FLOWFIELD INTERACTIONS IN
LOW ASPECT RATIO PIN-FIN ARRAYS

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Mechanical Engineering

by
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ABSTRACT

Pin-fin heat exchangers are frequently used in many important applications ranging from gas turbine airfoils to computer processors. While many studies have investigated pin-fin heat transfer, few studies have directly measured the flowfield in pin-fin arrays. The present work compares time-dependent flowfield measurements with time-mean heat transfer measurements to determine the relative influence of certain flowfield features on array heat transfer.

For single pin-fin rows, increasing Reynolds number decreased the wake closure position downstream of the pin, which corresponded with the position of maximum heat transfer. For multiple pin-fin rows, the wake closure position in the first row was influenced by streamwise spacing. For low Reynolds numbers, heat transfer in the initial rows was dominated by the horseshoe vortex. For high Reynolds number, heat transfer in the initial rows was influenced by the horseshoe vortex and generation of turbulence in the wake, close to the pin-fin. Despite localized differences in heat transfer when varying streamwise spacing, the row-averaged heat transfer in the initial rows of the array, was independent of streamwise spacing. In downstream rows, however, decreasing streamwise spacing (from 3.03 diameters to 2.16 diameters) was found to increase heat transfer because there was less streamwise distance for the decay of turbulence prior to the flow encountering the next row of pin-fins.

Decreasing spanwise spacing (from 3 diameters to 2 diameters) resulted in increased heat transfer for both low and high Reynolds numbers in multiple row pin-fin arrays. In the initial rows, close spanwise pin spacing resulted in a significant portion of the channel walls being disturbed by the horseshoe vortex that wrapped around the pins. In downstream rows, however, the vortex breakdown to turbulent flows resulted in increased heat transfer for close spacings. In addition, decreasing spanwise spacing (from 3 diameters to 2 diameters) resulted in a thinner wall layer resulting in increased heat transfer compared to wider spanwise spacings. For a given percentage decrease in either streamwise or spanwise spacing, a similar percentage increase in heat transfer was observed.

Improvements to pin-fin arrays may be possible through modifications to the pin-fin shape, the endwall shape, or to the pin-fin spacing. In the present work, a non-
uniform array was considered having variable streamwise pin-fin spacing. The non-uniform array performed as well as a closely spaced array at high Reynolds number. The important result was that improvements were realized through non-conventional spacing schemes. Further modifications that generate turbulence and prevent the streamwise decay of turbulence would be beneficial for array heat transfer based on the results presented in this work.
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<td>drag coefficient</td>
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<tr>
<td>$C_{D,p}$</td>
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<td>hydraulic diameter</td>
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<tr>
<td>$U,V,W$</td>
<td>time-mean (ensemble-averaged) velocity components</td>
</tr>
</tbody>
</table>
u', v', w' fluctuating velocity components \( u' = (\bar{U} - U) \)

or RMS velocity components \( u' = \langle u'u' \rangle^{0.5} \)

\( V_{\text{eff}} \) effective approach velocity

X, Y, Z streamwise, transverse, and wall-normal directions, respectively

X' streamwise coordinate referencing the pin-fin stagnation point as the origin, positive direction is aligned with the bulk flow

**Greek**

\( \alpha \) angle between LDV component #1 and global X-direction

\( \beta \) angle between LDV component #2 and global Y-direction

\( \delta \) boundary layer thickness

\( \delta_1 \) displacement thickness

\( \varepsilon \) emissivity; turbulent dissipation rate, error, equivalent surface roughness

\( \theta \) circumferential location on the pin-fin

\( \phi \) POD vector spatial function

\( \lambda \) POD temporal coefficient

\( \lambda_{c,i} \) swirling strength (Adrian et al. 2000)

\( \mu \) molecular dynamic viscosity

\( \nu \) molecular kinematic viscosity

\( \nu_T \) turbulent kinematic viscosity

\( \rho \) density

\( \omega \) specific dissipation rate, vorticity

**Subscripts**

\( \infty \) freestream value

\( c \) combined pin and endwall heat transfer

\( e \) endwall heat transfer only

\( m \) bulk averaged value

\( \text{max} \) maximum velocity through minimum flow area

\( n \) POD mode number

\( o \) unobstructed duct value

\( p \) pin heat-transfer only

\( \text{SGS} \) sub-grid scale

\( w \) value taken at the wall

**Superscripts**

\( + \) inner scaling coordinates

**Other**

\( = \) single over bar denotes row-averaged value

\( \bar{\bar{\cdot}} \) double over bar denotes array-averaged value

\( \sim \) tilde over a given variable denotes instantaneous value

\( < > \) angle brackets indicate ensemble average

\( \text{||} \) straight brackets indicate absolute value
ACKNOWLEDGEMENTS

First, I would like to thank my loving and supporting family. Mom and Dad, I could not have asked for a better life growing up. You have truly supported me through every new adventure and experience. I look forward to spending more time with you after graduation, since I will be so much closer to home! I have also been blessed with a brilliant and wonderful sister, Laura. It has been wonderful to grow and progress through graduate school with you. Second, I would like to thank my advisor, Dr. Karen Thole, for supporting, guiding, and encouraging me through these final stages of graduate school and through the initial stage of my career. I look forward to checking in with you to mention what a great advisor you were (are). Finally, I would like to thank my friends, colleagues, and lab-mates here at Penn State. Your camaraderie during the late nights in the lab always helped keep things in perspective. These friendships and memories will certainly last a lifetime.
CHAPTER 1:
INTRODUCTION

Staggered cylinders are used in many heat exchanger applications such as steam boilers, automotive radiators, and refrigerant evaporators. Typically the cylinders are either very long or very short relative to their diameter, as shown in Figure 1.1. Boilers are an example of a long-tube heat exchanger while automotive radiators and refrigerant evaporators are examples of short-tube (finned-tube) heat exchangers. While very long and very short cylinders are most common, there has been a growing interest in the case where the cylinder height and diameter are on the same order of magnitude. Heat exchangers having cylinder height-to-diameter ratio on the order of unity are referred to as pin-fin heat exchangers. Two common applications for pin-fin heat exchangers are gas turbine airfoil cooling and computer processor cooling, both of which are illustrated in Figure 1.2.

In any heat exchanger, the overall heat transfer capacity is of primary interest. Depending on the cylinder aspect ratio (height-to-diameter ratio, H/D), the mechanism for transferring heat may vary significantly. In long-tube heat exchangers, heat transfer occurs primarily on the cylinder surfaces while endwall effects are negligible. Conversely, in finned-tube heat exchangers, heat transfer on endwall surfaces is significant. In pin-fin heat exchangers, however, both the cylinder surface and endwall are important to the total convective heat transfer. Determining the performance in a pin-fin heat exchanger, therefore, requires knowledge of the local heat transfer coefficients on both endwalls and cylinder surfaces. Analysis of pin-fin heat exchangers is complicated because the local heat transfer on the cylinder and endwall surfaces vary strongly with position from the complex, turbulent flowfield. These complex flows are quite difficult to model and predict, so pin-fin heat transfer coefficients are typically obtained experimentally.

Most previous studies have investigated the heat transfer characteristics of pin-fin arrays by perturbing one or more of the following independent variables: Reynolds number, pin-fin spacing, pin-fin aspect ratio, flow incidence angle, pin-inclination angle, pin-fin fillets, duct convergence, duct rotation, and pin-fin shape. Empirical correlations have been developed to predict heat transfer as a function of these independent variables.
Very few studies, however, have directly measured the flowfield in pin-fin arrays. Knowledge of the flowfield behavior allows engineers to design pin-fin arrays that exploit the flow features which are most efficient at generating mixing and transferring thermal energy.

The present work investigates pin-fin flowfields using high resolution measurement techniques with the goal of understanding how the flow responds to changes in Reynolds number, pin-fin aspect ratio, and pin-fin spacing. Through the use of time-resolved, digital particle image velocimetry (TRDPIV), snapshots of the turbulent flowfield were captured at a high frequency to allow visualization of the turbulent eddies moving through the pin-fin array. To determine the effectiveness of these turbulent eddies in removing thermal energy, TRDPIV measurements were compared directly with heat transfer measurements. Heat transfer measurements were made using another high resolution technique, infrared (IR) thermography, which provided a spatially-resolved image of the level of heat transfer throughout the pin-fin array. These measurement techniques were applied to systematically determine the effects of Reynolds number, aspect ratio (H/D), streamwise spacing (X/D), and spanwise spacing (S/D).

The present work will systematically determine what pin-fin feature(s) is most important for maximizing heat transfer in pin-fins arrays. It is well known that the following features influence the level of heat transfer in pin-fin arrays:

- Geometric feature: Total convective surface area
- Geometric feature: Ratio of pin-fin-to-overall convective area
- Flowfield feature: Reynolds number
- Flowfield feature: Horseshoe vortex
- Flowfield feature: Vortex shedding (periodic unsteadiness)
- Flowfield feature: Turbulence (random unsteadiness)
- Flowfield feature: Three-dimensional effects

It is unknown, however, which features are primary and which are secondary effects. The goal of the present work is to identify the relative influence of the various feature(s) that influence(s) heat transfer in pin-fin arrays. Future studies may benefit from the
present work by targeting the primary heat transfer feature(s) through innovative pin-fin spacing or pin-fin shapes. Future studies may also perform more detailed experiments that target the important flowfield feature(s) with the goal of developing a more comprehensive theory on promoting heat transfer in pin-fin arrays.

The remainder of the present work will systematically determine which pin-fin feature listed in the previous discussion is most important for heat transfer in pin-fin arrays. A review of relevant literature is presented in Chapter 2. Chapter 3 discusses the experimental facilities and measurement techniques. Discussion of the results begins with the effects of Reynolds number in Chapter 4. Chapter 5 addresses that effect of pin-fin aspect ratio. The effects of streamwise and spanwise spacing are discussed in Chapter 6 and Chapter 7, respectively. While Chapter 4 through Chapter 7 discuss array heat transfer, the main focus is placed on the flowfield behavior. A more detailed analysis of heat transfer, friction factor, and array performance is given in Chapter 8. Chapter 9 discusses computational modeling considerations for pin-fin arrays. Finally, Chapter 10 provides a comprehensive summary of the work presented herein.
Figure 1.1. Short-tube cylinder bank from Webb (1980) (left) and long-tube cylinder bank from Incropera and Dewitt (2002) (right).

Figure 1.2. Examples of pin-fin heat exchangers where cylinder height-to-diameter ratio is on the order of unity.
CHAPTER 2: REVIEW OF RELEVANT LITERATURE

The following chapter discusses previous work pertaining to flow and heat transfer in pin-fin heat exchangers. Section 2.1 provides an introduction to the basic turbulent flows present in pin-fin heat exchangers. Section 2.2 reviews previous studies pertaining to pin-fin heat transfer and Section 2.3 discusses previous pin-fin flowfield studies. Finally, Section 2.4 will address the uniqueness of the present work.

2.1 Fundamentals Flows Found in Pin-Fin Arrays

The flow through pin-fin arrays may be considered as a combination of internal and external flows. The pin-fins are confined within a duct, creating the conditions of internal flow. The flow also crosses the pin-fins perpendicularly, creating the conditions of bluff-body flow. A third flow is present at the junction between the pin-fin and the endwall, similar to a wing-body junction flow. Despite the geometric simplicity of pin-fin arrays, there exists a mix of flowfield features (turbulent duct flow, bluff-body flow, and wing-body junction flow) that are rich in physics and are highly researched problems in fluid mechanics. Circular cylinders in crossflow are characterized by alternate vortex shedding in the wake, known as the Kármán vortex (KV) street. Wing-body junctions are characterized by the horseshoe vortex (HV) which wraps around the protruding body, forming the shape of a horseshoe. Both KVs and HVs are found in pin-fin arrays, as shown in Figure 2.1.

The circular cylinder in crossflow is one of the most fundamental cases studied in fluid mechanics. The flow over a cylinder is highly dependent on Reynolds number and may pass through many different flow regimes ranging from Stokes flow to fully-turbulent flow. For gas turbine applications, the Reynolds number of the flow across the pin-fins is typically between $Re_D = 3.0e3$ and $3.0e4$ (Han et al. 2000). In this range of Reynolds numbers, the flow across a single, infinitely long cylinder is characterized by transition to turbulence in the shear layers (Zdravkovich 1997). This regime is referred to as the TrSL regime, for *transition in shear layers*. Under low freestream turbulence conditions, a laminar boundary originates at the stagnation point, $\theta = 0^\circ$. As the flow
progresses along the cylinder, an adverse pressure gradient causes the flow to separate at \( \theta = 90^\circ \) from the stagnation point. Transition to turbulence occurs in the shear layer which emerges tangent to the cylinder at \( \theta = 90^\circ \). The distance required for transition to occur decreases with increasing Reynolds number. In effect, the shear layer becomes less stable with increasing Reynolds number. Consequently, the length of the recirculating region follows and decreases in size with increasing Reynolds number. These length scales approach a constant value near \( \text{Re}_D = 3.0 \times 10^4 \) and remain constant until the next regime change which occurs at \( \text{Re}_D = 1.0 \times 10^5 \) and transition to turbulence occurs in the boundary layer, referred to as the TrBL regime (Zdravkovich 1997).

Many factors have been shown to influence the canonical case of flow over a single, infinite cylinder under low freestream turbulence. Increasing freestream turbulence, for example, decreases the length of the recirculation region (Norberg 1986). Increased freestream turbulence causes greater instability in the shear layer, such that transition to turbulence occurs over a shorter distance, which corresponds to a shorter recirculation region. The flow around a single cylinder may also be influenced by blockage. Blockage in the wall-normal direction may be introduced by bounding the cylinder with two end-plates such that the cylinders have a finite length characterized by the aspect ratio, \( H/D \). For a single cylinder confined by two bounding walls, Szepessy and Bearman (1992) showed that a cylinder having \( H/D = 1.7 \) had stronger vortex shedding than \( H/D = 6.7 \). The shed vortices were more coherent across the height of the cylinder, i.e. more two-dimensional, for the \( H/D = 1.7 \) case in comparison with \( H/D = 6.7 \). Blockage may also be introduced in the cross-flow, or spanwise, direction. Blackburn (1994) showed that shed vortices were more coherent along the height of the cylinder when two walls, parallel with the flow direction, were drawn in closer to the cylinder. For example, 40% blockage showed more coherent vortices than a 5.6% blockage. For the high blockage ratio cases, the shed vortices were observed many diameters downstream while the low blockage ratio cases mixed out within a few diameters.

These fundamental studies on cylinders in crossflow show that the flow is sensitive to freestream turbulence, cylinder aspect ratio (H/D), and blockage in the spanwise direction, all of which are found in pin-fin arrays. Because the pin-fins are
mounted in a channel, the typical turbulence level is approximately 7% (from present measurements), much higher than typical external flow cases having low freestream turbulence (0.1%). And, pin-fins typically have H/D ~ O(1) such that blockage from the channel walls likely influences the structure of the near wake. Finally, adjacent pin-fins create blockage in the spanwise direction that may also influence the structure of the near wake.

Adding to the complexity of pin-fin flows is the presence of the horseshoe vortex (HV) system at the base of each pin-fin. Wing-body junction flows are created by a boundary layer that has developed over the “wing” and stagnates on the protruding “body”. The horseshoe vortex system develops as a result of a wall-normal velocity profile stagnating on a protruding body. As the flow approaches the protruding body, the lower momentum fluid near the wall is unable to overcome the adverse pressure gradient caused by the presence of the protruding body. The low momentum fluid separates from the wall and develops into a HV as shown in Figure 2.2. Praisner and Smith (2006a, b) showed that heat transfer on the endwall was caused by time-mean and fluctuating motion of the HV. The majority of heat transfer from the HV system occurred where the HV brought cool outer fluid into contact with the heated wall. This primary heat transfer mechanism had a large time-mean component of heat transfer. A secondary mechanism was observed in the occasional ejection of the secondary vortex (SV). The ejection of the SV contributed to a large amount of fluctuating heat transfer that also had a significant net heat flux in the time-mean. Both mechanisms, the HV entraining the core fluid and the ejection of the SV, contributed to the overall heat transfer at the stagnation region of the wing-body junction. Many other studies have shown that the size and position of the horseshoe vortex varies with the shape of incoming velocity profile, with incoming turbulence intensity, and with the Reynolds number of the approaching flow (Praisner and Smith 2006b; Devenport and Smith 1990; Paik et al. 2007; Baker 1991; Ozturk et al. 2008; Radomsky and Thole 2000). In pin-fin arrays, the horseshoe vortex is known to provide heat transfer enhancement by transporting the cooling flow into close proximity with the endwalls (Han et al. 2000).

In pin-fin arrays, regions of laminar, transitional, and fully-turbulent flow exist. The boundary layer is laminar on the pin-fin prior to separation. The shear layers
extending from the position of separation are transitional, and the wake flow is turbulent. Because of the presence of bounding walls (pin-fins and endwalls) the velocity fluctuations in turbulent regions are likely to be highly inhomogeneous and anisotropic. Furthermore, regions of separation, recirculation, and reattachment are present in the pin-wake and at the pin-endwall junction. For these reasons, it is quite difficult to predict pin-fin flows using numerical techniques. Many previous researchers have, therefore, relied on experiments to quantify heat transfer and pressure drop in pin-fin arrays.

2.2 Heat Transfer in Pin-Fin Arrays

Although the flow through pin-fin arrays is comprised of several basic turbulent flows, as discussed in Section 2.1, the resulting flowfield in pin-fin arrays is more complicated than a simple linear superposition of the basic flows. It was discovered in early pin-fin studies, that pin-fin arrays could not be interpolated between unobstructed ducts and long tube arrays (Metzger and Haley 1982; VanFossen 1982; Faulkner 1971). The remainder of this section discusses relevant studies pertaining to flow and heat transfer specific to pin-fin heat exchangers. The details of each pin-fin study reviewed are shown in Table 2.1.

Metzger et al. (1986) and VanFossen (1982) each developed a correlation for array-averaged heat transfer as a function of Reynolds number and pin-fin geometry (refer to Chapter 8 for the correlations). Armstrong and Winstanley (1988) showed that the correlations developed by Metzger et al. (1986) and VanFossen (1982) both predicted array average heat transfer to within 20% for pin-fins having H/D ≤ 3, 2 ≤ S/D ≤ 4, and 1.5 ≤ X/D ≤ 5. In general, these correlations showed that heat transfer increased with increasing Reynolds number, increasing H/D, decreasing S/D, and decreasing X/D.

In almost all pin-fin studies, including the work of Metzger et al. (1986) and VanFossen (1982), the array-averaged heat transfer coefficient is defined as the total heat transfer rate divided by the product of the total convective surface area and the driving temperature difference (refer to Chapter 3 and Chapter 8 for definitions). The array-averaged heat transfer coefficient, therefore, accounts for the amount of convective area for a given pin-fin array. The flowfield effects are then decoupled from surface area effects through the use of array-averaged heat transfer coefficients. Increasing H/D, for
example, may increase the total surface area but the array-averaged heat transfer coefficient accounts for the additional surface area which allows for a fair comparison with arrays having lower H/D. The convective surface area should, however, be considered when calculating the total heat removed from the pin-fin heat exchanger. Figure 2.3 differentiates between the array-averaged, combined (pin/endwall) heat transfer coefficient and the total heat removal for arrays having different total convective surface area. For the purposes of the present work, the goal was to determine the flowfield feature(s) that contributes to increasing heat transfer. The amount of surface area was accounted for in the definition of array-averaged, combined heat transfer coefficient and surface area was, therefore, eliminated from the list of pin-fin features in the previous discussion. The array-averaged correlations of Metzger et al. (1986) and VanFossen (1982) have provided a framework which shows the general trends for array-averaged heat transfer with changes in Reynolds number, H/D, S/D, and X/D. The flowfield effects resulting from changes in Reynolds number, H/D, S/D, and X/D, however, require further investigation.

The effects of aspect ratio (H/D) are a good example of how the ratio of pin-fin-to-overall convective surface area may influence array-averaged heat transfer. Both Brigham and VanFossen (1984) and Chyu et al. (2009) emphasized that array-averaged heat transfer augmentation increased for increasing H/D. Heat transfer augmentation is defined as the array-averaged heat transfer coefficient divided by the heat transfer coefficient in an unobstructed duct. It is important to note that the heat transfer coefficient in an unobstructed duct increases with increasing H/D, but at a different rate than the pin-finned duct. It was, therefore, more logical to compare the array-averaged heat transfer rather than array-averaged augmentation when varying H/D. The data of Brigham and VanFossen (1984) and Chyu et al. (2009) is shown in Figure 2.4. Brigham and VanFossen (1984) showed that array-averaged heat transfer was similar for $0.5 \leq H/D \leq 8$ in pin-fin arrays having $S/D = 4$ and $X/D = 4$. There was one noticeable difference in the data of Brigham and VanFossen (1984) where the pin-fin array having $H/D = 0.5$ showed higher heat transfer than $H/D > 0.5$ at low Reynolds number. The work of Chyu et al. (2009), however, showed increasing array-averaged heat transfer with increasing H/D for pin-fin arrays having $S/D = 2.5$, $X/D = 2.5$, and $2 \leq H/D \leq 4$. 
There were two possibilities that would explain the different effects of H/D at close and wide spacings. First, the pin-fin surface area comprises a greater fraction of the overall convective area for the closely spaced arrays (54% to 70% for H/D = 2 to 4, respectively) compared to the widely spaced arrays (17% to 29% for H/D = 2 to 4, respectively). If the pin-fin heat transfer coefficient was greater than that of the endwalls, then the ratio of pin-fin-to-overall surface area may contribute to the increase in heat transfer. Second, local flowfield effects may lead to a different heat transfer coefficient distribution on the pin-fins and endwalls. Chyu et al. (2009) showed that the ratio of pin-fin-to-overall convective area was more important than local flowfield effects for the closely spaced arrays. For the widely spaced arrays, the pin and endwall heat transfer were not measured separately, and the influence of flowfield effects in comparison with the ratio of pin-fin-to-overall surface area remained unclear.

The effects of spanwise (S/D) and streamwise (X/D) spacing are a good example of how flowfield effects may influence array-averaged heat transfer. Lyall et al. (2011) measured local pin and endwall heat transfer coefficients for a single row of pin-fins having various spanwise spacing S/D. Heat transfer on the pin-fins and on the endwall surfaces showed opposite trends when comparing heat transfer versus S/D. Decreasing S/D resulted in increased heat transfer on the endwall surfaces but decreased heat transfer on the pin-fins. The array-averaged, combined heat transfer increased with decreasing S/D. Ames et al. (2005) developed a correlation that describes the primary heat transfer mechanism on the pin-fin (stagnation point), and will be discussed in the next section. Lyall et al. (2011), however, made an interesting observation for the endwall heat transfer in the wake of a single row of pin-fins. Figure 2.5 shows the endwall Nusselt number for S/D = 2 and 4 at Re_D = 7.5e3 (Lyall et al. 2011). For S/D = 4, the heat transfer contours showed individual wakes downstream of each pin-fin. For S/D = 2, however, the endwall heat transfer contours in the wakes of adjacent pin-fins merged and a large enhancement was observed. Local flowfield effects clearly influenced the local heat transfer distribution on the endwall. It remained unclear, however, whether the enhancement observed for S/D = 2 was caused by three-dimensional effects (interaction of horseshoe vortex and wakes), wake shedding (periodic unsteadiness), or a breakdown of the Kármán vortex (random unsteadiness).
Lawson et al. (2011) investigated the effects of both S/D and X/D on local heat transfer coefficients for multiple row pin-fin arrays having H/D = 1. Figure 2.6 shows the row-by-row development in endwall heat transfer for X/D = 1.73 in comparison to X/D = 3.46. It should be noted that the ratio of pin-fin-to-overall convective area does not influence the endwall-averaged heat transfer and a fair comparison was made for varying S/D and X/D configurations. For a pin-fin array having S/D = 2 and X/D = 3.46, heat transfer peaked in the second row, decreased slightly, and increased to approach a constant value in the remainder of the array. When S/D = 2 and X/D = 1.73, however, heat transfer increased gradually through the third row and remained constant. And, the magnitude of heat transfer in the inner rows was much higher for X/D = 1.73 in comparison to X/D = 3.46. The row-by-row development of the S/D = 2, X/D = 1.73 case was in agreement with the generalized row-by-row development trend described by Metzger et al. (1986), shown in Figure 2.7. The difference between the X/D = 1.73 and 3.46 arrays in Figure 2.6, however, showed that there was a significant flowfield effect that caused higher heat transfer for X/D = 1.73 compared to X/D = 3.46. Recall that the effects of surface area have been decoupled from flowfield effects through the definition of heat transfer coefficient. In agreement with Lyall et al. (2011), Lawson et al. (2011) found that increasing S/D resulted in lower array-averaged heat transfer. It remained unclear, whether the enhancement observed for S/D = 2 was caused by wake shedding (periodic unsteadiness), interactions between adjacent wakes, or a breakdown of the Kármán vortex into turbulence.

In addition to localized measurements, many previous studies have investigated more realistic or innovative pin-fin arrays. To name a few examples, pin-fin heat transfer has been studied for ducts under rotation (Wright et al. 2008; Park et al. 2009), for inclined pin-fins (VanFossen 1982; Park et al. 2009), for various pin-fins shapes (Uzol and Camci 2005; Chyu et al. 2007; Goldstein et al. 1994; Metzger et al. 1984), for partial pin-fins (Arora and Abdel-Messeh 1990), and for pin-fins having fillets (Chyu 1990). The present work, however, focuses on idealized (uniformly spaced pin-fins in a constant height channel) pin-fin arrays to identify the primary heat transfer feature(s).
2.3 Pin-Fin Flowfield Measurements

Each of the previous studies in Section 2.2 provides, to some degree, an explanation of the flow physics governing the distribution of measured heat transfer coefficients. Few previous studies, however, have actually measured the flowfield to compare with the resulting heat transfer. Metzger and Haley (1982) and Simoneau and VanFossen (1984) measured the turbulence intensity through a pin-fin array using hot-wire anemometry and found a rough agreement with typical row-averaged heat transfer development. Specifically, turbulence intensity was low upstream of the pin-fin array. Turbulence increased and reached a maximum in the fourth row, followed by a decrease to a constant value in downstream rows. This result suggested that flow unsteadiness contributes to heat transfer. It was unclear, however, if periodic unsteadiness contributed to the measured turbulence intensity. If so, it was unclear whether periodic unsteadiness or turbulence correlated with heat transfer. This distinction between periodic unsteadiness and turbulence is very important in understanding the mechanism for heat transfer in pin-fin arrays. The present work will address the difference between the two motions and which is more influential on heat transfer.

Ames et al. (Ames et al. 2005; Ames and Dvorak 2006b, a; Ames et al. 2007) exhaustively studied a common pin-fin array having S/D = 2.5, X/D = 2.5, and H/D = 2.0. Ames et al. (2005) successfully correlated the local heat transfer at the pin-fin stagnation point with turbulence parameters obtained using hot-wire anemometry. The correlation for stagnation point heat transfer was approximated using

$$\frac{\text{Nu}_D}{\text{Re}^{0.5}_{De}} = 0.95 \left( 1 + 0.04 \left( \text{Tu} \text{Re}^{5/3}_{De} \left( \frac{D}{\text{Lu}} \right)^{1/3} \right) \right)$$  \hspace{1cm} (2.2)$$

where \(\text{Re}_{De}\) was the effective Reynolds number, \(\text{Tu}\) was turbulence intensity, and \(\text{Lu}\) was the dissipation length scale. The turbulence parameters were grouped into a non-dimensional parameter referred to as the TRL parameter:
The length scale used in $Re_{De}$ was the pin diameter and the velocity scale was the approach velocity, $V_{eff}$, approximated from the surface pressure distribution on the pin-fin:

$$U(\theta) = 1.81V_{eff} \sin(\theta) = \sqrt{\frac{p(\theta = 0^\circ) - p(\theta)}{2\rho}}$$ \hspace{1cm} (2.4)$$

The correlation by Ames et al. (2005) is the only study, to the author’s knowledge, where local heat transfer coefficients in a pin-fin array may be predicted empirically based on local turbulence parameters. This correlation shows that the mechanism for heat transfer at the stagnation point is a combination of the flow velocity scale, accounted for using $Re_{De}$, and the turbulence structure, accounted for using TRL. The correlation of equation 2.2 shows that increasing turbulence intensity leads to increased stagnation point heat transfer. Also, equation 2.2 suggests that decreasing the dissipation length scale increases heat transfer. Based on previous discussion, the correlation of Ames et al. (2005) suggests that heat transfer is augmented by highly unsteady flow with minimal turbulent length scale. It still remained unclear, however, what role periodic unsteadiness and turbulence have on heat transfer.

The study by Lyall et al. (2011) showed that increasing S/D increased the overall pin-fin heat transfer. To determine the mechanism driving heat transfer, the stagnation point heat transfer was extracted from the dataset of Lyall et al. (2011) and the correlation of equation 2.2 was applied. Turbulence intensity was approximated using:

$$Tu = 0.16 Re_{Db}^{-0.125}$$ \hspace{1cm} (2.5)$$

which is commonly used for estimating turbulence intensity at the center of a fully-developed, turbulent duct flow (FLUENT user manuals, www.ansys.com, 2012). Increasing S/D from 2 to 4, therefore, led to an increased and estimated decrease in Tu of
5% at the core of the duct. The approach velocity was estimated using the velocity at the core of the duct assuming a $1/7^{th}$ power law velocity profile. The resulting effective Reynolds number (to the 5/12 power), $Re_{de}^{5/12}$, increased by 18% when increasing S/D from 2 to 4. The dissipation length scale was assumed to be constant for the two cases, equal to the height of the channel. Figure 2.8 shows stagnation point heat transfer for Lyall et al. (2011) and Ames et al. (2005) as a function of TRL. It was found that the single pin-fin rows studied by Lyall et al. (2011) were in agreement with the correlation for stagnation point heat transfer. Increasing S/D from 2 to 4 led to a 24% increase in heat transfer at the stagnation point (and pin-fin midline, Z/H = 0). From this approximation, it can be concluded that, for a single row of pin-fins, increased pin-fin heat transfer with increasing S/D was caused by the increase in effective Reynolds number because turbulence intensity varied weakly with S/D and dissipation length scale was independent of S/D.

In addition to correlating stagnation point heat transfer with local turbulence parameters, Ames et al. (2005) showed that the wall-normal profile of (time-mean) streamwise velocity was in agreement with the law-of-the-wall across the viscous layer and most of the log-layer. The friction velocity, however, was much higher than that of an unobstructed channel which showed the enhancement of momentum transport in pin-fin arrays. For example, the skin friction coefficient was increased by 134% (low Reynolds number) and 53% (high Reynolds number) when compared to an unobstructed duct. Assuming the Reynolds’ analogy applies, the increased friction velocity was also indicative of enhanced heat transfer.

Follow-up studies from the stagnation point heat transfer study (Ames et al. 2005) included detailed measurements of static pressure (Ames and Dvorak 2006a), heat transfer (Ames et al. 2007), and numerical predictions (Ames and Dvorak 2006b; Delibra et al. 2010). The static pressure and endwall heat transfer distributions both showed evidence of the horseshoe vortex at the base of the pin-fin. Static pressure was decreased and endwall heat transfer was increased in the region of the horseshoe vortex.

The correlation developed by Ames et al. (2005), and shown in equation 2.2, is helpful for understanding the relationship between the turbulence parameters of the approaching flow and the heat transfer on pin-fin surfaces. However, to this point there
have been no studies presented that have related the instantaneous, *spatial* structure of the flowfield to the local heat transfer coefficients. Ozturk et al. (2008) and Uzol and Camci (2005) are the only two previous studies that have made spatially-resolved flowfield measurements for a pin-fin channel. Ozturk et al. (2008) used PIV to visualize the flowfield for a single, confined pin-fin. As such, there were no adjacent pin-fins in the spanwise or streamwise directions. It was found that the instantaneous structure of the horseshoe (HV) system was not symmetric about the duct centerline. The state of the HV system at one end did not correspond to the HV at the other end of the pin-fin (recall that the pin-fin usually spans the entire channel). In the time-average, the flow was symmetric about the channel centerline. Ozturk et al. (2008) also showed that the size and position of the HV system was strongly dependent on Reynolds number for the range $1.5e^3 \leq \text{Re}_D \leq 6.15e^3$. Increasing $\text{Re}_D$ led to a smaller HV located closer to the cylinder. Uzol and Camci (2005) compared circular and ellipse shaped pin-fins using PIV flowfield measurements and endwall heat transfer measurements. The ellipse shaped pins delivered less endwall heat transfer in the wake but outperformed circular pin-fins when both heat transfer and friction factor were considered. The flowfield measurements of Ozturk et al. (2008) and Uzol and Camci (2005) were sampled at low frequency such that the *time-dependent* spatial organization was not observed. While the work of Ozturk et al. (2008) and Uzol and Camci (2005) provided important insight into the instantaneous flowfield organization, it remained unclear which flowfield mechanisms contributed most to heat transfer.

While previous pin-fin flowfield studies provide some insight into the flow physics, it is worthwhile to mention several previous fundamental studies that investigate the mechanisms for heat transfer in turbulent, wall-bounded flows. It has been shown that the presence of streamwise vortices (such as the legs of the horseshoe vortex) contribute to increased momentum and thermal transport. Wroblewski and Eibeck (1991, 1992) showed that thermal transport is enhanced to a greater degree than momentum transport when considering a streamwise vortex embedded in an otherwise two-dimensional boundary layer. Assuming isotropic eddy-viscosity, the turbulent Prandtl number was found to vary significantly across the streamwise vortex, between $\text{Pr}_T = 0.2$ near the vortex core to $\text{Pr}_T = 1.0$ in the region of upwash and downwash (Wroblewski and
Eibeck 1991). It has also been shown that the turbulent Prandtl number varies across the wake for a single, infinite cylinder (Berajeklian and Mydlarski 2011). The flow in pin-fin channels can, therefore, be assumed to deviate, to some extent, from the Reynolds’ analogy because of the presence of horseshoe vortices and wake shedding.

In wall-bounded flows it is common to perform a conditional sampling, quadrant analysis based on the sign of \(u'\) and \(v'\) in the boundary layer near a wall. Note that the prime denotes the fluctuating component of streamwise velocity, for example \(u' = U - U\). The sign of \(u'\) and \(v'\) determines the sign of the Reynolds stress component \(u'v'\) and also provides insight in to the mechanisms contributing to turbulent momentum transport. For example, motions known as sweeps are classified when \(u' < 0, v' > 0\) and motions known as ejections are classified when \(u' < 0, v' > 0\) (Corino and Brodkey 1969; Wallace et al. 1972). The sweeps/ejections correspond to quadrants 4 and 2 (Q4/Q2), respectively, when considering the four possibilities for the sign of \(u'\) and \(v'\). The other two possibilities are outward/wallward interactions, which correspond to the reflection of high momentum fluid back to the freestream (outward interaction, Q1) and the reflection of low momentum fluid back towards the wall (wallward interaction, Q3). It has been shown that the sweep/ejections cause a large amount of turbulent momentum transport near the wall (Wallace et al. 1972). Sideridis et al. (1999) used this quadrant analysis to investigate heat transport in the wake of a cylinder placed above a flat surface (oriented so cylinder axis is in cross-flow direction). The authors found that the sweep/ejection (Q4/Q2) motions in the wall boundary layer did not vary significantly with streamwise distance downstream of the cylinder and, therefore, did not interact strongly with the cylinder wake. The interaction between the cylinder wake and the wall boundary layer was observed mainly in the outward/wallward (Q1/Q3) interactions, which did have a significant streamwise evolution. For pin-fin channels, it can also be assumed that sweep/ejection motions contribute to heat transfer on the endwall surfaces and there is likely an interaction between the cylinder wakes and the boundary layer on the endwalls where outward/wallward mechanisms may be important. It was the goal of the present work, however, to characterize the pin-fin wakes by measuring the flowfield at the channel mid-span. Subsequent time-mean heat transfer measurements were then
interpreted by considering the wake behavior. The topic of near-wall turbulent heat flux in pin-fin arrays, however, would be recommended for further investigation.

2.4 Uniqueness of Present Work

From this review of relevant studies, it is clear that pin-fin heat exchangers have been studied in great detail and there is certainly enough data available to design an effective pin-fin heat exchanger. The public literature, however, is lacking in detailed information on what flowfield features are most important for heat transfer which may allow for further innovations and improvements to standard pin-fin arrays. There has been only one account, by Ames et al. (2005), where local heat transfer was correlated with local turbulence parameters in the pin-fin array. This model for pin-fin stagnation point heat transfer, however, does not distinguish between periodic unsteadiness and turbulence. There have been no previous studies, to the author’s knowledge, that have captured the time-dependent spatial organization of the flowfield to correlate with local heat transfer coefficients in pin-fin arrays.

The present work will make use of a state-of-the-art measurement technique known as time-resolved, digital particle image velocimetry (TRDPIV). Using TRDPIV, the time-dependent flow will be captured in pin-fin arrays and comparisons will be made with local heat transfer coefficients. The present work will provide insight into what feature(s) of the pin-fin flowfield results in enhanced heat transfer.

While previously developed empirical correlations may be able to predict heat transfer in standard, uniformly spaced arrays, the present work will provide insight into the primary flowfield feature(s) that contribute to heat transfer. Future studies may benefit from this work by either developing innovative arrays that target the important flowfield features or by developing better semi-empirical models based on the important flowfield features identified in the present work.
# TABLE 2.1. SUMMARY OF PIN-FIN STUDIES RELEVANT TO THE PRESENT WORK

<table>
<thead>
<tr>
<th>Study</th>
<th>Re</th>
<th>H/D</th>
<th>S/D</th>
<th>X/D</th>
<th>Measurements/Results</th>
<th>Flowfield?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Al-Dabagh and Andrews 1992)</td>
<td>$4 \times 10^3$ to $3 \times 10^4$</td>
<td>0.7 to 2.2</td>
<td>2</td>
<td>1.5</td>
<td>Local Nu Array Nu</td>
<td>No</td>
</tr>
<tr>
<td>(Ames et al. 2005)</td>
<td>$3 \times 10^3$ to $3 \times 10^4$</td>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
<td>Local Pin Nu Local Pin $C_p$ Local Turbulence Parameters Array Friction Factor</td>
<td>Hotwire</td>
</tr>
<tr>
<td>(Ames and Dvorak 2006a)</td>
<td>$3 \times 10^3$ to $3 \times 10^4$</td>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
<td>Local $C_p$</td>
<td>No</td>
</tr>
<tr>
<td>(Ames and Dvorak 2006b)</td>
<td>$3 \times 10^3$ to $3 \times 10^4$</td>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
<td>Local Turbulence Parameters Energy Spectra CFD (RANS) Array Friction Factor</td>
<td>Hotwire CFD</td>
</tr>
<tr>
<td>(Ames et al. 2007)</td>
<td>$3 \times 10^3$ to $3 \times 10^4$</td>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
<td>Local Nu</td>
<td>No</td>
</tr>
<tr>
<td>(Armstrong and Winstanley 1988)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Review of Previous Studies</td>
<td>No</td>
</tr>
<tr>
<td>(Arora and Abdel-Messeh 1990)</td>
<td>$3 \times 10^3$ to $2.5 \times 10^4$</td>
<td>1.07</td>
<td>2.42</td>
<td>2.83</td>
<td>Row-averaged Nu Array Nu Friction Factor</td>
<td>No</td>
</tr>
<tr>
<td>(Bianchini et al. 2012)</td>
<td>$1.8 \times 10^4$</td>
<td>1</td>
<td>2.3</td>
<td>1.86</td>
<td>Local Nu PIV in between First Row Pins Array Friction Factor</td>
<td>PIV</td>
</tr>
<tr>
<td>(Brigham and VanFossen 1984)</td>
<td>$2 \times 10^3$ to $2 \times 10^4$</td>
<td>0.5 to 8</td>
<td>4</td>
<td>3.46</td>
<td>Array Nu</td>
<td>No</td>
</tr>
<tr>
<td>(Chyu 1990)</td>
<td>$5 \times 10^3$ to $3 \times 10^4$</td>
<td>1</td>
<td>2.5</td>
<td>2.5</td>
<td>Row-averaged Nu Array Nu Friction Factor</td>
<td>No</td>
</tr>
<tr>
<td>(Chyu et al. 1998)</td>
<td>$1 \times 10^4$ to $2.5 \times 10^4$</td>
<td>1</td>
<td>2.5</td>
<td>2.5</td>
<td>Row-averaged Nu Array Nu Endwall Streaklines Array Friction Factor</td>
<td>No</td>
</tr>
<tr>
<td>(Chyu et al. 1999)</td>
<td>$5 \times 10^3$ to $2.5 \times 10^4$</td>
<td>1</td>
<td>2.5</td>
<td>2.5</td>
<td>Row-averaged Nu (using mass transfer analogy)</td>
<td>No</td>
</tr>
<tr>
<td>(Chyu et al. 2007)</td>
<td>$1.2 \times 10^4$ to $1.9 \times 10^4$</td>
<td>1</td>
<td>2.5</td>
<td>2.5</td>
<td>Local Nu Array Friction Factor</td>
<td>No</td>
</tr>
<tr>
<td>(Chyu et al. 2009)</td>
<td>$1 \times 10^4$ to $3 \times 10^4$</td>
<td>2 to 4</td>
<td>2.5</td>
<td>2.5</td>
<td>Local Nu Row-averaged Nu Array Friction Factor</td>
<td>No</td>
</tr>
<tr>
<td>(Delibra et al. 2010)</td>
<td>$3 \times 10^3$ to $3 \times 10^4$</td>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
<td>CFD (LES and URANS)</td>
<td>CFD</td>
</tr>
<tr>
<td>(Lyal et al. 2011)</td>
<td>$5 \times 10^3$ to $2.5 \times 10^4$</td>
<td>1</td>
<td>2 - 8</td>
<td></td>
<td>Single Row Local Endwall Nu Local Pin Nu</td>
<td>No</td>
</tr>
<tr>
<td>Reference</td>
<td>Friction Factor</td>
<td>Exponent</td>
<td>Constants</td>
<td>Methodology</td>
<td>Measurement</td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>----------------</td>
<td>----------</td>
<td>-----------</td>
<td>-------------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>(Lawson et al. 2011)</td>
<td>5e3</td>
<td>1</td>
<td>2, 4</td>
<td>1.73, 3.46</td>
<td>Local Endwall Nu, Local Pin Nu, Array Friction Factor</td>
<td>No</td>
</tr>
<tr>
<td>(Metzger et al. 1986)</td>
<td>1e3 to 1e5</td>
<td>1</td>
<td>2.5</td>
<td>1.5 - 5</td>
<td>Row-average Nu</td>
<td>No</td>
</tr>
<tr>
<td>(Metzger and Haley 1982)</td>
<td>1.3e4 to 6.9e4</td>
<td>1</td>
<td>2.5</td>
<td>1.5, 2.5</td>
<td>Row-average Nu, Local Pin Nu, Row-average Pin Nu, Local Tu, Endwall Streaklines</td>
<td>Hotwire</td>
</tr>
<tr>
<td>(Metzger et al. 1986a)</td>
<td>1e3 to 1e5</td>
<td>1</td>
<td>2.5</td>
<td>1.5, 2.5</td>
<td>Row-average Nu, Array Friction Factor</td>
<td>No</td>
</tr>
<tr>
<td>(Metzger et al. 1982b)</td>
<td>2e3 to 6e4</td>
<td>1</td>
<td>2.5</td>
<td>1.5 - 5</td>
<td>Local Static Pressure, Array Friction Factor</td>
<td>No</td>
</tr>
<tr>
<td>(Metzger et al. 1984)</td>
<td>2e3 to 6e4</td>
<td>1</td>
<td>2.5</td>
<td>1.5, 2.5</td>
<td>Row-average Nu</td>
<td>No</td>
</tr>
<tr>
<td>(Park et al. 2009)</td>
<td>7e3</td>
<td>2 to 4</td>
<td>2.5</td>
<td>2.5</td>
<td>Local Nu (using mass transfer analogy)</td>
<td>No</td>
</tr>
<tr>
<td>(Simoneau and VanFossen 1984)</td>
<td>5e3 to 1.25e5</td>
<td>3.01</td>
<td>2.67</td>
<td>2.67</td>
<td>Row-average Nu, Local Tu</td>
<td>Hotwire</td>
</tr>
<tr>
<td>(Schwänen and Duggleby 2009, 2011, 2012)</td>
<td>1.3e4</td>
<td>1</td>
<td>2</td>
<td>Single Row</td>
<td>CFD (LES and URANS)</td>
<td>CFD</td>
</tr>
<tr>
<td>(Uzol and Camci 2005)</td>
<td>1.8e4 and 8.6e4</td>
<td>1.5</td>
<td>2</td>
<td>2</td>
<td>Local Nu, Local Static Pressure in Wake, PIV in Pin-Fin Wake, Array Friction Factor</td>
<td>PIV</td>
</tr>
<tr>
<td>(VanFossen 1982)</td>
<td>1e3 to 1e5</td>
<td>0.5, 2</td>
<td>2, 4</td>
<td>1.73, 3.46</td>
<td>Array Nu</td>
<td>No</td>
</tr>
</tbody>
</table>
Figure 2.1. Turbulent flows present in pin-fin arrays.

Figure 2.2. Schematic of horseshoe vortex system (Praisner and Smith 2006a).
**Figure 2.3.** Comparison of array-averaged heat transfer coefficient and total heat removed at the base (footprint) of the array. Taken from present work for pin-fin arrays having \(H/D = 1, S/D = 2, \text{Re}_D = 3.0e3\).
Figure 2.4. Effect of H/D on array averaged heat transfer (Brigham and VanFossen 1984; Chyu et al. 2009).
Figure 2.5. Effect of S/D on single row heat transfer for Re_D = 5.0e3 (Lyall et al. 2011).
Figure 2.6. Row-averaged endwall heat transfer comparing $X/D = 1.73$ and $X/D = 3.46$ at $Re_D = 2.5e3$ (Lawson et al. 2011).

Figure 2.7. Typical heat transfer development in pin-fin arrays having $H/D = 1$, $S/D = 2.5$ (Metzger et al. 1986).
Figure 2.8. Stagnation heat transfer as a function of TRL parameter.
CHAPTER 3:
EXPERIMENTAL FACILITY AND MEASUREMENT TECHNIQUES

This chapter discusses the experimental facilities and measurement techniques used to measure flow and heat transfer in pin-fin arrays. Section 3.1 discusses the design and operation of the experimental facilities and Sections 3.3 through 3.6 discuss the experimental measurement techniques. To ensure that the measurements taken were statistically significant, Section 3.7 discusses the experimental uncertainty associated with each measurement technique. Finally, Section 3.8 reports the benchmarking of the test facilities and validation of the measurement techniques.

3.1 Experimental Facilities

Two experimental facilities were used to measure flow and heat transfer in a pin-fin array. The flowfield facility was a large scale pin-fin channel having optical access for flowfield measurements using non-intrusive, optical measurement methods. The heat transfer facility was another large scale pin-fin channel having optical access in the infrared (IR) spectrum to allow for heat transfer measurements using IR thermography. The experimental facilities were designed at large scale to achieve high measurement resolution. A typical pin-fin channel on an aero-derivative gas turbine engine has channel height of approximately 1 mm (Han et al. 2000). The flowfield facility was, therefore, built to approximately 64x scale and the heat transfer facility was built to approximately 10x scale. Dynamic (geometric and kinematic) scaling was preserved when comparing the experimental setup with a typical pin-fin array in a gas turbine airfoil.

Both facilities were designed for ease of geometric changes through the use of interchangeable pin-fins and an adjustable channel height. The nominal channel heights were 63.5 mm and 9.5 mm for the flowfield and heat transfer facilities, respectively. Although the pin-fin diameter was varied for different test cases, the nominal pin-fin diameters were equal to that of the channel height, to give an aspect ratio (H/D) of unity. An H/D on the order of unity has been reported by industry to be relevant to gas turbine
applications in particular (Kohli 2010a). Both facilities used a very wide channel to minimize end effects. The ratio of channel-width-to-height were 17.8 and 53.3 for the flowfield and heat transfer facilities, respectively. Finally, both facilities had a constant cross sectional area such that the pin-fin duct was not converging or diverging.

Schematics of the flowfield and heat transfer facilities are shown in Figure 3.1 and Figure 3.2, respectively. The general layout of the two facilities were identical and described as follows. Both facilities operate at steady-state where flow is circulated around a closed loop. A centrifugal blower capable of 13 kPa and 0.66 m³/s supplied the differential pressure to circulate air through the facility. And, the flowrate of air was controlled using a variable-frequency, electric drive. The blower discharged air to a settling chamber, which was equipped with a splash plate to prevent jet formation in the measurement section. For the flowfield rig, there was a sharp-edged contraction upstream of the measurement section to promote transition to turbulence. In the heat transfer rig, a smooth, semi-circular transition was used to ensure uniform flow in the rig. In both cases, the choice of inlet contraction was validated during the benchmarking procedures. For the flowfield facility, the test section was 45 hydraulic diameters in length with 20 hydraulic diameters of entrance length to ensure fully-developed duct flow prior to entering the pin-fin array. For the heat transfer facility, the test section was 88 hydraulic diameters in length with 35 hydraulic diameters of entrance length. Flow then passed through a secondary settling chamber placed downstream of the measurement section before returning to the blower. The facilities also featured an inline flowmeter, to measure flowrate, and a heat exchanger, to remove excess heat and maintain steady-state operating conditions. The flowmeter was interchangeable such that an orifice plate was used for low flowrate tests and a venturi meter was used for high flowrate tests.

The main difference between the two facilities was the design of the measurement section. For the flowfield facility, the measurement region was constructed of glass and polycarbonate for the use of optical flowfield measurement techniques. The polycarbonate and glass provided optical access in the visible light spectrum. For the heat transfer facility, the measurement region was constructed of medium density fiberboard (MDF) with a cutout region for the use of optical heat transfer measurements. MDF was chosen for channel construction for its low thermal conductivity ($k = 0.124$
W/m·K) and for its high flatness tolerance. The cutout region allowed the installation of a Zinc-Selenide (ZnSe) window to provide optical access in the IR-spectrum. In both test facilities, the walls were hydro-dynamically smooth ($\varepsilon/D_h = 1\text{e-6}$).

During experiments, the local temperature and pressure in the test section portion of the facility were controlled in attempt to match atmospheric conditions ($20^\circ\text{C} \text{ and } 97.7 \text{ kPa}$). The temperature was regulated to match atmospheric conditions to prevent conduction which may change the bulk temperature along the axis of the duct. Similarly, the absolute pressure in the channel was regulated to match atmospheric conditions (zero gage pressure in the channel) to minimize leaks and to prevent deformation of the channel walls in the measurement section. Temperature was controlled by adjusting the flowrate of water through the heat exchanger, and gage pressure was controlled by adjusting a set of relief valves installed upstream and downstream of the blower.

