EVANESCENT FIELD COUPLED MICRODISK CAVITIES

A Thesis in

Physics

by

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Abstract

This dissertation focuses on studies of spontaneous and stimulated emissions from whispering gallery mode (WGM) (Ga, Al)As/GaAs based microdisk cavities that are evanescent field coupled with each other. Microdisks are of great interest as test beds to demonstrate concepts in the fields of cavity quantum electrodynamics (CQED) and novel optoelectronics devices. We employ molecular beam epitaxy (MBE) to prepare the samples that have epitaxial heterostructures of (Ga, Al)As/GaAs. Then we use clean-room processing techniques to fabricate the microdisk devices. We then perform static and dynamic optical spectroscopy measurements to investigate their optical properties.

The first experiment in this thesis describes external strain engineering of the microdisk lasers to lower the lasing thresholds. A stressed silicon nitride (SiNx) thin film was deposited on the microdisk surfaces to provide an external compressive strain. Lasing thresholds were decided with static light-in light-out intensity plot and charge life time dynamics.

The second and third experiment are both about evanescent field coupled microdisk cavities. We fabricated three types of twin coupled microdisk cavities: twin circular ones, twin elliptical ones coupled along the long axis and twin elliptical ones coupled along the short axis. Normal mode splitting was observed in these coupled cavities with a lower energy bonding mode and a higher energy antibonding mode. The second experiment characterizes the mode energy splitting as a function of different geometrical layout, excitation power and ambient temperature. We observed a clear
correlation between the mode energy splitting and the splitting mode intensities. The mode splitting increases as the intensities increase, and vice versa. This correlation is a result of the evanescent field coupling.

In the third experiment, we simulated the electromagnetic field patterns inside and outside the twin coupled microdisk cavities. In the simulation, we used a four-classe of symmetry to construct two nearly-degenerate bonding modes and two nearly-degenerate antibonding modes. The simulations predict directional and in-plane polarized emission from the bonding and the antibonding modes in the twin circular coupled microdisks. These predictions were confirmed by experiment. We also observed in-plane polarized emission patterns from the twin elliptical microdisks. However, these observations are not consistent with simulations.

The fourth experiment uses a new method to measure the cavity quality factor of the microdisk. We prepared optical fiber tapers that have a few micron diameter and placed the tapers in the vicinity of the microdisk rim so that they are coupled through evanescent field. We passed a white light spectrum through the taper coupled with the microdisk. The transmission spectra clear show dips caused by the absorption from the microdisk cavity modes. Linewidth of the dips are limited by the spectrometer resolution, but this still gives a lower limit of the microdisk cavity quality factor, which is around $1.9 \times 10^4$. 
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I dedicate this dissertation to my parents.
Chapter 1

Semiconductor Optical Microcavities

1.1 Introduction

Modern society, to a great extent, relies on semiconductor science and technology advances that are based on the manipulation of electrons. Transistors and integrated circuits (IC) are the most successful examples. On the other hand, photonics – a younger branch of semiconductor science and technology – centers around generating, controlling and detecting photons using semiconductor devices. Since the invention of the semiconductor diode laser in 1960[1], the low loss optical fiber in 1970[2], and the erbium-doped fiber amplifier (EDFA) in 1987[3, 4], photonics technology has evolved in a somewhat similar manner to transistor based IC technology. It has found wide applications in optical data recording, fiber optic telecommunications, displays/digital imaging, and biochemical synthesis. Single photonic devices keep shrinking in size while the application demands become more and more complex. Consequently, much effort in contemporary photonics technology is being directed at packing discrete photonic devices together to form integrated photonics and also to interconnect with electronic components.

The core of photonics technology consists of three key areas: lasers, waveguides and detectors. Optical microcavities are the basic components in both lasers and waveguides. In lasers, coherent photons resonate with modes of optical cavities and emit out as light sources. In waveguides, photons travel in the modes of optical cavities with a
limited loss. Hence, understanding optical cavities is critical in the context of photonics technology. Semiconductor optical cavities are of greater interest since they can be easily integrated with other semiconductor based electronic and photonic components.

While photonics technology dominates telecommunication applications, IC chips that are widely used in computers, cell phones, and other electronic devices are still constructed from the transistor based CMOS technology. However, electron charge based CMOS technology is facing the rapidly growing challenges of off-state leakage, heat dissipation, and variability in transistor behavior as feature sizes scale down[6]. Alternative solutions to CMOS include spintronics[7], which utilizes the spin property of electrons instead of charge and "optical computers" that replace (at least partially) electrons with photons in optoelectronic devices.[8]. In both cases, optical cavities are important. In spintronics, optical cavities can be used to efficiently couple circularly polarized photons to electron spins. In optical computers, optical cavities form basic components of photon manipulation.

Given the importance of optical microcavities, this dissertation focuses on the fundamental understanding of a certain type of semiconductor optical microcavity known as microdisks. With their high quality factor, easy integration with III-V group laser gain materials, and lateral light emission geometry, microdisks are potentially important in integrated photonics.[9, 40].

1.2 Types of semiconductor optical microcavity

Optical cavities, ideally, confine light without loss and with resonance frequencies at precise values. They also have a size-dependent resonance frequency spectrum, in
which the resonance frequencies distribute more sparsely as the cavity volume shrinks.
To this end, a microcavity has fewer resonance frequencies in a certain spectrum range
than a corresponding "macrocavity". Later on we shall see the motivation behind making
this microcavity clearly as the discussion goes further on. The cavity Q factor, which is
proportional to the confinement time in the units of the optical period, is introduced to
characterize the non ideal cavity. Together with the microcavity volume (V), or effective
microcavity volume (V_{eff}), these two values form a good set of benchmarks of optical
microcavity properties. Semiconductor microcavities are the focus of this dissertation.
A summary of representative semiconductor microcavities is given in Figure 1.1 and the
microcavities are organized by column according to the confinement method used and
by row according to the cavity Q factor. This dissertation focuses on whispering gallery
mode cavities (shown in the center column).

Fig. 1.1 Summary of semiconductor optical microcavities.[9]
We now briefly discuss each of the three different confinement type cavities. The left upper cells show a micropillar (micropost), which is a Fabry-Perot type cavity. Bragg mirrors, periods of two material layers with different refractive index, are usually placed at the upper stack and below to form vertical one dimensional cavity confinement. Figure 1.2 gives an example of the micropost cavities. Most commercial vertical-cavity surface-emission lasers (VCSELs) adopt similar cavity structure. The one in the lower row on the left is Fabry-Perot bulk optical cavity used in atomic physics experiments.

![Fig. 1.2 An illustration of a micropost microcavity where a single quantum dot is shown to emit a photon through the cavity top. The insert shows a scanning electron microscopy image of such a micropost cavity.][9]

In the center column, starting from left in the upper row is a microdisk cavity and a semiconductor polymer add/drop filter; and in the lower row is a microsphere and a microtoroid cavity. These are different from the vertical one dimension confinement in Fabry-Perot cavities: in a ray approximation, light travels along the side edge in the
whispering gallery cavities like a sound wave does in the real whispering gallery at the St. Paul Cathedral in London. This also enables the coupling between different cavities or the coupling in/out of the cavities through waveguides or tapered fibers. Also in this type of cavity, air-dielectric guiding provides both lateral and vertical confinement which usually yields higher Q factors than in Fabry-Perot cavities. Figure 1.3 shows an example of microdisk cavity.

Fig. 1.3 (a) Cartoon of a microdisk cavity. The arrow shows the light propagation direction. (b) Magnetic field profile in a microdisk simulated by finite-difference time domain method. (c) A scanning electron microscopy image shows a coupled microdisk cavity array.

Photonic crystal[10] cavities, in the right column, have attracted a great attention because of their ultra small cavity volume and high Q factor. Unlike the first two types of cavities, where light confinement is achieved with conservation of symmetry, defects that break pattern symmetry are intentionally introduced in photonic crystal cavities to
form defect modes. Patterns are usually periodic holes in dielectric membrane materials. Bragg reflection provides lateral confinement and air-dielectric guiding provides vertical confinement. An example is given in Figure 1.4

Fig. 1.4 Cross-sectional illustration of a photonic crystal defect microcavity. Hexagonal array of holes are forms in a thin membrane while one hole in the center is unetched to form the defect region. [9]

1.2.1 Cavity quality factor $Q$ and related quantities

In this section, we summarize a number of physical quantities including cavity finesse $F$ and cavity quality factor $Q$.

The essential definition of cavity $Q$ factor is:

$$Q = \omega \tau_{ph},$$

where $\omega$ is the cavity resonant mode frequency and $\tau_{ph}$ is the photon lifetime within the cavity. Since $\tau_{ph} = \frac{1}{\Delta \omega}$, where $\Delta \omega$ is the spectral full width at half maximum (FWHM),
this gives another expression of Q factor used more in experimental measurements:

\[ Q = \frac{\omega}{\Delta \omega}. \]  

(1.2)

The photon lifetime represents the energy decay rate of the cavity and sometimes a cavity decay length \( L_{ph} \) is defined through:

\[ \tau_{ph} = \frac{L_{ph}}{c/n_g}, \]  

(1.3)

where \( n_g \) is the group index of the mode within the cavity and \( c \) is the speed of light in vacuum. The Q factor of the cavity is written in terms of \( L_{ph} \) as:

\[ Q = \frac{2\pi n_g L_{ph}}{\lambda}. \]  

(1.4)

Since material absorption losses are often quoted in terms of a loss per unit length, equation 1.4 tells us how to compare the equivalent absorption-limited Q.

As we mentioned before, a cavity’s Q factor gives the number of cycles the optical field undergoes before its energy decay to a value that is \( 1/e \) time to its original value. Therefore we have:

\[ \frac{dU}{dt} = -\frac{\omega}{Q} U. \]  

(1.5)
where $U$ is the stored energy within the cavity, and $P_d = -\frac{dU}{dt}$ is the dissipated power. This gives another definition of $Q$:

$$Q = \omega \frac{U}{P_d}, \quad (1.6)$$

with $\omega = \frac{2\pi}{T}$, $T$ is the period of the field, equation 1.6 can be re-written as:

$$Q = 2\pi \frac{U}{U_{l,c}}, \quad (1.7)$$

where $U_{l,c}$ is the energy loss per cycle. For some of the traveling wave mode cavities, such as Fabry-Perots or whispering gallery ones, a cavity finesse $F$ is usually defined as:

$$F = \frac{U}{U_{l,rt}}, \quad (1.8)$$

where $U_{l,rt}$ is the energy loss per round trip length (where the round trip length is $2L$ for a Fabry-Perot cavity of length $L$ and $2\pi R$ for a whispering gallery cavity of radius $R$). The finesse is related to cavity Q factor by the ratio of $U_{l,c}$ to $U_{l,rt}$ with a $2\pi$ modulo. This ratio is the number of optical cycles within a round trip length $L_{rt}$, which is $L_{rt}/(\lambda/n_g)$. Thus we have:

$$F = \frac{Q \lambda}{2\pi n_g L_{rt}}, \quad (1.9)$$

Finally, with the equation 1.4 we can simply write the finesse as:

$$F = \frac{L_{ph}}{L_{rt}}, \quad (1.10)$$
1.3 Semiconductor cavity QED

1.3.1 Strong-coupling cavity QED

Strong coupling[11, 12, 13] in semiconductor cavity QED[14] context, means that a photon emitter (say an exciton) coherently interacts with a cavity mode for a meaningful time so that instead of emitting a photon to the surrounding in a irreversible way, the emitted photon can be re-absorbed by the exciton and emitted again and again. This kind of coupling, which results in a quantum energy shifting back and forth between the exciton and the cavity mode at the vacuum Rabi Frequency, is called strong coupling. To enter the strong coupling regime, the exciton mode-photon coupling rate $g$ needs to be larger than both the cavity loss rate $\kappa$ and photon emitter dipole decay rate $\gamma$. In particular, the ratio of $g$ to the larger of $\kappa$ and $\gamma$ approximately represents the number of Rabi oscillations that can take place before the effects of dissipation destroy coherent energy exchange. In most of the semiconductor cavity cases, loss in the system is found to be dominated by the optical cavity, with $\kappa > \gamma$. Hence the criterion for strong coupling can sometimes be approximated by $g > \kappa/4$. For example, the low temperature homogeneous line-width in self-assembled InAs QDs is typically a few $\mu$eV, corresponding to a QD dipole decay rate of $\gamma/2\pi \sim 1$ GHz[15]. To get a comparable cavity loss rate $\kappa/2\pi \sim 1$ GHz, for the $\lambda \sim 0.9-1.2$ $\mu$m emission wavelength, the corresponding optical mode cavity quality factor $Q$ is $\sim 10^5$ ($\kappa/2\pi=\omega/4\pi Q$). Therefore, it is desirable to achieve low loss cavity for strong coupling regime. On the other hand, the exciton mode-photon coupling rate $g$ can be written in terms of the oscillator strength as:
\[ g = \sqrt{\frac{e^2 f}{4\epsilon_0 n^2 V_{\text{eff}}}}, \]  

(1.11)

where \( f \) is the emitter oscillation strength and \( V_{\text{eff}} \) is the effective cavity mode volume defined as:

\[ V_{\text{eff}} = \frac{\int \epsilon(\vec{r})|\vec{E}(\vec{r})|^2 d^3r}{\epsilon(\vec{r}_{\text{max}})_{\text{max}}[|\vec{E}(\vec{r})|^2]}, \]  

(1.12)

where \( \vec{E}(\vec{r}) \) is the electric field at the location of the emitter.

It is also important to achieve small cavity mode volumes \( V_{\text{eff}} \) so that \( g \) is larger. It also needs to be pointed out that, as important as fabricating cavities with high Q factor and small cavity volume, choosing a photon emitter that has a higher oscillation strength has proven to be crucial for reaching strong coupling regime[16, 17, 54].

In spectroscopic experiments, coherent energy exchange of strong coupling results in anti-crossings between the photon emitter (exciton) and cavity mode (photon) dispersion relations and is characterized by the vacuum Rabi splitting. More specifically, a weak optical probe reveals that the cavity’s transmission and/or emitter’s photoluminescence spectra are split into two distinct peaks, which correspond to eigenfrequencies of the quantum entangled exciton-photon states. This splitting reaches its maximum at so-called zero detuning conditions: the photon emitter emits photons that have the same energy as the cavity mode photons.

Effects of strong coupling have been well demonstrated and thoroughly studied in high Q factor Fabry-Perot microcavities, such as micropillars. Observation of strong coupling of a single two-level solid state system with a photon, as realized by a single
quantum dot in a semiconductor microcavity, is reported with a vacuum Rabi splitting of about 140 $\mu$eV$^{[19]}$. However, strong coupling in whispering gallery cavities, which have even higher Q factors than Fabry-Perot ones, have only been shown recently. The relatively large cavity mode volume $V_{\text{eff}}$ of the whispering gallery cavities is thought to be a reason. Therefore, photon emitters that have higher oscillator strength, such as low indium content large InGaAs QDs and GaAs interface fluctuation QDs ($f \sim 100$) are chosen and play critical roles in achieving strong coupling in the whispering gallery cavities. One example is the report of a vacuum Rabi splitting about 200$\mu$eV using GaAs interface fluctuation QDs as photon emitters in a GaAs/AlGaAs microdisk microcavity$^{[54]}$. Photonic crystal microcavities can provide much smaller cavity mode volumes while possessing high Q factors. A vacuum Rabi splitting of about 170$\mu$eV is observed in a photonic crystal microcavity even using a self-assembled InAs QD with a smaller oscillation strength ($f \sim 10$)$^{[20]}$.

1.3.2 Enhancement and suppression of spontaneous emission: Purcell effect

Weak coupling, the opposite counterpart of strong coupling, refers to the irreversible spontaneous emission process when a photon emitter is coupled with a cavity. One apparent criterion is that the exciton mode-photon coupling rate $g$ is smaller than the larger one of the cavity loss rate $\kappa$ and photo emitter dipole decay rate $\gamma$. Another requirement, although is noted in the previous section and is satisfied in most cases, is that the cavity loss rate $\kappa$ is larger than photo emitter dipole decay rate $\gamma$: $\kappa > \gamma$. In other words, the escape time of spontaneous emission photons out of the cavity is shorter than the dipole decay time and hence the non coherent re-absorption is also negligible.
Purcell[21] first came to the conclusion that the spontaneous emission rate, when the photon emitter is *properly* coupled with a cavity: spectrally on resonance and spatially located at the antinode of the vacuum field with its dipole parallel to the vacuum electric field, is enhanced by a Purcell factor ($F_p$) defined as:

$$F_p = \frac{3}{4\pi^2} \left(\frac{\lambda}{n}\right)^3 \frac{Q}{V_{eff}}, \quad (1.13)$$

where $\lambda$ is the wavelength of the cavity mode. A larger Purcell factor gives more freedom in manipulation of spontaneous emission rate, which has become an important application of microcavities[22]. Similar to the situation in strong coupling regime, cavities with high Q factor and small cavity mode volume are advantageous in achieving high Purcell factors. However, the choice of emitter sets a upper limit on the Q factor to enter the weak coupling regime. As mentioned before, weak coupling requires $\kappa > \gamma$; the Purcell effect occurs when the emitter is in resonance with the cavity: in other words, the mode–cavity mode wavelength ($\lambda_c$) is the same as the emitter wavelength ($\lambda_e$): ($\lambda_c = \lambda_e = \lambda$).

Therefore with $\kappa \sim \Delta \lambda_c$ (cavity mode linewidth), $\gamma \sim \Delta \lambda_e$ (emitter linewidth):

$$Q = \frac{\lambda}{\Delta \lambda_c} < \frac{\lambda}{\Delta \lambda_e}. \quad (1.14)$$

Hence choosing a photon emitter that has a narrow linewidth[23, 24] lifts the upper limit for the Q factor needed to reach Purcell effect. With a larger Q factor, a higher Purcell factor is possible. For example, normal semiconductor QWs have linewidth $\sim 1$nm which corresponds to a Purcell factor $F_p \sim 1$, while a single self assembled InAs
QD has a linewidth < 0.1nm and the corresponding Purcell factor is at least about one order larger.

Time-resolved spectroscopy is usually employed in probing the Purcell effect by measuring the recombination time or decay time. This decay time, in a time dependent spontaneous emission intensity plot, is shortened when the photon emitter is in proper coupling with the cavity as a result of the Purcell effect. Femtosecond (150fs, 200fs) or small picosecond (1.5ps) pulse laser probes and picosecond time resolution streak cameras are needed in the experiments. Control measurements include the decay time measurement in (a) bulk QDs, (b) QDs embedded in a cavity but out of resonance with either spatial detuning or spectral detuning. Limited excitation power is also necessary so that only the resonant QDs are excited. Non-radiative surface recombination may also shorten the decay time which is not a generic result from Purcell effect. Emitters of certain materials, such as GaAs interface fluctuation QDs, that have longer surface recombination velocity and cavities that have high surface to volume ratios need cautious control measurements to rule out the effect of non-radiative recombination. The Purcell effect has been demonstrated in all three types of microcavities: In most cases, self-assembled InAs QDs are embedded in the cavities because of their narrow linewidth. Purcell factors measured with this time-resolved method are: $F_p \sim 5$ for microposts and $F_p \sim 15$ for microdisks. Estimated Purcell factors based on Q factor measurement and cavity mode volume estimates are typically much higher: $F_p \sim 32$ for microposts and $F_p \sim 190$ for microdisks.
1.4 Optical systems based on the coupling of microcavities, waveguides and fibers

1.4.1 Coupled microcavities and waveguides: filters in optical communications

Microcavities are becoming important in modern optical communication applications not only as laser sources but sometimes as passive filters that enable resonant transfer of optical power between two waveguides. This application has received considerable attention since the deployment of wavelength division multiplexed (WDM)[25, 26] light wave systems in long-distance transmission. In fiber-optics communication, WDM is a technology which multiplexes multiple optical carrier signals on a single optical fiber by using different wavelengths of laser light. This allows for a multiplication of capacity in addition to possible bidirectional communication through one strand of fiber. Hence a passive filter that has the ability to add or drop certain wavelength (channel) has a great application potential.

Whispering gallery cavities, as mentioned before, are relatively easy to couple with phase matched waveguides. Therefore, a filter that consists of a single whispering gallery microresonator sandwiched between two single-mode waveguides that are phase matched to certain microresonator modes can perform a function called channel add/drop in which a single channel is "dropped" with high extinction from a first wavelength and coupled with low loss to a second waveguide[27, 28, 29]. Microresonators, such as microdisks, with additional electrically controllable refractive index could lead to dynamic add/drop functions, tuning, ultrafast modulation and switching of resonant wavelength. Arrays of
these devices, thanks to the compact sizes of microresonators, could be fabricated and interconnected by fibers on common substrates to perform complex functions within the context of WDM.

1.4.2 Coupled microcavities: photonic molecules (PMs)

As optical cavities confine light spectrally at resonant frequencies and spatially inside cavities, variations of the geometrical shape and arrangement of cavities modify the electromagnetic field profiles and hence change the optical properties of the cavities. More complex structures are formed by coupling of microcavities, usually through the evanescent field, to modify and improve optical properties. For example, "heavy photon modes"–the modes with slow group velocity–[69] are expected in those structures and may serve as an "optical storage" for optical routing systems. A reduction of lasing threshold is predicted in certain patterns of photonic molecules[59] and a new mechanism of bistable lasing is presented based on diatomic photonic molecule structures[57]. Since the coupled cavity modes behave similar to the energy states in chemical molecules, these coupled cavity systems are often called photonic molecules. Among those cavities, there has been significant attention to whispering gallery cavities whose resonant modes spatial distribution can be described by analogy with the orbitals in a hydrogen atom.

We follow the standard quantum mechanics discussion of a simple chemical molecule case–the hydrogen molecule–and derive the two simplest eigenstates–bonding and antibonding states, which also are a good analogy to the resonant modes in photonic
molecules. The two eigenenergies are $E_-$ and $E_+$ and they are given as:

\[
E_+ = (-1 + \frac{2}{\rho} + \frac{\alpha - \gamma}{1 - S})E, \tag{1.15}
\]

\[
E_- = (-1 + \frac{2}{\rho} - \frac{\alpha + \gamma}{1 + S})E. \tag{1.16}
\]

where $E$ is the ground state energy of an isolated hydrogen atom or the resonant mode energy of an isolated cavity; $\rho = \frac{R}{a_0}$; $\alpha = \frac{A}{E}$; $\gamma = \frac{C}{E}$, and:

\[
A = E \times 2e^{-\rho}(1 + \rho), \tag{1.17}
\]

$A$ is called resonance integral.

\[
C = E \times \frac{2}{\rho}[1 - e^{-2\rho}(1 + \rho)], \tag{1.18}
\]

$C$ is called Coulomb integral.

\[
S = e^{-\rho}[1 + \rho + \frac{1}{3} \rho^2]. \tag{1.19}
\]

$S$ is called overlap integral. The eigenstate corresponding to $E_-$ is called a bonding state, and the one corresponding to $E_+$, an antibonding state.

Experimentally, the coupling of the resonant modes of individual microcavities in the biatomic-like PMs splits each mode into two modes. Bonding state modes have lower energies and antibonding state modes have higher energies. Equations 1.15 and 1.16 suggest that the energy splittings of bonding and antibonding states decrease with
increasing distance between two hydrogen atoms in the case of hydrogen molecules, or between two microcavities in the case of photonic molecules. Therefore, we characterize the coupling strength by the resonant modes energy splittings and expect to see anticrossing behavior in the resonant mode spectrum when two microcavities are brought closer and engaged into coupling. Most of the photonic molecules are formed by whispering gallery cavities because they are easier to couple with each other laterally through evanescent field. There are a few cases where Fabry-Perot cavities form photonic molecules through narrow waveguide channels. Microspheres made from polystyrene used to be the major microcavities to form photonic molecules. In that case, preliminary size screening has to be carried to minimize the size variation and special designed groove shape stages are needed for each different geometry arrangements. Recently, laterally coupled microdisk cavity systems are fabricated with the state-of-the-art processes in a more controllable way so that the cavity shape and arrangement are designed during lay out and implemented during fabrication. This dissertation will come back to this topic in more details.

1.5 Semiconductor lasers

1.5.1 Laser rate equation

For semiconductor lasers, the rate equations are often a pair of equations that describe the time evolution of the carrier number ($N$) and the cavity mode photon number ($N_p$). For the semiconductor material that we discuss in this dissertation, light emission occurs as a result of electron-hole recombination. We keep track of a single
carrier number $N$ and the rate of change of $N$ will be given by the difference between carrier generation process and carrier recombination process. For a semiconductor laser, current injection or optical pumping generates carriers at a rate $L$. Recombination processes include stimulated and spontaneous emission ($R_{st}$ and $R_{sp}$) and non radiative recombination ($R_{nr}$). The rate of change of carrier number $N$ is written as:

\[
\frac{dN}{dt} = L - (R_{nr} + R_{sp} + R_{st})V. \tag{1.20}
\]

where $V$ is the volume of the active region.

Similarly, the rate of change of $N_p$ will be given by difference in photon generation and photon removal processes. Stimulated emission and spontaneous emission generate photons, while cavity loss leads to photon removal. Hence we write the equation as:

\[
\frac{dN_p}{dt} = (R_{st} + \beta R_{sp})V - \frac{N_p}{\tau_{ph}}, \tag{1.21}
\]

where $\gamma_{ph} = 1/\tau_{ph}$ is the photon number loss rate from the cavity ($= \omega/Q$). $\beta$ is called the spontaneous emission coupling constant (mentioned in last section) and corresponds to the fraction of spontaneous emission emitted into cavity mode of interest.

Let us first consider stimulated recombination $R_{st}$. Stimulated emission generates photons with the presence of seed photons. Therefore:

\[
R_{st} = v_g g_t N_p, \tag{1.22}
\]
where $v_G$ is the group velocity of the cavity mode and $g_l$ is the gain per unit length. We can also define a gain per unit time $g$ as $g = v_g g_l$. Next, the spontaneous emission $R_{sp}$ is often taken to be a bi-particle process (electron-hole recombination), so that $R_{sp} = B N^2$, where $B$ is called bimolecular recombination rate. Non-radiative recombination is usually a combination of processes with varying power law dependence on $N$. One process that is important to microdisk lasers is surface recombination, which is often taken as $R_{sr} = AN$, where $A$ is some material dependent coefficient. Another process is Auger recombination, the transfer of kinetic energy from an electron-hole pair to another electron or hole. It is often taken as $R_A = CN^3$, where $C$ is called Auger recombination coefficient.

The rate equations we use to model the microdisk lasers are:

$$\frac{dN}{dt} = L - \left( \frac{N^{1.22}}{\tau_s} + \frac{N^2}{\tau_{sp}} \right) = g N_p,$$

\hspace{1cm} \text{(1.23)}

$$\frac{dN_p}{dt} = (g - \gamma_{ph}) N_p + \frac{\beta N^2}{\tau_{sp}'}.$$

\hspace{1cm} \text{(1.24)}

Here, we have assumed an $N^2$ dependence for radiative recombination, no Auger recombination, and have taken the surface recombination term to have a $N^{1.22}$ dependence[30]. Also in these equations, the coefficients in front of the $N$-dependent terms have been taken into lifetimes from rates. For example, $\tau_s$ is the surface recombination lifetime and $\tau_{sp}'$ is the Purcell-factor-modified spontaneous emission lifetime of the quantum dots.

