THE REDUCTION OF ROTORCRAFT POWER AND VIBRATION
USING OPTIMALLY CONTROLLED ACTIVE GURNEY FLAP

A Dissertation in
Aerospace Engineering
by
Eui Sung Bae

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The dissertation of Eui Sung Bae was reviewed and approved\textsuperscript{*} by the following:

Farhan Gandhi  
Professor of Aerospace Engineering  
Dissertation Advisor, Chair of Committee

Edward C. Smith  
Professor of Aerospace Engineering

Mark D. Maughmer  
Professor of Aerospace Engineering

Christopher D. Rahn  
Professor of Mechanical Engineering

George A. Lesieutre  
Professor of Aerospace Engineering  
Head of the Department of Aerospace Engineering

\textsuperscript{*}Signatures are on file in the Graduate School.
Abstract

The main topic of the present study is the application of active control scheme for the reduction of rotorcraft main rotor power reduction and vibratory load. When the helicopter is operated near its flight boundary, the required power and vibratory loads rapidly increases which impose a limit on the helicopter operation. Various methods were proposed and studied in order to achieve performance improvement under such operating condition. The effect of active control scheme was examined for its impact on the performance improvement and vibration reduction in the present study.

Numerical simulations are based on the UH-60A Blackhawk helicopter with an active Gurney flap spanning from 70\% R to 80\% R of the main rotor. For obtaining the aeroelastic response of the rotor blade, finite element method was used to represent elastic blade. The aerodynamic loads acting on the blade are provided by CFD based 2D lookup table. Prescribed wake model was used to resolve the induced inflow over the rotor disk. The unsteady aerodynamic behavior due to the higher harmonic actuation of active Gurney flap was resolved by the time-domain unsteady aerodynamic model.

The first part of preliminary study covers parametric study using Gurney flap. Starting with simple rigid blade representation of the rotor blade, the effect of 1/rev Gurney flap actuation was examined on three different gross weights. The effect of active Gurney flap width, the chordwise location of active Gurney flap, the effect of unsteady aerodynamic model, and the effect of 2/rev actuation frequency were examined. The second part of preliminary study was conducted with the elastic blade model to include the effect of torsion dynamics.

Performance improvement using active Gurney flap was examined for maximizing thrust capability at two flight speeds. 1/rev Gurney flap actuation increased the gross weight capability up to 1,000 lbs. Also, 1/rev actuation of Gurney flap increased maximum altitude limit of baseline rotor by 1,400 ft. Furthermore, it
was predicted that the maximum level flight speed can be increased by 30 knots with respect to that of the baseline rotor.

The effect of active Gurney flap on the vibration reduction was first examined at the stall condition. Using 1/rev actuation, in-plane vibratory force and moment can be reduced by 68% and 44%, respectively. The effects of higher harmonic frequencies were investigated at the high-speed cruise speed, and single frequency phase sweep was conducted to find the best phase angle that minimizes each vibratory components. 3/rev actuation yielded 36% reduction in in-plane vibratory moment. 74% reduction in vertical vibratory force was predicted with 4/rev actuation. With 5/rev actuation, 81% reduction in vertical vibratory load was observed. With the input-output information obtained from single frequency phase sweep, the plant model which correlates active control inputs to helicopter vibratory loads was constructed. Multicyclic controller was applied to the plant model, and 25% reduction in the cost function was reported. Vertical vibratory load was reduced by 51%, and inplane force and moment were reduced by 18%, 22%, respectively.
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\(\text{CFD}\) Computational Fluid Dynamics

\(a_\infty\) Speed of sound, \(a_\infty = 1119\ \text{ft/sec}\) for standard air at the sea level

\(a\) Lift curve slope

\(e\) Hinge offset

\(I_b\) Flapping inertia of the rotor blade, defined by \(I_b = \frac{1}{3}m_0R^3\)

\(m_0\) mass of rotor blade per unit length

\(M\) Mach number, defined by \(M = V/a_\infty\), Mass Matrix

\(\alpha\) Angle of attack

\(\alpha_{wd}\) The pitch attitude of fuselage, positive nose down

\(\phi_{wd}\) The roll attitude of fuselage, positive starboard down

\(\gamma\) Lock Number, defined by \(\gamma = \frac{\rho acR^4}{I_b}\)

\(\rho\) Density of air or density of isotropic beam
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Chapter 1

Introduction

1.1 Background

A rotorcraft is one of the major types of vertical take-off and landing (VTOL) aircrafts. Owing to its VTOL capability, a rotorcraft can perform unique missions such as high altitude rescue, fire fighting, and police patrols in the city. However, a rotorcraft has several shortcomings such as poor ride quality due to high vibration levels, short life times of structural components due to fatigue, high levels of noise, limited flight envelopes, and high operating costs. These shortcomings of rotorcraft mostly originate from its lift generation mechanism. The lift generation principle of a rotorcraft is different from that of the conventional fixed wing airplane. Helicopter rotor blades generate lift by rotating blades, while fixed wing airplanes generate lift by the propulsive force from jet engines. Due to the rotation of blades, each radial station of a rotor blade sees different incoming velocity. Thus, aerodynamic forces acting on the blade are proportional to the blade radial distance. Furthermore, combined with the forward flight velocity, velocity distribution around the rotor disk is further complicated. As a result, the aerodynamic environment of a rotorcraft becomes very complex, especially for forward flight condition.

A typical aerodynamic environment for the rotorcraft’s main rotor while flying forward is shown in Figure 1.1. On the advancing side \( (0^\circ < \Psi < 180^\circ) \), the forward flight velocity is added to the blade’s rotating velocity while on the retreating side \( (180^\circ < \Psi < 360^\circ) \), the forward flight velocity is subtracted from the blade’s
Due to the asymmetry of the velocity field in forward flight, the asymmetry of aerodynamic force can affect the vehicle attitude. Even after the cyclic control is applied to achieve roll balance, the resultant aerodynamic environment is still asymmetrical. For example, the compressibility effect is dominant on the advancing side due to high mach number of the tip. However, blade stall may be dominant and a reversed flow region may be observed on the retreating side if forward flight velocity is high. Considering the influence of the wake (Figure 1.2), the aerodynamic environment becomes even more complicated, especially during low-speed forward flight or transitional flight.

1.2 Literature Review

1.2.1 Helicopter Vibration

Structural vibration has been regarded as a parasitic problem of rotary wing aircraft. Interaction of the unique and complex aerodynamic environment with highly flexible structures transmits harmonic vibratory loads to fuselage through rotor hub. Such vibratory loads significantly affect crew/passenger comfort and fatigue,
the lifetime of a structural component, and reduce the crew efficiency in operation. For the UTTAS & AAH development program, less than 0.05g vibration at the blade passage frequency was considered to be a favorable level for helicopter. However, this vibration level requirement was not met even on the production versions of the Black Hawk and Apache, therefore the limit was increased to 0.1g at blade passage frequency for now. In order to achieve so called “jet-smooth ride”, the ultimate goal of reducing vibration levels to be less than 0.03g has been challenged by various groups since the 1960’s.

1.2.2 Passive Control for Helicopter Vibration Reduction

A variety of passive control devices were tested and used in helicopter mainly for vibration reduction. These passive devices can be grouped into major categories: Dynamic vibration absorber and the Vibration isolation mounts. The dynamic vibration absorber consists of a single degree of freedom system with a relatively small mass attached to a spring. The schematic of a dynamic vibration absorber is shown in Figure 1.3. If the natural frequency of the absorber is adjusted to excitation frequency, an opposing oscillation force will be generated which is in resonance with excitation frequency. Dynamic hub absorbers such as simple [1] or bifilar pendulums [2] has been studied, in particular the bifilar pendulum has been applied with success to the S-76 Helicopter. However, actual flight tests were required to determine the final design of these devices, which were very expensive, and therefore not pursued.
Vibration isolation device is another example of widely used passive control system. Pads of rubber or springs are installed between the vibrating systems in order to reduce the transmission of vibrating forces to the support structure. Furthermore, antiresonant principles were adopted for vibration reduction, and were proven to be successful [3–5]. However, in modern helicopters, elastomeric supports are used in place of conventional transmission mounting.

Passive control also includes passive devices designed by optimization techniques [6–10]. Structural and aerodynamic tailoring of rotor blade is a good example. In addition, composite tailoring (Figure 1.4) has also been conducted because composite material provides elastic couplings for potential optimal designs, as well as providing excellent opportunities for developing light weight/high stiffness structures [11–15].

The major benefit of passive control approach is that additional control power is not required for the passive control system. However, passive devices may result in a large increase of weight (up to 1% of gross weight) due to the additional masses
that may be used. Furthermore, complex mechanical parts are usually attached to
the rotating system or hub, which inevitably increase rotorcraft drag. The major
disadvantage of passive system is the lack of adaptability. Passive system is tuned
to or optimized for specific operating conditions, therefore, if there is a change in
flight conditions, rotational frequency, or system dynamics (for example, cargo,
fuel, and passenger), passive systems are likely to show limited performance.

1.2.3 Active Control for Helicopter Vibration Reduction

Comparing the principles of passive control devices and active control devices,
one can find a big difference between them; Passive control device works after
the vibration is generated, whereas active control generates force to counteract
the vibration at the source. Thus, the active control researches are primarily
aimed at the direct change of the excitation force. That is, the attempts were
made to control the structural response of rotor blade by modifying the unsteady
aerodynamic forces acting on rotor blade. Among various methods suggested,
Higher Harmonic Control (HHC) system seemed to be the most feasible solution.
In HHC control, servo-actuators are used to excite conventional swash plate in all
cyclic modes at the frequency of \(N_b/\text{rev}\) (Figure 1.5).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{hhc.png}
\caption{Schematics of Higher Harmonic Control}
\end{figure}

Due to the excitation on the entire cyclic mode, pitching oscillation at the three
frequencies, \((N_b - 1)/\text{rev}, N_b/\text{rev}, (N_b + 1)/\text{rev}\), will be generated in the rotating
frame. As a result of HHC blade pitch motions, additional unsteady aerodynamic
and oscillatory inertial loads will be generated, which are used to alleviate hub vibration. Therefore, the vibration propagating to the fuselage can be reduced at its source. Throughout many research attempts, for example, numerical simulations [16–20], full scale wind tunnel tests [21–27], and full-scale flight tests [28–30], remarkable vibration reduction has been demonstrated with HHC. The pitch input magnitude required for HHC was less than 3°, in general. Even though noticeable reduction in vibration could be achieved using HHC, two major disadvantages still remain:

- high actuation power required to excite the entire rotor blades at $N_b/\text{rev}$
- significant weight penalty associated with the hydraulic actuators to provide higher control authority

Moreover, an additional drawback comes from the fact that only single excitation frequency ($N_b/\text{rev}$) of the swash plate is available for reduction of the $N_b/\text{rev}$ vibration in the fixed frame. However, if HHC control needs to be used for other purposes such as stall alleviation or blade loads reduction, the excitation of entire blades at the other harmonics may be necessary. This is not possible using HHC, because non- $N_b/\text{rev}$ swashplate excitation would result in blade response close to that of dissimilar blades, which would create a large 1/rev in the fixed frame.

Individual Blade Control (IBC) concept was proposed as an extension of HHC control concept to overcome the limitation to single frequency ($N_b/\text{rev}$) swash plate excitation. In this system each blade is excited with actuator in the rotating frame, as compared to the HHC concept where the entire swash plate is actuated in the fixed frame. Therefore, an IBC system has the advantage of varying the blade excitation frequency, which enables these types of systems to be used for various applications such as noise reduction, lag damping augmentation, and stall flutter suppression. One important thing to note is that not only the actuation mechanism for each individual blade but also feedback control system of each blade is considered as an integral part of this concept. A conventional IBC concept, referred to as "Individual Blade Pitch Actuation", has been extensively studied to date. In this concept, hydraulic actuators are installed between pitch link and swash plate (Figure 1.6). The power to drive hydraulic actuator is provided with by slip ring unit.
Initially, the conventional IBC concept was developed for other purposes rather than vibration reduction [31]. An early study by Guinn [32] addressed a practical design consideration of conventional IBC concept to helicopter blade system. Significant research progress on conventional IBC concept was also made by Ham [33, 34]. In this system, hydraulic actuators were mounted on the pitch links to independently control each blade and the control system for the actuators was designed with the feedback signal from on-blade measurements to control the response of the blade first elastic flapwise mode of blade dynamics. Considerable effort has been made towards the practical implementation of IBC systems by the helicopter industry. The three major bodies of work in this area are:

- An IBC system composed of hydraulic actuators was developed and extensively tested both in wind tunnel and in flight on a BO-105 helicopter [35–39]. This 4-bladed hingeless rotor test was conducted for a single frequency, in an open loop configuration. Using a 2/rev blade pitch actuation, significant reduction of vibration and noise levels was achieved.

- Similar blade root actuation system was also installed on the Sikorsky UH-60 [40]. This system was developed under a joint research program by ZF Luftfahrttechnik (ZFL) in Germany, NASA Ames Research Center, and Sikorsky Aircraft Co.

- An experimental IBC system on CH-53G was developed, extensively tested, and certified by ZFL [41–44]. For this 6-bladed articulated rotor, more than
60% of vibration reduction was achieved with an open loop configuration using a $5/\text{rev}$ IBC input. The reduction in required rotor power as well as noise level was also demonstrated using $2/\text{rev}$ IBC inputs.

The IBC technique exhibits the possibilities for multiple objectives like vibration reduction, noise reduction and rotor performance enhancement. However, the conventional IBC using blade pitch actuation requires a heavy and complex hydraulic slip ring, which makes it impractical to implement on production helicopter. Besides, large actuation power is needed to pitch the entire blade with desired high frequencies. Moreover, conventional IBC concept is implemented on the top of the conventional swashplate, the airworthiness of the helicopter needs to be re-evaluated. In order to overcome such drawbacks, recent research effort was made to implement swashplateless IBC using an electro-mechanical actuator [45].

![Schematics of On-blade Trailing Edge Flap Control](image)

**Figure 1.7.** Schematics of On-blade Trailing Edge Flap Control

In addition to the limitations of conventional IBC concept mentioned before, conventional IBC changes only blade root pitch, thus tailoring of unsteady airloads is only possible along with the change in azimuth. Thus, on-blade actuation concepts were studied in conjunction with the use of smart materials as an actuator. Several methods were proposed and studied for on-blade actuation, for example, active trailing edge flaps, active twist, active tips and active tabs. Among these methods, active trailing edge flaps have been explored extensively for vibration reduction. In this method, trailing-edge flaps are deflected to generate additional unsteady aerodynamic loads which result in blade response changes. In general,
trailing-edge flap is applied to the small portion of the blade, usually at the outer board region (Figure 1.7). Therefore much smaller power is required to deflect small trailing edge flap rather than to change root pitch of entire blade (IBC) or to actuate swashplate (HHC). Furthermore, this active flap is separated from the primary control; thus it has little interference to airworthiness of the helicopter.

Servo flaps (moment flap) have been used for primary flight control. For example, use on-blade servo-flaps were adopted for blade pitch control in Kaman helicopters [46, 47]. The effect of servo-flaps on for vibration reduction was also investigated [48]. In this study, the flap deflection under steady and 1/rev frequency were actuated by way of swashplate, while 4/rev control was given in the rotating frame by electro-hydraulic actuators. An amplitude of 6° deflection at 4/rev exhibited significant vibration reduction.

For plain flaps (lift flaps; hereafter active flaps) extensive analytical studies [49–58] have been conducted. Analytic studies showed a comparable vibration reduction with respect to conventional IBC using moderate flap deflection. The experiments of active flaps for vibration were conducted on model-scale, four-bladed rotors. For the experiments at McDonnell Douglas [59–62], the flaps were actuated by mechanical devices (cam and cable linkage), while hydraulic actuators in rotating frame was used in the experiment at Sikorsky [63]. However, the weight and complexity of the actuation system still remains a major technical barrier hindering the implementation of these systems on production helicopters.

![Figure 1.8. Schematics of Active Twist Rotor](image)

Under the same IBC framework, an idea to control blade twist directly us-
ing smart materials was proposed, which is referred to as Active Twist Rotor (ATR) concept (Figure 1.8). The key component of this concept is smart materials, for example, Active Fiber Composite (AFC), Macro-Fiber Composite (MFC) and piezoelectric materials. Chen and Chopra [64, 65] implemented ATR concept with piezoelectric materials by embedding piezoelectric monolithic elements at $+45^\circ$ under the upper skin and $-45^\circ$ under the lower skin of the rotor blade. Blade tip twist amplitudes of 0.25° were achieved, and significant changes in the hub loads were also measured. Another way to induce blade twist is to use Active Fiber Composite (AFC) [66]. With the composite ply cured in a $+45^\circ/ -45^\circ$ orientation on the blade, actuation imposes a linear twist along the blade. Several research teams have tested the active blade twist concept using AFC on model scale rotors [67–71], demonstrated to achieve tip blade twists of up to $\pm1^\circ$.

One of the advantages of ATR concept is the simplicity of actuation mechanism compared to active trailing edge flap. ATR concept also has a benefit in that it does not increase profile drag compared to active trailing-edge flap deflection, which usually produce significant drag increment as a device cost. Even though ATR concept showed promising result for vibration reduction, the power requirement for the on-blade actuators is known to be far greater than that of active trailing edge flap.
1.2.4 Helicopter Main Rotor Power

The drag produced as a result of the lift generated by the main rotor blades can be categorized as either profile drag or induced drag. The sum of both drag components have a moment arm extended from the hub. This in-plane moment produced is counteracted by the torque provided by the engine to maintain the required main rotor angular velocity. This torque which is proportional to the power, changes with advancing ratio for given rotational velocity. Therefore, the flight envelope of helicopter may be limited by the engine torque available. For a given available engine torque, the flight envelope can be further increased or additional payload can be transported if rotorcraft required main rotor torque is reduced.

1.2.5 Passive Control for Main Rotor Power Reduction

Virtually no studies could be found for rotorcraft power reduction using passive control devices. One remarkable approach is multidisciplinary design optimization for rotor blade. In the approach, hover/forward flight power may be adopted as an objected function to minimize with other design variables such as blade radius, chord distribution, twist distribution, and blade natural frequencies.

1.2.6 Active Control for Main Rotor Power Reduction

As was seen in the vibration reduction section, tremendous effort has been devoted to the area of rotor vibration reduction using active control technologies. Similar efforts, but not as extensive as vibration reduction, have also been devoted to examining potential reductions in rotor power requirement through the use of active control.

The first active control technology that was considered for rotor power reduction was HHC, but its effect remains inconclusive. Some studies report negligible reduction [18,21,29,30] or even increase [16,19,22] in power, while a couple of studies have reported power reduction [20,24,72]. In Ref. 24, Shaw et al conducted a wind-tunnel test on a model 3-bladed articulated CH-47D rotor and observed power reductions of 6% at 135 kts and 4% at 160 kts using 2/rev inputs of 2° amplitude. In Ref. 20, Nguyen and Chopra examined power reductions on the
same model CH-47D rotor tested in Ref. 24, using a comprehensive analysis. Up to 3.8% power reduction was reported at high speeds using $2^\circ$, $2/rev$ inputs, but this was accompanied by a large increase in $2/rev$ in-plane blade root shear loads. The power reductions reported in both Ref. 24 and 20 do not account for actuation power requirements.

Jacklin et al examined the effect of $2/rev$ root pitch IBC for reducing the power of a full-scale BO-105 (4-bladed, hingeless) rotor in the 40x80 wind-tunnel at NASA Ames Research Center [36, 37]. At moderate speeds no power reductions were measured, but at high speeds ($\mu = 0.4$) power reductions of 4% with $1^\circ$ IBC amplitude and up to 7% with $2^\circ$ IBC amplitude were reported. However, the authors noted that when the actuation power is considered, the net gains are substantially reduced (to approximately 2.5%). In Ref. [73] Cheng et al. computationally examined the power reductions on a UH-60 type 4-bladed articulated helicopter using $2/rev$ root pitch IBC inputs. For a properly phased $2/rev$ input of amplitude $1^\circ$, power reductions of up to 1.5% for moderate gross weight and up to 3.8% for high gross weight were reported. These results were obtained using
a simplified structural model and a linear inflow model. In Ref. [74], however, the power reductions on the same configuration were significantly reduced (to the point of being virtually eliminated) when a free-wake model was used to represent the inflow around the rotor disk. Recently, the effect of a 2/rev root pitch IBC input on power reduction of a CH-53G was measured in flight tests [43, 44]. An IBC amplitude of 0.67°, with proper phasing, resulted in a measured torque reduction of 2% at 130 kts. However, the authors estimate that the net rotor power reduction after correction for trim would be of the order of 6%.

The effect of using trailing-edge flaps for power reduction on a BO-105 type 4-bladed hingeless rotor helicopter was examined, computationally, by Liu et al [58]. Unlike the previous root-pitch IBC studies which focused on 2/rev inputs, 2−5/rev inputs of the trailing-edge flap were used. At an advance ratio of 0.35 and moderate thrust, power reductions of 1.73% (single flap) to 1.76% (dual flaps) were reported, but the vibrations were increased by 100%. When the objective function considered both power and vibrations, the power reductions obtained were limited to 0.67% with dual flaps. Although slightly larger power reductions were obtained using a nonlinear algorithm, this algorithm could not be implemented in real-time due to the high computational cost. Larger power reductions of up to 1.46% were reported at higher thrust levels, increasing to 1.82% at very high thrust levels. Increasing the advance ratio to 0.4 resulted in possible power reductions of up to 4.04%. The reported flap deflection requirements were generally under 3°. An accurate assessment of the power reductions requires high-fidelity modeling of the trailing-edge flap drag. However, the drag was modeled very simplistically in their study (assumed to vary linearly with the magnitude of the flap deflection, without considering any dependence on the airfoil angle of attack or the Mach number).

A comparison of various active control technologies for rotor performance improvement was undertaken by Yeo[75]. His study considered leading-edge slats, variable droop leading edge, oscillatory jets, root pitch IBC, active twist, trailing edge flaps, and Gurney flaps, on an AH-64 with VR-12 airfoils. The effects of the active control technologies was added as increments in lift, drag and pitching moment to the VR-12 airfoil tables and the performance analysis was conducted using CAMRAD-II. The study showed that using leading-edge slats, variable droop leading edge, oscillatory jets and Gurney flaps, on the retreating side, could in-
crease the maximum blade loading, while $2/rev$ root pitch IBC, trailing-edge flap actuation and active twist moderately increased the rotor lift-to-drag ratio.

When assessing the active control technologies that have been mostly widely considered for rotorcraft applications thus far (HHC, root pitch IBC, active blade twist and active trailing-edge flaps), one of the factors that must be considered is the actuation force and power requirement. It is generally accepted that HHC actuation requirements are higher than those for root pitch IBC. Active blade twist actuation requirements can depend heavily on the rotor torsion frequency vis-à-vis the excitation frequency. One of the most attractive features of an actively controlled trailing-edge flap is the comparatively modest actuation force and power requirement. The flap generates aerodynamic moments to twist the blade, and by putting the aerodynamic forces to work, the energy to twist the blade does not have to be supplied directly by actuators. In contrast, for root pitch IBC the energy required to overcome the aerodynamic and inertial forces to dynamically feather the blade has to be supplied by the actuators, and for active twist rotors the energy to overcome aerodynamic, inertial and structural forces has to be supplied by the actuators. Although the actuation requirements for trailing-edge flaps are lower, the pressure differential on the upper and lower surfaces of the airfoil, manifesting itself as a “hinge moment” still needs to be overcome by the flap actuators. In this regard, a Gurney flap may be even more attractive for rotor active control. Gurney flap is a small flat plate, no greater than 2−3% of the airfoil chord, mounted normal to the lower surface of the airfoil near the trailing edge, (schematic sketch of Gurney flap is shown in Figure 1.9).
1.2.7 Gurney flap/MiTEs

Many researchers have examined and characterized the fundamental 2-D aerodynamic behavior of airfoils with Gurney flaps through wind-tunnel tests [75–84]. The wind-tunnel tests, carried out on a number of different airfoils such as the NACA-0012 [78,81,84], NACA-4412 [76], VR-12 [83], have measured the increase in lift, moment and drag coefficient associated with the Gurney flap deployment. The majority of the tests considered static Gurney flap deployment on a stationary airfoil [75–81], but some recent studies have also examined the behavior of Gurneys flaps on oscillating airfoils [82,83] as well as the behavior of an oscillating Gurney flap on a stationary airfoil [84]. Researchers have also carried out CFD simulations of the 2D behavior of airfoils with Gurney flaps [85–91], and the validation of these results with experimental data has varied from good to moderate. In contrast to the 2D studies listed above, a group of researchers at Stanford University has experimentally examined the use of multiple spanwise-segmented actively controlled Gurney flaps for flutter suppression on a finite wing [92–95].

The use of Gurney flaps on helicopter main rotor blades was first examined by Kentfield [96]. His study showed that by increasing the maximum lift coefficient using a fixed Gurney flap at the trailing edge of the blades, the rotor lift-to-drag ratio can be increased resulting in an improvement of rotor hover as well as cruise efficiency, and up to a 10% increase in the maximum gross weight. In 2003, Maughmer et al [97] first proposed using deployable Gurney flaps (referred to as Miniature Trailing-Edge Effectors or MiTEs in their study) that would be actuated when the base airfoil reached $C_{l_{\max}}$, allowing a reduction in power or an increase in maximum speed of a highly loaded rotor.

In this study, wind-tunnel tests were carried out on a S903 airfoil with static MiTE deflections and the lift and drag increments were measured and used in a simple rotor performance analysis. In Ref. 98, they used the CFD code OVERFLOW-2 to characterize the behavior of an oscillating VR-12 airfoil with a MiTE and showed reasonable comparison with experimental data from Ref. 83. Finally, in Ref. 99, the CFD data was used to synthesize a reduced order model based on the indicial response method for the unsteady behavior of a MiTE, and this was used in a rotor performance code to alleviate stall and showed increases in thrust of up to 10% or increase in maximum speed of up to 20%. While the studies by Kentfield [96] and
Maughmer et al [97–99] represented the Gurney flap behavior within a rotor blade-element framework, Min et al [100] took a different approach and calculated the effects of gurney deployment by directly solving the 3-D Navier-Stokes equations for the rotor.

**Figure 1.11.** 3D CFD grid used in Reference[99]
1.2.8 Summary of Literature Review

Significant efforts were made to reduce rotorcraft vibrations. Among such efforts were passive control schemes such as dynamic vibration absorber, elastomeric support, design optimization, and structural tailoring. Passive control schemes do not need actuation power, but its performance is limited to a specific operating condition. Therefore its performance cannot be guaranteed over off-design conditions.

In order to increase the adaptability of the design, active control schemes also have been extensively explored. HHC and IBC concepts exhibited good effects on vibration reduction, however, actuation power requirement for both methods are much higher for practical implementation on production helicopters. Concepts involving embedding of the smart materials on the blade surface, were also extensively researched, the premier example of this is the ATR concept. It has a benefit of a clean blade surface to minimize additional profile drag, however, significant power is needed to twist entire blade to the desired amount. As a part of the IBC concept, active control flap is extensively explored because of its good performance not only on vibration reduction, but also on power reduction and even noise reduction. However, practical implementation of an actuation mechanism is still a significant design challenge.

Gurney flap is a passive flow control device widely used in racing cars. Compared to its small size, Gurney flap shows a large increase in lift coefficient by simply attaching it to the pressure side of an airfoil. Various research efforts were made to understand the physics of Gurney flaps, but most of such efforts were confined to static aerodynamics, and 2-dimensional analysis. Recently, the research focus has moved to unsteady aerodynamics of Gurney flap/MiTEs and 3-dimensional analysis. Extending the Gurney flap concept to MiTEs (deployable Gurney flap), MiTEs have a potential to overcome the drawbacks of ACF such as high actuation power, actuator design, and hinge moment at neutral position.

There are a few studies of considering Gurney flap/MiTEs for rotorcraft application, and all of the research interest was aimed at performance improvement. In order to implement MiTEs into rotorcraft as vibration alleviation devices, further research on Higher Harmonic controller scheme, the aeroelastic coupling between blade and MiTEs, and the unsteady aerodynamics model for MiTEs are the key components.
1.3 Problem Statement and Objectives

The literature review reveals that the application of the active Gurney flap may be a viable solution for both performance improvement and vibration reduction. Compared to the on-blade trailing-edge flap and root pitch IBC control, the actuation effort is expected to be lower than those active control concepts. However, the increase in the drag coefficient incurred by a Gurney flap may limit the achievable performance improvement. If a Gurney flap can be implemented in an active manner, further performance improvement may be achieved by retracting the device when it is not necessary or when the drag penalty is too high. For vibration control, an active Gurney flap can be used to generate unsteady lift and pitching moment to alter the blade loads in the rotating frame to reduce hub vibratory loads in the fixed frame.

The research goals of present study are described as follows:

1. **Investigate the aerodynamic characteristics of Gurney flap and active Gurney flap**

   - For the preliminary study, the aerodynamic properties of Gurney flap was estimated using the combination of CFD solutions of different airfoil (VR-12 airfoil with 1% GF) and analytic equation \[?\]. The \(\Delta C_l\) from VR-12 with 1% GF was used as a reference point, and the lift increment of different size of Gurney flap was estimated by the analytic equation. However, as the analytic equation covers only the relationship between Gurney flap size, \(h\), and \(\Delta C_l\), thus the approximation model of \(\Delta C_d\) was proposed based on the various experimental data.

   - It was soon turned out that the revision of aerodynamic properties with high lift device be based on the right airfoil. The aerodynamic models used in the proceeding studies were not based on the correct aerodynamic properties. Therefore, *systematic efforts were made to revise the aerodynamic database by using CFD technology*. Various sizes of the Gurney flap were modeled at the trailing edge of SC-1094/R8 airfoil. The CFD solutions were obtained using 2D Navier-Stokes solver TURNS code developed in University of Maryland.
Following this, the CFD database were generated for other high-lift devices such as TEF (Trailing-edge Flap) and TEP (Trailing-edge Plate). In present study, the aerodynamic properties of Gurney flap were compared to other high-lift devices. Finally, the Gurney flap located at the upstream of the trailing edge were considered into the CFD database. In order to realize the active Gurney flap concept by imbedding the Gurney flap inside the airfoil when the device is not in use, the moving upstream is inevitable. 10\%c upstream from the trailing edge of SC-1094R8 airfoil CFD results was updated to the database.