Various instruments were used to monitor operating conditions in the test rig. Absolute pressure in the laboratory was measured for each experiment using a Setra Model 370 barometric pressure gage which had a digital readout. The line pressure at the venturi flowmeter was measured using a Meriam 2100 Series Smart Gage which also had a digital readout. All other pressures measurements were made using Setra Model 264 digital pressure transducers. Temperature measurements were made using type-E thermocouples. The voltage signals from pressure transducers and thermocouples were acquired and recorded using a National Instruments SCXI-1000 chassis with two SCXI-1100 signal conditioner modules. One signal conditioner was used in conjunction with an SCXI-1102 module for reading thermocouple voltages. The other signal conditioner was used in conjunction with an SCXI-1103 module for reading the higher voltage signals from the Setra pressure transducers. Pressure and temperature signals were monitored in real-time and recorded using LabView. For signals acquired using LabView, data was sampled over one minute at 1000 Hz sampling rate. Laser Doppler velocimetry (LDV) and time-resolved digital particle image velocimetry (TRDPIV) measurements were acquired, processed, and analyzed using their respective, standalone data acquisition hardware and software.
3.2 Characteristic Scales and Test Conditions

In pin-fin arrays there are several important length and velocity scales to consider. The characteristic length scales are the channel height (H), hydraulic diameter ($D_h$), and pin-fin diameter (D). It should be noted that, for an infinitely wide channel, the hydraulic diameter approaches a value of twice the channel height, $D_h \approx 2H$. For the present flowfield facility, $D_h = 1.89H$, and for the heat transfer facility, $D_h = 1.96H$. The characteristic velocity scales are the bulk velocity ($U_m$) and the minimum area velocity ($U_{max}$). The Reynolds numbers used to describe pin-fin flows are the duct Reynolds number,

$$ Re_{Dh} = \frac{U_m D_h}{\nu} $$

(3.1)

and the pin-fin Reynolds number,

$$ Re_D = \frac{U_{max} D}{\nu} $$

(3.2)

The relationship between the length scales, velocity scales and Reynolds numbers varies with the geometric arrangement of the pin-fins arrays. The ratio $D_h/D$ is linearly proportional to the pin-fin aspect ratio (H/D). The effect of H/D on the duct Reynolds number is illustrated in Figure 3.3. The ratio $U_{max}/U_m$ is linearly proportional to the spanwise spacing (S/D) for incompressible pin-fin flows. Figure 3.4 illustrates the effect of spanwise spacing on the characteristic velocity scales for pin-fin arrays. Included in Figure 3.4 is a single, infinite cylinders. For incompressible pin-fin flows, Figure 3.4 shows that increasing S/D will cause the minimum area velocity, $U_{max}$, to approach the bulk velocity, $U_m$. For single, infinite cylinders, however, the approaching freestream velocity is approximately equal to the freestream velocity at the widest portion of the cylinder (minimum flow area). Literature for single, infinite cylinders with spanwise blockage shows that integral parameters, such as drag coefficient, agree better with a single, infinite cylinder when using $U_{max}$ as the characteristic velocity (Zdravkovich
For all flowfield and heat transfer experiments, therefore, the pin-fin Reynolds number, $Re_D$, was matched. In Chapter 4, the effects of Reynolds number are investigated by considering $Re_D = 3.0e3$, $1.0e4$, and $2.0e4$ in a single row of pin-fins having $H/D = 2$, $S/D = 2.5$. In Chapter 5, the effects of aspect ratio are investigated by considering $Re_D = 3.0e3$ and $2.0e4$ in a single row of pin-fins having $S/D = 2.5$ where $H/D = 1$ and 2. In Chapter 6, the effects of streamwise spacing are investigated by considering $Re_D = 3.0e3$, $2.0e4$ in a seven row pin-fin array having $H/D = 1$, $S/D = 2$ where $X/D = 1.73$, $2.16$, $2.60$, $3.03$, and $3.46$. In Chapter 7, the effects of spanwise spacing are investigated by considering $Re_D = 3.0e3$, $2.0e4$ in a seven row pin-fin array having $H/D = 1$, $X/D = 2.16$ where $S/D = 2$, $2.5$, and $3$. In addition, spanwise spacing effects are investigated in pin-fin array having $H/D = 1$, $X/D = 3.03$ where $S/D = 2$, $2.5$, and $3$. In Chapter 8, array performance is investigated by comparing array heat transfer and friction factor measured from the geometries considered in Chapter 6 and Chapter 7. Also considered in Chapter 8, however, is the effect of matching $Re_{Dh}$ in comparison with $Re_D$ when assessing array heat transfer and friction factor.

### 3.3 Flowfield Measurement Techniques

Flowfield measurements were made using non-intrusive methods to prevent flow disruptions. Laser Doppler velocimetry (LDV) and time-resolved, digital particle image velocimetry (TRDPIV) techniques were used to capture instantaneous and mean velocity data. When using these techniques, tracer particle velocity was measured as an approximation to the carrier phase (air) velocity. Di-ethyl-hexyl-sebacat (DEHS), a common tracer particle used in air flows, was atomized to 1 μm droplets using a Laskin nozzle aerosol generator described in Raffel et al. (1998). The Stokes number for a DEHS particle was calculated to be several orders of magnitude below unity ($St = 1e-3$), validating the approximation that the DEHS particles follow the flow exactly. The particles were injected upstream of the inlet plenum which provided uniform seeding throughout the measurement section.

Although the majority of measurements were made using TRDPIV, LDV measurements were made during the initial benchmarking studies. LDV measurements were made with a three-component, fiber optic system. A 5 W Argon-ion laser was
passed through a beam separator to produce up to three beam pairs, where each beam pair measured one velocity component. Among each beam pair, one beam was phase-shifted by 40 MHz to allow the measurement of reversing flow. The fiber optic probes contained both transmitting and receiving optics and operated in a backscatter arrangement. The probes were fitted with a 750 mm lens which produced a measurement volume having major and minor diameters of 850 μm and 72 μm, respectively. For a duct Reynolds number of 2.0e4, the inner scaling of the probe volume had major and minor diameters of $y^* = 7$ and 0.7, respectively. For all LDV measurements, a minimum of 5000 samples (typically 10,000 samples were collected) were measured over at least 30 seconds for each component of velocity. During the benchmarking of the test facility, two-component LDV measurements were made in the channel. The two-component arrangement is shown in Figure 3.5 and the transform to a right-handed coordinate system is shown in equation 3.1 where $V_1, V_2$ are measured velocities and $U, V$ are transformed velocities.

$$
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
\cos \alpha & 0 \\
0 & \cos \gamma
\end{bmatrix} \begin{bmatrix}
U \\
V
\end{bmatrix}
$$

(3.1)

It should be noted that in equation 3.1, the inverse of the coefficient matrix is multiplied by the measured velocities to yield transformed velocities.

TRDPIV measurements were made by illuminating the DEHS tracer particles using a dual cavity 15 W Nd:YAG laser capable of firing at 10 kHz per laser cavity. Particle images were captured with a 2 kHz CMOS camera at 1024x1024 px. Figure 3.6 shows the orientation of the TRDPIV system when taking particle images of the near wake flow. The time delay between laser pulses was adjusted for each run to obtain a bulk particle displacement of approximately 16 px, which was less than one-quarter of the initial, 64x64 px, interrogation window size. Flow statistics were calculated over at least 1000 image pairs (typically 3000 image pairs) at 1024x1024 px resolution. For high Reynolds number tests, the sampling frequency was 1 kHz, and for low Reynolds number tests, the sampling frequency was 100 Hz. In each experiment, the flow crossed the
domain at least 35 times and the Nyquist frequency was at least 8.7 times greater than the measured vortex shedding frequency.

Images were processed using commercially available software, DaVis (www.LaVision.de, 2012). Signal-to-noise ratio was improved by performing a background subtraction. Images were then processed using a decreasing, multi-grid scheme whereby the first interrogation window was set to 64x64 px with 50% overlap and the final window was set to 16x16 px with 50% overlap. By default, the processing software halves the window size on intermediate passes. Images were masked to remove test section boundaries and non-illuminated areas. Interrogation windows were discarded on the initial pass if a given window was at least 30% masked and were discarded on all subsequent passes if a given window was at least 60% masked. A standard cross-correlation was used to determine displacement vectors amongst image pairs. A fractional window offset with window deformation was performed for intermediate and final passes. Window deformation was performed using a first-order, bilinear interpolation scheme according to the vector field of the previous pass. The final TRDPIV image processing scheme is shown in Table 3.1. Vector validation was performed after each pass using a 4-pass median filter similar to Nogueira et al. (1997) but with adjustable criteria for removal and re-insertion of possible spurious vectors. For each experiment, the vector validation scheme was checked to ensure that only spurious vectors were removed.

3.4 Static Pressure and Friction Factor Measurements

Static pressure measurements were made in flowfield facility to make use of the large, 64x scale. The pressure coefficients were measured on the pin-fin surface using flush-mounted static pressure taps. The pin-fins in the flowfield facility were made from polycarbonate tube stock having wall thickness of 1.59 mm. The pin-fin walls were drilled and instrumented with static pressure taps at varying circumferential and spanwise locations as shown in Figure 3.7. Although the majority of pin-fin tests used a 63.5 mm diameter pin, static pressure measurements were only made for the pin-fin case having H/D = 2, shown in Figure 3.7, where the pin-fin diameter is 31.75 mm. The inner diameter of the pressure tap was 0.8 mm. The angular resolution of the static pressure
taps was, therefore, 0.8% of the circumference of the 31.75 mm pin-fins. Pin-surface $C_p$ was calculated using equation 3.2.

$$C_p = \frac{P(\theta) - P(\theta = 0^\circ)}{\frac{1}{2} \rho U_m^2} \quad \text{or} \quad C_p = \frac{P(\theta) - P(\theta = 0^\circ)}{\frac{1}{2} \rho U_{\max}^2}$$ (3.2)

Friction factor measurements were made first in an unobstructed duct, but also in pin-fin channels. Unobstructed duct friction factor was calculated using equation 3.3 where $\Delta P$ is the measured pressure drop between two points separated by distance, $L$. Note that a factor of $1/4$ was applied to convert from Moody friction factor to a Fanning friction factor.

$$f_{L/Dh} = \frac{\Delta P}{2 \rho U_m^2} \frac{L}{D_h}$$ (3.3)

Pressure drop, $\Delta P$, was measured across nine pairs of flush-mounted pressure taps whose locations are shown schematically in Figure 3.8. Friction factor was calculated for pin-fin configurations using equation 3.4 where $N_{\text{rows}}$ was the number of streamwise rows, $\Delta P$ was the measured pressure drop, and $\Delta P_{\text{open}}$ was the estimated pressure drop due to the unobstructed part of the duct.

$$f_N = \frac{\Delta P - \Delta P_{\text{open}}}{2 \rho U_{\max}^2} \frac{1}{N_{\text{rows}}}$$ (3.4)

The estimated pressure drop due to the unobstructed portion of the duct was calculated from equation 3.5 where $L$ was the distance between pressure taps and $L_{\text{array}}$ was the length between the center of the first and last row of pin-fins.

$$\Delta P_{\text{open}} = f_v \left(2 \rho U_m^2 \left(\frac{D_h}{L - (L_{\text{array}} + 4D)}\right)\right)$$ (3.5)
It should be noted that 2 pin-diameters were included upstream and downstream of the pin-fin array, hence the 4D included in equation 3.5.

### 3.5 Pin-Fin Surface Heat Transfer Measurements

Similar to pin-fin static pressure measurements, pin-fin surface heat transfer measurements were made in the flowfield facility to obtain a high spatial resolution. Measurements were made on the pin-fin surface by wrapping an Inconel foil heater around the pin-fin to supply a constant heat flux boundary condition. The hollow region at the center of the pin-fin was filled with fiberglass insulation ($k = 0.043 \text{W/m}\cdot\text{K}$) to minimize conductive losses. Also, the polycarbonate shell used for constructing the pin-fins had low thermal conductivity ($k = 0.2 \text{W/m}\cdot\text{K}$) which minimized lateral conduction. Radiation losses were minimized since the Inconel foil has a low emissivity ($\varepsilon = 0.21$). Surface temperature measurements were made using E-type thermocouples secured to the backside of the heater using thermally conductive cement ($k = 1.1 \text{W/m}\cdot\text{K}$). As with static pressure measurements, holes were drilled in the polycarbonate shell of the pin-fin to allow access for thermocouple mounting on the heater. A picture of the instrumented heat transfer pin-fin, having $H/D = 2$ and diameter of 31.75 mm, is shown in Figure 3.9. Local heat transfer coefficients were calculated using equation 3.6 where area of the heater, $A$, was calculated as the area of the heater traces. It should be noted that the area of heater traces was not necessarily equal to $\pi DH$ because there was some unheated area between the heater traces.

$$h(\theta, Z) = \frac{I^2 R - q_{\text{loss}}}{A} \frac{1}{T(\theta, Z) - T_m}$$

Conduction losses were calculated using a one-dimensional approximation for heat transfer along the axis of the cylinder. Radiation losses were calculated by approximating the cylinder as a diffuse, gray body and the interior walls of the channel as a black body. The view factor used for each local temperature measurement was, therefore, unity since the cylinder has no line of sight to itself or any other heated objects.
The total heat loss, $q_{\text{loss}}$, was calculated as the sum of the calculated conduction and radiation losses. Typical heat loss values at the middle of the pin-fin were 5% of the heat dissipated by the foil heater. Near the channel walls, however, heat losses reached as high as 30%. Nusselt number was then calculated using equation 3.7 where the thermal conductivity of the fluid, $k_f$, was evaluated at the film temperature, $T_{\text{film}}$.

$$\text{Nu}_D(\theta, Z) = \frac{h(\theta, Z)D}{k_f} \quad (3.7)$$

### 3.6 Combined Endwall and Pin-Fin Heat Transfer Measurements

To compliment the pin-surface heat transfer measurements, infrared (IR) thermography was used to measure heat transfer along the endwall surfaces. The IR-thermography method used in the present work was based on the method used by Lyall et al. (2011) and Lawson et al. (2011). A description of the present IR-measurement method is presented in this section, and the reader is referred to Appendix A for a more detailed discussion on previous and present measurement methods. The IR thermography method was carried out in the heat transfer facility of Figure 3.2.

The test channel was constructed from medium-density fiberboard (MDF) for its low thermal conductivity and high flatness tolerances. The IR-thermography method used a constant heat flux boundary condition achieved by lining the top and bottom endwalls with thin, Inconel foil heaters. The top heater was custom made from three-strips of 1 mil thick Inconel while the bottom heater was a commercially available serpentine heater also made from an Inconel circuit but encased in Kapton. Figure 3.10 shows the commercially-available serpentine heater and the custom three-strip heater. The three-strip heater was fabricated by soldering three large strips of Inconel together in series using copper bus-bars, as shown in Figure 3.11. The copper bus-bars were soldered across the width of each heater strip to ensure uniform current flow along the length of the strip. The bus bars were soldered to the non-flow side of the heater strips and were set into a groove machined into the MDF endwalls to maintain a flush surface on the flow side.

Because optical access inside the heat transfer facility was unavailable, a portion of the top-facing channel wall was removed such that the back, or non-flow side of the
top heater was visible. Figure 3.10 shows the position of the cutout allowed optical access for making IR temperature measurements on the non-flow side of the channel. A schematic of the channel cross-section is shown in Figure 3.12. And, the exact positioning of the heaters, pin-fins, and the IR cutout in the facility is shown in Figure 3.13.

To accurately measure the heat transfer coefficients on the heater surface, any possible sources of heat loss from the channel were minimized. To minimize conduction through the MDF walls and into the surroundings, a layer of 50.8 mm thick insulating foam ($k = 0.02 \text{ W/m·K}$) was added to the MDF walls. To prevent natural convection from the non-flow side of the heater into the surroundings, a Zinc-Selenide (ZnSe) window was placed in the cutout to create a 6.4 mm air gap above the heater surface. ZnSe was chosen as the window material because it has high transmissivity in the IR spectrum, $\tau = 0.7$. The size of the air gap was designed to preserve symmetry in the thermal resistance to conduction through the top and bottom sides of the channel. It should be noted that the back side of the top heater surface was painted flat-black to create a highly emissive surface, $\varepsilon = 0.93$, for IR measurements. The flat-black coating created the potential for radiation losses to the surroundings (since the non-flow side of the heater has line-of-sight to the surroundings through the ZnSe window). Radiation losses, however, were included in the energy balance and will be discussed in more detail in a following discussion.

Copper pin-fins were installed in the channel using thermally conductive epoxy to minimize thermal contact resistance with the endwall heaters. Copper pin-fins were chosen such that the pin-fins were thermally active and contributed to the overall heat transfer of the channel. One example of previous studies using non-active pin-fin surfaces is the work by Metzger et al. (1982a) where the pin-fins were made from low thermal conductivity material (balsa wood) and did not contribute to the overall heat transfer. The copper pin-fins in the present work were electrically insulated from the heaters by the layer of Kapton film covering each heater. A close-up view of the junction between pin-fin and heater is shown in Figure 3.14. To facilitate disassembly of the test section, only pin-fins that were viewed through the ZnSe window used thermal epoxy on both ends. Those pin-fins outside the viewing window used thermal epoxy on the bottom
end and a non-solidifying thermally conductive paste on the top end of the pin-fin. The thermal paste had $k = 1\text{W/m}\cdot\text{K}$ and the thermal epoxy had $k = 4\text{W/m}\cdot\text{K}$.

Another reason for using copper pin-fins was to achieve a high fin-efficiency. A calculation showed that pin-fins in gas turbine engines have very high fin-efficiency, making copper an appropriate choice for engine-realistic thermal boundary conditions. The high fin-efficiency meant that the copper pin-fins had a Biot number much less than unity ($\text{Bi} = 5\times10^{-3}$). Because the IR images acquired the temperature at the base of each pin-fin, a lumped-model approximation was made possible because the pin-fin surface temperature was well approximated by the IR-measured, pin-fin base temperature. Previous methods had problems with lateral conduction along the heater surface and were unable to utilize the lumped-model approximation for calculating pin-fin heat transfer, as shown in Appendix A. The only other study in which independently measured heat transfer on thermally-active endwalls and pin-fins is the work of Chyu (1999) who used the mass transfer analogy, which relied on an assumed proportionality constant to compare mass and heat transfer coefficients. While transient methods have been used to independently measure pin-fin and endwall heat transfer at the same time (Chyu et al. 2007; Chyu et al. 2009), the present work is the only study which simultaneously measures pin-fin and endwall heat transfer coefficients for a steady-state, constant heat flux boundary condition (with constant temperature pin-fins). Refer to Appendix A for more discussion on the innovative test methods used in the present work to capture both pin-fin and endwall heat transfer simultaneously from an IR image.

The data reduction procedure began by defining the positions of each pin-fin visible in the IR images. Figure 3.15 shows IR images of the raw temperatures and the masks used to separate the regions corresponding to the pin-fin bases and the endwalls. An energy balance was performed at each pixel from the IR image where the pin-fin bases and endwalls were treated separately to account for the additional convective area underneath the pin-fin bases. The energy balance for a pixel corresponding to a location on the endwall is shown in Figure 3.16. And, the endwall Nusselt number was calculated using equation 3.8.
\[ \text{Nu}_{D,e}(X,Y) = \left( \frac{2q_{\text{heater}} - 2q_{\text{cond,norm}} - q_{\text{rad}}}{A_{\text{pixel}}(T_{\text{corrected}}(X,Y) - T_{m}(X))} \right) \frac{D}{k_f} \]  

(3.8)

In equation 3.8, \(q_{\text{heater}}\) was the heat generated by the Inconel heater within the pixel, \(q_{\text{cond,norm}}\) was the surface-normal conduction loss at each pixel, \(q_{\text{rad}}\) was the local radiation loss to the surroundings at each pixel, \(A_{\text{pixel}}\) was the area of each pixel on the heater surface, \(T_{\text{corrected}}(X,Y)\) was the local temperature corrected for conduction across the Kapton film separating the flow and non-flow sides of the heater, and the remaining terms are defined in the nomenclature. It should be noted that the bulk temperature, \(T_{m}(X)\), varied linearly with streamwise position as a function of the net heat into the flow from the upstream heated area. In equation 3.8, there was a factor of two applied to each heat loss/gain term, except for the radiation loss, to account for the top and bottom heaters. The radiation loss only occurred through the top of the heater, so a factor of one was applied in equation 3.8. The pin-fin heat transfer was calculated in a similar manner as the endwall heat transfer, with the exception of an additional area ratio term and an additional conduction term in the energy balance.

\[ \text{Nu}_{D,p}(X,Y) = \left( \frac{2q_{\text{heater}} - 2q'_{\text{cond,lateral}}A_{\text{lateral}} - 2q_{\text{cond,norm}} - q_{\text{rad}}}{A_{\text{pixel}}(T_{\text{corrected}}(X,Y) - T_{m}(X))} \right) \left( \frac{\pi D^2/4}{\pi DH} \right) \frac{D}{k_f} \]  

(3.9)

In Appendix A it was found that lateral conduction had no significant impact on calculating the row-averaged endwall heat transfer, but did have a significant impact on the row-averaged pin-fin heat transfer. Lateral conduction was, therefore, included for the pin-fins but not for the endwalls. In equation 3.9, \(q'_{\text{cond,lateral}}\) was the heat flux due to lateral conduction and \(A_{\text{lateral}}\) was the area through which lateral conduction occurred. The area ratio term in equation 3.9 accounted for the difference in the pin-fin base area and the pin-fin wetted area. Another subtlety in calculating pin-fin heat transfer was \(T_{\text{corrected}}(X,Y)\) accounted for the temperature change across the thermal epoxy in addition to the change across the Kapton film. For the endwall, \(T_{\text{corrected}}(X,Y)\) only accounted for the temperature change across the Kapton film.
It should be noted that the local endwall Nusselt number in equation 3.8 is designated with an e-subscript and the local pin-fin heat transfer of equation 3.9 is designated with a p-subscript. The combined pin-endwall Nusselt number was calculated using an area-weighted average shown in equation 3.10.

\[
\frac{\overline{Nu_{D,c}}}{A_{\text{pins}} + A_{\text{endwall}}} = \frac{A_{\text{pins}} \overline{Nu_{D,p}} + A_{\text{endwall}} \overline{Nu_{D,e}}}{A_{\text{pins}} + A_{\text{endwall}}}
\]

(3.10)

In equation 3.10, the combined Nusselt number is designated with a c-subscript and the over bars denote row-averaged quantities. Array-averages are denoted with double over-bars. The pin-fin area was defined using equation 3.11.

\[
A_{\text{pins}} = N_{\text{pins}} \pi DH
\]

(3.11)

The endwall area was defined using equation 3.12.

\[
A_{\text{endwall}} = 2N_{\text{rows}}XW - 2N_{\text{pins}} \frac{\pi D^2}{4}
\]

(3.12)

Previous methods accounted for surface-normal conduction losses using an array-averaged heat loss and neglected radiation losses (Lyall et al. 2011; Lawson et al. 2011). In the present work, heat loss predictions were made more accurate by considering local changes in driving temperature between the heater and surroundings. On the bottom of the channel, on the opposite side of the MDF wall, loss thermocouples were installed to calculate the conduction heat flux in the surface-normal direction. From Figure 3.12, and using the conduction of symmetry about the channel midline, the driving temperature for surface-normal conduction was the difference between heater temperature, a function of (X,Y), and the loss temperature, a function of (X). The maximum local conduction loss ranged between 9%, for low Reynolds number, and 2%, for high Reynolds number, relative to the heat dissipated by the heater. The surface-normal conduction was then
calculated assuming one-dimensional conduction to the surroundings using a network of thermal resistances in series.

Radiation losses from the non-flow side of the heater were estimated using gray-body exchange, where the heater surface had $\varepsilon = 0.93$, and the surroundings which were approximated as a blackbody at room temperature. Similar to conduction losses, radiation losses were calculated as a function of $(X,Y)$. Radiation exchange with the semi-transparent ZnSe window was also included on the non-flow side of the heater. Radiation exchange on the flow side of the channel, however, was negligible because of the small temperature differences on the flow-side. The largest temperature difference between the heater and surroundings occurred upstream or very far downstream of the pin-fins, as shown in Figure 3.15. Using position-dependent conduction losses did not show a significant difference with array-averaged conduction losses for integral quantities, but made local flow features more accentuated. Including radiation losses, however, were found to decrease endwall heat transfer by up to 20% in regions of high driving temperature differentials. Upstream of the pin-fin array, it was shown in Appendix A that including radiation losses result in good agreement with heat transfer in a fully-developed duct flow, to within 1%.

Lateral conduction, used for calculating pin-fin heat transfer, was calculated based on the local temperature gradient measured with the IR camera. A fourth-order accurate finite-volume calculation was performed at each pixel location to determine the net heat gain/loss due to conduction with the neighboring pixels. A fourth-order method was chosen to reduce the amount of noise in calculating the derivatives for lateral conduction. Lateral conduction was found to increase the row-averaged pin-fin heat transfer by up to 33%. Including lateral conduction resulted in better agreement of present pin-fin heat transfer coefficients when compared with previous studies. Refer to Appendix A for a detailed discussion on the lateral conduction analysis.

3.7 Uncertainty Analysis

To ensure that the measurement techniques produced results that were statistically significant, uncertainty was calculated for each measurement using the sequential perturbation technique described by Moffat (1988). Uncertainty calculations were taken
at 95% confidence level. Uncertainty in Re_Dh was within 0.9% and Re_D to within 3.0% for each facility. At Re_D = 3.0e3, uncertainty in friction factor was 23.7% in the flowfield rig (caused by very low differential pressures) and 9.6% for friction factor in the heat transfer rig. At high Reynolds numbers, uncertainty in friction factor was 6.4% in the flowfield facility and 10% in the heat transfer facility (caused by the range of pressure transducer required). Uncertainty in the pressure coefficient at the position θ = 180° measured at Re_D = 3.0e3 was 13.0% and was 5.7% at Re_D = 2.0e4. Uncertainty in pin-surface heat transfer measured at Re_D = 2.0e4 was 10.0%. In the heat transfer facility, the IR-measured heat transfer had 11.1% uncertainty at low Reynolds number and 6.7% uncertainty at high Reynolds number.

Uncertainty in LDV measurements were calculated using a 1% bias uncertainty for instantaneous measurements in velocity (Radomsky and Thole 2000). Then, precision uncertainty was calculated using a student’s T-distribution. Uncertainties in LDV measurements were 1.8% for $\bar{U}$ and 0.8% for $\bar{V}$. Note that uncertainty in instantaneous velocities, $\bar{U}$ and $\bar{V}$, is equal to that of time-mean velocities, U and V. The size of TRDPIV datasets were prohibitively large for using student’s T-distribution analysis for calculating precision uncertainty. For each TRDPIV measurement, however, convergence of turbulence statistics was achieved and precision uncertainty error were assumed to be negligible. Statistical convergence is discussed in Section 3.8. Uncertainties in TRDPIV measurements were estimated based on the maximum instantaneous displacement gradients. The maximum instantaneous displacement gradients were $(\partial U/\partial Y)_{max} = 0.4$ px/px and $(\partial V/\partial X)_{max} = 0.1$ px/px. The uncertainty in particle displacement for the present TRDPIV processing algorithm was $\varepsilon_u = 0.15$ px and $\varepsilon_v = 0.03$ px (Scarano and Riethmuller 2000; Raffel et al. 1998). The resulting TRDPIV uncertainty in TRDPIV-measured velocities were 2.0 % or $\bar{U}$ and 0.4% for $\bar{V}$. Because TRDPIV was used to calculate vorticity, the error in measured displacement gradients were quantified in addition to the error in particle displacement. The uncertainty in displacement gradients were $\mathcal{E}_{\partial U/\partial Y} = 0.042$ px/px and $\mathcal{E}_{\partial V/\partial X} = 0.006$ px/px. A summary of the errors in displacement and in displacement gradient associated with the TRDPIV measurements are shown in Table 3.2. From the error in measured
displacement gradients, the uncertainty in vorticity was 2.0% for low Reynolds number and 3.3% for high Reynolds number. Similarly, the uncertainty swirl strength was 2.0% and 3.3%. The in-plane, Reynolds stress components had an uncertainty of 8% for \( \langle u'u' \rangle \), 2% for \( \langle v'v' \rangle \), and 5.2% for \( \langle u'v' \rangle \).

A summary of the experimental uncertainty is shown in Table 3.3 for the flowfield facility and Table 3.4 for the heat transfer facility. For detailed error propagation calculations, refer to Appendix C.

### 3.8 Benchmarking the Test Facility and Measurement Methods

After constructing the flowfield facility, several benchmarking experiments were performed to ensure proper operation of the facility. It should be noted that the heat transfer facility has been benchmarked in previous studies (Lawson et al. 2011; Lyall et al. 2011). Unobstructed duct friction factor in the flowfield facility was found to agree well with published data as shown in Figure 3.17. Velocity uniformity was checked using two-component LDV to measure the streamwise velocity across the duct in the transverse, Y-direction. Flow uniformity measurements were taken at the duct midline, \( Z/H = 0 \). Normalized mean and RMS velocity were observed to have excellent uniformity, to within 2% of \( U_m \), across the center portion of the duct, as shown in Figure 3.18.

To validate the assumption that the flow is fully-developed, the inlet velocity profile was measured using two-component LDV. The mean and RMS streamwise velocity were measured in the empty duct and plotted in near wall-coordinates and compared with published data in Figure 3.19 and Figure 3.20, respectively. At high duct Reynolds number, \( \text{Re}_{Dh} = 45e3 \), good agreement with the law-of-the-wall was observed. At low duct Reynolds number, \( \text{Re}_{Dh} = 6.8e3 \), good agreement was also observed, but there was a slight overshoot in mean velocity near the buffer layer due to the transitional nature of the flow. For both low and high Reynolds numbers, the RMS streamwise velocity agreed well with published data (Harder and Tiederman 1991) and both cases showed a peak value of \( u^+ = 2.7 \) at \( Z^+ = 20 \). The wall shear stress was obtained from the LDV velocity profile measurements using a least-squares regression to match measured values in the log-layer to those calculated using the law-of-the wall. The resulting wall
shear stress was used to determine the unobstructed duct friction factor, which agreed with published, smooth pipe data (Colebrook 1939) to within 5.1% for both Reynolds numbers tested. As a result of these first benchmarking experiments, it was determined that the test facility was performing as designed.

After validating the unobstructed duct flow, a single row of pin-fins was installed in the test facility. The pin-fins had H/D = 2 and S/D = 2.5, while the diameter was 31.75 mm. Uniformity in streamwise and transverse velocity was measured across the duct. Two-component LDV measurements were taken downstream of the pin-fins, at X/D = 1.5 and Z/H = 0, to ensure that the flow between pin-fins showed periodicity. Periodicity in mean and RMS velocity along several pin-fin wakes was confirmed in Figure 3.21.

To validate the accuracy of the TRDPIV technique, it was required to prove convergence of turbulent statistics. The XY-plane in the pin-fin wake was chosen for TRDPIV validation due to high turbulence levels and large velocity gradients. This validation test was required to prove that the TRDPIV system was capable of capturing enough samples to allow convergence of turbulent statistics. To test the convergence of turbulence statistics, 2000 samples were collected in the pin-fin wake at a rate of 1000 Hz for Re_D = 2.0e4. Four subsets were taken from the 2000 samples to isolate the effect of sample size. In Figure 3.22, the RMS streamwise velocity is plotted across the pin-fin wake for various sample sizes. It was found that at least 1000 data points were required to obtain convergence. In all successive TRDPIV measurements, these results were used as a guideline for the minimum amount of samples to collect. In most cases, 3000 samples were collected for better frequency resolution in spectral analyses.

One of the main sources of error when processing TRDPIV data arises from the size of the interrogation windows with respect to the size of the wavelengths to be measured (Scarano and Riethmuller 2000). Smaller interrogation windows on the final pass are able to resolve smaller wavelengths, which is important in capturing the small turbulent scales. With smaller interrogations windows, however, less particle images will be included in the calculation of the displacement vector which can lead to poor signal-to-noise ratios and spurious data. As such, TRDPIV data was captured in the near wake region of a single row of pin-fins for Re_D = 2.0e4 and the data was processed with varying final pass interrogation windows. Final window sizes of 32x32, 16x16, and 8x8
were tested where each case used a 50% window overlap. At $Re_D = 2.0 \times 10^4$, 2000 samples were used to calculate RMS streamwise velocity which was then plotted across the wake as shown in Figure 3.23. It was found that the 32x32 px windows filtered out some of the small spatial wavelengths and, therefore, produced slightly lower (2%) RMS values along the shear layer in the pin-fin wake. This was confirmed in plotting instantaneous streamwise velocity at $t=0.000s$, shown in Figure 3.24. As final window size was reduced, higher frequency content was observed; however, it was unclear whether or not the high frequency content present in the 16x16 and 8x8 cases was due to spurious vectors. It was determined that 16x16 px windows with 50% overlap provided a good balance between capturing short spatial wavelengths while still including enough particle images to maintain good signal-to-noise ratio.

To conclude the benchmarking section, TRDPIV data was compared with LDV data. Again, measurements were taken across the pin-fin wake in a region of high velocity gradients and turbulence intensity. Figure 3.25 compares mean streamwise velocity and Figure 3.26 compares RMS streamwise velocity. The agreement between the LDV and TRDPIV measurements was within the uncertainty of the $U_{max}$ calculated from the duct flowrate which was 3.5% for each experiment. Considering the high turbulence intensity and large velocity gradients, this validation check proved that the two optical techniques produced consistent results. A final comparison between TRDPIV and LDV was performed by taking a time-series of transverse velocity one pin diameter downstream on the wake centerline. The results, shown in Figure 3.27, indicated that both TRDPIV and LDV are capable of resolving similar temporal scales. Each time series shows the large scale, vortex shedding fluctuations, and small scale, stochastic fluctuations.

From the validation exercises presented in this section, it was determined that the new test facility was operating properly. The flow was verified to be consistent with that of an infinitely wide, fully-developed duct. The optical measurement techniques were validated by showing good agreement between TRDPIV and two-component LDV in a region of high turbulence intensity and velocity gradients.
### TABLE 3.1. TRDPIV IMAGE PROCESSING SCHEME

<table>
<thead>
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<th>Pass</th>
<th>Window Size (px)</th>
<th>Overlap</th>
<th>Fractional Offset/</th>
<th>Image Deformation</th>
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<td>64x64</td>
<td>50%</td>
<td>No/No</td>
<td></td>
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<tr>
<td>2</td>
<td>32x32</td>
<td>50%</td>
<td>Yes/Yes</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>16x16</td>
<td>50%</td>
<td>Yes/Yes</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>16x16</td>
<td>50%</td>
<td>Yes/Yes</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 3.2. SUMMARY OF MEASUREMENT UNCERTAINTIES ASSOCIATED WITH TRDPIV DATA

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Re&lt;sub&gt;D&lt;/sub&gt; = 3.0e3</th>
<th>Re&lt;sub&gt;D&lt;/sub&gt; = 2.0e4</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε&lt;sub&gt;μ̅&lt;/sub&gt;</td>
<td>Error in streamwise velocity</td>
<td>0.15 px</td>
<td>0.15 px</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009 m/s)</td>
<td>(0.099 m/s)</td>
</tr>
<tr>
<td>ε&lt;sub&gt;ν̅&lt;/sub&gt;</td>
<td>Error in transverse velocity</td>
<td>0.03 px</td>
<td>0.03 px</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002 m/s)</td>
<td>(0.020 m/s)</td>
</tr>
<tr>
<td>ε&lt;sub&gt;UU/YY&lt;/sub&gt;\text{max}</td>
<td>Error in dU/dY velocity gradient</td>
<td>0.042 px/px</td>
<td>0.042 px/px</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.39 s&lt;sup&gt;-1&lt;/sup&gt;)</td>
<td>(26.25 s&lt;sup&gt;-1&lt;/sup&gt;)</td>
</tr>
<tr>
<td>ε&lt;sub&gt;UV/XX&lt;/sub&gt;\text{max}</td>
<td>Error in dU/dX velocity gradient</td>
<td>0.006 px/px</td>
<td>0.006 px/px</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.34 s&lt;sup&gt;-1&lt;/sup&gt;)</td>
<td>(3.75 s&lt;sup&gt;-1&lt;/sup&gt;)</td>
</tr>
</tbody>
</table>
### TABLE 3.3. SUMMARY OF EXPERIMENTAL UNCERTAINTY IN THE FLOWFIELD RIG (95% CONFIDENCE LEVEL)

<table>
<thead>
<tr>
<th></th>
<th>$Re_D = 3.0e3$</th>
<th>$Re_D = 2.0e4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_{Dh}$</td>
<td>0.6%</td>
<td>0.9%</td>
</tr>
<tr>
<td>$Re_D$</td>
<td>1.4%</td>
<td>1.5%</td>
</tr>
<tr>
<td>$U_m$</td>
<td>1.4%</td>
<td>1.5%</td>
</tr>
<tr>
<td>$U_{max}$</td>
<td>1.4%</td>
<td>1.5%</td>
</tr>
<tr>
<td>$f_{L/Dh}$</td>
<td>23.7%</td>
<td>6.4%</td>
</tr>
<tr>
<td>$C_p$</td>
<td>13.0%</td>
<td>5.7%</td>
</tr>
<tr>
<td>$Pin , Nu_{Dh}$</td>
<td>-</td>
<td>10.0%</td>
</tr>
<tr>
<td>LDV, $\bar{U}$</td>
<td>1.8%</td>
<td>0.7%</td>
</tr>
<tr>
<td>LDV, $\bar{V}$</td>
<td>0.8%</td>
<td>0.7%</td>
</tr>
<tr>
<td>PIV, $\tilde{U}$</td>
<td>2.0%</td>
<td>1.2%</td>
</tr>
<tr>
<td>PIV, $\tilde{V}$</td>
<td>0.4%</td>
<td>0.2%</td>
</tr>
<tr>
<td>PIV, $\theta_Z$</td>
<td>2.0%</td>
<td>3.4%</td>
</tr>
<tr>
<td>PIV, $\bar{\lambda}_{c,i}$</td>
<td>2.0%</td>
<td>3.4%</td>
</tr>
<tr>
<td>PIV, $&lt;u'u'&gt;$</td>
<td>8%</td>
<td>8%</td>
</tr>
<tr>
<td>PIV, $&lt;v'v'&gt;$</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>PIV, $&lt;u'v'&gt;$</td>
<td>5.2%</td>
<td>5.2%</td>
</tr>
</tbody>
</table>

### TABLE 3.4. SUMMARY OF EXPERIMENTAL UNCERTAINTY IN THE HEAT TRANSFER RIG (95% CONFIDENCE LEVEL)

<table>
<thead>
<tr>
<th></th>
<th>$Re_D = 3.0e3$</th>
<th>$Re_D = 2.0e4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_{Dh}$</td>
<td>0.8%</td>
<td>0.8%</td>
</tr>
<tr>
<td>$Re_D$</td>
<td>3.0%</td>
<td>3.0%</td>
</tr>
<tr>
<td>$U_m$</td>
<td>2.7%</td>
<td>2.7%</td>
</tr>
<tr>
<td>$U_{max}$</td>
<td>2.7%</td>
<td>2.7%</td>
</tr>
<tr>
<td>$f_{L/Dh}$</td>
<td>9.6%</td>
<td>10.0%</td>
</tr>
<tr>
<td>Endwall $Nu_{Dh}$</td>
<td>11.1%</td>
<td>6.7%</td>
</tr>
</tbody>
</table>
Figure 3.1. Schematic of flowfield facility.
Figure 3.2. Schematic of heat transfer facility.
Figure 3.3. Effect of H/D on duct Reynolds number in pin-fin arrays.
Figure 3.4. Effect of S/D on velocity scales in pin-fin arrays. A single, infinite cylinder is shown for comparison.
Figure 3.5. Two-component LDV orientation.
Figure 3.6. Schematic of the TRDPIV system oriented for near wake measurements.

Figure 3.7. Photographs of H/D = 2 pin-fin instrumented with static pressure taps.
Figure 3.8. Location of static pressure taps used to measure friction factor.

Figure 3.9. Photographs of H/D = 2 pin-fin instrumented with constant heat flux foil heater and surface mounted type-E thermocouples.
Figure 3.10. Bottom, serpentine heater (left) and top, three-strip heater (right) showing the position of the IR window cutout.

Figure 3.11. Orientation of three-strip heater within the measurement section.
Figure 3.12. Schematic of heat transfer facility measurement section (cross sectional view).

Figure 3.13. Schematic of heat transfer facility measurement section (top view).
Figure 3.14. Close-up view of attachment between heater and pin-fin.

Figure 3.15. Typical IR-measured temperatures, $T_{\text{Raw}}$, and the masks used to separate endwall temperatures, $T_{\text{Endwall}}$, from pin-fins, $T_{\text{Pins}}$. 
Figure 3.16. Energy balance for a given pixel location on the endwall (not under the base of a pin-fin).

Figure 3.17. Unobstructed duct friction factor plotted against published data.
Figure 3.18. Flow uniformity measurements across the center of the duct.

Figure 3.19. Inlet time-mean velocity profile in near-wall coordinates compared with the law-of-the-wall.
Figure 3.20. Inlet RMS velocity profile in near-wall coordinates compared with published data (Harder and Tiederman 1991).

Figure 3.21. Flow uniformity measurements. Normalized mean and RMS velocity downstream of the pin-fins, $Re_D = 10e3$. 
Figure 3.22. Statistical convergence of $u'$ across the pin-fin wake (16x16 pixel final window with 50% overlap).
Figure 3.23. Effect of final window size on $u'$ across the pin-fin wake (2000 sample average).

Figure 3.24. Effect of final window size on instantaneous velocity across the pin-fin wake (2000 sample average).
Figure 3.25. Normalized mean streamwise velocity showing comparison between LDV and TRDPIV at X/D=1.6, Z/H=0.

Figure 3.26. Normalized RMS streamwise velocity showing comparison between LDV and TRDPIV at X/D=1.6, Z/H=0.
Figure 3.27. Comparison of temporal resolution between LDV and TRDPIV for independent point measurements taken in the pin-fin wake.
CHAPTER 4:
EFFECTS OF REYNOLDS NUMBER

Depending on the application, pin-fin arrays in gas turbine engines commonly operate at Reynolds number conditions between $3.0 \times 10^3 < \text{Re}_D < 3.0 \times 10^4$ (Han et al. 2000). It is well known that increasing Reynolds number increases heat transfer in pin-fin arrays. Different applications require different overall heat transfer removal from the pin-fin array and, therefore, may target different Reynolds numbers. It remains unclear, however, what effects Reynolds number has on the local flowfield structure and, therefore, the local heat transfer coefficient distribution. The present chapter investigates how the Reynolds number dependence of the flowfield affects local heat transfer distributions.

In this chapter, the time-dependent flowfield was measured for a single row of pin-fins having $H/D=2$, $S/D=2.5$ at three Reynolds number conditions of $\text{Re}_D = 3.0 \times 10^3$, $1.0 \times 10^4$, and $2.0 \times 10^4$. Flowfield measurements were made in the wake and in the stagnation plane while static pressure measurements were made on the surface of the pin-fin. Comparisons of the flowfield were made with published heat transfer levels for a single pin-fin row (Lyall et al. 2011). This chapter will discuss the effects of Reynolds number on the pin-fin flowfield and what implications the flowfield effects have on local heat transfer levels.

4.1 Effect of Reynolds Number on Wake Closure Position

The pressure distribution at the pin midline ($Z/H = 0$) a single row of pin-fins was compared with a single, infinitely long cylinder (Thom 1933) and the first row of a seven row pin-fin array (Ames et al. 2005). Figure 4.1 and Figure 4.2 show the normalized static pressure distribution, $C_p$, measured at the pin midline for $\text{Re}_D = 3.0 \times 10^3$ and $2.0 \times 10^4$, respectively. The pin-fin $C_p$ distribution in Figure 4.1 and Figure 4.2 showed good agreement between a single pin-fin row and the first row of a seven row array from Ames et al. (2005). This result indicated that the presence of downstream rows, at least for pin-fins spaced at $X/D = 2.5$, does not significantly impact the mean pressure distribution at the pin midline of the first row. To compare the present results to a single infinite cylinder, $C_p$ was normalized using the mean velocity at the minimum flow area, $U_{\text{max}}$. 
The $C_p$ distribution for the present, single pin-fin row was significantly different than that of a single, infinite cylinder. Literature for single, infinite cylinders having spanwise blockage shows that neither $U_{\text{max}}$ nor $U_m$ normalization results in a collapse with that of a single, infinite cylinder (Zdravkovich 2003). Normalization using $U_{\text{max}}$, however, provides better agreement with a single, infinite cylinder and decouples the effect of spanwise blockage, to some extent (Zdravkovich 2003). Increased streamwise pressure gradients caused the different $C_p$ distributions when comparing a single pin-fin row with a single, infinite cylinder.

The effect of Reynolds number can be observed in Figure 4.3 which shows the $C_p$ distribution for the single pin-fin row at both $Re_D = 3.0e3$ and $2.0e4$. At low Reynolds number, $C_p$ reaches a local minimum around $\theta = 90^\circ$, recovers slightly, and levels off as $\theta$ approaches $180^\circ$. At high Reynolds number, however, $C_p$ forms two local minima at $\theta = 90^\circ$ and $180^\circ$. The second local minimum, at $\theta = 180^\circ$, was caused by the Kármán vortices (KV), which have low static pressure at their core, forming adjacent to the pin-fin. At low Reynolds number, the KV were forming downstream from the pin-fin rather than adjacent to the pin-fin, as observed at high Reynolds number. The $C_p$ distributions provided the first evidence in the present work that the characteristic length scale decreased for increasing Reynolds number.

The pressure coefficient was integrated around the circumference to give the pressure-drag coefficient, $C_{D,p}$. At $Re_D = 3.0e3$, $C_{D,p}$ was 0.80, and at $Re_D = 2.0e4$, $C_{D,p}$ was 0.86. The use of $U_{\text{max}}$ in calculating $C_{D,p}$ for the pin-fins did not agree with the $C_{D,p}$ for a single, infinite cylinder which is approximately 0.85 at $Re_D = 3.0e3$ and 1.1 at $Re_D = 2.0e4$. Again, the effects of spanwise blockage in the single pin-fin row caused discrepancy with the single, infinite cylinder (Zdravkovich 2003).

To visualize the topology of the local flowfield, LDV and TRDPIV measurements were made in the wake of the pin-fins at the channel midspan ($Z/H = 0$). Figure 4.4 shows contours of normalized streamwise velocity in the near-wake at $Re_D = 3.0e3$, $1.0e4$, and $2.0e4$. Both LDV and TRDPIV data is shown in Figure 4.4, where good agreement was observed. The wake closure position, $L_c/D$, was defined as the position of zero streamwise velocity along the wake axis (Zdravkovich 1997). As expected from the $C_p$ distributions, the wake closure position (indicated using white arrows in Figure 4.4)
was found to decrease with increasing Reynolds number. As discussed in Chapter 2, the wake closure position decreases as a result of increased instability in the shear layers with increasing Reynolds number.

The unsteadiness in the wake was quantified by plotting contours of the average in-plane, turbulent normal stress. Because two-component TRDPIV measurements were made in the X-Y plane, the available Reynolds stress components were \( \langle u'u' \rangle, \langle v'v' \rangle \), and \( \langle u'v' \rangle \). In the remainder of this work, the in-plane fluctuations are frequently quantified by taking the average of \( \langle u'u' \rangle \) and \( \langle v'v' \rangle \) as a measure of turbulent kinetic energy. If three component measurements had been made, the true turbulent kinetic energy could be calculated from \( \langle u'u' \rangle, \langle v'v' \rangle \), and \( \langle w'w' \rangle \). In Figure 4.5, contours of in-plane, turbulent normal stress are shown for Re\(_D\) = 3.0e3, 1.0e4, and 2.0e4. In general, turbulent normal stress contours were similar when comparing LDV and TRDPIV. At high Reynolds number, the difference in the maximum level of turbulent normal stress when comparing LDV and TRDPIV measurements was beyond the experimental uncertainty. The reason for the difference was unclear, but LDV and TRDPIV data were taken on separate occasions at two different scales (LDV had pin-fin diameter of 63.5 mm and TRDPIV had 31.75 mm pin-fin). The pin-fin having 31.75 mm was used to reduce the measurement uncertainty in the pin-fin pressure coefficient distribution, and TRDPIV measurements were made in the wake while the single row having 31.75 mm diameter was installed. All remaining TRDPIV measurements were taken at a pin-fin diameter of 63.5 mm to improve the resolution of the flow time scales. The main observation from the turbulent normal stress contours was that increasing Reynolds numbers led to increased fluctuations in the wake. The position of maximum fluctuations approached the pin-fin for increasing Reynolds number, in agreement with the wake closure position. The velocity fluctuations in the pin-fin wake were comprised of both periodic unsteadiness and turbulence. At high Reynolds numbers, the periodic unsteadiness increased which contributed to the increase in fluctuations observed in Figure 4.5 when compared to lower Reynolds numbers. The distinction between periodic unsteadiness and turbulence is discussed in more detail in Chapter 6.
Previous studies have reported that wake closure position and other characteristic length scales generally follow the vortex formation length, \( L_f/D \), where the Kármán vortices (KV) are fully-formed (Zdravkovich 1997). Although \( L_f/D \) is defined as the point of minimum \( C_p \) along the wake centerline, the point of maximum streamwise RMS, \( u' \), velocity is a good approximation (Zdravkovich 1997). To compare with public literature, \( L_f \) was calculated using the position of maximum \( u' \) along the wake axis.

Figure 4.6 shows the effect of Reynolds number on formation length, \( L_f/D \). Consistent with the \( C_p \) distributions and the wake closure position, \( L_c/D \), the \( L_f/D \) was found to decrease with increasing Reynolds number. The formation length for the present, single row of pin-fins agreed with single, infinite cylinder data at low Reynolds number. As Reynolds number increased, however, the single row of pin-fins showed decreased \( L_f/D \) relative to a single, infinite cylinder. Several key differences exist between the single row of pin-fins and the single, infinite cylinder. The pin-fin channel features elevated turbulence from wall shear, three-dimensional effects from having finite aspect ratio, a streamwise pressure gradient to move air through the duct, and additional pressure gradients from the presence of adjacent pin-fins. These differences contribute to the discrepancy in \( L_f/D \) between single row of pin-fins and single, infinite cylinder.

