INVESTMENT IN PROCESS CONTROL JUSTIFICATION FOR MULTI-STAGE SYSTEMS
USING ANALYTICAL MODEL

A Thesis in
Industrial Engineering

by
Mohammed Khaled Buqammaz

© 2012 Mohammed K. Buqammaz

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

May 2012
The thesis of Mohammed K. Buqammaz was reviewed and approved* by the following:

M. Jeya Chandra
Professor of Industrial Engineering
Thesis Adviser

Mosuk Chow
Associate Professor of Statistics

Paul Griffin
Professor of Industrial Engineering
Head of the Department of Industrial Engineering

*Signatures are on file in the Graduate School.
ABSTRACT

Process control and quality assurance are the composing elements of a quality management system that are used to ensure the quality of a product or a service. It is widely acknowledged that quality management systems improve the quality of the products and services produced. For the purpose of this research, the effect of process control element is investigated. Investment in process control activities is categorized under prevention cost section of the four quality costs categories. The investment in preventive activities assists to reduce the variance and deviation of the mean from the target value of the quality characteristic.

The main objective of this research is to quantify the effect of process control in a multi-stage system. The justification of investment in process control activities is based on analytical models, without and with process control, that evaluate costs affected by the presence of process control methods in a multi-stage systems, and these models help us to make optimal decisions.

In this research, the $\bar{X}$ chart will be used as the process control method, where several selected costs will be evaluated. The Net Present Value (NPV) method is used to compare the costs of a system without process control with those of a system with process control.

The fully developed analytical model including various costs is given in chapter two. In chapter three, numerical examples are provided to illustrate the implementation of the model. Finally in chapter four, conclusions and recommendations are presented.
## TABLE OF CONTENTS

LIST OF FIGURES........................................................................................................................................ v
LIST OF TABLES........................................................................................................................................ vi
ACKNOWLEDGMENTS.................................................................................................................................. vii

Chapter 1: INTRODUCTION........................................................................................................................... 1
  1.1 Literature review....................................................................................................................................... 1
  1.2 Cost of Quality (COQ)............................................................................................................................... 6
    1.2.1 Types of Quality Costs ..................................................................................................................... 6
    1.2.2 Process Control classification among Types of Quality Costs....................................................... 8
    1.2.3 Relationship between different Quality Costs ................................................................................. 9
  1.3 Objective of this Thesis ......................................................................................................................... 10

Chapter 2: COST ANALYSIS........................................................................................................................... 12
  2.1 Configuration of a Multi-Stage System .................................................................................................... 12
  2.2 Assumptions............................................................................................................................................. 13
  2.3 Probability of Type I and Type II Errors ............................................................................................... 16
  2.4 Stages of the Process in the Presence of Process Control ..................................................................... 17
  2.5 Costs Included in the Analytical Model ............................................................................................... 21
    2.5.1 Internal Failure Cost ....................................................................................................................... 22
    2.5.2 External Failure Cost ...................................................................................................................... 25
    2.5.3 Manufacturing Cost ........................................................................................................................ 29
    2.5.4 Inventory Cost .................................................................................................................................. 31
    2.5.5 Sampling Cost .................................................................................................................................. 32
    2.5.6 Out-of-control Signal Investigation Cost ....................................................................................... 32
    2.5.7 Assignable Cause Removal Cost .................................................................................................... 34
  2.6 Making Investment Decision ................................................................................................................ 35

Chapter 3: NUMERICAL EXAMPLES.............................................................................................................. 36
  3.1 Example 1 .................................................................................................................................................. 36
  3.2 Example 2 ................................................................................................................................................ 43

Chapter 4: CONCLUSIONS AND RECOMMENDATIONS ................................................................................. 50
  4.1 Conclusions ............................................................................................................................................. 50
  4.2 Recommendations .................................................................................................................................. 52

References ....................................................................................................................................................... 53
LIST OF FIGURES

Figure 2.1 Multi-Stage System Configuration ................................................................. 13
Figure 2.2 Stages of a cycle for the \( j^{th} \) stage of component/subassembly \( i \) ........................................... 19
Figure 2.3 Proportion of Defectives .............................................................................. 22
Figure 3.1 Example One Multi-Stage System Diagram .................................................. 37
### LIST OF TABLES

| Table 3. 1 Input Parameters | ................................................................................................................................. 37 |
| Table 3. 2 Other Input Parameters | ............................................................................................................................... 37 |
| Table 3. 3 Output Parameters for a System without Process Control | ......................................................................................................................... 38 |
| Table 3. 4 Output Parameters for a System with Process Control | .......................................................................................................................... 38 |
| Table 3. 5 Total Costs of a System without Process Control | ......................................................................................................................... 38 |
| Table 3. 6 Total Costs of a System with Process Control | .......................................................................................................................... 39 |
| Table 3. 7 Total Costs | ............................................................................................................................... 39 |
| Table 3. 8 Values of out-of-control means for different scenarios | .......................................................................................................................... 43 |
| Table 3. 9 Total Costs with variable $\mu_1$ values for a system without Process Control | ........................................ 43 |
| Table 3. 10 Total Costs with variable $\mu_1$ values for a system with Process Control | ........................................ 44 |
ACKNOWLEDGMENTS

First and foremost, I thank Allah (God), the Exalted, for blessing me with the strength and the power to successfully completing this thesis. I also would like to express my sincere appreciation and gratitude to my supervisor, Professor M. Jeya Chandra, for his immense support and encouragement during my research. In addition, I would like to thank Dr. Mosuk Chow and Professor Paul Griffin for giving me the time to read my thesis and provide me with their valuable feedback.

I extend my thanks to my friends, colleagues and everyone who helped to complete my thesis. I really appreciate your efforts and your contribution will not be forgotten.

Lastly, my thanks go to my beloved parents, my dear siblings, my esteemed family members and my cousin Omar, may his soul rest in peace in heaven, for always surrounding me by their love and support.
Chapter 1: INTRODUCTION

1.1 Literature review

Organizations, either in manufacturing or services industries, around the globe strive towards excellence because of the invaluable benefits those organizations receive in return, such as organization’s reputation and increasing market share, and hence organization expansion. In order to achieve highest level of excellence, programs have been developed to lead organizations to accomplish their goals. Programs that received special attention might include, but not limited to, Total Quality Management (TQM), Kaizen and Six Sigma. This trend towards business excellence programs was evident in late 1970’s and early 1980’s, when previously unchallenged American industries lost substantial market share in both US and world markets. To regain the competitive edge, companies began to adopt productivity improvement programs which had proven themselves successful (Kaynak, 2003). Establishing and sustaining excellence programs need evaluation and analyzing of costs associated with such projects. Cost of quality information can provide insights to organization’s management about operational improvement opportunities (Devasagayam, 2002). Besides benefits which the cost of quality model provides to organizations, quality costing serves the following purposes; a tool for gaining senior management commitment, as a means of preparing a case for a total quality management initiative, a tool for highlighting areas for improvement and as a means of providing estimates of the potential benefits to be gained through quality improvement (Porter and Rayner, 1991).

Several researchers have considered quality cost models to improve overall organizational performance and to provide a framework for evaluating the overall effectiveness of a firm’s
continuous improvement cycle. Omachonu et al. (2004) studied relationship between changes in the quality cost components and the level of quality in a manufacturing firm for three major input elements, material input, machine input and the company as a whole. They concluded that quality cost data can be used in an effort to be proactive, and to identify causes of the problem. Moreover, Desai (2008) examined the cost of quality scenario at small to medium enterprises (MSE). The company selected for the study was a small-scale general engineering firm engaged in manufacturing. The implementation of the cost of quality technique to the selected company resulted in reducing the total cost of quality by 24%, quality improvements of the products and reduction of customer complaints by 43% compared with previous year. In addition to the aforementioned cases, the following studies have successfully implemented cost of quality technique in various industries. For instance, Omurgonulsen (2009) examined the quality costs in seven leading Turkish food manufacturing firms; the author constructed and examined hypotheses that test the effects of the cost of conformance on the non-conformance costs. The study concluded that there is an inverse relation between conformance (prevention and appraisal) and non-conformance (internal and external failure) costs. It also confirmed that an increased expenditure on conformance activities causes reduction in external failure cost. Weeks (2002) offered a framework for understanding quality improvement efforts in the healthcare sector and depicted the process as an investment in quality improvements. Suthummanon et al. (2011) investigated the relationship between quality and cost of quality in a flower wholesale company. The authors developed the framework for the prevention-appraisal-failure (PAF) model from which the relationships between quality costs components were determined. The researchers concluded that conformance costs are inversely
proportional to non-conformance costs for all company’s inputs, conformance costs are directly proportional to quality for all company’s inputs and non-conformance costs are inversely proportional to the level of quality for all company’s inputs. Su et al. (2009) studied the trade-off relationship within quality costs in an auto parts manufacturing firm. In addition, Setijono et al. (2008) developed a methodology of quality cost measurement that has been applied for collecting, measuring, and reporting quality costs in a Swedish wood-flooring manufacturing company. The implementation of the study showed that the internal failure cost is the largest portion of the quality cost measurements and prevention cost is the smallest cost category. Also, the combination of failure costs are more than prevention and appraisal costs, which is consistent with the characteristics of quality costs. Moreover, Mukhapadhyay (2002) estimated the cost of quality in an Indian textile company, where the study’s framework considered several costs for a period of three financial years; the study indicated that an increased investment in preventive activities led to significant decline in failure costs, an increase in sales and reduction of idle machine hours. Additionally, Karg et al. (2009) conducted an empirical analysis to study the association between conformance quality and failure costs for open source software, where 32 open source projects have been included for the study. The study concluded that higher conformance quality is linked with lower failure costs. Continuing in this vein; Ramdeen et al. (2007) conducted a study to measure the cost of quality in a hotel restaurant operation over a period of two years. The results of the study revealed that increasing prevention expenditure contributed to reduction of the total cost of quality from 16% to 12% of sales and led to a significant decrease in appraisal and failure costs. Lastly, Schiffauerova et al. (2006) studied the quality programs in four large successful multinational
companies. Out of the four companies, only one company utilized the PAF model as a method to measure the total cost of quality. The study has indicated that the company using the PAF model has gained several benefits such as decreasing cost of quality components, improving the quality of the products and increasing customer’s satisfaction. On the other hand, the study suggested that other companies with different quality management initiatives will be gain more benefits by implementing a formal cost of quality model, since it will allow them to better identify target areas for cost reduction and quality improvement. Case studies presented above suggest that investing in prevention activities will improve products quality and will reduce total cost of quality. However, results inferred from previous studies are based on firm’s historical data that does not provide means for predicting the amount of money saved as the result of investment made in prevention activities. The methodology presented in this research is developed to predict the future returns from investing in prevention activities projects (i.e. Process Control), based on analytical models. Hence these models assist the higher management to make future decisions regarding investment in prevention activities.

