A Numerical Analysis of the Shear Key Cracking Problem in Adjacent Box Beam Bridges

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Zi Sang

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The thesis of Zi Sang was reviewed and approved by the following:

Maria Lopez de Murphy
Associate Professor of Civil Engineering
Thesis Advisor

Jubum Kim
Assistant Professor of Civil Engineering

Gordon Warn
Assistant Professor of Civil Engineering

Peggy A. Johnson
Professor of Civil and Environmental Engineering
Head of the Department of Civil and Environmental Engineering

Signatures are on file in the Graduate School
Abstract

This objective of this thesis is to evaluate the potential of selected shear key modifications in reducing tensile stress in the shear key region of adjacent box beam using numerical analysis. In order to meet the objective, a comprehensive literature review of the current design and construction practices was conducted. Shear key configuration, grouting material, post-tensioning, and bearing pad details were identified as four possible sources for cracking from the design perspective. A grillage analysis was conducted to determine the maximum moment and shear experienced by shear key. The information was then used in the finite element analysis to evaluate the potential of each shear key modification in reducing the tensile stress in the shear keys of adjacent box beam bridge. The most effective shear key modifications in reducing tensile stress, which was the main mechanism for cracking, were concluded at the end of the thesis.
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Chapter 1 - Introduction

1.1. Background Information

This thesis is originated from the Pennsylvania Department of Transportation research project work order 14 - determining more effective approaches for grouting shear keys of adjacent box beams. The project ended on May 30th, 2010. This thesis is an update on the existing PennDOT WO14 final report to incorporate the analysis information that was not included in that report.

The use of bridges built with adjacent precast, prestressed concrete box beams is favored by several State Departments of Transportation, due to the efficiency of their construction. Box beams are typically connected by shear keys with some type of transverse reinforcement (mild or prestressed) and are grouted. Although the structural performance of such bridges has been successful, a common problem that has been observed is shear key grout failure at early stages – cracking, see Figure 1.1. In many cases this cracking has resulted in water leakage, which could lead to corrosion problems both on the mild and prestressed reinforcement. Severe cracking can eventually affect the load transfer ability of the shear key unit. (Macioce et al. 2007)

Figure 1.1 Shear key cracking during construction (William Koller 2008)

A typical adjacent box beam bridge cross section is shown in Figure 1.2. A series of prestressed box beams are placed together next to each other. The box beams are joined together by a space of ¼” to ¾” filled with grouting material. The joint is called shear key as shown in the figure. Shear key is a unique and important structural component in adjacent box beam bridges. It acts as a connector between two adjacent box beams to transfer the loads across the entire superstructure. One advantage pertaining to adjacent box beam bridge is the small distribution factor, which means the vehicle load is spreading uniformly across the superstructure.
1.2. Objective

This thesis aims to evaluate the potential of shear key modifications in reducing tensile stress and thus cracking in the shear key region of adjacent box beam bridge. This evaluation will be conducted numerically based on experimental material characterization.

In order to meet this objective, the following tasks were conducted:

a) A literature review was conducted to identify possible shear key modifications that have the potential in reducing the likelihood of cracking. The review also looked into existing grillage analysis methods and finite element analysis used to model concrete bridge structures.

b) A grillage analysis was conducted to identify the maximum moment and shear experienced by the shear key under full live load. In addition, the distribution of loads in the bridge structure was examined.

c) Finite element analysis was conducted to evaluate the potential of each shear key modification in reducing stress level in the shear key under full live load.

1.3. Scope of research

In this research, a state of practice literature review was conducted to identify components that related to shear key cracking. Material characterization was conducted to provide the basic material properties needed for the numerical analysis. A typical PennDOT box beam bridge (80 feet span) was analyzed for the critical load cases that produced the maximum load effects in the shear key. Based on the results and the shear key modifications identified in literature review, numerical evaluation using finite element analysis was conducted to achieve the objective of the research.

1.4. Outline of thesis

Chapter 1 provides the introduction. Chapter 2 of the thesis presents the findings from a comprehensive literature review of the current practices and researches regarding shear key cracking. In Chapter 3, a grillage analysis is conducted to identify the maximum forces effect...
experienced by the shear key. The results from the grillage analysis are then used in finite element model to assess the effect of different modifications to shear key. The parametric study of the shear key modifications is discussed in Chapter 4. The final Chapter, Chapter 5, presents conclusions from the analysis and recommendations for future research.
Chapter 2 - Literature Review

2.1. Literature Review

A comprehensive literature review of current shear key practices was conducted. Sources consulted included the following databases: Compendex, Web of Science, Transportation Research Information System (TRIS), Transportation Research Board (TRB), Transportation Research in Progress (TRIP), National Technical Information System (NITS), American Society of Civil Engineers (ASCE), Precast/Prestressed Concrete Institute (PCI), American Concrete Institute (ACI), as well as State DOT’s database.

The goal of the literature review is to find feasible alternatives for state DOT’s to implement in the field. The findings from literature review will be closely compared with the current DOT’s practices. Based on the findings of the literature survey documented in the literature review and assessment report, the following list indicates the most likely sources for shear key cracking:

- Partial grouting depth and top tier shear key location (related to shear key geometry)
- Bearing details that induce relative beam deflection.
- Insufficient transverse post-tensioning reinforcement
- Inadequate strength and shrinkage incompatibility of grouting materials

2.1.1. Shear key geometry

In a study by Huckelbridge et al. in 1995, the finite element method was used to evaluate the stress state of the shear key when a truck tire rolled over the center of the box beam. He found that under the standard AASHTO HS-25 truck load the corners of the box beam bent inward toward the centerline of the box beam. The tensile stress associated with the strain eventually led to cracking of the shear key at the top portion. It can be concluded that the vertical location of the shear key has the potential to affect the performance of the system. In order to maintain the load transfer efficiency of the shear key, a mid-depth location between two adjacent box beams is recommended by other researchers (Miller et al. 1999).

Another study conducted by Kim et al. (2008) indicated that mid-tier shear key can control the relative deflection of two adjacent box beams when the bridge is loaded. They found that the top tier and bottom tier shear keys have a relative larger deflection as compared to the mid-depth shear configuration.

Grouting practices significantly affect the structural behavior of shear key. Two types of shear key grouting depth are used by state DOTs: partial depth and full depth. In a partial depth shear key, the grout only covers a portion of the total height of the box girder. It usually coincides with the size of the shear key when it is located on the top tier of box girder, as
shown Figure 2.1 a). Full depth shear keys have the entire space between two girders grouted, as shown in Figure 2.1b).

A study was conducted by the New York State Department of Transportation in 1996 to evaluate the performance of the full-depth shear key configuration in adjacent box girder bridges (Lall et al., 1998). Inspections were performed to 91 box girder bridges with full depth shear keys, built after 1992. The results from this study were compared to the results from a previous study conducted on box girder bridges using partial depth shear key (Tang 1992). Shear key with longitudinal cracking was found on 21 (23%) of the 91 inspected bridges with full depth shear key. In contrast, 54% of the bridges with partial depth shear key were found to have longitudinal cracking. Moreover, only 47 out of total 874 full depth shear keys were associated with deck cracking. Therefore, it was concluded that full-depth shear keys significantly reduced the shear key cracking. Figure 2.2 shows the categorized frequency of longitudinal deck cracking observed in this study.
Based on this literature review, at least two DOT’s have used full depth shear key configurations (Michigan State DOT and New York State DOT). According to the shear key details defined in BC-775M Standard Miscellaneous Prestress Details provided by PennDOT (Figure 2.3), PennDOT uses a partial depth shear key configuration (PennDOT 2007). Several sources found in this literature survey also indicated that full depth shear key is a possible solution to the shear key cracking problem (Lall et al. 1998; Badwan and Liang, 2007a, 2007b; Dong 2007; Hanna et al. 2007; Scott and Tremblay 2007; Attanayake and Aktan 2008, 2009).

![Shear Key Detail](image)

**Figure 2.3** PennDOT partial depth shear key details (PennDOT 2007)

### 2.1.2. Grouting material

Joints between box beams are filled with grouting material to transfer vertical shear and bending stresses induced by moving vehicles. Mechanical properties (e.g. stiffness, ductility, tensile strength, and shrinkage) of grouting materials as well as transverse shortening of concrete box beams may affect longitudinal cracking.

Nottingham (1995) pointed out that a high quality, low-shrinkage, impermeable, high bond, high early strength grout with good workability and low working temperature feature should be used to fill the joint. Gulyas et al. (1995) found that the closest material with these qualities was magnesium ammonium phosphate grout extended with pea gravel (Set 45 Hot Weather). They conducted a follow up study to compare Set 45 Hot Weather grout and a commonly used non-shrinkage grout. The results showed that the Set 45 Hot Weather developed higher bond strength when compared with the non-shrinkage grout. The composite shear, tensile, and compressive strength of Set 45 Hot Weather were also significantly higher than the conventional non-shrinkage grout. Another study conducted by Ohio DOT also confirmed the performance of Set 45 Hot Weather Grout by conducting full scale testing of an adjacent box girder bridge (Gulyas et al. 1995).
Epoxy based grout has been recommended by researchers as an alternative to improve shear key performance. Epoxy grout has the advantage in bond strength as compared to other types of grouting materials (El-Remaile et al. 1996, Miller et al. 1999).

Another material that can be used to grout the shear key is fiber reinforced concrete. It is apparent that the main cause of cracking is tensile stress. Fibers can be added to grout to control cracks. So far, based on the literature review, there has not been any research done on fiber reinforced concrete in terms of shear key application. However, it could be a possible solution to this problem.

2.1.3. Transverse reinforcement details

Currently PT tendons in PennDOT are placed near the top where the shear key recess is located, potentially leading to an uneven stress distribution across the section. Moreover, only a minimum tendon force of 30 kips is specified in the AASHTO Bridge Design Manual. Some state DOTs, including Michigan (Attanayake and Aktan 2008, 2009), recommend that tendons be placed either at the midheight if the beam depth is less than 27” or two sets of tendons at 1/3 point of beam depth if the beam is deeper, with the prestressing force being dependent upon live loads. For span length less than 50 feet, two tendons are placed at mid span, and one at each end of beam. In Japan, much heavier transverse reinforcement is used in combination with cast in place concrete in the keyway (Hanna et al. 2007). Figure 2.4 presents the transverse post-tensioning details used by New York State DOT. For span less than 50 feet, post-tensioning tendons are placed at three locations whereas for span greater than 50 feet post-tensioning tendons are placed at quarter points. In addition, NYSDOT places the tendons at the mid height of the box beam in combination with the use of mid tier shear key (NYSDOT 2008). Similarly, PennDOT uses almost the same transverse post-tensioning layout except no tendons are placed at the end span and the tendons are usually placed at the centroid of the shear key, which in most cases is not at the mid height of the box beam.

![Figure 2.4 NYSDOT transverse post-tensioning details (NYSDOT 2008)](image-url)
It is clear that this prescriptive design approach of specifying transverse reinforcement details is flawed. Shear key will crack when inadequate post-tensioning force is applied and the vehicle loading exceeds the service capacity of the prescribed reinforcement capacity. A performance based design approach is needed to prevent the shear key cracking from occurring. As shown in Figure 2.5, the spacing $S$ and post-tension force are to be determined using a performance based design approach.

![Image](image1.png)

Figure 2.5 Transverse post-tensioning position

In the study by Hanna (2007), a proposed formula to calculate the required post-tensioning force was suggested. The formula involved parameters such as bridge span length, loading, bridge width and skew angle. The formula was developed using grillage analysis. The formula can be presented using the chart, for a single HS25 truck load case, in Figure 2.6.

![Image](image2.png)

Figure 2.6 Post-tensioning force required based on analysis conducted by Hanna et. al (2007)
2.1.4. Bearing pad details

Bearing pads seem to be another contributor to the shear key cracking. Two sources have noted the effects of the bearing pad details on the relative deflection between girders. In a reflective cracking history graph provided by District Engineer William Koller (PennDOT District 1), it is clear that shear key cracking is initiating near the two ends of the box beams and propagating toward the mid span. Miller et al.’s full scale testing of a box beam bridge (1999) showed a similar crack pattern is observed as seen in Figure 2.7.

![Figure 2.7 Shear key cracking propagation in the full scale testing (Miller et al. 1999)](image)

The New York State DOT requires the bearing pad to be at least half the width of the box beam. However, a study (Lall et al. 1998) suggested that one half width of box beam is not adequate enough to provide lateral stability because the box beam can rotate along its longitudinal axis without restriction from such a short bearing pad. Therefore, they proposed two alternative bearing pad details as shown in Figure 2.8. The first alternative gives the box beam more restriction to rotate. The second alternative is to reduce the relative deflection between the two adjacent box beams as the bearing pad deforms.

