ESSAYS IN INTERNATIONAL TRADE AND MIGRATION

A Dissertation in Economics by Gary Lyn

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The dissertation of Gary Lyn was reviewed and approved* by the following:

Andrés Rodríguez-Clare  
Professor of Economics  
Chair of Committee  
Dissertation Advisor

Jonathan Eaton  
Professor of Economics

Stephen Yeaple  
Associate Professor of Economics

David Abler  
Professor of Agricultural, Environmental, Regional Economics and Demography

Vijay Krishna  
Distinguished Professor of Economics  
Director of Graduate Studies

*Signatures are on file in the Graduate School.
Abstract

This dissertation consists of three chapters.

Chapter 1: External Economies, and International Trade Redux: Comment (with Andrés Rodríguez-Clare)

Grossman and Rossi-Hansberg (2010, henceforth GRH) propose a novel way to think about the implications of international trade in the presence of national external economies at the industry level. Instead of perfect competition and two industries in the standard model, they assume Bertrand competition in a continuum of industries, and argue that the “pathologies” of the standard treatment eliminated. In particular, the equilibrium is typically unique with trade patterns consistent with “natural” comparative advantage. For high transportation costs, GRH posits a mixed strategy equilibrium in which firms in the country with the comparative advantage targets the world market by posting a pair of discrete prices in one state, and targets only the domestic market in the other state.

We first show that there is a profitable deviation to this conjectured equilibrium, and propose an alternative. We also provide a formal analysis of the equilibrium configurations under different transportation costs for a single industry that exhibits Marshallian externalities, and demonstrate that when transportation costs are low, the equilibrium is not necessarily unique. The implications are that trade patterns need not be consistent with natural comparative advantage, and, more importantly, a theoretical basis for industrial policy arises.
Chapter 2: Marshallian Externalities, Comparative Advantage, and International Trade (with Andrés Rodríguez-Clare)

There is strong evidence for the existence of external economies of scale that are limited in their industrial and geographical scope. What are the implications of these Marshallian externalities for the patterns of international trade, the welfare gains from trade, and industrial policy? The standard model in the literature assumes that firms engage in perfect competition and ignore the effect of their actions on industry output and productivity. This has the unfortunate implication that any assignment of industries across countries is consistent with equilibrium.

To avoid this predicament, we follow Grossman and Rossi-Hansberg (2010) and assume that firms in each industry engage in Bertrand competition and understand the implications of their decisions on industry output and productivity. We develop three main results.

First, we show that the indeterminacy of international trade patterns still persists for some industries when trade costs are low.

Second, we apply these results in a full general equilibrium analysis and reexamine the implications of Marshallian externalities for industrial policy. Our results indicate that the additional welfare gains from moving to the Pareto-superior equilibrium depend positively on the strength of Marshallian externalities and negatively on the strength of comparative advantage, and - using reasonable parameter estimates - are at most about 2%.

Finally, our framework allows us to ask whether Marshallian externalities lead to additional gains from trade. Our quantitative analysis indicates that this is indeed the case, and that Marshallian externalities increase overall gains from trade by around 50%.
Chapter 3: Brain Drain, Brain Return and the Role of on-the-Job Learning

This paper contributes to the literature on skilled migration by providing an alternative framework which examines the welfare effects of skilled emigration on a small developing country. In particular, we incorporate the fact that some skilled emigrants do return to the developing country after acquiring expertise abroad by way of on-the-job learning.

Ignoring return migration in our model yields results consistent with a “brain drain” for the developing country. Once we account for return migration the results are more nuanced. If the cost of return is sufficiently low to attract skilled emigrants with enhanced foreign expertise, then skilled emigration can lead to a “beneficial brain drain” for the developing country.
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Chapter 1

External Economies and International Trade Redux: Comment

1.1. Introduction

Grossman and Rossi-Hansberg (2010, henceforth GRH) propose a novel way to think about the implications of international trade in the presence of national external economies at the industry level. Instead of a model with two industries and perfect competition, as in the standard treatment in the literature, GRH postulate a model with a continuum of industries and Bertrand competition. These authors argue that with these alternative assumptions the “pathologies” of the standard treatment are eliminated. In particular, the equilibrium is typically unique with trade patterns consistent with “natural” comparative advantage. This is most clearly the case with frictionless trade, which the authors use to make their main points.

The authors then explore how the results change when trade costs are positive. They show that the previous results remain valid when trade costs are low enough, while for high enough trade costs the equilibrium entails no trade. However, there is a range of high trading costs for which there exists no equilibrium in pure strategies. For this case, GRH postulate a mixed-strategy equilibrium in which firms in the country with the comparative advantage mix over two strategies: the global strategy, in which firms target the world market with a pair of prices (one for the home market and one for the foreign market), and the domestic strategy, in which firms target only
the domestic market. Firms in the other country pursue a more “passive” strategy of always only targeting their domestic market.

In this note we first show that there is a profitable deviation to the mixed-strategy equilibrium postulated by GRH, and then propose an alternative set of strategies and establish that they constitute an equilibrium. The main difference is that firms in the country with the comparative advantage, when targeting the global market, randomize across a continuum of prices rather than posting a single price for the domestic market. However, this mixed-strategy equilibrium applies only to industries in which a country has a “superior” comparative advantage. Otherwise, firms in the country without comparative advantage and it profitable to deviate and target the global market. Characterizing the equilibrium for the case with intermediate trade costs and “non-superior” comparative advantage remains for future research.

As part of this note, we also provide a formal analysis of the equilibrium configurations under different trade costs for a single industry that exhibits Marshallian externalities. We confirm GRH’s result that for low trade costs there is complete specialization, for high enough trade costs there is no trade, and that there is a range of high trade costs for which there is no equilibrium in pure strategies. However, in contrast to GRH, we demonstrate that when trade costs are low, the equilibrium entails complete specialization, but is not necessarily unique. This implies that trade patterns need not be consistent with natural comparative advantage, and hence, as in the standard treatment in the literature, national external economies at the industry level still offer a theoretical basis for industrial policy. We explore these issues further in the subsequent Chapter.
1.2. Basic Assumptions

We consider a partial equilibrium version of the model presented by GRH. There are two countries, Home and Foreign, labor is the only factor of production, wages are exogenous and fixed at $w$ and $w^*$, and we focus on a single good with demand curves $x(p)$ and $x^*(p^*)$ that have a constant price elasticity of $\sigma$. The production technology has constant or increasing returns to scale due to external economies at the local level. In Home $a/A(X)$ units of labor are required to produce a unit of output, where $a > 0$ is an exogenous productivity parameter, $X$ is the total production of the good in Home net of trade cost, and $A(X)$ is non-decreasing and concave. Similarly, the labor unit requirement in Foreign is $a^*/A(X^*)$. There are $n, n^* = 2$ producers in Home and Foreign, respectively.\(^1\) Trade costs are of the “iceberg” type, so that delivering a unit of the good from one country to the other requires shipping $\tau > 1$ units. Markets are segmented, so that firms can set arbitrarily different prices across the two markets. Firms in each industry engage in Bertrand (price) competition in each market.

For most of our results, we will use the following assumption about the demand functions and economies of scale that are not in GRH:

**Assumption 1.2.1.** (a) Preferences are Cobb-Douglas, so $\sigma = 1$ and hence $x(p) = E/p$ and $x^*(p^*) = E^*/p^*$; (b) $A(X) = X^\phi$; and (c) $0 \leq \phi < 1/20 \leq \phi < 1/2$.

Also, for our analysis, it is sufficient to restrict the range of transportation costs as follows:

**Assumption 1.2.2.** Transportation costs $t_i$ are bounded above by $1/\phi$ for every $i$.

\(^1\)GRH assumes an arbitrary finite number of producers in each industry and country. For simplicity, we assume two firms.
Finally, for simplicity we restrict the analysis to the case in which demand is symmetric in the two countries, namely \( E = E^* \). Formally,

**Assumption 1.2.3.** Demand is symmetric in the two countries: \( E = E^* \). Without loss of generality we set \( E \) and \( E^* \) to one.

### 1.3. Equilibrium with Costly Trade

As in GRH, we study three types of equilibria: complete specialization (i.e., firms from one country supply both markets); mixed strategy equilibria (i.e., firms from one country randomize over which markets to serve and the price to charge, while firms from the other country offer to serve their own market at the autarky price); no trade (i.e., firms from each country serve only their own market). We start by considering the possibility of an equilibrium with complete specialization.

#### 1.3.1 Complete specialization equilibrium

If Home serves both markets, the equilibrium entails prices \( p \) and \( p^* = tp \), with \( p \) determined by\(^2\)

\[
p = \frac{w_a}{(x(p) + x^*(tp))^\phi}.
\]

(1.1)

To establish that this is an equilibrium, we need to consider the possible deviations by Home and Foreign firms.

A preliminary result is that the best that any firm can do is to shave prices \( p \) and \( p^* \), i.e., it is never optimal to charge strictly lower prices than \( p \) and \( p^* \). We can show

\(^2\)Given our assumption on demand, the assumption \( 0 \leq \phi < 1/2 \) provides a sufficient condition for the existence of \( p \). In principle, there could be an additional equilibrium at \( p = 0 \) if \((X)^\phi \) is not bounded above. In what follows we ignore this possibility.
that Assumption 1.2.2 (along with our assumption on demand) imply that the best possible deviation for a Home or Foreign firm entails shaving prices \( p \) and \( p^* \) (see the Appendix for a formal statement and proof of this result).

Consider now a deviation by a Home firm. Since prices \( p \) and \( p^* \) with \( p \) given by (1.1) imply that Home firms make zero profits in both markets, then Assumption 1.2.1(c) guarantees a Home firm cannot make positive profits with any alternative set of prices. So this establishes that there is no profitable deviation for a Home firm.

Turning to Foreign firms, we need to study two possible deviations: first, no firm from Foreign should find it optimal to take over the world market by undercutting Home firms in both markets, and second, no firm from Foreign should find it optimal to displace Home firms from the Foreign market. Writing \( x \) and \( x^* \) as shorthand for \( x(p) \) and \( x^*(tp) \), with \( p \) implicitly defined by (1.1), a sufficient condition for it to be unprofitable for Foreign firms to take over both markets is given by

\[
\left[ \frac{wa}{(x+x^*)^\phi} - \frac{w^*a^*t}{(x+x^*)^\phi} \right] x + \left[ \frac{wat}{(x+x^*)^\phi} - \frac{w^*a^*}{(x+x^*)^\phi} \right] x^* \leq 0. \tag{1.2} \]

In turn, a sufficient condition for it to be unprofitable for Foreign firms to displace Home firms from the Foreign market (only) is

\[
\left[ \frac{wat}{(x+x^*)^\phi} - \frac{w^*a^*}{(x^*)^\phi} \right] x^* \leq 0. \tag{1.3} \]

The previous arguments establish that if conditions (1.2) and (1.3) are satisfied for \( p \) given by the solution of (1.1), then there is an equilibrium with complete specialization in which Home firms serve both markets. Similarly, an equilibrium with complete specialization where Foreign serves both markets would have prices \( p \) and
\( p^* \), where \( p = tp^* \) and \( p^* \) is implicitly defined by

\[
p^* = \frac{w^*a^*}{(x(tp^*) + x^*(p^*))^{\phi}}.
\]

(1.4)

This is an equilibrium if the following two conditions are satisfied:

\[
\left[ \frac{w^*a^*t}{(x+x^*)^{\phi}} - \frac{wa}{(x+x^*)^{\phi}} \right] x + \left[ \frac{w^*a^*}{(x+x^*)^{\phi}} - \frac{wat}{(x+x^*)^{\phi}} \right] x^* \leq 0,
\]

(1.5)

\[
\left[ \frac{w^*a^*t}{(x+x^*)^{\phi}} - \frac{wa}{(x)^{\phi}} \right] x \leq 0.
\]

(1.6)

**Proposition 1.3.1.** Let \( \omega = w/w^* \) and \( \beta = a/a^* \). By Assumptions 1.2.1a and Assumption 1.2.3, conditions (1.2), (1.3), (1.5) and (1.6) can be written as

\[
\beta \omega \leq \frac{t+1/t}{2} \equiv g(t),
\]

(1.7)

\[
\beta \omega \leq t^{-1}(1+t)^{\phi} \equiv h(t)
\]

(1.8)

and

\[
\beta \omega \geq \frac{2}{1/t + t} \equiv g^*(t),
\]

(1.9)

\[
\beta \omega \geq t(1+t)^{-\phi} \equiv h^*(t),
\]

(1.10)

respectively. Let \( t_{\text{MAX}} \equiv 1/\phi \), and let \( \tilde{t} \) be implicitly defined by \( g(t) = h(t) \). Let \( t_{CS}(\beta \omega) \) and \( t_{CS}^*(\beta \omega) \) be implicitly defined by \( \beta \omega = h(t) \) and \( \beta \omega = h^*(t) \), respectively, and let \( t_0^*(\beta \omega) \) be implicitly defined by \( \beta \omega = g^*(\tilde{t}) \). Assume that \( \beta \omega < 1 \) (so that Home has a comparative advantage).\(^3\) There are two cases. Case a: Home has a strong comparative advantage, i.e., \( \beta \omega < g^*(\tilde{t}) \). Then for \( t \in [1, \min \{t_{CS}(\beta \omega), t_{\text{MAX}}\}] \) there

\(^3\)Note that \( \beta \omega < 1 \) implies \( wa < w^*a^* \). Since countries are symmetric, this is without loss of generality.
is a unique equilibrium with complete specialization, and this equilibrium has Home serving both markets (See Figure 1.1(a)). Case b: Home has a weak comparative advantage, i.e., \( g^*(\tilde{t}) \leq \beta \omega < 1 \). Then for \( t \in [1, t^*_0(\beta \omega)] \cup [t^*_{CS}(\beta \omega), t_{CS}(\beta \omega)] \) there is a unique equilibrium with complete specialization, and this equilibrium has Home serving both markets, whereas for \( t \in [t^*_0(\beta \omega), t^*_{CS}(\beta \omega)] \) there are two complete specialization equilibria, one with Home serving both markets, and another with Foreign serving both markets (See Figure 1.1(b)).

1.3.2 Equilibrium with no trade

Let’s now consider the conditions for there to be an equilibrium with no trade. This equilibrium would have the Home price \( p_A = \frac{w a}{(x(p_A))^\phi} \), and Foreign price \( p^*_A = \frac{w^* a^*}{(x^*(p^*_A))^\phi} \). The condition necessary for this to be an equilibrium is that neither Home nor Foreign firms find it profitable to sell in both markets. As explained above, Assumption 1.2.2 implies that the best deviation would be to charge the highest possible price while serving both markets. Thus, writing \( x_A \) and \( x^*_A \) as shorthand for \( x(p_A) \) and \( x^*(p^*_A) \), respectively, the condition that Home firms do not make profits from this deviation is

\[
\left[ \frac{w a}{(x_A)^\phi} - \frac{w a}{(x_A + x^*_A)^\phi} \right] x_A + \left[ \frac{w^* a^*}{(x^*_A)^\phi} - \frac{w a t}{(x_A + x^*_A)^\phi} \right] x^*_A \leq 0. \tag{1.11}
\]

If (1.11) is satisfied, an equilibrium with no trade would be immune to a deviation by a Home firm targeting both markets. Using Assumptions 1.2.1a and Assumption 1.2.3 this can be rewritten as

\[
t \geq \frac{2 \left( 1 + (\beta \omega)^{1/(1-\phi)} \right)^\phi - 1}{(\beta \omega)^{1/(1-\phi)}} \equiv t_{NT}(\beta \omega).
\]
Similarly, the condition necessary for Foreign firms not to make profits from a deviation to sell in both markets is given by

\[ t \geq \frac{2 \left(1 + (\beta \omega)^{-1/(1-\phi)}\right)^\phi - 1}{(\beta \omega)^{-1/(1-\phi)}} \equiv t^*_{NT}(\beta \omega). \]

It is easy to show that \( \beta \omega < 1 \) implies \( t_{NT}(\beta \omega) > t^*_NT(\beta \omega) \), and hence both conditions for non-tradability are satisfied if and only if \( t \geq t_{NT}(\beta \omega) \). This establishes the following result:

**Proposition 1.3.2.** Assume that \( \beta \omega < 1 \). An equilibrium with no trade exists if and only if

\[ t \geq t_{NT}(\beta \omega). \quad (1.12) \]

Since \( t_{NT}(.) \) and \( t^*_{NT}(.) \) are monotonic, their inverse is well defined. Letting \( l(t) \equiv (t_{NT})^{-1}(t) \) and \( l^*(t) \equiv (t^*_{NT})^{-1}(t) \), the conditions \( t \geq t_{NT}(\beta \omega) \) and \( t \geq t^*_{NT}(\beta \omega) \) are equivalent to \( \beta \omega \geq l(t) \) and \( \beta \omega \geq l^*(t) \). Figure 2 illustrates.

Figure 1.1: Complete specialization equilibria with low trade costs, \( \phi = 0.3 \).
1.3.3 Equilibrium with mixed strategies

GRH argue that there is a range of high transportation costs for there is no equilibrium in pure strategies. Our analysis confirms that this is indeed the case. The curve $l(t)$ is decreasing and intersects the horizontal line with $\beta \omega = 1$ at point $t_{NT}(1)$. It is readily verified that $t_{CS}(1) < t_{NT}(1)$. Moreover, as shown in the Appendix, the curve $h(t)$ is always below the curve $l(t)$, so $t_{CS}(\beta \omega) < t_{NT}(\beta \omega)$. This implies that, given $\beta \omega \leq 1$, there is no pure strategy equilibrium for $t \in (t_{CS}(\beta \omega), t_{NT}(\beta \omega))$. In other words, condition (1.2) is satisfied but conditions (1.3) and (1.11) are not – the violation of (1.3) implies that complete specialization in Home is not an equilibrium because Foreign firms would deviate to displace Home firms from their local market, and the violation of (1.11) implies that no trade is not an equilibrium because Home firms would deviate and seize both markets.

GRH argue that for this region of high trading costs there exists an equilibrium in which Home firms randomize between a strategy that leads to only sales in Home (the local strategy) and a strategy that ensures sales in both markets (the global strategy). The challenge in constructing such an equilibrium is that Home sales entail a profit while sales in Foreign entail a loss, so Home firms would be tempted to shave the Home price and charge a high price in Foreign, in that way capturing all the profits associated with local sales and avoiding the losses in the Foreign market. In fact, the equilibrium proposed by GRH can be shown to allow for a profitable deviation where a Home firm slightly shaves the Home price in the global strategy thereby appropriating all the profits in Home and making positive expected profits.
1.3.3.1 Profitable Deviation to the Mixed Strategy proposed by GRH

GRH propose an equilibrium in which Foreign firms do not export and charge a price \( p_A^* \) while Home firms mix between a local strategy (no export) with \( p_A \) and a global pricing strategy, where firms charge price \( p_A^* \) in Foreign and a price \( p_G \) in Home that satisfies \( \Phi(p_G) + \Phi^*(p_G) = 0 \), where \( \Phi(p_G) \) and \( \Phi^*(p_G) \) are defined by

\[
\Phi(p_G) = \left[ p_G - \frac{w a}{(x(p_G) + x^*(p_A^*))^\phi} \right] x(p_G)
\]

and

\[
\Phi^*(p_G) = \left[ p_A^* - \frac{w a t}{(x(p_G) + x^*(p_A^*))^\phi} \right] x^*(p_A^*).
\]

Figure 1.2: Equilibrium with no trade, \( \phi = 0.3 \)
As a first step, we show that \( \Phi^*(p_G) < 0 \), implying that Home firms make losses in Foreign. To see this, let

\[
\pi(p, p^*) \equiv \left[p - \frac{wa}{x(p) + x^*(p^*)^\phi}\right] x(p) + \left[p^* - \frac{w a t}{x(p) + x^*(p^*)^\phi}\right] x^*(p^*),
\]

and note that \( \Phi(p_G) + \Phi^*(p_G) = 0 \) can be written as \( \pi(p_G, p_A^*) = 0 \). Let’s imagine for a second that \( p_G = p_0 \), where \( p_0 \) is defined implicitly by

\[
p_0 = \frac{wa}{(x(p_0) + x^*(tp_0))^\phi}.
\]

In this case we would have \( \pi(p_0, tp_0) = 0 \) – if Home firms charged prices \( p_0 \) and \( tp_0 \) then they would indeed make zero profits. But the violation of condition (1.3) implies that \( tp_0 > p_A^* \), so charging \( tp_0 \) in Foreign cannot be part of an equilibrium. Instead, the proposed strategy is to charge \( p_G \) in Home and \( p_A^* \) in Foreign – with \( p_G = p_0 \), this means prices \( p_G \) in Home and \( p_A^* \) in Home, leading to profits \( \pi(p_0, p_A^*) \).

Our result that profits are increasing in prices (i.e., the best that a deviating firm can do is to shave current prices) implies that \( \pi_2 > 0 \), so \( \pi(p_0, tp_0) = 0 \) implies that \( \pi(p_0, p_A^*) < 0 \). It is easy to see that \( \pi(p_0, p_A^*) = \Phi(p_0) + \Phi^*(p_0) \), hence we can conclude that \( \Phi(p_0) + \Phi^*(p_0) < 0 \). But \( p_A^* < tp_0 \) implies that

\[
\Phi(p_0) \equiv \left[p_0 - \frac{wa}{(x(p_0) + x^*(p_A^*))^\phi}\right] x(p_0) > \left[p_0 - \frac{wa}{(x(p_0) + x^*(tp_0))^\phi}\right] x(p_0) = 0,
\]

hence \( \Phi(p_0) > 0 \). Combined with \( \Phi(p_0) + \Phi^*(p_0) < 0 \), we then conclude that \( \Phi^*(p_0) < 0 \).

Since \( p_G \) is defined by \( \pi(p_G, p_A^*) = 0 \) then the fact that \( \pi_1 > 0 \) implies that \( p_G > p_0 \). But since \( \Phi'' < 0 \), we finally conclude that \( \Phi^*(p_G) < 0 \).

