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THREE ESSAYS ON AGRICULTURAL CONTRACTING: INCORPORATING MARKET EQUILIBRIUM AND DYNAMICS INTO PRINCIPAL-AGENT MODELS

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ABSTRACT

Motivated by several contemporary problems existing in U.S. agriculture, this thesis investigates three particular issues concerning different aspects of agricultural contracting between growers and processors. Specifically, the first essay investigates the relationship between the use of various types of hog contracts and the performance of the spot market when information asymmetries exist with regard to observability of product quality. This essay contributes to the existing literature by embedding a principal-agent model of processor-producer behavior within a general equilibrium model of the hog market. Different from the results in most past studies, this essay concludes that contract supplies raise the expected spot market price and reduce the variance of spot market price under a formula-price contract. Moreover, this essay finds that both a formula-price contract and a cost-plus contract offer a greater profit to processors and a greater expected utility to growers relative to fixed-price or market-price contacts. The second essay discusses efficiency of broiler contracts out of concerns of growers' dissatisfaction with the existing relative-performance contracts. Specifically, this essay compares various relative-performance contracts with fixed-performance contracts in a dynamic setting, and discusses improvements of the static mixed-type relative-performance contract. Various theoretical specifications justify the superiority of relative-performance contracts both in a static setting and in a dynamic setting when common shocks dominate idiosyncratic shocks. In addition, a static two-pooled-tournament relative-performance contract is shown to improve both the processor's and the grower's welfare relative to the static single-tournament relative-performance contract. The third essay investigates the role of growers' reputation when an agricultural processor designs optimal incentives for better quality products in a two-period dynamic contract. In a dynamic model with no commitment by both parties, reputation effects embodied in the processor's posterior probability assessment of the grower's types (using Bayes' rule) reinforce the potential well-known ratchet effect. Based on the optimal dynamic contract with no commitment, the processor offers a direct reputation reward to the grower contingent on his past performance. This essay demonstrates that the optimal dynamic contract with a reputation reward would outperform contracts where no reputation reward is offered.

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INTRODUCTION

Background

Use of contracting in agriculture has increased significantly over the past several decades. A recent USDA survey find that contracting is common among all types of farms, accounting for 35 percent of total production, where over two-thirds of contract volume was marketing contracts and one-third was production contracts. Among the animals or livestock sectors, over 95 percent of the poultry sector is under contract, while 42.9 percent and 25.3 percent of the hog sector and cattle sectors are under contract. Another recent study on production contracts shows that 99 percent of broilers are produced under production contracts, while hogs and cattle are 33% and 14%, respectively (Hayenga et al. 2000).

Contracting motives for agricultural producers and processors have been studied in broad agricultural settings. Market assurances, product quality, and risk sharing are some of the major concerns in a contractual relationship (Hudson 2000). The cyclical and seasonal patterns of some animal productions, such as hogs and cattle, make supply assurance important to processors because supply shortages increase short-term costs. On the growers' side, growers may face the risk of not having a buyer when needed. Thus, acquisition of animals through contracting reduces exposure to the risk for both processors and growers. In addition, for growers, selling their animals via contracting with processors reduces price risks to growers relative to selling to the cash market.

Besides supply assurance, quality is another important factor to processors because high-quality animals can reduce processors' costs by affecting processing time and labor costs as well as the quantity of high-value fresh meat cut per animal (Hayenga et al. 2000). Traditionally, livestock and broilers are sold based on live weight or carcass weight without taking into account quality differentiation. However, these pricing methods have been inadequate at sending appropriate signals to producers regarding quality attributes (Hayenga et al. 2000). Thus, processors attempt to capture the highest quality animals via contracts and other marketing agreements. Through marketing contracts or production contracts, processors can provide incentives for better quality characteristics wanted in the animals they purchase from growers. In addition, under a production contract, processors are sometimes able to control the choice of genetic stocks, feeding programs, and management decisions on the production of contracted animals. However, unobservability of animal quality before delivery may prevent processors from contracting upon explicit quality characteristics. Instead, optimal incentives for better quality must be provided conditional on other observed variables, such as feed conversion ratio in the broiler industry, or lean yield in the hog industry.

While processors and producers are motivated by similar factors to use contracts across most agricultural sectors, contracting in each sector has its own features due to the special characteristics borne in its production process or historical development. In some specific sectors, for example, the increasing use of contracts between producers and packers has provoked controversy. As a result, processors and producers in each sector face different problems. The following section summarizes some of the existing issues in agricultural contracting that need to be addressed.

Effect of contracting on spot markets

The transition to contracting away from a traditional cash markets, such as in the pork and beef industries and many other agricultural sectors, has brought new issues and problems. Specifically, in the hog sector, the use by most large producers of formula pricing contracts,[1] which are based on a spot market price, has raised concerns that contracting makes the cash market thinner. Hence, some observers argue that formula pricing contracts reduce spot market prices and raise cash price volatility (Hayenga et al. 2000), although no empirical study has documented this concern in the hog sector. Similar effects may also take place in the cattle sector due to captive supplies of fed cattle through contracting and other types of marketing agreements.[2] While various empirical studies attempt to assess impacts of contracting on the spot market price in the fed cattle sector, existing studies find mixed results due to different data or model specifications. In addition, most existing studies use reduced-form estimation technique and do not account for asymmetric information of unobservable quality or grower types. Thus, results from these studies are likely to be incomplete or biased.

Relative-performance contracts

In contrast to the cattle and hog sectors, the broiler industry is free from problems concerning the spot market because the extensive use of contracting in this industry has led to the virtual disappearance of the spot market. Instead of basing payment on a spot price, the payment structure of modern broiler contracts usually consists of a fixed base payment and a variable bonus payment dependent on a grower's relative performance. The bonus payment is determined as a percentage of the difference between a grower's individual performance and the average performance of the grower's peers. However, many growers have complained about a bonus system that compares their productivity to others. Adopted in the early 1990s, various forms of legislation attempted to regulate

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[1] Formula price of hogs, for example, may be calculated using Iowa-Southern Minnesota weighted average price of 49-51% lean hogs (Hayenga et al. 2000).
[2] In February 2004, a federal jury found Tyson Fresh Meats guilty of using captive supplies to manipulate the spot price. Tyson's lawyers said they would appeal the ruling (The New York Times, Feb. 18, 2004).

broiler contracts in Minnesota, Wisconsin, and Kansas. Similarly, in North Carolina a bill was recently introduced that specifically prohibited payments to a grower based on his performance relative to other growers (Hayenga et al. 2000).

Reputations and long-term contracts

Specialized investments and contractual relationships often go hand in hand in a number of agricultural sectors, such as poultry, pork, egg, and processed or frozen fruits and vegetables, where agricultural producers must make long-term investments in specialized facilities and equipments. For example, poultry producers invest in single-use chicken houses that can not be readily converted to other uses on the expectation of continuing contracts. Apple, grape, and often fruit growers invest in stands of trees or vines that might also be tailored to a single use. In these cases, if the contractual relationship between grower and buyer is terminated, the grower may be left with liabilities that cannot be reimbursed. This types of problem, often called a "hold-up" problem, and others associated with the existence of specific investments has motivated the use of long-term contract productions (Williamson 1989). Large-scale processors extensively use long-term contracts with producers to expand their scale and this use is still expected to expand rapidly (Hayenga et al. 2000).

Over time, in a dynamic contractual relationship, reputation may play an important role in maintaining the processor-producer relationship. In a narrow sense, one's reputation is the history of his previous actions or history of measured performance. In line with Wilson (1985), a player's current reputation affects others' predictions of his current behavior and thereby affects their current actions, and the evolution of his reputation depends on the history of his actions. Thus, his optimal strategy must optimize the tradeoff between short-term consequences of his current action and long-term consequences due to the effect of his current action on his future reputation.

Many studies have discussed the role of reputation effects in various settings. For example, Wilson (1985) summarizes studies of reputation effects in various game-theoretic and market models with dynamic features and informational asymmetries among the participants. Goodhue (2000) tests hypotheses regarding relationships between long-term contracting and growers' reputation of consistent high quality in the California winegrape industry. Up to now, however, few studies have investigated the explicit effects of growers' reputations on optimal incentives when one designs a long-term dynamic contract.

Objectives and Brief Outlines of Three Essays

Given the diverse problems and issues existing in contemporary agricultural production, this thesis attempts to investigate three particular issues concerning different aspects of agricultural contracting. It consists of three independent essays, with each essay dealing with one issue concerning agricultural contracting.

Out of concerns that use of formula-price contracts reduces spot market prices and raises market volatility, the first essay investigates the relationship between contracting and the spot market in the hog sector. More specifically, the essay develops a multi-market equilibrium model and compares five different types of contracts in the hog industry, i.e., two fixed-price contracts, a market-price contract, a formula-price contract, and a cost-plus contract, in terms of their impacts on the spot market. Market equilibrium conditions are derived under five different contracting scenarios after incorporating a principal-agent model of individual producer-processor relationship into a market equilibrium model of contract and cash markets. In addition, the principal-agent model accounts for asymmetric information in terms of unobservable quality while embedding an endogenously determined cash market price into an individual processor-producer relationship. Based on the spot market-contract market equilibrium under each contract scenario, results are presented for a numerical example to simulate the impact of contracting on the hog spot market. While the model is applied to the hog sector, the same methodology can be applied to other sectors such as the cattle sector.

The second essay investigates efficiency of broiler contracts and compares performances of relative-performance contracts (RPC) and fixed-performance contracts (FPC) under both a static model and a two-period dynamic model. Two scenarios of the two-period dynamic relative performance contracts are investigated: the *current-period RPC* and the *previous-period RPC*, where the current-period RPC rewards bonuses to growers using the group average performance calculated in the same period as a standard, while the previous-period RPC rewards each grower by comparing his performance with the previous period's average performance of the same group of growers. Last but not the least, the essay investigates a static model in which processors may split growers into two tournaments by offering two pooled contracts. This portion of the essay allows the processor to respond to the hidden information of grower types. Under each scenario described above, asymmetric information in terms of unobservable types of growers and unobservable production effort are introduced into the model. The results and their policy implications are discussed in the final section of this essay.

The third essay studies the effects of growers' reputation in a two-period dynamic processor-grower relationship. Two scenarios of the two-period model are presented: a full-commitment model and a model with no commitment. Specifically, the full commitment contract requires that both parties are committed to the contract and the contract cannot be breached or renegotiated during the contracting period. The no-commitment contract assumes that neither the processor nor the grower can commit to an intertemporal scheme: i.e., the processor can revise the contract in the second period conditional on the grower's first period performance and the grower can quit the relationship at the end of each period. Under the no-commitment dynamic contract, a fully separating equilibrium, a semi-separating equilibrium, and a pooling equilibrium are established. In the no-commitment case, reputation effects reflect the existence of persistent asymmetric information and are embodied in the posterior probability assessment of the grower's types by the processor at the end of the first period. However, the analysis that follows in the third essay establishes that this type of reputation effect can only enhance incentives for deviation: i.e., imitating the behavior of the low-quality type yields future information rent to the high-quality grower type by sustaining the processor's belief that the grower might be of low-quality type. Based on the no-commitment dynamic contract, a reputation reward is then introduced into the model. In this case, one assumes that the grower's reputation is summarized by his past measured performance. Specifically, a reputation reward based on the grower's past performance is remunerated to the grower in the second period if the processor observes good performance at the end of the first period. To simplify the analysis further, the reputation reward is assumed to take the form of a lump sum payment. Under these assumptions, this essay attempts to demonstrate that the reputation reward contingent on the history of past performance provides incentives for the grower to invest effort in building a reputation for quality, and thereby, can improve both the processor and the grower's welfare, and result in a dominant equilibrium.

In each essay, the general framework of principal-agent models described in Mas-Collel, Whinston, and Green (1995) is used while incorporating various forms of asymmetric information. However, each model varies depending on the type of the contract studied.

Overview of Existing Literature

Existing literature about contracting theory and agricultural contracting is very broad. Mas-Collel, Whinston, and Green (1995) summarizes a basic contract design problem, which relies on principal-agent models to incorporate asymmetric information, including hidden action (moral hazard) and hidden information. A number of extensions, such as multiple agents, multidimensional effort, multiple signals, and long-term agency relationship, of the basic model have been studied in the literature. For example, Holmström (1982), Nalebuff and Stiglitz (1983), Green and Stokey (1983), and Malcomson (1986) examine cases in which multiple agents are hired. Bernheim and Whinston (1986), on the other hand, examine settings in which a single agent is hired by several principals. Radner (1985), Rogerson (1985), and Lambert (1983) examine situations in which the agency relationship is repeated over many periods. Malcomson and Spinnewyn (1988), and Fudenberg, Holmstrom, and Milgrom (1990) study implementation of long-term contracts via a sequence of short-term contracts. This list of literature is hardly exhaustive, with other good reviews of contracting theory found in Hart and Holmström (1987) and Salanié (1997).

On the other hand, most research in agricultural contracting has focused either on finding empirical evidence of contract efficiency or attempting theoretical approaches to investigate various contracting problems in agriculture. Most papers deal with contracts in the meat industries, such as pork, beef, and broilers, due to the extensive use of contracting in those sectors. For example, Azzam (1998), Elam (1992), Schroeder et al. (1993), and Ward, Koontz, and Schroeder (1998) investigate relationship between contracting and the cash market in the fed cattle industry and find mixed results. These studies, which employ different empirical techniques, data, and model specifications, fall short of providing a definitive description of impacts of contracting on the cash prices. Reduced-form estimation techniques ignore the fact that contract supplies and the spot market are endogenously related. Thus, the results from those studies are likely to be biased. Moreover, despite evidence to the contrary, none of them incorporates asymmetric information into their models, especially imperfectly observed quality differences in the spot market and contract market. Goodhue, Rausser, and Simon (1998), Knoeber and Thurman (1994), and Levy and Vukina (2001) examine efficiency of broiler contracts using empirical evidence, while Tsoulouhas and Vukina (2000) used a theoretical approach to compare a static relative performance contract and a fixed performance contract incorporating hidden actions by growers. All these studies have shed some light on the efficiency of broiler contracts; however, they are hardly exhaustive. A few studies have investigated reputation effects and long-term contracts in agriculture. For example, Goodhue et al. (2000) test hypotheses regarding long-term relationships between contracting and reputation of grape quality in the California winegrape industry. However, none of the reputation effect studies has adopted a principal-agent framework.

Contribution of the Thesis

Based on past studies, this thesis investigates three issues of agricultural contracting in various agricultural sectors. Because the three essays presented in this thesis are structurally independent, this thesis not only contributes to the general literature of agricultural contracting, but also provides a more thorough analysis on each specific topic. Particularly, the various approaches presented in the thesis extend the general literature of agricultural contracting to issues concerning contract market-spot market equilibrium, multi-agent relationships, and intertemporal contractual schemes. For each specific topic explored in the thesis, the analysis that follows presents plausible results that are generally consistent with what one observes in common practice, and thus, could be used to provide better policy guidelines.

The major contributions of each essay are briefly summarized in the following section.

(1) In the first essay, five different types of contracts are compared in terms of their diverse effects on the hog cash market. The contributions of this essay are threefold: First, this essay not only investigates the relationship between hog contracting and the hog spot market in particular, but also provides a general methodology for this type of problem. Different from most studies, this essay embeds a principal-agent model of processor-grower relationship within a general equilibrium market model in which the

endogenous relationship between contract supplies and the spot market is explicitly acknowledged. More specifically, a grower's participation constraint in a processor-grower relationship is endogenized by linking the grower's contracting decision to the general-equilibrium-determined spot price of hogs. Second, this essay contributes to the existing literature by incorporating asymmetric information in terms of imperfectly observed hog qualities. Third, the results established in this essay justify the dominant use of formula-price contracts in the hog sector and are consistent with what one observes in the real world.

(2) The second essay discusses efficiency of broiler contracts out of concerns of growers' dissatisfaction with existing relative-performance contracts. This essay first contributes to the literature by providing a more thorough and comprehensive analysis of broiler contracts with incorporation of both moral hazard and adverse selection. More specifically, no known study has investigated the mixed-type, multiple-pooled relative-performance contract and the previous-period dynamic relative-performance contract in the broiler sector. Second, this essay provides more definitive policy implications from the theoretical models. This essay demonstrates that, under certain conditions, relative-performance contracts perform better than fixed-performance contracts from the perspective of both growers' and processors' welfare. Hence, the various theoretical models presented in this essay justify the superiority of relative-performance contracts relative to fixed-performance contracts.

(3) The third essay investigates implications of growers' reputation on optimal incentives in a long-term dynamic contract. The results presented in this essay are generally consistent with the existing literature in dynamic contracts. However, several

major differences arise: Firstly, past studies have found mixed results concerning the existence of a separating equilibrium in a dynamic contract. In this essay, optimal conditions are explicitly derived for a separating equilibrium, a semi-separating equilibrium, and a pooling equilibrium under certain conditions. Further, conditions for optimality of a "handicapped" separating equilibrium are also investigated, where a "handicapped" separating equilibrium offers a single contract only to high-quality growers. Secondly, although many studies have discussed reputation effects in various settings, few studies have been known to explicitly investigate the effects on optimal incentives of a reputation reward contingent on past performance when one designs a long-term dynamic contract incorporating with asymmetric information among participating parties. The analysis in this essay demonstrates that introduction of a reputation reward in a long-term dynamic contract could improve the performance of the contract by providing more effective incentives, and thus, result in a dominant equilibrium relative to the case where no reputation reward is available.

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ESSAY I

SIMULATING THE IMPACTS OF CONTRACT SUPPLIES IN A SPOT MARKET-CONTRACT MARKET EQUILIBRIUM SETTING

1.1 Introduction

A major concern associated with the prevalence of contracting agricultural production is that increased livestock acquisitions under contracts or marketing agreements make the cash market thinner. While studies devoted to this issue show mixed results, several conclude that the increasing use of contracting in meat packing reduces spot market prices and makes cash prices more volatile. However, these studies and most others fail to account for differentiated product quality because of methodological and data-driven constraints. Different from most of the existing literature, this essay uses a structural model to analyze the impact of contracting on the spot market. The essay first develops a model within the principal-agent framework for each individual processor-producer relationship assuming quality differentiation exists in the contract market. Then the market equilibrium is derived via a general equilibrium model by aggregating individual processors' demand and producers' supply. Further, a sensitivity analysis is conducted by modifying model parameters indicating the extent of contracting to investigate the impact of contracting on the spot market.

1.2 Literature Review

Past Studies

Existing empirical studies concerning the impact of contracting in meat packing on spot prices find mixed results. Elam (1992) conducts an empirical study and concludes that increases in forward contracting shipments reduce the national monthly average Agricultural Marketing Service price of fed cattle; however, impacts of contract shipments on cash price within some states (Texas, Kansas, Colorado, and Nebraska) are mixed. Barkley and Schroeder (1996) present an empirical model of the beef packing sector and find that cash price variability is positively associated with the level of captive supplies. However, the impact of captive supplies on beef spot prices was not explicitly estimated in their empirical study. Ward, Koontz, and Schroeder (1996) develop an empirical model and report a negative relationship between captive supplies delivered from marketing agreements and forward contracted cattle and cash prices; however, the effects of packer fed cattle on spot cattle prices are mixed. In another study, Ward, Koontz, and Schroeder (1998) find that the effects of captive supplies on cash prices is ambiguous due to the shifts in both demand and supply in the spot market. Schroeder, Jones, Mintert, and Barkley (1993) analyze the impact of forward contracting on fed cattle cash price and find significant negative effects.

Theoretical studies that also investigate the impact of forward contracting on cash prices find similar mixed results. Azzam (1998) develops an equilibrium replacement model of cattle procurement, but finds ambiguous effects of captive supplies. He points out that negative effects of forward contracting on cash prices may not be plausible because comparative statics implied by the captive-supply-induced shifts in market demand and supply are not explicit. Further, existing empirical results based on reduced-form models without formal framework underlying the model are subject to criticism. Xia and Sexton (2004) analyze the implication of top-of-the-market clauses on cash prices using general equilibrium models and conclude that the marketing contracts with top-of-the-market (highest market price) clauses reduce cash market prices.

These studies, which employ different empirical techniques, data, and model specifications, fall short of providing a definitive description of impacts of contracting on the cash prices. Moreover, despite evidence to the contrary, none of them incorporates asymmetric information into their models, especially imperfectly observed quality differences in the spot market and contract market. Several studies summarized by Hayenga et al. (2000) report significant quality differences in hog quality sourced from contracts and cash market transaction. In addition, reflecting quality differentials, average contract prices are consistently higher than spot market prices (Hayenga et al. 2000, Buhr and Kunkel 1999).

As implied by the above discussion, most existing studies investigating the about relationship between contracting and spot markets concern the cattle industry. No study has been found to analyze the impact of contracting on the hog spot market. In fact, “captive supply” is a particular terminology used for the cattle market. However, the similarities in processing, producing, and the extent of contracting enable us to use methodologies used for the cattle market to analyze the hog market. Therefore, this essay will not only investigate the relationship between hog contracting and the hog spot market in particular, but also provide a general methodology for this type of problem.

Existing Marketing Contracts in the Hog Sector

Buhr and Kunkel (1999) summarize types of marketing contracts available in the hog sector[3]. The major types are formula-price, cost-plus, price-window, price-floor, fixed-basis, and fixed-price contracts. Table 1.1 duplicates the results on the 12 largest pork packers’ procurement methods according to Grimes and Meyer’s January 2000

[3] Similar description can also be found in Hayenga et al. (2000)

survey. Since packer-fed supplies account for only a very small portion of pork packers' procurement of hogs, it is excluded from this study. In addition, marketing contracts related to the futures market are also excluded because it is beyond the interest of this paper.

Table 1.1 Twelve largest U.S. pork packers' procurement methods

Pricing method	Percent (100%)
Formula (reported price plus some amount)	47.2
Fixed price tied to futures (i.e., a cash contract)	8.5
Fixed tied to feed prices, no ledger	3.3
Fixed tied to feed prices, with ledger maintained	9.0
Window risk sharing, no ledger	3.8
Window risk sharing, ledger maintained	0.8
Other (packer-owned)	1.7
Spot market purchases	25.7

Source: Based on Grimes and Meyer's January 2000 survey. Also see Hayenga et al (2000), p38.

Formula-price contracts are used as a mechanism to establish prices when large quantities of hogs are forward contracted with a packer. Formula pricing, which is based on spot market prices plus a price premium or discount, accounts for 47.2% of all procurement types. Formula prices, for example, may be calculated as the price for Iowa/S. Minnesota 47-49% lean hogs plus or minus a price differential and premium based on market differences such as location or quality of hogs. Some have argued that formula price contracts do not provide price protections as they will fluctuate along with the market price on which they are based. Therefore, statistical models have been used to test the hypothesis that formula price contracts reduce spot market prices and raise price volatility in the spot market. However, as discussed in the previous section, those results have been mixed depending on their data, estimation technique and model structure.

Cost-plus contracts specify a price based on feed costs, which comprise the greatest single cost of production. This price implicitly sets a minimum price level, so it provides risk protection in addition to quantity assurance and market access. These contracts may have a balancing clause where payments are made to contractors/processors when market prices are below the contract prices and vice versa.

Price-window contracts, in general, set a ceiling and floor price. When the market price falls within the ceiling and the floor, the hogs are exchanged at the market price. When the market price falls above the ceiling or below the floor price, the packer and the producer split the difference between the two prices.

Price-floor contracts set a minimum price. To compensate the packer for this protection, the producer places a portion of the hog price above a predetermined ceiling price in an account to carry through the low price periods. The performance of these contracts will resemble a long-term put option.

If there is a balancing account clause in the contracts, as in price floor contracts and some cost-plus contracts, these contracts must be modeled in a multi-period setting. Without accounting for time preferences, the balancing account only reallocates or smoothes producers' income over time to reduce income variability, but does not reduce any price risk. Thus, on average, the contract prices under these contracts can be expected to have similar effects as market prices.

To understand the relationship between contract supply and the spot market, this essay will focus on the following four types of contracts: 1) fixed price contracts, 2) market price contracts, 3) formula price contracts with quality premium, and 4) cost plus contracts with quality premium.

1.3 Objectives

The objective of this study is to investigate the relationship between use of contracting and the spot market in the hog industry using a theoretical model. To account for quality differentiation in the contract market, a principal-agent framework is used to model individual processor-producer relationships. In addition, we assume asymmetric information concerning unobservable hog quality in the contract market. For each type of contract, the market equilibrium is derived via a general equilibrium model by aggregating individual demand and supply. Further, in order to analyze the impact of use of contracting on the hog spot market, a sensitivity analysis is performed by modifying the model parameters indicating the extent of contracting in the model.

This essay contributes to the existing literature by embedding a principal-agent model of processor-producer equilibrium behavior within a general equilibrium model of the hog market. In a related way, it also contributes by endogenizing the producers' participation constraint by linking the producers' contracting decision to the general-equilibrium determined spot market price of hogs.

1.4 The Model

The model contains three stages. In stage I, packers compete for producers to whom they offer contracts, and each participating producer signs a contract with a packer. In stage II, each producer independently determines how many hogs to produce and deliver to the cash market. In stage III, when the cash market settles, each packer decides the quantity to purchase in the cash market and both the contract and cash markets clear. The process can be illustrated by the following figure.

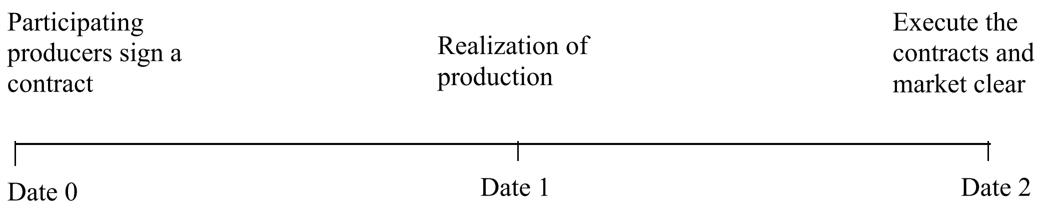

Figure 1.1 Timeline of the transaction

There are *N* homogenous producers and *M* homogenous processors in the pork sector, with *M*<<*N*. In the first stage, each producer decides either to sign a contract or not to sign a contract. Suppose in stage I, n_1^j producers sign a contract with packer *j*. For simplicity, we assume n_1^j is same for every packer *j*. Without loss of generality, we employ Xia and Sexton's (2004) assumption that each producer has a short-run supply function, $q = f(\varpi) = \varpi$, where *q* is the quantity of hogs produced by a producer and ϖ is the expected price the producer receives[4]. Each contract producer *i* independently produces a quantity of hogs q_0 based on the short-run supply function and sells a fixed proportion $\beta \in (0,1)$ of his hogs to a packer. Specifying the parameter β in this fashion

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[4] Xia and Sexton (2004) studied market price clause and captive supplies in the beef, not hog sector.

allows us to investigate the effect of captive supplies on the market equilibrium by modifying the value of the parameter β later on[5]. Thus, each packer j obtains $Q_1^j = n_1^j \beta q_0$ hogs from the contract market. On the other hand, those producers who do not participate in the contract independently decide to produce a quantity q_s, again based on the short-run supply function. Each packer converts procured hogs into a finished product according to a production function $g = g(Q \mid z)$, where z denotes the quality of hogs procured and z is only observable to producers before delivery. The production function is assumed to be concave in Q and z with $g_Q(Q \mid z) > 0$, $g_{QQ}(Q \mid z) \leq 0$ and $g_z(Q \mid z) > 0$, $g_{zz}(Q \mid z) \leq 0$, $g_{zQ}(Q \mid z) > 0$. Further, each packer incurs processing costs $h = h(Q \mid z)$ depending on the quality of hogs procured, with $h(.)$ being convex in Q and with $h_Q(Q \mid z) > 0$, $h_{QQ}(Q \mid z) > 0$, and $h_z(Q \mid z) < 0$.

Since the true hog quality is unobservable to packers before delivery, we assume that packers observe the market price of the finished products, such as fresh meat cuts, as an imperfect signal of the true quality of hogs delivered. More specifically, assume that the market price of the finished product is random based on a PDF $f(P| z)$ and a corresponding CDF $F(P| z)$, where P lies in the support Ω. It is assumed that the CDF $F(P| z)$ satisfies first-order stochastic dominance. In other words, if one supposes there are two quality levels $\{\underline{z}, \bar{z}\}$, then $F(P \mid \underline{z}) > F(P \mid \bar{z}\}$ in the sense that the expected market price of the finished product is higher when the quality of hogs is high than that

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[5] The parameter β can be thought of as the hedge ratio of an individual producer. Specifying the parameter β in this fashion not only allows for investigating the effect of captive supplies on the market equilibrium, but also significantly simplifies the analysis. In addition, specification of β guarantees the existence of a spot market.

when the quality of hogs is low. Note that there are some subtleties of timing or observational accuracy when one tries to interpret this variable *P*. One interpretation is that quality revelation and payment occur after delivery, and after market clearance. In this case, it may seem necessary to add a Date 3 in Figure 1 indicating the time of payment in the contract market. However, another interpretation is that quality is imperfectly observed upon delivery, and each packer uses it as a criterion for bonuses at the time of market clearance. This essay most closely follows this second interpretation by treating *P* as an imperfect signal that is closely related to the true quality of hogs delivered and is observable to packers upon delivery.

Since each packer purchases hogs from the cash market based on live weight basis or carcass weight basis, different qualities are not distinguished as precisely as in the contract market. To simplify the analysis, we assume that in the cash market only average quality if observed. Therefore, Akerlof's "lemons" argument applies and cash market prices would not provide sufficient incentives for hog producers to produce high-quality hogs. Hence, following the lemons argument, we assume that independent producers not participating in a contractual relationship will produce only low-quality hogs $\{\underline{z}\}$, while contracted producers will produce either high-quality or low-quality hogs depending on the individual contract. For simplicity, the quality of hogs available in the cash market is specified as the arithmetic mean of hog qualities sold by contracted producers and independent producers to the spot market[6].

[6] An alternative assumption is that the average market quality is the weighted average of hogs sold by contracted producers and independent producers to the spot market, however, it significantly complicates the analysis. Therefore, we only use the arithmetic mean in this essay as an illustration.

The unobservability of quality also plays an important role in how payoffs are structured. Since quality is only observable to producers, it cannot be explicitly contracted. In addition, in order to procure high-quality hogs from the contract-participating producers, processors must provide enough incentive to encourage high quality out of producers. Therefore, the contract price paid to producers by a processor must depend on the market price of the finished product, i.e., $w(P)$, which can be regarded as the imperfect quality signal.

According to Hayenga et al. (2000, p36), "… (Hog) quality affects processing time and labor costs as well as the quantity of high-value fresh meat cuts per hog. For example, each hog with excessive fat required more trimming and produces less lean meat. Conversely, a lean hog takes less time to process and produces a larger quantity of lean pork."

To simplify the analysis further, we assume that the output function of each packer is a linear function $g(Q_t \mid z) = \alpha_z Q_t$ with $\alpha_{\underline{z}} < \alpha_{\bar{z}}$ indicating the fact that high-quality hogs yields more finished product than low-quality hogs. The processing cost function for each packer takes a quadratic form $h(Q_t \mid z) = \frac{1}{2}\gamma_z (Q_t + \mu_t)^2$ where μ_t is a serially uncorrelated normally-distributed random variable with mean zero and variance σ_μ^2 affecting the processing cost function at time *t*. Additionally, it is assumed that $\gamma_{\bar{z}} < \gamma_{\underline{z}}$ reflecting the fact that low-quality hogs incur higher processing costs than high-quality hogs.

Contract producers have a time-invariant utility function $u(W)-v(z,q_0)$, where $W=\beta w(P)q_0+(1-\beta)p_t^s q_0$ represents the total revenue of each contract-participating producer from both the contract market and the cash market, and p_t^s is the spot market price at time *t*. However, for independent producers, the total revenue comes only from the spot market. That is, $W=p_t^s q_s$. Additionally, it is assumed that *u* is strictly concave in *W* with $u^{'}(W)>0$ and $u^{''}(W)<0$. Each producer incurs disutility according to the function $v(z,q_0)=c_z q_0^2$ with $c_{\underline{z}}<c_{\overline{z}}$. Producers' utility function is assumed to have the property of constant absolute risk aversion (CARA), $u(W)=1-\exp(-rW)$, where *r* is the Arrow-Pratt coefficient of absolute risk aversion. Then the expected utility $E[u(W)]$ is tantamount to

(1.1) $EW-\dfrac{1}{2}r\,\mathrm{var}(W)$,

where $\mathrm{var}(W)$ denotes the variance of *W*.

Given the assumptions made above, each packer maximizes its net profit:

(1.2)

$$\max_{w,q_2^j,n_1^j}\Pi=\int_{P\in\Omega}[Pg(Q_{1t}^j\mid z)-h(Q_{1t}^j\mid z)-w(P)Q_{1t}^j]dF(P\mid z)$$
$$+\int_{P\in\Omega}[Pg(q_{2t}^j\mid\widetilde{z})-h(q_{2t}^j\mid\widetilde{z})-p_t^s q_{2t}^j]dF(P\mid\widetilde{z})$$

subject to

(1.3)

$$\int_{P\in\Omega} E_{t-1}[u(\beta w(P)q_0 + (1-\beta)p_t^s q_0)]dF(P \mid z) - v(z,q_0) \geq E_{t-1}[u(p_t^s q_0)] - v(\underline{z},q_0), \quad \forall z \in \{\underline{z},\overline{z}\}$$

$$(1.4)\; z \in \arg\max_{\hat{z}} \int_{P\in\Omega} E_{t-1}[u(\beta w(P)q_0 + (1-\beta)p_t^s q_0)]dF(P \mid \hat{z}) - v(\hat{z},q_0), \qquad \forall z \in \{\underline{z},\overline{z}\}$$

where

E_{t-1} = Mathematical expectation operator of spot market price conditional on information available at time *t*-1,

$Q_{1t}^j = n_1^j \beta q_0$ hogs to be procured by packer *j* from the contract market,

q_{2t}^j = Hogs to be procured by packer *j* from the spot market,

$\widetilde{z}$ = average quality of hogs sold in the cash market,

p_t^s = Market price of hogs sold in the cash market at time *t*.

The individual rationality constraint (1.3) requires that the expected payoff to each producer participating the contract should be no less than that when he sells all his hogs to the cash market. Note that this cash market price will be determined by market equilibrium. The incentive compatibility constraint (1.4) ensures that under compensation schedule *w*(*P*) the producer's optimal quality choice is *z*.

As we have discussed in the section 1.2, we will focus on the following four types of hog marketing contracts: 1) Fixed-price contracts, 2) market-price contracts, 3) formula-price contracts with quality premium, and 4) cost-plus contracts with quality premium.

1.4.1 Fixed-price Contracts

First, suppose the processor optimally offers the producer a fixed price $w(P) = \underline{w}$ independent of P. Consequently, only the low quality $\underline{z}$ can be implemented. To see this, note that with this compensation scheme the producer's payoff is not affected by the quality of hogs contracted to the processor, so the producer will choose the lowest possible quality level to incur the lowest disutility. Therefore, the incentive compatibility constraint is satisfied. In addition, the producer must earn exactly the reservation utility because, otherwise, the processor can always reduce the reward until it reaches the producers' reservation utility level. Hence, the participation constraint (1.3) binds as well under the fixed-price contract.

Before we continue the analysis with the CARA utility function, we derive the market equilibrium with risk neutrality first.

a) *Producers are risk neutral*

Under risk neutrality, i.e., $u(W) = W$, the binding condition (1.3) becomes $E_{t-1}[\beta \underline{w} q_0 + (1-\beta) p_t^s q_0] - v(\underline{z}, q_0) = E_{t-1}[p_t^s q_0] - v(\underline{z}, q_0)$ from which we can solve the optimal contract price

(1.5) $\underline{w} = E_{t-1} p_t^s$.

Given the contract price and producers' short-run supply function, each contract producer produces

(1.6) $q_0 = \beta \underline{w} + (1-\beta) E_{t-1} p_t^s = E_{t-1} p_t^s$.

Similarly, independent producers choose to produce

(1.7) $q_s = E_{t-1} p_t^s$.

Hence, the packer's profit maximization problem becomes:

$$\max_{q_2^j, n_1^j} \Pi = \int_{P\in\Omega} [Pg(Q_{1t}^j \mid \underline{z}) - h(Q_{1t}^j \mid \underline{z}) - E_{t-1} p_t^s Q_1^j] dF(P \mid \underline{z}) + \int_{P\in\Omega} [Pg(q_{2t}^j \mid \underline{z}) - h(q_{2t}^j \mid \underline{z}) - p_t^s q_{2t}^j] dF(P \mid \underline{z})$$

Recall that $Q_{1t}^j = n_1^j \beta q_0$. The first-order conditions to this problem are:

(1.8) $\dfrac{\partial \Pi}{\partial q_{2t}^j} = \int_{P\in\Omega} [P\alpha_{\underline{z}} - \gamma_{\underline{z}} (q_{2t}^j + \mu_t) - p_t^s] dF(P \mid \underline{z}) = 0$, and

(1.9) $\dfrac{\partial \Pi}{\partial n_1^j} = \int_{P\in\Omega} [P\alpha_{\underline{z}} - \gamma_{\underline{z}} (Q_{1t}^j + \mu_t) - E_{t-1} p_t^s] dF(P \mid \underline{z}) = 0$.

From the conditions (1.8) and (1.9), we can derive the quantity of hogs to be procured by packer *j* from the spot market,

(1.10) $q_{2t}^j = \dfrac{\alpha_{\underline{z}} \int_{P\in\Omega} PdF(P \mid \underline{z}) - p_t^s}{\gamma_{\underline{z}}} - \mu_t = \dfrac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s}{\gamma_{\underline{z}}} - \mu_t$,

and the number of producers contracted with each packer,

(1.11) $n_1^j = \dfrac{\alpha_{\underline{z}} \int_{P\in\Omega} PdF(P \mid \underline{z}) - E_{t-1} p_t^s - \gamma_{\underline{z}} \mu_t}{\gamma_{\underline{z}} \beta q_0} = \dfrac{\alpha_{\underline{z}} P^{\underline{z}} - E_{t-1} p_t^s - \gamma_{\underline{z}} \mu_t}{\gamma_{\underline{z}} \beta q_0}$,

where $P^{\underline{z}}$ denotes the expected market price for the finished product by packers given low-quality hogs procured from both markets. Equation (1.10) shows that q_2^j is positively related to $P^{\underline{z}}$ and negatively related to the spot market price of hogs p_t^s. Note that in this model each producer signs a fixed proportion of their hogs with each packer, thus, packers can only adjust their demand for contract hogs by adjusting the number of producers contracted with each packer. The condition (1.11) confirms that if the expected cash market price goes up, a smaller number of producers will contract their hogs with the packers.

The market supply and demand in both the cash market and the contract market can be derived by aggregating individual demands and supplies. The market equilibrium then requires that supply equals demand in both the contract market and the cash market. Further, we assume that the contract market supply is perfectly elastic; therefore, we only need to solve the equilibrium spot market price. Specifically, the cash-market demand is $Q_{2d} = \sum_{j=1}^{M} q_{2t}^{j}(p_s \mid \underline{z}) = Mq_{2t}^{j}(p_t^s \mid \underline{z})$. Since $q_s = q_0$ in this case, the cash-market supply takes the form $Q_{2s} = Nq_0 - M\beta q_0 n_1^j (E_{t-1}p_t^s \mid \underline{z})$. Hence, the spot market clearing condition requires

(1.12) $Q_{2d} = Q_{2s} \Rightarrow Mq_{2t}^{j}(p_t^s \mid \underline{z}) = Nq_0 - M\beta q_0 n_1^j (E_{t-1}p_t^s \mid \underline{z})$.

Specifically, substituting (1.10) and (1.11) into the condition (1.12) yields

(1.13) $M[\frac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s}{\gamma_{\underline{z}}} - \mu_t] = Nq_0 - M\beta q_0 [\frac{\alpha_{\underline{z}} P^{\underline{z}} - E_{t-1}p_t^s - \gamma_{\underline{z}}\mu_t}{\gamma_{\underline{z}}\beta q_0}]$,

or,

(1.14) $\frac{M}{\gamma_{\underline{z}}}[p_t^s + E_{t-1}p_t^s] = \frac{2M\alpha_{\underline{z}} P^{\underline{z}}}{\gamma_{\underline{z}}} - Nq_0 - 2M\mu_t$.

Further, substituting (1.7) into (1.14) yields

(1.15) $\frac{M}{\gamma_{\underline{z}}}[p_t^s + E_{t-1}p_t^s] = \frac{2M\alpha_{\underline{z}}P^{\underline{z}}}{\gamma_{\underline{z}}} - NE_{t-1}p_t^s - 2M\mu_t$.

Applying the conditional expectations E_{t-1} to both sides of (1.15) and using the assumption that $E_{t-1}\mu_t = 0$, we can compute the expected spot-market price:

(1.16) $E_{t-1}p_t^s = \frac{2M\alpha_{\underline{z}}P^{\underline{z}}}{\gamma_{\underline{z}}} \Big/ (\frac{2M}{\gamma_{\underline{z}}} + N) = \frac{\alpha_{\underline{z}}P^{\underline{z}}}{1 + \frac{\gamma_{\underline{z}}}{2}\frac{N}{M}} = \frac{2\alpha_{\underline{z}}P^{\underline{z}}M}{2M + \gamma_{\underline{z}}N}$.

Note that the expected spot-market price decreases as the ratio of the number of producers to the number of packers, $\frac{N}{M}$, increases. In the limit, the expected spot market prices will be the lowest when the packer is a monopsonist (M = 1).

Hence, from (1.6), the optimal amount of hogs to be produced by the each producer equals to

(1.17) $q_0 = E_{t-1}p_t^s = \frac{2\alpha_{\underline{z}}P^{\underline{z}}M}{2M + \gamma_{\underline{z}}N}$.

Substituting (1.16) back into (1.15) solves for the spot market price:

(1.18) $p_t^s = \frac{2\alpha_{\underline{z}}P^{\underline{z}}M}{2M + \gamma_{\underline{z}}N} - 2\gamma_{\underline{z}}\mu_t$,

from which we can compute the time-independent conditional variance of the spot market price[7]:

(1.19) $\text{var}(p_t^s) = 4\gamma_{\underline{z}}^2\sigma_\mu^2$.

[7] The time independence of the variance arises from the stationarity of μ_t. If, instead, μ_t is non-stationary, the variance defined in (1.19) will change over time.

The number of hogs procured by packer *j* from the spot market, q_2^j, can be solved by substituting (1.18) into (1.10):

$$(1.20)\ q_{2t}^j = \frac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s}{\gamma_{\underline{z}}} - \mu_t = \frac{\alpha_{\underline{z}} P^{\underline{z}} N}{2M + \gamma_{\underline{z}} N} + \mu_t\ .$$

Similarly, substituting (1.16) into (1.11) yields

$$(1.21)\ n_1^j = \frac{\alpha_{\underline{z}} P^{\underline{z}} - E_{t-1} p_t^s - \gamma_{\underline{z}} \mu_t}{\gamma_{\underline{z}} \beta q_0} = \frac{N}{2\beta M} - \frac{\mu_t}{\beta q_0}.$$

In particular, if $\mu_t \equiv 0$, i.e., there is no uncertainty in packer's cost function, (1.21) becomes $n_1^j = \frac{N}{2\beta M}$. Thus, in order for the spot market to exist, the parameter β must be strictly greater than one half under expectation. That is, under risk neutrality, each contract producer will contract at least half of their hogs with a packer, given the contract price equals the spot-market price.

Under risk neutrality, it can also be verified that the producers are indifferent between selling their hogs to the contract market and selling to the cash market under expectation. Specifically, given the conditions (1.10)-(1.12),

$$(1.22)\ Q_{2s} = Nq_0 - M\beta q_0 n_1^j (E_{t-1} p_t^s \mid \underline{z}) = Nq_0 - M\beta q_0 [\frac{\alpha_{\underline{z}} P^{\underline{z}} - E_{t-1} p_t^s - \gamma_{\underline{z}} \mu_t}{\gamma_{\underline{z}} \beta q_0}]$$

$$= Nq_0 - M \frac{\alpha_{\underline{z}} P^{\underline{z}} - E_{t-1} p_t^s}{\gamma_{\underline{z}}} + M\mu_t = Nq_0 - Mq_{2t}^j (p_t^s \mid \underline{z}) + M \frac{E_{t-1} p_t^s - p_t^s}{\gamma_{\underline{z}}}.$$

Since $Q_{2d} = Mq_{2t}^j (p_t^s \mid \underline{z})$, the market clearing condition (1.12) requires

$$(1.23)\ Q_{2d} = Mq_{2t}^j(p_t^s \mid \underline{z}) = Nq_0 - Mq_{2t}^j(p_t^s \mid \underline{z}) + M\frac{E_{t-1}p_t^s - p_t^s}{\gamma_{\underline{z}}} = Q_{2s},$$

from which we can obtain

$$(1.24)\ Q_{2d} = \frac{Nq_0}{2} + M\frac{E_{t-1}p_t^s - p_t^s}{2\gamma_{\underline{z}}} = Q_{2s}.$$

Hence, taking expectation on both sides of (1.24) implies

$$(1.25)\ E_{t-1}Q_{2d} = E_{t-1}Q_{2s} = E_{t-1}Q_{1d} = E_{t-1}Q_{1s} = \frac{Nq_0}{2}.$$

Under this scheme, each packer obtains profit

$$\Pi_1^* = \int_{P\in\Omega}[Pg(Q_{1t}^j \mid \underline{z}) - h(Q_{1t}^j \mid \underline{z}) - E_{t-1}p_t^s Q_{1t}^j]dF(P \mid \underline{z})$$
$$+ \int_{P\in\Omega}[Pg(q_{2t}^j \mid \underline{z}) - h(q_{2t}^j \mid \underline{z}) - p_t^s q_{2t}^j]dF(P \mid \underline{z})$$
$$= [a_{\underline{z}}P^{\underline{z}}Q_{1t}^j - \frac{1}{2}\gamma_{\underline{z}}(Q_{1t}^j + \mu_t)^2 - E_{t-1}p_t^s Q_{1t}^j] + [a_{\underline{z}}P^{\underline{z}}q_{2t}^j - \frac{1}{2}\gamma_{\underline{z}}(q_{2t}^j + \mu_t)^2 - p_t^s q_{2t}^j]$$

$$= [a_{\underline{z}}P^{\underline{z}}Q_{1t}^j - \frac{1}{2}\gamma_{\underline{z}}(Q_{1t}^j)^2 - \frac{1}{2}\gamma_{\underline{z}}\mu_t^2 - \gamma_{\underline{z}}Q_{1t}^j\mu_t - E_{t-1}p_t^s Q_{1t}^j]$$
$$+ [a_{\underline{z}}P^{\underline{z}}q_{2t}^j - \frac{1}{2}\gamma_{\underline{z}}(q_{2t}^j)^2 - \frac{1}{2}\gamma_{\underline{z}}\mu_t^2 - \gamma_{\underline{z}}q_{2t}^j\mu_t - p_t^s q_{2t}^j]$$

$$= Q_{1t}^j[a_{\underline{z}}P^{\underline{z}} - \gamma_{\underline{z}}\mu_t - E_{t-1}p_t^s] - \frac{1}{2}\gamma_{\underline{z}}(Q_{1t}^j)^2 - \frac{1}{2}\gamma_{\underline{z}}\mu_t^2 + q_{2t}^j[a_{\underline{z}}P^{\underline{z}} - \gamma_{\underline{z}}\mu_t - p_t^s]$$
$$- \frac{1}{2}\gamma_{\underline{z}}(q_{2t}^j)^2 - \frac{1}{2}\gamma_{\underline{z}}\mu_t^2$$
$$= Q_{1t}^j[\gamma_{\underline{z}}Q_{1t}^j] - \frac{1}{2}\gamma_{\underline{z}}(Q_{1t}^j)^2 - \frac{1}{2}\gamma_{\underline{z}}\mu_t^2 + q_{2t}^j[\gamma_{\underline{z}}q_{2t}^j] - \frac{1}{2}\gamma_{\underline{z}}(q_{2t}^j)^2 - \frac{1}{2}\gamma_{\underline{z}}\mu_t^2$$

$$= [\frac{1}{2}\gamma_{\underline{z}}(\beta n_1^j q_0)^2 - \frac{1}{2}\gamma_{\underline{z}}\mu_t^2] + [\frac{1}{2}\gamma_{\underline{z}}(q_{2t}^j)^2 - \frac{1}{2}\gamma_{\underline{z}}\mu_t^2]$$

$$= [\frac{1}{2}\gamma_{\underline{z}}(\beta q_0)^2(\frac{N}{2\beta M} - \frac{\mu_t}{\beta q_0})^2 - \frac{1}{2}\gamma_{\underline{z}}\mu_t^2] + [\frac{1}{2}\gamma_{\underline{z}}(\frac{\alpha_{\underline{z}}P^{\underline{z}}N}{2M + \gamma_{\underline{z}}N} + \mu_t)^2 - \frac{1}{2}\gamma_{\underline{z}}\mu_t^2]$$

$$= [\frac{1}{8}\gamma_{\underline{z}}q_0^2(\frac{N}{M})^2 - \frac{1}{2}\frac{\gamma_{\underline{z}}q_0 N}{M}\mu_t] + [\frac{1}{2}\gamma_{\underline{z}}(\frac{\alpha_{\underline{z}}P^{\underline{z}}N}{2M + \gamma_{\underline{z}}N})^2 + \frac{\alpha_{\underline{z}}P^{\underline{z}}\gamma_{\underline{z}}N}{2M + \gamma_{\underline{z}}N}\mu_t],$$

from which we can compute the expected profit

(1.26)

$$E\Pi_1^* = \frac{1}{8}\gamma_{\underline{z}}q_0^2(\frac{N}{M})^2 + \frac{1}{2}\gamma_{\underline{z}}(\frac{\alpha_{\underline{z}}P^{\underline{z}}N}{2M + \gamma_{\underline{z}}N})^2 = \frac{1}{8}\gamma_{\underline{z}}(\frac{N}{M})^2(\frac{\alpha_{\underline{z}}P^{\underline{z}}}{1 + \frac{\gamma_{\underline{z}}N}{2M}})^2 + \frac{1}{2}\gamma_{\underline{z}}(\frac{\alpha_{\underline{z}}P^{\underline{z}}N}{2M + \gamma_{\underline{z}}N})^2$$

$$= \gamma_{\underline{z}}(\frac{\alpha_{\underline{z}}P^{\underline{z}}N}{2M + \gamma_{\underline{z}}N})^2,$$

and the variance of the profit

(1.27) $$Var(\Pi_1^*) = [\frac{\alpha_{\underline{z}}P^{\underline{z}}\gamma_{\underline{z}}N}{2M + \gamma_{\underline{z}}N} - \frac{1}{2}\frac{\gamma_{\underline{z}}q_0 N}{M}]\sigma_\mu^2 = [\frac{\alpha_{\underline{z}}P^{\underline{z}}\gamma_{\underline{z}}N}{2M + \gamma_{\underline{z}}N} - \frac{\alpha_{\underline{z}}P^{\underline{z}}\gamma_{\underline{z}}N}{2M + \gamma_{\underline{z}}N}]\sigma_\mu^2 = 0.$$

Note that the packers' profit is positively related to the ratio N/M. It reaches maximum when M=1 and N>>0. On the other hand, when there are a sufficiently large number of packers in the market to compete for a finite number of hog producers, the packers' net profits approach zero. In addition, under the scheme, the packers can perfectly eliminate the uncertainty in the profit by adjusting the demands from the contract market and the cash market.

b) *Producers are risk averse*

Recall that with constant absolute risk aversion, the expected utility $E[u(W)]$ is tantamount to $EW - \frac{1}{2} r \operatorname{var}(W)$. Given the fixed contract price $w(P) = \underline{w}$ and $W = \beta w(P) q_0 + (1-\beta) p_t^s q_0$, we have $EW = \beta \underline{w} q_0 + (1-\beta) E_{t-1} p_t^s q_0$, and $\operatorname{var}(W) = (1-\beta)^2 q_0^2 \operatorname{var}(p_t^s)$. Hence, the binding constraint (1.3) becomes

(1.28)

$$\int_{P \in \Omega} [(\beta \underline{w} q_0 + (1-\beta) E_{t-1} p_t^s q_0) - \frac{1}{2} r (1-\beta)^2 q_0^2 \operatorname{var}(p_t^s)] dF(P \mid \underline{z}) - v(\underline{z}, q_0)$$
$$= E_{t-1} p_t^s q_0 - \frac{1}{2} r q_0^2 \operatorname{var}(p_t^s) - v(\underline{z}, q_0),$$

from which we can compute the contract price

(1.29) $\underline{w} = E_{t-1} p_t^s - \frac{1}{2} r q_0 (2-\beta) \operatorname{var}(p_t^s)$.

Note that the constant contract price is positively related to the expected market price and is negatively related to the coefficient of risk aversion r and the variance of the market price. In other words, if producers are risk averse, packers can depress the contract price and make the producers indifferent between the contract market and the spot market. Moreover, the more volatile the spot market price is, the smaller the contract price can be offered by the packers.

Given the contract price and producers' short-run supply function, each contract producer produces

(1.30)

$$q_0 = \beta \underline{w} + (1-\beta)E_{t-1}p_t^s = \beta[E_{t-1}p_t^s - \frac{1}{2}rq_0(2-\beta)\,\mathrm{var}(p_t^s)] + (1-\beta)E_{t-1}p_t^s$$
$$= E_{t-1}p_t^s - \frac{1}{2}rq_0\beta(2-\beta)\,\mathrm{var}(p_t^s),$$

from which

(1.31) $$q_0 = \frac{E_{t-1}p_t^s}{1+\frac{1}{2}r\beta(2-\beta)\,\mathrm{var}(p_t^s)}.$$

Similarly, independent producers choose to produce

(1.32) $$q_s = E_{t-1}p_t^s.$$

Under these conditions, a packer maximizes its net profit:

$$\max_{q_2^j, n_1^j} \Pi = \int_{P\in\Omega} [Pg(Q_{1t}^j \mid \underline{z}) - h(Q_{1t}^j \mid \underline{z}) - (E_{t-1}p_t^s - \frac{1}{2}rq_0(2-\beta)\,\mathrm{var}(p_t^s))Q_{1t}^j]dF(P \mid \underline{z}) +$$
$$\int_{P\in\Omega}[Pg(q_{2t}^j \mid \underline{z}) - h(q_{2t}^j \mid \underline{z}) - p_t^s q_{2t}^j]dF(P \mid \underline{z})$$

The first-order conditions to this problem are:

(1.33) $$\frac{\partial \Pi}{\partial q_{2t}^j} = \int_{P\in\Omega}[P\alpha_{\underline{z}} - \gamma_{\underline{z}}(q_{2t}^j + \mu_t) - p_t^s]dF(P \mid \underline{z}) = 0$$, and

(1.34) $$\frac{\partial \Pi}{\partial n_1^j} = \int_{P\in\Omega}[P\alpha_{\underline{z}} - \gamma_{\underline{z}}(Q_{1t}^j + \mu_t) - (E_{t-1}p_t^s - \frac{1}{2}rq_0(2-\beta)\,\mathrm{var}(p_t^s))]dF(P \mid \underline{z}) = 0.$$

From condition (1.33), we can derive the quantity of hogs procured by packer *j* from the spot market,

(1.35) $$q_2^j = \frac{\alpha_{\underline{z}} \int_{P\in\Omega} PdF(P \mid \underline{z}) - p_t^s}{\gamma_{\underline{z}}} - \mu_t = \frac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s}{\gamma_{\underline{z}}} - \mu_t.$$

From condition (1.34), we can get the number of producers that each packer contracts with,

$$(1.36)\ n_1^j = \frac{\alpha_{\underline{z}} P^{\underline{z}} - E_{t-1} p_t^s + \frac{1}{2} r q_0 (2-\beta)\,\mathrm{var}(p_t^s) - \gamma_{\underline{z}} \mu_t}{\gamma_{\underline{z}} \beta q_0}.$$

Further, substituting (1.31) into (1.36) yields

(1.37)

$$n_1^j = \frac{\alpha_{\underline{z}} P^{\underline{z}} - E_{t-1} p_t^s + \dfrac{E_{t-1} p_t^s [\frac{1}{2} r (2-\beta)\,\mathrm{var}(p_t^s)]}{1+\frac{1}{2} r\beta(2-\beta)\,\mathrm{var}(p_t^s)}}{\gamma_{\underline{z}} \beta \dfrac{E_{t-1} p_t^s}{1+\frac{1}{2} r\beta(2-\beta)\,\mathrm{var}(p_t^s)}} - \frac{\mu_t}{\beta q_0}$$

$$= \frac{\alpha_{\underline{z}} P^{\underline{z}} - E_{t-1} p_t^s \{\dfrac{1+\frac{1}{2} r (2-\beta)(\beta-1)\,\mathrm{var}(p_t^s)}{1+\frac{1}{2} r\beta(2-\beta)\,\mathrm{var}(p_t^s)}\}}{\gamma_{\underline{z}} \beta \dfrac{E_{t-1} p_t^s}{1+\frac{1}{2} r\beta(2-\beta)\,\mathrm{var}(p_t^s)}} - \frac{\mu_t}{\beta q_0}$$

$$= \frac{\alpha_{\underline{z}} P^{\underline{z}} [1+\frac{1}{2} r\beta(2-\beta)\,\mathrm{var}(p_t^s)] - E_{t-1} p_t^s [1+\frac{1}{2} r(2-\beta)(\beta-1)\,\mathrm{var}(p_t^s)]}{\gamma_{\underline{z}} \beta E_{t-1} p_t^s} - \frac{\mu_t}{\beta q_0}.$$

To derive the market equilibrium, supply must equal demand in the spot market. Specifically, the spot market demand is

$$(1.38)\ Q_{2d} = M q_{2t}^j (p_t^s \mid \underline{z}),$$

and the spot market supply is

(1.39) $Q_{2s} = (N - Mn_1^j)q_s + M(1-\beta)q_0 n_1^j (E_{t-1}p_t^s \mid \underline{z})$.

