RELIABLE CONTROL AND ONLINE MONITORING DESIGN FOR NUCLEAR POWER PLANT WITH IRIS DEMONSTRATION

A Thesis in
Nuclear Engineering

by

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Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

May 2010
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ABSTRACT

Development and deployment of small-scale nuclear power reactors and their autonomous control and monitoring is part of the mission under the Global Nuclear Energy Partnership (GNEP) program. The goals for this project are to investigate, develop, and validate advanced methods for sensing, controlling, monitoring, and diagnosing the proposed small and medium sized export reactors (SMR) and apply it to the International Reactor Innovative & Secure (IRIS).

In this thesis, an online monitoring and reliable control system of nuclear power plants is presented. First, a reliable LQG controller is obtained by solving ARE and LMI and by using output feedback directly. The simulation results show that the LQG controller can maintain the system stable when sensors drift in allowable range and perform well in tracking the reference signal. Then, based on the dissipative port-controlled Hamiltonian theory, a nonlinear observer-based feedback dissipation controller for the pressure control of the helical-coil steam generator and a high gain online nonlinear state observer are designed. The asymptotical stability of both the observer-based feedback dissipation controller and the high gain online nonlinear state observer is guaranteed by the dissipation theory. The estimated states given by the state observer converge to the real value, and the nonlinear state observer also guarantees the stability and acceptable performance. The limitation of the reliable controller, the observer-based feedback dissipation controller and the high gain online state observer are also discussed. Recommendations for improvement and future research are discussed.
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ACKNOWLEDGEMENTS

My research is supported by a U.S. Department of Energy NERI-C grant with Pennsylvania State University and the University of Tennessee, under grant DE-FG07-07ID14895.

I would like to thank Dr. Upadhyaya of University of Tennessee and Dr. Doster of North Carolina State University for providing the Simulink models and Fortran Code of IRIS nuclear power plant. I also would like to thank Mr. Xin Jin, another graduate student in our group, for discussing problems with me. Dr. Edwards, thank you for providing me direction in my academic career and guidance in completing this work.

Last but not least, I would like to express my sincerest gratitude to my family. Without their support, I would not have been able to complete this study in The Pennsylvania State University.
Chapter 1

Introduction

1.1 Background and Motivation

Development and deployment of small-scale nuclear power reactors and their autonomous control and monitoring is part of the mission under the Global Nuclear Energy Partnership (GNEP) program. The goals for this project are to investigate, develop, and validate advanced methods for sensing, controlling, monitoring, and diagnosing the proposed small and medium sized export reactors (SMR) and apply it to the International Reactor Innovative & Secure (IRIS) [1]. IRIS is a smaller/medium power nuclear reactor designed by an international consortium of companies, laboratories and universities which includes over 20 members from 10 countries [2]. Westinghouse Science and Technology Center is responsible in developing the IRIS plant and in charge of the coordination of the international team. IRIS is in pre-application licensing with the NRC and safety testing for Design Certification (DC) is expected to be completed by 2010 with deployment in the 2015-2017 timeframe.

1.2 Thesis Organization

In this thesis, Chapter Two is an introduction to the IRIS nuclear power plant, including the component description, the mathematical model and Simulink model. Chapter Three is a description of the design steps taken that led to the reliable LQG control with sensor failure. Chapter Four introduces the dissipation Hamiltonian theory. Then, based on this theory, an observer-based output feedback dissipation controller is designed to stabilize the outlet pressure
of the Helical-Coil Steam Generator (HCSG). Furthermore, applying the dissipation Hamiltonian theory again, a nonlinear high gain state observer is designed to estimate the state variables of the reactor. Chapter Six provides a conclusion and ideas for future research.
Chapter 2

Mathematical and Simulation Model for IRIS

2.1 Introduction to IRIS

Details of the IRIS design and supporting analyses have been previously reported and the reader is directed to the listed references [3][4][5]. The purpose of this chapter is to provide a review of the IRIS characteristics. IRIS is a pressurized water reactor that utilizes an integral reactor coolant system layout. The IRIS reactor vessel houses not only the nuclear fuel and control rods, but also all the major reactor coolant system components including pumps, steam generators, pressurizer, control rod drive mechanisms and neutron reflector. The IRIS integral vessel is larger than a traditional PWR pressure vessel, but the size of the IRIS containment is a fraction of the size of corresponding loop reactors, resulting in a significant reduction in the overall size of the reactor plant. IRIS has been primarily focused on achieving design with innovative safety characteristics. The first line of defense in IRIS is to eliminate event initiators that could potentially lead to core damage. In IRIS, this concept is implemented through the “safety-by-design” approach, which can be simply described as “design the plant in such a way as to eliminate accidents from occurring, rather than coping with their consequences.” If it is not possible to eliminate certain accidents altogether, then the design inherently reduces their consequences and/or decreases their probability of occurring. The key difference in the IRIS “safety-by-design” approach from previous practice is that the integral reactor design is conducive to eliminating accidents, to a degree impossible in conventional loop-type reactors. The elimination of the large LOCAs, since no large primary generations of the reactor vessel or large loop piping exist, is only the most easily visible of the safety potential characteristics of
integral reactors. Many others are possible, but they must be carefully exploited through a design process that is kept focused on selecting design characteristics that are most amenable to eliminate accident initiating events.

2.1.1 IRIS Reactor

The IRIS core and fuel assemblies are similar to those of a loop type Westinghouse PWR design. An IRIS fuel assembly consists of 264 fuel rods with a 0.374 in. o.d. in a 17×17 square array. The central position is reserved for in-core instrumentation, and 24 positions have guide thimbles for the control rod. Low-power density is achieved by employing a core configuration consisting of 89 fuel assemblies with a 14-ft (4.267 m) active fuel height, and a nominal thermal power of 1000 MWt. The resulting average linear power density is about 75% of the AP600 value. The improved thermal margin provides increased operational flexibility, while enabling longer fuel cycles and increased overall plant capacity factors. Reactivity control is accomplished through solid burnable absorbers, control rods, and the use of a limited amount of soluble boron in the reactor coolant. The reduced use of soluble boron makes the moderator temperature coefficient more negative, thus increasing inherent safety. The core is designed for a 3–3.5-year cycle with half-core reload to optimize the overall fuel economics while maximizing the discharge burnup. In addition, a 4-year straight burn fuel cycle can also be implemented to improve the overall plant availability, but at the expense of a somewhat reduced discharge burnup.

The IRIS reactor coolant pumps are of a “spool type”, which has been used in marine applications, and are being designed and will soon be supplied for chemical plant applications requiring high flow rates and low developed head. The motor and pump consist of two concentric cylinders, where the outer ring is the stationary stator and the inner ring is the rotor that carries
high specific speed pump impellers. The spool type pump is located entirely within the reactor vessel, with only small penetrations for the electrical power cables and for water cooling supply and return. Further, significant qualification work has been completed on the use of high temperature motor windings. This and continued work on the bearing materials has the potential to eliminate even the need for cooling water and the associated piping penetrations through the RV. This pump compares very favorably to the typical canned motor RCPs, which have the pump/impeller extending through a large opening in the pressure boundary with the motor outside the RV. Consequently, the canned pump motor casing becomes part of the pressure boundary and is typically flanged and seal welded to the mating RV pressure boundary surface. All of this is eliminated in IRIS. In addition to the above advantages derived from its integral location, the spool pump geometric configuration maximizes the rotating inertia and these pumps have a high run-out flow capability. Both these attributes mitigate the consequences of LOCAs (Loss of Coolant Accidents). Because of their low developed head, spool pumps have never been candidates for nuclear applications. However, the IRIS integral RV configuration and low primary coolant pressure drop can accommodate these pumps and together with the assembly design conditions can take full advantage of their unique characteristics. The structure of the IRIS reactor is shown in Figure 2-1.
2.1.2 Helical-Coil Steam Generator

The IRIS SGs are once-through, helical-coil tube bundle design with the primary fluid outside the tubes. Eight steam generator modules are located in the annular space between the core barrel (outside diameter 2.85 m) and the reactor vessel (inside diameter 6.21 m). Each IRIS SG module consists of a central inner column which supports the tubes, the lower feed water header and the upper steam header. The enveloping outer diameter of the tube bundle is 1.64 m. Each SG has 656 tubes, and the tubes and headers are designed for the full external RCS pressure. The tubes are connected to the vertical sides of the lower feedwater header and the upper steam header. The SG is supported from the RV wall and the headers are bolted to the vessel from the inside of the feed inlet and steam outlet pipes. Figure 2-2 illustrates the IRIS helical coil SG upper steam discharge header and the tube bundle arrangement. The helical-coil tube bundle design is capable of accommodating thermal expansion without excessive mechanical stress, and has high resistance to flow-induced vibrations.
2.2 Mathematical Model of IRIS Nuclear Power Plant

A dynamic process is generally modeled as a distributed parameter system characterized by a set of partial differential equations. It is usually rather complicated to solve such a time dependent system with spatial variations. For this reason, lumped models are used to describe the dynamic behavior of the two major components of the IRIS nuclear power plant: reactor core and HCSG. In the lumped model, each lump has the same averaged properties, so the spatial dependence can be represented simply by the interaction between adjacent lumps.
2.2.1 Mathematical Model of Reactor

Figure 2-3. Schematic of the nodalization for the reactor core.

Figure 2-3 shows the nodalization to model the reactor core. The dynamic properties of the fuel rod and coolant are described by \( n \) lumped cells. The neutron kinetics and heat transfer dynamic are given as follows.
2.2.1.1 Neutron Kinetics

The Point kinetics model with six delayed neutron groups is used to describe the neutron kinetics for the IRIS reactor core. The dynamic equations are given as follows:

\[
\begin{align*}
\frac{dn_r}{dt} &= \frac{\delta \rho_r - \beta}{\Lambda} n_r + \sum_{i=1}^{6} \frac{\beta_i}{\Lambda} c_{ri} + \frac{\alpha_f}{\Lambda} n_r (T_f - T_{f0}) + \frac{\alpha_c}{\Lambda} n_r (T_{cav} - T_{cav0}) \\
\frac{dc_{ri}}{dt} &= \lambda_i (n_r - c_{ri}) \quad (i = 1, 2, \ldots, 6)
\end{align*}
\]  

(2-1)

where

\[ n_r = \text{the neutron density relative to density at rated condition} \]

\[ c_{ri} = \text{the precursor density relative to the density of the } i_{th} \text{ group delayed fission neutrons at rated condition}. \]

\[ \beta = \text{the fraction of all the delayed fission neutrons}. \]

\[ \beta_i = \text{the fraction of the } i_{th} \text{ group delayed fission neutrons}. \]

\[ \lambda_i = \text{the effective precursor radioactive decay constant of the } i_{th} \text{ group delayed fission neutrons}. \]

\[ \alpha_f = \text{the temperature feedback coefficient of the average fuel temperature}. \]

\[ \alpha_c = \text{the temperature feedback coefficient of the average coolant temperature}. \]

\[ T_f = \text{the average reactor fuel temperature}. \]

\[ T_{f0} = \text{the equilibrium average temperature of fuel}. \]

\[ T_{cav} = \text{the average temperature of the coolant}. \]

\[ T_{cav0} = \text{the equilibrium average temperature of the coolant}. \]
2.2.1.2 Heat Transfer in the Reactor Core

Some assumptions are given to provide a simple model of each lumped cell in Figure 2-3: (1) the fuel element is homogeneous, so one-dimensional thermo-dynamic equations can be achieved. (2) the thermal power is produced by the fission products and radiation process such as beta-radiation and gamma-radiation, but here the heat produced by the radiation process is neglected and (3) the temperature profiles of the fuel elements and the coolant inside the reactor core are assumed to be smooth. Thus a lumped parameter model for the $i_{th}$ cell can be given as follows:

\[
\begin{align*}
\frac{dT_{f-i}}{dt} &= -\frac{\Omega}{\mu_f} T_{f-i} + \frac{\Omega}{\mu_f} T_{cav-i} + \frac{P_0}{\mu_f} n_r \\
\frac{dT_{cav-i}}{dt} &= -\frac{2M + \Omega}{\mu_c} T_{cav-i} + \frac{\Omega}{\mu_c} T_{f-i} + \frac{2M}{\mu_c} T_{cin}
\end{align*}
\]

where,

- $T_{f-i}$ = the average fuel temperature of the $i_{th}$ cell.
- $T_{cav-i}$ = the average coolant temperature of the $i_{th}$ cell.
- $\Omega$ = the heat transfer coefficient between the fuel and coolant.
- $\mu_f$ = the total heat capacity of the fuel of the $i_{th}$ cell.
- $\mu_c$ = the total heat capacity of the coolant of the $i_{th}$ cell.
- $M$ = the value of the coolant flowrate times heat capacity of the coolant in the $i_{th}$ cell.
- $P_0$ = the rated power distributed in the $i_{th}$ cell.
2.2.2 Mathematical Model of Helical-Coil Steam Generator

To build the lumped parameter model of HCSG, in addition to the assumptions implied in a lumped model, the other major assumptions are as follows:

1) Only one pressure is used to characterize the superheated region.
2) The superheated vapor satisfies ideal gas law modified by an expansion coefficient.
3) The temperature of the second node in the subcooled region is equal to the saturated temperature.
4) The pressure drop between superheated region and saturated region is constant during any perturbation
5) The pressure drop between the saturated region and the subcooled region is constant during any perturbation
6) The steam quality in the boiling region can be assumed as a linear function of the axial coordinate so the density in the boiling region can be approximated as a function of steam pressure.
7) The steam generation rate assumes to be equal to the boiling rate.
8) The heat transfer coefficient for the superheated region, the saturated region and the subcooled region is assumed constant.