As a second step, we show that \( \Phi^*(p_G) < 0 \) implies that there exists a profitable
deviation to the proposed strategy. If the probability of choosing the local strategy is $q$, the expected profits made by a Home firm under the global strategy are $(\Phi(p_G) + \Phi^*(p_G)) \left( q + \frac{1-q}{2} \right) = 0$. Now consider a deviation to a pure strategy with price in Foreign equal to $p_A^*$ and the local price just below $p_G$, say at $p' = p_G - \varepsilon'$. The profits under the deviation are $q [\Phi(p') + \Phi^*(p')] + (1-q) [\Phi(p') + \Phi^*(p')/2]$. Since $p' \approx p_G$ then $\Phi(p') + \Phi^*(p') \approx 0$ and $\Phi^*(p') \approx -\Phi(p')$, hence profits under this deviation are close to $(1-q) \Phi(p')/2$, and this is positive. Intuitively, by charging a slightly lower price in the domestic market, a Home firm secures all the profits from Home sales while not incurring more losses in Foreign.

1.3.3.2 An Alternative Mixed Strategy

We now propose an alternative mixed strategy equilibrium that holds when Home has a “superior comparative advantage”, where we use “superior” rather than “strong” (used before) because the two concepts are different. We say that Home has a superior comparative advantage if $\beta \omega < l^*(\hat{t})$, where $\hat{t}$ is defined implicitly by $h(\hat{t}) = l^*(\hat{t})$ (see Figure 1.3).

Assume again that (1.2) is satisfied, whereas (1.3) and (1.11) are both violated. Let $\Phi(p)$ and $\Phi^*(p)$ be the profits made in Home and in Foreign by a Home firm that captures both markets selling at prices $p$ in Home and $p_A^*$ in Foreign, i.e.,

$$
\Phi(p) \equiv \left[ p - \frac{wa}{(x(p) + x^*(p_A^*))} \right] x(p)
$$

and

$$
\Phi^*(p) \equiv \left[ p_A^* - \frac{wat}{(x(p) + x^*(p_A^*))} \right] x^*(p_A^*).
$$

For this case, we propose the following equilibrium. Foreign firms price so as to
compete only for their domestic market – in particular, they set a prohibitively high price for exports and a local price of \( p_A^* \). Home firms pursue a mixed strategy: with probability \( q \) they charge a prohibitively high price for sales in Foreign and a local price of \( p_A \) and with probability \( 1 - q \), they contest both markets by shaving price \( p_A^* \) to capture the Foreign market, while setting a domestic price \( p \) that is drawn from the distribution

\[
F(p) = \frac{1}{M(p)} \int_s^p \zeta(y) M(y) dy \left/ \int_s^{p_A} \zeta(y) M(y) dy + \frac{M(p) - 1}{M(p)} \right.,
\]

(1.13)

with support \( p \in [s, p_A] \), where

\[
\zeta(y) = \frac{\Phi'(y) + \Phi'^*(y)}{\Phi(y)},
\]

and

\[
M(y) = \exp \left( \int_s^y \frac{\Phi'(t) + \Phi'^*(t)/2}{\Phi(t)} dt \right).
\]

It is easy to verify that \( F(s) = 0, F(p_A) = 1 \) and \( F'(p) > 0 \). The mixing probability \( q \) is given by,

\[
q = \left( 1 + \int_s^{p_A} \zeta(y) M(y) dy \right)^{-1}.
\]

(1.14)

Finally, \( s \) is determined implicitly by (1.14) and

\[
\Phi(s) + \left( q + \frac{1 - q}{2} \right) \Phi^*(s) = 0
\]

(1.15)

Formally,
Figure 1.3: Equilibrium with mixed strategies, $\phi = 0.3$

**Proposition 1.3.3.** Assume that Home has a superior comparative advantage, i.e., $\beta \omega < l_F(\hat{t})$, where $\hat{t}$ is defined implicitly by $h(\hat{t}) = t^*(\hat{t})$. For $t \in (t_{CS}(\beta \omega), t_{NT}(\beta \omega))$ the equilibrium entails Foreign firms charging $p_A^*$ in Foreign and making no sales in Home, and Home firms following a mixed strategy where with probability $q$ they follow the “local strategy” according to which they charge $p_A$ in Home and make no sales in Foreign and with probability $1 - q$ they follow the “global strategy” according to which they charge $p_A^*$ in Foreign and charge a price $p \in [s, p_A]$ in Home according to the distribution $F(p)$ in (1.13), with $q$ and $s$ satisfying (1.14) and (1.15).

**Proof.** We begin by deriving $F(p)$. Home firms earn zero profits when they pursue their local strategy in all states of nature. Thus, a Home firm pursuing the global
strategy should also expect zero profits. Moreover, for a Home firm to be willing to set prices \( p \) according to \( F(p) \), the expected profits for any \( p \in [s,p_A] \) should also be zero. To derive this expected profit given \( p \) in the global strategy, suppose first that the other Home firm pursues its local strategy. The profits are then \( \Phi(p) + \Phi^*(p) \). If the other firm pursues its global strategy, expected profits associated with a Home price of \( p \) are

\[
\begin{align*}
\Phi(p) + \Phi^*(p) \left(1 - F(p)\right) + \int_s^p \frac{\Phi^*(y) }{2} dF(y).
\end{align*}
\]

Thus, expected profits for a Home firm setting prices \( p \) and \( p_A \) when the other Home firm pursues the proposed mixed strategy are

\[
\Pi(p) \equiv q (\Phi(p) + \Phi^*(p)) + (1 - q) \left[ \Phi(p) + \frac{\Phi^*(p) }{2} \left(1 - F(p)\right) + \int_s^p \frac{\Phi^*(y) }{2} dF(y) \right].
\]

Our mixed strategy requires \( \Pi(p) = 0 \) for all \( p \in [s,p_A] \). Differentiating \( \Pi(p) \) with respect to \( p \), setting \( \Pi'(p) = 0 \) and solving for \( F'(p) \) yields

\[
(1 - q) F'(p) = q \frac{\Phi'(p) + \Phi'^*(p)}{\Phi(p)} + (1 - q) \left[ \frac{\Phi'(p) + \Phi'^*(p)/2}{\Phi(p)} \right] (1 - F(p)).
\]

The solution to this differential equation is

\[
F(p) = \frac{q}{1 - q} \int_s^p \frac{\zeta(y)M(y)}{M(p)} dy + 1 - \frac{M(s)}{M(p)}. \tag{1.16}
\]

Noting that \( M(s) = 1 \), setting \( F(p_A) = 1 \) and solving for \( q \) yields (1.14). Plugging this
back into (1.16) yields (1.13). Finally, we also need that $\Pi(s) = 0$. This implies that

$$\Phi(s) + \left(q + \frac{1-q}{2}\right)\Phi^*(s) = 0.$$  

This equation together with (1.14) can then be solved to yield the equilibrium value of $s$.

We need to study all possible deviations by Home and Foreign firms. A Foreign firm could deviate by going global, shaving prices $p_A$ and $p_A^*$. If both Home firms pursue their local strategy, which happens with probability $q^2$, the Foreign firm would capture both markets and make profits of

$$\Upsilon \equiv \left[ p_A - \frac{w^*a^*t}{(x(p_A) + x^*(p_A^*))^\beta} \right] x(p_A) + \left[ p_A^* - \frac{w^*a^*}{(x(p) + x^*(p_A^*))^\beta} \right] x^*(p_A^*).$$

Otherwise, the Foreign firm would simply sell in the local market and make zero profits. So we need to establish that $\Upsilon < 0$. One can readily verify that there exists a unique $\hat{t}$ such that for $\beta \omega < t_F(\hat{t})$ (Home has a superior comparative advantage), we have $t_{NT}^*(\beta \omega) < t_{CS}$ (Figure 3 illustrates this). Since our mixed strategy applies for $t \in (t_{CS}(\beta \omega), t_{NT}(\beta \omega))$, then we have $t > t_{NT}^*(\beta \omega)$ implies $\Upsilon < 0$.

To describe the possible deviations by Home firms, we use notation $p^* \lesssim p_A^*$ to mean a firm shaves $p_A^*$ (since a Home firm always makes losses in the Foreign market, firms will never want to charge a price lower than they need to capture this market). There are four possible types of pricing strategies by Home firms: (i) $p > p_A$ and $p^* > p_A^*$ (no entry- yields zero profits); (ii) $p \leq p_A$ and $p^* \lesssim p_A^*$ (competing for the global market); (iii) $p > p_A$ and $p^* \lesssim p_A^*$ (competing for foreign market only); and (iv) $p \leq p_A$ and $p^* > p_A^*$ (competing for domestic market only). But pricing strategy (i) strictly dominates pricing strategy (iii) since Home firms make losses on export sales.
This implies that we can rule out strategy (iii). The conjectured equilibrium above essentially considers mixing across a version of (iv) with \( p = p_A \) and \( p^* > p_A^* \), and (ii) as well as mixing within strategy (ii). Moreover, Home firms are indifferent between strategy \( p = p_A \) and \( p^* > p_A^* \) and (i) since in both strategies yield zero expected profits. Hence, our final step entails explicitly ruling out the version of strategy (iv) with \( p < p_A \) and \( p^* > p_A^* \) as a possible deviation.

First, note that if \( p < s \), the expected profits are

\[
q \left[ p - \frac{wA}{(x(p))^\phi} \right] x(p) + (1 - q) \Phi(p).
\]

We need this expression to be non-positive. But since this is increasing in \( p \) (recall that \( px(p) = 1 \) and that \( x(p)/(x(p))^\phi \) is decreasing by the assumption that \( \phi < 1/2 \)), it is enough to check that the expected profits of this type of deviation are non-positive for \( p \geq s \). For this case, the expected profits are

\[
\tilde{\Pi}(p) = q\Gamma(p) + (1 - q) (1 - F(p)) \Phi(p),
\]

where \( \Gamma(p) = \left[ p - \frac{wA}{(x(p))^\phi} \right] x(p) \). Since \( \Gamma(p_A) = 0 \) and \( F(p_A) = 1 \) then \( \tilde{\Pi}(p_A) = 0 \). We now show that \( \tilde{\Pi}'(p) \geq 0 \), implying that \( \tilde{\Pi}(p) \leq 0 \) for all \( p \). First, \( \Pi'(p) = 0 \) implies

\[
(1 - q) \Phi(p)F'(p) = q \left[ \Phi'(p) + \Phi''(p) \right] + (1 - q) \left[ \Phi'(p) + \Phi''(p)/2 \right] (1 - F(p)).
\]

Second,

\[
\tilde{\Pi}'(p) = q\Gamma'(p) + (1 - q) \Phi'(p) (1 - F(p)) - (1 - q) F'(p) \Phi(p).
\]
Combining these two expressions yields
\[ \tilde{\Pi}'(p) = q \left[ \Gamma'(p) - \Phi'(p) - \Phi''(p) \right] - (1 - q) \left( \Phi''(p) / 2 \right) (1 - F(p)). \]

One can easily verify that \( \Phi'(p) > 0 \), \( \Phi''(p) < 0 \) and \( \Gamma'(p) > 0 \). Hence, a sufficient condition for \( \tilde{\Pi}'(p) \geq 0 \) is that
\[ \Gamma'(p) - \Phi'(p) - \Phi''(p) \geq 0. \]

Simple differentiation reveals that \( \Gamma'(p) - \Phi'(p) - \Phi''(p) \geq 0 \) if and only if
\[ (1 - \phi) \left[ (1 + p/p_A^*)^{1+\phi} - 1 \right] \geq (1 - \phi t) p/p_A^*. \]

But \( t > 1 \) and \( \phi < 1/t \) so \( 0 < \phi < \phi t < 1 \), hence the previous inequality is satisfied under our assumptions. We conclude that \( \tilde{\Pi}(p) \leq 0 \) for any \( p \).

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4What happens if the condition for the good to be non-traded is almost satisfied, i.e., \( t \lesssim \tau_{NT}(\beta \omega) \) so that \( \Phi(p_A) + \Phi(p_A) \lesssim 0 \)? This implies that \( s \) and \( q \) satisfy \( s \lesssim p_A \) and \( q \lesssim 1 \). So the equilibrium transitions smoothly from the mixed strategy equilibrium in Proposition 3 to the pure strategy equilibrium with no trade in Proposition 2. What happens if the condition for complete specialization in Home to be an equilibrium is almost satisfied, i.e., \( t \gtrsim t_{CS}(\beta \omega) \) so that \( p_A^* = tp_0 \) where \( p_0 \) solves \( \Phi(p_0) = 0 \)? Our conjecture is that \( s \) and \( q \) satisfy \( s \gtrsim p_0 \) and \( q \gtrsim 0 \) because \( \zeta(y) \approx \infty \) for \( y \gtrsim p_0 \).
Chapter 2

Marshallian Externalities, Comparative Advantage and International Trade

2.1. Introduction

There is strong evidence for the existence of external economies of scale that are limited in their industrial and geographical scope.\(^1\) Such external economies of scale are commonly known as Marshallian or agglomeration externalities. The central idea is that the concentration of production in a particular location generates external benefits for firms in that location through knowledge spillovers, labor pooling, and close proximity of specialized suppliers. Classic examples include Silicon Valley software industry, Detroit car manufacturing, and Dalton carpets. More recent examples point to the potential significance of these externalities for international trade. For instance, Qiaotou, Wenzhou and Yanbu are all relatively small regions in China that account for 60% of world button production, 95% of world cigarette lighter production, and dominate global underwear production, respectively.\(^2\)

What are the implications of these Marshallian externalities for the patterns of international trade, the welfare gains from trade, and industrial policy? The standard model in the literature assumes that firms engage in perfect competition and ignore the effect of their actions on industry output and productivity. This has the unfortu-

\(^1\)See for example Caballero and Lyons (1989, 1990, 1992); Chan, Chen and Cheung (1995); Segura (1996); Henriksen, and Steen and Ulltveit-Moe (2001)).

\(^2\)See Krugman (2009) for a nice discussion of this.
nate implication that any assignment of industries across countries is consistent with equilibrium. To avoid this predicament, we follow Grossman and Rossi-Hansberg (2010, henceforth GRH) and assume that firms in each industry engage in Bertrand competition and understand the implications of their decisions on industry output and productivity. We develop three main results. First, we show that the indeterminacy of international trade patterns still persists for some industries when trade costs are low, and points to a potential role for industrial policy. Second, we follow up by applying these results in a full general equilibrium analysis, and reexamine the implications of Marshallian externalities for industrial policy. To assess the welfare importance of industrial policy, we construct a quantitative general equilibrium Ricardian trade model with Marshallian externalities. Our results indicate that the additional welfare gains from moving to the Pareto-superior equilibrium depend positively on the strength of Marshallian externalities and negatively on the strength of comparative advantage, and – using reasonable parameter estimates – are at most about 2%. Finally, our framework allows us to ask whether Marshallian externalities lead to additional gains from trade. Our quantitative analysis indicates that this is indeed the case, and that Marshallian externalities increase overall gains from trade by around 50%.

The standard approach to incorporate Marshallian externalities in an international trade model has been to assume that perfectly competitive firms take productivity as given, even though it depends positively on aggregate industry output (see Chipman, 1969). Typical results are the existence of multiple Pareto-rankable equilibria, the possibility that trade patterns may run counter to “natural” com-

\footnote{Note that the indeterminacy arises because of multiple equilibria. As such, we think of industrial policy as a policy in which a country attempts to select an equilibrium that leads to higher national welfare.}
parative advantage, and the possibility that some countries may lose from trade. An important implication was that Marshallian externalities provided a theoretical basis for infant-industry protection. For instance, Ethier (1982, henceforth Ethier) using a standard two country, two sector (one with constant returns to scale and the other with increasing returns to scale) Ricardian model formally confirmed the infant-industry argument.

GRH refer to the results that trade patterns can be inconsistent with “natural” comparative advantage and that a country can potentially lose from trade as “pathologies”. To avoid such “pathologies”, GRH propose an alternative equilibrium analysis in a Ricardian model of trade with Marshallian externalities. In particular, they follow Dornbush, Fischer and Samuelson (1977) and assume that there is a continuum of industries (instead of the two industries of the standard trade model), and they abandon perfect competition and assume instead that firms engage in Bertrand competition. Consistent with Bertrand competition, GRH assume that firms recognize the effect that they have on the scale of production and, through external economies of scale, the effect on their own productivity. They show that under frictionless trade the multiplicity of equilibria disappears, and the unique equilibrium entails specialization according to “natural” comparative advantage, as would have occurred in a standard constant return to scale framework. Moreover, they argue that if trade costs are low, the equilibrium is unique and entails complete specialization. In essence, GRH argues that, even with trade costs, the pattern of trade

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4In a Ricardian context, this means that the pattern of specialization can run counter to the ranking of relative (exogenous) productivities when measured at a common scale of production.

5See early work by Graham (1923), Ohlin (1933), Matthews (1949-50), Kemp (1964), Melvin (1969), and Markusen and Melvin (1982).

6For intermediate trade costs GRH show that there is no equilibrium in pure strategies, and they propose a mixed strategy equilibrium in which there is a probability that one country supplies the global market or sells only in the domestic market. In Lyn and Rodriguez-Clare (2012), we illustrate that there is a profitable deviation from the mixed strategy proposed by GRH and then
is consistent with “natural” comparative advantage, and there are always gains from trade. An important implication is that external economies of scale no longer seem to provide a theoretical foundation for industrial policy, calling into question the robustness of the argument for protection.

Consistent with GRH, we find that there are always gains from trade, but our results differ in that we find that trade patterns can indeed be “pathological” when trade costs are low. In Section 2.2 we provide an analysis of the equilibrium configurations under low trade costs for a single industry that exhibits Marshallian externalities using a simplified version of GRH. We find that GRH’s uniqueness result for the case of low trade costs relies on an implicit assumption of industry specific trade costs which are inversely related to the strength of comparative advantage, with basically zero trade costs for the “no comparative” advantage industry. Once we allow for more general “low” trade costs, multiple complete specialization equilibria arise for a set of industries with “weak” comparative advantage implying that trade patterns need not be consistent with “natural” comparative advantage. More importantly, the multiplicity hints at a potential role for industrial policy, by which we mean a policy where a country tries to select an equilibrium associated with a higher real income for itself.

In Section 2.3 we apply the results under partial equilibrium to examine the welfare implications of Marshallian externalities in general equilibrium. We characterize a set of complete specialization equilibria for the case of symmetric countries and low trade costs which are common across industries.\(^7\) In particular, we characterize a set of “disputed” industries for which each country has a relatively “weak” compar-

\(^7\)The assumption of trade costs which are constant across industries is a useful simplification for quantitative exercises and is commonly used in the trade literature. See, for instance, Dornbusch, Fischer and Samuelson (1977), Eaton and Kortum (2002), and Alvarez and Lucas (2007).
ative advantage. Importantly, note that an equilibrium in which a country produces a larger set of these industries is associated with a higher national welfare. Also, note that the trade cost at which this set is the largest is also the one for which the potential welfare gains from industrial policy are the biggest. We show that the set of “disputed” industries first monotonically increases with trade costs, and then shrinks as we approach the maximum trade cost under which there is complete specialization equilibrium. In that regard, our analysis highlights the trade cost for which the largest set of disputed industries occurs. This is the focus of our quantitative analyses exploring both the potential scope for industrial policy, as well as a first look at the quantitative importance of Marshallian externalities for the gains from trade.

To assess the importance of industrial policy, we move to a more general setting with potentially asymmetric countries, and focus on the trade cost associated with the largest set of disputed industries. We do so in order to get a sense of the maximum possible scope for industrial policy. We build a quantitative general equilibrium Ricardian trade model with Marshallian externalities using a probabilistic framework similar to that in Eaton and Kortum (2002, henceforth EK) and quantify the welfare implications of two extreme equilibria: one in which a country is arbitrarily assigned the production of all the disputed industries and one in which all those industries are assigned to the other country. In particular, we compute real wages for each equilibria which we then use to calculate the additional welfare gains from producing the entire set of disputed industries. We use three independent estimates for the two key parameters of our model which govern the strength of Marshallian externalities and that of comparative advantage. Our results indicate that the additional welfare gains from moving to the Pareto-superior equilibrium depend positively on the strength of Marshallian externalities and negatively on the strength of comparative advantage,
and – using reasonable parameter estimates – are at most about 2%.

By highlighting a theoretical basis for industrial policy, our analysis is broadly consistent with that of Ethier and the standard textbook approach to modelling static externalities in international trade. However, its implications are distinct in five main ways. First, unlike in Ethier, trade is always welfare improving. In that sense, the potential scope for industrial policy does not arise because of an attempt to avoid a bad equilibrium where a country loses from trade. Instead, in our framework, the incentive is to move to an equilibrium that entails additional welfare gains from trade. Second, Ethier argues that the case for protection applies to the smaller of two relatively similar sized economies, and is “less likely the greater the degree of increasing returns”. In our analysis, a potential scope for industrial policy applies equally to the case of two similar sized countries as it does to the case of a relatively small country and a large one. Moreover, the scope for industrial policy increases with the degree of increasing returns by raising the maximum trade cost for which multiple complete specialization equilibria apply, and thereby expanding the set of disputed industries for which industrial policy potentially applies. Third, in Ethier, a potential role for industrial policy arises in a world with frictionless trade. This is, however, not the case in our framework. In a world with frictionless trade there is no potential role for industrial policy. A potential role arises when trade costs between both countries are positive but not too high. Finally, we move beyond theory and embed our model in a quantitative framework to get a sense of the potential importance of industrial policy.

We go further by also investigating the potential importance of Marshallian externalities for the gains from trade. In particular, our framework allows us to ask and provide insights to new questions: do Marshallian externalities imply additional gains from trade? If so, how important are these externalities for the overall gains
from trade? Insights from the case of low trade costs with two symmetric coun-
tries suggest that Marshallian externalities do, in fact, imply larger gains from trade
over and above those predicted by a traditional constant returns to scale framework.
More importantly, a decomposition of these gains suggests the contribution can be
substantial. The median parameter estimates indicate Marshallian externalities can
account for approximately 35% of the overall gains from trade.

