

The Pennsylvania State University

The Graduate School

Department of Industrial and Manufacturing Engineering

**MULTIPLE SOURCING FOR DEPARTMENT OF DEFENSE SUPPLY CHAINS**

A Thesis in

Industrial Engineering

by

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Submitted in Partial Fulfillment  
of the Requirements  
for the Degree of

Master of Science

May 2011

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## **ABSTRACT**

Tactical operations and theater distribution in military supply chains play an important role in the success of a mission. Responsive and efficient delivery of supplies is essential to maintain equipment readiness, especially in combat operations. High uncertainty in demand and supply has a direct impact in readiness levels during combat military operations. Readiness levels are sensitive to sources of disruption, primarily from shortages, but also natural disasters, weather conditions, failure of communication and information systems, political instability, and terrorist attacks. This thesis measures uncertainty in a military supply chain from the brigade unit's perspective using exponentially distributed lead times, and investigates multiple sourcing as a strategy to improve readiness by reducing the expected supply lead time while increasing the order yield or percentage of order successfully received by brigade units. Multiple sourcing can potentially increase readiness by 70% - 90% and increasing order yield by 15%-21%. This work also proposes a process which contains past data to model supply lead times and determine the number of depots that supply a brigade unit along with the quantity of supplies to order while keeping the net order cost low. The solutions are presented using a Value Path Approach.

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## ACKNOWLEDGEMENTS

This thesis sets a very important milestone in my professional and personal development. There are many people that contributed directly or indirectly to the realization of this goal. Dr. Vittal Prabhu, my thesis advisor and Dr. Christopher Griffin, my co-advisor deserve my deepest gratitude. Their support, mentorship, continuous feedback and encouragement were very significant in the process.

I am also very thankful for all the professors of the Industrial and Manufacturing Engineering Department that were part of the successful completion of my Master's Degree. Their expertise, knowledge and experience were very valuable to my education and professional development.

In addition, I express my sincere gratitude to the special people that provided their unconditional support, guidance and sincere friendship: Jason Burrowes-Jones, Lourdes Medina, Josué Crespo, David Sudit, Aixa Cintrón, and all my friends. They made this experience a unique one and I am blessed to have them in my life. Thanks to all my teammates and fellow graduate students that enriched my learning experience and made it a very enjoyable one.

Finally, I want to thank my family, especially my parents Wilfredo Ríos and Isis M. Ramos, for their unconditional love, inspiration and courage to make this degree a reality. I dedicate my degree to them.

## Chapter 1

### **Introduction and Motivation**

An important topic of study in military supply chains is how to alleviate supply risks. During combat operations, disruption risks cause delays or sudden cancellations in the delivery of supplies to brigade units. Delays and interruptions in the delivery of spare parts result in a decrease of the brigade's readiness. In a military context, readiness can make the difference between success and failure of a mission since it measures a unit's capability of completing a mission from a military equipment standpoint.

It is a misconception to assume that selection of the supplier in closest proximity to the forward unit reduces uncertainty in the delivery of supplies (e.g., supplies will be received late or not received at all). Unfortunately, current US military doctrine dictates that each forward unit must receive supplies from exactly one responsible depot or Supply Support Activity (SSA). Scenarios such as recent attack(s) on one or more shipping channels (routes) may have an impact on the delivery of supplies; additionally, the depot itself may be attacked and no longer exist or be severely affected. Either scenario could result in delay or interruption of the delivery of supplies. A less extreme scenario includes delays caused by inventory shortage, in which the lead time is greater than the desired lead time of the forward unit. This may result in additional time-to-delivery of supplies from the depot selected or worst, re-routing of the order to another depot after the original order was placed. Any of these scenarios is likely to have a negative impact in readiness, causing additional delays in the completion of maintenance and service repairs for the unit's critical items. Hence, additional circumstances should be accounted for in the assessment of the risk of delays or interruption.

This thesis seeks to identify and account for risks associated with the delivery of supplies to a forward unit which include supply/demand risk and disruption risks. Properly capturing the risk associated with the delivery of supplies allows for the definition of a mathematical model to identify: i) the optimal selection of suppliers, and ii) the quantity of supplies to order from each, that minimizes the risk of delays and interruptions in the delivery of supplies. This document focuses on defining a risk measure to account for delays and disruptions under different scenarios and analyzing the results, such that the information can be used in a multi-criteria goal programming model to determine the optimal amounts that should be ordered such that the Army's requirements are satisfied while lowering uncertainty.

As an example, assume a brigade unit needs to submit an order of supplies. There are  $n$  depots from which supplies can be ordered, defined as  $D_1, D_2, \dots, D_n$ . Each depot  $D_i$  has quantities for  $p$  different items in inventory, namely  $I_1, I_2, \dots, I_p$ . Multiple scenarios are considered in the definition of a risk measure to assess the variability in the amount of commodity received by a brigade unit. Hence, in a scenario with  $n$  suppliers, the optimal amount of commodity can be found such should be ordered from each depot in order to maximize equipment readiness with minimum variability in the amount received.

The remainder of this thesis is organized as follows: Chapter 2 provides a comprehensive literature review including the characteristics of the Department of Defense (DoD) supply chain, a description of inventory management in the US DoD's supply chain, an overview of equipment readiness and previous research in the area of supplier selection models. Chapter 3 defines the course of action and the techniques applied towards this research by means of a process methodology.

Chapters 4 and 5 implement Steps 1 and 2 of the research methodology described in Chapter 3. The results prove multiple sourcing increases readiness by i) increasing the expected order yield, therefore increasing the percentage of Equipment on Hand and ii) reducing the

expected lead time for order delivery. Chapter 4 studies the factors affecting readiness in the brigade unit side of the DoD supply chain. Chapter 5 uses the readiness metrics from Chapter 4 to show multiple sourcing results in higher readiness levels for a brigade unit. Chapter 6 concludes this thesis document by analyzing results, illustrating the limitations of this research as well as and providing recommendations for future work.

## Chapter 2

### Literature Review

#### Introduction

Tactical operations and theater distribution in military supply chains play an important role in the success or failure of a mission. Responsive and efficient delivery of high priority commodities is essential to maintain readiness, especially under combat operations. A good example is the research performed by Peltz, Robbins, Giradini, Eden, Halliday, & Angers on Operation Iraqi Freedom. Among other findings, degradation of equipment readiness went from 90% to 70% for the 3<sup>rd</sup> Infantry Division; moreover, only 10% of the requests from the 101<sup>st</sup> Airborne Division were filled by Supply Support Activities (SSA) (Peltz et al, 2007). Although there was no evidence of operational effects during Operation Iraqi Freedom, lower levels of readiness affected the Army's assessment of risk. Higher risk parameters result in miscalculations and reassessment of plans, which could have a negative impact in the course of action in the face of more powerful enemies. Hence, proper evaluation of supply disruptions, the associated risk, readiness standards and process performance is essential when planning tactical operations and theater distribution. This research focuses on understanding the characteristics of the Department of Defense supply chain, its inventory and the sources of supply disruptions required to determine a new multiple sourcing policy that potentially reduces the risk of disruptions to maximize readiness.

Literature on multiple sourcing and supplier selection models covers a variety of scenarios and techniques. Empirical studies of commercial supply chain operations show that multiple sourcing in is an effective way to hedge against risk of supplier disruptions, including labor strikes, natural disasters, terrorism and political issues (Minner, 2003), (Kleindorfer & Saad,

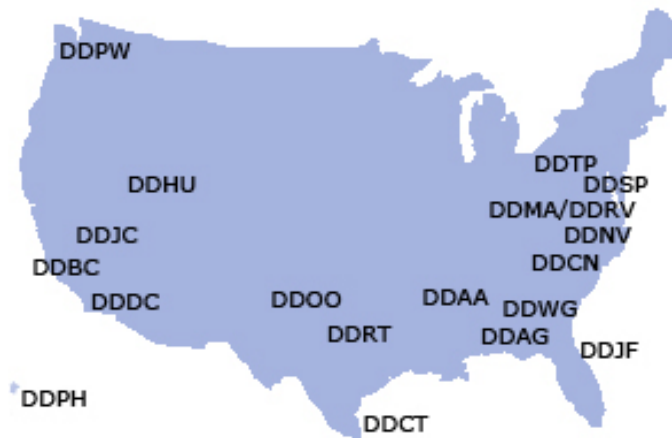


2005). However, the structure of military logistics in the US Army considers a single Supply Support Activities (SSA) for their brigade units. This literature review covers relevant topics for this thesis, including characterization of the Department of Defense supply chain in the combat operations, an overview of readiness, risks that affect readiness in a theater (combat) environment, and a discussion on multiple sourcing models in supply chain. The literature review ends with a description on how supplier selection models can be applied in military logistics to improve equipment readiness for spare parts that are readiness drivers as specified by the Army's Equipment Downtime Analyzer (Peltz, Robbins, Boren, & Wolff, 2002).

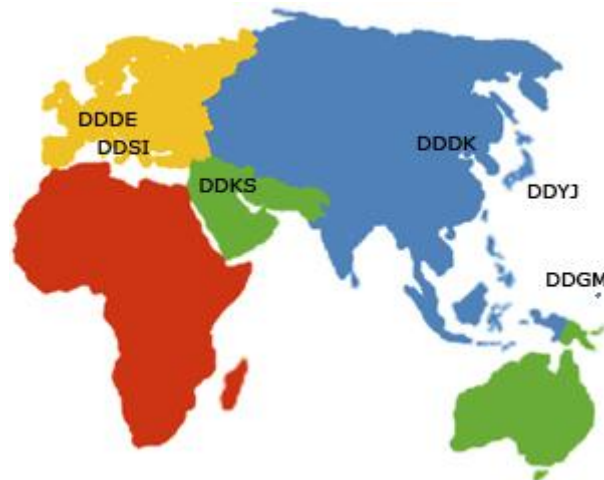
### **An overview of military supply chains**

The mission of the US DoD's is to provide an armed force to dissuade conflict and protect the security of the United States. The Secretary of Defense acts as the leader of the oldest and largest government agency and is the primary policy advisor to the President. It is also the country's largest employer, exceeding 1.4 million people on active duty, and 718,000 civilian personnel, 1.1 million serve in the National Guard and Reserve forces and there are over 2 million military retirees and their family members. The Department of Defense operates on a budget of approximately 550 billion dollars and its military departments include: Air Force, Army, Department of Homeland Security and the Navy (US Department of Defense ).

Out of the fifteen defense agencies, the Defense Logistics Agency (DLA) is responsible for supporting federal agencies as well as joint and allied forces in terms of logistics, acquisition and technical services. DLA is responsible for almost all the consumable items for military forces as well as the delivery of over 84% of the spare parts. There are 26 distribution centers around the globe. There are three principal defense supply centers. There are two distribution centers, one in Susquehanna, PA and one in San Joaquin, CA.



**Figure 2-1: Map of the DoD's locations in the Continental US (CONUS) - Adopted from (Defense Logistics Agency )**



**Figure 2-2: Map of DoD's distributions outside the Continental US (OCONUS) Adopted from (Defense Logistics Agency )**

### **A comparison of Commercial versus Military Supply Chains**

Most supply chains consist of a structure like the one shown in Figure 2-1. Stages are connected through exchanges of information, products and funds (Chopra & Meindl, 2007).



**Figure 2-3: A commercial supply chain**

The strategy of a supply chain depends largely on the demand and supplier uncertainty as well as in efficiency and responsiveness. A consumer goods supply chain like Wal-Mart's has a wide variety of products, from produce to garden supplies with low profit margin. The maturity of the products sold provides Walmart with low uncertainty in demand, making it easier to forecast demand and supply and to manage disruptions.

Technologies such as sales force automation are used to support a commercial supply chain strategy as it can provide a diverse set of tools, including: opportunity/ territory management applications, assessments on customer behaviors and value preferences, forecasting tools, and workflows to manage interactions with other stages of the supply chain. For example, Dell Computer tailors service levels according to customer value by using purchasing patterns to estimate the lifetime value of and identify their profitable customer segments. This information is used to design a responsive strategy that includes portals where high value customers can place and track orders, get technical support, and learn about technical specifications of different products. Dell also leverages technology to develop relationships with suppliers; 90% of the company's purchases are from its 33 most important suppliers. Close relationships with suppliers allow Dell to hold 6 days of inventory.

Diagrams 2-5 and 2-6 describe the Department of Defense supply chain; Figure 2-5 shows the structure during peacetime. Commanders report their readiness status via the Defense Readiness Reporting System (DRRS). This is a web-based architecture that allows the military to

exchange among others, information regarding personnel, equipment and supplies, serviceability and training proficiency using a computer, as shown in Figure 2-4.



**Figure 2-4: Computer equipment used in the military**

Brigade units request goods directly to their Supply Support Activity (SSA) and are delivered directly to the unit if available. Otherwise, a request or materials release order (MRO) is sent to the local Defense Distribution Center (DDC), resulting in a request sent to the National Inventory Control Point (NICP). Then NICP sends the request to the primary distribution sites (PDS). Commodities are sent from the PDS to the DDC and then delivered to the brigade unit through the SSA. Note that during peacetime, the structure of the supply chain is similar to a commercial supply chain like Wal-Mart.

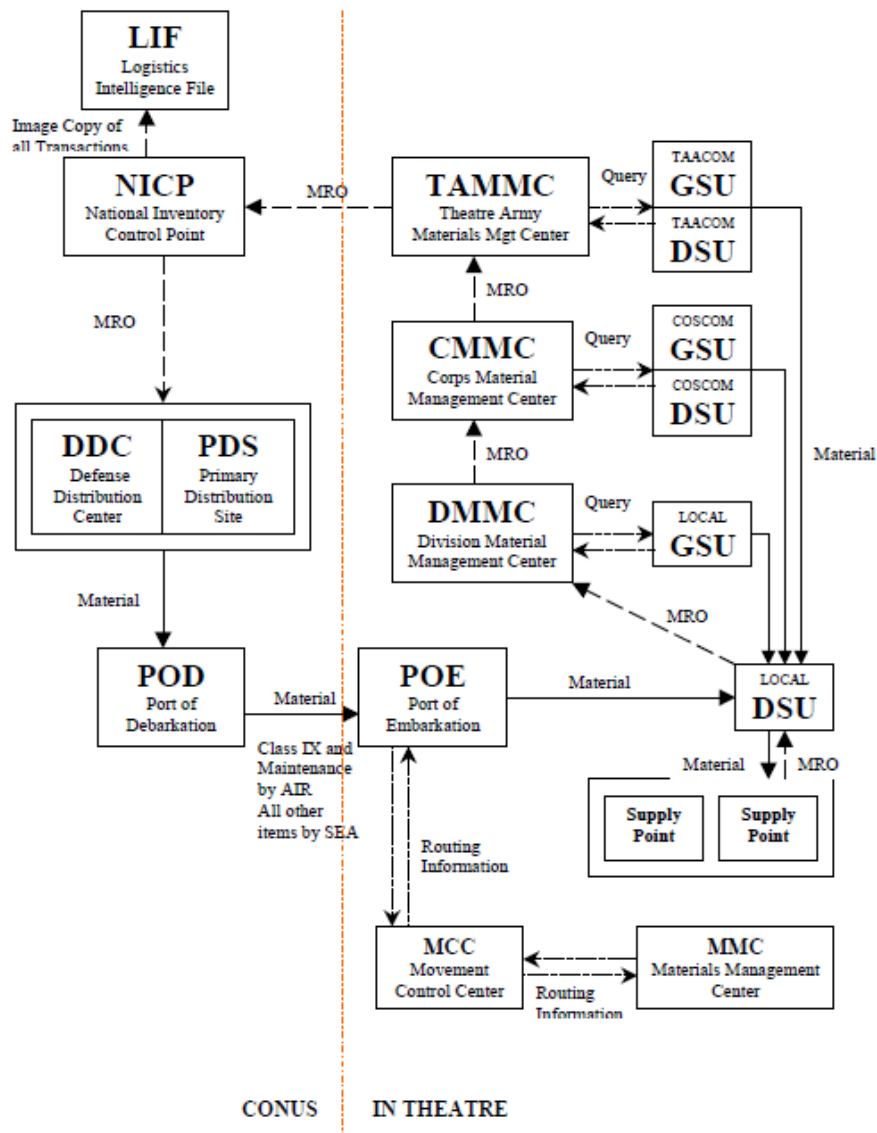


**Figure 2-5: DoD Supply Chain during peacetime**

However, during theater (combat) operations, the supply chain has a different structure since it has three levels of management:

- Theatre level
- Corps level - managing a branch of forces
- Division level – managing a smaller area within a theatre

Each one of the supply points are assigned to both a Direct Support Unit (DSU) and a General Supply Unit (GSU), as noted in Figure 2-6 (Lai, 2003). When a supply point submits an order, local DSU's are responsible for satisfying the request otherwise escalate the order to the Division Material Management Center (DMMC). Once the request is validated and prioritized, GSU's are queried and the order is escalated to the Army Corps Material Management Center (CMMC). Then the order is validated and prioritized again and the CMMC queries all DSU's and GSU's to fulfill the order. If the item is not found, then the order is routed to the Theatre Army Material Management Center (TAMMC) where is once again validated and prioritized. If no GSU's or DSU's can fulfill the order within the theater area, then the order is forwarded to the National Inventory Control Point (NICP) at the Continental US (CONUS). The order is sent to the closest US port and then shipped to the theatre port of embarkation. The Movement Control Center is responsible for moving the items in the most efficient way to the supply point.



**Figure 2-6: DoD Supply Chain in Theatre – Adopted from (Lai, 2003)**

Unlike a consumer goods supply chain, the Department of Defense supply chain's main characteristics include: diversity in supply, demand unpredictability, long and variable lead times, distributed locations mobile supply points, and unreliable inventory. Moreover, the Department of Defense' strategy is focused on readiness, or its ability to prepared for combat at any given time.

## Inventory Management in the DoD's Supply Chain

### Types of Supply

The supplies in the Department of Defense are organized in ten different classes. Supplies are stored in loads, according to their relevance. Operational loads are those used during peacetime and include items of Class I-V. Basic loads allow initiation of combat and include supplies of type I-V as well as type VIII. Prescribed loads are used during maintenance programs and include items of type II, IV, VIII and IX. This thesis focuses on spare parts that drive equipment readiness, which belong to class IX.

**Table 2-1: Types of supply in the Department of Defense (Army, 2000)**

Class	Description
I	Subsistence and commercially bottled water.
II	Clothing, individual equipment, tools, tool kits, tents, administrative and housekeeping type supplies, as well as unclassified maps.
III	POL includes bulk fuels and packaged products such as antifreeze.
IV	Construction items, including fortification and barrier materiel.
V	Ammunition of all types.
VI	Personal demand items (nonmilitary sales items) and gratuitous health and comfort pack items.
VII	Major end items, such as launchers, tanks, mobile maintenance shops, and vehicles.
VIII	Medical supplies, including repair parts for medical equipment.
IX	Materiel to support nonmilitary programs, such as agricultural and economic development, which are not included in supply classes I through IX.
X	Salvage, packaged water, captured enemy supplies.

## **Inventory issues**

As mentioned previously, the DoD's supply chain is characterized by diversity in supply, demand unpredictability, long and variable lead times, among others. This raises issues in terms of Inventory Management policies, such as: mobility overstocking, maintenance activities, and readiness. Storage space is limited to vehicles, including trucks, trailers and containers (Peltz et al, 2007) and the amount and size of supplies impacts directly a unit's mobility. Unpredictability in demand and supply typically results in overstocking of items that are not critical for the successful completion of a mission. Maintenance activities are troublesome for a unit because the demand for repair parts is unpredictable. The main issue in military supply chains is having the optimal inventory portfolio to guarantee the desired level of readiness. Readiness measures a unit's capability to sustain a mission and will be discussed in the next section.

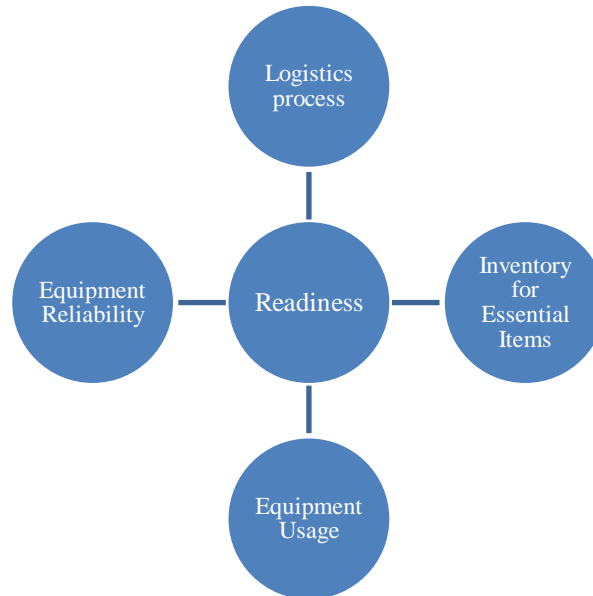
### **Readiness as a Cost Measure in Military Supply Chains**

While commercial supply chains look at financial costs to assess the performance of their policies, the DoD's supply chain's cost is defined in terms of 'sustainment cost', or more explicitly, readiness.

By definition, readiness represents an organization's capability to accomplish a mission. The Army assesses a unit's ability to accomplish its assigned core functions, namely C-level assessment, according to four different factors. The resulting Commander's Unit Status Report (CUSR) is used by high-level commanders and senior Armed Forces leaders to gather tactical-level capability, synchronize operational planning and resource management. In the context of the Army, readiness is an indicator of whether a unit is ready for deployment, if there are enough supplies, and whether equipment is sustainable and operational. Moreover, it considers the



impact of changes in the logistics support, performance, and failure rate. Readiness encapsulates inventory levels for essential items, equipment reliability, usage, and the logistics process to support the unit.



**Figure 2-7: Components of Readiness**

As mentioned previously, the Army C-level assessment considers four different factors: total available personnel, equipment and supplies on hand, equipment readiness and serviceability and unit training level proficiency (Department of the Army, 2010). Figures 2-8 and 2-9 display the methodology and metrics for C-level readiness assessment.

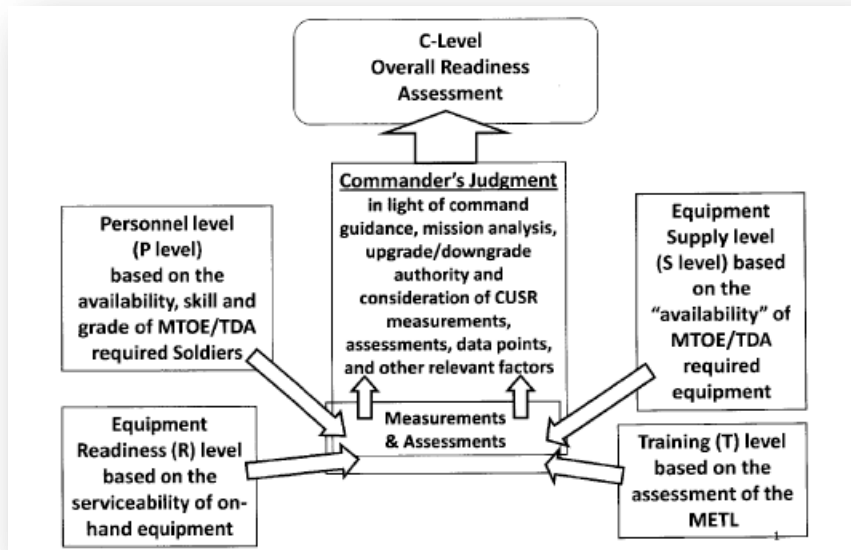


Figure 2-8: Methodology for unit readiness assessment (C-level) - Adopted from (Department of the Army, 2010)

<b>Overall Assessments</b>	<b>C-Level</b>
<b>Personnel Status Measurements</b>	<b>P-Level</b>
<b>EOH (Available) Status Measurements</b>	<b>S-Level</b>
<b>Equipment Readiness (Serviceability) Status Measurements</b>	<b>R-Level</b>
<b>Training Status Measurements</b>	<b>T-Level</b>

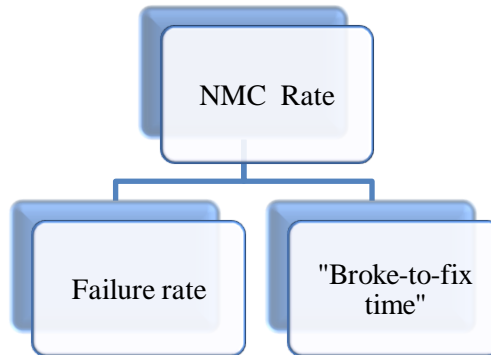
Figure 2-9: Metrics for unit readiness assessment (C-level) - Adopted from (Department of the Army, 2010)

In the context of this research, readiness is studied from two out of the four perspectives: equipment and supplies on hand (Equipment on Hand) and equipment readiness (ER). Equipment

on Hand (EOH) level compares the pacing items of mission essential equipment to the total mission essential equipment items in the unit's possession within 72 hours of mission execution. The higher the EOH level, the higher the inventory level at the time of mission execution. Equipment Readiness (ER) is defined and measured as the percent of time a given fleet that is Fully Mission Capable (FMC) from the 16<sup>th</sup> of each month to the 15<sup>th</sup> of the next one. The use of the Army's equipment varies significantly as there are long periods of no activity, short periods of high activity during training and tremendous activity when in combat. Equipment readiness is also known as 'pulse availability' as it is the percentage of time depends on the pulse (mission) time. The expression for equipment readiness is

$$ER = 1 - NMC$$

Where NMC is the percentage equipment is Non-Mission Capable. The NMC depends on two factors: equipment failure rate over a mission period (pulse) and the broke-to-fix time.



**Figure 2-10: Components of Non-Mission Capable Rate**

The broke-to-fix is the average time the equipment is down and relies on the logistics processes that include: diagnosis and parts ordering, awaiting parts, fix processing time, and part pickup and receipt. Bottlenecks in any of these segments increase the NMC rate; some examples are by delays in order fulfillment and lead times that are larger than the pulse times when units are deployed. The consistent pressure for units to maintain the expected readiness calls for small

failure rates and quick repairs, which requires equipment that is reliable, preventive and scheduled services, and parts that are available at the time maintenance or repairs are to be performed.