The unsteady lift model of reference [101] was implemented. The unsteady drag and unsteady pitching moment expressions were proposed. All unsteady aerodynamic expressions were fully integrated into the rotorcraft comprehensive analysis developed in the chapter 2.

2. Investigate the effect of the active Gurney flap on the main rotor performance improvement

- For this objective, the initial goal was to develop the aeroelastic analysis which adopts spanwise-segmented aerodynamic effectors. For the preliminary study, a simplified helicopter model with rigid blade flapping and rigid wake model. However, the aerodynamic effect of Gurney flap was based on several limiting assumptions. Optimization problem was formulated to find the control inputs for spanwise-segmented Gurney flaps so that the main rotor power is minimized while satisfying the trim.
- The aerodynamic models used in the preceding study [102] were not based on the correct aerodynamic properties. The aerodynamic properties of Gurney flap based on the CFD runs were incorporated into the analysis. Following this, the revision of the helicopter model was also made to better represent the UH-60 blackhawk helicopter by including nonlinear twist, airfoil section, and horizontal tail. The same optimization formulation was adopted to determine the control inputs of spanwise-segmented Gurney flaps [103].
- Final revisions were made including Fully coupled flap-lag-torsion rotating beam model [104], full fuselage aerodynamic properties of UH-60,
and unsteady aerodynamic model for upstream active Gurney flap. The open loop phase sweep under 1/rev and 2/rev frequencies were conducted at the edge of flight envelope of baseline helicopter.

3. Investigate the effect of the active Gurney flap on the main rotor vibration reduction

- When the helicopter is near the flight envelop or stall boundary, the vibratory loads are very high. With the aid of active Gurney flap actuated at 1/rev, the vibration reduction though stall alleviation will be examined.

- Similar to the performance improvement study, the open loop phase sweep will be conducted to identify the best phase angle. The closed loop multi-cyclic control scheme will be configured for vibration minimization.
1.4 Overview of Dissertation

This thesis consists of seven chapters, which are organized as follows:

1. The first chapter reviews the literatures on performance improvement and vibration reduction methodology.

2. The second chapter provides the detailed derivation on each component of comprehensive analysis model. The analytic model includes the fully coupled flap-lag-torsion elastic beam model, nonlinear aerodynamic model, and rotor inflow model.

3. The aerodynamic model for Gurney flap is discussed in detail in the third chapter. The changes in aerodynamic characteristics by Gurney flap are obtained with TURNS CFD solver, and presented in this chapter. Steady aerodynamic characteristics of Gurnet flap installed at the trailing-edge of an airfoil as well as at the 10%c upstream from the trailing-edge were presented in detail. Finally, the unsteady aerodynamic model is discussed. Unsteady lift is based on the reference [101], and the expressions for unsteady drag and pitching moment are presented.

4. In the chapter 4, the comprehensive analysis code developed in the chapter 2 was validated against UH-60A flight test data. Preliminary study using rigid blade model is conducted. The stall boundaries of different gross weights were identified, and 1/rev actuation of active Gurney flap was applied to alleviate stall. Short parametric studies including varying the width of active Gurney flap, changing the chordwise position of active Gurney flap were conducted. The influence of unsteady aerodynamic model and additional study for 2/rev actuation were conducted. The second part of the chapter 4 was conducted with elastic blade in order to examine the influence of torsion dynamics included in the analysis.

5. Performance improvement using single active Gurney flap was examined in the fifth chapter. Gross weight sweep, velocity sweep, and altitude sweep were performed to identify the flight envelope. Then, the active Gurney flap was used to extend the flight envelope. Actuation frequencies of 1/rev
and 2/rev were applied. The physics of power reduction mechanism will be examined in detail.

6. The effect of active Gurney flap was examined for vibration reduction in the chapter 6. The effect of active Gurney flap was examined for reducing vibratory loads at stalled condition where the excessive vibratory loads presents. With the application of 1/rev Gurney flap for configured for stall alleviation, the simultaneous alleviation of stall and vibration was examined. Vibration reduction through higher harmonic frequencies was studied. Actuation frequencies of 3, 4, and 5/rev were considered for this purpose. Actuation phase angle $\phi$ was varied to conduct off-line identification of T-matrix. With the identified T-matrix, closed-loop Higher Harmonic Controller was used to multi-cyclic control.

7. The last chapter covers the summary of the present study and future work will be presented.
Chapter 2

Analytic Model

The analysis of helicopter is multi-disciplinary in its nature - the complex interactions among aerodynamics, structural dynamics and control are present. Therefore the framework for rotorcraft analysis requires each discipline should have proper modelling fidelity as well as be coupled in “comprehensive” manner. The major sources of the formulations are from UMARC Theory Manual [104] and various Ph.D. works [105–107]. Instead of just following what has been done in previous references, extensive review on the comprehensive analysis methodology was conducted, and the re-design of the overall analysis procedure should be noted.

This chapter provides descriptions of a comprehensive analysis model of helicopter system with single rotor. The first section deals with the background of elastic blade model, followed by the description of the analytic model of the aerodynamic model. The inflow model and trim methodology will be discussed subsequently. The chapter will be closed with the formulation of the control algorithm to reduce the main rotor power and hub vibratory load.
2.1 Structural Model

2.1.1 Blade Undeformed frame

The coordinate system to describe the motion of the blade consists of a series of coordinate transformation. The coordinate transformation from the inertial frame (superscript I) to the rotor hub frame (superscript H) can be defined as follows.

\[
\begin{bmatrix}
I^H \\
J^H \\
K^H
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & \alpha_s \\
0 & 1 & -\phi_s \\
-\alpha_s & \phi_s & 1
\end{bmatrix}
\begin{bmatrix}
I^I \\
J^I \\
K^I
\end{bmatrix}
\] (2.1)

The coordinate transformation from the rotor hub frame (superscript H) to the rotating blade frame (superscript R) can be defined as follows.

\[
\begin{bmatrix}
I^R \\
J^R \\
K^R
\end{bmatrix} =
\begin{bmatrix}
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
I^H \\
J^H \\
K^H
\end{bmatrix}
\] (2.2)

For the blade with no precone angle ($\beta$), undeformed frame is same as rotating blade frame.

\[
\begin{bmatrix}
i \\
j \\
k
\end{bmatrix} =
\begin{bmatrix}
I^R \\
J^R \\
K^R
\end{bmatrix}
\] (2.3)

2.1.2 Blade Deformed frame

For accurate modelling of the behavior of elastic blade, the ability to represent moderately large deflection is essential. In present model, the strains are still assumed to be small with moderately large deflection. Under this assumption, nonlinear strain-displacement relationship can be established on the deformed configuration. As Bernoulli-Euler beam model preserves cross sections and maintains orthogonality of cross section to the elastic axis, the cross section geometry and section properties are still available even in the deformed condition. The strain-stress expression can be derived by satisfying the equilibrium under deformed configuration.

The deformed blade can be described in the deformed coordinate system. The
comparison between the undeformed and deformed coordinated systems is shown in Figure 2.1. A point P on the undeformed elastic axis undergoes deflections in \( u, v, \) and \( w \) in the undeformed frame and moves to a point \( P' \). The cross section that contains \( P' \) rotates \( \theta_1 \) about the deformed elastic axis. Figure 2.2 depicts the kinematics of the cross section that contains \( P' \).

Figure 2.2. Deformed cross-section

The total blade pitch \( \theta_1 \) can be defined as
\[ \theta_1 = \theta_0 + \hat{\phi} \]
\[ \theta_0 = \theta_{75} + \theta_{tw}(x) + \theta_1 c \cos \psi + \theta_1 s \sin \psi \quad (2.4) \]

\( \theta_0 \) refers to the rigid pitch input from swash plate and includes time-varying 1/rev input variation as well as pretwist \( \theta_{tw}(x) \). Note that \( \theta_{tw}(x) \) is a function that satisfies \( \theta_{tw}(0.75R) = 0 \). The elastic twist \( \hat{\phi} \) is defined as

\[ \hat{\phi} = \phi - \int_0^x \frac{\partial w}{\partial x} \frac{\partial^2 v}{\partial x^2} dx \quad (2.5) \]

The \( \phi \) can be interpreted as elastic twist about undeformed elastic axis while \( \hat{\phi} \) can be regarded as the elastic twist about the deformed elastic axis. This nonlinear kinematic effect is attributed to moderate deflection assumption.

The coordinate transformation from undeformed frame to deformed frame is given by

\[
\begin{align*}
\begin{bmatrix}
i \\
j \\
k
\end{bmatrix} & =
\begin{bmatrix}
i \\
j \\
k
\end{bmatrix} =
\begin{bmatrix}
i \\
j \\
k
\end{bmatrix}
\end{align*}
\]

The blade deformation can be determined by a sequence of Euler angles as shown in Figure 2.3. Successive rotations around of \( \bar{\zeta} \) around \( k \), \( \bar{\beta} \) around \( -j \), and \( \bar{\theta} \) around \( i \) will align unit vector of undeformed frame with that of deformed frame.

![Figure 2.3. Deformation and Euler angle](image-url)
Therefore, transformation matrix $T_{DU}$ is defined as

\[
T_{DU} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \bar{\theta} & \sin \bar{\theta} \\
0 & -\sin \bar{\theta} & \cos \bar{\theta}
\end{bmatrix} \begin{bmatrix}
\cos \bar{\beta} & 0 & -\sin \bar{\beta} \\
0 & 1 & 0 \\
\sin \bar{\beta} & 0 & \cos \bar{\beta}
\end{bmatrix} \begin{bmatrix}
\cos \bar{\zeta} & \sin \bar{\zeta} & 0 \\
-\sin \bar{\zeta} & \cos \bar{\zeta} & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (2.7)

The Euler angles appear in $T_{DU}$ can be approximated in terms of blade deformation as shown below.

\[
\cos \bar{\zeta} = \frac{\sqrt{1-v'^2-w'^2}}{\sqrt{1-w'^2}} \quad \sin \bar{\zeta} = \frac{v'}{\sqrt{1-w'^2}} \\
\cos \bar{\beta} = \sqrt{1-w'^2} \quad \sin \bar{\beta} = w' \\
\bar{\theta} = \theta_1
\] (2.8)

Substituting equation 2.8 into 2.7 and discarding higher order terms finally yields the transformation matrix $T_{DU}$ as

\[
T_{DU} = \begin{bmatrix}
1 - \frac{v'^2}{2} - \frac{w'^2}{2} \\
-v' \cos \theta_1 - w' \sin \theta_1 \\
v' \sin \theta_1 - w' \cos \theta_1
\end{bmatrix} \begin{bmatrix}
v' \\
(1 - \frac{v'^2}{2}) \cos \theta_1 - v' w' \sin \theta_1 \\
(1 - \frac{w'^2}{2}) \sin \theta_1 - v' w' \cos \theta_1
\end{bmatrix} \begin{bmatrix}
w' \\
(1 - \frac{w'^2}{2}) \sin \theta_1 \\
(1 - \frac{w'^2}{2}) \cos \theta_1
\end{bmatrix}
\] (2.9)

Note that $T_{DU}^{-1} = T_{DU}^T$ due to the orthogonality.

### 2.1.3 Nondimensionalization and Ordering Scheme

In general, the final expressions of complete nonlinear equations are subjected to scaling problem. For the remedy of this issue, nondimensionalization based on representative physical properties of system greatly improves the overall balance of the whole equation system. Furthermore, obtaining the solution using nondimensionalized equations is generally better than working with dimensionalized equations. The physical quantities used for nondimensionalization is listed in Table 2.1.

After performing nondimensionalization, it is important to assess the relative order between nondimensionalized quantities so that the higher order terms can be ruled out from the analysis for simplicity. In present analysis, the terms up to second orders $\epsilon^2$ are retained where $\epsilon$ denotes reference order of magnitude. Note that some third order terms are conserved in elastic torsion equation. The order of
Table 2.1. Parameters for nondimensionalization

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Reference Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>$R$</td>
</tr>
<tr>
<td>Time</td>
<td>$1/\Omega_{ref}$</td>
</tr>
<tr>
<td>Mass/Length</td>
<td>$m_0$</td>
</tr>
<tr>
<td>Velocity</td>
<td>$\omega R$</td>
</tr>
<tr>
<td>Acceleration</td>
<td>$\omega^2 R$</td>
</tr>
<tr>
<td>Force</td>
<td>$m_0\omega^2 R^2$</td>
</tr>
<tr>
<td>Moment</td>
<td>$m_0\omega^2 R^3$</td>
</tr>
<tr>
<td>Energy or Work</td>
<td>$m_0\omega^2 R^3$</td>
</tr>
</tbody>
</table>

Table 2.2. Order of magnitude of terms

<table>
<thead>
<tr>
<th>Terms List</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EA$</td>
<td>$O(\epsilon^{-2})$</td>
</tr>
<tr>
<td>$m_0\omega_{ref}^2 R^2$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$x$ $R$ $\bar{r}$, $\bar{r}<em>{CG}$, $\bar{m}</em>{CG}$, $m_0$, $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial \omega}$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$\mu$, $\cos\psi$, $\sin\psi$, $\theta_0$, $\theta_{tw}$, $\theta_{75}$, $\theta_{1c}$, $\theta_{1s}$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$EI_x$, $EI_y$, $EI_z$, $GJ$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$\frac{m_0\omega_{ref} R^4}{m_0\omega_{ref} R^4}$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$\frac{m_0\omega_{ref} R^4}{m_0\omega_{ref} R^4}$, $\frac{m_0\omega_{ref} R^4}{m_0\omega_{ref} R^4}$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$\alpha_s$, $\alpha_s$, $\lambda$, $\eta$, $\eta$, $\eta_T$, $\eta_T$,</td>
<td>$O(\epsilon)$</td>
</tr>
<tr>
<td>$\frac{m_0\omega_{ref} R^5}{m_0\omega_{ref} R^5}$, $\frac{m_0\omega_{ref} R^5}{m_0\omega_{ref} R^5}$</td>
<td>$O(\epsilon)$</td>
</tr>
<tr>
<td>$\frac{m_0\omega_{ref} R^6}{m_0\omega_{ref} R^6}$, $\frac{m_0\omega_{ref} R^6}{m_0\omega_{ref} R^6}$</td>
<td>$O(\epsilon^2)$</td>
</tr>
<tr>
<td>$\frac{m_0\omega_{ref} R^6}{m_0\omega_{ref} R^6}$, $\frac{m_0\omega_{ref} R^6}{m_0\omega_{ref} R^6}$</td>
<td>$O(\epsilon^2)$</td>
</tr>
</tbody>
</table>

The magnitude of nondimensionalized quantities used in present structural model are presented in Table 2.2.

Note that $m_0$ in Table 2.2 refers to the reference blade mass per unit length, defined by matching flapping inertia of actual blade, $I_\beta$, to that of the equivalent uniform blade.

$$m_0 = \frac{3I_\beta}{R^3} = \frac{3}{R^3} \int_0^R mr^2 dr$$  \hspace{1cm} (2.10)

Constant RPM was assumed throughout the present study, thus time “t” can be converted into azimuth angle “$\psi$” using reference rotational frequency, $\Omega_{ref}$. Time derivatives can be also converted in terms of azimuth angle as follows...
\[
\frac{d}{dt} = d\psi \frac{d}{d\psi} = \Omega_{\text{ref}} \frac{d}{d\psi} \\
\frac{d^2}{dt^2} = \frac{d^2\psi}{d\psi^2} \frac{d^2}{d\psi^2} = \Omega_{\text{ref}}^2 \frac{d^2}{d\psi^2}
\] (2.11)

2.1.4 Strain Energy of Rotating Beam

For elastic structure, the strain energy is stored when it undergoes deformation. Assuming the rotor blade as a long slender isotropic beam, uniaxial stress assumption \((\sigma_{yy} = \sigma_{yz} = \sigma_{zz} = 0)\) is considered to be valid. The relationship between stress and the classical engineering strains is shown below

\[
\sigma_{xx} = E\epsilon_{xx} \\
\sigma_{xn} = G\epsilon_{xn} \\
\sigma_{xz} = G\epsilon_{xz}
\] (2.12)

where \(\epsilon_{xx}\) is axial strain, and \(\epsilon_{xn}\) and \(\epsilon_{xz}\) are engineering shear strains. The strain energy expression of a reference blade can be formulated as

\[
U_B = \frac{1}{2} \int_0^R \left( \iint_A (\sigma_{xx}\epsilon_{xx} + \sigma_{xn}\epsilon_{xn} + \sigma_{xz}\epsilon_{xz}) d\eta d\zeta \right) dr
\] (2.13)

The nonlinearities caused by moderate beam deflections are reflected in these strain expressions. Note that additional contribution from rigid pitch control \(\theta_0\) is included in the strain expression.

\[
\epsilon_{xx} = u' + \frac{w'^2}{2} + \frac{\lambda_T\phi''}{2} - \lambda_T(\eta^2 + \zeta^2)(\theta_0'\phi' + \frac{\phi'^2}{2}) - v''[\eta\cos(\theta_0 + \dot{\phi}) - \zeta\sin(\theta_0 + \dot{\phi})] - w''[\eta\sin(\theta_0 + \dot{\phi}) + \zeta\cos(\theta_0 + \dot{\phi})]
\]
\[
\epsilon_{xn} = -(\zeta + \frac{\partial\lambda_T}{\partial\eta})\phi' \\
\epsilon_{xz} = -(\eta + \frac{\partial\lambda_T}{\partial\zeta})\phi'
\] (2.14)

From the relationship between elastic torsion in undeformed frame and deformed frame, the variable \(\phi'\) can be described in terms of \(\dot{\phi}'\). Differentiating equation 2.5 yield following expression

\[
\dot{\phi}' = \phi' - w'v''
\] (2.15)
The axial deflection, \( u \), can be decomposed into elastic axial deflection, \( u_e \), and kinematic foreshortening in axial direction, \( u_F \). Using this decomposition proposed in Reference [], the following relations can be used into strain energy expression.

\[
\begin{align*}
    u &= u_e + u_F = u_e - \frac{1}{2} \int_0^x (v'^2 + w'^2) \, dx \\
    u' &= u'_e - \frac{1}{2} (v'^2 + w'^2) \, dx 
\end{align*}
\]  

### 2.1.5 Kinetic Energy of Rotating Beam

When the blade is moving with a velocity, the kinetic energy is stored on the mass of the blade. Relative motion of the blade with respect to the hub defines the velocity for kinetic energy expression shown in equation 2.17. Note that no motion of the hub itself is assumed in the present analysis.

\[
T_B = \frac{1}{2} \int_0^R \left( \int_A \rho \vec{V}_b \cdot \vec{V}_b \, d\eta d\zeta \right) \, dr 
\]  

A generic point \( P(x, 0, 0) \) on the undeformed elastic axis moves to \( P'(x+u, v, w) \) after deformation, and the cross section that contains \( P' \) rotates \( \theta_1 \) about the deformed elastic axis. Denoting the position vector of a point on the blade after the deformation as \( \vec{r}' \), its coordinates in the undeformed frame is given as

\[
\vec{r}' = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}
\]  

where

\[
\begin{align*}
    x_1 &= x + u - \lambda_T \phi' - v'(y_1 - v) - w'(z_1 - w) \\
    y_1 &= v + (y_1 - v) \\
    z_1 &= w + (z_1 - w) 
\end{align*}
\]  

The terms \( (y_1 - v) \) and \( (z_1 - w) \) can be interpreted as the position of a generic point on the deformed cross section with respect to \( P' \) as shown below.

\[
\begin{align*}
    (y_1 - v) &= \eta \cos(\theta_0 + \hat{\phi}) - \zeta \sin(\theta_0 + \hat{\phi}) = \eta \cos \theta_1 - \zeta \sin \theta_1 \\
    (z_1 - w) &= \eta \sin(\theta_0 + \hat{\phi}) + \zeta \cos(\theta_0 + \hat{\phi}) = \eta \sin \theta_1 + \zeta \cos \theta_1 
\end{align*}
\]
By differentiating the position vector with respect to the fixed hub coordinate system, the velocity of a generic point in rotating frame is given by

\[
\vec{V}_b = \dot{\vec{r}} + \vec{\omega} \times \vec{r} = (\dot{x}_1 - \Omega y_1)i + (y_1 + \Omega x_1)j + \dot{z}_1k
\] (2.21)

Using chain rule, the first time derivatives of \(x_1, y_1,\) and \(z_1\) are obtained as

\[
\begin{align*}
\dot{x}_1 &= \dot{u} - \lambda_T \dot{\phi} - (\dot{v}' + w' \dot{\theta}_1)(y_1 - v) \\
&\quad - (\dot{w}' - v' \dot{\theta}_1)(z_1 - w) \\
\dot{y}_1 &= \dot{v} - \dot{\theta}_1(y_1 - v) \\
\dot{z}_1 &= \dot{w} + \dot{\theta}_1(z_1 - w)
\end{align*}
\] (2.22)

Furthermore, the time derivative of \(u, \dot{u},\) can be obtained by differentiating \(u\) shown in equation 2.16.

\[
\dot{u} = \dot{u}_c - \int_0^x (\dot{v}'v' + \dot{w}w')dx
\] (2.23)

### 2.1.6 Governing Equation of Rotating Beam

The equations of motion of rotating beam is derived by applying Hamilton’s variational principle. Hamilton’s principle states that \textit{true path of the conservative system is determined at a given time interval from \(t_1\) to \(t_2\) such that the time integral of the difference between potential and kinetic energy is a minimum}.

For nonconservative system, the generalized Hamilton’s principle can be presented as

\[
\delta \Pi = \int_{t_1}^{t_2} (\delta U - \delta T - \delta W)dt = 0
\] (2.24)

where \(\delta U\) and \(\delta T\) are the variation of strain energy and kinetic energy, respectively. The \(\delta W\) accounts for the virtual work done by the aerodynamic forces.

The variation of strain energy of the rotor blade, \(\delta U_B\), can be obtained by applying \textit{“\(\delta\) operator”} on equation 2.13.

\[
\delta U_B = \int_0^R \left( \int_A \sigma_{xx} \delta \epsilon_{xx} + \sigma_{xy} \delta \epsilon_{x\eta} + \sigma_{xz} \delta \epsilon_{x\zeta} \ d\eta d\zeta \right) \ dr
\] (2.25)
Substituting the stress-strain relationship shown in equation 2.12 will finally yield

\[
\delta U_B = \int_0^R \left( \int_A E \varepsilon_{xx} \delta \varepsilon_{xx} + G \varepsilon_{xy} \delta \varepsilon_{xy} + G \varepsilon_{xz} \delta \varepsilon_{xz} \, d\eta d\zeta \right) \, dr \quad (2.26)
\]

The variations of strain are obtained as follows

\[
\begin{align*}
\delta \varepsilon_{xx} &= \delta u' + v' \delta v' + w' \delta w' - \lambda_f \delta \phi'' + (\eta^2 + \zeta^2)(\theta_0' + \phi') \delta \phi' \\
&\quad - [\eta \cos(\theta_0 + \hat{\phi}) - \zeta \sin(\theta_0 + \hat{\phi})] (\delta v'' + w'' \delta \hat{\phi}) \\
&\quad - [\eta \sin(\theta_0 + \hat{\phi}) + \zeta \cos(\theta_0 + \hat{\phi})] (\delta w'' - v'' \delta \hat{\phi}) \\
\delta \varepsilon_{xy} &= - (\zeta + \frac{\partial \lambda_f}{\partial \eta}) \delta \phi' \\
\delta \varepsilon_{xz} &= - (\eta + \frac{\partial \lambda_f}{\partial \zeta}) \delta \phi'
\end{align*}
\]

(2.27)

In addition, the variations of \( \phi' \) and \( u' \) can be given by

\[
\begin{align*}
\delta \phi' &= \delta \hat{\phi}' + v'' \delta w' + w' \delta v'' \\
\delta u' &= \delta u' - v'' \delta v' + w' \delta w'
\end{align*}
\]

(2.28)

Substituting equations 2.27 and 2.28 into the variation of the strain energy expression, and applying ordering scheme discussed in section 2.1.3, the resulting expression for \( \delta U_B \) is

\[
\frac{\delta U_B}{m_0 \Omega_{ref}^2 R^3} = \int_0^1 \left( U_{u'} \delta u' + U_{v'} \delta v' + U_{w'} \delta w' + U_{v''} \delta v'' + U_{w''} \delta w'' + U_{\hat{\phi}} \delta \hat{\phi} + U_{\hat{\phi}'} \delta \hat{\phi}' + U_{\hat{\phi}''} \delta \hat{\phi}'' \right) dx
\]

(2.29)

Each component in the variation of strain energy is listed in Appendix A.

Similarly for the variation of strain energy, the variation of kinetic energy, \( \delta T_B \), is obtained with the application of variational operator on equation 2.17

\[
\delta T_B = \int_0^R \left( \int_A \rho \vec{V}_b \cdot \delta \vec{V}_b \, d\eta d\zeta \right) \, dr
\]

(2.30)

In order to convert the variation of velocity into the variation of displacement, the integration by parts were conducted for time. Then, the variation of kinetic
energy can be formulated as

\[
\frac{\delta T_B}{m_0 \Omega^2_{ref} R^3} = \int_0^1 \left( \int_A T_{x_1} \delta x_1 + T_{x_2} \delta x_2 + T_{x_3} \delta x_3 \, d\eta d\zeta \right) \, dx
\]  

(2.31)

where

\[
T_{x_1} = -\ddot{x}_1 + 2\dot{y}_1 + x_1
\]

\[
T_{y_1} = -\ddot{y}_1 - 2\dot{x}_1 + y_1
\]

\[
T_{z_1} = -\ddot{z}_1
\]

(2.32)

Note that the acceleration terms begin to appear, and are obtained by differentiating equation 2.22 once more. The acceleration terms will be

\[
\ddot{x}_1 = \ddot{u} - \lambda_T\ddot{\phi} - (\dddot{v} + w'\ddot{\theta}_1 + 2\dot{\theta}_1 \ddot{v}')(y_1 - v)
\]

\[
- (\dddot{w} - v'\ddot{\theta}_1 - 2\dot{\theta}_1 \ddot{v}')(z_1 - w)
\]

(2.33)

Also for the variation of displacement terms will be

\[
\delta x_1 = \delta u - \lambda_T \delta \phi' - \delta v'(\eta \cos \theta_1 - \zeta \cos \theta_1) - \delta w'(\eta \sin \theta_1 + \zeta \cos \theta_1)
\]

\[
- \delta \phi' (\eta \cos \theta_1 - \zeta \sin \theta_1) - \delta w'(\eta \sin \theta_1 + \zeta \cos \theta_1)
\]

\[
\delta y_1 = \delta v + \delta \phi'(\eta \cos \theta_1 - \zeta \sin \theta_1)
\]

\[
\delta z_1 = \delta w + \delta \phi'(\eta \sin \theta_1 + \zeta \cos \theta_1)
\]

(2.34)

Substituting velocity terms (equation 2.22), acceleration terms (equation 2.33), and variation of displacement (equation 2.34) into equation 2.31 and applying ordering scheme, the final expression will be obtained as

\[
\frac{\delta T_B}{m_0 \Omega^2_{ref} R^3} = \int_0^1 m(T_{u_1} \delta u_v + T_{v_1} \delta v + T_{w_1} \delta w + T_{v'1} \delta v' + T_{w'1} \delta w' + T_{\phi'1} \delta \phi + T_F) \, dx
\]  

(2.35)

Detailed expressions for the components in the variation of kinetic energy can be found in Appendix A.

The virtual work by nonconservative forces, \(\delta W\), is given by
\[
\frac{\delta W_B}{m_0 \Omega_{ref}^2 R^3} = \int_0^1 \left( F_u \delta u + F_v \delta v + F_w \delta w + M_{\hat{\phi}} \delta \hat{\phi} \right) dx \quad (2.36)
\]

\( F_u, F_v, F_w \) and \( M_{\hat{\phi}} \) represent the distributed loads in the undeformed frame. For the rotating beam, the distributed loads are primarily from aerodynamic forces, but not limited to - the forces from dampers can also be included. The formulation of aerodynamic forces will be discussed in detail in section 2.2.

For the sake of brevity, equations of motions in axial, lag, flap, and torsion are expressed in terms of the functionals derived from variation of strain energy (equation 2.29) and kinetic energy (equation 2.35). The final expressions of coupled flap-lag-torsion equation are presented as follows.

\[
\begin{align*}
-(U_{w'})' - T_{w} & = F_u \\
(U_{v'})'' - mT_v + m(T_{v'})' - m(F_A(x)v' + G_A(x)v')' & = F_v \\
(U_{w'})'' - (U_{w'})' - mT_w + m(T_{w'})' - m(F_A(x)w' + G_A(x)w')' & = F_w \\
(U_{\hat{\phi}}'' - (U_{\hat{\phi}}')' + U_{\hat{\phi}} - mT_{\hat{\phi}}) & = M_{\hat{\phi}}
\end{align*}
\]  

(2.37)

where \( F_A(x) \) and \( G_A(x) \) represents the effect of centrifugal force and Coriolis damping, respectively.

### 2.1.7 Discretization of Governing Equation

For the coupled set of equations of motions (equation 2.37) it is hard to obtain the solution by analytic method. Thus it is first necessary to reduce the original partial differential equation into coupled sets of ordinary differential equation. By assuming the solution of original PDE, \( w(x,t) \) as

\[
w(x,t) = F(x)G(t) \quad (2.38)
\]

then it is possible to decouple the PDE into ODEs of “spacial” variables and ODEs of “temporal” variables. Similarly, the degree of freedom of interest can be approximated using the spacial shape functions \( H(x) \) and generalized coordinate \( q(t) \) as shown below.
\[ w(x, t) = \sum_i H_i(x) q_i(t) = \vec{H}^T \vec{q} \quad (2.39) \]

With the application of this approximation, the spacial derivatives in the governing equations can be resolved with the differentials of spacial shape functions. Thus, the equation of motions eventually turn into the sets of ODEs in time.