![Figure 2.8 Bearing pad alternatives: a) Isolated bearing pads, b) shared bearing pad](image)

Based on a drawing provided by PennDOT (2008), the current Pennsylvania DOT specifies two bearing pads of a typical width of 10 inches under each box beam with an edge clearance of 5 inches. The details of the PennDOT bearing pad configuration is discussed in Section 4.2.6.
2.1.5 Numerical Analysis

Grillage analysis is one of the traditional analysis approaches used on bridge superstructure. The bridge superstructure is assumed to be a grid system with members representing the longitudinal and transverse property of the bridge structure. The grid structure is analyzed using commercial computer software.

Conventionally, the grillage analysis is done in such a way that the longitudinal members are representing each girder of the bridge and transverse members are representing the cross bracing in the bridge. However, this approach is designed for structure in which the members are discrete, in other words, the cross bracing and girders can be easily identified. For adjacent box beam bridge, the box beams are continuously connected together by the shear key. According to Hambly (1991), the grillage analysis of adjacent box beam bridge should assume equal spacing between longitudinal members and transverse members to allow uniform distribution of load. Moreover, the transverse member should represent the transverse structural properties of the adjacent box beam bridge.

Hambly also suggests that the shear key itself should be modeled as a hinge connection at the middle between two longitudinal members. I suspect Hambly models shear key as hinge because partial depth shear key, which was used at that time, does not transfer moment between box beams, instead it only transfers shear force across the bridge superstructure.

In a study by Attanayake and Aktan (2009) to solve the shear key problem, they propose a new way of performing the grillage analysis. The longitudinal members in their model represent composite beams which consist of halves of two adjacent box beams and the shear key in between them, as shown in Figure 2.9. The advantage of this approach is that it captures the interfacial behavior of the shear key. However, the results obtained from using this approach is with respect to the entire unit instead of the shear key itself.

![Figure 2.9 Composite box beam with shear key (Attanayake and Aktan, 2009)](image)

The torsional stiffness of the cross-section is usually calculated using a closed form formula. According to Hambly (1991), the torsional stiffness of a closed thin-walled section subjected to torque can be expressed as follows:
where $A_c$ is the area enclosed by the centerline of the box beam webs and flanges, $l$ is the length of each segment of the box beams (web and flange), and $t$ is their corresponding thickness. Marshall suggested the following expression to compute the torsional stiffness of bridge decks (since flange and/or web of box beams may be considered thick and open in torque). Equation 2.2 will be used to calculate the torsional stiffness of the elements considered in this analysis. When using Eq. 2.2, the $\sum \frac{l}{t}$ is taken as the sum of length thickness ratio of all the segments constructing the box beam. Moreover, the webs are neglected because they are already accounted for as the shear area of the longitudinal beam.

\[
J = J_o + J_c = \frac{bt^3}{3} + \frac{4A_c^2}{\sum \frac{l}{t}}
\]  
(Eq. 2.2)

In the literature review conducted, only one study has used finite element analysis in evaluating the stress in the shear key (Dong et. al 2007). In this study, the interest was to evaluate the effect of grouting material and shear key geometry in shear keys connecting decked prestressed concrete girders. In this study, a bridge analysis similar to a grillage analysis was conducted to obtain the moments and shears which were used in the finite element analysis. In the finite element analysis, only elastic material models were used in modeling the concrete and grouting materials. The materials analyzed were fictitious, usually assumed 0.5, 1 and 1.5 times the strength of concrete. Three different geometries were analyzed – full depth, partial depth, and diamond shear key. It was found full depth shear key tends to develop lower stresses using the same loading and material.
Dong et al. (2007) suggest that future research should conduct nonlinear analysis. Caution needs to be taken when using bridge analysis to obtain the loadings that will be used for the finite element analysis. In Dong et al.’s study, the shear key in their bridge analysis is assumed to have either fixed or hinge connections. It was suggested that the shear key connections is somewhere between fixed and hinge connection. In this thesis, the transverse grillage member will consist of several members to represents different sections in the box beam. For example, the shear key will be modeled as a member instead of a connection. The member will contain the section properties of the shear key cross section.

To expand on the study by Dong et al, other studies related to modeling nonlinear material properties and crack propagation were reviewed. Studies by Coronado and Lopez (2006, 2007, 2008, and 2010) provide a solid background for modeling cracks in concrete and fracture properties of interfaces. Todeschini et al. (1964) and Thorenfeldt et al. (1987)’s studies on normal and high strength concrete models were used in the nonlinear finite element analysis conducted in the thesis.

In the study by Coronado and Lopez (2006), a plastic-damage model from Abaqus is used to model the debonding failure at the interface of concrete and FRP. A thin damage band is modeled at the interface to have fracture energy of interface measured experimentally. Fracture energy, $G_F$ (energy, per unit area, needed to create and propagate a crack in a particular material or interface) is the area under the tension softening curve shown in Figure 2.11. The plastic-damage model can be used to predict the inelastic behavior of the concrete, grout and their interfaces, respectively. Moreover, the softening curve can be approximated using a linear model as shown in Figure 2.11c. The cohesive fracture energy is calculated as the total area under the load-LVDT curves obtained from the notched beam.

![Figure 2.11 Softening model for concrete (Coronado and Lopez 2006)](image-url)
2.2. Summary

In order to investigate the four factors identified above, two types of numerical models were developed. A grillage analysis of an adjacent box beam bridge will provide information on the global behavior (load transfer between adjacent box beams and force effects in the vicinity of the shear key region). The maximum moment and shear experience by shear key will be determined by the global analysis. This information will then be used in the local analysis. A finite element model will focus on details of the shear key connection; it will capture the local behavior (cracks and stress distribution on the shear key, effect of grouting materials, depth and geometry of shear key as well as the transverse post-tensioning details and bearing pad configurations). Based on the stress level and crack condition developed under each shear key modification, the best shear key alternative in reducing stress and cracks can be found.
Chapter 3. Grillage Analysis

3.1 Introduction

A grillage analysis was conducted to determine the load effects around the shear key region. Results including moment and shear forces obtained from the grillage analysis were used in the next phase of the analysis in which finite element analysis of an isolated shear key model was conducted, these FE analysis will be described in the next Chapter. The grillage converts the bridge deck structure into a network of rigidly connected beams, e.g. a network of skeletal members rigidly connected to each other at discrete nodes. Each element is given an equivalent bending and torsional rigidity to represent the portion of the deck which it replaces. Bending and torsional stiffnesses in every region of slab are assumed to be concentrated in nearest equivalent grillage beam.

A simply-supported, prototype bridge with the following characteristics was chosen for the grillage analysis (see Fig. 3.1). This bridge will represent a typical PennDOT long span adjacent box beam bridge of medium span (80 feet) with four vehicle lanes.

- Span length: 80 feet
- 12 box beams (AASHTO Standard BII-48) with 11 shear keys
- 1 inch-thick shear key, this geometry matches the experimental tests described in Chapter 2.
- Composite deck with 5.5” concrete overlay and typical PennDOT barrier (BD601m specifications)

![Figure 3.1 Cross-section of the bridge superstructure](image)

3.2 Loads

Two load types were considered in this study – dead load and live load. Dead load consisted of the self weight of box beams, concrete topping, and barriers of the bridge superstructure. Live load consisted of truck load and lane load per AASHTO LRFD specifications. The truck load was an HS25 design truck which consists of three axles of 10 kips, 40 kips, and 40 kips, as shown in Figure 3.2a. While the distance between the 40-kip and 10-kip axle loading can vary from 14 feet to 30 feet, 14 feet spacing was chosen as it produces the maximum load effect in a simply
supported beam. Another type of AASHTO live load was the design lane load of 0.64 kips per linear foot as shown in Figure 3.2b. The PennDOT’s P-82 truck overload (Fig. 3.2c) was also considered in the grillage analysis.

AASHTO bridge design guide Service I load combination (AASHTO Bridge Design Manual 2008) was considered for this grillage analysis. Multiple presence factor and dynamic impact factor were incorporated in the load cases.

![Figure 3.2 Live Loads: a) AASHTO HS25 design truck; b) AASHTO design lane load; c) PennDOT P-82 204-kip truck overload (PennDOT DM-4 2007)](image)

### 3.3 Modeling Description

#### 3.3.1 Grillage

In the grillage analysis the box beam system is treated as a grid system as shown in Figure 3.3. Twelve longitudinal grid lines coincide with the center lines of box beams represented in Figure 3.1. Each transverse member represents a portion of the box beam that acts as a transverse component, providing superstructure stiffness in transverse direction. The bending and torsional characteristics of the box beam plus 5.5 inch topping (assuming composite section) are assigned to these transverse members as well as longitudinal members. 5.5 inches pavement is the typical bridge roadway pavement thickness (PennDOT 2008). The two ends of the grid system are considered to be simply supported.
The mesh size is determined based on grillage analysis conventions. The transverse beam spacing must be sufficiently small to accurately model the distribution of loads and assignment of the point loads. If the spacing is too large, the grillage model may not adequately capture the transverse stiffness inherent to box beam bridges. It is recommended that the spacing of the transverse members be similar to the spacing of the longitudinal member spacing to allow a uniform distribution of loads as the adjacent box beam bridge superstructure acts like a continuous slab rather than discrete beams and cross bracings (Hambly 1991). Since the width of the box beam is 4 feet, the spacing between the longitudinal members is set to be 4 feet plus 1 inch to account for the width of the shear key. The spacing of the transverse members is set to be 4 feet.

### 3.3.2 Material Properties and Member Section Properties

Sectional properties of the longitudinal members were assigned according to the AASHTO/PCI bridge design standard (AASHTO/PCI Bridge Design Manual 1997) for box beam sections. Figure 3.4 shows the cross section of longitudinal members. The stiffness contribution of the barriers on the edge beams is considered in terms of shear area. The barrier was transformed to an equivalent rectangular section to simplify the calculation of section properties (see Figure 3.4b). Its flexural stiffness is not considered because it will make the edge beams disproportionately stiff; in grillage analysis, such stiff edge beams will not undergo any deflection and, therefore, will act as fixed supports on the two ends in the transverse direction (e.g. the bridge would appear to be supported on all 4 sides, which is incorrect). It should also be pointed out that
these barriers are usually discontinuous, non prestressed, and could be cracked. Therefore they don’t behave as elastic elements.

![Figure 3.4 Cross section of typical longitudinal members in the grillage analysis: a) Interior box beam; b) Exterior box beam (as-is and simplified rectangular section)](image)

The second moment of inertia of the section was calculated using conventional section analysis and assuming elastic behavior. Note that the center of gravity of the entire cross section is determined assuming that barriers are rigidly connected to the concrete overlay and box beams underneath, thus contributing to the torsional stiffness of the section. Therefore, the neutral axis should be located above mid-height of box beams. The torsional stiffness of the cross-section is calculated based on equations 2.2.

A 5.5 inch composite deck system is placed on top of the box beams. The topping is made of 4,000 psi concrete whereas the box beam is made of a higher strength concrete. Material characterization determined the compressive strength at 28 days of the concrete specimens tested to be 11,300 psi. This value was used in this grillage analysis to be consistent with the properties of the shear key sections tested in the laboratory (PennDOT Report 2010). When calculating the stiffness of the composite box beam section, the top layer of deck is transformed based on modulus ratio of two concrete materials. Table 3.1 summarizes the sectional properties used in the analysis.

<table>
<thead>
<tr>
<th>Table 3.1 Sectional properties of grillage elements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flexural Stiffness, I, (in^4)</strong></td>
</tr>
<tr>
<td>Members in longitudinal direction</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

The transverse members in the grid are modeled as shown in Figure 3.5. In between the longitudinal members, running along the y axis in Figure 3.3, a typical transverse member is divided into segments (see cross sections a1-b1, a2-b2, and a3-b3). Each segment represents a part of bridge cross section in the transverse direction. The segment that represents the box beam with the top and bottom flanges in the transverse direction is shown as section a1-b1 in Figure 3.5. Its flexural stiffness is calculated using the two flanges (with a void in-between). Table 3.1 shows this value for “two flanges”. The torsional stiffness of this segment is assumed to be the same as that of longitudinal members (as typically done in grillage analysis (Hambly 1991). The segment corresponding to the web section of the box beam has a rectangular cross section, shown in section a2-b2 of Figure 3.5. Section properties for the “web” are presented in Table 3-1. Similarly, the segment corresponding to the shear key also has a rectangular cross section. However its material properties are different (grout). Material properties obtained experimentally will be used for this segment. In addition, the shear key cross-section does not include the deck because the deck and shear key don’t act as a composite section.
The diaphragms inside the box beam are considered because their effect on the distribution of load is significant. The spacing and width of the diaphragms is based on PennDOT Bureau of Design standards BD651M. Figure 3.6 shows location, dimensions and cross section of and diaphragm dimensions. Properties of transverse members with diaphragms are presented in Table 3-1. They are calculated based on the cross sections shown in Figure 3.6. The solid concrete section (no void between flanges) where the diaphragm is present is represented as a web. The axial cross section area, shear area, and flexural stiffness in the transverse direction are calculated following the procedure discussed previously. The torsional stiffness is assumed to be the same as that of a longitudinal interior box beam, if we neglect the web effect of the diaphragm.
3.3.3. Live Load Combinations

One of the main objectives of conducting a grillage analysis was to determine the critical moment and shear forces in the shear key region that could be used in subsequent finite element analysis. To determine an appropriate range of bending moment and shear force in the transverse members, and thus shear key region, various load combinations and placement of live load were carefully considered. Seven different live load configurations expected to produce greater load effects are selected. They are summarized in Table 3.2, where uppercase letters represent AASHTO HS25 truck load, as shown in Figure 3.2, and lowercase letters for AASHTO design lane load or PennDOT P-82 at various location of bridge deck as illustrated in Figures 3.7 and 3.8.