Hence, the cash market clearing condition requires

(1.40)

$$\begin{aligned} Q_{2d} = Q_{2s} \Rightarrow Mq_{2t}^j(p_t^s \mid \underline{z}) &= (N - Mn_1^j)q_s + M(1-\beta)q_0 n_1^j (E_{t-1}p_t^s \mid \underline{z}) \\ &= Nq_s - Mn_1^j(E_{t-1}p_t^s \mid \underline{z})[q_s - (1-\beta)q_0] \end{aligned}.$$

Substituting (1.31), (1.32), (1.35), and (1.37) into (1.40) yields

(1.41)

$$M[\frac{\alpha_{\underline{z}}P^{\underline{z}} - p_t^s}{\gamma_{\underline{z}}} - \mu_t] = NE_{t-1}p_t^s - [E_{t-1}p_t^s - (1-\beta)\frac{E_{t-1}p_t^s}{1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)}]$$

$$M\{\frac{\alpha_{\underline{z}}P^{\underline{z}}[1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)] - E_{t-1}p_t^s[1+\frac{1}{2}r(2-\beta)(\beta-1)\operatorname{var}(p_t^s)]}{\gamma_{\underline{z}}\beta E_{t-1}p_t^s} - \frac{\mu_t}{\beta q_0}\}.$$

Moving the expected spot price and spot market price to the left side yields

(1.42)

$$M[\frac{\alpha_{\underline{z}}P^{\underline{z}} - p_t^s}{\gamma_{\underline{z}}}] - NE_{t-1}p_t^s + [E_{t-1}p_t^s - (1-\beta)\frac{E_{t-1}p_t^s}{1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)}]$$

$$M\frac{\alpha_{\underline{z}}P^{\underline{z}}[1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)] - E_{t-1}p_t^s[1+\frac{1}{2}r(2-\beta)(\beta-1)\operatorname{var}(p_t^s)]}{\gamma_{\underline{z}}\beta E_{t-1}p_t^s}$$

$$= M\mu_t\{1+\frac{1}{\beta q_0}[E_{t-1}p_t^s - (1-\beta)\frac{E_{t-1}p_t^s}{1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)}]\}.$$

Taking expectation E_{t-1} on both sides of (1.42) and applying the assumption $E_{t-1}\mu_t = 0$, (1.42) becomes

(1.43)

$$\frac{\alpha_z P^z}{\gamma_z} - \frac{E_{t-1}p_t^s}{\gamma_z} - \frac{NE_{t-1}p_t^s}{M} + [\frac{1+\frac{1}{2}r(2-\beta)\operatorname{var}(p_t^s)}{1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)}]$$

$$\frac{\alpha_z P^z[1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)] - E_{t-1}p_t^s[1+\frac{1}{2}r(2-\beta)(\beta-1)\operatorname{var}(p_t^s)]}{\gamma_z}$$

$$= \frac{\alpha_z P^z}{\gamma_z} - \frac{E_{t-1}p_t^s}{\gamma_z} - \frac{NE_{t-1}p_t^s}{M} + \frac{\alpha_z P^z}{\gamma_z}[\frac{1+\frac{1}{2}r(2-\beta)\operatorname{var}(p_t^s)}{1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)}][1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)]$$

$$-\frac{E_{t-1}p_t^s}{\gamma_z}[\frac{\frac{1}{2}r(2-\beta)\operatorname{var}(p_t^s)+1}{1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)}][1+\frac{1}{2}r(2-\beta)(\beta-1)\operatorname{var}(p_t^s)] = 0,$$

from which we can compute the expected spot market price

(1.44)

$E_{t-1}p_t^s =$

$$\frac{\frac{\alpha_z P^z}{\gamma_z} + \frac{\alpha_z P^z}{\gamma_z}[\frac{1+\frac{1}{2}r(2-\beta)\operatorname{var}(p_t^s)}{1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)}][1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)]}{\frac{1}{\gamma_z} + \frac{N}{M} + \frac{1}{\gamma_z}[\frac{\frac{1}{2}r(2-\beta)\operatorname{var}(p_t^s)+1}{1+\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)}][1+\frac{1}{2}r(2-\beta)(\beta-1)\operatorname{var}(p_t^s)]}$$

$$= \frac{\alpha_{\underline{z}} P^{\underline{z}} [\frac{1}{2} r(2-\beta)\,\mathrm{var}(p_t^s) + 2]}{1 + \frac{\gamma_{\underline{z}} N}{M} + [\frac{\frac{1}{2} r(2-\beta)\,\mathrm{var}(p_t^s) + 1}{1 + \frac{1}{2} r\beta(2-\beta)\,\mathrm{var}(p_t^s)}][1 + \frac{1}{2} r(2-\beta)(\beta-1)\,\mathrm{var}(p_t^s)]} .$$

Substituting (1.44) into (1.42) solves the spot market price,

(1.45) $$p_t^s = E_{t-1} p_t^s - \gamma_{\underline{z}} \{1 + \frac{1}{\beta q_0} [E_{t-1} p_t^s - (1-\beta) \frac{E_{t-1} p_t^s}{1 + \frac{1}{2} r\beta(2-\beta)\,\mathrm{var}(p_t^s)}]\} \mu_t .$$

Hence, we can compute the time-invariant variance of the spot market price,

(1.46)

$$\mathrm{var}(p_t^s) = \gamma_{\underline{z}}^2 \{1 + \frac{1}{\beta q_0} [E_{t-1} p_t^s - (1-\beta) \frac{E_{t-1} p_t^s}{1 + \frac{1}{2} r\beta(2-\beta)\,\mathrm{var}(p_t^s)}]\}^2 \sigma_\mu^2$$

$$= \gamma_{\underline{z}}^2 \{1 + \frac{E_{t-1} p_t^s}{q_0} [\frac{\frac{1}{2} r(2-\beta)\,\mathrm{var}(p_t^s) + 1}{1 + \frac{1}{2} r\beta(2-\beta)\,\mathrm{var}(p_t^s)}]\}^2 \sigma_\mu^2 \qquad \Big| \textit{substituting}\ (1.31)$$

$$= \gamma_{\underline{z}}^2 \{2 + \frac{1}{2} r(2-\beta)\,\mathrm{var}(p_t^s)\}^2 \sigma_\mu^2 \ .$$

Hence, the variance of spot market prices can be solved explicitly for each set of parameter values. Specifically, denote the variance $\mathrm{var}(p_t^s)$ as σ_p^2, we can solve it as

(1.47)

$$\mathrm{var}(p_t^s) = \sigma_p^2 = \sigma_p^2(\gamma_{\underline{z}}, r, \beta, \sigma_\mu^2)$$

$$= \frac{1 - 2\gamma_{\underline{z}}^2 \sigma_\mu^2 r(2-\beta) - \sqrt{[1 - 2\gamma_{\underline{z}}^2 \sigma_\mu^2 r(2-\beta)]^2 - 4\gamma_{\underline{z}}^4 \sigma_\mu^4 r^2 (2-\beta)^2}}{\frac{1}{2} \gamma_{\underline{z}}^2 \sigma_\mu^2 r^2 (2-\beta)^2}$$

The explicit equilibrium spot market price can be solved by substituting (1.31) and (1.47) into (1.45):

(1.48) $$p_t^s = \frac{\alpha_{\underline{z}} P^{\underline{z}}[\frac{1}{2}r(2-\beta)\sigma_p^2 + 2]}{1+\frac{\gamma_{\underline{z}} N}{M} + [\frac{\frac{1}{2}r(2-\beta)\sigma_p^2 + 1}{1+\frac{1}{2}r\beta(2-\beta)\sigma_p^2}][1+\frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]} - \gamma_{\underline{z}}\{2+\frac{1}{2}r(2-\beta)\sigma_p^2\}\mu_t$$

Further, the quantity of hogs procured by packer *j* from the spot market, q_2^j, and the number of producers that each packer contracts with, n_1^j, can be solved. Specifically, substituting (1.48) into (1.35) yields

(1.49)

$$q_2^j = \frac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s}{\gamma_{\underline{z}}} - \mu_t$$

$$= \frac{\alpha_{\underline{z}} P^{\underline{z}} - \dfrac{\alpha_{\underline{z}} P^{\underline{z}}[\frac{1}{2}r(2-\beta)\sigma_p^2 + 2]}{1+\frac{\gamma_{\underline{z}} N}{M} + [\frac{\frac{1}{2}r(2-\beta)\sigma_p^2 + 1}{1+\frac{1}{2}r\beta(2-\beta)\sigma_p^2}][1+\frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]} + \gamma_{\underline{z}}\{2+\frac{1}{2}r(2-\beta)\sigma_p^2\}\mu_t}{\gamma_{\underline{z}}} - \mu_t$$

$$= \frac{\alpha_{\underline{z}} P^{\underline{z}}}{\gamma_{\underline{z}}}\{1 - \frac{[\frac{1}{2}r(2-\beta)\sigma_p^2 + 2]}{1+\frac{\gamma_{\underline{z}} N}{M} + [\frac{\frac{1}{2}r(2-\beta)\sigma_p^2 + 1}{1+\frac{1}{2}r\beta(2-\beta)\sigma_p^2}][1+\frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]}\} + [1+\frac{1}{2}r(2-\beta)\sigma_p^2]\mu_t.$$

Denote the expectation of q_2^j as

(1.50) $$Eq_2^j = \frac{\alpha_{\underline{z}} P^{\underline{z}}}{\gamma_{\underline{z}}}\{1 - \frac{[\frac{1}{2}r(2-\beta)\sigma_p^2 + 2]}{1 + \frac{\gamma_{\underline{z}} N}{M} + [\frac{\frac{1}{2}r(2-\beta)\sigma_p^2 + 1}{1 + \frac{1}{2}r\beta(2-\beta)\sigma_p^2}][1 + \frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]}\}.$$

Similarly, substituting (1.31), (1.44), and (1.47) into (1.37) yields

(1.51)

$$n_1^j = \frac{\alpha_{\underline{z}} P^{\underline{z}}[1 + \frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)] - E_{t-1}p_t^s[1 + \frac{1}{2}r(2-\beta)(\beta-1)\operatorname{var}(p_t^s)]}{\gamma_{\underline{z}}\beta E_{t-1}p_t^s} - \frac{\mu_t}{\beta q_0}$$

$$= \frac{\alpha_{\underline{z}} P^{\underline{z}}[1 + \frac{1}{2}r\beta(2-\beta)\sigma_p^2]}{\gamma_{\underline{z}}\beta E_{t-1}p_t^s} - \frac{[1 + \frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]}{\gamma_{\underline{z}}\beta} - \frac{\mu_t}{\beta q_0}$$

$$= \frac{[1 + \frac{1}{2}r\beta(2-\beta)\sigma_p^2]\{1 + \frac{\gamma_{\underline{z}} N}{M} + [\frac{\frac{1}{2}r(2-\beta)\sigma_p^2 + 1}{1 + \frac{1}{2}r\beta(2-\beta)\sigma_p^2}][1 + \frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]\}}{\gamma_{\underline{z}}\beta[\frac{1}{2}r(2-\beta)\sigma_p^2 + 2]}$$

$$- \frac{[1 + \frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]}{\gamma_{\underline{z}}\beta} - \frac{\mu_t}{\beta q_0},$$

from which we can get the expected number of producers with which each packer signs a contract,

(1.52)

$$En_1^j = \frac{[1+\frac{1}{2}r\beta(2-\beta)\sigma_p^2]\{1+\frac{\gamma_{\underline{z}}N}{M}+[\frac{\frac{1}{2}r(2-\beta)\sigma_p^2+1}{1+\frac{1}{2}r\beta(2-\beta)\sigma_p^2}][1+\frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]\}}{\gamma_{\underline{z}}\beta[\frac{1}{2}r(2-\beta)\sigma_p^2+2]}$$

$$-\frac{[1+\frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]}{\gamma_{\underline{z}}\beta}.$$

Given the conditions (1.31), (1.49), and (1.52), each packer obtains net profit under this fixed price contract:

$$\Pi_2^* = \int_{P\in\Omega}[Pg(Q_{1t}^j\mid\underline{z})-h(Q_{1t}^j\mid\underline{z})-(E_{t-1}p_t^s-\frac{1}{2}rq_0(2-\beta)\operatorname{var}(p_t^s))Q_{1t}^j]dF(P\mid\underline{z})$$
$$+\int_{P\in\Omega}[Pg(q_{2t}^j\mid\underline{z})-h(q_{2t}^j\mid\underline{z})-p_t^s q_{2t}^j]dF(P\mid\underline{z})$$
$$=[a_{\underline{z}}P^{\underline{z}}Q_{1t}^j-\frac{1}{2}\gamma_{\underline{z}}(Q_{1t}^j+\mu_t)^2-(E_{t-1}p_t^s-\frac{1}{2}rq_0(2-\beta)\operatorname{var}(p_t^s))Q_{1t}^j]$$
$$+[a_{\underline{z}}P^{\underline{z}}q_{2t}^j-\frac{1}{2}\gamma_{\underline{z}}(q_{2t}^j+\mu_t)^2-p_t^s q_{2t}^j]$$
$$=[\frac{1}{2}\gamma_{\underline{z}}(Q_{1t}^j)^2-\frac{1}{2}\gamma_{\underline{z}}\mu_t^2]+[\frac{1}{2}\gamma_{\underline{z}}(q_{2t}^j)^2-\frac{1}{2}\gamma_{\underline{z}}\mu_t^2]$$

$$=[\frac{1}{2}\gamma_{\underline{z}}(\beta q_0)^2(\frac{[1+\frac{1}{2}r\beta(2-\beta)\sigma_p^2]\{1+\frac{\gamma_{\underline{z}}N}{M}+[\frac{\frac{1}{2}r(2-\beta)\sigma_p^2+1}{1+\frac{1}{2}r\beta(2-\beta)\sigma_p^2}][1+\frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]\}}{\gamma_{\underline{z}}\beta[\frac{1}{2}r(2-\beta)\sigma_p^2+2]}$$
$$-\frac{[1+\frac{1}{2}r(2-\beta)(\beta-1)\sigma_p^2]}{\gamma_{\underline{z}}\beta}-\frac{\mu_t}{\beta q_0})^2-\frac{1}{2}\gamma_{\underline{z}}\mu_t^2]$$

$$+\frac{1}{2}\gamma_{\underline{z}}[\frac{\alpha_{\underline{z}}P^{z}}{\gamma_{z}}(1-\frac{[\frac{1}{2}r(2-\beta)\sigma_{p}^{2}+2]}{1+\frac{\gamma_{\underline{z}}N}{M}+[\frac{\frac{1}{2}r(2-\beta)\sigma_{p}^{2}+1}{1+\frac{1}{2}r\beta(2-\beta)\sigma_{p}^{2}}][1+\frac{1}{2}r(2-\beta)(\beta-1)\sigma_{p}^{2}]})+(1+\frac{1}{2}r(2-\beta)\sigma_{p}^{2})\mu_{t}]^{2}$$

$$-\frac{1}{2}\gamma_{\underline{z}}\mu_{t}^{2}$$

$$=\frac{1}{2}\gamma_{\underline{z}}(\beta q_{0})^{2}[(En_{1}^{j})^{2}-2En_{1}^{j}\frac{\mu_{t}}{\beta q_{0}}]$$

$$+\frac{1}{2}\gamma_{\underline{z}}[(Eq_{2}^{j})^{2}+(1+\frac{1}{2}r(2-\beta)\sigma_{p}^{2})^{2}\mu_{t}^{2}+2Eq_{2}^{j}(1+\frac{1}{2}r(2-\beta)\sigma_{p}^{2})\mu_{t}]-\frac{1}{2}\gamma_{\underline{z}}\mu_{t}^{2},$$

from which we can computer the expected net profit

(1.53)

$$E\Pi_{2}^{*}=\frac{1}{2}\gamma_{\underline{z}}(\beta q_{0})^{2}(En_{1}^{j})^{2}+\frac{1}{2}\gamma_{\underline{z}}[(Eq_{2}^{j})^{2}+(1+\frac{1}{2}r(2-\beta)\sigma_{p}^{2})^{2}\sigma_{\mu}^{2}]-\frac{1}{2}\gamma_{\underline{z}}\sigma_{\mu}^{2},$$

and the variance of each packer's profit

(1.54) $$Var(\Pi_{2}^{*})=[\gamma_{\underline{z}}Eq_{2}^{j}(1+\frac{1}{2}r(2-\beta)\sigma_{p}^{2})-\gamma_{\underline{z}}\beta q_{0}En_{1}^{j}]^{2}\sigma_{\mu}^{2}.$$

1.4.2 Market-price Contracts

Under this contract, the contract price is set equal to the spot market price. That is, $w(P) = p_t^s$. Recall that, to implement a certain level of quality, the participation constraint (1.3) and the incentive compatibility constraint (1.4) must be satisfied. Specifically, (1.3) $\int_{P\in\Omega} E_{t-1}[u(\beta w(P)q_0 + (1-\beta)p_t^s q_0)]dF(P\mid z) - v(z,q_0) \ge E_{t-1}[u(p_t^s q_0)] - v(\underline{z},q_0)$, and

(1.4) $z \in \arg\max_{\hat{z}} \int_{P\in\Omega} E_{t-1}[u(\beta w(P)q_0 + (1-\beta)p_t^s q_0)]dF(P\mid \hat{z}) - v(\hat{z},\, q_0)\ ,\forall z \in \{\underline{z}, \overline{z}\}$.

Under this market price contract, however,

(1.55) $\int_{P\in\Omega} E_{t-1}[u(\beta p_t^s q_0 + (1-\beta)p_t^s q_0)]dF(P\mid \underline{z}) - v(\underline{z},q_0) \equiv E_{t-1}[u(p_t^s q_0)] - v(\underline{z},q_0)$,

and

(1.56)

$$\int_{P\in\Omega} E_{t-1}[u(\beta p_t^s q_0 + (1-\beta)p_t^s q_0)]dF(P\mid \overline{z}) - v(\overline{z},q_0) = E_{t-1}[u(p_t^s q_0)] - v(\overline{z},q_0)$$

$$< E_{t-1}[u(p_t^s q_0)] - v(\underline{z},q_0) .$$

Condition (1.55) states that under the market-price contract, a producer is indifferent between signing a low-quality contract with a packer and selling to the spot market, while condition (1.56) states that a producer would strictly prefer producing low-quality hogs to producing high-quality hogs. That is, producing high-quality hogs is not incentive compatible under the market-price contract. Combing the conditions (1.55) and (1.56) implies that under the market-price contract it is optimal for a producer to produce low-quality hogs only.

The analysis of this contract is similar to that described in section 1.4.1. Recall that with constant absolute risk aversion, the expected utility $E[u(W)]$ is tantamount to $EW - \frac{1}{2} r \operatorname{var}(W)$. Given the market-price contract $w(P) = p_t^s$ and

$W = \beta p_t^s q_0 + (1-\beta) p_t^s q_0 = p_t^s q_0$, we have $EW = E_{t-1} p_t^s q_0$, and $\text{var}(W) = q_0^2 \text{ var}(p_t^s)$.

Note that offering the market-price contract raises the variance of a producer's revenue relative to the constant contract price in section 1.4.1.

Given this contract, the packer maximizes his net profit:

$$\max_{q_2^j, n_1^j} \Pi = \int_{P\in\Omega} [Pg(Q_{1t}^j \mid \underline{z}) - h(Q_{1t}^j \mid \underline{z}) - p_t^s Q_{1t}^j] dF(P \mid \underline{z})$$
$$+ \int_{P\in\Omega} [Pg(q_{2t}^j \mid \underline{z}) - h(q_{2t}^j \mid \underline{z}) - p_t^s q_{2t}^j] dF(P \mid \underline{z})$$

The first order conditions to this problem are:

$$(1.57)\ \frac{\partial \Pi}{\partial q_{2t}^j} = \int_{P\in\Omega} [P\alpha_{\underline{z}} - \gamma_{\underline{z}}(q_{2t}^j + \mu_t) - p_t^s] dF(P \mid \underline{z}) = 0$$

$$(1.58)\ \frac{\partial \Pi}{\partial n_1^j} = \int_{P\in\Omega} [P\alpha_{\underline{z}} - \gamma_{\underline{z}}(Q_{1t}^j + \mu_t) - p_t^s] dF(P \mid \underline{z}) = 0$$

From condition (1.57), we can derive the quantity of hogs procured by packer j from the spot market,

$$(1.59)\ q_2^j = \frac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s}{\gamma_{\underline{z}}} - \mu_t .$$

From condition (1.58), we can determine the number of producers with which each packer contracts,

$$(1.60)\ n_1^j = \frac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s - \gamma_{\underline{z}} \mu_t}{\gamma_{\underline{z}} \beta q_0} .$$

Similar to the fixed-price contract under risk neutrality, under the market-price contract, each producer produces

(1.61) $q_0 = q_s = E_{t-1} p_t^s$.

The spot market equilibrium price can be derived by equating spot market demand and spot market supply. Specifically, the cash market clearing condition requires

$$(1.62)\ M[\frac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s}{\gamma_{\underline{z}}} - \mu_t] = Nq_0 - M\beta q_0 \frac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s - \gamma_{\underline{z}} \mu_t}{\gamma_{\underline{z}} \beta q_0}.$$

Moving the expected spot market price and spot market price to the right side, (1.62) becomes

$$(1.63)\ \frac{2\alpha_{\underline{z}} P^{\underline{z}}}{\gamma_{\underline{z}}} - 2\mu_t = \frac{N}{M} E_{t-1} p_t^s + \frac{2p_t^s}{\gamma_{\underline{z}}}.$$

Thus, taking expectation E_{t-1} on both sides of (1.63) and using the assumption $E_{t-1}\mu_t = 0$ solves the expected spot market price:

$$(1.64)\ E_{t-1} p_t^s = \frac{\frac{2\alpha_{\underline{z}} P^{\underline{z}}}{\gamma_{\underline{z}}}}{\frac{N}{M} + \frac{2}{\gamma_{\underline{z}}}} = \frac{2\alpha_{\underline{z}} P^{\underline{z}} M}{\gamma_{\underline{z}} N + 2M}.$$

Substituting (1.64) back into (1.63), we can obtain the spot market price

$$(1.65)\ p_t^s = \frac{2\alpha_{\underline{z}} P^{\underline{z}} M}{\gamma_{\underline{z}} N + 2M} - \gamma_{\underline{z}} \mu_t.$$

Hence, the time-independent variance of the spot market price is

$$(1.66)\ \mathrm{var}(p_t^s) = \gamma_{\underline{z}}^2 \sigma_\mu^2.$$

Further, the number of hogs procured by packer *j* from the spot market, q_2^j can be solved by substituting (1.65) into (1.59):

$$(1.67)\ q_{2t}^j = \frac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s}{\gamma_{\underline{z}}} - \mu_t = \frac{\alpha_{\underline{z}} P^{\underline{z}} N}{2M + \gamma_{\underline{z}} N}\ .$$

Substituting (1.65) into (1.60) yields

$$(1.68)\ n_1^j = \frac{\alpha_{\underline{z}} P^{\underline{z}} - p_t^s - \gamma_{\underline{z}} \mu_t}{\gamma_{\underline{z}} \beta q_0} = \frac{N}{2\beta M}.$$

Under this scheme, each packer obtains net profit

$$\Pi_3^* = \int_{P\in\Omega} [Pg(Q_{1t}^j \mid \underline{z}) - h(Q_{1t}^j \mid \underline{z}) - p_t^s Q_{1t}^j] dF(P \mid \underline{z})$$
$$+ \int_{P\in\Omega} [Pg(q_{2t}^j \mid \underline{z}) - h(q_{2t}^j \mid \underline{z}) - p_t^s q_{2t}^j] dF(P \mid \underline{z})$$
$$= [a_{\underline{z}} P^{\underline{z}} Q_{1t}^j - \frac{1}{2}\gamma_{\underline{z}} (Q_{1t}^j + \mu_t)^2 - p_t^s Q_{1t}^j] + [a_{\underline{z}} P^{\underline{z}} q_{2t}^j - \frac{1}{2}\gamma_{\underline{z}} (q_{2t}^j + \mu_t)^2 - p_t^s q_{2t}^j]$$

$$= [\frac{1}{2}\gamma_{\underline{z}} (\beta n_1^j q_0)^2 - \frac{1}{2}\gamma_{\underline{z}} \mu_t^2] + [\frac{1}{2}\gamma_{\underline{z}} (q_{2t}^j)^2 - \frac{1}{2}\gamma_{\underline{z}} \mu_t^2]$$

$$= [\frac{1}{2}\gamma_{\underline{z}} (\beta q_0)^2 (\frac{N}{2\beta M})^2 - \frac{1}{2}\gamma_{\underline{z}} \mu_t^2] + [\frac{1}{2}\gamma_{\underline{z}} (\frac{\alpha_{\underline{z}} P^{\underline{z}} N}{2M + \gamma_{\underline{z}} N})^2 - \frac{1}{2}\gamma_{\underline{z}} \mu_t^2]$$

$$= [\frac{1}{2}\gamma_{\underline{z}} (\frac{\alpha_{\underline{z}} P^{\underline{z}} N}{2M + \gamma_{\underline{z}} N})^2 - \frac{1}{2}\gamma_{\underline{z}} \mu_t^2] + [\frac{1}{2}\gamma_{\underline{z}} (\frac{\alpha_{\underline{z}} P^{\underline{z}} N}{2M + \gamma_{\underline{z}} N})^2 - \frac{1}{2}\gamma_{\underline{z}} \mu_t^2]$$

$$= \gamma_{\underline{z}} (\frac{\alpha_{\underline{z}} P^{\underline{z}} N}{2M + \gamma_{\underline{z}} N})^2 - \gamma_{\underline{z}} \sigma_\mu^2,$$

from which we can compute

(1.69) $E(\Pi_3^*) = \gamma_{\underline{z}}(\frac{\alpha_{\underline{z}} P^{\underline{z}} N}{2M + \gamma_{\underline{z}} N})^2 - \gamma_{\underline{z}} \sigma_{\mu}^2$,

and

(1.70) $\text{var}(\Pi_3^*) = 0$.

Similar to the fixed-price contract under risk neutrality, a packer can eliminate all the risk in the net profit by purchasing fixed amounts of hogs from both the contract market and the spot market. However, compared to the fixed-price contract under risk neutrality, a packer earns strictly less profit due to the uncertainty introduced by setting the contract price equal to the spot market price.

1.4.3 Formula-price Contracts with Quality Premium

From the previous sections, we have shown that a fixed-contract price or a market-price contract can only induce producers to produce low-quality hogs. However, under a formula-price contract with quality premium, packers can induce high-quality hogs from contract-participating producers.

Recall that we assume independent producers provide only low-quality hogs to the cash market, while contracted producers offer their hogs of a certain quality according to the contract. Thus, in this section, we assume that once a producer signs a formula-price contract with price premium, he will produce high-quality hogs only. However, for computational purposes, we assume the average quality of hogs in the cash market will be an arithmetic average of high quality and low quality. Specifically, (1.71) $\tilde{z} = \dfrac{\bar{z} + \underline{z}}{2}$.

The output function of each packer is still a linear function $g(Q_t \mid z) = \alpha_z Q_t$. Additionally, we assume the marginal product of finished hogs acquired from the spot market is

(1.72) $\alpha_{\tilde{z}} = \dfrac{\alpha_{\bar{z}} + \alpha_{\underline{z}}}{2}$

Similarly, the packers' processing cost still takes the form $h(Q_t \mid z) = \dfrac{1}{2}\gamma_z (Q_t + \mu_t)^2$ with $\gamma_{\bar{z}} < \gamma_{\tilde{z}} \le \gamma_{\underline{z}}$, where $\gamma_{\tilde{z}}$ is defined by

(1.73) $\gamma_{\tilde{z}} = \dfrac{\gamma_{\bar{z}} + \gamma_{\underline{z}}}{2}$.

To simplify the analysis further, we assume that the formula-price contract takes a linear form in terms of the market price of the finished product, *P*. More specifically, the contract price is

(1.74) $w(P) = p_t^s + a + bP$

Given these assumptions, each packer maximizes its net profit subject to each producer's participation constraint and incentive compatibility constraint. That is,

$$\max_{a,b,q_2^j,n_1^j} \Pi = \int_{P\in\Omega} [Pg(Q_{1t}^j \mid \bar{z}) - h(Q_{1t}^j \mid \bar{z}) - [p_t^s + a + bP]Q_{1t}^j] dF(P \mid \bar{z})$$
$$+ \int_{P\in\Omega} [Pg(q_{2t}^j \mid \tilde{z}) - h(q_{2t}^j \mid \tilde{z}) - p_t^s q_{2t}^j] dF(P \mid \tilde{z})$$

subject to

(1.75)

$$\int_{P\in\Omega} E_{t-1}[u(\beta q_0(p_t^s + a + bP) + (1-\beta)p_t^s q_0)]dF(P \mid \bar{z}) - v(\bar{z}, q_0) \geq E_{t-1}[u(p_t^s q_0)] - v(\underline{z}, q_0)$$

(1.76) $$\bar{z} \in \arg\max_{\hat{z}} \int_{P\in\Omega} E_{t-1}[u(\beta q_0(p_t^s + a + bP) + (1-\beta)p_t^s q_0)]dF(P \mid \hat{z}) - v(\hat{z},\, q_0) \quad .$$

Before deriving the first-order conditions, the parameters {a, b} in the contract price can be derived as follows. Given the contract price specified in (1.74), conditions (1.75) and (1.76) must be binding because, otherwise, the packer can always reduce the contract price until both of the constraints become equalities. Given each producer's gross revenue, $W = \beta q_0(p_t^s + a + bP) + (1-\beta)p_t^s q_0 = p_t^s q_0 + \beta q_0(a + bP)$, for any P we have $EW = E_{t-1}p_t^s q_0 + \beta q_0(a + bP)$, and $\text{var}(W) = q_0^2 \text{ var}(p_t^s)$.

Thus, the condition (1.75) is equivalent to

(1.77)

$$\int_{P\in\Omega} [E_{t-1}p_t^s q_0 + \beta q_0(a + bP) - \frac{1}{2}rq_0^2 \text{ var}(p_t^s)]dF(P \mid \bar{z}) - v(\bar{z}, q_0)$$
$$= E_{t-1}p_t^s q_0 - \frac{1}{2}rq_0^2 \text{ var}(p_t^s) - v(\underline{z}, q_0),$$

or,

(1.78) $a\beta q_0 + b\beta q_0 E[P \mid \bar{z}] = v(\bar{z}, q_0) - v(\underline{z}, q_0)$.

Similarly, the condition (1.76) becomes

(1.79)

$$\int_{P\in\Omega} [E_{t-1}p_t^s q_0 + \beta q_0(a+bP) - \frac{1}{2}rq_0^2 \operatorname{var}(p_t^s)]dF(P\mid\bar{z}) - v(\bar{z}, q_0)$$
$$= \int_{P\in\Omega} [E_{t-1}p_t^s q_0 + \beta q_0(a+bP) - \frac{1}{2}rq_0^2 \operatorname{var}(p_t^s)]dF(P\mid\underline{z}) - v(\underline{z}, q_0),$$

or,

(1.80) $\beta q_0 bE[P\mid\bar{z}] - \beta q_0 bE[P\mid\underline{z}] = v(\bar{z}, q_0) - v(\underline{z}, q_0).$

Thus, the parameters {a, b} in the contract price can be computed by the conditions (1.78) and (1.80). Precisely,

(1.81) $a = -\frac{[v(\bar{z}, q_0) - v(\underline{z}, q_0)]E[P\mid\underline{z}]}{\beta q_0(E[P\mid\bar{z}] - E[P\mid\underline{z}])} = -\frac{[v(\bar{z}, q_0) - v(\underline{z}, q_0)]P^{\underline{z}}}{\beta q_0(P^{\bar{z}} - P^{\underline{z}})}$, and

(1.82) $b = \frac{v(\bar{z}, q_0) - v(\underline{z}, q_0)}{\beta q_0(E[P\mid\bar{z}] - E[P\mid\underline{z}])} = \frac{v(\bar{z}, q_0) - v(\underline{z}, q_0)}{\beta q_0(P^{\bar{z}} - P^{\underline{z}})}$.

Substituting (1.81) and (1.82) into (1.74) yields the contract price

(1.83)

$$w(P) = p_t^s - \frac{[v(\bar{z}, q_0) - v(\underline{z}, q_0)]P^{\underline{z}}}{\beta q_0(P^{\bar{z}} - P^{\underline{z}})} + \frac{v(\bar{z}, q_0) - v(\underline{z}, q_0)}{\beta q_0(P^{\bar{z}} - P^{\underline{z}})}P$$
$$= p_t^s + \frac{[v(\bar{z}, q_0) - v(\underline{z}, q_0)]}{\beta q_0(P^{\bar{z}} - P^{\underline{z}})}(P - P^{\underline{z}}).$$

Note that the optimal contract price under the formula-price contract contains two parts: one part is the spot market price, which is used as a base price in the formula-price contract; the other part is the quality premium, which is positively related to the difference between the realized price of finished products and the expected price of low-quality finished products.

Furthermore, given $v(z, q_0) = c_z q_0^2$, the contract price can be written as

(1.84)

$$w(P) = p_t^s + \frac{c_{\bar{z}} q_0^2 - c_{\underline{z}} q_0^2}{\beta q_0 (P^{\bar{z}} - P^{\underline{z}})}(P - P^{\underline{z}}) = p_t^s + \frac{(c_{\bar{z}} - c_{\underline{z}}) q_0}{\beta (P^{\bar{z}} - P^{\underline{z}})}(P - P^{\underline{z}}).$$

Moreover, given the contract price (1.84) and producers' short-run supply function, each contract producer produces

(1.85)

$$q_0 = \beta E[w(P) \mid \bar{z}] + (1-\beta) E_{t-1} p_t^s = \beta E[p_t^s + \frac{(c_{\bar{z}} - c_{\underline{z}}) q_0}{\beta (P^{\bar{z}} - P^{\underline{z}})}(P - P^{\underline{z}}) \mid \bar{z}] + (1-\beta) E_{t-1} p_t^s$$
$$= E_{t-1} p_t^s + (c_{\bar{z}} - c_{\underline{z}}) q_0.$$

Hence,

(1.86) $$q_0 = \frac{E_{t-1} p_t^s}{1 - (c_{\bar{z}} - c_{\underline{z}})}.$$

Again, independent producers choose to produce

(1.87) $$q_s = E_{t-1} p_t^s.$$

Now, the first-order optimality conditions to this problem are ready to be derived.

First, the optimal quantity of hogs demanded from the spot market, q_{2t}^j, must satisfy

(1.88) $$\frac{\partial \Pi}{\partial q_{2t}^j} = \int_{P \in \Omega} [P \alpha_{\tilde{z}} - \gamma_{\tilde{z}} (q_{2t}^j + \mu_t) - p_t^s] dF(P \mid \tilde{z}) = 0 ,$$

from which

(1.89) $$q_{2t}^j = \frac{\alpha_{\tilde{z}} P^{\tilde{z}} - p_t^s}{\gamma_{\tilde{z}}} - \mu_t .$$

Second, the number of producers that each packer contracts with, n_1^j, must satisfy

(1.90)) $\frac{\partial L}{\partial n_1^j} = \int\limits_{P\in\Omega} [P\alpha_{\bar{z}}\beta q_0 - \gamma_{\bar{z}}\beta q_0(Q_{1t}^j + \mu_t) - \beta q_0(p_t^s + a + bP)]dF(P \mid \bar{z}) = 0,$

from which we can obtain

(1.91) $n_1^j = \frac{\alpha_{\bar{z}} P^{\bar{z}} - p_t^s - a - bE[P \mid \bar{z}] - \gamma_{\bar{z}}\mu_t}{\gamma_{\bar{z}}\beta q_0}$.

Given conditions (1.86), (1.87), (1.89), and (1.91), the spot market price can be obtained by setting market demand equal to market supply in the spot market. That is,

(1.92)

$$Q_{2s} = Q_{2d} \Rightarrow (N - Mn_1^j)q_s + M(1-\beta)q_0 n_1^j (E_{t-1}p_t^s \mid \bar{z})$$
$$= Nq_s - Mn_1^j(E_{t-1}p_t^s \mid \bar{z})[q_s - (1-\beta)q_0] = Mq_{2t}^j(p_t^s \mid \tilde{z}),$$

or, precisely,

(1.93)

$$NE_{t-1}p_t^s - M\frac{\alpha_{\bar{z}}P^{\bar{z}} - p_t^s - \frac{(c_{\bar{z}} - c_{\underline{z}})}{\beta}\frac{E_{t-1}p_t^s}{1-(c_{\bar{z}} - c_{\underline{z}})} - \gamma_{\bar{z}}\mu_t}{\gamma_{\bar{z}}\beta\frac{E_{t-1}p_t^s}{1-(c_{\bar{z}} - c_{\underline{z}})}}[E_{t-1}p_t^s - (1-\beta)\frac{E_{t-1}p_t^s}{1-(c_{\bar{z}} - c_{\underline{z}})}]$$
$$= M[\frac{\alpha_{\tilde{z}}P^{\tilde{z}} - p_t^s}{\gamma_{\tilde{z}}} - \mu_t].$$

Moving the expected spot market price and the spot market price to the left side, (1.93) becomes

(1.94)

$$\frac{N}{M}E_{t-1}p_t^s + [\frac{p_t^s}{\gamma_{\bar{z}}\beta} + \frac{(c_{\bar{z}}-c_{\underline{z}})E_{t-1}p_t^s}{\gamma_{\bar{z}}\beta^2[1-(c_{\bar{z}}-c_{\underline{z}})]}][\beta-(c_{\bar{z}}-c_{\underline{z}})] + \frac{p_t^s}{\gamma_{\tilde{z}}}$$

$$= \frac{\alpha_{\tilde{z}}P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \frac{\alpha_{\bar{z}}P^{\bar{z}}[\beta-(c_{\bar{z}}-c_{\underline{z}})]}{\gamma_{\bar{z}}\beta} - \mu_t - \frac{[\beta-(c_{\bar{z}}-c_{\underline{z}})]}{\beta}\mu_t .$$

Taking the expectation operator E_{t-1} on both sides of (1.94) and applying the assumption $E_{t-1}\mu_t = 0$, we can get the expected spot market price,

(1.95)

$$E_{t-1}p_t^s = \frac{\dfrac{\alpha_{\tilde{z}}P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \dfrac{\alpha_{\bar{z}}P^{\bar{z}}[\beta-(c_{\bar{z}}-c_{\underline{z}})]}{\gamma_{\bar{z}}\beta}}{\dfrac{N}{M} + \dfrac{1}{\gamma_{\tilde{z}}} + [\dfrac{1}{\gamma_{\bar{z}}\beta} + \dfrac{(c_{\bar{z}}-c_{\underline{z}})}{\gamma_{\bar{z}}\beta^2[1-(c_{\bar{z}}-c_{\underline{z}})]}][\beta-(c_{\bar{z}}-c_{\underline{z}})]}$$

$$= \frac{\dfrac{\alpha_{\tilde{z}}P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \dfrac{\alpha_{\bar{z}}P^{\bar{z}}[\beta-(c_{\bar{z}}-c_{\underline{z}})]}{\gamma_{\bar{z}}\beta}}{\dfrac{N}{M} + \dfrac{1}{\gamma_{\tilde{z}}} + [\dfrac{1}{\gamma_{\bar{z}}} - \dfrac{(1-\beta)(c_{\bar{z}}-c_{\underline{z}})^2}{\gamma_{\bar{z}}\beta^2[1-(c_{\bar{z}}-c_{\underline{z}})]}]} .$$

Substituting (1.95) back into (1.94) solves the spot market price,

(1.96)

$$p_t^s = \frac{\dfrac{\alpha_{\tilde{z}}P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \dfrac{\alpha_{\bar{z}}P^{\bar{z}}[\beta-(c_{\bar{z}}-c_{\underline{z}})]}{\gamma_{\bar{z}}\beta}}{\dfrac{N}{M} + \dfrac{1}{\gamma_{\tilde{z}}} + [\dfrac{1}{\gamma_{\bar{z}}} - \dfrac{(1-\beta)(c_{\bar{z}}-c_{\underline{z}})^2}{\gamma_{\bar{z}}\beta^2[1-(c_{\bar{z}}-c_{\underline{z}})]}]} - \frac{1+\dfrac{\beta-(c_{\bar{z}}-c_{\underline{z}})}{\beta}}{\dfrac{\beta-(c_{\bar{z}}-c_{\underline{z}})}{\gamma_{\bar{z}}\beta} + \dfrac{1}{\gamma_{\tilde{z}}}}\mu_t$$

$$= \frac{\dfrac{\alpha_{\tilde{z}}P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \dfrac{\alpha_{\bar{z}}P^{\bar{z}}[\beta-(c_{\bar{z}}-c_{\underline{z}})]}{\gamma_{\bar{z}}\beta}}{\dfrac{N}{M} + \dfrac{1}{\gamma_{\tilde{z}}} + [\dfrac{1}{\gamma_{\bar{z}}} - \dfrac{(1-\beta)(c_{\bar{z}}-c_{\underline{z}})^2}{\gamma_{\bar{z}}\beta^2[1-(c_{\bar{z}}-c_{\underline{z}})]}]} - \frac{\gamma_{\bar{z}}\gamma_{\tilde{z}}[2\beta-(c_{\bar{z}}-c_{\underline{z}})]}{\beta(\gamma_{\tilde{z}}+\gamma_{\bar{z}})-\gamma_{\tilde{z}}(c_{\bar{z}}-c_{\underline{z}})}\mu_t .$$

Hence, the variance of the spot market price can be computed as

(1.97) $$\operatorname{var}(p_t^s) = \{\frac{\gamma_{\bar{z}}\gamma_{\tilde{z}}[2\beta - (c_{\bar{z}} - c_{\underline{z}})]}{\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})}\}^2 \sigma_\mu^2 .$$

Substituting (1.96) into (1.89) yields the quantity of hogs demanded from the spot market by each packer,

(1.98)

$$q_{2t}^j = \frac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} - \frac{1}{\gamma_{\tilde{z}}}[\frac{\frac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \frac{\alpha_{\bar{z}} P^{\bar{z}}[\beta - (c_{\bar{z}} - c_{\underline{z}})]}{\gamma_{\bar{z}}\beta}}{\frac{N}{M} + \frac{1}{\gamma_{\tilde{z}}} + [\frac{1}{\gamma_{\bar{z}}} - \frac{(1-\beta)(c_{\bar{z}} - c_{\underline{z}})^2}{\gamma_{\bar{z}}\beta^2[1 - (c_{\bar{z}} - c_{\underline{z}})]}]} - \frac{\gamma_{\bar{z}}\gamma_{\tilde{z}}[2\beta - (c_{\bar{z}} - c_{\underline{z}})]}{\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})}\mu_t] - \mu_t$$

$$= \frac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} - \frac{1}{\gamma_{\tilde{z}}} \frac{\frac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \frac{\alpha_{\bar{z}} P^{\bar{z}}[\beta - (c_{\bar{z}} - c_{\underline{z}})]}{\gamma_{\bar{z}}\beta}}{\frac{N}{M} + \frac{1}{\gamma_{\tilde{z}}} + [\frac{1}{\gamma_{\bar{z}}} - \frac{(1-\beta)(c_{\bar{z}} - c_{\underline{z}})^2}{\gamma_{\bar{z}}\beta^2[1 - (c_{\bar{z}} - c_{\underline{z}})]}]} + \frac{(\gamma_{\tilde{z}} - \gamma_{\bar{z}})(c_{\bar{z}} - c_{\underline{z}} - \beta)}{\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})}\mu_t .$$

Similarly, substituting (1.81), (1.82), (1.86), and (1.96) into (1.91) yields the number of producers with which each packer signs a contract,

(1.99) $$n_1^j = \frac{\alpha_{\bar{z}} P^{\bar{z}} - p_t^s - a - bE[P \mid \bar{z}] - \gamma_{\bar{z}}\mu_t}{\gamma_{\bar{z}}\beta q_0} = \frac{\alpha_{\bar{z}} P^{\bar{z}} - p_t^s - \frac{(c_{\bar{z}} - c_{\underline{z}})q_0}{\beta} - \gamma_{\bar{z}}\mu_t}{\gamma_{\bar{z}}\beta q_0}$$

$$= \frac{\alpha_{\bar{z}} P^{\bar{z}} - E_{t-1}p_t^s + \frac{\gamma_{\bar{z}}\gamma_{\tilde{z}}[2\beta - (c_{\bar{z}} - c_{\underline{z}})]}{\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})}\mu_t - \frac{(c_{\bar{z}} - c_{\underline{z}})\frac{E_{t-1}p_t^s}{1 - (c_{\bar{z}} - c_{\underline{z}})}}{\beta} - \gamma_{\bar{z}}\mu_t}{\gamma_{\bar{z}}\beta \frac{E_{t-1}p_t^s}{1 - (c_{\bar{z}} - c_{\underline{z}})}}$$

$$= \frac{[1-(c_{\bar z}-c_{\underline z})](\alpha_{\bar z}P^{\bar z}-E_{t-1}p_t^s)\beta-(c_{\bar z}-c_{\underline z})E_{t-1}p_t^s}{\gamma_{\bar z}\beta^2 E_{t-1}p_t^s}+\frac{(\gamma_{\tilde z}-\gamma_{\bar z})[1-(c_{\bar z}-c_{\underline z})]}{[\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\tilde z}(c_{\bar z}-c_{\underline z})]E_{t-1}p_t^s}\mu_t$$

$$= \frac{\beta[1-(c_{\bar z}-c_{\underline z})]\alpha_{\bar z}P^{\bar z}}{\gamma_{\bar z}\beta^2 E_{t-1}p_t^s}-\frac{\beta+(1-\beta)(c_{\bar z}-c_{\underline z})}{\gamma_{\bar z}\beta^2}+\frac{(\gamma_{\tilde z}-\gamma_{\bar z})[1-(c_{\bar z}-c_{\underline z})]}{[\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\tilde z}(c_{\bar z}-c_{\underline z})]E_{t-1}p_t^s}\mu_t\,.$$

Further, we can compute each packer's profit under the formula-price contract,

$$\Pi_4^* = \int_{P\in\Omega}[Pg(Q_{1t}^j \mid \bar z)-h(Q_{1t}^j \mid \bar z)-(p_t^s+a+bP)Q_{1t}^j]dF(P\mid \bar z)$$

$$+\int_{P\in\Omega}[Pg(q_{2t}^j \mid \tilde z)-h(q_{2t}^j \mid \tilde z)-p_t^s q_{2t}^j]dF(P\mid \tilde z)$$

$$=[\frac{1}{2}\gamma_{\bar z}(\beta n_1^j q_0)^2-\frac{1}{2}\gamma_{\bar z}\mu_t^2]+[\frac{1}{2}\gamma_{\tilde z}(q_{2t}^j)^2-\frac{1}{2}\gamma_{\tilde z}\mu_t^2]$$

$$=\frac{1}{2}\gamma_{\bar z}(\beta q_0)^2[\frac{\beta[1-(c_{\bar z}-c_{\underline z})]\alpha_{\bar z}P^{\bar z}}{\gamma_{\bar z}\beta^2 E_{t-1}p_t^s}-\frac{\beta+(1-\beta)(c_{\bar z}-c_{\underline z})}{\gamma_{\bar z}\beta^2}+\frac{(\gamma_{\tilde z}-\gamma_{\bar z})[1-(c_{\bar z}-c_{\underline z})]}{[\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\tilde z}(c_{\bar z}-c_{\underline z})]E_{t-1}p_t^s}\mu_t]^2$$

$$-\frac{1}{2}\gamma_{\bar z}\mu_t^2+\frac{1}{2}\gamma_{\tilde z}[\frac{\alpha_{\tilde z}P^{\tilde z}}{\gamma_{\tilde z}}-\frac{1}{\gamma_{\tilde z}}E_{t-1}p_t^s+\frac{(\gamma_{\tilde z}-\gamma_{\bar z})(c_{\bar z}-c_{\underline z}-\beta)}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\tilde z}(c_{\bar z}-c_{\underline z})}\mu_t]^2-\frac{1}{2}\gamma_{\tilde z}\mu_t^2$$

$$=\frac{1}{2}\gamma_{\bar z}(\beta q_0)^2\{[\frac{\beta[1-(c_{\bar z}-c_{\underline z})]\alpha_{\bar z}P^{\bar z}}{\gamma_{\bar z}\beta^2 E_{t-1}p_t^s}-\frac{\beta+(1-\beta)(c_{\bar z}-c_{\underline z})}{\gamma_{\bar z}\beta^2}]^2+[\frac{(\gamma_{\tilde z}-\gamma_{\bar z})[1-(c_{\bar z}-c_{\underline z})]\mu_t}{[\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\tilde z}(c_{\bar z}-c_{\underline z})]E_{t-1}p_t^s}]^2$$

$$+2[\frac{\beta[1-(c_{\bar z}-c_{\underline z})]\alpha_{\bar z}P^{\bar z}}{\gamma_{\bar z}\beta^2 E_{t-1}p_t^s}-\frac{\beta+(1-\beta)(c_{\bar z}-c_{\underline z})}{\gamma_{\bar z}\beta^2}][\frac{(\gamma_{\tilde z}-\gamma_{\bar z})[1-(c_{\bar z}-c_{\underline z})]\mu_t}{[\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\tilde z}(c_{\bar z}-c_{\underline z})]E_{t-1}p_t^s}]\}$$

$$-\frac{1}{2}\gamma_{\bar z}\mu_t^2+\frac{1}{2}\gamma_{\tilde z}\{[\frac{\alpha_{\tilde z}P^{\tilde z}}{\gamma_{\tilde z}}-\frac{1}{\gamma_{\tilde z}}E_{t-1}p_t^s]^2+[\frac{(\gamma_{\tilde z}-\gamma_{\bar z})(c_{\bar z}-c_{\underline z}-\beta)}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\tilde z}(c_{\bar z}-c_{\underline z})}\mu_t]^2$$

$$+2[\frac{\alpha_{\tilde z}P^{\tilde z}}{\gamma_{\tilde z}}-\frac{1}{\gamma_{\tilde z}}E_{t-1}p_t^s][\frac{(\gamma_{\tilde z}-\gamma_{\bar z})(c_{\bar z}-c_{\underline z}-\beta)}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\tilde z}(c_{\bar z}-c_{\underline z})}\mu_t]\}-\frac{1}{2}\gamma_{\tilde z}\mu_t^2.$$

Hence, the expected profit under the formula price contract is

(1.100)

$$E(\Pi_4^*) = \frac{1}{2}\gamma_{\bar z}(\beta q_0)^2\{[\frac{\beta[1-(c_{\bar z}-c_{\underline z})]\alpha_{\bar z}P^{\bar z}}{\gamma_{\bar z}\beta^2 E_{t-1}p_t^s} - \frac{\beta+(1-\beta)(c_{\bar z}-c_{\underline z})}{\gamma_{\bar z}\beta^2}]^2$$
$$+[\frac{(\gamma_{\tilde z}-\gamma_{\bar z})[1-(c_{\bar z}-c_{\underline z})]}{[\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})]E_{t-1}p_t^s}]^2\sigma_\mu^2\}$$
$$-\frac{1}{2}\gamma_{\bar z}\sigma_\mu^2 + \frac{1}{2}\gamma_{\tilde z}\{[\frac{\alpha_{\tilde z}P^{\tilde z}}{\gamma_{\tilde z}} - \frac{1}{\gamma_{\tilde z}}E_{t-1}p_t^s]^2 + [\frac{(\gamma_{\tilde z}-\gamma_{\bar z})(c_{\bar z}-c_{\underline z}-\beta)}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})}]^2\sigma_\mu^2\} - \frac{1}{2}\gamma_{\tilde z}\sigma_\mu^2.$$

and variance of each processor's profit is

(1.101)

$$Var(\Pi_4^*) = \{\gamma_{\bar z}(\beta q_0)^2[\frac{\beta[1-(c_{\bar z}-c_{\underline z})]\alpha_{\bar z}P^{\bar z}}{\gamma_{\bar z}\beta^2 E_{t-1}p_t^s} - \frac{\beta+(1-\beta)(c_{\bar z}-c_{\underline z})}{\gamma_{\bar z}\beta^2}][\frac{(\gamma_{\tilde z}-\gamma_{\bar z})[1-(c_{\bar z}-c_{\underline z})]}{[\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})]E_{t-1}p_t^s}]$$
$$+\gamma_{\tilde z}[\frac{\alpha_{\tilde z}P^{\tilde z}}{\gamma_{\tilde z}} - \frac{1}{\gamma_{\tilde z}}E_{t-1}p_t^s][\frac{(\gamma_{\tilde z}-\gamma_{\bar z})(c_{\bar z}-c_{\underline z}-\beta)}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})}]\}^2\sigma_\mu^2$$

$$=\{[\frac{E_{t-1}p_t^s}{1-(c_{\bar z}-c_{\underline z})}]^2[\frac{\beta[1-(c_{\bar z}-c_{\underline z})]\alpha_{\bar z}P^{\bar z}}{E_{t-1}p_t^s} - \beta-(1-\beta)(c_{\bar z}-c_{\underline z})][\frac{(\gamma_{\tilde z}-\gamma_{\bar z})[1-(c_{\bar z}-c_{\underline z})]}{[\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})]E_{t-1}p_t^s}]$$
$$+\gamma_{\tilde z}[\frac{\alpha_{\tilde z}P^{\tilde z}}{\gamma_{\tilde z}} - \frac{1}{\gamma_{\tilde z}}E_{t-1}p_t^s][\frac{(\gamma_{\tilde z}-\gamma_{\bar z})(c_{\bar z}-c_{\underline z}-\beta)}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})}]\}^2\sigma_\mu^2$$

$$=\{\frac{\beta\alpha_{\bar z}P^{\bar z}(\gamma_{\tilde z}-\gamma_{\bar z})}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})} - \frac{E_{t-1}p_t^s}{1-(c_{\bar z}-c_{\underline z})}\frac{(\gamma_{\tilde z}-\gamma_{\bar z})[\beta+(1-\beta)(c_{\bar z}-c_{\underline z})]}{[\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})]}$$
$$+\alpha_{\tilde z}P^{\tilde z}[\frac{(\gamma_{\tilde z}-\gamma_{\bar z})(c_{\bar z}-c_{\underline z}-\beta)}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})}] - E_{t-1}p_t^s[\frac{(\gamma_{\tilde z}-\gamma_{\bar z})(c_{\bar z}-c_{\underline z}-\beta)}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})}]\}^2\sigma_\mu^2$$

$$=\{\frac{\beta\alpha_{\bar z}P^{\bar z}(\gamma_{\tilde z}-\gamma_{\bar z})}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})} + \alpha_{\tilde z}P^{\tilde z}[\frac{(\gamma_{\tilde z}-\gamma_{\bar z})(c_{\bar z}-c_{\underline z}-\beta)}{\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})}]$$
$$-\frac{E_{t-1}p_t^s}{1-(c_{\bar z}-c_{\underline z})}\frac{(\gamma_{\tilde z}-\gamma_{\bar z})[2(c_{\bar z}-c_{\underline z})-(c_{\bar z}-c_{\underline z})^2]}{[\beta(\gamma_{\tilde z}+\gamma_{\bar z})-\gamma_{\bar z}(c_{\bar z}-c_{\underline z})]}\}^2\sigma_\mu^2.$$

1.4.4 Cost-plus Contracts with Quality Premium

A cost-plus contract consists of the feed costs and some quality premium or discount. These contracts may also have a balancing clause where payments are made to contractors/processors when market prices are below the contract prices and vice versa. However, the balancing clause is ignored in the following analysis. In this section, the initial assumptions are that a processor wishes to implement high quality and the cost-plus contract takes a linear form:

(1.102) $w(P) = c_{\tilde{z}} + a + bP$,

where it is assumed that

(1.103) $c_{\tilde{z}} = \dfrac{c_{\underline{z}} + c_{\bar{z}}}{2}$,

represents the market average feed cost per unit of hog.

Thus, each packer solves the following problem:

$$\max_{a,b,q_2^j,n_1^j} \Pi = \int_{P\in\Omega} [Pg(Q_{1t}^j \mid \bar{z}) - h(Q_{1t}^j \mid \bar{z}) - [c_{\tilde{z}} + a + bP]Q_{1t}^j]dF(P \mid \bar{z})$$
$$+ \int_{P\in\Omega} [Pg(q_{2t}^j \mid \tilde{z}) - h(q_{2t}^j \mid \tilde{z}) - p_t^s q_{2t}^j]dF(P \mid \tilde{z})$$

subject to

(1.104)

$$\int_{P\in\Omega} E_{t-1}[u(\beta q_0(c_{\tilde{z}} + a + bP) + (1-\beta)p_t^s q_0)]dF(P \mid \bar{z}) - v(\bar{z}, q_0) \geq E_{t-1}[u(p_t^s q_0)] - v(\underline{z}, q_0) ,$$

and

(1.105) $\bar{z} \in \arg\max_{\hat{z}} \int_{P\in\Omega} E_{t-1}[u(\beta q_0(c_{\tilde{z}} + a + bP) + (1-\beta)p_t^s q_0)]dF(P \mid \hat{z}) - v(\hat{z}, q_0)$.

Similar to the formula-price contract, the constraints (1.104) and (1.105) can be used to derive the parameters {a, b} in the contract price. Again, given the price structure specified in (1.102), conditions (1.104) and (1.105) must be binding because, otherwise, the processor could always reduce the contract price until both become equalities. Given the utility function with constant absolute risk aversion and

$W = \beta q_0(c_{\tilde{z}} + a + bP) + (1-\beta) p_t^s q_0$, for any P we have

$EW = \beta q_0(c_{\tilde{z}} + a + bP) + (1-\beta) E_{t-1} p_t^s q_0$, and $\text{var}(W) = (1-\beta)^2 q_0^2 \text{ var}(p_t^s)$.

Thus, the condition (1.104) is equivalent to

(1.106)

$$\int_{P\in\Omega} [\beta q_0(c_{\tilde{z}} + a + bP) + (1-\beta) E_{t-1} p_t^s q_0 - \frac{1}{2} r(1-\beta)^2 q_0^2 \text{ var}(p_t^s)] dF(P \mid \bar{z}) - v(\bar{z}, q_0)$$
$$= E_{t-1} p_t^s q_0 - \frac{1}{2} r q_0^2 \text{ var}(p_t^s) - v(\underline{z}, q_0)$$

or,

(1.107) $$(c_{\tilde{z}} + a + bP^{\bar{z}}) = E_{t-1} p_t^s - \frac{1}{2} r(2-\beta) q_0 \text{ var}(p_t^s) + \frac{v(\bar{z}, q_0) - v(\underline{z}, q_0)}{\beta q_0}.$$

Similarly, the condition (1.105) becomes

(1.108)

$$\int_{P\in\Omega} [\beta q_0(c_{\tilde{z}} + a + bP) + (1-\beta) E_{t-1} p_t^s q_0 - \frac{1}{2} r(1-\beta)^2 q_0^2 \text{ var}(p_t^s)] dF(P \mid \bar{z}) - v(\bar{z}, q_0)$$

$$= \int_{P\in\Omega} [\beta q_0(c_{\tilde{z}} + a + bP) + (1-\beta) E_{t-1} p_t^s q_0 - \frac{1}{2} r(1-\beta)^2 q_0^2 \text{ var}(p_t^s)] dF(P \mid \underline{z}) - v(\underline{z}, q_0),$$

from which we can obtain

(1.109) $b\beta q_0[P^{\bar{z}} - P^{\underline{z}}] = v(\bar{z},\ q_0) - v(\underline{z},\ q_0)$.

Thus, the parameters {a, b} in the contract price can be solved as

(1.110) $a = E_{t-1}p_t^s - \frac{1}{2}r(2-\beta)q_0 \text{ var}(p_t^s) - c_{\tilde{z}} - \frac{[v(\bar{z},q_0) - v(\underline{z},q_0)]P^{\underline{z}}}{\beta q_0[P^{\bar{z}} - P^{\underline{z}}]}$, and

(1.111) $b = \frac{v(\bar{z},\ q_0) - v(\underline{z},\ q_0)}{\beta q_0(P^{\bar{z}} - P^{\underline{z}})}$.

Substituting (1.110) and (1.111) into (1.102) yields the contract price

(1.112)

$$w(P) = E_{t-1}p_t^s - \frac{1}{2}r(2-\beta)q_0 \text{ var}(p_t^s) - \frac{[v(\bar{z},q_0) - v(\underline{z},q_0)]P^{\underline{z}}}{\beta q_0[P^{\bar{z}} - P^{\underline{z}}]} + \frac{v(\bar{z},\ q_0) - v(\underline{z},\ q_0)}{\beta q_0(P^{\bar{z}} - P^{\underline{z}})}P$$

$$= E_{t-1}p_t^s - \frac{1}{2}r(2-\beta)q_0 \text{ var}(p_t^s) + \frac{[v(\bar{z},q_0) - v(\underline{z},q_0)]}{\beta q_0[P^{\bar{z}} - P^{\underline{z}}]}(P - P^{\underline{z}})\,.$$

Similar to the formula-price contract, the cost-plus contract price contains a base payment, which is constant under this contract, and a price premium that is positively related to the difference between the observed signal and the expected lowest possible signal. Note that the market average feed cost, $c_{\tilde{z}}$, is always cancelled out in the contract price if it is constant. Interestingly, the base payment under this contract takes the exactly same form as that under the fixed-price contract with risk aversion.

Given the contract price (1.112) and producers' short-run supply function, each contract producer produces

(1.113)

$$q_0 = \beta E[w(P) \mid \bar{z}] + (1-\beta)E_{t-1}p_t^s$$
$$= \beta E[E_{t-1}p_t^s - \frac{1}{2}r(2-\beta)q_0 \operatorname{var}(p_t^s) + \frac{[v(\bar{z}, q_0) - v(\underline{z}, q_0)]}{\beta q_0[P^{\bar{z}} - P^{\underline{z}}]}(P - P^{\underline{z}}) \mid \bar{z}] + (1-\beta)E_{t-1}p_t^s$$
$$= E_{t-1}p_t^s - \frac{1}{2}r\beta(2-\beta)q_0 \operatorname{var}(p_t^s) + (c_{\bar{z}} - c_{\underline{z}})q_0 .$$

Hence,

(1.114) $$q_0 = \frac{E_{t-1}p_t^s}{1 - (c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)} .$$

Again, independent producers choose to produce

(1.115) $q_s = E_{t-1}p_t^s$.

Turning to the first-order conditions to this problem, the optimal quantity of hogs demanded from the spot market, q_{2t}^j, must satisfy

(1.116) $$\frac{\partial L}{\partial q_{2t}^j} = \int_{P\in\Omega} [P\alpha_{\tilde{z}} - \gamma_{\underline{z}}(q_{2t}^j + \mu_t) - p_t^s]dF(P \mid \tilde{z}) = 0 ,$$

from which

(1.117) $$q_{2t}^j = \frac{\alpha_{\tilde{z}}P^{\tilde{z}} - p_t^s}{\gamma_{\tilde{z}}} - \mu_t .$$

The number of producers that each packer contracts with, n_1^j, must satisfy

(1.118) $$\frac{\partial L}{\partial n_1^j} = \int_{P\in\Omega}[P\alpha_{\tilde{z}}\beta q_0 - \gamma_{\tilde{z}}\beta q_0(Q_{1t}^j + \mu_t) - \beta q_0(c_{\tilde{z}} + a + bP)]dF(P \mid \bar{z}) = 0,$$

from which

(1.119) $n_1^j = \dfrac{\alpha_{\bar{z}} P^{\bar{z}} - c_{\tilde{z}} - a - bP^{\bar{z}} - \gamma_{\bar{z}}\mu_t}{\gamma_{\bar{z}}\beta q_0}$.

