SLIDING MODE/ $H_{\infty}$ CONTROL OF HYDROPOWER PLANTS

A Thesis in
Mechanical Engineering
by
Xibei Ding

© 2011 Xibei Ding

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

August 2011
The thesis of Xibei Ding was reviewed and approved* by the following:

Alok Sinha  
Professor of Mechanical Engineering  
Thesis Advisor

Christopher D. Rahn  
Professor of Mechanical Engineering

Karen A. Thole  
Professor of Mechanical Engineering  
Department Head of Mechanical and Nuclear Engineering

*Signatures are on file in the Graduate School
This thesis addresses the development of dynamic model and advanced controller design for entire Hydropower plant. Although hydropower has the best payback ratio and the highest efficiency in the all electricity generating method due to the constant upgrading of turbine, tunnel and reservoir design, most of the plants’ control systems are still using the technique developed in the 1960s. It limits the operating range of the system such as specific water flow rate and reservoir level. As energy demands are increasing every day, it is necessary to develop a more flexible control system for the plants to operate at different situations. The goal of this thesis is to design a nonlinear controller for the hydropower plant in order to improve its ability to handle different operating conditions. The work is more focused on the load frequency control. In order to achieve such goal, the detailed dynamic model of hydropower plant has been studied and presented, especially the turbine penstock transient response. Several advanced control algorithms have been implemented on the hydropower system, such as sliding mode control with $H_\infty$, feedback linearization with sliding mode control and second order sliding mode control. The simulation of each controller has been compared with traditional linear optimal controller. The results of this thesis show the benefits of these advanced controllers.
# TABLE OF CONTENTS

LIST OF FIGURES .............................................................................................................. vi

LIST OF TABLES ................................................................................................................. ix

ACKNOWLEDGEMENTS ...................................................................................................... x

Chapter 1 introduction...................................................................................................... 1
  1.1 Research Purpose and Aim ....................................................................................... 1
    1.1.1 Hydropower Background ................................................................................ 2
    1.1.2 Control Aspect ................................................................................................. 4
  1.2 Review of Previous Research ................................................................................... 7
    1.2.1 Dynamic Model .............................................................................................. 8
    1.2.2 Control Method .............................................................................................. 10
  1.3 Contents of this Thesis ............................................................................................ 12

Chapter 2 Hydropower Plant Dynamic Model ............................................................. 13
  2.1 Basic Concepts and Parameter, Variable List ......................................................... 13
  2.2 Kundur’s Dynamic Model ....................................................................................... 16
    2.2.1 Nonlinear Model............................................................................................... 16
    2.2.2 Linear Model .................................................................................................. 18
      1. Ideal .................................................................................................................. 18
      2. Non-ideal .......................................................................................................... 19
    2.2.3 Other Effects: ................................................................................................. 20
      1. Elastic Conduit Effect ....................................................................................... 20
      2. Surge Tank Effect ............................................................................................. 23
  2.3 Comparison with IEEE Model ................................................................................. 25
  2.4 Turbine-penstock dynamic simulation .................................................................... 27
  2.5 Hydropower Plant Other Components’ Dynamic Model ....................................... 30
    2.5.1 Gate Servo System ......................................................................................... 31
    2.5.2 Generator [51] ............................................................................................. 31
  2.6 New Turbine Model with Inner Torque Loss [2] ...................................................... 33

Chapter 3 Sliding Mode/ $H_{\infty}$ Control for Hydropower Plant .................................... 35
  3.1 Sliding Mode Control Theory .................................................................................. 35
  3.2 $H_{\infty}$ Control Theory .......................................................................................... 36
    3.2.1 Hydropower Plant Model .............................................................................. 37
    3.2.2 Sliding Mode Design ..................................................................................... 38
  3.3 Simulation Results ................................................................................................... 43
  3.4 Conclusion .............................................................................................................. 46
Chapter 4 Feedback Linearization with Sliding Mode Control for Hydropower Plant ........47

4.1 Feedback Linearization Theory .................................................................47
4.2 Design Detail ..............................................................................................48
  4.2.1 Hydropower Plant Model .................................................................48
  4.2.2 Feedback Linearization for Hydropower Plant Model ..................49
  4.2.3 Sliding Mode Control ........................................................................51
4.3 Simulation Results ...................................................................................53
4.4 Conclusion ................................................................................................56

Chapter 5 Second Order Sliding Mode Control for Hydropower Plant ........58

5.1 Second Order Sliding Mode Control Theory ........................................58
5.2 Controller Design Detail ..........................................................................60
  5.2.1 Basic Hydropower Plant Dynamic Model ......................................60
  5.2.2 First and Second Order Sliding Mode Systems ............................61
5.3 Simulation Results ....................................................................................65
5.4 Conclusion ................................................................................................69

Chapter 6 Conclusions and Future Work ..................................................71

  6.1 Thesis Conclusions ................................................................................71
  6.2 Future Work .............................................................................................72

Bibliography .....................................................................................................75

Appendix A New Turbine Model and Real Data Test ...................................83
Appendix B PI Tuning Procedures for Linear Controller .................................91
Appendix C Feedback Linearization ................................................................95
Appendix D Super-Twisting Algorithm ..............................................................99
Appendix E Steady State Dynamics After Reaching Sliding Surface ...........104
Appendix F $\mu$ Analysis and $\mu$ Synthesis on Hydropower Plant ..............105
LIST OF FIGURES

Figure 1-1: Global Energy Generation and Electricity Generation in 2005 [4]..........................3
Figure 1-2: Payback Ratio Comparison of Different Types of Energy Generation [5, 6].........3
Figure 1-3: Typical Hydropower Plant Scheme [7]..............................................................4
Figure 1-4: Block Diagram for Active Power Control of an Isolated System and Network
            Connected System [8]...........................................................................................5
Figure 1-5: Scheme of a Generator Connecting with Large Network, an Infinite Bus,
            through a Transformer. (Reactive Power Control) [9]............................................6
Figure 1-6: Scheme of Mechanical – hydraulic Governor [8].............................................7
Figure 1-7: PI Parameters Ranges from Hovey and Chaudhry [31]....................................10
Figure 2-1: Simple Scheme of Hydropower Plant [8].........................................................14
Figure 2-2: Illustration of Upstream and Downstream along Penstock............................21
Figure 2-3: Hydropower Plant with a Single Surge Tank and Riser Scheme [8].................24
Figure 2-4: Mechanical Power Change due to Gate Opening Change based on Linear
            Models..................................................................................................................28
Figure 2-5: Mechanical Power Change due to Gate Opening Change based on Nonlinear
            Models..................................................................................................................29
Figure 2-6: Linear and Nonlinear Model Comparison with Different Step Size Changes....30
Figure 3-1: Full State Feedback Control System.................................................................36
Figure 3-2: Block Diagram of Hydropower Plant under PI and Sliding Mode Control.......38
Figure 3-3: Full State Feedback Control System for Hydropower Plant.........................38
Figure 3-4: Boundary Layer of Sliding Hyperplane for Reducing Chattering [52].............43
Figure 3-5: Frequency Deviation $\Delta \omega$ versus Time under Constant Load Change ....45
Figure 3-6: Frequency Deviation $\Delta \omega$ with Parametric Uncertainties under $H_{\infty}$/Sliding
            Mode Control ........................................................................................................45
Figure 3-7: Frequency Deviation $\Delta \bar{\omega}$ with Parametric Uncertainties under PI Control........46

Figure 4-1: Responses Comparison under Three Control Methods........................................54

Figure 4-2: Nonlinear Controllers with and without Integral Feedback ................................54

Figure 4-3: Nonlinear Controller with Second Linear PI and Nonlinear Controller ..............56

Figure 5-1: Phase Portrait of the System under First and Second Order Sliding Mode
Control[60]..................................................................................................................59

Figure 5-2: Block Diagram for First Order and Second Order Sliding Mode Control of
Hydropower Plant .......................................................................................................61

Figure 5-3: Sliding Variable with Regular Sliding Mode Control (“sat” Function) and
Second Order Sliding Mode Control (both Methods) under Time Invariant
Disturbance ..................................................................................................................66

Figure 5-4: Second Order Sliding Mode Phase Trajectories from Method One and Two
under Time Invariant Disturbance ..............................................................................66

Figure 5-5: Frequency Change ($\Delta \bar{\omega}$) Responses from First Order, Second Order Sliding
Mode under Time Invariant Disturbance .....................................................................67

Figure 5-6: Controller Efforts from First Order, Second Order Sliding Mode under Time
Invariant Disturbance ..................................................................................................68

Figure 5-7: Frequency Change ($\Delta \bar{\omega}$) Responses from First Order, Second Order Sliding
Mode (method two) under System Uncertainties .........................................................69

Figure A-1: Efficiency vs. Flow Rate and Output Power from the Real Data....................86

Figure A-2: Impact Loss Comparison between Real Data and Curve Fitting Result ...........86

Figure A-3: Impact Loss vs. Flow Rate Shape Comparison between our test and paper
[2] Result .......................................................................................................................87

Figure A-4: The Power Output Comparison between Ideal Model, Real Data and New
Model ............................................................................................................................87

Figure A-5: Nonlinear Model with Updated Turbine Torque Model ..............................90

Figure B-1: Oscillation with Constant Amplitude with Ultimate Gain under Constant
Disturbance (0.03 p.u.)..............................................................................................91

Figure B-2: Frequency Change ($\Delta \bar{\omega}$) Responses with Ziegler-Nichols Method under
Constant Disturbance (0.03 p.u.) ..............................................................................92
Figure B-3: Frequency Change ($\Delta \bar{\omega}$) Responses with Method from [15] and [31] under Constant Disturbance (0.03 p.u.) ................................................................. 93

Figure B-4: Frequency Change ($\Delta \bar{\omega}$) Response with Further Tuning from Previous Method under Constant Disturbance (0.03 p.u.) ................................................................. 94

Figure D-1: Phase Portrait for Super-twisting Algorithm [60] ................................................................. 102

Figure F-1: Interconnection Structures for Nominal Plant with Uncertainties and Disturbance [63] ............................................................................................................. 105

Figure F-2: Block Diagram of Hydropower Plant ..................................................................................... 108

Figure F-3: Block Diagram of Hydropower Plant with Uncertainties Separated ......................... 109
LIST OF TABLES

Table 2-1: Parameters and Meanings in Turbine-Penstock Modeling........................................15
Table 2-2: Variable and Meanings in Turbine-Penstock Modeling..........................................15
Table 2-3: Parameter Used in the Following Simulation.........................................................27
Table 3-1: Parameter Used in the Simulation at Chapter 3....................................................44
Table 4-1: Parameters Used in Chapter 4 Simulations............................................................53
Table 5-1: Parameter Values for Controller Design of Method One......................................65
Table 5-2: Parameter Values for Controller Design of Method Two......................................65
Table B-1: Ziegler-Nichols Method..........................................................................................91
ACKNOWLEDGEMENTS

This work is sponsored by US Department of Energy grant under University Hydropower Research Program.

I would first like to thank my advisor, Dr. Alok Sinha. Thank you for your guidance and support from the beginning through my entire research years. I have obtained so much knowledge through our numerous discussions. Your encouragement and advice are greatly appreciated. It has been a pleasure to learn from you and work as your student.

Thank you to the members of the PSU/DOE Hydropower group for helping with my research. I had great experience of spending my graduate student life with you guys.

Appreciation is also expressed to Bill Colwill, Andy Ware and Gerry Russell from Weir American Hydro and Tom Colwill for the support and great suggestions for my research.

Personal gratitude is also expressed to Dr. Christopher Rahn for reading and comments on my thesis.

Thanks to the Department of Mechanical and Nuclear Engineering and Penn State University for the great graduate life. I have enjoyed every moment spent at Penn State. And the past two years really expanded my knowledge and experience.

Finally, I would like also to thank my parents who always support my decision and great advices through my life. I love you very much.
Chapter 1

Introduction

Hydro power plays a very important role in the electricity generating. It has the largest percentage of all types of renewable energy. It also has the highest efficiency and payback ratio in all types of electricity generation [1]. One goal of this thesis is to investigate the possible modern control techniques which can improve hydro power plants’ ability of handling different operating situations. Other goal of this thesis is to develop a system analysis computer model which can predict the dynamic response of the entire hydro power system with effects such as elastic pipe and surge tank. Such dynamic model should include wicket gate system, turbine-penstock and generator. Using the CFD model results, a more accurate dynamic model of hydraulic turbine should also be generated [2].

1.1 Research Purpose and Aim

Hydropower is a clean and renewable energy with high converting efficiency. In order to explore more hydro-energy for satisfying growing energy demand, we need to reduce the limitations of hydropower system. One of thresholds for the system is the control ability. A real case was brought at 2010 HydroVision International Conference [3]. Upgrading the old hydropower plant controller (a mechanical-hydraulic governor) made in 1960s to the modern digital controller, plant’s controllability is improved. It is able to generate at least additional $327,743.00 per year during the regular drought water conditions when it could not operate using old control system with only total cost of $639,000.00. In addition, the old controllers were not designed to handling system uncertainties, such as parameters change due to wear and tear. The
plant performance would decrease after prolonged use. Hence, by improving the control system, hydropower plant can provide more energy with economic profits. Following sections will provide more detail about hydropower system.

1.1.1 Hydropower Background

Human has been using hydropower for over 2000 years from the low power water wheel to massive Megawatts hydro generator. It is a clean and renewable energy. During the years, hydropower technology is gradually improved. Currently, it has the highest overall efficiency of all types of electricity generating. For example, hydropower has efficiency from 75% to 90% compared to 57% for modern gas turbine, 40% for nuclear plants, 5 to 15% for photovoltaic and 19 to 33% for wind turbine [1]. It also has the highest energy payback ratio which is defined as the sum of energy generated during its whole operating life over the sum of energy used during initial construction and later operation, maintenance and decommissioning. As shown in Figure 1-1, hydropower contributes more in the electricity generation compared to other forms of renewable energy combined. And Figure 1-2 shows hydro has the highest payback ratio. The other unique advantage of hydropower is its flexibility. Usually, most of the power plants have to run continuously after they are turned on. But hydropower can be switched on and off at any time as long as there is sufficient amount of water in the reservoir. Therefore, hydropower has advantages over most of electricity generation method and is a reliable, carbon-free, flexible, renewable energy.
Figure 1-1: Global Energy Generation and Electricity Generation in 2005[4].

Figure 1-2: Payback Ratio Comparison of Different Types of Energy Generation [5, 6].
Basically, a hydropower plant consists of reservoir, dam, penstock, turbine, generator, draft tube, and power house connected to the power network as shown in Figure 1-3. One of the most important components of the hydropower plant is the hydro-turbine. In general, there are two types of hydraulic turbine: impulse and reaction turbines. The impulse hydraulic turbine (Pelton) only uses the kinetic energy of water and the reaction turbine (Kaplan and Francis) combines the potential energy and kinetic energy. The selection of the type of turbine depends on the net head and plant’s other characteristics. The other important component is the wicket gate with control system.

![Typical Hydropower Plant Scheme](source: Environment Canada)

Figure 1-3: Typical Hydropower Plant Scheme [7].

1.1.2 Control Aspect

The energy produced by power system flowing through the transmission line can be defined into two groups: active power and reactive power [8]. The active power control is related to frequency control and the reactive power control is related to voltage control. Since relatively
constant frequency and voltage are very important for the quality of the power, active power control and reactive power control ability affect power system performance. The controller design for active power and reactive power can be separated. For active control, isolated systems and network systems have different design method as shown in Figure 1-4. Figure 1-5 shows the reactive power control block diagram.

Figure 1-4: Block Diagram for Active Power Control of an Isolated System and Network Connected System [8].
In this thesis, we focus on active power control or load frequency control on an isolated load system. The governor for hydropower plant system is evolving from mechanical-hydraulic governor to electrical-hydraulic governor and to the modern digital governor. In general, mechanical-hydraulic governor and electrical-hydraulic governor are essentially the same. Figure 1-6 shows a simple scheme of mechanical-hydraulic governor. Unlike mechanical-hydraulic governor using flyballs to sense speed, electrical-hydraulic governor uses sensors and electrical-hydraulic does not generate control input using linkage like mechanical-hydraulic governor.

Figure 1-5: Scheme of a Generator Connecting with Large Network, an Infinite Bus, through a Transformer. (Reactive Power Control) [9].
The disadvantage of mechanical and electrical hydraulic governor is that only very simple control algorithm can be implemented. As the requirements for governor system increase, modern control algorithm needs to be applied and digital controller should be used. The next section will review the existing literature about hydropower plant modeling and control methods.

1.2 Review of Previous Research

There are wide ranges of research on hydropower plant. It is hard to cover all the aspects. In this thesis, a brief literature review of hydropower plant model and hydropower plant control is presented.

Figure 1-6: Scheme of Mechanical–hydraulic Governor [8].
1.2.1 Dynamic Model

The study of dynamic model of hydropower plant can be separated into two time periods. Before the middle of 1990’s, most of the work has been concentrated on developing an accurate general model of the hydropower plant. After that, most of the efforts have been used on modifying the existing model or studying special cases.

In 1962, Oldenburger conducted frequency response tests on the turbines at the Apalachia Hydropower Plant of the Tennessee Valley Authority to check the accuracy of dynamic model for elastic penstock pipe [10]. The partial differential equations in time domain were transformed into ordinary differential equations in frequency domain and solved. The hyperbolic functions were used to represent the complicated mathematical model of the elastic pipe. The detailed solution of the differential equations can be found in [10]. Later, simpler low order models were studied to represent the same effect [11], [12]. The second order Padé approximation was used and the simplicity is essential for controller design. Hovey [13] presented a way to calculate the water starting time constant for the dynamic model. A relationship between the real gate position and ideal gate position is shown in [14] by Woodward in 1967.

During the 1970s, the more general case dynamic model was introduced compared to the ideal linear model, [15], [16]. The linear model is studied not just around rated values but also at other operating points which indicated linear model’s limitation. In 1974, Throne and Hill constructed Kaplan turbine dynamic model by considering the effects of blade angle [17]. During the 1990s, generator model was studied without considering the damping effect [18], [19]. A first order transfer function was used to represent the generator. In 1992 and 1994, two very important summaries of hydropower plant dynamic were made by IEEE [20] and Kundur [8]. They described models of gate system, turbine-penstock and generator from the simplest case to the situation with all effects including the linear and nonlinear. Although they seem to have different
models, essentially they are the same. We will show more details about them in Chapter 2. Also in 1994, a test procedure was developed by Hannett and Fardanesh [21] to find each parameter value which is needed to construct the dynamic model [21] based on the field tests conducted in the hydroelectric stations owned by the New York Power Authority. There are also several articles [22-25] about dynamic model of multiple turbines sharing a common penstock.

