The Pennsylvania State University

The Graduate School

College of the Liberal Arts

ESSAYS ON PRODUCTIVITY

AND SECTORAL DIFFERENCES

A Dissertation in

Economics

by

Alonso Alfaro Ureña

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

May 2012

The dissertation of Alonso Alfaro Ureña was reviewed and approved* by the following:

Andrés Rodriguez-Clare
Professor of Economics
Dissertation Adviser
Chair of Committee

Alexander Monge-Naranjo
Assistant Professor of Economics

Stephen R. Yeaple
Associate Professor of Economics

Edward C. Jaenicke
Associate Professor of Agricultural Economics

Robert C. Marshall
Professor of Economics
Head of the Department of Economics

*Signatures are on file in the Graduate School.

Abstract

This dissertation uses sectoral data in production and patents to provide explanations for previous findings and improves the measurement of technology diffusion and productivity differences across countries. Chapters 1 and 2 deal with measuring technology diffusion. In Chapter 1 I document stylized facts regarding technology diffusion across pairs of countries in different types of goods. I focus primarily on the composition of the patents that are requested by researchers from one country in any given destination. In the data, for a given origin, some types of ideas are more likely to be patented in richer, more technologically advanced countries, while other types of ideas are more likely to be patented in poorer, less advanced destinations. In chapter 2 I tackle the challenge of measuring international technology diffusion by proposing a model that allows for more precise estimation of bilateral diffusion rates between countries compared to previous literature using the data presented in Chapter 1. Due to the abstract nature of technology and, consequently, the limited amount of data that can be used this has been a particularly difficult task. In most cases it has been possible to measure how foreign research affects a country's productivity or even how good one country is at importing or exporting knowledge from abroad. Nevertheless, measurement of how much each partner contributes to the technological development of each country has not been performed successfully. My approach is to generalize an Eaton-Kortum model for diffusion of ideas to match empirical facts regarding patent data. In the model proposed the heterogeneity in the predicted data arises from the relative differences in technological progress of the pair of countries involved. I argue that this extension contributes to measuring international technology diffusion more precisely. This is possible since the generalization of the model requires the introduction of a few parameters, while allowing for a much larger data set to be used in the estimation.

Chapter 3 deals with a productivity question. Traditionally, variance decomposition exercises across countries result in total factor productivity (TFP) accounting for about two thirds of the variance in GDP per capita. In most cases a country is modeled as a one-sector economy. Recent work by Ferreira et al. (2008) that applies this methodology to 31 years of data shows that this result does not hold in the early seventies: disparities in physical and human capital were more relevant in explaining income differences. I expand the model to include agriculture, a sector assumed to be different in terms of the production function where land in particular plays an important role. I show that ignoring the existence of this sector can cause the importance of TFP to be overestimated due to the productivity of land being implicitly assigned to it, and that this overestimation fades away as the share of agriculture in production decreases. I show that part of the increase in the importance of TFP in explaining the variance of GDP per capita can be explained by the structural transformation that most economies have experienced recently, especially poorer ones.

Contents

List of Figures

List of Tables

Acknowledgements

I wish to thank Andrés Rodríguez-Clare for his invaluable guidance and support. I would also like to thank Alexander Monge-Naranjo for his support and useful discussions, as well as Pedro C. Ferreira, Diego Restuccia, Fernando A. Veloso and Stephen R. Yeaple, Ted Jaenicke, Jonathan Eaton, Neil Wallace, Adam Slawski, James Tybout, Felix Tintelnot, Milagro Saborío Rodríguez, Bernardo Díaz de Astarloa, Yosuke Igarashi, Gary Lyn, Minsoo Han and Byung Soo Lee for comments and suggestions.

To Luz, Amelia, my parents, and Rigo.

1

Stylized facts regarding technology diffusion

1.1 Motivation

Measuring international technology diffusion is a particularly difficult task since it entails measuring a good characterized, most relevantly, for being non-rival and partially non-excludable (see Romer, 1990). These characteristics make it hard to measure. A strand of the literature regarding diffusion solves this practical problem using patent data, one of the instances where the creation of new ideas leaves some trace in paper. Nevertheless, precise measures of bilateral diffusion rates are hard to obtain even then. For example Eaton and Kortum (1999) (henceforth EK) identify on average how good a country is at sending and receiving ideas, but it is not possible to properly identify proximity in ideas among specific pairs of countries. One of the objectives of this dissertation is to show that using sectoral patent data allows for a more precise identification of this bilateral diffusion rate. I perform an extension of the model proposed in EK that not only matches stylized facts regarding the patenting patterns in the world, but also allows me to estimate bilateral diffusion rates.

It is proper to put in context what diffusion means for this dissertation, in particular for Chapters 1 and 2. Diffusion is defined as the rate at which ideas for production are transferred from the country where they are originally discovered to any other country in the world. In the literature there are two main ways to model knowledge diffusion. On one hand, it is common to model knowledge as an aggregate measure (usually labeled A), and diffusion from i to n as the effect of A_i in A_n. For example in Lucas (2007) A_i in one economy is, for some exogenous reason, large and constantly expanding while the rest of the countries grow faster due to spillovers from the leading country. However, these spillovers are not specific to any country, they work in the same way for all destinations. In a similar fashion, Coe and Helpman (1995) use a measure of local and foreign research and development (R&D) to assess the effect of research done abroad on domestic total factor productivity (TFP). On the other hand there is a large amount of literature regarding this subject where diffusion is measured for specific types of technology. Caselli and Coleman (2001) explore the case for computers and Comin and Hobijn (2008) do it for other specific technologies. This dissertation studies the issue taking a stand somewhere in between these two approaches. Technology is thought of as an aggregate variable but diffusion occurs individually for each good while the patenting of ideas is a rational decision by the researchers who discover them.

One of the most important contributions of this dissertation is the use of sectoral data to study issues that have only been studied with aggregate data. For the first two chapters I use such data regarding patents, while I use production data for the last one. Regarding the first subject, little has been studied in the field regarding broad sectoral differences in the rates at which countries are able to use technology created in other countries. When the data is analyzed, it is clear that there are some important patterns in the patenting data. There are sectoral differences in the relative importance of patenting: there are some goods that tend to be patented more in some destinations and by some countries of origin. For example, the importance of chemical goods in the total number of patents requested is usually greater in destinations that are considered less developed by usual standards. The opposite happens with mechanical goods, they tend to be patented relatively less in those destinations. Those patterns are reversed when the destination country is more developed. It is relevant, then, to construct a model that can match these stylized facts. I claim that this model (which I present in the next Chapter) is able to explain the aforementioned patterns of patenting with a generalization of the EK model. This generalization requires only a few parameters to be added, while using a much larger data set. Therefore, the additional data can be used to estimate more precisely bilateral diffusion rates which is not possible to do using standard aggregate data.

This chapter will present stylized facts regarding patent data and some alternative explanations to the patterns observed in the data. Chapter 2 will present the model, while the third and last chapter will approach the issue of income differences with the use of sectoral differences using a separate framework.

1.2 Data and stylized facts

I use data from the World Intellectual Property Organization (WIPO) regarding the number of patents requested by individuals of country i in patent offices of country n. The data is classified into 38 categories depending on the type of good to which the idea applies to. However, I prefer to use an aggregation provided by the WIPO itself which classifies these finer categories into 6 groups: chemistry, electrical engineering, instruments, mechanical engineering, green energy and other fields.[1]

The data points that I use are an average of data from 2001-2005. One caveat of the data is that I cannot distinguish precisely patents that are requested in each member country of the European Patent Office (EPO). This is due to the fact that the patenting process for an idea originating anywhere in the world could start in an individual's country office or it could start in the European office. Whenever a process begins in the EPO this could represent the intention to protect the idea in several different countries which we cannot identify in the data. Therefore, all members of EPO are grouped into one country.[2]

I am most interested in the composition of the patenting pool in pairs of countries. Define the share of patents of type s that are requested in country n by inventors of country i as P_{ins}, and the total number of patents in that couple as $P_{in} = \sum_S P_{ins}$. Hence, the share of type s is

$$\zeta_{ins} = \frac{P_{ins}}{P_{in}}$$

Table 1.1 shows the difference in the composition of USA's domestic patents compared to the patents that USA's inventors request in Brazil. The share of chemistry goods is remarkably high, almost half of the total number of patents that inventors from the United States request in the Brazilian Patent Office are of this type. In the USPTO,[3] these "domestic" patents represent only a fourth of the total. Electrical engineering ideas are much more important in the total share for american inventors patenting domestically.

[1] The green energy category is not included in the categories by WIPO, but there are enough observations in three new classifications "solar energy," "wind energy technology" and "fuel cells technology" to create a new category for which I chose a convenient name.

[2] These countries are Austria, Belgium, Bulgaria, Switzerland, Cyprus, Czech Republic, Germany, Denmark, Estonia, Spain, Finland, France, United Kingdom, Greece, Hungary, Ireland, Iceland, Italy, Liechtenstein, Lithuania, Luxembourg, Latvia, Monaco, Netherlands, Norway, Poland, Portugal, Romania, Sweden, Slovenia, Slovakia, and Turkey. Other countries that became members of EPO after

Table 1.1: Patents by USA inventors in each destination country (shares).

	Brazil	United States
Electrical Engineering	**16.8%**	**34.5%**
Chemistry	**45.8%**	**25.0%**
Instruments	11.7%	17.2%
Mechanical Engineering	20.4%	15.4%
Other fields	5.0%	7.3%
Green Energy	0.3%	0.6 %

This interesting pattern emerges also when other countries are considered, as in Figure 1.1. Patents that originate in the United States for Chemistry and Mechanical engineering goods constitute a larger share in destination countries that are developing and that perform significantly less research than developed countries.

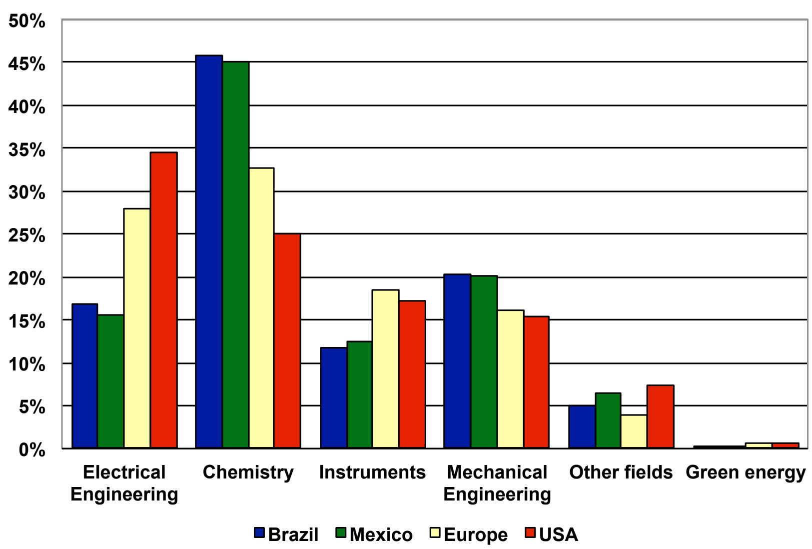


Figure 1.1: Share of USA patents in each destination ($\zeta_{\mathrm{USA}ns}$)

A more detailed analysis shows that the differences between the shares of patenting in different destination countries are statistically significant. In Table 1.2, I present several statistical tests that show that there is a strong relationship between the com-

2005 are not taken into account, which are Albania, Croatia, Macedonia, Malta, Serbia and San Marino.

[3]United States Patent and Trademark Office.

position of the patents requested at the USA Patent and Trademark Office (USPTO) by domestic agents and the composition of the patents requested in other research intensive destinations by the same agents. I take the six data points $\zeta_{\text{USAUSA}s}$ and compare them with the same six data points $\zeta_{\text{USA}ns}$ for each destination. A simple correlation coefficient, a Spearman ranking coefficient and the coefficient b_n of a simple regression $\zeta_{\text{USAUSA}s} = a + b_n * \zeta_{\text{USA}ns}$ are reported. The countries are ranked so that the country in the top is the one that produces more domestic patents (inventor's country of origin and patent office coincide), whereas the last country is the one that has the smallest number of domestic patents.[1]

Table 1.2: Descriptive statistics for patents originated in USA.

Country	Correlation	Spearman	Regression
JPN	0.94	0.94**	0.76**
EUR	0.92	0.80	0.88**
KOR	0.97	1.00***	0.74**
CHN	0.97	0.94**	0.83**
CAN	0.72	0.94**	0.68
BRA	0.62	0.71	0.47
MEX	0.60	0.60	0.47
Legend: $*p < 0.05; **p < 0.01; ***p < 0.001$			

This pattern is present when we take into account all countries that report receiving and sending patents in at least five types of goods (15 in total). To evaluate this claim, I run a regression for each type of good

$$\zeta_{ins} = a + b_s * \frac{Y_n}{Y_i}$$

and report coefficients b and c, where Y_n and Y_i are the GDP per capita of the country that receives and sends the patents respectively.

This regression suggests that Chemical goods are less important in the composition of the total patents whenever the GDP of the destination country is greater. The opposite happens for Electrical engineering, Instruments and Other fields. This same qualitative result is still valid when I use countries that receive and patent in all types

[1] Only seven countries are selected since few countries have information for all six sectors. If I exclude the Green Energy sector, the results are very similar, the patenting composition in the USA is less related to the composition in countries with less domestic patents are requested.

Table 1.3: Regression using countries that send and receive patents in all types.

	Chem.	Elec. Eng	Green Energy	Instrum.	Mech. Eng.	Other fields
$\frac{Y_n}{Y_i}$	-0.475***	.373***	0.009	0.082*	-0.077	0.089*
Legend: $*p < 0.05; **p < 0.01; ***p < 0.001$						

of goods (10 countries), in at least two types of goods (19 countries), and even when I take into consideration destinations and origins for which there is no information in one of the directions (30 countries). Performing this exercise in an even more limited set of countries like Australia, Brazil, Europe, and USA also yield the same results. Finally, for the countries that have information of domestic patents, a relative ratio of domestic patents is constructed and used (in logs) in the regression instead of the relative GDP data. The quantitative results are still the same.

1.3 A possible explanation to the data

I will describe in this section a possible explanation to the features of the data described above: is it that comparative advantage is driving these results? It could be that Brazil is particularly good in producing goods related to the chemical industry and that is why firms decide to patent proportionally more goods in that destination, while the comparative advantage of richer countries lies in electrical engineering.

To explore revealed comparative advantage in this section I require a different dataset for this analysis (much more disagreggated) than the one presented above (which is used for the model in Chapter 2) since patent data must be matched with other sources. The particular data used for this analysis is compiled by the European Patent Office in the Worldwide Patent Statistical Database (PATSTAT).[1] The data points contain information on patents applied for in the year 2005, and records the country of origin of the inventor,[2] the office where it is requested and the field according to the International Patent Classification (IPC). Also, an important characteristic of the dataset that I use is that whenever a patent is requested which applies to more than one field in the IPC coding, those are counted as one invention in each field. This causes the total number of patents in the dataset to exceed the actual number of filings.

[1] I was able to do the exact query to the data thanks to the financial support of Andrés Rodríguez-Clare and the kind technical help of Gianluca Tarasconi in Università di Bocconi (Lissoni et al. (2006)).

[2] The first person or company mentioned in the patent application.

The PATSTAT has the same problem of aggregation for European data, so members of the European Patent Office at the time are grouped as one country again. The problem with presenting the data in such way is that in some cases I could be double counting patents if there are requests in different offices.[1] In order to compare patent data with other sources of data I will convert the PATSTAT data from IPC to ISIC Revision 2[2] using a concordance developed by MERIT.[3] Table 1.4 presents the sectors used for this analysis.

Suppose that there is some reason for a country to have an advantage in the production of a specific type of good such as a technological advantage or consumer preferences specific to the country. This would cause those types of goods to be more profitable to patent in a destination. A higher profitability of a type of good must cause more patents to be requested in that destination by foreigners. Nevertheless, if this were true, domestic patents should also show the same pattern and more domestic patents should be requested for the more profitable goods. How do domestic patents compare to patents requested by foreigners?

I will compare two variables. The first one compares the relative importance of type "s" in the domestic patents (invented and patented in country "i") to the composition of the patents in country "n" by those same inventors. This variable tells us if a good is patented relatively more in a destination compared to the domestic market:

$$M_{ins} = \frac{\frac{P_{ins}}{P_{in}}}{\frac{P_{iis}}{P_{ii}}}. \tag{1.1}$$

This constitutes a measure for the composition of traded ideas. The other statistic that I construct is more closely related to the Revealed Comparative Advantage (RCA) used in trade literature. This "Patent RCA" will compute a ratio of the share of patents requested ***domestically*** for a type of good with respect to the corresponding share of

[1] I detected that some applications don't have a country of origin in the database. I made the assumption that these were all domestic patents because, for example, the numbers for Japan were too small. Besides Japan, the countries where domestic patents are significantly altered by this fix are Australia, Brazil, Hong Kong, South Africa, Singapore, New Zealand and Morocco.

[2] International Standard Industrial Classification of All Economic Activities, Rev.2

[3] Maastricht Economic Research Institute on Innovation and Technology. This concordance allows to convert IPC data to ISIC Revision 2, which is the coding used for national accounts. There are other concordances for patent data, for example one developed for the OECD by Daniel Johnson, but the conversion is done for ISIC Revision 3. On the other hand, the concordance that I was able to find regarding trade data was only available from SITC Revision 2 to ISIC Revision 2, and therefore limited which concordances I was able to use. The trade concordance I mention is developed by Marc Muendler, and can be found in his website.

Table 1.4: Classification for comparing trade and patent data (ISIC Rev. 2).

Code	Sector
100	Agriculture
310	Food, Beverages, Tobacco
320	Textiles, clothes, etc.
330	Wood and furniture
340	Paper, printing and publishing
351	Chemistry, pharmacy (combined with 352)
353	Oil refining (combined with 354)
355	Rubber and plastic products (combined with 356)
360	Stone, clay, and glass products
371	Ferrous basic metals
372	Non ferrous basic metals
381	Metal products
382	Computers, office machines and other machinery
383	Electric machines and electronics)
384	Shipbuilding, motor vehicles, aerospace, other transport
385	Instruments
390	Other industrial products
400	Utilities
500	Building and construction

patents requested for that type ***anywhere in the world***. It reveals for a given country how important is a type of good in terms of number of patents when compared to the world. Then, the Patent RCA can be defined as

$$PRCA_{ns} = \frac{\frac{P_{nns}}{P_{nn}}}{\frac{P_s}{P_{world}}} \tag{1.2}$$

I will assume that the universe of new ideas in the world is composed by the total number of ideas that are patented in domestic offices.[1] This assumes that any other request in other offices around the world do not constitute new ideas, but the desire to extract rents from the patent in additional destinations.

If a country that receives proportionally more patents (a higher M_{ins}) in some type of good because it has some comparative advantage in that good that causes higher profits for firms producing it, then this country should also have a high Patent RCA. If I use the data for the United States as the country of origin and I compare the Patent

[1]This is, all ideas that were created by individuals and were also patented in the country where those individuals are residents.

RCA to the $M_{USA,ns}$ ratio defined previously, there is no clear relationship that can be established.[1] The regression reported in Table 1.5 is $M_{USA,ns} = a + b * PRCA_{ns}$, where the result for the coefficient b is not significant, and the correlation coefficient between the two variables is only 0.0252.