### 4.2 Effect of Reynolds Number on Near Wake Dynamics

The instantaneous flowfield was captured with the TRDPIV system allowing for a detailed view of the near-wake dynamics in space and time. Figure 4.7 shows instantaneous streamtraces and normalized, in-plane velocity magnitude contours at \( \text{Re}_D = 3.0e3 \) and \( 2.0e4 \). Figure 4.7 shows the instantaneous flowfields corresponding to 0.1 shedding cycles for both Reynolds numbers. By visual inspection, the KV was about one cylinder diameter in size at \( \text{Re}_D = 3.0e3 \) and about 0.5 cylinder diameter in size at \( \text{Re}_D = 2.0e4 \). Consistent with the analysis in Section 4.1, the KV formed slightly downstream of the pin-fin at \( \text{Re}_D = 3.0e3 \) and formed closer to the pin-fin as Reynolds number increased.

Spectral analyses were performed at a point where large, periodic motions were observed in the time-signal, \( X/D = 1.6, Y/D = 0, Z/H = 0 \). The vortex shedding frequency was calculated by from the one-dimensional energy spectrum of the spanwise, \( V \)-velocity signal. TRDPIV data was sampled in regular intervals, so a standard fast-
Fourier-transform (FFT) was applied to the velocity signal. TRDPIV data was sampled at a minimum of 17 times the measured shedding frequency for a minimum of 50 shedding cycles. For LDV measurements, however, data was not sampled in regular intervals. To apply the FFT, LDV data was first down-sampled using nearest-neighbor interpolation to obtain a regularly-spaced time series. The frequency response of the down-sampled LDV data was at least 25 times that of the measured shedding frequency and data was collected over at least 300 shedding cycles. Both TRDPIV and LDV data were weighted using a Hanning window, although using a rectangular window showed no significant leakage at the measured shedding frequency.

Figure 4.8 shows the Strouhal frequency measured for the single row configuration. Also shown in Figure 4.8 are results from a single, infinitely long cylinder (Norberg 2003) and a multi-row configuration (Ames and Dvorak 2005). It should be noted that the pin-fin data makes use of the minimum area velocity, $U_{max}$, as the reference velocity in the definition of St. It was found that the present, single row Strouhal frequency matched that of single, infinite cylinders at $Re_D = 3.0e3$. At higher Reynolds numbers, however, the present results showed a decreasing Strouhal frequency. The present, single row Strouhal frequencies were lower than those reported by Ames and Dvorak (2005) for a multiple row pin-fin array. In multi-row arrays, it was likely that the presence of a downstream row imposed a constraint on the width of the near-wake which increased the shedding frequency.

4.3 Effect of Reynolds Number on Horseshoe Vortex Dynamics

As with typical wing-body junction flows, a horseshoe vortex (HV) was observed upstream of the pin-fins due to the variation in total pressure in the wall-normal direction from the incoming velocity profile. A schematic of the HV system was shown previously in Figure 2.2. The main HV is shown along with secondary (SV), tertiary (TV), and corner vortices (CV).

TRDPIV measurements were made in the stagnation plane, $\theta = 0^\circ$, to determine the effect of Reynolds number on the dynamics of the HV system. Figure 4.9 shows contours of mean streamwise velocity normalized by the bulk channel velocity. As Reynolds number increased, the velocity profile flattened and the HV moved closer to the
cylinder and decreased in size. The streamtraces in Figure 4.9 show a complete roll-up at lower Reynolds number due to a more stable HV in comparison to those at higher Reynolds numbers.

Because the HV was unstable at higher Reynolds numbers, it was difficult to identify the center of the HV from streamtraces of the mean velocity in Figure 4.9. To identify the location of the HV, the mean swirling strength, $\lambda_{ci}$, was determined by calculating the eigenvalues of the local two-dimensional velocity gradient tensor. Appendix D describes the procedures for calculating swirl strength. Swirling strength is a measure of vorticity that separates rotational motion from shearing motion based on the eigenvalues of the velocity gradient tensor (Adrian et al. 2000). Contours showing the local maxima of $\lambda_{ci}$ are shown in Figure 4.9 as the solid white lines. The location of the HV was then defined by the position of maximum $\lambda_{ci}$. The HV was determined to be located at $X/D = -0.80$, $-0.64$, and $-0.62$ for $Re_D = 3.0e3$, $10e3$, and $2.0e4$, respectively.

While previous research has shown the location of the HV to be a function of $\delta/D$, $Re_D$, and $Tu$, the measured HV locations agreed reasonably well with other wing-body junction flows (Praisner and Smith 2006b; Ozturk et al. 2008; Radomsky and Thole 2000; Devenport and Smith 1990).

For the Reynolds numbers considered in the present work, the HV system oscillated upstream of the obstruction in a quasi-periodic fashion. To quantify the unsteadiness of the HV system, the magnitude of in-plane, turbulent normal stress is shown in Figure 4.10. The peak in fluctuating velocity increased with Reynolds number which indicated a more unstable HV system at higher Reynolds numbers. The location of maximum fluctuations moved closer to the cylinder, which was consistent with the locations of maximum swirl strength in Figure 4.9.

Instantaneous realizations of the flowfield at $\theta = 0^\circ$ showed similar behavior to previous wing-body junction flow studies. Figure 4.11 shows instantaneous streamtraces and normalized, in-plane velocity magnitude for flow at $Re_D = 3.0e3$ and $2.0e4$. Instantaneous data is shown in time steps that correspond to the approximate turnover rate of the HV. At both Reynolds number conditions, the non-dimensional frame step was $St^{-1} = \Delta t U_{in} D^{-1} = 0.1$. The four vortices of the HV system depicted in Figure 2.2 were present at low and high Reynolds numbers. Figure 4.11 shows the ejection of the
SV as it became entrained by the HV. As the SV was ejected, an inrush of core fluid replaced the SV and was responsible for a portion of the mean endwall heat transfer (Praisner and Smith 2006a). The SV ejection was observed 0.95D upstream of the pin-fins at low Reynolds number and 0.80D upstream of the pin-fins at high Reynolds number. It was found, and will be shown in Section 4.4, that heat transfer was enhanced across the region between the SV ejection and the face of the pin-fin.

4.4 Effect of Reynolds Number on Heat Transfer

To this point, the data presented in Chapter 4 has shown the effect of Reynolds number on the flowfield in the near wake (development of the KV) and in the stagnation region (development of the HV). This section addresses how the flowfield behaviors, observed in the previous sections, relates to the local heat transfer distribution. To address this relationship, comparisons were made between the present flowfield measurements and surface heat transfer measurements conducted by other researchers.

Lyall et al. (2011) reported a significant increase in heat transfer in the near wake for a single-row of pin-fins having H/D = 1 and S/D = 2 relative to S/D = 4. Although the present measurements were made for H/D = 2 and S/D = 2.5, comparisons made with Lyall et al. (2011) were validated by the findings of Chyu et al. (2009) in which array heat transfer showed the same Reynolds number dependence when comparing H/D = 1 to H/D = 2. Figure 4.12 shows the endwall heat transfer measured by Lyall et al. (2011) for Reynolds numbers between 5.0e3 and 1.3e4. The position of maximum endwall heat transfer occurred downstream of the pin-fin, in the wake. For ReD = 5.0e3, the maximum heat transfer occurred at X/D = 3.8. At ReD = 1.3e4, the maximum heat transfer occurred at X/D = 1. As found with wake closure position and formation length, the length scale governing the position of maximum heat transfer decreased with increasing Reynolds number.

To compare formation length with the position of maximum heat transfer, Figure 4.12 includes solid white lines which correspond to the approximate formation length as measured with the present single row of cylinders. The formation length roughly correlates with the position of maximum heat transfer, suggesting that the Kármán vortex contributes, in some capacity, to increased heat transfer. It was unclear, however,
whether the Kármán vortex (periodic unsteadiness) contributed most to increased heat transfer or whether another flowfield feature (turbulence or other flowfield interaction) contributed most to heat transfer. Further experimentation is required to isolate the specific mechanism that caused a significant increase in heat transfer at X/D = 3.8 for a single row having S/D = 2 and Re_D = 5.0e3. Chapter 6, for example, will elaborate on the effects of periodic unsteadiness and turbulence on heat transfer.

In addition to enhanced heat transfer in the pin-fin wake, endwall heat transfer contours taken by Ames et al. (2007), shown in Figure 4.13, provide evidence that the horseshoe vortex contributes significantly to heat transfer. In the first row, the HV enhances heat transfer adjacent to the pin-fin between θ = 0° and 90° with a region of enhanced heat transfer extending tangent to θ = 90°. In the stagnation plane, the HV heat transfer footprint moves closer to the pin-fin as Reynolds number increases which is in agreement with the TRDPIV measurements of Section 4.3.

4.5 Summary of Reynolds Number Effects

In this chapter, the effects of Reynolds number were investigated for a single row of pin-fins having H/D = 2, S/D = 2.5. Reynolds number conditions of Re_D = 3.0e3, 1.0e4, and 2.0e4 were considered. It is well known that increasing Reynolds number results in increased heat transfer for pin-fin arrays. The goal of the present chapter, however, was to investigate the effect of Reynolds number on the local flowfield structure and on the local heat transfer distribution in pin-fin arrays.

It was found that increasing Reynolds number decreased the wake closure position. The decreased wake closure position was observed both in static pressure measurements on the pin-fin surface and in flowfield measurements taken in the wake at the channel midline (Z/H = 0). The position of maximum endwall heat transfer in the wake of a single row of pin-fins, from Lyall et al. (2011), correlated with the wake closure position and decreased with increasing Reynolds number. This result showed that the Kármán vortex contributed, in some capacity, to the local heat transfer distribution. Further experimentation is required, however, to determine the specific mechanism (periodic unsteadiness, turbulence, or three-dimensional effects) that contributed to the peak in heat transfer.
The horseshoe vortex was observed to decrease in size and move closer to the pin-fin with increasing Reynolds number, in agreement with previous studies. Heat transfer measurements, taken by Ames et al. (2007), showed that the size and position of the horseshoe vortex correlated with the local heat transfer distribution on the endwall in the first row of pin-fins. With increasing Reynolds number, the augmented heat transfer from the horseshoe vortex decreased in size and remained closer to the pin-fin than for low Reynolds numbers.

This chapter has shown that Reynolds number has a significant influence the local heat transfer distribution. Both the horseshoe vortex and the wake are affected by Reynolds number and both have an influence on the position of maximum heat transfer. In general, increasing Reynolds number results in a horseshoe vortex that moves closer to the pin-fin and a wake that forms closer to the pin-fin.

The present chapter provided the framework for the remainder of the present work. The effects of Reynolds number are significant, but further experimentation is required to determine the effects of periodic unsteadiness, turbulence, and three-dimensional effects. The contributions of periodic unsteadiness and turbulence are addressed in upcoming chapters, but the present measurements were unable to capture three-dimensional effects.
Figure 4.1. Pressure coefficient at the pin-fin midline for \( \text{Re}_D=3.0\times10^3 \).

Figure 4.2. Pressure coefficient at the pin-fin midline for \( \text{Re}_D=2.0\times10^4 \).
Figure 4.3. Effect of Reynolds number on pin-fin midline static pressure distribution.
Figure 4.4. Effect of Reynolds number on mean streamwise velocity in the wake of a single pin-fin row having $H/D=2$, $S/D=2.5$. 
Figure 4.5. Effect of Reynolds number on in-plane fluctuations in the wake of a single pin-fin row having H/D=2, S/D=2.5.
Figure 4.6. Effect of Reynolds number on formation length for a single row of pin-fins having H/D=2, S/D=2.5.
Figure 4.7. Effect of Reynolds number on near wake development. $Re_D=3.0e3$ shown on left, $Re_D=2.0e4$ shown on right.
Figure 4.8. Effect of Reynolds number on Strouhal frequency for a single row of pin-fins having H/D=2, S/D=2.5.
Figure 4.9. Effect of Reynolds number on position of horseshoe vortex for a single row of pin-fins having H/D=2, S/D=2.5.
Figure 4.10. Effect of Reynolds number on horseshoe vortex unsteadiness for a single row of pin-fins having H/D=2, S/D=2.5.
Figure 4.11. Effect of Reynolds number on instantaneous structure of the horseshoe vortex system for a single row of pin-fins having $H/D=2$, $S/D=2.5$. 
Figure 4.12. Effect of Reynolds number on endwall heat transfer for a single row of pin-fins having $H/D = 1$ and $S/D = 2$ (Lyall et al. 2011). Wake formation length is shown using solid white lines.
Figure 4.13. Effect of Reynolds number on significant features of endwall heat transfer in an array having H/D = 2, S/D = X/D = 2.5 (Ames et al. 2007).
CHAPTER 5:  
EFFECTS OF ASPECT RATIO

In a previous study by Chyu et al. (2009), it was found that array-averaged heat transfer augmentation increased for increasing aspect ratios (H/D) in the range of 2 ≤ H/D ≤ 4 and 2.7e3 ≤ ReD ≤ 14.3e4. Chyu et al. (2009) concluded that the ratio of pin-fin-to-overall convective surface area resulted in increased heat transfer for increasing H/D. This chapter will investigate whether flowfield effects also participated in the observed increase in heat transfer with increasing H/D.

The near wake flowfield was measured for two single row pin-fins having S/D = 2.5 with H/D = 1 and 2 since pin-fin aspect ratios H/D ~O(1) are typical in gas turbine applications (Kohli 2010a). Both H/D configurations were investigated for Reynolds numbers of Re_D = 3.0e3 and 2.0e4. Table 5.1 shows the characteristic velocity scales and Reynolds numbers for each configuration and Reynolds number. The near wake flowfield was measured using TRDPIV, and comparisons were made with local heat transfer measurements on the pin-fin surface.

5.1 Effect of Aspect Ratio on Time-Mean Flow

The time-mean velocity and turbulent normal stresses were first compared to determine the effects of H/D on the near wake. Figure 5.1 and Figure 5.2 show the mean flow and in-plane, turbulent normal stresses, respectively, in the pin-fin wake at Re_D = 3.0e3. TRDPIV data was taken at the channel midline (Z/H = 0). It should be noted that the TRDPIV domain was slightly different for the H/D = 1 and H/D = 2 cases, as observed in Figure 5.1 and Figure 5.2. For H/D = 1, the TRDPIV domain was centered on the pin-fin wake while the TRDPIV domain was centered between the pin-fins for H/D = 2. At low Reynolds number, there was a noticeable difference between H/D = 1 and H/D = 2 when comparing both the mean flow and the level of fluctuations. The mean flow showed a lower level of U/U_{max} in between the pins for H/D = 2 in comparison to H/D = 1. For each experiment in the present work, the average velocity through the minimum flow area (U_{max}) was held constant in order to match Re_D. Because the level of U/U_{max} was different for H/D = 1 and H/D = 2, there were three-dimensional effects present. To preserve continuity for this incompressible flow, the streamwise
velocity near the wall for the H/D = 1 case was lower than that of the H/D = 2. The reason for the difference in flowfield when comparing H/D = 1 and 2 was caused by the relative percentage of the duct height occupied by the horseshoe vortex. From the increased blockage at H/D = 1 relative to H/D = 2, the mean flow for H/D = 1 showed stronger flow acceleration around the pin-fin as shown in Figure 5.1 from the high flow velocity between the pin-fins. Increased velocity fluctuations were also observed for H/D = 1 and may have been caused by three-dimensional effects. Alternatively, the increased fluctuations may have been caused by the stronger shear layer at the channel middling for H/D = 1 resulting from the increased velocity gradient, dU/dY, compared to the H/D = 2 case.

Increasing Reynolds number to 2.0e4, there was less difference in the near wake flowfield than that of Re_D = 3.0e3. Figure 5.3 and Figure 5.4 show the time-mean flow and turbulent normal stress, respectively, in the pin-fin wake at Re_D = 2.0e4. The mean flow and fluctuating velocities were in good agreement for H/D = 1 and H/D = 2. As Reynolds number increases, it is well known that the incoming velocity profile becomes flatter with a sharper gradient at the wall. The analysis of the horseshoe vortex (HV) in Section 4.3 showed that the size of the HV decreased with increasing Reynolds number as a result of a flatter velocity profile. Because the velocity profile was flatter at high Reynolds number in comparison to low Reynolds number, the effect of blockage from the horseshoe vortex was less significant. As a result, the near wake flowfield showed similar structure for H/D = 1 and H/D = 2.

5.2 Effect of Aspect Ratio on Heat Transfer

To determine the effect of H/D on heat transfer, a single pin-fin was instrumented, as described in Chapter 3, to make surface heat transfer measurements both circumferentially and along the axis of the cylinder to create full-surface heat transfer contours. Only the high Reynolds number case was considered since buoyancy effects were non-negligible at low Reynolds numbers. Refer to Appendix B for further discussion on buoyancy effects.

Figure 5.5 shows a contour plot of heat transfer along the entire surface of the pin-fin for H/D = 1.0 and 2.0 at Re_D = 2.0e4. From Figure 5.5, the Nusselt number contours
were similar for H/D = 2.0 and H/D = 1.0. The horseshoe vortex (HV) was observed for both H/D cases at $\theta = 0^\circ$ and Z/H = $\pm 0.5$. The HV was of similar size for both H/D = 1.0 and 2.0. In agreement with the previous description of the incoming velocity profile, the HV occupied 20% of the duct height for H/D = 1 and 10% of the duct height for H/D = 2. The area-averaged heat transfer was $\text{Nu}_D = 96.8$ and 89.5 for H/D = 1 and 2, respectively. The percent difference between H/D = 1.0 and 2.0 was 7% and within the experimental uncertainty of 10%.

At high Reynolds number, it was shown in section 5.2 that flowfield effects at the channel midline did not vary significantly with H/D. Similarly, in the present section, the heat transfer around the pin-fin did not vary significantly with H/D. The results of this section showed that, at high Reynolds numbers, flowfield effects on local heat transfer coefficients were not important when varying H/D between 1 and 2.

5.3 Summary of Aspect Ratio Effects

The results of this chapter showed that the near wake flowfield was more dependent on H/D at low Reynolds numbers. Three-dimensional effects caused by blockage from the horseshoe vortex contributed to the flowfield dependence on Reynolds number. At high Reynolds numbers, the flowfield was independent of H/D because the horseshoe vortex was smaller than for low Reynolds numbers and because the streamwise velocity profile was flatter in the wall-normal direction than for low Reynolds numbers.

Chyu et al. (2009) showed that the array-averaged heat transfer increased with increasing H/D for pin-fin arrays having S/D = 2.5, X/D = 2.5 for $\text{Re}_D = 2.7\text{e}3$ and $14.3\text{e}4$. The pin-fin generally had a higher heat transfer coefficient than the endwall. Because increasing H/D resulted in an increased ratio of pin-fin-to-overall surface area, the array-averaged heat transfer increased with increasing H/D. The present work was in agreement with Chyu et al. (2009) because the pin-fin heat transfer coefficient did not vary significantly with H/D at high Reynolds numbers. The present work confirmed that H/D had no significant influence on the flowfield at high Reynolds numbers. At low Reynolds numbers, however, the flowfield was influenced by blockage from the horseshoe vortex. Similar to high Reynolds numbers, Chyu et al. (2009) showed that
flowfield effects were not as significant as the ratio of pin-fin-to-overall surface area for low Reynolds numbers.

In general, pin-fin arrays benefit from increasing H/D. The increased ratio of pin-fin-to-overall surface area influences the array-averaged heat transfer coefficients at close spanwise and streamwise spacings. In addition, increasing H/D increases the total surface area which results in greater overall heat transfer compared to shorter H/D configurations. As mentioned in Chapter 2, the goal of the present work was to decouple surface area effects from flowfield effects to determine what flowfield features contribute to heat transfer.

From this chapter, increasing H/D from 1 to 2 did not influence the flowfield in a manner that had a significant effect on the local heat transfer coefficients. The remainder of the present work focuses on first-order effects such as varying the streamwise and spanwise spacing.
### TABLE 5.1. EFFECT OF H/D ON THE VELOCITY SCALES AND REYNOLDS NUMBERS IN THE PIN-FIN CHANNEL

<table>
<thead>
<tr>
<th>H/D</th>
<th>D (mm)</th>
<th>S/D</th>
<th>Re_D</th>
<th>Re_Dh</th>
<th>U_{max} (m/s)</th>
<th>U_{m} (m/s)</th>
<th>U_{max}/U_{m}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63.5</td>
<td>2.5</td>
<td>3.0e3</td>
<td>3.6e3</td>
<td>0.75</td>
<td>0.45</td>
<td>1.667</td>
</tr>
<tr>
<td>2</td>
<td>31.75</td>
<td>2.5</td>
<td>3.0e3</td>
<td>7.2e3</td>
<td>0.75</td>
<td>0.45</td>
<td>1.667</td>
</tr>
<tr>
<td>1</td>
<td>63.5</td>
<td>2.5</td>
<td>2.0e4</td>
<td>2.4e4</td>
<td>5.00</td>
<td>3.00</td>
<td>1.667</td>
</tr>
<tr>
<td>2</td>
<td>31.75</td>
<td>2.5</td>
<td>2.0e4</td>
<td>4.8e4</td>
<td>5.00</td>
<td>3.00</td>
<td>1.667</td>
</tr>
</tbody>
</table>
Figure 5.1. Effect of H/D on time-mean velocity in a single pin-fin row having S/D = 2.5 at Re_D = 3.0e3.
Figure 5.2. Effect of H/D on turbulent normal stress in a single pin-fin row having S/D = 2.5 at Re_D = 3.0e3.
Figure 5.3. Effect of H/D on time-mean velocity in a single pin-fin row having S/D = 2.5 at Re_D = 2.0e4.
Figure 5.4. Effect of H/D on turbulent normal stress in a single pin-fin row having S/D = 2.5 at Re_D = 2.0e4.
Figure 5.5. Effect of H/D on surface heat transfer for a single pin-fin row having S/D = 2.5 at Re_D = 2.0e4.
CHAPTER 6:
EFFECTS OF STREAMWISE SPACING

Previous research has shown that the row-by-row development in heat transfer gradually increases to peak in the third or fourth row followed by a slight decline in downstream rows for arrays having a spanwise spacing of S/D = 2.5 (Metzger et al. 1982a). Lawson et al. (2011), however, varied the streamwise spacing, X/D, for an array having S/D = 2 and found that the location of peak heat transfer development does not always occur in the third or fourth row. For very distant X/D spacing, the heat transfer peaked in the second row and decreased in later rows (Lawson et al. 2011). In this chapter, the effect of X/D on the near wake flowfield was investigated to determine what feature(s) caused the geometry-specific, row-by-row development observed by Lawson et al. (2011).

The streamwise spacing was varied for a seven row pin-fin array having aspect ratio of H/D = 1.0, spanwise spacing of S/D = 2.0, and streamwise spacings of X/D = 1.73, 2.16, 2.60, 3.03, and 3.46. Reynolds number effects were considered by investigating Re_D = 3.0e3 and 2.0e4. The first and third row flowfields were also measured at the midspan of the channel (Z/H = 0). For the X/D = 1.73 and 3.46 cases, however, the flowfield was measured for every row, first through seventh. A single pin-fin row was investigated, for comparative purposes, having H/D = 1.0 and S/D = 2.0. By varying the streamwise pin-fin spacing, the degree of downstream obstruction was varied and the single pin-fin row provided a point for comparison by having no downstream obstruction.

6.1 Effect of Streamwise Spacing on Wake Closure Position

The effect of X/D on the near wake was first quantified using the wake closure position, L_c/D, which was defined in Chapter 4 as the position of zero streamwise velocity along the wake axis. Figure 6.1 and Figure 6.2 show the wake closure position as a function of X/D for Re_D = 3.0e3 and 2.0e4, respectively. The wake closure position measured for the single row case and the wake closure position for a single, infinite cylinder (Norberg 2003) are also included in Figure 6.1 and Figure 6.2. It should be noted that the wake closure position fell outside of the TRDPIV measurement domain for
several cases and was excluded from Figure 6.1 and Figure 6.2. It was found that wake closure position was dependent on Reynolds number, row position, and X/D spacing. In all cases, wake closure position was lower in the third row in comparison to the first row. Turbulence generated by upstream pin-fins caused the breakdown of the wake vortices. The instability in the shear layer caused the wake to form closer to the pin-fin in comparison with first row wakes. This phenomenon has been observed for single, infinite cylinders where increasing Reynolds number decreased the shear layer stability and resulted in a decreased wake closure position (Zdravkovich 1997). Similarly, this phenomenon has been shown for single, infinite cylinders under high freestream turbulence where the wake closure position was decreased relative to the low freestream turbulence case (Norberg 1986).

To better visualize the mean flow, Figure 6.3 and Figure 6.4 show the time-mean flowfield for Re_D = 3.0e3 and 2.0e4, respectively. The wake closure positions presented in Figure 6.1 and Figure 6.2 are indicated in Figure 6.3 and Figure 6.4 with dashed lines, allowing for comparison between different cases. In Figure 6.3, where Re_D = 3.0e3, the presence of the second row pin-fins was found to alter the first row wakes even for the widest streamwise spacing of X/D = 3.46. The wake was elongated for the first row of X/D = 3.46 in comparison with the single row of pin-fins because a favorable pressure gradient was created by the jet between second row pin-fins, which provided a suction force on the first row wake of X/D = 3.46. As X/D was reduced below 3.46, the wake closure position decreased, indicating that the adverse pressure gradients in front of second row pin-fins began to overcome the favorable pressure gradient between second row pin-fins. The wake closure position was nearly constant with X/D in the third row indicating the upstream disturbances influenced the vortex breakdown. The closure position for the third row varied between Lc/D = 1.33 and 1.42 for X/D ≥ 2.16 and decreased to Lc/D = 1.10 for X/D = 1.73.

The first row wakes at Re_D = 2.0e4, in Figure 6.4, showed no significant influence from the presence of the second row for X/D > 2.16. The wake closure position was between Lc/D = 1.23 and 1.32 for X/D > 2.16. At X/D = 2.16, however, the first row wake was elongated, to Lc/D = 1.61, from the favorable pressure gradient generated between second row pin-fins. For X/D = 1.73, the adverse pressure gradient from second
row pin-fins caused the first row wake to be shortened relative to that of $X/D = 2.16$. As with $Re_D = 3.0e3$, the third row wakes at $Re_D = 2.0e4$ were shortened relative to the first row wakes because of disturbances generated in upstream rows. And, the wake closure position was nearly constant with $X/D$ in the third row for $Re_D = 2.0e4$ as $L_c/D$ was between 1.03 and 1.17 for all $X/D$ spacings.

Time-mean velocity was extracted across the wake at the position of $X/D = 1$ and $Z/H = 0$ (because all measurements were made at the channel midline). To compare with a single, infinite cylinder, both the bulk velocity, $U_m$, and the minimum area velocity, $U_{\text{max}}$, were used to normalize the time-mean velocity. Figure 6.5 shows the effect of reference velocity on the time-mean, streamwise velocity for the single row of pin-fins at $Re_D = 3.0e3$. Included in Figure 6.5 is data for a single, infinite cylinder at a similar Reynolds number, $Re = 3.0e3$ (Norberg 1998). Neither $U_m$ nor $U_{\text{max}}$ resulted in a velocity profile that collapsed with the single, infinite cylinder data. Using $U_{\text{max}}$ as the reference velocity, however, gave better agreement with a single, infinite cylinder than using $U_m$ as the reference velocity.

The single pin-fin row having $H/D = 1$, $S/D = 2$ showed a narrowed wake in comparison to the single, infinite cylinder. The wake width is defined as the distance between the peak in streamwise fluctuating velocity, $u'$, in each shear layer (Norberg 1986). For qualitative comparison, however, the distance between shear layers can be estimated by visual inspection of regions of large velocity gradients in Figure 6.5. As flow crossed the single row of pin-fins, a jet formed between adjacent pin-fins that prevented the wake from expanding in the lateral direction which caused a narrowed wake for the pin-fin row in comparison to the single, infinite cylinder. It should be noted that the average of $U/U_m$ across the wake in Figure 6.5 did not equal unity because of three-dimensional effects (lower velocity near the walls). If the flow were averaged across all $Z/H$ locations instead of only at $Z/H = 0$, the average of $U/U_m$ would equal unity to satisfy continuity. Figure 6.6 shows the effects of streamwise spacing on the cross-wake profile of $U/U_{\text{max}}$ at low Reynolds number. In agreement with Figure 6.3, Figure 6.6 showed that decreasing streamwise spacing decreased the magnitude of the jet between pin-fins from the presence of the second row pin-fins. Decreasing $X/D$ caused a
narrower and also caused an increased magnitude of reversed flow along the wake axis (Y/D = 0) at X/D = 1.

At high Reynolds number, Re_D = 2.0e4, the cross-wake velocity of U/U_{max} profile was extracted at X/D = 1. Again, the effect of reference velocity was investigated to determine if there was Reynolds number dependence when using U_m or U_{max}. In Figure 6.7, neither U_m nor U_{max} provided a good match with the single, infinite cylinder data. The use of U_{max} under-predicted the velocity at Y/D = \pm 1 while the use of U_m over-predicted the velocity at Y/D = \pm 1. Comparing the low Reynolds number cases in Figure 6.5 and the high Reynolds number cases in Figure 6.7, it was found that the wake was more diffuse with increasing Reynolds number, as expected. This observation was also shown by comparing the single, infinite cylinder data in Figure 6.7 where increasing Reynolds number from Re = 8.0e3 to 4.0e4 resulted in a more diffuse wake. At low Reynolds number, the jet between pin-fins caused a narrowed wake in comparison to a single, infinite cylinder. At high Reynolds number, however, it was more difficult to determine the distance between shear layers. It will be shown in a following discussion that cross-wake profiles of turbulent normal stress that the single pin-fin row had a narrowed wake compared to the single, infinite cylinder because of the influence of the jet between pin-fins.

To investigate the effects of X/D on the near wake velocity profile at high Reynolds number, Figure 6.8 shows the cross-wake profile of U/U_{max} extracted at X/D = 1. The wake profiles had very similar shape for X/D \geq 2.60. For X/D = 2.16, the flow was actually accelerated to a greater local maximum at the positions Y/D \pm 0.6 when compared with X/D \geq 2.60. For X/D = 1.73, the presence of the second row pin-fins blocked the flow and caused reduced velocity magnitude across the wake at X/D = 1. Similar to low Reynolds number, the wake width decreased for X/D = 1.73 at high Reynolds number in comparison to wider spacings.

The time-mean vorticity shows the effect of turbulence generated in upstream rows on the size of the shear layers. Figure 6.9 shows the time-mean vorticity for Re_D = 3.0e3 and Figure 6.10 shows Re_D = 2.0e4. At low Reynolds number, the shear layer emerging from the first row extended several diameters downstream of the pin-fin and the length of the shear layers followed with the wake closure position. The shear layer
was very stable, indicated by the thin region of large vorticity magnitude, \( \omega_Z D/U_{\text{max}} = \pm 10 \), extending about one diameter downstream. In third row wakes, the shear layer was shorter and more diffuse than the first row because of the incoming disturbances from upstream rows. At high Reynolds number, the first row shear layers were quite diffuse, much more than the first row shear layers at low Reynolds number. As described previously, the stability of the shear layer was dependent on Reynolds number and turbulence level. Increases in both Reynolds number and turbulence level caused an increase in turbulent diffusion in the shear layer. An interesting result was found at the spacings \( X/D \leq 2.16 \) for \( \text{Re}_D = 2.0e4 \), where the shear layer was more stable (less diffuse) than \( X/D \geq 2.60 \), as indicated in the contours of vorticity in Figure 6.10.

To determine the effect of streamwise spacing on the level of velocity fluctuations in the wake, the in-plane components of the Reynolds stress tensor were calculated for each case. Figure 6.11 and Figure 6.12 show the average of the two in-plane turbulent normal stresses, \( \langle u'u' \rangle/U_{\text{max}}^2 \) and \( \langle v'v' \rangle/U_{\text{max}}^2 \), for \( \text{Re}_D = 3.0e3 \) and \( 2.0e4 \), respectively. For all cases, a band of increased turbulent normal stress was observed emerging tangent to the pin-fins as a result of Kelvin-Helmholtz instabilities along the shear layers. In some cases, regions of increased turbulent stress were also observed downstream of the pin-fins, along the wake axis, where Kármán vortices formed.

In Figure 6.11, at \( \text{Re}_D = 3.0e3 \), the first row of \( X/D = 3.46 \) showed similar levels of turbulent normal stress to the single row case. Decreasing the streamwise spacing resulted in decreased turbulent normal stress. In the third row at \( \text{Re}_D = 3.0e3 \), the magnitude of turbulent normal stress increased relative to the first row. Similar to the first row wakes, the magnitude of turbulent normal stress in the third row decreased with decreasing \( X/D \). When \( \text{Re}_D \) was increased to \( 2.0e4 \), the turbulent normal stress in Figure 6.12 indicated that elevated turbulence occurred closer to the pin-fins in comparison with \( \text{Re}_D = 3.0e3 \), in agreement with the wake closure position. The level of turbulent normal stress in the first row wakes showed no significant dependence on streamwise spacing for \( X/D \geq 2.60 \). When \( X/D \) was reduced to 2.16, however, the level of turbulent normal stress in the wake was significantly attenuated in comparison to \( X/D = 2.60 \). For \( X/D \geq 2.60 \), the magnitude of turbulent normal stress in the third row wakes was less than that of the first row wakes. It was found that the reduced fluctuations in the third row wakes at
high Reynolds number was caused by the disruption of vortex shedding, and this will be addressed in Section 6.2. The turbulent normal stress in the third row wake was also less dependent on X/D than the first row wakes. The most significant effect of streamwise spacing was observed in the attenuated turbulent normal stress in the first row of X/D = 2.16 in comparison to X/D = 2.60. A major regime change in the near wake flowfield caused the differences in turbulent normal stress in the first row wake between X/D = 2.16 and 2.60 which will also be discussed in more detail in Section 6.2.

Cross-wake profiles of the turbulent normal stress component $\langle u'u' \rangle / U_{\text{max}}^2$ were extracted at the position of constant X/D = 1 at low Reynolds number in Figure 6.13. The definition of wake width is illustrated in Figure 6.13 which shows that the wake width was reduced for the single row of pin-fins having H/D = 1, S/D = 2 in comparison to a single, infinite cylinder. As previously discussed, the jet between pin-fins caused the narrowed wake compared to a single, infinite cylinder. Also discussed in the analysis of time-mean velocity, the choice of reference velocity had an influence on the interpretation of the pin-fin data. In Figure 6.13, the level of $\langle u'u' \rangle / U_{\text{max}}^2$ was a factor of four lower than $\langle u'u' \rangle / U_{\text{m}}^2$ because $U_{\text{max}}/U_{\text{m}} = 2$ for the current spanwise spacing of S/D = 2. The interpretation of the pin-fin data, therefore, relied on the choice of normalizing velocity and made direct comparison of $\langle u'u' \rangle$ between pin-fins and single, infinite cylinders difficult. From Figure 6.13, normalization using $U_{\text{max}}$ gave better agreement with the single, infinite cylinder data. Regardless of the choice of reference velocity, the important conclusion to draw from Figure 6.13 was that the jet between pin-fins caused the a narrower wake when compared with a single, infinite cylinder.

The effect of streamwise spacing on each (available) component of the Reynolds stress tensor was investigated. Figure 6.14 shows the effect of X/D on $\langle u'u' \rangle / U_{\text{max}}^2$ at the position X/D = 1, Z/H = 0 at low Reynolds number. It was found that the wake width was wider at the position X/D = 1 for the pin-fin configuration having X/D = 3.46 in comparison to the single pin-fin row. This showed that the influence of the second row pin-fins in the X/D = 3.46 case not only elongated the wake, as observed in Figure 6.3, but also widened the wake relative to the single pin-fin row having no downstream
obstruction. The case having X/D = 3.03 also showed a wider wake than the single row case at X/D = 1. Further decreases in X/D, however, resulted in a narrowed wake.

Figure 6.15 shows the effect of X/D on $\langle v'v' \rangle / U_{max}^2$ at the position X/D = 1, Z/H = 0 at low Reynolds number. There was little influence of X/D on $\langle v'v' \rangle / U_{max}^2$ at X/D = 1 with the exception of the X/D = 1.73 case which showed increased $\langle v'v' \rangle / U_{max}^2$ relative to the other geometries. For X/D = 1.73, the flow path was more tortuous than for wider spacings because the flow had less distance to turn through the staggered pin-fins. At X/D = 1, the flow for the X/D = 1.73 had begun to turn towards the wake axis to pass through the second row of pin-fins. The flow from each side of the wake coincided along the wake axis and caused increased fluctuations in both streamwise and transverse directions as shown in both Figure 6.14 and Figure 6.15. Figure 6.16 shows the effect of X/D on $\langle u'u' \rangle / U_{max}^2$ at the position X/D = 1, Z/H = 0 at low Reynolds number. As X/D decreased, the peak in $\langle u'u' \rangle / U_{max}^2$ increased and the distance between the peaks moved closer together. The turbulent shear stress showed that turbulent mixing was increased at the position X/D = 1 for the X/D = 1.73 configuration in comparison to wider spacings.

Cross-wake profiles of the turbulent normal stress component $\langle u'u' \rangle / U_{max}^2$ were extracted at the position of constant X/D = 1 at high Reynolds number. Similar to the low Reynolds number analysis, Figure 6.17 shows that the wake width was reduced for the single row of pin-fins having H/D = 1, S/D = 2 in comparison to a single, infinite cylinder at high Reynolds number. Again, normalizing by $U_{max}$ gave better agreement with single, infinite cylinder data. An interesting anomaly was observed in the decrease of $\langle u'u' \rangle / U_{max}^2$ with increasing freestream turbulence for the single, infinite cylinder case. For low Reynolds numbers, Figure 6.13 showed that increasing freestream turbulence amplified the disturbances along the shear layers. For high Reynolds number, however, Figure 6.17 showed that increasing freestream turbulence attenuated the disturbances along the shear layers. No explanation was given for this anomaly (Zdravkovich 1997; Norberg 1986). Regardless, when comparing the present single pin-fin row to the single, infinite cylinder data, the local maxima of $\langle u'u' \rangle / U_{max}^2$ were reduced but the peaks were broadened. The jet between pin-fins was found to reduce the
wake width for low Reynolds numbers, and the same was true for high Reynolds numbers. For the single pin-fin row, the broader \( \langle u'u' \rangle / U_{\text{max}}^2 \) peak indicated more mixing along the shear layers in comparison to the single, infinite cylinder. As in previous discussions, the presence of wall-generated turbulence caused decreased stability of the shear layer, and a broadened peak of \( \langle u'u' \rangle / U_{\text{max}}^2 \), for the single row of pin-fins compared to a single, infinite cylinder under low freestream turbulence.

Figure 6.18 shows the effect of \( X/D \) on \( \langle u'u' \rangle / U_{\text{max}}^2 \) at the position \( X/D = 1, Z/H = 0 \) at high Reynolds number. The distance between local maxima of \( \langle u'u' \rangle / U_{\text{max}}^2 \) indicated that the \( X/D = 2.16 \) case had the widest wake at the position \( X/D = 1 \). The wake length and wake width showed evidence of coupling because, for low Reynolds number, the \( X/D = 3.46 \) case showed an elongated and widened wake in comparison to the single pin-fin row. Similarly, the present analysis showed that the \( X/D = 2.16 \) case showed an elongated and widened wake in comparison to the single pin-fin row at high Reynolds number. Decreasing \( X/D \) to 1.73 caused the wake to narrow compared with \( X/D = 2.16 \). Figure 6.19 shows the effect of \( X/D \) on \( \langle v'v' \rangle / U_{\text{max}}^2 \) at the position \( X/D = 1, Z/H = 0 \) at high Reynolds number. A significant decrease in \( \langle v'v' \rangle / U_{\text{max}}^2 \) was observed when \( X/D \) was reduced from 2.60 to 2.16. This result showed that the decrease in the turbulent normal stress in Figure 6.12 was driven by decreases in \( \langle v'v' \rangle / U_{\text{max}}^2 \) rather than \( \langle u'u' \rangle / U_{\text{max}}^2 \) when \( X/D \) was reduced from 2.60 to 2.16. Vortex shedding typically shows large excursions of cross-wake flow which contributed to \( \langle v'v' \rangle / U_{\text{max}}^2 \). The level of \( \langle v'v' \rangle / U_{\text{max}}^2 \) was, therefore, dependent on whether vortex shedding was present (increased \( \langle v'v' \rangle / U_{\text{max}}^2 \)) or attenuated (decreased \( \langle v'v' \rangle / U_{\text{max}}^2 \)). Figure 6.20 shows the effect of \( X/D \) on \( \langle u'v' \rangle / U_{\text{max}}^2 \) at the position \( X/D = 1, Z/H = 0 \) at high Reynolds number. The turbulent shear stress was significantly decreased at \( X/D = 1 \) for \( X/D \leq 2.16 \) in comparison with \( X/D \geq 2.60 \). Because vortex shedding is associated with significant cross-wake motion, a large amount of mixing occurs as high momentum fluid (outside the wake) is brought into regions of low momentum (inside the wake). The transverse
(Y-direction) transport of streamwise (X-direction) momentum contributed to increased \( \langle u'v' \rangle/U_{max}^2 \). The decrease in turbulent shear stress when X/D was reduced from 2.60 to 2.16 was, therefore, also dependent on whether vortex shedding was present or attenuated.

The effect of row position was investigated by extracting velocity profiles in the first and third row wakes at X/D = 1, where the local origin was located at the pin-fin axis in each row. At both low and high Reynolds number, the cases having X/D = 3.46, 2.60, and 1.73 were compared to represent a wide range of spacings. At low Reynolds number, Figure 6.21 shows that the time-mean flow in the third row wakes is less dependent on X/D than the first row wakes. The turbulence generated in upstream rows caused the wake of the third row to be less shear-layer-driven and more diffuse than the first row wakes. The turbulent shear stress in Figure 6.22 shows significantly increased mixing in comparison to the first row wake. The turbulent shear stress showed a broader peak in the third row wakes which supported the assertion that the third row was less shear-layer-driven than the first row wake. At high Reynolds number, Figure 6.23 shows less X/D dependence in the third row than the first row wakes, similar to at low Reynolds number. The first row wake having X/D = 1.73 showed the most sensitivity to the presence of downstream pin-fins as observed in the low velocity at Y/D = ±1 and high velocity at Y/D = ±0.5. The turbulent shear stress profiles at high Reynolds numbers were decreased in the third row for X/D = 3.46 and 2.60 relative to the first row. For X/D = 1.73, however, turbulent shear stress increased between first and third rows. For X/D = 3.46 and 2.60, vortex shedding was present in the first row wake. In the third row wake, disturbances from upstream rows caused decreased turbulent shear stress indicating that the vortex shedding motion was disrupted. For X/D = 1.73, however, shedding was not present in the first row wake and resulted in less turbulent shear stress than cases having vortex shedding. In the third row wake, however, disturbances generated in upstream rows were able to overcome the stabilizing influences (jet between pin-fins) in the shear layer and allow vortex shedding to occur for X/D = 1.73.
6.2  Effect of Streamwise Spacing on Instantaneous Flow Phenomena

To determine the nature of the regime change taking place at $Re_D = 2.0e4$ when $X/D$ was reduced from 2.60 to 2.16, the one-dimensional energy spectrum was calculated to determine the frequency content of the velocity fluctuations observed in Figure 6.12. To characterize the frequency content in the pin-fin flowfield, the streamwise velocity time signal was extracted at a point in the shear layer to be consistent with previous single cylinder studies (Dong et al. 2006; Prasad and Williamson 1997). The time signal was Hanning-windowed and a fast-Fourier-transform (FFT) was applied to obtain the one-dimensional energy spectrum. To reduce noise in the resulting energy spectra, several steps were taken. The velocity time signal was broken into 3 separate signals of 1000 data points. The energy spectra were calculated for each 1000-point signal and averaged together. By using three 1000-point signals instead of a single 3000-point signal, the frequency resolution was decreased but noise was significantly reduced at higher frequencies. Second, the time-mean TRDPIV data showed that flow in the pin-fin wake was approximately symmetric about the wake axis, so the energy spectrum was calculated on both sides of the wake axis and averaged together. The resulting energy spectra was calculated from six time signals, two positions on opposing sides of the wake which were each broken into three separate time signals. Appendix E shows the effects of averaging on the calculated energy spectra.

In Figure 6.25, the energy spectrum is shown for the case having $X/D = 2.16$ at $Re_D = 2.0e4$ where the interrogation points were $X/D = 1.5$, $Y/D = 0.4$ and $X/D = 1.5$, $Y/D = -0.4$. The energy spectrum showed a peak at the shedding frequency where $St = 0.21$. In Figure 6.26, the energy spectrum is shown for $X/D = 2.60$ at the interrogation points $X/D = 0.9$, $Y/D = 0.4$ and $X/D = 0.9$, $Y/D = -0.4$. A peak in the energy spectrum was observed at the shedding frequency, $St = 0.18$. The energy content at the shedding frequency, however, was two orders of magnitude lower for $X/D = 2.16$ in comparison to $X/D = 2.60$. From these energy spectra, a significant attenuation of the vortex shedding frequency caused the decreased turbulent normal stress that was observed in the first row wake when $X/D$ was reduced from 2.60 to 2.16.

Instantaneous flowfield snapshots confirmed the assertion that vortex shedding was attenuated when $X/D$ was reduced from 2.60 to 2.16 when $Re_D = 2.0e4$. The
The instantaneous near wake flow is shown for X/D = 2.16 in Figure 6.27 where Re_D = 2.0e4. The time between frames in Figure 6.27 was based on one-eighth of the vortex shedding period observed at the peak in the energy spectrum. The frame-step in Figure 6.27 was equivalent to an inverse Strouhal number of St^{-1} = (0.2)^{-1}(1/8) = 0.625. The instantaneous flowfield showed a large amount of random motion in the wake of the cylinder, but no coherent vortex shedding was observed. The random motion in the wake was caused by the small scale structures forming in the recirculation region of Figure 6.27. In addition, small scale structures that formed along the shear layers were usually advected out of the domain rather than entrained in the recirculating flow. For comparison, Figure 6.28 shows the instantaneous streamlines and vorticity magnitude in the first row wake for both X/D = 2.60 where Re_D = 2.0e4. The frame step in Figure 6.28 was also St^{-1} = 0.625. The near wake of the first row, X/D = 2.60 clearly showed that regular vortex shedding was present. In contrast with X/D = 2.16, the small scale structures generated along the shear layers for the X/D = 2.60 case were entrained by the vortex shedding motion and brought into the recirculating region.

To better characterize the behavior of the near wake flowfield, proper orthogonal decomposition (POD) was applied to the TRDPIV data. For a complete description of POD, the reader is referred to Aubry (1991), Chatterjee (2000), and Holmes et al. (1996). However, a brief description of POD will be given to describe the utility of the technique in the context of the pin-fin analysis. POD is similar to performing an FFT in that both tools may be used to calculate the energy content of certain modes of motion. The difference between POD and FFT, however, lies in the choice of basis functions. While FFT uses sinusoidal basis functions, the POD uses a set of orthogonal basis functions specific to the flowfield in question. The POD basis functions are optimized by sorting the modes in order of most energy content to least energy content, which is convenient for turbulent flow analysis. Simply stated, the first POD mode contains the most energy, the second mode contains the next greatest amount of energy, and so on. Like an FFT, the flow can be reconstructed from any number of the calculated POD modes. Using every single POD mode will exactly reproduce the original velocity field. Using only a few POD modes in the reconstruction will produce the best possible representation of the original velocity field for the number of modes used in the reconstruction (because the
basis functions are chosen based on optimality). The original velocity field is decomposed into a set of spatial basis functions, $\phi_U^n$ and $\phi_V^n$, with a corresponding set of temporal functions, $\lambda_n$.

$$
\tilde{u}(x, y, t) = U(x, y) + \sum_{n=1}^{N} \phi_U^n(x, y) \lambda_n(t)
$$

$$
\tilde{v}(x, y, t) = V(x, y) + \sum_{n=1}^{N} \phi_V^n(x, y) \lambda_n(t)
$$

(6.1)

In equation 6.1, the instantaneous velocity as a function of space and time is separated into the ensemble-averaged velocity (first term on right hand side) and fluctuating velocity (second term on right hand side). The fluctuating velocity is represented by a set of N-POD modes that include a vector spatial function and a scalar temporal function. In this regard the POD expands upon Reynolds-averaging by categorizing the fluctuating component of velocity into characteristic mode shapes that are sorted from most energetic to least energetic (in terms of fluctuating kinetic energy). It should be noted that there is a unique spatial function for each velocity component denoted using superscripts (hence vector spatial function). The temporal function is common to each velocity component in the decomposition (hence scalar temporal function). In the present work, several cases were analyzed with POD for differing number of POD modes (3000 maximum, limited by the number of PIV samples). It was found that the energy distribution converged at 2000 POD modes, which was then used in the analyses.

To calculate the spatial and temporal functions, several matrix operations were required. First, the instantaneous functions were re-written in matrix form using equation 6.2

$$
A = U\Sigma V^T
$$

(6.2)

where $A$ is an Nxm matrix where N is the number of samples and m is the number of data points. In the present work, there were 3000 PIV samples taken but only 2000 samples were used for POD, so $N = 2000$. Because the PIV images were 1024x1024 px, and the
grid spacing was 8x8 px, the vector fields had 128x128 = 16384 data points. The matrix A includes both velocity components measured (arranged in block form) such that m = 2x16384 = 32768. Stated simply, each PIV dataset (as a function of space and time) is represented by the matrix A. The remaining terms in equation 6.2 were the matrix U, an NxN orthogonal matrix, Σ, an Nxm matrix having all zeros except those along the diagonal, and V, an mxm orthogonal matrix. The T-superscript on the V matrix denotes the matrix transpose. It should be noted that the values along the diagonals of Σ, σ_n = Σ_{nn}, are the singular values of A and are arranged in decreasing order where σ_1 ≥ σ_2 ≥ … ≥ σ_N. Equation 6.2 may be solved using singular value decomposition (SVD) to determine U, Σ, and V (MATLAB has a built-in routine “[U, E, V] = svd(A)” to perform this operation). If we define Q = UΣ, then we can re-write equation 6.2 as

\[ A = QV^T = \sum_{n=1}^{N} q_n v_n^T \]  

(6.3)

where q_n is the nth row of Q and v_n is the nth column of V. As mentioned previously, n is the POD mode number. The temporal functions of equation 6.1 are represented by the Q matrix and the spatial functions are represented by the V matrix. Equations 6.2 and 6.3 show the matrix algebra behind the proper orthogonal decomposition defined in equation 6.1. For cases where m >> N (true for the present work and for most PIV data), the computational cost of performing an SVD may be reduced by indirectly solving for U, Σ, and V. As an alternative to directly computing the SVD, equation 6.2 may be solved using eigenvalue/eigenvector relationships, and is commonly referred to as “the method of snapshots” (Chatterjee 2000). In this method, the matrix U is solved for using U = AA^T. Next, the transpose of U is multiplied with A to give U^TA = ΣV^T. The commercial PIV software, DaVis, used in the present work employs the method of snapshots, and the reader is referred to Holmes et al. (1996) for a more complete description of the methods used in the DaVis software (www.Lavision.de, 2012).

