RF MAGNETIC FIELD, SPECIFIC ENERGY ABSORPTION RATE, AND SIGNAL TO NOISE RATIO IN MRI: EXPERIMENTS AND NUMERICAL CALCULATIONS WITH FINITE DIFFERENCE TIME DOMAIN METHOD

A Thesis in
Bioengineering
by
Wanzhan Liu

© 2005 Wanzhan Liu

Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

May 2005
The thesis of Wanzhan Liu was reviewed and approved* by the following:

Michael B. Smith  
Professor of Radiology  
Thesis Advisor  
Chair of Committee  

William J. Weiss  
Associate Professor of Bioengineering  

Nadine Barrie Smith  
Assistant Professor of Bioengineering  

Russell C. Scaduto, Jr  
Associate Professor of Physiology  

Herbert H. Lipowsky  
Professor of Bioengineering  
Head of the Department of Bioengineering  

*Signatures are on file in the Graduate School
ABSTRACT

When MRI moves towards higher fields for higher signal to noise ratio (SNR), one of the problems is that the complicated interaction between the radiofrequency (RF) field and the biological tissues degrades the performance of the system. The finite difference time domain (FDTD) numerical method for electromagnetism, verified by experiments, is a valuable tool to study the RF field in high field MRI. The RF magnetic (B₁) field distribution and the SNR for different end-ring/shield configurations in birdcage-type RF coils are examined numerically at 64 and 125 MHz and experimentally at 125 MHz. With a previously developed male body model, a new anatomically accurate female body model is created to study B₁ field distribution, SNR, and specific energy absorption rate (SAR) in different body types at 64 MHz and 128 MHz. The RF radiation loss, which is associated with SNR and SAR, in a surface coil, in a head size birdcage coil, and in a head size TEM coil loaded with phantoms at a frequency range from 64 MHz to 600 MHz is also evaluated numerically. It is found that a) the end-ring/shield configuration in a birdcage coil affects B₁ homogeneity and SNR in a head, b) loading a larger, more muscular subject results in significantly less homogeneous B₁⁺ distribution, lower SNR, higher SAR levels, and c) the radiation becomes significant at high fields and the interaction between the RF field and the dielectric material in the sample helps to reduce the radiation. These results provide useful information for RF design and MRI safety guideline at high fields.
### TABLE OF CONTENTS

**LIST OF FIGURES** .................................................................................................................. vii  
**LIST OF TABLES** ..................................................................................................................... xi  
**ACKNOWLEDGEMENTS** ......................................................................................................... xii  

Chapter 1  Introduction ................................................................................................................. 1  

1.1 NMR/MRI Basic....................................................................................................................... 3  
  1.1.1 Nuclei in magnetic fields................................................................................................. 3  
  1.1.2 Resonance, free induction decay, and relaxation .......................................................... 6  
  1.1.3 MRI imaging methods .................................................................................................... 13  
    1.1.3.1 Spatial encoding ...................................................................................................... 13  
    1.1.3.2 Imaging sequences ................................................................................................. 16  
      1.1.3.2.1 Spin echo .......................................................................................................... 16  
      1.1.3.2.2 Gradient echo ................................................................................................. 19  
  1.1.4 Signal to noise ratio ........................................................................................................ 20  
    1.1.4.1 Dependence on the field strength ........................................................................... 21  
    1.1.4.2 SNR at low fields .................................................................................................... 22  
    1.1.4.3 SNR at high fields .................................................................................................. 24  
  1.1.5 Hardware ....................................................................................................................... 24  
    1.1.5.1 Magnet .................................................................................................................. 25  
    1.1.5.2 Gradient coils ........................................................................................................ 26  
    1.1.5.3 RF coils ................................................................................................................ 26  
    1.1.5.4 Transmitter .......................................................................................................... 27  
    1.1.5.5 Receiver .............................................................................................................. 27  
    1.1.5.6 Computer ............................................................................................................. 28  
  1.1.6 RF coils: surface coils, birdcage coils, and TEM coils ................................................... 28  
  1.1.7 MRI safety ..................................................................................................................... 35  
    1.1.7.1 Static magnetic fields ............................................................................................ 35  
    1.1.7.2 Gradient magnetic fields ....................................................................................... 36  
    1.1.7.3 RF fields .............................................................................................................. 36  