2.2.2.1 Nodalization

Figure 2-4 shows the nodalization to model the dynamic behavior of HCSG system. Three regions, subcooled region, saturated region and superheated region, are used to characterize the significant difference of heat transfer and hydraulic behavior. In each region, two lumps with each volume are used to consider the axial temperature changes. Correspondingly, six metal
nodes are needed to describe the heat transfer from the primary side to the secondary side. For the two lumps of the secondary side in the saturated region, saturated temperature is maintained.

![Schematic of the nodalization for a helical steam generator.]

Figure 2-4. Schematic of the nodalization for a helical steam generator.

Tp1, Tp2, Tp3, Tp4, Tp5, Tp6 = Temperature of primary coolant

Tw1, Tw2, Tw3, Tw4, Tw5, Tw6 = Temperature of steam generator tubes

Ts1, Ts2, Tsat, Tsat, Tsc1, Tsc2 = Temperature of fluid on the steam generator tube side for superheated steam region, saturated region and subcooled region.

### 2.2.2.2 Primary Side Equation

The primary side temperature is given by:

\[
\frac{dT_{pi}}{dt} = a_1 \frac{T_{pi-1} - T_{pi}}{L_i} - a_2 (T_{wi} - T_{wi})
\]  

(2-3)

where

\[
L_i = \begin{cases} 
L_s & \text{for } i = 1,2 \\
L_b & \text{for } i = 3,4 \\
L_sc & \text{for } i = 5,6
\end{cases}
\]
\[ a_1 = \frac{C_{pp} W_p}{(A_{xs} C_p \rho)_p / 2} \]

\[ a_2 = \frac{h_{pm} P_{wp}}{(A_{xs} C_p \rho)_p / 2} \]

\( L_s \) = superheated length.

\( L_b \) = boiling length.

\( L_{sc} \) = subcooled length.

\( T \) = primary side temperature.

\( W_p \) = coolant flow rate.

\( C_p \) = specific heat.

\( \rho \) = density of the primary coolant.

\( A_{xs} \) = flow area.

\( h \) = heat transfer coefficient.

\( P_w \) = perimeter for heating.

In the above equations, subscript \( p \) and \( w \) refer to primary coolant and tube wall respectively.

### 2.2.2.3 Metal Equations

The metal temperature is given by:
\[
\frac{d}{dt} T_{wi} = a_3 (T_{pi} - T_{wi}) - a_{4i} (T_{wi} - T_{wi-1}) + (T_{wi} - T_{wi-1}) \frac{\dot{Z}_{i-1}}{L_i} + (T_{wi+1} - T_{wi}) \frac{\dot{Z}_i}{L_i} \\
\text{for } i = 1,2,3,4,5,6
\]

where

\[
a_3 = \frac{h_{pw} P_{wp}}{(\rho_w A_w c_{pw})}
\]

\[
a_{4i} = \begin{cases} 
\frac{h_{ws} P_{ws}}{(\rho_w A_w c_{pw})} & i = 1,2 \\
\frac{h_{wb} P_{wb}}{(\rho_w A_w c_{pw})} & i = 3,4 \\
\frac{h_{wb} P_{wb}}{(\rho_w A_w c_{pw})} & i = 5,6 
\end{cases}
\]

\[
\dot{Z}_0 = \dot{Z}_6 = 0 \\
\dot{Z}_1 = \dot{L}_s / 2 \\
\dot{Z}_2 = \dot{L}_s \\
\dot{Z}_3 = -(\dot{L}_{sc} + \dot{L}_b / 2) \\
\dot{Z}_4 = -\dot{L}_{sc} \\
\dot{Z}_5 = -\dot{L}_{sc} / 2
\]

where

\[
h_{ws}, h_{wb}, h_{wsc} = \text{heat transfer coefficient on the tube side for superheated steam region,}
\]

\[
saturate\text{d water region, and subcooled water region respectively}
\]

\[
P_{ws}, P_{wb}, P_{wsc} = \text{heating circumference on the tube side for superheated steam region,}
\]

\[
saturate\text{d water region, and subcooled water region respectively}
\]
2.2.2.4 Equations for the Superheated Region

The mass balance of the steam in the superheated steam nodes, node 1 and node 2, are given by:

\[
\dot{M}_{s1} = W_{21} - W_s \quad (2-5a)
\]

\[
\dot{M}_{s2} = W_b - W_{21} \quad (2-5b)
\]

where

\[M_s = \text{steam mass in the superheated region.}\]

\[W_s = \text{steam flow rate to turbine, which is an external constraint imposed by the controller.}\]

\[W_b = \text{steam production rate.}\]

The heat balance equations of the two superheated steam nodes, node 1 and node 2, are given by:

\[
\frac{d}{dt}(M_{s1}H_{s1} - P_s V_{s1}) = Q_{s1} + W_{21}H_{s2} - W_sH_{s1} - P_{s1}\dot{V}_{s1} \quad (2-6a)
\]

\[
\frac{d}{dt}(M_{s2}H_{s2} - P_s V_{s2}) = Q_{s2} + W_bH_g - W_{21}H_{s2} - P_{s2}\dot{V}_{s2} \quad (2-6b)
\]

where

\[M_s = \text{steam mass in the superheated region.}\]

\[P_s = \text{steam pressure in the superheated region.}\]

\[V_s = \text{steam volume in the superheated region.}\]
\( H_s \) = specific enthalpy of the steam.

\( Q_{s1}, Q_{s2} \) = heat transfer rate to the two superheated nodes.

\[ Q_{s1} = h_w s P_{ws} L_s (T_{w1} - T_{s1}) / 2 \]

\[ Q_{s2} = h_w s P_{ws} L_s (T_{w2} - T_{s2}) / 2 \]

Assuming the pressure loss in the superheated steam region is small, we have:

\[ P_s = P_{s1} = P_{s2} \]

Since specific enthalpy is a function of temperature and pressure, then we have:

\[
\dot{H}_s = \frac{\partial H_s}{\partial T_s} \dot{T}_s + \frac{\partial H_s}{\partial P_s} \dot{P}_s
\]  \hspace{1cm} (2-7)

and

\[
H_{s2} - H_{s1} = \frac{\partial H_s}{\partial T_s} (T_{s2} - T_{s1})
\]  \hspace{1cm} (2-8)

Combining with the mass balance equations and the expansion of the specific enthalpy, the energy balance equations can be rewritten as follows:

\[
M_{s1} C_{ps} \dot{T}_{s1} + C_{ps} (T_{s1} - T_{s2}) \dot{M}_{s1} + (M_{s1} \frac{\partial H_s}{\partial P_s} - V_{s1}) \dot{P}_s = Q_{s1} - W_s C_{ps} (T_{s1} - T_{s2})
\]  \hspace{1cm} (2-9)

\[
M_{s2} C_{ps} \dot{T}_{s2} + (M_{s2} \frac{\partial H_s}{\partial P_s} - V_{s2}) \dot{P}_s = Q_{s2} - W_s C_{ps} (T_{s2} - T_{sat})
\]  \hspace{1cm} (2-10)

The steam pressure in the superheated region can be described by compressibility adjusted ideal gas law, which is given by:
The time derivative of the steam pressure can then be determined by the following equation:

$$\dot{P}_s = \frac{Z_s R}{2M_{\text{stm}}} \left\{ M_s (T_{s1} + T_{s2}) + M_s (\dot{T}_{s1} + \dot{T}_{s2}) \right\} - P_s A_s \dot{L}_s \frac{1}{A_s L_s}$$

(2-12)

where

$$M_{\text{stm}} = \text{mole mass of steam.}$$

$$Z_s = \text{steam expansion coefficient.}$$

### 2.2.2.5 Equations for Boiling Region

The mass balance equation for the boiling region is given by:

$$\frac{d}{dt}(\rho_b A_s L_b) = W_{db} - W_b$$

(2-13)

If we notice

$$\frac{d\rho_b}{dt} = \frac{d\rho_b}{dP_{\text{sat}}} \frac{dP_{\text{sat}}}{dt}$$

then

$$A_s \rho_b \dot{L}_b + A_s L_b K_b \rho_{\text{sat}} = W_{db} - W_b$$

where

$$K_b = \frac{\partial \rho_b}{\partial P}$$
If we assume that the steam quality is a linear function of the axial position along the channel, then

\[
\bar{\rho}_b = \frac{\int_0^l \rho(x) \, dx}{\int_0^l dx} = \int_0^l v_f + xv_{fg} \, dx
\]

In the operation pressure range, we have:

\[
\bar{\rho}_b(P_s) = 1.61594 + 0.00552445P_s
\]

Therefore,

\[
\dot{L}_b = \{W_{db} - W_b - A_s L_b K_s \dot{P}_{sat}\} / (A_s \bar{\rho}_b)
\]

\[
W_b = h_{wb} U_{wb} \left( \frac{T_{wb3} + T_{wb4} - T_{sat}}{2} \right) L_b / h_{fg}
\]

\[
W_{db} = \text{flow rate leaving subcooled region to the saturated region.}
\]

\[
h_{fg} = \text{vaporization heat.}
\]

2.2.2.6 Equations for Subcooled Region

In analogy to the boiling region, the mass balance equation for the subcooled region can be given as follows:

\[
W_{db} = W_{fw} - \bar{\rho}_{sc} A_s \dot{L}_{sc} - A_s L_{sc} K_{sc} \dot{P}_{sc}
\]

where

\[
K_{sc} = \frac{\partial \bar{\rho}_{sc}}{\partial P_{sc}} = \frac{\partial}{\partial P_{sc}} (\rho_{fw} + \rho_f) / 2
\]

\[
W_{fw} = \text{feed water flow rate.}
\]
Heat balance equation for the subcooled region 1 is given by:

$$\frac{d(MC_pT)_{sc1}}{dt} - V_{sc1} \dot{P}_{sc} = h_{wsc} P_{wsc} L_{sc} (T_{w5} - T_{sc1}) + C_{psc} (W_{sc} T_{sc2} - W_{db} T_{sc1})$$  \hfill (2-14)

Since the outlet temperature of the first subcooled node can be approximated by the saturated temperature, then

$$\frac{A_s (\rho C_p)_{sc}}{2} [T_{sat} \dot{L}_{sc} + K_1 L_{sc} \dot{P}_{sat}] - \frac{A_s L_{sc}}{2} \dot{P}_{sc} = h_{wsc} P_{wsc} L_{sc} (T_{w5} - T_{sat}) / 2 + C_{psc} (W_{sc} T_{sc2} - W_{db} T_{sat})$$

where

$$K_1 = \frac{\partial T_{sat}}{\partial P}$$

Heat balance equation for the subcooled region 2:

$$\frac{A_s (\rho C_p)_{sc}}{2} [T_{sc2} \dot{L}_{sc} + L_{sc} \dot{P}_{sc2}] - \frac{A_s L_{sc}}{2} \dot{P}_{sc} = h_{wsc} U_{wsc} L_{sc} (T_{w6} - T_{sc2}) / 2 + C_{psc} (W_{fw} T_{fw} - W_{sc} T_{sc2})$$  \hfill (2-15)

After simplification, we obtain

$$\dot{L}_{sc2} = \left\{ 0.5 \times h_{wsc} P_{wsc} L_{sc} (T_{w6} - T_{sc2}) + C_{psc} (W_{fw} T_{fw} - W_{sc} T_{sc2}) + \frac{A_s L_{sc}}{2} \dot{P}_{sc} \right\} 2 / (A_s (\rho C_p)_{sc} - T_{sc2} \dot{L}_{sc}) \} / L_{sc}$$

If we assume $\dot{M}_{sc1} = \dot{M}_{sc2}$, then we have:

$$W_{sc} = (W_{fw} + W_{db}) / 2$$

Substituting the expression of $W_{sc}$ and $W_{db}$ into the heat balance equation for the subcooled region 1, we have:

$$\dot{L}_{sc} = \frac{1}{0.5(A_s (\rho C_p)_{sc} (T_{sc2} - T_{sat})) \times \left\{ 0.5 \times h_{wsc} P_{wsc} L_{sc} (T_{w5} - T_{sat}) + C_{psc} (W_{fw} T_{sc2} - W_{sc} T_{sat}) \right\} - \frac{A_s L_{sc}}{2} [K_{sc} C_{psc} (T_{sc2} - 2 T_{sat}) - 1] \dot{P}_{sc} + 0.5 A_s (\rho C_p)_{sc} K_1 L_{sc} \dot{P}_{sat}$$
Noticing the pressure relationship between $P_{sc}$, $P_{sat}$ and $P_s$, we have

$$P_{sc} = P_{sat} + \frac{1}{2}(\Delta P_{qph} + \Delta P_{spc})$$

$$P_{sat} = P_{ss} + \frac{1}{2}(\Delta P_{qph} + \Delta P_{spss})$$

where

$P_{sc}$ = pressure at the subcooled region.