Previous discussion indicated that quality costs can be grouped into four categories; first, for promoting quality as a business parameter; second, they give rise to performance measures and facilitating improvement activities; third, they provide a means for planning and controlling future quality costs; and fourth, they act as motivation (Omachonu et al., 2004). Furthermore, calculation of the cost of quality acts as an indicator of current efficiency of an organization. Reducing the cost of quality enables the improvement process to unfold without the burden of excessive additional costs (Gill, 2009). Lastly, integrating statistical process control into a
Manufacturing and services processes today usually involve more than one stage. With an emphasis on achieving satisfactory product and service quality, multistage process monitoring has become a necessity. Statistical Process Control (SPC) methods have been widely recognized as effective approaches for process monitoring, SPC utilizes statistical methods to monitor processes with an aim to maintain and improve the product quality, while decreasing the variance (Tsung, Li and Jin, 2008). The last decades have witnessed great progress in studying multi-stage statistical process control. Conventional control charts can be used to monitor stages of performance, only if quality measurements in every stage are indeed independent. Nevertheless, if stages have cascade property, that is, there may be some relationship between the quality measurements from current stage and those from upstream stages, advanced control charts, many of them are based on a regression model, need to be used (Ning, Shang and Tsung, 2009). A widely recognized chart that deals with cascading processes is the cause-selecting control chart. Sulek et al. (2006) used the cause-selecting chart in real grocery store process and compared it to the traditional Shewhart chart, where the cause-selecting chart outperformed the Shewhart chart in signaling abnormal patterns.

"Quality matters and it starts not at the factory floor but at the very top" (Dr Deming).

Therefore, in order to establish and sustain an integrated business excellence program, cooperative efforts of cross-functional teams, providing necessary resources (e.g. equipment, training), locating skillful personnel (e.g. quality managers) and allocating sufficient budgets are of paramount importance. Failing to meet business excellence program requirements might
undermine the importance of such programs in the minds of decision makers, and hence lose the benefits offered by those programs.

1.2 Cost of Quality (COQ)

The cost of quality is a term that's widely used and widely misunderstood. The "cost of quality" is not the price of creating a quality product or service. In fact, it is the cost of not creating a quality product or service. Every time work is redone, the cost of quality increases. Examples may include, reworking of a manufactured item, retesting of an assembly, rebuilding of a tool, correction of a bank statement and reworking of a service, such as the reprocessing of a loan operation or the replacement of a food order in a restaurant. In short, any cost that would not have been expended if quality were perfect, contributes to the cost of quality. Quality costs are the total of the costs incurred by investing in the prevention of non-conformance to requirements, appraising a product or service for conformance to requirements and failing to meet requirements (Campanella, 1999). A detailed description of the four categories of quality costs is given next.

1.2.1 Types of Quality Costs

(a) Prevention Costs

Those are the costs of all activities specifically designed to prevent poor quality in products or services. Examples of prevention costs may include new product review, quality planning, supplier capability surveys, process capability evaluations, quality improvement team meetings, quality improvement projects, quality education and training. According to a study conducted by Carr and Ponoemon (1994) to investigate the relationships among quality cost components,
they found that prevention activities cost component is the least expensive among the four quality cost components. However, prevention activities usually, receive the least amount of expenditure in most organizations in the United States (Diallo, 1995). Furthermore, studying the relationship between quality and quality cost revealed that by increasing prevention costs, the level of quality can be improved and failure costs will be decreased because of fewer errors (Omachonu et al., 2004). In fact, further investment in prevention activities will help to reduce appraisal costs as well (Vaxevanidis et al., 2008).

(b) Appraisal Costs

These costs are incurred to determine the degree of conformance to quality requirements, like pre-production verification; laboratory acceptance testing; incoming inspection and tests; in-process inspection and tests; final inspection and tests; field performance testing; inspection and test equipment; and record storage (Omurgonulsen, 2009). Appraisal activities costs receive the second largest amount of investments after prevention activities. In addition, keeping this component of quality costs at minimal level is preferred. However, exclusively investing in appraisal activities may lead to unacceptable costs and may affect organization’s reputation (Chauval and Andre, 1985).

(c) Failure Costs

These are the costs resulting from products or services not conforming to requirements or customer/user needs. Failure costs are divided into internal and external failure categories. Carr and Ponoemon (1994) studied the relationship among quality cost
components and found that the combination of internal and external failure costs is always higher than prevention and appraisal costs.

- **Internal Failure Costs**

  Failure costs occur prior to delivery or shipment of the product, or the furnishing of a service, to the customer. Examples are the costs of scrap, rework, re-inspection, re-testing, material review and downgrading (Desai, 2008). Among the four components of quality costs, internal failure cost is the most expensive (Carr and Ponoemon, 1994).

- **External Failure Costs**

  These are the costs occurring after delivery or shipment of the product, and during or after furnishing of a service, to the customer. Examples are the costs of processing customer complaints, customer returns, warranty claims and product recalls (Campanella 2000).

1.2.2 **Process Control classification among Types of Quality Costs**

From the description presented earlier of the four categories of quality costs, it can be seen that process control activities are classified under the prevention costs category since it seeks to eliminate the opportunity for quality defects. Moreover, case studies results confirmed that investment in prevention activities will result in reduction of failure costs and eventually reduction in appraisal costs.
1.2.3 Relationship between different Quality Costs

The exact relationship between the quality costs components is not easily determined, since it may change from one system to another depending on the nature of the business. The relationship between the four quality costs components have been investigated by several researchers. In general, the increase in appraisal and prevention expenditures is assumed to bring a decline in failure costs. The increase in the appraisal costs may lead to a reduction of the failure costs because the appraisal activities are designed to investigate and to find the defective product before it is delivered to the consumer. Greater expenditure on prevention would result in improved conformance and lower defects, which in turn, are likely to produce overall reduction in the total costs of quality because of significant savings in rework, scrap and warranty (Omurgonulsen, 2009).
1.3 Objective of this Thesis

In this research, the effect of investment in process control is investigated for systems consisting of multiple stages. Process control is categorized under prevention activities, and the investment in process control is classified under prevention cost branch of the four quality costs categories. The investment in preventive activities assists to reduce the variance and deviation of the mean from the target value of the quality characteristic.

The main objective of this research is to justify the investment in process control activities based on analytical models, without and with process control. This research evaluates costs affected by the presence of process control methods in multi-stage systems, and using these developed models optimal decisions can be made whether to invest in process control activities or not.

In this research, the $\bar{X}$ chart is used as the process control method. Several costs will be evaluated in the analytical model such as internal and external failure costs, sampling cost, out-of-control investigation cost, assignable cause removal cost, manufacturing cost and inventory cost. The Net Present Value (NPV) method is used to compare the costs of a system without process control against a system with process control to determine the return from utilizing process control and hence to justify the investment decision. Numerical examples are provided to illustrate the use of the developed analytical expressions, thus substantiating the investment decision. The analytical models can assist the decision makers in evaluating the effectiveness of the amount of investment they make and selecting optimal investment opportunities.
In chapter two, the analytical model is fully described with various costs included. Then in chapter three, numerical examples are provided to illustrate the implementation of the model. Finally in chapter four, conclusions and recommendations are presented.
Chapter 2: COST ANALYSIS

In chapter two, expressions required to build the analytical model are derived for multi-stage systems. These are developed first for a system that does not include any process control method and then for a system that includes process control. After deriving expressions for all costs, an optimal decision to invest in process control can be made by comparing various costs derived earlier against the initial investment that is required to implement the process control. The optimal investment decision is made after evaluation of the costs using net present value.

2.1 Configuration of a Multi-Stage System

Multi-stage system refers to a system consisting of multiple components, stations or stages required to finish the final product or service (Shi and Zhou, 2009). It is assumed that a multi-stage system provides the framework for developing the analytical model. The two-tuple (i,j) indicates the j\(^{th}\) stage of component /subassembly i, where i=1,2,........,N; j=1,2,........,n(i). The proportion of non-conforming and conforming components/subassemblies i at stage j can denoted as \(p(i,j)\) and \(1 - p(i,j)\) respectively. Let the batch size of finished products at the final stage of the system be Q (Zeng and Hayya, 2002). The system under consideration is depicted in the figure (2.1).
2.2 Assumptions

The assumptions used to derive expressions for various costs for each stage of a multi-stage system are listed below:

i- In a multi-stage system, stages are assumed to be independent of each other.

ii- The process generates quality characteristic, $X(i,j)$ at the $j^{th}$ stage of component/subassembly $i$, which is of nominal-the-best type. The quality characteristic, $X(i,j)$, has lower and upper specification limits that are $LSL(i,j)$ and $USL(i,j)$ respectively.

iii- The quality characteristic, $X(i,j)$, follows normal distribution with a mean $\mu(i,j)$ and a variance $\sigma^2(i,j)$ for the $j^{th}$ stage of component/subassembly $i$.

iv- The target values for the mean $\mu(i,j)$ and the variance $\sigma^2(i,j)$ are $\mu_0(i,j)$ and $\sigma^2_0(i,j)$ respectively.
The process starts with the mean of $X(i,j)$, $\mu(i,j)$, that equals the target value, $\mu_0(i,j)$ and the variance $\sigma^2(i,j)$ that equals the target value of $\sigma_0^2(i,j)$. After the expiration of $S(i,j)$ time units, the mean suddenly changes from $\mu_0(i,j)$ to $\mu_1(i,j)$ due to a single assignable cause. The time period $S(i,j)$ is a random variable that is exponentially distributed with a mean of $\frac{1}{\lambda}(i,j)$ for the $j^{th}$ stage of component/subassembly $i$.

