USING FORECAST UPDATES
AND RISK-SHARING AGREEMENTS
IN A THREE-ECHelon SUPPLY CHAIN

A Thesis in
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by
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Abstract

Originating from a research project on a real-world business situation facing a high-tech company, this dissertation explores coordination schemes that can be used to align the incentives of an original equipment manufacturer (OEM) and its contract manufacturer (CM). The coordination is mediated by information (i.e., demand forecast updates) sharing and risk (i.e., overstock of components) sharing.

Three major problems are studied: (1) two-component newsvendor model in which the CM needs to make ordering decisions on two complementary components at two different stages with new demand information being revealed in between; (2) an OEM-CM joint planning problem in which the CM makes its own purchasing decisions while the OEM can influence the CM’s decisions through information-sharing or risk-sharing mechanisms; and (3) a two-mode problem in which a company can split its freight between a slow mode and a fast mode and needs to improve its shipping decisions using demand forecast updates. The main focus in this dissertation is on the joint planning problem in a three-echelon supply chain with complementary product structure.

Two key issues in this research are how to model information sharing and how to characterize risk sharing. In the current literature, bivariate normal (BVN) distribution serves as the prevailing method of modeling demand forecast updates; however, we cannot find general closed-form solutions for the order quantities in the first and second stages in a two-stage problem with this formulation. In order to obtain closed-form solutions, we use a set of two related uniform distributions (U-U) to develop a new model.
The U-U formulation models demand in the second stage as a uniform random variable whose mean is unknown in the first stage, although its width is known in both stages. We demonstrate that the new model simplifies the solution procedure and is an accurate approximation for the BVN model. To capture risk sharing between the OEM and the CM, we propose a contract arrangement under which the OEM compensates the CM for components that are left over at the end of the selling season. Three contract variants, representing different levels of risk sharing, are considered in this dissertation.

Using the U-U model, we first study the CM’s planning problem in isolation and obtain an in-depth understanding of the CM’s ordering behaviors. Under both the BVN and the U-U model, the CM tends to increase the order quantity of the component with a longer lead time if better information becomes available at the second stage. In doing so, the CM can reduce expected shortage and improve its overall position. Then we investigate the OEM-CM joint planning problem based on different combinations of information sharing and risk sharing. A thorough examination of the interaction between information sharing and risk sharing reveals many interesting and useful insights for management. We find that information sharing is not necessarily a substitute for risk sharing. Through information sharing alone, both the OEM and the CM are better off; however, information sharing, when combined with risk sharing under certain agreements, could hurt the OEM’s performance. Finally, we apply the U-U model to the two-mode problem in which a company needs to tradeoff better information against premium shipping rates. Computational results suggest that better information helps to improve both service and cost performance under the U-U model.
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Chapter 1

Introduction

Among the elements of successful supply chain management, information sharing and risk and reward sharing are of special importance. Sharing information increases the transparency of the supply chain as firms can see more clearly the flow of raw materials, semi-finished products and finished goods in the pipeline. With superior knowledge of demand and supply, firms can adjust their production or distribution plans to better respond to customer requirements. For example, Collaborative Planning, Forecasting and Replenishment (CPFR) focuses on improving the partnership between manufacturers and distributors/retailers through shared information and frequent communication. Under CPFR, different parties of a supply chain work together to reach a single agreed-upon demand forecast instead of multiple forecasts. Not only is historical data being considered, events related to supply and sales (for instance, promotion plans) are also taken into account. As a result, forecast accuracy is greatly improved, which in turn improves customer satisfaction.

Risk and reward sharing serves as the incentive program, which ensures that every participant can benefit from getting in a relationship. In real-world business, it is not unusual to see one firm having objectives in conflict with those of another firm in the same supply chain because their cost structures are different. In order for partnerships to work, there must be a mechanism to align the interests of all relevant parties so that
they move in the same direction. For instance Dell, well known for its direct sales model, sets very strict conditions and requirements for its component suppliers to enforce its low-inventory and assemble-to-order policy. To conform to the delivery requirements set by Dell, its suppliers must maintain a high inventory level at a warehouse close to Dell’s assembly plant. Although this increases the total costs for the suppliers, they get a long-term commitment from a stable client that also agrees to provide them with substantial information about consumer demand.

The development of information technology makes information exchange much easier than ever before. Meanwhile the application of supply chain management techniques creates more room for performance improvement through risk and reward sharing than there used to be. A supply chain can be coordinated by information sharing or risk sharing. However, before making a commitment to coordination, many firms would like to know whether or not it is always beneficial to share information or risk and reward with supply chain partners, how to evaluate coordination, and how the two options interact with each other. The dissertation attempts to address these problems in a specific supply chain environment, namely, how sharing demand forecast updates and sharing component overstock risk can be used to achieve Pareto improvements so that all parties involved are better off.
1.1 Problem Motivation

1.1.1 A Description of Related Business Processes

The research is motivated by real-world business scenarios confronting an original equipment manufacturer (OEM) and its contract manufacturer (CM). The OEM procures critical subassemblies from its CM, which in turn procures components from its upstream suppliers. Some parts are common components shared by different types of subassemblies. Subassemblies are built into end products of different models with different margins. Currently, the multi-stage planning-commitment process is performed sequentially in a decentralized way: OEM-to-CM, and CM-to-suppliers, with each party making its own procurement and production decisions individually. On the upstream side of the supply chain, component ordering decisions and build directions for subassemblies are made by the CM; on the downstream side of the supply chain, the OEM determines, given the availability of subassemblies and other parts, how many units to make for each end product model. A planning-commitment-delivery cycle is shown in Figure 1.1.

The steps in this cycle are:

1. The OEM updates its demand forecast for finished products periodically to reflect changes in market conditions and shares the information with its contract manufacturer.

2. The CM checks availability of components needed to build subassemblies and sends information about committed component supply back to the OEM.
3. Based on its own demand forecasts and supply commitment from the CM, the OEM makes order commitments for subassemblies.

4. After receiving orders from the OEM, the CM places component purchasing orders with its upstream suppliers.

5. Suppliers prepare components as requisitioned and ship them to the CM. The lead time of some components can be as long as four months.

6. The CM builds subassemblies after components show up and delivers them to the OEM, who then builds subassemblies into end products.

In an ideal world without uncertainty, the above business processes would run perfectly. Unfortunately, the business world is far from certain, which poses many risks and challenges for companies. Several things complicate the situation pictured above. For one thing, both demand for end products and supply of components are stochastic. Supply commitment by the CM could deviate considerably from predicted material requirements set by the OEM. This is illustrated in Figures 1.2 and 1.3, which plot the
Fig. 1.2. An Example of Demand Forecast

Fig. 1.3. An Example of Supply Commitment
demand forecast and supply commitment, respectively, for three parts to be delivered in May 2002 using real data from a real OEM. As of March 2002, the predicted demand for part A was 7,218, and the committed supply was 6,901; as it came to April 2002, the predicted demand increased to 7,396 while the committed supply decreased sharply to 4,014. For parts B and C, the demand forecast and supply commitment moved in the same direction, but supply still fell short of demand. When both demand and supply exhibit uncertainty, ordering decisions are difficult to make. For another thing, product structure is not simple. Although each finished product has only one critical subassembly, a number of different components are required to build each type of subassembly, with some components as common input to various subassemblies. It is not straightforward how the product structure affects procurement decisions. Furthermore, due to double marginalization\footnote{Double marginalization means each company in a supply chain receives only a portion of the total product margin. As a result, the incentives of supply chain members may not be aligned.}, the incentives of the CM may be in conflict with those of the OEM. For example, in cases of common component shortage, different subassemblies will compete for scarce resources. Build directions preferred by one party may be undesirable to the other. It is far from clear how coordination mechanisms should be designed to achieve a win-win result. Finally, different components have different lead times, which means that components are ordered sequentially at different time points. As mentioned earlier, the OEM updates its demand forecast periodically and exchanges the information with the CM. It is of interest to investigate how the CM uses newly obtained information to determine ordering strategies.
1.1.2 Problems to be Addressed

Given the complexities inherent in the business situation, the OEM suggests an alternative planning process, which has the OEM as the sole planning control point, allowing a single planning-commitment pass across all stages. According to the OEM, at least two reasons may be used to justify this planning method. One reason claims that the OEM, compared with the CM, observes fluctuations of end-product demand more closely and has better information about changes in market conditions; the other reason argues that the OEM knows more than the CM about supply of critical components and possesses more power to obtain additional supply. Theoretically, this can bring benefits to the OEM and may help to streamline the operations along the supply chain. In practice, however, it is very difficult to implement because the CM, as an independent entity, has its own objective functions, which differ from those of the OEM. Between the current planning process and the alternative process suggested by the OEM, a variety of choices might exist to coordinate the planning process such that a win-win outcome can be reached. In the dissertation, we assume that the planning process will be decentralized and therefore study Pareto-improving mechanisms. More specifically, this dissertation seeks to understand, model, and solve the following questions:

- Should the OEM share demand forecast revisions with the CM? If so, how do information updates affect the CM’s component-ordering decisions and total costs?

- Should the OEM share component overage risk with the CM? If so, how much risk should the OEM take, and how does this affect the performance of the OEM?
• Is information sharing a substitute for risk sharing? How do they interact with each other? And how do different combinations of information sharing and risk sharing affect the performance of the OEM, the CM and the supply chain as a whole?

Besides the questions, we also study a two-mode problem in which a company can split its freight shipments between a slow mode and a fast mode and needs to improve its shipping decisions using demand forecast updates. The motivation for pursuing this problem is the problem structure, i.e., the two-stage decision making process with recourse which occurs after some additional information has been gathered. The managerial insights to be drawn from this research are relevant to a wide variety of business, manufacturing and retail alike.

1.2 Thesis Structure

The thesis consists of six chapters. This section describes briefly the topics to be covered in each chapter.

Chapter 1 is a general introduction, which explains research motivation and defines problems to be addressed. It stresses the importance of coordination through information sharing or risk and reward sharing. The business scenarios facing an OEM and its CM are described in detail by an illustration of the planning-commitment-delivery processes currently in use.

Chapter 2 reviews, categorizes, and critiques current literature related to this research. Discussion focuses on the following items: (1) What problems have been addressed in this field? How are they formulated or modeled? What are the strengths
and weaknesses of those models? (2) What are the key findings in previous research concerning supply chain coordination? and (3) How does this dissertation research differ from previous work?

An in-depth understanding of the planning issue of the CM is a prerequisite to tackling the OEM-CM planning problem. Therefore, Chapter 3 addresses the CM’s planning problem, which is formulated as what we call a single-period two-component newsvendor problem. A new forecast update model is developed to capture demand forecast revisions at two different stages. This formulation makes it possible to generate closed-form expressions for optimal order quantities and expected total costs. The solutions derived from this model can be used to approximate the solutions derived from the bivariate normal model commonly seen in the literature. With closed-form solutions, this chapter studies and compares the properties of optimal component ordering policies for the CM under three scenarios characterized by different levels of demand information revelation between component purchase decision points.

Chapter 4 moves on to the larger problem, namely, the joint OEM-CM planning problem. The OEM always shares its demand forecast with the CM at the time when the CM must order the first (long lead time) component, but it must decide whether or not to share an updated forecast when the CM must order the second component. Furthermore, while the OEM and CM both incur a portion of the margin-based opportunity cost if demand is not fully satisfied, the OEM can decide whether or not to share the component overage costs that the CM would otherwise fully bear. The expressions obtained in Chapter 3 are used to investigate the OEM’s decisions regarding sharing the forecast update and sharing overage costs. The solution to the joint planning problem can be
posed as a search for optimal values of incentives that critically impact the planning relationship between the OEM and the CM. Three types of contracts characterized by different levels of risk sharing are considered and compared in terms of their effects on OEM and CM costs and their efficiency in coordinating the supply chain.

Chapter 5 applies the new forecast update model developed in Chapter 3 to solve a two-mode problem in which a company has the option to choose between two different shipping modes. One is slower but cheaper; the other is faster but more expensive. Since demand forecasts can be revised as time progresses, the firm can improve its position by splitting freight into two shipments, one by a fast mode and the other by a slow mode. Although a higher shipping charge may be incurred with the fast mode, the firm can benefit from the more accurate forecast by reducing shortage cost and overage cost. Using closed-form solutions, this chapter investigates how shipping decisions and cost performance are affected by the tradeoff between potential benefits brought about by better information and extra costs resulting from higher shipping rates.

Chapter 6 concludes the dissertation with a recap of the problems being addressed and a summary of major findings in previous chapters. Managerial insights derived from this research are also summarized. In addition, this chapter discusses potential extensions of the models presented in the thesis and points out directions for future research.

1.3 Thesis Contributions

The dissertation has five major contributions:
1. We explore supply chain coordination between an OEM and its CM in a two-component setting with demand forecast updates, while most papers in this field deal with only one-product problems due to the complexity of adding more components or products to the models.

2. We propose a new method to model information updates. A set of two related uniform distributions are used to develop closed-form solutions. As demonstrated in the dissertation, the model can be applied to other two-stage problems with similar forecast updating structure.

3. Previous papers limited their discussion to the two extreme cases of forecast updating (either no information at all or perfect information in the recourse stage) because they were unable to develop closed-form solutions for the case of only some stage-two information. The addition of the solutions for the some stage-two information scenario in this dissertation enriches the research findings and makes it possible to study the problems being addressed in greater depth.

4. We study the interaction between information sharing and risk sharing in a supply chain context. A new type of incentive agreement is proposed and studied. While previous research tended to focus on only one aspect (either information sharing or risk sharing), this dissertation seeks to understand how the two combined together affect the performance of the OEM, the CM, and the supply chain as a whole.

5. Finally, the uniform-uniform model developed in this research serves as an accurate approximation for the bivariate normal model. The solution procedures under the new model make computation more efficient.
Chapter 2

Literature Review

As mentioned in Chapter 1, the thesis first studies the response of the contract manufacturer to a demand forecast update, and then explores the possibility of coordinating the two parties in the supply chain. In accordance with this purpose, Chapter 2 provides a comprehensive review and critique of the most related literature. Papers are classified into three categories: (1) the first group addresses demand forecast updating; (2) the second group deals with channel coordination and supply chain contract design; and (3) the last group studies the two-mode problem. Before we proceed, it must be stated clearly that the division is mainly for the convenience of exposition, and there are no absolute boundaries between different groups. Some papers may span two or more categories. For example, Donohue (2000) studies the efficiency of supply contracts for fashion goods with forecast updating; therefore, the discussion of any given paper could easily extend to any given group.

2.1 Demand Forecast Updating and Information Sharing

There are a number of papers dealing with demand forecast updates for various purposes and hence in different problem settings. Significant research efforts have been directed to inventory or production planning problems with information updates, such as Hertz and Schaffir (1960), Murray and Silver (1966), Change and Fyffe (1971), Hausman
and Peterson (1972), Crowston et al. (1973), Bitran et al. (1986), Bradford and Sugrue (1990), Lovejoy (1990), Fisher and Raman (1996), Eppen and Iyer (1997a), Gurnani and Tang (1999), Weng and Parlar (1999), etc. Because of its simplicity, the classical newsvendor model serves as the underlying cornerstone for most of these works.

In a general framework for updating forecast, the demand or its agent variables are assumed to follow a particular distribution with unknown parameters. Also the decision maker is assumed to know the empirical sales pattern for one selling season. A sales pattern “might be expressed by defining for each period an average fraction of total seasonal demands that occur in this period, or an average cumulative fraction of total seasonal demands that occur through that period” (Crowston et al., 1973). Initial estimates can be computed using historical data from previous seasons. As new information, in the form of either early sales or certain signals, becomes available, the parameter values are revised periodically between decision points to refine demand forecast, which is then used to improve production or inventory decisions.

Hertz and Schaffir (1960) propose a forecasting method for managing the inventories of styled products with highly seasonal demand. Ordering decisions are made on a weekly basis. By grouping items into product lines, they use cumulative normal curves to approximate the growth curve pattern of total sales of a product line for the entire season. The forecasts of total sales for each individual item and the length of the selling season are revised every week based on actual up-to-date sales and the predicted sales ratio. The expected marginal profit from ordering one additional unit of each item is then calculated using the probability of selling the unit by the end of the season. If the expected marginal profit is positive for one item, an additional order will be placed;
otherwise, no new order is placed. Their model has limited applicability since in practice many seasonal items must be ordered well before or only early in the selling season. Moreover, setup costs and fixed ordering costs may challenge the feasibility of weekly ordering.

Murray and Silver (1966) analyze the inventory problem of style goods using a Bayesian method. In their model, a vendor sells an item which has a finite selling period with a varying sales rate. The vendor has a limited number of instances to buy the item, but orders must be placed at discrete time points and acquisition costs vary depending on the time of purchasing. Sales probabilities of the item are captured by a subjective random variable whose distribution function parameters are modified adaptively by taking into account new information occurring between two ordering instances. Dynamic programming is used to find the optimal decision for each period. As the authors point out, their model can only be applied to the situations where all demand, either fulfilled or lost, can be registered. In cases where stockouts occur and demand is not fully recorded, their model does not work.

Hausman and Peterson (1972) study multi-product production scheduling for style goods with limited capacity and forecast revisions. Their problem formulation resembles that of Murray and Silver (1966) except that the latter considers a single product problem while the former considers multiple products competing for the same production facility. In addition, they give a different framework to model demand forecast revisions without explicitly considering early sales. They assume that no delivery is required until the end of the selling season. The entire horizon up to the final delivery is divided into many production periods. A revised forecast is available at the beginning of each period.
Based on the results reported in Hausman (1969), it is assumed that demand forecasts exhibit the Markovian property, and hence ratios of successive forecasts of total demand are mutually independent. The forecast ratio follows a lognormal distribution whose parameters can be estimated through an analysis of historical data. By conditioning on the value of forecast ratio, the minimum expected cost for each period can be computed recursively.

Change and Fyffe (1971) consider the estimation of forecast error. The selling period for a seasonal item is divided into \( n \) subperiods. The forecasted demand \( v_i \) for each subperiod \( i (i = 1, 2, \ldots, n) \) is expressed as the product of a weight, \( m_i (\sum_{i=1}^{n} m_i = 1) \), and total seasonal demand \( D \) plus a Gaussian noise \( e_i \), i.e., \( v_i = m_i D + e_i \). The weights are computed by least squares estimation and represent the sales pattern within a season. In a two-step process, the initial estimates for total demand and its variance are computed first through regression analysis on the basis of historical trend, economic indexes and other environmental data; then forecasting error and total seasonal demand are adjusted progressively by linear filtering in light of actual sales experience from the last subperiod.

The above papers essentially all deal with serial systems. In a serial system, the product structure is such that each stage during the course of the production has at most one predecessor and at most one successor. In contrast with a serial system, an assembly system is more complicated in that some stages have more than one predecessor, thus making component ordering decisions more difficult. Crowston et al. (1973) consider a multi-stage assembly system for a seasonal good with random demand. They adopted the formulation presented by Chang and Fyffe (1971) to model the relationship between subperiod demand and total seasonal demand; however, they employ a different approach.
to revise demand forecasts. Initial information about demand is characterized by a normal prior distribution. Given prior parameters, subperiod sales fraction and noise parameters, the mean and variance of the updated distribution of total demand are revised periodically after observing the actual cumulative demand. To solve the multi-stage problem, they develop four heuristics and compare their cost performance.

The production or inventory planning problem with forecast revision is commonly stated as a dynamic program with two state variables, one for inventory position and the other for new information about demand. For the problem with a single item, it is possible to find the optimal solution computationally. However, when it comes to the multi-product or multi-stage cases with complicating constraints, it may be computationally inefficient to find optimal solutions. That is why some papers (see Murray and Silver 1966, Hausman and Peterson 1972, Crowston et al. 1973, etc.) use heuristics to handle the more complex cases. Hartung (1973) simplifies the model using only one state variable based on certain assumptions. Historical sales data are analyzed and transformed into a sequence of numbers, which represent the cumulative purchases up to a period as fractions of the total seasonal sales. The control rule is to order up to a fraction of estimated total sales updated periodically.

Bitran et al. (1986) consider a multi-period capacitated production planning problem of style goods with demand forecast revision. The problem has a two-level hierarchical structure defined by product families and items. The changeover costs between families are so high that only one production run is allowed for each family, but the changeover cost from one item to another is trivial. They use a joint normal distribution and a Bayesian method to model demand forecast revisions. The problem is formulated
as a stochastic mixed integer program, which can not be solved analytically. The em-
phasis of their work is thus to develop an algorithm to approximate the optimal solution
by decomposition. The implication for management from their computation is that the
company should produce those products with flat forecast patterns in early periods and
delay the production of the products whose forecast errors decrease over time.

Bradford and Sugrue (1990) develop a Bayesian approach to a two-period style-
goods inventory problem. They assume demand follows a heterogeneous Poisson dis-
tribution. The rate parameter is gamma distributed. This structure gives a negative
binomial distribution for the demand. There are two ordering opportunities, one at the
beginning of period one and the other at the beginning of period two. Demand esti-
mates are aggregated by items, and sales information from period one is incorporated
into the decision process of period two. In finding the optimal ordering policies, they
use aggregate demand to update parameter values and then apply these to each item.

Lovejoy (1990) studies the ordering policies for some inventory problems with
uncertain demand distributions. The author shows that a simple myopic order-up-to-
policy based on a critical fractile can be optimal or near-optimal in some parameter
adaptive models. In order to develop a simple solution, a bivariate dynamic program is
first reduced to a dynamic program with a single state variable; then the latter is further
reduced to a static optimization problem. The solution to the static problem serves as
a myopic solution to the original one. The solution procedure is illustrated through the
analysis of a single-product, discounted, discrete-time inventory model with exponential
smoothing or Bayesian updates of distribution parameters. The author argues that a
simple operational decision may be less expensive to evaluate and implement and is more readily accepted by practitioners.

Eppen and Iyer (1997a) consider a multi-period fashion buying problem for a catalog merchandiser who has to place orders about nine months ahead of the selling season. The company has two options of distribution: catalog sales and retail outlets. Excess inventory originally acquired for catalog sales is allowed to be diverted to outlet stores, but not vice versa. The paper attempts to help the buyer decide how much to order originally for catalog sale and how much to divert to the other distribution option. They assume that the underlying demand is generated by a process satisfying four properties: family consistency, sum efficiency, stochastic dominance, and a monotone likelihood ratio property. There are two levels of uncertainty: one at the process or distribution level and the other at the parameter level. They developed a heuristic that combines the newsvendor model and the Bayesian model to update a distribution. The control policy can be derived on the basis of an order-up-to level and a dump-down-to level.

The forecast mechanisms used in most of the papers discussed thus far are based on Bayesian rules in a broad sense. In recent years, the bivariate normal distribution prevails in modeling information updates. Fisher and Raman (1996) consider production commitment decisions, with capacity and minimum lot size constraints, by a skiwear manufacturer who can arrange the production of a family of goods in two runs, one occurring before the selling season and the other after observation of early sales. Such a quick response system allows a greater portion of production to be postponed because the lead time is sufficiently reduced. Their seminal work proposes a new method, using the
bivariate normal distribution, for estimating the demand probability distribution. More specifically, the correlation between initial demand observed during the first period and total demand is captured by a bivariate normal distribution. To cope with the capacity and lot-size constraints, the large-scale optimization problem is decomposed by applying a Lagrangian method. In solving the subproblems, they develop a lower bound, based on the work of Nahmias and Schmidt (1984), to approximate the optimal solution.

Gurnani and Tang (1999) develop a model to address the optimal ordering policy for a retailer who has the option to order a single product at two instants before the selling season. The wholesale price at the second instant is unknown and could be higher or lower than the price originally set for stage one, depending on market conditions. This means that the retailer’s purchasing cost for the second period is uncertain at the first instant, but the retailer may gather market information in between to better describe the demand distribution. Therefore the buyer, when making ordering decisions at the first instant, must trade off the benefit of better information against possibly higher cost. They also use the bivariate normal distribution to model demand forecast revisions.

Some papers address the value of information updates directly or indirectly. Jones et al. (2001) explore a two-period production-scheduling problem with random yields and demands based on a practice of the seed corn industry. They show that the opportunity to produce in the second period after observing new information about yield or demand is quite valuable to the seed industry; however, they do not model demand forecast updates explicitly. Cachon and Fisher (2000) study the value of information sharing in a supply chain with one supplier and \( N \) identical retailers assuming stationary random demand from consumers. They quantify the value of information exchange between
channel members. They also investigate the benefits of shorter lead time and smaller batch size resulting from information sharing. A comparison of these values suggests that information sharing is significantly more valuable when it is used to accelerate and smooth the physical flow of goods.

While it is widely believed that “better” information, generally characterized by smaller demand variability, helps improve performance, Ridder et al. (1998) show this is not always true. Using a newsvendor model, they find that larger demand variability may lead to lower costs. Ferguson et al. (2003) also find that under certain conditions, demand information updating can actually hurt both the seller and the buyer. Although most papers model demand forecast revision directly, some researchers proposed alternative methods. For example, Van der Duyn Schouten et al. (1994) develop a Markov model to quantify the value of information provided to a retailer by a supplier who produces to order in fixed production cycles. With information updates about lead times, not about demand itself, the retailer can improve its cost considerably using a fixed order strategy (s, Q) with dynamic reorder points.

Most of the papers discussed thus far consider ordering the same item at two or more instants under a wholesaler-retailer context. In other words, their model assumes that the same product can be produced or procured twice at two instants. Though some papers consider multiple items, the problems can be reduced essentially to single-item problems by decomposition or relaxation of certain constraints like limited capacity. The model presented in this dissertation considers in an OEM-CM setting the optimal ordering decision of two complementary items with different lead times. The items ordered at the two instants are different, and one item can be ordered only once. Therefore,
the stage-two decision is constrained by the stage-one order quantity, which means that
the CM will never order the component with shorter lead time more than the one with
longer lead time. Furthermore, while most papers (Fisher and Raman 1996, Gurnani
and Tang 1999, Donohue 2000) do not give closed-form solutions for the case of some
information at stage two because of the inherent complexity, the uniform-uniform model
developed in this dissertation allows closed-form expressions for optimal order quantities.

2.2 Channel Coordination and Contract Design

Tsay et al. (1999) present an excellent review of literature concerning the mod-
eling of supply chain contracts. To classify a wide variety of contracts, they propose a
scheme based on contract clauses and suggest eight categories: specification of decision
rights, pricing, minimum purchase commitments, quantity flexibility, buyback or returns
policies, allocation rules, lead time and quality. In this chapter, we mainly focus on the
work most closely related to this dissertation, such as the papers dealing with pricing
and buyback policy.

Jeuland and Shugan (1983) explore the coordination problem in a manufacturer-
retailer-consumer channel. They present a simple model and demonstrate that channel
coordination can improve the total profit for the supply chain. Several coordination
mechanisms, such as contracts, joint ownership, and profit sharing, are discussed and
compared. Their analysis implies that profit sharing is the most efficient among these
mechanisms. As a method of profit sharing, a quantity discount model is studied in
detail. Under a discount schedule, the wholesale price varies based on the quantity being
purchased by the retailer. The larger the purchase quantity, the lower the per unit
wholesale price. They show that a quantity discount schedule can align the incentives of the channel members so that the supply chain optimum is realized.

Pasternack (1985) studies the effect of buyback policies on channel coordination in a single-manufacturer, single-retailer system with single product. It is shown that neither a policy allowing full credit and unlimited return nor a policy not allowing returns at all works in achieving system optimality. However, full credit with partial return works well. In a multi-retailer environment, partial credit and unlimited returns is the only choice to coordinate channel. Lariviere and Porteus (2001) analyze price-only contracts in a similar system. Instead of a return policy, they investigate the effect of wholesale pricing. According to their analysis, as relative variability measured by the coefficient of variation decreases, the manufacturer’s share of realized profit increases, but the retailer’s performance may deteriorate because the manufacturer can manipulate the wholesale price. Thus it may not be in the retailer’s interests to improve demand forecast accuracy.

Cachon and Zipkin (1999) study the inventory policies in a serial supply chain with one supplier and one retailer and compare the difference between global/cooperative and independent/competitive optimization. Their analysis indicates that competition reduces efficiency for the supply chain while cooperation improves efficiency. In the case of competitive supply chain, the system optimal solution can be achieved by adopting a linear transfer payment contract.

Iyer and Bergen (1997b) investigate the impact of quick response (QR) in a channel consisting of a manufacturer and a retailer. Quick response is an industry practice aimed at shortening lead time. Under QR, the retailer can place orders closer to the start of the selling season to take advantage of better demand information. Their research
suggests that a manufacturer may not benefit from QR. To achieve Pareto improvement, the retailer should make commitments in terms of service level, wholesale price, or volume to provide incentives for the manufacturer. Under these mechanisms, the supply chain can be effectively coordinated so that both parties benefit from quick response.

Emmons and Gilbert (1998) study return or buyback policies for catalogue goods in a single-period problem with one manufacturer and one retailer. They demonstrate that uncertainty tends to increase the retail price. They show that under certain conditions the manufacturer can improve its own profit by allowing the retailer to return excess stock. For a fixed wholesale price, both the manufacturer and the retailer can benefit from buyback policies.

Weng (1999) investigates the coordination in a supply chain consisting of one manufacturer and one distributor for short-life-cycle products. He finds that if demand is stochastic and price sensitive, a quantity discount schedule alone is not sufficient to achieve supply chain optimality. A two-part tariff policy (i.e., a fixed payment plus unit wholesale price) is sufficient in coordinating the supply chain. However, in order to realize the maximum channel profit, the unit wholesale price must be set equal to the manufacturer’s unit production cost and a fixed transfer payment must be made by the distributor to the manufacturer as a compensation. Clearly, this policy is difficult to implement in practice unless the two parties are vertically integrated into one entity.