$P_{sat}$ = pressure at the saturated region.

$\Delta P_{qph}$ = two phase pressure loss in the boiling region.

$\Delta P_{spc}$ = single phase pressure loss in the subcooled region.

$\Delta P_{spss}$ = single phase pressure loss in the superheated region.

### 2.2.2.7 Equations for the Pressure Controller

The secondary side pressure is maintained by regulating the steam flow rate. The steam flow rate satisfies the following equation:

$$\dot{W}_s = \frac{W_{s0}(1-C_u u) - W_s}{\tau_s}$$

(2.16)

where

$u$ = controller output.

$\tau$ = time constant.

$W_{s0}$ = initial steam flow rate on the secondary side.

$C_u$ = an adjustable parameter.

If a PI controller is used, the controller output has both the proportional part $u_1(t)$ and the integral part $u_2(t)$, which is given by:
\[ u_1(t) = k_1 \left( \frac{P_{bh}}{P_0} - \frac{P_{set}}{P_0} \right) \]

\[ \frac{du_2(t)}{dt} = k_2 \left( \frac{P_{bh}}{P_0} - \frac{P_{set}}{P_0} \right) \]

where

\[ k_1 = \text{the proportional gain.} \]

\[ k_2 = \text{the integral gain.} \]

\[ P_{bh} = \text{the turbine header pressure.} \]

\[ P_{set} = \text{the turbine header pressure set-point.} \]

\[ P_0 = \text{the nominal turbine header pressure.} \]

### 2.2.3 Mathematical Model of Steam Turbine

Steam is the medium which connects steam generator and steam turbine. After generated in the steam generators, passing through the steam line which connects the steam generators with steam turbine, steam enters the cylinder of the steam turbine. The steam pressure and flow rate into the steam turbine is adjusted by governing the high pressure control valve and high pressure bypass valve. In the steam turbine, the internal energy of the steam converts into rotary kinetic energy of the turbine rotor. Furthermore, the turbine rotor, as the prime mover, drives the rotor of the generator, converts the rotary kinetic energy into electricity energy then electricity energy is transported into the grid.

As mentioned above, the steam turbine is an important and complicated component of a plant unit. For different research purpose, a specific mathematical model with different complexity should be built. For example, if the steam expansion process, work process or changes of steam state in a steam turbine is the focus of study, then a number of specific
properties of steam turbine should be taken into account, such as type of steam turbine, and etc. Furthermore, the blade length and nozzle type determine how steam expands in a stage of the turbine. Due to lack of steam turbine specification, and because the purpose of this research is to study the responses of steam turbine in the various power conditions, a normalized and simplified model is designed. The turbine is modeled as follows:

First calculate the pressure loss coefficient due to the throttling of control valve

$$K_{TCV} = \frac{K}{T_{auV}^2}$$

$$T_{auV} = A_{TCV} (1 - b(1 - A_{TCV}))$$

where $K$ and $b$ are constants which are determined by the property of the control valve. $K_{TCV}$ is the pressure loss coefficient. $A_{TCV}$ is the valve position.

Then calculate the pressure loss:

$$P_{hdr} = P_{sg} - K_{TCV} \frac{W_s^2}{c \rho A^2}$$

where $P_{hdr}$ is the pressure of inlet steam of steam turbine. $P_{sg}$ is the pressure of outlet steam of steam generator. $W_s$ is the steam flow rate. $\rho$ is the density of steam. $A$ is the area of the pipe. $c$ is a constant.

The enthalpy drop is used to calculate the mechanical power of the steam turbine. Using the empirical formula, the enthalpy of the inlet steam of steam turbine and outlet steam of the steam turbine are calculated. Then, the mechanical power of steam turbine is obtained by the equation below:

$$W_{turb} = \frac{W_s \epsilon}{\kappa} (h_{in} - h_{out})$$
where $W_{turb}$ is the mechanical power of steam turbine. $h_{in}$ and $h_{out}$ are the enthalpy of inlet steam and enthalpy of outlet steam, respectively. $\varepsilon$ and $\kappa$ are constants.

2.3 Simulation Model of IRIS Nuclear Power Plant

The mathematical model described in Section 2.2 should be realized in a computer to do dynamic process simulation and controller design. The MATLAB/SIMULINK platform is chosen. MATLAB/SIMULINK® is an environment for multi-domain simulation and Model-Based Design for dynamic and embedded systems. It provides an interactive graphical environment and a customizable set of block libraries that let you design, simulate, implement, and test a variety of time-varying systems, including communications, controls, signal processing, video processing, and image processing [53].

The top layer of the SIMULINK model of IRIS is shown in figure 2-5.

![SIMULINK model of IRIS system.](image)
Chapter 3

Reliable LQG Controller Design with Sensor

3.1 Introduction to Reliable Control

Performance, reliability and safety of nuclear power plants depend upon valid and accurate sensor signals that measure plant condition for information display, health monitoring and control [6]. Measurable state variables and output variables such as pressure, power and temperature which are inputs of a controller are measured by different kinds of sensors. Because of rigorous surrounding environment in nuclear reactor core and the inherent deficiency, aging, sensors used in nuclear power plants can suffer from large calibration shifts, erratic and noisy output, response-time degradation and saturated output [7]. Due to the high requirement for safety, redundant sensors are installed in nuclear power plant to make sure reliable monitoring and stability in emergency condition. Spatially averaged time-dependent estimates are used for some parameters and furthermore fault detection and isolation (FDI) mechanism will delete some measurements given by sensors whose measurements are over the allowable bounds. However, if the sensors just drift in the allowable range or the FDI mechanism fails due to some other reasons, the control system should guarantee the stability of system. The reliable control is an effective approach to improve system reliability. A reliable controller is a controller with suitable structure to guarantee stability and satisfactory performance, not only when all control components are in operation, but also in the case when sensors, and actuators malfunction [8].

Reliable control for linear and nonlinear systems has been investigated by many researchers during the past two decades and corresponding calculation algorithms have been developed [9-15]. R. J. Veillette [9] presents a methodology for the design of modified linear-
quadratic (LQ) state feedback controls that can tolerate actuator outages, and presents the sufficient and necessary condition for the existence of such controller. Some LMI approaches are presented to design reliable controller. Li [10] presents an LMI algorithm to design reliable guaranteed cost controller for discrete-time systems with actuator failure. D. Zhang [11] et al. consider a more general condition, investigating the problem of reliable guaranteed cost control with multiple criteria constraints for a class of uncertain discrete-time systems subject to actuator faults. Reliable control for a nonlinear system has also been proposed. A Hamilton-Jacobi inequality (HJI) approach is employed by [12][13]. Yang et al. [14] present a reliable H control design for a nonlinear system with sensor and/or actuator faults. Liang et al. [13] study the reliable linear quadratic state feedback control problem for nonlinear systems with actuator faults which extend Veillette’s work on [9]. To avoid solving HJI, Huai-Ning Wu [8] proposes a reliable LQ fuzzy control design which can achieve the LQ performance for continuous-time nonlinear systems with actuator faults.

In this chapter, a methodology of reliable LQG controller design with direct output feedback of nuclear power plant model using the Simulink IRIS (International Reactor Innovative and Secure) model developed by Penn State University is presented. IRIS is a modular pressurized water reactor with an integral configuration. Compared with state feedback LQG controller, the reliable LQG controller proposed by G.H. Yang et al. [14] uses output feedback directly. The high order state estimator for a complex nonlinear system is not needed. This is an advantage of the reliable LQG controller. Because for a complex nonlinear system just like IRIS, constructing a state estimator may bring estimation error to the closed loop system and will enhance the complexity of system.
3.2 System Synthesis

3.2.1 Model Linearization

For reliable LQG control synthesis, a linear model with low order is desired to design a low order controller. In this paper, a linear model is first attained by using the control system toolbox of MATLAB, and then the minimal realization of the linear model is attained by removing uncontrollable and/or unobservable states. The minimal realized linear model is used to design the controller. Here, the order of the linear system is reduced from 28 to 22 with allowable $H_{\infty}$ error.

Sensor failure model which covers the cases of partial degradation and outage is considered in reliable LQG controller design. By solving algebraic Riccati equation and linear matrix inequality, reliable LQG control method yields a state space model controller which guarantees the whole system is stable when sensors suffer from shifts, erratic and noisy output, and with satisfactory response performance. Figure 3-1 shows the structure of the closed loop system.

Figure 3-1. Close loop system sketch.
3.2.2 Controller Design

The mathematical description of the IRIS system using linear state space model is presented as follows:

Linear model $\Sigma$:

$$
\Sigma: \dot{x}(t) = Ax(t) + Bu(t) + D_1 \omega(t) \\
y(t) = Cx(t) + D_2 \nu(t)
$$

(3-1)

where $x(t) \in \mathbb{R}^n$ is the state; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) \in \mathbb{R}^p$ is the measured output; $\omega(t)$ and $\nu(t)$ are zero mean white Gaussian noise processes with identical covariance matrix.

Furthermore, assume $\omega(t)$ and $\nu(t)$ is non-correlated, then

$$
E[\omega(t)^T \omega(t)] = V_1 \geq 0 \\
E[\nu(t)^T \nu(t)] = V_2 \geq 0 \\
E[\omega(t)^T \nu(t)] = 0
$$

(3-2)

Associated with this system is the cost function

$$
J = \lim E \left\{ \frac{1}{T} \int_0^T [x^T(t)R_1 x(t) + u^T(t)R_2 u(t)] \, dt \right\}
$$

(3-3)

where $R_1 \geq 0$, $R_2 > 0$, and $E\{\}$ denotes the expectation.

Reliable LQG controller $C$:

$$
C: \dot{\xi}(t) = F \xi(t) + G y^F(t) \\
u(t) = K \xi(t)
$$

(3-4)

where $\xi(t) \in \mathbb{R}^n$ is the controller state,

$$
y^F = diag(a_1, a_2, \ldots, a_m) y = ay \\
a_j \leq a_i \leq \bar{a}_i
$$

(3-5)
Equation 3-5 models sensor failures. When \( a_i = 1 \), the corresponding sensor works normally, when \( a_i \neq 1 \), the corresponding sensor does not work accurately. An allowable bound \( a_i, \bar{a}_i \) should be specified by designer. Applying the controller to system \( \Sigma \) described by Equation 3-1, the closed loop system \( CL \) is described by the following equation:

\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{\xi}(t)
\end{bmatrix} = \begin{bmatrix}
A & BK \\
GaC & F
\end{bmatrix} \begin{bmatrix}
x(t) \\
\xi(t)
\end{bmatrix} + \begin{bmatrix}
D_1 \\
GaD_2
\end{bmatrix} \omega(t)
\]

(3-6)

G.H. Yang et al. [9] have proposed the following theorem to solve the reliable LQG controller and the closed-loop system is stable when \( a \) is in the specified allowable bound.

Theorem: Consider the system \( \Sigma \) described by Equation 3-1. Suppose that

1. There exists a diagonal matrix \( R_0 > 0 \) such that the inequality \( R_0 \beta < V_2^{-1} \) holds.
2. The following Riccati equation has a stabilizing solution:

\[
A Q + Q A^T + V_1 - Q C^T [(I - \beta^2) V_2 + R_0^{-1} \beta^{-1}]^{-1} (I - \beta^3) C Q = 0
\]

(3-7)

3. There exist symmetric matrices \( X_1 > 0 \), \( X_0 > 0 \) and \( W \) such that the following inequalities hold:

\[
\begin{bmatrix}
A_0 X_1 + X_1 A_0^T + BW + W^T B^T + B_{10}B_{10}^T X_1 E_{1w}^T \\
E_{1w} X_1 \\
- I
\end{bmatrix} < 0
\]

(3-8)

where

\[
A_0 = A + Q E_{1w}^T E_{1w}, \quad B_{10} = \tilde{Q} C (V_0 + \beta R_0^{-1})^{-\frac{1}{2}}
\]
Then the corresponding reliable LQG controller is given by:

$$F = A_0 - G a_0 (I - V_2 \beta R_0)^{-1} C + BK$$

$$K = W X_1^{-1}$$

$$G = Q C^T [(I - \beta^2) V_2 + R_0^{-1} \beta]^{-1} a_0$$

With

$$a_0 = diag(a_{01}, a_{02}, \cdots, a_{0p})$$

The corresponding algorithm for the design given by this theorem is given as follows:

Step 1: Initialize $R_0$ satisfying $R_0 \beta < V_2^{-1}$ and solve Equation 7 for $Q$.