The length of the period in which the process mean is equal to its target value ($\mu(i,j) = \mu_0(i,j)$) is called the “in-control” period, however, when the mean changes from the target value ($\mu(i,j)$) to $\mu_1(i,j)$ the “out-of-control” period starts.

The process control method used is the $\bar{X}$ control chart, the purpose of which is to detect any change in the mean $\mu(i,j)$.

The lower control limit, $LCL(i,j)$, is set at $\bar{X}(i,j) - \frac{Z \sigma(i,j)}{\sqrt{n(i,j)}}$

The upper control limit, $UCL(i,j)$, is set at $\bar{X}(i,j) + \frac{Z \sigma(i,j)}{\sqrt{n(i,j)}}$

The center line (CL) is set at $\mu_0(i,j)$ which can be estimated by $\bar{X}(i,j)$, where

$$\bar{X}(i,j) = \frac{\sum_{k=1}^{n(i,j)} X_k(i,j)}{n(i,j)}$$  

(2.1)

In these expressions, $n(i,j)$ is the sample size, $Z_{\alpha/2} (i,j)$ is the Z-value obtained from the standard normal table and $\alpha$ is the probability of type I error, that is the probability of stopping the process while $\mu(i,j) = \mu_0(i,j)$.

Sample batches of size $n(i,j)$ are collected every $h(i,j)$ time units and the sample mean $\bar{X}(i,j)$ is calculated and plotted on the $\bar{X}$ control chart. The process is allowed to run as long as the plotted $\bar{X}(i,j)$ values are plotted within the control limits.
(UCL(i,j) and LCL(i,j)) or if an assignable cause is not detected when the sample mean falls outside the control limits.

The process is stopped only if any $\bar{X}(i,j)$ value falls outside the control limits because of an assignable cause. The cause of the out-of-control state is located, removed and the process is started again in its in-control state, which implies that the mean is set at the target value again ($\mu(i,j)=\mu_o(i,j)$).

In the absence of a $\bar{X}$ control chart, the shift in the mean cannot be detected.

One lot size consists of Q units produced at the final stage of the process, and the production rate for the $j^{th}$ stage of component/subassembly i of the process is $r(i,j)$ units per unit time.
2.3 Probability of Type I and Type II Errors

\[ P \{ \text{Type I Error} \} = P \{ \text{Stopping the process while it is in-control state} \} \]

Let \( \alpha(i, j) \) denote the Probability of Type I Error for the \( j^{\text{th}} \) stage of component /subassembly \( i \).

\[ P \{ \text{Type II Error} \} = P \{ \text{Running the process while it is in out-of-control state} \} \]

\[ = P \{ \text{LCL}(i,j) < \bar{X} < \text{UCL}(i,j) \| \mu(i,j) = \mu_1(i,j) \} \]

\[ = P \{ \mu_0(i,j) - Z_{\alpha(i,j)} \frac{\sigma_0(i,j)}{\sqrt{n(i,j)}} < \bar{X} < \mu_0(i,j) + Z_{\alpha(i,j)} \frac{\sigma_0(i,j)}{\sqrt{n(i,j)}} \| \mu(i,j) = \mu_1(i,j) \} \]

When the process goes out-of-control, the probability distribution of \( \bar{X}(i,j) \) is approximately normal with mean value of \( \mu_1(i,j) \) and variance of \( \frac{\sigma^2_0(i,j)}{n(i,j)} \), then

\[ P \{ \text{Type II Error} \} = P \left[ \frac{(\mu_0(i,j) - Z_{\alpha(i,j)} \frac{\sigma_0(i,j)}{\sqrt{n(i,j)}} - \mu_1(i,j))}{\frac{\sigma_0(i,j)}{\sqrt{n(i,j)}}} < Z < \frac{(\mu_0(i,j) + Z_{\alpha(i,j)} \frac{\sigma_0(i,j)}{\sqrt{n(i,j)}} - \mu_1(i,j))}{\frac{\sigma_0(i,j)}{\sqrt{n(i,j)}}} \right] \]

\[ = P \left[ (\mu_0(i,j) - \mu_1(i,j)) \cdot \frac{\sqrt{n(i,j)}}{\sigma_0(i,j)} - Z_{\alpha(i,j)} < Z < (\mu_0(i,j) - \mu_1(i,j)) \cdot \frac{\sqrt{n(i,j)}}{\sigma_0(i,j)} + Z_{\alpha(i,j)} \right] \]

\[ = P \left[ (\mu_0(i,j) - \mu_1(i,j)) \cdot \frac{\sqrt{n(i,j)}}{\sigma_0(i,j)} - Z_{\alpha(i,j)} < Z < (\mu_0(i,j) - \mu_1(i,j)) \cdot \frac{\sqrt{n(i,j)}}{\sigma_0(i,j)} + Z_{\alpha(i,j)} \right] \]

Let \( \beta(i, j) \) denote the Probability of Type II Error for the \( j^{\text{th}} \) stage of component /subassembly \( i \).
2.4 Stages of the Process in the Presence of Process Control

Based on Duncan (1956), the stages of a process when $\bar{X}$ chart is utilized are discussed in this section. At each stage of the process, the process goes through a cycle of time periods because of assumptions made earlier, and the following time periods are applicable to every stage of the multi-stage system. Those periods are:

(a) In-control period

(b) Out-of-control period until detection of the assignable cause

(c) Time interval to take a sample and interpret the results

(d) Time interval to remove the assignable cause

An out-of-control signal (a $\bar{X}$ value falling outside the control limits) may occur during the in-control period or during the out-of-control period. A false alarm occurs when a $\bar{X}$ value falls outside the control limits during the in-control period; investigation will not detect the assignable cause. As the process is operated during investigation, the mean length of the in-control period is still $\frac{1}{\lambda} (i,j)$. However, when an out-of-control signal occur during the out-of-control period, the investigation will lead to locating the assignable cause responsible for the out-of-control period and then only the process is stopped. According to the assumption numbered (ix), the assignable cause will be removed and the process is brought back to its in-control state.
The total length of the cycle is a random variable because the in-control period and the out-of-control period until detection of the assignable cause are random variables. The cycle renews itself probabilistically at every start and the lengths of the cycles are independent and identically distributed random variables. Therefore, this cycle is a Renewal Cycle and this stochastic process is a Renewal Process (Devasagayam, 2002). Let $E(CT)(i,j)$ be the expected length of a cycle for the $j^{th}$ stage of component/subassembly $i$. Now the expected length of a cycle can be derived, which is accomplished by determining the lengths of the three non-overlapping time segments that make one cycle which includes the in-control period, out-of-control period until detection of the assignable cause and time interval to remove the assignable cause.

(a) In-Control Period:

Throughout this period of the cycle, the process remains in-control state such that the mean of the quality characteristic equals the target mean ($\mu (i,j) = \mu_0(i,j)$). According to assumption (v), this period is a random variable that is exponentially distributed with a mean of $\frac{1}{\lambda (i,j)}$. Therefore,

$$E [\text{In-Control Period}] = \frac{1}{\lambda (i,j)}.$$ 

(b) Out-of-Control Period until detection of the assignable cause:

This period starts when the process goes out-of-control (process mean $\mu (i,j) = \mu_1(i,j)$), and ends when the user detects it because of a $\bar{X}(i,j)$ value falling outside the specified control
The expected length of the period until detection for the $j^{th}$ stage of component/subassembly $i$, which is denoted by $E(B)(i,j)$ is

$$E(B)(i,j) = \frac{h(i,j)}{(1-\beta(i,j))} \quad (2.3)$$

In the expression above, $h(i,j)$ is the interval length of that particular stage, $\frac{1}{(1-\beta(i,j))}$ is the mean number of $\bar{X}(i,j)$ values to be plotted before the first $\bar{X}(i,j)$ falls outside the control limits and $\beta(i,j)$ is probability of type II error of the $j^{th}$ stage of component/subassembly $i$ defined earlier.

There is an overlap between the in-control period and $E(B)(i,j)$. Let this overlap be $L(i,j)$ for the $j^{th}$ stage of component/subassembly $i$. This overlap between periods is illustrated in figure (2.2).

![Figure 2.2 Stages of a cycle for the $j^{th}$ stage of component/subassembly $i$](image)

Let the expected value of this period be $E(L)(i,j)$. This time interval needs to be subtracted from $E(B)(i,j)$ in order to determine the expected length of the out-of-control period until detection.
The value of $E(L)$ was found to be

$$E[L] = \frac{(1-(1+\lambda h)e^{-\lambda h})}{\lambda (1-e^{-\lambda h})}$$  \hspace{1cm} (2.4)$$

and hence the expected length of the out-of-control period, $E(BL)$, is

$$E(BL) = E(B) - E(L)$$

$$= \frac{h}{(1-\beta)} - \frac{(1-(1+\lambda h)e^{-\lambda h})}{\lambda (1-e^{-\lambda h})}$$  \hspace{1cm} (2.5)$$

Now generalizing for a system consisting of multi-stages,

$$E(BL)(i, j) = \frac{h(i,j)}{(1-\beta(i,j))} - \left[\frac{(1-(1+\lambda(i,j),h(i,j))e^{-\lambda(i,j) h(i,j))}}{\lambda(i,j) (1-e^{-\lambda(i,j) h(i,j))}}\right]$$  \hspace{1cm} (2.6)$$

(c) Time interval to take a sample and interpret the results:

The length of this time interval is assumed to be a constant $g(i, j)$ for the $j^{th}$ stage of component/subassembly $i$ multiplied by the sample size $n'(i, j)$ of that particular stage. Thus, this time interval can be obtained as $(g(i, j) * n'(i, j))$.

(d) Time Interval to find and the fix assignable cause:

The length of the time interval to remove an assignable cause at the $j^{th}$ stage of component/subassembly $i$, is assumed to be constant with a specified value $D(i, j)$. The total expected length of one cycle for the $j^{th}$ stage of component/subassembly $i$ is,

$$E(CT)(i, j) = \frac{1}{\lambda (i,j)} + \frac{h(i,j)}{(1-\beta(i,j))} - \left[\frac{(1-(1+\lambda(i,j),h(i,j))e^{-\lambda(i,j) h(i,j))}}{\lambda(i,j) (1-e^{-\lambda(i,j) h(i,j))}}\right] + (g(i, j) * n'(i, j)) + D(i,j)$$  \hspace{1cm} (2.7)$$
2.5 Costs Included in the Analytical Model

For this research effort several costs are included in the analytical model.