As an example, the procedure to derive equation of motion using the approximation in equation 2.39 is presented. Starting from the the variational expression using the Hamilton’s principle, we have

\[ \delta \Pi = \int_0^{2\pi} \left( \int_0^1 U w \delta w - T_w \delta \dot{w} - L_w \delta w \, dx \right) d\psi = 0 \quad (2.40) \]

Without losing the generality of the procedure, functionals can be assumed to be linear.

\[
\begin{align*}
U_w &= A_w w \\
T_w &= B_w \dot{w} \\
L_w &= C_w w 
\end{align*} \quad (2.41)
\]

Substituting the equation 2.39 into equation 2.40 then we get

\[
\delta \Pi = \int_0^{2\pi} \left( \int_0^1 A_w w \delta w + B_w \ddot{w} \delta w - C_w w \delta w \, dx \right) d\psi = 0
\]

\[ \int_0^{2\pi} \delta \vec{q}^T \left[ \left( \int_0^1 \vec{H} A_w \vec{H}^T \, dx \right) \vec{q} + \left( \int_0^1 \vec{H} B_w \vec{H}^T \, dx \right) \ddot{\vec{q}} - \left( \int_0^1 \vec{H} C_w \vec{H}^T \, dx \right) \vec{q} \right] d\psi = 0 \quad (2.42) \]

In order to satisfy \( \delta \Pi = 0 \) with respect to arbitrary variation in \( \delta \vec{q}^T \), the terms inside of the bracket in equation 2.42 should be zero. Finally for the equation of motion, we have

\[
\left( \int_0^1 \vec{H} A_w \vec{H}^T \, dx \right) \vec{q} + \left( \int_0^1 \vec{H} B_w \vec{H}^T \, dx \right) \ddot{\vec{q}} - \left( \int_0^1 \vec{H} C_w \vec{H}^T \, dx \right) \vec{q} = 0 \quad (2.43)
\]

Note that the governing equation becomes a second order ODE in time, and all of the spacial terms are condensed into integration terms.
The deflection of the beam can be described with 15 nodal degrees of freedom per element and their corresponding spatial shape functions. The nodal degree of freedom is given by

\[ q = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & v_1 & v_2 & v_1' & v_2' & w_1 & w_2 & w_1' & w_2' & \phi_1 & \phi_2 & \phi_2' \end{bmatrix}^T \] (2.44)

The axial degree of freedom, \( u_e \), can be expressed

\[ u_e = \sum_{i=1}^{4} H_{u_i} u_i = H_u^T \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \] (2.45)

where the shape functions \( H_{u_i} \) are

\[
H_u = \begin{bmatrix}
H_{u_1} \\
H_{u_2} \\
H_{u_3} \\
H_{u_4}
\end{bmatrix} = \begin{bmatrix}
-4.5s^3 + 9s^2 - 5.5s + 1 \\
13.5s^3 - 22.5s^2 + 9s \\
-13.5s^3 + 18s^2 - 4.5s \\
4.5s^3 - 4.5s^2 + s
\end{bmatrix}
\] (2.46)

The flap and lag degree of freedom can be represented with nodal degree of freedom and shape functions as shown below

\[
v = H_1 v_1 + H_2 v_1' + H_3 v_2 + H_4 v_2' = H^T \begin{bmatrix} v_1 \\ v_1' \\ v_2 \\ v_2' \end{bmatrix}
\] (2.47)

\[
w = H_1 w_1 + H_2 w_1' + H_3 w_2 + H_4 w_2' = H^T \begin{bmatrix} w_1 \\ w_1' \\ w_2 \\ w_2' \end{bmatrix}
\]

where \( H \) represents the shape functions for flap and lag bending displacements.
Note that the continuity of the slope is also imposed between elements for flap and lag bending displacement. The torsion degree of freedom can be approximated like

\[
\hat{\phi} = \sum_{i=1}^{3} H_{\phi_i} \hat{\phi}_i = H_{\phi}^T \begin{pmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \\ \hat{\phi}_3 \end{pmatrix}
\]

(2.49)

where the shape functions for torsional degree of freedom are given by

\[
H_{\phi} = \begin{pmatrix} H_{\phi_1} \\ H_{\phi_2} \\ H_{\phi_3} \end{pmatrix} = \begin{pmatrix} 2s^2 - 3s + 1 \\ -4s^2 + 4s \\ 2s^2 - s \end{pmatrix}
\]

(2.50)

Note that \( s \) is given by \( s = x/l_i \) and \( l_i \) represents the length of \( i \)-th element in the shape function expressions presented above.

Substituting the approximations for each degree of freedom \((u_e, v, w \text{ and } \hat{\phi})\) into coupled flap-lag-torsion equations of motion and conducting spacial integration will finally yield 15-by-15 elemental matrices. In general, first order terms constitutes linear mass, damping, and stiffness matrix. The equation of motion finally becomes

\[
M\ddot{q} + C\dot{q} + Kq = F(\psi, q, \dot{q})
\]

(2.51)

Note that due to the coupled nature of the governing equations of motion, the nonlinear terms are also present. Such high order, nonlinear terms are moved to the right hand side and are treated as an additional forcing function to the linear system matrices. The matrices of linear system matrices and additional forcing matrices are given in Appendix A for the sake of completeness.
2.2 Aerodynamic Model

The aerelastic response of rotor blade is dependent on the aerodynamic force. The aerodynamic force is also dependent on the structural motion. (fill it more)

2.2.1 Definition of Air Velocity

The velocity encountered at the blade section consists of two components, the wind velocity and the blade velocity. The wind velocity, $\vec{V}_w$, is the sum of the contributions from the vehicle forward flight speed and the rotor inflow.

$$\vec{V}_w = \mu \Omega R I^H - \lambda \Omega R K^H$$  \hspace{1cm} (2.52)

where $\mu$ and $\lambda$ refer to as advance ratio and inflow ratio, respectively. Applying the coordinate transformations defined on equations 2.1 and 2.2 in a row, the wind velocity in the undeformed frame can be obtained as follows.

$$\vec{V}_w = \mu \Omega R \cos \psi \mathbf{i} - \mu \Omega R \sin \psi \mathbf{j} - \lambda \Omega R \mathbf{k}$$  \hspace{1cm} (2.53)

The blade velocity at a generic point on the deformed cross section is derived in equation 2.21. Based on the strip theory, the aerodynamic force is obtained with the effective angle of attack at 25% chord or at 75% chord of the airfoil section. The angle of attack can be obtained from the velocity at the specific location along the chord line. The blade velocity at $(\eta_r,0)$, $\vec{V}_{b_r}$, has the components as shown below.

$$V_{b_x} = \dot{u} - (\dot{v}' + \dot{v}' \dot{\theta}_1) \eta_r \cos \theta_1 - (\dot{w}' - \dot{w}' \dot{\theta}_1) \eta_r \sin \theta_1 - \Omega (v + \eta_r \cos \theta_1)$$

$$V_{b_y} = \dot{v} - \dot{\theta}_1 \eta_r \sin \theta_1 - \Omega (x + u - v' \eta_r \cos \theta_1 - w' \eta_r \sin \theta_1)$$

$$V_{b_z} = \dot{w} + \dot{\theta}_1 \eta_r \cos \theta_1$$  \hspace{1cm} (2.54)

The resultant velocity at the undeformed frame can be obtained by the vectorial sum of equations 2.53 and 2.54.

$$\vec{V} = U_x \mathbf{i} + U_y \mathbf{j} + U_z \mathbf{k}$$

$$= (V_{b_x} - V_{w_x}) \mathbf{i} + (V_{b_y} - V_{w_y}) \mathbf{j} + (V_{b_z} - V_{w_z}) \mathbf{k}$$  \hspace{1cm} (2.55)
The blade airloads are calculated by the effective angle of attack, and the angle of attack is derived from the velocity in the deformed frame. Thus the velocity in the undeformed frame need to be transformed to the deformed frame. Using the transformation matrix $T_{DU}$ in equation 2.9, the resultant velocity in the rotating deformed frame is expressed as

$$\vec{V} = U_x i + U_y j + U_z k = U_R i^\xi + U_T j^n + U_P k^\xi$$ (2.56)

The component of resultant velocity in the deformed frame is finally obtained as

\[
\begin{align*}
\frac{U_R}{\Omega R} &= \dot{u} - v + v'(x + \mu \sin \psi) - \mu \cos \psi + \lambda w' \\
&\quad - \eta_r \cos \theta_0(1 + \phi') + \eta_r \sin \theta_0(\dot{\phi} - \dot{w}') \\
&\quad + v' \hat{\omega} + w' \hat{\omega} + \frac{1}{2} \mu \cos \psi(v'^2 + w'^2) \\
\frac{U_T}{\Omega R} &= \cos \theta_0[\dot{v} + u + \dot{\phi}(\lambda + w) + v'v + (x + \mu \sin \psi)(1 - \frac{v'^2}{2})] \\
&\quad + \mu \cos \psi(v' + \dot{\phi}w') + \sin \theta_0[\dot{w} + \lambda + \nu w' - \hat{\phi}w'] \\
&\quad -(x + \mu \sin \psi)(v'w' + \dot{\phi}) + \mu \cos \psi(w' - \hat{\phi}w')] \\
\frac{U_P}{\Omega R} &= \cos \theta_0[\dot{w} + \lambda + \nu w' - (x + \mu \sin \psi)(v'w' + \dot{\phi})] \\
&\quad + \mu \cos \psi(w' - \hat{\phi}w')] - \sin \theta_0[u + \dot{v} + v' \nu + \hat{\phi}(\dot{\omega} + \lambda) \\
&\quad + \mu \cos \psi(\nu' + \dot{\phi}w') + (x + \mu \sin \psi)(1 - \frac{v'^2}{2})] \\
&\quad + \eta_r(\dot{\theta}_0 + \dot{\phi} + \nu')
\end{align*}
\] (2.57)

2.2.2 Airloads Expressions - Circulatory Loads

The circulatory airloads are generated from the pressure difference across the airfoil, or the circulation of the airfoil. The circulatory airloads in the deformed frame can be written as

$$\begin{align*}
\bar{L} &= \frac{1}{2} \rho V^2 c C_l \\
\bar{D} &= \frac{1}{2} \rho V^2 c C_d \\
\bar{M} &= \frac{1}{2} \rho V^2 c^2 C_m
\end{align*}$$ (2.58)

where $V$ is the total incident velocity, and $C_l, C_d$ and $C_m$ are the aerodynamic
coefficients, respectively. Note that the $C_m$ is evaluated with respect to 25% chord of airfoil section. For simplicity, linear aerodynamic models together with compressibility correction factor, $\beta = \sqrt{1 - M^2}$, are frequently used, for example in reference [104–107]. However, without proper stall model, the airload prediction from the linear model can be unrealistically high when the angle of attack approaches extreme values. In order to account for the nonlinear stall characteristic as well as compressibility effect altogether, the use of aerodynamic database in the tabular form would be beneficial. In general, 2D form of aerodynamic coefficient tables generated from wind tunnel experiments or CFD analysis are widely adopted. In present study, the aerodynamic coefficients are given by Mach number ($M = \frac{V}{a_\infty}$) and $\alpha$ as follows

$$
\begin{align*}
C_l &= C_l(M, \alpha) \\
C_d &= C_d(M, \alpha) \\
C_m &= C_m(M, \alpha)
\end{align*}
$$

As the the aerodynamic loads in equation 2.58 are acting at the aerodynamic center, and $\bar{L}$ is perpendicular to its incident velocity, by its definition, the aerodynamic forces need to be resolved into normal force, chord force about the elastic axis, and along the deformed axis. Thus the aerodynamic load acting on the aerodynamic center can be transformed to the loads acting on the elastic axis as follows

$$
\begin{align*}
\bar{L}_u &= -\bar{D}\sin\Lambda \\
\bar{L}_w &= \bar{L}\cos\alpha + \bar{D}\sin\alpha \\
\bar{L}_v &= \bar{L}\sin\alpha - \bar{D}\cos\alpha \\
\bar{M}_\phi &= \bar{M} - c_dL_w
\end{align*}
$$

The $\bar{L}_u, \bar{L}_v, \bar{L}_w$, and $\bar{M}_\phi$ are the aerodynamic loads acting at the elastic axis along the deformed elastic axis. $\Lambda$ stands for the yawed angle of the rotor blade due to the radial velocity, $U_R$ shown in equation 2.57.

To be used in the variation form of the nonconservative forces and moments (equation 2.36), the airloads expressions in the equation 2.60 need to be normalized with the normalization factors discussed in the table 2.1. Note that the airloads expressions in the equation 2.60 are representing the sectional loads, thus sectional force and moment must be normalized with $m_0\Omega^2R$ and $m_0\Omega^2R^2$, respectively. Due
to the presence of the air density $\rho$ in the aerodynamic loads, the normalization using Lock number, which is defined by $\gamma = \frac{\rho \bar{a} \varepsilon R^4}{\bar{b}^2} = \frac{3\rho \bar{a} \varepsilon R}{m_0}$, is conducted. The nondimensionalized expressions for airloads are given by

$$
\begin{align*}
\bar{L}_u &= \frac{\gamma \bar{V}^2}{ba} \frac{e}{c} (-C_d \sin \Lambda) \\
\bar{L}_w &= \frac{\gamma \bar{V}^2}{ba} \frac{e}{c} (C_l \cos \alpha + C_d \sin \alpha) \\
\bar{L}_v &= \frac{\gamma \bar{V}^2}{ba} \frac{e}{c} (C_l \sin \alpha - C_d \cos \alpha) \\
\bar{M}_{\phi} &= \frac{\gamma \bar{V}^2}{ba} \frac{e}{c} \varepsilon C_{m} - \frac{e_d}{R} \bar{L}_w
\end{align*}
$$

(2.61)

2.2.3 Airloads Expressions - Non-circulatory Loads

The non-circulatory airloads (apparent mass term or virtual forces) are generated from the instantaneous motion of the airfoil. When the airfoil section undergoes the heave motion, $h$, and the pitch motion, $\alpha$, the expression for the non-circulatory loads can be found from the [108].

$$
\begin{align*}
L_{w}^{NC} &= L_2 + L_3 \\
&= \rho \pi b^2 (\ddot{h} - a_h \dot{b} \ddot{\alpha}) + \rho \pi b^2 \dot{U} \dot{\alpha} \\
M_{\phi}^{NC} &= a_h b L_2 - \left(\frac{1}{2} - a_h \right) b L_3 - \frac{\rho \pi b^4}{8} \dot{\alpha}
\end{align*}
$$

(2.62)

For the rotorcraft, the variables in equation 2.62 can be interpreted as

$$
\begin{align*}
U &= \Omega R (x + \mu \sin \psi) \\
a_h b &= -(\epsilon_d + 0.25c) \\
\ddot{h} &= -\ddot{w} \\
\ddot{\alpha} &= \ddot{\theta}_1 = \ddot{\theta}_0 + \ddot{\phi} \\
\dot{\alpha} &= \dot{\theta}_1 = \dot{\theta}_0 + \dot{\phi} \\
b &= 0.5c
\end{align*}
$$

(2.63)

Applying the normalization parameters for sectional forces and moments, the final expressions will be obtained as
\[
\frac{L_w^{NC}}{m_0\Omega^2 R} = \frac{7\pi c^2}{12a^2R} (-\ddot{w} + \frac{0.25c+e_d}{R} \dot{\theta}_1 + (x + \mu \sin \psi)\dot{\theta}_1)
\]

\[
\frac{M_{\phi}^{NC}}{m_0\Omega^2 R^2} = \frac{7\pi c^2}{12a^2R} \left( \frac{0.25c+e_d}{R} \ddot{w} - \left( \frac{0.25c+e_d}{R} \right)^2 \dot{\theta}_1 - \frac{0.5c+e_d}{R} (x + \mu \sin \psi)\dot{\theta}_1 - \frac{c^2}{3\Omega^2 R} \ddot{\theta}_1 \right)
\]

\[\text{2.64}\]

2.2.4 Implementation into Finite Element Framework

The normalized circulatory and non-circulatory airloads expressions obtained in equations 2.61 and 2.64 are substituted into virtual work expression for non-conservative force discussed in equation 2.36. As the circulatory airloads expressions (equation 2.61) are still in the deformed frame, thus it must be transformed into the undeformed frame. Applying the transformation derived in the equation 2.9, the airloads in the undeformed frame are given by

\[
\begin{pmatrix}
L_u \\
L_v \\
L_w \\
M_{\phi}
\end{pmatrix} = \begin{pmatrix}
T_{DU}^{-1} & 0_{3\times1} \\
0_{1\times3} & 1
\end{pmatrix} \begin{pmatrix}
\bar{L}_u \\
\bar{L}_v \\
\bar{L}_w \\
\bar{M}_{\phi}
\end{pmatrix}
\]

\[\text{2.65}\]

Note that the non-circulatory load components are acting in the undeformed frame, thus the virtual work expressions for the airloads are obtained as

\[
\frac{\delta W_{\text{aero}}}{m_0\Omega_{\text{ref}}^2 R^3} = \int_0^1 \left( L_u \delta u + L_v \delta v + (L_w + L_w^{NC}) \delta w + (M_{\phi} + M_{\phi}^{NC}) \delta \phi \right) dx
\]

\[\text{2.66}\]

2.3 Inflow Model

The formulation of the sectional aerodynamic loads in the previous section is obtained with the blade-element theory, which takes accounts for the additional components of induced velocity, \(v_i\). For example, the vertical velocity component, \(\nu_i\), in the equation 2.57 contains \(\lambda_i (= \frac{\nu_i}{\Omega R})\). Note that this component serves as a “correction” to 2D sectional angle of attack. In the present study, the magnitude
of $\lambda_i$ can be calculated from the momentum theory or the direct calculation from the lifting-line theory. In the momentum theory, the magnitude of $\lambda_i$ is balanced by the amount of the thrust. While in the view point of lifting-line theory, the strength of $\lambda_i$ can be obtained by summing the individual contributions from the vortex system, which extends behind the path of the each blade.

### 2.3.1 Momentum Theory Based Model

The momentum theory has long been used as a fundamental estimate of the magnitude of the inflow corresponding to the desired thrust level. The inflow equation in forward flight condition can be obtained as follows \[ [109] \]

$$
\lambda_i = \frac{C_T}{2\sqrt{\mu^2 + (\mu \tan \alpha_D + \lambda_i)^2}} \quad (2.67)
$$

Note that the inflow equation is nonlinear. In general, the numerical solution of equation 2.67 can be obtained by Newton-Raphson method \[ [109] \]. The momentum theory correlates the magnitude of the inflow with given thrust value in a global sense, thus the detailed information about the inflow distribution is unavailable through this model. Various modifications to the uniform inflow model were proposed by assuming the radial shape functions and/or azimuthal shape functions. This modification is referred to as “Linear Inflow Model”, which is still popular to date. The generic form of the linear inflow model is as follows.

$$
\lambda_i(x, \psi) = \lambda_i(1 + \kappa_x x \cos \psi + \kappa_y x \sin \psi) \quad (2.68)
$$

As can be seen, assumed shape of 1/rev harmonic variation as well as linear distribution in radial direction was adopted. The coefficients of various linear inflow models are summarized in Table 2.3. Note that the tip correction model needs to be applied when momentum theory model is used in rotor analysis.

### 2.3.2 Vortex theory based model

The momentum theory based model discussed in the previous section provides limited information on the inflow distribution - at most first order both on the radial and azimuthal direction. This level of inflow information is good enough
Table 2.3. Linear Inflow coefficients, $k_x$, $k_y$

<table>
<thead>
<tr>
<th>Source</th>
<th>$k_x$</th>
<th>$k_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coleman</td>
<td>$\tan(\chi/2)$</td>
<td>0</td>
</tr>
<tr>
<td>Drees</td>
<td>$4/3(1 - \cos\chi - 1.8\mu^2)/\sin\chi$</td>
<td>$-2\mu$</td>
</tr>
<tr>
<td>Payne</td>
<td>$4/3[\mu/\lambda/(1.2 + \mu/\lambda)]$</td>
<td>0</td>
</tr>
<tr>
<td>White&amp;Blake</td>
<td>$\sqrt{2}\sin(\chi)$</td>
<td>0</td>
</tr>
<tr>
<td>Pit&amp;Peters</td>
<td>$\frac{15\pi}{23}\tan(\chi/2)$</td>
<td>0</td>
</tr>
<tr>
<td>Howlett</td>
<td>$\sin^2\chi$</td>
<td>0</td>
</tr>
</tbody>
</table>

for performance prediction where the steady and 1/rev harmonic components has dominant effect. When the vibratory load prediction is conducted, then the higher harmonic components of inflow needs to be preserved for good prediction. In present study, the lifting line model is adopted to model the bound circulation converted from the magnitude of sectional lift.

Full 3D CFD analysis predicts good accuracy results to date. However, the grid generation and computational time to obtain a single solution (for example, a week using average performance clusters) posed as a barrier to its practical use. Similar to full 3D structural problem, the full order model was divided into “Inner Problem (2D model to evaluate $\Gamma$ considering compressibility, nonlinear stall, unsteady phenomena)” and “Outer Problem (The $\lambda_i$ obtained using $\Gamma$ (from inner problem) and the wake geometry)”

The inflow model using vortex theory corresponds to the outer problem. Based on the circulation $\Gamma$ from inner problem, lifting line model was generally used to construct 3D structure of rotor blade.

- Radial distribution of $\Gamma$ along aerodynamic center is designated as “Bound Vortex”
- The gradient of $\Gamma$ along radial direction, $\frac{\partial \Gamma}{\partial r}$, becomes “Trailer Vortex”
- The temporal variation of $\Gamma$, $\frac{\partial \Gamma}{\partial t}$, deposits its memory on its past path, which is referred to as “Shed Vortex”

It must be emphasized that the role of shed wake is almost identical to that of the 2D sectional unsteady model, thus care must be taken not to double account
the effect of both shed wake and unsteady model at the same time. In general, shed wake is suppressed from outer problem.

The wake geometry is obtained by marching a series of trailer vortices through wake age for (given) azimuth $\psi$.

$$\frac{\partial r}{\partial \psi} + \frac{\partial r}{\partial \zeta} = \frac{1}{\Omega} V(r)$$ (2.69)

$V(r)$ is working as a forcing function to PDE, and it contains the free-stream velocity and induced velocity. Note that induced velocity component is a function of wake geometry to drive $r$ as a force-free particle. $V(r)$ is evaluated using Biot-Savart law as shown below.

$$V(r) = \frac{1}{4\pi} \int_0^{\zeta_f} \frac{\Gamma \times (r - r(\zeta))}{|r - r(\zeta)|^3} d\zeta$$ (2.70)

Evaluating the whole systems of trailer vortices (i.e, full span vortex lattice model) is computationally extensive due to the consideration of interaction between all collocation points, $r$. Thus based on the observations from experiment, the simplification of wake shape is usually assumed.

- Near Wake portion - Affects the blade loading directly, thus full span lattice is maintained. Usually extends $15^\circ$ to $90^\circ$ of wake age

- Roll up Wake portion - In order to make smooth transition, the density of lattice reduces.

- Far Wake portion - Represents fully rolled up (strong) tip vortex.

Note that vortex wake model also involves semi-empiricism in its nature. The size of vortex core, the growth rate of that core affects the distortion of wake geometry and the inflow. These values needs to be tuned based on reference data or experience.

### 2.4 Blade Response

After performing spacial discretization using Finite Element Method, the governing equation is reduced to the set of ODEs. However, the size of general coordinates,
\( q \), rapidly increases with the number of element used. For example, assembly of 20 elements yields 186 nodes. With the first order form of time marching scheme shown in equation 2.71, then the number of variables doubles due to the structure of computational scheme. The total number of variables to be used for time marching finally becomes 372.

\[
\begin{bmatrix}
M & 0 \\
0 & -M
\end{bmatrix} \begin{bmatrix}
\ddot{q} \\
\dot{q}
\end{bmatrix} + \begin{bmatrix}
C & K \\
M & 0
\end{bmatrix} \begin{bmatrix}
\dot{q} \\
q
\end{bmatrix} = \begin{bmatrix}
F \\
0
\end{bmatrix}
\] (2.71)

Therefore, the use of the nodal DOFs becomes computationally expensive especially when repetitive computation is required (e.g., trim calculation).

### 2.4.1 Modal Reduction

In order to achieve the numerical efficiency, the modal reduction method discussed in Reference [104] is adopted in the present analysis. In modal reduction method, the system matrices are decomposed using modal matrix, \( \Phi \), which is a collection of each natural mode shape. The system equation of undamped, free system is given by

\[
M \ddot{q} + Kq = 0
\] (2.72)

assuming \( q = \exp^{i\omega t} \), the equation 2.72 becomes

\[
(-\omega^2 M + K) \exp^{i\omega t} = 0
\] (2.73)

Natural modes of rotor blade can be obtained from the eigen analysis on the equation \(-\omega^2 M + K = 0\). Once the eigenvalues of equation 2.73 are obtained, the associated eigenvectors, \( \Phi \), can be obtained. Therefore the global degree of freedom, \( q \), can be described as

\[
q(N_G \times 1) = \Phi(N_G \times m)\eta(m \times 1)
\] (2.74)

where \( N_G \) and \( m \) refer to the number of total degree of freedom of assembled system, and number of natural modes used to represent blade response. \( \eta \) represents the
modal coordinate corresponding to each mode. The variation of \( q \) can then becomes

\[
\delta q = \Phi \delta \eta
\]  

(2.75)

Recall the procedure used in equation 2.42, the \( \delta q^T \) must be pre-multiplied in front of equation 2.51. The original system (equation 2.51) becomes

\[
\ddot{\eta} + \ddot{\eta} + K \eta = \bar{F}(\psi, \eta, \dot{\eta})
\]  

(2.76)

where \( \bar{M}, \bar{C}, \bar{K}, \) and \( \bar{F} \) are given as

\[
\bar{M} = \Phi^T M \Phi \\
\bar{C} = \Phi^T C \Phi \\
\bar{K} = \Phi^T K \Phi \\
\bar{F}(\psi, \eta, \dot{\eta}) = \Phi^T F(\psi, \Phi \eta, \Phi \dot{\eta})
\]  

(2.77)

Owing to the orthogonality between the natural modes, the system matrices shown in equation 2.77 becomes diagonal matrix or close to it. Which in turn implies the system equation (equation 2.51) can be decoupled by the frequency (or eigenvalue) and its physical meaning can be identified by investigating the contribution of each modes on the response. Once the response of modal coordinate \( \eta \) is obtained, it is straight forward to recover the nodal displacement using equation 2.74. Together with the spacial shape functions discussed in previous section, the displacement at arbitrary location can be obtained. Note that modal reduction needs to be conducted only once, especially at the initialization stage of the whole computation sequence.

### 2.4.2 Periodic Response

The response of the rotor blade system described by equation 2.51 is periodic in its nature. Especially in the forward flight condition, 1/rev variation of the local free stream affects the aerodynamic loading to be periodic. Furthermore, 1/rev variation of imposed by pitch input contributes the periodicity of system matrices as well as forcing vector. In order to obtain the periodic response of equation 2.51 or 2.76, the most simple and widely used method is time-marching. Perform the
time-marching until the response becomes periodic. The benefit of time-marching method is its simplicity in formulation. The drawback is that there is no guarantee that the periodic response will be obtained. Note that the rotor blade forcing is primarily 1/rev - which is very close to the flap bending frequency of most of the rotor blade system (1.03/rev to 1.18/rev). Without proper (or sufficient) damping, the system tends to fall into the resonance phenomena. In general, the damping provided by the aerodynamic force stabilizes the response, however, it is well known that the pure articulated rotor is subjected to the lag instability due to the lack of aerodynamic damping in the lag mode. Without proper means of enhancing the damping, the use of time marching method may result in a difficulty in obtaining stable, periodic solution.

In order to bypass the above mentioned issue, the “expansion” based solution methodology is also used. Among various expansion method, it is worth to briefly describe Harmonic Balance Method (hereafter HBM) and Finite Element in Time (hereafter FEMT). For the HBM, the temporal term is expanded by the combination of cosine and sine functions with coefficients as shown in equation 2.78.

\[ q = a_0 + \sum_{n=1}^{N} a_n \cos n\psi + \sum_{n=1}^{N} a_n \sin n\psi \]  

(2.78)

Substituting equation 2.78 into the system (equation 2.51), minimize the norm
given by equation 2.79.

\[ \int_{0}^{2\pi} (M\ddot{q} + C\dot{q} + Kq - F)d\psi = 0 \]
\[ \int_{0}^{2\pi} (M\ddot{q} + C\dot{q} + Kq - F)\cos \psi d\psi = 0 \]
\[ \int_{0}^{2\pi} (M\ddot{q} + C\dot{q} + Kq - F)\cos 2\psi d\psi = 0 \]
\[ \vdots \]
\[ \int_{0}^{2\pi} (M\ddot{q} + C\dot{q} + Kq - F)\cos N\psi d\psi = 0 \]
\[ \int_{0}^{2\pi} (M\ddot{q} + C\dot{q} + Kq - F)\sin \psi d\psi = 0 \]
\[ \int_{0}^{2\pi} (M\ddot{q} + C\dot{q} + Kq - F)\sin 2\psi d\psi = 0 \]
\[ \vdots \]
\[ \int_{0}^{2\pi} (M\ddot{q} + C\dot{q} + Kq - F)\sin N\psi d\psi = 0 \]

(2.79)

Note that the ODEs in time have been converted into sets of nonlinear algebraic equations in terms of \( a_0 \), \( a_{nc}(n = 1, ..., N) \), and \( a_{ns}(n = 1, ..., N) \). The size of nonlinear algebraic equation depends on the number of variables to be expanded as well as the number of frequency to be used. For example, if 10 variables are expanded up to 10/rev frequency, the number of nonlinear algebraic equations is 210. Considering the numerical cost for evaluating Jacobian of 210-by-210 necessary during the solution process of nonlinear systems of equations, the application of HBM on the large and high frequency system may incur the additional computational cost compared to that of the time-marching method. FEMT is another way to obtain a solution using expansion. Expansion is performed using the temporal shape functions, \( H_{ti} \), and corresponding temporal node value, \( \bar{t}_i \).

\[ q = \sum_{i}^{N} H_{ti}(\psi)\bar{t}_i \]  

(2.80)

Substituting 2.80 into variational form given as

\[ \int_{0}^{2\pi} \delta q^T (M\ddot{q} + C\dot{q} + Kq - F)d\psi = 0 \]  

(2.81)
finally yields the discretized equation as shown below.

\[
\sum_{i=1}^{N} \int_{\psi_i}^{\psi_{i+1}} \delta \tilde{t}^T (\tilde{K} \tilde{t} - \tilde{F}) d\psi = 0 \quad (2.82)
\]

Note that \(\tilde{K}\) and \(\tilde{F}\) contains the components of temporal shape function mentioned in equation 2.80. Similarly for HBM, FEMT converts the systems of ODEs into the algebraic equations. However for FEMT, algebraic equations are functions of \(\tilde{t}\). The periodic condition can be imposed by assembling the first temporal node of Element 1 with the last temporal node of Element N. Further details about the FEMT can be found from Reference [104].