Table 1.5: Patent RCA as explanatory variable of the share of patenting from USA

Variable	Coefficient	(Std. Err.)
$PRCA_{ns}$	0.131	(0.398)
Intercept	1.301^{**}	(0.504)

Number of observations: 174

Could it be that Patent RCA does not reflect correctly the comparative advantage of the country? To test that hypothesis I use data from Commodity Trade Statistics (COMTRADE) to compute a Revealed Comparative Advantage index in the tradition of classical trade literature. This data is converted from SITC Revision 2 to ISIC Revision 2 using the concordance provided by Marc Muendler in his website. It is worth noting that the ISIC classifications used for the previous part correspond mostly to manufacturing sector. Therefore, I use those sectors to compute the aggregate trade values. The index is

$$RCA_{ns} = \frac{X_{ns}/X_n}{X_{world,s}/X_{world}}$$

where X is total exports and the indexes represent the usual elements.

I run a regression with the available data using this trade RCA with respect to Patent RCA in the following form $RCA_{ns} = a + b * PRCA_{ns}$ and the results are reported in Table 1.6, which show a significant and positive coefficient. The correlation coefficient between the two variables is 0.3315.[2]

Table 1.6: Patent RCA as explanatory variable of Trade RCA

Variable	Coefficient	(Std. Err.)
$PRCA_{ns}$	0.655^{**}	(0.099)
Intercept	0.543^{**}	(0.139)

Number of observations: 358

[1] There are some cases in which a country presents only a small sample of patents, causing the ratios estimated to be greatly inflated. I will use for the estimations presented here only the ten countries in which 14 or more types of goods (out of 19) are present in the data.

[2] I use in this case a total of 22 countries.

As an additional test, I run the regression $M_{USA,ns} = a + b * RCA_{ns}$, where the left hand side variable is the one that indicates the composition of USA patents in every destination. The results are reported in Table 1.7, which show that the relation between the two variables is not significantly different from zero. In this case the correlation coefficient is 0.0507.

Table 1.7: Trade RCA as explanatory variable of the share of patenting from USA

Variable	**Coefficient**	**(Std. Err.)**
RCA_{ns}	0.039	(0.0614)
Intercept	1.181**	(0.117)

Number of observations: 159

Therefore, there is evidence that the patterns observed in the patenting data are not related to revealed comparative advantage observed in the trade data. It is not the case that more patenting occurs because destination countries are just better at producing some goods, there are other reasons causing this.[1] The following Chapter proposes a model in which the patenting behavior across pairs of countries can be captured without the need of comparative advantage in sectors and the pattern is related to how technologically different the two countries involved are.

[1] A similar excercise to the one proposed in this section could be developed using Foreign Direct Investment (FDI) data. Currently not enough sectoral information is available for all countries (UNCTAD and OECD databases). It is, for example, particularly difficult to construct the data for the European countries (grouped as one country throughout this dissertation), and there are countries like Australia for which data is disaggregated by sectors but in a very coarse way. Manufacturing which is one of the finest categories that can be found for this country is a classification that includes most of the categories used here, so little can be done with this information.

2

A model for measuring bilateral technology diffusion

2.1 Introduction

The previous chapter of this dissertation introduced stylized facts regarding international technology diffusion. Salient facts include that the composition of patents varies by destination, and that this difference cannot be explained by revealed comparative advantage as usually defined in the trade literature. I will now present a model that attempts to provide an explanation to those stylized facts. This is, however, a stepping stone for the main purpose of the model. The crucial feature is that patterns of patenting provide additional information that will allow for a better measuring bilateral technology diffusion rates .

2.2 The model

This is a model of growth in which the creation of individual ideas by agents in the economy is explicitly modeled. In broad terms it can be summarized as follows: there are N countries in the world. Workers in each country must decide whether to work in research or in the production of goods. Ideas are produced by researchers at a given rate and after a new idea is born, they must decide whether to patent or not this idea in each of the N possible markets given the possible competing ideas that are available in that country. New ideas created anywhere could eventually become part of the stock of knowledge of each country by diffusion to that destination.

There are many elements to this model which need detailed explanation, especially the variables that the worker takes into account to make his choice, as well as the value of an idea after it has been created. I will present a version of the model that generalizes the EK model and I will later highlight where the differences and my own contributions lie.

2.2.1 Production

There are N countries in the world and S types of intermediate inputs, also referred to as goods henceforth. There is a continuum of these intermediates of size J, and this measure is partitioned in S types. The corresponding fraction of type s, where $s \in \{1, 2, \ldots, S\}$, is a measure of size J_s, thus $J = \sum_{s=1}^{S} J_s$. Each individual good is denoted j_s. These intermediate inputs, which are nontraded,[1] are combined to produce one tradeable, homogeneous final good, Y_{it}, with a constant returns to scale function

$$\ln\left(\frac{Y_{it}}{J}\right) = J^{-1} \sum_{s=1}^{S} \int_{j_s \in J_s} \ln[Z_{it}(j_s) X_{it}(j_s)] dj_s,$$

where $X_{it}(j_s)$ denotes the quantity of good j_s produced in country i at time t, while $Z_{it}(j_s)$ is the quality of that good. I assume for simplicity that the intermediate input j_s is produced using only labor in a linear relationship, $X_{it}(j_s) = L_{it}(j_s)$.

2.2.2 Research and new ideas

There are L_{it} workers in country i at time t. A fraction γ_{it} of those (endogenously) decide to work in research, and are able to produce ideas at a rate $\bar{\alpha}\alpha_{it}\gamma_{it}^{\beta} L_{it}$, where $\bar{\alpha}$ is a productivity parameter common to all countries. The parameter α_{it} represents the research productivity of country i at time t, while $\beta < 1$ represents the decline of productivity in research when more individuals are involved.[2]

New ideas have two characteristics: to which good does this idea apply, and the quality of the idea in question. Regarding the first, new idea could apply to any intermediate j_s with the same probability. This also implies that there is probability J_s/J that the idea will be of type type s. As for the latter characteristic, qualities are

[1] The nontradability assumption arises from the fact that to be able to sell a good in a world with property rights, it is necessary to produce the good yourself or license it for production. It is not possible to enter the market through exports without the proper license.

[2] This formulation implies that there is heterogeneity in the productivity of workers when engaged in research, see Phelps (1966). This functional form arises, for example, if the workers' productivity is drawn from a Pareto distribution as noted in Eaton and Kortum (1999).

randomly drawn from a type-specific distribution, so there is a technological difference across types of goods. If a new idea applies to a good of type s, its quality will be drawn from the Pareto distribution

$$G_s(q) = 1 - \left(q/\underline{q}\right)^{-\theta_s}.$$

2.2.3 Diffusion and accumulation of ideas

Whether an idea is good enough to be used in a given country and if so how profitable it is both depend on the stock of ideas already available. At any point in time in country i there are T_{it} ideas that can be used for production which have been discovered thanks to either domestic or foreign research and have diffused to country i. A new idea for production of good j_s that has diffused to a market has to compete with the ideas from that stock that apply to the same good.

These ideas in T_{it} were discovered in the past, and are part of the stock of knowledge because they have diffused to that country. This does not necessarily mean that they are used in production since they may be old ideas or newer but bad ideas. Diffusion only means that the knowledge exists in that market so that production could start if desired. EK refer to an idea that has been used as an adopted idea.

The diffusion lag of ideas among any pair of countries i and n, ρ_{in},[1] has an exponential distribution with parameter ϵ_{in}, $\Pr[\rho_{in} \leq x] = 1 - e^{-\epsilon_{in}x}$. Let $\dot{T}_{it}$ the rate at which ideas diffuse to country i at time t. Then the law of motion of the stock of ideas is

$$\dot{T}_{it} = J^{-1} \sum_{n=1}^{N} \epsilon_{in} \int_{-\infty}^{t} e^{-\epsilon_{in}(t-t')} \bar{\alpha} \alpha_{nt'} \gamma_{nt'}^{\beta} L_{nt'} dt'.$$

As I said, T_{it} is the number of ideas that can be used, but not necessarily the ideas that are used in practice. The technological frontier of the country is the distribution of the maximum draws from the Pareto distribution. Although the Pareto distribution considers only qualities $q \geq \underline{q}$, set $\bar{\alpha}\underline{q}^{\theta} = 1$. This allows the technology frontier to consider qualities $z > 0$,[2]

$$F_{ist}(z) = e^{-T_{it} z^{-\theta_s}}$$

which is the usual Fréchet distribution.

[1]In my notation the first country in a subscript pair will be the inventive country, where the idea was discovered, while the second is the country of destination, where the idea can be used for production.

[2]See Appendix B. Note that $T_{it} = \frac{\int_{-\infty}^{t} \dot{T}_{ns} ds}{\bar{\alpha}}$.

In this expression the size of θ_s determines the dispersion of the qualities of the goods of type s. The lower the value of this parameter, the greater the dispersion in the qualities. Also, $T_i > 0 \;\; \forall i \in N$, and this parameter determines the location of the distribution. A high value implies that country n will get high draws for all goods.[1] A country with a high T_i (due to high accumulated research from own and foreign sources) will have ideas with, on average, higher qualities used for production.

Note that the stock of knowledge of each country is not type specific. I will argue later on the chapter why I think this is a better modeling choice than allowing for knowledge to accumulate separately for each sector. For the moment just think that if a country is technologically advanced, this dominance will be reflected in all of the economy in the same proportion. Differences between the sectors rely exclusively in the dispersion parameters, θ_s.

2.2.4 Value of ideas

Ideas in this model can be thought of as the possibility to start up a firm. Once an idea to produce a good of type s with a certain quality q exists the agent that came up with this knowledge has to decide immediately whether to patent it or not in each of the N possible markets.[2] Since there is no trade in this model, it is not possible to sell intermediate goods to a particular country with a quality that has not diffused to that country.[3]

Requesting a patent in a market has one effect only: reducing the probability that an agent in the market can use the same idea for production, i.e. being imitated, denoted ι. If an individual is working in research and discovers a new idea, I will assume that he knows

- the quality of that idea,
- the type of good to which it applies,

[1]Figure 1 of Fieler (2009) exemplifies how different values for the parameter affect the distribution.

[2]The "Paris Convention for the Protection of Industrial Property," which governs the patenting process worldwide, states that after filing in one market, inventors have exactly one year to request patent protection in other markets where protection would also begin at the same date as the request at the original market (priority date). After this date, their ideas are not protected. Therefore, the assumption that all patent decisions must be made simultaneously seems reasonable, as argued also by EK.

[3]A patent prevents that a good to be "made, used, distributed or sold without the patent owner's consent" (WIPO). It is possibly more difficult in practice for goods that are patented to enter via imports since it has to be inspected in customs.

- whether the idea is the best idea for good j_s in his own domestic market. Only the ideas that constitute best ideas in the domestic market will be considered for patenting elsewhere,
- the technological distribution of all types of goods, controlled by θ_s,
- the stock of knowledge of each country T_{it},
- the rates at which ideas diffuse across countries ϵ_{in},
- the rate at which ideas can be imitated ι,
- the costs of requesting the patent f_{int} for a firm in country i requesting the protection for its technology in country n.

The patenting decision depends on the expected profits that the new potential firms have. A firm that owns an idea can reap profits only if the idea has diffused to a given market and no other agent there has copied the idea. If someone copies his idea or if the idea never diffuses, then the firm will have no profits from that market.

A firm that produces with quality q can only have positive profits if its technology is greater than the existing one, $q > z$. Whenever a new technology satisfies this condition, I will refer to it as a "**breakthrough**" (BT). As detailed before, researchers from i know whether the quality that they just discovered constitutes a BT in their domestic market. This availability of information is assumed to be costless, any researcher who discovers a new idea can obtain such information from its market (say, for example, from the domestic patent office).

Even when researchers know the quality of the idea that they just discovered q, its type s, whether it is BT in the domestic market, and the shape of the technology frontier for the type, they have no knowledge of the exact quality of the existing technology z. In Grossman-Helpman terms, researchers know that their work constitutes a step in the quality ladder, but not how big. This is, they know if $q - z > 0$, but they don't know the size of $q - z$. It can certainly happen that the quality of a new idea for a certain good is small enough that researchers decide not to request a patent; it could also be the case that even when a patent was requested, this decision turns out not to be a profitable one ex-post.

Regarding profits, suppose that the existing quality z in market n is known. If the final good is taken as a numeraire, the good j_s to which the idea applies has a demand equal to $\frac{Y_n}{Jp_n(j_s)}$ in market n. From the market structure, there is a unique unit cost of production which corresponds to the wage w faced by all firms. Assuming Bertrand competition, the innovating firm imposes a price $p = (q/z)w$, the highest price at which the existing technology is not profitable. Therefore, if in fact $q > z$ and the idea has diffused, starting production will lead to profits of $(1 - \frac{z}{q})\frac{Y_n}{J}$.

Firms do not actually know z, it is rather the value of expected profits the one that matters for the decision. Also, recall that the only effect that a patent has is to lower the probability of the idea being copied in a specific market. Therefore, a researcher will patent the idea if lowering this risk has more benefits than the cost of requesting the patent. It is then necessary to compute the ex-ante value of an idea discovered by the researchers.

When computing the profits of a firm that produces quality q it is necessary to account for the fact that there is a probability $e^{-(T_{n,t+t'} - T_{n,t})q^{-\theta_s}}$ that at time $t' > t$ this technology has become obsolete. Also, the expression $e^{-\iota(t'-t)}$ accounts for the possibility that the idea has been copied at period t', which occurs at a rate ι.[1] Therefore, the expected profits of the firm that produces the breakthrough quality q at time t given that the existing technology is z are

$$V_{inst}(q, z) = \int_0^\infty \left(1 - \frac{z}{q}\right) \frac{Y_{n,t+t'}}{J} e^{-(r+\iota)t'} (1 - e^{-\epsilon_{ni} t'}) e^{-(T_{n,t+t'} - T_{n,t})q^{-\theta_s}} dt' \quad (2.1)$$

whenever $q > z$ and zero otherwise.

As I said, the researcher that came up with quality q doesn't know exactly how different is q from the existing quality used in the market z. Therefore, the expected value of an idea will depend on the distribution of the qualities z as given by the technology frontier. It is worth noting that the diffusion process through which a new idea could become useful in another country is independent from each other, there is no good or type specific driving how fast ideas become available in other destinations.

In the case of the domestic market I assume, as I mentioned, that the inventor knows whether it is BT or not. The researcher also has some information regarding the likeliness that his own idea will also constitute a BT abroad.[2] For a BT idea in country n the expected value of an idea is

$$V_{inst,\mathrm{BT}}(q) = \int_0^q V_{inst}(z, q) dF^q_{nst}(z). \quad (2.2)$$

[1] The rate of copying will differ for ideas that are patented and those that are not. This possibility will be explained with more detail later on.

[2] Suppose that a country n has no domestic research and it only has technological communication (receives diffused ideas) from country i. If an idea is BT at country i it will also, for sure, be a BT idea in country n.

The superscript q in the cumulative distribution function denotes the distribution of qualities of type s conditional on the ideas being lower than q.[1]

For the foreign markets there is uncertainty on whether or not the idea is BT. If there is no information to determine whether the idea is BT or not, then the expected value of the idea is

$$V_{inst,\mathrm{NB}}(q) = \int_0^q V_{inst}(z,q)dF_{nst}(z) \tag{2.3}$$

where the distribution of competing qualities is not conditional.[2]

The likeliness that the idea is for sure a BT depends on the probability that the existing technology in country n was originally created in country i and, therefore, also constitutes a breakthrough in country n, denoted π_{ni}. Recall that all new ideas, no matter where they originate, are drawn from the Pareto distribution. Since T_n summarizes the total number of ideas available in country n, probability π_{ni} is just

[1] This conditional cumulative distribution is defined as

$$\begin{aligned} P[Z \leq z | Z \leq q] &= \frac{P[Z \leq z \wedge Z \leq q]}{P[Z \leq q]} \\ &= \frac{\int_0^z f(s)ds}{\int_0^q f(s)ds} \\ &= \frac{e^{-Tz^{-\theta}}}{e^{-Tq^{-\theta}}} \end{aligned}$$

[2] Suppose that we analyze a simple one period model in which there is no diffusion, probability of the idea being copied or outdated. The profits of the firm in country n can be explicitly computed as

$$\begin{aligned} & \int_0^q \left(1 - \frac{z}{q}\right) e^{-T_n z^{-\theta}} \theta T_n z^{-\theta-1} dz \\ = & \int_0^q e^{-T_n z^{-\theta}} \theta T_n z^{-\theta-1} dz - \int_0^q \frac{z}{q} e^{-T_n z^{-\theta}} \theta T_n z^{-\theta-1} dz \\ = & \int_b^\infty e^{-x} dx - \frac{1}{q T_n^{-1/\theta}} \int_b^\infty x^{-1/\theta} e^{-x} dx \\ = & \Gamma(1,b) - b^{1/\theta} \Gamma[(\theta-1)/\theta, b] \\ = & e^{-b} - b^{1/\theta} \Gamma[(\theta-1)/\theta, b] \\ = & e^{-b} \left(1 - e^b b^{1/\theta} \Gamma[(\theta-1)/\theta, b]\right) \end{aligned}$$

For the domestic market, this firm would have profits

$$\begin{aligned} & \int_0^q \left(1 - \frac{z}{q}\right) \frac{e^{-T_n z^{-\theta}}}{e^{-T_n q^{-\theta}}} \theta T_n z^{-\theta-1} dz \\ = & \left(1 - e^b b^{1/\theta} \Gamma[(\theta-1)/\theta, b]\right) \end{aligned}$$

The effect of the uncertainty of not knowing whether the idea is a breakthrough or not are summarized in the element $e^{-b} < 1$. This uncertainty causes the profits of the firm to be reduced in $1 - e^{-b}$.

the fraction of the ideas in T_n that originated in country i. Note that whenever $i = n$, $\pi_{int} = 1$ because I am assuming perfect knowledge of BT status in the domestic market.[1]

$$\pi_{int} = \begin{cases} \frac{J^{-1}\epsilon_{ni}\int_{-\infty}^{t}\int_{-\infty}^{u} e^{-\epsilon_{ni}(u-t')}\alpha_{it'}\gamma_{it'}^{\beta}L_{it'}dt'du}{T_n} & \text{if } i \neq n \\ 1 & \text{if i=n} \end{cases}$$

Therefore, taking into account the probability that an idea is breakthrough or not, the expected value of an idea q in country n is

$$V_{inst}(q) = \pi_{int}V_{inst,\mathrm{BT}}(q) + (1 - \pi_{int})V_{inst,\mathrm{NB}}(q).$$

As mentioned above, I will assume that the rate at which ideas are copied by agents in the destination market are different whenever there is a patent compared to whenever a patent has not been requested. Requesting a patent (and paying the cost associated to it) reduces the risk of patenting. Thus, there are two possible expected values of an idea, one whenever a patent has been requested and the rate is ι^{pat}, and one when the patent has not been requested, where the rate is ι^{not}. The difference of those expected values is the benefit of requesting a patent in country n. The cost, on the other hand, is going to be denoted f_{int}. The firm has the incentive to request a patent for the idea if the following inequality hods:

$$V_{inst}^{\mathrm{pat}}(q) - V_{inst}^{\mathrm{not}}(q) - f_{int} \geq 0 \tag{2.4}$$

The expression on the left hand side is the expected profits of a firm from country i considering patenting an idea of type s in country n at time t.