- Costs included in the model with or without process control:

  1. Internal Failure Cost
  2. External Failure Cost
  3. Manufacturing Cost
  4. Inventory Cost

- Costs included in the model only with process control:

  1. Sampling Cost
  2. Out-of-control Signal Investigation Cost
  3. Assignable Cause Removal Cost

In the analytical model, the internal and external failure costs, manufacturing cost and inventory cost rely on the values of the mean and variance of the quality characteristic, $X(i,j)$, at that particular stage of the process. In this research, the process is solely monitored by the $\bar{X}$ chart and therefore, the only difference between the processes with and without any process control is the mean of the quality characteristic $X(i,j)$. 
2.5.1 Internal Failure Cost

When items are produced with quality characteristic values outside the specification limits (USL(i,j) and LSL(i,j)), a cost is incurred due to the rejection of those items. This cost is called internal failure cost. In figure (2.3), an illustration of the proportion of undersized and oversized items is given.

![Figure 2.3 Proportion of Defectives](image)

The sum of the shaded areas under the normal distribution curve to the left of the lower specification limit and to the right of the upper specification limit in figure (2.3) represents
the total proportion of defective items for the j^{th} stage of component/subassembly i, \( p(i,j) \). Probabilistically, this is equal to

\[
p(i,j) = P[X < LSL(i,j)] + P[X > USL(i,j)]
\]

where \( Z \) is the standard normal variable. If the costs of rejection of undersized and oversized items for the j^{th} stage of component/subassembly i are \( C_u(i,j) \) and \( C_o(i,j) \) respectively, then the expected total cost of rejection per unit \( RC(i,j) \), for the j^{th} stage of component/subassembly i, is given by

\[
RC(i,j) = C_u(i,j) * P\left[Z < \frac{LSL(i,j)-\mu(i,j)}{\sigma(i,j)}\right] + C_o(i,j) * P\left[Z > \frac{USL(i,j)-\mu(i,j)}{\sigma(i,j)}\right]
\]

In the expression above, the actual value of the mean at j^{th} stage of component/subassembly i, \( \mu(i,j) \), depends on whether the process is in-control state or the out-of-control state and whether the \( \bar{X} \) control chart is employed to detect the shift in the mean or not. The total expected internal failure cost per year for a multi-stage system is derived below, first for a system without process control and then for a system with process control.

(a) For Multi-stage system without process control:

In a system consisting of multi-stages, the length of one production cycle in the j^{th} stage of component/subassembly i, which is the length of time required to machine one lot consisting of \( Q(i,j) \) items, is \( \frac{Q(i,j)}{r(i,j)} \) time units, where \( r(i,j) \) is the production rate at the j^{th} stage of component/subassembly i. According to assumption (v), the expected length of time the
process mean stays at $\mu_0(i,j)$ in each production cycle is $\frac{1}{\lambda}(i,j)$. During the remaining length of the production cycle, the mean stays at the out-of-control value, that is $\mu_1(i,j)$. Therefore, the total expected internal failure cost per year without process control for a multi-stage system, $\text{IFC}_{\text{NPC}}$, is given by,

$$\text{IFC}_{\text{NPC}} = N_\text{Q} \cdot \left( \left[ \sum_{i=1}^{N} \sum_{j=1}^{n(i)} \text{Min} \left( \frac{Q(i,j)}{r(i,j)}, \frac{1}{\lambda}(i,j) \right) \right] + \left[ \sum_{i=1}^{N} \sum_{j=1}^{n(i)} \text{Max} \left( \frac{Q(i,j)}{r(i,j)} - \frac{1}{\lambda}(i,j), 0 \right) \right] \right) \left( \frac{C_u(i,j)}{\sigma_0(i,j)} \right) + C_o(i,j) * \left( Z < \frac{\text{USL}(i,j) - \mu_0(i,j)}{\sigma_0(i,j)} \right) + \left( Z > \frac{\mu_1(i,j) - \text{LSL}(i,j)}{\sigma_0(i,j)} \right) \right) \right) (2.10)$$

where $N_\text{Q}$ is the total number of lots (batches) produced per year, $N$ is the number of components/subassemblies $i$, $n(i)$ is the number of stages for the $i$th component/subassembly and $Q(i,j)$ is the production quantity at the $j$th stage of component/subassembly $i$ to satisfy the demand of batch quantity $Q$, can be obtained as,

$$Q(i,j) = \frac{Q}{\prod_{j=1}^{n(i)} [1 - p(i,j)]} \quad (2.11)$$

$Q$ is the batch quantity of final product at final stage and $p(i,j)$ is proportion of defective items.

(b) For Multi-stage system with process control:

When the $\overline{X}$ chart is used to monitor the mean of the quality characteristic $X(i,j)$ at any stage of the process, the process goes through a cycle consisting of four distinct segments as described in section (2.4). However, the process at any particular stage operates only during the first two segments whose lengths are $\frac{1}{\lambda}(i,j)$ and $E(BL)(i,j)$. The expected length of time the
process mean stays at \( \mu_0(i,j) \) in each cycle at each stage of the process is \( \frac{1}{\lambda}(i,j) \) and the expected length of time the process mean is equal to \( \mu_1(i,j) \) is \( E(BL)(i,j) \). Thus, the total expected internal failure cost per year for a process with process control, IFC_{PC}, is given by,

\[
IFC_{PC} = NQ\left( \sum_{i=1}^{N} \sum_{j=1}^{n(i)} Q(i,j) \times \left[ \frac{1}{\lambda(i,j) + E(BL)(i,j)} \right] \times \left\{ C_u(i,j) + \frac{1}{\lambda(i,j) + E(BL)(i,j)} \right\} \right) \]

\( \text{2.12} \)

2.5.2 External Failure Cost

External failure cost come from costs associated with defects that are found after the customer receives the product. It is proven that products with quality characteristics having mean values as close as possible to the respective target values and variances kept at minimal levels, will perform better and last longer, hence minimizing the external failure costs to the producer and providing better products to the customers. Several researchers attempted to model the external failure cost. For instance, Taguchi developed the loss function where the expected loss value equals the economic loss to society due to failures after shipping the product. It is given by,

\[
E[L(x)] = K'[\sigma^2 + (\mu - \mu_o)^2]
\]

(2.13)

where \( K' \) is a constant, \( \sigma^2 \) is the process variance, \( \mu \) is the mean, and \( \mu_o \) is the target value of the quality characteristic. However, Taguchi’s attempt to convert the variance and the deviation of a particular product’s quality characteristic from its target value into dollars
through the use of $K'$ met with skepticism from the practitioners because of his simplistic definition of $K'$ (Devasagayam, 2002). Moreover, Deleveaux (1997) developed a failure time distribution that addresses both the individual performance of an item and the impact of proximity of means to the respective values on reliability, by modifying the traditional Weibull distribution. The following function estimates the lower bound of the reliability of an item at specified time $t$, and it is given by,

$$R(t) = e^{-t^{E}e^{-B C_{pm}^{m}}}$$  \hspace{1cm} (2.14)

where, $E$ and $B$ are constants estimated by using data on failure times. The capability index is defined as

$$C_{pm} = \frac{USL-LSL}{6\sqrt{\sigma^2 + (\mu-\mu_0)^2}}$$  \hspace{1cm} (2.15)

However, the aforementioned function developed by Deleveaux only estimates the lower bound of the reliability. Apart from the previously mentioned functions, a more recent function that was developed by Blue (2001) is used to estimate the reliability of items at time which is given by,

$$R(t) = \frac{1}{\sqrt{1+2B \sigma_p^2 t^\xi}} e^{-\left(U + \frac{B \sigma_p^2 \delta}{1+2B \sigma_p^2 t^\xi}\right)t^\xi}$$  \hspace{1cm} (2.16)

where $t$ is specified time $t$, constants $B$, $\xi$, and $U$ are to be estimated using data on failure times, $\sigma_p^2$ is the process variance. The value of $\delta$ can be obtained as,

$$\delta = \frac{(\mu-\mu_0)^2}{\sigma_p^2}$$  \hspace{1cm} (2.17)
where $\mu$ and $\mu_0$ are the actual and target means respectively.

From equations (2.22) and (2.23) it is noticed that as the mean, $\mu$, becomes closer to the target value, $\mu_0$, and the variance, $\sigma_p^2$, is reduced, the reliability at time $t$, $R(t)$, increases. As the reliability at time $t$ increases, the expected number of failures within time interval $t$ decreases, therefore, decreasing the external failure costs. Now, the expected number of failures of the product during the warranty period $[0,W]$ can be obtained from the reliability $R(t)$, as (Djamaludin et al., 1993),

$$\int_0^W \varphi(t)dt = [- \ln (R(t))]_0^W = - \ln R(W)$$

(2.18)

where $\varphi(t)$ is the failure rate of the product.

In general, the external failure cost/unit is given by,

$$EC = C_R \int_0^W \varphi(t)dt$$

(2.19)

where $C_R$ is the repair cost per unit. Now the external failure cost per year for a multi-stage processes without and with process control are derived.

(a) For Multi-stage system without process control:

Let $R_0(t)(i,j)$ be the reliability at time $t$ of an item in a batch at the $j^{th}$ stage of component/subassembly $i$ with a process mean equal to $\mu_0(i, j)$ and process variance $\sigma_p^2(i, j)$. Let $R_1(t)(i,j)$ be the reliability at time $t$ of an item in a batch at the same stage, with a process mean equal to $\mu_1(i, j)$ and process variance $\sigma_p^2(i, j)$. The value of $\delta(i, j)$ to be substituted in equation (2.22) is given by,
\[ \delta_0(i,j) = \frac{(\mu_0(i,j) - \mu_0(i,j))^2}{\sigma_0^2(i,j)} = 0 \]  

(2.20)

when \( \mu(i,j) = \mu_0(i,j) \) and

\[ \delta_1(i,j) = \frac{(\mu_1(i,j) - \mu_0(i,j))^2}{\sigma_1^2(i,j)} \]  

(2.21)

when \( \mu(i,j) = \mu_1(i,j) \).