Substituting the contract price (1.112) into (1.119) yields

$$(1.120)\ n_1^j = \frac{\alpha_{\bar{z}} P^{\bar{z}} - E_{t-1} p_t^s + \frac{1}{2} r(2-\beta) q_0 \operatorname{var}(p_t^s) - \frac{(c_{\bar{z}} - c_{\underline{z}}) q_0}{\beta} - \gamma_{\bar{z}}\mu_t}{\gamma_{\bar{z}}\beta q_0}.$$

Given the conditions (1.114), (1.115), (1.117), and (1.120), the spot market clearance requires that

(1.121)

$$Q_{2s} = Q_{2d} \Rightarrow Nq_s - Mn_1^j (E_{t-1} p_t^s \mid \bar{z})[q_s - (1-\beta)q_0] = Mq_{2t}^j (p_t^s \mid \tilde{z})$$

Substituting (1.114), (1.115), (1.117), and (1.120) into (1.121) and simplifying yields

(1.122)

$$\frac{N}{M} E_{t-1} p_t^s - \frac{\alpha_{\bar{z}} P^{\bar{z}} - E_{t-1} p_t^s + \dfrac{\frac{1}{2} r\beta(2-\beta) \operatorname{var}(p_t^s) E_{t-1} p_t^s - (c_{\bar{z}} - c_{\underline{z}}) E_{t-1} p_t^s}{\beta[1 - (c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2} r\beta(2-\beta)\operatorname{var}(p_t^s)]}] - \gamma_{\bar{z}}\mu_t}{\dfrac{\gamma_{\bar{z}}\beta E_{t-1} p_t^s}{1 - (c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2} r\beta(2-\beta)\operatorname{var}(p_t^s)}}$$

$$\times E_{t-1} p_t^s [\frac{\beta - (c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2} r\beta(2-\beta)\operatorname{var}(p_t^s)}{1 - (c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2} r\beta(2-\beta)\operatorname{var}(p_t^s)}] \quad = \quad \frac{\alpha_{\tilde{z}} P^{\tilde{z}} - p_t^s}{\gamma_{\tilde{z}}} - \mu_t .$$

Further, (1.122) can be written as

(1.123)

$$-\frac{[\alpha_{\bar{z}}P^{\bar{z}} - E_{t-1}p_t^s + \dfrac{[\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s) - (c_{\bar{z}} - c_{\underline{z}})]E_{t-1}p_t^s}{\beta[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)]} - \gamma_{\bar{z}}\mu_t][\beta - (c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)]}{\gamma_{\bar{z}}\beta}$$

$$+\frac{N}{M}E_{t-1}p_t^s \quad = \quad \frac{\alpha_{\tilde{z}}P^{\tilde{z}} - p_t^s}{\gamma_{\tilde{z}}} - \mu_t$$

Moving the expected spot market price and the spot market price to the left side, (1.123) becomes

(1.124)

$$\frac{E_{t-1}p_t^s[1 - \dfrac{[\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s) - (c_{\bar{z}} - c_{\underline{z}})]}{\beta[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)]}][\beta - (c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)]}{\gamma_{\bar{z}}\beta} + \frac{p_t^s}{\gamma_{\tilde{z}}} + \frac{N}{M}E_{t-1}p_t^s$$

$$= \frac{\alpha_{\tilde{z}}P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \frac{\alpha_{\bar{z}}P^{\bar{z}}[\beta - (c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)]}{\gamma_{\bar{z}}\beta} - [2 - \frac{(c_{\bar{z}} - c_{\underline{z}})}{\beta} + \frac{1}{2}r(2-\beta)\operatorname{var}(p_t^s)]\mu_t$$

Applying the expectation operator E_{t-1} on both sides of (1.124) and using the assumption, $E_{t-1}\mu_t = 0$, the expected spot market price is computed as

(1.125)

$$E_{t-1}p_t^s = \frac{\dfrac{\alpha_{\tilde{z}}P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \dfrac{\alpha_{\bar{z}}P^{\bar{z}}}{\gamma_{\bar{z}}}[1 - \dfrac{(c_{\bar{z}} - c_{\underline{z}})}{\beta} + \dfrac{1}{2}r(2-\beta)\operatorname{var}(p_t^s)]}{\dfrac{1}{\gamma_{\tilde{z}}} + \dfrac{N}{M} + (1 - \dfrac{\frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s) - (c_{\bar{z}} - c_{\underline{z}})}{\beta[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)]})\dfrac{[\beta - (c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\operatorname{var}(p_t^s)]}{\gamma_{\bar{z}}\beta}}.$$

Substituting (1.125) back into (1.124) solves the spot market price:

(1.126)

$$p_t^s = E_{t-1}p_t^s - \gamma_{\tilde{z}}[2 - \frac{(c_{\bar{z}} - c_{\underline{z}})}{\beta} + \frac{1}{2}r(2-\beta)\operatorname{var}(p_t^s)]\mu_t .$$

Hence, the time-invariant variance of the spot market price is

$$(1.127)\quad \operatorname{var}(p_t^s) = \gamma_{\bar{z}}^2[2 - \frac{(c_{\bar{z}} - c_{\underline{z}})}{\beta} + \frac{1}{2}r(2-\beta)\operatorname{var}(p_t^s)]^2\sigma_\mu^2 ,$$

from which we can solve the variance explicitly:

$$(1.128)\quad \operatorname{var}(p_t^s) = \sigma_p^2 = \sigma_p^2(\gamma_{\tilde{z}}, c_{\bar{z}}, c_{\underline{z}}, \beta, r, \sigma_\mu^2) .$$

Further, substituting (1.128) into (1.125) and (1.126) solves the explicit expected spot market price and the spot market price, respectively.

Given the spot market price, the quantity of hogs demanded from the spot market by each processor can be obtained by substituting (1.126) into (1.117):

$$(1.129)\quad q_{2t}^j = \frac{\alpha_{\tilde{z}}P^{\tilde{z}} - p_t^s}{\gamma_{\tilde{z}}} - \mu_t = \frac{\alpha_{\tilde{z}}P^{\tilde{z}} - E_{t-1}p_t^s}{\gamma_{\tilde{z}}} + [1 - \frac{(c_{\bar{z}} - c_{\underline{z}})}{\beta} + \frac{1}{2}r(2-\beta)\sigma_p^2]\mu_t .$$

Similarly, substituting (1.114), (1.125), and (1.128) into (1.120) yields the number of producers with which each packer signs a contract,

(1.130)

$$n_1^j = \frac{[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\sigma_p^2][\alpha_{\bar{z}}P^{\bar{z}} - E_{t-1}p_t^s + \frac{1}{2}r(2-\beta)q_0\sigma_p^2] - \frac{(c_{\bar{z}} - c_{\underline{z}})E_{t-1}p_t^s}{\beta}}{\gamma_{\bar{z}}\beta E_{t-1}p_t^s}$$

$$-\frac{[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\sigma_p^2]}{\beta E_{t-1}p_t^s}\mu_t .$$

Further, given this contract, each processor can obtain profit

(1.131)

$$\Pi_5^* = \int_{P\in\Omega}[Pg(Q_{1t}^j\,|\,\bar{z}) - h(Q_{1t}^j\,|\,\bar{z}) - (c_{\tilde{z}} + a + bP)Q_{1t}^j]dF(P\,|\,\bar{z}) + \int_{P\in\Omega}[Pg(q_{2t}^j\,|\,\tilde{z}) - h(q_{2t}^j\,|\,\tilde{z}) - p_t^s q_{2t}^j]dF(P\,|\,\tilde{z})$$

$$= [\frac{1}{2}\gamma_{\bar{z}}(\beta n_1^j q_0)^2 - \frac{1}{2}\gamma_{\bar{z}}\mu_t^2] + [\frac{1}{2}\gamma_{\tilde{z}}(q_{2t}^j)^2 - \frac{1}{2}\gamma_{\tilde{z}}\mu_t^2]$$

$$= \frac{1}{2}\gamma_{\bar{z}}(\beta q_0)^2\{\frac{[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\sigma_p^2][\alpha_{\bar{z}}P^{\bar{z}} - E_{t-1}p_t^s + \frac{1}{2}r(2-\beta)q_0\sigma_p^2] - \frac{(c_{\bar{z}} - c_{\underline{z}})E_{t-1}p_t^s}{\beta}}{\gamma_{\bar{z}}\beta E_{t-1}p_t^s}$$

$$-\frac{[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\sigma_p^2]}{\beta E_{t-1}p_t^s}\mu_t\}^2 - \frac{1}{2}\gamma_{\bar{z}}\mu_t^2 + \frac{1}{2}\gamma_{\tilde{z}}[\frac{\alpha_{\tilde{z}}P^{\tilde{z}} - E_{t-1}p_t^s}{\gamma_{\tilde{z}}} + (1 - \frac{(c_{\tilde{z}} - c_{\underline{z}})}{\beta} + \frac{1}{2}r(2-\beta)\sigma_p^2)\mu_t]^2 - \frac{1}{2}\gamma_{\tilde{z}}\mu_t^2$$

$$= \frac{1}{2}\gamma_{\bar{z}}(\beta q_0)^2\{[\frac{[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\sigma_p^2][\alpha_{\bar{z}}P^{\bar{z}} - E_{t-1}p_t^s + \frac{1}{2}r(2-\beta)q_0\sigma_p^2] - \frac{(c_{\bar{z}} - c_{\underline{z}})E_{t-1}p_t^s}{\beta}}{\gamma_{\bar{z}}\beta E_{t-1}p_t^s}]^2$$

$$+[\frac{[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\sigma_p^2]}{\beta E_{t-1}p_t^s}\mu_t]^2$$

$$-2[\frac{[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\sigma_p^2][\alpha_{\bar{z}}P^{\bar{z}} - E_{t-1}p_t^s + \frac{1}{2}r(2-\beta)q_0\sigma_p^2] - \frac{(c_{\bar{z}} - c_{\underline{z}})E_{t-1}p_t^s}{\beta}}{\gamma_{\bar{z}}\beta E_{t-1}p_t^s}][\frac{[1-(c_{\bar{z}} - c_{\underline{z}}) + \frac{1}{2}r\beta(2-\beta)\sigma_p^2]}{\beta E_{t-1}p_t^s}\mu_t]\}$$

$$+\frac{1}{2}\gamma_{\tilde{z}}\{[\frac{\alpha_{\tilde{z}}P^{\tilde{z}} - E_{t-1}p_t^s}{\gamma_{\tilde{z}}}]^2 + [(1 - \frac{(c_{\tilde{z}} - c_{\underline{z}})}{\beta} + \frac{1}{2}r(2-\beta)\sigma_p^2)\mu_t]^2 + 2\frac{\alpha_{\tilde{z}}P^{\tilde{z}} - E_{t-1}p_t^s}{\gamma_{\tilde{z}}}[1 - \frac{(c_{\tilde{z}} - c_{\underline{z}})}{\beta} + \frac{1}{2}r(2-\beta)\sigma_p^2]\mu_t\}$$

$$-\frac{1}{2}\gamma_{\tilde{z}}\mu_t^2 - \frac{1}{2}\gamma_{\bar{z}}\mu_t^2,$$

from which we can compute the expected profit

(1.132)

$$E\Pi_5^* = \frac{1}{2}\gamma_{\tilde{z}}(\beta q_0)^2\{[\frac{[1-(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2][\alpha_{\tilde{z}}P^{\tilde{z}}-E_{t-1}p_t^s+\frac{1}{2}r(2-\beta)q_0\sigma_p^2]-\frac{(c_{\tilde{z}}-c_{\underset{\sim}{z}})E_{t-1}p_t^s}{\beta}}{\gamma_{\tilde{z}}\beta E_{t-1}p_t^s}]^2$$

$$+[\frac{[1-(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2]}{\beta E_{t-1}p_t^s}]^2\sigma_\mu^2\}-\frac{1}{2}\gamma_{\tilde{z}}\sigma_\mu^2+\frac{1}{2}\gamma_{\tilde{z}}\{[\frac{\alpha_{\tilde{z}}P^{\tilde{z}}-E_{t-1}p_t^s}{\gamma_{\tilde{z}}}]^2+[(1-\frac{(c_{\tilde{z}}-c_{\underset{\sim}{z}})}{\beta}+\frac{1}{2}r(2-\beta)\sigma_p^2)]^2\sigma_\mu^2\}-\frac{1}{2}\gamma_{\tilde{z}}\sigma_\mu^2$$

$$=\frac{\{[1-(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2][\alpha_{\tilde{z}}P^{\tilde{z}}-E_{t-1}p_t^s+\frac{1}{2}r(2-\beta)q_0\sigma_p^2]-\frac{(c_{\tilde{z}}-c_{\underset{\sim}{z}})E_{t-1}p_t^s}{\beta}\}^2}{2\gamma_{\tilde{z}}[1-(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2]^2}$$

$$+\frac{1}{2}\gamma_{\tilde{z}}\{[\frac{\alpha_{\tilde{z}}P^{\tilde{z}}-E_{t-1}p_t^s}{\gamma_{\tilde{z}}}]^2+[(1-\frac{(c_{\tilde{z}}-c_{\underset{\sim}{z}})}{\beta}+\frac{1}{2}r(2-\beta)\sigma_p^2)]^2\sigma_\mu^2\}-\frac{1}{2}\gamma_{\tilde{z}}\sigma_\mu^2$$

$$=\frac{\{[1-(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2][\alpha_{\tilde{z}}P^{\tilde{z}}+\frac{1}{2}r(2-\beta)q_0\sigma_p^2]-E_{t-1}p_t^s[1+\frac{1-\beta}{\beta}(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2]\}^2}{2\gamma_{\tilde{z}}[1-(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2]^2}$$

$$+\frac{1}{2}\gamma_{\tilde{z}}\{[\frac{\alpha_{\tilde{z}}P^{\tilde{z}}-E_{t-1}p_t^s}{\gamma_{\tilde{z}}}]^2+[(1-\frac{(c_{\tilde{z}}-c_{\underset{\sim}{z}})}{\beta}+\frac{1}{2}r(2-\beta)\sigma_p^2)]^2\sigma_\mu^2\}-\frac{1}{2}\gamma_{\tilde{z}}\sigma_\mu^2$$

$$=\frac{\{\alpha_{\tilde{z}}P^{\tilde{z}}[1-(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2]-E_{t-1}p_t^s[1+\frac{1-\beta}{\beta}(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r(\beta-1)(2-\beta)\sigma_p^2]\}^2}{2\gamma_{\tilde{z}}[1-(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2]^2}$$

$$+\frac{1}{2}\gamma_{\tilde{z}}\{[\frac{\alpha_{\tilde{z}}P^{\tilde{z}}-E_{t-1}p_t^s}{\gamma_{\tilde{z}}}]^2+[1-\frac{(c_{\tilde{z}}-c_{\underset{\sim}{z}})}{\beta}+\frac{1}{2}r(2-\beta)\sigma_p^2]^2\sigma_\mu^2\}-\frac{1}{2}\gamma_{\tilde{z}}\sigma_\mu^2,$$

and the variance of each packer's profit

(1.133)

$$Var(\Pi_5^*)=\{-\gamma_{\tilde{z}}(\beta q_0)^2[\frac{[1-(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2][\alpha_{\tilde{z}}P^{\tilde{z}}-E_{t-1}p_t^s+\frac{1}{2}r(2-\beta)q_0\sigma_p^2]-\frac{(c_{\tilde{z}}-c_{\underset{\sim}{z}})E_{t-1}p_t^s}{\beta}}{\gamma_{\tilde{z}}\beta E_{t-1}p_t^s}]$$

$$[\frac{[1-(c_{\tilde{z}}-c_{\underset{\sim}{z}})+\frac{1}{2}r\beta(2-\beta)\sigma_p^2]}{\beta E_{t-1}p_t^s}]+\gamma_{\tilde{z}}\frac{\alpha_{\tilde{z}}P^{\tilde{z}}-E_{t-1}p_t^s}{\gamma_{\tilde{z}}}[1-\frac{(c_{\tilde{z}}-c_{\underset{\sim}{z}})}{\beta}+\frac{1}{2}r(2-\beta)\sigma_p^2]\}^2\sigma_\mu^2$$

$$=\{(\alpha_{\tilde{z}}P^{\tilde{z}}-E_{t-1}p_t^s)[1-\frac{(c_{\tilde{z}}-c_{\underset{\sim}{z}})}{\beta}+\frac{1}{2}r(2-\beta)\sigma_p^2]-\alpha_{\tilde{z}}P^{\tilde{z}}+E_{t-1}p_t^s-\frac{1}{2}r(2-\beta)q_0\sigma_p^2+\frac{(c_{\tilde{z}}-c_{\underset{\sim}{z}})q_0}{\beta}\}^2\sigma_\mu^2$$

.

1.5 A Numerical Example and Results

Based on the structural model for each type of contract formulated in the previous section, a numerical example is provided here to the various contracts' impacts on the spot market. Specifically, the following impacts are investigated: a) impact of the contract supply on the spot market price under each contract, b) impact of contract supply on producers' and packers' welfare under each contract, and c) impact of market power in terms of $\frac{N}{M}$ on the performance of each contract.

For simplicity, we assume that the randomness associated with the market price of the finished product, P, is governed by an exponential distribution function

(1.134) $f(P \mid z) = \frac{1}{z} e^{-P/z}, \; 0 \le P < \infty, \text{ and } z > 0$.

For the numerical example, the values of parameters $\{\bar{z}, \underline{z}, \beta, \alpha_{\bar{z}}, \alpha_{\underline{z}}, \gamma_{\bar{z}}, \gamma_{\underline{z}}, c_{\bar{z}}, c_{\underline{z}}, r, M, N, \sigma_{\mu}^2\}$ are described in Table 1.2.

Table 1.2 Parameters used in the numerical example

$\bar{z}$	4	$\underline{z}$	3
β	60%-95%	$c_{\underline{z}}$	0.1
$\alpha_{\bar{z}}$	0.5	M	10
$\alpha_{\underline{z}}$	0.4	N	20, 50, 100
$\gamma_{\bar{z}}$	0.2	σ_{μ}^2	0.5
$\gamma_{\underline{z}}$	0.3	r	0.1-2
$c_{\bar{z}}$	0.3		

Given these parameters, Table 1.3, Table 1.4, and Table 1.5 show the equilibrium prices and quantities from the numerical example with N=20, 50, 100, respectively. In addition, although the numerical example is conducted with the risk aversion parameter in the range 0.1 to 2, only the results for $r = 0.5$ are presented in each table. The results for other values of r are similar or exactly same under some contracts to those with r = 0.5. Moreover, the numbers shown in each table are expected values given those parameters.

1.5.1 Captive Supply and Spot Market Price

Since the term “captive supplies” is usually used in the beef sector, we use “contract supply” here to represent the total amount of hogs transacted through contracts. As we discussed in section 1.2, several empirical studies have reported a negative relationship between captive supplies delivered from marketing agreements and forward contracts (e.g., Elam 1992, Schroeder, Jones, Mintert, and Barkley 1993, Ward, Koontz, and Schroeder 1996); however, some have found ambiguous results (e.g., Azzam 1998, Ward, Koontz, and Schroeder 1998). These mixed results are partly due to different estimation techniques, data, and model specifications. More importantly, none of those empirical models has dealt with the endogeneity problem which arises from the mutual interaction between captive supplies and spot market prices. Thus, those statistical results are possibly biased. The results that follow are based on an equilibrium model that accounts for this endogeneity problem. Because of this reason, perhaps, the results contradict some of the previous findings.

1) *Fixed-price contracts*

a) *Fixed-price contracts under risk neutrality*

With risk neutrality, the contract price takes the form

(1.135) $\underline{w} = E_{t-1} p_t^s$

Under this contract, the expected spot market price is the lowest among all types of contracts except the market price contract. Moreover, both the contract supplies and the expected spot market price stay constant. Therefore, contract supplies do not have any causal effect on the expected spot market price. In addition, captive supplies do not affect the variance of the spot market price. Thus, by producing low-quality hogs, a risk-neutral producer is always indifferent between signing a contract and selling to the spot market regardless of values of β. Hence, for each β, a packer can minimize his risk by adjusting n_1^j ,the number of producers to sign a contract with, such that his expected demand from the contract market always equals that from the spot market. In other words, under this contract, each packer optimally acquires half of the hogs from the contract market and half from the spot market. As a result, the spot market equilibrium supply constitutes half of the total supply under expectation.

b) *Fixed- price contracts under risk aversion*

Under this contract, the contract price is specified as

(1.136) $\underline{w} = E_{t-1} p_t^s - \frac{1}{2} r q_0 (2-\beta) \operatorname{var}(p_t^s)$,

which varies according to the parameters r and β.

Figure 1.2 shows that both contract supplies and expected spot market prices decrease as β increases. Figure 1.3 demonstrates that contract supplies have a positive relationship with the expected spot market prices and the variance of the spot market prices. In addition, their relationship appears to be represented by a linear function. This effect can be explained as follows: As the parameter β increases, the processors have the incentive to raise the contract price to make the risk-averse producers indifferent between signing a contract and selling to the spot market. Increases in the contract price reduce the quantity demanded by each processor from the contract market and, hence,

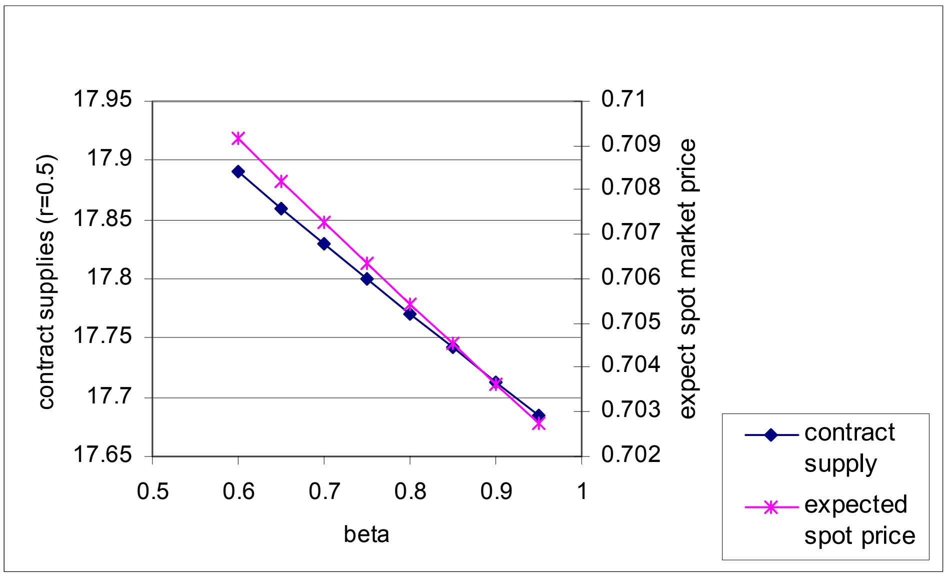

Figure 1.2 Impact of Beta on contract supplies and expected spot market price under the fixed-price contract with risk aversion

raise the quantity supplied to the spot market. Consequently, the quantity supplied to the spot market exceeds the quantity demanded from the spot market and the expected spot market price decreases. Therefore, as shown in Figure 1.3, contract supplies through the fixed price contract are positively related to the expected spot market price.

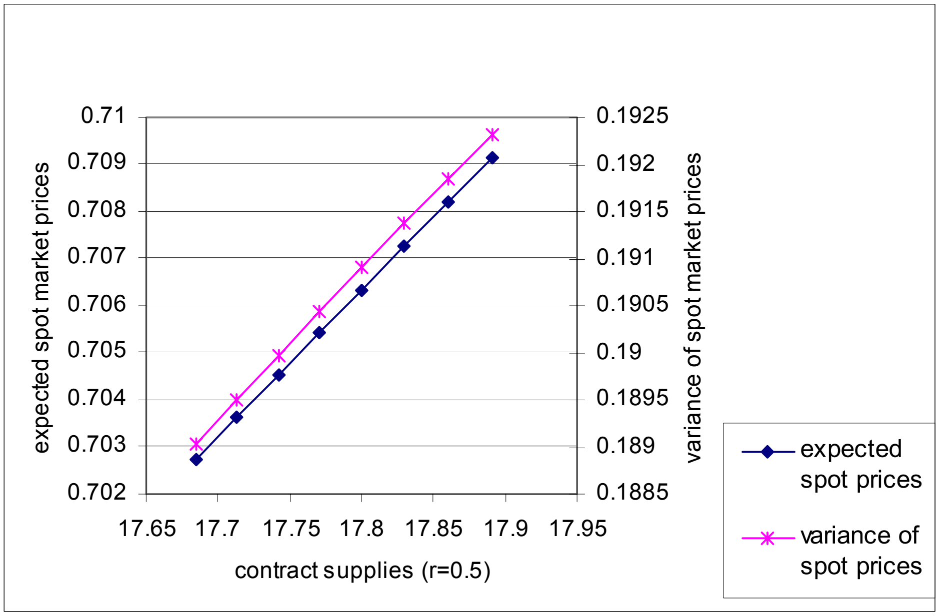

Figure 1.3 Contract supplies vs. expected spot market price and variance of spot market prices under the fixed-price contract with risk aversion

2) *Market-price contracts*

Similar to the fixed-price contract with risk neutrality, contract supplies through the market-price contract do not affect the expected spot market price and the variance of the spot market price. Recall that under the market-price contract, a contract producer is indifferent both *ex ante* and *ex post* between signing a contract and selling to the spot market, and strictly prefers to produce low-quality hogs regardless of the parameter β. Hence, given any value of β, a processor optimally purchases half of his hogs from the contract market and half from the spot market under expectation by adjusting n_1^j, the number of producers to sign a contract with. Consequently, contract supplies through the market-price contract do not affect the expected market price. Further, similar to the fixed-price contract under risk neutrality, contract supplies do not affect the variance of the spot market price. However, the market-price contract causes a smaller variance of spot market price relative to that under the fixed-price contract under risk neutrality.

3) *Formula-price contracts with premium*

Figure 1.4 shows that an increase in β raises both contract supplies and expected spot market prices. Figure 1.5 shows that contract supplies are positively related to the expected spot market price and are negatively related to the variance of the spot market price under the formula-price contract.

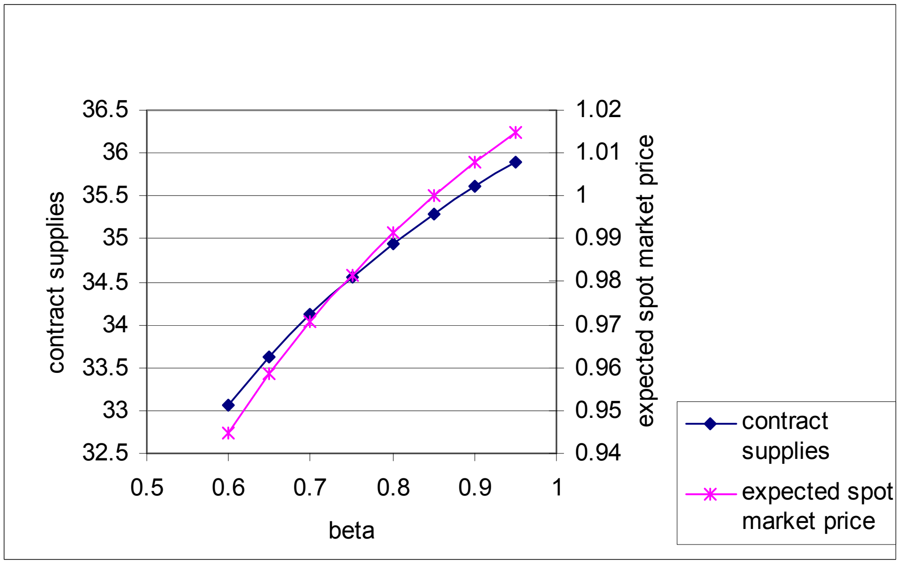

Figure 1.4 Impact of Beta on contract supplies and expected spot market price under the formula-price contract

These effects can be explained as follows: working through the participation constraint and the incentive compatibility constraint, as the parameter β increases, the packers reduce the contract price to make the contract producers indifferent between

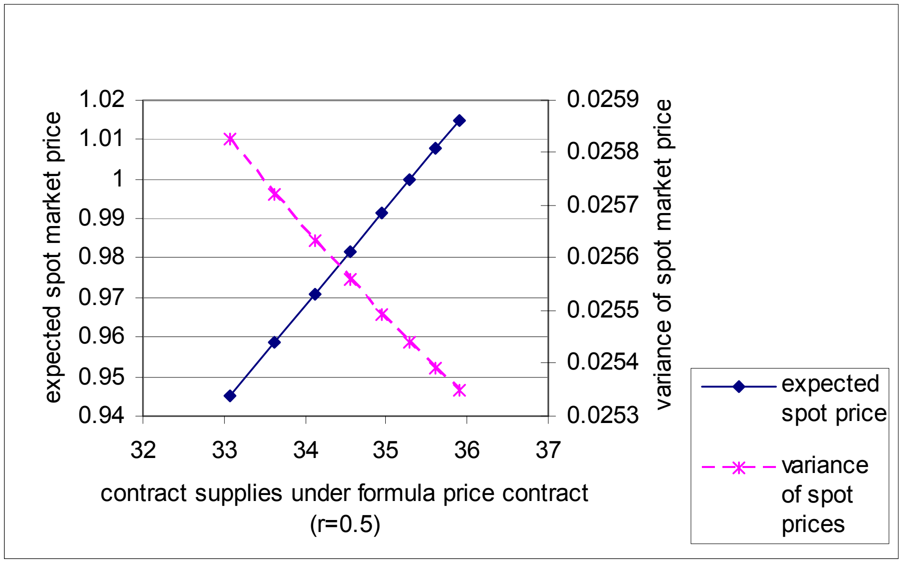

Figure 1.5 Captive supplies vs. expected spot market price and variance of spot market prices under the formula-price contract

signing a contract and selling to the spot market. Decreases in the contract price raise the quantity demanded by each processor from the contract market and, hence, reduce the quantity supplied to the spot market. As a result, the quantity demanded from the spot market exceeds the quantity supplied to the spot market and the expected spot market price increases. Further, each producer produces more hogs due to the increase in the expected spot market price and, additionally, contract producers raise their production more than independent producers. Hence, an increase in β raises the amount of total contract supply.

Note also that the expected market price under the formula-price contract is greater than those under the fixed-price contract and the market-price contract due to quality differences between the contract market and the cash market. Moreover, the formula-price contract causes the smallest variability of spot market prices among all types of contracts.

Another important property of this contract is that it makes the spot market thinner than the fixed-price contract and the market-price contract. Given the example shown in Table 1.3, spot market supply accounts for about 40.5%, on average, of total supply. Therefore, this effect of the formula-price contract is consistent with what has been observed in reality. However, instead of reducing spot market prices and making cash prices more volatile as claimed in several studies, these results show that the increased use of formula-price contracts raises expected spot market prices and reduces the variability of spot market prices. Thus, this result demonstrates that the endogeneity problem and asymmetric information concerning hog qualities that have not been taken into account in past studies play a critical role in determining the relationship between contracting and the spot market.

4) *Cost-plus contract with premium*

Similar to the formula-price contract, Figure 1.6 shows that both contract supply and expected spot market price increase as β increases. Further, increases in contract supply through the cost-plus contract raises the expected spot market price as well (Figure 1.7). However, unlike the formula-price contract, contract supplies under cost-plus contracts raise the variance of spot market prices as well.

Recall that in the section 1.4.4, the condition (1.112) implies that the parameter β is negatively related to the contract price given the parameters specified in Table 1.2. Thus, the contract price decreases as β increases. Thus, each packer purchases more hogs from the contract market and, hence, the quantity supplied to the spot market decreases. Consequently, the excess demand in the spot market drives up the equilibrium spot market price. Moreover, an increase in the expected spot market price raises the

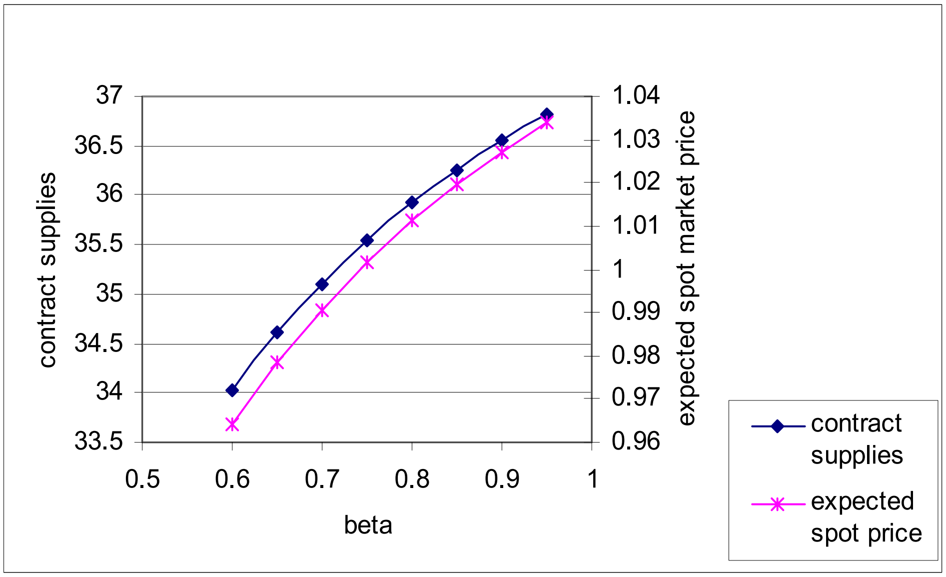

Figure 1.6 Impact of Beta on contract supplies and expected spot market price under the cost-plus contract

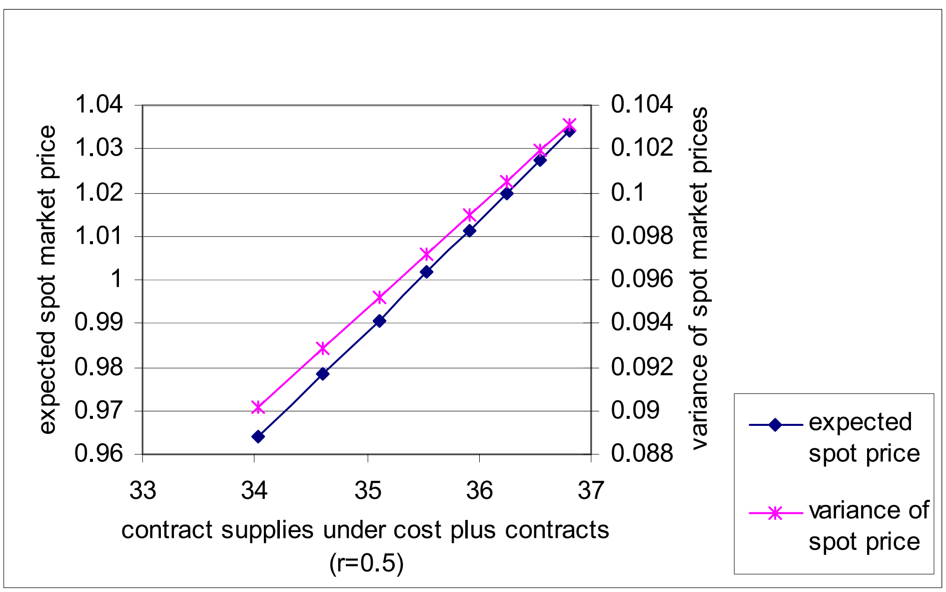

Figure 1.7 Contract supplies vs. expected spot market price and variance of spot market prices under the cost-plus contract

production by each producer and contract producers offer more hogs than independent producers. Thus, increases in β raise the total amount of contract supply as well.

Therefore, contract supplies through the cost-plus contract are positively related to the expected spot market price.

Note that for each r and β, the expected spot market price under a cost-plus contract is the greatest among all types of contracts. However, the variance of the spot market price is also greater than that under the formula-price contract and the market-price contract. Similar to the formula-price contract, the spot market becomes thinner under the cost plus contract. In addition, the spot market is the thinnest under the cost-plus contract among all types of contracts.

1.5.2 Packers' and Producers' Welfare

1) Fixed-price contract

a) Fixed-price contract under risk neutrality

Under the fixed-price contract, since changes in the parameter β do not affect the spot market price and contract supplies, packers' expected profit stays constant as β increases. For each r and β, packers obtain a relatively greater profit than producers. In addition, packers can eliminate all risk in their profit by adjusting the quantities demanded from the spot market and the contract market.

On the other hand, changes in β do not affect producers' expected utility, and contract producers earn the same expected utility as independent producers. However, changes in β have different effects on the variability of producers' incomes. Recall that a contract producer earns income $W = \beta w(P)q_0 + (1-\beta)p_t^s q_0$. Thus, as β increases, contract producers face a smaller variance of their income relative to independent producers. Therefore, under the fixed-price contract, a risk neutral contract producer prefers to contract more of his hogs with a packer.

b) Fixed-price contract under risk aversion

Figure 1.8 shows that increase in contract supplies not only raises packers' expected profit, but also raises the variance of packers' profit. Thus, an increase in β causes a tradeoff between packers' profit and its variance compared to the case under risk neutrality.

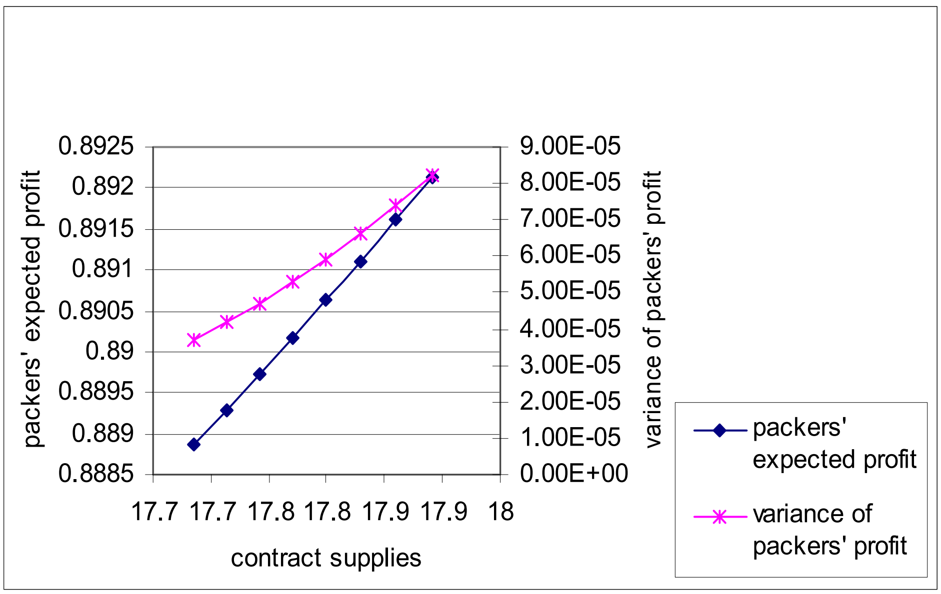

Figure 1.8 Contract supplies vs. packers' expected profit and variance of packers' profit under the fixed-price contract with risk aversion

On the other hand, as each contract producer signs a greater proportion of his hogs with a packer, total contract supply decreases and both contract producers and independent producers obtain a smaller expected utility. Hence, contract supply is positively related to producers' expected utility, and each contract producer prefers a small proportion of contracting under this contract. Further, since packers can depress the contract price as producers' degree of risk aversion increases, packers capture more surplus and, hence, contract producers earn a lower utility relative to independent producers under this contract. In addition, increases in contract supply raise the variance of producers' income. However, since the contract price is fixed for each r and β, contract producers face a relatively smaller variance of their income than independent producers. Figure 1.9 shows these impacts of contract supplies on both contract producers' and independent producers' profit.

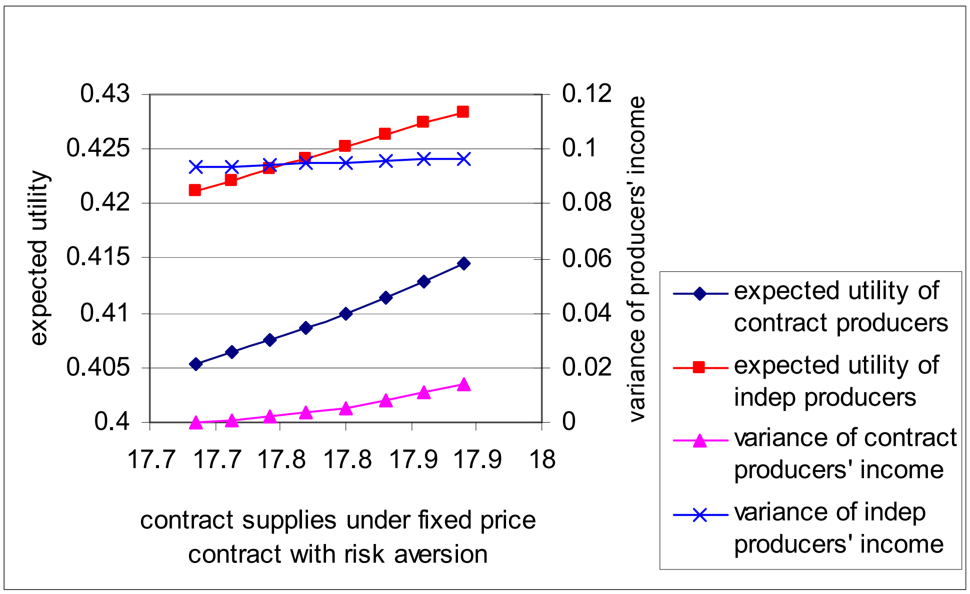

Figure 1.9 Contract supplies vs. producers' expected profit and variance of producers' income under the fixed-price contracts with risk aversion

2) Market-price contract

Similar to the fixed-price contract with risk neutrality, changes in β do not affect the amount of contract supplies, packers' profit, and producers' profit under the market-price contract. Additionally, as β increases, the variance of both contract producers' and independent producers' income stays constant. Compared to the fixed-price contract with risk neutrality, however, contract producers face a larger variance of income relative to independent producers under the market-price contract. Further, under the market-price contract, both packers and producers obtain smaller profit or utility relative to those under the fixed-price contract with risk neutrality; and packers earn the smallest profit among all types of contracts.

3) Formula-price contract with premium

Figure 1.10 shows that both packers' expected profit and variance of packers' profit increase as contract supplies increase. On the other hand, Figure 1.11 shows contract supply is positively related to producers' expected utility and variance of producers' income. Compared with independent producers, contract producers obtain a greater expected utility, but also face a greater variance of their income.

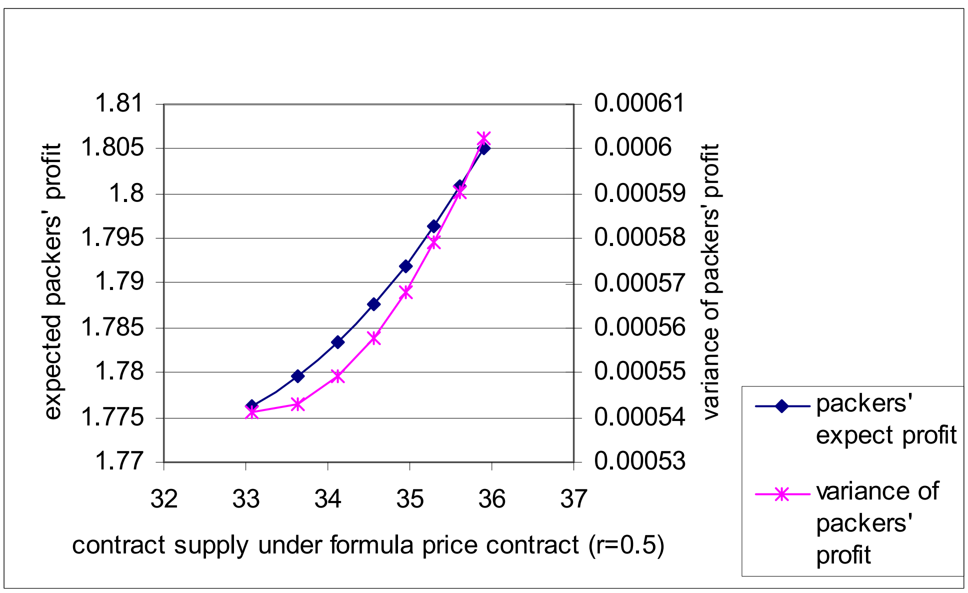

Figure 1.10 Contract supplies vs. packers' expected profit and variance of packers' profit under the formula-price contract

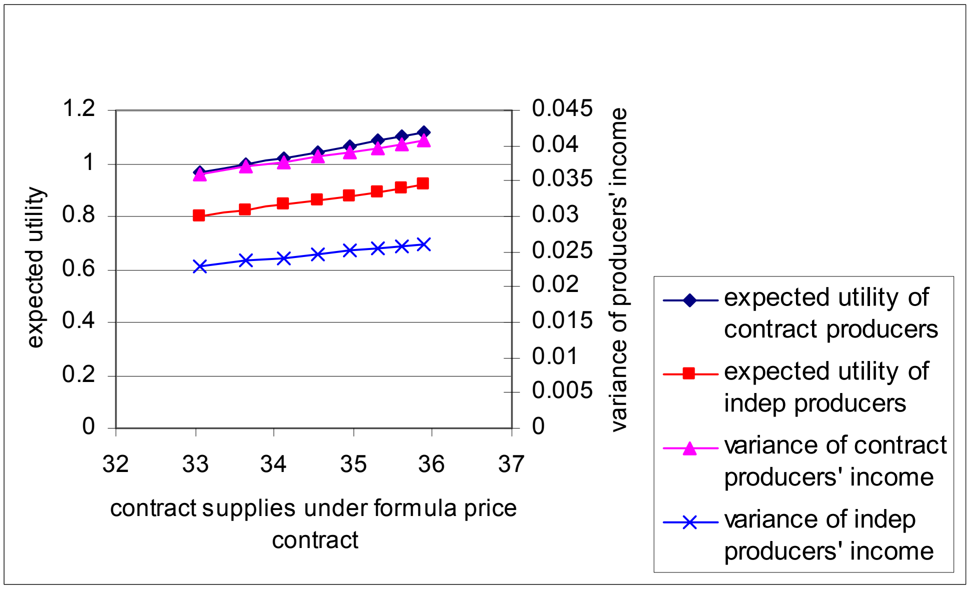

Figure 1.11 Contract supplies vs. producers' expected profit and variance of producers' income under the formula-price contract

Because packers can acquire high-quality hogs from the contract market, packers earn a greater profit than that under the fixed-price contract and the market-price contract due to greater profitability of high-quality hogs. Similarly, although producers incur high production costs by providing high-quality hogs to the market, both contract producers and independent producers can obtain a greater utility from high spot market prices and high contract prices. Risk-averse producers also benefit from low variance of spot market prices. In addition, given the short-run supply function, both contract producers and independent producers offer more hogs to the contract market and the spot market.

4) Cost-plus contract with premium

The performance of the cost-plus contract is very similar to the formula price contract. Figure 1.12 shows that increased contract supplies raise packers' profit and variance of packers' profit. In addition, the variance of packers' profit rises relatively

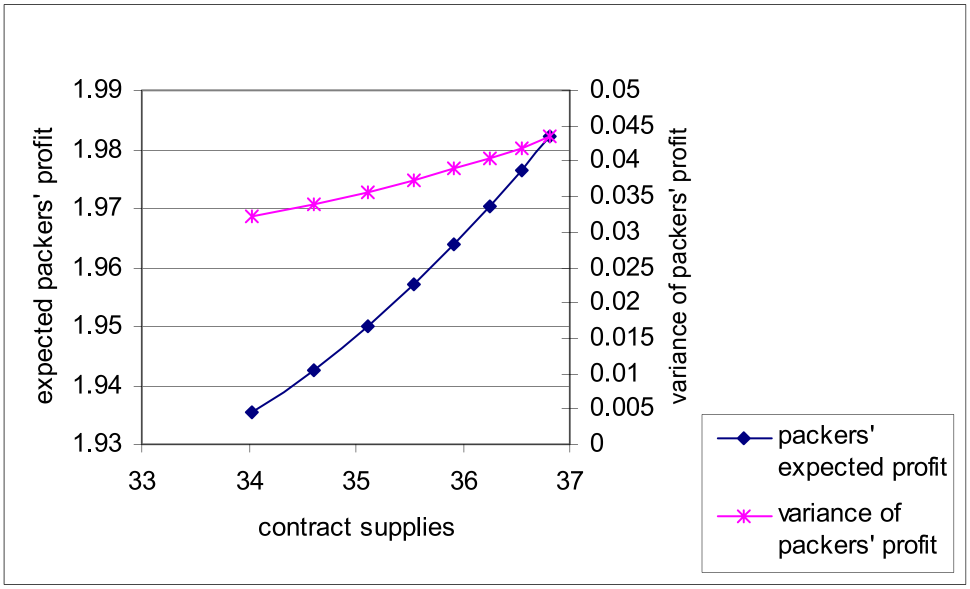

Figure 1.12 Contract supplies vs. packers’ expected profit and variance of packers’ profit under the cost-plus contract

slower than packers’ expected profit as contract supplies increase. The cost-plus contract offers the greatest profit to packers among all types of contracts. Compared to the formula price contract, however, packers incur a greater variance of profit.

Figure 1.13 shows that both contract producers’ and independent producers’ expected utilities increase as contract supplies increase. However, increased contract supplies raise the variance of independent producers’ income, while they reduce the variance of contract producers’ income. In addition, contract producers obtain a greater expected utility and a greater variance of income relative to independent producers for each level of contract supply. Compared to the formula-price contract, for each value of β and corresponding level of contract supply, contract producers earn a lower expected utility but face a smaller variance of income, while independent producers obtain a greater expected utility but face a greater variance of income.

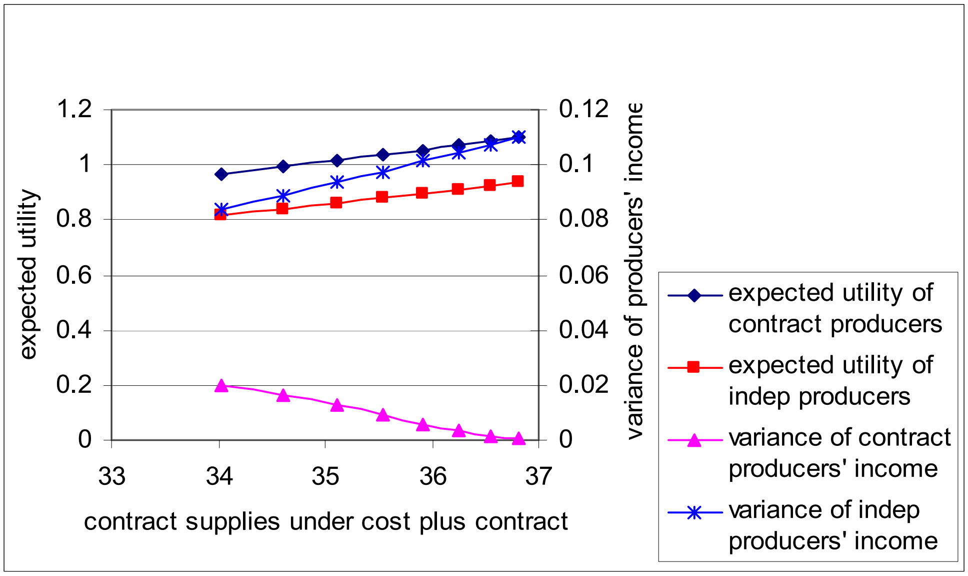

Figure 1.13 Contract supplies vs. producers' expected profit and variance of producers' income under the cost-plus contract

1.5.3 Impacts of Market Power

To demonstrate the effects of market power on the performance of the five types of contracts, one can vary, N/M, the ratio between the number of producers and the number of packers, given the same set of parameters. As N/M increases, packers gain more market power in the sense that they can manipulate the market equilibrium more significantly. Without loss of generality, fix the number of packers $M = 10$ and set the number of producers $N = 20, 50, 100$, as shown in Table 1.3, 1.4, and 1.5. Based on this example, the increase in packers' market power has the following effects:

1) Expected spot market prices

As shown in the tables, for each value of β under each type of contract, the expected spot market price is pushed down as N increases. However, the variance of spot market price stays unchanged. As a result, packers purchase more hogs from both the contract market and the spot market due to the lower prices and, hence, both the contract market and the spot market expand.

2) Packers' and producers' profit

As *N* increases, packers gain market power as buyers; hence, more surplus is captured by packers through both the contract market and the spot market. Thus, for each value of β under each type of contract, packers obtain a greater profit as *N* increases. However, each packer incurs a greater variance of profit under each contract as *N* increases. On the other hand, each producer earns a smaller expected utility due to the reduced spot market price and the reduced amount of hogs produced by each producer. However, each producer faces a smaller variance of income as well.

Based on the results from the sections 1.5.1 to 1.5.3, Table 1.6 summarizes the impacts of contract supplies on spot market price, packers' profit, and producers' utilities.

1.6 Conclusion and Discussion

This essay investigates the relationship between contracting and the spot market under four different types of contracts, including fixed-price contracts, market-price contracts, formula-price contracts, and cost-plus contracts. In addition, asymmetric information concerning unobservable hog qualities is introduced into the model. More precisely, it is assumed that producers have more information about the quality of their hogs than packers before delivery or slaughtering. For each contract, a principal-agent framework is used to derive the optimal conditions and the market equilibrium is derived by equating market demand to market supply in the spot market.

Based on the structural model described in section 1.4 and the numerical example in section 1.5, the major findings are summarized as follows:

First, different from the results in most past studies, this essay concludes that contract supplies raise the expected spot market price under the formula-price contract and reduce the variance of spot market price. Therefore, the results in these past studies are likely biased due to the endogenous relationship between contract supplies and the spot market price. In addition, contract supplies also have a positive relationship with the expected spot market price under the cost-plus contract; while contract supplies have no causal effect on the expected spot market price under the fixed-price contract and the market-price contract.

Second, several studies have reported that producers complain about formula-price contracts because they do not provide price protection. However, this essay finds that the formula-price contract offers the 2nd highest expected profit to packers, highest expected utility to contract producers, and the 2nd highest expected utility to independent producers relative to other contracts. Both packers and producers prefer the formula-price contract to the fixed-price contract or the market-price contact if asymmetric information about hog quality is taken into account. Compared to the cost-plus contract, the formula-price contract offers a smaller expected profit to packers and a lower expected utility to independent producers, but offers a greater expected utility to contract producers. In fact, performances of the cost-plus contract and the formula-price contract are similar and both are better than the fixed price contract and the market price contract.

Third, impacts of relative market power of packers and producers are simulated by varying the number of producers, *N*, in the market. We find that increases of packers' market power by raising *N* depress the expected market price and producers' expected utility, but raise packers' expected profit. However, the relative superiority of each contract is the same regardless of the relative market power of packers and producers.

Compared to the past studies, the contributions of this essay are twofold. First, this essay not only investigates the relationship between hog contracting and the hog spot market in particular, but also provides a general methodology for this type of problem. Methods in existing literatures dealing with this problem include various statistical models. However, different from most studies, this essay embeds a principal-agent model of processor-producer behavior within a general equilibrium model of the hog market. In the general equilibrium framework, this essay acknowledges the endogenous relationship between contract supplies and the spot market, which had been ignored, in general, in past studies. Second, this essay contributes to the existing literature by incorporating asymmetric information concerning hog qualities into the equilibrium model. Again, this has generally been ignored in past studies due to limitations of model structures. However, the results established in this essay demonstrate that both the endogeneity problem and asymmetric information play a critical role in determining the relationship between contracting and the spot market. In addition, the results found in this essay are consistent with what we observe in the real world and justify the dominant use of formula-price contracts in the hog sector.

This essay uses a simple structural approach to examine the relationship between contracting and the spot market. However, a few generalizations of the model could be made. For example, under the formula-price contract and the cost-plus contract, the market average quality of hogs is computed as the arithmetic mean of high quality and low quality. Instead, a weighted-average quality of hogs sold by contract producers and independent producers to the spot market could be used. Another generalization is that the quantity of hogs offered by each producer could be determined by maximizing each producer's expected utility instead of using the short-run supply function. However, any of these modifications would significantly complicate the analysis. Thus, we will not discuss them in this essay.

