THE IMPACT OF CONSUMER RETURNS ON MANUFACTURER’S INVESTMENTS, RETURNS POLICIES AND SALES CHANNEL DESIGN

A Dissertation in
Business Administration

by

Paolo Letizia

© 2012 Paolo Letizia

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

August 2012
The dissertation of Paolo Letizia was reviewed and approved* by the following:

Terry P. Harrison  
Professor of Supply Chain and Information Systems, Earl P. Strong Executive Education Professor in Business  
Co-Chair of Committee  
Dissertation Advisor

Kalyan Chatterjee  
Distinguished Professor of Economics and Management Science  
Co-Chair of Committee

Keith J. Crocker  
The William Elliot Chaired Professor of Insurance and Risk Management

Douglas J. Thomas  
Associate Professor of Supply Chain and Information Systems, Director of MBA Program

Susan H. Xu  
Professor of Management Science and Supply Chain Management

Gene Tyworth  
J. Coyle Endowed Professor of Supply Chain Management  
Department Chair of Supply Chain & Information Systems

*Signatures are on file in the Graduate School.
Abstract

Many companies consider the flow of consumer returns to be a nuisance and a source of reduced profits. Returns of products to the point of sale are continuously increasing, and the costs associated with acquiring, shipping, inspecting, refurbishing, and remarketing returned products can reach unsustainable levels for both manufacturers and retailers. Further, the majority of products are returned not because of product failure but rather because of a mismatch with consumer taste and needs, which suggests that there are several countermeasures that can be taken to reduce the flow of returns.

In this work we take the perspective of a manufacturer and investigate how the manufacturer can reduce the negative impact of returns on his profits through investments, returns policies, and design of the sales channel. Investments to discourage consumers from returning the product are relevant because most of the products are returned not for reasons of product performance or quality but for difficulty in the installation, configuration, and use of the product. The examples of manufacturers, who are active in providing simple and easy to follow instructions and in making their products more intuitive to use, abound. Returns policies adopted by manufacturers with their retailers are an ideal instrument to induce retailer’s collaboration in product value recovery. Especially for products that are time-sensitive in nature, it is necessary that retailers actively engage in inspecting and refurbishing the returned products rather than sending them back to the manufacturer. Finally, through optimal design of the sales channel, the manufacturer can attract consumers towards the sales channel where returns have a lower impact on the manufacturer’s profits. While the online channel allows the manufacturer to provide consumers with a higher product value, for instance through product variety, availability, and customization,
it poses the problem of uncertainty about product fit, which may trigger a product return. Investments, returns policies, and sales channel design are then three important tools for manufacturers to alleviate the negative impact of consumer returns on their profits. This work is articulated in four chapters and proposes practical rules for manufacturers to cope more efficiently with the problem of consumer returns.

Chapter 1 contains an introduction to the problem of consumer returns and gives a thorough review of the following three chapters.

Chapter 2 considers the problem of a manufacturer who is interested in having the retailer repackage and resell the products rather than send the products back to him. The manufacturer may take a costly hidden action to reduce the expected number of products returned by the consumer, but the returned products are privately observed by the retailer who has little incentive to engage in product recovery. We find that to discourage returns from the retailer the manufacturer should design an optimal returns policy which entails a full refund of the wholesale price for any returns, coupled with a bonus that is decreasing in the number of returns to the manufacturer.

Chapter 3 considers the inefficiencies in the product value recovery due to information asymmetry along the supply chain. We consider a manufacturer who faces consumer returns at the retail store and has to decide about a preventative investment before the new selling season starts, when the causes of returns are still uncertain. By the beginning of the selling season, the information about the causes of returns and thus about the effectiveness of the previous investment in reducing returns can be updated. We investigate how profits and investment are affected by the information held by retailer and manufacturer about the future product return rate. It is shown that when there is no or an asymmetric update of information about the product return rate, the manufacturer might opt for a less efficient product value recovery and, in some cases, he would forgo investing in the first place.

Finally, Chapter 4 considers the problem of the manufacturer to select the sales channel for his products. Through an online channel the manufacturer can offer
higher product value to the consumer than through the traditional retail channel, mainly because of product variety, availability and customization. However, there might be uncertainty about product fit and the manufacturer has to strategically control, through prices and a returns policy, the flow of consumers buying his product through the online channel vs. the retail channel. We find that when consumers prefer purchasing at the retail store, the manufacturer might decide to open anyway an online channel but just to induce the retailer to lower the selling price, thus boosting manufacturer’s demand and profits. When the salvage value that can be extracted from returns is sufficiently high, the manufacturer offers his products through both the retail and the online channel. Further, the manufacturer will adopt a liberal returns policy because in this way he can lead some of the consumers to switch towards an online purchase, which would alleviate the inefficiencies due to double marginalization occurring at the retail channel.
# Contents

List of Figures viii

List of Tables x

Acknowledgments xi

1 Introduction 1

2 Optimal Policies for Recovering the Value of Consumer Returns 7
   2.1 Introduction ................................................. 7
   2.2 The Model ................................................. 10
   2.3 First-Best Pareto Optimal Contracts .................... 15
   2.4 Second-Best Pareto Optimal Contracts ................. 16
   2.5 Discussion ............................................... 23
   2.6 An Example ............................................... 26
   2.7 Conclusions ............................................... 31

3 Investments to Reduce Consumer Returns under Information Asymmetry 36
   3.1 Introduction ............................................... 36
   3.2 Literature Review ......................................... 39
   3.3 Model Assumptions and Notation ....................... 41
   3.4 Retailer salvages the returned products ............. 43
      3.4.1 Full Information ..................................... 44
      3.4.2 Retailer privately informed ......................... 47
      3.4.3 Investment decision .................................. 50
   3.5 Manufacturer salvages the returned products ....... 52
      3.5.1 Full Information ..................................... 52
      3.5.2 Retailer privately informed ......................... 53
      3.5.3 Investment decision .................................. 55
   3.6 Endogenous reverse channel ............................ 55
   3.7 Extension - Manufacturer privately informed ....... 57
      3.7.1 Retailer salvages the returned products .......... 59
      3.7.2 Manufacturer salvages the returned products ...... 61
      3.7.3 Endogenous reverse channel ......................... 63
   3.8 Conclusions ............................................... 64
# The Impact of Consumer Returns on a Manufacturer Multichannel Strategy

## 4 The Impact of Consumer Returns on a Manufacturer Multichannel Strategy

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Introduction</td>
<td>69</td>
</tr>
<tr>
<td>4.2 Literature Review</td>
<td>69</td>
</tr>
<tr>
<td>4.3 The Model</td>
<td>72</td>
</tr>
<tr>
<td>4.3.1 Retailer</td>
<td>74</td>
</tr>
<tr>
<td>4.3.2 Consumers</td>
<td>74</td>
</tr>
<tr>
<td>4.3.3 Manufacturer</td>
<td>75</td>
</tr>
<tr>
<td>4.3.4 The Vertically Integrated Case</td>
<td>76</td>
</tr>
<tr>
<td>4.3.4.1 Manufacturer’s Optimization Problem</td>
<td>81</td>
</tr>
<tr>
<td>4.4 Analysis of Supply Chain Dual Channel Design</td>
<td>84</td>
</tr>
<tr>
<td>4.5 Conclusions</td>
<td>90</td>
</tr>
</tbody>
</table>

## Bibliography

| Appendix A | Proofs for Chapter 2 | 98 |
| Appendix B | Proofs for Chapter 3 | 101 |
| Appendix C | Proofs for Chapter 4 | 111 |
List of Figures

1.1 Costs breakdown for consumer returns. .................................. 2
2.1 Sequence of events. ........................................................... 14
2.2 Optimal allocation consisting of a decision, \( y(x) \), and a refund, \( r(x) \), implemented by the indirect mechanism, \( r(y) \). ....................... 18
2.3 Optimal number of returns to the manufacturer, \( y(x) \), in a first-best (dashed line) and a second-best (solid line) contract, for different values of the salvage value, \( s \). .................................................. 29
2.4 Optimal manufacturer returns policy, \( r(x) \), for different values of the salvage value, \( s \). .................................................. 30
2.5 Optimal manufacturer bonus, \( B(x) \), for different values of the salvage value, \( s \). .................................................. 31
2.6 Optimal manufacturer returns policy, \( r(y) \), for \( s = 0 \). ................. 31
3.1 Sequence of Events. ........................................................... 42
3.2 Manufacturer’s profits for \( 0 < v < \frac{\phi}{2} \), as a function of the manufacturer’s type, \( \gamma^l \). For \( \gamma^H \leq \bar{\gamma} \), any reduction of the product return rate is profitable, whereas for \( \bar{\gamma} < \gamma^H < \bar{\gamma} \), only a sufficiently high reduction of the product return rate (\( \gamma^L < \bar{\gamma} \)) is profitable. ....................... 46
3.3 Manufacturer’s expected profits in the first stage of the game as a function of \( \beta \), given in the second stage of the game a full information (squares) and a screening equilibrium (triangles), for the two cases (a) \( \gamma^H \leq \bar{\gamma} \) and (b) \( \bar{\gamma} < \gamma^H < \bar{\gamma} \), \( \gamma^L < \bar{\gamma} \). The values of the parameters in the two graphs are \( \phi = 100 \), \( v = 30 \), \( \gamma^L = 0.1 \), and \( \gamma^H = 0.5 \) in (a) and \( \gamma^H = 0.63 \) in (b). ....................... 50
3.4 Manufacturer’s profits in the second stage of the game when the manufacturer salvages returns and separates (two dots) and when the retailer salvages returns and the manufacturer pools (solid line) for the case $\gamma^H \leq \gamma$. The values of the parameters are: $\phi = 100$, $v = 30$, $\gamma^L = 0.1$ and $\gamma^H = 0.5$. 64

4.1 Consumer decision tree 75

4.2 Consumer expected utilities from purchasing the product at the retailer (solid line), and online (dashed line), when $m^H > 1$, $p_r \leq A$, and $v_z \leq v_x$. 79

4.3 Demand function at the retail channel when $m^H > 1$ 80

4.4 Demand function at the retail channel when $\mu < 1$ 81

4.5 Section on the plane $wp_o$ of the constraints $v^u_K \geq u^u_K$ (light gray) and $v^L_K \leq u^L_K$ (dark gray). The unshaded region corresponds to the retailer’s demand at the kink point $K$. We have set $f_o$ equal to its optimal value and used the following parameters values: $s = 0.25$, $c = 0.4$, $m = 0.8$, $\theta^H = 1.5$, $\theta^L = 0.5$. 87
List of Tables

2.1 Optimal values of $a$, $\mu$, and $w$ for different values of $s$. . . . . . . . . . . . 28
4.1 Demands at the retail and the online channels and returns at the online
channel for all possible cases. . . . . . . . . . . . . . . . . 82
Acknowledgments

I am heartily thankful and deeply indebted to all members of my committee for their guidance and support throughout the course of my Ph.D. studies. I am grateful to Dr. Terry P. Harrison for his thoughtful supervision and caring guidance. I thank Dr. Kalyan Chatterjee, especially for his continuous support in teaching me complex concepts of Game Theory. I am extremely grateful to Dr. Keith J. Crocker, a leading example during my academic studies: his contagious passion for research and strong commitment to the students will certainly inspire my future academic endeavours. I thank Dr. Douglas J. Thomas for keeping always a door open and sharing his sharp and intellect stimulating thoughts. I am grateful to Dr. Susan Xu, especially for her inspirational commitment to research, her continuous support and reassuring friendship.

I would like to show my deep gratitude to Alice Young, Jane Jones, Beth Bower, and Teresa Lehman because they were like a second family here and for their continuing encouragement and support. I am extremely grateful to Father Matthew Laffey and all the Catholic community at Penn State for their spiritual support and their prayers.

Most importantly, none of this would have been possible without the unconditional love and support of my wife. She shared every moment of this journey, a journey made at times of joy and at times of suffering, a journey where often going home was for me the only source of peace, thanks to her reassuring presence, support and love. It is only thanks to her that I could overcome the obstacles and realize the goal of the Ph.D. I would like also to thank my mother, brother and sister for their words of encouragement. Finally, I thank my two little boys, Francesco and Pietro, who without realizing it gave me the strength and courage to persevere and succeed.
Chapter 1

Introduction

While many consumers have grown accustomed to being able to bring unwanted merchandise back to the store for just about any reason, manufacturers and retailers have to cope with an increasing number of returned units. The figures are staggering. Recent estimates in the consumer electronics industry suggest that the rate of return for electronic devices ranges from 11% - 20% in the United States and between 2% - 9% in Europe (Steger et al. 2007). Other product categories exhibit even more daunting rates, up to 35% for fashion apparel (Toktay 2003). The return rates for products purchased over the Internet are at least 5% higher than the rates for the products purchased at a retail store (Ofek et al. 2011).

The costs associated with consumer returns can be considerable. The total cost of consumer electronics returns and repairs attributed to Europe and US consumers was estimated at $25.3 billion in 2007. The breakdown of these costs is reported in the graph in Figure 1.1, which highlights another disconcerting statistic: 20% of these costs are incurred for processing products that have “no trouble found”. In fact, only about 5% of consumer returns are truly defective (Lawton 2008); the majority of returns occur because of product mismatch with consumer needs and taste, or because of difficulty in product installation and use.

Given the problem of consumer returns, in this work we take the perspective of a manufacturer and investigate three possible actions the manufacturer can undertake to reduce the impact of product returns on his profits. First, as most of the returns are false failure, manufacturers are trying to prevent consumers from returning the product by providing easy and rapidly accessible information about product func-
tionality or by making their product more intuitive to install, configure and operate. These are investments that manufacturers may undertake to reduce the flow of units returned at the retail store. Second, manufacturers require that retailers take their own responsibilities in the value recovery for product returns. Therefore, the design of ad hoc returns policies is a relevant mechanism for the manufacturer to induce the retailer’s collaboration. Third, manufacturers have to carefully design the channel through which they want to sell their products, because consumer returns are significantly higher for e-commerce channels. In summary, this works explores the optimal investments, returns policies, and sales channel that a manufacturer must design and adopt while he is coping with the flow of consumer returns.

In chapter 2 we consider a manufacturer who must both select an optimal investment to reduce returns and design an optimal returns policy to induce the collaboration of his retailer in product value recovery. We assume that the manufacturer sells his product in a competitive market at a fixed, and known, price, and faces a random number of returns at the retail store. The retailer who receives the products returned by the consumer, and therefore is privately informed about the volume of returns,
may engage in a costly “refurbishment” to re-sell a returned product to consumers, or she may return the product to the manufacturer. The manufacturer in turn may take a costly, but hidden, investment that reduces the probability of product returns at the retail store. We characterize a Pareto optimal returns policy in this environment, which may be implemented by the use of a full refund of the purchase price to the retailer of any products returned to the manufacturer, coupled with a bonus paid to the retailer that is decreasing in the number of products actually returned.

The returns policy we characterize reflects a trade-off between providing the manufacturer with the incentive to take the costly action that reduces the number of expected returns, on the one hand, and the incentives retailers face to return the product to the manufacturer rather than to refurbish, on the other. The incentive scheme that we find analytically is adopted in practice by several companies, especially within the Electronics industry, where the manufacturer’s investment to reduce returns is relevant. For instance, Dell and HP have made large investments to develop a system of diagnostics for consumers who have an issue with the purchased product. The same companies provide their retailers with a full refund of the wholesale price plus a compensation which is decreasing in the number of returns to the manufacturer; such a policy is perfectly consistent with our analytical findings. Our research extends also to cases where the manufacturer’s investment is not relevant for reducing returns, as it may be for personal care products. In fact, when there is no hidden action by the manufacturer the returns policy makes the retailer internalize all the costs of returns and thus it consists of a refund of just the salvage value of returned products. As the salvage value for shampoo or other personal care products is supposedly very low, our findings can explain why Johnson & Johnson adopts a strict no-returns policy with its retailers. Finally, we also investigate cases where the retailer has a function of pure interface between the manufacturer and the market as it happens for manufacturers selling their products through a web portal like Amazon. When a consumer returns the product, whose sale was fulfilled by the manufacturer,
it is responsibility of the manufacturer himself to refund the consumer of the selling price. Such a returns policy can be explained by the fact that returns are observable to the manufacturer and therefore there is no more private information of the retailer.

In chapter 3 we consider a scenario where the manufacturer may undertake an investment under uncertainty that affects the product return rate and the outcome of the investment becomes known to either manufacturer or retailer before they start trading. The information asymmetry about the investment outcome, and thus about the future return rate, causes inefficiencies in the manufacturer’s profits. If the manufacturer knows the return rate, then he will have to signal it to the retailer whereas if the retailer knows the return rate, then the manufacturer will have to screen the private information of the retailer. The need to signal or screen the information about the product return rate causes inefficiencies to the manufacturer’s profits, but less so when the manufacturer’s contract consists of a wholesale price and a refund per return rather than just of a wholesale price. The opportunity to reduce these profit distortions leads the manufacturer to then accept returns from the retailer even though in some cases the retailer can extract a higher value from the returned units.

This research gives a possible explanation of why several manufacturers prefer taking the returned products back even though it would be more efficient to have the retailer salvaged the returned products. For example, when the EOS Rebel cameras by Canon are returned to Sears, the agreement is that the manufacturer takes responsibility for returns. However, Canon salvages the cameras through the third-party company Precison Camera, when the same salvage value for returned products could be obtained by Sears, thus avoiding the logistics costs incurred for sending products from Sears back to Canon. These inefficiencies in product recovery are quite common in the Electronics industry and our research shows that the underlying cause is the information asymmetry between manufacturer and retailer about the product return rate.

Finally, in chapter 4 we investigate another important trade-off for the manu-
manufacturer when he must cope with consumer returns. In fact, on one hand, the online channel allows the manufacturer to offer a high product value to the consumers through product availability, variety, and customization. On the other hand, products sold online cannot be physically inspected by the consumers; as a consequence, a mismatch between product and consumer preferences, which may trigger a product return, is likely to occur. While consumers are fully aware of their valuation for the product purchased at the store, the uncertainty about product fit for an online purchase can be mitigated by a liberal returns policy at the online channel. The selection of prices and returns policy then represents the instrument used by the manufacturer to allocate sales across the two channels.

We find that the manufacturer would offer only online sales when consumers expect higher value from the online channel, whereas he adopts a more complex strategy when consumers expect lower value from the online channel. In particular, if the salvage value for returned units is sufficiently low, the manufacturer uses the online channel as a sham: no online sales materialize but the retailer threatened by the online channel reduces the selling price, thus decreasing some of the inefficiencies due to double marginalization. If the salvage value for returned units is sufficiently high, however, the manufacturers sells the product through both the online and the retail channel, but softens the returns policy as opposed to the case of online channel only. The manufacturer adopts a more liberal returns policy because he may lead more consumers to switch towards the online channel and so partially reduce some of the inefficiencies of double marginalization occurring at the retail store.

In summary, this work investigates different strategies a manufacturer can adopt to cope with the expenses caused by the flow of consumer returns. By undertaking the right investments, the manufacturer may discourage consumers from returning the products. By designing the right returns policy, the manufacturer can lead the retailer to collaborate in the product value recovery. By designing the sales channel for his product, the manufacturer can lead consumers to make a purchase where the
impact of consumer returns is lower.

Notes

1 The data reported in Figure 1.1 are available at the following source: http://www.accenture.com/SiteCollectionDocuments/PDF/22701ReturnsRepairsRun04lr.pdf
Chapter 2

Optimal Policies for Recovering the Value of Consumer Returns

2.1. Introduction

There is a surprising variety in the policies adopted by manufacturers to deal with the products returned to their retailers by consumers. Some manufacturers, such as Johnson & Johnson, have a strict no-returns policy in which the retailer alone deals with the disposition of any consumer returns. In contrast, Dell and HP not only accept consumer returns from retailers, but also provide compensation to the retailers for these returns. Dell refunds the full wholesale price to the retailer for any returns and, for any number of returns below a prespecified cap, pays a bonus that is decreasing in the number of returns. HP also provides a full refund to retailers for returns but, in addition, sets aside a returns allowance, the amount of which is determined by a specified discount from the wholesale price on all sales, in a “holding account”. For each return to the manufacturer, HP deducts a constant fee from the account, the balance of which (either positive or negative) is paid to the retailer on a quarterly basis.

Finally, manufacturers selling their products through an Internet retailer such as Amazon.com generally fulfill their sales orders and accept all returns directly from the consumers. When a consumer wants to return his product, the Amazon “Online Returns Center” creates a return request that is forwarded to the manufacturer. In this case, the consumer is refunded directly by the manufacturer and the retailer acts simply as an intermediary between the manufacturer and the consumer. By
providing a model of optimal returns policies, this paper provides a rationale for why, in practice, manufacturers adopt different returns policies.

The use of returns policies by manufacturers to alleviate overstock problems by retailers facing random demand has been extensively studied and is well-understood.\textsuperscript{1} It has become increasingly apparent, however, that many manufacturer returns policies are motivated less by inventory management concerns than by the need to deal with the returns of products by consumers to retailers, particularly when the products involved have a short life cycle or are time-sensitive in nature.\textsuperscript{2} In such a setting, timely disposition of the returned products becomes paramount, and providing incentives for retailers to refurbish and resell the product, rather than returning it to the manufacturer who harvests a low salvage value from the return, becomes the overriding concern. This is the topic of our paper.

We consider an environment in which a manufacturer sells a product to a retailer, which is then resold to consumers in a competitive market at a fixed, and known, price. The consumer sales generate a number of returns to the retailer, which is a random variable, and the retailer may engage in costly “refurbishment” to re-sell a returned product to consumers, or it may return the product to the manufacturer. We assume that the manufacturer may take a costly, but hidden, action that reduces the probability of product returns, and that the actual number of returns is hidden information known only to the retailer. We characterize a Pareto optimal returns policy in this environment, which may be implemented by the use of a full refund of the purchase price to the retailer of any products returned to the manufacturer, coupled with a bonus paid to the retailer that is decreasing in the number of products actually returned.\textsuperscript{3}

The optimal returns policy reflects a trade-off between providing the manufacturer with the incentive to take the costly action that reduces the number of expected returns, on the one hand, and the incentives retailers face to return the product to the manufacturer rather than to refurbish, on the other. At one extreme, a pure sales con-
tract that entailed no payment to the retailer for returns to the manufacturer would provide the retailer with the incentive to engage in the economically efficient level of refurbishment, but such a contract would also provide the manufacturer with no incentive to take the costly action, leading to an excessive number of consumer returns. Alternatively, compensating the retailer for returns incentivizes the manufacturer to invest in reducing the number of returns from consumers, but such a policy also gives the retailer the incentive to send back too many returns to the manufacturer. The optimal returns policy involves a balancing of these competing efficiency effects.

This work builds on what has become a substantial literature on the problem of consumer returns and the appropriate design of reverse supply chains. Blackburn et al. (2004) report that a significant portion of the residual value of the returned products is eroded because of delays in the reverse supply chain. They suggest a strategy of preponement in which products with a high marginal value of time should have a responsive reverse chain involving an early disposition decision, in terms of testing, sorting and remanufacturing, of returns. Similarly, Guide et al. (2006) show that when the return rate and value decay of returns are both low, centralized evaluation is profitable, but when both are high it is profitable to increase the responsiveness of the reverse chain by placing the evaluation facility as early as possible, even at the retailer’s end. Finally, Ferguson et al. (2006) are concerned with the management of “false failure” returns by consumers, which is consistent with the problem we examine. They report that many companies, especially in the electronics industry, face a low percentage of returns with real functional or cosmetic defects, and that the majority of returns occur for other reasons including installation difficulties, product performance incompatible with consumer preferences, and buyer’s remorse. In order to provide retailers with the incentive to deal appropriately with the consumer returns, the authors recommend the implementation of a target rebate contract that pays the retailer a specific amount per unit of false failure returns below a target level. Our contribution is to derive formally a Pareto optimal returns policy in a reverse supply
chain setting in which early disposition of consumer returns is important.

The chapter proceeds as follows. In section 2.2 we describe the environment and the model, and in section 2.3 we characterize the (first-best) Pareto optimal returns in an environment without hidden information or actions. In section 2.4 we characterize the (second-best) Pareto optimal returns policy when manufacturers take a hidden action and the actual number of returns is hidden information possessed by the retailer. Section 2.5 provides a discussion of the results, and in section 2.6 we derive closed-form solutions to the optimal returns policy for a specific example. A final section contains concluding remarks.

2.2. The Model

Our setting consists of a risk-neutral manufacturer selling, through a risk-neutral retailer, a product to the end consumer in a market characterized by consumer returns. For simplicity and without loss of generality we select units to normalize this sale quantity to one. The products are first sold by the manufacturer to the retailer at the per unit wholesale price \( w \) which is determined below, then by the retailer to the end consumer in a competitive market at the exogenously determined per unit selling price \( p \). As commonly seen in practice, once the product is purchased by the consumer, the retailer’s returns policy allows the return of the product to the store for a full refund of the selling price, which is typically referred to as “no-question, money-back guarantee” and it is broadly applied by most major retailers in the United States. Once the products are returned, the retailer must decide whether to send them back to the manufacturer, or to expend resources to refurbish and resell the products to the consumers.

