of 10 object lengths from the spheroid. In order for the standard deviation of the random process to be an order of magnitude smaller than this deterministic value being estimated, then the ensemble should have close to 100 independent estimates and both the pressure and particle velocity channels should exhibit a SNR of 0dB. Dropping the ensemble population to 10 independent samples will require the SNR of both channels to increase up to approximately 10dB in order to keep the normalized random error at 0.1.

$$\sigma[\hat{\phi}_{pu}] = \frac{(1-\kappa_{pu}^2)^{1/2}}{|\kappa_{pu}|\sqrt{2n_d}} \quad 4.16$$
Figure 4-9: Standard deviation of the phase estimated from the cross-spectral estimator $\hat{G}_{pu}$ as a function of the number of independent samples contributing to the ensemble average. The relationship is plotted for several values of $\kappa_{pu}^2$ which annotate their respective lines.
Figure 4-10: Standard deviation of the phase estimate from the cross-spectral estimate $\hat{G}_{pu}$ as a function of the SNR of the pressure and velocity signals. It is assumed here that the two signals have identical values of SNR. The relationship is plotted for several values of $n_y$ which annotate their respective lines.
The coherence function is itself estimated from the data and is therefore subject to the same random error. The normalized random error of the coherence function estimate, $\hat{\kappa}_{pu}^2$, is a function of the true coherency $\kappa_{pu}$ and is given by Eq. 4.17 and plotted in Figure 4-12.

\[
\mathcal{E} \left[ \hat{\kappa}_{pu}^2 \right] = \frac{\sqrt{2} \left( 1 - \kappa_{pu}^2 \right)}{|\kappa_{pu}| \sqrt{n_d}}
\]  

4.17
The trends associated with the random errors discussed in this section can be used to determine what experimental parameters are required to insure a level of quality in a set of experimental measurements. In relation to the objective of this thesis, the relationship between the normalized error associated with estimation of the phase angle of the cross-spectrum and the SNR of the two channels is most relevant. It has been shown that the acoustic intensity estimates can be constructed by the cross-spectrum of

Figure 4-12: Normalized random error of the coherence function estimated from the cross-spectral estimates $\hat{G}_{pp}$, $\hat{G}_{uu}$, and $\hat{G}_{pu}$. The function is plotted against the number of independent samples contributing to the ensemble average. The relationship is plotted for several values of the true coherence function which annotate their respective lines.
the acoustic pressure and particle velocity sensor outputs. The objective of employing acoustic intensity estimates to infer the presence of a scattering body requires that coherence between the pressure and particle velocity channels be significant (SNR>0 in both channels) and to have $n_d >> 1$ in order to sustain an minimally acceptable level of normalized error.

4.3 Frequency Domain Estimators Employing Pressure Transducers and Accelerometers (p-a probes)

The measurement of underwater acoustic intensity can be made directly with a hydrophone and a sensor that measures the water particle velocity (i.e. velocity sensor such as a moving coil geophone). Such sensor packages are referred to as p-u probes and have successfully been developed\textsuperscript{10,11}. Sensor packages which employ two velocity sensors have also been developed by Bastyr\textsuperscript{3} to estimate acoustic intensity. The u-u probe employs a finite difference approximation to estimate the velocity gradient which is then related to acoustic pressure by the linearized equation of continuity. The estimate of acoustic pressure is then used in conjunction with the mean value of the signals from two velocity sensors to produce an estimate of acoustic intensity. Another well established technique to estimate acoustic intensity is by way of a pair of hydrophones. This $p-p$ probe employs a finite difference of the hydrophone output to estimate the pressure gradient between the two sensors. The pressure gradient is related to the particle velocity through the linearized Euler equation. This estimate of particle velocity is
combined with the mean value of the two hydrophone outputs to produce an estimate of acoustic intensity.

The techniques employing finite difference methods, \( p-p \) and \( u-u \) probes, are prone to frequency dependent errors. These errors arise from the fact that the sensor separation is fixed while the wavelength associated with the acoustic stimuli is variable. The mismatch between sensor separation and acoustic wavelength can cause two problems. The first type of error is due to the difference techniques and occurs when the acoustic wavelength is very large with respect to the separation between sensors. In this circumstance, the voltages of the two sensors are very nearly the same and their subtraction can cause the noise to become dominant. The second type of error is due to the \( s \) approximation of the gradient as the difference of the sensor output to the sensor separation. The observable is sinusoidal in nature but the sensor output is used for a linear model. This type of error is small if the sensor separation is very small with respect to the acoustic wavelength. The error increases as the sensor separation approaches one quarter of the acoustic wavelength. At this point, the linear model fails completely because the true spatial derivative of the observable has changed sign at least once between the two sensor locations and cannot be approximated by the first order model. In order to avoid these errors associated with \( p-p \) and \( u-u \) probes, it is advantageous to employ sensor packages having outputs whose voltage is directly proportional to acoustic pressure and particle velocity.

One objective of this dissertation is to investigate the acoustic intensity field scattered in the forward direction from a prolate spheroid. The theoretical scattering investigation performed in Chapter 3 indicates that the hypothesized phenomenon
manifests itself in the high-frequency scattering limit. This is achieved by either employing scattering bodies with very large dimensions or ensonifying the scattering body with high-frequencies. The latter of the two approaches was taken in order to employ scattering bodies that are readily handled during experimental investigations. A prolate spheroid scattering body (2m length and 10:1 aspect ration) was constructed from red oak and finished with a urethane varnish to approximate a rigid body. Constrained by the size of the chosen scattering body, the ensonifying frequencies need to be as high as 15kHz in order to approach the high-frequency scattering limit ($h > 30$). An acoustic sensor R&D company, Acoustech, was contracted to design and construct a suitable sensor package for these scattering studies. The contractor delivered a compact dual-axis intensity sensor of the $p$-$a$ probe variety as shown in Figure 4-13. The pressure transducer is a PZT-5A ring (APC model #421048). Its directivity pattern is nearly omnidirectional at frequencies up to 15kHz as shown in Figure 4-14. Its frequency averaged sensitivity is $-173$dB re $1V_{rms}/\mu Pa$ (with 27dB pre-amplifier) as shown in Figure 4-15. The $p$-$a$ sensor also employs ICP accelerometers (Ocean Sensor Technologies model AB1PN) which have a charge amplifier co-located within the sensor. The directivity and sensitivity of the accelerometers are given in Figure 4-16 through Figure 4-19. The accelerometers are oriented orthogonally to one another and are housed in a cylindrical structure located inside the PZT ring. The orthogonal dipole response provided by the accelerometers and the $360^\circ$ azimuthal response of the PZT ring allow for acoustic intensity to be estimated in the horizontal plane defined by the crossed dipoles.
Figure 4-13: Dual axis $p$-$a$ probe designed and constructed by Acoustech.

Figure 4-14: Directivity of the pressure sensor used in the $p$-$a$ probe.
Figure 4-15: Sensitivity of pressure sensor with preamplifier providing +27dB of gain.

Figure 4-16: Directivity of x-accelerometer in sensor package.
Figure 4-17: Sensitivity of x-accelerometers in sensor package (PCB Model 480E09 ICP signal conditioner S/N 23076)

Figure 4-18: Directivity of y-accelerometers in sensor package.
This inertial sensor package (p-a probe) produces voltages that are directly proportional to pressure and particle acceleration. The particle velocity is estimated by integrating the acceleration voltage over time. Performing this integration in the frequency domain is accomplished by multiplying the frequency domain values of acceleration by $\sqrt{-i\omega}$ since $e^{-i\omega t}$ has been adopted as the time dependence for all narrowband signals. The subsequent frequency domain estimates of pressure and velocity can then be used to construct estimates of acoustic intensity by the cross-spectral techniques presented in Section 4.2. A block diagram of the measurement model for the bi-axial intensity measurement is given in Figure 4-20.

Figure 4-19: Sensitivity of y-accelerometers in sensor package. (PCB Model 480E09 ICP signal conditioner S/N 23077)
Figure 4-20: Block diagram of measurement and estimation process with dual axis $p-a$ probes.
4.4 Analysis of Measurements Made with an Underwater p-a Probe in the Near Field of an Acoustic Source

In order to confirm satisfactory performance of the Acoustech p-a probe (S/N 2002-0005), a calibration test was conducted in the Acoustic Test Facility at the Applied Research Laboratory of The Pennsylvania State University. As illustrated in Figure 4-21, the test tank has approximate dimensions of 7.92m (26ft). by 5.33m (17.5ft) by 5.49m (18ft) and is equipped with two underwater viewing ports. The walls and bottom of the tank are lined with a sound absorbing rubber (Saper-T) in order to mitigate reverberation generated by those surfaces down to frequencies of 2-3kHz. The lining on the walls and bottom attenuate the acoustic energy but does not completely prevent reverberation. The free surface of the tank establishes a pressure-release boundary which is a significant source of reverberation and multipath. For these reasons, free-field conditions can only be approximated in the tank by the use of acoustic pulses of duration on the order of 2ms that enables the separation of the direct propagation path between a source and receiver from a surface reflected path.
The calibration test consisted of making single-axis intensity measurements in the near field of a spherical radiator and then estimating the acoustic intensity related quantities as presented in section 4.2 from the time-series measurements. In turn, these estimates could be compared to theoretical values and calibrated measurements to gain confidence in the sensor and subsequent data processing. An International Transducer Corporation ITC-1032 transducer having S/N 115 was employed for the test. This
transducer is a sphere of piezoelectric material (PZT) that is encased in a spherical shell of polyurethane. The outer dimension of the source is approximately 5 cm in diameter and was selected because of its small size. The size of the source is significant since this calibration test required a “compact source” to generate spherical wavefronts at frequencies between 3 kHz and 15 kHz. The ITC-1032 was calibrated using a USRD F33 transducer as a reference standard. The calibration consisted of measuring the transmit voltage response (TVR) of the ITC-1032 using a Hewlett Packard 89410A vector signal analyzer operating in a swept sine mode from 2 kHz to 10 kHz at 250 Hz steps. This calibration is made by measuring the pressure generated by the ITC-1032 at a range of 3.16 m and assuming that the wavefront is approximately planar. In order to verify that the ITC-1032 was operating as a pulsating sphere, a Polytec Scanning Vibrometer was employed to measure the radial velocity at the surface of the calibrated ITC-1032 as it was driven at frequencies between 2 kHz and 10 kHz. Knowing the diameter of the sphere, these measurements of surface velocity enable theoretical predictions of the acoustic field to be made using the model of a radially pulsating sphere. The boundary conditions require that the water particle velocity at the surface of the sphere match the radial velocity of the surface. After this calculation is performed, the acoustic intensity can be predicted at any observation under the assumption of free-field conditions. Comparing these calculations to measurements made by the p-a probe enables an assessment to be made regarding the ability to accurately estimate the quantities derived in earlier sections.