Another approach is to solve the periodic boundary value problem (hereafter PBVP). The idea of PBVP is based on the fact that the response at the end point \((\psi = 2\pi)\) of time-marching scheme is determined by the initial values given at \(\psi = 0\) and the system dynamics given by equation 2.71. Thus if the initial values are adjusted such that the initial values and the responses at the end point are matched within tolerance, then the solution of PBVP is said to be obtained. The numerical procedure to adjust initial values so that the periodicity is satisfied is referred to as “periodic shooting”. Further details about the periodic shooting can be found from Reference [110]. Considering the balance of the numerical cost and additional modification required to the equation 2.71, the periodic shooting method is chosen as a numerical method to obtain periodic response of the rotor blade.

### 2.5 Blade Loads

Once the response of the blade is obtained, the blade loads can be calculated using force summation method or reaction force method. In force summation method, the inertial force and aerodynamic force are integrated up to the point of interest. For obtaining the blade root loads, the integration is conducted up to the root of the blade.

\[
q = \sum_{i=1}^{N} H_{t_i}(\psi) \tilde{t}_i \quad (2.83)
\]
In reaction force method, the system element matrices are utilized to obtain the blade root loads.

\[ S = M\ddot{q} + C\dot{q} + Kq - F \]  \hspace{1cm} (2.84)

Note that the spacial integration is already conducted for each element, therefore the total loads is lumped at the both nodes of each element. Summing all of the elements will yield the blade root loads as

\[
S_x = \sum_{k=1}^{\text{# of nodes}} f_x^k \\
S_y = \sum_{k=1}^{\text{# of nodes}} f_y^k \\
S_z = \sum_{k=1}^{\text{# of nodes}} f_z^k \\
M_x = \sum_{k=1}^{\text{# of nodes}} m_x^k \\
M_y = \sum_{k=1}^{\text{# of nodes}} m_y^k - f_z^k \left( \sum_{n=1}^{k-1} l^n \right) \\
M_z = \sum_{k=1}^{\text{# of nodes}} m_z^k + f_y^k \left( \sum_{n=1}^{k-1} l^n \right) \hspace{1cm} (2.85)
\]

Unlike several previous works [105–107], the radial degree of freedom, \( u \), is considered in the FEM formulation, thus the reaction force method automatically covers all of the effect. Note that supplementary equations must be used if the radial degree of freedom is omitted in the analysis [105].

After evaluating the blade root loads, the hub loads can be obtained by summing up the root loads of each blade. Assuming the identical blade in the present study, the blade root loads of other blades can be obtained by shifting that of reference blade by \( 2\pi/N_b \), where \( N_b \) refers to the number of blades. Defining the azimuth angle of \( m^{th} \) blade as \( \psi_m = \psi + 2\pi/N_b(m - 1) \), then the expressions for
hub load can be given as

\[ F_X^H(\psi) = \sum_{m=1}^{N_b} S_x^m \cos \psi_m - S_y^m \sin \psi_m \]
\[ F_Y^H(\psi) = \sum_{m=1}^{N_b} S_x^m \sin \psi_m + S_y^m \cos \psi_m \]
\[ F_Z^H(\psi) = \sum_{m=1}^{N_b} S_z^m \]
\[ M_X^H(\psi) = \sum_{m=1}^{N_b} M_x^m \cos \psi_m - M_y^m \sin \psi_m \]
\[ M_Y^H(\psi) = \sum_{m=1}^{N_b} M_x^m \sin \psi_m + M_y^m \cos \psi_m \]
\[ M_Z^H(\psi) = \sum_{m=1}^{N_b} M_z^m \]

The hub loads are used for trim analysis and vibration analysis. For trim analysis, the steady components shown in equation 2.87 are provided into the set of equilibrium equations with which the rotor control inputs, fuselage attitude, and the tail rotor control input are solved for. The details of trim analysis will be discussed in the next section.

\[ F_X^0 = \frac{1}{2\pi} \int_{0}^{2\pi} F_X^H(\psi) d\psi \]
\[ F_Y^0 = \frac{1}{2\pi} \int_{0}^{2\pi} F_Y^H(\psi) d\psi \]
\[ F_Z^0 = \frac{1}{2\pi} \int_{0}^{2\pi} F_Z^H(\psi) d\psi \]
\[ M_X^0 = \frac{1}{2\pi} \int_{0}^{2\pi} M_X^H(\psi) d\psi \]
\[ M_Y^0 = \frac{1}{2\pi} \int_{0}^{2\pi} M_Y^H(\psi) d\psi \]
\[ M_Z^0 = \frac{1}{2\pi} \int_{0}^{2\pi} M_Z^H(\psi) d\psi \]

For vibration analysis, the \( N/\text{rev} \) components are obtained by performing fourier
analysis. The cosine components are given by equations 2.88.

\[
F_{NP}^{Xc} = \frac{1}{\pi} \int_0^{2\pi} F^H_X(\psi) \cos N\psi d\psi \\
F_{NP}^{Yc} = \frac{1}{\pi} \int_0^{2\pi} F^H_Y(\psi) \cos N\psi d\psi \\
F_{NP}^{Zc} = \frac{1}{\pi} \int_0^{2\pi} F^H_Z(\psi) \cos N\psi d\psi \\
M_{NP}^{Xc} = \frac{1}{\pi} \int_0^{2\pi} M^H_X(\psi) \cos N\psi d\psi \\
M_{NP}^{Yc} = \frac{1}{\pi} \int_0^{2\pi} M^H_Y(\psi) \cos N\psi d\psi \\
M_{NP}^{Zc} = \frac{1}{\pi} \int_0^{2\pi} M^H_Z(\psi) \cos N\psi d\psi
\text{(2.88)}
\]

Similarly for sine components, the expressions will yield,

\[
F_{NP}^{Xs} = \frac{1}{\pi} \int_0^{2\pi} F^H_X(\psi) \sin N\psi d\psi \\
F_{NP}^{Ys} = \frac{1}{\pi} \int_0^{2\pi} F^H_Y(\psi) \sin N\psi d\psi \\
F_{NP}^{Zs} = \frac{1}{\pi} \int_0^{2\pi} F^H_Z(\psi) \sin N\psi d\psi \\
M_{NP}^{Xs} = \frac{1}{\pi} \int_0^{2\pi} M^H_X(\psi) \sin N\psi d\psi \\
M_{NP}^{Ys} = \frac{1}{\pi} \int_0^{2\pi} M^H_Y(\psi) \sin N\psi d\psi \\
M_{NP}^{Zs} = \frac{1}{\pi} \int_0^{2\pi} M^H_Z(\psi) \sin N\psi d\psi
\text{(2.89)}
\]

Finally for the magnitude of \(N/\text{rev}\) hub loads can be obtained by

\[
F^N_X = \sqrt{F_{NP}^{Xc}^2 + F_{NP}^{Xs}^2} \\
F^N_Y = \sqrt{F_{NP}^{Yc}^2 + F_{NP}^{Ys}^2} \\
F^N_Z = \sqrt{F_{NP}^{Zc}^2 + F_{NP}^{Zs}^2} \\
M^N_X = \sqrt{M_{NP}^{Xc}^2 + M_{NP}^{Xs}^2} \\
M^N_Y = \sqrt{M_{NP}^{Yc}^2 + M_{NP}^{Ys}^2} \\
M^N_Z = \sqrt{M_{NP}^{Zc}^2 + M_{NP}^{Zs}^2}
\text{(2.90)}
\]
Note that only the integer multiples of \( N_b \) harmonics are transmitted to the fuselage if all the blades are identical or tracked. That is, the rotor acts as a filter for many harmonics.

### 2.6 Trim

For a given control inputs, the aeroelastic response of the rotor blade and its corresponding hub loads can be obtained using the methodology discussed in the previous section. However, the operational requirement is usually imposed on the resultant loads (for example, specifying the thrust value), thus it is necessary to find proper controls that satisfies the requirement. In a sense, this procedure can be regarded as an inverse control problem. Generally, two kinds of inverse control problem can be formulated for rotor blade - one is the wind tunnel trim and the other is propulsive trim.

#### 2.6.1 Definition of Flight Condition

In order to set up a specific flight condition, several input parameters maybe required. Some of which are dependent on one another, thus it is necessary to identify dominant parameters that can work as independent variable. For example, the Lock number \( \gamma \) can be treated as an input value, on the other hand, \( \gamma \) can be calculated from other input values such as air density \( (\rho) \), mass per unit length \( (m_0) \), radius, and so on.

In the present analysis, the front end inputs are gross weight \( (W) \), altitude \( (h) \), and flight speed \( (V_\infty) \). Note that \( \Omega, R, \) and \( I_b \) are already given from structural properties. From these front end inputs, the following nondimensional parameters can be derived

\[
\begin{align*}
\mu &= \frac{V_\infty}{\Omega R} \\
T_a &= f_1(h) \\
\rho &= f_2(h) \\
\gamma &= \frac{\rho a_{ref} c_{ref} R^4}{I_b} \\
a_\infty &= \sqrt{\gamma a R T_a} 
\end{align*}
\]

where \( \gamma_a \) and \( R_a \) refer to the ratio of specific heat for air and gas constant of air,
respectively. $T_a$ and $\rho$ are functions of altitude, and also affected by the air model. ICAO standard air model is widely used, but non-standard model is adopted to represent extreme situations such as “army hot day condition”.

2.6.2 Wind Tunnel Trim

The formulation for wind tunnel trim can be given as

\[
\begin{align*}
F_Z^0 &= C_{T_{req}} \\
M_X^0 &= 0 \\
M_Y^0 &= 0
\end{align*}
\] (2.92)

where the rotor shaft tilt, $\alpha_s$, is specified, and the trim variables are $\theta_0$, $\theta_1c$, and $\theta_1s$. Another variation of wind tunnel trim which treats $\alpha_s$ as a trim variable can be defined as

\[
\begin{align*}
F_X^0 &= -C_{Prop} \cos \alpha_s + C_W \sin \alpha_s \\
F_Z^0 &= C_{Prop} \sin \alpha_s + C_W \cos \alpha_s \\
M_X^0 &= 0 \\
M_Y^0 &= 0
\end{align*}
\] (2.93)

where $C_{Prop}$ and $C_W$ refer to propulsive force coefficient and weight coefficient respectively, and are defined as

\[
\begin{align*}
C_{Prop} &= \frac{F_{Prop}}{\rho \pi R^2 (\Omega R)^2} \\
C_W &= \frac{W}{\rho \pi R^2 (\Omega R)^2}
\end{align*}
\]

(2.94)

for steady-level flight, $F_{Prop}$ can be defined as $\frac{1}{2} \rho V_\infty^2 S$ where S is equivalent flat plate area for fuselage.

2.6.3 Propulsive Trim

A helicopter in free-flight condition is subject to equilibrium of 3 force and 3 moment dynamic balance equations for rigid body motion. Furthermore, 3 additional
equations describing angular rate are also required.

\[
\begin{align*}
\dot{u} &= -(wq - vr) + \frac{F_X}{M_a} - g \sin \theta \\
\dot{v} &= -(ur - wp) + \frac{F_Y}{M_a} + g \cos \theta \sin \phi \\
\dot{w} &= -(vp - uq) + \frac{F_Z}{M_a} + g \cos \theta \cos \phi \\
I_{xx}\dot{p} &= (I_{yy} - I_{zz})qr + I_{xz}(\dot{r} + pq) + \tilde{M}_X \\
I_{yy}\dot{q} &= (I_{zz} - I_{xx})rp + I_{xz}(r^2 - p^2) + \tilde{M}_Y \\
I_{zz}\dot{r} &= (I_{xx} - I_{yy})pq + I_{xz}(\dot{p} - qr) + \tilde{M}_Z \\
p &= \dot{\phi} - \dot{\Psi} \sin \theta \\
q &= \dot{\theta} \cos \phi + \dot{\Psi} \sin \phi \cos \theta \\
r &= -\dot{\theta} \sin \phi + \dot{\Psi} \cos \phi \cos \theta
\end{align*}
\]

In order to obtain solutions for 13 unknowns with 9 equations, 4 unknowns needs to be prescribed. In body fixed coordinate system, trimmed condition can be set as zero linear accelerations (\(\dot{u}=\dot{v}=\dot{w}=0\)), zero angular accelerations (\(\dot{p}=\dot{q}=\dot{r}=0\)), and zero angular rates (\(p=q=r=0\)). In addition, by imposing level flight condition (\(\theta=\phi=0\)), the dynamic equations are reduced to quasi-static equilibrium problem. Therefore, the forces and moments equilibrium becomes a set of algebraic equations in terms of trim variables.

\[
\begin{align*}
\tilde{F}_X &= D_{fus} + F_Y^0 \cos(\alpha_i + \alpha_{wl}) - F_Z^0 \sin(\alpha_i + \alpha_{wl}) \cos \phi_{wl} \\
\tilde{F}_Y &= Y_{fus} + F_Y^0 \cos \phi_{wl} + F_Z^0 \sin \phi_{wl} + T_{TR} \cos \phi_{wl} \cos \Gamma \\
\tilde{F}_Z &= F_Z^0 \cos(\alpha_i + \alpha_{wl}) \cos \phi_{wl} + F_X^0 \sin(\alpha_i + \alpha_{wl}) \\
&\quad - F_Y^0 \cos(\alpha_i + \alpha_{wl}) \sin \phi_{wl} - W + L_{fus} - L_{HT} + T_{TR} \sin \Gamma \\
\tilde{M}_X &= M_X^0 + M_{X_{fus}} + Y_{fus}(h \cos \phi_{wl} + y_{cg} \sin \phi_{wl}) \\
&\quad +(W - L_{fus}) \cos \alpha_{wl}(h \sin \phi_{wl} - y_{cg} \cos \phi_{wl}) \\
&\quad + D_{fus} \sin \alpha_{ul}(h \sin \phi_{wl} - y_{cg} \cos \phi_{wl}) + T_{TR}(h - z_{TR}) \cos \Gamma \\
\tilde{M}_Y &= M_Y^0 + M_{Y_{fus}} + (W - L_{fus})(h \sin \alpha_{wl} - x_{cg} \cos \alpha_{wl} \cos \phi_{wl}) \\
&\quad - D_{fus}(h \cos \alpha_{wl} + x_{cg} \sin \alpha_{wl} \cos \phi_{wl}) - L_{HT}(x_{cg} - x_{HT}) \\
&\quad + T_{TR}(x_{cg} - x_{TR}) \sin \Gamma \\
\tilde{M}_Z &= M_Z^0 + M_{Z_{fus}} + T_{TR}(x_{TR} - x_{cg}) \cos \Gamma - D_{fus} y_{cg} \cos \alpha_{wl} \\
&\quad +(W - L_{fus})(-x_{cg} \cos \alpha_{wl} \sin \phi_{wl} + y_{cg} \sin \alpha_{wl})
\end{align*}
\]
In order to obtain a solution of nonlinear algebraic equation set given in 2.96, 3 main rotor controls \((\theta_0, \theta_{1c}, \theta_{1s})\), 2 fuselage attitudes \((\alpha_{wl}, \phi_{wl})\), and tail rotor controls \((\theta_{TR})\) were iteratively adjusted to achieve equilibrium.

### 2.6.4 Iterative Trim Process

The nonlinear algebraic equations shown in 2.96 can be solved with Newton scheme. It is similar to the Newton-Rhapson method to solve inflow equation, but the Jacobian matrix need to be evaluated for updating the unknowns in vector. Defining the vector of trim variables as

\[
\Phi = [\theta_0 \theta_{1c} \theta_{1s} \alpha_{wl} \phi_{wl} \theta_{TR}]^T \tag{2.97}
\]

and, the Jacobian matrix is defined as

\[
J_{(6 \times 6)} = \begin{bmatrix}
\frac{\partial \bar{F}_X}{\partial \theta_0} & \frac{\partial \bar{F}_X}{\partial \theta_{1c}} & \frac{\partial \bar{F}_X}{\partial \theta_{1s}} & \frac{\partial \bar{F}_X}{\partial \alpha_{wl}} & \frac{\partial \bar{F}_X}{\partial \phi_{wl}} & \frac{\partial \bar{F}_X}{\partial \theta_{TR}} \\
\frac{\partial \bar{F}_Y}{\partial \theta_0} & \frac{\partial \bar{F}_Y}{\partial \theta_{1c}} & \frac{\partial \bar{F}_Y}{\partial \theta_{1s}} & \frac{\partial \bar{F}_Y}{\partial \alpha_{wl}} & \frac{\partial \bar{F}_Y}{\partial \phi_{wl}} & \frac{\partial \bar{F}_Y}{\partial \theta_{TR}} \\
\frac{\partial \bar{F}_Z}{\partial \theta_0} & \frac{\partial \bar{F}_Z}{\partial \theta_{1c}} & \frac{\partial \bar{F}_Z}{\partial \theta_{1s}} & \frac{\partial \bar{F}_Z}{\partial \alpha_{wl}} & \frac{\partial \bar{F}_Z}{\partial \phi_{wl}} & \frac{\partial \bar{F}_Z}{\partial \theta_{TR}} \\
\frac{\partial \bar{M}_X}{\partial \theta_0} & \frac{\partial \bar{M}_X}{\partial \theta_{1c}} & \frac{\partial \bar{M}_X}{\partial \theta_{1s}} & \frac{\partial \bar{M}_X}{\partial \alpha_{wl}} & \frac{\partial \bar{M}_X}{\partial \phi_{wl}} & \frac{\partial \bar{M}_X}{\partial \theta_{TR}} \\
\frac{\partial \bar{M}_Y}{\partial \theta_0} & \frac{\partial \bar{M}_Y}{\partial \theta_{1c}} & \frac{\partial \bar{M}_Y}{\partial \theta_{1s}} & \frac{\partial \bar{M}_Y}{\partial \alpha_{wl}} & \frac{\partial \bar{M}_Y}{\partial \phi_{wl}} & \frac{\partial \bar{M}_Y}{\partial \theta_{TR}} \\
\frac{\partial \bar{M}_Z}{\partial \theta_0} & \frac{\partial \bar{M}_Z}{\partial \theta_{1c}} & \frac{\partial \bar{M}_Z}{\partial \theta_{1s}} & \frac{\partial \bar{M}_Z}{\partial \alpha_{wl}} & \frac{\partial \bar{M}_Z}{\partial \phi_{wl}} & \frac{\partial \bar{M}_Z}{\partial \theta_{TR}} 
\end{bmatrix} \tag{2.98}
\]

Once the Jacobian matrix is evaluated with respect to the current trim variables at the step \(n\), \(\Phi_n\), the update process at the step \(n+1\) is given by

\[
\Phi_{n+1} = \Phi_n - J^{-1} \begin{bmatrix}
\bar{F}_X \\
\bar{F}_Y \\
\bar{F}_Z \\
\bar{M}_X \\
\bar{M}_Y \\
\bar{M}_Z 
\end{bmatrix}_n \tag{2.99}
\]
This update process continues until the norm defined in 2.100 drops down below the prescribed tolerance.

\[ \epsilon = \sqrt{\bar{F}_X^2 + \bar{F}_Y^2 + \bar{F}_Z^2 + M_X^2 + M_Y^2 + M_Z^2} \]  

(2.100)

### 2.6.5 Fuselage Aerodynamic Characteristics

The fuselage aerodynamic properties considered to be of minor influence on the trim solutions under steady, level flight condition, thus only the drag of fuselage is included in the previous studies. However, if the helicopter is operating at the high speed (thus high dynamic pressure) or the fuselage has certain aerodynamic characteristic, the effect of other aerodynamic characteristics may affect the trim solution. The contributions of fuselage aerodynamic loads can be further expressed as

\[
D_{fus} = \frac{1}{2} \rho V_\infty^2 C_{D_{fus}} \\
Y_{fus} = \frac{1}{2} \rho V_\infty^2 C_{Y_{fus}} \\
L_{fus} = \frac{1}{2} \rho V_\infty^2 C_{L_{fus}} \\
M_{X_{fus}} = \frac{1}{2} \rho V_\infty^2 C_{M_{X_{fus}}} \\
M_{Y_{fus}} = \frac{1}{2} \rho V_\infty^2 C_{M_{Y_{fus}}} \\
M_{Z_{fus}} = \frac{1}{2} \rho V_\infty^2 C_{M_{Z_{fus}}} 
\]

(2.101)

For the simplified model, only \( L_{fus} \) and \( D_{fus} \) is introduced in the trim equations, and given in reference [111]. However, the advanced model incorporates the full fuselage load expressions into the trim equations, which is obtained from reference [112]. Note that only lift, drag, and pitching moment varies with the fuselage pitch attitude. Due to the assumption of level flight (i.e., no sideslip angle), the roll moment, yaw moment, and side force has a constant value. The data for full fuselage aerodynamic coefficients are depicted in the figure 2.4 and 2.5

### 2.6.6 Horizontal Tail

The horizontal tail provides pitch damping especially in the high speed forward flight. The pitch damping is provided by attaching the tail with the negative
Figure 2.4. Fuselage Force

Figure 2.5. Fuselage Moment
incidence angle ($\epsilon$) such that it provides lift downwards.

\[
L_{HT} = \frac{1}{2} \rho V_{\infty}^2 S_{HT} C_L (\epsilon - \alpha_{WL}) \\
D_{HT} = \frac{1}{2} \rho V_{\infty}^2 S_{HT} C_D (\epsilon - \alpha_{WL})
\]  

(2.102)

where $C_L$ and $C_D$ are lift and drag coefficients of horizontal tail. $S_{HT}$ stands for the area of horizontal tail. Note that in the present study, the lift and drag of horizontal tail was included in the simplified model used in chapter 3. The advanced fidelity model used in the later chapters includes the effect of horizontal tail in the fuselage aerodynamic properties.

### 2.6.7 Tail Rotor

For propulsive trim, the anti-torque is required to counter act the steady torque from main rotor, $M_0^2$. For conventional tail rotor, anti-torque is provided by the thrust of tail rotor, $T_{TR}$. Normalizing $T_{TR}$ by $\rho \pi R_{TR}^2 (\Omega_{TR} R_{TR})^2$, the expression for thrust coefficient $C_{T_{TR}}$ is obtained as

\[
C_{T_{TR}} = \frac{\sigma_{T_{TR} a_{T_{TR}}} \theta_{T_{TR}}}{2} \left[ \frac{3}{2} \left( 1 + \frac{3}{2} \mu_{T_{TR}}^2 \right) - \frac{\lambda_{T_{TR}}}{2} \right]
\]  

(2.103)

where $\mu_{T_{TR}}$ refers to the advance ratio for the tail rotor, and are defined as

\[
\mu_{T_{TR}} = \frac{\Omega_{T_{TR}}}{\Omega} \frac{R_{T_{TR}}}{R} \mu
\]  

(2.104)

Tail rotor inflow $\lambda_{T_{TR}}$ is obtained by solving momentum equation shown in 2.105.

\[
\lambda_{T_{TR}} = \frac{C_{T_{TR}}}{2 \sqrt{\mu_{T_{TR}}^2 + \lambda_{T_{TR}}^2}}
\]  

(2.105)

Note that the equations 2.103 and 2.105 should be solved together for $\theta_{T_{TR}}$ given during the trim iteration.
Aerodynamics of Gurney Flap

3.1 Analytic Model for Gurney Flap

The analytic expression that captures primary characteristics of Gurney flap will be useful for rapid estimation of performance improvement achievable by Gurney flap. For tailing edge flap, the \( T_{10} \) function shown in equation 3.1 provides a first order estimate of the lift increment obtained by TEF.

\[
T_{10} = \sqrt{1 - c^2} + \cos^{-1} c \\
\Delta C_l = 2T_{10}\delta
\]  

(3.1)

where \( c \) denotes the distance from mid-chord to the flap hinge in terms of semi-chord, and \( \delta \) denotes TEF deflection. As reviewed in the chapter 1, the overall effect of the Gurney flap is well known from wind tunnel experiments and numerical analysis, however, a simple analytic expression similar to equation 3.1 can be hardly found.

3.1.1 Lift

One of the analytic expressions available from literature are shown in equation 3.2, used for performance calculation of rotor blade equipped with Gurney flap [96].

\[
\Delta C_l = 0.31858\left(\frac{h}{c}\right) - 0.07281\left(\frac{h}{c}\right)^2 + 0.00693\left(\frac{h}{c}\right)^3
\]  

(3.2)

where the \( h/c \) denotes the size of a Gurney flap. However, note that these
equations were derived from the test data on NACA 0020 airfoil, thus are strictly applicable to the specific airfoil only.

Recalling that the equation 3.1 is derived from thin-airfoil theory, Liu et al [113] made an attempt to formulate the theoretical prediction of the lift increment by the Gurney flap within the frame work of thin-airfoil theory. A generalized relationship between the Gurney flap height, \( h/c \), and the lift coefficient increment, \( \Delta C_l \) was proposed as

\[
\Delta C_l = q(Re) \sqrt{\frac{h}{c}}
\]  

(3.3)

where \( q(Re) \) is an empirical factor that reflects the influence of the airfoil characteristics based on the Reynolds number, which generally ranges from 2 to 5. All available published data in the literature were compared against the equation 3.3, and very good correlation between \( \frac{h}{c} \) and \( \Delta C_l/q \) should be noted.

The figure 3.1 compares the \( \Delta C_l \) prediction from the equation 3.2 and the equation 3.3. Considering the \( \Delta C_l \) from the equation 3.2 is a curve-fit from the experiments, the \( \Delta C_l \) predicted by the square-root law is in acceptable range.

### 3.1.2 Drag

The aerodynamic increment by each active Gurney flap segment is approximated by scaling the 1%e static MiTE table [97] with the analytic expression proposed in Reference [113]. The details of analytic expression which relates the lift increment to the size of the Gurney flap is discussed in detail in chapter 4. The drag increment is also modeled by the similar functional form as that of the lift increment. Based on the drag increment shown in figure 3.2, the correlation between Gurney flap size and drag increment of cambered airfoil becomes better when the exponent is 3/2. As the drag increment of 1% static Gurney flap size is known [97], it is also possible to estimate drag increment using 3/2 exponential law. The aerodynamic model used to simulated the lift and drag increment by Gurney flap is finally approximated as

\[
\Delta C_d = 0.135(C_d^{BSE})^{-\frac{3}{2}} \left(\frac{h}{c}\right)^{\frac{4}{3}}
\]  

(3.4)
\[ \Delta C_d = \Delta C_{d,1\%c} \times x^2 \]  

(3.5)

where \( x \) refers to the fraction of Gurney flap size with respect to 1\%c. For example, if the size of the Gurney flap is 2\%c, then \( x \) becomes 2. It should be noted that the 1\%c static MiTE table [97] is generated on the VR-12 airfoil, not on the SC-1095 airfoil. The increments or "deltas" in aerodynamic coefficients due to the deployment of the effectors are added to the aerodynamic coefficients of the baseline airfoil in an approach similar to that used by Yeo [114].

3.1.3 Pitching Moment

For the change in the pitching moment at the quarter chord, \( \Delta C_{m,0.25c} \) can be approximated as follows

\[ \Delta C_{m,0.25c} = -m(Re)\sqrt{\frac{h}{c}} \approx -0.25\Delta C_l \]  

(3.6)

The equation 3.6 was compared with wind tunnel experiment, and the comparison is depicted in the figure 3.3. The good correlation between experiment results and the equation 3.6 can be observed.

Note that the thin-airfoil theory can only provide lift and moment expressions.
Figure 3.1. Comparison of Equation 3.1 and Analytic $\Delta C_l$

Figure 3.2. The trend of drag increment versus Gurney flap size

Figure 3.3. $C_l$ change versus $C_m$ change [-]
3.2 CFD Analysis

The aerodynamic increments of the Gurney flap obtained from the experiment data becomes insufficient when those informations are used in rotorcraft analysis. It is due to the fact that the most of the experiments were conducted with narrow range of angle of attack as well as under the low Mach number condition only. For rotorcraft analysis, due to the wide variations in the aerodynamic conditions encountered by the rotor blade section, especially in forward flight case, the aerodynamic database that covers the combination of Mach number (0.0 \sim 1.0) and the angle of attack (\( -180^\circ \sim 180^\circ \)) would be essential. From the advancement in the CFD area, the aerodynamic table suitable for comprehensive analysis can be generated by performing series of runs to obtain the solution of Navier-Stokes equation around the airfoil at given combinations of Mach number and angle of attack [115], which now becomes one of the standard methods for making a 2D static airfoil table. The review of the current state-of-the art of the CFD technology applied for the rotorcraft is documented in the reference [116] in a comprehensive manner.