Table 1.3 A numerical example of the model (r =0.5, N=20)

r	beta	Ep	Varp	q_0	q_s	q_2	n_1	Contract supply
Fixed price contracts with risk neutrality								
0	0.6	0.92308	0.18	0.92308	0.92308	0.92308	1.6667	9.2308
0	0.65	0.92308	0.18	0.92308	0.92308	0.92308	1.5385	9.2308
0	0.7	0.92308	0.18	0.92308	0.92308	0.92308	1.4286	9.2308
0	0.75	0.92308	0.18	0.92308	0.92308	0.92308	1.3333	9.2308
0	0.8	0.92308	0.18	0.92308	0.92308	0.92308	1.25	9.2308
0	0.85	0.92308	0.18	0.92308	0.92308	0.92308	1.1765	9.2308
0	0.9	0.92308	0.18	0.92308	0.92308	0.92308	1.1111	9.2308
0	0.95	0.92308	0.18	0.92308	0.92308	0.92308	1.0526	9.2308
Fixed price contracts with risk aversion								
0.5	0.6	0.95478	0.19232	0.91772	0.95478	0.81739	1.8584	10.233
0.5	0.65	0.95348	0.19184	0.91497	0.95348	0.82174	1.7137	10.192
0.5	0.7	0.95219	0.19137	0.91246	0.95219	0.82603	1.5894	10.152
0.5	0.75	0.95092	0.1909	0.9102	0.95092	0.83026	1.4814	10.113
0.5	0.8	0.94967	0.19043	0.90817	0.94967	0.83443	1.3865	10.074
0.5	0.85	0.94844	0.18996	0.90636	0.94844	0.83853	1.3026	10.035
0.5	0.9	0.94722	0.1895	0.90479	0.94722	0.84259	1.2277	9.9976
0.5	0.95	0.94602	0.18904	0.90343	0.94602	0.84659	1.1605	9.9603
Market price contracts								
0.5	0.6	0.92308	0.045	0.92308	0.92308	0.92308	1.6667	9.2308
0.5	0.65	0.92308	0.045	0.92308	0.92308	0.92308	1.5385	9.2308
0.5	0.7	0.92308	0.045	0.92308	0.92308	0.92308	1.4286	9.2308
0.5	0.75	0.92308	0.045	0.92308	0.92308	0.92308	1.3333	9.2308
0.5	0.8	0.92308	0.045	0.92308	0.92308	0.92308	1.25	9.2308
0.5	0.85	0.92308	0.045	0.92308	0.92308	0.92308	1.1765	9.2308
0.5	0.9	0.92308	0.045	0.92308	0.92308	0.92308	1.1111	9.2308
0.5	0.95	0.92308	0.045	0.92308	0.92308	0.92308	1.0526	9.2308
Formula price contracts								
0.5	0.6	1.2093	0.025826	1.5117	1.2093	1.4627	1.581	14.339
0.5	0.65	1.2252	0.02572	1.5315	1.2252	1.3993	1.525	15.181
0.5	0.7	1.2393	0.025632	1.5492	1.2393	1.3427	1.4665	15.903
0.5	0.75	1.252	0.025558	1.5651	1.252	1.2918	1.4083	16.531
0.5	0.8	1.2635	0.025494	1.5794	1.2635	1.2459	1.3519	17.082
0.5	0.85	1.2739	0.02544	1.5924	1.2739	1.2043	1.298	17.57
0.5	0.9	1.2834	0.025392	1.6042	1.2834	1.1664	1.247	18.005
0.5	0.95	1.2921	0.02535	1.6151	1.2921	1.1318	1.199	18.396
Cost plus contracts								
0.5	0.6	1.2324	0.090122	1.5049	1.2324	1.3705	1.7358	15.673
0.5	0.65	1.2491	0.092842	1.5226	1.2491	1.3037	1.6679	16.507
0.5	0.7	1.2638	0.095181	1.5381	1.2638	1.2447	1.5989	17.215
0.5	0.75	1.2769	0.097209	1.5519	1.2769	1.1926	1.5313	17.823
0.5	0.8	1.2884	0.098978	1.5641	1.2884	1.1462	1.4664	18.349
0.5	0.85	1.2988	0.10053	1.5751	1.2988	1.1049	1.4048	18.808
0.5	0.9	1.308	0.1019	1.585	1.308	1.068	1.3466	19.21
0.5	0.95	1.3163	0.10312	1.5941	1.3163	1.035	1.2919	19.565

Table 1.3 (Cont.)

r	beta	spot supply	packer profit	Varprof (packer)	utility1 (contract producer)	varInc1	utility2 (indep. Producer)	varInc2
Fixed price contracts with risk neutrality								
0	0.6	9.2308	0.25562	0	0.76686	0.02454	0.76686	0.15337
0	0.65	9.2308	0.25562	0	0.76686	0.018788	0.76686	0.15337
0	0.7	9.2308	0.25562	0	0.76686	0.013804	0.76686	0.15337
0	0.75	9.2308	0.25562	0	0.76686	0.0095858	0.76686	0.15337
0	0.8	9.2308	0.25562	0	0.76686	0.0061349	0.76686	0.15337
0	0.85	9.2308	0.25562	0	0.76686	0.0034509	0.76686	0.15337
0	0.9	9.2308	0.25562	0	0.76686	0.0015337	0.76686	0.15337
0	0.95	9.2308	0.25562	0	0.76686	0.00038343	0.76686	0.15337
Fixed price contracts with risk aversion								
0.5	0.6	8.1739	0.26773	0.0010246	0.75151	0.025916	0.77662	0.17532
0.5	0.65	8.2174	0.26714	0.00093658	0.74853	0.019674	0.7746	0.17441
0.5	0.7	8.2603	0.26657	0.00085442	0.74575	0.01434	0.77262	0.17351
0.5	0.75	8.3026	0.26601	0.00077774	0.74314	0.0098845	0.77067	0.17262
0.5	0.8	8.3443	0.26547	0.00070619	0.74072	0.0062824	0.76875	0.17174
0.5	0.85	8.3853	0.26495	0.00063946	0.73847	0.0035112	0.76686	0.17088
0.5	0.9	8.4259	0.26444	0.00057727	0.73639	0.0015513	0.765	0.17003
0.5	0.95	8.4659	0.26395	0.00051934	0.73448	0.00038574	0.76317	0.16919
Market price contracts								
0.5	0.6	9.2308	0.10562	0	0.75728	0.038343	0.75728	0.038343
0.5	0.65	9.2308	0.10562	0	0.75728	0.038343	0.75728	0.038343
0.5	0.7	9.2308	0.10562	0	0.75728	0.038343	0.75728	0.038343
0.5	0.75	9.2308	0.10562	0	0.75728	0.038343	0.75728	0.038343
0.5	0.8	9.2308	0.10562	0	0.75728	0.038343	0.75728	0.038343
0.5	0.85	9.2308	0.10562	0	0.75728	0.038343	0.75728	0.038343
0.5	0.9	9.2308	0.10562	0	0.75728	0.038343	0.75728	0.038343
0.5	0.95	9.2308	0.10562	0	0.75728	0.038343	0.75728	0.038343
Formula price contracts								
0.5	0.6	14.627	0.362	1.72E-05	1.5848	0.059016	1.3068	0.03777
0.5	0.65	13.993	0.36417	3.39E-05	1.6267	0.060323	1.3413	0.038607
0.5	0.7	13.427	0.3672	5.35E-05	1.6645	0.061513	1.3725	0.039369
0.5	0.75	12.918	0.37079	7.48E-05	1.6989	0.062601	1.4008	0.040064
0.5	0.8	12.459	0.37474	9.71E-05	1.7303	0.063596	1.4267	0.040701
0.5	0.85	12.043	0.37889	0.00011991	1.7589	0.064509	1.4503	0.041285
0.5	0.9	11.664	0.38316	0.00014276	1.7852	0.065349	1.4719	0.041823
0.5	0.95	11.318	0.38745	0.00016541	1.8094	0.066124	1.4919	0.042319
Cost plus contracts								
0.5	0.6	13.705	0.51088	0.0027558	1.5771	0.032655	1.3327	0.13688
0.5	0.65	13.037	0.51765	0.0044452	1.6162	0.026366	1.368	0.14485
0.5	0.7	12.447	0.52474	0.0063165	1.651	0.020267	1.3995	0.15203
0.5	0.75	11.926	0.53188	0.0082892	1.6822	0.014632	1.4277	0.15849
0.5	0.8	11.462	0.53891	0.010303	1.7101	0.0096858	1.453	0.16431
0.5	0.85	11.049	0.5457	0.012313	1.7352	0.0056117	1.4757	0.16958
0.5	0.9	10.68	0.55219	0.014288	1.758	0.0025601	1.4962	0.17434
0.5	0.95	10.35	0.55835	0.016204	1.7786	0.00065509	1.5146	0.17866

Table 1.4 A numerical example of the model (r =0.5, N=50)

r	beta	Ep	Varp	q_0	q_s	q_2	n_1	Contract supply
Fixed price contracts with risk neutrality								
0	0.6	0.6857	0.18	0.6857	0.6857	1.7143	4.1667	17.143
0	0.65	0.6857	0.18	0.6857	0.6857	1.7143	3.8462	17.143
0	0.7	0.6857	0.18	0.6857	0.6857	1.7143	3.5714	17.143
0	0.75	0.6857	0.18	0.6857	0.6857	1.7143	3.3333	17.143
0	0.8	0.6857	0.18	0.6857	0.6857	1.7143	3.125	17.143
0	0.85	0.6857	0.18	0.6857	0.6857	1.7143	2.9412	17.143
0	0.9	0.6857	0.18	0.6857	0.6857	1.7143	2.7778	17.143
0	0.95	0.6857	0.18	0.6857	0.6857	1.7143	2.6316	17.143
Fixed price contracts with risk aversion								
0.5	0.6	0.7092	0.19232	0.6816	0.7092	1.6362	4.3747	17.891
0.5	0.65	0.7082	0.19184	0.6796	0.7082	1.6393	4.0431	17.86
0.5	0.7	0.7073	0.19137	0.6778	0.7073	1.6425	3.7581	17.83
0.5	0.75	0.7063	0.1909	0.6761	0.7063	1.6455	3.5103	17.8
0.5	0.8	0.7054	0.19043	0.6746	0.7054	1.6486	3.2928	17.77
0.5	0.85	0.7045	0.18996	0.6733	0.7045	1.6516	3.1001	17.742
0.5	0.9	0.7036	0.1895	0.6721	0.7036	1.6546	2.9283	17.713
0.5	0.95	0.7028	0.18904	0.6711	0.7028	1.6575	2.7739	17.685
Market price contracts								
0.5	0.6	0.6857	0.045	0.6857	0.6857	1.7143	4.1667	17.143
0.5	0.65	0.6857	0.045	0.6857	0.6857	1.7143	3.8462	17.143
0.5	0.7	0.6857	0.045	0.6857	0.6857	1.7143	3.5714	17.143
0.5	0.75	0.6857	0.045	0.6857	0.6857	1.7143	3.3333	17.143
0.5	0.8	0.6857	0.045	0.6857	0.6857	1.7143	3.125	17.143
0.5	0.85	0.6857	0.045	0.6857	0.6857	1.7143	2.9412	17.143
0.5	0.9	0.6857	0.045	0.6857	0.6857	1.7143	2.7778	17.143
0.5	0.95	0.6857	0.045	0.6857	0.6857	1.7143	2.6316	17.143
Formula price contracts								
0.5	0.6	0.9449	0.0258	1.1812	0.9449	2.5202	4.6658	33.067
0.5	0.65	0.9587	0.0257	1.1984	0.9587	2.4653	4.3174	33.629
0.5	0.7	0.9708	0.0256	1.2135	0.9708	2.4167	4.017	34.123
0.5	0.75	0.9816	0.0256	1.2270	0.9816	2.3736	3.7555	34.56
0.5	0.8	0.9913	0.0255	1.2391	0.9913	2.335	3.5258	34.949
0.5	0.85	0.9999	0.0254	1.2499	0.9999	2.3003	3.3225	35.299
0.5	0.9	1.0078	0.0254	1.2597	1.0078	2.2689	3.1413	35.614
0.5	0.95	1.0149	0.0254	1.2686	1.0149	2.2403	2.9787	35.9
Cost plus contracts								
0.5	0.6	0.9640	0.0901	1.1772	0.9640	2.4438	4.8186	34.035
0.5	0.65	0.9783	0.0928	1.1925	0.9783	2.3869	4.465	34.609
0.5	0.7	0.9907	0.0952	1.2057	0.9907	2.3373	4.1595	35.106
0.5	0.75	1.0016	0.0972	1.2173	1.0016	2.2937	3.8927	35.539
0.5	0.8	1.0112	0.0990	1.2275	1.0112	2.2553	3.6577	35.919
0.5	0.85	1.0197	0.1005	1.2366	1.0197	2.2213	3.4491	36.254
0.5	0.9	1.0272	0.1019	1.2448	1.0272	2.191	3.2625	36.55
0.5	0.95	1.0340	0.1031	1.2522	1.0340	2.164	3.0945	36.813

Table 1.4 (Cont.)

r	beta	spot supply	packer profit	Varprof (packer)	utility1 (contract producer)	varInc1	utility2 (indep. Producer)	varInc2
Fixed price contracts with risk neutrality								
0	0.6	17.143	0.88163	0	0.42318	1.354E-02	0.4232	0.0846
0	0.65	17.143	0.88163	0	0.42318	1.037E-02	0.4232	0.0846
0	0.7	17.143	0.88163	0	0.42318	7.617E-03	0.4232	0.0846
0	0.75	17.143	0.88163	0	0.42318	5.290E-03	0.4232	0.0846
0	0.8	17.143	0.88163	0	0.42318	3.386E-03	0.4232	0.0846
0	0.85	17.143	0.88163	0	0.42318	1.904E-03	0.4232	0.0846
0	0.9	17.143	0.88163	0	0.42318	8.464E-04	0.4232	0.0846
0	0.95	17.143	0.88163	0	0.42318	2.116E-04	0.4232	0.0846
Fixed price contracts with risk aversion								
0.5	0.6	16.362	0.89214	8.24E-05	0.41457	1.430E-02	0.4284	0.0967
0.5	0.65	16.393	0.89162	7.39E-05	0.41295	1.085E-02	0.4273	0.0962
0.5	0.7	16.425	0.89111	6.62E-05	0.41144	7.912E-03	0.4263	0.0957
0.5	0.75	16.455	0.89063	5.92E-05	0.41002	5.454E-03	0.4252	0.0952
0.5	0.8	16.486	0.89016	5.29E-05	0.40871	3.466E-03	0.4242	0.0948
0.5	0.85	16.516	0.88972	4.71E-05	0.40748	1.938E-03	0.4232	0.0943
0.5	0.9	16.546	0.88928	4.19E-05	0.40635	8.560E-04	0.4221	0.0938
0.5	0.95	16.575	0.88886	3.72E-05	0.4053	2.129E-04	0.4211	0.0934
Market price contracts								
0.5	0.6	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
0.5	0.65	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
0.5	0.7	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
0.5	0.75	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
0.5	0.8	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
0.5	0.85	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
0.5	0.9	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
0.5	0.95	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
Formula price contracts								
0.5	0.6	25.202	1.7763	5.413E-04	0.96761	0.036032	0.7979	0.0231
0.5	0.65	24.653	1.7796	5.431E-04	0.99601	0.036936	0.8213	0.0236
0.5	0.7	24.167	1.7834	5.491E-04	1.0214	0.037746	0.8422	0.0242
0.5	0.75	23.736	1.7876	5.577E-04	1.0443	0.038478	0.8610	0.0246
0.5	0.8	23.35	1.7919	5.678E-04	1.0649	0.03914	0.8781	0.0251
0.5	0.85	23.003	1.7963	5.789E-04	1.0837	0.039743	0.8935	0.0254
0.5	0.9	22.689	1.8008	5.905E-04	1.1008	0.040294	0.9076	0.0258
0.5	0.95	22.403	1.8051	6.023E-04	1.1164	0.040799	0.9205	0.0261
Cost plus contracts								
0.5	0.6	24.438	1.9354	0.0323	0.96506	1.998E-02	0.8155	0.0838
0.5	0.65	23.869	1.9427	0.0339	0.99136	1.617E-02	0.8391	0.0889
0.5	0.7	23.373	1.95	0.0356	1.0145	1.245E-02	0.8600	0.0934
0.5	0.75	22.937	1.9571	0.0372	1.035	9.003E-03	0.8785	0.0975
0.5	0.8	22.553	1.964	0.0389	1.0533	5.966E-03	0.8949	0.1012
0.5	0.85	22.213	1.9705	0.0404	1.0696	3.459E-03	0.9097	0.1045
0.5	0.9	21.91	1.9766	0.0419	1.0843	1.579E-03	0.9228	0.1075
0.5	0.95	21.64	1.9823	0.0434	1.0976	4.043E-04	0.9347	0.1103

Table 1.5 A numerical example of the model (r =0.5, N=100)

r	beta	Ep	Varp	q_0	q_s	q_2	n_1	Contract supply
Fixed price contracts with risk neutrality								
0	0.6	0.48	0.18	0.48	0.48	2.4	8.3333	24
0	0.65	0.48	0.18	0.48	0.48	2.4	7.6923	24
0	0.7	0.48	0.18	0.48	0.48	2.4	7.1429	24
0	0.75	0.48	0.18	0.48	0.48	2.4	6.6667	24
0	0.8	0.48	0.18	0.48	0.48	2.4	6.25	24
0	0.85	0.48	0.18	0.48	0.48	2.4	5.8824	24
0	0.9	0.48	0.18	0.48	0.48	2.4	5.5556	24
0	0.95	0.48	0.18	0.48	0.48	2.4	5.2632	24
Fixed price contracts with risk aversion								
0.5	0.6	0.4963	0.1923	0.4771	0.4963	2.3456	8.5685	24.526
0.5	0.65	0.4957	0.1918	0.4757	0.4957	2.3477	7.9255	24.504
0.5	0.7	0.4950	0.1914	0.4744	0.4950	2.3499	7.3726	24.482
0.5	0.75	0.4944	0.1909	0.4732	0.4944	2.3520	6.8919	24.461
0.5	0.8	0.4938	0.1904	0.4722	0.4938	2.3541	6.4699	24.44
0.5	0.85	0.4932	0.1900	0.4713	0.4932	2.3562	6.0960	24.42
0.5	0.9	0.4925	0.1895	0.4705	0.4925	2.3582	5.7625	24.399
0.5	0.95	0.4919	0.1890	0.4698	0.4919	2.3603	5.4628	24.38
Market price contracts								
0.5	0.6	0.48	0.045	0.48	0.48	2.4	8.3333	24
0.5	0.65	0.48	0.045	0.48	0.48	2.4	7.6923	24
0.5	0.7	0.48	0.045	0.48	0.48	2.4	7.1429	24
0.5	0.75	0.48	0.045	0.48	0.48	2.4	6.6667	24
0.5	0.8	0.48	0.045	0.48	0.48	2.4	6.25	24
0.5	0.85	0.48	0.045	0.48	0.48	2.4	5.8824	24
0.5	0.9	0.48	0.045	0.48	0.48	2.4	5.5556	24
0.5	0.95	0.48	0.045	0.48	0.48	2.4	5.2632	24
Formula price contracts								
0.5	0.6	0.6926	0.0258	0.8657	0.6926	3.5297	9.8072	50.942
0.5	0.65	0.7036	0.0257	0.8795	0.7036	3.4855	8.9712	51.288
0.5	0.7	0.7133	0.0256	0.8916	0.7133	3.4469	8.2678	51.6
0.5	0.75	0.7218	0.0256	0.9022	0.7218	3.4129	7.6675	51.882
0.5	0.8	0.7293	0.0255	0.9116	0.7293	3.3827	7.1490	52.139
0.5	0.85	0.7361	0.0254	0.9201	0.7361	3.3557	6.6966	52.372
0.5	0.9	0.7421	0.0254	0.9277	0.7421	3.3314	6.2983	52.585
0.5	0.95	0.7476	0.0254	0.9346	0.7476	3.3095	5.9450	52.781
Cost plus contracts								
0.5	0.6	0.7073	0.0901	0.8637	0.7073	3.4706	9.9566	51.6
0.5	0.65	0.7186	0.0928	0.8760	0.7186	3.4256	9.1269	51.966
0.5	0.7	0.7283	0.0952	0.8864	0.7283	3.3866	8.4271	52.291
0.5	0.75	0.7368	0.0972	0.8955	0.7368	3.3527	7.8284	52.579
0.5	0.8	0.7443	0.0990	0.9035	0.7443	3.3230	7.3099	52.835
0.5	0.85	0.7508	0.1005	0.9105	0.7508	3.2968	6.8562	53.064
0.5	0.9	0.7566	0.1019	0.9168	0.7566	3.2736	6.4555	53.268
0.5	0.95	0.7618	0.1031	0.9225	0.7618	3.2530	6.0988	53.45

Table 1.5 (Cont.)

r	beta	spot supply	packer profit	Varprof (packer)	utility1 (contract producer)	varInc1	utility2 (indep. Producer)	varInc2
Fixed price contracts with risk neutrality								
0	0.6	24	1.728	0	0.2074	6.636E-03	0.2074	0.04147
0	0.65	24	1.728	0	0.2074	5.080E-03	0.2074	0.04147
0	0.7	24	1.728	0	0.2074	3.733E-03	0.2074	0.04147
0	0.75	24	1.728	0	0.2074	2.592E-03	0.2074	0.04147
0	0.8	24	1.728	0	0.2074	1.659E-03	0.2074	0.04147
0	0.85	24	1.728	0	0.2074	9.331E-04	0.2074	0.04147
0	0.9	24	1.728	0	0.2074	4.147E-04	0.2074	0.04147
0	0.95	24	1.728	0	0.2074	1.037E-04	0.2074	0.04147
Fixed price contracts with risk aversion								
0.5	0.6	23.456	1.738	1.163E-04	0.2031	7.003E-03	0.2099	0.04738
0.5	0.65	23.477	1.7375	1.096E-04	0.2023	5.317E-03	0.2093	0.04714
0.5	0.7	23.499	1.737	1.028E-04	0.2016	3.876E-03	0.2088	0.04690
0.5	0.75	23.52	1.7365	9.608E-05	0.2009	2.672E-03	0.2083	0.04666
0.5	0.8	23.541	1.7361	8.938E-05	0.2002	1.698E-03	0.2078	0.04643
0.5	0.85	23.562	1.7356	8.277E-05	0.1997	9.493E-04	0.2073	0.04620
0.5	0.9	23.582	1.7352	7.627E-05	0.1991	4.194E-04	0.2068	0.04597
0.5	0.95	23.603	1.7348	6.992E-05	0.1986	1.043E-04	0.2064	0.04575
Market price contracts								
0.5	0.6	24	1.578	0	0.2048	1.037E-02	0.2048	0.01037
0.5	0.65	24	1.578	0	0.2048	1.037E-02	0.2048	0.01037
0.5	0.7	24	1.578	0	0.2048	1.037E-02	0.2048	0.01037
0.5	0.75	24	1.578	0	0.2048	1.037E-02	0.2048	0.01037
0.5	0.8	24	1.578	0	0.2048	1.037E-02	0.2048	0.01037
0.5	0.85	24	1.578	0	0.2048	1.037E-02	0.2048	0.01037
0.5	0.9	24	1.578	0	0.2048	1.037E-02	0.2048	0.01037
0.5	0.95	24	1.578	0	0.2048	1.037E-02	0.2048	0.01037
Formula price contracts								
0.5	0.6	35.297	4.0414	1.724E-03	0.5198	1.936E-02	0.4286	0.01239
0.5	0.65	34.855	4.038	1.603E-03	0.5365	1.990E-02	0.4424	0.01273
0.5	0.7	34.469	4.0367	1.513E-03	0.5514	2.038E-02	0.4546	0.01304
0.5	0.75	34.129	4.0367	1.444E-03	0.5646	2.080E-02	0.4655	0.01331
0.5	0.8	33.827	4.0377	1.389E-03	0.5765	2.119E-02	0.4753	0.01356
0.5	0.85	33.557	4.0394	1.346E-03	0.5872	2.154E-02	0.4842	0.01378
0.5	0.9	33.314	4.0414	1.311E-03	0.5970	2.185E-02	0.4922	0.01399
0.5	0.95	33.095	4.0438	1.282E-03	0.6058	2.214E-02	0.4995	0.01417
Cost plus contracts								
0.5	0.6	34.706	4.1987	9.082E-02	0.5195	1.076E-02	0.4390	0.04509
0.5	0.65	34.256	4.2	8.803E-02	0.5349	8.727E-03	0.4528	0.04794
0.5	0.7	33.866	4.2027	8.606E-02	0.5484	6.731E-03	0.4648	0.05049
0.5	0.75	33.527	4.2061	8.465E-02	0.5602	4.872E-03	0.4754	0.05278
0.5	0.8	33.23	4.2098	8.363E-02	0.5706	3.232E-03	0.4848	0.05483
0.5	0.85	32.968	4.2137	8.290E-02	0.5799	1.875E-03	0.4932	0.05667
0.5	0.9	32.736	4.2176	8.237E-02	0.5882	8.566E-04	0.5006	0.05833
0.5	0.95	32.53	4.2214	8.199E-02	0.5957	2.194E-04	0.5073	0.05984

Table 1.6 Summary of impacts of contract supplies under each contract

	Expected Spot market price	Variance of spot price	Packer profit	Variance of packers' profit	Expected utility of contract producers	Variance of contract producers' income	Expected utility of indep producers	Variance of indep producers' income
Fixed-price with risk neutrality	*No change and lowest*	*No change 2nd highest*	*No change and 2nd lowest*	*No change and lowest*	*No change and 3rd lowest*	*Decrease with beta and lowest*	*No change*	*No change*
Fixed-price with risk aversion	*Positive and 2nd lowest*	*Positive and highest*	*Positive and 3rd lowest*	*Positive and 2nd lowest*	*Positive and lowest*	*Positive and 2nd lowest*	*positive*	*positive*
Market-price contract	*No change and lowest*	*No change 2nd lowest*	*No change and lowest*	*No change and lowest*	*No change and 2nd lowest*	*No change and 2nd highest*	*No change and lowest*	*No change and lowest*
Formula-price contract	*Positive 2nd highest*	*Negative and lowest*	*Positive 2nd highest*	*Positive 2nd highest*	*Positive and highest*	*Positive and highest*	*Positive and 2nd highest*	*Positive and 2nd lowest*
Cost-plus contract	*Positive and highest*	*Positive 3rd lowest*	*Positive and highest*	*Positive and highest*	*Positive and 2nd highest*	*Negative and 3rd highest*	*Positive and highest*	*positive*

Notes:

1. "No change" indicates that contract supplies have no effect on the variable listed in the column heading. "Positive" indicates that contract supplies have a positive relationship with that variable; "negative" indicates a negative relationship.

2. The order (ranking) is based on the relative magnitude of variable listed in the column heading for all five contract scenarios. If no order is indicated, relative rankings are indeterminate. The shaded boxes reflect the two most preferred rankings.

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ESSAY II

STATIC AND DYNAMIC EFFICIENCY OF POOLED BROILER CONTRACTS: RELATIVE PERFORMANCE CONTRACTS VS. FIXED PERFORMANCE CONTRACTS

2.1 Introduction

From modest beginnings prior to World War II, the U.S. broiler industry grew into one of the most integrated of all agricultural sectors. Annual per capita consumption of broilers in the United States increased more than 100-fold from 0.7 pounds in 1935 to 72 pounds in 1997 (Martinez 1999). In 1950, 95 percent of broiler producers were independent. Today, the U.S. broiler industry is one of several agricultural sectors that extensively employ contracts as a method of vertical coordination between processors and producers and more than 95 percent of chickens are grown under contract (Martinez 1999). Contracts and vertical integration played an important role in the adoption of new technology and the coordination of production with consumer preferences for quality and consistency. In the 1950s, large capital requirements, coupled with declining, highly variable broiler prices, made broiler production a risky business. Consequently, large feed companies established broiler production contracts with growers. As the market for high-quality broilers grew, poultry processors replaced feed companies as integrators or contractors. According to Perry, Banker, and Green (1999), product quality, standardization, product consistency, identification, and risk reduction and risk management in the production process are among the benefits from contracting that accrue directly to processors.

Most major processors now control the vertical stages in the broiler industry, from breeders to market-ready products, through production contracts or vertical integration. These processor-integrators, such as Tyson Foods, Inc., breed the parent stock, produce hatching eggs, and hatch the eggs. They then provide baby chicks, feed, and veterinary services to the contracted growers who raise the chicks. Growers provide the chicken houses and labor. After the chickens grow to the market weight, the grown broilers are harvested, slaughtered, and dressed for market or processed further (Martinez 1999).

A broiler production contract usually contains three types of compensation for grower service: a) a base payment, b) a performance payment, and c) a disaster payment (Perry, Banker, and Green 1999). The base payment is a fixed fee per pound of live meat produced. The performance payment is a bonus or punishment based on the difference between an individual grower's settlement cost and either the average settlement costs of all contractor flocks or a fixed settlement cost. Broiler contracts with average settlement costs are usually called relative-performance contracts (RPCs), while those with a fixed settlement cost are called fixed-performance contracts (FPCs). Feed conversion ratio (feed used per pound of broiler produced) is often used as a proxy for settlement costs. A high feed conversion ratio indicates low settlement costs and better grower performance. According to Knoeber and Thurman (1995), broiler production contracts changed in 1984 to relative-performance contracts from rank-order tournaments in which growers were rewarded solely based on their ordinal ranking in a production tournament.

While a majority of poultry contract growers may be satisfied with most aspects of their contractual arrangements, many are dissatisfied with at least one aspect of their contractual arrangements (Hayenga et al. 2000). According to Hayenga et al., their complaints focus primarily on the system that bases their bonus on how their performance compares to other growers. Many broiler growers complain that relative-performance payments are biased and unfair and are generally opposed to a contract in which their payments depend on how their neighbors perform. For example, consecutive flocks grown by the same grower, while having similar production costs, can receive substantially different bonus payments depending on the performance of other growers in the settlement group. Growers have expressed exasperation over this form of remuneration since they have no way of anticipating how large their payments will be (Hayenga et al. 2000). Therefore, some states, such as North Carolina, have considered legislation that prohibits the use of relative-performance contracts. Various forms of legislation aimed at regulating broiler contracts were also passed in Minnesota, Kansas and Wisconsin (Tsoulouhas and Vukina, 2000).

Thus, this essay attempts to compare the optimal incentives of relative-performance contracts and fixed-performance contracts in both a static model and a two-period dynamic model. Asymmetric information in terms of both unobservable grower abilities and unobservable production effort is introduced into the model. Further, the last part of the analysis investigates the case where a processor institutes two tournaments in a single period as a means to mitigate the adverse selection problem. The final section of this essay discusses policy implications.

2.2 Literature Review

Broiler production and relative-performance contracts

Broiler production is usually coordinated by processors or integrators. Processors' payments to the growers under contract are based on relative performance of each grower. Following Levy and Vukina (2001, 2002), a typical payment function under a relative-performance contract can be constructed as follows: $w_i = \alpha + \beta[x_i - \frac{1}{n}\sum_{j=1}^{n} x_j]$,

where α represents the base payment, β represents the bonus payment, x_i is live output of chickens produced by grower *i*, and w_i is the payment received by grower *i*. The term $\frac{1}{n}\sum_{j=1}^{n} x_j$ is the peer average performance. In a few analytic papers, such as Roe and Wu (2003) and Tsoulouhas and Vukina (2000), grower *i*'s performance is excluded from the calculation of the peer average. However, because results from past studies show that this assumption does not significantly affect the contract performance when the number of growers in the competition is large, the peer average will be based on all *n* growers throughout this essay. If the average performance is replaced by a fixed number, the contract becomes a "fixed-performance contract". Relative-performance contracts assume that flocks in different farms within the same group are grown under relatively homogenous conditions. Moreover, these contracts require that the calculation of the group average performance includes growers whose flocks where harvested at approximately the same time, so that they are all exposed to the same influence of common stochastic factors. The essence of the contract settlement is the elimination of the common production risk from the responsibility of a grower through relative-performance mechanism. However, as indicated above, the reward to an individual grower will be substantially different when heterogeneous growers are in the same comparison group and the group composition continually changes. Thus, unobservable grower heterogeneity introduces new risks (Goodhue 2000).

Past Studies

Several recent papers have studied relative-performance incentives or tournament contracts. Goodhue (2000) models an adverse selection problem with two unobservable types in a broiler production setting. She finds that in, the presence of unobservable types of growers, the average performance is not a sufficient statistic[8] for the vector of individual outputs, so the average output cannot be used to calculate an optimal sharing rule. She also concludes that by controlling inputs a processor can reduce information rents it must pay to high productivity growers. However, relative-performance incentives are not explicitly modeled in her paper. Tsoulouhas and Vukina (2000) compare a relative-performance broiler contract with a fixed-performance contract in the presence of moral hazard. Under the assumption that common production risk dominates group composition risk, they conclude that the enforcement of fixed-performance standards absent any regulations on the piece-rate bonus will result in less income insurance and welfare to the growers and reduce integrator welfare as well. Further, social surplus is reduced because integrator welfare is reduced and grower welfare is unchanged. In contrast, replacing relative-performance with fixed-performance contracts accompanied by a correctly specified piece-rate bonus can increase grower welfare and may or may not reduce social surplus. However, integrator welfare is unambiguously reduced.

Che and Yoo (2001) argue that, relative to joint performance evaluation in teams, the relative-performance evaluation scheme is not optimal in the repeated setting because it is susceptible to collusion. As the authors indicate, joint performance evaluation performs better when workers work closely and the relationship among workers has a long life span. On the other hand, relative-performance evaluation works better when workers have a short-term relationship. In the context of the broiler industry, each grower is rewarded based on his relative-performance among a group of growers whose broilers are harvested approximately at the same time. Timing issues, therefore, ensure that composition of the comparison group changes over time, and hence, makes growers'

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[8] Further discussion can be found in Holmström (1982).

relationships very short. Further, the lack of repeated interaction among a fixed group of growers naturally prevents them from colluding with each other.

A recent paper by Roe and Wu (2003) compares relative-performance contracts and fixed-performance contract in a two-period dynamic model following steps similar to those described in Meyers and Vickers (1997). Roe and Wu conclude that banning tournaments can increase total surplus by mitigating the well-known ratchet effect only in a dynamic model, while banning tournaments can never be welfare improving in a static setting. In many ways, the model presented in this essay is a generalization of Roe and Wu (2003). However, significant differences arise in the model specification and interpretation of fixed-performance contracts and relative-performance contracts.

Several empirical studies have also tested the properties of broiler contracts. Knoeber and Thurman (1994) use a sample of 75 growers from 1,174 flocks under contract from November 1981 to December 1985 to test three predictions from the theory of tournaments: (1) changes in the level of prizes that leave prize differentials unchanged should not affect performance; (2) in mixed tournaments, more able players should choose less risky strategies; and (3) processors should attempt to handicap players of unequal ability or reduce mixing to avoid the disincentive effects of mixed tournaments. They find that the data are consistent with each of their predictions. Knoeber and Thurman (1995) conduct another study to test the efficiency of broiler contracts using the same data set. They compare the actual payment series under tournament contracts with the “contract without tournaments” and “no contracts” by calculating the ratio of simulated standard deviation to actual standard deviation. They conclude that relative-performance production contracts reduce grower’s risk and shift 97% of risks, including price risk and common production risk, from growers to integrators. Goodhue et al. (1998) find evidence to support the hypotheses that high-ability growers receive larger, more frequent, and more consistent flock assignments. Hegde and Vukina (2002) use a sample of 1,366 growers and 8,041 flocks to compare welfare of the contracts with no market-price clause from June 1984 to December 1985 and contracts with a market-price clause from July 1995 to July 1997. They find that contracts that include the market-price clause are welfare superior on a payment per-pound basis as compared to those without the market-price clause. However, for total per-flock payments, their results depend on the grower attitude toward risk. Levy and Vukina (2001) conduct a similar study to compare the league composition effects in broiler contracts. By regressing growers' production costs on grower dummy variables and time dummy variables, they find strong evidence of heterogeneity of grower types and existence of large common production shocks. Further, they compare the welfare of simple piece-rate contracts and relative-performance contracts with fixed leagues and random leagues in both a single tournament and a sequence of tournaments and conclude that RPC can outperform FPC only in a dynamic setting under certain conditions.

An earlier group of studies analyze rank-order tournaments contracts. As discussed above, rank-order tournament contracts are similar to relative-performance contracts since they both provide incentives for agents to compete among a group of agents. Therefore, the methodology used in those studies provides some insights into analysis of relative-performance contracts. Lazear and Rosen (1981) analyze the efficiency of a rank-order contract for a finite contest of risk-neutral agents by considering a tournament contract in the labor market. However, they consider only a special case of rank-order contract rather than a whole class of such contracts. Green and Stokey (1983) study the efficiency of a rank-order tournament contract with or without a common shock. They find that for a finite number of agents, in the absence of common shocks, the use of tournaments is dominated by optimal independent contracts. For a large group of agents or when the distribution of the common shock is very diffuse, a rank-order tournament dominates independent contracts. Malcomson (1986) establishes, for any given piece-rate contract, that there exists an equivalent rank-order contract with the same outcome. This finding implies that there exists a first-best rank-order contract for a contest among an infinite number of risk-neutral agents under dual information asymmetry. A recent paper by Yun (1997) provides a more comprehensive analysis of rank-order contracts. Yun analyzes the efficiency of the rank-order contracts for a finite number of risk-neutral agents under both moral hazard and adverse selection and shows that the set of first-best rank-order contracts has the following properties: (i) the first-best contract of each type should penalize less than some critical fraction of contestants, where the fraction is never greater than one half; (ii) The critical fraction is smaller and penalty larger for the contests of higher ability types; and (iii) although both penalty-giving and prize-giving contracts work equally well as effort schemes, a penalty-giving contract is better than a prize-giving contract in inducing self-selection among different types of agents.

2.3 Objectives

The primary objectives of this essay are to investigate the efficiency of broiler-industry-style relative-performance contracts in the presence of asymmetric information and to compare various relative-performance contracts with fixed-performance contracts.

Optimal incentives will be derived under relative-performance contracts and fixed-performance contracts with both moral hazard and adverse selection. The moral hazard reflects the fact that growers choose an unobservable effort level after the contract is signed, while the adverse selection reflects the fact that heterogeneous unobservable ability types of growers exist before the contract is signed. This essay compares relative-performance contracts with fixed-performance contracts with respect to their optimal incentives in both a static model and a two-period dynamic model. Two specific scenarios of the two-period dynamic relative-performance contracts are investigated: the *current-period RPC* and the *previous-period RPC*. More precisely, the current-period RPC rewards bonuses to growers using the group average performance in the current period as a standard, while the previous-period RPC rewards each grower by comparing his performance with the average performance of the same group of growers in the previous period.

This essay's contributions to the literature stem from its general methodology and its policy implications. As discussed above, most existing literature on relative-performance contracts assumes either moral hazard or adverse selection in a static setting. The only analysis of dynamic relative-performance contracts was presented by Roe and Wu (2003), which was based on Meyers and Vickers (1997). However, Roe and Wu's interpretation of fixed-performance contracts and relative-performance contracts has significant differences from broiler contracts being used in the real world. This essay, which incorporates with both moral hazard and adverse selection, not only compares various relative-performance contracts with fixed-performance contracts in a dynamic setting, but also discusses improvements of the static mixed-type relative-performance contract. Thus, compared with existing literature, this essay provides a more thorough, more comprehensive, and more practical analysis of broiler contracts. The second contribution of this paper lies in the policy implications of the theoretical results. In spite of growers' complaints about the contemporaneous relative-performance contracts used in the broiler industry, the various theoretical specifications in this essay largely justify the popularity and superiority of relative-performance contracts relative to fixed-performance contracts. This essay shows that, under certain conditions, relative-performance contracts perform better than fixed-performance contracts from the perspective of growers' and processors' welfares.

This essay develops a general model that is applied to three related cases: The first case compares a static relative-performance contract and a fixed-performance contract. The second case extends the static model into a two-period full-commitment model and a two-period dynamic model. Two specific sub-cases of the two-period dynamic RPC are then investigated in detail: the *current-period RPC* and the *previous-period RPC*. The third case investigates two pooled tournaments in a static setting. Model results from all cases and their policy implications are discussed in the final section.

2.4 The Model

In general, a payment schedule of broiler contracts contains a base payment and a bonus or discount payment based on growers' relative-performance. In this essay, we will adopt the setup described in Roe and Wu (2003) with some simplifying modifications. However, significant differences in the assumptions underlying the payment schedules for the fixed-performance contract and the relative-performance contract lead to results that are substantially different to Roe and Wu's.

As described in the previous section, a typical payment function for grower *i* at time *t* under a relative-performance contract can be constructed as:

$$(2.1)\quad w_{it} = \alpha + \beta[x_{it} - \frac{1}{n}\sum_{j=1}^{n} x_{jt}].$$

The calculation of the group's average performance includes all growers whose flocks were harvested at approximately the same time. It is assumed that each grower produces only one flock in each period throughout the essay. Further, it is assumed that the live output produced by each grower is given by $x_{it} = x(e_{it}, a_i, z_t, u_{it})$, where e_{it} is grower *i*'s effort exerted in period *t*, a_i is grower *i*'s ability realized before the contract is signed, z_t is the common shock borne by all growers in period *t*, and u_{it} is grower *i*'s idiosyncratic risk in period *t*. It is assumed that u_{it} is an i.i.d normally distributed random variable across growers and periods with mean zero and variance σ_u^2, while a_i is uniformly distributed in the range $[\overline{a}, \underline{a}]$ with $0 < \underline{a} < \overline{a} < \infty$.[9] Additionally, for the moment, z_t is an i.i.d normal random variable across periods with mean zero and variance σ_z^2. A more complicated specification of z_t will be discussed below in a two-period dynamic model. Recall that we assume both growers' abilities and efforts are not

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[9] Note that there is significant difference between the interpretation of ability *a* in Roe and Wu (2003) and Meyers and Vickers (1997) and that in this essay. More precisely, Roe and Wu (2003) and Meyers and Vickers (1997) treat *a* as a random variable drawn after the contract is offered. Instead, this essay treats *a* as a random variable drawn before the contract is offered. Thus, growers' ability *a*, which is unobservable to the processor, is deterministic after the contract is offered and its distribution function is known to both the processor and the growers.

directly observable by the processor. However, the distributions specified above are public information to both the processor and the growers.

In particular, the following output structure is usually used in the existing literature

(2.2) $x_{it} = a_i + e_{it} + z_t + u_{it}$.

Hence, the variance of x_{it} is $\text{var}(x_{it}) = \sigma_z^2 + \sigma_u^2$ and the covariance between any x_{it} and x_{jt} is $\text{cov}(x_{it}, x_{jt}) = \sigma_z^2$.[10]

Note that effort only affects the mean of the output in this structure. This significantly simplifies the analysis in the following sections. However, a more complicated structure in which effort affects both the mean and the variance of the output could be used, but is not investigated in this essay due to length limitations. In addition, each grower's ability does not change over time.

The processor is risk neutral and has a profit function, $\pi_t(x,w) = \sum_{i=1}^{n}(x_{it} - w_{it})$, where w_{it} is specified in (2.1). Each grower with ability a_i has a time-separable utility function $U_{it}(w_{it}, e_{it}, a_i) = u(w_{it}) - C(e_{it}, a_i)$, where the utility function is strictly concave and the disutility function takes the form $C(e_{it}, a_i) = \frac{1}{2a_i} e_{it}^2$. Further, we adopt the commonly used assumption that growers' utility function has the property of constant absolute risk aversion, $u(w_{it}) = -\exp(-rw_{it})$, where r is the Arrow-Pratt coefficient of absolute risk aversion. Thus the expected utility $E_t[U_{it}(\cdot)]$ is tantamount to

(2.3) $E_t[U_{it}(\cdot)] \propto Ew_{it} - \frac{1}{2} r\,\text{var}(w_{it}) - \frac{1}{2a_i} e_{it}^2$.

[10] Due to the assumption that growers' ability a is a random variable drawn before the contract is offered, the corresponding variances and covariances are different from those specified in Roe and Wu (2003).

Throughout the essay, we use E_t to represent the mathematical expectation operator conditional on information available at the beginning of period *t*.

Note that, in this setup, growers differ in their disutilities of effort. Lower ability types incur higher costs relative to higher ability types for a same level of effort. In addition, marginal disutility of effort decreases with ability as well.

We adopt the notational convention of writing $\mathbf{a}_{-i} = (a_1,...a_{i-1}, a_{i+1},...a_n)$, $\mathbf{a} = (a_i, \mathbf{a}_{-i})$, $\mathbf{e}_{-it} = (e_{1t},...e_{i-1,t}, e_{i+1,t},...e_{nt})$, $\mathbf{e}_t = (e_{it}, \mathbf{e}_{-it})$, and $\mathbf{x}_{-it} = (x_{1t}, x_{2t},..x_{i-1,t}, x_{i+1,t},..x_{nt})$, $\mathbf{x}_t = (x_{it}, \mathbf{x}_{-it})$ throughout the essay.

2.4.1 A Static Model with One Tournament

In this case, a processor signs a one-period contract with *n* growers. The processor observes only the live output from each grower in the group and uses either a relative-performance contract or a fixed-performance contract to reward the growers. Thus, the contract offered to all growers specifies a payment schedule depending on $\{\alpha, \beta, \mathbf{x}\}$. In the static model, the subscript *t* will be omitted for all variables.

Given the assumptions described above, the processor maximizes its expected profits subjected to incentive-compatibility constraints and growers' participation constraints. Since only one contract is offered to all growers regardless of their abilities in one period, the processor must offer a pooling contract across all ability levels.

Thus, the processor solves the problem:

(2.4) $\max\limits_{\alpha,\beta} E_a \{\sum_{i=1}^{n} (Ex_i - Ew_i)\}$

subject to

(2.5) $E_a[EU_i] = E_a[Ew_i - \frac{1}{2} r \operatorname{var}(w_i) - \frac{1}{2a_i} e_i^2] \geq 0$,

(2.6) $e_i \in \arg\max \{Ew_i - \frac{1}{2} r \operatorname{var}(w_i) - \frac{1}{2a_i} e_i^2\}, \qquad \forall i$.

The participation condition (2.5) states that an average-ability grower obtains his reservation utility zero under the pooling contract offered by the processor, while the incentive-compatibility constraint (2.6) requires that each grower optimally chooses his effort by maximizing the expected utility.

Standard results from contract theory[11] require that the participation constraint (2.5) is always binding because otherwise, the processor can always reduce the payment to the growers until it reaches their reservation utility level. Following Roe and Wu (2003) and Meyers and Vickers (1997), given the binding participation constraint, the processor's objective can be transformed into maximizing the total welfare obtained by the processor and all growers. Precisely, denote the expected total welfare obtained by the processor and all growers as

(2.7)

$$W = E_a\{\sum_{i=1}^{n}(Ex_i - Ew_i)\} = E_a\{\sum_{i=1}^{n}(Ex_i - (\frac{1}{2} r \operatorname{var}(w_i) + \frac{1}{2a_i} e_i^2)\}$$
$$= E_a\{\sum_{i=1}^{n}(Ex_i - \frac{1}{2} r \operatorname{var}(w_i) - \frac{1}{2a_i} e_i^2)\}$$

Thus, the optimal contract chosen by maximizing (2.7) will be Pareto optimal. However, we should note that maximization of the total welfare W is equivalent to maximizing the processor's expected profit only if the participation constraint is binding. For cases

[11] Good references on this topic include Mas-Collel, Whinston, and Green (1995) and Salanie (1997).

where the two concepts are not equivalent, later discussed in the section 2.4.3, we assume that the processor always maximizes his expected profit rather than total welfare.

1) *Fixed-performance contract*

To first investigate the optimal incentives under a fixed-performance contract, denote the optimal contract as $C_F = \{\alpha_F, \beta_F\}$. Assuming the fixed standard used in the contract is s, the payment to each grower (2.1) becomes

(2.8) $w_i = \alpha_F + \beta_F[x_i - s], \forall i$.

Hence,

(2.9) $Ew_i = \alpha_F + \beta_F[a_i + e_i - s]$, and

(2.10) $\mathrm{var}(w_i) = \beta_F^2 \,\mathrm{var}(x_i) = \beta_F^2(\sigma_z^2 + \sigma_u^2)$.

Substituting (2.9) and (2.10) into the problem (2.4) - (2.6), the processor's problem becomes

(2.11)

$$W_F = \max_{\alpha_F, \beta_F} E_a\{\sum_{i=1}^n (Ex_i - \frac{1}{2} r \,\mathrm{var}(w_i) - \frac{1}{2a_i} e_i^2)\},$$

subject to (2.12) $E_a[EU_i] = E_a[\alpha_F + \beta_F[a_i + e_i - s] - \frac{1}{2} r\beta_F^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_i^2] \geq 0$,

(2.13) $e_i \in \arg\max\{\alpha_F + \beta_F[a_i + e_i - s] - \frac{1}{2} r\beta_F^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_i^2]\}, \forall i$.

First, from the incentive-compatibility constraint (2.13), each grower's optimal effort must satisfy

(2.14) $e_i = a_i \beta_F$

Second, the participation constraint (2.12) must be binding because otherwise the processor can always reduce the base payment under it reaches each grower's reservation utility level. Hence, using (2.7), the processor's problem is equivalent to

(2.15)

$$W_F = \max_{\alpha_F, \beta_F} E_a \{ \sum_{i=1}^{n} (Ex_i - \frac{1}{2} r \,\mathrm{var}(w_i) - \frac{1}{2a_i} e_i^2) \}$$
$$= \max_{\alpha_F, \beta_F} E_a \{ \sum_{i=1}^{n} (a_i + e_i - \frac{1}{2} r \beta_F^2 (\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_i^2) \}.$$

Since a_i is assumed to be uniformly distributed in the range $[\bar{a}, \underline{a}]$ with $0 < \underline{a} < \bar{a} < \infty$, we denote the population mean of a_i as $a_m = \frac{\bar{a} + \underline{a}}{2}$. Substituting (2.14) into (2.15) and taking expectation with respected to a yields

(2.16) $W_F = \max_{\alpha_F, \beta_F} \sum_{i=1}^{n} (a_m + a_m \beta_F - \frac{1}{2} r \beta_F^2 (\sigma_z^2 + \sigma_u^2) - \frac{a_m}{2} \beta_F^2)$

Taking the derivative with respect to β_F yields

(2.17) $\frac{\partial W_F}{\partial \beta_F} = n(a_m - r\beta_F (\sigma_z^2 + \sigma_u^2) - a_m \beta_F) = 0$,

from which we can compute the optimal choice of β_F,

(2.18) $\beta_F = \dfrac{1}{1 + \dfrac{r}{a_m}(\sigma_z^2 + \sigma_u^2)}$

Several characteristics are apparent from examining the bonus payment β_F.[12] First, since only one contract is offered to all growers regardless of their abilities, the bonus payment β_F is same for all possible levels of grower abilities. Second, since growers bear all production uncertainty under the fixed-performance contract, both the common

[12] The form of the bonus β_F is also similar to results in Levy and Vukina (2002).

shock and the idiosyncratic shock affect the bonus payment. Specifically, the bonus payment decreases with the variance of either of the random shocks. Third, the bonus payment is positively related to the average ability level in the group and negatively related to growers' risk aversion, however, the fixed standard s specified in the contract does not affect the bonus payment.

From the binding participation constraint (2.12), the base payment can be solved as

(2.19)

$$\alpha_F = \frac{1}{2} r\beta_F^2(\sigma_z^2+\sigma_u^2) + \frac{1}{2} a_m \beta_F^2 - \beta_F[a_m + a_m\beta_F - s]$$
$$= \frac{1}{2}\beta_F^2[r(\sigma_z^2+\sigma_u^2) - a_m] + \beta_F[s - a_m]$$
$$= \frac{[r(\sigma_z^2+\sigma_u^2) - a_m]}{2[1+\frac{r}{a_m}(\sigma_z^2+\sigma_u^2)]^2} + \frac{s - a_m}{1+\frac{r}{a_m}(\sigma_z^2+\sigma_u^2)}.$$

While parameters affect the base payment in a complicated fashion, it is worth noting that the fixed standard s is positively related to the base payment due to the binding participation constraint.

Further, the total welfare under the optimal fixed-performance contract can be computed,

(2.20)

$$W_F = \sum_{i=1}^n (a_m + a_m\beta_F - \frac{1}{2} r\beta_F^2(\sigma_z^2+\sigma_u^2) - \frac{a_m}{2}\beta_F^2)$$
$$= n(a_m + a_m\beta_F - \frac{1}{2} a_m\beta_F^2[1+\frac{r}{a_m}(\sigma_z^2+\sigma_u^2)])$$
$$= n(a_m + a_m\beta_F - \frac{1}{2} a_m\beta_F) = na_m(1+\frac{1}{2}\beta_F) = na_m[1+\frac{1}{2(1+\frac{r}{a_m}(\sigma_z^2+\sigma_u^2))}].$$

2) *Relative-performance contract*

Under a relative-performance contract, the processor uses the peer average performance as a standard to reward each grower. Denote the optimal contract as $C_R = \{\alpha_R, \beta_R\}$. Hence, the payment to each grower becomes

(2.21) $w_i = \alpha_R + \beta_R [x_i - \frac{1}{n}\sum_{j=1}^{n} x_j] = \alpha_R + \beta_R [x_i - \bar{x}], \quad \forall i$.

Hence,

(2.22) $Ew_i = \alpha_R + \beta_R [a_i + e_i - \frac{1}{n}\sum_{j=1}^{n}(a_j + e_j)]$, and

(2.23)

$$\begin{aligned}
\mathrm{var}(w_i) &= \beta_R^2 \,\mathrm{var}(x_i - \frac{1}{n}\sum_{j=1}^{n} x_j]) = \beta_R^2 \,\mathrm{var}(\frac{n-1}{n} x_i - \frac{1}{n}\sum_{j=1,\, j\neq i}^{n} x_j]) \\
&= \beta_R^2 [\frac{(n-1)^2}{n^2}\mathrm{var}(x_i) + \frac{n-1}{n^2}\mathrm{var}(x_j) - 2\frac{n-1}{n}\frac{1}{n}(n-1)\,\mathrm{cov}(x_i, x_j) \\
&+ 2\frac{1}{n^2}\binom{n-1}{2}\mathrm{cov}(x_{j\neq i}, x_{k\neq i})] \\
&= \beta_R^2 [(\frac{(n-1)^2}{n^2} + \frac{n-1}{n^2})(\sigma_z^2 + \sigma_u^2) - 2\frac{(n-1)^2}{n^2}\sigma_z^2 + \frac{(n-1)(n-2)}{n^2}\sigma_z^2] = \beta_R^2 \frac{n-1}{n}\sigma_u^2
\end{aligned}$$

Note that the variance of each grower payment depends only on the idiosyncratic shock without being affected by the common shock.

Substituting (2.22) and (2.23) into the problem (2.4) - (2.6), the processor's problem becomes

(2.24) $W_R = \max_{\alpha_R, \beta_R} E_a \{\sum_{i=1}^{n}(Ex_i - \frac{1}{2} r\,\mathrm{var}(w_i) - \frac{1}{2a_i}e_i^2)\}$

subject to

(2.25) $E_a[EU_i] = E_a[\alpha_R + \beta_R[\frac{n-1}{n}(a_i + e_i) - \frac{1}{n}\sum_{j=1, j\neq i}^{n}(a_j + e_j)] - \frac{1}{2} r \beta_R^2 \frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_i^2] \geq 0$

(2.26) $e_i \in \arg\max\{\alpha_R + \beta_R[\frac{n-1}{n}(a_i + e_i) - \frac{1}{n}\sum_{j=1, j\neq i}^{n}(a_j + e_j)] - \frac{1}{2}r\beta_R^2 \frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_i^2\}$, $\forall i$

From the incentive-compatibility constraint (2.26), each grower chooses the optimal effort such that

(2.27) $e_i = \frac{n-1}{n}a_i\beta_R$.

Hence, substituting (2.27) into (2.7), the total welfare under the contract is

(2.28)

$$W_R = \max_{\alpha_R, \beta_R} E_a\{\sum_{i=1}^{n}(Ex_i - \frac{1}{2}r\,\mathrm{var}(w_i) - \frac{1}{2a_i}e_i^2)\} = E_a\{\sum_{i=1}^{n}(a_i + e_i - \frac{1}{2}r\beta_R^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_i^2)\}$$

$$= \sum_{i=1}^{n}(a_m + \frac{n-1}{n}a_m\beta_R - \frac{1}{2}r\beta_R^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2}(\frac{n-1}{n})^2 a_m\beta_R^2)$$

Taking derivative with respect to β_R yields

(2.29) $\frac{\partial W_R}{\partial \beta_R} = n(\frac{n-1}{n}a_m - r\beta_R\frac{n-1}{n}\sigma_u^2 - (\frac{n-1}{n})^2 a_m\,\beta_R) = 0$,

from which

(2.30) $\beta_R = \frac{a_m}{\frac{n-1}{n}a_m + r\sigma_u^2}$

The most prominent feature of this bonus payment is that it is independent of the common shock. As a matter of fact, this is one of the most critical reasons that researchers favor relative-performance contracts under certain circumstances discussed below.

Further, the participation constraint (2.25) must be binding because otherwise the processor can always reduce the base payment until it reaches each grower's reservation utility level. Thus, the base payment can be computed as

(2.31)

$$\alpha_R = \frac{1}{2} r\beta_R^2 \frac{n-1}{n}\sigma_u^2 + \frac{1}{2}(\frac{n-1}{n})^2 a_m\, \beta_R^2 - \beta_R[a_m + \frac{n-1}{n} a_m\beta_R - \frac{1}{n}\sum_{j=1}^n (a_m + \frac{n-1}{n} a_m\beta_R)]$$
$$= \frac{1}{2} r\beta_R^2 \frac{n-1}{n}\sigma_u^2 + \frac{1}{2}(\frac{n-1}{n})^2 a_m\, \beta_R^2 = \frac{1}{2}\frac{n-1}{n}\beta_R^2 (r\sigma_u^2 + \frac{n-1}{n} a_m) = \frac{a_m}{2[1 + \frac{r}{a_m}\frac{n}{n-1}\sigma_u^2]}.$$

Hence, we can compute the total welfare under the optimal relative-performance contract:

(2.32)

$$W_R = \sum_{i=1}^n (a_m + \frac{n-1}{n} a_m\beta_R - \frac{1}{2} r\beta_R^2 \frac{n-1}{n}\sigma_u^2 - \frac{1}{2}(\frac{n-1}{n})^2 a_m\, \beta_R^2)$$
$$= n[a_m + \frac{n-1}{n} a_m\beta_R - \frac{1}{2}\frac{n-1}{n}\beta_R^2 (r\sigma_u^2 + \frac{n-1}{n} a_m)]$$
$$= n[a_m + \frac{n-1}{n} a_m\beta_R - \frac{1}{2}\frac{n-1}{n}\frac{(a_m)^2}{(r\sigma_u^2 + \frac{n-1}{n} a_m)}] = n[a_m + \frac{1}{2}\frac{n-1}{n} a_m\beta_R]$$
$$= n a_m [1 + \frac{1}{2(1 + \frac{r}{a_m}\frac{n}{n-1}\sigma_u^2)}].$$

Comparing (2.20) and (2.32), the two welfare expressions, yields the following proposition:

Proposition 1: *a*) $W_R > W_F$ if $\sigma_z^2 > \frac{1}{n-1}\sigma_u^2$, *b*) $W_R < W_F$ if $\sigma_z^2 < \frac{1}{n-1}\sigma_u^2$, *c*) $W_R = W_F$ if $\sigma_z^2 = \frac{1}{n-1}\sigma_u^2$, and *d*) $W_R > W_F$ if $n \to \infty$.

The proof is straightforward. From (2.20) and (2.32), we can compute

$$W_R - W_F = n a_m [1 + \frac{1}{2(1 + \frac{r}{a_m}\frac{n}{n-1}\sigma_u^2)}] - n a_m [1 + \frac{1}{2(1 + \frac{r}{a_m}(\sigma_z^2 + \sigma_u^2))}]$$
$$= n a_m [\frac{1}{2(1 + \frac{r}{a_m}\frac{n}{n-1}\sigma_u^2)} - \frac{1}{2(1 + \frac{r}{a_m}(\sigma_z^2 + \sigma_u^2))}].$$

Thus, it is equivalent to compare

$$W_R - W_F \propto \frac{r}{a_m}(\sigma_z^2 + \sigma_u^2) - \frac{r}{a_m}\frac{n}{n-1}\sigma_u^2 = \frac{r}{a_m}[\sigma_z^2 - \frac{1}{n-1}\sigma_u^2].$$

Therefore, a) $W_R > W_F$ if $\sigma_z^2 > \frac{1}{n-1}\sigma_u^2$, b) $W_R < W_F$ if $\sigma_z^2 < \frac{1}{n-1}\sigma_u^2$, c) $W_R = W_F$ if $\sigma_z^2 = \frac{1}{n-1}\sigma_u^2$. Similarly, if $n \to \infty$, $W_R - W_F \propto \frac{r}{a_m}[\sigma_z^2 - \frac{1}{n-1}\sigma_u^2] \xrightarrow{n\to\infty} \frac{r}{a_m}\sigma_z^2 > 0$.

This proposition is a standard result in the literature of relative-performance contracts and rank-tournament contracts and is consistent with the literature specific to broiler contracts (e.g., Levy and Vukina 2001). Intuitively, the proposition states that the relative-performance contract performs better than the fixed-performance contract when the common shock dominates the idiosyncratic shock because comparing one grower's performance with other growers approximately at the same time completely eliminates the common production shock borne by all growers. On the other hand, the fixed-performance contract is better when the idiosyncratic shock dominates the common shock because under this condition, comparing a grower's performance to a fixed standard reduces his variance of income relative to that under the relative-performance contract.

In addition, it is easy to verify that the optimal bonus has properties similar to total welfare. The following corollary summarizes these properties without further proof.

Corollary 1.1: *a)* $\beta_R > \beta_F$ if $\sigma_z^2 > \frac{1}{n-1}\sigma_u^2$, *b)* $\beta_R < \beta_F$ if $\sigma_z^2 < \frac{1}{n-1}\sigma_u^2$, *c)* $\beta_R = \beta_F$ if $\sigma_z^2 = \frac{1}{n-1}\sigma_u^2$, and *d)* $\beta_R > \beta_F$ if $n \to \infty$.

The corollary says that when the common shock dominates, not only does the static RPC improve total welfare relative to the static FPC, but also it offers a greater bonus than that under the static FPC.

2.4.2 Two-period Models

The static model is extended to include two time periods in this section. In a dynamic context, ratchet effects might exist due to the presence of asymmetric information.[13] Thus, the optimal contract provided by the processor must account for this potential effect and adjust the intertemporal incentives accordingly. This section consists of three related parts. The first part simply discusses the optimal two-period contracts under full-commitment by the processor and growers. The second part investigates a current-period dynamic relative-performance contract and a fixed-performance contract where neither the processor nor growers can commit to an intertemporal scheme. In this second part, the relative standard used in the contract is the peer average performance in the current period. While the terms and payments schedules in actual contracts are much more complex than those specified in this part, the current-period dynamic relative-performance contract has been widely used in the broiler industry[14]. Therefore, readers should be aware that the model formulated here is highly stylized relative to actual broilers contracts. The third part further extends the model and investigates a dynamic previous-period relative-performance contract and the fixed-performance contract. Here, the term previous-period relative-performance contract is used to indicate that the relative

[13] Freixas, Guesnerie, and Tirole (1985) states that ratchet effects induce firms to underproduce to avoid more demanding schedule in the future as the central planner revises the scheme over time to take into account information provided by the firm's performance.
[14] Good examples of broilers contracts include Tyson Richmond broiler contract, Pilgrim Pride Contract, ConAgra broiler contract, MBA Broiler contract, etc.

standard used in the contract is the peer average performance in the previous period. Although this particular type of contract has not been explicitly used in the broiler industry, we examine this scenario here for the following two purposes: One is to correspond to the concept of all-period ban of relative-performance contracts as defined in Roe and Wu (2003) [15]; the other is because it would be natural to assume that, if current-period tournaments were banned, producers may still use data on past performance to set a fixed standard.

Further, it is assumed that the common shock takes the simple form of a stationary process in the dynamic context:

(2.33) $z_t = \phi z_{t-1} + \varepsilon_t$, $|\phi| < 1$, where $\varepsilon_t \sim i.i.d.N(0, \sigma_\varepsilon^2)$.

With this specification, it is straightforward to verify $z_t \sim N(0, \sigma_z^2)$, where $\sigma_z^2 = \dfrac{\sigma_\varepsilon^2}{1-\phi^2}$,

and $\text{cov}(z_t, z_{t-1}) = \dfrac{\phi\sigma_\varepsilon^2}{1-\phi^2}$. Note that given the stationary process, the relationship between outputs in two periods is similar to that described in Roe and Wu (2003), except that we exclude the possibility of autocorrelation between abilities.

2.4.2.1 Two-period Contracts Under Full Commitment

Before we proceed to the dynamic model, we investigate the optimal two-period contracts under full-commitment . Two conditions describe full-commitment : On one hand, the processor promises beforehand not to use information revealed in the first period to modify the contract in the second period. On the other hand, growers promise

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[15] Roe and Wu (2003) define all-periods ban, in a two-period model, as disallowing the processor of using information concerning player j from any period to develop contract parameters for player i.

not to breach the contract during the contract period. Under full-commitment , since the processor cannot apply information revealed in the first period to the contract in the second period, the optimal contracts in each of the two periods are independent and are exact replications of the static contract in each period. Therefore, no dynamic effect exists in the case.

Specifically, under the relative-performance contract, the processor offers the contract $C_R = \{\alpha_R, \beta_R\}$ in each period, with α_R and β_R specified by (2.31) and (2.30), respectively. Assuming both the processor and the growers discount their profit or utility by a factor δ , the total two-period welfare under the relative-performance contract is

(2.34)

$$W_R^F = (1+\delta)na_m[1+\frac{1}{2(1+\frac{r}{a_m}\frac{n}{n-1}\sigma_u^2)}],$$

where the superscript F denotes full-commitment and the subscript R denotes relative-performance contract.

Similarly, under the fixed-performance contract, the processor offers the contract $C_F = \{\alpha_F, \beta_F\}$ in each period, with α_F and β_F specified by (2.19) and (2.18), respectively. Hence, the total two-period welfare under the fixed-performance contract is

(2.35) $$W_F^F = (1+\delta)na_m[1+\frac{1}{2(1+\frac{r}{a_m}(\sigma_z^2+\sigma_u^2))}],$$

where the superscript F denotes full-commitment and the subscript F denotes fixed-performance contract. Note that Proposition 1 applies to the full-commitment two-period contract as well.