The laser vibrometer was placed several inches from the surface of an underwater viewing port in the tank such that the laser beam illuminated a hexagonal reflective patch
(edge length of 2mm) that was affixed to the surface of the ITC-1032. The optical sensor of the vibrometer was oriented such that the laser beam illuminated the patch and was normal to the surface of ITC-1032. The transducer was driven with a $1V_{\text{rms}}$ sinusoidal pulse which was passed through an Instruments, Inc. model-L6 power amplifier (+40dB gain). The results of the experiment are presented in Figure 4-22. The TVR predicted by the theoretical formulation of a pulsating sphere is in good agreement with the TVR measured with a reference hydrophone. While the trend of the curves is consistent, the root cause of the 3-4dB offset has not been identified. This result allows the near field of the ITC-1032 to be probed with the $p-a$ intensity probe with an expectation that the wavefront is truly spherical at frequencies as high as 10kHz. Direct comparisons can then be made between the intensity estimates and theoretical predictions.
The ICP signal conditioners and preamplifiers whose amplitude and phase response was incorporated into the sensor calibration data presented in section 4.3 were not available for this experiment. An identical model number ICP signal conditioner from PCB Piezotronics was used for the x-axis accelerometer while no preamplifier was used with the hydrophone. Only the x-axis of the dual-axis p-a probe was employed for this near field calibration due to constrained access to the test facility and required assets. The $p-a_x$ probe was calibrated in the new configuration with the calibrated ITC-1032 between frequencies of 1kHz and 10kHz at a range of 3.16m. The sensitivities of the

Figure 4-22: Estimation of TVR by means of the Laser Doppler Vibrometer and by calibration with the USRD F33 reference hydrophone.
probes and the relative phase between the hydrophone and x-axis accelerometer are shown in Figure 4-23, Figure 4-24, and Figure 4-25. These calibration values will be used to correct the raw sensor measurements thereby allowing the acoustical quantities to be estimated correctly.

Figure 4-23: Measured FFVS for the hydrophone of the dual axis p-a probe by means of ITC-1032 transducer. No preamplifier was employed for the hydrophone.
Figure 4-24: Measured FFVS for the $x$-channel accelerometer of the dual axis $p$-$a$ probe by means of ITC-1032 transducer. The accelerometer was connected to PCB Model 480E09 ICP signal conditioner (S/N 5663).

Figure 4-25: Measured phase difference between hydrophone and $a_x$ accelerometer. No preamplifier was employed for the hydrophone. The accelerometer was connected to PCB Model 480E09 ICP signal conditioner (S/N 5633).
The \( p-a \) probe was located at five ranges from the ITC-1032 during the test. At the two farthest ranges, 3.16m and 2.0m, the probe was suspended by its cantilever to a depth of 2.0m (6ft. 7in) below the water surface. At this depth, the \( p-a \) probe resided on the horizontal line-of-sight of the ITC-1032. Three closer test ranges of 26cm, 51cm, and 102cm required that the ITC-1032 and the \( p-a \) probe be rigged to have a standoff of 0.71m (28in) from an aluminum I-beam because the supports on the test facility’s rail system could not provide these separations. The I-beam was then lowered into the test tank by a single hydraulic support. The small separations of the source and receiver make it more difficult to replicate free-field conditions because the I-beam causes a reflector to be located in close proximity of both the source and receiver. While rigging the \( p-a \) probe on the I-beam, the accelerometer was manually rotated to align the maximum response of the dipole pattern with the ITC-1032 by means of an index mark on the sensor housing provided by the manufacturer. While every attempt was made to establish an accurate test geometry, azimuthal or vertical misalignment may impact the measurements at the smallest separations.

At each of the range separations, a 1Vrms 2ms CW pulse was transmitted from the ITC-1032 from a power amplifier that provided +40dB of gain. The pulses were transmitted with center frequencies of 3kHz, 5kHz, and 10kHz. The sensor output was routed directly into a TEAC RX-8000 (S/N 885030) data recorder along with the 1Vrms transmit signal. Approximately 200 pulses were recorded at each range-frequency combination. The TEAC recorder is known to employ individual 16bit analog-to-digital converters for each input channel which sample simultaneously at 48kHz with no zero
phase shift in the passband of the anti-alias filter whose affects become appreciable at 20kHz.

The time series from the hydrophone and accelerometer are shown in Figure 4-26 for a 3kHz pulse and in Figure 4-27 for a 5kHz pulse for a single record at three ranges. The relative phase between the two channels can be seen to move closer to 90° as the range to the source increases. In a pure monochromatic plane wave, the pressure and particle acceleration are 90° out of phase so that pressure and particle velocity are in-phase. This observable phase shift indicates that the sensor is responding to the near-field of the source where the acoustic wavefront is not planar. It is worthwhile to note that the sensor was mounted on the I-beam, the time series does not show any significant artifact (i.e. multipath or reverberation) for the 3ms after the direct pulse ends. While there is no observable phase difference between the three ranges in Figure 4-27, the amplitude of the signal is larger than that presented in Figure 4-26. This is because the RMS voltage of the transmit waveform was kept constant while the frequency was increased from 3kHz to 5kHz. The TVR of the ITC-1032 can be seen to increase by 10dB over this span by Figure 4-22 while the sensitivity of the sensors have remained relatively flat in Figure 4-23 and Figure 4-24.
Figure 4-26: Sample time series at different source-receiver separations: 26cm (top), 102cm (middle), 316cm (bottom). Esonification frequency was 3kHz.
Figure 4-27: Sample time series at different source-receiver separations: 26cm (top), 102cm (middle), 316cm (bottom). Esonification frequency was 5kHz.
For each of the pulses, a 96 point data window was taken in the center of the received pulse. A typical example regarding the selection of the 96 point window is shown in Figure 4-28 as the region highlighted in red. The same data window was applied to the pulse at all three frequencies.

Figure 4-28: Application of gate to data record. A data segment of 96pts for the 5kHz pulse was windowed and operated upon by a 96pt FFT. (top) Voltage signal from the hydrophone. (bottom) Voltage signal from the accelerometer.

The autospectra for the pressure and x-accelerometer channels are presented in Figure 4-29 and Figure 4-30 for 3.16m range data records at 3kHz and 5kHz, respectively. The figures show the spectra of the 96 point data record which contain the 2ms pulse. Also shown are the autospectra for 96 and 1024 point data records taken prior to the arrival of the pulse that employed the same sized DFT. These represent the background noise in the digitized signal and provide insight into the signal-to-noise ratio.
The 96 point data window provides a 500Hz analysis bandwidth. The data prior to the pulse arrival was processed to illustrate the noise level in the analysis bandwidth. The 3kHz, 5kHz, and 10kHz all fall on an analysis bin of the 96 point DFT and therefore incurred no processing losses. The noise spectra are seen to be white out to the cutoff frequency of the TEAC’s anti-aliasing filter at 20kHz. A 60Hz signal is present in both spectra, but appears to only affect the noise level in the 3kHz signal spectra for the pressure transducer because of leakage from the data window. The pressure transducer is being operated in a voltage mode (in the absence of a charge amplifier) and therefore exhibits a high impedance which makes it susceptible to noise. The accelerometer has a built-in charge amplifier which presents a low impedance to the recorder and is therefore not as susceptible to those same noise sources. Overall, the signals are exhibiting SNR values on the order of 40dB and 50dB for the 3kHz and 5kHz signals respectively. The increase in SNR over frequency is only due to the increase in the TVR of the source with frequency shown in Figure 4-22 since the sensitivity of the hydrophone and accelerometer are relatively flat and the noise in the digital time series is white in the vicinity of the signals.
Figure 4-29: Autospectra of the pressure transducer (top) and accelerometer (bottom) during 3kHz pulse and prior to pulse arrival when sensor was 316cm from ITC-1032. Spectra have been generated using a single data record. (red): Autospectra of 96pt windowed signal with 96pt DFT; (lt blue): Autospectra of 96pt windowed noise record with 96pt DFT; (dk blue): Autospectra of 1024pt windowed noise record with 1024pt DFT.
Figure 4-30: Autospectra of the pressure transducer (top) and accelerometer (bottom) during 5kHz pulse and prior to pulse arrival when sensor was 316cm from ITC-1032. Spectra have been generated using a single data record. (red): Autospectra of 96pt windowed signal with 96pt DFT; (lt blue): Autospectra of 96pt windowed noise record with 96pt DFT; (dk blue): Autospectra of 1024pt windowed noise record with 1024pt DFT.
The cross-spectrum of the \textit{p-a} probe is generated to estimate the acoustic intensity associated with the wavefield. The active acoustic intensity was computed at the face of the \textit{p-a} probe. The propagation loss attributed to spherical spreading has been removed to estimate the intensity at 1m from the ITC-1032. A scatterplot of all 200 estimates from each range-frequency pair is plotted in Figure 4-31 together with the sound pressure level (SPL) at the face of the ITC-1032. The SPL is estimated from the TVR and the 100Vrms transmit signal (1Vrms+40dB gain). There is good agreement between the data sets in Figure 4-31 indicating that the intensity sensor is faithfully predicting the active intensity. Alternatively, it could be said that the SPL is in good agreement with the active intensity indicating that the wavefield is approximately planar and that the power factor angle is close to zero.
Another way to examine the data is to plot the active intensity versus the normalized quantity $kr$ as done in Figure 4-32. This method separates each frequency-range combination and shows that the data are aligned along a line which indicates a reduction of 6dB for each doubling of range and is attributed to the $10 \log_{10} r^2$ reduction in intensity due to spherical spreading. The three frequencies are shifted from each other because of the TVR of the ITC-1032. If the transmit voltage had been controlled so as to

Figure 4-31: Scatterplot of active acoustic intensity at 1m from the face of the of ITC-1032 generated by removing the propagation loss associated with the active acoustic intensity estimate made at the $p-a$ probe. The ITC-1032 was driven with a 1Vrms CW pulse which was amplified by +40dB. The scatterplot has been plotted on top of the output for the ITC-1032 (TVR+40dB).
maintain a constant SPL when the frequency was changed, the data would collapse to a single line that represents a frequency independent propagation loss.

Since approximately 200 data records were obtained for each range-frequency combination, the statistics of each ensemble can be examined. Individual estimates were normalized as errors about an ensemble mean, \( \frac{\hat{x} - \bar{x}}{\bar{x}} \), and are presented in Figure 4-33 as histograms. The ensemble means and standard deviations are presented in Table 4-1 each frequency-range permutation as well. Since the estimates used to produce the histograms have been normalized by the mean values of the ensembles and since the mean value varies across ensemble, the histograms do not depict the same trends as the values in Table 4-1. Instead the spread of the histograms follows the ratio of the standard deviation to the mean for each ensemble (i.e., the normalized random error). The histograms imply that the estimates all reside within 10% of the mean value. The frequency-range permutations result in a set of 15 ensembles whose SNR should increase with frequency (i.e., higher TVR values) or reduces with increasing source-receiver separation (i.e., greater propagation loss). If the random errors presented in the previous section were dominant in these measurements, one would expect the normalized random error in each permutation to decrease with increasing SNR. While this trend is visible in the histograms as the standard deviation of the histogram decreases as range decreases or frequency increases it is not sufficient to declare that the random errors are dominant.
Figure 4-32: Scatterplot of active acoustic intensity at face of sensor. Black lines are results of a least-square fit of the data points, the slope of the resulting lines are indicated in the legend.
Figure 4-33: Histogram of the normalized error associated with the active intensity estimates made from approximately 200 records at each frequency and range. The $y$-axes of the histograms have been normalized by the number of samples.
The reactive intensity estimates at the face of the $p-a$ probe are displayed as a scatterplot in Figure 4-34 as a function of the non-dimensional parameter $kr$. The slope of lines implied by the data clusters is greater than that observed in Figure 4-32 because the magnitude of the reactive intensity decreases with the cube of the propagation distance from a spherical source instead of the square as does the active intensity (e.g. Table 1-2). The wide scatter of the data points in Figure 4-34 indicates that the estimates exhibit a large variance which can be observed in the histograms of the normalized estimates presented in Figure 4-35. The histograms alone imply that the normalized random error of some ensembles reaches as high as 25-50%. The actual mean and standard deviation of the ensembles are presented in Table 4-2.