To address some of these challenges, Peltz, Robbins, Boren, & Wolff proposed Equipment Downtime Analyzer (EDA) as an effort to monitor and develop strategies to reduce the amount of time the equipment spends in a segment (Peltz, et al, 2002). The system includes performance metrics that connect all processes to provide feedback on improvement initiatives. Some of these performance metrics include daily NMC reports, Cross-Functional repair records and Detailed Unit Exercise History reports. Daily NMC reports helps decision makers identify situations that affect readiness negatively, while Cross-Functional reports aids in proper assessment of bottleneck processes and logistics response problems. Moreover, the information in the EDA can be used to determine parts that are critical in the improvement of the overall readiness, by considering the number of times a component appears in an order as well the number of days a component appears in a deadline report. Some of the main contributions of this system include real time identification of key parts that drive readiness as well as the key drivers, such as removal rates and supplier problems. Hence, the information about battalions from the EDA enables logisticians to provide assessments on those processes having a direct impact directly on NMC rates. Some readiness improvements include: development of purchasing strategies to reduce the uncertainty on delivery of spare parts that drive readiness, while maximizing utilization of space.

### **Risks affecting the Army's Supply Chain**

Two broad categories impact supply chain design and management: risk arising from coordination of supply and demand and disruption risks (Kleindorfer & Saad, 2005). This section

includes sources of risk from the Army's military operations, combining information from procedures for unit leaders and observations from Operation Iraqi Freedom (OIF) and suggests the definition of a risk measure for delivery of spare parts.

### **Risk associated with coordination of supply and demand**

The Army's supply chain is characterized by high instability in demand and supply. Therefore a highly responsive strategy is required to sustain the desired readiness levels, especially during wartime. Below we enlist the major sources of supply/demand coordination risk

- Lack of communication devices
- Delays and errors in order shipments
- Underestimates on demand for trucks and spare parts
- Lack of tools for dynamic planning and monitoring of supplies

Major findings from Operation Iraqi Freedom suggest highlight that lack of communication devices hindered equipment readiness, hence the sustainment of military operations. As the number MRO's increased, problems in communication and IT systems delayed replenishment requests and prevented orders from being processed in the supply system. This resulted in a decrease in both satisfaction levels and fill rates. For the 101<sup>st</sup> Airborne Division, the satisfaction rate decreased approximately 45%, while the fill rate went down to approximately 10 percent.

The research also highlights orders for spare parts were reduced due to "distributed battlefield, extended supply lines and frequent movement" (Pelz et al, 2007). Moreover, misalignments between preparation practices at the Points of Embarkation and demands that exceeded the CONUS distribution center's capacity resulted in error in the delivery of supplies.

Another issue that affected negatively tactical operations was the poor estimation of demand for trucks and spare parts. During the deployment planning process, truck suppliers were considered individually rather than as elements of an aggregated package. This resulted in order quantities lower than required given a lack of understanding on the impact of truck supplies in overall combat capabilities (Peltz et al, 2007). The stock requirements for spare parts were negatively affected by the Army's failure to adjust to wartime operations, long lead times and supplier disruptions. This poor estimation of commodities is the result of: lack of tools for integrated dynamic planning, monitoring and evaluation of sustainability.

### **Disruption Risks**

Disruption risks, as Kleindorfer and Saad describe in their paper (Kleindorfer & Saad, 2005) refers to the uncertainty in normal operation and consists of: operational risks, discontinuities in supply, natural hazards, terrorist attacks and political instability. Some disruption risks associated with the Army's supply chain during wartime include:

- Disruptions in equipment due to terrain conditions
- Sudden weather changes
- Mobility of units resulting in increased lead time
- Enemy attacks

When performing theatre operations, geographical conditions are the main source of disruptions in supply, as delivery of supplies is negatively impacted by weather and ground conditions, exposure to ambushes and dispersion of trucks. The Army's organizational supply field manual specifies the procedures to be taken under several types of environment, exposing different characteristics in terms of weather and directly impacting the mobility of trucks

delivering supplies. Shipments performed during the night require more idling and vehicles using lower gears, slowing delivery. Under nuclear, biological or chemical warfare, disruptions are greater because of slower travel speeds, greater loading times and incorrect diagnoses of equipment failures, degradation in equipment ammunition and poor performance in decontamination of equipment. Delivery of supplies for jungle operations are disrupted by the additional effort in securing the supply route from ambushes, mines and other infiltration. Urban operations suffer from interruptions in communications devices, refugees, obstacles (mines, snipers, ambushes) and may require armed convoys. Desert operations suffer from poor navigation, vulnerabilities to attacks and wide dispersion. When delivering supplies to operations in cold weather and mountains, shipments focus in mobility and dispersion at the expense of local security. Deliveries of supplies in arctic conditions are threatened by dramatic changes in temperature, visibility and mobility. In addition there are few roads or alternate transportation networks.

Using information on previous events, such as Operation Iraqi Freedom and the knowledge on what drives and impacts readiness as well as risk in the delivery of supplies to brigade units, an inference can be made. Under combat conditions, the risks associated with cancellation and delay in the delivery of spare parts essential for equipment readiness is high and can hinder the success of military operations. These risks lower equipment readiness required for sustainment, therefore resulting in inaccurate assessments of the potential of a mission as well as sudden changes of plans. As seen in the next section, diversification of suppliers is an alternative to hedge these risks and is discussed in the next section of this literature review.

## **Supplier Selection Models**

### **Previous Research**

Supplier selection is a high priority activity in purchasing activities in a commercial supply chain. Companies have incentives to use a single vendor or supplier in their supply chain strategy, including strong partnerships, reduction in coordination efforts to ensure on time deliveries, provision of higher quality levels, and more attractive prices in terms of quantity discounts (Minner, 2003). However, with the increasing trend in global sourcing and more competitiveness between suppliers in developing and developed countries, single supplier models also present multiple risks. Some examples of supplier risk for conventional supply chains are applicable to military supply chains and include: coordination of supply and demand, disruptions due to operational malfunctions, discontinuity of supply, terrorist attacks, political issues, capacity restrictions and lead time uncertainty. Multiple sourcing can be applied to hedge against these risks. In multiple sourcing, a portfolio of suppliers is built such that the tradeoff between competitiveness, risk and cost are kept at optimal levels.

Researchers have addressed supplier selection models in supply chain risk management in a variety of ways. Tang discusses several quantitative methods to address supply chain risk, including supplier selection and supplier order allocation (Tang, 2006). Pan et al presented a technique to estimate lead time distribution parameters for multiple sourcing using order statistics. They considered three different lead time distributions: uniform, exponential and normal (Pan et al, 1991). Supplier selection problems have been solved in a variety of ways; most of them incorporate Linear Programming, Mixed-Integer Linear Programming, Multi-criteria Optimization and Non-Linear Programming. Anthony and Buffa (Anthony & Buffa, 1977) developed a model for Strategic Purchasing Scheduling that included price and storage cost. A



mixed-integer programming model was applied by Narasimhan and Stoyhoff to reduce the procurement portfolio for a group of vendors (Narasimhan & Stoyhoff, 1986). Weber and Current study the relationship between factors in supplier selection using multi-criteria linear programming (Weber & Current, 1993). Benton developed a non-linear problem and a heuristic procedure that accounts for multiple items and suppliers, resource limitations and quantity discount (Benton, 1991). Ghosypour and O'Brien developed both a non-linear model as well as multi-objective goal programming model that minimizes the total annual purchasing cost (Ghosypour & O'Brien, 2001). Their model requires computing the combinations of suppliers, solve the model for each combination of suppliers that satisfy the demand requirements and identify the optimal portfolio of suppliers. The optimal policy for a (s,Q) model with Erlang lead times and deterministic demand is proposed by Kim, Sun, He and Hayya by applying linear regression and experimental design, hence providing decision makers with a heuristic that reduces inventory costs (Kim, Sun, He, & Hayya, 2004). Erdem, Fadiloğlu and Özekici present a supplier selection model based on the EOQ by looking at the supplier yield. In their research, they define the supplier capacity as a random variable and study the effects under two scenarios: modeling the supplier capacity with a uniform distribution and using an exponential distribution. Their results show that under exponentially distributed capacities, the optimal number of suppliers increases as their expected capacity decreases.

The models presented by these researchers are primarily focused in finding the optimal portfolio by reducing the purchasing cost based on the Economic Order Quantity (EOQ). In commercial supply chains, the optimal order quantity is determined according to order costs, inventory holding costs, demand and lead time. However, in a setting for theater operations, the optimal portfolio of SSA's for a brigade unit is that one maintaining a certain level of equipment readiness with limited storage space at minimum risk of disruptions/cancellations. The amount of commodity a specific brigade unit orders depends on demand, storage capacity, and lead time

constraints. Research from Peltz et al in this regard show the main concern is to minimize the storage capacity while having enough inventory to maintain a readiness level while covering for short disruptions in replenishments (Peltz et al, 2007). Decision makers seek to maximize the utilization of a fixed capacity, which consists of trucks, trailers and containers. Hence, the tradeoff between space utilization and readiness is used to determine the appropriate capacity assigned to a unit. Some of the efforts made by researchers in improving equipment readiness in the Army include identification of spare parts that drive readiness by means of the Equipment Downtime Analyzer (Peltz et al, 2002) such that decision-makers and logisticians can develop strategies to aggregate orders for common parts that result in the largest increase in equipment readiness.

### **Conclusions**

This chapter provided an overview of the Army's supply chain, its associated risks, and different approaches taken by researchers using supplier selection models to manage risk in supply chains. Within the context of the army, there are risks associated with coordination of supply and demand as well as disruption risks. This is due to the nature of theatre operations, where the uncertainty in demand is high, units are in constant movement and high equipment readiness is critical at all times. Although at the time a unit is assigned a single Supply Support Activity (SSA), the uncertainty in lead times reported by researchers in the field suggests risk can be reduced if multiple suppliers are considered.

Research in supplier selection models, is focused in the minimization of risk while simultaneously minimizing the total purchasing cost using the EOQ model. However, as literature in the Army's supply chain suggest, the main concern for decision-makers is to reduce

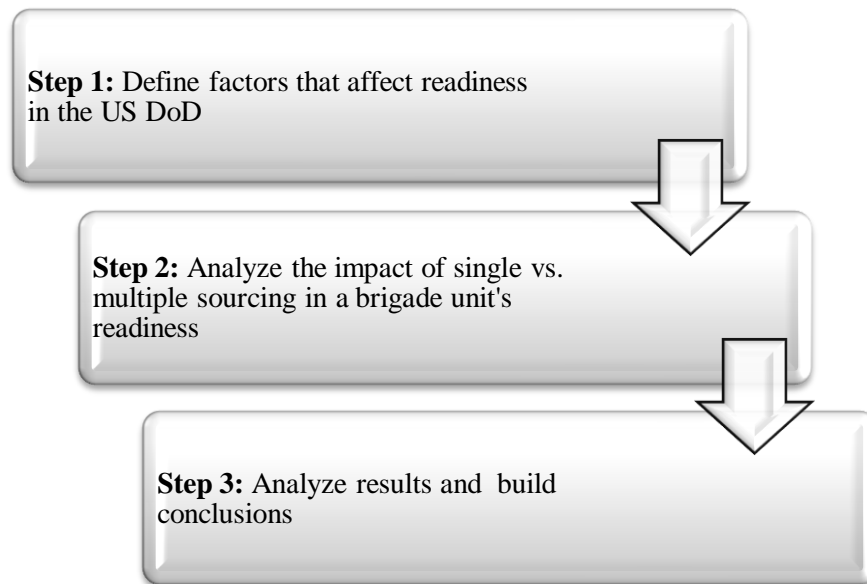
maximize the equipment readiness, which requires maximizing the Equipment on Hand and the Equipment Readiness levels.

The high uncertainty in demand and supply of the Army's supply chain for spare parts requires a strategy focused in responsiveness. Decision making models that measure the supplier risk and incorporate multiple suppliers can contribute to this responsiveness. This is achieved by reduction of uncertainty in lead times and amount of supplies received at a given time, therefore reducing repair times, increasing readiness and enabling decision-makers to make accurate assessments with regard to mission capabilities.

## Chapter 3

### Research Methodology

The models developed in this research consider a scenario with a single unit placing an order of  $x_{\text{req}}$  for a single commodity when in combat mode. This Chapter focuses on describing a framework to assess whether the current policy of ordering commodities from a single supplier should be altered. The first step consists of defining the factors that characterize sustainability costs. This step also studies how lead time and supply yield in orders to a single supplier can affect readiness, provided that readiness is the most important metric in the DoD. The second step consists of an analysis of equipment readiness under multiple suppliers, assuming that at any given time one or more suppliers might be unavailable. Using this information, a risk measure is defined in the third step and an optimization model is formulated.



**Figure 3-1 Research Methodology**

## Methodology

### Step 1

The expressions for two of the factors considered in the Army's assessment of readiness are defined in Step 1. From previous literature, this research will consider readiness from two perspectives:

- Equipment on Hand - % of equipment in Inventory within 72 hours of a mission
- Equipment Readiness - % of time equipment is readily available, dependent on
  - broke-to-fix time
  - failure rate of the spare part

The formula to compute readiness in the military context is presented. Some assumptions taken in this step are:

- Considering the 'broke-to-fix time' as the lead time for order delivery.
- The order amount from a supplier depends on the failure rate of the equipment as well as the number of units of equipment the brigade uses. This information is assumed to come from the Equipment Downtime Analyzer (EDA)
- The lead time for order arrival of  $x_{req}$  follows an exponential distribution with parameter  $\lambda$  and expectation  $1/\lambda$
- Defining a yield measure,  $Y$ , as the ratio of the amount of commodity received to the amount ordered  $x_{req}$
- When an order of  $x_{req}$  is placed, the order yield  $Y$  represents the Equipment on Hand (EOH) level at lead time  $l^*$ .

## **Step 2**

This step studies the behavior of EOH and equipment readiness under single and dual sourcing. Moreover, these factors are analyzed when an order of  $x_{req}$  is split amongst multiple suppliers. The results multiple experiments show that as the number of suppliers increases, the expected EOH level increases and the Expected lead time for order arrival decreases. For each scenario, the percentage improvement in the expected order yield and expected lead time are compared against the percentage increase in purchasing costs in order to identify the optimal number of depots.

## **Step 3**

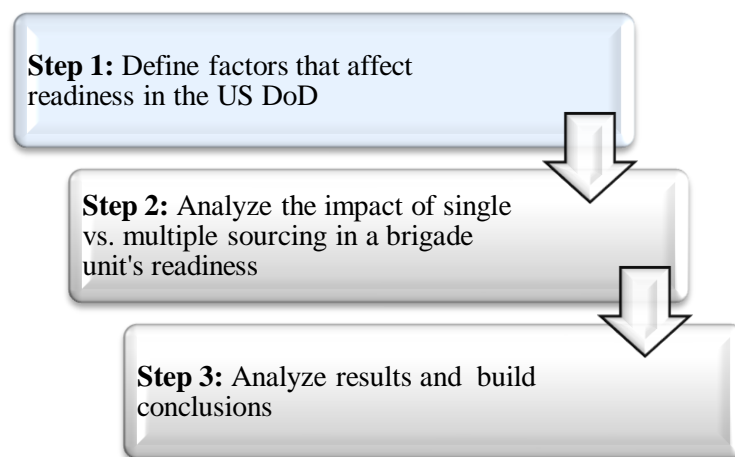
The results from the numerical results in Step 2 are presented and discussed. Conclusions, contributions and future work are stated.

## Chapter 4

### A Cost Measure for the Department of Defense Supply Chain

#### Introduction

This chapter models the sustainment cost a brigade units incurs when ordering supplies of Class IX items in terms from both the brigade unit and the depot's point of view. First, a review of costs in commercial as well as military supply chains is presented. As a second step a description on how orders for spare parts are placed and received is provided. In addition, the impact of increasing lead times and decreasing order yield values in sustainment costs is illustrated using examples. After understanding the effect of order yield and large lead times in equipment readiness, The equations that describe for Equipment on Hand levels and equipment readiness for brigade units in theatre combat operations are stated. This fulfills the requirements for Step 1.



**Figure 4-1: An overview of Chapter #4**

### Costs in the front end of a commercial supply chain: EOQ Model

Conventional supply chains seek economies of scale by applying the EOQ model in their inventory policy. In case both demand and lead time are deterministic, the following assumptions hold:

- A single commodity is to be purchased
- There is a fixed setup cost of  $\$a$  to penalize frequent purchasing
- An inventory cost per unit of  $\$h$  is charged to penalize excessive inventory
- There is a steady demand of  $d$  units

The optimal order quantity  $Q^*$  is computed by (Ravindran, Phillips, & Solberg, 1987):

$$Q^* = \sqrt{\frac{2ad}{h}}$$

If the demand or lead time happens to be stochastic, the following assumptions apply:

- Demand is stochastic with known probability distribution
- Lead time may be stochastic with known distribution
- Shortages may occur, and no delivery of partial orders

This formula is easy to implement and provides a great deal of robustness to errors in the estimation of its parameters; an error margin of 100% in any of its parameters produces only 41% error in the result (Ravindran, Phillips, & Solberg, 1987). Based on the economic order quantity, additional information on the inventory policy can be computed, such as optimal order frequency, cycle inventory, number of orders per year, annual order and holding costs, average flow time and order costs. A table showing these computations is shown below.



**Table 4-1 – Relevant parameters in the EOQ formula**

Metric	Formula
Optimal Order Frequency	$n^* = \frac{D}{Q^*} = \sqrt[2]{\frac{DC}{2A}}$
Cycle Inventory	$\frac{Q^*}{2}$
Number of orders per year	$\frac{D}{Q^*}$
Annual ordering and holding costs	$\frac{D}{Q^*}A + \frac{Q^*}{2}IC$
Average flow time	$\frac{Q^*}{2D}$
Total variable cost	$\sqrt[3]{2ADh}$

There are two main reasons why the EOQ model is not applicable in a scenario where a brigade unit orders supplies from a depot. First, the assumptions for the EOQ model are not held; the optimal order quantity  $Q$  for the first period is computed at the first time period  $T$  by considering expenses and revenues throughout the entire future of the ordering process. The demand for class IX items is highly uncertain as it depends on the nature of the mission and other factors, which implies the order process is not constant. Therefore, a single period needs to be considered at the time an order is placed. There is no evidence of a setup order cost in the literature and no inventory holding costs. However, the main reason why the EOQ model is not applicable is because in the scenario under study, cost is defined in terms of sustainability in combat operations rather as a total purchasing cost, which is discussed next.

## **Cost in the Department of Defense Supply Chain: Sustainment Cost**

While conventional supply chains consider the total purchasing cost, military research considers three cost factors in the Army:

- Readiness (equipment readiness)
- Maintenance footprint
- Equipment maintenance costs

As defined previously, equipment readiness is the percentage of time equipment is available. Maintenance footprint is the capacity in terms of equipment and personnel at each echelon and combines equipment reliability, maintainability and support. Equipment maintenance costs include maintenance and spare parts personnel, contract maintenance, and costs associated to management of the maintenance system and accounted for 12% of the Army's budget in 1999 (Peltz E. , 2003). This research focuses in defining cost in terms of readiness and extends the concept to both Equipment on Hand (EOH) and Equipment Readiness (ER).

### **Readiness as the principal cost measure**

#### ***Equipment on Hand***

Equipment on Hand (EOH) or percentage fill level compares the pacing items of mission essential equipment to the total mission essential equipment items in the unit's possession within 72 hours of mission execution. Pacing items include major weapon systems, aircraft, and other equipment items that are central to the organization's ability to perform its core functions/designed capabilities, and are subject to continuous monitoring and management at all

levels of command (Department of the Army, 2010). Therefore, they are critical to a mission's success.

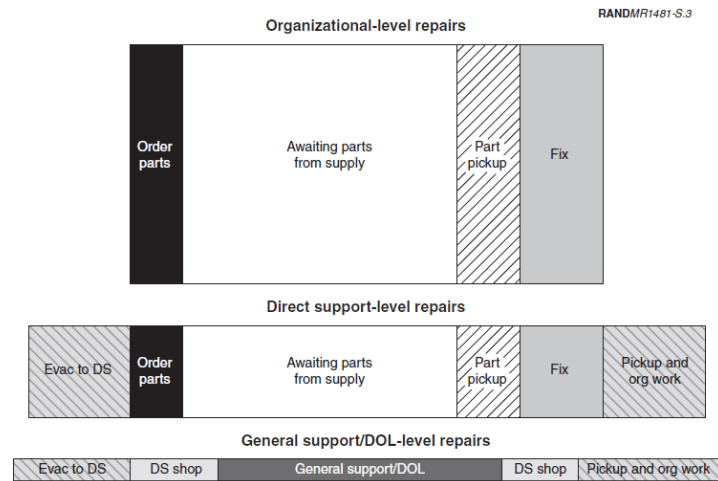
Army Regulation 220-1 states that the EOH or percentage fill is computed by dividing available equipment by the required quantity as specified in the modification table of organization and equipment (MTOE).

### ***Equipment Readiness***

Peltz defines low mission availability or 'equipment readiness' as the primary critical cost of poor sustainability in the Army (Peltz E. , 2003). Two factors that heavily influence pulse availability are the time required for repairs, namely 'broke-to-fix time' and the reliability of the equipment or 'failure rate'. As mentioned in Chapter 2, pulse availability is defined as 1 - Non-Mission Capable rate (NMC).

$$\text{Readiness} = 1 - \text{NMC} = 1 - (\text{broke-to-fix time})(\text{failure rate})$$

While the failure rate is a measure of a system's reliability and provides an idea of how frequently spare parts are to be ordered during combat operations, the broke-to-fix time relies on the logistics processes such as: diagnosis and parts ordering, awaiting parts, fix processing time, and part pickup and receipt (Peltz, Robbins, Boren, & Wolff, 2002). A breakdown of the broke to fix time is shown in the Figure 4-2.



**Figure 4-2: Breakdown of the Broke-to-fix processes – Adopted from (Peltz, Robbins, Boren, & Wolff, 2002)**

### Sustainment Cost: Readiness Assumptions

This research assumes sustainment cost depends only in terms of readiness. Therefore cost increases as the order yield and equipment readiness levels decrease. For the context of this research, the Equipment on Hand level is computed as the percentage of an order of  $x_{req}$  successfully delivered to the forward unit, also defined as order yield:

$$Y = \frac{y}{x_{req}}$$

Where  $y$  is the amount of commodity received. The larger the value  $y$ , the closer  $Y$  is to 1.

Equipment readiness was previously defined as

$$ER = 1 - (\text{broke-to-fix time})(\text{failure rate})$$

To simplify the problem, equipment readiness is computed for a single commodity (tank, truck, other) that requires a single spare part for repair. Methods to produce reliable estimates on broke-to-fix time are unavailable (Peltz E. , 2003); therefore the broke-to-fix time depends

completely on the lead time for a specific part ordered. This means that i) diagnosis and parts ordering are performed on the same day the part is ordered and ii) parts pickup, receipt and fix processing are performed the same day the order for the part is received. Moreover, in order to compute the failure rate, this research assumes values are extracted from the Equipment Downtime Analyzer (Peltz, Robbins, Boren, & Wolff, 2002) and provide an estimate on the demand for a specific commodity to be ordered. An illustrative example is provided below.

***Example #1***

Assuming there is a combat mission with duration 15 days and a failure rate of 3/system/30 days. An order for spare parts has a mean lead time of 2 days. Then the failure rate for the system over a pulse is computed as

$$\text{Failure rate} = \frac{3 \text{ failures}}{30 \text{ days}} \left( \frac{15 \text{ days}}{1 \text{ pulse}} \right) = \frac{1.5 \text{ failures}}{15 \text{ day pulse}}$$

Moreover,

$$\text{ER} = 1 - \left( (2) \left( \frac{1.5}{15} \right) \right) = 1 - \frac{3}{15} = 0.8 \text{ or } 80\% \text{ readiness}$$

The information from the example shows that under the assumptions stated previously a lead time of 2 days for an order of spare parts to arrive results in a readiness of 80% per system.

## Modeling Class IX parts ordering in theatre combat operations

### An expression for Equipment Readiness (ER) based on exponentially distributed lead times

Sustainment costs due to equipment readiness depend on the lead time and failure rate of the commodity to be ordered. Sustainment cost increases linearly as the number of suppliers increases, as shown in Figure 4-3.

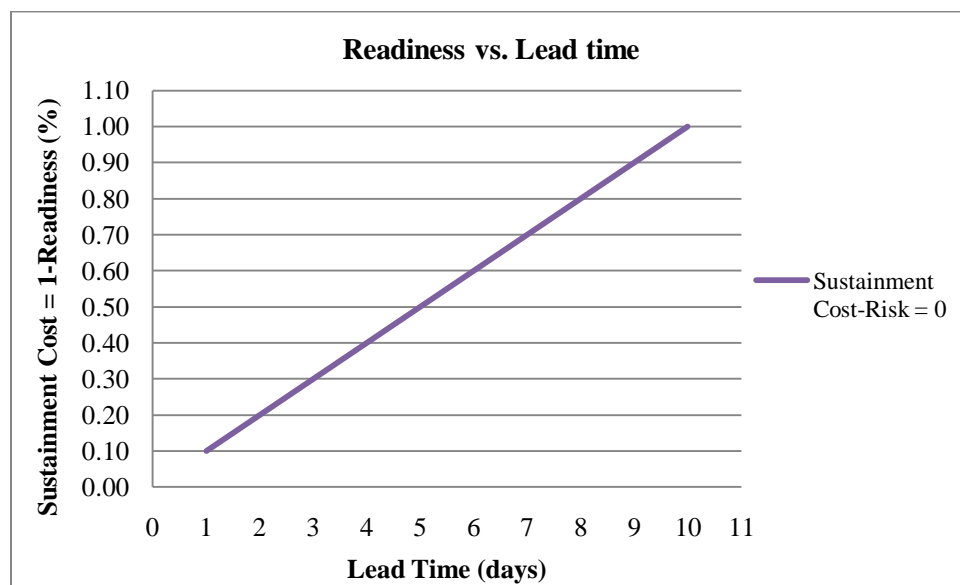


Figure 4-3: The relationship between sustainment costs and lead time

#### Example #2

(Continued from Example #1) Assume that, instead of a lead time of 2 days, the lead time becomes 4 days. The effect in sustainment cost is:

$$cost = \frac{1.5(4)}{15} = \frac{6}{15}$$

$$readiness = 1 - \frac{6}{15} = \frac{9}{15} = 60\%$$

Therefore, an increase in lead time by 2 days results in 20% difference in readiness for a pulse.

Assume the lead time  $L_A$  for order arrival from a depot to the current location of the brigade unit follows an exponential distribution with rates  $\lambda$ . The properties of the exponential distribution, the expected value and variance of the lead time  $L$  are:

$$E[L_A] = \frac{1}{\lambda} = \frac{x_{Max}}{k}$$

$$\text{Var} [L_A] = \frac{1}{\lambda^2}$$

Therefore, under the assumption that the broke-to-fix time equals the lead time for order arrival, the expression for Equipment Readiness when the lead time is exponentially distributed becomes:

$$ER = 1 - \left( \frac{1}{\lambda} \right) (\text{failure rate}) \quad (i)$$

The table below shows readiness computations for a depot following an exponential distribution for order lead time with rate  $\lambda=5$ . Note that the range of equipment readiness values goes from 44 to 91%.