The following loading cases were considered to produce maximum load effects: Case 1, a HS25 truck near the edge beam at the mid span of the bridge; Case 3, two HS trucks on opposite edges at the mid span of the bridge; Case 4; and Case 6, which adds the design lane load to the edge lane of the bridge. These four cases would likely produce the maximum negative moments in the transverse members, or shear keys, at the mid span. Case 2, a HS25 truck at the center of the bridge, and Case 5, a HS 25 truck at the center of bridge plus two design lane load occupying the two interior lanes would produce the largest positive moment in the shear key at mid span of the bridge. Last, the case in which a truck overload (P-82), as shown in Figure 3.2, is placed on the edge lane of the bridge (Case 7) was also investigated.
Table 3.2 Load cases considered in this study

<table>
<thead>
<tr>
<th>Load Case No.</th>
<th>Live load configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D</td>
</tr>
<tr>
<td>2</td>
<td>E</td>
</tr>
<tr>
<td>3</td>
<td>D, F</td>
</tr>
<tr>
<td>4</td>
<td>D + a</td>
</tr>
<tr>
<td>5</td>
<td>E + b, c</td>
</tr>
<tr>
<td>6</td>
<td>D, F + a, d</td>
</tr>
<tr>
<td>7</td>
<td>PennDOT P-82 on edge beam</td>
</tr>
</tbody>
</table>

3.4 Results and Discussion

Bending moments and shear forces from the grillage analysis were examined. The longitudinal members carried larger bending moments compared to the transverse members, as expected. Figure 3.9 shows the bending moment on the transverse member across mid span (see Fig. 3-3) when the system is subjected to load case 7 (Other load cases are shown in Appendix B). The figure shows bending moments expressed in terms of the node location in the transverse direction from one end of the bridge to another. Note that there is a discontinuity in moment magnitude where the transverse beam intersects with longitudinal beam members. This is expected, as in the Figure 3.9, the torsional moment of the longitudinal beam becomes part of the moment equilibrium about the global x-axis (x-axis in Figure 3.3): the sum of bending moments on the transverse beam and torsional moments on the longitudinal beam equal to zero. This check verifies the validity of the grillage model developed.
The transverse beam presents rigidity in discrete locations while in reality there is no physical beam in that direction. Therefore, the bending moments on the shear key region should be averaged out. Figure 3.10 shows the transverse moments in terms of shear key locations from one end of cross section to another in y-axis (y-axis in Figure 3.3), averages are also indicated. A similar procedure is applied when plotting the shear forces in the shear key.
The results of grillage analysis are summarized in Figures 3.11 and 3.12 where the average moment and shear force are plotted at each shear key location for all the load cases. It can be observed that two load cases, LC5 and LC2, create the largest maximum positive moments (which creates tension stresses at the bottom of the shear key) for shear key locations 3 to 10. The largest magnitude is found at mid span (shear key 6) under LC5, where a HS 25 truck at the center of bridge plus two design lane loads occupy the two interior lanes. This result is tabulated in Table 3.3. For shear keys located near the exterior beam (1, 2, 10, 11), LC4 controls however it magnitude is four times smaller than the one obtained under LC5. The maximum negative moment, which creates tension stresses at the top of the shear key was found to be created by LC6 (two HS trucks on opposite edges at the mid span of the bridge plus lane load) on shear key 6 (midspan). This result is tabulated in Table 3.3.

The maximum shear forces are found to be influenced by the position of the design truck. For each load case, the maximum shear force is located in the shear key closest to the position of the truck. The largest magnitude corresponds to LC4 where truck and lane loads were placed near the exterior beams.
Figure 3.11 Average moment in shear keys for all load cases

Figure 3.12 Average shear force in shear keys for all load cases
To summarize, the maximum load effects are determined as follows:

- Max positive moment = 2,701 kip-in, shear = 0 kips (load case 5), shear key 6 (midspan)
- Max negative moment = -1,470 kip-in, shear = 0 kips (load case 6), shear key 6 (midspan)
- Max shear = 26.7 kips, moment = 306 kip-in (load case 4), shear key 3 (edge)

<table>
<thead>
<tr>
<th>Case number</th>
<th>Maximum Moment (kip-in)</th>
<th>Location (nth shear key)</th>
<th>Maximum shear force (kip)</th>
<th>Location (nth shear key)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-508</td>
<td>6</td>
<td>23</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2393</td>
<td>6</td>
<td>24.3</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>-1340</td>
<td>6</td>
<td>20.3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>650</td>
<td>2</td>
<td>26.7</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>2700</td>
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<tr>
<td>6</td>
<td>-1471</td>
<td>6</td>
<td>23.5</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>-931</td>
<td>7</td>
<td>23</td>
<td>3</td>
</tr>
</tbody>
</table>

These three load cases will represent the behavior of shear keys at mid span and the edge of the bridge, thus providing an appropriate framework to explore the effects of connection modifications (geometry, material, transverse post-tensioning). Results from this analysis will be presented in the next Chapter.

3.5 Distribution Factor

In addition to identifying the maximum moment and shear, the distribution of loads among box beams are examined by calculating the distribution factor of each box beam at mid span. The distribution factor is defined as the moment or shear experienced by each box beam divided by the total moment or total shear experienced by all 12 box beams. It represents the percentage of the load that is distributed into each box beam. The distribution factor determined using grillage analysis is then compared with the ones determined using formulas from AASHTO Bridge Design Specifications. Only the three most critical cases are evaluated in the comparison. The calculation of AASHTO distribution factor is presented in Appendix C. Note that AASHTO only provides the formula for calculating the maximum distribution factor.
### Table 3.4 Distribution Factor for Load Case 4

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Moment D.F.</th>
<th>Moment D.F. (AASHTO)</th>
<th>Shear D.F.</th>
<th>Shear D.F. (AASHTO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.09</td>
<td>0.19</td>
<td>0.08</td>
<td>0.54</td>
</tr>
<tr>
<td>2</td>
<td>0.09</td>
<td>0.17</td>
<td>0.08</td>
<td>0.43</td>
</tr>
<tr>
<td>3</td>
<td>0.09</td>
<td>0.17</td>
<td>0.08</td>
<td>0.43</td>
</tr>
<tr>
<td>4</td>
<td>0.09</td>
<td>0.17</td>
<td>0.09</td>
<td>0.43</td>
</tr>
<tr>
<td>5</td>
<td>0.08</td>
<td>0.17</td>
<td>0.08</td>
<td>0.43</td>
</tr>
<tr>
<td>6</td>
<td>0.08</td>
<td>0.17</td>
<td>0.09</td>
<td>0.43</td>
</tr>
<tr>
<td>7</td>
<td>0.08</td>
<td>0.17</td>
<td>0.09</td>
<td>0.43</td>
</tr>
<tr>
<td>8</td>
<td>0.08</td>
<td>0.17</td>
<td>0.09</td>
<td>0.43</td>
</tr>
<tr>
<td>9</td>
<td>0.08</td>
<td>0.17</td>
<td>0.08</td>
<td>0.43</td>
</tr>
<tr>
<td>10</td>
<td>0.08</td>
<td>0.17</td>
<td>0.08</td>
<td>0.43</td>
</tr>
<tr>
<td>11</td>
<td>0.08</td>
<td>0.17</td>
<td>0.08</td>
<td>0.43</td>
</tr>
<tr>
<td>12</td>
<td>0.08</td>
<td>0.19</td>
<td>0.06</td>
<td>0.54</td>
</tr>
</tbody>
</table>

### Table 3.5 Distribution Factor for Load Case 5

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Moment D.F.</th>
<th>Moment D.F. (AASHTO)</th>
<th>Shear D.F.</th>
<th>Shear D.F. (AASHTO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08</td>
<td>0.27</td>
<td>0.08</td>
<td>0.55</td>
</tr>
<tr>
<td>2</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>3</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>4</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>5</td>
<td>0.08</td>
<td>0.26</td>
<td>0.08</td>
<td>0.45</td>
</tr>
<tr>
<td>6</td>
<td>0.08</td>
<td>0.26</td>
<td>0.07</td>
<td>0.45</td>
</tr>
<tr>
<td>7</td>
<td>0.08</td>
<td>0.26</td>
<td>0.07</td>
<td>0.45</td>
</tr>
<tr>
<td>8</td>
<td>0.08</td>
<td>0.26</td>
<td>0.08</td>
<td>0.45</td>
</tr>
<tr>
<td>9</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>10</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>11</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>12</td>
<td>0.08</td>
<td>0.27</td>
<td>0.08</td>
<td>0.55</td>
</tr>
</tbody>
</table>
As shown in Table 3.4, 3.5, and 3.6, the distribution factors for moment and shear calculated using grillage analysis are well below the AASHTO design value. Moreover, the distribution factors for all the box beams are nearly the same and constant regardless of load cases. Recall that the AASHTO distribution factor assumes partial depth shear key (hinge behavior). At the time AASHTO published the formulas for distribution factor, only cementitious grout were being used in the field. The results in Table 3.4, 3.5, and 3.6 reveal that the load transfer ability of adjacent box beam bridge using full depth epoxy grout is superior compared to the one using partial depth cementitious grout. See Appendix D for calculations of distribution factors.

### Table 3.6 Distribution Factor for Load Case 6

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Moment D.F.</th>
<th>Moment D.F. (AASHTO)</th>
<th>Shear D.F.</th>
<th>Shear D.F. (AASHTO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08</td>
<td>0.27</td>
<td>0.07</td>
<td>0.55</td>
</tr>
<tr>
<td>2</td>
<td>0.08</td>
<td>0.26</td>
<td>0.08</td>
<td>0.45</td>
</tr>
<tr>
<td>3</td>
<td>0.08</td>
<td>0.26</td>
<td>0.08</td>
<td>0.45</td>
</tr>
<tr>
<td>4</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>5</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>6</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>7</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>8</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>9</td>
<td>0.08</td>
<td>0.26</td>
<td>0.09</td>
<td>0.45</td>
</tr>
<tr>
<td>10</td>
<td>0.08</td>
<td>0.26</td>
<td>0.08</td>
<td>0.45</td>
</tr>
<tr>
<td>11</td>
<td>0.08</td>
<td>0.26</td>
<td>0.08</td>
<td>0.45</td>
</tr>
<tr>
<td>12</td>
<td>0.08</td>
<td>0.27</td>
<td>0.07</td>
<td>0.55</td>
</tr>
</tbody>
</table>
Chapter 4. Finite Element Analysis

4.1. Modeling of the shear key test

4.1.1. Background information: experimental tests of shear key and material characterization

A shear test was developed in the PennDOT project for the purpose of verifying the results from finite element analysis. The shear test is essentially made of a shear key specimen, which is fixed at one side and restrained from moving horizontally on the other side, as shown in Figure 4.1. In this section, the predicted patterns as well as the peak loads obtained through finite element analysis are compared with the experimentally obtained results. The results show that the finite element methods used in this study is valid for evaluating the performance of shear key modifications.

A few important features of the test setup are highlighted in Figure 4.1 below, as they will determine geometry and boundary conditions of the developed models. The detailed configuration of the shear test is described in Section 2.3.4 of Chapter 2 of PennDOT WO14 final report (2010). Figure 4.1a shows the dimensions of the shear key specimen and surrounding supports. The shear key specimen is placed into a steel socket (right side of the specimen) as shown in Figure 4.1b. The steel socket consists of two angle plates gripping onto the shear key flange on each side of the specimen and one top plate to prevent vertical movement of the specimen. Its main purpose is to serve as a fixed support to the shear key specimen. On the free end of the specimen (left side of the specimen), a steel beam is placed to restrain the specimen movement in the horizontal direction. The load is applied as a downward point load by the hydraulic actuator onto a 2 inches wide steel plate (Figure 4.1a).

![Figure 4.1 a) Front view and b) 3-dimensional rendering of the experimental shear test](image-url)
In addition, material characterization of the concrete and grouting materials was conducted in this project. The tests conducted are listed in Table 4.1.