Pasternack (2002) compares outright sale with consignment (revenue sharing) in a single-period newsvendor problem. The retailer can either purchase a product from the manufacturer at a higher price or rent the product at a lower price. In the latter, the buyer has to share revenue with the manufacturer. Given fixed pricing structure, channel
coordination may be achieved using this revenue sharing scheme, but the manufacturer’s expected profit is likely to decrease without compensation from the buyer.

Tomlin (2003) investigates the impact of price-only contracts on capacity investment decisions in a supply chain comprising a manufacturer and a supplier. He proves that there exist a class of price-only contracts which coordinate the supply chain. The optimal contract for the manufacturer is a quantity-premium schedule if the supplier’s reservation profit is below a certain level. Under this arrangement, the per unit wholesale price increases with the order size. The manufacturer prefers simple piecewise-linear quantity-premium contracts to linear contracts.

The aforementioned papers generally analyze a simple supply chain composed of one manufacturer/wholesaler and one retailer. Gerchak and Wang (2002) compare two different coordination schemes between an assembly firm and its suppliers. One scheme is vendor-managed inventory with revenue sharing; the other is wholesale pricing. Their study shows that channel coordination can be achieved if the assembly firm agrees to share with the suppliers both revenue and overage cost for unsold components. They also find that revenue sharing is better than wholesale pricing in coordinating the supply chain if there are multiple suppliers.

Some researchers have considered channel or supply chain coordination with demand forecast updates (see Weng and Parlar 1999, Donohue 2000, Ferguson et al. 2003, Tsay 1999, Cachon and Lariviere 2001). Weng and Parlar (1999) explore prior-sale discounted sales promotions. They use the binomial distribution to model demand which is similar to the idea of sales potential proposed by Murray and Silver (1966). Suppose there are $n$ potential customers in a market for a product. The probability for each
customer to buy the product is $p$. Thus the total demand is characterized by a binomial distribution with parameters $(n, p)$. To coordinate the supply chain, the seller initiates a promotion in the form of prior-sale discount to induce early order commitment from customers. Offering a discount can increase the number of committed buyers and reduce demand variance. If there are $x$ customers who make early order commitment before the season to benefit from the discounted price, then the demand base is reduced from $n$ to $(n - x)$. This information is used to update the demand distribution. Since the posterior distribution has a variance smaller than the prior distribution, the seller can make better production or order decisions.

Donohue (2000) examines how a pricing scheme (i.e., specifying wholesale prices and return prices) can be used to achieve supply chain coordination between a manufacturer and a distributor under two production modes with forecast updating. The first production mode is cheaper than the second one, but it requires longer lead time. A general model, instead of a specific distribution, is used to deal with demand forecast updates. The author shows that supply chain optimality can be achieved if the wholesale prices and return prices are specified appropriately. However, the pricing conditions for optimality vary depending on the extent of information revelation between the two periods as well as the manufacturer’s access to forecast information.

Ferguson et al. (2003) investigate commitment decisions of an end-product manufacturer with partial information updates. The manufacturer has two choices of when to commit an order from its component supplier. The supplier arranges production based on the demand forecast from the manufacturer. Under an early commitment, the manufacturer must place its order before the supplier commit production resources; under a
delayed commitment, the manufacturer waits to order until observing new demand information and after the supplier begins production. In order to model partial information updates, they assume that the random variable, \( Z \), for total demand is composed of two independent variables, \( X \) and \( Y \) \((Z = X + Y)\). \( X \) represents the uncertainty resolved before the manufacturer places orders, while \( Y \) represents residual demand uncertainty. A parameter, \( \rho = \frac{\text{Var}(X)}{\text{Var}(X) + \text{Var}(Y)} \), is introduced to characterize additional information.

They compare the performance and order preferences of the buyer and the seller under four supply chain structures defined by who possesses the power to set the transfer price.

Eppen and Iyer (1997b) explore “backup agreements” in which a retailer makes an order commitment prior to the selling season and buys from the manufacturer a fraction of the committed quantity immediately. Purchasing of the remaining order can be delayed to give the buyer more flexibility, but a penalty cost will be charged for any committed unit the buyer does not purchase later on. Their analysis shows that backup agreement can improve expected profits substantially for both parties.

Tsay (1999) examines the quantity flexibility (QF) contract as a mechanism to align the incentives of the seller and the buyer. The QF contract specifies ranges by which purchasing and supply can deviate from the commitment. The planning-commitment process resembles that described in Chapter 1, but the paper does not model forecast updates explicitly. It is shown that the QF contract is capable of motivating the buyer and the supplier to improve supply chain efficiency.

Cachon and Lariviere (2001) study a contracting game between a manufacturer and a supplier. The manufacturer buys from the supplier a critical component and the latter makes capacity decisions. On the one hand, a contract is proposed by the
manufacturer, and the supplier needs to decide whether to accept it or not; on the other hand, the manufacturer needs to decide whether to share the demand forecast with the supplier or not. Under each information scenario, they consider two cases: (1) forced compliance and (2) voluntary compliance. Under forced compliance, the supplier has no freedom in making capacity decisions; under voluntary compliance, the supplier can make the capacity decisions as it wishes. They demonstrate that the contract compliance regime can significantly influence the analysis and outcome of the contracting game and it is always in the manufacturer’s interest to share information with the supplier when the demand forecast is high. They conclude that firm commitments cannot always align incentives in this setting.

Barnes-Schuster et al. (2002) study the role of options and coordination in a single-supplier, single-buyer system using a two-period model with correlated demands. In their model, the buyer needs to make order commitments for both periods and can buy options to purchase additional products in the second period. Before exercising options, the buyer can use information about realized demand in the first period to update the demand distribution for the second period. On the other hand, the supplier, the Stackelberg leader in this model, possesses the right to determine wholesale prices, option prices, and exercising prices and to schedule production. According to their analysis, channel coordination cannot be achieved under linear pricing arrangement. However, return policies, when combined with linear pricing, could be used to coordinate the supply chain.

Petruzzi and Dada (1999) examine an extension of the newsvendor problem in which the stocking policy and pricing policy are made simultaneously. By incorporating
pricing into the newsvendor problem, they analyze the interaction between operations and marketing within the same company. They do not consider channel coordination in their model.

Our research differs from previous work on channel coordination because we address a problem with complementary component structure and use a new method to model forecast updates as discussed in Chapter 3.

2.3 Mode Splits and Emergency Replenishments

Papers in this line of research essentially investigate the tradeoff between the benefits brought about by emergency ordering and consequential extra costs for expedition. Some papers that most relate to our research have been discussed in Section 2.1, like those by Fisher and Raman (1996) and Gurnani and Tang (1999). In this section, we discuss other work related to the two-mode problem.

Blumenfeld et al. (1985) evaluate the trade-off between freight expediting and inventory costs in a periodic review inventory system consisting of a supplier and a manufacturing plant. Their results indicate that a small safety stock can reduce freight expediting costs dramatically by reducing the likelihood of stockout. They also find that the expediting charge does not have much influence on the optimal trade-off.

Significant research efforts on the two-mode problem have been directed to the development of an approximate algorithm which simplifies the solution procedure. Moinzadeh and Nahmias (1988) present an approximate model for a continuous review inventory system with two supply modes. To incorporate emergency ordering, they modify the standard $(Q, R)$ policy to allow for two different lot sizes $(Q_1, Q_2)$ and two different
reorder levels \((R_1, R_2)\). They develop a procedure to determine the parameter values and use simulation to verify the heuristic policy. Tagaras and Vlachos (2001) investigate the two-mode problem in a periodic review inventory system. They also propose an improved heuristic optimization algorithm. They show that the option of emergency orders can reduce costs and improve the service level substantially. Moinzadeh and Schmidt (1991) propose an \((S - 1, S)\) policy under which \(S\) is the maximum stocking level, and an order is placed at the supplier every time a unit is demanded. Johansen and Thorstenson (1998) present similar work, but they deal with an inventory system with Poisson demand.

Yan et al. (2003) study the optimal ordering policy in a dual-supplier system with demand forecast updates. They derive expressions for the order quantities under each mode and evaluate the benefit of demand information updating. However, their demand forecast model is rather complex in that their demand distributions are expressed as functions of the forecast and the variance reduction parameter requires another discrete probability distribution. The managerial decision rules for computing optimal order quantities are not easily expressed.

Finally, as pointed out in Chapter 1, our research differs from previous research in many aspects. While most work in this field deals with only one-product problems, we explore coordinated production planning in a two-component setting with demand forecast updates. Also we propose a new method to model information updates which makes it possible to develop closed-form solutions. The addition of the solutions for the case of some stage-two information in this dissertation enriches the research findings.
An in-depth understanding is developed concerning the interaction between information sharing and risk sharing in a supply chain context.
Chapter 3

Forecast Updating and the
Two-Component Newsvendor Problem

As described in Chapter 1, the OEM sells to its customers a final product, which is built from a semi-finished product supplied by the CM. In order to manufacture products for the OEM, the CM must order two components that have different lead times. The CM uses an assemble-to-order system and is responsible for the procurement of both components. All components unused at the end of the selling season are salvaged by the CM based on market-determined salvage values.

This chapter studies the CM’s planning problem in isolation to understand the critical aspects of the problem of purchasing complementary components when information may be available to reduce uncertainty in demand between procurement decisions. Since two components are involved in the simplified model (see Figure 3.1), the CM’s problem is referred to as the two-component newsvendor (TCNV) problem hereafter. The most important aspect of this problem is that two complementary components with different lead times must be purchased under demand uncertainty regarding the product into which they are assembled. The two component purchase decisions can therefore be made, in general, at two different points in time, referred to hereafter as stage one—the latest point in time at which the purchase decision for the longer-lead-time component may be made—and stage two—the latest point in time at which the second purchase decision may be made. This chapter formulates and analyzes the TCNV first by modeling
demand as normally distributed and correlated with a normally distributed information signal. Since we cannot obtain closed-form solutions for the purchase decisions in this bivariate normal setting, this chapter demonstrates that the model can be approximated very closely by a uniform-demand model, which also makes it possible to obtain results for a “some information” case in closed form.

This chapter is organized as follows. Section 3.1 formulates the CM’s planning problem as a two-component newsvendor problem and proposes a new forecast update model using two related uniform distributions. Section 3.2 investigates the CM’s ordering behaviors using closed-form expressions derived under the new model. Section 3.3 describes how the new model can be used to approximate the bivariate normal model and evaluates the performance of the approximation. Section 3.4 summarizes research findings and presents major conclusions from this work.
3.1 Two-Component Newsvendor Model (TCNV)

Suppose the contract manufacturer must procure two components—component 1 (with unit cost $c_1$) and component 2 (with unit cost $c_2$)—to build a product for which demand $D$ is uncertain. Component 1 has a longer lead time and is therefore procured earlier than component 2; thus, sequential ordering decisions are made by the CM, with new demand information developing between the two decisions. Without loss of generality, the sum of per unit component cost is set equal to one, i.e., $c_1 + c_2 = 1$. The CM sells completed units to the OEM for $1 + m_c$ (i.e., at a markup rate $m_c$)\(^1\). The per unit salvage loss or overage cost of a component $i$ is $l_i$ ($i = 1, 2$). The CM seeks to minimize the sum of shortage cost (lost margin times unmet demand) and overage cost (lost value due to units left over). The beginning of the planning horizon is labeled as $t_1$. At this point in time, the CM must place an order, $Q_1$, for component 1. At some future point $t_2$, the demand forecast is updated, and the CM must place an order, $Q_2$, for component 2. The $Q_2$ decision is influenced not only by the $Q_1$ decision, but also by the update parameter whose value remains unknown until stage two. It is obvious that the CM will never order more of component 2 than component 1, i.e., $Q_2 \leq Q_1$.

The problem can be formulated as a two-stage dynamic program. Assume information updating is captured by a random variable $I$, whose realization determines the stage-two distribution of demand. The conditional $p.d.f.$ and $c.d.f.$ of demand given $I$

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\(^1\)To generalize, it may further be assumed that $m_c$ captures both markup and goodwill cost, thus encompassing the total cost of not fulfilling a unit of demand.
are denoted by $f_{D|I}$ and $F_{D|I}$. The CM’s cost function is

$$
TC^c(Q_1, Q_2) = m_c(D - Q_2)^+ + l_1(Q_1 - \min(D, Q_2))^+ + l_2(Q_2 - D)^+. \quad (3.1)
$$

The first term on the right-hand-side represents the underage cost; the second and third terms represent the overage costs of components 1 and 2, respectively. By conditioning on information variable $I$ and demand $D$, the optimization problem for the CM can be formulated as follows:

$$
\min_{0 \leq Q_1} \left\{ \mathbb{E}_I \left[ \min_{0 \leq Q_2 \leq Q_1} \mathbb{E}_{D|I} (TC^c) \right] \right\}. \quad (P1)
$$

Problem (P1) can be solved using backwards substitution by first finding the solution for the stage-two order quantity $Q_2$ and then substituting $Q_2^*$ into the stage-one problem to solve for $Q_1$.

### 3.1.1 Solving the Stage-Two Problem

Because $Q_2$ is a function of $Q_1$ and $I$, the CM first needs to solve for the optimal order quantity at stage two, $Q_2^*$, given $Q_1$ and $I$. The optimization problem at stage two is

$$
\min_{0 \leq Q_2 \leq Q_1} \mathbb{E}_{D|I} (TC^c) = \int_0^{+\infty} \left[ m_c(D - Q_2)^+ \\
+ l_1(Q_1 - \min(D, Q_2))^+ \\
+ l_2(Q_2 - D)^+ \right] f_{D|I} dD. \quad (P2)
$$
Problem (P2) can be easily solved because the objective at stage two is a convex function of $Q_2$.

**Theorem 1.** $E_{D|I}(TC^c)$ is a convex function of $Q_2$.

**Proof.** The method used by Fisher and Raman (1996) is adopted here to prove the theorem. Define

$$TC^c_0 = m_c(D - Q_2)^+ + (l_1 + l_2)(Q_2 - D)^+.$$  \hfill (3.2)

Since $Q_1$ is no less than $Q_2$, it is obvious that $TC^c$ can be rewritten as

$$TC^c = m_c(D - Q_2)^+ + (l_1 + l_2)(Q_2 - D)^+ + l_1(Q_1 - Q_2)$$
$$= TC^c_0 + l_1(Q_1 - Q_2).$$  \hfill (3.3)

Clearly, $TC^c_0$ is a convex function of $Q_2$. Note that $TC^c$ equals $TC^c_0$ plus a linear term of $Q_2$. Thus it is also a convex function of $Q_2$. $E_{D|I}(TC^c)$ is no more than the expected value of $TC^c$ on demand $D$, given new information $I$. Since the expectation operation preserves convexity, $E_{D|I}(TC^c)$ is also convex in $Q_2$. \hfill \qed

Thus $Q_2^*$ can be obtained by solving the first-order condition:

$$\frac{dE_{D|I}(TC^c)}{dQ_2} = -l_1 + (l_1 + l_2)F_{D|I}(Q_2) - m[1 - F_{D|I}(Q_2)] = 0.$$  \hfill (3.4)

Let $\alpha_2$ be the stage-two critical fractile. It follows that

$$\alpha_2 = F_{D|I}(Q_2^*) = \frac{m_c + l_1}{m_c + l_1 + l_2}.$$  \hfill (3.4)
This implies that under the complementary product structure the per unit underage cost of component 2 is the sum of product margin plus the per unit overage cost of component 1. In other words, if the CM underbuys one unit of component 2, it will not only miss the opportunity to make a profit in the amount of $m_c$, but will also have to salvage one unit of component 1 which results in a loss of $l_1$. Define the optimal unconstrained stage-two order quantity as

$$\beta_2 = F^{-1}_{D|I}(\alpha_2).$$

(3.5)

The optimal order quantity of component 2 therefore equals the smaller value of $Q_1$ and $\beta_2$, i.e.,

$$Q^*_2 = \min(Q_1, \beta_2).$$

### 3.1.2 Some Properties of the Stage-One Problem

With the expression for $Q^*_2$ developed in previous section, this section proceeds to the next step of finding the optimal order quantity for component 1. Before formulating and solving the CM’s optimization problem at stage one, this section first presents some preliminary results.

Let $TC^c_2$ denote the minimum value of the stage-two cost function $\mathbb{E}_{D|I}(TC^c)$:

$$TC^c_2 = \min_{0 \leq Q_2 \leq Q_1} \mathbb{E}_{D|I}(TC^c).$$

(3.6)

**Theorem 2.** $TC^c_2$ is convex on $Q_1 \geq 0$. 

Proof. According to Theorem 1, $\mathbb{E}_{D|I}(TC^c)$ is a convex function of $Q_2$. $TC^c_2$ is the minimum value of $\mathbb{E}_{D|I}(TC^c)$ subject to the constraint $0 \leq Q_2 \leq Q_1$. Because the minimum value of a convex function subject to linear constraints is convex in the right-hand side of the constraints, it follows that $TC^c_2$ is a convex function of $Q_1$. \qed

Theorem 2 indicates that the optimal expected total cost for the CM at stage two, $TC^c_2$, is a convex function of the stage-one order quantity. The stage-one optimization problem can be formulated by conditioning on information variable $I$.

$$\min_{0 \leq Q_1} \left\{ \mathbb{E}_I \left[ TC^c_2 \equiv \min_{0 \leq Q_2 \leq Q_1} \mathbb{E}_{D|I}(TC^c) \right] \right\}$$ (P3)

The objective function at stage one also exhibits nice properties as suggested by Theorem 3, which facilitates the solution procedure.

**Theorem 3.** The CM’s cost function at stage one, $\mathbb{E}_I(TC^c_2)$, is convex in $Q_1$.

Proof. According to Theorem 2, $TC^c_2$ is convex in $Q_1$. Note that the CM’s cost function at stage one is simply the expected value of $TC^c_2$ on $I$. Hence, $\mathbb{E}_I(TC^c_2)$ is also a convex function of $Q_1$ since expectation preserves convexity. \qed

The results discussed thus far are independent of the specific forms of demand distribution. However, in order to study the properties of stage-one ordering behaviors, specific distributions must be employed. One of the key issues in the TCNV problem is how to model demand forecast updates between the two ordering instants. The sections that follow investigates the CM’s ordering behavior and cost performance using specific distributions.
3.1.3 TCNV with Normal Demand and Correlated Demand-Related Information

Suppose demand $D$ at stage one follows a normal distribution with mean $\mu_1$ and standard deviation $\sigma_1$. Between the two ordering instants, the CM can get some new information, characterized by random variable $X$, to update the demand forecast. Without loss of generality, assume the information or signal variable $X$, with p.d.f. $g_X(\cdot)$ and c.d.f. $G_X(\cdot)$, follows the standard normal distribution. The correlation coefficient between $D$ and $X$ is $\rho$ ($0 \leq \rho \leq 1$). A realization, $x$, of the signal can be used to update the p.d.f. of demand at stage two to reflect newly-obtained information. Demand at stage two also follows a normal distribution with mean $\mu_2 = \mu_1 + \rho \sigma_1 x$ and standard deviation $\sigma_2 = \sigma_1 \sqrt{1 - \rho^2}$. The bigger the $\rho$, the more information the CM has at stage two because the variance is smaller. Though the variance at stage two is known at stage one, the mean at stage two remains unknown until the information variable $X$ is realized. Let $f_{D|X}(\cdot)$ and $F_{D|X}(\cdot)$ be the conditional p.d.f. and c.d.f. of demand, given the realization of $X$. Using the above notation, the problem of finding optimal order quantity for component 1 can be formulated as:

$$
\min_{0 \leq Q_1} \mathbb{E}(TC_2^C) = \int_{-\infty}^{+\infty} \int_0^{+\infty} [m_c(D - Q_2)^+] \nonumber \\
+ l_1(Q_1 - \min(D, Q_2))^+ 

+ l_2(Q_2 - D)^+] f_{D|X}^{(D)}(D) g_X(x) \, dD \, dx. 
\text{(P4)}$$
Again, the three terms within brackets represent underage cost, component 1 underage and component 2 underage respectively. The stage-two ordering level is determined as:

$$\beta_2 = F_{D|X}^{-1}(\alpha_2) = \mu_1 + \rho \sigma_1 x + Z_{\alpha_2} \sigma_1 \sqrt{1 - \rho^2},$$

where $Z_{\alpha_2}$ is the standard normal score. Let $\phi(\cdot)$ and $\Phi(\cdot)$ be the p.d.f. and c.d.f. of the Standard Normal distribution. By substituting $Q_2^* = \min(Q_1, \beta_2)$ into the cost function, Problem (P4) can be rewritten as Problem (P5).

$$\begin{align*}
\min_{0 \leq Q_1} E(TC_2^C) &= \int_{-\infty}^{x'} \sigma_2 \left[ - (m_c + l_1) Z_{\alpha_2} + l_1 (Q_1 - \mu_2) / \sigma_2 \
+ (m_c + l_1 + l_2) [\phi(Z_{\alpha_2}) + \alpha_2 Z_{\alpha_2}] \right] g_X(x) dx \\
&\quad + \int_{x'}^{+\infty} \sigma_2 \left[ - m_c (Q_1 - \mu_2) / \sigma_2 \
+ (m_c + l_1 + l_2) \left[ \phi((Q_1 - \mu_2) / \sigma_2) \right] \right] g_X(x) dx.
\end{align*}$$

(P5)

where

$$x' = \frac{Q_1 - \mu_1 - Z_{\alpha_2} \sigma_1 \sqrt{1 - \rho^2}}{\rho \sigma_1},$$

the cutoff point at which $Q_1 = \beta_2$ (see Appendix A for details).

The cost function in Problem (P5) exhibits convexity within the domain of $Q_1$ because the second-order derivative is non-negative (see the proof of Theorem 4). Hence there exists a $Q_1^*$ such that the expected total cost is minimized. Although we cannot find a closed-form expression for $Q_1^*$ for $0 < \rho < 1$, it is possible to investigate how $Q_1^*$ behaves as $\rho$ changes.
**Theorem 4.** Under the bivariate normal distribution, more stage two information leads to a larger optimal order for component 1, i.e., \( dQ_1^*/d\rho \geq 0 \).

**Proof.** Define \( h = \frac{Q_1 - \mu_2}{\sigma_2} = \frac{Q_1 - \mu_1 - \rho \sigma_1 x}{\sigma_1 \sqrt{1 - \rho^2}} \).

The first-order derivative of the cost function in Problem (P5) w.r.t. \( Q_1 \) is

\[
\frac{d\mathbb{E}[TC_{c2}^c]}{dQ_1} = \int_{-\infty}^{x'} l_1 g_X(x) \, dx + \int_{x'}^{+\infty} [(m_c + l_1 + l_2) \Phi(h) - m_c] g_X(x) \, dx. \tag{3.7}
\]

The second-order derivative is

\[
\frac{d^2\mathbb{E}[TC_{c2}^c]}{dQ_1^2} = \frac{m_c + l_1 + l_2}{\sigma_1 \sqrt{1 - \rho^2}} \int_{x'}^{+\infty} \phi(h) g_X(x) \, dx > 0. \tag{3.8}
\]

The first-order condition implicitly defines a function between \( Q_1^* \) and \( \rho \). Differentiating equation (3.7) with respect to \( \rho \), we obtain:

\[
\frac{d^2\mathbb{E}[TC_{c2}^c]}{dQ_1 \, d\rho} = (m_c + l_1 + l_2) \int_{x'}^{+\infty} [\partial F_{D|X}(Q_1) / \partial \rho] g_X(x) \, dx. \tag{3.9}
\]

Lemma 5.1 in Brown and Lee (1997) states that for any values of \( a \) and \( b \),

\[
\int_{a}^{b} \frac{\partial F_{D|X}(Q_1)}{\partial \rho} g_X(x) \, dx = \left[ \exp(-\hat{b}) - \exp(-\hat{a}) \right] / 2\pi \sqrt{1 - \rho^2} \tag{3.10}
\]
where

\[
\hat{a} = \left( Q_1^2 + \hat{a}^2 - 2\hat{a}Q_1\rho \right) / \left[ 2(1 - \rho^2) \right]
\]  
(3.11)

\[
\hat{b} = \left( Q_1^2 + b^2 - 2bQ_1\rho \right) / \left[ 2(1 - \rho^2) \right]
\]  
(3.12)

\[
\hat{a} = \left( Q_1 - (\mu + \rho\sigma x) / (\sigma \sqrt{1 - \rho^2}) \right)
\]  
(3.14)

\[
\hat{b} = (b - \mu) / \sigma.
\]  
(3.16)

It follows that

\[
\frac{d^2 E[T_{C_2}]}{dQ_1 d\rho} < 0.
\]

Thus we have

\[
\frac{dQ_1^*}{d\rho} = - \frac{d^2 E[T_{C_2}]}{dQ_1 d\rho} / \frac{d^2 E[T_{C_2}]}{dQ_1^2} > 0.
\]

The implication of Theorem 4 is that the CM tends to order more component 1 if more demand information is available at stage two.
At the two end points where \( \rho = 0 \) or 1, Problem (P5) reduces to a traditional newsvendor problem with normal distribution. Define

\[
\alpha_n = \frac{m_c}{m_c + l_1 + l_2},
\]

(3.17)

\[
\alpha_p = \frac{m_c}{m_c + l_1}.
\]

(3.18)

The fraction \( \alpha_n \) is actually the critical fractile for the case where no information updates are available at stage two \( (\rho = 0) \), and \( \alpha_p \) is for the case where exact demand becomes known at stage two \( (\rho = 1) \). The optimal solutions for these two cases are given by

\[
Q^*_1|\rho=0 = \mu_1 + Z_{\alpha_n} \sigma_1,
\]

(3.19)

and

\[
Q^*_1|\rho=1 = \mu_1 + Z_{\alpha_p} \sigma_1.
\]

(3.20)

### 3.1.4 TCNV with Uniform Demand and Uniform Mean

As stated in the previous section, we did not find closed-form solutions for the BVN model if \( 0 < \rho < 1 \). Now consider another approach to model demand forecast updates. Suppose demand \( D \) follows at stage two \( (\text{time } t_2) \) a uniform distribution \( D \sim U[\mu - \delta, \mu + \delta] \). At stage one \( (\text{time } t_1) \), we assume that the width parameter \( \delta \) is known, but the stage-two mean, \( \mu \), is a random variable with probability distribution \( \mu \sim U[\hat{\mu} - \delta, \hat{\mu} + \delta] \). \( \hat{\mu} \) is the expectation of \( \mu \) at stage one. Let \( \underline{\mu} = \hat{\mu} - \delta \) and \( \overline{\mu} = \hat{\mu} + \delta \) denote the lower and upper limit of \( \mu \), respectively. To facilitate exposition, we introduce
\(0 \leq \epsilon \leq 1\) and define \(\delta = \hat{\mu} \epsilon / 2\). With this formulation, the demand distribution at stage one when the CM places order for component 1 follows a symmetric triangular distribution with lower and upper limits \(\hat{\mu}(1 - \epsilon)\).

**Theorem 5.** If stage-two demand follows a uniform distribution \(U(\mu - \delta, \mu + \delta)\) with mean \(\mu\) uniformly distributed over \((\hat{\mu} - \delta, \hat{\mu} + \delta)\), stage-one demand follows a symmetric triangular distribution \(\text{tri}(\hat{\mu}(1 - \epsilon), \hat{\mu}(1 + \epsilon), \hat{\mu})\), where \(\epsilon \in [0, 1]\) and \(\delta = \hat{\mu} \epsilon / 2\).

**Proof.** Given a random variable \(Y \sim U(\mu - \delta, \mu + \delta)\), the minimum and maximum values of \(Y\) are \(\hat{\mu} - 2\delta\) and \(\hat{\mu} + 2\delta\), respectively. The probability \(P(Y = y)\) can be calculated as follows.

If \(\hat{\mu} - 2\delta \leq y \leq \hat{\mu}\), then

\[
P(Y = y) = \int_{\mu}^{y + \delta} \left( \frac{1}{(\mu + \delta) - (\mu - \delta)} \right) \left( \frac{1}{(\hat{\mu} + \delta) - (\hat{\mu} - \delta)} \right) dY
\]

\[
= \frac{y - (\hat{\mu} - 2\delta)}{4\delta^2}.
\]

(3.21)

If \(\hat{\mu} \leq y \leq \hat{\mu} + 2\delta\), then

\[
P(Y = y) = \int_{y - \delta}^{\hat{\mu}} \left( \frac{1}{(\mu + \delta) - (\mu - \delta)} \right) \left( \frac{1}{(\hat{\mu} + \delta) - (\hat{\mu} - \delta)} \right) dY
\]

\[
= \frac{(\hat{\mu} + 2\delta) - y}{4\delta^2}.
\]

(3.22)

Equations (3.21) and (3.22) are the probability density functions for the triangular distribution with parameters \((\hat{\mu}(1 - \epsilon), \hat{\mu}(1 + \epsilon), \hat{\mu})\).
New information helps to refine a demand forecast. With \( \mu \) becoming known at stage two, demand follows a uniform distribution when the CM orders component 2. Because the uniform-uniform structure is used to model forecast updating, this model is named the U-U model. Three scenarios are considered based on the degrees of demand information revelation at stage two.

1. *No stage two information (NSTI)*, in which the demand forecast is not updated prior to the component 2 order. This case simplifies to a traditional newsvendor formulation.

2. *Some stage two information (SSTI)*, in which the forecast is updated before the component 2 order, but there is still some demand uncertainty when this order is placed. In order to derive closed-form solutions in this demand setting, we choose a specific level of demand revelation between decision stages.

3. *Perfect stage two information (PSTI)*, in which the exact demand value is known at stage 2 when component 2 is ordered. This is equivalent to the situation where component 2 has no salvage loss. Note that this does not mean all demand is satisfied, since there may be a shortage of component 1. As with NSTI, this case reduces to a traditional newsvendor formulation.