Lead time L	Failure rate	NMC	ER
4.12	0.07	0.27	73%
4.82	0.07	0.32	68%
1.34	0.07	0.09	91%
1.15	0.07	0.08	92%
8.39	0.07	0.56	44%
3.57	0.07	0.24	76%
4.04	0.07	0.27	73%
2.18	0.07	0.15	85%
1.72	0.07	0.11	89%
1.41	0.07	0.09	91%

An assumption made from the supplier's point of view is that a depot  $i$  receiving an order will fulfill the complete request of  $x_{iMax}$  units; therefore, no partial orders are allowed. For an order of amount  $x_{iMax}$ , let  $L_i$  be a random variable that represents the lead time of an order of  $x_{iMax}$ . In this research,  $L_i$  follows an exponential distribution with parameter  $\lambda_i$  and mean  $1/\lambda_i$ . For the purpose of this study, the value  $\lambda_i$  will depend on:

- Amount ordered - a larger expected lead time mean  $1/\lambda_i$  occurs as a consequence of a greater amount of commodity to be ordered from a supplier. The larger the amount, the Then the rate of the distribution of  $L_i$  is represented as

$$\lambda_i = \frac{k_i}{x_{iMax}}, k_i \text{ constant or performance measure}$$

- Disruption risk index – this is a combination of the relative distance from a depot to a forward unit combined with the ‘riskiness’ of the environment. This value can be affected by sudden events, such as attacks, environmental disasters, weather conditions, and other. Then the rate  $\lambda_i$  is defined as:

$$\lambda_i = \frac{k_i}{\delta_i}$$

The probability that supplier  $i$  will deliver amount  $x_{ij} = x_{iMax}$  in time  $t^*$  is given by:

$$p(L_i = t) = 1 - e^{-\lambda_i t}$$

For an amount  $x_{Max}$  requested by a brigade unit, define  $y_i$  as a random variable representing the amount received by supplier  $i$ . Moreover, let  $Y$  be a discrete random variable representing the order yield or the percentage of the order received by a unit. Note that this expression is similar to the fill rate as it measures the proportion of customer demand that is satisfied from the depot’s inventory (Chopra & Meindl, 2007). However, this calculation depends on the probability that a supplier will be available as well as the lead time distribution of the depot. For the current policy of a single supplier, the order yield  $Y$  depends on the lead time distribution of the depot and can take the values of:

$$Y = \begin{cases} \frac{y_i}{x_{iMax}} = 1 & P(L_i \leq t^*) = 1 - e^{-\lambda_i t^*} \\ 0 & P(L_i > t^*) = e^{-\lambda_i t^*} \end{cases} \quad (ii)$$



## Conclusions

Chapter #4 defined a cost measure for the Department of Defense supply chain. As opposed to commercial supply chains where the cost is defined in terms of the total purchasing costs, the DoD's supply chain measures cost in terms of operational readiness. The expressions for the Equipment on Hand level and Equipment Readiness and order yield Y level are stated in Table 4-2. These expressions will be used in Chapter #5 to evaluate the effect of multiple sourcing.

**Table 4-2 – Expressions for the DoD's supply chain cost metrics at the brigade unit's stage**

Equipment Readiness (ER)	$ER = 1 - \left( \left( \frac{1}{\lambda} \right) (\text{failure rate}) \right)$
Order yield Y	$Y = \begin{cases} \frac{y_i}{x_{iMax}} = 1 & P(L_i \leq t^*) = 1 - e^{-\lambda t^*} \\ 0 & P(L_i > t^*) = e^{-\lambda t^*} \end{cases}$

## Chapter 5

### Multiple Sourcing and Sustainment Costs

#### Introduction

Chapter 5 presents the advantages of multiple sourcing vs. single sourcing at the front end of the Army's supply chain when in combat operations. Sustainment costs are analyzed for single orders placed by a brigade unit under variations of exponential lead times and order yield. The process is repeated for multiple orders placed by a brigade unit when depots exhibit similar and different values of lead time and order yield. Improvements in the total expected cost under multiple sourcing is justified. Finally, the effect of the number of suppliers in operational readiness is discussed, and the hence completing the requirements for Step 2.

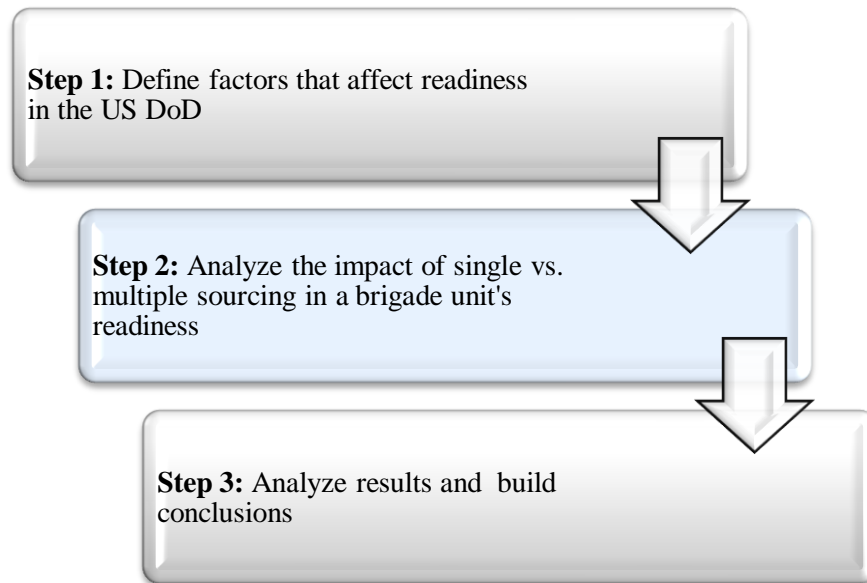


Figure 5-1: Steps covered in Chapter 5

## Single Sourcing vs. dual sourcing for exponentially distributed lead times

### Single sourcing under exponentially distributed lead times dependent on order size

Single sourcing increases the risk of delays and cancellations of orders in scenarios where the lead time depends on the amount of commodity purchased. On the other hand, ordering from multiple suppliers reduces lead time variability while increasing inventory yield. Consider a single unit that is required to order 150 units of supply from A. The lead time  $L_A$  the depot to the current location of the brigade unit follows an exponential distribution with rates  $\lambda_A$ . The relationship between the average lead time  $\lambda_A$  to the amount ordered  $X_{iMax}$  is given by:

$$\lambda_i = \frac{k_i}{X_{iMax}}, k_i \text{ constant, } i \in \{A\}$$

Using the properties of the exponential distribution, the expected value and variance of the lead time  $L_A$  is:

$$E[L_A] = \frac{1}{\lambda_A} = \frac{x_{BMax}}{k_A}$$

$$\text{Var}[L_A] = \frac{1}{\lambda_A^2}$$

The lead time distribution is computed for  $k_i = 50$  Equipment readiness is computed as in Equation #1 from Chapter #4 with  $f = 1$  failure/system/15 day mission and a penalty  $\alpha = \lambda_A f = 0$ .

**Table 5-1: Lead time expectation, variance and readiness computations for a single supplier scenario**

Option	Order Amount	Rate $\lambda$	Average Lead Time (days)	Failure rate (days/pulse)	Sustainment Cost	Readiness
$S_A$	150	0.333	3	0.07	20%	80%

Intuitively, a decision maker will notice supplier A is risky as the mean lead time is 1/5 of the pulse duration. Moreover, the projection on equipment readiness for this mission is lower than

the required 90% in the Army. Scenarios #1 and #2 describe the behavior of dual sourcing when the lead time for order arrival is exponentially distributed with rate  $\lambda$  dependent on the order amount  $x_{req}$ . Scenario #1 assumes the order is equally split among 2 suppliers, whereas Scenario #2 considers a scenario where one of the suppliers has 2/3 of the order while the second supplier has 1/3 of the order.

**Scenario #1: Order arrival rate dependent on the amount of commodity requested and order is distributed equally among suppliers**

If an order of  $x_{req}$  is split evenly among two suppliers, risk of order delays is reduced and expected order yield level increases. Since the rate for order arrival is dependent on the amount requested and both suppliers have same parameter  $k_i$ , then depots A and B have the same characteristics, as shown in Figure 5-2:

**Table 5-2: Lead time expectation when a single order is split equally in 2 suppliers**

Option	Order Amount	Rate $\lambda$	Average Lead Time (days)
S <sub>A</sub>	75	0.667	1.5
S <sub>B</sub>	75	0.667	1.5

The combined lead time  $L = \max(L_A, L_B)$  is the total time until the last order arrives. By conditioning on who takes longer to deliver the order (Ross, 2007), the expected lead time is computed as:

$$E(L) = E(L_A|L_B < L_A) p(L_B < L_A) + E(L_B|L_A < L_B) p(L_A < L_B)$$

$$E(L) = \frac{1}{\lambda_A} \left( \frac{\lambda_B}{\lambda_A + \lambda_B} \right) + \frac{1}{\lambda_B} \left( \frac{\lambda_A}{\lambda_A + \lambda_B} \right)$$

$$E(L) = \frac{1}{.667} \left( \frac{.667}{.667 + .667} \right) + \frac{1}{.667} \left( \frac{.667}{.667 + .667} \right) = \frac{2}{2(.667)} = 1.5 \text{ days}$$

Therefore, the combined lead time has an expectation and standard deviation of 1.5 days when an order is split amongst two suppliers. This is approximately a 50% reduction in the mean lead time. A reduction in the mean lead time results in an increase in the unit's operational readiness projections under the assumptions from the previous chapter where it was stated that the 'broke-to-fix' time equals the lead time. The table below summarizes the computations for projected equipment readiness under the three scenarios.

**Table 5-3: Expectation, variance and readiness results for single and dual sourcing under exponentially distributed lead times with rates dependent on the quantity ordered**

Option	Order Amount	Rate $\lambda$	Average Lead Time (days)	Failure rate (days/pulse)	Sustainment Cost	Readiness
$S_A$	150	0.333	3	0.10	30%	70%
$S_A + S_B$	75 for each supplier	0.667	1.50	0.10	14.9%	85.1%

The results indicate therefore indicate that when the mean and standard deviation of exponentially distributed lead times depend on the amount of commodity purchased, an improvement in equipment readiness can be achieved by splitting an order in equal parts.

***Single sourcing vs. dual sourcing effects in average Order Yield***

The results from the previous section show that incorporating more than one supplier reduces the expectation and variability of the order lead time. Multiple sourcing further increases the expectation of the Equipment in Hand and reduces its variability, hence improving the performance of a unit.

Order arrival is critical in determining daily values for readiness and equipment-on-hand. Single sourcing increases the risk of not complying with Army's requirements due to uncertainty during combat operations. Some instances include: high failure rates, expected lead times that are greater than pulse durations, supplier disruptions, and demand uncertainty. This uncertainty leads

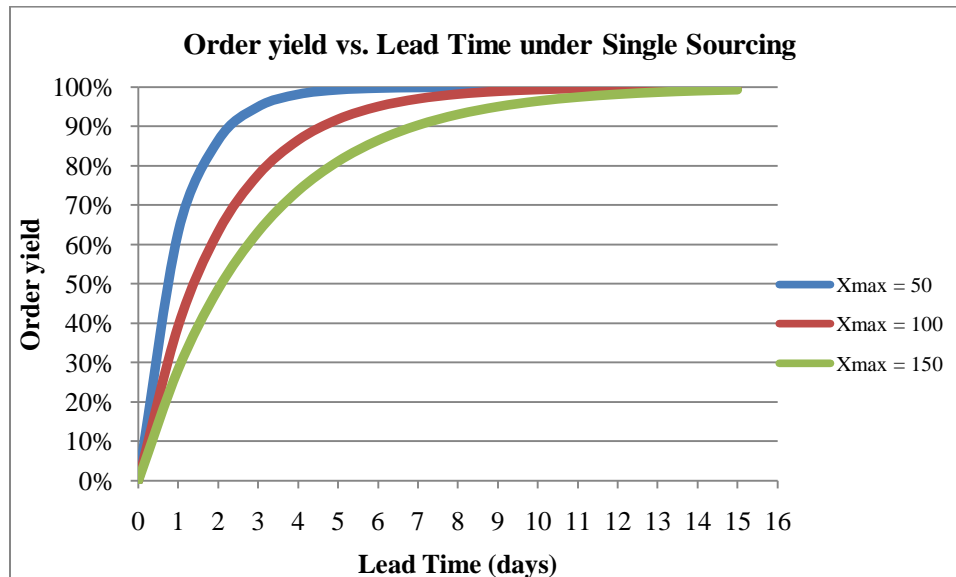
forward units to overstock in order to increase equipment readiness, therefore reducing the unit's mobility given limitations in space.

In single sourcing, the order yield and expected yield over time can be computed using the expression defined in Chapter 4 for and the definition of expectation for discrete random variables:

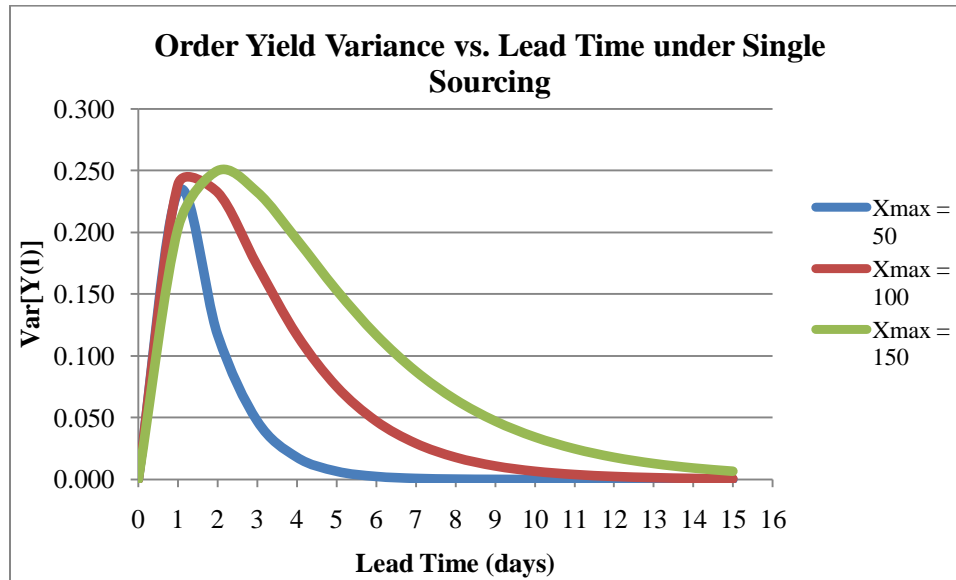
$$Y(L_i) = \begin{cases} \frac{y_i}{x_{iMax}} = 1 & P(L_i \leq t^*) = 1 - e^{-\lambda t^*} \\ 0 & P(L_i > t^*) = e^{-\lambda t^*} \end{cases}$$

$$E[Y(L_i)] = \frac{y_i}{x_{iMax}} * (1 - e^{-\lambda t^*}) + 0 * (e^{-\lambda t^*}) = 1 - e^{-\lambda t^*}$$

The chart below illustrates the effect of single sourcing in the expected order yield over a time period of 15 days for different order sizes, using the same parameter  $K_i = 50$  from previous examples. As expected, order yield increases exponentially with time. Moreover, at any given point in time, order yield increases as the order size increases. Recall Scenario #1 states the rate  $\lambda$  for the supplier depends on the order size.



**Figure 5-2: Order Yield with respect to exponential lead time under single supplier scenario with rate dependent on amount ordered (Scenario #1)**



**Figure 5-3 Variability of Order Yield with respect to exponential lead time under single supplier (Scenario #1)**

The assumptions for Scenario #1 provide decision makers with two alternatives in order to increase order yield: decreasing the amount ordered or to allow for greater lead times. However, these alternatives come at a cost. Smaller order sizes result in lower equipment on hand values of order yield available and shortages. Greater lead times impose equipment readiness levels that are under the Army’s expectations. The table below shows readiness calculations for orders delivered by three different depots with parameters  $\lambda=1/3$  and  $\lambda=1/2$  and  $\lambda=1/5$ . Results show that the larger the value of lead time rate, the greater the decrease in equipment readiness.

**Table 5-4: Effect of lead time in equipment readiness for a equipment with failure rate = .10 and  $\lambda=1/2$**

Readiness computations for $\lambda=1/2$			
Lead Time (days)	Failure rate	NMC	ER
1.65	0.10	0.16	84%
1.93	0.10	0.19	81%
0.53	0.10	0.05	95%
0.46	0.10	0.05	95%
3.35	0.10	0.34	66%
1.43	0.10	0.14	86%
1.62	0.10	0.16	84%
0.87	0.10	0.09	91%
0.69	0.10	0.07	93%
0.56	0.10	0.06	94%

**Table 5-5: Effect of lead time in equipment readiness for a equipment with failure rate = .10 and  $\lambda=1/3$**

Readiness computations for $\lambda=1/3$			
Lead Time (days)	Failure rate	NMC	ER
1.04	0.10	0.10	90%
10.34	0.10	1.03	-3%
3.85	0.10	0.39	61%
9.23	0.10	0.92	8%
7.00	0.10	0.70	30%
0.58	0.10	0.06	94%
1.09	0.10	0.11	89%
3.45	0.10	0.34	66%
0.15	0.10	0.02	98%
10.11	0.10	1.01	-1%

**Table 5-6: Effect of lead time in equipment readiness for a equipment with failure rate = .10 and  $\lambda=1/10$**

Readiness computations for $\lambda=1/10$			
Lead Time (days)	Failure rate	NMC	ER
2.86	0.10	0.29	71%
13.66	0.10	1.37	-37%
6.81	0.10	0.68	32%
3.58	0.10	0.36	64%
1.16	0.10	0.12	88%
0.42	0.10	0.04	96%
6.03	0.10	0.60	40%
19.76	0.10	1.98	-98%
19.02	0.10	1.90	-90%
13.57	0.10	1.36	-36%

Table 5-5 and 5-6 show that for a failure rate of .10, a lead time exceeding 10 days results in a negative value of equipment readiness and the unit is incapable to sustain in a combat operation. The arithmetic mean for the randomly generated lead times and equipment readiness is computed for all three scenarios and summarized in Figure 5-7. The maximum value of 87% is obtained for the random values generated from the distribution with mean = 2 days (therefore  $\lambda=1/2$ ) while the



mean value of 13% readiness is obtained when the rate  $\lambda=1/10$ . Note that, the all three scenarios have ER values lower than the minimum readiness of 90%.

**Table 5-7: Plot of arithmetic mean lead time (days) vs. Equipment Readiness for parameter  $\lambda=1/2, 1/3, 1/10$**

$\lambda$	Lead Time (days)	ER
1/2	1.31	87%
1/3	4.68	53%
1/10	8.69	13%

Dual sourcing increases expected order yield while reducing the lead time for order arrival. Two depots, 1 and 2, with parameters  $\lambda_1$  and  $\lambda_2$  have an order yield distribution  $Y(l^*)$  as:

$$Y(l^*) = \begin{cases} 0 & e^{-\lambda_1 l^*} * e^{-\lambda_2 l^*} \\ Y_1 & 1 - e^{-\lambda_1 l^*} (e^{-\lambda_2 l^*}) \\ Y_2 & e^{-\lambda_1 l^*} (1 - e^{-\lambda_2 l^*}) \\ Y_1 + Y_2 & (1 - e^{-\lambda_1 l^*})(1 - e^{-\lambda_2 l^*}) \end{cases}$$

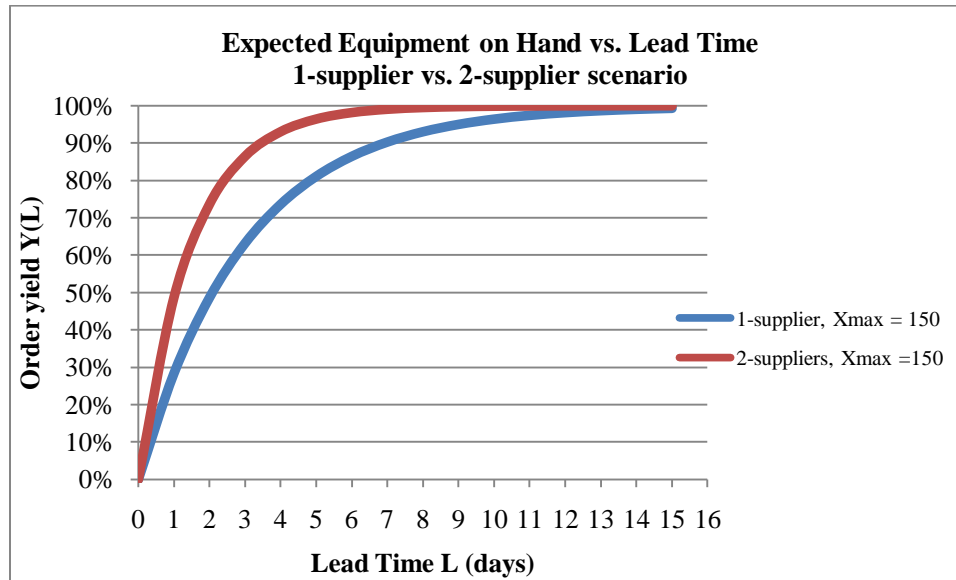
Using the distribution of  $Y(l^*)$ , we can compute the expectation:

$$E[Y(l^*)] = Y_1 * (1 - e^{-\lambda_1 l^*} (e^{-\lambda_2 l^*})) + Y_2 * (e^{-\lambda_1 l^*} (1 - e^{-\lambda_2 l^*})) + (Y_1 + Y_2) * (1 - e^{-\lambda_1 l^*})(1 - e^{-\lambda_2 l^*})$$

The variance is computed as:

$$\begin{aligned} Var[Y(l^*)] &= E[Y(l^*)^2] - [E[Y(l^*)]]^2 \\ &= Y_1^2 * (1 - e^{-\lambda_1 l^*} (e^{-\lambda_2 l^*})) + Y_2^2 * (e^{-\lambda_1 l^*} (1 - e^{-\lambda_2 l^*})) \\ &\quad + (Y_1 + Y_2)^2 (1 - e^{-\lambda_1 l^*})(1 - e^{-\lambda_2 l^*}) \\ &\quad - \left[ Y_1 * (1 - e^{-\lambda_1 l^*} (e^{-\lambda_2 l^*})) + Y_2 * (e^{-\lambda_1 l^*} (1 - e^{-\lambda_2 l^*})) + (Y_1 + Y_2) \right. \\ &\quad \left. * (1 - e^{-\lambda_1 l^*})(1 - e^{-\lambda_2 l^*}) \right]^2 \end{aligned}$$

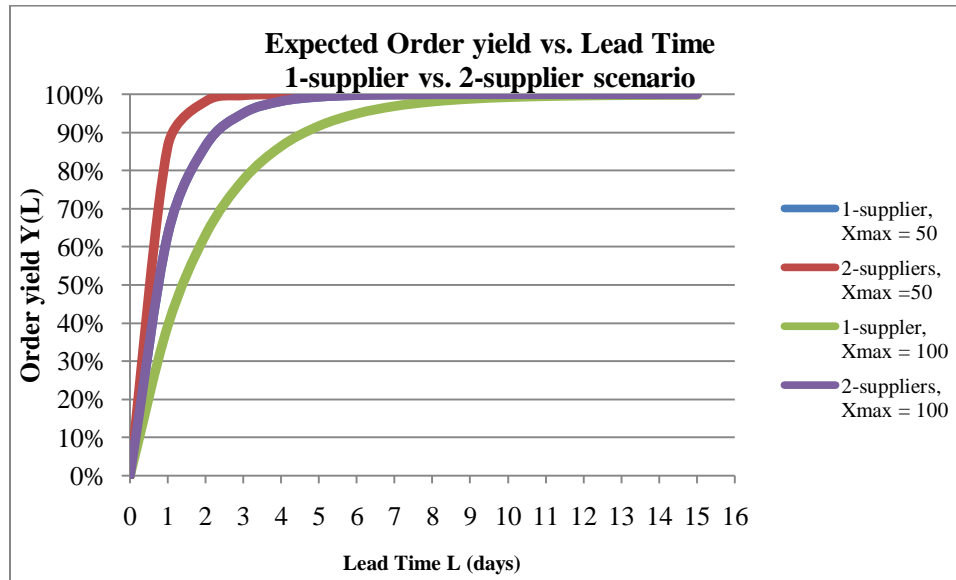
The chart below compares the order yield results for the 1-supplier vs. 2-supplier scenario discussed previously with parameters  $K_i=50$  and  $x_{iMax} = 150$ .



**Figure 5-4: Expected order yield computations under single and 2-supplier sourcing for an order of 150 units under exponential lead times**

The results show that a by splitting an order in two suppliers with similar characteristics, an expected yield of 93.1% can be obtained 4 days for an order of 150 units. On the other hand, it takes a lead time of 7 days to exceed 90% yield for an order of the same size when a single supplier is used. From the information in Table 5-4, this represents an increase in the projected readiness of approximately 30%.

The same results apply when looking order amounts studied previously. For  $x_{Max} = 50$  and  $x_{Max} = 100$ . For an order of 50, the expected order yield becomes 98% for a lead time of 2 days. For an order of 100, the expected order yield exceeds 90% within a lead time of 3 days.



**Figure 5-5: Expected yield computations under single and 2-supplier sourcing for orders of 50 and 100 units under exponential lead times**

Similarly, increasing the number of suppliers reduces the variance in the order yield for a required amount. Figure 5-6 displays the variance of the order yield levels under both single and dual sourcing policy. While for a single supplier the variance can go as high as 0.25 for a lead time of 2 days, the lead time when the order is distributed among 2 suppliers does not exceed 0.12. Under a single supplier policy, a decision maker would have to consider that an order takes 7 days to arrive in order to consider a variability of 0.10. The same value of 0.10 is achieved when the same amount  $x_{req}$  is distributed among 2 suppliers.

The same result holds when considering larger and lower order amounts; as the number of suppliers increase, the variance decreases. As shown in Figure 5-7, the variability under 2-supplier policy with similar rates does not exceed 0.13 for  $x_{req} = 50$  and  $x_{req} = 100$ . Note that as the amount ordered  $x_{req}$  is larger, the rate at which the variance decreases becomes smaller. Smaller variances imply there is more certainty that the amount received is closer to the expected value. More certainty in the amount received is an incentive for commanders to place orders that

are closer to the demand forecast, therefore making better use of space and reducing excess inventory.