It can be seen from (2.26) and (2.27) that \( \delta_0(i,j) < \delta_1(i,j) \) and hence \( R_0(t)(i,j) > R_1(t)(i,j) \). Let \( EC_0(i,j) \) and \( EC_1(i,j) \) be the external failure costs per item during in-control and out-of-control periods respectively at the \( j \)th stage of component/subassembly \( i \), as per equation (2.25), computed using \( R_0(t)(i,j) \) and \( R_1(t)(i,j) \) respectively. Now, the total expected external failure cost per year is given by,

\[
EFC_{NPC} = NQ \left( \sum_{i=1}^{N} \sum_{j=1}^{n(i)} \text{Min} \left( \frac{Q(i,j)}{r(i,j)} , \frac{1}{\lambda} (i,j) \right) \ast r(i,j) \ast EC_0(i,j) + \sum_{i=1}^{N} \sum_{j=1}^{n(i)} \text{Max} \left( \frac{Q(i,j)}{r(i,j)} , \frac{1}{\lambda} (i,j) \right) \ast \frac{1}{\lambda} (i,j), 0 \right) \ast r(i,j) \ast EC_1(i,j) \right)
\]

(2.22)

(b) For Multi-stage system with process control:

Now, for a system with process control and using the same rationale employed to derive an expression for a system without process control, the total expected external failure cost per year can be obtained as,

\[
EFC_{PC} = NQ \left( \sum_{i=1}^{N} \sum_{j=1}^{n(i)} Q(i,j) \left[ \frac{1}{\lambda(i,j) + E(BL)(i,j)} \ast EC_0(i,j) + \frac{E(BL)(i,j)}{\lambda(i,j) + E(BL)(i,j)} \ast EC_1(i,j) \right] \right)
\]

(2.23)
2.5.3 Manufacturing Cost

Let the batch quantity of final products at the final stage be denoted as Q. The service level of demand for final products shall be satisfied from the components/subassemblies replenished at all stages. It is assumed that the defective units are not reworked at any stage. As the proportion of defectives at the \( j \textsuperscript{th} \) stage of component/subassembly \( i \) is \( p(i, j) \), the quantity replenished for each batch at the \( j \textsuperscript{th} \) stage of component/subassembly \( i \), considering the variation of demand and service level can be obtained as (Lee, 2004)

\[
M(i,j) = Q(i, j) + k(i, j)\sigma_D(i, j)\sqrt{\tau(i, j)}
\]  
(2.24)

\[
= \frac{Q}{\prod^n_{g=j} [1-p(i,g)]} + k(i, j) \sigma_D(i, j) \frac{Q}{\sqrt{d(i,j)\prod^n_{g=j} [1-p(i,g)]}}
\]  
(2.25)

where \( M(i,j) \) is the quantity replenished for each batch at the \( j \textsuperscript{th} \) stage of component/subassembly \( i \), \( p(i, g) \) is the proportion of defectives at the \( j \textsuperscript{th} \) stage of component/subassembly \( i \), \( \tau(i, j) \) is the replenishment time at the \( j \textsuperscript{th} \) stage of component/subassembly \( i \), and \( k(i,j) \) is the safety stock factor for each stage. This can be determined by

\[
k(i, j) = \Phi^{-1}[L(i, j)] = \Phi\{Q_0(i,j) \leq M(i,j)\}
\]  
(2.26)

where \( L(i, j) \) is the service level for each batch at the \( j \textsuperscript{th} \) stage of component/subassembly \( i \). \( Q_0(i,j) \) is the demand of quantity for batches at each stage that is normally distributed with mean of \( d(i,j) \) and standard deviation of \( \sigma_D(i,j) \).
As the manufacturing cost per unit per unit at the $j^{th}$ stage of component/subassembly $i$ is $C_M(i, j)$, the total manufacturing cost per year can be obtained as,

$$TC_M = N_Q \times \left[ \sum_{i=1}^{N} \sum_{j=1}^{n(i)} C_M(i, j) \right]$$

(2.27)

(a) For Multi-stage system without process control:

When no process control method is employed to monitor the process in a multi-stage system, the proportion of defective items, $p(i, g)$, at the $j^{th}$ stage of component/subassembly $i$ to be substituted in equation (2.31) is obtained as,

$$p(i, g) = \left( \left( \frac{M \ln\left(\frac{Q(i,j)}{\tau(i,j) \Lambda(i,j)}\right)}{\left(\frac{Q(i,j)}{\tau(i,j)}\right) \Lambda(i,j)} \right) \times \left\{ P \left[ Z < \frac{ULS(i,j)-\mu_0(i,j)}{\sigma_0(i,j)} \right] + P \left[ Z > \frac{ULS(i,j)-\mu_1(i,j)}{\sigma_0(i,j)} \right] \right\} \right)$$

(2.28)

(b) For Multi-stage system with process control:

In the presence of process control method to monitor a multi-stage process, the proportion of defective items, $p(i, g)$, at the $j^{th}$ stage of component/subassembly $i$ to be substituted in equation (2.25) is obtained as,

$$p(i, g) = \left( \left( \frac{1}{Z(i,j)+E(\text{BL})(i,j)} \right) \times \left\{ P \left[ Z < \frac{ULS(i,j)-\mu_0(i,j)}{\sigma_0(i,j)} \right] + P \left[ Z > \frac{ULS(i,j)-\mu_1(i,j)}{\sigma_0(i,j)} \right] \right\} \right)$$

(2.29)
2.5.4 Inventory Cost

This cost includes all the expenses incurred because of carrying inventory. Let the production rate at the \( j^{th} \) stage of component/subassembly \( i \) be \( r(i, j) \). The production time and the maximum inventory level are denoted as \( \omega(i,j) \) and \( y(i, j) \) respectively. The production time, \( \omega(i,j) \), is given by (Lee, Chandra, Delevaux, 1997),

\[
\omega(l, j) = \frac{Q}{r(l, j) \prod_{g=j}^{n(i)} [1 - p(l, g)]}
\]

(2.30)

The maximum inventory level at the \( j^{th} \) stage of component/subassembly \( i \) can be obtained as,

\[
y(i, j) = \{r(i, j) - d(i, j)\} \times \omega(i, j)
\]

(2.31)

As the inventory cost per unit of average inventory is \( H \), the total inventory cost per year is obtained as,

\[
TC_H = N Q \left[ \frac{H}{2} \sum_{i=1}^{N} \sum_{j=1}^{n(i)} \left\{ r(i, j) - \frac{d}{\prod_{g=j+1}^{n(i)} [1 - p(l, g)]} \right\} \times \frac{Q}{r(l, j) \prod_{g=j}^{n(i)} [1 - p(l, g)]} \right]
\]

(2.32)

where, \( d \) is the rate of demand.

(a) For Multi-stage system without process control:

For a multi-stage system with no process control method employed to monitor the processes, the proportion of defective items, \( p(i, g) \), to be substituted in equation (2.31) is obtained as per equation (2.27).
(b) For Multi-stage system with process control:

For a system that uses process control methods to monitor the processes in a multi-stage system, the proportion of defective items, \( p(i, g) \), to be substituted in equation (2.31) can be obtained as per equation (2.28).

### 2.5.5 Sampling Cost

Sampling cost is one of the costs that occur only when \( \bar{X} \) control chart is used as the process control method. It is assumed that the cost of sampling consists of a fixed cost of \( $a_1(i, j) \), which is the fixed sampling cost per batch for the \( j \)th stage of component/subassembly \( i \), which is independent of the sample size, \( n'(i, j) \) of that particular stage, and a variable cost of \( $a_2(i, j) \) per item at this same stage. Then the total expected sampling cost per year can be obtained by,

\[
SC_{PC} = N_Q \times \left[ \sum_{i=1}^{N} \sum_{j=1}^{n(i)} (a_1(i, j) + a_2(i, j)n'(i, j)) \frac{Q(i, j)}{r(i, j)h(i, j)} \right] \tag{2.32}
\]

where, \( h(i, j) \) is the time between successive sample batches at the \( j \)th stage of component/subassembly \( i \).

### 2.5.6 Out-of-control Signal Investigation Cost

This cost also is incurred only when \( \bar{X} \) chart is used to monitor the process. Let the cost incurred for every out-of-control signal be \( C_1(i, j) \) at the \( j \)th stage of component/subassembly \( i \). An out-of-control signal is \( \bar{X}(i, j) \) value that falls outside the specified control limits. After the process mean shifts from \( \mu_o(i, j) \) to \( \mu_1(i, j) \), only one \( \bar{X}(i, j) \) value falls outside the control
limits, as the assignable cause will be detected and the process will be brought back to its in-control state. The number of out-of-control signals during the in-control period, which are referred to as false alarms, can be obtained as follows.

Let the M be the number of false alarms per cycle. Then,

\[ E[\text{number of false alarms per cycle}] = E[M] = \sum_{m=0}^{\infty} (m \alpha) \ P[mh < s < (m + 1)h] \]

\[ = \alpha \sum_{m=0}^{\infty} m \int_{mh}^{(m+1)h} \lambda e^{-\lambda s} \ ds \]

\[ E[M] = \alpha \sum_{m=0}^{\infty} m \left[ e^{-mh\lambda} - e^{-(m+1)h\lambda} \right] \]

\[ = \alpha \sum_{m=0}^{\infty} m e^{-mh\lambda} - \alpha \sum_{m=0}^{\infty} m e^{-(m+1)h\lambda} \] \hspace{1cm} (2.34)

In the aforementioned equation,

\[ \alpha \sum_{m=0}^{\infty} m e^{-mh\lambda} = \alpha \sum_{m=0}^{\infty} m (e^{-\lambda h})^m \]

\[ = \frac{\alpha e^{-\lambda h}}{(1-e^{-\lambda h})^2} \] \hspace{1cm} (2.35)

and

\[ \alpha \sum_{m=0}^{\infty} m e^{-(m+1)h\lambda} = \alpha e^{-\lambda h} \sum_{m=0}^{\infty} m e^{-mh\lambda} \]

\[ = \alpha e^{-\lambda h} \left[ \frac{e^{-\lambda h}}{(1-e^{-\lambda h})^2} \right] \] \hspace{1cm} (2.36)

Now, equation (2.40) can be simplified to

\[ E[M] = \frac{\alpha e^{-\lambda h}}{(1-e^{-\lambda h})^2} - \alpha e^{-\lambda h} \left[ \frac{e^{-\lambda h}}{(1-e^{-\lambda h})^2} \right] \]

\[ = \frac{\alpha e^{-\lambda h}}{1-e^{-\lambda h}} \] \hspace{1cm} (2.37)
The total expected number of out-of-control signals in a cycle is,

\[ \text{NOC} = E[M] + 1 \]

Therefore, the generalized formula for the total expected cost per year of investigating out-of-control signals in a multi-stage system is given by,

\[
\text{TOC}_{PC} = N_Q \sum_{i=1}^{N} \sum_{j=1}^{r(i,j)} C_1(i,j) \frac{Q(i,j)}{r(i,j) E(CT)(i,j)} (E(M)(i,j) + 1) \tag{2.38}
\]

where, \( E(M)(i,j) \) is the expected number of false alarms per cycle at the \( j \)th stage of component/subassembly \( i \), and \( E(CT)(i,j) \) is the length of one cycle at the \( j \)th stage of component/subassembly \( i \), as defined in equation (2.14).