Table 4.1 Tests used for material characterization

<table>
<thead>
<tr>
<th>Material Property Test</th>
<th>Grouting Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM C 579 Compressive Strength</td>
<td></td>
</tr>
<tr>
<td>ASTM C 580 Flexural Strength and Modulus of Elasticity</td>
<td></td>
</tr>
<tr>
<td>ASTM C1609 Flexural parameters (FRC only)</td>
<td></td>
</tr>
<tr>
<td>ASTM C531 Coefficient of Thermal Expansion</td>
<td></td>
</tr>
<tr>
<td>AASHTO TP 60 Coeff. Of thermal expansion for concrete</td>
<td></td>
</tr>
<tr>
<td>Concrete-Grouting Interface</td>
<td></td>
</tr>
<tr>
<td>ASTM C 496 Splitting Tensile Test</td>
<td></td>
</tr>
<tr>
<td>ASTM C 882 Bond to concrete</td>
<td></td>
</tr>
<tr>
<td>ACI 446 Fracture Toughness Test</td>
<td></td>
</tr>
</tbody>
</table>

(Note: the results for each specific material are presented in Table 4.2 and Table 4.7)

The thesis is mainly about the analysis conducted in the PennDOT project. The detailed information on the experimental tests and material characterization can be found in the PennDOT WO14 final report (2010).

4.1.2 Geometry and boundary conditions

Using this information, a 2-Dimensional FE model was created using ABAQUS. The geometry of the model follows the actual dimension of the specimen tested. Specimen thickness was defined as 4.75 inches (8 in were assumed for the concrete region inside the steel socket). Two boundary conditions were investigated to evaluate the support conditions in the experimental test setup. They are shown in Figure 4.2. Neither model represents the real boundary condition of the test setup as shown in Figure 4.1: 1) the concrete can still adjust itself in the steel socket after loading starts; 2) the contact between the concrete and the steel socket should not be modeled as boundary condition, instead surface to surface interaction should be assigned. However, the parameters required by the surface to surface interaction model is missing and, therefore, the above boundary conditions are assigned. Results show that the modeling approach shown in Figure 4.2 is sufficiently accurate in terms of crack pattern and peak failure load. The reaction provided by the steel beam on the left side of the specimen is modeled assuming a restraint of horizontal movement ($u_1 = 0$). The support on the right side of the specimen required further study. Due to the possibility of localized slip or deformations between the concrete and the steel socket, two conditions were evaluated: the first model, see Figure 4.2a, assumes that the steel socket provides full restraint of displacement in the
horizontal and vertical direction as well as in-plane rotation (therefore \( u_1 = 0 \), \( u_2 = 0 \), and \( u_3 = 0 \)); the second model, see Figure 4.2b, includes the possibility of deformation of the concrete, except in the contact regions with the steel plates. Results from these two models showed that that the behavior of the shear key specimen is not significantly influenced by these two types of boundary conditions. Therefore, the first boundary condition, shown in Figure 4.2a, will be used due to its simplicity for all models described in this section.

![Figure 4.2 Two FE models of the shear test (boundary conditions differ)](image)

4.1.3. Material models

A plastic-damage model (available in ABAQUS) is used to predict the constitutive behavior of the concrete and grouting materials, epoxy and cementitious, as well as their respective interfaces. This approach assumes that compressive crushing and tensile cracking are the main failure mechanisms of these materials. Both of these phenomena are the result of microcracking. Tensile cracking and compressive crushing are interpreted as a local damage effect controlled by a yield function, which defines their onset and evolution. Particular details of the mathematical implementation of these ideas to model quasi-brittle materials are given by Lubliner et al. (1989) and Lee and Fenves (1998).

Input parameters required for this type of plastic-damage model are: compressive stress-strain relationship, elastic modulus, tensile strength, dilation angle, and fracture energy. Details of this implementation are described next:
Compressive behavior: The stress-strain curve of the cementitious grout under uni-axial compression is assumed to follow a parabolic function, shown in Equation 4.1 below, as defined by Todeschini et al. (1964).

\[ f_c = \frac{2f'_c (\varepsilon / \varepsilon_0)}{1 + (\varepsilon / \varepsilon_0)^2} \]  

(Eq. 4.1)

\[ f'^*_c = 0.9f'_c, \quad \varepsilon_c = 1.71 \frac{f'_c}{E_c} \]

The compressive stress-strain relationship of the concrete and epoxy grout is calculated based on the formula developed by Thorenfeldt et al. (1987) shown below:

\[ \frac{f_c}{f'_c} = \frac{n (\varepsilon_c / \varepsilon_0)}{n-1 + (\varepsilon_c / \varepsilon_0)^n k} \]  

(Eq. 4.2)

\[ n = 0.8 + \left( \frac{f'_c}{2500} \right), \quad k = 1 \]

\[ \varepsilon_0 = \frac{f'_c}{E_c} \left( \frac{n}{n-1} \right) \]

These functions have been proven to be appropriate for the modeling of concrete structures. It is expected that they can also represent the behavior of the epoxy and cementitious grout in compression. Figure 4.3 shows the compression stress-strain curves for epoxy, concrete and cementitious grout. Table 4.1 shows the maximum compressive strength values for all the materials used in the numerical models.
Table 4.2 Material properties used in the numerical models

<table>
<thead>
<tr>
<th>Material</th>
<th>Concrete</th>
<th>Epoxy grout</th>
<th>Concrete-epoxy interface</th>
<th>Cementitious grout</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (lb•s²/in.)</td>
<td>0.00022487</td>
<td>0.00022487</td>
<td>0.00022487</td>
<td>0.00022487</td>
</tr>
<tr>
<td>Compressive strength (psi)</td>
<td>11329</td>
<td>10036</td>
<td>11329</td>
<td>4474</td>
</tr>
<tr>
<td>Splitting tensile strength (psi)</td>
<td>741</td>
<td>1325</td>
<td>730</td>
<td>100.3</td>
</tr>
<tr>
<td>Modulus of elasticity (psi)</td>
<td>5.26 x 10⁶</td>
<td>1.46 x 10⁶</td>
<td>5.26 x 10⁶</td>
<td>8.37 x 10⁵</td>
</tr>
<tr>
<td>Fracture energy (lb/in.)</td>
<td>0.9022</td>
<td>1.613</td>
<td>0.6652</td>
<td>0.05082</td>
</tr>
</tbody>
</table>

(Note: the bolded values were calculated using methods explained below, the rest of the values were obtained experimentally)

**Elastic modulus:** The elastic modulus of the grout (cementitious and epoxy) was determined experimentally. The values presented in Table 4.2 were used in the numerical models. For the concrete, epoxy grout, and concrete-grout interfaces, the elastic modulus was determined analytically using ACI Committee 363 proposed equation for high strength concrete:

\[ E_c = 40,000\sqrt{f'_c} + 1.0 \times 10^6 \]  

where \( f'_c \) is the uniaxial compressive strength in psi.

**Tensile strength:** Experimental results from the splitting tensile tests were used as tensile strength of the concrete, grouting materials and the concrete-grout interfaces. Table 1 shows the values used in the numerical models.

**Dilation angle:** Based on previous numerical work conducted by the PI’s research group (Coronado and Lopez 2010), the dilation angle was assumed to be 38 degrees for all materials.

**Fracture energy:** Table 4.1 shows the fracture energy values used in the numerical models. It should be pointed out that all values were obtained experimentally, except for the fracture energy of the epoxy grout which was calculated based on the relative strength of the epoxy vs. concrete. The concept of “damage band” has been used to model the behavior of the interface between concrete and epoxy in adhesively bonded fiber reinforced polymer (FRP) repairs (Coronado and Lopez, 2007, 2008, 2010). In the numerical model of the shear test, it will be used to characterize interface between concrete and the grouting material. Physically, a region where damage or cracking occurs between two dissimilar materials can be represented as a damage band. This “band” has similar fracture properties as plain concrete but its fracture energy and tensile strength could be different. Experimentally, it was found that the shear tests of epoxy grouted specimens failed in the concrete region prior to any failure in the grout. Therefore for the modeling of this type of failure, the damage band was placed on the concrete side in the vicinity of the epoxy shear key. Figure 4.4a) shows the dimensions of the damage...
band used in the numerical model of the shear test. When modeling the cementitious grout, the damage band was not assigned as a region in the model because the cementitious grout is weaker than plain concrete (its fracture energy is two orders of magnitude smaller than that of plain concrete) and therefore its properties will control failure. Test observations were consistent with this assumption: cracks occurred at the shear key interface (edge) between the two materials. Therefore, the shear key was modeled using the material properties of the cementitious grout and the fracture energy of the concrete-cement interface. The numerical model of the cement grouted shear test specimen is shown in Figure 4.5b). Another alternative could be adding a thin layer at the interface between cementitious grout and concrete. The fracture energy of the interface was used in the thin layer. And the fracture energy of pure cementitious grout was used for the shear key region. However, the fracture energy of pure cementitious grout was not available and the complicated shear key geometry could lead to potential problems with meshing with the addition of the thin layer. Moreover, the results from using interface fracture energy for the entire shear key region demonstrated good correlation in terms of crack patterns.

![Diagram of shear test specimen](image)

**Figure 4.4 Locations of material property used in the epoxy grout models**

### 4.1.4. Elements and mesh

The element type used for all materials of the shear test model is a 4-node plane strain (CPE4R) element, where each node consists of two DOF as shown in Figure 4.5. A plane strain element is selected because the strain in the out-of-plane direction is assumed to be uniform; the
thickness in the out of plane direction is significantly large (4.75 inch.) with respect to the dimension of the model in the in-plane directions.

![Node bilinear element with coordinates](image)

**Figure 4.5 Nodes bilinear element**

The R in the element type designation indicates reduced integration. Reduced integration uses a lower-order integration to form the element stiffness, thereby reducing running time. For example, CPE4 uses 4 integration points; whereas CPE4R uses 1 integration point. Consequently, element assembly is approximately 4 times more costly for CPE4 than for CPE4R. Element distortion control and hour glass control is enabled to help with convergence because 1 integration point tends to cause the element to distort in such a way that the strains calculated at the integration to be all zero, which, in turn, may lead to uncontrolled distortion of the mesh.

Meshing of the model is shown in Figure 4.6 Areas of interest in the model have a finer mesh. Region 1 may experience stress concentration due to the concentrated load, region 2 represents the damage band where the possibility of crack occurrence is high, and region 3 is the shear key. All three regions are given a fine mesh. Region 4 is an area of less interest; therefore the meshing of region 4 is coarser and tries to accommodate the meshing of other regions. The typical mesh size for fine mesh in the model is about 0.2 inches; and that of the coarse mesh is around 0.5 – 1 inches. In addition, the mesh in the model is arranged so that it is composed of rectangles for the most part or four nodes elements, as shown in region 1. The meshes in region 2 and 3 are non-rectangular, because of the irregular geometry of the shear key.
In the model of epoxy grout shear key, the bond between the epoxy grout and concrete is very strong. Therefore, the surfaces of epoxy grout and concrete are modeled using a TIE command, which means that all degrees of freedom at the interface are locked together. As proved by the experimental data, any crack that occurred in the epoxy grouted specimens went through the epoxy grout – concrete interface as if they have perfect bond (PennDOT WO14 Final Report 2010). Same constraints are used in the cement grout model. However, due to the large differences in orders of magnitude of the fracture energies of the concrete and the cementitious grout, crack is expected to occur within the shear key following its edge.

4.1.5 Results and discussion

Results from the numerical modeling of one epoxy and one cementitious (no fibers) grouted specimens are presented in this section. Only the first cementitious test will be used for comparison since results from the second test appear to be influenced by the surface preparation of the shear key.

The concrete damage plasticity model from Abaqus is used in all the models. Damage is represented by the magnitude of plastic strain. By plotting the plastic strain distribution of the model, the cracking pattern of the model can be displayed.

Epoxy grout specimen

The crack sequence on the FE model of the epoxy grout specimen is characterized by three stages, shown in Figure 4.7. In stage 1 the applied load increases from zero to 76 kips (in the numerical model). In this initial stage, plastic strain develops as the load increases. The applied load creates shear forces that produce a maximum principal tensile stress in a diagonal direction in the specimen. Just before the first major crack occurs, a sloped strip of plastic strain
concentration develops near the fixed supported edge in the model. Minor cracks forms in the vertical direction initiating from the bottom of the support. This type of crack was observed during the experimental tests and is captured by the numerical model, as shown in Figure 4.7a. The load at which this crack occurred experimentally is 10% smaller than the numerical one. The second stage of the crack sequence occurs at an applied load of 85.5 kips (in the numerical model). A major crack is initiated at the loading point and propagates diagonally toward the bottom support, as shown in Figure 4.7b). This main crack was also observed in the experimental test at a smaller load level, see Table 4.3. At this load level the crack width of the main crack is small and a few additional shear cracks develop at the top corner near the support. The last stage of the crack sequence (stage 3) is the opening of the main crack while another crack propagates from the loading point toward the bottom support on the top region of the shear key, see Figure 4.7c. This failure mechanism occurs at a load level of 115.7 kips. Note that the experimental test was stopped at 90 kips as the specimen underwent large deformation and approached the clearance at the bottom of test setup.