Examples of the stage-one and stage-two demand distributions for the three scenarios are given in Figure 3.2. For all three scenarios, demand follows at stage one the same triangular distribution \( \text{tri}(50,150,100) \). For NSTI, the demand distribution remains unchanged at stage two since no new information is available to revise the forecast. For SSTI, the stage-two demand distribution follows a uniform distribution with
width 50, but the mean of the uniform distribution can be anywhere between 75 and 125. The dashed line shows an example of the stage-two distribution for $\mu = 90$. For PSTI, the precise demand becomes known, and therefore the demand distribution reduces to a single point in the figure.

The cost formulation for NSTI and PSTI is simply a newsvendor model with triangular distribution (see Appendix B for details); however, the formulation of the CM’s cost function under SSTI is not that straightforward. The first thing that needs to be resolved is to find the stage-two critical ordering level $\beta_2$, which can be calculated using the inverse function of the uniform distribution and the results developed earlier in this chapter:

$$
\beta_2 = F^{-1}_{D|\mu}(\alpha_2) = \mu + (2\alpha_2 - 1)\delta.
$$

(3.23)

Let $\underline{\beta_2}$ and $\overline{\beta_2}$ be the minimum and maximum values of $\beta_2$, respectively. Given $Q_1$, the CM’s cost function under SSTI can be formulated by conditioning on $\mu$.

1. If $Q_1 \leq \underline{\beta_2}$, the CM will order $Q_2$ equal to $Q_1$. Therefore, its cost expression in this case is the same as that for NSTI:

$$
\mathbb{E}(TC^c_{SSTI}) = \mathbb{E}(TC^c_{NSTI}).
$$

(3.24)
Fig. 3.2. Example of demand distributions under NSTI, SSTI, and PSTI
2. If $\beta_2 < Q_1 \leq \hat{\mu}$, the CM’s cost expression is

$$
E(TC_{SSTI}^c) = \int_{\mu}^{\bar{\mu}'} \left[ \int_{\mu-\delta}^{\beta_2} [l_1(Q_1 - D) + l_2(\beta_2 - D)] f_{D|\mu}(D) dD \\
+ \int_{\beta_2}^{\mu+\delta} [m_c(D - \beta_2) + l_1(Q_1 - \beta_2)]
\times f_{D|\mu}(D) dD \right] g_{\mu}(\mu) d\mu
\\
+ \int_{\mu}^{Q_1+\delta} \left[ \int_{\mu-\delta}^{\beta_2} [l_1 + l_2](Q_1 - D)] f_{D|\mu}(D) dD \\
+ \int_{\beta_2}^{\mu+\delta} [m_c(D - Q_1)] f_{D|\mu}(D) dD \right] g_{\mu}(\mu) d\mu
\\
+ \int_{Q_1}^{\bar{\mu}} \left[ \int_{\mu-\delta}^{\mu+\delta} [m_c(D - Q_1)] f_{D|\mu}(D) dD \right] g_{\mu}(\mu) d\mu.
$$

(3.25)

3. If $\hat{\mu} < Q_1 \leq \bar{\beta}_2$, the CM’s cost expression is

$$
E(TC_{SSTI}^c) = \int_{\mu}^{\bar{\mu}'} \left[ \int_{\mu-\delta}^{\beta_2} [l_1(Q_1 - D) + l_2(\beta_2 - D)] f_{D|\mu}(D) dD \\
+ \int_{\beta_2}^{\mu+\delta} [m_c(D - \beta_2) + l_1(Q_1 - \beta_2)]
\times f_{D|\mu}(D) dD \right] g_{\mu}(\mu) d\mu
\\
+ \int_{\mu}^{Q_1+\delta} \left[ \int_{\mu-\delta}^{\beta_2} [(l_1 + l_2)(Q_1 - D)] f_{D|\mu}(D) dD \\
+ \int_{\beta_2}^{\mu+\delta} [m_c(D - Q_1)] f_{D|\mu}(D) dD \right] g_{\mu}(\mu) d\mu.
$$

(3.26)
4. If $\overline{\beta_2} < Q_1$, the CM will always have $Q_2$ equal to $\beta_2$. Its cost expression is

$$E(TC_{SSTI}^c) = \int_{\mu}^{\overline{\mu}} \left[ \int_{\mu-\delta}^{\mu} \left[ l_1(Q_1 - D) + l_2(\beta_2 - D) \right] f_{D|\mu}(D) dD ight. \\
+ \left. \int_{\beta_2}^{\mu+\delta} \left[ m_c(D - \beta_2) + l_1(Q_1 - \beta_2) \right] \right] f_{D|\mu}(D) dD \right] g_\mu(\mu) d\mu.$$  (3.27)

Although the CM’s cost function is piecewise under the uniform-uniform formulation, it is still convex in $Q_1$ according to Theorem 3. Closed-form expressions for the optimal solution $Q_1^*$ can be obtained by solving the first-order conditions, which makes it possible to analytically examine the properties of optimal ordering decisions.

### 3.2 Analytical Results

This section presents some analytical results based on the U-U version of the two-component newsvendor problem.

#### 3.2.1 Ordering Decisions

Let $Q_{1, NSTI}^*$ be the optimal order quantity of component 1 under NSTI, $Q_{1, SSTI}^*$ and $Q_{1, PSTI}^*$ are defined similarly. Under NSTI and PSTI, the optimal order quantity for component 1 can be calculated using the solution to the classic newsvendor problem:

$$Q_{1, NSTI}^* = \begin{cases} 
\hat{\mu}(1 - \epsilon) + \hat{\mu} \epsilon \sqrt{2\alpha_n} & \text{if } \alpha_n \leq 0.5; \\
\hat{\mu}(1 + \epsilon) - \hat{\mu} \epsilon \sqrt{2(1 - \alpha_n)} & \text{if } \alpha_n > 0.5.
\end{cases}$$  (3.28)
Under SSTI, the $Q^*_1$ decision is complicated by the order quantity of component 2, which depends on the relationship between $\beta_2$ and $Q_1^*$:

\[
Q^*_{1,PSTI} = \begin{cases} 
\hat{\mu}(1 - \epsilon) + \hat{\mu} \epsilon \sqrt{2\alpha_p} & \text{if } \alpha_p \leq 0.5; \\
\hat{\mu}(1 + \epsilon) - \hat{\mu} \epsilon \sqrt{2(1 - \alpha_p)} & \text{if } \alpha_p > 0.5.
\end{cases} \tag{3.29}
\]

Under SSTI, the $Q_1$ decision is complicated by the order quantity of component 2, which depends on the relationship between $\beta_2$ and $Q_1^*$:

\[
Q^*_{1,SSTI} = \begin{cases} 
Q^*_{1,NSTI} & \text{if } 0 \leq \alpha_n \leq \frac{\alpha_2}{2}; \\
\hat{\mu} \left[ 1 + \left( \frac{\alpha_2}{2} + \frac{\alpha_n}{\alpha_2} - 1 \right) \epsilon \right] & \text{if } \frac{\alpha_2}{2} < \alpha_n \leq \alpha_2 - \frac{\alpha_2}{2}; \\
\hat{\mu} \left[ 1 + \left( \alpha_2 - \sqrt{2(\alpha_2 - \alpha_n)} \right) \epsilon \right] & \text{if } \alpha_2 - \frac{\alpha_2}{2} < \alpha_n \leq \alpha_2.
\end{cases} \tag{3.30}
\]

Note that there are four expressions for the total cost but only three expressions for $Q^*_1$. This is because the last total cost expression is monotonically increasing in $Q_1$ (the first-order derivative is $l_1 \geq 0$). Therefore, the optimal order quantity in this case is simply $\beta_2$, which is captured by the third expression for $Q^*_1$. This makes sense because the CM, knowing the maximum feasible order quantity for component 2 is $\beta_2$, will never order more of component 1 than that.

Under the U-U model, $Q^*_1$ has the same property as it does under the BVN model.

**Theorem 6.** The optimal order quantity of component 1 increases as the CM has better information at stage two, $Q^*_1,NSTI \leq Q^*_1,SSTI \leq Q^*_1,PSTI$.

**Proof.** This proof consists of two parts. The first part shows that $Q^*_1,NSTI \leq Q^*_1,SSTI$; the second part proves that $Q^*_1,SSTI \leq Q^*_1,PSTI$. 
Part I: $Q^*_{1,NSTI} \leq Q^*_{1,SSTI}$

Clearly, this is equivalent to showing that

$$Q^*_{1,SSTI} - Q^*_{1,NSTI} \geq 0.$$  \hfill (3.31)

According to equation (3.30), $Q^*_{1,SSTI}$ can take one of three forms depending on the relationship between $\alpha_n$ and $\alpha_2$. If $0 \leq \alpha_n \leq \frac{\alpha_2^2}{2}$, then $Q^*_{1,SSTI} = Q^*_{1,NSTI}$ and inequality (3.31) is true. Next it is shown that the relationship also holds when $Q^*_{1,SSTI}$ takes other forms.

1. $\frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2 - \frac{\alpha_2^2}{2}$

   In this case, $\alpha_n \leq \alpha_2 - \frac{\alpha_2^2}{2}$. Since $\alpha_2 - \frac{\alpha_2^2}{2} = \frac{1-(1-\alpha_2)^2}{2} \leq \frac{1}{2}$, $\alpha_n$ is also no greater than $\frac{1}{2}$. So only the expression for $Q^*_{1,NSTI}$ when $\alpha_n \leq 0.5$ should be considered.

   Substitute the appropriate expressions into inequality (3.31) and simplify:

   $$Q^*_{1,SSTI} - Q^*_{1,NSTI} = \hat{\mu} \epsilon \left[ \frac{\alpha_2}{2} + \frac{\alpha_n}{\alpha_2} - \sqrt{2\alpha_n} \right] \geq 0.$$ 

   The above inequality is equivalent to

   $$\frac{\alpha_2}{2} + \frac{\alpha_n}{\alpha_2} - \sqrt{2\alpha_n} \geq 0.$$ 

   Rearrange,

   $$\frac{\alpha_2}{2} + \frac{\alpha_n}{\alpha_2} \geq \sqrt{2\alpha_n}.$$
The above inequality holds because

\[ \frac{\alpha_2}{2} + \frac{\alpha_n}{\alpha_2} \geq 2 \sqrt{\left( \frac{\alpha_2}{2} \right) \left( \frac{\alpha_n}{\alpha_2} \right)} = \sqrt{2\alpha_n}. \]

This shows that \( Q_{1,SSTI}^* - Q_{1,NSTI}^* \geq 0 \) for \( \frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2 - \frac{\alpha_2^2}{2} \).

2. \( \alpha_2 - \frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2 \)

There are two subcases to consider because \( Q_{1,NSTI}^* \) can take either of the two forms depending on whether \( \alpha_n \) is greater than 0.5 or not.

(a) \( \alpha_n \leq 0.5 \)

Substitute the appropriate expressions of \( Q_{1,NSTI}^* \) and \( Q_{1,SSTI}^* \) into inequality (3.31) and simplify:

\[ Q_{1,SSTI}^* - Q_{1,NSTI}^* = \hat{\mu} \left[ \alpha_2 - \sqrt{2(\alpha_2 - \alpha_n)} + 1 - \sqrt{2\alpha_n} \right] \geq 0. \]

The above inequality is equivalent to

\[ \alpha_2 - \sqrt{2(\alpha_2 - \alpha_n)} + 1 - \sqrt{2\alpha_n} \geq 0. \]

Rearrange,

\[ 1 + \alpha_2 \geq \sqrt{2\alpha_n} + \sqrt{2(\alpha_2 - \alpha_n)}. \]

Note that \( 1 \geq \sqrt{2\alpha_n} \) and \( \alpha_2 \geq \sqrt{2(\alpha_2 - \alpha_n)} \), which follows from the conditions \( \alpha_n \leq 0.5 \) and \( \alpha_2 - \frac{\alpha_2^2}{2} < \alpha_n \), respectively. This means that the
left-hand-side of the above inequality is no less than the right-hand-side. This shows that inequality (3.31) holds in this case.

(b) $\alpha_n > 0.5$

Again, substitute the appropriate expressions of $Q_{1,NSTI}^*$ and $Q_{1,SSTI}^*$ into inequality (3.31) and simplify:

$$Q_{1,SSTI}^* - Q_{1,NSTI}^* = \hat{\mu} \epsilon \left[ \alpha_2 - \sqrt{2(\alpha_2 - \alpha_n)} - 1 + \sqrt{2(1-\alpha_n)} \right] \geq 0.$$  

The above inequality is equivalent to

$$\alpha_2 - \sqrt{2(\alpha_2 - \alpha_n)} - 1 + \sqrt{2(1-\alpha_n)} \geq 0. \quad (3.32)$$

The first-order derivative of the left-hand-side w.r.t. $\alpha_2$ is $1 - \frac{1}{\sqrt{2(\alpha_2 - \alpha_n)}}$, which is $\leq 0$. To see this, use the condition $\alpha_2 \geq \sqrt{2(\alpha_2 - \alpha_n)}$:

$$1 - \frac{1}{\sqrt{2(\alpha_2 - \alpha_n)}} \leq 1 - \frac{1}{\alpha_2} \leq 0.$$ 

Therefore, the left-hand-side of inequality (3.32) is decreasing in $\alpha_2$. At the point of $\alpha_2 = 1$, it reaches the minimum value 0. This again means inequality (3.31) is true for $\alpha_n > 0.5$.

From the above analysis, it follows that $Q_{1,SSTI}^* - Q_{1,NSTI}^* \geq 0$ is true for all possible values of $\alpha_n$ and $\alpha_2$. 
Part II: $Q^*_{1,SSTI} \leq Q^*_{1,PSTI}$

This is equivalent to showing that

$$Q^*_{1,PSTI} - Q^*_{1,SSTI} \geq 0.$$  \hspace{1cm} (3.33)

Note that $\alpha_p = \alpha_n/\alpha_2$. Again, it will be shown that the above inequality is true for all possible values of $\alpha_n$ and $\alpha_2$.

1. $\alpha_n \leq \frac{\alpha_2}{2}$

   If $\alpha_n \leq \frac{\alpha_2}{2}$, then $\alpha_p = \frac{\alpha_n}{\alpha_2} \leq \frac{1}{2}$. Substitute $Q^*_{1,SSTI} = Q^*_{1,NSTI|\alpha_n \leq 0.5}$ and $Q^*_{1,PSTI|\alpha_p \leq 0.5}$ into inequality (3.33) and simplify:

   $$Q^*_{1,PSTI} - Q^*_{1,SSTI} = \hat{\mu} \epsilon \left[ \sqrt{2\alpha_p} - \sqrt{2\alpha_n} \right] \geq 0.$$

   This is equivalent to

   $$\sqrt{\frac{2\alpha_n}{\alpha_2}} \geq \sqrt{2\alpha_n}.$$

   Clearly the above inequality is true because $\alpha_2 \leq 1$. This proves that $Q^*_{1,PSTI} - Q^*_{1,SSTI} \geq 0$ for $\alpha_n \leq \frac{\alpha_2}{2}$.

2. $\frac{\alpha_2}{2} < \alpha_n \leq \alpha_2 - \frac{\alpha_2}{2}$

   In this case, $Q^*_{1,PSTI}$ can take either of the two forms depending on whether $\alpha_p$ is greater than 0.5 or not. Note that $\alpha_p \leq 0.5$ is the same as $\alpha_n \leq \frac{\alpha_2}{2}$ and $\alpha_p > 0.5$ is the same as $\alpha_n > \frac{\alpha_2}{2}$. There are two subcases to be considered.
(a) \( \alpha_n \leq \frac{\alpha_2}{2} \)

Substitute the appropriate expressions of \( Q^*_{1,PSTI} \) and \( Q^*_{1,SSTI} \) into inequality (3.33) and simplify:

\[
Q^*_{1,PSTI} - Q^*_{1,SSTI} = \hat{\mu} \epsilon \left[ -1 - \sqrt{2\alpha_p} - \left( \frac{\alpha_2}{2} + \frac{\alpha_n}{\alpha_2} - 1 \right) \right] \geq 0.
\]

The above inequality simplifies to

\[
\sqrt{\frac{2\alpha_n}{\alpha_2}} \geq \frac{\alpha_2}{2} + \frac{\alpha_n}{\alpha_2}.
\]

Square both sides and rearrange,

\[
8\alpha_n\alpha_2 \geq (\alpha_2^2 + 2\alpha_n)^2. \tag{3.34}
\]

According to the conditions \( \frac{\alpha_2}{2} < \alpha_n \) and \( \alpha_n \leq \frac{\alpha_2}{2} \), it follows that \( \alpha_2^2 \leq 2\alpha_n \leq \alpha_2 \). The left-hand-side of the above inequality is no less than \( 8\alpha_n(2\alpha_n) = 16\alpha_n^2 \); the right-hand-side is no greater than \( (2\alpha_n + 2\alpha_n)^2 = 16\alpha_n^2 \). This means that inequality (3.34) is true given the above conditions, which in turn shows that \( Q^*_{1,PSTI} \geq Q^*_{1,SSTI} \) in this case.

(b) \( \alpha_n > \frac{\alpha_2}{2} \)

Again, substitute the appropriate expressions of \( Q^*_{1,PSTI} \) and \( Q^*_{1,SSTI} \) into inequality (3.33) and simplify:

\[
Q^*_{1,PSTI} - Q^*_{1,SSTI} = \hat{\mu} \epsilon \left[ 1 - \sqrt{2(1 - \alpha_p)} - \left( \frac{\alpha_2}{2} + \frac{\alpha_n}{\alpha_2} - 1 \right) \right] \geq 0.
\]
The above inequality reduces to

\[ 2 - \left( \frac{\alpha_2}{2} + \frac{\alpha_n}{\alpha_2} \right) \geq \sqrt{2 \left( 1 - \frac{\alpha_n}{\alpha_2} \right)}. \]  

(3.35)

It follows from the boundary conditions that \( \frac{1}{2} < \frac{\alpha_n}{\alpha_2} \leq 1 - \frac{\alpha_2}{2} \). The left-hand-side of inequality (3.35) is no less than 1, and the right-hand-side is no greater than 1. Therefore, inequality (3.35) is true, which shows that \( Q_{1,PSTI}^* - Q_{1,SSTI}^* \geq 0 \) in this case.

3. \( \alpha_n > \alpha_2 - \frac{\alpha_2^2}{2} \)

Since \( \alpha_2 - \frac{\alpha_2^2}{2} \geq \frac{\alpha_2}{2} \), \( \alpha_n \) must be greater than \( \frac{\alpha_2}{2} \) too. Therefore only the expression of \( Q_{1,PSTI}^* \) for \( \alpha_p > 0.5 \) should be considered. Substitute the appropriate expressions of \( Q_{1,PSTI}^* \) and \( Q_{1,SSTI}^* \) into inequality (3.33) and simplify:

\[ Q_{1,PSTI}^* - Q_{1,SSTI}^* = \hat{\mu} e \left[ 1 - \sqrt{2(1-\alpha_p)} - \left( \alpha_2 - \sqrt{2(\alpha_2 - \alpha_n)} \right) \right] \geq 0. \]

This reduces to

\[ 1 - \sqrt{2(1 - \frac{\alpha_n}{\alpha_2})} \geq \alpha_2 - \sqrt{2(\alpha_2 - \alpha_n)}. \]  

(3.36)
Both sides of the above inequality are non-negative, which can be easily verified by using the boundary conditions. Thus it follows that

\[
1 - \sqrt{2(1 - \frac{\alpha_n}{\alpha_2})} = \frac{\sqrt{\alpha_2} - \sqrt{2(\alpha_2 - \alpha_n)}}{\sqrt{\alpha_2}} \\
\geq \sqrt{\alpha_2} - \sqrt{2(\alpha_2 - \alpha_n)} \\
\geq \alpha_2 - \sqrt{2(\alpha_2 - \alpha_n)}.
\]

This shows the left-hand-side of inequality (3.36) is always greater than or equal to the right-hand-side. Therefore, inequality (3.33) must be true in this case.

The above analysis suggests that \( Q^*_{1,PSTI} - Q^*_{1,SSTI} \geq 0 \) holds under all possible conditions. Therefore, the results from both parts combined together show that \( Q^*_1,NSTI \leq Q^*_1,SSTI \leq Q^*_1,PSTI \). This ends the proof.

Under NSTI, the two components are ordered in squared sets. In other words, the order quantities of the two components are always the same. However, under SSTI this is no longer true because the \( Q_2 \) decision is delayed to take advantage of a more accurate forecast at stage two. The order quantity of component 2, which cannot be determined until the revelation of \( \mu \), is a random variable at stage one. It is of interest to investigate the gap between \( Q^*_1 \) and the expected value of \( Q^*_2 \), which can be formulated the same
way as total cost. The expressions of $E(Q^*_2)$ are given by

$$E(Q^*_2) = \begin{cases} 
Q^*_{1,SSTI} = Q^*_{1,NSTI} & \text{if } 0 \leq \alpha_n \leq \frac{\alpha_2}{2}; \\
\frac{1}{8} \hat{\mu} \left[ 8 - \left( \alpha_2^2 - 4\alpha_n - 4\alpha_2 + 4 \left( \frac{\alpha_n}{\alpha_2} \right)^2 - 8 \left( \frac{\alpha_n}{\alpha_2} \right) + 8 \right) \right] & \text{if } \alpha_2^2/2 < \alpha_n \leq \alpha_2 - \frac{\alpha_2^2}{2}; \\
\frac{1}{2} \hat{\mu} [2 + (2\alpha_n - 1)\epsilon] & \text{if } \alpha_2 - \frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2.
\end{cases}$$

(3.37)

With closed-form expressions for $Q^*_1$ and $E(Q^*_2)$, the gap between them can be easily found:

$$E(Q^*_1 - Q^*_2) = \begin{cases} 
0 & \text{if } 0 \leq \alpha_n \leq \frac{\alpha_2}{2}; \\
\frac{1}{8} \hat{\mu} \epsilon \left[ 1 - \sqrt{2(\alpha_2 - \alpha_n)} \right] & \text{if } \alpha_2 - \frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2.
\end{cases}$$

(3.38)

Proposition 1. Under SSTI the gap between the optimal order quantities of the two components widens as $\alpha_n$ increases and narrows as $\alpha_2$ increases.
Proof. The partial derivatives of $E(Q_1^* - Q_2^*)$ w.r.t. $\alpha_n$ are

$$\frac{\partial E(Q_1^* - Q_2^*)}{\partial \alpha_n} = \begin{cases} 0 & \text{if } 0 \leq \alpha_n \leq \frac{\alpha_2^2}{2}; \\ \frac{1}{2} \hat{\mu} \epsilon \left( \frac{1}{\alpha_2} \right) \left[ 2 \left( \frac{\alpha_n}{\alpha_2} \right) - \alpha_2 \right] & \text{if } \frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2 - \frac{\alpha_2^2}{2}; \\ \hat{\mu} \epsilon \left[ \frac{1 - \sqrt{2(\alpha_2 - \alpha_n)}}{\sqrt{2(\alpha_2 - \alpha_n)}} \right] & \text{if } \alpha_2 - \frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2. \end{cases}$$

The partial derivatives of $E(Q_1^* - Q_2^*)$ w.r.t. $\alpha_2$ are

$$\frac{\partial E(Q_1^* - Q_2^*)}{\partial \alpha_2} = \begin{cases} 0 & \text{if } 0 \leq \alpha_n \leq \frac{\alpha_2^2}{2}; \\ -\frac{1}{4} \hat{\mu} \epsilon \left[ 2 \left( \frac{\alpha_n}{\alpha_2} \right) - \alpha_2 \right] \left[ \frac{2\alpha_n}{\alpha_2^2} + 1 \right] & \text{if } \frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2 - \frac{\alpha_2^2}{2}; \\ -\hat{\mu} \epsilon \left[ \frac{1 - \sqrt{2(\alpha_2 - \alpha_n)}}{\sqrt{2(\alpha_2 - \alpha_n)}} \right] & \text{if } \alpha_2 - \frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2. \end{cases}$$

By the boundary conditions, $\partial E(Q_1^* - Q_2^*)/\partial \alpha_n$ is non-negative and $\partial E(Q_1^* - Q_2^*)/\partial \alpha_2$ is non-positive. Therefore the gap between the optimal order quantities of the two components increases with $\alpha_n$ and decreases with $\alpha_2$ on average. □

Given any $Q_1$, the probability of $Q_2^* = Q_1$ under SSTI is simply the probability of $\beta_2 > Q_1$. Using the expressions $\beta_2 = \mu + (2\alpha_2 - 1)\delta$ and $\delta = \hat{\mu} \epsilon / 2$, it then follows
that

\[ P(Q_2^* = Q_1) = P(\beta_2 > Q_1) \]

\[ = P[\mu + (2\alpha_2 - 1)\delta > Q_1] \]

\[ = P[\mu > Q_1 - (2\alpha_2 - 1)\delta] \]

\[ = \int_{Q_1-(2\alpha_2-1)\delta}^{\infty} \frac{1}{\mu\epsilon} d\mu \]

\[ = \alpha_2 + \frac{1}{\epsilon} - \frac{Q_1}{\mu\epsilon}. \quad (3.39) \]

Next, substitute \( Q_{1,SSTI}^* \) into the above equation and simplify. The probability of ordering the same quantity for both components is

\[
P(Q_2^* = Q_1^*) = \begin{cases} 
1 & \text{if } 0 \leq \alpha_n \leq \frac{\alpha_2}{2}; \\
1 + \frac{\alpha_2}{2} - \frac{\alpha_n}{\alpha_2} & \text{if } \frac{\alpha_2}{2} < \alpha_n \leq \alpha_2 - \frac{\alpha_2}{2}; \\
\sqrt{2(\alpha_2 - \alpha_n)} & \text{if } \alpha_2 - \frac{\alpha_2}{2} < \alpha_n \leq \alpha_2. 
\end{cases} \quad (3.40)\]

3.2.2 Expressions for Total Costs

Substitute \( Q_1^* \)'s into the cost functions to obtain expressions for optimal expected total cost under all three scenarios:

\[
E(TC)^*_{NSTI} = \begin{cases} 
\frac{1}{3}(3 - 2\sqrt{2\alpha_n})m_\epsilon \hat{\mu}\epsilon & \text{if } \alpha_n \leq 0.5; \\
\frac{1}{3} \left[ 3 - 2\sqrt{2(1 - \alpha_n)} \right] (l_1 + l_2)\epsilon \hat{\mu} & \text{if } \alpha_n > 0.5.
\end{cases} \quad (3.41)\]
\[
E(TC)^*_{\text{PSTI}} = \begin{cases} 
\frac{1}{3}(3 - 2\sqrt{2\alpha_p})m_c\hat{\mu}\epsilon & \text{if } \alpha_p \leq 0.5; \\
\frac{1}{3} \left[ 3 - 2\sqrt{2(1 - \alpha_p)} \right] l_1\hat{\mu}\epsilon & \text{if } \alpha_p > 0.5.
\end{cases}
\]

(3.42)

\[
E(TC)^*_{\text{NSTI}} = \\
\begin{cases} 
\frac{1}{3}(3 - 2\sqrt{2\alpha_p})m_c\hat{\mu}\epsilon & \text{if } 0 \leq \alpha_n \leq \frac{\alpha_2^2}{2}; \\
\frac{1}{24\alpha_2}\hat{\mu}\epsilon(m_c + l_1 + l_2) \left[ -12\alpha_n^2 + 24\alpha_n\alpha_2 - 12\alpha_n\alpha_2^2 + \alpha_2^4 \right] & \text{if } \frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2 - \frac{\alpha_2^2}{2}; \\
\frac{1}{6}\hat{\mu}\epsilon \left[ 3\alpha_2(2l_1 + l_2) - 4l_1\sqrt{2(\alpha_2 - \alpha_n)} \right] & \text{if } \alpha_2 - \frac{\alpha_2^2}{2} < \alpha_n \leq \alpha_2.
\end{cases}
\]

(3.43)

The SSTI solutions can be used to approximate the solutions under the bivariate normal model with \(\rho\)'s other than 0 or 1. This simplifies the solution procedure without sacrificing performance as explained in the next section.

### 3.3 Approximating Normal-Distribution TCNV Solutions

This section demonstrates that the U-U model, if designed appropriately, can approximate the BVN model accurately in terms of optimal order quantity and cost. The closed-form solutions, which we could not find under the BVN formulation for \(\rho \in (0,1)\), are used to study channel coordination in next chapter. In approximating normal-distribution TCNV solutions, we fit first moments of the stage-one distributions (normal and triangular), and then find the correlation value (\(\hat{\rho}\)) against which the uniform-demand SSTI solution is most closely mapped onto the normal-demand model.
3.3.1 Matching Moments of Stage-One Demand Distributions

To approximate the BVN model, it is necessary to fit the mean and end points of the triangular distribution. The stage-one mean is fit simply by setting \( \hat{\mu} = \mu_1 \). To express the width of the triangle distribution in terms of \( \sigma_1 \), let

\[
\hat{\mu}_\epsilon = Z\lambda \sigma_1,
\]  

(3.44)

where \( 0.5 \leq \lambda \leq 1 \), and \( Z \) is the standard normal score. Thus the end points of the triangular distribution are \( (\mu_1 \pm Z\lambda \sigma_1) \).

