PATTERN SHAPING WITH LENSES AND GRATED SURFACES

A Thesis in

Electrical Engineering

by

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Abstract

This thesis is comprised of two distinct studies in the shaping of radiation patterns. The first part involves a comparative study of hyperbolic, profiled and flat lenses of varying diameters with a focus on achieving high gain, narrow beamwidth and wide-angle scan performance. The second part entails the design of a grated dielectric surface that produces an isoflux pattern when irradiated from below. The geometries involved were simulated using GEMS, a commercially available parallel Finite Difference Time Domain (FDTD) electromagnetics solver.

Contribution to Knowledge:

- Utilizing a parallel FDTD EM solver to conduct a comparative study of microwave lenses
- Exciting a leaky-wave antenna with an arbitrary radiator located underneath the structure as opposed to a waveguide and coupler located at one end
# TABLE OF CONTENTS

LIST OF FIGURES .....................................................................................................................v

LIST OF TABLES ..........................................................................................................................ix

ACKNOWLEDGEMENTS ...........................................................................................................x

## 1 Comparative Study of Hyperbolic, Profiled and Flat lenses .............................................1

1.1 Introduction and Motivation ...............................................................................................1

1.2 Hyperbolic Lens .................................................................................................................3

1.3 Profiled Lens .......................................................................................................................8

1.4 Flat Lens .............................................................................................................................12

1.5 Results and Interpretations ...............................................................................................17

1.6 Hyperbolic, Profiled and Flat Lens Conclusion and Future Work ....................................21

1.7 Large Lenses ......................................................................................................................22

1.8 Large Lens Results and Interpretations ............................................................................30

1.9 Large Lens Conclusion and Future Work ...........................................................................33

## 2 Grated Dielectric Surfaces ................................................................................................35

2.1 Introduction and Motivation ...............................................................................................35

2.2 Uniform Dielectric Slab Waveguides ...............................................................................38

2.3 Theoretical Background of Leaky-Wave Antennas ..........................................................41

2.4 Heterogeneous Grated Dielectric Surface .........................................................................43

2.5 Heterogeneous Grated Dielectric Surface Conclusion and Future Work .......................51

2.6 Homogeneous Grated Dielectric Surface .........................................................................52

2.7 Spatial Filter Overlay .........................................................................................................56
## REFERENCES

- 1.1 Broadband zone plate lens from transformation optics
- 1.2 Quarter-wave zone plate lens
- 1.3 Cross section view of hyperbolic lens
- 1.4 \(\text{yx, isometric, zx and zy views of hyperbolic lens}
- 1.5 Method of excitation
- 1.6 Spatial distribution (Amplitude of \(E_x\)) of hyperbolic lens for \(\theta = 0^\circ, 5^\circ, 10^\circ\) and \(15^\circ\)
- 1.7 Spatial distribution (Amplitude of \(E_x\)) of hyperbolic lens for \(\theta = 20^\circ, 25^\circ, 30^\circ\) and \(35^\circ\)
- 1.8 Spatial distribution (Amplitude of \(E_x\)) of hyperbolic lens for \(\theta = 40^\circ, 45^\circ, 50^\circ\) and \(55^\circ\)
- 1.9 Profiled lens with paths of interest
- 1.10 \(\text{yx, isometric, zx, and zy views of profiled lens}
- 1.11 Spatial distribution (Amplitude of \(E_x\)) of profiled lens for \(\theta = 0^\circ, 5^\circ, 10^\circ\) and \(15^\circ\)
- 1.12 Spatial distribution (Amplitude of \(E_x\)) of profiled lens for \(\theta = 20^\circ, 25^\circ, 30^\circ\) and \(35^\circ\)
- 1.13 Spatial distribution (Amplitude of \(E_x\)) of profiled lens for \(\theta = 40^\circ, 45^\circ, 50^\circ\) and \(55^\circ\)
- 1.14 Flat lens with paths of interest
- 1.15 \(\text{yx, isometric, zx and zy views of flat lens}
- 1.16 Spatial distribution (Amplitude of \(E_x\)) of flat lens for \(\theta = 0^\circ, 5^\circ, 10^\circ\) and \(15^\circ\)
- 1.17 Spatial distribution (Amplitude of \(E_x\)) of flat lens for \(\theta = 20^\circ, 25^\circ, 30^\circ\) and \(35^\circ\)
- 1.18 Spatial distribution (Amplitude of \(E_x\)) of flat lens for \(\theta = 40^\circ, 45^\circ, 50^\circ\) and \(55^\circ\)
- 1.19 Bandwidth performance of hyperbolic, profiled and flat lenses
- 1.20 Scan performance of hyperbolic, profiled and flat lenses
1.21 Focal region length..................................................................................................................19
1.22 Focal region length as a function of angle of incidence.........................................................20
1.23 Focal region length as a function of frequency.......................................................................21
1.24 $yx$, isometric, $zx$ and $zy$ views of large hyperbolic lens....................................................23
1.25 Spatial distribution (Amplitude of $E_x$) of large hyperbolic lens for $\theta = 0^\circ, 5^\circ, 10^\circ$ and
    $15^\circ$.........................................................................................................................................23
1.26 Spatial distribution (Amplitude of $E_x$) of large hyperbolic lens for $\theta = 20^\circ, 25^\circ, 30^\circ$ and
    $35^\circ$.........................................................................................................................................24
1.27 Spatial distribution (Amplitude of $E_x$) of large hyperbolic lens for $\theta = 40^\circ, 45^\circ, 50^\circ$ and
    $55^\circ$.........................................................................................................................................24
1.28 $yx$, isometric, $zx$ and $zy$ views of large profiled lens............................................................26
1.29 Spatial distribution (Amplitude of $E_x$) of large profiled lens for $\theta = 0^\circ, 5^\circ, 10^\circ$ and
    $15^\circ$.........................................................................................................................................26
1.30 Spatial distribution (Amplitude of $E_x$) of large profiled lens for $\theta = 20^\circ, 25^\circ, 30^\circ$ and
    $35^\circ$.........................................................................................................................................27
1.31 Spatial distribution (Amplitude of $E_x$) of large profiled lens for $\theta = 40^\circ, 45^\circ, 50^\circ$ and
    $55^\circ$.........................................................................................................................................27
1.32 $yx$, isometric, $zx$ and $zy$ views of large flat lens.................................................................28
1.33 Spatial distribution (Amplitude of $E_x$) of large flat lens for $\theta = 0^\circ, 5^\circ, 10^\circ$ and
    $15^\circ$.........................................................................................................................................29
1.34 Spatial distribution (Amplitude of $E_x$) of large flat lens for $\theta = 20^\circ, 25^\circ, 30^\circ$ and
    $35^\circ$.........................................................................................................................................29
1.35 Spatial distribution (Amplitude of $E_x$) of large flat lens for $\theta = 40^\circ$, $45^\circ$, $50^\circ$ and $55^\circ$ …………………………………………………………………………………………………30
1.36 Spatial distribution (Amplitude of $E_x$) of large flat lens (10 regions) for $\theta = 0^\circ$, $10^\circ$, $20^\circ$ and $30^\circ$ …………………………………………………………………………………………………31
1.37 Bandwidth performance of large hyperbolic, large profiled and large flat lenses………32
1.38 Scan performance of large hyperbolic, large profiled and large flat lenses………………33
2.1 Desired operation of design…………………………………………………………………35
2.2 Periodic SiC microstructure by Greffet et al……………………………………………35
2.3 Periodic Nickel structure by Kemme et al……………………………………………36
2.4 Grated dielectric antenna designed by Antar et al……………………………………37
2.5 Homogeneous dielectric leaky-wave antenna by Shanjia and Xinzhang……………38
2.6 Even and odd mode field distributions in a dielectric slab waveguide………………39
2.7 Real components of wave number…………………………………………………………40
2.8 Heterogeneous dielectric grating under investigation…………………………………43
2.9 Heterogeneous grating profile……………………………………………………………43
2.10 Corrugated antenna of finite width………………………………………………………44
2.11 Dimensions of planar half-wave dipole………………………………………………45
2.12 $\phi$ –cuts of interest above heterogeneous grated dielectric…………………………46
2.13 $\phi = 0^\circ$ cut far-field radiation patterns for $2\lambda_0$ through $8\lambda_0$…………………………46
2.14 $\phi = 0^\circ$ cut far-field radiation patterns for $10\lambda_0$ through $16\lambda_0$…………………………47
2.15 $\phi = 0^\circ$ cut far-field radiation patterns for $18\lambda_0$ and $20\lambda_0$…………………………47
2.16 Effect of varying aspect ratio on far-field radiation patterns………………………49
2.17 Positioning and orientation of three dipoles of random excitation…………………..50
2.18 Heterogeneous grated surface excited by a random array..............................51
2.19 Homogeneous dielectric grating under investigation..............................52
2.20 Homogeneous grating profile.................................................................53
2.21 φ = 0° cut far-field radiation patterns for 2λ₀ through 8λ₀.........................53
2.22 φ = 0° cut far-field radiation patterns for 10λ₀ through 16λ₀...................54
2.23 φ = 0° cut far-field radiation patterns for 18λ₀ and 20λ₀........................54
2.24 Effect of varying aspect ratio on far-field radiation patterns..................55
2.25 Homogeneous grated surface excited by random array............................56
2.26 Spatial filter configurations 1, 2, 3 and 4..............................................57
2.27 Far-field performance of grated surface and spatial filter combination........58
2.28 Grated surface and spatial filter with random excitation runs 1, 2, 3, and 4....60
2.29 Grated surface and spatial filter with random excitation runs 5, 6 and 7.......61