<table>
<thead>
<tr>
<th>Crack designation</th>
<th>Load level in FE model (kips)</th>
<th>Load level in the test (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>76</td>
<td>67.3</td>
</tr>
<tr>
<td>2</td>
<td>85.5</td>
<td>76.3</td>
</tr>
<tr>
<td>3</td>
<td>115.7</td>
<td>90.1</td>
</tr>
</tbody>
</table>
Cementitious grout specimen

The crack sequence on the FE model of cementitious grout model can be described in two stages. In the first one, cracks start to form at the interface between the concrete and the shear key at a very early loading level (8.6 kips). Experimentally, cracks at such early stage will be of the size of hairlines, making it difficult to detect with the eye. However the experimental response shows a change in stiffness around 5 kips of load. At the load level of 60.7 kips, the FE model predicts that the specimen fails by total debonding of the interface between the concrete and the shear key. At this second stage, shear cracks develop with increasing loads, as shown in Figure 4.8b. The FE results indicate that this type of failure is brittle in nature. Similarly, in the experimental tests, no major cracks were found by visual inspection up to a load level of 38.1 kips when a major crack initiated at the bottom of the shear key region and propagated in a sudden manner along the interface. The experimental test was stopped at this point so photographs could be taken of the cracks without further damaging the shear key region.
Table 4.4 Crack sequence of the cementitious grout model

<table>
<thead>
<tr>
<th>Crack designation</th>
<th>Load level in FE model (kips)</th>
<th>Load level in the first test (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.6</td>
<td>&lt;5</td>
</tr>
<tr>
<td>2</td>
<td>60.7</td>
<td>&gt;38.1</td>
</tr>
</tbody>
</table>

The cementitious grout model showed all major cracks occurring inside the shear key. The adjacent concrete does not fail. Moreover, the cracks in shear key region generally follow the interface (edge) of the shear key. Top and bottom cracks are joined diagonally through the shear key. This damage pattern is very similar to the one observed in the experimental test, as will described in more detail next.
4.1.6 Modeling remarks

Results from the numerical models developed in this section were compared with the experimental observations from the shear tests of concrete-grout specimens. The FE models were able to predict the differences in failure modes observed in epoxy and cementitious grout specimens as well as the experimentally observed crack sequence and patterns, and strength levels. Therefore, the modeling approach discussed in this section will be extended to model the shear keys of the bridge model developed in Chapter 3.

4.2. Parametric Study of Shear Key Modifications

4.2.1 Modeling approach

The modeling approach developed in Section 4.2 is used in this section to model the shear key region between adjacent precast box girders. Moments and shear forces obtained from the grillage analysis developed in chapter 3 is used as external loading for this region. The effects of changes in the shear key configuration, grouting material, post-tensioning, and bearing pad positioning are evaluated. The numerical model of an isolated shear key was created following the shear key geometry in accordance to PennDOT’s specifications (PennDOT BC775M). Details of the geometry, loading and material properties used are described next. Note that a partial
depth shear key with cementitious grout was used as the basic case for comparison with other models. Figure 4.10 shows this shear key under a combination of positive shear force and positive moment.

Figure 4.10 Numerical model on an isolated partial depth shear key with cementitious grout under a combination of shear and moment

**Geometry**

Since the objective of this model is to assess the effect of different shear key modifications on the shear key region between adjacent precast concrete box girders, the geometry of this model reflects the geometry of the prototype bridge discussed in Chapter 3. The same box beam (BII-48) used in the grillage analysis is modeled as shown in Figure 1 (right): two halves of the box beams are connected by a shear key in the middle. The concrete box girders are assumed solid. There are two reasons for this modeling approach: 1) the maximum moments and shear occurs at mid span (as described in Chapter 3) where there is a diaphragm, as shown in Figure 3.6; and 2) without modeling the steel reinforcement, the web of the box beam may undergo large deformations and most likely cracks under shear and flexural effects. It should be pointed out that the main objective of this analytical evaluation is to accurately model the shear key region, not the entire precast box girder.

**Boundary conditions and loading**

To recreate the state of stresses in the shear key of the real bridge, the moments and shear forces from the grillage analysis are applied to the geometry described above using a statically determinate system. The left edge of the model is assumed fixed (u1=0, u2=0, and u3=0). Loading is applied on the right edge of the model by a combination of axial stress distribution and vertical shear stress distribution that creates a resultant moment and shear at the shear key location that matches the combinations of moment and shear obtained from the grillage
analysis. Recall that in the grillage analysis, the maximum moment and shear forces are calculated at the shear key location as shown in Figure 4.11a.

**Element type and meshing**

As used in the shear test models described in section 4.2, a 4-node bilinear plane strain element (CPE4R) is used in this model. Each shear key model represents one isolated shear key with a thickness in the transverse direction of 48 inches, based on the grillage analysis developed in Chapter 3. The large majority of the elements are rectangular to increase the accuracy of the numerical model. The concrete region in the vicinity of the shear key (Region 1 in Figure 4.11a) as well as the shear key itself are considered areas of interest where cracks are likely to occur, therefore a fine mesh (0.2 inches) is assigned to these regions. Other regions are assigned a coarser mesh (0.5 inches for region 2).

![Mesh configuration of the shear key FE model](image1)

**Figure 4.11 a) Mesh configuration of the shear key FE model b) Material used in the shear key FE model**

**Material properties**

After verifying the validity of the material models used in the concrete-grout shear key specimens, the same material properties were used for the shear key model in the real bridge. The material properties are listed in Table 4.1. Figure 4.11b shows the assignment of the material properties. For models using epoxy grout, a damage band is used (1.5 in thick) in the region enclosed by the shear key contour and the dotted lines. Material properties for this damage band are listed in Table 4.1. For models using cementitious grout, no damage band is assigned as discussed in section 4.2. The shear key is assigned material properties of either epoxy or cementitious grout. These properties are listed in Table 4.1. Plain concrete properties are assigned to the remaining regions of the numerical model, also listed in Table 4.1.
4.2.2 Analysis of typical PennDOT shear keys

To better understand the effect of each possible shear key modification (shear key configuration, grouting material, post-tensioning, and bearing pad positioning) on the shear key behavior, each individual effect is evaluated independently. Then, one optimum combination of effects is presented for comparison with the current PennDOT shear key specifications. Thus, the first step of this analytical study is to examine the state of stresses (and possibility of cracking) of shear keys designed with current PennDOT design procedures.

Load cases

Based on the results from the grillage analysis, three load cases were chosen to represent the critical combinations of moment and shear force that act on the shear keys of a typical bridge (described in chapter 3). The three load cases create the maximum negative moment, the maximum positive moment, and the maximum shear on shear keys. The magnitudes of the load cases are presented in the following table. All three load effects happen at the mid span of the bridge. Their transverse locations are illustrated in Figure 4.13 shown below and listed in Table 4.4.

![Figure 4.12 Location of shear keys along the transverse direction of the bridge](image)

<table>
<thead>
<tr>
<th>Table 4.5 Load cases used in shear key FE model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
</tr>
<tr>
<td>Load case 5, Max M-</td>
</tr>
<tr>
<td>Load case 6, Max M+</td>
</tr>
<tr>
<td>Load case 4, Max V</td>
</tr>
</tbody>
</table>

An FE model of a partial depth shear key with cementitious grout is evaluated under these three load cases. Material properties used are listed in Table 4.2. Results indicate that the shear key cracks under all three load cases. The predicted crack pattern for all three load cases is shown in Figure 4.14. These results also show the locations of crack initiation on the shear key, which coincide with the regions of higher tensile stresses (before cracking). As shown in Figure 4.14, under maximum positive moment, cracking propagates along the middle of shear key from the bottom toward the deck of the bridge; whereas under negative moment cracking starts from top to bottom along the centerline of the shear key. In both load cases, the shear key is completely damaged and not able to retain any residual strength. Under a combination of
maximum shear and corresponding moment, cracking also propagates from the bottom, but the crack follows the geometry of the shear key, similar to a debonding failure. Based on the experimental tests of shear keys with cementitious grout, it can be expected that the geometry of the shear key could still provide a locking effect on the box beams, if they are restrained laterally.

Figure 4.13 Crack patterns of a partial depth shear key with cementitious grout under the three maximum load cases

To avoid converge problems in the numerical models, the applied stresses that create the resultant moment and shear are applied in 20 load increments. This is the default number for load increment in Abaqus. Under the three load cases described here, the numerical models indicated that cracking occurred at a very early stage. The shear key cracked at 10% of the maximum negative moment; cracking occurred at 5% of the maximum positive moment and at 40% of maximum shear force. These results confirm that the shear forces and moments generated by full live load on the prototype bridge analyzed in this study are very likely to produce cracking on the corresponding shear keys.
### Discussion

Results described in this section indicate that a partial depth shear key with cementitious grout is very likely to crack under the load cases developed in this study. These results, however, do not explain the reason why shear keys may fail before the bridge is subjected to full live load. They do indicate that live load does contribute to the cracking of these elements. Shear key modifications (configuration, grouting material, post-tensioning, and bearing pad positioning) are explored in the next sections to evaluate the likelihood of avoiding (or minimizing) cracking in the shear keys.

#### 4.2.3. Grouting material

**Epoxy grout**

First, shear key models using the combination of partial depth and epoxy grout were analyzed for the three loading cases described previously (Max M+, Max M-, Max V). Results show that the shear key cracks under maximum positive and negative moments. No cracking is observed under the maximum shear force. As expected, under the maximum negative moment, the shear key cracks from the top and propagates toward the bottom whereas under maximum positive moment, the cracks propagates from bottom toward the top, as shown in Figure 4.15. In both cases, shear keys are completely damaged and not able to retain any residual strength. It was observed that the epoxy grouted shear keys cracked in the damage band, as observed in the experimental tests and modeled in section 4.2.

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Percent of load at failure</th>
<th>Maximum principal stress at applied load (psi)</th>
<th>Top of shear key</th>
<th>Bottom of shear key</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum -M</td>
<td>10%</td>
<td>101 (tension)</td>
<td>260 (compression)</td>
<td></td>
</tr>
<tr>
<td>Maximum +M</td>
<td>5%</td>
<td>124 (compression)</td>
<td>103 (tension)</td>
<td></td>
</tr>
<tr>
<td>Maximum V</td>
<td>40%</td>
<td>201 (compression)</td>
<td>103 (tension)</td>
<td></td>
</tr>
</tbody>
</table>
Numerical results indicate that the partial depth shear key with epoxy grout cracks at a later stage, compared to the case with cementitious grout. The shear key cracks at 80% of the maximum negative moment and at 35% of the maximum positive moment. Therefore the use of epoxy grout can be successful at preventing cracking due to shear under full live load conditions. For bending, even though it cannot be shown to prevent cracking under full live load, the % of maximum bending moment that it can withstand before cracking is at least 7 times higher than for the cementitious grout. For the maximum shear force, maximum in plane principal stresses generated on the shear key are 793 psi in tension (at the bottom of the shear key) and 273 psi in compression (at the top of the shear key), which are 59.8% of the tensile strength of the epoxy grout and 2.7% of the compressive strength of the epoxy grout, respectively.

<table>
<thead>
<tr>
<th>Load case</th>
<th>Percent of load at failure</th>
<th>Maximum principal stress at applied load (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-M</td>
<td>&gt;80%</td>
<td>Top of shear key 760 (tension) 2230 (compression)</td>
</tr>
<tr>
<td>+M</td>
<td>35%</td>
<td>645 (compression) 889 (tension)</td>
</tr>
<tr>
<td>V</td>
<td>No crack</td>
<td>273 (compression) 793 (tension)</td>
</tr>
</tbody>
</table>

**Fiber reinforced cementitious grout**

The same models substituted with fiber reinforced cementitious grout are evaluated. The material properties of the fiber reinforced cementitious grout used in the model is listed in Table 4.7. All three load cases show cracking. However, they crack at a later stage compared with the cementitious grout. The fiber reinforced cementitious grout cracks at 20% of the maximum negative moment and at 10% of the maximum positive moment. At the maximum
shear, the model just starts to crack. The maximum tensile stresses developed in each load case prior to failure are around 368 psi which is close to the splitting tensile strength of the material. Overall, the performance of fiber reinforced cementitious grout is somewhat superior compared with cementitious grout.