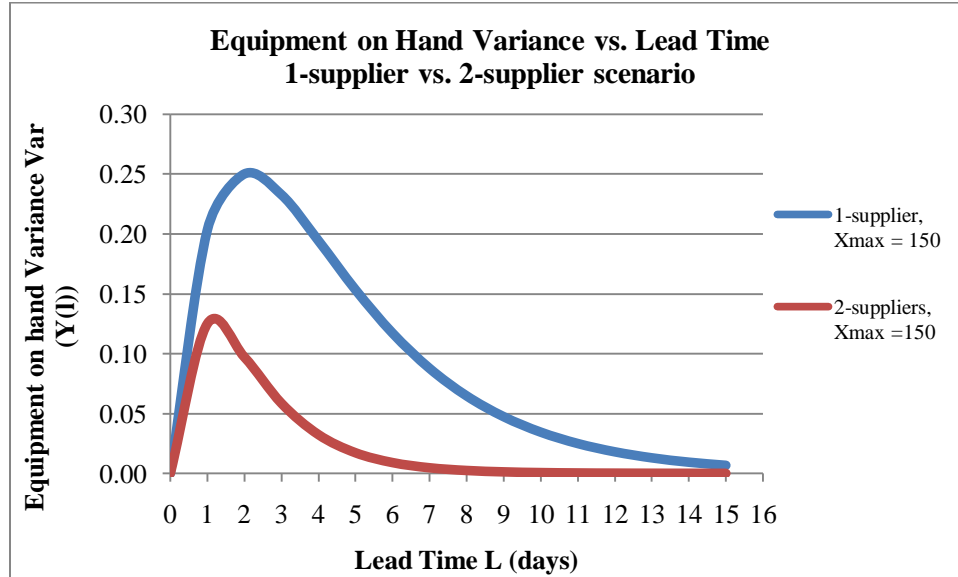


Figure 5-6: Order yield computations under single and 2-supplier sourcing for  $X_{req} = 150$  units under exponential lead times

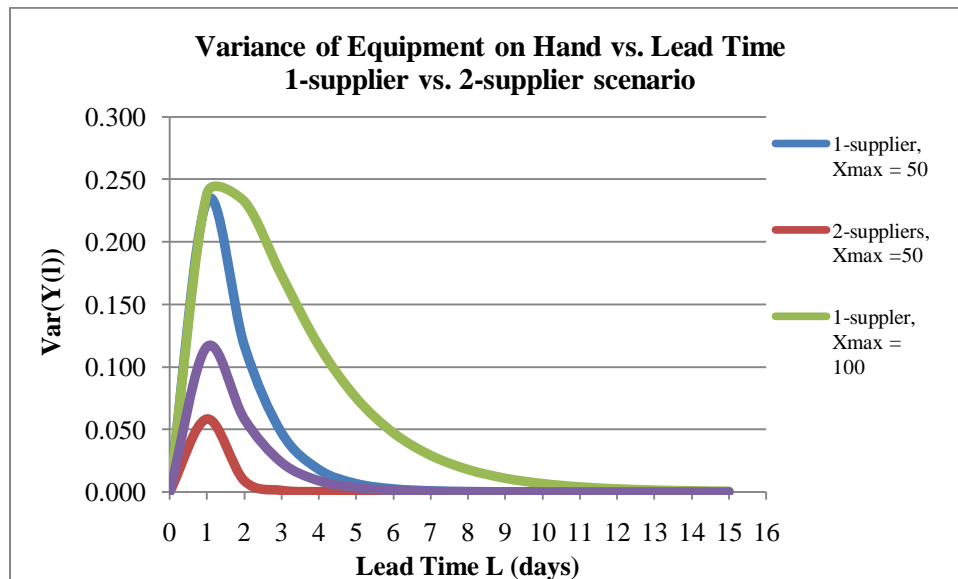


Figure 5-7: Variance of  $Y(l)$  under different order amounts vs. time

**Scenario #2: Order arrival rate dependent on the amount of commodity requested and order is distributed in different proportions among suppliers**

This scenario considers two suppliers A and B with same parameter  $k_i$ . Supplier A has 2/3 of the order of  $x_{req}$ , while supplier B has 1/3 of the order. The distribution of lead time for order arrival is shown in Table 5-5:

**Table 5-8: Lead time expectation when a single order is split equally in 2 suppliers**

Option	Order Amount	Rate $\lambda$	Average Lead Time (days)
S <sub>A</sub>	100	0.5	2 days
S <sub>B</sub>	50	1	1 day

The combined lead time  $L = \max(L_A, L_B)$  is the total time until the last order arrives. By conditioning on who takes longer to deliver the order (Ross, 2007), the expected lead time is computed as:

$$E(L) = E(L_A|L_B < L_A) p(L_B < L_A) + E(L_B|L_A < L_B) p(L_A < L_B)$$

$$E(L) = \frac{1}{\lambda_A} \left( \frac{\lambda_B}{\lambda_A + \lambda_B} \right) + \frac{1}{\lambda_B} \left( \frac{\lambda_A}{\lambda_A + \lambda_B} \right)$$

$$E(L) = \frac{1}{.5} \left( \frac{1}{.1+.5} \right) + \frac{1}{1} \left( \frac{.5}{.1+.5} \right) = 1.333 + .3333 = 1.667 \text{ days} \approx \frac{5}{3} \text{ days}$$

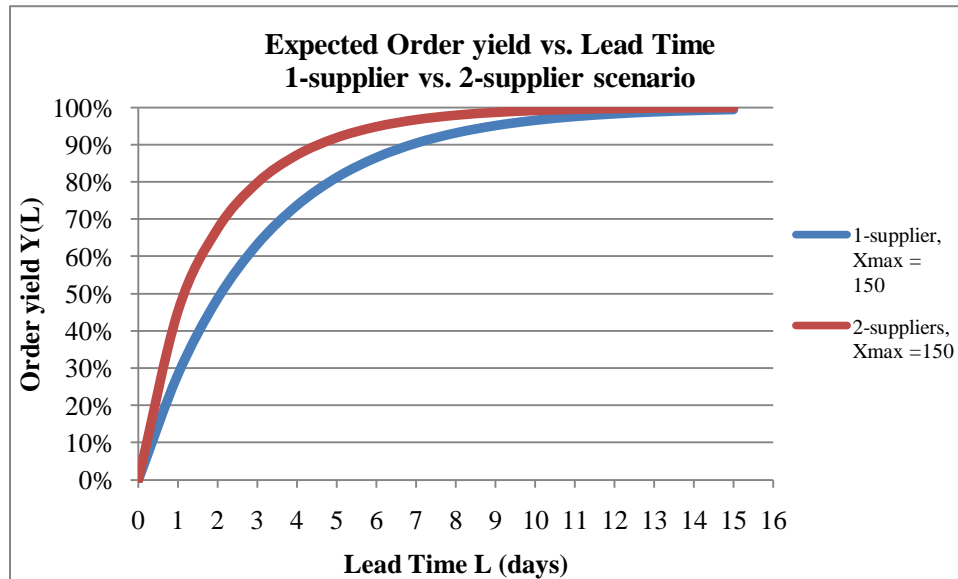
Therefore, the combined lead time has an expectation and standard deviation of 1.667 days when an order is split amongst two suppliers. This is approximately a 44% reduction in the mean lead time.

**Table 5-9: Expectation, variance and readiness results for single and dual sourcing under for Scenario #2**

Option	Order Amount	Rate $\lambda$	Average Lead Time (days)	Failure rate (days/pulse)	Sustainment Cost	Readiness
S <sub>A</sub>	150	0.333	3	0.10	30%	70%
S <sub>A</sub> + S <sub>B</sub>	75 for each supplier	0.600	1.667	0.10	16.7%	83.3%

Therefore, under exponentially distributed lead times dependent on the amount ordered, splitting the order in unequal amounts still reduces the expected lead time given the relationship between the expected lead time and the amount ordered. However, the solution is not optimal when compared to Scenario #1.

When comparing the Order yield levels, Scenario #2 leads to the same result as Scenario #1. Under dual sourcing, the expected order yield increases since the probability that at least one of the suppliers will be available increases. While under single sourcing it takes a lead time of 6 days to exceed 86%, it takes 4 days to exceed the same value under dual sourcing. However, the order yield level when the order is equally split among depots A and B exceeds 85% at a lead time of 3 days.



**Figure 5-8: Order yield Levels for single vs. dual sourcing for Scenario #2**

When considering different order sizes, note that for smaller orders, the expected order yield level is the highest when the order size is the smallest (50) and the number of suppliers is the largest (2). The results for variance also show that dual sourcing results in more certainty in

the amount of commodity received as illustrated in. Lower variances occur under dual sourcing; the lower the order amount, the lower the variance.

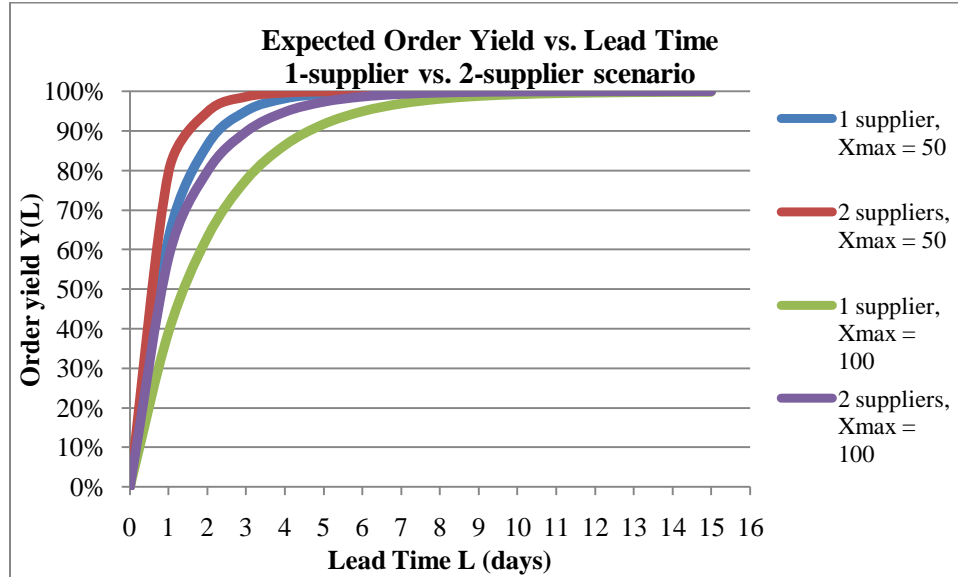


Figure 5-9: Expected order yield for single and dual sourcing for exponentially distributed lead times and uneven order amounts.

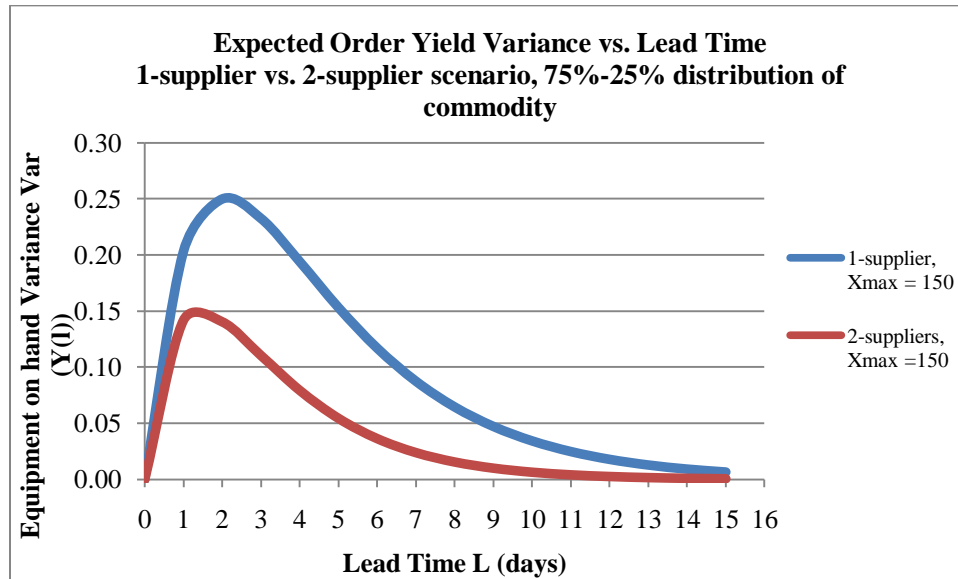
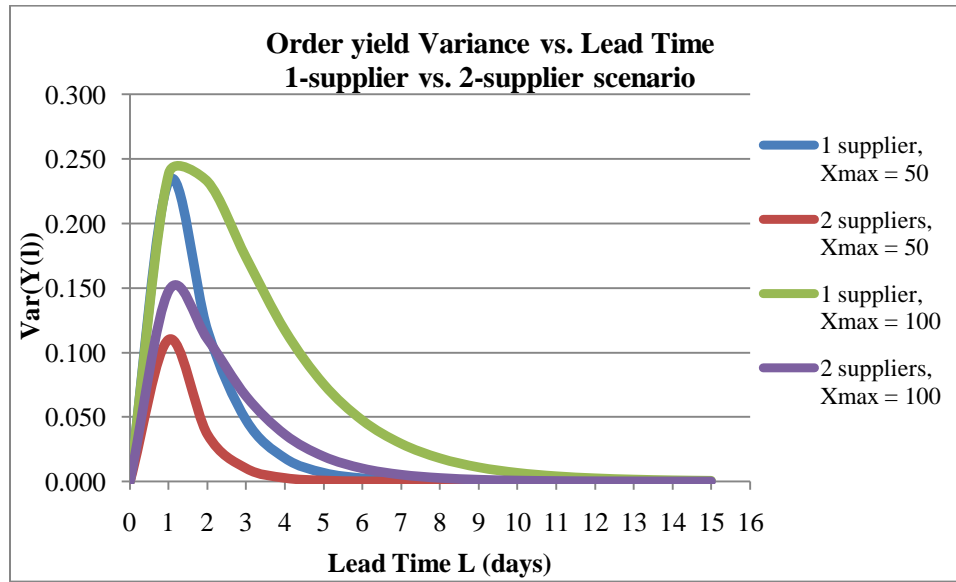


Figure 5-10: Variance for an order amount of  $x_{req} = 150$  under Scenario #2

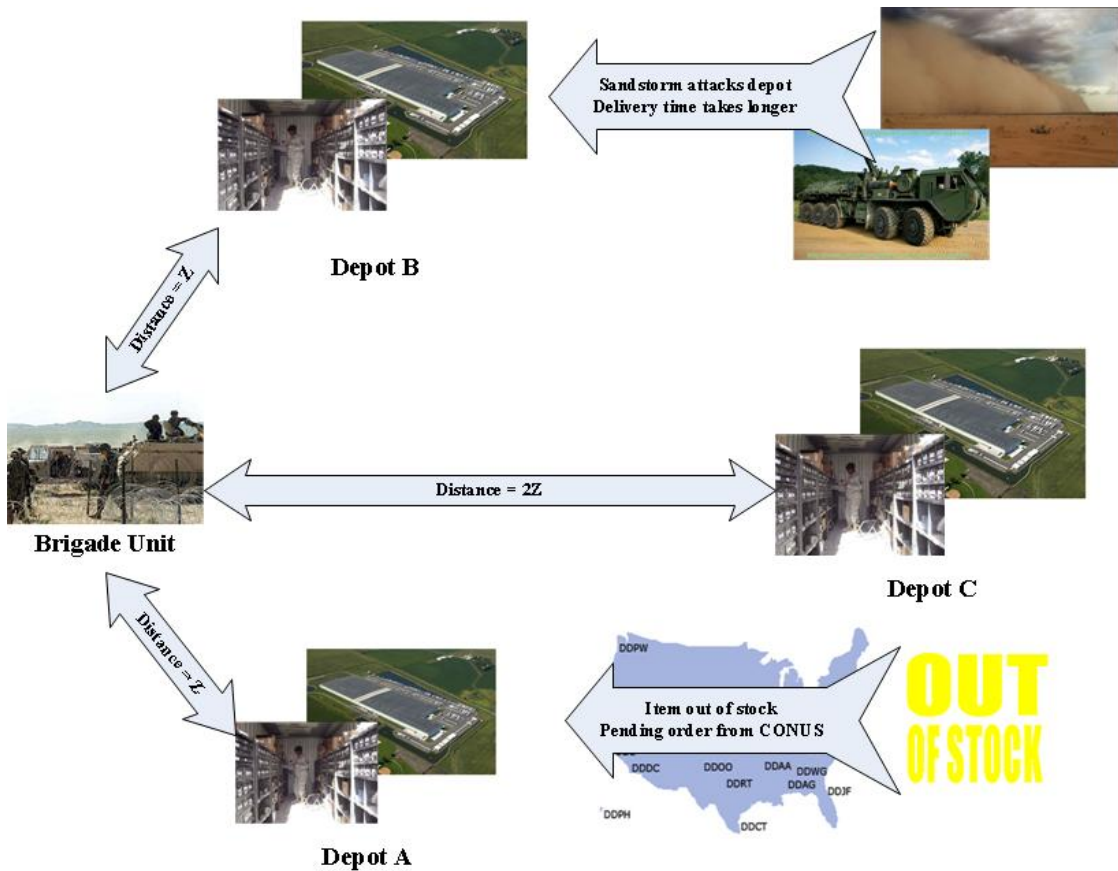


**Figure 5-11: Variance of Order Yield for orders of 50 and 100 under Scenario #2**

**Single sourcing vs. Dual Sourcing under exponentially distributed lead times dependent  $\lambda$  dependent historical data**

Scenarios #1 and #2 were analyzed under the assumption that lead times for order arrivals follow an exponential distribution with parameter  $\lambda$  dependent on the order amount. This analysis focuses in scenarios where the parameter  $\lambda$  for order arrival is independent of the order amount. For example, a brigade unit in combat operations at a foreign country may choose to order a specific commodity from any of three depots, namely A, B and C. Depot A and B are relatively close to the unit, while the distance of between depot C and the unit is twice as much as the distance between A or B and the unit. Depot A has high activity and has requested parts from the Continental US in the last few days; therefore at risk of running out of stock for the item requested. Supplier B is located in an area that is vulnerable to sandstorms. This makes it difficult for the depot to deliver supplies on time and results in medium lead time variability. C has not experienced any disruptions at the time.





**Figure 5-12: Single supplier with three depots with lead time distributions that are dependent on factors such as distance, shortages and disruptions**

In the example described and illustrated in Figure 5-12, the expected lead time depends on several factors, including distance, supplier risk (shortages) and disruption risk (environmental conditions, terrorist attacks, political issues and other). A way to express the rate  $\lambda$  for the lead time between a depot and a brigade unit is to relate it two parameters,  $k_i$  and  $\delta_i$ . Then the rate can be instantly computed based on the current and projections on the status of a depot relative to its location to the unit.

$$\lambda_i = \frac{k_i}{\delta_i}$$

***Single sourcing vs. dual sourcing and lead time variability***

Consider a single unit that is required to order 150 units of supply from A. The lead time  $L_A$  the depot to the current location of the brigade unit follows an exponential distribution with rate  $\lambda_A$ . The relationship between the average lead time  $\lambda_A$  to the amount ordered  $X_{iMax}$  is given by:

$$\lambda_i = \frac{K_i}{\delta_i}, K_i \text{ and } \delta_i \text{ constant, } i \in \{A\}$$

Using the properties of the exponential distribution, the expected value and variance of the lead time  $L_A$  is:

$$E[L_A] = \frac{1}{\lambda_A} = \frac{\delta_i}{K_A}$$

$$\text{Var}[L_A] = \frac{1}{\lambda_A^2}$$

The lead time distribution is computed for  $K_i = 50$  and  $\delta_i = 250$ ;  $\lambda = 1/5$ . Equipment readiness is computed as in Equation #1 from Chapter #4 with  $f = 1$  failure/system/15 day mission and a penalty  $\alpha = \lambda_A f = 0$ .

**Table 5-10: Lead time expectation, variance and readiness computations for a single supplier scenario**

Option	Order Amount	Rate $\lambda$	Average Lead Time (days)	Failure rate (days/pulse)	Sustainment Cost	Readiness
$S_A$	150	0.200	5	0.05	25%	75%

Note supplier A is risky as the mean lead time is 1/3 of the mission duration. Moreover, the projection on equipment readiness for this mission is lower than the required 90% in the Army. The next section shows how dual sourcing reduces the expected lead time and order yield.

**Scenario #3: Order arrival with distinct rates and equally distributed order amounts**

To improve equipment readiness, a decision maker could split an order of  $x_{req}$  among two suppliers, A and B with distinct characteristics, as shown below:

**Table 5-11: Lead time expectation when a single order is split equally in 2 suppliers**

Option	Order Amount	Rate $\lambda$	Average Lead Time (days)
$S_A$	75	0.20	5
$S_B$	75	.50	2

The combined lead time  $L = \max(L_A, L_B)$  is the total time until the last order arrives. As seen in Scenarios #1 and #2, expected lead time is computed as:

$$E(L) = E(L_A | L_B < L_A) p(L_B < L_A) + E(L_B | L_A < L_B) p(L_A < L_B)$$

$$E(L) = \frac{1}{\lambda_A} \left( \frac{\lambda_B}{\lambda_A + \lambda_B} \right) + \frac{1}{\lambda_B} \left( \frac{\lambda_A}{\lambda_A + \lambda_B} \right)$$

$$E(L) = \frac{1}{.50} \left( \frac{.20}{.20 + .50} \right) + \frac{1}{.200} \left( \frac{.50}{.20 + .50} \right) = 4.15 \text{ days}$$

Therefore, the combined lead time has an expectation and standard deviation of 4.15 days when an order is split amongst two suppliers. This is approximately a 17% reduction in the mean lead time. Note that the resulting lead time expectation tends to be closer to the maximum expected lead time; therefore, depots with lower values of  $\lambda$  dominate the solution when two suppliers are combined. The table below summarizes the computations for projected equipment readiness under the three scenarios.

**Table 5-12: Equipment readiness according to lead time distributions**

Option	Order Amount	Rate $\lambda$	Average Lead Time (days)	Failure rate (days/pulse)	Sustainment Cost (NMC)	ER
$S_A$	150	0.200	5	0.05	30%	70%
$S_B$	150	1	1	0.05	5%	95%
$S_A + S_B$	75 for each supplier	0.241	4.15	0.05	20.75%	79.25%

### Effects in average Order Yield

As in Scenarios #1 and #2, the expressions from Chapter 4 were used to compute the expectation and variance of the order yield for a single supplier. The graphs on 5-13 and 5-14 show the effect of expected lead time in order yield. As in previous results, depots with smaller lead times for order delivery result in greater values of order yield and lower deviations in the amount received. Note also that the expected yield for an order of amount  $x_{\text{req}} = 150$  is the same as for an order of amount  $x_{\text{req}}=50$  since the parameter  $\lambda$  is independent of the order amount

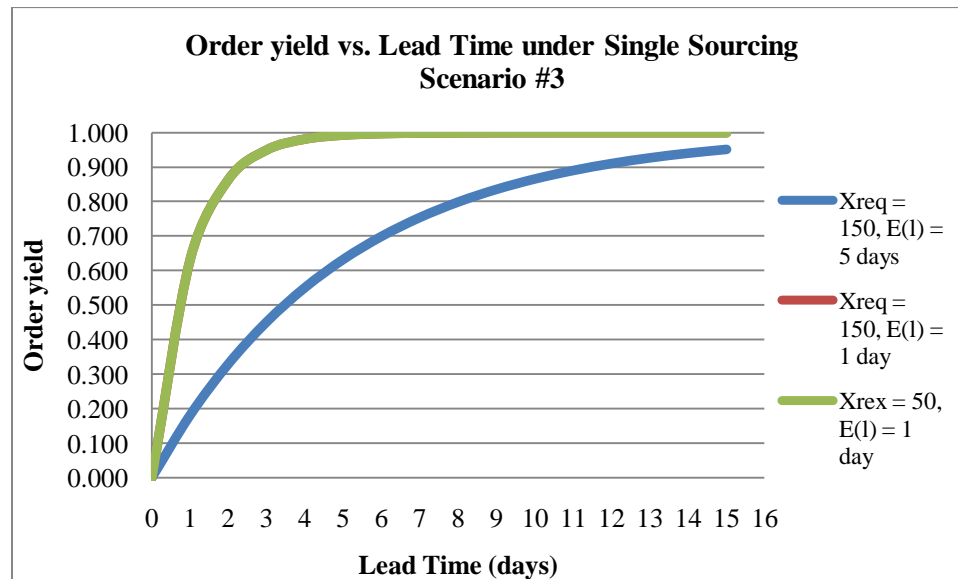
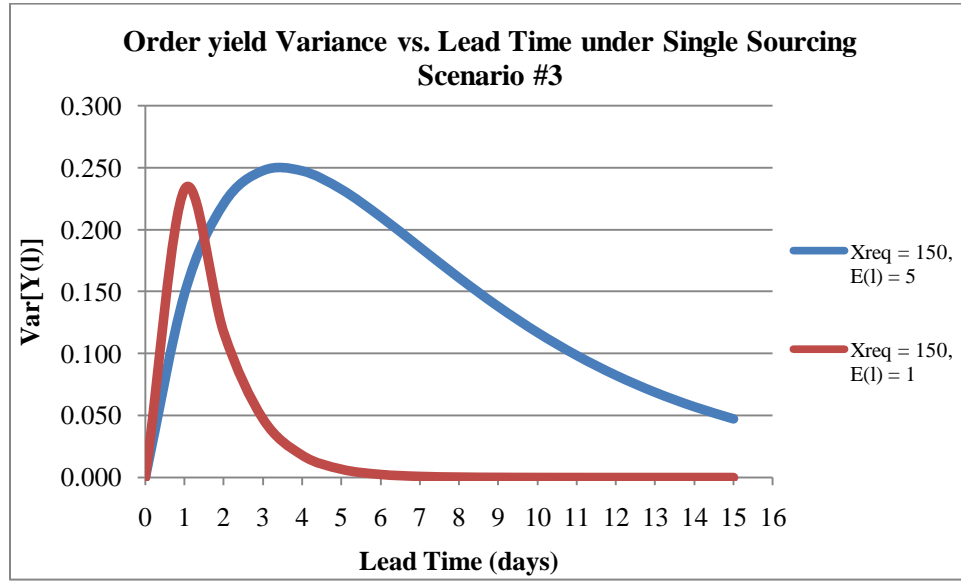