---

<table>
<thead>
<tr>
<th>Mean</th>
<th>Rng=316cm</th>
<th>Rng=200cm</th>
<th>Rng=102cm</th>
<th>Rng=51cm</th>
<th>Rng=26cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq=3kHz</td>
<td>2.3e-5</td>
<td>6.6e-5</td>
<td>2.5e-4</td>
<td>7.6e-4</td>
<td>3.6e-3</td>
</tr>
<tr>
<td>Freq=5kHz</td>
<td>2.0e-4</td>
<td>4.8e-4</td>
<td>1.6e-3</td>
<td>6.9e-3</td>
<td>2.7e-2</td>
</tr>
<tr>
<td>Freq=10kHz</td>
<td>3.6e-3</td>
<td>1.0e-2</td>
<td>3.1e-2</td>
<td>1.2e-1</td>
<td>5.3e-1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Std. Dev.</th>
<th>Rng=316cm</th>
<th>Rng=200cm</th>
<th>Rng=102cm</th>
<th>Rng=51cm</th>
<th>Rng=26cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq=3kHz</td>
<td>1.2e-6</td>
<td>4.9e-7</td>
<td>3.3e-6</td>
<td>3.2e-5</td>
<td>4.1e-5</td>
</tr>
<tr>
<td>Freq=5kHz</td>
<td>5.9e-6</td>
<td>8.2e-6</td>
<td>1.5e-5</td>
<td>3.6e-5</td>
<td>2.2e-4</td>
</tr>
<tr>
<td>Freq=10kHz</td>
<td>3.3e-5</td>
<td>7.3e-5</td>
<td>1.4e-4</td>
<td>1.2e-3</td>
<td>1.6e-3</td>
</tr>
</tbody>
</table>

---

Table 4-1: Ensemble means (top) and standard deviations (bottom) for the active intensity estimates presented in Figure 4-32 and Figure 4-33. Units of values are Watts/m$^2$. 
Figure 4-34: Scatterplot of the reactive intensity at the face of the p-α probe across frequency and source-receiver separation. Black lines are results of a least-square fit of the data points, the slope of the resulting lines are indicated in the legend.
Figure 4-35: Histogram of the normalized error associated with the reactive intensity estimates made from approximately 200 records at each frequency and range. The y-axes of the histograms have been normalized by the number of samples.
The estimates associated with the magnitude of the complex intensity are presented in Figure 4-36, Figure 4-37, and Table 4-3. The trends are similar to those of the active acoustic intensity except at low frequencies and close ranges where the power factor angle becomes appreciable.
Figure 4-36: Scatterplot of the magnitude of complex acoustic intensity at the face of sensor. Black lines are results of a least-square fit of the data points, the slope of the resulting lines are indicated in the legend.
Figure 4-37: Histogram of the normalized error associated with the magnitude of the complex intensity estimates made from approximately 200 records at each frequency and range. The y-axes of the histograms have been normalized by the number of samples.
The estimates associated with the magnitude of the complex intensity are presented in Figure 4-38, Figure 4-39, and Table 4-4. It can be seen that the experimental data in the scatterplot follows the general trend of the theoretical curve. The estimates follow a similar trend as the earlier plots with respect to increasing SNR and that the standard deviation in Table 4-4 rarely exceeds 1°. However, because the mean value of the phase angle is small at high frequencies and long ranges, the normalized random error depicted by the histograms appears to be exceedingly large.

Table 4-3: Ensemble means (top) and standard deviations (bottom) for the magnitude of the complex intensity estimates presented in Figure 4-34 and Figure 4-35. Units of values are Watts/m².

<table>
<thead>
<tr>
<th></th>
<th>Rng=316cm</th>
<th>Rng=200cm</th>
<th>Rng=102cm</th>
<th>Rng=51cm</th>
<th>Rng=26cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Freq=3kHz</td>
<td>2.3e-5</td>
<td>6.6e-5</td>
<td>2.5e-4</td>
<td>7.8e-4</td>
<td>4.0e-3</td>
</tr>
<tr>
<td>Freq=5kHz</td>
<td>2.0e-4</td>
<td>4.8e-4</td>
<td>1.6e-3</td>
<td>7.0e-3</td>
<td>2.7e-2</td>
</tr>
<tr>
<td>Freq=10kHz</td>
<td>3.6e-3</td>
<td>1.0e-2</td>
<td>3.1e-2</td>
<td>1.2e-1</td>
<td>5.3e-1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Std. Dev.</th>
<th>Rng=316cm</th>
<th>Rng=200cm</th>
<th>Rng=102cm</th>
<th>Rng=51cm</th>
<th>Rng=26cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq=3kHz</td>
<td>1.2e-6</td>
<td>5.2e-7</td>
<td>2.8e-6</td>
<td>3.9e-5</td>
<td>3.6e-5</td>
</tr>
<tr>
<td>Freq=5kHz</td>
<td>6.1e-6</td>
<td>7.6e-6</td>
<td>1.4e-5</td>
<td>5.5e-5</td>
<td>2.2e-4</td>
</tr>
<tr>
<td>Freq=10kHz</td>
<td>3.3e-5</td>
<td>7.3e-5</td>
<td>1.4e-4</td>
<td>1.1e-3</td>
<td>1.6e-3</td>
</tr>
</tbody>
</table>
Figure 4-38: Scatterplot of the phase of the complex intensity estimate made at the face of the $p-a$ probe across frequency and source-receiver separation. The theoretical value for the relative phase is plotted in black for reference.
Figure 4-39: Histogram of the normalized error associated with the phase of the complex intensity estimates made from approximately 200 records at each frequency and range. The y-axes of the histograms have been normalized by the number of samples.
Table 4-4

Table 4-4: Ensemble means (top) and standard deviations (bottom) for the phase of the complex intensity estimates presented in Figure 4-34 and Figure 4-35. Units of values are degrees.

<table>
<thead>
<tr>
<th></th>
<th>Rng=316cm</th>
<th>Rng=200cm</th>
<th>Rng=102cm</th>
<th>Rng=51cm</th>
<th>Rng=26cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Freq=3kHz</td>
<td>-3.6</td>
<td>-4.8</td>
<td>8.8</td>
<td>12</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Freq=5kHz</td>
<td>4.1</td>
<td>4.7</td>
<td>-3.9</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td>Freq=10kHz</td>
<td>-6.9</td>
<td>-9.5</td>
<td>-5.5</td>
<td>-1.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Rng=316cm</th>
<th>Rng=200cm</th>
<th>Rng=102cm</th>
<th>Rng=51cm</th>
<th>Rng=26cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev. Freq=3kHz</td>
<td>2.1</td>
<td>1.4</td>
<td>.95</td>
<td>2.3</td>
<td>.7</td>
</tr>
<tr>
<td></td>
<td>Freq=5kHz</td>
<td>1.1</td>
<td>.95</td>
<td>.32</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Freq=10kHz</td>
<td>.76</td>
<td>.26</td>
<td>.3</td>
<td>.73</td>
</tr>
</tbody>
</table>

4.5 Summary

This chapter has presented the complex acoustic intensity processing stream associated with a novel \( p-a \) intensity probe. The random errors associated with estimating the cross-spectrum and its parameters have also been presented in order to determine what SNR conditions are required of the sensor channels in order to infer the presence of a scattering body by way of the deterministic intensities predicted in Chapter 3. In addition, a calibration experiment was conducted in the near field of a spherical source to confirm that the selected acoustic intensity sensor was able to measure the active and reactive intensities as well as the power factor angle associated with the spherical wave. While the estimates of phase angle exhibited error when compared to the known theoretical value, the standard deviation of the estimates were smaller than the
deterministic values predicted in Chapter 3. In support of the objective of this thesis, it is therefore reasonable to expect that the sensor and sensor processing stream should be able to generate estimates that can differentiate between 0° of phase angle when no spheroid is present and 3-5° when the spheroid is present under similar SNR conditions.
Chapter 5

Experimental Measurement of Acoustic Intensity in the Forward Scatter Direction

5.1 Introduction

The theoretical investigation presented in Chapter 3 concluded that when a rigid prolate spheroid is placed in a steady-state plane wave field, a phase difference between pressure and particle velocity arises. The phase difference is localized to the forward scatter direction as the acoustic shadow is formed at high frequencies and in the direction of specular reflection as the scattered signal interferes with the incident wavefield. At large values of $h = 20$ to $40$ when the plane wave was incident from a direction of $\left( \theta_{inc} = 60^\circ, \phi_{inc} = 0^\circ \right)$, calculations predicted $\phi_{pu} \sim 3^\circ$ of phase difference in the forward scatter region at ranges of 20m-30m from a 10:1 prolate spheroid having a dimension of $L_0 = 2m$. Furthermore, the calibration experiments discussed in Section 4.4 concluded that the high-frequency $p-a$ probe developed by Acoustech would respond to such small phase differences at moderately high values of signal-to-noise ratios (~40dB). While it was not conclusive that measurements were accurate enough to estimate the absolute phase difference within $5^\circ$ of error when $\phi_{pu} < 10^\circ$, it is believed that the sensor would allow measurement of the relative phase difference of $\phi_{pu}$ for the case when the spheroid
was not in the field and the case when the spheroid was positioned between the acoustic source and sensor. This belief is supported by the fact that while the estimates of $\varphi_{pu}$ were biased at several values of $kr$ in Figure 4-38 the variance of those estimates were an order of magnitude smaller than the mean value. This statement is supported by the histograms in Figure 4-39 which are annotated with the ensemble mean and standard deviation. Two field experiment were designed and conducted at 1) a flooded quarry in Jacksonville, PA; and 2) the Seneca Lake Sonar Test Facility in Dresden, NY. The objective of these experiments was to determine if perturbations to the total pressure $P$ and intensity vectors $I_e$, $I$, and $\tilde{Q}$ are observable with the dual-axis p-a probe described in Section 4.3 when a 10:1 aspect ratio prolate spheroid is placed between the acoustic source and sensor.