2.4.2.2 Dynamic Fixed-performance Contract (FPC) and Relative-performance Contract (RPC)

In this section, it is assumed that the processor is not fully committed in the second period. Thus, the processor optimally adjusts the second-period incentives using information acquired at the end of the first period. Two scenarios of relative-performance contract will be investigated: current-period RPC and previous-period RPC. Under the current-period relative-performance contract, the relative standard is computed by averaging the performance of growers contracted in the same period; while under the previous-period RPC, the processor uses growers' average performance in the previous period to reward each grower. We still use the same fixed standard in both periods under the fixed-performance contract. In addition, it is assumed that the same growers are under contract in both periods in a two-period model throughout this section.

Given the output structure (2.2) and the distributions of the random shocks, the joint distribution of output $\mathbf{x}$ is

$$(2.36)\quad \mathbf{x} = \begin{pmatrix} x_{11} \\ \vdots \\ x_{n1} \\ x_{12} \\ \vdots \\ x_{n2} \end{pmatrix} \sim N\left[\begin{pmatrix} a_1 + e_{11} \\ \vdots \\ a_n + e_{n1} \\ a_1 + e_{12} \\ \vdots \\ a_n + e_{n1} \end{pmatrix}, \sigma_z^2 \begin{pmatrix} \tau & 1 & \cdots & 1 & \phi & \phi & \cdots & \phi \\ 1 & \tau & \cdots & 1 & \phi & \phi & \cdots & \phi \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & 1 & \cdots & \tau & \phi & \phi & \cdots & \phi \\ \phi & \phi & \cdots & \phi & \tau & 1 & \cdots & 1 \\ \phi & \phi & \cdots & \phi & 1 & \tau & \cdots & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & 1 \\ \phi & \phi & \cdots & \phi & 1 & 1 & \cdots & \tau \end{pmatrix}\right],$$

where $\tau = \dfrac{\sigma_z^2 + \sigma_u^2}{\sigma_z^2}$.

Hence, we can compute the following expressions[16]:

(2.37)

$$E[x_{i2} \mid x_{11}, x_{21}, ..., x_{n1}] = a_i + e_{i2} + \Sigma_{12}\Sigma_{22}^{-1}(\mathbf{x}_1 - \boldsymbol{\mu}_1)$$

$$= a_i + e_{i2} + (\phi, \phi, ..., \phi)\begin{pmatrix} \tau & 1 & \cdots & 1 \\ 1 & \tau & \cdots & 1 \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 1 & \cdots & \tau \end{pmatrix}_{n\times n}^{-1} \begin{pmatrix} x_{11} - \mu_{11} \\ \vdots \\ x_{n1} - \mu_{n1} \end{pmatrix}$$

$$= a_i + e_{i2} + \frac{\phi(\tau - 1)}{\tau(\tau + n - 2) - (n-1)} \sum_{j=1}^{n} (x_{j1} - \mu_{j1})$$

$$= a_i + e_{i2} + \frac{\phi(\tau - 1)}{\tau(\tau + n - 2) - (n-1)} \sum_{j=1}^{n} (z_1 + u_{j1}),$$

(2.38)

$$\text{var}[x_{i2} \mid x_{11}, x_{21}, ..., x_{n1}] = \sigma_z^2[\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}]$$

$$= \sigma_z^2[\tau - (\phi, \phi, ..., \phi)\begin{pmatrix} \tau & 1 & \cdots & 1 \\ 1 & \tau & \cdots & 1 \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 1 & \cdots & \tau \end{pmatrix}_{n\times n}^{-1} \begin{pmatrix} \phi \\ \vdots \\ \phi \end{pmatrix}]$$

$$= \sigma_z^2(\tau - \frac{n\phi^2(\tau - 1)}{\tau(\tau + n - 2) - (n-1)}),$$

where

$\Sigma_{11}, \Sigma_{12}, \Sigma_{22}$ are partitions of the covariance matrix of the random vector

$(x_{i2}, x_{11}, x_{21}, ..., x_{n1})$,

$\mathbf{x}_1$ = a column random vector containing $(x_{11}, x_{21}, ..., x_{n1})$,

[16] The formulas used in the following calculation can be found in Greene (2000), p.86-87.

$\boldsymbol{\mu}_1 = (\mu_{11},...\mu_{n1})^{'}$ = a column vector containing the mean of the random vector

$(x_{11}, x_{21},..., x_{n1})$, and

$$\begin{pmatrix} \tau & 1 & \cdots & 1 \\ 1 & \tau & \cdots & 1 \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 1 & \cdots & \tau \end{pmatrix}_{n\times n}^{-1} = \frac{1}{\tau(\tau+n-2)-(n-1)} \begin{pmatrix} \tau+n-2 & -1 & \cdots & -1 \\ -1 & \tau+n-2 & \cdots & -1 \\ \vdots & \vdots & \cdots & \vdots \\ -1 & -1 & \cdots & \tau+n-2 \end{pmatrix}_{n\times n}.$$

Similarly,

(2.39)

$$\text{var}[x_{i2}, x_{j2} \mid x_{11}, x_{21},..., x_{n1}] = \sigma_z^2[\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}]$$

$$= \sigma_z^2[\begin{pmatrix} \tau & 1 \\ 1 & \tau \end{pmatrix} - (\begin{matrix} \phi & \cdots & \phi \\ \phi & \cdots & \phi \end{matrix}) \begin{pmatrix} \tau & 1 & \cdots & 1 \\ 1 & \tau & \cdots & 1 \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}_{n\times n}^{-1} \begin{pmatrix} \phi & \phi \\ \vdots & \vdots \\ \phi & \phi \end{pmatrix}]$$

$$= \sigma_z^2 \begin{pmatrix} \tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)} & 1 - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)} \\ 1 - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)} & \tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)} \end{pmatrix},$$

where

$\Sigma_{11}, \Sigma_{12}, \Sigma_{22}$ are partitions of the covariance matrix of the random vector

$(x_{i2}, x_{j2}, x_{11}, x_{21},..., x_{n1})$.

Hence,

(2.40) $\text{cov}[x_{i2}, x_{j2} \mid x_{11}, x_{21},..., x_{n1}] = \sigma_z^2(1 - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})$

2.4.2.2.1 A Dynamic Fixed-performance Contract

Under the two-period dynamic fixed-performance contract without full-commitment , the optimal incentives can be formulated backward using a dynamic programming approach. Additionally, since growers' outputs are correlated in the two periods under the assumption (2.33), the processor and growers take expectations of the second-period rewards and outputs conditional on the first period outputs.

A) *Second-period schemes*

Denote the second-period optimal contract as $C_{F2} = \{\alpha_{F2}, \beta_{F2}\}$. Again, we assume the fixed standard used to reward growers is *s* in both periods. Hence, the payment to each grower in the second period becomes

(2.41) $w_{i2} = \alpha_{F2} + \beta_{F2}[x_{i2} - s], \quad \forall i$.

Hence,

(2.42) $E_2[w_{i2} \mid x_{11},...,x_{n1}] = \alpha_{F2} + \beta_{F2}[a_i + e_{i2} + \dfrac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)} \sum_{j=1}^{n} (z_1 + u_{j1}) - s]$,

and

(2.43) $\text{var}(w_{i2} \mid x_{11},...,x_{n1}) = \beta_{F2}^2 \text{ var}(x_{i2} - s \mid x_{11},...,x_{n1}) = \beta_{F2}^2 \text{ var}(x_{i2} \mid x_{11},...,x_{n1})$

$$= \beta_{F2}^2[\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}].$$

Similar to the static model, the processor solves the following problem

(2.44) $\max\limits_{\alpha_{F2},\beta_{F2}} E_a\{\sum_{i=1}^{n} (E_2 x_{i2} - \dfrac{1}{2} r \text{ var}(w_{i2}) - \dfrac{1}{2a_i} e_{i2}^2) \mid x_{11},...x_{n1}\}$,

subject to

(2.45)

$$E_a[E_2U_{i2}] = E_a[\alpha_{F2} + \beta_{F2}[a_i + e_{i2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}) - s]$$
$$-\frac{1}{2}r\beta_{F2}^2\sigma_z^2[\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}] - \frac{1}{2a_i}e_{i2}^2] \geq 0.$$

Similarly, the incentive-compatibility constraint becomes

(2.46)

$$e_{i2} \in \arg\max\{\alpha_{F2} + \beta_{F2}[a_i + e_{i2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}) - s]$$
$$-\frac{1}{2}r\beta_{F2}^2\sigma_z^2[\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}] - \frac{1}{2a_i}e_{i2}^2\}, \qquad \forall i.$$

From the constraint (2.46), each grower chooses the optimal effort such that

(2.47) $e_{i2} = a_i\beta_{F2}$.

Thus, the total welfare in the second period conditional on outputs in the first period is

(2.48)

$$W_{F2} = \max_{\alpha_{F2},\beta_{F2}} E_a\{\sum_{i=1}^{n}(E_2x_{i2} - \frac{1}{2}r\,\mathrm{var}(w_{i2}) - \frac{1}{2a_i}e_{i2}^2 \mid x_{11},...x_{n1}\}$$

$$= E_a\{\sum_{i=1}^{n}(a_i + e_{i2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}) - \frac{1}{2}r\beta_{F2}^2\sigma_z^2[\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}] - \frac{1}{2a_i}e_{i2}^2)\}$$
$$= \sum_{i=1}^{n}(a_m + a_m\beta_{F2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}) - \frac{1}{2}r\beta_{F2}^2\sigma_z^2[\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}] - \frac{1}{2}a_m\beta_{F2}^2).$$

Differentiating (2.48) with respect to β_{F2} yields

(2.49) $$\frac{\partial W_{F2}}{\partial \beta_{F2}} = n(a_m - r\beta_{F2}\sigma_z^2[\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}] - a_m\beta_{F2}) = 0,$$

from which

(2.50) $$\beta_{F2} = \frac{a_m}{a_m + r\sigma_z^2[\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}]}.$$

From the binding participation constraint (2.45), we can obtain the optimal base payment,

(2.51)

$$\alpha_{F2} = -\beta_{F2}[a_m + a_m\beta_{F2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) - s]$$
$$+\frac{1}{2}r\beta_{F2}^2\sigma_z^2[\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}] + \frac{1}{2}a_m\beta_{F2}^2$$
$$= -\beta_{F2}[a_m + a_m\beta_{F2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) - s]$$
$$+\frac{1}{2}\beta_{F2}^2\sigma_z^2[r(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) + a_m]$$
$$= -\beta_{F2}[a_m + a_m\beta_{F2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) - s] + \frac{1}{2}a_m\beta_{F2}$$
$$= -\frac{1}{2}a_m\beta_{F2} - a_m\beta_{F2}^2 - \beta_{F2}[\frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) - s].$$

Further, the total welfare in the second period under the fixed-performance contract can be computed as:

(2.52)

$$W_{F2} = \sum_{i=1}^{n}(a_m + a_m\beta_{F2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1})$$
$$-\frac{1}{2}r\beta_{F2}^2\sigma_z^2[\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}] - \frac{1}{2}a_m\beta_{F2}^2)$$
$$= n(a_m + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) + \frac{1}{2}a_m\beta_{F2})$$
$$= na_m[1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]}] + \frac{n\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}).$$

(B) *First-period Schemes*

Denote the first-period optimal contract as $C_{F1} = \{\alpha_{F1}, \beta_{F1}\}$. At the beginning of the first period, the processor chooses the optimal bonus and the base payment for the first period by maximizing the total two-period welfare. Similarly to the second-period reward, the first-period reward to each grower takes the form,

(2.53) $w_{i1} = \alpha_{F1} + \beta_{F1}[x_{i1} - s], \quad \forall i$.

Hence,

(2.54) $E_1[w_{i1}] = \alpha_{F1} + \beta_{F1}[a_i + e_{i1} - s]$, and

(2.55) $\text{var}(w_{i1}) = \beta_{F1}^2 \text{ var}(x_{i1} - s) = \beta_{F1}^2 \text{ var}(x_{i1}) = \beta_{F1}^2(\sigma_z^2 + \sigma_u^2)$.

Let W_F^D denote the two-period total welfare under the dynamic fixed-performance contract and W_{F1} denote the first-period welfare. The processor solves the following problem in the first period:

(2.56)

$$
\begin{aligned}
W_F^D &= \max_{\alpha_{F1},\beta_{F1}} \{W_{F1} + \delta E_1(W_{F2} \mid x_{11},\ldots,x_{n1})\} \\
&= \max_{\alpha_{F1},\beta_{F1}} \{E_a[\textstyle\sum_{i=1}^n (E_1 x_{i1} - \frac{1}{2} r \text{ var}(w_{i1}) - \frac{1}{2a_i} e_{i1}^2)] + \delta E_1(W_{F2} \mid x_{11},\ldots,x_{n1})\} \\
&= \max_{\alpha_{F1},\beta_{F1}} \{E_a[\textstyle\sum_{i=1}^n (a_i + e_{i1} - \frac{1}{2} r \beta_{F1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_{i1}^2)] + \delta E_1(W_{F2} \mid x_{11},\ldots,x_{n1})\}
\end{aligned}
$$

subject to

(2.57)

$$
\begin{aligned}
&E_a[E_1 U_{i1} + \delta E_1(E_2 U_{i2})] = E_a[E_1 w_{i1} - \frac{1}{2} r \text{ var}(w_{i1}) - \frac{1}{2a_i} e_{i1}^2] + \delta E_1[E_a(E_2 U_{i2})] \\
&= E_a[\alpha_{F1} + \beta_{F1}[a_i + e_{i1} - s] - \frac{1}{2} r \beta_{F1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_{i1}^2] + \delta E_1[E_a(E_2 U_{i2})] \ge 0,
\end{aligned}
$$

and the incentive-compatibility constraint,

(2.58)

$$
e_{i1} \in \arg\max \{\alpha_{F1} + \beta_{F1}[a_i + e_{i1} - s] - \frac{1}{2} r \beta_{F1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_{i1}^2 + \delta E_1[E_2 U_{i2}]\}, \ \forall i.
$$

From the constraint (2.58), the optimal effort in the first period must satisfy

(2.59) $e_{i1} = a_i \beta_{F1}$.

Substituting (2.59) into (2.56) yields,

(2.60)

$$W_F^D = \max_{\alpha_{F1},\beta_{F1}} \{E_a[\sum_{i=1}^n (a_i + e_{i1} - \frac{1}{2} r\beta_{F1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_{i1}^2)] + \delta E_1(W_{F2} \mid x_{11},...,x_{n1})\}$$

$$= \max_{\alpha_{F1},\beta_{F1}} \{n(a_m + a_m \beta_{F1} - \frac{1}{2} r\beta_{F1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2} a_m \beta_{F1}^2) + \delta E_1(W_{F2} \mid x_{11},...,x_{n1})\}.$$

Differentiating (2.60) with respect to β_{F1} yields the optimal condition for the bonus in the first-period contract,

(2.61) $\dfrac{\partial W_F^D}{\partial \beta_{F1}} = n(a_m - r\beta_{F1}(\sigma_z^2 + \sigma_u^2) - a_m \beta_{F1}) = 0$,

or, more precisely,

(2.62) $\beta_{F1} = \dfrac{a_m}{a_m + r(\sigma_z^2 + \sigma_u^2)}$.

The base payment can also be computed by plugging (2.62) into the binding constraint (2.57), that is,

$$\alpha_{F1} + \beta_{F1}[a_m + a_m\beta_{F1} - s] - \frac{1}{2} r\beta_{F1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2} a_m \beta_{F1}^2 + \delta E_1[E_a(E_2 U_{i2})] = 0,$$

from which we can solve

(2.63)

$$\alpha_{F1} = -\beta_{F1}[a_m + a_m\beta_{F1} - s] + \frac{1}{2} r\beta_{F1}^2(\sigma_z^2 + \sigma_u^2) + \frac{1}{2} a_m \beta_{F1}^2$$

$$= -\beta_{F1}[a_m + a_m\beta_{F1} - s] + \frac{1}{2} a_m \beta_{F1} = s\beta_{F1} - a_m\beta_{F1}^2 - \frac{1}{2} a_m \beta_{F1}.$$

Note that we used the result $E_a[E_2U_{i2}]=0$ from the second-period scheme in the above calculation.

Further, we can obtain the expected two-period total welfare under the dynamic fixed-performance contract,

(2.64)

$$W_F^D = n(a_m + a_m\beta_{F1} - \frac{1}{2}r\beta_{F1}^2(\sigma_z^2+\sigma_u^2) - \frac{1}{2}a_m\beta_{F1}^2) + \delta E_1(W_{F2} \mid x_{11},...,x_{n1})$$

$$= na_m(1+\frac{1}{2}\beta_{F1}) + \delta E_1[na_m(1+\frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$+\frac{n\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1})]$$

$$= na_m(1+\frac{a_m}{2[a_m + r(\sigma_z^2+\sigma_u^2)]}) + \delta na_m(1+\frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]}).$$

The following proposition compares the total welfare under the dynamic FPC with that under the full-commitment FPC given by (2.35).

Proposition 2: The total welfare under the two-period dynamic FPC exceeds that under the full-commitment FPC. That is, $W_F^D > W_F^F$.

The proof is straightforward. Recall that $\tau = \frac{\sigma_z^2+\sigma_u^2}{\sigma_z^2} > 1$, hence, the following term in (2.64) has the property:

$$\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) = \sigma_z^2+\sigma_u^2 - \frac{\sigma_z^2 n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)} < \sigma_z^2+\sigma_u^2$$

Thus,

$$W_F^D = na_m(1+\frac{a_m}{2[a_m + r(\sigma_z^2+\sigma_u^2)]}) + \delta na_m(1+\frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$> (1+\delta)na_m(1+\frac{a_m}{2[a_m + r(\sigma_z^2+\sigma_u^2)]}) = W_F^F .$$

Intuitively, under the dynamic FPC, the processor can obtain more information from growers' first-period performance and raise his expected profit by using the information to provide the second-period incentives.

2.4.2.2.2 A Dynamic Current-period Relative-performance Contract

We investigate a dynamic current-period relative-performance contract in this section. More precisely, the relative standard specified in this contract is the average current-period performance of all contract growers. The two-period dynamic relative-performance contract can be solved in the similar fashion to that in the previous section. Since growers' outputs are correlated in the two periods under the assumption (2.33), the processor and growers take expectations of the second-period rewards and outputs conditional on the first period outputs.

A) *Second-period schemes*

Denote the second-period optimal contract as $C_{R2} = \{\alpha_{R2}, \beta_{R2}\}$. Each grower's payment in the second period becomes

(2.65) $w_{i2} = \alpha_{R2} + \beta_{R2}[x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}], \quad \forall i$

Hence,

(2.66) $E_2[w_{i2} \mid x_{11},...,x_{n1}] = \alpha_{R2} + \beta_{R2}[a_i + e_{i2} - \frac{1}{n}\sum_{j=1}^{n}(a_j + e_{j2})]$, and

(2.67)

$$\text{var}(w_{i2} \mid x_{11},...,x_{n1}) = \beta_{R2}^2 \text{ var}(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2} \mid x_{11},...,x_{n1})$$

$$= \beta_{R2}^2 \text{ var}(\frac{n-1}{n} x_{i2} - \frac{1}{n}\sum_{j=1,j\neq i}^{n} x_{j2} \mid x_{11},...,x_{n1})$$

$$= \beta_{R2}^2[\frac{(n-1)^2}{n^2}\text{var}(x_{i2} \mid x_{11},...,x_{n1}) + \frac{n-1}{n^2}\text{var}(x_{j2} \mid x_{11},...,x_{n1}) - 2\frac{n-1}{n}\frac{1}{n}(n-1)\text{cov}(x_{i2}, x_{j2} \mid x_{11},...,x_{n1})$$

$$+ 2\frac{1}{n^2}\binom{n-1}{2}\text{cov}(x_{j\neq i,2}, x_{k\neq i,2} \mid x_{11},...,x_{n1})]$$

$$= \beta_{R2}^2[\frac{n-1}{n}\text{var}(x_{i2} \mid x_{11},...,x_{n1}) - \frac{n-1}{n}\text{cov}(x_{i2}, x_{j2} \mid x_{11},...,x_{n1})]$$

$$= \beta_{R2}^2 \frac{n-1}{n}\sigma_z^2[(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) - (1 - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]$$

$$= \beta_{R2}^2 \frac{n-1}{n}\sigma_z^2(\tau-1) = \beta_{R2}^2 \frac{n-1}{n}\sigma_u^2.$$

Note that the last equality results from $\tau = \frac{\sigma_z^2 + \sigma_u^2}{\sigma_z^2}$. In addition, the variance of the second period payment depends only on the idiosyncratic shock without being affected by the common shock.

Similar to the static model, the processor solves

(2.68) $\max\limits_{\alpha_{R2},\beta_{R2}} E_a\{\sum_{i=1}^{n}(E_2 x_{i2} - \frac{1}{2}r\text{ var}(w_{i2}) - \frac{1}{2a_i}e_{i2}^2) \mid x_{11},...x_{n1}\}$

subject to

(2.69)

$$E_a[E_2 U_{i2} \mid x_{11},...x_{n1}] = E_a[\alpha_{R2} + \beta_{R2}[a_i + e_{i2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1})$$

$$- \frac{1}{n}\sum_{j=1}^{n}(a_j + e_{j2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}))] - \frac{1}{2}r\beta_{R2}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i2}^2]$$

$$= E_a[\alpha_{R2} + \beta_{R2}[a_i + e_{i2} - \frac{1}{n}\sum_{j=1}^{n}(a_j + e_{j2})] - \frac{1}{2}r\beta_{R2}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i2}^2] \geq 0.$$

Similarly, the incentive-compatibility constraint becomes

(2.70)

$$e_{i2} \in \arg\max\{\alpha_{R2} + \beta_{R2}[a_i + e_{i2} - \frac{1}{n}\sum_{j=1}^{n}(a_j + e_{j2})] - \frac{1}{2}r\beta_{R2}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i2}^2\}$$
$$= \alpha_{R2} + \beta_{R2}[\frac{n-1}{n}(a_i + e_{i2}) - \frac{1}{n}\sum_{j=1,j\neq i}^{n}(a_j + e_{j2})] - \frac{1}{2}r\beta_{R2}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i2}^2, \quad \forall i.$$

From the constraint (2.70), the optimal effort from each grower must satisfy

(2.71) $e_{i2} = \dfrac{n-1}{n}a_i\beta_{R2}$.

Thus, the second-period welfare conditional on outputs in the first period is

(2.72)

$$W_{R2} = \max_{\alpha_{R2},\beta_{R2}} E_a\{\sum_{i=1}^{n}(E_2 x_{i2} - \frac{1}{2}r\,\mathrm{var}(w_{i2}) - \frac{1}{2a_i}e_{i2}^2 \mid x_{11},...x_{n1}\}$$
$$= E_a\{\sum_{i=1}^{n}(a_i + e_{i2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}) - \frac{1}{2}r\beta_{R2}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i2}^2)\}$$

$$= \sum_{i=1}^{n}(a_m + \frac{n-1}{n}a_m\beta_{R2} - \frac{1}{2}r\beta_{R2}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2}(\frac{n-1}{n})^2 a_m\,\beta_{R2}^2 + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1})).$$

Differentiating (2.72) with respect to β_{R2} yields,

(2.73) $\dfrac{\partial W_{R2}}{\partial \beta_{R2}} = n(\dfrac{n-1}{n}a_m - r\beta_{R2}\dfrac{n-1}{n}\sigma_u^2 - (\dfrac{n-1}{n})^2 a_m\,\beta_{R2}) = 0$,

from which

(2.74) $\beta_{R2} = \dfrac{a_m}{\frac{n-1}{n}a_m + r\sigma_u^2}$.

From the binding participation constraint (2.69), we can obtain the optimal base payment,

(2.75)

$$\alpha_{R2} = \frac{1}{2} r\beta_{R2}^2 \frac{n-1}{n}\sigma_u^2 + \frac{1}{2}(\frac{n-1}{n})^2 a_m \beta_{R2}^2 = \frac{1}{2}\frac{n-1}{n}\beta_{R2}^2[r\sigma_u^2 + \frac{n-1}{n}a_m]$$

$$= \frac{1}{2}\frac{n-1}{n}\frac{(a_m)^2}{r\sigma_u^2 + \frac{n-1}{n}a_m} = \frac{a_m}{2[1+\frac{r}{a_m}\frac{n}{n-1}\sigma_u^2]}.$$

Further, the total welfare in the second period under the relative-performance contract can be computed as:

(2.76)

$$W_{R2} = \sum_{i=1}^{n}(a_m + \frac{n-1}{n}a_m\beta_{R2} - \frac{1}{2}r\beta_{R2}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2}(\frac{n-1}{n})^2 a_m\beta_{R2}^2$$
$$+ \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}))$$

$$= n(a_m + \frac{1}{2}a_m\frac{n-1}{n}\beta_{R2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}))$$
$$= na_m[1+\frac{1}{2(1+\frac{r}{a_m}\frac{n}{n-1}\sigma_u^2)}] + \frac{n\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}).$$

(B) *First-period schemes*

Denote the first-period optimal contract as $C_{R1} = \{\alpha_{R1}, \beta_{R1}\}$. The first-period reward to each grower takes the form,

(2.77) $w_{i1} = \alpha_{R1} + \beta_{R1}[x_{i1} - \frac{1}{n}\sum_{j=1}^{n}x_{j1}]$, $\quad \forall i$.

Hence,

(2.78) $E_1[w_{i1}] = \alpha_{R1} + \beta_{R1}[a_i + e_{i1} - \frac{1}{n}\sum_{j=1}^{n}(a_j + e_{j1})]$, and

(2.79)

$$\text{var}(w_{i1}) = \beta_{R1}^2 \,\text{var}(x_{i1} - \frac{1}{n}\sum_{j=1}^{n}x_{j1}) = \beta_{R1}^2\,\text{var}(\frac{n-1}{n}x_{i1} - \frac{1}{n}\sum_{j=1, j\neq i}^{n}x_{j1})$$

$$= \beta_{R1}^2[\frac{(n-1)^2}{n^2}\text{var}(x_{i1}) + \frac{n-1}{n^2}\text{var}(x_{j1}) - 2\frac{n-1}{n}\frac{1}{n}(n-1)\text{cov}(x_{i1}, x_{j1}) + 2\frac{1}{n^2}\binom{n-1}{2}\text{cov}(x_{j\neq i,1}, x_{k\neq i,1})]$$

$$= \beta_{R1}^2[\frac{n-1}{n}\text{var}(x_{i2}) - \frac{n-1}{n}\text{cov}(x_{i1}, x_{j1}]$$

$$= \beta_{R1}^2\frac{n-1}{n}(\sigma_z^2 + \sigma_u^2 - \sigma_z^2) = \beta_{R1}^2\frac{n-1}{n}\sigma_u^2.$$

Similar to the dynamic fixed-performance contract, the processor chooses the first-period optimal bonus and the base payment by maximizing the total two-period welfare. Specifically, let W_R^S denote the two-period total welfare under the current-period relative-performance contract, where the superscript S stands for current-period and the subscript R stands for relative-performance contract, and W_{R1} denote the first-period welfare. The processor solves the following problem in the first period:

(2.80)

$$W_R^S = \max_{\alpha_{R1},\beta_{R1}}\{W_{R1} + \delta E_1(W_{R2} \mid x_{11},...,x_{n1})\}$$

$$= \max_{\alpha_{R1},\beta_{R1}}\{E_a[\sum_{i=1}^n(E_1 x_{i1} - \frac{1}{2}r\,\text{var}(w_{i1}) - \frac{1}{2a_i}e_{i1}^2)] + \delta E_1(W_{R2} \mid x_{11},...,x_{n1})\}$$

$$= \max_{\alpha_{R1},\beta_{R1}}\{E_a[\sum_{i=1}^n(a_i + e_{i1} - \frac{1}{2}r\beta_{R1}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i1}^2)] + \delta E_1(W_{R2} \mid x_{11},...,x_{n1})\}$$

subject to

(2.81)

$$E_a[E_1 U_{i1} + \delta E_1(E_2 U_{i2})] = E_a[E_1 w_{i1} - \frac{1}{2}r\,\text{var}(w_{i1}) - \frac{1}{2a_i}e_{i1}^2] + \delta E_1[E_a(E_2 U_{i2})]$$

$$= E_a[\alpha_{R1} + \beta_{R1}[a_i + e_{i1} - \frac{1}{n}\sum_{j=1}^n(a_j + e_{j1})] - \frac{1}{2}r\beta_{R1}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i1}^2] + \delta E_1[E_a(E_2 U_{i2})]$$

$$= E_a[\alpha_{R1} + \beta_{R1}[\frac{n-1}{n}(a_i + e_{i1}) - \frac{1}{n}\sum_{j=1, j\neq i}^n(a_j + e_{j1})] - \frac{1}{2}r\beta_{R1}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i1}^2]$$

$$+ \delta E_1[E_a(E_2 U_{i2})] \geq 0,$$

and the incentive-compatibility constraint,

(2.82)

$$e_{i1} \in \arg\max\{\alpha_{R1} + \beta_{R1}[\frac{n-1}{n}(a_i + e_{i1}) - \frac{1}{n}\sum_{j=1,j\neq i}^{n}(a_j + e_{j1})] - \frac{1}{2}r\beta_{R1}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i1}^2 + \delta E_1[E_2U_{i2}]\}$$

, $\forall i$.

From the constraint (2.83), the optimal effort in the first period must satisfy

(2.83) $e_{i1} = \frac{n-1}{n}a_i\beta_{R1}$.

Substituting (2.82) into (2.80) yields,

(2.84)

$$W_R^S = \max_{\alpha_{R1},\beta_{R1}} E_a[\sum_{i=1}^{n}(a_i + e_{i1} - \frac{1}{2}r\beta_{R1}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i1}^2)] + \delta E_1(W_{R2} \mid x_{11},...,x_{n1})$$
$$= \max_{\alpha_{R1},\beta_{R1}} E_a[\sum_{i=1}^{n}(a_i + \frac{n-1}{n}a_i\beta_{R1} - \frac{1}{2}r\beta_{R1}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2}a_i(\frac{n-1}{n}\beta_{R1})^2)] + \delta E_1(W_{R2} \mid x_{11},...,x_{n1})$$
$$= \max_{\alpha_{R1},\beta_{R1}} n(a_m + \frac{n-1}{n}a_m\beta_{R1} - \frac{1}{2}r\beta_{R1}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2}a_m(\frac{n-1}{n}\beta_{R1})^2) + \delta E_1(W_{R2} \mid x_{11},...,x_{n1}).$$

Differentiating (2.84) with respect to β_{R1} yields the optimal condition for the bonus in the first-period contract,

(2.85) $\frac{\partial W_R^S}{\partial \beta_{R1}} = n(\frac{n-1}{n}a_m - r\beta_{R1}\frac{n-1}{n}\sigma_u^2 - a_m(\frac{n-1}{n})^2\beta_{R1}) = 0$,

or, more precisely,

(2.86) $\beta_{R1} = \frac{a_m}{\frac{n-1}{n}a_m + r\sigma_u^2}$.

The base payment can also be computed by substituting (2.86) into the binding constraint (2.81), that is,

$$E_a[\alpha_{R1} + \beta_{R1}[\frac{n-1}{n}(a_i + e_{i1}) - \frac{1}{n}\sum_{j=1,j\neq i}^{n}(a_j + e_{j1})] - \frac{1}{2}r\beta_{R1}^2\frac{n-1}{n}\sigma_u^2 - \frac{1}{2a_i}e_{i1}^2]$$
$$+ \delta E_1[E_a(E_2U_{i2})]$$

$$= \alpha_{R1} - \frac{1}{2} r\beta_{R1}^2 \frac{n-1}{n}\sigma_u^2 - \frac{1}{2} a_m (\frac{n-1}{n}\beta_{R1})^2 + \delta E_1[E_a(E_2 U_{i2})]$$

$$= \alpha_{R1} - \frac{1}{2}\frac{n-1}{n}\frac{a_m^2}{r\sigma_u^2 + \frac{n-1}{n}a_m} = 0,$$

from which we can obtain

(2.87)

$$\alpha_{R1} = \frac{a_m}{2[1 + \frac{r}{a_m}\frac{n}{n-1}\sigma_u^2]}.$$

Note that we used the result $E_a(E_2 U_{i2}) = 0$ from the second-period scheme in the above calculation.

Further, we can obtain the expected two-period total welfare,

(2.88)

$$W_R^S = n(a_m + \frac{n-1}{n} a_m \beta_{R1} - \frac{1}{2} r\beta_{R1}^2 \frac{n-1}{n}\sigma_u^2 - \frac{1}{2} a_m (\frac{n-1}{n}\beta_{R1})^2) + \delta E_1(W_{R2} \mid x_{11}, ..., x_{n1})$$

$$= n(a_m + \frac{1}{2}\frac{n-1}{n} a_m \beta_{R1}) + \delta E_1(na_m[1 + \frac{1}{2(1 + \frac{r}{a_m}\frac{n}{n-1}\sigma_u^2)}] + \frac{n\phi(\tau - 1)}{\tau(\tau + n - 2) - (n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}))$$

$$= na_m(1 + \frac{1}{2(1 + \frac{r}{a_m}\frac{n}{n-1}\sigma_u^2)}) + \delta na_m[1 + \frac{1}{2(1 + \frac{r}{a_m}\frac{n}{n-1}\sigma_u^2)}] = (1+\delta)na_m(1 + \frac{1}{2(1 + \frac{r}{a_m}\frac{n}{n-1}\sigma_u^2)}).$$

First, note that the two-period total welfare under this dynamic current-period RPC is exactly same as that under the full-commitment RPC given by (2.34) and is exactly a repetition of the static RPC. That is, the intertemporal relationship between the incentives in the two periods does not alter the optimal choice of rewards offered by the processor and the optimal efforts provided by growers. Thus, under the dynamic current-period RPC, both the processor and growers are myopic. This result is a special feature of the current-period relative-performance contract.

Second, we can compare performance of the dynamic current-period RPC with the dynamic FPC. However, it is not straightforward to show whether one is superior to the other under certain conditions. We summarize some plausible results in the following proposition.

Proposition 3: *a*) $W_R^S < W_F^D$ if $\sigma_z^2 \le \frac{1}{n-1}\sigma_u^2$, *b*) $W_R^S > W_F^D$ if $\sigma_z^2 >> \frac{1}{n-1}\sigma_u^2$.

Proof: Part *a* is straightforward. From (2.64),

$$W_F^D = na_m(1+\frac{a_m}{2[a_m + r(\sigma_z^2+\sigma_u^2)]}) + \delta na_m(1+\frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$> (1+\delta)na_m(1+\frac{a_m}{2[a_m + r(\sigma_z^2+\sigma_u^2)]}).$$

Thus, similar to Proposition 1, $W_R^S < W_F^D$ if $\sigma_z^2 \le \frac{1}{n-1}\sigma_u^2$.

However, we could only provide some intuition for part *b*. That is, only if the variance of common shocks is sufficiently greater than that of the idiosyncratic shocks would the dynamic RPC perform better than the dynamic FPC.

Under the current-period relative-performance contract, comparing one grower's performance to others' completely eliminates the common uncertainty without being affected by their intertemporal relationship. Consequently, the optimal dynamic current-period RPC mimics a sequence of optimal static RPC although the second period incentives under this contract do account for the growers' first-period information.

2.4.2.2.3 A Dynamic Previous-period Relative-performance Contract

In this case, instead of using the average performance in current period as a standard, the previous-period relative-performance contract rewards each grower by comparing his performance with the previous-period average performance of the same group of growers. As discussed above, this scenario corresponds to the concept of an all-period ban defined in Roe and Wu (2003). Later on, when the performance of the dynamic FPC is compared to the dynamic previous-period RPC, readers could think of the possibility of eliminating the dynamic previous-period RPC as an all-period ban of RPC. Finally, to investigate the dynamic effects on the optimal incentives, it is necessary to assume that the processor signs a contract with the same group of growers in both periods.

The previous-period dynamic relative-performance contract can be solved in the similar dynamic programming approach used in previous sections.

A) *Second-period schemes*

Denote the second-period optimal contract as $C_{L2} = \{\alpha_{L2}, \beta_{L2}\}$ where the subscript denotes the last or previous period. Using group average performance in the last period as a standard, the processor rewards each grower

(2.89) $w_{i2} = \alpha_{L2} + \beta_{L2}[x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}], \quad \forall i$.

Note that the performance of grower *i* in the previous period is not excluded from the calculation of the group average.

Hence,

(2.90)

$$E_2[w_{i2} \mid x_{11},...,x_{n1}] = \alpha_{L2} + \beta_{L2}[a_i + e_{i2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}) - \frac{1}{n}\sum_{j=1}^{n} x_{j1}],$$

and

(2.91)

$$\text{var}(w_{i2} \mid x_{11},...,x_{n1}) = \beta_{L2}^2 \,\text{var}(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j1} \mid x_{11},...,x_{n1}) = \beta_{L2}^2 \,\text{var}(x_{i2} \mid x_{11},...,x_{n1})$$

$$= \beta_{L2}^2 \sigma_z^2 [\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}].$$

Similar to the static model, the processor solves

(2.92) $$\max_{\alpha_{L2},\beta_{L2}} E_a\{\sum_{i=1}^{n}(E_2 x_{i2} - \frac{1}{2} r \,\text{var}(w_{i2}) - \frac{1}{2a_i} e_{i2}^2) \mid x_{11},...x_{n1}\}$$

subject to

(2.93)

$$E_a[E_2 U_{i2} \mid x_{11},...x_{n1}] = E_a\{\alpha_{L2} + \beta_{L2}[a_i + e_{i2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}) - \frac{1}{n}\sum_{j=1}^{n} x_{j1}]$$
$$- \frac{1}{2} r \beta_{L2}^2 \sigma_z^2 (\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) - \frac{1}{2a_i} e_{i2}^2\} \geq 0.$$

The optimal effort must satisfy the incentive-compatibility constraint

(2.94)

$$e_{i2} \in \arg\max\{\alpha_{L2} + \beta_{L2}[a_i + e_{i2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1 + u_{j1}) - \frac{1}{n}\sum_{j=1}^{n} x_{j1}]$$
$$- \frac{1}{2} r \beta_{L2}^2 \sigma_z^2 (\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) - \frac{1}{2a_i} e_{i2}^2\}, \qquad \forall i.$$

The first order condition to the constraint (2.94) gives

(2.95) $e_{i2} = a_i \beta_{L2}$.

Thus, the second-period welfare conditional on outputs in the first period is

(2.96)

$$W_{L2} = \max_{\alpha_{L2},\beta_{L2}} E_a\{\sum_{i=1}^{n}(E_2 x_{i2} - \frac{1}{2} r\,\mathrm{var}(w_{i2}) - \frac{1}{2a_i} e_{i2}^2 \mid x_{11},...x_{n1}\}$$

$$= E_a\{\sum_{i=1}^{n}(a_i + e_{i2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) - \frac{1}{2} r\beta_{L2}^2\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) - \frac{1}{2a_i} e_{i2}^2)\}$$

$$= \sum_{i=1}^{n}(a_m + a_m\beta_{L2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) - \frac{1}{2} r\beta_{L2}^2\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) - \frac{1}{2} a_m\,\beta_{R2}^2).$$

Differentiating (2.96) with respect to β_{L2} yields,

(2.97) $$\frac{\partial W_{L2}}{\partial \beta_{L2}} = n[a_m - r\beta_{R2}\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) - a_m\,\beta_{L2}] = 0\,,$$

from which

(2.98) $$\beta_{L2} = \frac{a_m}{a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})}\,.$$

The optimal base payment can be obtained from the binding participation constraint

(2.93):

(2.99)

$$\alpha_{L2} = -\beta_{L2}[a_m + a_m\beta_{L2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) - \frac{1}{n}\sum_{j=1}^{n}x_{j1}]$$
$$+\frac{1}{2} r\beta_{L2}^2\sigma_z^2[\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}] + \frac{1}{2} a_m\beta_{L2}^2$$

$$= -\beta_{L2}[a_m + a_m\beta_{L2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) - \frac{1}{n}\sum_{j=1}^{n}x_{j1}]$$
$$+\frac{1}{2}\beta_{L2}^2\sigma_z^2[r(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) + a_m]$$
$$= -\beta_{L2}[a_m + a_m\beta_{L2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) - \frac{1}{n}\sum_{j=1}^{n}x_{j1}] + \frac{1}{2} a_m\beta_{L2}$$
$$= -\frac{1}{2} a_m\beta_{L2} - a_m\beta_{L2}^2 - \beta_{L2}[\frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)}\sum_{j=1}^{n}(z_1+u_{j1}) - \frac{1}{n}\sum_{j=1}^{n}x_{j1}].$$

The second-period total welfare can also be obtained:

(2.100)

$$W_{L2} = \sum_{i=1}^{n} (a_m + a_m \beta_{L2} + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)} \sum_{j=1}^{n} (z_1 + u_{j1})$$
$$-\frac{1}{2} r \beta_{L2}^2 \sigma_z^2 [\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}] - \frac{1}{2} a_m \beta_{L2}^2)$$
$$= n(a_m + \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)} \sum_{j=1}^{n} (z_1 + u_{j1}) + \frac{1}{2} a_m \beta_{L2})$$
$$= n a_m [1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]}] + \frac{n\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)} \sum_{j=1}^{n} (z_1 + u_{j1}) .$$

(B) *First-period schemes*

Denote the first-period optimal contract as $C_{L1} = \{\alpha_{L1}, \beta_{L1}\}$. However, at the beginning of the first period, the processor does not have the information of growers' performance in the previous period. Thus, for simplicity, we assume that the same fixed standard *s* used in the fixed-performance contract will be adopted for the first-period contract of the dynamic previous-period RPC.

Under this assumption, each grower receives a reward in the first period,

(2.101) $w_{i1} = \alpha_{L1} + \beta_{L1}[x_{i1} - s], \quad \forall i$.

Hence,

(2.102) $E_1[w_{i1}] = \alpha_{L1} + \beta_{L1}[a_i + e_{i1} - s]$, and

(2.103) $\text{var}(w_{i1}) = \beta_{L1}^2 \,\text{var}(x_{i1} - s) = \beta_{L1}^2 \,\text{var}(x_{i1}) = \beta_{L1}^2 (\sigma_z^2 + \sigma_u^2)$.

Let W_R^L denote the two-period total welfare under the previous-period RPC, where the superscript *L* stands for previous-period and the subscript *R* stands for relative-performance contract, and W_{L1} denote the first-period welfare. The processor chooses the first-period optimal incentives by maximizing the total two-period welfare. Specifically,

(2.104)

$$W_R^L = \max_{\alpha_{L1},\beta_{L1}} \{W_{L1} + \delta E_1(W_{L2} \mid x_{11},...,x_{n1})\}$$
$$= \max_{\alpha_{L1},\beta_{L1}} \{E_a[\sum_{i=1}^n (E_1 x_{i1} - \frac{1}{2} r \operatorname{var}(w_{i1}) - \frac{1}{2a_i} e_{i1}^2)] + \delta E_1(W_{L2} \mid x_{11},...,x_{n1})\}$$
$$= \max_{\alpha_{L1},\beta_{L1}} \{E_a[\sum_{i=1}^n (a_i + e_{i1} - \frac{1}{2} r\beta_{L1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_{i1}^2)] + \delta E_1(W_{L2} \mid x_{11},...,x_{n1})\},$$

subject to

(2.105)

$$E_a[E_1 U_{i1} + \delta E_1(E_2 U_{i2} \mid x_{11},...,x_{n1})] = E_a[E_1 w_{i1} - \frac{1}{2} r \operatorname{var}(w_{i1}) - \frac{1}{2a_i} e_{i1}^2 + \delta E_1(E_2 U_{i2})]$$
$$= E_a\{\alpha_{L1} + \beta_{L1}[a_i + e_{i1} - s] - \frac{1}{2} r\beta_{L1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_{i1}^2 + \delta E_1[\alpha_{L2} + \beta_{L2}(a_i + e_{i2}$$
$$+ \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)} \sum_{j=1}^n (z_1 + u_{j1}) - \frac{1}{n} \sum_{j=1}^n x_{j1})$$
$$- \frac{1}{2} r\beta_{L2}^2 \sigma_z^2 (\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) - \frac{1}{2a_i} e_{i2}^2]\} \geq 0.$$

and the incentive-compatibility constraint,

(2.106)

$$e_{i1} \in \arg\max \{\alpha_{L1} + \beta_{L1}[a_i + e_{i1} - s] - \frac{1}{2} r\beta_{L1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_{i1}^2 + \delta E_1[\alpha_{L2} + \beta_{L2}(a_i + e_{i2}$$
$$+ \frac{\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)} \sum_{j=1}^n (z_1 + u_{j1}) - \frac{1}{n} \sum_{j=1}^n x_{j1}) - \frac{1}{2} r\beta_{L2}^2 \sigma_z^2 (\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)}) - \frac{1}{2a_i} e_{i2}^2]\}$$
$, \forall i$.

Recall that in the above expression, $x_{j1} = a_j + e_{j1} + z_1 + u_{j1}$, $\forall j \in [1, n]$.

Thus, incentive constraint (2.106) requires that the first-period optimal effort satisfy

(2.107) $e_{i1} = a_i(\beta_{L1} - \frac{1}{n} \delta\beta_{L2})$.

Substituting (2.107) into (2.104) yields,

(2.108) $W_R^L = \max_{\alpha_{L1},\beta_{L1}} \{E_a[\sum_{i=1}^n (a_i + e_{i1} - \frac{1}{2} r\beta_{L1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_{i1}^2)] + \delta E_1(W_{L2} \mid x_{11},...,x_{n1})\}$

$$= \max_{\alpha_{L1},\beta_{L1}} \{E_a[\sum_{i=1}^{n}(a_i + e_{i1} - \frac{1}{2} r\beta_{L1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2a_i} e_{i1}^2)] +$$

$$\delta E_1[na_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]}) + \frac{n\phi(\tau-1)}{\tau(\tau+n-2)-(n-1)} \sum_{j=1}^{n}(z_1 + u_{j1})]\}$$

$$= \max_{\alpha_{L1},\beta_{L1}} \{n(a_m + a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2}) - \frac{1}{2} r\beta_{L1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2} a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2})^2) +$$

$$\delta na_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})\}.$$

Differentiating (2.108) with respect to β_{L1} yields

(2.109) $$\frac{\partial W_R^L}{\partial \beta_{L1}} = n(a_m - r\beta_{L1}(\sigma_z^2 + \sigma_u^2) - a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2})) = 0,$$

from which,

(2.110) $$\beta_{L1} = \frac{a_m + a_m\delta\beta_{L2}/n}{a_m + r(\sigma_z^2 + \sigma_u^2)}.$$

Hence, the binding participation constraint (2.105) can be written as

(2.111)

$$\alpha_{L1} + \beta_{L1}[a_m + a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2}) - s] - \frac{1}{2} r\beta_{L1}^2(\sigma_z^2 + \sigma_u^2) - \frac{1}{2} a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2})^2 + \delta E_1[E_a(E_2U_{i2})] = 0,$$

from which we can obtain the first-period base payment,

(2.112)

$$\alpha_{L1} = -\beta_{L1}[a_m + a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2}) - s] + \frac{1}{2} r\beta_{L1}^2(\sigma_z^2 + \sigma_u^2) + \frac{1}{2} a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2})^2 - \delta E_1[E_a(E_2U_{i2})]$$

$$= -\beta_{L1}[a_m + a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2}) - s] + \frac{1}{2} r\beta_{L1}^2(\sigma_z^2 + \sigma_u^2) + \frac{1}{2} a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2})^2.$$

Note that we used the result $E_a[E_2U_{i2} \mid x_{11},...x_{n1}] = 0$ from the second-period scheme in the above calculation.

Further, the expected two-period total welfare can be calculated:

(2.113)

$$W_R^L = n[a_m + a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2}) - \frac{1}{2}r\beta_{L1}^2(\sigma_z^2+\sigma_u^2) - \frac{1}{2}a_m(\beta_{L1} - \frac{1}{n}\delta\beta_{L2})^2] +$$

$$\delta n a_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$= n[a_m + a_m\beta_{L1} - \frac{1}{n}a_m\delta\beta_{L2} - \frac{1}{2}r\beta_{L1}^2(\sigma_z^2+\sigma_u^2) - \frac{1}{2}a_m(\beta_{L1}^2 + (\frac{1}{n}\delta\beta_{L2})^2 - 2\frac{1}{n}\delta\beta_{L1}\beta_{L2})] +$$

$$\delta n a_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$= n[a_m + a_m\beta_{L1} - \frac{1}{n}a_m\delta\beta_{L2} - \frac{1}{2}\beta_{L1}^2[r(\sigma_z^2+\sigma_u^2)+a_m] - \frac{1}{2}a_m(\frac{1}{n}\delta\beta_{L2})^2 + \frac{1}{n}a_m\delta\beta_{L1}\beta_{L2}] +$$

$$\delta n a_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$= n[a_m + a_m\beta_{L1} - \frac{1}{n}a_m\delta\beta_{L2} - \frac{1}{2}[a_m + \frac{1}{n}a_m\delta\beta_{L2}]\beta_{L1} - \frac{1}{2}a_m(\frac{1}{n}\delta\beta_{L2})^2 + \frac{1}{n}a_m\delta\beta_{L1}\beta_{L2}] +$$

$$\delta n a_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$= n[a_m + \frac{1}{2}a_m\beta_{L1} - \frac{1}{n}a_m\delta\beta_{L2} + \frac{1}{2n}a_m\delta\beta_{L1}\beta_{L2} - \frac{1}{2}a_m(\frac{1}{n}\delta\beta_{L2})^2]$$

$$+ \delta n a_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$= n a_m[1 + \frac{1}{2}\frac{a_m + a_m\delta\beta_{L2}/n}{a_m + r(\sigma_z^2+\sigma_u^2)} - \frac{1}{n}\delta\beta_{L2} + \frac{1}{2}\frac{a_m + a_m\delta\beta_{L2}/n}{a_m + r(\sigma_z^2+\sigma_u^2)}\frac{1}{n}\delta\beta_{L2} - \frac{1}{2}(\frac{1}{n}\delta\beta_{L2})^2]$$

$$+ \delta n a_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$= n a_m[1 + \frac{a_m}{2[a_m + r(\sigma_z^2+\sigma_u^2)]} + \frac{1}{2}\frac{a_m}{a_m + r(\sigma_z^2+\sigma_u^2)}\frac{1}{n}\delta\beta_{L2} - \frac{1}{n}\delta\beta_{L2} + \frac{1}{2}\frac{a_m}{a_m + r(\sigma_z^2+\sigma_u^2)}$$

$$(1 + \frac{1}{n}\delta\beta_{L2})\frac{1}{n}\delta\beta_{L2} - \frac{1}{2}(\frac{1}{n}\delta\beta_{L2})^2] + \delta n a_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$= na_m[1 + \frac{a_m}{2[a_m + r(\sigma_z^2 + \sigma_u^2)]} + \frac{a_m}{a_m + r(\sigma_z^2 + \sigma_u^2)}\frac{1}{n}\delta\beta_{L2} - \frac{1}{n}\delta\beta_{L2} +$$

$$+ \frac{1}{2}\frac{a_m}{a_m + r(\sigma_z^2 + \sigma_u^2)}(\frac{1}{n}\delta\beta_{L2})^2 - \frac{1}{2}(\frac{1}{n}\delta\beta_{L2})^2] + \delta na_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$= na_m[1 + \frac{a_m}{2[a_m + r(\sigma_z^2 + \sigma_u^2)]} - \frac{1}{n}\delta\beta_{L2}[\frac{r(\sigma_z^2 + \sigma_u^2)}{a_m + r(\sigma_z^2 + \sigma_u^2)}] - \frac{1}{2}(\frac{1}{n}\delta\beta_{L2})^2[\frac{r(\sigma_z^2 + \sigma_u^2)}{a_m + r(\sigma_z^2 + \sigma_u^2)}]]$$

$$+ \delta na_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$= na_m[1 + \frac{a_m}{2[a_m + r(\sigma_z^2 + \sigma_u^2)]} - \frac{1}{2}\frac{r(\sigma_z^2 + \sigma_u^2)}{a_m + r(\sigma_z^2 + \sigma_u^2)}[(\frac{1}{n}\delta\beta_{L2})^2 + 2\frac{1}{n}\delta\beta_{L2}]]$$

$$+ \delta na_m(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})$$

$$= na_m[1 + \frac{a_m}{2[a_m + r(\sigma_z^2 + \sigma_u^2)]} + \delta(1 + \frac{a_m}{2[a_m + r\sigma_z^2(\tau - \frac{n\phi^2(\tau-1)}{\tau(\tau+n-2)-(n-1)})]})]$$

$$- na_m\frac{r(\sigma_z^2 + \sigma_u^2)}{2[a_m + r(\sigma_z^2 + \sigma_u^2)]}[(\frac{1}{n}\delta\beta_{L2} + 1)^2 - 1].$$

The following proposition compares the total welfare under the dynamic previous-period RPC with that under the dynamic FPC.

Proposition 4: The total welfare under the dynamic FPC exceeds that under the dynamic previous-period RPC. Precisely, $W_F^D > W_R^L$.

Proof: The proof is straightforward.

Since the last term in (2.113), $na_m\frac{r(\sigma_z^2 + \sigma_u^2)}{2[a_m + r(\sigma_z^2 + \sigma_u^2)]}[(\frac{1}{n}\delta\beta_{L2} + 1)^2 - 1]$, is always positive, comparing (2.113) with (2.64) concludes the proposition.

This proposition and the results on which it is based, lead to two general comments about the dynamic previous-period contract: First, under the previous-period dynamic

RPC, the optimal condition (2.107) indicates that growers tend to exert less effort in the first period when offered the same bonus as in a static RPC. In turn, from (2.110), the processor has to offer a greater bonus in the first period to induce more effort from growers. This result is the manifestation of the ratchet effect that discourages growers to provide efforts in the first period because they anticipate the processor would use their first-period performance as a standard for their second-period performance. Second, it is assumed that the processor adopts a fixed-performance contract in the first period because no information is available about the growers' performance before the first period. This assumption contributes to Proposition 4. However, if instead a current-period RPC is used in the first period under this contract, the relative superiority of the dynamic FPC and the dynamic previous-period RPC will depend on the relative magnitude of σ_z^2, σ_u^2, and possibly other parameters.

2.4.3 A Static Model with Two Pooled Tournaments

Knoeber and Thurman (1994) indicate that there are efficiency costs to mixing growers of unequal ability under tournament contracts. In this section, we assume that the processor constructs two tournaments in one period, and provides two separate contracts, with each contract targeted to a different group of growers based on their abilities.

Recall that growers' ability is uniformly distributed in the range $[\underline{a}, \overline{a}]$. Suppose the processor offers two contracts $C_G = \{\alpha_G, \beta_G\}$ and $C_B = \{\alpha_B, \beta_B\}$, where the contract C_G is offered to high-ability growers type with $a_i \in [\hat{a}, \overline{a}]$ and the contract C_B is offered to low-quality growers type with $a_i \in [\underline{a}, \hat{a}]$. Throughout this section, *G* denotes the high-ability group and *B* denotes the low-ability group.

Hence, it can be easily verified that, for every grower *i*,

(2.114) $p^B = prob(a_i \in [\underline{a}, \hat{a}]) = \dfrac{\hat{a} - \underline{a}}{\overline{a} - \underline{a}}$,

(2.115) $p^G = prob(a_i \in [\hat{a}, \overline{a}]) = \dfrac{\overline{a} - \hat{a}}{\overline{a} - \underline{a}}$,

(2.116) $a_m^B = E(a_i \mid a_i \in [\underline{a}, \hat{a}]) = \dfrac{\hat{a} + \underline{a}}{2}$, and

(2.117) $a_m^G = E(a_i \mid a_i \in [\hat{a}, \overline{a}]) = \dfrac{\hat{a} + \overline{a}}{2}$.

The processor rewards each grower in the high-ability group,

(2.118) $w_i^G = \alpha_G + \beta_G [x_i^G - \overline{x}^G]$,

where we define

(2.119) $\overline{x}^G = \dfrac{1}{n^G} \sum_G x_j^G$, and

(2.120) $n^G = nprob(a_i \in [\hat{a}, \overline{a}]) = n \dfrac{\overline{a} - \hat{a}}{\overline{a} - \underline{a}} = np^G$.

Here we use $\sum_G x_j$ to denote the sum of outputs produced by the high-ability group.

Hence,

(2.121) $Ew_i^G = \alpha_G + \beta_G [a_i^G + e_i^G - \dfrac{1}{n^G} \sum_G (a_j^G + e_j^G)]$, and

(2.122)

$$\mathrm{var}(w_i^G) = \beta_G^2 \,\mathrm{var}(x_i^G - \bar{x}^G) = \beta_G^2 \,\mathrm{var}(\frac{n^G - 1}{n^G} x_i^G - \frac{1}{n^G}\sum\nolimits_{G,j\neq i} x_j^G)$$

$$= \beta_G^2 [(\frac{n^G-1}{n^G})^2 \,\mathrm{var}(x_i^G) + \frac{n^G-1}{(n^G)^2}\mathrm{var}(x_j^G) - 2\frac{n^G-1}{n^G}\frac{1}{n^G}(n^G-1)\,\mathrm{cov}(x_i^G, x_j^G)$$

$$+ 2\frac{1}{(n^G)^2}\frac{(n^G-1)(n^G-2)}{2}\mathrm{cov}(x_{j\neq i}^G, x_{k\neq i}^G)]$$

$$= \beta_G^2[((\frac{n^G-1}{n^G})^2 + \frac{n^G-1}{(n^G)^2})(\sigma_z^2 + \sigma_u^2) - 2(\frac{n^G-1}{n^G})^2\sigma_z^2 + \frac{(n^G-1)(n^G-2)}{(n^G)^2}\sigma_z^2] = \beta_G^2 \frac{n^G-1}{n^G}\sigma_u^2 .$$

Similarly, the payment to each grower in the low-ability group are given by

(2.123) $w_i^B = \alpha_B + \beta_B [x_i^B - \bar{x}^B]$,

where

(2.124) $\bar{x}^B = \frac{1}{n^B}\sum\nolimits_B x_j^B$, and

(2.125) $n^B = nprob(a_i \in [\underline{a}, \hat{a}]) = n\frac{\hat{a} - \underline{a}}{\bar{a} - \underline{a}} = np^B$.

Hence, we can compute

(2.126) $Ew_i^B = \alpha_B + \beta_B [a_i^B + e_i^B - \frac{1}{n^B}\sum\nolimits_B (a_j^B + e_j^B)]$, and

(2.127) $\mathrm{var}(w_i^B) = \beta_B^2 \,\mathrm{var}(x_i^B - \bar{x}^B) = \beta_B^2 \frac{n^B - 1}{n^B}\sigma_u^2$.

Thus, the processor solves the following problem

(2.128)

$$\overline{W}_R = \max_{\alpha_B, \beta_B, \alpha_G, \beta_G, \hat{a}} \{E_a[\sum\nolimits_G (Ex_i^G - Ew_i^G) \mid a_i \in [\hat{a}, \bar{a}]] + E_a[\sum\nolimits_B (Ex_i^B - Ew_i^B) \mid a_i \in [\underline{a}, \hat{a}]]\}$$

However, in this case, maximization of the total welfare is different from maximizing the processor's expected profit. Thus, instead of maximizing the total welfare, we assume that the processor maximizes his expected profit.

The optimal contracts must satisfy the following set of constraints: First, the participation constraints must be satisfied:

(2.129)

$$E_a[EU_i^G \mid a_i \in [\hat{a},\overline{a}]] = E_a[Ew_i^G - \frac{1}{2a_i^G}(e_i^G)^2 - \frac{1}{2} r \operatorname{var}(w_i^G) \mid a_i \in [\hat{a},\overline{a}]]$$

$$= E_a[\alpha_G + \beta_G[a_i^G + e_i^G - \frac{1}{n^G}\sum\nolimits_G (a_j^G + e_j^G)] - \frac{1}{2a_i^G}(e_i^G)^2 - \frac{1}{2} r\beta_G^2 \frac{n^G - 1}{n^G}\sigma_u^2 \mid a_i \in [\hat{a},\overline{a}]] \geq 0,$$

(2.130)

$$E_a[EU_i^B \mid a_i \in [\underline{a},\hat{a}]] = E_a[Ew_i^B - \frac{1}{2a_i^B}(e_i^B)^2 - \frac{1}{2} r \operatorname{var}(w_i^B) \mid a_i \in [\underline{a},\hat{a}]]$$

$$= E_a[\alpha_B + \beta_B[a_i^B + e_i^B - \frac{1}{n^B}\sum\nolimits_B (a_j^B + e_j^B)] - \frac{1}{2a_i^B}(e_i^B)^2 - \frac{1}{2} r\beta_B^2 \frac{n^B - 1}{n^B}\sigma_u^2 \mid a_i \in [\underline{a},\hat{a}]] \geq 0.$$

Second, the optimal effort of each grower in the high-ability group must satisfy the incentive-compatibility constraint,

(2.131)

$$e_i^G \in \arg\max\{\alpha_G + \beta_G[a_i^G + e_i^G - \frac{1}{n^G}\sum\nolimits_G (a_j^G + e_j^G)] - \frac{1}{2a_i^G}(e_i^G)^2 - \frac{1}{2} r\beta_G^2 \frac{n^G - 1}{n^G}\sigma_u^2\},\ \forall i \in G.$$

Similarly, for the low-ability group,

(2.132)

$$e_i^B \in \arg\max\{\alpha_B + \beta_B[a_i^B + e_i^B - \frac{1}{n^B}\sum\nolimits_B (a_j^B + e_j^B)] - \frac{1}{2a_i^B}(e_i^B)^2 - \frac{1}{2} r\beta_B^2 \frac{n^B - 1}{n^B}\sigma_u^2\},\ \forall i \in B$$

In addition to the above constraints, the optimal contracts must satisfy another pair of incentive-compatibility constraints. More specifically, under the optimal contracts, it must be optimal for each grower type to choose his own league rather than the other league. Before formulating these constraints, additional notations must be defined.