By highlighting Marshallian externalities as a potentially important channel of
gains from trade, our work is also related to the international trade literature that
focuses on quantifying the contribution of a particular margin to the overall gains
from trade; see for instance recent work by Broda and Weinstein (2006), and Feenstra
and Kee (2008), Goldberg et al (2009), and Feenstra and Weinstein (2009). While
this literature has made significant progress in highlighting new margins of gains, the
implications for the size of the total gains from trade has not changed (see Arkolakis
et. al., 2010). In contrast, our analysis indicates that Marshallian externalities may
not only be a significant margin of gains, but also one which has important impli-
cations for measuring the overall gains from trade. In this regard, our preliminary
results seem to provide some support for the widespread perception among trade
economists that the gains from trade are larger than those predicted by traditional
quantitative trade models.8

Finally, we demonstrate that in the absence of trade costs the model readily ex-
tends to a multicountry setting, and yields a simple intuitive expression for the gains
from trade. In particular, the welfare gains from trade depend only on the expend-
diture share on domestically produced goods and the two key parameters governing
the strength of both comparative advantage and Marshallian externalities.

8See Arkolakis et al (2008) for a discussion about this.
2.2. The Model

Our model is a simplified version of GRH. In particular, we assume Cobb-Douglas preferences (as was done in the earlier literature) instead of the more general constant elasticity of substitution (CES) preferences used in GRH. Also, for expositional simplicity, we assume a particular functional form for the channel through which Marshallian externalities operate. We start first by outlining the general environment. Next we proceed with a partial equilibrium analysis in which we focus on a particular industry and characterize equilibria for the case of low trade costs (see Lyn and Rodriguez-Clare (2012) for a full analysis for different levels of trade costs). Finally, in a full general equilibrium analysis, we characterize a set of complete specialization equilibria for low trade costs in order to gain insights for our quantitative analysis in the following section.

2.2.1 The General Environment

There are two countries, Home \((H)\) and Foreign \((F)\), and a continuum of industries/goods indexed \(v \in [0,1]\). Preferences in each country \(i = H,F\) are identical and uniform Cobb-Douglas with associated industry demand \(x_i (p(v)) = D_i / p_i (v)\) where \(D_i\) is the aggregate expenditure in country \(i\), and \(p_i (v)\) is the price in country \(i\) and industry \(v\).

Labor is the only factor of production and is inelastically supplied. Labor can move freely across industries within a country, but is immobile across countries. We denote by \(L_i\) and \(w_i\) the labor supply and wage in country \(i\). The production technology has constant or increasing returns to scale due to external economies at the local industry level. In country \(i\) it takes \(a_i (v) / (X_i (v))^\theta\) units of labor to produce a unit of output, where \(a_i (v) > 0\) is an exogenous productivity parameter, \(X_i (v)\) is the
total production of the good in country $i$ net of trade cost, and $\phi$ is the parameter which governs the strength of Marshallian externalities. In each industry and in each country there are two producers in Home and Foreign, $m_H(v) = m_F(v) = 2$. Markets are segmented, so that firms can set arbitrarily different prices across the two markets. Firms in each industry engage in Bertrand (price) competition in each market: setting a price above the minimum price leads to no sales; setting a price below all other prices allows the firm to capture the entire market; and setting a price equal to the minimum price implies that the market is shared among all those firms that set this same price.

Trade costs are of the “iceberg” type, so that delivering a unit of the good from one country to the other requires shipping $\tau(v) \geq 1$ units.

We make the following restriction on the the parameter which governs the strength of Marshallian externalities:

**Assumption 2.2.1.** $0 \leq \phi < 1/2$.

Note that if $\phi = 0$ then the technology exhibits constant returns to scale, and the standard results obtain. In particular, the equilibrium entails complete specialization in Home for $w_{HaH}(v) \tau(v) \leq w_{FaF}(v)$, complete specialization in Foreign for $w_{HaH}(v) \geq w_{FaF}(v) \tau(v)$, and the equilibrium entails no trade otherwise.

Also, for reasons explained below in the subsection on partial equilibrium, we also restrict the range of trade costs as follows:

**Assumption 2.2.2.** Trade costs $\tau(v)$ are bounded above by $1/\phi$ for every $v$.

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9As in GRH, we can accommodate a finite number of firms in each industry and country. However, doing so complicates the intermediate trade costs analysis without adding any interesting insights. Hence, we simply assume two firms from the onset.

10In principle, one could also consider the case of integrated markets. However, we follow GRH and assume the simpler case of segmented markets.
2.2.2 Partial Equilibrium

In this subsection we focus on a particular industry, and treat wages as exogenous and fixed at $w_H$ and $w_F$. For simplicity we restrict the analysis to the case in which demand is symmetric in the two countries, namely $D_H = D_F$. Formally,

**Assumption 2.2.3.** Demand is symmetric in the two countries: $D_H = D_F$. Without loss of generality we set $D_H$ and $D_F$ to one.

2.2.2.1 Autarky

Consider the autarky equilibrium in Home. Market equilibrium under autarky requires that production be equal to demand $X_H = x_H(p_H)$, while Bertrand competition leads to average cost pricing, $p_H = w_H a_H / (X_H)$. These two equations imply

$$p_A^H = w_H a_H / (x_H(p_A^H)) \phi. \tag{2.1}$$

A sufficient condition for existence is that the demand curve be steeper than the supply curve, that is,

$$\phi < 1. \tag{2.2}$$

This condition is guaranteed by Assumption 2.2.1. In principle, there could be an additional equilibrium at $p_A^H = 0$ if $(X_H)^\phi$ is not bounded above. In what follows we ignore this possibility.

Consider the allocation with $p_A^H$ as determined by (2.1) and imagine that a firm deviates by charging a price $p_H$ slightly lower than $p_A^H$. The deviant would capture
the whole market and make profits of

\[ \pi(p_H) = \left( p_H - \frac{w_H a_H}{(x_H(p_H))^\phi} \right) x_H(p_H). \]

But since \( p_H^A = w_H a_H / (x_H(p_H^A))^\phi \), then it is easy to show that (2.2) implies \( \pi(p_H) < 0 \) for all \( p_H < p_H^A \). We henceforth use \( p_H^A \) to denote the solution to (2.1). Under Assumption 2.2.3 this is simply \( p_H^A = (w_H a_H)^{1/(1-\phi)} \).

### 2.2.2.2 Frictionless Trade

GRH show that Bertrand competition gives rise to a unique equilibrium in which the pattern of trade is governed by Ricardian comparative advantage. In particular, Home will export the good if \( w_H a_H < w_F a_F \), and the opposite will occur if \( w_H a_H > w_F a_F \). Without loss of generality, we henceforth assume that \( w_H a_H < w_F a_F \), so that Home has a comparative advantage in the good under consideration. In equilibrium, Home firms will sell at average cost in both the Home and Foreign markets, so the equilibrium price in both markets is determined by:

\[ p_H^{FT} = \frac{w_H a_H}{(x_H(p_H^{FT}) + x_F(p_H^{FT}))^\phi}. \]

Firms make zero profits, and no firm can profitably deviate – in particular, the assumption that \( \phi < 1/2 \) implies that any firm in Home or Foreign would make losses by charging a price lower than \( p_H^{FT} \). Moreover, the assumption that \( \phi < 1/2 \) also implies that the equilibrium is unique.

Can there be an equilibrium in which Foreign firms dominate the industry, selling in both markets? The answer is no. To see this, note that such an equilibrium would
have a price $p_F^{FT}$ in both markets given implicitly by

$$p_F^{FT} = \frac{w_F a_F}{(x_H(p_F^{FT}) + x_F(p_F^{FT}))^\phi}.$$  

A Home firm could shave this price, capture both markets, achieve economies of scale that lead to productivity $(x_H(p_F^{FT}) + x_F(p_F^{FT}))^\phi/a_H$, and achieve cost

$$\frac{w_H a_H}{(x_H(p_F^{FT}) + x_F(p_F^{FT}))^\phi}.$$  

This is lower than $p_F^{FT}$ by the starting assumption that $w_H a_H < w_F a_F$, so the deviation is profitable.

### 2.2.2.3 Costly Trade

There are three types of equilibria: complete specialization (i.e., firms from one country supply both markets); mixed strategy equilibria (i.e., firms from one country randomize over which markets to serve and the price to charge, while firms from the other country offer to serve their own market at the autarky price); no trade (i.e., firms from each country serve only their own market). For the purposes of this paper we consider only the possibility of an equilibrium with complete specialization.

#### 2.2.2.3.1 Complete specialization equilibrium

If Home serves both markets, the equilibrium entails prices $p_H$ and $p_F = \tau p_H$, with $p_H$ determined by

$$p_H = \frac{w_H a_H}{(x_H(p_H) + x_F(\tau p_H))^\phi}. \quad (2.3)$$  

To establish that this is an equilibrium, we need to consider the possible deviations by Home and Foreign firms.
A preliminary result is that the best that any firm can do is to shave prices $p_H$ and $p_F$, i.e., it is never optimal to charge strictly lower prices than $p_H$ and $p_F$. Recall that in autarky and under frictionless trade this is guaranteed by $\phi < 1/2$. But in the presence of trade costs, we need a more stringent condition. This is because there is an additional gain to a firm in lowering the domestic price, because now the economies of scale lead to lower costs that can also be exploited in exports. We can show that $\phi < 1/2$ and $\tau < 1/\phi$ (guaranteed by Assumptions 2.2.1 and 2.2.2, respectively) imply that the best possible deviation for a Home or Foreign firm entails shaving prices $p_H$ and $p_F$ (see the Appendix for a formal statement and proof of this result).

Consider now a deviation by a Home firm. Since prices $p_H$ and $p_F$ with $p_H$ given by (2.3) imply that Home firms make zero profits in both markets, then a Home firm cannot make positive profits with any alternative set of prices. So this establishes that there is no profitable deviation for a Home firm.

Turning to Foreign firms, we need to study two possible deviations: first, no firm from Foreign should find it optimal to take over the world market by undercutting Home firms in both markets, and second, no firm from Foreign should find it optimal to displace Home firms from the Foreign market. Writing $x_H$ and $x_F$ as shorthand for $x_H(p_H)$ and $x_F(\tau p_H)$, with $p_H$ implicitly defined by (2.3), a sufficient condition for it to be unprofitable for Foreign firms to take over both markets is given by

$$\left[ \frac{w_H a_H}{(x_H + x_F)^{\phi}} - \frac{w_F a_F \tau}{(x_H + x_F)^{\phi}} \right] x_H + \left[ \frac{w_H a_H \tau}{(x_H + x_F)^{\phi}} - \frac{w_F a_F}{(x_H + x_F)^{\phi}} \right] x_F \leq 0.$$ 

This is equivalent to

$$\frac{a_H}{a_F} \leq \frac{w_F \tau x_H + x_F}{w_H x_H + \tau x_F}. \quad (2.4)$$
In turn, a sufficient condition for it to be unprofitable for Foreign firms to displace Home firms from the Foreign market (only) is

\[
\left[ \frac{w_H a_H \tau}{(x_H + x_F)^\phi} - \frac{w_F a_F}{(x_F)^\phi} \right] x_F \leq 0. \tag{2.5}
\]

This is equivalent to

\[
\frac{a_H}{a_F} \leq \frac{w_F (x_H + x_F)^\phi}{w_H} \frac{(x_F)^\phi}{\tau}. \tag{2.6}
\]

The second term on the RHS of (2.6), \( \frac{(x_H + x_F)^\phi}{\tau} \), captures the \textit{trade cost-scale effect} trade-off: the larger the benefits of economies of scale from capturing both markets, the larger the trade cost needs to be to effectively protect Foreign firms in their domestic market.

The previous arguments establish that if conditions (2.4) and (2.6) are satisfied for \( p_H \) given by the solution of (2.3), then there is an equilibrium with complete specialization in which Home firms serve both markets. Similarly, an equilibrium with complete specialization where Foreign serves both markets would have prices \( p_H \) and \( p_F \), where \( p_H = \tau p_F \) and \( p_F \) is implicitly defined by

\[
p_F = \frac{w_F a_F}{(x_H(\tau p_F) + x_F(p_F))^\phi}. \tag{2.7}
\]

This is an equilibrium if the following two conditions are satisfied:

\[
\frac{a_H}{a_F} \geq \frac{w_F \tau x_H(\tau p_F) + x_F(p_F)}{w_H x_H(\tau p_F) + \tau x_F(p_F)}, \tag{2.8}
\]

\[
\frac{a_H}{a_F} \geq \frac{w_F \tau}{w_H (x_H(\tau p_F) + x_F(p_F))^\phi / (x_H(\tau p_F))^\phi}. \tag{2.9}
\]

If trade costs are low, i.e., if \( \tau \) close to 1, condition (2.4) implies condition (2.6)
and condition (2.8) implies (2.9). In other words, a deviation to serve the domestic market only is never profitable when trade costs are low, since such a deviation implies high costs due to small scale but small benefits from being able to save on trade costs.

Let \( \omega = w_H/w_F \) and \( \beta = a_H/a_F \). Our assumption above that \( a_H w_H < a_F w_F \) (so that Home has a comparative advantage) implies that \( \beta \omega < 1 \). Using Assumption 2.2.3, so that \( x_H(p_H) = 1/p_H \) and \( x_F(p_F) = 1/p_F \), then conditions (2.4) and (2.6) with \( p_F = \tau p_H \), can be written as

\[
\beta \omega \leq \frac{\tau x_H(p_H) + x_F(\tau p_H)}{x_H(p_H) + \tau x_F(\tau p_H)} = \frac{\tau + 1/\tau}{2} \equiv g_H(\tau) \tag{2.10}
\]

and

\[
\beta \omega \leq \frac{(x_H(p_H) + x_F(\tau p_H))^\phi}{\tau} / (x_F(\tau p_H))^\phi = \tau^{-1} (1 + \tau)^\phi \equiv h_H(\tau). \tag{2.11}
\]

Similarly, conditions (2.8) and (2.9) with \( p_H = \tau p_F \) can be written as

\[
\beta \omega \geq \frac{2}{1/\tau + \tau} \equiv g_F(\tau) \tag{2.12}
\]

and

\[
\beta \omega \geq \tau (1 + \tau)^{-\phi} \equiv h_F(\tau). \tag{2.13}
\]

Note that \( g_F(\tau) = 1/g_H(\tau) \) and \( h_F(\tau) = 1/h_H(\tau) \). For future reference we refer to this as “symmetry”. Moreover, note that \( g_F(1) = 1 \) and \( g_F(\tau) < 1 \) for \( \tau > 1 \).

To proceed, we need some additional notation. Let \( \tau_{MAX} \equiv 1/\phi \). Let \( \tilde{\tau} \) be implicitly defined by \( g_F(\tau) = h_F(\tau) \), let \( \tau_{H}^{CS}(\beta \omega) \) and \( \tau_{F}^{CS}(\beta \omega) \) be implicitly defined by \( \beta \omega = h_H(\tau) \) and \( \beta \omega = h_F(\tau) \), respectively, and let \( \tau_{F}^{0}(\beta \omega) \) be implicitly defined by
\( \beta \omega = g_F(\tau) \). We need to consider two cases: strong and weak comparative advantage. We say that Home has a “strong comparative advantage” if \( \beta \omega < g_F(\tau) \). We say that Home has a “weak comparative advantage” if \( g_F(\tau) \leq \beta \omega < 1 \). For an analogous analysis, see Figure 1.1 in Chapter 1.

**Proposition 2.2.1.** Assume that \( \beta \omega < 1 \). There are two cases. Case a: Home has a strong comparative advantage, i.e., \( \beta \omega < g_F(\tilde{\tau}) \). Then for \( \tau \in [1, \min\{\tau_{CS}^S(\beta \omega), \tau_{MAX}\}] \) there is a unique equilibrium with complete specialization, and this equilibrium has Home serving both markets. Case b: Home has a weak comparative advantage, i.e., \( g_F(\tilde{\tau}) \leq \beta \omega < 1 \). Then for \( \tau \in [1, \tau_F^0(\beta \omega)] \cup [\tau_{FC}^C(\beta \omega), \tau_{CS}^C(\beta \omega)] \) there is a unique equilibrium with complete specialization, and this equilibrium has Home serving both markets, whereas for \( \tau \in [\tau_F^0(\beta \omega), \tau_{FC}^C(\beta \omega)] \) there are two complete specialization equilibria, one with Home serving both markets, and another with Foreign serving both markets.

### 2.2.3 General Equilibrium

In this section, we consider the entire set of industries and characterize, formally, a set of complete specialization equilibria for symmetric countries when trade costs are low. The results imply that the multiplicity applies to a set of “disputed” industries (industries for which both countries have a relatively weak comparative advantage), and hints at a possible role for industrial policy. We now proceed to characterize the full general equilibrium. For any industry \( v \), define \( z_i \equiv 1/a_i^\alpha \) where \( \alpha \equiv 1/(1 - \phi) \) and \( z_i \) is the productivity of firms in country \( i \). Note that \( \alpha \geq 1 \) \((\phi \geq 0)\) with \( \alpha = 1 \) \((\phi = 0)\) resulting in the special case of constant returns to scale. For convenience we...

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\(^{11}\)To see why we define \( z_i \equiv 1/a_i^\alpha \Leftrightarrow a_i \equiv 1/z_i^{1/\alpha} \), first recall there are \( n_i \geq 2 \) producers/firms in each industry and country. Firms (indexed by \( m \)) in a particular industry \( v \) and country, say
use $\alpha$ instead of $\phi$ in what follows. Let $T_i$ represent country $i$’s state of technology. For expositional simplicity, we make the following additional assumptions which we use throughout this section.

**Assumption 2.2.4.** Countries are symmetric: $T_H = T_F$ and $L_H = L_F$. Without loss of generality we set $T_H$, $T_F$, $L_H$ and $L_F$ to one.

Notice that symmetry here implies $L_H = L_F$, and is distinct from our assumption of symmetric demand, $D_H = D_F$, in our partial analysis. For our general equilibrium analysis, we essentially dispense with Assumption 2.2.3 (demand is symmetric) in favor of Assumption 2.2.4. We do this, for the most part, because in general equilibrium, we allow relative wages to be endogenously determined. We also assume a country specific parametric distributional structure for industry productivity similar to that used in EK. In particular,

**Assumption 2.2.5.** In any country $i$, the productivity for each industry is independently drawn from a country specific Frechet distribution

Home, have access to identical production technology

$$X_H^m = \left[\left(\frac{1}{z_H}\right)^{1/\alpha} \left(X_H^{\alpha-1}/\alpha\right)\right] L_H^m$$

where $X_H = \sum_m X_H^m$ and $L_H = \sum_m L_H^m$. Assuming firms take wages as given, unit cost is given by

$$w_H = \frac{w_H a_H}{(z_H)^{1/\alpha} \left(X_H^{\alpha-1}/\alpha\right)}$$

But $a_H \equiv 1/z_H^{1/\alpha}$ and $\phi \equiv (\alpha - 1)/\alpha$, so we have

$$w_H a_H = \frac{w_H a_H}{(X_H)^{\phi}}$$

In particular, production exhibits national increasing returns to scale at the local industry level, and is given by

$$X_H = z_H L_H^\phi.$$
As in EK, $T_i$ represents country $i$’s absolute advantage across the continuum of industries, whereas $\theta$ determines comparative advantage within the continuum. While in principle one does not need a probabilistic framework for the theoretical analysis on two symmetric countries, we present it here for convenience since it becomes relevant to our quantitative exercises later on.

2.2.3.1 Complete Specialization Equilibria

Define $\beta (v) \equiv a_H (v) / a_F (v)$. Order industries such that $\beta (v)$ is continuous and strictly increasing in $v$. Importantly, note that the analogs of $g_H (.), g_F (.), h_H (.)$ and $h_F (.)$ from our partial analysis are: $g_H (\omega, \tau) \equiv \left[ \frac{\tau + \omega - 1}{1 + \omega - 1} \right]$; $g_F (\omega, \tau) \equiv \left[ \frac{1 + \omega - 1}{1/\tau + \omega - 1} \right]$; $h_H (\omega, \tau) \equiv \frac{1}{\tau} \left[ \frac{\tau + \omega - 1}{\omega - 1} \right]^\phi$; and $h_F (\omega, \tau) \equiv \tau \left[ \frac{1}{1 + \omega - 1/\tau} \right]^\phi$. Note also that our partial equilibrium analysis focuses on a particular industry $v$. The difference here is that we consider the entire set of industries. Hence, our conditions for complete specialization by the Home and Foreign countries modified by the appropriate $g_i (.)$ and $h_i (.)$ functions (along with the fact that $\beta$ is a function of $v$) still apply here, namely, (2.10) and (2.11) for the Home country, and (2.12) and (2.13) for the Foreign one.

In Proposition 2.2.1, we showed that there are multiple complete specialization equilibria when trade costs are low and comparative advantage is weak. Here we characterize the set of “disputed” industries for which multiple complete specialization equilibria apply. In order to give a rough sketch of the idea behind the multiplicity of complete specialization equilibria when considering all industries, imagine for a moment relative wages are fixed. Also, let $v_H^0$ and $v_F^0$ solve (2.10) and (2.12) when it holds with equality. Importantly, we argue that for trade costs low enough, (2.11)
and (2.13) are also satisfied for \(v^0_H\) and \(v^0_F\) respectively. Moreover, \(v^0_H > v^0_F\) along with the fact that \(\beta(v)\) is increasing implies an overlapping range of industries for which the conditions for complete specialization in each country are simultaneously satisfied (we establish later that this is in fact the case). Before proceeding further, we first define the concept of a complete specialization allocation.

**Definition 2.2.2.** A complete specialization allocation (CSA), \(\tilde{v}\), is an allocation where all goods \(v \leq \tilde{v}\) are produced only by the Home country and all goods \(v > \tilde{v}\) are produced by the Foreign country.