5.2 Fabrication of 10:1 Prolate Spheroid

The testing was first to be attempted at a flooded quarry with divers deploying the experimental apparatus. The manual deployment of the scattering body required it to be manageable in air and in water by one or two individuals. In order to accomplish this while allowing a reduced frequency of $h > 20$ at frequencies near 10kHz, the prolate spheroid was selected to be 2m in length. The approximation to a rigid body was not deemed critical to the experiment and therefore the spheroid was constructed from laminations of red oak. In order to simplify transportation and storage of the 2m long spheroid, the spheroid was constructed in two halves. The form for the prolate spheroid was constructed in a Computer Aided Design (CAD) environment as shown in Figure 5-
1. The computer model was then downloaded to a Computer Numerically Controlled (CNC) lathe which cut the shape as shown in Figure 5-2. The nose of each half-spheroid was then finished by hand. The flat ends of each of the two halves were fitted with a stainless steel joining plate which had a hole at the center of the face and tapped with opposite facing threads allowing a 1.25in long 1-12UNF stud to join the two halves together. Two pairs of holes were drilled in the edges of the two joining plates at 180º apart and tapped to accept 5/16-18 eybolts. These mounting points would allow the spheroid to either be hung or suspended at its midpoint. In addition, two 1/4-20 stainless steel threaded inserts were epoxied into the spheroid 180º apart one-third of the way in from either end to accept eyebolts allowing guys to be attached if necessary during deployment. The spheroid was then finished with spar varnish to give it a finish that was both waterproof and durable. The completed spheroid is shown in Figure 5-3.
Figure 5-1: CAD drawings of 10:1 prolate spheroid.
Figure 5-2: Construction of wooden 10:1 prolate spheroid.
5.3 Initial Testing at Flooded Quarry in Jacksonville, PA

The scattering experiment was to be conducted in a flooded quarry in Jacksonville, PA that is pictured in Figure 5-4. Permission to conduct the experiments was obtained from Hanson Aggregates who own and operate the quarry. The quarry is approximately 12m deep by 100m wide by 150m long. The quarry is fed by rising ground water and as a result has a sharp thermocline at a depth of 4m. The surface layer was ~25°C during testing and the water temperature immediately below the thermocline...
dropped to ~10°C. The test equipment on the wet side of the experiment was deployed manually by Pennsylvania State University science & research divers. The experiment geometry was controlled by placing anchors at the bottom of the quarry and then floating the test assets to the middle of the water column. The dual-axis \( p-a \) probe described in Section 4.3 was employed along with the prolate spheroid as the scattering body and a Lubell LL-9162 transducer as the source. The geometry for the initial experiment called for the source to be 10m from the target and the sensor to be another 3m along the line-of-sight between the source and spheroid. The wooden scattering body was approximately 13.6kg (30lbs) buoyant and could therefore be suspended directly from bow shackles located at its midpoint as shown in Figure 5-5. The \( p-a \) probe was tethered to the anchor on one side of the stainless steel cage and a 22.7kg (50lb) lift bag was inflated to provide buoyancy and to keep the line taut as shown in Figure 5-6. The \( p-a \) probe was rigged such that the cage could be rotated without twisting the lines thereby allowing a diver to manually align one set of dipoles towards the scattering body and the source. A DSP Technology, Inc. Model 20-42 (S/N 11612) signal analysis station was used to generate the transmit signal and record the three data channels from the \( p-a \) probe. A power amplifier designed for automobile stereo systems was used to provide adequate sound pressure levels in the water. A block diagram for the test equipment is provided in Figure 5-7.
Figure 5-4: Flooded quarry in Jacksonville, PA which was the site of initial dual-axis intensity sensor testing.

Figure 5-5: 10:1 Prolate spheroid scattering body suspended in flooded quarry.
Figure 5-6: Deployment of dual-axis \( p-a \) probe and transducer.
The shallow depth of the quarry made it difficult to conduct long baseline experiments which would ensure that scattering from the spheroid is in the far-field of the source and the sensor is in the far-field of the spheroid. The desire to receive signals without interference from boundary reflections limits the pulse lengths to several milliseconds in duration at close ranges. Banks of deep cycle marine batteries were used to power both the data acquisition system and the power amplifier for the transducer. The batteries provided about 4hrs worth of experiment time. Security at the quarry was inadequate to leave test equipment deployed in the quarry overnight on the weekend. Noise levels were too high during the weekday due to the operation of heavy machinery. For these reasons, it was decided to limit the use of the quarry for shakedown testing and to resolve deployment and data collection issues.

Figure 5-7: Block diagram of equipment setup for quarry test.
5.4 Testing at Seneca Lake Sonar Test Facility in Dresden, NY

A second series of underwater tests with the prolate spheroid were conducted at the Seneca Lake Sonar Test Facility in Dresden, NY from 15-19 September 2003. The experiments were conducted on the Systems Measurement Platform (SMP) which has a catamaran-type configuration and a typical draft of 3m (10ft). The SMP is held in position by a four-point bottom moor to minimize movement. The assets were deployed along the inside of one pontoon leg of the SMP according to Figure 5-8. The spheroid was deployed from the SMP using a series of pipe stringers to lower the spheroid to a fixed depth and at a known orientation as shown in Figure 5-9. A broadside incidence angle of \( \left( \theta_{inc} = 90^\circ, \varphi_{inc} = 0^\circ \right) \) was chosen for the in-water experiment geometry rather than \( \left( \theta_{inc} = 60^\circ, \varphi_{inc} = 0^\circ \right) \) due to the ease of experimental setup. The rigid mounting of the spheroid reduced uncertainties in the position and orientation of the scattering body. Because the theoretical results presented in Chapter 3 were calculated with an incident angle of \( \left( \theta_{inc} = 60^\circ, \varphi_{inc} = 0^\circ \right) \), the calculations were repeated for the incident angle of \( \left( \theta_{inc} = 90^\circ, \varphi_{inc} = 0^\circ \right) \). The active intensity field, reactive intensity field, and the power factor angle are presented in Figure 5-10 through Figure 5-12 for the \( \hat{\xi} \) direction and in Figure 5-13 through Figure 5-15 for the \( \hat{\eta} \) direction. At the SMP, the water depth was approximately 140m (460ft). A conductivity-temperature-depth (CTD) sensor was deployed from the SMP each morning of the experiment to generate the sound speed profiles shown in Figure 5-16. A current Doppler profiler (CDP) is normally deployed
during experiment days to measure the subsurface currents but was unavailable during this test period.

Figure 5-8: Planned experimental geometry of scattering study aboard SMP at Seneca Lake Sonar Test Facility. Predicted arrival times for the direct path and surface reflections are indicated.
Figure 5-9: Deployment of spheroid rigidly mounted to pipe stringers.
Figure 5-10: Theoretical prediction of total active acoustic intensity, $I$, in the $\hat{\xi}$ direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of $(\theta_{inc} = 90^\circ, \varphi_{inc} = 0^\circ)$. 
Figure 5-11: Theoretical prediction of total reactive acoustic intensity, $Q$, in the $\hat{\xi}$ direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of $(\theta_{inc} = 90^\circ, \varphi_{inc} = 0^\circ)$. 

- $\vec{k}_{inc}$
Figure 5-12: Theoretical prediction of magnitude of the power factor angle, $\varphi_{pu}$ in the $\xi$ direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of $(\theta_{inc} = 90^\circ, \varphi_{inc} = 0^\circ)$. 

$\vec{K}_{inc}$
Figure 5-13: Theoretical prediction of total active acoustic intensity, $I$, in the $\hat{n}$ direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of $(\theta_{inc} = 90^\circ, \varphi_{inc} = 0^\circ)$. 

\[ \vec{k}_{inc} \]
Figure 5-14: Theoretical prediction of total reactive acoustic intensity, $Q$, in the $\hat{\eta}$ direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of $(\theta_{inc} = 90^\circ, \phi_{inc} = 0^\circ)$. 
Figure 5-15: Theoretical prediction of magnitude of the power factor angle, $\varphi_{pu}$ in the $\hat{n}$ direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of $\left(\theta_{inc} = 90^\circ, \varphi_{inc} = 0^\circ\right)$. 
The source used in the experiments was an International Transducer Corporation ITC-1007 16.5cm (6.5in.) diameter spherical transducer whose nominal resonance frequency is 11.5kHz and whose advertised transmit voltage response is given in Figure 5-17. The depth of the experiment was set for a nominal value of 46m (150ft) in order to provide a relatively large time window during which free-field conditions (i.e. no boundary reflections or boundary reverberation) could be approximated. The ITC-1007 source is omnidirectional at our nominal test frequency of 10kHz and was rigged from a

Figure 5-16: Sound speed profiles estimated from CTD measurements during week of 15 September 2003.
spanner bar to prevent any rotation or undue motion of the source. However, the spanner was deployed from a portable winch to the test depth and it is conceivable that subsurface currents could drag it off the test geometry. The dual axis p-a probes developed by Acoustech had not yet been subjected to a depth in excess of 12m raising some concern about its survivability. Additionally, each of the probe signals were carried topside by a 100m length of RG-174 coaxial cable which does not possess any underwater performance specifications from the manufacturer. Due to the concern for sensor or cable failure, two dual axis p-a probes were deployed as a vertical array as pictured in Figure 5-18. The lower sensor bore the serial number 2002-4 and was of similar construction to the lower probe bearing serial number 2002-5 whose calibration was discussed in Section 4.3. Since the second p-a probe was fielded as a backup sensor, the midpoint of the vertical array was deployed at the same depth as the source and scattering body. Therefore neither sensor was intentionally placed directly on the line-of-sight with the spheroid and the source. In addition, the sensor array was lowered to depth by wire cable from a portable winch. The sensor array was rigged with a sash weight to steady the sensor and the sensor was attached to the wire cable with a shackle that was free to rotate. In an effort to keep the sensor from being dragged by subsurface currents, tag lines were attached to the sensor cage and tied off on the pontoon. However this did not guarantee that the sensor was oriented with one axis along the line-of-sight nor could it prevent the sensor from rotating during or between experiments. A block diagram for the experiment instrumentation is shown in Figure 5-19. The signals were generated by a DSP Technology, Inc. Model 20-40 (S/N 11612) Siglab signal analysis station and routed to the Seneca Lake facility for amplification and transmission on the ITC-1007
transducer. The output of the vertical array of sensors ran to the deck of the SMP through a bundle of six lines of RG-174 coaxial cable. The accelerometer channels were routed to ICP signal conditioners while the hydrophone output was routed to preamplifiers. The output from one sensor was routed to the Siglab analysis station for real-time monitoring of sensors and experiment. The six channels of sensor output and the source signal were routed to the TEAC RX-800 (S/N 885030) recorder for archival and post-processing.

Midway through the tests, the hydrophone signal on p-a probe S/N 2002-4 failed and as a result the data from that sensor was deemed untrustworthy and will not be presented here. Calibration tests performed at the Acoustic Test Facility (ATF) at The Pennsylvania State University Applied Research Laboratory (PSU/ARL) subsequent to the field experiment verified that p-a probe S/N 2002-4 had failed but that the measured responses of p-a probe S/N 2002-5 matched those taken prior to the field experiment. All data presented here will therefore be from p-a probe S/N 2002-5 and will have been reduced from the data collected on a TEAC RX-800 (S/N 885030) recorder.
Figure 5-17: Nominal transmit voltage response of ITC-1007 spherical transducer. Data provided by International Transducer Corporation, Santa Barbara, CA.
Figure 5-18: Vertical array of dual axis \( p-a \) probes deployed for scattering tests. Upper sensor was S/N 2002-4 and lower sensor was S/N 2002-5.
In order to determine the time window with which a signal from the source would arrive at the receiver with no interference from boundary reflections or other secondary reflectors, a 1ms 8kHz pulse was transmitted from the source and received at the sensor when the spheroid was not in the water column. A single data record from the pressure sensor is shown in the upper portion of Figure 5-20 and Figure 5-21 for a 1ms and 35ms CW pulse, while the lower portion depicts the mean square value of the hydrophone voltage from an ensemble of 20 such data records. The signal arriving along the direct path is dominant and the surface reflection arrives at the sensor approximately 40ms after the direct signal. The pulse length for the scattering studies was therefore chosen to be 35ms in length to maximize the amount of signal while emulating free-field conditions.