Since each grower's reward is associated with the difference between his performance with the group average performance, one grower's deviation from choosing his own league would also affect the average performance of the group which he actually chooses. Thus, if one high-ability grower *i* chooses the low-ability league, one defines the average performance of the low-ability group as

(2.133) $\bar{x}^{BG} = \frac{1}{n^B+1}(x_i^G + \sum_B x_j^B)$

Thus, the deviating grower receives reward

(2.134) $w_i^{GD} = \alpha_B + \beta_B[x_i^G - \bar{x}^{BG}]$.

Consequently, the expected payoff and variance of a deviating high-ability grower are

(2.135) $Ew_i^{GD} = \alpha_B + \beta_B[a_i^G + e_i^G - \frac{1}{n^B+1}(a_i^G + e_i^G + \sum_B (a_i^B + e_i^B))]$, and

(2.136)

$$\mathrm{var}(w_i^{GD}) = \beta_B^2 \,\mathrm{var}(x_i^G - \bar{x}^{BG}) = \beta_B^2 \,\mathrm{var}(\frac{n^B}{n^B+1}x_i^G - \frac{1}{n^B+1}\sum_B x_j^B)$$

$$= \beta_B^2[(\frac{n^B}{n^B+1})^2 \,\mathrm{var}(x_i^G) + \frac{n^B}{(n^B+1)^2}\mathrm{var}(x_j^G) - 2\frac{n^B}{n^B+1}\frac{n^B}{n^B+1}\mathrm{cov}(x_i^G, x_j^B)$$
$$+ 2\frac{1}{(n^B+1)^2}\frac{n^B(n^B-1)}{2}\mathrm{cov}(x_j^B, x_k^B)]$$

$$= \beta_B^2[((\frac{n^B}{n^B+1})^2 + \frac{n^B}{(n^B+1)^2})(\sigma_z^2 + \sigma_u^2) - 2(\frac{n^B}{n^B+1})^2\sigma_z^2 + \frac{n^B(n^B-1)}{(n^B+1)^2}\sigma_z^2] = \beta_B^2 \frac{n^B}{n^B+1}\sigma_u^2.$$

Hence, a deviating high-ability grower can obtain expected utility

(2.137)

$$EU_i^{GD} = Ew_i^{GD} - \frac{1}{2a_i^G}(e_i^{GD})^2 - \frac{1}{2}r\,\mathrm{var}(w_i^{GD})$$

$$= \alpha_B + \beta_B[a_i^G + e_i^{GD} - \frac{1}{n^B+1}(a_i^G + e_i^{GD} + \sum_B (a_i^B + e_i^B))] - \frac{1}{2a_i^G}(e_i^{GD})^2 - \frac{1}{2}r\beta_B^2\frac{n^B}{n^B+1}\sigma_u^2.$$

In addition, the deviating high-ability grower optimally chooses the optimal effort by maximizing (2.137). That is,

(2.138)

$$e_i^{GD} \in \arg\max\{\alpha_B + \beta_B[a_i^G + e_i^{GD} - \frac{1}{n^B+1}(a_i^G + e_i^{GD} + \sum_B (a_i^B + e_i^B))] - \frac{1}{2a_i^G}(e_i^{GD})^2$$

$$-\frac{1}{2}r\beta_B^2\frac{n^B}{n^B+1}\sigma_u^2\},$$

from which

(2.139) $e_i^{GD} = \frac{n^B}{n^B+1}a_i^G\beta_B$.

On the other hand, if one low-ability grower *i* chooses the high-ability league, we define the average performance of the high-ability group as

(2.140) $\bar{x}^{GB} = \frac{1}{n^G+1}(x_i^B + \sum_G x_j^G)$.

Thus, the deviating low-ability grower receives reward

(2.141) $w_i^{BD} = \alpha_G + \beta_G[x_i^B - \bar{x}^{GB}]$.

Similarly, we can compute the expected payoff and variance of a deviating grower,

(2.142) $Ew_i^{BD} = \alpha_G + \beta_G[a_i^B + e_i^B - \frac{1}{n^G+1}(a_i^B + e_i^B + \sum_G (a_i^G + e_i^G))]$, and

(2.143) $\mathrm{var}(w_i^{BD}) = \beta_G^2\,\mathrm{var}(x_i^B - \bar{x}^{GB}) = \beta_G^2\,\mathrm{var}(\frac{n^G}{n^G+1}x_i^B - \frac{1}{n^G+1}\sum_G x_j^G)$

$$= \beta_G^2[(\frac{n^G}{n^G+1})^2 \operatorname{var}(x_i^B) + \frac{n^G}{(n^G+1)^2}\operatorname{var}(x_j^G) - 2\frac{n^G}{n^G+1}\frac{n^G}{n^G+1}\operatorname{cov}(x_i^B, x_j^G)$$

$$+2\frac{1}{(n^G+1)^2}\frac{n^G(n^G-1)}{2}\operatorname{cov}(x_j^G, x_k^G)]$$

$$= \beta_G^2[((\frac{n^G}{n^G+1})^2 + \frac{n^G}{(n^G+1)^2})(\sigma_z^2+\sigma_u^2) - 2(\frac{n^G}{n^G+1})^2\sigma_z^2 + \frac{n^G(n^G-1)}{(n^G+1)^2}\sigma_z^2] = \beta_G^2\frac{n^G}{n^G+1}\sigma_u^2 .$$

Further, the deviating low-ability grower must optimally choose optimal effort by maximizing

(2.144)

$$e_i^{BD} \in \arg\max\{\alpha_G + \beta_G[a_i^B + e_i^{BD} - \frac{1}{n^G+1}(a_i^B + e_i^{BD} + \sum\nolimits_G (a_i^G + e_i^G))] - \frac{1}{2a_i^B}(e_i^{BD})^2$$
$$-\frac{1}{2}r\beta_G^2\frac{n^G}{n^G+1}\sigma_u^2\},$$

from which

(2.145) $e_i^{BD} = \frac{n^G}{n^G+1}a_i^B\beta_G$.

Now, the additional incentive-compatibility constraints can be formulated. Since the processor offers a pooling contract for each of the two groups of growers, each incentive-compatibility constraint must be fulfilled under the expectation of corresponding grower abilities in that group. In other words, under the optimal contracts, a grower of average ability in one group must prefer his own contract to that designed for the other group. More precisely, for the high-ability group,

(2.146)

$$E_G\{\alpha_G+\beta_G[a_i^G+e_i^G-\frac{1}{n^G}\sum_G(a_j^G+e_j^G)]-\frac{1}{2a_i^G}(e_i^G)^2-\frac{1}{2}r\beta_G^2\frac{n^G-1}{n^G}\sigma_u^2\}\geq$$

$$E_G\{\alpha_B+\beta_B[a_i^G+e_i^{GD}-\frac{1}{n^B+1}(a_i^G+e_i^{GD}+\sum_B(a_j^B+e_j^B))]-\frac{1}{2a_i^G}(e_i^{GD})^2-\frac{1}{2}r\beta_B^2\frac{n^B}{n^B+1}\sigma_u^2\}.$$

Similarly, for the low-ability group,

(2.147)

$$E_B\{\alpha_B+\beta_B[a_i^B+e_i^B-\frac{1}{n^B}\sum_B(a_j^B+e_j^B)]-\frac{1}{2a_i^B}(e_i^B)^2-\frac{1}{2}r\beta_B^2\frac{n^B-1}{n^B}\sigma_u^2\}\geq$$

$$E_B\{\alpha_G+\beta_G[a_i^B+e_i^{BD}-\frac{1}{n^G+1}(a_i^B+e_i^{BD}+\sum_G(a_j^G+e_j^G))]-\frac{1}{2a_i^B}(e_i^{BD})^2-\frac{1}{2}r\beta_G^2\frac{n^G}{n^G+1}\sigma_u^2\}.$$

Thus, the processor solves the problem (2.128) subject to the constraints (2.129)-(2.132) and (2.146)-(2.147).

From the incentive-compatibility constraint (2.131) we can obtain the condition for optimal effort exerted by each grower in the high-ability group,

(2.148) $e_i^G=\frac{n^G-1}{n^G}a_i^G\beta_G,\ \forall i\in G.$

Similarly, for growers in the low-ability group,

(2.149) $e_i^B=\frac{n^B-1}{n^B}a_i^B\beta_B,\ \forall i\in B.$

Without loss of generality, we will adopt the standard results from contract theory (good references see footnote 1) for simplification of computations. Specifically, we assume that the participation constraint (2.129) and the incentive constraint (2.147) are not binding.

Thus, from the binding participation constraint (2.130), we can obtain

(2.150) $E_a[Ew_i^B\mid a_i\in[\underline{a},\hat{a}]]=E_a[\frac{1}{2a_i^B}(e_i^B)^2+\frac{1}{2}r\,\mathrm{var}(w_i^B)\mid a_i\in[\underline{a},\hat{a}]].$

From the binding incentive constraint (2.146),

(2.151)

$$E_a[Ew_i^G - \frac{1}{2a_i^G}(e_i^G)^2 - \frac{1}{2} r\,\mathrm{var}(w_i^G) \mid a_i \in [\hat{a}, \overline{a}]]$$

$$= E_a\{\alpha_G + \beta_G[a_i^G + e_i^G - \frac{1}{n^G}\sum_G (a_j^G + e_j^G)] - \frac{1}{2a_i^G}(e_i^G)^2 - \frac{1}{2} r\beta_G^2 \frac{n^G - 1}{n^G}\sigma_u^2\}$$

$$= E_a\{\alpha_B + \beta_B[a_i^G + e_i^{GD} - \frac{1}{n^B+1}(a_i^G + e_i^{GD} + \sum_B (a_j^B + e_j^B))] - \frac{1}{2a_i^G}(e_i^{GD})^2 - \frac{1}{2} r\beta_B^2 \frac{n^B}{n^B+1}\sigma_u^2\}$$

$$= E_a\{\alpha_B + \beta_B \frac{n^B}{n^B+1}(a_i^G + e_i^{GD}) - \beta_B \frac{1}{n^B+1}(\sum_B (a_j^B + e_j^B)) - \frac{1}{2a_i^G}(e_i^{GD})^2 - \frac{1}{2} r\beta_B^2 \frac{n^B}{n^B+1}\sigma_u^2\}$$

$$= E_a[\alpha_B + \beta_B[a_i^B + e_i^B - \frac{1}{n^B}\sum_B (a_j^B + e_j^B)] - \frac{1}{2a_i^B}(e_i^B)^2 - \frac{1}{2} r\beta_B^2 \frac{n^B - 1}{n^B}\sigma_u^2$$

$$- \beta_B[a_i^B + e_i^B - \frac{1}{n^B}\sum_B (a_j^B + e_j^B)] + \frac{1}{2a_i^B}(e_i^B)^2 + \frac{1}{2} r\beta_B^2 \frac{n^B - 1}{n^B}\sigma_u^2$$

$$+ \beta_B \frac{n^B}{n^B+1}(a_i^G + e_i^{GD}) - \beta_B \frac{1}{n^B+1}(\sum_B (a_j^B + e_j^B)) - \frac{1}{2a_i^G}(e_i^{GD})^2 - \frac{1}{2} r\beta_B^2 \frac{n^B}{n^B+1}\sigma_u^2]$$

$$= \beta_B \frac{n^B}{n^B+1}(a_i^G + e_i^{GD}) - \beta_B(a_i^B + e_i^B) + \beta_B \frac{1}{n^B}\sum_B (a_j^B + e_j^B) - \beta_B \frac{1}{n^B+1}(\sum_B (a_j^B + e_j^B)$$

$$+ \frac{1}{2a_i^B}(e_i^B)^2 - \frac{1}{2a_i^G}(e_i^{GD})^2 + \frac{1}{2} r\beta_B^2 \frac{n^B - 1}{n^B}\sigma_u^2 - \frac{1}{2} r\beta_B^2 \frac{n^B}{n^B+1}\sigma_u^2]$$

$$= \beta_B \frac{n^B}{n^B+1}(a_m^G + \frac{n^B}{n^B+1}a_m^G\beta_B) - \beta_B(a_m^B + \frac{n^B - 1}{n^B}a_m^B\beta_B)$$

$$+ \beta_B \frac{1}{n^B}\sum_B (a_m^B + \frac{n^B - 1}{n^B}a_m^B\beta_B) - \beta_B \frac{1}{n^B+1}(\sum_B (a_m^B + \frac{n^B - 1}{n^B}a_m^B\beta_B)$$

$$+ \frac{1}{2}a_m^B(\frac{n^B - 1}{n^B}\beta_B)^2 - \frac{1}{2}a_m^G(\frac{n^B}{n^B+1}\beta_B)^2 + \frac{1}{2} r\beta_B^2 \frac{n^B - 1}{n^B}\sigma_u^2 - \frac{1}{2} r\beta_B^2 \frac{n^B}{n^B+1}\sigma_u^2$$

$$= \beta_B[\frac{n^B}{n^B+1}(a_m^G + \frac{n^B}{n^B+1}a_m^G\beta_B) - (a_m^B + \frac{n^B - 1}{n^B}a_m^B\beta_B)] + \beta_B \frac{1}{n^B+1}(a_m^B + \frac{n^B - 1}{n^B}a_m^B\beta_B)$$

$$+ \frac{1}{2}(\beta_B)^2[a_m^B(\frac{n^B - 1}{n^B})^2 - a_m^G(\frac{n^B}{n^B+1})^2] - \frac{1}{2} r\beta_B^2\sigma_u^2 \frac{1}{n^B(n^B+1)}$$

$$= \beta_B \frac{n^B}{n^B+1}(a_m^G + \frac{n^B}{n^B+1}a_m^G\beta_B) - \beta_B \frac{n^B}{n^B+1}(a_m^B + \frac{n^B - 1}{n^B}a_m^B\beta_B)$$

$$+ \frac{1}{2}(\beta_B)^2[a_m^B(\frac{n^B - 1}{n^B})^2 - a_m^G(\frac{n^B}{n^B+1})^2] - \frac{1}{2} r\beta_B^2\sigma_u^2 \frac{1}{n^B(n^B+1)}$$

$$= \frac{n^B}{n^B+1} a_m^G \beta_B + a_m^G (\frac{n^B}{n^B+1}\beta_B)^2 - \frac{n^B}{n^B+1} a_m^B \beta_B - \frac{n^B-1}{n^B+1} a_m^B (\beta_B)^2$$
$$+\frac{1}{2}(\frac{n^B-1}{n^B})^2 a_m^B (\beta_B)^2 - \frac{1}{2}(\frac{n^B}{n^B+1})^2 a_m^G (\beta_B)^2 - \frac{1}{2} r \beta_B^2 \sigma_u^2 \frac{1}{n^B(n^B+1)}$$
$$= \frac{n^B}{n^B+1}\beta_B (a_m^G - a_m^B) + \frac{1}{2}(\beta_B)^2 [a_m^G (\frac{n^B}{n^B+1})^2 - a_m^B \frac{(n^B)^2+1}{(n^B)^2}\frac{n^B-1}{(n^B+1)}] - \frac{1}{2} r \beta_B^2 \sigma_u^2 \frac{1}{n^B(n^B+1)}.$$

Thus, we can obtain

(2.152)

$$E_a[Ew_i^G \mid a_i \in [\hat{a},\overline{a}]] = E_a[\frac{1}{2a_i^G}(e_i^G)^2 + \frac{1}{2} r \operatorname{var}(w_i^G) \mid a_i \in [\hat{a},\overline{a}]] +$$
$$\frac{n^B}{n^B+1}\beta_B (a_m^G - a_m^B) + \frac{1}{2}(\beta_B)^2 [a_m^G (\frac{n^B}{n^B+1})^2 - a_m^B \frac{(n^B)^2+1}{(n^B)^2}\frac{n^B-1}{(n^B+1)}] - \frac{1}{2} r \beta_B^2 \sigma_u^2 \frac{1}{n^B(n^B+1)}.$$

For simplicity, for large n^B, we can approximate the above expression as

(2.153)

$$E_a[Ew_i^G \mid a_i \in [\hat{a},\overline{a}]] \xrightarrow{n^B \; l\arg e} E_a[\frac{1}{2a_i^G}(e_i^G)^2 + \frac{1}{2} r \operatorname{var}(w_i^G) \mid a_i \in [\hat{a},\overline{a}]]$$
$$+ (a_m^G - a_m^B)[\beta_B + \frac{1}{2}(\beta_B)^2].$$

The last term in the above expression is the information rent received by high-ability type growers.

Substituting (2.150) and (2.153) into (2.128), the processor's expected profit becomes,

(2.154)

$$\overline{W}_R = \max_{\alpha_B,\beta_B,\alpha_G,\beta_G,\hat{a}} \{E_a[\textstyle\sum_G (Ex_i^G - Ew_i^G) \mid a_i \in [\hat{a},\overline{a}]] + E_a[\textstyle\sum_B (Ex_i^B - Ew_i^B) \mid a_i \in [\underline{a},\hat{a}]]\}$$
$$= E_a[\textstyle\sum_G (Ex_i^G - \frac{1}{2a_i^G}(e_i^G)^2 - \frac{1}{2} r \operatorname{var}(w_i^G) - (a_m^G - a_m^B)[\beta_B + \frac{1}{2}(\beta_B)^2]) \mid a_i \in [\hat{a},\overline{a}]]$$
$$+ E_a[\textstyle\sum_B (Ex_i^B - \frac{1}{2a_i^G}(e_i^G)^2 - \frac{1}{2} r \operatorname{var}(w_i^G)) \mid a_i \in [\underline{a},\hat{a}]]$$

$$= E_a[\sum_G (a_i^G + \frac{n^G - 1}{n^G} a_i^G \beta_G - \frac{1}{2} a_i^G (\frac{n^G - 1}{n^G} \beta_G)^2 - \frac{1}{2} r \beta_G^2 \frac{n^G - 1}{n^G} \sigma_u^2) \mid a_i \in [\hat{a}, \overline{a}]]$$

$$+ E_a[\sum_B (a_i^B + \frac{n^B - 1}{n^B} a_i^B \beta_B - \frac{1}{2} a_i^B (\frac{n^B - 1}{n^B} \beta_B)^2 - \frac{1}{2} r \beta_B^2 \frac{n^B - 1}{n^B} \sigma_u^2) \mid a_i \in [\underline{a}, \hat{a}]]$$

$$- n^G (a_m^G - a_m^B)[\beta_B + \frac{1}{2}(\beta_B)^2]$$

$$= n^G (a_m^G + \frac{n^G - 1}{n^G} a_m^G \beta_G - \frac{1}{2} a_m^G (\frac{n^G - 1}{n^G} \beta_G)^2 - \frac{1}{2} r \beta_G^2 \frac{n^G - 1}{n^G} \sigma_u^2)$$

$$+ n^B (a_m^B + \frac{n^B - 1}{n^B} a_m^B \beta_B - \frac{1}{2} a_m^B (\frac{n^B - 1}{n^B} \beta_B)^2 - \frac{1}{2} r \beta_B^2 \frac{n^B - 1}{n^B} \sigma_u^2) - n^G (a_m^G - a_m^B)[\beta_B + \frac{1}{2}(\beta_B)^2].$$

Differentiating (2.154) with respect to β_G yields,

(2.155) $$\frac{\partial \overline{W}_R}{\partial \beta_G} = n^G (\frac{n^G - 1}{n^G} a_m^G - a_m^G (\frac{n^G - 1}{n^G})^2 \beta_G - r \beta_G \frac{n^G - 1}{n^G} \sigma_u^2) = 0,$$

from which

(2.156) $$\beta_G = \frac{a_m^G}{a_m^G \frac{n^G - 1}{n^G} + r \sigma_u^2} = \frac{1}{\frac{n^G - 1}{n^G} + \frac{r}{a_m^G} \sigma_u^2}.$$

Similarly, differentiating (2.154) with respect to β_B yields,

(2.157)

$$\frac{\partial \overline{W}_R}{\partial \beta_B} = n^B (\frac{n^B - 1}{n^B} a_m^B - a_m^B (\frac{n^B - 1}{n^B})^2 \beta_B - r \beta_B \frac{n^B - 1}{n^B} \sigma_u^2) - n^G (a_m^G - a_m^B)[1 + \beta_B] = 0,$$

from which

(2.158) $$\beta_B = \frac{a_m^B - \frac{n^G}{(n^B - 1)}(a_m^G - a_m^B)}{\frac{n^B - 1}{n^B} a_m^B + r \sigma_u^2 + \frac{n^G}{n^B - 1}(a_m^G - a_m^B)}.$$

In addition, the base payment in each contract can be solved using the participation constraints (2.130) and the incentive-compatibility constraints (2.146) or (2.153), respectively. Specifically, for the low-ability group, the base payment is given by

(2.159)

$$\alpha_B = -\beta_B[a_m^B + \frac{n^B - 1}{n^B} a_m^B \beta_B - \frac{1}{n^B}\sum\nolimits_B (a_m^B + \frac{n^B-1}{n^B} a_m^B \beta_B)] + \frac{1}{2} a_m^B (\frac{n^B-1}{n^B}\beta_B)^2 + \frac{1}{2} r\beta_B^2 \frac{n^B-1}{n^B}\sigma_u^2$$

$$= \frac{1}{2}\frac{n^B-1}{n^B} a_m^B \beta_B = \frac{1}{2}\frac{n^B-1}{n^B} \frac{a_m^B - \frac{n^G}{(n^B-1)}(a_m^G - a_m^B)}{\frac{n^B-1}{n^B} a_m^B + r\sigma_u^2 + \frac{n^G}{n^B-1}(a_m^G - a_m^B)}.$$

For the high-ability group, the base payment, for n^B large, can be approximated by

(2.160)

$$\alpha_G = \frac{1}{2} a_m^G (\frac{n^G-1}{n^G}\beta_G)^2 + \frac{1}{2} r\beta_G^2 \frac{n^G-1}{n^G}\sigma_u^2 + (a_m^G - a_m^B)[\beta_B + \frac{1}{2}(\beta_B)^2]$$

$$= \frac{1}{2}\frac{n^G-1}{n^G} a_m^G \beta_G + (a_m^G - a_m^B)[\beta_B + \frac{1}{2}(\beta_B)^2].$$

Note that, in order to induce a separating equilibrium for the two groups of growers, the difference between the highest ability level $\overline{a}$ and the lowest ability level $\underline{a}$ must be sufficiently large. To see this, if $\overline{a} - \underline{a}$ is small, then $a_m^G - a_m^B = \frac{\overline{a} + \hat{a}}{2} - \frac{\hat{a} + \underline{a}}{2} = \frac{\overline{a} - \underline{a}}{2}$ is small. Comparing (2.159) and (2.160) shows that the base payment for the high-ability group α_G is close to that for the low-ability group α_B when *n* is large. Similarly, from (2.156) and (2.158), the bonus for the high-ability group β_G is close to that for the low-ability group β_B as well. In addition, from (2.151), if $\overline{a} - \underline{a}$ is small, the incentive-compatibility constraint (2.146) for the high-ability group could be violated because the absolute value of the negative term in (2.151) could exceed sum of the positive terms. In other words, the high-ability type growers would prefer to choose the contract designed for the low-ability type growers rather than their own contract. As a result, only a single pooling contract could be sustained if $\overline{a} - \underline{a}$ is too small.

Further, the processor can optimally choose a separating ability level $\hat{a}$ by maximizing the expected profit (2.154). However, we omit it here due to the tedious computation. Instead, we assume that the processor assigns an arbitrary ability level to divide the growers into two groups. As an illustration, suppose that the processor chooses $\hat{a} = \frac{\overline{a} + \underline{a}}{2}$.

Under this assumption, we can compute

(2.161) $n^G = n^B = \frac{1}{2} n$,

(2.162) $a_m^G = \frac{3}{4}\overline{a} + \frac{1}{4}\underline{a}$, and

(2.163) $a_m^B = \frac{3}{4}\underline{a} + \frac{1}{4}\overline{a}$.

Hence, assuming that *n* is sufficiently large, the optimal contract for the high-ability group can be written as,

(2.164) $\beta_G = \dfrac{1}{\dfrac{n^G - 1}{n^G} + \dfrac{r}{a_m^G}\sigma_u^2} \cong \dfrac{a_m^G}{a_m^G + r\sigma_u^2} = \dfrac{1}{1 + \dfrac{4r}{3\overline{a} + \underline{a}}\sigma_u^2}$, and

(2.165)

$$\alpha_G = \frac{1}{2}\frac{n^G - 1}{n^G} a_m^G \beta_G + (a_m^G - a_m^B)[\beta_B + \frac{1}{2}(\beta_B)^2]$$
$$\cong \frac{1}{8}(3\overline{a} + \underline{a})\beta_G + \frac{\overline{a} - \underline{a}}{2}[\beta_B + \frac{1}{2}(\beta_B)^2].$$

Similarly, for the low-ability group,

$$(2.166)\quad \beta_B = \frac{a_m^B - \frac{n^G}{(n^B-1)}(a_m^G - a_m^B)}{\frac{n^B-1}{n^B}a_m^B + r\sigma_u^2 + \frac{n^G}{n^B-1}(a_m^G - a_m^B)} \cong \frac{2a_m^B - a_m^G}{a_m^G + r\sigma_u^2} = \frac{5\underline{a} - \overline{a}}{3\overline{a} + \underline{a} + 4r\sigma_u^2}\text{, and}$$

$$(2.167)\quad \alpha_B = \frac{1}{2}\frac{n^B-1}{n^B}a_m^B\beta_B \cong \frac{1}{8}(3\underline{a} + \overline{a})\beta_B .$$

It can be easily verified that $\beta_B < \beta_G$ and $\alpha_B < \alpha_G$.[17] In other words, the processor offers greater base payment and bonus to the high-ability group than those to the low-ability group. Consequently, growers belonging to the low-ability group would prefer their own contract C_B to the contract C_G because they would incur greater penalty if they would have joined the high-ability league. On the other hand, high-ability growers would also prefer the contract C_G to the contract C_B because they would receive a smaller bonus if they would have joined the low-ability league. At the optimum, the optimal contract for the high-ability group offers a positive information rent through the base payment α_G, which makes an average high-ability grower indifferent between choosing the contract C_G and choosing C_B. In addition, the optimal base payment to the high-ability group guarantees that it is sufficiently small such that an average low-ability grower would not deviate and choose the contract C_G.

[17] To show $\beta_B < \beta_G$, let $\tilde{\beta}_B = \frac{a_m^B}{\frac{n^B-1}{n^B}a_m^B + r\sigma_u^2} = \frac{1}{\frac{n^B-1}{n^B} + \frac{r}{a_m^B}\sigma_u^2}$. Then $\tilde{\beta}_B > \beta_B$. Since $a_m^B < a_m^G$, $\tilde{\beta}_B = \frac{1}{\frac{n^B-1}{n^B} + \frac{r}{a_m^B}\sigma_u^2} < \beta_G = \frac{1}{\frac{n^G-1}{n^G} + \frac{r}{a_m^G}\sigma_u^2}$. Therefore, $\beta_B < \beta_G$. Hence, $\alpha_B < \alpha_G$ as well.

Given the assumption $\hat{a} = \frac{\overline{a} + \underline{a}}{2}$ and conditions (2.164)-(2.167), we can compute the processor's expected profit under the two-tournament scheme.

(2.168)

$$\overline{W_R} = n^G(a_m^G + \frac{n^G - 1}{n^G} a_m^G \beta_G - \frac{1}{2} a_m^G (\frac{n^G - 1}{n^G}\beta_G)^2 - \frac{1}{2} r\beta_G^2 \frac{n^G - 1}{n^G}\sigma_u^2)$$
$$+ n^B(a_m^B + \frac{n^B - 1}{n^B} a_m^B \beta_B - \frac{1}{2} a_m^B (\frac{n^B - 1}{n^B}\beta_B)^2 - \frac{1}{2} r\beta_B^2 \frac{n^B - 1}{n^B}\sigma_u^2) - n^G(a_m^G - a_m^B)[\beta_B + \frac{1}{2}(\beta_B)^2]$$

$$\cong n^G(a_m^G + \frac{1}{2} a_m^G \beta_G) + n^B\{a_m^B + a_m^B\beta_B - (a_m^G - a_m^B)\beta_B - \frac{1}{2}\beta_B^2[a_m^B + r\sigma_u^2 + (a_m^G - a_m^B)]\}$$
$$= n^G(a_m^G + \frac{1}{2} a_m^G \beta_G) + n^B\{a_m^B + (2a_m^B - a_m^G)\beta_B - \frac{1}{2}\beta_B^2[a_m^G + r\sigma_u^2]\}$$
$$= n^G(a_m^G + \frac{1}{2} a_m^G \beta_G) + n^B[a_m^B + \frac{1}{2}(2a_m^B - a_m^G)\beta_B]$$
$$= \frac{1}{2} n(\frac{3\overline{a} + \underline{a}}{4})(1 + \frac{1}{2[1 + \frac{4}{(3\overline{a} + \underline{a})} r\sigma_u^2]}) + \frac{1}{2} n(\frac{3\underline{a} + \overline{a}}{4})(1 + \frac{1}{2}(\frac{5\underline{a} - \overline{a}}{3\underline{a} + \overline{a}})\frac{5\underline{a} - \overline{a}}{3\overline{a} + \underline{a} + 4r\sigma_u^2})$$
$$= \frac{1}{2} n(\frac{3\overline{a} + \underline{a}}{4})(1 + \frac{1}{2[1 + \frac{4}{(3\overline{a} + \underline{a})} r\sigma_u^2]}) + \frac{1}{2} n(\frac{3\underline{a} + \overline{a}}{4})(1 + (\frac{5\underline{a} - \overline{a}}{3\underline{a} + \overline{a}})^2 \frac{1}{2[1 + \frac{4}{3\underline{a} + \overline{a}} r\sigma_u^2]}).$$

Further, we can compare the expected profit (2.168) under the two-tournament RPC scheme with that under the one-tournament RPC given by (2.32). However, to compare these two schemes involves some manipulation.

First, we define the following function:

(2.169) $f(y) = ny(1 + \frac{1}{2[1 + \frac{1}{y} r\sigma_u^2]})$, for $y \in [\underline{a}, \overline{a}]$.

From this function, one can define the following:

$$f^{'}(y)=n+\frac{2n[1+\frac{1}{y}r\sigma_u^2]+ny\frac{2}{y^2}r\sigma_u^2}{4[1+\frac{1}{y}r\sigma_u^2]^2}=n+\frac{n[1+\frac{2r\sigma_u^2}{y}]}{2[1+\frac{r\sigma_u^2}{y}]^2}>0\text{, and}$$

$$f^{''}(y)=\frac{-n\frac{4r\sigma_u^2}{y^2}[1+\frac{r\sigma_u^2}{y}]^2+4n[1+\frac{2r\sigma_u^2}{y}][1+\frac{r\sigma_u^2}{y}]\frac{r\sigma_u^2}{y^2}}{4[1+\frac{r\sigma_u^2}{y}]^4}=\frac{n(r\sigma_u^2)^2}{y^3[1+\frac{r\sigma_u^2}{y}]^3}>0.$$

Therefore, $f(y)$ is strictly convex in y. From properties of convexity, it must be true that

$(1-\lambda)f(y_1)+\lambda f(y_2)\ge f[(1-\lambda)y_1+\lambda y_2]$ for $0<\lambda<1$. Thus, define $\lambda=\frac{1}{2}$,

$y_1=\frac{3\overline{a}+\underline{a}}{4}$, and $y_2=\frac{3\underline{a}+\overline{a}}{4}$, we can obtain

(2.170)

$$\overline{W}_R^{'}=\frac{1}{2}n(\frac{3\overline{a}+\underline{a}}{4})(1+\frac{1}{2[1+\frac{4}{(3\overline{a}+\underline{a})}r\sigma_u^2]})+\frac{1}{2}n(\frac{3\underline{a}+\overline{a}}{4})(1+\frac{1}{2[1+\frac{4}{3\underline{a}+\overline{a}}r\sigma_u^2]})>$$

$$n\frac{\overline{a}+\underline{a}}{2}[1+\frac{1}{2(1+\frac{2r}{\overline{a}+\underline{a}}\sigma_u^2)}]=W_R.$$

However, since $\frac{5\underline{a}-\overline{a}}{3\underline{a}+\overline{a}}=\frac{3\underline{a}+\overline{a}+2(\underline{a}-\overline{a})}{3\underline{a}+\overline{a}}=1-\frac{2(\overline{a}-\underline{a})}{3\underline{a}+\overline{a}}<1$, it is easy to verify that

$\overline{W}_R^{'}>\overline{W}_R$. Therefore, given the assumption $\hat{a}=\frac{\overline{a}+\underline{a}}{2}$, $\overline{W}_R>W_R$ only if $\frac{5\underline{a}-\overline{a}}{3\underline{a}+\overline{a}}$ is close to one, or equivalently, $\overline{a}-\underline{a}$ is sufficiently small. However, recall that $\overline{a}-\underline{a}$ cannot be too small because otherwise, only a single pooling contract could be sustained.

Consequently, given $\hat{a}=\frac{\overline{a}+\underline{a}}{2}$, if $\overline{a}-\underline{a}$ is not sufficiently small such that $\overline{W}_R>W_R$, then the separating point $\hat{a}$ is not optimal for the processor. Instead, the processor should use a greater $\hat{a}$ as the separating point for the two groups. By using a greater $\hat{a}$, the processor reduces the total bonus paid to the high-ability group and hence, raises total expected profit. To understand this, we should attribute the profit improvement to more information under the two-tournament scheme. More precisely, relative to the one-tournament RPC, growers reveal more information concerning their abilities when two tournaments are offered in one period. Thus, the processor can take advantage of the new information and exploit more profit from growers. A numerical example will be necessary to find the explicit optimal separating point. However, it is not discussed further in this essay due to length restrictions.

2.5 Conclusion and Discussion

Several papers have discussed broiler contracts out of concerns of growers' dissatisfaction with the existing relative-performance contracts and have compared RPC with an alternative FPC either in the static setting or a dynamic setting. However, these studies draw different conclusions about the relative superiority of RPC and FPC due to different assumptions, different model structure, or different data. To better understand broiler contracts, this essay not only compares relative-performance contracts with fixed-performance contracts in both a static setting and a dynamic setting, but it also discusses improvements of the static relative-performance contract. More specifically, a static RPC and a FPC are formulated in the first part of the essay. Based on the static model, a two-period full-commitment model is constructed as well. The second part of the model consists of three types of two-period dynamic contracts: a dynamic fixed-performance contract, a dynamic current-period relative-performance contract, and a dynamic previous-period relative-performance contract. Then the following comparisons are drawn: the dynamic FPC with the full-commitment FPC; the dynamic current-period RPC with the dynamic FPC; and the dynamic previous-period RPC with the dynamic FPC. The last part of the model develops a static two-tournament RPC and compares it with the static RPC.

Comparisons between various scenarios of RPC and FPC are summarized in Table 2.1. Major findings include the following five general results:

First, under the static RPC and FPC, the efficiency results depend on the relative magnitude of the common shocks and idiosyncratic shocks. Specifically, the static RPC performs better if the common shock is sufficiently large, while the static FPC is better if the idiosyncratic shock dominates. This result is consistent with most of the previous studies except Roe and Wu (2003), who find that banning RPC in a static model can never increase total surplus. Their results are different because of their model specifications: in particular, the formulation and interpretation of the payment schedules and the assumptions of the random variables in the output structure contribute to their results.

Since the full-commitment contracts are exactly a sequence of static contracts, the full-commitment RPC and the full-commitment FPC have the same properties as the static contracts.

Second, the dynamic FPC performs better than the full-commitment FPC because under the dynamic FPC, the processor improves the second-period contract by taking advantage of the new information acquired at the end of the first period. By providing a greater bonus in the second period under the dynamic FPC, the processor induces more efforts from the growers, and hence, increases total welfare.

Third, regardless of the autocorrelation of common shocks in the two periods, the dynamic current-period RPC eliminates the contemporary common shocks. Thus, the dynamic RPC is exactly a repetition of the static RPC. Comparing the dynamic current-period RPC with the dynamic FPC indicates that the dynamic current-period RPC performs better than the dynamic FPC only if the common shock is sufficiently large, and vice versa. However, Proposition 3 demonstrates that the FPC becomes more beneficial in the sense that the dynamic FPC is favored against relative-performance contracts under more circumstances relative to the static FPC. In other words, in a dynamic setting, a FPC becomes more effective at gathering information and improving the efficiency of the incentives relative to the static case.

Fourth, the dynamic FPC performs better than the dynamic previous-period RPC under any conditions. In addition, under this contract, significant ratchet effects are present in the sense that growers exert less effort in the first period in anticipation of a higher standard in the second period based on their first-period performance. In turn, at the equilibrium, the processor must offer a greater bonus in the first period to induce more effort. However, readers should be reminded that the assumption of the first-period FPC under the dynamic previous-period RPC is critical to lead to the conclusion. If, instead, a static RPC is adopted in the first period under the dynamic previous-period RPC, the efficiency results will depend on the stochastic shocks. In particular, if a static RPC is used, the dynamic previous-period RPC would perform better than the dynamic FPC if the common shock is sufficiently large.

Finally, the last part of the model contains a static two-tournament RPC. Under this contract, the processor offers both a greater bonus and a greater base payment to high-ability growers than to low-ability growers. Intuitively, the large bonus for the high-ability group prevents the low-ability group from shirking because it becomes a large penalty if a low-ability grower deviates. On the other hand, a small bonus for the low-ability group prevents the high-ability group from shirking because if a grower in the high-ability group deviates, not only would he make less direct profit, but he would lose the positive information rents paid to the high-ability group. Further, the results suggest that the two-tournament RPC can improve the processor's expected profit relative to the static RPC because the processor can provide more efficient incentives by differentiating growers of different abilities and, hence, extract a greater profit from high-ability growers.

Compared to past studies, this essay provides a more thorough and comprehensive analysis of broiler contracts. In particular, the dynamic previous-period RPC and the two-tournament static RPC have not been investigated in the existing literature.

The results in this essay provide some important policy implications and practical guidelines. First, except for the dynamic previous-period RPC, comparisons between relative-performance contracts and fixed-performance contracts under each scenario justify the superiority of relative-performance contracts both in a static setting and in a dynamic setting when common shocks dominate idiosyncratic shocks. Roe and Wu (2003) corroborate this result. As for the dynamic previous-period RPC, it could still perform better than the dynamic FPC if the first-period contract is specified with a current-period RPC. However, unlike Roe and Wu (2003), this essay does not account for the possibility of changing bargaining powers of growers in future periods as their abilities are revealed in previous periods. Therefore, in the principal-agent framework, the results from this essay cannot demonstrate the favorability of one contract against the other from growers' point of view because growers always receive their expected reservation utility under each type of contract. In the real world, however, growers possibly have bargaining power due to competition among processors. We have shown that relative-performance contracts improve total welfare when the common shock dominates and, thus, growers could capture a share of the surplus and still favor relative-performance contracts against fixed-performance contracts.

On the other hand, however, readers should know that the contracts derived in this essay are still highly stylized versions of actual broiler contracts. For example, payments to growers in actual broiler contracts are usually based on the feed conversion ratio. Here we use growers' output as a substitute. Second, in the real world, growers are different not only in terms of their ability, but also in terms of their production capacity, flock sizes, or number of flocks assigned in each period. In addition, in this stylized model, it is necessary to assume a fixed league composition in the dynamic setting in order to investigate the ratchet effect. An analysis of random league compositions, as happens in the real world, would seriously complicate the analysis. However, all of these potential extensions are beyond the specific interests of this essay.

Table 2.1 Comparisons of fixed-performance contracts and relative-performance contracts

		Static FPC			Static RPC			Full Commitment FPC			Full Commitment RPC		
		α_F	β_F	W_F	α_R	β_R	W_R	α_F	β_F	W_F^F	α_R	β_R	W_R^F
Static FPC	α_F				n/a			=			n/a		
	β_F					$<^*$			=			$<^*$	
	W_F						$<^*$						$<^*$
Static RPC	α_R							n/a			=		
	β_R								$^*>$			=	
	W_R												
Full Commitment FPC	α_F										n/a		
	β_F											$<^*$	
	W_F^F												$<^*$
Full Commitment RPC	α_R												
	β_R												
	W_R^F												
Dynamic FPC	α_{F1}												
	β_{F1}												
	α_{F2}												
	β_{F2}												
	W_F^D												
Dynamic current-period RPC	α_{R1}												
	β_{R1}												
	α_{R2}												
	β_{R2}												
	W_R^S												

Table 2.1 (Cont.)

		Dynamic FPC					Dynamic current-period RPC					Dynamic previous-period RPC				
		α_{F1}	β_{F1}	α_{F2}	β_{F2}	W_F^D	α_{R1}	β_{R1}	α_{R2}	β_{R2}	W_R^S	α_{L1}	β_{L1}	α_{L2}	β_{L2}	W_R^L
Static FPC	α_F	=		>			n/a		n/a			n/a		n/a		
	β_F		=		<			$<^*$		$<^*$			<		<	
	W_F															
Static RPC	α_R						=		=							
	β_R							=		=						
	W_R															
Full Commitment FPC	α_F	=		>												
	β_F		=		<											
	W_F^F					<										
Full Commitment RPC	α_R						=		=							
	β_R							=		=						
	W_R^F										=					
Dynamic FPC	α_{F1}						n/a					n/a				
	β_{F1}							$<^{**}$					<			
	α_{F2}								n/a					n/a		
	β_{F2}									n/a					=	
	W_F^D										$<^{**}$					>
Dynamic current-period RPC	α_{R1}															
	β_{R1}															
	α_{R2}															
	β_{R2}															
	W_R^S															

Notes:
a) Each cell in the table compares the corresponding parameter in the second column and the corresponding parameter in the second row. For example, a " < " sign means that the corresponding parameter in the second column is less than that in the second row.
b) The cells with one asterisk (*) depend on the relative magnitude of the common shock and the idiosyncratic shock. The explicit conditions are derived in Proposition 1 and Corollary 1.1. The cells with (**) depend on the condition derived in Proposition 3.
c) The matrix in the table is symmetric except the last scenario, i.e., Dynamic previous-period RPC. Thus, only the upper triangle of the table is filled.
d) We use the symbol "n/a" to indicate that these cells are indeterminate and use empty cells to indicate that these are irrelevant or not the interest of this paper.

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ESSAY III

DYNAMIC CONTRACTS WITH ADVERSE SELECTION: INCORPORATING GROWER REPUTATION INTO CONTRACT DESIGN

3.1 Introduction

This essay investigates the role of growers' reputation when an agricultural processor designs optimal incentives for high quality products in a dynamic contracting context. Due to the characteristics borne in agricultural production, agricultural contracts have special features different from those in other industries. For example, agricultural producers often require large capital investments in land, agricultural stock, equipment, and facilities that make the processor-producer relationship specific. As a result, large-scale processors, such as those in the frozen or canned fruits and vegetables, extensively use long-term contracts with producers. Many industry observers believe that reputation for quality plays an important role in maintaining the processor-producer relationship. For example, Caspers (2000) reports that pork contractors invest much effort in building up reputation capital in areas were contract production is widespread. Goodhue et al. (2000) test hypotheses regarding long-term relationships between contracting and reputation of grape quality in the California winegrape industry. They point out that a grower's reputation for consistent grape quality is necessary for him to receive a high price for future harvests and that the grape grower may not recover his costs through resale if his farm's reputation is harmed. Despite these reports and observations, few studies have taken into account reputation effects when one designs a long-term agricultural contract.

Wilson (1985) summarizes past studies of reputation effects in various contexts. In game-theoretic and market models, one's reputation is usually defined as the history of his previously observed actions or measured performance. Operationally, it is usually summarized by a derived posterior probability assessment by his opponent, which is used to predict the player's future actions. He indicates that "at least four ingredients are necessary to enable a role for reputations. (1) There must be several players in the game, and (2) at least one player has some private information that persists over time. This player (3) is likely to take several actions in sequence, and (4) is unable to commit in advance to the sequence of actions he will take (Wilson, 1985, p.29)." According to Wilson, a player's optimal strategy in the presence of reputation effects must take into account the following chain of reasoning: his current reputation affects others' predictions of his current behavior and thereby affects their current actions. Thus, his optimal strategies must be not only the best choice in the current immediate decision, but also the best longer-run decision which takes into consideration of the effect of his current decision on his future reputation. Further, others' current actions will be affected as well anticipating the player's long-term consequences. Since the four necessary components are contained in a dynamic principal-agent problem with adverse selection, reputation effects could be sustained to provide more effective incentives in such a context.

To address this issue, a two-period principal-agent model is used in this essay. Since commitment to the intertemporal contract terms is crucial to the optimal incentives, this essay is organized in the following manner. The first section of the model derives properties of a long-term contract with full commitment by both parties. The second section establishes a long-term dynamic contract with no commitment to the intertemporal schemes. The third section introduces a reputation reward contingent on observation of high quality and investigates its effect on the dynamics of optimal incentives. The final section concludes and discusses potential extensions of the model.

3.2 Literature Review

Reputations effects in games and markets

Wilson (1985) differentiates past studies of reputation effects in two groups, game-theoretic models and market models. Among all the game-theoretic models, the chain-store game (Selten, 1978; Kreps and Wilson, 1982; Milgrom and Roberts, 1982), the sequential bargaining game (Fudenberg and Tirole, 1983), and the repeated prisoners' dilemma game (Kreps et al. 1982) are among the most classic representations of reputation effects in repeated games. In these studies, a player's reputation is summarized in his opponent's beliefs about his type. The key ingredient of these studies is that players would be likely to incur short-term costs to build up reputation that yield favorable long-term consequences when he is patient and his planning horizon is long. However, none of these studies is formulated within the principal-agent framework.

Another group of models examines the role of reputation effects in markets. Shapiro (1982) examines how a profit-maximizing firm chooses product quality in an environment where consumers, who cannot observe quality before purchase, use product reputation as a criterion for quality. Since reputation adjustment can reward high quality production only with a lag, Shapiro shows that the firm will not find it profitable to provide as high a quality as under perfect information. Shapiro (1983) derives an equilibrium price-quality schedule for markets in which product quality is unobservable to consumers before purchase. He argues that high-quality products should be paid a price premium for compensating sellers for their investment in reputation. Similarly, Allen (1984) studies the role of reputations in a competitive market where product quality is unobservable and finds that there exist equilibria where price is equal to average cost but greater than marginal cost. As Wilson (1985) indicates, however, most of these studies do not explicitly specify the source of the reputation effects; they simply assume that they are present.

Several papers concerning reputation and quality are also found in an agricultural setting. For example, Worth (1999) develops a model of how food firms determine the quality of their output in the presence of reputation of product quality. Quagrainie, McCluskey, and Loureiro (2001) adopt a dynamic multiple-indicator model to test the relationship between reputation of quality and price premium for Washington apples and find that price premiums are good indicators of reputation. Goodhue et al. (2000) test hypotheses regarding long-term relationships between contracting and reputation of grape quality in the California winegrape industry. A hedonic pricing study conducted by Schamel (2002) finds significant association between California wine prices and winery reputation indicators.

Repeated agency problems

Many papers have investigated dynamics of repeated agency models in the presence of asymmetric information. One set of studies focuses on multi-period agency models with moral hazard in either a finite or an infinite horizon. For example, Rubinstein and Yaari (1983) and Radner (1985) study an infinitely repeated problem in which neither principal nor agent discounts the future. They show that in this case there exists an optimal contract that yields both the principal and the agent the same expected utilities as they would have received in the first-best case. Thus, inefficiencies due to moral hazard that arise in static settings are completely overcome in this case. Radner (1985) shows that if the discount rates are close to one, the first-best solution is approximately achievable. Lambert (1983) and Rogerson (1985) examine qualitative features of the optimal contract with discounting. Lambert develops properties of the optimal contract in a finite horizon model using the first-order approach, while Rogerson (1985) examines the relationship between wages and effort for any two successive periods in a repeated problem with discounting. Both papers show that history plays an essential role in a repeated relationship and the optimal contract in any period will depend on the entire previous history of the relationship.

In contrast, another set of studies investigates multi-period contracts in the presence of adverse selection. For example, Freixas, Guesnerie, and Tirole (1985) study the dynamics of a linear contract and demonstrate that ratchet effects exist in the presence of hidden information. Laffont and Tirole (1988) study a two-period principal-agent model with unobservable agents' abilities. However, unlike Freixas, Guesnerie, and Tirole (1985), they demonstrate that with a continuum of types, for any first-period incentive schemes, there exists no fully separating continuation equilibrium. Hosios and Peters (1989) examine a two-period insurance contract and show that, in the absence of discounting, no fully separating equilibrium can be sustained.

A few other papers also investigate relationships between short-term contracts and long-term contracts. Specifically, these papers deal with spot implementability of a long-term contract via a sequence of short-term contracts. For example, Fudenberg, Holmström, and Milgrom (1990) use a multiperiod principal-agent model to illustrate that an optimal long-term contract can be implemented by a sequence of short-term contracts under certain conditions. Spear and Srivastava (1987) analyze an optimal contract in an infinitely repeated agency model in which both principal and agent discount the future. They show that the multi-period problem can be reduced to a static variational problem and a simple stationary representation of the dynamic optimal contract exists. Malcomson and Spinnewyn (1988) also show that under certain conditions repeated short-term contracts implement long-term contracts and that linking payoffs in one period to outcomes in previous periods does not improve the tradeoff between incentives and risk sharing. Rey and Salanié (1996) concluded that a sequence of short-term contracts could be as efficient as long-term renegotiation-proof contracts in the presence of adverse selection if renegotiation is always possible. However, spot contracting will be efficient under much more restrictive assumptions.

Although few studies have directly investigated reputation effects in the principal-agent framework, those studies about repeated agency problems can shed some light on this type of problem. In some sense, the analysis that follows in this essay is a synthesis of those studies with hidden information and yet incorporates with the notion of reputation effects in an agricultural context.

3.3 Objectives

This main objective of this essay is to investigate the role of growers' reputation when an agricultural processor designs optimal incentives for better quality products in a two-period dynamic contract. When the grower's type, as reflected by product quality, is unobservable to the processor, adverse selection would be likely to occur in a processor-producer relationship if no effective incentives are provided. In addition, in the absence of commitment to intertemporal contract terms by both parties, the existence of hidden information that persists over time and a grower's sequential choices of actions enable a role for reputation effects in the two-period dynamic contract. Thus, optimal incentives in such a contract must take into consideration not only the adverse consequences of hidden information in the short term, but also its intertemporal consequences in the longer term.

The first section of the essay develops a two-period full-commitment model, which requires that both parties be committed to the contract terms and that the contract cannot be breached or renegotiated during the contracting period. This model serves as a baseline. Then a two-period dynamic model with no commitment is developed. Specifically, the no-commitment contract assumes that neither the processor nor the grower can commit to an intertemporal scheme. In other words, the processor can revise the contract in the second period conditional on the grower's first-period performance and the grower can quit the relationship at the end of each period. Under this contract, optimal conditions for a fully separating equilibrium, a semi-separating equilibrium, and a pooling equilibrium are established. In this case, reputation effects are embodied in the posterior probability assessment (Bayes' rule) of the grower's types by the processor at the end of the first period. Anticipating the processor's strategies, the high-quality grower type chooses to build up his reputation by either imitating the low-quality type or revealing his true type, whichever is favorable. In fact, imitating the dominant behavior of a low-quality type yields future information rents to the high-quality type by sustaining the processor's belief that the grower might be of low-quality type.

Based on the no-commitment dynamic model, the third section incorporates a reputation reward contingent on the grower's past performance into the model. More specifically, a reputation reward is remunerated to the grower in the second period if the processor observes good performance at the end of the first period. To simplify the analysis further, the reputation reward is assumed to take the form of a lump sum payment. Under these assumptions, this essay demonstrates that the reputation reward contingent on the grower's history of performance provides incentives for the grower to invest effort in building a reputation for high quality and, thereby, could improve both the processor's and the grower's welfare and result in a dominant equilibrium. The final section of the essay concludes and discusses potential policy implications.

This essay contributes to the related literature in the following aspects: Firstly, in contrast with some of the past studies that rule out existence of a fully separating equilibrium in a dynamic contract, this essay establishes optimal conditions for a fully separating equilibrium, a semi-separating equilibrium, and a pooling equilibrium under certain conditions. Moreover, conditions for optimality of a "handicapped" separating equilibrium, in which a single contract is offered to the high-quality grower type, are also investigated. Secondly, although many studies have discussed reputation effects in various game-theoretic settings, few studies have explicitly investigated reputation effects in a principal-agent framework with asymmetric information, and virtually no study models reputation rewards contingent on past observed performance in a dynamic contract. Thus, this essay addresses a question of both theoretical interest and practical importance.

3.4 The model

For simplicity, it is assumed that there are only two time periods t=1, 2. Although the following model is more like a short-term contract as defined in Rey and Salanie (1996), extending the model to a longer term is straightforward. Growers are heterogeneous in terms of their capability to produce high-quality products. For example, grower differences include, among others, production technology, management skills, and soil conditions that can be sustained over time as long as grower types are not fully revealed to the processor. Let $\theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}$ denote the two possible quality types of the growers with $\underline{\theta} < \overline{\theta}$. The processor cannot observe θ, but has some prior belief $f(\theta)$ that the proportion of low-quality type $\underline{\theta}$ is $1 - r_1$ and that of high-quality type $\overline{\theta}$ is r_1. At the beginning of each period *t*, the grower privately chooses an action, e_t, to improve the quality of his products, which is only observable to the grower. For example, in the process of winegrape production, this action may include pruning, irrigating, and pest management, among other managerial actions. Thus, the observed or realized quality q_t of the grower's products is determined by $q_t = q_t(\theta, e_t)$. For simplicity, we assume that no uncertainty is involved in the production process. In particular, the quality structure is governed by the following

(3.1) $P_t = q_t(\theta, e_t) = \theta e_t$.

The processor can observe the realized quality of the finished products produced by the grower, but she cannot distinguish the effects of the grower's type θ and his effort e_t on improving quality. It is assumed that the processor can sell the product at price, $P_t = q_t$.

Recall that the grower's quality type θ can be sustained over time as long as this information is not fully revealed, which is a necessary condition to incur reputational effects in a dynamic context. Since the quality structure is deterministic, in each period each grower type can set a specific target of realized quality given an optimally chosen effort level.

The processor is risk neutral and has a profit function, $\pi_t(P_t, w_t) = P_t - w_t$, where w_t is the reward to the grower at period *t*. Each grower type θ has a time-separable utility function $U_t(w_t, e_t, \theta) = u(w_t) - g(e_t, \theta)$, where $g(e_t, \theta) = v(e_t)/\theta$. From (3.1), the utility function is equivalent to $U_t(w_t, e_t, \theta) = u(w_t) - v(P_t/\theta)/\theta$. It is assumed that *u* is strictly concave in w_t with $u^{'}(w_t) > 0$ and $u^{"}(w_t) < 0$; and *v* is strictly convex in e_t: $v_e > 0, v_{ee} > 0$ and $v(0) = 0$. Hence, we know that $g_e(e_t, \theta) > 0$, $g_\theta(e_t, \theta) < 0$, and $g_{e\theta}(e_t, \theta) < 0$. Note that in this setup growers differ in their disutility of effort and marginal contribution of effort to realized quality. The low-quality type incurs higher costs relative to the high-quality type for a same level of effort. In addition, the marginal disutility of efforts decreases with θ, i.e., decreases with grower abilities.

3.4.1 Two-period Full-Commitment Contract

In this case, since both parties are committed to a two-period contract, the contract cannot be breached or renegotiated during the contracting period. An alternative interpretation of full commitment is that the processor promises at date one to an intertemporal incentive scheme and commits not to use the information revealed by the grower in the first period during the second period. Hence, reputation has no effect on the optimal incentives to revealing the grower's private information. In addition, in the absence of an intertemporal incentive problem, the reward in one period does not depend on outcomes occurred in the previous period.

a) *Optimal contract under perfect information*

Under perfect information, the processor can perfectly observe the grower's type and the incentive problem is absent. Since the two periods are independent, the processor would solve, in each period *t*, for each type $\theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}$,

(3.2) $\max_{w(\theta),e(\theta)} Z_t(\theta) = P_t(\theta) - w_t(\theta) = \theta e_t - w_t$

subject to $U_t = u(w_t) - v(e_t)/\theta \geq u_0$.

The Lagrangian for this problem is

$$\max_{w_t, e_t} L = (\theta e_t - w_t) + \lambda(u(w_t) - v(e_t)/\theta - u_0),$$

and the first order conditions are

(3.2) $\frac{\partial L}{\partial e_t} = \theta - \frac{1}{\theta}\lambda v' = 0$,

(3.3) $\frac{\partial L}{\partial w_t} = -1 + \lambda u' = 0$, and

(3.4) $u(w_t) - v(e_t)/\theta \geq u_0$.

From (3.3) and the concavity of $u(w_t)$, we know

(3.5) $\lambda = 1/u' > 0$,

which confirms that the participation constraint (3.4) is binding. Thus, from (3.4), the optimal reward to the grower is

(3.6) $w_t^*(\theta) = u^{-1}(u_0 + v(e_t^*)/\theta), \quad \theta \in \{\underline{\theta}, \overline{\theta}\}$.

Then, from (3.2) and (3.3), we can obtain the optimal level of effort for each grower type $\theta \in \{\underline{\theta}, \overline{\theta}\}$, $e_t^*(\theta)$:

(3.7) $e_t^* \in \arg\{v'(e_t) = \theta^2 u'(w_t)\}$, or,

(3.8) $\dfrac{v'(e_t^*)}{u'(w_t^*)} = \theta^2$.

Thus, given the assumptions of the utility function and disutility of efforts, condition (3.8) states that the optimal level of effort for each type θ, $e^*(\theta)$, increases with θ. In other words, the optimal contract requires that less effort be demanded from the low-quality type. In addition, from (3.6), the grower of each quality type obtains the reservation utility u_0 in both periods.

Let $Z^*(\theta) = \max\limits_{w(\theta),e(\theta)} Z_t(\theta)$. Then in each period, the processor can obtain net profit $Z^*(\overline{\theta}) = \overline{\theta}e^*(\overline{\theta}) - w^*(\overline{\theta})$ from the grower type $\overline{\theta}$, and $Z^*(\underline{\theta}) = \underline{\theta}e^*(\underline{\theta}) - w^*(\underline{\theta})$ from the grower type $\underline{\theta}$. In addition, it can be verified that $Z^*(\overline{\theta}) > Z^*(\underline{\theta})$. This relationship is illustrated in Figure 3.1. Note that since $P(\theta) = \theta e = 0$ when $e = 0$, the processor's net profits for each $\theta \in \{\underline{\theta}, \overline{\theta}\}$ are exactly the distances from the origin O to the point A and point B on the vertical axis respectively. Given strict concavity of $u(w)$

and convexity of $v(e)$ and the processor's profit function, clearly, OB>OA, or, $Z^*(\overline{\theta}) > Z^*(\underline{\theta})$.

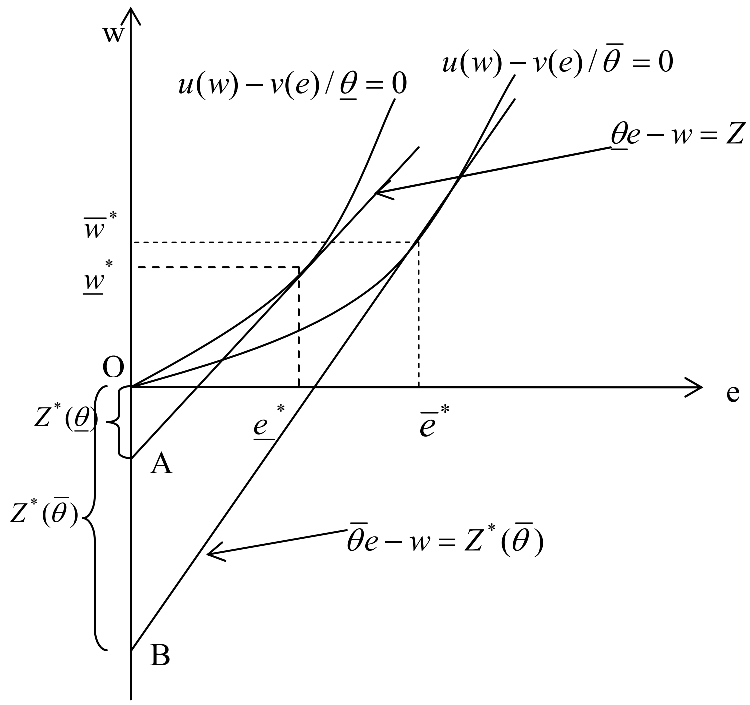

Figure 3.1 The optimal contract with perfect information

Thus, in a full-commitment contract under perfect information, the optimal contract will mimic a sequence of optimal static contracts. The static contract in every period is exactly same and independent over time. For future reference, denote this perfect information contract $C^* = \{C_L^*, C_H^*\}$, where $C_L^* = \{w^*(\underline{\theta}), e^*(\underline{\theta})\}$ and $C_H^* = \{w^*(\overline{\theta}), e^*(\overline{\theta})\}$. To simplify the notation further, let $\underline{w}^* = w^*(\underline{\theta})$, $\underline{e}^* = e^*(\underline{\theta})$ and $\overline{w}^* = w^*(\overline{\theta}), \overline{e}^* = e^*(\overline{\theta})$. Thus, the optimal contract under perfect information can also be written as $C_L^* = \{\underline{w}^*, \underline{e}^*\}$ and $C_H^* = \{\overline{w}^*, \overline{e}^*\}$.

b) *A full-commitment contract with asymmetric information*

With asymmetric information, incentive constraints must be imposed to have the grower's type truthfully revealed. Since the processor commits not to use the information revealed by the grower in the first period during the second period, the optimal contracts in two periods are independent. Thus, in each period *t*, the processor maximizes its expected net profit subject to the participation constraint and the incentive constraints. Again, let $Z_t(\theta) = \theta e_t(\theta) - w_t(\theta)$. Thus, in each period *t*, the processor maximizes

(3.9) $\Pi_t = r_1 Z_t(\overline{\theta}) + (1-r_1) Z_t(\underline{\theta}) = r_1[\overline{\theta} e_t(\overline{\theta}) - w_t(\overline{\theta})] + (1-r_1)[\underline{\theta} e_t(\underline{\theta}) - w_t(\underline{\theta})]$.