After the mid 1990s, the focus of studies has been shifted to improving each component in the model and more detailed system modeling for specific plant. For example, in 1995, a system model is developed to simulate the dynamics of a hydroelectric plant with two surge tanks, one above the turbine and the other one below the turbine [26]. And in 2004, a model is studied for a plant with severely leaky wicket gate [27]. Konidaris and Tegopoulos modified the model by adding network interaction and draft tube surge effects [28]. In 2005, big refinements were made to IEEE model on several components. A hill chart look-up table was added to the turbine output power and another look-up table was added to transform the real gate position to ideal gate position in the equation and the first order filter block was used to include unsteady effects from gate movement [29]. A modification on Kundur’s turbine model was made in [30]. The new model is obtained through frequency response tests. Recently, a new hydraulic turbine model has been developed with consideration of inner torque loss. Instead of looking up the hill curve which is generated by CFD simulation, a dynamic model is developed from CFD simulation results [2]. The new model considers the loss of mechanical friction loss, hydraulic and volume loss and most important, the impact loss.
1.2.2 Control Method

Similar to the history of dynamic model development, the control method for hydropower plant can also be divided into two categories: PID linear controller for mechanical or electrical hydraulic governor and modern control algorithms for digital control system.

Before 1960’s, mechanical-hydraulic governors were widely used. For mechanical-hydraulic governor, controller characteristics are represented by temporary droop and permanent droop. In the 1970’s, electrical-hydraulic governors employing PID controller became popular. Paynter and Hovey studied the stable region of mechanical-hydraulic governor without considering permanent droop [13]. Later, Chaudhry improved the controller by including the above missing parts. Figure 1-7 shows the regions found by Hovey (1) and Chaudhry (2). Thorne and Hill [31] developed PID controller, but did not really explain the derivative gain. In 1979, Hagihara summarized all above methods and explained how to find derivative gain by applying Routh-Hurwitz Criterion [32]. IEEE published an article about how to select PI controller coefficients based on system parameters for Steam Power plant and Hydropower plant [15].

![Figure 1-7: PI Parameters Ranges from Hovey and Chaudhry][1]

[1]: https://example.com/image.png
PID controller is not without limitations. It can not handle complex nonlinear system. It also has poor ability of handling changes in system caused by wear and tear over the time. PID control gains were generated from linearized model about one operating point. In order to control plant in all situations, “Gain Schedule PID” was introduced. The control gain switches from one setting to other one based on gate position and speed error magnitude [33]. The optimal gain was generated by implementing LQR technique. During the 1980’s, several studies about variable structure control have been conducted [34], [35], [36]. New technique about how to construct sliding surface has been introduced, such as LQR with pole assignment. The advantage of this method is very robust and insensitive to system uncertainties. Later, more modern control methods have been studied for hydropower plant, such as Robust Control [37], [38], Adaptive Control [39], Fuzzy Logic [40], [41] and Artificial Neural Networks [42], [43]. A very interesting controller design approach was introduced in [37]. The author separated the hydropower plant model into two parts. One is the linear model and the other part represents the nonlinear characteristics of the turbine. Then the nonlinear characteristics were treated as system uncertainties and robust control was brought in to handle that. Reference [38] applied robust control technique to minimize the effects of system uncertainties, such as parametric uncertainties and noise effects. Adaptive control, fuzzy logic and artificial neural networks were usually implemented to handle the situation when the plant is interconnected to the large scale electric power systems which involve large capacity transmission networks, cable networks and long distance interconnections [39],[44]. Unfortunately, most of the controller designs were based on linear model. There are a couple of papers on the application of feedback linearization; however, the controller designs were more focused on the generator aspect [45], [46].
1.3 Contents of this Thesis

Both linear and nonlinear dynamic models have been considered. The elastic pipe and water compressibility effects and surge tank effects also have been included. The real testing data has been used to test the new turbine torque inner loss model.

Sliding mode with $H_\infty$ technique has been used for the controller design for the linear model. Then with feedback linearization, above method is used to design a controller for nonlinear model. Second order sliding mode and $\mu$ analysis have used to improve the controller performance.

The thesis is organized as follow:

Chapter 2 presents detailed linear and nonlinear models with and without special effects such as elastic pipe, compressibility of water and surge tank based on the [8]. A comparison is made between this and IEEE model. A new turbine torque model is also presented.

Chapter 3 gives detailed procedures for designing a sliding mode controller with $H_\infty$ technique for hydropower plant. The simulation results from different control algorithms show the advantage of above method.

Chapter 4 applies feedback linearization to the nonlinear hydropower plant model, and then the similar technique as Chapter 3 is used to design the controller. The simulation results of both this method and PI controller are compared.

Chapter 5 discusses the application of second order sliding mode control. Two controller design approaches are proposed to implement the second order sliding mode control. Simulation results with such techniques are compared with those from regular sliding mode.

Chapter 6 summarizes the results from each chapter and presents the future works, such as reactive power control.
Chapter 2

Hydropower Plant Dynamic Model

This chapter presents the complete nonlinear and linear hydropower plant dynamic model. Hydropower plant is usually consists of reservoir, penstock, electric-hydraulic servo system, turbine and generator. Some plants with long penstock might have one or multiple surge tanks to reduce water hammer effect. The turbine and penstock dynamic model is the most important part of the system modelling. There are three types of turbine, Pelton, Francis and Kaplan. In this chapter, Francis turbine model is described in detail. In general, there are two types of turbine penstock models. One is simplified nonlinear analytical model, which treats the hydro turbine model as a valve model. The other one is based on prototype characteristics. Since model based on the valve model is widely used in the power system simulation software such as PSS/E made by Siemens, this model will be described in detail and used in later chapters for controller design. In this chapter, the turbine-penstock model will be studied first. It includes the nonlinear and linearized ideal model, with and without elastic penstock effect, and surge tank effect. The basic dynamic models of hydropower plant’s servo gate system and generator will also be covered in this chapter.

2.1 Basic Concepts and Parameter, Variable List

In hydro-turbine study, the variables are usually normalized by their rated values. It is very convenient and useful, since it becomes unit less (per unit) which helps on different unit systems and can be used on different types of hydro turbines. The new values can be calculated by multiplying the per unit values with the new rated values. The rated values are the values used
during turbine design. They should be same or close to the real plant values where the turbine is going to be installed. Therefore, the rated values are the same as initial steady-state values (net water head), such as \( h_0 = h_r \). If the rated values are not the same as real plant values, they can be easily converted to the new values in the model. In this thesis, real plant values are assumed to be same as rated values.

Since the hydropower plant dynamic model is very complicated and different authors used different notations, the standard notations and their meanings for turbine penstock modeling in this thesis are presented as follows:

In general, all the scalar variables are presented by lower case symboles; all the matrix variables are presented as captial case and all the vector variables are presented by bolded font. The variables with upper bar mean the normalized values. The variables with \( \Delta \) mean the difference between current values and nominal or operating values.
Table 2-1: Parameters and Meanings in Turbine-Penstock Modeling.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_g )</td>
<td>Acceleration from gravity</td>
</tr>
<tr>
<td>( A_p, A_{tu} )</td>
<td>Cross section area at penstock, tunnel</td>
</tr>
<tr>
<td>( l_p )</td>
<td>Penstock pipe Length</td>
</tr>
<tr>
<td>( f )</td>
<td>Pipe thickness</td>
</tr>
<tr>
<td>( d_p, d_{tu} )</td>
<td>Diameter of pipe at penstock, tunnel</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Water density</td>
</tr>
<tr>
<td>( k )</td>
<td>Water bulk modulus</td>
</tr>
<tr>
<td>( E )</td>
<td>Young’s Modulus</td>
</tr>
<tr>
<td>( k_u, k_p )</td>
<td>Constant Parameter for water velocity and turbine power</td>
</tr>
<tr>
<td>( t_{wp}, t_{wtu} )</td>
<td>Water starting time constant (at penstock, tunnel)</td>
</tr>
<tr>
<td>( t_s )</td>
<td>Surge tank time constant</td>
</tr>
<tr>
<td>( t_{ep}, t_{etu} )</td>
<td>Elastic time constant (at penstock, tunnel)</td>
</tr>
<tr>
<td>( a_t )</td>
<td>Turbine gain</td>
</tr>
<tr>
<td>( z_p, z_{tu} )</td>
<td>Hydraulic surge impedance at conduit (at penstock, tunnel)</td>
</tr>
<tr>
<td>( m = 2h )</td>
<td>Generator inertia</td>
</tr>
<tr>
<td>( d_s )</td>
<td>Generator damping coefficient</td>
</tr>
</tbody>
</table>

Table 2-2: Variable and Meanings in Turbine-Penstock Modeling.

<table>
<thead>
<tr>
<th>System Variables</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p, p_r )</td>
<td>Turbine output power and rated power</td>
</tr>
<tr>
<td>( p_m )</td>
<td>Turbine mechanical power</td>
</tr>
<tr>
<td>( p_n )</td>
<td>No load condition power</td>
</tr>
<tr>
<td>( p_e )</td>
<td>Disturbance power</td>
</tr>
<tr>
<td>( g, g_r )</td>
<td>Main gate movement, rated value</td>
</tr>
<tr>
<td>( x_e )</td>
<td>Pilot servo movement</td>
</tr>
<tr>
<td>( u_t, u_{nt}, u_r )</td>
<td>Water velocity at turbine, no load value and rated value</td>
</tr>
<tr>
<td>( u_s, u_{riser}, u_p, u_{tu} )</td>
<td>Water velocity at surge tank and riser, penstock and tunnel</td>
</tr>
<tr>
<td>( q_t, q_r )</td>
<td>Water flow rate at turbine, rated value</td>
</tr>
<tr>
<td>( h_p, h_t, h_r )</td>
<td>Water head at penstock, turbine and rated value</td>
</tr>
<tr>
<td>( h_s, h_{riser}, h_w )</td>
<td>Water head at surge tank, riser and reservoir</td>
</tr>
<tr>
<td>( h_0 )</td>
<td>Water head initial steady state value (same as rated value)</td>
</tr>
<tr>
<td>( u_0 )</td>
<td>Water velocity initial steady state value (same as rated value)</td>
</tr>
<tr>
<td>( a )</td>
<td>Wave velocity</td>
</tr>
</tbody>
</table>
2.2 Kundur’s Dynamic Model

This model treats the hydro-turbine as a valve [8]. The details are presented in following sections.

2.2.1 Nonlinear Model

The hydro-turbine penstock dynamic model is derived from three basic equations with certain assumptions:

Velocity of water in the penstock:

\[ u_t = k_u g \sqrt{h_t} \]  \hspace{1cm} (2.1)

Turbine output power:

\[ p = k_p h_t u_t \]  \hspace{1cm} (2.2)

Acceleration of water column:

\[ \frac{du_t}{dt} = \frac{a_g}{l_p} h_p = -\frac{a_g}{l_p} (h_t - h_0) \]  \hspace{1cm} (2.3)

where \( h_0 \) is water head initial steady state value.

In the above equations, the hydraulic resistance is neglected. The penstock is assumed to be rigid and water is incompressible. The turbine is also considered as an orifice valve, for which

| \( \omega \) | Generator frequency (speed) |
| \( f_p, f_w \) | Friction coefficients of penstock and tunnel |
| \( f_r, f_0 \) | Friction coefficients (IEEE model) |
| \( D_t \) | Damping effect of turbine (IEEE model) |
the water velocity varies with the gate opening position and square root of the water head. The turbine output power is proportional to the product of head and water flow.

Normalizing equations (2.1)-(2.3), with respect to rated values,

\[
\frac{u_t}{u_r} = \frac{g}{g_r} \left( \frac{h_t}{h_r} \right)^{0.5} = \overline{u_t} = \left[ \frac{h_t}{h_r} \right]^{0.5}
\]  

(2.4)

\[
\frac{p}{p_r} = \frac{u_t}{u_r} \frac{h_t}{h_r} = \overline{p} = \overline{u_t} \overline{h_t}
\]  

(2.5)

\[
\frac{d}{dt} \left( \frac{u_t}{u_r} \right) = -\frac{a_g}{l_p} \frac{h_t}{h_r} \left( h_t - h_{t0} \right) = \frac{d\overline{u_t}}{dt} = \frac{1}{t_{wp}} (\overline{h_t} - \overline{h_t})
\]  

(2.6)

where \( t_{wp} = \frac{l_p u_t}{a_g h_r} = \frac{l_p q_r}{a_g A_p h_r} \) as the water starting time constant.

The mechanical power output \( p_m \) of turbine is

\[
p_m = p - p_{nl}
\]  

(2.7)

where \( p_{nl} \) is the power loss of the turbine from friction as \( p_{nl} = u_{nl} h_t \) and \( u_{nl} \) is a fixed value called no-load water velocity. Therefore, the normalized mechanical power is expressed as:

\[
\frac{p_m}{p_r} = \frac{p}{p_r} - \frac{p_{nl}}{p_r} = \overline{p_m} = (\overline{u_t} - \overline{u_{nl}}) \overline{h_t}
\]  

(2.8)

In [8], there are two additional constant coefficients. One is \( a_t \) which converts the actual gate position to the effective gate position. The other one converts the power from the turbine rated power based to that of the generator volt-ampere base as \((\text{Turbine MW rating})/(\text{Generator MVA rating})\).
2.2.2 Linear Model

1. Ideal

Linearizing the water velocity equation (2.1) at an operating point (usually the rated value)

\[ \Delta u_i = \frac{\partial u_i}{\partial h_i} \Delta h_i + \frac{\partial u_i}{\partial g} \Delta g = k_u \frac{g_r}{2\sqrt{h_r}} \Delta h_i + k_u \sqrt{h_r} \Delta g \]  

(2.9)

Normalization (2.9) with respect to the initial steady-state values (the rated value)

\[ u_r = k_u g_r \sqrt{h_r} \]  

yields

\[ \frac{\Delta u_i}{u_r} = \frac{\Delta h_i}{2h_r} + \frac{\Delta g}{g_r} \Delta u_r = \frac{1}{2} \Delta h_i + \Delta g \]  

(2.10)

Similar to the above procedures, the linearization and normalization of turbine mechanical power equation (2.2) without considering no load condition loss \((p = p_m)\) are shown as

\[ \Delta p_m = \frac{\partial p_m}{\partial h_i} \Delta h_i + \frac{\partial p_m}{\partial u_i} \Delta u_i = k_p u_r \Delta h_i + k_p u_r \Delta u_i \]  

(2.11)

Dividing (2.11) by \(p_r = k_p h_i u_r\),

\[ \frac{\Delta p_m}{p_r} = \frac{\Delta h_i}{h_r} + \frac{\Delta u_i}{u_r} = \frac{\Delta p_m}{p_r} = \Delta h_i + \Delta u_i \]  

(2.12)

Linearizing (2.3)

\[ \left(\rho A_p \right) \frac{d\Delta u_i}{dt} = -A_p \left(\rho_A \right) \Delta h_i \]  

(2.13)

\(\rho A_p\) is the mass of water in the conduit (penstock) and \(\rho_A \Delta h_i\) is the incremental change in pressure at turbine gate.
Dividing both sides of (2.13) by \( A_p x_g h_r u_r \), the acceleration equation (2.13) in normalized form can be expressed as:

\[
\frac{l_r u_r}{a_g h_r} \frac{d}{dt} \left( \frac{\Delta u_r}{u_r} \right) = -\frac{\Delta h_r}{h_r} = t_{wp} \frac{d\Delta u_r}{dt} = -\Delta h_r
\]

(2.14)

where \( t_{wp} \) is the same as nonlinear model water starting time constant.

Combining (2.10, 12, 14), the transfer function between turbine power and gate position can be expressed as:

\[
\frac{\Delta \bar{p}_m}{\Delta \bar{g}} = \frac{1-t_{wp} s}{1+\frac{1}{2} t_{wp} s}
\]

(2.15)

Therefore, (2.15) is the ideal classical transfer function of a hydraulic turbine between power output and main gate movement.

2. Non-ideal

Unlike direct linearization, the non-ideal turbine's transfer function can be expressed as:

\[
\Delta \bar{u}_r = a_{11} \Delta \bar{h}_r + a_{12} \Delta \bar{\omega} + a_{13} \Delta \bar{g}
\]

(2.16)

\[
\Delta \bar{p}_m = a_{21} \Delta \bar{h}_r + a_{22} \Delta \bar{\omega} + a_{23} \Delta \bar{g}
\]

(2.17)

where \( a_{ij} \) is the partial derivative of flow or power with respect to head, speed deviation and gate opening. \( \Delta \bar{\omega} \) is the per unit speed deviation. Similarly, for Kaplan Turbine with adjustable blade angle, there will be additional terms for change of blade angle [37]. Usually, the speed deviation in p.u. is very small, especially connected to a large system. Equation (2.16) and (2.17) can be reduced as:

\[
\Delta \bar{u}_r = a_{11} \Delta \bar{h}_r + a_{13} \Delta \bar{g}
\]

(2.18)
\[ \Delta \tilde{\rho}_m = a_{21} \Delta \tilde{h}_r + a_{23} \Delta \tilde{g} \]  
\hspace{1cm} \text{(2.19)}

Combining (2.18, 19), the transfer function between turbine power and gate position is obtained as:

\[ \frac{\Delta \tilde{\rho}_m}{\Delta \tilde{g}} = a_{23} \frac{1 + (a_{11} - a_{13} a_{21}/a_{23}) f_{wp} s}{1 + a_{11} f_{wp} s} \]  
\hspace{1cm} \text{(2.20)}

Therefore, the ideal turbine model is a special case where \( a_{11} = 0.5, a_{13} = 1, a_{21} = 1.5, \) \( a_{23} = 1. \)

### 2.2.3 Other Effects:

In general, there are two types of possibly important effects in the turbine-penstock dynamic model. One is the effect from elastic pipe and water compression. The other one is the surge tank effect for some plant having long penstock.

#### 1. Elastic Conduit Effect

When the servo gate suddenly decreases it opening, there will be upstream water flow caused by presusre wave. This water flow will expand a part of the conduit wall. There are two basic equations (compressible fluid in a uniform elastic pipe) dealing with the elastic column and water compressibility.

Netwon's second law:

\[ \frac{\partial u}{\partial t} = -a_g \frac{\partial h}{\partial \xi} \]  
\hspace{1cm} \text{(2.21)}

Continuity equation:
\[
\frac{\partial u}{\partial x} = -\alpha \frac{\partial h}{\partial t}
\]

(2.22)

where \( \alpha = \rho \alpha g \left( \frac{1}{k} + \frac{d_p}{Ef} \right) \), and \( x \) indicated the distance between two points along the conduit.