In our microdisk cavities, carrier generation is accomplished through optical pumping, where the measured quantity is the pump power incident on the sample surface,
$P_{inc}$. $L$ is related to $P_{inc}$ through:

$$L = \frac{P_{inc} \eta_{abs} \eta_{int}}{E_{ph,pump} \frac{A_m}{A_{pump}}}.$$  \hspace{1cm} (1.25)

where $\eta_{abs}$ is the fraction of incident pump power that is absorbed, $\eta_{int}$ is the internal efficiency of carrier generation, $E_{ph,pump}$ is the energy per pump photon, $A_m$ is the modal area, and $A_{pump}$ is the pump beam area. Basically, $P_{inc} \eta_{abs} A_m/A_{pump}$ gives the absorbed pump power by the disk, dividing by $E_{ph,pump}$ converts this to an absorbed photon number rate, and multiplying by $\eta_{int}$ converts this to carrier generation rate.

The surface recombination lifetime $\tau_s$ is taken as:

$$\tau_s = \frac{1}{2(2\pi R \rho_{A,QD}) v_s}.$$  \hspace{1cm} (1.26)

Here, $\rho_{A,QD}$ is the areal quantum dot density, a quantity estimated by the material growers, so that $2\pi R \rho_{A,QD}$ gives a linear QD density along the perimeter of the device. $v_s$ is the surface recombination velocity, which in simulations is taken as a fit parameter often times, varies with materials.

The gain per unit time $g$ is taken to have the form:

$$g = g'(N - N_{tr}).$$  \hspace{1cm} (1.27)

where $g'$ is the differential gain and $N_{tr} = \rho_{A,QD} A_m$ is the transparency carrier number. $g'$ is taken to be the maximum modal gain if all QD ground states interacting with the cavity mode are inverted divided by the total number of QD states.
We thus solve the rate equations in steady state to give us the steady state photon number \( N_{p,ss} \) as a function of pump power. The collected laser power \( L_{out} \) is related to \( N_{p,ss} \) through:

\[
L_{out} = \eta_{coll} E_{ph} \gamma_{ph} N_{p,ss},
\]

where \( E_{ph} \) is the emitted photon energy \( (E_{ph} = \hbar \omega) \) and \( \eta_{coll} \) is the collection efficiency. Finally, we have:

\[
\eta_{coll} = \varepsilon \frac{E_{ph,pump}}{E_{ph}} \eta_{int},
\]

where \( \varepsilon \) is the laser’s differential efficiency and \( E_{ph,pump}/E_{ph} \) is the ratio of the energies of the pump and emission photons.

### 1.6 Whispering Gallery modes microdisk cavities

Whispering Gallery modes cavities, especially the microdisk ones, are the main cavities utilized in the experiments and simulations discussed in this dissertation. We now give a simple analytic solution of the microdisk cavities and take note of one special modal property of the microdisk cavities—standing wave whispering gallery modes as follows.

#### 1.6.1 Theories of microdisk cavity modes

Whispering gallery modes (WGMs) are classified in terms of their polarization (TE or TM), radial order \( (p) \), and azimuthal number \( (m) \). As microdisks are often optically thin, only the first order TE and TM modes are considered. Unlike microspheres,
where the WGMs can be analytically solved, microdisk modes do not have an analytic solution. An approximated analytic solution is presented as follows[31].

We start with Maxwell’s equations in a charge-free, current-free medium:

\[ \nabla \times \mathbf{E} = -i\omega \mu_0 \mathbf{H}, \] (1.30)

\[ \nabla \times \mathbf{H} = +i\omega n^2 \epsilon_0 \mathbf{E}, \] (1.31)

\[ \nabla \cdot (n^2 \epsilon_0 \mathbf{E}) = 0, \] (1.32)

\[ \nabla \cdot \mu_0 \mathbf{H} = 0. \] (1.33)

Wave equations are derived as:

\[ \nabla^2 \mathbf{E} + \frac{n^2 \omega^2}{c^2} \mathbf{E} = 0, \] (1.34)

\[ \nabla^2 \mathbf{H} + \frac{n^2 \omega^2}{c^2} \mathbf{H} = 0. \] (1.35)

As the form of equations 1.34 and 1.35 is the same, we use a vector field \( \mathbf{F} \) to stand for either \( \mathbf{E} \) or \( \mathbf{H} \). Then we write this in cylindrical coordinates \((\rho, \phi, z)\):

\[ \left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} + (\frac{n\omega}{c})^2 \right) \mathbf{F} = 0. \] (1.36)

We now apply the major approximation, which is to separate the modes into TE and TM polarizations. Then \( \mathbf{F} \) has the field components \( \{E_\rho, E_\phi, H_z\} \) and \( \{H_\rho, H_\phi, E_z\} \) respectively. Thus \( F_z \) is \( H_z \) (\( E_z \)) for TE (TM) modes. We separate variables with
The radial solution Φ(ρ) has the form:

\[ \Phi(\rho) = \begin{cases} 
  J_m\left(\frac{\omega}{c}\bar{n}\rho\right), & \rho \leq R, \\
  J_m\left(\frac{\omega}{c}\bar{n}R\right)\exp\left(-\alpha(\rho-R)\right), & \rho \geq R, 
\end{cases} \]  

(1.40)

The decay constant α is given as \( \alpha = \frac{\omega}{c}\left(\bar{n}^2 - n_0^2\right)^{1/2} \) (\( n_0 = 1 \) for an air-clad disk); the azimuthal mode number \( m \) is determined by the boundary conditions on the field at \( \rho = R \). This gives the transcendental equation:

\[ \frac{\omega}{c}\bar{n}J_{m+1}\left(\frac{\omega}{c}\bar{n}R\right) = \left(\frac{\omega}{R} + \eta\alpha\right)J_m\left(\frac{\omega}{c}\bar{n}R\right), \]  

(1.41)

where \( \eta = \bar{n}^2/n_0^2 \) for a TE mode and \( \eta = 1 \) for a TM mode.
A rough estimate of \( m \) is made to require \( m \) wavelengths fit in the circumference of the disk. This is stated as:

\[
m \frac{\lambda_m}{n_g} = 2\pi R, \tag{1.42}
\]

where \( \lambda_m \) is the resonant wavelength of mode \( m \), and \( n_g \) is the group index of the waveguide mode. The free spectral range (FSR), which gives the separation between adjacent modes is:

\[
\Delta \nu = \nu_{m+1} - \nu_m = \frac{c}{2\pi R n_g}. \tag{1.43}
\]

### 1.6.2 Standing wave whispering gallery modes

In realistic devices, the high-Q modes are not traveling waves but are instead standing waves. Surface roughness couples the forward and backward propagating disk modes coherently so that standing waves are formed in the cavities. This standing wave scenario posts some important modifications on the cavity related properties of the real microdisk cavities. Since actually there are two degenerate modes, the physical values, such as the spontaneous emission coupling constant \( \beta \), which are usually acquired experimentally, are double the real values for each of these modes. Also, standing wave microdisks have approximately half the volume of the traveling wave microdisks, so that the coupling rate \( g \) between a single quantum dot and a single photon in a standing wave cavity mode is expected to be \( \sqrt{2} \) times that when the quantum dot is coupled to a traveling wave cavity mode.
We present a simple analysis of this coupling here. Maxwell’s equation for the electric field vector in the microdisk structure is

\[ \nabla^2 \mathbf{E} - \mu_0 (\varepsilon^0 + \delta \varepsilon) \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0. \]  

(1.44)

where \( \mu_0 \) is the permeability of free space, \( \varepsilon^0 \) is the dielectric function for the ideal microdisk and \( \delta \varepsilon \) is the dielectric perturbation that is the source of mode coupling between the cw and ccw modes. The modes of the ideal structure are written as \( \mathbf{E}_j^0(r, t) = \mathbf{E}_j^0(r) \exp(i\omega_j t) \), and are solutions of equation 1.44 with \( \delta \varepsilon = 0 \). Trial solutions to the real structure, that is, to equation 1.44 with \( \delta \varepsilon \neq 0 \) are written as

\[ \mathbf{E}(r, t) = e^{-i\omega_0 t} \sum_j a_j(t) \mathbf{E}_j^0(r). \]  

(1.45)

Plugging the trial solutions into 1.44, keeping only terms up to first order, and utilizing mode orthogonality, we arrive at the coupled mode equations

\[ \frac{d a_k}{d t} + i \Delta \omega_k a_k(t) = i \sum_j \beta_{jk} a_j(t), \]  

(1.46)

\[ \beta_{jk} = \frac{\omega_0}{2} \frac{\int \delta \varepsilon \mathbf{E}_j^0(r) \mathbf{E}_k^0(r)^* \, dr}{\int \varepsilon^0 | \mathbf{E}_k^0(r) |^2 \, dr}. \]  

(1.47)

We now explicitly assume that only two modes (the cw and ccw modes of a given polarization (TE and TM), azimuthal mode number \( m \), and radial mode number \( P \)) are involved, and that the amplitude of the backscattering rates are equal, so that |
\( \beta_{cw,ccw} = |\beta_{ccw,cw}| = |\beta| \). The coupled mode equations then read as

\[
\frac{da_{cw}}{dt} = -i \nabla \omega a_{cw}(t) + i |\beta| e^{i\varepsilon} a_{ccw}(t),
\]

(1.48)

\[
\frac{da_{ccw}}{dt} = -i \nabla \omega a_{ccw}(t) + i |\beta| e^{-i\varepsilon} a_{cw}(T),
\]

(1.49)

where we have taken \( \beta = |\beta| e^{i\varepsilon} \). These equations represent the time evolution of the two mode amplitudes \((a_{cw}, a_{ccw})\) of an isolated system, without loss or coupling to an external waveguide. These two coupled equations can be uncoupled by introducing the variables \( a_{sw,1} \) and \( a_{sw,2} \), which represent the standing wave mode amplitudes:

\[
a_{sw,1} = \frac{1}{\sqrt{2}} (a_{cw} + e^{i\varepsilon} a_{ccw}),
\]

(1.50)

\[
a_{sw,2} = \frac{1}{\sqrt{2}} (a_{cw} - e^{i\varepsilon} a_{ccw}).
\]

(1.51)

For an ideal microdisk, \( a_{cw} \) and \( a_{ccw} \) have an azimuthal spatial dependence of \( e^{im\Phi} \) (where \( m \) is the azimuthal mode number and is a nonzero integer), so that \( a_{sw,1} \) and \( a_{sw,2} \) will have an azimuthal spatial dependence being a function of phase \( \varepsilon \) of the back scattering parameter \( \beta \). Rewriting the coupled mode equation in terms of the standing wave mode amplitude, we have:

\[
\frac{da_{sw,1}}{dt} = -i \Delta \omega a_{sw,1}(t) + i |\beta| a_{sw,1}(t),
\]

(1.52)

\[
\frac{da_{sw,2}}{dt} = -i \Delta \omega a_{sw,2}(t) - i |\beta| a_{sw,2}(t).
\]

(1.53)
Thus, we see that the standing wave modes resonate at frequencies $\pm |\beta|$ detuned from the original resonance frequency.

### 1.7 Finite-difference time-domain and finite element simulation

The whispering-gallery mode cavities studied in this dissertation are numerically investigated through the finite-domain time-difference (FDTD) method[32]. The FDTD method discretizes Maxwell’s equation, replacing derivatives with finite differences that are accurate to second order. One of the commonly used implementations is based on the Yee algorithm. The Yee algorithm[33] is a discretized solution to the Maxwell’s curl equations for both the electric and magnetic fields in time and space. One way to construct the Yee algorithm is to follow a grid in which every electric field component is surrounded by four magnetic field components, and vice versa. This arrangement satisfies the Faraday’s and Ampere’s laws as well as Gauss’s divergence laws.

Maxwell’s curl equations for an isotropic medium are:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}, \quad (1.54)$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}. \quad (1.55)$$

These can be written as six scalar equations in Cartesian coordinates,

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu} \left( \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right), \quad (1.56)$$
\[
\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \left( \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right),
\] (1.57)

\[
\frac{\partial H_z}{\partial t} = \frac{1}{\mu} \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right),
\] (1.58)

\[
\frac{\partial E_x}{\partial t} = \frac{1}{\epsilon} \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x \right),
\] (1.59)

\[
\frac{\partial E_y}{\partial t} = \frac{1}{\epsilon} \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y \right),
\] (1.60)

\[
\frac{\partial E_z}{\partial t} = \frac{1}{\epsilon} \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z \right).
\] (1.61)

Yee defines the grid coordinates \((i,j,k)\) as,

\[(i, j, k) = (i \Delta x, j \Delta y, k \Delta z),\] (1.62)

where \(\Delta x, \Delta y, \text{ and } \Delta z\) are the actual grid separations. Any function of space and time is written as,

\[F^n(i, j, k) = F(i \Delta x, j \Delta y, k \Delta z),\] (1.63)

where \(\Delta t\) is the time increment, \(n\) is the time index and \(\delta = \Delta x = \Delta y = \Delta z\).

The spatial and temporal derivatives of \(F\) are written using central finite difference
approximations as,

\[ \frac{\partial F^n(i, j, k)}{\partial x} = \frac{F^n(i + 1/2, j, k) - F^n(i - 1/2, j, k)}{\delta}, \quad (1.64) \]

\[ \frac{\partial F^n(i, j, k)}{\partial t} = \frac{F^{n+1/2}(i, j, k) - F^{n-1/2}(i, j, k)}{\Delta t}, \quad (1.65) \]

Equations 1.64 and 1.65 are applied to the six scalar equations 1.55, resulting in six coupled explicit finite difference equations,

\[ E^{n+1}_x(i + 1/2, j, k) = A_{i+1/2,j,k}E^n_x(i + 1/2, j, k) + B_{i+1/2,j,k}[H^{n+1/2}_x(i + 1/2, j + 1/2, k) \]
\[ - H^{n+1/2}_z(i + 1/2, j - 1/2, k) + H^{n+1/2}_y(i + 1/2, j, k - 1/2) \]
\[ - H^{n+1/2}_y(i + 1/2, j, k + 1/2)], \quad (1.66) \]

\[ E^{n+1}_y(i, j + 1/2, k) = A_{i,j+1/2,k}E^n_y(i, j + 1/2, k) + B_{i,j+1/2,k}[H^{n+1/2}_x(i, j + 1/2, k + 1/2) \]
\[ - H^{n+1/2}_x(i, j + 1/2, k - 1/2) + H^{n+1/2}_x(i - 1/2, j + 1/2, k) \]
\[ - H^{n+1/2}_z(i + 1/2, j + 1/2, k)], \quad (1.67) \]
\[ E^{n+1}_z(i, j, k + 1/2) = A_{i,j,k+1/2}E^n_z(i, j, k + 1/2) + B_{i,j,k+1/2}[H^{n+1/2}_y(i + 1/2, j, k + 1/2) - H^{n+1/2}_y(i - 1/2, j, k + 1/2) + H^{n+1/2}_x(i, j - 1/2, k + 1/2) - H^{n+1/2}_x(i, j + 1/2, k + 1/2)], \]  

\[ H^{n+1/2}_x(i, j + 1/2, k + 1/2) = H^{n-1/2}_x(i, j + 1/2, k + 1/2) + \frac{\Delta t}{\mu \delta}[E^n_y(i, j + 1/2, k) - E^n_y(i + 1/2, j, k) + E^n_z(i, j, k + 1/2) - E^n_z(i, j + 1/2, k + 1/2)], \]  

\[ H^{n+1/2}_y(i + 1/2, j, k + 1/2) = H^{n-1/2}_y(i + 1/2, j, k + 1/2) + \frac{\Delta t}{\mu \delta}[E^n_z(i + 1, j, k) - E^n_z(i + 1/2, j, k) + E^n_x(i, j + 1/2, k) - E^n_x(i + 1/2, j + 1/2, k)], \]  

\[ H^{n+1/2}_z(i + 1/2, j + 1/2, k) = H^{n-1/2}_z(i + 1/2, j + 1/2, k) + \frac{\Delta t}{\mu \delta}[E^n_x(i + 1/2, j, k) + E^n_y(i, j + 1/2, k) - E^n_y(i + 1, j + 1/2, k)], \]
where the terms A and B are given by,

\[ A_{i,j,k} = 1 - \frac{\sigma(i,j,k)}{\epsilon(i,j,k)}, \] (1.72)

\[ B_{i,j,k} = \frac{\Delta t}{\epsilon(i,j,k)\delta}. \] (1.73)

In all of the finite difference equations the components of \( \vec{E} \) and \( \vec{H} \) are located within a single unit cell in the three-dimensional lattice. \( \vec{E} \) and \( \vec{H} \) are evaluated at alternate half time steps using Equations 1.66-1.71, such that all field components are calculated in each time step \( \Delta t \).
2.1 Introduction

The experiments in this dissertation utilize methods in the following three areas: molecular beam epitaxy (MBE) semiconductor sample growth, cleanroom semiconductor processing and optical spectroscopy characterization. General descriptions about each major experimental method in those areas are given in this chapter as all these experimental methods serve active and important roles not only in scientific research but in state-of-art manufacturing and engineering.

2.2 Molecular beam epitaxy (MBE) and its components

2.2.1 Overview

Molecular beam epitaxy (MBE) along with metal organic chemical vapor epitaxy (MOCVD) are the two most widely used epitaxial crystal growth methods in semiconductor materials research and engineering. Both methods, especially MBE, give precise control of atomic layer-by-layer construction of thin films in semiconductor heterostructures fabrication. MBE was firstly developed by Cho and Arthur at Bell Labs in the late 1960s particularly as an application to III-V compound semiconductors[34]. In MBE, fluxes of constituent elements and dopants are generated and then they react on the
single crystal substrate surface to form ordered films. The growth rates, crystal composition and doping level of ordered films are well controlled by the relative arrival rates of the constituent elements and dopants through direct and indirect flux monitoring methods such as flux ion gauge reading and reflection high energy electron diffraction (RHEED) oscillation. Also the low growth rate (a few monolayer/sec) with ultra high vacuum ($10^{-10}$ torr) chamber ambient and elevated substrate growth temperature (a few hundred °C) guarantee high quality crystal growth. A representative scheme of an MBE system and a picture of the MBE system used in this dissertation are shown in Figure 2.1.

As shown in the Figure 2.1, a standard state-of-art MBE system should have at least the following components: a matrix of chambers that are constructed of polished stainless steel with low vapor pressure and interconnected with UHV transfer chambers, effusion cells loaded with high purity growth elements and equipped with temperature controlled power sources, vacuum pumps, substrate loading/unloading/transferring robotic systems and RHEED systems.

### 2.2.2 Vacuum pumps and chambers

Both the stainless steel chambers and vacuum pumps are designed to maintain the UHV condition in MBE systems, which is critical for high quality growth. The mean free paths of molecules and atoms heated from the effusion cells are longer than the distance from the cells to the substrate in UHV conditions. Hence the arrival rates of the molecules and atoms are solely determined by the thermal conditions of the cells without the influence of collisions during the diffusion paths. This serves as the core
Fig. 2.1 A schematic diagram of an MBE system and a picture of Applied EPI/Veeco 930/620 MBE systems discussed in this dissertation.
mechanism of MBE growth. Also, under the UHV conditions, impurity levels from the residual gases in the chambers are minimized and in situ surface monitoring using RHEED is possible.

Typical vacuum pumps used for MBE systems can be categorized into two types according to their pumping abilities. One type is for UHV pumping and the other is for chamber roughing. Ion pumps and cryopumps are the most commonly used UHV pumps. Ion pumps operate by ionizing gas residues to break chemical bonds and trapping them inside magnetically confined cathodes. They are capable of achieving UHV conditions as low as $10^{-11}$ torr and operate quietly without motion of physical parts. The drawback of ion pumps is their relatively low pumping rate, especially for the low molecule weight gases like hydrogen, which makes them unsuitable for pumping vacuum systems that have extra water vapor. Also since ion pumps do not actually remove the residues from vacuum systems, maintenance jobs such as changing the cathodes are necessary. Cryopumps, on the other hand, work with a different mechanism. Using high purity helium gas to cool the cold head inserted in a chamber to a few Kelvin, cryopumps utilize cold trap mechanism by either condensing or dragging the gas residues on or near the cold head surfaces to lower the chamber pressure. Different from ion pumps, cryopumps are good at pumping under extra water vapor circumstances with cold head temperatures around a few Kelvin, cryotrapping even hydrogen molecules. Similar to ion pumps, all the residues are not removed from the vacuum systems either. They are just trapped on the cold head surfaces without contributing to the chamber pressure. Thus one also expects that cryopumps saturate at some working time length and need certain maintenance. This type of maintenance is called regeneration, which essentially heats up
the cold head and uses other pumps to remove the residues on the cold head away from
the vacuum systems. One note about the cryopumps is that since a compressor is needed
to cool the cold heads, cryopumps work at a much higher noise and vibration level than
ion pumps. Turbo pumps are sometimes used in UHV systems too. But the highest
vacuum level they are able to achieve is limited to $10^{-9}$ torr. Pumps for roughing can
only achieve much lower vacuum ($10^{-3}$ torr). Since neither ion pumps nor cryopumps
has efficient pumping abilities at this vacuum level, we use the roughing pumps to first
pump down the vacuum systems to a level where the UHV pumps work efficiently.
Although mechanical pumps can be used, they have the danger of oil contamination
in the chambers. Hence, oil free dry roughing pumps are typically employed in most
state-of-art MBE systems.

In our Veeco/Applied EPI 620 and 930 systems, both ion pumps and cryopumps
are used as UHV pumps. Specifically, the 930 system has two ion pumps for its growth
and buffer chambers. The 620 system has a cryopump connected to the growth chamber
and a turbo pump to the buffer chamber. A scroll pump serves as a roughing pump and
a few sorption pumps cooled by liquid nitrogen are also used as an alternative.

### 2.2.3 Effusion cells

Other important components of MBE systems are effusion cells whose temper-
\at\ure is controlled by power supplies and feedback control. As mentioned before, the
crystal growth rates of MBE methods are mainly determined by the temperatures of
effusion cell sources. Therefore precise heating and temperature control of the effusion
cells are critical. Single-channel effusion cells, conical in shape, are at present frequently
used as sources of the main molecular and atomic beams in MBE systems. Conical shape is proven to achieve better angular and surface distribution of the beam fluxes. A tungsten filament is usually used as a heat source and a crucible loaded with source material is placed inside a cell for a uniform heating. For some certain types of materials, such as gallium or aluminum, dual filament effusion cells are used to deliberately generate temperature differences at different parts of the crucibles to form "hot lip" or "cold lip" cells. For example, when used for gallium or indium, "hot lip" cells prevent material condensation at the lips of the crucibles and are proven to reduce the formation of oval defects at the growth surface. While for aluminum, a "cold lip" cell helps to prevent melted aluminum overflowing out of the crucible and cracking the cell. This also helps to maintain a more stable and uniform aluminum flux. Fig. 2.2 gives an illustration of this kind of material condensation happens in gallium and indium cells. Pyrolytical boron nitride (PBN) is commonly used as the crucible materials due to its good mechanical, thermal and chemical properties. It is perfect to use in a range of high temperature because of the special characteristic of increasing its tensile strength as the temperature rises. The thermal conductivity of PBN is very good: 0.17 cal/cm-sec-K at 500°C. It also has good chemical stability and resistance to corrosion. Temperatures in effusion cells are monitored with W-Re based thermocouples. These refractory alloys are suitable for operation at elevated temperatures. Effusion cells designed as specified above have a operating temperature range up to 1400°C, which allows short outgassing sequences at 1600°C. These temperatures are more than adequate for common III-V material evaporation.
In situ RHEED has become a standard and important part of state-of-art MBE systems to not only monitor real-time growth surface but calibrate crystal growth rates through RHEED intensity oscillation. Under the conditions which are usually employed in MBE, a high energy beam of electrons in the range of 5-40 keV is directed at low angle (1° – 3°) to the surface. The de Broglie wavelength of these electrons is therefore in the range 0.17-0.06 Å and the penetration of the beam into the surface is low, being restricted to the outmost atomic layers. Therefore the reflected electron beams serve as a good probe of crystal surface profiles. The diffraction process in RHEED is in general not a true reflection, as the scattered beam does not leave the substrate from the same surface where the incident beam entered. Most surfaces are rough and the diffraction pattern is produced in transmission through surface asperities. In this case the diffraction pattern exhibits many spotty features. For diffraction on smooth surfaces the RHEED pattern is usually constituted of elongated streaks which are normal to the shadow edge. This
case is closer to the true reflection diffraction and can be explained by considering that
the penetration of the electron beam into the solid surface is restricted to the uppermost
layer of the crystal. In other words, spotty or streaky RHEED patterns indicate the
surface smoothness of a crystal surface.

RHEED diffraction patterns can be correlated with periodic surface atom re-
arrangements, which in many cases differ from those in bulk crystals. These kind of
rearrangements are called surface reconstructions and usually denoted by a convention
described by Wood. For example a surface structure denoted by GaAs (001)-(m×n)
means that a GaAs crystal is oriented with the [001] direction normal to the surface and
has a surface structure, due to reconstruction, whose unit mesh is m×n times larger than
the underlying bulk structure. Such surface meshes may be centered, in which case the
notion would be GaAs (001)-c(m×n). If the mesh is rotated so that its principal axes are
not aligned with those of the underlaying bulk then the rotation angle is also included.

It has become apparent that when MBE growth is initiated, the intensity of
RHEED patterns shows time-dependent oscillations that are directly related to growth
rates and alloy compositions. As shown in the Fig 2.3, the RHEED intensity oscillation
period corresponds to the growth of a single monolayer. By comparing the growth rate
differences from different alloy materials, we may determine the composition of ternary
materials (such as Ga$_{1-x}$Al$_x$As) by:

\[
x = \frac{R(\text{ternary}) - R(\text{binary})}{R(\text{ternary})}.
\]

(2.1)

where R() is the growth rate.
Fig. 2.3 Development of monolayer coverage during layer-by-layer growth in relation to RHEED specular beam intensity oscillation. [36]
2.3 Electron beam lithography

2.3.1 Overview

Lithography transfers design patterns from a mask or CAD files to the lithography resist layer coated on a wafer surface. These patterns may be further transferred on to the wafers as structures through steps such as etching and lift-off. Wavelengths of light sources used in photolithography set limits of resolution sizes and hence feature sizes. As features sizes decrease, more complicated optical systems are needed to overcome the diffraction effects. Electron beams are also utilized since an electron’s de Broglie wavelength is much smaller. Lithography resists are basically light or electron sensitive materials whose chemical properties change under certain doses of light or electrons. Developers make use of this property change so that the exposed parts of resists are either developed away (positive resist) or intact (negative resist).

2.3.2 Electron beam lithography system components

Electron beam lithography or ebeam lithography is used more often in research and development applications for its flexibility and reasonable patterning speed for feature sizes smaller than 200nm. The set up of an ebeam lithography machine is similar to a scanning electron microscope (SEM): an electron beam of energy 10 keV-100 keV is formed and scanned at a controlled rate over the resist surface. The part of the ebeam lithography system that forms the electron beam is normally referred to as the column. An ebeam column typically consists of an electron source, two or more electron lenses,
a mechanism for deflecting the beam, a blanker for turning the beam on and off, a stigmator for correcting any astigmatism in the beam, aperture for helping to define the beam, alignment systems for centering the beam in the column, and finally, an electron detector for assisting with focusing and locating marks on the samples. Fig. 2.4 shows the Leica EBPG-5HR ebeam system used in this dissertation.