Figure 5-19: Block diagram of instrumentation for Seneca Lake scattering experiment.
Figure 5-20: (top) Data record from pressure sensor with 1ms 8kHz pulse transmitted from ITC-1007. (bottom) Mean square voltage level from pressure sensor over 20 records. The source to receiver separation is 38m (125ft).

Figure 5-21: (top) Data record from pressure sensor with 35ms 8kHz pulse transmitted from ITC-1007. (bottom) Mean square voltage level from pressure sensor over 20 records. The source to receiver separation is 38m (125ft).
The time series of the sensor recordings were processed in the same manner as the data presented in Section 4.4; however, both axes of the p-a probe were recorded and processed using the calibration values presented in Section 4.3. The longer pulse length employed during the Seneca Lake tests enables a larger DFT to be employed relative to that used to process the data in Section 4.4. Three frequencies were chosen for the signals in the Seneca Lake experiment: 8kHz, 10kHz, and 12kHz. The frequency selection was bounded on the low end by the TVR of the ITC-1007 transducer and on the high end by the p-a probe response. A windowed CW pulse was employed to collect scattering data that was free from multipath and reverberation. This source signal would be best able to emulate the free-field conditions under which the theoretical results were obtained. In addition, data was collected with a steady-state continuous wave (CW) as the source signal in order to collect data in the presence of interference due to multipath and reverberation. A DFT size of 1200pts was chosen in order for these frequencies to fall at the center of a DFT analysis bin. Coupled with the 48kHz sample rate of the TEAC recorder, this DFT size would generate a 40Hz analysis binwidth causing the three signal frequencies to reside in bin #200, 250, and 300 respectively. This DFT size was employed for both the CW signals as well as the 35ms CW pulses. The steady state signals were processed by breaking the recorded time series into non-overlapping data records whose length corresponds to the DFT size. The pulsed signals were processed by detecting the leading edge of the first arrival and windowing the data record for the DFT. A time series from the three channels of the p-a probe is given in Figure 5-22 for the 53m (175ft) source/receiver separation and illustrates where the 1200pt window was extracted.
from the 35ms pulse. The autospectra for the three data records shown in Figure 5-22 are shown in Figure 5-23 through Figure 5-25 together with the autospectra of the recorded noise. The autospectra for the noise was constructed by processing the data record prior to the record containing the signal. No analysis was performed to determine the source of the background noise in the recorded data, but the flat nature of the spectrum would lead one to suspect that the noise is predominantly due to the electronics of the measurement system rather than ambient acoustic noise. The number of data records processed for each source/receiver separation, frequency, and signal type are listed in Table 5-1.
Figure 5-22: Time series for the three channels of the p-a probe for a 35msec 10kHz pulse for which the 1200pt processing window is highlighted in red. (top) Voltage signal from the hydrophone. (middle) Voltage signal from the x-axis accelerometer. (bottom) Voltage signal from the y-axis accelerometer. The source to receiver separation is 53m (175ft).
Figure 5-23: Autospectra from a single data record for the pressure transducer channel for the 35ms 10kHz CW pulse with src/rcv separation of 53m (175ft). Spectra corresponding to the background noise recorded on 16 Sept (cyan) and 18 Sept (blue) are also presented in the same analysis bandwidth for comparison.
Figure 5-24: Autospectra from a single data record for the x-axis accelerometer channel for the 35ms 10kHz CW pulse with src/rcv separation of 53m (175ft). Spectra corresponding to the background noise recorded on 16 Sept (cyan) and 18 Sept (blue) are also presented in the same analysis bandwidth for comparison.
Figure 5-25: Autospectra from a single data record for the $y$-axis accelerometer channel for the 35ms 10kHz CW pulse with src/rcv separation of 53m (ft). Spectra corresponding to the background noise recorded on 16 Sept (cyan) and 18 Sept (blue) are also presented in the same analysis bandwidth for comparison.

Table 5-1: Number of 1200pt data records for each Seneca Lake Experiment permutation and the actual source to receiver separations.

<table>
<thead>
<tr>
<th>SRC/RCV Separation</th>
<th>Frequency</th>
<th>38m (125ft)</th>
<th>53m (175ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State CW</td>
<td>8kHz</td>
<td>5629</td>
<td>2434</td>
</tr>
<tr>
<td></td>
<td>10kHz</td>
<td>2819</td>
<td>3589</td>
</tr>
<tr>
<td></td>
<td>12kHz</td>
<td>3635</td>
<td>3878</td>
</tr>
<tr>
<td>CW Pulse</td>
<td>8kHz</td>
<td>30</td>
<td>536</td>
</tr>
<tr>
<td></td>
<td>10kHz</td>
<td>388</td>
<td>414</td>
</tr>
<tr>
<td></td>
<td>12kHz</td>
<td>416</td>
<td>525</td>
</tr>
</tbody>
</table>
Frequency domain estimates of the equivalent plane wave intensity

\[ I_{\text{equiv}} = \frac{P^2}{2 \rho c}, \]

total active intensity \( I_r \), and total reactive intensity \( Q_r \), were constructed from the data records listed in Table 5-1. In addition, the bearings associated with the active intensity, \( \theta_{Ixy} \), and reactive intensity, \( \theta_{Qxy} \), were computed using Eq. 5.1 and Eq. 5.2. As presented in Eq. 2.21, the bearing for the active acoustic intensity, \( \theta_I \), represents the direction from which acoustic energy is arriving. Analysis of Eq. 2.23 reveals that the bearing of the reactive intensity vector, \( \theta_Q \), is determined by the spatial gradient of the mean square pressure field from. Time series of these estimates are presented in Figure 5-26 for the 10kHz steady state signal and in Figure 5-27 for the 10kHz CW pulses.

\[ \theta_{Ixy} = \tan^{-1} \left( \frac{I_y}{I_x} \right) = \tan^{-1} \left( \frac{\text{Re}(I_{xy})}{\text{Re}(I_{cx})} \right) \]

\[ \theta_{Qxy} = \tan^{-1} \left( \frac{Q_y}{Q_x} \right) = \tan^{-1} \left( \frac{\text{Im}(I_{xy})}{\text{Im}(I_{cx})} \right) \]
Figure 5-26: Frequency domain estimates of $I_{equiv}$, $I_t$, $Q_t$, $\theta_{txy}$, and $\theta_{gyy}$ across 1200pt data records for the 10kHz steady state signal corresponding to a src/rcv separation of 38m (125ft) relative to the $xy$ coordinate frame. The time on the abscissa represents the time of the data record from which the estimate was made. The spheroid was present during data recording.
Figure 5-27: Frequency domain estimates of $I_{\text{eqv}}$, $I_t$, $Q_t$, $\theta_{xy}$, and $\theta_{Qxy}$ across 1200pt data records for the 10kHz pulsed CW signal corresponding to a src/rcv separation of 38m (125ft) relative to the $xy$ coordinate frame. The time on the abscissa represents the time of the data record from which the estimate was made. The spheroid was present during data recording.
Figure 5-26 and Figure 5-27 both indicate that the axes of the p-a probe were not oriented to be radial and tangential to the line-of-sight between the source and receiver. The intensity estimates, $I_x \hat{x}$ and $I_y \hat{y}$, for each data record were rotated in the horizontal plane by the active intensity bearing $\theta_{xy}$ in order to construct estimates from a pair of crossed dipoles corresponding to the radial and tangential directions in the analytical vector calculations, $I_x \hat{e}_x$ and $I_y \hat{e}_y$, presented in Chapter 3. The rotation was performed according to Eq. 5.3 and employed the active intensity bearing estimate from that data record (i.e. no filtering) to construct intensity estimates in a rotated frame having unit vectors $\hat{a}$ and $\hat{b}$ shown in Figure 5-28. The rotated estimates of $I_{\text{equiv}}$, total active intensity $I_t$, total reactive intensity $Q_t$, $\theta_{\text{lab}}$, and $\theta_{\text{lab}}$ are presented in Figure 5-29 and Figure 5-30 for comparison against Figure 5-26 and Figure 5-27. As expected, the total intensities are invariant to the rotation since they represent the magnitude of the intensity vectors rather than either component.

$$
\begin{bmatrix}
I_{ca} \\
I_{cb}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_{xy} & \sin \theta_{xy} \\
-\sin \theta_{xy} & \cos \theta_{xy}
\end{bmatrix}
\begin{bmatrix}
I_{cx} \\
I_{cy}
\end{bmatrix}
$$

5.3
Figure 5-28: Geometry of the rotated intensity vectors in the $ab$ frame relative to the original $xy$ frame and the incident signal.
Figure 5-29: Frequency domain estimates of $I_{\text{equiv}}$, $I_t$, $Q_t$, $\theta_{\text{lab}}$, and $\theta_{\text{Qlb}}$ across 1200pt data records for the 10kHz steady state signal corresponding to a src/rcv separation of 38m (125ft) relative to the $ab$ coordinate frame. The time on the abscissa represents the time of the data record from which the estimate was made. The spheroid was present during data recording.
Figure 5-30: Frequency domain estimates of $I_{\text{equiv}}$, $I_t$, $Q_t$, $\theta_{ab}$, and $\theta_{Qab}$ across 1200pt data records for the 10kHz pulsed CW signal corresponding to a src/rcv separation of 38m (125ft) relative to the $ab$ coordinate frame. The time on the abscissa represents the time of the data record from which the estimate was made. The spheroid was present during data recording.
The ensembles of intensity estimates made in the rotated coordinate frame were used to construct histograms of the various intensity metrics in Table 5-2. Two of the cases listed in Table 5-1 are discussed in this chapter while the balance of the data is deferred to Appendix B. During the data analysis, concerns arose regarding the application of the calibration values presented in Chapter 4. These concerns will be discussed below. The possibility that the calibration values are not representative of the in-situ behavior of the sensor prohibits the data in the histograms from being presented as acoustic intensity. The histograms are therefore not labeled in quantities of acoustic intensity but instead are presented in terms of the corresponding auto-spectra and cross-spectra quantities identified in Table 5-2. In order to draw some conclusions regarding the integrity of the calibration values, the spectral values have been computed using the calibration values of $M_p$, $M_{ax}$, $\phi_{scal}$, $M_{ay}$, and $\phi_{ycal}$ which were estimated at the ARL Test Facility and presented in Figure 4-15, Figure 4-17, and Figure 4-19. The autospectra of the hydrophone voltage output, $G_{pp}$, is converted to $\tilde{G}_{pp}$ according to Eq. 5.4. Denoting the cross-spectra between voltage time series of the hydrophones and accelerometers as $G_{pax}$ and $G_{pay}$, the calibration values are applied to these cross-spectra according to Eq. 5.6 and Eq. 5.5 for the assumed time dependence of $e^{-i\omega t}$. If the calibration values are assumed to be accurate, $\tilde{G}_{pp}$ can be identified as the equivalent plane wave intensity while $\tilde{G}_{pax}$ and $\tilde{G}_{pay}$ can be identified as the complex intensity estimates in the $\hat{x}$ and $\hat{y}$ directions. Even though the application of the calibrations might allow the units of $\tilde{G}_{pp}$, $\tilde{G}_{pax}$, and $\tilde{G}_{pay}$ to be identified as $Watts/m^2$ rather than in
terms of voltages, it would be inappropriate to unequivocally identify these quantities as acoustic intensities due to the concerns regarding the in-situ behavior of the sensors at Seneca Lake. Many of the quantities presented in these figures are based upon \( \tilde{G}_{pax} \) and \( \tilde{G}_{pab} \) which are cross-spectra that have been generated by rotating \( \tilde{G}_{pax} \) and \( \tilde{G}_{pay} \) to \( \hat{a} \) and \( \hat{b} \) directions according to Eq. 5.3.