In order to determine the appropriate value for \( \lambda \), we next consider the optimization problem which, given mean \( \mu_1 \) and standard deviation \( \sigma_1 \) of the normal distribution, attempts to minimize the distance between the optimal order quantities obtained in the two different models. Let \( Q^*_T \) and \( Q^*_N \) be the optimal solutions to the traditional newsvendor problem with triangular distribution and normal distribution, respectively. Given critical fractile \( \alpha \) \((0 \leq \alpha \leq 1)\), the optimal order quantity under the triangular distribution is

\[
Q^*_T = \begin{cases} 
\mu_1 + Z\lambda \sigma_1(\sqrt{2\alpha} - 1) & \text{if } \alpha \leq 0.5; \\
\mu_1 + Z\lambda \sigma_1 \left[1 - \sqrt{2(1 - \alpha)}\right] & \text{if } \alpha > 0.5.
\end{cases}
\]  

(3.45)

The optimal order quantity for the normal demand case is

\[
Q^*_N = \mu_1 + Z\alpha \sigma_1.
\]  

(3.46)
Let $\triangle Q^*$ be the distance between the above two solutions:

$$\triangle Q^* = |Q_T^* - Q_N^*| = \begin{cases} 
\sigma_1 |(\sqrt{2\alpha} - 1)Z_\lambda - Z_\alpha| & \text{if } \alpha \leq 0.5; \\
\sigma_1 \left|\left[1 - \sqrt{2(1-\alpha)}\right]Z_\lambda - Z_\alpha\right| & \text{if } \alpha > 0.5.
\end{cases} \quad (3.47)$$

The optimization problem we want to solve is as follows:

$$\min_{0.5 \leq \lambda \leq 1} \int_0^1 \triangle Q^* d\alpha. \quad (P6)$$

By substituting equation (3.44) into Problem (P6) and using the fact $\triangle Q^*|_{\alpha = 1 - \alpha} = \triangle Q^*|_{1-\alpha}$, Problem (P6) can be rewritten as

$$\min_{0.5 \leq \lambda \leq 1} \int_{0.5}^1 2\sigma_1 \left|\left[1 - \sqrt{2(1-\alpha)}\right]Z_\lambda - Z_\alpha\right| d\alpha. \quad (P7)$$

Given any $\sigma_1$, Problem (P7) contains only one variable, $\lambda$, and can be solved numerically using a line search algorithm. The search for optimal $\lambda$ converges at 0.989977,\(^2\) i.e., $\lambda^* \approx 0.99$. For convenience, the end points of the triangular distribution at stage one are fixed at $(\mu_1 \pm Z_{0.99}\sigma_1)$, which are the 1% and 99% critical points of the normal distribution (see Figure 3.3). From equation (3.44), it follows that

$$\epsilon = Z_{0.99}\sigma_1/\mu_1. \quad (3.48)$$

---

\(^2\)This is the result obtained under the convergence criterion that the interval containing $\lambda^*$ is shorter than $10^{-6}$. 
Fig. 3.3. Determining the end points of triangular distribution
3.3.2 Mapping Uniform-Demand SSTI to the Normal-Demand Model

In order to find a $Q_1$ solution for any $\rho$, a value for $\rho$ must be found against which the SSTI solution can be mapped onto the BVN curve. Let this value be $\hat{\rho}$. By equating the stage-two variances from both models, it follows that

$$\frac{1}{12}(\hat{\mu} \epsilon)^2 = \sigma_1^2 (1 - \rho^2).$$

(3.49)

Substitute equation (3.48) and $\hat{\mu} = \mu_1$ into the above equation to solve for $\hat{\rho}$:

$$\hat{\rho} = \sqrt{1 - \frac{Z_{99}^2}{12}} = 0.7410.$$  

(3.50)

3.3.3 Approximation Scheme

With these results, the analytical solutions under the U-U model can now be used to approximate the solutions under the BVN model. The approximation procedure works as follows.

1. Find $\epsilon$.

   Given $m_c, l_1, l_2, \mu_1$, and $\sigma_1$, first find the relative width, $\epsilon$, using equation (3.48).

2. Calculate $Q_1^*$ in the U-U model.

   With $\epsilon$ obtained in the previous step, equations (3.28) to (3.30) are used to calculate the optimal order quantities and optimal expected total cost for NSTI, PSTI and SSTI.
3. Compute $Q_1^*$ at the two end points of the BVN model.

Since the BVN model reduces to a classic newsvendor problem at the two end points, the solutions can be obtained using equations (3.19) and (3.20).

4. Interpolate $Q_1^*$ for $0 < \rho < 1$.

Given any $\rho$ between 0 and 1, $Q_1^*$ can be approximated through interpolation using either the solutions at two end points of the BVN model (2-point interpolation) or those two points plus the solution to SSTI (3-point interpolation). Figure 3.4 shows an example of the approximation using both methods.

The 2-point approach works as follows:

$$\hat{Q}_1^* = Q_1^*|_{\rho=0} + \rho (Q_1^*|_{\rho=1} - Q_1^*|_{\rho=0}).$$

The 3-point approach works as follows:

$$\hat{Q}_1^* = \begin{cases} 
Q_1^*|_{\rho=0} + \rho (Q_1^*|_{\rho=1} - Q_1^*|_{\rho=0}) & \text{if } 0 < \rho \leq 0.741; \\
Q_1,SSTI + (Q_1^*|_{\rho=1} - Q_1^*|_{\rho=0}) \left(\frac{\rho - 0.741}{1 - 0.741}\right) & \text{if } 0.741 < \rho < 1. 
\end{cases}$$

In order to examine the performance of interpolation, the optimal $Q_1^*$ under the BVN model is calculated using Newton’s method. Numerical integration is used in computing the first-order and second-order derivatives.

5. Evaluate the performance of the approximation.

The performance of the approximation can be measured by comparing the total
cost at \( Q_1^* \) and interpolated \( \hat{Q}_1^* \)'s. Percentage error is calculated to judge the goodness of approximation.

\[
PE = \frac{TC(\hat{Q}_1^*) - TC(Q_1^*)}{TC(Q_1^*)} \times 100\%.
\]

The performance of approximation is examined in 29,700 cases based on different combinations of margin, salvage loss, and uncertainty (accounted for by \( m_c, l_1, l_2, \) and \( \sigma_1/\mu_1 \), respectively). Table 3.1 summarizes the computational results for all 29,700 cases and gives the average, standard deviation, and maximum of percentage errors for both total cost and order quantity. Compared with the 2-point approach, the 3-point approach makes a significant difference. For example, the highest and average percentage errors observed for total cost are 2.18% and 0.08% from the 3-point interpolation in contrast to 11.75% and 0.84% from the 2-point approach. These results also indicate that the solution easily obtained from 3-point interpolation accurately approximates the exact solution under BVN.

Table 3.1. A Summarization of Percentage Errors from Approximation
(Total Number of Cases Examined:29,700)

<table>
<thead>
<tr>
<th></th>
<th>3-pt Approach</th>
<th>2-pt Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AVG.</td>
<td>S.D.</td>
</tr>
<tr>
<td>Total Cost</td>
<td>0.08%</td>
<td>0.13%</td>
</tr>
<tr>
<td>Order Quantity</td>
<td>0.61%</td>
<td>0.79%</td>
</tr>
</tbody>
</table>

(\( m_c = 0.1 : 0.1 : 2, l_1 = 0.1 : 0.1 : 0.9, l_2 = 0.1 : 0.1 : (1 - l_1), \mu_1 = 100, \sigma_1 = 10 : 10 : 30, \rho = 0 : 0.1 : 1 \))
Fig. 3.4. Approximate the optimal normal-demand order quantity.
3.4 Conclusion

So far the thesis has focused on forecast updating and the CM’s ordering behavior. A new model using a uniform-uniform (U-U) distribution was developed to capture demand forecast updates. The U-U model serves as a good approximation for the bivariate normal model. Under the U-U model, closed-form expressions can be found for optimal order quantity and optimal total cost. Analytical results show that with complementary product structure, the CM will increase the order quantity of component 1 if better information becomes available at the point when the second component is ordered. Although this may increase the overage cost of component 1, overall cost can be reduced.

The chapter that follows will explore the coordination between the OEM and the CM. It is an extension from the CM’s problem, the TCNV, outward to a larger supply-chain problem. The closed-form solutions of the U-U model are used to develop some important properties regarding the relative value of contract mechanisms and information sharing.
Chapter 4

The OEM-CM Joint Planning Problem
with Information Sharing and Risk Sharing

This chapter investigates channel coordination between the OEM and the CM, which is an outward extension of the TCNV problem from the previous chapter. While the TCNV focuses on the CM’s ordering behavior alone, this chapter studies the interaction between both parties under various schemes differentiated by the means and degree of coordination. The media for collaboration include information sharing and risk sharing. In the bigger picture of OEM-CM joint planning, the OEM serves as the sole source of demand information for the CM and always shares its demand forecast with the CM at the time when the CM must order the first (long lead time) component but must decide whether or not to share an updated forecast when the CM must order the second component. Furthermore, while the OEM and CM both incur a portion of the margin-based opportunity cost if demand is not fully satisfied, the OEM can decide whether or not to share the component overage cost that the CM would otherwise fully bear. In this chapter, three types of contracts characterized by different levels of risk sharing are considered and compared in terms of their performance and efficiency in coordinating the supply chain. The U-U model and closed-form expressions developed in Chapter 3 are used to investigate the OEM’s decisions regarding sharing forecast updates and sharing overage cost. Computational experiments show that similar results can be observed under the bivariate normal model.
This chapter is organized as follows. Section 4.1 presents an introduction of the joint planning problem and describes the risk-sharing agreements. Section 4.2 formulates the cost functions for the OEM and the CM. Section 4.3 investigates the interaction between forecast update sharing and component overage risk sharing. Section 4.4 studies some properties of the risk-sharing agreements. Section 4.5 presents computational results. Section 4.6 concludes this chapter by summarizing major results and managerial insights.

4.1 Introduction

The OEM sells to its clients a final product, the assembly of which requires a semi-finished product from the CM. In order to make the subassembly, the CM needs to order two components that have different lead times. The CM uses an assemble-to-order system and is responsible for the procurement of both components. All components unused at the end of the selling season are salvaged by the CM. There are market determined salvage values for components as well as an industry standard markup. This chapter adopts some of the notations from the previous chapter. Assume component 1 is the long lead time component, with unit cost $c_1$, and $c_2 = 1 - c_1$. Since the problem is considered only within a single selling season, components left over incur salvage losses for a certain fraction of the original cost. The per unit salvage loss is denoted by $l_i$ for component $i$ ($i = 1, 2$). The OEM’s selling price is $p = 1 + m_o + m_c$, where $m_c$ is the markup paid to the CM, and thus $m_o$ is the OEM’s profit per unit.

If the OEM bears no risk for component overage, it certainly would prefer that the CM maintain high inventory levels for both components so that it does not forfeit any
sales opportunities. Unfortunately, the CM may not do so because it seeks to minimize its own costs including the cost of oversupply of components and the opportunity cost from its portion of the lost margin caused by product undersupply. This causes the OEM’s and CM’s interests to be in conflict, meaning that component ordering decisions desired by the CM may not be favored by the OEM. Assume that although the OEM in this supply chain has ceded the authority for component ordering decisions to the CM, the OEM can influence the CM’s decisions through two mechanisms: (1) control of the demand forecast information and (2) offering contract terms to the CM whereby the OEM absorbs some fraction of the CM’s overage cost, a control mechanism called a \textit{salvage offset contract}, which is explained in detail later in this section.

It is assumed that the OEM always shares the long range demand forecast, based on which the CM can make a procurement decision for component 1. However, the OEM may or may not share revised forecast with the CM depending on which choice is in its best interests. If the OEM declares that the forecast update will not be shared, the CM will order component 2 equal to component 1. Otherwise the CM will have access to new demand information when it orders component 2.

Similarly, the OEM needs to decide whether it should share component overage risk with the CM to ensure part supply or not. Consider the coordination scheme under which the OEM shares risk with the CM and compensates the latter for components that are left over at the end of the selling season. More specifically the OEM absorbs a fraction, $\gamma_i$ ($0 \leq \gamma_i \leq 1$), of the salvage loss $l_i$ for unused component $i$ ($i = 1$ or 2). This is called a $\gamma$-contract or salvage offset contract because $\gamma_1$ and $\gamma_2$ are the contract parameters to be set by the OEM and offered to the CM.
Three variants of the salvage offset contract as described next are studied.

1. **Single-$\gamma$ (SG) contract**, under which the OEM compensates the CM for unused component 1 only\(^1\) but not component 2. To ensure the supply of key parts and mitigate the CM’s risk exposure, the OEM agrees to bear a portion of overage cost for the more critical, yet more expensive components with longer lead times, yet not for the relatively cheaper component with shorter lead time. The single-$\gamma$ contract captures the characteristics of industry practice very well.

2. **Single-$\gamma$, squared-sets (SS) contract**, under which the OEM compensates the CM for each pair of unused components but not for unmatched parts. This means that the CM will be compensated for salvage loss only when both component 1 and component 2 are left over. For every pair of surplus components, the OEM will pay the CM the amount of $\gamma(l_1 + l_2)$ ($0 \leq \gamma \leq 1$). If at the end of the season it turns out that there are extra component 1, but component 2 is used up, then the CM will not get paid at all for any loss from salvaging component 1. The reason to propose such an agreement is that the SG contract tends to induce the CM to buy too much component 1 and possibly not enough of component 2. In case the demand falls between $Q_1$ and $Q_2$, the OEM would end up missing sales while at the same time still having to absorb a portion of salvage loss for component 1.

With a squared-set $\gamma$-contract, this problem may be resolved.

---

\(^1\)This is justifiable from a practical point of view because according to our discussions with a large high-tech manufacturer, generally the planning problem lies in inadequacy of component 1, the more expensive component with longer lead time.
3. *Dual-γ (DG) contract*, under which the OEM offers an amount of $\gamma_i l_i$ to the CM for any unit of surplus component $i$, even if it is not matched by its counterpart. In other words, the OEM is willing to take a portion $\gamma_i$ of the loss from salvaging any unused part $i$. The OEM has to specify the value of both $\gamma_1$ and $\gamma_2$ before the CM makes ordering decision for component 1. Under such an agreement, once the OEM specifies $\gamma_1$ and $\gamma_2$, their value cannot be changed throughout the period.

To summarize, the sequence of events is defined by two steps. First, the OEM, understanding that the CM optimizes its own cost function, $TC^c$, chooses values for $\gamma_1$ and $\gamma_2$ as well as whether or not to share the forecast update. Next, given this contract and demand forecast decision, the CM determines the order quantity of component 1 and component 2 sequentially. Thus, the OEM has two dependent choices: (1) should the OEM share the forecast update? and (2) should the OEM offer a risk-sharing contract?

4.2 The Model

This section presents a general cost model for the OEM and the CM under different contracts. Because both the SG contract and the SS contract are variants of the DG contract, this section first demonstrates cost formulations under the DG contract. Cost functions under other contracts can then be derived in a similar manner. Essentially, the total cost of the OEM, $TC^o$, and that of the CM, $TC^c$, can be defined as functions of $Q_1$ and $Q_2$ given $\gamma_1$ and $\gamma_2$. Note that $Q_2^*$ is a function of $\beta_2$ which in turn is a function of the stage-two critical fractile $\alpha_2$. By viewing $\alpha_2$ as a proxy variable for $Q_2$, $TC^o$ and $TC^c$ can then be formulated as functions of $Q_1$ and $\alpha_2$. 
Without loss of generality, let $G_I$ be the c.d.f. of the information variable $I$ at stage one and $F_{D|I}(\cdot)$ be the c.d.f. of demand at stage two. Define $A$ to be the set of cases where $Q_1 \leq \beta_2 = F_{D|I}^{-1}(\alpha_2)$. In other words, $A$ includes all the cases where the CM orders squared-sets for the two components. Thus, \( \bar{A} \) is the set of cases where $Q_1 > \beta_2 = Q_2^*$. To facilitate analysis, expressions for expected unit shortage and overage are developed first. Then, cost parameters can be applied to obtain expressions for $TC^O$ and $TC^C$.

The expected unit shortage, $U$, is given by

$$U(Q_1, \alpha_2) = \int_A \mathbb{E}_{D|I}(D - Q_1)^+ dG_I + \int_{\bar{A}} \mathbb{E}_{D|I}(D - \beta_2)^+ dG_I. \quad (4.1)$$

Let $O$ denote the expected overage of component 2. If the CM orders squared-sets, the overage of component 2 is the same as that for component 1; otherwise, it is $((\beta_2 - D))^+$. Thus, we have

$$O(Q_1, \alpha_2) = \int_A \mathbb{E}_{D|I}(Q_1 - D)^+ dG_I + \int_{\bar{A}} \mathbb{E}_{D|I}((\beta_2 - D))^+ dG_I. \quad (4.2)$$

The gap between the order quantities for component 1 and component 2, denoted by $V$, can be formulated as

$$V(Q_1, \alpha_2) = \int_{\bar{A}} (Q_1 - \beta_2)^+ dG_I. \quad (4.3)$$

Under the DG contract, the OEM shares risk with the CM by absorbing an overage cost in the amount of $\gamma_i l_i$ for each surplus unit of component $i$. The CM’s expected total
cost, \( TC^c \), is therefore

\[
TC^c = m_U + [(1 - \gamma_1)l_1 + (1 - \gamma_2)l_2]O + [(1 - \gamma_1)l_1]V. \tag{4.4}
\]

Note that for the OEM, expected unit shortage and overage are the same as those for the CM. So \( U, O, \) and \( V \) remain unchanged. The OEM’s expected total cost is

\[
TC^O = m_U + (\gamma_1 l_1 + \gamma_2 l_2)O + \gamma_1 l_1 V. \tag{4.5}
\]

Equations (4.4) and (4.5) are the cost functions under the DG contract. Under the SG contract, the OEM agrees to absorb some portion, \( \gamma_1 \), of the overage cost for extra component 1 but not for component 2. The cost functions are

\[
TC^c = m_U + [(1 - \gamma)l_1 + l_2]O + [(1 - \gamma_1)l_1]V, \tag{4.6}
\]

and

\[
TC^O = m_U + \gamma_1 l_1 O + \gamma_1 l_1 V. \tag{4.7}
\]

Under the SS contract, there is only a single \( \gamma \) which applies to each pair of remaining components. The CM bears full overage risk for any unmatched component 1. The cost functions are

\[
TC^c = m_U + (1 - \gamma)(l_1 + l_2)O + l_1 V, \tag{4.8}
\]
and
\[
TC^o = m_o U + \gamma (l_1 + l_2) O.
\] (4.9)

The CM’s optimization problem is to minimize \(TC^c\) over \(Q_1\) and \(\alpha_2\) given \(\gamma_1\) and \(\gamma_2\). The OEM’s objective is to minimize \(TC^o\) over \(\gamma_1\) and \(\gamma_2\) assuming the CM’s optimal \(Q_1\) and \(\alpha_2\).

4.3 Interaction between the OEM and the CM

In order to analyze the properties of optimal order quantity and expected total costs, this section first develops some intermediate results about the partial derivatives of \(U\), \(O\), and \(V\) with regard to \(Q_1\) and \(\alpha_2\). These results are then used to derive results later in this chapter.

4.3.1 Intermediaries

The partial derivatives of expected underage w.r.t. \(Q_1\) and \(\alpha_2\) are
\[
\frac{\partial U}{\partial Q_1} = - \int_A F_{D|I}(Q_1) dG_I, \tag{4.10}
\]
and
\[
\frac{\partial U}{\partial \alpha_2} = \int_A \frac{F_{D|I}(Q_1) \partial \beta_2}{\partial \alpha_2} dG_I = -(1 - \alpha_2) \int_A \frac{\partial \beta_2}{\partial \alpha_2} dG_I. \tag{4.11}
\]

The partial derivatives of expected overage for component 2 w.r.t. \(Q_1\) and \(\alpha_2\) are
\[
\frac{\partial O}{\partial Q_1} = \int_A F_{D|I}(Q_1) dG_I, \tag{4.12}
\]
and

\[ \frac{\partial O}{\partial \alpha_2} = \int_{A} F_{D|I} (\beta_2) \frac{\partial \beta_2}{\partial \alpha_2} dG_I = \alpha_2 \int_{A} \frac{\partial \beta_2}{\partial \alpha_2} dG_I. \]  

(4.13)

The partial derivatives of \( V \) w.r.t. \( Q_1 \) and \( \alpha_2 \) are

\[ \frac{\partial V}{\partial Q_1} = \int_{A} dG_I = P(\overline{A}) \]  

(4.14)

where \( P(\overline{A}) \) is the probability of not ordering squared-sets and equals \( P(\beta_2 < Q_1) \), and

\[ \frac{\partial V}{\partial \alpha_2} = - \int_{A} \frac{\partial \beta_2}{\partial \alpha_2} dG_I. \]  

(4.15)

Given \( \gamma_1 \) and \( \gamma_2 \), the critical fractiles for the CM can be defined the same way as they were in last chapter:

\[ \alpha_n^c = \frac{m_c}{m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2}, \]  

(4.16)

\[ \alpha_p^c = \frac{m_c}{m_c + (1 - \gamma_1)l_1}, \]  

(4.17)

\[ \alpha_2^c = \frac{m_c + (1 - \gamma_1)l_1}{m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2}. \]  

(4.18)
The derivatives of $TC^c$ w.r.t. $Q_1$ and $\alpha_2$ are

\[
\frac{\partial TC^c}{\partial Q_1} = -m_c P(A) + m_c \int_A F_{D|I}(Q_1) dG_I + [(1 - \gamma_1)l_1
\]

\[
+ (1 - \gamma_2)l_2] \int_A F_{D|I}(Q_1) dG_I + (1 - \gamma_1)l_1[1 - P(A)]
\]

\[
= -[m_c + (1 - \gamma_1)l_1]P(A)
\]

\[
+ [m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2] \int_A F_{D|I}(Q_1) dG_I + (1 - \gamma_1)l_1
\]

\[
= [m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2]
\]

\[
\times \left[ -\alpha_2^c P(A) + \int_A F_{D|I}(Q_1) dG_I + (\alpha_2^c - \alpha_n^c) \right], \quad (4.19)
\]

and

\[
\frac{\partial TC^c}{\partial \alpha_2} = \left[ -m_c + \alpha_2 [m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2] - (1 - \gamma_1)l_1 \right]
\]

\[
\times \int_A \frac{\partial \beta_2}{\partial \alpha_2} dG_I
\]

\[
= [m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2][\alpha_2 - \alpha_2^c] \int_A \frac{\partial \beta_2}{\partial \alpha_2} dG_I. \quad (4.20)
\]

For the OEM, the derivatives can be obtained similarly. Define the following fractiles as if the OEM were to solve for the optimal order quantities based on its own cost parameters:

\[
\alpha_n^o = \frac{m_o}{m_o + \gamma_1l_1 + \gamma_2l_2}, \quad (4.21)
\]

\[
\alpha_p^o = \frac{m_o}{m_o + \gamma_1l_1}, \quad (4.22)
\]

\[
\alpha_2^o = \frac{m_o + \gamma_1l_1}{m_o + \gamma_1l_1 + \gamma_2l_2}. \quad (4.23)
\]
The derivatives of $TC^o$ w.r.t. $Q_1$ and $\alpha_2$ are

\[
\frac{\partial TC^o}{\partial Q_1} = -m_oP(A) + m_o \int_A F_{D|I}(Q_1) \, dG_I \\
(\gamma_1 l_1 + \gamma_2 l_2) \int_A F_{D|I}(Q_1) \, dG_I + \gamma_1 l_1 [1 - P(A)] \\
= -(m_o + \gamma_1 l_1)P(A) \\
+ (m_o + \gamma_1 l_1 + \gamma_2 l_2) \int_A F_{D|I}(Q_1) \, dG_I + \gamma_1 l_1 \\
= (m_o + \gamma_1 l_1 + \gamma_2 l_2) \\
\times \left[ -\alpha_2^c P(A) + \int_A F_{D|I}(Q_1) \, dG_I + (\alpha_2^o - \alpha_2^o) \right], \quad (4.24)
\]

and

\[
\frac{\partial TC^o}{\partial \alpha_2} = [-m_o + \alpha_2(m_o + \gamma_1 l_1 + \gamma_2 l_2) - \gamma_1 l_1] \int_A \frac{\partial \beta_2}{\partial \alpha_2} \, dG_I \\
= (m_o + \gamma_1 l_1 + \gamma_2 l_2)(\alpha_2 - \alpha_2^o) \int_A \frac{\partial \beta_2}{\partial \alpha_2} \, dG_I. \quad (4.25)
\]

The interpretations of equations (4.19) and (4.24) are quite meaningful. The marginal benefit of ordering one additional unit of component 1 depends on three things: (1) if the CM orders squared sets and ends up with demand in excess of $Q_1$, then an additional unit of component 1 can help to bring underage cost down by $m_c$ for the CM and $m_o$ for the OEM; (2) if the CM orders squared sets but ends up with demand $D$ lower than $Q_1$, then an extra unit of component 1 increases overage cost by $(1 - \gamma_1)l_1 + (1 - \gamma_2)l_2$ for the CM and $\gamma_1 l_1 + \gamma_2 l_2$ for the OEM; and (3) if the CM orders less of component 2 than component 1, then having an additional unit of component 1 does not help at all because this adds more overage cost by $(1 - \gamma_1)l_1$ for the CM and $\gamma_1 l_1$ for the OEM.
4.3.2 Stage-Two Ordering Behavior

This part explores how various contracts may impact the CM’s ordering decision for component 2 under SSTI.

Under the SG contract, the stage-two fractile is

$$\alpha_2^c = \frac{m_c + (1 - \gamma_1)l_1}{m_c + (1 - \gamma_1)l_1 + l_2}.$$

Under the SS contract, the stage-two fractile is

$$\alpha_2^c = \frac{m_c + l_1}{m_c + (1 - \gamma)(l_1 + l_2)}.$$

Under the DG contract, the stage-two fractile is

$$\alpha_2^c = \frac{m_c + (1 - \gamma_1)l_1}{m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2}.$$

A comparison of these expressions indicates that given $\gamma_1 = \gamma$ the stage-two critical fractiles under the last two types of contracts (SS and DG) are always higher than that under the SG contract. The ordering of the critical fractiles under the SS and DG contracts depends on the value of $\gamma_2$. One thing worth mentioning is that the unit overage cost of component 2 under the SS contract is $(1 - \gamma)l_2 - \gamma l_1$. This implies that although the CM incurs a loss $(1 - \gamma)l_2$ for any extra unit of component 2, it can recover a portion of the overage cost in the amount of $\gamma l_1$ from salvaging a surplus unit of component 1. Therefore, the CM will always order squared sets if $(1 - \gamma)l_2 - \gamma l_1 < 0$, i.e., $\gamma > l_2/(l_1 + l_2)$. 
4.3.3 Some Properties of Optimal Solutions

According to Chapter 3, $TC^c$ is a convex function of $Q_1$ and $Q_2$ (or $\alpha_2$). The first-order conditions must hold at the CM’s optimal order quantities:

$$\frac{\partial TC^c}{\partial \alpha_2} = 0; \quad (4.26)$$

$$\frac{\partial TC^c}{\partial Q_1} = 0. \quad (4.27)$$

Equation (4.26) enforces the optimal ordering policy at stage two, and equation (4.27) ensures that the marginal total cost from buying one more unit of component 1 is zero. Substituting equations (4.20) and (4.19) into (4.26) and (4.27), it is easy to see that the optimal solutions must satisfy the following conditions

$$\alpha_2^* = \alpha_2^c; \quad (4.28)$$

$$\int_A F_D I(Q_1^*) dG_I = -[1 - P(A)] \alpha_2^c + \alpha_n^c. \quad (4.29)$$

By offering a risk-sharing agreement, the OEM intends to drive up the order quantities of both components so as to reduce the occurrence of stockout. Does the CM always react favorably to the OEM’s offer? It is of interest to investigate the changes in $Q_1$ and $Q_2$ brought about by $\gamma$’s.

**Theorem 7.** Given the salvage offset contract provided by the OEM,

(1) the CM’s critical fractile at stage two, $\alpha_2^c$, in non-increasing in $\gamma_1$ and non-decreasing in $\gamma_2$.
(2) the CM’s optimal order quantity of component 1, $Q^*_1$, is non-decreasing in both $\gamma_1$ and $\gamma_2$.

Proof. The first half of Theorem 7 follows from the partial derivatives of $\alpha_c^2$:

$$\frac{\partial \alpha^c_2}{\partial \gamma_1} = \frac{-l_1 l_2 (1 - \gamma_2)}{[m_c + (1 - \gamma_1) l_1 + (1 - \gamma_2) l_2]^2} \leq 0,$$

(4.30)

$$\frac{\partial \alpha^c_2}{\partial \gamma_2} = \frac{[m_c + (1 - \gamma_1) l_1] l_2}{[m_c + (1 - \gamma_1) l_1 + (1 - \gamma_2) l_2]^2} \geq 0.$$  

(4.31)

To prove the second half of Theorem 7, differentiate both sides of equation (4.29) w.r.t. $\gamma_1$:

$$\int_A f_{D|I}(Q^*_1) \left( \frac{\partial Q^*_1}{\partial \gamma_1} \right) dG_I = -[1 - P(A)] \frac{\partial \alpha^c_2}{\partial \gamma_1} + \frac{\partial \alpha^c_n}{\partial \gamma_1}.$$ 

(4.32)

The derivative of $Q^*_1$ w.r.t. $\gamma_1$ is therefore

$$\frac{\partial Q^*_1}{\partial \gamma_1} = \frac{-[1 - P(A)] \frac{\partial \alpha^c_2}{\partial \gamma_1} + \frac{\partial \alpha^c_n}{\partial \gamma_1}}{\int_A f_{D|I}(Q^*_1) dG_I}.$$  

(4.33)

Note that the derivatives of $\alpha^c_n$ w.r.t. $\gamma_1$ and $\gamma_2$ are both non-negative:

$$\frac{\partial \alpha^c_n}{\partial \gamma_1} = \frac{m_c l_1}{[m_c + (1 - \gamma_1) l_1 + (1 - \gamma_2) l_2]^2} \geq 0,$$

(4.33)

$$\frac{\partial \alpha^c_n}{\partial \gamma_2} = \frac{m_c l_2}{[m_c + (1 - \gamma_1) l_1 + (1 - \gamma_2) l_2]^2} \geq 0.$$  

(4.34)
Since $\partial \alpha^C_2/\partial \gamma_1 \leq 0$ and $\partial \alpha^C_n/\partial \gamma_1 \geq 0$, the numerator of the right-hand-side term in equation (4.32) is non-negative. The denominator is a positive term. Therefore, $\partial Q^*_1/\partial \gamma_1$ is greater than or equal to 0. This indicates that $Q^*_1$ is non-decreasing in $\gamma_1$.