Among all these components, the electron source is of the most importance and complexity. The electron beam is generated from a conducting material either by heating it to a point where the electrons have sufficient energy to overcome the work function barrier of the conductor (thermionic sources) or by applying an electric field sufficiently strong that they tunnel through the barrier (field emission sources). In the past, an electrically heated loop of tungsten wire was typically employed as a standard thermionic electron source. Although tungsten has the ability to withstand high temperatures without melting or evaporating, this source is not very bright and also has a large energy spread caused by the very high operating temperature (2700 K). Recently, lanthanum hexaboride has become the cathode of choice. It has a very low work function, a high brightness that is obtained at an operating temperature of around 1800 K. The beam current delivered by the thermionic sources depends on the temperature of the cathode. Higher temperatures can deliver greater beam current, but the tradeoff is an exponentially decreasing lifetime due to thermal evaporation of the cathode material.

A field emission source typically consists of a tungsten needle sharpened to a point, with a radius less than 1 µm. The sharp tip helps provide the extremely high electric fields needed to pull electrons out of metal. Although cold field emission sources have become common in electron microscopes, they have seen little use in ebeam electron
Fig. 2.4 Leica EBPG-5HR ebeam system used in this dissertation at the Penn State nanofab facility[37].
sources due to their instability with regard to short term noise as well as long term drift, which is a much more serious problem for lithography than microscopy. Fluctuation of emission current occurs when atoms are adsorbed onto the surface of the tip and are ionized by the electron beam and subsequently accelerated back into the tip. To minimize the current fluctuation, the electron source must be operated in a UHV environment, $10^{-10}$ torr or better.

Electron lenses in principle behave the same as optical lenses, however electrons are focused either by electrostatic forces or magnetic forces. A magnetic lens is formed from two circularly symmetric iron (or some other high permeability material) pole pieces with a copper winding in between. Electrostatic lenses have worse aberrations than magnetic lenses, so they are not as commonly used.

### 2.3.3 Secondary electron scattering and proximity effect

Although ebeam lithography systems are capable of forming extremely fine probes, more complex effects are needed to be considered when electrons, usually of high energy, hit the resist layers. As the electrons penetrate the resist, they experience many small angle scattering events (forward scattering), which tend to broaden the initial beam diameter. As the electrons penetrate through the resist into the substrate, they occasionally undergo large angle scattering events (back scattering). The back scattering electrons cause the proximity effect, where the dose that a pattern feature receives is affected by electrons scattering from other features nearby. During this process the electrons are continuously slowing down, producing a cascade of low voltage electrons called secondary electrons. Much of the primary electron energy is dissipated in the form of
secondary electrons with energy from 2 to 50 eV. They are responsible for the bulk of the actual resist exposure process. Since they only have a range of a few nm deep in resist, they contribute little to the proximity effect. The major effect of the secondary electron on patterning features is an effective widening of the beam diameter by roughly 10 nm. The minimal practical resolution of 20 nm in the highest resolution electron beam system comes in large part from this effect. The bias effect that is seen in positive resist systems, where the exposed features develop larger than the size they were normally written, also has contributions from secondary electrons.

The net result of the electron scattering discussed in the previous paragraph is that the dose delivered by the electron beam tool is not confined to the shapes that ebeam systems write. For example, a narrow line between two large exposed areas tends to be over exposed by receiving many scattered electrons and developed away in positive resist scenario. Proximity effect is an important effect in ebeam lithography in the sense that if not avoided or depressed certain designs are not possible. If a pattern has fairly uniform density and linewidth, one only needs to adjust the overall dose till the pattern comes out the proper size. Multilevel resists, in which a thin top layer is sensitive to electrons and the pattern developed in it is transferred by dry etching into a thicker underlying layer, reduce the forward scattering effect. Higher beam voltages, from 50 kV to 100 kV or more, minimize forward scattering. Conversely, by going to very low beam energies, where the electron range is smaller than the minimum feature size, the proximity effect can be eliminated. Other techniques are developed to correct proximity effect by modulating either the dose (dose modulation) or the size (pattern biasing) for each individual shape in the patterns. A third technique (GHOST) uses a defocused
ebeam to mimic the shape of the backscatter distribution so that the pattern will have a roughly uniform background dose after primary writing. However, GHOST does not properly correct for forward scattering.

2.4 Plasma process related etching and thin film deposition

2.4.1 Overview

Lithography defines patterns and etching transfers them into actual structures in wafers. Etching can be carried out in either a ”wet” or ”dry” environment. Wet etching involves the use of liquid etchants. The wafers are usually immersed in the etchant solution and the exposed material is etched mostly by chemical processes. Dry etching involves the use of gas-phase etchants in a plasma. Here the etching usually takes place by a combination of chemical and physical processes. Because a plasma is involved, dry etching is often called ”plasma etching.” Two important concepts in etching processes are etch selectivity and directionality. Etch selectivity is the ratio of the etch rates of the different materials in an etch process. When the etch rates of the mask and the underlying substrate are near zero, while the etch rate of the film is appreciable, then the etch selectivity of the film with respect to both the mask and the substrate is high, which is normally a desired situation. Etch directionality is a measure of the relative etch rate in different directions, usually vertical versus lateral. Isotropic etching occurs when etch rates are the same in all directions and the etch distance laterally under each side of the mask is the same as vertically under the mask opening. In anisotropic etching, less etching occurs laterally. Etch directionality is usually related to physical effects in
etching such as ion bombardment and sputtering. More physical etch processes tend to be more directional and less selective. On the other hand, more chemical etch processes are more selective and less directional. Vertical etching is of more importance in this dissertation as it maintains lithographically defined feature sizes as they are transferred by etching into underlying films. Hence, plasma etching, which is a more physical process, is discussed in more details in the following sections.

### 2.4.2 Plasma etching systems

A basic plasma etch system is shown schematically in Fig. 2.5. In these etch systems low-pressure (1 mtorr - 1 torr) gases are used in the chamber. By applying a high electric field across two electrodes, some of the gas atoms are ionized, producing positive ions and free electrons and creating a plasma. The energy is supplied by an RF generator, usually operating at 13.56 MHz in compliance with FCC regulations. Due to the difference in mobility of the electrons and the ions, a voltage bias develops between the plasma and the electrodes. Initially, the more mobile electrons are lost to the electrodes at a faster rate than the slower ions. This results in the plasma being biased positively with respect to the electrodes. For a symmetric RF plasma system, with the two electrodes of equal area, the voltage distribution that develops across the electrodes is also symmetric with voltage drops near the two electrodes. If one of the electrodes is made much smaller in area, then the voltage distribution becomes asymmetric with a much larger voltage drop occurring from the plasma to the smaller electrode. In standard plasma etch systems, the plasma density (the concentration of ions or electrons in the main plasma, which are virtually equal) and the ion energy as it travels across the sheath
to strike the wafer are closely coupled. As the RF power increases, more ions are created, increasing the density, and the sheath voltage increases as well, increasing the ion energy.

Fig. 2.5 A basic plasma etch system is shown schematically, which contains a pair of electrodes powered by an RF generator, gas outlets/inlets and vacuum systems.

One standard plasma etch system is reactive ion etching (RIE) system. In this type of system, wafers sit on the smaller electrode, which is usually the lower electrode. The lower electrode can be made physically smaller than the upper electrode, and the upper electrode can be the one that is grounded and electrically connected to the chamber walls. This makes the upper electrode effectively much larger. A much larger voltage drop occurs from the plasma to the wafers, in the 100-700 V range, resulting in more energetic ion bombardment of the wafer. With stronger ion bombardment, more directional etching can be achieved. High density plasma (HDP) systems are another type of plasma etch system that are replacing the RIE ones and becoming popular in recent years. These include electron cyclotron resonance (ECR) and inductively coupled plasma
(ICP) sources. The ICP one is becoming even more popular due to its much simpler design and equipment. In ICP systems, very high plasma densities are produced compared to the standard RIE systems. In standard RIE systems, increasing the power increases the plasma density, allowing for higher etch rates, but also increases the sheath bias and ion energy, leading to more substrate damage. With the decoupling of the plasma density and ion energy, many ions can be produced at low pressures with low ion energies in ICP systems. As a result, ion bombardment damage can be kept low while maintaining high etch rates and good anisotropic etching. In addition, a lower ion energy usually gives better selectivity than higher ion energy. Fig. 2.6 shows comparison between RIE and ICP systems in a schematic view.

Fig. 2.6 Comparison between RIE and ICP systems is shown schematically. The left one is RIE system where only one RF generator is used, compared with the right ICP system, a separate coil is used to generate very high density plasma while maintaining low ion energies[38].
2.4.3 Plasma etching mechanism

The two main types of species involved in plasma etching are the reactive neutral chemical species and the ions. It is the reactive neutral species—free radicals in many cases, but sometimes other reactive species such as Cl$_2$—which are primarily responsible for the chemical component in plasma etch processes. It is the ions that are responsible for the physical component. In most cases of plasma etching, they work together in a synergistic manner.

Chemical etching of materials in plasma system is commonly done by free radicals. Free radicals are electrically neutral species that have incomplete bonding; that is, they have unpaired electrons. Fluorine free radical, F, and the neutral CF$_3$ free radical are two examples. Both of these can be produced by the reaction between free electrons in the plasma and CF$_4$:

$$e^- + CF_4 \rightarrow CF_3 + F + e^-$$

Because of their incomplete bonding structure, free radicals are highly reactive chemical species. The idea in plasma etching is for the reactive neutral species to react with the material to be etched. The byproduct should be a volatile species that leaves the surfaces. Gas additives can be used to increase the production of the reactive etch species and thereby increase the chemical etching rate. For example, oxygen is often added to a CF$_4$ plasma. The oxygen reacts with dissociated CF$_4$ species (CF$_3$ or CF$_2$, for example), which reduces the recombination of those species with F, thereby increasing the amount of free F.
The other main species that participate in plasma etching are the ions that are present. Because of the voltage drop between the plasma and each electrode, positive ions will be accelerated toward each electrode. Since wafers are placed on one of the electrodes, the ionic species such as \( \text{Cl}^+ \) will be accelerated to the wafer where they strike the surface, resulting in the more physical component of etching. The flux of ions toward the wafer surface is much more directional than flux of neutral free radicals because of the directionality of the electric field from the plasma to the surface of the wafer. Thus the etching of this physical process will be much more directional and more anisotropic.

Another way that ions participate in plasma etch processes is through ion-enhanced etching. A classic example of ion-enhanced etching is the etching rate of silicon as \( \text{XeF}_2 \) gas and \( \text{Ar}^+ \) ions are introduced to the surface. First \( \text{XeF}_2 \) gas is introduced into the chamber and very little etching occurs. Then an \( \text{Ar}^+ \) ion beam is directed toward the surface and the etch rate increases ten times. Finally the \( \text{XeF}_2 \) gas is turned off so that only physical ion etching occurs, and the etch rate becomes almost zero. The chemical and physical species work in a synergistic manner. In this type of etch process, good selectivity can be obtained because of the chemical component and processes along with very anisotropic etch profiles. This is a desired etching scenario, thus gas chemistries and etch conditions are usually chosen to promote ion-enhanced etching and to suppress independent chemical and physics etching. There have been many explanations and mechanisms proposed for this type of cooperative etch process. In most of the models the ion bombardment enhances one of the steps of the chemical etch process, such as surface adsorption, etching reaction and formation of byproduct, or removal of
byproduct or unreacted etchant. One way, for example, the ion bombardment helps the chemical process is by causing damage, such as breaking bonds, to the surface and makes the surface more apt to chemically react with the radical. The ion bombardment could also accelerate the formation of volatile byproducts. Another possibility is that the ion bombardment may dislodge or sputter away etch byproducts which would otherwise tend to stay on the surface and impede the etch process. In a particular ion-enhanced etch process, one or more of the above mentioned mechanisms may be occurring.

2.4.4 Plasma-enhanced CVD thin film deposition

Thin film deposition, in standard VLSI technologies, is an independent area of processing technology. Chemical vapor deposition and physical vapor deposition are two major types. Among those types of different methods, low-pressure CVD (LPCVD), plasma-enhanced CVD (PECVD), evaporation and sputtering are widely used. In this dissertation, only plasma process related CVD methods are discussed.

There are restrictions on the temperature to which GaAs substrates can be exposed when depositing a film. In this dissertation, silicon nitride film needs to be deposited on GaAs substrates. Since the growth temperature of GaAs is only about 550 °C, the deposition temperature must be much lower than this. To achieve good film quality and reasonable deposition rates at low deposition temperature, plasma-enhanced CVD method is needed. A plasma source is used in addition to a thermal source to provide the energy needed for the chemical reactions to occur during the deposition. Plasma supplies additional energy to the reactant gases so that the reactions needed for deposition can occur at much lower temperature. For example PECVD deposition of silicon
nitride on GaAs substrate is done at 100 °C. PECVD also allows more film’s properties (stress, composition, etc.) to be easily altered. A detailed example and discussion about the engineering of silicon nitride film stress and hence the engineering of microdisk laser properties through PECVD will be given in chapter 3.

A typical PECVD system has a similar configuration to the RIE etching system discussed before. In plasma etching, the energetic species created in the plasma are used to etch materials on the wafer. In plasma-enhanced deposition, the energetic species are used to help form the films on top of wafers. The wafers are placed on the lower plate electrode. Heat can be supplied to the wafers through a heater beneath the electrode. The reactant gases, such as silane, nitrogen and argon for the deposition of silicon nitride, are fed through inlets. A plasma is sustained between the lower and upper electrodes. This is formed by applying a high electric field, often at 13.56 MHz, to a low pressure gas (between 50 mtorr and 5 torr), creating ions and free electrons. The plasma is sustained when high-energy electrons strike and ionize atoms and molecules. In the plasma, interactions with the high-energy electrons cause the reactant gases to dissociate and ionize into a variety of species. These include ionized and excited molecules (or atoms), neutral molecules and neutral and ionized fragments of broken-up molecules, including free radicals. These species are extremely reactive, and with other species are absorbed onto the wafer surface, migrate, interact, rearrange, and chemically recombine to form film. Also the bombardment of ions and electrons transfer even more energy to the species on the surface, breaking chemical bonds for example, and further enhance the different surface processes and reactions. The net result from the plasma-enhanced
fragmentation, free radical generation, and ion bombardment is that the surface processes and deposition can occur at much lower temperatures than in nonplasma systems.
Chapter 3

External strain engineering of the static and dynamic optical response of (Al, Ga)As/GaAs microdisk lasers

3.1 Motivation

Microdisks are of great interest as a testbed of high spontaneous emission factor low threshold lasers[40] and some concepts of quantum information processing.[41, 42, 43, 44] Recent studies show they provide unexpected ways of controlling electron spin coherence via light-matter interactions in confined geometries, suggesting new routes towards spin-based quantum information processing. [39] Also their small dimensions (a few microns in diameter) and mushroom like free standing structure make possible the engineering of certain physical properties—such as strain. Compressively strained quantum wells (QWs) are considered advantageous as active layers in semiconductor lasers for bringing the quasi-Fermi levels closer to and more symmetric across the band gaps so that both the TE mode polarized material gain \(g\) and the differential gain \(dg/dN\) are enhanced.[45] Although lasing characteristics of microdisk lasers have been extensively studied, the influence of strain has only been demonstrated in compressively-strained InGaAsP[46] or InGaAs QWs embedded in microdisks where strain comes from the lattice constant difference between well and barrier materials. External strain, which has been utilized in some other low dimensional systems such as strained silicon in 90nm CMOS technology[47] and tuning band gaps of carbon nanotubes[48], remains an open
topic for microdisks. Here we study external strain influence on (Al, Ga)As/GaAs microdisk lasing characteristics. The microdisks in this study are engineered with different strain by encapsulating the surfaces with SiN$_x$ thin films that have different stress through PECVD method.

### 3.2 PECVD deposition of stressed silicon nitride thin films

Each microdisk in this study contains five 10-nm-thick GaAs QWs isolated by 10-nm-thick Al$_{0.31}$Ga$_{0.69}$As barriers and is supported by a narrow Al$_{0.81}$Ga$_{0.19}$As pedestal. The diameter and thickness of the microdisk region are $\sim 3\mu$m and $\sim 120$nm. Patterning methods of the microdisks have been described elsewhere.[49] The process flow is finalized by encapsulating the sample with a 40 nm SiN$_x$ thin film using electron cyclotron resonance plasma enhanced chemical vapor deposition (ECR-PECVD) to stabilize the sulfide passivation layer on the disk surfaces[50] and also to exert mechanical strain in our case. The SiH$_4$ : N$_2$ : Ar ratio is set as 10.1 : 8 : 3 and the sample temperature is $\sim 100$ °C in the SiN$_x$ deposition. Different radio frequency powers are coupled to the plasma beam to deposit SiN$_x$ films with different build-in compressive strain. [51] Fig. 3.1 shows the deposition rate and index of refractive of SiN$_x$ film dependence on PECVD radio frequency power. Note that the deposition rate decreases with increasing power below $\sim 500$ W and then stabilizes. The index of refractive is roughly constant at powers lower than $\sim 400$ W and drops when the power gets higher. We also note that the peaks of QW spontaneous emission from microdisks encapsulated with SiN$_x$ deposited at higher power become broader in line width and quenched in intensity while the energy redshifts. The QW emission from the microdisks totally diminishes on the
samples with SiN$_x$ deposited at 500 W and higher powers. However the QW emission from unprocessed bulk region is independent of SiN$_x$ deposition power. For example, shown in Fig. 3.2, QW emission is quenched more from the undercut disk than from the mesa with same 700 W silicon nitride film. These suggest the QWs embedded in the microdisks experience strain applied by the SiN$_x$ films encapsulating the disks. The increasing of QW width because of the squeezing effect of the compressive strained film may explain the emission peak line width broadening and energy redshifting.

Fig. 3.1 Deposition rate and index of refractive of SiN$_x$ film versus PECVD radio frequency power are plotted. The blue cycle marks the power range used to deposit the SiN$_x$ film discussed in this chapter.
Fig. 3.2 QW emission is quenched more from the undercut microdisks than from mesas under the same power deposited silicon nitride film. The measurement is performed at 10 K.
3.3 External compressive strain in microdisk lasers induced by stressed film

We collect polarization-resolved spontaneous emission from the side of a microdisk in the far field. The spectra of samples with SiN$_x$ deposited at 300 W, 350 W and 400 W are shown from bottom to top in Fig. 3.3. TE and TM stand for polarization parallel and perpendicular to the microdisk plane. The degeneracy of TE and TM polarized emission is usually broken in bulk QWs because of the splitting of heavy holes (HHs) and light holes (LHs) in valence band due to quantum confinement. TE polarized emission is preferred by selection rules for recombination between electrons in conduction band and HHs or LHs. The transition matrix elements for QWs (C-HH, C-LH) are (1/2, 1/6) for TE polarization and (0, 2/3) for TM polarization. Hence, TE polarization is more favored by recombination between conduction band and heavy hole while TM is more favored by conduction band and light hole. However, here the degeneracy is restored in spectrum of the samples with 300 W and 350 W deposited films: both TE and TM polarized emission have similar intensity. In the sample with 400 W deposited film TE polarized emission is again preferred over TM. We explain these by the interplay between the tensile strain from the free standing shape of microdisks[52] and the compressive strain from the encapsulating strained SiN$_x$ films: for samples with lower compressive strained films (300 W and 350 W ones) the net strain is the tensile strain from the disk shape and the degeneracy is restored by the tensile strain effect; for samples with higher compressively strained films (400 W) the net strain is the compressive strain from the
strained films and the degeneracy is broken again by the compressive strain effect. Fig. 3.4 shows how the band splitting changes with different external strains.

### 3.4 Steady-state and dynamic lasing responses under external strain

We perform steady-state photoluminescence (PL) measurements to study stimulated and spontaneous emission from microdisks with the similar set-up discussed elsewhere.[49] Fig. 3.5 and Fig. 3.6 show the low temperature (T~6 K) excitation power dependent spectra of lasing modes in a smaller microdisk and in a larger one on the sample with 400 W deposited SiNx film. Single mode lasing is observed in the smaller microdisk (Fig. 3.5) as the mode spacing increases with decreasing disk diameter[53] so that only one mode exists in the spontaneous emission gain region (1.51 eV~1.56 eV). The mode is identified as whispering-gallery mode (WGM) TE_{23,1} using FDTD methods. In the larger microdisk, we observed power-dependent mode hopping between adjacent modes as the power is increased. (Fig. 3.6). These two modes are identified as TE_{19,2} and TE_{23,1} from high energy to low energy.

Fig. 3.7 focuses on lasing characteristics of the smaller microdisks that show the single mode (TE_{23,1}) lasing in three samples with SiNx films deposited at different powers: Fig. 3.7(a)(b) are from the sample with 300 W power deposited film; Fig. 3.7(c)(d) and Fig. 3.7(e)(f) compared to 350 W and 400 W respectively. Fig. 3.7(a), Fig. 3.7(c) and Fig. 3.7(e) show log-log plots of the laser emission intensity in this mode as a function of excitation intensity I. The laser thresholds are clearly indicated by kinks in plots of Fig. 3.7(c) (~728 W/cm²) and Fig. 3.7(e) (~288 W/cm²). However no clear threshold is observed in Fig. 3.7(a) where the laser emission intensity increases almost
Fig. 3.3 Polarization-resolved far field emission spectra from the side of a single microdisk on samples encapsulated by SiN$_x$ films with different strain are shown. The power used on those samples is 300 W, 350 W and 400 W from bottom to top respectively. TE and TM stand for polarization parallel and perpendicular to the microdisk plane. The measurement is performed at 10 K.
Fig. 3.4 An illustration showing how QW band splitting changes under different external strains in microdisks.

linearly in the log-log scale with excitation power until gets saturated at $\sim 600 \text{ W/cm}^2$.

Fig. 3.7(b), Fig. 3.7(d) and Fig. 3.7(f) show the power dependence of the energy and the full width at half maximum (FWHM) of the TE$^{23,1}$ laser mode of microdisk lasers in three samples. Below and around the thresholds, we observe spectral narrowing of the mode as the excitation power increases in Fig. 3.7(d) and Fig. 3.7(f). This suggests that the FWHMs of the modes are limited by the absorption of the QWs in weak excitation regions below the transparency thresholds which just lie below the lasing thresholds.[42]

Once the excitation power passes the transparency thresholds, stimulated emission starts dominating over spontaneous emission as the source of photons coupled into the cavity modes. On the other hand, heating of the microdisk becomes pronounced and eventually increases FWHM of the modes. However we do not observe spectral narrowing in Fig.
Fig. 3.5 Excitation power dependence of the emission spectrum of a single smaller microdisk showing single mode $\text{TE}_{23,1}$ (triangle) lasing at $T=6$ K.

3.7(b). The redshifts of mode energy at higher excitation power also arise because of microdisk heating.

The dynamical response from microdisks is measured with time-resolved PL. The optical pumping is carried out with 150 fs width pulsed excitations at 1.67 eV. The PL is spectrally resolved with a monochromator and temporally resolved using a streak camera whose resolution is 2 ps. Fig. 3.8 shows temporal evolution of one microdisk laser mode at two pumping powers. Both the rise and decay of the emission become faster with higher power. The decay time or the charge life time is defined as $(1/e)$th time constant of the decay trail [marked with a arrow in (a)] by exponential fitting to the data. Figs. 3.9(a), (b), (c) show the power dependent decay time plots from three microdisks with different strain. All three decay times keep decreasing with increasing
powers and become relatively constant above certain powers. This is due to the increased stimulated emission rate which dominates the emission above lasing threshold. Thus the lasing threshold should just occur around the pumping power where decay time becomes constant. For the sample that has the highest compressive strain (400W) and the one has the medium compressive strain (350W) the thresholds are \( \sim 180 \text{ W/cm}^2 \) and \( \sim 800 \text{ W/cm}^2 \) respectively which are relatively consistent with the values from steady-state PL: \( \sim 288 \text{ W/cm}^2 \) and \( \sim 728 \text{ W/cm}^2 \). Also for the sample that has tensile strain (300W) the threshold is \( \sim 120 \text{ W/cm}^2 \), suggesting it is below the measurement range in steady-state PL measurement. Higher TE mode gain of compressively strain QWs explains why sample with the highest compressive strain shows lower threshold. Carrier diffusion towards disk edge in free standing tensile strain microdisk may explain why it also shows
Fig. 3.7 (a), (c), (e) Power dependence of lasing intensity in the TE$_{23,1}$ mode plotted on log-log scales. The arrows indicate the laser thresholds. (b), (d), (f) Power dependence of the energy (open circle) and the FWHM (open square) of the same lasing mode. Three PECVD powers are used to deposit SiN$_x$ films with different strain: (a)(b) 300 W, (c)(d) 350 W, (e)(f) 400 W.
lower threshold since a free standing disk shape causes a tensile strain gradient starting from the supporting post edge and increasing towards disk edge. Therefore, there is also a QW bandgap gradient which becomes smaller towards the disk edge and makes more carriers diffuse towards disk edge where whispering gallery modes are located.

![Temporal evolution of one laser mode emission at two different pumping powers at T=30K. The arrow marks the decay trail.](image)

**Fig. 3.8** Temporal evolution of one laser mode emission at two different pumping powers at T=30K. The arrow marks the decay trail.

### 3.5 Conclusion

In summary, we have demonstrated that the encapsulation of lattice-matched (Al, Ga)As/ GaAs microdisk laser with controllable compressive stress SiNx film allows systematic strain engineering. Polarization-resolved spontaneous emission shows that GaAs QWs in the active region of the microdisks experience strain from both the SiNx and
Fig. 3.9 Power dependent decay time (or charge life time) plots on three microdisk samples with different strain. The PECVD power for them are (a): 300W, (b): 350W and (c): 400W. The measurement is performed at 10 K.
the free standing disk shape. We performed steady-state and dynamic optical characteristics measurements on three microdisk samples that have net strain changing from tensile strain to compressive strain. Laser thresholds first increase possibly with weakening effect of carrier diffusion towards disk edge and then decrease with higher mode gain from compressively strained QWs.
Chapter 4

Normal mode splitting characterization
in twin coupled microdisk cavities

4.1 Introduction and motivation

The splitting of normal optical modes is a signature of strong interaction of light-matter/light-light in optical systems. Examples include the strong coupling between atoms/quantum dots and optical cavities, or the strong coupling between optical cavities. The strong coupling between optical cavities is a light-light interaction based on the evanescent electromagnetic (EM) fields of the cavities. Experimental realizations of strong coupling between a single quantum dot and a single mode in a cavity have demonstrated striking predictions of cavity quantum electrodynamics (CQED)[54]. In parallel, this provides motivation to observe strong coupling between a pair of single photons in coupled optical cavities. A recent study proposes microdisk cavities as one of the candidate schemes to realize such strong coupling[55]. The normal mode splitting due to the strong coupling of the whispering gallery mode photons has recently been observed in twin coupled microdisk cavities[57, 56, 58]. Our goal was to further study the mode splitting dependence on other physical variables in the coupled cavity systems. We perform a systematic characterization of the normal mode splitting as a function of cavity geometrical layout, excitation power and ambient temperature. We demonstrate
a clear correlation between the splitting mode intensity and the mode energy splitting and we attribute this to the evanescent field nature of the coupling.