To demonstrate the utility of identifying certain flow modes, POD was applied to the first row wake for both the X/D = 2.16 and 2.60 cases at Re_D = 2.0e4. The first four POD modes are shown in Figure 6.29 for X/D = 2.16 and Figure 6.30 for X/D = 2.60.
The POD analysis for the case having X/D = 2.60 is described first because the flowfield organization was less complex than that of X/D = 2.16. Inspection of the POD modes for X/D = 2.60 showed that the modes were sorted from most to least energetic (by definition of the POD). Because the temporal functions in Figure 6.30 were on the same order of magnitude for modes 1 through 4, the amount of energy content was dependent on the area-averaged magnitude of the spatial functions. From inspection of Figure 6.30, the first two modes clearly contained the most energy. It should be noted that the exact calculation of the energy contained in a given mode requires a triple integral, over each spatial dimension and over time. Because the first two modes contained the most energy, it was expected that these modes represented vortex shedding because vortex shedding contained the most energy in the one-dimensional energy spectrum. This may seem incorrect because the spatial functions for the first two POD modes in Figure 6.30 resembled a large vortex along the centerline of the wake. However, one must remember that the POD modes are a decomposition of the fluctuating velocity and the POD modes do not include the energy content of the advection from the mean flowfield (in some literature, the time-mean flowfield is referred to as POD mode zero). The reconstructed velocity field would be a superposition of the POD modes together with the time-mean flowfield. The temporal functions of Figure 6.30 showed that modes 1 and 2 were approximately sinusoidal which indicated that the vortex shedding process was periodic and repeatable. In comparison, the temporal functions for X/D = 2.16 in Figure 6.29 were irregular which indicated that periodic vortex shedding was not present. The spatial functions in for X/D = 2.16 were also quite different from that of X/D = 2.60. For X/D = 2.16, POD modes 1 and 4 had similar structure while POD modes 2 and 3 showed activity along the shear layers. In addition, the magnitude of the spatial functions for X/D = 2.16 were much less than for X/D = 2.60. For example, the first POD mode showed a factor of three increase in the peak magnitude of the spatial function for X/D = 2.60 compared with X/D = 2.16. The POD analysis showed that vortex shedding was dominant and periodic for X/D = 2.60 while shedding was attenuated and non-periodic for X/D = 2.16. The POD analysis was in agreement with the energy spectra analysis which showed a significantly attenuated energy content at the shedding frequency for X/D = 2.16 in comparison to X/D = 2.60.
The temporal functions of the first four POD modes for X/D = 2.16, in Figure 6.29, appeared similar to one another and each function was random. In comparison, the first two temporal functions at X/D = 2.60, in Figure 6.30, were sinusoidal while the third and fourth temporal functions were random. The difference in temporal functions showed that the large scale motions were random for X/D = 2.16 while there was a repeatable sinusoidal motion present in the X/D = 2.60 case, corresponding to vortex shedding.

The flowfield was approximated using the first two POD modes for the X/D = 2.60 case in Figure 6.31. The POD-approximation was shown in Figure 6.31 at the same time-steps as the original velocity field in Figure 6.28. From Figure 6.31, the first two POD modes showed the classic vortex shedding motion for cylinders in crossflow (Cantwell and Coles 1983). In the first four time-steps of Figure 6.31, a Kármán vortex (KV) was observed developing from the top half of the pin-fin. And, in time-steps five through eight, a KV was observed developing from the bottom half of the pin-fin. Each KV detached from the pin-fin as a result of cross-wake flow induced by the formation of the opposing KV. The top KV detached in the fourth frame and the bottom KV detached in the eighth frame. One benefit of this decomposition is that the remaining modes 3 through 2000 may be used to visualize all motions except for the vortex shedding motion. Figure 6.32 shows the random motions in the flow represented by the superposition of modes 3 through 2000 for the X/D = 2.60 case at Re_D = 2.0e4. Shear layer eddies were observed emerging tangent to the pin-fins in Figure 6.32. It was interesting to note that the shear layer trajectory followed the KV development. For example, the fourth frame in Figure 6.32 shows shear layer eddies emerging from the top of the pin-fin and sweeping downward across the wake from the motion of the KV in Figure 6.31. The same phenomena was observed on the bottom half of the pin-fin in the eighth frame of Figure 6.32.

It was found that cases having strong vortex shedding contained a large amount of fluctuating energy in the first two POD modes. To assess the effects of X/D on the organization of the flowfield, the cumulative distribution of energy contained in the POD modes was plotted as a function of mode number. Figure 6.33 shows the cumulative POD energy for the first row wakes at Re_D = 3.0e3. The first 1000 POD modes are
shown in Figure 6.33 because the cumulative POD energy was nearly 100% within the first 1000 POD modes, and the POD modes 1001-2000 contributed very little to the fluctuating energy. From Figure 6.33, the cumulative POD energy distribution had a similar profile for all X/D cases. At high Reynolds number, however, Figure 6.34 showed a significant change in the profile of the cumulative POD energy between X/D = 2.60 and 2.16. This result supports the assertion that vortex shedding was attenuated for high Reynolds number when X/D was reduced from 2.60 to 2.16.

To better quantify the effect of Reynolds number on the first row wake, the first four POD modes are shown in Figure 6.35 for the configuration having X/D = 2.60 at low Reynolds number. Similar to the high Reynolds number flow in Figure 6.30, the first two POD modes showed a large vortex along the wake axis. The temporal functions in Figure 6.35, however, were quasi-periodic and had a largely varying magnitude over time. For X/D = 2.60 at high Reynolds number, the temporal functions of POD modes 1 and 2 were sinusoidal and periodic. This analysis showed that vortex shedding was not regular for X/D = 2.60 at low Reynolds number. Another difference between low and high Reynolds number was the contribution from POD modes 3 and 4. At low Reynolds number, in Figure 6.35, POD modes 3 and 4 showed three to four well-defined structures along the shear layers. At high Reynolds number, in Figure 6.30, there were only one or two structures along the shear layers in POD modes 3 and 4. And, the structures along the shear layers were larger for the high Reynolds number case. POD modes 3 and 4 were used to approximate the flowfield in Figure 6.36 for both low and high Reynolds number. In Figure 6.36, only POD modes 3 and 4 were used in the reconstruction and the time-mean flowfield was not included to better illustrate the eddy motion. Instantaneous contours of vorticity are shown in Figure 6.36 in increments of St⁻¹ = 0.117 for low Reynolds number and St⁻¹ = 0.157. A different time-step, St⁻¹, was chosen to illustrate the shear layer motions when compared with St⁻¹ = 0.625 used to illustrate vortex shedding motions because the shear layer motions occurred at a higher frequency than the vortex shedding motions (Dong et al. 2006; Prasad and Williamson 1997). Figure 6.36 shows a significant difference between POD modes 3 and 4 for low and high Reynolds number. At low Reynolds number, POD modes 3 and 4 corresponded to shear layer eddies. At high Reynolds number, however, POD modes 3 and 4 did not resemble those
shear layer eddies that were observed in Figure 6.32. Instead, the motions from POD modes 3 and 4 corresponded to a structure having length scale between the Kármán and shear layer vortices.

In the third row, the cumulative POD energy distribution was very similar when comparing $Re_D = 3.0e3$ and $Re_D = 2.0e4$. Figure 6.37 shows the third row energy distribution at $Re_D = 3.0e3$ and Figure 6.38 shows the third row energy distribution at $Re_D = 2.0e4$. In addition to the effects of Reynolds number and streamwise spacing discussed for the first row wakes, the POD energy distributions of Figure 6.37 and Figure 6.38 showed that row position also had a significant influence on the flowfield organization. In the third row wakes, for both low and high Reynolds number, there was more energy contained in the first two modes when $X/D \geq 2.60$ in comparison to $X/D \leq 2.16$. Yet, the energy content in the first two modes was less than that of the first row, high Reynolds number case. To determine what mechanism caused the row-dependence on the POD energy distribution, Figure 6.39 shows the first four POD modes for the third row wake of the configuration having $X/D = 2.60$ where $Re_D = 3.0e3$. Figure 6.40 shows the first four POD modes for the third row wake of the configuration having $X/D = 2.60$ where $Re_D = 2.0e4$. For both low and high Reynolds numbers, the first two POD modes corresponded to vortex shedding and the third and fourth modes corresponded to motion in the shear layers. Inspection of the temporal functions yielded an important result because the first two modes showed a quasi-periodic motion for both low and high Reynolds numbers. The quasi-periodic motion was similar to that observed for the first row wake of the $X/D = 2.60$ case at low Reynolds number. And, the quasi-periodic temporal functions indicated that vortex shedding was occasionally disrupted by the upstream wakes.

Decreasing streamwise spacing to $X/D = 2.16$ in the third row showed evidence that vortex shedding was attenuated at both low and high Reynolds number by inspection of the cumulative POD energy distributions in Figure 6.37 and Figure 6.38. The first two POD modes, however, showed very similar spatial and temporal functions to that of the $X/D = 2.60$ case. At $X/D \leq 2.16$ in the third row, vortex shedding was disrupted by upstream wakes similar to $X/D \geq 2.60$. The different trend observed in the cumulative POD energy distributions in Figure 6.37 and Figure 6.38 was caused by the vortex
shedding modes contributing less to the total energy content of the flowfield for X/D ≤ 2.16 compared to X/D ≥ 2.60. To classify the effects of Reynolds number, X/D, and row position on the flowfield organization, the temporal functions from the first few POD modes were inspected for each measured flowfield. The shedding motion was classified as attenuated (for flows resembling Figure 6.29), periodic (for flows resembling Figure 6.30), and quasi-periodic (for flows resembling Figure 6.35 and Figure 6.39). The results are summarized in Table 6.1. It was found that all third row wakes had quasi-periodic shedding, independent of the streamwise spacing and Reynolds number. In the first row, shedding was always attenuated for X/D ≤ 2.16, independent of Reynolds number. For X/D ≥ 2.60, however, the first row wake was quasi-periodic for Re_D = 3.0e3 and periodic for Re_D = 2.0e4.

The POD analysis in this section has provided a means of interpreting the organization of the near wake flow and a means of separating the periodic vortex shedding motion from turbulence. The detailed knowledge of the flowfield organization allowed for meaningful interpretation of local heat transfer measurements.

6.3 Effect of Streamwise Spacing on Pin-Fin Heat Transfer

The heat transfer coefficients were measured on the pin-fin surface for the first, third, and fifth rows of the X/D = 2.16 and 2.60 cases. As discussed in Chapter 3, pin-fin surface heat transfer measurements were made on an insulated pin-fin with a constant heat flux surface. Thermocouple measurements were made at the heater surface to determine the local pin-fin heat transfer coefficients along the pin-fin axis and along the circumference. Because buoyancy effects were present at low Reynolds numbers, pin-surface heat transfer was only measured at Re_D = 2.0e4 where buoyancy effect were negligible. Figure 6.41 shows contours of the local Nusselt number on the pin-surface for X/D = 2.16 (left side) and 2.60 (right side). The Nusselt number contours showed similar distribution when comparing X/D = 2.16 and 2.60. For both streamwise spacings, there was enhanced heat transfer at Z/H = ±0.5 and θ = 0° from the presence of the horseshoe vortex. Heat transfer decreased as flow progressed from the stagnation region, θ = 0°, to the trailing side of the pin-fin from the growth of the thermal boundary layer along the
pin-fin surface. The heat transfer minimum occurred near $\theta = 120^\circ$ in each case and heat transfer recovered beyond $\theta = 120^\circ$.

The area-averaged Nusselt number is shown in Figure 6.42. The pin-fins having streamwise spacing of $X/D = 2.16$ and 2.60 were within the experimental uncertainty (10.0%) for each row considered. Consistent with previous studies, the heat transfer on the pin-fin surface increased between the first and third row but showed no significant change between the third and fifth rows (Metzger and Haley 1982; Lawson et al. 2011; Ames et al. 2005). The results of Figure 6.42 showed agreement, to within the experimental uncertainty, for the present cases having $X/D = 2.16$ and 2.60. The present results were also in agreement with data taken by Lawson et al. (2011) who measured pin-fin heat transfer for an array having $H/D = 1$, $S/D = 2$, $X/D = 1.73$ at $Re_D = 2.5e4$. This result showed that pin-fin heat transfer was similar for configurations having $H/D = 1$, $S/D = 2$, and $X/D = 1.73, 2.16$, and 2.60 at high Reynolds number. From Figure 6.11, the case having $X/D = 1.73$ showed that the level of velocity fluctuations increased to the third row where the flow was sufficiently developed. Downstream of the third row, there was no additional increase in velocity fluctuations. Similarly, pin-fin heat transfer reached a plateau beyond the third row. It will be discussed in the next section that turbulence contributes more to heat transfer than periodic motion from vortex shedding. In the context of the present pin-fin surface analysis, pin-fin heat transfer reached a plateau because the level of turbulence became fully-developed by the third row for high Reynolds numbers.

Figure 6.43 shows the circumferential heat transfer distribution extracted at the pin-fin midline, $Z/H = 0$. In the third and fifth rows, stagnation point heat transfer was increased relative to the first row for both $X/D = 2.16$ and 2.60. Again, heat transfer decreased as flow progressed across the cylinder towards the trailing edge at $\theta = 180^\circ$. For the Reynolds number considered, $Re_D = 2.0e4$, separation occurs between $\theta = 90^\circ$ and $100^\circ$ for a single, infinite cylinder (Zdravkovich 1997). From the circumferential heat transfer distributions of Figure 6.43, a local heat transfer minimum occurred at $\theta = 90^\circ$ for the first row of $X/D = 2.60$ but occurred at $\theta = 120^\circ$ for the first row of $X/D = 2.16$. There was clearly a localized difference in heat transfer around the circumference of the pin-fin caused by streamwise spacing.
In the POD analysis, a significant change in the first row flowfield was observed when X/D was reduced from 2.60 (periodic shedding) to 2.16 (attenuated shedding) at Re_D = 2.0e4. To determine the relative influence of periodic and random unsteadiness, the POD analysis was used to separate the large scale, periodic motions from turbulence. The entire 3000 sample TRDPIV dataset was reconstructed, first, using the POD modes that represented vortex shedding and, second, using all remaining POD modes. After the flow was reconstructed, the in-plane, turbulent normal stress was calculated for the vortex-shedding-POD-approximation and for the residual-POD-approximation. Separating the large scale motion from the turbulence using POD was slightly different than an unsteady, Reynolds-averaged turbulence model. The POD analysis includes the true contribution of momentum transport from turbulence that is imposed on the large scale motions. An unsteady Reynolds-averaged turbulence model, however, relies on modeling the turbulent transport that affects the formation of the large scale motion. To compare X/D = 2.16 and 2.60, Figure 6.44 shows the total amount of in-plane, turbulent normal stress decomposed into the vortex-shedding-component and the residual component. It was clear that the periodic unsteadiness was much greater for X/D = 2.60 in comparison to X/D = 2.16 from the presence of periodic vortex shedding. The level of turbulence was more similar than the periodic unsteadiness when comparing X/D = 2.16 and X/D = 2.60. There was a significant difference however, in the amount of turbulence near the trailing edge of the pin-fins. It was found that the level of turbulence near the trailing side of the pin-fin increased for the X/D = 2.60 case in comparison to the 2.16 case. In effect, the presence of periodic shedding brought turbulent eddies closer to the surface of the pin-fin for X/D = 2.60 compared to X/D = 2.16 which resulted in an increase in heat transfer on the trailing side of the pin-fin for X/D = 2.60.

6.4 Effect of Streamwise Spacing on Endwall Heat Transfer

Endwall heat transfer was measured in the present work for X/D = 2.16, 2.60, and 3.03 at Re_D = 3.0e3 and 2.0e4. The endwall heat transfer was measured using IR thermography as described in Chapter 3. The resulting spatially-resolved heat transfer contours are shown in Figure 6.45 for Re_D = 3.0e3. In the first row, the horseshoe vortex was observed wrapping around the pin-fin and extending several diameters downstream.
The heat transfer from the horseshoe vortex was the most significant feature in the first row. In all downstream rows, however, the horseshoe vortex was not as distinguished as in the first row. As flow crossed through the second row pin-fins, the heat transfer levels increased for each X/D spacing. For X/D = 2.16, heat transfer continued to increase through the fourth or fifth row before reaching a plateau. For X/D = 2.60 and 3.03, however, heat transfer reached a maximum around the third row.

Row-averaged endwall heat transfer is shown in Figure 6.46 and heat transfer augmentation is shown in Figure 6.47 for low Reynolds number flow. Recall that the effects of surface area are decoupled when considering endwall heat transfer separate from pin-fin heat transfer. In a similar heat transfer study, Lawson et al. (2011) showed the effect of X/D on the row-averaged heat transfer development for pin-fin arrays having H/D = 1, S/D = 2, and X/D of 1.73 and 3.46. It should be noted that the data of Lawson et al. (2011) in Figure 6.46 was taken at a Re_D = 5.0e3 and, therefore, showed higher heat transfer and lower heat transfer augmentation than the present data which was taken at a Reynolds number of Re_D = 3.0e3. For the present work, Figure 6.46 and Figure 6.47 showed that the heat transfer levels in the first three rows were within the uncertainty for each X/D case except for the second row heat transfer of X/D = 3.03, which was higher than that of X/D = 2.16 and 2.60. This showed that neither the horseshoe vortex nor the wake flow had a significant influence on heat transfer in the first three rows for low Reynolds number flow.

In the fourth through the seventh rows, decreasing X/D resulted in increased heat transfer. Although heat transfer was within the uncertainty when comparing X/D = 2.16 and 2.60, there was a significant difference between X/D = 2.16 and 3.03. It was found that the level of velocity fluctuations (both periodic and random), shown in Figure 6.11, decreased with decreasing X/D for low Reynolds number flow. The level of velocity fluctuations, therefore, did not correlate with increased heat transfer observed at closer X/D spacings. It was known, however, that vortex shedding motions contributed to a significant portion of the fluctuating energy. The vortex shedding motion was separated from the total fluctuating velocity using POD.

Figure 6.48 shows the effect of X/D on the level of turbulence (periodic motions removed) in the third row wake at low Reynolds number. To more easily interpret the
distribution of turbulence, data was extracted at constant \( Y/D = 0 \), as shown in Figure 6.49. Figure 6.49 uses an alternate coordinate system, \( X' \), which represents the distance from the leading edge of the fourth row pin-fin. From Figure 6.49, it was clear that streamwise spacing had an influence on the level of turbulence prior to encountering the leading edge of the fourth pin-fin row. Stated simply, the turbulence generated in the wake experienced a certain amount of decay before crossing the next row of pin-fins. Decreasing streamwise spacing reduced the amount of decay which correlated with increased endwall heat transfer. In fact, for \( X/D = 1.73 \), Figure 6.49 showed that turbulence was still being generated prior to the flow crossing the leading edge of the fourth row of pin-fins. By process of elimination, the present section has shown that the horseshoe vortex, wake shedding, and periodic unsteadiness did not correlate with endwall heat transfer. Turbulence did correlate with endwall heat transfer and proved to be the important flowfield feature contributing to heat transfer.

Local endwall heat transfer contours are shown in Figure 6.50 for high Reynolds number. As with low Reynolds number, the horseshoe vortex was observed in the heat transfer contours of the first row. In agreement with Chapter 4, inspection of the heat transfer contours showed that the size of the horseshoe vortex decreased with increasing Reynolds number. When Reynolds number was increased to \( \text{Re}_D = 2.0 \times 10^4 \), it was shown that the first row wake closure position was slightly elongated for \( X/D = 2.16 \) in comparison to \( X/D = 2.60 \). In addition, vortex shedding was the dominant motion for \( X/D \geq 2.60 \) in the first row wake but was attenuated for \( X/D \leq 2.16 \). The near wake flow organization was reflected in the local heat transfer contours in Figure 6.50. Specifically, a close-up view of the first row wake in Figure 6.51 illustrates the difference in heat transfer in the first row wake when comparing \( X/D = 2.16 \) and \( X/D = 2.60 \). In agreement with the analysis of Chapter 4, the position of maximum heat transfer followed the wake closure position. For \( X/D = 2.16 \), the wake closure position was elongated relative to \( X/D = 2.60 \) and the position of maximum heat transfer for \( X/D = 2.16 \) was displaced slightly downstream of the position of maximum heat transfer observed for \( X/D = 2.60 \).

The row-averaged endwall heat transfer is shown in Figure 6.52 and endwall augmentation is shown in Figure 6.53 for high Reynolds number. The row-by-row heat transfer development showed very similar trends to the low Reynolds number cases. The
first two rows showed no significant difference in heat transfer when X/D was varied between 2.16 and 3.03. This indicated that the horseshoe vortex and the near wake flow in the first two rows was unaffected by changes in X/D. In the third through seventh rows, decreasing X/D resulted in increased heat transfer. From the heat transfer contours in Figure 6.50, it was deduced that the horseshoe vortex was unaffected by X/D in the developed portion of the array, as was observed in the developing part of the array. Inspection of the fluctuating velocity (both periodic and random) contours of Figure 6.12 showed that the level of fluctuating velocity did not correlate with heat transfer. As with low Reynolds number, therefore, the horseshoe vortex, wake shedding, and periodic unsteadiness did not correlate with endwall heat transfer. The effect of X/D on turbulent motions (periodic motions removed) is shown in Figure 6.54 for the third row wake of Re_D = 2.0e4. For ease of interpretation, the random contribution of the fluctuating velocity was extracted at constant Y/D = 0 and included in Figure 6.49. As with low Reynolds numbers, turbulence correlated with endwall heat transfer and reduced streamwise spacing allowed less distance for decay and resulted in increased heat transfer.

From the results of the present section and Section 6.3, the endwall heat transfer had a greater response to the level of turbulent motions than the pin-fins. For example, the fifth row pin-fin showed 51% higher heat transfer than the first row pin-fin for the array having H/D = 1, S/D = 2, X/D = 2.16 at high Reynolds number. The endwall heat transfer, however, showed a 72% increase in the fifth row relative to the first row and a 310% increase relative to an unobstructed channel. To make a fair comparison when quantifying the response to turbulence, the endwall heat transfer in the fifth row was compared to the heat transfer in an unobstructed duct because the first row endwall heat transfer was influenced by the flowfield features (horseshoe vortex) induced by the presence of the pin-fins. For the first row pin-fin heat transfer, it has been shown that the average midline heat transfer was approximately equal to the area-averaged pin-fin heat transfer across the entire surface. So, the flowfield features induced by the presence of the endwalls did not cause a significant bias to the pin-fin heat transfer in the first row. As such, it was fair to state that the presence of turbulent motions caused a 51% increase for pin-fins and a 310% increase for the endwalls. This result was crucial to the goal of
the present work. Turbulence caused a greater augmentation of heat transfer on the endwalls than the pin-fins. Stated simply, the flowfield feature that provided the greatest heat transfer enhancement was the interaction of the turbulence with the thermal boundary layer on the endwalls.

6.5 Heat Transfer for a Non-Uniformly Spaced Pin-Fin Array

A pin-fin array was designed having variable streamwise spacing such that the first rows were more widely spaced than downstream rows. The distance between pin-fin rows, in order, was X/D = 3.46, 3.03, 2.60, 2.16, 2.16, and 1.73. A schematic of the non-uniform array is shown in Figure 6.55 to illustrate the spacing. The aspect ratio was held constant at H/D = 1, and the spanwise spacing was held constant at S/D = 2. This pin-fin configuration is referred to as the “non-uniform” configuration in the following discussion. The objective of investigating the non-uniform array was to determine if a standard, uniformly spaced pin-fin array could be improved upon through variable streamwise spacing. In previous analyses, the heat transfer in the first two rows was independent of X/D. In downstream rows, heat transfer responded to closer streamwise spacing. The non-uniform array combined widely spaced rows in the initial portion of the array and closely spaced rows in the developed portion of the array. If the trends and observations in previous analyses applied to the non-uniform array, then the array-averaged heat transfer observed for the uniform cases having X/D = 2.16 could be achieved while extending the length of the array by 16%.

Figure 6.56 compares the spatially-resolved, endwall heat transfer coefficients for four cases: the X/D = 2.16, 2.60, 3.03 and the non-uniform array at Re_D = 3.0e3. The horseshoe vortex extended about two pin-fin diameters downstream for the non-uniform array, similar to the widest uniform array which had X/D = 3.03. Increasing the distance between the first and second rows from 3.03 diameters (widest uniform array) to 3.46 diameters (non-uniform array) had no significant impact on the downstream extent of the horseshoe vortex. By inspection of Figure 6.56, heat transfer was reduced in the third through seventh rows for the non-uniform array in comparison to the X/D = 2.16 array. Figure 6.57 shows the row-averaged, endwall augmentation development for the uniform and non-uniform arrays at Re_D = 3.0e3. In agreement with the uniform arrays, the non-
uniform array showed no significant difference in heat transfer for the first two rows. In the third through seventh rows, however, the non-uniform array showed lower heat transfer than that of the $X/D = 2.16$ array. At low Reynolds numbers, therefore, the trends and observations in the previous analyses did not apply to the developed portion of the non-uniform array. To better understand the differences between the uniform and non-uniform arrays, the high Reynolds number case will be discussed first followed by a summary of the differences between uniform and non-uniform arrays.

Figure 6.58 compares the spatially-resolved, endwall heat transfer coefficients for the uniform and non-uniform arrays at $Re_D = 2.0e4$. From inspection of Figure 6.58, the heat transfer in the non-uniform array at high Reynolds number was more similar to the uniform $X/D = 2.16$ array than for low Reynolds number flow. The row-averaged, endwall augmentation is shown in Figure 6.59 for the high Reynolds number case. It was observed, and expected, that heat transfer would be similar for the non-uniform and uniform cases in the first two rows. In the third row and beyond, however, the non-uniformly spaced array was within the uncertainty of the array having $X/D = 2.16$. High Reynolds number flows were, therefore, not adversely affected by having wide spacing in the first few rows when the developed portion of the array had close streamwise spacing.

Further investigation is required to determine the exact cause of the Reynolds number dependence observed in the non-uniform arrays. The results of the present work, however, suggest that low Reynolds number flows relied on close $X/D$ spacing in the first few rows to achieve the maximum level of turbulence. For example, the uniform array having $X/D = 2.16$ likely outperformed the non-uniform array because the close spacing was required in the first few rows to transition the flow to fully-turbulent. For high Reynolds numbers, the flow was more dependent on the row position than on the spacing to transition to fully-turbulent. For example, high Reynolds number flows may require only one or two rows (with arbitrary $X/D$) to become fully-turbulent. At high Reynolds numbers, the non-uniform array had the same heat transfer (to within the uncertainty) as the uniform array having $X/D = 2.16$ and extended the coverage by 16% in the streamwise direction.
6.6 Summary of Streamwise Spacing Effects

The results of this chapter have shown that the flowfield and heat transfer are strongly dependent on streamwise spacing. Heat transfer measurements showed that the first two rows were independent of X/D, but heat transfer in the developed portion of the array increased with decreasing X/D. A process of elimination approach was taken to deduce which flowfield feature(s) contributed to increasing heat transfer with decreasing X/D. Recall from previous discussions that surface area effects have been decoupled from the flowfield effects by measuring the endwall heat transfer separate from the pin-fin heat transfer. Surface area considerations become important when considering array-averaged, combined (pin/endwall) heat transfer. Surface area therefore, did not contribute to increasing heat transfer with decreasing X/D.

As with all turbulent internal flows, increasing Reynolds number was found to increase heat transfer through increased turbulent mixing. For uniformly spaced arrays, the present chapter showed that the effects of streamwise spacing were similar for both low and high Reynolds numbers. For the uniform arrays having $2.16 \leq X/D \leq 3.03$, heat transfer in the first two rows were independent of X/D for both low and high Reynolds numbers. In the developed portion of the array, heat transfer increased for decreasing X/D for both low and high Reynolds numbers. The percent increase in heat transfer for rows 3-7 when reducing X/D from 3.03 to 2.16 was 17% at low Reynolds number and 19% at high Reynolds numbers. For non-uniform arrays, however, there was a significant difference for low and high Reynolds numbers. At low Reynolds number, the non-uniform array had lower heat transfer than the uniform array having X/D = 2.16. At high Reynolds number, the non-uniform array had similar heat transfer to the uniform array having X/D = 2.16. Further experimentation is required, to determine the exact cause of the Reynolds number dependence in non-uniform arrays. For the purposes of the present work in identifying the flowfield feature(s) that contributes to heat transfer, Reynolds number effects were simply identified as significant when considering non-uniformly spaced arrays.

The horseshoe vortex was not a major contributor to increased heat transfer for closer X/D spacings. Although the horseshoe vortex contributed to significant heat transfer in the first row, there was no observed dependence of the horseshoe vortex on
X/D. The horseshoe vortex was much smaller in the developed portion of the array compared with the first row.

The presence or attenuation of periodic vortex shedding did not correlate with heat transfer. For example, there was no difference in row-averaged endwall heat transfer for X/D = 2.60 (which showed coherent, periodic shedding) and X/D = 2.16 (which showed attenuated shedding).

A correlation was found, in the developed portion of the array, when the turbulent fluctuations were separated from the vortex shedding. Using proper orthogonal decomposition (POD), the level of turbulence was found to decay with increased streamwise spacing by the next row of pins. Decreasing streamwise spacing allowed less distance for turbulent decay which resulted in increased heat transfer. An interesting observation was that the turbulent motions had stronger interaction (caused higher heat transfer augmentation) on the endwall surfaces than on the pin-fins.

It should be noted that the present section makes the assertion that turbulence at the channel midline influence the heat transfer at the endwalls. Three-dimensional effects also influenced heat transfer. Interactions between the horseshoe vortex and the wake, for example, contribute to enhanced heat transfer. The present flowfield measurements at the channel midline were unable to resolve such three-dimensional effects. The main result of the present chapter was the correlation between increased turbulent motions observed at the channel midline and increased endwall heat transfer for decreasing streamwise spacing.
TABLE 6.1. EFFECT OF X/D ON VORTEX SHEDDING BEHAVIOR IN THE WAKE OF FIRST AND THIRD ROWS

<table>
<thead>
<tr>
<th>X/D</th>
<th>$\text{Re}_D = 3.0e3$</th>
<th>$\text{Re}_D = 2.0e4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Row 1</td>
<td>Row 3</td>
</tr>
<tr>
<td>1.73</td>
<td>Attenuated</td>
<td>Quasi-periodic</td>
</tr>
<tr>
<td>2.16</td>
<td>Attenuated</td>
<td>Quasi-periodic</td>
</tr>
<tr>
<td>2.60</td>
<td>Quasi-periodic</td>
<td>Quasi-periodic</td>
</tr>
<tr>
<td>3.03</td>
<td>Quasi-periodic</td>
<td>Quasi-periodic</td>
</tr>
<tr>
<td>3.46</td>
<td>Quasi-periodic</td>
<td>Quasi-periodic</td>
</tr>
</tbody>
</table>
Figure 6.1. Wake closure position, $L_c/D$, as a function of $X/D$ for the first and third row wakes at $Re_D = 3.0e3$.

Figure 6.2. Wake closure position, $L_c/D$, as a function of $X/D$ for the first and third row wakes at $Re_D = 2.0e4$. 
Figure 6.3. Effect of $X/D$ on time-mean flow for $Re_D = 3.0e3$. Wake closure position shown with dashed lines where applicable.
Figure 6.4. Effect of X/D on time-mean flow for Re_D = 2.0e4. Wake closure position shown with dashed lines where applicable.
Figure 6.5. Effect of reference velocity on cross-wake profile of streamwise velocity for low Reynolds number flow at constant X/D = 1, Z/H = 0.
Figure 6.6. Effect of X/D on cross-wake profile of U/U\textsubscript{max} at X/D = 1, Z/H = 0 for pin-fins having H/D = 1, S/D = 2, Re\textsubscript{D} = 3.0e3.
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Figure 6.15. Effect of $X/D$ on $<v'v'>/U_{\text{max}}^2$ in the first row wake at $X/D = 1$, $Z/H = 0$ for $Re_D = 3.0e3$.

Figure 6.16. Effect of $X/D$ on $<u'v'>/U_{\text{max}}^2$ in the first row wake at $X/D = 1$, $Z/H = 0$ for $Re_D = 3.0e3$. 
Figure 6.17. Effect of reference velocity on cross-wake profiles of \(<u'u'>\) for high Reynolds number flow at constant \(X/D = 1, Z/H = 0\).
Figure 6.18. Effect of X/D on $\langle u'u' \rangle / U_{\max}^2$ in the first row wake at X/D = 1, Z/H = 0 for Re_D = 2.0e4.
Figure 6.19. Effect of X/D on $<v'v'>/U_{\text{max}}^2$ in the first row wake at X/D = 1, Z/H = 0 for Re_D = 2.0e4.

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Figure 6.34. Effect of X/D on the cumulative energy content in the first 1000 POD modes for the first row wake at Re_D = 2.0e4.
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Figure 6.40. First four POD modes for the third row wake of X/D = 2.60 at Re_D = 2.0e3.
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H/D = 1, S/D = 2, X/D = 2.16 and 2.60 at Re_D = 2.0e4. The periodic 
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**Figure 6.52.** Effect of X/D on row-averaged endwall heat transfer for Re_D = 2.0e4.

**Figure 6.53.** Effect of X/D on row-averaged endwall heat transfer augmentation for Re_D = 2.0e4.
Figure 6.54. Effect of X/D on turbulence in the third row wake (POD modes 3-2000) for H/D = 1, S/D = 2, and Re_D = 2.0e4.

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Figure 6.56. Endwall heat transfer for uniform and non-uniform arrays at $Re_D = 3.0e3$. 
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\[
\frac{Nu_{Dh,e}}{Nu_o}
\]
Figure 6.58. Endwall heat transfer for uniform and non-uniform arrays at $Re_D = 2.0e4$. 
Figure 6.59. Row-averaged endwall heat transfer for uniform and non-uniform arrays at $Re_D = 2.0\times10^4$. 

\[
\frac{Nu_{Dh,e}}{Nu_o}
\]
CHAPTER 7: EFFECTS OF SPANWISE SPACING

Spanwise spacing, S/D, determines the open area fraction throughout the array and, therefore, plays an important role in array heat transfer. As discussed in Chapter 3, the characteristic velocity scales are dependent on spanwise spacing. Decreasing spanwise spacing, for example, leads to increased acceleration of the bulk flow through the minimum flow area. As discussed in Chapter 3, the literature for single, infinite cylinders with spanwise blockage shows that integral parameters, such as drag coefficient, agree better with a single, infinite cylinder when using $U_{\text{max}}$ as the characteristic velocity (Zdravkovich 2003). Also, previous studies have shown that pin-fin heat transfer scales with $Re_D$, which is based on $U_{\text{max}}$ (Metzger et al. 1986). Flowfield and heat transfer measurements were, therefore, carried out by maintaining a constant $Re_D$ (and $U_{\text{max}}$) when varying spanwise spacing in the present chapter.

In Chapter 2, the effects of surface area were discussed and it was mentioned that flowfield effects may be decoupled from surface area effects using area-averaged heat transfer coefficients. When calculating array-averaged, combined heat transfer, the total convective area is accounted for. When calculating endwall heat transfer separate from pin-fin heat transfer, the ratio of pin-fin-to-overall convective area is accounted for. The present chapter only discusses endwall and pin-fin heat transfer data such that surface area effects are considered independently. Surface area effects are discussed in Chapter 8 where array-averaged, combined heat transfer data is presented.

In addition to affecting the characteristic velocity scales, the spanwise spacing also determines the heat transfer coverage from the pin-fin wakes. For example, the pin-fin wakes are spaced far apart for wide spanwise spacings and array heat transfer approaches that of an empty duct. Closer spanwise spacings brings the pin-fin wakes closer together and increases the heat transfer coverage from the wakes. Although decreasing the S/D ratio typically results in increased heat transfer, array pressure drop also increases. In practical applications both heat transfer and pressure drop would be considered, and will be discussed in Chapter 8. The objective of this chapter was to investigate the effects of spanwise spacing on the pin-fin flowfield and heat transfer.
In Chapter 6, it was found that the downstream obstruction played an important role in flowfield organization and heat transfer. To account for the effects of downstream obstruction on the near wake, the present chapter investigates the effects of spanwise spacing for various streamwise spacings. First, an unconstrained near wake was considered by investigating a single row of pin-fins having S/D = 1.5, 2, 2.5, and 3. Second, downstream obstruction was then introduced by considering a seven-row array having streamwise spacing X/D = 3.03 where the spanwise spacing was varied between S/D = 2, 2.5, and 3. Finally, the streamwise spacing was decreased further to X/D = 2.16 and the spanwise spacings considered were S/D = 2, 2.5, and 3. For the single pin-fin row, a Reynolds number of $Re_D = 2.0e4$ was considered. For the multi-row cases, two Reynolds number conditions were considered, $Re_D = 3.0e3$ and $2.0e4$. In all cases, the pin-fin aspect ratio was H/D = 1. Table 7.1 summarizes the flow velocities and Reynolds numbers for each spanwise spacing considered in the chapter.

7.1 Effect of Spanwise Spacing on Flowfield in a Single Row of Pin-Fins

TRDPIV measurements were made in the near wake of a single row of pin-fins having various S/D ratios at $Re_D = 2.0e4$. Figure 7.1 shows the effect of spanwise spacing on the time-mean flow in the near wake. From Figure 7.1, it was found that the near wake had similar structure for S/D $\geq 2$. For S/D = 1.5, however, the near wake was asymmetric and the wake was elongated relative to S/D $\geq 2$. Because the wake was asymmetric, TRDPIV measurements were taken in the near wake of two adjacent pin-fins such that Figure 7.1 includes two TRDPIV experiments where the domains are overlapping one another. Asymmetry in the wake was observed by Bradshaw (1965) for a single row of long tubes having S/D < 2. In agreement with the present work, Bradshaw (1965) showed that wakes were biased to form alternating, wide and narrow wakes for spanwise spacings of S/D < 2. The wake asymmetry for S/D = 1.5 is clearly shown in Figure 7.2 by the time-mean velocity profile extracted across the wake for constant X/D = 2, Z/H = 0.

For S/D $\geq 2$, the length of the wake is indicated in Figure 7.1 using dashed lines at the position of zero $U/U_{max}$ along the wake centerline. There was no clear trend of wake closure position with spanwise spacing. The wake closure position decreased from Lc/D
= 1.27 to 1.08 for S/D = 2 and 2.5, respectively. When spanwise spacing was increased from S/D = 2.5 to 3, however, Lc/D increased from 1.08 to 1.15, respectively. For each case having S/D ≥ 2, the wake length was less than that of a single, infinite cylinder at high Reynolds number which has Lc/D = 1.5 (Norberg 2003). Inspection of the isocontour having U/U_{max} = 0.4 in Figure 7.1 showed that increasing S/D decreased the streamwise distance for the wake to mix out and return to a uniform cross-wake velocity. Alternatively, Figure 7.1 was re-plotted by normalizing velocity by the bulk velocity, U_m, in Figure 7.3. Contours of U/U_m showed that increasing spanwise spacing reduced the amount of distance for the flow to return to a value of U/U_m = 1. This analysis warrants further discussion, following that of Chapter 6, regarding the appropriate velocity scale for normalizing flowfield data. Normalizing the flowfield with U_{max} in Figure 7.1 highlighted the differences in the measured velocity when varying S/D because U_{max} was held constant for each case. In Figure 7.3, interpreting the measured velocity required knowledge of the bulk velocity for each spanwise spacing. Normalizing by U_m in Figure 7.3, however, had the advantage of distinguishing the influence of the jet between pin-fins on the wake flow. A spanwise spacing of S/D = 3, for example, showed less influence from the jet between pin-fins than S/D = 1.5. It was determined that using U_{max} for contour plots was desired for ease of interpreting the measured quantities since U_{max} was held constant for each spanwise spacing considered. For line-plots, U_m and U_{max} were used judiciously to best illustrate the information being presented. The remaining figures in the present work follow this convention where each choice of reference velocity is clearly indicated in each case.

The time-mean, streamwise velocity was extracted at the position of constant X/D = 1, Z/H = 0 in Figure 7.4 using U_m normalization. For increasing spanwise spacing, the flow was more uniform across the wake because U_{max} approached U_m. As expected, the wake profile approached that of a single, infinite cylinder for increasing S/D. Increasing spanwise spacing from S/D = 2.5 to 3, however, resulted in little change of the cross-wake profile of U/U_{in} and further increases in spanwise spacing may not allow the pin-fin data to collapse entirely with the single, infinite cylinder data. The pin-fins flowfields were subject to wall-generated turbulence and three-dimensional effects that led to differences with the single, infinite cylinder data. For comparison, the effect of spanwise
spacing is shown using $U_{\text{max}}$ normalization in Figure 7.5. Normalization using $U_{\text{max}}$ showed better agreement among the pin-fin data than $U_m$ normalization at the position $Y/D = \pm 1$ where $U/U_{\text{max}}$ was approximately unity. When using $U_{\text{max}}$ normalization, however, it was difficult to infer the reduced influence of the jet between pin-fins for increasing $S/D$.

Figure 7.6 shows the effect of $S/D$ on the in-plane, turbulent normal stress in the near wake for $Re_D = 2.0e4$. From Figure 7.6, it was found that the in-plane, turbulent normal stresses increased with increasing $S/D$. For closer spanwise spacing, the jet between pin-fins had a stabilizing influence on the wake that reduced fluctuations. Figure 7.7 shows the effect of $S/D$ on $\left< u'u' \right>/U_{\text{max}}^2$ at the position $X/D = 1$, $Z/H = 0$ for $Re_D = 2.0e4$. The pin-fin data showed a similar magnitude of $\left< u'u' \right>/U_{\text{max}}^2$ in the shear layers when compared with the single, infinite cylinder. Increasing spanwise spacing resulted in increased $\left< u'u' \right>/U_{\text{max}}^2$, consistent with the decreased stabilizing influence from the jet between pin-fins. The distance between peaks in local maxima of $\left< u'u' \right>/U_{\text{max}}^2$ showed that the wake width was dependent on spanwise spacing. The pin-fin row having $S/D = 1.5$ showed the widest wake, even wider than the single, infinite cylinder. The fundamentally different (asymmetric) wake structure, however, precluded $S/D = 1.5$ from direct comparisons with the pin-fins having $S/D \geq 2$ or the single, infinite cylinder. As discussed in Chapter 6, the jet between pin-fins (usually) caused narrowed wake for pin-fins in comparison to a single, infinite cylinder. In the present section, for those cases having $S/D \geq 2$, increasing spanwise spacing resulted in an increased wake width that approached the width of the single, infinite cylinder wake. Further testing is required, however, to determine if further increases in $S/D$ would result in a wake width that collapses with the single, infinite cylinder. A broader peak in $\left< u'u' \right>/U_{\text{max}}^2$ was observed for the pin-fin data when compared with the single, infinite cylinder data indicating a more diffuse shear layer. Even for the widest pin-fin spacing, $S/D = 3$, the broadened peak in $\left< u'u' \right>/U_{\text{max}}^2$ showed the flow around a pin-fin had fundamental differences from a single, infinite cylinder. Turbulence generated at the channel walls and three-dimensional effects were present for the pin-fins and caused the differences in
the cross-wake streamwise velocity and turbulent stress profiles when compared to a single, infinite cylinder. Note that $U_m$ normalization resulted in turbulent stress values that were much higher than that of a single, infinite cylinder, because the turbulent stress increased with the square of $U_{\text{max}}/U_m$. For $S/D = 2$, for example, the $\langle u'u' \rangle/U_m^2$ is a factor of four higher than $\langle u'u' \rangle/U_{\text{max}}^2$. Normalization using $U_m$ was, therefore, not included in this turbulent stress analysis.

Figure 7.8 shows the effect of $S/D$ on $\langle v'v' \rangle/U_{\text{max}}^2$ at the position $X/D = 1$, $Z/H = 0$ for $Re_D = 2.0e4$. Increasing spanwise spacing generally resulted in increased $\langle v'v' \rangle/U_{\text{max}}^2$ at the position $X/D = 1$ with the exception of the $S/D = 3$ case showing lower magnitude $\langle v'v' \rangle/U_{\text{max}}^2$ than $S/D = 2.5$. The discrepancy was caused by the slightly longer wake closure position for $S/D = 3$, $L_c/D = 1.15$, in comparison to $S/D = 2.5$, $L_c/D = 1.08$. The position of maximum fluctuations showed good agreement with the wake closure position, so the $\langle v'v' \rangle/U_{\text{max}}^2$ was increased for $S/D = 2.5$ at the position $X/D = 1$. The elevated $\langle v'v' \rangle/U_{\text{max}}^2$ was caused by wake mixing and increased $\langle v'v' \rangle/U_{\text{max}}^2$ for $S/D \geq 2$ in comparison to $S/D = 1.5$ was attributed to the shortened wake closure position. The Kármán vortices formed closer to the pin-fin for $S/D = 2$ and breakdown to turbulence, subsequently, occurred closer to the pin-fin in comparison to $S/D = 1.5$. Figure 7.9 shows the effect of $S/D$ on $\langle u'u' \rangle/U_{\text{max}}^2$ at the position $X/D = 1$, $Z/H = 0$ for $Re_D = 2.0e4$. Turbulent shear stress increased with increasing spanwise spacing, in agreement with the profiles of $\langle u'u' \rangle/U_{\text{max}}^2$. The level of turbulent shear stress at $X/D = 1$ indicated the effectiveness of coherent structures (Kármán and shear layer vortices) in transporting streamwise momentum across the wake in the transverse direction. Comparing the $S/D = 2.5$ and $S/D = 3$ cases, the profiles of $\langle u'u' \rangle/U_{\text{max}}^2$ showed increased turbulent shear stress along the top shear layer for $S/D = 2.5$ but increased turbulent shear stress along the bottom shear layer for $S/D = 3$. Both $S/D = 2.5$ and 3, however, had increased $\langle u'u' \rangle/U_{\text{max}}^2$ relative to $S/D = 2$ which showed that the stabilizing effect on the wake had more influence between $S/D = 2$ and $S/D = 2.5$ and less
influence between S/D = 2.5 and 3. For S/D = 2, the jet between pin-fins reduced the amount of transport of streamwise momentum in the transverse direction in comparison to S/D = 2.5 and 3.

Inspection of the time-dependent velocity field revealed that the reduced fluctuations for S/D = 1.5 relative to S/D ≥ 2 in Figure 7.6 were caused by a fundamental difference in the vortex shedding behavior. Figure 7.10 shows instantaneous snapshots of the flowfield for S/D = 1.5 (the bottom pin-fin of Figure 7.6). The snapshots of Figure 7.10 are shown in increments of St⁻¹ = (0.2)⁻¹(1/8) = 0.625. Figure 7.10 shows two large vortices forming along the shear layers and advecting downstream. The motion shown in Figure 7.10 was significantly different from the classic vortex shedding pattern, but still showed signs of “vortex shedding” because of the alternating formation and downstream advection of vortices that originated along the shear layers.

Although alternating formation of large scale vortices was observed for S/D = 1.5, proper orthogonal decomposition (POD) showed a distinct change in flowfield behavior for S/D = 1.5 in comparison with S/D ≥ 2 for the present single pin-fin rows. Figure 7.11 shows the first four POD modes for the single row having S/D = 1.5 (the bottom pin-fin of Figure 7.6). Modes 1 through 3 showed evidence of a large vortex along the wake axis while mode 4 showed activity along the shear layers. For single rows having S/D = 2, it was shown in Chapter 6 that POD modes 1 and 2 combined to create the vortex shedding motion. For the present case having S/D = 1.5, however, the combination of POD modes 1 through 3 contributed to the formation and advection of Kármán vortices. There was less regularity in vortex shedding for S/D = 1.5 in comparison to S/D = 2 indicated by the temporal modes of Figure 7.11. The cumulative POD energy distribution is shown in Figure 7.12 for all spanwise spacings considered and the energy content in the first two POD modes is summarized in Figure 7.13. From the cumulative POD energy distribution, the differences between S/D = 1.5 and S/D ≥ 2 were highlighted. The effect of the jet between pin-fins had a stabilizing influence on the wake of the pin-fins having S/D = 1.5 and the flow was more shear-layer-driven. In comparison, those cases having S/D ≥ 2 were dominated by vortex shedding. Both the top and bottom pin-fins of the S/D = 1.5 case in Figure 7.1 showed reduced energy in the lower order modes in comparison
to $S/D \geq 2$. For both asymmetric wakes in Figure 7.1, therefore, the jet between pin-fins had a significant influence on the wake.

### 7.2 Effect of Spanwise Spacing on Heat Transfer in a Single Row of Pin-Fins

To correlate near wake flow with endwall heat transfer, the present TRDPIV measurements were compared with Lyall et al. (2011) who measured endwall heat transfer for several single row pin-fin configurations having $H/D = 1.0$ and $2.0 \leq S/D \leq 8.0$. From the present TRDPIV measurements, increasing $S/D$ increased the level of fluctuating velocity in the near wake for a constant $Re_D = 2.0e4$. At the position $X/D = 1.5$, $Y/D = 0$, $Z/H = 0$ the level of in-plane, turbulent normal stress increased by 9% and 33% relative to $S/D = 2$ when $S/D$ was increased to 2.5 and to 3.0, respectively. Endwall heat transfer from Lyall et al. (2011) was also extracted at $X/D = 1.5$ and $Y/D = 0$ for a constant $Re_D = 1.75e4$. Endwall $Nu_D$ only varied to within 5%, relative to $S/D = 2$, when $S/D$ was increased to 4 and to 8 which indicated that the overall level of velocity fluctuations did not appreciably increase heat transfer along the endwall.

To determine whether periodic unsteadiness or turbulence correlated with heat transfer, proper orthogonal decomposition was used to separate the periodic from the random velocity fluctuations in the wake. Figure 7.14 shows the random unsteadiness contributions to the in-plane, turbulent normal stress for each spanwise spacing considered. Although the overall level of velocity fluctuations increased with increasing $S/D$, the level of turbulence showed no trend with increasing spanwise spacing for $2 \leq S/D \leq 3$. In agreement with Chapter 6, the level of periodic unsteadiness did not correlate with the heat transfer.