In each period, the grower earns at least the reservation utility u_0:

(3.10) $U_t = u(w_t) - v(e_t)/\theta \geq u_0, \quad \forall \theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}$.

Given the two distinct types, the optimal contract requires that the grower type $\underline{\theta}$ produces at quality level, $P(\underline{\theta}) = \underline{\theta} e(\underline{\theta})$ (recall that market price is set equal to the observed quality) and receives $w(\underline{\theta})$, while the grower type $\overline{\theta}$ produces at quality level $P(\overline{\theta}) = \overline{\theta} e(\overline{\theta})$ and receives $w(\overline{\theta})$. Denote this full-commitment contract with asymmetric information $C^F = \{C_L^F, C_H^F\}$, where $C_L^F = \{w^F(\underline{\theta}), e^F(\underline{\theta})\}$ and $C_H^F = \{w^F(\overline{\theta}), e^F(\overline{\theta})\}$. To simplify the notations, let $\underline{w}^F = w^F(\underline{\theta})$, $\underline{e}^F = e^F(\underline{\theta})$, $\overline{w}^F = w^F(\overline{\theta})$, $\overline{e}^F = e^F(\overline{\theta})$, $\overline{P}^F = P^F(\overline{\theta})$, and $\underline{P}^F = P^F(\underline{\theta})$. The superscript F is omitted in the following section.

Thus, to prevent deviation, the following incentive constraints must be satisfied

(3.11) $u(\underline{w}) - v(\underline{e})/\underline{\theta} \geq u(\overline{w}) - v(\overline{P}/\underline{\theta})/\underline{\theta}$,

(3.12) $u(\overline{w}) - v(\overline{e})/\overline{\theta} \geq u(\underline{w}) - v(\underline{P}/\overline{\theta})/\overline{\theta}$.

This condition is equivalent to the following condition for each type $\theta \in \{\underline{\theta}, \overline{\theta}\}$:

(3.13) $\theta \in \arg\sup_{\hat{\theta}} u(w(\hat{\theta})) - v(\hat{P} / \theta) / \theta) = u(w(\hat{\theta})) - v(\hat{\theta} e(\hat{\theta}) / \theta) / \theta$.

The above condition can be interpreted as follows: If each grower type θ is asked to report his type to the processor, the optimal incentives require that it is optimal for each grower to truthfully report his type. This condition is consistent with the direct revelation principle.

Thus, the processor would solve, in each period *t*,

(3.9) $\max_{e(\theta), w(\theta)} \Pi_t = r_1 Z_t(\overline{\theta}) + (1 - r_1) Z_t(\underline{\theta}) = r_1[\overline{\theta}\overline{e}_t - \overline{w}_t] + (1 - r_1)[\underline{\theta}\underline{e}_t - \underline{w}_t]$

subject to

(3.10) $U_t = u(w_t) - v(e_t) / \theta \geq u_0, \quad \forall \theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}$

(3.11) $u(\underline{w}) - v(\underline{e}) / \underline{\theta} \geq u(\overline{w}) - v(\overline{P} / \underline{\theta}) / \underline{\theta}$

(3.12) $u(\overline{w}) - v(\overline{e}) / \overline{\theta} \geq u(\underline{w}) - v(\underline{P} / \overline{\theta}) / \overline{\theta}$

To help solve the above problem, the following results are first derived to simplify the problem.

Full-commitment result (1): An optimal contract must be such that (i) $P(\theta)$ increases with θ and (ii) $w(\theta)$ increases with θ .

The proof is straightforward. Given the distinct growers types, $\underline{\theta}$ and $\overline{\theta}$, with $\underline{\theta} < \overline{\theta}$, (3.11) and (3.12) are equivalent to:

(3.11') $u(\underline{w}) - v(\underline{e}) / \underline{\theta} \geq u(\overline{w}) - v(\overline{e}\,\overline{\theta} / \underline{\theta}) / \underline{\theta}$.

(3.12') $u(\overline{w}) - v(\overline{e})/\overline{\theta} \geq u(\underline{w}) - v(\underline{e}\underline{\theta}/\overline{\theta})/\overline{\theta}$.

Thus, summing up (3.11') and (3.12') implies:

(3.13) $v(\underline{e}\underline{\theta}/\overline{\theta})/\overline{\theta} - v(\underline{e})/\underline{\theta} \geq v(\overline{e})/\overline{\theta} - v(\overline{e}\,\overline{\theta}/\underline{\theta})/\underline{\theta}$.

Define the function $g(e) = v(\underline{\theta}e/\overline{\theta})/\overline{\theta} - v(e)/\underline{\theta}$, thus, $g(\underline{e}) = v(\underline{\theta}\underline{e}/\overline{\theta})/\overline{\theta} - v(\underline{e})/\underline{\theta}$, and $g(\overline{P}/\underline{\theta}) = v(\overline{e})/\overline{\theta} - v(\overline{P}/\underline{\theta})/\underline{\theta}$.

From the convexity of $v(e)$, it must be true that

$g'(e) = v'(\underline{\theta}e/\overline{\theta})\frac{\underline{\theta}}{\overline{\theta}^2} - v'(e)/\underline{\theta} = \underline{\theta}[v'(\underline{\theta}e/\overline{\theta})/\overline{\theta}^2 - v'(e)/\underline{\theta}^2] < 0$ since $\underline{\theta} < \overline{\theta}$. Hence,

from (3.13), we know that $g(\underline{e}) \geq g(\overline{P}/\underline{\theta})$. Therefore, given $g'(e) < 0$, it must be true that $\underline{e} \leq \overline{P}/\underline{\theta}$, that is, $\underline{P} = \underline{e}\underline{\theta} \leq \overline{P} = \overline{e}\,\overline{\theta}$ for $\underline{\theta} < \overline{\theta}$. In fact, $\overline{P}$ must be strictly greater than $\underline{P}$ for $\underline{\theta} < \overline{\theta}$ for a separating equilibrium because, otherwise, it will be a pooling equilibrium. Hence, from (3.12'), we have $u(\overline{w}) - u(\underline{w}) \geq v(\overline{e})/\overline{\theta} - v(\underline{e}\underline{\theta}/\overline{\theta})/\overline{\theta} > 0$, from which $\overline{w} > \underline{w}$.

Recall that $P = \theta e$ is the realized quality given the grower's type and his effort choice. Result (1) states that the optimal choices of realized quality increase with θ. As a result, the optimal rewards increase with θ as well. This condition makes separating equilibrium possible.

Full-commitment result (2): The participation constraint for type $\overline{\theta}$ (3.12) is not binding.

To see this, (3.10) and (3.12) implies

$$u(\overline{w}) - v(\overline{e})/\overline{\theta} \geq u(\underline{w}) - v(\underline{P}/\overline{\theta})/\overline{\theta} \geq u(\underline{w}) - v(\underline{e})/\underline{\theta} \geq u_0$$

because $\overline{\theta} \geq \underline{\theta}$ and $\underline{P}/\overline{\theta} < \underline{e}$.

That is, the participation constraint for type $\overline{\theta}$ is satisfied. Since the low-quality type, $\underline{\theta}$, earns at least the reservation utility, u_0, the participation constraint for type $\overline{\theta}$ does not affect the optimal solution to this problem. Hence, it is not binding. Moreover, an optimal contract in this problem must have the participation constraint for type $\underline{\theta}$ binding, i.e., $u(\underline{w}) - v(\underline{e})/\underline{\theta} = u_0$. Otherwise, the processor can always reduce the reward for this grower type until it reaches his reservation utility.

Now, let us derive the optimal choice of effort exerted by each grower type. Ignoring the participation constraint for the high-quality type $\overline{\theta}$, let λ, μ_L, and μ_H denote the Lagrangian multipliers for (3.10), (3.11), and (3.12). Then the Lagrangian for the above problem (3.9)-(3.12) is

$$\begin{aligned} L &= (1-r_1)[\underline{e}\underline{\theta} - \underline{w}] + r_1[\overline{e}\overline{\theta} - \overline{w}] + \lambda(u(\underline{w}) - v(\underline{e})/\underline{\theta} - u_0) \\ &+ \mu_L(u(\underline{w}) - v(\underline{e})/\underline{\theta} - u(\overline{w}) + v(\overline{P}/\underline{\theta})/\underline{\theta}) \\ &+ \mu_H(u(\overline{w}) - v(\overline{e})/\overline{\theta} \geq u(\underline{w}) - v(\underline{P}/\overline{\theta})/\overline{\theta}) \end{aligned}$$

The first order conditions to this problem are:

(3.14) $$\frac{\partial L}{\partial \underline{e}} = (1-r_1)\underline{\theta} - \lambda v'(\underline{e})/\underline{\theta} - \mu_L v'(\underline{e})/\underline{\theta} + \mu_H(\underline{\theta}/\overline{\theta})v'(\underline{P}/\overline{\theta})/\overline{\theta} = 0,$$

(3.15) $$\frac{\partial L}{\partial \overline{e}} = r_1\overline{\theta} + \mu_L(\overline{\theta}/\underline{\theta})v'(\overline{P}/\underline{\theta})/\underline{\theta} - \mu_H v'(\overline{e})/\overline{\theta} = 0,$$

(3.16) $$\frac{\partial L}{\partial \underline{w}} = -(1-r_1) + \lambda u'(\underline{w}) + \mu_L u'(\underline{w}) - \mu_H u'(\underline{w}) = 0,$$

(3.17) $\frac{\partial L}{\partial \overline{w}} = -r_1 - \mu_L u'(\overline{w}) + \mu_H u'(\overline{w}) = 0$.

From (3.17), we know that $\mu_H > 0$ since $r_1 > 0$ and $u' > 0$. Hence, the incentive compatibility constraint (3.12) is binding.

Further, since $\overline{P} > \underline{P}$, it is not possible for both incentive compatibility constraints to be binding. To show that, one can assume the contrary. If both constraints are binding, then from (3.11') and (3.12'), we can get

(3.18) $v(\overline{e}\,\overline{\theta} / \underline{\theta}) / \underline{\theta} - v(\underline{e}) / \underline{\theta} = u(\overline{w}) - u(\underline{w}) = v(\overline{e}) / \overline{\theta} - v(\underline{e}\underline{\theta} / \overline{\theta}) / \overline{\theta}$.

Since $\underline{\theta} < \overline{\theta}$, and $v(\overline{e}\,\overline{\theta} / \underline{\theta}) - v(\underline{e}) \geq v(\overline{e}) - v(\underline{e}\underline{\theta} / \overline{\theta})$ due to strict convexity of $v(e)$, (3.18) is not possible. Therefore, the incentive compatibility constraint (3.11) must be strict inequality, which implies $\mu_L = 0$.

Substituting $\mu_L = 0$ into (3.15) and (3.17) gives

(3.19) $\overline{e} \in \arg\{\overline{\theta}^2 u'(\overline{w}) = v'(\overline{e})\}$,

which coincides with the optimal condition for type $\overline{\theta}$ under perfect information (equation 3.7).

Substituting $\mu_L = 0$ into (3.15) and (3.16) yields, respectively,

(3.20) $\mu_H = r_1 \overline{\theta}^2 / v'(\overline{e})$, and

(3.21) $\lambda = \frac{1 - r_1}{u'(\underline{w})} + \mu_H$.

Then the optimal effort choice for grower type $\underline{\theta}$ can be solved by substituting (3.20) and (3.21) into (3.14):

$(1-r_1)\underline{\theta}-[\frac{1-r_1}{u'(\underline{w})}+\mu_H]v'(\underline{e})/\underline{\theta}+\mu_H(\underline{\theta}/\overline{\theta})v'(\underline{P}/\overline{\theta})/\overline{\theta}=0\}$, which implies

$(1-r_1)\underline{\theta}-\frac{1-r_1}{u'(\underline{w})}v'(\underline{e})/\underline{\theta}+\mu_H[(\underline{\theta}/\overline{\theta})v'(\underline{P}/\overline{\theta})/\overline{\theta}-v'(\underline{e})/\underline{\theta}]=0\}$, or,

$(1-r_1)\underline{\theta}[1-\frac{v'(\underline{e})}{u'(\underline{w})\underline{\theta}^2}]+\frac{r_1}{\underline{\theta}v'(\overline{e})}[\underline{\theta}^2v'(\underline{P}/\overline{\theta})-\overline{\theta}^2v'(\underline{e})]=0$.

Hence, the optimal effort choice for grower type $\underline{\theta}$ is given by

(3.22) $\underline{e}\in\arg\{(1-r_1)\underline{\theta}[1-\frac{v'(\underline{e})}{u'(\underline{w})\underline{\theta}^2}]+\frac{r_1}{\underline{\theta}v'(\overline{e})}[\underline{\theta}^2v'(\underline{P}/\overline{\theta})-\overline{\theta}^2v'(\underline{e})]=0\}$.

Since $\underline{\theta}^2v'(\underline{P}/\overline{\theta})-\overline{\theta}^2v'(\underline{e})<0$, it must be true that

$\frac{v'(\underline{e})}{u'(\underline{w})\underline{\theta}^2}<1$, which implies $\underline{e}<\underline{e}^*$, where $\underline{e}^*$ is the optimal choice of effort for grower type $\underline{\theta}$ under perfect information.

Further, since the participation constraint for the grower type $\underline{\theta}$ is binding, he earns exactly the reservation utility u_0 at the equilibrium, i.e.,

(3.23) $u(\underline{w})-v(\underline{e})/\underline{\theta}=u_0$

from which we can solve the optimal reward to the grower type $\underline{\theta}$.

Then from (3.12),

(3.24) $u(\overline{w})-v(\overline{e})/\overline{\theta}=u(\underline{w})-v(\underline{P}/\overline{\theta})/\overline{\theta}=u(\underline{w})-v(\underline{e})/\underline{\theta}+v(\underline{e})/\underline{\theta}-v(\underline{P}/\overline{\theta})/\overline{\theta}$

$$= u_0 + v(\underline{e}) / \underline{\theta} - v(\underline{P} / \overline{\theta}) / \overline{\theta}$$

Since $v(\underline{e}) / \underline{\theta} - v(\underline{P} / \overline{\theta}) / \overline{\theta} > 0$, the grower type $\overline{\theta}$ earns strictly positive information rents from the optimal contract under asymmetric information. Thus, this optimal full commitment contract under asymmetric information can be written as $C^F = \{C_L^F, C_H^F\}$ with $C_L^F = \{\underline{w}^F, \underline{e}^F\}$ and $C_H^F = \{\overline{w}^F, \overline{e}^F\}$, where $\{\underline{w}^F, \underline{e}^F\}$ is given by conditions (3.22) and (3.23), while $\{\overline{w}^F, \overline{e}^F\}$ is given by conditions (3.19) and (3.24).

To summarize, if the processor commits itself in the first period not to use the information revealed by the grower in the following period, and the grower commits to the two-period contract and cannot breach the relationship, the optimal incentives with full commitment mimic the same static contract in both periods. Further, commitment by the processor eliminates the possibility of incorporating reputation effects into the optimal incentives. Therefore, under the assumption of full commitment, growers' reputation of quality does not affect the dynamics of the optimal contract with asymmetric information.

3.4.2 Two-period Dynamic Contracts with No Commitment

In this case, it is assumed that neither the processor nor the grower can commit to an intertemporal incentive scheme. Thus, the processor chooses the optimal incentive scheme in the second period conditional on the grower's first-period performance. The grower cannot commit to the two-period contract and can quit the relationship at the end of the first period. We assume that the grower can obtain his reservation utility if he quits.

Recall that grower type $\overline{\theta}$ is the high-quality type and $\underline{\theta}$ is the low-quality type. If the grower type $\overline{\theta}$ deviates and pretends to be the grower type $\underline{\theta}$, i.e., choose the target market price $P(\underline{\theta})$, he would earn relatively less profit in the first period and enjoy positive information rent in the second period. In the following analysis, we exclude the possibility that the low-quality type would mimic the high-quality ability because this strategy always generates a loss to the low-quality type.

Specifically, the processor's strategy consists of incentives schemes $\{w_1(P_1), w_2(P_1, P_2, w_1)\}$ and the grower's strategy is a sequence of decisions of the effort levels $\{e_1(w_1, \theta), e_2(w_1, w_2, \theta, e_1\}$. Denote the set of feasible contract as $C_1 = \{C_{1L}, C_{1H}\}$ and $C_2 = \{C_{2L}, C_{2H}\}$ where $C_{1j} = \{w_{1j,}e_{1j}\}$ and $C_{2j} = \{w_{2j,}e_{2j}\}$ for $j \in \{L, H\}$. These optimal strategies must form a perfect Bayesian equilibrium such that (i) e_2 is optimal for the grower given w_2, (ii) w_2 maximizes the processor's expected profit given its belief about θ, $f_2(\theta \mid w_1, e_1)$, in the second period, (iii) e_1 is optimal for the grower given w_1 and the second-period incentive schemes, (iv) w_1 maximizes the processor's expected profit given its belief about θ, $f_1(\theta)$, in the first period and the second-period strategies, and (v) the processor's second-period belief $f_2(\theta \mid w_1, e_1)$ is derived from the first-period belief $f_1(\theta)$ and the grower's first period strategy using Bayes' rule.

From the analysis described in the first case, an optimal static contract in each period requires each grower type has a fixed target quality level, i.e., $q(\overline{\theta})$ and $q(\underline{\theta})$, which correspond to the market prices $P(\overline{\theta})$ and $P(\underline{\theta})$, respectively. Hence, three types of continuation equilibria could potentially be sustained:

(i) *Separating equilibrium*: The high-quality grower type $\overline{\theta}$ chooses $P(\overline{\theta})$ and the low-quality grower type $\underline{\theta}$ chooses $P(\underline{\theta})$. Then the processor's second-period belief becomes $r_2(w_1(\theta), P(\overline{\theta})) = 1$ and $r_2(w_1(\theta), P(\underline{\theta})) = 0$.

(ii) *Pooling equilibrium*: Both grower types choose $P(\underline{\theta})$ in the first period. The processor updates her second-period belief about grower types such that $r_2(w_1(\theta), P(\underline{\theta})) = r_1$, and $r_2(w_1(\theta), P(\overline{\theta})) = 1$. That is, if $P(\underline{\theta})$ is observed, the processor cannot obtain the grower's true type by observing his first-period performance, so the Bayesian updating results in the exactly same distribution of grower types in the second period as in the first period. If $P(\overline{\theta})$ is observed instead, then the processor updates its belief such that the grower type is of high-quality type.

(iii) *Semi-separating equilibrium*: If the high-quality type randomizes over $P_1(\underline{\theta})$ and $P_1(\overline{\theta})$, then the processor updates its belief using Bayes' rule. Let π be the probability that the grower type $\overline{\theta}$ chooses the contract designed for the grower type $\underline{\theta}$. Then, the processor's second period belief becomes

(3.25) $\hat{r}_2(P_1(\underline{\theta}), \pi) = \dfrac{r_1\pi}{r_1\pi + 1 - r_1} < r_1$ and $r_2(P_1(\overline{\theta}), \pi) = 1$.

Thus, for a given belief of grower types in the second period, denote the processor's second-period net profit as $W_2(r_2)$ and the first period net profit as $W_1(r_1, C_{1L}, C_{1H})$. Similarly, define the grower's second period utility as $U_2(C_{2i}(r_2) \mid \theta_i)$ and the first period utility $U_1(C_{1i} \mid \theta_i)$, for $i \in \{L, H\}$. Note that we use $\theta_L = \underline{\theta}$ and $\theta_H = \overline{\theta}$ here. From now on, these notations might be used interchangeably for notational simplification.

Now, let us take a close look at the second period incentives schemes.

Second-period incentive schemes

As have discussed in the previous section, the second period incentive schemes depend on the grower's first-period performance and the processor's second period belief about grower types. The optimal incentives can be derived following the same procedure as described in the full commitment case. In this case, without loss of generality, we normalize the reservation utility for all grower types u_0 to be zero.

Given the processor's belief about the grower types r_2 in the second period, the processor solves the following problem:

$$\max_{e_2(\theta), w_2(\theta)} W_2(r_2) = r_2 Z_2(\overline{\theta}) + (1-r_2) Z_2(\underline{\theta}) = r_2[\overline{\theta} e_{2H} - w_{2H}] + (1-r_2)[\underline{\theta} e_{2L} - w_{2L}]$$

subject to

(3.26) $U_2 = u(w_2) - v(e_2)/\theta \geq 0, \quad \forall \theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}$.

(3.27) $u(w_{2L}) - v(e_{2L})/\underline{\theta} \geq u(w_{2H}) - v(e_{2H}\overline{\theta}/\underline{\theta})/\underline{\theta}$.

(3.28) $u(w_{2H}) - v(e_{2H})/\overline{\theta} \geq u(w_{2L}) - v(e_{2L}\underline{\theta}/\overline{\theta})/\overline{\theta}$.

Following the procedure as in the full commitment case, we can derive the optimal contract for the second period contingent on the processor's belief about r_2. Specifically, there could exist three different types of equilibria given different values of r_2.

(1) *Separating*: If the first period equilibrium is separating, i.e., $r_2(w_1(\theta), P(\overline{\theta})) = 1$ and $r_2(w_1(\theta), P(\underline{\theta})) = 0$, then second-period equilibrium is exactly same as the optimal contract under perfect information. In other words, once the grower's true type is

revealed in the first period, the grower's private information concerning their quality becomes public. Hence, the processor can offer a contract that provides the reservation utility to the grower of each type and extract all surplus from the grower. Thus, in a dynamic two-period contract, if the first-period contract is separating, the optimal contract for the second period is $C_2 = \{C_{2L}, C_{2H}\}$, where $C_{2L} = (\underline{w}^*, \underline{e}^*)$ and $C_{2H} = (\overline{w}^*, \overline{e}^*)$ as in the perfect information contract. Recall that the optimal contract C_2 requires $U_2(C_{2L} \mid \theta_L) = U_2(C_{2H} \mid \theta_H) = 0$.

(2)*Pooling*: If both growers types pool in the first period and choose $P_1(\underline{\theta})$, the processor adopts the same distribution of grower types as the prior distribution, i.e., $r_2 = r_1$. As a result, the optimal contract for the second period is same as the full-commitment contract. That is, $C_2^P = \{C_{2L}^P, C_{2H}^P\}$, where $C_{2L}^P = (\underline{w}^F, \underline{e}^F)$ and $C_{2H}^p = (\overline{w}^F, \overline{e}^F)$. The superscript P stands for a pooling continuation equilibrium when the first-period contract is fully concealing.

(3) *Semi-separating*: If the high-quality type randomizes over $P_1(\underline{\theta})$ and $P_1(\overline{\theta})$ in the first period, then the processor updates its belief using Bayes' rule and solves the second-period problem given r_2 specified by (3.25).

Given any value of r_2, the optimal contract can be solved using the similar procedure described in the full-commitment contract. More precisely, the optimal contract must satisfy the following four conditions:

(3.29) $e_{2H} \in \arg\{\overline{\theta}^2 u'(w_{2H}) = v'(e_{2H})\}$.

(3.30) $e_{2L} \in \arg\{(1-r_2)\underline{\theta}[1-\frac{v'(e_{2L})}{u'(w_{2L})\underline{\theta}^2}]+\frac{r_2}{\underline{\theta}v'(e_{2H})}[\underline{\theta}^2 v'(e_{2L}\underline{\theta}/\overline{\theta})-\overline{\theta}^2 v'(e_{2L})]=0\}$.

(3.31) $u(w_{2L})-v(e_{2L})/\underline{\theta}=0$.

(3.32)

$$u(w_{2H})-v(e_{2H})/\overline{\theta}=u(w_{2L})-v(e_{2L}\underline{\theta}/\overline{\theta})/\overline{\theta}=u(w_{2L})-v(e_{2L})/\underline{\theta}+v(e_{2L})/\underline{\theta}-v(e_{2L}\underline{\theta}/\overline{\theta})/\overline{\theta}$$
$$=v(e_{2L})/\underline{\theta}-v(e_{2L}\underline{\theta}/\overline{\theta})/\overline{\theta}>0.$$

Hence, if the grower type $\overline{\theta}$ deviates in the first period and pools with the grower type $\underline{\theta}$ or randomizes, he obtains positive information rents in the second period:

(3.33) $I_{2H}(r_2)=v(e_{2L})/\underline{\theta}-v(e_{2L}\underline{\theta}/\overline{\theta})/\overline{\theta}>0$.

Note that the low-quality type always obtains his reservation utility in the second period independent of the processor's belief of grower types. Therefore, there is no incentive for the low-quality type to deviate in the first period. In other words, the low-quality type always chooses his own contract in the first period. On the other hand, the high-quality grower obtains a greater payoff in the second period by mimicking a low-quality type in the first period. In addition, the more likely the processor believes that the grower is of low-quality type (smaller value of r_2), the greater payoff the high-quality grower could obtain in the second period. This result confirms the assumption that only the high-quality type has incentives to pool with the low-quality type or to randomize. This finding leads to the following lemma.

Lemma 1: $U_2(C_{2H}(r_2)\,|\,\overline{\theta})=I_{2H}(r_2)$ decreases in r_2.

Proof: Taking the derivative of (3.33) with respect to r_2, we have

$$\frac{\partial}{\partial r_2} I_{2H}(r_2) = \frac{\partial}{\partial r_2} v(e_{2L}) / \underline{\theta} - \frac{\partial}{\partial r_2} v(e_{2L} \underline{\theta} / \overline{\theta}) / \overline{\theta} = \frac{v'(e_{2L})}{\underline{\theta}} \frac{\partial e_{2L}}{\partial r_2} - \frac{v'(e_{2L} \underline{\theta} / \overline{\theta})}{\overline{\theta}} \frac{\partial e_{2L}}{\partial r_2} \frac{\underline{\theta}}{\overline{\theta}}$$

$$= \underline{\theta} [\frac{v'(e_{2L})}{\underline{\theta}^2} - \frac{v'(e_{2L} \underline{\theta} / \overline{\theta})}{\overline{\theta}^2}] \frac{\partial e_{2L}}{\partial r_2}$$

Since $\underline{\theta} < \overline{\theta}$ and $v''(e) > 0$, it must be true that $\frac{v'(e_{2L})}{\underline{\theta}^2} - \frac{v'(e_{2L} \underline{\theta} / \overline{\theta})}{\overline{\theta}^2} > 0$. Thus, the sign of $\frac{\partial e_{2L}}{\partial r_2}$ becomes the main issue.

From (3.30), since $\underline{\theta}^2 v'(e_{2L} \underline{\theta} / \overline{\theta}) - \overline{\theta}^2 v'(e_{2L}) < 0$, the greater r_2 is, the greater the expression $1 - \frac{v'(e_{2L})}{u'(w_{2L}) \underline{\theta}^2}$ must be. That is, the expression $\frac{v'(e_{2L})}{u'(w_{2L}) \underline{\theta}^2}$ must be smaller.

Hence, e_{2L} decrease in r_2, or $\frac{\partial e_{2L}}{\partial r_2} < 0$. In addition, $U_2(C_{2H}(r_2) \mid \overline{\theta}) = I_{2H}(r_2)$ is maximized at $r_2 = 0$. This lemma can be better understood in Figure 3.2.

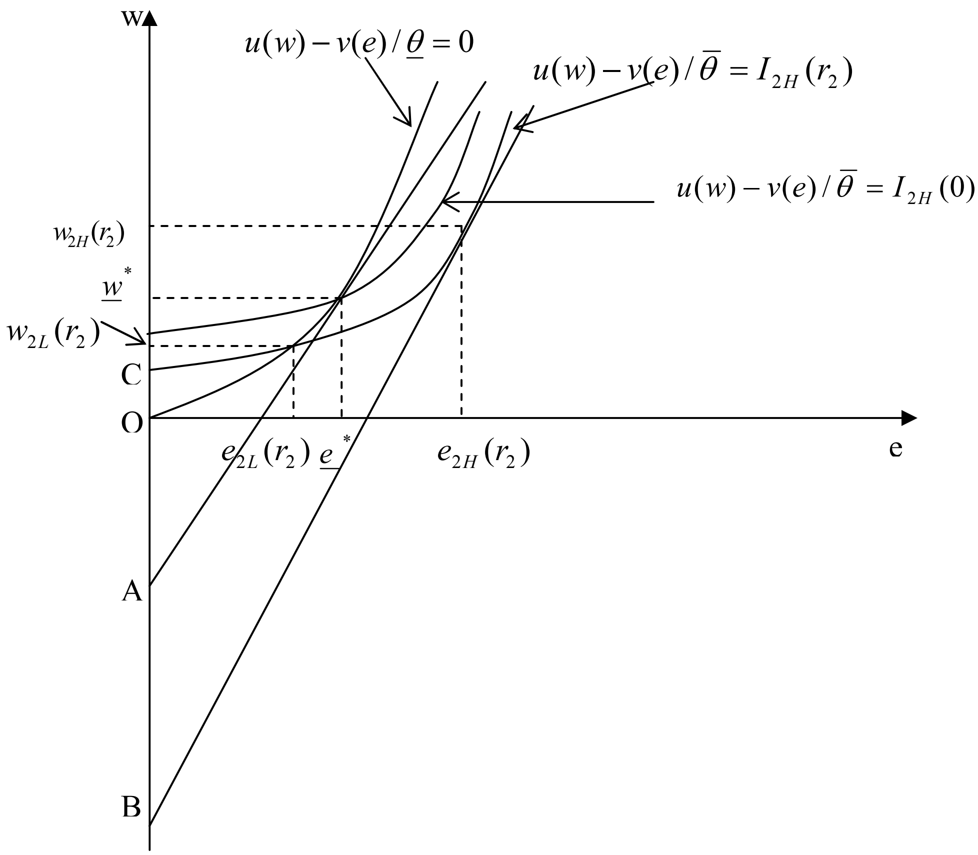

Figure 3.2 Illustration of Effects of r_2 on high-quality type's information rents

Since condition (3.28) is binding, that is, $U_2(C_{2H}(r_2)\mid\overline{\theta})=U_2(C_{2L}(r_2)\mid\overline{\theta})$, for any given r_2, the optimal contract for the high-quality type must be on the indifference curve that intersects with the indifference curve $U_2(C_{2L}\mid\underline{\theta})=u(w)-v(e)/\underline{\theta}=0$ through point $(w_{2L}(r_2),e_{2L}(r_2))$. Since $v(0)=0$, the information rent $I_{2H}(r_2)$ is exactly the distance from the origin to the point C on the vertical axis. Hence, if r_2 decreases, the contract for the low-quality type $(w_{2L}(r_2),e_{2L}(r_2))$ moves along the indifference curve $U_2(C_{2L}\mid\underline{\theta})=u(w)-v(e)/\underline{\theta}=0$ toward point $(\underline{w}^*,\underline{e}^*)$, which represents the optimal contract for the low-quality type under perfect information. That is, in the limit, when $r_2=0$, the contract C_{2L} converges to the perfect information contract $C_L^*=(\underline{w}^*,\underline{e}^*)$ at which the information rent for the high-quality type is maximized. Therefore, $U_2(C_{2H}(r_2)\,|\,\overline{\theta}) = I_{2H}(r_2)$ decreases in r_2. Intuitively, this lemma states that the information rent for the high-quality grower type increases as the processor believes that the grower is more likely to be a low-quality type.

Since the processor's second-period net profit depends on its belief about growers' types in the second period, denote $Z_2(\overline{\theta}) = \overline{\theta} e_{2H}(r_2) - w_{2H}(r_2)$ and $Z_2(\underline{\theta}) = \underline{\theta} e_{2L}(r_2) - w_{2L}(r_2)$. Then $W_2^*(r_2) = \max\limits_{e_2(\theta), w_2(\theta)} W_2(r_2) = r_2 Z_2(\overline{\theta}) + (1-r_2) Z_2(\underline{\theta})$ is the maximum second-period net profit contingent on r_2. It can be shown that the processor's second period net profit increases in r_2. This result is demonstrated in the following Lemma.

Lemma 2: $W_2^*(r_2)$ increases in r_2 and is convex in r_2.

Proof: Using the Envelope theorem, $\frac{d}{dr_2} W_2^*(r_2) = Z_2(\overline{\theta}) - Z_2(\underline{\theta})$. Thus, it is sufficient to show $Z_2(\overline{\theta}) > Z_2(\underline{\theta})$ for any r_2. This can be illustrated in Figure 3.2. As indicated above, the distance OA is the net profit the processor can obtain from the low-quality grower under perfect information. As r_2 decreases, the contract for the low-quality type $(w_{2L}(r_2), e_{2L}(r_2))$ moves along the indifference curve $U_2(C_{2L}\,|\,\underline{\theta}) = 0$ toward the point $(\underline{w}^*, \underline{e}^*)$. Thus, for any r_2 ($0 \le r_2 \le 1$), the maximum profit the processor can obtain from the low-quality grower is $Z^*(\underline{\theta})$ at the point $(\underline{w}^*, \underline{e}^*)$. In other words, $Z^*(\underline{\theta}) \ge Z_2(\underline{\theta})$ for any r_2 (equality for $r_2 = 0$). On the other hand, since (3.28) is binding and (3.27) is not binding, the indifference curve of the high-quality type must intersect with the indifference curve $U_2(C_{2L} \mid \underline{\theta}) = 0$ through point $(w_{2L}(r_2), e_{2L}(r_2))$. As r_2 decreases, in the limit, the indifference curve for the high-quality type must cut the curve $U_2(C_{2L} \mid \underline{\theta}) = 0$ through the point $(\underline{w}^*, \underline{e}^*)$, where the high-quality type obtains maximum informational rents and the processor acquires the minimum net profit from the high-quality grower type. Since the optimal contract $C_2 = \{C_{2L}, C_{2H}\}$ requires that the high-quality type be indifferent between C_{2H} and C_{2L} and the low-quality type strictly prefer C_{2L} to C_{2H} (condition (3.27) and (3.28)), the optimal contract C_{2H} must be located in the region below the curve $u(w) - v(e)/\underline{\theta} = 0$ and above the curve (actually on the curve at the equilibrium) $u(w) - v(e)/\overline{\theta} = I_{2H}(0)$. Since the slope of the processor's iso-profit line is greater for type $\overline{\theta}$ than for type $\underline{\theta}$, thus, it must be true that $\min_{r_2} Z_2(\overline{\theta}) > Z^*(\underline{\theta}) = \max_{r_2} Z_2(\underline{\theta})$. Thus, $Z_2(\overline{\theta}) > Z_2(\underline{\theta})$ for any r_2 and $W_2^*(r_2)$ increases in r_2.

To show that $W_2^*(r_2)$ is convex in r_2, it is sufficient to prove

$$\frac{\partial^2 W_2^*(r_2)}{\partial r_2^2} = \frac{\partial Z_2(\overline{\theta})}{\partial r_2} - \frac{\partial Z_2(\underline{\theta})}{\partial r_2} = (\overline{\theta}\frac{\partial e_{2H}}{\partial r_2} - \frac{\partial w_{2H}}{\partial r_2}) - (\underline{\theta}\frac{\partial e_{2L}}{\partial r_2} - \frac{\partial w_{2L}}{\partial r_2}) > 0.$$

From condition (3.32) and Lemma 1, we know that

$$\frac{\partial I_{2H}(r_2)}{\partial r_2} = u'(w_{2H})\frac{\partial w_{2H}}{\partial r_2} - v'(e_{2H})\frac{\partial e_{2H}}{\partial r_2}/\overline{\theta} = \frac{1}{\overline{\theta}^2}[\overline{\theta}^2 u'(w_{2H})\frac{\partial w_{2H}}{\partial r_2} - v'(e_{2H})\overline{\theta}\frac{\partial e_{2H}}{\partial r_2}] < 0.$$

Since, from condition (3.29), $\overline{\theta}^2 u'(w_{2H}) = v'(e_{2H})$, it must be true that

$$\overline{\theta}\frac{\partial e_{2H}}{\partial r_2} - \frac{\partial w_{2H}}{\partial r_2} > 0.$$

Further, from condition (3.31),

$$u'(w_{2L})\frac{\partial w_{2L}}{\partial r_2} - v'(e_{2L})\frac{\partial e_{2L}}{\partial r_2}/\underline{\theta} = 0,$$

or, $$\underline{\theta}^2 u'(w_{2L})\frac{\partial w_{2L}}{\partial r_2} - v'(e_{2L})\underline{\theta}\frac{\partial e_{2L}}{\partial r_2} = 0.$$

From condition (3.30), we know that

$$\frac{\partial w_{2L}}{\partial r_2} \bigg/ \underline{\theta}\frac{\partial e_{2L}}{\partial r_2} = \frac{v'(e_{2L})}{\underline{\theta}^2 u'(w_{2L})} < 1.$$

In addition, we have shown in Lemma 1 that $\frac{\partial e_{2L}}{\partial r_2} < 0$ and hence, $\frac{\partial w_{2L}}{\partial r_2} < 0$, therefore,

$$\underline{\theta}\frac{\partial e_{2L}}{\partial r_2} - \frac{\partial w_{2L}}{\partial r_2} < 0.$$

Combining these conditions yields the result that $W_2^*(r_2)$ is convex in r_2.

Now, let us turn to the first-period incentive schemes.

First-period incentive schemes

In the first period, the processor maximizes its expected payoff subject to the grower's participation constraints and incentive compatibility constraints. Since the processor cannot commit not to use the first-period information revealed by the growers to revise the second-period contract, the incentive compatibility constraints must take into account the effect of first-period decisions on the second-period payoff.

For any first-period contract, $C_1 = \{C_{1L}, C_{1H}\}$, let $V_H(C_{1H} \mid \overline{\theta})$ denote the two-period payoff to the grower type $\overline{\theta}$ if $P_1(\overline{\theta})$ is observed, i.e., if the high-quality grower chooses his own contract C_{1H} in the first period. Recall that if $P_1(\overline{\theta})$ is observed, the processor updates its belief such that $r_2(w_1(\overline{\theta}), P_1(\overline{\theta})) = 1$. Thus,

(3.34) $V_H(C_{1H} \mid \overline{\theta}) = U_1(C_{1H} \mid \overline{\theta}) + \delta U_2(C_H^* \mid \overline{\theta}) = U_1(C_{1H} \mid \overline{\theta})$.

Note that if the first-period contract is fully revealing, the second-period contract is same as the perfect information contract under which the high-quality grower obtains his reservation utility zero (i.e., $U_2(C_H^* \mid \overline{\theta}) = 0$).

Similarly, denote $V_H(C_{1L}, \pi \mid \overline{\theta})$ as the two-period payoff to the grower type $\overline{\theta}$ if $P_1(\underline{\theta})$ is observed and the high-quality grower type chooses the contract designed for the low-quality grower type with probability π. Thus,

(3.35) $V_H(C_{1L}, \pi \mid \overline{\theta}) = U_1(C_{1L} \mid \overline{\theta}) + \delta I_{2H}(\hat{r}_2)$,

where, from (3.25), $\hat{r}_2(P_1(\underline{\theta}), \pi) = \dfrac{r_1 \pi}{r_1 \pi + 1 - r_1}$.

Now the grower's equilibrium strategy $\hat{\pi}$ must be optimal for him given the processor's belief. In other words, the grower must be indifferent between revealing his true type and mimicking the other type at the equilibrium given the optimal $\hat{\pi}$. Therefore, the equilibrium strategy must satisfy the following condition:

(3.36) $V_H(C_{1H} \mid \overline{\theta}) = V_H(C_{1L}, \hat{\pi} \mid \overline{\theta})$.

Recall that in the full-commitment contract (also the static contract), the incentive constraint for the high-quality type must be binding, i.e., $U_1(C_H^F \mid \overline{\theta}) = U_1(C_L^F \mid \overline{\theta})$. However, in the dynamic setting, the contract $C^F = \{C_L^F, C_H^F\}$ can only result in $V_H(C_{1H} \mid \overline{\theta}) < V_H(C_{1L}, \hat{\pi} \mid \overline{\theta})$ because $I_{2H}(\hat{r}_2) > 0$ for $0 \le r_2 < 1$. That is, the high-quality type always gains from mimicking the low-quality type if the optimal static contract is offered in the dynamic setting. Therefore, the static contract C^F cannot be an optimal separating equilibrium in a dynamic context.

Given continuity of $V_H(C_{1L},\pi\,|\,\overline{\theta})$ in π and condition (3.36), three types of equilibrium could be sustained:

(3.37) Separating equilibrium if: $V_H(C_{1H}\,|\,\overline{\theta}) \geq V_H(C_{1L},0\,|\,\overline{\theta})$,

(3.38) Pooling equilibrium if: $V_H(C_{1H}\,|\,\overline{\theta}) \leq V_H(C_{1L},1\,|\,\overline{\theta})$, and

(3.39) Semi-separating equilibrium if $V_H(C_{1H}\,|\,\overline{\theta}) = V_H(C_{1L},\hat{\pi}\,|\,\overline{\theta})$ for some $\hat{\pi}$.

From (3.35), $V_H(C_{1L},0\,|\,\overline{\theta}) = U_1(C_{1L}\,|\,\overline{\theta}) + \delta I_{2H}(0)$. From Lemma 1 and Figure 3.2, we know that the information rent for the high-quality grower type $I_{2H}(r_2)$ is maximized at $r_2 = 0$. Thus,

(3.40) $I_{2H}(0) = v(\underline{e}^*)/\underline{\theta} - v(\underline{e}^*\underline{\theta}/\overline{\theta})/\overline{\theta}$.

Condition (3.37) then requires that

(3.41) $U_1(C_{1H}\,|\,\overline{\theta}) \geq U_1(C_{1L}\,|\,\overline{\theta}) + \delta I_{2H}(0)$.

Similarly, from (3.33) we can get

(3.42) $I_{2H}(r_1) = v(\underline{e}^F)/\underline{\theta} - v(\underline{e}^F\underline{\theta}/\overline{\theta})/\overline{\theta}$.

Thus, condition (3.38) is equivalent to

(3.43) $U_1(C_{1H}\,|\,\overline{\theta}) \leq U_1(C_{1L}\,|\,\overline{\theta}) + \delta I_{2H}(r_1)$.

Note that, from Lemma 1, $I_{2H}(r_1) < I_{2H}(\hat{r}_2) < I_{2H}(0)$, therefore, conditions (3.37), (3.38), and (3.39) are mutually exclusive.

For each type of equilibrium, the processor maximizes its discounted expected two-period payoff subject to the participation constraints and incentive compatibility constraints for both grower types.

Let ψ denote the probability of a grower choosing contract C_{1L} given the contract $C_1 = \{C_{1L}, C_{1H}\}$. Since only the high-quality type has incentive to deviate, then for any π, $\psi = r_1\pi(C_1) + 1 - r_1$. Thus, the processor's two-period net profit is

(3.44) $W_1(r_1, C_{1L}, C_{1H}) = (1-\psi)[\overline{\theta}e_{1H} - w_{1H} + \delta W_2(1)] + \psi[\underline{\theta}e_{1L} - w_{1L} + \delta W_2(\hat{r}_2)]$.

First-period separating equilibrium

First, let us focus on the separating equilibrium. In a separating equilibrium, the high-quality grower chooses his own contract with probability 1, or $\pi = 0$. Thus, to induce a separating equilibrium, the processor solves the following problem:

$$\max_{e_{1H}, w_{1H}, e_{1L}, w_{1L}} W_1(r_1, C_{1L}, C_{1H}) = r_1[\overline{\theta}e_{1H} - w_{1H} + \delta W_2(1)] + (1-r_1)[\underline{\theta}e_{1L} - w_{1L} + \delta W_2(0)]$$

subject to

(3.45) $U_1 = u(w_{1i}) - v(e_{1i})/\theta_i \geq 0, \quad \forall i \in \{L, H\}$

(3.41) $U_1(C_{1H} \mid \overline{\theta}) \geq U_1(C_{1L} \mid \overline{\theta}) + \delta I_{2H}(0)$, and

(3.46) $U_1(C_{1L} \mid \underline{\theta}) \geq U_1(C_{1H} \mid \underline{\theta})$.

Conditions (3.41) and (3.46) state that each grower type prefers his own contract to the contract designed for the other type. Note that from (3.46) the low-quality quality type always chooses the contracts that he most prefers in the short run because if he mimics the high-quality type in the first period, the processor will only offer the contract C_H^* under which the low-quality type makes loss.

Following the same procedure described in the previous sections, the optimal contract must satisfy the following conditions:

(i) The participation constraint for the low-quality grower type must be binding, i.e., $u(w_{1L}) - v(e_{1L})/\underline{\theta} = 0$,

(ii) The incentive compatibility constraint for the high-quality type is binding, i.e., $U_1(C_{1H} \mid \bar{\theta}) = U_1(C_{1L} \mid \bar{\theta}) + \delta I_{2H}(0)$, and

(iii) The low-quality type strictly prefers his own contract to the contract designed for the high-quality type, i.e., $U_1(C_{1L} \mid \underline{\theta}) > U_1(C_{1H} \mid \underline{\theta})$.

The feasible set of contracts can be demonstrated in Figure 3.3. First, since the low-quality type always chooses his own contract and obtains the reservation utility, the feasible set of contracts for the low-quality type must be the segment from the origin to the point $(\underline{w}^*, \underline{e}^*)$ on his indifference curve $u(w) - v(e)/\underline{\theta} = 0$. Denote S_{1L} as this set. Given any contract for the low-quality type $C_{1L} = (w_{1L}, e_{1L})$ in the set S_{1L}, properties of the optimal contract (i)-(iii) require that the feasible contract for the high-quality type must be in the region below the low-quality indifference curve $u(w) - v(e)/\underline{\theta} = 0$ and above the high-quality indifference curve H2 in Figure 3.3. More specifically, from condition (ii), the optimal contract for the high-quality type must be located on the indifference curve H2. Note that the indifference curve H1 intersects with the low-quality indifference curve $u(w) - v(e)/\underline{\theta} = 0$ through the point (w_{1L}, e_{1L}) and the distance between H1 and H2 is exactly $\delta I_{2H}(0)$. Hence, for any given contract $C_{1L} = (w_{1L}, e_{1L})$, the contract for the high-quality type $C_{1H} = (w_{1H}, e_{1H})$ always satisfies conditions (3.41), (3.45), (3.46) and properties (i)-(iii) at the equilibrium. Thus, the problem boils down to solving the optimal contract for the low-quality type. Once the optimal contract for the low-quality type is determined, it is straightforward to find the optimal contract for the high-quality type.

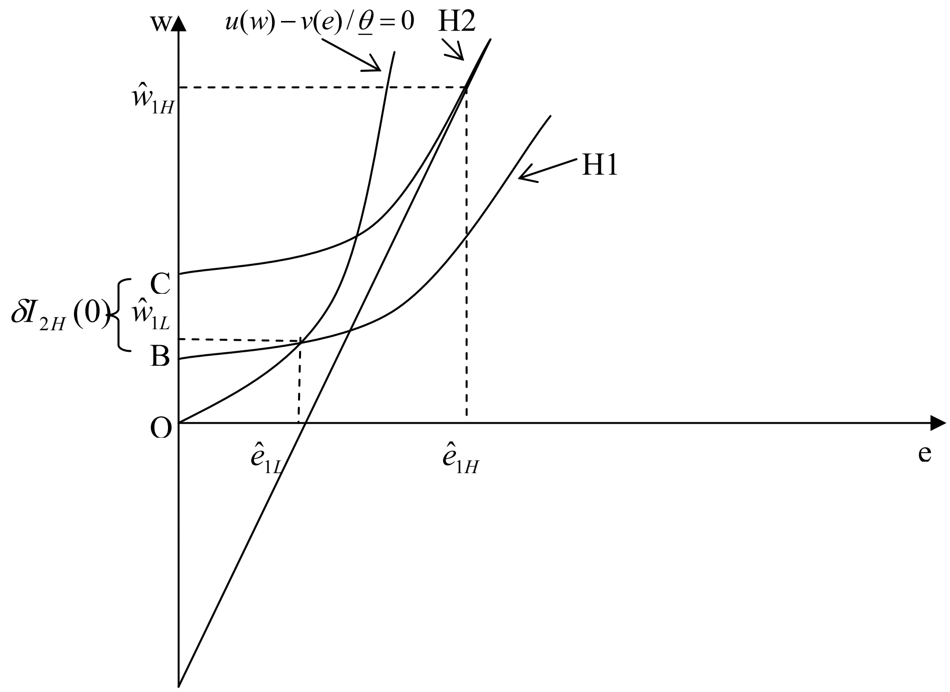

Figure 3.3 A first-period separating equilibrium

The optimal first-period contract can be solved in the similar manner as in the previous sections. Ignoring the participation constraint for the high-quality type and the incentive compatibility constraint (3.46), let λ and μ_H denote the Lagrangian multipliers for conditions (3.45) and (3.41) respectively, thus, the Lagrangian for the problem is:

$$L = r_1[\overline{\theta}e_{1H} - w_{1H} + \delta W_2(1)] + (1-r_1)[\underline{\theta}e_{1L} - w_{1L} + \delta W_2(0)] + \lambda[u(w_{1L}) - v(e_{1L})/\underline{\theta}]$$
$$+ \mu_H[u(w_{1H}) - v(e_{1H})/\overline{\theta} - u(w_{1L}) + v(e_{1L}\underline{\theta}/\overline{\theta})/\overline{\theta} - \delta I_{2H}(0)]$$

The first order conditions are:

$$(3.47)\quad \frac{\partial L}{\partial e_{1H}} = r_1\overline{\theta} - \mu_H v'(e_{1H})/\overline{\theta} = 0,$$

(3.48) $\frac{\partial L}{\partial w_{1H}} = -r_1 + \mu_H u'(w_{1H}) = 0$,

(3.49) $\frac{\partial L}{\partial e_{1L}} = (1-r_1)\underline{\theta} - \lambda v'(e_{1L})/\underline{\theta} + \mu_H(\underline{\theta}/\overline{\theta})v'(e_{1L}\underline{\theta}/\overline{\theta})/\overline{\theta} = 0$, and

(3.50) $\frac{\partial L}{\partial w_{1L}} = -(1-r_1) + \lambda u'(w_{1L}) - \mu_H u'(w_{1L}) = 0$.

Thus, the optimal contract $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$ is given by the following conditions:

(3.51) $e_{1H} \in \arg\{\overline{\theta}^2 u'(w_{1H}) = v'(e_{1H})\}$,

(3.52) $e_{1L} \in \arg\{(1-r_1)\underline{\theta}[1 - \frac{v'(e_{1L})}{u'(w_{1L})\underline{\theta}^2}] + \frac{r_1}{\underline{\theta}v'(e_{1H})}[\underline{\theta}^2 v'(e_{1L}\underline{\theta}/\overline{\theta}) - \overline{\theta}^2 v'(e_{1L})] = 0\}$,

(3.53) $u(w_{1L}) - v(e_{1L})/\underline{\theta} = 0$, and

(3.54)

$$u(w_{1H}) - v(e_{1H})/\overline{\theta} = u(w_{1L}) - v(e_{1L}\underline{\theta}/\overline{\theta})/\overline{\theta} + \delta I_{2H}(0) = u(w_{1L}) - v(e_{1L})/\underline{\theta}$$
$$+ v(e_{1L})/\underline{\theta} - v(e_{1L}\underline{\theta}/\overline{\theta})/\overline{\theta} + \delta I_{2H}(0) = v(e_{1L})/\underline{\theta} - v(e_{1L}\underline{\theta}/\overline{\theta})/\overline{\theta} + \delta I_{2H}(0) > 0.$$

The optimal contract is demonstrated in Figure 3.3. Denote this contract as $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$ where $\hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L})$ and $\hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H})$. Given the first order conditions (3.51)-(3.54), the contract $\hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L})$ and $\hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H})$ illustrated in Figure 3.3 constitutes a separating equilibrium. In fact, assuming that both grower types participate in the first period, the optimal separating equilibrium contract $\hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L})$ and $\hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H})$ is unique for a given prior belief r_1. Given this contract, the low-quality type strictly prefers the contract $\hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L})$ to $\hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H})$ both in a one-period static contract and in a two-period dynamic contract, while the high-quality type strictly prefers the contract $\hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H})$ to $\hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L})$ in the short run (i.e., in a one-period contract) and is indifferent in the two-period dynamic context. Thus, at the separating equilibrium, the low-quality type chooses his own contract $\hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L})$ in the first period and will be offered $\hat{C}_{2L} = (\underline{w}^*, \underline{e}^*)$ in the second period, and earns zero payoff in two periods. Similarly, the high-quality type chooses $\hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H})$ in the first period and obtains positive payoff $v(\hat{e}_{1L})/\underline{\theta} - v(\hat{e}_{1L}\underline{\theta}/\overline{\theta})/\overline{\theta} + \delta I_{2H}(0) > 0$, and will be offered $\hat{C}_{2H} = (\overline{w}^*, \overline{e}^*)$ in the second period. If the high-quality type deviates and chooses $\hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L})$ in the first period, he will earn $v(\hat{e}_{1L})/\underline{\theta} - v(\hat{e}_{1L}\underline{\theta}/\overline{\theta})/\overline{\theta}$ in the first period and earn information rent $I_{2H}(0)$ in the second period, which makes him indifferent between $\hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L})$ and $\hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H})$. Note that the contract $\hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H})$ is the tangent point between the processor's iso-profit line and the high-quality type's indifference curve H2 in Figure 3.3, therefore, the optimal contract for the high-quality type is efficient. Additionally, as in the full-commitment contract with asymmetric information, the processor offers the low-quality type a contract that is suboptimal in order to reduce the information rent paid to the high-quality type.

However, besides this separating equilibrium, other separating equilibria might also exist. In particular, define $C^0 = (w, e) = (0,0)$ as the null contract (i.e., a grower type does not sign the contract at all if a null contract is offered). The contract $\widetilde{C}_1 = \{\widetilde{C}_{1L}, \widetilde{C}_{1H}\} = \{C^0, \widetilde{C}_{1H}\}$ establishes another separating equilibrium, where $\widetilde{C}_{1H} = (\widetilde{w}_{1H}, \widetilde{e}_{1H})$ in Figure 3.4 is the tangent point between the high-quality type's indifference curve H1 and the processor's iso-profit line for the high-quality type. This contract is actually the limit of the separating contract $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$ as r_1 approaches 1. Since the low-quality type will not participate in the first period given this contract, this outcome is called "*handicapped separating equilibrium*". More specifically, the contract $\widetilde{C}_{1H} = (\widetilde{w}_{1H}, \widetilde{e}_{1H})$ must satisfy the following conditions:

(3.55) $\widetilde{e}_{1H} \in \arg\{\overline{\theta}^2 u'(\widetilde{w}_{1H}) = v'(\widetilde{e}_{1H})\}$, and

(3.56) $u(\widetilde{w}_{1H}) - v(\widetilde{e}_{1H})/\overline{\theta} = \delta I_{2H}(0)$.

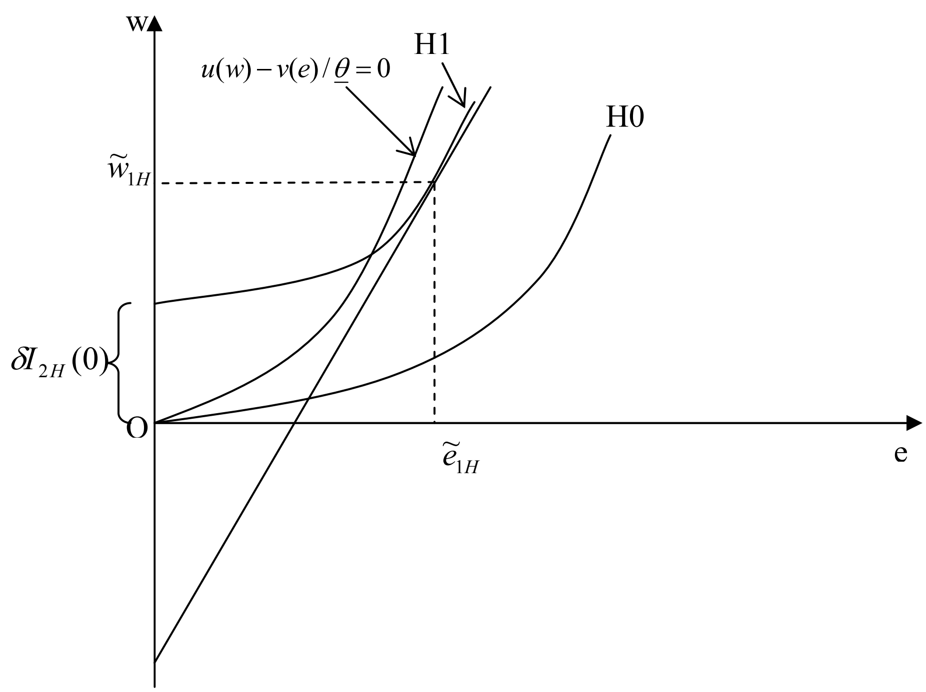

Figure 3.4 A handicapped separating equilibrium

Under this contract, the low-quality type strictly prefers the null contract C^0 to the contract $\widetilde{C}_{1H}$ because he could make losses in both periods if he chooses $\widetilde{C}_{1H}$ in the first period. Note that the low-quality type also strictly prefers C^0 to $\widetilde{C}_{1H}$ in a one-period static contract. Similarly, by choosing $\widetilde{C}_{1H}$ in the first period, the two-period payoff to the high-quality type is $U_1(\widetilde{C}_{1H} \mid \overline{\theta}) + \delta U_2(C_H^* \mid \overline{\theta}) = U_1(\widetilde{C}_{1H} \mid \overline{\theta}) = \delta I_{2H}(0)$, while by choosing C^0, he obtains $U_1(C^0 \mid \overline{\theta}) + \delta U_2(C_L^* \mid \overline{\theta}) = \delta U_2(C_L^* \mid \overline{\theta}) = \delta I_{2H}(0)$. Hence, the high-quality type is indifferent between the contract $\widetilde{C}_{1H}$ and C^0. Therefore, $\widetilde{C}_1 = \{\widetilde{C}_{1L}, \widetilde{C}_{1H}\} = \{C^0, \widetilde{C}_{1H}\}$ constitutes another separating equilibrium.

Given the two separating equilibria, the processor must offer the one that maximizes her net profit. That is, the optimal contract maximizes the maximum of $W_1(r_1, \hat{C}_{1L}, \hat{C}_{1H})$ and $W_1(r_1, C^0, \widetilde{C}_{1H})$. The results are summarized in the following proposition.

Proposition 1: There exists two possible separating equilibria to the dynamic contract: $\widetilde{C}_1 = \{C^0, \widetilde{C}_{1H}\}$ and $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$. In addition, there exists a r_1^* such that for $r_1 < r_1^*$, the optimal separating equilibrium is $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$, while for $r_1 > r_1^*$, the optimal separating equilibrium is $\widetilde{C}_1 = \{C^0, \widetilde{C}_{1H}\}$.

Proof: If the contract $\widetilde{C}_1 = \{C^0, \widetilde{C}_{1H}\}$ is offered, the processor obtains net profit

(3.57) $W_1^*(r_1, C^0, \widetilde{C}_{1H}) = r_1[\overline{\theta}\widetilde{e}_{1H} - \widetilde{w}_{1H} + \delta W_2(1)] + (1 - r_1)[\delta W_2(0)]$.

On the other hand, if the contract $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$ is offered, the processor earns

(3.58) $W_1^*(r_1, \hat{C}_{1L}, \hat{C}_{1H}) = r_1[\overline{\theta}\hat{e}_{1H} - \hat{w}_{1H} + \delta W_2(1)] + (1 - r_1)[\underline{\theta}\hat{e}_{1L} - \hat{w}_{1L} + \delta W_2(0)]$.