They are partial differential equations and can be solved by using waving equation and characteristic method in reference [48]. Since the control aspect is the goal here, the equations are solved by using Laplace transformation technique. After normalization with respect to the rated head \( (h_r) \) and rated flow \( (q_r) \), the solution are:

\[
\tilde{h}_2 = \tilde{h}_1 \text{sech}(t_{ep} s) - z_p \tilde{q}_2 \tanh(t_{ep} s)
\]

(2.23)

\[
\tilde{q}_1 = \tilde{q}_2 \cosh(t_{ep} s) + \frac{1}{z_p} \tilde{h}_2 \sinh(t_{ep} s)
\]

(2.24)

where \( t_{ep} \) is the elastic time constant of a penstock defined as \( \frac{t_p}{\sqrt{a_g \alpha}} \), \( z_p \) is the normalized value of hydraulic surge impedance of the conduit which is calculated as \( z_p = \frac{1}{A_p \sqrt{a_g \alpha} h_r} \) for penstock and \( t_{wp} = z_p t_{ep} \). Subscripts 1 and 2 indicate the value at upstream and downstream as shown at Figure 2-2. More detail about the solving procedures are shown in [10].

Figure 2-2: Illustration of Upstream and Downstream along Penstock
If we assume that the conduit’s cross section area is constant, the normalized flow equals the normalized water velocity \((\bar{q} = \bar{u})\).

Then, equations (2.23) and (2.24) reduce to

\[
\bar{h}_2 = \bar{h}_1 \text{sech}(\ell_{ep, s}) - z_p \bar{u}_2 \tanh(\ell_{ep, s}) \tag{2.25}
\]

\[
\bar{u}_1 = \bar{u}_2 \cosh(\ell_{ep, s}) + \frac{1}{z_p} \bar{h}_2 \sinh(\ell_{ep, s}) \tag{2.26}
\]

To include conduit friction, \(-\phi \bar{u}_2\) needs to be added to (2.25) right hand side, where \(\phi = 2k_j |\mu_{e0}|\).

Taking the deviations from steady state values of head and water velocity, equations (2.25) and (2.26) can be expressed as [8]:

\[
\Delta \bar{h}_2 = \Delta \bar{h}_1 \text{sech}(\ell_{ep, s}) - z_p \Delta \bar{u}_2 \tanh(\ell_{ep, s}) \tag{2.27}
\]

\[
\Delta \bar{u}_1 = \Delta \bar{u}_2 \cosh(\ell_{ep, s}) + \frac{1}{z_p} \Delta \bar{h}_2 \sinh(\ell_{ep, s}) \tag{2.28}
\]

where \(\Delta \bar{h}_1 = \bar{h}_i - \bar{h}_0\) and \(\Delta \bar{u}_1 = \bar{u}_i - \bar{u}_0\).

Usually, the plant has a constant upstream water head (reservoir water level), therefore, \(\Delta \bar{h}_1\) will be 0. For the system without a surge tank, the water reservoir is upstream and the hydro-turbine is downstream. Then, (2.27) will become \(\Delta \bar{h}_1 = -z_p \Delta \bar{u}_i \tanh(\ell_{ep, s})\), since it is very difficult to deal with hyperbolic function, the following lumped-parameter approximation is used to solve this task.

\[
\tanh(\ell_{ep, s}) = \frac{1 - e^{-2\ell_{ep, s}}}{1 + e^{-2\ell_{ep, s}}} \approx 1 + \frac{2st_{ep}}{(2n-1)\pi} \left[ 1 + \left( \frac{2st_{ep}}{(2n-1)\pi} \right)^2 \right] \tag{2.29}
\]
Usually, $n$ is set to be 1, 2 or 3. Since $e^{-t_\text{ep}}$ can be expressed in Matlab Simulink as a time delay, hyperbolic function can also be expressed accurately.

Combining with (2.15 and 2.27), we can have transfer function representing a ideal turbine-penstock model with elastic pipe and water compression effects.

\[
\frac{\Delta p_m}{\Delta g} = \frac{1 - z_p \tanh(t_{ep} s)}{1 + \frac{1}{2} z_p \tanh(t_{ep} s)}
\]  

(2.30)

Similar to nonlinear model, there is also a conversion factor $a_e$ which transfers real gate opening to effective gate opening and another converting factor for transferring turbine power based value to generator power based value as $(\text{Turbine MW rating})/(\text{Generator MVA rating})$.

2. **Surge Tank Effect**

One or multiple surge tanks are usually added to the long penstock in order to reduce the water hammer effect. The surge tank converts the kinetic energy to potential energy and it acts like a system connector. It connects two separated upstream and downstream systems. Figure 2-3 shows a single surge tank system. The first part is reservoir-short tunnel-surge tank. The second part is surge tank-penstock-turbine. The connection between them is the surge tank with riser as shown in Figure 2-3.
First system (reservoir-short tunnel-surge tank)

Based on equation (2.27) and component of hydropower plant as shown in Figure 2-3, the dynamic in first system can be expressed as:

\[
\Delta h_{\text{riser}} = \Delta h_w \ \text{sech}(t_{etu}s) - z_{tu} \Delta u_{tu} \ \text{tanh}(t_{etu}s)
\]

where \( \Delta h_{\text{riser}} \) is the riser water head change, \( \Delta h_w \) is the water head in the reservoir, \( \Delta u_{tu} \) is the water velocity change in the tunnel and \( t_{etu} \) is elastic time constant of the tunnel, \( z_{tu} \) is the hydraulic surge impedance of tunnel.

Surge tank

Based on the continuity equation, the flow rates and velocities at the connection part [8] (surge tank with riser) can be expressed as:

\[
\Delta \bar{u}_{tu} = (\Delta \bar{u}_s + \Delta \bar{u}_{\text{riser}}) + \Delta \bar{u}_p
\]

where \( \Delta \bar{u}_s \) is the surge tank water velocity change and \( \Delta \bar{u}_p \) is the water velocity change of penstock.

The relationship between head in the riser and water velocity can be expressed as [49]:

Figure 2-3: Hydropower Plant with a Single Surge Tank and Riser Scheme [8].


\[
(\Delta \bar{u}_s + \Delta \bar{u}_{\text{riser}}) = s t_s \Delta \bar{h}_{\text{riser}}
\]  

(2.33)

where \( t_s \) is the surge tank time constant.

**Second system (surge tank-penstock-turbine)**

Based on equation (2.27) and (2.28) corresponding to hydropower plant’s components as shown in Figure 2-3, the dynamic in second system can be experssed as:

\[
\Delta \bar{h}_t = \Delta \bar{h}_\text{riser} \ \sech(t_{ep} s) - \frac{z_p}{z_p} \Delta \bar{u}_t \ \tanh(t_{ep} s)
\]  

(2.34)

\[
\Delta \bar{u}_p = \Delta \bar{u}_t \ \cosh(t_{ep} s) + \frac{1}{z_p} \Delta \bar{h}_t \ \sinh(t_{ep} s)
\]  

(2.35)

By plug (2.32 and 33) into (2.31), the transfer function between \( \Delta \bar{h}_{\text{riser}} \) and \( \Delta \bar{u}_p \) can be expressed as:

\[
\frac{\Delta \bar{h}_{\text{riser}}}{\Delta \bar{u}_p} = -\frac{z_{tu} \ \tanh(t_{etu} s)}{1 + z_{tu} \ \tanh(t_{etu} s) \ \frac{1}{s \ s}} = TF_1(s)
\]  

(2.36)

Plug (2.36) into the second system relationship (2.34 and 2.35) with \( \tanh(x) = \frac{\sinh(x)}{\cosh(x)} \) and \( \sech(x) = (\cosh(x))^{-1} \), the transfer function between \( \Delta \bar{u}_t \) and \( \Delta \bar{h}_t \) is as:

\[
\Delta \bar{u}_t = -\frac{1 + TF_1(s)/z_p \ \tanh(t_{ep} s)}{TF_1(s) + z_p \ \tanh(t_{ep} s)} \Delta \bar{h}_t = TF(s) \Delta \bar{h}_t
\]  

(2.37)

Therefore, (2.37) is the transfer function between water velocity and head at turbine with considering elastic conduit wall, water compressibility and single surge tank.

**2.3 Comparison with IEEE Model**

The IEEE models [20] are fundamentally the same as the models presented in Kundur's book with several minor differences.
1. In the turbine mechanical power output, IEEE model not only considers the no-load condition, but also involves the damping effects of the turbine as a function of gate opening. Therefore, (2.7) is modified as:

\[
\bar{p}_m = \bar{h}_r (\bar{u}_r - \bar{u}_{nl}) - D_r \bar{h} \Delta \bar{\omega}
\]

(2.38)

2. Not like models presented in the Kundur's book which consider converting real gate position and effective gate position and converting turbine based power to voltage-ampere based for generator power separately, IEEE models put them together as following:

\[
a_r = \frac{\text{MW (rating)}}{\text{MVA (based)}} \frac{1}{\bar{h}_r (q_r - q_{nl})}
\]

(2.39)

3. IEEE models never consider the ideal model for linear models. It directly linearizes the turbine power output equation, which can be expressed as non-ideal model form in Kundur's book if not including turbine damping effect.

\[
\frac{\Delta \bar{p}_m}{\Delta \bar{g}} = \frac{1 - (\bar{u}_0 - \bar{u}_m) y_{wp} s}{1 + \frac{1}{2} t_{wp} s} = a_{23} \frac{1 + (a_{11} - a_{13} a_{21}/a_{23}) y_{wp} s}{1 + a_{11} t_{wp} s}
\]

(2.40)

where \( a_{11} = \frac{1}{2}, \ a_{13} = 1, \ a_{21} = \frac{1}{2} + \bar{u}_0 - \bar{u}_{nl}, \ a_{23} = 1 \).

4. Another difference is the expression of friction as \( f_r = f_p \bar{u}_r^2 \) compared with \( f_r = -\phi \bar{u}_r \).

5. The most differences between these two models are in the elastic pipe, water compressibility and surge tank effect modeling.

For IEEE model, the hydro-turbine penstock dynamic model for system with elastic pipe and single surge tank can be expressed as:

Dynamics of the tunnel and surge tank:

\[
\Delta \bar{u}_{tu} = \bar{h}_0 - f_p \bar{u}_{tu} \bar{u}_{tu} |\bar{u}_{tu}| - \varepsilon_{tu} \tanh (t_{euu} s) \bar{\eta}_{tu}
\]

(2.41)

Dynamics of surge tank:
\[
\bar{h}_{\text{riser}} = \frac{1}{\bar{t}_s} \bar{u}_s - f_0 \bar{u}_s \left| \bar{u}_s \right|
\]

(2.42)

\[
\bar{u}_s = \bar{u}_{iu} - \bar{u}_s
\]

(2.43)

Dynamics of penstock and hydro-turbine:

\[
\bar{h}_t = \bar{h}_{\text{riser}} - f_p \bar{u}_t^2 - z_p \tanh(t_{ep} s) \bar{u}_{iu}
\]

(2.44)

Eq(2.42 and 43) are the same as (2.32 and 33) in Kundur's model. But (2.41) and (2.44) do not consider the effect of \( \text{sech}(t,s) \) at the first term of right hand side like Kundur's model.

### 2.4 Turbine-penstock dynamic simulation

This section will show the simulations of mechanical power change due to the gate opening change for different types of models. The linear and nonlinear model results were compared.

#### Table 2-3: Parameter Used in the Following Simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{ep} )</td>
<td>0.42</td>
<td>Second</td>
</tr>
<tr>
<td>( t_s )</td>
<td>138.22</td>
<td>Second</td>
</tr>
<tr>
<td>( t_{wp} )</td>
<td>5.78</td>
<td>Second</td>
</tr>
<tr>
<td>( t_{wtu} )</td>
<td>1.77</td>
<td>Second</td>
</tr>
<tr>
<td>( z_p )</td>
<td>4.187</td>
<td>Unit less</td>
</tr>
<tr>
<td>( f_p )</td>
<td>0.046</td>
<td>P.U.</td>
</tr>
<tr>
<td>( f_{tu} )</td>
<td>0.0138</td>
<td>P.U.</td>
</tr>
<tr>
<td>( \bar{u}_{nl} )</td>
<td>0.0538</td>
<td>P.U.</td>
</tr>
<tr>
<td>( a_t )</td>
<td>1</td>
<td>P.U.</td>
</tr>
</tbody>
</table>
Different types of linear model simulations based on Kundur’s book are plotted in Figure 2-4. The simulations are conducted with the gate opening increases 0.1 p.u. It shows the slow oscillation due to the surge tank (color magenta and black). The zoom in part clearly shows the differences between the ideal linear model and ideal linear model with friction. It also shows the oscillation associated with the elastic pipe and water compression effects.

Figure 2-4: Mechanical Power Change due to Gate Opening Change based on Linear Models.
Figure 2-5 shows simulation results of mechanical power change with different types of nonlinear models due to gate opening increased 0.1 p.u. The zoom-in part in the left shows the effects from elastic pipe and water compression. The slow oscillation shows the surge tank effect. The simulation results are very similar to linear models.
Mechanical power change responses due to different gate opening changes (0.01 and 0.1 p.u.) are plotted in Figure 2-6 for linear and nonlinear model with elastic pipe effect and water compression ($n = 1$) effects. Clearly, as the step size increases, linear and nonlinear models have different responses. Therefore, a nonlinear controller is very important for improving plant operation ranges.

2.5 Hydropower Plant Other Components’ Dynamic Model

Other important components of hydropower plant are wicket gate servo system and generator. Both of them can be expressed in very simple first order transfer functions. The generator model is presented in the mechanical side of view based on the swing equation.
2.5.1 Gate Servo System

Since the hydropower plant requires high force to move the wicket gate, the whole system usually contain two cascaded systems [50]. One is pilot actuator/servo system and the other one is the main gate servo system. The relationship between the input signal \( u \) (volt.) and pilot actuator movement change \( \Delta \bar{x}_e \) is:

\[
\frac{\Delta \bar{x}_e}{u} = \frac{1}{t_p s + 1}
\]

(2.45)

The relationship between the pilot actuator output \( \Delta \bar{x}_e \) and gate servo (gate) change position \( \Delta \bar{g} \) is:

\[
\frac{\Delta \bar{g}}{\Delta \bar{x}_e} = \frac{1}{t_e s + 1}
\]

(2.46)

where \( t_p \) and \( t_e \) are the time constant for the pilot actuator and main gate servo.

2.5.2 Generator [51]

Based on Newton's second law, the equation used to represent the relationship among generator's rotor's angle, angular acceleration and the torque of the shaft of generator set is as [51]:

\[
J \alpha = T_m - T_e - T_{damping}
\]

(2.47)

where \( J \) is the moment of inertia of the rotor of the generator set; \( \alpha \) is the angular acceleration of the rotor; \( T_m \) is the mechanical torque on the shaft; \( T_e \) is the electromagnetic torque of the generator (external load) and \( T_{damping} \) is the damping torque which is proportional to the variations of angular speed.
After normalization, (2.47) can be written as:

\[
\frac{2h}{2\pi f_{\text{base}}} \frac{d^2 \delta}{dt^2} = \bar{T}_m - \bar{T}_e - \bar{T}_{\text{damping}}
\]

(2.48)

where \( h \) and \( t \) are in seconds, \( h \) is the generator inertia and \( \bar{T}_m, \bar{T}_e \) and \( \bar{T}_{\text{damping}} \) are in per unit of value. For engineering application, power values usually are used instead of torque values using: \( T = \frac{p}{\omega} \). The base value \( P_0 \) of per unit-power is the power rating with unit of KVA. Thus, (2.48) is transformed as:

\[
\frac{2h}{2\pi f_{\text{base}}} \frac{d^2 \delta}{dt^2} = \bar{p}_m - \bar{p}_e - \bar{p}_{\text{damping}}
\]

(2.49)

where damping power is expressed as \( p_{\text{damping}} = \frac{d_d}{2\pi f_{\text{base}}} \frac{d\delta}{dt} \) and \( d_d \) is the damping coefficient and is constant and radian as its per unit value.

But in our case, the angular speed also is expressed in per-unit value as:

\[
\bar{\omega} = \frac{\omega}{\omega_{\text{base}}} \Rightarrow \frac{d\bar{\omega}}{dt} = \frac{1}{\omega_{\text{base}}} \frac{d\omega}{dt} = \frac{1}{\omega_{\text{base}}} \frac{d^2 \delta}{dt^2}
\]

(2.50)

where \( \omega_{\text{base}} = 2\pi f_{\text{base}} \)

So (2.49) can be written in per-unit value as:

\[
2h \bar{\omega} = \bar{p}_m - \bar{p}_e - d_d \bar{\omega} = m \frac{d\bar{\omega}}{dt}
\]

(2.51)

where all the values are in per-unit value and usually \( 2h \) is replaced by \( m \)

For now, \( \bar{p}_e \) is treated as a constant, but in the future it will be expressed as:

\[
p_e(\delta) = \frac{E_q V_f}{x_d} \sin(\delta) + \left( \frac{1}{x_q} - \frac{1}{x_d} \right) \frac{V_f^2}{2} \sin(2\delta)
\]

(2.52)
where $E_q$ is the transient EMF in the quadrature axis of the generator and is constant. $V_t$ is the amplitude of the generator terminal voltage. $x_d, x_q$ are the stator windings' self inductive reactances of axes $d$ and $q$.

The generator model for hydro-turbine penstock linear model based on [8] is shown as following:

In frequency control studies, the torque is usually expressed as mechanical power $T_m$ and electrical power $T_e$ by using $p = \omega T$ as following:

\[
\begin{align*}
\{ p &= p_0 + \Delta p \\
T &= T_0 + \Delta T \Rightarrow \\
\omega &= \omega_0 + \Delta \omega
\end{align*}
\]

\[
\Delta p + p_0 = (\omega_0 + \Delta \omega)(T_0 + \Delta T)
\]

(2.53.a, b)

\[
\Delta p = \omega_0 \Delta T + T_0 \Delta \omega,
\]

\[
\Delta p_m - \Delta p_i = \omega_0 (\Delta T_m - \Delta T_i) + (T_{m0} - T_{i0}) \Delta \omega.
\]

Since $T_{m0} = T_{i0}$ and $\omega_0 = 1$, $\Delta p_m - \Delta p_i = \Delta T_m - \Delta T_i$.

$\Delta p_i$ is the electrical load change which usually consists of resistive loads change and frequency related load change. $\Delta p_i = \Delta p_e + d_\Delta \omega$. Therefore, the block diagram of generator model is present as:

The linear generator model:

\[
\Delta p_m - \Delta p_e = \frac{1}{m} \frac{d \Delta \omega}{dt} + d_\Delta \omega
\]

(2.54)

### 2.6 New Turbine Model with Inner Torque Loss [2]

There have been several hydropower system dynamic studied in the past. But none of them consider the inner loss the turbine. So the turbine is assumed to perform at the highest efficiency in those models. As computer power increasing, CFD has been widely used in turbine design and simulation. But integrating turbine CFD into a dynamic system is very complex. Therefore, if there is a way can interpret the CFD results or measured data (efficiency curve of
the turbine) with simple functions, then it can be easily integrated into the dynamic system to give more realistic simulation results. It can also help with designing control system.