When considering the average heat transfer on the endwall for a single row of pin-fins, Lyall et al. (2011) showed that increasing $S/D$ decreased heat transfer. It was just shown that heat transfer was independent of $S/D$. The average heat transfer on the endwall, however, decreased with increasing $S/D$ simply because the pin-fins were spaced farther apart. Stated simply, the augmentation on the endwall surfaces in the wake of the pin-fins remains constant but bringing the pin-fins closer together reduces the amount of undisturbed endwall.
For the present single row pin-fins having $1.5 \leq S/D \leq 3$, the pin-surface heat transfer was measured at $Re_D = 2.0e4$. Figure 7.15 shows the effect of $S/D$ on local heat transfer across the surface of the pin-fin. The thermocouple measurement locations in Figure 7.15 are indicated with black dots. For the cases having $S/D \geq 2$, symmetry was assumed about $\theta = 0^\circ$ and $Z/H = 0$ and the contours are mirrored about these two planes. Because of the asymmetry observed for $S/D = 1.5$, however, measurements were made about the entire circumference and the contours in Figure 7.15 are only mirrored about $Z/H = 0$. From Figure 7.15, increased pin-fin heat transfer was observed for increasing spanwise spacing, in agreement with Lyall et al. (2011). Consistent with the pin-fin heat transfer measurements presented in Chapter 5 and Chapter 6, the heat transfer contours in Figure 7.15 showed augmentation from the horseshoe vortex at the positions $\theta = 0^\circ$ and $Z/H = \pm 0.5$. Similar to previous analyses, heat transfer was reduced as the flow progressed around the pin-fin as a result of the growth of the thermal boundary layer.

The pin-fin heat transfer was integrated over the entire pin-surface to determine area-averaged heat transfer. Figure 7.16 shows the effect of $S/D$ on the area-averaged, pin-surface heat transfer for a single row at $Re_D = 2.0e4$. As shown in the heat transfer contours, increasing $S/D$ resulted in increased pin-fin heat transfer. The present results agreed, to within the experimental uncertainty, with previously published data that integrated heat transfer around the pin-fin midline (Ames and Dvorak 2005; Lyall et al. 2011). This result showed that the overall pin-fin heat transfer coefficient was well-approximated using the integrated heat transfer coefficient around the pin-fin midline.

Figure 7.17 shows the effect of spanwise spacing on the local Nusselt number extracted around the pin-fin midline, $Z/H = 0$. As mentioned previously, heat transfer data was taken around the entire circumference for $S/D = 1.5$. Figure 7.17, therefore, shows two sets of data for $S/D = 1.5$ where one set is labeled “folded” for the circumferential positions $\theta < 0^\circ$. Each case showed high heat transfer at the stagnation point. As flow progressed around the pin-fin, the thermal boundary layer grew and heat transfer decreased. Despite the various levels of flow acceleration (dependent on $S/D$), heat transfer at $\theta = 90^\circ$ was within 10% for each spanwise spacing considered. The heat transfer distributions collapsed at $\theta = 90^\circ$, which was likely caused by the constant minimum area velocity, $U_{max}$. On the trailing side of the pin-fin, the local heat transfer
recovered to nearly the stagnation point heat transfer for $S/D \geq 2.0$. The recovery for $S/D = 1.5$, however, was much less significant than that of $S/D \geq 2.0$. This result showed that the elongated wake for $S/D = 1.5$, caused decreased heat transfer on the trailing side of the pin-fin in comparison to $S/D \geq 2$.

Ames et al. (2005) showed that the stagnation point heat transfer is a function of a turbulence parameter, referred to as TRL, which includes turbulence intensity, approaching Reynolds number, and dissipation length scale. In Chapter 2, the correlation of Ames et al. (2005) was used to show that increased pin-fin heat transfer for increasing $S/D$ in a single row of pin-fins was primarily caused by the increased approaching velocity. When maintaining a constant $Re_D$, increasing $S/D$ increased the bulk velocity and, therefore, the approaching velocity. For the present results, the increased area-averaged pin-fin heat transfer for increasing $S/D$ was attributed to the increased approaching velocity. The elongated wake for $S/D = 1.5$, however, had an adverse affect on the local heat transfer (recovery) on the trailing side of the pin-fin.

### 7.3 Effect of Spanwise Spacing on Flowfield for Wide Streamwise Spacing

For a single row of pin-fins, where the wake was completely unconstrained, it was found that the wakes became biased and asymmetric when $S/D$ was reduced from 2 to 1.5. In gas turbine applications, however, $S/D \geq 2$ is common for pin-fin arrays (Kohli 2010b). For the remaining analysis of the effects of spanwise spacing, only $S/D = 2, 2.5, \text{and } 3$ were considered. In this section, the effects of $S/D$ on the flowfield were quantified for pin-fin arrays having $H/D = 1, X/D = 3.03$. By adding downstream pin-fins at a spacing of $X/D = 3.03$, the wake was constrained to a greater degree than the single row cases in Section 7.1. Effects of Reynolds number were considered by investigating two Reynolds number flow conditions, $Re_D = 3.0e3$ and $2.0e4$. Note that the single row study in Section 7.1 only considered $Re_D = 2.0e4$.

Figure 7.18 shows the time-mean velocity for three cases having $S/D = 2, 2.5, \text{and } 3$ at constant $H/D = 1, X/D = 3.03$, and $Re_D = 3.0e3$. From Figure 7.18, the wake closure position, $L_c/D$, was indicated using dashed lines at the position of zero streamwise velocity along the wake axis. Figure 7.19 shows the wake closure position for the first and third row wakes. In Chapter 6, the wake closure position for pin-fin arrays having
S/D = 2 was influenced by the second row pin-fins at low Reynolds numbers for X/D ≤ 3.46. In the present section, where X/D = 3.03, the wake of the first row pin-fin was affected by the second row pin-fins for S/D = 2. As S/D increased from 2.5 to 3, Lc/D decreased slightly in the first row. In the third row wakes, however, wake closure position increased slightly between S/D = 2.5 and 3. The wake closure position, therefore, does not appear to converge for 2 ≤ S/D ≤ 3 at streamwise spacing of X/D = 3.03 and Re_D = 3.0e3. It is expected that the Lc/D approaches a constant value with further increases in S/D. The first and third row wakes would presumably approach different Lc/D values because the third row is influenced by the wake of the first row for the spacing X/D = 3.03. Further testing is required to determine the S/D spacing where the wake closure position approaches a constant value in the first and third row wakes.

In Figure 7.20, the time-mean, streamwise velocity was extracted across the wake at constant X/D = 1, Z/H = 0 and normalized using U_m. As expected, Figure 7.20 showed that the jet effect between pin-fins in the first row became less significant for increasing S/D. Even for the widest spanwise spacing, S/D = 3, the jet effect was observed when comparing to the single, infinite cylinder as shown at the position Y/D = ±1. As in the discussion on wake closure position, the velocity profiles showed that the wake was not converged when increasing to S/D = 3, and further increases in S/D are required to determine if the velocity profile collapses with the single, infinite cylinder data. The effect of row position was quantified by extracting the time-mean velocity in the third row wakes, again, one pin-fin diameter downstream of the pin-fin axis. Figure 7.21 shows that the wake is more diffuse than a single, infinite cylinder. As discussed in Chapter 6, disturbances generated in upstream rows caused a more diffuse third row wake. While increasing S/D from 2 to 2.5 showed a reduced influence of the jet between pin-fins, increasing S/D from 2.5 to 3 showed similar velocity profiles. It was difficult to determine if further increases in S/D would alter the third row wake because the velocity profiles in Figure 7.21 converged but the wake length in Figure 7.19 increased when S/D was increased from 2.5 to 3.

Figure 7.22 shows the effect of spanwise spacing on contours of in-plane, turbulent normal stress using U_max normalization in the first and third row wake at low Reynolds number. Increasing S/D generally resulted in increased turbulent fluctuations,
although the first row wake having S/D = 2.5 showed lower turbulent normal stress than S/D = 3. Increasing turbulent normal stress with increased S/D was observed for the single row of pin-fins at high Reynolds number in Section 7.1 because of the reduced stabilizing influence of the jet between pin-fins. Figure 7.23 shows the turbulent stress component $\langle u'u' \rangle / U_{\text{max}}^2$ plotted across the first row wake at X/D = 1.0, Z/H = 0. Consistent with previous analyses, the wake width was reduced for pin-fins having 2 ≤ S/D ≤ 3 in comparison to a single, infinite cylinder as observed by the distance between peaks of $\langle u'u' \rangle / U_{\text{max}}^2$ across the wake. Increasing S/D did not influence the wake width for the first row wakes having X/D = 3.03. Similarly, the wake width was unaffected by S/D in the third row wakes as shown in Figure 7.24. Consistent with previous analyses, the third row wakes showed more diffuse shear layers by the broadened peak of $\langle u'u' \rangle / U_{\text{max}}^2$ relative to a single, infinite cylinder caused by turbulence from upstream rows. By the third row, the flow was sufficiently turbulent as shown by the increased magnitude of $\langle u'u' \rangle / U_{\text{max}}^2$ relative that of the first row wake in Figure 7.23.

The jet between pin-fins was found to have a significant influence on the flowfield for a single pin-fin row at high Reynolds number and for a pin-fin array having X/D = 3.03 at low Reynolds number. The jet between pin-fins was, again, found to have a significant influence on the flowfield when Reynolds number was increased from $\text{Re}_D = 3.0e3$ to $2.0e4$ for pin-fins having H/D = 1, X/D = 3.03. Figure 7.25 shows the effect of spanwise spacing on time-mean velocity at $\text{Re}_D = 2.0e4$. In the previous discussion, for the single pin-fin row, there was no clear relationship of wake closure position with spanwise spacing. For the multiple row array having X/D = 3.03, however, the first and third row wakes showed decreasing $L_c/D$ with increasing S/D. The wake closure position is summarized in Figure 7.26 for high Reynolds flow for the single row cases, for the first rows having X/D = 3.03, and for the third rows having X/D = 3.03. From Figure 7.26 the $L_c/D$ was less than that of the single, infinite cylinders for each spanwise spacing considered. This result was shown in Chapter 6 where the first row wake for cases having X/D ≥ 2.60 were unaffected by the second row of pin-fins and the wake closure position was reduced relative to a single, infinite cylinder. At high Reynolds numbers, the wake closure position decreased for pin-fins relative to a single, infinite cylinder.
because of turbulence generated at the walls of the channel. Low Reynolds number wakes were impervious to the turbulence generated at the walls from the stronger influence of the jet between pin-fins (observed in cross-wake profiles of streamwise velocity). In general, Figure 7.26 shows that the wake closure position decreases for increasing S/D in high Reynolds number flows when the downstream pin-fins do not influence the wake formation. At high Reynolds numbers, the stabilizing influence of the jet for decreasing S/D values increased the wake closure position. At low Reynolds numbers, in Figure 7.19, the stabilizing influence of the jet was observed in time-mean velocity profiles for decreasing S/D. The wake closure position was difficult to interpret for low Reynolds numbers, however, because the downstream pin-fins were influencing the wake in addition to effects of the jet between pin-fins.

The time-mean velocity was extracted at the point X/D = 1, Z/H = 0 and normalized using U_m to determine the effect of S/D on the velocity profile for arrays having H/D = 1, X/D = 3.03 at high Reynolds number. Figure 7.27 shows that the first row wake approached the single, infinite cylinder flow at the position Y/D = ±1 for increasing S/D. When compared with the low Reynolds number flow in Figure 7.20, the high Reynolds number wake in the first row showed better agreement with the single, infinite cylinder data. The influence of the jet between pin-fins was, therefore, not only dependent on S/D, but also on Reynolds number. Increasing Reynolds number and increasing S/D both led to reduced influence of the jet between pin-fins. At the position Y/D = 0, however, increasing S/D led to further deviation from a single, infinite cylinder. For high Reynolds number, it was determined that increases in S/D would not cause a pin-fin wake to collapse with a single, infinite cylinder wake. Referring back to the single pin-fin row at high Reynolds number, Figure 7.4 shows that increasing S/D led to further deviation from a single, infinite cylinder at the position Y/D = 0. For low Reynolds numbers, in Figure 7.20, it was possible that further increases in S/D would allow the velocity profile to collapse with a single, infinite cylinder. Figure 7.28 shows the velocity profile in the third row wakes for the X/D = 3.03 arrays at high Reynolds number. Similar to the third row wake at low Reynolds number, the velocity profiles collapsed when increasing S/D from 2.5 to 3 in the third row wake at high Reynolds number.
Figure 7.29 shows the effect of spanwise spacing on the in-plane, turbulent normal stress at $\text{Re}_D = 2.0\times10^4$. As with the single pin-fin rows at $\text{Re}_D = 2.0\times10^4$, increasing S/D resulted in increased turbulent fluctuations. The first row wakes showed a high level of turbulent normal stress, consistent with Chapter 6 for cases having strong, periodic vortex shedding ($X/D \geq 2.60$). Both first and third row wakes showed increased turbulent normal stress with increasing S/D as a result of the decreased influence of the jet between pin-fins. Also, as described in Chapter 6, the level of fluctuations in the third row wakes were lower than that of the first row wakes at high Reynolds number as a result of upstream turbulence disrupting the vortex shedding process in the third row wakes. It is important to note that the first row wakes showed higher turbulent normal stress than third row wakes because of contributions from coherent, periodic vortex shedding. The contours of Figure 7.29 should not be interpreted as a measure of turbulence intensity. POD analyses in Chapter 6 showed the majority of the fluctuating energy was attributed to the periodic vortex shedding motion observed in the first row (for $X/D \geq 2.60$). In the third rows, however, upstream disturbances facilitated the breakdown of the Kármán vortices in the third row wakes which resulted in reduced turbulent normal stresses when compared with first row wakes. The third rows, therefore, had a more random and disorganized wake than the first rows.

Figure 7.30 shows the cross-wake profile of $\langle u'u' \rangle / U_{\text{max}}^2$ at the position $X/D = 1$, $Z/H = 0$ for the first row wakes. The level of maximum $\langle u'u' \rangle / U_{\text{max}}^2$ for each spanwise spacing considered was lower than the corresponding single pin-fin row in Figure 7.7. The single pin-fin row showed a 12%, 6%, and 11% increase in $\langle u'u' \rangle / U_{\text{max}}^2$ when compared to the first row having $X/D = 3.03$ for S/D = 2, 2.5, and 3, respectively. The presence of the second row pin-fins, therefore, influenced the level of $\langle u'u' \rangle / U_{\text{max}}^2$ in the first row wake. Figure 7.31 shows the cross-wake profile of $\langle u'u' \rangle / U_{\text{max}}^2$ at the position $X/D = 1$, $Z/H = 0$ for the third row wakes. The third row wakes were more diffuse than the first row wakes. The level of peak $\langle u'u' \rangle / U_{\text{max}}^2$ in the third row wakes decreased in comparison to the first row wakes because vortex shedding was disrupted by upstream turbulence. The level of $\langle u'u' \rangle / U_{\text{max}}^2$ in the region between pin-fins, $Y/D = \pm 1$, increased
in the third row wakes relative to the first row wakes. Similar to low Reynolds number flows, the flow was sufficiently turbulent in the third row wake.

Performing POD on the near wake, it was found that the distribution of energy showed no significant difference with increasing S/D when the streamwise spacing was $X/D = 3.03$. Figure 7.32 shows the cumulative POD energy distribution for the first row wakes at $Re_D = 3.0e3$. As discussed in Chapter 6, the lower order POD modes for first row wakes at low Reynolds numbers indicated a shear-layer-driven flow. There was no clear trend observed in the POD energy distribution in the first row wake when increasing S/D for low Reynolds number flow. Figure 7.33 shows the cumulative POD energy distribution for the first row wakes at $Re_D = 2.0e4$. Similar to the single pin-fin rows at high Reynolds number, decreasing S/D showed a reduction in the percentage of energy contained in the first two, shedding modes. The jet between pin-fins had a stabilizing influence on the near wake that reduced the percentage of fluctuating energy contained in the vortex shedding motion. The decrease in turbulent normal stress observed for decreasing S/D was, therefore, caused by reducing the percentage of energy contained in the vortex shedding motion. The energy content of the first two POD modes is summarized in Figure 7.34 for the first row wakes in pin-fin arrays having $X/D = 3.03$ at both low and high Reynolds number.

Figure 7.35 and Figure 7.36 show the cumulative POD energy distribution in the third row wake for $Re_D = 3.0e3$ and $2.0e4$, respectively. In the third row wakes, vortices were formed and readily broken down for both low and high Reynolds numbers. In comparison, the flow was shear-layer-driven in the first row for low Reynolds numbers and the flow was dominated by periodic shedding in the first row for high Reynolds numbers. There was no clear trend in the POD energy distributions with increasing spanwise spacing for both low and high Reynolds numbers. The mechanism of vortex formation and breakdown in the third row wake was, therefore, independent of Reynolds number and independent of spanwise spacing. The energy content in the first two POD modes is summarized in Figure 7.37.
7.4 Effect of Spanwise Spacing on Heat Transfer for Wide Streamwise Spacing

Contours of local endwall heat transfer heat transfer are shown in Figure 7.38 for \( \text{Re}_D = 3.0 \times 10^3 \). Consistent with previous studies, the endwall heat transfer was increased for decreasing S/D (Lyall et al. 2011; Lawson et al. 2011). The horseshoe vortex (HV) was a distinguishing feature in the first row. The HV was observed to wrap around the pin-fin and extend about two pin diameters downstream. In downstream rows, the heat transfer contours showed less distinguishing flow features because of the high level of turbulent mixing. The HV occupied a much smaller area in downstream rows in comparison to the first row. In the third row and beyond, individual wakes were observed in the heat transfer contours for S/D = 2.5 and 3 while no individual pin-fin wakes were observed for S/D = 2.

Figure 7.39 shows the effect of spanwise spacing on the laterally-averaged endwall heat transfer. Figure 7.40 shows the laterally-averaged endwall heat transfer augmentation. From Figure 7.38 through Figure 7.40, the case having S/D = 2 showed increased heat transfer between adjacent pin-fins in the first row when compared with S/D = 2.5 and 3. At the position X/D = 0, for example, the laterally averaged Nu\(_Dh\) was 19.2% higher for S/D = 2 relative to S/D = 2.5. For S/D = 2, the horseshoe vortices from two neighboring pin-fins in the first row coincided at the mid-pitch location (Y/D = ±0.5 for the S/D = 2 case). But, for S/D = 2.5 and 3, the horseshoe vortices of adjacent pin-fins did not coincide and this allowed the incoming thermal boundary layer to develop and cause poor heat transfer at the mid-pitch location.

In addition to having higher heat transfer in the first row, the pin-fin array having S/D = 2 showed higher heat transfer in rows 2 through 7. In rows 2-7, however, the horseshoe vortex was not as distinguished as in the first row, and it was likely that another pin-fin feature was causing increased heat transfer for decreased S/D. The contribution of periodic and random unsteadiness was compared as a function of spanwise spacing. Figure 7.41 shows the turbulence in the third row wake as a function of S/D at low Reynolds number. Unlike Chapter 6, the increased turbulence did not correlate with decreased heat transfer. Increasing S/D resulted in increased turbulence, yet heat transfer was observed to decrease. When considering S/D effects, therefore, another flowfield feature was contributing to heat transfer. For a single row of pin-fins, it
was found that decreasing S/D increased heat transfer because there was simply less undisturbed endwall area. In the developed portion of multi-row arrays, however, the flow is sufficiently turbulent. Inspection of the time-mean streamwise flow at the channel midline, in Figure 7.18, showed that increasing S/D resulted in increased velocities between the pin-fins at the position X/D = 0.5, Y/D ± 1 relative to the closer S/D spacings. To preserve continuity, the configurations having increased S/D must have lower velocity closer to the wall compared to the closer S/D cases. To estimate the influence of three-dimensional effects, the velocity gradient at the wall increased by 15% when velocity at the centerline increased by 20% (which was observed when increasing S/D from 2 to 3) assuming a 1/7 power law for the velocity profile. The steeper velocity gradient at the wall for the wider spacing results in higher endwall heat transfer.

Increasing Reynolds number to 2.0e4 resulted in significantly different local heat transfer distributions than the low Reynolds number flows. In Figure 7.42, the effect of spanwise spacing on the endwall heat transfer is shown. At high Reynolds numbers, the HV occupied a smaller area around the pin-fin when compared with low Reynolds numbers, in agreement with the analysis of Chapter 4 where the HV became smaller and moved closer to the pin-fin as Reynolds number increased from ReD = 3.0e3 to 2.0e4. Each individual wake was observed in the heat transfer contours of Figure 7.42 for all rows and all spanwise spacings considered.

The effect of spanwise spacing on laterally-averaged endwall heat transfer is shown in Figure 7.43 for ReD = 2.0e4. Similarly, laterally-averaged endwall heat transfer augmentation is shown in Figure 7.44. In comparison with low Reynolds number flow, the local heat transfer levels showed less dependence on spanwise spacing at high Reynolds number. In fact, the average endwall heat transfer for rows 1 through 7 was within the experimental uncertainty for 2 ≤ S/D ≤ 3 at high Reynolds number. In Chapter 6, changes in streamwise spacing were also more significant at low Reynolds number.

Similar to low Reynolds numbers, the level of turbulence in the wakes increased with increasing S/D. Again, three-dimensional effects may have counteracted the increase in turbulence and resulted in decreased heat transfer for increasing S/D.
7.5 Effect of Spanwise Spacing on Flowfield for Close Streamwise Spacing

The streamwise pin-fin spacing was decreased to $X/D = 2.16$ to investigate the effects of spanwise spacing when the near wake was constrained by pin-fins in close proximity. In Chapter 6, the near wake flow was significantly affected when $X/D$ was reduced to 2.16. At high Reynolds numbers, the constraint on the wake caused a shear-layer-driven flow rather than a flow dominated by periodic shedding. At low Reynolds number, the effects of $X/D$ were not as significant as for high Reynolds number. Decreasing $X/D$ to 2.16 decreased the wake closure position and decreased the level of velocity fluctuations in the first row wake when compared with $X/D = 2.60$. In this section, the effect of spanwise spacing was investigated for pin-fin arrays having $H/D = 1$, $X/D = 2.16$, and $S/D = 2, 2.5,$ and 3.

The effect of $S/D$ on the time-mean velocity is shown in Figure 7.45 for the pin-fin arrays having $X/D = 2.16$ at low Reynolds number. The wake closure position, $L_c/D$, is indicated in Figure 7.45 with dashed lines and summarized in Figure 7.46. Increasing $S/D$ resulted in increased $L_c/D$ for both first and third row wakes. Previously, in Section 7.3, Figure 7.19 showed that increasing $S/D$ decreased $L_c/D$ in the first row when $X/D = 3.03$. With a more constrained wake, where $X/D = 2.16$, it was not surprising that increasing $S/D$ resulted in increased $L_c/D$. For the staggered arrangement of pin-fins, the effect of the second row of pin-fins decreases with increasing $S/D$. Even with a close streamwise spacing of $X/D = 2.16$, increased spanwise spacing reduced the influence of the second row of pin-fins enough to allow $L_c/D$ to increase.

Figure 7.47 shows the effect of $S/D$ on time-mean, streamwise velocity extracted at the position $X/D = 1, Z/H = 0$ for the first row of the pin-fin arrays having $X/D = 2.16$ at low Reynolds number. The effect of the jet was significant, even at the reduced streamwise spacing of $X/D = 2.16$. Increasing $S/D$ from 2 to 2.5 showed a decreased influence of the jet between pin-fins. The velocity profiles collapsed, however, between $S/D = 2.5$ and $S/D = 3$. It was previously shown, for $X/D = 3.03$, that the velocity profiles did not collapse when increasing $S/D$ from 2.5 to 3. The jet between pin-fins was only able to overcome the presence of the second pin-fin row for $S/D = 2$. The effect of second row pin-fins was also observed in the shape of the velocity profile when compared with $X/D = 3.03$. The local maximum at $Y/D = \pm 0.5$ was more pronounced.
for the cases having $X/D = 2.16$ in comparison to $X/D = 3.03$. The jet between pin-fins was blocked by the second row for $X/D = 2.16$ which caused a reduction in velocity observed at $Y/D = \pm 1$ and made the maxima at $Y/D = \pm 0.7$ more pronounced than that of $X/D = 3.03$. Figure 7.48 shows that the fourth row pin-fins influenced the velocity profile for $X/D = 2.16$. Similar to the first row wakes, the velocity maxima at $Y/D = \pm 0.7$ for the third row wakes were more pronounced for $X/D = 2.16$ in comparison to $X/D = 3.03$.

The effect of $S/D$ on the in-plane, turbulent normal stress is shown in Figure 7.49 for the first and third rows of the pin-fin arrays having $X/D = 2.16$ at low Reynolds number. Increasing $S/D$ led to increased turbulent fluctuations because of the reduced influence of the jet between pin-fins. Previously, for pin-fin arrays having $X/D = 3.03$, there was no clear trend of turbulent normal stress with increasing $S/D$ in the first row but increasing $S/D$ increased fluctuations in the third row. Figure 7.50 shows the effect of $S/D$ on $\langle u'u' \rangle/U_{max}^2$ extracted at the position $X/D = 1$, $Z/H = 0$ for the first row of the pin-fin arrays having $X/D = 2.16$. In comparison to $X/D = 3.03$, in Figure 7.23, the presence of second row pin-fins caused a reduction in $\langle u'u' \rangle/U_{max}^2$ for $X/D = 2.16$. Also, the second row pin-fins caused a narrowed wake for $X/D = 2.16$ in comparison to $X/D = 3.03$. Figure 7.51 shows the effect of $S/D$ on $\langle u'u' \rangle/U_{max}^2$ extracted at the position $X/D = 1$, $Z/H = 0$ for the third row of the pin-fin arrays having $X/D = 2.16$. Similar to the first row, the level of $\langle u'u' \rangle/U_{max}^2$ was reduced from the fourth row pin-fins for $X/D = 2.16$, in Figure 7.51, when compared to $X/D = 3.03$, in Figure 7.24. The wake width was similar when comparing the third row wakes for $X/D = 2.16$ to that of $X/D = 3.03$. For both first and third row wakes, the wake width was reduced for $S/D = 2$ in comparison to $S/D = 2.5$ and 3.

The effect of $S/D$ on the time-mean velocity is shown in Figure 7.52 for the pin-fin arrays having $X/D = 2.16$ at high Reynolds number. The wake closure position, $L_c/D$ is indicated in Figure 7.52 with dashed lines and summarized in Figure 7.53. There was little influence of $S/D$ on $L_c/D$ at high Reynolds number. In previous discussions, wake closure length was determined by the stabilizing influence of the jet between pin-fins (increased $L_c/D$) or by the blockage from downstream pin-fins (decreased $L_c/D$). The
two influences affecting $L_c/D$ balanced for the case having $X/D = 2.16$ at high Reynolds number.

Figure 7.54 shows the effect of $S/D$ on time-mean, streamwise velocity extracted at the position $X/D = 1$, $Z/H = 0$ for the first row of the pin-fin arrays having $X/D = 2.16$ at high Reynolds number. It was previously shown that, for $X/D = 3.03$, the velocity profiles did not collapse when increasing $S/D$ from 2.5 to 3. Similar to the low Reynolds number case for $X/D = 2.16$, the second row of pin-fins, had an influence on the jet formation between first row pin-fins for $X/D = 2.16$ and the velocity profiles collapsed for $S/D = 2.5$ and 3. In the third row, Figure 7.55 shows good agreement in the velocity profiles of $S/D = 2.5$ and 3, consistent with the $X/D = 3.03$ cases at high Reynolds number. Unlike the first row wakes, the velocity profile was flat between $Y/D = \pm 0.7$ and $Y/D = \pm 1$. There was less influence of the fourth row pin-fins on the third row wakes than the second row pin-fins on the first row wakes.

The effect of $S/D$ on the in-plane, turbulent normal stress is shown in Figure 7.56 for the first and third rows of the pin-fin arrays having $X/D = 2.16$ at high Reynolds number. As with the pin-fin arrays having $X/D = 3.03$, increasing $S/D$ led to increased turbulent fluctuations because of the reduced influence of the jet between pin-fins. Figure 7.57 shows the effect of $S/D$ on $\langle u'u' \rangle/U_{max}^2$ extracted at the position $X/D = 1$, $Z/H = 0$ for the first row of the pin-fin arrays having $X/D = 2.16$. In comparison to $X/D = 3.03$, in Figure 7.30, the presence of second row pin-fins caused a reduction in $\langle u'u' \rangle/U_{max}^2$ for $X/D = 2.16$. Figure 7.58 shows the effect of $S/D$ on $\langle u'u' \rangle/U_{max}^2$ extracted at the position $X/D = 1$, $Z/H = 0$ for the third row of the pin-fin arrays having $X/D = 2.16$. The level of $\langle u'u' \rangle/U_{max}^2$ in the wake of the $X/D = 2.16$ pin-fin arrays was comparable to that of $X/D = 3.03$.

POD was applied to the first and third row wakes to better compare the flowfield organization for the pin-fin arrays having wide spanwise spacing, $X/D = 3.03$, and close spanwise spacing, $X/D = 2.16$. The cumulative energy distribution of the POD modes of the first row wakes are shown in Figure 7.59 and Figure 7.60 for $Re_D = 3.0e3$ and $2.0e4$, respectively. For low Reynolds number, Figure 7.59 shows that the POD energy distribution was similar for $2 \leq S/D \leq 3$. For each spanwise spacing considered, the flow
was shear-layer-driven. As Reynolds number was increased to \( \text{Re}_D = 2.0e4 \), however, a difference in flowfield organization was observed between \( S/D = 2.5 \) and 3. For \( S/D = 2 \) and 2.5, the cumulative POD energy distribution in Figure 7.60 was similar and showed a shear-layer-driven flow. For \( S/D = 3 \), however, the energy in the first two POD modes was increased relative to \( S/D = 2 \) and 2.5 as shown in Figure 7.61. The POD energy distribution for \( S/D = 3 \) suggested that there was more energy content in the vortex shedding modes. Inspection of the first four POD modes, shown in Figure 7.62, showed that the first two modes corresponded to vortex shedding motion. The temporal modes showed a periodic signal, but the amplitude varied significantly with time which indicated that vortex shedding was intermittent, or quasi-periodic. It is expected that further increases in \( S/D \) would reduce the influence of the second row pin-fins on the first row wake and the flow would become dominated by periodic vortex shedding. For the case having \( S/D = 3 \), \( X/D = 2.16 \), the POD energy distribution showed that the flow was on the limit between having a first row wake influenced by second row pin-fins and having a wake not influenced by second row pin-fins.

Figure 7.63 and Figure 7.64 show the cumulative POD energy distribution in the third row wakes for \( \text{Re}_D = 3.0e3 \) and 2.0e4, respectively. For both low and high Reynolds numbers, Figure 7.65 shows that the energy distributions were less dependent on spanwise spacing when compared with the first row wakes. For all spanwise spacings considered, vortices were formed and readily broken down in the third row wakes at both low and high Reynolds numbers.

### 7.6 Effect of Spanwise Spacing on Heat Transfer for Close Streamwise Spacing

Endwall heat transfer is shown for the low Reynolds number cases having \( H/D = 1, X/D = 2.16 \) in Figure 7.66. The heat transfer contours showed similar features as the data from Section 7.3 where the streamwise spacing was \( X/D = 3.03 \). In the first row, the horseshoe vortex (HV) from two adjacent pin-fins coincided at the mid-pitch location \((Y/D = \pm 0.5)\) for the spanwise spacing \( S/D = 2 \). As \( S/D \) increased, however, the HV from two adjacent pin-fins did not coincide and the fully-developed thermal boundary layer was able to continue undisturbed. In the remainder of the array, heat transfer was increased for decreasing spanwise spacing. The laterally-averaged endwall heat transfer
is shown in Figure 7.67, and the laterally-averaged endwall augmentation is shown in Figure 7.68. Similar to when $X/D = 3.03$, first row heat transfer was higher for $S/D = 2$ in comparison with $S/D = 2.5$ and 3. In contrast to $X/D = 3.03$, however, second row heat transfer was within the experimental uncertainty for $2 \leq S/D \leq 3$ for $X/D = 2.16$. When $X/D = 3.03$, both the first and second row heat transfer were higher for $S/D = 2$ compared with $S/D = 2.5$ and 3. The third row showed no difference between $S/D = 2$ and 2.5 when $X/D = 3.03$ and $X/D = 2.16$. The most notable result from the present analysis was observed in the fourth rows and beyond. Spanwise spacing effects were more significant for $X/D = 2.16$ than $X/D = 3.03$ for low Reynolds number flow. The fourth through seventh rows showed an average increase of 16% for $S/D = 2$ relative to $S/D = 2.5$ when $X/D = 2.16$. In comparison, the percent increase was only 12% when $X/D = 3.03$.

At high Reynolds number, there was a significant effect of spanwise spacing on the flowfield organization for the pin-fin arrays having $X/D = 2.16$. Specifically, the flow was shear-layer-driven for $S/D \leq 2.5$ and intermittent vortex shedding was observed for $S/D = 3$. To determine the influence of vortex shedding on downstream heat transfer, the spatially-resolved endwall heat transfer is shown in Figure 7.69. From inspection of the endwall heat transfer contours, there was not a significant difference in the heat transfer pattern on the endwall behind the first row of pin-fins when comparing $S/D = 2.5$ and 3, where the flowfield change was observed. A close-up view of the first row heat transfer contours in Figure 7.70 shows that the heat transfer coefficients were not influenced by the wake organization. The laterally-averaged endwall heat transfer is shown in Figure 7.71 and the laterally-averaged endwall augmentation is shown in Figure 7.72. As observed previously when investigating the effects of $S/D$ for $X/D = 3.03$, high Reynolds number flow showed less dependence on $S/D$ than low Reynolds number flow when $X/D = 2.16$. In each row, the average endwall heat transfer was within the experimental uncertainty when comparing $S/D = 2$ and 2.5. Furthermore, only in the second and third rows was there a discernible difference between $S/D = 2$ and 3.
7.7 Summary of Effects of Spanwise Spacing on Heat Transfer

The results of this chapter have shown that the flowfield and heat transfer are strongly dependent on spanwise spacing. Decreasing spanwise spacing resulted in increased heat transfer. In the previous chapter, the initial rows were independent of streamwise spacing. In the present chapter, however, decreased spanwise spacing resulted in increased heat transfer in the initial rows. Bringing the pin-fins closer together in the spanwise direction resulted in more undisturbed endwall in the initial rows. In the developed rows, the flow was sufficiently turbulent and there was no undisturbed endwall surface. Heat transfer was found to increase for decreasing S/D despite a decreasing turbulence levels observed at the channel midline. It was determined that three-dimensional effects were present. The present flowfield measurements at the channel midline were unable to resolve three-dimensional effects and further investigation is required.
# Table 7.1. Effect of S/D on the Velocity Scales and Reynolds Numbers in the Pin-Fin Channel, H/D = 1

<table>
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<th>S/D</th>
<th>Contraction Ratio ($U_{max}/U_m$)</th>
<th>$Re_D$</th>
<th>$Re_{Dh}$</th>
<th>$U_{max}$ (m/s)</th>
<th>$U_m$ (m/s)</th>
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<td>3.0e3</td>
<td>0.75</td>
<td>0.38</td>
</tr>
<tr>
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<td>1.667</td>
<td>3.0e3</td>
<td>3.6e3</td>
<td>0.75</td>
<td>0.45</td>
</tr>
<tr>
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<td>1.5</td>
<td>3.0e3</td>
<td>4.0e3</td>
<td>0.75</td>
<td>0.50</td>
</tr>
<tr>
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<td>1.3e4</td>
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</tr>
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<td>2.7e4</td>
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Figure 7.1. Effect of S/D on time-mean flow in near wake of a single pin-fin row having H/D = 1 where Re_D = 2.0e4.
Figure 7.2. Asymmetry in time-mean velocity across the wake at X/D = 2 for a single pin-fin row having H/D = 1, S/D = 1.5 where Re<sub>D</sub> = 2.0e4.
Figure 7.3. Effect of S/D on time-mean flow, using $U_m$ normalization, in near wake of a single pin-fin row having H/D = 1 where $Re_D = 2.0e4$. 
Figure 7.4. Effect of S/D on $U/U_m$ at $X/D = 1$ for a single row of pin-fins having $H/D = 1$ where $Re_D = 2.0e4$. 
Figure 7.5. Effect of S/D on $U/U_{\text{max}}$ at X/D = 1 for a single row of pin-fins having H/D = 1 where Re_D = 2.0e4.
Figure 7.6. Effect of S/D on in-plane, normal turbulent stress in near wake of a single pin-fin row having H/D = 1 where Re$_D$ = 2.0e4.
Figure 7.7. Effect of S/D on $<u' u'>/U^2_{max}$ at constant $X/D = 1$ for a single row of pin-fins having $H/D = 1$ where $Re_D = 2.0e4$. 
Refer to Figure 7.7 for legend

Figure 7.8. Effect of S/D on $<v'v'>/U^2_{\text{max}}$ at constant X/D = 1 for a single row of pin-fins having H/D = 1 where Re_D = 2.0e4.

Figure 7.9. Effect of S/D on $<u'u'>/U^2_{\text{max}}$ at constant X/D = 1 for a single row of pin-fins having H/D = 1 where Re_D = 2.0e4.
Figure 7.10. Instantaneous flow for a single row of pin-fins having S/D = 1.5 (bottom pin-fin of Figure 7.4) at Re_D = 2.0e4.
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**Figure 7.12.** Effect of S/D on the cumulative POD energy distribution in the wake of a single pin-fin row having H/D = 1 where Re_D = 2.0e4.

**Figure 7.13.** Effect of S/D on first two POD modes in the wake of single pin-fin rows at Re_D = 2.0e4.
Figure 7.14. Effect of S/D on turbulence for single pin-fin rows having H/D = 1 where Re_D = 2.0e4.
Figure 7.15. Effect of S/D on pin-surface heat transfer for a single pin-fin row having H/D = 1 where $Re_D = 2.0\times10^4$. 
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Figure 7.17. Effect of S/D on local heat transfer at pin midline for a single pin-fin row having H/D = 1 where Re_D = 2.0e4.
Figure 7.18. Effect of S/D on time-mean flow in first and third wakes of a pin-fin array having H/D = 1, X/D = 3.03 where \( \text{Re}_D = 3.0e3 \).
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Figure 7.20. Effect of S/D on velocity profile in the first row wake at X/D = 1, Z/H = 0 for pin-fins having H/D = 1, X/D = 3.03, Re_D = 3.0e3.
Figure 7.21. Effect of S/D on velocity profile in the third row wake at X/D = 1, Z/H = 0 for pin-fins having H/D = 1, X/D = 3.03, Re_D = 3.0e3.
Figure 7.22. Effect of S/D on average, turbulent normal stress in first and third wakes of a pin-fin array having H/D = 1, X/D = 3.03 where Re_D = 3.0e3.
Figure 7.23. Effect of S/D on $<u'u'>/U_{max}$ at X/D = 1, Z/H = 0 for the first row wake of a pin-fin array having H/D = 1, X/D = 3.03 where Re_D = 3.0e3.
Figure 7.24. Effect of S/D on $\langle u'u'\rangle/U_{\text{max}}^2$ at X/D = 1, Z/H = 0 for the third row wake of a pin-fin array having H/D = 1, X/D = 3.03 where $Re_D = 3.0e3$. 

Refer to Figure 7.23 for legend.
Figure 7.25. Effect of S/D on time-mean flow in first and third wakes of a pin-fin array having H/D = 1, X/D = 3.03 where $Re_D = 2.0e4$. 

![Flow Diagram]
Figure 7.26. Effect of S/D, row position, and downstream obstruction on wake length for pin-fin arrays having H/D = 1, X/D = 3.03 where Re_D = 2.0e4.
Figure 7.27. Effect of S/D on velocity profile in the first row wake at X/D = 1, Z/H = 0 for pin-fins having H/D = 1, X/D = 3.03, Re_D = 2.0e4.
Figure 7.28. Effect of S/D on velocity profile in the third row wake at X/D = 1, Z/H = 0 for pin-fins having H/D = 1, X/D = 3.03, Re_D = 2.0e4.
Figure 7.29. Effect of S/D on average, turbulent normal stress in first and third wakes of a pin-fin array having H/D = 1, X/D = 3.03 where Re_D = 2.0e4.
Figure 7.30. Effect of S/D on $<u'u'>/U_{max}$ at X/D = 1, Z/H = 0 for the first row wake of a pin-fin array having H/D = 1, X/D = 3.03 where $Re_D = 2.0e4$. 
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Figure 7.32. Effect of S/D on the POD energy distribution in the first row wake of a pin-fin array row having H/D = 1, X/D = 3.03 where \( \text{Re}_D = 3.0 \times 10^3 \).
Figure 7.33. Effect of S/D on the POD energy distribution in the first row wake of a pin-fin array row having H/D = 1, X/D = 3.03 where Re_D = 2.0e4.

Figure 7.34. Effect of S/D on energy content of first two POD modes in the first row wakes of pin-fin arrays having X/D = 3.03.
Figure 7.35. Effect of S/D on the POD energy distribution in the third row wake of a pin-fin array row having H/D = 1, X/D = 3.03 where Re_D = 3.0e3.

Figure 7.36. Effect of S/D on the POD energy distribution in the third row wake of a pin-fin array row having H/D = 1, X/D = 3.03 where Re_D = 2.0e4.
Figure 7.37. Effect of $S/D$ on energy content of first two POD modes in the third row wakes of pin-fin arrays having $X/D = 3.03$. 
Figure 7.38. Effect of S/D on endwall heat transfer for pin-fin arrays having H/D = 1, X/D = 3.03 where Re_D = 3.0e3.
Figure 7.39. Effect of S/D on laterally averaged endwall heat transfer for pin-fin arrays having H/D = 1, X/D = 3.03 where Re_D = 3.0e3.

Figure 7.40. Effect of S/D on laterally averaged endwall augmentation for pin-fin arrays having H/D = 1, X/D = 3.03 where Re_D = 3.0e3.
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Figure 7.64. Effect of S/D on the POD energy distribution in the third row wake of a pin-fin array row having H/D = 1, X/D = 2.16 where Re_D = 2.0e4.
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Figure 7.66. Effect of S/D on endwall heat transfer for pin-fin arrays having $H/D = 1$, $X/D = 2.16$ where $Re_D = 3.0e3$. 
Figure 7.67. Effect of S/D on laterally averaged endwall heat transfer for pin-fin arrays having H/D = 1, X/D = 2.16 where Re_D = 3.0e3.

Figure 7.68. Effect of S/D on laterally averaged endwall augmentation for pin-fin arrays having H/D = 1, X/D = 2.16 where Re_D = 3.0e3.
Figure 7.69. Effect of S/D on endwall heat transfer for pin-fin arrays having H/D = 1, X/D = 2.16 where Re_D = 2.0e4.
Figure 7.70. Effect of S/D on heat transfer in first row wake for arrays having X/D = 2.16. Shedding was partially attenuated for S/D = 3, X/D = 2.16, yet the heat transfer contours are similar.

Figure 7.71. Effect of S/D on laterally averaged endwall heat transfer for pin-fin arrays having H/D = 1, X/D = 2.16 where Re_D = 2.0e4.
Figure 7.72. Effect of S/D on laterally averaged endwall augmentation for pin-fin arrays having H/D = 1, X/D = 2.16 where Re_D = 2.0e4.
CHAPTER 8:
HEAT TRANSFER AND ARRAY PERFORMANCE

In previous chapters, detailed flowfield and local heat transfer measurements were presented for various pin-fin geometries and Reynolds number conditions. In the design of pin-fin arrays, however, empirical correlations are commonly used to interpolate between experimental data points to provide a prediction of heat exchanger performance at the targeted pin-fin geometry and Reynolds number condition. In addition, empirical correlations provide insight into the physical mechanisms responsible for heat transfer. The Reynolds number exponent, for example, describes the influence of Reynolds number on array-averaged heat transfer. In Section 8.1, the array-averaged heat transfer data for the present work will be compared with previously developed empirical correlations to determine how Reynolds number and other parameters, such as pin-fin spacing, influence array-averaged heat transfer. Similarly, Section 8.2 evaluates an existing correlation for pin-fin stagnation point heat transfer.

As mentioned in Chapter 3, the IR thermography method used in the present work allowed the simultaneous measurement of the pin-fin and endwall surfaces. And, the present work is one of only a few studies which measure pin-fin and endwall heat transfer simultaneously with heated pin-fins and endwalls. Section 8.3 addresses the independent contribution of the pin-fin and endwall surfaces to the overall, or combined, heat transfer in the array.

In addition to independently measuring pin-fin and endwall heat transfer, the use of IR thermography allowed for row-by-row heat transfer averaging. Section 8.4 presents correlations for pin-fin and endwall heat transfer on a row-averaged basis. The row-averaged empirical constants were used to determine the effect of row-position on pin-fin and endwall heat transfer.

The design of pin-fin heat exchangers depends on the array pressure drop in addition to heat transfer. Array pressure drop was non-dimensionalized to a friction factor. For each geometric configuration and Reynolds number condition of the present work, friction factor measurements were taken across the pin-fin array as described in Chapter 3. In general, reducing pin-fin friction factors is desirable for gas turbine engines since the pumping power required for turbine cooling is a parasitic loss on the engine. In
some cases, however, the differential pressure between the inlet and exit of the pin-fin channel are specified by the system design to provide the desired coolant. In this case, it is more desirable to use a pin-fin configuration having a friction factor that is appropriate for the allotted pressure differential in comparison to using the pin-fin configuration with the lowest friction factor. Regardless of the targeted pin-fin friction factor, an accurate empirical correlation is important for pin-fin design. In Section 8.5, the previously developed friction factor correlations are evaluated and a new correlation is developed. As with the heat transfer analysis, the empirical correlations were used to interpret which parameters governed the friction factor.

Finally, array performance was computed for each test case by normalizing heat transfer data with an equivalent pumping power, proportional to the cube root of pressure drop (Han et al. 2000). The resulting array performance index is commonly used as a means of comparing different pin-fin geometries with one another while accounting for both heat transfer and pressure drop. In Section 8.6, the array performance index is used to describe how independent changes in both streamwise and spanwise pin-fin spacing affect both heat transfer and pressure drop.

### 8.1 Array-Averaged, Combined Heat Transfer

In this section the array-averaged, combined heat transfer was calculated for each pin-fin geometry in the present work. The definition of combined heat transfer was shown previously in equation 3.10 and refers to the area-weighted average of the pin-fin and endwall heat transfer for the entire array. As mentioned in Chapter 1, the array-averaged heat transfer coefficient effectively decouples the effect of the total surface area from the flowfield effects. It was also mentioned in Chapter 1, however, that the ratio of pin-fin-to-overall surface area may influence the array-averaged heat transfer. For example, assuming the pin-fin heat transfer coefficient is higher than the endwalls, then the array-averaged heat transfer will be higher for an array having closer S/D and X/D because the ratio of pin-fin-to-overall surface area is higher. The present section, only considers array-averaged heat transfer. Section 8.3 addresses the pin-fin and endwall heat transfer separately for the purpose of decoupling the ratio of pin-fin-to-overall surface area from the flowfield effects.
The present array-averaged, combined heat transfer was predicted using several existing correlations. Table 8.1 summarizes the limits of the pin-fin configurations that were used to develop each of the existing correlations. In a 1988 review paper, Armstrong and Winstanley (1988) showed that correlations developed by VanFossen (1982) and Metzger et al. (1986) both accurately predicted array-averaged, combined heat transfer for pin-fins having $0.5 \leq H/D \leq 3$, $2 \leq S/D \leq 4$, $1.5 \leq X/D \leq 5$. Note that the correlations were developed using the parameters shown in Table 8.1 and Armstrong and Winstanley (1988) applied the correlations to a larger set of data to recommend an improved set of correlation limits. The correlation of VanFossen (1982) makes use of a flow-volume length scale, shown in equation 8.1, to account for changes in $H/D$, $S/D$, and $X/D$. The flow-volume length scale is also illustrated in Figure 8.1.

$$\frac{L_{4V/A}}{D} = \frac{(H/D)[4(X/D)(S/D) - \pi]}{2(X/D)(S/D) + \pi(H/D - 0.5)} \quad (8.1)$$

The Nusselt number and Reynolds number definitions used by VanFossen (1982) are shown in equation 8.2 and equation 8.3, respectively, and the array-averaged, combined heat transfer correlation is given in equation 8.4.

$$Nu_{4V/A} = Nu_D \frac{L_{4V/A}}{D} = Nu_D \frac{(H/D)[4(X/D)(S/D) - \pi]}{2(X/D)(S/D) + \pi(H/D - 0.5)} \quad (8.2)$$

$$Re_{4V/A} = Re_D \left( \frac{(H/D)[4(X/D)(S/D) - \pi]}{2(X/D)(S/D) + \pi(H/D - 0.5)} \right) \left( \frac{(S/D - 1)}{(S/D) - (\pi/4)(X/D)^{-1}} \right) \quad (8.3)$$

$$Nu_{4V/A} = 0.153 Re_{4V/A}^{0.685} \quad (8.4)$$

When predicting heat transfer using an empirical correlation, Armstrong and Winstanley (1988) corrected the measured heat transfer data to account for the number of rows in an array. A weighting factor was calculated using the generalized row-average heat transfer development observed by Metzger et al. (1986), which is shown in Figure 2.7. The weighting factor normalized the heat transfer to match that of a ten row array. For
example, a four-row array required a heat transfer increase to match the heat transfer in a ten-row array. For the present data, no correction was applied since the seven-row arrays used in the present work had negligible difference when compared with ten-row arrays.

Figure 8.2 shows array-averaged heat transfer measured in the present work in comparison with the correlation of VanFossen (1982). In general, good agreement was observed with the VanFossen (1982) correlation. The largest discrepancy was observed for those cases having $S/D = 2$ at $Re_D = 3e3$. As a measure of correlation fit, the mean-squared-error (MSE) and the coefficient of determination ($R^2$) were computed to be $MSE = 2.19$ and $R^2 = 0.81$. The coefficient of determination was described by Colban et al. (2011) who note that there is no widely accepted value of $R^2$ that classifies a curve-fit as being acceptable. Rather, the $R^2$ criteria depends on the application. In general, a better correlation fit will result in MSE approaching zero and an $R^2$ value approaching unity.

The Reynolds number exponent of 0.685 in equation 8.4 fell between 0.556 (Grimison 1937), which is the value for long-tube data having $S/D = 2$ and $X/D = 2$, and 0.8 (McAdams 1942), which is the value for empty channel flow. The Reynolds number exponent, therefore, showed that contributions from the pin-fins and from the endwalls were both important to array-averaged, combined heat transfer.