Similarly, the derivative of $Q^*_1$ w.r.t. $\gamma_2$ is

$$\frac{\partial Q^*_1}{\partial \gamma_2} = \frac{-[1 - P(A)] \frac{\partial \alpha^C_2}{\partial \gamma_2} + \frac{\partial \alpha^C_n}{\partial \gamma_2}}{\int_A f_{D|I}(Q^*_1) dG_I}.$$ 

Substitute equations (4.33) and (4.34) into the above expression and simplify. It follows that

$$\frac{\partial Q^*_1}{\partial \gamma_2} = \left[-[1 - P(A)] \alpha^C_2 + \alpha^C_n\right]$$

$$\times \left[\frac{1}{\int_A f_{D|I}(Q^*_1) dG_I} \left[\frac{l_2}{m_c + (1 - \gamma_1) l_1 + (1 - \gamma_2) l_2}\right]\right]. \quad (4.35)$$

From equation (4.29), the term $-[1 - P(A)] \alpha^C_2 + \alpha^C_n$ must be non-negative at the optimal solution. Hence, $\partial Q^*_1/\partial \gamma_2$ is non-negative. This shows that $Q^*_1$ is non-decreasing in $\gamma_2$. \[\square\]

According to Theorem 7, if the OEM agrees to share overage risk of component 1, then the CM will buy more of component 1, but may choose to decrease the order quantity of component 2 because $\alpha^C_2$ becomes smaller. Meanwhile, if the OEM agrees to assume a portion of the overage risk for component 2, then the CM will purchase more for both components.
Now it is clear that $Q^*_1$ is positively related with $\gamma$'s. From the standpoint of the OEM, it would like to know how its cost performance is affected by changes in $Q^*_1$.

**Theorem 8.** At the OEM’s optimal choices of $\gamma_1$ and $\gamma_2$, the OEM’s total cost would be reduced by increasing $Q^*_1$, i.e., $\partial TC^o/\partial Q^*_1 \leq 0$. Furthermore, the total supply chain cost would be reduced by increasing $Q^*_1$.

**Proof.** If $\gamma_1^*$ and $\gamma_2^*$ are the optimal decisions for the OEM, the first-order conditions must hold:

$$\frac{\partial TC^o}{\partial \gamma_1} = \frac{\partial TC^o}{\partial Q^*_1} \frac{\partial Q^*_1}{\partial \gamma_1} + \frac{\partial TC^o}{\partial \alpha_2} \frac{\partial \alpha_2}{\partial \gamma_1} + l_1(O + V) = 0,$$  \hspace{1cm} (4.36)

$$\frac{\partial TC^o}{\partial \gamma_2} = \frac{\partial TC^o}{\partial Q^*_1} \frac{\partial Q^*_1}{\partial \gamma_2} + \frac{\partial TC^o}{\partial \alpha_2} \frac{\partial \alpha_2}{\partial \gamma_2} + l_2O = 0.$$ \hspace{1cm} (4.37)

Suppose in some cases, $\partial TC^o/\partial Q^*_1 > 0$ at the point $(\gamma_1^*, \gamma_2^*)$. Consider the directional derivative in $[1, k]$ for $\alpha_2$:

$$\frac{1}{k+1} \frac{\partial \alpha^C_2}{\partial \gamma_1} + \frac{k}{k+1} \frac{\partial \alpha^C_2}{\partial \gamma_2} = \frac{l_2[k(m_c + (1 - \gamma_1)l_1) - l_1(1 - \gamma_2)]}{(k+1)[m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2]^2}.$$ \hspace{1cm} (4.38)

With $k = \frac{l_1(1 - \gamma_2)}{m_c + (1 - \gamma_1)l_1}$, the above directional derivative is 0, meaning as we move in that direction, $\alpha_2$ is unchanged. The directional derivative for OEM’s total cost at
this point is therefore determined only by the change in cost due to changes in \( Q^*_1 \),

\[
\frac{1}{k+1} \left[ \frac{\partial TC^o}{\partial Q_1^*} \left( \frac{\partial Q^*_1}{\partial \gamma_1} + k \frac{\partial Q^*_2}{\partial \gamma_2} \right) + \frac{\partial TC^o}{\partial \alpha_2} \left( \frac{\partial \alpha^c_2}{\partial \gamma_1} + k \frac{\partial \alpha^c_2}{\partial \gamma_2} \right) + [l_1(O + V) + kl_2O] \right]
\]

\[
= \frac{1}{k+1} \frac{\partial TC^o}{\partial Q_1^*} \left[ \frac{1}{\int_A f_{D|J}(Q^*_1) \, dG_I} \right]
\]

\[
\times \left[ -[1 - P(A)] \left( \frac{\partial \alpha^c_2}{\partial \gamma_1} + k \frac{\partial \alpha^c_2}{\partial \gamma_2} \right) + \left( \frac{\partial \alpha^c_2}{\partial \gamma_1} + k \frac{\partial \alpha^c_2}{\partial \gamma_2} \right) \right]
\]

\[
+ \frac{1}{k+1}[l_1(O + V) + kl_2O]
\]

\[
= \frac{1}{k+1} \frac{\partial TC^o}{\partial Q_1^*} \left( \frac{1}{\int_A f_{D|J}(Q^*_1) \, dG_I} \right) + \left( \frac{\partial \alpha^c_2}{\partial \gamma_1} + k \frac{\partial \alpha^c_2}{\partial \gamma_2} \right)
\]

\[
+ \frac{1}{k+1}[l_1(O + V) + kl_2O] > 0. \tag{4.39}
\]

By supposition, \( \frac{\partial TC^o}{\partial Q_1^*} > 0 \). This means that this directional derivative is strictly positive, contradicting the optimality of \( \gamma^*_1 \) and \( \gamma^*_2 \). Therefore, at \( \gamma^*_1 \) and \( \gamma^*_2 \), it must be true that \( \frac{\partial TC^o}{\partial Q_1^*} \leq 0 \).

To establish the second part of the theorem, note that the total supply chain cost is equal to the sum of the CM’s and OEM’s costs, thus

\[
\frac{\partial TC^{sc}}{\partial Q_1^*} = \frac{\partial TC^o}{\partial Q_1^*} + \frac{\partial TC^c}{\partial Q_1^*}
\]

\[
= \frac{\partial TC^o}{\partial Q_1^*} + 0 \leq 0.
\]

\( \square \)
As shown previously, a \( \gamma \)-contract can induce the CM to buy more of component 1. Theorem 8 implies that even at the optimal \( \gamma \)'s, the CM still underbuys from OEM and supply chain perspectives.

4.3.4 Analytical Results from the U-U Model

In the previous section, a general model was presented, and some properties about the interaction between the OEM and the CM were identified. However, with the general model, no closed-form expressions could be found for \( Q_1^* \), \( E(Q_2^*) \) as well as expected underage and overage. No general conclusion can be drawn about the relationship between expected shortage and the quality of information at stage two. Although it seems that better information results in less underage, this is not always true. For example, suppose demand distribution at stage one is

\[
D = \begin{cases} 
1 & \text{with probability } \frac{2}{5}; \\
2 & \text{with probability } \frac{1}{5}; \\
3 & \text{with probability } \frac{2}{5}.
\end{cases}
\]

Assume under SSTI the demand distribution at stage two will be one of the two situations: (1) either 1 or 2 with probability \( \frac{2}{3} \) or \( \frac{1}{3} \) respectively; (2) exactly 3 with probability 1. For the cost parameters \( m_c = 2, l_1 = 0, l_2 = 1 \), \( Q_1^* \) under NSTI is 3 and \( Q_2^* \) is also 3. This means that under NSTI there will be no shortages. Under SSTI, \( Q_1^* \) is 3 since there is no salvage loss associated with component one; however, \( Q_2^* \) can be as low as 1 if revised demand distribution at stage two is in the first situation. This
will result in an expected shortage of $1/5$, demonstrating that sharing the update can increase expected shortages.

In this section, we use the U-U model to derive closed-form expressions for expected shortage and overage under all three information scenarios, namely, NSTI, SSTI, and PSTI. We show that better information reduces expected shortage under the U-U model. The unit shortage, overage, and gap between $Q_1$ and $Q_2$ are:

$$U_{NSTI} = \begin{cases} 
\frac{1}{3} \hat{\mu} \epsilon \left[ 3 - (3 - \alpha^c_n) \sqrt{2\alpha^c_n} \right] & \text{if } \alpha^c_n \leq 0.5; \\
\frac{1}{3} \hat{\mu} \epsilon (1 - \alpha^c_n) \sqrt{2(1 - \alpha^c_n)} & \text{if } \alpha^c_n > 0.5.
\end{cases} \quad (4.40)$$

$$O_{NSTI} = \begin{cases} 
\frac{1}{3} \hat{\mu} \epsilon \alpha^c_n \sqrt{2\alpha^c_n} & \text{if } \alpha^c_n \leq 0.5; \\
\frac{1}{3} \hat{\mu} \epsilon \left[ 3 - (2 + \alpha^c_n) \sqrt{2(1 - \alpha^c_n)} \right] & \text{if } \alpha^c_n > 0.5.
\end{cases} \quad (4.41)$$
The expressions for shortage and overage under PSTI can be obtained by simply replacing \( \alpha^c_n \) with \( \alpha^c_p \) in the above equations. Under SSTI, we have

\[
U_{SSTI} = \begin{cases} 
U_{NSTI} & \text{if } 0 \leq \alpha^c_n \leq \frac{(\alpha^c_2)^2}{2}; \\
\frac{1}{24} \hat{\mu} \epsilon \left[ (\alpha^c_2)^2 (3 - 2\alpha^c_2) + 12 (1 - \alpha^c_n)(1 - \alpha^c_2) + 12 \left( 1 - \frac{\alpha^c_n}{\alpha^c_2} \right)^2 \right] & \text{if } (\alpha^c_2)^2 / 2 < \alpha^c_n \leq \alpha^c_2 - \frac{(\alpha^c_2)^2}{2}; \\
\frac{1}{24} \hat{\mu} \epsilon \left[ 3(1 - \alpha^c_2)(1 + \alpha^c_2 - 2\alpha^c_n) + 2(\alpha^c_2 - \alpha^c_n)^2 \right] & \text{if } \alpha^c_2 - \alpha^c_n^2 / 2 < \alpha^c_n \leq \alpha^c_2. 
\end{cases}
\] (4.42)

\[
O_{SSTI} = \begin{cases} 
O_{NSTI} & \text{if } 0 \leq \alpha^c_n \leq \frac{(\alpha^c_2)^2}{2}; \\
\frac{1}{12} \hat{\mu} \epsilon \left[ \alpha^c_2 (6\alpha^c_n - (\alpha^c_2)^2) \right] & \text{if } (\alpha^c_2)^2 / 2 < \alpha^c_n \leq \alpha^c_2 - \frac{(\alpha^c_2)^2}{2}; \\
\frac{1}{6} \hat{\mu} \epsilon \left[ 3\alpha^c_2 (2\alpha^c_n - \alpha^c_2) + [2(\alpha^c_2 - \alpha^c_n)]^2 \right] & \text{if } \alpha^c_2 - \alpha^c_n^2 / 2 < \alpha^c_n \leq \alpha^c_2. 
\end{cases}
\] (4.43)
Using the above expressions, we can establish the following proposition.

**Proposition 2.** When the CM orders optimally, expected units short is decreasing in improved information, \( U_{PSTI} \leq U_{SSTI} \leq U_{NSTI} \).

*Proof.* Note that under PSTI no shortage will be caused by a lack of component 2 since perfect information is available at stage two. In other words, shortage depends on the order quantity of component 1 only. Under SSTI, the order quantity of component 2 may be less than \( Q_{1,PSTI}^* \) and shortage depends on both \( Q_{1,SSTI}^* \) and \( Q_{2}^* \). As shown previously, \( Q_{1,SSTI}^* \leq Q_{1,PSTI}^* \). This implies that the shortage under PSTI is always less than that under SSTI, which shows the first part of Proposition 2, i.e., \( U_{PSTI} \leq U_{SSTI} \). We next show that \( U_{SSTI} \leq U_{NSTI} \). Clearly, this is equivalent to showing that

\[
U_{SSTI} - U_{NSTI} \leq 0 \tag{4.45}
\]

If \( 0 \leq \alpha_n \leq \frac{\alpha_2^2}{2} \), we have \( U_{SSTI} = U_{NSTI} \) and inequality (4.45) is true. Next we prove that it also holds in other cases.
1. \( \frac{\alpha^2}{2} < \alpha_n \leq \alpha_2 - \frac{\alpha^2}{2} \)

Substitute the appropriate expressions into inequality (4.45) and simplify.

\[
U_{SSTI} - U_{NSTI} = \frac{1}{24} \hat{\mu} \left[ 12 \left( 1 - \frac{\alpha^2}{\alpha_2} \right)^2 - 2 \alpha^3_2 
+ 12 \alpha_n \alpha_2 - 12 + (24 - 8 \alpha_n) \sqrt{2 \alpha_n - 24 \alpha_n} \right] \leq 0.
\]

Using previous results \( \frac{\alpha^2}{2} + \frac{\alpha_n}{\alpha_2} \geq \sqrt{2 \alpha_n} \), it follows that

\[
12 \left( 1 - \frac{\alpha^2}{\alpha_2} \right)^2 - 2 \alpha^3_2
+ 12 \alpha_n \alpha_2 - 12 + (24 - 8 \alpha_n) \sqrt{2 \alpha_n - 24 \alpha_n}
\leq 12 \left( 1 - \sqrt{2 \alpha_n} \right)^2 - 2 \alpha^3_2
+ 12 \alpha_n \alpha_2 - 12 + (24 - 8 \alpha_n) \sqrt{2 \alpha_n - 24 \alpha_n}
= - 2 \alpha^3_2 + 12 \alpha_n \alpha_2 - 8 \alpha_n \sqrt{2 \alpha_n}.
\]

The last expression is decreasing in \( \alpha_n \) since its derivative w.r.t. \( \alpha_n \) is \( 12 (\alpha_2 - \sqrt{2 \alpha_n}) < 0 \). At the point of \( \alpha_n = \alpha^2_2/2 \), it reaches the maximum value 0. This shows that \( U_{SSTI} - U_{NSTI} \leq 0 \) for \( \frac{\alpha^2}{2} < \alpha_n \leq \alpha_2 - \frac{\alpha^2}{2} \).

2. \( \alpha_2 - \frac{\alpha^2}{2} < \alpha_n \leq \alpha_2 \)

Two subcases are to be considered.

(a) \( \alpha_n \leq 0.5 \)

Substitute the appropriate expressions into the left-hand-side of inequality
(4.45) and simplify.

\[
USSTI - U NSTI = \frac{1}{6} \mu_e \left[ 3(1 - \alpha_2)(1 + \alpha_2 - 2\alpha_n) + \left( \sqrt{2(\alpha_2 - \alpha_n)} \right)^3 - [6 - (6 - 2\alpha_n)\sqrt{2\alpha_n}] \right].
\]

Using the result \( \sqrt{2(\alpha_2 - \alpha_n)} \leq \alpha_2 \), we have

\[
3(1 - \alpha_2)(1 + \alpha_2 - 2\alpha_n) + \left( \sqrt{2(\alpha_2 - \alpha_n)} \right)^3 - [6 - (6 - 2\alpha_n)\sqrt{2\alpha_n}] \\
\leq 3(1 - \alpha_2)(1 + \alpha_2 - 2\alpha_n) + \alpha_2^3 - [6 - (6 - 2\alpha_n)\sqrt{2\alpha_n}] \\
= (\alpha_2 - \sqrt{2\alpha_n})^3 - 3(1 - \sqrt{2\alpha_n})^2 - 3\alpha_2^2(1 - \sqrt{2\alpha_n}).
\]

The last expression is non-positive because \( \alpha_2 < \sqrt{2\alpha_n} \) and \( \alpha_n \leq 0.5 \). This means that the left-hand-side of inequality 4.45 is non-positive in this case.

(b) \( \alpha_n > 0.5 \)

Again, substitute the appropriate expressions into the left-hand-side of inequality (4.45) and simplify.

\[
USSTI - U NSTI = \frac{1}{6} \mu_e \left[ 3(1 - \alpha_2)(1 + \alpha_2 - 2\alpha_n) + \left( \sqrt{2(\alpha_2 - \alpha_n)} \right)^3 - [\sqrt{2(1 - \alpha_n)}]^3 \right].
\]
The first-order derivative of the above expression w.r.t. $\alpha_2$ is 
\[ \frac{1}{2} \hat{\mu} \epsilon \left( \sqrt{2(\alpha_2 - \alpha_n)} - 2(\alpha_2 - \alpha_n) \right), \] which is non-negative because $\alpha_n > 0.5$. This means $U_{SSTI} - U_{NSTI}$ is increasing in $\alpha_2$. At the point of $\alpha_2 = 1$, it attains the maximum value 0. This proves that $U_{SSTI} - U_{NSTI} \leq 0$ is true for $\alpha_n > 0.5$.

The above analysis shows $U_{PSTI} \leq U_{SSTI} \leq U_{NSTI}$ and expected units short is decreasing in improved information. This ends the proof.

Thus, for any fixed risk-sharing agreement, choosing to share the forecast update will result in fewer shortages for the OEM. This does not necessarily mean that it is in the best interest of the OEM to share the information. As we will see below, that will depend on the choices of $\gamma_i$. It is worth recalling here that when there is no risk-sharing agreement, the OEM’s cost consists only of shortages. As the following proposition indicates, for this no-risk-sharing situation, both the CM and OEM are better off when the forecast update is shared.

**Proposition 3.** In the U-U demand case, when there is no risk-sharing agreement, it is Pareto-improving for the OEM to share the updated forecast.

With the closed-form expressions derived from the U-U model, we can further investigate the CM’s reactions to the OEM’s offering of $\gamma$’s. Clearly the order quantities of component 1 under NSTI and PSTI are non-decreasing in $\gamma_1$ and $\gamma_2$. Under SSTI, we obtain the following result.

**Theorem 9.** Given the DG contract under SSTI, the optimal order quantity of component 1, $Q_{1,SSTI}^*$, is non-decreasing in $\gamma_1$ and $\gamma_2$. 
Proof. We essentially need to determine the sign of the first-order derivatives of $Q^*_1$ with regard to $\gamma$’s.

1. For $\gamma_1$, we have

   (a) $0 \leq \alpha_n^c \leq \frac{(\alpha_1^c)^2}{2}$

   $$\frac{\partial Q^*_{1,SSTI}}{\partial \gamma_1} = \hat{\mu} \epsilon \left( \frac{1}{\sqrt{2\alpha_n^c}} \right) \frac{\partial \alpha_n^c}{\partial \gamma_1} \geq 0.$$ 

   (b) $\frac{(\alpha_2^c)^2}{2} < \alpha_n^c \leq \alpha_2^c - \frac{(\alpha_2^c)^2}{2}$

   $$\frac{\partial Q^*_{1,SSTI}}{\partial \gamma_1} = \hat{\mu} d_1 \left[ \frac{m_c}{[m_c + (1 - \gamma_1)l_1]^2} - \frac{(1 - \gamma_2)l_2}{2[m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2]^2} \right].$$

   The right-hand-side of the above equation is greater than 0 which follows from the condition $\frac{(\alpha_2^c)^2}{2} < \alpha_n^c$. Therefore, $Q^*_{1,SSTI}$ in this case increases with $\gamma_1$.

   (c) $\alpha_2^c - \frac{(\alpha_2^c)^2}{2} < \alpha_n^c \leq \alpha_2^c$

   $$\frac{\partial Q^*_{1,SSTI}}{\partial \gamma_1} = \frac{\partial Q^*_{1,SSTI}}{\partial \alpha_n^c} \frac{\partial \alpha_n^c}{\partial \gamma_1} + \frac{\partial Q^*_{1,SSTI}}{\partial \alpha_2^c} \frac{\partial \alpha_2^c}{\partial \gamma_1}$$

   $$= \hat{\mu} \epsilon \left[ \frac{(1 - \gamma_2)l_1 l_2 [1 - \sqrt{2(\alpha_2^c - \alpha_n^c)}] + m_c l_1}{\sqrt{2(\alpha_2^c - \alpha_n^c)[m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2]^2}} \right]$$

   $$\geq 0.$$ 

   This shows that $Q^*_{1,SSTI}$ is non-decreasing in $\gamma_1$. 

2. For $\gamma_2$, we have

(a) $0 \leq \alpha_n^c \leq \frac{(\alpha_n^c)^2}{2}$

\[ \frac{\partial Q_{1,SSTI}^*}{\partial \gamma_2} = \hat{\mu} \epsilon \left( \frac{1}{2\alpha_n^c} \right) \frac{\partial \alpha_n^c}{\partial \gamma_2} \geq 0. \]

(b) $\frac{(\alpha_n^c)^2}{2} < \alpha_n^c \leq \alpha_n^c - \frac{(\alpha_n^c)^2}{2}$

\[ \frac{\partial Q_{1,SSTI}^*}{\partial \gamma_2} = \hat{\mu} \epsilon \left[ \frac{[m_c + (1 - \gamma_1)l_1]l_2}{2[m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2]^2} \right] \geq 0. \]

(c) $\alpha_2^c - \frac{(\alpha_2^c)^2}{2} < \alpha_n^c \leq \alpha_2^c$

\[ \frac{\partial Q_{1,SSTI}^*}{\partial \gamma_2} = \frac{\partial Q_{1,SSTI}^*}{\partial \alpha_n^c} \frac{\partial \alpha_n^c}{\partial \gamma_2} + \frac{\partial Q_{1,SSTI}^*}{\partial \alpha_2^c} \frac{\partial \alpha_2^c}{\partial \gamma_2} \]

\[ = \hat{\mu} \epsilon \left[ \frac{(1 - \gamma_1)l_1l_2[\sqrt{2(\alpha_2^c - \alpha_n^c)} - 1] + m_cl_2\sqrt{2(\alpha_2^c - \alpha_n^c)}}{\sqrt{2(\alpha_2^c - \alpha_n^c)[m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2]^2}} \right]. \]

The right-hand-side of the above equation is greater than 0, which follows from the condition $\alpha_2^c - \frac{(\alpha_2^c)^2}{2} < \alpha_n^c$. Hence, $Q_{1,SSTI}^*$ increases with $\gamma_2$ in this case.

\[ \square \]

Although $\gamma$'s increase the order quantity of component 1 under SSTI, this is not necessarily true for component 2. It is obvious that $Q_{2}^*$ increases with $\gamma_2$; however, as
\( \gamma_1 \) increases, \( \alpha_{c_2}^c \) becomes smaller, which in turn can bring down the order quantity of component 2. It is of interest to investigate how \( \gamma \)'s affect the gap, \( V_{\text{SSTI}}^* \), between \( Q_{1}^* \) and \( Q_{2}^* \).

**Theorem 10.** Given the DG contract under SSTI, the gap between \( Q_{1}^*, \text{SSTI} \) and \( E(Q_{2}^*) \) never shrinks as \( \gamma_1 \) increases and never widens as \( \gamma_2 \) increases.

**Proof.** From the results in the previous chapter, \( \partial V_{\text{SSTI}} / \partial \alpha_{c_2}^c \leq 0 \) and \( \partial V_{\text{SSTI}} / \partial \alpha_{n}^c \geq 0 \). Next we need to determine the sign of the first-order derivatives of \( V_{\text{SSTI}} \) with regard to \( \gamma \)'s.

1. For \( \gamma_1 \), we have

\[
\frac{\partial V_{\text{SSTI}}}{\partial \gamma_1} = \frac{\partial V_{\text{SSTI}}}{\partial \alpha_n^c} \frac{\partial \alpha_n^c}{\partial \gamma_1} + \frac{\partial V_{\text{SSTI}}}{\partial \alpha_2^c} \frac{\partial \alpha_2^c}{\partial \gamma_1}.
\]

Note that \( \frac{\partial \alpha_2^c}{\partial \gamma_1} \leq 0 \) and \( \frac{\partial \alpha_n^c}{\partial \gamma_1} \geq 0 \). Therefore, \( \partial V_{\text{SSTI}} / \partial \gamma_1 \geq 0 \), which means that the gap between the order quantities for the two components does not shrink as \( \gamma_1 \) increases.

2. For \( \gamma_2 \), we consider the derivatives in three cases.

   (a) \( 0 \leq \alpha_n^c \leq \frac{(\alpha_2^c)^2}{2} \)

   In this case, the CM always orders squared sets and there is no difference between \( Q_{1}^* \) and \( Q_{2}^* \). The proposition is obviously true.

   (b) \( \frac{(\alpha_2^c)^2}{2} < \alpha_n^c \leq \alpha_2^c - \frac{(\alpha_2^c)^2}{2} \)

   \[
   \frac{\partial V_{\text{SSTI}}}{\partial \gamma_2} = -\frac{1}{4} \tilde{\mu} \epsilon \left[ 2 \left( \frac{\alpha_n^c}{\alpha_2^c} \right) - \alpha_2^c \right] \frac{\partial \alpha_2^c}{\partial \gamma_2} \leq 0.
   \]
The derivative is non-positive, which implies that the gap is non-increasing in $\gamma_2$.

(c) $\alpha_2^c - \frac{(\alpha_2^c)^2}{2} < \alpha_n^c \leq \alpha_2^c$.

\[
\frac{\partial V_{SSTI}}{\partial \gamma_2} = -\mu \epsilon \left[ 1 - \sqrt{\frac{2(\alpha_2^c - \alpha_n^c)}{\sqrt{2}(\alpha_2^c - \alpha_n^c)^2}} \right] \left[ \frac{(1 - \gamma_1)l_1l_2}{[m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2]^2} \right] \leq 0.
\]

The derivative is again non-positive because $2(\alpha_2^c - \alpha_n^c) < 1$, which implies that the gap is non-increasing in $\gamma_2$.

From these analysis, we know that the gap between $Q_{1,SSTI}^*$ and $\mathbb{E}(Q_{2}^*)$ never narrows as $\gamma_1$ increases and never widens as $\gamma_2$ increases. This ends the proof.

Finally, although Theorems 9 and 10 are based on the DG contract, the results for $\gamma_1$ are still valid for the SG contract because the latter is just a special case of the former. There are some important managerial insights from this. If the OEM shares information while at the same time shares overage risk for component 1 only, its cost performance may deteriorate. As discussed previously the CM tends to order more of component 1 and may order less of component 2 if the OEM were to offer an SG contract. In the case of demand falling between $Q_1$ and $Q_2$, there will be more lost sales resulting from a lower $Q_2$. Meanwhile the OEM has to bear more overage cost because the gap between $Q_1$ and $Q_2$ becomes larger. The effects of $\gamma_1$ on both sides put together could therefore worsen the OEM’s position.
4.4 An Analysis of the Salvage Offset Contract

As discussed earlier in Chapter 2, the salvage offset contract is different than most of the supply chain agreements that have appeared in literature, such as pricing, buyback, quantity flexibility or options contracts. The difference between salvage offset and pricing, quantity flexibility, or options is obvious. Under a buyback contract, the seller agrees to repurchase unused or unsold items that it originally distributed to the buyer, who then sells to end customers, in an attempt to induce the buyer to keep as much inventory as possible. The salvage offset agreement distinguishes itself from the buyback agreement in that the OEM is willing to share component overage risk though it may not necessarily be the supplier of the components. This section investigates the properties of the salvage offset contract.

4.4.1 Contract Equivalence

Clearly the SG contract is a special case of the DG contract. It can be shown (see Proposition 4) that under certain conditions the SS contract is actually equivalent to a $\gamma_2$-only contract, under which the OEM agrees to share the overage cost of component 2 only.

**Proposition 4.** The squared-sets contract (SS) with parameter $\gamma \leq l_2/(l_1 + l_2)$ is equivalent to a dual-$\gamma$ (DG) contract with parameters $\gamma_1 = 0$ and $\gamma_2 = \gamma(l_1 + l_2)/l_2$.

**Proof.** Substitute $\gamma_1 = 0$ and $\gamma_2 = \gamma(l_1 + l_2)/l_2$ into equations (4.4) and (4.5) and we get exactly the same cost functions as in equations (4.8) and (4.9). Note that unit shortage and overage are determined by the optimal order quantities, which can be expressed
as functions of the critical fractiles. Under the DG contract, the critical fractiles are
\[ \alpha_{c_2} = \frac{m_c + (1 - \gamma_1)l_1}{m_c + (1 - \gamma_2)l_2} \] and \[ \alpha_{c_n} = \frac{m_c + (1 - \gamma_2)l_2}{m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2} \]. By substituting \( \gamma_1 = 0 \) and \( \gamma_2 = \gamma(l_1 + l_2)/l_2 \) into these expressions, we have
\[ \alpha_{c_2} = \frac{m_c + l_1}{m_c + (1 - \gamma)(l_1 + l_2)} \] and \[ \alpha_{c_n} = \frac{m_c + (1 - \gamma)(l_1 + l_2)}{m_c + (1 - \gamma)(l_1 + l_2)} \], which are the critical fractiles under the SS contract. This means that for any SS contract with parameter \( \gamma \), there always exists a DG contract with parameters \((0, \gamma(l_1 + l_2)/l_2)\) which performs equally as the former.

With the results from Proposition 4, it is easy to see that for the OEM the DG contract always outperforms the SS contract as long as \( \gamma \leq \frac{l_2}{l_1 + l_2} \). If the OEM allows \( \gamma_2 \) to be greater than 1, then this result can be extended to any SS contract.