Reference [2] proposed a method which can generate a considerable simple function for turbine torque including inner loss. Appendix A shows the detail about this method, how to integrate this new method into our existing system and real data testing results.
Chapter 3

Sliding Mode/ $H_\infty$ Control for Hydropower Plant

In this chapter, a new control method for hydropower system: sliding mode control (SMC) blended with robust optimal $H_\infty$ theory, which is inspired by studies conducted in vibration area [52,53]. Sliding model control is well known as its robust and insensitive to external disturbance and system uncertainty. But the matching condition must be satisfied in order for it to provide the best control performance. This new control method has no assumptions on whether the matching conditions are satisfied. The $H_\infty$ control provides the optimal choice of sliding surface and also minimizes the impact of external disturbances. Therefore, this hybrid method can provide the best sliding mode control for nonmatching condition system such as the hydropower system.

3.1 Sliding Mode Control Theory

The fundamental concepts of sliding mode control theory can be found in [52],[54],[55]. Generally, there are two steps for sliding mode control design.

First step is to determine the switching hyperplane which is defined as follows:

$$ s = g x(t) $$

(3.1)

where $x$ is the state vector and $g$ is a vector which needs to be determined. It should be noted that there is only one sliding hyperplane as there is only one control input, $u$ which controls the movement of pilot actuator.

Second step is to determine the control law.
The control input signal $u$ which is the input voltage for the electric-hydraulic servo system is generated to achieve the following condition:

$$ss < 0$$ (3.2)

The condition (3.2) ensures that the system will reach the sliding hyperplane and remain on it.

### 3.2 $H_\infty$ Control Theory

Usually, a state space model with disturbance can be partitioned as shown in Figure 3-1 in order to implement full state feedback $H_\infty$ control.

$$\begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ I & 0 & 0 \end{bmatrix}$$

where $M_{sl}$ as the state transfer matrix and $k$ as the feedback gain for the controller. Therefore,

$$\begin{cases}
\dot{x} = Ax + B_1d + B_2u \\
e = C_1x + D_{12}u \\
y = x
\end{cases}$$ (3.3)

Figure 3-1: Full State Feedback Control System.
Applying $H_{x}$ control method, the minimum value of $\gamma$ can be found for $\|T_{ed}\| < \gamma$ where $T_{ed}$ is the transfer function between disturbance $d$ and effect error $e$. Therefore, it provides the minimization of the effect of the disturbance.

### 3.2.1 Hydropower Plant Model

Based on detailed description of hydropower plant model in Chapter 2, the ideal linear state space model can be generated by combining following equations:

Turbine velocity and output power dynamic equations:

\[
\Delta \bar{u}_t = \frac{1}{2} \Delta \bar{h}_t + \Delta \bar{g} \tag{3.4}
\]

\[
\Delta \bar{p}_m = \Delta \bar{h}_t + \Delta \bar{u}_t \tag{3.5}
\]

Penstock and wicket gate dynamic equations:

\[
t_w \frac{d\Delta \bar{u}_t}{dt} = -\Delta \bar{h}_t \tag{3.6}
\]

Generator dynamic equation:

\[
\Delta \bar{p}_m - \Delta \bar{p}_e = \frac{1}{m} \frac{d\Delta \bar{\omega}}{dt} + d_d \Delta \bar{\omega} \tag{3.7}
\]

Wicket system dynamic equations:

\[
\frac{\Delta \bar{\omega}}{\Delta \bar{v}_e} = \frac{1}{1_p s + 1} \tag{3.8}
\]

\[
\frac{\Delta \bar{g}}{\Delta \bar{v}_e} = \frac{1}{1_s s + 1} \tag{3.9}
\]

The states are:
All states can be easily measured. Therefore, full state feedback sliding mode control will be designed. The state-space model for the system with controller is generated as:

$$\dot{x} = Ax + bu + d\Delta \ddot{p}_e$$

(3.11)

where

$$A = \begin{bmatrix}
-\frac{1}{t_p} & 0 & 0 & -\frac{1}{t_p'} \\
\frac{1}{t_g} & -\frac{1}{t_g} & 0 & 0 \\
-\frac{2}{t_g} & \left(\frac{2}{t_w} + \frac{2}{t_g}\right) & -\frac{2}{t_w} & 0 \\
0 & 0 & \frac{1}{m} & -\frac{d_e}{m}
\end{bmatrix}$$

and

$$b = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}$$

and

$$d = \begin{bmatrix}
m
\end{bmatrix}.$$ 

(3.12.a,b,c)

### 3.2.2 Sliding Mode Design

The matching condition is expressed as

$$\text{rank}([b, d]) = \text{rank}([b])$$

(3.13)
Since vector \( b \) and \( d \) are independent, matching condition is not satisfied as shown in equation (3.13) [56]. \( H_u \) control method is applied with sliding mode control to minimize the nonmatching condition effect. An integral term is also added to the state space model to reduce the steady state error for a constant disturbance. The new states are:

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
\end{bmatrix} = \begin{bmatrix}
\Delta \tilde{\omega} dt \\
\Delta \tilde{x}_x \\
\Delta \tilde{\eta} \\
\Delta \tilde{p}_m \\
\Delta \tilde{\sigma}
\end{bmatrix}
\]

(3.14)

And the new parameters matrix are

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
0 & -1/t_p & 0 & 0 & -1/t_p r_p \\
0 & -1/t_g & 0 & 0 & 0 \\
0 & 2/t_g (2/t_w + 2/t_g) & -2/t_w & 0 & 0 \\
0 & 0 & 0 & 1/m & -d_d/m
\end{bmatrix}
\]

\[
b = \begin{bmatrix}
0 \\
-1/t_p \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
d = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
-1/m
\end{bmatrix}
\]

(3.15,a,b,c)

It will be assumed that \( |\Delta \tilde{p}_e| \leq q_u \), where \( q_u \) is the upper bound of the load disturbance.

The design steps are similar as shown in previous sections.

Step 1. Constructing the sliding surface

The following similarity transformation is applied to the state vector \( x(t) \) as:

\[
q(t) = H x(t)
\]

(3.16)

where \( H = [N, b]^T \) and \( N \) are composed of basis vectors of the null space of \( b^T \). Combining (3.11 and 3.16).

\[
\dot{q} = \dot{\tilde{\omega}} q(t) + \tilde{b} u(t) + \tilde{d} \Delta \tilde{p}_e
\]

(3.17)

where \( \tilde{A} = HAH^{-1} \), \( \tilde{b} = Hb \) and \( \tilde{d} = Hd \).
Because of the definition of $H$ matrix, (3.16) can be written as:

$$\dot{q} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{b}_r \end{bmatrix} u + \begin{bmatrix} d_1 \\ \ddot{d}_2 \end{bmatrix} \Delta \bar{p}_e$$

(3.18)

where $q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$

It should be noted that $q_1$ is a 4 dimensional vector where $q_2$ is a scalar. From (3.18),

$$\dot{q}_1 = \bar{A}_{11}q_1 + \bar{A}_{12}q_2 + d_1 \Delta \bar{p}_e$$

(3.19)

Here, $q_1$ and $q_2$ are viewed as states and input for the construction of sliding hyperplane via full state feedback

$$q_2 = -k q_1$$

(3.20)

where $k$ is the state feedback gain vector. In this case, sliding hyperplane becomes

$$s_i = [k \ 1] q$$

(3.21)

Using (3.16 and 3.21), sliding hyperplane vector $g$ is defined as:

$$g = [k \ 1] H$$

(3.22)

Because the matching condition is not satisfied, $d_1 \neq 0$ and disturbance will affect the system response on the sliding hyperplane [56]. The objective is to select the state feedback gain $k$ or equivalently the sliding hyperplane vector $g$ via $H_\infty$ technique such that the effect of disturbance is minimized. For this purpose, the block diagram, Figure 3-3, is developed, where the matrix $M_{sl}$ is based on (3.23):
where \( \gamma > 0 \).

Utilizing the expression of \( e \) from (3.23 and 24),

\[
J = \frac{1}{2} \int_0^\infty \left( e^T e - \gamma^2 \Delta p_e^T \Delta p_e \right) dt
\]

(3.25)

where \( Q_{11} = C_1^T C_1 \), \( Q_{12} = C_1^T D_{12} \), \( Q_{22} = D_{12}^T D_{12} \).

It should be noted that
\( e^T e = q^T \begin{bmatrix} C_1^T \\ D_{12} \end{bmatrix} \begin{bmatrix} C_1 \\ D_{12} \end{bmatrix} x = x \bar{Q} x \) \hspace{1cm} (3.26)

where \( \bar{Q} = H^T \begin{bmatrix} C_1^T \\ D_{12} \end{bmatrix} \begin{bmatrix} C_1 \\ D_{12} \end{bmatrix} H \)

The gain matrix \( k \) for the minimum value of \( J \) is \([52]\)

\[ k = Q_{22}^{-1} \left( \bar{A}_{12} P + Q_{12}^T \right) \] \hspace{1cm} (3.27)

where \( P \) is the unique, symmetric, positive semi definite solution of the following ARE (Algebraic Riccati Equation):

\[ P(\bar{A}_{11} - \bar{A}_{12} Q_{22}^{-1} \bar{A}_{12}^T) + (\bar{A}_{12}^T - Q_{12} Q_{22}^{-1} \bar{A}_{12}^T) P - P(\bar{A}_{12} Q_{22}^{-1} \bar{A}_{12} - \gamma^{-2} d_1 d_1^T) P + (Q_{11} - Q_{12} Q_{22}^{-1} Q_{12}^T) = 0 \] \hspace{1cm} (3.28)

MATLAB Robust Control Toolbox is used to solve (3.28).

Step 2. Determine the control law to satisfy the reaching condition \( s_t \dot{s}_t < 0 \).

From (3.1),

\[ \dot{s}_t = g\dot{x} = g(\lambda x + bu + d\Delta \bar{p}_e) \] \hspace{1cm} (3.29)

Since \( s_t \) is a scalar,

\[ s_t \dot{s}_t = s_t (g \lambda x + gb u + gd \Delta \bar{p}_e) \] \hspace{1cm} (3.30)

The input signal \( u \) can be partitioned into two parts.

\[ u = u_{eq} + u_{un} \] \hspace{1cm} (3.31)

where \( g \lambda x + gb u_{eq} = 0 \)

Therefore,

\[ u_{eq} = -(gb)^{-1} (g \lambda x) \] \hspace{1cm} (3.32)

And, \( u_{un} \) is used to make sure that the condition (3.2) is satisfied; i.e.,

\[ s_t \dot{s}_t = s_t (gb u_{un} + gd \Delta \bar{p}_e) < 0 \] \hspace{1cm} (3.33)
Or

\[ u_{un} = -(gb)^{-1}e_u \text{ sgn}(s_i) \]  

(3.34)

where \( e_u = |gd\| \eta + \eta \) and \( \eta > 0 \). Since the initial condition is zero for our system, the size of \( \eta \) does not effect the performance of the controller.

In order to eliminate the chattering effect, \( \text{sgn}(s_i) \) is replaced by a saturation function [52].

![Diagram](figure3-4.png)

Figure 3-4: Boundary Layer of Sliding Hyperplane for Reducing Chattering [52].

\[
\text{sat}(s_i) = \begin{cases} 
\text{sgn}(s_i) & \text{if } |s_i| > \rho \\
\|s_i\|/\rho & \text{otherwise}
\end{cases}
\]  

(3.35)

### 3.3 Simulation Results

In the simulation, model parameters are shown in Table 3-1. The controller is designed with \( q_u = 0.5 \), \( \eta = 0.05 \), and \( \rho = 0.05 \). Simulation is conducted with a step load change
\( \Delta P_e = 0.03 \text{ ( pu) } \) and weight matrix used in (3.25) as \( \overline{Q} = \begin{bmatrix} 250 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 250 \end{bmatrix} \) to minimize deviations in generator speed and change in the power output.

Table 3-1: Parameter Used in the Simulation at Chapter 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_w )</td>
<td>1.3</td>
<td>Second</td>
</tr>
<tr>
<td>( t_g )</td>
<td>0.2</td>
<td>Second</td>
</tr>
<tr>
<td>( t_p )</td>
<td>0.02</td>
<td>Second</td>
</tr>
<tr>
<td>( r_p )</td>
<td>0.541</td>
<td>Unit less</td>
</tr>
<tr>
<td>( m )</td>
<td>6</td>
<td>Second</td>
</tr>
<tr>
<td>( d_d )</td>
<td>1</td>
<td>Second</td>
</tr>
</tbody>
</table>

In Figure 3-5, \( \Delta \omega \) is plotted for three types of controllers: PI, \( H_\infty \) /sliding mode control and LQR control (\( r=1 \)) for which the objective function is defined as:

\[
J = \frac{1}{2} \int_0^\infty (x^T \overline{Q} x + ru^2) dt
\]  (3.36)

The PI controller:

\[
u = k_i \int_0^\infty \Delta \omega dt + k_{pc} \Delta \omega
\]  (3.37)

The proportional gain \( k_{pc} \) and integral gain \( k_i \) values were initially selected based on method recommended in [31] and [15]. Then, a further trial and error tuning was performed to achieve a shortest settling time response. The detail of tuning PI controller coefficients are shown in Appendix B.

It is clear that the sliding mode blended with \( H_\infty \) method gives the shortest responding time and lowest overshoot value as shown in Figure 3-5. When the system is controlled by \( H_\infty \) /sliding mode method, the response of the system is quite insensitive to parametric unceratinties. Figure 3-6 shows that system response under \( H_\infty \) /sliding mode for -20% deviation in turbine
model parameter $t_w$ and -10% difference on all other parameters ($t_g$, $t_p$, $m$ and $d_d$). Note that the robustness of the traditional PI controller (3.38) is significantly less as shown in Figure 3-7.

Figure 3-5: Frequency Deviation $\Delta \omega$ versus Time under Constant Load Change

Figure 3-6: Frequency Deviation $\Delta \omega$ with Parametric Uncertainties under $H_\infty$/Sliding Mode Control
A sliding mode controller blended with $H_{\infty}$ full state feedback control has been developed for governing hydropower plant in this chapter based on the linear ideal model. The sliding mode hyperplane is constructed based on the dynamic model of the plant by full state feedback $H_{\infty}$ theory. The saturation function is used to eliminate the chattering effect. The performance of this control method in the presence of disturbance and parametric uncertainties was evaluated in MATLAB/Simulink software. The results from simulation have shown the advantage which this method brings to the system response. The next steps will be applying this method on nonlinear model of hydropower plant by using feedback linearization which is presented in the next chapter.

Figure 3-7: Frequency Deviation $\Delta \bar{\omega}$ with Parametric Uncertainties under PI Control
Chapter 4

Feedback Linearization with Sliding Mode Control for Hydropower Plant

In this chapter, we design the frequency controller by using feedback linearization and sliding mode control. The input-output feedback linearization is applied to deal with the nonlinearity of the system and then sliding mode control is implemented to handle system uncertainties with a robust control performance. Due to the non-matching condition of hydropower plant model, a sliding mode control blended with $H_{\infty}$ method is used for selecting sliding hyperplane as shown in the previous chapter.

4.1 Feedback Linearization Theory

The central ideal of feedback linearization is to algebraically transform the nonlinear system to a linear system without making any approximations. Then applying control algorithm to that linear system, the control input for the actual nonlinear system is generated. More details on feedback linearization can be found in [57] and [58].

\[
\dot{x} = f(x) + g_s(x)u \Rightarrow \dot{z} = Az + Bv
\]

(4.1)

where \( z = T(x) \) and \( u = \alpha + \beta v \)
4.2 Design Detail

4.2.1 Hydropower Plant Model

Based on detailed description of hydropower plant model in Chapter 2, the ideal nonlinear state space model can be generated by following equations:

Turbine velocity and output power dynamic equations:

\[ \ddot{u}_t = \ddot{g} \sqrt{\bar{h}_t} \]  \hspace{1cm} (4.2)

\[ \bar{p}_m = \bar{h}_t (\bar{u}_t - \bar{u}_m) \]  \hspace{1cm} (4.3)

Penstock dynamic equation:

\[ \ddot{u}_t = \frac{1}{t_w s} \ddot{h}_p = \frac{1}{t_w s} (\bar{h}_0 - \bar{h}_t) \]  \hspace{1cm} (4.4)

Generator dynamic equation:

\[ 2h_0 \ddot{\omega} = \bar{p}_m - \bar{p}_e - d_\omega \omega = m \frac{d\ddot{\omega}}{dt} \]  \hspace{1cm} (4.5)

Wicket system dynamic equations:

\[ \frac{\Delta \bar{g}}{u} = \frac{1}{t_p s + 1} \]  \hspace{1cm} (4.6)

\[ \frac{\Delta \ddot{g}}{\Delta \bar{X}_e} = \frac{1}{t_\gamma s + 1} \]  \hspace{1cm} (4.7)

From (4.2), (4.3) and (4.4), the turbine-penstock dynamic can be expressed slightly different as:

\[ s \ddot{u}_t = \frac{1}{t_w} (\bar{h}_0 - \left( \frac{\bar{u}_t}{\ddot{g}} \right)^2) \]  \hspace{1cm} (4.8)

\[ \bar{h}_t = \left( \frac{\bar{u}_t}{\ddot{g}} \right)^2 \]  \hspace{1cm} (4.9)
\[ p_m = \frac{(\bar{\mu} - \mu_{nl})(\bar{\mu})^2}{g} \]  

(4.10)

Combinding (4.5-10), the entire system can be represented by four states: \( x_1 = \Delta \bar{\omega} \), \( x_2 = \bar{\mu}_t \), \( x_3 = \bar{g} \), and \( x_4 = \Delta \bar{\chi} \), where \( \Delta \bar{\omega} = \bar{\omega} - \bar{\omega}_{\text{ref}} \). The entire system is expressed as:

\[
\begin{align*}
\dot{x}_1 &= \frac{1}{m} \frac{x^2}{x^3} \bar{\mu}_{nl} \frac{x^2}{x^3} \frac{d_d}{d_d} \frac{x_1}{m} - \frac{d_d}{m} \bar{\omega}_{\text{ref}} \frac{\bar{p}_e}{m} \\
\dot{x}_2 &= -\frac{1}{t_w} \frac{x_2}{x_3} \bar{h}_0 \\
\dot{x}_3 &= \frac{1}{t_g} x_4 - \frac{1}{t_g} x_3 + \bar{g}_0 \\
\dot{x}_4 &= -\frac{1}{t_p} x_4 + \frac{1}{t_p} u 
\end{align*}
\]

(4.11)

### 4.2.2 Feedback Linearization for Hydropower Plant Model

Since the nonlinear model of system is not in the chain of integral format, it needs to be transferred to new states by using input state feedback linearization. If the disturbance \( \bar{p}_e \) is constant, not time depended and combined with \( x \) terms, the state space of hydro-turbine system can be expressed in the classic standard format:

\[
\dot{x} = f(x) + g_s(x)u
\]