Rather than using a flow-volume length scale that accounts for changes in $H/D$, $S/D$, and $X/D$, Metzger et al. (1986) correlated array-averaged heat transfer as a function of $Re_D$ and $X/D$ as shown in equation 8.5.

$$Nu_D = 0.135 Re_D^{0.69} (X/D)^{-0.34}$$

The effects of spanwise spacing are captured with $Re_D$ because the velocity scale, $U_{max}$, is dependent on $S/D$. Figure 8.3 shows good agreement between the present pin-fin configurations and the correlation of Metzger et al. (1986). The present configurations were predicted with better accuracy using the simple correlation of Metzger et al. (1986) than that of VanFossen (1982). The correlation fit metrics were $MSE = 1.35$ and $R^2 = 0.84$. To compare the correlation fit metrics between the various correlations, Figure 8.4 and Figure 8.5 show the MSE and $R^2$ values, respectively, for each correlation. It should be noted that Figure 8.4 and Figure 8.5 include two only datasets, the present work and a
compilation of data including the present work. The compilation of data is summarized in Table 8.2 and will be discussed in more detail later in this section.

The Reynolds number exponent of equation 8.5 was 0.69, almost equal to that of VanFossen (1982) despite the different length scale used in the definition of Reynolds number. The exponent on the streamwise spacing term was -0.34. Consistent with Chapter 6 of the present work, the X/D exponent showed that decreasing streamwise spacing resulted in increased heat transfer. From the exponents in equation 8.5, a simple hand calculation can provide a first-order approximation of Reynolds number and streamwise spacing effects. For example, doubling Reynolds number increases heat transfer by 61% while halving streamwise spacing increases heat transfer by 27%. In Chapter 7 of the present work, it was found that decreasing spanwise spacing increased heat transfer for a constant Re_D and the effects of spanwise spacing were more significant at lower Reynolds numbers. It was interesting that Metzger et al. (1986) did not include a term to account for S/D effects. In the next section, the effects of S/D are included in heat transfer correlations to capture the S/D dependence observed in the present work.

Armstrong and Winstanley (1988) reviewed a third correlation for array-averaged heat transfer developed by Faulkner (1971). The correlation of Faulkner (1971) used a length scale similar to that of VanFossen (1982) to account for changes in H/D, S/D, and X/D. In fact, Faulkner (1971) used the same flow-volume Reynolds number as VanFossen (1982). Faulkner’s (1971) correlation was formulated differently, however, to resemble a blend between long-tube data and empty duct data. One advantage of Faulkner’s (1971) correlation was that heat transfer approached empty duct heat transfer for very wide spanwise spacings. The Nusselt number was calculated using equation 8.6, and the correlation is shown in equation 8.7.

\[
\text{Nu}_{\text{Faulkner}} = \text{Nu}_D \frac{4(X/D)(H/D)(S/D-1)}{2(X/D)(S/D) + \pi(H/D-0.5)}
\]

\[
\text{Nu}_{\text{Faulkner}} = [0.023 + 4.143 \exp \left( -3.094 \left( \frac{S}{D} \right)^{-1} + 0.89 \left( \frac{S}{D} / D \right)^{0.5075} \right) \text{Re}^{-0.2946} \text{Pr}^{0.8} \text{Re}_{AV}^{0.8} \text{Pr}^{1/3}]
\]
Armstrong and Winstanley (1988) recommended the Faulkner (1971) correlation for pin-fin arrays having $H/D > 3$, $S/D < 4$, $1.5 \leq X/D \leq 3.4$. The present data fell within the range of cylinder spacings used to develop the Faulkner (1971) correlation. Figure 8.6, however, shows that the present heat transfer measurements were over-predicted by the correlation of Faulkner (1971), mainly because the pin-fin aspect ratio was $H/D = 1$ and outside of the limits recommended by Armstrong and Winstanley (1988). The correlation fit parameters were $\text{MSE} = 1.64$ and $R^2 = 0.77$, as shown in Figure 8.4 and Figure 8.5, which indicated that the Faulkner (1971) correlation did not perform as well as Metzger et al. (1986) or VanFossen (1982).

There were two terms in the Faulkner (1971) correlation having Reynolds number exponents of 0.8 and 0.51. Faulkner (1971), therefore, assumed a Reynolds number exponent of 0.51 for the contribution from long-tube cylinders and an exponent of 0.8 for the contribution from the empty channel.

In 2009, Chyu et al. (2009) presented several correlations specific to pin-fin arrays having $S/D = 2.5$, $X/D = 2.5$ and pin-fin aspect ratios of $H/D = 1, 2, 3$, and 4. The correlations used by Chyu et al. (2009) are shown in equations 8.8 through 8.11.

\[
\begin{align*}
\text{Nu}_D &= 0.14 \text{Re}_D^{0.65} \quad \text{for } H/D = 1 \\
\text{Nu}_D &= 0.061 \text{Re}_D^{0.727} \quad \text{for } H/D = 2 \\
\text{Nu}_D &= 0.188 \text{Re}_D^{0.626} \quad \text{for } H/D = 3 \\
\text{Nu}_D &= 0.110 \text{Re}_D^{0.708} \quad \text{for } H/D = 4
\end{align*}
\]

It should be noted that Chyu et al. (2009) presented different correlation constants in the publication manuscript than shown in equations 8.8 through 8.11. The published constants yielded results that were not self-consistent with data in the manuscript. It was believed that there was confusion on the choice of Reynolds number and Nusselt number, so equations 8.8 through 8.11 were obtained by extracting data from the manuscript of Chyu et al. (2009) and using a least-squares regression to obtain the correlation constants. The resulting equations 8.8 through 8.11 produced data that was consistent with Chyu et
al. (2009) and produced good agreement with the present results. When equations 8.8 through 8.11 were applied to the data of Chyu et al. (2009), the resulting correlation fit parameters were MSE = 0.51 and \( R^2 = 0.98 \). Figure 8.7 shows the array-averaged, combined heat transfer for the present work in comparison to the correlation of Chyu et al. (2009). The correlation fit metrics were MSE = 1.10 and \( R^2 = 0.89 \) when applied to the present data. Although the correlation of Chyu et al. (2009) showed the best correlation fit metrics when applied to the present work, as shown in Figure 8.4 and Figure 8.5, it is important to note that the present data only considered H/D = 1 and, therefore, only used equation 8.8. In the work of Chyu et al. (2009), however, two different experimental techniques were used. The mass transfer analogy was used for H/D = 1 (Chyu et al. 1999) and transient methods were used for H/D = 2, 3, and 4 (Chyu et al. 2009). As such, the Chyu et al. (2009) correlation was validated against a wider range of H/D values to properly evaluate its performance, and will be addressed in a following discussion.

The Reynolds number exponents in equations 8.8 through 8.11 varied between 0.626 and 0.727 and showed no clear trend with increasing aspect ratio. For increasing H/D, it would be expected that the Reynolds number exponent approach the value for long-tube data, which is 0.556 for S/D = 2 and X/D = 2 (Grimison 1937). Better results may be achieved using an additional term to account for the effects of H/D, similar to equation 8.5, rather than using a separate equation for each H/D investigated.

To this point, only the present data has been applied to the correlations of VanFossen (1982), Metzger et al (1986), Faulkner (1971), and Chyu et al. (2009). Since the review of Armstrong and Winstanley (1988), however, there have been many pin-fin publications that present array-averaged heat transfer data. And, no study has collectively applied more recent data to the existing correlations in equations 8.2 through 8.11. As such, data taken after 1988 was predicted using the existing correlations for array-averaged, combined heat transfer. The studies used to evaluate the existing correlations are shown in Table 8.2. It should be noted that the data of Metzger et al. (1982a) was included, despite being pre-1988, because the dataset includes a wide range of X/D spacings, between 1.5 ≤ X/D ≤ 5. The resulting correlation fit metrics for the compiled data are shown in Figure 8.4 and Figure 8.5.
The correlation of VanFossen (1982) predicted heat transfer well for H/D = 1, but had difficulty with H/D > 1, as shown in Figure 8.8. In fact, a follow-up study by Simoneau and VanFossen (1984) showed that heat transfer was constant for 0.5 ≤ H/D ≤ 2 and increased for H/D > 2 when using the flow-volume length scale defined by VanFossen (1982). The flow-volume length scale, therefore, did not adequately capture H/D effects for large values. The correlation of Metzger et al. (1986), shown in Figure 8.9, gave the best array-averaged heat transfer prediction for the wide range of geometries considered in the compilation of recent data in Table 8.2. It was interesting that the correlation of Metzger et al. (1986) produced better results than VanFossen (1982) considering pin-fin aspect ratio was not accounted for in the Metzger correlation. Figure 8.10 shows the correlation of Faulkner (1971), which also performed well over a wide range of geometries. The largest error in the Faulkner (1971) predictions was for a pin-fin array having H/D = 1, S/D = 2.5, X/D = 1.5 (Metzger et al. 1982a). Finally, Figure 8.11 shows the correlation of Chyu et al. (2009). The correlation of Chyu et al. (2009) had the most difficulty in predicting heat transfer for H/D = 2.2, S/D = 2, X/D = 1.5 (Al-Dabagh and Andrews 1992) and H/D = 1, S/D = 2.5, X/D = 1.5 (Metzger et al. 1982a).

From this analysis of array-averaged, combined heat transfer, it was found that the correlation of Metzger et al (1986) gave the best predictions for a compilation of a wide range of pin-fin geometries. The Metzger et al (1986) correlation had a Reynolds number dependence of \( Re_D^{0.69} \) and a streamwise spacing dependence of \( (X/D)^{-0.34} \). Array-averaged heat transfer, therefore, was more dependent on \( Re_D \) than streamwise spacing.

### 8.2 Correlation for Pin-Fin Stagnation Point Heat Transfer

It has been shown that array-averaged heat transfer depends on Reynolds number, streamwise spacing, and spanwise spacing. Few studies have been able to predict local heat transfer in the pin-fin array. Ames et al. (2005), however, were able to predict the local stagnation point heat transfer on the pin-fin surfaces using the TRL parameter (turbulence intensity, Reynolds number, and dissipation length scale). In Chapter 2, the TRL parameter was estimated to determine the effects of S/D on pin-fin heat transfer. In the present section, the TRL parameter was calculated from TRDPIV data. The
measurement locations for calculating the TRL parameter are shown in Figure 8.12. The
dissipation rate was first calculated by fitting the measured, one-dimensional energy
spectrum to the following equation:

$$E(k) = 1.62 \left( \frac{18}{55} \right) \varepsilon^{2/3} k^{-5/3}$$  \hspace{1cm} (8.12)$$

where $E(k)$ is the energy content at a given wavenumber, $k$ is the wavenumber, and $\varepsilon$ is
the dissipation rate. Equation 8.12 was only applied to the inertial subrange. The
dissipation length scale was then calculated using equation 8.1.

$$L_u = \frac{1.5|u'|^3}{\varepsilon}$$  \hspace{1cm} (8.13)$$

The turbulence intensity at the stagnation point was estimated to include decay between
the measurement location and the stagnation point using

$$Tu(X) = \frac{1}{1 + \frac{X}{Tu(0)}} \frac{X}{2L_u}$$  \hspace{1cm} (8.14)$$

where $Tu(0)$ is the turbulence intensity at the measurement location, $X$ is the distance
between the measurement location and stagnation point, and $Tu(X)$ is the turbulence
intensity at the stagnation point. The effective Reynolds number was the only value in
the TRL parameter that was estimated because static pressure measurements on the pin-
fin were not available. The effective Reynolds number was interpolated from the data of
Ames et al. (2005). For example, the effective Reynolds number was given by Ames et
al. (2005) as a function of row number at $Re_D = 3.0e4$. Because the present work made
measurements at $Re_D = 2.0e4$, the effective Reynolds number throughout the array of
Ames et al. (2005) was multiplied by $2/3$ for use with the present data. The present work
used a seven row array having $H/D = 1$, $S/D = 2$, $X/D = 2.16$ while Ames et al. (2005)
used a five row array having \( H/D = 2, S/D = 2.5, X/D = 2.5 \). Figure 8.13 shows the present results for pin-fin stagnation point heat transfer. It should be noted that the pin-fin stagnation point heat transfer was measured for the heated pin-fin test case where the inlet velocity was used as the reference temperature. From Figure 8.13, the present results follow the trend of Ames et al. (2005), but fall slightly below the correlation by about 10%. A possible source of uncertainty was the use of TRDPIV to measure the turbulent dissipation rate and dissipation length scale. Specifically, the limited number of samples with TRDPIV data caused more noise in the energy spectra than typical hot-wire measurements. Nonetheless, the correlation of Ames et al. (2005) and the present results showed that stagnation point heat transfer increased with increasing TRL parameter. Increasing turbulence intensity, decreasing the dissipation length scale, and increasing the approaching velocity all contributed to increased stagnation point heat transfer. The correlation of Ames et al. (2005), in equation 2.3, showed that turbulence intensity had an exponent of 1, the approaching Reynolds number had an exponent of 5/24, and dissipation length scale had an exponent of -1/3. The exponents of equation 2.3 showed that increases of turbulence intensity were most important, followed by decreases in dissipation length scale, and finally by increases in approaching Reynolds number.

In agreement with previous chapters, the correlation of Ames et al. (2005) showed that heat transfer (on the pin-fin surface) was strongly dependent on the local turbulence level. In previous chapters, turbulence was separated from periodic unsteadiness and turbulence was found to correlate with increased heat transfer. The correlation of Ames et al. (2005), however, does not distinguish between periodic unsteadiness and turbulence in the definition of turbulence intensity.

8.3 Contribution from Pin-Fin and Endwall Surfaces

The major advantage of using IR thermography was the independent and simultaneous measurement of pin-surface and endwall heat transfer. Previous chapters have shown good agreement between the present endwall heat transfer measurements and that of previous studies. To compare the present pin-fin heat transfer with previous studies, Figure 8.14 shows the array-averaged heat transfer contribution from only the pin-fins. Good agreement was observed between the present study and that of previous
pin-fin heat transfer measurements. The present measurements did, however, show slightly lower pin-fin heat transfer than other previous studies. The magnitude of the pin-fin heat transfer coefficient was a result of the choice of reference temperature in the definition of heat transfer coefficient. When using the IR thermography method, the endwalls were thermally active and heat transfer coefficients on the pin-fin surface were calculated using the bulk temperature as defined in Chapter 3. In most previous experiments, however, the pin-fin surface is the only thermally active surface and the inlet temperature is used as the reference temperature (Lyall et al. 2011; Lawson et al. 2011; Ames et al. 2005; Metzger and Haley 1982). Chyu et al. (1999) measured pin-fin mass transfer with (thermally) active walls, but the mass transfer analogy used an empirically determined constant to convert mass transfer coefficients to heat transfer coefficients. Al Dabagh and Andrews (1992) also had thermally active endwalls when measuring pin-fin heat transfer, but the experimental setup may have introduced significant lateral conduction which would influence the measured pin-fin heat transfer. In the present work, pin-fin heat transfer was investigated in two separate experiments. In one experiment, the endwalls were heated and the bulk temperature was used as the reference temperature. In another experiment, only the pin-fin was heated and the inlet temperature was used as the reference temperature.

To further emphasize the choice of reference temperature and endwall boundary condition, Figure 8.15 shows the pin-fin heat transfer for the case having H/D = 1, S/D = 2, X/D = 2.16 at Re$_D$ = 2.0e4. Both a constant heat flux pin-fin (using inlet temperature as a reference) and a constant heat flux endwall with coppers pins (using bulk temperature as a reference) were included in Figure 8.15. From Figure 8.15, the use of a heated endwall resulted in significantly lower heat transfer. The lower heat transfer when using thermally active endwalls explains why the present measurements were lower than previous studies in Figure 8.14. For further discussion on calculating the pin-fin heat transfer coefficient using the IR thermography method, refer to Appendix A.

Using the present dataset, several correlations were developed to describe heat transfer as a function of Reynolds number, streamwise spacing, and spanwise spacing. Correlations were developed for the overall/combined heat transfer, for the pin-fin surfaces only, and for the endwall surfaces only. The correlation of Metzger et al.
(1986), discussed in Section 8.1, provided the best results for a wide range of Reynolds numbers and geometries. Also, the form of the Metzger et al. (1986) equation was very simple to compute as shown in equation 8.15.

\[ \text{Nu}_D = C_1 \text{Re}_D^{m_1}(X/D)^{m_2} \quad (8.15) \]

Using the skeleton form of the Metzger et al. (1986) equation, the constants \( C_1, m_1, \) and \( m_2 \) were determined using a least-squares regression, and the resulting correlation constants and correlation fit metrics are shown in Table 8.3. The present data was accurately predicted using these correlations because \( R^2 \) value never fell below 0.95 and MSE never exceeded 0.8. For array-averaged heat transfer on the pin-fins, endwalls, and combined, the correlation constants were similar. The Reynolds number exponent was between 0.57 and 0.59 and the streamwise spacing exponent was -0.20 for each configuration. The correlation constants developed for the present work showed a lower Reynolds number exponent than 0.69, given by Metzger et al. (1982a). A similar analysis was performed using a modified version of the Metzger et al. (1986) equation shown below in equation 8.1.

\[ \text{Nu}_D = C_1 \text{Re}_D^{m_1}(X/D)^{m_2}(S/D)^{m_3} \quad (8.16) \]

The present data was applied to equation 8.1 and the constants \( C_1, m_1, m_2, \) and \( m_3, \) were determined using a least-squares regression. The resulting correlation constants and correlation fit metrics are shown in Table 8.3. Again, the correlation fit metrics showed good agreement with the present data where the maximum MSE value was 0.72 and the minimum \( R^2 \) value was 0.96 when considering combined, endwall, and pin-fin heat transfer. For array-averaged, combined heat transfer, the correlation constants showed a Reynolds number exponent of 0.57, a streamwise spacing exponent of -0.2, and a spanwise spacing exponent of -0.24. This result showed that the effects of spanwise spacing were slightly more influential on array-averaged, combined heat transfer than streamwise spacing. For array-averaged, endwall heat transfer, the correlation constants showed a Reynolds number exponent of 0.56, a streamwise spacing exponent of -0.2, and
a spanwise spacing exponent of -0.26. For array-averaged, pin-fin heat transfer, the correlation constants showed a Reynolds number exponent of 0.58, a streamwise spacing exponent of -0.26, and a spanwise spacing exponent of -0.26. The important result from these correlations was the similar exponent on streamwise and spanwise spacing. Consider endwall heat transfer for example, a 25% reduction in streamwise spacing results in a 6% increase in endwall heat transfer (averaged over the entire seven rows). When reducing spanwise spacing by 25%, endwall heat transfer was increased by 8% (averaged over the entire seven rows).

8.4 Row-Averaged Heat Transfer

In gas turbine applications, conduction along the channel walls may result in a constant temperature boundary condition or a mixed boundary condition, depending on the wall Biot number, at the interface between the wetted area and the coolant. Figure 8.16 illustrates the difference in boundary conditions for the present experimental setup and that of a gas turbine pin-fin channel (showing constant wall temperature). In the experimental setup, the use of a constant heat flux boundary condition was chosen for simplicity. One important consequence of the boundary conditions in Figure 8.16 is the diminishing driving temperature difference between the walls and the fluid for the engine application (Busche et al. 2012). Stated simply, the cooling capacity diminishes as flow progresses through the array. The rate at which the cooling capacity diminishes depends on heat transfer as a function of streamwise location. It is, therefore, important to accurately predict the row-by-row heat transfer development to properly account for the diminishing cooling capacity as the flow progresses. The current method used to account for row effects is to apply a weighting factor given by Metzger et al. (1986), and shown previously in Figure 2.7. However, the generalized weighting factor has very large uncertainty caused by pin-fin spacing and Reynolds number effects. The goal of this section was to improve upon the accuracy of the generalized row-weighting function in addition to investigating the physics associated with row effects.

For each row, the pin-fin and endwall Nusselt number was cast into the form shown in equation 8.15. The MSE was then minimized by iterating for the correlation constants in equation 8.15. The resulting correlation constants for endwall heat transfer
as a function of row position are shown in Table 8.4 and the constants for pin-fin heat transfer are shown in Table 8.5. Inspecting the correlation constants provided insight into the heat transfer mechanisms present in each row.

The Reynolds number exponent ranged between 0.54 and 0.64 for both the pin-fins and endwalls. There was no clear dependence of Reynolds number exponent on row position. The streamwise spacing exponent showed a dependence on row position. For the endwall surfaces the first row showed a streamwise spacing exponent of zero which indicated that the first row was independent of streamwise spacing. In the second row, the exponent increased to 0.16, which indicated that increased streamwise spacing resulted in increased heat transfer. Similar to the first row, the endwall heat transfer in the third row had streamwise spacing exponent near zero at a value of -0.05. In the fourth row and beyond, the streamwise spacing exponent was between -0.31 and -0.40. The endwall heat transfer was, therefore, strongly dependent on streamwise spacing for the fourth row and beyond and heat transfer increased for decreasing X/D because of the negative sign on the streamwise spacing exponent.

A similar analysis was performed by inspecting the correlation constants for the pin-fin surfaces. As mentioned previously, the Reynolds number exponent for pin-fin heat transfer showed no clear dependence on row position. The first two rows showed that heat transfer increased with increasing streamwise spacing from the positive sign of the streamwise spacing exponent. In the third row and beyond, the streamwise spacing exponent was negative and decreasing X/D resulted in increased heat transfer. For third row pin-fin heat transfer, the largest streamwise spacing exponent was observed at a value of -0.50. This result showed that the third row pin-fin surface was the position in the pin-fin array that was most influenced by the streamwise spacing.

The row-averaged correlations for pin-fin and endwall heat transfer confirmed the flow mechanisms responsible for heat transfer observed Chapter 6 and Chapter 7. Heat transfer in the first two rows was governed by a different mechanism than heat transfer in the third through seventh rows. The present analysis showed that streamwise spacing had little influence on heat transfer in the first two rows. This was observed in the previous chapters with the exception that low Reynolds numbers benefit from wider streamwise spacing to utilize the horseshoe vortex. In downstream rows, heat transfer benefits from
the breakdown of vortices into turbulence. The present analysis shows that decreasing streamwise spacing in downstream rows increases heat transfer on both pin-fins and endwalls. Decreased streamwise spacing has been shown to facilitate the breakdown of vortices into turbulence. Most notable was the comparison of a non-uniform array with a uniform array having $X/D = 2.16$ at low Reynolds numbers. In the rows 4-6 of the non-uniform array, the streamwise spacing between pin-fins was $X/D = 2.16$ yet heat transfer was reduced for the non-uniform array. The breakdown of vortices to turbulence was facilitated for the uniform array having $X/D = 2.16$ because the flow was more turbulent in upstream rows in comparison to the non-uniform array which had a more organized flow in upstream rows (because $X/D \geq 2.60$ in the first three rows).

The non-uniform pin-fin geometry provided an opportunity to test the row-averaged correlations, which were developed using only uniformly spaced pin-fin arrays. Figure 8.17 and Figure 8.18 show the predicted heat transfer on the endwalls and pin-fins, respectively, for a low Reynolds number case having $Re_D = 3.0e3$. Similarly, Figure 8.19 and Figure 8.20 show the predicted heat transfer on the endwalls and pin-fins, respectively, for a high Reynolds number case having $Re_D = 2.0e4$. For better comparison, the percent difference of the predicted heat transfer relative to the measured value is shown for each row, each surface, and each Reynolds number in Figure 8.21. Six of twenty-eight predictions were beyond the experimental uncertainty of 9.3%. No prediction exceeded 16% difference relative to the measured value.

For additional validation, the endwall heat transfer measured by Ames et al. (2007) was predicted using the present row-averaged, endwall heat transfer correlations. Ames et al. (2007) measured endwall heat transfer for a seven row pin-fin array having $H/D = 2$, $S/D = 2.5$, $X/D = 2.5$. It should be noted that only the endwalls were heated and the pin-fins were adiabatic. Figure 8.22 shows the predicted endwall heat transfer of Ames et al. (2007) at $Re_D = 3.0e3$, Figure 8.23 at $Re_D = 1.0e4$, and Figure 8.24 at $Re_D = 3.0e4$. The predictions showed a similar row-averaged trend when compared with the measurements of Ames et al. (2007). The percent difference of the predicted endwall heat transfer relative to the measured heat transfer of Ames et al. (2007) is shown in Figure 8.25. The average percent difference for the low Reynolds number case was 21%, which was quite higher than the measurement uncertainty quoted by Ames et al. (2007)
of 12% for \( Re_D = 3.0 \times 10^3 \). In Figure 8.25, The two higher Reynolds numbers showed an average percent difference to within the experimental uncertainty. Although the row-by-row trends were captured well by the present endwall correlations, the array-averaged magnitude varied with Reynolds number when applied to the data of Ames et al. (2007). It was unclear if the variation in Reynolds number was caused by a different aspect ratio (Ames et al. (2007) used \( H/D = 2 \) and the present work used \( H/D = 1 \)), or if the variation was caused by differences in the experimental setups (Ames et al. (2007) used adiabatic pin-fins and the present work used thermally active pin-fins).

As a first-order approximation, the row-averaged correlations summarized in Table 8.4 and Table 8.5 were able to capture the row-by-row heat transfer development trends for both a uniformly spaced array (Ames et al. 2007) and a non-uniform array having variable streamwise spacing. The correlations developed in this section were unable to predict the magnitude, but predicted the row dependence of endwall heat transfer at low Reynolds number by Ames et al. (2007).

### 8.5 Array Friction Factor

In the review of Armstrong and Winstanley (1988), it was found that friction factor in pin-fin arrays was accurately predicted using a correlation developed by Metzger et al. (1982b) shown in equations 8.17 and 8.18.

\[
\begin{align*}
    f_N &= 0.317 R e_D^{-0.132} \text{ for } 10^3 \leq Re_D \leq 10^4 \\
    f_N &= 1.76 R e_D^{-0.318} \text{ for } 10^4 \leq Re_D \leq 10^5
\end{align*}
\]

(8.17)  
(8.18)

Note that the definition of friction factor used in equations 8.17 and 8.18 was shown previously in Chapter 3. The friction factor correlations in equations 8.17 and 8.18 were developed for pin-fins having \( H/D = 1, \ S/D = 2.5, \ 1.5 \leq X/D \leq 5 \). Armstrong and Winstanley (1988), however, showed the correlations to be applicable for \( 0.5 \leq H/D \leq 6, \ 2 \leq S/D \leq 4 \). Figure 8.26 shows the performance of the Metzger et al. (1982b) correlation when applied to a compilation of data taken in recent years. The geometric configurations of the data in Figure 8.26, shown in Table 8.6, were within the limits suggested by Armstrong and Winstanley (1988). The correlation fit parameters
corresponding to Figure 8.26 were $\text{MSE} = 0.02$ and $R^2 = 0.55$. The $R^2$ value was much lower than that observed for heat transfer correlations. Figure 8.27 shows friction factor plotted against Reynolds number. It was found that the largest discrepancy occurred in the dataset of Lawson et al (2011) for a case having $\text{Re}_D = 5\times10^3$, $S/D = 4$, $X/D = 3.46$. The wide spanwise spacing of this configuration, $S/D = 4$, was on the border of the correlation limits recommended by Armstrong and Winstanley (1988). Several other cases are marked on Figure 8.27 to illustrate which geometric configurations fell outside the ±20% bands. Clearly, there were spanwise spacing effects unaccounted for in the Metzger et al. (1982b) correlation.

Similar to the previous correlations developed in this chapter, a new correlation for friction factor was developed to take streamwise spacing into account. The form of the friction factor equation was adopted from equation 8.15 and the constants were determined using a least-squares regression from the comprehensive dataset shown in Table 8.6. The resulting correlation for friction factor is shown in equation 8.19.

$$f_N = 0.78 \text{Re}^{0.24}_D (X/D)^{0.06}$$

The correlation fit metrics were $\text{MSE} = 0.002$ and $R^2 = 0.69$. The predicted and measured friction factors are shown in Figure 8.28. Although the prediction of friction factor was an improvement compared with the correlation of Metzger et al. (1982b), the streamwise spacing exponent was nearly zero and indicated a weak dependence with changes in $X/D$. The outliers in Figure 8.28, which are marked on the figure, had $S/D = 4$ and were much wider than the other cases considered in Table 8.6. As such, the form of friction factor in equation 8.19 was modified to include effects of spanwise spacing rather than streamwise spacing. Good agreement between measured and predicted friction factors was accomplished using equation 8.20.

$$f_N = 1.38 \text{Re}^{0.25}_D (S/D)^{0.44}$$

Figure 8.29 shows the predicted and measured array friction factors where improvement over Figure 8.28 was observed. The correlation fit metrics when using equation 8.20
were \( \text{MSE} = 0.001 \) and \( R^2 = 0.86 \). This correlation showed substantial improvement in the \( R^2 \) value when compared with the correlation of Metzger et al. (1982b) and the correlation in equation 8.19. Again, the data used to determine the correlation constants of equation 8.20 is shown in Table 8.6. The correlation of equation 8.20 showed that array friction factor was dependent on both Reynolds number and spanwise spacing. Unlike array heat transfer, however, friction factor was dependent on spanwise spacing rather than streamwise spacing. The effect of spacing on friction factor was more important than the effect of Reynolds number as observed in the exponents of equation 8.20.

8.6 Effect of Streamwise and Spanwise Spacing on Array Performance

To quantify array performance, both heat transfer and pressure drop must be accounted for in each pin-fin configuration and Reynolds number condition. A convenient means of quantify array performance is to normalize heat transfer augmentation by the cube-root of friction factor augmentation (Han et al. 2000). Using the cube-root of friction factor augmentation is a way to estimate the additional pumping power required when adding pin-fins to an empty duct. And, heat transfer augmentation is the additional heat transfer realized by adding pin-fins to an empty duct. An important distinction must be made among the definitions in friction factor used for pin-fin arrays. In Section 8.5, the definition of \( f_N \) was used, which normalizes pressure drop using \( 0.5\rho U_{\text{max}}^2 N_{\text{rows}} \). The definition used by Metzger et al. (1982b), allows friction factor data to collapse well when plotted as a function of Reynolds number. Another definition of friction factor may be used, \( f_{L/Dh} \), which is more standard for computing a duct friction factor. The definition of \( f_{L/Dh} \) normalizes pressure drop using \( 0.5\rho U_m^2 (L/D_h) \) where \( L/D_h \) is the non-dimensional length of the pin-fin array. The two friction factor normalizations were explained, in detail, in Chapter 3. It was logical, therefore, to use \( f_{L/Dh} \) because the goal was to compare the performance of one configuration to another based on the non-dimensional length of the array rather than on the number of pin-fin rows in the array. Similarly, the various definitions of Nusselt number were considered to give the most appropriate comparison between different geometries. Using the hydraulic diameter was the appropriate choice since the unobstructed duct heat transfer was calculated based on
hydraulic diameter. As such, \( \text{Nu}_{\text{Dh}} \) was used to calculate array performance rather than \( \text{Nu}_D \) or \( \text{Nu}_{4V/A} \). The resulting form of array performance is shown in equation 8.21.

\[
\frac{\text{Nu}/\text{Nu}_0}{\left(f/f_0\right)^{1/3}} = \frac{\text{Nu}_{\text{Dh}}/\text{Nu}_0}{\left(f_{L/Dh}/f_0\right)^{1/3}} = \frac{\text{Nu}_{\text{Dh}}}{f_{L/Dh}} \frac{4\left(0.078 \text{Re}_{\text{Dh}}^{-0.25}\right)}{0.023 \text{Re}_{\text{Dh}}^{0.8} \text{Pr}^{0.4}} \quad (8.21)
\]

The double over-bar in equation 8.21 designated array-averaged heat transfer. Friction factor is defined as array-averaged, so there was no double over-bar needed for \( f_{L/Dh} \).

The array performance was quantified for the present work in addition to that of Ames et al. (2005; 2007) and Lawson et al. (2011). The geometric parameters from each set of data are shown in Table 8.2 and Table 8.6. Recall that the present work makes use of pin-fin heat transfer measurements obtained from IR thermography such that the endwalls were thermally active. The data of Ames et al. (2005; 2007) and Lawson et al. (2011) both used IR thermography to capture endwall heat transfer coefficients, but both measured pin-fin heat transfer in a separate experiment where the endwalls were not thermally active. It was shown in Section 8.3 that the boundary conditions and reference temperature significantly impact the measured pin-fin heat transfer coefficient. Despite the difference in experimental procedures, good agreement was observed between the present work and that of Ames et al. (2005; 2007) and Lawson et al. (2011). When array performance was plotted as a function of \( \text{Re}_D \), shown in Figure 8.30, all data considered fell to within ±20% of a least-squares curve-fit. The case having \( H/D = 1, S/D = 4, X/D = 3.46 \) showed the highest performance across \( 5.0 \times 10^3 \leq \text{Re}_D \leq 3.0 \times 10^4 \). The wide pin-fin spacing resulted in an increase in array performance when compared with close spacings. It is known that wide pin-fin spacing generally results in reduced heat transfer when compared with closer pin-fin spacing. The increase array performance for \( H/D = 1, S/D = 4, X/D = 3.46 \) was, therefore, caused by the savings in pressure drop outweighing the reduction in heat transfer. Uzol and Camci (2005) showed a similar result when comparing circular pin-fins to ellipse-shaped pin-fins. Uzol and Camci (2005) showed that the array performance was increased for ellipse shaped pin-fins because the pressure drop savings outweighed the heat transfer reduction when compared to circular pin-fins. The data of Uzol and Camci (2005) was not included in the present analysis because heat
transfer data was only obtained in a small portion of the array as opposed to across the entire array, and a fair comparison could not be made with the present data.

To better quantify the differences in array performance between various cases, array performance was presented with heat transfer on the Y-axis and friction factor on the X-axis. Plotting data in this manner resembles a performance map where increased array performance is found for data points in the upper-left quadrant, as shown in Figure 8.31. Increasing pin-fin spacing showed a reduction in friction factor in Figure 8.31, as the curves became offset to the left for increased pin-fin spacing. Increasing spacing also resulted in reduced heat transfer, and the curves shifted down for increased spacing. It is important to note that the data of Figure 8.31 was taken at constant Re_D, which means that U_max was held constant. As discussed in Chapter 7, the bulk velocity increases with increasing spanwise spacing as an artifact of maintaining a constant U_max. While it is advantageous to have higher bulk velocity, the widened pin-fin spacing generally results in a reduction of heat transfer when compared with closer pin-fin spacing because of reduced coverage from adjacent pin-fins.

Matching U_max was the logical choice for the flowfield measurements in previous chapters because integral parameters, such as drag coefficient, show better agreement with a single, infinite cylinder when U_max is used as the characteristic velocity scale. And, in the context of this section, plotting array performance for constant Re_D (U_max) is helpful for turbine design, assuming the designer wishes to specify the characteristic Reynolds number at the minimum flow area. It may be desired, however, to select a pin-fin design based on a Reynolds number that characterizes the approaching flow in the unobstructed portion of the array. To compare pin-fin arrays based on Re_Dh, all heat transfer and friction factor measurements were carried out at constant Re_Dh (U_m) in addition to constant Re_D (U_max). Refer to Chapter 3 for a detailed explanation on the characteristic scales and Reynolds numbers in pin-fin arrays. Figure 8.32 shows the array performance map for the same pin-fin configurations as Figure 8.31 except using a constant duct Reynolds number among the various geometries. The effects of increased pin-fin spacing were more pronounced in Figure 8.32 when Re_Dh was held constant in comparison to constant Re_D. Specifically, heat transfer was reduced to a greater degree for increase spacing when Re_Dh was held constant in comparison to constant Re_D.
The array performance maps of Figure 8.31 and Figure 8.32 are useful for turbine design because they provide a convenient means of targeting a specific Nusselt number and friction factor for a wide range of geometries. Several other useful observations were made from Figure 8.31 and Figure 8.32. The cases where S/D = 2, X/D = 3.03 and S/D = 2.5, X/D = 2.16 had nearly overlapping curves on the performance plots. The case having S/D = 2.5, X/D = 2.16 showed both increased heat transfer and decreased friction factor when U_{max} was held constant. This observation shows that the use of S/D = 2.5, X/D = 2.16 will outperform an array having S/D = 2, X/D = 3.03. Similarly, the cases having S/D = 2.5, X/D = 3.03 and S/D = 3, X/D = 2.16 showed nearly overlapping performance curves. In this case, the case having S/D = 2.5, X/D = 3.03 delivered higher heat transfer and lower friction factor at all Reynolds numbers when U_{max} was held constant.

Frequently, performance maps are used to obtain general trends when controlling for a single, independent variable. In the present work, Figure 8.31 and Figure 8.32 were used to identify performance envelopes for constant S/D, lines of constant Re_{Dh}, lines of constant X/D and Reynolds number, and lines of constant X/D and S/D. For example, the performance envelopes for pin-fin arrays having constant S/D are shown in Figure 8.33 where duct Reynolds number was held constant. In general, these performance envelopes show that increasing spanwise spacing reduces heat transfer and friction factor. Once can visualize that an ideal pin-fin geometry would break from this pattern by providing increased heat transfer and decreased pressure drop to shift the performance envelope to the upper-left direction. Lines of constant duct Reynolds number are illustrated in Figure 8.34. The lines of constant Reynolds number indicate the sensitivity of friction factor and heat transfer to changes in the pin-fin geometry. For example, at low Reynolds numbers, the shallow slope indicated that array friction factor is more dependent on array geometry than array heat transfer. The slope of the constant Re_{Dh} lines increased with increasing Re_{Dh}, which indicated that array heat transfer became more dependent on the array geometry with increasing duct Reynolds number. Lines of constant X/D and Re_{Dh} are shown in Figure 8.35. Another interpretation for Figure 8.35 is the variation of S/D while X/D and Re_{Dh} were constant. The lines of Figure 8.35 had high curvature at high Reynolds numbers and low curvature at low Reynolds numbers.
At high Reynolds numbers, increasing S/D from 2 to 2.5 resulted in similar heat transfer but reduced friction factor. The curvature in the line showed that an increase from S/D from 2.5 to 3 resulted in significantly reduced heat transfer compared to the change between S/D = 2 to 2.5. Finally, lines of constant X/D and S/D are shown in Figure 8.36. Here, the effect of Reynolds number is isolated by moving along a given line. The slope of the line increases for increasing Reynolds number. At higher Reynolds numbers, therefore, changes in Reynolds number resulted in large changes in array heat transfer with small changes in array friction factor. At low Reynolds numbers, changes in Reynolds number resulted in large changes in friction factor with small changes in array heat transfer. Lines of constant S/D and X/D, provide an important result for turbine design if the duct Reynolds number varies during operation. The operational Reynolds number may be selected based on whether the turbine design requires a more uniform heat transfer (operating at low Reynolds numbers is ideal) or more uniform friction factor (operating at high Reynolds numbers is ideal).

Array performance was predicted using the heat transfer correlation of Metzger et al. (1986) and the friction factor correlation of Metzger et al. (1982b). Figure 8.37 shows that the trends in the array performance map are not accurately captured at low Reynolds numbers. An improvement in predicting the array performance map was realized using the heat transfer correlation of Metzger et al. (1986) but using the friction factor correlation of equation 8.20. Figure 8.38 shows better prediction of the array performance trends than that of Figure 8.37.

8.7 Summary of Array Performance and Empirical Correlations

In this chapter, it was found that effects of pin-fin Reynolds number, streamwise spacing, and spanwise spacing were important in predicting heat transfer. The effects of streamwise and spanwise spacing had similar effect on heat transfer. For example, reducing streamwise and spanwise spacing by 25% resulted in increased endwall heat transfer (averaged across all seven rows) by 6% and 8%, respectively.

Using the present data, several correlations were developed for row-averaged heat transfer on either the pin-fins or endwall surfaces. In agreement with previous chapters,
it was found that rows in the developed portion of the array were more strongly influenced by streamwise spacing than that of the initial rows.

Array performance was quantified for each pin-fin configuration using heat transfer augmentation normalized by the cube-root of friction factor augmentation. It was found that the majority of the pin-fin arrays had similar array performance. However, a previous study by Lawson et al. (2011) showed the highest array performance in a configuration having H/D = 1, S/D = 4, X/D = 3.46. The wide spacing resulted in reduced heat transfer compared with more closely spaced arrays, but the wide spacing provided a large savings in pressure drop which boosted the array performance. Several array performance maps were developed to provide convenient tools for assessing trends in pin-fin heat transfer and pressure drop. For a given streamwise and spanwise spacing, for example, increasing Reynolds number resulted in significant increases heat transfer with little influence on friction factor. For a given streamwise spacing and Reynolds number, increasing spanwise spacing resulted in lower heat transfer and friction factor, but the decreases were more significant at higher Reynolds numbers. Finally, for a given spanwise spacing and Reynolds number, increasing streamwise spacing resulted in decreased heat transfer and friction factor, but the decreases were more significant at higher Reynolds numbers and closer spanwise spacings.
### TABLE 8.1. SUMMARY OF LIMITS USED TO DEVELOP CORRELATIONS OF ARRAY-AVERAGED, COMBINED HEAT TRANSFER

<table>
<thead>
<tr>
<th>Correlation</th>
<th>H/D</th>
<th>S/D</th>
<th>X/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>VanFossen (1982) - Equation 8.4</td>
<td>0.5 to 2</td>
<td>2 to 4</td>
<td>1.732 to 3.46</td>
</tr>
<tr>
<td>Metzger et al. (1986) - Equation 8.5</td>
<td>1</td>
<td>2.5</td>
<td>1.5 to 5</td>
</tr>
<tr>
<td>Faulkner (1971) – Equation 8.7</td>
<td>0.17 to 7.7</td>
<td>1.5 to 10</td>
<td>1.3 to 8.66</td>
</tr>
<tr>
<td>Chyu et al. (2009) - Equations 8.8 through 8.11</td>
<td>1 to 4</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

### TABLE 8.2. SUMMARY OF DATA USED TO EVALUATE EXISTING ARRAY-AVERAGED, COMBINED HEAT TRANSFER CORRELATIONS

<table>
<thead>
<tr>
<th>Study</th>
<th>H/D</th>
<th>S/D</th>
<th>X/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al Dabagah and Andrews (1991)</td>
<td>0.7 to 2.2</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>Ames et al. (2007)</td>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Chyu et al. (1999)</td>
<td>1</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Chyu et al. (2009)</td>
<td>2 to 4</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Cowan et al. (2010)</td>
<td>1</td>
<td>2.828</td>
<td>1.414</td>
</tr>
<tr>
<td>Lawson et al. (2011)</td>
<td>1</td>
<td>2 to 4</td>
<td>1.73 to 3.46</td>
</tr>
<tr>
<td>Metzger et al. (1982a)</td>
<td>1</td>
<td>2.5</td>
<td>1.5 to 5</td>
</tr>
<tr>
<td>Present Work</td>
<td>1</td>
<td>2 to 3</td>
<td>2.16 to 3.03</td>
</tr>
</tbody>
</table>

### TABLE 8.3. CORRELATION CONSTANTS FOR ARRAY-AVERAGED HEAT TRANSFER FOR THE PRESENT DATASET

**Correlation Form:** \( \text{Nu}_D = C_1 \text{Re}_C^{m_1} (X/D)^{m_2} \)

<table>
<thead>
<tr>
<th>Surface</th>
<th>C_1</th>
<th>m_1</th>
<th>m_2</th>
<th>MSE</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined, Array-Averaged</td>
<td>0.30</td>
<td>0.58</td>
<td>-0.20</td>
<td>0.62</td>
<td>0.97</td>
</tr>
<tr>
<td>Endwall, Array-Averaged</td>
<td>0.33</td>
<td>0.57</td>
<td>-0.20</td>
<td>0.59</td>
<td>0.97</td>
</tr>
<tr>
<td>Pin-Fins, Array-Averaged</td>
<td>0.27</td>
<td>0.59</td>
<td>-0.20</td>
<td>0.80</td>
<td>0.95</td>
</tr>
</tbody>
</table>

**Correlation Form:** \( \text{Nu}_D = C_1 \text{Re}_C^{m_1} (X/D)^{m_2} (S/D)^{m_3} \)

<table>
<thead>
<tr>
<th>Surface</th>
<th>C_1</th>
<th>m_1</th>
<th>m_2</th>
<th>m_3</th>
<th>MSE</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined, Array-Averaged</td>
<td>0.41</td>
<td>0.57</td>
<td>-0.20</td>
<td>-0.24</td>
<td>0.52</td>
<td>0.98</td>
</tr>
<tr>
<td>Endwall, Array-Averaged</td>
<td>0.46</td>
<td>0.56</td>
<td>-0.20</td>
<td>-0.26</td>
<td>0.46</td>
<td>0.98</td>
</tr>
<tr>
<td>Pin-Fins, Array-Averaged</td>
<td>0.40</td>
<td>0.58</td>
<td>-0.26</td>
<td>-0.26</td>
<td>0.72</td>
<td>0.96</td>
</tr>
</tbody>
</table>
### TABLE 8.4. ROW-AVERAGED ENDWALL HEAT TRANSFER CORRELATION
### CONSTANTS FOR THE PRESENT DATASET

<table>
<thead>
<tr>
<th>Surface</th>
<th>$C_1$</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>MSE</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1 Endwall</td>
<td>0.13</td>
<td>0.60</td>
<td>0.00</td>
<td>0.30</td>
<td>0.98</td>
</tr>
<tr>
<td>Row 2 Endwall</td>
<td>0.11</td>
<td>0.64</td>
<td>0.16</td>
<td>0.57</td>
<td>0.97</td>
</tr>
<tr>
<td>Row 3 Endwall</td>
<td>0.26</td>
<td>0.59</td>
<td>-0.05</td>
<td>0.82</td>
<td>0.95</td>
</tr>
<tr>
<td>Row 4 Endwall</td>
<td>0.41</td>
<td>0.57</td>
<td>-0.32</td>
<td>0.68</td>
<td>0.97</td>
</tr>
<tr>
<td>Row 5 Endwall</td>
<td>0.53</td>
<td>0.55</td>
<td>-0.40</td>
<td>0.64</td>
<td>0.97</td>
</tr>
<tr>
<td>Row 6 Endwall</td>
<td>0.50</td>
<td>0.55</td>
<td>-0.36</td>
<td>0.67</td>
<td>0.97</td>
</tr>
<tr>
<td>Row 7 Endwall</td>
<td>0.47</td>
<td>0.55</td>
<td>-0.31</td>
<td>0.70</td>
<td>0.96</td>
</tr>
</tbody>
</table>

### Correlation Form:

\[ \text{Nu}_D = C_1 \text{Re}_D^{m_1} (X/D)^{m_2} \]

### TABLE 8.5. ROW-AVERAGED PIN-FIN HEAT TRANSFER CORRELATION
### CONSTANTS FOR THE PRESENT DATASET

<table>
<thead>
<tr>
<th>Surface</th>
<th>$C_1$</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>MSE</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1 Pin-Fin</td>
<td>0.14</td>
<td>0.60</td>
<td>0.20</td>
<td>0.71</td>
<td>0.94</td>
</tr>
<tr>
<td>Row 2 Pin-Fin</td>
<td>0.33</td>
<td>0.54</td>
<td>0.09</td>
<td>0.86</td>
<td>0.93</td>
</tr>
<tr>
<td>Row 3 Pin-Fin</td>
<td>0.37</td>
<td>0.60</td>
<td>-0.50</td>
<td>1.28</td>
<td>0.90</td>
</tr>
<tr>
<td>Row 4 Pin-Fin</td>
<td>0.21</td>
<td>0.64</td>
<td>-0.39</td>
<td>0.85</td>
<td>0.95</td>
</tr>
<tr>
<td>Row 5 Pin-Fin</td>
<td>0.32</td>
<td>0.60</td>
<td>-0.46</td>
<td>0.85</td>
<td>0.95</td>
</tr>
<tr>
<td>Row 6 Pin-Fin</td>
<td>0.21</td>
<td>0.61</td>
<td>-0.13</td>
<td>0.84</td>
<td>0.94</td>
</tr>
<tr>
<td>Row 7 Pin-Fin</td>
<td>0.23</td>
<td>0.61</td>
<td>-0.25</td>
<td>0.80</td>
<td>0.95</td>
</tr>
</tbody>
</table>

### Correlation Form:

\[ \text{Nu}_D = C_1 \text{Re}_D^{m_1} (X/D)^{m_2} \]

### TABLE 8.6. SUMMARY OF DATA USED TO EVALUATE EXISTING FRICTION FACTOR CORRELATIONS

<table>
<thead>
<tr>
<th>Study</th>
<th>H/D</th>
<th>S/D</th>
<th>X/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ames et al. (2007)</td>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Cowan et al. (2010)</td>
<td>1</td>
<td>2.828</td>
<td>1.414</td>
</tr>
<tr>
<td>Lawson et al. (2011)</td>
<td>1</td>
<td>2 to 4</td>
<td>1.73 to 3.46</td>
</tr>
<tr>
<td>Metzger et al. (1982a)</td>
<td>1</td>
<td>2.5</td>
<td>1.5 to 5</td>
</tr>
<tr>
<td>Present Work</td>
<td>1</td>
<td>2 to 3</td>
<td>2.16 to 3.03</td>
</tr>
</tbody>
</table>
Figure 8.1. Definition of flow-volume length scale used by VanFossen (1982).

Flow Volume Length Scale: \[ L_{\text{V/A}} = \frac{4V}{A} = \frac{\frac{H}{D} \left[ 4 \left( \frac{X}{D} \frac{S}{D} \right) - \pi \right]}{2 \left( \frac{X}{D} \frac{S}{D} \right) + \pi \left( \frac{H}{D} - 0.5 \right)} \]
Figure 8.2. Array-averaged heat transfer of the present pin-fin configurations in comparison with the correlation of Van Fossen (1982).

Figure 8.3. Array-averaged heat transfer of the present pin-fin configurations in comparison with the correlation of Metzger et al. (1986).
Figure 8.4. Mean-squared-error (MSE) of four correlations applied to the present data and to a compilation of previous data.

Figure 8.5. Coefficient of determination ($R^2$) of four correlations applied to the present data and to a compilation of previous data.
Figure 8.6. Array-averaged heat transfer of the present pin-fin configurations in comparison with the correlation of Faulkner (1971).

Figure 8.7. Array-averaged heat transfer of the present pin-fin configurations in comparison with the correlation of Chyu et al. (2009).
Figure 8.8. Prediction of array-averaged heat transfer using the correlation of VanFossen (1982).

Note the use of equation 8.2 for Nusselt number.
Figure 8.9. Prediction of array-averaged heat transfer using the correlation of Metzger et al. (1986).