4.4.2 Conditions for Supply Chain-Optimal Contracts

The cost function for the supply chain can be formulated as

\[ TC^{sc} = (m_o + m_c)U + (l_1 + l_2)O + l_1V. \] (4.48)

In order to align the CM’s decision with the optimal order policy for the entire supply chain, the salvage offset offered by the OEM must be such that the CM’s critical fractiles equal those for the supply chain.

Proposition 5. There always exists a contract \((\hat{\gamma}_1, \hat{\gamma}_2)\) under which supply chain optimality can be achieved:

\[ \hat{\gamma}_1 = \hat{\gamma}_2 = \frac{m_o}{m_c + m_o}. \] (4.49)
Proof. By definition, the critical fractiles for the CM and the supply chain are

\[
\alpha_c = \frac{m_c}{m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2},
\]

\[
\alpha_c^2 = \frac{m_c + (1 - \gamma_1)l_1}{m_c + (1 - \gamma_1)l_1 + (1 - \gamma_2)l_2},
\]

\[
\alpha_{sc} = \frac{m_c + m_o}{m_c + m_o + l_1 + l_2},
\]

\[
\alpha_{sc}^2 = \frac{m_c + m_o + l_1}{m_c + m_o + l_1 + l_2}.
\]

Under all three scenarios, the optimal order quantity can be expressed as a function of the critical fractiles. Therefore, if the critical fractiles for the CM and the supply chain are the same, the quantity the CM wants to order will be the same as the quantity that minimizes the expected total cost for the supply chain as a whole. Solving the following equations for \(\gamma_1\) and \(\gamma_2\), we obtain the results stated in Proposition 5.

\[
\alpha_c = \alpha_{sc}, \quad (4.50)
\]

\[
\alpha_c^2 = \alpha_{sc}^2. \quad (4.51)
\]

The above analysis shows that the \(\gamma\) value leading to supply chain optimality is equal to the OEM’s share of the total profit margin for the supply chain. Under this contract, the order quantity favorable for the CM is also desired by the supply chain. This is consistent with the intuition that risk and return must be in parity. Essentially
salvage offset contracts are intended to induce the CM to purchase more components than it does without the offer, which means higher risk for the CM. From the CM’s standpoint, it grabs only a portion, $m_c/(m_o + m_c)$, of the total profit for every item sold to the end customer. Corresponding to this, the CM would like the OEM to assume part of the overage risk until its own risk level matches the return. Although theoretically there exists such a contract, the OEM may not offer it because its cost performance may actually deteriorate.

4.5 Computational Results

As discussed earlier, there are two types of coordination. One is based on the sharing of demand forecast updates; the other is mediated by risk-sharing agreements. This section presents computational results and compares the two types of coordination as well as the performance of various contracts within a specific information scenario. The analysis mainly focuses on the comparison of various contracts under NSTI and SSTI. The results for PSTI are not presented here for two reasons. First, in reality it is almost impossible to have perfect information about demand; second, it does not make sense for the OEM to offer the SS or DG contract under PSTI because the risk of overbuying component 2 is completely resolved by having perfect demand information at stage two.

4.5.1 Contract Efficiency and Percentage Cost Improvement

Tables 4.1 to 4.3 summarize computational results and provide an overview of contract efficiency, percentage reduction in $TC^o$ and $TC^c$ under different coordination
schemes based on 10,125 cases. The abbreviation “NC” in the tables stands for “No Contract”, and “PC” means perfect coordination or centralization. The degree of coordination is defined by two dimensions: (1) information sharing and (2) risk sharing. On one extreme lies no stage-two information sharing and no salvage-offset contract (NSTI/NC); on the other extreme lies sharing stage-two information and perfect coordination (SSTI/PC) as if the OEM and CM were one entity. Between these two extremes there are several coordination schemes characterized by different combinations of information scenario and contract. For example, “NSTI/SG” means no stage-two information sharing and the presence of the SG contract; “SSTI/SS” refers to stage-two information sharing and the presence of the SS contract.

Contract Efficiency measures the effectiveness of each coordination scheme in terms of improving the total welfare of the supply chain. The supply chain total cost is calculated as the sum of OEM cost and CM cost ($TC_{sc} = TC^o + TC^c$), assuming optimal $\gamma$'s for the OEM, if applicable, and optimal order quantities for the CM. The supply chain cost under “NSTI/NC” is used as the starting point and the improvement brought about by “SSTI/PC” is used as the benchmark. Thus, the largest possible improvement in supply chain cost is $TC_{sc}^{NSTI/NC} - TC_{sc}^{SSTI/PC}$. The improvement brought about by each coordination mechanism is the difference between $TC_{sc}^{NSTI/NC}$ and $TC_{sc}$ under that specific scheme. Contract efficiency is then measured by the ratio of the improvement from a specific coordination scheme against the possible biggest improvement. For example, the efficiency of the DG contract under NSTI is

$$\frac{TC_{sc}^{NSTI/NC} - TC_{sc}^{NSTI/DG}}{TC_{sc}^{NSTI/NC} - TC_{sc}^{SSTI/PC}} \times 100\%.$$
In general the following equation is used to calculate contract efficiency:

\[
\text{Contract Efficiency} = \frac{TC_{sc}^{NSTI/NC} - TC_{sc}^{Info/Contract}}{TC_{sc}^{NSTI/NC} - TC_{sc}^{SSTI/PC}} \times 100\%,
\]

where “Info/Contract” refers to a specific coordination structure.

Percentage cost improvement for the OEM is defined as the reduction in the OEM’s total cost under a specific scheme compared with its total cost under “NSTI/NC”.

The following equation is used to calculate the improvement:

\[
\text{OEM Percentage Cost Reduction} = \frac{TC_{oem}^{NSTI/NC} - TC_{oem}^{Info/Contract}}{TC_{oem}^{NSTI/NC}} \times 100\%.
\]

Percentage cost improvement for the CM is defined in a similar way:

\[
\text{CM Percentage Cost Reduction} = \frac{TC_{cm}^{NSTI/NC} - TC_{cm}^{Info/Contract}}{TC_{cm}^{NSTI/NC}} \times 100\%.
\]

Unsurprisingly, “SSTI/DG” is the most efficient coordination scheme which attains more than 64% (with a median of 70.22%) of the possible biggest improvement on average. Generally a contract under SSTI is substantially more efficient than under NSTI. By sharing information and risk, both the OEM and the CM enjoy a dramatic reduction in their total costs. It is very interesting that the CM’s position could be hurt by centralization. Although the OEM’s situation becomes much better with centralized planning, its improvement is achieved at the expense of the CM. Therefore, if the OEM wants to take control of the CM’s planning, it must compensate the CM to make sure that the CM’s situation would not get worse at the very least.
<table>
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<th></th>
<th>NSTI/NC</th>
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<th>NSTI/SS</th>
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</tr>
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<td>65.23%</td>
</tr>
<tr>
<td><strong>3rd Quartile</strong></td>
<td>–</td>
<td>29.37%</td>
<td>52.68%</td>
<td>52.68%</td>
<td>87.86%</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>–</td>
<td>99.63%</td>
<td>99.68%</td>
<td>99.68%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>SSTI/NC</th>
<th>SSTI/SG</th>
<th>SSTI/SS</th>
<th>SSTI/DG</th>
<th>SSTI/PC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>39.46%</td>
<td>55.36%</td>
<td>60.30%</td>
<td>64.41%</td>
<td>100.00%</td>
</tr>
<tr>
<td><strong>S.D.</strong></td>
<td>31.79%</td>
<td>24.73%</td>
<td>27.95%</td>
<td>28.52%</td>
<td>0.00%</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td><strong>1st Quartile</strong></td>
<td>8.56%</td>
<td>40.98%</td>
<td>39.81%</td>
<td>44.44%</td>
<td>100.00%</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>35.51%</td>
<td>55.21%</td>
<td>66.20%</td>
<td>70.22%</td>
<td>100.00%</td>
</tr>
<tr>
<td><strong>3rd Quartile</strong></td>
<td>66.82%</td>
<td>74.42%</td>
<td>83.59%</td>
<td>89.27%</td>
<td>100.00%</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

\( m_o = 0.1 : 0.1 : 1.5, m_c = 0.1 : 0.1 : 1.5, l_1 = 0.1 : 0.1 : 0.9, l_2 = 0.1 : 0.1 : (1 - l_1), \bar{\mu} = 100, \epsilon = 0.698. \)

4.5.2 Non-substitutability of Risk Sharing by Information Sharing

By offering information to the CM, the OEM may improve its own cost performance if the information turns out to be valuable to the CM. This may lead one to think that the OEM is less willing to absorb the risk of component overstocking after providing the CM with more information about demand. In other words, the OEM may use information sharing to replace risk sharing, however, numerical study shows that this is not necessarily true. The example in Table 4.4 indicates that information is not necessarily a substitute for incentive alignment. In this example, the OEM can improve its position by going from NSTI to SSTI without employing a risk sharing agreement. By sharing overage risk, the OEM can further improve its position. According to this computation, the OEM is willing to share some overage risk under NSTI by offering a
Table 4.2. Percentage Cost Reduction for the OEM under U-U Model

<table>
<thead>
<tr>
<th></th>
<th>NSTI/NC</th>
<th>NSTI/SG</th>
<th>NSTI/SS</th>
<th>NSTI/DG</th>
<th>NSTI/PC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>–</td>
<td>4.59%</td>
<td>6.91%</td>
<td>6.91%</td>
<td>46.18%</td>
</tr>
<tr>
<td>S.D.</td>
<td>–</td>
<td>9.28%</td>
<td>13.71%</td>
<td>13.71%</td>
<td>19.31%</td>
</tr>
<tr>
<td>Minimum</td>
<td>–</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>4.55%</td>
</tr>
<tr>
<td>1st Quartile</td>
<td>–</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>32.06%</td>
</tr>
<tr>
<td>Median</td>
<td>–</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>48.10%</td>
</tr>
<tr>
<td>3rd Quartile</td>
<td>–</td>
<td>4.53%</td>
<td>6.09%</td>
<td>6.09%</td>
<td>60.18%</td>
</tr>
<tr>
<td>Maximum</td>
<td>–</td>
<td>55.23%</td>
<td>68.90%</td>
<td>68.90%</td>
<td>93.63%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>SSTI/NC</th>
<th>SSTI/SG</th>
<th>SSTI/SS</th>
<th>SSTI/DG</th>
<th>SSTI/PC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>11.72%</td>
<td>15.87%</td>
<td>17.82%</td>
<td>18.80%</td>
<td>56.59%</td>
</tr>
<tr>
<td>S.D.</td>
<td>10.49%</td>
<td>10.27%</td>
<td>13.06%</td>
<td>13.88%</td>
<td>18.25%</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>6.82%</td>
</tr>
<tr>
<td>1st Quartile</td>
<td>2.38%</td>
<td>7.56%</td>
<td>7.08%</td>
<td>7.69%</td>
<td>44.21%</td>
</tr>
<tr>
<td>Median</td>
<td>9.52%</td>
<td>15.32%</td>
<td>15.91%</td>
<td>16.66%</td>
<td>59.16%</td>
</tr>
<tr>
<td>3rd Quartile</td>
<td>18.37%</td>
<td>22.52%</td>
<td>26.15%</td>
<td>27.27%</td>
<td>70.58%</td>
</tr>
<tr>
<td>Maximum</td>
<td>43.75%</td>
<td>50.17%</td>
<td>61.88%</td>
<td>72.97%</td>
<td>96.06%</td>
</tr>
</tbody>
</table>

(m_0 = 0.1 : 1.5, m_c = 0.1 : 1.5, l_1 = 0.1 : 0.9, l_2 = 0.1 : 0.1 : (1 - l_1), \hat{\mu} = 100, \epsilon = 0.698.)

Table 4.3. Percentage Cost Reduction for the CM under U-U Model

<table>
<thead>
<tr>
<th></th>
<th>NSTI/NC</th>
<th>NSTI/SG</th>
<th>NSTI/SS</th>
<th>NSTI/DG</th>
<th>NSTI/PC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>–</td>
<td>6.72%</td>
<td>11.11%</td>
<td>11.11%</td>
<td>-22.29%</td>
</tr>
<tr>
<td>S.D.</td>
<td>–</td>
<td>11.56%</td>
<td>18.11%</td>
<td>18.11%</td>
<td>37.56%</td>
</tr>
<tr>
<td>Minimum</td>
<td>–</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-244.20%</td>
</tr>
<tr>
<td>1st Quartile</td>
<td>–</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-21.82%</td>
</tr>
<tr>
<td>Median</td>
<td>–</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-9.12%</td>
</tr>
<tr>
<td>3rd Quartile</td>
<td>–</td>
<td>10.02%</td>
<td>18.28%</td>
<td>18.28%</td>
<td>-3.32%</td>
</tr>
<tr>
<td>Maximum</td>
<td>–</td>
<td>58.97%</td>
<td>78.30%</td>
<td>78.30%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>SSTI/NC</th>
<th>SSTI/SG</th>
<th>SSTI/SS</th>
<th>SSTI/DG</th>
<th>SSTI/PC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>4.50%</td>
<td>10.79%</td>
<td>14.27%</td>
<td>15.72%</td>
<td>-17.56%</td>
</tr>
<tr>
<td>S.D.</td>
<td>5.02%</td>
<td>9.76%</td>
<td>15.98%</td>
<td>17.38%</td>
<td>39.63%</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-257.60%</td>
</tr>
<tr>
<td>1st Quartile</td>
<td>0.27%</td>
<td>2.60%</td>
<td>2.20%</td>
<td>2.60%</td>
<td>-18.95%</td>
</tr>
<tr>
<td>Median</td>
<td>2.41%</td>
<td>8.71%</td>
<td>8.37%</td>
<td>9.27%</td>
<td>-4.62%</td>
</tr>
<tr>
<td>3rd Quartile</td>
<td>7.57%</td>
<td>16.10%</td>
<td>18.80%</td>
<td>22.11%</td>
<td>1.73%</td>
</tr>
<tr>
<td>Maximum</td>
<td>17.49%</td>
<td>49.05%</td>
<td>71.38%</td>
<td>79.87%</td>
<td>17.42%</td>
</tr>
</tbody>
</table>

(m_0 = 0.1 : 0.1 : 1.5, m_c = 0.1 : 0.1 : 1.5, l_1 = 0.1 : 0.9, l_2 = 0.1 : 0.1 : (1 - l_1), \hat{\mu} = 100, \epsilon = 0.698.)
DG contract with parameters (0.07,0.12). Interestingly enough, under SSTI, the OEM would like to assume even more overage risk as indicated by the optimal $\gamma$'s (0.13, 0.13). The implication of this counter-intuitive observation is that even if the OEM has shared information with the CM, it may still benefit from sharing overage cost. Moreover, the OEM may want to absorb more risk with information sharing than without information sharing.

Table 4.4. Non-substitutability of Risk Sharing by Information Sharing

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$Q_1^*$</th>
<th>$TC_{sc}$</th>
<th>$TC_{cm}$</th>
<th>$TC_{oem}$</th>
<th>$\gamma_1^*$</th>
<th>$\gamma_2^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSTI/NC</td>
<td>97.63</td>
<td>30.24</td>
<td>17.39</td>
<td>12.85</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td>NSTI/DG</td>
<td>99.53</td>
<td>29.29</td>
<td>16.50</td>
<td>12.79</td>
<td>0.07</td>
<td>0.12</td>
</tr>
<tr>
<td>SSTI/NC</td>
<td>102.39</td>
<td>28.23</td>
<td>16.76</td>
<td>11.47</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>SSTI/DG</td>
<td>105.39</td>
<td>26.94</td>
<td>15.56</td>
<td>11.38</td>
<td>0.13</td>
<td>0.13</td>
</tr>
</tbody>
</table>

$(m_o = 1.0, m_c = 0.7, l_1 = 0.3, l_2 = 0.5, \hat{\mu} = 100, \epsilon = 0.698)$

4.5.3 A Comparison of $\gamma$-Contracts under NSTI

Under NSTI, all three contracts are studied and compared (see Figure 4.1). The curve labeled “NC” represents the OEM’s optimal expected total cost in the absence of salvage offset agreements. Other curves are labeled similarly. As shown in the graph, when the margin is very small, the OEM is reluctant to take any overage risk. In other words, no salvage offset contract is offered to the CM. This is reflected on the left-hand side of the figure by the convergence of the four curves. As the markup, $m_o$, increases, the
OEM becomes willing to assume overage risk. Compared with the OEM’s performance without any contract offer, all three contracts can improve the OEM’s position.

**Observation 1.** *For the OEM, the SS contract and DG contract always equally perform under NSTI. Both of them dominate the SG contract.*

The curve for the SG contract always lies between the curve without contract and the curve for the SS or DG contract. When the salvage loss of component 1 is small enough, the SG contract does little to lower the OEM’s total cost. On the other hand, when $l_1$ is big, the SG contract can greatly reduce the OEM’s cost. However, in most cases, the improvement under the SG contract is not as dramatic as that under the other two types of agreements. This implies that although the SG contract may be an option for the OEM to reduce shortage cost, it is certainly not the best choice. Therefore, if forecast updates are not shared between the two parties, the OEM should consider the SS or DG contracts rather than the SG contract.

**4.5.4 A Comparison of $\gamma$-Contracts under SSTI**

As was the case with NSTI, the OEM is reluctant to offer a salvage offset contract if margin is very small (see Figure 4.2). As margin grows, the OEM becomes willing to share overage risk with the CM. These contracts exhibit different properties in terms of improving OEM’s performance. The contract structure, together with availability of new demand information at stage two, complicates the CM’s ordering behavior, which makes it difficult for the OEM to determine $\gamma$’s.

The DG contract always outperforms the SG and SS contract under SSTI. Since the SG contract only offers salvage offset for component 1, the CM does not get any
Fig. 4.1. OEM Cost under Different $\gamma$-Contracts (NSTI)
Fig. 4.2. OEM Cost under Different $\gamma$-Contracts (SSTI)
compensation for holding extra component 2. This can cause under-production of the sub-assembly and hence the final product. Therefore, the DG contract does better than the SG contract in improving the OEM’s total cost. With the SS contract, the CM can possibly recover a portion of the salvage loss for both components. However, this occurs only when the components are left over in pairs. Although the SS contract ties with the DG contract under NSTI, it can no longer catch up with the latter under SSTI due to higher overage cost resulting from the CM’s overbuy of component 2. The relative performance of the SG and SS contract depends on the values of the parameters. It is possible that the OEM prefers the SG contract to the SS contract while the CM prefers the other way around, or vice versa. In the case presented in Table 4.5, the OEM prefers the SG contract to the SS contract while the CM prefers the latter to the former.

Table 4.5. OEM-CM Contract Preference under SSTI

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$Q_1^*$</th>
<th>$TC_{cm}$</th>
<th>$TC_{oem}$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC</td>
<td>110.50</td>
<td>8.78</td>
<td>14.35</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>SG</td>
<td>130.18</td>
<td>4.79</td>
<td>10.08</td>
<td>0.71</td>
<td>0.00</td>
</tr>
<tr>
<td>SS</td>
<td>128.38</td>
<td>4.51</td>
<td>10.81</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>DG</td>
<td>130.20</td>
<td>4.75</td>
<td>10.07</td>
<td>0.70</td>
<td>0.05</td>
</tr>
</tbody>
</table>

($m_o = 2, m_c = 0.5, l_1 = 0.24, l_2 = 0.06, \mu = 100, \epsilon = 0.698$)

In summary, the DG contract is the best choice among all three types of contracts for the OEM regardless of the availability of demand forecast updates.
4.5.5 A Comparison across Information Scenarios

It is assumed in this study that information sharing does not incur any additional cost for the OEM or the CM. For the same risk-sharing agreement, the CM is always better off by having a more accurate demand forecast at stage two. Does the same result apply to the OEM or should the OEM always share forecast updates with the CM? If the OEM does not share overage risk with the CM, the answer is “yes”. However, in the presence of salvage offset contract, information sharing may not always be the right choice for the OEM. Computational results indicate that the OEM may incur a loss by sharing information with the CM under the SG or SS contract, as illustrated by the examples in Tables 4.6 and 4.7.

Table 4.6. An Example of Risk/Information Interaction under U-U Model

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$Q_1^*$</th>
<th>$TC_{sc}$</th>
<th>$TC_{cm}$</th>
<th>$TC_{oem}$</th>
<th>$\gamma_1^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSTI/NC</td>
<td>67.51</td>
<td>21.62</td>
<td>4.49</td>
<td>17.13</td>
<td></td>
</tr>
<tr>
<td>NSTI/SG</td>
<td>96.01</td>
<td>14.09</td>
<td>2.59</td>
<td>11.50</td>
<td>0.95</td>
</tr>
<tr>
<td>SSTI/NC</td>
<td>67.51</td>
<td>21.62</td>
<td>4.49</td>
<td>17.13</td>
<td></td>
</tr>
<tr>
<td>SSTI/SG</td>
<td>91.67</td>
<td>15.00</td>
<td>3.00</td>
<td>12.00</td>
<td>0.84</td>
</tr>
</tbody>
</table>

($m_o = 0.5, m_c = 0.1, l_1 = 0.5, l_2 = 0.1, \mu = 100, \epsilon = 0.698$)

Table 4.6 is for the U-U model while Table 4.7 is for the BVN model. They also suggest that given the same set of parameters, the results generated from the two different models are very similar.

Table 4.8 contains an example where there exists conflict between the OEM’s and the CM’s preferences. In this example, the OEM does not want to share information
Table 4.7. An Example of Risk/Information Interaction under BVN Model

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$Q^*$</th>
<th>$TC^{sc}$</th>
<th>$TC^{cm}$</th>
<th>$TC^{oem}$</th>
<th>$γ^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ρ = 0/NC$</td>
<td>67.97</td>
<td>21.85</td>
<td>4.74</td>
<td>17.11</td>
<td></td>
</tr>
<tr>
<td>$ρ = 0/SG$</td>
<td>95.81</td>
<td>14.50</td>
<td>2.67</td>
<td>11.84</td>
<td>0.95</td>
</tr>
<tr>
<td>$ρ = 0.741/NC$</td>
<td>68.03</td>
<td>21.82</td>
<td>4.74</td>
<td>17.09</td>
<td></td>
</tr>
<tr>
<td>$ρ = 0.741/SG$</td>
<td>91.87</td>
<td>15.33</td>
<td>3.05</td>
<td>12.28</td>
<td>0.83</td>
</tr>
</tbody>
</table>

$(m_o = 0.5, m_c = 0.1, l_1 = 0.5, l_2 = 0.1, μ_1 = 100, σ_1 = 30)$

Table 4.8. OEM-CM Preference Conflict under U-U Model

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$Q^*$</th>
<th>$TC^{sc}$</th>
<th>$TC^{cm}$</th>
<th>$TC^{oem}$</th>
<th>$γ^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSTI/NC</td>
<td>74.35</td>
<td>38.35</td>
<td>4.04</td>
<td>34.31</td>
<td></td>
</tr>
<tr>
<td>NSTI/SG</td>
<td>100.00</td>
<td>19.77</td>
<td>2.33</td>
<td>17.45</td>
<td>1.00</td>
</tr>
<tr>
<td>SSSI/NC</td>
<td>74.35</td>
<td>38.35</td>
<td>4.04</td>
<td>34.31</td>
<td></td>
</tr>
<tr>
<td>SSSI/SG</td>
<td>108.34</td>
<td>20.87</td>
<td>2.27</td>
<td>18.60</td>
<td>0.93</td>
</tr>
</tbody>
</table>

$(m_o = 1.2, m_c = 0.1, l_1 = 0.3, l_2 = 0.1, μ = 100, ε = 0.698)$

Table 4.9. OEM-CM Preference Conflict under BVN Model

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$Q^*$</th>
<th>$TC^{sc}$</th>
<th>$TC^{cm}$</th>
<th>$TC^{oem}$</th>
<th>$γ^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ρ = 0/NC$</td>
<td>74.75</td>
<td>38.52</td>
<td>4.20</td>
<td>34.32</td>
<td></td>
</tr>
<tr>
<td>$ρ = 0/SG$</td>
<td>100.00</td>
<td>20.35</td>
<td>2.39</td>
<td>17.95</td>
<td>1.00</td>
</tr>
<tr>
<td>$ρ = 0.741/NC$</td>
<td>75.02</td>
<td>38.29</td>
<td>4.19</td>
<td>34.10</td>
<td></td>
</tr>
<tr>
<td>$ρ = 0.741/SG$</td>
<td>109.22</td>
<td>20.78</td>
<td>2.24</td>
<td>18.54</td>
<td>0.93</td>
</tr>
</tbody>
</table>

$(m_o = 1.2, m_c = 0.1, l_1 = 0.3, l_2 = 0.1, μ_1 = 100, σ_1 = 30)$

Table 4.10. Statistics of Cases with Unexpected Outcomes

<table>
<thead>
<tr>
<th>(Total Number of Cases Examined: 10, 125)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TC^{oem}$</td>
</tr>
<tr>
<td>SG</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>870</td>
</tr>
<tr>
<td>1626</td>
</tr>
<tr>
<td>8.59%</td>
</tr>
<tr>
<td>16.06%</td>
</tr>
</tbody>
</table>
under the SG contract, whereas the CM prefers information sharing. Table 4.9 shows similar results under the BVN model.

A computational study also revealed an unexpected outcome where $TC^o$ or $TC^c$ under NSTI performs better than under SSSI. Table 4.10 presents a summary for such cases. Although sharing information may result in higher total cost for the OEM under the SG or SS contract, no similar examples were observed under the DG contract. As a general principle, the OEM should first consider information-mediated coordination. Both the OEM and the CM are better off by having a better demand forecast at stage two. After sharing information with the CM, the OEM may consider a higher level of coordination by offering risk sharing agreements depending on its margin level. Among the several contract structures, the dual-$\gamma$ (DG) contract is the best and safest choice. The single-$\gamma$ (SG) or squared-sets (SS) contract should be used very carefully.

### 4.6 Conclusions

This chapter studied the OEM-CM planning problem thoroughly and investigated the interaction between information sharing and risk sharing. The following insights can be drawn from the results developed and presented in the previous sections.

- Given the salvage offset contract provided by the OEM, the CM tends to increase the order quantity of component 1, but the order quantity of component 2 could move up or down.
• The gap between the order quantity of the two components becomes larger as the OEM increases its risk exposure on component 1; the gap shrinks as the OEM assumes more overage cost for component 2.

• There always exists a contract under which supply chain optimality can be achieved; however, the OEM may not offer this contract to protect its own interests.

• Information sharing is not necessarily a substitute for risk sharing. By sharing information and risk, both the OEM and the CM benefit. Among all available coordination schemes, “SSTI/DG” is the most efficient one.

• For the OEM, the DG contract always outperforms the SG or SS contract. Information sharing, when combined with risk sharing under the SG or SS contract, could hurt the OEM’s performance.
Chapter 5

Improving Shipping Decisions
with Demand Forecast Updates

5.1 Introduction

This chapter investigates the two-mode problem in which a company employs two transportation modes to ship one product from the place of manufacture to the market where the product is sold. Of the two modes, one (e.g., air transport) is faster but more expensive than the other (e.g., ocean shipping). A number of companies are adopting such practices. For example, to avoid high labor expenses, many U.S. high-tech companies have outsourced the production and assembly of their products to China, India, and other countries with cheap labor and ship the finished products to the places of sales. Although they incur large freight cost, the huge savings in labor cost still justify these practices. When transporting finished products, they can use either air or ocean shipping or both depending on their budgets, delivery requirements, or other considerations. Thus, one problem they need to answer is how to divide the freight between two different modes of transportation.

Not only do high-tech companies face such problems, retail companies also need to efficiently arrange the shipment of their goods to achieve a balance between cost and speed. A good example is Wal-Mart, who purchases a lot of merchandise from China. To ensure supply for holiday sales at the end of each year, it needs to ship huge volumes
of goods from China to the U.S. well in advance of the selling season since usually these goods are transported by ocean. Sometimes certain goods may sell so well that they want to arrange a second shipment after observing the occurrence of early sales. In such cases, shipping time becomes critical, and air transport may be used to expedite the delivery in order not to miss the sales opportunity. This problem is also of interest to some companies whose products are produced domestically because they sometimes need to make a choice between air and motor transport.

If there are no demand forecast updates or early sales cannot be observed before the second shipment, companies will probably choose the slow mode so as to save shipping costs. However, on some occasions it is possible for these companies to get better demand information closer to the point of time when actual sales occur. In other words, companies can revise their demand forecasts as time progresses and new information becomes available. They can improve their position by splitting the freight into two shipments, one by a fast mode and the other by a slow mode. Although higher shipping costs may be incurred with the faster mode, they can benefit from the more accurate forecast by reducing shortage cost and overage cost. In this chapter we investigate how the interaction between information updates and shipping rates affect shipping decisions (like the quantities to be shipped by each mode) and cost performance. There are some papers, such as those by Fisher and Raman (1996), and Gurnani and Tang (1999), which investigate problems that are very similar to the problem addressed in this research. However, these papers used bivariate normal distribution to model demand forecast updating and did not offer closed-form solutions. In this chapter, the
uniform-uniform (U-U) model developed earlier in the dissertation is applied to derive both analytical and computational results.

The organization of this chapter is as follows. Section 5.2 formulates the two-mode problem using the U-U model developed in Chapter 3. Section 5.3 presents analytical results based on the closed-form solutions. Section 5.4 demonstrates that the U-U model again can be used to well approximate the bivariate normal model in solving the two-mode problem. Section 5.5 presents computational results and derives managerial insights. Section 5.6 concludes this chapter by summing up major research findings.