2.5.7 Assignable Cause Removal Cost

It is assumed that a fixed cost of \( C_2(i,j)($/cycle) \) is incurred to remove the assignable cause at the \( j \)th stage of component/subassembly \( i \). This cost is incurred once in every cycle, and the total expected cost to remove the assignable cause per year is given by,

\[
\text{ACC}_{PC} = N_Q \sum_{i=1}^{N} \sum_{j=1}^{r(i,j)} C_2(i,j) \frac{Q(i,j)}{r(i,j) E(CT)(i,j)} \tag{2.39}
\]

To summarize, the total expected cost per year for a process without process control is,

\[
\text{TC}_{NPC} = \text{IFC}_{NPC} + \text{EFC}_{NPC} + \text{TC}_{MNPC} + \text{TC}_{HNPC} \tag{2.40}
\]

where \( \text{IFC}_{NPC} \), \( \text{EFC}_{NPC} \), \( \text{TC}_{MNPC} \) and \( \text{TC}_{HNPC} \) are given in equations (2.10), (2.22), (2.26) and (2.31) respectively.

The total expected cost per year for a process with process control is,
where IFC\(_{PC}\), EFC\(_{PC}\), SC\(_{PC}\), TOC\(_{PC}\), ACC\(_{PC}\), TC\(_{MPC}\) and TC\(_{HPC}\) are given in equations (2.12), (2.23), (2.26), (2.31), (2.32), (2.38) and (2.39) respectively.

### 2.6 Making Investment Decision

After determining the total annual costs, TC\(_{NPC}\) and TC\(_{PC}\), from which the annual return is calculated, the decision to invest in a process control to monitor a multi-stage system performance can be evaluated by using the net present value. Let \( I \) be the one-time investment made at time zero to establish the process control method. In addition, let \( C_a \) be the cost of maintaining the process control method per year. Then the net present value of incremental benefit of using the process control method is given by,

\[
NPV = -I + \sum_{k=1}^{K} (TC_{NPC} - TC_{PC} - C_a)(1 + i)^{-k}
\]  

(2.42)

where, \( i \) is the minimum attractive rate of return and \( K \) is the planning horizon. The investment in process control method should be approved, if NPV in equation (2.48) is greater than zero.

In the next chapter, numerical examples are provided to further illustrate the use of various expressions developed in building the analytical model.
Chapter 3: NUMERICAL EXAMPLES

In chapter three, numerical examples are presented to illustrate the model developed in this research.

3.1 Example 1
Consider the manufacturing process of a product that consists of two subassemblies labeled as subassembly one and subassembly two. Subassembly one has two stages marked as (1, 1) and (1, 2). Subassembly two has two stages marked as (2, 1) and (2, 2). The batch quantity of final products, \( Q \), is 220 units. The inventory cost per unit of averaged inventory, \( H \), is 10. The following values are assumed for all stages, probability of type I error, \( \alpha = 0.05 \); sample size, \( n' = 5 \) units (for all stages); time required to remove the assignable cause, \( D = 0.5 \) hour; constants to be used in the reliability function are, \( B = 1; \ U = 1; \ \xi = 1 \); repair cost per unit, \( C_R = $10 \); warranty period, \( W = 2 \) years; demand rate at the final stage of the final product, \( d = 145 \) units/day; \( N_Q = 50, a_1 = 5, a_2 = 0.1, g = 10 \), planning horizon (K) = 5 years and Interest rate, \( i = 10\% \). Other values of input parameters are assumed and provided in tables below. The diagram of the process is depicted in figure (3.1). It is assumed that the process control investment costs $100,000 and the annual cost of maintaining the system is $12,000.
Figure 3.1 Example One Multi-Stage System Diagram

![Diagram](image)

Table 3.1 Input Parameters

<table>
<thead>
<tr>
<th>Stage</th>
<th>$\frac{1}{\lambda}(i,j)$</th>
<th>$\mu_1(i,j)$</th>
<th>$\mu_0(i,j)$</th>
<th>$LSL(i,j)$</th>
<th>$USL(i,j)$</th>
<th>$\sigma_o(i,j)$</th>
<th>$C_A(i,j)$</th>
<th>$C_o(i,j)$</th>
<th>$r(i,j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1)</td>
<td>1</td>
<td>3.7</td>
<td>3</td>
<td>1.5</td>
<td>4.5</td>
<td>0.5</td>
<td>6</td>
<td>4</td>
<td>160</td>
</tr>
<tr>
<td>(1, 2)</td>
<td>1</td>
<td>6.1</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>0.8</td>
<td>5</td>
<td>3</td>
<td>155</td>
</tr>
<tr>
<td>(2, 1)</td>
<td>1</td>
<td>4.9</td>
<td>4</td>
<td>2.5</td>
<td>5.5</td>
<td>0.6</td>
<td>5</td>
<td>3</td>
<td>165</td>
</tr>
<tr>
<td>(2, 2)</td>
<td>0.75</td>
<td>2.4</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0.2</td>
<td>6</td>
<td>4</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 3.2 Other Input Parameters

<table>
<thead>
<tr>
<th>Stage</th>
<th>$L(i,j)$</th>
<th>$k(i,j)$</th>
<th>$\sigma_D(i,j)$</th>
<th>$h(i,j)$</th>
<th>$C_1(i,j)$</th>
<th>$C_2(i,j)$</th>
<th>$C_M(i,j)$</th>
<th>$LCL(i,j)$</th>
<th>$UCL(i,j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1)</td>
<td>0.95</td>
<td>1.645</td>
<td>4</td>
<td>0.333</td>
<td>15</td>
<td>15</td>
<td>7</td>
<td>2.4</td>
<td>3.6</td>
</tr>
<tr>
<td>(1, 2)</td>
<td>0.95</td>
<td>1.645</td>
<td>4</td>
<td>0.5</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>4.0</td>
<td>6.0</td>
</tr>
<tr>
<td>(2, 1)</td>
<td>0.95</td>
<td>1.645</td>
<td>4</td>
<td>0.667</td>
<td>15</td>
<td>15</td>
<td>5</td>
<td>3.2</td>
<td>4.8</td>
</tr>
<tr>
<td>(2, 2)</td>
<td>0.95</td>
<td>1.645</td>
<td>4</td>
<td>0.333</td>
<td>10</td>
<td>10</td>
<td>8</td>
<td>1.7</td>
<td>2.3</td>
</tr>
</tbody>
</table>
Expressions developed in chapter 2 are used to calculate various costs and hence the total costs for systems with and without process control are obtained. The calculated costs per year are provided in the table below:

<table>
<thead>
<tr>
<th>Stage</th>
<th>Q(i,j)</th>
<th>P(i,j)</th>
<th>d(i,j)</th>
<th>M(i,j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1)</td>
<td>274</td>
<td>0.0247</td>
<td>159</td>
<td>282</td>
</tr>
<tr>
<td>(1, 2)</td>
<td>267</td>
<td>0.0629</td>
<td>155</td>
<td>275</td>
</tr>
<tr>
<td>(2, 1)</td>
<td>269</td>
<td>0.0705</td>
<td>156</td>
<td>278</td>
</tr>
<tr>
<td>(2, 2)</td>
<td>252</td>
<td>0.0007</td>
<td>147</td>
<td>261</td>
</tr>
</tbody>
</table>

Table 3.3 Output Parameters for a System without Process Control

<table>
<thead>
<tr>
<th>Stage</th>
<th>Q(i,j)</th>
<th>P(i,j)</th>
<th>d(i,j)</th>
<th>M(i,j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1)</td>
<td>267</td>
<td>0.0151</td>
<td>155</td>
<td>276</td>
</tr>
<tr>
<td>(1, 2)</td>
<td>263</td>
<td>0.0504</td>
<td>153</td>
<td>272</td>
</tr>
<tr>
<td>(2, 1)</td>
<td>266</td>
<td>0.0595</td>
<td>154</td>
<td>275</td>
</tr>
<tr>
<td>(2, 2)</td>
<td>250</td>
<td>0.0005</td>
<td>145</td>
<td>259</td>
</tr>
</tbody>
</table>

Table 3.4 Output Parameters for a System with Process Control

Expressions developed in chapter 2 are used to calculate various costs and hence the total costs for systems with and without process control are obtained. The calculated costs per year are provided in the table below:

<table>
<thead>
<tr>
<th>Stage</th>
<th>$IC_{NPC}(i,j)$</th>
<th>$EFC_{NPC}(i,j)$</th>
<th>$TC_{MNPC}(i,j)$</th>
<th>$TC_{HNPC}(i,j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1)</td>
<td>$1,392$</td>
<td>$215,480$</td>
<td>$98,769$</td>
<td>$572$</td>
</tr>
<tr>
<td>(1, 2)</td>
<td>$2,640$</td>
<td>$259,073$</td>
<td>$137,717$</td>
<td>$111$</td>
</tr>
<tr>
<td>(2, 1)</td>
<td>$2,995$</td>
<td>$233,834$</td>
<td>$69,447$</td>
<td>$3,627$</td>
</tr>
<tr>
<td>(2, 2)</td>
<td>$36$</td>
<td>$153,352$</td>
<td>$103,528$</td>
<td>$2,042$</td>
</tr>
<tr>
<td>Total</td>
<td>$7,063$</td>
<td>$861,739$</td>
<td>$409,460$</td>
<td>$6,352$</td>
</tr>
</tbody>
</table>

Table 3.5 Total Costs of a System without Process Control
Now, net present value to evaluate the benefits gained from employing process control, is computed as

\[ NPV = -100,000 + \sum_{k=1}^{5} (1,284,613 - 1,299,693 - 12,000) (1 + 0.1)^{-k} \]

\[ = 188,234 \]

Since the value of the NPV is greater than zero, the investment in process control for this system is justified.
- Example Analysis:

At first glance one will notice that the total cost for a system with process control ($1,229,693) is less than the total cost for a system without process control ($1,284,613), although employing process control in a system will incur further costs such sampling cost, Out-of-control Signal Investigation Cost and Assignable Cause Removal Cost. So, what drives the total cost for a system with process control down? And what are the implications that can be inferred from these results? Also, will there be any other benefits gained from implementing process control besides reducing total cost? Those questions will be addressed in the following discussion.