Is is important to note at this point where the specific contributions of this model lie compared to Eaton and Kortum (1999). The introduction of different types of goods which are different in the parameter θ_s is a crucial generalization in this model, EK presents a one type version. Additionally, the assumption that researchers know whether their own ideas constitute a BT idea in the domestic market is of relevance. In the EK version researchers only know the quality of the idea that they produced q, but are unaware of the quality leap that it represents. In that setup, it could happen that researchers request domestically a patent for an idea that has a lower quality than the existing technology in that market. In my setup, this possibility is forgone for the domestic market.

[1]If the existing technology in n originated in i but the quality overtaken by the innovation in market i originated elsewhere (for example, in a third country k), this also necessarily implies that the prevailing quality in country n is still lower than the breakthrough innovation in i.

2.2.5 Equilibrium

First, note that profits are increasing in the quality of the good. It is straightforward to realize that given Equation (2.4), there exists a cutoff value $\bar{q}_{inst}$ such that ideas of higher quality are patented, whereas ideas of lower quality are not patented.

Given parameters ϵ_{in}, θ_s, J_s, β, ι_{pat}, ι_{not}, f_{in}, an equilibrium in this dynamic framework is a vector of γ_{it} and $\bar{q}_{inst}$ such that

- researchers patent to maximize profits
- workers maximize expected income by choosing in which sector to work.

The share of workers that engage in research γ_{it} closes the model. Note that given that an idea could apply randomly to any type of good s, the value of an idea produced by a researcher from i in country n before knowing its quality is

$$V_{int} = \sum_{s=1}^{S} \frac{J_s}{J} \left\{ \int_0^{\bar{q}^{inst}} V_{inst}^{\text{not}}(q) dF(q)_{ist} + \int_{\bar{q}_{inst}}^{\infty} [V_{inst}^{\text{pat}}(q) - f_{in}] dF(q)_{ist} \right\} \tag{2.5}$$

When all possible destination markets are considered, then the value of an idea is

$$V_{it} = \sum_{n=1}^{N} V_{int} \tag{2.6}$$

A fraction of workers decide to engage in research activities so that for the marginal worker the value of performing research is exactly the same as the wage that he would get in the production of goods. Hence, we must know the ex-ante value of research. When workers make the decision on which sector to work at, they are unaware of both the quality and type of ideas that they will eventually discover.

In equilibrium the marginal worker must be indifferent between doing research and working in the production of intermediates. Therefore, the share of workers that do research, γ_i should solve

$$\alpha_{it} \beta \gamma_{it}^{\beta-1} V_{it} = w_{it} \tag{2.7}$$

The total number of new ideas available for patenting in country i at time t is given by

$$\frac{1}{T_{it}} \alpha_{it} \gamma_{it}^{\beta} L_{it} \tag{2.8}$$

This accounts for the fact that only a fraction $\frac{1}{T_{it}}$ of new ideas will be actually ever considered for patenting since only domestic BT ideas are considered for patenting in the rest of the world.

2.2.6 Steady state

This model introduces different assumptions regarding the patenting decision process that firms go through. Nevertheless, besides accounting for different types of goods it is the same as the EK framework regarding the accumulation of knowledge and diffusion. Therefore, the same conditions required for a steady state are required in this framework. Namely, for the state variables T_{it} to grow at a constant common rate g I will assume that

- workforces are constant,
- productivity of each country is proportional to the stock of knowledge[1]

$$\alpha_{it} = \alpha T_{it},$$

- patenting costs are proportional to output, $f_{int} = f_{in} Y_{nt}$,
- interest rate r is constant across countries and time.[2]

The labor market closes the equilibrium in this economy: a fraction of the total labor force available in the country choose to work in research given the value of the research performed by the marginal researcher and the wage paid to him. The solution to the labor market is denoted in steady state by the shares γ_i. This is the share of workers that will do research in country i.

[1]This assumption and the previous one imply endogenous growth. It is possible, as EK do, to find a steady state equilibrium with semi-endogenous growth in which the productivity of each country takes the general form

$$\alpha_{it} = \alpha \left(\frac{T_{it}}{\bar{T}_t} \right) \bar{T}_t^{\eta}$$

where $\bar{T}_t = \sum_{i=1}^{N} T_{it}$. For semi-endogenous growth the growth of population $g_L > 0$ and $\eta < 1$, while $\eta = 1$ for the case analyzed here. It is worth noting that there must be some long run relationship between the productivities of the countries in order to reach a steady state. Otherwise, the gap between stocks of knowledge would increase forever, so a steady state would be impossible.

[2]Although not explicitly modeled, in this general equilibrium model the interest rate is an equilibrium object that depends on intertemporal preferences.

For a steady state productivities in all countries should grow at the same rate g. As shown in EK, there are two sets of equations that need to be solved simultaneously. The first is

$$\alpha_i \beta \gamma_i^{\beta-1} V_i = w_i,$$

where V_i denotes the expected value of a new idea in country i. The second set is

$$g = \frac{\dot{T}_n}{T_n} = \frac{\alpha}{J} \sum_{i=1}^{N} \frac{\epsilon_{in}}{\epsilon_{in} + g} \frac{T_i}{T_n} \gamma_i^{\beta} L_i \qquad n = i \dots N.$$

Growth must be equal in all countries, and the share of individuals engaging in research must match in equilibrium the relative ratios of the stock of knowledge for each country.[1]3

2.2.7 Implications of the model

It can be shown that the distribution of the quality of BT ideas is Fréchet, with the same parameters as the technology frontier of the country where they originate.[2] Since for each destination market there is a cutoff quality $\bar{q}_{inst}$ above which ideas will be patented, the patenting rate of ideas of type s in country n by researchers of i is defined as

[1] In this setup, A_i can be defined as the total factor productivity of economy i. It is the average productivity of all the ideas in the economy, and it is defined as $e^{\frac{\psi}{J}\sum_{s=1}^{S}\frac{J_s}{\theta_s}} T_i^{\frac{1}{J}\sum_{s=1}^{S}\frac{J_s}{\theta_s}}$, where ψ is Euler's constant. Hence given g (the growth of T's), then $g_A = \frac{\dot{A}_n}{A_n} = g\sum_{s=1}^{S}\frac{J_s}{J}\frac{1}{\theta_s}$.

[2] Intuitively, it makes sense since the Fréchet distribution is the max of a large number of draws from the Pareto distribution. Since new ideas are distributed Pareto, $G(q)$, the number of ideas that will actually be considered for patenting are only those that are greater than the technology frontier which is distributed Fréchet, $F(z)$. Recall that the Pareto has a support $[\underline{q}, \infty]$. Then, these new ideas that are actual breakthroughs are distributed

$$\begin{aligned}
\Pr[Q \leq q | Z \leq Q] &= \frac{\Pr[Q \leq q \wedge Z \leq Q]}{P[Z \leq Q]} \\
&= \frac{\int_{\underline{q}}^{q} \int_0^s f(t) g(s) dt ds}{\int_{\underline{q}}^{\infty} \int_0^s f(t) g(s) dt ds} \\
&= \frac{\int_{\underline{q}}^{q} F(s) g(s) ds}{\int_{\underline{q}}^{\infty} F(s) g(s) ds} \\
&= \frac{\int_{\underline{q}}^{q} \frac{1}{\underline{q}^{-\theta}} e^{-Ts^{-\theta}} \theta s^{-\theta-1} ds}{\int_{\underline{q}}^{\infty} \frac{1}{\underline{q}^{-\theta}} e^{-Ts^{-\theta}} \theta s^{-\theta-1} ds} \\
&= \frac{e^{-Tq^{-\theta}} - e^{-T\underline{q}^{-\theta}}}{1 - e^{-T\underline{q}^{-\theta}}}
\end{aligned}$$

Given the normalization proposed, $\underline{q} \to 0$, thus the resulting distribution is $P[Q \leq q | Z \leq Q] = e^{-Tq^{-\theta}}$.

$$P_{ins} = \frac{J_s}{J}\frac{1}{T_i}\alpha_i\gamma_i^{\beta}L_i[1 - F_{is}(\bar{q}_{ins})]$$

There is a relevant implication of having more than one type of good in the way that it has been presented in this model. The differences in the technology parameters θ generate variation in the patenting rates for different types of goods across pairs of countries. To see why this is the case, define the following variable

$$\zeta_{ins} \equiv \frac{J_s[1 - F_{is}(\bar{q}_{ins})]}{\sum_{s=1}^{S} J_s[1 - F_{is}(\bar{q}_{ins})]},$$

which is simply the share of patents of country i in country n for type s.[1]

Proposition 1 Patenting rates for destination countries with different T_n

Assume that there is an inventing country i and two destination countries n and n', and there are $s = 1, \ldots S$ types of goods with different technological parameters θ_s. Assume also that the destination countries have different stocks of knowledge, $T_n > T_{n'}$, but are otherwise identical. Then, for any pair of types of goods s, s', the ratio

$$\frac{\frac{\zeta_{ins}}{\zeta_{ins'}}}{\frac{\zeta_{in's}}{\zeta_{in's'}}} = \begin{cases} > 1 & for \;\; \theta_s < \theta_{s'} \\ < 1 & for \;\; \theta_s > \theta_{s'} \end{cases} \tag{2.9}$$

Proof: See Appendix A.

A greater stock of knowledge in the destination country causes less relative patenting in the goods with the lowest dispersion (high value for θ_s) and more patenting in the good with the highest dispersion (low value for θ_s).

In a model without technological differences ($\theta_s = \theta$) the variable ζ_{ins} would remain constant for all relative values of stock of knowledge $\frac{T_i}{T_n}$. This means that the relative importance of type s goods in the overall patents requested by country i in any destination would be the same, changes in the parameters that determine the patenting cutoff decision would only change the absolute number of patents but not the relative composition of that pool.

The key driving force for the result is that the types of ideas with lower dispersion are more sensitive to changes in the cutoffs, which are directly affected by diffusion rates. Since there are smaller markups in those goods, they are patented much less than more disperse goods in absolute terms. Hence, changes in patenting cutoffs cause the

[1] This is equivalent to the variable defined in Chapter 1 for analizing the data.

percentage increase in the patents requested for goods with low dispersion to increase much more than the increase in goods with high dispersion due to the curvature of the Fréchet distribution.

2.3 Alternative modeling options

The two characteristics of my model that make it stand apart from Eaton and Kortum (1999) are (a) the introduction of different types of goods, and (b) different assumptions regarding the patenting decisions made by researchers. I will explore an alternative modeling framework in which I keep characteristic (a) but drop the assumptions implied in (b). Therefore, in this alternative version of the model the patenting decision process is exactly the same as in Eaton and Kortum (1999). This means that the researchers have to make all patenting decisions at once for every market in the world without knowing if the technology created constitutes a breakthrough or not. Also, the patenting rate is calculated using the Pareto distribution, since all new ideas in all countries are drawn from this distribution.

If no other assumption is made besides the introduction of different types of goods, then the patenting composition of country i's ideas remains unaltered for different destination markets. Even though the quality cutoffs are different for different types, the patenting ratios will adjust proportionally and ultimately will not depend on any characteristic of the country that receives the innovations. To see why this is the case note (as shown in Appendix A) that by relabeling using the variable $b_s = T_n \bar{q}^{-\theta_s}$, which is constant in steady state, the behavior of relative shares between types s and s' in my model can be explained by a ratio like

$$\frac{1 - e^{-\frac{T_i}{T_n}\bar{b}_{nis}}}{1 - e^{-\frac{T_i}{T_n}\bar{b}_{nis'}}}$$

where the bar denotes the cutoff in b corresponding to the cutoff in q. This is due to the Fréchet distribution of the new, patentable ideas in the domestic country. If, on the other hand, I assume that all ideas were patentable (without prior knowledge of BT status in the domestic market), then this ratio would be simply

$$\frac{\bar{b}_{nis}}{\bar{b}_{nis'}}.$$

Even though the absolute number of patents between a pair of countries will depend on the relative stocks of knowledge, the composition of the patents will not depend on it and there would not be any pattern in the patenting ratios.

It is therefore necessary to introduce some specific assumption in the EK model to obtain similar results to the ones present in my model. I will explore another possible assumption which is simpler compared to assuming a different process for patent decision-making. Assume that there are differences in the demand for different types of goods in different economies. For example, suppose that production in country i takes the following form:

$$\ln\left(\frac{Y_{nt}}{M}\right) = S^{-1}\sum_{s=1}^{S}\lambda_{ns}J_s^{-1}\int \ln[Z_n(j_s)X_n(j_s)]dj_s,$$

where the λ_{ns}'s represent a weight given to type s in country n. The rest of the notation is exactly the same as in my original model. Note that I have assumed throughout that $\lambda_{ns} = 1 \ \ \forall s$.

Given that competition is Bertrand, the purchases of the most advanced version of a good j_s that is prices $p(j_s)$ are $q(j_s) = \frac{\lambda_{ns}Y_n}{p(j_s)\sum_{\tau=1}^{S}J_\tau}$. This leads to firm profits of $(1-\frac{z}{q})\frac{\lambda_{ns}Y_n}{\sum_{\tau=1}^{S}J_\tau}$.[1] This different weight on the profits coming from the introduction of $\lambda_{n\tau}$'s causes the expected profits of a firm producing quality q to differ in the same proportion, i.e., the expected profits will be a proportion $\frac{\lambda_{ns}M}{\sum_{\tau=1}^{S}J_\tau}$ of the profits in the original model.

2.3.1 The effect of different production functions for each country.

The only consequence of the introduction of this weights in the production function is that the cutoff for patenting qualities will differ. If in country n there is a type of good s for which $(1-\frac{z}{q})\frac{\lambda_{ns}M}{\sum_{\tau=1}^{S}J_\tau} > 1$, this sector will have a lower quality cutoff and more patents will be requested in this sector. The opposite (a higher quality cutoff and lower patenting rate) will occur to the types of goods with a ratio lower than 1. Therefore this variation in the model could explain why in the data there is proportionally more patenting for some types of goods in some countries. Nevertheless, this same effect should be present for all possible origins of ideas and even domestic patents. It does not explain why the compositional pattern is different depending on the specific pair of countries involved.

On the contrary, the data shows that, for example, the patterns of patenting shown by USA firms in foreign countries is different from the patterns shown by domestic firms in those destination markets. This feature of the data is indeed captured in my model: the composition of total patenting differs by the characteristics of the country

[1] In the model without preference weights the profits are simply $(1-\frac{z}{q})\frac{Y_n}{M}$.

where ideas originate. Thus, having only differences in the production functions is not sufficient to explain the features of the data and replicate the results of the original model.

2.3.2 A sector specific T_{ns}.

To explore another possible explanation for the results derived from the model assume that instead of having a unique process of accumulation of ideas for one country I allow for the stock of knowledge to differ for each type of good. This means that if a country devotes a significant amount of resources to research for type s then this should reflect in a type-specific stock of knowledge T_{nst}. A country would devote a significant amount of resources if, for example, the country-specific production function gives a larger weight to these type of goods. A similar result would arise if the country had type-specific productivities for research.

To introduce how the stock of knowledge accumulates, suppose initially that there is only one country n, and let $\dot{T}_{ns}$ the rate at which new ideas for sector s are created thanks to research performed in that country. Ideas can come from the same sector or from other sectors. The following expression takes into account these possibilities.

$$\dot{T}_{ns} = \sum_{\sigma=1}^{S} \psi(\sigma, s) \frac{\bar{\alpha}\alpha_{n\sigma}\gamma_{i\sigma}^{\beta} L_i}{\sum_{\varsigma=1}^{S} \psi(\varsigma, \sigma) J_{\varsigma}}$$

where

$$\psi(a, b) = \begin{cases} 1 & \text{if } a = b \\ \Psi & \text{if } a \neq b \end{cases}$$

where $\Psi \in [0, 1]$ is a measure of how directed the research can be. Suppose that $\Psi = 0$, then $\dot{T}_{ns} = \frac{1}{J_s}\alpha_{ns}\gamma_{is}^{\beta} L_i$. This means that the research performed in sector s produces exclusively ideas that can be used in this sector, and no ideas produced from other sector contribute to the total stock of ideas of sector s. On the other hand, suppose that $\Psi = 1$. This other extreme is equivalent to assuming that $\dot{T}_{ns} = \frac{1}{J}\sum_{\sigma=1}^{S} \bar{\alpha}\alpha_{n\sigma}\gamma_{i\sigma}^{\beta} L_i$. This expression has exactly the same value for all sectors. It states that all of the ideas created in the country apply randomly to any good, no matter which sector it belongs to (which is my preferred modeling option as presented before).

Having defined how ideas could be used by different sectors in one country, I will proceed to expand this definition to include the possibility of ideas flowing across countries and at different points in time.

$$\dot{T}_{nst} = \sum_{i=1}^{N} \epsilon_{ins} \int_{-\infty}^{t} e^{-\epsilon_{ins}(t-t')} \sum_{\sigma=1}^{S} \psi(\sigma, s) \frac{\bar{\alpha}\alpha_{nt\sigma}\gamma_{it\sigma}^{\beta} L_{it}}{\sum_{\varsigma=1}^{S} \psi(\varsigma, \sigma) J_{\varsigma}} dt'. \tag{2.10}$$

This element helps us characterize the technological frontier for each type of good, which is a very important object for the analysis done in this dissertation. As in the model presented before, the technological frontier for each type of good can be summarized as

$$F_{nst}(z) = e^{-T_{nts} z^{-\theta_s}} \tag{2.11}$$

which is the usual Fréchet distribution.

Given that research could have different expected values depending on how useful it is in different sectors, an equilibrium allocation of research could be found for each sector. A greater value of an idea in sector s will cause more research to be done in that sector.

Suppose that two countries are exactly the same in terms of the production function (no difference in the weights for each sector) but have different ratios for the stock of knowledge due to how directed research can be and the sector specific research productivities. Suppose that there are only two types of goods, τ and τ', and that in country n sector τ has a larger weight than in country n'. For example, if country n accumulates more ideas in sector τ this means that $\frac{T_{n\tau}}{T_{n\tau'}} > \frac{T_{n'\tau}}{T_{n'\tau'}}$. This could occur if country n is "closer" to countries that devote more research resources to sector τ than the countries "closer" to n'. In this model closer means that there is faster diffusion of ideas between the countries.

In the simplest of the scenarios, the cutoff quality above which ideas would be patented can be defined as the value of $\bar{b}_{nit\tau} = T_n(\bar{q}_{nit\tau})^{-\theta_\tau}$ that solves

$$e^{-b}\left(1 - e^{b} b^{1/\theta} \Gamma[(\theta-1)/\theta, b]\right) = f_{nit}.$$

This means that $\bar{b}$'s are the same no matter the destination country. This implies

$$T_{n\tau} q_{n\tau}^{-\theta_\tau} = T_{n'\tau} q_{n'\tau}^{-\theta_\tau} \quad ; \quad T_{n\tau'} q_{n\tau'}^{-\theta_{\tau'}} = T_{n'\tau'} q_{n'\tau'}^{-\theta_{\tau'}}.$$

This in turn implies that necessarily

$$\frac{q_{n\tau}^{-\theta_\tau}}{q_{n\tau'}^{-\theta_{\tau'}}} < \frac{q_{n'\tau}^{-\theta_\tau}}{q_{n'\tau'}^{-\theta_{\tau'}}}.$$

This means that there is going to be relatively less patenting in types of goods τ in country n.[1] This is precisely the types of goods in which country n has more accumulated knowledge compared to country n'. The direct consequence is that an advantage in terms of knowledge for a type of good has a negative effect in the patenting rate in that country. Following Fieler (2009) it is reasonable to think that richer countries have an advantage in technologically complex goods (those with a high dispersion in qualities). This version of the model would imply that there would be less patents requested in richer countries for technologically advanced goods. In terms of patenting ratios, it is a "bad thing" to have a lot of accumulated experience. In contrast, my model implies that relatively more patents would be requested for these types of goods in a country with a higher stock of knowledge.