4.2 Fabrication of the twin coupled microdisk cavities

All the samples fabricated in this study are grown by MBE methods. They contain two layers of (Al, Ga)As/GaAs heterostructures. One layer is the optical disk layer containing six 4.2 nm GaAs quantum wells with interface fluctuation quantum dots (IFQDs). The IFQDs are formed with 2 minute growth interruptions at the each interface of quantum wells and barriers. Another 500 nm Al$_{0.8}$GaAs bottom layer is designed to form the post that supports the disk layer after the processing. Also a very thin layer of GaAs (2 nm) is grown between the bottom post layer and the top optical disk layer to smooth the interface and prevent the high aluminum content layer from being oxidized and hence to maintain a better optical quality.

In order to achieve interdisk spacing in the range of $\sim$200 nm, we use electron beam lithography to pattern the coupled twin microdisk designs. Dual layers of ebeam resists are used with a thick 5.5% copolymer P(MMA/MAA) bottom layer and a thin 3% 950 MW PMMA layer on the top. Spin speed curves suggest the thickness of the bottom layer is about 200 nm and the top layer is about 100 nm. The reason to use this dual-layer resist method is two fold. First this method suppresses the proximity effect in ebeam writing (discussed in chapter 2). The thin imaging PMMA layer gives sufficient resolution. Second is this dual-layer resist is very useful for metal lift-off which is the next step in the fabrication. The bottom layer copolymer P(MMA/MAA) tends to develop more than the top PMMA layer under the same developing condition, which
leaves a undercut structure of resists after developing. With this kind of structure, it is much easier for the metal layer to be lifted off.

Before the real ebeam writing, a 15 nm metal layer is thermally evaporated onto the ebeam resist coated wafer. Since the undoped GaAs is not a good conductor, the thin metal layer serves as a conducting layer that prevents the electrons from accumulating on the wafer surface during ebeam writing. Aluminum is chosen for our process since it can be easily removed after ebeam writing using standard photolithography developer CD-26, which is essentially tetramethylammonium hydroxide water solution and does no harm to the GaAs based wafers. Only thermal evaporation is used since the electrons from e-gun evaporation will expose the ebeam resist layers on the wafers.

Dose calibrations are often required before writing actual patterns to determine the optimized dose. This is carried out through writing the same die of the patterns at different dose—the amount of electrons on a unit area—and the smallest one that gives good patterns is used in actual writing. Because of the proximity effect, doses change with patterns, resists and conducting layers. Hence it is desirable to keep them the same when writing multiple wafers using same dose. The regular dose for the twin microdisk writing with the Al/PMMA/Copolymer coating on the Leica EBGH 5B ebeam system is about $550\mu$C/cm$^2$.

A three step developing method is used after the ebeam writing. First the wafer is immersed in CD-26 for about 2 minutes to remove aluminum conducting layer (Fig.4.1(a)). Note that since CD-26 is an alkali, it also reacts with the $\text{Al}_{0.8}\text{GaAs}$ post layer, so the wafer cannot be immersed in it for too long. Following this, an MIBK:IPA=1:1 developer is used to develop the imaging resist PMMA to show the
features as well as the bottom layer resist for 60 seconds. Finally, a 15 second or so developing in IPA undercuts the bottom copolymer layer (Fig.4.1(b)). After developing, we use an optical microscope to examine the resist and make sure all the patterns come out correctly.

We need to transfer the patterns on the resist into wafer, and this is realized with dry etching using metal hard masks. A 50 nm thick aluminum layer is thermally evaporated uniformly on the wafer so that at the region where the resist is already developed away aluminum sticks directly on the wafer surface and elsewhere aluminum coats on the remaining resist (Fig.4.1(c)). Then we use photoresist remover 1165 to dissolve the remaining resist underneath the aluminum layer so that the aluminum layer is lifted off by the dissolved resists (Fig.4.1(d)). Therefore we transfer the patterns from the resist to the aluminum and hard masks are formed. One can heat up the remover 1165 to speed up lift-off process since it is not volatile. Usually wafers are face down during lift-off: this keeps the peeling off metal layer from staying on the wafer surface. Ultrasonic shaking can be used if needed at the end of the lift-off process. It is important to rinse away the remaining remover 1165 with IPA instead of DI water first, because remover 1165 is an alkali in water and reacts with both the aluminum hard mask and the Al$_{0.8}$GaAs post layer. Finally, we rinse the wafer with DI water and blow it dry with N$_2$ gas.

Plasma dry etching follows to complete the transformation of patterns into wafers. For regular III-V semiconductor, Cl$_2$/Ar mixed gas is used in the RIE etching system. As discussed in chapter 2, RIE dry etch gives pretty straight vertical side walls which is desirable to maintain the narrow interdisk spacing patterned through ebeam lithography.
Aluminum hard mask gives good resistance to the Cl$_2$/Ar mix gas RIE dry etching. Once the mesas are formed, we strip off the aluminum with CD-26 again. Finally, we use (NH$_4$)$_2$S/S solution to selectively etch away Al$_{0.8}$GaAs over GaAs so that posts/disks are formed. The sulfur atoms also passivate the wafer surface by forming As-S and Ga-S bondings. Hence, the non-radiative recombination caused by the dangling bonding of As/Ga atoms is strongly suppressed. To prevent the S atoms in the S-As bonds from being replaced by O atoms from air, a thin silicon nitride film (60 nm) is encapsulated on the surface of wafer deposited by PECVD. The deposition temperature is as low as 100°C so that the crystal property of the wafers does not change. Fig. 4.2 shows a representative twin coupled microdisks taken by SEM during the process after the disks and posts are formed.
4.3 Bonding and antibonding modes in twin coupled microdisks

In a single microdisk, two degenerate modes form a real mode discussed in chapter 1. In contrast, a pair of twin coupled microdisks have more modes induced by the coupling. In this section, we first investigate the mode property in twin coupled microdisks with FDTD simulations. Then we demonstrate experimental results that agree with the simulations.

A general description about the FDTD simulation has been discussed in chapter 1. Since chapter 5 focuses in more detail on FDTD simulations, we will describe the simulation methods later. We use symmetry conditions that fall into four classes. These determine the number of the modes and the mode field distribution. Fig. 4.3 shows the near field distribution of the z component of the magnetic field ($H_z$) showing the four
symmetry classes. Hence, there are totally four modes in the twin microdisks and each pattern shows the $H_z$ for one mode.

In the simulation, we use a coordinate system where the disks lie in the x-y plane, \( \hat{x} \) is parallel to the coupling axis, and the origin is midway between the disk centers. We name the four modes according to their field symmetry along the x and y axes: ee (even along both \( \hat{x} \) and \( \hat{y} \)), oe (odd along \( \hat{x} \), even along \( \hat{y} \)), eo (even along \( \hat{x} \), odd along \( \hat{y} \)), and oo (odd along both \( \hat{x} \) and \( \hat{y} \)). Since the $H_z$ patterns are similar to the electron wavefunction patterns in a diatomic molecule, we borrow the names of "bonding" and "antibonding". The first two modes—ee and oe—are nearly degenerate bonding modes, while the other two—eo and oo—are nearly degenerate antibonding modes[59, 58]. The simulation by Ref. [59] show that the splitting between the nearly-degenerate modes is at least two magnitude smaller than the normal mode splitting.

We now compare the theoretical expectation with experiment. The emission spectra from twin coupled microdisks indeed reveal two modes (Fig. 4.4 (b)) compared with only one from a single microdisk (Fig. 4.4 (a)). We interpret the lower energy peak as a bonding mode and the higher energy one as an antibonding mode. We note that—according to the simulation mentioned above—each mode in the spectrum is actually two nearly-degenerate modes.

We have found that this degeneracy is sometimes resolved—probably due to extrinsic factors, such as the light back scattering produced by fabrication defects[60]. For instance, Fig. 4.5 shows two special cases of the mode behavior in the twin coupled microdisk cavities. Here, the degeneracy of the bonding mode (4.5 (b)) or the antibonding (4.5 (a)) mode is removed, resulting in a small splitting in either the bonding mode or the
Fig. 4.3 Near field $H_z$ component patterns for TE(5,1) mode in twin coupled circular disks simulated through FDTD method.

Fig. 4.4 Lasing spectra from a single microdisk (a) and a pair of twin microdisks (b). Bonding and antibonding modes with a 5nm mode splitting.
antibonding mode or in both modes for some cases. In agreement with the simulation[59], this splitting is far smaller than the normal mode splitting. However, the experimentally observed values are much larger than predicted.

In general, the antibonding mode has a higher Q factor than the bonding mode in the ideal "cold cavity" scenario. This is considered to be due to the large penetration of the modal electric field into the interdisk spacing and a smaller scattering loss for the antibonding mode having a node at the center of the interdisk spacing. However, if we consider the effects to Q factors from the gain material, which is basically the case of a real laser, the antibonding mode does not necessarily have a higher Q. For example, the absorption of the gain material, which tends to lower the Q factor, varies with wavelength/energy. For GaAs based quantum wells, the absorption increases/decreases

Fig. 4.5 Cases of bonding (b)/antibonding (a) mode degeneracy removal marked by the arrows.
with energy/wavelength. Thus, for GaAs quantum well based twin microdisk lasers, the Q factors of antibonding and bonding modes are about the same according to our study.

### 4.4 Normal mode splitting dependence on geometry layout of the twin microdisks

As shown in the previous section, mode splitting is a result of the EM field pattern modification induced by the geometrical coupling of the twin disks. The magnitude of the mode splitting can be regarded as an indicator of the coupling strength of the twin disks. For example, one expects that the coupling strength increases as the interdisk spacing decreases since the evanescent field decays exponentially outside the cavity. So, the mode splitting from a pair of twin disks that has a smaller spacing is expected to be larger. This has been proven both theoretically and experimentally. Fig. 4.6 is an example.

To further investigate the mode splitting dependence on other geometry elements, we fabricated three types of coupled twin microdisks. They are coupled circular (C), elliptical coupled along the long axis (E_L) and along the short axis (E_S) all with the same designed spacing 200 nm illustrated in Fig. 4.7. An FESEM survey shows that the actual spacing varies from 185 nm to 225 nm. Fig. 4.8 summarizes the mode splittings for the three different types of twin microdisks patterned on the same wafer. The data show significant spread in mode splittings even within a particular type of geometry, suggesting contributions from statistical variations in the interdisk spacing as well as microdisk shape in the patterned devices. However, even within this statistical variation, the twin circular (type C) appear to show larger mode splitting than the twin
Fig. 4.6 Mode splitting from twin disks with 200-300 nm spacing is smaller than the ones with 100-200 nm spacing.

Fig. 4.7 Three types of twin microdisks. C: circular, $E_L$: elliptical coupled along the long axis, $E_S$: elliptical coupled along the short axis.
elliptical ones (type $E_L$, $E_S$). We do not detect statistically significant differences in mode splitting for elliptical disks coupled along different symmetry axes.

Fig. 4.8 Summary of the mode splitting of the three types of twin microdisks. C: circular, $E_L$: elliptical coupled along the long axis, $E_S$: elliptical coupled along the short axis.

4.5 Normal mode splitting dependence on excitation power and ambient temperature

Normal mode splitting in the twin coupled microdisk cavities is further characterized with excitation power dependent measurements. Different from the regular normal mode splitting, which is induced by the coupling of light and matter, such as an atom and a cavity or a quantum dot and a cavity, the normal mode splitting of the coupled cavities relies on the interaction of the evanescent field outside the cavities. We expect
that this special type of coupling should show some dependence on the evanescent field intensity by varying the excitation power. Fig. 4.9(a) and (b) show two representative excitation power dependent mode behaviors in our study. We observe both behaviors in all three types of twin coupled microdisk cavities. Note that the excitation power in this study is set to be above the lasing threshold so that stimulated emission provides strong enough evanescent field intensity outside the cavities. In Fig. 4.9(a), both the bonding and the antibonding modes red shift as the excitation power increases. Also, the bonding mode intensity increases as the antibonding mode is quenched with the increasing excitation power. The mode energy red shifting is probably a result of local heating. We attribute the mode intensity variation to a combination of effects: the carrier concentration/temperature dependence of the gain profile and mode competition. The result is that the gain spectrum changes its shape with increasing carrier concentration/temperature so that the gain becomes higher at lower energy in the energy range of the bonding/antibonding modes. Thus, the lower energy bonding mode has a higher gain than the higher energy antibonding mode and becomes dominant. Fig. 4.9(b) shows a special case where the bonding mode degeneracy is removed. As the excitation power increases, instead the antibonding mode becomes dominant. We attribute this to the mode competition between the two non-degenerate bonding modes: compared with the case in Fig. 4.9(a), here in the case of Fig. 4.9(b), the gain has to support two modes, which lowers the intensity of each bonding mode.

We further study the normal mode splitting as a function of the excitation power. The results for the case in Fig. 4.9(a) are shown in Fig. 4.9(c)(d). Fig. 4.9(c) shows that the mode splitting value increases from $\sim 1.8$ meV to $\sim 2.2$ meV with high excitation
power as the bonding and the antibonding mode energies red shift. In Fig. 4.9(d), we plot both the mode splitting and the product of the bonding and the antibonding mode intensities as functions of the excitation power. The product of the intensities reflects the number of photons that are engaged in the bonding and the antibonding modes, which are the direct result of the evanescent field coupling. We observe a clear correlation between the intensity product and the mode splitting (Fig. 4.9(d)). The plateaus in the mode splitting curve around \( \sim 0.85 \, \text{kW/cm}^2 \) and \( \sim 1.7 \, \text{kW/cm}^2 \) are probably due to the local heating effects. As we will demonstrate later, higher temperature weakens the evanescent field coupling and narrows the normal mode splitting. At even higher excitation power (\( \gg 2.6 \, \text{kw/cm}^2 \)), the antibonding mode diminishes completely because of the combined effects of gain profile and mode competition. Hence, the normal mode splitting disappears even if the coupling still exists. The mode splitting value for the case of the Figure 2(b) also shows similar increasing trend with increasing excitation power until at higher excitation power (\( \gg 5.1 \, \text{kW/cm}^2 \)), where the local heating effect dominates and the mode intensities saturates, the mode splitting value starts to decrease.

We also study the temperature dependent normal mode splitting by varying the temperature from 11K to 49K at a constant excitation power above the lasing threshold. Fig. 4.10(a) shows the mode spectra at each temperature. A similar mode intensity variation to Fig. 4.9(a) is observed: as the temperature increases, the bonding mode becomes stronger while the antibonding mode is weakened. This is still a result of the temperature dependent gain profile change. More importantly, the antibonding mode totally disappears at 42K. The mode splitting value is plotted as a function of temperature in Fig. 4.10(c)(d). It decreases from \( \sim 2.3 \, \text{meV} \) to \( \sim 1.8 \, \text{meV} \) with increasing
temperature from 11K to 35K. Again, Fig. 4.10(d) shows a correlation between the mode splitting value and the product of the bonding mode and the antibonding mode intensities as they both decrease with increasing temperature. We attribute the decreasing of the mode intensity to the non-radiative recombination of carriers because of the GaAs surface states, which quenches the emission of the gain medium at higher temperature.

4.6 Conclusion

In this chapter, we discussed the study of three types of twin coupled microdisks: circular, elliptical coupled along the long axis, and elliptical coupled along the short axis. Each disk in a pair has a narrow interdisk spacing around 200 nm. Light-light interaction induced normal mode splitting based on evanescent electromagnetic (EM) field coupling is observed in all three types of coupled microdisk cavities. Evanescent EM field coupling of the WGMs from each microdisk cavity modifies the EM field patterns and essentially forms new modes, or "supermodes". The spatial energy distributions (spatial distributions of Poyning vector amplitudes) of the supermodes’ EM field are like the electron wavefunction patterns of bonding and antibonding states in a diatomic molecule, such as hydrogen. Hence, the lower energy mode in the normal mode splitting of the twin coupled cavities is called bonding mode and the higher energy one is named antibonding mode. In this physical picture, photons from the twin coupled cavities behave like electrons in hydrogen molecules.

We performed a systematic characterization of the normal mode splitting as functions of cavity geometrical layout, excitation power and ambient temperature, and showed a clear correlation between the splitting mode intensity and the amount of mode
energy splitting. The energy splitting increases with higher excitation power on the coupled cavities as the intensities of the modes increase. The splitting decreases with rising temperature as the intensities of the modes quench due to the non-radiative recombination.

The type of coupled microdisk cavities discussed here should serve as a good test scheme to demonstrate strong coupling between single photons from coupled cavities. The observation of such a coupling would constitute a new regime for photons in quantum optical systems to create deterministic sources of entangled photon pairs and ways to implement quantum logic gates between strongly coupled single photons.
Fig. 4.9 (a)(b) Two representative cases of power dependent mode splitting behavior. (c) Summary of the amount of mode energy splitting and the mode energy for the case (a). (d) Summary showing the correlation between the amount of mode energy splitting and the product of the bonding mode and the antibonding mode intensities as functions of the excitation power for the case (a). The measurement is performed at 10 K.
Fig. 4.10 (a) A representative case of ambient temperature dependent mode splitting behavior. The antibonding mode disappears at 42K and higher temperatures. (b) Summary of the amount of mode energy splitting and the mode energy for the case (a). (c) Summary showing the correlation between the amount of mode energy splitting and the product of the bonding mode and the antibonding mode intensities as functions of the ambient temperature for the case (a).
Chapter 5

FDTD analysis and experiment of in-plane polarized and directional emission in twin microdisk photonic molecules

5.1 Introduction and motivation

High-Q whispering gallery (WG)-mode resonators, such as microdisks, microrings, microspheres and microtoroids made from semiconductor, silica and polymer, are of interest in a variety of photonic applications as narrow bandwidth add/drop filters[61, 62], ultra-low threshold lasers[63] and high sensitivity chemical/biological sensors[64, 65]. A wide free spectral range (FSR), or in other words single mode operation, of the resonators is desirable for those applications[66]. Shrinking the scale of microcavity resonators is one straightforward way to increase the cavity FSR due to the WG mode property. However, as the cavity scale shrinks, so do the saturation level of filters and the power output level of lasers based on these cavities. Another fundamental disadvantage is the lack of emission directionality in resonator structures with circular symmetry. These two drawbacks of the conventional WG mode resonators hinder their further application deployments.

Photonic molecules (PM)[57, 56, 58, 67, 68, 69] –clusters of evanescent electromagnetic (EM) field coupled optical microcavities– are expected to be an alternative to overcome those drawbacks along with other proposals, such as deformed shape resonators[66].
Coupled-resonator optical waveguides (CROWs) [70, 71]—linear chains of side-coupled microcavities have been studied for the aim of achieving low-loss or even loss-less waveguide bends. In order to design low-loss CROWs, it is important to understand the supermodes induced by the coupling in PMs. The simplest case of a PM, a pair of twin circular microdisks, has been demonstrated to show splitting of modes in the form of the bonding and the antibonding supermodes with four classes of symmetry [57, 56, 58, 59]. They are also expected to have directional emission patterns [66, 59] and dramatically increased Q factors [72] as a result of the EM field modification induced by the coupling. Previous simulations have suggested that the achievement of these benefits requires very careful control over the mutual coupling between individual resonators.

In this chapter, we perform FDTD simulations on both twin circular PMs and elliptical PMs with four classes of symmetry matching conditions of the bonding and the antibonding modes. Our simulations predict directional in-plane polarized emission patterns from circular PMs. These predictions are confirmed by experiment. We also observe directional and in-plane polarized emission patterns from twin elliptical PMs. However, these observations are not consistent with simulation.

5.2 FDTD simulation set up

Our simulation focuses on three classes of PMs: (a) twin circular PMs, (b) twin elliptical PMs coupled along the long axis and (c) twin elliptical PMs coupled along the short axis. The circular microdisks are in radius of 1 µm. The elliptical ones have a short axis length of 2 µm with the short axis length to the long axis length ratio 0.75. Each microdisk in a pair is separated by a gap of width 200 nm from rim to rim. The
thickness of the microdisks is 120 nm. Since the TE WG modes are dominant in the thin microdisks, we only search for the TE polarized emission in the following analysis with 3-D FDTD method. The simulation region is realized by a $200 \times 200 \times 50$ matrix where all physical parameters are discretized in each unit cell. Three matrices are used to store the conductivity, permittivity and permeability and are initialized according to the geometrical layout. Three matrices are used to store the EM field components ($E_x$, $E_y$ and $H_z$).

The implementation of discretizing Maxwell’s curl equations is usually based on the Yee algorithm solving for both the electric and magnetic fields in time and space. The Yee algorithm follows a grid in which every electric component is surrounded by four circulating magnetic field components, and vice versa. Specifically for the TE modes in the microdisk resonators we have the following discrete equations:

$$E_x^{N+1}(i, j, k) = (1 - \frac{\sigma(i, j, k)}{\epsilon(i, j, k)}) E_x^N(i, j, k)$$
$$+ \frac{\Delta t}{\epsilon(i, j, k) \delta} [H_z^M(i, j, k) - H_z^M(i, j - 1, k)] \tag{5.1}$$

$$E_y^{N+1}(i, j, k) = (1 - \frac{\sigma(i, j, k)}{\epsilon(i, j, k)}) E_y^N(i, j, k)$$
$$+ \frac{\Delta t}{\epsilon(i, j, k) \delta} [H_z^M(i, j, k) - H_z^M(i + 1, j, k)] \tag{5.2}$$

$$H_z^M(i, j, k) = H_z^{M-1}(i, j, k) + \frac{\Delta t}{\mu(i, j, k) \delta} [E_y^N(i, j, k)$$
$$- E_y^N(i + 1, j, k) + E_x^N(i, j + 1, k) - E_x^N(i, j, k)] \tag{5.3}$$
where i, j, k are unit cell index variables for the matrices; \( \sigma(i, j, k) \), \( \epsilon(i, j, k) \) and \( \mu(i, j, k) \) are the conductivity, permittivity and permeability at each unit cell; M and N are time index variables with \( M = N + 1/2 \); \( \Delta t \) is the time increment; \( \delta = \Delta x = \Delta y = \Delta z \) is the actual grid separation.

A Gaussian profile pulsed source is placed along one of the microdisks' rim to start the EM field evolution towards a standing wave WG mode by initializing the EM field components in the corresponding unit cells. The wavelength of the pulsed source is chosen to support the TE(5,1) WG mode in 1 \( \mu \text{m} \) radius circular microdisks. Perfectly matched layer (PML) boundary conditions are applied along the boundary of the simulation region.

We search for four classes of EM field pattern symmetry in the PMs (ee, eo, oe, oo according to the field symmetry along the \( \hat{x}, \hat{y} \) axes) by imposing corresponding symmetry matching conditions in simulation. We use a coordinate system where the disks lie in \( xy \) plane, \( \hat{x} \) is parallel to the coupling axis and the origin is midway between the disk centers. Hence for example, the matching condition for ee mode is that the field pattern (\( H_z \) for TE modes, \( E_z \) for TM modes) is even along both \( \hat{x} \) and \( \hat{y} \) axes and for oe it is odd along \( \hat{x} \), even along \( \hat{y} \), etc. In the simulation, the symmetry matching conditions are:

\[
H_z(i, j, k) = H_z(-i, j, k) \quad (5.4)
\]
\[
H_z(i, j, k) = H_z(i, -j, k) \quad (5.5)
\]
for ee, and

\[ H_z(i, j, k) = H_z(-i, j, k) \]  \hspace{1cm} (5.6)  \\
\[ H_z(i, j, k) = -H_z(i, -j, k) \]  \hspace{1cm} (5.7)

for oe, etc.

According to the symmetry along \( \hat{y} \), ee and oe are two nearly degenerate bonding modes; eo and oo are two nearly degenerate antibonding modes[58, 59].

For the TE(5,1) mode in our study, there is only one magnetic field component, which is perpendicular to the disk plane, \( H_z \). Fig. 5.1 shows the far field \( H_z \) patterns for all four classes of the bonding and the antibonding modes. To study the polarization resolved emission, in-plane electric field components along \( \hat{x} \) and \( \hat{y} \) are also simulated. The intensity of in-plane polarized TE mode emission along \( \hat{x}(\hat{y}) \) is proportional to the product of the \( x(y) \) electric component with the \( z \) magnetic component.

### 5.3 Simulation results

For the TE(5,1) mode in our study, there is only one magnetic field component, which is perpendicular to the disk plane, \( H_z \). Fig. 5.1 shows the far field \( H_z \) patterns for all four classes of the bonding and the antibonding modes. To study the polarization resolved emission, in-plane electric field components along \( \hat{x} \) and \( \hat{y} \) are also simulated. The intensity of in-plane polarized TE mode emission along \( \hat{x}(\hat{y}) \) is proportional to the product of the \( x(y) \) electric component with the \( z \) magnetic component.
Fig. 5.1 Far field $H_z$ field component emission patterns from a twin circular PM in four classes of bonding and antibonding modes.

Fig. 5.2 and Fig. 5.3 show the electric field component and the intensity patterns of in-plane polarized light emission along $\hat{x}$ and $\hat{y}$ for the ee bonding mode and the eo antibonding mode. In Fig. 5.2 for the ee bonding mode, the in-plane x-polarized light emission is stronger than the y-polarized light in intensity. Similarly in Fig. 5.3 for the eo antibonding mode, the in-plane y-polarized light emission is stronger than the x-polarized light in intensity. In other words, the ee bonding mode is more x-polarized while the eo antibonding mode is more y-polarized. For the other two classes of bonding and antibonding modes oe and oo, the preference of in-plane polarization is not as apparent as the ee and the eo modes. One possible explanation is that the geometrical set up in this study, for example the ratio of the gap $w$ to the disk radius $a$, is not at the optimized values for the oe and the oo modes to achieve maximum in-plane polarization.
Fig. 5.2 Far field $E_x$ (a), $E_y$ (b) field component and in-plane x-polarized (c), y-polarized (d) light emission patterns from a twin circular PM in the ee bonding mode.

We also perform similar simulation that imposes the four-class symmetry matching conditions on the twin elliptical PMs, whose coupling axes are either along the long axis or the short axis. We note that the partial removal of the standing wave WG mode degeneracy even in a single elliptical disk may produce more classes of symmetry and hence bonding/antibonding modes in the case of twin elliptical PMs. Therefore the four-class symmetry conditions that apply on the twin circular PMs may not be valid for the twin elliptical PMs. We are not aware of any studies on the twin elliptical PMs in terms of the number of symmetry class. Fig. 5.4 shows the x and the y-polarized light intensity patterns for two (ee, oo) of the four classes of bonding and antibonding modes in the twin elliptical PMs coupled along the long axis. As we suspect, this simulation does not give any preferred in-plane polarization for either of the bonding or the antibonding modes in
the twin elliptical PMs coupled along either the long or the short axis. Note that in Fig. 4 we rotate the coordinate system counter-clockwise 45° to compare with the experiment results, which have the polarization axes along the \( \hat{x} \) and \( \hat{y} \) in this coordinate.

Directional light emission from the twin circular PMs is also predicted by the simulation. Fig. 5.5 shows the directional far field light emission patterns for the x-polarized ee bonding mode and the y-polarized eo antibonding mode in the twin circular PMs. If the contributions from the y-polarized emission for the ee mode and the x-polarized emission for the eo mode are also included, the far field directional emission patterns agree with the simulation results by other methods[59]. No such apparent directional emission patterns are found in the other two modes (oe and oo). This may still be because that the coupling geometrical set up is not at the optimized values.
for those two modes to achieve apparent directional emission. For example, a previous study predicts that for the ee and the eo modes, maximized directional emission occurs at $w/a \approx 0.4$, while for the oe and the oo it occurs at $w/a \approx 1.1$ and 2.2 respectively [59]. In our FDTD simulation, for practical consideration, $w/a$ is 0.2.