LIST OF TABLES
1.1 Geometric properties of profiled lens.......................................................9
1.2 Dielectric and geometric properties of flat lens.......................................14
1.3 Geometric properties of large profiled lens.............................................25
1.4 Dielectric and geometric properties of large flat lens...............................28
2.1 Heterogeneous grated surface initial design parameters..........................44
2.2 Dipole orientation and phase information for three dipole array..............50
2.3 Homogeneous grated surface initial design parameters............................53
2.4 Initial spatial filter parameters..............................................................57
2.5 Dipole orientation and phase information for runs 1, 2 and 3....................58
2.6 Dipole orientation and phase information for runs 4 and 5......................59
2.7 Dipole orientation and phase information for run 6...............................59
2.8 Dipole orientation and phase information for run 7...............................60
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1. Comparative Study of Hyperbolic, Profiled and Flat Lenses

1.1 Introduction and Motivation

The typical lens encountered in optics transmits and refracts light, resulting in the convergence or divergence of a beam. The main difference between these lenses and those covered in this discussion is the frequency of operation; specifically, the lenses we investigate are designed to operate in the GHz vs. 100s of THz regime. However, the basic physics of the lenses, which remains unchanged, will be examined in the following section from the microwave point of view. The underlying mechanism that creates a focal point on the exiting side of a lens is based on the enforcement of equivalent electrical lengths regardless of the physical path. The two most direct ways to do this are to either create a lens comprised of radially varying heterogeneous materials (in the case of a cylindrical flat lens), or to design a profiled lens that utilizes a single material, but varies the thickness of each successive region.

The lens design that inspired this study was based on one described by Hao et al. [1] (see Figure 1.1), where the theory of Transformation Optics was used to compress a conventional hyperbolic lens into a flat, zone plate lens with varying effective dielectric parameters. This lens was claimed to have a superior bandwidth performance (for $20 \text{ GHz} < f < 40\text{GHz}$) when compared to a conventional quarter-wave Fresnel zone plate lens proposed by Petosa et al. [2].

Though it is not immediately apparent from Figure 1.1, both of these lenses utilize a homogenous dielectric disk perforated with holes of varying diameter which, in turn, give rise to varying effective dielectric constants.
Figure 1.1. Development and rough schematic of a zone plate lens proposed by Hao et al..

Figure 1.2. Quarter-wave zone plate lens proposed by Petosa et al..

Compared to these lenses, the structures covered in this study are simple and include a hyperbolic, profiled and flat lens whose design properties are described first followed by their
simulation results. The importance of this work is that prior to the introduction of parallel FDTD EM solvers, such a comparative study as this, would not have been possible due to the inherent computational intensiveness of the simulations.

1.2 Hyperbolic Lens

The first lens we investigated, inspired by Cagnon et al. in [3], had a hyperbolic geometry with a diameter \( D = 63.5 \) mm, focal length \( F = D/4 = 15.875 \), \( \varepsilon_r = 2.6 \) and a central design frequency \( f_0 = 30.0 \) GHz. The corresponding values in terms of free-space wavelength for \( D \) and \( F \) are \( 6.35\lambda_0 \) and \( 1.5875\lambda_0 \), respectively. The particular dimensions of the lens and its relative permittivity were chosen to facilitate an immediate comparison to the performance of the lens described in [1]. In common with all focusing lenses, the hyperbolic lens design needs to ensure that the electrical path length of all incident rays are equivalent regardless of physical path length which it achieves by utilizing a curved profile. A mathematically rigorous explanation of this will be presented in the next section on profiled lenses, which may be viewed as a “coarse” hyperbolic lens. The hyperbolic profile of the lens is described by

\[
R(\theta) = \frac{(\sqrt{\varepsilon_r} - 1)F}{\sqrt{\varepsilon_r \cos(\theta) - 1}} \tag{1.1}
\]

and is illustrated in Figure 1.3.
Starting with \(D\), \(F\) and \(\varepsilon_r\) it is possible to find the thickness of the lens \(t\), which turns out to be approximately \(2\lambda_0\). Different views of the hyperbolic lens are shown in Figure 1.4.
The first area of interest, for all of the lenses in this study, was their scan capabilities at the central design frequency of 30 GHz, *i.e.*, their ability to maintain a compact distribution in the focal region as the incidence angle was progressively increased. To evaluate this capability, GEMS, a parallel FDTD EM Solver, was used. The lens was excited by a plane wave traveling in the \( \text{zy}\)-plane with an E-field intensity of 1 V/m, polarized in the \( x\)-direction, as shown in Figure 1.5.

![Figure 1.5. Method of excitation.](image)

The angle of incidence, \( \theta \), is the angle formed between the unit vector normal to the top of the lens, \(+z\)-axis in this case, and the poynting vector of the incident wave. The position of the transverse test plane for all of the lens designs was directly under the lens, bisecting it along the \( x\)-axis, with its longest edge oriented in the \( z\)-direction. The width of the plane is equal to that of the diameter of the lenses, 6.35\( \lambda_0 \), and to ensure that the majority of the focal region was captured, while still minimizing the computational domain of the simulations, a height of 7.00\( \lambda_0 \) was used. It should be noted that normally when lenses are fabricated and tested their
performance is gauged with a horn antenna placed in the focal region of the lens; however, this is not done here for the sake of simplifying the numerical modeling of the lenses.

The scan performance of the hyperbolic lens can be found in Figures 1.6, 1.7 and 1.8. We see that as the angle of incidence moves off the z-axis so does the primary focal region. This is to be expected because the rays are encountering a single dielectric material and their incidence angles and exiting angles relative to the normal formed by the surface, should be identical. Not until the angle of incidence surpasses 45°, in Figure 1.8, do we start to see severe degradation of the focal region. The relationship between the angle of incidence and the averaged magnitude of the E-field inside of the focal region, as well as bandwidth is discussed for this, and all of the other lenses, in a following section.

Figure 1.6. Spatial distribution (Amplitude of \( E_x \)) of hyperbolic lens for \( \theta = 0°, 5°, 10° \) and 15°.
Figure 1.7. Spatial distribution (Amplitude of $E_x$) of hyperbolic lens for $\theta = 20^\circ$, $25^\circ$, $30^\circ$ and $35^\circ$.