Figure 5-13: Order yield with respect to exponential lead time under single supplier scenario with rate dependent on amount ordered (Scenario #1)



**Figure 5-14 Variability of Order yield with respect to exponential lead time under single supplier (Scenario #3)**

Using the computations of expected yield in a 2-supplier scenario with the rates from Scenario #3 results in a distribution of  $Y(l^*)$  is:

$$Y(l^*) = \begin{cases} 0 & e^{-.2l^*} * e^{-1l^*} \\ 75 & 1 - e^{-.2l^*} (e^{-1l^*}) \\ 75 & e^{-.2l^*} (1 - e^{-1l^*}) \\ 150 & (1 - e^{-.2l^*})(1 - e^{-1l^*}) \end{cases}$$

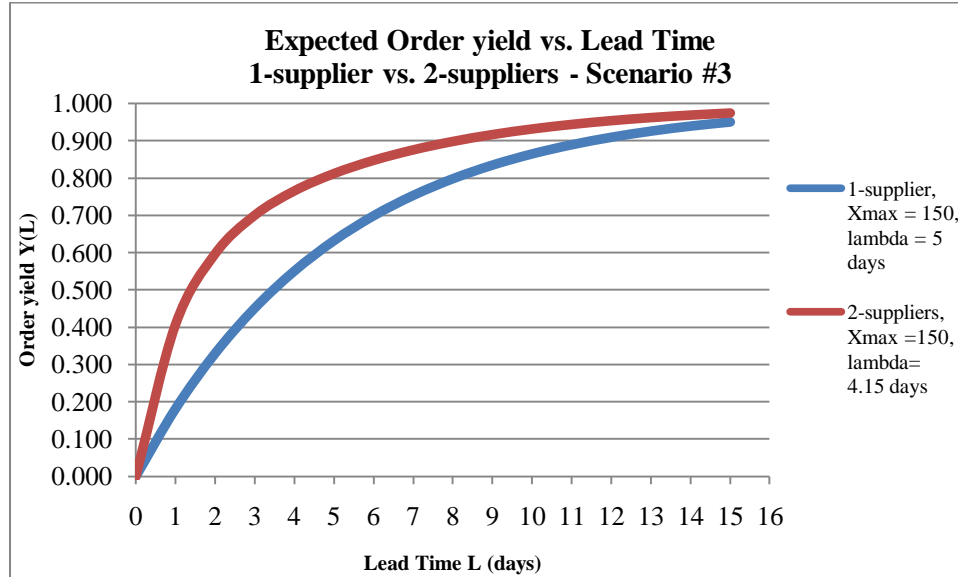
Using the distribution of  $Y(t^*)$ , we can compute the expectation:

$$E[Y(l^*)] = 75 * (1 - e^{-.2l^*} (e^{-1l^*})) + 75 * (e^{-.2l^*} (1 - e^{-1l^*})) + 150 * (1 - e^{-.2l^*})(1 - e^{-1l^*})$$

The variance is computed as:

$$\begin{aligned} Var[Y(l^*)] &= E[Y(l^*)^2] - [E[Y(l^*)]]^2 \\ &= 75^2 * (1 - e^{-.2l^*} (e^{-1l^*})) + 75^2 * (e^{-.2l^*} (1 - e^{-1l^*})) \\ &\quad + (150)^2 (1 - e^{-.2l^*})(1 - e^{-1l^*}) \\ &\quad - [75 * (1 - e^{-.2l^*} (e^{-1l^*})) + 75 * (e^{-.2l^*} (1 - e^{-1l^*})) + 150 \\ &\quad * (1 - e^{-.2l^*})(1 - e^{-1l^*})]^2 \end{aligned}$$

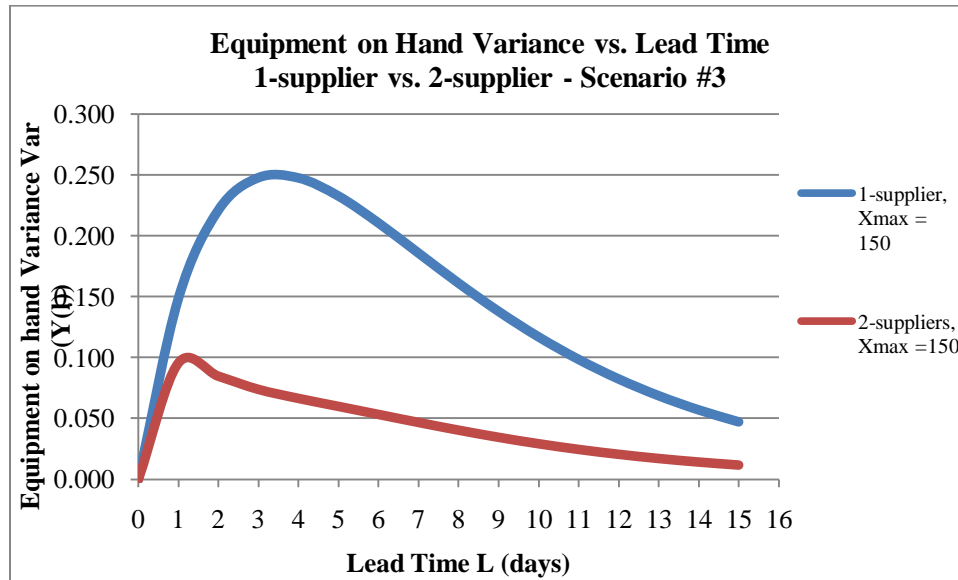
The chart below compares the order yield results for the 1-supplier vs. 2-supplier scenario discussed previously with parameters  $\lambda_A$  and  $\lambda_B$ .



**Figure 5-15: Expected order yield computations under single and 2-supplier sourcing for an order of 150 units under exponential lead times**

The results show that by splitting an order in two suppliers with similar characteristics, an expected yield of 90% can be obtained 8 days for an order of 150 units. On the other hand, it takes a lead time of 12 days to exceed 90% yield for an order of the same size when a single supplier is used with  $\lambda=2$ .

Similarly, increasing the number of suppliers reduces the variance in the order yield for a required amount. While for a single supplier the variance can go as high as 0.25 for a lead time of 2 days, the lead time when the order is distributed amongst 2 suppliers does not exceed 0.12. Under a single supplier policy, a decision maker would have to consider that an order takes 7 days to arrive in order to consider a variability of 0.10. The same value of 0.10 is achieved when the same amount  $x_{req}$  is distributed among 2 suppliers.



**Figure 5-16: Order yield computations under single and 2-supplier sourcing for  $X_{req} = 150$  units under exponential lead times**

The same result holds for other quantities; as the number of suppliers increase, the variance decreases. The variability under 2-supplier policy with similar rates does not exceed 0.13 for  $x_{req} = 50$  and  $x_{req} = 100$ . In addition, the results show that when the amount ordered  $x_{req}$  is larger, the rate at which the variance decreases becomes smaller. A low variance of order yield means that the forward unit can be more certain that the amount received will be closer to their estimates for a given lead time. More certainty in the amount of commodity received results in lower safety stock.

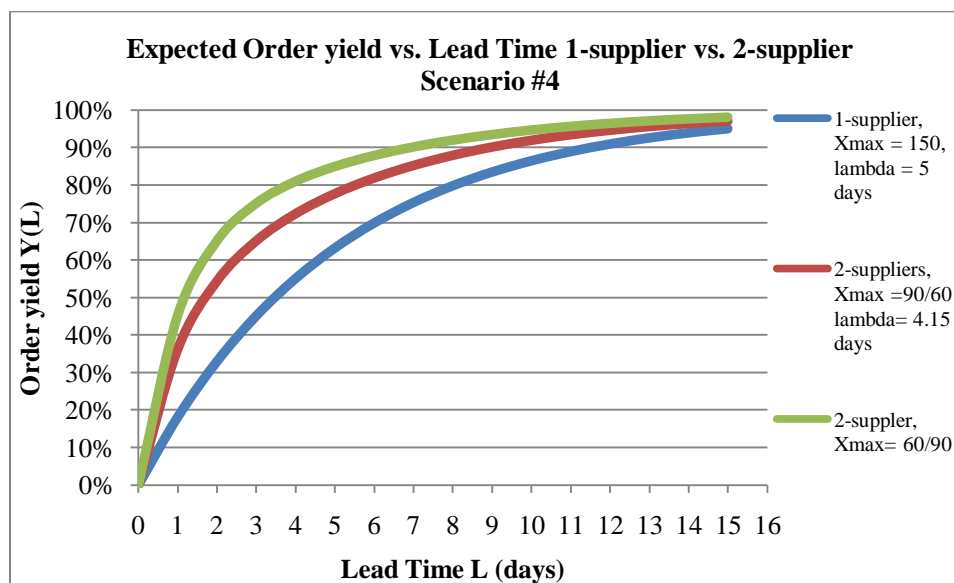
**Scenario #4: Order arrival with distinct rates  $\lambda$  dependent historical data and different order amounts**

A last scenario to compare single and dual sourcing considers an order of  $x_{req}$  to be split by assigning 60% of the order to a depot and the remaining 40% to the other while maintaining the same rated for lead time distribution as in Scenario #3. The information is summarized in Table 5-10.

**Table 5-13: Equipment readiness according to lead time distributions**

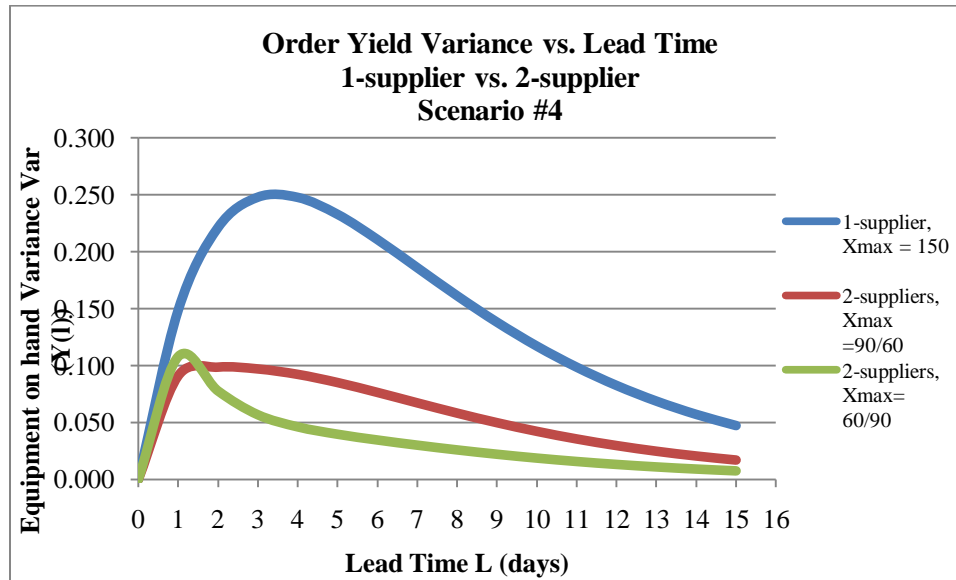
Option	Order Amount	Rate $\lambda$	Average Lead Time (days)	Failure rate (days/pulse)	ER	Readiness
$S_A$	150	0.200	5	0.05	30%	70%
$S_B$	150	1	1	0.05	5%	95%
$S_A + S_B$	90 for $S_A$ , 60 for $S_B$	0.241	4.15	0.05	20.75%	79.25%
$S_A + S_B$	60 for $S_A$ , 90 for $S_B$	0.241	4.15	0.05	20.75%	79.25%

As opposed to Scenario #2 where the lead time for order delivery of a supplier was dependent on the order amount, ordering different amounts from each supplier in Scenario #4 does not change equipment readiness. However, Figures 5-17 and 5-18 show that when an order is split among two suppliers, assigning the a greater portion of the order to the depot with the smallest expected lead time results in greater values of order yield and lower variance.



**Figure 5-17: Expected order yield - single sourcing vs. multiple sourcing for Scenario #4**





**Figure 5-18: Variance of order yield – Single sourcing vs. multiple sourcing, Scenario #3**

The summary of results in Table 5-11 show that when a brigade unit is has a primary supplier with high lead time for order delivery, increasing the number of suppliers yields better results; however, the expected lead time and order yield level is not better than the supplier with minimum lead time variability. In a dual sourcing policy, the best results are found when the larger portion of the order is requested from the supplier with the smallest lead time expectation. Under dual sourcing, the depot with the largest parameter  $\lambda$  influences the resulting lead time calculation.

**Table 5-14: Summary of results for Single vs. Dual Sourcing when the lead time distribution is independent of the order amount**

Option	Order Amount	Rate $\lambda$	E[L] (days)	Failure rate (days/pulse)	NMC	ER	E[Y(t)]	Var[Y(t)]
S <sub>A</sub>	150	0.200	5	0.05	30%	70%	.6692	.3536
S <sub>B</sub>	150	1	1	0.05	5%	95%	.9011	.0865
S <sub>A</sub> + S <sub>B</sub>	75 for S <sub>A</sub> , 75 for S <sub>B</sub>	0.241	4.15	0.05	20.75%	79.25%	.7852	.1898
S <sub>A</sub> + S <sub>B</sub>	90 for S <sub>A</sub> , 60 for S <sub>B</sub>	0.241	4.15	0.05	20.75%	79.25%	.7620	.2197
S <sub>A</sub> + S <sub>B</sub>	60 for S <sub>A</sub> , 90 for S <sub>B</sub>	0.241	4.15	0.05	20.75%	79.25%	.8084	.1620

## A model for multiple suppliers

### Description

The previous section explained how dual sourcing benefits units by improving the order yield and reducing lead time of order arrivals. This section extends this approach for multiple suppliers by providing a model to compute the expected yield and lead time in a multiple supplier scenario.

Let  $S$  be supplier set with  $n$  suppliers; let  $D \subseteq S$  be the suppliers that deliver commodities at time and  $N \subseteq S$  be the subset that fail to deliver their order due to disruptions or shortages. Then  $D \cup N = S$  and  $D \cap N = \emptyset$ . Let  $i$  denote index of the supplier from which commodities are received,  $i \in D \in \{1, 2, \dots, m\}$ ,  $m < n$ . Let  $x_i$  be the amount of commodity ordered from supplier  $i$  and the total demand for a unit is  $X_{\text{Max}}$ . Then  $L_i$  is an exponential random variable with parameter  $\lambda_i$  representing the lead time an order of amount  $x_{i\text{Max}}$  and  $y_i$  is the amount received from supplier  $i$ .

Assuming no partial orders are allowed,  $y_i = x_{i\text{Max}}$ . The yield  $Y(L_i)$  is the percentage of order delivered by supplier  $i$  at time  $t^*$ :

$$Y(L_i) = \begin{cases} \frac{y_i}{x_{\text{Max}}} & P(L_i \leq t^*) = 1 - e^{-\lambda_i t^*} \\ 0 & P(L_i > t^*) = e^{-\lambda_i t^*} \end{cases}$$

An expression to model the probability that  $n$  suppliers will deliver their order is:

$$p(N(t^*) = n) = \prod_{i \in D} 1 - e^{-\lambda_i t^*} \prod_{j \in N} e^{-\lambda_j t^*}$$

Then the number of order arrivals at a lead time  $t^*$  number of events  $N(t^*) = n$  depends on the subset  $D$  representing those suppliers capable of deliver their order of amount  $x_{i\text{Max}}$ .

A counting process is used to represent the total number of order arrivals by lead time  $l^*$  under multiple sourcing. The order yield  $Y(l)$  is modeled as a compound Poisson process where  $[N(l), l \geq 0]$  is a Poisson process, where  $[Y(l)_i, i \geq 1]$  is a family of independent and identically distributed random variables, also independent of  $[N(t), t \geq 0]$ . A complete description on Poisson Processes and Compound Poisson processes is discussed by Ross (Ross, 2007).

$$Y(l^*) = \sum_{\substack{i=1 \\ i \in D}}^{N(l^*)} Y_i(l^*), \quad 0 \leq Y(l^*) \leq l$$

In a realistic scenario where supplier and disruption risks are present and each supplier is either available or unavailable at a lead time  $l^*$ , the set  $S$  results generates  $2^n$  combinations of available/unavailable suppliers. Let  $k$  represent the index of a combination of suppliers in set  $S$ . For each  $k$  of the  $2^n$  combinations,  $Y_k(l^*)$  is computed as:

$$E[Y_k(l^*)] = \sum_{\substack{i=1 \\ i \in D}}^m Y_{ik}(l^*) = \sum_{\substack{i=1 \\ i \in D}}^m Y_{ik} * (1 - e^{-\lambda_i l^*})$$

The expected order yield  $Y(l^*)$  is computed for each time instant by

$$E[Y(l^*)] = \frac{\sum_{k=1}^{2^n} Y_k(l^*)}{2^n} \text{ for each value of } l^*$$

Therefore,  $E[Y(l^*)]$  represents the expected yield for all possible combinations of  $n$  suppliers over time. The variance  $\text{Var}[Y]$  is computed as:

$$\begin{aligned} \text{Var}[Y(l^*)] &= E[Y(l^*)^2] - [E[Y(l^*)]]^2 \\ &= \frac{\sum_{k=1}^{2^n} (y_k(l^*))^2 p(y_k(l^*))}{2^n} - \left( \frac{\sum_{k=1}^{2^n} Y_k(l^*)}{2^n} \right)^2 \end{aligned}$$

and depends on all possible combinations of available/unavailable suppliers  $2^n$ . To capture the expectation and standard deviation across all lead time  $l$ ,

$$E[Y] = E[Y(l^*)] = \sum_{i=0}^l \frac{\sum_{k=1}^{2^n} Y_k(l^*)}{2^n}$$

$$std[Y] = \sqrt{\sum_{i=0}^l \frac{\sum_{k=1}^{2^n} (y_k(l^*))^2 p(y_k(l^*))}{2^n} - \left( \frac{\sum_{k=1}^{2^n} Y_k(l^*)}{2^n} \right)^2}$$

The expected lead time in a multiple supplier scenario is  $E[L]$ ,  $L = \max\{L_{(1)}, L_{(2)}, \dots, L_{(m)}\}$ ,  $m = n$ . The expectation of  $L$  required conditioning on the rank ordering of  $L_i$ , hence:

$$E[\max L] = \frac{1}{\lambda} = \sum_{(i) \neq (j) \neq \dots \neq (n)} E[\max L \mid L_{(i)} < \dots < L_{(n)}] P(L_{(i)} < \dots < L_{(n)})$$

### Example #1

For a three supplier scenario, the expected lead time  $E[L]$  is:

$$E[\max\{L_1, L_2, L_3\}] = \frac{1}{\lambda} = \sum_{(i) \neq (j) \neq \dots \neq (n)} E[\max L \mid L_{(i)} < \dots < L_{(n)}] P(L_{(i)} < \dots < L_{(n)})$$

$$= \sum_{(i) \neq (j) \neq (k)} \frac{1}{\lambda_{(k)}} P(L_{(i)} < L_{(j)} < L_{(k)})$$

and

$$P(L_{(i)} < L_{(j)} < L_{(k)}) = \left( \frac{\lambda_{(i)}}{\lambda_{(i)} + \lambda_{(j)} + \lambda_{(k)}} \right) \left( \frac{\lambda_{(j)}}{\lambda_{(j)} + \lambda_{(k)}} \right)$$

$$\text{Var}[\max L] = 1/\lambda^2$$

Hence, the rate of  $L$  is given by

$$\lambda = \frac{1}{\sum_{(i) \neq (j) \neq \dots \neq (m)} E[\max L \mid L_{(i)} < \dots < L_{(m)}] P(L_{(i)} < \dots < L_{(m)})}$$

## Numerical Results

The advantages of multiple sourcing can be shown using numerical data. Scenarios #1 through #4 are extended for an  $n$  number of suppliers, where  $1 \leq n \leq 10$ . Each iteration of  $n$  generates a set  $S$  with  $2^n$  combinations of available/unavailable suppliers with same parameter  $\lambda$  from which the expected order yield  $E[Y]$ , the variance  $\text{Var}[Y]$  and lead time  $E[L]$  are computed.

### Scenario #1: Multiple sourcing, equally distributed $x_{\text{req}}$ and suppliers with same parameter $\lambda$ dependent on the amount of commodity ordered

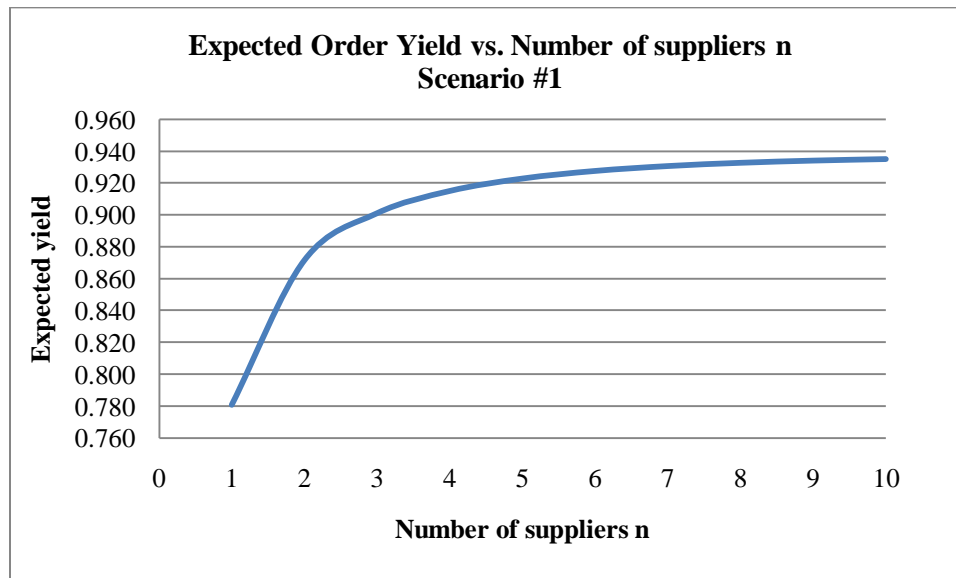
The simplest case considers multiple suppliers with same parameter  $\lambda$ . The results are shown below. The expected lead time  $E(L)$  equals the expected lead time for any single supplier since all suppliers have same parameter  $\lambda$ . Moreover, as the number of suppliers increases, the amount ordered decreases while the expected order yield increases and the variance of the order yield decreases.

**Table 5-15: Expected Order Yield, Variance of Order Yield and Expected Lead Time for  $n$  suppliers,  $n$  from 1 to 10,  $\lambda$  dependent on order amount for Scenario #1**

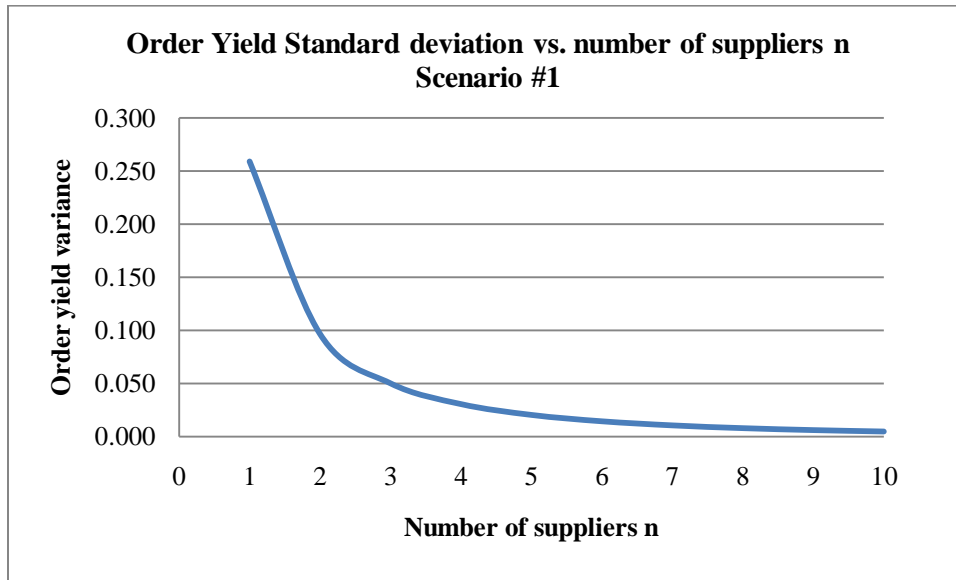
$n$	$E(Y)$	Std $E(Y)$	$E(L) = \text{Std } E(L)$
1	0.781	0.259	3.00
2	0.872	0.096	1.50
3	0.901	0.050	1.00
4	0.915	0.030	0.75
5	0.923	0.020	0.60
6	0.928	0.014	0.50
7	0.931	0.010	0.43
8	0.933	0.008	0.37
9	0.934	0.006	0.33
10	0.935	0.005	0.30

A graphical interpretation of Table 5-5 is provided in Figures 5-6 through 5-8. Figure 5-6 displays the Expected order on Hand vs. number of suppliers, while Figure 5-7 and 5-8 illustrate

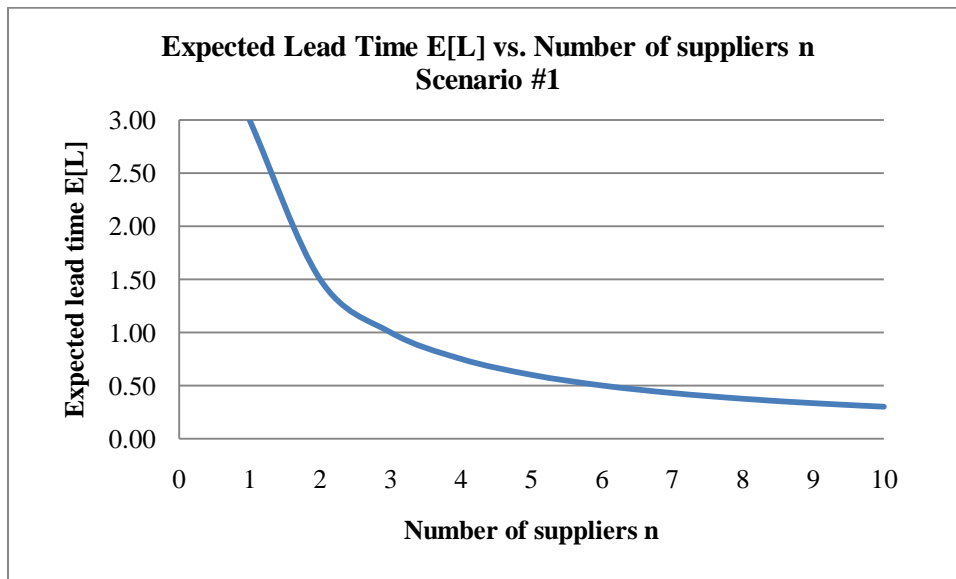
the standard deviation and Expected Lead time for the information. The percentage improvement in the expected order yield becomes less than 0.5% when the number of suppliers exceeds 6. Figure 5-7 displays the standard deviation vs. number of suppliers for an amount of 15 units. The improvement in the standard deviation becomes lower than 0.6 as the number of suppliers exceeds 6. Moreover, as the number of suppliers exceeds 6, the improvement in lead time is lower than 10%.