(4.9)

where

\[
f(x) = \begin{bmatrix}
\frac{1}{m} \frac{x^2}{x^3} \bar{\mu}_{nl} \frac{x^2}{x^3} \frac{d_d}{d_d} x_1 - \frac{d_d}{m} \bar{\omega}_{\text{ref}} \frac{\bar{p}_e}{m} \\
-\frac{1}{t_w} \frac{x_2}{x_3} \bar{h}_0 \\
\frac{1}{t_g} x_4 - \frac{1}{t_g} x_3 + \bar{g}_0 \\
-\frac{1}{t_p} x_4 
\end{bmatrix}
\]

and

\[
g_s(x) = \begin{bmatrix}
0 \\
0 \\
0 \\
\frac{1}{t_p}
\end{bmatrix}
\]

(4.10.a,b)
In order to achieve the feedback linearization mentioned above, there must exist a
diffeomorphism \( T(x) \) such that the new states can be obtained as:

\[
\begin{align*}
\begin{bmatrix} 
z_{d1} \\
z_{d2} \\
z_{d3} \\
z_{d4} \\
\end{bmatrix} &= \begin{bmatrix} 
T_1(x) \\
T_2(x) \\
T_3(x) \\
T_4(x) \\
\end{bmatrix} = T(x) \\
\end{align*}
\] (4.11)

Under the equivalent linearizable condition \( \frac{\partial T_4}{\partial x} g_s \neq 0 \), the following linear state equations are
obtained:

\[
\begin{align*}
\dot{z}_{d1} &= z_{d2} \\
\dot{z}_{d2} &= z_{d3} \\
\dot{z}_{d3} &= z_{d4} \\
\dot{z}_{d4} &= v + \frac{d_d^3 \times \overline{p}_e}{m_w}
\end{align*}
\] (4.12)

where

\[
v = (-\frac{\alpha}{\beta} + \frac{1}{\beta} u), \quad \beta = \frac{1}{\frac{\partial T_4}{\partial x} g_s(x)} \quad \text{and} \quad \alpha = -[\frac{\partial T_4}{\partial x} f(x)]/\beta
\] (4.13.a,b,c)

States \( z_{d2}, z_{d3} \) and \( z_{d4} \) are not known because of unknown constant disturbance \( \overline{p}_e \). Therefore,
they are split into known parts \( z_2, z_3 \) and \( z_4 \) and unknown parts \( d_2, d_3 \) and \( d_4 \).

\[
\begin{align*}
z_{d1} &= z_1 + d_1 \\
z_{d2} &= z_2 + d_2 \\
z_{d3} &= z_3 + d_3 \\
z_{d4} &= z_4 + d_4
\end{align*}
\] (4.14.a,b,c,d)

where

\[
\begin{align*}
z_1 &= \frac{m}{t_w} x_1 - \overline{u}_n l x_2 + \frac{1}{2} x_2^2 \\
z_2 &= \frac{\partial T_1}{\partial x} f = \frac{\overline{h}_o x_2}{t_w} - \frac{d_d x_1}{t_w} - \frac{h_o \overline{p}_n l}{t_w} - \frac{d_d \overline{\omega}_r}{t_w} \\
z_3 &= \frac{\overline{h}_o^2}{t_w^2} x_2 + \frac{\overline{h}_o x_2^2}{t_w^3} + \frac{d_d^2 \overline{\omega}_r}{m_w t_w} + \frac{d_d^2 x_1}{m_w t_w} - \frac{d_d^2 x_2}{m_w t_w^2} + \frac{d_d^2 \overline{p}_e}{m_w t_w^3}
\end{align*}
\] (4.15.a,b,c,d)
\[ z_{d4} = \frac{\partial z_{d3}}{\partial x} f(x) \]

The details about how to find \( z_1 \) are shown in Appendix C.

\[ d_1 = 0, \quad d_2 = -\frac{\vec{p}_e}{t_w}, \quad d_3 = \frac{d_4^2 \vec{p}_e}{m_t w}, \quad d_4 = -\frac{d_5^2 \vec{p}_e}{m^2_t w}, \quad d_5 = \frac{d_6^3 \vec{p}_e}{m_t w} \]  

(4.16.a, b, c, d, e)

And the feedback linearizable condition is satisfied:

\[ \frac{\partial T_4}{\partial x} g_s = -\frac{2 \chi_5^2 (m\bar{\eta}_0 - d_{10} t_w + d_d t_w x_2)}{m_t t_w x_3^3} \neq 0 \]

Therefore, the state-space model is expressed as:

\[ \begin{align*}
\dot{x}_1 &= z_2 + d_2 \\
\dot{x}_2 &= z_3 + d_3 \\
\dot{x}_3 &= z_4 + d_4 \\
\dot{x}_4 &= v + d_5
\end{align*} \]  

(4.17)

### 4.2.3 Sliding Mode Control

Similar to Chapter 3, there are two steps for sliding mode control design [52, 54] as follows.

First, determine the sliding hyperplane. The sliding surface of the system is defined as

\[ s_f = g_z = 0. \]  

(4.18)

\( g \) is the sliding hyperplane vector which needs to be constructed and \( z \) is the state vector after feedback linearization.

Second, determine the control law to achieve the reaching condition

\[ s_f \dot{s}_f < 0 \]  

(4.19)

Then, new state space equations (4.17) for hydropower plant can be also written as:

\[ \dot{x} = Ax + bv + c\vec{p}_e \]  

(4.20)

where
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad c = \begin{bmatrix} -\frac{1}{t_w} & \frac{d_d}{m t_w} & -\frac{d_d^2}{m^2 t_w} & \frac{d_d^3}{m^3 t_w} \end{bmatrix}^T$$

(4.21)

Clearly, \( b \) and \( c \) are independent and the matching condition [56] is not satisfied. There will be influence from the external disturbance after the system reaches the sliding hyperplane. The \( g \) in (4.18) can be constructed based on the \( H_\infty \) method introduced in Chapter 3 in order to reduce the effect to a minimum. Let the sliding hyperplane be

$$g = [g_1 \quad g_2 \quad g_3 \quad g_4]$$

(4.22)

In order to make sure \( s_l \hat{s}_l < 0 \), the control input \( u \) is obtained as follows:

$$u = -\frac{\beta}{g_4} [v_{eq} + k \text{sgn}(s_l)]$$

(4.23)

where \( k \geq D_2 + D_3 + D_4 + D_5 + \eta \) and \( D_2, D_3, D_4 \) and \( D_5 \) are the upper bounds of \( |g_1 d_2|, |g_2 d_3|, |g_3 d_4| \) and \( |g_4 d_5| \), respectively. \( \eta \) is a constant small positive value which controls the reaching speed to the sliding surface.

In order to obtain a better steady state error result, an integral term is added to the sliding model control design as shown above. The new sliding surface is defined as:

$$s_I = g_{11} \int_0^t z_1 dt + g_{12} z_1 + g_{13} z_2 + g_{14} z_3 + g_{15} z_4$$

(4.25)

where \( g_{11}, g_{12}, g_{13}, g_{14}, g_{15} \) is also constructed by using \( H_\infty \) technique as described previously.

In order to make sure \( s_I \hat{s}_I < 0 \), the control input \( u \) is obtained as follows:

$$u = -\frac{\beta}{g_{15}} [-v_{eql} + k_I \text{sgn}(s_I)]$$

(4.26)

where \( k_I \) is a constant small positive value which controls the reaching speed to the sliding surface.

$$v_{eql} = -\beta z_1 + g_{12} z_2 + g_{13} z_3 + g_{14} z_4 + g_{15} z_5$$

(4.27)
where $k_1 \geq D_{I2} + D_{I3} + D_{I4} + D_{I5} + \eta$ and $D_{I2}$, $D_{I3}$, $D_{I4}$ and $D_{I5}$ are the upper bounds of $|g_{I2}d_2|$, $|g_{I2}d_3|$, $|g_{I3}d_4|$ and $|g_{I4}d_5|$, respectively. $\eta$ is a constant small positive value which controls the reaching speed to the sliding surface.

4.3 Simulation Results

Table 4-1: Parameters Used in Chapter 4 Simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_p$</td>
<td>0.02</td>
<td>Second</td>
</tr>
<tr>
<td>$t_g$</td>
<td>0.2</td>
<td>Second</td>
</tr>
<tr>
<td>$t_w$</td>
<td>1.3</td>
<td>Second</td>
</tr>
<tr>
<td>$m$</td>
<td>6</td>
<td>Second</td>
</tr>
<tr>
<td>$d_d$</td>
<td>1</td>
<td>Second</td>
</tr>
<tr>
<td>$\bar{p}_e$</td>
<td>0.05</td>
<td>P.U.</td>
</tr>
<tr>
<td>$u_{nl}$</td>
<td>0.068</td>
<td>P.U.</td>
</tr>
<tr>
<td>$h_0$</td>
<td>1</td>
<td>P.U.</td>
</tr>
<tr>
<td>$\omega_{ref}$</td>
<td>0.2</td>
<td>P.U.</td>
</tr>
</tbody>
</table>

Three different control methods have been applied to the nonlinear hydropower system model. They are nonlinear controllers with and without integral feedback, Eq. (4.23) and (4.26), and the linear PI controller described as follows:

$$u = k_i \int x_1 dt + k_{pc} x_1$$  \hspace{1cm} (4.28)

The proportional gain $k_{pc}$ and integral gain $k_i$ values are the same as Chapter 3.

Results are shown in Figure 4-1 for comparison purpose. Clearly, two nonlinear controllers which are discussed in the previous section have the fast response time to a steady state value compared to the linear PI controller. However, none of them brings the frequency change back to zero which is the goal of load frequency control. The steady state error ($x_1 = \Delta \tilde{\omega}$) from nonlinear
controller is significantly reduced by having integral action. But, it does not go to zero because integral action only guarantees that $z_1$ goes to zero, Figure 4-2. Note that $z_1$ and $x_1$ are related by Eq. (4.15.a), and therefore, $z_1 = 0$ does not imply $x_1 = 0$ because of the presence of $x_2$.

Figure 4-1: Responses Comparison under Three Control Methods.

Figure 4-2: Nonlinear Controllers with and without Integral Feedback.
For the above reason, in order to achieve the load frequency control goal, a linear PI controller (4.28) is added after the system reaches the steady state. Figure 4-3 shows the simulation result. However, the performance of nonlinear controller with linear PI controller is not promising. It takes similar amount of time as single linear PI controller in Figure 4-1. In order to keep good performance of the first nonlinear controller, a second nonlinear controller is developed. The actual disturbance $\bar{p}_e$ value can be estimated based on the steady state values of $x_1, x_2, and x_3$. The first state equation in (4.9) leads to the following equation for disturbance estimation in steady state:

$$\bar{p}_e = \frac{x_3^2}{x_3^3} - \bar{u}_d \frac{x_3^2}{x_3^3} - d_d x_1 - d_d \bar{\omega}_{ref}$$  \hspace{1cm} (4.29)

Since the disturbance $\bar{p}_e$ is known via Eq. (4.29), all the final desired values of $x$ can be calculated from Eq. (4.9) with $x = 0$ and $x_1 = 0$. These desired values of $x$ are used to obtain the desired final value of $z_{d1}$ which is denoted as $z_{d1f}$. Then the simple input feedback linearization control is used to steer the previous steady state value to the final desired value $z_{d1f}$ as follows.

$$u = \alpha + \beta v$$  \hspace{1cm} (4.30)

$$v = -d_5 + \delta_1 z_{d4} + \delta_3 z_{d3} + \delta_2 z_{d2} + \delta_1 (z_{d1} - z_{d1f})$$  \hspace{1cm} (4.31)

where feedback gains $\delta_{1-4}$ are chosen to ensure that system (4.20) is stable. It can be easily seen that $z_{d1}$ will reach $z_{d1f}$ in steady state, and the corresponding frequency error $x_1$ is guaranteed to be zero in steady state.

A digital simulation method is applied to detect steady state and automatically switch to the second controller (4.30). The simulation time is divided into many very small steps. During each time step, MATLAB ODE45 is used to calculate the final values which also are the initial
conditions for the next step. Then the latest 5 frequency change values are averaged and compared with the next one. If the difference is smaller than the predefined tolerance, the system reaches steady state and the second controller (4.30) is turned on. From the simulation (Figure 4-3), two-stage nonlinear controller yields significantly better performance compared to the nonlinear controller with linear PI controller.

![Graph showing frequency change over time for different controllers](image)

Figure 4-3: Nonlinear Controller with Second Linear PI and Nonlinear Controller.

### 4.4 Conclusion

In this chapter, a nonlinear controller has been developed for hydraulic power plant governing system. Input state feedback linearization is used to transform nonlinear system dynamic equations to an equivalent linear system. Sliding mode control method is then implemented on such a linear system. The sliding hyperplane is constructed based on full state feedback $H_{\infty}$ theory to reduce the disturbance effects. The performance of the control method has been evaluated via MATLAB software in the presence of disturbance. Compared with traditional
linear PI controller, the nonlinear controller brings significant improvement to the governing system for frequency control. It is shown that a novel two-stage nonlinear controller with integral feedback can bring the error in frequency to zero in a very short time.
Chapter 5

Second Order Sliding Mode Control for Hydropower Plant

Although the traditional sliding mode control can provide robust responses to system uncertainties and external disturbances with simple control laws, there is an issue with it. The unwanted high frequency oscillation of the system trajectory, which is called chattering effects is often associated with sliding mode control. This fast switching can potentially excite the plant natural mode to destabilize the system and cost high mechanical wear and tear, which are not desirable.

5.1 Second Order Sliding Mode Control Theory

There are different ways to solve that disadvantage, such as replacing the “sgn” function with a saturation function [52]. Higher order sliding mode control (HOSM) is another method and was introduced by Arie Levant (formerly L.V.Levantovsky) in 1980s and has drawn great attentions to the control community, because it can not only reduce the chattering effect, but also remain the robustness feature from the first order sliding mode. As shown in Chapter 3, the first order sliding mode acts on the first derivative of the sliding manifold, similarly HOSM acts on higher derivative of the sliding manifold. Unlike the first order sliding mode, the higher order sliding mode controller, say $m^{th}$ order ($m > 1$), is designed to make sure not only $s = 0$ but also $\dot{s} = \ldots = s^{(m-1)} = 0$ in finite time [60]. Therefore, the control input will be a continuous function after the integration, since the discontinuous action is applied on $s^m$. 
Among the higher order sliding mode control methods, second order sliding mode control is the easiest one to be implemented, because utilizing the higher order sliding mode control requires more information. For example, to design an \( m^{th} \) order \((m > 1)\) sliding mode controller requires information from \( s, \dot{s}, \cdots, s^{m-1} \). In the second order sliding mode control algorithms, super-twisting is an exception. It only requires the knowledge of \( s \). It is designed for systems which have relative degree of one. (Relative degree \( n \) means that sliding variable \( s \) has to be differentiated \( n \) times before the input \( u \) appears [61]) As illustrated in Chapter 3, the hydropower plant with sliding mode control has relative degree of one. Therefore, our first order sliding mode controller for the hydropower plant (as shown in Chapter 3) can be converted to super-twisting controller with only few changes.

We have studied super-twisting algorithm and applied it to hydropower plant for frequency control. The sliding hyperplane vector is designed following the method introduced in

Figure 5-1: Phase Portrait of the System under First and Second Order Sliding Mode Control[60]
Chapter 3. Then two modified controller design procedures have been proposed and tested. The simulations of both methods and regular first order sliding mode control with saturation function were conducted. The results of frequency change responses and control efforts were compared between the second order and regular sliding mode. Two second order sliding mode controllers were also compared and one of them was chosen as the better controller design approach. From the simulation, it is clear that the second order sliding mode dramatically reduces the chattering and still keeps the robustness from the regular sliding mode, especially under a large disturbance.

5.2 Controller Design Detail

5.2.1 Basic Hydropower Plant Dynamic Model

Let us recall the state space model of hydropower plant in Chapter 3.

\[
\dot{x} = Ax + bu + d\Delta P_e
\]  (5.1)

where

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
0 & -\frac{1}{t_p} & 0 & 0 & -\frac{1}{t_p r_p} \\
0 & \frac{1}{t_g} & -\frac{1}{t_g} & 0 & 0 \\
0 & -\frac{2}{t_g} & \left(\frac{2}{t_w} + \frac{2}{t_g}\right) & -\frac{2}{t_w} & 0 \\
0 & 0 & 0 & \frac{1}{m} & -\frac{d_d}{m}
\end{bmatrix}
\]

and

\[
x = \begin{bmatrix} x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \end{bmatrix} = \begin{bmatrix} t_q \Delta \dot{m} dt \\
\Delta x_e \\
\Delta x_f \\
\Delta \dot{P}_m \\
\Delta \omega \end{bmatrix}
\]  (5.3)

and

\[
b = \begin{bmatrix} 0 \\
-\frac{1}{t_p} \\
0 \\
0 \\
0 \end{bmatrix} \text{ and } d = \begin{bmatrix} 0 \\
0 \\
0 \\
0 \\
-\frac{1}{m} \end{bmatrix}
\]  (5.2 a,b,c)
5.2.2 First and Second Order Sliding Mode Systems

1. Sliding hyperplane design

The sliding hyperplane is expressed as

\[ s_t = gx \]  \hspace{1cm} (5.4)  

where the row vector \( g \) is determined by minimizing the effect of external disturbance via \( H_\infty \) control technique which is the same as shown in Chapter 3.

![Block Diagram for First Order and Second Order Sliding Mode Control of Hydropower Plant.](image)

2. Controller Design

Here we will show three different controller design methods, which include the first order sliding mode control and the second order sliding mode with two different methods. The more detail about each controller are provided as follows.

a. First Order Sliding Mode Controller [52]

\[ \dot{s}_t = gx = g(Ax + bu + d\Delta p_e) \]  \hspace{1cm} (5.5)  

In order to satisfy the reaching condition \( s_t \dot{s}_t < 0 \),
\[ u(t) = u_{eq}(t) + u_{un}(t) \quad (5.6) \]

where

\[ u_{eq}(t) = -(gb)^{-1} gAx \quad (5.7) \]

\[ u_{un}(t) = -\eta sgn(s_I) \quad (5.8) \]

In order to reduce the chattering effect, usually a saturation function \( \text{sat} \) is used instead of \( \text{sgn} \) in (5.8):

\[ \text{sat}(s_I) = \begin{cases} 
\text{sgn}(s_I) & \text{if } |s_I| > \rho \\
\frac{s_I}{\rho} & \text{if } |s_I| \leq \rho
\end{cases} \quad (5.9) \]

where \( \rho \) is the boundary layer thickness. However, such method does not have a good chattering reduction performance under large disturbance, since \( s_I \) value often goes outside of the boundary layer. Although a larger thickness of the boundary layer can fix that problem, the robustness and accuracy of the system are compromised. More results are shown in the simulation section.

b. Second Order Sliding Mode Controller

The super-twisting algorithm is selected for second order sliding mode control study on hydropower plant, because of its special features compared to other types of second order sliding mode control algorithms, such as that super-twisting algorithm does not require information of \( \dot{s} \).