Figure 1.8. Spatial distribution (Amplitude of $E_x$) of hyperbolic lens for $\theta = 40^\circ$, $45^\circ$, $50^\circ$ and $55^\circ$. 
1.3 Profiled Lens Design

The second lens of interest was a conventional profiled lens, whose parameters were identical to those of the hyperbolic lens, namely a diameter $D = 63.5$ mm, focal length $F = D/4 = 15.875$ mm and a central design frequency $f_0 = 30.0$ GHz. The key geometric feature that differentiates this type of lens from the hyperbolic lens is that instead of possessing a smooth surface along its incident face, the profiled lens is comprised of distinct regions with varying thicknesses that mimic the curved profile by staircasing. Not unexpectedly, it is necessary to choose certain design parameters of the lens from the outset, as for instance, the dielectric material, the number of distinct regions and the thickness of either the outermost or the central region. Through trial and error it was determined that six distinct regions would suffice. A relative permittivity $\varepsilon_r = 4.0$ was chosen owing to its widespread availability, and lower level of computational intensiveness required to simulate it. With regards to setting the thickness of the outermost region it was important to keep in mind that the thickness of the entire lens itself was not as important as the thickness differential that existed between contiguous regions, insofar as the functionality of the lens was concerned. Thus, the thinnest possible thickness was assigned to the outer region in order to minimize the overall lens thickness.

![Figure 1.9. Profiled lens with paths of interest.](image)
To make this lens work properly we need to ascertain that the electrical lengths of path \( l \) (represented by the red arrows) and path \( n \) (represented by the blue arrows) in Figure 1.9 are equivalent. The thickness of region \( n \) is predefined and we need to determine the thickness of region \( l \). We start by equating the paths:

\[
t_1 k_m + l_1 k_0 = (t_1 - t_n) k_0 + t_n k_m + l_n k_0
\]

(1.2)

where \( k_m = \frac{2\pi}{\lambda_m}, \lambda_m = \frac{\lambda_0}{\sqrt{\varepsilon_{r,m}}}, l_n = \sqrt{l_{n1}^2 + R^2}, \) and \( R \) is the distance between the center of the lens and the exact point within each region where we wish to guarantee the equivalent electrical path lengths. Solving for \( t_1 \) we get:

\[
t_1 = \frac{t_n(k_m - k_0) + l_n k_0 - l_1 k_0}{(k_m - k_0)} = \frac{t_n(\sqrt{\varepsilon_{r,m} - 1}) + l_n - l_1}{(\sqrt{\varepsilon_{r,m} - 1})}
\]

(1.3)

From (1.3) we can see that the relative permittivity of the material used to fabricate the lens does affect the thickness of the central region. Specifically, the larger the relative permittivity of the material is, the thinner the central region needs to be. There is a tradeoff; however, in that it is desirable to keep reflections from the incident face of the lens to a minimum and the easiest way to do this is to ensure a good impedance match between the lens and free space. The same method described above to find the thickness of the central region was also used to find the thickness of the remaining regions. Region \( l \) corresponds to the area in the center of the lens, while region 6 corresponds to the area that constitutes the outermost region (see Figure 1.10).

<table>
<thead>
<tr>
<th>region</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t(\lambda_0) )</td>
<td>1.51</td>
<td>1.39</td>
<td>1.13</td>
<td>0.84</td>
<td>0.52</td>
<td>0.20</td>
</tr>
<tr>
<td>( \text{width}(\lambda_0) )</td>
<td>0.289</td>
<td>0.577</td>
<td>0.577</td>
<td>0.577</td>
<td>0.577</td>
<td>0.577</td>
</tr>
</tbody>
</table>
The simulation results for this lens, when excited by a plane wave as discussed earlier, are displayed in Figures 1.11, 1.12 and 1.13.

Figure 1.11. Spatial distribution (Amplitude of $E_x$) of profiled lens for $\theta = 0^\circ$, $5^\circ$, $10^\circ$ and $15^\circ$. 
Figure 1.12. Spatial distribution (Amplitude of $E_x$) of profiled lens for $\theta = 20^\circ$, $25^\circ$, $30^\circ$ and $35^\circ$.

Figure 1.13. Spatial distribution (Amplitude of $E_x$) of profiled lens for $\theta = 40^\circ$, $45^\circ$, $50^\circ$ and $55^\circ$. 
Just as with the hyperbolic lens we see that the focal region essentially follows the angle of incidence as it increases, and once again there is no severe degradation of this region until the angle of incidence exceeds 45°.

1.4 Flat Lens Design

The final configuration of interest was a flat lens with a diameter \( D = 63.5 \text{ mm} \), focal length \( F = D/4 = 15.875 \text{ mm} \), and a thickness \( t = 9.00 \text{ mm} \), which operates at a central design frequency \( f_0 = 30.0 \text{ GHz} \). The corresponding values of \( D \), \( F \) and \( t \) are then 6.35\( \lambda_0 \), 1.5875\( \lambda_0 \) and 0.90\( \lambda_0 \), respectively. Since the profile of this particular lens is flat, its material parameters (permittivities) must have an appropriate radial variation in order to focus an incoming wave, by converting the plane wave into a spherically converging one. To facilitate a direct comparison to the aforementioned profiled lens, six concentric rings were used to emulate the six regions of varying thickness. It should be mentioned that increasing the number of rings serves to improve the overall performance in terms of scanning and focusing capabilities. This occurs because the greater the number of distinct regions present in the lens, the greater is its ability to equate different path lengths. A more visual way of approaching this is by thinking of each distinct dielectric region as being comprised of a material found throughout the lens but with a different thickness. Therefore, as one increases the number of regions, the thickness differential between contiguous regions decreases and in turn gives rise to a “smoother” lens that essentially mimics the hyperbolic lens we discussed earlier. Just as before, if a particular thickness was assigned to the outermost region for the profiled lens, the permittivity for the outermost ring also had to be predefined. A relative permittivity of \( \varepsilon_r = 1.1 \) was chosen to represent the smallest realistic
dielectric available (foam), so as to minimize the relative permittivity of the central region, which would keep the reflection at the interface low.

The method used to assign a unique relative permittivity to each region of the flat lens is based on the criterion of equivalent electrical path lengths regardless of physical path.

![Figure 1.14. Flat lens schematic with paths of interest.](image)

Just as before, for the lens to function properly it is necessary to ascertain that the electrical lengths of path 1 (represented by the red arrows) and path n (represented by the blue arrows) are equal. The permittivity of the dielectric in region n is predefined, and we need to determine the dielectric parameters of region 1. We start by equating the paths:

\[ k_1 t + k_o l_1 = k_n t + k_o l_n \]  \hspace{1cm} (1.4)

where \( k_n = \frac{2\pi}{\lambda_n} \), \( \lambda_n = \frac{\lambda_o}{\sqrt{\varepsilon_{\varepsilon, n}}} \), \( l_n = \sqrt{l_1^2 + R'^2} \), and \( R' \) is the distance between the center of the lens and the exact point within each region we wish to guarantee equivalent electrical path lengths.

Since our objective is to solve for \( \varepsilon_{\varepsilon, 1} \), we must first solve for \( k_1 \):
\[ k_1 = \frac{k_n t + k_o l_n - k_n l_1}{t} \]  \hspace{1cm} (1.5)

which leads to:

\[ \lambda_1 = \frac{t}{\frac{1}{\lambda_n} (l_n - l_1) + \frac{1}{\lambda_o} l_n - l_1 + \sqrt{\varepsilon_{r,n}' t}} \]  \hspace{1cm} (1.6)

From (1.6) we deduce that:

\[ \varepsilon_{r,1} = \left( \frac{l_n - l_1 + \sqrt{\varepsilon_{r,n}' t}}{t} \right)^2 = \left( \frac{\sqrt{l_1^2 + R^2} - l_1 + \sqrt{\varepsilon_{r,n}' t}}{t} \right)^2 \]  \hspace{1cm} (1.7)

which defines the relative permittivity of region 1. The method described above is also used to determine the dielectric parameters of the remaining five regions that are shown in Table 1.2. Region 1 corresponds to the region in the center of the lens while region 6 is the outermost region (see Figure 1.15).