Step 2: Minimize $tr(X_0)$ subject to LMI constraints.

Step 3: If step 2 is not feasible, then change $R_0$ and go to step 1.

This algorithm can be performed effectively by using MATLAB.

The controller obtained by this algorithm is with the same order as the plant. To make the controller easy to realize, a reduced-order controller is desired. The same method which is used to reduce the plant model is used to reduce the order of controller.
3.3 Simulation

According to the theorem given above, a reliable LQG controller which can be tolerant of ±50% sensors drift is designed. Applying the reliable LQG controller to the nonlinear system, the following running condition is simulated:

Case I: at t=500s, the nuclear power reference signal steps from 100% nominal value to 90%, at t=800s, nuclear power reference signal steps from 90% to 100%, at t=1000s, nuclear power sensor drifts 20% with slope of 1%/sec, at t=1500s, the sensor recovers from drift. Figure 3-2 shows the dynamic response of the closed loop system and Figure 3-3 shows the two control effects: steam generator secondary side feedwater flow rate and control rod reactivity. In the normal case, the controller can make the output of the nonlinear system follow the reference signal with the steady state error less than 0.5%. The response time of nuclear power is 1sec. When nuclear power sensors cannot provide accurate signals, the reliable LQG controller can still maintain output-steam pressure reference signal with steady state error less than 1% and maintain the system stable.

Figure 3-2. Dynamic response of case I.
Figure 3-3. Control effects of case I.

Case II: at $t=500s$, steam pressure reference signal step from 100\% nominal value to 90\%, at $t=1000s$, steam pressure sensor drifts 20\% with slope of -1\%/sec, at $t=1500s$, the sensor recovers from the drift. Figure 3-4 shows the dynamic response of the closed-loop system and Figure 3-5 shows the two control effects. Just like case I, in normal operation, the controller can make the output of the nonlinear system follow the reference signal with the steady state error less than 0.5\%. The response time of steam pressure is 50 sec. When the steam pressure sensor cannot provide an accurate signal, the reliable LQG controller can still maintain output-nuclear power reference signal with steady state error less than 1\% and maintain the system stable.
3.4 Summary and discussion

A reliable LQG controller is obtained by solving ARE and LMI. Then, the order of the controller is reduced to 9 by using Hankel norm approximation method. From the simulation
result, the LQG controller can maintain the system stable when sensors drift in allowable range
and perform well in tracking the reference signal. However, compared with robust control, for
example, $H_{\infty}$ robust control, the reliable LQG does not perform well in the noise rejection. So, the
future work will focus on enhancing the ability to reject noise of reliable LQG control.
Chapter 4

Nonlinear-Observed Feedback Dissipation Controller Design for HCSG Pressure Control

4.1 Introduction

The Steam Generators used in commercial pressurized water reactor power plants are U-tube steam generators (UTSG) which are different than helical-coil steam generators (HCSG) used in the IRIS system. For UTSG, water level control is one of the main control issues. To guarantee the sufficient cooling of the nuclear reactor, and to avoid the damage of steam turbine blades, the water level of steam generator should be maintained in a proper region, which is important for the operational safety of the whole nuclear power plant system. Water level control for UTSG is not easy to deal with. Several factors which lead to the difficulties in designing an effective water level control system for UTSG were summarized by Kothare [23]. These factors include: 1). the nonlinear plant characteristic arisen by the highly nonlinear plant dynamics; 2). the nonminimum-phase plant characteristics arisen by the so called “swell and shrink” effects [23]; 3). the unreliability of sensor measurement at low powers; 4). the constraints of the feedwater flow rate arisen by the feedwater system. Meanwhile, the temperature and flow rate of both primary loop coolant and secondary loop feedwater, the main steam flow rate have direct effect on the water level. The main steam flow rate depends on the electrical load of the generator while the temperature and flow rate of primary loop coolant depends on the operation power of the nuclear reactor. So, the water level controller of the nuclear steam generator has feedwater flow rate and temperature, coolant flow rate and temperature, and main steam flow rate as input. It has the level control signal of the steam generator as output. The feedwater flow rate should be
regulated to stabilize the proper water level whenever there is a load change or operational power change. Although the control algorithms of the steam generator level controller now still use a PI (Proportional and Integral) and application of new control algorithms is restricted because of the safety consideration, many references present many different controllers designed by using different methods and validated by computer simulation. A PID water level controller tuned by the model prediction control (MPC) approach is proposed by Na [24]. Due to the rapid development of artificial intelligence and high performance computers, some intelligent controllers are applied for the steam generator water level control. Cho [25] proposed a stability-guaranteed fuzzy logic controller for nuclear steam generators. Na presented another fuzzy logic water level controller tuned by genetic algorithm. Neural network also is applied to water level control of UTSG. Munasinghe [26] designed an adaptive neurofuzzy controller to regulate UTSG water level.

In recent years, port-controlled Hamiltonian (PCH) system proposed by Maschke and Van Der Schaft has been investigated by many researchers and many achievements are obtained. PCH is an effective method to design a controller with dissipation which guarantees the stability of the system. Indeed, the PCH system has clear physical meaning and is widely used in the engineering field. The Hamiltonian function of the PCH system is considered as the total energy and can serve as a Lyapunov function. The PCH methodology has been used in many engineering systems such as mechanical systems and power systems and so on. Many references investigate the generalized Hamiltonian realization (GHR) of a single machine and multi-machine power system and disturbance attenuation control. Liu [27] investigated how to express multi-machine multi-load power systems in a dissipative Hamiltonian realization form. Wang [28] proposed an energy-based L2-disturbance attenuation control for multi-machine power systems which are expressed in dissipative Hamiltonian realization. In some cases, not all state variables of the systems are measurable, Leon-Morales [29] proposed an observer-based controller using
Hamiltonian approach to control a synchronous generator. Generally, applying the Hamiltonian function method to express the system concerned into a Hamiltonian system with dissipation which is called dissipative Hamiltonian realization involves two steps. The first step is to express the system into a generalized Hamiltonian system, i.e., obtain the GHR. The second step is to eliminate the non-dissipative part of GHR by state feedback to get a Hamiltonian system with dissipation. However, to express a system as a GHR, the output and the Hamiltonian function should meet some matching conditions and furthermore it is not guaranteed that all of the state variables are measurable, these two conditions make the controller design using Hamiltonian function method difficult. To solve this problem, Dong [30][33] proposed a novel method called observer-based feedback dissipation (OBFD) and then Dong used this OBFD theorem to design a power controller for a low temperature nuclear reactor. OBFD is an effective method which can express complicated nonlinear systems in the form of GHR. Another advantage of OBFD is that controller design using OBFD method does not depend on the parameters of state functions closely except the input matrix and output matrix which makes the design easier.

In this chapter, based on OBFD method, the pressure control of HCSG is considered. HCSG which is used in the IRIS nuclear power plant is different than UTSG greatly in principle and structure. Feedwater undergoes three different state regions: subcooled, boiling and superheated after going into the helical-coil tube. There is not a “water level” with clear physical concept in HCSG. In other words, “water level” in HCSG is not measurable. Here “water level” in HCSG is defined as the summation of the length of subcooled region and boiling region. The control objective for the HCSG is to maintain the pressure of outlet steam to be a constant, while maintaining the length of the three different state regions within acceptable boundaries.

OBFD is a model-based method. So in this chapter, a simple nonlinear state space model for HCSG of IRIS is derived first according to basic energy balance and mass balance. Then
using OBFD method, a single input single output (SISO) pressure controller for HCSG is designed. Simulation results show that the control performance is high.

### 4.2 Nonlinear State Space Model of HCSG

The steam turbine is an important and complicated component of a plant unit. For different purpose, a specific mathematical model with different complexity should be built. Two dynamic models are available for IRIS power plant systems which are developed by the North Carolina State University [35] and the University of Tennessee [35]. Generally, these two models consider a lot of detailed characteristics and consist of partial differential equations and ordinary differential equations in which many state variables couple with each other. Usually, iterative algorithms or finite difference methods are employed to solve the coupled differential equations. The degree of the two models is higher than 20. All of these lead the two existing models to have high fidelity and are suitable for the test and evaluation of a controller but not suitable for model-based controller design.

Observer-based feedback dissipation (OBFD) is a model-based controller design method. A simplified minimal dynamic state space model is needed. In this Chapter, based on the high order mathematical model of HCSG[35] developed by University of Tennessee and a mathematical model of UTSG[36] which is specified for nonlinear $H_{\infty}$ controller design, a simplified low order lumped parameter model is developed and used to describe the HCSG dynamic behavior. Each lump has the same averaged properties. This model is based on basic mass and energy conservation and described by ordinary differential equations which are only suitable for control system design. Because feedwater suffers three different states inside the tubes of the secondary side of the HCSG which are subcooled water first, then two-phase boiling steam and water mixture and finally superheated steam, correspondingly, three lumps are used to
characterize the significant difference of heat transfer and hydraulic behavior. Figure 4-1 shows the specific nodalization. In addition to the assumptions implied in a lumped model, the following major assumptions are made to simplify the formulation of the model.

Figure 4-1. Lump model sketch of HCSG.

(1). The superheated vapor satisfies ideal gas equation modified by an expansion coefficient.

(2). The pressure drop between superheated region and boiling region is constant during any perturbation.

(3). The pressure drop between the boiling region and subcooled region is constant during any perturbation.

(4). The two-phase flow inside the boiling region is homogenous. The flow quality of the homogenous two-phase flow is a linear function of the axial coordinate along the tube until the end of the boiling region.

(5). The steam generation rate is assumed to be equal to the boiling rate.

(6). The heat transfer coefficient for the superheated region, the boiling region and the subcooled region is constant.
Remark:

The real flow condition in the helical tube is complicated, especially in the two-phase region. To reduce the order of the lumped state space model, Homogenous two-phase flow model is considered here. Homogenous two-phase flow model is the simplest two-phase flow model which makes it suitable for controller design. The linear steam quality with respect to the axial coordinate along the tube until the end of the boiling region can be derived based on homogeneous two-phase flow model.

Here, the nomenclature used to describe this nonlinear state space model is given in Appendix A

4.2.1 Subcooled Region

Assume that the density of water inside the subcooled region is a constant and the flow area $A_{sc}$ is a constant. The mass balance equation for the subcooled region is

$$\frac{dM_{sc}}{dt} = W_{fwsc} - W_{sc} \quad (4-1)$$

$$M_{sc} = \rho_{sc} A_{sc} L_{sc} \quad (4-2)$$

The energy balance equation for the subcooled region is

$$\frac{d(h_{sc} M_{sc})}{dt} = h_{fw} W_{fw} - h_{sc} W_{sc} + Q_{sc} \quad (4-3)$$

Note that $H_{sc} = h_{sc} M_{sc}$, the energy balance equation can be rewritten as:

$$\frac{dH_{sc}}{dt} = - \frac{H_{sc}}{M_{sc}} W_{sc} + Q_{sc} + h_{fw} W_{fw} \quad (4-4)$$

Substitute Eq. (4-2) into Eq. (4-1), the mass balance equation can be rewritten as:
\[
\frac{dL_{sc}}{dt} = \frac{W_{fw} - W_{sc}}{\rho_{sc} A_{sc}} \tag{4-5}
\]

4.2.2 Boiling Region

The mass balance equation for the boiling region is

\[
\frac{dM_b}{dt} = W_{sc} - W_{g} \tag{4-6}
\]

\[
M_b = \bar{\rho}_b A_b L_b \tag{4-7}
\]

According to the assumption (4), the flow quality \(x\) of the homogenous two-phase flow in the boiling region is a linear function of the length \(L_b\) along the boiling region. At the beginning of boiling region, \(x=0\), at the end of the boiling region, \(x=1\). This assumption yields an average flow quality \(\langle x \rangle\) of the boiling region as

\[
\langle x \rangle = 0.5 \tag{4-8}
\]

The energy balance equation for the boiling region is

\[
\frac{d(h_b M_b)}{dt} = h_{sc} W_{sc} - h_g W_{g} + Q_b \tag{4-9}
\]

\[
h_b = h_f + 0.5h_{fg} \tag{4-10}
\]

\[
h_g = h_f + h_{fg} \tag{4-11}
\]