As such, we consider an allocation \(\tilde{v} \in [v^0_F, v^0_H]\) such that for goods \(v \leq \tilde{v}\) production is concentrated in the Home country, and for goods \(v > \tilde{v}\) production is concentrated in the Foreign one. In that regard, we consider equilibria in which each country gets a share of the disputed industries (and in the extreme case all or none of the disputed industries) such that increasing \(\tilde{v}\) entails expanding the partitioned set of industries which the Home country produces and vice-versa.\(^{12}\) Hence, Home country’s domestic sales and export sales are \(\tilde{v}w_H L_H\) and \(\tilde{v}w_F L_F\) respectively (note that, in equilibrium, income and expenditure in the Home and Foreign countries are \(w_H L_H\) and \(w_F L_F\), respectively (we formally define a complete specialization equilibrium in what follows). Full employment in Home requires

\[
w_H L_H = \tilde{v}w_H L_H + \tilde{v}w_F L_F. \tag{2.15}
\]

\(^{12}\)Since the industries are ordered with decreasing comparative advantage for Home country, any other non-partitioned assignment is strictly dominated in terms of efficiency. Hence, we confine ourselves to connected sets so that complete specialization equilibria can be characterized by a single \(\tilde{v}\).
From Assumption 2.2.4, we have $L_H = L_F = 1$ implies
\[
\omega = \omega(\nu) \equiv \frac{\nu}{1 - \nu}.
\] (2.16)

Using Assumption 2.2.4 ($T_H = T_F = 1$), Assumption 2.2.5 (the productivity distribution is Fretchet), and bearing the ordering of goods in mind, one can derive
\[
\beta(v) = \left(\frac{v}{1 - v}\right)^{1/\alpha \theta}.
\] (2.17)

Clearly $\beta(.)$ is strictly increasing. We now define formally a complete specialization equilibrium.

**Definition 2.2.3.** A complete specialization equilibrium (CSE) is a CSA which satisfies the appropriately modified versions of (2.10)-(2.13) for all goods, that is, a CSA in which firms in neither country have an incentive to deviate and take over their home markets or both markets; and labor markets clear so that $\omega = \omega(\nu)$.

Let $v^g_H(\tau)$ be the solution to
\[
\beta(v) \omega(v) = g_H(\omega(v), \tau)
\] (2.18)
and $v^h_H(\tau)$ be the solution to
\[
\beta(v) \omega(v) = h_H(\omega(v), \tau)
\] (2.19)

Similarly, let $v^g_F(\tau)$ be the solution to
\[
\beta(v) \omega(v) = g_F(\omega(v), \tau)
\] (2.20)
Figure 2.1: Set of complete specialization equilibria with low trade costs.

\[ \beta(v) \omega(v) = h_F(\omega(v), \tau) \]

Importantly, note that symmetry implies \( v^g_H(\tau) = v(\tau) \) and \( v^g_F(\tau) = 1 - v(\tau) \), and \( v^h_H(\tau) = v(\tau) \) and \( v^h_F(\tau) = 1 - v(\tau) \). We exploit this property throughout the proof of Proposition 2.2.4 in the Appendix.

**Proposition 2.2.4.** There exists a unique \( \bar{\tau} \) that satisfies \( v^g_H(\bar{\tau}) = v^h_H(\bar{\tau}) \) (and also \( v^g_F(\bar{\tau}) = v^h_F(\bar{\tau}) \)) and there exists a unique \( \tilde{\tau} \) satisfying \( v^h_H(\tilde{\tau}) = v^h_F(\tilde{\tau}) = 1/2 \). For \( \tau \leq \bar{\tau} \), any CSA \( \bar{v} \in [v^g_F(\tau), v^g_H(\tau)] \) is a CSE, and for \( \tau \in (\bar{\tau}, \tilde{\tau}] \), any CSA \( \bar{v} \in [v^h_F(\tau), v^h_H(\tau)] \) is a CSE.
2.3. Welfare

A interesting implication of Proposition 2.2.4 is that as we initially increase trade costs, the set of disputed industries for which multiple complete specialization equilibria apply expands. Beyond $\tilde{\tau}$, the set shrinks. Figure 2.1 illustrates. More importantly, the existence of multiple equilibria with respect to this disputed set suggests a potential role for industrial policy. We explore this further in a more general setting in what follows, fixing our attention on complete specialization equilibria for $\tau = \tilde{\tau}$. As illustrated in Figure 2.1, the largest possible set of disputed industries occurs at $\tilde{\tau}$, and since our objective is to gauge the maximum possible role for industrial policy, our quantitative exercise hones in on the case $\tau = \tilde{\tau}$. Given this, it seems sufficient to restrict our analysis to the case $\tau \leq \tilde{\tau}$ in which the global-deviation conditions are the binding constraints.

In the section on welfare that immediately follows we explore this potential for industrial policy by considering the welfare implications of two extreme equilibria: one in which all the disputed industries are produced by the Home country, and the other in which the converse occurs. More importantly, we also explore quantitatively how important external economies of scale are for the overall gains from trade.

Recall from the previous subsection that for $\tau \leq \tilde{\tau}$, satisfying the global-deviation conditions for both Home and Foreign implied the respective local-deviation conditions were also satisfied. From Proposition 2.2.4 we know that the set of equilibria is given by $\tilde{\tau} \in [v^g_F(\tau), v^g_H(\tau)]$. For the subsection on industrial policy we restrict the analysis to the two extreme equilibria in this set, namely $\tilde{\tau} = v^g_F(\tau)$ and $\tilde{\tau} = v^g_H(\tau)$. We refer to the equilibrium with $\tilde{\tau} = v^g_F(\tau)$ as the $i$ equilibrium. Here we relax the assumptions that the level of technology, $T_i$, and the size of the labor force, $L_i$, are the same across countries. In our quantitative exercise we check to verify that the
local-deviation conditions are satisfied.

Let \( \pi_{HH} \) and \( \pi_{FF} \) be the share of expenditure devoted to local production in Home and Foreign, respectively. Our assumption on preferences implies \( \pi_{HH} = \tilde{v} \) and \( \pi_{FF} = 1 - \tilde{v} \). The following proposition outlines key objects of our welfare analysis, namely, real wages in each country. Let \( P_i \) be the appropriate price index in country \( i \).

**Proposition 2.3.1.** Real wage in country \( i \) is given by

\[
\frac{w_i}{P_i} = \left( \frac{T_i}{\eta^\theta} \right)^{1/\theta} L_i^{\alpha - 1} \pi_{ii}^{-1/\theta} \left( \frac{\Upsilon_i}{w_i L_i} \right)^{\alpha - 1} \tag{2.22}
\]

where

\[
\Upsilon_H \equiv \Upsilon_H (\tilde{v}) = \left( \frac{w_H L_H + w_F L_F}{\tau} \right)^{\pi_{HH}} \left( \frac{\left( \frac{\beta (\tilde{v}) \omega (\tilde{v})}{\tau} \right)^{\alpha/(\alpha - 1)}}{w_H L_H / \tau + w_F L_F} \right)^{1 - \pi_{HH}} \tag{2.23}
\]

and

\[
\Upsilon_F \equiv \Upsilon_F (\tilde{v}) = \left( \frac{w_H L_H / \tau + w_F L_F}{\tau} \right)^{\pi_{FF}} \left( \frac{1}{\tau \left( \frac{\beta (\tilde{v}) \omega (\tilde{v})}{\tau} \right)^{\alpha/(\alpha - 1)}} \right)^{1 - \pi_{FF}} \tag{2.24}
\]

The first two terms in (2.22) capture the effect of technology and size on real wages, respectively. The third term is the gains from trade through comparative advantage, and, as we explain below, the last term captures the gains from trade arising through economies of scale.

Let \( w_i^A / P_i^A \) be the autarky real wage in country \( i \).
Corollary 2.3.2. The gains from trade in country \( i \) are

\[
\frac{w_i}{P_i} = \frac{w_i^A}{P_i^A} = \pi^{-1/\theta}_{ii} \left( \frac{\Upsilon_i}{w_i L_i} \right)^{\alpha-1}
\] (2.25)

where \( \Upsilon_i \) is given by (2.23) and (2.24) for \( i = H \) and \( F \), respectively.

The expression in (2.25) highlights the two channels for gains from trade in our model: the gains from comparative advantage \((\pi^{-1/\theta}_{ii})\), and the gains from economies of scale \((\Upsilon_i/w_i L_i)^{\alpha-1}\). The first term requires no explanation (see EK and Arkolakis et al (2010)). The second term represents the additional gains through economies of scale associated with concentrating global production in a single location. In turn, this term can be decomposed into two parts. Focusing on Home, the first part, \( (\frac{w_{HLH}+w_{LF}}{\tau}w_{HLH})^{(\alpha-1)\pi_{HH}} \), captures the gains from economies of scale associated with the expansion of industries \( \frac{w_{HLH}+w_{LF}}{\tau}w_{HLH} \) at Home for \( \nu \leq \tilde{\nu} \), while the second part, \( \left[ \left( \frac{(\beta(\tilde{\nu})\omega(\tilde{\nu}))^{\alpha}}{\tau} \right) \left( \frac{w_{HLH}+w_{LF}}{\tau}w_{HLH} \right)^{\alpha-1} \right]^{1-\pi_{HH}} \), captures those gains associated with larger scale of industries at Foreign relative to Home in autarky by \( \frac{w_{HLH}+w_{LF}}{\tau}w_{HLH} \) for \( \nu \geq \tilde{\nu} \), with the adjustment term \( \left( \frac{(\beta(\tilde{\nu})\omega(\tilde{\nu}))^{\alpha}}{\tau} \right) \) arising because of the difference in unit costs at the cut-off \( \nu = \tilde{\nu} \). We exploit these expressions in our welfare analyses on industrial policy and gains from trade below.

2.3.1 Industrial Policy

We think of industrial policy as that which moves the economy to a superior equilibrium. In particular, industrial policy in Home would be aimed at switching from the \( F \) to the \( H \) equilibrium, and the opposite would be the case for industrial policy in Foreign. The question we are interested in here is the following: How large is the increase in the real wage for Home (Foreign) associated with a switch from the \( F \)
(H) to the H (F) equilibrium?

We now present a quantitative exercise to shed some light on this question. We need to set values for parameters governing the strength of Marshallian externalities (α) and the strength of comparative advantage (θ). For the external economies of scale parameter, we use implied estimates from three independent studies: Antweiler and Trefler (2002) general equilibrium approach using data on 71 countries (α = 1.054); Fuss and Gupta (1981) analysis using Canadian data (α ≡ 1/(1 − φ) = 1.15); and Paul and Siegel (1999) partial equilibrium approach using industry level US manufacturing data (α ≡ 1/(1 − φ) = 1.3). For the comparative advantage parameter we use three estimates for θ coming from EK, namely 3.6, 8.28 and 12.86.

The results are reported in the tables below. Not surprisingly, the set of disputed industries increases with trade costs, and the trade cost associated with the largest set of disputed industries increases with the scale parameter (recall that we restrict our attention to τ = ˜τ, refer to Figure 2.1). In all cases, we compute the trade cost associated with the largest set of disputed industries, ˜τ, and analyze the welfare implications of the two extreme equilibria. For symmetric countries, we need only examine the welfare implications of giving all the disputed industries to the Home country. In all other cases we use H and F to indicate whether we are considering the equilibrium in which all the disputed goods production go to either Home or Foreign respectively. Importantly, note that a lower θ implies greater variability of productivity across the entire set of industries, and thus stronger forces of com-

\footnote{Without loss of generality, consider the Home country. Let \( w^i_H / P^i_H \) be the real wage in Home country for equilibrium \( i = H, F \). Then the potential welfare gains for Home from producing all the disputed industries relative to producing none is simply

\[
\frac{w^F_H / P^F_H - w^H_H / P^H_H}{w^F_H / P^F_H} \times 100.
\]
parative advantage. In contrast a higher $\alpha$ implies stronger external economies of scale.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>1.05</th>
<th>1.15</th>
<th>1.30</th>
</tr>
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<tbody>
<tr>
<td>$\theta$</td>
<td>3.60</td>
<td>3.60</td>
<td>3.60</td>
</tr>
<tr>
<td>$\tilde{\tau}$</td>
<td>1.04</td>
<td>1.10</td>
<td>1.19</td>
</tr>
<tr>
<td>Scope Indust. Pol. %</td>
<td>0.06</td>
<td>0.40</td>
<td>1.46</td>
</tr>
</tbody>
</table>

Table 2.1: Gains from Disputed Industries (Symmetric Countries)

Table 2.1 indicates that there are additional gains from trade associated with producing the entire set of disputed industries and as a result gives us a measure of the potential scope for industrial policy. The magnitudes of these additional gains range from a negligible 0.06% to at most about 2%, with the potential importance increasing strongly with the strength of Marshallian externalities ($\alpha$), and decreasing weakly with the strength of comparative advantage ($1/\theta$). Essentially, a higher $\alpha$ or a higher $\theta$ increases the scope for industrial policy by expanding the set of disputed industries. The former does so by expanding the range of low trade costs for which industrial policy applies and as a result raises the low trade cost associated with the largest set of disputed industries ($\tilde{\tau}$).

Might asymmetries alter the basic result above? In Table 2.2, we explore technology asymmetry by assuming the Home country has on average superior technology, and analyze the welfare implications using only the highest implied external economies of scale and comparative advantage parameter estimates so as to focus on the maximum possible scope for industrial policy. The results indicate that the additional welfare gains for both countries do not diverge much from the case of
Table 2.2: Home has Superior Technology \((T_H = 2, T_F = 1)\)

<table>
<thead>
<tr>
<th></th>
<th>(H)</th>
<th>(F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gains from disputed industries %</td>
<td>1.48</td>
<td>1.55</td>
</tr>
<tr>
<td>Share of disputed industries %</td>
<td>1.36</td>
<td>1.36</td>
</tr>
</tbody>
</table>

\(\theta = 12.86, \alpha = 1.30\)

Table 2.3: Home has a Larger Labor Force \((L_H = 2, L_F = 1)\)

<table>
<thead>
<tr>
<th></th>
<th>(H)</th>
<th>(F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gains from disputed industries %</td>
<td>0.70</td>
<td>1.39</td>
</tr>
<tr>
<td>Share of disputed industries %</td>
<td>0.66</td>
<td>0.66</td>
</tr>
</tbody>
</table>

\(\theta = 12.86, \alpha = 1.30\)

symmetric countries, that is, additional welfare gains of 1.48% for the Home country and 1.55% for the Foreign one, with the disputed industries accounting for 1.36% of all industries.

In Table 2.3, we explore the possibility when one country has a larger labor force. The additional welfare gains from producing the set of disputed industries are approximately two times higher for the small country, 1.39% versus 0.70% for the large one (here again we use only the highest Marshallian externalities and comparative advantage parameter estimates). However, the maximum potential gains are still within the range implied by our benchmark case of symmetric countries.

While, in principle, there appears to be a potential role for industrial policy that results from the indeterminacy of trade patterns for a set of weak comparative advantage industries in the presence of low trade costs, quantitatively the scope for such a role appears to be modest.
2.3.2 The Gains from Trade

In this subsection, we ask: how do Marshallian externalities affect the overall gains from trade? We do a decomposition of gains from trade implied by the expression (2.25) using the parameter estimates from the previous section for the case of symmetric countries with low trade costs. Next, we show that the model readily extends to a multicountry setting when there are no barriers to trade, and yields interesting insights regarding the gains from trade.

2.3.2.1 Decomposition of the Gains from Trade: A First Look

In the previous subsection, we identified two sources of gains from trade: the gains from comparative advantage \( (\pi_{ii}^{−1/\theta}) \); and the gains from external economies of scale \( (\frac{\Upsilon_i}{w_iL_i})^{\alpha−1} \). Also, note that, given trade shares, accounting for Marshallian externalities imply larger gains from trade over and above those of a traditional constant returns framework, which captures only the gains from comparative advantage. A decomposition of these gains for the case of symmetric countries is reported in Table 2.4. The fourth row reports the overall gains from trade in percentage terms, whereas the last two rows report the contribution of comparative advantage and Marshallian externalities to the overall gains from trade for the case of low trade costs, \( \tau = \tilde{\tau} \). We focus on the “natural” equilibrium in which each country produces exactly one half of the entire set disputed industries, that is, \( \tilde{v} = 1/2 \).

In Table 2.4 we see that the contribution of external economies of scale to the overall gains from trade ranges from approximately 7% (strongest comparative adv-

---

14In particular, we compute \( \frac{w_i/P_i - w_A^A/P_A^A}{w_i^A/P_i^A} \times 100 \), where \( w_i/P_i \) is the real wage in the equilibrium with trade and \( w_i^A/P_i^A \) is the autarky real wage for country \( i \).

15We calculate the contribution of Marshallian externalities to the overall gains from trade by computing \( \ln \left( \frac{\Upsilon_i/w_iL_i}{(w_i/P_i)} \right) / \ln \left( \frac{(w_i/P_i)}{(w_i^A/P_i^A)} \right) \).
vantage, weakest Marshallian externalities) to 64% (weakest comparative advantage, strongest Marshallian externalities). Interestingly, our middle range estimates of the two key parameters ($\theta = 8.28$, $\alpha = 1.18$), imply the contribution can be substantial, with Marshallian externalities accounting for roughly 35% of the overall gains from trade of 14%.

2.3.2.2 Gains from Marshallian Externalities: A Multicountry Framework

In our partial analysis, we have already established that in the absence of trade costs there exists a unique equilibrium in which the patterns of specialization are consistent with comparative advantage. In the Appendix we demonstrate that the case of costless trade can be readily generalized to multiple countries in which the gains from trade for any country $n$ can be calculated using a simple formula depending only on the expenditure share on domestically produced goods and the two key parameters governing the strength of comparative advantage and Marshallian externalities.\(^\text{16}\)

\(^\text{16}\)Formally,

\[
\begin{array}{cccccc}
\alpha & 1.05 & 1.15 & 1.30 \\
\theta & 3.60 & 8.28 & 12.86 & 3.60 & 8.28 & 12.86 & 3.60 & 8.28 & 12.86 \\
\tilde{\tau} & 1.04 & 1.12 & 1.18 \\
Total \% & 22.95 & 10.27 & 7.03 & 26.13 & 13.68 & 10.52 & 31.81 & 19.29 & 16.12 \\
Comp. Adv. \% & 93.19 & 85.61 & 79.29 & 82.95 & 65.30 & 53.91 & 69.72 & 47.46 & 36.06 \\
Marsh. Ext. \% & 6.81 & 14.39 & 20.71 & 17.05 & 34.70 & 46.09 & 30.28 & 52.54 & 63.94 \\
\end{array}
\]

Table 2.4: Gains from Trade (Symmetric Countries with Low Trade Costs)

\[^{16}\text{One can verify that this case can also be readily extended to more general CES preferences and its associated demand with constant price elasticity } \sigma > 1. \text{ The only additional restriction required in this case is } \sigma < 1 + \lambda \theta \text{, where } \lambda \equiv \alpha (1 - \sigma) + \sigma.\]
Proposition 2.3.3. Under frictionless trade, the gains from trade for any country \( n \) are

\[
\frac{w_n/P_n}{w^A_n/P^A_n} = \pi_{nn}^{1/\theta} \pi_n^{-(\alpha-1)}. \tag{2.26}
\]

Here again the first term captures the gains from comparative advantage, while the second that from Marshallian externalities. Equation (2.26) has an interesting implication. As is consistent with a the standard EK-type model with no economies of scale, the overall gains from trade depend primarily on a country’s expenditure share on its own goods, and as such can vary across countries. However, a simple decomposition of the gains illustrate that, given trade shares, the contribution of each channel to the total gains from trade is constant across countries. Formally,

Corollary 2.3.4. For any country \( n \), comparative advantage and Marshallian externalities account for shares of the overall gains from trade \( \frac{1}{1+(\alpha-1)\theta} \) and \( \frac{(\alpha-1)\theta}{1+(\alpha-1)\theta} \), respectively.

Table 2.5 reports a decomposition of the gains from trade using corollary along with the parameter estimates from the previous subsection. Here the results indicate that Marshallian externalities account for at least about 15% and at most approximately 79% of the total gains from trade. Interestingly, the median parameter estimates suggests a contribution of more than a half of the overall gains from trade.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>1.05</th>
<th>1.15</th>
<th>1.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>3.60 8.28 12.86</td>
<td>3.60 8.28 12.86</td>
<td>3.60 8.28 12.86</td>
</tr>
<tr>
<td>Comp. Adv. %</td>
<td>84.75 70.72 60.86</td>
<td>64.94 44.60 34.14</td>
<td>48.08 28.70 20.59</td>
</tr>
<tr>
<td>Marsh. Ext. %</td>
<td>15.25 29.28 39.14</td>
<td>35.06 55.40 65.86</td>
<td>51.92 71.30 79.41</td>
</tr>
</tbody>
</table>

Table 2.5: Gains from Trade (Frictionless Trade)
2.4. Concluding Remarks

In this paper we provide insights to longstanding questions regarding external economies of scale and its implications for the patterns of international trade, the gains from trade, and a role for industrial policy. Our paper contributes to the literature by revisiting the implications of trade costs in a new game theoretic framework by GRH designed mainly to overturn the indeterminacy of trade patterns associated with the prevalence of multiple equilibria in the early literature.