<table>
<thead>
<tr>
<th>region</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_r )</td>
<td>8.67</td>
<td>8.02</td>
<td>6.39</td>
<td>4.40</td>
<td>2.54</td>
<td>1.10</td>
</tr>
<tr>
<td>width(( \lambda_o ))</td>
<td>0.289</td>
<td>0.577</td>
<td>0.577</td>
<td>0.577</td>
<td>0.577</td>
<td>0.577</td>
</tr>
</tbody>
</table>
In Figure 1.16, 1.17 and 1.18 we see that the spatial distribution of the focal region maintains the trend that was seen in both the hyperbolic and profiled lenses. One apparent difference; however, is that the length of the primary focal region, a parameter that will be discussed in the following section, is noticeably smaller than that of the previous two lenses. It should be noted that the flat lens’ thickness is about half that of the other two lenses and it would stand to reason, from the ray-tracing exercises done above, that the length of the primary focal region should also be approximately half that of the other two lenses.
Figure 1.16. Spatial distribution (Amplitude of $E_x$) of flat lens for $\theta = 0^\circ$, $5^\circ$, $10^\circ$ and $15^\circ$.

Figure 1.17. Spatial distribution (Amplitude of $E_x$) of flat lens for $\theta = 20^\circ$, $25^\circ$, $30^\circ$ and $35^\circ$. 
1.5 Results and Interpretations

Using the plane wave excitation results we were able to directly compare the bandwidth and scanning capabilities of each lens, as is shown in Figures 1.19 and 1.20 below.
From Figure 1.19 it can be seen that the directivity increases with frequency regardless of the lens used. This is to be expected, since the effective aperture size of the lens increases as the frequency is increased, which results in increased directivity, as it does with other antenna apertures. Next, in Figure 1.20, it can be seen that on the whole, directivity decreases as the angle of incidence increases. This trend is most pronounced when the angle of incidence exceeds 30°, especially for the profiled and flat lenses. The reason for this behavior is that as the angle of incidence increases, the number of paths with equivalent electrical lengths decreases. The net result is the introduction of other focal regions, which manifest themselves as a “stretching” of the area around the primary focal region. This reduces the amount of energy found within the primary focal region, and consequently causes a decrease in directivity.
GEMS provides a contour map with 22 distinct regions representing an averaged value of the amplitude of $E_x$ in that particular area. The length of the region with the greatest magnitude is defined as the focal region length. There are a few exceptions to this, such as Figure 1.17 when $\theta = 30^\circ$, where the region with the greatest magnitude is not where the focal region is
actually located. Therefore, it is important to estimate where the focal point is expected to approximately exist for a particular angle of incidence. By tracking the focal region length (see Figure 1.22 and 1.23) one is able to support the bandwidth and scanning performances shown in Figures 1.19 and 1.20, respectively, as well as demonstrate the resolving capabilities (ability to discriminate between multiple targets) of the lenses.

![Focal Region Length vs. angle of incidence for f = 30 GHz](image)

Figure 1.22. Focal region length as a function of angle of incidence.
1.6 Hyperbolic, Profiled and Flat Lens Conclusion and Future Work

The purpose of this study was to compare the scan and bandwidth performance of the three different lenses that were designed to operate in the millimeter band. The first was a typical hyperbolic lens with a diameter $D = 63.5$ mm, focal length $F = D/4 = 15.875$, both of from Yang et al., $\varepsilon_r = 2.6$ and a central design frequency $f_0 = 30.0$ GHz. The second configuration was a profiled lens, which could be thought of as a discretized version of the hyperbolic lens. It shared the same diameter and focal length as the previous lens, but it was comprised of a material with a relative permittivity of 4.0. Also, the number of regions used to form this lens was based on the results of simulating lenses of varying regions and determining at what point any additional regions would add little or nothing to the overall performance of the lens. The thickness of each region was determined by enforcing the condition that all rays that pass through the lens are to have the same electrical length regardless of physical path. Lastly, the flat lens, which shared the
same diameter and focal length as the other lenses, was comprised of a number of different
dielectrics whose parameters were determined by enforcing the aforementioned path-length
condition. Also, each region had a thickness of 9.00 mm, which was the same thickness as the
lens investigated by Yang et al. By illuminating the surface of each lens with a plane wave for
different angles of incidences and frequencies and then studying the resultant radiation patterns on
test planes, it was possible to gauge the desired scan and bandwidth performance. The ability to
carry out such resource intensive microwave lens simulations is a recent development in
computational electromagnetics and is the main contribution to knowledge of this particular
study.

In terms of future work, we feel that the most immediate effort worth pursuing is the
design of a matching layer that would enclose the lenses designed in this study. Such a layer
would increase the bandwidth and scanning performance of the lens by reducing the impedance
mismatch that appears at the air/surface interface. Also, a study into the effects of varying the
thickness on the formation of the primary focal region could prove worthwhile. In this study we
noted that the focal region length was approximately that of the thickness of the thickest section
of any particular lens, which means very thin lenses could be designed and tested.

1.7 Large Lenses

In an attempt to see how the diameter affected the scan capability and bandwidth
performance of the different type of lenses, they were reexamined, but this time with twice the
diameter, i.e., with $D = 12.7 \lambda_0$. The central design frequency was once again $f_0 = 30.0$ GHz,
and the relationship between the focal length and the diameter ($F/D = 1/4$) was preserved as well.
These lens designs are collectively referred to as the “Large” lenses. Just as in the previous
section, the first lens of interest was hyperbolic, whose profile is described by (1.1). The large hyperbolic lens was comprised of the same relative permittivity as that of the previous hyperbolic lens, $\varepsilon_r = 2.66$, and had a maximum thickness of $3.95\lambda_0$. Four different views of the object are displayed in Figure 1.24. All of the large lenses were excited in the same fashion as their small counterparts. The scan performance of the large hyperbolic lens can be found in Figure 1.25, 1.26 and 1.27.

![Figure 1.24](image)

Figure 1.24. (a) $yx$, (b) isometric, (c) $zx$ and (d) $zy$.

![Figure 1.25](image)

Figure 1.25. Spatial distribution (Amplitude of $E_x$) of large hyperbolic lens for $\theta = 0^\circ$, $5^\circ$, $10^\circ$ and $15^\circ$. 

23
Figure 1.26. Spatial distribution (Amplitude of $E_x$) of large hyperbolic lens for $\theta = 20^\circ$, $25^\circ$, $30^\circ$ and $35^\circ$.

Figure 1.27. Spatial distribution (Amplitude of $E_x$) of large hyperbolic lens for $\theta = 40^\circ$, $45^\circ$, $50^\circ$ and $55^\circ$. 
The most notable difference between this new large hyperbolic lens and the previous one is the apparent stretching of primary focal region. As discussed earlier, the thickness of the lens itself plays a direct role in the length of the primary focal region and this lens is nearly $4\lambda_0$ thick. We also notice the introduction of a secondary focal region for angles of incidence greater than and including $30^\circ$. Just as in the previous section the comparative scanning and bandwidth performances of all the lens will appear in the following section.

Next, a large profiled lens with eleven regions of distinct thicknesses, as opposed to six, was modeled. Its geometric properties and different views are detailed in Table 1.3 and Figure 1.28, respectively.

<table>
<thead>
<tr>
<th>region</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>thickness ($\lambda_0$)</td>
<td>2.79</td>
<td>2.73</td>
<td>2.58</td>
<td>2.36</td>
<td>2.10</td>
<td>1.81</td>
<td>1.50</td>
<td>1.19</td>
<td>0.86</td>
<td>0.53</td>
<td>0.20</td>
</tr>
<tr>
<td>width ($\lambda_0$)</td>
<td>0.3025</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
</tr>
</tbody>
</table>
Figure 1.28. (a) $yx$, (b) isometric, (c) $zx$ and (d) $zy$.

Figure 1.29: Spatial distribution (Amplitude of $E_x$) of large profiled lens for $\theta = 0^\circ$, $5^\circ$, $10^\circ$ and $15^\circ$. 
Figure 1.30. Spatial distribution (Amplitude of $E_x$) of large profiled lens for $\theta = 20^\circ$, 25°, 30° and 35°.

Figure 1.31. Spatial distribution (Amplitude of $E_x$) of large profiled lens for $\theta = 40^\circ$, 45°, 50° and 55°.
Compared to its smaller counterpart, the large profiled lens exhibits the same general scanning behavior. Also, except for the case when that angle of incidence equals 35° (see Figure 1.30), there is not a pronounced, secondary focal region like the one seen for the large hyperbolic lens.

Lastly, a large flat lens also comprised of eleven distinct regions (for direct comparison with the large profiled lens), was modeled, and its properties and views can be found in Table 1.4 and Figure 1.32, respectively.