Figure 8.10. Prediction of array-averaged heat transfer using the correlation of Faulkner (1971).
Figure 8.11. Prediction of array-averaged heat transfer using the correlation of Chyu et al. (2009).

Figure 8.12. Position of measurement locations used in calculating TRL parameter.
Figure 8.13. Effect of TRL on pin-fin stagnation point heat transfer in rows 1, 3, and 5 for H/D = 1, S/D = 2, X/D = 2.16 at Re_D = 2.0e4.

Figure 8.14. Present, array-averaged pin-fin heat transfer taken from IR images and compared with previous studies.
Heated Pin-Fin Only
Reference Temperature = Inlet Temperature
Endwalls and Pin-Fins Heated
Reference Temperature = Bulk Temperature

Figure 8.15. Effect of boundary conditions and choice of reference temperature on pin-fin heat transfer coefficients.
Figure 8.16. Illustration of different boundary conditions for present experimental setup and gas turbine application.
Figure 8.17. Predicting row-averaged endwall heat transfer in a non-uniformly spaced array using present correlations, $Re_D = 3.0e3$.

Figure 8.18. Predicting row-averaged pin-fin heat transfer in a non-uniformly spaced array using present correlations, $Re_D = 3.0e3$. 
Figure 8.19. Predicting row-averaged endwall heat transfer in a non-uniformly spaced array using present correlations, $Re_D = 2.0e4$.

Figure 8.20. Predicting row-averaged pin-fin heat transfer in a non-uniformly spaced array using present correlations, $Re_D = 2.0e4$. 
Figure 8.21. Percent difference from heat transfer measured in the non-uniform array.

Figure 8.22. Predicting row-averaged endwall heat transfer for the data of Ames et al. (2005) at $Re_D = 3.0e3$. 
Figure 8.23. Predicting row-averaged endwall heat transfer for the data of Ames et al. (2005) at $Re_D = 1.0e4$. 

Figure 8.24. Predicting row-averaged endwall heat transfer for the data of Ames et al. (2005) at $Re_D = 2.0e4$. 

$Nu_{D,e}$
Figure 8.25. Percent difference from heat transfer measured by Ames et al. (2007)

Figure 8.26. Prediction of array-averaged friction factor using the correlation of Metzger et al. (1982b).
Present Work
Ames et al. (2007)
Cowan and Tafti (2010)
Lawson et al. (2011)
Correlation - Metzger et al. (1982b)

Figure 8.27. Array-averaged friction factor as a function of Reynolds number using the correlation of Metzger et al. (1982b).

Measured $f_N$ - Predicted
$S/D = 4, X/D = 3.46$
$Re_D = 3.05e4$

Figure 8.28. Prediction of array-averaged friction factor as a function of Reynolds number and streamwise spacing ($X/D$).
Figure 8.29. Prediction of array-averaged friction factor as a function of Reynolds number and spanwise spacing (S/D).
Figure 8.30. Array performance index for the present dataset as a function of $Re_D$ (pin-fin diameter length scale).
Figure 8.31. Performance map of $\text{Nu}_{Dh,c}$ versus $f_{L/Dh}$ for each geometric configuration in the present work where $U_{\text{max}}$ was matched (constant $Re_D$).
Figure 8.32. Performance map of $\mathrm{Nu}_{Dh}$ versus $f_{L/Dh}$ for each geometric configuration in the present work where $U_m$ was matched (constant $Re_{Dh}$).

Figure 8.33. Performance envelopes superimposed on the performance map where $U_m$ was matched (constant $Re_{Dh}$).
Figure 8.34. Lines of constant $Re_{Dh}$ superimposed on the performance map where $U_m$ was matched (constant $Re_{Dh}$).

Figure 8.35. Lines of constant $X/D$ and constant $Re_{Dh}$ superimposed on the performance map where $U_m$ was matched (constant $Re_{Dh}$).
Figure 8.36. Lines of constant X/D and constant S/D superimposed on the performance map where \( U_m \) was matched (constant \( \text{Re}_{Dh} \)).

Figure 8.37. Predicted performance map using correlations of Metzger et al. (1986) for heat transfer and Metzger et al. (1982b) for friction factor.
Figure 8.38. Predicted performance map using correlations of Metzger et al. (1986) for heat transfer and equation 8.20 for friction factor.
CHAPTER 9:
NUMERICAL MODELING CONSIDERATIONS

In this chapter, turbulence modeling predictions of pin-fin flowfields and surface heat transfer were performed to investigate the applicability of several turbulence models that come standard with ANSYS FLUENT 13.0 (www.ansys.com, 2012). The SST k-ω (SST) eddy viscosity turbulence model, developed by Menter (1994), was used for both steady and unsteady solutions of the Reynolds-Averaged Navier-Stokes (RANS) equations. Both steady RANS and unsteady RANS (URANS) calculations were carried out in a single pin-fin row having H/D = 2, S/D = 2.5 to quantify the effect of turbulence modeling on predicting the near wake flowfield and pin-fin surface heat transfer. The single pin-fin row was chosen to compare RANS and URANS models where vortex shedding was present. Moreover, it was of interest to compare an industry standard (RANS) with what might better be used to simulate these flows. Flowfield comparisons were made for Reynolds numbers of Re_D = 3.0e3 and 2.0e4, while heat transfer comparisons were made at Re_D = 2.0e4.

In a separate calculation, Large Eddy Simulation (LES) was applied to a seven row pin-fin array having H/D = 1, S/D = 2, X/D = 2.16. The LES model was chosen for the full pin-fin array because experiments showed that a streamwise spacing of X/D = 2.16 caused attenuated vortex shedding in the first row, while vortex shedding was present in the third row. The objective was to determine the effect of turbulence modeling, using a higher-order model, in a pin-fin flowfield that showed a row-dependent flowfield. LES simulations utilized the Smagorinsky-Lilly sub-grid scale model (Smagorinsky 1963) to model sub-grid velocity fluctuations. Flowfield and heat transfer comparisons were made with experiments at a Reynolds number of Re_D = 2.0e4.

As described in Chapter 3, flowfield data was obtained using time-resolved, digital particle image velocimetry (TRDPIV). Heat transfer measurements were made on both the pin-fin and endwall surfaces using IR thermography. In this chapter, numerical solutions were evaluated using the experimental uncertainty as a criteria for accurate prediction.
9.1 RANS and URANS Solution Procedures

Numerical solution of the incompressible RANS equations was accomplished using ANSYS FLUENT 13.0, a commercially available, finite-volume solver (www.ansys.com, 2012). A schematic of the computational domain and boundary conditions used for RANS and URANS simulations is shown in Figure 9.1. The length scale used for the numerical solutions was based on a pin-fin diameter of 63.5 mm. As mentioned the geometric configuration was a single pin-fin row having H/D = 2 and S/D = 2.5. The Reynolds numbers considered were Re_D = 3.0e3 and 2.0e4. The values used for each boundary condition are summarized in Table 9.1. For steady calculations, the sides of the domain used a symmetry boundary condition. While, for unsteady calculations, the sides of the domain used a periodic boundary condition to allow developing vortices to propagate into and out of the domain. To match heat transfer experiments for the pin-fin heat transfer tests of Chapter 3, a no-slip, zero-heat flux boundary condition was applied to the endwall surface. A no-slip, constant heat flux boundary condition was applied to pin-fin surface. The inlet velocity boundary condition was calculated separately for a two-dimensional, empty channel having length of 40 hydraulic diameters. The two dimensional, empty channel was solved using the SST-RANS model and the flow variables at the exit of the two-dimensional, empty channel (velocity, pressure, temperature, transport scalars) were applied to the inlet of the pin-fin domain. Solving for the inlet conditions was advantageous because the fully-developed turbulent kinetic energy and turbulent dissipation profiles were applied to the inlet in addition to the fully-developed velocity profile. The outlet of the computational domain was modeled as a constant pressure outlet. Only half the duct was modeled using a symmetry boundary condition along the channel midline.

The governing equations (continuity, X/Y/Z momentum, turbulent kinetic energy, specific turbulent dissipation, and energy) were discretized using second order schemes for all solution variables including pressure, velocities, and transport scalars. It should be noted that turbulent heat transport was modeled using the Reynolds’ analogy with turbulent momentum transport (default when using ANSYS FLUENT 13.0). The turbulent Prandtl number was assumed to be a constant Pr_T = 0.85. To confirm that the Reynolds’ analogy holds true in pin-fin arrays, simultaneous velocity and temperature
measurements would be required throughout the flowfield, and was beyond the scope of this work. Second order, upwinding schemes were used for convective terms in the governing equations. In unsteady calculations, a second order implicit formulation was used for time advancement. Pressure-velocity coupling was performed using a coupled solver in which all solution variables were solved for simultaneously. This pressure-velocity coupling algorithm has been preferred by other researchers in situations that require full resolution of the viscous layer when using RANS models (Walters and Cokljat 2008). In the present work, the SST model required full resolution of the viscous layer and the coupled solver converged in cases where other schemes (the SIMPLE scheme, for example) would not. For steady calculations, the solution was considered converged when normalized residuals fell below 1e-5. Similarly, for unsteady calculations, each time step was considered converged when the normalized residuals fell below 1e-5. Approximately 5 to 10 sub-iterations were required for each time step to converge for unsteady calculations.

In unsteady calculations, the second-order time discretization used was fully-implicit and, therefore, conditionally stable. As such, there was no limit on the maximum time step size to maintain numerical stability. To accurately resolve the vortex shedding motions, the time step was set to 3.21e-4 s, or 1/215 of a lift cycle based on the expected Strouhal number of 0.2 (Zdravkovich 1997). The resulting cell Courant numbers for this time step were less than Co = 2 everywhere, with 87.8% of cells having Co < 1. To expedite unsteady calculations, an artificial disturbance was introduced by applying a rotating wall boundary condition to the pin-fin surface. After approximately 200 time steps, the rotating wall boundary condition was changed to a stationary wall boundary condition and vortex shedding began to occur. Without introducing an artificial disturbance, the solution required many thousands of time steps for vortex shedding to initiate, grow, and oscillate. After the initial rotating wall disturbance, the solution was advanced approximately 2.0e4 time steps to allow flow to cross the domain several times. When it was determined that the fluctuating lift on the cylinder appeared to be statistically stationary, flow statistics were obtained for 5.0e3 time steps. Figure 9.2 shows the statistical convergence of a typical unsteady calculation. About 10 days of
wall clock time were required to complete 2.5e4 time steps for the URANS calculation on a 16-node computer cluster using the grid having 6.16e5 cells.

A grid sensitivity analysis was performed to ensure adequate resolution of the flowfield. Grid sensitivity was checked using the steady, SST k-ω turbulence model for a single row of pin-fins at Re_D = 2.0e4. The grids used in the grid sensitivity analysis are shown in Figure 9.3. In order of most coarse resolution to most fine resolutions, the grids contained 8.98e4 (coarse), 3.38e5 (medium), and 6.16e5 (fine) cells. Histograms of y^+ for all wall-adjacent cells, in Figure 9.4, showed that all wall-adjacent cells were below y^+ = 5. For the finest grid resolution, all cells were below y^+ = 1.31.

Table 9.2 shows a summary of the grid refinement study. The pin-fin drag coefficient, C_D, and wake closure length, L_c/D, were calculated for each grid. As in previous chapters, the wake closure length was defined as the position of zero streamwise velocity along the wake axis. The pin-fin drag coefficient, C_D, and wake closure length, L_c/D, approached constant values of 0.74 and 3.95, respectively. The local streamwise velocity was plotted along the wake centerline for each of the three grids in Figure 9.5. When refining the grid from coarse to medium, agreement to within 11% of U_max was observed, which was beyond the convergence criteria. When refining the grid from medium to fine, however, agreement to within 2% of U_max was observed. All subsequent RANS and URANS predictions were completed using the fine grid having 6.16e5 cells.

### 9.2 Modeling Near Wake Flow with RANS and URANS

The main advantage of RANS modeling is the low computational cost when compared with URANS and LES. The difference between RANS and URANS is the inclusion of the unsteady terms in each of the governing equations for URANS modeling. It should be noted that the RANS equations are ensemble-averaged, not time-averaged. The ensemble-averaged equations, therefore, permit unsteadiness provided that the unsteady disturbance(s) occur on a time scale much longer than the dissipative time scale. For industrial applications, the use of unsteady models is often cost prohibitive so RANS models are commonly used. For a seven row pin-fin array having H/D = 2, S/D = 2.5, and X/D = 2.5, Ames and Dvorak (2006b) showed that the standard k-ε, RNG k-ε, and realizable k-ε RANS models predicted array friction factor to within 5% at Re_D = 3.0e3
number but under-predicted array friction factor by 14-57% at \( \text{Re}_D = 3.0 \times 10^4 \). The friction factor prediction at high Reynolds number was dependent on the choice of turbulence model. Ames and Dvorak (2006b) showed that there was less dependence on the turbulence model when predicting array heat transfer. Each \( k-\varepsilon \) model under-predicted array heat transfer by 40% at \( \text{Re}_D = 3.0 \times 10^3 \) and by 33% at \( \text{Re}_D = 3.0 \times 10^4 \). Ames and Dvorak (2006b) mention that the size of the recirculation region was not predicted accurately using RANS models, leading to discrepancies with experimental data. Andreini et al. (2004), however, showed good agreement (to within 6%) between \( k-\varepsilon \) based models and experimentally measured endwall heat transfer. There was a wide range in the accuracy of pin-fin heat transfer predictions, between 5-25% of experimentally measured heat transfer. Other RANS models have been applied to pin-fin arrays, such as \( k-\omega \) based models (Bianchini et al. 2012) and Reynolds-stress models (Su et al. 2007). The SST \( k-\omega \) model was applied to a realistic pin-fin channel geometry which included turning effects and duct convergence (Bianchini et al. 2012). The spanwise-averaged heat transfer coefficients varied by up to 33% and friction factor varied by up to 57% when compared with experiments. Using a Reynolds-stress model produced good regionally-averaged results, to within 13% of experiments (Su et al. 2007). Previous pin-fin studies have shown that RANS models usually have significant uncertainty associated with predicting friction factor and heat transfer in pin-fin arrays.

For the present work, the goal was to compare RANS and URANS models in resolving the near wake flow and pin-fin heat transfer. Schwänen and Duggleby (2012) used the SST \( k-\omega \) model in the URANS equations to predict flow across a single row of pin-fins having \( H/D = 1, S/D = 2 \). The URANS solution shows good agreement with experiments for spanwise-averaged local heat transfer in the pin-fin wake, to within 5%. In a study by Ünal et al. (2010), the flow over a single, infinite cylinder was simulated using several different turbulence models at a Reynolds number of \( \text{Re} = 4.0 \times 10^4 \). The turbulence models used by Ünal et al. (2010) were: Wilcox \( k-\omega \), SST \( k-\omega \), realizable \( k-\varepsilon \), and Spalart-Allmaras. Each turbulence model was applied to the URANS equations and the SST \( k-\omega \) model clearly outperformed the other models in predicting the mean flow.
topology in the near wake. From the work of Schwänen and Duggleby (2012) and Ünal et al. (2010), it was decided that the SST $k-\omega$ model provided the best candidate for simulating the flow across a single row of pin-fins.

Friction factor was calculated from the RANS and URANS solutions for flow across a single row of pin-fins having $H/D = 2$, $S/D = 2.5$. Figure 9.6 shows the array friction factor for RANS, URANS, and experiments. The URANS prediction at $Re_D = 2.0e4$ was the only case that did not agree with experiments to within the uncertainty bands shown in Figure 9.6. In general, the pin-fin friction factor showed little difference when using RANS or URANS.

A comparison was made at the channel mid-line (symmetry plane) between the RANS solution, URANS solution, and experimental data. Time-resolved digital particle image velocimetry (TRDPIV) was used to experimentally measure the near wake flow, as described in Chapter 3 at the channel midline, $Z/H = 0$. Figure 9.7 shows the time-mean streamwise velocity in the wake for RANS, URANS and TRDPIV at $Re_D = 3.0e3$. The non-dimensional wake closure length, $L_c/D$, was calculated for each case in Figure 9.7 by locating the position of zero streamwise velocity along the wake axis. The wake closure position is indicated in Figure 9.7 using dashed lines. The wake closure length was $L_c/D = 4.02$, 1.73, and 2.14 for RANS, URANS, and experiments, respectively. The RANS solution over-predicted the length of the recirculating region by 88% comparison to experiments. The over-prediction of the recirculating region in a bluff-body wake is a classic example of the shortcomings of RANS turbulence modeling which have no means of capturing the time-dependent, periodic unsteadiness from vortex shedding. While the URANS solution showed improvement over the RANS solution, the length of the recirculation region was under-predicted by 19% compared with experiments. In previous chapters and in the public literature, the wake closure length was found to correspond to the amount of diffusion in the shear layers (Zdravkovich 1997). Increasing Reynolds number, for example, leads to increased instability in the shear layers which causes more turbulent diffusion than at low Reynolds numbers (Zdravkovich 1997). The URANS solution over-predicted the turbulent diffusion in the shear layers which caused the wake closure length to decrease. Figure 9.7 also showed an increase in the maximum observed streamwise velocity at the channel midline row RANS and URANS in
comparison to experiments. Because predictions were carried out to match experiments, the bulk velocity was equivalent for each case in Figure 9.7. The increased velocity at the channel midline, therefore, was caused by three-dimensional effects. For RANS and URANS, there was a greater displacement thickness at the wall compared with experiments. The difference in displacement thickness caused the difference in velocity at the channel midline.

Figure 9.8 shows the time-mean streamwise velocity for Re_D = 2.0e4. The wake closure length was L_c/D = 3.96, 1.42, and 1.24 for RANS, URANS, and experiments, respectively. As with Re_D = 3.0e3, the RANS model severely over-predicted the wake closure length at high Reynolds number, by 219% relative to the experimental value. The URANS model, however, over-predicted the wake closure length by 15% at high Reynolds number where wake closure length was under-predicted at low Reynolds numbers. This result showed that the URANS model was not capturing the Reynolds number dependence on turbulent diffusivity in the shear layers. Use of a Detached-Eddy Simulation (DES) model may better predict the flow physics at separation and in the shear layers. DES models have been shown in previous studies to improve prediction of separated flows in comparison to RANS and URANS (Paterson and Peltier 2005). The experimental uncertainty in the position of the wake closure length was less than 1% of the cylinder diameter because of the high spatial resolution of TRDPIV measurements. The prediction of wake closure length was, therefore, entirely caused by uncertainty associated with the turbulence models. From the wake closure length analysis, the URANS model was clearly the more appropriate choice for resolving the near wake flowfield when compared with RANS. A summary of the comparison between RANS, URANS, and experimentally measured wake closure length is shown in Table 9.3.

The effect of turbulence modeling on the time-dependent flow was quantified using snapshots of the flowfield taken at certain phase angles of the vortex shedding cycle. The vortex shedding frequency was experimentally measured to be St = 0.17. The URANS predictions were in good agreement, where St = 0.18. Using the measured and predicted shedding frequencies, Figure 9.9 shows the instantaneous flow in the near wake for URANS and experiments at certain stages of the vortex shedding cycle. The snapshots in Figure 9.9 were taken in increments of one-eighth of the vortex shedding
cycle, $St^1 = 0.625$. From Figure 9.9, the effect of turbulence modeling was observed in the smoothed URANS velocity field compared to the turbulent flowfield in experiments. The over-prediction of the wake closure length, discussed previously, was also observed in the time-dependent flow because the Kármán vortices formed closer to the pin-fin in experiments compared to the URANS model. The time history of transverse velocity was extracted at the point $X/D = 1.6, Y/D = 0$ for both experiments and the URANS model. Figure 9.10 shows the instantaneous transverse velocity as a function of time. The effect of turbulence modeling was observed in the smooth, nearly sinusoidal signal for URANS. The experimental data showed turbulent fluctuations superimposed on the sinusoidal shedding frequency.

In Chapter 6, proper orthogonal decomposition (POD) was used to approximate the flowfield using the most energetic modes of the decomposition. For cases that admitted vortex shedding, the first two modes were usually associated with the vortex shedding motion and contained up to 70% of the overall fluctuating energy. In the present chapter, POD was used to isolate the vortex shedding motion from the instantaneous TRDPIV flowfield to compare with the URANS-predicted flowfield. The measured flowfield was approximated using the first two POD modes and compared with the URANS flowfield. The streamlines in Figure 9.11 showed more curvature in the POD-approximated flowfield where the Kármán vortex formed. Figure 9.11 shows the instantaneous flow in the near wake for URANS and the POD-approximated flowfield. By isolating the vortex shedding motions from the smaller-scale turbulence motions, the POD-approximated flowfield more closely resembled the URANS flowfield than the measured TRDPIV flowfield of Figure 9.9. Comparing the POD-approximated flowfield and the URANS prediction provided insight into how the experimental and modeled flowfields differed from one another. The most prominent difference between the measured and predicted flowfields was increased the transverse velocity sweeping across the wake observed in the experiments. Figure 9.11 was marked with arrows to show the increased cross-wake motion for experiments in comparison with the URANS modeled flowfield. The development of the near wake was dependent on the formation and shedding of the Kármán vortices, and the strength of the induced cross-wake motions had a significant impact on the flowfield. Although the agreement between experiments and
URANS predictions were quite good in the similar organization of the time-dependent flow, the comparison between POD-approximated and URANS-predicted flowfields has shown where the predictions break down.

9.3 Modeling Pin-Fin Heat Transfer with RANS and URANS

Experiments were performed in the present work, as described in Chapter 3, to measure the local heat transfer distribution on the pin-fin surface, both circumferentially and along the pin-fin axis. The spatially-resolved, time-mean heat transfer on the pin-fin surface provided a means for comparing the ability of RANS and URANS models to predict local heat transfer coefficients on the pin-fin surface. To minimize free convection effects, heat transfer measurements were only performed at a Reynolds number of $Re_D = 2.0e4$. Comparisons between RANS, URANS, and experiments were only performed at $Re_D = 2.0e4$.

Figure 9.12 shows the spatially-resolved Nusselt number across the pin-fin surface for RANS, URANS, and experimental data. Upon inspection, it was found that the experimental data was much more uniform across the pin-fin surface than the RANS and URANS models. Heat transfer varied between $0.43 \leq Nu_D Re_D^{-0.5} \leq 0.82$ for the experimental data, but varied between $0.24 \leq Nu_D Re_D^{-0.5} \leq 0.96$ for RANS and URANS model. Specifically, the stagnation heat transfer was over-predicted by both RANS and URANS models when compared with experimental data. Table 9.4 shows the RANS model over-predicted stagnation heat transfer by 15.6% and the URANS model over-predicted stagnation heat transfer by 20.6%.

For comparison, the governing equations were solved for laminar flow despite the obvious presence of wall-generated turbulence in the duct and wake turbulence downstream of the pin-fin. It was possible that the RANS and URANS equations were over-predicting heat transfer based on the false assumption that the boundary layer near the stagnation point was turbulent. Using the laminar model, stagnation heat transfer was in better agreement with experiments than RANS and URANS because $Nu_D$ was only over-predicted by 10.8% at the stagnation point, shown in Table 9.4. It should be noted that pin-fin heat transfer had an experimental uncertainty around 10%, so the laminar flow prediction was just beyond the limits of the uncertainty.
The local heat transfer distribution was extracted at the pin-fin midline to compare experiments with RANS, URANS, and the laminar model. Figure 9.13 shows the local circumferential heat transfer distribution at the pin-fin midline, $Z/H = 0$. As expected, the URANS model performed better than the RANS model in predicting heat transfer on the trailing side ($\theta > 90^\circ$) of the pin-fin because the large scale shedding motions were captured, to a certain extent, with URANS. As mentioned, the laminar model predicts stagnation heat transfer with accuracy than RANS and URANS, but the laminar model was also found to predict the trailing side heat transfer with reasonable accuracy. It should be noted that LES computations by Cowan et al. (2010) for a similar geometry showed under-predicted heat transfer at the stagnation point, followed by a sharp rise in heat transfer within the first $\theta = 10^\circ$ around the pin-fin.

The overall heat transfer was calculated for each case by performing an area-weighted average across the entire pin-fin surface. The results are summarized in Table 9.4. The RANS, URANS, and laminar cases all predicted the overall pin-fin heat transfer to within the experimental uncertainty. The RANS model under-predicted the overall, pin-fin heat transfer by 9.1% and the URANS model under-predicted the overall, pin-fin heat transfer by 3.5%. Interestingly, the laminar model was within 1% of experiments; however, it was clearly inappropriate to solve the entire pin-fin flow domain using the laminar flow equations where separation and wake turbulence were present.

From this analysis, it was clear that the URANS model predicted pin-fin flow and heat transfer with better accuracy than the RANS model. For the present case of a single pin-fin row, an improvement could be made with the URANS model by modifying the solution to include a transition model or by using DES. Furthermore, for multi-row arrays, the pin-fin heat transfer in downstream rows would likely necessitate full RANS treatment because of the turbulence generated in upstream rows.

9.4 LES Solution Procedures

While previous researchers have used LES to predict flow and heat transfer in pin-fin arrays, none have made detailed comparisons with experimental flowfield data (Cowan et al. 2010; Delibra et al. 2010; Schwänen and Duggleby 2011, 2012). The goal of the present section was to evaluate the performance of LES by comparing with
TRDPIV data in a row-dependent pin-fin flowfield. Section 9.2 and Section 9.3 showed that using URANS reduced the uncertainty of numerical predictions when compared with RANS models in a flowfield that admits vortex shedding. It was observed in Chapter 6, that some pin-fin arrays showed attenuated shedding in the first row from the presence of downstream pin-fins. Specifically, the pin-fin configuration having H/D = 1, S/D = 2, X/D = 2.16 showed attenuated shedding in the first row wake. The third row wake showed evidence of quasi-periodic vortex shedding that was occasionally disrupted from upstream wakes. It was decided to use LES to model the flowfield in the X/D = 2.16 array where a wide variety of flow physics (attenuated shedding and quasi-periodic shedding) were observed in the experimental data.

The Smagorinsky-Lilly sub-grid scale model was used in the LES solution with options for dynamic stress and dynamic energy flux (www.ansys.com, 2012). Discretization of the momentum and energy equations was accomplished using second order, bounded-central differencing. Pressure-velocity coupling was accomplished using the PISO scheme. The time advancement scheme was second-order implicit using a non-iterative time-advancement. The time step was set to 1e-4 s which corresponded to 1/690 of a shedding cycle assuming a Strouhal number of St = 0.2. The local Courant numbers resulting from a time step of 1e-4 s were below unity in 95.5% of the total cell count, indicating an appropriate choice of time step. The flow was allowed to develop for 2e4 time steps, corresponding to 2 s of flow time before sampling data. Similar to URANS solutions, Figure 9.14 shows the lift coefficient on the first row pin-fin as a function of time which was used to validate a statistically steady solution after 2 s of flow time. Statistics were calculated over an additional 1e4 time steps for a total of 3e4 time steps corresponding to 3 s of flow time. The LES simulation was carried out on different computational clusters having between 8 and 20 nodes. The overall time required to reach 3e4 time steps on a 12.1e6 cell mesh was about 20 days of wall clock time.

The flow domain and boundary conditions used for the LES solutions are shown in Figure 9.15. And, the values for each boundary condition are summarized in Table 9.5. The inlet boundary conditions were taken from the SST-RANS solution of flow through a fully-developed, two-dimensional channel. The resulting velocity, temperature, and transport scalars were taken from the exit of the two-dimensional
channel and applied to the inlet of the LES domain. It is important to note that the turbulent transport scalars were applied to the inlet as a function of wall distance, but the temperature at the inlet was specified at a constant $T = 300$ K. In the experiments, there were only a few diameters of heated area upstream of the pin-fins, so it was not desired to apply a fully-thermally-developed temperature profile to the inlet. To simulate instantaneous velocity fluctuations at the inlet, the inlet $k$-$\omega$ profile was used as the input to a spectral synthesizer algorithm (www.ansys.com, 2012). A no slip, constant heat flux boundary condition was applied to the pin-fins and endwalls. It is important to note that the RANS and URANS models had only heated pin-fins while the present section had heated endwalls and heated pin-fins to match the experimental setup. For the multi-row array to be solved using LES, experiments were performed in the heat transfer facility where the endwalls and pin-fins were both thermally active. Periodic boundary conditions were used on the sides of the domain to allow developing vortices to propagate into and out of the domain. While the RANS and URANS solutions made use of symmetry about the channel midline, the LES solution modeled the entire channel to capture three-dimensional effects across the channel midline.

The mesh used for the LES simulations featured structured cells near the walls and unstructured cells in the core of the duct. Figure 9.16 shows a close-up view of the structured/unstructured mesh used for LES solutions. The structured/unstructured mesh was chosen to achieve high accuracy in near-wall regions with the flexibility of using an unstructured mesh in the core of the duct. Kim (2004) showed that flow over a single, infinite cylinder at a Reynolds number of $Re = 1.0e4$ was in good agreement with experiments when using a fully unstructured mesh for a finite-volume, LES solution computed using the same software as in the present work. As with RANS and URANS solutions, a grid sensitivity analysis was performed for the LES solution. Two meshes were created having $5.35e6$ and $12.1e6$ cells. The time-averaged streamwise velocity was extracted along the wake centerline behind the first row of pin-fins in Figure 9.17. The maximum observed difference in streamwise velocity for grids having $5.35e6$ and $12.1e6$ was 2%, relative to $U_{\text{max}}$. As such, the mesh with $12.1e6$ cells was used for the final LES solution.
9.5 Modeling a Seven Row Pin-Fin Array with LES

The time-mean flowfield at the channel midline is shown in Figure 9.18 for both LES and experiments. Overall there was relatively good agreement between the LES simulations and experiments. Additionally, the LES solution was improved relative to the URANS and RANS simulations previously described. The wake closure position is indicated on Figure 9.18 using dashed lines at the position of zero streamwise velocity along the wake axis. In the first row, the predicted wake closure length was in good agreement with experiments. The wake closure length in the first row was $L_c/D = 1.59$ for the LES solution while the experimental wake closure length was $L_c/D = 1.62$. In the third row, $L_c/D = 0.93$ for LES and $L_c/D = 1.12$ for experiments.

To determine the effect of turbulence modeling on the fluctuating velocity in the wake, the in-plane components of the Reynolds stress tensor were calculated for LES and experiments. Figure 9.19 shows the turbulent normal stress component $<u'u'>/U_{max}^2$, Figure 9.20 shows the turbulent normal stress component $<v'v'>/U_{max}^2$, and Figure 9.21, shows the turbulent shear stress component $<u'v'>/U_{max}^2$. From Figure 9.19, regions of high turbulent normal stress in the streamwise direction, $<u'u'>/U_{max}^2$, were observed along the shear layers. In the first row, however, the LES under-predicted the level of $<u'u'>/U_{max}^2$. In the third row, there was better agreement between LES and experiments for $<u'u'>/U_{max}^2$ along the shear layer. From Figure 9.20, there were regions of high turbulent normal stress in the transverse direction, $<v'v'>/U_{max}^2$, along the wake axis. Large scale motions crossing the wake contributed to high $<v'v'>/U_{max}^2$ along the wake axis. From Figure 9.21, there were regions of elevated turbulent shear stress along the shear layers. As with $<u'u'>/U_{max}^2$, turbulent shear stress was under-predicted in the LES computation in the first row shear layers when compared with experiments. Regions of increased turbulent shear stress were caused by coherent structures forming along the shear layers that transported high momentum fluid into the wake and low momentum fluid out of the wake.

Inspection of the time-dependent LES flowfield showed that there were less fluctuations along the shear layer than experiments because the shear layer eddies in the LES flowfield were not resolved. Figure 9.22 shows the time-dependent flow in the first row wake for LES and experiments. The time step in Figure 9.22 was equal to one-
eighth of the vortex shedding frequency, $St^{-1} = 0.625$. Clearly, the experimental data shows shear layer eddies emerging tangent to the pin-fin as observed by the discrete packets of vorticity in the shear layers. The LES solution, however, showed a long, thin band of vorticity along the shear layer rather than individual shear layer eddies. Figure 9.23 shows an example of the instantaneous sub-grid turbulent kinetic energy. From Figure 9.23 it was clear that the LES algorithm was filtering the shear layer eddies and relying on sub-grid turbulence modeling to capture the turbulent viscosity in the first row shear layers.

In the third row wake, each component of in-plane turbulent stress was predicted with better accuracy than in the first row wake. As with the first row wake, the instantaneous sub-grid turbulent kinetic energy showed a significant modeling of sub-grid turbulence along the third row shear layers. While the first row wake was dominated by shear layer flow, the third row wake showed significant contributions from large scale motions. This was proven using proper orthogonal decomposition (POD) in Chapter 6. The LES solution, therefore, had difficulty in predicting the level of turbulent stresses when the wake was dominated by shear layer flow. In the third row, where large scale motions were present, the LES solution better predicted the turbulent stresses. The time-dependent flowfield in the third row wake is shown in Figure 9.24 for LES and experiments. From Figure 9.24, the presence of large scale motions was observed in both LES and experiments. In the experimental data, there were more small-scale motions relative to LES as observed in the contours of instantaneous vorticity. From the instantaneous sub-grid scale turbulent kinetic energy contours of Figure 9.23, much of the small-scale motion observed in experiments was filtered out by the sub-grid scale model.

Heat transfer predictions from LES were compared with experiments by first considering pin-fin heat transfer. It has been mentioned several times that heat transfer coefficients on the pin-fin surface were calculated using two different boundary conditions approaching the pin-fin. In one experiment, the pin-fin was heated using a constant heat flux surface with adiabatic walls surrounding the pin fin. Measurements were made using an array of thermocouples installed under the constant heat flux surface. In this case, only the pin-fin was thermally active. Heat transfer coefficients were calculated using the inlet channel temperature as the reference temperature. In the second
type of experiment, the endwall and pin-fin heat transfer were measured simultaneously using a constant heat flux endwall with copper pin-fins. IR thermography was used to capture local temperatures of the endwalls and the pin-fin bases. The pin-fin heat transfer was then calculated using the lumped model approximation because the pin-fin Biot number was much less than unity, $Bi = 5e^{-3}$. Moreover, the pin efficiency was nearly unity. In the second experiment, the reference temperature was the bulk temperature, calculated from the first law as a function of streamwise position in the array based on how much heat was added upstream of the current position. The LES predictions more closely simulated the second experimental setup where both the pin-fins and endwalls were heated and the bulk temperature from the first law was used as the reference temperature. Although the second experimental conditions were most closely matched to that of the LES, it is important to recognize that the conduction was not simulated in the pin but rather a constant heat flux around the pin surface was assumed.

Figure 9.25 shows the area-averaged pin-fin heat transfer as a function of row number for LES and for the two experimental methods. The LES predictions were only in agreement with the experimental uncertainty of the second and third row pin-fins for the case where the endwalls and pin-fins were heated. The array-averaged, pin-fin heat transfer calculated from LES was 11% over-predicted compared with the heated pin-fin and endwall case. The LES solution showed a 15% under-prediction in array-averaged, pin-fin heat transfer compared with having only the pin-fin heated. Although the LES solution was unable to capture array-averaged, pin-fin heat transfer to within the experimental uncertainty, the row-averaged pin-fin heat transfer showed a similar development to the experiments having heated endwalls and pin-fins. From the pin-fin heat transfer predictions, the choice of thermal boundary conditions and the choice of reference temperature were found to influence the measured pin-fin heat transfer. By heating both pin-fins and endwalls and using the bulk temperature as a reference, the experimentally measured pin-fin heat transfer was 23% lower than having only the pin-fin heated and using the inlet temperature as a reference. The LES predictions validated the effect of boundary condition and reference temperature on pin-fin heat transfer.

Endwall heat transfer was also compared with the LES solution (note that the pin heat transfer was not included in this average). The row-averaged endwall heat transfer
is shown in Figure 9.26 and the local heat transfer distribution is shown in Figure 9.27. It was found that row-averaged endwall heat transfer was predicted to within the experimental uncertainty only for the first row. The remaining rows of the array were over-predicted by LES. The array-averaged, endwall heat transfer was over-predicted by 20%. Despite over-predicting heat transfer by 20%, the LES solution captured the general topology of the local heat transfer coefficients in Figure 9.27. A specific feature that LES was able to capture was the local minimum in heat transfer observed in experimental data just downstream of the minimum flow area in any given row. One source of discrepancy was observed where the LES solution showed an enlarged horseshoe vortex region when compared with experiments. The LES-predicted horseshoe vortex was larger and showed higher local heat transfer in each row. Delibra et al. (2010) performed computations in a pin-fin array having H/D = 2, S/D = 2.5, X/D = 2.5 using a hybrid RANS/LES model at Re_D = 3.0e4. Similar to the present work, Delibra et al. (2010), observed over-predicted endwall heat transfer at the horseshoe vortex as shown in Figure 9.28. While the present work over-predicted the array-averaged, endwall heat transfer by 20%, Delibra et al. (2010) under-predicted heat transfer by 20%. Cowan et al. (2010) predicted array-averaged, combined heat transfer to within 5% of the Metzger et al. (1986) correlation using LES for the geometry of H/D = 1, S/D = 2.818, X/D = 1.414. From the present work and previous studies, LES predictions typically show accuracy to within 20% of experiments for array heat transfer.

9.6 Summary of Numerical Modeling Considerations

This chapter has shown the effects of using RANS and URANS in a single pin-fin row where vortex shedding was present. The classic over-prediction of the wake closure length was observed for the RANS solution. The URANS solution was able to capture the large scale, vortex shedding motion which provided a substantial increase in prediction accuracy of the wake closure length. Experiments showed that the developing Kármán vortices were able to sweep flow across the wake to a greater extent than the Kármán vortices predicted with URANS. This observation demonstrated the effect of turbulence modeling on the time-resolved flow in the near wake. Modeling pin-fin heat transfer using URANS agreed in the time-mean with experimental data, but the local heat
transfer distribution showed significant differences. The stagnation region was over-predicted with RANS and URANS, and the wake region was under-predicted. Because URANS was able to capture the vortex shedding motions, heat transfer on the downstream side of the pin-fin was predicted with better accuracy than RANS.

LES was applied to a pin-fin array where experiments showed attenuated vortex shedding in the first row wake. It was found that shear layer eddies in the first row were captured by the sub-grid scale model. The Reynolds stress components were modeled with better accuracy in the third row wake where large scale motions were present as opposed to the shear-layer-dominated first row wake.

Heat transfer results from the LES predictions provided important physical insight into the experimental measurement of pin-fin heat transfer coefficients. Experimental data was obtained in one case having only thermally active pin-fins and in another case having both pin-fins and endwalls thermally active. In the case having only heated pin-fins, the inlet temperature was used as the reference temperature. In the case having heated endwalls and pin-fins, the bulk temperature was used as the reference temperature. In the LES predictions, the endwalls and pin-fins were heated to approximate the latter experimental setup. The LES predictions showed better agreement with experimental setup having heated endwalls and pin-fins. This result showed that the choice of thermal boundary conditions and reference temperature had a significant influence on the pin-fin heat transfer coefficient and requires further study. The use of heated endwalls and pin-fins in conjunction with the bulk temperature reference resulted in a 23% decrease in the experimentally measured, array-averaged, pin-fin heat transfer coefficient. This difference was beyond the experimental uncertainty. The LES results validated that the difference in boundary conditions and reference temperature contributed to the different in experimentally measured pin-fin heat transfer.

In summary, the design of pin-fin arrays for industrial use would benefit from using URANS modeling rather than RANS in configurations where vortex shedding is present. The heat transfer contribution from vortex shedding is a significant effect in pin-fin arrays, and should be modeled. The pin-fin heat transfer was predicted to 3.5% for URANS, while RANS models showed 9% difference with experiments. While LES showed good agreement with flowfield and heat transfer measurements, the array-
averaged heat transfer on the endwalls was over-predicted by 20% using the Smagorinsky-Lilly subgrid-scale model and a constant turbulent Prandtl number of 0.85. Further investigation is required to determine whether a different solution procedure would provide a better prediction of heat transfer in pin-fin arrays.
TABLE 9.1. BOUNDARY CONDITIONS USED FOR RANS AND URANS SOLUTIONS AT Re_D = 3.0e3 AND 2.0e4

<table>
<thead>
<tr>
<th>Surface</th>
<th>Type</th>
<th>Value(s)</th>
<th>Re_D = 3.0e3</th>
<th>Re_D = 2.0e4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endwall</td>
<td>Wall</td>
<td>q&quot; = 0 W/m²</td>
<td>q&quot; = 0 W/m²</td>
<td></td>
</tr>
<tr>
<td>Inlet</td>
<td>Velocity Inlet</td>
<td>U_m = 0.414 m/s</td>
<td>U_m = 2.77 m/s</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>U_max = 0.690 m/s</td>
<td>U_max = 4.62 m/s</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T = 300 K</td>
<td>T = 300 K</td>
<td></td>
</tr>
<tr>
<td>Outlet</td>
<td>Constant Pressure</td>
<td>P_abs = 1.01e5 Pa</td>
<td>P_abs = 1.01e5 Pa</td>
<td></td>
</tr>
<tr>
<td>Pin-Fin</td>
<td>Wall</td>
<td>q&quot; = 116 W/m²</td>
<td>q&quot; = 300 W/m²</td>
<td></td>
</tr>
<tr>
<td>Sides</td>
<td>Symmetry (RANS)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Periodic (URANS)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>Symmetry</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 9.2. SUMMARY OF GRID SENSITIVITY ANALYSIS PERFORMED AT Re_D = 2.0e4 FOR THE RANS AND URANS GRIDS

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Max y⁺</th>
<th>Ave y⁺</th>
<th>Cells</th>
<th>C_D</th>
<th>L_c/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>3.29</td>
<td>1.37</td>
<td>89,856</td>
<td>0.73</td>
<td>3.50</td>
</tr>
<tr>
<td>Medium</td>
<td>2.09</td>
<td>0.82</td>
<td>338,415</td>
<td>0.74</td>
<td>3.94</td>
</tr>
<tr>
<td>Fine</td>
<td>1.31</td>
<td>0.52</td>
<td>616,044</td>
<td>0.74</td>
<td>3.95</td>
</tr>
</tbody>
</table>

TABLE 9.3. EFFECT OF TURBULENCE MODELING ON WAKE CLOSURE LENGTH

<table>
<thead>
<tr>
<th>Case</th>
<th>L_c/D</th>
<th>% Error in L_c/D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Re_D = 3.0e3</td>
<td>Re_D = 2.0e4</td>
</tr>
<tr>
<td>RANS SST</td>
<td>4.02</td>
<td>3.95</td>
</tr>
<tr>
<td>URANS SST</td>
<td>1.73</td>
<td>1.42</td>
</tr>
<tr>
<td>TRDPIV</td>
<td>2.14</td>
<td>1.24</td>
</tr>
</tbody>
</table>

TABLE 9.4. EFFECT OF TURBULENCE MODELING ON PIN-FIN HEAT TRANSFER AT Re_D = 2.0e4

<table>
<thead>
<tr>
<th>Case</th>
<th>Nu_D at θ = 0°</th>
<th>% Error in Nu_D at θ = 0°</th>
<th>Nu_D</th>
<th>% Error in Nu_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>RANS SST</td>
<td>128.5</td>
<td>15.6%</td>
<td>83.5</td>
<td>-9.1%</td>
</tr>
<tr>
<td>URANS SST</td>
<td>134.1</td>
<td>20.6%</td>
<td>88.7</td>
<td>-3.5%</td>
</tr>
<tr>
<td>Laminar</td>
<td>123.2</td>
<td>10.8%</td>
<td>91.5</td>
<td>-0.4%</td>
</tr>
<tr>
<td>Experiments</td>
<td>111.2</td>
<td>-</td>
<td>91.9</td>
<td>-</td>
</tr>
</tbody>
</table>
### TABLE 9.5. BOUNDARY CONDITIONS USED FOR LES SOLUTIONS AT $Re_D = 2.0\text{e}4$

<table>
<thead>
<tr>
<th>Surface</th>
<th>Type</th>
<th>Value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endwall</td>
<td>Wall</td>
<td>$q'' = 1000 \text{ W/m}^2$</td>
</tr>
<tr>
<td>Inlet</td>
<td>Velocity Inlet</td>
<td>$U_m = 2.30 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$U_{max} = 4.60 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$U, V, k, \omega = f(Z)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$T = 300 \text{ K}$</td>
</tr>
<tr>
<td>Outlet</td>
<td>Constant Pressure</td>
<td>$P_{abs} = 1.01\text{e}5 \text{ Pa}$</td>
</tr>
<tr>
<td>Pin-Fin</td>
<td>Wall</td>
<td>$q'' = 500 \text{ W/m}^2$</td>
</tr>
<tr>
<td>Sides</td>
<td>Symmetry (RANS)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Periodic (URANS)</td>
<td>-</td>
</tr>
<tr>
<td>Top</td>
<td>Symmetry</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 9.1. Computational domain and boundary conditions used for RANS and URANS simulations in a single pin-fin row having H/D = 2, S/D = 2.5.

Figure 9.2. Typical time-history showing statistical convergence of a URANS simulation.
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Figure 9.4. Local $y^+$ histograms showing refinement level during grid sensitivity analysis for RANS and URANS simulations.
Figure 9.5. Mean streamwise velocity taken along wake centerline showing grid sensitivity for the RANS-SST simulation.

Figure 9.6. Effect of turbulence modeling on prediction of friction factor in a single pin-fin row.
Figure 9.7. Effect of turbulence modeling on predicting the time-mean flowfield at $Re_D = 3.0e3$. 
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Figure 9.15. Computational domain and boundary conditions used for LES simulations in a seven row array having H/D = 1, S/D = 2, X/D = 2.16.
Figure 9.16. Close-up view of structured/unstructured mesh used for LES simulations.

Figure 9.17. Time-mean streamwise velocity along the wake axis behind the first row showing grid sensitivity study for LES simulations.
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Figure 9.19. Comparison between LES and TRDPIV of turbulent normal stress, \( \langle u'u' \rangle \), at the channel midline.
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Figure 9.21. Comparison between LES and TRDPIV of turbulent shear stress, $\langle u'v' \rangle$, at the channel midline.
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Figure 9.24. Effect of turbulence modeling on time-dependent flowfield in the third row wake.
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Figure 9.26. Effect of turbulence modeling on row-averaged endwall heat transfer development.
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Figure 9.28. Computations of Delibra et al. (2010) using LES/RANS where horseshoe vortex heat transfer was over-predicted.
CHAPTER 10:
CONCLUSIONS

This document has systematically investigated the effects of Reynolds number, aspect ratio (H/D), streamwise spacing (X/D), and spanwise spacing (S/D) in pin-fin arrays. Time-dependent flowfield measurements were compared with time-mean heat transfer measurements to determine the relative influence of the flowfield feature(s) that contributed to increases in heat transfer.

10.1 Relative Influence of Flowfield Features on Heat Transfer

In Chapter 1, a list of potential pin-fin features that contribute to heat transfer in pin-fin arrays was introduced. The pin-fin features that may contribute to heat transfer were surface area and flowfield effects. Surface area effects were decoupled from flowfield effects by considering the pin-fin and endwall heat transfer independently and by measuring local heat transfer coefficients. The present work considers the flowfield features contributing to enhancing the area-averaged heat transfer coefficient.

Flowfield effects included Reynolds number, three-dimensionality, periodic unsteadiness, turbulence, and wake effects. The effects of Reynolds number were considered throughout the present work when varying aspect ratio, streamwise spacing, and spanwise spacing. In general, it is well known that increasing Reynolds number results in increased heat transfer. The present work, however, investigated the effects of Reynolds number on the flowfield and on the local heat transfer distribution. In a single pin-fin row, it was found that increasing Reynolds number resulted in a shortened wake closure position. The shortened wake closure position correlated with the location of maximum endwall heat transfer in the pin-fin wake. It was found that turbulence levels that were isolated from the periodic shedding correlated with the level of heat transfer in multiple-row arrays, while periodic unsteadiness did not correlate with heat transfer. The shortened wake closure position, therefore, determined the position at which the wakes broke down to turbulence.

The horseshoe vortex was found to be an important flow feature for heat transfer in the first row and particularly for low Reynolds numbers. In the second through seventh rows, however, augmented heat transfer from the horseshoe vortex was confined to a smaller area around the pin-fin. Increasing Reynolds number was found to decrease the size of the horseshoe vortex.
vortex in a single row as observed in flowfield results. In addition, the increased unsteadiness of the flow in the second through seventh rows decreased the size of the horseshoe vortex as observed in heat transfer results. The horseshoe vortex, while important in the first row, showed no dependence on streamwise or spanwise spacing in the developed portion of the array.

Wake shedding was observed in the first row wakes for high Reynolds number flows and streamwise spacings of $X/D \geq 2.60$. When $X/D$ was reduced to 2.16, however, vortex shedding was attenuated. The presence or attenuation of vortex shedding was found to influence local heat transfer coefficients. For example, heat transfer was increased on the trailing side of the pin-fin for $X/D = 2.60$ because vortex shedding brought turbulent eddies into closer proximity with the pin-fin surface when compared to $X/D = 2.16$. The row-averaged heat transfer, however, was independent of $X/D$ in the first row which showed that periodic unsteadiness had no significant effect on heat transfer.

In the developed portion of the array, turbulence was generated in the pin-fin shear layers at each row. For increased streamwise spacing, the turbulence decreased before encountering the next row of pin-fins. increased turbulence correlated with increased heat transfer. Decreasing streamwise spacing allowed less distance for turbulence to decay and heat transfer increased. For both low and high Reynolds numbers, the level of turbulence correlated with heat transfer. Turbulence provided a greater enhancement on the endwalls than on the pin-fins. In comparison to unobstructed duct flow, increasing the level of turbulence increased heat transfer on the endwall by 310% yet only increased heat transfer by 51% on the pin-fin. The influence of turbulence on the thermal boundary layer at the endwall was found to be the most significant feature contributing to heat transfer in pin-fin arrays.

The same flowfield features that contribute to heat transfer in the initial rows when varying streamwise spacing applied when varying spanwise spacing. At low Reynolds numbers, the horseshoe vortex dominated heat transfer in the first row when varying spanwise spacing. Decreasing spanwise spacing brought the horseshoe vortex from adjacent pin-fins closer together and reduced the amount of undisturbed thermal boundary layer. At high Reynolds numbers, the horseshoe vortex was important in the first row in addition to wake mixing. Decreasing spanwise spacing at high Reynolds numbers also reduced the amount of undisturbed thermal boundary layer in the first row. In the developed portion of the array, the flow was sufficiently turbulent and there was no undisturbed boundary layer. It was observed that increasing $S/D$ may
have resulted in a thickened wall layer caused by three-dimensional effects. Increased S/D resulted in increased turbulence yet heat transfer was observed to decrease. The thickened wall layer may have counteracted the turbulence observed at the channel midline. Further investigation is necessary to determine the cause of decreased heat transfer for increasing S/D.