1.2 Basic Electromagnetic Theory and Resonant Circuits ............................................................ 39  
  1.2.1 Electromagnetic spectrum ............................................................................................. 40  
  1.2.2 Maxwell’s equations ...................................................................................................... 41  
  1.2.3 Electromagnetic fields in media .................................................................................... 44  
  1.2.4 Energy and Poynting theorem ...................................................................................... 46  
  1.2.5 Principle of reciprocity ................................................................................................ 47  
  1.2.6 Resonant circuits ......................................................................................................... 50  

1.3 Finite Difference Time Domain (FDTD) Method for Electromagnetism .................................. 52  
  1.3.1 Formula ....................................................................................................................... 52  
  1.3.2 Boundary conditions ................................................................................................... 57  
    1.3.2.1 Liao boundary condition ....................................................................................... 58
LIST OF FIGURES

Figure 1.1 Precession of a proton with magnetic moment $\mu$ in a static field $B_0$. ..........5

Figure 1.2 The net magnetism $M$ resulting from the spin population difference between the lower energy state and the higher energy state in an external field $B_0$. .........................................................................................................................6

Figure 1.3 $B_{1+}$ tips $M$ from the z direction to the x-y plane by $\alpha$. .........................8

Figure 1.4 Free Induction Decay (FID) signal.............................................................9

Figure 1.5 The recovery curve of $M_z$ with a time constant $T1$ after a 90° pulse........10

Figure 1.6 After a 90° RF pulse, the transverse component of net magnetization, $M_{xy}$, decays with a time constant $T2$. ......................................................................................................................12

Figure 1.7 Slice selection gradient $G_z$, phase encoding gradient $G_y$, and frequency encoding gradient $G_x$. Arrows represent the phase of magnetization in the four voxels in slice 2. See text for more details....................................................14

Figure 1.8 Time course of a spin echo imaging sequence. Horizontal lines next to $G_y$ indicate different phase encoding gradient strengths used for sequential scans. ..................................................................................................................18

Figure 1.9 Time course of a gradient echo imaging sequence. Horizontal lines next to $G_y$ indicate different phase encoding gradient strengths used for sequential scans. ...................................................................................................20

Figure 1.10 A simplified MRI system. ........................................................................25

Figure 1.11 The circuit of a surface coil that has four fixed tuning capacitors $C$, an adjustable matching capacitor $C_m$, and an adjustable tuning capacitor $C_t$ (a) and the decrease of the magnetic field along the axis of a surface coil with a radius of $R$ (b).................................................................................................................29

Figure 1.12 A schematic diagram (a) and the lumped element equivalent circuit (b) of an unshielded low-pass birdcage coil. $L_s$ is the inductance of an end-ring segment, $L_g$ is the inductance of a leg, and $C$ is the value of a tuning capacitor.........................................................................................................................31

Figure 1.13 The magnetic field distribution of mode 1 in an empty birdcage coil driven with one source at 125 MHz (FDTD simulation)..................................32
Figure 1.14 A sketch of a TEM coil (a) shown with a cross section (b). The coil is tuned to the desired frequency with the adjustable inner conductor length. ........34

Figure 1.15 Electromagnetic spectrum. .................................................................40

Figure 1.16 The polarization of electric dipole moments in a media without (a) and with (b) the presence of an external electric field $E$. ............................................45

Figure 1.17 The diagram (a) and the impedance change with frequency (b) of a series resonant circuit. The impedance of the circuit is defined as $V/I$..............50

Figure 1.18 The diagram (a) and the impedance change with frequency (b) of a parallel resonant circuit. The impedance of the circuit is defined as $V/I_l$............51