Table 1.4. Dielectric and Geometric Properties of large flat Lens.

<table>
<thead>
<tr>
<th>region</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_r$</td>
<td>26.1</td>
<td>25.5</td>
<td>23.7</td>
<td>20.9</td>
<td>17.7</td>
<td>14.2</td>
<td>10.8</td>
<td>7.7</td>
<td>4.9</td>
<td>2.7</td>
<td>1.1</td>
</tr>
<tr>
<td>width ($\lambda_o$)</td>
<td>0.3025</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
<td>0.605</td>
</tr>
</tbody>
</table>

Figure 1.32. (a) yx, (b) isometric, (c) zx and (d) zy.
Figure 1.33. Spatial distribution (Amplitude of $E_x$) of large flat lens for $\theta = 0^\circ$, 5°, 10° and 15°.

Figure 1.34. Spatial distribution (Amplitude of $E_x$) of large flat lens for $\theta = 20^\circ$, 25°, 30° and 35°.
Figure 1.35. Spatial distribution (Amplitude of $E_x$) of large flat lens for $\theta = 40^\circ, 45^\circ, 50^\circ$ and $55^\circ$.

The large flat lens scanning performance deteriorates dramatically for angles of incidence greater than and including 25°. This is particularly surprising considering the superior performance that the previous flat lens offered (see Figure 1.16, 1.17 and 1.18). It should be pointed out, that though there does appear to be the presence of a secondary focal region (see Figure 1.33) next to the primary one, this is of little importance considering that when the angle of incidence exceeds 30° any semblance of a focal region ceases to exist. This odd behavior is discussed more thoroughly in the following section.

1.8 Large Lens Results and Interpretations

With the exception of the existence of secondary focal regions, the scan performance of the large lenses mirrors the behaviors of the small lenses. After discovering the unexpected behavior of the large flat lens we decided to alter the number of regions used and record the effects. Now,
keeping in mind that the counterpart of the flat lens is the profiled lens, and that by reducing the number of regions one would essentially be producing a “coarser” lens, it would be reasonable to expect that the scan performance would degrade as the number of regions decreased and vice versa. This assumption was found to be correct by comparing the performances of large lenses comprised of 6, 8 and 10 regions, but failed for 11 regions. In Figure 1.36 the scan performance of the 10 region lens shows the expected behavior of a flat lens, which is noticeably different than that found in Figures 1.33 and 1.34.

Figure 1.36: Spatial distribution (Amplitude of $E_x$) of large flat lens (10 regions) for $\theta=0^\circ$, $10^\circ$, $20^\circ$ and $30^\circ$.

At this moment we have no explanation for the large flat lens performance, and feel that this phenomenon requires further investigation.

In terms of bandwidth, the large lens’ performance, on the whole, obeys the relationship between frequency and directivity that was discussed previously (see Figure 1.37). There are;
however, anomalies in the overall performance of the large flat lens, as can be seen in Figure 1.34, which reveal themselves in both Figure 1.37 and 1.38. The cause of this erratic behavior is due to the inadequate formation of a single primary focal region. In other words, instead of having a single focal region of the type found in Figure 1.36 for normal incidence, there are two distinct regions, indicating that energy is not being properly focused. This behavior was seen in 6, 8 and 10 region lenses as well. Much like the scan performance of the large flat lens, this odd behavior is currently inexplicable and also requires further investigation.

Figure 1.37. Bandwidth performance of large hyperbolic, large profiled and large flat lenses.
1.9 Large Lens Conclusion and Future Work

In an attempt to determine the relationship between lens diameter and scan/bandwidth performance, we doubled the diameter of our previous designs and monitored the effects of exciting them with plane waves at various angles of incidence. On the whole, the scan and bandwidth performance for these lenses mirrored that of their smaller counterparts, but with one noticeable exception: the large flat lens. This lens produced unusual radiation patterns in the focal plane, which in turn gave rise to poor bandwidth and scan performance.

Considering the behavior exhibited by the large flat lens we feel this is the most pertinent future direction of study. Also, much like the previous set of lenses, in an attempt to increase directivity, it would prove worthwhile to develop a matching layer for these lenses as well.
Lastly, a study focusing on the relationship between the number of regions used in a particular lens and its scan/bandwidth performance is advised.
2. **Grated Dielectric Surfaces**

2.1 **Introduction and Motivation**

The motivation for investigating this design is to generate an isoflux radiation pattern in the millimeter band, with lobes located as from broadside as possible, when the underside of the surface is excited by a source with random phase and orientation (see Figure 2.1).

![Figure 2.1. Desired operation of design.](image)

In the infrared regime there are already surfaces such as periodic gratings [4] (see Figure 2.2) that produce an isoflux pattern when heated to a particular temperature and then left to radiate. The periodicity of this particular surface was $0.55\lambda_0$, and the depth of each grating was $0.025\lambda_0$.

![Figure 2.2. Periodic SiC microstructure designed by Greffet et al.](image)
Another design intended to work in the infrared was proposed by Kemme et al. [5]. This structure, shown in Figure 2.3, is also periodic, but is fabricated from nickel, which has unique electrical properties \( n = 0.05 + i2.0 \) in the frequency range of interest (3.0 to 12.0μm).

Figure 2.3. Periodic nickel structure designed by Kemme et al..

Both of these designs rely on the phenomenon of photon/plasmon coupling, which takes place at the air/surface interface. This coupling only occurs at certain wavelengths, and enables the surface to generate highly directional emissions, which manifest themselves as an isoflux pattern in the far-field. The region of interest for our research; however, is not in the infrared, but is instead in the microwave regime. As would be expected, the materials previously mentioned are not the best suited for microwave frequencies, owing to their lossy behaviors, though the periodic structures themselves could still be potentially implemented in a design. Given this fact, another set of structures, namely leaky-wave antennas with their ability to produce highly directional and narrow beams off of broadside, were also investigated.

The first leaky-wave antenna we studied was designed by Antar et al. [6], and is displayed in Figure 2.4.
Figure 2.4. Grated dielectric antenna proposed by Antar et al..

The center design frequency for this structure was 30.0 GHz, and had a periodicity of 0.4 \( \lambda_0 \). Aside from the method of excitation (a waveguide source located at one of the ends of the surface) and design frequency, this leaky-wave antenna differs from the designs proposed by Greffet and Kemme, in that it is made of two distinct materials. The base or film of the structure is Alumina (\( \varepsilon_r = 9.8 \) at 30.0 GHz) while the grating is comprised of Aluminum Titanate (\( \varepsilon_r = 30 \) at 30.0 GHz). This heterogeneous design will be discussed further in the following section. The last candidate design, displayed in Figure 2.5, was described by Shanjia and Xinzhang [7], who proposed a leaky-wave antenna comprised of a single dielectric material (\( \varepsilon_r = 2.10 \)), which produced a lobe at approximately 60° from broadside at \( f = 38.0 \text{ GHz} \). Of all the possible designs, this particular one was deemed to be the most practical, and received the most detailed study. Beyond meeting the design criteria of producing an isoflux far-field pattern the importance of this research is that it is the first, to our knowledge, to study the effects of exciting leaky-wave structures from below with an arbitrary radiator as opposed to the conventional method of exciting the structure with a waveguide/coupler from one end.
2.2 Uniform Dielectric Slab Waveguides

To develop an understanding of how leaky-wave antennas work, one must first gain some familiarity with the phenomenon of field propagation in a dielectric waveguide. In common with all waveguides, the purpose of the dielectric guide is to contain energy within the structure and propagate it in a given direction. What is unique to dielectric waveguides is that it accomplishes this by bouncing the wave back and forth between the upper and lower interfaces, at an angle of incidence greater than that of the critical angle. When this happens, the field outside the dielectric forms evanescent waves and all the real energy is reflected and contained within the waveguide [8]. In common with PEC waveguides, the dielectric waveguide can also support modal fields, known as surface waves. However, in contrast to the modal characteristics of PEC guides, the number of “bound” modes in open dielectric waveguides is very limited, and they are complemented by a continuous spectrum of modes not present in closed PEC guides. The bound mode that is of most interest in dielectric waveguides is TM$_0$, which is its dominant mode with a cutoff frequency of zero, implying that regardless of the frequency of operation, the TM$_0$ mode...
will propagate unattenuated within the waveguide. It should be noted that TM\(_0\) is an odd mode whose field distribution is shown in Figure 2.6.