Table 4.8 Material property of fiber reinforced cementitious grout used in the model

<table>
<thead>
<tr>
<th>Material</th>
<th>Fiber reinforced Cementitious grout</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (lb•s²/in.)</td>
<td>0.00022487</td>
</tr>
<tr>
<td>Compressive strength (psi)</td>
<td>4606</td>
</tr>
<tr>
<td>Splitting tensile strength (psi)</td>
<td>331</td>
</tr>
<tr>
<td>Modulus of elasticity (psi)</td>
<td>1.12 x 10⁶</td>
</tr>
<tr>
<td>Fracture energy (lb/in.)</td>
<td>0.2815</td>
</tr>
</tbody>
</table>

(Note: the values are obtained experimentally)

Table 4.9 Performance of partial depth shear key using fiber reinforced cementitious grout

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Percent of Load at failure</th>
<th>Maximum principal stress at applied load (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Top of shear key</td>
</tr>
<tr>
<td>Maximum -M</td>
<td>20%</td>
<td>351 (tension)</td>
</tr>
<tr>
<td>Maximum +M</td>
<td>10%</td>
<td>351 (compression)</td>
</tr>
<tr>
<td>Maximum V</td>
<td>100%</td>
<td>305 (compression)</td>
</tr>
</tbody>
</table>

Discussion

Based on the results obtained in this section, it is shown that the use of epoxy grout improves the performance of the shear key under full live load. It can be inferred that it probably also improves the performance of the shear key under other load effects (such as thermal). However, the higher stress concentrations found on the partial depth geometry prior to cracking indicate that changes in shear key configuration may also improve performance and therefore decrease the likelihood of cracking.

4.2.4 Shear key configuration

Full depth

The behavior of the FE models of full depth shear keys with cementitious and epoxy grouts were evaluated under the three loading cases. The results show that under the maximum positive moment (Max M+), the cementitious grout cracks whereas the epoxy grout does not. Figure 4.16 shows the crack pattern of the full depth shear key with cementitious grout. For the cementitious grout, using partial depth or full depth does not prevent cracking, under maximum positive moment, however the % of the applied moment at cracking increases from 5% to 40%. For the epoxy grout, using a full depth shear key avoids cracking. The shear key can
withstand the maximum positive moment without cracking. Maximum in plane principal stresses generated on the shear key were 302 psi tension (at the bottom of the shear key) and 298 psi compression (at the top of the shear key), which are 22.8% of the tensile strength of grout and 3.0% of compressive strength of grout.

Figure 4.15 Plastic strain (crack patterns) of cementitious grout full depth shear key

**Location of key way**

The location of the key way (top tier vs mid depth) and its effect on the behavior of shear key under full live load was explored. FE models of mid depth shear key with cementitious and epoxy grouts were analyzed and the results are summarized in Table 4.9. Both top tier and mid depth shear keys with cementitious grout crack at 40% of the maximum positive moment. Neither top tier nor mid depth shear keys with epoxy grout crack under the maximum positive moment. Stress distributions for these two epoxy cases was further evaluated. Maximum stresses were found to be about 302 psi at the bottom of the shear key regardless of shear key locations. These results suggest that when using a full depth grouted shear key, the location of key way does not affect its performance.

<table>
<thead>
<tr>
<th>Load case</th>
<th>Percent of load at failure</th>
<th>Maximum principal stress at applied load (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Top of shear key</td>
</tr>
<tr>
<td>Full depth epoxy grout</td>
<td>No crack</td>
<td>298 (compression)</td>
</tr>
<tr>
<td>Mid depth epoxy grout</td>
<td>No crack</td>
<td>300 (compression)</td>
</tr>
<tr>
<td>Full depth cementitious grout</td>
<td>40%</td>
<td>127 (compression)</td>
</tr>
<tr>
<td>Mid depth cementitious grout</td>
<td>40%</td>
<td>139 (compression)</td>
</tr>
</tbody>
</table>
**Shear key width**

FE models of two full depth epoxy shear keys with two different key widths (¼” and 1”) were evaluated. The results indicate that neither of the shear keys cracks under the maximum positive moment. Moreover, the maximum principal stresses, which occur at the bottom of both keys, are very close in magnitude (see Table 4.10), indicating that a change in the width of a full depth shear key does not affect its performance. It is assumed that the epoxy grout can flow in such a narrow key way and form good bond with the concrete surface. Experimental testing is needed to confirm the validity of this idea.

<table>
<thead>
<tr>
<th></th>
<th>Percent of load at failure</th>
<th>Max principal Stress at the maximum load (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Top of shear key</td>
</tr>
<tr>
<td>¼” full depth epoxy grout</td>
<td>No crack</td>
<td>294 (compression)</td>
</tr>
<tr>
<td>1” full depth epoxy grout</td>
<td>No crack</td>
<td>298 (compression)</td>
</tr>
</tbody>
</table>

**4.2.5 Post-tensioning effect**

**Post-tensioning**

Based on current PennDOT specifications (PennDOT BD651M), the transverse post-tensioning layout of the prototype bridge was defined as shown in Figure 4.16a. The prototype bridge has a span length of 80 feet and no skew (90 degree skew angle), therefore its transverse post-tensioning details follows the plan B layout from PennDOT BD651M with an end void zone of 16 feet. According to PennDOT BC775M, each post-tensioning force represents a ½” 270ksi poly-strand tendon, post-tensioned up to at least 30 kips and placed at the centroid of the shear key (Figure 4.16b). It is noted that the end diaphragm is not specified to have a transverse post-tensioning tendon.
The transverse post-tensioning tendons are expected to provide a confinement effect to the box beams and shear keys. In particular, properly designed transverse post-tensioning could counter-balance part of the tensile stresses developed in the shear key at early construction stages (such as temperature and shrinkage) as well as during service conditions under full live load. Therefore, in order to use this information in the FE shear key model described previously, the stress distribution developed in the shear key due to this transverse post-tensioning was determined.

Because the post-tensioning force is not applied at the centroid of the box girder, the post-tensioning stresses vary linearly through the shear key cross section. Two FE models were generated to evaluate the post-tensioning effect: The first 2-D model looks at the distribution of the compressive stresses over the entire bridge. This model is used to evaluate the areas of confinement due to the post-tensioning forces. For an isolated shear key, such as the one used in this study, the variation of the stresses due to the eccentricity of the post-tensioning force and geometry of the shear key are further explored with a second FE model.

**FE model of two adjacent box girders subjected to a post-tensioning force**

Because current PennDOT shear keys specifications indicate partial depth and a post-tensioning tendon located at the centroid of shear key, the variations of stresses along the shear key height needed to be evaluated. Figure 4.20 shows the FE model of two adjacent box beams connected with a shear key. The concrete box beams are assumed to be fully solid to represent behavior on a diaphragm. A post-tensioning force is applied as point load at the centerline of the shear key. The same modeling details from previous FE analysis are incorporated in this model.
The results, shown in Figure 4.17, indicate that compressive stresses fan out from the loading point toward the shear key. However, only the top portion of the box beam experiences the confining effects of the post-tensioning. The highest compressive stress in the shear key region was found to be 44.3 psi. The magnitude of this compressive stress is smaller than the tensile stress created by the vehicle loading (live load). Moreover, the top portion of the shear key exhibits a negligible amount of compression. Therefore, it can be concluded that the current PennDOT transverse post-tensioning details provide limited confinement of the shear key region.

![Figure 4.17 Maximum negative principal stress of a partial depth cementitious grout shear key model under a single post-tensioning force](image)

When a full depth shear key with epoxy grout is used, the distribution of the maximum principal stresses is similar than the one observed for partial depth shear key, as shown in Figure 4.18. The magnitude of stresses does change: compressive stresses developed in the shear key at the top (7.6 psi) and tensile stresses are developed at the bottom (1.25 psi). Under positive moment, the tensile stress induced by the post-tensioning tendons may help the initiation of cracks. Moreover, the compressive stress developed at the top is insignificant compared with the stresses developed by vehicle loadings. This finding suggests that a higher post-tensioning force should be applied at the centroid of the entire cross section to effectively confine the shear key region.

![Figure 4.18 Maximum negative principal stress of a full depth epoxy grout shear key model under one post-tensioning force](image)
As shown by the two models, the full depth feature does not alter the stress distribution induced by the post-tensioning. In other terms, the one tendon post-tensioning is creating compressive stress throughout the entire partial depth shear key. Using a full depth shear does not get benefit from post-tensioning. However, if it is desired to increase the post-tensioning force and to have the post-tensioning tendon at the centroid of the box beam, a full depth shear key should be implemented for reasons shown in the following section.

**Effect of three post-tensioning tendons in a full depth epoxy grout shear key**

In order to create a uniform compressive stress across the entire box beam section, three post-tensioning tendons are provided as shown in Figure 4.19. Each post-tensioning tendon is jacked up to 30kips. The results show that the entire out of plane cross-section of the shear key is undergoing the same compressive stress (10.9 psi.)

![Figure 4.19 Maximum negative principal stress of a full depth epoxy grout shear key model under three post-tensioning force](image)

The magnitude of post-tensioning should be calculated based on the stress level developed by vehicle load. If the stress exceeds the strength of the grouting material, post-tensioning should be calculated to balance the tensile stress so that crack does not occur. Based on the magnitude of the post-tensioning force calculated, the spacing and layers of the post-tensioning tendons can then be determined. It is recommended to longitudinally spread the transverse post-tensioning tendons uniformly. Current PennDOT bridge design standards specify no tendons at the end span of the bridge for span length larger than 55 feet. Crack patterns provided by PennDOT and other studies (Miller et. al 1999) show cracks initiating at the end of span.

**FE model of bridge superstructure with post-tensioning forces**

The 2D FE model of the bridge superstructure, including the post-tensioning forces defined previously is shown in Figure 4.20. Each arrow represents a force of 30 kips. Plane stress
elements (CPS4) are used since the thickness of the model is small compared to in-plane
dimensions of the model. It is assumed that the entire model is made of plain concrete.
Sections that are light gray represent the void in the box beam or the flanges (thickness = 11
inches) whereas the darker gray sections represent solid section in the box beam, due to
intermediate and end diaphragms (thickness = 33 inches). The concrete deck is not included in
the model because the transverse post-tensioning is put in place before the placement of deck.
Post-tensioning forces are applied at the centerlines of the intermediate diaphragms. This FE
model is artificially pinned at one point ($u_1=0$ and $u_2=0$) to avoid rigid body deformations. A mesh
size of 5 inches is used for the entire model.

Figure 4.20 2-D FE model of bridge superstructure with transverse post-tensioning

Figure 4.21 shows the distribution of normal stress in the y direction ($s_{22}$) across the
superstructure of the bridge. The compression stress ranges from 0 to 163.5 psi (stress
concentration at the loading point). The stress distribution of the FE model shows that post-
tensioning forces produce confinement only in the region near the loading points. The area of
influence can be approximated on average to two adjacent box girders in the transverse (y)
direction and a maximum width of 4 feet. This area experiences a range of compression stress
between 15 psi and 45 psi. In regions further away from the loading points, the compressive
stress distribution becomes more uniform. Shear keys in this region were found to be subjected
to 3.5 psi of compressive stress on average. In particular, for the two full depth shear keys
evaluated using the load cases from the grillage analysis (shear keys 3 and 6, see location in
Figure 4.12); the average compressive stresses were 4.2 psi and 3.7 psi respectively. It can be concluded that the post-tensioning layout used in the prototype bridge, which follows current PennDOT specifications, has a limited confinement effect on the overall bridge. Only shear keys in the immediate proximity to the loading points may benefit from this compressive force.

One finding from this analysis is that the end spans of the bridge do not undergo compressive stress, as shown in Figure 4.21 (top). This phenomenon raises concerns because in the crack history diagram provided by PennDOT and the ones from other study, cracks were observed to initiate from the end span of the bridge. The transverse post-tensioning is not providing the
necessary confinement to the box beams. Therefore, other models are run with one more post-tensioning going through the end spans, see Figure 4.21 (bottom).

As shown by the analysis, the current post-tensioning is inadequate for confinement purpose. A method for determining the amount of post-tensioning required needs to be established. It is expected that the spacing of tendons should be inversely proportional to the stress developed and post-tensioning force is directly proportional to the stress developed. To verify, a parametric study of different transverse post-tensioning configurations was conducted. The two variables defining a transverse post-tensioning layout are the spacing of the tendons and post-tensioning force. The post-tensioning force can be expressed indirectly by the number of tendons at each post-tensioning location. The parametric study was conducted using the already established finite element model. The variation of spacing is achieved by adding more loading points along the edge of the bridge superstructure as shown in Figure 4.20. The number of tendons is reflected in terms of the post-tensioning forces applied at the edge in Figure 4.20. After running the analysis, the results are presented in Table 4.11 and 4.12.