Figure 1.19 E components (green) and B components (red) in a Yee cell..............57

Figure 2.1 Rung currents that produce a homogeneous $B_1$ field in an infinitely long cylinder. Green arrows indicate the direction and the numbers are magnitudes of the currents.................................................................66

Figure 2.2 The current distribution in a birdcage coil. Green arrows indicate the direction and the numbers are magnitudes of the currents. ...............................67

Figure 2.3 Current return paths in the RF coil used by Wen et al. (1994). Green arrows indicate the direction and the numbers are magnitudes of the currents....68

Figure 2.4 Half of the birdcage coil with 1) a cylindrical shield (Conventional), 2) a shield with annular extensions to closely shield the end-rings (Surrounding Shield), 3) a shield with annular extensions connecting the rungs to the shield (Solid Connection), and 4) thin wires connecting the rungs to the shield (Thin Wire Connection).................................................................70

Figure 2.5 The axial (left), sagittal (middle), and coronal (right) planes in the computer model of the conventional birdcage coil loaded with a human head. The dashed lines indicate the location of the axial plane on which the homogeneity of the field and $SNR$ were calculated.................................71

Figure 2.6 Normalized $B_1^+$ on the central axial plane in the unloaded coil with the four different end-ring/shield configurations defined in Figure 2.4. $B_1^+$ at the center is normalized to 1.................................................................74

Figure 2.7 Normalized $B1^+$ on the central axial plane in the head with the four different end-ring/shield configurations defined in Figure 2.4. $B1^+$ at the center is normalized to 1.................................................................75
Figure 2.8 Normalized $|B_1|$ on the central coronal plane in the coil with four different end-ring/shield configurations and loaded with the human head model at 64 MHz and 125 MHz. $|B_1|$ at the center is normalized to 1. The black line inside the coil indicates the position of the head. .................................76

Figure 3.1 Four 12-rung low-pass linear birdcage-type coils with four different end-ring/shield configurations. ..................................................................................................................87

Figure 3.2 A schematic diagram of the matching circuit for the four configurations. .................................................................................................................................................89

Figure 3.3 Transmission return loss curves for the four coils. To include all the modes, the curves for the conventional configuration and the surrounding shield configuration span over 270 MHz while those for the solid connection configuration and the thin wire connection configuration span over 60 MHz .... 90

Figure 3.4 Image slices of a subject’s head in the four coils. Only the central slices are shown here. .............................................................................................................................................93

Figure 3.5 The central axial image slice of the oil phantom for the four coils. The circle of the dashed line, which has a diameter about 70% of that of the coil, indicates the outer boundary of the area within which the homogeneity was calculated. ........................................................................................................................................94

Figure 4.1 Coronal (a), sagittal (b), and axial (c) slices of subject 1 (left) and subject 2 (right) through the center of the coil in the rung-on-plane orientation where two coil rungs and the subject’s two arms are on the same plane and an axial slice (d) of the rung-off-plane orientation where the coil is rotated half of the distance between two adjacent rungs from the rung-on-plane orientation so no rungs are on the same plane with subject’s arms. ........... 105

Figure 4.2 Location of the drive ports in the conventional quadrature excitation and in the four-port excitation with subject 1 in the rung-on-plane orientation and in the rung-off-plane orientation..................................................................................................................110

Figure 4.3 The distribution of $B_1^+$ on the central axial plane in the empty coils and in the coils loaded with subject 1 in the rung-on-plane orientation for the conventional quadrature excitation, the four-port excitation, and the ideal excitation at 128 MHz. The $B_1^+$ at the center of the coil is normalized to 1.96 µT, the field strength necessary to produce a 3-ms 90° rectangular RF pulse. $B_1^+$ above 8 µT is expressed as 8 µT..............................................................................................................113

Figure 4.4 The distribution of the flip angle for maximum signal of a reconstructed gradient echo image with a 3-ms rectangular RF pulse on the central axial plane with the four-port excitation..................................................................................115
Figure 4.5 The local SAR distribution in the axial, coronal, and sagittal plane that through the location of the $SAR_L$ at 128 MHz. Both subjects were loaded in the rung-on-plane orientation and the coil was driven with four-port excitation. Linear gray scale is from 0 (black) to 21.23 W/kg (white). Values above 21.23 W/kg are shown with the same (white) color.