![Figure 2.6. Even and odd mode field distributions in a dielectric slab waveguide (Source: C. Balanis, Advanced Engineering Electromagnetics, 1989).](image)

Knowing how to determine the phase and attenuation constants for waves above a dielectric slab waveguide is useful, even though a corrugated surface does not have a uniform height or permittivity owing to the perturbations on its surface. In [8] we find that by solving equations (2.1) and (2.2) below:

\[
-\frac{\varepsilon_0}{\varepsilon_d} (\beta_{yd} h) \cot (\beta_{yd} h) = \alpha_{yd} h \quad \text{TM}^z (\text{even}) \tag{2.1}
\]

\[
\frac{\varepsilon_0}{\varepsilon_d} (\beta_{yd} h) \tan (\beta_{yd} h) = \alpha_{yd} h \quad \text{TM}^z (\text{odd}) \tag{2.2}
\]

\[
\beta_{yd}^2 + \beta_z^2 = \beta_d^2 = \omega^2 \mu_d \varepsilon_d \Rightarrow \beta_{yd}^2 = \beta_d^2 - \beta_z^2 = \omega^2 \mu_d \varepsilon_d - \beta_z^2 \quad \text{TM}^z (\text{even and odd}) \tag{2.3}
\]

it is possible to determine the phase constant \(\beta\), as well as the attenuation constant \(\alpha\), both of which appear in the propagation constant as follows:
\[ \gamma = \alpha + j\beta \quad (2.4) \]

Having discussed how these parameters can be found, we now consider how they affect the behaviors of the waves themselves. By analyzing

\[ \beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2 \quad (2.5) \]

for different values of \( \beta, \beta_x, \beta_y \) and \( \beta_z \), one can deduce the nature of the wave above the structure. For this particular application, only two of the three components are of interest, and to remain consistent with Figure 2.6, we choose to ignore the \( x \)-component of the propagation constant and (2.5) is reduced to:

\[ \beta^2 = \beta_y^2 + \beta_z^2 \quad (2.6) \]

In Figure 2.7 the real components of (2.6) are shown in their vector form in addition to the expected launching angle \( \theta \) of the wave.

There are two possible behaviors for a wave in this situation. The first one involves the category where \( \beta_y \) is purely imaginary, \( i.e., \) the real component of \( \beta_y \) equals zero, and \( \beta_z \) is greater than the free space wave number \( \beta_0 \). In this case, clearly, the only remaining component of the real wavenumber is the one associated with the \( z \)-direction, demonstrating that the wave travels purely in the \( z \)-direction. This type of wave is called a surface wave, because it remains bound to
the surface of the structure. The second possible behavior, and the one that is of interest in this study, is the case when the real components of both $\beta_y$ and $\beta_z$ are greater than zero, but less than $\beta_0$. When this occurs, the wave is no longer bound to the surface and is essentially launched at an angle above the surface. Such a wave is referred to as the *leaky-wave*. The launch angle of the leaky-wave is determined by a trigonometric relationship between $\beta_y$ and $\beta_z$, namely:

$$\theta = \tan^{-1} \left( \frac{\text{Re} (\beta_y)}{\text{Re} (\beta_z)} \right)$$  \hspace{1cm} (2.7)

The second parameter of interest is the attenuation constant, which is the real component of the propagation constant. Like all waves radiating from passive surfaces, the attenuation constant is expected to have a negative value so that the wave does not increase in strength as it leaves the surface. When solving for $\alpha_{y0}$ and $\alpha_{z0}$ using conventional methods, a surprising phenomenon is encountered: the $z$-component of the attenuation constant is negative, as is expected, but the $y$-component is not. This behavior is not an error in the calculation, but would be erroneous if we were to assume that this growing behavior continues to infinity, and the radiation condition is violated. What actually happens; however, is that this growth only occurs in the region that is within a cone defined by the launch angle, and the wave starts to decay in the vertical direction, as conventional physics says it should, beyond this point.

Having discussed the importance of knowing the wave numbers and attenuation constants of waves above a surface, the next step is to provide an overview of how these parameters are calculated for leaky-wave antennas.

### 2.3 Theoretical Background of Leaky-Wave Antennas

We now turn to leaky-wave antennas, and provide a theoretical background of these structures to help develop an understanding of how they function. Schwering *et al.* [9] and
Jackson et al. [10] offer a thorough discussion involving the underlying mechanisms that govern the behavior of a leaky-wave antennas, and we provide a brief summary in what follows. The corrugated dielectric surface is comprised of two distinct regions: the unperturbed structure (the base which the teeth rest on) and the periodic layer (the teeth themselves). The radiation from dielectric leaky-wave antennas is affected by the periodic perturbation of the waves (induced by the grated surface) that are guided by the uniform part of structure. As expected, radiation only occurs in preferred directions and is directly affected by the unperturbed structure, and the periodic perturbations. Just as in a normal dielectric waveguide, the wave must decay exponentially as it propagates along the unperturbed structure and will, in turn, only exist with a significant magnitude over a finite length of the antenna. The behavior of this wave along the surface of the antenna manifests itself in the far field as a beam with a width proportional to the decay constant of the wave. This decay constant is often referred to as the leakage constant because it can be thought of as a measure of the rate of energy leakage from the surface.

Therefore, two key values that need to be kept in mind when designing a leaky-wave antenna are the phase and leakage constants. In order to determine the phase and leakage constant parameters of a leaky-wave antenna, we can employ the effective dielectric constant (EDC) method, as well as techniques used in the analysis of optical periodic couplers. Additionally, the radiation patterns can be determined from the field distributions on the antenna aperture by performing a spatial Fourier transform. This solution methodology is covered extensively in [9].

At this point it is important to reiterate that for this particular study the surface is not illuminated by a waveguide located at one of its edges, but is instead excited by a radiator with random phase and orientation. As a result, any discussion involving the determination of such
values as launch angle, leakage constants, etc., for dielectric leaky-wave antennas can only be employed as rough guides for predicting the expected performance of our particular design. With this in mind, our contention is that the $\text{TM}_0$ of the structure, which is normally excited by the $\text{TE}_{10}$ of a rectangular waveguide with the aid of a coupler from one end of a dielectric guide, can also be excited by this unique source located beneath the structure.

### 2.4 Heterogeneous Grated Dielectric Surface

The first leaky-wave structure we simulated was that proposed by Antar et al. [6]. As mentioned earlier, this was a grated dielectric surface, depicted in Figure 2.4, which comprised of Alumina (the unperturbed region), collectively referred to herein as the film of the antenna, and Alumina Titanate (the periodic layer).

![Figure 2.8. Heterogeneous dielectric grating under investigation.](image)

![Figure 2.9. Heterogeneous grating profile.](image)
Table 2.1. Heterogeneous grated surface initial design parameters.

<table>
<thead>
<tr>
<th>$\lambda_0$ (mm)</th>
<th>$L(\lambda_0)$</th>
<th>$d(\lambda_0)$</th>
<th>$w(\lambda_0)$</th>
<th>$d_1(\lambda_0)$</th>
<th>$t_g(\lambda_0)$</th>
<th>$t_f(\lambda_0)$</th>
<th>$\varepsilon_g$</th>
<th>$\varepsilon_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>2→20</td>
<td>0.40</td>
<td>2.0</td>
<td>0.16</td>
<td>0.045</td>
<td>0.1016</td>
<td>30.0</td>
<td>9.8</td>
</tr>
</tbody>
</table>

With the exception of $d$ and $d_1$, all of the design parameters found in Table 2.1 are from the final optimized design found in [6]. Also, a width of $2.0\lambda_0$ was chosen because it meets the criterion of being larger than $\lambda_0$, which ensures that $\phi$ (see Figure 2.8), is relatively small. For this case, the wave will propagate down the antenna and not bounce back and forth between the edges resulting in the creation of a standing wave. Also, a width of $2.0\lambda_0$ was chosen to keep the computational intensiveness of the simulation to a minimum.