5.4 In plane polarized stimulated emission from twin coupled microdisks

In-plane polarized light emission is examined by adding a linear polarizer in the detection path after the objective but before the optical fiber coupler. The light emission runs through a three meter long multi-mode optical fiber to the spectrometer. The optical fiber is not polarization maintaining. The light emission–after being linear polarized–resolved by the linear polarizer–is not polarized when it enters the spectrometer entrance.
Fig. 5.5 Far field directional light emission patterns for the $x$-polarized $ee$ bonding mode (a) and the $y$-polarized $eo$ antibonding mode (b) in a twin circular PM.

Therefore even if the spectrometer has some certain type of polarization dependence, it does not destroy our results since we essentially only measure the intensity of a non-polarized light after we resolve its linear polarization. A 0.70 numerical aperture (NA) objective excites disks and collects emission with a $\sim10\mu m$ diameter spot size. With the disk diameter $\sim2\mu m$ and careful positioning of the PM in the center of the beam, the pumping spot should excite the whole PM instead of exciting one or part of the disks locally. An objective with such a high NA offers a wide collecting angle that is capable of collecting light emission out from the disk edge in the disk plane. Since the NA is defined as:

$$NA = n \sin \theta$$  \hspace{1cm} (5.8)

where $n$ is the index of refraction of the medium in which the lens is working (1.0 in this case), and $\theta$ is the half-angle of the maximum cone of light that can enter or exit the lens. Thus the collecting angle is $2\theta=89^\circ$.

Microdisk lasers emit light from side in disk plane with propagation direction perpendicular to disk rim. Only TE-polarized (polarized in disk plane) WG modes are
supported in thin microdisks and the bonding and antibonding modes in our PMs are TE supermodes. Therefore the light emission from them is solely polarized in the disk plane and has no out-of-plane polarization components. So the in-plane light polarization can be mapped out by rotating the linear polarizer whose polarization axis is then rotated accordingly in the image plane. Fig. 5.6 shows that the stimulated emission from a twin circular PM has distinct in-plane polarization parallel and perpendicular to the coupling axis for the bonding and antibonding modes. Fig. 5.7 shows the ratio of the bonding mode intensity to the antibonding mode intensity as a function of the linear polarizer angle for the two types of PMs we studied. The data for the coupled circular disks (Fig. 5.7(a)) are in agreement with the predictions of our FDTD simulations: the bonding mode has a stronger intensity along \( \hat{x} \) whereas the antibonding mode has a stronger intensity along \( \hat{y} \). However, the data from the coupled elliptical disks (Fig. 5.7(b)) do not agree with the simulations which predict no significant polarization in either case. Elliptical disk PMs coupled along the long axis show a very pronounced polarization at 45° and 135° with the coupling axis (Fig. 5.7(b)). Further, for elliptical disk PMs coupled along the short axis, we do not observe a clear correlation between the intensity ratio and the linear polarizer angle: although one set of data again suggest preferential polarization at 0° and 90° from the coupling axis, we have found that the intensity ratio oscillation amplitude changes from one pair of disks to another in our measurements. Also in some of the PMs, even though the intensity ratio oscillates with the linear polarizer angle, the intensity of the bonding mode is always greater than that of the antibonding mode. This may be because of variations in the pumping spot that change the relative intensity of the two types of modes.[57, 58]
Fig. 5.6 Polarization-resolved lasing spectra from twin circular coupled microdisks. The polarization directions are noted by the arrows.

5.5 CCD imaging of far field emission pattern from twin coupled microdisks

To have a direct feeling of the in-plane polarized lasing emission from the coupled twin microdisks, we use a CCD camera that is sensitive to the lasing wavelength following the linear polarizer in the detection light path to acquire far field emission pattern. Fig. 5.8 shows the layout of the optical measurement set up. Fig. 5.9 shows the far field emission patterns taken with the CCD camera mounted with a linear polarizer along different polarization directions for the twin circular microdisk Fig. 5.9(a)(b) and the elliptical microdisks coupled along the long axis Fig. 5.9(c)(d). For the circular ones, the images show that vertically polarized light intensity is more concentrated along the
Fig. 5.7 The intensity ratio vs. the linear polarizer angle in the twin circular coupled microdisks (a), elliptical coupled microdisks along the long axis (b).
Fig. 5.8 A schematic diagram showing the optical measurement layout discussed in this chapter.
coupling axis and horizontally polarized light is more concentrated perpendicular to the coupling axis. This also agrees with the simulation results. For the elliptical ones coupled along the long axis, the concentration patterns are not very clear, although they do show a trend of concentrating perpendicular to the polarization directions.

Fig. 5.9 Far field emission patterns from the twin coupled circular microdisks (a)(b) and the elliptical coupled microdisks along the long axis (c)(d) for two perpendicular polarization directions marked by arrows.

5.6 Conclusion

In summary, a numerical simulation of the EM field and in-plane polarized light emission patterns in the twin circular and the twin elliptical PMs is performed through the FDTD method. Four-class of symmetry in the form of bonding (ee, oe) and antibonding (eo, oo) modes is applied to the simulation by using the EM field pattern symmetry matching conditions. In-plane polarized and directional light emission from
the twin circular PMs is demonstrated in simulation and examined in experiment, where the ee bonding mode polarizes parallel to the coupling axis and the eo antibonding mode polarizes perpendicular to it. Limitation of this four-class symmetry appears when simulation on the twin elliptical PMs fails to yield the in-plane polarized light emission observed in experiment, where the bonding mode polarizes $45^\circ$ counter-clockwise to the coupling axis and the antibonding mode polarizes $135^\circ$ counter-clockwise to it in the twin elliptical PMs coupled along the long axis. Those coupled along the short axis do not show any systematic polarization in experiment.

Demonstration of in-plane polarized and directional light emission through EM field pattern design from the simplest PM case makes the photonic applications of PMs as essential components feasible and practical. Although PMs composed of deformed shape resonators, such as the elliptical ones, still need further study to understand the number of symmetry class and so on to make manipulation of their modal properties possible.
Chapter 6

Near field optical probing of microdisk cavity properties through coupled taper fiber

6.1 Introduction and motivation

An optical objective is used to both excite microdisk lasers and collect the emission from them in the study of the chapter 3 and 4. This "free space" collection of emission from a microdisk is a function of several factors: it depends on the position and NA of the objective, the emission pattern and quality factor (Q) of the resonant mode and most importantly optical losses from the microdisk. Essentially the emission that the objective collects is the light leaking out of the microdisk cavity because of the losses. Ideally there is only one type of optical loss: radiation due to the radial tunneling of light from the disk periphery. In actual microdisk devices, scattering loss from surface roughness at the disk edge and material absorption claim a major part of the optical losses. Especially in the III-V group material microdisks which have a high refractive index (3.5), any light that is collected by the free space methods is the result of the scattering loss of WG resonant mode because of the surface roughness. This ends up with a situation that the more perfect the disks are made (less surface roughness), and further that higher Q factor is improved, the more difficult it becomes to collect emission efficiently out of the cavities.
A tapered fiber/fiber taper offers an attractive alternative to efficiently and directly couple light out of microdisk cavities through a different mechanism. Instead of relying on the non-directional scattering from surface roughness and weak intrinsic radiation of the microdisk, the fiber taper couples its resonant mode with the WG resonant mode in the disk through the evanescent field in the interaction region of the taper and the disk and hence provides a means to couple the emission out of the disk. This collection method has been demonstrated to be efficient with small diameter microdisks where the phase matching between the silica optical fiber and microdisk is not as limiting. While the fiber taper does load the cavity mode and thus degrades its Q factor, this is a ”good loss” in the sense that it can be efficiently coupled into the fiber and collected. This actually maintains a high Q factor while obtaining a high coupling efficiency at the same time.

The fiber taper technique is also advantageous for excitation. Since the free-space excitation through an objective pumps the microdisk more or less uniformly, the gain material which is not spatially aligned with the WG modes is also excited. This degrades the mode Q factor because of the unnecessary material absorption/scattering. On the other hand, the fiber taper only excites a limited part of the gain material which is at the edge of the microdisk. The cavity Q factor is increased by reducing gain material related contribution to absorption/scattering loss with a limited part of gain material excited. This in turn reduces the required optical gain to achieve threshold. More importantly, since WG modes are located at the edge, gain material aligns well with the resonant modes spatially. Therefore, the transparency component that increases the threshold is suppressed with the limited gain material excitation, while the gain component that
decreases the threshold is about the same in the optical taper excitation case as the free space excitation case. A usually much lower lasing threshold is expected with this fiber taper pumping and collecting approach.

6.2 Theory of fiber taper coupling with a microdisk cavity

6.2.1 Traveling wave mode resonator

We first consider the coupling between a single mode waveguide and a single mode of a cavity. In the context of this dissertation, this would be the coupling between the forward propagating mode of a waveguide—a single mode (SM) fiber taper and the clockwise propagating WG mode of a microdisk resonator. The cavity’s intrinsic loss rate is $\gamma_0$, and its loss rate into the waveguide is $\gamma_e$. The waveguide input field is labeled $s$, and it couples to a cavity mode of amplitude $a_c w$. The transmitted field past the cavity is $t$, with $s$ and $t$ normalized to power, and $a_c w$ normalized to energy.

The time evolution of the mode amplitude $a_c w$ is given by:

$$\frac{d a_c w}{dt} = i \omega_0 a_c w - \frac{\gamma_T}{2} a_c w + k_e s,$$  \hspace{1cm} (6.1)

where $\gamma_T$ is the total energy decay rate of the cavity mode (equal to $\gamma_0 + \gamma_e$ above), and $k_e$ is the waveguide-resonator coupling coefficient. The above equation states that the mode amplitude $a_c w$ oscillates in time with a frequency $\omega_0$, decays with a loss rate $\gamma_T$, and is driven by an input field $s$ with coupling coefficient $k_e$.

The transmitted signal $t$ has a contribution from the portion of the input signal $s$ that does not couple into the cavity, and a contribution from the signal coupled out
of the cavity. We thus expect \( t = \alpha_1 s + \alpha_2 a_c w \), where \( \alpha_1 \) and \( \alpha_2 \) are coefficients to be determined. With the assumption that there are no other inputs and losses except the ones specified above, we use power conservation to determine the two coefficients. Here we equate the power transfer into the cavity with the change in the cavity’s internal energy plus the dissipated power and we have:

\[
|s|^2 - |t|^2 = \frac{d|a_c w|^2}{dt} + \gamma_0 |a_c w|^2. \tag{6.2}
\]

Plugging into equation 6.1 along with \( t = \alpha_1 s + \alpha_2 a_c w \) yields three equations for the variables \( \alpha_1 \) and \( \alpha_2 \) (which are complex). We have:

\[
1 - |\alpha_1|^2 = 0, \tag{6.3}
\]

\[
|\alpha_2|^2 = \gamma_e, \tag{6.4}
\]

\[
-\alpha_1 \alpha_2^{*} = k_e. \tag{6.5}
\]

One simple set of trial solution is \( \alpha_1 = -1,\alpha_2 = k_e^{*} \) (with \( |k_e|^2 = \gamma_e \)), giving the transmitted signal as:

\[
t = -s + k_e^{*}a_c w. \tag{6.6}
\]

We now add a loss term representing parasitic coupling between the waveguide and resonator (for example, coupling-induced scattering into radiation modes) and now we have \( \gamma_T = \gamma_0 + \gamma_e + \gamma_p \). We can find the steady state transmission through the waveguide by \( T = \mid \frac{s}{t} \mid^2 \):

\[
T = \frac{\gamma_e - (\gamma_0 + \gamma_p) - 2i\Delta \omega}{\gamma_e + (\gamma_0 + \gamma_p) + 2i\Delta \omega}. \tag{6.7}
\]
where $\Delta \omega = \omega - \omega_0$ is the difference between the drive frequency and the cavity resonance frequency. As a function of $\omega$, $T$ is a Lorentzian centered at $\omega = \omega_0$.

On resonance ($\Delta \omega = 0$), we can rewrite the equation as:

$$T = \left(\frac{1 - \kappa}{1 + \kappa}\right)^2,$$

(6.8)

where $\kappa$ is called coupling parameters, and is defined as

$$\kappa = \frac{\gamma_e}{\gamma_0 + \gamma_p},$$

(6.9)

$\kappa$ is the ratio of coupling into waveguide with coupling into intrinsic and parasitic loss channels.

Experimentally, we always measure $Q_T = \omega_0/\gamma_T$, although we have control over $\gamma_e$ by controlling the taper-cavity separation. In practice, we can increase the taper-cavity separation to the point that $\gamma_e$ is quite small, giving us an estimate of the cold-cavity Q factor $Q_i$ with the assumption that $\gamma_p$ also becomes quite small as the separation becomes large. Without changing the taper-cavity separation, we can get an estimate of $Q_{i+p}$, the $Q$ due to intrinsic decay and parasitic waveguide-cavity coupling, by knowing $\kappa$, or equivalently, the transmission depth on resonance. In particular,

$$Q_{i+p} = \frac{\omega_0}{\gamma_0 + \gamma_p} = \frac{\omega_0}{\gamma_0 + \gamma_r + \gamma_p} \cdot \frac{\gamma_0 + \gamma_e + \gamma_p}{\gamma_0 + \gamma_p} = Q_T(1 + \kappa).$$

(6.10)

We also define another important parameter called the ideality $I_i$, which is the ratio of the coupling into the waveguide mode of interest with the coupling into all waveguide
channels.

\[ I = \frac{\gamma_e}{\gamma_e + \gamma_p} \]  

(6.11)

So far, we only consider the cavity as passive. Next, let us consider the case of an emitter within the cavity. A fraction \( \beta \) of the emitter’s spontaneous emission will be coupled into the cavity mode of interest. The fraction of these photons that are then coupled into the waveguide mode of interest is given by the parameter \( \eta_0 \), with

\[ \eta_0 = \frac{\gamma_e}{\gamma_e + \gamma_0 + \gamma_p}, \]  

(6.12)

The ratio of the total (loaded) \( Q_T \) to the intrinsic \( Q_i \) can be written in terms of \( \eta_0 \) and \( I \) as,

\[ \frac{Q_T}{Q_i} = \frac{\gamma_0}{\gamma_0 + \gamma_e + \gamma_p} = \frac{\gamma_0 + \gamma_e + \gamma_p}{\gamma_0 + \gamma_e + \gamma_p} - \frac{\gamma_e + \gamma_p}{\gamma_0 + \gamma_e + \gamma_p} \]

\[ = 1 - \frac{\gamma_e}{\gamma_e + \gamma_0 + \gamma_p} \cdot \frac{\gamma_e + \gamma_p}{\gamma_e} \]

\[ = 1 - \frac{\eta_0}{I}. \]  

(6.13)

6.2.2 Standing wave mode resonator

We now consider the real case where the resonator supports a standing wave mode rather than a traveling wave mode. This is true in the WG mode microdisk cavities studied in the dissertation. The main difference is that the standing wave mode decays equally into the forward and backward propagating modes of the waveguides. Still considering \( \gamma_e \) as the loss rate into the forward propagating waveguide mode, the
total loss rate $\gamma_T = 2\gamma_e + \gamma_0 + \gamma_p$. The formula for the normalized transmission is then,

$$T = \left| \frac{\gamma_e - (\gamma_e + \gamma_0 + \gamma_p) - 2i\Delta\omega}{\gamma_e + (\gamma_e + \gamma_0 + \gamma_p) + 2i\Delta\omega} \right|^2. \quad (6.14)$$

On resonance ($\Delta\omega = 0$), we can rewrite this equation as:

$$T = \left( \frac{1 - \kappa}{1 + \kappa} \right)^2, \quad (6.15)$$

where the coupling parameter $\kappa$ is now written as:

$$\kappa = \frac{\gamma_e}{\gamma_e + \gamma_0 + \gamma_p}. \quad (6.16)$$

Note that in the standing wave case, $\kappa$ is restricted to $\kappa \leq 1$.

In coupling to a standing wave mode, there is now a reflected signal coming out of the input port, with the normalized reflection $R$ given as $R = \left| \frac{r_s}{s} \right|^2$. In this equation, $r = k_e^s a_s w$, where $a_s w$ is the standing wave mode amplitude. We then arrive at:

$$R = \left| \frac{2\gamma_e}{\gamma_e + (\gamma_e + \gamma_0 + \gamma_p) + 2i\Delta\omega} \right|^2. \quad (6.17)$$

At resonance, this is rewritten in terms of $\kappa$ as:

$$R = \frac{4\kappa^2}{(1 + \kappa)^2}. \quad (6.18)$$
Finally, we consider the parameter $\eta_0$. Assuming that photons are only collected from one of the waveguide modes, it is defined as:

$$\eta_0 = \frac{\eta_e}{2\gamma_e + \gamma_0 + \gamma_p}, \quad (6.19)$$

and can again be rewritten in terms of the coupling parameter $\kappa$ as:

$$\eta_0 = \frac{1}{1 + 1/\kappa}. \quad (6.20)$$

Recall that in 6.16 $\kappa$ is defined to be $\leq 1$. Hence, $\eta_0 \leq 50\%$, which make sense because the cavity mode equally decays into the forwards and backwards channels of the waveguide, so that at most 50% of the cavity photons can be collected out of any one channel.

6.3 Passive measurement of microdisk cavity quality factor through fiber taper

6.3.1 Preparation of single mode optical fiber taper

A single mode optical fiber is heated and stretched to form the fiber taper. The silica cladding diameter of the fiber is around 125$\mu$m and after heating up and stretching, the diameter of the cladding and core is tapered down to a few microns. We set up a pulling rig to prepare the taper in a controllable way, with a propane torch to heat up the fiber. The flame temperature of a propane torch can reach 1995 °C. At this temperature the coating layer of the fiber (usually nylon) burns away, and the silica cladding and core of the fiber melts and can be stretched. Since the fiber is tapered down with the
cladding layer intact, the loss of the fiber taper caused by the shape change does not increase dramatically. With the total length of the tapered region around a few mm, the fiber still confines light quite well, which is essential for the measurement. The taper—usually has a diameter around a few µm—is carefully positioned very close (less than 500 nm) to the rim of a microdisk cavity so that it overlaps with the evanescent field of the cavity where the coupling theory discussed in this chapter is valid.

6.3.2 Passive measurement set up and results

A white light source is coupled into the fiber. It runs through the fiber and interacts with the microdisk cavity at the taper region. The total quality factor $Q_T$ is extracted from the linewidth (FWHM) of the dip ($\gamma_T$) in the transmission/absorption spectrum. Fig. 6.1 is a picture of the experimental set up. The white light beam is collimated and then focused into the single mode fiber. It runs through the taper and at the other end of the fiber, with a bare fiber coupler and an SMA to FC converter, is sent into the spectrometer.

Fig. 6.2 shows a transmission spectrum exhibiting clear dips are shown. The linewidth (FWHM) of the dip is about 0.078nm, probably limited by our spectrometer resolution of about 0.07nm. This gives us a lower limit of the $Q$ factor of the microdisk of around $1.9 \times 10^4$. 
Fig. 6.1 Experiment set up of the passive quality factor measurement with a fiber taper.

Fig. 6.2 Transmission spectrum of the passive Q factor measurement, where two dips from cavity absorption are clearly shown.
Chapter 7

Conclusion

7.1 Summary

In this thesis, we have presented a series of experiments and simulations that revolve around semiconductor microdisk cavities in the context of semiconductor cavity quantum electrodynamics (CQED), semiconductor lasers and resonator property engineering. State-of-art cleanroom process fabrication techniques and MBE sample growth method enable us to investigate interactions of light and matter, light and light in semiconductor optical cavities and to some level even to manipulate these interactions. Modern photonics technology utilizes both interactions to construct lasers, waveguides and filters. CQED, an important physics topic in photonics also focuses on both interactions. The studies in this thesis can be summarized into two parts, although they are not mutually exclusive.

1. External strain engineering of (Al, Ga)As/GaAs microdisk lasers; bimodal lasing in twin (Al, Ga)As/GaAs microdisk photonic molecules. The light-matter interaction-here the coupling between the GaAs quantum wells/quantum dots and microdisk cavities-is the basic mechanism of these two experiments. (chapter 3 and part of chapter 4 and 5)
2. Normal mode splitting in twin coupled GaAs microdisk cavities; polarized and directional emission from twin coupled GaAs microdisk cavities (photonic molecules). Although the photons in these experiments are generated as a result of the light-matter interaction, light-light interaction—the coupling of the optical cavities—is the basic mechanism. (chapter 6 and part of chapter 4 and 5)

In chapter 3, we utilized the stress of the SiN$_x$ thin film to externally engineer the strain of the free standing microdisk lasers. Compressive strain was introduced to decrease the TE mode lasing threshold since the TE polarized recombination between the conduction band and the heavy hole is preferred under compressive strain. The threshold is decided by light-in vs. light-out $cw$ intensity plot and by the charge life time in dynamic measurement.

In the future, one should measure the exact stress values inside the SiN$_x$ thin film directly. This will help engineer the strain effect in the devices in a more controllable way. Also, more devices can be fabricated to find out the optimized condition of achieving even lower lasing threshold.

In chapter 4, we fabricated a type of coupled semiconductor cavities (photonic molecules), which are composed of two microdisks side coupled through evanescent field. Normal mode splitting—a signature of strong coupling—is observed. FDTD simulation of the EM field distribution shows diatom molecule electron wavefunction like patterns. Hence the lower energy mode in the mode splitting is named as bonding mode and the higher energy mode is named as antibonding mode. Systematic characterization of the normal mode splitting energy as functions of geometrical layout, excitation power and
ambient temperature is performed. A correlation between the mode splitting energy and the mode intensities is clearly shown in the excitation power and temperature dependent measurements. This can explained by the nature of the evanescent field coupling as the coupling depends on the field intensity.

Future work about this part of the dissertation may include more investigation about the geometrical dependence of the mode splitting. For example, how would the splitting vary in a more complicated coupled cavity systems, such as a circular array of microdisks? Another interesting work probably is to include the strong coupling between a quantum dot and a cavity photon. Hence, a cavity photon can strongly couple with a quantum dot either inside its own cavity or with a quantum dot in the other cavity, and vice versa.

In chapter 5, another aspect of the coupled cavities is studied—the modification of the EM field inside and outside the cavities induced by the broken symmetry. We perform FDTD simulation of the EM field patterns in the twin coupled microdisk cavities. And in-plane polarized light emission patterns are acquired from the EM field component patterns. The simulation suggests directional and in-plane polarized light emission from some of the twin coupled microdisks. This in-plane polarized emission is examined in experiment. For example, the bonding mode polarizes along the coupling axis and the antibonding mode polarizes perpendicular to it in the twin circular coupled microdisks. Therefore, directional lasing can be easily achieved through proper geometrical layout design, which brings a lot flexibility in designing and fabrication.

One immediate future work for this chapter is to understand the field symmetry patterns in the coupled elliptical disks. Since the four symmetry class field pattern of
coupled circular disks is not valid. Experimental results suggest a pair of symmetry axes that rotate 45 °C from the coupling axis. This may because that the standing wave degeneracy condition is partially removed in elliptical disks.

In chapter 6, we adopt a new method to measure the quality factor of microdisk cavity passively. A fiber taper is prepared from a single mode fiber and positioned in the evanescent field vicinity of the microdisk. Transmission/absorption spectrum is resolved with a dip in the spectrum. The total quality factor is derived from the linewidth of the dip with $Q_T=\omega_0/\gamma_T$. Limited by the resolution of our spectrometer, the estimated intrinsic Q factor of the microdisk should be greater than $1 \times 10^4$. Active excitation and collection of stimulated emission through taper is still under way.

The fiber taper technique discussed in chapter 6 is very helpful to understand the EM field patterns in coupled cavities discussed in chapter 5. Since the taper–if placed properly at different vicinities of the coupled cavities–provides a method to monitor the EM field intensity insitu. Hence, field distribution patterns in complicated coupled cavity systems can be mapped out to understand and engineer "super modes" induced by coupling.

7.2 Future directions

7.2.1 Towards photonic clusters and CROWs

The work in chapter 5 suggests that one can manipulate and engineer complicated photonics devices that have special functions. They are feasible by simply forming
Fig. 7.1 An SEM picture of a cyclic photonic cluster formed by 16 single elliptical microdisks.

degemtical arrays of single cavities with proper EM field design. Multiple function devices require relatively large number of cavities to provide enough power and robustness. One potential device that has been proposed and studied is the coupled resonator optical waveguide (CROW). CROWs can be regarded as a simple form of photonic clusters since they are linear in most cases. Other forms of photonic clusters, like the one that we patterned (Fig. 7.1), may demonstrate much more complicated mode behavior. FDTD simulation of several different mode field distribution with certain symmetry configuration is shown in Fig. 7.2 for this kind of photonic clusters. However one expects to find more symmetry classes when the number of cavity dramatically increase in photonic clusters.

Experimentally the mode behavior could be even more complicated. For example, Fig. 7.3 shows the variation of mode behaviors of the kind of photonic cluster shown in Fig. 7.1. As the excitation profile changes from a local pumping of a few cavities to a global one of the whole cluster, the number of optical modes actually increases. This
may suggest that as the cluster is excited as a whole, the induced coupling between the cavities brings degeneracy.

The work discussed in chapter 6 may provide a convenient way to investigate the EM field pattern in photonic cluster of complex geometries. One can run tapered fiber through different parts of the cluster and collect light emission from those parts locally at the same excitation profile. With this method, one could map out the light emission patterns from photonic clusters and understand the EM field distribution patterns in whole.

7.2.2 Magnetic microdisk lasers towards spin storage and manipulation in optical cavities

Another potential interesting direction is to hybridize magnetic (ferromagnetic) materials with optical materials through MBE sample growth and fabricate microdisk lasers that also show ferromagnetism in situ. One of the motivations is to manipulate spins in optical cavities with a weak magnetic field or in zero field. Recent research shows that electron spin coherence time is enhanced at certain energies in a microdisk cavity. Therefore the in situ magnetic field could manipulate the spins that are confined in the cavity. Fig. 7.4 shows our approach to that with a double layer disk structure. The sample is grown by MBE with the common microdisk structure plus the top ferromagnetic disk layer of GaMnAs designed to provide fringe magnetic field for the bottom optical disk layer.

Initial characterization shows that the device lases and has ferromagnetism with a Curie temperature around 45 K. Spin related measurements are still under way.
Fig. 7.2 FDTD simulation results of EM field patterns in the cyclic photonic cluster shown in Fig. 7.1.