The number of returns by consumers to the retailer is assumed to be a random variable, \( x \), distributed on the interval \([\underline{x}, \overline{x}]\) according to the distribution \( F(x | a) \) where \( a \) is an action taken by the manufacturer that affects the probability distribution
of returns. The action $a$ is known only by the manufacturer, and is therefore a \textit{hidden action}. We assume that $F$ is a concave function of the action $a$ and that higher values of the action $a$ shift the distribution of the number of returns to the left in the sense of first order stochastic dominance.\footnote{One possibility would be to think of the manufacturer’s action $a$ as an investment that increases product quality, thereby reducing the potential for product failure and the associated return of the product to the retailer. However, as noted earlier, the majority of consumer returns in practice appear to occur for reasons not directly related to product failure (Ferguson et al. 2006). These \textit{false failure} returns may reflect either a mismatch of product attributes with consumer needs and tastes, or perhaps a difficulty in product installation and use. To mitigate this problem, some manufacturers have focused on providing the consumer with easy and rapidly accessible information about product functionality. For example, Vizio Inc., a TV maker, has enclosed in its instruction booklet a one-page guide that helps the consumer to quickly set up the basic configuration of the product. Other manufacturers have taken a number of actions to ameliorate the effects of the technical complexity of their product in order to solve issues related to its hookup, use, or operation. For example, Philips in 2000 embarked on a program called “Initial Experience Predictor”, the specific objective of which was to predict and thus improve the ease of use of Philips products. Since 2000, Philips has reduced returns by more than 500,000 units and saved more than $\$100$ million dollars per year (Sciarrotta 2003). In a similar vein, disk-drive maker Seagate Technology did away with installation CDs in Fall 2007 when it launched a consumer line of digital storage products called OneTouch in which the installation software came preloaded onto the device. Many companies have also increased service support with web and call center enhancements and launched advertising campaigns that encourage the customer to contact the manufacturer before returning the product to the retail store. For instance, in 2007 Sharp Corp. launched an added help service for customers who purchased high-end Aquos TVs. Still active, the service}
allows members to obtain free telephone and Internet advice, including how to set up the equipment, with an option for Saturday in-home service (Lawton 2008).

We assume that the value of returns, \( x \), when realized, is privately observed by the retailer, and therefore is hidden information. Thus, the manufacturer cannot observe the realization, \( x \), of returns to the retail store, while the retailer is not aware of the manufacturer’s action, \( a \), affecting the distribution of consumer returns. What the manufacturer actually observes is the number of products, \( y \), that the retailer chooses to send back to the manufacturer. After returning \( y \) to the manufacturer, the retailer refurbishes and resells the remaining \( (x - y) \) products at the same selling price as the new ones.\(^9\) Refurbishment is costly, however, and so the retailer incurs the convex refurbishment cost \( c(x - y) \). A contract consists of a wholesale price, \( w \), and a refund to the retailer, \( r(y) \), the latter contingent on the amount of returns \( y \) sent by the retailer back to the manufacturer.\(^10\)

The retailer’s profit function is given by

\[
\Pi_R \equiv (1 - y)p - w + r(y) - c(x - y),
\]

where \( c(x - y) \) represents the cost incurred by the retailer to resell the refurbished products. We assume that \( c(0) = c'(0) = 0 \), \( c' > 0 \), and \( c'' > 0 \), so that the cost incurred by the retailer is increasing and convex in the amount of products resold.

There are several different interpretations of \( c(x - y) \) which are consistent with our model. The first is that it may reflect a formal refurbishment cost as a consequence of resources expended by the retailer to bring the returned product back to an as new condition and to resell it at the price \( p \). For example, Sears Holdings Corp. categorizes returns into those that are serviceable and not serviceable, and the retailer takes responsibility for refurbishing and reselling serviceable items such as TVs and DVD players. In addition to the correction of repairable defects, refurbishment may entail the costly inspection and testing of the consumer returns to determine suitability for
resale. An interesting case is provided by Motorola who in the interests of sustainability eliminated the plastic packaging for its cable set-up box. An unintended side effect was that the retailer could no longer determine easily whether the product had been used, and so as a result the returns now required costly inspection and testing prior to resale.

A second interpretation applies in the case of false failure returns having no functional problems. Such returns nonetheless must be restocked, and may also require repackaging, to be resold at the as-new price \( p \). Indeed, it is a common practice in the electronics industry for the manufacturer to provide the retailer with repackaging supplies to expedite resale (Stock et al. 2006). Moreover, after-sales costs incurred by some retailers to avoid consumer returns by effectively reselling the product to the original purchaser are also relevant. For example, the Geek Squad support plan of Best Buy, who offers in-home technical service to those consumers who purchased a product and are not sure of how to install, configure, or use the product. Best-Buy declared that the support program was effective in reducing returns of home-theater systems by 10% and of PCs by 40% (Lawton 2008).

Finally, one could also think of \( c(x - y) \) as the discount from the as-new price of \( p \) required to resell the product returned by the consumer. One often sees “opened box” products selling in retail establishments at substantial discounts simply to move the item off the floor. In each of these cases, the convexity of \( c \) may reflect simple bottlenecks in the refurbishment, inspecting or restocking technologies. Alternatively, it could be that returned products arrive at the retail store in different quality conditions, after which the retailer sorts the returns with respect to their quality and expends resources on the easiest-to-refurbish (and, hence, less costly) returns first. Similarly, the discount from the new price required to resell “opened box” products may be increasing and convex in the quality of the returned item, with those having only the seal broken in the box requiring only a small discount while those having obviously been used requiring a much larger mark down.
The manufacturer’s profit function is given by

$$\Pi_M \equiv w - r(y) - h(a) + sy,$$  \hspace{1cm} (2.2)

where $h(a)$ represents the cost incurred by the manufacturer in exerting the action $a$, and $s$ is the exogenously-determined residual value of any product returned to the manufacturer, where $p > s \geq 0$. We assume that $h$ is strictly increasing and convex, and that $h(0) = h'(0) = 0$.

We now turn to the informational structure of the model, as illustrated by the sequence of events reported in Figure 2.1. At time 0, the contract $\{w, r(y)\}$ is assigned by a social planner, a process we describe in more detail below. At time 1, the manufacturer takes the hidden action $a$. At time 2, the retailer privately observes the realization of the number of returns $x$ at her store, so that $x$ constitutes hidden information. Next, at time 3, the retailer selects the number, $y$, of returned products to send back to the manufacturer, and resells the remaining $(x - y)$ refurbished products at the selling price of $p$. Finally, at time 4, the contract is implemented and the manufacturer is paid $w$ for the products sold to the retailer, and the retailer receives
the refund for returns, \(r(y)\), based on her previously selected value of \(y\).

2.3. First-Best Pareto Optimal Contracts

Before proceeding, we characterize as a benchmark for comparison an optimal contract were the number of consumer returns, \(x\), and the manufacturer’s action, \(a\), publicly observable. In this setting, we may think of a contract being composed of a wholesale price, \(w\), a stipulated manufacturer’s action, \(a\), and a returns policy, \(\{r(x), y(x)\}\) that specifies for each level of consumer returns a payment to the retailer and a number of returns that should be sent back to the manufacturer.\(^{11}\) The contracts we characterize are termed “first-best” because there are no informational constraints (regarding the observability of \(x\) and \(a\)) facing the manufacturer and retailer, in contrast to the scenario examined in section 2.4.

Our approach to characterizing a Pareto optimal contract is to employ the notion of the social planner, which is the standard tool of normative analysis in economics.\(^{12}\) The social planner is assumed to be omniscient, in the sense that it knows every agent’s preferences, as well as omnipotent, having the ability (at time 0 in Figure 2.1) to dictatorially assign contracts to the agents in the economy.\(^{13}\) It is also assumed that the social planner is guided by the Pareto criterion, so that the planner would not assign a contract that was Pareto dominated by another. As shown by Samuelson (1954), the solution to the social planner’s problem can be characterized by a straightforward constrained optimization problem, which in our environment may be written as maximizing the expected profit of the manufacturer

\[
\max_{a,w,r(x),y(x)} \int_\Xi \Pi_M(r(x), y(x), a, w)f(x|a)dx
\]

subject to an expected profit constraint of the retailer
\[
\int_{x} \Pi_R(r(x), y(x), a, w)f(x|a)dx \geq \Pi_R,
\]
where \( f \) is the density function associated with the distribution \( F \).

A solution to this maximization problem characterizes the Pareto optimal contract for a particular level of retailer expected profit, and the full range of Pareto optimal contracts is obtained by varying \( \Pi_R \). In the analysis that follows, we will in the interests of simplicity set \( \Pi_R = 0 \), and we will refer to this as a participation constraint.

**Proposition 2.3.1.** A first-best Pareto optimal contract solves the necessary conditions

(i) \( c'(x - y(x)) = p - s \);

(ii) \( \int_{x} \left[ h'f + (p - s)y(x)f_a \right] dx = 0 \); and

(iii) \( \int_{x} \Pi_R f dx = 0 \).

The first condition of Proposition 2.3.1 characterizes the first-best number of consumer returns that the retailer should forward to the manufacturer, and condition (ii) characterizes the first-best action that should be taken by the manufacturer to reduce consumer returns, which is also in this case the action that maximizes the total expected profits of the supply chain. The final condition characterizes the optimal payments, \( w \) and \( r(x) \).\(^{14}\)

**2.4. Second-Best Pareto Optimal Contracts**

We now turn to the characterization of Pareto optimal contracts when the social planner faces an environment in which the action taken by the manufacturer, \( a \), is a hidden action the value of which is known only to the manufacturer, and the realization of consumer returns, \( x \), is hidden information known only to the retailer. Since the social planner is constrained by the same informational asymmetries that face the agents in the economy, the contracts assigned by the social planner cannot
depend directly on $a$ and $x$, which are not publicly observable. Accordingly, in this setting the social planner faces constraints that were not present in the first-best analysis above, and so the Pareto optimal contracts characterized in this section are necessarily second-best. We begin by considering the effect on the social planner’s problem of the retailer’s hidden information regarding the actual number of consumer returns, $x$. In contrast to the first-best benchmark where the social planner could observe $x$ and assign $y$ accordingly to Proposition 2.3.1(i), in this section the number of returns to the manufacturer is necessarily chosen by the retailer who possesses private information on the actual number of consumer returns.

When presented with the returns policy, $r(y)$, the retailer must decide which level of returns maximizes her profit. With reference to Figure 2.2, the retailer selects the level of returns at which the retailer’s indifference curve in $(r, y)$ space, labeled as $\Pi_R(x)$, is tangent to the refund function $r(y)$. Since the retailer’s indifference curve depends on the actual level of consumer returns, $x$, which is privately known by the retailer, we know that, given $r(y)$, a retailer having consumer returns of $x$ will select the level of returns $y(x)$. In this fashion, the social planner may use a returns policy to influence the number of returns sent back to the manufacturer.

In this environment with hidden information, the standard solution technique is to apply the Revelation Principle (Myerson, 1979). The approach is as follows. We begin by decomposing the contract $r(y)$ into its constituent parts represented by the allocation \{$r(x), y(x)$\}, where we are recognizing that the refund and returns selected by the retailer depend on that retailer’s level of consumer returns, $x$, which we shall refer to as retailer “type”. By the Revelation Principle, there is no loss of generality in restricting our search for an efficient contract to those resulting from the application of a “Direct Revelation Mechanism” in which each retailer announces her type to be $\hat{x}$, and receives from the social planner the allocation \{$r(\hat{x}), y(\hat{x})$\} that satisfies the condition

$$\Pi_R(r(x), y(x), w | x) \geq \Pi_R(r(\hat{x}), y(\hat{x}), w | x), \quad \forall x, \hat{x} \in [\underline{x}, \overline{x}] . \quad (2.3)$$
Equation (2.3), which is often referred to as the “truth-telling” condition, guarantees that the retailer who is privately informed of the realization of returns, $x$, would always weakly prefer the contract $\{r(x), y(x), w\}$ to the contract $\{r(x'), y(x'), w\}$ for every $x' \neq x$. In other words, the contract chosen by the social planner provides the incentive for the privately-informed retailer to truthfully reveal her type, so that a retailer of type $x$ receives the allocation $\{r(x), y(x)\}$. Once the efficient allocations are characterized, we can recover the refund policy of interest, $r(y)$, by inverting $y(x)$ and substituting the result into $r(x)$. The truth-telling condition (2.3) guarantees that, when faced with the returns policy $r(y)$, the type $x$ retailer selects the level of manufacturer returns $y(x)$ (and receives the associated refund $r(x)$) as depicted in Figure 2.2.

We may write the profit of the $x$-type retailer who reports $\hat{x}$ and receives the
allocation \{r(\hat{x}), y(\hat{x})\} as

\[ \Pi_R(r(\hat{x}), y(\hat{x}), w | x) = (p - w) - py(\hat{x}) + r(\hat{x}) - c(x - y(\hat{x})) \]

and condition (2.3) requires that this be maximized at \( \hat{x} = x \). The first order condition for the profit-maximizing choice of \( \hat{x} \) is given by

\[ \frac{d\Pi_R}{d\hat{x}} = y'(\hat{x}) \left[ c'(x - y(\hat{x})) - p \right] + r'(\hat{x}) = 0 \] (2.4)

at \( \hat{x} = x \). As long as (2.4) is satisfied, we know that the retailer truthfully reports her type, so we may write her profit as \( \Pi_R(r(x), y(x), w | x) = (p - w) - py(x) + r(x) - c(x - y(x)) \). Taking the total derivative of \( \Pi_R \) with respect to \( x \) and substituting from (2.4) yields

\[ \frac{d\Pi_R}{dx} = -c'(x - y) \] (2.5)

which reflects the information rents that must be paid to induce the retailer to report her type truthfully in the direct revelation mechanism. Put differently, any returns policy selected by the social planner must satisfy the incentive compatibility constraint (2.5) in order to ensure that the truth-telling condition (2.3) required by the retailer’s hidden information is satisfied.

The second order condition associated with (2.4) may be expressed as

\[ \frac{\partial}{\partial x} \left( \frac{\partial \Pi_R}{\partial y} \right) \frac{dy}{dx} = c''(x - y)y'(x) \geq 0, \] (2.6)

which, given the convexity of \( c(x - y) \), implies that \( y(x) \) must be monotonically increasing. As a result, an incentive compatible contract will induce the retailer to send a higher number of products back to the manufacturer as the number of returns she receives by the consumers increases.

A (second-best) Pareto optimal-contract is characterized by a solution to the prob-
lem that maximizes the expected profits of the manufacturer

$$\max_{r(x), g(x), a, w} \int_{\mathbb{R}} \Pi_M(r(x), y(x), a, w) f(x|a) dx,$$

(2.7)

where $f$ is the density function associated with the distribution $F$, subject to the incentive compatibility constraint (2.5), a participation constraint for the retailer, and a delegation constraint reflecting the optimal choice of the hidden action $a$ by the manufacturer.

Notice that the contract is selected before the retailer observes the realization of $x$; hence, the participation constraint has to grant the retailer an ex ante profit greater or equal than her reservation profit, which we assume here, without loss of generality, to be zero. Therefore, the participation constraint is given by

$$\int_{\mathbb{R}} \Pi_R(r(x), y(x), w) f(x|a) dx \geq 0.$$  

(2.8)

Because the manufacturer selects the hidden action, $a$, without observing the realization of $x$, he will choose an action to maximize his expected profits

$$\max_a \int_{\mathbb{R}} \Pi_M(r(x), y(x), a, w) f(x|a) dx.$$  

(2.9)

The first order condition associated with the maximization problem (2.9) yields

$$\int_{\mathbb{R}} [\Pi_M f_a - h'(a)f] dx = 0,$$

(2.10)

which equates the expected marginal cost of taking the action with its expected marginal benefit, the latter consisting of the increase in the manufacturer’s profits as the actual returns are reduced through the investment $a$. Equation (2.10) is the delegation constraint, reflecting the fact that the social planner is forced to delegate the choice of $a$ to the manufacturer because it is a hidden action. The second order condi-
tion associated with the maximization problem (2.9) requires \( \int \frac{sy(x) - r(x)}{-h''(a)} f_{aa} dx - h''(a) \leq 0 \). We will return to this issue below.

In order to set up the optimization problem, we follow the approach of Guesnerie and Laffont (1984) and begin by performing a change of variables. Solving the retailer’s profit function for the refund yields 
\[
  r(x) = \Pi_R(x) - (1 - y(x))p + w + c(x - y(x)),
\]
which upon substitution into the manufacturer’s profit yields
\[
  \Pi_M = (1 - y(x))p - \Pi_R(x) - c(x - y(x)) - h(a) + sy(x).
\]

This substitution permits one to solve the maximization problem (2.7) subject to constraints (2.5), (2.8), and (2.10), by writing the Hamiltonian as
\[
  H = \Pi_M f + \phi(x) \frac{d\Pi_R}{dx} + \mu \left[ \Pi_M f_a - h' f \right] + \lambda \Pi_R f,
\]
where \( y \) is the control variable, \( \Pi_R(x) \) is the state variable, \( \mu \) and \( \lambda \) are the Lagrange multipliers associated with constraints (2.10) and (2.8), respectively, and \( \phi(x) \) is the costate variable for the equation of motion (2.5).

**Theorem 2.4.1.** A solution \{\( \Pi_R(x), y(x), \phi(x), \mu, w, a \)\} to the optimal control problem solves the following necessary conditions:

(i) \(-p + s + c'(f + mf_a) + \phi c'' = 0;\)

(ii) \(\phi(x) = \mu F_a;\)

(iii) \(\int \frac{\Pi_R(r(x), y(x), w)f(x | a)}{dx} dx = 0;\)

(iv) \(\int \frac{\Pi_R + yp + c - sy}{dx} f_a dx + \mu \int \frac{\Pi_R + yp + c - sy}{dx} f_{aa} dx + h' + \mu h'' = 0;\)

(v) \(\int \frac{\Pi_R + yp + c(x - y) - sy}{dx} f_a dx + \mu h' = 0;\) and

(vi) \(\frac{d\Pi_R}{dx} = -c'(x - y).\)
Conditions (i) and (ii) of the Theorem characterize the optimal number of returns sent by the retailer to the manufacturer, $y(x)$; (iii) represents the binding ex-ante participation constraint of the retailer, which characterizes the optimal wholesale price, $w$; (iv) represents the optimal choice of action, $a$; (v) is the delegation constraint; and (vi) is the equation of motion for the state variable $\Pi_R$, which determines the marginal information rent received by the retailer that is required in order to satisfy the incentive (“truth-telling”) constraint.

Now, we may recover the refund function $r(x)$. The equation of motion (vi) governing the path of the retailer profit state variable implies that

$$\Pi_R(x) = -\int_{\pi}^{x} c'(t - y(t))dt + \text{constant.} \quad (2.13)$$

Since the total surplus is divided between the manufacturer and the retailer, one may write

$$(1 - y)p + sy - c - h = \Pi_M(x) + \Pi_R(x),$$

which, upon substitution of (2.2) and (2.13) yields

$$r(x) = py(x) + c(x - y(x)) - \int_{\pi}^{x} c'(t - y(t))dt + \text{constant,} \quad (2.14)$$

where the term $(w - p)$ has been included in the constant. We have some flexibility in selecting a constant to normalize this expression. In doing so, we note that we may decompose the refund function as

$$r(x) = wy(x) + B(x), \quad (2.15)$$

where $wy(x)$ is a full refund from the manufacturer to the retailer for returns, and $B(x)$ is a bonus paid to the retailer. We will normalize payoffs so that the bonus associated with the highest possible level of consumer returns, $\pi$, is zero, so that
\( r(\pi) = wy(\pi). \) This leads to the following result.

**Proposition 2.4.2.** The optimal refund is given by

\[
  r(x) = py(x) + c(x - y(x)) - \int_{\pi}^{x} c'(t - y(t)) dt - (p - w)y(\pi) - c(\pi - y(\pi)). \tag{2.16}
\]

Finally, turning to the second order condition for the maximization problem (2.9) that governs the choice of the action \( a, \) integrating by parts and noting that \( F_{aa}(\pi) = F_{aa}(x) = 0 \) yields

\[
  \int_{\pi}^{x} \left[ sy'(x) - r'(x) \right] F_{aa} dx + h''(a) \geq 0.
\]

From (2.1) we know that the refund is given by \( r(x) = py(x) + w - p + c(x - y(x)) + \Pi_R(x). \)

Differentiating \( r(x) \) in conjunction with the equation of motion (2.5) yields \( r'(x) = y'(x)(p - c'(x - y)), \) so that we may write the second order condition as

\[
  \int_{\pi}^{x} \left[ (s - p + c'(x - y))y'(x) \right] F_{aa} dx + h''(a) \geq 0.
\]

Since \( F \) is assumed to be a concave function of \( a \) and \( y' > 0 \) by (2.6), this second order condition holds as long as the term in brackets is negative, so that \( c' < p - s, \) which implies that the retailer sends too many returns back to the manufacturer.

### 2.5. Discussion

For a given level of consumer returns, \( x, \) the first-best Pareto optimal number of products that the retailer should send back to the manufacturer is characterized by part (i) of Proposition 2.3.1. From part (i) of the Theorem, we know that the second-best Pareto optimal level of returns in our environment with a hidden action and hidden information is given by

\[
  c'(x - y(x)) = p - s - \frac{\phi c''}{f + \mu f_a}.
\]
By part (ii) of the Theorem, we know that \( \phi \) equals zero at \( \bar{x} \) and \( \bar{\pi} \), resulting in the first-best level of returns, but otherwise the number of returns to the manufacturer is distinctly second-best.

The optimal returns policy reflects a tension between the hidden action and hidden information aspects of the environment. To see this, first consider the extreme case in which the hidden action is irrelevant, so that \( F_a = 0 \). This implies \( \phi = 0 \) for every \( x \), so that \( y(x) \) is first-best. The optimal returns policy in this setting is given by the following Proposition.

**Proposition 2.5.1.** In a setting with only hidden information and no hidden action, an optimal returns policy is given by

\[
    r(y) = sy. 
\]

Proposition 2.5.1 characterizes the optimal returns policy when the manufacturer’s action does not influence the number of consumer returns, which makes the retailer internalize the salvage value of any returns to the manufacturer. In this case, the optimal returns policy is to pay the retailer a refund reflecting the salvage value of the returns to the manufacturer, which in the case of a zero salvage value reduces to an *outright sale contract* with no payment to the retailer for returns.\(^{18}\) The case of Johnson & Johnson mentioned earlier represents an example of products (such as shampoo) in which the manufacturer has no relevant actions to control the flow of consumer returns to the retail store. As the salvage value of products returned to Johnson & Johnson can be reasonably assumed to be zero, Proposition 2.5.1 explains why this manufacturer enforces the returns policy with its retailers to a zero refund policy.\(^{19}\)

Consider now the other extreme in which there is a relevant hidden action (\( F_a > 0 \)), but no hidden information. Recall that the truth-telling constraint occasioned by the hidden information resulted in a constraint on the profit path of the retailer
represented by the equation of motion \((vi)\) in Theorem 2.4.1.\(^{20}\) But, if \(x\) is publicly observable, there is no such constraint on the profit path of the retailer, so that the costate variable \(\phi\) is uniformly zero and the number of returns to the manufacturer is first-best.

**Proposition 2.5.2.** In a setting without hidden information and only a hidden action, the manufacturer requires that the retailer send back the first-best number of consumer returns, and a corresponding optimal returns policy is given by

\[
r(y) = py.
\]

To draw out the implications of Proposition 2.5.2, let \(w = p - k\), where \(k\) is a constant to be determined, and let \(r(y) = wy\). Substituting into (2.1) and (2.2) yields

\[
\Pi_R = k - c(x - y); \quad \text{and}
\]

\[
\Pi_M = p - py + sy - k - h(a).
\]

Recalling that \(y\) is first-best in this setting, it follows from Proposition 2.3.1 that the number of products refurbished by the retailer, \(x - y\), is constant. Thus, if we set \(k = c(x - y)\), we effectively have a consignment contract in which the manufacturer pays the retailer a fixed amount to place the product on the shelf and manage the consumer returns, while the manufacturer receives all of the revenues from sales to consumers as well as bears the full cost of any consumer returns. This is the scenario faced by manufacturers who sell their products through the Amazon web portal. The manufacturer in this setting receives all returns directly from consumers, the retailer has no private information and a consignment contract is optimal.

To summarize, in the case of no hidden action (Proposition 2.5.1), a pure sale contract in which the retailer has no recourse for sending returns back to the manufacturer would be optimal (in the case of zero salvage value), whereas in the case
of no hidden information (Proposition 2.5.2) a pure consignment contract in which the retailer never takes title to the product and forwards all consumer returns to the manufacturer would be optimal.

In a more general setting, the second-best nature of returns, $y$, reflects an efficiency trade-off between the hidden action and hidden information aspects of the problem, which can be seen most clearly in the case of zero salvage value ($s = 0$). In this case, a pure sale contract with no credit for returns to the manufacturer ($r(y) = 0$) gives the retailer the incentive to send the first-best level of returns back to the manufacturer, but this contract also gives the manufacturer no incentive to take the costly hidden action which leads to an inefficiently high level of consumer returns, $x$. Alternatively, the payment of a refund to retailers for returns ($r(y) > 0$) incentivizes the manufacturer to take the costly action, $a$, that reduces the expected number of consumer returns, but this payment also distorts the retailer’s return incentives. The (second-best) efficient contract reflects a balancing of these competing efficiency concerns.