From the proof of super-twisting algorithm in Appendix D, it is clear that convergence conditions (5.12) and (5.15) are not derived from traditional control theories, but more from mathematical aspects. Therefore, it would be very difficult to design a controller directly following those conditions. Here, we showed and tested two methods for designing the second order sliding mode controller. The first one is based on conditions in (5.12) and (5.15) with little modification (partitioning the controller like regular sliding mode controller design) in order to find the necessary parameter values. The second method is to choose controller parameters by
1. Modified second order sliding mode controller

\[ s_i = g\mathbf{x} = g(A\mathbf{x} + bu + d\Delta p_e) = a(t) + b(t)\mu \tag{5.10} \]

where

\[ a(t) = gAx + gd\Delta p_e \quad \text{and} \quad b(t) = gb \tag{5.11 a,b} \]

It is assumed that there are positive constants \((C, K_m, K_M, U_M, q)\) such that

\[ |\dot{\mathbf{x}}| + U_M |\dot{\mathbf{x}}| \leq C \tag{5.12 a} \]

\[ 0 \leq K_m \leq b(t, s) \leq K_M \tag{5.12 b} \]

\[ |a/b| < U_M \tag{5.12 c} \]

\[ 0 < q < 1 \tag{5.12 d} \]

The control input is designed as

\[ u = -\lambda \frac{|\mathbf{s}|}{q} \frac{1}{2} \operatorname{sgn}(s_i) + u_1 \tag{5.13} \]

where

\[ u_1 = \begin{cases} 
-\mu & |\mu| > U_M \\
-\alpha \operatorname{sgn}(s_i) |\mu| & |\mu| \leq U_M 
\end{cases} \tag{5.14} \]

and \(\gamma, \alpha\) are selected to satisfy following conditions.

\[ \alpha > C/K_m \tag{5.15 a} \]

\[ \lambda > \sqrt{\frac{2}{(K_m\alpha - C)(K_m\alpha + C)K_M(1+q)}} \tag{5.15 b} \]

It would be very hard to find the boundary values in (5.11) for \(a(t)\) defined by equation (5.11 a). A little modification is made in order to find boundary values easily. If the control input
is partitioned into two parts [52], $u_{eq}$ same as (5.7) and $u_{sec}$ is the controller output generated by the second order sliding mode controller; that is,

$$u(t) = u_{eq}(t) + u_{sec}(t)$$

(5.16)

In this case,

$$\dot{s}_I = a_{new}(t) + b_{new}(t)u_{sec}$$

(5.17)

where

$$a_{new} = gd\Delta \tilde{p}_e \text{ and } b_{new} = gb$$

(5.18 a, b)

It should be noted that

$$\dot{a}_{new} = gA\Delta \tilde{p}_e \text{ and } \dot{b}_{new} = 0$$

(5.19 a, b)

Now, if the disturbance function’s $(\Delta \tilde{p}_e)$ general shape and magnitude boundary are known, those positive constants in (5.12) can be easily determined.

2. Second order sliding mode controller design from tuning

As mentioned above, the convergence conditions in (5.12) and (5.15) are very conservative. Reference [61, 62] also had same conclusion. If we set the value of $U_M$ to positive infinite, then the controller will be the same as (5.13) with

$$\dot{u}_I = -\alpha \text{sgn}(s_I)$$

(5.20)

Therefore, only two parameters $\lambda$ and $\alpha$ need to be found. The tuning method is similar to PID design, but simpler. Initially, $\lambda$ and $\alpha$ are set to be small values. First $\lambda$ is increased until the system response is nearly not changing; then the $\alpha$ value is increased. After that, more trial and error tuning is required to achieve the best response. Similar controller design approaches were also presented in [61, 62]. The value of $\lambda$ is very important, since it affects the system steady state error, settling time, control effort and chattering. Large $\lambda$ will decrease the steady state error, have fast response and require less control effort. But a small $\lambda$ will lead less
chattering phenomenon. Therefore, the value selection of $\lambda$ depends on the real application situation, because of above trade-off.

5.3 Simulation Results

Simulations were conducted based on both methods with time invariant disturbance (a step function). The disturbance has magnitude of 0.4 (p.u.). The initial condition is set to be $[0, 0, 0, 0, -0.05]$. All the hydropower plant’s parameters values are the same as Chapter 3.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Time Invariant Disturbance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_m$</td>
<td>2490</td>
</tr>
<tr>
<td>$K_M$</td>
<td>2510</td>
</tr>
<tr>
<td>$C$</td>
<td>3</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
</tr>
<tr>
<td>$q$</td>
<td>0.5</td>
</tr>
<tr>
<td>$U_M$</td>
<td>2</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.0859</td>
</tr>
</tbody>
</table>

Table 5-2: Parameter Values for Controller Design of Method Two.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Time Invariant Disturbance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.05</td>
</tr>
</tbody>
</table>
As shown in Figure 5-3, the sliding value $s_l$ keeps jumping outside of boundary layer under the large disturbance. Therefore, the saturation function does not have good chattering reduction for a large disturbance without sacrificing the robustness and accuracy of the system by increasing the boundary layer width. From the plots, second order sliding mode control with method two has less oscillation, which matches the same result from control effort comparison.

Figure 5-4: Second Order Sliding Mode Phase Trajectories from Method One and Two under Time Invariant Disturbance.
Phase trajectories for super-twisting under time invariant disturbance is shown in Figure 5-4. \( s \) and \( \dot{s} \) both go to the origin in finite time in both methods. Compared to method one, \( \dot{s} \) values in method two are much smaller, which proves method two is less conservative than method one.

![Frequency Change Responses](image)

Figure 5-5: Frequency Change (\( \Delta \omega \)) Responses from First Order, Second Order Sliding Mode under Time Invariant Disturbance.

Frequency change responses under a step disturbance (magnitude of 0.4 (p.u.)) with first order and second order (under both methods) sliding mode are plotted in Figure 5-5. Both controllers are able to bring the frequency change back to zero because of the integral terms in (5.3) under the time invariant disturbance and response shapes are very similar. The plots also show that both second order sliding mode controllers actually have better performance than the regular sliding mode controller with smaller overshoot and shorter settling time. Compared to the first method, the second method has a slightly poor performance. This is basically a trade-off between reducing chattering effect and increasing response performance during tuning.
Control efforts from both types of controllers are plotted in Figure 5-6. Both methods’ controllers have smaller magnitudes than regular sliding mode controller. From the zoom-in part on the left, it is indubitable that the second order sliding mode under method one has higher oscillation than the first order sliding mode controller, which is the opposite of what we desire. The controller outputs from method two and first order sliding mode are shown on the right. The zoom-in part illustrates that the second order sliding mode has much less oscillation than the regular sliding mode control. Therefore, method two can reduce the chattering effect and is used for robust study.
Second order sliding mode with system parametric uncertainties under a step disturbance was conducted for testing its robustness. The results were compared to the ones from regular sliding mode controller as shown in Figure 5-7. They are similar to each other and the second order sliding mode even has fewer variations under system parametric uncertainties. From the results in Chapter 3 and above comparison, this second order sliding mode controller (method two) retains all the robustness property of the regular sliding mode and reduces the chattering effects.

5.4 Conclusion

Two methods of second order sliding mode control have been studied for super-twisting algorithm. One is based on the convergent condition in [60] with a slightly modification. The other one is to obtain controller parameters from tuning. The sliding hyperplane is constructed in the same way as shown in Chapter 3. The simulation results show that both methods have less
control efforts than regular sliding mode control. However, the method one is not able to reduce the chattering effects. But the method two can achieve that and keeps the robustness of regular sliding mode as shown in Figure 5-6 and 7. Method two also brings additional practical benefit during implementation, because of its simple tuning procedures. Compared to using saturation function to reduce the chattering effect [52], second order sliding mode control is able to handle large disturbance without sacrificing system robustness and accuracy. It should be noted that the second order sliding mode control is only designed for reducing the chattering effects. It cannot deal with the non-matching conditions.
Chapter 6

Conclusions and Future Work

6.1 Thesis Conclusions

As mentioned in the introduction, the goal of this thesis is to construct detailed dynamic model for the entire hydropower plant and develop advanced modern controllers. Hydropower plant usually consists of reservoir, dam, penstock and turbine with generator. The entire plant model is constructed by analyzing each component and combining their dynamic models together. A hydropower plant has nonlinear characteristics. Although many existing plants have very high efficiency turbines, the control systems are still based on linear control algorithm developed in 1960s such as mechanical-hydraulic governors. A linear controller implemented on a nonlinear system significantly decreases the operation range of the plant and performance under system uncertainties due to wear and tear. Thus, hydropower plants are in need of modern controllers to provide fast stable and robust performance. In this thesis, sliding mode control is primarily applied on the hydropower plant. The simulation results show promising performance of such controllers.

A sliding mode controller blended with $H_\infty$ control technique has been developed for governing hydropower plant based on the linear ideal model. The sliding mode hyperplane has been constructed based on the dynamic model of the plant by full state feedback $H_\infty$ theory. The results from simulation have shown the advantages which this method brings to the system response, such as fast response, less overshoot and robustness.

Then, this method has been applied on a nonlinear model of the hydropower plant by using feedback linearization to handle the nonlinear characteristics of the plant. The nonlinear
controller performance is compared with traditional linear PI controller. It is found that this approach brings significant improvement to the governing system for frequency control. It is shown that a novel two-stage nonlinear controller with integral feedback can bring the error in frequency to zero in a very short time.

One issue with sliding mode control is its high frequency chattering effect. In this thesis, second order sliding mode control has been investigated to solve such issue. Two design methods of second order sliding mode control have been studied. From the simulation, one of these methods can achieve the control goal and keep the robustness of regular sliding mode. Compared to using saturation function, this method can be more robust and accurate under large disturbances.

6.2 Future Work

The future work which can be extended from this thesis is listed below. It involves both modeling and controller design.

Controller design based on more complicated dynamic model

Currently, all types of controllers designed in this thesis did not consider the elastic pipe and water compression effect as well as the effect from surge tank, although they have been described in the Chapter 2. From Figure 2-4 and 5, both effects create small oscillations to the power output from the system during the transient period. It would be very beneficial to involve such effects for the controller design.

More tests on new turbine model

Appendix A gives detail about the new turbine torque model introduced in [2]. The appendix also presents a comparison between real data and this new model. The result shows that the new model can be used to represent the real turbines more accurately. However, this is only
the preliminary investigation of this new model. More evaluations are required, especially from industrial aspect, since it would bring much more complexity to the dynamic model of the hydropower plant.

**μ analysis on hydropower plant**

In Chapters 3 and 4, sliding mode control is applied to provide a robust performance of the system under parametric uncertainties and external disturbances. More work can be done on analyzing those effects by using μ analysis, which is less conservative than the small gain theory which the usual way to deal with uncertainties. Appendix F gives an introduction about how to apply μ analysis and μ synthesis on hydropower plant.

**Reactive power control**

Another control area in hydropower industry is the reactive power control. Unlike the active power or real power, reactive power appears when the voltage and current are not changing in a synchronized manner. Reactive power demand changes will affect output voltage of the generator. Like frequency in active power control, voltage in reactive power is also an important factor to judge if the power plant can provide quality energy to the grid. Unlike active power control which works on wicket gate system to regulate flow to increase or decrease the power base on the demand, reactive power control is acting on the exciter which controls generators output voltage and reactive power as shown in Figure 1-5. Although, active power control and reactive power control are usually treated separately, it would be very helpful to combine the control actions together to create one controller for the plant.

**Grid and load model**

In our simulation, change in the grid load is simply represented by step or sine function. However, in most of the papers, the grid is treated as infinite buses, which assume that the plant connects to a very large network. A more detailed and accurate grid and load models will bring
our simulation closer to the real world situation and will serve as a better test for our controller’s performance.
Bibliography


*Transactions of the ASME*, Vol. 64, pp. 759-768, 1942


[64] MATLAB, *Robust Toolbox*, MathWorks,

< http://saba.kntu.ac.ir/eecd/ecourses/Robust%2086/Robust%20Toolbox.pdf >

[65] MATLAB, *LMI Toolbox*, MathWorks,

[66] MATLAB, *Mu Toolbox*, MathWorks,

Appendix A

New Turbine Model and Real Data Test

The new turbine model is introduced in [2]. This appendix gives more detail about how to construct the new model, how to integrate this model to our existing system and provide a real data test on such model.

In the past, the turbine torque model only involves the loss from no load condition which is the friction loss for the plant.

\[ p_m = \gamma q_l h_t - \gamma q_{nl} h_t \]  
\[ (A.1) \]

where \( p_m \) is the mechanical power output from the turbine (\( kw \)), \( q_l, q_{nl} \) are the flow rate at turbine and at no-load condition(\( m^3/s \)), \( h_t \) is the water head at turbine (\( m \)), \( \gamma \) is the water special density (9.81kN/\( m^3 \)).

The new model considers several other inner energy losses as follows:

\[ p_m = \gamma q_l h_t - \Delta p_{ml} - \Delta p_h - \Delta p_v - \Delta p_i \]  
\[ (A.2) \]

where \( \Delta p_{ml} \) is the loss of the power due to the mechanical friction (\( kw \)), \( \Delta p_h \) is the loss power due to hydraulic loss (\( kw \)), \( \Delta p_v \) is the loss of power due to the leakage (\( kw \)) and \( \Delta p_i \) is the power loss due to water impact at the non-optimal condition such as the loss at inlet of the turbine runner and vortex band of draft tube (\( kw \)).

**Hydraulic loss:**

Hydraulic loss is mainly the continuous loss in the runner with the basic form of \( kq^2/2a_g \).

Therefore, it can be represented as:
where \( k_h \) is the coefficient of the flow channel loss characteristic, which relates with the flow passage components of the hydro-turbine.

**Volume loss:**

\[
\Delta p_v = \gamma k_v q_i h_i
\]  

(A.4)

where \( k_v \) is the coefficient of the volume loss, which is affected by the sealing method of the turbine. Its value is about 0.0025 to 0.005 for modern large turbine.

**Impact loss:**

As mentioned above, the impact loss is caused by working at non-optimal condition. Therefore, it depends on the deviation from the maximum efficiency point. And the function is represented as:

\[
\Delta p_i = F \left( q_i - q_0 \right)^2
\]  

(A.5)

where \( F \) is the function of variable \( (q_i - q_0)^2 \) and \( q_0 \) is the flow at the maximum efficiency point.

**Mechanical loss:**

At the non-load condition, there is no power output from the system, since all the power is used to compensate for the loss of mechanical friction. Therefore, we can set \( p_m \) to zero and with the knowledge of above losses (\( \Delta p_h \), \( \Delta p_v \) and \( \Delta p_i \)), the mechanical loss function can be defined as:

\[
0 = \gamma q_{nl} h_{nl} - \Delta p_{ml} - \Delta p_h - \Delta p_v - \Delta p_i \Rightarrow \Delta p_{ml} = \gamma q_{nl} h_{nl} - k_h q_{nl}^3 - \gamma k_v q_{nl} h_{nl} - F \left( q_i - q_{nl} \right)^2
\]  

(A.6)

where \( h_{nl} \approx h_i \). This equation is used for integrating the new turbine model to the system in the later part.

The mechanical loss can also be calculated at maximum efficiency point as:
\[ \Delta p_{mi} = \gamma q_i h_i - k_h q_i^3 - \gamma k_v q_i h_i - p_{mi} \]  

(A.7)

where \( p_{mi} \) is the power output at the maximum efficiency point. The mechanical loss depends on the rotation speed, which is nearly constant in the practical operation. Therefore, it can be treated as a constant.

**\( k_h \) calculation:**

The impact loss is zero at the maximum efficiency point and has the same values for the same deviation away from the maximum efficiency point. So two points on the efficiency curve at some head can be selected, whose flow satisfies the following:

\[
\begin{align*}
q_1 &= q_i + \Delta q \\
q_2 &= q_i - \Delta q
\end{align*}
\]

(A.8)

where \( \Delta q \) is usually chosen between 0.1 and 0.01 of \( q_i \), which is the rated value of flow rate.

Since impact losses for both cases are equal, \( k_h \) can be calculated as:

\[
k_h = \frac{\gamma h_i (q_1 - q_2)(1-k_v)-p_{m1}+p_{m2}}{q_1^3 - q_2^3}
\]

(A.9)

where \( p_{m1} \) and \( p_{m2} \) are the corresponding power output to \( q_1 \) and \( q_2 \).

**\( F \) function construction:**

Rearranging (A.2), the impact loss is calculated as:

\[
\Delta p_i = \gamma q_i h_i - \Delta p_{m1} - \Delta p_h - \Delta p_v - p_m
\]

(A.10)

Then a curve fitting method is applied to obtain the function.

**Real data testing:**

A turbine performance data is provided by Weir America Hydro. The following figures show the results and comparison.
Figure A-1: Efficiency vs. Flow Rate and Output Power from the Real Data

Figure A-2: Impact Loss Comparison between Real Data and Curve Fitting Result

Figure A-2 shows the curve fitting results. MATLAB “fminsearch” is used to obtain an exponential like function. From the plot, the following function fits the real data very well:

$$\Delta p_i = F\left( (q_i - q_f)^2 \right) = 5.88 \times 10^3 \times e^{0.49 \times 10^{-3} \times (q_i - q_f)^2} - 5.88 \times 10^3 \times e^{-0.58 \times 10^{-3} \times (q_i - q_f)^2}$$  \hfill (A.11)
Figure A-3: Impact Loss vs. Flow Rate Shape Comparison between our test and paper [2] Result

Figure A-3 presents the comparison between paper result [2] and our testing result. Since we have different data, we cannot directly compare both results. However, in a qualitative sense, our testing result matches the paper predicted results.

Figure A-4: The Power Output Comparison between Ideal Model, Real Data and New Model

Integration with Existing Model:
Insert (A.3), (A.4), (A.5) and (A.6) to (A.2), then the new torque can be calculated as:

\[ p_m = \gamma h_i (q_i - q_{nl})(1 - k_r) - k_h (q_i^3 - q_{nl}^3) - \{F[(q_i - q)^2] - F[(q_i - q_{nl})^2] \} \tag{A.12} \]

Above equation is normalized with respect to the turbine rated value \( p_r \) (known) and times \( h_r \), \( q_r \) (known) on the both numerator and denominator of the right hand side.

\[ \frac{p_m}{p_r} = \frac{h_i}{h_r} \left( \frac{q_i - q_{nl}}{q_r} \right) \left( 1 - k_r \right) - \frac{q_i^3 - q_{nl}^3}{q_r^3} \frac{q_i}{q_r} \frac{h_i}{h_r} k_h - \frac{1}{p_r} F[(q_i - q)^2] + \frac{1}{p_r} F[(q_i - q_{nl})^2] \tag{A.13} \]

To simplify above equation:

\[ \bar{p}_m = A_r \bar{h}_i (\bar{q}_i - \bar{q}_{nl})(1 - k_r) - A_r k_h (\bar{q}_i^3 - \bar{q}_{nl}^3) - \frac{1}{p_r} F[(\bar{q}_i - \bar{q})^2] + \frac{1}{p_r} F[(\bar{q}_i - \bar{q}_{nl})^2] \tag{A.14} \]

where \( A_r = \frac{h_i}{p_r} q_r \), \( k_h = k_h q_i^2 \).