Figure 5.1 The model of the surface coil with a spherical phantom in the whole problem region. Power radiated out of box S is calculated. The origin is set at the center of the phantom and the axis of the coil is in the x direction.

Figure 5.2 The model of the volume coil-sample system in the whole problem region (a) and the close look of the spherical phantom in the birdcage (b) and the TEM coil (c) with half of the coil removed to show the details of coil structure. The capacitors in the end-rings of the birdcage coil are not shown here. Power radiated out of box S is calculated. The origin is set at the center of the phantom and the coil axis is in the z direction.

Figure 5.3 The percentage of input power radiated by a surface coil loaded with sample 1 (blue) and sample 2 (red).

Figure 5.4 The percentage of input power radiated by a head-size birdcage coil (square) and TEM coil (triangle) loaded with two different samples versus frequency. Blue lines represent sample 1 and red lines represent sample 2.

Figure 5.5 Snapshots of E field magnitude and radiation pattern in the central x-z plane of the surface coil loaded with sample 1 (a, c) and with sample 2 (b, d) at 600 MHz. The field is normalized such that the maximum E field magnitude is equal to 1. The white circle in (a, b) indicates the location of the sample. The radiation pattern is the plot of gain (dBi) versus $\theta$, which is defined in Figure 5.1. The dashed line indicates the direction of maximum gain.

Figure 5.6 Snapshots of E field magnitude and radiation pattern in the central x-z plane for the birdcage coil at 170 MHz (a, c) and for the TEM coil at 345 MHz (b, d). Both coils are loaded with sample 1. The field is normalized such that the maximum E field magnitude is equal to 1. The radiation pattern is the plot of gain (dBi) versus $\theta$, which is defined in Figure 5.2. The dashed line indicates the direction of maximum gain.
LIST OF TABLES

Table 1-1 T1 and T2 Relaxation time for different tissues at 1.5 T. ...........................11

Table 1-2 SAR limits in the normal mode of MRI. ..........................................................37

Table 2-1 Normalization factor, $V$, total absorbed power ($V^2 P_{abs}$), SNR, homogeneity in the empty coil, and homogeneity in the coil loaded with a human head for the four different end-ring/shield configurations at 64 MHz and 125 MHz. ...........................................................................................................77

Table 3-1 Capacitor values in the rungs of the four coils. .............................................91

Table 3-2 Q values, RF power $P$ in dB, homogeneity in the oil phantom, and SNR in the head for the four coils. The homogeneity is defined on the central axial plane within 70% of the coil radius as the percentage of the area that has image signal intensity deviation within 10% of the average image signal intensity. The RF power is measured for a 3.2 ms 90 degree gauss pulse during head imaging and normalized such that the power used by the conventional coil is equal to 0 dB. .........................................................................................95

Table 4-1 Conductivity $\sigma$, relative permittivity $\varepsilon_r$, and the mass density $\rho$ assigned to each tissue at different frequencies.................................................................107

Table 4-2 The average flip angle for maximal signal contributing to a reconstructed gradient echo image, homogeneity in the central axial plane, the total absorbed power in the body ($P_{abs}$), whole-body average SAR ($SAR_w$), maximum local SAR in one gram of tissue ($SAR_L$), and ISNR in the central axial plane in different subjects with different orientations of a body-size birdcage coil driven by different methods at 64 MHz and 128 MHz. All results are calculated for a 3-ms 90° rectangular RF pulse with a 100% duty cycle..................................................................................................................114

Table 5-1 The percentage of the input power radiated, $e$, calculated by taking the ratio of power radiated over the sum of power radiated and the power dissipated in the sample, for a surface coil loaded with different samples at different frequencies $f$. .........................................................................................................................133

Table 5-2 The percentage of the input power radiated by a high-pass birdcage coil and by a TEM coil loaded with different samples at different frequencies $f$............135
ACKNOWLEDGEMENTS

To Dr. Michael Smith for his patience, supporting, and directing and to other committee members: Dr. William Weiss, Dr. Nadine Barrie Smith, and Dr. Russell Scaduto Jr. for the guidance.