Fig. 7.3 Variation of mode behaviors at different excitation profiles in the cyclic photonic cluster shown in Fig. 7.1.
Fig. 7.4 An SEM picture showing the double disk layer structure to realize the magnetic microdisk lasers. The top layer is made by GaMnAs and separated by a 250nm AlGaAs post region.
Appendix A

Source code of FDTD method used in simulation of microdisk cavity

%***********************************************************************
% Fundamental constants
%***********************************************************************

cc=2.99792458e8;               %speed of light in free space
muz=4.0*pi*1.0e-7;            %permeability of free space
epsz=1.0/(cc*cc*muz);        %permittivity of free space

%***********************************************************************
% Grid parameters
%***********************************************************************
nmax=2;                        %total number of time steps
dx=1e-8;                       %space increment of cubic lattice
dt=dx/(2.0*cc);               %time step

% Add metal disk

diam=100;                     % diameter of disk
\texttt{rad=diam/2.0; \% radius of disk}

\texttt{thickness=10; \% thickness of disk}

\texttt{ie=140; \% number of grid cells in x-direction}

\texttt{je=140; \% number of grid cells in y-direction}

\texttt{ke=50; \% number of grid cells in z-direction}

\texttt{io=30; \% location of observed point}

\texttt{jo=je/2; \% location of observed point}

\texttt{ko=ke/2; \% location of observed point}

\texttt{ib=ie+1;}

\texttt{jb=je+1;}

\texttt{kb=ke+1;}

\texttt{ib2=ie-1;}

\texttt{jb2=je-1;}

\texttt{kb2=ke-1;}

\texttt{\% is=12; \% location of z-directed current source}

\texttt{\% js=je/2; \% location of z-directed current source}

\texttt{\% ks=ke/2; \% location of z-directed current source}
Hz_op=[];  %record Hz at observed point as a function of time

kobs=ke/2;

%***********************************************************************
% Mode parameters
%***********************************************************************

m_min=3;
m_max=3;

n_M=m_max-m_min+1;

for m=m_min:m_max
    M(m)=m;
end

for m=m_min:m_max
    phase_theta(m)=0;
    % phase_theta(m)=2*pi*rand;
end

x_M_1(3)=6.3802;
x_M_1(4)=7.5883;
x_M_1(5)=8.7715;
x_M_1(6)=9.9361;
%***********************************************************************
% Material parameters
%***********************************************************************

media=2;

eps=[1.0 10.24];
sig=[0.0 0.0];
mur=[1.0 1.0];
sim=[0.0 0.0];

%***********************************************************************
% Updating coefficients
%***********************************************************************

for i=1:media
    eaf(i) =dt*sig(i)/(2.0*epsz*eps(i)); % material
camat(i)=(1.0-eaf(i))/(1.0+eaf(i)); % ca in different material

cbmat(i)=dt/epsz/eps(i)/dx/(1.0+eaf(i)); % cb in different material

haf(i)=dt*sim(i)/(2.0*muz*mur(i));

damat(i)=(1.0-haf(i))/(1.0+haf(i)); % da in different material

dbmat(i)=dt/muz/mur(i)/dx/(1.0+haf(i)); % db in different material

end

%**************************************************************************
% Geometry specification (main grid)
%**************************************************************************

% Initialize entire main grid to free space

c(1:ib,1:jb,1:kb)=camat(1);

cb(1:ib,1:jb,1:kb)=cbmat(1);

da(1:ib,1:jb,1:kb)=damat(1);

db(1:ib,1:jb,1:kb)=dbmat(1);

icenter=ie/2; % i-coordinate of disk’s center
jcenter=je/2; % j-coordinate of disk’s center
kcenter=ke/2; % k-coordinate of disk’s center
for i=1:ie
    for j=1:je
        for k=1:ke
            dist2=(i+0.5-icenter)^2 + (j-jcenter)^2;
            dist=abs(k-kcenter);
            if dist2 <= rad^2 && dist <= thickness/2
                ca(i,j,k)=camat(2);
                cb(i,j,k)=cbmat(2);
            end
        end
    end
end

%***********************************************************************
% Field arrays
%***********************************************************************
Hz0=1;
epsilon=0.00001;
Hx = zeros(ie-1,je,ke);    Ex = zeros(ie,je-1,ke-1);
Hy = zeros(ie,je-1,ke);    Ey = zeros(ie-1,je,ke-1);
Hz = zeros(ie,je,ke-1);    Ez = zeros(ie-1,je-1,ke);
mode_Hz = zeros(ie,je);
mode_Hz_final = zeros(ie,je);

for i=1:ie
    for j=1:je
        for k=kcenter-1:kcenter+1
            dist2=(i+0.5-icenter)^2 + (j-jcenter)^2;
            theta=atan2((j-jcenter),(-i+icenter+epsilon));
            % offset i by epsilon to prevent infinity

            % reverse i (x axis) to fit io
            if dist2 <= rad^2
                initial_mode=0;
                for m=m_min:m_max
                    initial_mode = initial_mode + besselj(M(m),x_M_1(m)*sqrt(dist2)/rad)*cos(M(m)*theta+phase_theta(m)); %
                end
                Hz(i,j,k) = Hz0 * initial_mode/n_M;
                % pure TE modes for initial condition
        end
    end
end
end

dist2=(i-icenter)^2 + (j+0.5-jcenter)^2;

if dist2 <= rad^2

initial_mode=0;

for m=m_min:m_max

initial_mode = initial_mode + besselj(M(m),x_M_1(m) *sqrt(dist2)/rad)*cos(M(m)*theta+phase_theta(m));

end

Hz(i,j,k) = Hz0 * initial_mode/n_M;

% pure TE modes for initial condition

end

end

end

end

%*************************************************************
% Parameter of PML
%*************************************************************

% auxiliary variables

coeff_a = dt/epsz;

coeff_b = dt/muz;

coeff_A = coeff_a/dx;%dt/(epsz*dx);

coeff_B = coeff_b/dx;%dt/(muz*dx);
N = 8; % number of cells of PML - N > 2

R_0 = 0.0001; % factor of reflection (0.01%)

% Polynomial grading (P. 306 Taflove book)
% linear (m_poly_PML=1); parabolic (m_poly_PML=2);
3 <= m <= 4 optimal

m_poly_PML = 3;

thickness_PML = N*dx; % total thickness of PML

sig_m = - epsz*cc*log(R_0)/(2/(m_poly_PML+1) %thickness_PML);

% conductivity of PML

sig_0(1) = sig_m/((m_poly_PML+1)*2^(m_poly_PML+1) %N^m_poly_PML);

% conductivity of PML at L=0

i_L=1;

for L=1:N-1 % conductivity for Exy Exz Eyx Eyz Ezx Ezy
    i_L=i_L+1;
    sig_0(i_L) = sig_0(1)*((2*L +1)^(m_poly_PML+1) - (2*L - 1)^(m_poly_PML+1))
                ^((m_poly_PML+1));
end

i_L=0;
for L=0.5:1:N-0.5

% conductivity for magnetic layer Hxy Hxz Hyx Hyz Hzx Hzy

i_L=i_L+1;

sig_0_m(i_L) = (sig_0(1)*((2*L +1)^(m_poly_PML+1) - (2*L - 1)^(m_poly_PML+1)))*muz/epsz;

end

%***********************************************************************
% stability condition
%***********************************************************************

EstbPML1 = 2*pi*epsz/(nmax*dt);

if (sig_0(1)>=EstbPML1)
    disp('stability condition error!!!')
    return
end

sig_0 = 1 - sig_0*coeff_a;

sig_0_m= 1 - sig_0_m*coeff_b;

%***********************************************************************
% Initial PML - lr->side left and right, fb->front and back, tb-> top and bottom
%***********************************************************************

Exy_lr = zeros(2*N+ie,2*N,2*N+ke-1);

Exz_lr = zeros(2*N+ie,2*N,2*N+ke-1);

Exy_fb = zeros(2*N,je-1,2*N+ke-1);

Exz_fb = zeros(2*N,je-1,2*N+ke-1);
Exy_tb = zeros(ie, je-1, 2*N);
Exz_tb = zeros(ie, je-1, 2*N);

Eyz_lr = zeros(2*N+ie-1, 2*N, 2*N+ke-1);
Eyx_lr = zeros(2*N+ie-1, 2*N, 2*N+ke-1);
Eyz_fb = zeros(2*N, je-1, 2*N+ke-1);
Eyx_fb = zeros(2*N, je-1, 2*N+ke-1);
Eyz_tb = zeros(ie-1, je, 2*N);
yx_tb = zeros(ie-1, je, 2*N);

Ezx_lr = zeros(2*N+ie-1, 2*N, 2*N+ke);
Ezy_lr = zeros(2*N+ie-1, 2*N, 2*N+ke);
Ezx_fb = zeros(2*N, je-1, 2*N+ke);
Ezy_fb = zeros(2*N, je-1, 2*N+ke);
Ezx_tb = zeros(ie-1, je-1, 2*N);
Ezy_tb = zeros(ie-1, je-1, 2*N);

Hxy_lr = zeros(2*N+ie-1, 2*N, 2*N+ke);
Hxz_lr = zeros(2*N+ie-1, 2*N, 2*N+ke);
Hxy_fb = zeros(2*N, je, 2*N+ke);
Hxz_fb = zeros(2*N, je, 2*N+ke);
Hxy_tb = zeros(ie-1, je, 2*N);
Hxz_tb = zeros(ie-1, je, 2*N);
Hyz_lr = zeros(2*N+ie,2*N,2*N+ke);
Hyx_lr = zeros(2*N+ie,2*N,2*N+ke);
Hyz_fb = zeros(2*N,je-1,2*N+ke);
Hyx_fb = zeros(2*N,je-1,2*N+ke);
Hyz_tb = zeros(ie,je-1,2*N);
Hyx_tb = zeros(ie,je-1,2*N);

Hxz_lr = zeros(2*N+ie,2*N,2*N+ke-1);
Hzy_lr = zeros(2*N+ie,2*N,2*N+ke-1);
Hxz_fb = zeros(2*N,je,2*N+ke-1);
Hzy_fb = zeros(2*N,je,2*N+ke-1);
Hxz_tb = zeros(ie,je,2*N);
Hzy_tb = zeros(ie,je,2*N);

%***********************************************************************
% Movie initialization
%***********************************************************************

plot_max=0.3;
plot_min=-0.3;

tview(:,:,)=Hz(:,:,kobs);
svview(:,:,)=Hz(:,:,jo,:);
subplot('position',[0.15 0.45 0.7 0.45]),pcolor(tview');
shading flat;
caxis([plot_min plot_max]);
colorbar;
axis image;
title(['Ey(i,j,k=5), time step = 0']);
xlabel('i coordinate');
ylabel('j coordinate');

subplot('position',[0.15 0.10 0.7 0.25]),pcolor(sview');
shading flat;
caxis([plot_min plot_max]);
colorbar;
axis image;
title(['Ey(i,j=13,k), time step = 0']);
xlabel('i coordinate');
ylabel('k coordinate');

rect=get(gcf,'Position');
rect(1:2)=[0 0];

M=moviein(nmax/2,gcf,rect);
for n=1:nmax

% Update electric fields

% calculation of Ex

for i=1:ie
    for j=1:je-1
        for k=1:ke-1
            Ex(i,j,k) = ca(i,j,k).*Ex(i,j,k)+...cb(i,j,k).*(Hz(i,j+1,k) - Hz(i,j,k) - Hy(i,j,k+1) + Hy(i,j,k));
        end
    end
end

% calculation of Ey
for i=1:ie-1
    for j=1:je
        for k=1:ke-1
            Ey(i,j,k) = ca(i,j,k).*Ey(i,j,k)+...
            cb(i,j,k).*(Hx(i,j,k+1) - Hx(i,j,k) - Hz(i+1,j,k) + Hz(i, j,k));
        end
    end
end

% calculation of Ez
%**********************************************************************
for i=1:ie-1
    for j=1:je-1
        for k=1:ke
            Ez(i,j,k) = ca(i,j,k).*Ez(i,j,k)+...
            cb(i,j,k).*(Hy(i+1,j,k) - Hy(i,j,k) - Hx(i,j+1,k) + Hx(i, j,k));
        end
    end
end

% Set the PML absorbing boundary condition
%**********************************************************************
for i=1:2*N+ie
  for k=1:2*N+ke-1
    for j=1:N-1
      Exy_lr(i,j,k) = sig_0(N-j+1)*Exy_lr(i,j,k) + coeff_A
      *(Hzx_lr(i,j+1,k) - Hzx_lr(i,j,k) + Hzy_lr(i,j+1,k) - Hzy_lr(i,j,k));
    end
    for j=N+2:2*N
      Exy_lr(i,j,k) = sig_0(j-N)*Exy_lr(i,j,k) + coeff_A
      *(Hzx_lr(i,j,k) - Hzx_lr(i,j-1,k) + Hzy_lr(i,j,k) - Hzy_lr(i,j-1,k));
    end
  end
end
for k=1:2*N+ke-1
  for i=1:N
    Exy_lr(i,N,k) = sig_0(N-j+1)*Exy_lr(i,N,k) + coeff_A
    *(Hzx_fb(i,1,k) - Hzx_lr(i,N,k) + Hzy_fb(i,1,k) - Hzy_lr(i,N,k));
    Exy_lr(i,N+1,k) = sig_0(j-N)*Exy_lr(i,N+1,k) + coeff_A
    *(Hzx_lr(i,N+1,k) - Hzx_fb(i,je,k) + Hzy_lr(i,N+1,k) - Hzy_fb(i,je,k));
  end
for i=N+ie+1:2*N+ie
    Exy_lr(i,N,k) = sig_0(1)*Exy_lr(i,N,k) + coeff_A
    *(Hzx_fb(i-ie,1,k) - Hxz_lr(i,N,k) + Hzy_fb(i-ie,1,k) - Hzy_lr(i,N,k));
    Exy_lr(i,N+1,k) = sig_0(1)*Exy_lr(i,N+1,k) + coeff_A
    *(Hzx_lr(i,N+1,k) - Hxz_fb(i-ie,je,k) + Hzy_lr(i,N+1,k) - Hzy_fb(i-ie,je,k));
end
end

for i=N+1:N+ie
    for k=1:N
        Exy_lr(i,N,k) = sig_0(1)*Exy_lr(i,N,k) + coeff_A
        *(Hzx_tb(i-N,1,k) - Hxz_lr(i,N,k) + Hzy_tb(i-N,1,k) - Hzy_lr(i,N,k));
        Exy_lr(i,N+1,k) = sig_0(1)*Exy_lr(i,N+1,k) + coeff_A
        *(Hzx_lr(i,N+1,k) - Hxz_tb(i-N,je,k) + Hzy_lr(i,N+1,k) - Hzy_tb(i-N,je,k));
    end
    for k=N+1:N+ke-1
        Exy_lr(i,N,k) = sig_0(1)*Exy_lr(i,N,k) + coeff_A
        *(Hz(i-N,1,k-N) - Hxz_lr(i,N,k) - Hzy_lr(i,N,k));
    end
end
for k=N+1:N+ke-1
    Exy_lr(i,N,k) = sig_0(1)*Exy_lr(i,N,k) + coeff_A
    *(Hz(i-N,1,k-N) - Hxz_lr(i,N,k) - Hzy_lr(i,N,k));
Exy_lr(i,N+1,k) = sig_0(1)*Exy_lr(i,N+1,k) + coeff_A *(Hzx_lr(i,N+1,k) + Hzy_lr(i,N+1,k) - Hz(i-N,je,k-N));
end

for k=N+ke:2*N+ke-1
Exy_lr(i,N,k) = sig_0(1)*Exy_lr(i,N,k) + coeff_A *(Hzx_tb(i-N,1,k-ke+1) - Hzx_lr(i,N,k) + Hzy_tb(i-N,1,k-ke+1) - Hzy_lr(i,N,k));
Exy_lr(i,N+1,k) = sig_0(1)*Exy_lr(i,N+1,k) + coeff_A *(Hzx_lr(i,N+1,k) - Hzx_tb(i-N,je,k-ke+1) + Hzy_lr(i,N+1,k) -
Hzy_tb(i-N,je,k-ke+1));
end
end

%***********************************************************************
% Exy_fb - back and front
%***********************************************************************

for i=1:2*N
    for j=1:je-1
        for k=1:2*N+ke-1
            Exy_fb(i,j,k) = Exy_fb(i,j,k) + coeff_A *(Hzx_fb(i,j+1,k) - Hzx_fb(i,j,k) + Hzy_fb(i,j+1,k) - Hzy_fb(i,j,k));
        end
    end
end
end
%***********************************************************************
%Exy_tb - bottom and top
%***********************************************************************

for i=1:ie
    for j=1:je-1
        for k=1:2*N
            Exy_tb(i,j,k) = Exy_tb(i,j,k) + coeff_A*(Hzx_tb(i,j+1,k) - Hzx_tb(i,j,k) + Hzy_tb(i,j+1,k) - Hzy_tb(i,j,k));
        end
    end
end

%***********************************************************************
%Exz_lr - left and right
%***********************************************************************

for i=1:2*N+ie
    for j=1:2*N %j=1:N j=N+1:2*N
        for k=1:N
            Exz_lr(i,j,k) = sig_0(N-k+1)*Exz_lr(i,j,k) - coeff_A*(Hyz_lr(i,j,k+1) - Hyz_lr(i,j,k) + Hyx_lr(i,j,k+1) - Hyx_lr(i,j,k));
        end
    end
    for k=N+1:N+ke-1
        Exz_lr(i,j,k) = Exz_lr(i,j,k) - coeff_A*(Hyz_lr(i,j,k+1) - Hyz_lr(i,j,k) + Hyx_lr(i,j,k+1) - Hyx_lr(i,j,k));
    end
end
for k=N+ke:2*N+ke-1
    Exz_lr(i,j,k) = sig_0(k-N-ke+1)*Exz_lr(i,j,k) - coeff_A
    *(Hyz_lr(i,j,k+1) - Hyz_lr(i,j,k) + Hyx_lr(i,j,k+1) - Hyx_lr(i,j,k));
end
end

%***********************************************************************
%Exz_fb - back and front
%***********************************************************************
for i=1:2*N %i=1:N i=N+1:2*N
    for j=1:je-1
        for k=1:N
            Exz_fb(i,j,k) = sig_0(N+1-k)*Exz_fb(i,j,k) - coeff_A
            *(Hyz_fb(i,j,k+1) - Hyz_fb(i,j,k) + Hyx_fb(i,j,k+1) - Hyx_fb(i,j,k));
        end
        for k=N+1:N+ke-1 %
            Exz_fb(i,j,k) = Exz_fb(i,j,k) - coeff_A
            *(Hyz_fb(i,j,k+1) - Hyz_fb(i,j,k) + Hyx_fb(i,j,k+1) - Hyx_fb(i,j,k));
        end
        for k=N+ke:2*N+ke-1 %
            Exz_fb(i,j,k) = sig_0(k-N-ke+1)*Exz_fb(i,j,k) - coeff_A
            *(Hyz_fb(i,j,k+1) - Hyz_fb(i,j,k) + Hyx_fb(i,j,k+1) - Hyx_fb(i,j,k));
        end
    end
end

end
%***********************************************************************
%Exz_tb - bottom and top
%***********************************************************************

for i=1:ie
    for j=1:je-1
        Exz_tb(i,j,N) = sig_0(1)*Exz_tb(i,j,N) - coeff_A*(Hy(i,j,1) - Hyz_tb(i,j,N) - Hyx_tb(i,j,N));
        Exz_tb(i,j,N+1) = sig_0(1)*Exz_tb(i,j,N+1) - coeff_A*(Hyz_tb(i,j,N+1) + Hyx_tb(i,j,N+1) - Hy(i,j,ke));
        for k=1:N-1 %
            Exz_tb(i,j,k) = sig_0(N+1-k)*Exz_tb(i,j,k) - coeff_A*(Hyz_tb(i,j,k+1) - Hyz_tb(i,j,k) + Hyx_tb(i,j,k+1) - Hyx_tb(i,j,k));
        end
        for k=N+2:2*N %
            Exz_tb(i,j,k) = sig_0(k-N)*Exz_tb(i,j,k) - coeff_A*(Hyz_tb(i,j,k) - Hyz_tb(i,j,k-1) + Hyx_tb(i,j,k) - Hyx_tb(i,j,k-1));
        end
    end
end

%***********************************************************************
%Eyz_lr - left and right

%*******************************************************************************

for i=1:2*N+ie-1

for j=1:2*N %j=1:N j=N+1:2*N

for k=1:N %

Eyz_lr(i,j,k) = sig_0(N+1-k)*Eyz_lr(i,j,k) + coeff_A

*(Hxy_lr(i,j,k+1) - Hxy_lr(i,j,k) + Hxz_lr(i,j,k+1) - Hxz_lr(i,j,k));
end

for k=N+1:N+ke-1 %

Eyz_lr(i,j,k) = Eyz_lr(i,j,k) + coeff_A

*(Hxy_lr(i,j,k+1) - Hxy_lr(i,j,k) + Hxz_lr(i,j,k+1) - Hxz_lr(i,j,k));
end

for k=N+ke:2*N+ke-1 %

Eyz_lr(i,j,k) = sig_0(k-N-ke+1)*Eyz_lr(i,j,k) + coeff_A

*(Hxy_lr(i,j,k+1) - Hxy_lr(i,j,k) + Hxz_lr(i,j,k+1) - Hxz_lr(i,j,k));
end

end

end

%*******************************************************************************

%Eyz_fb - back and front

%*******************************************************************************

for i=1:2*N %i=1:N i=N+1:2*N

for j=1:je
for k=1:N 
    \[ Eyz_{fb}(i,j,k) = \text{sig}_0(N+1-k) \cdot Eyz_{fb}(i,j,k) + \text{coeff}_A \times (Hxy_{fb}(i,j,k+1) - Hxy_{fb}(i,j,k) + Hxz_{fb}(i,j,k+1) - Hxz_{fb}(i,j,k)); \]
end

for k=N+1:N+ke-1 
    \[ Eyz_{fb}(i,j,k) = Eyz_{fb}(i,j,k) + \text{coeff}_A \times (Hxy_{fb}(i,j,k+1) - Hxy_{fb}(i,j,k) + Hxz_{fb}(i,j,k+1) - Hxz_{fb}(i,j,k)); \]
end

for k=ke+N:2*N+ke-1 
    \[ Eyz_{fb}(i,j,k) = \text{sig}_0(k-ke-N+1) \cdot Eyz_{fb}(i,j,k) + \text{coeff}_A \times (Hxy_{fb}(i,j,k+1) - Hxy_{fb}(i,j,k) + Hxz_{fb}(i,j,k+1) - Hxz_{fb}(i,j,k)); \]
end

end

end

%***********************************************************************
%Eyz_{tb} - bottom and top
%***********************************************************************

for i=1:ie-1
    for j=1:je
        \[ Eyz_{tb}(i,j,N) = \text{sig}_0(1) \cdot Eyz_{tb}(i,j,N) + \text{coeff}_A \times (Hx(i,j,1) - Hxy_{tb}(i,j,N) - Hxz_{tb}(i,j,N)); \]
        \[ Eyz_{tb}(i,j,N+1) = \text{sig}_0(1) \cdot Eyz_{tb}(i,j,N+1) + \text{coeff}_A \times (Hxy_{tb}(i,j,N+1) + Hxz_{tb}(i,j,N+1) - Hx(i,j,ke)); \]
    end
end

end
for k=1:N-1%
    Eyz_tb(i,j,k) = sig_0(N+1-k)*Eyz_tb(i,j,k) + coeff_A
    *(Hxy_tb(i,j,k+1) - Hxy_tb(i,j,k) + Hxz_tb(i,j,k+1) - Hxz_tb(i,j,k));
end
for k=N+2:2*N %
    Eyz_tb(i,j,k) = sig_0(k-N)*Eyz_tb(i,j,k) + coeff_A
    *(Hxy_tb(i,j,k) - Hxy_tb(i,j,k-1) + Hxz_tb(i,j,k) - Hxz_tb(i,j,k-1));
end
end
end

%***********************************************************************
%Eyx_lr - left and right
%***********************************************************************
for j=1:2*N %j=1:N j=N+1:2*N
    for k=1:2*N+ke-1 %
        for i=1:N
            Eyx_lr(i,j,k) = sig_0(N+1-i)*Eyx_lr(i,j,k) - coeff_A
            *(Hzx_lr(i+1,j,k) - Hxz_lr(i,j,k) + Hzy_lr(i+1,j,k) - Hzy_lr(i,j,k));
        end
    end
    for i=N+1:N+ie-1 %
        Eyx_lr(i,j,k) = Eyx_lr(i,j,k) - coeff_A
        *(Hzx_lr(i+1,j,k) - Hxz_lr(i,j,k) + Hzy_lr(i+1,j,k) - Hzy_lr(i,j,k));
    end
end
for i=N+ie:2*N+ie-1 %

   Eyx_lr(i,j,k) = sig_0(i-N-ie+1)*Eyx_lr(i,j,k) - coeff_A
   *(Hzx_lr(i+1,j,k) - Hzx_lr(i,j,k) + Hzy_lr(i+1,j,k) - Hzy_lr(i,j,k));

end

end

end

%***********************************************************************
%Eyx_fb - back and front
%***********************************************************************

for j=1:je

   for k=1:N %

      Eyx_fb(N,j,k) = sig_0(1)*Eyx_fb(N,j,k) - coeff_A
      *(Hzx_tb(1,j,k) - Hzx_fb(N,j,k) + Hzy_tb(1,j,k) - Hzy_fb(N,j,k));

      Eyx_fb(N+1,j,k) = sig_0(1)*Eyx_fb(N+1,j,k) - coeff_A
      *(Hxz Tb(1,j,k) - Hzx_tb(ie,j,k) + Hzy_tb(N+1,j,k) - Hzy_tb(ie,j,k));

   end

   for k=N+1:N+ke-1 %

      Eyx_fb(N,j,k) = sig_0(1)*Eyx_fb(N,j,k) - coeff_A
      *(Hz(1,j,k-N) - Hzx_fb(N,j,k) - Hzy_fb(N,j,k));

      Eyx_fb(N+1,j,k) = sig_0(1)*Eyx_fb(N+1,j,k) - coeff_A
      *(Hzx_fb(N+1,j,k) + Hzy_fb(N+1,j,k) - Hz(ie,j,k-N));

   end

end
for k=N+ke:2*N+ke-1 %
    Eyx_fb(N,j,k) = sig_0(1)*Eyx_fb(N,j,k) - coeff_A
    *(Hzx_tb(1,j,k-ke+1) - Hzx_fb(N,j,k) + Hzy_tb(1,j,k-ke+1) -
    Hzy_fb(N,j,k));
    Eyx_fb(N+1,j,k) = sig_0(1)*Eyx_fb(N+1,j,k) - coeff_A
    *(Hzx_fb(N+1,j,k) - Hzx_tb(ie,j,k-ke+1) + Hzy_fb(N+1,j,k) -
    Hzy_tb(ie,j,k-ke+1));
end
for k=1:2*N+ke-1 %
    for i=1:N-1
        Eyx_fb(i,j,k) = sig_0(N+1-i)*Eyx_fb(i,j,k) - coeff_A
        *(Hzx_fb(i+1,j,k) - Hzx_fb(i,j,k) + Hzy_fb(i+1,j,k) - Hzy_fb(i,j,k));
    end
    for i=N+2:2*N
        Eyx_fb(i,j,k) = sig_0(i-N)*Eyx_fb(i,j,k) - coeff_A
        *(Hzx_fb(i,j,k) - Hzx_fb(i-1,j,k) + Hzy_fb(i,j,k) - Hzy_fb(i-1,j,k));
    end
end

%***********************************************************************
%Eyx_tb - bottom and top
%***********************************************************************
for i=1:ie-1
for j=1:je
    for k=1:2*N %k=1:N k+N+1:2*N -
        Eyx_tb(i,j,k) = Eyx_tb(i,j,k) - coeff_A
        *(Hzx_tb(i+1,j,k) - Hzx_tb(i,j,k) + Hzy_tb(i+1,j,k) - Hzy_tb(i,j,k));
    end
end
end