Noticed that

\[
\frac{d\bar{\rho}_b}{dt} = \frac{d\bar{\rho}_b}{dP_b} \frac{dP_b}{dt} \tag{4-12}
\]

Substitute Eq. 4-7 and 4-12 into Eq. 4-6, the mass balance can be rewritten as:
\[
\frac{dL_b}{dt} = \frac{W_{sc} - W_g - A_b L_b K_b \frac{dP_b}{dt}}{A_b \overline{P}_b}
\]

Where

\[
K_b = \frac{\partial \overline{P}_b}{\partial P_b}
\]

In the operation pressure range, we have

\[
\overline{P}_b(P_b) = 1.61594 + 0.00552445 P_b
\]

According to Eq. 4-10, \( h_b \) is a constant. Rewrite the Eq. 4-9 as

\[
\frac{dM_b}{dt} = \frac{h_{sc} W_{sc} - h_g W_g + Q_b}{h_b}
\]

### 5.2.3 Superheated Region

The mass balance equation for the superheat region is

\[
\frac{dM_s}{dt} = W_g - W_s
\]

\[
M_s = \rho_s A_s L_s
\]

The energy balance equation for the superheat region is

\[
\frac{d(M_s h_s - P_s V_s)}{dt} = Q_s + W_g h_g - W_s h_s - P_s \dot{V}_s
\]

Since specific enthalpy is a function of temperature and pressure, then

\[
\frac{dh_s}{dt} = \frac{\partial h_s}{\partial T_s} \dot{T}_s + \frac{\partial h_s}{\partial P_s} \dot{P}_s
\]

Combine Eq. 4-17, 4-18, 4-19 and 4-20 together, the energy balance equation can be written as:
\[
M \frac{\partial h_s}{\partial T_s} \dot{T}_s + \left( M \frac{\partial h_s}{\partial P_s} - V_s \right) \dot{P}_s = Q_s + W_b \left( h_s - h_g \right)
\]  

(4-21)

The steam pressure in the superheated region can be described by the compressibility adjusted ideal gas law, which is given by:

\[
P_{sV} = \frac{Z_s^* R M_s T_s}{M}
\]  

(4-22)

Rewrite Eq. 4-22 as:

\[
P_s = \frac{Z_s^* R M_s T_s}{M L_s A_s}
\]  

(4-23)

Differentiate Eq. 4-23 on both sides:

\[
\dot{P}_s = \frac{Z_s^* R}{M L_s A_s} \left( \dot{M}_s T_s + M_s \dot{T}_s - P_s A_s \dot{L}_s \right)
\]  

(4-24)

At steady state, \( W_g \approx W_s \), simplify Eq. 4-24 and Eq. 4-13

\[
\dot{P}_s = \frac{Z_s^* R}{M L_s A_s} \left( M_s \dot{T}_s - P_s A_s \dot{L}_s \right)
\]  

(4-25)

\[
\frac{dL_b}{dt} = \frac{A_b L_b K_b dP_b}{\dot{\rho}_b}
\]  

(4-26)

Because

\[
L_{total} = L_s + L_b + L_{sc}
\]  

(4-27)

Substitute Eq. 4-5 and Eq. 4-13 into 5-28 to arrive at Eq. 4-29

\[
\dot{L}_s = -\dot{L}_b - \dot{L}_{sc}
\]  

(4-28)

Substitute Eq. 4-5 and 4-13 into 4-28
\[\dot{L}_s = \left\{ \frac{W_{fw} - W_{sc}}{\rho_{sc} A_{sc}} - \frac{A_h L_h K_{sb} dP_{sb}}{d\bar{P}_b} \right\} \]

Combining Eq. 4-21, 4-25 and 4-29 together finally the Eq. 4-30 is derived.

\[
\hat{P}_s = \frac{Z_s^* R \left( Q_s + W_b (h_s - h_g) \right)}{\frac{ML_s A_s \frac{\partial H}{\partial T_s}}{1 + \frac{Z_s^* R \left( \frac{\partial H}{\partial P_s} - V_s \right)}{ML_s A_s \frac{\partial H}{\partial T_s}} + \frac{Z_s^* R P_s L_{sb} K_{sb}}{ML_s \bar{P}_b}}}.
\]

### 4.2.4 Nonlinear State Space Model

Choose \( x = \begin{bmatrix} P_s & M_b & H_{sc} \end{bmatrix}^T \) as the state variables, control input is \( u = W_{fw} \). Eq. 4-4, 4-16 and 4-30 form a nonlinear state space model of HCSG.

\[
\frac{d}{dt} \begin{bmatrix} P_s \\ M_b \\ H_{sc} \end{bmatrix} = \begin{bmatrix} f_1(x) \\ h_s W_{sc} - h_g W_c + Q_b \\ -H_{sc} \frac{W_{sc} + Q_{sc}}{M_{sc}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ h_{fw} \end{bmatrix} W_{fw}
\]

Where
\[
\frac{Z_s^* R \left( Q_s + W_b (h_s - h_g) \right)}{ML_s A_s \frac{\partial H_s}{\partial T_s}}
\]
\[
f_1(x) = \frac{Z_s^* R \left( M_s \frac{\partial H_s}{\partial P_s} - V_s \right) + Z_s^* R P_s L_b K_b}{1 + ML_s A_s \frac{\partial H_s}{\partial T_s}} \frac{ML_s \bar{P}_b}{ML_s \bar{P}_b}
\]

Denote \( x_r = \begin{bmatrix} P_{sr} \\ M_{br} \\ H_{scr} \end{bmatrix} \) as the state variables in the rated power, express \( x = \begin{bmatrix} P_s \\ M_b \\ H_{sc} \end{bmatrix} \) as \( x = x_r + \delta x \), where \( \delta x = \begin{bmatrix} \delta P_s \\ \delta M_b \\ \delta H_{sc} \end{bmatrix} \), rewrite 4-31 as:

\[
\frac{d}{dt} \begin{bmatrix} \delta P_s \\ \delta M_b \\ \delta H_{sc} \end{bmatrix} = \begin{bmatrix} f_1(x) \\ \frac{h_{sc} W_{sc} - h_g W_g + Q_b}{h_b} \\ -\frac{H_{scr} + \delta H_{sc}}{M_{sc}} W_{sc} + Q_{sc} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ W_{fw} \end{bmatrix}
\]

Remark:

Here the state variables are changed from the real value of \( \begin{bmatrix} P_s \\ M_b \\ H_{sc} \end{bmatrix} \) to the relative change of them, correspondingly, the state functions need to be modified with respected to the new state variables. However, because OBFD has NO close relation with the state functions which will be shown in the next section, so clear state functions will be helpful for the OBFD controller but not necessary if it is difficult to get them. So here the state functions are still given in the original form.

### 4.3 Observer-Based Feedback Dissipation

First, several concepts and definitions are presented.

*Definition 4-1 [31]: Hamiltonian system*
An autonomous dynamic system

\[ \dot{x} = f(x), \quad x \in \mathbb{R}^n \]  

(4-34)

with \( f(x_0) = 0 \) for some \( x_0 \in \mathbb{R}^n \), is called a generalized Hamiltonian system (GHS) if the dynamic system expressed by Eq. 4-34 can be expressed as

\[ \dot{x} = T(x) \nabla H(x) \]  

(4-35)

Where \( T(x) \in \mathbb{R}^{n \times n} \) is a real matrix that is called the structure matrix and \( H(\bullet): \mathbb{R}^n \rightarrow \mathbb{R}^+ \) \((\mathbb{R}^+ = [0, \infty))\), is a smooth function, and is called the Hamiltonian function, represents the total stored energy, and

\[ \nabla H(x) = \left( \frac{\partial H(x)}{\partial x} \right)^T \]

Based on the Definition 4-1, the concept of dissipative system is given as follows

**Definition 4-2 [31]: Hamiltonian realization**

If the structure matrix \( T(x) \) of the dynamic Eq. 4-35 can be expressed as

\[ T(x) = J(x) - R(x) \]

with skew-symmetric \( J(x) \) and symmetric nonnegative \( R(x) \), then system Eq. 4-34 is said to be dissipative, and Eq. 4-35 is the dissipative generalized Hamiltonian realization of the system in Eq. 4-34. Moreover, the system in Eq. 4-34 is said to be strict dissipative if \( R(x) \) is positive, and Eq. 4-35 is the corresponding strict dissipative generalized Hamiltonian realization.

From Eq. 4-31, the nonlinear dynamics of the HCGS is essentially a sample of the single-input-single-output (SISO) nonlinear systems taking the form as
\begin{align}
\begin{cases}
x = f(x) + g(x)u \\
y = h(x)
\end{cases}
\tag{4-36}
\end{align}

Where $x \in R^n$ is the state vector, $u \in R$ is the control input and $y \in R$ is the output. To express an nonlinear system taking the form of Eq. 4-36 as a generalized Hamiltonian function, an important and useful lemma is given as follows:

**Lemma 4-1[32]:**

For an arbitrary given nonlinear system taking the form of Eq. 4-36, if there exists a Hamiltonian function $H(x)$ such that $\forall x \in R^n \setminus \{O\}$,

\begin{align}
\begin{cases}
\nabla(x) \neq O \\
L_{f(x)}H(x) \neq 0
\end{cases}
\tag{4-37}
\end{align}

Where $L_{f(x)}H(x) = \frac{\partial H(x)}{\partial x} f(x)$ is the Lie derivative of $H(x)$ along $f(x)$, then the system Eq. 4-36 can be converted to the following generalized Hamiltonian realization.

\begin{align}
\dot{x} = [J(x) - R(x) + S(x)] \nabla H(x)
\tag{4-38}
\end{align}

Where $J(x)$ is a skew-symmetric matrix, $R(x)$ and $S(x)$ are all symmetric semi-positive, and the three matrices satisfy

\begin{align}
J(x) &= \frac{1}{2\|\nabla H(x)\|_2^2} \left[ f(x)[\nabla H(x)]^T - \nabla H(x)f^T(x) \right]
\tag{4-39}
\\
R(x) &= \frac{1}{4\|\nabla H(x)\|_2^2} \left[ f(x)f(x)^T + \nabla H(x)[\nabla H(x)]^T \right]
\tag{4-40}
\\
S(x) &= \frac{1}{4\|\nabla H(x)\|_2^2} \left[ f(x) + \nabla H(x) \right] \left[ f(x) + \nabla H(x) \right]^T
\tag{4-41}
\end{align}

respectively.

Proof: please refer to Appendix A of reference 32.
From Lemma 4-1, the generalized Hamiltonian realization for the nonlinear state-space of HCSG Eq. 4-31 can be written as

\[
\frac{dx}{dt} = [J(x) - R(x) + S(x)]\nabla H(x) + g(x)u \\
y = h(x)
\]

(4-42)

Where \( x \) is the state variable, \( u \in R \) is the control input, \( h(x) = c^T x \), \( H(x) \) is the Hamiltonian function which is chosen as

\[
H(x) = \frac{1}{2} \delta P_s^2
\]

(4-43)

\( J(x), R(x) \) and \( S(x) \) satisfy Eq. 4-39, 4-40, 4-41 respectively. Usually, to get the generalized Hamiltonian system, a matching condition between output and Hamiltonian function should be satisfied which is:

\[
h(x) = g(x)\nabla H(x)
\]

(4-44)

However, for the system of concern in this Chapter, this matching condition is not satisfied:

\[
h(x) = \delta P_s \neq 0 = g(x)\nabla H(x)
\]

(4-45)

To solve this problem, the OBFD method proposed by Dong\(^{10}\) is introduced. Consider the generalized Hamiltonian SISO system taking the form

\[
\frac{dx}{dt} = [J(x) - R(x) + S(x)]\nabla H(x) + g(x)u \\
y = I^T(x)\nabla H(x)
\]

(4-46)

Where \( x \in R^n \) is the state vector, \( u \in R \) is the control input and \( y \in R \) is the output with \( I(x) \neq g(x), H(x) \) is the Hamiltonian function. \( J(x) \) is a skew-symmetric matrix, \( R(x) \) and \( S(x) \) are all symmetric semi-positive.

Suppose the system Eq. 4-36 has a state-observer taking the form
\[
\begin{align*}
\frac{d\hat{x}}{dt} &= [J_\theta(\hat{x})^\top R_\theta(x)]\nabla H(x) + K^T(x)[y - I^T(x)\nabla H(x)] \\
\end{align*}
\]

(4-47)

Where \( \hat{x} \in \mathbb{R}^n \) is the observation of the state vector, \( K(\hat{x}) \) is the observer gain matrix, \( J_\theta(\hat{x}) \) is a skew-symmetric matrix, and \( R_\theta(\hat{x}) \) is symmetric semi-positive.