To Dr. Christopher Collins for being such a great mentor and such a great friend.

To everybody at the Center for NRM research lab in Hershey for making it such a nice place to work.

To my family, Mom, Dad, Wanshun, and Wanzhong, for emotional and financial supporting.

This work is dedicated to my parents
Chapter 1

Introduction
Nuclear magnetic resonance spectroscopy (NMR or MRS) and nuclear magnetic resonance imaging (MRI) technologies are widely used in scientific research and clinical medicine. MRS can be used to non-invasively detect signals from various atomic nuclei in the body and with these signals MRI can be used to create images of the body tissues with excellent resolution.

Reported the first time in condensed matter by Felix Bloch (1946) and by Edward Purcell (1946) independently, developed by Raymond Damadian (1971; 1976), Paul Lauterbur (1973), Richard Ernst (1966; Kumar et al., 1975), Peter Mansfield (Garroway et al., 1974; Mansfield, 1977) and other researchers, NMR/MRI technology is now one of the most powerful tools in medical imaging and scientific research. Five Nobel prizes have been awarded to research related to NMR/MRI and the field, which combines medicine, biology, physics, mathematics, computer science, and other fields, and is still growing.

The focus of this work is on the radiofrequency (RF) design in high field MRI. The ultimate goals are to solve the technical difficulties encountered in high field MRI, such as the RF magnetic field ($B_1$) inhomogeneity and the RF radiation loss, to optimize the SNR benefits of high fields, and to provide critical information for safety guideline such as specific energy absorption rate (SAR) levels. The finite difference time domain (FDTD) numerical method for electromagnetics, which is validated in many research fields including NMR/MRI (Yee, 1966; Kunz and Luebbers, 1993; Guy et al., 1999; Taflove and Hagness, 2000; Alecci et al., 2001; Vaughan et al., 2001; Krug II et al., 2002; Koulouridis and Nikita, 2004), is used throughout this work to calculate parameters that are very difficult to measure experimentally or to calculate analytically.
In this chapter, a brief overview of NMR/MRI principles, some topics in electromagnetism that are related to NMR/MRI, and the theory of FDTD will be provided. The reader should refer to the work of Abragam (1961) for more detailed theory of NMR, Fukushima and Roeder (1981) and Chen and Hoult (1989) for MRI hardware, Haacke et al. (1999) for MR imaging methods, Jackson (1999) and Lorrain and Corson’s (1970) for electromagnetic theory, and Kunz and Luebbers (1993) and Taflove and Hagness (2000) for FDTD.

1.1 NMR/MRI Basic

1.1.1 Nuclei in magnetic fields

Nuclei possess spin, a fundamental property like electrical charge or mass. Nuclei of different atoms possess different spin. All nuclei of hydrogen atoms of atomic mass 1 (proton) have a spin of $\frac{1}{2}$. All carbon atoms of atomic mass 12 have a spin of 0. All carbon atoms of atomic mass 13 have a spin of $\frac{1}{2}$. Just as electrical charge can be + or -, all these spins can also be + or -.