The returns policies of Dell and HP are consistent with this general case, as both exhibit the decomposition (2.15) associated with an optimal policy in the presence of hidden information and a hidden action. Both provide a full refund of the wholesale price to the retailer, as well as a bonus schedule that is decreasing in the number of returns to the manufacturer. In the case of Dell, the decreasing bonus is formally part of the policy, while HP achieves the same result by debiting the returns allowance account for the products returned by the retailer.

2.6. An Example

A closed-form solution for the manufacturer’s optimal returns policy can be derived only for explicit forms of the distribution $F$ and of the costs $c$ and $h$. In this section we derive the expression of the returns policy for a specific example.
Let the selling price be $p = 0.1$. The number of returns $x$ are assumed to vary in the interval $[0.1, 0.2]$ according to the distribution $F(x \mid a) = \frac{(x-0.1)(x-0.1+a)}{a+0.1}$, where $a > 0$. Simple differentiation yields $f(x \mid a) = \frac{2(10x-1+5a)}{10a+1}$, $F_a(x \mid a) = -\frac{2(10x-1)(5x-1)}{(10a+1)^2}$, and $f_a(x \mid a) = \frac{10(3-20x)}{(10a+1)^2}$. Note that the support of $x$ does not vary with $a$ as the following conditions are satisfied, $F_a(0.1 \mid a) = F_a(0.2 \mid a) = 0$. Also, $F_{aa} < 0$ so that the second order condition for the choice of $a$ is satisfied as long as $y(x)$ is greater than the first-best value, which we will demonstrate below. Since $F_a(x \mid a) \geq 0 \ \forall x \in [0.1, 0.2]$, it follows that higher values of the manufacturer’s investment, $a$, decreases the probability of products being returned to the retailer. Finally, we let $c(x − y) = \frac{(x-y)^2}{2}$ and $h(a) = \frac{a^3}{3}$.

We first find the expression of the optimal number of returns to the manufacturer. Conditions $(i)$ and $(ii)$ of the Theorem imply

$$y(x) = x - 0.1 + s + \Gamma,$$  \hspace{1cm} (2.17)

where $\Gamma = \frac{\mu F_a}{f + \mu f_a}$ is the distortion from first-best. The second order condition as given by (2.6), $y'(x) = 1 + \Gamma' \geq 0$, can be verified only after obtaining the optimal values of $a$ and $\mu$. From Proposition 2.4.2, the expression of the optimal refund in this particular case is given by

$$r(x) = \frac{\Gamma^2}{2} + s\Gamma + \int_{0.2}^{x} \Gamma dx + s(x - 0.2) + w(0.1 + s).$$ \hspace{1cm} (2.18)

Conditions $(iv)$ and $(v)$ of the Theorem can be solved simultaneously to obtain, for a given salvage value, $s$, the optimal values of $a$ and $\mu$. Condition $(iv)$ reduces to

$$\int_{0.1}^{0.2} \left[ (0.1 - s)x + \frac{\Gamma^2}{2} \right] f_a dx + \mu \int_{0.1}^{0.2} \left[ \frac{\Gamma^2}{2} + \int_{0.2}^{x} \Gamma dx \right] f_a dx + a^2 + 2\mu a = 0,$$ \hspace{1cm} (2.19)
whereas condition \((v)\) yields

\[
\int_{0.1}^{0.2} \left[ \frac{I^2}{2} + (w - s)(s + 0.1) + \int_{0.2}^{x} \Gamma dx \right] f_\alpha dx + a^2 = 0. \tag{2.20}
\]

The optimal wholesale price, \(w\), is obtained through condition \((iii)\) of the Theorem as follows

\[
w = \frac{0.05(s + 0.7)(0.3 - s) + \int_{0.1}^{0.2} [(s - 0.1)x + \int_{0.2}^{x} \Gamma dx] f(x | a) dx}{0.1(0.9 - s)}. \tag{2.21}
\]

The optimal values of \(a\), \(\mu\) and \(w\), for different values of \(s\) are reported in Table 2.1. It is apparent from the values of \(a\) that it is efficient for the manufacturer to invest less intensively in reducing returns from consumers for higher values of the salvage value. A similar effect of an increasing salvage value on the manufacturer’s investment is reflected by the Lagrangian multiplier, \(\mu\), which reflects the shadow cost of the delegation constraint associated with the selection of the optimal \(a\) by the manufacturer. The decreasing values of \(\mu\) for higher \(s\) correspond to the lower impact that the optimal investment, \(a\), has on the expected profits of the manufacturer as the salvage value increases. The last column of Table 2.1 shows the optimal wholesale price. As established by the Theorem, the only role of the wholesale price is to extract all the surplus from the retailer and reduce her expected profits to zero. In particular,

<table>
<thead>
<tr>
<th>(s)</th>
<th>(a)</th>
<th>(\mu)</th>
<th>(w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.01104440075</td>
<td>0.05579087873</td>
<td>0.09809168503</td>
</tr>
<tr>
<td>0.01</td>
<td>0.01072397905</td>
<td>0.0517719273</td>
<td>0.0987599453</td>
</tr>
<tr>
<td>0.02</td>
<td>0.01037599011</td>
<td>0.04761738306</td>
<td>0.09932694264</td>
</tr>
<tr>
<td>0.03</td>
<td>0.00999378945</td>
<td>0.04328944719</td>
<td>0.09979533069</td>
</tr>
<tr>
<td>0.04</td>
<td>0.00956790174</td>
<td>0.03876483044</td>
<td>0.10015985860</td>
</tr>
<tr>
<td>0.05</td>
<td>0.00908430261</td>
<td>0.03400257203</td>
<td>0.1004175670</td>
</tr>
<tr>
<td>0.06</td>
<td>0.00852032256</td>
<td>0.02894307643</td>
<td>0.1005682360</td>
</tr>
<tr>
<td>0.07</td>
<td>0.00783557272</td>
<td>0.02349141426</td>
<td>0.1005963700</td>
</tr>
<tr>
<td>0.08</td>
<td>0.00694574301</td>
<td>0.021747357856</td>
<td>0.10051234760</td>
</tr>
</tbody>
</table>

Table 2.1: Optimal values of \(a\), \(\mu\), and \(w\) for different values of \(s\).
notice that for $s \geq 0.04$, the wholesale price becomes higher than the selling price of 0.1. As a consequence, the retailer incurs a loss when selling to the consumer, but she balances this loss with the profit earned from the refund received when sending the products back to the manufacturer.

In Figure 2.3 we report, for different values of $s$, the number of returns, $y(x)$, associated with the optimal returns policy. We contrast the optimal $y(x)$ in a decentralized system with those that would result in a (first-best) vertically integrated setting and find that, except for values of $x$ at the boundary of the support, $y(x)$ is greater than first-best. The distortion, $\Gamma$, resulting from the hidden action of the manufacturer and the hidden information of the retailer is non-negative for all $x$. Notice that, as required by the second order condition (2.6), $y(x)$ is monotonically increasing in $x$.

![Figure 2.3: Optimal number of returns to the manufacturer, $y(x)$, in a first-best (dashed line) and a second-best (solid line) contract, for different values of the salvage value, $s$.](image)

The manufacturer’s optimal refund, $r(x)$, for different values of the salvage value, $s$, is reported in Figure 2.4. The refund is increasing in $x$ and is higher for higher values of $s$, so that the manufacturer adopts a more generous pay-back to the retailer for the returns to his plant when he can extract more value from them through the
higher salvage value. The monotonicity of $r(x)$ could be mistakenly interpreted as an increasing reward to the retailer for returning products. However, $r(x)$ can be decomposed, as noted by (2.15), into a refund of the wholesale price, $wy$, and a bonus meant to induce the retailer to refurbish and then resell the returned products. Figure 2.5 depicts this bonus for different values of $s$ and shows that it is indeed decreasing in $x$. Furthermore, for higher values of $s$, the bonus is lower, which reflects the manufacturer’s tendency to give a lower incentive to the retailer for reselling the returned product when the manufacturer can recover value through a higher salvage value.

Finally, having reported the efficient allocations, $y(x)$ and $r(x)$, we turn back to the main goal of this chapter, which is to characterize the optimal returns policy, $r(y)$. To derive the returns policy as a function of $y$, we first invert the function $y(x)^{21}$, and then substitute the resulting expression into the function $r(x)$. In Figure 2.6 we report the optimal returns policy as a function of $y$ for the case of $s = 0$. 

Figure 2.4: Optimal manufacturer returns policy, $r(x)$, for different values of the salvage value, $s$. 
Figure 2.5: Optimal manufacturer bonus, $B(x)$, for different values of the salvage value, $s$.

Figure 2.6: Optimal manufacturer returns policy, $r(y)$, for $s = 0$.

### 2.7. Conclusions

While previous research has focused on the design of manufacturer’s returns policies as a tool to alleviate the overstock problem that occurs in a decentralized supply
chain when retailers face an uncertain demand, this paper is concerned with returns of a different nature. As a result of the liberal returns policies generally adopted by retailers, consumers often return products with no quality or performance issues, and the key to value recovery is timely processing and resale by the retailer. An effective returns policy must provide the retailers with the incentive to refurbish and resell the returned products, when appropriate, rather than sending the returns back to the manufacturer who harvests a low salvage value. This paper has characterized such a policy, the implications of which we have shown to be consistent with many of the returns policies observed in practice.

We consider an environment in which a manufacturer may take a hidden action that reduces the expected number of consumer returns, which when realized is privately known by the retailer. The retailer may either send a returned product back to the manufacturer, or she may engage in costly refurbishment to resell the product to consumers. The manufacturer’s returns policy affects the retailer’s decision, and we show a Pareto optimal returns policy entails a full refund of the purchase price to retailers for any products returned to the manufacturer, as well as a bonus paid to the retailer that is decreasing in the number of manufacturer returns. We also find that, as the salvage value received by the manufacturer increases, the bonus paid to the retailer is reduced, and the number of returns to the manufacturer increases.

Our primary managerial insight is that, in the presence of hidden action and hidden information, the optimal returns policy in a decentralized supply chain cannot achieve the level of efficiency achievable by a centralized supply chain, as neither the action taken by the manufacturer to reduce returns nor the number of returns to the manufacturer by the retailer are first-best. Nonetheless, we find the design of a returns policy is an important determinant of a manufacturer’s ability to increase the responsiveness of the reverse supply chain.
Notes

1 The most famous article in this regard is that of Pasternack (1985) who, in the context of the single period inventory (“newsvendor”) problem with random demand, demonstrates that an appropriately designed “buy back” contract can allow the manufacturer in a decentralized supply chain setting to achieve the vertically integrated (“first-best”) outcome. More recently, Arya and Mittendorf (2004) derive an optimal returns policy in a setting in which the retailer has private information about consumers’ valuation of the product, and they demonstrate that returns policies may serve as a useful tool for eliciting the retailer’s private information. In a similar vein, Taylor and Xiao (2009) examine the role of rebates and returns policies in providing the incentive for retailers to obtain private information on consumers’ demand through costly forecasting.

2 A good example is the case of consumer electronics in which Lawton (2008) notes that “consumers bring back to the store 11% to 20% of all electronic goods they purchase, with the highest return rates for wireless phones, GPS units, MP3 players, and wireless networking gear”. The reasons noted for the return by the consumers were “no trouble found” (68%), “buyer’s remorse” (27%), and product defect (5%). Fueled by the adoption of liberal returns policies at most major retailers, consumers have responded by returning products for just about any reason.

3 Our approach is similar to that employed by Crocker and Slemrod (2007) who characterize an optimal contract in an earnings management setting with both a hidden action and hidden information. That work, in turn, draws on the seminal work on hidden actions by Holmstrom (1979) as well as the most recent analysis of costly state falsification as a problem of hidden information by Crocker and Morgan (1998). In each of these cases, the characterization of the optimal contract is accomplished without any a priori restrictions on the functional form of the agreement.

4 While both of these articles advocate a responsive reverse supply chain, neither considers the monetary incentives that would need to be provided to the retailer to realize this design. In other words, they do not examine optimal returns policies.

5 Indeed, the contract suggested by Ferguson et al. (2006) can be viewed as a step-function approximation of the optimal bonus we derive, which is decreasing in the number of returns to the manufacturer.

6 Note that we are distinguishing our approach from the overstock literature by assuming that the retailer can always sell all that she wishes in the competitive market at the competitive price of $p$. As a result, the retailer will never experience an overstock problem in the forward supply chain, which permits us to focus exclusively on the appropriate incentives, through an optimal returns policy, in the reverse supply chain.

7 As we show below, the assumption of concavity for $F$ will be required to satisfy the second order condition associated with the optimal choice of the action $a$ by the manufacturer.

8 Note that the support for the random variable $x$ does not change with $a$, which implies that $F_a(x) = F_a(y) = 0$.

9 The majority of returned products have no functional or cosmetic defect and, after visual inspection and repackaging, can be resold as new products (Guide et al. 2006, p. 1202).
In the analysis below, we demonstrate that the role of \( w \) is to extract as much profit as possible for the manufacturer from the channel, while \( r \) determines the efficiency of the channel.

Note that, for a given \( \{r(x), y(x)\} \) we can construct the returns policy \( r(y) \) by inverting \( y(x) \) to obtain \( x(y) \) and substituting the result into \( r(x) \).

The fundamental difference between normative (efficiency) and positive (equilibrium) analysis can be easily demonstrated with reference to the “Edgeworth box” exchange economy, which is a staple of undergraduate microeconomics courses. When characterizing the class of Pareto optimal allocations, the exercise performed is to examine the problem faced by a fictitious social planner that has the power to assign to the agents any allocation in the box, but is constrained by the resource constraints of the economy, that is, by the dimensions of the box. The planner is assumed to be omniscient, in the sense that she knows the preferences of the agents, and she is guided by the Pareto criterion. In this setting, the social planner would always assign an allocation on the contract curve (defined as the locus of the tangencies of the agents’ indifference curves) so that there would be no unexploited gains from trade.

In contrast, an equilibrium analysis would proceed by examining the problem faced by a fictitious Walrasian auctioneer whose only power is to announce prices, after which the agents announce the trades that they would be willing to make at those prices. The process stops when the auctioneer arrives at prices that when announced result in trades that match, thereby clearing the market. As it turns out, this also results in an allocation that is on the contract curve. This confluence of the efficiency and equilibrium results forms the basis for the fundamental welfare theorems of economics (see Tresch 1981, p. 9).


Any \( w \) and \( r(x) \) that satisfies (iii) will solve the optimization problem. To see this, note that a pointwise optimization to determine \( w \) yields the first order condition \( \lambda = 1 \), just as did the first order condition for \( r \). The reason for this indeterminacy is that, in the first-best setting, the only role played by both \( r \) and \( w \) is to extract profit from the retailer to ensure (iii), so having both tools is redundant. When we move to the second-best setting, however, the role of \( r \) changes substantially, as it will have an incentive effect that influences the retailer’s decision regarding the number of consumer returns to send back to the manufacturer.

This is, for example, the approach adopted by Crocker and Snow (1986, p. 322) in characterizing Pareto optimal contracts in an insurance market in which customers have hidden information regarding their probabilities of suffering an insurable loss. A similar approach is used in Stiglitz’s examination (in Auerbach and Feldstein 1987, p. 996) of optimal tax policies when workers possess hidden information regarding their productivities. This point is also emphasized in Laffont (1986): “When the relevant information for allocating resources is decentralized, it is important to take into account the incentives that economic agents have to reveal their private information. Indeed, it would not be legitimate to criticise an allocation of resources that could be improved upon by a (social) planner having complete information without constraining him to use mechanisms capable of extracting the information that he does not possess” (p. 146).

From (2.1) we know that the equation of an \( x \)-type retailer’s indifference curve associated with the profit level \( \Pi_R \) is given by \( r = (y - 1)p + w + c(x - y) + \Pi_R \).

Fudenberg and Tirole (1991), p. 258. Note that the monotonicity of \( y(x) \) guarantees its invert-
ibility.

18 Note that the wholesale price, \( w \), is still determined by the ex ante zero profit condition for the retailer, which is part (iii) of Theorem 2.4.1.

19 An interesting exception in this policy is provided for products that are nationally discontinued, in which case Johnson & Johnson takes back the products from the retailer and gives full credit for them. Since the decision to discontinue a product is the choice of the manufacturer, this policy is consistent with our results and causes the manufacturer to internalize the effects of its decision.

20 Put differently, (vi) characterizes the information rents that must be paid to the retailer in order that information is truthfully revealed in the direct revelation mechanism.

21 The monotonicity of \( y(x) \) guarantees the existence of the inverse function. See also note 17.
Chapter 3

Investments to Reduce Consumer Returns under Information Asymmetry

3.1. Introduction

When HDTV maker Vizio Inc. first launched its high-definition set, it faced a consistent number of returns to its retailer, Best Buy. The TV maker, presuming that consumers were returning the product because of difficulty in connecting the TV set to a high-definition source, added instructions on the package and included within the instruction booklet a quick-start guide to the product basic set-up. The main reason for returns, however, was quite different. Consumers, used to old cables with different connectors, did not realize that a high-definition cable was required. According to Best-Buy the “unnecessary” returns of the HDTV would have persisted even after Vizio’s preventative efforts (Lawton, 2008).

The flow of consumer returns has become a serious concern for manufacturers and retailers. Recent estimates in the consumer electronics industry suggest that the rate of return for electronic devices ranges from 11% – 20%, for a total cost of $14 billion every year in the United States alone (Steger et al. 2007). Lenient returns policies at most major retailers allow consumers to return unwanted merchandise back to the store for almost any reason – consumers can buy a product, evaluate it, and, if the product does not meet their expectations, they can return it to the store. In fact, most returns are not related to issues of product quality, but rather to product mismatch with customer needs and tastes or to difficulty in product installation and use (these are called false failure returns – Ferguson et al. 2006).
In an attempt to reduce false failure returns, manufacturers have begun to undertake investments in improving instructions, increasing the quality and availability of after-sales support, and even redesigning the product to simplify the technical complexity imposed on the consumer. As the Vizio example highlights, however, one issue with these investments is that, at the time of the investment decision, there is considerable uncertainty about its effectiveness. The investment of the manufacturer can be ineffective and leave the high return rate of the product unchanged or it can be effective and reduce the return rate of the product. Despite the uncertainty of the manufacturer about the effectiveness of his investment, the retailer, thanks to her proximity to the market, may be aware, after observing the investment, of how the product return rate will be affected. In particular, a specialized retailer like Best Buy, with product expertise and an intimate understanding of her customers and their needs, can predict whether or not the manufacturer’s investment will reduce the rate of the returns.

We consider a scenario where the manufacturer can undertake an investment to reduce returns but is uncertain about its outcome, whereas the retailer, after observing the manufacturer’s investment, is aware of the return rate of the product as a result of the investment. The information asymmetry along the supply chain about the product return rate is supported by the fact that the manufacturer demands a higher payment for a low return rate product, which provides an incentive to the retailer to pretend the product return rate at her store is always high. The manufacturer has to design an optimal contract that aligns his incentives with those of the informed retailer.

The contract of the manufacturer has a different structure, depending on whether the retailer or the manufacturer salvages the returned products. If the retailer is responsible for recovering the value of the returned units, the manufacturer’s contract entails only a wholesale price whereas, if the manufacturer salvages the returned units, it entails both a wholesale price and a refund per return. Assuming that both the
manufacturer and the retailer have access to an outlet for recovering the value of returned units, the reverse channel is considered efficient when returned products are recovered by the channel member who can extract the highest salvage value. In this chapter we investigate how the information asymmetry between the manufacturer and the retailer about the product return rate affects the manufacturer’s profits, the manufacturer’s investment, and the efficient selection of the reverse channel.

We find that, when the retailer is responsible for salvaging returns, the manufacturer is unable, through a wholesale price contract, to screen the retailer’s information. As a consequence, the manufacturer’s profits suffer from distortions. When the manufacturer is responsible for salvaging returns, however, the contract, which consists of both a wholesale price and a refund, allows the manufacturer to screen the product return rate, which results in less distorted profits than in the previous case. In both reverse channel structures the manufacturer might forgo investment because of the asymmetry of information, but the probability of investment is higher when the manufacturer rather than the retailer takes responsibility for salvaging returns. The different distortions in the manufacturer’s profits might lead to inefficiencies in the selection of the reverse channel: the manufacturer might decide to take responsibility for salvaging returns even though the retailer can extract a higher value from a returned unit.

We extend our analysis to the case where the manufacturer rather than the retailer is privately informed about the impact of his investment on the product return rate. This reverse information asymmetry along the supply chain is motivated by both the fact that more and more manufacturers are taking steps to establish a direct contact with their end-consumers, which would make manufacturers likely to learn about the effectiveness of their investment, and the fact that several retailers are increasingly adopting “no questions, money-back guarantee” kinds of returns policy, which would preclude retailers from learning why products are being returned. When the manufacturer is privately informed about the product return rate, he has an
incentive to pretend his product will exhibit a low return rate. The manufacturer then has to signal his information to the retailer through an optimal contract. We show that because of the information asymmetry, the manufacturer would still incur distortions in his profits as long as the retailer is responsible for salvaging returns; however, when the manufacturer salvages returns he can efficiently signal his product return rate to the retailer, so that he makes the same optimal investment and obtain the same level of profits as under a full information scenario. The manufacturer then has an extra motivation to take responsibility for salvaging returns and to be more active in learning about the impact of his investment on returns.

### 3.2. Literature Review

There is a growing body of literature on the management of consumer returns. The research has initially focused on the design of optimal returns policies at the retail store in order to govern the flow of consumer returns. While a penalty per return certainly has the effect of discouraging the consumer from returning the product, it may also have the side effect of discouraging the consumer from purchasing the product in the first place. Davis et al. (1995) find that when the retailer has a sufficiently high salvage value, a “money-back guarantee” policy is more profitable than a no-refund policy. Che (1996) shows that a full refund policy is optimal when consumers are highly risk-averse or when the retailer procurement costs are high. Su (2009) shows that full refunds are suboptimal for the retailer and finds that the refund to consumers for a returned unit should equal the salvage value. Shulman et al. (2009) develop a detailed understanding of how a retailer can use price, restocking fee, and information to affect consumer purchases and returns. In our work, we omit from consideration the selection of the optimal returns policy at the store. We assume a full refund policy and consider the drop in returns as driven not by the returns policy of the retailer but by the investment of the manufacturer.
Although cases of manufacturers actively taking countermeasures against returns are broadly observed in practice (Lawton 2008), this topic has received little attention in the academic literature. An exception is the work of Ferguson et al. (2006), where the authors study how a manufacturer, through a target rebate contract, can induce the retailer to exert an effort aimed at reducing false failure returns. More recently, Crocker and Letizia (2011) have studied the optimal returns policy that a manufacturer could use to induce the retailer to resell the returned products rather than send them back to the manufacturer. Both these articles, however, assume that returns are salvaged by the manufacturer, whereas false failure returns could alternatively be salvaged by the retailer. The article by Shulman et al. (2010) considers the possibility of both reverse channel structures for returns and shows that in some cases the manufacturer may find it more convenient to handle returns himself, even if the retailer can extract a higher value from a returned unit. This result follows from the ability of the manufacturer to more effectively influence the retailer’s returns policy (which can ration the manufacturer’s volume of sales) only through the joint use of a wholesale price and a refund for returns. In our setting, such inefficiency in the product value recovery arises from the asymmetric information between manufacturer and retailer about the outcome of the manufacturer’s investment (i.e., whether the investment yields a high or low product return rate).

Our chapter contributes to the large body of literature on information sharing in supply chains with independent players. A thorough review of this stream of research is provided by Chen (2003). In our setting the manufacturer may have an information disadvantage about the future product return rate with respect to the retailer, which leads to the manufacturer trying to screen the retailer’s information. Screening games have been largely used in a supply chain context, where the typical scenario involves a supplier trying to screen demand information from a downstream firm (Arya and Mittendorf, 2004; Taylor and Xiao, 2009). However, to the best of our knowledge, this is the first work where the information being screened is the returned demand
rather than the demand itself. Our work considers also the reverse case where the manufacturer has an information advantage about the future product return rate over the retailer, so that the manufacturer has to signal his information to the retailer. In particular, we consider the investment which then determines the private information of the manufacturer as endogenous. Baiman et al. (2001) follow a similar approach and consider the case where a supplier decides first on an investment in production technology and then has to signal the quality of his product to the manufacturer. Anand and Goyal (2009) study the case of an incumbent retailer who first decides on acquiring private information about market demand, and then signals it to a competitor through the information leakage of a common supplier. In our work we consider both screening and signalling games, as we compare the manufacturer’s profits when either the manufacturer or the retailer has an information advantage. Chu (1992) studies a scenario where the manufacturer has private information on demand and compares the two cases where the manufacturer signals demand to the retailer through advertising and price, and the retailer screens demand through slotting allowances. In our paper the manufacturer always proposes the contract to the retailer and, depending on the informational structure, he either screens or signals through the contract the return rate of his product.