10.2 Recommendations and Future Work

The present results have shown that pin-fin heat transfer is dependent on Reynolds number, aspect ratio, streamwise spacing, and spanwise spacing. There was no single pin-fin array which provided both maximum heat transfer and minimum pressure drop.

In attempt to improve upon uniformly spaced arrays, a non-uniform array was considered in the present work where streamwise spacing varied within the pin-fin array. It was observed throughout the present work that the initial rows were independent of streamwise spacing and the developed portion of the array showed higher heat transfer for closer streamwise spacing. To exploit these observations, the non-uniform array had wide streamwise spacing in the initial rows and close streamwise spacing in the developed portion. At low Reynolds number, the uniform array having H/D = 1, S/D = 2, X/D = 2.16 outperformed the non-uniform array. At low Reynolds number, this result showed that close streamwise spacing was required in the initial rows to transition the flow to fully-turbulent. For high Reynolds number flows, however, the non-uniformly spaced array was found to provide similar heat transfer to the uniformly spaced array having H/D = 1, S/D = 2, X/D = 2.16. High Reynolds number flows were less sensitive to the streamwise spacing in the initial rows and heat transfer enhancement was mainly dependent on the streamwise spacing of the rows in the developed portion of the array. The non-uniform array provided an improvement over the uniform array having H/D = 1, S/D = 2, X/D = 2.16 because a similar level of heat transfer was observed and the total streamwise coverage of the non-uniform array was extended by 16%. Stated simply, improvements were made to the conventional, uniformly spaced array by using a non-uniform streamwise spacing (wide spacing in initial rows and decreased streamwise spacing in later rows) for high Reynolds number flows.

For pin-fin designs that require a balance of high heat transfer and low pressure drop, the arrays having S/D = 2.5 are recommended. For arrays having S/D = 2.5, adjacent pin-fins are closer together (beneficial for heat transfer) than arrays having S/D = 3 and the streamwise pressure gradients are lower (beneficial for pressure drop) than arrays having S/D = 2. The array
having $H/D = 1$, $S/D = 2.5$, $X/D = 2.16$ provided increased heat transfer and similar pressure drop to the pin-fin array having $H/D = 1$, $S/D = 2$, $X/D = 3.03$. Similarly, the array having $H/D = 1$, $S/D = 2.5$, $X/D = 3.03$ provided similar heat transfer and reduced pressure drop compared to the pin-fin array having $H/D = 1$, $S/D = 3$, $X/D = 2.16$.

From a numerical modeling standpoint, the use of unsteady RANS turbulence modeling improved numerical predictions when compared to steady RANS in a pin-fin configuration where vortex shedding was present. It is, therefore, recommended that URANS models are used in cases where vortex shedding was present, such as $X/D \geq 2.60$ at high Reynolds number. Although periodic shedding did not directly influence heat transfer, periodic shedding was shown in the present work to influence the generation of turbulence which did influence heat transfer. Large Eddy Simulation did not provide a significant increase in accuracy considering the increased solution time required when compared to RANS and URANS.

For future studies, improvements could be made to the pin-fin array to target the heat transfer mechanisms identified in the present work. In the first row, the horseshoe vortex was a significant flowfield feature for both low and high Reynolds numbers. Modifications to the pin-fin spacing, pin-fin shape, or endwall shape may aim to stimulate the horseshoe vortex. Modifications to the endwall shape, for example, could increase the level of heat transfer or extend the coverage of the horseshoe vortex. For low Reynolds number flows, future studies may target a means to expedite the transition to turbulence in the wake. For example, the non-uniform array in the present work could be flipped such that the first few rows have very close streamwise spacing. The close streamwise spacing in the first two rows may stimulate the transition to turbulence. For both low and high Reynolds numbers, future studies may investigate a means of enhancing the generation of turbulence in the developed portion of the array. Modifications to the pin-fin shape, endwall shape, or pin-fin arrangement could stimulate heat transfer through increased turbulence.
REFERENCES


APPENDIX A:
DETAILS OF THE IR THERMOGRAPHY TECHNIQUE

This appendix serves as supplementary discussion to Section 3.6 in which the IR thermography technique was described for calculating pin-fin and endwall heat transfer coefficients in the present work. Although the present IR-measurement method was based on that of previous work (Lyall et al. 2011; Lawson et al. 2011), several changes were made to the experimental method and to the data reduction technique.

The method was first developed by Lyall (2006). Instead of the three-strip heater design used in the present work, Lyall (2006) used a commercially purchased heater which consisted of a Kapton-encased serpentine Inconel circuit backed with a thin copper layer such that the heater had a stacking order of Kapton, Inconel, Kapton, and copper (from flow-side to non-flow side). These heaters were very durable and were able to withstand many tear-down and rebuild cycles before needing replacement. The copper backing was needed to smooth out the discontinuities in heat flux, as shown in Figure A.1, that would otherwise exist if the heater was only comprised of the serpentine Inconel circuit. The copper backing resulted in a mixed boundary condition that was nearly constant heat flux, but permitted some lateral conduction. The presence of lateral conduction had a smoothing effect on the surface temperature distribution and, therefore, the measured heat transfer coefficient distribution. For row- and array-averaged heat transfer, the presence of lateral conduction was acceptable and results were in agreement with public literature (Lyall et al. 2011; Lawson et al. 2011). However, the main advantage of using the IR-measurement technique is to distinguish the local flow features that contribute to heat transfer. And, when using a copper backing, the heat transfer distribution is too smeared to distinguish individual flow features. In fact, using the three-strip heater design reduced the amount of lateral conduction by a factor of 27. Figure A.3 shows the effect of lateral conduction on the measured endwall heat transfer distribution. Using the three-strip heater minimized the amount of lateral conduction and better accentuated the flow features.

In the initial development of the test method, Lyall (2006) attempted to embed a thermocouple in one of the copper pin-fins that was installed on the copper-backed heaters. Because the pin-fin Biot number was much less than unity (Bi = 5e-3), the goal was to use a lumped-capacitance analysis to measure the pin-fin heat transfer coefficient. Lyall (2006) showed that the pin-fin heat transfer could not be measured accurately because of lateral
conduction along the heater. At the time, there was no way to account for the amount of lateral conduction, so the energy balance on the pin-fin could not be modified to include the additional heat loss/gain from lateral conduction. The solution to the lumped-model problem was to measure the temperature on a constant heat flux surface wrapped around an insulated pin-fin, much like the method used in the present work in Section 3.5. The constant heat flux pin produced results that were in agreement with public literature, but the number of experiments was doubled since the pin-fin and endwall heat transfer were measured separately. The present work made use of a three-strip heater with no copper backing. The three-strip design minimized lateral conduction and made possible the lumped-capacitance pin-fin approximation. To utilize the lumped-capacitance model, the present work used two innovative developments. First, the IR images were used to acquire the pin-fin surface temperature because the pin-fin base temperature (measured with IR) was approximately equal to the pin-fin surface temperature for pin-fins having low Biot number. Second, the net heat into the pin-fins was corrected using a finite-volume calculation for lateral conduction exchange between the endwall and the pin-fin. Although lateral conduction was minimized using the three-strip heater design, it was necessary to account for lateral conduction to obtain favorable agreement with previous literature.

One downside to using the three-strip heater was that the heaters were destroyed upon disassembly of the test section. Removing and replacing the damaged heater strips was justified by the improved spatial resolution and the ability to capture pin-fin and endwall heat transfer simultaneously.

A.1 Lateral Conduction Calculation

The lateral conduction along the heater surface was accounted for using a finite-volume calculation. Because the IR thermography method produces the spatially-resolved temperature field on the heater surface, it was very convenient to calculate the amount of lateral conduction between adjacent pixels. The lateral conduction was calculated using fourth-order central differencing for pixels in the innermost rows of the IR images. And, a second-order central differencing scheme was used for the second-to-last pixels in from each edge of the image. The row of pixels along the edge were neglected and lateral conduction was not accounted for at these outer-most pixels. The lateral conduction into each pixel in the inner rows was calculated using
where $\Delta x$ is the length of each pixel, $\Delta y$ is the width of each pixel, $\Delta z$ is the thickness of the heater, $i$ is the column-index of the pixel, and $j$ is the row-index of the pixel. It should be emphasized again that the lateral conduction in equation A.1 represents a heat source such that a positive value will increase the amount of heat into a pixel and a negative value will decrease the amount of heat into a pixel. The second-order finite-volume scheme is shown in equation A.2 and also represents a heat source at a given pixel.
To quantify the effects of conduction, data was taken for a seven row pin-fin array having $H/D = 1$, $S/D = 2$, $X/D = 1.73$ using a copper-backed heater and using a three-strip heater. In all cases, the Reynolds number was set to $Re_D = 5.0 \times 10^3$. It was found that a large amount of noise was introduced when calculating the derivatives in the conduction calculation, especially for the copper-backed heater. Figure A.4 shows the local $Nu_D$ distribution on the endwall when lateral conduction is included in the energy balance. Using a three-strip heater reduced the amount of lateral conduction which, subsequently, reduced the amount of noise introduced from the calculating lateral conduction. Figure A.5 shows the row-averaged endwall Nusselt number calculated with and without lateral conduction. For the three-strip heater, endwall heat transfer calculation only resulted in a 2.5% difference between including lateral conduction and neglecting lateral conduction. In the remainder of the present work, lateral conduction was neglected for endwall heat transfer because including lateral conduction introduced a significant amount of noise, especially at low heat transfer. And, including lateral conduction only resulted in a 2.5% difference in the array-average in comparison to neglecting lateral conduction. Comparison of row-averaged endwall heat transfer with previous studies showed good agreement in Figure A.6 when lateral conduction was not included for the present work.

Figure A.7 shows the effect of lateral conduction on row-averaged pin-fin heat transfer. For the three-strip heater, there was a 21% difference between the array-average pin heat transfer calculated without conduction and with conduction. Figure A.8 shows that including lateral conduction for pin-fin heat transfer in the present work results in good agreement with previous
studies for the first row of pin-fins. For the remainder of the present work, lateral conduction was included for all pin-fin heat transfer data when using the IR thermography method.

A.2 Effects of Local Surface-Normal Conduction and Radiation Losses

In the work of Lyall et al. (2011) and Lawson et al. (2011), the surface energy balance only accounted for an array-averaged conduction loss term in the surface-normal direction. However, there exists a large variation in temperature across the heater surface. Including position-dependent conduction losses was implemented in the present work because hotter portions of the heater surface may have over-predicted heat transfer and cooler portions of the heater surface may have under-predicted heat transfer. To demonstrate the effect of local variations in conduction losses, three calculations were performed. The laterally averaged endwall heat transfer augmentation was calculated using an average conduction loss, using spatially-dependent conduction losses, and using both spatially-dependent conduction and radiation losses. The losses were calculated for a pin-fin array having H/D = 1, S/D = 2, X/D = 2.16 and Re_D = 3.0e3 (a geometry was chosen that had a large amount area visible upstream of the pin-fin array). The resulting endwall augmentation is shown in as a function of streamwise location in Figure A.9. Including spatially dependent surface-normal conduction did not show a significant change in laterally averaged heat transfer. Including radiation losses, however, decreased augmentation by 20% upstream of the pin-fin array. And, including radiation resulted in laterally averaged endwall augmentation within 1% of unity, which was expected for a fully-developed unobstructed duct flow.

A.3 Effect of Area-Weighting

Another subtle difference between the present work and that of Lyall et al. (2011) and Lawson et al. (2011) was the definition of endwall Nusselt number. The present work used the energy balance in equation 3.8 to calculate endwall Nusselt number. But, Lyall et al. (2011) and Lawson et al. (2011) included an additional area ratio to account for the wetted area of the pin-fins.
\[
\text{Nu}_{D,e}(X,Y) = \left( \frac{2q^*_{\text{heater}} - 2q^*_{\text{cond,ave}}}{T_{\text{corrected}}(X,Y) - T_m(X)} \right) \left( \frac{N_{\text{rows}}XW}{A_{\text{pins}} + A_{\text{endwall}}} \right) \left( \frac{D}{k_f} \right)
\] (A.3)

The effect of using the different convention for \( \text{Nu}_{D,e} \) is shown in Figure A.10. Using equation 3.8, the data of Lawson et al. (2011) agreed to within 5% of that of Metzger et al. (1986). Using equation A.3, however, there was 15% difference with Metzger et al. (1986). It should be noted that the endwall data of Lyall et al. (2011) and Lawson et al. (2011) has been corrected in all other plots in the present work to cancel the area ratio term equation A.3.

### A.4 Pseudo-Code for Data Reduction

The algorithm for the data reduction code is expressed using pseudo-code in this section.

1) Read 5 IR images acquired during experiment.
2) Average 5 images together to give \( T_{\text{raw}} \) at the non-flow side of the heater.
3) Transform \( T_{\text{raw}} \) to remove skew from non-perpendicular camera view.
4) Crop \( T_{\text{raw}} \) to include only region of interest.
5) Define geometry and create a mask for the pin-fins (spacing, aspect ratio, pin-positions, etc…).
6) Define test conditions (Reynolds numbers, ambient conditions, etc…).
7) Calculate \( T_m \) as a function of X-position (linear increase from first law analysis).
8) Calculate \( T_{\text{loss}} \) as a function of X-position (interpolate using a polynomial function based on the 3 loss thermocouples in the test rig).
9) For endwall pixels, correct \( T_{\text{raw}} \) to account for 1-dimensional conduction through the Kapton film.
10) For pin-fin pixels, correct \( T_{\text{raw}} \) to account for 1-dimensional conduction through the Kapton film and through the thermal epoxy at the base of the pin-fin.
   
   Note: Steps 10 and 11 give \( T_{\text{corrected}} \).
11) Calculate 1-dimensional conduction loss through the endwalls. This will be a function of \( X,Y \) depending on the driving temperature of the current pixel (\( T_{\text{raw}} \)) to the loss thermocouples temperature (\( T_{\text{loss}} \)).
12) Calculate radiation loss at the current pixel. Include radiation exchange with ZnSe window and with surroundings. This will use a driving temperature difference between $T_{\text{raw}}$ and $T_{\text{room}}$. The ZnSe window can be approximated at the average temperature between $T_{\text{raw}}$ and $T_{\text{room}}$.

13) Calculate lateral conduction using the finite-volume method. Driving temperature differences will be based on local temperature distribution on the heater.

14) Calculate heat transfer coefficients for endwall including heat in, surface-normal conduction, and radiation (no lateral conduction). The driving temperature difference will be between $T_{\text{corrected}}$ and $T_m$.

15) Calculate heat transfer coefficients for pin-fin including heat in, surface-normal conduction, radiation, and lateral conduction. The driving temperature difference will be between $T_{\text{corrected}}$ and $T_m$.

16) Calculate combined heat transfer coefficients using area-weighted averaged of pin-fins and endwalls.

17) Display all information of interest and save workspace variables.
Figure A.1. Discontinuities in surface heat flux when a serpentine heater does not feature a copper backing.

Figure A.2. Present work featured a three-strip heater which prevented discontinuities in heat flux and also minimized lateral conduction.
Figure A.3. Effect of heater thermal conductivity on local heat transfer distribution.
Figure A.4. Effect of heater thermal conductivity on local heat transfer distribution when lateral conduction is included in surface energy balance.
Figure A.5. Effect of lateral conduction on row-averaged endwall-fin heat transfer for $H/D = 1$, $S/D = 2$, $X/D = 1.73$ at $Re_D = 5.0e3$.

Figure A.6. Effect of heater thermal conductivity on row-averaged endwall heat transfer for $Re_D = 5.0e3$. 
Figure A.7. Effect of lateral conduction on row-averaged pin-fin heat transfer for H/D = 1, S/D = 2, X/D = 1.73 at Re_D = 5.0e3.

Figure A.8. Summary of first row pin-fin heat transfer from previous studies.
Figure A.9. Effect of spatially-dependent conduction and radiation losses on laterally averaged endwall augmentation.

Figure A.10. Effect of area-weighting used in previous studies to calculate endwall heat transfer.
APPENDIX B:
BUOYANCY EFFECTS

When measuring pin-fin surface heat transfer, buoyancy effects were non-negligible at low Reynolds number conditions. With increasing scale of the test facility, the characteristic velocities decreases. In the present work, the 64x flowfield facility had very low characteristic velocities on the order of 1 m/s. To determine influence of buoyancy effects, a hand calculation was performed to determine the magnitude of free convection from the pin-fins.

The pin-fin in the 64x facility was approximated as an infinitely long cylinder. As shown in Figure 3.1, the 64x flowfield facility was oriented such that the pin-fin channel was aligned vertically and the pin-fins were oriented horizontally. For a single, infinitely long cylinder oriented horizontally, the free calculation was calculated using equation B.1 from Incropera and Dewitt (2002).

\[
\overline{\text{Nu}}_{D,p} = \left[ 0.60 + \frac{0.387 \text{Ra}^{1/6}_D}{\left(1 + (0.559/\text{Pr})^{9/16}\right)^{9/27}} \right]^2 \text{ for } \text{Ra}_D < \text{le}^{12} \tag{B.1}
\]

Experiments were typically performed using 20°C temperature differential between the pin-fin surface and the bulk temperature. The resulting Rayleigh number was $\text{Re}_D = 4.46e5$ and the resulting pin-fin Nusselt number was $\overline{\text{Nu}}_{D,p} = 12$. From previous studies, the pin-fin Nusselt number was $\overline{\text{Nu}}_{D,p} \approx 40$ at low Reynolds number ($\text{Re}_D = 5.0e3$) and was as $\overline{\text{Nu}}_{D,p} \approx 100$ at high Reynolds number ($\text{Re}_D = 2.5e4$). From this hand calculation free convection was clearly on the same order of magnitude as forced convection. All experiments were, therefore, performed at high Reynolds number ($\text{Re}_D = 2.0e4$) in the 64x flowfield facility.
APPENDIX C:

UNCERTAINTY CALCULATIONS

For each measurement presented, the total measurement uncertainty was calculated to determine whether or not the resolution of the measurement was statistically significant. The partial derivative (sequential perturbation) method described by Moffat (1988) was used to calculate total measurement uncertainty. The sequential perturbation method was used to calculate uncertainty in Reynolds number, flow velocity, friction factor, surface heat transfer, and local velocities collected using both LDV and TRDPIV.

C.1 Instrumentation Bias and Precision Uncertainty

The uncertainty in any derived quantity, such as Reynolds number or mass flowrate, is dependent on the cumulative uncertainty associated with each measurement required for that derived quantity. For more details on estimating uncertainty, the reader is referred to Mechanical Measurements (Beckwith et al. 2007). For each instrument, the bias and precision uncertainty was calculated at a 95% confidence level. For bias uncertainty, it was assumed that the manufacturer specifications were at 95% confidence. Precision uncertainty was calculated using a two-sided student’s T-distribution at 95% confidence for 11 samples. The resulting equation for precision uncertainty was:

\[ u_{\text{precision}} = 2.228 \frac{S_x}{\sqrt{10}} \]

where \( S_x \) is the sample standard deviation. For single sample measurements, such as using a hand-held voltmeter, the precision uncertainty at 95% confidence was estimated using:

\[ u_{\text{precision}} = 1.96\sigma_e \]

where \( \sigma_e \) is an estimate of the population standard deviation. According to Beckwith et al. (2007), the estimated population standard deviation should be chosen based on knowledge of the system. For example, the voltage across a surface heater remains nearly constant for the duration
of an experiment since a highly accurate current-controlled power source is delivering power to the heater. The precision uncertainty in measuring voltage drop across the heater will most likely be dominated by operator error in making a good reading of voltage. A 1% precision uncertainty at 95% confidence was deemed reasonable for voltage measurements in the present work. A summary of the bias and precision uncertainty are shown in Table C.1 for each instrument used in the present work. Note that many of the instruments were found to have negligible precision uncertainty. The remaining sections will present details on uncertainty propagation for each measured quantity.

C.2 Uncertainty in Reynolds Number

Depending on the test being performed, either a Venturi nozzle or an orifice plate was used to measure a differential pressure that corresponds to a standard flowrate as shown in equation C.3.

\[
\text{SCFM} = \frac{5.9816 \cdot d^2 \cdot K \cdot Y \cdot \sqrt{\Delta P} \cdot \frac{2.703 \cdot P_L \cdot SG}{460 + T_L}}{2.703 \cdot 14.7 \cdot SG} \frac{460 + T_b}{460 + T_b} \tag{C.3}
\]

In equation C.3, the variables d (in), K, and Y are specific to the metering device. SG is the specific gravity of the fluid which is 1.0, P_L (in w/c) is the measured line pressure, T_L (°F) is measured line temperature, T_b (°F) is base temperature which is 60°F, and ΔP (in w/c) is pressure drop across the metering device. This standard flowrate was converted to actual volumetric flowrate using:

\[
\text{ACFM} = \text{SCFM} \frac{460 + T_m}{35.4(P_{\text{atm}} + P_g)} \tag{C.4}
\]

where \(T_m\) (°F) is bulk air temperature, \(P_{\text{atm}}\) (psia) is the measured atmospheric pressure, and \(P_g\) (psig) is gage pressure in the measurement section. Both \(T_m\) and \(P_g\) were measured in the test section, just upstream of the pin-fins. Mass flowrate was calculated using:
where \( \rho \) (kg/s) was calculated using the Ideal gas law from the local temperature \( (T_m) \) and local absolute pressure \( (P_{atm}+P_g) \) in the measurement section. The uncertainty in mass flowrate was simply the root-sum-square of the uncertainty introduced by each measured quantity. Equation C.6 shows the total uncertainty in mass flowrate resulting from the uncertainty in each measured quantity used in equations C.3 through C.5.

\[
\frac{\partial \dot{m}}{\partial T_L} u_{T_L} + \frac{\partial \dot{m}}{\partial T_m} u_{T_m} + \frac{\partial \dot{m}}{\partial P_L} u_{P_L}\]

To calculate the uncertainty in mass flowrate shown in equation C.6, one could derive each of the partial differential equations from equations C.3 through C.5. Alternatively, the same results may be achieved numerically by perturbing each measured quantity by the associated uncertainty and recording the change in mass flowrate. From mass flowrate, the bulk channel velocity was calculated using:

\[
U_m = \frac{\dot{m}}{\rho WH}
\]

where \( W \) (m) is the channel width and \( H \) (m) is the channel height. For the flowfield facility, uncertainty in both \( W \) and \( H \) was taken to be 1/16 in. For the heat transfer facility, uncertainty in \( W \) was taken to be 0.031 in and uncertainty in \( H \) was taken to be 0.031 in. Using the sequential perturbation method, uncertainty in bulk velocity was calculated using:

\[
\frac{\partial U_m}{\partial \dot{m}} u_{\dot{m}} + \frac{\partial U_m}{\partial W} u_{W} + \frac{\partial U_m}{\partial H} u_{H}
\]

where uncertainty in \( \rho \) was neglected. Finally, Reynolds number was calculated using:  

\[
\text{Re} = \frac{U_m D}{\nu}
\]
\[
Re_{Dh} = \frac{U_m D_h}{\nu}
\]  

(C.9)

And, the sequential perturbation method applied to equation C.9 gives:

\[
u_{Re_{Dh}} = \sqrt{\left( \frac{\partial Re_{Dh}}{\partial U_m} u_{Um} \right)^2 + \left( \frac{\partial Re_{Dh}}{\partial D_h} u_{Dh} \right)^2}
\]  

(C.10)

As previously mentioned, the uncertainty in each derived quantity of equations C.6, C.8, and C.11 was calculated using a numerical perturbation. This was easily achieved using a the same spreadsheet for calculating flowrate. First, the nominal measurement values are inserted into the spreadsheet to calculate the nominal Reynolds number. Second, each measured quantity was perturbed (offset) by the uncertainty associated with that measurement. The change in derived quantities such as mass flowrate, velocity, and Reynolds number was then recorded. Finally, the root-sum-square of these deltas was computed to give the total uncertainty in each derived quantity. The uncertainty in mass flowrate, velocity, and Reynolds number is shown for several representative test cases in Table C.2 through Table C.5. It was found that the largest sources of uncertainty in flowrate were the pressure drop across the flowmeter, \(\Delta P\), and the channel height, \(H\).

### C.3 Uncertainty in Pressure Coefficient and Friction Factor

The measurement procedures for pressure coefficient and friction factor are presented in Section 3.4. The uncertainty in pressure coefficient was calculated using:

\[
u_{Cp} = \sqrt{\left( \frac{\partial C_p}{\partial \Delta P} u_{\Delta P} \right)^2 + \left( \frac{\partial C_p}{\partial U_{max}} u_{U_{max}} \right)^2}
\]  

(C.11)

The uncertainty in friction factor was calculated using:
Table C.8 and Table C.9 show the uncertainty in friction factor for the heat transfer facility at low and high Reynolds numbers, respectively. Table C.10 shows the uncertainty in pressure coefficient on the pin-fin surface.

**C.4 Uncertainty in Heat Transfer**

When calculating the uncertainty in pin-fin surface heat transfer, several steps were required. First, uncertainty in heat flux was calculated using:

\[
\begin{align*}
    u_{fDh} &= \left( \frac{\partial f}{\partial \Delta P} u_{\Delta P} \right)^2 + \left( \frac{\partial f}{\partial U_m} u_{U_m} \right)^2 + \\
    &+ \left( \frac{\partial f}{\partial U_{max}} u_{U_{max}} \right)^2 + \left( \frac{\partial f}{\partial Re_{Dh}} u_{Re_{Dh}} \right)^2 + \\
    &+ \left( \frac{\partial f}{\partial H} u_H \right)^2 + \left( \frac{\partial f}{\partial W} u_W \right)^2 + \left( \frac{\partial f}{\partial D} u_D \right)^2 
\end{align*}
\]  

Equations C.12 and C.13 were also used to calculate uncertainty in IR-measured surface heat transfer. Because the IR-images were calibrated to surface-mounted thermocouples, the uncertainty in the IR-measured temperatures were assumed to be equal to the uncertainty of the E-type thermocouples. The uncertainty in pin-fin heat transfer (measured with thermocouples) is shown in Table C.11 and the uncertainty in endwall heat transfer (measured with IR thermography) is shown in Table C.12.
**TABLE C.1. BIAS AND PRECISION UNCERTAINTY AT 95% CONFIDENCE LEVEL OF FREQUENTLY USED INSTRUMENTATION**

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Bias Uncertainty</th>
<th>Precision Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-E Thermocouple</td>
<td>0.2°C</td>
<td>≈ 0</td>
</tr>
<tr>
<td>Setra 264 Differential</td>
<td>1% of full scale</td>
<td>≈ 0</td>
</tr>
<tr>
<td>Pressure Transducer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Setra 370 Barometer</td>
<td>0.02% of full scale</td>
<td>≈ 0</td>
</tr>
<tr>
<td>Meriam 2100 SMART gage</td>
<td>0.1% of full scale</td>
<td>≈ 0</td>
</tr>
<tr>
<td>Fluke 75III Multimeter</td>
<td>0.3% of reading</td>
<td>1% of reading</td>
</tr>
<tr>
<td>IR Camera</td>
<td>0.2°C</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE C.2. UNCERTAINTY IN MASS FLOWRATE, VELOCITY, AND REYNOLDS NUMBER FOR THE FLOWFIELD FACILITY AT LOW REYNOLDS NUMBER**

<table>
<thead>
<tr>
<th></th>
<th>m (kg/s)</th>
<th>U_m (m/s)</th>
<th>U_max (m/s)</th>
<th>Re_{Dh}</th>
<th>Re_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Value</td>
<td>3.08e-2</td>
<td>0.37</td>
<td>0.75</td>
<td>2864</td>
<td>3025</td>
</tr>
<tr>
<td>((\partial/\partial \Delta p)(u_{\Delta p}))^2</td>
<td>1.76e-4</td>
<td>0.002</td>
<td>0.004</td>
<td>16.4</td>
<td>17.3</td>
</tr>
<tr>
<td>((\partial/\partial T_L)(u_{T_L}))^2</td>
<td>1.04e-5</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>((\partial/\partial T_m)(u_{T_m}))^2</td>
<td>2.08e-5</td>
<td>0</td>
<td>0.001</td>
<td>1.5</td>
<td>1.6</td>
</tr>
<tr>
<td>((\partial/\partial P_{atm})(u_{P_{atm}}))^2</td>
<td>6.49e-5</td>
<td>0.001</td>
<td>0.002</td>
<td>6.0</td>
<td>6.4</td>
</tr>
<tr>
<td>((\partial/\partial P_g)(u_{P_g}))^2</td>
<td>3.92e-6</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>((\partial/\partial P_L)(u_{P_L}))^2</td>
<td>1.96e-7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>((\partial/\partial W)(u_W))^2</td>
<td>0</td>
<td>0</td>
<td>0.001</td>
<td>1.9</td>
<td>2.1</td>
</tr>
<tr>
<td>((\partial/\partial H)(u_H))^2</td>
<td>0</td>
<td>0.005</td>
<td>0.009</td>
<td>1.9</td>
<td>37.3</td>
</tr>
<tr>
<td>((\partial/\partial D)(u_D))^2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6.0</td>
</tr>
<tr>
<td>Total Uncertainty</td>
<td>1.89e-4</td>
<td>0.01</td>
<td>0.01</td>
<td>17.8</td>
<td>42.2</td>
</tr>
<tr>
<td>% Uncertainty</td>
<td>0.6%</td>
<td>1.4%</td>
<td>1.4%</td>
<td>0.6%</td>
<td>1.4%</td>
</tr>
</tbody>
</table>
### TABLE C.3. UNCERTAINTY IN MASS FLOWRATE, VELOCITY, AND REYNOLDS NUMBER FOR THE FLOWFIELD FACILITY AT HIGH REYNOLDS NUMBER

<table>
<thead>
<tr>
<th>Nominal Value</th>
<th>m (kg/s)</th>
<th>U_m (m/s)</th>
<th>U_max (m/s)</th>
<th>Re_{Dh}</th>
<th>Re_D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.04e-1</td>
<td>2.46</td>
<td>4.93</td>
<td>18985</td>
<td>20051</td>
</tr>
<tr>
<td>[ (∂/∂Δp)(u_{AP}) ]^2</td>
<td>1.68e-3</td>
<td>0.02</td>
<td>0.04</td>
<td>156.3</td>
<td>165.0</td>
</tr>
<tr>
<td>[ (∂/∂T_L)(u_{TL}) ]^2</td>
<td>6.90e-5</td>
<td>0</td>
<td>0</td>
<td>6.4</td>
<td>6.8</td>
</tr>
<tr>
<td>[ (∂/∂T_m)(u_{Tm}) ]^2</td>
<td>1.38e-4</td>
<td>0</td>
<td>0</td>
<td>10.1</td>
<td>10.7</td>
</tr>
<tr>
<td>[ (∂/∂P_{atm})(u_{Patm}) ]^2</td>
<td>4.29e-4</td>
<td>0.01</td>
<td>0.01</td>
<td>40.2</td>
<td>42.4</td>
</tr>
<tr>
<td>[ (∂/∂P_g)(u_{Pg}) ]^2</td>
<td>2.59e-5</td>
<td>0</td>
<td>0</td>
<td>2.4</td>
<td>2.6</td>
</tr>
<tr>
<td>[ (∂/∂P_L)(u_{PL}) ]^2</td>
<td>1.30e-6</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>[ (∂/∂W)(u_{W}) ]^2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>12.6</td>
<td>14.1</td>
</tr>
<tr>
<td>[ (∂/∂H)(u_{H}) ]^2</td>
<td>0</td>
<td>0.03</td>
<td>0.06</td>
<td>12.6</td>
<td>247.5</td>
</tr>
<tr>
<td>[ (∂/∂D)(u_{D}) ]^2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>40.1</td>
</tr>
<tr>
<td>Total Uncertainty</td>
<td>1.74e-3</td>
<td>0.04</td>
<td>0.07</td>
<td>163</td>
<td>304</td>
</tr>
<tr>
<td>% Uncertainty</td>
<td>0.9%</td>
<td>1.5%</td>
<td>1.5%</td>
<td>0.9%</td>
<td>1.5%</td>
</tr>
</tbody>
</table>

### TABLE C.4. UNCERTAINTY IN MASS FLOWRATE, VELOCITY, AND REYNOLDS NUMBER FOR THE HEAT TRANSFER FACILITY AT LOW REYNOLDS NUMBER

<table>
<thead>
<tr>
<th>Nominal Value</th>
<th>m (kg/s)</th>
<th>U_m (m/s)</th>
<th>U_max (m/s)</th>
<th>Re_{Dh}</th>
<th>Re_D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.37e-2</td>
<td>2.43</td>
<td>4.86</td>
<td>2963</td>
<td>3018</td>
</tr>
<tr>
<td>[ (∂/∂Δp)(u_{AP}) ]^2</td>
<td>1.03e-4</td>
<td>0.02</td>
<td>0.04</td>
<td>22.3</td>
<td>22.7</td>
</tr>
<tr>
<td>[ (∂/∂T_L)(u_{TL}) ]^2</td>
<td>4.60e-6</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>[ (∂/∂T_m)(u_{Tm}) ]^2</td>
<td>9.35e-6</td>
<td>0</td>
<td>0</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>[ (∂/∂P_{atm})(u_{Patm}) ]^2</td>
<td>2.90e-5</td>
<td>0.01</td>
<td>0.01</td>
<td>6.3</td>
<td>6.4</td>
</tr>
<tr>
<td>[ (∂/∂P_g)(u_{Pg}) ]^2</td>
<td>1.75e-6</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>[ (∂/∂P_L)(u_{PL}) ]^2</td>
<td>8.73e-8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[ (∂/∂W)(u_{W}) ]^2</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>4.5</td>
<td>4.7</td>
</tr>
<tr>
<td>[ (∂/∂H)(u_{H}) ]^2</td>
<td>0</td>
<td>0.06</td>
<td>0.13</td>
<td>4.5</td>
<td>78.4</td>
</tr>
<tr>
<td>[ (∂/∂D)(u_{D}) ]^2</td>
<td>4.60e-6</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>39.2</td>
</tr>
<tr>
<td>Total Uncertainty</td>
<td>1.07e-4</td>
<td>0.1</td>
<td>0.1</td>
<td>23.7</td>
<td>90.9</td>
</tr>
<tr>
<td>% Uncertainty</td>
<td>0.8%</td>
<td>2.7%</td>
<td>2.7%</td>
<td>0.8%</td>
<td>3.0%</td>
</tr>
</tbody>
</table>

### TABLE C.5. UNCERTAINTY IN MASS FLOWRATE, VELOCITY AND REYNOLDS NUMBER FOR THE HEAT TRANSFER FACILITY AT HIGH REYNOLDS NUMBER
<table>
<thead>
<tr>
<th>Nominal Value</th>
<th>m (kg/s)</th>
<th>U_m (m/s)</th>
<th>U_max (m/s)</th>
<th>Re_{Dh}</th>
<th>Re_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.96e-2</td>
<td>15.96</td>
<td>31.92</td>
<td>19485</td>
<td>19850</td>
<td></td>
</tr>
</tbody>
</table>

\[
\left( \frac{\partial}{\partial \Delta p} \right) (u_{\Delta p})^2 = 7.15e-4 \\
\left( \frac{\partial}{\partial T_L} \right) (u_{TL})^2 = 2.91e-5 \\
\left( \frac{\partial}{\partial U_m} \right) (u_{Um})^2 = 6.15e-5 \\
\left( \frac{\partial}{\partial Re_{Dh}} \right) (u_{Re_{Dh}})^2 = 1.89e-4 \\
\left( \frac{\partial}{\partial P_{atm}} \right) (u_{P_{atm}})^2 = 1.14e-5 \\
\left( \frac{\partial}{\partial P_g} \right) (u_{P_g})^2 = 5.69e-7 \\
\left( \frac{\partial}{\partial P_L} \right) (u_{P_L})^2 = 0 \\
\left( \frac{\partial}{\partial H} \right) (u_H)^2 = 0 \\
\left( \frac{\partial}{\partial D} \right) (u_D)^2 = 0 \\
\frac{\text{Total Uncertainty}}{} = 7.43e-4 \\
\% \text{ Uncertainty} = 0.8\% \\
\]

**TABLE C.6. UNCERTAINTY IN FRICTION FACTOR FOR THE FLOWFIELD FACILITY AT LOW REYNOLDS NUMBER**

<table>
<thead>
<tr>
<th>Nominal Value</th>
<th>f_{L/Dh}</th>
<th>f_N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43</td>
<td>0.099</td>
<td>0.028</td>
</tr>
<tr>
<td>0.12</td>
<td>0.024</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.03</td>
</tr>
</tbody>
</table>

\% Uncertainty = 23.5\%
TABLE C.7. UNCERTAINTY IN FRICTION FACTOR FOR THE FLOWFIELD FACILITY AT HIGH REYNOLDS NUMBER

<table>
<thead>
<tr>
<th></th>
<th>$f_{L/Dh}$</th>
<th>$f_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal Value</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.28</td>
<td>0.08</td>
</tr>
<tr>
<td>$[(\partial/\partial \Delta p)(u_{\Delta p})]^2$</td>
<td>0.006</td>
<td>0.002</td>
</tr>
<tr>
<td>$[(\partial/\partial U_m)(u_{U_m})]^2$</td>
<td>0.016</td>
<td>0.000</td>
</tr>
<tr>
<td>$[(\partial/\partial U_{max})(u_{U_{max}})]^2$</td>
<td>0.000</td>
<td>0.004</td>
</tr>
<tr>
<td>$[(\partial/\partial Re_{Dh})(u_{Re_{Dh}})]^2$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$[(\partial/\partial H)(u_H)]^2$</td>
<td>0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>$[(\partial/\partial W)(u_W)]^2$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$[(\partial/\partial D)(u_D)]^2$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Total Uncertainty</strong></td>
<td>0.018</td>
<td>0.004</td>
</tr>
<tr>
<td><strong>% Uncertainty</strong></td>
<td><strong>6.4%</strong></td>
<td><strong>5.6%</strong></td>
</tr>
</tbody>
</table>

TABLE C.8. UNCERTAINTY IN FRICTION FACTOR FOR THE HEAT TRANSFER FACILITY AT LOW REYNOLDS NUMBER

<table>
<thead>
<tr>
<th></th>
<th>$f_{L/Dh}$</th>
<th>$f_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal Value</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>0.52</td>
</tr>
<tr>
<td>$[(\partial/\partial \Delta p)(u_{\Delta p})]^2$</td>
<td>0.006</td>
<td>0.002</td>
</tr>
<tr>
<td>$[(\partial/\partial U_m)(u_{U_m})]^2$</td>
<td>0.047</td>
<td>0.001</td>
</tr>
<tr>
<td>$[(\partial/\partial U_{max})(u_{U_{max}})]^2$</td>
<td>0.000</td>
<td>0.012</td>
</tr>
<tr>
<td>$[(\partial/\partial Re_{Dh})(u_{Re_{Dh}})]^2$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$[(\partial/\partial H)(u_H)]^2$</td>
<td>0.015</td>
<td>0.000</td>
</tr>
<tr>
<td>$[(\partial/\partial W)(u_W)]^2$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$[(\partial/\partial D)(u_D)]^2$</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Total Uncertainty</strong></td>
<td>0.049</td>
<td>0.012</td>
</tr>
<tr>
<td><strong>% Uncertainty</strong></td>
<td><strong>9.6%</strong></td>
<td><strong>8.5%</strong></td>
</tr>
</tbody>
</table>
### TABLE C.9. UNCERTAINTY IN FRICTION FACTOR FOR THE HEAT TRANSFER FACILITY AT HIGH REYNOLDS NUMBER

<table>
<thead>
<tr>
<th></th>
<th>$f_{L/Dh}$</th>
<th>$f_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Value</td>
<td>0.30</td>
<td>0.08</td>
</tr>
<tr>
<td>$[(\partial/\partial \Delta p)(u_{\Delta p})]^2$</td>
<td>0.010</td>
<td>0.003</td>
</tr>
<tr>
<td>$[(\partial/\partial U_m)(u_{U_m})]^2$</td>
<td>0.024</td>
<td>0.000</td>
</tr>
<tr>
<td>$[(\partial/\partial U_{\text{max}})(u_{U_{\text{max}}})]^2$</td>
<td>0.004</td>
<td>0.006</td>
</tr>
<tr>
<td>$[(\partial/\partial \text{Re}<em>{Dh})(u</em>{\text{Re}_{Dh}})]^2$</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>$[(\partial/\partial H)(u_{H})]^2$</td>
<td>0.013</td>
<td>0.001</td>
</tr>
<tr>
<td>$[(\partial/\partial W)(u_{W})]^2$</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>$[(\partial/\partial D)(u_{D})]^2$</td>
<td>0.003</td>
<td>0.001</td>
</tr>
<tr>
<td><strong>Total Uncertainty</strong></td>
<td>0.030</td>
<td>0.007</td>
</tr>
<tr>
<td><strong>% Uncertainty</strong></td>
<td>10.0%</td>
<td>8.4%</td>
</tr>
</tbody>
</table>

### TABLE C.10. UNCERTAINTY IN PRESSURE COEFFICIENT

<table>
<thead>
<tr>
<th></th>
<th>$C_p$ at $\theta = 180^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{Re}_D = 2.0e4$</td>
</tr>
<tr>
<td>Nominal Value</td>
<td>-1.17</td>
</tr>
<tr>
<td>$[(\partial/\partial \Delta p)(u_{\Delta p})]^2$</td>
<td>0.032</td>
</tr>
<tr>
<td>$[(\partial/\partial U_{\text{max}})(u_{U_{\text{max}}})]^2$</td>
<td>0.005</td>
</tr>
<tr>
<td><strong>Total Uncertainty</strong></td>
<td>0.191</td>
</tr>
<tr>
<td><strong>% Uncertainty</strong></td>
<td>13.0%</td>
</tr>
</tbody>
</table>
### TABLE C.11. UNCERTAINTY IN PIN-FIN HEAT TRANSFER

<table>
<thead>
<tr>
<th>Nominal Value</th>
<th>Nu&lt;sub&gt;D&lt;/sub&gt; at θ = 0°</th>
</tr>
</thead>
<tbody>
<tr>
<td>[[(∂/∂Ti)(uTi)]&lt;sup&gt;2&lt;/sup&gt;</td>
<td>7.98</td>
</tr>
<tr>
<td>[[(∂/∂Tsurf)(uTsurf)]&lt;sup&gt;2&lt;/sup&gt;</td>
<td>7.98</td>
</tr>
<tr>
<td>[[(∂/∂q''in)(uq''in)]&lt;sup&gt;2&lt;/sup&gt;</td>
<td>5.90</td>
</tr>
<tr>
<td><strong>Total Uncertainty</strong></td>
<td><strong>54.6</strong></td>
</tr>
<tr>
<td><strong>% Uncertainty</strong></td>
<td><strong>10.0%</strong></td>
</tr>
</tbody>
</table>

### TABLE C.12. UNCERTAINTY IN ENDWALL HEAT TRANSFER

<table>
<thead>
<tr>
<th>Nominal Value</th>
<th>Nu&lt;sub&gt;Dh&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re&lt;sub&gt;D&lt;/sub&gt; = 3.0e3</td>
<td>Re&lt;sub&gt;D&lt;/sub&gt; = 2.0e4</td>
</tr>
<tr>
<td>[[(∂/∂Ti)(uTi)]&lt;sup&gt;2&lt;/sup&gt;</td>
<td>8.46</td>
</tr>
<tr>
<td>[[(∂/∂Tsurf)(uTsurf)]&lt;sup&gt;2&lt;/sup&gt;</td>
<td>7.62</td>
</tr>
<tr>
<td>[[(∂/∂q''in)(uq''in)]&lt;sup&gt;2&lt;/sup&gt;</td>
<td>4.76</td>
</tr>
<tr>
<td><strong>Total Uncertainty</strong></td>
<td><strong>4.6</strong></td>
</tr>
<tr>
<td><strong>% Uncertainty</strong></td>
<td><strong>11.1%</strong></td>
</tr>
</tbody>
</table>
APPENDIX D:
SWIRL STRENGTH CALCULATIONS

Adrian et al. (2000) discuss several techniques for analyzing turbulent flowfields. Among the various techniques is the calculation of swirl strength or swirl, \( \lambda_{c,i} \). The swirl strength is similar to vorticity because the velocity gradient tensor is used to calculate swirl. Swirl is different from vorticity, however, because swirl removes pure shearing motion and leaves only swirling motions. Vorticity can be used to identify turbulent eddies, but results can be misleading because vorticity includes shearing motions in addition to swirling motions. Swirl and vorticity have the same units of \( s^{-1} \) although the software package used with the TRDPIV system (www.LaVision.de, 2012) calculated swirl-strength-squared, giving units of \( s^{-2} \).

To calculate swirl, the eigenvalues of the local velocity gradient tensor are calculated:

\[
\begin{vmatrix}
\frac{d\tilde{U}}{dx} - \lambda & \frac{d\tilde{U}}{dy} \\
\frac{d\tilde{V}}{dx} & \frac{d\tilde{V}}{dy} - \lambda
\end{vmatrix} = \lambda^2 - \left( \frac{d\tilde{U}}{dx} + \frac{d\tilde{V}}{dy} \right) \lambda - \frac{d\tilde{U}}{dx} \frac{d\tilde{V}}{dy} = 0
\]

(C.1)

Solving the quadratic expression in equation C.1 will produce either a pair of real eigenvalues or a pair of complex conjugates. A swirling motion exists when the eigenvalues are complex conjugates of the form \( \lambda_{c,r} \pm i\lambda_{c,i} \) where \( \lambda_{c,r} \) is the real portion and \( \lambda_{c,i} \) is the imaginary portion. Assuming the eigenvalues are complex conjugates, the quadratic formula gives \( \lambda_{c,r} = \frac{-b}{2a} \) and \( \lambda_{c,i} = \frac{\pm(b^2 - 4ac)^{0.5}}{2a} \). The swirl strength of a given eddy in the filtered flowfield is equal to \( \lambda_{c,i} \).

As mentioned, the TRDPIV software used in the present work (www.LaVision.de, 2012) has the capability of calculating swirl strength (squared). Matlab or Tecplot could also be used to calculate swirl strength. The user must simply calculate the velocity gradient tensor using a second-order finite difference (or higher-order scheme, if desired). Then, a calculation must be performed to see if the eigenvalues of the velocity gradient tensors are complex conjugates. Finally, in regions where the eigenvalues are complex conjugates, the swirl strength may be computed using the above procedures.
APPENDIX E: ENERGY SPECTRA CALCULATIONS

The TRDPIV system used in the present work had the capability of resolving the flow at high sampling rate, 1 kHz for 1024 x 1024 px resolution. Because of the high sampling rate, it was possible to compute energy spectra from TRDPIV data. The Nyquist frequency of 500 Hz was well above the typical vortex shedding frequency of 15 Hz at \( \text{Re}_D = 2.0e4 \) (worst case scenario). Furthermore, for a single, infinite cylinder the frequency associated with shear layer eddies was approximately 150 Hz at \( \text{Re}_D = 2.0e4 \). Because the TRDPIV system had enough frequency resolution to capture important flow features, spectral analysis was performed on the time-dependent data. One advantage of using TRDPIV to calculate energy spectra is the spatial resolution of the flowfield. A single TRDPIV test case produced up to 1024 x 1024 time signals of U- and V-velocity, one time signal for each pixel in the domain. A single TRDPIV test case can, therefore, produce the one-dimensional energy spectrum of U- and V-velocity at each point in the domain. For hot-wire or LDV, however, a new signal must be captured at each point where the energy spectrum is desired. The major disadvantage of calculating energy spectra using TRDPIV is the relatively small amount of samples. In the present work, the TRDPIV system captured 3000 samples for each test case. While 3000 samples was proven to be adequate for convergence of turbulence statistics (see Section 3.8) the energy spectra produced from the TRDPIV data had a high degree of noise. To reduce the amount of noise from calculated TRDPIV spectra, several operations were applied to the data. The 3000-point time signal was broken into three 1000-point time signals. The energy spectra were calculated for each 1000-point time signal and averaged together. Figure E.1 shows the energy spectrum for a single 3000-point time series and Figure E.2 shows the averaged energy spectrum for three 1000-point time signals. The interrogation point was taken at \( X/D = 1.6, Y/D = 0.4 \) for the case having \( H/D = 1, S/D = 2, X/D = 2.16 \) at \( \text{Re}_D = 2.0e4 \). From Figure E.2, it was apparent that using three 1000-point time signals decreased resolution of lower frequencies, but reduced noise significantly. In addition, the symmetry condition about the wake axis was used to further reduce noise in energy spectra. Because each pin-fin wake was approximately symmetric about the wake axis, two interrogation points were collected on either side of the wake. For example a point at \( X/D = 1.6, Y/D = 0.4 \) had approximately the same frequency content as \( X/D = 1.6, Y/D = -0.4 \). Using two opposing interrogation positions, the number of calculated energy spectra was
doubled, from three 1000-point time series to six 1000-point time series. Figure E.3 shows the calculated energy spectrum averaged using six 1000-point time series. In Figure E.3, a distinct peak was found at the shedding frequency near \( St = 0.2 \).
Figure E.1. Energy spectrum calculated from a single 3000-point time series at a point in the shear layer for $S/D = 2$, $X/D = 2.16$, $Re_D = 2.0e4$.

Figure E.2. Energy spectrum calculated from the average of three 1000-point time series at a point in the shear layer for $S/D = 2$, $X/D = 2.16$, $Re_D = 2.0e4$. 
Figure E.3. Energy spectrum calculated from the average of six 1000-point time series at one point in the shear layer and at the symmetric position across the wake for S/D = 2, X/D = 2.16, Re_D = 2.0e4.
VITA

Jason K. Ostanek

Jason Ostanek graduated from Virginia Polytechnic Institute and State University (Virginia Tech) in May 2006 with a Bachelor of Science degree in Mechanical Engineering. Jason worked as an undergraduate research assistant for Dr. Karen Thole in the Experimental and Computational Convection Laboratory (ExCCL) while at Virginia Tech. Upon graduation, Jason moved with Dr. Thole and the rest of the ExCCL facilities and students to Pennsylvania State University (Penn State). Jason graduated, in May 2008, from Penn State with a Master’s degree in Mechanical Engineering. During graduate school, Jason was awarded a Science, Math, and Research for Transformation (SMART) fellowship with the Department of Defense. The Naval Surface Warfare Center, Carderock Division (NSWCCD) in Philadelphia, PA was the research laboratory which sponsored Jason’s SMART fellowship. Under the SMART program, Jason completed his Doctor of Philosophy degree in Mechanical Engineering in May, 2012. Upon graduation with his PhD, Jason will begin work with NSWCCD in Philadelphia, PA.