5.2 Problem Formulation and Modeling

Suppose a company manufactures one product in its production site and needs to transport the final product to another place for sale. This chapter considers such a problem in a single selling season framework. The margin from selling one unit of product is $m$. There are two transport methods, one faster than the other. The per-unit shipping rate for the slow mode is $r_1$ and that for the fast mode is $r_2$. It is reasonable to assume that $r_1 \leq r_2 \leq m$. To facilitate exposition, define the difference between the shipping rates for the two modes as $r = r_2 - r_1$. Because the two modes require different shipping time, the company does not have to ship all its products simultaneously. Instead it can arrange the shipments at two different points in time. The products transported via the slow mode are dispatched earlier at time $t_1$ while those shipped via the fast mode are not dispatched until time $t_2$. Although the company can choose to make several shipments using the same mode, it is assumed here for simplicity that only one shipment is made for each mode; however, both shipments must arrive before the selling season starts.
One reason to postpone the second shipment is that possibly more information will become available. The company can use the newly obtained information to revise its demand forecast and make adjustments to its production and shipment plan. If demand is predicted to be at the high end, then the company can produce and ship additional units to avoid shortage costs; if the demand is predicted to be at the low end, then there is no need to produce or ship more units. Since the loss incurred from salvaging final products is higher than that from semi-finished products, it is not unusual to see many firms choosing to delay the assembly of final products as late as possible. At the end of the selling season, the company disposes of all its unsold products, and for each unit, a salvage loss in the amount of $l$ is incurred.

The company thus needs to make two decisions about shipment quantity. Let $W_1$ be the shipment quantity for the slow mode (stage-one decision) and $W_2$ denote the fast mode (stage-two decision). This chapter investigates the company’s quantity decisions and performances under three scenarios: (1) no stage-two information (NSTI), (2) some stage-two information (SSTI) and (3) perfect stage-two information (PSTI). The problem can be formulated as a two-stage dynamic program. The optimal shipment quantity at stage two ($W_2^*$) given $W_1$ and information realization is solved first conditionally. Then the optimal stage-two expression $W_2^*$ is substituted into the stage-one objective function to solve for the optimal shipment quantity at stage one ($W_1^*$).

5.2.1 Stage-two Decision

Assume information updating is captured by a random variable $I$, whose realization determines the stage-two distribution of demand. The conditional $p.d.f.$ and $c.d.f.$
of demand given \( I \) are denoted by \( f_{D|I} \) and \( F_{D|I} \). Because \( W_2 \) is a function of \( W_1 \) and \( I \), the company first needs to find the solution for optimal order quantity \( W_2^* \) given \( W_1 \) and \( I \). Define the cost function:

\[
TC = m[D - (W_1 + W_2)]^+ + l[(W_1 + W_2) - D]^+ + r_1 W_1 + r_2 W_2 - r_1 D. \tag{5.1}
\]

Note that the above cost formulation reflects the deviation from the maximal profit \((mD)\) that would be realized if the company were to know the exact demand at stage one. The first term \( m[D - (W_1 + W_2)]^+ \) represents the underage cost, the second term \( l[(W_1 + W_2) - D]^+ \) stands for the overage cost, and the last piece \( r_1 W_1 + r_2 W_2 - r_1 D \) captures the additional transportation cost to be paid by the company. The optimization problem at stage two can be formulated as follows.

\[
\min_{0 \leq W_2} \mathbb{E}_{D|I}[TC] = \int_{0}^{W_1+W_2} [l[(W_1 + W_2) - D]
\]

\[
+ r_1(W_1-D) + r_2W_2] f_{D|I} dD
\]

\[
+ \int_{W_1+W_2}^{+\infty} [m[D - (W_1 + W_2)]
\]

\[
+ r_1(W_1-D) + r_2W_2] f_{D|I} dD.
\]

(PS)

The objective function in the above problem is convex in \( W_2 \) (see Theorem 11).

**Theorem 11.** \( \mathbb{E}_{D|I}[TC] \) is convex on \( W_2 \geq 0 \).
Proof. We essentially need to show that the second-order derivative of $E_{D|I}(TC)$ w.r.t. $W_2$ is non-negative. The first-order derivative is

$$\frac{dE_{D|I}(TC)}{dW_2} = (m + l) F_{D|I}(W_1 + W_2) - m + r_2. \quad (5.2)$$

The second-order derivative is

$$\frac{d^2 E_{D|I}(TC)}{dW_2^2} = (m + l) f_{D|I}(W_1 + W_2) \geq 0. \quad (5.3)$$

Because $m$, $l$, and $f_{D|I}$ are all non-negative, the second-order derivative is non-negative. This shows that $E_{D|I}(TC)$ is convex in $W_2$.

Solve the first-order condition, and we can obtain $W_2^*$. The stage-two critical fractile is

$$a_2 = \frac{m - r_2}{m + l}.$$

Define the stage-two critical shipment quantity as

$$B_2 = F_{D|I}^{-1}(a_2). \quad (5.4)$$

The optimal shipment quantity at stage two is therefore

$$W_2^* = \max[(B_2 - W_1), 0].$$
By substituting $W_2^*$ into equation (5.1) and taking the expectation with regard to the information variable $I$, we can formulate the stage-one optimization problem as follows.

$$\min_{0 \leq W_1} \mathbb{E}_I \left[ TC(W_1, W_2^*) \right]. \quad (P9)$$

The solution to Problem (P9) must satisfy the first-order condition; however, no general closed-form solution can be found because the form of the solution depends on the distribution of the information variable $I$. Next, we apply the U-U model developed in previous chapters to Problem (P9) to obtain closed-form solutions.

### 5.2.2 Applying the U-U Model to the Two-Mode Problem

As a recap (see Chapter 3 for details), with the U-U formulation, demand $D$ follows a symmetric triangular distribution with parameters $(\hat{\mu}(1 - \epsilon), \hat{\mu}(1 + \epsilon), \hat{\mu})$. At stage two, as new information becomes available, demand follows a uniform distribution $D \sim U[\mu - \delta, \mu + \delta]$ where $\delta = \hat{\mu}\epsilon/2$. The width parameter $\delta$ is known at stage one, but the value of the stage-two mean $\mu$ is not revealed until stage two. $\mu$ follows a uniform distribution at stage one, i.e., $\mu \sim U[\hat{\mu} - \delta, \hat{\mu} + \delta]$. Thus $\hat{\mu}$ is the expectation of $\mu$ at stage one, and $\underline{\mu} = \hat{\mu} - \delta$ and $\overline{\mu} = \hat{\mu} + \delta$ are the lower and upper limits of $\mu$, respectively.

The stage-two critical shipment quantity $B_2$ is

$$B_2 = F_{D|\mu}^{-1}(a_2) = \mu + (2a_2 - 1)\delta. \quad (5.5)$$

Let $\underline{B_2}$ and $\overline{B_2}$ be the minimum and maximum value of $B_2$, respectively. Given $W_1$, we can formulate expected total cost under SSTI by conditioning on $\mu$. Again,
the total cost is expressed as the deviation of the total profit from the profit in the
perfect-information case where the company knows the exact demand at stage one.

1. If $W_1 < B_2$, the company will order $W_2$ equal to $(B_2 - W_1)$ for any $\mu$. Therefore
its cost expression in this case is

$$
E(TC_{SSTI}) = \int_{\mu}^{\mu'} \left[ \int_{\mu-\delta}^{B_2} [(B_2 - D) - r_1 D] f_{D|\mu}(D) dD \\
+ \int_{B_2}^{\mu+\delta} [m(D - B_2) - r_1 D] f_{D|\mu}(D) dD \\
+ r_1 W_1 + r_2 (B_2 - W_1) \right] g_{\mu}(\mu) d\mu.
$$

(5.6)

2. If $B_2 \leq W_1 < \hat{\mu}$, the cost expression is

$$
E(TC_{SSTI}) = \int_{\mu}^{\mu'} \left[ \int_{\mu-\delta}^{W_1} [(l + r_1)(W_1 - D)] f_{D|\mu}(D) dD \\
+ \int_{W_1}^{\mu+\delta} [(m - r_1)(D - W_1)] f_{D|\mu}(D) dD \right] g_{\mu}(\mu) d\mu \\
+ \int_{\mu}^{\mu'} \left[ \int_{\mu-\delta}^{B_2} [(B_2 - D) - r_1 D] f_{D|\mu}(D) dD \\
+ \int_{B_2}^{\mu+\delta} [m(D - B_2) - r_1 D] f_{D|\mu}(D) dD \\
+ r_1 W_1 + r_2 (B_2 - W_1) \right] g_{\mu}(\mu) d\mu.
$$

(5.7)
3. If $\hat{\mu} \leq W_1 < B_2$, the cost expression is

$$
\mathbb{E}(TC_{SSTI}) = \int_{\mu}^{W_1 - \delta} \left[ \int_{\mu - \delta}^{\mu + \delta} ((l + r_1)(W_1 - D)]f_{D\mid\mu}(D) dD \right] g_\mu(\mu) d\mu
$$

$$
+ \int_{W_1 - \delta}^{\mu'} \left[ \int_{\mu - \delta}^{W_1 - \delta} ((l + r_1)(W_1 - D)]f_{D\mid\mu}(D) dD \right] g_\mu(\mu) d\mu
$$

$$
+ \int_{W_1}^{\mu + \delta} [m - r_1](D - W_1)]f_{D\mid\mu}(D) dD g_\mu(\mu) d\mu
$$

$$
+ \int_{\mu}^{\mu'} \left[ \int_{\mu - \delta}^{B_2 - \delta} [l(B_2 - D) - r_1D]f_{D\mid\mu}(D) dD \right] g_\mu(\mu) d\mu
$$

$$
+ \int_{B_2}^{\mu + \delta} [m(D - B_2) - r_1D]f_{D\mid\mu}(D) dD g_\mu(\mu) d\mu
$$

$$
+ r_1W_1 + r_2(B_2 - W_1) g_\mu(\mu) d\mu.
$$

(5.8)

4. If $B_2 \leq W_1$, the company will always have $W_2 = 0$ for any $\mu$. This case then reduces to NSTI, and its cost expression is

$$
\mathbb{E}(TC_{SSTI}) = \mathbb{E}(TC_{NSTI}).
$$

(5.9)

Although the cost function is piecewise, it is convex in $W_1$ because expectation operations preserve convexity. The optimal solution $W_1^*$ can be obtained by solving the first-order conditions.
5.3 Analytical Results

5.3.1 Optimal Shipment Quantity at Stage One

Let \( W^*_{1,NSTI} \) be the optimal shipment quantity under NSTI. \( W^*_{1,SSTI} \) and \( W^*_{1,PSTI} \) are the optimal shipment quantity at stage one under SSTI and PSTI, respectively. Under NSTI and PSTI, the two-mode problem reduces to the traditional newsvendor problem, and solutions can be found easily. Define

\[
a_n = \frac{m - r_1}{m + l},
\]

\[
a_p = \frac{r_2 - r_1}{l + r_2}.
\]

The optimal shipment quantity under NSTI and PSTI are given by

\[
W^*_{1,NSTI} = \begin{cases} 
\hat{\mu} \left[ (1 - \epsilon) + \epsilon \sqrt{2a_n} \right] & \text{if } a_n \leq 0.5; \\
\hat{\mu} \left[ (1 + \epsilon) - \epsilon \sqrt{2(1 - a_n)} \right] & \text{if } a_n > 0.5.
\end{cases}
\] (5.10)

\[
W^*_{1,PSTI} = \begin{cases} 
\hat{\mu} \left[ (1 - \epsilon) + \epsilon \sqrt{2a_p} \right] & \text{if } a_p \leq 0.5; \\
\hat{\mu} \left[ (1 + \epsilon) - \epsilon \sqrt{2(1 - a_p)} \right] & \text{if } a_p > 0.5.
\end{cases}
\] (5.11)
There are four expressions for the total cost under SSTI, but only three expressions for \( W^*_1 \). This is because the first expression is monotonically decreasing in \( W_1 \) (the first-order derivative is \( r_1 - r_2 \leq 0 \)). Therefore, the optimal shipment quantity in this case is simply \( B_2 \), which is captured by the second expression for \( W^*_1 \). This makes sense because the company, knowing the total shipment quantity is at least \( B_2 \), has no reason to ship less than that at stage one.

**Theorem 12.** With the U-U model, the optimal shipment quantity at stage one decreases as the company has better information at stage two, \( W^*_{1,NSTI} \geq W^*_{1,SSTI} \geq W^*_{1,PSTI} \).

**Proof.** This proof consists of two parts. The first part shows that \( W^*_{1,NSTI} \geq W^*_{1,SSTI} \); the second part proves that \( W^*_{1,SSTI} \geq W^*_{1,PSTI} \).

**Part I:** \( W^*_{1,NSTI} \geq W^*_{1,SSTI} \)

Note that \( W^*_{1,NSTI} \geq W^*_{1,SSTI} \) is equivalent to

\[
W^*_{1,NSTI} - W^*_{1,SSTI} \geq 0.
\]
According to equation (5.12), $W_{1,SSTI}^*$ expression can take one of three forms depending on the relationship between $a_n$ and $a_2$. If $\frac{1+2a_2-a_2^2}{2} \leq a_n$, Theorem 12 is true because the stage-one shipment quantities under NSTI and SSTI are the same, i.e., $W_{1,SSTI}^* = W_{1,NSTI}^*$. Next it is shown that the proposition also holds when

$\left(a_2 \leq a_n < \frac{1+a_2^2}{2}\right)$ or $\left(\frac{1+a_2^2}{2} \leq a_n < \frac{1+2a_2-a_2^2}{2}\right)$.

1. $a_2 \leq a_n < \frac{1+a_2^2}{2}$

There are two subcases to consider depending on the value of $a_n$.

(a) $a_n \leq 0.5$

Substitute the appropriate expressions of $W_{1,NSTI}^*$ and $W_{1,SSTI}^*$ into inequality (5.13) and simplify:

$W_{1,NSTI}^* - W_{1,SSTI}^* = \hat{\mu} \epsilon \left[\sqrt{2a_n} - a_2 - \sqrt{2(a_n - a_2)}\right] \geq 0$.

The above inequality is equivalent to

$\sqrt{2a_n} - a_2 - \sqrt{2(a_n - a_2)} \geq 0$.

Rearrange,

$\sqrt{2a_n} - a_2 \geq \sqrt{2(a_n - a_2)}$.

Square both sides and simplify,

$1 + \frac{a_2^2}{2} \geq 2a_n$. 
If $a_n \leq 0.5$, the right-hand-side of the above inequality is no greater than 1. The left-hand-side is no less than 1 because $a_2$ is non-negative. Therefore the above result is true, which proves that Theorem 12 holds for $a_n \leq 0.5$.

(b) $a_n > 0.5$

Again, substitute the appropriate expressions of $W^*_{1,NSTI}$ and $W^*_{1,SSTI}$ into inequality (5.13) and simplify:

$$W^*_{1,NSTI} - W^*_{1,SSTI} = \hat{\mu} \left[ 1 - \sqrt{2(1-a_n)} + (1-a_2) - \sqrt{2(a_n-a_2)} \right] \geq 0.$$  

The above inequality is equivalent to

$$1 - \sqrt{2(1-a_n)} + (1-a_2) - \sqrt{2(a_n-a_2)} \geq 0.$$  

Rearrange,

$$1 - \sqrt{2(1-a_n)} > \sqrt{2(a_n-a_2)} - (1-a_2).$$  

The left-hand-side of the above inequality is greater than 0 because $a_n > 0.5$. The right-hand-side is less than 0, which is shown below using the boundary condition that $a_n < \frac{1+a^2_2}{2}$:

$$\sqrt{2(a_n-a_2)} - (1-a_2) < 0,$$

$$\sqrt{2(a_n-a_2)} < (1-a_2),$$

$$2(a_n-a_2) < (1-a_2)^2,$$

$$a_n < \frac{1+a^2_2}{2}.$$  

Therefore, \(1 - \sqrt{2(1 - a_n)}\) is greater than \(\sqrt{2(a_n - a_2) - (1 - a_2)}\), and \(W_{1,NSTI}^* - W_{1,SSTI}^* \geq 0\). This shows that Theorem 12 is true for \(0.5 < a_n < \frac{1 + a_2^2}{2} \).

2. \(\frac{1 + a_2^2}{2} \leq a_n < \frac{1 + 2a_2 - a_2^2}{2}\)

In this case, \(a_n > \frac{1 + a_2^2}{2} \geq 0.5\). So only the expression for \(W_{1,NSTI}^*\) when \(a_n > 0.5\) should be considered. Substitute the appropriate expressions into inequality (5.13) and simplify:

\[
W_{1,NSTI}^* - W_{1,SSTI}^* = \mu \epsilon \left[ 1 - \sqrt{2(1 - a_n)} + \frac{1 - a_2}{2} - \frac{a_n - a_2}{1 - a_2} \right] \geq 0.
\]

The above inequality is equivalent to

\[
1 - \sqrt{2(1 - a_n)} + \frac{1 - a_2}{2} - \frac{a_n - a_2}{1 - a_2} \geq 0.
\]

Simplify and rearrange,

\[
\frac{1 - a_n}{1 - a_2} + \frac{1 - a_2}{2} \geq \sqrt{2(1 - a_n)}. \]

The above inequality holds because

\[
\frac{1 - a_n}{1 - a_2} + \frac{1 - a_2}{2} \geq 2 \sqrt{\frac{1 - a_n}{1 - a_2}} \left( \frac{1 - a_2}{2} \right) = \sqrt{2(1 - a_n)}.
\]

This again shows that \(W_{1,NSTI}^* - W_{1,SSTI}^* \geq 0\) for \(\frac{1 + a_2^2}{2} \leq a_n < \frac{1 + 2a_2 - a_2^2}{2} \).
Part II: $W^*_{1,SSTI} \geq W^*_{1,PSTI}$

The inequality $W^*_{1,SSTI} \geq W^*_{1,PSTI}$ can be rewritten as

$$W^*_{1,SSTI} - W^*_{1,PSTI} \geq 0. \tag{5.14}$$

Note that $a_p = \frac{a_n-a_2}{1-a_2}$ and $a_p < a_n$. It is easy to verify that $W^*_{1,NSTI} \geq W^*_{1,PSTI}$ if $\frac{1+2a_2-a_2^2}{2} \leq a_n$. Theorem 12 is true because $W^*_{1,SSTI} = W^*_{1,NSTI} \geq W^*_{1,PSTI}$. Next it is shown that the result also holds in other cases.

1. $a_2 \leq a_n < \frac{1+a_2}{2}$

   In this case, $a_n < \frac{1+a_2}{2} \leq \frac{1+a_2}{2}$. Note that $a_p \leq 0.5$ is equivalent to $a_n \leq \frac{1+a_2}{2}$ and $a_p > 0.5$ is equivalent to $a_n > \frac{1+a_2}{2}$. Therefore $W^*_{1,PSTI}$ must take the expression only for $a_p \leq 0.5$ in this case. Substitute the appropriate expressions into inequality (5.14) and simplify:

$$W^*_{1,SSTI} - W^*_{1,PSTI} = \hat{\mu} \epsilon \left[ a_2 + \sqrt{2(a_n-a_2)} - \sqrt{2a_p} \right] \geq 0.$$ 

This is equivalent to

$$a_2 + \sqrt{2(a_n-a_2)} - \sqrt{2 \left( \frac{a_n-a_2}{1-a_2} \right)} \geq 0.$$ 

The first-order derivative of the left-hand-side in the above inequality w.r.t. $a_n$ is

$$\frac{1}{\sqrt{2(a_n-a_2)}} \left( 1 - \frac{1}{\sqrt{1-a_2}} \right),$$

which is non-positive. At the point of $a_n = \frac{1+a_2^2}{2}$,
the left-hand-side attains its minimum $1 - \sqrt{1 - a^2}$, which is non-negative. This shows that Theorem 12 is true in this case.

2. \[ \frac{1 + a^2}{2} \leq a_n < \frac{1 + 2a^2 - a^2}{2} \]

There are two subcases to consider depending on the value of $a_p$.

(a) $a_p \leq 0.5$, i.e., $a_n \leq \frac{1 + a^2}{2}$

Substitute the appropriate expressions of $W_{1,SSTI}^*$ and $W_{1,PSTI}^*$ into inequality (5.14) and simplify:

\[ W_{1,SSTI}^* - W_{1,PSTI}^* = \hat{\mu} \epsilon \left( \frac{a_n - a^2}{1 - a^2} - \frac{1 - a^2}{2} + 1 - \sqrt{2a_p} \right) \geq 0. \]

It is easy to see that the left-hand-side of the above inequality is non-negative because $\frac{a_n - a^2}{1 - a^2} \geq \frac{1 - a^2}{2}$ and $1 - \sqrt{2a_p} \geq 0$. Hence $W_{1,SSTI}^*$ is no less than $W_{1,PSTI}^*$ in this case.

(b) $a_p > 0.5$, i.e., $a_n > \frac{1 + a^2}{2}$

Again, substitute the appropriate expressions of $W_{1,SSTI}^*$ and $W_{1,PSTI}^*$ into inequality (5.14) and simplify:

\[ W_{1,SSTI}^* - W_{1,PSTI}^* = \hat{\mu} \epsilon \left[ \frac{a_n - a^2}{1 - a^2} - \frac{1 - a^2}{2} - 1 + \sqrt{2 \left( \frac{1 - a_n}{1 - a^2} \right) } \right] \geq 0. \]

The first-order derivative of the left-hand-side in the above inequality w.r.t. $a_n$ is

\[ \frac{1}{1 - a^2} \left[ 1 - \frac{1}{\sqrt{2(1 - a_p)}} \right] , \text{ which is non-positive because } a_p > 0.5. \] At the
point of $a_n = \frac{1 + 2a_2 - a_2^2}{2}$, the left-hand-side attains its minimum $\sqrt{1 - a_2} - (1 - a_2) \geq 0$. Therefore, $W^*_{1,SSTI}$ is again no less than $W^*_{1,PSTI}$ in this case.

The above analysis combined together shows that Theorem 12 is true for all feasible values of $a_n$ and $a_2$. \qed

### 5.3.2 Expected Shipment Quantity at Stage Two

Under NSTI, there is only one shipment, and $W_2$ is zero. Under SSTI and PSTI, however, the company can arrange a second shipment after the revelation of new information. The expected shipment quantities at stage two are

$$
E(W^*_{2,SSTI}) = \begin{cases} 
\frac{1}{2} \hat{\mu} \epsilon \left[1 - \sqrt{2(a_n - a_2)}\right]^2 & \text{if } a_2 \leq a_n < \frac{1 + a_2^2}{2}; \\
\frac{1}{8} \hat{\mu} \epsilon \left[1 + a_2 - 2 \left(\frac{a_n - a_2}{1 - a_2}\right)\right]^2 & \text{if } \frac{1 + a_2^2}{2} \leq a_n < \frac{1 + 2a_2 - a_2^2}{2}; \\
0 & \text{if } \frac{1 + 2a_2 - a_2^2}{2} \leq a_n. 
\end{cases}
$$

$$
E(W^*_{2,PSTI}) = \begin{cases} 
\frac{1}{2} \hat{\mu} \epsilon \left[3 + (a_p - 3)\sqrt{2a_p}\right] & \text{if } a_p \leq 0.5; \\
\frac{1}{3} \hat{\mu} \epsilon (1 - a_p)\sqrt{2(1 - a_p)} & \text{if } a_p > 0.5.
\end{cases}
$$

### 5.3.3 Expected Underage

Under PSTI, there is no shortage at all because the company knows the exact demand at stage two and can always arrange a shipment to meet any demand that is not satisfied from the first shipment. Under NSTI and SSTI, however, a certain portion of
the demand may not be satisfied from inventory on hand and hence results in lost sales.

The expected shortage under NSTI is

\[
U_{NSTI} = \begin{cases} 
\frac{1}{3} \hat{\mu} \epsilon [3 + (a_n - 3) \sqrt{2a_n}] & \text{if } a_n \leq 0.5; \\
\frac{1}{3} \hat{\mu} \epsilon (1 - a_n) \sqrt{2(1 - a_n)} & \text{if } a_n > 0.5.
\end{cases}
\]

The expected shortage under SSTI is

\[
U_{SSTI} = \begin{cases} 
\frac{1}{6} \hat{\mu} \epsilon \left[3(1 - a_2^2) - 6a_n(1 - a_2) - [2(a_n - a_2)]^2\right] & \text{if } a_2 \leq a_n < \frac{1 + a_2^2}{2}; \\
\frac{1}{12} \hat{\mu} \epsilon (1 - a_2) \left[6(1 - a_n) - (1 - a_2)^2\right] & \text{if } \frac{1 + a_2^2}{2} \leq a_n < \frac{1 + 2a_2 - a_2^2}{2}; \\
U_{NSTI} & \text{if } \frac{1 + 2a_2 - a_2^2}{2} \leq a_n.
\end{cases}
\]

5.3.4 Expected Overage

In any of the three cases, it is possible for the company to ship too many products.

The expected overage under NSTI is

\[
O_{NSTI} = \begin{cases} 
\frac{1}{3} \hat{\mu} \epsilon a_n \sqrt{2a_n} & \text{if } a_n \leq 0.5; \\
\frac{1}{3} \hat{\mu} \epsilon \left[3 - (2 + a_n) \sqrt{2(1 - a_n)}\right] & \text{if } a_n > 0.5.
\end{cases}
\]

The expected overage under PSTI is

\[
O_{PSTI} = \begin{cases} 
\frac{1}{3} \hat{\mu} \epsilon a_p \sqrt{2a_p} & \text{if } a_p \leq 0.5; \\
\frac{1}{3} \hat{\mu} \epsilon \left[3 - (2 + a_p) \sqrt{2(1 - a_p)}\right] & \text{if } a_p > 0.5.
\end{cases}
\]
The expected overage under SSTI is

\[
O_{SSTI} = \begin{cases} 
\frac{1}{6} \bar{\mu} \epsilon \left[ 3a_2(2a_n - a_2) + +[2(a_n - a_2)]^2 \right] & \text{if } a_2 \leq a_n < (1 + a_2^2)/2; \\
\frac{1}{2} \bar{\mu} \epsilon \left[ 12a_n a_2 + (2a_2 + 1)(1 - a_2)^2 + 12 \left( \frac{a_n - a_2}{1-a_2} \right)^2 \right] & \text{if } (1 + a_2^2)/2 \leq a_n < (1 + 2a_2 - a_2^2)/2; \\
O_{NSTI} & \text{if } (1 + 2a_2 - a_2^2)/2 \leq a_n.
\end{cases}
\]

5.3.5 Optimal Expected Total Cost

By substituting optimal shipment quantities into cost functions, we can obtain closed-form expressions for optimal expected total costs in all three information scenarios.

The optimal total cost under NSTI is

\[
E(\text{TC})^*_{NSTI} = \begin{cases} 
\frac{1}{3} \bar{\mu} \epsilon (m - r_1)[3 - 2\sqrt{2a_n}] & \text{if } a_n \leq 0.5; \\
\frac{1}{3} \bar{\mu} \epsilon (l + r_1)[3 - 2\sqrt{2(1-a_n)}] & \text{if } a_n > 0.5.
\end{cases}
\]

The optimal total cost under PSTI is

\[
E(\text{TC})^*_{PSTI} = \begin{cases} 
\frac{1}{3} \bar{\mu} \epsilon (r_2 - r_1)[3 - 2\sqrt{2a_p}] & \text{if } a_p \leq 0.5; \\
\frac{1}{3} \bar{\mu} \epsilon (l + r_1)[3 - 2\sqrt{2(1-a_p)}] & \text{if } a_p > 0.5.
\end{cases}
\]
The optimal total cost under SSTI is

\[
E(TC)^*_{SSTI} = \begin{cases} 
\frac{1}{6} \hat{\mu} \epsilon (m + l) \left[ 3(1 - a_2)(2a_n - a_2) - 2(2a_n - a_2)^3 \right] 
& \text{if } a_2 \leq a_n < \frac{1 + a_2^2}{2}; \\
\frac{1}{24} \hat{\mu} \epsilon (m + l) \left[ (1 - a_2)^3 + 12(a_n - a_2^2) \left( \frac{1 - a_n}{1 - a_2} \right) \right] 
& \text{if } \frac{1 + a_2^2}{2} \leq a_n < \frac{1 + 2a_2 - a_2^2}{2}; \\
E(TC)^*_{NSTI} 
& \text{if } \frac{1 + 2a_2 - a_2^2}{2} \leq a_n.
\end{cases}
\]

5.4 Approximating the Bivariate Normal Model

The SSTI solutions obtained above can be used to approximate the solutions under the bivariate normal (BVN) model for any \( \rho \) other than 0 or 1 as was done in Chapter 3. With the BVN model, demand \( D \) follows a normal distribution with mean \( \mu_1 \) and S.D. \( \sigma_1 \) at stage one. New information, characterized by random variable \( X \), is revealed at the beginning of stage two to update the demand forecast. The information variable \( X \) follows the standard normal distribution, with \( p.d.f. g_X(\cdot) \) and \( c.d.f. G_X(\cdot) \).

The correlation coefficient between \( D \) and \( X \) is \( \rho \) (\( 0 \leq \rho \leq 1 \)). According to the standard results of BVN distribution, demand at stage two also follows a normal distribution with mean \( \mu_2 = \mu_1 + \rho \sigma_1 x \) and S.D. \( \sigma_2 = \sigma_1 \sqrt{1 - \rho^2} \). Let \( f_{D|X}(\cdot) \) and \( F_{D|X}(\cdot) \) be the conditional \( p.d.f. \) and \( c.d.f. \) of demand, given the realization of \( X \). Using the results \( B_2 = \mu_1 + \rho \sigma_1 x + Z_{a_2^2} \sigma_1 \sqrt{1 - \rho^2} \) and \( W_2^* = \max[(B_2 - W_1), 0] \), the problem of solving
for the optimal shipment quantity at stage one (i.e., \( W_1^* \)) can be formulated as follows.

\[
\min_{0 \leq W_1} \mathbb{E}(TC) = \int_{x'}^{x''} \left[ \int_{-\infty}^{W_1} \left[ (l + r_1)(W_1 - D) \right] f_D(X)(D) dD \right] dD \\
\quad + \int_{W_1}^{+\infty} \left[ (m - r_1)(D - W_1) \right] f_D(X)(D) dD g(x) dx \\
\quad + \int_{x'}^{+\infty} \left[ \int_{-\infty}^{B_2} \left[ (l + r_1)(B_2 - D) \right] f_D(X)(D) dD \right] dD \\
\quad + \int_{B_2}^{+\infty} \left[ (m - r_1)(D - B_2) \right] f_D(X)(D) dD \\
\quad + (r_2 - r_1)(B_2 - W_1) \right] g(x) dx 
\]

\[(P10)\]

where \( x' = \frac{W_1 - \mu_1 - Z_{a_2} \sigma_1 \sqrt{1 - \rho^2}}{\rho \sigma_1} \), the point at which \( B_2 = W_1 \).