Due to the high computational costs of CFD runs, it would be beneficial to reduce the number of Mach number and Angle of attack combination, however, a set of Mach number and Angle of Attack that representing the trimmed condition should be included. An example of the aerodynamic table is depicted in the figure 3.4. The hatched region represents the combination of Mach number and Angle of Attack under trimmed condition.
3.2.1 TURNS CFD solver and validation

The solution to the compressible, viscous flow around the SC-1094R8 airfoil was obtained using TURNS code [117]. Navier-Stokes equation was solved using finite difference upwind numerical scheme. Inviscid fluxes are discretized by Roe’s upwind scheme while viscous fluxes are discretized using 2nd order central difference scheme. MUSCL (Monotone Upstream-centered Scheme for the Conservation Laws) approach was used in junction with TVD (Total Variation Diminishing) limiter to obtain up to the third order spacial accuracy. LU-SGS (Lower-Upper-Symmetric Gauss-Seidel) scheme is used to resolve temporal terms on the LHS in an implicit manner. Single block, C-type structured grid was used in TURNS solver. In order to resolve boundary layer, the height of the first grid cell at the airfoil surface is set to 5x10^{-5}. Spallart-Allmaras One equation model [118] was used for turbulence modeling. Fully turbulent boundary layers are assumed with chord Reynolds number of 4.8x10^6.

3.2.1.1 Validation with SC-1094R8 airfoil

In order to check the validity of the CFD solver, two validation studies were conducted for “clean” SC-1094R8 airfoil and VR-12 airfoil with Gurney flap, respectively. The validation results of CFD solver for SC-1094R8 airfoil are depicted in the figures 3.7 through 3.12. The experiment data used for validation of SC-1094R8 airfoil is obtained from reference [119]. $C_l$, $C_d$, and $C_m$ at Mach number 0.4 and 0.6 were presented in figures 3.7 through 3.12. Good correlation with experiment results are observed.

3.2.1.2 Validation with VR-12 with Gurney flap Experiment

Next, an attempt was made for the validation of CFD solution of the airfoil equipped with Gurney flap against experiment. Unfortunately, majority of the wind tunnel test results available in the literatures (for example, references [75–82,84]) were conducted at the low Mach number (or incompressible flow). Thus, comparison between the incompressible wind tunnel test results and the solution from compressible CFD solver will not be proper for validation.

In Ref. 83, wind tunnel test was conducted for VR-12 with 1.5%c Gurney
flap at Mach number 0.2 to 0.4 as a part of VDLE (Variable Droop Leading Edge) experiment. The Mach number range of this experiment is considered to be suitable for compressible solver validation, thus the validation of CFD solver used in present study was made with this test. The wind tunnel test was conducted by pitching airfoil under specified reduced frequency with fixed Gurney flap. For validation, test data at very low reduced frequency were compared with CFD results under the assumption that \( k=0.002 \) effectively represent “steady” airfoil properties. In order to rule out compressibility limit issue, CFD results at Mach number 0.4 was considered throughout this section. The study conducted in Ref. 88 includes the validation on the same data set, thus the CFD results of Ref. 88 were presented for the comparison.

Figures 3.13 and 3.14 show \( C_l \) versus \( \alpha \) for clean VR-12 and VR-12 with 1.5%c Gurney flap, respectively. The predictions for maximum \( C_l \) from both CFD analysis are higher than that of experiment for clean VR-12 configuration. Maximum \( C_l \) predictions for Gurney flap cases are closer to the test data than clean airfoil. However present TURNS result slightly overpredicts maximum \( C_l \) for Gurney flap configuration. Note that 2\(^\circ\) to 3\(^\circ\) difference in static stall angle can be observed between CFD results and experiment for both configurations.

Figures 3.15 and 3.16 show \( C_d \) versus \( \alpha \) for Clean VR-12 and VR-12 with 1.5%c Gurney flap, respectively. At low angle of attack \( C_d \) predictions from present TURNS compare well with Ref. 88. Both CFD results predict higher drag than that of experiment. However, the measured \( C_d \) data appears to be extremely low and even negative for the clean airfoil. No further explanation was given either at Refs. 83and 88 on this issue. At high angle of attack both CFD still compares well while both CFD result in underprediction compared to the test data. This is mainly due to the overprediction of stall angle by CFD compared to experiment.

Figures 3.17 and 3.18 shows \( C_m \) versus \( \alpha \) for Clean VR-12and VR-12 with 1.5%c Gurney flap, respectively. \( C_m \) was evaluated at 25% of airfoil chord. Constant offset of \( C_m \) between CFD predictions and experiment is noticeable for clean as well as Gurney flap configuration. This offset strongly implies the presence of additional trailing edge tab in the experiment model. It was unable to figure out the details from Ref. 83, thus the geometry used in present study was based on pure VR-12 airfoil shape. Same geometry was also used in CFD analysis in Ref. 88.
Nevertheless, it should be highlighted that CFD can predict offset of $C_m$ curve by Gurney flap.

### 3.2.2 CFD Analysis for Gurney flap at the Trailing Edge

The single block grid used for the trailing-edge Gurney flap is shown in Figure 3.5. The Gurney flap is modeled by shifting the branch-cut downward to the bottom of the Gurney flap and solid wall boundary condition was imposed on the inserted elements. Following the suggestion in the reference [120], 20 grid points per 1%c GF were used on the Gurney flap portion. The Gurney flap sizes considered in the present study are 0.5%, 1.0%, 1.5%, and 2.0%c.

The lift coefficients with 0.5, 1, 1.5, and 2%c Gurney flap amplitudes at Mach numbers 0.4 and 0.6 are shown in Figs. 3.19 and 3.20, respectively. As the height of the Gurney flap increases, the increase in the maximum $C_l$ can be clearly observed for both Mach numbers. With 2%c Gurney flap, maximum $C_l$ can be increased by 23% at $M=0.4$, and 45% at $M=0.6$ compared to the maximum $C_l$ of baseline SC-1094R8 airfoil.

The drag coefficients with 0.5, 1, 1.5, and 2%c Gurney flap amplitudes at Mach numbers 0.4 and 0.6 are shown in Figs. 3.21 and 3.22, respectively. Compared to TEFs, very large increases in $C_d$ are observed even at low to moderate angles of attack, and the drag increment continues to remain higher at angles of attack.
approaching stall, and beyond. At M=0.4, a 2%c Gurney flap results in increments in drag coefficient of 0.014 at $\alpha=6^\circ$ (100% increase over the baseline), and 0.033 at $\alpha=10^\circ$ (121% increase over the baseline). At the higher Mach number, M=0.6, the 2%c Gurney flap is seen to produce a drag increment of 0.041 at $\alpha=4^\circ$ (196% increase over the baseline).

The pitching moment coefficients for the different Gurney flap amplitudes are shown in Figs. 3.23 and 3.24 for the two Mach numbers. As the size of Gurney flap increases, larger nose-down pitching moment can be observed. From the figures, two effects of Gurney flap on baseline airfoil can be observed. One is the chord effect induced by the shift of aerodynamic center, which is reflected in the change of the slope of the pitching moment curves relative to the baseline airfoil. Note that the pitching moment slope does not vary with the size of Gurney flap. The other is the camber effect, as can be seen from the parallel shift of the pitching moment curves for increasing Gurney flap size.

3.2.3 CFD Analysis for 10%c upstream Gurney flap

The single block grid used for the upstream Gurney flap is shown in Fig. 3.6. The Gurney flap is modeled by inserting the upstream Gurney flap geometry at the lower surface of an airfoil. The grid was refined with elliptic smoother based on Laplace solver to resolve skewness present around the upstream Gurney flap. For upstream Gurney flap, the sizes considered are 1.0% and 2.0%c.

The lift coefficients with 1%c and 2%c 10%c upstream Gurney flap amplitudes at Mach numbers 0.4 and 0.6 are shown in Figs. 3.25 and 3.26, respectively. As the height of the Gurney flap increases, the increase in the maximum $C_l$ can be clearly observed for both Mach numbers. With 2%c Gurney flap, maximum $C_l$ can be increased by 13% at M=0.4, and 23.4% at M=0.6 compared to the maximum $C_l$ of baseline SC-1094R8 airfoil.

The drag coefficients with 1%c and 2%c 10%c upstream Gurney flap amplitudes at Mach numbers 0.4 and 0.6 are shown in Figs. 3.27 and 3.28, respectively. Very large increases in $C_d$ are observed even at low to moderate angles of attack, and the drag increment continues to remain higher at angles of attack approaching stall, and beyond. At M=0.4, a 2%c Gurney flap results in increments in drag coefficient.
Figure 3.6. CFD grid for 10\%c upstream Gurney flap

coefficient of 0.016 at $\alpha=6^\circ$ (115\% increase over the baseline), and 0.023 at $\alpha=10^\circ$ (82\% increase over the baseline). At the higher Mach number, $M=0.6$, the 2\%c Gurney flap is seen to produce a drag increment of 0.036 at $\alpha=4^\circ$ (174\% increase over the baseline).

The pitching moment coefficients for the different Gurney flap amplitudes are shown in Figs. 3.29 and 3.30 for the two Mach numbers. As the size of Gurney flap increases, nose-down pitching moment is increased. Compared to the Gurney flap at the trailing edge, the amount of nose-down pitching moment is smaller for the same Gurney flap input. As seen from the previous section, the upstream movement also shifts the circulation pocket where the pressure difference is generated. As the moment arm is reduced with respect to the aerodynamic center, increase in the nose-down pitching moment becomes smaller than that of the Gurney flap located at the trailing edge.

3.3 The Unsteady Aerodynamic Model

The unsteady aerodynamic model of the reference [101] is implemented. Note that the unsteady models developed in references [121, 122] was integrated into basic model of flapping rigid blade with cosine inflow model. The unsteady variation of the airload was only presented in reference [101]. In present study, the unsteady
aerodynamic model will be integrated into the high-fidelity model, which has the elastic blade beam model, non-uniform inflow distribution by prescribe wake, and high-fidelity model of UH-60 helicopter including the fuselage aerodynamic properties, as explained in the chapter 2.

The unsteady dynamic effect of Gurney flap actuation is given in figures 3.31 and 3.32. In the legend of figures, “CFD(JIM)” refers to the CFD analysis for VR-7 from reference [123]. For UH-60, k=0.25 corresponds to the 4/rev frequency if Gurney flap is installed at 50%R on the hovering rotor. Note that reduced frequency k needs to be evaluated per azimuth in case of forward flight. As can be seen from the figures, the vortex effect due to the upstream Gurney flap actuation becomes more pronounced at the high reduced frequency.

The unsteady drag equation was proposed based on the static drag coefficient variation of SC-1094R8 airfoil. The expression for unsteady drag is given in equation 3.7

$$\Delta C_d = 2.0\left(\frac{h_{\text{eff}}}{c}\right)^{\frac{4}{3}}$$

where the h refers to the instantaneous height as derived in the reference [101]. Figures 3.33 and 3.34 show the unsteady drag variation predicted by equation 3.7 and unsteady CFD results from reference [123]. Considering the simplicity of the expression for $\Delta C_d$, good agreement between CFD data is observed for the reduced frequency value considered.

For the unsteady moment, the unsteady pitching moment change can be related with the same form as equation 3.6. For better correlation with CFD data, the coefficient -0.25 was adjusted to best match the CFD data. The expression for unsteady pitching moment variation is given in equation 3.8.

$$\Delta C_m = -0.19\Delta C_{l_{\text{unsteady}}}$$

where $C_{l_{\text{unsteady}}}$ refers to the unsteady $\Delta C_l$ expression derived in the reference [101]. The unsteady pitching moment prediction from simple equation given in equation 3.8 is compared against CFD data of reference [123] in figures 3.35 and 3.36. Due to the limit of simple equation, the detailed characteristics like a sharp peak due to the vortex moment cannot be captured. However, the moment
prediction of equation 3.8 predicts overall trend as well as the mean slope of the airload variation.
Figure 3.7. Lift Coefficient of SC-1094R8 airfoil at M=0.4

Figure 3.8. Lift Coefficient of SC-1094R8 airfoil at M=0.6
Figure 3.9. Drag Coefficient of SC-1094R8 airfoil at $M=0.4$

Figure 3.10. Drag Coefficient of SC-1094R8 airfoil at $M=0.6$
Figure 3.11. Pitching Moment Coefficient of SC-1094R8 airfoil at M=0.4

Figure 3.12. Pitching Moment Coefficient of SC-1094R8 airfoil at M=0.6
Figure 3.13. Lift Coefficient of Clean VR-12

Figure 3.14. Lift Coefficient of VR-12 with 1.5%c GF
Figure 3.15. Drag Coefficient of Clean VR-12

Figure 3.16. Drag Coefficient of VR-12 with 1.5%c GF
Figure 3.17. Moment Coefficient of Clean VR-12

Figure 3.18. Moment Coefficient of VR-12 with 1.5%c GF
Figure 3.19. Lift Coefficient of GF at M=0.4

Figure 3.20. Lift Coefficient of GF at M=0.6
Figure 3.21. Drag Coefficient of GF at M=0.4

Figure 3.22. Drag Coefficient of GF at M=0.6
Figure 3.23. Pitching Moment Coefficient of GF at M=0.4

Figure 3.24. Pitching Moment Coefficient of GF at M=0.6
Figure 3.25. Lift Coefficient of upstream GF at M=0.4

Figure 3.26. Lift Coefficient of upstream GF at M=0.6
Figure 3.27. Drag Coefficient of upstream GF at M=0.4

Figure 3.28. Drag Coefficient of upstream GF at M=0.6
Figure 3.29. Pitching Moment Coefficient of upstream GF at M=0.4

Figure 3.30. Pitching Moment Coefficient of upstream GF at M=0.6
Figure 3.31. Unsteady $\Delta C_l$ at $k=0.25$

Figure 3.32. Unsteady $\Delta C_l$ at $k=1.0$
Figure 3.33. Unsteady $\Delta C_d$ at $k=0.25$

Figure 3.34. Unsteady $\Delta C_d$ at $k=1.0$
Figure 3.35. Unsteady $\Delta C_m$ at $k=0.25$

Figure 3.36. Unsteady $\Delta C_m$ at $k=1.0$
Chapter 4

Preliminary Study

In the present chapter, a preliminary study is conducted to examine the effect of an active Gurney flap on the rotor. Two different rotor models are considered. The first part of this chapter uses a rigid blade model. The effect of active Gurney flap span, 10%c upstream location, and the influence of unsteady aerodynamics are examined using the rigid blade model. Additional studies considering gross weight variation and 2/rev actuation are also presented.

The second part of the chapter considers a fully coupled flap-lag-torsion elastic blade model discussed in chapter 2. This has the ability to capture the influence of the pitching moment increment, $\Delta C_m$, due to active Gurney flap deployment in twisting the rotor blade elastically. On the elastic blade model, the effect of span of Gurney flap, and the influence of unsteady aerodynamic model were also considered.
4.1 Baseline Helicopter Model

UH-60A Black Hawk helicopter is chosen as the baseline configuration which has a single main rotor and a tail rotor. The main rotor has a 3° built-in shaft tilt so that excessive nose down attitude can be avoided in high speed forward flight. The main rotor has 4 blades and each has 26.83 ft radius. The main rotor has nominal chord length of 1.73 ft, and reference RPM is 258 or 27 rad/sec. The details on the aerodynamic properties of the airfoil sections used in the UH-60A main rotor are available in references [124, 125]. The inflow of the main rotor is obtained using a prescribed wake. The aerodynamic characteristics are obtained from 2D quasi-steady tables. Note that the effect of unsteady aerodynamics of baseline airfoils are not considered in the present study although the unsteady effect introduced by active Gurney flap motions are considered. The tail rotor has a $-18^\circ$ twist and a cant angle of $20^\circ$, but the twist of tail rotor is not included in the present study. The thrust of tail rotor is calculated using the simple equation discussed in the chapter 2. Recall that the uniform inflow model is applied for the tail rotor thrust calculation.
4.2 Part I : Rigid Blade Model

The torque of rotorcraft is determined from steady component of hub yaw moment, $M^0_z$. Therefore, the 0/rev (steady component) and 1/rev component of blade response have primary effect on the rotorcraft power while higher harmonic components have little influence on the performance calculation. The rigid blade model is known to be good for capturing the primary physics of rotorcraft performance to a satisfactory level [111,126]. Furthermore, due to the low computational cost compared to that of the full elastic blade model, the rigid blade model is attractive for parametric design study which requires lots of runs to be made per system parameter change. In the present section, the rigid flapping blade model was simulated by retaining only the first flap mode (rigid flapping mode) from modal reduction. As a result, all of the deformation but rigid flapping rotation is effectively suppressed inside the FEM solutions.

The main rotor power and trim variables were compared against the flight test available from References [127]. The weight coefficient of $C_w/\sigma = 0.079$ was chosen as a reference configuration. Propulsive trim was conducted at a given flight speed. The flight speed was increased until the helicopter can no longer satisfy the balance of trim equations.

The gross weight of helicopter corresponds to 18,300 lbs at sea level condition. Figure 4.1 shows the power loading prediction from the rigid blade analysis and that from flight test data. Good correlation can be observed, but overprediction in high speed regime ($\mu>0.35$) is noted. Rotor Controls are compared with flight test, and depicted in figures 4.2 through 4.4. Collective pitch shows excellent correlation with test data while both of cyclic pitch input show constant offset from test data. The overall trend of rotor controls with respect to the flight speed is well captured using rigid blade model. Fuselage attitude angles are compared against those from flight test in figures 4.5 and 4.6. Note that fuselage pitch attitude is defined as "positive nose up", and roll attitude is defined as "positive advancing side down." Overprediction of 1 to 2 degrees in pitch attitude is observed, but the overall trends with increase in flight speed is well captured. However, the roll attitude shows large difference especially at advance ratio greater than 0.25.
Figure 4.1. Power Loading prediction for UH-60A

Figure 4.2. Collective Pitch prediction for UH-60A
Figure 4.3. Lateral Cyclic Pitch prediction for UH-60A

Figure 4.4. Longitudinal Cyclic Pitch prediction for UH-60A
Figure 4.5. Fuselage Pitch Attitude prediction for UH-60A

Figure 4.6. Fuselage Roll Attitude prediction for UH-60A
4.2.1 Velocity Sweep of Baseline Helicopter

From the previous studies on the application of high lift devices for performance improvement \[102, 103, 126, 128\], the identification of flight envelope is important because the benefit of high lift device appears near the edge of flight envelope. In order to examine the flight boundary, three different gross weight of 16,000 lbs, 18,300 lbs, and 22,000 lbs were examined while increasing the forward flight speed until the helicopter can be trimmed. Altitude was set at 8,000 ft, where the air density value is very similar to “army hot day condition (4,000 ft, 95°F).” Figure 4.7 depicts the main power curve with respect to the flight speed at a different gross weight. At a gross weight of 22,000 lbs, the maximum forward flight is limited up to 108 knots, while the helicopter can be trimmed up to 171 knot with the other two gross weight levels considered (assuming available power). In order to check whether the rotor is in stall or not, the angle of attack distributions at the highest flight speed condition of each gross weight were examined, and shown in figures 4.8 through 4.10. Note that the upper limit of the angle of attack contour plot is limited to 15°, which represents nominal value of the stall angle at low Mach number regime. It can be observed from the figures 4.9 and 4.10 that the angle of attack over the retreating side of the rotor disk is greater than 15°, while the angle of attack distribution shown in figure 4.8 does not exceed 15°.
Figure 4.7. Main Rotor Power at Different Gross weights (8,000 ft)

Figure 4.8. Angle of Attack Distribution at GW=16,000 lbs
Figure 4.9. Angle of Attack Distribution at GW=18,300 lbs

Figure 4.10. Angle of Attack Distribution at GW=22,000 lbs
4.2.2 1/rev phase sweep at stall velocity

From the previous section, the stalled condition of each gross weight was examined with angle of attack distribution. When the rotor is close to the stall condition, the collective pitch was maximized to increase the thrust of the rotor while large amount of negative longitudinal cyclic pitch was required to maintain the roll balance especially at the high speed condition. Therefore, if active control by way of active Gurney flap actuation can reduce both the collective pitch and longitudinal cyclic pitch requirement of the baseline stalled rotor, the rotor can be recovered to normal operating condition.

The active Gurney flap can produce this type of aerodynamic effect due to its ability to increase $C_{l,max}$. In the configuration considered, the Gurney flap actuation height cannot be a negative value, thus the deployment of Gurney flap can only add the lift with respect to the baseline airfoil. In other words, the application of Gurney flap can reduce the collective pitch requirement for a given thrust level. Furthermore, if the Gurney flap is actuated under 1/rev harmonic function, the cyclic pitch requirement can be also altered. The deployment schedule of $h/c$ is shown in equation 4.1.

$$h_{c} (\psi) = \bar{h}_{max} \frac{2}{2} + \bar{h}_{max} \sin(\psi - \phi_{1P})$$ \hspace{1cm} (4.1)

Where $\bar{h}_{max}$ denotes the maximum amplitude of Gurney flap height normalized by the reference chord, and $\phi_{1P}$ denotes the phase angle of 1/rev actuation. Figure 4.11 shows the active Gurney flap control input for different values of 1/rev phase angle, $\phi_{1P}$. Note that $\phi_{1P} = 180^\circ$ corresponds to maximum deployment of active Gurney flap deployment on the retreating side ($\psi = 270^\circ$).

In the present section, the Gurney flap spanning from 70%$R$ to 80%$R$ was actuated under 1/rev frequency with varying $\phi_{1P}$. maximum amplitude of Gurney flap, $\bar{h}_{max}$ is set to 2%c. The Gurney flap is attached at the trailing edge of airfoil section. Quasi-steady airload model is used for the Gurney flap at the trailing edge. The rotor was re-trimmed with Gurney flap activated under 1/rev frequency with a specified phase angle.
Figure 4.11. Active Gurney flap deployment schedule with different $\phi_{1P}$
4.2.2.1 Gross Weight 16,000 lbs

Active Gurney flap 1/rev phase sweep conducted at 171 knots are shown in the figure 4.12. The main rotor power of baseline helicopter is 3264 HP, and the minimum power achievable with 1/rev Gurney flap actuation is 3278 HP, which yields 0.4% increase in main rotor power. This implies that the drag penalty due to active Gurney flap deployment appears to be greater than the benefits due to alleviating what was a mildly stalled condition. The angle of attack distribution of Gurney flap at the best phase angle, $\phi_{1P} = 180^\circ$, is shown in figure 4.13. Comparing with the angle of attack distribution depicted in figure 4.8, the angle of attack is reduced over the retreating side. The baseline lift distribution, $M^2\bar{c}C_l$, is presented in figure 4.14, and the difference between the lift distribution of Gurney flap with $\phi_{1P} = 180^\circ$ and that of baseline configuration. As can be seen in figure 4.15, the lift is added primarily on the front and back of the rotor disk while off loading the outer rim of the rotor disk. The drag distribution of the baseline configuration and the difference between the minimum power phase and the baseline are shown in figures 4.16 and 4.17, respectively. Overall decrease of the drag can be observed, while the increase in drag around the advancing side and $\Psi = 225^\circ$. The increase in drag around the “band” is due to the drag penalty of Gurney flap.
Figure 4.12. Phase Sweep at 1/rev Frequency, GW=16,000 lbs

Figure 4.13. Angle of Attack Distribution at $\phi_{1P}=180^\circ$
Figure 4.14. Baseline Lift Distribution, $M^2 \bar{C}_l$

Figure 4.15. Difference in $M^2 \bar{C}_l$ (GF-Baseline)
Figure 4.16. Baseline Drag Distribution, $M^2 \bar{C}_d$

Figure 4.17. Difference in $M^2 \bar{C}_d$ (GF-Baseline)
4.2.2.2 Gross Weight 18,300 lbs

1/rev phase sweep was conducted at a higher gross weight of 18,300 lbs. 1/rev phase angle versus main rotor power is depicted in figure 4.18. The main rotor power is reduced by 2.3% at phase angle of 180°. Comparing the angle of attack distribution in figure 4.19 with that of the baseline shown in figure 4.9, the recovery of retreating blade stall can be observed. The baseline lift distribution is depicted in figure 4.20. Due to the high angles of attack around retreating side, the lift distribution is more concentrated around the front and back of the rotor disk. Gurney flap actuation results in unloading of the blade tips at the front and rear of the disk, a reduction in negative lift at the advancing blade tip and an increase in lift over the annulus where the active Gurney flap is present (Figure 4.21). The effect of unloading the tip can be observed from the drag difference plot shown in figure 4.23. Note that the baseline drag distribution has high drag concentrated at the tip of the fourth quadrant of the rotor disk. By applying the Gurney flap which provides maximum lift at the retreating side, the drag at the outer rim is substantially reduced, which led to the power reduction.
Figure 4.18. Phase Sweep at 1/rev Frequency, GW=18,300 lbs

Figure 4.19. Angle of Attack Distribution at φ₁P=180°
Figure 4.20. Baseline Lift Distribution, $M^2 \bar{c}Cl$

Figure 4.21. Difference in $M^2 \bar{c}Cl$ (GF-Baseline)
Figure 4.22. Baseline Drag Distribution, $M^2 \bar{C}_d$

Figure 4.23. Difference in $M^2 \bar{C}_d$ (GF-Baseline)
4.2.2.3 Gross Weight 22,000 lbs

Unlike the previous two gross weight cases, the maximum forward flight speed is limited to 110 knots ($\mu = 0.25$) at this gross weight level. At this flight speed, the Mach number at the advancing side and the retreating side are 0.763 and 0.566, respectively. Considering the hover tip Mach number of baseline helicopter ($M_{tip}^{hover} = 0.65$), the roll imbalance tends to be lower than that from the high speed condition. Thus, the longitudinal cyclic pitch requirement is lower than that is required at the high speed condition. Thus the stall is primarily caused due to the increase in collective pitch to sustain high gross weight.

The variation of main rotor power with respect to the change of the 1/rev phase angle is depicted in figure 4.24. Unlike previous cases, power reduction can be observed over all of the phase angle range. Similar to the previous cases, the minimum power is achieved at a phase angle around $180^\circ$. The maximum reduction of 7.4% is noted. Similar to the 18,300 lbs case, the recovery of retreating blade stall is clearly observed when comparing the angle of attack contours of baseline (figure 4.10) and best phase angle Gurney flap case (figure 4.25).

Figure 4.26 represents stalled lift distribution of baseline rotor. As angle of attack is high over the retreating side, the lift is concentrated mostly on the front and back of the rotor disk. The effect of Gurney flap operated with $\phi_{1P} = 180^\circ$ is shown in figure 4.27. For a given thrust level to be satisfied, the lift increment from Gurney flap generally reduces the lift requirement over the outer rim of the rotor. The collective pitch is reduced from 11 deg for the baseline to 10 deg with Gurney flap actuation.

The drag distribution of the baseline and its difference between minimum power case and the baseline are shown in figures 4.28 and 4.29, respectively. Overall reduction of drag can be observed, and high drag concentrated around the tip of the fourth quadrant is reduced due to the unloading at the tip region.
Figure 4.24. Phase Sweep at 1/rev Frequency, GW=22,000 lbs

Figure 4.25. Angle of Attack Distribution at $\phi_{1P}=180^\circ$
Figure 4.26. Baseline Lift Distribution, $M^2 \bar{c}C_l$

Figure 4.27. Difference in $M^2 \bar{c}C_l$ (GF-Baseline)
Baseline Drag distribution

Figure 4.28. Baseline Drag Distribution, $M^2\bar{C}_d$

Difference in Drag (GF−Baseline)

Figure 4.29. Difference in $M^2\bar{C}_d$ (GF-Baseline)
4.2.3 Effect of Gurney flap Span

For the on-blade aerodynamic actuator, the span of aerodynamic devices are one of the important design parameter due to the compromise between the efficiency of the device (lift capability) and the cost of the device (drag penalty). Various span of Gurney flap have been studied, which ranges from 2%\( R \) strip [129] to whole span of the rotor blade [99]. In the present section, the effect of increasing the span of Gurney flap will be examined. For UH-60A rotor blade, the SC-1094R8 airfoil spans from 49%\( R \) to 83%\( R \), and the Gurney flap aerodynamic characteristics were evaluated on the SC-1094R8 airfoil as discussed in the chapter 3. The actuation authority of the aerodynamic device is proportional to the dynamic pressure, thus it would be advantageous to place Gurney flap closer to the tip. Thus, the outer edge of Gurney flap is held fixed at 80%\( R \) while the inner edge was varied from 75%\( R \) to 50%\( R \), with 5%\( R \) interval. The Gurney flap is applied at the stall speed of 108 knot with gross weight of 22,000 lbs, and is actuated under 1/rev frequency with phase angle of 180\( ^\circ \), where the minimum power occurs.

Figure 4.30 shows the main rotor power reduction with change in the inboard limit of Gurney flap. Due to the stall recovery realized by the Gurney flap, main rotor power is lower than that of the baseline for the range of inboard location considered. It is observed that as the span of Gurney flap increases, the main rotor power is reduced up to 9% of the baseline power, as depicted in figure 4.31.

The trim control settings of baseline, 10%\( R \) span Gurney flap, and 30%\( R \) span Gurney flap are presented in the table 4.1. Reduction in the rotor controls can be observed. As discussed previously, the Gurney flap actuated under 1/rev frequency with 180° phase angle increases the mean lift and provides the lift augmentation at the retreating side. For the 30%\( R \) span Gurney flap configuration, the collective pitch is reduced by 1.4° due to the mean lift augmentation, and the longitudinal cyclic pitch requirement is reduced by 1° due to the lift augmentation at the retreating side.