**Figure 5-19 Expected order yield  $E[Y(l)]$  vs. n, n from 1 to 10**



**Figure 5-20: Order yield Standard deviation  $\text{std}[Y(l)]$  vs. n, n from 1 to 10**



**Figure 5-21: Expected Lead time  $E[L]$  vs. n, n from 1 to 10**

**Scenario #2: Exponentially distributed lead times with rate  $\lambda$  dependent on amount ordered**  
 $x_{iMax}, \frac{2x_{req}}{n}$  units ordered from principal supplier  $\frac{x_{req}-2\frac{x_{req}}{n}}{n-1}$  units requested from the remaining  
to n-1 suppliers

After considering the scenario where the same amount of commodity is ordered from each supplier (hence the same distribution of lead time for each supplier) a second analysis is performed when an order of a larger amount is ordered from one supplier and the remaining portion is split equally amongst the remaining ones. For this part of the study, we split an order of  $x_{req}$  and amongst a number of suppliers  $i$  by assigning an amount of  $2\frac{x_{req}}{i}$  to one supplier and  $\frac{x_{req}-2\frac{x_{req}}{i}}{i-1}$  to the remaining  $i-1$  suppliers.

Table 5-13 summarizes the results for Scenario #2. The maximum expected order yield is found when  $n=10$  for a value of 93.3%. The minimum standard deviation is also found when  $n=10$ , for a value of 0.01.

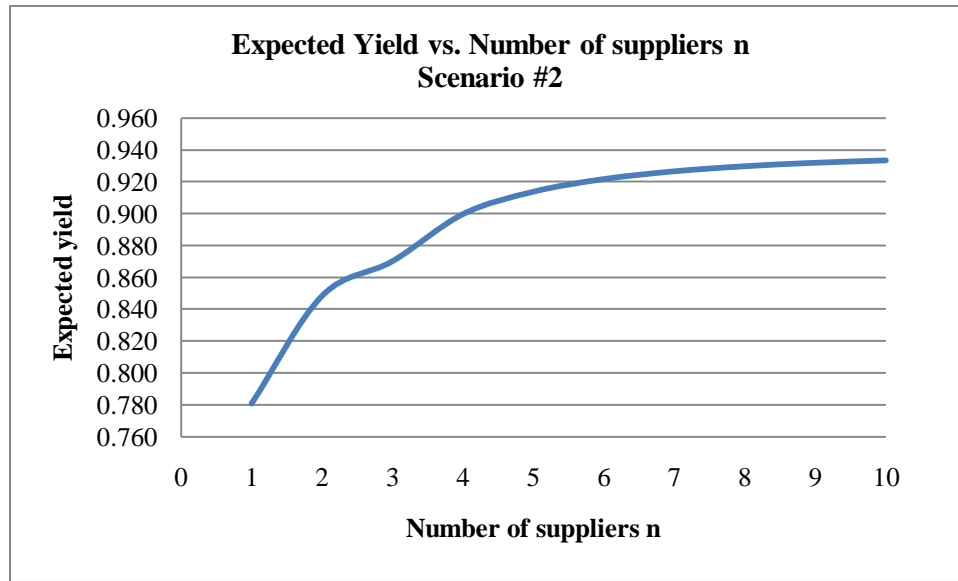
**Table 5-16: Results for Scenario #2: Expectation of order yield across time, standard deviation of order yield and variance of lead time for n depots**

n	E(Y)	Std(Y)	E(L) = var(L)
1	0.781	0.259	3
2	0.848	0.154	1.88
3	0.87	0.122	1.07
4	0.9	0.07	0.71
5	0.914	0.044	0.55
6	0.922	0.03	0.45
7	0.927	0.022	0.38
8	0.93	0.016	0.34
9	0.932	0.012	0.3
10	0.933	0.01	0.27

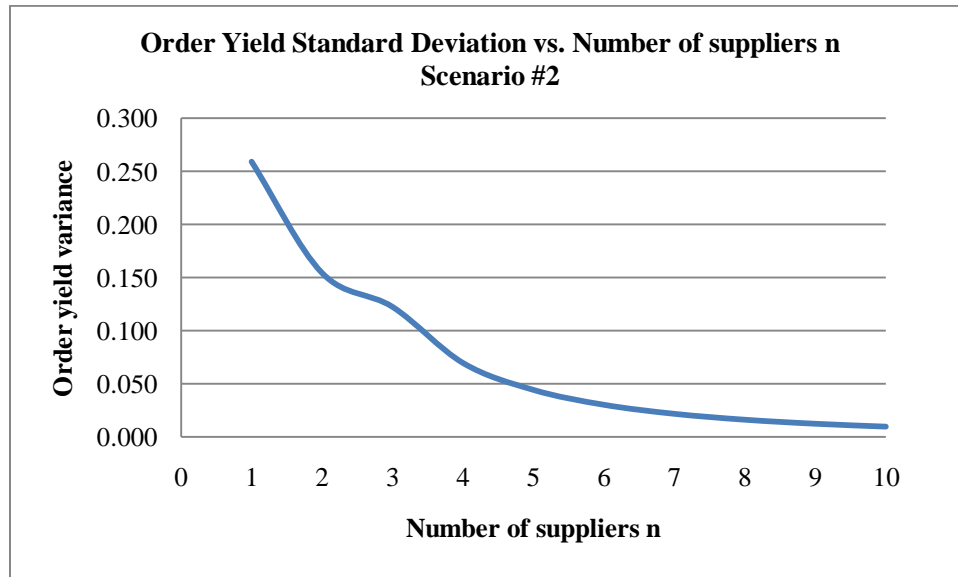
The graphical representations are illustrated in Figures 5-22 through 5-24. As it happens in Scenario #1, when the number of depots increases, the expected readiness improves and the variability in the amount of commodity received is reduced. When comparing the results of Scenario #1 against Scenario #2, the expected readiness and readiness, the values from Scenario



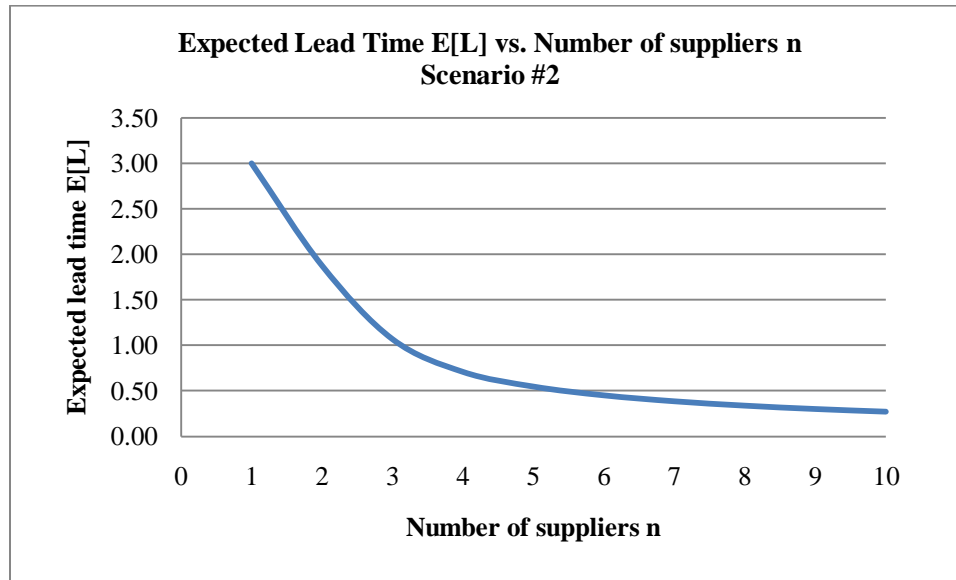
#2 are slightly lower than Scenario #1. However, the expected lead time in Scenario #2 is slightly lower than in Scenario #1. This is due to the fact the first supplier has twice as much amount of commodity ordered as the rest of the depots.



**Figure 5-22: Expected order yield vs. number of suppliers for exponentially distributed lead times and one supplier having twice as much commodity as any of the n-1 suppliers**



**Figure 5-23: Expected order yield vs. number of suppliers for exponentially distributed lead times and one supplier having twice as much commodity as any of the n-1 suppliers**



**Figure 5-24: Expected lead time vs. number of suppliers for exponentially distributed lead times and one supplier having twice as much commodity as any of the n-1 suppliers**

**Conclusions: Scenarios #1 and #2**

The results from Scenarios #1 and #2 show that increasing the number of suppliers reduces the overall expected lead time while increasing the expected order yield or Order Yield. The expected order yield is higher when an order of  $x_{req}$  is uniformly split among n suppliers. Moreover, splitting the order of  $x_{req}$  uniformly results in lower standard deviation.

**Table 5-17 Summary of results for Scenarios #1 and #2**

n	Scenario #1			Scenario #2		
	E(L) = var(L)	E(Y)	Std(Y)	E(L) = var(L)	E(Y)	Std(Y)
1	3.00	0.781	0.259	3	0.781	0.259
2	1.50	0.872	0.096	1.88	0.848	0.154
3	1.00	0.901	0.050	1.07	0.87	0.122
4	0.75	0.915	0.030	0.71	0.9	0.07
5	0.60	0.923	0.020	0.55	0.914	0.044
6	0.50	0.928	0.014	0.45	0.922	0.03
7	0.43	0.931	0.010	0.38	0.927	0.022
8	0.37	0.933	0.008	0.34	0.93	0.016
9	0.33	0.934	0.006	0.3	0.932	0.012
10	0.30	0.935	0.005	0.27	0.933	0.01

As seen previously, the expression describing the amount of commodity a forward unit receives is given by:

$$y(l^*) = \sum_{\substack{i=1 \\ i \in D}}^n E[y_i(l^*)] = \sum_{\substack{i=1 \\ i \in D}}^n x_i * (1 - e^{-\lambda_i l^*}) = \sum_{\substack{i=1 \\ i \in D}}^n x_i * (1 - e^{-\frac{k_i}{x_{iMax}} l^*})$$

As the number of suppliers  $n \rightarrow \infty$ , the amount to order from each supplier is reduced; hence

$x_{iMax} \rightarrow 0$ . As this occurs,  $-\frac{k_i}{x_{iMax}} l^* \rightarrow -\infty$  and  $e^{-\frac{k_i}{x_{iMax}} l^*} \rightarrow 0$ . This results in:

$$\lim_{m \rightarrow \infty} \sum_{i=1}^m x_{iMax} \left(1 - e^{-\frac{k_i}{x_{iMax}} l^*}\right) \approx \sum_{i=1}^m x_{iMax}$$

Then, using the definition of order yield, as  $n \rightarrow \infty$ ,

$$Y(l^*) = \frac{y(l^*)}{X_{req}} = \frac{\sum_{i=1}^m x_{iMax}}{X_{req}} \approx 100\%$$

Hence, we can conclude that the readiness of a forward unit increases with the number of suppliers capable of delivering orders at a time  $t^*$ .

### **Scenario #3: Multiple sourcing with equally distributed $x_{req}$ and suppliers with distinct parameter $\lambda$ dependent on historical information**

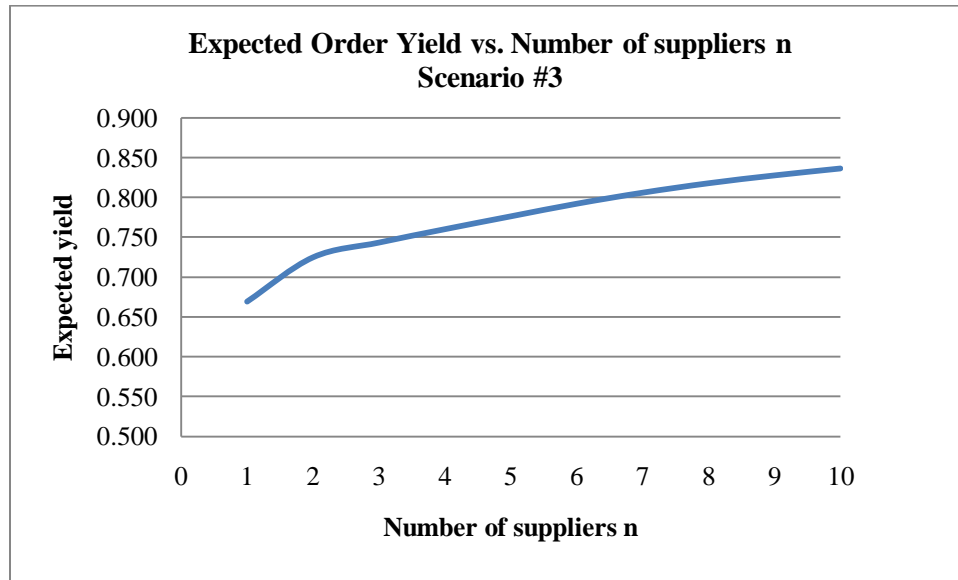
Table 5-15 displays randomly generated values of order lead time delivery in order to study the effect of multiple sourcing on depots with independent exponential lead time distributions. These randomly generated suppliers are sorted in descending order of expected lead time prior to making any computations to study how the lead time decreases with every iteration a supplier with lower expected lead time is included. The results on mean order yield over time, standard deviation of order yield over time and expectation of lead time  $E[L]$  for  $n$  suppliers are shown in Table 5-16 and plotted in Figures 5-25 through 5-27.

**Table 5-18: Sample supplier data for scenario #3**

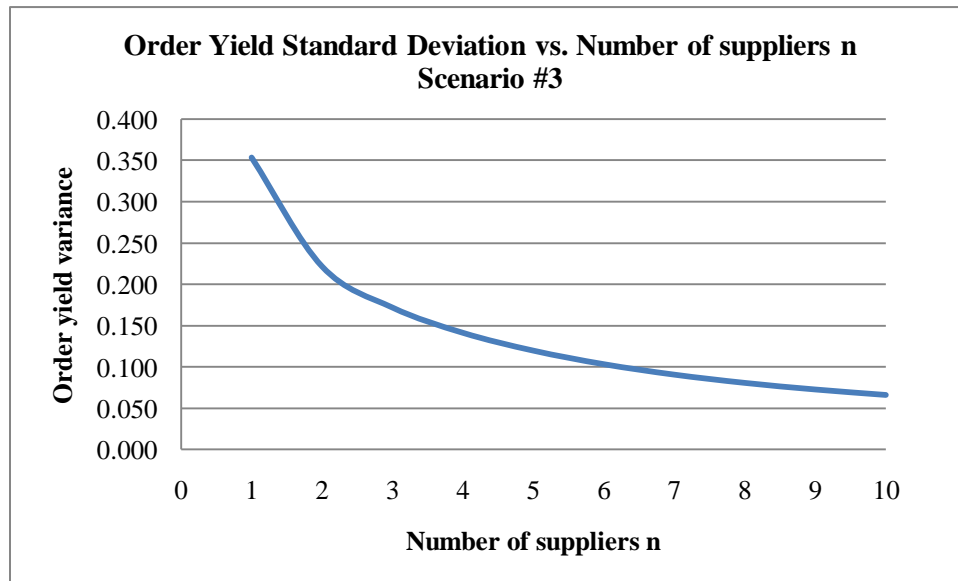
Supplier	$k_i$	$\delta_i$	$\lambda_i$	$1/\lambda_i$
1	10	30	0.200	5
2	10	25	0.333	3
3	10	10	0.333	3
4	100	150	0.400	2.5
5	100	300	0.500	2
6	100	75	0.667	1.5
7	500	450	0.833	1.2
8	500	600	1.000	1
9	500	1000	1.111	0.9
10	500	2500	1.333	0.75

**Table 5-19: Results for Scenario #3: Expectation of order yield across time, standard deviation of order yield and variance of lead time for n depots**

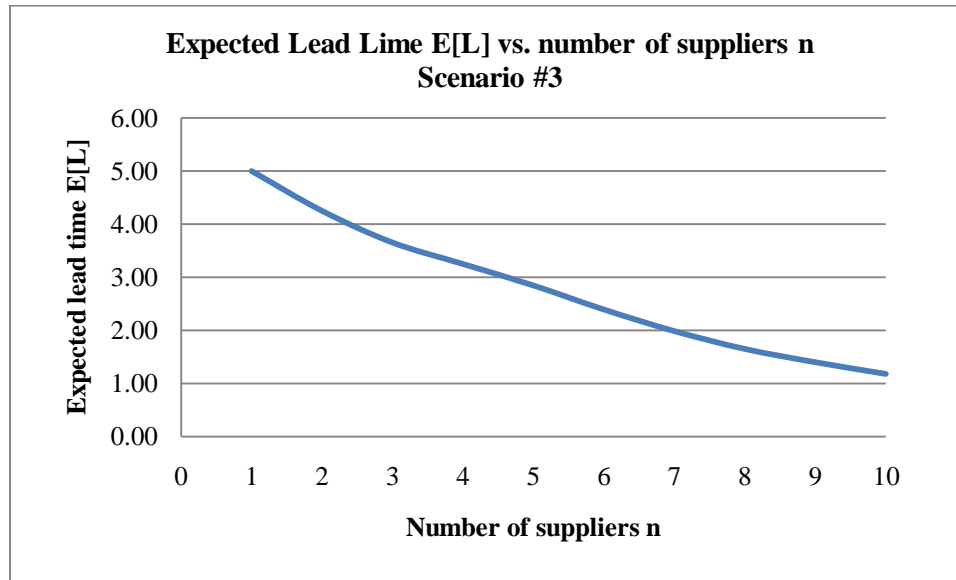
n	E(Y)	Std(Y)	E(L) = var(L)
1	0.669	0.354	5.00
2	0.725	0.222	4.25
3	0.743	0.172	3.65
4	0.76	0.141	3.25
5	0.776	0.12	2.85
6	0.792	0.103	2.39
7	0.806	0.091	1.99
8	0.818	0.081	1.65
9	0.828	0.073	1.40
10	0.837	0.066	1.18



**Figure 5-25: Expected order yield vs. number of suppliers for independent exponentially distributed lead times and equal amounts ordered from each supplier**



**Figure 5-26: Standard deviation across time of order yield vs. number of suppliers for independent exponentially distributed lead times and equal amounts ordered from each supplier**



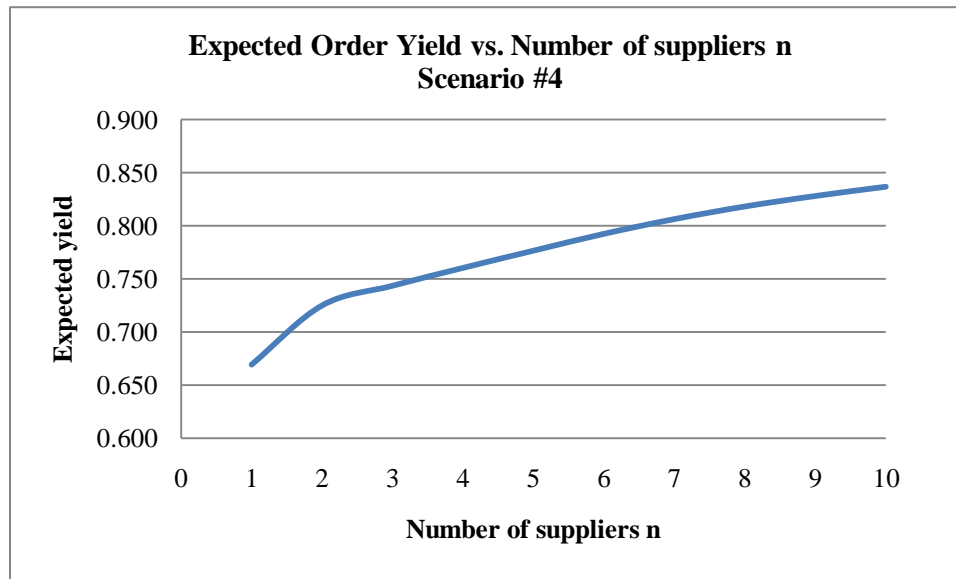
**Figure 5-27: Expected lead time vs. number of suppliers for independent exponentially distributed lead times and equal amounts ordered from each supplier**

**Scenario #4: Independent exponentially distributed lead times,  $\frac{2x_{req}}{n}$  units ordered from principal supplier  $\frac{x_{req}-2\frac{x_{req}}{n}}{n-1}$  units requested from the remaining to n-1 suppliers**

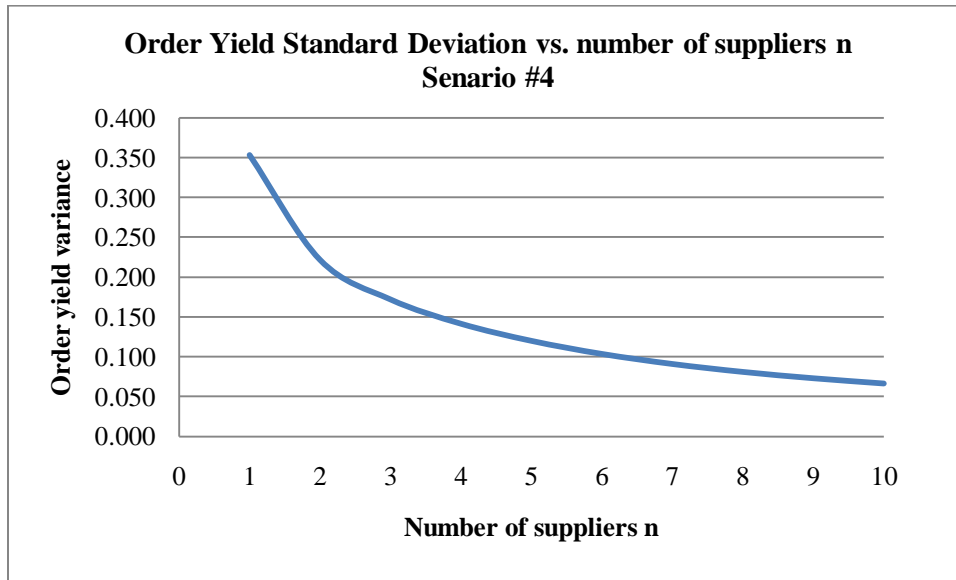
Scenario #4 uses the data values from Scenario #3 to study the effect of multiple sourcing on depots with independent exponential lead time distributions when one of the suppliers is assigned more commodity than the others. Similar to Scenario #2, each iteration of  $n$  in  $1 \leq n \leq 10$ , an order of  $x_{req}$  is split between of suppliers  $i$  by assigning to the principal supplier (Supplier 1) an amount of  $2\frac{x_{req}}{i}$  and  $\frac{x_{req}-2\frac{x_{req}}{i}}{i-1}$  to the remaining  $i-1$  suppliers. The results on mean order yield over time, standard deviation of order yield over time and expectation of lead time  $E[L]$  for  $n$  suppliers are shown in Table 5-17 and plotted in Figures 5-28 through 5-30.

**Table 5-20: Mean order yield, Standard Deviation and Expected lead time for n number of suppliers,  $1 \leq n \leq 10$  with independent exponentially distributed lead time**

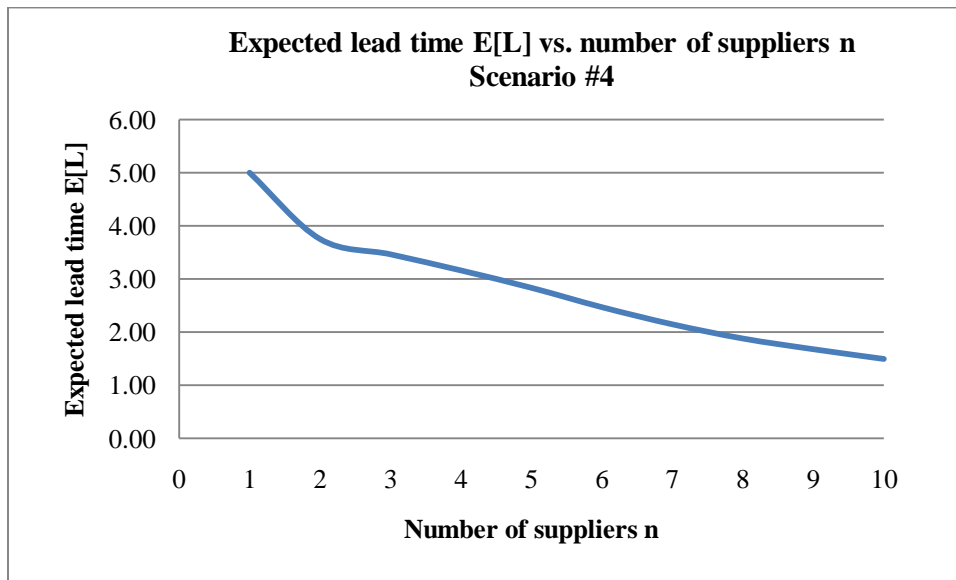
n	E(Y)	Std(Y)	E(L) = var(L)
1	0.669	0.354	5.00
2	0.697	0.274	4.25
3	0.759	0.236	3.65
4	0.803	0.177	3.25
5	0.830	0.141	2.85
6	0.848	0.118	2.39
7	0.861	0.101	1.99
8	0.870	0.088	1.65
9	0.878	0.079	1.40
10	0.884	0.071	1.18



**Figure 5-28: Expected order yield vs. number of suppliers for independent exponentially distributed lead times and unequal amounts ordered from each supplier**



**Figure 5-29 Order yield standard deviation across time vs. number of suppliers for independent exponentially distributed lead times and unequal amounts ordered from each supplier**



**Figure 5-30: Expected lead time vs. number of suppliers for independent exponentially distributed lead times and unequal amounts ordered from each supplier**



**Conclusions: Scenarios #3 and #4**

The results from Scenarios #3 and #4 are consistent the previous two and summarized in. Brigade units can reduce the risk of delays and cancellations due to supplier and disruption risk by incorporating multiple suppliers. This results in greater expected order yield at any time  $t$  as well as an increased value of equipment readiness due to smaller expected lead times for order arrival. As in previous results, the mean value of the expected order yield over time is higher when an order of  $x_{req}$  is uniformly split among  $n$  suppliers. However, when the rate  $\lambda$  of the lead time distribution is independent of the amount requested  $x_{iMax}$ , ordering a larger portion from the principal supplier or the supplier with maximum lead time distribution results in lower standard deviation when compared to a scenario where the orders are uniform. Therefore, if a unit seeks to get more certainty in both the expected lead time and the deviation of expected order on hand, then it would be beneficial to order a larger portion from the primary supplier rather than ordering the same amount from all suppliers. If the unit’s primary goal is to maximize expected order yield and reduce the expected lead time, then the order can be split equally among  $n$  suppliers.