Table 4.12 Average stress developed in shear key versus number of intermediate diaphragms and width of 48 feet

<table>
<thead>
<tr>
<th>Number of intermediate diaphragms</th>
<th>Average compressive stresses developed in shear key (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.70</td>
</tr>
<tr>
<td>3</td>
<td>3.53</td>
</tr>
<tr>
<td>7</td>
<td>7.13</td>
</tr>
</tbody>
</table>

Table 4.13 Average stress developed in shear key versus post-tensioning force with 3 intermediate diaphragms and width of 48 feet

<table>
<thead>
<tr>
<th>Post-tensioning force for each diaphragm (k/f)</th>
<th>Average compressive stresses developed in shear key (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3.53</td>
</tr>
<tr>
<td>18</td>
<td>10.64</td>
</tr>
</tbody>
</table>

Table 4.14 Average stress developed in shear key versus spacing of the post-tensioning with 3 intermediate diaphragms and 6 k/f post-tensioning

<table>
<thead>
<tr>
<th>Bridge Width (ft)</th>
<th>Average compressive stresses developed in shear key (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>3.53</td>
</tr>
<tr>
<td>24</td>
<td>3.60</td>
</tr>
</tbody>
</table>

Just as expected, reducing the spacing of tendons results in increasing stress and increasing the number of tendons results in increasing stress. Increasing post-tensioning force at each
diaphragm will increase the compressive stress proportionally. And Table 4.13 shows that the increasing of bridge width does not result in increasing compressive stress. Unfortunately, the finite element analysis conducted here does not cover enough range of bridge width. The following relationship should be valid,

\[ \sigma_{avg,sk} = \frac{P(n+1)}{L*D} \]  

(Eq. 3.1)

where \( P \) is the total prestressing force in each diaphragm after all losses, \( n \) is the number of intermediate diaphragms, \( L \) is the span length, and \( D \) is the depth of the box beam. The assumptions are full depth shear key with end span post-tensioning.

This relationship can be used as a basis for calculating how much prestressing is required to prevent shear key crack. Usually the number of intermediate diaphragm is known. Based on the span length and beam depth, the amount of post-tensioning can be calculated according to the grouting materials used and the load analysis. Also the assumption is that the diaphragms are the main mechanism for transferring load across the bridge superstructure. For bridge of 80 feet with one post-tensioning at quarter points and BII-48 box beam, using Equation 3.1, the average compressive stress in the shear key is 4.7 psi, which is close to the result obtained from the finite element analysis (4.2 and 3.7 psi). In order to reduce the maximum tensile stresses in the shear key below the tensile strength of the cementitious grout, approximately 50 times the amount of post-tensioning used by PennDOT is required.

In design, the corresponding strength reduction factor should be applied to the effect of the post-tensioning tendons. In this thesis, these factors will not be considered. Alternatively, equation developed by Hanna et. al (2007) can also be used to calculate the required prestressing.

4.2.6. Bearing pad

Model details

To analyze the effect of bearing pad on shear key, the following finite element is developed. Same model details of box beams and shear key from the shear test model are incorporated in this model. Full depth epoxy grout is used for the model. The new component in this model is the bearing pad. The bearing pad is modeled using simple spring elements which are able to mimic the shear stiffness and compressive stiffness of the bearing. The shear modulus of the bearing pad is assumed to be 135 psi, which is selected from the recommended range (80 psi – 189 psi) from PennDOT DM4. Based on the selected shear modulus of the bearing pad, the shear stiffness and effective compressive modulus of the bearing pad are calculated (AASHTO 14.6.3.1-2 and C14.6.3.1-2). The shear stiffness of the bearing pad is determined to be 13.7 kip per inch whereas the modulus of elasticity of the bearing pad is 12.98 ksi.
The two box beams modeled are located on the edge. One side of the box beam is full fixed. One wheel load of 25 kips with a width of 12 inches is applied as uniformly distributed load at two locations one at a time. One location is at the centerline of exterior box beam, and the other is the shear key. Two bearing pad alternatives for adjacent box beam bridge are evaluated with the FE model, see Figure 4.23. The width of bearing pad in both alternatives is 24 inches, the thickness is 1 inch. The contact between the box beam and the bearing pad is modeled as tie connection. In this model, the material properties of epoxy grout is used in the shear key region.

The graphic results of the distribution of maximum principal stress are plotted in Figure 4.23. The maximum principal stress in each case can be found in Table 4.15. Shear key in case 2 develops the highest tensile stress as expected because, with the absence of support below the shear key, the shear key is able to deflect more under the same level of loading. Depending on the level of the truck load, the stress can exceed the tensile strength of the grouting material and leads to cracking. This qualitative study of the effect of the bearing pad configuration suggests putting bearing pad below shear key can greatly reduce the stress developed.

Based on the request from the PennDOT technical advisor, William Koller, a model incorporating the current PennDOT district 1 standard bearing pad details was evaluated. According to the standard, two 10” wide bearing pads are placed under each box beam with an edge clearance of 5”, see Figure 24a. The result shows that the maximum tensile stress develops near the bottom of the shear key. The magnitude of the maximum tensile stress is 132 psi, which is an improvement compared to the bearing pad alternative 1 shown in Figure 4.23a. However, bearing pad alternative 2, in which the bearing pads are placed right under shear key, is still superior compared to the current PennDOT practice in terms of stress level at the same loading conditions.
Figure 4.23 Maximum principal stress distributions of the PennDOT bearing pad practice and four other cases

Table 4.15 Maximum principal stress in the shear key developed in 4 cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Maximum principal stress in the shear key (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>15.1</td>
</tr>
<tr>
<td>Case 2</td>
<td>341</td>
</tr>
<tr>
<td>Case 3</td>
<td>16.4</td>
</tr>
<tr>
<td>Case 4</td>
<td>60.5</td>
</tr>
<tr>
<td>Current PennDOT practice</td>
<td>132</td>
</tr>
</tbody>
</table>
### 4.2.7. Table summary

Table 4.16 Summary of modifications based on the findings from FE modeling

<table>
<thead>
<tr>
<th>Modification</th>
<th>Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Load cases</strong></td>
<td>- Maximum positive moment is the most critical load case. Crack propagates from bottom of shear key toward top.</td>
</tr>
<tr>
<td><strong>Material</strong></td>
<td></td>
</tr>
<tr>
<td>Cementitious grout</td>
<td>- Cementitious grout cracks in all cases.</td>
</tr>
<tr>
<td></td>
<td>- It has weak bond strength, crack tends to develop along the interface between cementitious grout and concrete.</td>
</tr>
<tr>
<td>Epoxy grout</td>
<td>- Epoxy grout does not crack except for the partial depth shear key case.</td>
</tr>
<tr>
<td></td>
<td>- It has very strong bond strength and fracture properties. Crack is likely to occur in the concrete when failure happens.</td>
</tr>
<tr>
<td>Fiber reinforced cementitious grout</td>
<td>- It has better bond strength compared with cementitious grout.</td>
</tr>
<tr>
<td></td>
<td>- It still cracks in most cases except when full depth shear key is used.</td>
</tr>
<tr>
<td><strong>Configuration</strong></td>
<td></td>
</tr>
<tr>
<td>Grouting depth</td>
<td>- Full depth shear key relieves the stress developed in shear key and therefore is more effective in preventing crack than partial depth shear key.</td>
</tr>
<tr>
<td>Location</td>
<td>- Location of shear key does not affect the performance of the shear key significantly</td>
</tr>
<tr>
<td>Width</td>
<td>- For epoxy grout, the width of shear key is not affecting the performance of the shear key as long as the grout can flow through key way and form good bond with concrete.</td>
</tr>
<tr>
<td>Post-tensioning</td>
<td>- The current PennDOT specified post-tensioning is insufficient in resisting cracks.</td>
</tr>
<tr>
<td></td>
<td>- Higher post-tensioning force is recommended to help with the cracking problem.</td>
</tr>
<tr>
<td></td>
<td>- It is also recommended to put post-tensioning through the mid height of the box beam to create uniform compressive stress.</td>
</tr>
<tr>
<td>Bearing pad</td>
<td>- Putting bearing pad right under the weakest point, namely the shear key, can reduce the stress developed in the shear key under the same loading.</td>
</tr>
</tbody>
</table>
Chapter 5. Conclusions and Recommendations

5.1 Conclusions

The objective of the thesis was to evaluate the potential of each shear key modification in reducing the tensile stress level developed by vehicle loadings. The objective was met by conducting the two levels of analysis, a grillage analysis and a finite element analysis.

The major findings are summarized in Table 4.1. Compared to the current PennDOT practice, full depth shear key should be able to reduce the tensile stresses developed in the shear key. Mid tier shear key does not improve over the full depth top tier shear key. When using epoxy grout, the width of the shear key is insignificant as long as the epoxy grout can flow into the shear key during construction. Epoxy grout does increase the ability of the shear key in carrying stress by approximately 1000%. Epoxy grout tends to develop good bond between adjacent concrete box beams. Good bond allows a uniform transfer of stresses. On the contrary, cementitious grout tends to crack in most configurations and under all load cases.

Post-tensioning develops compressive stresses that could help reducing the tensile stress in the shear key when the bridge is carrying vehicle loads. Bearing pad placed directly under the shear key developed the least tensile stress in the shear key compared to the other alternatives.

The effect of the post-tensioning can be calculated based on the simple expression developed in the thesis. Based on this expression and vehicle loading, the required post-tensioning force can be determined.

5.2 Recommendations for future research

A grillage analysis with partial depth shear key should be analyzed and compared with the full depth shear key grillage model. The results may suggest which alternative is better for practice.

In the finite element analysis of the shear test, the pure fracture energy of the grouting material should be measured and used for more accurate correlation between analysis and experimental results.

In the parametric study of the post-tensioning effect, wider range of span lengths and bridge width should be studied to give a complete picture of the post-tensioning configuration on stress distribution.
Appendix A – Calculation of Grillage Analysis Section Properties

**Interior Longitudinal Beam**

Transformation coefficient: \( n = \frac{\sqrt{4000}}{10000} = 0.6325 \)

Axial area: \( A = A_{box\ beam} + A_{pavement} = 752.5 + 5.5 \times 48 \times 0.6325 = 919.5 \text{ in}^2 \)

Neutral axis: \( \bar{y} = \frac{A_{box\ beam} \times \bar{y}_{box\ beam} + A_{pavement} \times \bar{y}_{pavement}}{A} = \frac{752.5 \times 16.33 + 167 \times (33 + 5.5/2)}{919.5} = 19.86 \text{ in} \)

Flexural stiffness: \( I = I_{box\ beam} + I_{pavement} = 110,499 + 752.5 \times (19.86 - 16.33)^2 + 0.6325 \times \frac{5.5^3 \times 48}{12} + 167 \times (33 + \frac{5.5}{2} - 19.86)^2 = 162,463 \text{ in}^4 \)

Torsional stiffness: \( J = J_{open} + J_{closed} = \left[ \frac{48 \times 5.5^3}{3} + \frac{48 \times 5.5^3}{3} \right] + \left[ \frac{48 \times (33 - 5.5)^2 \times 2}{\frac{5.5}{2} \times \frac{5.5}{2} \times \frac{5.5}{2}} \right] = 5,324 + 199,650 = 204,974 \text{ in}^4 \)

Shear area: \( a_s = 5 \times 33 \times 2 = 330 \text{ in}^2 \)

**Exterior Longitudinal Beam**
Axial area: \( A = A + A_{\text{barrier}} = 919.5 + 21 \times 42 \times 0.6325 = 1,477 \text{ in}^2 \)

Assuming the barrier does not significantly change the flexural stiffness of the exterior box beam,

Neutral axis: \( \bar{y} = 19.86 \text{ in} \)

Flexural stiffness: \( I = 162,463 \text{ in}^4 \)

Torsional stiffness: \( J = 204,974 \text{ in}^4 \)

Shear area: \( a_s = 330 + 21 \times 42 \times 0.6325 = 878 \text{ in}^2 \)

\textit{Transverse beam – flange section}

Axial area: \( A = 48 \times 11 + 48 \times 5.5 \times 0.6325 = 695 \text{ in}^2 \)

Neutral axis: \( \bar{y} = \frac{48 \times 5.5 \times 0.6325 \times (33 + 5.5/2)}{695} + 48 \times 5.5 \times (33 - 5.5/2) + 48 \times 5.5 \times 5.5/2 = 21.12 \text{ in} \)

Flexural stiffness: \( I = \frac{48 \times 5.5^3}{12} \times 2.6325 + 0.6325 \times \left[ 48 \times 5.5 \times \left( 33 + \frac{5.5}{2} - 21.12 \right)^2 \right] + 48 \times 5.5 \times \left( 33 - \frac{5.5}{2} - 21.12 \right)^2 + 48 \times 5.5 \times \left( 21.12 - \frac{5.5}{2} \right)^2 = 1,752 + 35,740 + 22,006 + 89,089 = 148,587 \text{ in}^4 \)

Torsional stiffness: \( J = 204,974 \text{ in}^4 \)