Note that as the contract $(\hat{w}_{1L}, \hat{e}_{1L})$ moves toward to (0, 0) along the indifference curve $u(w) - v(e)/\underline{\theta} = 0$, the high-quality type's indifference curve shifts down accordingly. Thus, the optimal condition (3.55) requires that the processor's iso-profit line for the high-quality type shifts towards southeast until it is tangent to the high-quality type's indifference curve. Therefore, the processor obtains greater profit from the high-quality type as $(\hat{w}_{1L}, \hat{e}_{1L})$ moves toward the origin (0, 0), precisely, $\bar{\theta}\tilde{e}_{1H} - \tilde{w}_{1H} > \bar{\theta}\hat{e}_{1H} - \hat{w}_{1H}$. Hence, there must exist a r_1^* such that $W_1^*(r_1^*, \hat{C}_{1L}, \hat{C}_{1H}) = W_1^*(r_1^*, C^0, \tilde{C}_{1H})$. Thus, for $r_1 < r_1^*$, $W_1^*(r_1^*, \hat{C}_{1L}, \hat{C}_{1H}) > W_1^*(r_1^*, C^0, \tilde{C}_{1H})$, the processor will offer the optimal separating equilibrium $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$, while for $r_1 > r_1^*$, $W_1^*(r_1^*, \hat{C}_{1L}, \hat{C}_{1H}) < W_1^*(r_1^*, C^0, \tilde{C}_{1H})$ and the optimal separating equilibrium is $\tilde{C}_1 = \{C^0, \tilde{C}_{1H}\}$.

Intuitively, Proposition 1 states that the separating contract $\tilde{C}_1$ would dominate $\hat{C}_1$ when the processor believes that a large proportion of the growers are of high-quality type. Thus, it is less costly for the processor if it only offers a contract to the high-quality type and handicaps the low-quality type. On the contrary, if the processor believes that the proportion of high-quality type is sufficiently small, then it would be better off by offering the separating contract $\hat{C}_1$. In addition, we could show that the processor's two-period profit increases with the proportion of the high-quality type. This result is summarized in the following corollary.

Corollary 1.1: In a separating equilibrium, $W_1(r_1, C_{1L}, C_{1H})$ increases with r_1.

When $r_1 < r_1^*$, the optimal separating equilibrium $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$ dominates. As r_1 increases, the optimal contract $\hat{C}_{1L}$ moves toward the origin (0,0) along the low-quality indifference curve $u(w) - v(e)/\underline{\theta} = 0$, while the contract $\hat{C}_{1H}$ shifts down toward the perfect information contract C_H^*. Thus, as r_1 increases, the processor would acquire a greater profit from the high-quality type by paying less information rents, and obtain a smaller profit from the low-quality type without changing the payoff to the low-quality grower type. Since a change in $(\hat{w}_{1L}, \hat{e}_{1L})$ along the low-quality type's indifference curve would only incur a second-order effect on the processor's profit, while a corresponding change in $(\hat{w}_{1H}, \hat{e}_{1H})$ would have a first-order effect on the processor's profit, the processor's net profit increases as r_1 increases. That is, $W_1(r_1, \hat{C}_{1L}, \hat{C}_{1H})$ increases with r_1.

Up to now, we have assumed that for any contract (w, P)[18], it is always true that $P - w \geq 0$, i.e., on the right side of the 45 degree line in the (w, P) space. However, restricting positive profit reduces the set of feasible contracts. Specifically, given the optimal contract $\hat{C}_{1L}$, the separating contract $\hat{C}_{1H} = (\hat{w}_{1H}, \hat{P}_{1H})$ in (w, P) space may become infeasible for δ sufficiently large because it would lie to the left side of the zero-profit line $w = P$. A similar argument could be made for the handicapped separating equilibrium $\{C^0, \widetilde{C}_{1H}\}$. An example of an infeasible separating contract is illustrated in Figure 3.5. These arguments are provided in the following corollary without further proof.

Corollary 1.2: These exists a δ^* such that for $\delta > \delta^*$, the separating equilibrium $\hat{C}_1 = \{\hat{C}_{1H}, \hat{C}_{1L}\}$ becomes infeasible.

Although the value of δ^* cannot be precisely determined, the intuition behind this corollary is that if growers are patient (i.e., δ large), it becomes too costly for the processor to induce a separating equilibrium in the first period. When growers are patient, the processor is better off by providing a pooling contract or a semi-separating contract instead of a fully separating contract.

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[18] Here, we use the contract space (w, P) instead of (w, e). Recall that $P = q = \theta e$. From now on, we may use these two alternative spaces interchangeably.

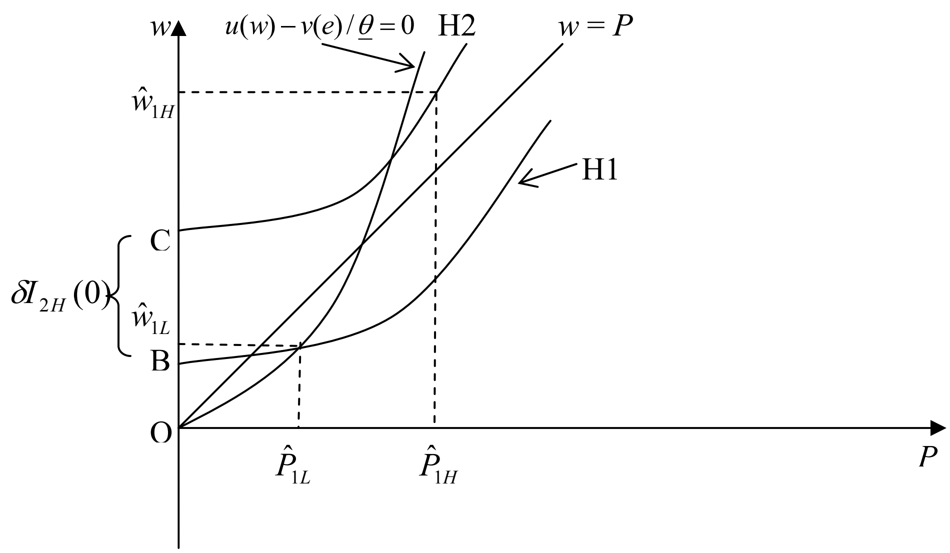

Figure 3.5 Illustration of an infeasible separating equilibrium

For the separating equilibrium to be stable, we need to check if these contracts would be dominated by other contracts.

First, we consider the contract $\widetilde{C}_1 = \{C^0, \widetilde{C}_{1H}\}$, given $r_1 > r_1^*$.

Since the low-quality type always chooses the contract that he most prefers in the short run and earns exactly the reservation utility in each period, any pooling equilibrium must be located on the low-quality type's zero-utility indifference curve. Suppose there exists a pooling equilibrium $\{C_1^p, C_1^p\}$ on the low-quality type's indifference curve $u(w) - v(P/\underline{\theta})/\underline{\theta} = 0$, where $C_1^p = (w^p, P^p)$. Here, we change the contract space into (w, P) instead of using (w, e) to denote the pooling contract. For a pooling contract, both growers produce at the same quality (equivalently, price) level, but each grower type incurs a different level of effort and disutility. Under this contract, the processor obtains net profit

(3.59) $W_1^*(r_1, C_1^p, C_1^p) = P^p - w^p + \delta W_2(r_1)$.

From (3.57), $W_1^*(r_1, C^0, \widetilde{C}_{1H}) = r_1[\widetilde{P}_{1H} - \widetilde{w}_{1H} + \delta W_2(1)] + (1-r_1)[\delta W_2(0)]$. Since $C_1^p = (w^p, P^p)$ lies on the low-quality type's indifference curve $u(w) - v(P/\underline{\theta})/\underline{\theta} = 0$ and $\widetilde{C}_{1H}$ is located to the right side of that curve, it must be true that $\widetilde{P}_{1H} - \widetilde{w}_{1H} > P^p - w^p$. In addition, we know from Lemma 2 that, for any r_1, $0 < r_1 < 1$, $W_2(r_1) < r_1 W_2(1) + (1-r_1) W_2(0)$ due to convexity of $W_2(\,)$. Thus, for r_1 sufficiently large, $W_1^*(r_1, C^0, \widetilde{C}_{1H}) > W_1^*(r_1, C_1^p, C_1^p)$. Alternatively, for any given r_1, the difference $\widetilde{Z}_H - Z^p = (\widetilde{P}_{1H} - \widetilde{w}_{1H}) - (P^p - w^p)$ must be sufficiently large for the handicapped separating equilibrium to be dominant. In general, this requires that the difference between $\overline{\theta}$ and $\underline{\theta}$ is sufficiently large. For a given $\underline{\theta}$, a larger value of $\overline{\theta}$ would make the high-quality type's indifference curves less steep everywhere. Since the optimal contract for the high-quality type is the point where the processor's iso-profit line is tangent to the high-quality type's indifference curve, a flatter indifference curve of the high-quality type would make the processor's iso-profit line cut the vertical axis at a point even farther away from the origin. Recall that the distance from the origin to the intersection point is exactly the processor's net profit. Thus, the difference $\widetilde{Z}_H - Z^p = (\widetilde{P}_{1H} - \widetilde{w}_{1H}) - (P^p - w^p)$ increases with the difference between $\overline{\theta}$ and $\underline{\theta}$[19]. Note that increases in the difference between $\overline{\theta}$ and $\underline{\theta}$ would raise the high-quality type's information rent $\delta I_{2H}(0)$ as well. However, it is straightforward to show that the net increase in the processor's profit is always positive as the difference between $\overline{\theta}$ and $\underline{\theta}$ increases. Briefly, fix a $\underline{\theta}$, as $\overline{\theta}$ increases, $\widetilde{Z}_H - Z^p$ monotonically increases while the

[19] The exact relationship between $\widetilde{Z}_H - Z^p = (\widetilde{P}_{1H} - \widetilde{w}_{1H}) - (P^p - w^p)$ and difference between $\overline{\theta}$ and $\underline{\theta}$ would depend on the functional forms of the grower's utility function *u*( ) and the disutility function *v*( ).

information rent $I_{2H}(0)$ approaches a constant $v(e^{*})/\underline{\theta}$. Therefore, for a sufficiently large $\overline{\theta}$, increases in $\tilde{Z}_H - Z^p$ exceed those in $I_{2H}(0)$.

A similar argument could be made for the separating contract $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$ for r_1 small. From (3.58),

$W_1^*(r_1, \hat{C}_{1L}, \hat{C}_{1H}) = r_1[\overline{\theta}\hat{e}_{1H} - \hat{w}_{1H} + \delta W_2(1)] + (1-r_1)[\underline{\theta}\hat{e}_{1L} - \hat{w}_{1L} + \delta W_2(0)]$. For a pooling contract located on the low-quality type's zero-utility indifference curve between the origin and the point $\hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L})$, it is straightforward to show that $P^p - w^p < \underline{\theta}\hat{e}_{1L} - \hat{w}_{1L} < \overline{\theta}\hat{e}_{1H} - \hat{w}_{1H}$. In addition, from convexity of $W_2()$, for any r_1, $0 < r_1 < 1$, $W_2(r_1) < r_1 W_2(1) + (1-r_1)W_2(0)$. Therefore, $W_1^*(r_1, \hat{C}_{1L}, \hat{C}_{1H}) > W_1^*(r_1, C_1^p, C_1^p)$. Similarly, for a pooling contract located on the low-quality type's zero-utility indifference curve between the point $\hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L})$ and $C_L^* = (\underline{w}^*, \underline{e}^*)$, $\underline{\theta}\hat{e}_{1L} - \hat{w}_{1L} < P^p - w^p < \overline{\theta}\hat{e}_{1H} - \hat{w}_{1H}$. Since the separating contract $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$ dominates the handicapped separating equilibrium only if r_1 is sufficiently small, then, for the contract $\hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\}$ to be optimal, it must be true that the difference $\tilde{Z}_H - Z^p = (\overline{\theta}\hat{e}_{1H} - \hat{w}_{1H}) - (P^p - w^p)$ is sufficiently large. Following a similar argument used for the handicapped separating contract, the difference between $\overline{\theta}$ and $\underline{\theta}$ must be sufficiently large. Combining these arguments and Corollary 1.2 yields the following corollary.

Corollary 1.3: When the difference between $\overline{\theta}$ and $\underline{\theta}$ is sufficiently large and δ is sufficiently small, there exists a fully separating equilibrium.

The intuition behind this corollary is similar to that of corollary 1.2. For large values of δ (close to 1), the high-quality grower type is very patient and it is prohibitively costly for the processor to distinguish the grower types. For δ sufficiently small and the difference between $\overline{\theta}$ and $\underline{\theta}$ sufficiently large, not only is it less costly for the processor to distinguish the grower type, but also the high-quality grower type would intend to distinguish himself from the low-quality type under the optimal contract.

First-period semi-separating equilibrium

Using similar procedures as described in the previous section, a semi-separating equilibrium could be established.

Let $C_1^s = \{C_{1L}^s, C_{1H}^s\}$ be the semi-separating contract, where $C_{1i}^s = (w_{1i}^s, e_{1i}^s)$ for $i \in \{L, H\}$. Recall that for the contract to be semi-separating, the condition (3.39) must be satisfied. Specifically, the condition (3.39) is equivalent to

(3.60) $U_1(C_{1H}^s \mid \overline{\theta}) = U_1(C_{1L}^s \mid \overline{\theta}) + \delta U_2(C_{2H}^s(\hat{r}_2) \mid \overline{\theta})) = U_1(C_{1H}^s \mid \overline{\theta}) + \delta I_{2H}(\hat{r}_2)$,

where $\hat{r}_2(P_1(\underline{\theta}), \pi) = \dfrac{r_1 \pi}{r_1 \pi + 1 - r_1}$, and π is the probability that the grower type $\overline{\theta}$ chooses the contract designed for the grower type $\underline{\theta}$ in the first period. In addition, the semi-separating contract for the low-quality type must lie on his zero-utility indifference curve, $u(w) - v(e)/\underline{\theta} = 0$. Thus, the processor solves the following problem:

$$\max_{w_{1i}^s, e_{1i}^s \pi} W_1(r_1, \hat{\pi}, C_{1L}^s, C_{1H}^s) = (1-\psi)[\overline{\theta} e_{1H}^s - w_{1H}^s + \delta W_2(1)] + \psi[\underline{\theta} e_{1L}^s - w_{1L}^s + \delta W_2(\hat{r}_2)]$$

subject to

(3.60) $U_1(C_{1H}^s \mid \overline{\theta}) = U_1(C_{1L}^s \mid \overline{\theta}) + \delta I_{2H}(\hat{r}_2)$, and

(3.61) $u(w^s_{1L}) - v(e^s_{1L})/\underline{\theta} = 0$,

where $\psi = r_1\pi + 1 - r_1$.

Let λ and μ_H denote the Lagrangian multipliers for (3.61) and (3.60). Then the Lagrangian for the problem above is

$$L = (1-\psi)[\overline{\theta}e^s_{1H} - w^s_{1H} + \delta W_2(1)] + \psi[\underline{\theta}e^s_{1L} - w^s_{1L} + \delta W_2(\hat{r}_2)]$$
$$+ \lambda(u(w^s_{1L}) - v(e^s_{1L})/\underline{\theta}) + \mu_H[u(w^s_{1H}) - v(e^s_{1H})/\overline{\theta} - u(w^s_{1L}) + v(P^s_{1L}/\overline{\theta})/\overline{\theta} - \delta I_{2H}(\hat{r}_2)].$$

The first order conditions are:

(3.62) $\dfrac{\partial L}{\partial e^s_{1H}} = (1-\psi)\overline{\theta} - \mu_H v'(e^s_{1H})/\overline{\theta} = 0$.

(3.63) $\dfrac{\partial L}{\partial w^s_{1H}} = -(1-\psi) + \mu_H u'(w^s_{1H}) = 0$.

(3.64) $\dfrac{\partial L}{\partial e^s_{1L}} = \psi\underline{\theta} - \lambda v'(e^s_{1L})/\underline{\theta} + \mu_H(\underline{\theta}/\overline{\theta})v'(P^s_{1L}/\overline{\theta})/\overline{\theta} = 0$.

(3.65) $\dfrac{\partial L}{\partial w^s_{1L}} = -\psi + \lambda u'(w^s_{1L}) - \mu_H u'(w^s_{1L}) = 0$.

(3.66)

$$\frac{\partial L}{\partial \pi} = -r_1[\overline{\theta}e^s_{1H} - w^s_{1H} + \delta W_2(1)] + r_1[\underline{\theta}e^s_{1L} - w^s_{1L}] + r_1\delta W_2(\hat{r}_2) + \psi\delta\hat{r}_2'[Z_{2H}(\hat{r}_2) - Z_{2L}(\hat{r}_2)]$$
$$- \mu_H \delta I'_{2H}(\hat{r}_2) = 0,$$

where in the condition (3.66),

$$\hat{r}_2' = \frac{\partial \hat{r}_2}{\partial \pi} = \frac{r_1(1-r_1)}{(r_1\pi + 1 - r_1)^2},$$

$Z_{2H}(\hat{r}_2) = \overline{\theta}e_{2H}(\hat{r}_2) - w_{2H}(\hat{r}_2)$, and

$Z_{2L}(\hat{r}_2) = \underline{\theta} e_{2L}(\hat{r}_2) - w_{2L}(\hat{r}_2)$.

From (3.62) and (3.63), we can derive the optimal level of effort for the high-quality type:

(3.67) $e^s_{1H} \in \arg\{\overline{\theta}^2 u'(w^s_{1H}) = v'(e^s_{1H})\}$.

From (3.65), we can get

(3.68) $\lambda = \dfrac{\psi}{u'(w^s_{1L})} + \mu_H$, and

from (3.62), we can get

(3.69) $\mu_H = \dfrac{(1-\psi)\overline{\theta}^2}{v'(e^s_{1H})}$.

Substituting (3.68) and (3.69) into (3.64) yields:

(3.70) $e^s_{1L} \in \arg\{\psi\underline{\theta}[1 - \dfrac{v'(e^s_{1L})}{u'(w^s_{1L})\underline{\theta}^2}] + \dfrac{1-\psi}{\underline{\theta} v'(e^s_{1H})}[\underline{\theta}^2 v'(e^s_{1L}/\overline{\theta}) - \overline{\theta}^2 v'(e^s_{1L})] = 0\}$.

Further, since the conditions (3.60) and (3.61) are always equalities, we have

(3.71) $U_1(C^s_{1H} \mid \overline{\theta}) = u(w^s_{1H}) - v(e^s_{1H})/\overline{\theta} = v(e^s_{1L})/\underline{\theta} - v(P^s_{1L}/\overline{\theta})/\overline{\theta} + \delta I_{2H}(\hat{r}_2)$, and

(3.72) $U_1(C^s_{1L} \mid \underline{\theta}) = u(w^s_{1L}) - v(e^s_{1L})/\underline{\theta} = 0$.

Finally, substituting conditions (3.67), (3.70)-(3.72) into the condition (3.66) solves the optimal strategy $\hat{\pi}$ for the high-quality grower type.

This optimal semi-separating contract $C^s_1 = \{C^s_{1L}, C^s_{1H}\}$ is illustrated in Figure 3.6.

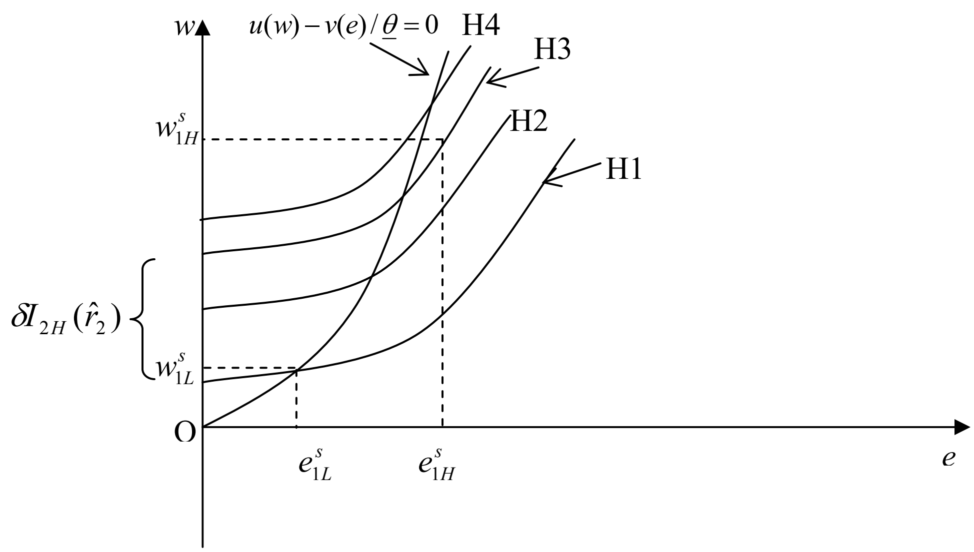

Figure 3.6 A first-period semi-separating equilibrium

In Figure 3.6, H1-H4 are the high-quality type's indifference curves, where the distances between the curve H1 and the curves H2, H3, and H4 are $\delta I_{2H}(r_1)$, $\delta I_{2H}(\hat{r}_2)$, and $\delta I_{2H}(0)$, respectively and $\delta I_{2H}(r_1) < \delta I_{2H}(\hat{r}_2) < \delta I_{2H}(0)$. From conditions (3.37)-(3.39), a semi-separating contract $C_1^s = \{C_{1L}^s, C_{1H}^s\}$ must satisfy

(3.73) $U(C_{1L}^s \mid \overline{\theta}) + \delta I_{2H}(r_1) \le U(C_{1H}^s \mid \overline{\theta}) \le U(C_{1L}^s \mid \overline{\theta}) + \delta I_{2H}(0)$.

Thus, given that the optimal contract C_{1L}^s is located on the low-quality type's indifference curve $u(w_{1L}^s) - v(e_{1L}^s)/\underline{\theta} = 0$, the optimal contract C_{1H}^s must lie on a indifference curve, with the curve H3 as an illustration, that is above the indifference curve H2 and below H4. Therefore, there exists a $\hat{\pi}$ such that $U(C_{1H}^s \mid \overline{\theta}) = U(C_{1L}^s \mid \overline{\theta}) + \delta I_{2H}(\hat{r}_2)$, i.e., a semi-separating equilibrium.

In addition, the following remark establishes the relationship between r_1 and $\hat{\pi}$.

Remark 1: The optimal strategy $\hat{\pi}$ in a semi-separating equilibrium increases with r_1.

Recall that the exact relationship between r_1 and $\hat{\pi}$ is governed by the conditions (3.66), (3.67), (3.70)-(3.72), especially the condition (3.66). However, to determine their exact relationship is not a trivial task analytically. The intuition behind this remark is as follows: As r_1 increases, i.e., the processor believes that a larger proportion of growers is of high-quality type, the optimal contract C_{1H}^s would extract more surplus from the high-quality grower type by rewarding less information rent. In the limit, when r_1 is one, the processor would only offer the optimal contract $(\overline{w}^*, \overline{e}^*)$ under which the high-quality grower type earns exactly his reservation utility zero. Thus, as r_1 increases, the high-quality type is better off mimicking the low-quality type in the first period and earns more information rent. Therefore, in a semi-separating equilibrium, the high-quality grower type is more likely to choose the contract that is designed for the low-quality type, that is, $\hat{\pi}$ increases as r_1 increases. Similarly, in the other direction, as r_1 decreases, the optimal contract the processor would offer becomes closer to the contract $(\underline{w}^*, \underline{e}^*)$. In the limit, when r_1 approaches zero, the processor would offer the contract $(\underline{w}^*, \underline{e}^*)$ under which the high-quality grower type obtains the maximum information rent. Thus, the high-quality type is less likely to mimic a low-quality type as r_1 decreases, i.e., $\hat{\pi}$ decreases as r_1 decreases.

To guarantee that the semi-separating equilibrium could be sustained, we also need to compare the semi-separating equilibrium with other potential equilibria. However, it is not trivial to determine the relationship between the semi-separating equilibrium with other potential equilibria analytically without specifying the functional forms of $u(\,)$ and $v(\,)$.

Remark 2: For r_1 sufficiently small, a semi-separating contract C_1^s might dominate the fully separating contract $\hat{C}_1$.

Recall conditions (3.52) and (3.70),

$$(3.52)\ \hat{e}_{1L} \in \arg\{(1-r_1)\underline{\theta}[1-\frac{v'(\hat{e}_{1L})}{u'(\hat{w}_{1L})\underline{\theta}^2}]+\frac{r_1}{\underline{\theta}v'(\hat{e}_{1H})}[\underline{\theta}^2 v'(\hat{e}_{1L}\underline{\theta}/\overline{\theta})-\overline{\theta}^2 v'(\hat{e}_{1L})]=0\}.$$

$$(3.70)\ e_{1L}^s \in \arg\{\psi\underline{\theta}[1-\frac{v'(e_{1L}^s)}{u'(w_{1L}^s)\underline{\theta}^2}]+\frac{1-\psi}{\underline{\theta}v'(e_{1H}^s)}[\underline{\theta}^2 v'(e_{1L}^s/\overline{\theta})-\overline{\theta}^2 v'(e_{1L}^s)]=0\}.$$

Since $1-\psi = r_1(1-\pi) \le r_1$, the change from r_1 in the separating equilibrium $\hat{C}_1$ to $1-\psi$ in the semi-separating contract C_1^s results in the similar consequences to that from r_1 to $1-\psi$ in the separating equilibrium $\hat{C}_1$. Thus, given the contract $\hat{C}_{1L}$ as the reference point, the contract C_{1L}^s is located closer to the perfect information contract $(\underline{w}^*, \underline{e}^*)$ on the low-quality type's indifference curve $u(w)-v(e)/\underline{\theta}=0$. Since $C_L^* = (\underline{w}^*, \underline{e}^*)$ maximizes the processor's profit acquired from the low-quality grower type, the processor always obtains less profit from the low-quality type under $\hat{C}_{1L}$ than that under C_{1L}^s, or precisely, $\underline{\theta}\hat{e}_{1L} - \hat{w}_{1L} < \underline{\theta}e_{1L}^s - w_{1L}^s$. On the other hand, using Figure 3.7, since the optimal semi-separating contract C_{1H}^s lies between the high-quality type's indifference curves H2 and H4, the high-quality type obtains less information rent under C_{1H}^s than that under the contract $\hat{C}_{1H}$. Thus, the processor earns less profit under the separating contract $\hat{C}_{1H}$ than that under C_{1H}^s, i.e., $\underline{\theta}\hat{e}_{1H} - \hat{w}_{1H} < \underline{\theta}e_{1H}^s - w_{1H}^s$.

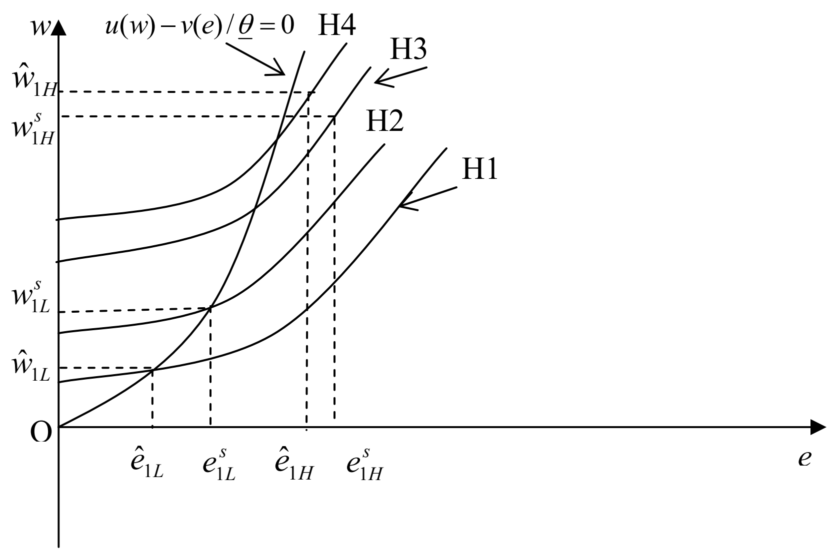

Figure 3.7 A semi-separating equilibrium dominates a separating equilibrium

From the first-period separating contract, the maximum profit the processor obtains from the contract $\hat{C}_1$ is

$$W_1(r_1,\hat{C}_{1L},\hat{C}_{1H}) = r_1[\overline{\theta}e_{1H} - w_{1H} + \delta W_2(1)] + (1-r_1)[\underline{\theta}e_{1L} - w_{1L} + \delta W_2(0)].$$

From the first-period semi-separating contract, the maximum profit the processor obtains is

$$W_1(r_1,\hat{\pi},C_{1L}^s,C_{1H}^s) = r_1(1-\hat{\pi})[\overline{\theta}e_{1H}^s - w_{1H}^s + \delta W_2(1)] + (1-r_1+r_1\hat{\pi})[\underline{\theta}e_{1L}^s - w_{1L}^s + \delta W_2(\hat{r}_2)]$$

As r_1 decreases, the high-quality grower type is less likely to deviate, thus, the term $r_1\hat{\pi}$ in $W_1(r_1,\hat{\pi},C_{1L}^s,C_{1H}^s)$ becomes negligible as r_1 becomes sufficiently small. Therefore, for r_1 sufficiently small, it is possible that $W_1(r_1,\hat{\pi},C_{1L}^s,C_{1H}^s) > W_1(r_1,\hat{C}_{1L},\hat{C}_{1H})$, i.e., the semi-separating contract C_1^s dominates the fully separating contract $\hat{C}_1$.

Further, similar arguments used for the separating equilibrium could also be applied to the semi-separating equilibrium. Specifically,

Remark 3: For δ sufficiently large and the difference between $\overline{\theta}$ and $\underline{\theta}$ sufficiently small, the fully separating equilibrium $\hat{C}_1$ and the semi-separating equilibrium C_1^s are dominated by a pooling equilibrium.

Unfortunately, without making more assumptions, the exact nature of the relationship among these potential equilibria could not be explicitly determined. The intuition behind this remark is that when δ is sufficiently large and the difference between $\overline{\theta}$ and $\underline{\theta}$ is sufficiently small, both the semi-separating contract and the fully separating contract would become infeasible and it would become too costly for the processor to distinguish the grower types by offering such contracts. Under these conditions, the only possible equilibrium would be a pooling equilibrium.

3.4.3 Reputation Rewards

In the previous section, reputation effects are embodied in the posterior probability assessment (using Bayes' rule) of the grower's types by the processor at the end of the first period. Anticipating the processor's strategies, the high-quality grower type chooses to build up his reputation by either imitating the low-quality type or revealing his true type, whichever is favorable. Under this scheme, however, imitating the dominant behavior of a low-quality type yields greater future information rents to the high-quality type. Therefore, the reputation effect (updating beliefs about the grower type using Bayes' rule) encourages the high-quality grower to conceal his type, that is, reinforces the potential ratchet effects.

In this section, we assume that at the beginning of each period, a reputation, R_t, of the grower is formed from his past observed performance. Thus, the processor will not only update its beliefs about the grower types by observing the grower's past contract choice, but also can offer a direct reward to the grower contingent on the observed performance of the grower. Accumulation of the grower's reputation is assumed to be based on an exogenous rule $R_t = \beta q_{t-1} + (1-\beta) R_{t-1}$ with $0 \le \beta \le 1$, and the grower's initial reputation is R_0. To simplify the analysis further, a special case of the example $R_t = q_{t-1}$ with $R_0 = 0$ is used to demonstrate the effects of the reputation reward on the dynamic contract.

Accumulation of reputation can be interpreted differently given different values of β. When β is small, i.e., very close to zero, the latest period quality does not provide much contribution to the grower's reputation. This situation could occur under some circumstances such that the processor already has a long-term relationship before this contract and the grower's reputation has almost converged to a constant by the latest period. In this case, including reputation effects in the contract would not improve much on the optimal incentives. On the other hand, when β is large, the latest period quality is crucial for the grower's reputation in the current period. Thus, stronger incentives can be provided by the processor when reputation of growers is incorporated into the contract. $R_t = q_{t-1}$ is a special case of this example when setting $\beta = 1$.

Specifically, the processor offers the grower some extra reputation rewards, $s_t(R_t)$, when it observes R_t from the previous periods. If the processor observes $P(\bar{\theta})$ in the first period, the reward in the second period will be $s(P(\bar{\theta}))$, while if the processor observes $P(\underline{\theta})$ in the first period, the reward in the second period will be $s(P(\underline{\theta}))$.

Moreover, we assume that the reputational rewards take the form of lump-sum payments or constants. In particular, the reward $s(P(\underline{\theta}))$ is normalized to be zero.

Similar to the previous section, the processor maximizes its expected profit subject to the participation constraints and incentive compatibility constraints for both grower types in both periods. However, to simplify the analysis, only a separating equilibrium will be discussed. First, let us investigate the second-period incentive scheme.

Second-period incentive schemes with reputation rewards

The second-period incentive scheme stays the same as that in the previous section since the reputation rewards do not affect the grower's participation constraints and incentive compatibility constraints in the second period. In a separating equilibrium, the private information concerning the grower's types becomes perfect information in the second period. Let $s_{2H} = s_2(P(\theta))$, then for each grower type, $\theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}$, the processor offers the contract $C_2^R = \{C_{2L}^R, C_{2H}^R\}$, where $C_{2L}^R = (\underline{w}^*, \underline{e}^*)$ and $C_{2H}^R = (\overline{w}^* + s_{2H}, e_{2H}^*)$. In addition, the optimal contract C_{2H}^R satisfies $e_{2H}^* \in \arg\{v'(e_{2H}) = \overline{\theta}^2 u'(\overline{w}^* + s_{2H})\}$. Note that the reputation reward changes the high-quality type's equilibrium behavior in the second period.

Recall that in a separating equilibrium, if the high-quality grower type deviates in the first period, he obtains the information rent $I_{2H}(0) = v(\underline{e}^*)/\underline{\theta} - v(\underline{e}^*\underline{\theta}/\overline{\theta})/\overline{\theta} > 0$ in the second period. Similarly, if the low-quality grower type deviates and chooses the contract designed for the high-quality type in the first period, then he will make loss in the second period, (3.74) $I_{2L}(1) = u(\overline{w}^*) - v(\overline{e}^*\overline{\theta}/\underline{\theta})/\underline{\theta} = v(\overline{e}^*)/\overline{\theta} - v(\overline{e}^*\overline{\theta}/\underline{\theta})/\underline{\theta} < 0$.[20]

In addition, it can be shown that $I_{2H}(0) < -I_{2L}(1)$.

First-period incentive schemes with reputation rewards

In the first period, the processor must maximize the two-period expected profit to find a separating equilibrium. Then the processor maximizes the following:

$$\max_{e_{1H}, w_{1H}, e_{1L}, w_{1L}} W_1(r_1, C_{1L}, C_{1H}) = r_1[\overline{\theta}e_{1H} - w_{1H} + \delta W_2(1) - \delta s_{2H}] + (1 - r_1)[\underline{\theta}e_{1L} - w_{1L} + \delta W_2(0)]$$

The participation constraints take the following form:

(3.75) $U_1 = u(w_{1i}) - v(e_{1i})/\theta_i \geq 0, \quad \forall i \in \{L, H\}$.

However, risk aversion brings about some complications in the formulation of incentive compatibility constraints. To induce a separating equilibrium in the first period, the incentive compatibility constraints for the high-quality grower type must satisfy

(3.76) $U_1(C_{1H} \mid \overline{\theta}) + \delta \hat{s}_{2H}(\overline{\theta}) \geq U_1(C_{1L} \mid \overline{\theta}) + \delta I_{2H}(0)$.

This constraint states that at the equilibrium, the high-quality grower type must prefer revealing his true type to mimicking the low-quality grower type. Note that the extra reward s_{2H} in the processor's profit $W_1(r_1, C_{1L}, C_{1H})$ is given in monetary units, while $\hat{s}_{2H}(\overline{\theta})$ is the equivalent amount in the units of the high-quality type's utility. More specifically, $\hat{s}_{2H}(\overline{\theta}) = u(w_{2H} + s_{2H}) - u(w_{2H})$. If, instead, the growers are risk neutral, then $\hat{s}_{2H}(\overline{\theta}) \equiv s_{2H}$.

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[20] In the previous section in the absence of reputation rewards, the possibility that the low-quality type mimics the high-quality type is excluded because the low-quality type would make loss if he does so. However, in the presence of reputation rewards, certain conditions, which will be elaborated later in the text, would be required to exclude this possibility.

Due to risk aversion, the same amount of monetary reward results in different utility measures for different grower type. Thus, for the low-quality type, the incentive compatibility constraint must satisfy

(3.77) $U_1(C_{1L} | \underline{\theta}) \geq U_1(C_{1H} | \underline{\theta}) + \delta \hat{s}_{2H}(\underline{\theta}) + \delta I_{2L}(1)$.

This constraint states that the low-quality type prefers revealing his true type than mimicking the high-quality type. The term $\hat{s}_{2H}(\underline{\theta})$ represents the equivalent measure of the monetary reward in the units of the low-quality type's utility, and $I_{2L}(1)$ denotes the loss the low-quality type would make if he mimics the high-quality type in the first period. However, there is a little relaxation of the notations here because the two terms on the right hand side, $\delta \hat{s}_{2H}(\underline{\theta})$ and $\delta I_{2L}(1)$ cannot add together directly due to risk aversion. For the moment, we use the current formulation because the condition (3.77) will be modified later.

Recall that in the previous section the low-quality type always chooses the contract that he prefers in the short run because he always makes loss if he deviates. From the condition (3.77), if $\delta \hat{s}_{2H} < -\delta I_{2L}(1)$, or $\hat{s}_{2H}$ is sufficiently small, then the low-quality type has no incentive to deviate in the first period. In other words, only when the extra reward is sufficient large would the low-quality type deviate. Therefore, for the moment, we assume $\delta \hat{s}_{2H} < -\delta I_{2L}(1)$. Thus, the incentive compatibility constraints (3.77) is equivalent to

(3.78) $U_1(C_{1L} | \underline{\theta}) \geq U_1(C_{1H} | \underline{\theta})$.

Hence, similarly to the previous section, ignoring the participation constraint for the high-quality type and the incentive condition (3.78), the Lagrangian for this problem is

(3.79)

$$L = r_1[\overline{\theta}e_{1H} - w_{1H} + \delta W_2(1) - \delta s_{2H}] + (1-r_1)[\underline{\theta}e_{1L} - w_{1L} + \delta W_2(0)]$$
$$+ \lambda[u(w_{1L}) - v(e_{1L})/\underline{\theta}] + \mu_H[u(w_{1H}) - v(e_{1H})/\overline{\theta} + \delta s_{2H}(\overline{\theta}) - u(w_{1L}) + v(e_{1L}\underline{\theta}/\overline{\theta})/\overline{\theta} - \delta I_{2H}(0)].$$

Denote the optimal contract as $C_1^R = \{C_{1L}^R, C_{1H}^R\}$, where $C_{1L}^R = (w_{1L}^R, e_{1L}^R)$ and $C_{1H}^R = (w_{1H}^R, e_{1H}^R)$. Thus, the first order conditions are:

(3.80) $\dfrac{\partial L}{\partial e_{1H}} = r_1\overline{\theta} - \mu_H v'(e_{1H})/\overline{\theta} = 0$,

(3.81) $\dfrac{\partial L}{\partial w_{1H}} = -r_1 + \mu_H u'(w_{1H}) = 0$,

(3.82) $\dfrac{\partial L}{\partial e_{1L}} = (1-r_1)\underline{\theta} - \lambda v'(e_{1L})/\underline{\theta} + \mu_H(\underline{\theta}/\overline{\theta}^2)v'(e_{1L}\underline{\theta}/\overline{\theta}) = 0$, and

(3.83) $\dfrac{\partial L}{\partial w_{1L}} = -(1-r_1) + \lambda u'(w_{1L}) - \mu_H u'(w_{1L}) = 0$.

Following the similar procedures in the previous section, the optimal contract can be solved as the following:

(3.84) $e_{1H} \in \arg\{v'(e_{1H}) = \overline{\theta}^2 u'(w_{1H})\}$,

(3.85) $e_{1L} \in \arg\{(1-r_1)\underline{\theta}[1 - \dfrac{v'(e_{1L})}{\underline{\theta}^2 u'(w_{1L})}] + \dfrac{r_1}{\underline{\theta}v'(e_{1H})}[\underline{\theta}^2 v'(e_{1L}\underline{\theta}/\overline{\theta}) - \overline{\theta}^2 v'(e_{1L})] = 0\}$,

(3.86) $u(w_{1L}) - v(e_{1L})/\underline{\theta} = 0$, and

(3.87) $u(w_{1H}) - v(e_{1H})/\overline{\theta} = v(e_{1L})/\underline{\theta} - v(e_{1L}\underline{\theta}/\overline{\theta})/\overline{\theta} + \delta I_{2H}(0) - \delta s_{2H}(\overline{\theta})$.

Note that the condition (3.85) implies that $\dfrac{v'(e_{1L})}{\underline{\theta}^2 u'(w_{1L})} < 1$ for $r_1 > 0$. In words, the optimal effort choice of the grower type $\underline{\theta}$ is less than that under perfect information.

Comparing the optimal contract C_1^R and the optimal contract $\hat{C}_1$ in the previous section, the optimal contract C_1^R simply requires that the processor takes a portion of the high-quality grower type's wage from the first period and promises to pay the grower in the second period if high quality is actually observed. However, due to risk aversion, the reputation reward also affects the optimal choices of efforts in the first period and hence the optimal contract.

(i) *Growers are risk neutral*

If the growers are risk neutral, i.e., $u(w) = w$, then $\hat{s}_{2H}(\overline{\theta}) = s_{2H}$. Thus, from condition (3.84), the change in w_{1H} does not affect the optimal choice of effort for the high-quality grower type. Hence, from condition (3.85), the optimal choice of effort for the low-quality grower type stays constant. In summary, under the assumption of risk neutrality, the optimal contract C_1^R only changes the high-quality type's payoff given the reputation reward without affecting the optimal contract for low-quality grower type and the processor's two-period expected profit. Note that to guarantee that this contract is indeed fully revealing, the reputation rewards must satisfy (3.75) and (3.77), or in words, the reputation rewards must be sufficiently small such that the high-quality type participates in the first period and the low-quality type has no incentive to deviate given the reputation reward.

(ii) *Growers are risk averse*

If growers are risk averse, given any positive reputation reward s_{2H} for observed high quality, decreases in the optimal wage w_{1H} requires that the optimal effort e_{1H} increases from the condition (3.84). Hence, from the conditions (3.85) and (3.86), both the optimal effort e_{1L} and the optimal wage w_{1L} for the low-quality type increase. This effect is illustrated in Figure 3.8.

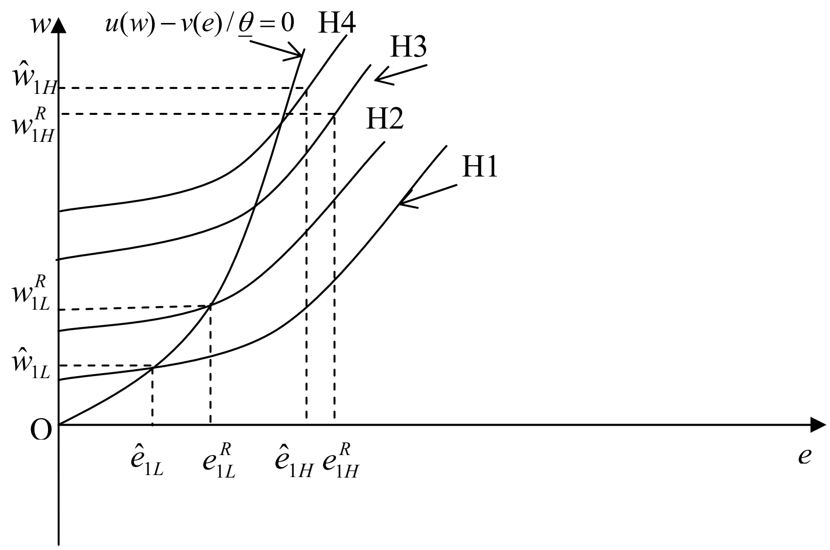

Figure 3.8 Illustration of the effects of the reputation reward

Note that the effect of increases in e_{1H} on the optimal contract C_{1L}^{R} is similar to that of decreases in r_1. Since the positive reputation reward reduces the optimal w_{1H} and raises the corresponding optimal effort e_{1H}, the optimal contract for the low-quality type, C_{1L}^{R}, must move upward along the low-quality type's zero-utility indifference curve as illustrated in Figure 3.8. The effects of the reputation rewards are summarized in the following proposition.

Proposition 2: There exists some reputation reward, s_{2H}^{*}, such that the separating equilibrium C_1^{R} would dominate the contract $\hat{C}_1$.

Proof: As discussed above, the introduction of the reputation reward for high quality reduces the optimal wage w_{1H} and raises the optimal effort e_{1H} for the high-quality grower type. Hence, the processor can obtain more profit from the high-quality grower type in the short run. Since the optimal contract C_{1L}^R moves upward along the low-quality indifference curve $u(w_{1L}) - v(e_{1L})/\underline{\theta} = 0$, the processor makes more profit from the low-quality grower as well. In addition, using the Envelope theorem, a small change in (w_{1L}, e_{1L}) that keeps the low-quality type's utility constant only has a second-order effect on the processor's profit, while a small change in (w_{1H}, e_{1H}) has a first-order effect on the processor's profit. In the second period, since the high-quality grower type adjusts his efforts according to the optimal contract C_{2H}^R, the processor obtains less profit in the second period. In fact, the processor's gain from the high-quality type in the first period completely offsets its loss in the second period because the processor simply takes δs_{2H} from the high-quality type and rewards him s_{2H} in the second period. Therefore, using the optimal revealing contract $\hat{C}_{1H}$ as a reference point, the processor is better off or at least as well off by offering the reputation reward in the two-period contract duration.

On the other hand, from the grower's perspective, the high-quality grower type also prefers the contract C_1^R to $\hat{C}_1$ for a sufficiently small s_{2H}. Again, using $\hat{C}_{1H}$ as a reference point, since $\hat{w}_{1H} > \overline{w}^*$ (recall that the second-period separating equilibrium offers the high-quality grower type the perfect information contract C_H^*), for a sufficiently small reputation reward s_{2H}, $u(\hat{w}_{1H}) - u(\hat{w}_{1H} - s_{2H}) < u(\overline{w}^* + s_{2H}) - u(\overline{w}^*)$. In words, the high-quality grower type would value the reward more in the second period than in the first period due to risk aversion. Thus, the high-quality grower type gains from the reputation reward, while the low-quality grower type is indifferent between the two contracts. Therefore, there exists some reputation rewards such that the separating equilibrium with direct reputation rewards contingent on observed performance, C_1^R, dominates the separating contract in the absence of the reputation rewards, $\hat{C}_1$.

Note that Proposition 2 applies only when both the separating equilibrium $\hat{C}_1$ and C_1^R are feasible, and these contracts are feasible only when δ is sufficiently small and the difference between $\overline{\theta}$ and $\underline{\theta}$ is sufficiently large. In addition, if r_1 is large, a "handicapped" separating equilibrium which only offers a contract to the high-quality grower type may become dominant. While not included in this essay, a similar statement to Proposition 2 could be made for the "handicapped" separating equilibrium.

Recall that the reputation reward must be sufficiently small such that the low-quality grower type has no incentive to deviate. If the reputation reward is large, not only would the high-quality grower type prefer to reveal his true type, but also the low-quality type would prefer to mimic the high-quality type. Thus, large reputation rewards would bring another set of equilibria. However, these potential cases are beyond the scope of this essay.

Remark 4: Effects of the direct reputational rewards would be more significant if the model is extended to a longer-term context. In addition, both the grower and the processor would benefit more from the direct reputation rewards as the contract duration increases.

Taking the fully separating equilibrium $\hat{C}_1$ as a reference point, recall that to induce a separating equilibrium, the optimal payment to the high-quality type in the first period must include the information rent he would obtain in the second period if he deviates in the first period. As the contract duration increases, the optimal payment to the high-quality type in the first period would become prohibitively large and the processor would be reluctant to pay the grower to have his true type revealed. In contrast, with the reputation rewards contingent on the grower's past performance, the potential large information rents in the first period under the contract $\hat{C}_1$ could be broken down and be distributed into the remaining contract periods. More precisely, as the number of contract periods approaches infinity, there would exist some reputation reward to the high-quality type such that the optimal first-period dynamic contract C_1^R would converge to the optimal static contract C^F if the processor promises to pay the reputation reward every period in which good performance is observed. In other words, if the processor promises to pay the reputation reward whenever good performance is observed and that promise is credible, the optimal incentive scheme in the static contract could result in a fully separating equilibrium in the dynamic context when the number of contract periods is large. Following similar arguments used for the two-period case, for a sufficiently small reputation reward to the high-quality type, both the processor and the grower would be better off with the direct reputation reward in the long run.

3.5 Conclusion and Discussion

This essay investigates the implications of growers' reputation when a processor designs a two-period dynamic contract with asymmetric information. The optimal strategies of the processor and the grower form a perfect Bayesian equilibrium. Under full commitment by both parties, growers' reputation has no effect on the optimal incentives. Hence, the optimal two-period contract mimics a sequence of optimal static contracts in the contract period. However, with no commitment by both parties, the optimal dynamic contract is rather complex. Since grower types are assumed unobservable to the processor, a potential ratchet effect would occur in a dynamic context that would prevent the grower from revealing his true type in the first period. In other words, the grower would tend to conceal his true type in the first period due to concerns that the processor would extract more of his surplus in the second period after his true type were revealed in the first period. Thus, to induce the grower to reveal his true type, the optimal contract must specify a payment for the first period such that it consists of information rents the grower could obtain in both periods. Moreover, the reputation effects embodied in the processor's posterior probability assessment about grower types reinforce the potential ratchet effect when the processor updates its beliefs of the grower's type based on the grower's past performance using Bayes' rule. More precisely, if the high-quality grower type conceals his type or randomizes in the first period, the processor would believe that it is less likely that the grower is a high-quality type. Consequently, the high-quality type obtains a greater payoff in the second period from deviating in the first period. In the limit, the processor believes the grower is a low-quality type and only offers a contract to the low-quality type under which the high-quality type realizes the maximum information rent.

Further, the optimal contract that could be sustained depends on growers' time preferences and differences between the two grower types. Proposition 1 establishes that a separating equilibrium could be sustained only if the discount factor is sufficiently small and the difference between the grower quality types is sufficiently large. In addition, a "handicapped" separating equilibrium would dominate the fully separating equilibrium when probability of high-quality growers is large. For a sufficiently large discount factor (the grower is patient) and sufficiently small difference between the grower types, it would become too costly for the processor to have the growers' private information revealed. Hence, a pooling equilibrium would dominate the separating equilibrium. Unfortunately, the exact nature of the relationship among the separating equilibrium, the pooling equilibrium, and the semi-separating equilibrium could be not explicitly determined without making further assumptions about functional forms of the grower's utility function and disutility function.

Based on the optimal dynamic contract with no commitment, the processor could offer a direct reputation reward to the grower in the second period if good performance (i.e., realized high quality) is observed at the end of the first period. Proposition 2 demonstrates that both the processor and the grower can gain from the direct reputation reward. Thus, the optimal dynamic contract with the reputation reward would dominate contracts without reputation rewards. Moreover, effects of the reputation reward would become more significant in the longer-term dynamic contract.

The results presented in the essay are in general consistent with the existing literature in dynamic contracts. However, several major differences exist: Firstly, past studies have found mixed results about existence of a separating equilibrium under different assumptions. For example, Hosios and Peters (1989) show that no fully separating equilibrium exists in a dynamic insurance contract with two types. Laffont and Tirole (1988) conclude similar results with continuous agent types. On the other hand, Freixas, Guesnerie, and Tirole (1985) derive optimal conditions for a separating equilibrium in a linear dynamic contract. In this essay, we not only derive optimal conditions for a separating equilibrium, a semi-separating equilibrium, and a pooling equilibrium, but also discuss the optimality of a "handicapped" separating equilibrium. Secondly, this essay introduces a direct reputation reward contingent on past performance that has not previously been analyzed in a dynamic principal-agent framework. The analysis presented in the text demonstrates that introduction of a direct reputation reward would provide more effective incentive schemes, and thus result in a dominant dynamic contract relative to those contracts without the reputation reward.

However, the analysis presented in this essay is far from exhaustive. Several generalizations of the model would be interesting for future research. First, the two-period model could be extended to allow for more than two periods. Extending the model to a longer term would make the effects of the reputation rewards more significant relative to the dynamic contract in the absence of reputation rewards. Second, uncertainties of realized quality or the production process could be incorporated into the model. Recall that we assume a deterministic production function for each grower type. However, introduction of uncertainties would significantly complicate the updating process of the processor's beliefs about grower types. Third, more complicated structures of reputation accumulation could be used in the model. We could expect that a different structure of the reputation rewards would have different impacts on the optimal dynamic contract.

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THESIS SUMMARY

This thesis studies three issues of agricultural contracting in various sectors. Specifically, the first essay deals with the relationship between contracting and the spot market in the hog sector due to hog producers' complaints about the potentially distorting effects of hog marketing contracts. The second essay investigates the efficiency of broiler contracts and compares performance of relative-performance contracts and fixed-performance contracts because of growers' concerns of unfairness of relative performance contracts. The third essay studies the effects of growers' reputation on the dynamics of optimal incentives with asymmetric information because of the importance of long-term contractual relationships in many agricultural settings. Each of the three structurally independent essays not only contributes to the general literature of agricultural contracting, but also provides a more thorough and more practical analysis on each specific topic. The following section briefly summarizes the major results of the three essays and discusses their contributions.

Essay 1 investigates the relationship between contracting and the spot market under five different types of contracts, including two fixed-price contracts, a market-price contract, a formula-price contract, and a cost-plus contract. In addition, asymmetric information concerning unobservable hog qualities is introduced into the model. This essay contributes to the existing literature by embedding a principal-agent model of processor-producer equilibrium behavior within a general equilibrium model of the hog market. In a related way, it also contributes by endogenizing the producers' participation constraint by linking the producers' contracting decision to the general-equilibrium-determined spot market price of hogs.

Different from the results in most past studies, this essay concludes that increased contract supplies raise the expected spot market price under the formula-price contract and reduce the variance of spot market price. Indeed, if differentiated quality is a feature of the hog market, and if the contract market is endogenously linked to the spot market, then existing empirical studies are likely to be biased. Under these assumptions, this essay finds that both the formula-price contract and the cost-plus contract offer a greater profit to processors and a greater expected utility to growers relative to the fixed-price contract or the market-price contact. Both processors and producers prefer the formula-price contract to the fixed-price contract or the market-price contact if asymmetric information about hog quality is taken into account. Finally, increases in processors' market power, simulated by raising the number of growers relative to the number of processors, depress the expected market price and growers' expected utility, but raise processors' expected profit. However, the relative superiority of each contract is the same regardless of processors' market power.

Essay 2 discusses efficiency of broiler contracts out of concerns of growers' dissatisfaction with the existing relative performance contracts (RPCs). The primary objectives of this essay are to investigate the efficiency of broiler-industry-style relative-performance contracts in the presence of asymmetric information and to compare various relative-performance contracts with fixed-performance contracts (FPCs). This essay, which incorporates with both moral hazard and adverse selection, contributes to the literature by comparing various relative-performance contracts with fixed-performance contracts in a dynamic setting and by analyzing improvements to a static mixed-type relative-performance contract.

In spite of growers' complaints about the contemporaneous relative-performance contracts used in the broiler industry, the various theoretical specifications in this essay largely justify the popularity and superiority of relative-performance contracts relative to fixed-performance contracts. Some of the major findings are highlighted as follows: First, efficiency of the static RPC or FPC and efficiency of the full-commitment RPC or FPC depend on the relative magnitude of common shocks and idiosyncratic shocks. More specifically, the static RPC or the full-commitment RPC performs better if the common shock is sufficiently large, while the static FPC or the full commitment FPC is better if the idiosyncratic shock dominates. This result is consistent with most previous studies. Second, the dynamic current-period RPC eliminates the contemporary common shocks regardless of the autocorrelation of common shocks in two periods and performs better than the dynamic FPC if the common shock is sufficiently large. Third, the dynamic FPC outperforms the dynamic previous-period RPC under the assumption that the dynamic previous-period RPC uses a fixed-performance contract in the first period. In addition, under the previous-period RPC, growers tend to exert less effort in the first period anticipating a higher standard in the second period, which is the well-known ratchet effect. Finally, this essay demonstrates that a static two-pooled-tournament RPC could improve both the processor's and the growers' welfare relative to the static single-tournament RPC.

Compared with existing literature, this essay provides a more thorough, more comprehensive, and more practical analysis of broiler contracts. Except for the dynamic previous-period RPC, comparisons between relative-performance contracts and fixed-performance contracts under each scenario justify the superiority of relative-performance contracts both in a static setting and in a dynamic setting when common shocks dominate idiosyncratic shocks.

Essay 3 investigates the role of growers' reputation when an agricultural processor designs optimal incentives for high quality products in a two-period dynamic contract. Due to unobservability of grower quality types and absence of commitment to intertemporal contract terms by both parties, reputation effects play a role in the dynamic contract. Thus, optimal incentives in such a contract must take into consideration not only the adverse consequences of hidden information in the short term, but also its intertemporal consequences in the longer term.

A two-period full-commitment contract, which requires that both parties be committed to the contract terms and the contract cannot be breached or renegotiated during the contracting period, is developed first as a baseline. Under full commitment by both parties, the optimal two-period contract mimics a sequence of optimal static contracts during the contract period. However, in a two-period dynamic model with no commitment, where neither the processor nor the grower could commit to an intertemporal scheme, three types of equilibria could potentially be sustained: a fully separating equilibrium, a semi-separating equilibrium, and a pooling equilibrium. In these cases, grower reputations are embodied in the posterior probability assessment of the grower's type by the processor at the end of the first period. Anticipating the processor's strategies, the high-quality grower type chooses to build up his reputation by either imitating the low-quality type or revealing his true type, whichever is favorable.

However, reputation effects reinforce the potential ratchet effect when the processor updates her beliefs of the grower's type based on the grower's past performance using Bayes' rule. More precisely, imitating the dominant behavior of a low-quality type yields future information rents to the high-quality type by sustaining the processor's belief that the grower might be of low-quality type. Thereby the reputation effects reflected in the posterior probability of grower types encourage deviation of the high-quality grower type. Further, the optimal contract that could be sustained depends on growers' time preferences and differences between the two grower types. In general, a separating equilibrium could be sustained only if the discount factor is sufficiently small and the difference between the grower quality types is sufficiently large. For a sufficiently large discount factor (i.e., when the grower is patient) and sufficiently small difference between the grower types, a pooling equilibrium would dominate the separating equilibrium or the semi-separating equilibrium.

Based on the optimal dynamic contract with no commitment, the processor offers a direct reputation reward to the grower in the second period if good performance is observed at the end of the first period. This essay demonstrates that the optimal dynamic contract with the reputation reward would dominate similar contracts without a reputation reward. Moreover, both the processor and the grower would increasingly benefit from the reputation reward the longer the contract duration is.

The results presented in this essay are in general consistent with the existing literature in dynamic contracts. However, several major differences exist. Firstly, past studies have found mixed results about existence of a separating equilibrium under various assumptions. In this essay, optimal conditions are derived for the following types of potential equilibria: a separating equilibrium, a semi-separating equilibrium, a pooling equilibrium. Secondly, this essay introduces a direct reputation reward contingent on past performance that has not been previously analyzed in a dynamic principal-agent framework. The analysis presented in the text demonstrates that introduction of a direct reputation reward would provide more effective incentive schemes, and thus, result in a dominant dynamic contract relative to that without the reputation reward.

As indicated, this thesis discusses three contracting issues in various agricultural sectors and provides a more thorough and more practical analysis on each topic. Since these essays are structurally independent, their policy implications are, on one hand, sector- or industry-specific. Specifically, in the first essay, the general equilibrium analysis of the relationship between contracting and spot market largely justifies the dominant use of formula-price contracts in the hog sector under certain conditions. Therefore, the distorting effects observed in the spot market, if they do exist, are likely from different sources such as hog processors' monopsonistic or oligopolistic pricing mechanisms rather than simply because of large contract supplies. In other words, if policy regulations could prevent hog processors as buyers from employing their market power, formula-price contracts could still benefit both processors and hog producers. In the second essay, comparisons of various forms of relative-performance and fixed-performance contracts demonstrate that relative-performance contracts perform better than fixed-performance contracts from the perspectives of both processors and growers if common productions shocks dominate idiosyncratic shocks. Thus, regardless of growers' complaints about relative-performance contracts, growers could be better off, on average, under relative-performance contracts relative to fixed-performance contracts.

The third essay demonstrates the practicality of a direct reputation reward in a long-term dynamic contract employed in various agricultural sectors such as processed vegetable and winegrape industries.

On the other hand, each of these essays provides a general methodology that could be applied to other sectors in which a similar context exists. For example, the analysis between contracting and the spot market could be applied as well to other sectors such as the cattle sector. The analysis of relative performance contracts that are widely employed in the labor market could also be applied to any sectors where a single principal contracts with multiple agents with relatively uniform products. Finally, the optimal dynamic contract with direct reputation rewards could be used in sectors in which processors and producers maintain long-term relationships.

Nevertheless, the analysis presented in this thesis is hardly exhaustive. Moreover, readers should realize that the various contracts analyzed in this thesis are still highly stylized and they only mimic the real world as closely as they could with the restriction of analytic tractability. Despite its limitations and simplifications, the analysis presented in this thesis sheds light on contemporary policy-making issues in various agricultural sectors and provides some theoretical guidelines for designing effective agricultural contracts.

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