**Theorem 4-1[30]:**

Consider the generalized Hamiltonian system in Eq. 4-46 with a state-observer taking the form of Eq. 4-47, and suppose

\[
\begin{align*}
\nabla(x) &\neq 0, \quad L_{f(x)}H(x) \neq 0, \quad x \neq 0 \\
\nabla(x) &= 0, \quad x = 0
\end{align*}
\]

If there exists an observer gain matrix \( K(\hat{x}) \) and positive definite matrix \( T(\hat{x}) \) and \( R_\theta(\hat{x}) \) such that

\[
\begin{align*}
R_\theta(\hat{x})^\top [I(x)K(x) + K^T(x)I^T(x)] > O \\
\end{align*}
\]

and

\[
\begin{align*}
T(\hat{x}) + R(x) - S(x) - \delta_m g(x)g^T(x) - \\
\frac{1}{4}I(x)K(\hat{x})^\top \left[ R_\theta(\hat{x}) + \frac{1}{2}[I(x)K(x) + K^T(x)I^T(x)] \right]^{-1}K^T(x)I^T(x) > O
\end{align*}
\]

(4-49)

where the positive \( \delta_m \) satisfies:

\[
\delta(x, \hat{x}) \leq \delta_m \left| L_{f(x)}H(x) \right|
\]

and

\[
\delta(x, \hat{x}) = \frac{[\nabla H(x)]^T T(\hat{x}) \nabla H(x)}{L_{f(x)}H(x)} - \frac{[\nabla H(x)]^T T(x) \nabla H(x)}{L_{f(\hat{x})}H(\hat{x})}
\]

then the following control
\begin{align*}
    u(\hat{x}) &= -\frac{[\nabla H(\hat{x})]^T T(x) \nabla H(x)}{L_{f(x)} H(\hat{x})},
\end{align*}

guarantee asymptotically stability of the close-loop systems.

Proof: please see reference 30

Remark:

If the state functions are simple, \( R(x) \) and \( S(x) \) can be calculated directly, then
by solving the matrix inequality Eq. 4-48 and Eq. 4-49, the OBFD controller is obtained
directly. If the state functions are very complicated, whose \( R(x) \) and \( S(x) \) can not
computed easily and obtained directly, then the parameters of the observer usually are
obtained by experience and validated by simulation for such a complicated system.

5.4 Simulation and Result

The OBFD dissipative controller is applied to stabilize the pressure of the outlet steam of
the HCSG. The HCSG model is developed by University of Tennessee using SIMULINK. This
whole IRIS model consists of a 22\textsuperscript{nd} order HCSG, a 2\textsuperscript{nd} order point kinetics with one delayed
neutron group, and 4th order heat exchange of reactor core with temperature feedback from
lumped fuel and coolant temperature calculation [34] which were added at the Pennsylvania State
University.

\( J_{\theta}(\hat{x}), \ R_{\theta}(\hat{x}), \ K(\hat{x}) \) and \( T(\hat{x}) \) is chosen as
The following conditions are simulated:

Case I: in the first 1000 seconds, the IRIS nuclear power plant is at normal 100% rated power operation condition, and the pressure of output steam is at its rated value. At t=1000 seconds, suddenly, four primary flow pumps (Total is 8) fail. The OBFD controller should maintain the pressure of outlet steam at its rated value to protect against damage to the steam turbine. At t=2000 seconds, the four failed primary flow pumps recover from the accident. Figure 4-2 shows the pressure response in case I. The fluctuation of pressure is nearly 0 in this emergency condition. Figure 4-3 shows the length change within the tubes of HCSG. Consider the summation of length of boiling region and subcooled region as the concept of “water level”. When half of primary flow pumps fail, the length of boiling region first increases a little, then decreases quickly, this is because the “swell and shrink” effect also exists in the HCSG. Similarly, when the failed pumps recover, the length of boiling region first decreases a little, then increases quickly. Figure 4-4 shows the control effect of feedwater flow rate.

Case II: in the first 1000 seconds, the IRIS nuclear power plant is at normal 100% rated power operation condition, and the pressure of output steam is at its rated value. At t=1000 seconds, the power of IRIS reactor is stepped from 100% rated power to 50% rated power. At t=2000 seconds, the power of IRIS reactor steps from 50% rated power to 100% rated power.

Figure 4-5, 4-6, 4-7 give the simulation result in case II. Simulation results show that the outlet pressure is nearly unchanged. The OBFD controller also stabilizes the pressure very well in transient process.
Figure 4-2. Pressure response of HCSG: Case I

Figure 4-3. Length change of Lsc & Lb : Case I.
Figure 4-4. Feedwater flow rate: Case I

Figure 4-5. Pressure response of HCSG: Case II
Figure 4-6. Length change of $L_{sc}$ and $L_b$: Case II

Figure 4-7. Feedwater flow rate: Case II
4.5 Summary

Stabilization of pressure of the outlet steam in the HCSG is important for operation and safety of the IRIS nuclear power plant. Using the level control of current U-tube steam generator for reference, a nonlinear observer-based feedback dissipation controller for the pressure control of HCSG is presented in this chapter. The first step of the OBFD is to model the HCSG in the form of a simplified nonlinear state space model, and then based-on this simplified model, OBFD is designed. Numerical simulation is done by applying the OBFD to a SIMULINK IRIS model which is higher order. The simulation results show that the OBFD can well stabilize the pressure of HCSG.

Furthermore, because the state space model used to design the OBFD controller is simple and with low order, Uncertainties are introduced in the process of model simplification. So an OBFD with large stable margin is expected. If $R(x)$ and $S(x)$ can be calculated directly, then by solving the matrix inequality Eq. 4-48 and Eq. 4-49, and choosing a sufficient positive solution, the OBFD controller is obtained directly and can guarantee enough stable margin. But usually, for a complicated system, the matrix of $R(x)$ and $S(x)$ can not be calculated directly, then the matrix of the observer can only be obtained by experience and validated by simulation. That is the limitation of the OBFD.
Chapter 5

Online State Observer Design Based on Dissipation-Based High Gain Filter

5.1 Introduction

The control and operation of complex systems, such as nuclear power plants, critically rely on monitoring of the state variables. Knowing the system states is necessary to solve many control theory problems, for example, stabilizing a system using state feedback. Usually, sensor systems are applied to sample and monitor the system state variables, for example, temperature, pressure. Unfortunately, in most practical cases, many physical states of the system cannot be determined by direct observation, i.e. the fuel temperature of reactor core, the concentration of delayed neutrons, while in some cases, some state variables cannot be interpreted as explicit physical variables. To solve this problem, state observers are built to reconstruct some desired states which are unmeasurable. State observer is a system that models a real system in order to provide an estimate of its internal state, given measurements of the input and output of the real system. It is typically a computer-implemented mathematical model. The observer is called asymptotically stable if the estimated states converge to the real state when the time goes to infinity. For linear system, the Kalman filter (KF) is the most widely used state observer in practical engineering [38]. Specifically, in nuclear engineering field, the application of KF in monitoring and fault detection and isolation (FDI) for nuclear reactor is also done [39][40][41]. However, since nuclear power plant systems are highly nonlinear, the KF filter cannot achieve an acceptable performance. State observer for nonlinear system is needed. For a nonlinear system, the state observer design is more complicated than that for a linear system. The observer design
for a nonlinear system has been an active research topic for decades. In general, there are two main approaches [51]. In the first approach [42][43][44], nonlinear systems are brought into a so-called observer form by means of a nonlinear state transformation. This special form yields a completely linear error dynamic of state such that linear observer theory can be applied. However, for this approach, the condition for achieving the desired state coordinate transformation are in general difficult to satisfy, which requires the solution of a set of partial differential equations. More recently, significant research efforts have been directed to alleviate the difficulty by developing more generalized state transformation procedures and more generalized canonical forms so that the larger class of nonlinear system can be dealt with [45][46]. On the other hand, the second approach is concerned with designing observer for nonlinear system without the need of state transformation. The high-gain observers in [47][48][49][50] are attractive because of their robustness. Available design techniques include pole-placement approach [47], Riccati equation approach [48], Lyapunov equation approach [49] and dissipation approach [50].

In this chapter, a nonlinear observer for the IRIS reactor is realized based on the dissipation-based high gain filter (DHGF) theory proposed by Dong [50]. Some basic concepts about dissipation theory and Hamiltonian realization are introduced in Chapter 4. These concepts will be recalled here to design the state-observer.

5.2 Methodology

5.2.1 Problem Formulation

Consider the nonlinear systems taking the form as [50]
\[
\begin{aligned}
\dot{x}(t) &= f(x) \\
y &= h(x)
\end{aligned}
\]  
(5-1)

where \( x(t) \in \mathbb{R}^n \) is the state, \( y \in \mathbb{R}^m \) is the system output and both the functions \( f(\bullet) : \mathbb{R}^n \to \mathbb{R}^n \) and \( h(\bullet) : \mathbb{R}^n \to \mathbb{R}^m \) are smooth. Suppose system (5-1) has the following state-observer

\[
\dot{\hat{x}} = f(x) + K(y - y)
\]  
(5-2)

where \( \hat{x} \in \mathbb{R}^n \) is the state-observation, \( y \in \mathbb{R}^m \) is the system output, \( K \in \mathbb{R}^n \) is the observer gain, and \( \hat{y} = h(x) \).

Define the observation error vector as

\[ e = x - \hat{x} \]  
(5-3)

and the error dynamics can be written as

\[ \dot{e} = f(x) - f(\hat{x}) - K(y - y) \]  
(5-4)

From the theory of Taylor series expansion, it is clear that

\[ f(x) = f(\hat{x}) + f_\epsilon(e) \]  
(5-5)

and

\[ y = \hat{y} + h_\epsilon(e) \]  
(5-6)

where \( f_\epsilon(\bullet) : \mathbb{R}^n \to \mathbb{R}^n \) and \( h_\epsilon(\bullet) : \mathbb{R}^n \to \mathbb{R}^m \) are both smooth functions. Therefore error dynamics Equation (5-4) can be rewritten as

\[ \dot{e} = f_\epsilon(e) - Kh_\epsilon(e) \]  
(5-7)

In order to design an asymptotic state-observer for nuclear reactors, it is necessary to formulate and solve the following two problems:

**Problem 5-1:** How to choose the observer gain \( K \) so that state-observer (5-2) is asymptotically stable, i.e.
The actual dynamics of nuclear reactors is very complex, and the mathematical model used in controller design for nuclear reactor can only approximately describe the actual reactor dynamics near some given operating point. There must be exterior disturbances and model uncertainty, and then it is quite necessary to solve the following problem 2:

Problem 5-2: How to evaluate the robustness of the state-observer?

5.2.1 Design and Robustness Analysis of DHGF

In order to present the design of the DHGF, the concept of zero-state detectability is introduced as follows:

Definition 5-1[52]: Consider the nonlinear systems taking the form as

\[
\begin{cases}
\dot{\xi} = \Phi(\xi) \\
\eta = \Theta(\xi)
\end{cases}
\]  

(5-8)

where and \( \xi \in \mathbb{R}^n \) is the state-vector, \( \eta \in \mathbb{R}^m \) is the system output, and both the functions \( \Phi(\bullet): \mathbb{R}^n \rightarrow \mathbb{R}^n \) and \( \Theta(\bullet): \mathbb{R}^n \rightarrow \mathbb{R}^m \) are smooth functions. System 5-8 is called zero-state detectable if \( \forall \eta(\forall t \geq 0) \) implies \( \lim_{t \to \infty} \xi = 0 \).

Based on the definition of zero-state observability, Theorem 5-1 is given as follows, which gives the solution of Problem 5-1.

Theorem 5-1: Consider the nonlinear systems described by dynamic 5-1 with state-observer taking the form as Equation 5-2. Define

\[
e_y = y - \hat{y}
\]

(5-9)
and assume that there exists a smooth function $H(\bullet): \mathbb{R}^m \to \mathbb{R}^+$ such that $H(e_y)$ is only minimal at $e_y = 0$ and

$$e_y = e_y L \left[ \frac{\partial H(e_y)}{e} \right]^T$$

(5-10)

where $L \in \mathbb{R}^{m \times m}$ is a given matrix. Moreover, suppose that the error dynamics 5-7 with the output defined as $e_y$ is zero-state detectable. Therefore the state-observer 5-2 is asymptotically stable if

$$K = \frac{1}{\varepsilon} L$$

(5-11)

where $\varepsilon$ is a small enough given positive scalar.

Proof: From 5-6, we have

$$e_y = h_\varepsilon(e)$$

Therefore the composite function $H \circ h_\varepsilon(\bullet): \mathbb{R}^n \to \mathbb{R}^+$ can be chosen as a Hamiltonian function.