The angle of attack distribution of 30%\( R \) span configuration is presented in figure 4.32. Comparing with figure 4.25, it can be observed that the angles of attack over the retreating side are further reduced. The difference of lift distribution is depicted in the figure 4.33. The lift increase from 50%\( R \) to 80%\( R \) is clearly seen, which represents the direct lift augmentation supplied by the 30%\( R \) wide Gurney
Table 4.1. Trim solution (GW=22,000 lbs, 108 knots)

<table>
<thead>
<tr>
<th>Trim Control</th>
<th>Baseline</th>
<th>GF (70%R - 80%R)</th>
<th>GF (50%R - 80%R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>11.0</td>
<td>10.1</td>
<td>9.61</td>
</tr>
<tr>
<td>$\theta_{IC}$</td>
<td>4.16</td>
<td>3.66</td>
<td>3.61</td>
</tr>
<tr>
<td>$\theta_{LS}$</td>
<td>-9.14</td>
<td>-8.48</td>
<td>-8.11</td>
</tr>
<tr>
<td>$\alpha_{WL}$</td>
<td>-0.77</td>
<td>-0.43</td>
<td>-0.39</td>
</tr>
<tr>
<td>$\phi_{WL}$</td>
<td>-0.72</td>
<td>-0.70</td>
<td>-0.72</td>
</tr>
<tr>
<td>MR Power (HP)</td>
<td>1842</td>
<td>1705</td>
<td>1673</td>
</tr>
<tr>
<td>Reduction</td>
<td>-</td>
<td>7.4%</td>
<td>9%</td>
</tr>
</tbody>
</table>

flap. The offloading of the rotor lift is observed around the tip region owing to the reduction of collective pitch.
Figure 4.30. Inboard Location of Gurney flap versus Main Rotor Power

Figure 4.31. Inboard Location of Gurney flap versus Reduction of Main Rotor Power
Figure 4.32. Angle of Attack Distribution (30%R span)

Figure 4.33. Difference in $M^2 c C_l$ (30%R span)
4.2.4 Effect of Gurney flap chordwise location

The examination of the performance improvement was conducted with the Gurney flap placed at the trailing edge of SC-1094R8 airfoil in the previous sections. However, the actual implementation of the Gurney flap as an active control device requires the space to retract the Gurney flap inside of an airfoil. Therefore, moving the Gurney flap location upstream from the trailing edge is inevitable. Note that the lift increment is reduced as the Gurney flap is moved upstream of the trailing edge for a given Gurney flap height. In order to retain the similar lift increment, an upstream Gurney flap needs to be deployed with a greater amplitude than that of the Gurney flap at 100%c. The Gurney flap at the trailing-edge represents the upper limit of the benefits achievable and the upper limit in the rotor performance improvement. In the present section, an upstream Gurney flap located at the 90%c of an airfoil is examined and compared to the Gurney flap at the trailing edge for performance improvement.

A 1/rev phase sweep was conducted using 10%c upstream Gurney flap at the same condition as previous section - a gross weight of 22,000 lbs, a flight speed of 108 knots, and an altitude of 8,000 ft. Figure 4.34 depicts the variation of the main rotor power with the change of phase angle $\phi_{1P}$. 1/rev actuation of upstream Gurney flap with 180$^\circ$ reduces the main rotor power by 6%. Note that the power reduction of 7.4% is achieved with Gurney flap at the trailing edge.

The section drag $M^2\bar{c}C_d$ is related to the main rotor power. The section drag at the 75%R around rotor revolution is presented in figure 4.35. The section drag of the baseline rotor, which is in stalled condition, is very high especially over the retreating side. By applying the Gurney flap, the section drag level drops down 50% of baseline peak value owing to the reduction in the collective pitch requirement.

The difference in lift distribution between baseline and upstream Gurney flap actuated with $\phi_{1P} = 180^\circ$ is shown in figure 4.36. The unloading of the main rotor blade tips and augmentation of the lift by upstream Gurney flap are observed, which is consistent with all of the previous cases. Figure 4.37 depicts the sectional drag difference with respect to that of the baseline at stalled condition. Due to the reduction of the lift at the tip, which implies the angle of attack is reduced, reduction in drag in the tip region can be observed especially over the tip region.
of the retreating side.
Figure 4.34. Phase Sweep with Gurney flap at 100\%c and 90\%c

Figure 4.35. Section Drag $M^2\bar{c}C_d$ at 75\%R versus azimuth angle $\Psi$
Figure 4.36. Difference in $M^2 \bar{c} C_l$ with 10\%c upstream Gurney flap

Figure 4.37. Difference in $M^2 \bar{c} C_d$ with 10\%c upstream Gurney flap
4.2.5 Effect of Unsteady Aerodynamic Model

When the angle of attack of an airfoil is changed in time, airloads corresponding to the change of input usually have a phase lag as well as amplitude attenuation (relative to quasi-static aerodynamic model). In other words, the airloads require a certain time to build up following the input. In general, such an unsteady effect is a function of oscillation frequency of the input, $\omega$.

For the airfoil equipped with Gurney flap, the pressure distribution near the trailing edge portion is changed with the height of the Gurney flap. Therefore, the lift increment obtained with Gurney flap becomes a function of Gurney flap height as discussed in the chapter 3. If the height of Gurney flap is actuated at a certain frequency, the lift change is at the same frequency. Furthermore, unsteady effects are present as well. As examined in the chapter 3, the unsteady effect becomes more pronounced when the chordwise position of active Gurney flap is moved upstream of the trailing edge of the airfoil. In the present section, the effect of active Gurney flap unsteady aerodynamics is examined. For the rotor power reduction in stalled condition, the unsteady aerodynamic model discussed in the chapter 3 is integrated into rotor analysis. The 10% $c$ upstream Gurney flap is actuated with 1/rev frequency, equivalent to the mean reduced frequency $\tilde{k} = 0.075$, while varying the phase angle.

Figure 4.38 presents the phase angle versus main rotor power at the gross weight 22,000 lbs and flight speed of 108 knots. For comparison, 1/rev phase sweep results of quasi-steady model of 10%$c$ upstream Gurney flap is also presented. Comparing with the quasi-steady aerodynamic model, the power reduction with the unsteady aerodynamic model are lower than that of the quasi-steady model. The minimum power is 20 HP higher than the quasi-steady case, or about 1% increase in main rotor power.

The change in power can be related with the variation of section drag $M^2 \tilde{c} C_d$. The section drag at the 75%$R$ around one rotor revolution is shown in figure 4.39. By actuating the upstream Gurney flap under 1/rev with phase angle of 180°, the section drag reduces by over 20% from baseline peak value.

The difference in lift and drag with respect to that of the baseline airloads are depicted in figures 4.40 and 4.41, respectively. The lift distribution shows similar pattern as that of quasi-steady aerodynamic model shown in figure 4.36, and the
lift is unloaded around the outer rim. For the drag difference, the increase in drag is observed from \( \psi = 0^\circ \) to \( 270^\circ \), which is due to the drag increase caused by the unsteady aerodynamic model discussed in the chapter 3.
Figure 4.38. Phase Sweep with Gurney flap at 90% c, with and without unsteady effect

Figure 4.39. Section Drag $M^2\bar{c}C_d$ at 75% R versus azimuth angle $\Psi$
Figure 4.40. Difference in $\bar{M}^2\bar{c}C_l$ with 10\%c upstream Gurney flap, unsteady model

Figure 4.41. Difference in $\bar{M}^2\bar{c}C_d$ with 10\%c upstream Gurney flap, unsteady model
4.2.6 Effect of Gross Weight

Three different gross weight conditions were analyzed for identifying the maximum forward speed in the previous sections. At the highest gross weight of 22,000 lbs, the maximum forward flight speed at which the aircraft could be trimmed was 108 knots while the maximum forward flight speed is 171 knots for the gross weight of 16,000 lbs. When the rotor is stalled at the maximum flight speed, the Gurney flap was shown to alleviate stall and can reduce the main rotor power up to 7%.

In the present section, the rotor stall behavior and its recovery using Gurney flap will be examined from a different aspect - the increase in main rotor thrust capability will be examined. Given the flight speed and altitude, the gross weight of helicopter is increased until it can be trimmed. Considering the propulsive force is function of flight speed, the propulsive force can be assumed to be constant. Therefore, the maximum gross weight effectively represents the maximum thrust level that can be achieved with the rotor. Flight speed of 150 knots and altitude of 8,000 ft were chosen as a reference condition. Three different aerodynamic models of Gurney flap discussed in previous sections were used for overall comparison. Based on the results of previous sections, the actuation phase angle was fixed to $180^\circ$ where the maximum reduction is obtained under 1/rev actuation frequency.

Figure 4.42 shows the Main rotor power versus gross weight sweep for the baseline helicopter and various active Gurney flap models. The baseline rotor stalls at a gross weight of 19,600 lbs, while the maximum thrust of the rotor with active Gurney flap is further increased by 1,500 lbs (active Gurney flap at the trailing edge, quasi-steady aero), and by 1,000 lbs (active Gurney flap at 90\%c, unsteady aero). Note that when quasi-steady aerodynamics is used Gurney flap starts to exhibit the benefit if the gross weight of helicopter is greater than 17,000 lbs. When the performance prediction includes the effect of unsteady aerodynamics, benefit can be observed from gross weight of 19,000 lbs upward.

The trim controls of baseline and three different Gurney flap models are compared in the table 4.2. The comparison was made at the gross weight of 19,600 lbs where the baseline rotor stalls. Main rotor power can be reduced by 6.1\% when the active Gurney flap at the trailing edge is used. The large reductions in collective pitch should be noted, which are direct consequences of the mean lift augmentation. When the active Gurney flap is moved to 10\%c upstream, the power
Table 4.2. Trim solution (GW=19,600 lbs, 150 knots, 8,000 ft)

<table>
<thead>
<tr>
<th>Trim Control</th>
<th>Baseline</th>
<th>GF@100%c</th>
<th>GF@90%c</th>
<th>GF@90%c (Unsteady)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>13.5</td>
<td>12.4</td>
<td>12.7</td>
<td>12.6</td>
</tr>
<tr>
<td>$\theta_{IC}$</td>
<td>5.44</td>
<td>4.80</td>
<td>4.90</td>
<td>4.89</td>
</tr>
<tr>
<td>$\theta_{IS}$</td>
<td>-11.7</td>
<td>-10.6</td>
<td>-10.9</td>
<td>-10.9</td>
</tr>
<tr>
<td>$\alpha_{WL}$</td>
<td>2.65</td>
<td>3.15</td>
<td>3.06</td>
<td>3.08</td>
</tr>
<tr>
<td>$\phi_{WL}$</td>
<td>-1.80</td>
<td>-2.01</td>
<td>-1.98</td>
<td>-2.03</td>
</tr>
<tr>
<td>MR Power (HP)</td>
<td>2558</td>
<td>2403</td>
<td>2434</td>
<td>2466</td>
</tr>
<tr>
<td>Reduction</td>
<td>-</td>
<td>6.1%</td>
<td>4.85%</td>
<td>3.60%</td>
</tr>
</tbody>
</table>

Reduction is 4.9%. Furthermore, if the unsteady effect is being accounted for, the power reduction goes down below 3.6%. As discussed in the previous sections, the active Gurney flap at the trailing edge represents the “ideal” upper limit, while the upstream active Gurney flap with unsteady effect represents the level in practice.
Figure 4.42. Gross Weight Sweep at Altitude 8,000 ft, V=150 knots
Table 4.3. Trim solution (GW=22,000 lbs, 108 knots, 8,000 ft)

<table>
<thead>
<tr>
<th>Trim Control</th>
<th>Baseline</th>
<th>GF@TE($\phi_{1P} = 180^\circ$)</th>
<th>GF@TE($\phi_{2P} = 300^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>11.0</td>
<td>10.1</td>
<td>10.0</td>
</tr>
<tr>
<td>$\theta_{1C}$</td>
<td>4.16</td>
<td>3.66</td>
<td>3.64</td>
</tr>
<tr>
<td>$\theta_{1S}$</td>
<td>-9.14</td>
<td>-8.48</td>
<td>-8.71</td>
</tr>
<tr>
<td>$\alpha_{WL}$</td>
<td>-0.77</td>
<td>-0.43</td>
<td>-0.43</td>
</tr>
<tr>
<td>$\phi_{WL}$</td>
<td>-0.72</td>
<td>-0.70</td>
<td>-0.67</td>
</tr>
<tr>
<td>MR Power (HP)</td>
<td>1842</td>
<td>1705</td>
<td>1712</td>
</tr>
<tr>
<td>Reduction</td>
<td>-</td>
<td>7.4%</td>
<td>7.1%</td>
</tr>
</tbody>
</table>

4.2.7 Effect of 2/rev actuation frequency

The 2/rev actuation frequency is known to be beneficial for performance improvement. As reviewed in the chapter 1, it was demonstrated on the wind tunnel test and flight test that the properly phased 2/rev actuation frequency can reduce the main rotor power up to 6%. The power reduction mechanism of 2/rev frequency was explained by the redistributing (spreading) of airloads over retreating side to relieve the thrust requirement [130]. In the previous sections, it is observed that the airloads generated under 1/rev Gurney flap input interacts with the rotor control. Similarly for 2/rev input, the airloads generated by 2/rev frequency introduce interference to the main rotor control as can be seen from the table 4.3.

In the present section, the effect of 2/rev actuation of Gurney flap on the main rotor power reduction will be examined. The control input of Gurney flap height is given in equation 4.2.

$$\frac{h}{c}(\psi) = \frac{\bar{h}_{\text{max}}}{2} + \frac{\bar{h}_{\text{max}}}{2} \sin(2\psi - \phi_{2P}) \quad (4.2)$$

The 2/rev phase angle $\phi_{2P}$ will be changed while satisfying the force and moment equilibrium equations at the stall boundary of gross weight 22,000 lbs. The flight speed is 180 knots, and the altitude is 8,000ft. The three different Gurney flap model discussed in the previous sections are used.

The 2/rev phase angle versus the main rotor power is shown in figure 4.43. All three Gurney flap model reduce the main rotor power. Furthermore, the main rotor power variation is less sensitive with respect to the change of 2/rev phase angle compared to that of the 1/rev phase sweep results as shown in figs 4.24, 4.34,
and 4.38. The maximum power reductions achieved are 7\% (for Gurney flap at 100\%c), 4.9\% (for Gurney flap at 90\%c), and 2.55\% (for Gurney flap at 90\%c with unsteady effect). In order to examine the effect of 2/rev frequency in detail, Gurney flap at 100\%c case was chosen due to the similar power reduction observed for that of the 1/rev actuation case. As the power reduction is similar, the effect from the different actuation frequencies will be clearly revealed.

The angle of attack distribution around the rotor disk is shown in figure 4.44. The area where the angle of attack is greater than 15° is reduced compared to that of the baseline. Note that overall angle of attack distribution is similar to that of 1/rev best phase shown in figure 4.25. The lift difference between the baseline and Gurney flap actuated under 2/rev frequency with $\phi_{2P} = 300^\circ$ is presented in figure 4.45. The lift is primarily augmented at the front and rear of the rotor disk. Note that the lift is not augmented at the advancing and retreating side. Comparing with the lift difference shown in figure 4.27, the 1/rev actuation frequency with $\phi_{1P} = 180^\circ$ augments lift on the advancing and retreating side as well and in particular the retreating side. Figure 4.46 shows the drag difference between the baseline and Gurney flap actuated under 2/rev frequency with $\phi_{2P} = 300^\circ$. The reduction of drag at the tip region is observed, which is similar to that is observed in figure 4.29. However, the drag increase due to the Gurney flap deployment is observed especially at the first quadrant of the rotor disk.
**Figure 4.43.** Phase Sweep at 2/rev Frequency, GW=22,000 lbs

**Figure 4.44.** Angle of Attack Distribution at $\phi_{2P}=300^\circ$
Figure 4.45. Difference in $M^2 \bar{C}_l$ with Gurney flap at 100%c, 2/rev

Figure 4.46. Difference in $M^2 \bar{C}_d$ with Gurney flap at 100%c, 2/rev
4.3 Part II: Elastic Blade Model

The rigid blade model has been widely adopted for performance calculation. As the performance calculation primarily deals with the steady component of rotor force, the rigid blade model that can capture the blade response up to the 1/rev frequency is sufficient for capturing low frequency behavior associated with the performance calculation. On the other hand, the high frequency contents in multiples of $N_b/\text{rev}$ component are imposed on top of the thrust for $N_b$ bladed rotor. These higher harmonic hub loads from the rotor are transferred to the fuselage, thus contributes to the vibratory load of helicopter. Note that not only $N_b/\text{rev}$ component but also $(N_b - 1)/\text{rev}$ and $(N_b + 1)/\text{rev}$ components in the rotating frame contribute to the $N_b/\text{rev}$ component in the hub frame. For accurate calculation of the hub vibratory loads, the structural analysis needs to capture these high frequency contents in the rotating frame. Note that one of important high frequency content comes from torsion and it interacts with the aerodynamic pitching moment from the airfoil.

For validation of analysis model, full vehicle propulsive trim was conducted for the baseline helicopter model with elastic blade model. Main rotor power and trim control settings of the flight test [127] were compared. The weight coefficient of $C_w/\sigma = 0.079$ was chosen as a reference condition. Furthermore, the analysis results using RCAS from reference [128] as well as the rigid blade analysis from section 4.2 are included for comparison.

The power loading, $\frac{C_p}{\sigma}$, predictions from the elastic blade model, RCAS, and the rigid blade model are presented in figure 4.47. Excellent correlation can be observed for the advance ratio greater than 0.15. Comparing with the predictions using rigid blade model, the elastic blade model predicts lower power. Figures 4.48 through 4.50 shows the correlation of trim controls prediction with flight test data. About 3 degrees of offset from the flight test data is observed for collective pitch. For the elastic blade model, higher collective pitch input is required to compensate the reduction in section angle of attack due to the nose-down elastic torsion deformation, which is caused by the aerodynamic pitching moment exerted on the blade section. The lateral cyclic pitch prediction is shown in figure 4.50. Although there is 2 to 3 degree offset compared to the test data, the trend of $\theta_{1C}$ variation is well captured. The longitudinal cyclic pitch prediction compared with the flight
test data is presented in figure 4.50. At lower advance ratio, about 2 degrees of offset from the test data is observed at lower advance ratio. However with the advance ratio greater than 0.2, the longitudinal cyclic pitch prediction shows good correlation with the flight test data.

Figures 4.51 and 4.52 depict the fuselage pitch and roll angle of fuselage attitude. The trends of each variation along with the flight speed is well captured, but overprediction of 1 to 2 degrees in pitch attitude is noted. Furthermore, large difference in roll attitude is observed especially at the advance ratio greater than 0.25.
Figure 4.47. Power Loading prediction for UH-60A

Figure 4.48. Collective Pitch prediction for UH-60A
Figure 4.49. Lateral Cyclic Pitch prediction for UH-60A

Figure 4.50. Longitudinal Cyclic Pitch prediction for UH-60A
Figure 4.51. Fuselage Pitch Attitude prediction for UH-60A

Figure 4.52. Fuselage Roll Attitude prediction for UH-60A
4.3.1 Velocity Sweep (22,000 lbs, 8,000ft)

The influence of elastic blade model is examined at the heavy gross weight, high altitude condition (gross weight of 22,000 lbs and altitude of 8,000 ft). As discussed in the preliminary study using rigid blade, the high lift device shows performance benefit near the edge of flight envelope. In the present section, the stall velocity will be identified by increasing the flight velocity until the trim solution can no longer be obtained. Figure 4.53 shows the main rotor power curve with respect to the flight speed. For comparison, the rigid blade results of chapter 4.2 is also presented. The elastic blade model predicts maximum velocity of 132 knot while the rigid blade stalls at 108 knots. Note that main rotor power predicted using the rigid blade is higher than that of the elastic blade by about 200 HP.

The rotor disk angle of attack at the stall velocity, 132 knot, is depicted in figure 4.54. It can be seen that the retreating side is subjected to stall. Note that the contour level of angle of attack is limited to 15°, which represents the stall angle of the airfoil used in Black Hawk main rotor at low Mach number. The lift distribution is shown in figure 4.55. Due to significant stall over the retreating side, the lift is concentrated on the front and back of the rotor disk. Figure 4.56 shows the drag distribution at the stalled condition. Similar to the rigid blade result discussed in part one, the drag at outer rim on the retreating side (4th quadrant) is very high.
Figure 4.53. Flight Speed versus Main Rotor Power at Gross Weight 22,000 lbs

Figure 4.54. Angle of Attack distribution at Stall Velocity, 132 knots
Figure 4.55. Lift Distribution $M^2 \bar{c} C_l$ at 132 knots

Figure 4.56. Drag Distribution $M^2 \bar{c} C_d$ at 132 knots
4.3.2 1/rev phase sweep at stall velocity

When the baseline rotor is stalled at the maximum speed, large amount of collective pitch and longitudinal cyclic pitch are required for the roll balance of the rotor. As seen from the rigid blade study, the rotor pitch control inputs of the near-stall baseline rotor can be reduced by the application of active Gurney flap. When the 1/rev phase angle $\phi_1$ is adjusted to $180^\circ$, the active Gurney flap augments mean lift as well as the lift on the retreating side. However, the rigid blade model used in section 4.2 does not include the torsional dynamics. Thus the effect of the pitching moment imposed by active Gurney flap actuation was not accounted for. The present elastic blade model captures the torsion dynamics, and the effect of the pitching moment generated by active Gurney flap.

In the present section, the same layout of the Gurney flap as considered in section 4.2 was adopted. The Gurney flap spans from 70\%$R$ to 80\%$R$ was actuated under 1/rev frequency while changing actuation phase angle $\phi_1$. The maximum amplitude of Gurney flap, $\bar{h}_{\text{max}}$ is limited to 2\%$c$. Note that only the 10\%$c$ upstream Gurney flap with unsteady model is used. Given the 1/rev airloads variation produced by unsteady Gurney flap actuation, the rotor was re-trimmed to satisfy trim at a given flight condition.

The 1/rev phase angle versus main rotor power is shown in figure 4.57. Considering the best phase identified from part one, the phase sweep was conducted for the phase angle varying from $120^\circ$ to $240^\circ$. The minimum power is achieved when the phase angle is $180^\circ$, and the power reduction predicted is 5\%. Figure 4.58 depicts the angle of attack distribution with the Gurney flap actuation input. The reduction of stalled area is observed, especially around $\psi = 315^\circ$. The difference in lift distribution is presented in figure 4.59. Unloading of the outer rim of the rotor disk can be observed, which is similar to the rigid blade analysis discussed in figure 4.40. Overall reduction due to the unloading can be observed in figure 4.59, especially around the tip region.
**Figure 4.57.** 1/rev phase angle versus main rotor power

**Figure 4.58.** Angle of Attack distribution, $\phi_{1P} = 180^\circ$
Figure 4.59. Difference in $M^2\bar{c}C_l, \phi_{1P} = 180^\circ$

Figure 4.60. Difference in $M^2\bar{c}C_d, \phi_{1P} = 180^\circ$
4.3.3 Effect of Gurney flap Span

In section 4.2, the effect of Gurney flap width was examined, and with the rigid blade model it was shown that increasing Gurney flap width results in a progressive reduction in the main rotor power. However with the elastic blade model, the nose-down pitching moment induced by Gurney flap deployment can negate the lift increment by Gurney flap, thus it is necessary to check the effect of increasing Gurney flap width capability for further benefit in performance improvement. In the present section, the outer edge of Gurney flap is held fixed at 80\%R, and the inner edge of the Gurney flap was changed from 75\%R to 50\%R, with 5\%R interval. The Gurney flap is applied at the stall speed of 132 knots with gross weight of 22,000 lbs, and is actuated under 1/rev frequency with phase angle of 180°, where maximum power reductions are observed.

The main rotor power change with the change in Gurney flap width is shown in figure 4.61. Owing to the stall recovery with Gurney flap, Gurney flap shows lower power than that of the baseline power for the range of inboard location considered. Note that the main rotor power is decreased when the inboard point is at 70\%R. Contrary to the results from section 4.2.3, further increasing the Gurney flap span increases main rotor power. Figure 4.62 depicts the percentage of reduction with respect to the change of the Gurney flap span. The main rotor power is reduced up to 5\% of the baseline power, but the performance benefit is reduced as the Gurney flap width is greater than 10\%R.

The angle of attack distribution of 30\%R width configuration is presented in figure 4.63. Comparing with figure 4.58, it can be observed that the angle of attack over the retreating side is further reduced especially around $\psi = 315^\circ$. However, the drag increase by wider Gurney flap span increases drag inboard of the rotor disk, as can be seen from figure 4.64 (compared to figure 4.58), which lead to the power increase compared to 10\%R width configuration.
Figure 4.61. Inboard Location of Gurney flap versus Main Rotor Power

Figure 4.62. Inboard Location of Gurney flap versus Reduction of Main Rotor Power
Figure 4.63. Angle of Attack Distribution (30%R width)

Figure 4.64. Difference in $M^2\bar{c}C_d$ (30%R span-10%R span)
4.3.4 Effect of 2/rev actuation frequency

The effect of 2/rev actuation frequency was examined in section 4.2.7. 2/rev actuation frequency also reduces main rotor power by moving the lift inboard. In the present section, the effect of 2/rev actuation is examined on the torsionally compliant blade. An upstream Gurney flap at 90\%c, extending from 70 to 80\%R is actuated under 2/rev frequency while changing the phase angle $\phi_{2P}$.

The variation of the main rotor power with respect to the change of 2/rev phase angle is given in figure 4.65. The initial attempt was made with 2/rev phase angles of $\phi_{2P} = [300^\circ, 330^\circ, 360^\circ]$, which covers the best phase angle of power reduction for the rigid blade model. Surprisingly, no power reduction was observed with these phase angles, thus second attempt was made with $\phi_{2P} = [150^\circ, 180^\circ, 210^\circ]$. The amount of power reduction obtained with 2/rev actuation with $\phi_{2P} = 180^\circ$ is 4.6\% which is similar to that is obtained with 1/rev actuation.

Figure 4.66 shows the angle of attack distribution around the rotor disk with $\phi_{2P} = 180^\circ$. Overall distribution looks similar to that of the 1/rev best phase case. The lift difference between the baseline and Gurney flap actuated under 2/rev frequency with $\phi_{2P} = 180^\circ$ is given in figure 4.67. The rotor disk is unloaded especially around the advancing. Note that lift augmentation by Gurney flap is clearly seen in the second quadrant. Note that the lift is not augmented at the retreating side while 1/rev actuation frequency with $\phi_{1P} = 180^\circ$ augments around the retreating side. Figure 4.68 depicts the drag difference between the baseline and Gurney flap actuated under 2/rev frequency with $\phi_{2P} = 180^\circ$. The drag is reduced at the tip region of retreating side. However, the drag increase due to the Gurney flap deployment is observed especially at the second quadrant of the rotor disk. This is due to the deployment schedule of 2/rev frequency, which exhibits the maximum amplitude at $\psi = 135^\circ$. The active control input of Gurney flap actuation and tip torsion responses (Baseline and Gurney flap actuation with $\phi_{2P} = 180^\circ$) are depicted in figure 4.69. The tip torsion deflection of active Gurney flap with $\phi_{2P} = 180^\circ$ exhibits larger nose-down tip torsion deflection at the second quadrant, while similar torsion deflections are observed over the retreating side.
Figure 4.65. Phase Sweep at 2/rev Frequency, GW=22,000 lbs

Figure 4.66. Angle of Attack Distribution at $\phi_2=300^\circ$
Figure 4.67. Difference in $M^2\tilde{c}C_l$ with Gurney flap at 90%$c$, 2/rev

Figure 4.68. Difference in $M^2\tilde{c}C_d$ with Gurney flap at 90%$c$, 2/rev
Figure 4.69. 2/rev actuation input and tip torsion response
The key parameters that determine the operating condition of a helicopter are flight speed, gross weight, and altitude. The limiting values of these parameters define the flight envelope of a helicopter. When the helicopter is approaching its flight envelope boundary, the rotor begins to experience stall where the rotor power requirement increases tremendously. In order to extend the flight envelope, various methods were proposed to enhance the performance at high speed, high gross weight, and high altitude operating conditions. Recently, the TEP (Trailing Edge Plate) was examined for performance improvement in reference [128], and performance improvement predicted is promising.

In the present chapter, performance improvement using single active Gurney flap will be examined in various aspects. For a given flight speed and altitude, the gross weight will be changed to increase the thrust generated by rotor. After identifying the thrust limit of the baseline helicopter, the effect of active Gurney flap on increasing the thrust limit of baseline rotor will be examined as well as the power reduction near the baseline aircraft envelop boundary. Next, for a given gross weight and flight speed the flight altitude will be increased from the sea level until the baseline helicopter can no longer be trimmed. The ability to increase the maximum altitude of baseline rotor using active Gurney flap will be investigated. The chapter will be closed with the application of active Gurney flap for increasing maximum flight speed of baseline helicopter.
5.1 Power Reduction and Maximizing Thrust Capability at 130 knots

In order to examine the thrust capability of a helicopter, the gross weight of a helicopter is increased until the rotor can be trimmed. For present section, the flight speed of 130 knots and altitude of 8,000 ft were selected as a reference operating condition. Figure 5.1 shows the variation of main rotor power with respect to the change of 1/rev phase angle. The active Gurney flap is actuated under 1/rev frequency when the baseline rotor is stalled at the gross weight of 22,050 lbs. As seen from the chapter 4, the minimum power is obtained when the phase angle is $180^\circ$, which provides mean lift augmentation as well as lift augmentation at the retreating side. Based on the results presented in the chapter 4 and the phase sweep results presented in the figure 5.1, the phase angle of $180^\circ$ may be assumed to provide the best power reduction under 1/rev actuation without loss of generality. Therefore, the 1/rev phase angle will be fixed to $180^\circ$ for the rest of the present chapter.

The gross weight sweep of UH-60A helicopter with and without active Gurney flap is presented in figure 5.2. The helicopter equipped with Gurney flap does not show any benefit in main rotor power when the gross weight of helicopter is lower than 21,000 lbs. However, the baseline rotor stalls at the 22,050 lbs, the helicopter with active Gurney flap can increase the gross weight limit by 950 lbs.

The angle of attack distribution of baseline is shown in figure 5.3. Most of the retreating side is in a stalled state, which is identified from the region where the angle of attack is greater than $15^\circ$. The inboard of retreating side is subjected to stalled condition. The highest angle of attack value is $25^\circ$. When active Gurney flap is applied, the angle of attack is reduced significantly over the entire retreating side as shown in figure 5.4. Figure 5.4 represents the difference in angle of attack distribution by subtracting baseline angle of distribution from that of 1/rev Gurney flap actuation with $\phi_{1P} = 180^\circ$. The reduction of angle of attack about 2 to 4 degrees is seen over most of the retreating side.