2.4 Simulations

As mentioned above, for a steady state equilibrium two equations must be solved, the labor market equilibrium and the equal growth of the stock of knowledge of all economies. Introducing any given set of parameters into the program allows it to find values of γ_i and $\frac{T_n}{T_i}$ that solve simultaneously those equations. Given those equilibrium values, the model predicts a number of patents requested for each bilateral pair of countries and type of good.

When this model is simulated, it is possible to replicate the findings of Section 1.2. In order to relate the simulated data to the implications of Proposition 1, I will assume four countries that are identical in the bilateral diffusion rates, $\epsilon_{in} = 0.01$ whenever $i \neq n$ and $\epsilon_{in} = 0.3585$ whenever $i = n$. This allows for the value of the stock of knowledge to depend almost exclusively on domestic research, but researchers do have incentives to pantent in foreign countries. To allow for different values of the stock of knowledge T_i, I will allow the sizes of each country to vary, with the populations shown in Table 2.1.

Regarding the patenting cost, I use for the simulated values the share of GDP that a domestic USA patent request cost in the data used in EK, which is $2.3x10^{-10}$. I assume that the rates at which ideas are copied are $i^{\text{not}} = 0.45$ and $i^{\text{pat}} = 0.37$, and the coefficient that determines heterogeneity in research $\beta = 0.18$. Additionaly, the interest rate is assumed to be $r = 0.07$, while the growth of TFP $g_A = 0.018$ by

[1] Recall that, given the assumptions, $q_{n\tau}^{-\theta_\tau}$ represents the share of new ideas that is patented for type τ in country n.

Table 2.1: Size of countries in simulated model, millions

Country	1	2	3	4
Population	100	500	850	1000

assumption. Finally, I assume that the sizes of the sectors are the same, $J_s = 0.5$ million, and the only difference is the technological parameter θ_s. I assume the values presented in Table 2.2. I also present the results of a similar exercise to that of Section 1.2 For each sector, I run the regression

$$\zeta_{ins} = a + b_s * \frac{T_n}{T_i}$$

and present the parameter b_s in the same table. All of them are significant at the 5% level.

Table 2.2: Techonological parameter of sectors in simulated model

Sector	a	b	c
θ_s	1.25	3	6
b_s	0.0416	-0.0045	-0.0371

These results are in line with the prediction of Proposition 1, differences in the relative ratio of stocks of knowledge of the destination causes different effects for different types of goods. In particular, for the good with the greatest dispersion (denoted “a”), inventors want to patent relatively more in countries with a high stock of knowledge, while the opposite happens for good “c” where the rate of patenting is greater whenever T_n is lower.

It is possible, however, that whenever you look at data, the behavior predicted by the proposition is not directly observable. This could occur when the assumptions of Proposition 1 are not satisfied: countries differ not only in the stock of knowledge but also in diffusion rates or patenting cutoffs. These are two characteristics that make it possibly harder for inventors of one country to patent their inventions in a given destination if the bilateral diffusion is low or if the cost of patenting is high.

Nevertheless, different values for these two characteristics that determine bilateral patenting rates are consistent with the implications of Proposition 1. To understand why, note that a higher stock of knowledge in a destination implies that it is harder to patent in that location, competition with existing technologies is greater, so expected profits will be smaller, and a lower number of patents will be requested. Whenever that happens, the goods with qualities that are less disperse are more sensitive, causing a decrease in the relative share of patenting for that good. This also occurs whenever there is lower bilateral diffusion or patenting cutoffs are greater and therefore expected profits are lower. In those cases, less patenting is expected in the good with lower diffusion.

To see how important these issues could be I will present a simulation with the data generated using the values from EK. I will assume as data the patenting costs and the sizes of the population, but I will also take as data the estimated values for diffusion rates, heterogeneity of research and imitation rates. I will assume three sectors, one that has a technology dispersion equal to the one estimated in the paper, while the other two have dispersions that are greater and lower in a similar proportion to the first one.[1] The interest rate and the growth of TFP is assumed to be the same as in the previous simulation. Table 2.3 presents the assumed parameters.[2]

In Table 2.4 we can observe two important features. First, even when the sizes of all sectors are the same, the relative shares of patenting differ in a given destination, and this is exclusively due to the role of the technology dispersion parameter. Second, there are differences in the behavior depending on the characteristics of each country. Take for example the patents requested in Germany and France by the researchers in the rest of the countries. The behavior seen matches the predictions of the proposition. France has a relative stock of knowledge higher than that of Germany, and the shares of patenting are greater for the good with the highest dispersion (Good 1), while the opposite is true for the good with the lowest dispersion.

However, this behavior in the simulated results is not the same that we observe when U.K and Japan are analized. In those cases, the higher cost of patenting and the

[1]Note that the simulation does not yield the same results as in EK due to different assumptions regarding the production function of intermediates and the patenting assumptions introduced in this model. Also note that the bilateral diffusion rates are constructed assuming that $\epsilon_{in} = \epsilon_{i\cdot} * \epsilon_{\cdot n} * \epsilon_d$, where the first element is the rate at which a country sends ideas, the second one is the rate at which it receives ideas, and the third element is equal to one if $i \neq n$, and takes a value of 17.7 whenever $i = n$. Hence, there are 11 parameters used to construct the 25 bilateral diffusion rates.

[2]EK make a distinction between the diffusion rates at home and abroad for the empirical exercise. I follow those assumed parameters in this simulatio.

Table 2.3: Data used in simulation of EK.

ϵ_{in}	Germany	France	U.K.	Japan	USA
Germany	3.13	0.20	0.07	0.10	0.09
France	0.05	1.09	0.02	0.03	0.03
U.K.	0.11	0.13	0.72	0.06	0.06
Japan	0.22	0.26	0.08	2.28	0.12
USA	0.04	0.05	0.01	0.02	0.37
$f_{in}/f_{\text{USA, USA}}$	Germany	France	U.K.	Japan	USA
Germany	2.57	3.24	5.54	5.50	2.44
France	2.57	3.24	5.54	5.50	2.47
U.K.	2.57	3.24	5.54	5.50	1.00
Japan	7.39	9.94	18.55	11.05	3.03
USA	2.57	3.24	5.54	5.50	1.00

	Good 1	Good 2	Good 3
θ_τ	1.15	1.87	2.59
J_τ	0.37	0.37	0.37

β	0.18
$\iota^{\text{a}}_{\text{not}}$	0.24
$\iota^{\text{a}}_{\text{pat}}$	0.25
$\iota^{\text{d}}_{\text{not}}$	0.42
$\iota^{\text{d}}_{\text{pat}}$	0.23

Table 2.4: Patenting shares of simulated model.

	Germany	France	U.K.	Japan	USA
T_n/T_{USA}	0.86	0.90	0.62	0.75	1

Good 1	Germany	France	U.K.	Japan	USA
Germany	33.4	34.9	35.8	35.7	35.2
France	35.4	33.5	39.0	37.8	36.5
U.K.	35.4	35.7	33.5	37.0	35.2
Japan	36.0	36.5	39.6	33.6	35.5
USA	35.5	35.8	40.2	38.5	33.4
Good 2					
Germany	33.3	33.2	33.2	33.2	33.2
France	33.2	33.3	32.9	33.0	33.1
U.K.	33.2	33.2	33.3	33.1	33.2
Japan	33.2	33.1	32.8	33.3	33.2
USA	33.2	33.2	32.8	33.0	33.3
Good 3					
Germany	33.2	31.9	31.0	31.1	31.6
France	31.4	33.2	28.1	29.2	30.4
U.K.	31.4	31.2	33.2	30.0	31.6
Japan	30.9	30.4	27.6	33.1	31.3
USA	31.3	31.0	27.0	28.6	33.3

lower rates of diffusion cause less patenting overall, and this has the discussed effect of increasing the patenting share of the good with the lowest dispersion. In fact, this effect is so strong that dominates, and whe I run the regression presented above of the shares and relative stocks of knowledge, the coefficient has the opposite signs, as can be seen in Table 2.5.

Table 2.5: Simulated sectors

Good	1	2	3
θ_s	1.15	1.87	2.59
b_s	-0.0312	0.0020	0.0292

As the simulations presented showed, the dispersion parameters of different sectors play an important role on the patenting behavior across different types of goods, and this particular generalization of the model allows for the use of a larger data set. There is a significant advantage of using this model in terms of estimating the parameters. In a model with only one technological sector there are $N * N$ patent data points that can be used. It is impossible, for example, to estimate the $N * N$ diffusion rates, even in the case when productivity and research data is used as it is in the EK paper. With the generalization of the model to include different types, it is now possible to have $N * N * S$ data points available for the estimation. In my case, this is an increase of six times the number of data points.

2.5 Estimation procedure

To estimate the parameters of the model model I will make some assumptions similar to the ones made for the simulations. I will take the values of the interest rate of $r = 0.07$ and mean growth of productivity $g_A = 0.018$ as given. Also, the imitation rate of ideas when a patent is not requested is assumed to be $\iota^{\text{not}} = 0.415$. Finally, the domestic diffusion rate of USA is normalized so that it matches a mean lag of adoption of 2.5. The data required for the estimation is the size of the working population of each country, L_i and the data for the patenting cost relative to each countrys's output.[1]

[1] As in the simulation, I use for the cost of patenting the share of GDP that a domestic USA patent request cost in the data used in EK, which is $2.3x10^{-10}$.

I need to estimate the rest of the parameters of the model, I will refer to them as the vector Ω. These parameters are the country-pair diffusion rates (ϵ_{in}), and the technology parameters for each type of good (θ_s). Also, size of each type of good J_s, research heterogeneity β and rate at which ideas are copied ι_{pat}.

The procedure of estimation is as follows:

- Begin with a value Ω_0 for the parameters that we want to estimate. Given those parameters, an equilibrium to the labor market can be found. This is, values γ_i^0 are found, and these are consistent with T_i/T_n values that solve the growth equations.

- Given Ω_0 and its corresponding γ_i^0, the model predicts values for the patents requested in each combination of countries and types of goods. Hence, the share of patents for each pair of countries in each type of good can be computed. The equation for the number of patents is $\hat{P}_{ins} = \alpha \gamma_i^{\beta} L_i[1 - F_{is}(\bar{q}_{ins})]$.

- The value $\hat{P}^0_{ins}$ is compared to the actual data P_{ins}. If they are not the same, new parameters Ω_1 are selected, and the process starts again.

- The objective of the process is to minimize the square of the residuals using a standard nonlinear least squares routine. The residuals are defined

$$F = \frac{\hat{P}_{ins} - P_{ins}}{P_{ins}}$$

Whenever this procedure is applied to a simulated model it is successful in recovering the parameters that were used to simulate the data. Hence, it is possible to apply this same procedure to data sets of patents to recover real values for the parameters.

2.6 Results and discussion

I will now present the results of using this model and the estimation procedure detailed above with the data from WIPO. I performed an estimation for three countries: China, Europe and USA.[1]

[1] I was not able to include Japan (the other country with a lot of patenting activity) in this estimation because the data for domestic patents of Japan must be adjusted since it is unusually high. EK note that there is about 4.9 times more domestic patents for Japan since there are fewer inventive claims in each patent request. If I were to apply the same adjustment, the Japanese domestic patents would be less than the patents requested by those same individuals in the USA office. Further analysis must be done to determine the correct adjustment for this data.

Table 2.6: Estimation results.

ϵ_{in}	China	Europe	USA
China	0.073	0.053	0.055
Europe	0.076	0.385	0.102
USA	0.074	0.110	0.363

	Chem.	Elec. Eng.	Green En.	Instrum.	Mech. Eng.	Other
θ_τ	2.2	1.7	2.7	2.1	2.3	1.8
J_τ (tens of millions)	41.8	27.5	1.3	18.7	29.2	5.2
β	0.36					
ι_{not}	0.25					

Even though this is a small set of countries, the purpose of this exercise is to show that it is possible to identify bilateral diffusion rates and that the features of the data described in Chapter 1 are in line with the predictions of Proposition 1. The values of θ_s are the lowest for Electrical engineering, Instruments and Other goods. Those three are sectors for which ideas are more likely to be patented in richer, more technologically advanced countries. According to the Proposition, goods with a greater dispersion (lower value of θ_s) are the ones that will be patented more in those destinations. The opposite happens with Chemicals and Mechanical Engineering, those are (according to the stylized facts) patented more in poorer, less technologically advanced countries, and those are precisely the two goods (aside from Green energy) that have the lowest dispersion (highest value of θ_s).

The diffusion rates tell different stories for each country. USA and Europe are very similar in terms of sending and receiving ideas from the other countries. Also, domestic diffusion is very important in these two countries. The story is different, however, for a country like China. Domestic diffusion is not the main source of ideas in this market, and ideas from the other two countries are equally as important. Also, China is a more efficient recipient of ideas than it is a sender of ideas. All of these features are in line with what we should expect for this large developing country where technology development becomes more and more important every year.

This estimation can be improved with additional data. I am assuming the same relative patenting cost because I do not have this information available. Additionally, EK use the share of population in research and production information as additional sources of information. In my case I use the patent data as the only source, which is not the data set that the aforementioned paper trusts the most in terms of weights in the estimation. It is also a natural step to include more countries, but the quality and availability of that information as well as the computation burden of expanding the set of countries is an important obstacle to overcome.

3

Explaining income differences with a two-sector economy

3.1 Introduction

Why does output per worker differ so much among countries? The growth literature has approached this question in several ways, and most of the current discussion is based on the work of Mankiw et al. (1992), Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999) who study the relevance of both total factor productivity (TFP) and the availability of factors of production in each country. A simplified view of the production of a country measured on a per worker basis is $y = AX$, where X represents the factors (usually physical and human capital) and A is the TFP component. The result of a common exercise of variance decomposition of this equation gives a very well known result, namely that about two thirds of the differences in gross domestic output (GDP) per worker are accounted for by differences in TFP.

The same methodology was used in Ferreira et al. (2008) (FPV), but instead of doing the estimation with one year of data as it is usually done, the authors extended the study to 31 years, from the early seventies until the year 2000. In each year, a separate variance decomposition exercise was done. They found that TFP differences were not as important in the seventies and most of the differences of GDP per worker came from disparities in the level of factors (physical and human capital). As time passed, the disparities in terms of TFP grew larger and today can explain about two thirds of the differences observed in output per worker, the traditional result found in the literature. The goal of this chapter is to explore whether the results in FPV can be explained by taking into account the structural transformation that the same economies have experienced when the size of the agriculture sector decreased over time.

The share of the agriculture sector was much larger for most of the countries in 1970, specially the poorer ones. This is relevant because the production function in agriculture is significantly different from the one used in the non-agriculture sector (defined as manufacture and services): agriculture uses land intensively, a factor that is not taken into account in the traditional one-sector growth model. Also, this sector uses a small amount of capital. The use of the traditional one-sector growth model implies that the contribution of land to total output is assigned to the TFP component (A) and not to the factors (X). The effect is an overestimation of the TFP component for poorer countries, which typically have a larger agriculture sector. This could lead to the finding of FPV, in which the differences in TFP among countries are not as large in 1970 as one would expect, when agriculture was much more important especially in the poorer countries.

A numerical example with actual data of Thailand for 1985 will illustrate how this overestimation takes place. Think of the traditional production function in the one sector model with capital (K) and human capital (H):

$$Y = AK^{\alpha}H^{1-\alpha}$$

The actual values of factors available are $K = 500$, $H = 0.13$, while the level of output is $Y = 180$.[1] Assume a commonly used value for the parameter of the function, $\alpha = 1/3$. As usual, the A can be measured as a residual from this equation, which turns out to be $A = 88.1$.

Now assume that the true production structure for the economy is better represented by two functions, one for agriculture $Y_a = A_a R^{\alpha_a} H_a^{1-\alpha_a}$ (where R represents land) and one for non-agriculture $Y_n = A_n K^{\alpha_n} H_n^{1-\alpha_n}$. These two equations imply that agriculture uses land instead of physical capital. Land has a value of $R = 18$ (millions of hectares of arable land),[2] and let us assume first that human capital is assigned efficiently among sectors, and that the values of parameters chosen are $\alpha_a = \alpha_n = 1/3$ (land and capital are used with the same intensity in each sector). These two assumptions imply equilibrium values $H_a = 0.01$ and $H_n = 0.12$. Data from FAO, which will be

[1]Output datum comes from Penn World Table, while capital is constructed using the investment series from the same source. Both series are in millions of 1985 PPP dollars. Human Capital is constructed using the Barro and Lee (2000) database on education levels following classical literature. Specifics on how physical and human capital series are constructed are detailed in Section 3.3.

[2]Datum taken from World Development Indicators of the World Bank.

discussed thoroughly below, tell us that $Y_a = 15$ and $Y_n = 165$. Therefore, following the same logic as before, the TFP component can be measured in each sector separately as residuals. For these particular data and assumptions, $A_n = 86.1$ and $A_a = 110.1$. This simple example shows how the TFP measure in non-agriculture could be overestimated by about 2.3% if land, a critical factor in the production of agricultural goods, is not taken into account in the production function of this country. This overestimation becomes less important when the size of the agriculture sector relative to total output becomes smaller, which is exactly the process that has occurred in the recent decades for most countries.

Córdoba and Ripoll (2006) (CR) show that failing to consider the agriculture sector may overestimate the productivity of a poor economy. Vollrath (2009) also emphasizes the importance of having more than one sector to account more accurately for the differences in TFP between countries. Nevertheless, none of these studies does an intertemporal comparison of their findings because the data that is required is not available. Chanda and Dalgaard (2008) provide a similar motivation for the exercise that they perform: the effect that considering agriculture has on the measure of TFP. The main advantage of their work is that it uses actual data for the allocation of resources, which are not available for a significant part of the years that I study in this chapter. The authors stress the role of differing efficiencies among sectors as an explanation to why including agriculture helps in measuring more appropriately TFP, while Vollrath (2009) focuses on misallocation of resources as a source of income differences between different countries. In this chapter I follow more closely the work of CR, assuming that factors are allocated efficiently and focusing on how the measurement of TFP is affected by not accounting for the agricultural sector.

This chapter is intended to be exploratory of some important issues regarding sectoral transformation. I show that not accounting for more than one sector causes overestimation of the increase in the importance of TFP in explaining GDP per worker differences over time. In particular, the adjustment for agriculture proposed in this dissertation is between 5%-7% in the early seventies, while this same adjustment is only around 2% in the late nineties. This decrease in the adjustment is masked in a one-sector model as an increase in the importance of TFP in explaining differences in income. I will also show that when the non-agriculture sector is considered separately and a variance decomposition exercise is done, the importance of TFP has the opposite effect to what is observed in a one-sector model for the aggregate economy, it has decreased over time.