3.3. Model Assumptions and Notation

We consider a two-echelon supply chain, with a manufacturer selling a product to the end-consumer through his retailer. The manufacturer and the retailer face the linear downward-sloping demand function, \( D(p) = \phi - p \), but a fixed percentage \( \gamma^H \) of the demand, \( \gamma^H \in (0, 1) \), is returned by the end-consumer to the retailer for a full refund of the selling price, \( p \). From the retail store, the product can follow one of two reverse channels, where either the manufacturer or the retailer salvages its residual value. Both the manufacturer and the retailer are risk-neutral expected profit maximizers.
Given the high return rate of his product, the manufacturer might decide to undertake an investment \(i > 0\) with the aim of reducing the percentage of returned demand from \(\gamma^H\) to \(\gamma^L\), \(\gamma^H > \gamma^L \geq 0\). Hereupon, we refer to the manufacturer whose retailer faces a high product return rate, \(\gamma^H\), as the high-type manufacturer, whereas we refer to the manufacturer whose retailer faces a low product return rate, \(\gamma^L\), as the low-type manufacturer. At the time of the investment decision, the manufacturer is uncertain whether the investment will reduce the return rate of his product at the store. Given the investment \(i\), the manufacturer assesses with probability \(\beta(i)\) that his product return rate will be low (i.e., equal to \(\gamma^L\)). We assume that \(\beta(0) = 0\), \(\lim_{i \to 0^+} \beta'(i) = +\infty\) and that \(\beta(i)\) is strictly increasing and strictly concave in \(i\).

The sequence of events is illustrated in Figure 3.1. At the beginning (denoted date 1), the manufacturer selects an investment \(i\), which determines his belief, \(\beta(i)\), that the product return rate will be low. At date 2, the retailer observes the manufacturer’s investment, and, given her detailed understanding of customers needs and of the causes of returns, she can exactly predict the impact of the manufacturer’s investment on the product return rate. In other words, at date 2, after observing the investment \(i\), the retailer knows the manufacturer’s type. At date 3, the manufacturer offers a contract to the retailer, which depends on the reverse channel where the returned products are salvaged. If returns are salvaged by the retailer, then the manufacturer’s contract consists of just a wholesale price, \(w\), whereas, if returns are salvaged by the manufacturer, then the manufacturer’s contract consists of both a wholesale price, \(w\), and a refund to the retailer, \(b\), per return. We assume that the reverse channel for

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Manufacturer decides on investment (i)</td>
</tr>
<tr>
<td>2</td>
<td>The informational structure is determined</td>
</tr>
<tr>
<td>3</td>
<td>Manufacturer offers contract to retailer</td>
</tr>
<tr>
<td>4</td>
<td>Retailer selects selling price</td>
</tr>
<tr>
<td>5</td>
<td>Market is cleared</td>
</tr>
</tbody>
</table>
returned products is exogenously given; that is, when the manufacturer has to decide about his investment (date 1), both the manufacturer and the retailer have already established who will salvage the returned products. (In section 3.6 we relax this assumption and consider the case of an endogenous reverse channel selection). Given the manufacturer’s contract, at date 4, the retailer selects the selling price, \( p \), which determines the demand \( D(p) \) of products sold, and, depending on the manufacturer’s type, the number of returns to the retail store, \( \gamma^j D(p) \). These returns are then salvaged at a value \( v, v > 0 \), in the exogenously-given reverse channel. Finally, at date 5, the market is cleared and the profits of the manufacturer and the retailer are determined.

To derive the manufacturer’s optimal investment and profits we have to solve a two-stage game. By backward induction, we first derive the equilibrium of the trade between the manufacturer and the retailer in the second stage of the game (from date 3 on in Figure 3.1), and then we derive the manufacturer’s decision about the optimal investment in the first stage of the game (at date 1 in Figure 3.1). In particular, the second stage of the game is a screening game, where the manufacturer designs a contract that induces the retailer to reveal her private information about the actual manufacturer’s type. We solve the two-stage game considering in section 3.4 that the returned products are salvaged by the retailer, and in section 3.5 that they are salvaged by the manufacturer.

We use the following notation throughout the paper. The two subscripts \( M \) and \( R \) denote the manufacturer and the retailer, respectively, whereas the superscript \( j, j \in \{H, L\} \), denotes the type of the manufacturer which can be high or low, respectively.

### 3.4. Retailer salvages the returned products

When the retailer salvages the returned products, the manufacturer’s contract consists just of the wholesale price, \( w \). The retailer is the only channel member who takes responsibility for the returns; she gives a full refund of the selling price, \( p \), to the
consumer who returns the product and then extracts a residual value, \( v \), from the returned product. We derive the equilibrium of the trade between manufacturer and retailer (i.e., the second stage of the game) first for the benchmark case of full information in section 3.4.1, and then for the case of information asymmetry in section 3.4.2. Finally, we derive the manufacturer’s optimal investment in section 3.4.3.

### 3.4.1 Full Information

Under full information about the manufacturer’s type, the trade between manufacturer and retailer reduces to a Stackelberg game, where the manufacturer offers the contract, \( w \), to the retailer, and the retailer selects the selling price, \( p \). The reaction function of the retailer to a given wholesale price solves the following optimization problem

\[
\max_p \Pi^j_R(p) = (\phi - p) \left[ (p - w) - \gamma^j(p - v) \right],
\]

which yields \( p^*j(w) = \frac{\phi}{2} + \frac{w - v\gamma^j}{2(1 - \gamma^j)} \). Given the retailer’s reaction function, the manufacturer optimizes his profits by solving

\[
\max_w \Pi^j_M(w) = (\phi - \Pi^j_R(p^*j(w)))w
\]

s.t. \( \Pi^j_R(p^*j(w)) \geq 0 \),

where the manufacturer guarantees to the retailer a level of profits not lower than her outside option, normalized here to zero. The manufacturer’s problem in (3.2) is solved by the optimal wholesale price

\[
w^*j = \frac{\phi(1 - \gamma^j) + v\gamma^j}{2}.
\]

Optimal retail price and equilibrium manufacturer’s profits are then given by

\[
p^*j = \frac{3}{4} \phi - \frac{v\gamma^j}{4(1 - \gamma^j)} \quad \text{and}
\]
\[ \Pi^*_M = \frac{[\phi(1 - \gamma^j) + v\gamma^j]^2}{8(1 - \gamma^j)}. \] (3.5)

At equilibrium the low-type manufacturer charges the retailer with a higher wholesale price than the high-type manufacturer.\(^5\) Therefore, the manufacturer requires a higher payment from the retailer and realizes a higher contribution margin per unit of sale for the product with lower return rate. However, a reduction of the product return rate from \(\gamma^H\) to \(\gamma^L\) leads the retailer to increase the equilibrium selling price, thus penalizing the quantity demand of the manufacturer.\(^6\) These two opposing effects - raising the manufacturer’s contribution margin per sale and dropping the manufacturer’s volume of sales - might lead the high-type manufacturer to choose not to invest in reducing the high return rate of the product. In particular, the increase of the selling price by the retailer becomes critical to the manufacturer when the salvage value, \(v\), or the initial return rate, \(\gamma^H\), are sufficiently high. In fact, when \(v\) is sufficiently high, the retailer substantially increases the selling price in response to a reduction of the return rate because she can extract a high recovery value for a lower portion of returns. Likewise, when the initial return rate, \(\gamma^H\), is sufficiently high, the retailer selects such a low selling price \(p^*_H\) that the high-type manufacturer would not be willing to change the resulting large demand. These considerations lead to the following result.

**Lemma 3.4.1.** When the retailer salvages returns, under full information about the product return rate, \(\Pi^*_M > \Pi^*_L \Rightarrow \gamma^H < \gamma = \frac{\phi(\phi - 2v)}{(\phi - v)^2}\).

Lemma 3.4.1 asserts that an upper bound on the initial return rate, \(\gamma^H < \bar{\gamma}\), is a necessary but not sufficient condition for the high-type manufacturer to profit from a reduction of the product return rate. Notice that, as \(\gamma^H \in (0, 1)\), Lemma 3.4.1 also imposes an upper bound on the salvage value, \(v < \frac{\phi}{2}\), which confirms the previous intuition that the high-type manufacturer can benefit from a reduction of the product return rate only for limited values of the salvage value, \(v\), and of the
initial product return rate, $\gamma^H$. Reporting the $j$-type manufacturer’s profits, $\Pi^*_j$, as a function of the manufacturer’s type, $\gamma^j$, we can notice that $\Pi^*_j$ is strictly convex\(^7\) and non-monotonic in $\gamma^j$ (see Figure 3.2): $\Pi^*_j$ decreases in $\gamma^j$ for $\gamma^j \leq \bar{\gamma} = \frac{\phi - 2v}{\phi - v}$ and increases in $\gamma^j$ for $\bar{\gamma} < \gamma^j < \overline{\gamma}$. When the initial return rate, $\gamma^H$, is sufficiently low

\[
\begin{align*}
\Pi^*_j
\end{align*}
\]

Figure 3.2: Manufacturer’s profits for $0 < v < \frac{\phi}{2}$, as a function of the manufacturer’s type, $\gamma^j$. For $\gamma^H \leq \bar{\gamma}$, any reduction of the product return rate is profitable, whereas for $\bar{\gamma} < \gamma^H < \overline{\gamma}$, only a sufficiently high reduction of the product return rate ($\gamma^L < \tilde{\gamma}$) is profitable.

($\gamma^H \leq \bar{\gamma}$), the manufacturer would always profit from a reduction of the product return rate ($\forall \gamma^L < \gamma^H$) because the resulting drop in quantity demand would have a smaller impact on the manufacturer’s profits than the resulting increase in the contribution margin. However, when the initial return rate is sufficiently high ($\bar{\gamma} < \gamma^H < \overline{\gamma}$), a reduction of the product return rate would be profitable to the manufacturer only if it is sufficiently large ($\gamma^L < \tilde{\gamma}$); in other words, the manufacturer needs a sufficiently high increase in the contribution margin to benefit from a reduced return rate.

We focus on cases where under full information the reduction of the product return rate is profitable to the high-type manufacturer ($\Pi^*_H > \Pi^*_M$ which is equivalent to either $\gamma^H \leq \bar{\gamma}$ or $\bar{\gamma} < \gamma^H < \overline{\gamma}$ and $\gamma^L < \tilde{\gamma}$), and investigate how the information asymmetry about the product return rate affects the manufacturer’s profitability and
his willingness to invest in reducing returns.

3.4.2 Retailer privately informed

When the retailer is privately informed about the manufacturer’s type, the second stage of the game is a screening game, where the manufacturer is interested in inducing the retailer to reveal the true product return rate. In fact, the retailer has an incentive to pretend she is facing a high return rate of the manufacturer’s product, because, as it was shown in the full information case, the manufacturer would require a higher payment for a low return rate product. To screen the retailer’s private information, the manufacturer offers a menu of two contracts to the retailer, one for each manufacturer’s type: $w^L_c$ and $w^H_c$. Following standard contract theory, we focus, without loss of generality, on a menu of contracts that are incentive-compatible and satisfy the retailer’s participation constraints. While the manufacturer, uncertain of his type, optimizes his profits in expected terms\(^9\), the retailer, aware of the manufacturer’s type, reacts to the wholesale price contract with the same optimal response $p^{*j}(w)$ she selected under full information. The manufacturer’s problem can be written as follows

$$\max_{(w^H_c; w^L_c)} \beta \Pi^L_M(w^L_c) + (1 - \beta) \Pi^H_M(w^H_c)$$

s.t.

$$\Pi^H_R(p^{*H}(w^H_c)) \geq \Pi^H_R(p^{*H}(w^L_c))$$  \hspace{1cm} (3.6) $$\Pi^L_R(p^{*L}(w^L_c)) \geq \Pi^L_R(p^{*L}(w^H_c))$$  \hspace{1cm} (3.7) $$\Pi^H_R(p^{*H}(w^H_c)) \geq 0$$  \hspace{1cm} (3.8) $$\Pi^L_R(p^{*L}(w^L_c)) \geq 0.$$  \hspace{1cm} (3.9)

where $\Pi^j_R(p)$ and $\Pi^j_M(w), j \in \{H, L\}$, are given by (3.1) and (3.2), respectively. Inequalities (3.6) and (3.7) represent the incentive-compatibility constraints, whereas (3.8) and (3.9) represent the retailer’s participation constraints. As the retailer’s
profit decreases in the manufacturer’s wholesale price, (3.6) implies that the whole-
sale price for a high return rate product has to be lower than the wholesale price
for a low return rate product, that is \( w^H_c \leq w^L_c \). This inequality corresponds to the
intuitive result found under full information that the low-type manufacturer demands
a higher payment than the high-type manufacturer. However, because the retailer
facing a low return rate of the product has an incentive to pretend the product return
rate is high, the manufacturer has to contrast this incentive by raising the wholesale
price of a high-type; thus (3.7) implies \( w^H_c \geq w^L_c \). The two incentive-compatibility
constraints then result in an equal wholesale price for either type of manufacturer,
that is \( w^H_c = w^L_c \).

Lemma 3.4.2. When the retailer salvages returns, in a screening equilibrium the
manufacturer proposes one wholesale price contract, \( w^*_c \), to the retailer and is therefore
unable to screen his type.

The inability of the manufacturer to screen the manufacturer’s type causes se-
vere distortions of his expected profits as compared to the benchmark case of full
information. In fact, under full information the manufacturer was solving one of two
different optimization problems, depending on his type, which was leading to the se-
lection of an optimal wholesale price, different per type (\( w^*_j \) as given by (3.2)). When
the retailer is privately informed about the manufacturer’s type, the manufacturer is
unable to discriminate through one wholesale price between the high and low type
and has to weigh the odds of a realization of one type versus the other. More for-
mally, under full information, the expected profits\(^{10}\) of the manufacturer are given by
\( \beta \max_{w^L} \Pi^L_M(w^L) + (1 - \beta) \max_{w^H} \Pi^H_M(w^H) \), whereas when the retailer is privately informed
the expected profits of the manufacturer are given by \( \max_{w^c} \beta \Pi^L_M(w^c) + (1 - \beta) \Pi^H_M(w^c) \),
which are clearly sub-optimal given that the concave functions \( \Pi^L_M(w) \) and \( \Pi^H_M(w) \)
are maximized for a different value of \( w \). The manufacturer’s profits in a screening
equilibrium correspond for \( \beta = 0 \) to \( \Pi^*_M \) and for \( \beta = 1 \) to \( \Pi^L_M \); however for \( \beta \in (0,1) \),
they are lower than the expected full information profits $\left( \beta \Pi^*_{M} + (1 - \beta) \Pi^*_{H} \right)$. The following result establishes that, in a screening equilibrium, because of the distortions in his profits, the manufacturer may not benefit from a reduction of the product return rate.

**Proposition 3.4.3.** When the retailer salvages returns, in a screening equilibrium the manufacturer’s expected profits are strictly convex in $\beta$ and for $\gamma^H \leq \bar{\gamma}$ they are strictly increasing in $\beta$. However, for $\bar{\gamma} < \gamma^H < \bar{\gamma}$ and $\gamma^L < \bar{\gamma}$, the manufacturer’s expected profits, $\Pi^*_{M_{c}}$, are non-monotonic in $\beta$ and there exists a threshold, $\beta_R$, such that the manufacturer is expected to benefit from a reduction of the product return rate only if $\beta > \beta_R$.

When under full information the manufacturer’s profits function is strictly decreasing in $\gamma^j \ (\gamma^H \leq \bar{\gamma})$, the information asymmetry, while distorting the manufacturer’s expected profits, maintains them to a level above $\Pi^*_{M}$. When under full information the manufacturer’s profits function is non-monotonic in $\gamma^j \ (\bar{\gamma} < \gamma^H < \bar{\gamma}$ and $\gamma^L < \bar{\gamma}$), however, the information asymmetry decreases the manufacturer’s profits and, for a range of beliefs $\beta$, limits them to a level below $\Pi^*_{M}$. The manufacturer then has no interest in a reduction of the product return rate if its realization is not sufficiently probable. In Figure 3 we have reported the manufacturer’s expected profits as a function of $\beta$ under full and asymmetric information for the two cases, $\gamma^H \leq \bar{\gamma}$ in graph (a), and $\bar{\gamma} < \gamma^H < \bar{\gamma}$, $\gamma^L < \bar{\gamma}$ in graph (b). Even though under full information the reduction of the product return rate was profitable to the manufacturer, the private information of the retailer can render such a reduction unprofitable. In fact, in graph (b), a scarce belief $\beta$ in a successful reduction of returns causes the manufacturer’s expected profits to drop below its initial level, $\Pi^*_{M}$: the manufacturer is guaranteed to benefit from a reduction of the product return rate only for a sufficiently high belief on a successful outcome of his investment, $\beta > \beta_R$. 
Figure 3.3: Manufacturer’s expected profits in the first stage of the game as a function of $\beta$, given in the second stage of the game a full information (squares) and a screening equilibrium (triangles), for the two cases (a) $\gamma^H \leq \bar{\gamma}$ and (b) $\bar{\gamma} < \gamma^H < \overline{\gamma}, \gamma^L < \tilde{\gamma}$. The values of the parameters in the two graphs are $\phi = 100$, $v = 30$, $\gamma^L = 0.1$, and $\gamma^H = 0.5$ in (a) and $\gamma^H = 0.63$ in (b).

3.4.3 Investment decision

Having derived the equilibrium for the second stage of the game, we turn now to the analysis of the first stage of the game, where the manufacturer decides on the optimal investment, $i^*$. When the second stage of the game corresponds to the full information
case, the optimal investment can be computed by solving the maximization problem,

$$\max_{i \geq 0} \beta(i) \Pi_M^L + (1 - \beta(i)) \Pi_M^H - i.$$  (3.10)

The objective function in (3.10) is equal to $\Pi_M^H$ for $i = 0$ and is strictly concave in $i$; hence, there always exists a unique optimal investment, $i^* > 0$, given by the first-order condition, $\beta'(i^*)(\Pi_M^L - \Pi_M^H) = 1$, which generates higher expected profits to the manufacturer than his current level $\Pi_M^H$. The manufacturer then will always invest in reducing returns in the first stage of the game when both the manufacturer and the retailer are aware of the manufacturer’s type in the second stage of the game.

When the second stage of the game corresponds to the case of private information of the retailer, the manufacturer has to solve

$$\max_{i \geq 0} \Pi_{Mc}(i) - i,$$  (3.11)

where $\Pi_{Mc}(i)$ is a function of the investment $i$ through the belief function $\beta(i)$. For $\gamma < \gamma^H < \bar{\gamma}$, and if the belief function $\beta(i)$ has a very low marginal rate, problem (3.11) would only be solved by $i^* = 0$, which results in no preventative investment from the manufacturer. Even when problem (3.11) is solved by a positive $i^*$, then it should be verified that the expected manufacturer’s profits at optimum, $\Pi_{Mc}(i^*) - i^*$, are higher than $\Pi_M^H$. In particular this condition implies necessarily that the optimal investment $i^*$ generates a belief $\beta(i^*) > \beta_R$. The following corollary is a direct consequence of Proposition 3.4.3 and provides a sufficient condition to the manufacturer to undertake a positive preventative investment.

**Corollary 3.4.4.** When the retailer salvages returns, in a screening equilibrium if $\gamma^H \leq \bar{\gamma}$ then the manufacturer undertakes a positive investment $i^*$.

Even though under full information the manufacturer would benefit from a reduction of the product return rate, the retailer’s private information about it is guar-
anteed to motivate the manufacturer to invest in reducing returns only for a limited value of $\gamma^H$. If the initial return rate is instead sufficiently high and there is no sufficient belief in a successful outcome of the investment, the information asymmetry about the product return rate might lead the manufacturer to forgo investment.

3.5. Manufacturer salvages the returned products

When the manufacturer salvages the returned products, the manufacturer’s contract consists of both a wholesale price, $w$, and a refund for the returned products, $b \leq w$. The manufacturer takes responsibility for the returned products and salvages them at a value $\nu$ (see Footnote 4). Following the same structure of the previous section, we first derive, by backward induction, the equilibrium of the trade between manufacturer and retailer (i.e., the second stage of the game) for both the benchmark case of full information in section 3.5.1 and for the case of information asymmetry in section 3.5.2. Then, we obtain the manufacturer’s optimal investment in section 3.5.3.

3.5.1 Full Information

Under full information about the manufacturer’s type, the second stage of the game is again a Stackelberg game, where the $j$-type manufacturer offers the contract $(w, b)$ as the Stackelberg leader and the retailer selects the selling price, $p$, as the Stackelberg follower. The reaction function of the retailer to a given pair $(w, b)$ solves the following optimization problem,

$$
max_p \Pi^j_R(p) = (\phi - p) \left[ (p - w) - \gamma^j(p - b) \right],
$$

(3.12)
which yields $p^{*j}(w, b) = \phi + \frac{w - b\gamma_j}{2(1 - \gamma_j)}$. Given the retailer’s reaction function, the $j$-type manufacturer selects his contract by solving the problem,

$$
\max_{w, b} \Pi_M(w, b) = (\phi - p^{*j}(w, b)) \left[ w - \gamma_j(b - v) \right]
$$

s.t. $b \leq w$

$$
\Pi_R(p^{*j}(w, b)) \geq 0.
$$

The objective function in (3.13) is concave (but not strictly so) in $w$ and $b$. As a consequence, the optimal contract of the $j$-type manufacturer is not unique but consists of a menu of equally profitable contracts

$$
w^{*j} = \phi(1 - \gamma_j) + \gamma_j(2b^{*j} - v), \quad b^{*j} \leq \frac{\phi}{2} - \frac{v\gamma_j}{2(1 - \gamma_j)}.
$$

One may notice that for $b^{*j} = 0$ the optimal wholesale price, $w^{*j}$, corresponds exactly to the one obtained when the retailer was responsible for salvaging returns. Therefore, the manufacturer attains the same profit (salvage value $v$ being equal), whether he or the retailer is responsible for salvaging returns. However, the manufacturer has now gained in flexibility because, when he salvages returns, he can equivalently select any combination of wholesale price and refund as long as (3.14) is satisfied.

### 3.5.2 Retailer privately informed

When the retailer is privately informed about the manufacturer’s type, the manufacturer screens her private information through a menu of two contracts: $(w_c^H, b_c^H)$ for the high-type and $(w_c^L, b_c^L)$ for the low-type. The manufacturer attracts the retailer facing a high product return rate by requiring a high payment for his products up-front but guaranteeing a full refund for the returned products. The retailer facing a low product return rate, however, is offered a contract which entails the payment of a
low wholesale price upfront but only a partial refund for the returned products. The following Proposition derives the screening contract.

**Proposition 3.5.1.** When the retailer salvages returns, in a screening equilibrium the manufacturer proposes the two contracts \((w^*_c, b^*_c),\) and \((w^*_c, b^*_c)\) such that \(w^*_c = b^*_c\) (detailed in Appendix), and \(w^*_c = w^*_c(1-\gamma^L) + b^*_c\gamma^L,\) \(\forall b^*_c \geq \frac{w^*_c(2-\gamma^H-\gamma^L)-2\delta(1-\gamma^H)}{\gamma^H-\gamma^L} \leq b^*_c < w^*_c.\)

Notice from Proposition 3.5.1 that to screen the retailer’s private information, the manufacturer discriminates between his types by offering a partial refund contract to the low-type and a full refund contract to the high-type such that \(b^*_c < w^*_c < w^*_c = b^*_c.\) Proposition 3.5.1 confirms the previous intuition that the manufacturer will charge a lower wholesale price to the retailer facing a low product return rate but will also tighten his returns policy. Even though the manufacturer is now able to screen the retailer’s private information, the efficiency cost of the retailer’s information rent drops the expected manufacturer’s profits below the full information levels. It is readily checked that, as in the case of retailer salvaging returns, the manufacturer’s expected profits in the screening equilibrium are again strictly convex in \(\beta\) and, depending on the value of \(\gamma^H,\) they might be either increasing or non-monotonic in \(\beta.\) However, the manufacturer’s ability to screen the retailer’s information allows him to achieve a higher level of profits than in the previous case of retailer salvaging returns. As a consequence, the manufacturer will find it profitable to reduce the product return rate for a larger range of the belief \(\beta\) than in the case of retailer salvaging returns.

**Lemma 3.5.2.** When the manufacturer salvages returns, in a screening equilibrium, for \(\gamma^H \leq \overline{\gamma}\) the manufacturer expects to benefit from a reduction of the product return rate \(\forall \beta.\) However, for \(\overline{\gamma} < \gamma^H < \overline{\gamma}, \gamma^L < \overline{\gamma},\) there exists a threshold, \(\beta_M < \beta_R,\) such that the manufacturer expects to benefit from a reduction of the product return rate only if \(\beta > \beta_M.\)
Lemma 3.5.2 confirms the intuition that the manufacturer has an extra motivation to reduce the product return rate when he rather than the retailer is responsible for salvaging returns. Salvage value being equal, if the effect of the investment on product returns is privately known by the retailer, the two reverse channels offer different expectations of profits to the high-type manufacturer; thus they imply a different willingness of the manufacturer to undertake the investment itself.

3.5.3 Investment decision

The derivation of the optimal investment when the manufacturer salvages returns is similar to the previous case of the retailer salvaging returns. In a full information equilibrium, the optimal investment is identical to the one derived in section 3.4.3, whereas in a screening equilibrium, a similar result to Corollary 3.4.4 applies here and guarantees the existence of a positive investment, \( i^* \), only when \( \gamma^H \leq \gamma \). However, when the manufacturer is responsible, for salvaging returns he is willing to invest in reducing returns for a larger range of beliefs than in the case of retailer salvaging returns.

3.6. Endogenous reverse channel

In this section we relax the assumption of exogenous reverse channel and analyze the case where manufacturer and retailer decide on which channel member should be responsible for salvaging the returned products: the selection of the contract structure \(- w \text{ or } (w, b)\) - offered by the manufacturer to the retailer becomes endogenous to the model. Previously, we used \( v \) to represent the salvage value no matter who was salvaging returns. Here, we must distinguish the value that can be extracted from the returned product by the manufacturer from the one that can be extracted by the retailer. Let \( v_R \) and \( v_M \) denote the salvage values of the returned product when the
retailer or the manufacturer salvages it, respectively.