A nucleus with a spin produces a magnetic field around it, which is called its nuclear magnetic moment. How strong the field is depends on the mass, charge and the rate of spin of the nucleus. A vector $\mathbf{\mu}$ is used to represent the magnetic moment, which can be imagined as a field of a microscopic bar magnet produced by a spinning charged sphere. As a compass lines up with the earth’s magnetic field, the nuclear magnetic moment in an external field will align to the field. Depending on its spin state, the
magnetic moment can align either parallel or anti-parallel to the field. For example, the proton has two possible spin states, $+\frac{1}{2}$ and $-\frac{1}{2}$. These two spin states determine the nuclei’s two energy states in an external magnetic field: a lower energy state ($+\frac{1}{2}$) and a higher energy state ($-\frac{1}{2}$). In the lower energy state, the nuclear magnetic moment is parallel to the external field and in the higher energy state it is anti-parallel to the external field. According to quantum mechanics, the moment is not align exactly with the magnetic field, but rather is tilted at an angle. Because of this angle, the nuclear magnetic moment is not static in the external field but precesses about the field’s axis. This precession is analogous to the motion of a spinning gyroscope in the presence of gravity (Figure 1.1) and how fast it precesses is determined by the Larmor equation

$$\omega = \gamma B_0$$  \[1.1\]

where $\omega$ is the frequency of precession (radians/s), and $\gamma$ is the gyromagnetic ratio (radians/s/Tesla), $B_0$ is the magnitude of the external static field, $B_0$. Eq. 1.1 shows that the stronger the external magnetic field, the faster the precession. The gyromagnetic ratio, $\gamma$, is a constant that is different for different types of nuclei. For $^1$H, the gyromagnetic ratio equals to $2.675\times10^8$ radians/s/Tesla. For $^{31}$P, it is $1.082\times10^8$ radians/s/Tesla.
Often in MRI, the word “proton” is used interchangeably with the phrase “hydrogen nucleus.” The field strength determines at what frequency the protons precess. The distribution of protons between energy states is determined by Boltzmann statistics. The population of spins at the lower energy level is slightly larger than that at the higher energy level (1 proton out of about 100,000 at 1.5 T). The net magnetism, $M$, produced by this spin population difference is in the direction of spin magnetism of the lower energy state — parallel to the external field (Figure 1.2). The small difference of the population between two energy states makes MRS/MRI a low signal-to-noise ratio (SNR) technique but since there is a large amount of protons in the body, there are still enough signals. The difference of the population increases with the external field strength $B_0$, which means the SNR of MRS/MRI system can be improved by using higher magnetic fields. More about SNR will be discussed in section 1.1.4.

Figure 1.1 Precession of a proton with magnetic moment $\mu$ in a static field $B_0$. 

\[ \begin{align*} 
\mu & \quad B_0 \\
\text{Proton} & \\
\end{align*} \]
Traditionally a Cartesian coordinate system is defined so that the z-axis is parallel to the external magnetic field $B_0$. So at equilibrium, the $z$ component of the net magnetization equals $M$, and the $x$, $y$ components of the net magnetization are zero, i.e., $M_z = M$, $M_{xy} = 0$.

For the weak net magnetization $M$ in the $z$ direction to be detected, it needs to be separated from the strong static field $B_0$. The RF magnetic field $B_1$ is introduced for that purpose. $B_1$ is quite different from the static field $B_0$. The amplitude of $B_1$ is in the order of microTesla. Its direction is ideally perpendicular to that of $B_0$ field, which is always

![Figure 1.2 The net magnetism $M$ resulting from the spin population difference between the lower energy state and the higher energy state in an external field $B_0$.](image)
along z-axis. One component of it, $B_1^+$, is rotating about the z-axis in the same direction of the spin precession at the Larmor frequency in the x-y plane. Imagining one stands on a precessing nucleus, since $B_1^+$ is rotating at the same speed as the nuclear precession, it looks still to him/her. If the x-axis and y-axis are also rotating at the same speed, the RF field will also be still in this reference frame. This x-y-z frame is called the rotating-frame and the traditional static frame is called the laboratory-frame. The benefit of the rotating frame is that in this frame, $B_1^+$ is static and the nuclei are not seen to precess about the z-axis. The net magnetization $M$ will precess around $B_1^+$ in the rotating frame just as it precesses around $B_0$ in laboratory frame (Figure 1.3). The frequency of the rotation, $\omega$, depends on the amplitude of $B_1^+$, $B_1^+$,