A final issue that this dissertation wants to address is the need for GDP sectoral data measured at international prices. Without this kind of information many relevant research questions in the growth literature cannot be addressed empirically. For this chapter I solve this issue by creating a series using the only year for which this information is available, and extending the observations using the growth of the real GDP in each sector.

Section 3.2 introduces the basic structure of the economy that will be used throughout the chapter. Section 3.3 discusses the data that is used and particularly how the series of sectoral GDP at international prices is generated. Section 3.4 presents three different exercises that illustrate how the integration of agriculture into the model has relevant implications for the role of TFP in explaining income differences, and section 3.5 concludes.

3.2 The basic structure of the economy

The production of the economy is represented by two sectors, agriculture (a) and non-agriculture (n). The production of sector j in country i at year t is represented by a Cobb-Douglas production function with differing factor shares across sectors.

$$Y_{ait} = \tilde{A}_{ait} R_{ait}^{\delta} K_{ait}^{\gamma} H_{ait}^{1-\gamma-\delta}, \tag{3.1}$$

$$Y_{nit} = \tilde{A}_{nit} R_{nit}^{\beta} K_{nit}^{\alpha} H_{nit}^{1-\alpha-\beta}. \tag{3.2}$$

The inputs used in this economy are land (R_{jit}), capital (K_{jit}) and human capital (H_{jit}) for $j = a, n$, while $\tilde{A}_{jit}$ represents the TFP component of sector j. The value of total production can be expressed as

$$Y_{it} = p_{at}^{*} Y_{ait} + p_{nt}^{*} Y_{nit}$$

where p_{jt}^{*} is the international price of sector j at year t. The only way to perform a cross-country comparison exercise is by valuing each country's production using the same price vector. Since this vector is unique, these prices do not provide any additional source of variation of income, so it is possible to rewrite the expressions of sectoral production including a "price adjusted" TFP component, $A_{jit} \equiv p_{jit}^{*} \tilde{A}_{jit}$. The value of production in each sector is then

$$p_{at}^{*} Y_{ait} = A_{ait} R_{ait}^{\delta} K_{ait}^{\gamma} H_{ait}^{1-\gamma-\delta}, \tag{3.3}$$

$$p_{nt}^{*} Y_{nit} = A_{nit} R_{nit}^{\beta} K_{nit}^{\alpha} H_{nit}^{1-\alpha-\beta}. \tag{3.4}$$

I will also refer to the shorter notation $F_{ait} \equiv R_{ait}^{\delta} K_{ait}^{\gamma} H_{ait}^{1-\gamma-\delta}$ and $F_{nit} \equiv R_{nit}^{\beta} K_{nit}^{\alpha} H_{nit}^{1-\alpha-\beta}$ for ease of exposition later on.

3.3 The data

The most important reason why there are few papers discussing the possible relevance of structural transformation in explaining cross-country information is the lack of comparable data: production of agriculture and non-agriculture measured using the same prices for every country.[1] The Penn World Table (PWT) has data on GDP measured at international prices, but there is no sectoral information available. Without comparable data it is impossible to develop empirical exercises like the one proposed in this dissertation.

To my knowledge, the only source for sectoral GDP information is a database constructed by the Food and Agriculture Organization (FAO) for 1985 documented in Prasada Rao (1993). This database deliberately uses the PWT methodology to estimate the GDP in agriculture for a set of countries. My goal is to construct a complete panel using these data as the basic information. Even when this panel deals satisfactorily with the issue of using a uniform price vector for all countries, as Caselli (2005) notes, it does not use the same price level as the PWT, so the databases are not perfectly comparable. For the purposes of this dissertation, an adjustment must be done first.[2]

I construct a data set (denoted "NEW") that includes total GDP as well as sectoral GDP (agriculture and non-agriculture) using the study from FAO, the information available from national accounts in the World Development Indicators (WDI) and the data from PWT. This data set is constructed so that it satisfies the following three requirements.

1. The ratios of GDP in agriculture across countries for 1985 match the corresponding ratios from the FAO database. The difference between the *levels* in the "NEW" and the original FAO databases is the adjustment in prices that will be described later on.

2. The growth rate of agricultural GDP corresponds to the growth rate of that sector's real GDP at local prices (from national accounts). This assumption is

[1] A traditional source of data for structural transformation research is the Groningen 10-sector database, but this data is not measured using international prices.

[2] This is required so that the information of the non-agricultural sector can be inferred from the total and agricultural GDP's.

needed to extend the panel to the rest of the years. It is reasonable since we want to identify the changes in production and this must be independent of the level of prices used.[1]

3. For every year the share of agriculture in total GDP of the USA in the new database is equal to the share of agriculture in GDP measured at current domestic prices (from WDI). This requirement is needed because in the "NEW" database the price vector used must correspond to that of the PWT. Since PWT prices are quantity-weighted, the price vector of rich countries is very close to the international price vector.[2] Therefore, the USA domestic prices at every year must be very similar to those used in the PWT,[3] which implies that the sectoral shares of total GDP must also be of similar magnitude. Remember that the only difference between the FAO and PWT measures is the level of the prices chosen, so any adjustment done must be the same one for every single country. In this case the data of the USA for each year provides the information for rescaling all the data generated from requirements 1 and 2. This is the same adjustment to

[1]From the WDI database of the World Bank I obtain information on sectoral GDP (agriculture, services and industry) in constant local currency units. I aggregate services and industry into the nonagriculture sector. This method of using an internationally comparable data source for a base year and imputing the growths of the variables from national accounts is the same method used in the construction of the series of the PWT. Summers et al. (2002) use it to construct the components of GDP measuring it on the expenditure side. Duarte and Restuccia (2006) construct a database using this same idea, in which sectoral production per worker in a set of countries (measured not in actual levels, but relative to the USA) is generated for three sectors. In their work they don't obtain a measure of each sector's GDP.

It is important to note a mismatch between the exact data that I need, and the available data. Ideally, agriculture has the same definition across all databases, but this is not the case. In the FAO 1985 database the agriculture sector is defined as "Agriculture and Livestock Production" (Code 011) and "Hunting, Trapping and Game Propagation" (Code 030) from the International Standard Industrial Classification (ISIC) of Economic Activities by the United Nations Rev.1. This excludes forestry, fishing and agricultural services from the study.

Both the WDI database and the labor statistics that I use define the agriculture sector as classifications 1-5 in ISIC Rev.3. This means that they include the classifications 014 (Agricultural services), 020 (Forestry), and 050 (Fishing) that I would need to exclude to have an exact match between the databases.

[2]See for example Hill (2000).

[3]Summers et al. (2002) indicate that the series of real GDP per capita is a chain series, and they claim that "The merit of RGDPCH (what makes it the recommended intertemporal GDP time series) is the fact that its growth rate for any period is based upon international prices most closely allied with the period." This means that for every year, this measure takes into account the changes in relative international prices.

the price level that Caselli (2005) uses for 1985.[1]

The procedure to construct the database according to these criteria can be summarized as follows. From 1, the data from the FAO study provides the GDP per worker in agriculture for every country i in 1985. Using agricultural labor it is possible to obtain the total production of this sector, which I will call $\mathcal{Y}^{FAO}_{a,i,1985}$. These data can be extended for any year t using criteria 2 and obtain $\mathcal{Y}^{FAO}_{a,i,t}$, which is the agricultural GDP using the constant FAO international prices. The series that will be obtained after the price adjustment, which will be denoted $\mathcal{Y}^{NEW}_{a,i,t}$, has to satisfy

$$\frac{\mathcal{Y}^{NEW}_{a,i,t}}{\mathcal{Y}^{NEW}_{a,j,t}} = \frac{\mathcal{Y}^{FAO}_{a,i,t}}{\mathcal{Y}^{FAO}_{a,j,t}} \qquad \forall i, j, t. \tag{3.5}$$

To obtain the "NEW" data, the level of the series $\mathcal{Y}^{FAO}_{a,i,t}$ needs to be adjusted to match the price level in PWT. As outlined in 3, the data from the USA will be used to rescale the data for the rest of the countries. For the case of the USA, take the data from national accounts at **current** domestic prices at year t, $\mathcal{Y}^{WDI}_{a,USA,t}$. Criterion 3 is $\frac{\mathcal{Y}^{NEW}_{a,USA,t}}{\mathcal{Y}^{NEW}_{USA,t}} \approx \frac{\mathcal{Y}^{WDI}_{a,USA,t}}{\mathcal{Y}^{WDI}_{USA,t}}$ $\forall t$, where the lack of subscript a denotes Total GDP.[2] This information can be compared with the data obtained using 1 and 2, $\mathcal{Y}^{FAO}_{a,USA,t}$. The ratio of those two values is the adjustment that has to be done for every country at year t. The adjustment is then

$$\frac{\mathcal{Y}^{NEW}_{a,USA,t}}{\mathcal{Y}^{FAO}_{a,USA,t}} = \frac{\frac{\mathcal{Y}^{WDI}_{a,USA,t}}{\mathcal{Y}^{WDI}_{USA,t}}}{\frac{\mathcal{Y}^{FAO}_{a,USA,t}}{\mathcal{Y}^{NEW}_{USA,t}}}. \tag{3.6}$$

In equation (3.6) the right hand side is the ratio of two numbers for which I have data: the share of agriculture in national accounts at current domestic prices for the USA (in the numerator), and the ratio of agriculture measured at the FAO prices with respect to total GDP measured at PWT prices (in the denominator). The value in (3.6) changes through time because the numerator is a nominal measure, while the denominator is a real measure.

Given the assumption that PWT prices match USA current domestic prices, this number should be exactly one if FAO and PWT prices were also the same. Since they

[1]The ratios of GDP in agriculture across countries for all years will be exactly the same for a series that satisfies only requirements 1. and 2. if it is compared to the "NEW" series, which satisfies all three. This is the case due to the adjustment done to satisfy requirement 3. is the same for all countries in a particular year.

[2]Note that $\mathcal{Y}^{NEW}_{i,USA}$ is aggregate GDP of the USA from PWT.

are not the same, the observations for every country and every year can be derived from

$$y^{NEW}_{a,i,t} = y^{FAO}_{a,i,t} \frac{y^{NEW}_{USA,t}}{y^{FAO}_{a,USA,t}} \frac{y^{WDI}_{a,USA,t}}{y^{WDI}_{USA,t}}. \tag{3.7}$$

When the relative price of agriculture in the USA is greater the adjustment must be larger and, therefore, the measure of production of the agriculture sector in the new database must also be higher. This process takes into account the importance that agriculture has in a specific year relative to the rest of production. How important this sector is depends on the relative prices in the USA.

It is evident in the data for the USA that during the early seventies there was a steep increase in the relative price of agriculture. The share of the agriculture sector in nominal GDP increased from 3.6% in 1972 to 4.8% in 1973, and this causes the adjustment factor that I calculate to be much higher in 1973 compared to the previous years. This increase is due to a significant spike in the global demand[1] for agricultural goods which caused the prices of agricultural commodities to go up significantly (Peters et al. (2009)). After that period, the absolute prices of agriculture have remained relatively constant while the rest of the goods of the economy have had increases in prices. The first shock (increase in demand of agricultural goods) causes the adjustment to increase dramatically for 1973, and the subsequent increase of the prices of the other goods in the economy cause the adjustment to decrease during the following decades, as can be observed in Table 3.1.

Having a $y^{NEW}_{a,i,t}$ for every country at every point in time, it is now possible to obtain the corresponding production of the non-agriculture sector ($y^{NEW}_{n,i,t}$) as a residual using the data from PWT.[2]

$$y^{NEW}_{it} = y^{NEW}_{a,i,t} + y^{NEW}_{n,i,t},$$

where y^{NEW}_{it} is aggregate GDP from PWT.

Appendix D shows how the data generated in this new database compares with other sources of data, namely the real and nominal shares of the agriculture sector in national accounts (WDI). I present as the first year 1973 because that is the year where

[1] Among other reasons, the increase in purchases of centralized economies and production shortfalls are given as explanations for this.

[2] The numbers for 1985 don't exactly match the numbers obtained by Caselli (2005) because the labor information is not consistent with the one that he uses. He uses the data that comes in the FAO study, but this data is only available for one year. There are slight differences between this data and the data that I use with more differences appearing in poorer countries, but those differences in the databases are not systematic.

Table 3.1: Adjustment to convert observations from FAO prices to PWT prices

Year	$\frac{y^{NEW}_{a,USA,t}}{y^{FAO}_{a,USA,t}}$
1971	2.35
1972	2.59
1973	3.58
1974	3.26
1975	2.90
1976	2.81
1977	2.72
1978	3.15
1979	3.23
1980	2.76
1985	1.91
1990	1.91
1995	1.61
1999	1.07

the adjustment shown previously is the greatest, making the size of the agriculture sector large. It is, therefore, important to notice that when the shares of agriculture are significantly big in 1973 (for example, India, Ghana, Bangladesh), the share in the new database is smaller than the nominal share. It is also true that in most cases this share in the new database decreases significantly fast, more than it does in the nominal GDP database. This is due to the fact that prices have decreased more rapidly in the USA than in most countries. Nevertheless, this is a feature consistent with the way in which international prices are calculated, giving more weight to rich countries. If some other international prices were available where more weight is given to poor countries, this decrease would be much slower.

To do the variance decomposition, which must be done on a per-worker basis, it is necessary to have sectoral information on employment. There are limited sources of sectoral labor data, especially for a long span of time. The most complete database regarding agriculture and non-agriculture sector is available in the Food and Agriculture

Organization Statistical Databases (FAOSTAT) website (Population Division (2007)).[1] This database comprises information for 197 countries and territories on sectoral economically active population (which includes all individuals engaged in production, even when this production is not intended for sale in a market, e.g. own consumption).

There are other sources of data needed regarding the inputs in the economy. The physical capital series is constructed using the investment series from from the PWT.[2] I follow the procedure detailed in the FPV paper (the perpetual inventory method) to obtain the physical capital series. I use the values that they assume for the parameters needed, such as the growth of technological process and depreciation rate.[3] I also use data on population growth from the same source as they do to obtain the initial level of investment. From there on, the series is built using each year's investment and the corresponding depreciation of the existing capital. It is worth mentioning that for most countries the investment series has information starting in 1950, and the rest of the countries that are used have information starting in 1960. The authors argue that this is a long enough period of time to avoid the initial values of investment to have an effect on the series.

The data on land that is generally used in the literature is "arable land," obtained from the WDI. Human Capital for each country at each point in time, H_{it}, also follows FPV, which in turn follows the functional form and calibrated parameters by Bils and Klenow (2000):

$$H_{it} = L_{it} h_{it} \tag{3.8}$$

where

$$h_{it} = e^{\frac{\theta}{1-\zeta} s_{it}^{1-\zeta}} \tag{3.9}$$

In particular, ζ provides the curvature of the returns of education with respect to schooling levels. θ is a parameter chosen so that the mean of the Mincerian returns matches the evidence found in international data by Psacharopoulos (1994). The data on s_{it}, the number of years of schooling,[4] is obtained from the Barro and Lee (2000)

[1] Accessed in May 2009

[2] For this dissertation PWT 6.1 is used for comparability with FPV. The results remain valid when PWT 6.3 is used.

[3] The depreciation is assumed to be 3.5%, the growth of technological process is 1.53%. The growth of the population for each country (n) is the average between 1960 and 1999, where the population comes directly from the PWT. For the initial year it is assumed that $K_0 = I_0/[(1+g)(1+n)-(1-\delta)]$, where K_0 is the initial capital level, calculated as an average of the first five years observed. The series is almost identical, except that they also include 2000 as part of their sample.

[4] I use the attainment years of the population over 15 years.

(BL) database. The data is interpolated to obtain a yearly frequency, and the constants used in equations (3.8) and (3.9) are the same used in the FPV paper.[1] For total employment I use the Population Division (2007) database.

Given the restricted availability of some data sources (particularly sectoral information), the final sample is of 53 countries (for a detailed list, see Appendix C). One of the most restrictive databases for my purposes is the WDI where the information of GDP in local currency is obtained. The years used in the sample are 1971-1999. The BL database includes information up to 1999, and for 1970 there are some countries which lack information in the WDI. Another restrictive and crucial database is the FAO study. Even when more sources were available for other data, the rarity of work in comparable international data at the sectoral level makes the task of increasing the sample nearly impossible. Future additional research on the subject depends on the expansion of such type of study for more countries and more recent years.

Before introducing the main results of the chapter I will briefly discuss the data that I use in relation to a well known result in growth literature, concretely the variance decomposition exercise that has been performed extensively. For the rest of the chapter I will name the contribution of element P to the variance decomposition as $\Phi_P \equiv \frac{cov(\ln y, \ln P)}{var(\ln y)}$, in which the elements are defined following the tradition of Hall and Jones (1999) and Klenow and Rodriguez-Clare (1997). For the initial exercises the two possible components are TFP (A) and factors (X). The results for 78 countries for which I have aggregate data are presented in Table 3.2, which show exactly the same behavior that is observed in the FPV paper. The choice of the parameter for capital of their one-sector growth model is $\alpha = 0.4$ to stay as close as possible as FPV's setup.

Table 3.2: Variance decomposition with 78 countries

Year	1971	1972	1973	1974	1975	1980	1985	1990	1995	1999
Φ_A	0.379	0.394	0.412	0.409	0.407	0.451	0.456	0.502	0.533	0.534
Φ_X	0.621	0.606	0.588	0.591	0.593	0.549	0.544	0.498	0.467	0.466

When doing exactly the same variance decomposition as FPV but with the smaller sample (due to the lack of sectoral information for several countries), I obtain a similar behavior of the estimated parameters as they do, an increase in the importance of TFP. The result presented in Table 3.3 corresponds to my limited sample of 53 countries,[2]

[1] The parameters used in the paper are $\zeta = 0.58$ and $\theta = 0.32$.

[2] The countries that are removed from the sample are Australia, Benin, Barbados, Botswana,

Table 3.3: Variance decomposition with 53 countries

Year	1971	1972	1973	1974	1975	1980	1985	1990	1995	1999
Φ_A	0.422	0.435	0.463	0.456	0.443	0.454	0.447	0.477	0.488	0.479
Φ_X	0.578	0.565	0.537	0.544	0.557	0.546	0.553	0.523	0.512	0.521

It is clear that the selection of the countries affects the quantitative results. Interestingly, even when an increase of the Φ_A is still present in the data, this component fails to become the most important one in the recent data, which is not consistent with the traditionally accepted result that TFP accounts for about two thirds of the variance of GDP per capita. I want to acknowledge that the different sample used doesn't allow for a quantitative comparison to the results in FPV or to the classical literature. Nevertheless, both in the larger sample (similar to FPV's) and in my reduced sample I find that there is an increase in the importance of TFP, which is the trend that motivates the current research.

The goal of this dissertation is to identify if this increase in the importance of TFP in explaining the variance of GDP per worker can be accounted for by including the agriculture sector. These last results represent my baseline. The exercises in the following section are meant to provide insights about why including the agriculture sector can play a role in the model, and the results obtained can be comparable to the ones presented in Table 3.3.