In the main, we make three points. First, as is consistent with the early literature, we show that trade patterns are indeterminate for a set of “weak” comparative advantage industries in the presence of Marshallian externalities and low trade costs. More importantly, we demonstrate that the multiple equilibria associated with this set of “disputed” industries implies trade patterns need not be consistent with “natural” comparative advantage. Second, we show that the multiple Pareto-rankable equilibria associated with the set of “disputed” industries also provides a motive for industrial policy. We follow up with a quantitative exploration of its potential importance. The quantitative evidence suggests modest welfare gains of at most about 2%. Finally, our framework allows us to ask whether Marshallian externalities lead to additional gains from trade. Our analysis indicates that this is indeed the case. In particular, using the median parameter estimates, our quantitative results imply that Marshallian externalities can account for approximately 35% of the overall gains from trade.
Chapter 3

Brain Drain, Brain Return and On-the-Job Learning

3.1. Introduction

Is skilled migration from a less developed economy to a more developed one detrimental to the sending country?\(^1\) Early work by Berry and Soligo (1969) and Bhagwati and Hamada (1974) have suggested the answer to the former is yes. More recent work has suggested the answer may be more nuanced. For instance, Mountford (1997) (extended in a growth framework by Beine et al (2001)) has suggested that opportunities for migration to a more developed country may increase the incentives of persons in the less developed country to acquire more formal education (the “incentive effect”) and as a result translate into a higher proportion of “highly educated” individuals in the sending country.\(^2\)

Another source of benefit highlighted has been that of return migration. Christian Dustmann along with other authors have done considerable work on return migration and have argued this phenomenon has received little attention despite “the fact that many migrants return to their home countries after having spent a number of years in

\(^1\)For documented evidence of several major receiving countries relaxing immigration policies in favor of the highly skilled see Kapur and McHale (2005).

\(^2\)In other words skilled emigration from a developing country may translate into a "beneficial brain drain".

50
Moreover, recent case studies have suggested that a few highly skilled returnees can have a significant impact on the country of origin, and that returning emigrants often engage in entrepreneurial activity. While much of the scholarly efforts on return migration have focused on providing possible motivations for return (as well of estimates of return), recent work by Mayr and Peri (2009) has attempted to account for return migration and its underlying implications when doing a “brain drain” type analysis. Their results suggest that accounting for return may not only amplify the incentive effect, but could overall translate into higher average labor productivity for the source country. Other benefits highlighted which may mitigate the effects of the brain drain are remittances (Ozden and Schiff (2006)), and diaspora network effects (Gould (1994), and Rauch (1999)).

In this paper we revisit theoretically the welfare implications of skilled emigration on a small developing country. In particular, we take seriously the fact that immigration policies are becoming increasingly more favorable towards the highly skilled, and that emigrants often return to their home country after acquiring expertise overseas. In contrast to most of the existing literature, we abstract from the acquisition of formal education and instead investigate the role of on-the-job learning when considering the welfare effects on the developing country. Moreover, our framework allows us to not only analyse the welfare effects on workers, but also the welfare effects on managers/firm leaders, and ultimately the effect on per capita income.

Our paper is similar in spirit to Mayr and Peri (2008) and Zakharenko (2008),

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5See also Zakharenko (2008).
6Note that there is a large body of literature investigating and highlighting the importance of on-the-job learning (see for example Rosen (1972), empirical work by Mincer (1988) pg 26 Human Capital (Becker))
in that we also take return migration seriously when analysing the implications of skilled emigration on a developing country. It is distinct from Mayr and Peri (2008), in that, we abstract from formal education, which is central to their analysis, so as to highlight the role of on-the-job experience. More importantly, return migration in their framework arises via an exogenous premium paid to the returnee in the developing country. Whereas, in our model return is a natural result as experienced immigrant entrepreneurs (with superior knowledge) want to take advantage of lower wages in their home country. Moreover, the span of control model we use allows us to consider the welfare implications for not only workers but also firms profitability.

Zakharenko (2008) uses a pairwise matching framework with a notion of learning similar to ours. However, in his framework the cost of learning is somewhat internalized via Nash Bargaining for the surplus produced by the match. Whereas, in our model learning is an externality which arises from the interaction of managers and workers. While return is also a natural result in his framework, the model lacks analytical tractability and the resulting intuitive appeal evident in both Mayr and Peri (2008) and our framework.

We conduct our analysis in two distinct ways. First, we conduct what we refer to as a standard brain drain type analysis (albeit in an alternative modeling framework) as is done in most of the “new brain drain” literature. In particular, we do not account for return migration (which is in large part treated as negligible in much of the new brain drain literature), and investigate the welfare implications of skilled migration from both a small developing country to a large developed one while highlighting the role of on-the-job learning. Next, we extend the model to account for

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7The focus of Mayr and Peri’s analysis is on the "incentive effect" which in their model is amplified by return migration. However, Schiff et al (2005) argue that the "incentive effect" may be non-existent or at the very least exaggerated in a general equilibrium setting.
the fact that some skilled individuals do return to the home country after acquiring foreign expertise, and analyze the implications of “brain return”. In doing so, we also provide a rationale for return migration, in that, after acquiring expertise overseas, highly skilled migrant entrepreneurs return so as to take advantage of lower wages in the home country. However, poor institutions fraught with heavy government bureaucracy may potentially make it difficult for the migrant to realize these returns. We model this potential cost as a tax on the profits of the returnee, and analyze the policy implications of reducing this tax on the welfare of the developing country.

We use a variant of the Lucas (1978) type span of control model in an overlapping generations framework (similar to that used in Monge-Naranjo (2008)). As such, our model has two broad classes of individuals: an entrepreneurial class; and a working class. The standard brain drain type analysis of emigration from a small developing country suggests that, in equilibrium, the emigration of young talent leads to lower wages, higher profits and lower per capita income in the developing country (relative to autarky). As such, there is increased inequality within the developing country, as well as increased inequality across countries (divergence). In addition, if migration flows are unrestricted, the prospect of learning from more capable managers in the developed country prompts higher emigration flows of young talent from the developing country, and as a result exacerbates inequality within the developing country as well as across countries (amplifies the brain drain).

When the model is extended to allow for endogenous return, the outcome for both countries depend critically on how costly it is for emigrant entrepreneurs to return to their country of origin. If the cost is too high, then no emigrants will return and the result from the standard brain drain analysis follows. However, if the cost of return is low enough such that emigrant entrepreneurs find it profitable to return, then we consider two cases: one in which there is “incomplete learning spillovers” from
returning managers, and the other in which there is “complete learning spillovers” from returnees. In the case of the former, wages rise, profits fall and per capita income in the developing country rises (brain gain via brain return). In the case of the latter, if emigrants learn fully from their overseas bosses and this knowledge completely diffuses to their workers upon return, then the small developing country may eventually catch up the developed one.

3.2. The Model

Consider an infinite horizon OLG economy in which individuals live for two periods. Time is discrete and there exists one consumption good. The utility of an individual born at time $t$ that consumes $c_t^t$ and $c_{t+1}^t$ in periods $t$ and $t + 1$ is

$$U_t = c_t^t + \beta c_{t+1}^t$$

where $\beta \in (0, 1)$.

Cohorts are ex-ante identical and represent a continuum of measure one. As in Monge-Naranjo (2008), we assume, an exogenous fraction $\omega \in (0,1)$ are potential managers/firm leaders, and $1 - \omega$ are workers. By making this assumption, we abstract from the acquisition of formal education, which is explicitly modeled in Mayr and Peri (2008) and much of the new brain drain literature, so as to focus on the role “on the job learning”. In that regard, the economy can be viewed as being populated by two classes of individuals: the highly educated and the less educated class.

To the extent that formal schooling is related to entrepreneurial capability, the role of formal education can be viewed as the capacity of individuals to absorb
knowledge by learning on the job, thereby increasing the possibility of becoming a manager in the future. A more literal interpretation is that skilled individuals in our economy are ones that are endowed with entrepreneurial ability which can be nurtured by way of on-the-job learning. Workers work for both periods of life, while potential managers work in the first period and decide whether to be a practicing manager in the second period. We consider here a general learning which is simply the result of interaction between active managers and young potential managers, that is, learning by association (similar to that in Chari and Hopenhayn (1991)). As such, each young potential manager learns from their immediate manager. In the case of autarky (no migration), we assume young potential managers learn "fully" from their immediate managers, so that in the end the young potential manager has a skill level on par with that of his current manager. Hence, $2 - \omega$ is the minimum mass of workers, and $\omega$ is the maximum mass of managers. Let $n_s = N_w/N_m = (2 - \omega)/\omega > 1$ be the aggregate ratio of workers-per-firm if all potential managers choose to be practicing managers, where $N_w$ and $N_m$ are the total measure of workers and managers, respectively.

A firm consists of a team of one manager and $n$ units of labor such that a team with a manager of skill level $z$ produces output $y = zn^{\alpha}$, where $\alpha \in (0,1)$ is the manager’s span of control. In that regard, a potential manager must decide between remaining a worker and becoming a manager in the second period of his life. If he decides to be a manager, he must choose the amount of labor to hire.

In the immediate subsection, we characterize equilibrium in autarky. Next, we consider the welfare implications of skilled emigration on a small developing country without accounting for return, which is consistent with a standard brain drain type analysis. However, here we isolate and investigate the possible role of on-the-job

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8The manager being the residual claimant.
learning. Finally, we enrich the model to allow for both permanent as well as temporary migration (brain return) and simultaneously examine the welfare implications.

3.2.1 Autarky

Assuming no growth in the economy along with “full” learning suggests the problem becomes a static one. The potential manager in the second period of life decides whether to remain a worker or become a practicing manager. Hence, the agent chooses to be a manager if and only if the profits from being a manager is at least as large as the wage rate from being a worker. As such, the individual’s problem as a manager is given by

\[
\pi(z, w) = \max_n \{zn^\alpha - wn\}
\]

and the first order condition is

\[
\alpha zn^{\alpha - 1} = w \Rightarrow n^*(z, w) = \left[\frac{\alpha z}{w}\right]^{\frac{1}{1-\alpha}}
\]

Substituting this into the profit function yields

\[
\pi(z, w) = \alpha \frac{\alpha}{1-\alpha} (1-\alpha) z^{\frac{1}{1-\alpha}} w^{\frac{\alpha}{1-\alpha}}
\]

So a potential manager chooses to become a practicing manager if and only if

\[
\pi(z, w) \geq w
\]

For now, we assume all managers have identical skill level \(z > 0\) and all potential managers are strictly better off becoming active managers as opposed to remaining
workers, that is, \( n_s > \frac{\alpha}{1-\alpha} \) (the aggregate supply of workers per firm exceeds a certain threshold).

**Condition 3.2.1.** \( \frac{2 - \omega}{\omega} > \frac{\alpha}{1-\alpha} > 1.9 \)

Market clearing requires the demand for workers per firm is equal to the supply of workers per firm, that is, \( n^*(z, w) = n_s \). As such, the equilibrium wage rate is

\[
w^* = \alpha z n_s^{\alpha - 1}
\]

and the equilibrium profit per manager is

\[
\pi^* = (1 - \alpha) z n_s^\alpha
\]

Let \( Y \) and \( C \) be aggregate output and aggregate consumption respectively, and \( \bar{y} \) be per capita output. Then, \( Y, C \) and \( \bar{y} \) are given by

\[
Y = z \omega^{1-\alpha} (2 - \omega)^\alpha \equiv C
\]

and

\[
\bar{y} = \frac{1}{2} z \omega^{1-\alpha} (2 - \omega)^\alpha
\]

respectively.\(^{10}\)

\(^9\)The last inequality ensures that the managerial span of control is high enough (\( \alpha > 1/2 \)) which is consistent with the value used frequently in the literature (\( \alpha = 0.85 \)). See for example Atkeson and Kehoe (2005), and Restuccia and Rogerson (2008).

\(^{10}\)Note that \( \pi(z, w) \geq w \) can now be equivalently expressed as \( \frac{1}{w} \geq \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \).
3.2.2 Skilled Emigration from a Small Developing Country

In this subsection, we examine the implications of skilled (young potential managers) emigration from a small developing (home) country to a large developed (foreign) one. By small, we mean that immigration does not affect the wages and profits in the developed country.\textsuperscript{11} For simplicity, we assume that managers in the developing country have identical skill level $z_h$. Similarly, managers in the foreign country have identical knowledge level $z_f$. Developing suggests that foreign managers have superior managerial expertise such that $z_f > z_h$.

Suppose an exogenous quota $p \in (0, \omega)$ of young potential managers in the developing country are allowed to immigrate to a more developed country. Then young potential managers in the first period of life decide whether to emigrate or stay in the home country. Young potential managers which stay receive the home wages in the first period of life, and learn fully from the active managers there, so that at the end of the first period mature potential managers attain knowledge level $z_h$.\textsuperscript{12} After learning young potential managers must decide whether to remain workers or become active managers in the second period of life (occupational choice). If they choose to migrate, then they will receive the wages in the foreign country in the first period of life, and learn from the foreign managers. The extent of learning by the immigrant in the foreign country will be allowed to vary, and as such may not necessarily be full. Let $z_c$ be the knowledge of the immigrant potential manager after learning. Then $z_c \in [z_h, z_f]$ suggests the immigrant can learn no less than he would have if he had stayed in the home country.

For a given migration quota $p$, a young potential manager will migrate if the

\textsuperscript{11}This assumption is commonly used in the "new brain drain" literature.
\textsuperscript{12}Young potential managers in the foreign (developed) country (those who are foreign nationals) also learn fully from the active managers there.
expected payoff from migrating exceeds that from staying at home. From the case of autarky highlighted above, it is clear that the movement will affect equilibrium wages and profits in the home country by changing the relative supply of managers and workers. Hence, if $x_e$ workers leave, then the supply of workers at home in the first period of opening to migration is $2 - \omega - x_e$. However, in the second period after migration there will be a shortage of old potential managers $(\omega - x_e)$. Consequently, the aggregate supply of labor per firm has to account for both effects.

Define the expected payoff of a young potential manager from a developing country leaving (migrating), and staying-home as $V^L_h$, and $V^S_h$ respectively ($V^S_f$ is the expected payoff of a young potential manager in the foreign country). The payoff functions are then

$$V^L_h = (1 - \tau_1)w_f + \beta \max\{\pi_f(z_c, w_f), w_f\}$$

$$V^S_h = w_h + \beta \max\{\pi_h(z_h, w_h), w_h\}$$

$$V^S_f = w_f + \beta \pi_f(z_f, w_f)$$

where $\tau_1$ is a migration tax associated with the cost of migration.

**Definition 3.2.2.** Let a steady-state equilibrium (for given knowledge levels in both countries $z_f > z_h$, and learning with $z_c \in [z_h, z_f]$) with stable migration flows $x_e^* \in [0, p]$, wages and profits $w_h^*, w_f^*, \pi^*(z_h, w_h), \pi^*(z_c, w_f)$ be such that:

1. Managers maximize profits given wages and their knowledge level in both countries.

2. Occupational choice by agents is optimal.
3. Migration flows satisfy $V^L_h \leq V^S_h$, $x_e \geq 0$ and $(V^L_h - V^S_h) x_e = 0$; or $V^L_h > V^S_h$ and $x_e = p > 0$.

4. Labor markets clear in both countries.

The young potential manager will choose to migrate as long as his expected lifetime payoff from migrating is greater than his expected lifetime utility from staying home

$$V^L_h = (1 - \tau_1) w_f + \beta \max \{\pi_f(z_c, w_f), w_f\} > w_h + \beta \max \{\pi_h(z_f, w_h), w_h\} = V^S_h \quad (3.1)$$

We have in mind some stable migration flow $x_e \leq p$ every period. Hence, the wages and profits in the home country are given by

$$w_h = \alpha z_h [n^h_s]^\alpha - 1$$

and

$$\pi_h(z_h, w_h) = (1 - \alpha) z_h [n^h_s]^\alpha$$

where $n^h_s = (2 - \omega - x_e)/\omega$.

Also, the wages and profits of the immigrant in the foreign country can be written as

$$(1 - \tau_1) w_f = (1 - \tau_1) w^A_f = (1 - \tau_1) \alpha z_f n^\alpha_s - 1$$

and

$$\pi_f(z_c, w_f) = c\pi^A_f(z_f, w_f) = c (1 - \alpha) z_f n^\alpha_s$$
where $c = \left(\frac{z_c}{z_f}\right)^{1/(1-\alpha)}$. Note that $c$ measures the extent of learning, and $c \in [b, 1]$ where $b = \left(\frac{z_h}{z_f}\right)^{1/(1-\alpha)}$ is a measure of the relative development of the developing country to that of the developed one.

Note that below we establish that there is some threshold level $\bar{p}$ at which the equation (3.1) holds with equality, that is, $V^L_h(c) = V^S_h(p)$, where $V^L_h(c) = (1-\tau_1)w^A_f + \beta \max\left\{c\pi^A_f(z_f, w^A_f), w^A_f\right\}$ and $V^S_h(p) = w_h(p) + \beta \pi_h(z_h, w_h(p))$. Moreover, for any $p \leq \bar{p}$ the equilibrium entails one in which the migration quota is binding.

Before we proceed, it is instructive to note that a strictly positive measure of young potential managers will find it profitable to migrate as long as $V^L_h(c) > V^S_h(0)$. Denote $\hat{V}^L_h(c) = w^A_f + \beta \max\left\{c\pi^A_f(z_f, w^A_f), w^A_f\right\}$, and $V^S_h(0) = w^A_h + \beta \max\left\{\pi^A_h(z_h, w^A_h), w^A_h\right\}$.

Then the following condition guarantees that the tax on migration is low enough such that at least a strictly positive measure of young potential managers find it optimal to migrate to the developed country, that is, $\tau_1$ is low enough such that $V^L_h > V^S_h(0)$.

**Condition 3.2.3.** $\tau_1 < \tau_1$ where $\tau_1 = (1/w^A_f) \left[\hat{V}^L_h(c) - V^S_h(0)\right] > 0$.\(^{13}\)

**Proposition 3.2.4.** For any $z_f > z_h$, $\tau_1$ and $c \in [b, 1]$ consistent with conditions 3.2.1 and 3.2.3 and for $\beta \geq \beta^*$ (see below), there exists a unique migration threshold level $\bar{p} \in (0, \omega)$. Moreover, for any $p \leq \bar{p}$ the unique equilibrium migration level is $x^*_e = p$. Whereas, for any $p > \bar{p}$ the unique equilibrium migration level is $x^*_e = \bar{p}$.

**Corollary 3.2.5.** Wages in the home country falls relative to autarky such that $w^A_f > w^A_h > w_h(x^*_e)$. While profits in the home country rises relative to autarky ($\pi_h(z_h, w_h(x^*_e)) > \pi_h(z_h, w_h)$). Hence, inequality within the developing country increases. Suppose also that $p > \bar{p}$. Then the prospect of learning prompts higher

\(^{13}\)For the case $c < \theta$ (immigrant potential managers learning is not sufficient for them to choose to be active managers in the foreign country), the condition is simply $\tau_1 < (1+\beta) - [1+\theta\beta^{-1}]b^{1-\alpha}$. Otherwise ($c \geq \theta$), the condition is given by $\tau_1 < \beta\theta^{-1}(c - b^{1-\alpha}) + \alpha(1-b^{1-\alpha})$.\)
equilibrium migration flows, that is, if \( x_e^* = \bar{p} > 0 \), then \( \frac{dp}{dc} \geq 0 \) (if \( c \geq \theta, \frac{dc}{dx} > 0 \)).

**Proposition 3.2.6.** Skilled migration flows lowers per capita income in the small developing country relative to autarky, and as a result increases inequality across countries. Moreover, if the migration quota is non-binding and \( c \geq \theta \), then learning amplifies inequality across countries i.e. \( \frac{y_h}{y_f} \) falls relative to autarky.

**Proof.** Suppose \( p > \bar{p} \). In equilibrium migration flows are \( x_e^* = \bar{p} \), and per capita income in the developing country is given by \( y_h(x_e^*) = \frac{1}{2} z_h(1-x_e^*)^{-1}(\omega-x_e^*)^{1-\alpha}(2-\omega-x_e^*)^{\alpha} \). Then \( \frac{dy_h}{dx_e} < 0 \) implies that \( y_h(x_e^*) < y_f^A \). Since \( y_f(x_e^*) = y_f^A \), then \( y_h(x_e^*)/y_f(x_e^*) < y_h^A/y_f^A \). In addition, if \( c \geq \theta \), \( \frac{dx_e}{dc} > 0 \). Then both \( \frac{dx_e}{dc} > 0 \) and \( \frac{dy_h}{dx_e} < 0 \) imply \( \frac{dy_h}{dc} < 0 \). \( \square \)

### 3.2.3 Accounting for Return: Skilled Emigration from a Small Developing Country

In order to integrate emigration and return, while also exploring some possible policy implications, we assume that there is a tax on return, \( \tau_2 \), which may be due to poor institutions (examples are corruption, bureaucracy, nepotism, e.t.c.) which creates an unfavorable business environment for the returning entrepreneur.\(^{15}\) Define \( \tilde{V}_h^L = \max \{ \max \{ \pi_f(z_c, w_f^A), w_f^A \}, (1-\tau_2)\pi_h(z_c, w_h) \} \). Then the payoff functions for migrating, and staying are given by

\[ \text{max} \{ \pi_f(z_c, w_f^A), w_f^A \}, (1-\tau_2)\pi_h(z_c, w_h) \}

\(^{14}\)Note that \( \theta = (\alpha/(1-\alpha))(\omega/(2-\omega)) \) is the cutoff learning derived from equating \( \pi_f(z_c, w_f^A) \) and \( w_f^A \).

\(^{15}\)In contrast, Mayr and Peri (2008) assume a uniform cost of staying in the foreign country in both periods. However, in their framework they impose an exogenous premium paid to returnees (which essentially amplifies the returns to schooling at home for returnees). Whereas, in our model, return is a natural outcome as mature potential firm leaders would like to take advantage of lower wages at home (recall the standard brain drain analysis in which wage inequality across countries increases). However, a poor institutional environment in the home country makes it costly for the potential returnee to benefit from the low wages at home.
\[ V_h^L(c) = (1 - \tau_1)w_f^A + \beta \tilde{V}_h^L \]

\[ V_h^S = w_h(x_e, x_r) + \beta \pi_h(z_h, w_h) \]

where \( x_e \in [0, p) \) is the measure of young potential managers who migrate, and \( x_r \in [0, x_e] \) is the measure of returnees.