The previous analysis of the two-stage game highlighted that when there is full information about the product return rate, the manufacturer earns the same profits no matter where returns are salvaged so long as the product salvage values were equal (i.e., \( v_R = v_M \), in the current notation). Further, as the manufacturer’s profits increase in the salvage value, the returned products are salvaged by the channel member who can extract the largest value; i.e., the reverse channel selection is efficient. However, when the information is asymmetric, the expected profits of the manufacturer suffer from a distortion that is more pronounced when the retailer rather than the manufacturer salvages returns. The following Proposition establishes that the efficient selection of the reverse channel is not guaranteed when the information about the product return rate is asymmetric.

**Proposition 3.6.1.** (i) The efficiency of the reverse channel selection is guaranteed when either \( v_R < v_M \) or there is full information about the product return rate. (ii) When the retailer is privately informed about the product return rate, \( v_R > v_M \) and the difference in salvage value, \( v_R - v_M \), is not too large, the manufacturer may inefficiently take responsibility for salvaging returns.

Proposition 3.6.1 shows another downside of the information asymmetry about the product return rate: the manufacturer can decide to take responsibility for salvaging the returned products even though he is less efficient than the retailer in recovering the value of returns.

Shulman et al. (2010) show that a similar inefficient recovery of the consumer returns value can occur under uncertainty of both the manufacturer and the retailer about the product return rate. The authors consider a different business context, however, where the retailer selects both the selling price and a restocking fee for the consumer returning the product. The manufacturer wants the retailer not to impose a high fee to the consumer because it will negatively affect the consumer’s willingness...
to purchase, causing a drop in quantity demanded. However, the manufacturer is effective in inducing the retailer to set the optimal return penalty through the joint use of a wholesale price and a refund for returns. As a consequence, the manufacturer may accept salvaging returns himself, even in cases where the retailer has access to a more efficient outlet for the product value recovery. In our work, the (potential) deterrent to returns is the manufacturer’s investment rather than the retailer’s restocking fee. We highlight that even with a liberal returns policy, which implies no fee to the consumer returning the product to the store, inefficiencies in the reverse channel may still occur because of the asymmetric information about the post-investment product return rate.

3.7. Extension - Manufacturer privately informed

In this section we extend the previous analysis to consider the case where the manufacturer may undertake an investment whose impact on the product return rate is initially uncertain but then becomes known to the manufacturer. While the manufacturer learns whether his investment will yield a low or high return rate of the product, the retailer, after observing the manufacturer’s investment, is uncertain about its effectiveness in reducing returns. The motivation for this extension of the model is based on the business case of Philips Consumer Electronics (Sciarrotta 2003). Through its customer service the company figured that its MP3 player was returned because of difficulty in product use. Uncertain of the correct action to take, Philips focused on simplifying the software installation for the MP3 player, and then, thanks to the collaboration of a local retailer that allowed the company to interview a sample of customers who bought the enhanced product, could make sure that its intervention was in the right direction. When Philips started to sell its enhanced product to Wal-Mart, the retailer, because of very liberal returns policies (e.g., “no question, money-back guarantee” policies) and limited interaction with its customers, could not
predict whether the consumers would have returned the MP3 player at a lower rate than before. Then, at the moment of trade while the manufacturer was informed about the product return rate, the retailer was uncertain about it.

The sequence of events for this reverse case of information asymmetry is the same as the one reported in Figure 3.1, the only difference being at date 2. When at date 2 the retailer observes the manufacturer’s investment, she has uncertainty about the future product return rate: the retailer assesses with probability $\alpha(i)$ that the product return rate will be low. We assume that $\alpha(0) = 0$, $\lim_{i \to 0^+} \alpha'(i) = +\infty$ and that $\alpha(i)$ is strictly increasing and strictly concave in $i$. The manufacturer, however, has resolved his uncertainty about the product return rate at date 2 and therefore he knows his type at the same date.

When the privately informed manufacturer at date 3 offers a contract to the uncertain retailer, he signals his private information about the product return rate through the contract parameters. The resulting dynamic game of incomplete information requires the derivation of the Perfect Bayesian Nash Equilibria (PBNE), where the retailer’s belief, after observing the contract, can be either a function of the manufacturer’s contract (separating equilibrium) or independent of it (pooling equilibrium). In particular, denoting with $\hat{\alpha}$ the belief of the retailer after observing the contract $(w, b)$ offered by the manufacturer to the retailer, the two PBNE imply the following Bayesian updating of the retailer’s belief:

(i) In a separating equilibrium $\hat{\alpha}(w^L, b^L) = 1$ and $\hat{\alpha}(w^H, b^H) = 0$, where $(w^H, b^H) \neq (w^L, b^L)$;

(ii) In a pooling equilibrium $\hat{\alpha}(w^L, b^L) = \hat{\alpha}(w^H, b^H) = \alpha^{11}$, where $(w^H, b^H) = (w^L, b^L)$.

Similarly to the previous analysis we solve the two-stage game, considering in section 3.7.1 that the returned products are salvaged by the retailer and in section 3.7.2 that they are salvaged by the manufacturer. In section 3.7.3 we comment on
the case of endogenous reverse channel. As additional notation we use the subscript \( s \) to denote the separating equilibrium, and the subscript \( p \) to denote the pooling equilibrium.

### 3.7.1 Retailer salvages the returned products

When the manufacturer is privately informed about his type, he uses the wholesale-price contract \( w \) as a signal of his type. To solve this signalling game we need to derive the separating and pooling equilibria as follows.

**The Separating Equilibrium.** In a separating equilibrium, the manufacturer charges the retailer with a different wholesale price, depending on his type, and the retailer’s inferences about the manufacturer’s type are a function of the charged price. As the solution of the full information case shows, the high-type manufacturer sets a lower wholesale price than the low-type manufacturer; hence, in a reasonable belief structure\(^{12}\), the probability that the retailer ascribes to the manufacturer being high-type decreases in the manufacturer’s wholesale price. However, the pair of wholesale prices selected by the manufacturer under full information will not allow the retailer to discriminate between the two types under private information of the manufacturer. The following Proposition establishes how a separation of the two manufacturer’s types can be achieved.

**Proposition 3.7.1.** When the retailer salvages returns, a separating PBNE equilibrium exists and has the following features:

(i) The high-type manufacturer selects the same wholesale price as in the full information case \((w^*_H = w^H)\), whereas the low-type manufacturer selects a higher wholesale price \((w^*_L > w^L)\).

(ii) The retailer holds beliefs \( \hat{\alpha} = \begin{cases} 1 & \text{if } w \geq w^*_L \\ 0 & \text{otherwise} \end{cases} \).
(iii) Either type of manufacturer attains profits equal to the full information profits of the high-type manufacturer ($\Pi^*_M = \Pi^*_L = \Pi^*_H$).

As the low-type manufacturer has no incentive to mimic the high-type (i.e., pretending that the product return rate is high even though it is low), the latter can credibly reveal his type by charging the retailer with the same wholesale price as the one chosen under full information. To give evidence of a low return rate of the product, however, the low-type manufacturer is required to select a higher wholesale price than the one he selected under full information. In the end, the low-type manufacturer eliminates any mimicking incentive by selecting a wholesale price which reduces his profits to the same level of the high-type manufacturer’s profits. In a separating equilibrium, the benefits of a reduction of returns are completely absorbed by the necessity for the manufacturer to credibly reveal his type.

The Pooling Equilibrium. In a pooling equilibrium, the manufacturer charges the retailer with the same wholesale price, no matter his type. The retailer cannot unravel any information about the product return rate from the manufacturer’s contract and thus she sticks with her prior beliefs. The retailer remains uncertain for any wholesale price higher than the one at equilibrium, whereas she interprets a lower wholesale price as conclusive evidence of a high product return rate. Given this belief structure, the following Proposition derives the manufacturer’s profits in a pooling equilibrium.

Proposition 3.7.2. When the retailer salvages returns, a pooling PBNE exists and has the following features:

(i) Either type of manufacturer selects the same wholesale price, $w^*_p$, which is between the high- and low-type full information optimal wholesale prices.

(ii) The retailer holds posterior belief, $\hat{\alpha} = \left\{ \begin{array}{ll} \alpha & \text{if } w \geq w^*_p \\ 0 & \text{otherwise} \end{array} \right.$.
(iii) Either type of manufacturer attains the same profits, $\Pi^*_M$, which are strictly convex in $\alpha$, strictly increasing in $\alpha$ for $\gamma^H \leq \tilde{\gamma}$, and non-monotonic in $\alpha$ for $\gamma < \gamma^H < \tilde{\gamma}$, $\gamma^L < \tilde{\gamma}$.

It is clear that the selling price selected by the retailer at equilibrium converges, for $\alpha$ approaching 0 (1), to the selling price selected under full information for a high-type (low-type) manufacturer. However, when the retailer is uncertain about the product return rate she distorts the selling price upwards so that at equilibrium the selling price is increasing and concave in $\alpha$. The distortion of the selling price causes in turn a distortion of the manufacturer’s wholesale price downwards, so that the manufacturer’s profits turn out to be convex in $\alpha$. The retailer’s uncertainty has a similar effect on the manufacturer’s profits in a pooling equilibrium than the manufacturer’s uncertainty has in a screening equilibrium. In both equilibria, the manufacturer offers only one contract to the retailer, but in the screening equilibrium it was the manufacturer whereas in a pooling equilibrium it is the retailer who selects the price by weighing the odds of a low rather than high product return rate. It can be easily verified that for $\gamma^H \leq \tilde{\gamma}$, $\Pi^*_M$ is strictly increasing in $\alpha$, whereas, for $\gamma < \gamma^H < \tilde{\gamma}$ and $\gamma^L < \tilde{\gamma}$, $\Pi^*_M$ is non-monotonic in $\alpha$. Then, similarly to the result in Proposition 3.4.3, the manufacturer will find it profitable to achieve a reduction of the product return rate only if the retailer’s prior belief about a materialization of $\gamma^L$ is sufficiently high.

3.7.2 Manufacturer salvages the returned products

When the manufacturer is privately informed about his type and salvages the returned products, he signals his private information to the retailer through the contract $(w, b)$. To obtain a solution of the signalling game, we derive below the separating and pooling equilibria.

The Separating Equilibrium. The manufacturer, no matter his type, has an
incentive to pretend his product will yield a low return rate; hence, the retailer would take any high-type contract from menu (3.14) as evidence of a high-type manufacturer. However, when the retailer is offered a low-type contract from the same menu, she cannot discern between the two types of manufacturer. The following Proposition shows that the separation of the manufacturer’s types can be achieved through a proper choice of the refund $b$, and it implies no penalization of the full information profits.

**Proposition 3.7.3.** When the manufacturer salvages returns, a separating PBNE exists and has the following features:

(i) The $j$-type manufacturer sets $(w^*_j, b^*_j)$ satisfying (3.14) but with $b^*_j \geq \bar{b}$ (detailed in Appendix).

(ii) The retailer holds posterior belief, $\hat{\alpha} = \begin{cases} 1 & \text{if } (w, b) \text{ satisfies (3.14) for } j = L \text{ and } b \geq \bar{b} \\ 0 & \text{otherwise} \end{cases}$.

(iii) Either type of manufacturer attains the same profits as in the full information case.

If the high-type manufacturer offers a low-type contract from the menu of optimal full information contracts, then his profits will decrease in the refund $b$. As a consequence, for a sufficiently high value of the refund, the high-type manufacturer prefers being correctly identified by the retailer to mimicking the low-type manufacturer. The latter, then, can both separate and select one of his full information optimal contracts. In a separating equilibrium, the manufacturer’s private information, thanks to the flexibility of two signals (the wholesale price and the refund), does not cause any distortion to the profits attained under full information. Further, the equivalence of the manufacturer’s profits in a full information and separating equilibrium suggests that the manufacturer’s willingness to undertake an investment to reduce returns does not change because of the information held by the retailer about the return rate of the product, as long as the manufacturer is aware of it.
The Pooling Equilibrium. When the retailer, after observing the contract \((w,b)\), remains uncertain about the manufacturer’s type, a pooling equilibrium is sustained only if the high-type manufacturer, with the contract of a low-type, realizes profits above his full information levels. The following Lemma establishes that this is not the case.

Lemma 3.7.4. When the manufacturer salvages returns, a pooling equilibrium does not exist.

The low-type manufacturer would offer, by pooling, a full refund to the retailer for returns. Such a contract is too costly to the high-type manufacturer, who cannot afford reimbursing the entire wholesale price for a high amount of returns. The high-type manufacturer prefers separating rather than pooling, so that he can earn profits equal to those under full information. In the end, a pooling equilibrium cannot be sustained.\(^{13}\)

3.7.3 Endogenous reverse channel

When the selection of the reverse channel is endogenous to the model there is again the possibility that the manufacturer assigns inefficiently the responsibility for salvaging returns. In Figure 3.4 we have represented the separating equilibrium (corresponding to the two levels \(\Pi^H_M\) and \(\Pi^L_M\)) when the manufacturer salvages returns and the pooling equilibrium when the retailer salvages returns. Depending on the outcome of his investment, the retailer’s belief \(\alpha(i)\), and the difference between the salvage values in the two reverse channels, \(v_M - v_R\), the allocation of responsibility for salvaging returns might be again inefficient.

If the product return rate has remained high even after the investment was taken, salvage value being equal, the manufacturer is better off to pool and leave the responsibility for salvaging returns to the retailer. The manufacturer will prefer such an allocation even when he is more efficient at salvaging returns than the retailer.
Figure 3.4: Manufacturer’s profits in the second stage of the game when the manufacturer salvages returns and separates (two dots) and when the retailer salvages returns and the manufacturer pools (solid line) for the case $\gamma^H \leq \bar{\gamma}$. The values of the parameters are: $\phi = 100$, $v = 30$, $\gamma^L = 0.1$ and $\gamma^H = 0.5$.

$(v_M - v_R > 0)$ as long as the advantage of salvage value is sufficiently low. If the product return rate has become low as a result of the investment, salvage value being equal, the manufacturer is better off to separate and take himself the responsibility for returns. The manufacturer will select this allocation of responsibility for returns even when he is less efficient than the retailer in salvaging returns $(v_M - v_R < 0)$, as long as the difference in salvage value is sufficiently low.

### 3.8. Conclusions

The flow of consumer returns is viewed as a nuisance by manufacturers and retailers. Products are often returned not because of defects, but because of difficulties consumers face with the installation, configuration and use of the purchased product, or the product simply not meeting their expectations. Through an investment, the manufacturer may reduce the probability of a product return, but this investment can expose the manufacturer to a disadvantage of information with respect to the
A retailer, particularly a specialized retailer with deep understanding of her customers' needs, is generally able to predict the impact of the manufacturer's investment on returns. The retailer who observes the manufacturer's investment and knows its impact on product returns has an incentive to pretend that the investment is ineffective and the product return rate has remained high, because the manufacturer demands a lower wholesale price for a product with high return rate. In this work we have studied how the information held by the retailer about the product return rate affects the investment of the manufacturer, the profits of the manufacturer, and the efficient selection of the reverse channel structure where returns are salvaged.

We have found that the private information of the retailer about the product return rate creates distortions in the profits and the investment of the manufacturer, and in some cases, it might lead the manufacturer to even forgo investment. The distortions in the manufacturer's profits are less severe when the manufacturer (rather than the retailer) is responsible for salvaging returns, because the contract, consisting of both a wholesale price and a refund per return (rather than just a wholesale price) allows the manufacturer to screen the private information of the retailer. As a consequence, under information asymmetry about the product return rate, the manufacturer is more willing to invest in reducing returns when he rather than the retailer has the responsibility for salvaging returns. The different distortions of the manufacturer's profits under the two contracts, which entail different structures of the reverse channel, can cause inefficiency in the selection of the outlet where the value of the returned products is recovered: the manufacturer might take responsibility for salvaging returns even though the retailer can extract a higher value from returns. We have extended our analysis to the case where the manufacturer is privately informed about the product return rate as a result of his investment. Although there are still equilibria which might entail no investment and an inefficient selection of the reverse channel, when the manufacturer salvages returns he can achieve the same profits as under a full information scenario. Therefore, the manufacturer has an extra
motivation to understand the impact of his investment on returns and, in case this impact is positive, to take responsibility for salvaging the returned products.

There are several directions in which our research can be extended. For instance, given the perfect quality of the returned products, it is common practice for some retailers to resell the products in “open box” (Strauss 2007), in which case the salvage value of the product would be a specific fraction of the selling price, rather than a fixed market value, \( v \). The retailer in this case would keep the selling price constant, no matter the manufacturer’s type, thus the information asymmetry about the product return rate is expected to cause lower distortions to the manufacturer’s profits. Further research in this direction could confirm these findings, and encourage the practice of “open box” for recovering the value of the returned products at the retail store. Another possible avenue of research may consider retailer’s investments (or efforts) to reduce returns, for instance in terms of extra-time spent with consumers to explain the features of the product and thus to avoid a mismatch of the product with the consumer preferences (which most likely would prevent the product from returning). Our modeling framework can be easily adapted to address these extensions.

In sum, this chapter highlights the inefficiencies in profits, investments and product value recovery caused by the information asymmetry about the product return rate. Given the persistent cost that returns imply to both manufacturers and retailers, our research reinforces the intuition that all supply chain members can greatly benefit from first learning the reasons why consumers return the products and then find appropriate ways to credibly share the acquired information along the supply chain.
Notes

1 For instance, Philips Consumer Electronics is reported to work with pilot retailers and conduct marketing research with 400-plus consumers to learn whether the company investment in product ease of use would yield the desired outcome of returns reduction (Sciarrotta 2003).

2 We assume two levels (“types”) for the product return rate: high and low. Laffont and Martimort (2002, p. 134) assert that the consideration of a continuum of types would generate very few additional insights. In this chapter we consider and solve dynamic games of incomplete information, which become easily intractable if a continuum of types is assumed (Bolton and Dewatripont 2005, p. 125). A recent work involving dynamic games of incomplete information with the consideration of binary types is the one from Anand and Goyal (2009) about information management under leakage in a supply chain.

3 A positive value of \( v \) reflects the ability of either the manufacturer or the retailer to resell the returned product at a higher price than the logistics cost of remarketing it. Our results, however, would not qualitatively change if \( v \leq 0 \).

4 As the reverse channel is exogenous, there is no need at this time to denote differently the value of the returned product, depending on whether the manufacturer or the retailer salvages the product. We introduce this differentiation of the salvage value in section 3.6, when we consider the case of endogenous reverse channel selection.

5 From (3.3) it follows \( w^*L - w^*H = \frac{(\phi - v)(\gamma^H - \gamma^L)}{2} > 0 \).

6 From (3.4) it follows \( p^*L - p^*H = \frac{v(\gamma^H - \gamma^L)}{4(1 - \gamma^H)(1 - \gamma^L)} > 0 \).

7 \( \frac{\partial^2 \Pi^*M}{\partial \gamma^2} = \frac{v^2}{4(1 - \gamma^2)} > 0 \).

8 By solving the inequality \( \Pi^*_M > \Pi^*_H \) for \( \gamma^L \) we obtain \( \gamma^L < \tilde{\gamma} = 1 - \frac{v^2}{(1 - \gamma^H)(\phi - v)} \).

9 Note that as the manufacturer’s investment is taken in the first stage of the game, in the second stage of the game the function \( \beta(i) \) is a scalar which we indicate here with \( \beta \).

10 By expected profits under full information we mean the profits the manufacturer expects to achieve at date 1 (in Figure 3.1), when he is uncertain about the product return rate, given that at date 3 he and the retailer are both informed about whether the product return rate will be low or high.

11 The same considerations made for \( \beta \) in note 9 apply also here to \( \alpha \).

12 Although the retailer’s off-equilibrium beliefs are arbitrary in principle, we restrict our analysis to belief structures that are monotonic in the manufacturer’s signal. Anand and Goyal (2009) adopt a similar approach in considering the entrant retailer’s beliefs about the market demand being high as monotonic in the incumbent retailer’s order quantity.

13 As an additional note, it should be mentioned that the separating and pooling equilibria derived for the two reverse channels satisfy the Intuitive Criterion of Cho and Kreps (1987). In fact, even though for a range of wholesale prices either type of manufacturer would benefit from being identified as a low-type, there is no unilateral deviation of one type from the equilibrium path that is equilibrium-
dominated for the other type. As a consequence, both the separating and pooling equilibria derived are supported by reasonable beliefs of the retailer.
Chapter 4
The Impact of Consumer Returns on a Manufacturer Multichannel Strategy

4.1. Introduction

With the advent of e-commerce and the increasing access of consumers to the Internet, it is apparent that a manufacturer cannot ignore the opportunity of selling his products directly through the online channel in addition to the traditional retail channel. Products sold through an online channel may be produced according to a make-to-order strategy which allows the manufacturer to meet the preferences of a larger number of consumers than a make-to-stock strategy. In particular, a make-to-order manufacturing strategy allows the manufacturer to provide the consumers with a higher product availability and variety, and also, as it is becoming increasingly common in practice, it allows customization of the products. Consumers purchasing online may be able to extract a higher value from the product than those purchasing at the retail store. In fact, because of greater horizontal differentiation, consumers purchasing online are more likely to find a fit to their specific preferences.

The advantages offered by the online outlet, however, do not come free of any undesirable side effect. When purchasing a product online, a consumer is uncertain about product fit and, after receiving the product, might decide to return it to the manufacturer. The problem of consumer returns is particularly acute in product categories where consumers need to “touch and feel” the product to determine how well it fits their tastes and needs. For example, it is very difficult for a consumer purchasing a jewel to ascertain how well it would fit with a particular dress. In a
similar vein, the selection of a pair of shoes might result in a mismatch with the consumer preferences when there is no opportunity to actually wear the shoes before purchasing them. Similar reasoning can be extended to other product categories, such as furniture, fashion apparel, sporting goods, art works, etc.

By contrast, retailers do offer the possibility to physically inspect the product before making a purchase and the associated uncertainty about product fit can be almost completely resolved. Recent industry surveys show in fact that return rates for the product categories considered are still high when the purchase occurs through the Internet channel as opposed to when a consumer buys the same product at the store.\footnote{1} Further, products for which consumers require more physical inspection to determine fit are sold less online compared with products whose attributes and specifications can easily be communicated digitally.\footnote{2}

In this paper we consider an important and relevant trade-off for the manufacturer in the design of his product sales channel: an online (or direct) channel would provide consumers with a higher product marginal value than the retail channel, but the retail channel would not pose any problem of consumer returns as opposed to the online channel. An important role in this scenario is played by the manufacturer’s returns policy in the direct channel. A lenient returns policy in fact would mitigate the consumer uncertainty about product fit but could also potentially result in a large amount of products returned with a small residual value. The adoption of a specific returns policy by the manufacturer is then an important element to account for in the design of the sales channel. Consider the following example from the furniture industry. Two American manufacturers of lamps and furniture, Modernica and Graypants, have both a different sales channel structure and returns policy. Modernica sells its products through a dual channel structure, counting hundreds of dealers throughout the U.S., and offers a very lenient returns policy to the consumers for their online purchases; that is a full refund for returns within three months from the purchase date but no refund for the shipping of the product back to the manufacturer. Graypants,
however, sells only through the online channel but charges consumers with a restocking fee of 15% of the value of the purchased product in addition to giving zero refund for the shipping costs mentioned above. Our research gives a possible explanation of why manufacturers having a different sales channel structure might adopt different returns policies. More specifically, the main research objectives of this chapter can be summarized through the following research questions:

1. What is the optimal sales channel design for a manufacturer when the online channel offers more product value but also more uncertainty than a retail channel?

2. How is the returns policy related to the sales channel structure?

We find that the design of the sales channel depends on the product marginal value the consumer expects to obtain from the online channel and on the salvage value that can be extracted from the consumer returns. When the online expected marginal value is higher, because of limited uncertainty about product fit, then the manufacturer will never sell his products at the store. However, when consumers expect to obtain higher product value from the store, because of too high product fit uncertainty at the direct channel, then the manufacturer finds it profitable to open a sales channel even though no sales would occur there. In fact, in this case, the manufacturer uses the direct channel just as a mechanism to threaten the retailer and therefore lead her to lower the selling price, ultimately increasing the manufacturer’s volume of sales at the store, hence his profits.

Further, when both channels are active the manufacturer softens his returns policy as opposed to the case when only the online channel is active. The generous returns policy indeed attracts consumers towards the direct channel and reduces the inefficiencies that occur at the decentralized retail channel because of double marginalization. This manufacturer strategy could explain why in our previous example from the industry Graypants who sells only online adopts more restrictive returns policies than
Modernica, who sells through both a retail and an online channel.

The remaining of this chapter is organized as follows. The next section summarizes the relevant literature. This is followed by a description of the basic model setup and the analysis of the vertically integrated case in section 4.3. In section 4.4 we analyze the manufacturer’s design of the sales channel when the retailer is an independent firm. Concluding remarks are given in section 4.5. All proofs and technical details are reported in Appendix C.

4.2. Literature Review

Our work is primarily related to two streams of research. The first examines the implications of selling through a dual versus a single channel, and the second relates to the design of returns policies in a market characterized by consumer returns. Below we review each of these streams of research and their interactions with the current research.