3.4 Estimating the importance of agriculture

I am going to analyze different approaches to study the issue of structural transformation by including the agriculture sector in the traditional one-sector model and how those approaches relate to the findings in FPV. The main goal of this section is to obtain results that are comparable with the findings in that paper. To achieve this, I will rewrite the production function of the economy composed of two sectors in such a way that a variance decomposition exercise can be done, which is similar to what CR do. The final expression for the production function will only differ from the one sector model in that it includes an additional component, the adjustment for having

Switzerland, Fiji, Guyana, Ireland, Iceland, Israel, Jamaica, Jordan, Lesotho, Mozambique, Mauritius, Nicaragua, New Zealand, Panama, Syrian Arab Republic, Togo, Trinidad and Tobago, Tanzania, Uganda, Uruguay, Zambia.

an agriculture sector. Expressing the production of the economy in terms of the non-agriculture sector allows for a comparison with the results in the traditional models.

Assume that all the resources in the economy are allocated to the non-agriculture sector. Then the production of the economy would be represented by equation (3.10).

$$Y_{it} \quad = \quad A_{nit} R_{it}^{\beta} K_{it}^{\alpha} H_{it}^{1-\alpha-\beta} = A_{nit} \mathcal{F}_{nit}. \tag{3.10}$$

I assume a value of $\beta \approx 0$ for the calculations, and this translates into the usual one sector model that considers physical and human capital as the only inputs for production. Now, dropping the latest assumption of only one relevant sector this expression will remain useful, as shown next, in representing the two sector economy in a simple form, and will make the results comparable to previous research.

One important assumption that must be made before proceeding is that the factors are allocated efficiently in the two sectors, which is a common assumption in the growth literature. The main reason why it must be made is that there is no data on the sectoral allocation of factors, in particular for capital. Other solutions have been proposed in the literature, for example, Vollrath (2009) uses a capital series developed by Crego et al. (1998), but this series lacks some of the later years of our sample and the observations are in domestic prices.

The assumption of efficient allocation of resources between the two sectors implies the choice of a vector of prices, which may or may not be the same as the international prices at which production is valued for international comparisons. For example, resources may be allocated considering domestic prices. To relate the data generated by the efficiency assumption and the information at which production is valued for international comparison it is necessary to include in the notation a ratio of two sets of prices, one at which production is valued (international prices, denoted by a star superscript) and the prices assumed to be relevant for factor allocation. This ratio will be called λ_{it}, which takes different values for each country (i) and year (t),

$$\lambda_{it} = \frac{\frac{p^*_{ait}}{p^*_{nit}}}{\frac{p_{ait}}{p_{nit}}}.$$

The use of PWT data and some other source of GDP series that uses a different set of prices for valuating production (e.g. domestic prices) allows this ratio to be identified, which is useful for the estimation. If both sets of prices are the same, $\lambda_{it} = 1 \quad \forall i, t$.

To proceed with cross country comparisons, the allocation rules must be expressed in international prices. Notice that the choice of a set of prices for the allocation of resources will determine the value of λ and how much of a resource is used in each sector. Dropping time and country subscripts the conditions are

$$\lambda\alpha\frac{A_nF_n}{K_n}=\gamma\frac{A_aF_a}{K_a} \quad ; \quad \lambda\beta\frac{A_nF_n}{R_n}=\delta\frac{A_aF_a}{R_a} \quad ; \quad \lambda(1-\alpha-\beta)\frac{A_nF_n}{H_n}=(1-\gamma-\delta)\frac{A_aF_a}{H_a}.$$

It is now possible to rewrite the equation that represents the whole economy in a simplified form. Start with the production of the economy composed of the two sectors

$$\begin{aligned} Y &= A_aF_a+A_nF_n \\ &= A_nF_n\left[\lambda\left(\beta\frac{R}{R_n}+\alpha\frac{K}{K_n}+(1-\alpha-\beta)\frac{H}{H_n}\right)+(1-\lambda)\right]. \end{aligned} \tag{3.11}$$

Equation (3.11) uses the equilibrium conditions in the allocation of resources. I will rewrite this equation so that it includes the expression in (3.10), and the production function becomes comparable with a model in which all the factors are used in the unique non-agriculture sector. The value of the difference between this new expression and (3.10) will determine how important is the inclusion of the agriculture when measuring aggregate production.

$$\begin{aligned} Y = \; & A_n\mathcal{F}_n\Bigg\{\lambda\Bigg[\beta\left(\frac{R_n}{R}\right)^{\beta-1}\left(\frac{K_n}{K}\right)^{\alpha}\left(\frac{H_n}{H}\right)^{1-\alpha-\beta}+\alpha\left(\frac{R_n}{R}\right)^{\beta}\left(\frac{K_n}{K}\right)^{\alpha-1}\left(\frac{H_n}{H}\right)^{1-\alpha-\beta} \\ & +(1-\alpha-\beta)\left(\frac{R_n}{R}\right)^{\beta}\left(\frac{K_n}{K}\right)^{\alpha}\left(\frac{H_n}{H}\right)^{(1-\alpha-\beta)-1}\Bigg]+(1-\lambda)\left[\left(\frac{R_n}{R}\right)^{\beta}\left(\frac{K_n}{K}\right)^{\alpha}\left(\frac{H_n}{H}\right)^{1-\alpha-\beta}\right]\Bigg\} \end{aligned} \tag{3.12}$$

Equation (3.13) can be simply expressed as

$$\begin{aligned} Y &= A_n\mathcal{F}_n\left\{\lambda Z_n+(1-\lambda)\left(\frac{F_n}{\mathcal{F}_n}\right)\right\} \\ Y &= A_n\mathcal{F}_n\Psi_n \end{aligned} \tag{3.13}$$

where Ψ_n is the expression between brackets in equation (3.13) and

$$Z_n=\left(\frac{R_n}{R}\right)^{\beta}\left(\frac{K_n}{K}\right)^{\alpha}\left(\frac{H_n}{H}\right)^{1-\alpha-\beta}+\alpha\left(\frac{R_n}{R}\right)^{\beta}\left(\frac{K_n}{K}\right)^{\alpha}\left(\frac{H_n}{H}\right)^{1-\alpha-\beta}+(1-\alpha-\beta)\left(\frac{R_n}{R}\right)^{\beta}\left(\frac{K_n}{K}\right)^{\alpha}\left(\frac{H_n}{H}\right)^{(1-\alpha-\beta)}.$$

Z_n reveals information regarding the importance of the non-agriculture sector in total GDP. If a country had a nonexistent agriculture sector, all the resources of the economy would be allocated to the non-agriculture sector, and the value of Z_n would be 1. This component measures how big is the bias in TFP induced by ignoring the existence of the agriculture sector. This component was introduced in CR's setup. The importance of the size of Z_n with respect to the bias in TFP depends on λ.

A numerical example will clarify the implications of this component. Consider a country with $\lambda = 1$ and assume values for a low productivity, agriculture oriented country in year t, $Z_{nt} = 1.5$ and $A_{nt} = 100$. Disregarding the adjustment for the agriculture sector leads to an overestimation of the TFP component by 50%, a "naive" measure of TFP (using a one-sector model) would be $\hat{A}_{nt} = 150$. The changes on the value of this parameter through time are also key to understanding the results presented next. Imagine that s years later $A_{n,t+s}$ is still 100, but $Z_{n,t+s} = 1.25$. The traditional formula that ignores the second sector would result in $\hat{A}_{nt} = 150$ and $\hat{A}n, t+s = 125$. Although there was no real decrease in TFP, the measure obtained with the traditional model indicates a lower value for this component. The decrease in the importance of agriculture (implied in the lower adjustment value Z_n) would be disguised as a widening of the differences of TFP. This widening effect would cause this component to be able to explain more of the GDP per worker differences when a variance decomposition is done.

When $\lambda = 1$, Ψ_n collapses to the value of Z_n. If, on the other hand, the ratio of prices differs significantly from the unit value, the importance of each component of Ψ_n will differ, and how the relative prices change through time will also play a role in the estimation of TFP.

The variance decomposition exercise can be done in a similar way to how it has been presented in the literature. Output is defined as

$$y \equiv \frac{Y}{L} = \bar{A}\bar{X}\bar{\Psi}$$

where

$$\bar{A} = A_n^{1/(1-\alpha)}; \qquad \bar{\Psi} = \Psi_n^{1/(1-\alpha)}; \qquad \bar{X} = \left(\frac{R}{hL}\right)^{\beta/(1-\alpha)} \left(\frac{K}{Y}\right)^{\alpha/(1-\alpha)} h$$

The decomposition is then

$$\Phi_{\bar{X}} = \frac{cov(\ln y, \ln \bar{X})}{var(\ln y)}; \qquad \Phi_{\bar{A}} = \frac{cov(\ln y, \ln \bar{A})}{var(\ln y)}; \qquad \Phi_{\bar{\Psi}} = \frac{cov(\ln y, \ln \bar{\Psi})}{var(\ln y)}.$$

3.4.1 Analysis of $\lambda_{it} = 1 \quad \forall i, t$.

As a first exercise, I will analyze the case in which the relative prices are the same both for the valuation of output and the allocation of resources in equilibrium, PWT prices. This is close to the analysis done by CR and is more closely related to what is presented in FPV, the only difference with respect to a one sector model is the inclusion of agriculture. The expressions used for the variance decomposition are the same as the ones that were introduced immediately above but using Z instead of Ψ.

CR use parameters for their baseline estimation $\beta \approx 0$, $\alpha = 1/3$, $\gamma \approx 0$ and $\delta = 1/3$. I will use instead a value of $\alpha = 0.4$, as I mentioned before, to make the results as comparable as possible to those of FPV. These values imply that the parameters for human capital are 0.6 everywhere in the economy, and capital is used in the non-agriculture sector with the same intensity as land is used in the agriculture sector. This also implies that $K_n \approx K$, and $R_n \approx 0$, which means that all the capital is allocated to the non-agriculture sector, while none of the land is used in this sector. The other implication, that virtually no capital is used in agriculture, is an assumption best suited for those countries in which this sector is rather important in terms of total production, which are the poorer and the ones that rely much more in workers to extract the production from the land. I have data available for L_n, but I will assume that this factor is also assigned efficiently through the economy, which is not a key assumption for the results obtained in this section and makes possible to rewrite production as in (3.13).

Since a more developed country has a greater relative importance of the non-agriculture sector compared to a poor country, its Z_n value would be smaller. Thus, it is expected that the coefficient Φ_Z that I estimate is negative. The estimations done by CR are based only on one year, but it is relevant to know if those results would vary with changes in sectoral composition observed in the world in the last decades. The summarized results are presented in Table 3.3. On average, the relative importance of the Z_n component decreased over time. Most countries have experienced a growth of the non-agriculture sector relative to the size of the whole economy, making this adjustment smaller in magnitude in later years. While this is true both for rich and poor countries, the magnitudes of reduction vary greatly for different countries, and these help explain in part the differences in GDP per worker as seen next.

The result of the decomposition is that around 3% of the increase in the importance of TFP comes from the adjustment for having more than one sector, as shown in the decrease of the importance of the component Z_n in explaining GDP per worker. In fact, if the years 1971 and 1972 are not considered,[1] that number is over 5%. The most relevant idea that can be extracted from Table 3.4 is that the increase observed in the coefficient of TFP in Table 3.3 (the traditional one sector model) is no longer present

[1] In this years the share of agriculture in the USA at domestic prices is significantly small, as explained before.

Table 3.4: Variance decomposition using two sectors

Year	$\Phi_{\bar{A}}$	$\Phi_{\bar{X}}$	$\Phi_{\bar{Z}}$
1971	0.469	0.578	-0.047
1972	0.485	0.565	-0.050
1973	0.532	0.537	-0.069
1974	0.519	0.544	-0.063
1975	0.495	0.557	-0.052
1976	0.500	0.551	-0.051
1977	0.497	0.552	-0.050
1978	0.495	0.563	-0.058
1979	0.500	0.559	-0.059
1980	0.502	0.546	-0.048
1981	0.483	0.558	-0.041
1982	0.464	0.572	-0.035
1983	0.462	0.570	-0.032
1984	0.481	0.554	-0.035
1985	0.474	0.553	-0.028
1986	0.478	0.549	-0.028
1987	0.493	0.536	-0.029
1988	0.498	0.533	-0.032
1989	0.503	0.531	-0.033
1990	0.511	0.523	-0.034
1991	0.498	0.531	-0.029
1992	0.505	0.522	-0.027
1993	0.500	0.528	-0.028
1994	0.542	0.485	-0.027
1995	0.515	0.512	-0.027
1996	0.508	0.524	-0.032
1997	0.505	0.521	-0.026
1998	0.501	0.519	-0.020
1999	0.496	0.521	-0.017

with the new estimation. If, again, the first two years are dropped, there is even a decrease in the importance of this component.

Appendix E contains graphs of a small sample of countries in which it can be observed how the importance of Z_n can explain part of the changes in TFP compared to the when only one sector is considered. For a country like India the contribution is significant, about 11.1% of the TFP observed in 1973 with a one-sector growth model can be accounted for by the adjustment of the agriculture sector in 1973, and this adjustment becomes only 1.5% in 1999. This implies that if a unique production function is used to infer the TFP component for India, almost 10% of the productivity increase observed can be explained by including another sector (agriculture) into the model.[1] Less dramatic examples are El Salvador and Korea, countries in which the contribution of this adjustment through time is around 4% and 2.4% respectively. A table in Appendix F summarizes the results for 1973 (the year where the effect of agriculture is greater) and 1999 in my sample for each country.

Taking into account the agriculture sector eliminates the clear increase in the importance of TFP in explaining GDP per worker, the results even suggest that the importance of TFP has decreased.

3.4.2 Analysis of $\lambda_{it} \neq 1$.

Let's analyze a more general setup in which the allocation of resources does not occur according to the international prices. For the previous exercise the relevant prices in each country are the international prices used in PWT, which is a very strong assumption. The implication of this assumption is that factors are allocated in equilibrium using those international prices, and in the case of labor, most of it is allocated to the non-agriculture sector.[2] The reason for this result is that the size of this sector is very large relative to total GDP. Appendix G shows the difference between equilibrium labor from the model and the actual allocation of labor observed from the data for 1985. For most of the countries, especially the poorer ones, a much higher labor share is predicted for the non-agriculture sector than the one in the data.

[1] The change in productivity is positive in some countries, and negative for some. Nevertheless, it is important to keep in mind that the information in the variance decomposition is related to the differences between countries in a specific year, and not the changes that countries experience over time.

[2] An implicit assumption of the model used so far is that the human capital of all the workers is the same. Therefore, the allocation of workers and the allocation of human capital are, in percentages, the exact same measure.

Hsieh and Klenow (2007) show that although investment rates seem correlated with income when it is measured at international prices, the shares are fairly constant when local prices are considered. When measured at local prices, the agriculture share of GDP is usually larger in poorer countries when compared to the equivalent measure at international prices. With that feature in mind, it is possible to rewrite the equations using the assumption that factors are allocated efficiently between the two sectors, but with this allocation being based in local prices instead of international prices. It is key to emphasize again that these local prices determine how resources are allocated, but don't change how output is valued.

Assume that for each country the prices that determine the allocation of resources in equilibrium are the prices used in each country's series of national accounts in real terms. Changes in the relative size of agriculture provide information on how the shares of real sectoral GDP have changed within each individual country. Therefore, for each country at each point in time there is a specific ratio of domestic prices (which are constant) and the international prices. Following the notation introduced before, the λ ratio includes two sets of prices, a star superscript denotes the international prices (PWT), as opposed to the local constant prices used in national accounts (WDI). Since these prices represent the only difference between the production measured using the two distinct sets of prices, it is possible to extract the values of λ_{it} using

$$\lambda_{it} = \frac{\frac{p^*_{ait} Y_{ait}}{p^*_{nit} Y_{nit}}}{\frac{p_{ai} Y_{ait}}{p_{ni} Y_{nit}}}$$

where $p^*_{jit} Y_{jit}$ represents the total GDP of sector j at year t in country i in the database generated in Section 3 of this document (at international prices). The corresponding expression $p_{ji} Y_{jit}$ represents the total real GDP of sector j at year t in country i from national accounts (WDI).

A value of λ_{it} smaller than 1 implies that the relative price of agriculture at international prices is smaller than the relative price of agriculture at constant domestic prices. This is the usual case for poorer countries, as Duarte and Restuccia (2006) show, agricultural relative prices fall as countries develop.

The Z_n is now calculated using local prices which usually results in a higher value for poorer countries which in turn have a much larger agriculture sector. Note, nevertheless, that the expression Ψ_n is a weighted average between this Z_n and another component which is the importance of factors assigned to the non-agriculture sector relative to the overall factors available in the economy. The estimated values of λ are smaller for these poorer, more agriculture oriented countries. The result is that the larger Z_n's receive a lower weight, so the effect of including this adjustment is canceled. In fact, when the estimation is done, the result is that Ψ_n helps explain GDP differences - richer countries have a larger Ψ_n. The implication of this alternative adjustment to the production function is that the TFP component has the opposite "bias" compared to the result in the previous section. TFP in poorer countries must be underestimated in the economy given the assumptions of this section.[1]

The results are shown in Table 3.5.

Table 3.5: Variance decomposition with allocation according to domestic prices

Year	1971	1972	1973	1974	1975	1980	1985	1990	1995	1999
$\Phi_{\bar{A}}$	0.394	0.425	0.513	0.485	0.449	0.436	0.378	0.436	0.427	0.382
$\Phi_{\bar{X}}$	0.551	0.539	0.513	0.520	0.532	0.537	0.546	0.519	0.508	0.521
$\Phi_{\bar{\psi}}$	0.055	0.036	-0.026	-0.005	0.019	0.027	0.076	0.045	0.065	0.098

Note that the increase in the importance of TFP is no longer present. Furthermore, if the first two years of the sample are not considered, from 1974 to 1999 there is a significant decrease in the importance of this component. This result provides some supporting evidence for the findings from the specific case $\lambda = 1$, in which the increase in the importance of TFP was not as strong as in the one-sector model.[2]

If the countries are analyzed separately, I find that the λ's decrease for most countries because international prices have decreased faster than the domestic prices in most cases. This causes a greater weight to be given to $\frac{F_n}{\mathcal{F}_n}$ in equation (3.13). Since this component is more important in richer countries, Ψ contributes in a greater measure to explain income differences. This means that under the assumption that local prices are the ones that determine the allocation of resources in the economy, the initial importance that is given to the adjustment for having an agriculture sector subsides and allows the factors in the non-agriculture sector to play a more important role in the latter years.

Even when the effect of the adjustment is the opposite as it was when a value of $\lambda = 1$ is assumed (overestimation of TFP in poor countries), the implication for the im-

[1] Two countries, Nepal and Papua New Guinea, cannot be used in this estimation due to lack of data.

[2] Once more, due to the small sample used in this exercise I do not claim that the absolute values of the importance of TFP contradict the traditional results in the literature. What I do want to emphasize is the change observed through time of this measure which turns to be, at best, constant.

portance of TFP in explaining GDP per worker through time is the same: it decreases. This result provides additional supporting evidence for arguing that the results in the FPV paper can be explained by expanding the model to include agriculture. When the economy has more than one sector the result is that TFP's importance has not increased even when considering different efficiency conditions, and specially when the non-agriculture sector is considered separately.