The trim control setting are given in table 5.1. Collective pitch was reduced by $0.5^\circ$ and lateral cyclic pitch is reduced by $1.3^\circ$. With the lift augmentation from active Gurney flap, the main rotor control is adjusted to match given thrust.
Table 5.1. Trim solution (GW=22,050 lbs, 130 knots)

<table>
<thead>
<tr>
<th>Trim Control</th>
<th>Baseline</th>
<th>GF(ϕ₁P = 180°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ₀</td>
<td>15.7</td>
<td>15.2</td>
</tr>
<tr>
<td>θ₁C</td>
<td>5.86</td>
<td>4.58</td>
</tr>
<tr>
<td>θ₁S</td>
<td>-9.22</td>
<td>-8.39</td>
</tr>
<tr>
<td>α₆WL</td>
<td>-0.12</td>
<td>0.49</td>
</tr>
<tr>
<td>φ₆WL</td>
<td>-1.07</td>
<td>-1.24</td>
</tr>
<tr>
<td>MR Power (HP)</td>
<td>2031</td>
<td>1932</td>
</tr>
<tr>
<td>Reduction</td>
<td>-</td>
<td>4.9%</td>
</tr>
</tbody>
</table>

level determined by the flight speed and the gross weight. Note that the fuselage attitude does not show large variation, which implies the steady component of rotor roll and pitch does not change between two configurations, and only the main rotor torque is reduced.

The baseline lift distribution is depicted in figure 5.5. Due to the high angles of attack at the retreating side, the lift distribution is more concentrated at the front and rear of the rotor disk. From figure 5.6, it is observed that the rotor disk is unloaded around the tip region at the front and rear of the disk after applying active Gurney flap. On the other hand, lift is increased both the advancing and retreating sides. Radial distribution at ψ = 45° and 315° is depicted in figure 5.7. The section lift increase due to active Gurney flap deployment can be observed.

Figure 5.8 shows the baseline drag disk distribution around the rotor disk. The high angles of attack observed at the retreating side is responsible for high drag observed near the tip of the fourth quadrant of the rotor disk. Note that due to the low dynamic pressure inboard of the rotor, the resultant drag around inboard region is lower even though inboard portion of the retreating side shows angles of attack greater than 15°. The function of active Gurney flap is also shown in figure 5.9. In addition, figure 5.10 depicts drag distribution along radius at ψ = 45° and 315°. At ψ = 45°, drag penalty due to active Gurney flap deployment can be observed where the active Gurney flap is implemented. However at ψ = 315°, the drag increment of active Gurney flap is less than that of advancing side. Note that the change in trim controls due to stall alleviation also affects the aerodynamic loads. The high drag region at the tip is reduced on the retreating side (due to the stall alleviation; figure 5.4) and at the rear of the disk (due to the lift unloading;
figure 5.6) at the tip is reduced. Due to the high Mach number at the tip region, even small amount of change in angle of attack can reduce the drag to a greater degree.

Figure 5.11 shows tip torsion of the baseline rotor and active Gurney flap actuation with $\phi_P = 180^\circ$. On the advancing side, active Gurney flap actuation shows larger nose-down twist while at the retreating side the amount of tip torsion is similar to that of the baseline.
Figure 5.1. 1/rev Phase sweep at 130 knots, gross weight 22,000 lbs

Figure 5.2. Gross weight sweep at 130 knots, altitude 8,000 ft
Figure 5.3. Angle of attack distribution of baseline rotor

Figure 5.4. Difference in angle of attack distribution
Figure 5.5. Baseline Lift Distribution $M^2 \bar{c} C_l$

Figure 5.6. Difference in $M^2 \bar{c} C_l$
Figure 5.7. Radial Lift Distribution at $\psi = 45^\circ, 315^\circ$

Figure 5.8. Baseline Drag Distribution $M^2 c C_d$
Figure 5.9. Difference in $M^2 c C_d$

Figure 5.10. Radial Drag Distribution at $\psi = 45^\circ, 315^\circ$
Figure 5.11. Azimuth angle versus Tip Torsion
Table 5.2. Trim solution (GW=23,510 lbs, 90 knots)

<table>
<thead>
<tr>
<th>Trim Control</th>
<th>Baseline</th>
<th>GF($\phi_{1P} = 180^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>14.0</td>
<td>13.1</td>
</tr>
<tr>
<td>$\theta_{1C}$</td>
<td>5.73</td>
<td>4.32</td>
</tr>
<tr>
<td>$\theta_{1S}$</td>
<td>-8.02</td>
<td>-7.27</td>
</tr>
<tr>
<td>$\alpha_{WL}$</td>
<td>-2.62</td>
<td>-2.09</td>
</tr>
<tr>
<td>$\phi_{WL}$</td>
<td>-0.762</td>
<td>-0.731</td>
</tr>
<tr>
<td>MR Power (HP)</td>
<td>1762</td>
<td>1563</td>
</tr>
<tr>
<td>Reduction</td>
<td>-</td>
<td>11.3%</td>
</tr>
</tbody>
</table>

5.2 Power Reduction and Maximizing Thrust Capability at 90 knots

The gross weight sweep was conducted at lower flight speed, 90 knots. Due to the lower flight speed, the asymmetry of the rotor flow will be reduced, thus the requirement to keep roll balance becomes less pronounced than the high speed flight condition. As a result, the rotor can produce higher thrust level than that of the high speed condition. The variation of main rotor power with respect to the change of 1/rev phase angle is presented in figure 5.12. Similar to the figure 5.1, 1/rev phase angle ($\phi_{1P}$) of $180^\circ$ yields minimum power. Figure 5.13 shows the gross weight sweep conducted at 90 knots and altitude of 8,000 ft. Similar observations can be made with the results discussed in section 5.1. The baseline rotor can be trimmed up to a maximum gross weight of 23,510 lbs while the active Gurney flap as increase in maximum gross weight of 990 lbs is possible. When the Gurney flap is used at the gross weight of 23,510 lbs, the main rotor power can be reduced by 11.3%.

The trim control setting are presented in table 5.2. Collective pitch was reduced by 0.9° and lateral cyclic pitch is reduced by 1.4°. The reduction of collective pitch is related to the reduction of mean rotor lift provided while the reduction in lateral cyclic pitch is related with the reduction in lift at the front and back of the rotor disk. With the lift augmentation from active Gurney flap, the main rotor control is adjusted to match given thrust level determined by the flight speed and the gross weight. Note that the fuselage attitude does not show large variation, which
implies the steady component of rotor roll and pitch does not change between two configurations, and only the main rotor torque is reduced.

Angle of attack distribution of baseline rotor is shown in figure 5.14. The inboard of retreating side is subjected to stalled condition. The highest angle of attack value is 26$^\circ$. Figure 5.15 represents the baseline angle of attack distribution and the difference in angle of attack distribution by subtracting baseline angle of distribution from that of 1/rev Gurney flap actuation with $\phi_{1P} = 180^\circ$, respectively. The reduction of angle of attack about 2 to 4 degrees over the retreating side is evident. The lift distribution of baseline rotor is presented in figure 5.16. The area where negative lift appears in the advancing side is reduced compared to the higher speed cased discussed in the previous section. However, due to the high thrust requirement, the lift distribution is concentrated at the front and back of the rotor disk. Figure 5.17 depicts the lift difference between baseline and 1/rev active Gurney flap actuated with $\phi_{1P} = 180^\circ$. Owing to the additional lift augmentation provided by 1/rev active Gurney flap actuation, lift is augmented over the retreating side between 70 and 80% $R$ at the tip region. As discussed in the previous section, the reduction in the tip region is related to the reduction of the rotor control. Baseline drag distribution is shown in figure 5.18. Rapid increase of the drag can be observed in the tip portion of the retreating side. With the actuation of active Gurney flap, the high drag seen on the tip region of retreating side is reduced as can be seen from the drag difference shown in figure 5.9. Note that the highest drag observed around $\psi = 300^\circ$ is reduced by the stall alleviation owing to actuation of the active Gurney flap.
Figure 5.12. 1/rev Phase sweep at 90 knots, gross weight 23,510 lbs

Figure 5.13. Gross weight sweep at 90 knots, altitude 8,000 ft
Figure 5.14. Angle of attack distribution of baseline rotor

Figure 5.15. Difference in angle of attack distribution
Figure 5.16. Baseline Lift Distribution $M^2 \bar{c} C_l$

Figure 5.17. Difference in $M^2 \bar{c} C_l$
Figure 5.18. Baseline Drag Distribution $M^2\bar{c}C_d$

Figure 5.19. Difference in $M^2\bar{c}C_d$
5.3 Flight Envelope: Altitude

According to the standard atmosphere properties, the temperature is changed with the altitude variation. Due to the change in temperature, the value of air density is changed, which affects the dynamic pressure and thus the airloads acting on the rotor blade. Furthermore, the speed of sound is decreased when the altitude increases, the effective Mach number is increased for the same forward flight speed. Therefore, the performance of the helicopter will be limited when the helicopter is operated at the high altitude. Furthermore, the change in air density affects the lock number, which defines the ratio between aerodynamic force and inertia force of the blade. Note that the comprehensive rotorcraft model discussed in the chapter 2 adopts the lock number as one of the key parameter in normalizing the governing equation.

For altitude sweep, gross weight of 22,000 lbs and flight speed of 90 knots were chosen as reference operating condition. The helicopter is first trimmed at the sea level. And then, the altitude is increased until the trim solution can no longer be obtained. As mentioned in the section 5.1, the 1/rev phase angle of Gurney flap actuation, $\phi_{1P}$, is 180°. The altitude versus main rotor power is presented in figure 5.20. The baseline rotor can be trimmed up the altitude of 10,060 ft while the rotor with active Gurney flap can operate at 1,440 ft higher. At the altitude where the baseline rotor stalls, the main rotor power is reduced by 11.4% with the aid of Gurney flap. At this flight condition, the baseline rotor performance is limited by stall. Table 5.3 shows the trim control settings for baseline rotor and 1/rev active Gurney flap actuation. Collective pitch was reduced by 0.9° and lateral cyclic pitch is reduced by 1.2°.

Applying active Gurney flap, the angle of attack can be reduced especially over the retreating side as can be observed from figure 5.21. The angle of attack is reduced almost all over the rotor disk except around $\psi = 90^\circ$.

The vertical forces of the baseline and the configuration with active Gurney flap are depicted in figure 5.22. As the rotor hub is acting as filter, the steady component (thrust) and multiples of 4/rev components appears at the hub. Note that the mean values of hub vertical forces are nearly same, indicating both rotors are trimmed to the same thrust. It is interesting to point out that 4/rev component
Table 5.3. Trim solution (Altitude=10,060 ft, 90 knots)

<table>
<thead>
<tr>
<th>Trim Control</th>
<th>Baseline</th>
<th>GF($\phi_{1P} = 180^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>13.8</td>
<td>12.9</td>
</tr>
<tr>
<td>$\theta_{1C}$</td>
<td>5.57</td>
<td>4.32</td>
</tr>
<tr>
<td>$\theta_{1S}$</td>
<td>-7.9</td>
<td>-7.1</td>
</tr>
<tr>
<td>$\alpha_{WL}$</td>
<td>-2.55</td>
<td>-1.95</td>
</tr>
<tr>
<td>$\phi_{WL}$</td>
<td>-0.85</td>
<td>-0.81</td>
</tr>
<tr>
<td>MR Power (HP)</td>
<td>1670</td>
<td>1480</td>
</tr>
<tr>
<td>Reduction</td>
<td>-</td>
<td>11.4%</td>
</tr>
</tbody>
</table>

is reduced when active Gurney flap is applied, and this topic will be examined in detail in the chapter 6. Figure 5.23 shows the yaw moment at the hub. The steady component of yaw moment is the rotor torque, and can be converted into horse power. The mean value of yaw moment is reduced when the active Gurney flap is used, which in turn shows the power reduction realized with active Gurney flap.
Figure 5.20. Altitude sweep at 90 knots, gross weight 22,000 lbs

Figure 5.21. Difference in angle of attack distribution
Figure 5.22. Vertical Force at the hub, 10,060 ft

Figure 5.23. Yaw Moment at the hub, 10,060 ft
5.4 Flight Envelope: Maximum Speed

Increasing the maximum level flight speed has been a big challenge to both the aerodynamicist and dynamicist in rotorcraft engineering. The maximum speed is limited by two major barriers inherent in rotorcraft aerodynamic environment. One is the compressibility limit at the advancing side and the other is blade stall in the retreating side. On the advancing side, as the flight speed increases, the local Mach number at the tip region is approaching to 1.0. At the higher Mach number regime, the drag bucket becomes narrower than that of subsonic condition, thus the airfoil should be operating in the drag bucket otherwise excessive drag penalty will be a limiting factor. On the other hand, the lower dynamic pressure due to the velocity difference on the retreating side requires larger $C_{l_{\text{max}}}$ in order to keep roll balance of the rotor.

In the present section, the active Gurney flap is applied to extend the maximum speed limit of the baseline. Gross weight of 22,000 lbs and altitude 8,000 ft was chosen as a reference operating condition. For a given actuation phase angle of 180°, the flight speed was increased until the trim solution is no longer obtained. Figure 5.24 compares the main rotor power of the baseline rotor and the rotor with 1/rev active Gurney flap. The baseline rotor stalls at 132 knots. It is remarkable that the 1/rev actuation of active Gurney flap increases the maximum speed limit by 28 knots. When the active Gurney flap is used, 5% reduction in the main rotor power is observed at the baseline stall speed. As detailed discussion on the power reduction at the baseline stall speed was already given in the chapter 4, the maximum speed capability will be examined in the present section.

Table 5.4 shows the trim control settings for baseline rotor, 1/rev active Gurney flap actuation at 132 knot, and 1/rev active Gurney flap actuation at 160 knots. Collective pitch was reduced by 0.5° and corresponding power reduction is 5%. With the active Gurney flap actuation, the helicopter can fly with higher speed. Note that at the maximum velocity, the collective pitch is increase by 3° and corresponding power increase is 36%.

Figure 5.25 shows the section lift coefficient variation at $r/R = 75\%$. The comparison was made at the maximum speed of both configurations. The maximum lift coefficient of baseline rotor is 1.47 while the active Gurney flap with $\phi_{1P} = 180^\circ$
Table 5.4. Trim solution (GW=22,000 lbs, 8,000 ft)

<table>
<thead>
<tr>
<th>Trim Control</th>
<th>Baseline (132 knots)</th>
<th>GF (132 knots)</th>
<th>GF (160 knots)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>15.9</td>
<td>15.4</td>
<td>19.1</td>
</tr>
<tr>
<td>$\theta_{1C}$</td>
<td>5.99</td>
<td>4.69</td>
<td>6.55</td>
</tr>
<tr>
<td>$\theta_{1S}$</td>
<td>-9.23</td>
<td>-8.52</td>
<td>-9.23</td>
</tr>
<tr>
<td>$\alpha_{WL}$</td>
<td>0.0</td>
<td>0.59</td>
<td>2.59</td>
</tr>
<tr>
<td>$\phi_{WL}$</td>
<td>-1.1</td>
<td>-1.24</td>
<td>-1.69</td>
</tr>
<tr>
<td>MR Power (HP)</td>
<td>2081</td>
<td>1976</td>
<td>2839</td>
</tr>
<tr>
<td>Reduction</td>
<td>-</td>
<td>5%</td>
<td>-36.4%</td>
</tr>
</tbody>
</table>

can increase maximum lift coefficient up to 1.8. The pitching moment variation at the same radial location is depicted in figure 5.26. Note that the retreating blade stall of the baseline rotor is clearly shown by the large negative peak at $\psi = 325^\circ$. The pitching moment variation of the rotor with active Gurney flap does not show large negative peaks as baseline rotor, but the active Gurney flap shifts the pitching moment down around -0.04, Note that this offset can be related with the average Gurney flap height - $1%c$ in present analysis. The drag coefficient variation is presented in figure 5.27. The drag peak present at $\psi = 315^\circ$ seems to be a sign of retreating blade stall. The drag coefficient becomes lower than that of the baseline case especially over the fourth quadrant of the rotor disk, while the increase in drag coefficient is observed from the first to the third quadrant of the rotor disk.
Figure 5.24. Velocity sweep at 8,000 ft, gross weight = 22,000 lbs

Figure 5.25. Section Lift Coefficient at 75%R
Figure 5.26. Section Pitching Coefficient at 75% $R$

Figure 5.27. Section Drag Coefficient at 75% $R$
5.5 Effect of Torsional Stiffness

The previous simulation results were all based on the torsionally compliant blade of which the 1\textsuperscript{st} torsional frequency is 4.3/rev. For flexible blade, aerodynamic pitching moment causes elastic torsion deformation, and the angle of attack is affected by elastic torsion deformation. The effect of an active Gurney flap on torsionally rigid blade will be examined by increasing the first torsion frequency of the baseline helicopter from 4.3/rev to 7.3/rev. The effect of torsionally stiff blade was examined at a flight speed of 130 knots and at a altitude of 8,000 ft. The gross weight of a helicopter is increased from 17,000 lbs until it can be trimmed. The phase angle of 1/rev actuation is set to 180°.

The gross weight sweep of UH-60A helicopter with and without active Gurney flap is depicted in the figure 5.28. When the gross weight of the helicopter is less than 20,000 lbs, main rotor with an active Gurney flap actuation requires more power than the baseline rotor. When the baseline rotor stalls at 20,900 lbs, the main rotor power is reduced by 4.74% with 1/rev actuation of active Gurney flap. Note that the gross weight capability can be further increased by 600 lbs using 1/rev active Gurney flap actuation.

When an active Gurney flap is actuated, the angle of attack is reduced as shown in figure 5.29. Figure 5.29 represents the difference in angle of attack distribution by subtracting baseline angle of attack distribution from that of 1/rev Gurney flap actuation. It is observed that the angle of attack over the retreating side is reduced about 2 degrees while virtually no change in angle of attack is observed in the advancing side.

Trim solutions at gross weight 20,900 lbs are presented in table 5.5. The main rotor control input is reduced about 0.6° to 0.8° for the stiff rotor case. Due to the torsional stiffness, the influence of additional pitching moment imposed by active Gurney flap becomes negligible, while the lift augmentation from active Gurney flap actuation is more pronounced than that of the torsionally compliant rotor. Similar to the results discussed in section 5.1, the fuselage attitude does not show large variation, which implies the steady component of rotor roll and pitch does not change between two configurations, and only the main rotor torque is reduced.

The tip torsion response of the stiff rotor (7.3/rev) are presented in figure 5.30.
Comparing with the tip torsion response of torsionally soft rotor (4.3/rev) shown in figure 5.11, the soft rotor shows larger tip torsion deflection due to the torsional flexibility while the stiff rotor shows much smaller tip torsion deflection. Note that the gross weight where helicopter stalls is different for soft rotor (4.3/rev) and stiff rotor (7.3/rev).
Figure 5.28. Gross weight sweep at 130 knots, altitude 8,000 ft

Figure 5.29. Difference in angle of attack distribution
Table 5.5. Trim solution (GW=20,900 lbs, 130 knots, 8,000 ft)

<table>
<thead>
<tr>
<th>Trim Control</th>
<th>Baseline (7.3/rev)</th>
<th>GF ($\phi_{1P} = 180^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>11.8</td>
<td>11.0</td>
</tr>
<tr>
<td>$\theta_{1C}$</td>
<td>5.49</td>
<td>4.7</td>
</tr>
<tr>
<td>$\theta_{1S}$</td>
<td>-9.96</td>
<td>-9.35</td>
</tr>
<tr>
<td>$\alpha_{WL}$</td>
<td>0.0</td>
<td>0.6</td>
</tr>
<tr>
<td>$\phi_{WL}$</td>
<td>-1.201</td>
<td>-1.297</td>
</tr>
<tr>
<td>MR Power (HP)</td>
<td>1836</td>
<td>1749</td>
</tr>
<tr>
<td>Reduction</td>
<td>-</td>
<td>4.74%</td>
</tr>
</tbody>
</table>

Figure 5.30. Elastic tip torsion comparison
Vibration Reduction using Active Gurney Flap

Vibration has been regraded as an inherent problem of helicopter. Flexible structure interacting with highly complex aerodynamics generates high frequency vibratory load on the rotor blade. In the rotating frame, \((N_b - 1)/\text{rev}, N_b/\text{rev}, \text{and } (N_b+1)/\text{rev}\) frequencies are the important frequency where \(N_b\) refers to the number of rotor blade. Assuming the perfectly identical, and tracked rotor, the summation of the each rotor blade root load yields steady component imposed with \(N_b/\text{rev}\) forces and moments. Note that the hub acts as a filter which passes only the multiples of \(N_b/\text{rev}\) frequencies to the fuselage. The vibratory loads are affecting the crew/passenger comfort, fatigue life of mechanical parts, and crew efficiency during the operation. As reviewed from chapter 1, various methodologies were studied, tested and implemented on the helicopter to reduce the vibratory loads. However the vibration reduction methods used in practice are moderately satisfactory - the ultimate goal of jet smooth ride (<0.05g) has not yet been realized.

In the present chapter, the application of active Gurney flap for vibration reduction will be examined. As active Gurney flap functions as an on-blade actuator, the forces and moment to be used for control purpose, or control authority will be affected by external aerodynamic environment. The vibration reduction through stall alleviation will be examined. The other methods which makes use of high frequency actuation to generate high frequency airloads will be examined.
6.1 Vibration Reduction by Stall Alleviation

When the helicopter operates near the flight boundary, the retreating blade stall is accompanied by high vibratory loads. In the chapter 5, the hub vertical force was examined for baseline rotor and the rotor with Gurney flap. In figure 5.22, the mean level of hub vertical force was checked to ensure the rotor was trimmed to the same thrust level. On the other hand, it was observed that the magnitude of 4/rev component was decreased with the use of the active Gurney flap. This shows the possibility of the vibration reduction through stall alleviation using active Gurney flap. In present section, the maximum thrust at 90 knots case will be examined for the vibration reduction. Recall that the baseline helicopter stalls at the gross weight of 23,510 lbs and altitude is 8,000 ft.

Figure 6.1 shows the hub vertical force. Similar to figure 5.22, the mean values of vertical components are nearly same as the gross weight. Comparing the magnitude of 4/rev components, active Gurney flap reduces the magnitude of vertical vibratory load. The variation of 4/rev vertical vibratory load with respect to the gross weight change is depicted in figure 6.2. 24% reduction in vertical vibratory force is predicted. The in-plane force is defined by $\sqrt{F_x^{4P^2} + F_y^{4P^2}}$, and the variation of 4/rev in-plane force with respect to the gross weight is depicted in figure 6.3. A remarkable reduction of 68% should be noted. Similarly, 4/rev in-plane moment is defined as $\sqrt{M_x^{4P^2} + M_y^{4P^2}}$, and the in-plane moments of baseline rotor and rotor with active Gurney flap are presented in figure 6.4. As can be seen from the figure, hub in-plane moment is reduced by 44%. From these results, clearly the stall alleviation owing to the active Gurney flap also reduces the high vibration encountered near the flight envelope. Note that 1/rev actuation frequency was used for stall alleviation, thus the airload generated by Gurney flap has no influence on the higher harmonic components in $N_b/rev$ and its multiples. Therefore, the vibratory load at this condition is reduced due to the trim change, not by the harmonic cancellation.
Figure 6.1. Hub Vertical Force at gross weight 23,510 lbs

Figure 6.2. Gross weight versus 4/rev Vertical Force
Figure 6.3. Gross weight versus 4/rev Inplane Force

Figure 6.4. Gross weight versus 4/rev Inplane Moment
6.2 Vibration Reduction with Higher Frequency

Vibration reduction through stall alleviation is examined in the preceding section. It was demonstrated that recovering the rotor from stalled condition reduces vibration associated with stalled condition up to 68% for the 4/rev in-plane force. As discussed, the forcing in \((N_b - 1)/rev\), \(N_b/rev\), and \((N_b + 1)/rev\) frequency in the rotating frame primarily contributes to the \(N_b/rev\) hub loads, thus it would be possible to reduce the magnitude of \(N_b/rev\) loads by generating counteracting force. Recall from the literature survey on the active control methodology discussed in the chapter 1, the conventional HHC (Higher Harmonic Control) exerts counteracting force through the swash plate actuation in \(N_b/rev\) frequency, in the fixed frame. Such a forcing given in the fixed frame can generate the forcing in \((N_b - 1)/rev\), \(N_b/rev\), and \((N_b - 1)/rev\) frequency in the rotating frame, but the actual control authority realized in the rotating frame will be limited. In order to increase the control authority, it would be beneficial to generate the counteracting force in the rotating frame. By generating the counteracting forcing in \((N_b - 1)/rev\), \(N_b/rev\), and \((N_b + 1)/rev\) frequency in the rotating frame, the higher frequency forcing that introduces vibratory loads to the system can be cancelled from the source.

In the present section, the vibratory loads of baseline helicopter will be controlled through the active Gurney flap actuated with higher harmonic frequency, 3/rev, 4/rev, and 5/rev.

6.2.1 Vibration Level of Baseline helicopter

The prediction of vibratory loads has been regarded as a challenging problem in rotorcraft engineering. Lots of efforts were made to improve the vibratory loads prediction. One of such efforts is “vibratory load workshop” held in 1996 [131]. The flight test data of Lynx helicopter at 158 knots were used as a reference, and the vibration level at the same flight condition was predicted by the comprehensive analysis code of various participants. The prediction results showed a very large scatter, in terms of both magnitude and phase compared to the flight test data.

The vibration level of baseline helicopter is examined in the present section. For validation purpose, the vibratory loads prediction were compared against two
different rotorcraft comprehensive analysis code, RCAS [128] and UMARC [132]. The flight condition was set up with gross weight 18,300 lbs and altitude at sea level (consistent with references [128,132]). Figure 6.5 compares the in-plane vibratory loads predictions. The over prediction of present analysis is recognized, but care must be taken for interpreting the results. The analysis model of present study does not include lag dampers in the model. Note that RCAS is multibody dynamics code, and all of the complex mechanical devices around the root is included in the model. The in-plane moment prediction was compared against other analysis in figure 6.6. The in-plane moment prediction shows similar level as that of RCAS. The magnitude of 4/rev hub vertical force is presented in figure 6.7. Similar level of hub vertical force is predicted compared to RCAS while UMARC predicts too low vertical vibration level especially at high speed. Note that the low vibration level predicted by present analysis is due to the prescribed wake model at low speed regime. At low speed, the wake structure deformation due to the interaction between vortex filaments is present in the low speed regime, thus the use of free-wake model will improve the prediction. For inflow model, RCAS uses dynamic inflow model and UMARC uses free-wake model in the results discussed above.

For setting up the baseline vibration level of the present section, the flight condition of the validation case (gross weight of 17,330 lbs and altitude 2,300 ft) discussed in the chapter 4 is adopted. The flight speed chosen for vibration reduction is 130 knots. At this speed, helicopter is not subjected to stall, but significant vibratory loads due to high speed flight are present. The individual components of 4/rev hub loads at 130 knots are depicted in figure 6.8.
Figure 6.5. Flight Speed versus $4/\text{rev}$ Inplane Force

Figure 6.6. Flight Speed versus $4/\text{rev}$ Inplane Moment
Figure 6.7. Flight Speed versus 4/rev Vertical Force

Figure 6.8. Individual Components of Vibratory Loads at 130 knots
6.2.2 Single Frequency Phase Sweep

In the present section, the variation in vibratory loads at 130 knots will be examined using single frequency active Gurney flap actuation inputs. The baseline rotor was actuated with active Gurney flap spanning from 70% $R$ to 80% $R$. The active Gurney flap is actuated with the control input described in equation 6.1

$$\frac{\bar{h}}{c}(\psi) = \bar{h}(\psi) = \frac{\bar{h}_{\text{max}}}{2} + \frac{\bar{h}_{\text{max}}}{2} \sin(n\psi - \phi_{nP})$$ (6.1)

where $\bar{h}$ refers to the instantaneous Gurney flap height normalized by nominal chord length, $c$. In order to see the effect of active Gurney flap amplitude on the vibratory loads, three different amplitudes of $\bar{h}_{\text{max}}$ are considered - 0.5%, 1% and 2%. As the baseline helicopter has 4 blades, thus 3/rev, 4/rev and 5/rev frequencies in the rotating frame are important for vibration control purpose. $\phi_{nP}$ will be varied to change the location where the maximum deployment occurs. Note that the unsteady aerodynamic effect will become more pronounced.

6.2.2.1 Actuation with 3/rev frequency

The effect of 3/rev actuation of active Gurney flap is examined for reducing vibratory loads. Variation of 4/rev in-plane force versus 3/rev actuation phase angle is depicted in figure 6.9. The best phase that minimizes 4/rev in-plane force is at 60°, and the in-plane force is reduced by 15% with $h/c=2\%$. Variation of 4/rev vertical force is given in figure 6.10. 4/rev vertical force is reduced by 24% using 1% $c$ Gurney flap amplitude. The best phase that achieves maximum reduction is at 150°. Figure 6.11 depicts the variation of 4/rev in-plane moments with respect to the 3/rev phase angle change. 4/rev in-plane moment force is reduced up to 36% when active Gurney flap is actuated with $\bar{h}_{\text{max}} = 2\% c$ and $\phi_{3P} = 90°$. Note that the best phase angle of in-plane force and in-plane moment are similar, thus those two components can be reduced at the cost of increasing vertical vibratory load. Conversely, using $\bar{h}_{\text{max}} = 1\% c$ and $\phi_{3P} = 150°$, 24% reduction in 4/rev vertical loads can be achieved along with a 15% reduction in 4/rev in-plane moment and without a significant penalty to 4/rev in-plane force.
6.2.2.2 Actuation with 4/rev frequency

The active Gurney flap is actuated under 4/rev frequency, and its effect on reducing the in-plane force, vertical force, and in-plane moment is examined in present section. From the figure 6.12, it is observed that the in-plane force is not reduced by 4/rev actuation. By reducing the Gurney flap amplitude, the maximum variation of in-plane force is reduced. Although no reduction is possible with 4/rev actuation, the minimum increase occurs at the phase angle of 270° for all of Gurney flap amplitude considered. With $\bar{h}_{\text{max}} = 0.5\%$, the 4/rev in-plane force is increased by 2%. Higher $\bar{h}_{\text{max}}$ of 2% yield 17% increase in in-plane force. With the 4/rev actuation, the vertical force is reduced by 74% with $\bar{h}_{\text{max}} = 2\%$. As can be seen from figure 6.13, this reduction occurs at $\phi_{4P} = 240^\circ$. Note the effect on 4/rev vertical force with increasing active Gurney flap amplitude appears linear in nature. Figure 6.14 depicts 4/rev in-plane moment versus variations in 4/rev actuation phase. The best phase is at 270°, which yields 24% reduction using $\bar{h}_{\text{max}} = 2\%$. For the 4/rev actuation frequency, the best frequencies lies around 270°, thus the 2% amplitude of Gurney flap actuation can lead to maximum reduction in overall vibratory loads (743 lbs in 4/rev vertical force and 540 lbs-ft) at the cost of increasing in-plane force by 300 lbs.