**Table 5-21 Summary of results for Scenarios #3 and #4**

n	E(L) = var(L)	Scenario #3		Scenario #4	
		E(Y) (Scenario	Std(Y)	E(Y)	Std(Y)
1	5.00	0.669	0.354	0.669	0.354
2	4.25	0.697	0.274	0.725	0.222
3	3.65	0.759	0.236	0.743	0.172
4	3.25	0.803	0.177	0.76	0.141
5	2.85	0.830	0.141	0.776	0.12
6	2.39	0.848	0.118	0.792	0.103
7	1.99	0.861	0.101	0.806	0.091
8	1.65	0.870	0.088	0.818	0.081
9	1.40	0.878	0.079	0.828	0.073
10	1.18	0.884	0.071	0.837	0.066

Scenarios #3 and #4 provide a more general approach to measure the risk of delays and supplier disruptions when in combat operations. Assuming there are  $n$  suppliers with same

parameter  $\lambda$ , since the probability  $p(Y = 0) = (e^{-\lambda t})^n$ . As  $n \rightarrow \infty$ ,  $p(Y = 0) \rightarrow 0$  and the  $p(Y > 0) \rightarrow 1$ ; therefore,  $E[Y] \rightarrow 1$  for  $n \gg N$ . Moreover, if the rate at which orders are received is dependent on  $x_{iMax}$ , then as  $n \rightarrow \infty$ ,  $x_{iMax} \rightarrow 0$ ; therefore  $e^{-\frac{k_i}{x_{iMax}} t^*} \rightarrow 0$ .

### **Identifying the optimal number of suppliers**

An interesting yet relevant question in multiple supplier selection models is determining the optimal number of suppliers. This section proposes a model that compares the percentage of expected shortage and the percentage reduction in expected lead time as the number increases against the percentage increase in purchasing cost for each scenario under study.

As an assumption, the purchasing cost increases linearly as the number of depots increases. In the absence of data regarding the purchasing costs for spare parts at the brigade unit's stage of the supply chain, an order cost of \$13.26 (RAND Corporation, 2004) and a mean unit cost of \$1.10 are assumed.

Using the previous computations on the mean order yield over time and the expected lead time for  $n$  depots,  $1 \leq n \leq 10$ , the percentage of expected shortage and the percentage improvement in expected lead time are computed as:

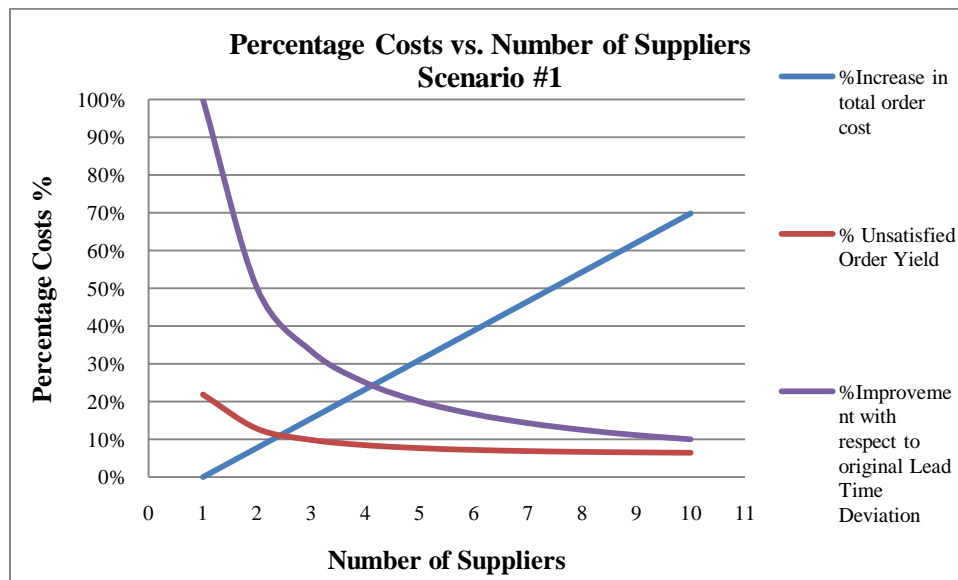
$$\% \text{ of expected shortage} = 1 - E[Y]$$

$$\% \text{ improvement in expected lead time} = \frac{E[L(n)]}{E[L(n=1)]}$$

Where  $E[L(n)]$  is the expected lead time for  $n$  number of suppliers and  $E[L(n=1)]$  is the expected lead time of the primary supplier.

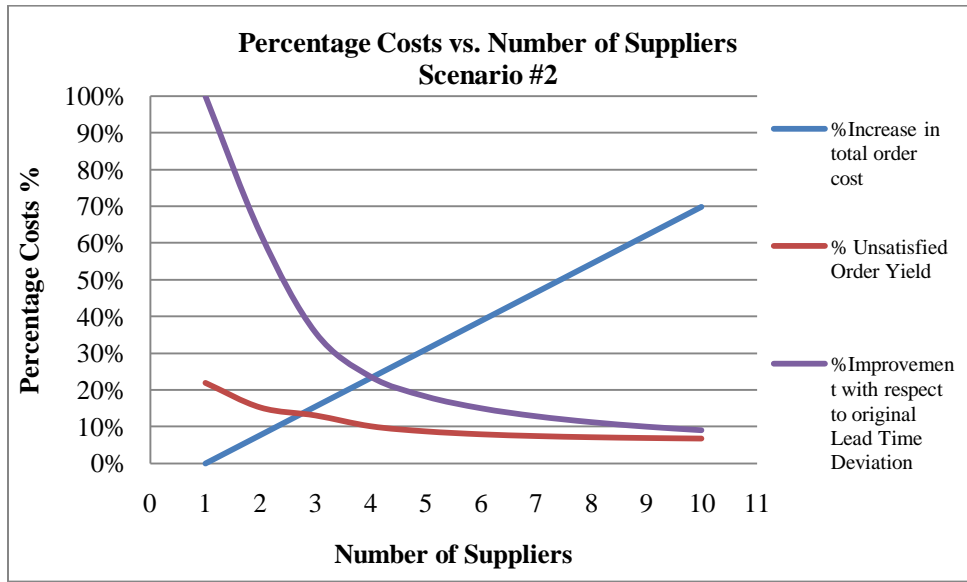
Figures 5-31 to 5-34 show the plots for all scenarios. The results show that there is no common point between the percentage increase in total cost, the percentage of unsatisfied yield and the percentage improvement in lead time deviation. For all scenarios the unsatisfied yield

curve and the cost line intersect at some point between  $n=2$  suppliers and  $n=3$  suppliers. For Scenarios #1 and #2, the intersection between the purchasing cost line and the lead time deviation occurs approximately at  $n=4$ . However, for Scenarios #3 and #4, the lines intersect at a point between  $n=6$  and  $n=7$ . The slight deviation in the Unsatisfied between  $n=2$  and  $n=3$  occurs due to the fact the lead times are in descending order. When  $n=2$ , the expected lead time is computed for 2 suppliers with  $E[L] = \{5 \text{ days}, 3 \text{ days}\}$ , whereas when  $n=3$ ,  $E[L]$  is computed for 3 suppliers with lead times  $\{5 \text{ days}, 3 \text{ days}, 3 \text{ days}\}$ .

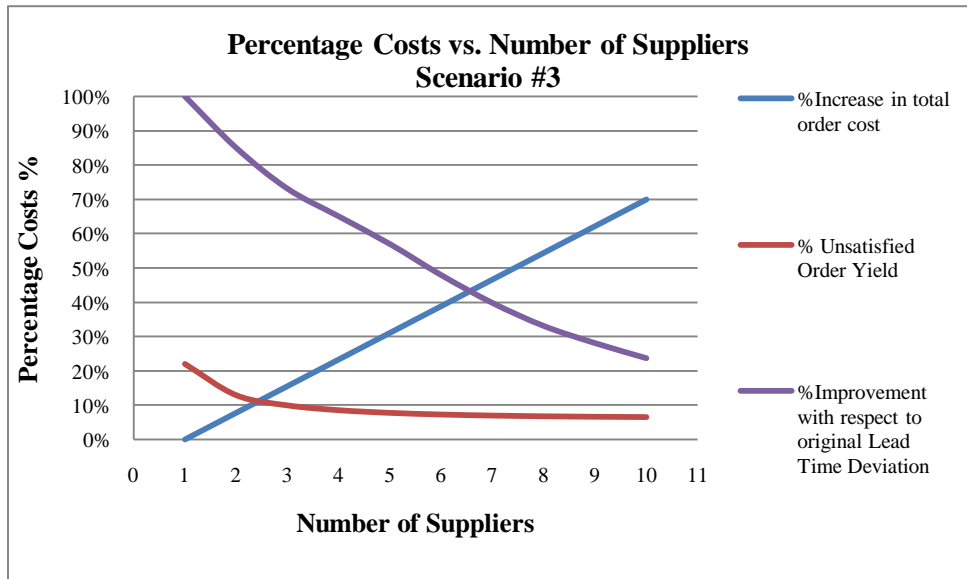


**Figure 5-31: Optimal Number of Suppliers - Scenario #1**

Figure 5-35 shows the curves of unsatisfied order yield for all Scenarios. Note that the slope of the curve starts to decrease as the number of suppliers goes from  $n=2$  to  $n=3$ . Figure 5-36 shows that for Scenarios #3 and #4, the expected lead time appears to have linear behavior.



**Figure 5-32: Optimal Number of Suppliers - Scenario #2**



**Figure 5-33: Optimal Number of Suppliers - Scenario #3**

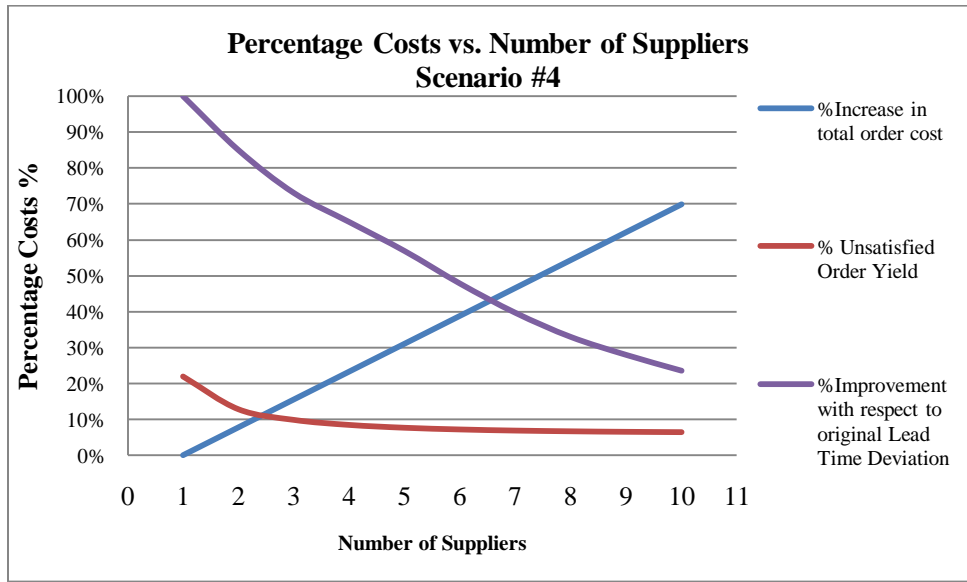


Figure 5-34: Optimal Number of Suppliers - Scenario #4

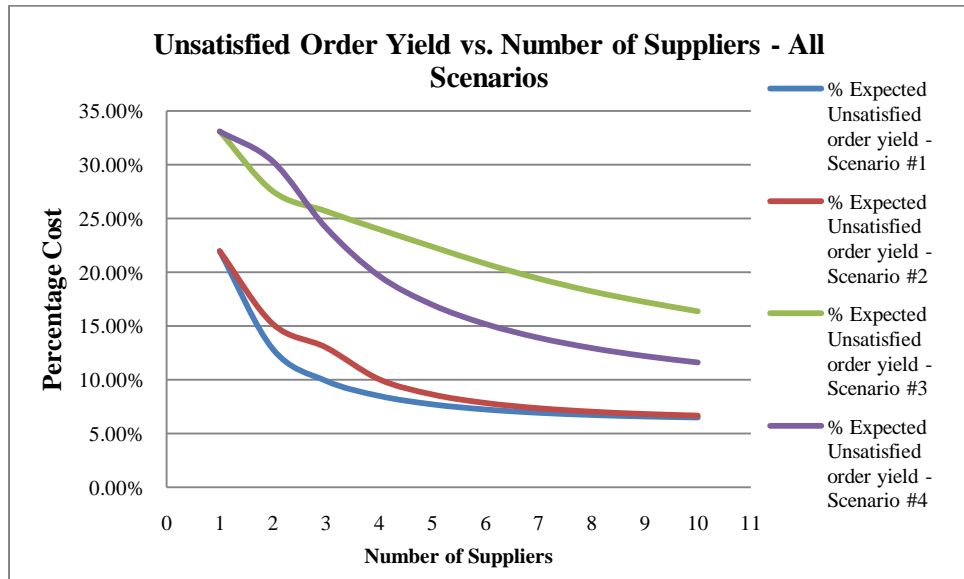
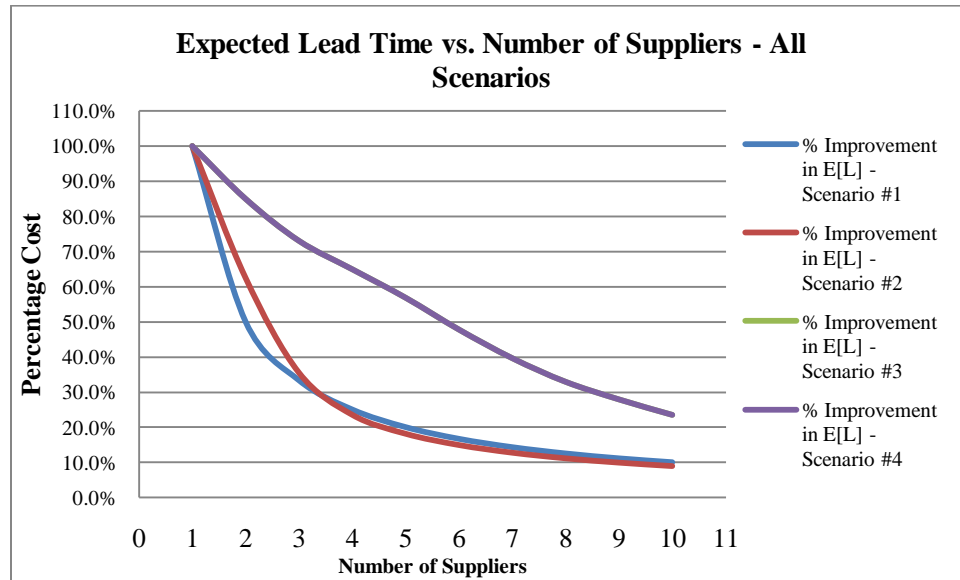


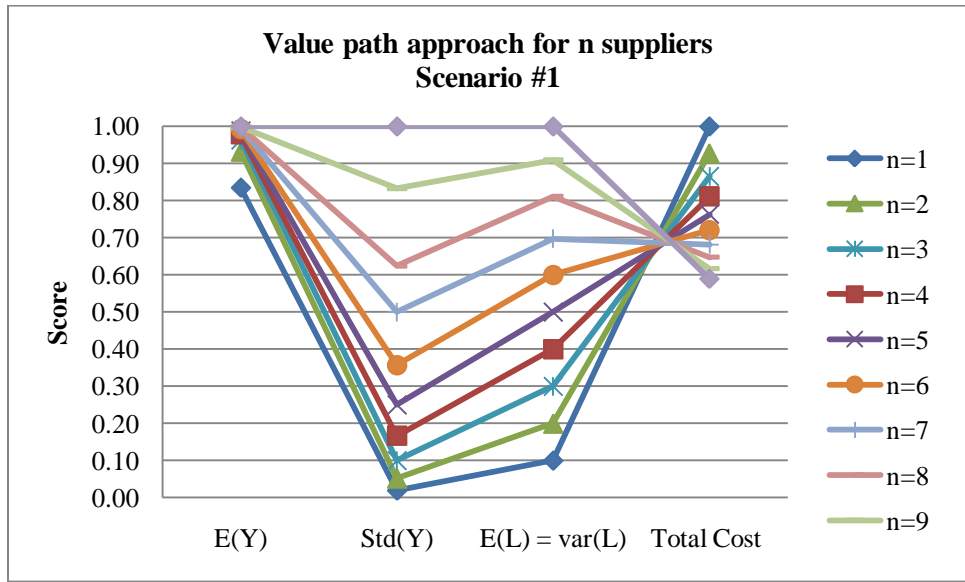
Figure 5-35: Unsatisfied Order Yield vs. Number of Suppliers for all Scenarios



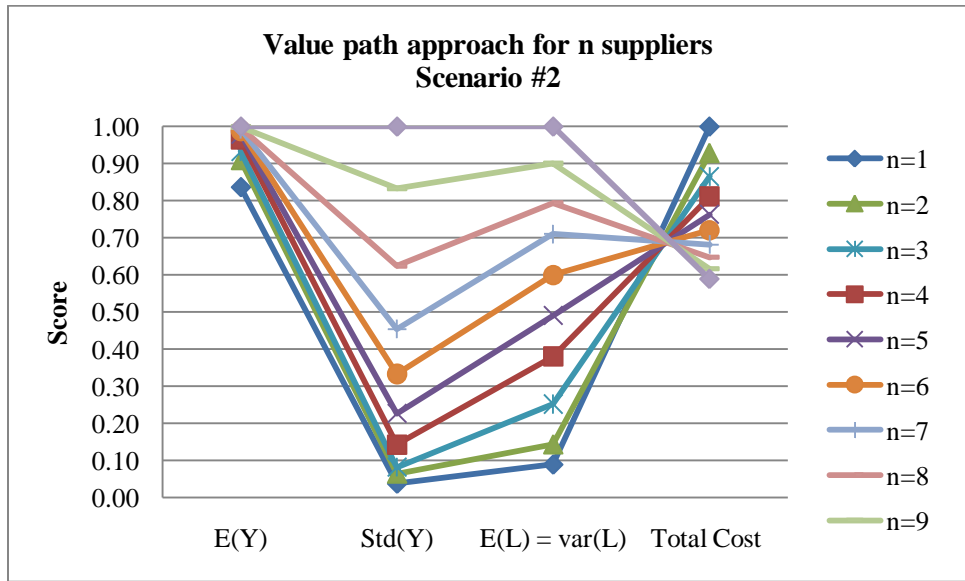
**Figure 5-36: Unsatisfied Order Yield vs. Number of Suppliers for all Scenarios**

### **Value Path Approach to Determine the Optimal Number of suppliers n**

Another strategy that can be used to determine the optimal number of suppliers is the Value Path Approach in Multi-criteria optimization. This method incorporates scaling using ideal values and results in a graphical representation that allows for a tradeoff analysis. More information is provided on Schilling's paper (Schilling et al, 1983). The diagrams from 5-35 through 5-38 show the results for all 4 scenarios. Although all solutions are non-dominated, the decision maker can make choices according to his/her preferences. If he/she does not prioritize purchasing cost, then he /she might be more inclined to order from a large number of suppliers. However, if the commander gives a high priority to purchasing costs, then he/she might prefer to select a smaller number of suppliers. A value of  $n=4$  could be an optimal solution across all scenarios when considering an expected order yield whose score is greater than 0.9 and whose score for total cost is greater than 0.8.



**Figure 5-37: Value Path Approach for Scenario #1**



**Figure 5-38: Value Path Approach for Scenario #2**

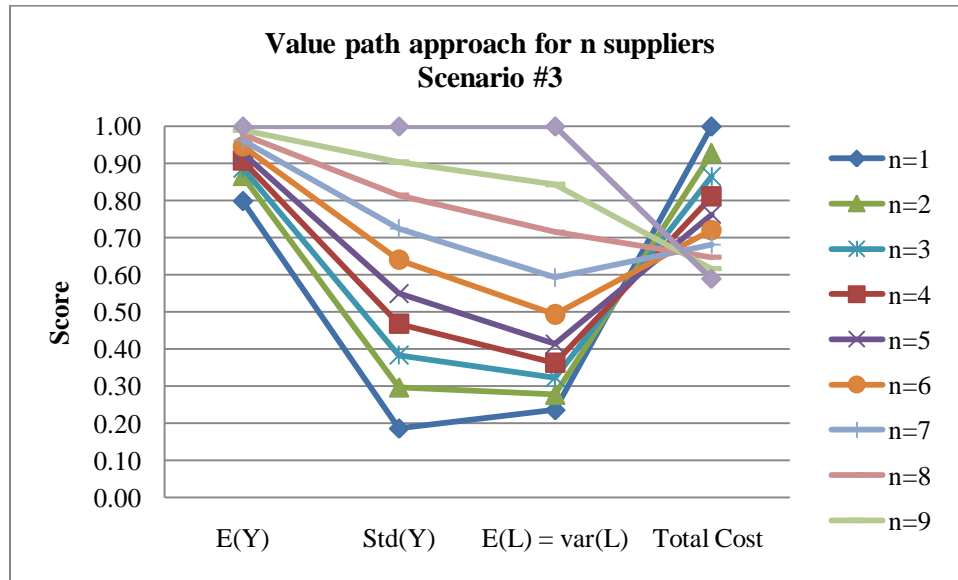


Figure 5-39: Value Path Approach for Scenario #3

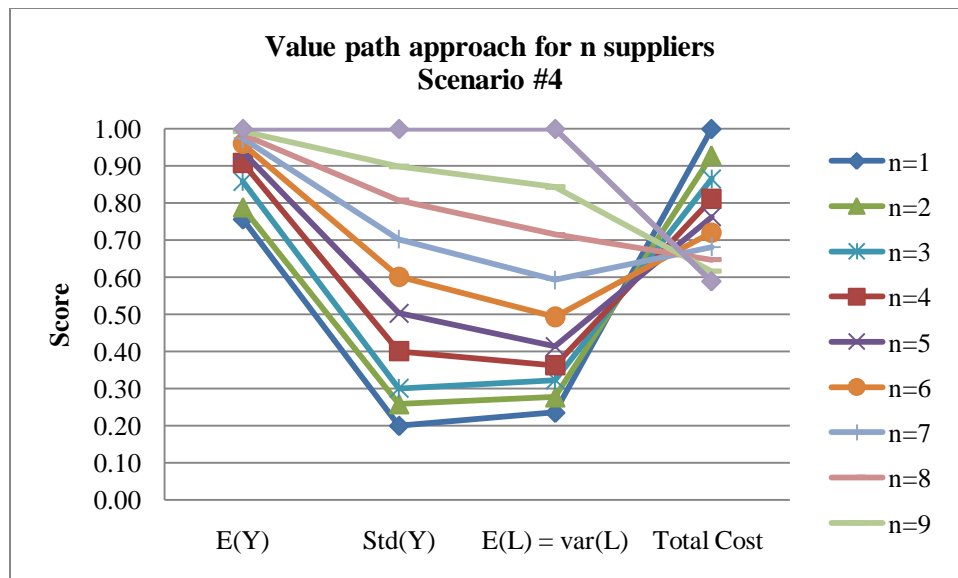


Figure 5-40: Value Path Approach for Scenario #4

### Conclusions

Chapter #5 justifies the benefits of multiple sourcing over single sourcing at the brigade unit stage of the DoD's supply chain and provides a methodology to determine the optimal



number of suppliers. Analysis of the lead time distributions and expected order yield for different scenarios show that, when the lead time distribution depends on the amount ordered, splitting an order evenly among multiple suppliers results in greater expected yields, lower variability in the amount received and lower expected lead times when compared with orders that . However, when the rate  $\lambda$  of the lead time distribution is independent of the amount requested  $x_{iMax}$ , ordering a larger portion from the principal supplier or the supplier with maximum lead time distribution results in lower standard deviation when compared to a scenario where the orders are uniform.

In the context of this research, readiness is measured in terms of the Expected Order Yield and the Expected lead time and lead time variability. A reduction in lead time expectation and deviation results in higher values of equipment readiness. In summary, the numerical results show that multiple sourcing increases the expected order yield by 15%-21% and the order lead time by 70%-90%

**Table 5-22: Overall improvement in % order yield and expected lead time across all scenarios**

Scenario	%Improvement order yield	%Improvement Lead time
1	15%	90%
2	15%	91%
3	17%	76%
4	21%	70%

The improvements in order yield and lead time expectation were compared against the purchasing costs for spare parts in order to determine the optimal number of suppliers n. For all scenarios the expected shortage and the cost curve found a common point between n=2 and n=3 suppliers. Scenarios #1 and #2 had intersection points between expected lead time curves and the cost curve approximately at n=4; whereas Scenarios #3 and #4 had such intersection point approximately at n=7.

## Chapter 6

### Conclusions and Future Work

#### Conclusions

This thesis steps outside the current military doctrine by studying the effect of multiple sourcing vs. single sourcing under a variety of scenarios. As opposed to commercial supply chains, the DoD's supply chain measures cost at the brigade unit side in terms of readiness; the lower the readiness value, the higher the cost. This research focuses in the commander's assessment of a unit's readiness by considering the unit's Equipment Readiness (ER) versus the risk of delays and cancellations of orders. ER is measured in terms of the lead time for order arrival for a given supplier under a single sourcing policy.

Assuming the unit is in a combat operation, the risk of delays and disruptions in orders placed by a forward unit to a Supply Support Activity (SSA) was captured using exponentially distributed lead times with parameter  $\lambda$ . An assessment of current military doctrine was made by comparing the expected lead time, expected order yield, and order yield deviation under four different scenarios were considered. Two scenarios had exponentially distributed lead times with parameter dependent on the amount ordered, while the latter 2 considered independent exponentially distributed lead times.