\[
\tilde{G}_{pp} = \frac{G_{pp}}{M_p^2 \rho c} e^{i\varphi_{cal}}
\]

\[
\tilde{G}_{pax} = \frac{G_{pax}}{M_p M_{ax} (-i\omega)} e^{i\varphi_{cal}}
\]

\[
\tilde{G}_{pay} = \frac{G_{pay}}{M_p M_{ay} (-i\omega)} e^{i\varphi_{cal}}
\]
Table 5-2: Acoustic field indicators and metrics employed to quantify disturbance to the acoustic field due to presence of prolate spheroid.

<table>
<thead>
<tr>
<th>Description</th>
<th>Metric</th>
<th>Cross-Spectral Quantity</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent plane wave intensity</td>
<td>$\frac{P^2}{2\rho c}$</td>
<td>$\tilde{G}_{pp}$</td>
<td>$\frac{W}{m^2}$</td>
</tr>
<tr>
<td>Vector components of active intensity</td>
<td>$I_a$</td>
<td>$\text{Re}(\tilde{G}_{pa})$</td>
<td>$\frac{W}{m^2}$</td>
</tr>
<tr>
<td></td>
<td>$I_b$</td>
<td>$\text{Re}(\tilde{G}_{pb})$</td>
<td>$\frac{W}{m^2}$</td>
</tr>
<tr>
<td>Vector components of reactive intensity</td>
<td>$O_a$</td>
<td>$\text{Im}(\tilde{G}_{pa})$</td>
<td>$\frac{W}{m^2}$</td>
</tr>
<tr>
<td></td>
<td>$O_b$</td>
<td>$\text{Im}(\tilde{G}_{pb})$</td>
<td>$\frac{W}{m^2}$</td>
</tr>
<tr>
<td>Total active and reactive intensity</td>
<td>$I_t = \sqrt{I_a^2 + I_b^2}$</td>
<td>$\sqrt{\text{Re}(\tilde{G}<em>{pa})^2 + \text{Re}(\tilde{G}</em>{pb})^2}$</td>
<td>$\frac{W}{m^2}$</td>
</tr>
<tr>
<td></td>
<td>$O_t = \sqrt{O_a^2 + O_b^2}$</td>
<td>$\sqrt{\text{Im}(\tilde{G}<em>{pa})^2 + \text{Im}(\tilde{G}</em>{pb})^2}$</td>
<td>$\frac{W}{m^2}$</td>
</tr>
<tr>
<td>Magnitudes of vector components of complex intensity</td>
<td>$</td>
<td>I_{ca}</td>
<td>= \sqrt{I_a^2 + O_a^2}$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>I_{cb}</td>
<td>= \sqrt{I_b^2 + O_b^2}$</td>
</tr>
<tr>
<td>Power factor angle associated with vector components of intensity</td>
<td>$\varphi_{pa}$</td>
<td>$\angle \tilde{G}_{pa}$</td>
<td>deg.</td>
</tr>
<tr>
<td></td>
<td>$\varphi_{pb}$</td>
<td>$\angle \tilde{G}_{pb}$</td>
<td>deg.</td>
</tr>
<tr>
<td>Bearings of active and reactive intensities</td>
<td>$\theta_{fab}$</td>
<td>$\tan^{-1}\left(\frac{\text{Re}(\tilde{G}<em>{pa})}{\text{Re}(\tilde{G}</em>{pb})}\right)$</td>
<td>deg.</td>
</tr>
<tr>
<td></td>
<td>$\theta_{Qab}$</td>
<td>$\tan^{-1}\left(\frac{\text{Im}(\tilde{G}<em>{pa})}{\text{Im}(\tilde{G}</em>{pb})}\right)$</td>
<td>deg.</td>
</tr>
<tr>
<td>Normalized vector components of active intensity</td>
<td>$I_{norm,a} = \frac{I_a}{</td>
<td>I_{ca}</td>
<td>} = \cos \varphi_{pa}$</td>
</tr>
<tr>
<td></td>
<td>$I_{norm,b} = \frac{I_b}{</td>
<td>I_{cb}</td>
<td>} = \cos \varphi_{pb}$</td>
</tr>
<tr>
<td>Normalized vector components of reactive intensity</td>
<td>$Q_{norm,a} = \frac{O_a}{</td>
<td>I_{ca}</td>
<td>} = \sin \varphi_{pa}$</td>
</tr>
<tr>
<td></td>
<td>$Q_{norm,b} = \frac{O_b}{</td>
<td>I_{cb}</td>
<td>} = \sin \varphi_{pb}$</td>
</tr>
<tr>
<td>Normalized total active and reactive intensity</td>
<td>$I_{norm} = \frac{\sqrt{I_a^2 + I_b^2}}{</td>
<td>I_{ca}</td>
<td>^2 +</td>
</tr>
<tr>
<td></td>
<td>$Q_{norm} = \frac{\sqrt{O_a^2 + O_b^2}}{</td>
<td>I_{ca}</td>
<td>^2 +</td>
</tr>
</tbody>
</table>
The histograms associated with a 10kHz steady-state CW signal as well as a 35ms CW pulse. The source to receiver (src/rcv) separation was 38m (125ft) for the data sets presented in Figure 5-31 through Figure Error! Reference source not found.. In these figures, the data in red indicate the test condition for which there was no obstacle between the source and receiver. The data presented in blue indicates the test condition in which the 10:1 prolate spheroid resides on the line-of-sight between the source and receiver. The autospectum corresponding to the equivalent plane wave intensity estimated with the scalar pressure sensor channel of the biaxial intensity sensor is presented in Figure 5-31 and Figure 5-32. The ITC-1007 source was driven with a variable voltage such that the sound pressure level was 190dB r. 1µPa@1m. The required drive voltage was determined with the TVR of the source and verified using a calibrated reference hydrophone located in the water. If the calibration values of the hydrophone are presumed correct, then the mean equivalent plane wave intensity can be estimated at 7.5 mW/m² for the steady state CW signal from the data in Figure 5-31 and 6.5 mW/m² for the CW pulses based upon the data in Figure 5-32. If the propagation loss from a range of 38m is removed, the received levels would predict that the source level was on the order of 

\[ 10 \cdot \log_{10} \left( \frac{0.0075 \text{ mW/m}^2}{I_{\text{ref}}} \right) + 20 \cdot \log_{10} \left( \frac{38 \text{ m}}{1 \text{ m}} \right) = 192 \text{ dB r. } I_{\text{ref}} \] 

\[ I_{\text{ref}} = 6.67 \cdot 10^{-19} \text{ W/m}^2 \text{ and } 191 \text{ dB r. } I_{\text{ref}} \] respectively. These values are in respectable agreement with those determined from the calibration phone. It should be noted that the equivalent plane wave intensity exhibits no appreciable sensitivity to the presence of the spheroid.
Figure 5-31: Histograms of $\tilde{G}_{pp}$ for src/rcv separation for 10kHz steady state CW at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent.