It is well established in the supply chain literature that trade in a bilateral monopoly, where a single manufacturer sells his product through a single retailer, suffers from the inefficiencies of double marginalization (Spengler 1950). The opportunity for a manufacturer to sell through a direct channel therefore seems appealing as this source of inefficiency would be eliminated. The consumer’s acceptance of the online channel, however, is not so widespread that a manufacturer might think of not selling through the retailer. The problem of selling through both channels then is relevant and deserves large academic attention. Chiang et al. (2003) investigate the manufacturer’s problem of whether to sell direct over the Internet, exclusively through a retailer, or through a hybrid of both approaches when the consumer’s acceptance for the direct channel is lower than the one for the retail channel. They find that when the consumer’s acceptances for the two channels are comparable, the manufacturer sets the prices so that no online sales materialize; however, the presence
of the direct channel leads the retailer to lower the selling price, thus reducing the inefficiencies of double marginalization to the benefit of the manufacturer’s profits. In a similar vein, Kumar and Ruan (2002) find that by complementing the retail channel with a direct online channel a manufacturer can price discriminate between store loyal and brand loyal consumers, and this discrimination in turn allows the manufacturer to achieve a higher contribution margin with the latter category of consumers. Another way to discriminate the two channels is along the dimension of both price and sales effort, as proposed by Tsay and Agrawal (2004). However, consumers might research the product in one channel but make the purchase to the other. This issue of free-riding coupled again with the double marginalization create inefficiencies in the manufacturer’s sales channel design and might create scenarios where paradoxically the manufacturer would prefer selling through only the retailer whereas the retailer would prefer that the manufacturer opens a direct channel in parallel. Our research explores two new dimensions along which a manufacturer is required to design his sales channel: the product marginal value and the uncertainty about product fit. This trade-off faced by the manufacturer in the sales channel design is unexplored in the academic literature and our research aims at covering this gap.

In the marketing literature the impact of consumer returns on the retailer’s multichannel strategy design has been recently addressed by the work of Ofek et al. (2011). Our work, however, considers the manufacturer’s multichannel strategy as opposed to the retailer’s one, which is relevant especially considering the recent tendency of several manufacturers to open an online channel in addition to the retail channel. For instance, companies like Procter & Gamble, Levis Strauss, and Mattel have been recently reported to open an eStore, that is a hub of the manufacturer’s brands which puts the manufacturer in direct competition with the retailer (Galante 2010).

With respect to the stream of research on consumer returns, it should be mentioned that there is a growing body of literature on this topic in both the areas of Marketing and Supply Chain Management. The research which is relevant for this
chapter is the one focused on the design of optimal returns policies at the point of sale. The generally accepted theory is that a penalty per unit return has the effect of discouraging the consumer from returning the product, but it may also have the side effect of discouraging the consumer from purchasing the product in the first place. Davis et al. (1995) find that when the retailer has a sufficiently high salvage value, a money-back guarantee policy is more profitable than a no-refund policy. Che (1996) shows that a full refund policy is optimal when consumers are highly risk-averse or when the retailer procurement costs are high. Su (2009) shows that full refunds are suboptimal for the retailer and finds that the refund to consumers for a returned unit should equal the salvage value. Shulman et al. (2009) develop a detailed understanding of how a retailer can use price, restocking fee, and information to affect consumer purchases and returns. In this chapter, we consider for the first time in the literature how the returns policy in one channel is related to the structure of the sales channel. The interaction between the two channels in a market where returns are an issue is the novel element of this research.

4.3. The Model

4.3.1 Retailer

The retailer offers a horizontally differentiated product, which can come in a number of sizes, designs, or models such that consumers have to determine which variants fits their needs or tastes best. Vertical quality is assumed to be identical for products. When a consumer buys a product at the retailer, he solves any uncertainty about product fit through the physical inspection of the product, which can be realized only at the retailer. For this reason, there are no consumer returns at the retail store.³

The retailer acts as a Stackelberg follower in that she pays the wholesale price, $w$, to the manufacturer and then sells the product at an optimally selected selling price,
4.3.2 Consumers

Consumers can buy a product from either the retail or the online channel. Consumers are heterogeneous with respect to their willingness to pay for the product purchased at the retailer, \( v \), which is assumed uniformly distributed in \([0, 1]\). Each consumer, whose valuation for the product purchased at the retail store is \( v \), values \( \theta^j v, j \in \{H, L\} \), the product purchased at the online channel. The parameter \( \theta^j \) represents the marginal value of the product at the moment of the online purchase and its value is realized only when the product is inspected by the consumer: \( \theta^j \) can be equal to \( \theta^H > 1 \) with probability \( m \) if the product matches the consumer preferences, or equal to \( \theta^L < \theta^H \) with probability \( 1 - m \) when the product does not match the consumer preferences. Once the value of the product purchased online is realized, the consumer decides whether to keep the product or to return it for a refund. Denoting with \( p_o \) the selling price of the product sold online, with \( f_o \) the penalty charged to the consumer returning the product previously purchased online, the consumer must make a purchasing decision according to the decision tree in Figure 4.1.

![Figure 4.1: Consumer decision tree](image-url)
The competition between the two channels then depends not only on the prices but also on the value that the consumer can obtain from the product and on the penalty for a return, which mitigates the uncertainty about product fit in the online channel. Notice that we are limiting the scope of this research to search goods and do not consider experience goods, so that returns do not occur at the retailer. In fact, with search goods any uncertainty about product fit is resolved as long as the consumer has the opportunity to physically inspect the product.

4.3.3 Manufacturer

The manufacturer has to select the channel through which he sells his product. The manufacturer incurs a per unit production cost of $c$ which is independent of the channel where the product is sold.\textsuperscript{5} Sales at the online channel can be realized only if the marginal value of the product is higher than its marginal cost. In particular, we focus on the case where the marginal value of the product dominates the production cost even when there is a mismatch with consumer preferences, so that $c < \theta^L$. The manufacturer sells his product through the online channel at a price $p_o$ and implements a returns policy based on a penalty $f_o$ for each product returned by the consumer. We assume that the manufacturer has the ability to salvage the returned product at the value $s$, with $s < c$, which implies that any product return is necessarily costly to the manufacturer.

4.3.4 The Vertically Integrated Case

We start our analysis by considering the case where the manufacturer owns both the retail and the online channels. The decision variables for the manufacturer then are the two selling prices, $p_r$ and $p_o$, and the penalty, $f_o$. To derive the demand faced by each channel we have to compare the utility obtained by the consumer in purchasing the manufacturer’s product from the store with the utility obtained in purchasing
from online. It is clear that the consumer utility from store purchasing is given by $U_r = v - p_r$. When purchasing through the online channel, however, the consumer faces uncertainty about product fit. The consumer decides whether to buy the product by taking into account that with probability $m$ the product will match his preferences, realizing a consumer surplus of $\theta_H v - p_o$, whereas with probability $1 - m$ the product will not match his preferences, realizing a consumer surplus of $\theta_L v - p_o$. Once the purchased product is received and inspected, the consumer makes a second decision, which is whether to keep or to return the purchased product (see Figure 4.1). The purchased product which matches the consumer preferences is kept by the consumer if $v > T^H = \frac{\theta_H v - p_o}{\theta_H}$, and returned otherwise. Likewise, the product which does not match the consumer preferences is kept by the consumer if $v > T^L = \frac{\theta_L v - p_o}{\theta_L}$, and returned otherwise. The two thresholds $T^L$ and $T^H$, with $T^L > T^H$, divide the market of consumers making an online purchase into three segments. For $v \leq T^H$ consumers prefer not purchasing any product because their expected utility is negative and equal to $-f_o$. We refer to the consumers belonging to this market segment as to consumers never purchasing the product. For $T^H \leq v \leq T^L$, a portion $m$ of the consumers who buy the product will then keep it, whereas the remaining portion $1 - m$ of the consumers will return it; this second market segment consists therefore of consumers potentially returning the product. Finally, for $v > T^L$, no consumer will return the purchased product, no matter its misfit with consumers preferences; therefore we may refer to these consumers as to consumers never returning the product. The expected utility from an online purchase (denoted as $E[U_O]$) is then given by the following piece-wise linear function:

$$E[U_O] = \begin{cases} 
-f_o, & v \leq T^H \\
 m(\theta_H v - p_o) - (1 - m)f_o, & T^H \leq v \leq T^L \\
 \mu v - p_o, & v \geq T^L
\end{cases},$$

(4.1)
where $\mu = m\theta^H + (1 - m)\theta^L$ is the expected marginal value of the product purchased online (i.e., before product physical inspection). From expression (4.1) it is apparent that an increasing value for the penalty $f_o$ negatively affects both demand and number of returns. In fact, the number of consumers potentially returning the product decreases with the penalty $f_o$.\(^6\)

Our objective is to derive the demands at the online channel, $Q_o$, at the retail channel, $Q_r$, and the number of returns at the online channel, $R_o$. Let $v_z = \frac{mp_o + (1 - m)f_o - p_r}{m\theta^H - 1}$ and $v_x = p_o - p_r^m\theta^H$ denote the two threshold values at which consumers potentially and never returning the product, respectively, are indifferent between purchasing at either channel, and let $A = \frac{p_o + \frac{1 - m}{m}f_o}{\theta^H}$ denote the value at which consumers potentially returning the product expect to obtain zero utility from an online purchase.

Consider the first case where uncertainty about product fit is so limited that the expected marginal value of the online product matching the consumer preferences is higher than the marginal value of the retail product, that is $m\theta^H > 1$, and consequently, $\mu > 1$. Under these conditions a dual channel is allowed only for a sufficiently low selling price at the retail store, i.e., $p_r \leq A$ (see Figure 4.2).

Demands and number of returns when $m\theta^H > 1$ and $p_r \leq A$ then are given by the following expressions:

\[
\begin{align*}
Q_o &= 1 - v_z \\
Q_r &= v_z - p_r \\
R_o &= (1 - m) [T^L - v_z] \\
\end{align*}
\]

if $v_z \leq v_x$, and by

\[
\begin{align*}
Q_o &= 1 - v_x \\
Q_r &= v_x - p_r \\
R_o &= 0 \\
\end{align*}
\]

if $v_z \geq v_x$, (4.2)

where $v_z$ as represented in Figure 4.2 can become $\geq v_x$, because a sufficiently high penalty $f_o$ makes the consumers potentially returning the product switch from the
online to the retail channel. When the uncertainty about product fit remains low, \( m\theta^H > 1 \), but the retail selling price is sufficiently high, \( p_r \geq A \), then it is clear that no consumer will find it profitable to purchase at the retail channel, as the online channel offers higher product value with little uncertainty about product fit. However, the manufacturer may realize sales of his product through a dual channel by sufficiently decreasing the retail selling price, \( p_r \leq A \). The first four lines of Table 4.1 provide the expressions of the two demands and of the number of returns when \( m\theta^H > 1 \).

By deriving the demand function for the retail store we notice that there may exist a point at which the demand elasticity changes. Indeed, consider the case where for a low retail selling price the online channel attracts the purchases of only those consumers never returning the product. Then, by increasing the retail selling price, all of these consumers will switch to the online channel, corresponding to the segment \( BK \) in Figure 4.3. A further increase of \( p_r \) leads even the consumers potentially returning the product to prefer the online to the retail channel, which corresponds to the segment \( AK \). The point \( K \) in Figure 4.3 then is a “kink” point where the de-
mand function becomes more elastic as a consequence of losing a more price-sensitive category of consumers.

When the uncertainty about product fit for an online purchase increases, the expected marginal value of the matching online product may become lower than the value of the retail product \( (m \theta^H < 1) \), but the product overall expected marginal value may remain still higher, that is \( \mu > 1 \). The expressions of demand at either channel and of the number of returns at the online channel for this case are reported in the lines 5–8 of Table 4.1. The retail demand function can be derived with an approach similar to the previous case.

Finally, the third case occurs when the consumer uncertainty about product fit is so high that the product expected marginal value at the online channel is outweighed by the product value at the retail channel, that is \( \mu < 1 \). The demands and the number of returns in this third case are reported in rows 9–12 of Table 4.1. It is interesting to notice that for \( \mu < 1 \), the retail channel may monopolize the market if the retail selling price is sufficiently low \( (p_r \leq A) \). A first “kink” point in the retail demand function will correspond then to a level of the selling price that makes some of the consumers
potentially returning the product switch to the online channel. However, there is also a second “kink” point corresponding to a level of the selling price that makes even those consumers never returning the product switch to the online market. The demand function for $\mu < 1$ is represented in Figure 4.4, where the two “kink” points are denoted by $K_1$ and $K_2$.

![Figure 4.4: Demand function at the retail channel when $\mu < 1$.](image)

### 4.3.4.1 Manufacturer’s Optimization Problem

Given the demands at both channels and the number of returns at the online channel, the manufacturer has to optimally select the selling prices $p_r$ and $p_o$, and the penalty $f_o$ according to the maximization problem

$$\max_{p_o,p_r,f_o} \Pi_M = Q_o(p_o - c) + R_o(f_o + s - p_o) + Q_r(p_r - c).$$  \hspace{1cm} (4.3)
The prices and the penalty in turn determine the optimal design of channels through which the manufacturer will sell his product. The following result establishes a first condition under which the manufacturer decides to sell exclusively over the Internet.

**Proposition 4.3.1.** In a vertically integrated system, for \( \mu > 1 \) the manufacturer will keep only the direct channel active.

Proposition 4.3.1 confirms the intuition that when the expected marginal value for the product is higher at the online than at the retail channel, the manufacturer will optimally decide to shut down the latter. Basically, the manufacturer adjusts the retail selling price, \( p_r \), so that no consumer will benefit from purchasing the product at the store. Even though returns might be an issue, the manufacturer will control the flow of returns by strategically adjusting the returns policy to the salvage value that can be extracted from the returned products. In particular, the manufacturer will raise the penalty for a returned unit whose salvage value is low, so that no consumer will prefer returning to keeping the purchased product. However, for a salvage value sufficiently high, the manufacturer will decrease the penalty per return, and consequently a portion of consumers who suffer from product misfit will opt for

<table>
<thead>
<tr>
<th>( Q_r )</th>
<th>( Q_o )</th>
<th>( R_o )</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_x - p_r )</td>
<td>( 1 - v_x )</td>
<td>( 1 - v_x )</td>
<td>( 1 - m)(T - v_x) )</td>
</tr>
<tr>
<td>( 0 )</td>
<td>( 1 - A )</td>
<td>( 1 - A )</td>
<td>( 1 - m)(T - A) )</td>
</tr>
<tr>
<td>( v_x - p_r )</td>
<td>( 1 - v_x )</td>
<td>( 0 )</td>
<td>( m\theta^H &gt; 1 \Rightarrow \mu &gt; 1 )</td>
</tr>
<tr>
<td>( v_x - v_z )</td>
<td>( 1 - v_x + v_z - A )</td>
<td>( (1 - m)(v_z - A) )</td>
<td>( p_r \leq A )</td>
</tr>
<tr>
<td>( 0 )</td>
<td>( 1 - A )</td>
<td>( (1 - m)(T - A) )</td>
<td>( v_z \leq v_x )</td>
</tr>
<tr>
<td>( 1 - v_z )</td>
<td>( v_z - A )</td>
<td>( (1 - m)(v_z - A) )</td>
<td>( v_z \geq v_x )</td>
</tr>
</tbody>
</table>

Table 4.1: Demands at the retail and the online channels and returns at the online channel for all possible cases.
returning the mismatched product. We summarize these observations in the following Corollary.

**Corollary 4.3.2.** For $\mu > 1$, there is a threshold for the salvage value, $\bar{s} = \frac{c \theta L}{\mu}$, such that for $s > \bar{s}$ the manufacturer finds it profitable to allow returns at the online channel.

Even though consumer returns imply clearly a cost, the liberal returns policy creates an effect of market expansion which affects positively the manufacturers profits. A lower penalty implies an increase in both demand and returns but the sufficiently high salvage value makes the benefits of the demand increase outweigh the losses caused by the returns flow. Notice also that the higher is the expected product value, $\mu$, and the lower is the threshold $\bar{s}$, which reflects the fact that with lower uncertainty about product fit (and so less returns) the manufacturer is willing to soften his returns policy and to allow returns even when he is less efficient in product recovery.

We turn now to analyze the case where there is high uncertainty about product fit in the online purchases so that the product expected marginal value is lower over the Internet than at the store (i.e., $\mu < 1$). In this case, the manufacturer might also prefer keeping active the retail channel in addition to the online channel. The design of the sales channel will be again a function of the salvage value $s$ as suggested by the following result.

**Proposition 4.3.3.** In a vertically integrated system, for $\mu < 1$ the manufacturer will keep only the retail channel active if $s \leq c \frac{1-m \theta H}{1-m}$, both retail and online channels active if $c \frac{1-m \theta H}{1-m} < s < \frac{1-\theta H}{1-m}$, and only the online channel active if $s \geq \frac{1-\theta H}{1-m}$.

When consumers expect a lower product value at the online than at the retail channel, the manufacturer selects the first-best value for the retail selling price, i.e., $p^*_r = \frac{1+c_2}{2}$, whereas he manipulates the selling price and the penalty in the direct channel in order to realize additional profits. Both price and penalty are selected
high by the manufacturer for $s \leq c^{1-m\theta H_{1-m}}$ so that no consumer finds the direct channel attractive. When the salvage value increases, the decrease of the selling price and of the penalty make an increasing portion of consumers switch from the retail to the online channel. In the end, a sufficiently high salvage value leads the manufacturer to activate exclusively the online channel. Notice once more that the threshold values of $s$ that make the manufacturer switch from a retail-based sales channel to an online-based sales channel decrease with $m$, which confirms the intuition that for lower uncertainty about product fit the manufacturer tends more towards activating the online channel.

### 4.4. Analysis of Supply Chain Dual Channel Design

We consider now the case where the manufacturer has to design the channel through which to sell his products assuming that the retailer is an independent profit-maximizer firm. The manufacturer and the retailer engage in a Stackelberg game where the manufacturer acts as a leader and sets the wholesale price, $w$, for the retail sales, the selling price, $p_o$, and the penalty, $f_o$, for the direct sales, whereas the retailer acts as a Stackelberg follower and optimally selects the selling price, $p_r$, for the retail sales.

To avoid considering unrealistic cases where the retailer might have an incentive to purchase the products from the online channel, we require that the wholesale price be higher than the direct channel price, that is $w \leq p_o$.

The response function of the retailer for a given wholesale price depends on the retailer’s quantity demand, $Q_r$, as given by one of the entries in Table 4.1. The retailer solves the maximization problem

$$
\max_{p_r} Q_r(p_r - c),
$$

(4.4)
and determines a reaction function \( p_r^* \) to the manufacturer decision variables; in other words, the selling price \( p_r^* \) is a function of \( w, p_o, \) and \( f_o \).

For the analysis of the Stackelberg game it is crucial to recognize the piece-wise linear nature of the demand function at the retail store. When there is little uncertainty about product fit, \( m\theta^H > 1 \), we consider the demand function reported in Figure 4.3. When the retail demand corresponds to the segment \( BK \), all the consumers would purchase the manufacturer’s product through the retailer except for a small fraction of consumers never returning the product. The retailer’s reaction function is equal to \( p_r^*_{BK} = \frac{w_o}{2} + \frac{w_o}{2p_r} \), but only under a certain set of constraints. In particular, by denoting with \( v^*_{BK} \) and \( v^*_{BK} \) the expressions of \( v_z(p^*_{BK}) \) and of \( v_x(p^*_{BK}) \), respectively, the manufacturer’s optimization problem when the retail demand corresponds to the segment \( BK \) is given by:

\[
\max_{w, p_o, f_o} (1 - v^*_{BK})(p_o - c) + (v^*_{BK} - p^*_{BK})(w - c)
\]

\[
\text{s.t. } \begin{align*}
p^*_{BK} & \leq A \\
v^*_{BK} & \leq v^*_{BK} \\
p_o & \geq w
\end{align*}
\]  

(4.5)

When the retail demand function corresponds to the segment \( AK \) in Figure 4.3, then all of the consumers never returning the product have switched to the online channel and the consumers potentially returning the product are following the same path as the retail selling price increases. The retailer reaction function in this case is equal to \( p_r^*_{AK} = w + \frac{4}{2} \), and using a similar notation as in (4.5), the manufacturer’s problem
when the retail demand is in the segment $\overline{AK}$ can be expressed as

$$
\begin{align*}
\max_{w,p_o,f_o} & \quad (1 - v_z^{\overline{AK}})(p_o - c) + (v_z^{\overline{AK}} - p_{r*,\overline{AK}})(w - c) + \\
& \quad (1 - m)(T^L - v_z^{\overline{AK}})(f_o + s - p_o) \\
\text{s.t.} & \quad p_{r*,\overline{AK}} \leq A \\
& \quad v_z^{\overline{AK}} \leq v_z^{\overline{AK}} \\
& \quad p_o \geq w
\end{align*}
$$

(4.6)

Finally, there is the case where the manufacturer sets $(w, p_o, f_o)$ in neither of the previous regions, which corresponds to the optimal retail price being located at the kink point $K$, that is $p_{r*K} = p_o - (\mu - 1)T^L$. The manufacturer then will solve the following optimization problem:

$$
\begin{align*}
\max_{w,p_o,f_o} & \quad (1 - T^L)(p_o - c) + (T^L - p_{r*K}^{*})(w - c) \\
\text{s.t.} & \quad p_{r*K} \leq A \\
& \quad v_x^{\overline{BK}} \geq v_x^{\overline{BK}} \\
& \quad v_x^{\overline{AK}} \geq v_x^{\overline{AK}} \\
& \quad p_o \geq w
\end{align*}
$$

(4.7)

In Figure 4.5 we represent a section on the plane $wp_o$ of the manufacturer’s constraints, $v_x^{\overline{BK}} \geq v_x^{\overline{BK}}$ as in (4.5) and $v_x^{\overline{AK}} \leq v_x^{\overline{AK}}$ as in (4.6). The unshaded region corresponds to the complement of the two previous constraints, that is $v_x^{\overline{BK}} \leq v_x^{\overline{BK}}$ and $v_x^{\overline{AK}} \geq v_x^{\overline{AK}}$ as in (4.7). The unshaded region then represents the region where the manufacturer selects the optimal decision variables when the retail demand is at the kink point $K$.

The manufacturer was not willing to keep the retail channel open when he owned it; product sales would materialize exclusively through the direct channel. Now, the question is whether the fact that the retailer is an independent firm would somehow change the manufacturer’s sales channel design. It is well known that a decentral-
Figure 4.5: Section on the plane $wp_o$ of the constraints $v_{x}^{BK} \geq v_{x}^{BK}$ (light gray) and $v_{z}^{AK} \leq v_{z}^{BK}$ (dark gray). The unshaded region corresponds to the retailer’s demand at the kink point $K$. We have set $f_o$ equal to its optimal value and used the following parameters values: $s = 0.25, c = 0.4, m = 0.8, \theta^H = 1.5, \theta^L = 0.5$.

The unshaded retail channel brings inefficiency to the manufacturer’s profits as opposed to the vertically integrated one, because of double marginalization. Therefore, our intuition would tell us that the previous decision of the manufacturer to shut down the retail channel would be reinforced when the retailer is an independent profit-maximizer firm. The next result confirms this intuition.

**Proposition 4.4.1.** In a decentralized retail channel, for $m\theta^H > 1$ the manufacturer selects $w, p_o, f_o$ such that the retail channel has no demand.

The manufacturer then does not allow the retailer to have demand along the segment $BK$ because by charging a high wholesale price he forces the retailer to raise the selling price and move her demand to the segment $AK$. However, the manufacturer finds it profitable to push the retailer’s selling price to the extreme $A$ of the same segment so that the retail demand becomes equal to zero. The retail channel then is shut down and the manufacturer sells only through the online channel. Therefore, the manufacturer achieves the same profits as the ones obtained in the vertically integrated system.
This same reasoning about the profits inefficiencies caused by the decentralized retail channel apply also in the second case we have considered, where there is higher uncertainty about product fit but overall the consumers still perceive a higher expected marginal value of the product by purchasing online than at the store, i.e., $m\theta^H < 1$ but $\mu > 1$. In fact, also in this case the manufacturer opted to shut down the retail channel in a vertically integrated system and the decentralized retail channel cannot have any effect other than reinforcing the same sales channel design.

**Corollary 4.4.2.** In a decentralized retail channel, for $m\theta^H < 1$ and $\mu > 1$ the manufacturer shuts down the retail channel.

When $m\theta^H < 1$ but $\mu > 1$ the manufacturer adopts a different strategy to shut down the retailer. For a sufficiently high salvage value ($s \geq c^L \frac{\mu}{\mu}$) the manufacturer has to keep the retail price even higher than before, because he has to make those consumers potentially returning the product choose for the online channel. However, this category of consumers now has a marginal value for the online product which is dominated by the value for the retail product ($m\theta^H < 1$); therefore the manufacturer will have to charge the retailer such a high wholesale price that the resulting retail selling price will prevent those consumers from purchasing from the retail store. However, for a low salvage value ($s \leq c^L \frac{\mu}{\mu}$) the manufacturer enforces the returns policy and has to deal only with consumers never returning the product: the equilibrium is identical to the previous one as in both cases the expected marginal value of the online product is higher than the marginal value of the in-store product, i.e., $\mu > 1$.