**Definition 3.2.7.** Let a steady-state equilibrium (for given knowledge levels in both countries \( z_f > z_h \), and learning with \( z_c \in [z_h, z_f] \)) with stable migration flows \( x_e^* \in [0, p] \) and return flows \( x_r^* \in [0, x_e^*] \), wages and profits \( w_h^*, w_f^*, \pi^*(z_h, w_h), \pi^*(z_c, w_f), \pi^*(z_c, w_h) \) be such that:

1. Managers maximize profits given wages and their knowledge level in both countries.
2. Occupational choice by agents is optimal.
3. Migration flows satisfy \( V_h^L \leq V_h^S \), \( x_e^* \geq 0 \) and \((V_h^L - V_h^S) \neq 0\), or \( V_h^L > V_h^S \), \( x_e^* > 0 \); and \((1 - \tau_2)\pi^*(z_c, w_h) \leq \pi^*(z_c, w_f) \), \( x_r^* \geq 0 \) and \([(1 - \tau_2)\pi^*(z_c, w_h) - \pi^*(z_c, w_f)]x_r^* = 0\), or \((1 - \tau_2)\pi^*(z_c, w_h) > \pi^*(z_c, w_f) \), and \( x_r^* = x_e^* > 0 \).
4. Labor markets clear in both countries.

We first consider the case in which there are incomplete learning spillovers from returning managers. By that we mean that returnees operate at the knowledge level acquired overseas, however, their workers can learn no more from them than they would have from a manager who stayed home. This can be viewed as a barrier to
knowledge diffusion via learning between the returnee and the young potential managers who work for them. As such, we understate the potential impact of returnees. Finally, we consider the case of complete learning spillovers, i.e., the case in which young potential managers learn fully from their returnee bosses.

3.2.3.1 Incomplete Learning from Returning Managers

Since there is incomplete learning spillovers from returnee, the equilibrium entails one in which two types of managers co-exist in the home country: $\omega - x_e$ managers with skill level $z_h$; and $x_r$ returning entrepreneurs with knowledge level $z_c$. Market clearing requires

\[
(\omega - x_e)n^*(z_h, w_h) + x_r n^*(z_c, w_h) = 2 - \omega - x_e
\]

\[
\Rightarrow w_h = \alpha z_h \left[ n^h (x_e, x_r) \right]^{\alpha - 1}
\]

where $n^h (x_e, x_r) = (2 - \omega - x_e) / (2 - \omega - x_e + cb^{-1} x_r)$. Also, profits for the home and returnee managers are given by

\[
\pi_h(z_h, w_h) = (1 - \alpha)z_h \left[ n^h (x_e, x_r) \right]^{\alpha}
\]

and

\[
\pi_h(z_c, w_h) = (1 - \alpha)cb^{-1} z_h \left[ n^h (x_e, x_r) \right]^{\alpha}
\]

respectively.

Then young potential managers will emigrate from the small developing country
as long as $V^L_L(c) \geq V^S_h$, that is,

$$V^L_L(c) = (1 - \tau_1)w_f + \beta V^L_L \geq w_h(x_e, x_r) + \beta \pi_h(z_h, w_h) = V^S_h.$$  

However, note that conditional on migrating the young potential managers (in the second period of life) will return as long as

$$(1 - \tau_2)\pi_h(z_c, w_h) \geq \pi_f(z_c, w^A_f) \quad (3.2)$$

Taking (3.2) to hold with equality, we can derive

$$x_r(x_e) = (\tilde{\tau}_2 - b)c^{-1}\omega + (b - \tilde{\tau}_2 n_s^{-1})c^{-1}x_e \quad (3.3)$$

where $\tilde{\tau}_2 = (1 - \tau_2)^{1/\alpha}$.

From (3.3), it is evident that if the managerial expertise of the managers in the developing country is high enough relative to that in the developed country ($b > \tilde{\tau}_2 n_s^{-1}$), then the propensity to return is increasing in the level of emigration from the developing country ($\frac{dx_r}{dx_e} > 0$).\footnote{{A sufficient condition is $b > n_s^{-1}$}} For the analysis which follows, we restrict our attention to the case in which $b > \tilde{\tau}_2 n_s^{-1}$. Moreover, $\frac{dx_r}{d\tau_2} < 0$ suggests the propensity to return is decreasing in the tax on return.

Define return as

$$x_r = \begin{cases} x_r(x_e) & \text{if } x_e > 0 \\ 0 & \text{otherwise} \end{cases}$$

and $x_r \leq x_e$. Then the latter restriction implies that the form of the return function depends on the level of the tax rate on return ($\tau_2$). As such, we can redefine return as follows:
(i) if $\tau_2 < 1 - b^\alpha$, then
\[
x_r = \begin{cases} 
  x_r(x_e) & \text{if } x_e > \hat{x}_e \\
  x_e & \text{otherwise}
\end{cases}
\]
(3.4)

where $\hat{x}_e = \left[ (\tilde{\tau}_2 - b) c^{-1} / (1 - (b - \tilde{\tau}_2 n_s^{-1}) c^{-1}) \right] \omega$.

(ii) if $\tau_2 > 1 - b^\alpha$, then
\[
x_r = \begin{cases} 
  x_r(x_e) & \text{if } x_e > \tilde{x}_e \\
  0 & \text{otherwise}
\end{cases}
\]
(3.5)

where $\tilde{x}_e = \left[ (b - \tilde{\tau}_2) / (b - \tilde{\tau}_2 n_s^{-1}) \right] \omega$.

For simplicity, we consider below the cases in which the tax on return ($\tau_2$) is either zero or one, and the tax on migration ($\tau_1$) is zero. We also circumvent complications associated with optimal occupational decisions in the developing country which may arise due to return by imposing the following condition.

**Condition 3.2.8.** $(1 - \omega) / cb^{-1} \omega > \alpha / (1 - \alpha) > 1.$

Note also that, in the absence of any cost associated with return ($\tau_2 = 0$), a strictly positive measure of young potential managers will find it profitable to return (conditional on emigrating) as long as $\pi_h(z_c, w_h(0, 0)) > \pi_f(z_c, w_f^A)$. Condition 3.2.8 also guarantees that this is always the case. The following proposition establishes the existence of an equilibrium with return migration.

**Proposition 3.2.9.** Assume $\tau_1 = 0$ and Condition 3.2.8 holds. For any $z_f > z_h$ and $c \in (b, 1] : (i)$ if $\tau_2 = 0$ and $p \leq \tilde{p} = \hat{x}_e$, there exists a unique equilibrium $x_r^* = x_e^* = p$.

---

17 This condition can alternatively be thought of requiring the developing country to be sufficiently developed, that is, $b > \left( \frac{\omega}{1 - \omega} \right) \frac{\alpha}{1 - \alpha}$.

18 Note that $\tau_2 = 0$ implies $\tilde{x}_e > 0$. 
(ii) if \( \tau_2 = 1 \) and \( p \leq \bar{p} = \bar{x}_e \), then there exists a unique equilibrium such that \( x_e^* = p \) and \( x_r^* = 0 \).

**Corollary 3.2.10.** In an equilibrium with return migration, wages rise and profits fall in the developing country. In the no return equilibrium, the converse is true.

**Proposition 3.2.11.** In an equilibrium with return migration, per capita income in the developing country increases, and as a result lowers inequality across countries (brain gain). In the no return equilibrium, the converse is true (brain drain).

**Proof.** Note that \( \frac{d\bar{y}_h(x_e)}{dx_e} > 0 \) implies that \( \bar{y}_h(x_e^*) > \bar{y}_h^A \). Since \( \bar{y}_f(x_e^*) > \bar{y}_f^A \), then \( \frac{\bar{y}_h(x_e^*)}{\bar{y}_f(x_e^*)} > \frac{\bar{y}_h^A}{\bar{y}_f^A} \). The latter result follows from Proposition 3.2.6.

Proposition 3.2.11 suggests that policy measures which reduces the cost associated with return may lead to overall gains from skilled emigration for the developing country via the expertise acquired abroad (brain gain via brain return).

### 3.2.3.2 Complete Learning From Returning Managers

If young potential managers in the developing country learn fully from returning managers with superior expertise acquired overseas, then the long run equilibrium entails no emigration. In particular, the developing country will converge to the developed one infinite time with the amount of catch-up limited only by the extent of learning overseas by the returning emigrant. For simplicity, we present the transition path for the simple case in which there is no emigration nor return tax, and there is full learning by emigrants overseas. In particular, let \( \tau_2 = \tau_1 = 0 \) and \( c = 1 \). We use as our starting point the equilibrium in which there are incomplete learning spillovers (i.e., there are no learning spillovers other than \( z_h \) for young potential managers working for a returnee with \( z_f \)), and consider a stable transition path when we allow for complete learning spillovers.
Definition 3.2.12. A stable transition path is one in which migration and return flows satisfy: \( V^L_h \leq V^S_h, \ x_e \geq 0 \) and \( (V^L_h - V^S_h) x_e = 0 \), or \( V^L_h > V^S_h, \ x_e > 0 \); and \( (1 - \tau_2) \pi(z_c, w_h) \leq \pi(z_c, w_f) \), \( x_r \geq 0 \) and \( [(1 - \tau_2) \pi(z_c, w_h) - \pi(z_c, w_f)] x_r = 0 \), or \( (1 - \tau_2) \pi(z_c, w_h) > \pi(z_c, w_f) \), and \( x_r = x_e > 0 \).

As shown in the previous subsection, the equilibrium with incomplete learning spillovers entails one in which two types of managers coexists in developing country: \( \omega - x_e \) managers with skill level \( z_h \), and \( x_r \) returning entrepreneurs with knowledge level \( z_f \). Moreover, the payoffs from leave and stay are

\[
V^L_h(c) = w_f^A + \beta \max\{\pi(z_f, w_f^A), \pi(z_f, w_h(x_e, x_r))\}
\]

and

\[
V^S_h = w_h(x_e, x_r) + \beta \pi_h(z_h, w_h(x_e, x_r)),
\]

respectively.

And

\[
w_h = \alpha z_h \left[ n_h^h(x_e, x_r) \right]^{\alpha - 1},
\]

\[
\pi(z_h, w_h) = (1 - \alpha) z_h \left[ n_h^h(x_e, x_r) \right]^\alpha,
\]

\[
\pi(z_f, w_h) = (1 - \alpha) b^{-1} z_h \left[ n_h^h(x_e, x_r) \right]^\alpha,
\]

where \( n_h^h(x_e, x_r) = (2 - \omega - x_e)/(\omega - x_e + b^{-1} x_r), \ x_e \in [0,p) \), and \( x_r \in [0,x_e]. \)

Let \( p \) solve \( \pi(z_f, w_f^A) = \pi(z_f, w_h(p,p)) \), and consider the equilibrium with return \( x^*_r = x^*_e = p \). Here we have \( w_f = w_h(p,p) \) and \( \pi(z_f, w_f^A) = \pi(z_f, w_h(p,p)) > \).
The latter implies an incentive for emigration is always present, and in this equilibrium the migration quota is binding. Moreover, we have

\[ n_h^h(x_e^s, x_r^s) = \frac{2 - \omega - \bar{p}}{\omega - \bar{p} + b^{-1}\bar{p}} \]

Suppose now we allow young potential managers to learn fully from their returnee bosses (acquire \( z_f \)). Then the value of staying becomes

\[ V_h^S = w_h(x_e, x_r) + \beta[q \pi_h(z_h, w_h(x_e, x_r)) + (1 - q) \pi(z_f, w_h(x_e, x_r))] \]

where

\[ q = \frac{\text{measure of active managers with } z_h}{\text{total measure of managers}} \]

If we start from this equilibrium, then next period (call it period 1) after allowing for learning we have \( \omega - x_1^e - \bar{p} \) managers with \( z_h \) and \( \bar{p} + x_1^r \) managers with \( z_f \). We consider a stable transition path in which there is indifference between staying or returning \( \pi(z_f, w_1^f) = \pi(z_f, w_h(x_e, x_r)) \) in the second period after migration, or equivalently, one in which the the adjustments are such that \( n_h^h \) is unchanged. In particular, we have \( x_1^e = \bar{p} \). Hence, we require \( x_1^r \) to be such that this holds. \( n_h^h \) unchanged implies we also have \( w_1^f = w_h \) along the adjustment path. So

\[ n_h^{h1} = \frac{2 - \omega - \bar{p}}{\omega - \bar{p} + b^{-1}(\bar{p} + x_1^r)} \]

Since

\[ n_h^{h0} = \frac{2 - \omega - \bar{p}}{\omega - \bar{p} + b^{-1}\bar{p}} \]
indifference requires \( x^1_r = b \bar{p} \). Similarly, period 2 we have

\[
\begin{align*}
    n_{s2} & = \frac{2 - \omega - \bar{p}}{\omega - \bar{p} - (\bar{p} + x^1_r) + b^{-1}(\bar{p} + x^1_r + x^2_r)} \\
    & = \frac{2 - \omega - \bar{p}}{\omega - \bar{p} - (\bar{p} + b \bar{p}) + b^{-1}(\bar{p} + b \bar{p} + x^2_r)} \\
\end{align*}
\]

Indifference again requires \( x^2_r = b^2 \bar{p} \). Iterating to period \( n \), we have indifference requires \( x^n_r = b^n \bar{p} \). Notice that as \( n \to \infty \), \( x^n_r \to 0 \) and \( q \to 0 \). So the developing country converges to the level of development of the developed one. Moreover, \( q \to 0 \) implies \( V^S_h \to V^L_h \), which, in turn, implies \( x_e \to 0 \), so that the long run equilibrium entails no emigration.

### 3.3. Conclusion

Scholarly work over the last half century has attempted to understand of the economic effects of the brain drain. This paper contributes to this literature by providing a alternative framework to simultaneously examine the effects of skilled migration and return for a developing country. In particular, we incorporate the fact that some skilled emigrants do return to the developing country after acquiring expertise abroad by way of on-the-job learning, and consider the welfare implications of this for a small developing country. Our results suggest that if we treat return migration as negligible (as is done in most of the brain drain literature), then skilled emigration lowers wages, increases profits, lowers per capita income, and increases inequality within the developing country. Moreover, if migration non-binding, then on-the-job learning exacerbates the inequality across countries. This result is consistent with
early work in the literature, and would suggest that developing countries need to implement restrictive policies in order to stem the flow of skilled emigrants.

Once we account for return migration, the answer is more nuanced. In that, whether the developing country receives any gains from skilled emigration depends critically on its ability to attract returnees with valuable foreign expertise by lowering the cost associated with return. If the cost of return is low enough, then skilled emigration leads to a rise in wages, fall in profits, increase in per capita income and a decrease in inequality within the developing country as well as across countries even in the absence of incomplete knowledge spillovers from returning entrepreneurs. In other words, skilled emigration leads to a beneficial brain drain by allowing the developing country to benefit from superior foreign expertise acquired abroad. Importantly, once we allow for complete knowledge spillovers from returning managers, if the emigrant learns fully from her overseas bosses, then the developing country eventually catches up the developed one.

Interestingly, our results suggest that a policy approach which seeks to restrict skilled emigration flows may not be welfare maximizing. A better approach may be to institute policies which lowers the cost of return for skilled emigrants who have acquired enhanced expertise overseas.
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Appendix A

Proofs for Chapter 1

Profits are increasing in prices

It can be readily verified that with our assumptions on demand and technology, along with the assumption $\phi < 1/2$ is sufficient to imply that profits are increasing in the price for a firm that sells in a single market. Now consider the case of a firm that sells in both markets. A Home firm that sells at prices $p$ and $p^*$ in Home and Foreign makes profits of

$$\pi(p, p^*) \equiv \left[p - \frac{wa}{A(x(p) + x^*(p^*))}\right] x(p) + \left[p^* - \frac{wat}{A(x(p) + x^*(p^*))}\right] x^*(p^*).$$

Simple differentiation reveals that, $\pi_2(p, p^*) > 0$ if $\phi < 1$, but $\pi_1(p, p^*) > 0$ requires a more stringent condition, namely $\phi < t \frac{x(p) + x^*(p^*)}{x(p) + tx^*(p^*)}$. A sufficient condition here is that $t < 1/\phi$, which is stated above as Assumption 1.2.2.

**Proof of Proposition 1.3.1.** Define $t_{MAX} = 1/\phi$ and $\phi_{MAX} = 1/2$ (recall Assumption 1.2.1c requires $\phi \leq 1/2$). Note that $g^*(t) = 1/g(t)$ and $h^*(t) = 1/h(t)$. For future reference we refer to this as ”symmetry”. To prove the existence of $\tilde{t}$ and $t_{CS}$ we use two results: first, that $g(1) = 1$ and $g(t)$ is increasing for $t > 1$, and second that $h(1) > 1$ and $h(t)$ is decreasing for $t \geq 1$ with $h(t_{MAX}) < 1$. To prove the first result, note that $2g'(t) = 1 - 1/t^2 > 0$ implies $g'(t) > 0$. To prove the second result, note that $\frac{\partial h(t)}{\partial \ln t} = \frac{t}{t+1} - 1$. Since $\phi < 1$ (Assumption 1.2.1c), then this is negative. Also, note that $h(t_{MAX}) = t_{MAX}^{-1}(1 + t_{MAX})^\phi = \phi (1 + 1/\phi)^\phi$. Since $\phi (1 + 1/\phi)^\phi$ is increasing in
\( \phi \), to prove that \( h_H(t_{MAX}) < 1 \) for any \( \phi \in (0, \phi_{MAX}] \) it is sufficient to show that 
\( \phi_{MAX} (1 + 1/\phi_{MAX})^{\phi_{MAX}} < 1 \). But this is clearly satisfied. These two results along with the continuity of both \( g(.) \) and \( h(.) \) imply that there exists a unique \( \tilde{t} \) which is higher than 1, with \( g(\tilde{t}) > 1 \). Moreover, for any \( \beta \omega \in [h(t_{MAX}), 1) \) there exists a unique \( t^*_{HS} \). Symmetry implies that \( \tilde{t} \) uniquely satisfies \( g^*(t) = h^*(t) \) and \( g^*(\tilde{t}) < 1 \). Symmetry also implies that \( g^*(1) = 1 \) and \( g^*(t) \) is decreasing for \( t > 1 \), and second that \( h^*(1) < 1 \) and \( h^*(t) \) is increasing for \( t \geq 1 \) with \( h^*(t_{MAX}) > 1 \). One can also verify that there exists a unique \( t \) such that \( h(t) = h^*(t) = 1 \) and \( t > \tilde{t} \). The result then follows.

\[ \square \]

**High Transportation Costs**

We now establish that for any good \( \beta \omega \leq 1 \) there exists a range of transportation costs for which no pure strategy in which production is either concentrated in a single country nor one in which there is domestic only production can be sustained as an equilibrium. Recall first that by Proposition 1.3.2 we know that an equilibrium with no trade exists if and only if \( \beta \omega \geq l(t) \). As established above, \( l(t) \) is decreasing and intersects the horizontal line with \( \beta \omega = 1 \) at point \( t_{NT}(1) = 2^{1+\phi} - 1 \). It is readily verified that \( t_{CS}(1) < t_{NT}(1) \). To see this recall that \( t_{CS}(1) \) is defined implicitly by 
\[ 1 = h(t) \equiv \frac{(1+t)^\phi}{t}. \]
Since \( h(.) \) is strictly decreasing, to show that \( t_{NT}(1) > t_{CS}(1) \), it is sufficient to show that \( 1 > \frac{(1+t_{NT}(1))^\phi}{t_{NT}(1)} \Leftrightarrow t_{NT}(1) > (1 + t_{NT}(1))^\phi \), or \( 2^{1+\phi} - 1 > (2^{1+\phi})^\phi \). But this is satisfied for all \( \phi < 1/2 \), a restriction satisfied by Assumption 1.2.1c.

We now establish that the curve \( h(t) \) is always below the curve \( l(t) \), so that \( t_{CS}(\beta \omega) < t_{NT}(\beta \omega) \). This further implies that for any relevant \( \beta \omega \leq 1 \) there is no pure strategy equilibrium for \( t \in (t_{CS}(\beta \omega), t_{NT}(\beta \omega)) \). As mentioned above our analysis is restricted to the range of \( t \) that satisfies Assumption 1.2.2, i.e., \( t < 1/\phi \).
Define $t_{MAX} \equiv 1/\phi$. Assumption 1.2.2 implies our analysis is relevant for any $t \in [1, t_{MAX}]$. We now proceed to establish that $l(t) > h(t)$ for all $t \in [t_{NT}, t_{MAX}]$. We do this in three steps: first, we first show that $l(t_{MAX}) > h(t_{MAX})$, second, we establish that $l'(t) \leq h'(t) \leq 0$ for all $t \in (1, t_{MAX})$, and third, we establish the final result using steps one and two.

Step 1: Since $l(.)$ is decreasing then $l(t_{MAX}) > h(t_{MAX})$ is equivalent to $1 < \phi l(h(1/\phi))$, which in turn is equivalent to

$$1 < \frac{2 \left( 1 + \left( \phi (1 + 1/\phi) \right)^{1/(1-\phi)} \right)^{\phi} - 1}{\left( \phi (1 + 1/\phi) \right)^{1/(1-\phi)} / \phi}.$$ 

It can be verified that this inequality is satisfied for $0 \leq \phi \leq 1/2$.