- Effect of process control on internal failure cost:

While a system without process control doesn’t focus on monitoring process’s quality characteristic mean values in an effort to detect unusual trends during production operations, detecting the process when it goes out-of-control will be a tough task. On the other hand, a system with process control keeps an eye on quality characteristic mean values, and if any unusual trend occurs, the process is stopped, sources of variation are investigated, the existing sources are rectified and then process is continued. The effect of process control is expressed in equation (2.12), where process is only allowed to run as long as the process is in an in-control condition that is during the time period \(\left(\frac{1}{\lambda}\right)(i,j)\). This effect is quantified in the form of the ratio \(\frac{1}{X}\), and once an out-of-control signal is detected, the process will be stopped immediately limiting the amount of parts manufactured during the out-of-control period which
is only allowed to run during the time period $E(BL)(i,j)$. The amount of the parts produced is quantified in the form of the ratio $\frac{E(BL)(i,j)}{\lambda(i,j)+E(BL)(i,j)}$. In addition, the amount of production quantity at the $j^{th}$ stage of component/ subassembly $i$, $Q(i,j)$, is also affected by the presence of process control. A system with process control will have smaller proportion of defective items, $p(i,j)$, compared to a system without process control, according to equations (2.27) and (2.28), since process control only allows a process to run during in-control-conditions. Therefore, fewer items are required to be produced to feed successive stage(s) resulting in smaller internal failure cost.

- **Effect of process control on external failure cost:**

  The effect of process control on external failure cost can be seen evidently in equation (2.23), where process control tries to keep the process running as long as process remains in-control conditions, which implies that more items are produced with the mean closer to the specified target mean and hence items are produced with better reliability and hence less external failure cost is incurred.

- **Effect of process control on manufacturing cost:**

  The manufacturing cost is a function of the quantity replenished for each batch at the $j^{th}$ stage of component/ subassembly $i$. This is equal to $M(i,j)$, which according to equation (2.24) is a function of the amount of production quantity at the $j^{th}$ stage of component/ subassembly $i$, $Q(i,j)$, and the square root of replenishment time at the $j^{th}$ stage of component/ subassembly $i$, $T(i,j)$. As those values increase the quantity $M(i,j)$ will increase as a result. Employing
process control works on keeping $Q(i,j)$ and $T(i,j)$ at lowest possible values by minimizing the proportion of defective items. This result implies that process control will bolster the production process by providing the amount of required items in shorter replenishment time period with fewer amounts of rejected items and hence decreasing manufacturing cost.

- **Effect of process control on inventory cost:**

  The inventory cost is a function of the demand rate at the $j^{th}$ stage of component/subassembly $i$, $d(i,j)$, and the production time, $\omega(i,j)$ as illustrated in equation (2.31). As mentioned previously, employing process control tends to lower the values of $d(i,j)$ and $\omega(i,j)$ since it minimizes the proportion of defective items that causes $d(i,j)$ and $\omega(i,j)$ to go higher. However, for two systems with the same inputs, the system with process control will have higher maximum inventory level, $y(i,j)$, since process control tends to produce more items with less defectives. This will result in higher inventory level and hence higher inventory cost. This issue can be fixed by adjusting the production rate, $r(i,j)$, to meet required inventory level. This result can be utilized to satisfy just-in-time orders which imply that by adopting process control technique, the system with process control will be capable of satisfying the demand in shorter time period. Finally, although employing process control to a system will cause higher inventory costs, the overall effect of process control will reduce the total cost.
3.2 Example 2

Using the same input parameters used in the first example, the effect of different values of the out-of-control mean, $\mu_1$, is investigated. The effect of different values of out-of-control mean is selected because this parameter has a significant effect on various costs included in the analytical model developed in this research effort, keeping in mind that the main purpose of process control is to minimize the presence of out-of-control mean during operation. The values specified for $\mu_1$ for the four scenarios considered in this example are provided in table (3.8) below. An increment of 0.2 inches is added for each successive stage from the preceding one.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>3.7</td>
<td>3.9</td>
<td>4.1</td>
<td>4.3</td>
</tr>
<tr>
<td>(1,2)</td>
<td>6.1</td>
<td>6.3</td>
<td>6.5</td>
<td>6.7</td>
</tr>
<tr>
<td>(2,1)</td>
<td>4.9</td>
<td>5.1</td>
<td>5.3</td>
<td>5.5</td>
</tr>
<tr>
<td>(2,2)</td>
<td>2.4</td>
<td>2.6</td>
<td>2.8</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table 3.8 Values of out-of-control means for different scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$IFC_{NPC}(i,j)$</th>
<th>$EFC_{NPC}(i,j)$</th>
<th>$TC_{MNPC}(i,j)$</th>
<th>$TC_{HNPC}(i,j)$</th>
<th>$SC_{NPC}(i,j)$</th>
<th>$TOC_{NPC}(i,j)$</th>
<th>$ACC_{NPC}(i,j)$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$7,063$</td>
<td>$861,739$</td>
<td>$409,460$</td>
<td>$6,352$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$1,284,613$</td>
</tr>
<tr>
<td>2</td>
<td>$12,294$</td>
<td>$981,393$</td>
<td>$422,849$</td>
<td>$7,283$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$1,423,819$</td>
</tr>
<tr>
<td>3</td>
<td>$23,948$</td>
<td>$1,180,836$</td>
<td>$454,628$</td>
<td>$11,207$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$1,670,619$</td>
</tr>
<tr>
<td>4</td>
<td>$45,919$</td>
<td>$1,509,358$</td>
<td>$517,051$</td>
<td>$40,837$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$2,113,165$</td>
</tr>
</tbody>
</table>

Table 3.9 Total Costs with variable $\mu_1$ values for a system without Process Control
- Example Analysis:
  - Internal Failure Cost:

  After running the model for different values of out-of-control mean, $\mu_1$, it is noticed from results presented in tables (3.9) and (3.10) that an increase in the value of out-of-control mean will lead to an increased internal failure cost, regardless of whether the system is employing process control or not. This effect can be traced to equation (2.10) for a system without process control, where the term $\{ C_u(i,j) \times P[Z<\frac{(LSL(i,j)-\mu_1(i,j))}{\sigma_0(i,j)}] + C_o(i,j) \times P[Z>\frac{(USL(i,j)-\mu_1(i,j))}{\sigma_0(i,j)}] \}$ is causing the increase in internal failure cost, as out-of-control mean increases. Another term that might affect the internal failure cost is given by the combined effect of the term $[Min \left( \frac{Q(i,j)}{r(i,j)} , \frac{1}{\lambda} (i,j) \right)]$ and $[Max \left( \frac{Q(i,j)}{r(i,j)} - \frac{1}{\lambda} (i,j), 0 \right)]$ in equation (2.10). Increased values of out-of-control mean will cause the value of $Q(i,j)$ to increase according to equations (2.11) and (2.27) and hence increase the internal failure cost. However, this applies only when the ratio $\frac{Q(i,j)}{r(i,j)}$ is larger than $\frac{1}{\lambda} (i,j)$. 

<table>
<thead>
<tr>
<th>Scenario</th>
<th>IFC&lt;sub&gt;PC&lt;/sub&gt;(i,j)</th>
<th>EFC&lt;sub&gt;PC&lt;/sub&gt;(i,j)</th>
<th>TC&lt;sub&gt;MPC&lt;/sub&gt;(i,j)</th>
<th>TC&lt;sub&gt;HPC&lt;/sub&gt;(i,j)</th>
<th>SC&lt;sub&gt;PC&lt;/sub&gt;(i,j)</th>
<th>TOC&lt;sub&gt;PC&lt;/sub&gt;(i,j)</th>
<th>ACC&lt;sub&gt;PC&lt;/sub&gt;(i,j)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5,452</td>
<td>$808,530</td>
<td>$404,705</td>
<td>$9,426</td>
<td>$1,396</td>
<td>$87</td>
<td>$96</td>
<td>$1,229,693</td>
</tr>
<tr>
<td>2</td>
<td>$8,872</td>
<td>$885,452</td>
<td>$414,379</td>
<td>$12,899</td>
<td>$1,422</td>
<td>$94</td>
<td>$105</td>
<td>$1,323,223</td>
</tr>
<tr>
<td>3</td>
<td>$16,520</td>
<td>$1,012,382</td>
<td>$436,192</td>
<td>$22,829</td>
<td>$1,439</td>
<td>$112</td>
<td>$124</td>
<td>$1,489,598</td>
</tr>
<tr>
<td>4</td>
<td>$33,065</td>
<td>$1,241,777</td>
<td>$483,869</td>
<td>$57,642</td>
<td>$1,455</td>
<td>$128</td>
<td>$156</td>
<td>$1,818,092</td>
</tr>
</tbody>
</table>