Finally, we analyze the last case, where the uncertainty about product fit in the online channel is so high that consumers expect to obtain higher marginal value from a purchase at the store than over the Internet, i.e., $\mu < 1$. As it has been illustrated in Figure 4.4, the retail demand function now consists of three segments and two kink points. The manufacturer then will have to solve a total of five optimization problems corresponding to the retail demand varying along one of the three segments or being
equal to one of the two kink points. The procedure is completely similar to the one shown above. We provide below the intuition for the results.

When the retailer’s demand is a point of the segment $BK_1$ in Figure 4.4, the direct channel has no sales, as the retail selling price is too low for letting consumers switch to the direct channel. The retail demand is the same as in a simple manufacturer/retailer supply chain, $Q_r = 1 - p_r$, and the manufacturer’s profits bear the associated inefficiencies of the “double marginalization”. At the kink point $K_1$, even though the only active sales channel is still the retailer, something starts to change in the manufacturer’s channel design strategy. In fact, because the retail selling price has increased, consumers become more inclined towards the direct channel. To keep the sales of the whole market, the retailer has to further lower the selling price which in turn boosts the manufacturer’s demand as opposed to the previous case of double marginalization. The role of the direct channel is then to threaten the retailer in order to partially recover the profits inefficiencies due to the double marginalization. As the retail demand proceeds towards the segment $K_1K_2$ the same threat becomes more pronounced. The manufacturer has now a double advantage: reducing the retailer’s margin to his benefit and open the direct channel to a portion of the consumers potentially returning the product. Once more the manufacturer’s strategy depends on the value that can be extracted from the returned products.

**Proposition 4.4.3.** For $\mu < 1$ there is a threshold for the salvage value, $\tilde{s}$, such that

(i) for $s < \tilde{s}$, the manufacturer does not sell his product through the direct channel. However, the presence of the direct channel leads the retailer to reduce the selling price, thus decreasing the inefficiencies of the “double marginalization”.

(ii) for $s \geq \tilde{s}$, the manufacturer opens partially the direct channel to the consumers potentially returning the product.

When the salvage value for returned products is low the manufacturer does not find it convenient to open the direct channel because the retailer’s drop in the selling
price gives the manufacturer the highest level of profits. However, when the salvage value for returned products becomes sufficiently high, the manufacturer can reach even a larger market through the dual channel design, while the costs of product returns are mitigated by the high value recovery.

The inefficiencies of the double marginalization have also another effect, this time on the manufacturer’s returns policy at the online channel. In fact, it is readily checked that \( \tilde{s} < \bar{s} \), which implies that as opposed to the case of only direct channel, with a dual channel the manufacturer softens the returns policy even when his ability of product recovery is lower. The returns policy is used then by the manufacturer as an instrument to reduce even more the inefficiencies of the double marginalization by attracting some of the consumers towards the direct channel.

**Proposition 4.4.4.** For \( \mu < 1 \) the manufacturer selects a penalty \( f_o \) in a dual channel which is lower than the one selected in a direct channel only.

Finally, when the retailer’s demand lies at the kink point \( K_2 \) or along the segment \( CK_2 \) the retail selling price is so high that only the consumers never returning the product would make a purchase from the direct channel. However, the manufacturer faces the same market size as if only the retail channel is active and wants thus to exploit the higher marginal value consumers obtain from an in-store purchase. For this reason, the manufacturer adjusts the selling prices so that all sales materialize at the store.

### 4.5. Conclusions

Consumers are increasingly accustomed to both being offered a sufficiently large product variety and being provided with almost unlimited product availability. It is not uncommon that consumers may want to be able to customize their products, in order to fully satisfy each individual preference. These characteristics of the market can be
fulfilled only through a make-to-order strategy that forces the manufacturer to consider selling his products through an online channel rather than, or in addition to, a traditional retail channel. On the other hand, especially for those product categories for which a physical inspection of the product is important to understand product fit, the online channel suffers, sometimes seriously, from the flow of product returns. A consumer who makes a purchase over the Internet may receive a product that does not match his preferences as expected, and therefore the consumer may decide to return the product purchased to the point of sale. In this chapter we investigate the tradeoff a manufacturer faces between the provision of high product value to the consumer (for instance through product variety, availability and customization) and the uncertainty of the consumer about product fit in designing the channel of products sales.

Our results show that a manufacturer would sell just through an online channel when consumers expect to receive more value from that channel, thanks to limited uncertainty about product fit. When instead the uncertainty increases to a level that the product value is higher at the store rather than over the Internet, then the manufacturer might adopt different strategies depending on the salvage value that can be extracted from the returned units. More specifically, when a manufacturer has limited ability to recover the value of product return, he would open a direct channel but only as a sham: the pure presence of the direct channel represents a threat for the retailer, who, in the interest of retaining the whole market, is willing to decrease the retail selling price to the benefit of the manufacturer’s profits. However, when the manufacturer’s ability to recover the residual value of product returns is sufficiently high, the manufacturer considers the option of opening the direct channel and uses the returns policy as an instrument for attracting part of the consumers towards an online purchase. The reason why the manufacturer offers generous returns policies is that by switching part of the sales to the online channel the inefficiencies of the double marginalization at the decentralized retail channel can be partially reduced.
There are several directions in which this research can be extended. For instance, consumer returns occur typically among experienced goods, in that to solve product uncertainty about product fit consumers need to use and thus to have a certain experience with the product. With experience goods then the physical inspection of the product at the store would not be sufficient to completely prevent the consumer from returning the product. Therefore, a promising avenue of research would be to consider consumer returns at both the online and the retail channels. Notice, however, that the inclusion of returns at the retail channel might be challenging to investigate, as it would require comparing two piece-wise utility functions as opposed to a linear and piece-wise utility function considered in this chapter. Another possible extension to the current research would be to consider a different production/logistics cost for the two channels. In this case, it would be reasonable to assume that the production/logistics cost is higher at the online channel than at the retail channel, since the manufacturer would be exposed to the additional expenses of producing individually tailored products and shipping individual orders when selling through the online channel.

Finally, this research sheds light on the optimal strategies a manufacturer should follow in designing the sales channel in a market characterized by consumer returns. In particular, it points out that because of consumer returns the manufacturer may refuse to offer more value to consumers through a make-to-order manufacturing strategy and stick to a more traditional make-to-stock manufacturing strategy. Returns policies have a fundamental role in the manufacturer’s design of the sales channel and they must be designed after a careful assessment of the value that can be extracted from the returned product. Returns policies and channel structure are then highly interrelated.
Notes

1 From different sources (http://www.internetretailer.com/2011/08/31/many-happy-returns, http://www.internetretailer.com/2010/03/31/get-back http://www.linkedin.com/answers/technology/e-commerce/TCH_ECM/548315-12529698) it results that for hard goods, casual apparel, high-tech products, and fashion apparel the return rate at the retailer is 5-9%, 12-18%, 15-20%, and 35%, respectively whereas the Internet return rate for the same product categories is as high as 8-10%, 20-30%, 28-35%, and 30-50%, respectively.

2 According to Forrester Research (Mulpuru 2008), the percentage of category sales that take place online is as high as 45% for computer hardware and software; 24% for books, music, and video; and 18% for consumer electronics, but is only 11% for jewelry, 10% for apparel and footwear, 9% for home furnishings, and 9% for cosmetics and fragrances.

3 The assumption of no consumer returns at the store can be also justified as the result of a normalization of the consumer uncertainty about product fit. More specifically, given that the Internet channel exhibits higher return rates than the retail channel, we normalize to zero the uncertainty about product fit for the product purchased at the store.

4 Notice that in case of a product match with the consumer preferences the online channel provides the consumer with a higher marginal value than the retail channel.

5 Even though the logistics cost might be different across the retail and the online channels, here we want to focus on the selection of the channel by the manufacturer according to the two dimensions of marginal product value and uncertainty about product fit.

6 The effect of the returns policy on market expansion as well as on returns volume has already been investigated in the marketing literature (Shulman et al. 2010).
Bibliography


Steger, T., Sprague, B., D. Douthit. 2007. Big trouble with no trouble found: how consumer electronics firms confront the high cost of consumer returns. Internal report *Accenture Communications & High-Tech*.


Appendix A

Proofs for Chapter 2

Proof of Proposition 2.3.1. The characterization of \( r(x) \) and \( y(x) \) involves functional optimization with an isoperimetric constraint, which is examined in Kamien and Schwartz (1991, section 16) and Takayama (1991, p. 651). The Hamiltonian expression may be written as

\[
H = \Pi_M f + \lambda \Pi_R f
\]

where \( \lambda \) is an undetermined (Lagrange) multiplier. The first order necessary conditions for a solution are \( H_y = H_r = 0 \), where

\[
H_y = sf + \lambda [-p + c'(x - y)] f; \quad \text{and} \quad H_r = -f + \lambda f.
\]

The second condition implies \( \lambda = 1 \), which implies part \((iii)\) of the Proposition and, upon substitution into the first condition, yields part \((i)\).

The optimal value of \( a \) is determined by the point-wise maximization

\[
\max_a \int_{\mathbb{E}} \Pi_M f \, dx + \lambda \int_{\mathbb{E}} \Pi_R f \, dx
\]

which, since \( \lambda = 1 \), reduces to

\[
\max_a \int_{\mathbb{E}} (\Pi_M + \Pi_R) f \, dx
\]

so that the choice of manufacturer action maximizes expected total profit. The first
order condition for this maximization may be written as

\[ \int_{\xi}^{\xi} \left[ \left( \frac{d\Pi_M}{da} + \frac{d\Pi_R}{da} \right) f + (\Pi_M + \Pi_R) f_a \right] dx = 0. \]

Simplifying this expression, while noting that \( \int_{\xi}^{\xi} f_a dx = 0 \) and that \( c(x - y) \) is a constant by condition (i), yields condition (ii).

**Proof of Theorem 2.4.1.** Substituting expression (2.11) and the equation of motion (2.5), the Hamiltonian in (2.12) results in

\[ H = \left[ (1 - y(x))p - \Pi_R - c(x - y(x)) - h(a) + sy \right] f - \phi(x)c'(x - y(x)) \\
+ \mu \left[ (1 - y(x))p - \Pi_R - c(x - y(x)) - h(a) + sy \right] f_a - h'(a)f + \lambda \Pi_R f. \] (A.2)

The Pontryagin necessary conditions for a maximum require that \( \frac{d\phi}{dx} = -\frac{\partial H}{\partial \Pi_R} \) and \( \frac{\partial H}{\partial y} = 0 \), so that we obtain, respectively

\[ \frac{d\phi(x)}{dx} = (1 - \lambda)f + \mu f_a, \text{ and} \quad (A.3) \]
\[ [-p + c'(x - y(x)) + s] (f + \mu f_a) + \phi(x)c''(x - y(x)) = 0. \] (A.4)

The first order condition associated with the optimal choice of \( w \) is

\[ \int_{\xi}^{\xi} \left[ \frac{\partial \Pi_M}{\partial w}(f + \mu f_a) + \lambda \frac{\partial \Pi_R}{\partial w} f \right] dx = 0, \] (A.5)

which, after simplification and noting that \( \int_{\xi}^{\xi} f_a dx = 0 \), yields \( \lambda = 1 \). As a consequence of the retailer’s binding participation constraint (2.8), the Pontryagin condition (A.3) reduces to \( \frac{d\phi(x)}{dx} = \mu f_a \). Integrating this last expression, in conjunction with the transversality condition, \( \phi(x) = 0 \), implies that \( \phi(x) = \mu F_a \). Finally, the first order
condition associated with the optimal choice of \( a \) is given by

\[
\int_{\bar{x}}^{\bar{x}} \left[ \frac{\partial \Pi_M}{\partial a} f + (\Pi_M + \lambda \Pi_R) f_a + \mu \left( \frac{\partial \Pi_M}{\partial a} - h' \right) f_a + \Pi_M f_{aa} - h'' f \right] dx = 0, \quad (A.6)
\]

which after some computation yields condition \((iv)\).

Finally, applying again \( \int_{\bar{x}}^{\bar{x}} f_a dx = 0 \), condition \((v)\) is the result of constraint \((2.10)\), whereas condition \((vi)\) is the equation of motion given by \((2.5)\).

**Proof of Proposition 2.5.1.** From conditions \((i)\) and \((ii)\) of Theorem 2.4.1, the function \( y(x) \) is defined by the condition \( c'(x - y) - p + s = 0 \), and an application of the implicit function rule yields \( y'(x) = 1 \). Total differentiation of \((2.1)\) with respect to \( x \) yields

\[
\frac{d \Pi_R(x)}{dx} = -py' + r'(y)y' - c'(x - y)(1 - y'). \quad (A.7)
\]

Substituting this result into condition \((vi)\) of Theorem 2.4.1, and recalling that \( y'(x) = 1 \), we obtain

\[
-p + r'(y) - c'(x - y) = 0
\]

which implies that \( r'(y) = s \). Normalizing the returns policy so that the retailer receives a payment of zero when there are no returns to the manufacturer yields the desired result. \( \square \)

**Proof of Proposition 2.5.2.** Since \( \phi(x) = 0 \), from part \((i)\) of Theorem 2.4.1 we know that \( y(x) \) is first-best. Since the manufacturer’s choice of \( a \) satisfies \((2.10)\), upon substitution of \( \Pi_M \) from \((2.10)\) we obtain

\[
\int_{\bar{x}}^{\bar{x}} \left[ (r(y) - sy) f_a + h' f \right] dx = 0. \quad (A.9)
\]

Setting \( r(y) = py \) reduces equation \((A.9)\) to condition \((ii)\) of Proposition 2.3.1, thus yielding a first-best choice of \( a \). \( \square \)
Appendix B

Proofs for Chapter 3

Proof of Lemma 3.4.1. From (3.5) we solve $\Pi_M^L - \Pi_M^H$ for $\gamma^L$ and obtain $\gamma^L < \tilde{\gamma} = 1 - \frac{v^2}{(1 - \gamma^H)(\phi - v)^2}$. As $\gamma^L \geq 0$ it follows $\tilde{\gamma} > 0$, which implies $(1 - \gamma^H)(\phi - v)^2 - v^2 > 0$. Solving the last inequality for $\gamma^H$ gives the result of Lemma 3.4.1, $\gamma^H < \bar{\gamma} = \frac{\phi(\phi - 2v)}{(\phi - v)^2}$.

Proof of Lemma 3.4.2. In a screening equilibrium, the retailer reacts to a given wholesale price, $w$, as in the case of full information, and thus he selects $p^*(w) = \frac{\phi}{2} + \frac{w - v}{2(1 - \gamma^H)}$. The manufacturer’s maximization problem can be expressed as

$$\max_{\{w_c^H, w_c^L\}} \Pi_{Mc} = \frac{\beta}{2(1 - \gamma^L)} [\phi(1 - \gamma^L) + v\gamma^L - w_c^L] w_c^L + \frac{1 - \beta}{2(1 - \gamma^H)} [\phi(1 - \gamma^H) + v\gamma^H - w_c^H] w_c^H$$

s.t. 

$$[\phi(1 - \gamma^H) + v\gamma^H - w_c^H]^2 \geq [\phi(1 - \gamma^H) + v\gamma^H - w_c^H]^2$$ (B.1)

$$[\phi(1 - \gamma^L) + v\gamma^L - w_c^L]^2 \geq [\phi(1 - \gamma^L) + v\gamma^L - w_c^L]^2$$ (B.2)

$$[\phi(1 - \gamma^H) + v\gamma^H - w_c^H]^2 \geq 0$$ (B.3)

$$[\phi(1 - \gamma^L) + v\gamma^L - w_c^L]^2 \geq 0.$$ (B.4)

The two terms $[\phi(1 - \gamma^H) + v\gamma^H - w_c^H]$ and $[\phi(1 - \gamma^L) + v\gamma^L - w_c^L]$ represent the demand of a high-type and a low-type manufacturer, respectively, and, as a consequence, they must be $\geq 0$. As $\gamma^L < \gamma^H$ and $v < \phi$, from $\phi(1 - \gamma^H) + v\gamma^H - w_c^H \geq 0$ it follows $\phi(1 - \gamma^L) + v\gamma^L - w_c^L \geq 0$, so that constraint (B.2) reduces to $w_c^L \leq w_c^H$. From $\phi(1 - \gamma^H) + v\gamma^H - w_c^H \geq 0$ and $w_c^L \leq w_c^H$ it follows $\phi(1 - \gamma^H) + v\gamma^H - w_c^L \geq 0$. Then constraint (B.1) reduces to $w_c^H \leq w_c^L$. Since the two participation constraints are always satisfied, the manufacturer’s maximization problem is equivalent to the unconstrained
problem

\[ \max_{w_c} \Pi_{Mc} = \frac{\beta}{2(1 - \gamma^L)} \left[ \phi(1 - \gamma^L) + v\gamma^L - w_c \right] w_c + \frac{1 - \beta}{2(1 - \gamma^H)} \left[ \phi(1 - \gamma^H) + v\gamma^H - w_c \right] w_c, \]

which yields \( w_c^* = \frac{\phi(1 - \gamma^H) + \gamma^H(1 - \gamma^L) - \beta v(\gamma^H - \gamma^L)}{2(1 - \gamma^H)(1 - \gamma^L)} > 0. \) Accordingly, the manufacturer’s profits in a screening equilibrium are given by \( \Pi_{Mc}^* = \left\{ \frac{\phi(1 - \gamma^H) + \gamma^H(1 - \gamma^L) - \beta v(\gamma^H - \gamma^L)}{8(1 - \gamma^H)(1 - \gamma^L)(1 - \beta)\gamma^L} \right\}^2. \)

\[ \square \]

**Proof of Proposition 3.4.3.** Computing \( \frac{\partial^2 \Pi_{Mc}^*}{\partial \beta^2} = \frac{(\gamma^H - \gamma^L)^2(\phi - \gamma^L)(1 - \gamma^L)}{4(1 - \gamma^H)(1 - \gamma^L)^2} > 0, \) it follows that \( \Pi_{Mc}^* \) is convex in \( \beta. \) Also, we evaluate that the sign of the term \( [\phi(1 - \gamma^H) + v\gamma^H - 2v] \) in \( \left. \frac{\partial \Pi_{Mc}^*}{\partial \beta} \right|_{\beta=0} = \frac{[\phi(1 - \gamma^H) + v\gamma^H - 2v] \phi(1 - \gamma^H) + v\gamma^H}{8(1 - \gamma^H)(1 - \gamma^L)} \) is \( \geq 0 \) as long as \( \gamma^H \leq \gamma. \) Since \( \left. \frac{\partial \Pi_{Mc}^*}{\partial \beta} \right|_{\beta=1} = \frac{[\phi(1 - \gamma^L) + v\gamma^L - 2\gamma] \phi(1 - \gamma^L) + v\gamma^L}{8(1 - \gamma^L)} > 0 \) (notice that \( [\phi(1 - \gamma^L) + v\gamma^L - 2\gamma] > 0 \) follows from \( \Pi_{Mc}^L - \Pi_{Mc}^H > 0 \)), it follows that \( \Pi_{Mc}^* \) is strictly increasing for \( \gamma^H \leq \gamma \) and non-monotonic for \( \gamma < \gamma^H < \overline{\gamma}, \gamma^L < \gamma. \)

Finally, we solve the inequality \( \Pi_{Mc}^* - \Pi_{Mc}^H > 0 \) for \( \beta \) and we obtain \( \beta > \beta_R = -\frac{[\phi(1 - \gamma^L) + v\gamma^L - 2\gamma] \phi(1 - \gamma^L) + v\gamma^L}{(\gamma^H - \gamma^L)^2}, \) where \( \beta_R \in (0, 1) \) for \( \gamma < \gamma^H < \overline{\gamma}. \)

\[ \square \]

**Proof of corollary 3.4.4.** The manufacturer’s profits in the screening equilibrium are given by \( \Pi_{Mc}^* = \left\{ \frac{\phi(1 - \gamma^H) + \gamma^H(1 - \gamma^L) - \beta(\gamma^H - \gamma^L)}{8(1 - \gamma^H)(1 - \gamma^L)(1 - \beta)\gamma^L} \right\}^2, \) and they are dependent on the investment \( i \) through the probability function \( \beta(i). \) The manufacturer decides on the optimal investment by solving

\[ \max_{i \geq 0} \Pi_{Mc}^* - i. \] (B.5)

\[ 1 \text{Alternatively, to solve the manufacturer’s maximization problem one may observe that the feasible region identified by inequalities (B.1) - (B.4) is not convex but it is included in the region } (w_{c1}^L, w_{c2}^L) \ni w_{c1}^L \geq w_{c2}^L. \text{ It is straightforward then to show that the constraint } w_{c1}^H \geq w_{c2}^L \text{ is binding at optimum.} \]
For $i = 0$ it follows $\Pi_{Mc}^* - i = \Pi_M^H$. We observe that $\lim_{i \to 0^+} \frac{d(\Pi_{Mc}^* - i)}{di} = \left(\frac{\partial \Pi_{Mc}^*}{\partial \beta} \mid \beta = 0\right) \lim_{i \to 0^+} \beta'(i) - 1 = +\infty$, which, in conjunction with the continuity of $\frac{d(\Pi_{Mc}^* - i)}{di}$, implies $\Pi_{Mc}^* - i = -\infty$. As $\Pi_{Mc}^* - i$ is continuous in $i$ and the $\lim_{i \to +\infty} \Pi_{Mc}^* - i = -\infty$, by the mean value theorem, $\exists$ a compact interval, $[i_1, i_2]$, in which $\Pi_{Mc}^* - i > \Pi_M^H$ at least one $i > a \ni \Pi_{Mc}^* - i = \Pi_M^H$. Define $\tilde{i} = \max_{i > 0} \{\Pi_{Mc}^* - i = \Pi_M^H\}$ and consider the compact interval $[a, \tilde{i}]$. The Theorem of Bolzano-Weierstrass applies to the continuous function $\Pi_{Mc}^* - i$, in the compact interval $[i_1, i_2]$ so that the manufacturer’s problem in (B.5) attains at least an optimal solution in $[i_1, i_2]$. \hfill $\Box$

**Proof of Proposition 3.5.1.** In a screening equilibrium, the manufacturer has to solve the following optimization problem

$$
\max_{\{(w^H_c, b^H_c), (w^L_c, b^L_c)\}} \Pi_{Mc} = \beta \left[ \phi - p^L(w^L_c, b^L_c) \right] \left[ w^L_c - (b^L_c - v) \gamma^H \right] + (1 - \beta) \left[ \phi - p^H(w^H_c, b^H_c) \right] \left[ w^H_c - (b^H_c - v) \gamma^L \right]
\text{s.t.} \quad \Pi_H^R(p^H(w^H_c, b^H_c)) \geq \Pi_H^R(p^H(w^L_c, b^L_c)) \quad \Pi_L^R(p^L(w^L_c, b^L_c)) \geq \Pi_L^R(p^L(w^H_c, b^H_c)) \quad b^L_c \leq w^L_c \quad b^H_c \leq w^H_c,
$$

where the two participation constraints of the retailer are excluded because they are redundant, $\Pi^R(p)$ is given by (3.12), and $p^j(w, b)$ is the same as under full information, i.e., $p^j(w, b) = \frac{\phi}{2} + \frac{w - b^j}{2(1 - \gamma^j)}$. The two incentive-compatible constraints (first two inequalities) of problem (B.6) are given by

$$
\left[ \phi(1 - \gamma^H) - w^H_c + b^H_c \gamma^H \right] \geq \left[ \phi(1 - \gamma^L) - w^L_c + b^L_c \gamma^H \right] \quad \left[ \phi(1 - \gamma^L) - w^L_c + b^L_c \gamma^L \right] \geq \left[ \phi(1 - \gamma^L) - w^H_c + b^H_c \gamma^L \right],
$$

(B.7) (B.8)
The two terms, \( \phi(1 - \gamma^H) - w_c^H + b_c^H \gamma^H \) and \( \phi(1 - \gamma^L) - w_c^L + b_c^L \gamma^L \) are proportional (by a factor \( \frac{1}{2} \)) to the demand of a low-type and a high-type manufacturer, respectively; hence, they must be \( \geq 0 \). Since \( \phi(1 - \gamma^H) < \phi(1 - \gamma^L) \) it follows \( \phi(1 - \gamma^L) - w_c^H + b_c^H \gamma^L \geq 0 \) so that the two terms squared in constraint (B.8) are both \( \geq 0 \) and constraint (B.8) reduces to \( [w_c^H - w_c^L + \gamma^L(b_c^L - b_c^H)] \geq 0 \). We find the optimal solution of problem (B.6) by omitting constraint (B.7) but we will check \textit{ex post} that the omitted constraint is satisfied by the optimal solution. Consider the Lagrangian function \( \mathcal{L} = \Pi_{Mc} + \lambda_1 [w_c^H - w_c^L + \gamma^L(b_c^L - b_c^H)] - \lambda_2 (b_c^H - w_c^H) - \lambda_3 (b_c^L - w_c^L) \). The first order conditions are given by

\[
\frac{\partial \mathcal{L}}{\partial w_c^L} = 0 \Rightarrow (1 - \beta) \left[ \phi(1 - \gamma^H) - 2w_c^L + 2\gamma^L b_c^L - \gamma^H \right] + \lambda_1 + \lambda_2 = 0
\]
\[
\frac{\partial \mathcal{L}}{\partial w_c^H} = 0 \Rightarrow \frac{\beta(1 - \gamma^L) - 2w_c^H + 2\gamma^L b_c^H - \gamma^L}{2(1 - \gamma^L)} - \lambda_1 + \lambda_3 = 0
\]
\[
\frac{\partial \mathcal{L}}{\partial b_c^H} = 0 \Rightarrow -\frac{(1 - \beta) \gamma^H \left[ \phi(1 - \gamma^H) - 2w_c^H + 2\gamma^H b_c^H - \gamma^H \right] - \lambda_1 \gamma^L - \lambda_2 = 0}
\]
\[
\frac{\partial \mathcal{L}}{\partial b_c^L} = 0 \Rightarrow -\frac{\beta \gamma^L \left[ \phi(1 - \gamma^L) - 2w_c^L + 2\gamma^L b_c^L - \gamma^L \right] + \lambda_1 \gamma^L - \lambda_3 = 0}
\]

The solution is given by the values of the multipliers \( \lambda_1 > 0, \lambda_2 > 0, \) and \( \lambda_3 = 0 \). The optimal solution is given then by \( w_c^*H = b_c^*H = \frac{\phi}{2} - \frac{\mu}{2} \frac{(1 - \beta) \gamma^H + \beta \gamma^L}{1 - (1 - \beta) \gamma^H - \beta \gamma^L} \), and \( w_c^*L = w_c^H (1 - \gamma^L) + b_c^*L \gamma^L \), \( \forall b_c^*L \leq w_c^*H \). Substituting this solution into constraint (B.7) we obtain \( w_c^*H (2 - \gamma^L - \gamma^H) - b_c^*L (\gamma^H - \gamma^L) \leq 2\phi(1 - \gamma^H) \) which is always strictly satisfied by \( b_c^*L = w_c^*H \). By continuity of the last inequality in \( b_c^*L \), the manufacturer can always offer an optimal contract for the low-type manufacturer such that \( b_c^*L < w_c^*H \), which implies \( b_c^*L < w_c^L < w_c^*L = b_c^*H \), so that the manufacturer will be able to screen the retailer’s private information about his type. Denoting with \( \mu \) the expected return rate, \( \mu = \beta \gamma^L + (1 - \beta) \gamma^H \), the optimal manufacturer’s profits are given by \( \Pi_{Mc}^* = \frac{[\phi(1 - \mu) + \gamma^H]^2}{8(1 - \mu)} \).