Figure 5-32: Histograms of $\tilde{G}_{pp}$ for a 10kHz CW pulse at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Histograms corresponding to the cross-spectra related to the active acoustic intensity in the \( \hat{a} \) and \( \hat{b} \) directions are presented for the two cases in Figure 5-33 and Figure 5-34. The co-spectra in the \( \hat{a} \) direction along the line of sight between the source and receiver does exhibit some sensitivity to the presence of the spheroid since the mean values of the two histograms are separated by approximately 2mW/m\(^2\). If the co-spectra is interpreted as the active intensity, then the mean active intensity when the spheroid is not present is approximately 4.5mW/m\(^2\) (190dB r. \( I_{ref} \)) for the steady state signal and 3mW/m\(^2\) for the CW pulses. The fact that the active intensity for the CW pulses does not match the equivalent plane wave intensity when the spheroid is absent raises questions regarding the accuracy of the measurements in the \( \hat{a} \) and \( \hat{b} \) directions or the calibration values. Since the receiver was in the far field of the source and it had been determined that the CW pulses could be recorded without any interference there is no physical reason for the two measurements to differ. This difference can most easily be explained by a calibration error in the magnitude of the sensitivity or relative phase between the hydrophone and accelerometer. In order for the active and equivalent plane wave intensities to differ by 40\% (i.e. \( \cos \phi_{pu} = .4 \)) due only to a nonzero power factor angle, the power factor angle must be \( \phi_{pu} = 1.15 \text{rad} \) or \( \phi_{pu} = 66^\circ \). This amount of phase shift is too significant for hand-waving. It is also surprising that when the spheroid is placed along the line of sight, the active acoustic intensity in the \( \hat{a} \) direction decreases by almost 3dB when the computational results predict that the active acoustic intensity should be reduced by no more than 1dB. The reduction in variance of the estimates when the spheroid is present is also strikingly apparent. The measurement with and without the
spheroid was separated by 4-6 hours on the same day without raising the probe from the water. It is possible that either the subsurface currents changed over that time period and caused the response plane of the biaxial probe to be tilted out of the horizontal plane or that rotation of the sensor may have caused it to become fouled on the tag line. The calculations have assumed that the pitch of the sensor is identically zero. Deviations from this assumption may cause the vertical sensitivity of the sensors to play a role in the measurements, but this would only modify the sensitivity values and not the relative phase between the hydrophone and accelerometer. If the sensor was in motion at the end of its tether (i.e. a pendulum), it is not unreasonable to think that it may have moved and caused the sensor to be located off its intended location on the line-of-sight. Given the scope of the line required to place the sensor at depth, a tilt of the tether as small as 2.5° from vertical could put the sensors 2m off of their desired location in the horizontal plane. Error in the experimental setup may have also placed the sensor at an incorrect depth. In fact the experimental geometry placed the midpoint of the two probe array on the line-of-sight. In the absence of other positional errors, this would place the two sensors 0.3m above and below the line-of-sight. An analytical calculation of the total active intensity field in the $\hat{\zeta}$ direction was performed in the $yz$ (vertical) plane that is orthogonal to the line-of-sight and is presented in Figure 5-35. It can be seen that the active intensity can vary by $\pm 0.5$ dB over a distance of 0.5m from the line-of-sight. The placement of the sensor in a region of destructive interference rather than constructive interference would explain the decrease in active intensity observed in the histograms when the spheroid was present. Motion of the sensor when the spheroid was not present may also have contributed to a higher variance in the data when the spheroid was not
present. The active intensity in the \( \hat{b} \) direction is negligible for both the steady-state and gated signals due to the digital coordinate rotation. Since \( I_b = 0 \), the histograms for the total active intensity \( I \) takes the same shape as \( I_a \). The bearing associated with the active acoustic intensity is identically zero for each estimate indicating that the \( \hat{a} \) direction coincides with the direction the acoustic energy is flowing from.
Figure 5-33: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz steady state CW at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure 5-34: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz CW pulse at a src/rev separation of 38m (125ft). From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure 5-35: Total active intensity in the $\hat{\xi}$ direction for a 10kHz plane wave incident upon a 10:1 rigid prolate spheroid at an incident angle of $(\theta_{inc} = 90^\circ, \varphi_{inc} = 0^\circ)$. The computational plane lies in the yz (vertical) plane perpendicular to the incident wavenumber vector which resides in the horizontal plane. The scale of the plot is in dB relative to the incident plane wave intensity.
The histograms in Figure 5-36 and Figure 5-37 depict a statistical difference in the estimates of the quad spectra in the \( \hat{a} \) and \( \hat{b} \) directions. For both the pulsed and steady-state signals, the quad spectrum in the \( \hat{a} \) direction appears to be more sensitive to the presence of the spheroid than that the quad spectra in the \( \hat{b} \) direction. The greater sensitivity is attributed to the fact that the mean values of the \( \hat{a} \) histograms are further apart than the mean values depicted in the \( \hat{b} \) histograms. The reason for this is not clear since the theoretical predictions for the case of broadside incidence would lead one to expect that the quad spectrum in the \( \hat{b} \) direction should be 10-15dB higher than that in the \( \hat{a} \) direction. The experimental measurements indicate that the quad spectrum in the \( \hat{a} \) direction is an order of magnitude larger than the \( \hat{b} \) quad spectrum. It is also interesting that the quad spectrum in the \( \hat{a} \) direction changed sign when the scattering body was present. A sign change in reactive intensity implies that the spatial gradient of the mean square pressure changed sign and that a vector oriented in the direction of decreasing acoustic pressure would subsequently be oriented in the direction of increasing acoustic pressure. Because the active and reactive intensities can change sign, the field indicators representing the magnitudes of the active and reactive intensities lose this directional information. Even though the sign of the spatial gradient was lost when constructing the magnitude of the total quad spectrum vector, this information appears to have been retained in the bearing associated with the reactive intensity vector. When the scattering body is not present, the bearing angle is approximately 0°. It is possible that this is due to a large positive pressure gradient in the \( \hat{a} \) direction. After the scattering
body is inserted into the field, the pressure gradient may have reoriented itself to the negative \( \hat{a} \) direction since the bearing associated with the quad-spectra shifts to 180°.
Figure 5-36: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{\text{paa}}$ and $\tilde{G}_{\text{pub}}$ for a 10kHz steady state CW at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure 5-37: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{pa a}$ and $\tilde{G}_{pub}$ for a 10kHz CW pulse at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
The histograms corresponding to the magnitudes and phases of the cross-spectra are shown in Figure 5-38 and Figure 5-39. The magnitudes of the complex acoustic intensity in the $\hat{a}$ direction have slightly different mean values compared to the $\hat{a}$ active acoustic intensity. The phase angle associated with the $\hat{a}$ cross-spectrum indicates that a phase shift of almost 1 radian or 60° between the pressure and particle velocity was observed when the scattering body was placed between the source and receiver. Such a significant phase difference is difficult to justify when the theoretical calculation predicted <5° of phase difference. It is again worthwhile to point out that in the absence of the spheroid, the phase angle of the $\hat{a}$ cross-spectrum should be zero for the case of an incident plane wave. The magnitude of the complex acoustic intensity in the $\hat{b}$ direction corresponds directly to the $\hat{b}$ quad-spectrum shown in Figure 5-36 and Figure 5-37 since the phase angle associated with the cross-spectra is $-\pi/2$. 
Figure 5-38: Histograms of magnitudes and phase angles of $\tilde{G}_{pa}$ and $\tilde{G}_{pb}$ for a 10kHz steady state CW at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure 5-39: Histograms of magnitudes and phase angles of $\tilde{G}_{\text{pas}}$ and $\tilde{G}_{\text{pab}}$ for a 10kHz CW pulse at a src/rcv separation of 38m (125ft). From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
and in

The last four figures contain the histograms for the normalized field indicators which represent the fraction of the cross-spectra that is attributed to the co-spectra and quad-spectra. These indicators are bounded on the interval \([-1 1]\) and are unitless. The data for the normalized co-spectra metrics are given in Figure 5-40 and Figure 5-41 while the data for the normalized quad-spectra metrics is presented in Figure 5-42 and Figure 5-43. If the sensor calibration was accurate, the phase angle of the cross-spectrum should be zero when the acoustic field is a pure plane wave. If this was the case, the appropriate normalized co-spectrum metric would be unity while the normalized quad-spectrum would be zero when the spheroid was absent. It is therefore surprising that the normalized co-spectrum estimates which reside closer to unity are associated with measurements made in the presence of a spheroid. Because these indicators involve non-linear transforms of the phase angle, any error in the sensor’s phase calibration could shift the blue points to the left while shifting the red points to the right (i.e. a \(-10^\circ\) change in \(\varphi_{pu}\)).
Figure 5-40: Histograms of normalized co-spectra for 10kHz steady state CW at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure 5-41: Histograms of normalized co-spectra for 10kHz CW pulse at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure 5-42: Histograms of normalized quad-spectra estimates for 10kHz steady state CW at a src/recv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure 5–43: Histograms of normalized quad-spectra estimates for 10kHz CW pulse at a src/rcv separation of 38m (125ft). Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
5.5 Summary

This chapter has presented the results of an experimental effort to determine if an underwater acoustic intensity sensor is able to measure a perturbation in the acoustic field that is not observable with corresponding measurements from a scalar pressure sensor alone. The histograms of the hydrophone autospectra do not indicate any sensitivity to the presence of the scattering body while the cross-speactral metrics exhibited a significant statistical difference when the spheroid was present. The experimental results cannot be directly compared to the theoretical results due to the questionable sensor calibration. It is not clear if the difficulties associated with data interpretation are exclusively due to incorrect calibration values or if perhaps the biaxial intensity sensor was not operating correctly. A post-test calibration of the sensor resulted in calibration values that were identical to those obtained prior to the field experiment. Therefore it is not believed that the sensor failed during the experiment. The non-zero power factor angles that were measured in conditions approximating free-field condition in the far-field of the source indicates a significant factor in the experiment may have been overlooked. However, since the integrity of the sensor was verified after the experiment, it is believed that the relative changes in the spectra are valid and that the cross-spectra of the hydrophone and accelerometer contains information regarding the presence of the spheroid that cannot be extracted from the autospectrum of the hydrophone. Because the scope of the experimental investigation was limited to a comparison of the scalar pressure sensor data to the data collected with the novel, underwater biaxial intensity sensor, the
objectives of the experimental portion of the thesis have been accomplished in a relative sense.
Chapter 6
Concluding Remarks

6.1 Summary

The problem considered by this body of work is the application of underwater acoustic intensity concepts to the area of underwater scattering measurements. The specific objective of this research has been to theoretically and experimentally determine if the measurement of the acoustic intensity field provided new or more reliable information regarding the presence of a scattering body of those obtained with scalar pressure sensors alone.

The theoretical work conducted in support of the objective was successful in computing the total acoustic intensity field generated by the scattering of a steady-state, harmonic plane wave by a 10:1 rigid prolate spheroid. The computations predicted that in the forward scatter direction, the $\hat{\eta}$ reactive intensity would have values of 25-30dB below the incident acoustic intensity when the scattering body was present. This same variable would also be identically zero in the absence of the spheroid because the acoustic field would be composed only of the plane wave. Similarly, the power factor angle associated with the $\hat{\eta}$ complex intensity should shift from 0° to 4° when the spheroid is placed in the steady-state field. In contrast, the $\hat{\xi}$ active intensity or pressure field exhibited fluctuations of ±0.5dB rel. to incident acoustic intensity due to the
scattering of the spheroid. The $\hat{\eta}$ intensities were therefore more promising indicators of the presence of the spheroid.

Experimental work conducted in support of the thesis objective was performed in order to determine if a novel biaxial underwater acoustic intensity sensor could measure perturbations in the acoustic intensity field made by a spheroid. A model spheroid was placed on the line-of-sight between the source and sensor to emulate the rigid body from the theoretical work. A significant amount of data was collected during the experiment allowing the intensity estimates to be presented as histograms. The histograms allowed the investigator to be certain that any perturbations attributed to the presence of the spheroid were not due to random error. The two data sets discussed in Chapter 5.4 permit the conclusion that the pressure sensor estimates did not exhibit any statistically significant difference when the spheroid was placed on the line-of-sight between the source and receiver. In contrast, acoustic intensity estimates from the same data sets did exhibit a significant change between the presence and absence of the spheroid. The reactive intensity, power factor angle, and tangential active acoustic intensity estimates did not agree with the theoretical predictions. The difference between the theoretical and experimental results is attributed to the intensity sensor and experimental conditions which do not match the ideal conditions used to generate the theoretical results. This mismatch does not diminish the experimental results because the scope of the research was limited to a comparison of the intensity estimates in the presence and absence of the spheroid rather than a comparison of the intensity estimates absolute reference intensity.
6.2 Future Work

Future work regarding this research should investigate the reasons for the mismatch between the theoretical and experimental work. In order to perform a comprehensive verification of the theoretical results, constraints that were imposed upon this investigation should be removed. The assets available for this investigation constrained the scattering body to be man-portable. This led to the selection of the 10:1 fineness ratio prolate spheroid with major axis of 2m. The experiment, therefore, had to be performed at frequencies above 10kHz in order to achieve the values of reduced frequencies at which theoretical work indicated the phenomenon manifested. The measurement of acoustic intensity is very difficult at these frequencies. The spatial scale of the experiments should be increased by an order of magnitude so as to work with scattering bodies that have dimensions on the order of 20m-50. Increasing the size of the scattering body would permit the measurement of acoustic intensity at 500Hz-1kHz. This is a practical frequency range for the inertial measurement of acoustic particle velocity. One drawback is that the area in which the experiment would be conducted would also have to increase in dimension to replicate a unbounded medium.

Since it is unlikely that a intensity sensor will be constructed that can match the performance of the one used in the theoretical computations, it may be advantageous to use repeat the theoretical work with a model of an imperfect sensor. The modeling should also consider the significance of placing the scattering body in a waveguide or in an underwater acoustic channel. Real environments may generate vertical and horizontal gradients in the acoustic field that could interfere with the ability to observe the
perturbations caused by the scattering body. Such a study would help determine how robust the phenomenon is to imperfect sensors and environments.