%***********************************************************************
% Ezx_lr - left and right
%***********************************************************************

for j=1:2*N %j=1:N j=N+1:2*N
    for k=1:2*N+ke 
        for i=1:N 
            Ezx_lr(i,j,k) = sig_0(N+1-i)*Ezx_lr(i,j,k) + coeff_A
            *(Hyz_lr(i+1,j,k) - Hyz_lr(i,j,k) + Hyx_lr(i+1,j,k) - Hyx_lr(i,j,k));
        end
        for i=N+1:N+ie-1 
            Ezx_lr(i,j,k) = Ezx_lr(i,j,k) + coeff_A
            *(Hyz_lr(i+1,j,k) - Hyz_lr(i,j,k) + Hyx_lr(i+1,j,k) - Hyx_lr(i,j,k));
        end
        for i=N+ie:2*N+ie-1 
            Ezx_lr(i,j,k) = sig_0(i-N-ie+1)*Ezx_lr(i,j,k) + coeff_A
            *(Hyz_lr(i+1,j,k) - Hyz_lr(i,j,k) + Hyx_lr(i+1,j,k) - Hyx_lr(i,j,k));
        end
end
for j=1:je-1
    for k=1:N %
        Ezx_fb(N,j,k) = sig_0(1)*Ezx_fb(N,j,k) + coeff_A
        *(Hyz_tb(1,j,k) - Hyz_fb(N,j,k) + Hyx_tb(1,j,k) - Hyx_fb(N,j,k));
    end
    for k=N+1:N+ke %
        Ezx_fb(N,j,k) = sig_0(1)*Ezx_fb(N,j,k) + coeff_A
        *(Hyz_fb(N+1,j,k) - Hyz_tb(ie,j,k) + Hyx_fb(N+1,j,k) -
        Hyx_tb(ie,j,k));
    end
    for k=N+ke+1:2*N+ke %
        Ezx_fb(N,j,k) = sig_0(1)*Ezx_fb(N,j,k) + coeff_A
        *(Hyz_tb(1,j,k-ke) - Hyz_fb(N,j,k) + Hyx_tb(1,j,k-ke) -
    end
end
end

%***********************************************************************
%Ezx_fb - back and front
%***********************************************************************

for j=1:je-1
    for k=1:N %
        Ezx_fb(N,j,k) = sig_0(1)*Ezx_fb(N,j,k) + coeff_A
        *(Hyz_tb(1,j,k) - Hyz_fb(N,j,k) + Hyx_tb(1,j,k) - Hyx_fb(N,j,k));
    end
    for k=N+1:N+ke %
        Ezx_fb(N+1,j,k) = sig_0(1)*Ezx_fb(N+1,j,k) + coeff_A
        *(Hyz_fb(N+1,j,k) - Hyz_tb(ie,j,k) + Hyx_fb(N+1,j,k) -
        Hyx_tb(ie,j,k));
    end
    for k=N+ke+1:2*N+ke %
        Ezx_fb(N+1,j,k) = sig_0(1)*Ezx_fb(N+1,j,k) + coeff_A
        *(Hyz_tb(1,j,k-ke) - Hyz_fb(N+1,j,k) + Hyx_tb(1,j,k-ke) -
    end
Hyx_fb(N,j,k));

Ezx_fb(N+1,j,k) = sig_0(1)*Ezx_fb(N+1,j,k) + coeff_A
*(Hyz_fb(N+1,j,k) - Hyz_tb(i_e,j,k-ke) + Hyx_fb(N+1,j,k) -
Hyx_tb(i_e,j,k-ke));

end

for k=1:2*N+ke %
for i=1:N-1

Ezx_fb(i,j,k) = sig_0(N+1-i)*Ezx_fb(i,j,k) + coeff_A
*(Hyz_fb(i+1,j,k) - Hyz_fb(i,j,k) + Hyx_fb(i+1,j,k) - Hyx_fb(i,j,k));
end

for i=N+2:2*N %

Ezx_fb(i,j,k) = sig_0(i-N)*Ezx_fb(i,j,k) + coeff_A
*(Hyz_fb(i,j,k) - Hyz_fb(i-1,j,k) + Hyx_fb(i,j,k) - Hyx_fb(i-1,j,k));
end
end
end

%**********************************************************************
%Ezx_tb - bottom and top
%**********************************************************************

for i=1:ie-1

for j=1:je-1

for k=1:2*N %k=1:N k=N+1:2*N

Ezx_tb(i,j,k) = Ezx_tb(i,j,k) + coeff_A


\[(Hyz_{tb}(i+1,j,k) - Hyz_{tb}(i,j,k) + Hyx_{tb}(i+1,j,k) - Hyx_{tb}(i,j,k));\]

end

end

end

\%

%***********************************************************************
%

%Ezy_{lr} - left and right
%

%***********************************************************************

for i=1:2*N+ie-1

for k=1:2*N+ke

for j=1:N-1

Ezy_{lr}(i,j,k) = sig_0(N+1-j)*Ezy_{lr}(i,j,k) - coeff_A

*(Hxy_{lr}(i,j+1,k) - Hxy_{lr}(i,j,k) + Hxz_{lr}(i,j+1,k) - Hxz_{lr}(i,j,k));

end

for j=N+2:2*N

Ezy_{lr}(i,j,k) = sig_0(j-N)*Ezy_{lr}(i,j,k) - coeff_A

*(Hxy_{lr}(i,j,k) - Hxy_{lr}(i,j-1,k) + Hxz_{lr}(i,j,k) - Hxz_{lr}(i,j-1,k));

end

end

end

for k=1:2*N+ke

for i=1:N

Ezy_{lr}(i,N,k) = sig_0(1)*Ezy_{lr}(i,N,k) - coeff_A

\]
\[ *(Hxy_{fb}(i,1,k) - Hxy_{lr}(i,N,k) + Hxz_{fb}(i,1,k) - Hxz_{lr}(i,N,k)); \]

\[ Ezy_{lr}(i,N+1,k) = \text{sig}_0(1)*Ezy_{lr}(i,N+1,k) - \text{coeff}_A \]

\[ *(Hxy_{lr}(i,N+1,k) - Hxy_{fb}(i,je,k) + Hxz_{lr}(i,N+1,k) - Hxz_{fb}(i,je,k)); \]

end

for \( i=N+ie:2*N+ie-1 \)

\[ Ezy_{lr}(i,N,k) = \text{sig}_0(1)*Ezy_{lr}(i,N,k) - \text{coeff}_A \]

\[ *(Hxy_{fb}(i-ie+1,1,k) - Hxy_{lr}(i,N,k) + Hxz_{fb}(i-ie+1,1,k) - Hxz_{lr}(i,N,k)); \]

\[ Ezy_{lr}(i,N+1,k) = \text{sig}_0(1)*Ezy_{lr}(i,N+1,k) - \text{coeff}_A \]

\[ *(Hxy_{lr}(i,N+1,k) - Hxy_{fb}(i-ie+1,je,k) + Hxz_{lr}(i,N+1,k) - Hxz_{fb}(i-ie+1,je,k)); \]

end

end

for \( i=N+1:N+ie-1 \)

for \( k=1:N \)

\[ Ezy_{lr}(i,N,k) = \text{sig}_0(1)*Ezy_{lr}(i,N,k) - \text{coeff}_A \]

\[ *(Hxy_{tb}(i-N,1,k) - Hxy_{lr}(i,N,k) + Hxz_{tb}(i-N,1,k) - Hxz_{lr}(i,N,k)); \]

\[ Ezy_{lr}(i,N+1,k) = \text{sig}_0(1)*Ezy_{lr}(i,N+1,k) - \text{coeff}_A \]

\[ *(Hxy_{lr}(i,N+1,k) - Hxy_{tb}(i-N,je,k) + Hxz_{lr}(i,N+1,k) - Hxz_{tb}(i-N,je,k)); \]

end
for k=N+1:N+ke
  Ezy_lr(i,N,k) = sig_0(1)*Ezy_lr(i,N,k) - coeff_A
  *(Hx(i-N,1,k-N) - Hxy_lr(i,N,k) - Hxz_lr(i,N,k));
  Ezy_lr(i,N+1,k) = sig_0(1)*Ezy_lr(i,N+1,k) - coeff_A
  *(Hxy_lr(i,N+1,k) + Hxz_lr(i,N+1,k) - Hx(i-N,je,k-N));
end
for k=N+ke+1:2*N+ke
  Ezy_lr(i,N,k) = sig_0(1)*Ezy_lr(i,N,k) - coeff_A
  *(Hxy_tb(i-N,1,k-ke) - Hxy_lr(i,N,k) + Hxz_tb(i-N,1,k-ke) - Hxz_lr(i,N,k));
  Ezy_lr(i,N+1,k) = sig_0(1)*Ezy_lr(i,N+1,k) - coeff_A
  *(Hxy_lr(i,N+1,k) - Hxy_tb(i-N,je,k-ke) + Hxz_lr(i,N+1,k) - Hxz_tb(i-N,je,k-ke));
end
end

%***********************************************************************
%Ezy_fb - back and front
%***********************************************************************

for i=1:2*N %i=1:N i=N+1:2*N
  for j=1:je-1
    for k=1:2*N+ke %
      Ezy_fb(i,j,k) = Ezy_fb(i,j,k) - coeff_A
      *(Hxy_fb(i,j+1,k) - Hxy_fb(i,j,k) + Hxz_fb(i,j+1,k) - Hxz_fb(i,j,k));
    end
  end
end

%***********************************************************************
%Ezy_fb - back and front
%***********************************************************************
for i=1:ie-1
    for j=1:je-1
        for k=1:2*N %k=1:N k=N+1:2*N
            Ezy_tb(i,j,k) = Ezy_tb(i,j,k) - coeff_A
                *(Hxy_tb(i,j+1,k) - Hxy_tb(i,j,k) + Hxz_tb(i,j+1,k) - Hxz_tb(i,j,k));
        end
    end
end

for i=1:2*N+ie-1
    for k=1:2*N+ke
        Hxy_lr(i,1,k) = sig_0_m(N)*Hxy_lr(i,1,k) - coeff_B
            *(Ezx_lr(i,1,k) + Ezy_lr(i,1,k));% parede metalica
        Hxy_lr(i,2*N,k) = sig_0_m(N)*Hxy_lr(i,2*N,k) - coeff_B
            *( - Ezx_lr(i,2*N,k) - Ezy_lr(i,2*N,k));%metalica wall
for j=2:N
    Hxy_lr(i,j,k) = sig_0_m(N-j+1)*Hxy_lr(i,j,k) - coeff_B*(Ezx_lr(i,j,k) - Ezx_lr(i,j-1,k) + Ezy_lr(i,j,k) - Ezy_lr(i,j-1,k));
end
for j=N+1:2*N-1
    Hxy_lr(i,j,k) = sig_0_m(j-N)*Hxy_lr(i,j,k) - coeff_B*(Ezx_lr(i,j+1,k) - Ezx_lr(i,j,k) + Ezy_lr(i,j+1,k) - Ezy_lr(i,j,k));
end
end
end

%***********************************************************************
%Hxy_fb - back and front
%***********************************************************************
for k=1:2*N+ke
    for i=1:N
        Hxy_fb(i,1,k) = Hxy_fb(i,1,k) - coeff_B*(Ezx_fb(i,1,k) - Ezx_lr(i,N,k) + Ezy_fb(i,1,k) - Ezy_lr(i,N,k));
    end
    for i=N+1:2*N
        Hxy_fb(i,je,k) = Hxy_fb(i,je,k) - coeff_B*(Ezx_lr(i,N+1,k) - Ezx_fb(i,je-1,k) + Ezy_lr(i,N+1,k) - Ezy_fb(i,je-1,k));
    end
end
for i=N+1:2*N
    Hxy_fb(i,1,k) = Hxy_fb(i,1,k) - coeff_B
\[(Ezx_{fb}(i,1,k) - Ezx_{lr}(i+ie-1,N,k) + Ezy_{fb}(i,1,k) - Ezy_{lr}(i+ie-1,N,k));
Hxy_{fb}(i,je,k) = Hxy_{fb}(i,je,k) - coeff_B
\]
\[(Ezx_{lr}(i+ie-1,N+1,k) - Ezx_{fb}(i,je-1,k) + Ezy_{lr}(i+ie-1,N+1,k) - Ezy_{fb}(i,je-1,k));
\]
end
for i=1:2*N %i=1:N i=N+1:2*N
for j=2:je-1
    Hxy_{fb}(i,j,k) = Hxy_{fb}(i,j,k) - coeff_B
    \[(Ezx_{fb}(i,j,k) - Ezx_{fb}(i,j-1,k) + Ezy_{fb}(i,j,k) - Ezy_{fb}(i,j-1,k));
\]
end
end
end

%***********************************************************************
%Hxy_{tb} - bottom and top
%***********************************************************************
for i=1:ie-1
for k=1:N
    Hxy_{tb}(i,1,k) = Hxy_{tb}(i,1,k) - coeff_B
    \[(Ezx_{tb}(i,1,k) - Ezx_{lr}(N+i,N,k) + Ezy_{tb}(i,1,k) - Ezy_{lr}(N+i,N,k));
Hxy_{tb}(i,je,k) = Hxy_{tb}(i,je,k) - coeff_B
    \[(Ezx_{lr}(N+i,N+1,k) - Ezx_{tb}(i,je-1,k) + Ezy_{lr}(N+i,N+1,k) - Ezy_{tb}(i,je-1,k));
Ezy_{tb}(i,je-1,k));
\]
for k=N+1:2*N 
    Hxy_tb(i,1,k) = Hxy_tb(i,1,k) - coeff_B * (Ezx_tb(i,1,k) - Ezx_lr(i+N,N,k+ke) + Ezy_tb(i,1,k) - Ezy_lr(i+N,N,k+ke));
    Hxy_tb(i,je,k) = Hxy_tb(i,je,k) - coeff_B * (Ezx_lr(i+N,N+1,k+ke) - Ezx_tb(i,je-1,k) + Ezy_lr(i+N,N+1,k+ke) - Ezy_tb(i,je-1,k));
end
for k=1:2*N %k=1:N k=N+1:2*N
    for j=2:je-1
        Hxy_tb(i,j,k) = Hxy_tb(i,j,k) - coeff_B * (Ezx_tb(i,j,k) - Ezx_tb(i,j-1,k) + Ezy_tb(i,j,k) - Ezy_tb(i,j-1,k));
    end
end
end

%**********************************************************************
%Hxz_lr - left and right
%**********************************************************************

for i=1:2*N+ie-1
    for j=1:2*N %j=1:N j=N+1:2*N -
        Hxz_lr(i,j,1) = sig_0_m(N)*Hxz_lr(i,j,1) + coeff_B * (Eyz_lr(i,j,1) + Eyx_lr(i,j,1)) + coeff_B * (Eyz_lr(i,j,1) + Eyx_lr(i,j,1));
    end
end

%metalic wall
\[
\text{Hxz}_{lr}(i,j,2N+ke) = \text{sig}_0_m(N) \times \text{Hxz}_{lr}(i,j,2N+ke) + \text{coeff}_B \\
\times (- \text{Eyz}_{lr}(i,j,2N+ke-1) - \text{Eyx}_{lr}(i,j,2N+ke-1)); \text{\% metallic wall}
\]

for \( k=2:N \)

\[
\text{Hxz}_{lr}(i,j,k) = \text{sig}_0_m(N-k+1) \times \text{Hxz}_{lr}(i,j,k) + \text{coeff}_B \\
\times (\text{Eyz}_{lr}(i,j,k) - \text{Eyz}_{lr}(i,j,k-1) - \text{Eyx}_{lr}(i,j,k) - \text{Eyx}_{lr}(i,j,k-1));
\]

end

for \( k=N+1:ke+N \)

\[
\text{Hxz}_{lr}(i,j,k) = \text{Hxz}_{lr}(i,j,k) + \text{coeff}_B \\
\times (\text{Eyz}_{lr}(i,j,k) - \text{Eyz}_{lr}(i,j,k-1) - \text{Eyx}_{lr}(i,j,k) - \text{Eyx}_{lr}(i,j,k-1));
\]

end

for \( k=ke+N+1:2N+ke-1 \)

\[
\text{Hxz}_{lr}(i,j,k) = \text{sig}_0_m(k-ke-N) \times \text{Hxz}_{lr}(i,j,k) + \text{coeff}_B \\
\times (\text{Eyz}_{lr}(i,j,k) - \text{Eyz}_{lr}(i,j,k-1) - \text{Eyx}_{lr}(i,j,k) - \text{Eyx}_{lr}(i,j,k-1));
\]

end

end

end

%***********************************************************************
%Hxz_fb - back and front
%***********************************************************************

for \( i=1:2N \) \( %i=1:N \) \( i=N+1:2N \)

for \( j=1:je \)

\[
\text{Hxz}_{fb}(i,j,1) = \text{sig}_0_m(N) \times \text{Hxz}_{fb}(i,j,1) + \text{coeff}_B \\
\times (\text{Eyz}_{fb}(i,j,1) + \text{Eyx}_{fb}(i,j,1)); \text{\% metallic wall}
\]

end

end
Hxz_fb(i, j, 2*N+ke) = sig_0_m(N)*Hxz_fb(i, j, 2*N+ke) + coeff_B

*( - Eyz_fb(i, j, 2*N+ke-1) - Eyx_fb(i, j, 2*N+ke-1)); \% metallic wall

for k=2:N 

Hxz_fb(i, j, k) = sig_0_m(N+1-k)*Hxz_fb(i, j, k) + coeff_B

*(Eyz_fb(i, j, k) - Eyz_fb(i, j, k-1) + Eyx_fb(i, j, k) - Eyx_fb(i, j, k-1));

end

for k=N+1:N+ke 

Hxz_fb(i, j, k) = Hxz_fb(i, j, k) + coeff_B

*(Eyz_fb(i, j, k) - Eyz_fb(i, j, k-1) + Eyx_fb(i, j, k) - Eyx_fb(i, j, k-1));

end

for k=N+ke+1:2*N+ke-1 

Hxz_fb(i, j, k) = sig_0_m(k-N-ke)*Hxz_fb(i, j, k) + coeff_B

*(Eyz_fb(i, j, k) - Eyz_fb(i, j, k-1) + Eyx_fb(i, j, k) - Eyx_fb(i, j, k-1));

end

end

end

\%**********************************************************************
\%Hxz_tb - bottom and top
\%**********************************************************************

for i=1:ie-1

for j=1:je 

Hxz_tb(i, j, 1) = sig_0_m(N)*Hxz_tb(i, j, 1) + coeff_B

*(Eyz_tb(i, j, 1) + Eyx_tb(i, j, 1)); \% metallic wall
Hxz_tb(i,j,2*N) = sig_0_m(N)*Hxz_tb(i,j,2*N) + coeff_B
* ( - Eyz_tb(i,j,2*N) - Eyx_tb(i,j,2*N));%metalic wall

for k=2:N %
    Hzx_tb(i,j,k) = sig_0_m(N+1-k)*Hxz_tb(i,j,k) + coeff_B
    *(Eyz_tb(i,j,k) - Eyz_tb(i,j,k-1) + Eyx_tb(i,j,k) - Eyx_tb(i,j,k-1));
end

for k=N+1:2*N-1 %
    Hzx_tb(i,j,k) = sig_0_m(k-N)*Hxz_tb(i,j,k) + coeff_B
    *(Eyz_tb(i,j,k+1) - Eyz_tb(i,j,k) + Eyx_tb(i,j,k+1) - Eyx_tb(i,j,k));
end
end
end

%***********************************************************************
%Hyz_lr - left and right
%***********************************************************************

for i=1:2*N+ie

    for j=1:2*N %j=1:N j=N+1:2*N
        Hyz_lr(i,j,1) = sig_0_m(N)*Hyz_lr(i,j,1) - coeff_B
        *(Exy_lr(i,j,1) + Exz_lr(i,j,1));% metalic wall
        Hyz_lr(i,j,2*N+ke) = sig_0_m(N)*Hyz_lr(i,j,2*N+ke) - coeff_B
        *( - Exy_lr(i,j,2*N+ke-1) - Exz_lr(i,j,2*N+ke-1));% metalic wall
        for k=2:N %
            Hyz_lr(i,j,k) = sig_0_m(N-k+1)*Hyz_lr(i,j,k) - coeff_B
        end
    end
end
end

%***********************************************************************
*(Exy_lr(i,j,k) - Exy_lr(i,j,k-1) + Exz_lr(i,j,k) - Exz_lr(i,j,k-1));
end
for k=N+1:ke+N 
\( \text{Hyz}_{lr}(i,j,k) = \text{Hyz}_{lr}(i,j,k) - \text{coeff}_B \\
*(Exy_lr(i,j,k) - Exy_lr(i,j,k-1) + Exz_lr(i,j,k) - Exz_lr(i,j,k-1)); \)
end
for k=ke+N+1:2*N+ke-1 
\( \text{Hyz}_{lr}(i,j,k) = \text{sig}_0_m(k-ke-N)\text{Hyz}_{lr}(i,j,k) - \text{coeff}_B \\
*(Exy_lr(i,j,k) - Exy_lr(i,j,k-1) + Exz_lr(i,j,k) - Exz_lr(i,j,k-1)); \)
end
end
end

%***********************************************************************
%Hyz_fb - back and front
%***********************************************************************

for i=1:2*N %i=1:N i=N+1:2*N
for j=1:je-1 
\( \text{Hyz}_{fb}(i,j,1) = \text{sig}_0_m(N)\text{Hyz}_{fb}(i,j,1) - \text{coeff}_B \\
*(Exy_fb(i,j,1) + Exz_fb(i,j,1)); \text{metallic wall} \\
\( \text{Hyz}_{fb}(i,j,2*N+ke) = \text{sig}_0_m(N)\text{Hyz}_{fb}(i,j,2*N+ke) - \text{coeff}_B \\
*( - Exy_fb(i,j,2*N+ke-1) - Exz_fb(i,j,2*N+ke-1)); \text{metallic wall} \\
for k=2:N 
\( \text{Hyz}_{fb}(i,j,k) = \text{sig}_0_m(N+1-k)\text{Hyz}_{fb}(i,j,k) - \text{coeff}_B \\
*( - Exy_fb(i,j,2*N+ke-1) - Exz_fb(i,j,2*N+ke-1)); \text{metallic wall} \\
\)
*(Exy_fb(i,j,k) - Exy_fb(i,j,k-1) + Exz_fb(i,j,k) - Exz_fb(i,j,k-1));
end

for k=N+1:N+ke %
   Hyz_fb(i,j,k) = Hyz_fb(i,j,k) - coeff_B *(Exy_fb(i,j,k) - Exy_fb(i,j,k-1) + Exz_fb(i,j,k) - Exz_fb(i,j,k-1));
end

for k=N+ke+1:2*N+ke-1 %
   Hyz_fb(i,j,k) = sig_0_m(k-N-ke)*Hyz_fb(i,j,k) - coeff_B *(Exy_fb(i,j,k) - Exy_fb(i,j,k-1) + Exz_fb(i,j,k) - Exz_fb(i,j,k-1));
end
end

%**********************************************************************
%Hyz_tb - bottom and top
%**********************************************************************

for i=1:ie
   for j=1:je-1 %
      Hyz_tb(i,j,1) = sig_0_m(N)*Hyz_tb(i,j,1) - coeff_B *(Exy_tb(i,j,1) + Exz_tb(i,j,1));% metalic wall
      Hyz_tb(i,j,2*N) = sig_0_m(N)*Hyz_tb(i,j,2*N) - coeff_B *( - Exy_tb(i,j,2*N) - Exz_tb(i,j,2*N));%metalic wall
      for k=2:N %
         Hyz_tb(i,j,k) = sig_0_m(N+1-k)*Hyz_tb(i,j,k) - coeff_B
      end
   end
end
*(Exy_tb(i,j,k) - Exy_tb(i,j,k-1) + Exz_tb(i,j,k) - Exz_tb(i,j,k-1));
end
for k=N+1:2*N-1 
Hyz_tb(i,j,k) = sig_0_m(k-N)*Hyz_tb(i,j,k) - coeff_B 
*(Exy_tb(i,j,k+1) - Exy_tb(i,j,k) + Exz_tb(i,j,k+1) - Exz_tb(i,j,k));
end
end
end

%%%***********************************************************************
%%%Hyx_lr - left and right
%%%***********************************************************************
for j=1:2*N %j=1:N j=N+1:2*N
    for k=1:2*N+ke %
        Hyx_lr(1,j,k) = sig_0_m(N)*Hyx_lr(1,j,k) + coeff_B
        *(Ezx_lr(1,j,k) + Ezy_lr(1,j,k)); % metallic wall
        Hyx_lr(2*N+ie,j,k) = sig_0_m(N)*Hyx_lr(2*N+ie,j,k) + coeff_B
        *( - Ezx_lr(2*N+ie-1,j,k) - Ezy_lr(2*N+ie-1,j,k));%metallic wall
        for i=2:N %
            Hyx_lr(i,j,k) = sig_0_m(N-i+1)*Hyx_lr(i,j,k) + coeff_B
            *(Ezx_lr(i,j,k) - Ezx_lr(i-1,j,k) + Ezy_lr(i,j,k) - Ezy_lr(i-1,j,k));
        end
        for i=N+1:ie+N %
            Hyx_lr(i,j,k) = Hyx_lr(i,j,k) + coeff_B
        end
    end
end
*(Ezx_lr(i,j,k) - Ezx_lr(i-1,j,k) + Ezy_lr(i,j,k) - Ezy_lr(i-1,j,k));
end
for i=N+ie+1:2*N+ie-1 *
Hyx_lr(i,j,k) = sig_0_m(i-N-ie)*Hyx_lr(i,j,k) + coeff_B
*(Ezx_lr(i,j,k) - Ezx_lr(i-1,j,k) + Ezy_lr(i,j,k) - Ezy_lr(i-1,j,k));
end
end

%***********************************************************************
%Hyx_fb - back and front
%***********************************************************************
for j=1:je-1
for k=1:2*N+ke
Hyx_fb(1,j,k) = sig_0_m(N)*Hyx_fb(1,j,k) + coeff_B
*(Ezx_fb(1,j,k) + Ezy_fb(1,j,k));% metalic wall
Hyx_fb(2*N,j,k) = sig_0_m(N)*Hyx_fb(2*N,j,k) + coeff_B
*( - Ezx_fb(2*N,j,k) - Ezy_fb(2*N,j,k));%metalic wall
for i=2:N
Hyx_fb(i,j,k) = sig_0_m(N+1-i)*Hyx_fb(i,j,k) + coeff_B
*(Ezx_fb(i,j,k) - Ezx_fb(i-1,j,k) + Ezy_fb(i,j,k) - Ezy_fb(i-1,j,k));
end
for i=N+1:2*N-1
Hyx_fb(i,j,k) = sig_0_m(i-N)*Hyx_fb(i,j,k) + coeff_B
*

*(Ezx_fb(i+1,j,k) - Ezx_fb(i,j,k) + Ezy_fb(i+1,j,k) - Ezy_fb(i,j,k));
end
end
end