From Lemma 4-1, the GHR of both system 5-1 and the state-observer 5-2 can be represented respectively as

$$\dot{x} = [J(x,e_y) - R(x,e_y) + S(x,e_y)] \left[ \frac{\partial H(e_y)}{\partial x} \right]^T$$

(5-12)

and

$$\dot{x} = [J(x,e_y) - R(x,e_y) + S(x,e_y)] \left[ \frac{\partial H(e_y)}{\partial x} \right]^T + K(y - y)$$

(5-13)

where
and moreover if we replace the variable \( x \) by \( \hat{x} \), then we can get the expression for the matrices \( J(\hat{x}, e_y) \), \( R(\hat{x}, e_y) \) and \( S(\hat{x}, e_y) \).

From 5-3

\[
\frac{\partial H(e_y)}{\partial x} = \frac{\partial H(e_y)}{\partial e} \frac{\partial e}{\partial x} = \frac{\partial H(e_y)}{\partial e}
\]

and

\[
\frac{\partial H(e_y)}{\partial \hat{x}} = \frac{\partial H(e_y)}{\partial e} \frac{\partial e}{\partial \hat{x}} = -\frac{\partial H(e_y)}{\partial e}
\]

subtract (5-13) from (5-20), and then the GHR of the error dynamics (5-7) can be written as:

\[
\dot{e} = [J_e(x, \hat{x}, e_y) - R_e(x, x, e_y) + S_e(x, x, e_y)] \left[ \frac{\partial H(e_y)}{\partial e} \right]^T + Ke_y
\]  

(5-17)

where

\[
J_e(x, \hat{x}, e_y) = J(x, e_y) + J(x, e_y)
\]  

(5-18)
\[ R_\epsilon(x, \dot{x}, e_y) = R(x, e_y) + R(x, e_y) \]  \hspace{1cm} (5-19) \\

and

\[ S_\epsilon(x, \dot{x}, e_y) = S(x, e_y) + S(x, e_y) \]  \hspace{1cm} (5-20) \\

based on the (5-10), (5-11), differentiate Hamiltonian function \( H(e_y) \) along the trajectory given by error dynamics (5-17).

\[
\frac{dH(e_y)}{dt} = \frac{\partial H(e_y)}{\partial e_y} \frac{de}{dt} \\
= \frac{\partial H(e_y)}{\partial e_y} \left[ S_\epsilon(x, \dot{x}, e_y) - R_\epsilon(x, x, e_y) - \epsilon^{-1} L L^T \right] \frac{\partial H(e_y)}{\partial e_y}^T \\
= -\left\| \nabla_e H(e_y) \right\|_{S_y}^2 + \left\| \nabla_e H(e_y) \right\|_{S_y}^2 - \epsilon^{-1} \left\| \nabla_e H(e_y) \right\|_{M}^2
\]

where

\[
\nabla_e H(e_y) = \left[ \frac{\partial H(e_y)}{\partial e_y} \right]^T
\]

and

\[
M = L L^T
\]

from condition (5-10), it is clear that if \( e_y \neq o \), then

\[
\nabla_e H(e_y) \neq o
\]

and furthermore from (5-21), there exists an enough small positive scalar \( \epsilon \) such that

\[
\left\| \nabla_e H(e_y) \right\|_{S_y}^2 - \epsilon^{-1} \left\| \nabla_e H(e_y) \right\|_{M}^2 < 0
\]

i.e.

\[
\frac{dH(e_y)}{dt} < 0
\]

If \( e_y = o \), then from the definition of zero-state detectability,
\[
\lim_{t \to \infty} e = 0
\]

Therefore if conditions (5-10) and (5-11) are both satisfied, then the state-observer (5-2) is asymptotically stable for small enough positive scalar \( \varepsilon \).

The robustness analysis of the DHGF is given in the reference [50].

5.3 Application in IRIS Nuclear Power Plant

5.3.1 State Space Model of IRIS Core for State-Observer Design

In this section, the DHGF for monitoring the IRIS reactor state will be realized. The dynamic model which is used to design the DHGF is based on Equations (2-1) and (2-2). Generally, to make the estimation more accurate, the corresponding dynamic model should give more information about the real physical process. However, since the dynamic model which is used to design the DHGF should satisfy some requirements such as zero-state detectability and note that some lower-order models also give acceptable and reasonable description for the dynamic process, so here, some reasonable simplifications are made. Note that the one-delayed neutron group is essentially an equivalent delayed neutron group whose concentration and decay constant are both the average values of those corresponding to the six delayed neutron group, so the one delayed neutron group point kinetic is used here. Then, treat the a whole fuel rod as a cell, the dynamic model which is used for DHGF design is given as:
\[
\begin{align*}
\frac{dn_r}{dt} &= \frac{\delta \rho_r - \beta}{\Lambda} n_r + \sum_{i=1}^{6} \frac{\beta_i}{\Lambda} c_{r_i} + \frac{\alpha_r n_r}{\Lambda} (T_f - T_{f0}) + \frac{\alpha_c n_r}{\Lambda} (T_{\text{cav}} - T_{\text{cav0}}) \\
\frac{dc_r}{dt} &= \lambda (n_r - c_r) \\
\frac{dT_f}{dt} &= -\frac{\Omega}{\mu_f} T_f + \frac{\Omega}{\mu_f} T_{\text{cav}} + \frac{P_0}{n_r} n_r \\
\frac{dT_{\text{cav}}}{dt} &= -\frac{2M + \Omega}{\mu_c} T_{\text{cav}} + \frac{\Omega}{\mu_c} T_f + \frac{2M}{\mu_c} T_{\text{cin}} 
\end{align*}
\] (5-23)

The meaning of each parameter is the same as that given in (2-1) and (2-2).

Here, to make the dynamic model meet the requirement of zero-state detectability, rewrite the nonlinear state space model (5-23) in the incremental form. Choose the state vector as

\[ x = [\delta n_r \quad \delta c_r \quad \delta T_f \quad \delta T_{\text{cav}}], \]

and then dynamic (5-23) can be written as

\[ \dot{x} = f(x, u) = f(x, \delta \rho_r) \] (5-24)

\[
\begin{bmatrix}
-\frac{\beta}{\Lambda} \delta n_r + \frac{\beta}{\Lambda} \delta c_r + \frac{\alpha_f n_{r0}}{\Lambda} \delta T_f + \frac{\alpha_c n_{r0}}{\Lambda} \delta T_{\text{cav}} + \frac{n_{r0}}{\Lambda} \delta \rho_r \\
\quad + \frac{\alpha_f}{\Lambda} n_r \delta T_f + \frac{\alpha_c}{\Lambda} n_r \delta T_{\text{cav}} + \frac{1}{\Lambda} \delta n_r \delta \rho_r \\
\end{bmatrix} \\
= \frac{\lambda}{\Lambda} \delta n_r - \lambda \delta c_r \\
-\frac{\Omega}{\mu_f} \delta T_f + \frac{\Omega}{\mu_f} \delta T_{\text{cav}} + \frac{P_0}{n_r} \delta n_r \\
-\frac{2M + \Omega}{\mu_c} \delta T_{\text{cav}} + \frac{\Omega}{\mu_c} \delta T_f + \frac{2M}{\mu_c} \delta T_{\text{cin}}
\]

where,

\[
\begin{align*}
u &= \delta \rho_r \\
\delta n_r &= n_r - n_{r0} \\
\delta c_r &= c_r - c_{r0} \\
\delta T_f &= T_f - T_{f0} \\
\delta T_{\text{cav}} &= T_{\text{cav}} - T_{\text{cav0}}
\end{align*}
\]
In this differential equation, only $\delta n_r$ and $\delta T_{\text{cav}}$ are measurable, let

$$ y = h(x) = [\delta n_r, \delta T_{\text{cav}}] $$

so,

$$ e_y = [\delta n_r - \hat{\delta n}_r, \delta T_{\text{cav}} - \hat{\delta T}_{\text{cav}}] $$

In the following, the zero-state observability of observation error dynamics (5-7) will be determined. Choose the state-observer gain matrix as (5-11), and then substitute $e_y = o$ to (5-7), we have

$$ 0 = \frac{\beta}{\Lambda} (\delta c_r - \hat{\delta c}_r) + \frac{\alpha_f (n_{\text{ro}} + \delta n_r)}{\Lambda} (\delta T_f - \hat{\delta T}_f) + \frac{\alpha_f (n_{\text{ro}} + \delta n_r)}{\Lambda} (\delta \rho_r - \hat{\delta \rho}_r) $$

(5-25)

$$ \frac{d(\delta c_r - \hat{\delta c}_r)}{dt} = -\lambda (\delta c_r - \hat{\delta c}_r) $$

$$ \frac{d(\delta T_f - \hat{\delta T}_f)}{dt} = -\frac{\Omega}{\mu_f} (\delta T_f - \hat{\delta T}_f) $$

$$ 0 = \frac{\Omega}{\mu_c} (\delta T_f - \hat{\delta T}_f) + \frac{2M}{\mu_c} (\delta T_{\text{cin}} - \hat{\delta T}_{\text{cin}}) $$

From the 4th equation of (5-25), if we want to get $\delta T_f - \hat{\delta T}_f = 0$, then the $\delta T_{\text{cin}} - \hat{\delta T}_{\text{cin}} = 0$ is required. Since $T_{\text{cin}}$ is the core inlet temperature of the coolant, it is measurable. So, to meet the zero-state detectability requirement, here, let $\hat{\delta T}_{\text{cin}}$ is always equal to $\delta T_{\text{cin}}$. Then we get:

$$ \delta T_f - \hat{\delta T}_f = 0 $$

(5-26)

Substitute (5-26) to (5-25),

$$ 0 = \frac{\beta}{\Lambda} (\delta c_r - \hat{\delta c}_r) + \frac{\alpha_f (n_{\text{ro}} + \delta n_r)}{\Lambda} (\delta \rho_r - \hat{\delta \rho}_r) $$

(5-27)

$$ \frac{d(\delta c_r - \hat{\delta c}_r)}{dt} = -\lambda (\delta c_r - \hat{\delta c}_r) $$
Since \( \delta \rho \) is the input reactivity to the real plant and \( \hat{\delta} \rho \) is the input reactivity to the plant, so \( \delta \rho = \hat{\delta} \rho = u \), then we get

\[
\delta c_r - \hat{\delta} c_r = 0 \tag{5-28}
\]

From equation (5-26), (5-28), we can conclude that if \( e_y = o \), then \( \lim_{t \to \infty} e = o \). i.e. error dynamic (5-7) is zero-detectable. Therefore the DHFD derived in the Section 2 can be utilized to observe the state of the IRIS reactor.

Choose the Hamiltonian function as

\[
H(e_y) = \frac{1}{2} [k_1 (\delta n_r - \hat{\delta} n_r)^2 + k_2 (\delta T_{\text{can}} - \hat{\delta} T_{\text{can}})^2],
\]

then

\[
e_y = L^T \left[ \frac{\partial H(e_y)}{\partial e} \right]
\]

where

\[
L = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_2 \end{bmatrix},
\]

\( k_1 \) and \( k_2 \) are two positive numbers. Recall Equation (5-11), the observer gain \( K = \frac{1}{\varepsilon} L \) is chosen by adjusting the \( \varepsilon \). Here, we will directly choose \( k_1 \) and \( k_2 \).

### 5.3.2 Numerical Simulation

Applying this DHGF into the simulation model discussed in Chapter 2. The HCSG pressure controller is the OBFD controller. The simulation result by choosing two different sets of \( (k_1, k_2) \) are compared to illustrate the influence of the gain \( K \) in state observer Equation (5-4).
Three different dynamic processes are simulated to show the performance of the designed DHGF. Here, the values of state variables given by a high order model of IRIS (6-delayed group neutron and 4 core cells) will be considered as the real measurement. And the calculation values given by a low order model (1-delayed group neutron and 1 core cell) will be compared with estimation given by the DHGF.

Case I: reference power step from 1.0 to 0.8 in 20 seconds and the plant parameters used for the estimation is correct.

Figure 5-1, 5-2, 5-3, 5-4 shows the transient response of the four state variables. The relative power $n_r$ and average coolant temperature $T_{cav}$ are measurable while precursor density $c_r$ and average fuel temperature $T_f$ are not measurable. $c_r$ and $T_f$ will be estimated by $n_r$ and $T_{cav}$. Different choice of set $(k_1, k_2)$ are also shown. From Figure 5-1 and Figure 5-2, there is a difference between $n_r$, $c_r$ calculated by the 1-delay group point kinetic model and $n_r$, $c_r$ calculated by the 6-delay group point kinetic model. The estimated $n_r$, $c_r$ calculated by the DHGF follows the real measurement quite well. Figure 5-3 and Figure 5-4 show that as the gain increases, the tracking performance becomes better. The high gain can effectively reduce the tracking error in dynamic process.
Figure 5-1. Transient response of \( n_r \) in case I.

Figure 5-2. Transient response of \( c_r \) in case I.
Case II: Reactor coolant pump (RCP) overspeeds 1.02 with observer with accurate plant parameters.