Lastly the theoretical framework for this acoustic intensity study has assumed that the incident signal and the scattered signal reside at the same frequency for destructive interference. If relative motion of the source, receiver, or scatterer is allowed, the incident and scattered signal may have different frequencies. In certain cases, the two signals may be resolved during frequency domain processing and not interfere.
Bibliography


Appendix A

Computational Details with Scattering from a Rigid Prolate Spheroid

A.1 Introduction

The numerical techniques employed to solve for the scattered acoustic field in Chapter 3 are founded in the ability to exactly solve the prolate spheroid harmonics at reduced frequencies of $h$ that approach 50. This appendix discusses how the numerical codes developed by Van Buren and Boisvert were employed to solve for the field values presented in Chapter 3.

The calculation and summation of the partial waves was performed in MATLAB using double precision arithmetic. In order to calculate any individual partial wave in Eq. 3.18 and Eq. 3.22, the prolate spheroidal angle function, the radial function, as well as their derivatives had to be computed. Van Buren and Boisvert at the Naval Undersea Warfare Center Division Newport have recently developed techniques to accurately calculate the prolate spheroidal harmonics and have implemented those techniques in FORTRAN 90 using quad-precision arithmetic. These FORTRAN subroutines are grouped into a single package referred to as PROFCN which is able to compute the spheroidal harmonics of order $m = 100$ with $h > 100$ and a specified minimum number of accurate digits for the radial spheroidal calculations. This FORTRAN package is intended to supersede the FORTRAN 77 packages PRAD and PANG which were more limited in their ability to evaluate the prolate spheroidal functions over their parameter
The FORTRAN code for the PROFCN package was made available to the author by Van Buren and was employed to compute the analytical results presented in Chapter 3.

The PROFCN subroutine package is designed to be called from a main program written in FORTRAN 90 and will return the values of the prolate spheroidal functions corresponding to the requested order $m$, degree $n$, reduced frequency $h$, and spheroidal coordinate $\xi$ or $\eta$. In order to call PROFCN from MATLAB, a FORTRAN wrapper was constructed to pass address locations of variables between the MATLAB environment and the FORTRAN subroutine. The FORTRAN code was then compiled under MATLAB with a Lahey-Fujitsu Fortran 95 compiler as a MEX file which results in a dynamic linked library under the Windows operating system. The advantage of this approach is that the ability to compute the spheroidal functions with quad-precision arithmetic under Lahey Fortran was retained. The MATLAB scripts and functions which call PROFCN are passed a double-precision floating point number. The quad-precision was only needed to ensure accurate calculations of the spheroidal functions and was not needed to perform the inner and outer summations in Eq. 3.18 and Eq. 3.22.

The MATLAB script written to perform the computations in Chapter 3 accepts input parameters from the user regarding the rigid spheroid, the incident plane wave, and the computation grid in the $(\xi, \eta, \varphi)$ coordinate system. The inner and outer summations were generally carried out to a value of $m = n = 50$ over the domain of the computational grid. The computations were not carried within a body length of the spheroid because the investigation was principally concerned with the far-field behavior of the acoustic
intensity field. After the partial waves were evaluated, the results were periodically analyzed to ensure that the summation had converged sufficiently and that additional partial waves contributed <<10% to the field value at those locations. These procedures enabled $p(\xi, \eta, \varphi)$ and $\bar{u}(\xi, \eta, \varphi)$ to be evaluated over the computational domain which enabled the complex intensity to be computed via Eq. 2.28.

In order to adequately present the results as the images shown in Chapter 3, a regular Cartesian grid was constructed within the computational domain and the corresponding acoustic quantity was estimated by linearly interpolating the field field values computed on the regular $(\xi, \eta, \varphi)$ grid. The computational grid was dense enough to allow this procedure and not introduce significant error. Scatterplots of the field values on the $(\xi, \eta, \varphi)$ were compared to images constructed on the regular Cartesian grid to ensure that anomalies were not introduced.
Appendix B

Histograms of Cross-Spectral Estimates from Seneca Lake Experiment

The following figures present additional data that were collected during the experiment at Seneca Lake. Due to the volume of data, these figures were not included in Chapter 5. As discussed in Chapter 5, the biaxial sensor does not appear to respond in the same manner as it did in the ARL Anechoic Test Facility and therefore the calibration values may not be accurate. For this reason the figures are not labeled in quantities of acoustic intensity and instead are presented in terms of their auto-spectra and cross-spectra. The spectral values presented here have been computed using the calibration values of \( M_p \), \( M_{ax} \), \( \phi_{xcal} \), \( M_{ay} \), and \( \phi_{ycal} \) which were estimated at the ARL Test Facility and presented in Figure 4-15, Figure 4-17, and Figure 4-19. The autospectra of the hydrophone voltage output, \( G_{pp} \), is converted to \( \tilde{G}_{pp} \) according to Eq. B.1. Denoting the cross-spectra between voltage time series of the hydrophones and accelerometers as \( pax \) and \( pay \), the calibration values are applied to these cross-spectra according to Eq. B.2 and Eq. B.3 for the assumed time dependence of \( e^{-i\omega t} \). If the calibration values are accurate, \( \tilde{G}_{pp} \) could be identified as the equivalent plane wave intensity while \( \tilde{G}_{pax} \) and \( \tilde{G}_{pay} \) could be identified as the complex intensity estimates in the \( \hat{x} \) and \( \hat{y} \) directions. Even though the application of the calibrations might allow the units of \( \tilde{G}_{pp} \), \( \tilde{G}_{pax} \), and \( \tilde{G}_{pay} \) to be identified as \( \text{Watts}/m^2 \) rather than in terms of voltages, it would be
inappropriate to identify these quantities as acoustic intensities due to concerns about the in-situ behavior of the sensors at Seneca Lake. These figures are documented here only for completeness. Many of the quantities presented in these figures are based upon $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ which are cross-spectra that have been generated by rotating $\tilde{G}_{pax}$ and $\tilde{G}_{pay}$ to $\hat{a}$ and $\hat{b}$ directions in a manner similar to Eq. 5.3. In all figures, the blue histograms represent estimates made when the spheroid was present while the red histograms are estimates when the spheroid is not present.

\[
\tilde{G}_{pp} = \frac{G_{pp}}{M_p^2 \rho c}
\]

\[
\tilde{G}_{pax} = \frac{G_{pax}}{M_p M_{ax} (-i\omega)} e^{i\phi_{peal}}
\]

\[
\tilde{G}_{pay} = \frac{G_{pay}}{M_p M_{ay} (-i\omega)} e^{i\phi_{peal}}
\]
Figure B-1: Histograms of $\tilde{G}_{pp}$ for 8kHz steady state CW at a src/rcv separation of 38m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-2: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 8kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-3: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 8kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-4: Histograms of magnitudes and phase angles of $\tilde{G}_{pa}$ and $\tilde{G}_{pb}$ for 8kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-5: Histograms of $\tilde{G}_{pp}$ for 12kHz steady state CW at a src/rcv separation of 38m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-6: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-7: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-8: Histograms of magnitudes and phase angles of $\tilde{G}_{pa}$ and $\tilde{G}_{pb}$ for 12kHz steady state CW at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-9: Histograms of $\tilde{G}_{pp}$ for 8kHz CW pulse at a src/rcv separation of 38m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-10: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for an 8kHz CW pulse at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-11: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pub}$ for 8kHz CW pulse at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-12: Histograms of magnitudes and phase angles of $\tilde{G}_{pa}$ and $\tilde{G}_{pb}$ for 8kHz CW pulse at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-13: Histograms of $\hat{G}_{pp}$ for 12kHz CW pulse at a src/rev separation of 38m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-14: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 12kHz CW pulse at a src/rev separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-15: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz CW pulse at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-16: Histograms of magnitudes and phase angles of $\tilde{G}_\text{paa}$ and $\tilde{G}_\text{pab}$ for 12kHz CW pulse at a src/rcv separation of 38m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-17: Histograms of $\tilde{G}_{pp}$ for 8kHz steady state CW at a src/rcv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-18: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for an 8kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-19: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 8kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-20: Histograms of magnitudes and phase angles of $\tilde{G}_{\text{paa}}$ and $\tilde{G}_{\text{pab}}$ for 8kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-21: Histograms of $\tilde{G}_{pp}$ for 10kHz steady state CW at a src/rcv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-22: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-23: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 10kHz steady state CW at a src/rev separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-24: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 10kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-25: Histograms of $\tilde{G}_{pp}$ for 12kHz steady state CW at a src/rcv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-26: Histograms of quantities derived from the co-spectra of $\tilde{G}_{\text{paa}}$ and $\tilde{G}_{\text{pab}}$ for a 12kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-27: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-28: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz steady state CW at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-29: Histograms of $\tilde{G}_{pp}$ for 8kHz CW pulse at a src/rcv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-30: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 8kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-31: Histograms of quantities derived from the quad-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 8kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-32: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 8kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-33: Histograms of $\tilde{G}_{pp}$ for 10kHz CW pulse at a src/rev separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-34: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 10kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-35: Histograms of quantities derived from the quad-spectra of $\hat{G}_{paa}$ and $\hat{G}_{pab}$ for 10kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-36: Histograms of magnitudes and phase angles of $\tilde{G}_{\text{paa}}$ and $\tilde{G}_{\text{pab}}$ for 10kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-37: Histograms of $\tilde{G}_{pp}$ for 12kHz CW pulse at a src/recv separation of 53m. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-38: Histograms of quantities derived from the co-spectra of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for a 12kHz CW pulse at a src/rev separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-39: Histograms of quantities derived from the quad-spectra of $G_{paa}$ and $G_{pab}$ for 12kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude in the $a$ direction, the magnitude in the $b$ direction, their vector sum, and the bearing of the vector sum in degrees. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
Figure B-40: Histograms of magnitudes and phase angles of $\tilde{G}_{paa}$ and $\tilde{G}_{pab}$ for 12kHz CW pulse at a src/rcv separation of 53m. From top to bottom the histograms represent the magnitude and phase angle in the $a$ direction followed by magnitude and phase angle in the $b$ direction. Phase angles are presented in radians. Blue and red histograms respectively represent estimates made when the spheroid was present and absent.
VITA

Brian Richard Rapids

Brian Rapids received his Bachelor of Science in physics from Worcester Polytechnic Institute in Worcester, MA in 1994. He continued his studies as a research assistant in the Geoacoustics Laboratory in the Applied Marine Physics Department of the Rosenstiel School of Marine and Atmospheric Science at the University of Miami in Miami, FL. The Master of Science degree was awarded to him in 1996 for his work in small scale tomographic studies of marine sediments. From there he went on to work at the Advanced Systems Division of Sanders, A Lockheed Martin Company in Manchester, NH as a systems engineer. His work there included the development of prototype optical and acoustic anti-submarine warfare and mine countermeasure systems. In 1999, Brian left Sanders to pursue his PhD work in sonar signal processing and underwater acoustics as a student at the Pennsylvania State University while working at the Applied Research Laboratory developing advanced guidance and control technology for the Anti-Torpedo Torpedo.