%**********************************************************************
%Hyx_tb - bottom and top
%**********************************************************************

for j=1:je-1
    for k=1:N 
        Hyx_tb(1,j,k) = Hyx_tb(1,j,k) + coeff_B
        *(Ezx_tb(1,j,k) - Ezx_fb(N,j,k) + Ezy_tb(1,j,k) - Ezy_fb(N,j,k));
        Hyx_tb(ie,j,k) = Hyx_tb(ie,j,k) + coeff_B
        *(Ezx_fb(N+1,j,k) - Ezx_tb(ie-1,j,k) + Ezy_fb(N+1,j,k) - Ezy_tb(ie-1,j,k));
    end
    for k=N+1:2*N 
        Hyx_tb(1,j,k) = Hyx_tb(1,j,k) + coeff_B
        *(Ezx_tb(1,j,k) - Ezx_fb(N,j,k+ke) + Ezy_tb(1,j,k) - Ezy_fb(N,j,k+ke));
        Hyx_tb(ie,j,k) = Hyx_tb(ie,j,k) + coeff_B
        *(Ezx_fb(N+1,j,k+ke) - Ezx_tb(ie-1,j,k) + Ezy_fb(N+1,j,k+ke) - Ezy_tb(ie-1,j,k));
    end
end

end

for k=N+1:2*N 
    Hyx_tb(1,j,k) = Hyx_tb(1,j,k) + coeff_B
    *(Ezx_tb(1,j,k) - Ezx_fb(N,j,k+ke) + Ezy_tb(1,j,k) - Ezy_fb(N,j,k+ke));
    Hyx_tb(ie,j,k) = Hyx_tb(ie,j,k) + coeff_B
    *(Ezx_fb(N+1,j,k+ke) - Ezx_tb(ie-1,j,k) + Ezy_fb(N+1,j,k+ke) - Ezy_tb(ie-1,j,k));
end
for k=1:2*N %k=1:N k=N+1:2*N
    for i=2:ie-1 
        Hyx_tb(i,j,k) = Hyx_tb(i,j,k) + coeff_B
        *(Ezx_tb(i,j,k) - Ezx_tb(i-1,j,k) + Ezy_tb(i,j,k) - Ezy_tb(i-1,j,k));
    end
    end
end

%***********************************************************************
%Hzx_lr - left and right
%***********************************************************************

for j=1:2*N %j=1:N j=N+1:2*N
    for k=1:2*N+ke-1 
        Hzx_lr(1,j,k) = sig_0_m(N)*Hzx_lr(1,j,k) - coeff_B
        *(Eyz_lr(1,j,k) + Eyx_lr(1,j,k));
        Hzx_lr(2*N+ie,j,k) = sig_0_m(N)*Hzx_lr(2*N+ie,j,k) - coeff_B
        *( - Eyz_lr(2*N+ie-1,j,k) - Eyx_lr(2*N+ie-1,j,k));
        for i=2:N%
            Hzx_lr(i,j,k) = sig_0_m(N-i+1)*Hzx_lr(i,j,k) - coeff_B
            *(Eyz_lr(i,j,k) - Eyz_lr(i-1,j,k) + Eyx_lr(i,j,k) - Eyx_lr(i-1,j,k));
        end
    end
    for i=N+1:ie+N%
        Hzx_lr(i,j,k) = Hzx_lr(i,j,k) - coeff_B
        *(Eyz_lr(i,j,k) - Eyz_lr(i-1,j,k) + Eyx_lr(i,j,k) - Eyx_lr(i-1,j,k));
    end
end
end

for i=N+ie+1:2*N+ie-1
    Hzx_lr(i,j,k) = sig_0_m(i-N-ie)*Hzx_lr(i,j,k) - coeff_B 
    *(Eyz_lr(i,j,k) - Eyz_lr(i-1,j,k) + Eyx_lr(i,j,k) - Eyx_lr(i-1,j,k));
end

end

%***********************************************************************
% Hzx_fb - back and front
%***********************************************************************

for j=1:je
    for k=1:2*N+ke-1
        Hzx_fb(1,j,k) = sig_0_m(N)*Hzx_fb(1,j,k) - coeff_B 
        *(Eyz_fb(1,j,k) + Eyx_fb(1,j,k));% metallic wall
        Hzx_fb(2*N,j,k) = sig_0_m(N)*Hzx_fb(2*N,j,k) - coeff_B 
        *( - Eyz_fb(2*N,j,k) - Eyx_fb(2*N,j,k));% meetalic wall
        for i=2:N
            Hzx_fb(i,j,k) = sig_0_m(N+1-i)*Hzx_fb(i,j,k) - coeff_B 
            *(Eyz_fb(i,j,k) - Eyz_fb(i-1,j,k) + Eyx_fb(i,j,k) - Eyx_fb(i-1,j,k));
        end
    end
    for i=N+1:2*N-1
        Hzx_fb(i,j,k) = sig_0_m(i-N)*Hzx_fb(i,j,k) - coeff_B 
        *(Eyz_fb(i+1,j,k) - Eyz_fb(i,j,k) + Eyx_fb(i+1,j,k) - Eyx_fb(i,j,k));
    end
end
for j=1:je
    for k=1:N %
        Hzx_tb(1,j,k) = Hzx_tb(1,j,k) - coeff_B
        *(Eyz_tb(1,j,k) - Eyz_fb(N,j,k) + Eyx_tb(1,j,k) - Eyx_fb(N,j,k));
        Hzx_tb(ie,j,k) = Hzx_tb(ie,j,k) - coeff_B
        *(Eyz_fb(N+1,j,k) - Eyz_tb(ie-1,j,k) + Eyx_fb(N+1,j,k) - Eyx_tb(ie-1,j,k));

    end
    for k=N+1:2*N %
        Hzx_tb(1,j,k) = Hzx_tb(1,j,k) - coeff_B
        *(Eyz_tb(1,j,k) - Eyz_fb(N,j,k+ke-1) + Eyx_tb(1,j,k) - Eyx_fb(N,j,k+ke-1));
        Hzx_tb(ie,j,k) = Hzx_tb(ie,j,k) - coeff_B
        *(Eyz_fb(N+1,j,k+ke-1) - Eyz_tb(ie-1,j,k) + Eyx_fb(N+1,j,k+ke-1) - Eyx_tb(ie-1,j,k));

    end
    for k=1:2*N \%k=1:N k=N+1:2*N
for i=2:ie-1
    Hxz_tb(i,j,k) = Hxz_tb(i,j,k) - coeff_B
    *(Eyz_tb(i,j,k) - Eyz_tb(i-1,j,k) + Eyx_tb(i,j,k) - Eyx_tb(i-1,j,k));
end
end

%***********************************************************************
%Hzy_lr - left and right - presente em toda a PML
%***********************************************************************

for i=1:2*N+ie
    for k=1:2*N+ke-1
        Hzy_lr(i,1,k) = sig_0_m(N)*Hzy_lr(i,1,k) + coeff_B
        *(Exy_lr(i,1,k) + Exz_lr(i,1,k));%metallic wall
        Hzy_lr(i,2*N,k) = sig_0_m(N)*Hzy_lr(i,2*N,k) + coeff_B
        *( - Exy_lr(i,2*N,k) - Exz_lr(i,2*N,k));%metallic wall
        for j=2:N
            Hzy_lr(i,j,k) = sig_0_m(N-j+1)*Hzy_lr(i,j,k) + coeff_B
            *(Exy_lr(i,j,k) - Exy_lr(i,j-1,k) + Exz_lr(i,j,k) - Exz_lr(i,j-1,k));
        end
        for j=N+1:2*N-1
            Hzy_lr(i,j,k) = sig_0_m(j-N)*Hzy_lr(i,j,k) + coeff_B
            *(Exy_lr(i,j+1,k) - Exy_lr(i,j,k) + Exz_lr(i,j+1,k) - Exz_lr(i,j,k));
        end
    end
end
for k=1:2*N+ke-1 %
    for i=1:N
        Hzy_fb(i,1,k) = Hzy_fb(i,1,k) + coeff_B
        *(Exy_fb(i,1,k) - Exy_lr(i,N,k) + Exz_fb(i,1,k) - Exz_lr(i,N,k));
        Hzy_fb(i,je,k) = Hzy_fb(i,je,k) + coeff_B
        *(Exy_lr(i,N+1,k) - Exy_fb(i,je-1,k) + Exz_lr(i,N+1,k) - Exz_fb(i,je-1,k));
    end
    for i=N+1:2*N %i=1:N i=N+1:2*N
        Hzy_fb(i,1,k) = Hzy_fb(i,1,k) + coeff_B
        *(Exy_fb(i,1,k) - Exy_lr(i+ie,N,k) + Exz_fb(i,1,k) - Exz_lr(i+ie,N,k));
        Hzy_fb(i,je,k) = Hzy_fb(i,je,k) + coeff_B
        *(Exy_lr(i+ie,N+1,k) - Exy_fb(i,je-1,k) + Exz_lr(i+ie,N+1,k)
        - Exz_fb(i,je-1,k));
    end
    for i=1:2*N %i=1:N i=N+1:2*N
        for j=2:je-1
            Hzy_fb(i,j,k) = Hzy_fb(i,j,k) + coeff_B
        end
    end
\[ \text{Exy}_fb(i,j,k) - \text{Exy}_fb(i,j-1,k) + \text{Exz}_fb(i,j,k) - \text{Exz}_fb(i,j-1,k); \]

\begin{align*}
&\text{end} \\
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\end{align*}

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for k=1:2*N %k=1:N k=N+1:2*N
  for j=2:je-1 %
  Hzy_tb(i,j,k) = Hzy_tb(i,j,k) + coeff_B
  *(Exy_tb(i,j,k) - Exy_tb(i,j-1,k) + Exz_tb(i,j,k) - Exz_tb(i,j-1,k));
  end
  end
end

%***********************************************************************
% Mesh knots: domain of study - contact with PML
%***********************************************************************
%***********************************************************************
% Calculation of Hx
%***********************************************************************
for i=1:ie-1
  Hx(i,1,1) = Hx(i,1,1) + db(i,1,1).*((Ey(i,1,1) - (Eyx_tb(i,1,N) + Eyz_tb(i,1,N)) - Ez(i,1,1) + (Ezx_lr(i+N,N,N+1) + Ezy_lr(i+N,N,N+1))) + Ez(i,1,1) + (Ezx_lr(i+N,N,N+1) + Ezy_lr(i+N,N,N+1)));
  Hx(i,1,ke) = Hx(i,1,ke) + db(i,1,ke).*((Ey(i,1,1) - (Eyx_tb(i,1,N) + Eyz_tb(i,1,N)) - Ez(i,1,1) + (Ezx_lr(i+N,N,ke+N) + Ezy_lr(i+N,N,ke+N))) + Ez(i,1,ke) + (Ezx_lr(i+N,N,ke+N) + Ezy_lr(i+N,N,ke+N)));
  Hx(i,je,1) = Hx(i,je,1) + db(i,je,1).*((Ey(i,1,1) - (Eyx_tb(i,1,N) + Eyz_tb(i,1,N)) - Ez(i,1,1) + (Ezx_lr(i+N,N,je) + Ezy_lr(i+N,N,je+N))) + Ez(i,1,1) + (Ezx_lr(i+N,N,je+N) + Ezy_lr(i+N,N,je+N)));
end

\[ H_x(i,je,ke) = H_x(i,je,ke) + d_b(i,je,ke) \cdot (E_{yx\_tb}(i,je,N+1) + \\
E_{yz\_tb}(i,je,N+1) - E_y(i,je,ke-1) - (E_{zx\_lr}(i+N,N+1,ke+N) + \\
E_{zy\_lr}(i+N,N+1,ke+N)) + E_z(i,je-1,ke)); \]

\[ \text{for } k=2:ke-1 \]

\[ H_x(i,1,k) = H_x(i,1,k) + d_b(i,1,k) \cdot (E_y(i,1,k) - E_y(i,1,k-1) - \\
E_z(i,1,k) + (E_{zx\_lr}(i+N,N,k+N) + E_{zy\_lr}(i+N,N,k+N))); \]

\[ H_x(i,je,k) = H_x(i,je,k) + d_b(i,je,k) \cdot (E_y(i,je,k) - E_y(i,je,k-1) - \\
(E_{zx\_lr}(i+N,N+1,k+N) + E_{zy\_lr}(i+N,N+1,k+N)) + E_z(i,je-1,k)); \]

\[ \text{end} \]

\[ \text{for } j=2:je-1 \]

\[ H_x(i,j,1) = H_x(i,j,1) + d_b(i,j,1) \cdot (E_y(i,j,1) - (E_{zx\_fb}(N,j,N+1) + \\
E_{zy\_fb}(N,j,N+1)) - E_x(i,j,1) + (E_{xy\_tb}(i,j,N) + E_{xz\_tb}(i,j,N))); \]

\[ H_x(i,j,ke) = H_x(i,j,ke) + d_b(i,j,ke) \cdot (E_y(i,j,ke) + E_y(i,j,ke-1) - \\
E_z(i,j,ke) + E_z(i,j-1,ke)); \]

\[ \text{end} \]

\[ \text{end} \]

%******************************************************************************
% Calculation of Hy
%******************************************************************************

\[ \text{for } j=1:je-1 \]

\[ H_y(1,j,1) = H_y(1,j,1) + d_b(1,j,1) \cdot (E_z(1,j,1) - (E_{zx\_fb}(N,j,N+1) + \\
E_{zy\_fb}(N,j,N+1)) - E_x(1,j,1) + (E_{xy\_tb}(1,j,N) + E_{zx\_tb}(1,j,N)))); \]
\[
\begin{align*}
\text{Hy}(1,j,ke) &= \text{Hy}(1,j,ke) + \text{db}(1,j,ke) \cdot (\text{Ez}(1,j,ke) - (\text{Ezx_fb}(N,j,N+ke) \\
&\quad + \text{Ezy_fb}(N,j,N+ke)) - (\text{Exy_tb}(1,j,N+1) + \text{Exz_tb}(1,j,N+1))) \\
&\quad + \text{Ex}(1,j,ke-1)); \\
\text{Hy}(ie,j,1) &= \text{Hy}(ie,j,1) + \text{db}(ie,j,1) \cdot (\text{Ezx_fb}(N+1,j,N+1) + \\
&\quad \text{Ezy_fb}(N+1,j,N+1) - \text{Ez}(ie-1,j,1) - \text{Ex}(ie,j,1) + (\text{Exy_tb}(ie,j,N) \\
&\quad + \text{Exz_tb}(ie,j,N))); \\
\text{Hy}(ie,j,ke) &= \text{Hy}(ie,j,ke) + \text{db}(ie,j,ke) \cdot (\text{Ezx_fb}(N+1,j,N+ke) + \\
&\quad \text{Ezy_fb}(N+1,j,N+ke) - \text{Ez}(ie-1,j,ke) - (\text{Exy_tb}(ie,j,N+1) + \\
&\quad \text{Exz_tb}(ie,j,N+1)) + \text{Ex}(ie,j,ke-1)); \\
\text{for} \ k = 2:ke-1 \\
\text{Hy}(1,j,k) &= \text{Hy}(1,j,k) + \text{db}(1,j,k) \cdot (\text{Ez}(1,j,k) - (\text{Ezx_fb}(N,j,N+k) \\
&\quad + \text{Ezy_fb}(N,j,N+k)) - \text{Ex}(1,j,k) + \text{Ex}(1,j,k-1)); \\
\text{Hy}(ie,j,k) &= \text{Hy}(ie,j,k) + \text{db}(ie,j,k) \cdot (\text{Ezx_fb}(N+1,j,N+k) + \\
&\quad \text{Ezy_fb}(N+1,j,N+k) - \text{Ez}(ie-1,j,k) - \text{Ex}(ie,j,k) + \text{Ex}(ie,j,k-1)); \\
\text{end} \\
\text{for} \ i = 2:ie-1 \\
\text{Hy}(i,j,1) &= \text{Hy}(i,j,1) + \text{db}(i,j,1) \cdot (\text{Ez}(i,j,1) - \text{Ez}(i-1,j,1) - \text{Ex}(i,j,1) \\
&\quad + (\text{Exy_tb}(i,j,N) + \text{Exz_tb}(i,j,N))); \\
\text{Hy}(i,j,ke) &= \text{Hy}(i,j,ke) + \text{db}(i,j,ke) \cdot (\text{Ez}(i,j,ke) - \text{Ez}(i-1,j,ke) - \\
&\quad (\text{Exy_tb}(i,j,N+1) + \text{Exz_tb}(i,j,N+1)) + \text{Ex}(i,j,ke-1)); \\
\text{end} \\
\text{end} \\
\%**********************************************************************
\end{align*}
\]
% Calculation of Hz

%**************************************************************************

for k=1:ke-1

Hz(1,1,k) = Hz(1,1,k) + db(1,1,k).*((Ex(1,1,k) - (Exy_lr(N+1,N,N+k) + Exz_lr(N+1,N,N+k)) - Ey(1,1,k) + (Eyx_fb(N,1,N+k) + Eyz_fb(N,1,N+k)));

Hz(1,je,k) = Hz(1,je,k) + db(1,je,k).*((Exy_lr(N+1,N+1,N+k) + Exz_lr(N+1,N+1,N+k) - Ex(1,je-1,k) - Ey(1,je,k) + (Eyx_fb(N,je,N+k) + Eyz_fb(N,je,N+k)));

Hz(ie,1,k) = Hz(ie,1,k) + db(ie,1,k).*((Ex(ie,1,k) - (Exy_lr(N+ie,N,N+k) + Exz_lr(N+ie,N,N+k)) - (Eyx_fb(N+1,1,N+k) + Eyz_fb(N+1,1,N+k) + Ey(ie-1,1,k));

Hz(ie,je,k) = Hz(ie,je,k) + db(ie,je,k).*((Exy_lr(N+ie,N+1,N+k) + Exz_lr(N+ie,N+1,N+k) - Ex(ie,je-1,k) - (Eyx_fb(N+1,je,N+k) + Eyz_fb(N+1,je,N+k)) + Ey(ie-1,je,k));

for j=2:je-1

Hz(1,j,k) = Hz(1,j,k) + db(1,j,k).*((Ex(1,j,k) - Ex(1,j-1,k) - Ey(1,j,k) + (Eyx_fb(N,j,N+k) + Eyz_fb(N,j,N+k)));

Hz(ie,j,k) = Hz(ie,j,k) + db(ie,j,k).*((Ex(ie,j,k) - Ex(ie,j-1,k) - (Eyx_fb(N+1,j,N+k) + Eyz_fb(N+1,j,N+k) + Ey(ie-1,j,k));

end

for i=2:ie-1
Hz(i,1,k) = Hz(i,1,k) + db(i,1,k).*(Ex(i,1,k) - (Exy_lr(N+i,N,N+k) + Exz_lr(N+i,N,N+k)) - Ey(i,1,k) + Ey(i-1,1,k));
Hz(i,je,k) = Hz(i,je,k) + db(i,je,k).*(Exy_lr(N+i,N+1,N+k) + Exz_lr(N+i,N+1,N+k) - Ex(i,je-1,k) - Ey(i,je,k) + Ey(i-1,je,k));
end
end

%***********************************************************************
% Update magnetic fields - without contact with PML!
%***********************************************************************
%***********************************************************************
%***********************************************************************
% Hx - without contact with PML!
%***********************************************************************
%***********************************************************************
for i=1:ie-1
for j=2:je-1
for k=2:ke-1
Hx(i,j,k) = Hx(i,j,k) + db(i,j,k).*(Ey(i,j,k) - Ey(i,j,k-1) - Ez(i,j,k) + Ez(i,j-1,k));
end
end
end

%***********************************************************************
%Hy - without contact with PML!
%***********************************************************************
for i=2:ie-1
    for j=1:je-1
        for k=2:ke-1
            \( \text{Hy}(i,j,k) = \text{Hy}(i,j,k) + d_b(i,j,k) \cdot (E_z(i,j,k) - E_z(i-1,j,k) - E_x(i,j,k) + E_x(i,j,k-1)) \);
        end
    end
end

%***********************************************************************
%Hz - without contact with PML!
%***********************************************************************

for i=2:ie-1
    for j=2:je-1
        for k=1:ke-1
            \( \text{Hz}(i,j,k) = \text{Hz}(i,j,k) + d_b(i,j,k) \cdot (E_x(i,j,k) - E_x(i,j-1,k) - E_y(i,j,k) + E_y(i-1,j,k)) \);
        end
    end
end

Hz_op = [Hz_op; Hz(io,jo,kobs)];

%***********************************************************************
% Save to mode distribution
for i=1:ie
    for j=1:je
        mode_Hz(i,j) = mode_Hz(i,j) + Hz(i,j,kobs);
    end
end

% Visualize fields

if mod(n,2)==0;
    timestep=int2str(n);
    tview(:,:,kobs)=Hz(:,:,kobs);
    sview(:,jo,:)=Hz(:,jo,:);
    subplot('position',[0.15 0.45 0.7 0.45]),pcolor(tview);
    shading flat;
    caxis([plot_min plot_max]);
    colorbar;
    axis image;
    title(['Ey(i,j,k=5), time step = ',timestep]);
    xlabel('i coordinate');
ylabel('j coordinate');

subplot('position',[0.15 0.10 0.7 0.25]),pcolor(sview');
shading flat;
caxis([plot_min plot_max]);
colorbar;
axis image;
title(['Ey(i,j=13,k), time step = ',timestep]);
xlabel('i coordinate');
ylabel('k coordinate');

nn=n/2;
M(:,nn)=getframe(gcf,rect);

%**********************************************************************
% Creating the individual images
%**********************************************************************

Image = getframe(gcf,rect);

% get current pseudo color image

directory = 'images/';

% The next four lines parse and assemble file
number = num2str(n);

    % names for the current image (Note the need
extension = '.bmp';

    % for a directory called "images"
filename = [directory,number,extension];

imwrite(Image.cdata,eval('filename'), 'bmp');

end;

%***********************************************************************
% END TIME-STEPPING LOOP
%***********************************************************************

end

%***********************************************************************
% FIND MODE FREQUENCY BY DOING FFT
%***********************************************************************

F = fft(Hz_op,1024);

Pyy = F.* conj(F) / 1024;

    % The power spectrum, a measurement of the power
% at various frequencies

\[ f = \frac{1}{dt} \times (0:512)/1024; \]

% Graph the first 257 points (the other 255 points are redundant)
% on a meaningful frequency axis

g = f';

% plot(f,Pyy(1:513))
% title('Frequency content of y')

%***********************************************************************

% FIND MODE DISTRIBUTION
%***********************************************************************

for i=1:ie
    for j=1:je
        mode_Hz_final(i,j) = Hz(i,j,kobs);
    end
end

for i=1:ie
    for j=1:je
        mode_dist(i,j) = mode_Hz(i,j) / nmax;
    end
end
% Plot mode distribution

% tview(:,:,)=mode_dist(:,:,);

subplot('position',[0.15 0.45 0.7 0.45]),pcolor(mode_dist');
shading flat;
caxis([plot_min plot_max]);
colorbar;
axis image;
title(['Ey(i,j,k=5), time step = ',timestep]);
xlabel('i coordinate');
ylabel('j coordinate');

% Save the results

save('mode_dist.txt','mode_dist','-ASCII')
save('mode_Hz_final.txt','mode_Hz_final','-ASCII')
save('g.txt','g','-ASCII')
save('Pyy.txt','Pyy','-ASCII')
save('Hz_op.txt','Hz_op','-ASCII')

% movie(gcf,M,0,10,rect);
Appendix B

Process recipes used for microdisk fabrication

B.1 Photolithography and ebeam lithography

B.1.1 Photolithography for positive imaging resist

1. Clean wafer with acetone and IPA, rinse with DI water.

2. Dehydrate 5 minutes at 120 degree C, cool down 3 minutes.

3. Spin on HMDS at 4000 rpm, spin dry.

4. Spin on photo resist, usually 1800 series, at 4000 rpm, 1000 rpm, 40 seconds.

5. Bake at 105 degree C, 2 minutes, cool down 1 minute.

6. Expose 3 seconds with channel 1 (365nm) of the MA6 aligner, 12 mW, hard contact.

7. Develop with CD-26 developer for around 1 minute, see clear patterns.

8. Rinse off the developer and blow dry with N2 gas.

9. Examine the photo resist patterns under optical microscope.

B.1.2 Ebeam lithography with dual layer resist coating for lift-off process

1. Clean wafer with acetone and IPA, rinse with DI water.

2. Dehydrate 5 minutes at 120 degree C, cool down 3 minutes.
3. Spin on copolymer P(MMA/MAA) 5.5%, spin at 2000 rpm, 60 seconds.

4. Bake at 180 degree C for 3 minutes, cool down 30 seconds.

5. Spin on PMMA 3% at 4000 rpm for 60 seconds.

6. Bake at 180 degree C for 3 minutes.

7. Deposit 15 nm aluminum as a conducting layer for insulating samples.

8. Ready for ebeam writing.

9. Immerse exposed wafer in CD-26 for 2 minutes to remove aluminum.


11. Immerse in (MIBK:IPA=1:1) developer for 60 seconds.

12. Immerse in IPA for 15 seconds. (this step is for forming the undercut of the copolymer resist, hence timing decides the undercut and is critical.)


14. Examine the ebeam resist patterns under optical microscope.

B.1.3 Ebeam lithography with negative resist CAN014 2.8 cp

1. Clean wafer with acetone and IPA, rinse with DI water.

2. Dehydrate 5 minutes at 120 degree C, cool down 3 minutes.

3. Prime with HMDS and spin on OEBR-CAN014 (2.8 cp) at 2000 rpm for 70 seconds.

4. Pre bake at 100 degree C for 90 seconds.
5. Thermal deposit 10 nm gold as conducting layer.

6. Ready for ebeam writing.

7. Post exposure bake at 110 degree C for 90 seconds.

8. Remove gold with gold etchant, 1 minute.

9. Develop in CD-26, 1 minute, rinse with DI water.

10. Post developing bake at 100 degree C for 1 minute.

**B.2 Etching and deposition**

**B.2.1 Reactive ion etch (RIE) with Cl$_2$/Ar for Al$_{0.8}$GaAs**

- Cl$_2$ flow rate: 20 sccm
- Ar flow rate: 20 sccm
- pressure: 10 mtorr
- power: 250 W
- temperature: 25 °C
- etching speed: 250 nm/min
B.2.2 ECR-PECVD deposition of SiN$_x$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiH$_4$ flow rate</td>
<td>10.1 sccm</td>
</tr>
<tr>
<td>N$_2$ flow rate</td>
<td>8 sccm</td>
</tr>
<tr>
<td>Ar flow rate</td>
<td>3 sccm</td>
</tr>
<tr>
<td>Pressure</td>
<td>4 mtorr</td>
</tr>
<tr>
<td>Temperature</td>
<td>100°C</td>
</tr>
<tr>
<td>Power (rf2)</td>
<td>300-400 W</td>
</tr>
<tr>
<td>Upper magnet</td>
<td>173</td>
</tr>
<tr>
<td>Lower magnet</td>
<td>24</td>
</tr>
</tbody>
</table>

B.2.3 Bromine based wet etching

See Dr. Wei-hua Wang’s thesis.
Appendix C

Useful and processed sample list
for microdisk related projects

1. 060803A, B: magnetic microdisk sample, 100 nm GaMnAs + 10 nm HT GaAs + 250 nm AlGaAs post + six 4.2 nm QWs IFQDs disk layer 120 nm + 500 nm AlGaAs post + 200 nm GaAs buffer. Measured Tc 45 K for B and 35 K for A. Used for magnetic microdisk laser project.

2. 060615A, B: microdisk sample, six 4.2 nm QWs and IFQDs, 500 nm AlGaAs post. Used for coupled microdisk project.

3. 060131A, B, 060201A: microdisk sample, five 4.2 nm QWs and IFQDs, 500 nm AlGaAs post. Used for coupled microdisk project.

4. 050407A, B, C: Delta doped Mn microdisk samples. A: undoped, five 10 nm GaAs QWs. B: delta doped Mn only in the middle QW. C: delta doped Mn in every five QWs. Delta dope means opening Mn shutter for only 3 seconds. Used for spin noise project.

5. 041215A: microdisk sample, five 10 nm QWs. Used for external strain engineering project.
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[73] J. Lin, A. Leven, N. G. Weimann, Y. Yang, R. F. Kopf, R. Reyes, Y. K. Chen, and 
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