6.2.2.3 Actuation with 5/rev frequency

In the present section, the ability of 5/rev actuation of active Gurney flap was examined for reducing hub vibratory loads. From figure 6.15, the 5/rev actuation does not show any reduction in 4/rev in-plane force. The best phase angle that minimizes the inplane force penalty is at 30°. When $\bar{h}_{\text{max}}$ of 0.5% is used, the penalty in in-plane force is negligible. However, it should be noted that the inplane force increases with the Gurney flap amplitude. For the vertical force, the reduction achieved is 81% using 5/rev actuation with 1% amplitude at the phase angle of 330°, as seen from figure 6.16. Figure 6.17 depicts the variation of 4/rev inplane moment with respect to the phase angle sweep. 17% reduction is observed with $\bar{h}_{\text{max}} = 1\%$ and 5/rev phase angle between 60° to 90°. Similar to the 3/rev phase sweep results, the best phase angle of in-plane force and in-plane moment are in variance with that of the vertical force. 4/rev vertical force of 81% possible with
\( \bar{h}_{\text{max}} = 1\% \) at the cost of 27\% increase in inplane force and 17\% increase inplane moment.
Figure 6.9. Effect of $\phi_{3P}$ on 4/rev In-plane Force

Figure 6.10. Effect of $\phi_{3P}$ on 4/rev Vertical Force

Figure 6.11. Effect of $\phi_{3P}$ on 4/rev In-plane Moment
Figure 6.12. Effect of $\phi_{4P}$ on 4/rev In-plane Force

Figure 6.13. Effect of $\phi_{4P}$ on 4/rev Vertical Force

Figure 6.14. Effect of $\phi_{4P}$ on 4/rev In-plane Moment
Figure 6.15. Effect of $\phi_{5P}$ on 4/rev In-plane Force

Figure 6.16. Effect of $\phi_{5P}$ on 4/rev Vertical Force

Figure 6.17. Effect of $\phi_{5P}$ on 4/rev In-plane Moment
6.2.3 Multicyclic Higher Harmonic Control

The single frequency sweep provides an information about the vibratory loads behavior with respect to the active Gurney flap control inputs. The reduction depends on the amplitude of active Gurney flap input. Furthermore, the best phase angle can vary for each component of vibratory loads. In order to achieve the best performance under such circumstance, the control methodology to properly adjust the conflicts inherent in the system becomes essential.

T-matrix approach, one of the standard methods widely used for higher harmonic control, will be used to examine the ability to reduce vibratory load in an optimal manner. For the linear system, T-matrix approach is known to work well. Even for moderately nonlinear system, T-matrix shows satisfactory performance in reducing vibratory loads [107].

6.2.3.1 Plant Model

The T-matrix approach discussed in the reference [133] is based on the quasi-static model of the actual helicopter. This quasi-static model defines the input-output relationship between the harmonics of control inputs and that of the system responses to be measured. The linear plant model is given in equation 6.2

\[
\begin{align*}
    z &= \begin{bmatrix} F_x^{4P} & F_y^{4P} & F_z^{4P} & M_x^{4P} & M_y^{4P} \end{bmatrix}^T \\
    \bar{h} &= \begin{bmatrix} \bar{h}_{3C} & \bar{h}_{3S} & \bar{h}_{4C} & \bar{h}_{4S} & \bar{h}_{5C} & \bar{h}_{5S} \end{bmatrix}^T \\
    z &= z_0 + T\bar{h}
\end{align*}
\]  

(6.2)

where \( z \) is a vector that contains the different components of 4/rev vibratory loads, and \( \bar{h} \) is a vector that consists of harmonic components of active Gurney flap input. Note that \( z_0 \) refers to the uncontrolled vibration level.

The identification of T-matrix can be done by least square fit of measured data using equation 6.3.

\[
\begin{align*}
    \theta_i &= [\bar{h}_i^T, 1]^T \\
    \Theta_N &= [\theta_1, \theta_2, ..., \theta_N]^T \\
    Z_N &= [z_1, z_2, ..., z_N]^T \\
    T &= Z_N \Theta_N^T [\Theta_N \Theta_N^T]^{-1}
\end{align*}
\]  

(6.3)
where $\theta$ represents the vector consists of active Gurney flap inputs and constant input. $\Theta_N$ and $Z_N$ refer to measured N data points of control inputs and vibratory loads, respectively.

On the other hand, T-matrix can be constructed by perturbing the active Gurney flap input vector one-by-one as shown in equation 6.4.

$$T_{ij} = \frac{\partial z_i}{\partial h_j} \tag{6.4}$$

### 6.2.3.2 Controller

The controller model from reference [107] is used in the present study. The objective function to be minimized is defined in equation 6.5

$$J = z^T W_z z + \bar{h}^T W_h \bar{h}$$

$$z = [F_x^{4P} \ F_y^{4P} \ F_z^{4P} \ M_x^{4P} \ M_y^{4P}]^T$$

$$\bar{h} = [\bar{h}_3C \ \bar{h}_3S \ \bar{h}_4C \ \bar{h}_4S \ \bar{h}_5C \ \bar{h}_5S]^T \tag{6.5}$$

$W_z$ and $W_h$ are the diagonal matrices which adjust the weighting factor for the components to be controlled. In the present study, $W_z$ and $W_h$ are both identity matrices. From the optimal control theory, the optimal input to minimize the objective function can be analytically derived, and is given in equation 6.6

$$D = T^T W_z T + W_h$$

$$\bar{h}_{optimal} = -D^{-1} T^T W_z z_0 \tag{6.6}$$

Note that T matrix in equation 6.6 represents the dynamics of the plant, and the variation of the vibratory hub loads to the control input vector h. If the plant is perfectly linear, then multicyclic HHC controller converges to the optimum solution in a single step. On the other hand, if the plant has nonlinearity, an iterative procedure is necessary to find the minima of the objective function. The iterative procedure can be defined by modifying the optimal solution vector for the linear system ($\bar{h}_{optimal}$) and is given in equation 6.7
\[ \tilde{h}_n = -D^{-1} T^T W_z (z_{n-1} - T \tilde{h}_{n-1}) \]
\[ \Delta \tilde{h}_n = -D^{-1} (T^T W_z z_{n-1} - W_h \tilde{h}_{n-1}) \]  \hspace{1cm} (6.7)

Under the assumption of the moderate nonlinearity of the system considered in the present study, the iterative method was applied to update the optimal inputs that minimizes the objective function. However, the iterative scheme based on the classic linear controller frequently produces non-physical inputs that violate the constraints of the system. In order to prevent such issues, the use of relaxation factors is recommended. Equation 6.8 shows the relaxation factors used in the current controller.

\[ \tilde{h}_n = \tilde{h}_{n-1} + \alpha \Delta \tilde{h}_n \]
\[ W_h = \beta W_h \]  \hspace{1cm} (6.8)

\( \alpha \) and \( \beta \) both prevent rapid jumps during the control update, thus helping with the stability of the iterative scheme. In present study, \( \alpha = 0.1 \) and \( \beta = 0.00001 \) were used.

6.2.3.3 Vibration Reduction using HHC Controller

Higher harmonic controller discussed in section X is implemented and used for vibration reduction at gross weight of 17,330 lbs, 130 knots, and 2,300 ft. For evaluating current vibratory load \( z_n \), either the linear plant model \( (z = z_0 + T\tilde{h}, \text{ in equation 6.2}) \) or the actual nonlinear plant \( (z = f(\tilde{h})) \) can be used. Note that T-matrix is also used to determine the gain in control update as shown in equation 6.7. For the linear plant model, the single frequency phase sweep results shown in previous sections were used to construct T-matrix information. When nonlinear plant model is used, the T-matrix is obtained by perturbing control input vector \( \tilde{h} \) one component-by-one component.

The components of vibratory loads are compared with/without control in figure 6.18. Note that all of the components are reduced with multi-cyclic control inputs. When the current vibratory loads are evaluated using linear plant model, the similar reductions of 25% for all components were observed. If the actual
nonlinear plant model is used to evaluate current vibratory loads, the largest reduction of 51% is observed for vertical force component. 18%, and 22% reductions are observed for inplane force and moment, respectively.

\[
CostFunction = 100 \times \frac{\sqrt{z^T W_z z}}{\sqrt{z_0^T W_z z_0}} \tag{6.9}
\]

Figure 6.19 shows the history of cost function value defined in equation 6.9. Cost function value of 100 represents that of the baseline, and using HHC controller, similar reduction of 25% is observed for both plant models.

Figure 6.20 depicts the higher harmonic input of active Gurney flap that 51% reduction in vertical force. Note that 3/rev component and 5/rev component are dominant while 4/rev component has smaller magnitude than those two components.

The basic principle of vibration reduction using active Gurney flap is to reduce the vibratory loads in the rotating frame, which is the source of hub vibratory loads. For the vertical vibratory loads, the 4/rev component in rotating frame contributes to the hub vertical loads while other frequencies are cancelled out. Figure 6.21 shows the 4/rev component of vertical root force. As can be seen from the figure, multicycle actuation of active Gurney flap reduces the vertical loads in the rotating frame, thus lead to 51% reduction in the hub vibratory loads.
Figure 6.18. Comparison of vibratory loads components

Figure 6.19. Convergence history of cost function
Figure 6.20. Multicyclic active Gurney flap inputs

Figure 6.21. Harmonic Components of Vertical Root Loads
Concluding Remarks

7.1 Summary and conclusions

In the present study, the application of active Gurney flap for performance enhancement and vibration reduction was examined. Broad range of research activities were conducted including

- Development of numerical tool for rotorcraft comprehensive analysis
- CFD analysis for Gurney flap at the 100\%c an 90\%c
- Implementation of unsteady lift of upstream Gurney flap
- Integration of full unsteady aerodynamic model into comprehensive analysis

With the established numerical code for rotorcraft, parametric studies were conducted to examine the effect of active Gurney flap on the performance improvement in chapter 4. Parametric studies with rigid blade model covered broad topics, and the physics of performance enhancement using Gurney flap is examined in a greater detail. The integration of unsteady aerodynamic model was one of the important goal for the rigid blade analysis. The parametric study was extended with the use of elastic blade model, and different physical phenomena was observed with high fidelity model. For example, the effect of increasing the width of active Gurney flap turn out to be different between rigid blade and elastic blade.

The conclusions from part I of the chapter 4 are summarized as follows
• 1/rev actuation of active Gurney flap alleviates stall when phase angle $\phi_{1P}$ is 180°

• Velocity sweep conducted with gross weights of 16,000 lbs, 18,300 lbs, and 22,000 lbs at altitude of 8,000 ft. With 1/rev actuation of Gurney flap, the main rotor power is increased by 0.4% when gross weight is 16,000 lbs. For higher gross weights of 18,300 lbs and 22,000 lbs, the main rotor power is reduced by 2.3% and 7.4%, respectively.

• The effect of active Gurney flap span is examined at gross weight of 22,000 lbs. The 30%R span active Gurney flap yielded 9.4% main rotor power reduction.

• The effect of chordwise location of active Gurney flap span is examined at gross weight of 22,000 lbs. With the active Gurney flap located at 90%c, 6% reduction in main rotor power is observed.

• The effect of unsteady aerodynamic model is examined at gross weight of 22,000 lbs with active Gurney flap located at 90%c. When the unsteady aerodynamic effect is accounted for, 5% reduction in main rotor power is observed.

• The effect of gross weight is examined at flight speed of 150 knots. The baseline rotor stalls at when gross weight is 19,600 lbs. With the active Gurney flap actuation, the main rotor power at 19,600 lbs can be reduced by 3.6% when the active Gurney flap at 90%c with unsteady aerodynamic model is used. Active Gurney flap actuation can increase the gross weight by 1,000 lbs.

• The effect of 2/rev actuation frequency is examined at gross weight of 22,000 lbs. The maximum power reductions achieved are 7% (for Gurney flap at 100%c), 4.9% (for Gurney flap at 90%c), and 2.55% (for Gurney flap at 90%c with unsteady effect)

The conclusions from part II of the chapter 4 are summarized as follows
• Velocity sweep conducted at gross weight of 22,000 lbs at altitude of 8,000 ft. The effect of elastic blade model is accounted for. When the elastic blade model is used, the maximum velocity is 132 knots while the maximum velocity of rigid blade model is 108 knots.

• When 1/rev actuation of Gurney flap is applied at stalled condition, the main rotor power is reduced by 5%.

• The effect of active Gurney flap span is examined at gross weight of 22,000 lbs. Increasing the span of active Gurney flap does not yield progressive reduction in main rotor power. 10%R span shows best reduction of 5%.

• The effect of 2/rev actuation frequency is examined at gross weight of 22,000 lbs. The maximum power reductions achieved is 4.6%.

With the fundamental understanding from preliminary studies, the performance enhancement using active Gurney flap was examined in detail. The 1/rev active Gurney flap was used for maximizing thrust capability at 130 knots and 90 knots, increasing the maximum altitude, and increasing maximum level flight. The stall alleviation was realized by augmenting additional lift from active Gurney flap while maintaining trim. Which in turn implies that the trim control can be changed so that the main rotor power is reduced, due to the additional lift provided by active Gurney flap actuation.

The conclusions from chapter 5 is summarized as follows

• Power reduction and maximizing thrust capability is examined at a flight speed of 130 knots. With 1/rev actuation of active Gurney flap, the main rotor power is reduced by 4.9% when the baseline rotor is stalled. The gross weight can be increased by 950 lbs.

• Power reduction and maximizing thrust capability is examined at a flight speed of 90 knots. With 1/rev actuation of active Gurney flap, the main rotor power is reduced by 11.3% when the baseline rotor is stalled. The gross weight can be increased by 990 lbs.
• Extension of altitude is examined with 1/rev actuation of active Gurney flap. The main rotor power is reduced by 11.4%. The maximum altitude is increased by 1,440 ft.

• Maximum level flight speed can be extended by 28 knots using 1/rec actuation of active Gurney flap.

In the last part of the present study, the effect of active Gurney flap on the vibration reduction was examined. Near the stall boundary, not only the main rotor power, but also the vibratory loads are rapidly increasing. Vibration reduction through stall alleviation was examined at high altitude and high gross weight condition discussed in chapter 4. It was demonstrated that the excessive vibratory loads encountered at the stall condition can be relived by stall alleviation realized by Gurney flap actuation. The vibration reduction using higher harmonic frequencies were studied. Single frequency phase sweep was conducted to identify the effect of each single frequency on the vibratory loads. The present study was closed with the demonstration of multicyclic higher harmonic control to reduce the vibratory loads up to a greater extent.

The conclusions from chapter 6 is summarized as follows

• Vibration reduction by stall alleviation is examined at 90 knots. With 1/rev actuation of active Gurney flap at stalled condition, 4/rev vertical force can be reduced by 24%. 4/rev inplane force and inplane moments are reduced by 67.6%, 44%, respectively.

• Single frequency phase sweep is conducted with 3/rev frequency. 4/rev inplane force is reduced by 15% with $\tilde{h}_{\text{max}} = 2\%, \phi_{3P} = 60^\circ$. 4/rev vertical force is reduced by 24% with $\tilde{h}_{\text{max}} = 1\%, \phi_{3P} = 150^\circ$. 4/rev inplane moment is reduced by 36% with $\tilde{h}_{\text{max}} = 2\%, \phi_{3P} = 90^\circ$.

• Single frequency phase sweep is conducted with 4/rev frequency. 4/rev inplane force does not reduce with the combination of phase angle and amplitude considered. 4/rev vertical force is reduced by 74% with $\tilde{h}_{\text{max}} = 2\%, \phi_{4P} = 240^\circ$. 4/rev inplane moment is reduced by 24% with $\tilde{h}_{\text{max}} = 2\%, \phi_{4P} = 270^\circ$. 
• Single frequency phase sweep is conducted with 5/rev frequency. 4/rev inplane force does not reduce with the combination of phase angle and amplitude considered. 4/rev vertical force is reduced by 81% with $h_{\text{max}} = 1\%$, $\phi_5 = 330^\circ$. 4/rev inplane moment is reduced by 17% with $h_{\text{max}} = 1\%$, $\phi_4 = 60^\circ$ to $90^\circ$.

• Multicyclic Higher Harmonic Controller is used to reduce main rotor vibratory loads. 25% reduction in objective function is observed. For individual components, 4/rev vertical force is reduced by 51%, 4/rev inplane force is reduced by 18%, and 4/rev inplane moment is reduced by 22%.

7.2 Proposed works for the future

Comprehensive analysis of rotorcraft was conducted in present study to examine the effect of active Gurney flap on the performance improvement and vibration reduction. The following topics would be recommend for further investigation.

• Implementation of refined unsteady aerodynamics

The unsteady aerodynamic model for active Gurney flap was considered in the present study, but it includes the exact expression for the unsteady lift only. The increment of unsteady drag and unsteady pitching moment is estimated using simple equations. More rigorous methods - indicial functions are need to be established for proper treatment of unsteady airloads. Due to the limitation of the time, the indicial functions for unsteady drag and unsteady moment were not able to accomplish. Note that unsteady CFD simulation results are very important in determining the parameters in the indicial functions.

• Coupling with 3D full CFD analysis

The unsteady aerodynamics model has a limitation in its nature due to the 2D assumption. The 3D effect is partially accounted for by the lifting-line and wake model, but the influence of radial flow can not be modelled with classic lifting line model. As the computation power increases day by day, and parallel computing is more affordable than before. Thus coupling of
rotorcraft comprehensive analysis with full 3D CFD would be the right direction to proceed.

- Examine different helicopter
  In present study, UH-60A Black Hawk helicopter was used as a baseline configuration, which has articulated rotor. It would be interesting to examine the different rotor configuration. Bo-105 helicopter would be a good example. Furthermore, this helicopter has been extensively studied for vibration reduction using active control devices, thus it would be beneficial to examine the effect of active Gurney flap on Bo-105 helicopter.

- Vibration Control Scheme
  In the present study, the multicyclic controller was configured to reduce the vibratory loads of UH-60A helicopter. Considering the large reductions reported in the literature, it would be necessary to examine the vibration control scheme further in detail. In present study, simple fixed gain controller was adopted which is known to work better with linear plant model. Implementation of adaptive control scheme will be beneficial for controlling nonlinear plant like a helicopter.

- Noise Reduction using active Gurney flap
  Plain trailing edge flap demonstrated its ability to reduce noise. Active Gurney flap has an ability to change the aerodynamic loadings over the rotor disk, thus active Gurney flap may be used in a similar manner as that of trailing edge flap. Thus application of active Gurney flap for the noise reduction purpose would be necessary.

- Studies on the actuation scheme
  The present study did not addressed the actuation scheme. For practical implementation, how to implement the active device is an important issue.

- Experiments for practical implementation
  Wind tunnel test of wing section equipped with active Gurney flap is recently conducted. For practical implementation on the rotor, hover test and full
scale test needs to be conducted with active Gurney flap installed. However, this step seems to need more time.
Elemental matrices of Beam Model

A.1 Variation of Strain Energy

The variational form of strain energy is shown below

\[ \frac{\delta U_B}{m_0\Omega_0^2 R^3} = \int_0^1 (U_{uu} \delta u' + U_{uv} \delta v' + U_{uv'} \delta v'' + U_{ww} \delta w' + U_{ww'} \delta w'') \]

+ \ U_\phi \delta \phi + U_\phi' \delta \phi' + U_\phi'' \delta \phi'') dx

where

\[ U_{uu} = EA \left[ u_e' + k_A^2 \theta_0' (\phi' + w' v') + k_A^2 \phi'^2 \right] \]

\[ \quad - EAc_A \left[ v'' \cos(\theta_0 + \phi) + w'' \sin(\theta_0 + \phi) \right] \]

\[ U_{vv} = 0 \]

\[ U_{vv'} = \ v''(EI_x \cos^2 \theta_0 + EI_y \sin^2 \theta_0) + w''(EI_x - EI_y) \cos \theta_0 \sin \theta_0 \]

\[ \quad - EAc_A u_e' \cos(\theta_0 + \phi) - \phi' EB_2 \theta_0' \cos \theta_0 \]

\[ \quad + w'' \phi (EI_x - EI_y) \cos 2\theta_0 - v'' \phi (EI_x - EI_y) \sin 2\theta_0 \]

\[ \quad + (GJ + EB_1 \theta_0' \phi' + EAc_A^2 \theta_0' \phi') w'' \]

\[ U_{ww} = \ (GJ + EB_1 \theta_0' \phi' + EAc_A^2 \theta_0' \phi') w'' \]

\[ U_{ww'} = \ w''(EI_x \sin^2 \theta_0 + EI_y \cos^2 \theta_0) + v''(EI_x - EI_y) \cos \theta_0 \sin \theta_0 \]

\[ \quad - EAc_A u_e' \sin(\theta_0 + \phi) - \phi' EB_2 \theta_0' \sin \theta_0 \]

\[ \quad + w'' \phi (EI_x - EI_y) \sin 2\theta_0 + v'' \phi (EI_x - EI_y) \cos 2\theta_0 \]
\[ U_{\phi} = (w'w - v'v)(E_{I_2} - E_{I_3}) \sin \theta_0 \cos \theta_0 + v''w''(E_{I_2} - E_{I_3}) \cos 2\theta_0 \]

\[ U_{\phi'} = GJ(\dot{\phi} + w'\dot{\phi'}) + EB_1 \theta_0^2 \dot{\phi}' + E A k_c A^2 (\theta_0' + \dot{\phi}) u_w' \]

\[ -EB_2 \theta_0'(v'' \cos \theta_0 + w'' \sin \theta_0) \]

\[ U_{\phi''} = EC_1 \dot{\phi}'' + EC_2 (w'' \cos \theta_0 - v'' \sin \theta_0) \]

### A.2 Variation of Kinetic Energy

The variational form of kinetic energy is shown below

\[
\frac{\delta T_B}{m_0 \Omega^2_{ref} R^3} = \int_0^1 m (T_u \delta u_e + T_\nu \delta v + T_\nu' \delta v' + T_w \delta w + T_w' \delta w' + T_\phi \delta \phi + T_F) dx
\]

where

\[
T_u = x + u_e + 2 \dot{v} - \ddot{u}_e
\]

\[
T_\nu = e_g (\cos \theta_0 + \hat{\theta}_0 \sin \theta_0) + v - \dot{\phi} e_g \sin \theta_0 + 2 \dot{v} e_g \cos \theta_0 + 2 \dot{w} e_g \sin \theta_0 - \dot{v} - \ddot{\phi} e_g \sin \theta_0 - 2 \dot{u}_e + 2 \int_0^\xi (v' \dot{v}' + w' \dot{w}') \ d\xi
\]

\[
T_\nu' = -e_g x (\sin \theta_0 - \dot{\phi} \sin \theta_0) - 2 e_g \dot{v} \cos \theta_0
\]

\[
T_w = -\hat{\theta}_0 e_g \cos \theta_0 - \dot{w} - \dot{\phi} e_g \cos \theta_0
\]

\[
T_w' = -e_g x (\sin \theta_0 - \dot{\phi} \cos \theta_0) - 2 e_g \dot{v} \sin \theta_0
\]

\[
T_\phi = -k_{m2}^2 (\hat{\theta}_0 + \dot{\phi}) + x e_g (v' \sin \theta_0 - w' \cos \theta_0) - (k_{m2}^2 - k_{m1}^2) (\cos \theta_0 \sin \theta_0 + \dot{\phi} \cos 2 \theta_0)
\]

\[
-2 \dot{e}_g \sin \theta_0 + e_g (\dot{v} \sin \theta_0 - \dot{w} \cos \theta_0)
\]

\[
T_F = -(x + 2 \dot{v}) \int_0^\xi (v' \dot{v}' + w' \dot{w}') \ d\xi
\]

The traction terms, \( \int_0^1 m T_F dx \), present in the kinetic energy expression can be further simplified as follows.

\[
\int_0^1 m_x (\int_0^x v' \dot{v}' + w' \dot{w}' \ d\xi) \ dx = \int_0^1 \int_x^1 m_x d\xi (v' \dot{v}' + w' \dot{w}') \ dx
\]

\[= \int_0^1 F_A(x) (v' \dot{v}' + w' \dot{w}') \ dx \]

(A.4)

where \( F_A(x) = \int_x^1 m_x d\xi \) denotes the effect of centrifugal force. Similarly,
\[
\int_0^1 2m \dot{v} \left( \int_0^x v' \delta v' + w' \delta w' \, dx \right) \, dx = \int_0^1 \int_0^1 2m \dot{v} d\xi \left( v' \delta v' + w' \delta w' \right) \, dx \\
= \int_0^1 G_A(x)(v' \delta v' + w' \delta w') \, dx
\]

(A.5)

where \( G_A(x) = \int_x^1 2m \dot{v} d\xi \) represents the effect of Coriolis damping.

### A.3 Linear Mass Matrix

Linear mass matrix can be partitioned by

\[
\begin{bmatrix}
M_{uu} & M_{uv} & M_{uw} & M_{u\phi} \\
M_{vu} & M_{vv} & M_{vw} & M_{v\phi} \\
M_{wu} & M_{wv} & M_{ww} & M_{w\phi} \\
M_{\phi u} & M_{\phi v} & M_{\phi w} & M_{\phi \phi}
\end{bmatrix}
\]

where the each submatrices can be give as

\[
M_{uu} = \int_0^1 m H_u H_u^T \, ds \\
M_{uv} = M_{vu} = 0 \\
M_{uw} = M_{wu} = 0 \\
M_{u\phi} = M_{\phi u} = 0 \\
M_{vv} = \int_0^1 m H H^T \, ds \\
M_{vw} = M_{wv} = 0 \\
M_{v\phi} = M_{\phi v} = -\int_0^1 m e_g \sin \theta_0 H H_{\phi}^T \, ds \\
M_{ww} = \int_0^1 m H H^T \, ds \\
M_{w\phi} = M_{\phi w} = \int_0^1 m e_g \cos \theta_0 H H_{\phi}^T \, ds \\
M_{\phi \phi} = \int_0^1 m k m_2 H_{\phi} H_{\phi}^T \, ds
\]

(A.6)
A.4 Linear Damping Matrix

Linear damping matrix can be partitioned by

\[
\begin{bmatrix}
C_{uu} & C_{uv} & C_{uw} & C_{\hat{u}\phi} \\
C_{vu} & C_{vv} & C_{vw} & C_{\hat{v}\phi} \\
C_{wu} & C_{wv} & C_{ww} & C_{\hat{w}\phi} \\
C_{\hat{u}\phi} & C_{\hat{v}\phi} & C_{\hat{w}\phi} & C_{\hat{\phi}\hat{\phi}}
\end{bmatrix}
\]  
(A.7)

where the each submatrices can be give as

\[
C_{uu} = 0 \\
C_{uv} = -C_{vu} = -\int_0^1 2m\Omega H_u H^T \, ds \\
C_{uw} = C_{wu} = 0 \\
C_{\hat{u}\phi} = C_{\hat{\phi}\hat{u}} = 0 \\
C_{vv} = \int_0^1 2m e_g \Omega \cos \theta_0 H H^T \, ds - \int_0^1 2m e_g \Omega \cos \theta_0 H H^T \, ds \\
C_{vw} = -C_{wv} = -\int_0^1 2m e_g \Omega \sin \theta_0 H H^T \, ds \\
C_{\hat{v}\phi} = C_{\hat{\phi}\hat{v}} = 0 \\
C_{ww} = 0 \\
C_{\hat{w}\phi} = C_{\hat{\phi}\hat{w}} = 0 \\
C_{\hat{\phi}\hat{\phi}} = 0
\]  
(A.8)

A.5 Linear Stiffness Matrix

Linear damping matrix can be partitioned by

\[
\begin{bmatrix}
K_{uu} & K_{uv} & K_{uw} & K_{\hat{u}\phi} \\
K_{vu} & K_{vv} & K_{vw} & K_{\hat{v}\phi} \\
K_{wu} & K_{wv} & K_{ww} & K_{\hat{w}\phi} \\
K_{\hat{u}\phi} & K_{\hat{v}\phi} & K_{\hat{w}\phi} & K_{\hat{\phi}\hat{\phi}}
\end{bmatrix}
\]  
(A.9)

where the each submatrices can be give as
\[ K_{uu} = \int_0^1 EAH_u'H_u'^T \, ds \]

\[ K_{uv} = K_{vu} = -\int_0^1 EAE_A \cos \theta_0 H_u'H''_u' \, ds \]

\[ K_{uw} = K_{wu} = -\int_0^1 EAE_A \sin \theta_0 H_u'H''_u' \, ds \]

\[ K_{u\phi} = K_{\phi u} = \int_0^1 EAk_A^2 \theta_0'H_u'H_{\phi}' \, ds \]

\[ K_{vv} = \int_0^1 FAH'H''_T + (EI_y \sin^2 \theta_0 + EI_z \cos^2 \theta_0)H''H''_T - m\Omega^2 HH''_T \, ds \]  

(A.10)

\[ K_{vw} = K_{wv} = -\int_0^1 (EI_z - EI_y) \sin \theta_0 \cos \theta_0 H''H''_T \, ds \]

\[ K_{v\phi} = K_{\phi v} = \int_0^1 m\Omega^2 e_g \sin \theta_0 HH_{\phi}' + x\Omega^2 e_g \sin \theta_0 H'H_{\phi}' \, ds \]

\[ K_{w\phi} = K_{\phi w} = 0 \]

\[ K_{\phi \phi} = 0 \]
Bibliography


Vita
Eui Sung Bae

Education

- Ph.D. at The Pennsylvania State University (2006-2012)
- M.S. at The Seoul National University (2003-2005)
- B.S. at The Seoul National University (1996-2003)

Publication