3.4.3 Analyzing sectors separately

One last exercise that can be performed is based on the novelty that sectoral production valued at international prices were obtained in Section 3. A natural step is to do the traditional variance decomposition for each sector separately. Note that when analyzing one sector the other becomes irrelevant, it is as if, for example, the world were only composed of agricultural production. The analysis is similar to that in a one-sector model and, therefore, relative prices become irrelevant. Then, the production of each sector can be represented with equations (3.14) and (3.15) respectively.

$$y_n \equiv \frac{Y_n}{L_n} = (\tilde{A}_n)^{\frac{1}{1-\alpha}} \left(\frac{R_n}{hL_n}\right)^{\frac{\beta}{1-\alpha}} \left(\frac{K_n}{Y_n}\right)^{\frac{\alpha}{1-\alpha}} h \tag{3.14}$$

$$y_a \equiv \frac{Y_a}{L_a} = (\tilde{A}_a)^{\frac{1}{1-\gamma}} \left(\frac{R_a}{hL_a}\right)^{\frac{\delta}{1-\gamma}} \left(\frac{K_a}{Y_a}\right)^{\frac{\gamma}{1-\gamma}} h \tag{3.15}$$

Analogous to the exercise done at the end of the previous section, it is possible to measure the importance of TFP and the factors (Φ_A and Φ_X), but in this case the analysis is done for both agriculture and non-agriculture separately. The variance decomposition of the agriculture sector is shown in Table 3.6, and the corresponding exercise for the non-agriculture sector is shown in Table 3.7.

Table 3.6: Variance decomposition in agriculture

Year	1971	1972	1973	1974	1975	1980	1985	1990	1995	1999
Φ_{Aa}	0.706	0.703	0.713	0.714	0.723	0.737	0.753	0.764	0.776	0.777
Φ_{Xa}	0.294	0.297	0.287	0.286	0.277	0.263	0.247	0.236	0.224	0.223

When the agriculture sector is considered separately, the values of each component's contribution have the same qualitative behavior as in the traditional one-sector growth model. In terms of quantitative values, the importance of the TFP is much greater compared to the aggregate measure. Remarkably, the non-agriculture sector

Table 3.7: Variance decomposition in non-agriculture

Year	1971	1972	1973	1974	1975	1980	1985	1990	1995	1999
Φ_{An}	0.480	0.489	0.480	0.448	0.430	0.304	0.221	0.304	0.280	0.281
Φ_{Xn}	0.520	0.511	0.520	0.552	0.570	0.696	0.779	0.696	0.720	0.719

has a totally different behavior. The TFP component has been steadily decreasing it's importance in explaining GDP per worker difference.

Lagakos and Waugh (2009) show how a simple two sector model that includes subsistence consumption results in differences in sectoral productivity between countries with different income levels. Their model predicts that these differences will be greater in agriculture, the sector that provides the subsistence consumption. This is consistent with my finding of a large contribution of the TFP component in explaining differences in GDP per worker in this sector. Also, even when the variance of GDP per worker in agriculture increases 60% in the sample years, the covariance of product and factors does not increase as significantly (only 22%), which implies that, in recent years, factors[1] are able to explain less of the differences in GDP per worker. Lagakos and Waugh (2009) argue that as the aggregate income of countries increase, inefficient workers leave the agriculture sector, increasing the productivity per worker for those countries.[2]

The result in non-agriculture is interesting because the effect goes in the opposite direction as the findings of FPV. During the sample period, the covariance between non-agriculture production and the factors assigned to this sector increase by 116%, while the variance of the production only increase by 56%. This causes the contribution of factors to explaining sectoral income differences to increase dramatically as is observed on Table 3.7. The main issue that this data brings up to discussion is that measuring the TFP with a one sector model affects the measurement of TFP (in particular the one in non-agriculture, the sector that is assumed to represent the economy in these models).

Much of the variance of GDP per worker can be related to TFP of the agriculture sector,[3] rather than to differences in the same component for the non-agriculture sector.

[1]In this case land and human capital due to the fact that efficient allocation of factors assigns virtually no capital and almost all land to this sector.

[2]In their setup, the price of agriculture should also fall, while in the model presented here the prices at which production is valued are assumed to be international prices.

[3]As Caselli (2005) discusses, the variance of the agriculture production is very large compared to the aggregate, and even more when compared to the non-agriculture sector. He argues that the

Duarte and Restuccia (2006) show that labor productivity differences are significant in agriculture and services, while they are of smaller importance in manufacture. Developing countries have been able to catch up in manufacture productivity, a result is consistent with the lower importance of TFP in the variance decomposition that I do for the non-agriculture sector. The results presented in this section provide additional evidence that considering two different production functions results in the importance of TFP and factors in explaining sectoral income differences to have different magnitudes and effects through time, and these differences must not obviated.

3.5 Conclusion

The result in FPV that the importance of TFP has increased over time can be explained in my sample by considering more than one sector and adjusting the decomposition for the size of the agriculture sector. The result of the estimation done is that this importance decreases (or at best remains constant) through time. Even further, the results of a separate sectoral variance decomposition for non-agriculture are much more stark: countries have become more similar in terms of TFP and a greater part of the variance can be explained by factors. This evidence is also supported by a model that considers local prices instead of international prices in the allocation of factors in the economy.

Given these results, any concern caused by divergence in terms of TFP observed in FPV should be focused on the agriculture sector, rather than being generalized to the whole economy. It is also the case that more variation in aggregate GDP can be explained by variation in the production of the agriculture sector, as Caselli (2005) notes. In the case of the production of non-agricultural goods, the accumulation of physical and human capital play a much more important role than the one perceived when looking at the aggregate statistics. These factors are very significant in explaining differences in GDP per worker of this sector and have become more important with time.

The results presented here rely on accounting only for one more sector, agriculture sector. It is possible that a more complete explanation could be given if other sectors are considered. For example, Duarte and Restuccia (2006) note that the main reason why some countries have stagnated is that the productivity in the service sector has not increased as much as in the USA. Nevertheless, without comparable data (valued

agriculture sector is the critical source of dispersion in aggregate per worker income.

at international prices) of the different sectors in the economies, such studies are not possible.

A caveat that should be considered is the possibility of measurement error in the production data used in the present chapter due mainly to two reasons. First, the definition of the agriculture sector in the data from Prasada Rao (1993) is more restricted than what is desirable. Second, the data generated for years other than 1985 is subject to very strong assumptions. For my calculations I have approximated a time series of the GDP per worker in agriculture and non-agriculture. The development of a database that extends the work done by the International Comparison Program and the PWT so that data on comparable production measures (hopefully for several sectors) can be used would, without doubt, benefit the quantity and quality of the research that could be done. A deeper understanding of why countries are different in each particular sector could help understand much better the aggregate results and allow for the design of specific policies focused on reducing the prevailing income gaps in the world.

The extension of the database should also allow the inclusion of more countries in the analysis. Compared to the number of existing countries, the estimations done include only a small fraction of them. It is even a limited sample when compared to the database used in the FPV database. The inclusion of more countries could make the results more reliable as representative of the world. As shown at the end of Section 3, having a smaller sample of countries for the decomposition makes the results vary quantitatively.

Future research should also focus on the issue of the allocation of resources in the economy. In the present dissertation the assumption is that the factors of production are allocated efficiently in the different sectors of the economy. Vollrath (2009) argues that the misallocation of resources can explain a significant percentage of GDP differences. To quantify how these results vary through time it is necessary to have at least a series of sectoral physical capital measured at international prices, which is data that is not yet available. A relevant question that such an exercise could answer is whether the convergence in terms of TFP that countries have experienced in the non-agriculture sector shown in Section 4 can be explained with a better allocation of factors in poorer countries.

Another issue that must be addressed is the choice of the prices used in PWT. Even though these prices are taken as given in this dissertation, the choice is not irrelevant to the results. Hill (2000) argues that the way in which the Purchasing Power Parity (PPP) statistics used in PWT are calculated overestimates the incomes of countries that have relative prices significantly different from the reference prices. Since the price vector that is used in the current PPP measures (using the Geary-Khamis method) is similar to that of a rich country, the overestimation would occur in poorer countries, and the differences between the output of the countries would tend to be underestimated. If it were the case that countries have become more similar in terms of domestic prices, this bias induced by the reference price vector would be reduced. This is an additional plausible explanation for the findings of FPV, and the inclusion of a more balanced price vector into the model presented here could provide a much closer representation of how different components can explain cross country differences.

Appendix A

Proof of Proposition 1: The effect of T_n in patenting

Note that by defining the variable $b_s = T_n \bar{q}^{-\theta_s}$, it is possible to show that Equation (2.2) can be written as

$$\frac{V_{inst,\mathrm{BT}}(q)}{Y_{nt}} = (Jg)^{-1} e^{-b_s} \left(1 - e^{b_s}(b_s)^{1/\theta_s}\Gamma[(\theta_s - 1)/\theta_s, b_s]\right) \{\Psi(\hat{r}_{in}/g, b_s) - \Psi[(\hat{r}_{in} + \epsilon_{in})/g, b_s]\}$$

where

$$\hat{r}_{in} \equiv r + \iota_{in} - g_Y$$

and $\Psi(a/g, Tc) = g \int_0^\infty e^{-as} e^{-be^{gs}} ds$. Here g_Y represents the growth of Y_{nt} and g the growth of T_{nt}.

Similarly, when there is uncertainty that the idea constitutes a breakthrough in market n, Equation (2.3) can be written as

$$\frac{V_{inst,\mathrm{NB}}(q)}{Y_{nt}} = (Jg)^{-1} \left(1 - e^{b_s}(b_s)^{1/\theta_s}\Gamma[(\theta_s - 1)/\theta_s, b_s]\right) \{\Psi(\hat{r}_{in}/g, b_s) - \Psi[(\hat{r}_{in} + \epsilon_{in})/g, b_s]\}$$

Then,

$$\frac{V^l_{inst}(b)}{Y_{nt}} = \pi_{int} \frac{V^l_{inst,\mathrm{BT}}(b)}{Y_{nt}} + (1 - \pi_{int}) \frac{V^l_{inst,\mathrm{NB}}(b)}{Y_{nt}}$$

where $l = \mathrm{pat}, \mathrm{not}$.

The thresholds are then defined by

$$\frac{V^{\mathrm{pat}}_{inst}(b)}{Y_{nt}} - \frac{V^{\mathrm{not}}_{inst}(b)}{Y_{nt}} = f_{in} \tag{A.1}$$

I will name the left hand side of this equation $\bigvee_{inst}(b)$.

Preliminary result: $\frac{d\bar{b}_{ins}}{d\theta_s} < 0$

We know that in country n the cutoff for type of good s created in country i ($\bar{b}_{ins}$) is defined as the value of b that makes the following expression to hold with equality

$$\bigvee_{ins} (\bar{b}_{ins}, \theta_s, T_n) \geq f_{in}$$

To analyze what is the effect of a change in θ_s in the cutoff value $\tilde{b}_{ins}$, solve the following expression

$$\frac{\partial \bigvee_{ins}}{\partial \theta_s} d\theta_s + \frac{\partial \bigvee_{ins}}{\partial \bar{b}_{ins}} d\bar{b}_{ins} = 0$$

The solution is

$$\frac{d\bar{b}_{ins}}{d\theta_s} = -\frac{\frac{\partial \bigvee_{ins}}{\partial \theta_s}}{\frac{\partial \bigvee_{ins}}{\partial \bar{b}_{ins}}}$$

The expression is always negative[1] because the numerator is always negative,

$$\begin{aligned} \frac{\partial \bigvee_{ins}}{\partial \theta_s} \quad = \quad & -(Jg)^{-1} \left\{ \left[\Psi(\hat{r}_{in}^{\text{pat}}/g, b_s) - \Psi\left((\hat{r}_{in}^{\text{pat}} + \epsilon_{in})/g, Tc_\tau \right) \right] \right. \\ & \left. - \left[\Psi(\hat{r}_{in}^{\text{not}}/g, \bar{b}_{ins}) - \Psi\left((\hat{r}_{in}^{\text{not}} + \epsilon_{in})/g, \bar{b}_{ins} \right) \right] \right\} * \\ & \frac{e^{-\bar{b}_{ins}}}{\theta_s^2} G_{2,3}^{3,0} \left(\bar{b}_{ins} \middle| \begin{matrix} & 1 + \frac{1}{\theta_s}, & 1 + \frac{1}{\theta_s} \\ 1, & \frac{1}{\theta_s}, & \frac{1}{\theta_s}\left(\pi_{int} e^{T_n c} + (1 - \pi_{int}) \right) \end{matrix} \right) \end{aligned} \quad \text{(A.2)}$$

and the denominator is also negative. Even though it is not easily solvable, recall that the value of an idea is increasing with q, and therefore is decreasing with b.

Now, given this preliminary result, when we consider just two goods. Take the derivative with respect to x of the following ratio

$$\frac{\partial}{\partial x} \frac{1 - e^{-ax}}{1 - e^{-bx}} = \frac{e^{x(b-a)}[b(1 - e^{ax}) - a(1 - e^{bx})]}{(e^{bx} - 1)^2}$$

It is easy to show that this is a positive number if $b > a$. Then, we know exactly what happens with $\zeta_{int\tau}$ by knowing what happens with the following ratio:

$$\frac{1 - e^{-\frac{T_i}{T_n}\bar{b}_{ins}}}{1 - e^{-\frac{T_i}{T_n}\bar{b}_{ins'}}}$$

[1]This relies on Meijer's G Function, $G_{2,3}^{3,0} \left(\bar{b}_{ins} \middle| \begin{matrix} & 1 + \frac{1}{\theta_s}, & 1 + \frac{1}{\theta_s} \\ 1, & \frac{1}{\theta_s}, & \frac{1}{\theta_s} \end{matrix} \right)$ being positive, which is the case for $\bar{b}_{ins} > 0$ and values $\theta_s > 1$

Therefore, if I assume that $\theta_s > \theta_{s'}$, we know then that $\bar{b}_{ins} < \bar{b}_{ins'}$, it is immediate that this fraction is increasing with T_i/T_n. Hence, a lower stock of knowledge in the destination country causes more patenting in the good with the lowest dispersion (higher θ) and less patenting in the good with the highest.

Appendix B

Technological frontier

Recall that

$$\dot{T}_{nt} = M^{-1} \sum_{i=1}^{N} \epsilon_{in} \int_{-\infty}^{t} e^{-\epsilon_{in}(t-t')} \bar{\alpha} \alpha_{it'} \gamma_{it'}^{\beta} L_{it'} dt'$$

For an input of type s and quality z, new ideas will be adopted at a rate $\dot{T}_{nt} \left(z/\underline{q} \right)^{-\theta}$. Following Eaton and Kortum (2009), normalize $\bar{\alpha}\underline{q}^{\theta} = 1$. Taking the limit as $\underline{q} \to 0$ (and $\bar{\alpha} \to \infty$), it is possible to consider all qualities such that $z \in (0, \infty)$. Name $\dot{T}'_{nt} = M^{-1} \sum_{i=1}^{N} \epsilon_{in} \int_{-\infty}^{t} e^{-\epsilon_{in}(t-s)} \alpha_{it's} \gamma_{it'}^{\beta} L_{it'} ds$.

Therefore, the rate of adoption of ideas can be defined now as $\dot{T}'_{nt} z^{-\theta_s}$. Given that the probability of no adoption in $[t, t+dt]$ is then $e^{-\dot{T}'_{nt} z^{-\theta_s}}$, it is possible to solve for the the technological frontier of type s as

$$F_{is}(z; t) = e^{-T_{ist} z^{-\theta_s}}$$

For details on this derivation see Eaton and Kortum (1999).

Appendix C

List of countries in the estimation

Argentina, Austria, Bangladesh, Bolivia, Brazil, Canada, Chile, Cameroon, Colombia, Costa Rica, Denmark, Dominican Republic, Ecuador, Spain, Finland, France, United Kingdom, Ghana, Greece, Guatemala, Honduras, Indonesia, India, Iran, Islamic Rep., Italy, Japan, Kenya, Korea, Rep., Sri Lanka, Mexico, Mali, Malawi, Malaysia, Niger, Netherlands, Norway, Nepal, Pakistan, Peru, Philippines, Papua New Guinea, Portugal, Paraguay, Rwanda, Senegal, El Salvador, Sweden, Thailand, Turkey, United States, Venezuela, RB, South Africa, Zimbabwe.

Appendix D

Data for the agriculture sector

(Table in the following page)

Table D.1: Share of agriculture in GDP - 3 different data sources

	1973			1985			1999		
Country	NEW	WDIreal	WDInom	NEW	WDIreal	WDInom	NEW	WDIreal	WDInom
ARG	15.9	5.7	12.0	8.9	6.0	7.6	5.2	5.8	4.7
AUT	5.8	2.8	7.6	2.9	2.6	4.3	1.4	2.2	2.2
BGD	21.3	38.6	56.6	7.9	31.4	32.8	2.8	25.3	26.2
BOL	17.2	14.3	21.0	9.5	17.2	20.6	6.1	15.3	15.1
BRA	12.5	5.9	13.4	6.3	5.9	11.5	3.2	5.9	5.5
CAN	6.9	3.6	5.6	2.7	2.8	3.5	1.3	2.4	2.5
CHL	8.3	4.3	6.7	6.1	5.8	7.6	2.6	4.6	6.2
CMR	2.1	23.9	30.8	0.7	12.5	21.6	0.7	22.1	24.4
COL	13.4	16.9	24.6	6.2	15.9	17.5	2.7	11.1	14.0
CRI	18.7	14.6	20.2	8.5	12.7	11.9	5.2	11.9	10.5
DNK	5.2	1.8	6.3	3.7	2.2	5.0	2.3	2.5	2.4
DOM	12.8	20.0	22.2	5.3	16.3	13.1	1.9	11.6	11.4
ECU	18.5	18.1	20.2	7.0	14.8	13.4	3.0	10.9	11.7
ESP	7.3	4.6	10.5	3.8	4.4	6.2	2.2	4.3	4.5
FIN	4.8	7.0	11.0	2.1	5.5	8.2	0.7	3.4	3.4
FRA	6.5	3.9	7.9	3.2	3.4	4.3	1.6	3.0	3.0
GBR	2.3	1.2	2.9	1.4	1.4	1.8	0.6	1.1	1.2
GHA	36.6	51.2	49.0	15.5	51.6	44.9	10.5	41.1	35.8
GRC	9.7	9.0	14.3	4.9	8.7	11.6	2.1	7.2	7.1
GTM	12.1	17.7	27.9	5.5	16.2	25.9	3.1	14.2	23.0
HND	20.0	22.6	30.9	8.3	17.3	21.9	4.9	15.1	15.9
IDN	21.3	32.5	38.2	7.2	22.6	23.2	2.8	16.1	19.6
IND	24.2	42.1	43.7	10.1	35.0	31.2	3.8	25.0	25.0
IRN	5.1	8.2	15.0	4.7	12.6	19.4	2.3	14.5	15.1
ITA	7.0	3.4	8.1	3.2	2.8	4.7	1.7	3.0	3.0
JPN	2.1	4.6	6.2	0.7	2.9	3.3	0.2	1.8	1.9
KEN	30.1	36.4	35.5	13.4	32.4	32.6	7.7	29.8	32.4
KOR	6.7	19.0	26.7	1.9	11.1	13.5	0.4	5.2	5.2
LKA	17.2	26.1	27.3	8.0	23.2	27.7	3.1	15.4	20.7
MEX	6.6	6.1	12.1	2.9	5.1	10.1	1.3	4.0	4.7
MLI	36.8	39.5	57.5	18.5	38.3	40.3	21.6	46.9	46.5
MWI	55.3	32.8	41.6	24.8	29.4	42.9	14.9	37.8	37.8
MYS	14.5	26.1	27.2	5.6	18.6	19.9	1.2	8.8	10.8
NER	27.0	37.2	60.3	12.2	33.8	36.7	7.2	40.7	40.7
NLD	5.4	2.0	5.2	3.7	2.6	4.1	2.2	2.7	2.7
NOR	3.2	3.6	5.2	1.2	2.5	3.3	0.6	2.2	2.4
NPL	29.1	55.3	66.7	12.2	49.1	51.7	5.3	38.3	41.3
PAK	29.8	36.9	36.0	11.8	27.4	28.5	5.4	25.4	27.0
PER	6.5	8.7	n.d.	3.5	8.2	n.d.	2.9	10.1	n.d.
PHL	18.2	27.4	30.6	8.3	24.7	24.6	3.8	20.0	17.1
PNG	22.4	n.d.	29.5	14.3	35.8	37.0	9.8	39.8	36.1
PRT	4.5	7.0	28.2	2.4	6.9	15.4	0.7	4.1	4.0
PRY	22.0	22.9	37.7	11.2	20.5	28.9	8.4	23.0	17.6
RWA	34.3	36.3	61.0	18.8	37.0	41.8	12.2	43.9	41.9
SEN	18.9	22.6	23.8	9.2	22.7	23.2	4.6	19.0	19.0
SLV	11.7	17.8	39.8	5.8	19.0	26.9	2.1	14.2	11.2
SWE	3.8	3.2	5.1	1.7	2.7	4.5	0.7	2.0	2.2
THA	20.7	24.5	27.7	8.4	19.1	15.8	2.4	10.1	9.4
TUR	17.8	25.7	35.0	7.0	18.9	20.3	2.5	13.2	11.5
USA	4.8	1.2	4.8	2.4	1.2	2.4	1.2	1.1	1.2
VEN	3.9	4.4	5.4	2.7	5.7	6.3	1.3	5.5	4.7
ZAF	5.5	3.2	7.2	2.7	3.3	5.2	1.3	3.3	3.5
ZWE	16.5	17.0	16.6	6.6	19.0	22.7	4.7	17.8	19.4

Appendix E

Productivities for a sample of countries

Figure E.1: TFP's for El Salvador.