The cost function in Problem (P10) can be rewritten as

\[
\mathbb{E}(TC) = \int_{x'}^{x''} \sigma_2 \left[ - (m - r_1)(W_1 - \mu_2) / \sigma_2 
\quad + (m + l) \left[ \phi((W_1 - \mu_2) / \sigma_2) \right] \right] g(x) dx \\
\quad + \int_{x'}^{+\infty} \sigma_2 \left[ - (m - r_1)Z_{a_2} + (r_2 - r_1)[Z_{a_2} - (W_1 - \mu_2) / \sigma_2] \right] \\
\quad + (m + l) \left[ \phi(Z_{a_2}) + a_2 Z_{a_2} \right] g(x) dx. 
\]

\[(5.15)\]

We cannot obtain a general closed-form solution to the above problem. However, Newton’s method can be applied to find the optimal \( W_1^* \) under the BVN model using
numerical integration. Even though, it is not so straightforward for practitioners in industry to implement the solution procedure. By adopting an approximation algorithm similar to that used in Chapter 3, the solutions can be found relatively easily.

Given any $\rho$ between 0 and 1, $W^*_1$ under the BVN model can be approximated through interpolation using either the two end-point solutions from the BVN model (2-point interpolation) or the two end-point solutions plus the SSTI solution $W^*_{1,SSTI}$ (3-point interpolation), as illustrated in Figure 5.1.

The 2-point approach works as follows:

$$\hat{W}^*_1 = W^*_1|_{\rho=0} + \rho (W^*_1|_{\rho=1} - W^*_1|_{\rho=0}).$$

The 3-point approach works as follows:

$$\hat{W}^*_1 = \begin{cases} 
W^*_1|_{\rho=0} + \rho (W^*_{1,SSTI} - W^*_1|_{\rho=0}) & \text{if } 0 < \rho \leq 0.741; \\
W^*_1,SSTI + (W^*_1|_{\rho=1} - W^*_1,SSTI) \left( \frac{\rho - 0.741}{1 - 0.741} \right) & \text{if } 0.741 < \rho < 1.
\end{cases}$$

The performance of the approximation is measured by comparing the total cost at $W^*_1$ and the interpolated $\hat{W}^*_1$'s. Percentage error is calculated to judge the goodness of approximation.

$$PE = \frac{TC(\hat{W}^*_1) - TC(W^*_1)}{TC(W^*_1)} \times 100\%.$$ 

We investigated 34,650 cases based on different combinations of margin, salvage loss, shipping rates and uncertainty (accounted for by $m$, $l$, $r_1$, $r_2$, and $\sigma_1/\mu_1$, respectively). Table 5.1 summarizes the computational results for all 34,650 cases and gives
Stage-one Shipment Quantity ($W_1^*\left(\rho, L, r_1, r_2, \mu_1, \sigma_1\right)$)

Fig. 5.1. Approximate the Optimal Shipment Quantity for BVN Model
the average, standard deviation, and maximum of absolute percentage errors for both total cost and stage-one shipment quantity. Compared with the 2-point approach, the 3-point approach performs substantially better. For example, the highest and average cost percentage errors observed are 1.05% and 0.08% for the 3-point interpolation in contrast with 6.80% and 0.95% for the 2-point approach.

Table 5.1. Approximation Performance for the Two-Mode Problem (Total Number of Cases Examined:34,650)

<table>
<thead>
<tr>
<th></th>
<th>3-pt Approach</th>
<th>2-pt Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AVG. S.D. MAX.</td>
<td>AVG. S.D. MAX.</td>
</tr>
<tr>
<td>Total Cost</td>
<td>0.08% 0.11% 1.05%</td>
<td>0.95% 1.06% 6.80%</td>
</tr>
<tr>
<td>$W^*_1$</td>
<td>0.67% 0.83% 6.52%</td>
<td>2.71% 2.77% 15.02%</td>
</tr>
</tbody>
</table>

($m = 0.6 : 0.1 : 1.2, l = 0.2 : 0.1 : 0.8, r_1 = 0.1 : 0.1 : 0.4, r_2 = (r_1 + 0.1) : 0.1 : min((m - 0.1), 1), \sigma_1 = 10 : 10 : 30, \rho = 0 : 0.1 : 1.)$

5.5 Computational Results and Managerial Insights

5.5.1 The Impact of Information Updates

The expected shipment quantity at stage two, shortage, and overage can all be expressed as functions of $a_n$ and $a_2$. Therefore, in order to investigate the influence of information updates, computational experiments can be performed to compare $E(W^*_2)$, $U$, and $O$ across all three scenarios over the grid of $a_n$ and $a_2$. We examined 500,500 cases ($a_2 = 0.001 : 0.001 : 1, a_n = a_2 : 0.001 : 1$), and the computation suggested the following results.
Observation 2. With the U-U model,

(1) The expected optimal shipment quantity at stage two increases as the company has better information at stage two, \( E(W_{2,NSTI}^*) \leq E(W_{2,SSTI}^*) \leq E(W_{2,PSTI}^*) \).

(2) The ratio of the stage-one shipment quantity to the expected total shipment quantity (i.e., \( \frac{W_1^*}{W_1^* + E(W_2^*)} \)) decreases as the company has better information at stage two.

(3) Both the expected unit underage and overage decrease as the company has better information at stage two.

These results are illustrated by Figures 5.2 through 5.5. In these figures, the dashed line stands for NSTI, the solid line for SSTI and the dotted line for PSTI. In Figure 5.2, the expected shipment quantity at stage two decreases under SSTI and PSTI as \( a_n \) goes up. Given \( a_2 \), \( E(W_2^*) \) initially drops steeply as \( a_n \) grows; as \( a_n \) further increases, the curves flatten out until they finally converge to 0. Under NSTI, there is only one shipment and \( E(W_2^*) \) is therefore 0. Across all 500,500 cases, the curves are ordered with PSTI at the top, SSTI in the middle, and NSTI at the bottom. In other words, the expected shipment quantity at the second instant goes up with better demand information.

In Figure 5.3, the ratio of stage-one shipment quantity to the total shipment quantity \( (W_1^*/W_1^* + E(W_2^*)) \) increases under SSTI and PSTI as \( a_n \) increases. For fixed \( a_2 \), the ratio sharply increases as \( a_n \) initially grows; after \( a_n \) reaches certain level, the curves flatten out, finally converging to 1. Under NSTI, there is no shipment at stage two, and the ratio is simply 1. The graph indicates that the curves are ordered with NSTI at the top, SSTI in the middle, and PSTI at the bottom. In other words,
Fig. 5.2. The Impact of Information on Stage-two Shipment Quantity
Fig. 5.3. The Impact of Information on the Ratio of Shipment Quantity
the ratio of units shipped at stage one to all units shipped at both instants put together decreases as more demand information becomes available at stage two. This makes sense because the company tends to postpone its shipment decision to take advantage of the newly revealed information. Recall that Theorem 12 suggests that $W_1$ decreases with information and computation shows that $E(W_2^*)$ moves in the opposite direction. As a result, the ratio of $W_1^*/[W_1^* + E(W_2^*)]$ becomes smaller if better information is available at stage two.

Figure 5.4 shows the expected shortage decreases under NSTI and SSTI as $a_n$ increases. For fixed $a_2$, the expected shortage decreases as $a_n$ grows and finally converges to 0 as $a_n$ approaches 1. The graph indicates that the curves are ordered with NSTI at the top, SSTI in the middle, and PSTI at the bottom. Under PSTI, the company can always meet demand by arranging a second shipment if necessary. Hence, the expected shortage under PSTI is always 0. The inference from the observation is that better information improves service by reducing stockouts.

Figure 5.5 shows the expected overage increases under all three scenarios as $a_n$ increases. For fixed $a_2$, the expected overage slowly increases as $a_n$ initially grows. After $a_n$ gets to some point, the overage begins to increase sharply. The graph indicates that the curves are ordered with NSTI at the top, SSTI in the middle, and PSTI at the bottom. The inference to be drawn from the observation is that better information helps to reduce unnecessary inventory.

Based on these observations, it follows that the company’s cost performance is improved by having better information at stage two because of less stockout and lower overage.
Fig. 5.4. The Impact of Information on Shortage
Fig. 5.5. The Impact of Information on Overage

\[ \mu_{\hat{a}} = 100 \]
\[ \epsilon = 0.6979 \]
\[ a_2 = 0.5 \]
5.5.2 The Impact of Rate Gap on Shipment Quantity

For the two-mode problem, the trade-off is between information and shipping charges. The previous section investigated how information affects shipment decisions. This section explores how the rate gap between two transportation modes influences the company’s shipping decisions.

In Figure 5.6, the ratio of stage-one shipment quantity to the total shipment quantity \((W^*_1/[W^*_1 + E(W^*_2)])\) increases under SSTI and PSTI as the rate gap \((r_2 - r_1)\) grows. For fixed \(m, l\) and \(r_1\), the ratio increases sharply as the gap initially widens; after the gap reaches a certain level, the curves gradually flatten out until they finally converge to the curve for NSTI. This suggests that the company’s shipping decisions are quite sensitive to the changes in shipping rates. If the shipping rates for the two modes are the same, the company just ships the minimum quantity required. As the stage-two shipping rate grows beyond the cheaper one, the company responds by shifting a substantial fraction of the total shipment quantity to the slow mode to avoid a higher transportation cost. When the gap reaches a certain level, the company becomes less sensitive to further rises in the shipping rate for the fast mode because most products are shipped via the slow mode.

5.6 Conclusions

This chapter investigated the two-mode problem in which a company needs to make choices between fast and slow transportation modes. The uniform-uniform (U-U)
Fig. 5.6. The Impact of the Gap between Shipping Rates
model was applied to formulate and analyze the problem. The shipment decisions and performance under all three scenarios (NSTI, SSTI, PSTI) were studied and compared.

Under the U-U model, the optimal shipment quantity at stage one decreases as the company has access to more information at stage two. Computational results indicate that the expected optimal shipment quantity at stage two, however, increases as better information becomes available. Consequently, the ratio of the stage-one shipment quantity to the expected total shipment quantity decreases with information quality at stage two. Information tends to improve both service and cost performance because both the expected underage and overage decrease as the company has better stage-two information.

Other things unchanged, the ratio of stage-one shipment quantity to the expected total shipment quantity increases as the gap between shipping rates for the two modes widens. The company’s shipping decisions initially are quite sensitive to any rise in the shipping rate for the fast mode. After the gap reaches certain level, the company becomes less sensitive to further rate increase.

The solutions derived from the U-U model can be used to approximate the solutions from the BVN model. This simplifies the solution procedure and makes it easier for industry practitioners to implement. An examination of 34,650 cases suggested that the approximation performs very well.
Chapter 6

Conclusions and Directions for Future Research

Information sharing and risk sharing, both critical to successful supply chain management, are important mechanisms for coordination between companies. Inspired by a real-world business situation facing an OEM and its CM, this dissertation studied the joint planning problem in a three-echelon supply chain with complementary product structure.

The CM’s planning problem was formulated as a two-component newsvendor (TCNV) model in which the CM needs to make purchasing decisions on two complementary components with different lead times. A new forecast update model using a set of two related uniform (U-U) distributions was developed. Closed-form solutions were derived in the U-U demand case. Based on the new model, an algorithm was developed to approximate the solutions under the widely used bivariate normal (BVN) model. The approximation scheme simplifies the solution procedure without substantial loss of performance as indicated by computational results. We show that under both the BVN and the U-U model, the CM increases the order quantity of the component with a longer lead time if better information becomes available at the second ordering stage. In doing so, the CM can reduce expected shortage and reduce costs.

The OEM-CM joint planning problem is an extension of the TCNV problem and has been the main focus in the dissertation. Due to double marginalization and
different risk exposure, the incentives of the CM are not necessarily in line with those of the OEM. The OEM serves as the sole source of demand information in this larger problem. Although the OEM cannot control the CM’s decisions concerning component ordering, it can influence the CM’s decisions through two mechanisms: (1) control of the demand forecast information and (2) offering *salvage offset* contracts to the CM whereby the OEM absorbs some fraction of the CM’s overage cost. Three contract arrangements (single-gamma, squared-sets, and dual-gamma contracts), characterized by different levels of risk sharing, were considered and examined in the dissertation. A thorough investigation of the interaction between information sharing and risk sharing generated many interesting and important insights.

- The presence of the salvage offset contract induces the CM to increase the order quantity of component 1, but the order quantity of component 2 could move up or down.

- The gap between the order quantity of the two components becomes larger as the OEM increases its risk exposure on component 1; the gap shrinks as the OEM assumes more overage cost for component 2.

- Although there always exists a contract under which supply chain optimality can be achieved, the OEM will typically not offer this contract because its own interests could be hurt.

- Information sharing is not necessarily a substitute for risk sharing. By sharing information and risk, both the OEM and the CM benefit. Among all available
coordination schemes excluding the extreme case of PSTI, “SSTI/DG” is the most efficient one in terms of improving the performance of the supply chain.

- For the OEM, the DG contract always outperforms the SG or SS contract. Information sharing, when combined with risk sharing under the SG and SS contracts, could hurt the OEM’s performance.

Chapter 5 applied the U-U model developed in Chapter 3 to solve a two-mode problem in which a company has the option to choose between two different shipping modes. One is slower but cheaper; the other is faster but more expensive. Therefore the tradeoff is between potential benefits brought about by better information and extra costs resulting from higher shipping rates. Analysis shows that under the U-U model, the optimal shipment quantity at stage one decreases as the company has access to better information at stage two. Computational results suggest that the expected optimal shipment quantity at stage two, however, increases as better information becomes available. Consequently, the ratio of stage-one shipment quantity to the expected total shipment quantity decreases with information quality at stage two. Information tends to improve both service and cost performance because both the expected underage and overage decrease as the company has better stage-two information. Other things being equal, the ratio of stage-one shipment quantity to the total shipment quantity increases as the gap between shipping rates for the two modes widens. The company’s shipping decisions initially are quite sensitive to any rise in the shipping rate for the fast mode. After the gap reaches a certain level, the company becomes indifferent to further rate
hikes. Again, the computational results in this chapter suggested that the U-U model is a good approximation for the BVN model.

For future research, the models presented in this dissertation can be further developed in many directions. The most straightforward extension of the two-component newsvendor model is a multi-period planning problem. In a single-period problem, the CM simply salvages all components that are left over at the end of the selling season. In the case of multiple planning horizons, however, the CM can consider carrying surplus components over to the next selling season. Instead of incurring a salvage loss, an inventory cost will be charged. When making ordering decisions, the CM thus needs to take its inventory position into account.

\[
\begin{array}{cccc}
\text{Component} & \text{Suppliers} & \text{CM} & \text{OEM} \\
A_1 & S_1 & E_1 \\
A_2 & S_2 & E_2 \\
\end{array}
\]

Fig. 6.1. An Extension of the OEM-CM Planning Problem
Another option is to develop the OEM-CM planning problem into a multi-component and multi-product model with component commonality. An example is presented in Figure 6.1. In this model, the OEM uses a CM to assemble two semi-finished products $S_1$ and $S_2$ that are customized by the OEM into finished products $E_1$ and $E_2$, respectively. Only one unit of component $A_1$ is required to produce one unit of $S_1$, whereas one unit of both $A_1$ and $A_2$ are needed to produce one unit of $S_2$. Order quantities of $A_1$ and $A_2$ are $Q_1$ and $Q_2$, respectively, while production quantities of $S_1$ and $S_2$ are $X_1$ and $X_2$, respectively. The OEM and the CM both seek to minimize their own expected total costs. This model extends the OEM-CM joint planning problem beyond a procurement focus and into providing build directions for the CM that jointly optimize its objective along with that of the OEM.

As a third choice, random yield can be incorporated into our models. This makes the problem more interesting and even more complicated because the company may get only a fraction of what was ordered. There are two decision stages: stage one, at which time the decision maker determines the procurement quantity for each component; and stage two, at which time the decision maker, given the actual supply of components, determines the quantity of sub-assemblies or end products to build.

In addition, a new contract arrangement of margin bonus can be used to enhance the coordination between the OEM and the CM. Under this arrangement, the OEM not only shares overage risk with the CM, but also shares rewards. Besides salvage offset for surplus components, a bonus could be awarded by the OEM to the CM for any unit of end-item sold to the customer. Because sales bonuses provide the CM with even more benefits and incentives than does salvage offset, this coordination mechanism (salvage
offset plus sales bonus) has the potential to perform even better than gamma-contracts at both the level of the supply chain and the level of the individual company. Finally, the uniform-uniform model can be used to solve some two-stage problems with similar structures of demand forecast updating.
Appendix A

TCNV Cost Formulation under the BVN Model

Under the bivariate normal distribution, we first model the stage-two cost function assuming the realization of information variable \( x \) and then take the expectation of stage-two total costs on \( x \) to formulate the stage-one cost function.

A.1 The Stage-two Cost Functions

Note that the order quantity of component 2, \( Q_2 \), is determined by \( Q_1 \) and the stage-two critical order level \( \beta_2 \). Given distribution parameters \((\mu_1, \sigma_1, \text{ and } \rho)\) and cost parameters \((m_c, l_1, \text{ and } l_2)\), the realization of \( x \) determines \( \beta_2 \).

\[
\beta_2 = \mu_1 + \rho \sigma_1 x + Z_{\alpha_2} \sigma_1 \sqrt{1 - \rho^2}.
\] (A.1)

Clearly, \( \beta_2 \) is an increasing function of \( X \). Given \( Q_1 \), define

\[
x' = \frac{Q_1 - \mu_1 - Z_{\alpha_2} \sigma_1 \sqrt{1 - \rho^2}}{\rho \sigma_1}.
\]

It is easy to verify that \( \beta_2 \) will be less than the stage-one order quantity \( Q_1 \) if \( x \) is below the threshold value \( x' \); otherwise \( \beta_2 \) will be greater than \( Q_1 \). The following two cases are to be considered.
**Case I:** If $Q_1 > \beta_2$, we have $Q_2^* = \beta_2$.

1. If $D < \beta_2$, the overage cost is

$$TC = l_1(Q_1 - D) + l_2(\beta_2 - D).$$

2. Else, if $\beta_2 \leq D$, the overage and underage costs are

$$TC = mc(D - \beta_2) + l_1(Q_1 - \beta_2).$$

The conditional expected total costs in this case is

$$TC(Q_1|Q_1 > \beta_2) = \int_{-\infty}^{\beta_2} [l_1(Q_1 - D) + l_2(\beta_2 - D)]f_{D|X}(D) dD + \int_{\beta_2}^{+\infty} [mc(D - \beta_2) + l_1(Q_1 - \beta_2)]f_{D|X}(D) dD,$$  \hspace{1em} (A.2)

where $f_{D|X}(D) = \frac{1}{\sigma_2 \sqrt{2\pi}} \exp \left[ -\frac{(D - \mu_2)^2}{2\sigma_2^2} \right]$.

This can be re-written as follows

$$TC(Q_1|Q_1 > \beta_2) = mc(\mu_2 - \beta_2) + l_1(Q_1 - \beta_2)$$

$$+ \sigma_2 (mc + l_1 + l_2) \left[ \phi \left( \frac{\beta_2 - \mu_2}{\sigma_2} \right) \right]$$

$$+ \frac{\beta_2 - \mu_2}{\sigma_2} \Phi \left( \frac{\beta_2 - \mu_2}{\sigma_2} \right),$$  \hspace{1em} (A.3)

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the p.d.f. and c.d.f. for the Standard Normal distribution.
Substitute $\beta_2 = \mu_2 + Z_{\alpha_2}\sigma_2$ into the above equation,

$$TC(Q_1|Q_1 > \beta_2) = \sigma_2 \left[-(m_c + l_1)Z_{\alpha_2} + l_1 \left(\frac{Q_1 - \mu_2}{\sigma_2}\right)ight] + (m_c + l_1 + l_2) \left[\phi(Z_{\alpha_2}) + \alpha_2Z_{\alpha_2}\right]. \quad (A.4)$$

**Case II:** If $Q_1 \leq \beta_2$, we have $Q_2^* = Q_1$.

1. If $D < Q_1$, the overage cost is

   $$TC = (l_1 + l_2)(Q_1 - D).$$

2. Else, if $D \geq Q_1$, the underage cost is

   $$TC = m_c(D - Q_1).$$

The conditional expected total cost in this case is

$$TC(Q_1|Q_1 \leq \beta_2) = \int_{-\infty}^{Q_1} [(l_1 + l_2)(Q_1 - D)]f_{D|X}(D)\,dD + \int_{Q_1}^{+\infty} [m_c(D - Q_1)]f_{D|X}(D)\,dD. \quad (A.5)$$

Using the standardization transformation, the above equation can be expressed as

$$TC(Q_1|Q_1 \leq \beta_2) = m_c(\mu_2 - Q_1) + \sigma_2(m_c + l_1 + l_2) \times \left[\phi\left(\frac{Q_1 - \mu_2}{\sigma_2}\right) + \frac{Q_1 - \mu_2}{\sigma_2} \Phi\left(\frac{Q_1 - \mu_2}{\sigma_2}\right)\right]. \quad (A.6)$$
This can be re-written as follows

\[
TC(Q_1|Q_1 \leq \beta_2) = \sigma_2 \left[ -m_c \left( \frac{Q_1 - \mu_2}{\sigma_2} \right) + (m_c + l_1 + l_2) \right. \\
\times \left. \left( \phi \left( \frac{Q_1 - \mu_2}{\sigma_2} \right) + \frac{Q_1 - \mu_2}{\sigma_2} \Phi \left( \frac{Q_1 - \mu_2}{\sigma_2} \right) \right) \right]. \quad (A.7)
\]

\section*{A.2 The Stage-One Cost Function}

We can formulate the stage-one total cost by taking the expectation of stage-two total costs on \(x\). The stage-one expected total cost under the bivariate normal model is given by

\[
E[TC(Q_1)] = \int_{-\infty}^{x'} TC(Q_1|Q_1 \geq \beta_2) g_X(x) \, dx \\
+ \int_{x'}^{+\infty} TC(Q_1|Q_1 \leq \beta_2) g_X(x) \, dx \quad (A.8)
\]

where \(x' = \frac{Q_1 - \mu_1 - Z_{\alpha_2} \sigma_1}{\rho \sigma_1} \sqrt{1 - \rho^2}\) and \(g_X\) is the p.d.f. for the information variable \(X\).
Appendix B

TCNV Cost Formulation under the U-U Model

B.1 No Stage Two Information (NSTI)

In this scenario, demand follows a triangular distribution at stage one. Since no new information about demand is available at stage two, demand distribution remains unchanged when the CM orders component 2. Because $Q_1 = Q_2$, this problem can be solved by the newsvendor solution. The CM’s objective function is

\[
E[TC^c(Q_1)] = \int_0^{Q_1} (l_1 + l_2)(Q_1 - D)f(D)dD + \int_{Q_1}^{+\infty} m_c(D - Q_1)f(D)dD. \quad (B.1)
\]

The first term represents overage cost while the second term represents underage cost. The critical fractile is

\[
\alpha_n = \frac{m_c}{m_c + l_1 + l_2}. \quad (B.2)
\]

Hence, $Q^*_1 = F^{-1}(\alpha_n)$. For the U-U model, let $f_l(D)$ and $f_r(D)$ be the left and right half of the triangular p.d.f., respectively.
If $\alpha_n < 0.5$, $Q_1$ is less than $\hat{\mu}$. The CM’s cost function is

$$E(TC^C) = \int_{\hat{\mu}(1-\epsilon)}^{\hat{\mu}} [(l_1 + l_2)(\hat{\mu} - D)] f_1(D) dD + \int_{\hat{\mu}}^{\hat{\mu}} [m_c(\hat{\mu} - Q_1)] f_1(D) dD + \int_{\hat{\mu}}^{\hat{\mu}(1+\epsilon)} [m_c(\hat{\mu} - D)] f_1(D) dD.$$  \hspace{1cm} (B.3)

Otherwise, $Q_1 \geq \hat{\mu}$. The CM’s cost function is

$$E(TC^C) = \int_{\hat{\mu}(1-\epsilon)}^{\hat{\mu}} [(l_1 + l_2)\hat{\mu} - (D)] f_1(D) dD + \int_{\hat{\mu}}^{\hat{\mu}} [m_c(D - Q_1)] f_1(D) dD + \int_{\hat{\mu}}^{\hat{\mu}(1+\epsilon)} [m_c(D - Q_1)] f_1(D) dD.$$  \hspace{1cm} (B.4)

By solving the first-order condition, we obtain the optimal order quantity:

$$Q_{1,NSTI}^* = \begin{cases} 
\hat{\mu}(1 - \epsilon) + \hat{\mu}\epsilon \sqrt{2\alpha_n} & \text{if } \alpha_n \leq 0.5; \\
\hat{\mu}(1 + \epsilon) - \hat{\mu}\epsilon \sqrt{2(1 - \alpha_n)} & \text{if } \alpha_n > 0.5. 
\end{cases}$$  \hspace{1cm} (B.5)

Substitute $Q_{1,NSTI}^*$ into $TC^C$ to get the optimal expected total cost:

$$E(TC)^*_{NSTI} = \begin{cases} 
\frac{1}{3}(3 - 2\sqrt{2\alpha_n})m_c\hat{\mu}\epsilon & \text{if } \alpha_n \leq 0.5; \\
\frac{1}{3} \left[3 - 2\sqrt{2(1 - \alpha_n)}\right] (l_1 + l_2)\hat{\mu}\epsilon & \text{if } \alpha_n > 0.5. 
\end{cases}$$  \hspace{1cm} (B.6)
B.2 Perfect Stage Two Information (PSTI)

In this scenario, demand follows a triangular distribution at stage one; however, demand becomes known at stage two, which means the CM knows the exact realized demand. If demand turns out to be less than $Q_1$, the CM orders component 2 equal to the realized demand; otherwise, the CM simply sets $Q_2$ equal to $Q_1$. So the CM will never have component 2 overage cost. This is no more than a traditional newsvendor problem with a single product. The CM’s objective function is

$$\mathbb{E}[TC^c(Q_1)] = \int_0^{Q_1} l_1(Q_1 - D)f(D) dD + \int_{Q_1}^{+\infty} m_c(D - Q_1)f(D) dD.$$ (B.7)

The first term represents overage cost while the second term represents underage cost. The critical fractile is

$$\alpha_p = \frac{m_c}{m_c + l_1}.$$ (B.8)

Therefore, $Q_1^* = F^{-1}(\alpha_p)$. If $\alpha_p < 0.5$, $Q_1 < \hat{\mu}$. With the U-U formulation, the CM’s cost function is

$$\mathbb{E}(TC^c) = \int_{\hat{\mu}(1-\epsilon)}^{Q_1} l_1(Q_1 - D)f_l(D) dD$$
$$+ \int_{\hat{\mu}}^{Q_1} m_c(D - Q_1)f_l(D) dD$$
$$+ \int_{\hat{\mu}(1+\epsilon)}^{\hat{\mu}} m_c(D - Q_1)f_r(D) dD.$$ (B.9)
Otherwise, $Q_1 \geq \hat{\mu}$. The CM’s cost function is

$$
\mathbb{E}(TC^c) = \int_{\hat{\mu}(1-\epsilon)}^{\hat{\mu}} l_1(\hat{\mu} - D)f_l(D) \, dD + \int_{\hat{\mu}}^{Q_1} l_1(Q_1 - \hat{\mu})f_r(D) \, dD + \int_{Q_1}^{\hat{\mu}(1+\epsilon)} m_c(D - Q_1)f_r(D) \, dD. \tag{B.10}
$$

The optimal order quantity is obtained by solving the first-order condition.

$$
Q_{1,PSTI}^* = \begin{cases} 
\hat{\mu}(1 - \epsilon) + \hat{\mu} \epsilon \sqrt{2\alpha_p} & \text{if } \alpha_p \leq 0.5; \\
\hat{\mu}(1 + \epsilon) - \hat{\mu} \epsilon \sqrt{2(1 - \alpha_p)} & \text{if } \alpha_p > 0.5.
\end{cases} \tag{B.11}
$$

Substitute $Q_{1,PSTI}^*$ into the cost function, and the CM’s optimal expected total cost under $PSTI$ is

$$
\mathbb{E}(TC)^*_{PSTI} = \begin{cases} 
\frac{1}{3} (3 - 2\sqrt{2\alpha_p}) m_c \hat{\mu} \epsilon & \text{if } \alpha_p \leq 0.5; \\
\frac{1}{3} \left[ 3 - 2\sqrt{2(1 - \alpha_p)} \right] l_1 \hat{\mu} \epsilon & \text{if } \alpha_p > 0.5.
\end{cases} \tag{B.12}
$$
References


Vita

Xueyi Zhang was born in Hunan province, China on January 3, 1976. He received his B.S. degree in Economics in 1997 and his M.S. degree in Management in 2000 from Beijing Technology & Business University where he was the recipient of merit-based scholarships for seven consecutive years. In 1997, he won the Beijing Municipal Distinguished College Student Award and graduated with honor. He worked at the Department of Logistics Management in the Lenovo Computer Co. Ltd. from December 1999 to May 2000.

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