Step 2: Now we proceed to show that $l'(t) \leq h'(t) \iff |h'(t)| \leq |l'(t)|$ for all $t \in (1, t_{MAX})$. Totally differentiating $\frac{2(1+y)^{(1/(1-\phi))}}{y^{1/(1-\phi)}} - 1 = t$, we have

$$l'(t) = -\frac{(1-\phi) y}{t - \phi \left( \frac{1+ty^{1/(1-\phi)}}{1+y^{1/(1-\phi)}} \right)}.$$ 

Similarly, we have $h'(t) = -\left( \frac{1}{t} - \frac{\phi}{1+t} \right) y$. Hence, $|h'(t)| \leq |l'(t)|$ if

$$\frac{1}{t} - \frac{\phi}{1+t} \leq \frac{1 - \phi}{t - \phi \left( \frac{1+ty^{1/(1-\phi)}}{1+y^{1/(1-\phi)}} \right)}.$$ 

A sufficient condition for this is

$$\phi \leq \frac{1}{t}$$

which is clearly satisfied for $t = t_{MAX}$. The result then follows.
Step 3: We now establish that $l(t) > h(t)$ for all $t \in [t_{NT}, t_{MAX}]$. From the analysis above along with Step 1, we already know that $l(t_{NT}) > h(t_{NT})$ and $l(t_{MAX}) > h(t_{MAX})$. Suppose by contradiction there exists $t' \in (t_{NT}, t_{MAX})$ such that $l(t') = h(t')$. Then $l(t) \leq h(t)$ (by Step 2) along with $l(t_{NT}) > h(t_{NT})$ implies $l(t_{MAX}) < h(t_{MAX})$. A contradiction. Hence, the result follows.
Appendix B

Proofs for Chapter 2

Profits are increasing in prices

It can be readily verified that with our assumptions on demand and technology, \( \phi < 1 \) (by Assumption 2.2.1) is sufficient to imply that profits are increasing in the price for a firm that sells in a single market. Now consider the case of a firm that sells in both markets. A Home firm that sells at prices \( p \) and \( p^* \) in Home and Foreign makes profits of

\[
\pi(p_H, p_F) \equiv \left[ p_H - \frac{w_H a_H}{A(x_H(p_H) + x_F(p_F))} \right] x_H(p_H) + \left[ p_F - \frac{w_H a_H \tau}{A(x_H(p_H) + x_F(p_F))} \right] x_F(p_F).
\]

Simple differentiation reveals that, \( \pi_2(p_H, p_F) > 0 \) if \( \phi < 1 \), but \( \pi_1(p_H, p_F) > 0 \) requires a more stringent condition, namely \( \phi < \tau \frac{x_H(p_H) + x_F(p_F)}{x_H(p_H) + x_F(p_F)} \). A sufficient condition here is that \( \tau < 1/\phi \), which is stated above as Assumption 2.2.2.

Proof of Proposition 2.2.1. Define \( \tau_{MAX} = 1/\phi \) and \( \phi_{MAX} = 1/2 \) (recall Assumption 2.2.1 requires \( \phi \leq 1/2 \)). To prove the existence of \( \tilde{\tau} \) and \( \tau_{HS}^C \) we use two results: first, that \( g_H(1) = 1 \) and \( g_H(\tau) \) is increasing for \( \tau > 1 \), and second that \( h_H(1) > 1 \) and \( h_H(\tau) \) is decreasing for \( \tau \geq 1 \) with \( h_H(\tau_{MAX}) < 1 \). To prove the first result, note that \( 2g_H(\tau) = 1 - 1/\tau^2 > 0 \) implies \( g_H(\tau) > 0 \). To prove the second result, note that \( \frac{\partial \ln h_H(\tau)}{\partial \ln \tau} = \phi \frac{\tau}{1+\tau} - 1 \). Since \( \phi < 1 \) (by Assumption 2.2.1), then this is negative.

Also, note that \( h_H(\tau_{MAX}) = \tau_{MAX}^{-1} (1 + \tau_{MAX})^\phi = \phi (1 + 1/\phi)^\phi \). Since \( \phi (1 + 1/\phi)^\phi \) is increasing in \( \phi \), to prove that \( h_H(\tau_{MAX}) < 1 \) for any \( \phi \in (0, \phi_{MAX}] \) it is sufficient
to show that $\phi_{\text{MAX}} (1 + 1/\phi_{\text{MAX}})^{\phi_{\text{MAX}}} < 1$. But this is clearly satisfied. These two results along with the continuity of both $g_H(.)$ and $h_H(.)$ imply that there exists a unique $\tilde{\tau}$ which is higher than 1, with $g_H(\tilde{\tau}) > 1$. Moreover, for any $\beta \omega \in [h_H(\tau_{\text{MAX}}), 1)$ there exists a unique $\tau_{H}^{CS}$. Symmetry implies that $\tilde{\tau}$ uniquely satisfies $g_F(\tau) = h_F(\tau)$ and $g_F(\tilde{\tau}) < 1$. Symmetry also implies that $g_F(1) = 1$ and $g_F(\tau)$ is decreasing for $\tau > 1$, and second that $h_F(1) < 1$ and $h_F(\tau)$ is increasing for $\tau \geq 1$ with $h_F(\tau_{\text{MAX}}) > 1$.

One can also verify that there exists a unique $\tau$ such that $h_H(\tau) = h_F(\tau) = 1$ and $\tau > \tilde{\tau}$.

We now need to establish that for every $\phi$ the ranges for both cases a and b exist, i.e., $h_H(\tau_{\text{MAX}}) < g_F(\tilde{\tau}) < 1$. We have already shown above that the second inequality holds. Hence, we need to show that for every $\phi$ we have $h_H(\tau_{\text{MAX}}) < g_F(\tilde{\tau})$. Recall that $h_H(\tau_{\text{MAX}}) = \frac{1}{\tau_{\text{MAX}}} (1 + \tau_{\text{MAX}})^{\phi} = \phi (1 + 1/\phi)^{\phi}$ and $\tilde{\tau}$ is implicitly defined by $g_F(\tau) = h_F(\tau)$ or equivalently $\frac{1}{1/\tau + \tau} = (1 + \tau)^{-\phi}$. So we need to show that for all $\phi \in [0, \phi_{\text{MAX}}]$ we have $h_H(\tau_{\text{MAX}}) < g_F(\tilde{\tau})$ or $\phi (1 + 1/\phi)^{\phi} < g_F(\tilde{\tau})$. Define $\tilde{\tau}_{\text{MAX}}$ as that which implicitly solves $\frac{2}{1/\tau + \tau} = \tau (1 + \tau)^{-\phi_{\text{MAX}}}$. Note that $\phi (1 + 1/\phi)^{\phi}$ is increasing in $\phi$. Also, note that $\tilde{\tau}$ is increasing in $\phi$ implies $g_F(\tilde{\tau})$ is decreasing in $\phi$. Hence, it is sufficient to show $\phi_{\text{MAX}} (1 + 1/\phi_{\text{MAX}})^{\phi_{\text{MAX}}} < g_F(\tilde{\tau}_{\text{MAX}})$. One can then readily verify that this is satisfied. The result then follows, that is, for any $\phi \in [0, 1/2]$ there exists a range for which both case (a), $g_F(\tilde{\tau}) > \beta \omega \geq h_H(\tau_{\text{MAX}})$, and (b), $1 > \beta \omega \geq g_F(\tilde{\tau})$, apply.

The results for cases a and b then follow.

\begin{lemma}
\label{lem:vH_vF}
The functions $v_H^0(\tau)$ and $v_F^0(\tau)$ exist (this entails existence and uniqueness of a solution in $v$ to (2.18) and (2.20) respectively), $v_H^0(\tau)$ is increasing and $v_F^0(\tau)$ is decreasing, and for any $\tau > 1$ we have $v_H^0(\tau) > 1/2 > v_F^0(\tau)$.
\end{lemma}

\begin{proof}[Proof of Lemma 1]
Recall that $v_H^0(\tau)$ and $v_F^0(\tau)$ are implicitly defined by $\beta(v)/\omega(v) = $
\end{proof}
$g_H(\omega(v), \tau)$ and $\beta(v) \omega(v) = g_F(\omega(v), \tau)$. Consider first $v^g_H(\tau)$. We have $g_H(\omega(v), \tau) = \frac{\tau + (1/v - 1)/\tau}{1 + 1/v - 1} = \tau v + (1 - v)/\tau = v(\tau - 1/\tau) + 1/\tau$, hence $\beta(v) \omega(v) = g_H(\omega(v), \tau)$ is equivalent to

$$(1/v - 1)^{-1/\xi} = v(\tau - 1/\tau) + 1/\tau,$$

where $\xi \equiv \alpha \theta / (1 + \alpha \theta)$. Both the LHS and RHS are increasing in $v$ (since $\tau > 1/\tau$). To show that there exists a solution, note that for $v = 1/2$ we have the RHS > LHS, whereas for $v = 1$ the LHS is infinite while the RHS is $\tau$, so LHS > RHS. This along with the continuity of the LHS and the RHS guarantees existence. For uniqueness, note that the derivative of the LHS is

$$\frac{1}{\xi} (1/v - 1)^{-1/\xi - 1} \frac{1}{v^2} = \frac{1}{\xi} (1/v - 1)^{-1/\xi} \frac{1}{v(1-v)}$$

while the derivative of the RHS is $\tau - 1/\tau$. Then we require

$$\frac{1}{\xi} (1/v - 1)^{-1/\xi} \frac{1}{v(1-v)} > \tau - 1/\tau$$

Since both the LHS and the RHS are increasing in $v$, we know that any point of intersection occurs at $v \in (1/2, 1)$. In fact, any intersection satisfies $(1/v - 1)^{-1/\xi} = v(\tau - 1/\tau) + 1/\tau$. Evaluating the LHS derivative at such an intersection yields

$$\frac{1}{\xi} [v(\tau - 1/\tau) + 1/\tau] \frac{1}{v(1-v)} = \frac{1}{\xi (1-v)} (\tau - 1/\tau) + \frac{1/\tau}{v(1-v)}$$

Clearly,

$$\frac{1}{\xi (1-v)} (\tau - 1/\tau) + \frac{1/\tau}{v(1-v)} > \tau - 1/\tau$$

establishing uniqueness.

Now we proceed to show that $v^g_H(\tau)$ is increasing. Note that the RHS, $v(\tau -$
1/τ + 1/τ, is increasing in τ. This is obvious because the derivative w.r.t. τ is
v − (1/τ²) (1 − v), and since v > 1/2 and τ > 1 then this is positive. The result then
follows from the implicit function theorem.

The fact that \(v_P^\beta(\tau)\) exists, is unique, and is strictly decreasing follows directly
from symmetry. In particular, symmetry implies \(v_P^\beta(\tau) = 1 - v_H^\beta(\tau)\). The result then
follows.

Finally, for a given \(\tau\), it clearly follows that \(v_H^\beta(\tau) > 1/2 > v_P^\beta(\tau)\).

Let \(\tilde{\tau}\) be defined implicitly by \(v_H^\beta(\tilde{\tau}) = v_H^\beta(\tau)\). Note that if \(v_H^\beta(\tilde{\tau}) = v_H^\beta(\tau)\) then
by symmetry we have \(v_H^\beta(\tilde{\tau}) = v_F^\beta(\tilde{\tau})\). Then it is easy to see that for \(\tau \leq \tilde{\tau}\) any CSA
\(\tilde{v} \in [v_F^\beta(\tau), v_H^\beta(\tau)]\) is also a CSE. This follows because for \(\tau \leq \tilde{\tau}\) we have \(v_H^\beta(\tau) < v_H^\beta(\tau)\)
and \(v_H^\beta(\tau) > v_P^\beta(\tau)\), hence conditions (2.10) and (2.12) imply conditions (2.11) and
(2.13). Before establishing this, we first establish the existence of \(v_H^\beta(\tau)\) and \(v_F^\beta(\tau)\).

**Lemma B.0.2.** The functions \(v_H^\beta(\tau)\) and \(v_F^\beta(\tau)\) exist (this entails existence and
uniqueness of a solution in \(v\) to (2.19) and (2.21)), \(v_H^\beta(\tau)\) is decreasing and \(v_F^\beta(\tau)\) is
increasing in \(\tau\).

**Proof of Lemma 2.** Consider first \(v_H^\beta(\tau)\). Since \(h_H(\omega(v), \tau) = \frac{1}{\tau} \left[\frac{\tau + (1/\nu - 1)}{(1/\nu - 1)}\right]^{(\alpha - 1)/\alpha} = \frac{1}{\tau} \left[1 + \frac{\tau v}{1 - v}\right]^{\phi}\), the fact that \(v_H^\beta(\tau)\) is implicitly defined by \(\beta(v)\omega(v) = h_H(\omega(v), \tau)\) is
equivalent to \(v_H^\beta(\tau)\) being implicitly defined by

\[
(1/\nu - 1)^{-1/\xi} = \frac{1}{\tau} \left[1 + \frac{\tau v}{1 - v}\right]^{\phi}
\]

or

\[
\left(\frac{v}{(1 - v)^{1 - \xi}}\right)^{1/\xi} = \frac{1}{\tau} [(\tau - 1) v + 1]^{\phi}
\]

Both the LHS and RHS are increasing in \(v\) (since \(\tau > 1\) and \(\xi \phi < 1\)). To show that
there exists a solution, note that for \(v = 0\) we have the \(RHS > LHS\), whereas for
\( v = 1 \) the \( \text{LHS} \) is infinite while the \( \text{RHS} \) is \( 1/\tau^{1-\phi} \), so \( \text{LHS} > \text{RHS} \). This along with the continuity of both the \( \text{LHS} \) and the \( \text{RHS} \) guarantees existence. For uniqueness, note that the derivative of the \( \text{LHS} \) is

\[
\frac{1}{\xi} \left( \frac{v}{(1 - v)^{1-\xi\phi}} \right)^{1/\xi} \frac{1}{v} \frac{1 - \xi \phi v}{1 - v}
\]

while the derivative of the \( \text{RHS} \) is

\[
\phi \frac{1}{\tau} [(\tau - 1) v + 1]^\phi \frac{\tau - 1}{(\tau - 1) v + 1}
\]

Then we require

\[
\frac{1}{\xi} \left( \frac{v}{(1 - v)^{1-\xi\phi}} \right)^{1/\xi} \frac{1}{v} \frac{1 - \xi \phi v}{1 - v} > \phi \frac{1}{\tau} [(\tau - 1) v + 1]^\phi \frac{\tau - 1}{(\tau - 1) v + 1}
\]

Any intersection satisfies \( \left( \frac{v}{(1 - v)^{1-\xi\phi}} \right)^{1/\xi} = \frac{1}{\phi} [(\tau - 1) v + 1]^\phi \). Evaluating the \( \text{LHS} \) derivative at such an intersection implies we require

\[
\frac{1}{\xi} \frac{1}{\tau} [(\tau - 1) v + 1]^\phi \frac{1}{v} \frac{1 - \xi \phi v}{1 - v} > \phi \frac{1}{\tau} [(\tau - 1) v + 1]^\phi \frac{\tau - 1}{(\tau - 1) v + 1}
\]

or

\[
\frac{1}{\xi} \frac{1}{\tau} \frac{1}{v} \frac{1 - \xi \phi v}{1 - v} > \phi \frac{\tau - 1}{\tau} \frac{1}{(\tau - 1) v + 1}
\]

which is clearly satisfied. Uniqueness then follows.

Now we proceed to show that \( v_H^\phi(\tau) \) is decreasing. Note that the \( \text{RHS} \), \( \frac{1}{\tau} [(\tau - 1) v + 1]^\phi \), is decreasing in \( \tau \). The result then follows from the implicit function theorem.

The fact that \( v_F^\phi(\tau) \) exists, is unique, and is increasing follows directly from symmetry. In particular, symmetry implies \( v_H^\phi(\tau) = 1 - v_F^\phi(\tau) \). The result then fol-
Proof of Proposition 2.2.4. Note that the implicit function \( v_H^g (\tau) \) and \( v_F^g (\tau) \) characterize the limiting goods for which the global-deviation condition for both Home and Foreign are satisfied respectively, while (2.19) and (2.21) capture the limiting goods for which the local-deviation condition is also satisfied. Consider first Home country. Recall that \( v_H^g (\tau) \) is the implicit function which solves

\[
(1/v - 1)^{-1/\xi} = \frac{\tau v}{1 - v} + 1
\]

where \( \xi = \alpha \theta / (1 + \alpha \theta) \) and \( \phi = (\alpha - 1) / \alpha \).

Also, recall that \( v_H^f (\tau) \) is the implicit function which solves

\[
(1/v - 1)^{-1/\xi} = \tau v + \frac{1 - v}{\tau}.
\]

We proceed as follows. First, we establish the existence of a unique \( \hat{\tau} \) satisfying \( v_H^h (\hat{\tau}) = 1/2 \). Second, we show that \( \tau_{MAX} > \hat{\tau} \). Third, we show there exists a unique \( \bar{\tau} \) satisfying \( v_H^g (\bar{\tau}) = v_H^h (\bar{\tau}) \) and \( \bar{\tau} < \hat{\tau} \). Finally, we establish that for the relevant range of trade costs, any CSA within the two extreme cases is also a CSE.

Note that to show there exists a unique \( \hat{\tau} \) satisfying \( v_H^h (\hat{\tau}) = 1/2 \) is equivalent to establishing there exists a unique \( \hat{\tau} \) satisfying \( \tau = [\tau + 1]^{\phi} \). But this follows from Assumption 2.2.1 (\( \phi \leq 1/2 \)). To show that \( \tau_{MAX} > \hat{\tau} \), it sufficient to show that \( \tau_{MAX} > \hat{\tau}_{MAX} \) where \( \tau_{MAX} \) implicitly solves \( \tau = [\tau + 1]^{\phi_{MAX}} \). But this is clearly satisfied. The fact that \( \hat{\tau} \) also satisfies \( v_H^h (\hat{\tau}) = v_F^h (\hat{\tau}) = 1/2 \) follows directly from symmetry. In particular, symmetry implies \( v_H^g (\tau) = 1 - v_F^g (\tau) \). The result then follows.
Evaluating the first and second equations above at $\tau = 1$ yields $(v_H^1 / (1 - v_H^0))^{1/\xi} = [v_H^1 / (1 - v_H^0) + 1]^{\phi}$ and $(v_H^2 / (1 - v_H^0))^{1/\xi} = 1$ respectively. Since $[v_H^1 / (1 - v_H^0) + 1]^{\phi} > 1$, we have $v_H^1 (1) > v_H^0 (1) = 1/2$. Also, $v_H^0 (1) = 1/2$ and $v_H^0 (.)$ strictly increasing (by lemma 1) imply $v_H^0 (\tilde{\tau}) > 1/2$. So we have $v_H^1 (\tilde{\tau}) > v_H^0 (\tilde{\tau}) = 1/2$. Since, both $v_H^1 (.)$ and $v_H^0 (.)$ are continuous there exists $\tilde{\tau}$ and $\tau < \tilde{\tau}$. Uniqueness follows from $\frac{dv_H^0 (\tau)}{d\tau} > 0$ (by Lemma B.0.1) and $\frac{dv_H^1 (\tau)}{d\tau} < 0$ (by Lemma B.0.2). The fact that $\tilde{\tau}$ also satisfies $v_F^0 (\tilde{\tau}) = v_F^1 (\tilde{\tau})$ and $v_F^1 (\tau) < v_F^0 (\tau)$ for any $\tau < \tilde{\tau}$ follows directly from symmetry. In particular, symmetry implies $v_F^1 (\tau) = 1 - v_F^0 (\tau)$. The result then follows.

We now proceed to show that for any $\tau \leq \tilde{\tau}$, any allocation $\tilde{v} \in [v_F^0 (\tau), v_H^1 (\tau)]$ with $\tilde{\omega} \equiv \omega(\tilde{v})$ is a CSE, i.e., it satisfies (2.10)-(2.13). In particular, $\tilde{v}$ satisfies $\beta (\tilde{v}) \tilde{\omega} \geq g_H (\tilde{\omega}, \tau)$, $\beta (\tilde{v}) \tilde{\omega} \leq h_H (\tilde{\omega}, \tau)$, $\beta (\tilde{v}) \tilde{\omega} \geq g_F (\tilde{\omega}, \tau)$ and $\beta (\tilde{v}) \tilde{\omega} \geq h_F (\tilde{\omega}, \tau)$. Consider the first of these conditions: $\beta (\tilde{v}) \tilde{\omega} \leq g_H (\tilde{\omega}, \tau)$. In Lemma B.0.1, we showed that at $v = 1/2$ we have $RHS > LHS$ and there exists a unique intersection at $v = v_F^0 (\tau)$. So for $v \leq v_H^0 (\tau)$ we must have $RHS \geq LHS$, i.e., $\beta (v) \omega (v) \leq g_H (\omega (v), \tau)$ for any $1/2 \leq v \leq v_F^0 (\tau)$. Moreover, the fact that both the $LHS$ and $RHS$ are strictly increasing and continuous on $[0, 1)$ implies for any $0 \leq v \leq v_F^0 (\tau)$ it must also be the case that $\beta (v) \omega (v) \leq g_H (\omega (v), \tau)$. Similarly, symmetry implies for any $1/2 \leq v \geq v_F^0 (\tau)$ we have $\beta (v) \omega (v) \geq g_F (\omega (v), \tau)$ and by extension this is also the case for any $0 \geq v \geq v_F^0 (\tau)$. Hence, for any $\tilde{v} \in [v_F^0 (\tau), v_H^1 (\tau)]$, we have $\tilde{\omega}^{-1} g_F (\tilde{\omega}, \tau) \leq \beta (\tilde{v}) \leq \tilde{\omega}^{-1} g_H (\tilde{\omega}, \tau)$.

Consider now, the first local-deviation condition: $\beta (v) \omega (v) \leq h_H (\omega (v), \tau)$. We know that $v_H^1 (\tau)$ is implicitly defined by

$$(1/v - 1)^{-1/\xi} = \frac{1}{\tau} \left[ 1 + \frac{\tau v}{1 - v} \right]^{\phi}$$

For $v = 1/2$ we have $LHS$ is 1, while the $RHS$ is $\frac{1}{2} [1 + \tau]^{\phi}$. So for the relevant range of low trade costs along with the restriction $\alpha < 2$, we have $RHS > LHS$. 