![Figure 2.10. Corrugated antenna of finite width [9].](image)

Due to the underlying periodic nature of this particular design, the period $d$ plays an important role in the far-field performance of the aperture. For this design, $d$ would remain fixed at $0.40\lambda_0$, but $d_1$ would vary. This meant that the aspect ratio (AR), which is defined as $d_1/d$, would also vary. The optimized design proposed in [6] had an AR of 0.24, which did not produce the desired isoflux pattern when simulated in GEMS. In light of this, a decision was made to start with an AR of 0.4, a value closer the AR suggested in [9].

The method of excitation, at least in the preliminary stages, was chosen to be a single half-wave planar dipole placed $0.15\lambda_0$ below the structure at its geometric center, whose parameters
can be found in Figure 2.11. A half-wave planar dipole was used in our simulations, to circumvent unnecessary computational intensiveness that would be introduced if we were to employ a normal wire dipole instead, because our simulations would then require a fine mesh to capture the minute details of the dipole itself. Also, a half wave dipole emulates the nearly isotropic behavior that needs to be simulated to capture the behavior of our unique radiator.

![Figure 2.11. Dimensions of planar half-wave dipole.](image)

Having settled on an acceptable aspect ratio, a number of different lengths of the grated surface were simulated in order to determine for what choice of length the truncation effect becomes negligible. It is to be expected that for shorter lengths (2, 4, 6 and even $8 \lambda_0$) the leaky-waves are essentially not provided an adequate length to effectively leave the surface of the aperture and are reflected back from the longitudinal edges. The truncation effect arises from the fact that the waves encounter an impedance mismatch at the ends of the grated surface. This leads to an undesirable behavior, insofar as generating an isoflux radiation pattern is concerned, since the pattern now contains many side lobes that corrupt the desired pattern characteristics we seek.
this particular design, a length of $10\lambda_0$ was found to be the minimum acceptable structural length. Polar plots demonstrating this behavior are shown in Figure 2.13 and Figure 2.14. Because the structure is symmetric about the $x$-axis, and only the space above the surface was of interest, only $\phi = 0^\circ$ cuts with $0^\circ \leq \theta \leq 90^\circ$ are displayed (see Figure 2.12).

Figure 2.12. $\phi$ cuts of interest above heterogeneous grated dielectric.

Figure 2.13. $\phi = 0^\circ$ cut far field radiation patterns for $2\lambda_0$ through $8\lambda_0$. 
Figure 2.14. $\phi = 0^\circ$ cut far field radiation patterns for $10\lambda_0$ through $16\lambda_0$.

Figure 2.15. $\phi = 0^\circ$ cut far field radiation patterns for $18\lambda_0$ and $20\lambda_0$. 


We see that even for a relatively short length of $2\lambda_0$, the structure is effectively displacing the radiation away from broadside and out towards values of $\theta$ (the angle formed between the $+z$ and $+x$-axis) greater than $30^\circ$. As we progressively increase the length of the structure from $2\lambda_0$ to $12\lambda_0$, we notice that the main lobe shift towards the $+x$-axis and its level of directivity increases from around $8 \text{ dB}$ to $10 \text{ dB}$. For lengths from $12\lambda_0$ to $20\lambda_0$ we see marginal improvements in the location of the main lobe and its level of directivity. This tells us that at around $L = 12\lambda_0$, virtually all of the leaky-waves generated on the surface of the structure have adequate room to leave the surface and hence no longer suffer from a truncation effect beyond that point. It is important to keep in mind that this was the minimum acceptable length for a single excitation element. Later on in this study we find that the acceptable structural length is affected by the length spanned by the radiating elements.

Having established the minimum acceptable length for the grated surface, the next parameter that we needed to determine was the aspect ratio. As mentioned earlier, the aspect ratio is defined as the width of the grooves, $d_1$, divided by the grating periodicity, $d$. The previous far-field patterns were for the case where the aspect ratio was 0.4. Considering the range of typical aspect ratios used in other grated dielectric leaky-wave antennas [9], it was decided that aspect ratios of both 0.5 and 0.6 would be worthwhile to investigate. The results of these investigations are shown in Figure 2.16 where the lengths of the structure are $10\lambda_0$ and $20\lambda_0$. 
As stated before, in the initial simulations, a single half-wave planar dipole oriented in the $y$-direction (the direction that was shown to give rise to the most desirable main lobes in the $zx$-plane) was used. The next step in the design process involved using an array of half-wave dipoles of random orientation (within the $yx$-plane) and phase. This step was undertaken because the actual behavior of the radiator that we are attempting to emulate in this study is one of both random excitation and phase. A sample orientation of the three dipoles used in this type of excitation can be found in Figure 2.17. Also, the exact orientation and phase for each dipole used in the four different runs can be found in Table 2.2.
The results of these simulations can be seen in Figure 2.18, where with the exception of run 1, the structure is able to distribute much of the radiation away from broadside. For this particular run, there appears to be two main lobes, one situated along the zenith-direction with a maximum of 5 dB and another one at $\theta = 70^\circ$, with a maximum of 6 dB. This appears to be a direct result of the orientation of two of the three dipoles involved which were oriented along the $x$-axis. This is a known subpar orientation, with respect to far-field performance, to excite this particular structure. The remaining runs; however, demonstrate that this structure can create the
desired isoflux pattern when excited using three planar half-wave dipoles of random orientation and phase.

Figure 2.18. Heterogeneous grated surface excited with 3 planar dipoles with random orientation and phase.

2.5 Heterogeneous Grated Dielectric Surface Conclusion and Future Directions

The purpose of this study was to develop a structure that would generate an isoflux radiation pattern in the millimeter band, when the underside of the surface was excited by a source with random phase and orientation. Due to the radiation patterns produced by leaky-wave antennas [6], [7], [9] and [10], these structures were investigated and later modeled in GEMS, a parallel FDTD solver. Through experimentation with the length, and aspect ratios of the grated surface, when excited by a single plane half-wave dipole, an optimal design was finally determined. With this same design, an array of dipoles with random orientation and phase were used to better represent the actual unique radiator of interest. These simulations revealed that,
given a relatively small span of radiators, the structure could still produce the desired isoflux pattern.

The most immediate addition to this research would be the introduction of more dipoles. This would allow for the determination of the exact span the current structural length could handle in terms of producing the desired far field pattern. Also, experimentation with the thickness of both the film and grating could reveal reductions in necessary structure length. Lastly, testing the performance of the surface over a band of frequencies and not just one in particular would make the design far more robust and in turn far more practical.

2.6 Homogeneous Dielectric Grated Surface

The second grated surface design of interest was proposed by Shanjia et al. [7], and is shown in Figures 2.19 and 2.20. As with the previous design, this surface was originally used as a leaky-wave antenna, but with one major exception in that it is comprised of a single low-density dielectric material. This feature is attractive from the point of view of easier and less expensive fabrication. The homogeneous design is about four times thicker than the previous one, and all of its relevant design parameters can be found in Table 2.3.

![Homogeneous dielectric grating under investigation.](image)
Figure 2.20. Homogeneous grating profile.

Table 2.3. Homogeneous grated surface initial design parameter.

<table>
<thead>
<tr>
<th>$\lambda_0$(mm)</th>
<th>L($\lambda_0$)</th>
<th>d($\lambda_0$)</th>
<th>w($\lambda_0$)</th>
<th>d_1($\lambda_0$)</th>
<th>t_g($\lambda_0$)</th>
<th>t_s($\lambda_0$)</th>
<th>$\varepsilon_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.00</td>
<td>2→20</td>
<td>0.5</td>
<td>2.5</td>
<td>0.25</td>
<td>0.15</td>
<td>0.5</td>
<td>2.1</td>
</tr>
</tbody>
</table>

During the preliminary stages, the homogeneous structures of various lengths were excited by a single half-wave planar dipole oriented in the $y$-direction, as shown in Figure 2.11. The results of these simulations are provided in Figures 2.21 through 2.23.

Figure 2.21. $\varphi = 0^\circ$ cut far field radiation patterns for $2\lambda_0$ through $8\lambda_0$. 
Figure 2.22. $\phi = 0^\circ$ cut far field radiation patterns for $10\lambda_0$ through $16\lambda_0$.