Shear area: \( a_s = \frac{E}{G} \cdot \frac{d_w}{lh(d_t^3 + d_b^3)} \cdot l = 2 \times (1 + 0.15) \times \frac{5^3 \times (11^3 + 5.5^3)}{48 \times (33 - 5.5/2) \times (11^3 + 5.5^3) + 48^2 \times 5^3} = 8.4 \text{ in}^2 \)
Transverse beam – web section

Axial area: \( A = 48 \times 33 + 48 \times 5.5 \times 0.6325 = 1751 \text{ in}^2 \)

Neutral axis: \( \bar{y} = \frac{48 \times 33 \times 16.5 + 48 \times 5.5 \times 0.6325 \times (33 + 5.5/2)}{1751} = 18.33 \text{ in} \)

Flexural stiffness: \( I = \frac{48 \times 33^3}{12} + \frac{48 \times 5.5^3}{12} \times 0.6325 + 48 \times 5.5 \times \left( 18.33 - \frac{33}{2} \right)^2 + 48 \times 5.5 \times \left( 33 + \frac{5.5}{2} - 18.33 \right)^2 = 144,169 + 85,417 = 229,586 \text{ in}^4 \)

Torsional stiffness: \( J = 48 \times 33^3 \times \left[ \frac{1}{3} - 0.21 \times \frac{33}{48} \times \left( 1 - \frac{33}{12 \times 48} \right) \right] = 330,585 \text{ in}^4 \)

Shear area: \( a_s = 33 \times 48 = 1584 \text{ in}^2 \)

Transverse beam – shear key

Axial area: \( A = 1536 \text{ in}^2 \)

Flexural stiffness: \( I = \frac{48 \times 33^3}{12} = 131,072 \text{ in}^4 \)

Torsional stiffness: \( J = 307,712 \text{ in}^4 \)
Shear area: $a_s = 1584 \text{ in}^2$

*Transverse beam – intermediate diaphragm*

Axial area: $A = 695 + 12 \times 22 = 959 \text{ in}^2$

Flexural stiffness: $l \approx 148,587 + \frac{12 \times 22^3}{12} = 159,235 \text{ in}^4$

Torsional stiffness: $J \approx 204,974 \text{ in}^4$

Shear area: $a_s = 33 \times 12 = 396 \text{ in}^2$

*Transverse beam – end span diaphragm*

Axial area: $A = 695 + 33 \times 22 = 1,421 \text{ in}^2$

Flexural stiffness: $l \approx 148,587 + \frac{33 \times 22^3}{12} = 177,869 \text{ in}^4$

Torsional stiffness: $J \approx 204,974 \text{ in}^4$

Shear area: $a_s = 33 \times 33 = 1,089 \text{ in}^2$
Appendix B – Grillage Analysis Results

In this appendix, the bending moment and shear plots at the mid span of the bridge from the grillage analysis for all 7 load cases are presented. The location number on the x-axis of the plots represents the node number along the transverse direction at the mid span of the bridge. Note: Nodes 5-8, 17-20, 29-32, 41-44, 53-56, 65-68, 77-80, 89-92, 101-104, 113-116, and 125-128 correspond to the nodes for shear key 1 to 11.
i) Case 1

Bending moment (kip-in)

Shear (kips)

location along transverse direction
ii) Case 2

![Graph showing bending moment and shear distribution along the transverse direction for HS25 Truck.]
iii) Case 3

Bending Moment (kip-in)

Shear (kips)

location along transverse direction
iv) Case 4

Lane Load
HS25 Truck

Bending Moment (kip-in)

location along transverse direction

Shear (kips)

location along transverse direction
v) Case 5

Lane Load

H525 Truck

<table>
<thead>
<tr>
<th>Location along transverse direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 20 40 60 80 100 120 140</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bending Moment (kip·in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000 2500 2000 1500 1000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shear (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 30 15 0 -15 -30 -45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>location along transverse direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 20 40 60 80 100 120 140</td>
</tr>
</tbody>
</table>
vi) Case 6

Location along transverse direction

Bending Moment (kip-in)

Shear (kips)
vii) Case 7

Overload Truck

location along transverse direction

Bending Moment (kip-in)

Shear (kips)

location along transverse direction
Appendix C – Calculations of Distribution Factor

1. Moment distribution factor for interior beam (AASHTO 2008 Table 4.6.2.2.2b-1)
   a) When one design lane is loaded,
   
   \[ g = k \left( \frac{b}{33.3L} \right)^{0.5} \left( \frac{I}{J} \right)^{0.25} \]

   \[ k = 2.5(N_b)^{-0.2} \geq 1.5 \]

   Beam width, \( 35 \leq b \leq 60 \)
   Span length, \( 20 \leq L \leq 120 \)
   Number of beams, \( 5 \leq N_b \leq 20 \)

   \[ \frac{I}{J} = 0.5, N_b = 12, L = 80, b = 48 \]

   \[ k = 1.52 \rightarrow g = 0.17 \]

   b) When two or more design lanes are loaded,
   
   \[ g = k \left( \frac{b}{305} \right)^{0.6} \left( \frac{b}{12.0L} \right)^{0.2} \left( \frac{I}{J} \right)^{0.06} \]

   \[ k = 1.52 \rightarrow g = 0.26 \]

2. Moment distribution factor for exterior beam (AASHTO 2008 Table 4.6.2.2.2d-1)
   a) When one design lane is loaded,
   
   \[ g = e \cdot g_{\text{interior}} \]

   \[ e = 1.125 + \frac{d_e}{30} \geq 1.0 \]

   Horizontal distance from the centerline of the exterior web of exterior beam at the deck level to the interior edge of curb or traffic barrier (ft.), \( d_e = 0 \text{ ft} \)

   \[ e = 1.125 \rightarrow g = 0.19 \]
b) When two or more design lanes are loaded,

\[ g = e \cdot g_{interior} \]

\[ e = 1.04 + \frac{d_e}{25} \geq 1.0 \]

\[ e = 1.04 \rightarrow g = 0.27 \]

3. Shear distribution factor for interior beam (AASHTO 2008 Table 4.6.2.2.3a-1)
   
   a) When one design lane is loaded,

\[ g = \left( \frac{b}{130L} \right)^{0.15} \left( \frac{l}{f} \right)^{0.05} \]

Beam width, \(35 \leq b \leq 60\)

Span length, \(20 \leq L \leq 120\)

Number of beams, \(5 \leq N_b \leq 20\)

Torsional stiffness, \(25,000 \leq I \leq 610,000\)

Flexural stiffness, \(40,000 \leq l \leq 610,000\)

\[ \frac{l}{f} = 0.5, N_b = 12, L = 80, b = 48 \]

\[ g = 0.43 \]

b) When two or more design lanes are loaded,

\[ g = \left( \frac{b}{156} \right)^{0.4} \left( \frac{b}{12.0L} \right)^{0.1} \left( \frac{l}{f} \right)^{0.05} \left( \frac{b}{48} \right) \]

\[ \frac{b}{48} \geq 1.0 \]

\[ g = 0.45 \]

4. Shear distribution factor for exterior beam (AASHTO 2008 Table 4.6.2.2.3b-1)
a) When one design lane is loaded,

\[ g = e \cdot g_{\text{interior}} \]

\[ e = 1.25 + \frac{d_e}{20} \geq 1.0 \]

Horizontal distance from the centerline of the exterior web of exterior beam at the deck level to the interior edge of curb or traffic barrier (ft.), \( d_e \leq 2.0 \)

Beam width, \( 35 \leq b \leq 60 \)

\[ e = 1.25 \rightarrow g = 0.55 \]

b) When two or more design lanes are loaded,

\[ g = e \cdot g_{\text{interior}} \cdot \left(\frac{b}{48}\right) \]

\[ \left(\frac{b}{48}\right) \leq 1.0 \]

\[ e = 1 + \left(\frac{d_e + \frac{b}{12} - 2.0}{40}\right)^{0.5} \geq 1.0 \]

\[ e = 1.22 \rightarrow g = 0.55 \]
## Appendix D – Finite Element Analysis Summary

<table>
<thead>
<tr>
<th>Grouting depth</th>
<th>Location</th>
<th>Width</th>
<th>Material</th>
<th>Load Case</th>
<th>% of loading when crack occurs</th>
<th>Highest Tensile Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>full depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>Cementitious grout</td>
<td>Max -M</td>
<td>75%</td>
<td>102(top)</td>
</tr>
<tr>
<td>full depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>cementitious grout</td>
<td>Max +M</td>
<td>40%</td>
<td>102(bottom)</td>
</tr>
<tr>
<td>full depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>cementitious grout</td>
<td>Max V</td>
<td>No crack</td>
<td>45 (bottom)</td>
</tr>
<tr>
<td>full depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>epoxy grout</td>
<td>Max -M</td>
<td>No crack</td>
<td>163(top)</td>
</tr>
<tr>
<td>full depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>epoxy grout</td>
<td>Max +M</td>
<td>No crack</td>
<td>302(bottom)</td>
</tr>
<tr>
<td>full depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>FRC grout</td>
<td>Max V</td>
<td>No crack</td>
<td>41(bottom)</td>
</tr>
<tr>
<td>full depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>FRC grout</td>
<td>Max +M</td>
<td>No crack</td>
<td>165(top)</td>
</tr>
<tr>
<td>full depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>FRC grout</td>
<td>Max V</td>
<td>No crack</td>
<td>307(bottom)</td>
</tr>
<tr>
<td>full depth</td>
<td>mid tier</td>
<td>1&quot;</td>
<td>cementitious grout</td>
<td>Max -M</td>
<td>75%</td>
<td>102(top)</td>
</tr>
<tr>
<td>full depth</td>
<td>mid tier</td>
<td>1&quot;</td>
<td>cementitious grout</td>
<td>Max +M</td>
<td>40%</td>
<td>102(bottom)</td>
</tr>
<tr>
<td>full depth</td>
<td>mid tier</td>
<td>1&quot;</td>
<td>cementitious grout</td>
<td>Max V</td>
<td>No crack</td>
<td>45(bottom)</td>
</tr>
<tr>
<td>full depth</td>
<td>mid tier</td>
<td>1&quot;</td>
<td>epoxy grout</td>
<td>Max -M</td>
<td>No crack</td>
<td>163(top)</td>
</tr>
<tr>
<td>full depth</td>
<td>mid tier</td>
<td>1&quot;</td>
<td>epoxy grout</td>
<td>Max +M</td>
<td>No crack</td>
<td>303(bottom)</td>
</tr>
<tr>
<td>full depth</td>
<td>mid tier</td>
<td>1&quot;</td>
<td>epoxy grout</td>
<td>Max V</td>
<td>No crack</td>
<td>41(bottom)</td>
</tr>
<tr>
<td>partial depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>Cementitious grout</td>
<td>Max -M</td>
<td>10%</td>
<td>101(top)</td>
</tr>
<tr>
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<td>top tier</td>
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<td>Max +M</td>
<td>5%</td>
<td>103(bottom)</td>
</tr>
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<td>partial depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>cementitious grout</td>
<td>Max V</td>
<td>40%</td>
<td>103(bottom)</td>
</tr>
<tr>
<td>partial depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>epoxy grout</td>
<td>Max -M</td>
<td>80%</td>
<td>760(top)</td>
</tr>
<tr>
<td>Grouting depth</td>
<td>Location</td>
<td>Width</td>
<td>Material</td>
<td>Load Case</td>
<td>% of loading when crack occurs</td>
<td>Highest Tensile Stress (psi)</td>
</tr>
<tr>
<td>---------------</td>
<td>----------</td>
<td>-------</td>
<td>---------------</td>
<td>-----------</td>
<td>-------------------------------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>partial depth</td>
<td>top tier</td>
<td>1&quot;</td>
<td>epoxy grout</td>
<td>Max +M</td>
<td>35%</td>
<td>889(bottom)</td>
</tr>
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<td>top tier</td>
<td>1&quot;</td>
<td>epoxy grout</td>
<td>Max V</td>
<td>No crack</td>
<td>793(bottom)</td>
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<tr>
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<td>1&quot;</td>
<td>FRC grout</td>
<td>Max -M</td>
<td>20%</td>
<td>351(top)</td>
</tr>
<tr>
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<td>top tier</td>
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<td>FRC grout</td>
<td>Max +M</td>
<td>10%</td>
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<tr>
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<td>1&quot;</td>
<td>FRC grout</td>
<td>Max V</td>
<td>100%</td>
<td>372(bottom)</td>
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<td>epoxy grout</td>
<td>Max +M</td>
<td>No crack</td>
<td>308(bottom)</td>
</tr>
</tbody>
</table>
References
ACI Committee 446 (2007). “Fracture Toughness Testing of Concrete.” American Concrete Institute, Farmington Hills, Michigan.


Hanna, K. E., Morcous, G., and Tadros, M. K. (2008). "Non-Post-Tensioned Transverse Design and Detailing of Adjacent Box Beam Bridges." NCBC Concrete Bridge Conference, St. Louis, MO.