Based on the [8], the relationships among flow rate, turbine head and ideal power (without friction loss) are

\[ p = \gamma q_i h_i \tag{A.15} \]

\[ q_i = k_{aq} g \sqrt{h_i} \tag{A.16} \]

and after normalization them with respect to \( p_r \), \( h_r \), \( q_r \), \( g_r \).

\[ \frac{p}{p_r} = \bar{p} = \frac{q_i}{q_r} \frac{h_i}{h_r} = \bar{q}_i \bar{h}_i \tag{A.17} \]

\[ \frac{q_i}{q_r} = \bar{q}_i = \frac{g}{g_r} \sqrt{\frac{h_i}{h_r}} = \bar{g} \sqrt{\bar{h}_i} \tag{A.18} \]

And for the ideal penstock, the relationship between flow and head is

\[ \frac{du_i}{dt} = -\frac{a_s}{l_p} (h_i - h_0) = \frac{a}{l_p} h_p \tag{A.19} \]
where \( h_p \) is the water head at penstock (this dynamic head which will be zeros at the steady state). Normalization with respect to rated values yields.

\[
\frac{d}{dt} \left( u_r - u_e \right) = -\frac{a_g h_r}{l_p} \left( h_t - h_0 \right) = -\frac{1}{t_w} (\bar{h}_t - \bar{h}_0) = \frac{1}{t_w} \bar{h}_p
\]

(A.20)

where \( t_w = \frac{l_p u_r}{a_g h_r} = \frac{l_p q_r}{a_g A_p h_r} \) and usually \( h_r = h_0 \).

Therefore the water head at turbine admission can be calculated:

\[
\bar{h}_t = \bar{h}_0 - \bar{h}_p
\]

(A.21)

\[
\bar{p}_m = A_p \gamma (\bar{h}_0 - \bar{h}_p) (\bar{q} - \bar{q}_{nl})(1 - k_x) - A_p k_h (\bar{q}_i^3 - \bar{q}_{nl}^3)
\]

\[- \frac{1}{p_r} F[(q_i - q_t)^2] + \frac{1}{p_r} F[(q_i - q_{nl}^2)] \]

(A.22)

Note: No-load flow value is determined at the rated head in study, but in the real world practice the head might be different. However, constant energy method can be used to calculate the new no-load flow.

\[
\gamma q_{nl-r} h_r = \gamma q_{nl-x} h_x
\]

(A.23)

where "-x" indicated at any head.

\[
q_{nl-x} = \frac{h_r}{h_x} q_{nl-r} = \frac{h_r}{h_0} q_{nl-r} = \Delta H q_{nl-r}
\]

(A.24)

The same concept is applied to optimal flow \( q_i \)

\[
q_{i-x} = \Delta H q_{i-r}
\]

(A.25)

Therefore,

\[
\bar{p}_m = A_p \gamma (\bar{h}_0 - \bar{h}_p) (\bar{q}_i - \Delta H q_{nl-r})(1 - k_x) - A_p k_h (\bar{q}_i^3 - \Delta H^3 q_{nl-r}^3)
\]

\[- \frac{1}{p_r} F[(\Delta H q_{i-r} - q_t)^2] + \frac{1}{p_r} F[(\Delta H q_{i-r} - \Delta H q_{nl-r}^2)] \]

(A.26)
Since all the rated values, no-load flow and optimal flow at rated values are constant as well as steady head and coefficients at the operating; the turbine output power only depends on water flow rate and dynamic head caused by penstock.

In conclusion, the following equations describe the new turbine-penstock dynamics (also see Figure A-5):

\[ \bar{q}_t = \bar{h}_t \]  \hspace{1cm} (A.18)

\[ \frac{d\bar{q}_t}{dt} = \frac{d\bar{h}_t}{dt} = -\frac{1}{t_w} (\bar{h}_t - \bar{h}_0) = \frac{1}{t_w} \bar{h}_p \]  \hspace{1cm} (A.20)

\[ \bar{p}_m = A_r (\bar{h}_0 - \bar{h}_p)(\bar{q}_t - \Delta H\bar{q}_{nl-r})(1 - k_r) - A_r k_r (\bar{q}_t^3 - \Delta H^3\bar{q}_{nl-r}^3) \]  \hspace{1cm} (A.26)

\[ -\frac{1}{P_r} F[(\Delta Hq_{i-r} - q_t)^2] + \frac{1}{P_r} F[(\Delta Hq_{i-r} - \Delta Hq_{nl-r})^2] \]

---

Figure A-5: Nonlinear Model with Updated Turbine Torque Model.
Appendix B

PI Tuning Procedures for Linear Controller

This appendix shows the procedures to find the $k_{pc}$ and $k_i$ for PI controller which is used for comparison between advance controller and linear PI controller.

First, the PI controller parameters are selected based on Ziegler-Nichols method [59]. The first value needs to be determined is the ultimate gain $k_u$, which is the propositional gain “$k_{pc}$” with zero integral gain. This gain will make the system have constant amplitude oscillation. The oscillation period $T_u$ is used to calculate $k_{pc}$ and $k_i$ as shown in Table B-1.

Table B-1: Ziegler-Nichols Method

<table>
<thead>
<tr>
<th>Control Type</th>
<th>$k_{pc}$</th>
<th>$k_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>$k_u / 2.2$</td>
<td>$1.2 k_{pc} / T_u$</td>
</tr>
</tbody>
</table>

Figure B-1 shows the system oscillation response with only proportional gain $k_u$.

Figure B-1: Oscillation with Constant Amplitude with Ultimate Gain under Constant Disturbance (0.03 p.u.)
Therefore, the coefficients’ values are:

\[
\begin{align*}
    k_u &= 0.2185 \\
    T_u &= 6.16 \\
    k_i &= 0.019
\end{align*}
\]  \hspace{1cm} (B.1)

From the simulation result shown in Figure B-2, the controller’s performance is not very promising. The settling time is too long. Later, a new method is applied based on [15] and [31].

The coefficients, values can be found as following:

\[
\begin{align*}
    T_r &= 5t_w \\
    \delta &= 2.5t_w / M \\
    k_{pc} &= \frac{1}{\delta} \\
    k_i &= \frac{1}{\delta T_r}
\end{align*}
\]  \hspace{1cm} (B.2)

with our plant’s parameters (in Chapter 3): \( t_w = 1.3 \) and \( M = 6 \), the coefficients are:

\[
\begin{align*}
    k_{pc} &= 1.8462 \\
    k_i &= 0.2840
\end{align*}
\]  \hspace{1cm} (B.3)

Simulation result from above method is shown in Figure B-3.
Compared with Zigeler-Nichols method, this method provides much better performance. After further tuning, controller gains for the best settling time are:

\[
\begin{align*}
    k_{pc} &= 1.8462 \\
    k_i &= 0.42
\end{align*}
\]  

(B.4)

Simulation result from above method is shown in Figure B-4.
Figure B-4: Frequency Change ($\Delta \omega$) Response with Further Tuning from Previous Method under Constant Disturbance (0.03 p.u.)
Appendix C

Feedback Linearization

Recalling from Chapter 4, the hydropower plant model can be represented in the classic standard form as:

\[ \dot{x} = f(x) + g_s(x)u \]  \hspace{1cm} (C.1)

where

\[
f = \begin{bmatrix}
\frac{1}{m} x_2^2 - \frac{\bar{u}_{nl}}{m} x_2^2 - \frac{d_d}{m} x_1 - \frac{d_d}{m} \bar{w}_{ref} - \frac{\bar{p}_e}{m} \\
- \frac{1}{t_w} x_2^2 + \frac{1}{t_w} h_0 \\
\frac{1}{t_g} x_4 - \frac{1}{t_g} x_3 + \frac{\bar{g}_0}{t_g} \\
- \frac{1}{t_p} x_4
\end{bmatrix}
\]

and

\[
g_s = \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\]

For implementing feedback linearization, we need to construct the vector fields: \( g_s \), \( ad_t g_s \), \ldots, \( ad_t^{n-1} g_s \) for the hydropower plant model (n=4).

\[
g_s = \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}, \quad ad_t g_s = \begin{bmatrix}
0 \\
0 \\
1 \\
\frac{1}{t_p}
\end{bmatrix}, \quad ad_t^2 g_s = ad_t(ad_t g_s) = \begin{bmatrix}
2x_2^3 (\bar{u}_{nl} - x_2) \\
mt_g t_p x_3^2 \\
2x_2^2 \\
t_g t_p t_w x_3^2 \\
t_g + t_p \\
t_g^2 t_p^2 \\
1
\end{bmatrix}
\]
\[
\text{ad}_f^3 g_s = \text{ad}_f (\text{ad}_f^2 g_s) = \begin{bmatrix}
0 & \frac{4x_2 \bar{u}_l - 6x_2^2}{m} & \frac{-6x_2^2 (\bar{u}_l - x_3)}{m} & 0 \\
0 & \frac{4x_2}{t_p x_3^3} - \frac{6x_2^2}{t_p x_3^4} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \times \begin{bmatrix}
\frac{1}{x_3^2} & \frac{-\bar{u}_l}{m} & \frac{\bar{d}_p^3}{m} & -x_3 \\
\frac{2x_3}{t_p x_3^3} & \frac{-2x_3}{t_p x_3^4} & 0 & 0 \\
\frac{2x_3}{t_p x_3^3} & \frac{-2x_3}{t_p x_3^4} & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
C.3,a,b,c
\]

\[\text{ad}_f^3 g_s\] is a very big 4x1 matrix.

\[\text{ad}_f^i g_s\] is called Lie bracket of \(f\) and \(g_s\), where \(f\) and \(g_s\) are two vector fields on \(\mathbb{R}^n\) and is defined as [57]:

\[\left[ f, g_s \right] = \nabla g_s \times f - \nabla f \times g_s\]  \hspace{1cm} (C.4)

where \(\nabla f = \frac{\partial f}{\partial x}\) and \(\nabla g_s = \frac{\partial g_s}{\partial x}\). Therefore, \(\text{ad}_f^0 g_s = g_s\) and \(\text{ad}_f^i g_s = \left[ f, \text{ad}_f^{i-1} g_s \right]\), for \(i = 1, 2\ldots[57]\)

Assume the input-state linearizable conditions are satisfied, and then the new states can be constructed.

Since the new states have following properties:

\[
\begin{align*}
\dot{z}_{d1} &= z_{d2} \\
\dot{z}_{d2} &= z_{d3} \\
\dot{z}_{d3} &= z_{d4} \\
\dot{z}_{d4} &= v
\end{align*}
\]  \hspace{1cm} (C.5)

\[
\begin{bmatrix}
\dot{z}_{d1} \\
\dot{z}_{d2} \\
\dot{z}_{d3} \\
\dot{z}_{d4}
\end{bmatrix} = \frac{\partial \bar{z}_{d1}}{\partial x} \times \begin{bmatrix}
\text{ad}_f^0 g_s & \text{ad}_f g_s & \text{ad}_f^2 g_s & \text{ad}_f^3 g_s
\end{bmatrix} = [0 \ 0 \ 0 \ N]
\]
where \( N \) is not zero.

Based on expression of \( ad_t g_s \) and \( ad_t^2 g_s \), \( \frac{\partial z_{d1}}{\partial x_3} = 0 \) and \( \frac{\partial z_{d1}}{\partial x_4} = 0 \). Let \( z_{d1} = \frac{m}{t_w} x_1 - q_{al} x_2 + \frac{1}{2} x_2^2 \),

then \( \frac{\partial z_{d1}}{\partial x} = \begin{bmatrix} M \frac{t_w}{M} - (\bar{a}_{al} - x_2) & 0 & 0 \end{bmatrix} \).

Since \( ad_t^3 g \) is very complicated, only the result is shown:

\[
\frac{\partial z_{d1} \times ad_t^3 g_s}{\partial x} = -\frac{2x_2^2 (m\bar{h}_0 - d_d\bar{a}_{al}t_w + d_d t_w x_2)}{mt_w^2 t_{p,2}^2 x_3^3} \neq 0
\]

(C.6)

Therefore, \( z_{d1} \) satisfies the feedback linearization condition.

Following input feedback linearization method, the new states can be split into known and unknown parts as shown in (4.14).

\[
L^0 z_{d1} = z_{d1} = z_1
\]

\[
L^1 z_{d1} = \left( \frac{\bar{h}_0 x_2}{t_w} - \frac{d_d x_1}{t_w} - \frac{d_d \bar{a}_{al} \bar{\theta}_{ref}}{t_w} \right) - \frac{\bar{p}_{\bar{e}}}{t_w} = z_{d2} = z_2 + (- \frac{\bar{p}_{\bar{e}}}{t_w})
\]

\[
L^2 z_{d1} = \left( \frac{\bar{h}_0^2}{t_w^2} + \frac{\bar{h}_0 x_3^2}{t_w x_3^2} + \frac{d_d \bar{a}_{al}^2 \bar{\theta}_{ref}}{mt_w^2} + \frac{d_d^2 x_1 x_2}{mt_w x_3^5} + \frac{d_d^2 \bar{a}_{al}^2 x_2^2}{mt_w x_3^5} + \frac{d_d^2 \bar{a}_{al} \bar{\theta}_{ref}}{mt_w x_3} + \frac{d_d^2 \bar{p}_{\bar{e}}}{mt_w} \right) = z_{d3} = z_3 + (\frac{d_d \bar{p}_{\bar{e}}}{mt_w})
\]

(C.7.a,b,c)

The next \( L^3 z_{d1} \) is in a very complicated form and \( L^4 z_{d1} \) could be even more complicated. But they can both be expressed with a known term and an uncertain disturbance term as

\[
L^3 z_{d1} = z_{d4} = z_4 + (\frac{d_d^2 \bar{p}_{\bar{e}}}{mt_w})
\]

\[
L^4 z_{d1} = z_{d5} = (\frac{\alpha}{\beta} + \frac{u}{\beta}) + (\frac{d_d^3 \bar{p}_{\bar{e}}}{mt_w}) + v + (\frac{d_d^3 \bar{p}_{\bar{e}}}{mt_w})
\]

(C.7.d,e)
where $z_1, z_2, z_3$ and $z_4$ are known value compared with $z_d1, z_d2, z_d3$ and $z_d4$ as them with disturbance; $\alpha = -\frac{L_\epsilon^4 z_1}{L_\epsilon^3 z_1}$ and $\beta = \frac{1}{L_\epsilon^3 z_1}$

$L_\epsilon z_{d1}$ is called Lie derivative of $z_{d1}$ with respect to $f$ and is defined as: $L_\epsilon z_{d1} = \nabla z_{d1} f$, which is a simply a directional derivative of $z_{d1}$ along direction of vector $f$ [57]. Therefore, Lie derivative has following properties: $L_\epsilon^0 z_{d1} = z_{d1}, L_\epsilon^i z_{d1} = L_\epsilon \left( L_\epsilon^{i-1} z_{d1} \right) = \nabla \left( L_\epsilon^{i-1} z_{d1} \right) f$ and $L_\epsilon L_\epsilon z_{d1} = \nabla (L_\epsilon z_{d1}) g_x [57]$. 
Appendix D

Super-Twisting Algorithm

The super twisting algorithm is designed for the system which has relative degree one. (Control input $u$ appears after first derivative of the sliding surface $s$.) It guarantees that $s$ converge to zero in a finite time. This proof is based on procedures shown in [60].

Let us have following definitions.

$$s = gx \text{ and } \dot{s} = a(t) + b(t)u$$  \hspace{0.5cm} (D.1)

There are positive constants: $C, K_m, K_M, U_M, q$, such that:

$$|\dot{s} + U_M| \leq C, \ 0 \leq K_m \leq b(t) \leq K_M, \ |u/b| < U_M, \ 0 < q < 1$$  \hspace{0.5cm} (D.2)

The control input is designed as

$$u = -\gamma |s|^{1/2} \text{sgn}(s) + u_1$$  \hspace{0.5cm} (D.3)

where $u_1 = \begin{cases} -u, & |u| > U_M \\ -\alpha \text{sgn}(s)|u| \leq U_M \end{cases}$ and $\gamma, \alpha$ are selected to satisfy following conditions.

$$\alpha > C/K_m, \ \lambda > \frac{2}{\sqrt{(K_m \alpha - C)(K_M (1 + q))}}$$  \hspace{0.5cm} (D.4)

Note: $\frac{d}{dt}|s| = \dot{s} \text{sgn}(s)$.

There are two steps to prove that this algorithm guarantees that $s \to 0$ in a finite time. The first step is to show that controller output $u$ will enter the segment $[-U_M, U_M]$ in a finite time and stay there with the above conditions. The second step is to prove $s \to 0$ and $\dot{s} \to 0$ in finite time with above condition when $|u| \leq U_M$.

**Step One**
When $|u| > U_M$, $\dot{u}_1 = -u$ and $\ddot{u} = -\frac{1}{2} \lambda \dot{s}|s|^{-1/2} - u$. In order to achieve $|u| \leq U_M$, we need to prove $u\ddot{u} < 0$. Let us first show $\dot{s}u > 0$ with $|u| > U_M$.

Since $|a/b| < qU_M, |u| > U_M, b > 0$, then $-qU_M < a/b < qU_M$ and $-bqU_M < a < bqU_M$.

For $u > 0$, $-bquU_M < au < bquU_M \Rightarrow \dot{s}u = a(t)u + b(t)u^2 > -bquU_M + bu^2 > 0$

For $u < 0$, $bquU_M < au < -bquU_M \Rightarrow \dot{s}u = a(t)u + b(t)u^2 > bquU_M + bu^2 > 0$

Therefore, $\dot{s}u > 0$ with $|u| > U_M$

When $\dot{s}u > 0 \Rightarrow u\ddot{u} = -\frac{1}{2} \lambda (\dot{s}|s|^{1/2} - u^2) < 0$ because $\lambda$, $\dot{s}$, $|s|^{1/2}$ and $u^2$ are all positive.

Since $\dot{s}u > 0$, $\dot{s}$ and $u$ have the same sign.

$$\ddot{u} = -\frac{1}{2} \lambda \dot{s}|s|^{1/2} - u = -\left(\frac{1}{2} \lambda \dot{s}|s|^{1/2} + u\right) \Rightarrow |\dot{u}| = \left|\left(\frac{1}{2} \lambda \dot{s}|s|^{1/2} + u\right)\right| > |u| > U_M$$

Therefore, for $|u| > U_M$ with the required condition, the following relationships are achieved.

$$|u| > U_M \Rightarrow \begin{cases} \ddot{u} = -\frac{1}{2} \lambda \dot{s}|s|^{1/2} - u \\ \dot{s}u > 0 \Rightarrow |\dot{u}| > |u| > U_M \end{cases}$$

Then, $|u| \leq U_M$ will be established in a finite time. Therefore, we only need to prove that $\dot{s}$ and $u$ converge to zero in the case of $|u| \leq U_M$.