$$\omega = \gamma B_1^+ \quad \text{[1.2]}$$

The rotation about $B_1^+$ is also called tipping. As long as the $B_1^+$ field is applied, the net magnetization will keep tipping. When the $B_1^+$ stops, the tipping of the magnetization vector stops. If the $B_1^+$ field is of constant magnitude and lasts time $\tau$, the tipping angle, $\alpha$, will be,

$$\alpha = \omega \tau = \gamma B_1^+ \tau \quad \text{[1.3]}$$

$B_1^+$ determines how fast the tipping will be and the duration of $B_1^+$ determines the total tipping angle of the pulse. Since the duration of $B_1^+$ usually is very short (several milliseconds), this field is called the RF pulse with the final tipping angle in its name. For example, a 90° RF pulse could be used to tip the net magnetization down 90 degrees
from the z direction into the x-y plane. A 180° pulse could be used to tip the net magnetization down 180 degrees from the z direction to the –z direction.

If the net magnetization \( \mathbf{M} \) is tipped down into the x-y plane with a 90° pulse and then the RF pulse is turned off, \( \mathbf{M} \) will rotate about the z direction in x-y plane and \( \mathbf{M}_{xy} \) is used to represent this x-y component of net magnetization. If a coil is put in the x-y plane, the rotating \( \mathbf{M}_{xy} \) will cause a changing net magnetic flux through this coil and according to Faraday’s law, it will induce an oscillating current in this coil. This current is the MR signal.

Figure 1.3 \( \mathbf{B}_{1+} \) tips \( \mathbf{M} \) from the z direction to the x-y plane by \( \alpha \).
Before an RF pulse was added to protons, the system was in an equilibrium state. The net magnetization at that time was \( M \) in the \( z \) direction. The RF pulse broke the balance and tipped \( M \) to the x-y plane. If the RF pulse is turned off, the whole system will return to the equilibrium state. That means the \( z \) component of the net magnetization, \( M_z \), will increase from 0 to its equilibrium value \( M \). At the same time the x-y component of the net magnetization \( M_{xy} \) will decay from \( M \) to equilibrium value, 0. Since the MRI signal is proportional to the strength of the x-y component of the net magnetization, \( M_{xy} \), it will also decay to zero. This decaying MRI signal is called free induction decay (FID) (Figure 1.4).

![Free Induction Decay (FID) signal](image)

**Figure 1.4** Free Induction Decay (FID) signal.
A time constant, T1 relaxation time, is used to describe how fast the z component of net magnetization, \( M_z \), will recover from 0 to the equilibrium value \( M \). The return of \( M_z \) to equilibrium value is an exponential function of time,

\[
|M_z| = |M| \left(1 - e^{-t/T_1}\right)
\]

[1.4]

The curve of this equation is shown in Figure 1.5. At time \( t = 0 \), \( |M_z| = 0 \); at \( t = T_1 \), \( |M_z| = |M| \times (1-1/e) \approx 0.63|M| \); at \( t = 5T_1 \), \( |M_z| \approx |M| \). That is, T1 is defined as the time for \( |M_z| \) to increase from 0 to 0.63\(|M|\). So the smaller is the T1, the faster the recovery of \( M_z \).

Figure 1.5 The recovery curve of \( M_z \) with a time constant T1 after a 90° pulse.
T1 is also called the longitudinal or spin-lattice relaxation time. Different tissues have different T1 values. T1 for different tissues at a field strength of 1.5 T are listed in Table 1-1 (Webb, 2003). In general, the T1 of diseased and damaged tissue is longer than that of the corresponding healthy tissue and T1 increases with the static field strength.

As T1 relaxation time is used to describe how fast $M_z$ goes back to its equilibrium value $M$, another time constant, T2 relaxation time, is used to describe how fast, after a 90° pulse, the x-y component $M_{xy}$ decay from $M$ to its equilibrium value $0$:

$$\left|M_{xy}\right| = \left|M\right|e^{\left(-\frac{t}{T2}\right)} \quad [1.5]$$

Table 1-1 T1 and T2 Relaxation time for different tissues at 1.5 T.