Figure 2.23. $\phi = 0^\circ$ cut far field radiation patterns for $18\lambda_0$ through $20\lambda_0$. 
Unlike the previous heterogeneous structure, the structural length that appears to limit the truncation effects is not 10 or 12$\lambda_0$, but is around 18 or 20$\lambda_0$ instead. The reason for this is that the effective dielectric constant of the groove and free-space interface for the heterogeneous design is greater than that of the homogeneous one. This leads to a greater leakage constant meaning less structural length is required to correct for the truncation effect.

Having determined the appropriate length for the case where the excitation is a single half-wave planar dipole, the next step was to see how the aspect ratio of the structure affected its far-field performance (see Figure 2.24).

![Figure 2.24. Effect of varying the aspect ratio on homogeneous grated surface.](image)

We note, in comparing Fig. 2.24 and Figure 2.16, that the aspect ratio has a minimal effect on the far-field performance for this particular design, when it is excited by a single half-wave planar dipole. Next, just as in the previous case, the structure was excited by three dipoles of the
same orientation and phase as those found in Table 2.2. From Figure 2.25 we see that a large portion of the radiation does in fact radiate in the broadside. This undesirable behavior was a problem that was not present in the heterogeneous design and required a solution that did not involve using high density dielectrics.

![Figure 2.25: Homogeneous grated surface excited with 3 planar dipoles with random orientation and phase.](image)

### 2.7 Spatial Filter Overlay

Our proposed solution to this problem was a spatial filter, meant to suppress the undesirable components of the E-field that were radiating from the grated surface. It was determined that the components that contributed to the broadside patterns, were primarily polarized in the longitudinal (x-direction), and could in turn be suppressed by an array of uniformly spaced PEC strips, with their primary axis also oriented in the longitudinal direction. In theory, this would allow for the y-polarized components of the E-field to pass through relatively unperturbed while simultaneously reflecting the unwanted x-polarized components. In order to determine the best possible configuration, a number of candidate designs were proposed,
each with its own unique fill factor. The fill factor was defined as the ratio of the area of the PEC strips to the total area of the grated surface, as seen in the yx-plane. The initial gap width between the PEC strips was set at $0.08\lambda_0$. The parameters of each configuration as well as their design can be found Table 2.4 and Figure 2.26, respectively.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of PEC strips</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>PEC strip width($\lambda_0$)</td>
<td>1.21</td>
<td>0.78</td>
<td>0.565</td>
<td>0.436</td>
</tr>
<tr>
<td>Fill Factor</td>
<td>0.968</td>
<td>0.936</td>
<td>0.904</td>
<td>0.872</td>
</tr>
</tbody>
</table>

![Table 2.4. Initial spatial filter parameters.](image)

Figure 2.26.(a) Configuration 1, (b) configuration 2, (c) configuration 3 and (d) configuration 4.

Each of the proposed designs were then placed $0.0625\lambda_0$ above the homogeneous dielectric grated surface, and then the entire assembly was excited by a planar dipole oriented in the y-
direction, positioned 0.15\( \lambda_0 \) below the dielectric grated surface. All of the candidate designs managed to suppress the undesirable broadside radiation, but the one with the best performance was configuration 2.

![Figure 2.27. Grated surface and various spatial polarizer performance.](image)

Finally, different numbers of radiators were introduced to determine the span (total length covered by the array of radiators) at which the isoflux pattern could be preserved. The exact orientation and phase of each dipole in each run can be found in Tables 2.5, 2.6, 2.7 and 2.8.

Table 2.5. Dipole orientation and phase information for runs 1, 2 and 3.

<table>
<thead>
<tr>
<th>Run</th>
<th>Dipole</th>
<th>( \theta(\degree) )</th>
<th>( \phi(\degree) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>154</td>
<td>283</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>108</td>
<td>116</td>
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<td>348</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>66</td>
<td>263</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>80</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10</td>
<td>236</td>
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<tr>
<td></td>
<td>3</td>
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<td>96</td>
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<tr>
<td></td>
<td>5</td>
<td>117</td>
<td>175</td>
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**Table 2.6. Dipole orientation and phase information for runs 4 and 5.**

<table>
<thead>
<tr>
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<th>Dipole</th>
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<th>φ(°)</th>
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<tr>
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<td>3</td>
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<td>5</td>
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<tr>
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<td>6</td>
<td>175</td>
<td>71</td>
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<tr>
<td></td>
<td>7</td>
<td>170</td>
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<tr>
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<tr>
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<td></td>
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<td>227</td>
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</tbody>
</table>

**Table 2.7. Dipole orientation and phase information for run 6.**

<table>
<thead>
<tr>
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<th>θ(°)</th>
<th>φ(°)</th>
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<tbody>
<tr>
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<td></td>
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<td>174</td>
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<tr>
<td></td>
<td>11</td>
<td>142</td>
<td>37</td>
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</table>
Table 2.8. Dipole orientation and phase information for run 7.

<table>
<thead>
<tr>
<th>Run</th>
</tr>
</thead>
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<tr>
<td>Dipole</td>
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<td>9</td>
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<tr>
<td>10</td>
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<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>13</td>
</tr>
</tbody>
</table>

Figure 2.28: Grated surface and spatial polarizer performance for run 1, 2, 3 and 4.

From Figure 2.28 we say that for a single dipole excitation of random orientation and phase, run 1, the grated surface and spatial polarizer combination produces an isoflux radiation pattern with a pronounced narrow beamwidth. Also from Figure 2.26, it appears as the number
of dipoles introduced into the simulation increases, the farther away from this ideal isoflux pattern the far-field pattern becomes. There does appear to be an exception to this generality, when we observe the far-field pattern for run 6 in Figure 2.29 where a noticeable main lobe exists at around 80°. However, the fact that these results are only for one set of random orientations and phases, no exact conclusions can be drawn other than the observation that as the span of the elements used to radiate the structure increases, the more erratic and less ideal the far-field pattern in the plane of interest becomes.

Figure 2.29. Grated surface and spatial polarizer performance for run 5, 6 and 7.

2.8 Homogeneous Grated Dielectric Surface Conclusion and Future Directions

Just as with the heterogeneous grated dielectric surface, the design goal for this surface was to generate an isoflux radiation pattern in the millimeter band, with lobes as far off of broadside as possible, when excited from beneath by a source with random phase and orientation (see Figure 2.1). Following the design methodology that was developed for the heterogeneous
dielectric structure, a generic aspect ratio was chosen and a number of different structural lengths were simulated for single half-wave planar dipole excitations. After determining the appropriate length, the most effective aspect ratio was also determined. Next, for four different cases, this proposed structure was excited by an array of dipoles at random orientations and phases in an attempt to emulate the actual behavior of the primary radiator. At this point, it became apparent that unlike the heterogeneous case, this low-density surface could not effectively move large portions of radiation away from broadside, which prompted an investigation into a spatial filter. A number of fill factors where experimented with to determine which one produced the most desirable pattern, and once having determined this parameter, the structure and spatial filter polarization combination were excited with a number of different dipole arrays. These simulations revealed that, as expected, an inverse relationship existed between the span of half-wave dipole radiators and the quality of the isoflux radiation pattern in that as the span increased, the quality of the desired pattern decreased. As mentioned earlier, beyond attempting to meet the design criteria of producing a far-field isoflux radiation pattern, this is the first study, to our knowledge, to investigate the effects of exciting leaky-wave antennas from below with an arbitrary radiator as opposed to the conventional method of excitation, namely a waveguide/couple configuration positioned at one end.

Future directions of this study include experimenting with design parameters of both the homogeneous dielectric structure and the spatial filter overlay. For example, we feel that it would be advantageous to design a type of frequency selective surface (FSS), in place of the simple spatial filter that is discussed in this paper, that would allow for more accurate beam steering will simultaneously being able to deal with an arbitrary span of radiators. Another avenue of study would be to vary the height of both the gratings and the substrate to see what
effects this has on far-field performance. Lastly, due to the superior performance the heterogeneous structure had in terms of required structural length to mitigate truncation effects, it would prove beneficial to experiment with permittivities that are greater than 2.1, but are yet still commercially available and inexpensive.
REFERENCES