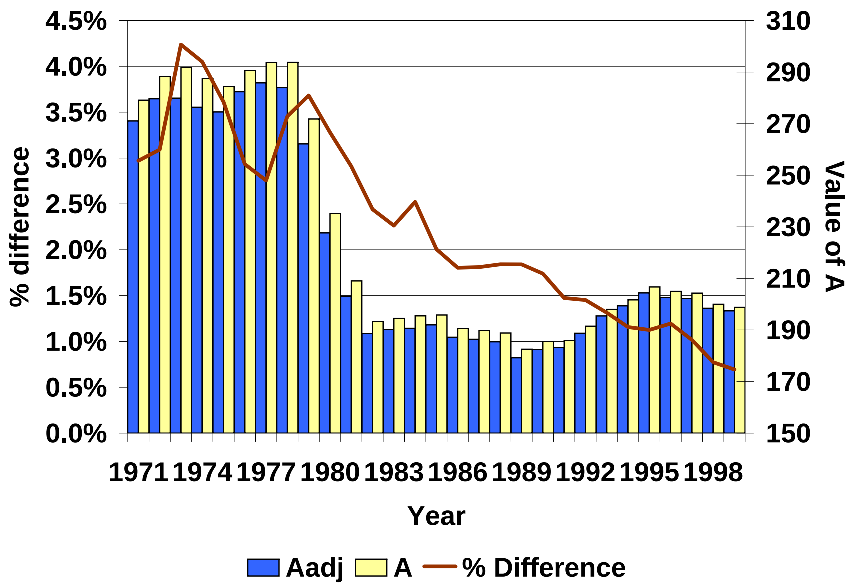


Figure E.2: TFP's for India.

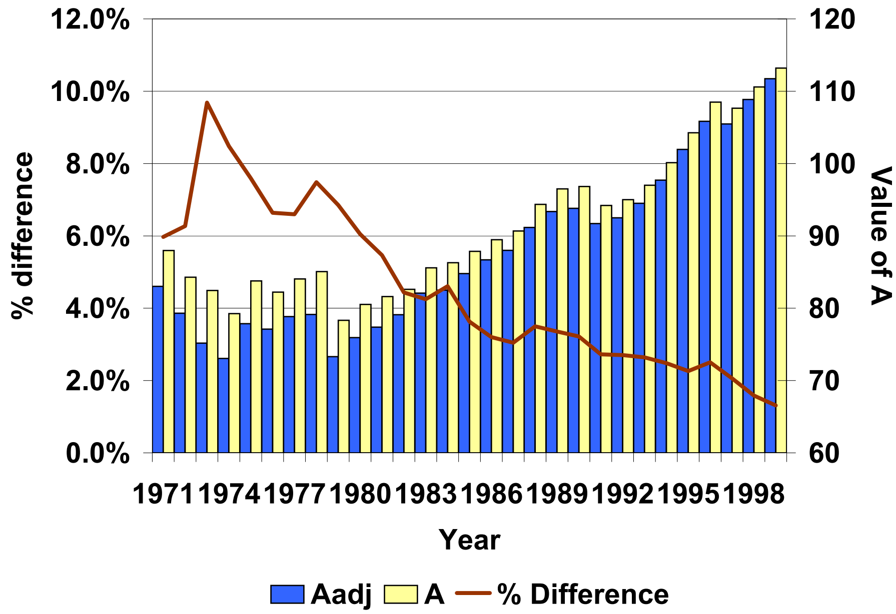


Figure E.3: TFP's for Korea.

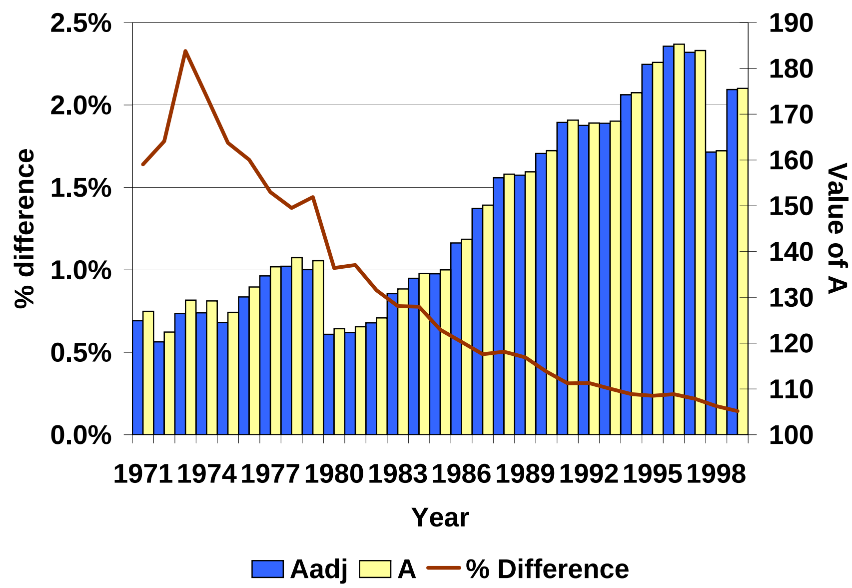
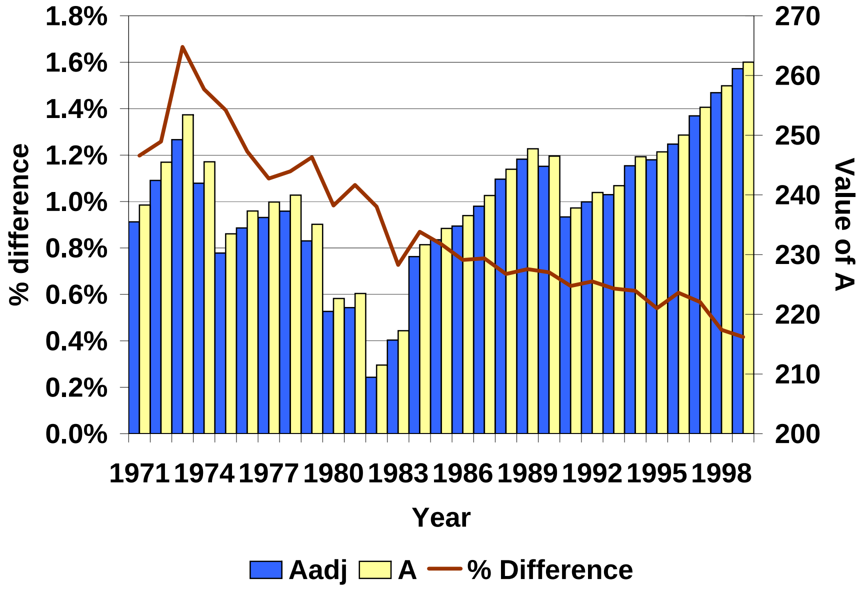


Figure E.4: TFP's for USA.



Appendix F

Adjustment required in a two sector model

(Table in the following page)

Table F.1: Z adjustment for agriculture

Country	Z_{1973}	Z_{1999}	Country	Z_{1973}	Z_{1999}
ARG	1.061	1.018	KOR	1.023	1.001
AUT	1.020	1.005	LKA	1.066	1.011
BGD	1.085	1.010	MEX	1.023	1.004
BOL	1.066	1.021	MLI	1.173	1.086
BRA	1.046	1.011	MWI	1.331	1.056
CAN	1.024	1.004	MYS	1.054	1.004
CHL	1.030	1.009	NER	1.114	1.025
CMR	1.007	1.002	NLD	1.019	1.007
COL	1.050	1.009	NOR	1.011	1.002
CRI	1.073	1.018	NPL	1.126	1.018
DNK	1.018	1.008	PAK	1.130	1.019
DOM	1.047	1.006	PER	1.023	1.010
ECU	1.072	1.010	PHL	1.070	1.013
ESP	1.026	1.007	PNG	1.091	1.035
FIN	1.017	1.002	PRT	1.016	1.002
FRA	1.023	1.005	PRY	1.089	1.030
GBR	1.008	1.002	RWA	1.157	1.045
GHA	1.172	1.038	SEN	1.074	1.016
GRC	1.035	1.007	SLV	1.043	1.007
GTM	1.045	1.010	SWE	1.013	1.002
HND	1.079	1.017	THA	1.082	1.008
IDN	1.085	1.009	TUR	1.069	1.009
IND	1.100	1.013	USA	1.017	1.004
IRN	1.017	1.008	VEN	1.013	1.004
ITA	1.025	1.006	ZAF	1.019	1.004
JPN	1.007	1.001	ZWE	1.063	1.016
KEN	1.131	1.027			

Appendix G

Labor data

(Table in the following page)

Table G.1: Labor share of the non-agriculture sector in 1985

Country	Share L_n Model	Share L_n Data	Country	Share L_n Model	Share L_n Data
ARG	0.90	0.87	KOR	0.98	0.72
AUT	0.97	0.91	LKA	0.91	0.50
BGD	0.91	0.31	MEX	0.97	0.68
BOL	0.90	0.50	MLI	0.80	0.13
BRA	0.93	0.70	MWI	0.73	0.13
CAN	0.97	0.95	MYS	0.94	0.66
CHL	0.93	0.80	NER	0.87	0.09
CMR	0.99	0.29	NLD	0.96	0.95
COL	0.93	0.66	NOR	0.99	0.93
CRI	0.91	0.69	NPL	0.87	0.06
DNK	0.96	0.94	PAK	0.87	0.42
DOM	0.94	0.71	PER	0.96	0.62
ECU	0.92	0.63	PHL	0.91	0.51
ESP	0.96	0.85	PNG	0.84	0.19
FIN	0.98	0.90	PRT	0.97	0.78
FRA	0.96	0.93	PRY	0.88	0.58
GBR	0.98	0.98	RWA	0.80	0.08
GHA	0.83	0.40	SEN	0.90	0.21
GRC	0.95	0.73	SLV	0.94	0.60
GTM	0.94	0.47	SWE	0.98	0.95
HND	0.91	0.51	THA	0.91	0.33
IDN	0.92	0.44	TUR	0.92	0.43
IND	0.89	0.33	USA	0.97	0.97
IRN	0.95	0.64	VEN	0.97	0.87
ITA	0.96	0.89	ZAF	0.97	0.85
JPN	0.99	0.91	ZWE	0.93	0.30
KEN	0.85	0.19			

Bibliography

Barro, Robert J. and Jong-Wha Lee, "International Data on Educational Attainment Updates and Implications," NBER Working Papers 7911, National Bureau of Economic Research, Inc September 2000. 37, 45

Bils, Mark and Peter J. Klenow, "Does Schooling Cause Growth?," *American Economic Review*, December 2000, *90* (5), 1160–1183. 45

Caselli, Francesco, "Accounting for Cross-Country Income Differences," in Philippe Aghion and Steven Durlauf, eds., *Handbook of Economic Growth*, 2005. 40, 42, 43, 57, 58

__ and Wilbur John Coleman, "Cross-Country Technology Diffusion: The Case of Computers," *American Economic Review*, May 2001, *91* (2), 328–335. 2

Chanda, Areendam and Carl-Johan Dalgaard, "Dual Economies and International Total Factor Productivity Differences: Channelling the Impact from Institutions, Trade, and Geography," *Economica*, 2008, *75* (300), 629–661. 38

Coe, David T. and Elhanan Helpman, "International R&D spillovers," *European Economic Review*, May 1995, *39* (5), 859–887. 2

Comin, Diego A. and Bart Hobijn, "An Exploration of Technology Diffusion," Harvard Business School Working Papers 08-093, Harvard Business School March 2008. 2

Córdoba, Juan Carlos and Marla Ripoll, "Agriculture and Aggregation," Working Papers 371, University of Pittsburgh, Department of Economics November 2006. 38

Crego, Al, Donald Larson, Rita Butzer, and Yair Mundlak, "A new database on investment and capital for agriculture and manufacturing," Policy Research Working Paper Series 2013, The World Bank November 1998. 48

Division, United Nations Population, "World Population Prospects: 2006 Revision Vol. 1: Comprehensive Tables," 2007. 45, 46

Duarte, Margarida and Diego Restuccia, "The Role of the Structural Transformation in Aggregate Productivity," 2006 Meeting Papers 415, Society for Economic Dynamics December 2006. 41, 54, 58

Eaton, Jonathan and Samuel Kortum, "International Technology Diffusion: Theory and Measurement," *International Economic Review*, 1999, *40* (3), 537–570. 1, 12, 18, 23, 64

__ and __ , "Technology in the Global Economy: A Framework for Quantitative Analysis," 2009. Unpublished manuscript. 64

Ferreira, Pedro, Samuel Pessoa, and Fernando Veloso, "The Evolution of International Output Differences (1970-2000): From Factors to Productivity," *Topics in Macroeconomics*, 2008, *8* (1), 1578–1578. 36

Fieler, Ana Cecilia, "Non-Homotheticity and Bilateral Trade: Evidence and a Quantitative Explanation," 2009. Unpublished manuscript. 14, 27

Hall, Robert E. and Charles I. Jones, "Why Do Some Countries Produce So Much More Output Per Worker Than Others?," *The Quarterly Journal of Economics*, February 1999, *114* (1), 83–116. 36, 46

Hill, Robert J., "Measuring substitution bias in international comparisons based on additive purchasing power parity methods," *European Economic Review*, January 2000, *44* (1), 145–162. 41, 59

Hsieh, Chang-Tai and Peter J. Klenow, "Relative Prices and Relative Prosperity," *American Economic Review*, June 2007, *97* (3), 562–585. 53

Klenow, Peter J. and Andres Rodriguez-Clare, "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?," *NBER Macroeconomics Annual*, 1997, *12*, 73–103. 36, 46

Lagakos, David and Michael E. Waugh, "Specialization, Economic Development and Aggregate Productivity Differences," 2009. Unpublished manuscript. 57

Lissoni, Francesco, Gianluca Tarasconi, and Bulat Sanditov, "The KEINS Database on Academic Inventors: Methodology and Contents," 2006. CESPRI Working Paper 181, Università Bocconi. 6

Lucas, Robert E., "Trade and the Diffusion of the Industrial Revolution," NBER Working Papers 13286, National Bureau of Economic Research, Inc August 2007. 1

Mankiw, N. Gregory, David Romer, and David N. Weil, "A Contribution to the Empirics of Economic Growth," *The Quarterly Journal of Economics*, 1992, *107* (2), 407–437. 36

Peters, May, Suchada Langley, and Paul Westcott, "Agricultural Commodity Price Spikes in the 1970s and 1990s: Valuable Lessons for Today," *Amber Waves*, March 2009, *7* (1), 16–23. 43

Phelps, E. S., "Models of Technical Progress and the Golden Rule of Research," *The Review of Economic Studies*, 1966, *33* (2), pp. 133–145. 12

Psacharopoulos, George, "Returns to investment in education: A global update," *World Development*, September 1994, *22* (9), 1325–1343. 45

Rao, D.S. Prasada, "Intercountry comparisons of agricultural output and productivity," Fao Economic and Social Development Paper 112, FAO, Rome 1993. 40, 59

Romer, Paul M, "Endogenous Technological Change," *Journal of Political Economy*, October 1990, *98* (5), S71–102. 1

Summers, Robert, Alan Heston, and Betina Atten, "Penn World Table Version 6.1," 2002. 41

Vollrath, Dietrich, "How important are dual economy effects for aggregate productivity?," *Journal of Development Economics*, March 2009, *88* (2), 325–334. 38, 48, 59

Zhou, Eve Y. and Bob Stembridge, "Patented in China, the Present and Future State of Innovation in China," Study 2010.

Alonso Alfaro Ureña

Contact Information

Departamento de Investigación
Banco Central de Costa Rica

Voice: + (506) 2243-3333
E-mail: alfaroua@bccr.fi.cr, alonso.alfaro@gmail.com

Research Interests

International trade, growth, development.

Education

Pennsylvania State University, Pennsylvania, USA.

Ph.D. Student, Economics (expected graduation date: May 2012)
M.A. in Economics (August 2010)

Universidad de Costa Rica, San José, Costa Rica.

B.Sc., Economics (Honors), February, 2005.

Honors and Awards

Fulbright Scholarship, 2006-2008

Best Grade Point Average, Escuela de Economía, Universidad de Costa Rica, 2001 & 2003.

Academic Experience

Pennsylvania State University, Pennsylvania USA

Research Assistant

- For Prof. Andrés Rodríguez-Clare **Summer, 2008**
- For Prof. Alexander Monge-Naranjo **Summer 2010, Summer and Fall 2011**

Teaching Experience

Pennsylvania State University, Pennsylvania USA

Summer Instructor
Undergraduate Intermediate Macroeconomics **Summer of 2009 and 2011**

Recitation Leader
Undergraduate Intermediate Microeconomics, Intermediate Macroeconomics and Introduction to Econometrics. **Fall 2007 - Spring 2010**

Teaching assistant
Undergraduate International Trade, Industrial Organization, Introductory Microeconomics General Education Project. **Fall 2006 - Spring 2011**

Computer Skills

Statistical Packages: Eviews, Stata, Matlab.

Professional Experience

Consejeros Económicos y Financieros, S.A. San José, Costa Rica.
Research assistant **January, 2004 - December, 2005**

Banco Improsa, San José, Costa Rica.
Credit Analyst **January, 2006 - July, 2006**

Banco Central de Costa Rica San José, Costa Rica.
Researcher **January, 2011 -**