Moreover, from Lemma B.0.2 there exists a unique intersection at \( v = v^b_H(\tau) \). Hence, for any \( v \leq v^b_H(\tau) \) we have \( \text{RHS} \geq \text{LHS} \) implies that \( \beta(v) \omega(v) \leq h_H(\omega(v), \tau) \) for any \( 1/2 \leq v \leq v^b_H(\tau) \). Similarly, symmetry implies for any \( 1/2 \geq v \geq v^b_F(\tau) \) we have \( \beta(v) \omega(v) \geq h_F(\omega(v), \tau) \). Furthermore, for any \( \tau \leq \bar{\tau} \), we have already established that \( v^b_H(\tau) \geq v^b_H(\bar{\tau}) \geq 1/2 \geq v^b_F(\tau) \geq v^b_F(\bar{\tau}) \). Thus, for any \( \tilde{\nu} \in [v^b_H(\tau), v^b_H(\bar{\tau})] \), we have \( \tilde{\omega}^{-1} h_F(\tilde{\omega}, \tau) \leq \tilde{\omega}^{-1} g_F(\tilde{\omega}, \tau) \leq \tilde{\beta} \leq \tilde{\omega}^{-1} g_H(\tilde{\omega}, \tau) \leq \tilde{\omega}^{-1} h_H(\tilde{\omega}, \tau) \). So by Definition 2.2.3, \( \tilde{\nu} \) is a CSE.

Finally, we have already established that there exists a unique solution to \( v^b_H(\bar{\tau}) = 1/2 \). Moreover, since \( v^b_H(\tau) \) is increasing and \( v^b_H(1) = 1/2 \) then with \( \bar{\tau} \) defined by \( v^b_H(\tau) = v^b_H(\bar{\tau}) \) we must have that \( \bar{\tau} > \bar{\tau} \), and \( v^b_H(\tau) > v^b_H(\bar{\tau}) \) for \( \tau \in (\bar{\tau}, \bar{\tau}) \). Symmetry implies the corresponding result, i.e., for any \( \tau \in (\bar{\tau}, \bar{\tau}) \), \( v^b_H(\tau) < v^b_F(\tau) \). Also note that we have already established above that for \( \tau \leq \bar{\tau} \) and \( v^b_H(\tau) \geq \tilde{\nu} \geq v^b_F(\tau) \) we have \( \tilde{\omega}^{-1} g_F(\tilde{\omega}, \tau) \leq \tilde{\beta} \leq \tilde{\omega}^{-1} h_H(\tilde{\omega}, \tau) \). By Lemma B.0.1, we see that this also applies for any \( \tau > 1 \). Similarly, for any \( \tau \leq \bar{\tau} \) and \( v^b_H(\tau) \geq \tilde{\nu} \geq v^b_F(\tau) \) we already know that \( \tilde{\omega}^{-1} h_F(\tilde{\omega}, \tau) \leq \tilde{\beta} \leq \tilde{\omega}^{-1} h_H(\tilde{\omega}, \tau) \). That this also applies for any \( \tau \leq \bar{\tau} \) follows from Definition 2.2.3 and the fact that for any \( \tau \leq \bar{\tau} \) we have \( v^b_H(\tau) \geq 1/2 \geq v^b_F(\tau) \). Moreover, for any \( \tau \in (\bar{\tau}, \bar{\tau}) \), we have \( v^b_H(\tau) \geq v^b_H(\bar{\tau}) \geq 1/2 \geq v^b_F(\tau) \geq v^b_F(\bar{\tau}) \). Hence, for any \( \tau \in (\bar{\tau}, \bar{\tau}) \) and \( \tilde{\nu} \in [v^b_F(\tau), v^b_F(\bar{\tau})] \), we have \( v^b_H(\tau) \geq v^b_H(\bar{\tau}) \geq \tilde{\nu} \geq v^b_F(\tau) \geq v^b_F(\bar{\tau}) \) implies \( \tilde{\omega}^{-1} g_F(\tilde{\omega}, \tau) \leq \tilde{\omega}^{-1} g_F(\tilde{\omega}, \tau) \leq \tilde{\beta} \leq \tilde{\omega}^{-1} h_H(\tilde{\omega}, \tau) \leq \tilde{\omega}^{-1} g_F(\tilde{\omega}, \tau) \). The fact that \( \tilde{\nu} \) is a CSE then follows from Definition 2.2.3

\[ \square \]

**Real Wages and the Gains from Trade**

**Proof of Proposition 2.3.1.** Allowing for asymmetries in technology and labor force across countries, we can derive a more general version of the relative productivity function, i.e.

\[ \beta(v) = \left( \frac{T_F}{T_H} \right)^{1/\alpha} \left( \frac{v}{1-v} \right)^{1/\alpha} \]  

(B.1)
Let $l \equiv L_H / L_F$. Note also that the more general versions of the global-deviation conditions imply: $g_H (\omega, \tau) \equiv \frac{\tau + (\omega l)^{-1}}{1 + (\omega l)^{-1}}$; and $g_F (\omega, \tau) \equiv \frac{1 + (\omega l)^{-1}}{1 + (\omega l)^{-1}}$. Using (B.1), we can rewrite (2.18) and (2.20) so that each is implicitly solved by $v_H$ and $v_F$, respectively. Importantly, note that $v_H$ and $v_F$ are also the share of goods that Home firms produce in the extreme cases in which all the disputed industries are allocated to the Home and Foreign country respectively. Note that (B.1) implies

$$\tilde{v} = \frac{T_H}{T_H + T_F \beta - \alpha \theta}.$$  

From Proposition 2.2.4 we know that the set of equilibria is given by $\tilde{v} \in [v^g_F (\tau), v^g_H (\tau)]$. In particular, the necessary global-deviation conditions for complete specialization are satisfied, that is, $\omega (\tilde{v})^{-1} g_F (\omega (\tilde{v}), \tau) \leq \beta (\tilde{v}) \leq \omega (\tilde{v})^{-1} g_H (\omega (\tilde{v}), \tau)$. Consider first the Home country. Then (B.2) along with $\pi_{HH} = \tilde{v}$ implies

$$\pi_{HH} = \frac{T_H w_H^{-\alpha \theta}}{\Phi H}$$

where $\Phi_H = \left[ T_H w_H^{-\alpha \theta} + T_F w_F^{-\alpha \theta} (\beta (\tilde{v}) \omega (\tilde{v}))^{-\alpha \theta} \right]$.  

Let $p_{ni} (\tilde{v})$ be the price of good $v$ if $i$ sells to $n$ ($i, n = H, F$). Note that since we consider only complete specialization, any country which sells the good supplies the

\footnote{In our quantitative exercise we also verify that no Home or Foreign firms have an incentive to target their domestic market only, i.e. the local deviation conditions for complete specialization are also satisfied. Both provide a sufficient condition for the existence of a complete specialization equilibrium.}
world market. Hence, for any industry \( v \) we have either price pairs\(^2\)

\[
p_{HH} = \frac{w_{HA}}{[x_H(p_{HH}) + x_F(\tau p_{HH})]^{\phi}} \quad \text{and} \quad p_{FH} = p_{HH}^\tau
\]
or

\[
p_{FF} = \frac{w_{FA}}{[x_H(\tau p_{FF}) + x_F(p_{FF})]^{\phi}} \quad \text{and} \quad p_{HF} = p_{FF}^\tau
\]

where \( x_H(p) = \frac{w_H L_H}{p} \) and \( x_F(p) = \frac{w_F L_F}{p} \).

The associated price index for Home is then given by\(^3\)

\[
P_H = \eta (\chi_H)^{-(\alpha-1)} (\Phi_H)^{-1/\theta}
\]

\(^2\)In particular, the relevant prices are

\[
p_{HH}(v) = \frac{(w_H a_H(v))^\alpha}{(w_H L_H + w_F L_F / \tau)^{(\alpha-1)}} = \frac{1}{z_H(v)} \frac{(w_H)^\alpha}{(w_H L_H + w_F L_F / \tau)^{(\alpha-1)}}
\]

and

\[
p_{FF}(v) = \frac{(w_F a_F(v))^\alpha}{(w_H L_H / \tau + w_F L_F)^{(\alpha-1)}} = \frac{1}{z_F(v)} \frac{(w_F)^\alpha}{(w_H L_H / \tau + w_F L_F)^{(\alpha-1)}}.
\]

\(^3\)To see this note first that

\[
P_H = \exp \left\{ \int_0^v \ln p_{HH}(v) \, dv + \int_v^1 \ln p_{HF}(v) \, dv \right\}.
\]

Using

\[
p_{HH}(v) = \frac{(w_H a_H(v))^\alpha}{(w_H L_H + w_F L_F / \tau)^{(\alpha-1)}} = \frac{1}{z_H(v)} \frac{(w_H)^\alpha}{(w_H L_H + w_F L_F / \tau)^{(\alpha-1)}}
\]

and

\[
p_{FF}(v) = \frac{(w_F a_F(v))^\alpha}{(w_H L_H / \tau + w_F L_F)^{(\alpha-1)}} = \frac{1}{z_F(v)} \frac{(w_F)^\alpha}{(w_H L_H / \tau + w_F L_F)^{(\alpha-1)}}
\]

along with (2.14), we can derive the price index as noted in (B.4).
where $\eta = e^{-\gamma/\theta}$, $\gamma$ is Euler’s constant, and

$$\Upsilon_H \equiv (w_H L_H + w_F L_F / \tau)^{\pi_{HH}} \left( \left( \frac{\beta (\tilde{\nu}) \omega (\tilde{\nu})}{\tau} \right)^{1/(\alpha-1)} (w_H L_H / \tau + w_F L_F) \right)^{1-\pi_{HH}}$$

(B.4) implies $\Phi_H = (P_H / \eta)^{-\theta} (\Upsilon_H)^{-(\alpha-1)\theta}$. Substituting into (B.3) yields real wages

$$\frac{w_H}{P_H} = \left( \frac{T_H}{\eta^\theta} \right)^{1/\theta} \frac{\Upsilon_H}{w_H} \left( \frac{\alpha}{\pi_{HH}} \right)^{(\alpha-1)}.$$

Analogously, we can derive real wages in Foreign.

\[ \square \]

**Proof of Corollary 2.3.2.** In autarky,

$$\frac{w_H^A}{P_H^A} = \left( \frac{T_H}{\eta^\theta} \right)^{1/\theta} (L_H)^{(\alpha-1)}.$$

Therefore, gains from trade are

$$\frac{w_H / P_H}{w_H^A / P_H^A} = \frac{\Upsilon_H}{w_H L_H} \left( \frac{\alpha}{\pi_{HH}} \right)^{(\alpha-1)}.$$

Similarly, we can also derive the gains from trade in Foreign.

\[ \square \]

**Multiple Countries and Frictionless Trade**

We already know that with frictionless trade there exists a unique equilibrium in which the patterns of trade are consistent with natural comparative advantage. The difference here is that we consider $K \geq 2$ countries. Here $K$ can be large, but finite. As noted in our general environment, preferences are uniform Cobb-Douglas with its associated demand

$$x_i(v) = \frac{D_i}{p_i(v)} \quad (B.5)$$
where $D_i$ is aggregate expenditure by country $i$.

The stability condition in (2.2) holds, namely

$$\left(\frac{\alpha - 1}{\alpha}\right) < 1$$

and firms in each industry engage in Bertrand competition on the world market. Before proceeding to the proof we first formally restate the definition of an equilibrium.

**Definition B.0.3.** Given country size $L_i$ and country specific productivity distribution $F_i(z)$, an equilibrium with frictionless trade consists of prices $p_{ni}(v)$, $w_i$, and quantities $x_i(v)$ such that: (a) industry markets clear; (b) firms in each industry engage price competition in all markets simultaneously; and (c) labor market clears.

**Proof of Proposition 2.3.3.** Price competition amongst firms in any domestic industry implies average cost pricing, that is

$$p_{ni}(v) = \frac{w_i}{z_i(v)^{1/\alpha} X_i(v)^{(\alpha - 1)/\alpha}}$$  \quad \text{(B.6)}$$

Note that if country $i$ supplies the world market, then $X_i(v) = \sum_{n=1}^{K} x_n(p_{ni}(v)) = \sum_{n=1}^{K} \frac{D_n}{p_{ni}(v)}$. Moreover, the actual prices paid for good $v$ is the lowest from all sources $i$

$$p_n(v) = \min \{p_{ni}(v) : i = 1, ..., K\}$$  \quad \text{(B.7)}$$

Complete specialization along with industry market clearing imply

$$p_{ni}(v) = \frac{w_i^0}{z_i(v) \left( \sum_{k=1}^{K} \frac{w_k L_k}{w_i} \right)^{\alpha - 1}}.$$
Using the assumption that the productivity distribution is Fretchet, we can derive the distribution of prices that \( i \) presents to \( n \)

\[
G_{ni}(p) = 1 - e^{-T_i w_i^{-\alpha \theta} \left( \sum_{k=1}^{K} w_k L_k \right)^{\alpha - 1} \frac{\theta}{p^\theta}}. \tag{B.8}
\]

From (B.8) we can derive the price distribution for which country \( n \) actually buys some good \( j \), that is, the lowest price of a good in country \( n \) will be less than \( p \) unless each source’s price is greater than \( p \)

\[
G_n(p) = 1 - \Pi_{i=1}^{K} [1 - G_{ni}(p)]
\]

So

\[
G_n(p) = 1 - e^{-\Phi_n p^\theta}
\]

where \( \Phi_n = \sum_{i=1}^{K} T_i w_i^{-\alpha \theta} \left( \sum_{k=1}^{K} w_k L_k \right)^{\alpha - 1} \frac{\theta}{p^\theta} \).

Hence, the probability \( n \) buys from \( i \) is given by

\[
\pi_{ni} = \Pr \{ p_{ni} \leq \min \{ p_{ns} : s \neq i \} \}
= \int_{0}^{\infty} \Pi_{s \neq i} [1 - G_{ns}(p)] \, dG_{ni}(p)
\]

So

\[
\pi_{ni} = \frac{T_i w_i^{-\alpha \theta} \left( \sum_{k=1}^{K} w_k L_k \right)^{\alpha - 1} \frac{\theta}{p^\theta}}{\Phi_n} = \frac{T_i w_i^{-\alpha \theta}}{\sum_{i=1}^{K} T_i w_i^{-\alpha \theta}}
\]

\( ^4 \)In autarky the distribution of prices is given by \( G_n(p; \{ n \}) = 1 - e^{-T_n w_n^{-\alpha \theta} (w_n L_n)^{\alpha - 1} p^\theta} \).

Hence, it depends not only on the state of country \( n \)’s technology discounted by its input cost as in EK, but also, because of external economies, on the market size of \( n \).
With costless trade $\pi_{ni}$ is the same for all $n$, and is also independent of $v$. Hence, $\pi_{ni}$ is also the share of goods that any country $n$ buys from country $i$. Moreover, the price of a good country $n$ actually buys from $i$ has the distribution $G_n(p)$ so that $\pi_{ni}$ is also the share of expenditure by $n$ on goods produced in $i$, that is

$$\pi_{ni} = \frac{X_{ni}}{X_n} = \frac{T_iw_i^{-\alpha\theta} \left( \sum_{k=1}^{K} w_k L_k \right)^{(\alpha-1)\theta}}{\sum_{k\in\Gamma} T_k w_k^{-\alpha\theta}}$$ (B.9)

Also, the associated price index for is

$$P_n = \eta \Phi_n^{-1/\theta}$$ (B.10)

where $\eta = e^{-\gamma/\theta}$ and $\gamma$ is Euler’s constant.

In equilibrium, labor market clearing/trade balance implies

$$w_i L_i = \sum_{n=1}^{K} \pi_{ni} X_n$$ (B.11)

Equation (B.9) along with (B.11) yield an expression for relative wages

$$\frac{w_i}{w_n} = \left( \frac{T_i/L_i}{T_n/L_n} \right)^{1/(1+\alpha\theta)}$$ (B.12)

Using (B.9) and (B.10) we can derive real wages

$$\frac{w_n}{P_n} = \left( \frac{T_n/\eta}{\eta} \right)^{1/\theta} \pi_n^{-1/\theta} \left( \sum_{k=1}^{K} \frac{w_k L_k}{w_n L_n} \right)^{\alpha-1}$$
In autarky real wage is given by

\[ \omega_n^A = \frac{w_n^A}{P_n^A} = \left( \frac{T_n}{\eta^A} \right)^{1/\theta} L_n^{\alpha-1} \]

Hence, gains from trade are

\[ \frac{w_n/P_n}{w_n^A/P_n^A} = \pi_{nn}^{-1/\theta} \left( \sum_{k=1}^{K} \frac{w_k L_k}{w_n L_n} \right)^{\alpha-1} \]

Finally, both (B.9) and (B.12) imply

\[ \pi_{nn} = \sum_{k=1}^{K} \frac{w_n L_n}{w_k L_k}. \]

The result then follows.

Proof of Corollary 2.3.4. From (2.26) we can then simply derive the contribution of comparative advantage and Marshallian externalities using \( \ln \left( \frac{-1/\theta}{\pi_{nn}/\pi_{nn}^{-(\alpha-1)}} \right) \) and

\[ \frac{\ln \left( \frac{-\alpha-1}{\pi_{nn}/\pi_{nn}^{-(\alpha-1)}} \right)}{\ln \left( \frac{-1/\theta}{\pi_{nn}/\pi_{nn}^{-(\alpha-1)}} \right)} \]

respectively. The result then follows.
Appendix C

Proofs for Chapter 3

Proof of Proposition 3.2.4. By Condition 3.2.3, young potential managers find it optimal to migrate, and if unrestricted they will do so until \( V_L^h(c) = V_S^h(x_e^*) \), that is,

\[
(1 - \tau_1)\alpha z_f n_s^{\alpha - 1} + \beta \max\{(1 - \alpha)c z_f n_s^{\alpha}, \alpha z_f n_s^{\alpha - 1}\} \\
\equiv \alpha z_h [n^n_h(x_e^*)]^{\alpha - 1} + (1 - \alpha) \beta z_h \left[n^n_s(x_e^*)\right]^\alpha
\]

where \( n^n_h(x_e^*) = (2 - \omega - x_e^*)/(\omega - x_e^*) \). Note that Condition 3.2.1 implies all remaining mature home potential managers choose to be active managers i.e. \((2 - \omega - x_e)/(\omega - x_e) > \alpha/(1 - \alpha) \). We now establish that there is indeed a unique \( p \) satisfying \( V_L^h(c) = V_S^h(p) \).

Define

\[
L(x_e) = \{(1 - \tau_1)\alpha z_f n_s^{\alpha - 1} + \beta \max\{(1 - \alpha)c z_f n_s^{\alpha}, \alpha z_f n_s^{\alpha - 1}\}\} \\
- \left\{\alpha z_h [n^n_h(x_e^*)]^{\alpha - 1} + (1 - \alpha) \beta z_h \left[n^n_s(x_e^*)\right]^\alpha\right\}
\]

Then there are basically two cases to consider: (i) talented immigrants learn enough to find it optimal to be an active manager in the foreign country when mature \( c \geq \theta \) where \( \theta = (\alpha/(1 - \alpha))(\omega/(2 - \omega)) < 1 \) (by Condition 3.2.1); and (ii) talented immigrants learning is such that it is optimal for them to remain workers when mature \((c < \theta) \).

For both cases we can show that given Condition 3.2.3, \( L(0) > 0 \). Also, we can show the \( \lim_{x_e \to \omega} L(x_e) = -\infty \). Then by the continuity of \( L(x_e) \) and the Intermediate Value
Theorem, there exists an $\bar{p}$ such that $L(\bar{p}) = 0$. Moreover, $\frac{dL(x_e)}{dx_e} < 0$ for $\beta \geq \beta^*$ where $\beta^* = \frac{1}{n^h(x_e)}$ establishes uniqueness. The results then follow.

Proof of Proposition 3.2.11. (i) Assume $\tau_1 = \tau_2 = 0$. Define $L(x_e, x_r) \equiv V_h^L(x_e, x_r) - V_h^S(x_e, x_r)$ where $x_r$ is given by (3.4). There are two cases to consider: (a) $c \geq \tilde{\theta}$; and $c < \tilde{\theta}$. For both cases we can show that given Conditions 3.2.3 and 3.2.8, $L(0, 0) > 0$. Also, $\tau_2 = 0$ implies $\pi_h(z_c, w_h(0, 0)) > \pi_f(z_c, w_f^A)$, so there exists a unique $\hat{x}_e$ such that $\pi_h(z_c, w_h(\hat{x}_e, \hat{x}_e)) = \pi_f(z_c, w_f^A)$. We now establish that $L(\hat{x}_e, \hat{x}_e) > 0$. To see this, first note that $\pi_h(z_c, w_h(\hat{x}_e, \hat{x}_e)) = \pi_f(z_c, w_f^A)$ implies that $w_h(\hat{x}_e, \hat{x}_e) = w_f^A$. This along with $\pi_h(z_c, w_h(\hat{x}_e, \hat{x}_e)) > \pi_h(z_h, w_h(\hat{x}_e, \hat{x}_e))$ implies $L(\hat{x}_e, \hat{x}_e) > 0$. Finally, simple differentiation yields $\frac{dL(x_e)}{dx_e} < 0$. Then for any $p \leq \hat{x}_e$, the existence of a unique equilibrium $x_e^* = x_r^* = p$ follows from the continuity of $L(.)$.

(ii) Assume now that $\tau_1 = 0$ and $\tau_2 = 1$. The proof then follows directly from Proposition 3.2.4.
VITA
Gary Lyn

Contact Information
608 Kern Graduate Building Office: (814) 865-1108
Department of Economics Cell: (814) 777-1582
The Pennsylvania State University Email: gary.lyn@psu.edu
University Park, PA 16802

Education
• Ph.D. in Economics 2012 The Pennsylvania State University
• M.Sc. in Economics 2006 University of the West Indies, Jamaica
• B.Sc. in Economics 2004 University of the West Indies, Jamaica

Research Interests

Working Papers
• Lyn, G., A. Rodríguez-Clare. 2012. Marshallian Externalities, Comparative Advantage and International Trade, working paper.
• Lyn, G., A. Rodríguez-Clare. 2012. External Economies, and International Trade Redux: Comment, working paper.
• Lyn, G. 2012. Brain Drain, Brain Return and on-the-job Learning, working paper.