The results from Chapter 5 show that multiple sourcing can reduce costs at the brigade unit stage of the DoD's supply chain by increasing the order yield expectation and reducing order delivery time. Assuming there are  $n$  suppliers with same parameter  $\lambda$ , the expected order yield increases since the probability  $p(Y = 0) = (e^{-\lambda l})^n$ . As  $n \rightarrow \infty$ ,  $p(Y = 0) \rightarrow 0$  and  $p(Y > 0) \rightarrow 1$ . Moreover, if the rate at which orders are received is dependent on  $x_{iMax}$ , then as  $n \rightarrow$

$\infty$ ,  $x_{iMax} \rightarrow 0$ ; therefore  $e^{-\frac{k_i}{x_{iMax}} l^*} \rightarrow 0$ . Numerical results show that order yield levels increased

at a faster rate for those scenarios where the lead time distributions were dependent on the amount ordered when compared to those who did not. Moreover, when the lead time distribution is independent of the amount ordered, the expected order yield level is the same regardless of the order size. Finally, when the primary depot has a high risk of delays and cancellations, multiple sourcing improves equipment readiness as it results in smaller expected lead times.

### **Contributions**

The first contribution of this thesis to the field is an assessment of the DoD's single supplier policy at the brigade unit side of the supply chain. Risk was and measured for each supplier using exponentially distributed lead times. Four different scenarios were defined according to two main categories: suppliers having exponentially distributed lead times dependent on the amount ordered  $x_{req}$  and suppliers having exponentially distributed lead times that are independent of the quantity ordered. Under each scenario, the expected order yield and expected lead time were analyzed for different values of the number of suppliers  $n$ . Numerical results show that multiple sourcing improves the readiness of military operations. Order yield was improved by 15%-21%, while the lead time improved by 70%-91%. In addition, this research provides two approaches to identify the optimal number of suppliers for a given unit. The first one is to compare the improvement in order yield and expected lead time for order arrival to the purchasing cost. The second incorporates a Value Path Approach to compare the values of expected lead time for order arrival as well as the expectation and deviation of the order yield vs. the total purchasing cost  $n$  suppliers. This could serve as a tool that enables commanders to make smarter order purchasing decisions while integrating this information with powerful IT systems such as NetUSR.

## **Future Work**

The models developed in this thesis require further testing and validation on a realistic scenario. Additional layers of complexity may include considering the effect of orders from multiple units in the overall risk of delays and cancellations as well as crossover within suppliers. Another scenario worth considering is the effect of crossover or when shortages results in order requests among depots.

In addition, the results from this research could serve in defining a risk parameter  $\beta$  for delays or supplier disruptions, similar to the Capital Asset Pricing Model (CAPM). An approach to this could be to divide the lead time variance by the pooled lead time variance for  $n$  suppliers. This could prove useful in the definition of a multi-criteria linear programming/goal programming model to find the optimal policy to minimize sustainment costs for a given unit. In addition, these results can be tested when the supplier/disruption risk assume a normal distribution of lead times.

## Appendix A

### Program Code to Model the distribution of Expectation and Variance of Equipment-on-Hand for n suppliers

#### plotExpectedEquipmentOnHand

```
% Wilmarie Rios
% Penn State University
% February 17, 2011
% Thesis Research IE 600

% plotExpectedEquipmentOnHand executes getExpectedReadiness to gather the distributions
% of expected readiness for a number of depots starting from 1 to n at
% at times from 0 to 10 units.
% The resulting distributions are stored in a matrix and plotted to observe
% how the readiness changes as the number of suppliers changes

close all
clear all

n = 2;
t = 15;
k = 50;
xRequired = 150;

legIndex = zeros(n,1);
legText = [];

expectedCommodity = zeros(n, t+1);
varianceCommodity = zeros(n, t+1);
E = zeros(n,1);
V = zeros(n,1);

%subplot(2,1,1);
for i=1:n

    % [expectedCommodity(i,:) varianceCommodity(i,:)] = computeEoHScenario1(i,
    xRequired, k, t);
    % [expectedCommodity(i,:) varianceCommodity(i,:)] = computeEoHScenario2(i, t);
    % [expectedCommodity(i,:) varianceCommodity(i,:)] = computeEoHScenario3(i,
    xRequired, k, sig, t);
    % [expectedCommodity(i,:) varianceCommodity(i,:)] = computeEoHScenario4(i,
    xRequired, k, sig, t);
    E(i,1) = mean(expectedCommodity(i,:));
    V(i,1) = mean(sqrt(varianceCommodity(i,:)));
    %plot(0:10, 1-(expectedCommodity(i,:)));
    %legText = [legText; ['nSuppliers = ', num2str(i)]];
    %hold all

end

% title(['Distribution of Expected Readiness for Given Number of Suppliers at Times 0 <=
t <=10']);
% h = legend(legText);

%subplot(2,1,2);
```

```
%plot(1:7, 1-(expectedCommodity(1:7,2)));
```

## **computeEoHScenario1.m**

```
% Wilmarie Rios
% Penn State University
% February 17, 2011
% Thesis Research IE 600

% function computeEoHScenario1 computes the distributions of expected Equipment on Hand
% for a number of depots equal to nSup. at times from 0 to 10
% units.
% The delivery times for each supplier are exponentially distributed with a
% rate that depends on the amount ordered. Note that this means
% larger the amounts requested will have a lower the rate,
% hence higher the mean time for delivery. An initial order of 150 is
% placed to be split in equal parts amongst nSuppliers to get identical
% distributions.

function [E]= computeEoHScenario1(nSuppliers, xReq, k, t)

nCombinations = 2^(nSuppliers);

xMax = struct;
lambda = struct;

%compute the rates for each supplier
for (i=1:nSuppliers)
    xMax.(['s', num2str(i)]) = xReq/nSuppliers;
    lambda.(['s', num2str(i)]) = k/xMax.(['s', num2str(i)]);
    mean.(['s', num2str(i)]) = 1/lambda.(['s', num2str(i)])
end

%sets the end time interval to be studied
timeLength = t;

%Generates a binary matrix, where '1' denotes supplier i delivers commodity
%and '0' denoted supplier i does not deliver commodity. This is used in the
%computations of partial commodity and probability from each supplier

D = zeros(nCombinations,1);
for i=1:nCombinations
    D(i,1) = i-1;
end
B = dec2bin(D);
Y = zeros(nCombinations,timeLength+1);
P = zeros(nCombinations,timeLength+1);

% Computes the distribution Y(t*) of commodity at a given time as well
% as it's corresponding probability
for t=0:timeLength
    for scenario =1:nCombinations
        totalCommodity = 0;
        totalProbability = 1;
        for s = 1:nSuppliers
            number = str2num(B(scenario,s));
            totalCommodity = totalCommodity + (number * xMax(1).(['s', num2str(s)]));
            pFailure = exp(-1*lambda(1).(['s', num2str(s)])*t);

            if number == 0
                totalProbability = totalProbability*pFailure;
            else
                totalProbability = totalProbability * (1-pFailure);
            end
        end
    end
end
```

```

        end
    end
    Y(scenario,t+1) = totalCommodity;
    P(scenario,t+1) = totalProbability;
end

end

% computes E[R(t*)] for each time instant t*
% from 0 to timelength

Z = P.*Y/xReq;
Y2 = (Y/xReq).*(Y/xReq);
Z2 = P.*Y2;
E = zeros(1,timeLength);
E2 = zeros(1,timeLength);

W = zeros(nSuppliers,1);
for i=1:nSuppliers
    W(i,1) = sum(Z(i,:));
end

for i=1:timeLength+1
    E(1,i) = sum(Z(:,i));
    E2(1,i) = sum(Z2(:,i));
end
V = E2 - (E.*E);

```

## computeEoHScenario2.m

```

% Wilmarie Rios
% Penn State University
% February 17, 2011
% Thesis Research IE 600

% function getExpectedReadiness computes the distributions of expected readiness
% for a number of depots equal to nSup. at times from 0 to 10
% units.
% The delivery times for each supplier are exponentially distributed with a
% rate that depends on the amount ordered. Note that this means
% larger the amounts requested will have a lower the rate,
% hence higher the mean time for delivery. An initial order of 15000 is
% placed to be split in equal parts amongst nSuppliers to get identical
% distributions.

%function [E Z P Y E2 Y2 Z2 V]= getExpectedReadiness(nSup)
function [E V xMax lambda]= computeEoHScenario2(nSup, t)
xReq = 150;

nSuppliers = nSup;
nCombinations = 2^(nSuppliers);
k = 50;
xMax = struct;
lambda = struct;

%compute the rates for each supplier
if nSuppliers ==1
    xMax.(['s', num2str(1)]) = xReq;
    lambda.(['s', num2str(1)]) = k/xMax.(['s', num2str(1)]);
    mean.(['s', num2str(1)]) = 1/lambda.(['s', num2str(1)]);

```

```

elseif nSuppliers == 2
    xMax.(['s', num2str(1)]) = (3/4)* xReq;
    lambda.(['s', num2str(1)]) = k/xMax.(['s', num2str(1)]);
    mean.(['s', num2str(1)]) = 1/lambda.(['s', num2str(1)])

    xMax.(['s', num2str(2)]) = (1/4)* xReq;
    lambda.(['s', num2str(2)]) = k/xMax.(['s', num2str(2)]);
    mean.(['s', num2str(2)]) = 1/lambda.(['s', num2str(2)])
else
    for (i=1:nSuppliers)

        if i==1
            xMax.(['s', num2str(i)]) = 2*(xReq/nSuppliers);
        elseif i>1
            xMax.(['s', num2str(i)]) = (xReq-(2*(xReq/nSuppliers))) / (nSuppliers-1);
        end

        lambda.(['s', num2str(i)]) = k/xMax.(['s', num2str(i)]);
        mean.(['s', num2str(i)]) = 1/lambda.(['s', num2str(i)]);
    end
end

%sets the end time interval to be studied
timeLength = t;

%Generates a binary matrix, where '1' denotes supplier i delivers commodity
%and '0' denoted supplier i does not deliver commodity. This is used in the
%computations of partial commodity and probability from each supplier

D = zeros(nCombinations,1);
for i=1:nCombinations
    D(i,1) = i-1;
end

B = dec2bin(D);
Y = zeros(nCombinations,timeLength+1);
P = zeros(nCombinations,timeLength+1);

% Computes the distribution Y(t*) of commodity at a given time as well
% as it's corresponding probability
for t=0:timeLength
    for scenario =1:nCombinations
        totalCommodity = 0;
        totalProbability = 1;
        for s = 1:nSuppliers
            number = str2num(B(scenario,s));
            totalCommodity = totalCommodity + (number * xMax(1).(['s', num2str(s)]));
            pFailure = exp(-1*lambda(1).(['s', num2str(s)])*t);

            if number == 0
                totalProbability = totalProbability*pFailure;
            else
                totalProbability = totalProbability *(1-pFailure);
            end
        end
        Y(scenario,t+1) = totalCommodity;
        P(scenario,t+1) = totalProbability;
    end
end

end

% computes the expected readiness E[R(t*)] for each time instant t*
% from 0 to timelength

Z = P.*Y/xReq;
Y2 = (Y/xReq).*(Y/xReq);

```



```

Z2 = P.*Y2;
E = zeros(1,timeLength);
E2 = zeros(1,timeLength);
for i=1:timeLength+1
    E(1,i) = sum(Z(:,i));
    E2(1,i) = sum(Z2(:,i));
end
V = E2 - (E.*E);

```

### computeEoHScenario3.m

```

% Wilmarie Rios
% Penn State University
% February 17, 2011
% Thesis Research IE 600

% function computeEoHScenario1 computes the distributions of expected Equipment on Hand
% for a number of depots equal to nSup. at times from 0 to 10
% units.
% The delivery times for each supplier are exponentially distributed with a
% rate that depends on the amount ordered. Note that this means
% larger the amounts requested will have a lower the rate,
% hence higher the mean time for delivery. An initial order of 150 is
% placed to be split in equal parts amongst nSuppliers to get identical
% distributions.

function [E V]= computeEoHScenario3(nSuppliers, xReq, k, sig, t)

nCombinations = 2^(nSuppliers);

sig = 1./sig;
lambdaAll = k.*sig;
lambdaAll = sort(lambdaAll);

xMax = struct;
lambda = struct;

%compute the rates for each supplier
for (i=1:nSuppliers)
    xMax.(['s', num2str(i)]) = xReq/nSuppliers;
    lambda.(['s', num2str(i)]) =lambdaAll(i);
    mean.(['s', num2str(i)]) = 1/lambda.(['s', num2str(i)]);
end

%sets the end time interval to be studied
timeLength = t;

%Generates a binary matrix, where '1' denotes supplier i delivers commodity
%and '0' denoted supplier i does not deliver commodity. This is used in the
%computations of partial commodity and probability from each supplier

D = zeros(nCombinations,1);
for i=1:nCombinations
    D(i,1) = i-1;
end
B = dec2bin(D);
Y = zeros(nCombinations,timeLength+1);
P = zeros(nCombinations,timeLength+1);

% Computes the distribution Y(t*) of commodity at a given time as well
% as it's corresponding probability
for t=0:timeLength

```

```

for scenario =1:nCombinations
    totalCommodity = 0;
    totalProbability = 1;
    for s = 1:nSuppliers
        number = str2num(B(scenario,s));
        totalCommodity = totalCommodity + (number * xMax(1).(['s', num2str(s)]));
        pFailure = exp(-1*lambda(1).(['s', num2str(s)])*t);

        if number == 0
            totalProbability = totalProbability*pFailure;
        else
            totalProbability = totalProbability * (1-pFailure);
        end
    end
    Y(scenario,t+1) = totalCommodity;
    P(scenario,t+1) = totalProbability;
end

end

% computes E[R(t*)] for each time instant t*
% from 0 to timelength

Z = P.*Y/xReq;
Y2 = (Y/xReq).*(Y/xReq);
Z2 = P.*Y2;
E = zeros(1,timeLength);
E2 = zeros(1,timeLength);

W = zeros(nSuppliers,1);
for i=1:nSuppliers
    W(i,1) = sum(Z(i,:));
end

for i=1:timeLength+1
    E(1,i) = sum(Z(:,i));
    E2(1,i) = sum(Z2(:,i));
end
V = E2 - (E.*E);

```

## computeEoHScenario4.m

```

% Wilmarie Rios
% Penn State University
% February 21, 2011
% Thesis Research IE 600

% function computeEoHScenario1 computes the distributions of expected Equipment on Hand
% for a number of depots equal to nSup. at times from 0 to 10
% units.
% The delivery times for each supplier are exponentially distributed with a
% rate that depends on the amount ordered. Note that this means
% larger the amounts requested will have a lower the rate,
% hence higher the mean time for delivery. An initial order of 150 is
% placed to be split in equal parts amongst nSuppliers to get identical
% distributions.

function [E V xMax]= computeEoHScenario4(nSuppliers, xReq, k, sig, t)

nCombinations = 2^(nSuppliers);

```

```

xMax = struct;
lambda = struct;

sig = 1./sig;
lambdaAll = k.*sig;
lambdaAll = sort(lambdaAll);

%compute the rates for each supplier
% for (i=1:nSuppliers)
%     xMax.(['s', num2str(i)]) = xOrderArray(i);
%     lambda.(['s', num2str(i)]) = lArray(1,i);
%     mean.(['s', num2str(i)]) = 1/lambda.(['s', num2str(i)])
% end

if nSuppliers ==1
    xMax.(['s', num2str(1)]) = xReq;
    lambda.(['s', num2str(1)]) = lambdaAll(1);
    mean.(['s', num2str(1)]) = 1/lambda.(['s', num2str(1)]);

elseif nSuppliers == 2
    xMax.(['s', num2str(1)]) = (3/4)* xReq;
    lambda.(['s', num2str(1)]) = lambdaAll(1);
    mean.(['s', num2str(1)]) = 1/lambda.(['s', num2str(1)]);

    xMax.(['s', num2str(2)]) = (1/4)* xReq;
    lambda.(['s', num2str(2)]) = lambdaAll(2);
    mean.(['s', num2str(2)]) = 1/lambda.(['s', num2str(2)]);

else
    for (i=1:nSuppliers)

        if i==1
            xMax.(['s', num2str(i)]) = 2*(xReq/nSuppliers);
            lambda.(['s', num2str(i)]) = lambdaAll(i);
            mean.(['s', num2str(i)]) = 1/lambda.(['s', num2str(i)]);
        elseif i>1
            xMax.(['s', num2str(i)]) = (xReq-(2*(xReq/nSuppliers)))/(nSuppliers-1);
            lambda.(['s', num2str(i)]) = k(i)/sig(i);
            mean.(['s', num2str(i)]) = 1/lambda.(['s', num2str(i)])
        end

    end

end

end

%sets the end time interval to be studied
timeLength = t;

%Generates a binary matrix, where '1' denotes supplier i delivers commodity
%and '0' denoted supplier i does not deliver commodity. This is used in the
%computations of partial commodity and probability from each supplier

D = zeros(nCombinations,1);
for i=1:nCombinations
    D(i,1) = i-1;
end
B = dec2bin(D);
Y = zeros(nCombinations,timeLength+1);
P = zeros(nCombinations,timeLength+1);

% Computes the distribution Y(t*) of commodity at a given time as well
% as it's corresponding probability
for t=0:timeLength
    for scenario =1:nCombinations

```

```

totalCommodity = 0;
totalProbability = 1;
for s = 1:nSuppliers
    number = str2num(B(scenario,s));
    totalCommodity = totalCommodity + (number * xMax(1).(['s', num2str(s)]));
    pFailure = exp(-1*lambda(1).(['s', num2str(s)])*t);

    if number == 0
        totalProbability = totalProbability*pFailure;
    else
        totalProbability = totalProbability * (1-pFailure);
    end
end
Y(scenario,t+1) = totalCommodity;
P(scenario,t+1) = totalProbability;
end

end

% computes E[R(t*)] for each time instant t*
% from 0 to timelength

Z = P.*Y/xReq;
Y2 = (Y/xReq).*(Y/xReq);
Z2 = P.*Y2;
E = zeros(1,timeLength);
E2 = zeros(1,timeLength);

W = zeros(nSuppliers,1);
for i=1:nSuppliers
    W(i,1) = sum(Z(i,:));
end

for i=1:timeLength+1
    E(1,i) = sum(Z(:,i));
    E2(1,i) = sum(Z2(:,i));
end
V = E2 - (E.*E);

```

## Appendix B

### Program code to compute the expected lead time for n suppliers with exponentially distributed lead times dependent on the amount of commodity ordered

#### computeAllLeadTimes.m

```
% Wilmarie Rios
% Penn State University
% April 17, 2011
% Thesis Research IE 600
% This script (computeAllLeadTimes.m) executes computeLeadTime to compute the
% expected lead time for a number of suppliers with exponential lead time
% distributions with parameter lambda. The array of Expected lead times for nSuppliers is
% held in variable LeadTimes

nSuppliers = 6;
LeadTimes = zeros(1,nSuppliers);

for i=1:nSuppliers
    [L] = computeLeadTime(150,i,50);
    LeadTimes(1,i) = L;
End
```

#### computeLeadTimesSc1.m

```
% Wilmarie Rios
% Penn State University
% April 19, 2011
% Thesis Research IE 600
% computeLeadTime computes the expected lead time
% for a number of suppliers with exponential lead time
% distributions with parameter lambda dependent on
% a constant value k and the amount ordered.
% The Expected lead time is held in variable L

function [L Ei Pi] = computeLeadTime(xReq,nSuppliers,k)
lambdaAll = zeros(1,nSuppliers);
lambdaAll(1,1:nSuppliers) = k/(xReq/nSuppliers);
combinations = perms(lambdaAll);

nCombinations = length(combinations);
Ei = zeros(nCombinations, 1);
Pi = zeros(nCombinations, 1);

for h = 1:nCombinations
    p=1;
    for i=1:nSuppliers
        a = combinations(h,i)/sum(combinations(h,i:nSuppliers));
        p = p*a;
        if i==nSuppliers
            Pi(h,1) = p;
            p = 1;
        end
    end
end
```

```

        end
    end
    Ei(h,1) = 1/combinations(h,1);
end

L = sum(Ei.*Pi);
End

```

## computeLeadTimesSc2.m

```

% Wilmarie Rios
% Penn State University
% April 19, 2011
% Thesis Research IE 600
% computeLeadTime computes the expected lead time
% for a number of suppliers with exponential lead time
% distributions with parameter lambda dependent on
% a constant value k and the amount ordered .
% The Expected lead time is held in variable L

function [L Ei Pi] = computeLeadTimeSc2(xReq,nSuppliers,k)
lambdaAll = zeros(1,nSuppliers);
if nSuppliers ==1
    lambdaAll(1) = k/xReq;
elseif nSuppliers == 2
    lambdaAll(1) = k/((3/4)*xReq);
    lambdaAll(2) = k/((1/4)* xReq);
else
    for (i=1:nSuppliers)
        if i==1
            x = 2*(xReq/nSuppliers);
        elseif i>1
            x = (xReq-(2*(xReq/nSuppliers))) / (nSuppliers-1);
        end

        lambdaAll(i) = k/x;
    end
end

%lambdaAll(1,1:nSuppliers) = k/(xReq/nSuppliers);
combinations = perms(lambdaAll);

nCombinations = length(combinations);
Ei = zeros(nCombinations, 1);
Pi = zeros(nCombinations, 1);

for h = 1:nCombinations
    p=1;
    for i=1:nSuppliers
        a = combinations(h,i)/sum(combinations(h,i:nSuppliers));
        p = p*a;
        if i==nSuppliers
            Pi(h,1) = p;
            p = 1;
        end
    end
    Ei(h,1) = 1/combinations(h,1);
end

```

```
L = sum(Ei.*Pi);
end
```

### **computeLeadTimesSc3.m (Same values for Scenario #4)**

```
% Wilmarie Rios
% Penn State University
% April 19, 2011
% Thesis Research IE 600
% computeLeadTime computes the expected lead time
% for a number of suppliers with exponential lead time
% distributions with parameter lambda dependent on
% a constant value k and the amount ordered .
% The Expected lead time is held in variable L

function [L Ei Pi] = computeLeadTimeSc3(nSuppliers,lambdaAll)

combinations = perms(lambdaAll);
nCombinations = length(combinations);
Ei = zeros(nCombinations, 1);
Pi = zeros(nCombinations, 1);

for h = 1:nCombinations
    p=1;
    for i=1:nSuppliers
        a = combinations(h,i)/sum(combinations(h,i:nSuppliers));
        p = p*a;
        if i==nSuppliers
            Pi(h,1) = p;
            p = 1;
        end
    end
    Ei(h,1) = 1/combinations(h,1);
end

L = sum(Ei.*Pi);
end
```

## Bibliography

Anthony, T. F., & Buffa, F. P. (1977). Strategic Purchasing Scheduling . *Journal of Purchasing and Materials Management* , 27-31.

Army, D. o. (2000, April 14). Organizational Supply and Services for Unit Leaders. Washington , DC, USA: .

Benton, W. (1991). Quantity Discount Decision Under Conditions of Multiple Items, Multiple Suppliers and Resource Limitation. *International Journal of Production Research* , 1953-1961.

Chopra, S., & Meindl, P. (2007). *Supply Chain Management: Strategy, Planning and Operation*. . Upper Saddle River, NJ: : Prentice Hall.

Defense Logistics Agency . (n.d.). *DLA Disribution*. Retrieved April 23, 2011, from Defense Logistics Agency DIstribution: <http://www.ddc.dla.mil/Sites/default.aspx>

Department of the Army. (2010). *Army Unit Status Reporting and Force Registration - Consolidated Policies*. Washington DC: Department of the Army.

Erdem, A. S., Fadiloğlu, M. M., & Özekici, S. (2005). An EOQ Model with Multiple Suppliers and Random Capacity. *Naval Research Logistics* , 101-114.

Ghodsypour, S., & O'Brien, C. (2001). The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint. *International Journal of Production Economics* , 15-27.

GSP and Associates LLC. (n.d.). A Strategic Model for Sales Force Automation: The Value Chain Concept.



Kim, J., Sun, D., He, X., & Hayya, J. (2004). The (s,Q) Inventory Model with Erlang Lead Time and Deterministic demand. *Naval Research Logistics* , 906-923.

Kleindorfer, P., & Saad, G. (2005). Managing Disruption Risks in Supply Chains. *Production and Operations Management* , 53-68.

Lai, E. (2003, May). An Analysis of the Department of Defense Supply Chain: Potential Applications of the Auto-ID Center Technology to Improve Readiness.

Minner, S. (2003). Multiple-supplier inventory models in supply chain management: A review. *International Journal of Production Economics* , 265-279.

Moore, D., & Fearson, H. (1973). Computer-assisted Decision Making in Purchasing. *Journal of Purchasing* , 5-25.

Narasimhan, R., & Stoyhoff, K. (1986). Optimising Aggregate Procurement Allocation Decisions. *Journal of Purchasing and Materials Management* , 23-30.

Pan, A., Ramasesh, R., Hayya, J., & Ord, K. (1991). Multiple Sourcing: The determination of lead times. *Operations Research Letters* , 1-7.

Peltz, E. (2003). *Equipment Sustainment Requirements for the Transforming Army* . RAND Arroyo Center .

Peltz, E., Robbins, M., Boren, P., & Wolff, M. (2002). *Diagnosing the Army's Equipment Readiness*.

Peltz, E., Robbins, M., Giradini, K., Eden, R., Halliday, J., & Angers, J. (2007). *Sustainment of Army Forces in Operation Iraqi Freedom: Major Findings and Recommendations*. Santa Monica, CA: RAND Corporation.

RAND Corporation. (2004). *Dollar Cost Banding: A New Algorithm for Computing Inventory Levels for Army Supply Support Activities*. Santa Monica, CA: RAND Corporation.

Ravindran, A., Phillips, D., & Solberg, J. (1987). Chapter 8: Inventory Models. In *Operations Research: Principles and Practice* (pp. 343-354).

Ross, S. (2007). *Introduction to probability models*. Burlington, MA: Elsevier.

Schilling, D., & al, e. (1983). An Approach to the Display and Analysis of Multi-Objective Problems. *Socio-Economic Planning Sciences* , 57-63.

Schilling, D., Reville, C., & Cohon, J. (1983). An Approach to the Display and Analysis of Multi-Objective Problems. *Socio-Economic Planning Sciences* , 57-63.

Tang, C. (2006). Perspectives in Supply Chain Risk Management. *International Journal of Production Economic* , 452-483.

U.S. General Accounting Office. (1992). *Defense Inventory: Cost Factors Used to Manage Secondary Items*. National Security and International Affairs Division, Washington, DC.

United States Department of Defense . (2011). *United States Department of Defense Fiscal Year 2012 Budget Request* .

US Department of Defense . (n.d.). *The Official Home of the Department of Defense* . Retrieved March 2011 15, 2011, from <http://www.defense.gov/OrgChart/office.aspx?id=3>

Weber, C., & Current, J. (1993). A Multiobjective Approach to Vendor Selection. *European Journal of Operations Research* , 173-184.