**Proof of lemma 3.5.2.** The proof about the monotonicity of the manufacturer’s
expected profits in $\gamma^j$ is similar to the proof of Proposition 3.4.3. Here we show that $\beta_M < \beta_R$. Given the manufacturer’s expected profits when the manufacturer is responsible for salvaging returns, $\Pi^*_M = \frac{[\phi(1-\mu)+v_M]^2}{8(1-\mu)}$, $\beta_M$ solves $\Pi^*_M = \Pi^*_M$ and is given by $\beta_M = -\frac{[\phi(1-\gamma^H)+v_M^R]2}{(\phi-v)^2(1-\gamma^H)(\gamma^H-\gamma^L)}$. The expression of $\beta_R$ is provided by the proof of Proposition 3.4.3, and allows us to evaluate $\beta_M - \beta_R = A[\phi(1-\gamma^H)+v_M^R]2\gamma^H\phi(1-\gamma^H)+v_M^R-2v]2(1-\gamma^H)(\gamma^H-\gamma^L)^2$, where $A = (1-\gamma^H)(1-\gamma^L)\phi^2-2v(1-\gamma^H)(1-\gamma^L)\phi+\phi^2(\gamma^H\gamma^L-\gamma^H-\gamma^L)$. From $\Pi^*_M < \Pi^*_{R}$ it follows $A > 0$; thus for $\gamma < \gamma^H < \gamma$ it follows $\beta_M - \beta_R < 0$. □

**Proof of Proposition 3.6.1.** We remind that the salvage value of a returned unit is denoted by $v_M$ and $v_R$, depending on whether the manufacturer or the retailer salvages returns, respectively. In the second stage of the game the manufacturer proposes a contract by selecting endogenously the reverse channel for returns. Under full information the manufacturer’s type is known; hence the $j$–type manufacturer’s profits correspond to $\Pi^*_{M,j} = \frac{[\phi(1-\gamma^j)+v_M^j\gamma^j]}{8(1-\gamma^j)}$ if the manufacturer salvages returns and to $\Pi^*_{M,R,j} = \frac{[\phi(1-\gamma^j)+v_R^j\gamma^j]}{8(1-\gamma^j)}$ if the retailer salvages returns, where we have used an additional super-script $M$ or $R$ to discriminate between the two reverse channels. Given that for $v_M = v_R$ it follows $\Pi^*_{M,j} = \Pi^*_{M,R,j}$ and that $\Pi^*_{M,j}$ and $\Pi^*_{M,R,j}$ are increasing in the salvage value, it is clear that under full information the selection of the reverse channel is efficient.

Consider now a scenario where the retailer is privately informed and the manufacturer is uncertain about the product return rate. The manufacturer’s expected profits are given by $\Pi^*_M = \frac{[\phi(1-\gamma^H)+v^R^M\gamma^R]}{8(1-\gamma^H)}(1-\gamma^L-\beta v_R^M\gamma^L)^2$, when the retailer salvages returns and by $\Pi^*_M = \frac{[\phi(1-\gamma^H)+v^M\gamma^M]}{8(1-\mu)}$, when the manufacturer salvages returns (we remind that $\mu = \gamma^L + (1-\gamma)^H$). For $v_M = v_R = v$ and $\gamma^H \leq \gamma$ we verify that $\Pi^*_M > \Pi^*_{R}$. In fact, solving $\Pi^*_M = \Pi^*_{R}$ for $\beta$ we obtain $\beta_1 = 0$, $\beta_2 = 1$, and $\beta_3 = -\frac{[\phi(1-\gamma^H)+v^M\gamma^M]}{(\gamma^H-\gamma^L)(\gamma^H-\gamma^L)}$, where $\beta_3 < 0$ for $\gamma^H > \gamma$. Then, either $\Pi^*_M > \Pi^*_{R}$, $\forall \beta \in (0,1)$ or vice-versa, $\Pi^*_M < \Pi^*_{R}$, $\forall \beta \in (0,1)$. Con-
considering the differentials $\left. \frac{d\Pi^M_{M\tilde{c}}}{d\beta} \right|_{\beta=0} = \frac{B}{8(1-\gamma_H)^2}$, and $\left. \frac{d\Pi^R_{M\tilde{c}}}{d\beta} \right|_{\beta=0} = \frac{B}{8(1-\gamma_H)(1-\gamma_L)}$, where $B = \left[ \phi(1-\gamma_H) + v\gamma_H \right] \left[ \phi(1-\gamma_H) + v\gamma_H - 2v \right] (\gamma_H - \gamma_L) > 0$, it follows $\left. \frac{d\Pi^M_{M\tilde{c}}}{d\beta} \right|_{\beta=0} > \left. \frac{d\Pi^R_{M\tilde{c}}}{d\beta} \right|_{\beta=0}$. Therefore, for $v_M = v_R = v$ and $\gamma_H \leq \gamma$ it is verified that $\Pi^M_{M\tilde{c}} > \Pi^R_{M\tilde{c}}, \forall \beta \in (0,1)$. For $v_M > v_R$, as $\Pi^M_{M\tilde{c}}$ increases in $v_M$ and $\Pi^R_{M\tilde{c}}$ does not depend on $v_M$, it follows again $\Pi^M_{M\tilde{c}} > \Pi^R_{M\tilde{c}}, \forall \beta \in [0,1]$. Therefore, the reverse channel is efficient for $v_M > v_R$. However, for $v_M < v_R$, as $\Pi^M_{M\tilde{c}}$ increases in $v_M$, if $v_M$ is not too much lower than $v_R$, $\exists$ a set $\Gamma \subset (0,1) \ni \forall \beta \in \Gamma$, $\Pi^M_{M\tilde{c}} < \Pi^R_{M\tilde{c}}$. If the optimal investment $i^*$ generates a belief $\beta(i^*) \in \Gamma$, then the reverse channel is inefficient. A similar proof would apply to the case $\bar{\gamma} < \gamma_H < \bar{\gamma}, \gamma_L < \bar{\gamma}$. □

**Proof of Proposition 3.7.1.** As the high-type manufacturer benefits from mimicking the low-type manufacturer but not vice-versa, the low-type manufacturer needs to separate by solving the following optimization problem

$$
\max_{w_s^L} \left( \phi - p^L(w_s^L) \right) w_s^L \\
\text{s.t.} \quad \left( \phi - p^L(w_s^L) \right) w_s^L \leq \Pi^H_{M} \\
\Pi^L_{R}(p^L(w_s^L)) \geq 0,
$$

where $p^L(w) = \frac{\phi}{2} + \frac{w-v\gamma_L}{2(1-\gamma_L)}$, $\Pi^H_{M}$ is given by (3.5) for $j = H$, and $\Pi^L_{R}(p)$ by (3.1) for $j = L$. Constraint (B.10) prevents the high-type manufacturer from mimicking a low-type manufacturer and constraint (B.11) is the retailer’s participation constraint. Notice that (B.10) is binding because the high-type manufacturer can achieve higher profits by mimicking a low-type manufacturer, whereas (B.11) is always satisfied because $\Pi^L_{R}(p^L(w_s^L))$ is a square. Therefore, the optimal wholesale price solves the equation associated with constraint (B.10), $w_{s}^L = w_{s}^L + \sqrt{(1-\gamma_H)(\gamma_H-\gamma_L)(\phi-v)^2(1-\gamma_H)(1-\gamma_L)-v^2}$. Therefore, $w_{s}^L$ is the required separating equilibrium, according to the retailer’s pos-
terior belief, \( \hat{\alpha} = \begin{cases} 
1 & \text{if } w \geq w^*_s \\
0 & \text{otherwise} 
\end{cases} \). □

**Proof of Proposition 3.7.2.** In a pooling equilibrium, after observing the manufacturer’s contract, the retailer maintains her priors and sets the selling price by solving

\[
\max_{p_p} \Pi_{R_p}(p) = (\phi - p_p) [(p_p - w) - \mu(p_p - v)]
\]  

(B.12)

Given the unique optimal solution to problem (B.12), \( p^*_p(w) = \phi \frac{1}{2} + \frac{w - \mu}{2(1 - \mu)} \), either type of manufacturer solves

\[
\max_{w_p} (\phi - p^*_p(w_p)) \#_p \\
s.t. \quad \Pi_{R_p}(p^*_p(w_p)) \geq 0.
\]

(B.13)

Noting that the participation constraint is always satisfied (since \( \Pi_{R_p}(p^*_p(w_p)) \) is a square), the solution to (B.13) is given by \( w^*_p = \phi \frac{1}{2} \frac{1}{1 - \mu} + \frac{\mu}{2(1 - \mu)} \).

Given the retailer’s posterior belief, \( \hat{\alpha} = \begin{cases} 
\alpha & \text{if } w \geq w^*_p \\
0 & \text{otherwise} 
\end{cases} \), \( w^*_p \) is the required pooling equilibrium. The corresponding selling price and the manufacturer’s profits are given by \( p^*_p = \frac{3\phi}{4} - \frac{\mu v}{4(1 - \mu)} \) and \( \Pi^*_M = \frac{[\phi(1 - \mu) + \mu v]^2}{8(1 - \mu)} \), respectively. Substituting the expression of \( \mu \) we evaluate

\[
\frac{\partial^2 \Pi^*_M}{\partial \alpha^2} = \frac{[\phi(1 - \mu) + \mu v]^2}{4(1 - \alpha \gamma - (1 - \alpha) \gamma_H \gamma_L)^3} > 0, \quad \frac{\partial \Pi^*_M}{\partial \alpha} \bigg|_{\alpha = 0} = \frac{[\phi(1 - \gamma_H) + \mu v](\phi(1 - \gamma_H) + \mu v \gamma_H - 2v)(\gamma_H - \gamma_L)}{8(1 - \gamma_H)^2},
\]

which is \( \geq 0 \) for \( \gamma^H \leq \gamma_L \), and

\[
\frac{\partial \Pi^*_M}{\partial \alpha} \bigg|_{\alpha = 1} = \frac{[\phi(1 - \gamma_H) + \mu v \gamma_L](\phi(1 - \gamma_L) + \mu v \gamma_L - 2v)(\gamma_H - \gamma_L)}{8(1 - \gamma_L)^2} > 0.
\]

Then it is proved that \( \Pi^*_M \) is strictly convex in \( \alpha \), strictly increasing in \( \alpha \) for \( \gamma^H < \gamma_L \), and non-monotonic in \( \alpha \) for \( \gamma_L < \gamma < \gamma_H \). □

**Proof of Proposition 3.7.3.** The high-type manufacturer benefits from mimicking a low-type but not vice-versa. Thus, the low-type manufacturer has to separate from
the high-type by solving

\[
\max_{w^L_s, b^L_s} \left[ \phi - p^L_s(w^L_s, b^L_s) \right] \left[ w^L_s - \gamma^L L b^L_s - v \right] \tag{B.14}
\]

s.t. \[
\max_{w^L_s, b^L_s} \left[ \phi - p^L_s(w^L_s, b^L_s) \right] \left[ w^L_s - \gamma^H L b^L_s - v \right] \leq \Pi^M_s \tag{B.15}
\]

\[
\Pi^f_{R}(p^L_s(w^L_s, b^L_s)) \geq 0 \tag{B.16}
\]

\[
b^L_s \leq w^L_s, \tag{B.17}
\]

where constraint (B.15) guarantees that a high-type manufacturer does not benefit form mimicking a low-type. The solution under full information, \( w^L_s = \phi \left( 1 - \gamma^L \right) + \gamma^L \left( 2b^L_s - v \right) \), satisfies constraint (B.15) for \( b^L_s \geq b^L_s = \left( 1 - \gamma^H \right) \left( 1 - \gamma^L \right) \phi^2 - \left( \gamma^H - \gamma^L \left( 1 - \gamma^H \right) \right) v^2 \). The separating equilibrium is then given by \( (w^L_s, b^L_s) \) satisfying menu (3.14), where \( b^L_s \leq b^L_s \leq \frac{\phi}{2} - \frac{v^L_s}{2(1-\gamma^L)} \) and the retailer holding posterior belief as specified in Proposition 3.7.3. As a consequence, the menu of contracts in a separating equilibrium is a subset of the menu of contracts in a full information equilibrium. □

**Proof of Lemma 3.7.4.** After observing the manufacturer’s contract \((w, b)\), the retailer maintains her priors and selects the selling price by solving

\[
\max_{p} \Pi_{R}(p) = (\phi - p) [(p - w) - \mu(p - b)] \tag{B.18}
\]

Given the unique optimal solution to problem (B.18), \( p^*_p(w, b) = \frac{\phi}{2} + \frac{w - bp}{v^L_s} \), the high-type manufacturer solves

\[
\max_{w^H_p, b^H_p} \Pi^M_p(p^*_p(w^H_p, b^H_p)) = \left[ \phi - p^*_p(w^H_p, b^H_p) \right] \left[ w^H_p - \gamma^H (b^H_p - v) \right] \tag{B.19}
\]

s.t. \[
\Pi_{R}(p^*_p(w^H_p, b^H_p)) \geq 0 \]

\[
b^H_p \leq w^H_p
\]

\[
b^H_p, w^H_p \geq 0
\]
Noting that $\Pi_{Rp}(p^*_p(w^H_p, b^H_p))$ is a square (and so the retailer’s participation constraint is redundant), the Lagrangian function of (B.19) is given by $\mathcal{L} = \Pi_{Mp}^H(p^*_p(w^H_p, b^H_p)) - \lambda_1(b^H_p - w^H_p) + \lambda_2 b^H_p + \lambda_3 w^H_p$. The first order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial w^H_p} = 0 \Rightarrow \frac{\phi(1 - \mu) + b^H_p \mu + (b^H_p - v) \gamma_H - 2w^H_p}{2(1 - \mu)} + \lambda_1 + \lambda_3 = 0$$

$$\frac{\partial \mathcal{L}}{\partial b^H_p} = 0 \Rightarrow \frac{\gamma_H [-\phi(1 - \mu) + w^H_p \mu + \mu v - 2b^H_p] + \mu w^H_p}{2(1 - \mu)} - \lambda_1 + \lambda_2 = 0.$$
identified as a high-type. In fact, $\Pi^H_{M_p}(p^*(w^L_p, b^L_p)) - \Pi^H_M = -\frac{[v(\gamma^H_p - \gamma^L_p)]^2}{8(1-\gamma^L_p)^2(1-\gamma^H_p)} \leq 0$. A pooling equilibrium then cannot be sustained. \qed
Appendix C

Proofs for Chapter 4

Proof of Proposition 4.3.1. Consider the first row of Table 4.1. We must solve
the manufacturer’s problem as in (4.3) where the expressions of $Q_o$, $Q_r$, $R_o$ and the
constraints are the ones reported in Table 4.1. The Lagrangian for the manufacturer’s
maximization problem is equal to $L = \Pi_M - \lambda_1(p_r - A) - \lambda_2(v_z - v_x)$. One solution
is given for $s \geq c_{\frac{\theta L}{\mu}}$ by $\lambda_1 > 0$ which corresponds to having the first constraint binding,
and having a zero demand at the retail channel. Another solution is given for
$s \leq c_{\frac{\theta L}{\mu}}$ which corresponds to having both constraints binding, hence having again
zero demand at the retail channel. The same procedure could be repeated for the
first 8 rows of Table 4.1 and it could be verified that in all the cases where $\mu > 1$ holds, the manufacturer never decides to activate the retail channel. □

Proof of Corollary 4.3.2. When $m\theta_H > 1$ consider again the Lagrangian $L = \Pi_M - \lambda_1(p_r - A) - \lambda_2(v_z - v_x)$. The solution corresponds to $p_r = A$ for $s \geq c_{\frac{\theta L}{\mu}}$, whereas
it corresponds to $p_r = A$ and $v_z = v_x$ for $s \leq c_{\frac{\theta L}{\mu}}$. When $m\theta_H < 1$ but $\mu > 1$, the
solution is given by $v_z = v_x$ for $s \geq c_{\frac{\theta L}{\mu}}$ and by $p_r = A$, $v_z = v_x$, and $f_o = \frac{\mu - \theta L}{\mu} p_o$ for
$s \leq c_{\frac{\theta L}{\mu}}$. □

Proof of Proposition 4.3.3. When $\mu < 1$ the problem to solve follows:

\[
\begin{align*}
\max_{p_o,f_o,w} \quad & \Pi_M = Q_o(p_o - c) + Q_r(p_r - c) + R_o(f_o + s - p_o) \\
\text{s.t.} \quad & p_r \geq A \\
& v_x \geq v_z \\
& \frac{\mu - \theta L}{\mu} p_o \geq f_o.
\end{align*}
\] (C.1)
and the associated Lagrangian is given by $L = \Pi_M - \lambda_1(A - p_r) - \lambda_2(v_z - v_x) - \lambda_3(f_o - \frac{\mu - \theta}{\mu} p_o)$. A first solution is given when $s \geq \frac{c(1-m\theta_H)}{1-m}$ and the second constraint is binding, that is $v_z = v_x$. The demand at the retailer however, $Q_r = \frac{1-m\theta_H - s(1-m)}{2(1-m\theta_H)}$ becomes zero for $s = \frac{1-m\theta_H}{1-m}$. Another solution is given when both the first and second constraints are binding, which implies that the demand at the online channel, $Q_o = 0$. 

Proof of Proposition 4.4.1. The proof consists of solving problems (4.5), (4.6), and (4.7). The Lagrangian associated to problem (4.5) is given by $L = (1 - v_x^B)(p_o - c) + (v_x^B - p^B_r)(w - c) - \lambda_1(p^B_r - A) - \lambda_2(v_x^B - v_x^B) - \lambda_3(w - p_o)$. The solution is obtained for $\lambda_2 > 0$ and yields $w^* = \frac{1}{2} \left(\frac{1}{2}\right), p^*_o = \frac{\mu + c}{2}, f^*_o = \frac{(\mu - \theta) + (\mu + c)}{2\mu},$ and $p^*_r = w$. The corresponding manufacturer’s profits are $\Pi^*_M = \frac{(\mu - c)^2}{4\mu}$. Problem (4.6) has solution completely equal to the vertically integrated case and the profits are given by $\Pi^*_{AK} = \Pi^*_M \text{ for } s \leq \frac{\theta_L}{\mu}$, and by $\Pi^*_{AK} = \frac{(1-m)[2\mu^2 - 2c\theta_L s + m\theta_H \theta^2_L] + (c-m\theta_H)^2 \theta_L}{4m\theta_H \theta^2_L} \times \Pi^*_M \text{ for } s \leq \frac{\theta_L}{\mu}$. Problem (4.7) has Lagrangian $L = (1 - T^L)(p_o - c) + (T^L - p^*_r)(w - c) - \lambda_1(p^*_r - A) - \lambda_2(v^*_z - v^*_x) - \lambda_3(w - p_o)$, which is solved for $\lambda_1 > 0, \lambda_2 > 0, \lambda_3 > 0$, yielding $w^* = \frac{\mu + c}{2}, p^*_o = \frac{\mu + c}{2}, f^*_o = \frac{\mu - \theta}{\mu} p^*_o,$ and again $\Pi^*_{M} = \Pi^*_{MK}$. The solution is equivalent to the one obtained for the vertically integrated system.

Proof of Corollary 4.4.2. The proof is completely similar to the one shown for Proposition 4.4.1 and is therefore omitted. 

Proof of Proposition 4.4.3. Part (i) of Proposition 4.4.3 is demonstrated by solving the manufacturer’s optimization problem when the retailer’s demand is at the kink point $K_1$ (see Figure 4.4). When the retail demand belongs to the segment $B K_1$, the retailer solves $\max_{p_r}(1 - p_r)(p_r - w)$, which yields $p^*_r = \frac{1 + w}{2}$. When the retail
demand belongs to segment $K_1K_2$, the retailer solves $\max_{p_r} (1-v_z)(p_r-w)$, which yields $p^*_{K_1K_2} = \frac{w+1-m\theta_H+m\rho_0+f_0(1-m)}{2}$. When the retail demand function is at the kink point $K_1$ then $p_r = A$ and the demands and returns are given by: $Q_r = 1-A$, $Q_o = R_o = 0$. The manufacturer’s optimization problem then can be written as

$$\max_{p_o,f_o,w} (1-A)(w-c)$$

s.t. $p_o \geq w$

$$p_{K_1K_2} \geq A$$

$$p_{K_1K_2} \leq A$$

$$\frac{\mu-\theta_L}{\mu}p_o \geq f_o.$$  \hspace{1cm} (C.2)

The solution requires the third constraint binding and yields: $w^* = \frac{1+c}{2}$, $f_o^* = \frac{v_z}{m}$, $p_o^* = -\frac{1-m}{m}f_o^* + \frac{\theta_H(3+c-2m\theta_H)}{2(1-m\theta_H)}$, which implies $p^*_{K_1} = A = \frac{3+c-2m\theta_H}{4-2m\theta_H} < \frac{3+c}{4}$. The retailer then charges a selling price which is lower than the one of double marginalization. The manufacturer’s profits are also higher than in the double marginalization case and equal to: $\Pi^K_M = \frac{(1-c)^2}{4(2-m\theta_H)}$.

When the retailer’s demand is along the segment $K_1K_2$ then the we have $Q_r = 1-v_z$, $Q_o = v_z - A$ and $R_o = (1-m)(v_z - A)$. The retailer solves $\max_{p_r} (1-v_z)(p_r-w)$ which yields $p^*_{K_1K_2} = \frac{w+1-m\theta_H+m\rho_0+f_0(1-m)}{2}$. The manufacturer then solves the problem:

$$\max_{p_o,f_o,w} (1-v_z)(w-c) + (v_z - A)(p_o - c) + (1-m)(v_z - A)(f_o + s - p_o)$$

s.t. $p_o \geq w$

$$p^*_{K_1K_2} \geq A$$

$$v_x \leq v_{K_1K_2}$$

$$\frac{\mu-\theta_L}{\mu}p_o \geq f_o.$$  \hspace{1cm} (C.3)

There is an interior solution for $s \geq \tilde{s} = \frac{(1-m\theta_H)(2c-m\theta_H)}{(1-m)(2-m\theta_H)}$ and the associated profits are strictly higher than $\Pi^K_M$. For $s \leq \tilde{s}$ the solution binds at the second constraint.
so that \( p^*_{rK_1K_2} = A \) and the associated profits are equal to \( \Pi^*_M \).

**Proof of Proposition 4.4.4.** It is straightforward to show that \( \tilde{s} < \bar{s} \) and so the penalty in the decentralized channel, \( f^*_{oK_1K_2} = \frac{m(\theta H^{-1}) + c - s}{2(1 - m)} \), is dominated by the penalty of the online channel only, \( f^*_o = \frac{\mu - \theta L - s + c}{2} \). \( \square \)
VITA
Paolo Letizia

Contact Information
463A Business Building
Department of Supply Chain & Information Systems
The Smeal College of Business
The Pennsylvania State University
University Park, PA 16802

Email: pletizia@psu.edu
Work: (814) 865-0607
Cell: (814) 777-1582

Education
• Ph.D. in Business Administration 2012 The Pennsylvania State University
• Master in Supply Chain Management 2006 Bordeaux Business School, France
• B.S. in Electrical Engineering 2001 University of Pavia, Italy

Research Interests
Closed Loop Supply Chain, Sales Channel Design, Sustainable Operations, Supply Chain Management.

Working Papers