**Step Two**

For $|u| \leq U_M$ and $s \neq 0$,

$$\ddot{s} = \dot{a} + bu + b\left[-\lambda |s|^{1/2} \dot{s} - \alpha \text{sgn}(s)\right] = \dot{a} + bu - \frac{1}{2} \lambda b \frac{\dot{s}}{|s|^{1/2}} - ba \text{sgn}(s) = (\dot{a} + bu) - b\left[\frac{1}{2} \lambda \frac{\dot{s}}{|s|^{1/2}} + \alpha \text{sgn}(s)\right]$$

(D.7)

Based on (1.14)
For the first quadrant of $s - \dot{s}$ plane

With $s > 0$, $\dot{s} > 0 \Rightarrow \left(\frac{1}{2} \lambda \frac{\dot{s}}{|\dot{s}|^{1/2}} + \alpha \text{sgn}(s)\right) > 0$, then

$$\ddot{s} \in \left[-C - K_m \left(\frac{1}{2} \lambda \frac{\dot{s}}{|\dot{s}|^{1/2}} + \alpha \text{sgn}(s)\right), C - K_m \left(\frac{1}{2} \lambda \frac{\dot{s}}{|\dot{s}|^{1/2}} + \alpha \text{sgn}(s)\right)\right]$$

Let us define $\ddot{s}_u = -(K_m \alpha - C) = C - K_m \alpha$, a majorant curve and $\alpha > C/K_m \Rightarrow C - K_m \alpha < 0$

Since $0 > C - K_m \alpha > C - K_m \left(\frac{1}{2} \lambda \frac{\dot{s}}{|\dot{s}|^{1/2}} + \alpha \text{sgn}(s)\right)$, then $\ddot{s}_u = C - K_m \alpha$ converges slower than the real trajectory. Let us say it crosses $\ddot{s} = 0$ at $s_{um}$; then $s_{um}$ is the limit for real trajectory crossing $\ddot{s} = 0$ value.

Let us solve the ODE $\ddot{s}_u = C - K_m \alpha$ with initial condition $s_{ui} = 0, \dot{s}_{ui} = \ddot{s}_0$ and final value $s_{uf} = s_{um}, \dot{s}_{uf} = 0$ to find an expression for $\ddot{s}(0)$.

Let $\ddot{s}_u = -M_1$ where $M_1 = K_m \alpha - C$, then

$$\ddot{s}_u = -M_1 \Rightarrow \dot{s}_u = -M_1 s_u + M_2 \Rightarrow s_u = -\frac{1}{2} M_1 s_u^2 + M_2 s_u + M_3$$

with $s_{uf} = s_{um}, \dot{s}_{uf} = 0$, $M_2$ and $M_3$ can be expressed as:

$$\ddot{s}_u = -M_1 s_u + M_2 \Rightarrow 0 = -M_1 s_{um} + M_2 \Rightarrow M_2 = M_1 s_{um}$$

and

$$s_u = -\frac{1}{2} M_1 s_u^2 + M_2 s_u + M_3 \Rightarrow s_{um} = -\frac{1}{2} M_1 s_{um}^2 + M_2 s_{um} + M_3 \Rightarrow M_3 = s_{um} + \frac{1}{2} M_1 s_{um}^2 - M_2 s_{um}$$

Plug $M_2$ and $M_3$ values into $s_{um}$ and $\dot{s}_{um}$.
\[
s_u = \frac{1}{2} M_1 s_u^2 + M_1 s_{um} s_u + s_{um} + \frac{1}{2} M_1 s_{um}^2 - M_2 s_{um} = -\frac{M_1}{2} (s_u - s_{um})^2 + s_{um} \text{ and}
\]
\[
\dot{s}_u = -M_1 s_u + M_1 s_{um} = -M_1 (s_u - s_{um}) \text{ then } s_u = -\frac{M_1}{2} (s_u - s_{um})^2 + s_{um} = s_{um} - \frac{\dot{s}_u}{2(K_m \alpha - C)} \text{ for } s > 0, \dot{s} > 0
\]

with \( s_{ul} = 0, \dot{s}_{ul} = \dot{s}_0, \) \( s_u = s_{um} - \frac{\dot{s}_u}{2(K_m \alpha - C)} \Rightarrow 0 = s_{um} - \frac{\dot{s}_0}{2(K_m \alpha - C)} \Rightarrow \dot{s}_0 = 2(K_m \alpha - C)s_{um} \)

For the fourth quadrant of \( s - \dot{s} \) plane

Let us define the rest of the majorant curve for \( s > 0 \) and \( \dot{s} < 0 \)

It is easy to see the maximum negative bound of \( \dot{s}_u \), from (1.16) is \( \dot{s}_u \geq -\frac{2}{\lambda} (\frac{C}{K_m} + \alpha)s_{um}^{1/2} \) as shown in Figure D-1

\[ \text{Figure D-1: Phase Portrait for Super-twisting Algorithm [60].} \]

Let \( \dot{s}_{um} = -\frac{2}{\lambda} (\frac{C}{K_m} + \alpha)s_{um}^{1/2} \), when \( \dot{s}_{um} = 0 \). Then the corresponding \( \dot{s}_u \) can be expressed as:

\[
\dot{s}_u = -C - K_m \left( \frac{1}{2} \lambda \frac{\dot{s}}{s^{1/2}} + \alpha \right)
\]
The majorant curve can be constructed as a rectangular shape as shown in Figure D-1. Since $\dot{s} < 0$, $s$ is decreasing. Therefore, the real trajectory is inside of this majorant curve.

In order to achieve convergence, the following condition has to be satisfied.

$$\left|\frac{s_{um}}{s_0}\right|^2 < 1 \Rightarrow \left(\frac{2(K_m\alpha + C)^2}{\lambda^2 K_m^2 (K_m\alpha - C)}\right) < 1$$

(D.11)

So, the majorant curve is converging to zero in finite time as well as the real trajectory.

Unfortunately, this condition is not sufficient enough and does not consider possible 1-sliding mode keeping $u = \pm U_M$.

From (D.1) and (D.2) with $|a| \leq U_M$, $|\dot{s}| = b|a/b + u| \leq K_M (1 + q) U_M$ and $|\dot{s}| = b|a/b + u| \geq K_m (1 - q) U_M$. Then the following condition has to be satisfied.

$$\left|\frac{\dot{s}_{um}}{\dot{s}_0}\right|^2 < \frac{K_m (1 - q) U_M}{K_M (1 + q) U_M} = \frac{K_m (1 - q)}{K_M (1 + q)} < 1$$

(D.12)

This condition ensures that $u \neq \pm U_M$. Combining (D.11) and (D.12) gives the final condition coincides (D.4).
Appendix E

Steady State Dynamics After Reaching Sliding Surface

This Appendix shows the detail of steady state results from Matching and non-matching condition for sliding mode control [56].

A linear system with disturbance state space model and sliding surface is expressed as:

\[
\begin{align*}
\dot{x} &= Ax + Bu + Dv \\
0 &= s = gx
\end{align*}
\]

(E.1.a,b)

Based on sliding mode control theory (both first order and higher order), the sliding manifold \( s \) will reach zero in finite time. Therefore, \( s = 0 = gx \Rightarrow \dot{g} = 0 \). The control input at the steady state or acting on sliding surface can be calculated as:

\[
g\dot{x} = g(Ax + Bu_s + Dv) = 0 \Rightarrow u_s = -(gB)^{-1}g(Ax + Dv)
\]

(E.2)

By combining (E.1.a,b) and (E.2), the system dynamic can be expressed as:

\[
\dot{x} = Ax + Bu + Dv = Ax + Dv - B(gB)^{-1}g(Ax + Dv) = N(Ax + Dv)
\]

(E.3)

where \( N = (I - B(gB)^{-1}g) \). If the matching condition, (3.13) is satisfied, the disturbance will disappear because \( (I - B(gB)^{-1}g)D = 0 \) and dynamic order will be decreased. If the matching condition is not satisfied, there will be influence from disturbance acting as an input after the system reaches the sliding surface.

\[
\dot{x} = A_{new}x + B_{new}v
\]

(E.4)

where \( A_{new} = [(I - B(gB)^{-1}g)A \) and \( B_{new} = [(I - B(gB)^{-1}g)D \).

One of the methods to deal with this issue is to apply \( H_{\infty} \) technique to minimize the effect from disturbance as shown in Chapters 3 and 4.
Appendix F

μ Analysis and μ Synthesis on Hydropower Plant

The small gain theorem is a very good way to handle the system uncertainties in order to provide a robust and stable performance. It assumes that the uncertainty matrix is in a general form. However, the uncertainty matrix usually has a block diagonal structure. Therefore, the result from small gain theorem will be highly conservative. The structured singular value μ is used to address such issue. Here, we only show the basic definitions and general steps about how to apply such method on hydropower plant.

Basic definitions of structured singular value [52], [63]:

1. Uncertainty matrix:

\[
\Delta = \text{diag} [\delta_1 I_{r_1} \ldots \delta_S I_{r_S} \Delta_1 \ldots \Delta_F]
\]

where \( \delta_i \) is the structured uncertainty and \( r_i \) represents how many times it repeated. \( \Delta_j \) is the unstructured uncertainty. All the external disturbances can also be presented by \( \Delta \) through fictitious perturbations as shown in Figure F-1.

Figure F-1: Interconnection Structures for Nominal Plant with Uncertainties and Disturbance [63].
2. Structured singular value of $M$:

$$
\mu_\Delta(M) = \begin{cases} 
0 & \text{if } \det(I - M\Delta) \neq 0 \text{ for any } \Delta \in \Delta \\
\min \{\overline{\sigma}(\Delta); \Delta \in \Delta, \det(I - M\Delta) = 0\} & \text{otherwise}
\end{cases}
$$

3. Upper boundary of $\mu_\Delta(M)$

$$
\mu_\Delta(M) \leq \inf_{D \in \mathbf{D}} \overline{\sigma}(DMD^{-1}) \text{ or } \mu_\Delta(M) = \inf_{D \in \mathbf{D}} \overline{\sigma}(DMD^{-1}) \text{ if } 2S + F \leq 3
$$

where $D = \text{diag}[D_1, \ldots, D_S, d_1I_{m_1}, \ldots, d_{F-1}I_{m_{F-1}}, I_{m_F}]$ which is corresponding to the structure of $\Delta$.

The system has robust performance if and only if $\mu_\Delta(M) \leq 1$. Therefore, we need to determine $D$ which minimize the value of $\overline{\sigma}(DMD^{-1})$. The $\mu$ synthesis with $D - K$ iteration is used to construct such controller.

$\mu$ Synthesis Procedures [52]:

1. Initial set $D = I$
2. Hold this $D(s)$ and find a $K(s)$ to minimize $\|DMD^{-1}\|_\infty$. If $\|DMD^{-1}\|_\infty \leq 1$, stop.
   (MATLAB Command hinfopt, hinfsyn, hinfric or hinflmi)
3. Hold this $K(s)$ and find a $D(s)$ to minimize $\|DMD^{-1}\|_\infty$, which is the same to calculate $\mu_\Delta(M)$. (MATLAB Command ssv)
4. Use curve fitting method to construct a low-order transfer function to approximately represent $D_{\text{new}}(s)$. (MATLAB Command fitd)
5. Go back to step 2 until the condition is satisfied. (MATLAB Command augd)

$\mu$ Synthesis for Hydropower Plant:

The linear model for hydropower plant without any specified control method can be expressed as:
\[
\begin{align*}
\Delta \dot{c}_e &= -\frac{1}{t_p} \Delta x_e - \frac{1}{t_p} u \\
\Delta \dot{g} &= \frac{1}{t_g} \Delta x_e - \frac{1}{t_g} \Delta g \\
\Delta p_m &= -\frac{2}{t_g} \Delta x_e + \frac{2}{t_w} \Delta g + \frac{2}{t_g} \Delta g - \frac{2}{t_w} \Delta p_m \\
\Delta \dot{\omega} &= \frac{d_d}{m} \Delta \omega + \frac{1}{m} \Delta p_m - \frac{1}{m} \Delta p_e
\end{align*}
\]

Define the states as:
\[
\begin{align*}
x_1 &= \Delta x_e \\
x_2 &= \Delta g \\
x_3 &= \Delta p_m \\
x_4 &= \Delta \omega
\end{align*}
\]

The system parameters with uncertainties are defined as:
\[
\begin{align*}
t_p &= \bar{t}_p + w_p \delta_p \\
t_g &= \bar{t}_g + w_g \delta_g \\
t_w &= \bar{t}_w + w_w \delta_w \\
d &= \bar{d} + w_d \delta_d \\
m &= \bar{m} + w_m \delta_m
\end{align*}
\]

where \( \bar{t}_p, \bar{t}_g, \bar{t}_w, \bar{d} \) and \( \bar{m} \) are the nominal values; \( w_p, w_g, w_w, w_d \) and \( w_m \) are the weights to make the magnitudes of uncertainties \( \delta_p, \delta_g, \delta_w, \delta_d \) and \( \delta_m \) are less than one. Therefore, \( \frac{w_p}{\bar{t}_p}, \frac{w_g}{\bar{t}_g}, \frac{w_w}{\bar{t}_w}, \frac{w_d}{\bar{d}} \) and \( \frac{w_m}{\bar{m}} \) represent percentages of uncertainties in the parameters.
The parameter uncertainties can be separated from the nominal values based on the method shown in [52] and [63].

Figure F-2: Block Diagram of Hydropower Plant
If we use the full state feedback, where \( y = x \), the rearranged matrix based on Figure F-3 is as:
$$\dot{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \\ z \\ y \end{bmatrix}} = \begin{bmatrix} - \frac{1}{\bar{t}_p} & 0 & 0 & 0 & \frac{1}{\bar{t}_p} & 0 & 0 & 0 & 0 & - \frac{1}{\bar{t}_p} \\ \frac{1}{\bar{t}_g} & - \frac{1}{\bar{t}_g} & 0 & 0 & 0 & \frac{1}{\bar{t}_g} & 0 & 0 & 0 & 0 \\ - \frac{2}{\bar{t}_g} & \frac{2}{\bar{t}_g} & \frac{2}{\bar{t}_w} & - \frac{2}{\bar{t}_w} & 0 & 0 & - \frac{2}{\bar{t}_g} & \frac{2}{\bar{t}_w} & 0 & 0 & 0 \\ - \frac{w_p}{\bar{t}_p} & 0 & 0 & 0 & \frac{w_p}{\bar{t}_p} & 0 & 0 & 0 & 0 & 0 & - \frac{w_p}{\bar{t}_p} \\ \frac{w_g}{\bar{t}_g} & - \frac{w_g}{\bar{t}_g} & 0 & 0 & 0 & \frac{w_g}{\bar{t}_w} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{w_w}{\bar{t}_w} & - \frac{w_w}{\bar{t}_w} & 0 & 0 & 0 & \frac{w_g}{\bar{t}_w} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{w_d}{\bar{t}_d} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ q_2 \\ q_4 \\ q_6 \\ q_8 \\ q_{10} \\ \Delta p_e \\ u \end{bmatrix}$$

where $z$ is the output ($\Delta \omega$) we are interested. And we can partition the above matrix into two poles system.

$$a = \begin{bmatrix} - \frac{1}{\bar{t}_p} & 0 & 0 & 0 \\ \frac{1}{\bar{t}_g} & - \frac{1}{\bar{t}_g} & 0 & 0 \\ - \frac{2}{\bar{t}_g} & \frac{2}{\bar{t}_g} & \frac{2}{\bar{t}_w} & - \frac{2}{\bar{t}_w} \\ 0 & 0 & \frac{1}{\bar{t}_m} & - \frac{d_d}{\bar{t}_m} \end{bmatrix}$$

$$b_1 = \begin{bmatrix} \frac{1}{\bar{t}_p} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}$$

$$b_2 = \begin{bmatrix} - \frac{1}{\bar{t}_p} \\ 0 \\ \frac{1}{\bar{t}_g} \\ - \frac{2}{\bar{t}_g} \frac{2}{\bar{t}_w} \\ \frac{1}{\bar{t}_m} - \frac{d_d}{\bar{t}_m} \end{bmatrix}$$
\[
\begin{bmatrix}
-\frac{w_p}{\ell_p} & 0 & 0 & 0 \\
\frac{w_g}{\ell_g} & -\frac{w_g}{\ell_g} & 0 & 0 \\
0 & \frac{w_w}{\ell_w} & -\frac{w_w}{\ell_w} & 0 \\
0 & 0 & 0 & \frac{w_d}{\ell_w} \\
0 & 0 & \frac{w_m}{\ell_w} & -\frac{w_m \times \tilde{d}_d}{\ell_w} \\
0 & 0 & 0 & \frac{w_m}{\ell_w} \\
\end{bmatrix},
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{w_p}{\ell_p} & 0 & 0 & 0 & 0 \\
0 & \frac{w_g}{\ell_g} & 0 & 0 & 0 \\
0 & 0 & \frac{w_w}{\ell_w} & 0 & 0 \\
0 & 0 & 0 & \frac{w_m}{\ell_w} & -\frac{w_m \times \tilde{d}_d}{\ell_w} \\
0 & 0 & 0 & \frac{w_m}{\ell_w} & 0 \\
\end{bmatrix},
\begin{bmatrix}
\frac{w_p}{\ell_p} \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix},
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
q_2 \\
q_4 \\
q_6 \\
q_8 \\
q_{10} \\
\Delta p_e \\
\end{bmatrix} =
\begin{bmatrix}
\delta_p & 0 & 0 & 0 & 0 \\
0 & \delta_g & 0 & 0 & 0 \\
0 & 0 & \delta_w & 0 & 0 \\
0 & 0 & 0 & \delta_d & 0 \\
0 & 0 & 0 & \delta_m & 0 \\
0 & 0 & 0 & 0 & \Delta_d \\
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_3 \\
q_5 \\
q_7 \\
q_9 \\
z \\
\end{bmatrix}
\]

Since we do not have any repeated uncertainty, the \( D \) can be constructed as:
Then based on the \( \mu \) synthesis procedures mentioned previously, the controller can be designed.

Because the hydropower plant model is a singular case because \( d_{21} \) is zero as shown in (F.5), “hinfric” or “hinflmi” is used to minimize \( \|DMD^{-1}\|_\infty \).

MATLAB also has built in functions to handle \( \mu \) synthesis, \( D-K \) iteration, such as “dkit or musyn”. More details can be found in [64],[65] and [66].