Table 3.10 Total Costs with variable $\mu_1$ values for a system with Process Control
On the other hand, increased values of out-of-control mean for a system with process control can be seen through equation (2.12). Several expressions of equation (2.12) vary as out-of-control mean varies. First, in the term \(\frac{1}{\lambda(i,j)+E(BL)(i,j)}\) which represents the proportion of time the process stays in in-control conditions, the value of the expected length of the out-of-control period for the \(j^{th}\) stage of component/subassembly \(i\), \(E(BL)(i,j)\), which depends on the value \(1 - \beta(i,j)\). Here \(\beta(i,j)\) is probability of type II error of the \(j^{th}\) stage of component/subassembly \(i\). The value \(1 - \beta(i,j)\) increases as the out-of-control mean value increases, hence causing \(E(BL)(i,j)\) to decrease. Thus, increasing the proportion of time the process operates in in-control conditions. Secondly, the term \(\frac{E(BL)(i,j)}{\lambda(i,j)+E(BL)(i,j)}\) which represents the proportion of time the process stays in out-of-control conditions, is also influenced by the presence of \((BL)(i,j)\). The value \(E(BL)(i,j)\) decreases as out-of-control mean increases, therefore decreasing the proportion of time the process operates in out-of-control conditions. Another term that is effected by the out-of-control mean values for a system with process control is given by \((C_u(i,j) + P[Z<\frac{LSL(i,j) - \mu_1(i,j)}{\sigma_0(i,j)}] + C_o(i,j) + P[Z>\frac{USL(i,j) - \mu_1(i,j)}{\sigma_0(i,j)}])\). It can be seen that this quantity increases as out-of-control mean increases. Though the three quantities mentioned above will contribute to increasing the internal failure cost, this increase is lesser in a system with process control compared to a system without process control. Lastly, the value of \(Q(i,j)\) given in equation (2.12) will increase as out-of-control mean increases according to equations (2.11) and (2.28), thereby causing internal failure cost to increase.
External Failure Cost:

By merely looking at tables (3.9) and (3.10), it is noticed that external failure cost increases as the out-of-control mean increases for a system with or without process control. This effect is quantified in equations (2.22) and (2.23) for systems without and with process control respectively. For instance, external failure cost for a system without process control is given by (2.23)

$$E_{FC \text{- NPC}} = N_Q \times \left( \sum_{i=1}^{N} \sum_{j=1}^{n(i)} \text{Min} \left( \frac{Q(i,j)}{r(i,j)} , \frac{1}{\lambda} (i,j) \right) \right) \times r(i,j) \times E_{C0(i,j)} \times \sum_{i=1}^{N} \sum_{j=1}^{n(i)} \text{Max} \left( \frac{Q(i,j)}{r(i,j)} - \frac{1}{\lambda} (i,j), 0 \right) \times r(i,j) \times E_{C1(i,j)}$$

In the expression given above, the values of external failure cost per item, $E_{C0(i,j)}$ and $E_{C1(i,j)}$, of the $j^{th}$ stage of component/subassembly $i$, is dependent on the value of out-of-control mean, $\mu_1$. According to equations (2.16), (2.17), (2.18) and (2.19), as the value of the out-of-control mean increases, the reliability of the manufactured item decreases, hence resulting in an increasingly number of expected failures during the warranty period which naturally will increase the external failure costs. The combined effect of the terms $[\text{Min} \left( \frac{Q(i,j)}{r(i,j)} , \frac{1}{\lambda} (i,j) \right)]$ and $[\text{Max} \left( \frac{Q(i,j)}{r(i,j)} - \frac{1}{\lambda} (i,j), 0 \right)]$ in equation (2.22) is the same as explained in the previous section for equation (2.10). The effect is to increase the external failure cost, however, this applies only when the ratio $\frac{Q(i,j)}{r(i,j)}$ is greater than $\frac{1}{\lambda} (i,j)$.

For a system with process control, the external failure cost is given by equation (2.23),
The out-of-control mean of the $j^{th}$ stage of component/subassembly $i$, effects the quantities
\[ \frac{\gamma(i,j)}{\theta(i,j)+E(BL)(i,j)} \text{ and } \frac{E(BL)(i,j)}{\theta(i,j)+E(BL)(i,j)}. \]
Increased values of out-of-control mean for the $j^{th}$ stage of component/subassembly $i$, will increase \[ \frac{\gamma(i,j)}{\theta(i,j)+E(BL)(i,j)} \] and decrease \[ \frac{E(BL)(i,j)}{\theta(i,j)+E(BL)(i,j)}. \] In addition, the presence of $Q(i,j)$ will tend to increase the external failure cost as the value of out-of-control mean increases. However, the presence of process control will work on minimizing the time interval during which the process operates with the out-of-control mean. Therefore, the combined effect of the three quantities discussed above will increase the external failure cost but in slower rate than a system without process control.

- **Manufacturing Cost:**

The manufacturing cost is given by equation (2.26),
\[ TC_M = N_Q \times \left[ \sum_{i=1}^{N} \sum_{j=1}^{n(i)} C_M(i,j) M(i,j) \right] \]
In the expression above, the value of the quantity replenished for each batch at the $j^{th}$ stage of component/subassembly $i$, $M(i,j)$, will increase as the value of out-of-control mean increases according to equation (2.25). However, the increase in this quantity in a system with process control is less than the one in a system without process control. This finding can be seen by comparing the values of manufacturing cost for both systems given in tables (3.9) and (3.10).
• **Inventory Cost:**

The inventory cost is given by equation (2.31)

\[
TC_H = N_Q \sum_{i=1}^{N} \sum_{j=1}^{n(i)} \left( r(i,j) - d(i,j) \right) \frac{Q(i,j)}{r(i,j)}
\]

The two quantities that are affected by the increase of the out-of-control mean values are the demand rate, \(d(i,j)\), and the production quantity, \(Q(i,j)\), at the \(j^{th}\) stage of component/subassembly \(i\). The increase in the values of out-of-control mean will increase the values of both quantities, \(d(i,j)\) and \(Q(i,j)\), thereby increasing the inventory costs. By comparing the inventory costs for systems with and without process control, it was found that the system with process control will incur higher inventory cost. The explanation of this trend is provided in section “Effect of process control on inventory cost” of the analysis part of example one.

• **Sampling Cost:**

The sampling cost is presented in equation (2.32),

\[
SC_{PC} = N_Q \sum_{i=1}^{N} \sum_{j=1}^{n(i)} \left( a_1(i,j) + a_2(i,j) \frac{n'(i,j)}{r(i,j)h(i,j)} \right) \frac{Q(i,j)}{r(i,j)h(i,j)}
\]

The only quantity that is affected by increasing values of out-of-control mean in the expression above is \(Q(i,j)\). As out-of-control mean value increases, the value of \(Q(i,j)\) increases leading to greater sampling cost.

• **Out-of-control Signal Investigation Cost:**

This cost is calculated according to equation (2.38),
From Table (3.10), it is evident that the cost of investigating out-of-control signal increases as the out-of-control means value increases. This trend occurs due to the fact that as the out-of-control mean value increases, the quantity $Q(i,j)$ increases as explained earlier, and the expected length of cycle, $E(CT)(i,j)$ for the $j^{th}$ stage of component/subassembly $i$, given by equation (2.8) will decrease, resulting in increasing values of out-of-control signal investigation cost.

- Assignable Cause Removal Cost:

The cost of removing assignable cause was found to follow the same trend of the sampling cost and the out-of-control signal investigation cost. This cost is given by equation (2.39)

\[
TOC_{PC} = N_{\alpha}^* \sum_{i=1}^{N} \sum_{j=1}^{n(i)} C_1(i,j) \frac{Q(i,j)}{r(i,j) E(CT)(i,j)} \left( E(M)(i,j) + 1 \right)
\]

\[
ACC_{PC} = N_{\alpha}^* \sum_{i=1}^{N} \sum_{j=1}^{n(i)} C_2(i,j) * \frac{Q(i,j)}{r(i,j) E(CT)(i,j)}
\]

In the expression above, the two quantities that are affected by the increasing values of out-of-control mean are $Q(i,j)$ and $E(CT)(i,j)$. As the out-of-control mean value increases the value of $Q(i,j)$ increases while the value of $E(CT)(i,j)$ decreases, resulting in an increase in the assignable cause removal cost.
Chapter 4: CONCLUSIONS AND RECOMMENDATIONS

In this chapter, conclusions drawn from finding of this research effort are presented, followed by some proposed recommendations for future extensions.

4.1 Conclusions

This research was originally devoted to derive analytical models that explain the effect of adopting process control in a multi-stage system, hence assisting decision makers to make optimal decisions regarding investment in process control. Two main scenarios were considered while planning to develop the analytical models with several costs. The first scenario was a system without process control that includes the following costs; internal failure cost, external failure cost, manufacturing cost and inventory cost. The second scenario was developed for a system with process control that includes three more costs in addition to the four previous in a system without process control. The three costs are; sampling cost, investigating out-of-control signal cost and assignable cause removal cost. It is to be mentioned here that the three previous costs are incurred only in a system with process control.

In chapter three, numerical examples were presented to illustrate the use of the developed expressions. First example demonstrates the analytical models derived for both systems with their corresponding costs considered for this research. After obtaining the total cost for each system, the return on investment is determined by subtracting the total cost of the system with process control from the total cost of the system without process control. Then the net present value is utilized to make a decision regarding investment in process control,
which was found to be optimal. Therefore, the investment decision was justified. In the second example, the effect of different values of the out-of-control mean, $\mu_1$, was studied to determine the effect of that factor with respect to different costs considered in this research.

Below are the most important findings that were concluded from this research:

- Although employing process control to a system will incur additional costs, these additional costs are marginal in comparison to the amount of money saved due to process control application. This result concurs with the theory in the literature.

- The values of the common costs (External, Internal, Manufacturing and Inventory costs) between a system without process control and a system with process control were found to be greater in the system without process control. But the inventory cost was found greater in a system with process control. This finding can be utilized to adjust future planning or can be used as an advantage to satisfy just-in-time orders.

- Varying the values of the out-of-control mean revealed important information regarding the trend of various costs considered in this research. Although costs for both systems witnessed an increasing trend, system with process control costs increased at slower rate due to process control which minimized the effect of out-of-control mean, $\mu_1$. 

4.2 Recommendations

In this section some suggestions are provided for future work to extend the findings in the area of process control research. Below are some ideas that can be further developed for future work extensions:

- There are other costs that can be affected by the presence of process control that need to be investigated. These costs might include but not limited to Stock-out cost, Backlog cost and Delay cost.

- In addition to the $\bar{X}$ chart to monitor the performance of a process, consider using the $R$ chart to monitor the process variance needs to be considered as well.

- In this research, stages were considered to be independent. It is recommended to consider the case when stages are dependent. In this case, cascading property exists and hence other regression based charts are used to monitor the process such as the cause selecting control chart.

- This research considered systems consisting of multi-stages. An extension to this type of processes would be to consider system consisting of multi-levels.
References