Same as in case I, Figure 5-5, 5-6, 5-7, 5-8 shows the transient response of the four state variables. The same conclusion can be made that there is error between the estimation of low order model and high order model during the dynamic process. Comparatively, when the state
observer gain $K$ is big enough, the DHGF can estimate the state valuables very well by using a low order model.

Figure 5-5. Transient response of $n_r$ in case II.

Figure 5-6. Transient response of $c_r$ in case II.
Case III: RCP overspeed 1.02 with observer with inaccurate plant parameters

In case III, the robustness of DHGF is checked. Because in many cases, many coefficients of real dynamic process cannot be determined very accurately, in other words, there exists uncertainty. So, in case III, the performance of DHGF with inaccurate plant parameters is simulated. A high gain $K$ is chosen.
The following parameters are changed:

Table 5-1. Inaccurate parameters of DHGF for Simulation of Case III.

<table>
<thead>
<tr>
<th></th>
<th>$\mu_f$</th>
<th>$\mu_c$</th>
<th>$\alpha_f$</th>
<th>$\alpha_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value used in Plant</td>
<td>0.4519</td>
<td>0.7357</td>
<td>-1.6500e-005</td>
<td>-4.0000e-004</td>
</tr>
<tr>
<td>Value used in Observer</td>
<td>0.5423</td>
<td>0.8828</td>
<td>-1.4850e-005</td>
<td>-3.6000e-004</td>
</tr>
</tbody>
</table>

Figures 5-9, 5-10, 5-11, 5-12 show the transient response of the four state variables. The performance of DHGF is still good. While some parameters are changed, the low order model results in a steady state error.

Figure 5-9. Transient response of $n_r$ in case III.
Figure 5-10. Transient response of $C_r$ in case III.

Figure 5-11. Transient response of $T_f$ in case III.
As we can see from the numerical simulation results, the DHGF can provide asymptotically stable state-observer in both the three cases. Moreover, the variation of system output leads to the change of output error $e_y = y - \hat{y}$, which makes the observed state changes according to the variation of the state vector. According to the Theorem 5-1, if $\varepsilon$ is small enough, in other words, $k_1$ and $k_2$ are big enough, then the DHGF is asymptotically stable. This guarantees that the existence of output error $e_y$ results in the trend that observation error $e$ becoming smaller, which in turn causes the output difference to be smaller. Based on the concept of zero-state detectability given by Definition 5-2, it is clear that the state-observation error can be satisfactory small after a transition period.

5.4 Conclusion

In this chapter, based on dissipation theory, a DHGF nonlinear state observer is designed and applied into the IRIS reactor. Three different conditions are simulated. Simulation results
show that the performance of DHGF is better than a low order model which is used to estimate state values. And when the observer gain $K$ is chosen big enough, then the observer is asymptotically stable and the estimated state values track the real state values well.
Chapter 6
Conclusion and Suggestion

6.1 Summary of Results

This thesis has demonstrated the design of a MIMO reliable LQG controller design for IRIS power plant system. Then applying dissipation Hamiltonian theory, an observer-based output feedback dissipation controller and a dissipation-based high gain filter are designed and applied into the IRIS power plant.

Chapter 1 introduced the background and motivation of this thesis.

Chapter 2 presented the mathematical models of IRIS reactor core, helical-coil steam generator and steam generator. These mathematical models are developed according to the physical laws, i.e. neutron kinetics, heat transfer. Then, the mathematical model is implemented in MATLAB/SIMULINK environment. This computer simulation model is used to determine the dynamic response, the controller design and online state-observer design.

Chapter 3 introduces a detailed procedure to design a reliable LQG controller. The reliable LQG controller is obtained by solving ARE and LMI. Then, the order of controller is reduced to 9 by using Hankel norm approximation method. From the simulation result, the reliable LQG controller can maintain the system stable when sensors drift in allowable range and perform well in tracking reference signal.

Chapter 4 introduces dissipative Hamiltonian theory. Several basic definitions are given. Then by referring the model used in level control of current U-tube steam generator, a nonlinear observer-based feedback dissipation controller for the pressure control of HCSG is presented in
this chapter. The simulation results show that the OBFD can well stabilize the pressure of HCSG and the OBFD controller can guarantee enough stable margins.

Chapter 5 introduces an online state-observer mechanism for the nonlinear reactor model. First, since the dynamic model used to do the observer design should satisfy the zero-state detectability and the simulation model cannot be used because it is complicated and does not meet the requirement for the zero-state detectability, a new and simple dynamic model used to the observer design need to be derived. Then based on this model, applying the dissipation-based high gain filter theorem, the state observer is designed. The simulation shows that the estimation given by this state observer tracks the real value well.

### 6.2 Future Work

The following works are recommended for the future research.

1). The reliable LQG controller can maintain the system stable when sensors drift in allowable range and perform well in tracking reference signal. However, compared with robust control, for example, \( H_\infty \) robust control, the reliable LQG does not perform well in the noise rejection. So, the future work will focus on enhancing the ability to reject noise of reliable LQG control.

2). For a complicated system, the observer matrix of the OBFD can only be obtained by experience and validated by simulation. That is the limitation of the OBFD. So, how to choose an effective matrix of the observer is recommended as the future research.

3). The DHGF is an effective state observer design method. The key step of this method is to write the dynamic process in a model which satisfies the zero-state detectability. However, it is not easy to realize this step, especially for a nonlinear and complicated dynamic process. How to find an effective way to express the dynamic process which can be used to do the DHGF
design is recommended for the future work. Furthermore, Applying DHGF into the state estimation of HCSG is recommended for future research since a nonlinear state space model is given in Chapter 4.
Reference


Appendix A

Nomenclatures For OBFD Nonlinear State Space Model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{sc}$</td>
<td>Flow area of subcooled region</td>
</tr>
<tr>
<td>$A_b$</td>
<td>Cross section of boiling region</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Flow area of super heated region</td>
</tr>
<tr>
<td>$H_d$</td>
<td>Total enthalpy of water in the subcooled region</td>
</tr>
<tr>
<td>$h_d$</td>
<td>Enthalpy of liquid water in the subcooled region</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Enthalpy of liquid water in the boiling region</td>
</tr>
<tr>
<td>$h_g$</td>
<td>Enthalpy of steam in the boiling region</td>
</tr>
<tr>
<td>$h_{fg}$</td>
<td>Latent heat of vaporization</td>
</tr>
<tr>
<td>$h_{fe}$</td>
<td>Enthalpy of feedwater</td>
</tr>
<tr>
<td>$h_b$</td>
<td>Average enthalpy of the two-phase flow in the boiling region</td>
</tr>
<tr>
<td>$h_s$</td>
<td>Enthalpy of the steam in the super heated region</td>
</tr>
<tr>
<td>$L_{sc}$</td>
<td>Length of subcooled region</td>
</tr>
<tr>
<td>$L_b$</td>
<td>Length of boiling region</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Length of super heated region</td>
</tr>
<tr>
<td>$L_{total}$</td>
<td>Length of tube</td>
</tr>
<tr>
<td>$M_{sc}$</td>
<td>Mass of the water in the subcooled region</td>
</tr>
<tr>
<td>$M_b$</td>
<td>Mass of the water in the boiling region</td>
</tr>
<tr>
<td>$M_s$</td>
<td>Mass of the water in the super heated region</td>
</tr>
<tr>
<td>$P_{sc}$</td>
<td>Pressure of steam in the super heated region</td>
</tr>
<tr>
<td>$P_{sat}$</td>
<td>Pressure of saturated steam</td>
</tr>
<tr>
<td>$P_b$</td>
<td>Pressure of the water in the subcooled region</td>
</tr>
<tr>
<td>$Q_{sc}$</td>
<td>Heat transferred from metal wall to the second side in the subcooled region</td>
</tr>
</tbody>
</table>
\( Q_b \) Heat transferred from metal wall to the second side in the boiling region
\( Q_s \) Heat transferred from metal wall to the second side in the super heated region
\( W_{sc} \) Water flowrate leaving the subcooled region
\( W_b \) Steam flowrate leaving the boiling region
\( W_s \) Steam flowrate leaving the super heated region
\( W_{fw} \) Feedwater flowrate
\( V_{sc} \) Volume of subcooled region
\( V_b \) Volume of boiling region
\( V_s \) Volume of super heated region
\( \rho_{sc} \) Water density inside the subcooled region
\( \bar{\rho}_b \) Average density of water and steam inside the boiling region
\( \rho_s \) Steam density inside the superheated region
\( x \) Flow quality of two-phase fluid inside the boiling region
\( \langle x \rangle \) Average flow quality of two-phase fluid inside the boiling region
\( Z_s^* \) Adjusting expansion coefficient for steam inside the superheated region
\( R \) Ideal gas constant
\( M \) Mole mass of steam
\( H_{sc} \) Total Enthalpy of water inside the downcomer.
Appendix B

Reliable LQG MATLAB Code

```matlab
%model_reduction_new
clear all
clc

load ABCD

A=Ar;
B=Br;
C=Cr;
D=Dr
n=rank(A);

I2=eye(2);

V1=1e4*eye(n);
V1=1000*eye(n);
%V1=B*B';
%R1=V1;
%R1=0.1*eye(n);
R1=0.01*eye(n);
V2=0.99*I2;

R2=diag([0.1 10]);
R2=diag([0.001 1])
%R2=diag([0.1 100]);
%R2=diag([10 1]);
%R2=diag([0.5 1]);

alpha0=diag([1 0.5]);
%alpha0=diag([1 1]);
beta=diag([0 1]);
%beta=diag([0 0.5]);

R0=diag([1 0.021]);
%R0=diag([1 1]);
disp('eig(inv(V2)-R0*beta)=')
eig(inv(V2)-R0*beta)
pause

temp0=C'*inv((I2-beta^2)*V2+inv(R0)*beta)*(I2-beta^2)*C;
Q=are(A',temp0,V1);
eig(Q)
```
```matlab
eig(A' - temp0*Q)
disp('eigenvalue of ATrans-temp0*Q')

pause

% Step 3
E1inf = (I2 - beta^0.5*R0^0.5*V2*beta^0.5*R0^0.5)^(-0.5)*beta^0.5*R0^0.5*C;
V0 = inv(inv(V2) - beta*R0);
Cbar = (I2 + V0*beta*R0)*C;
A0 = A + Q'*E1inf'*E1inf;
B10 = Q*Cbar'*((V0 + beta*inv(R0)))^(-0.5);

temp1 = [R1^0.5; zeros(2, n)];
temp2 = [zeros(n, 2); R2^0.5];

%Solve LMI
setlmis([]);
X0 = lmivar(1, [n+2 1]);
X1 = lmivar(1, [n 1]);
W = lmivar(2, [2 n]);

lmiterm([-1 1 1 X1], 1, 1);
% LMI #1: X1
lmiterm([-2 1 1 X0], 1, 1);
% LMI #2: X0
lmiterm([1 1 1 X1], A0, 1, 's');
% LMI #1: A0*X1 + X1*A0'
lmiterm([1 1 1 W], B, 1, 's');
% LMI #1: B*W + W'*B'
lmiterm([1 1 1 0], B10*B10');
% LMI #1: B10*B10'
lmiterm([1 2 1 X1], E1inf, 1);
% LMI #1: E1inf*X1
lmiterm([1 2 2 0], -I2);
% LMI #1: -I2
lmiterm([2 1 1 X0], 1, -1);
% LMI #2: -X0
lmiterm([2 2 1 X1], 1, temp1');
% LMI #2: X1*temp1'
lmiterm([2 2 1 -W], 1, temp2');
% LMI #2: W'*temp2'
lmiterm([2 2 2 X1], 1, -1);
% LMI #2: -X1

IRIS_LMI = getlmis;

%c = mat2dec(IRIS_LMI, eye(n+1));
c = mat2dec(IRIS_LMI, eye(n+2), zeros(n, n), zeros(2, n));
options = [1e-5, 0, 0, 0, 0];
[copt, xopt] = mincx(IRIS_LMI, c, options);

X0opt = dec2mat(IRIS_LMI, xopt, X0);
X1opt = dec2mat(IRIS_LMI, xopt, X1);
Wopt = dec2mat(IRIS_LMI, xopt, W);

Jmax = trace(X0opt) + trace(R1*Q);
G = Q*C'*inv((I2 - beta^2)*V2 + inv(R0)*beta)*inv(alpha0);
K = Wopt*inv(X1opt);
F = A0 - G*alpha0*inv(I2 - V2*beta*R0)*C+B*K;
```
Ggcc=G;
Kgcc=K;
Fgcc=F;

save('FGK','G','K','F');

for i=0:0.1:1
    alpha=diag([1 i]);
    Ae=[A B*K;G*alpha*C F];
    Be=[D]
    disp('Eigenvalue of Ae')
    eig(Ae)
end