<table>
<thead>
<tr>
<th>Tissue</th>
<th>T1 (ms)</th>
<th>T2 (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat</td>
<td>260</td>
<td>80</td>
</tr>
<tr>
<td>Muscle</td>
<td>870</td>
<td>45</td>
</tr>
<tr>
<td>Liver</td>
<td>500</td>
<td>40</td>
</tr>
<tr>
<td>Gray matter</td>
<td>900</td>
<td>100</td>
</tr>
<tr>
<td>White matter</td>
<td>780</td>
<td>90</td>
</tr>
<tr>
<td>Cerebrospinal fluid</td>
<td>2400</td>
<td>160</td>
</tr>
</tbody>
</table>

Figure 1.6 shows the plot of the equation. At time $t = 0$, $\left|M_{xy}\right| = |M|$; at time $t = T2$, $\left|M_{xy}\right| = |M|*1/e \approx 0.37|M|$; at time $t = 5T2$, $\left|M_{xy}\right| \approx 0$. So T2 determines the time the magnetization $M_{xy}$ needs to decrease from maximum value $|M|$ to 0.37$|M|$. The smaller is the T2, the faster the decrease of $M_{xy}$. 
T2 is also called transverse or spin-spin relaxation time. Different tissues also have different T2 values. T2 values for some tissues at 1.5 T are listed in Table 1-1. Usually, T2 of the tissue is much less than T1 and never exceeds T1.

Figure 1.6 After a 90° RF pulse, the transverse component of net magnetization, $M_{xy}$, decays with a time constant T2.
1.1.3 MRI imaging methods

1.1.3.1 Spatial encoding

The FID signal induced by the net magnetization $M_{xy}$ can be detected with an RF receiving coil in the x-y plane. If the spatial information of body tissues could be encoded into the FID signal, an image can then be reconstructed from it. The typical encoding methods include slice selection, frequency encoding, and phase encoding.

Supposing a patient is put in a magnetic field $B_0$ and two cross sections of his/her head at different locations, one slice cross the patient’s eyes (slice 1) and another cross the patient’s nose (slice 2) (Figure 1.7), are to be imaged. If the patient’s whole head is in the same $B_0$, and an RF field of the Larmor frequency $\omega = \gamma B_0$ is added, all protons in the patient’s head will be excited and contribute to the FID signal. It is not possible to tell which part of the FID signal is from slice 1 and which part from slice 2. If these two slices are in different static magnetic fields, however, they will be excited at different Larmor frequencies then be selectively imaged.
For example, if slice 1 is in a magnetic field $B_0 + \Delta B$ and slice 2 is in a field $B_0 - \Delta B$ and if an RF field with frequency of $\gamma |B_0 + \Delta B|$ is added, only slice 1 will be excited. If an RF field with frequency of $\gamma |B_0 - \Delta B|$ is added, only slice 2 will be excited. Therefore, different slices can be selected by using different RF frequencies. This technique is called slice selection. During the experiment, a weak linear gradient magnetic field $G_z$ along the z-axis is added to the $B_0$ field so that the total magnetic field in the z direction will be $B_0 + G_z$. The gradient magnetic field $G_z$ is a linear function of z, so every slice along the z
direction is in a magnetic field of a different strength and can be imaged with corresponding Larmor frequency.

Now the signal from different slices can be separated by the application of $G_z$. The signals from the different locations inside one slice are still needed to separate. Phase encoding and frequency encoding techniques are used to do that (Figure 1.7). After the slice selection, all protons in this slice are precessing at the same Larmor frequency. Now if a linear gradient magnetic field is added along the y-axis, the protons along the y direction will experience different magnetic fields and thus have different Larmor frequencies. After a time delay, they will have different phases after the gradient field is turned off. Therefore, the phases of protons include the spatial information through the application of gradient magnetic field $G_y$. This technique is called phase encoding. If the slice is considered as a matrix, now the different rows have different phases after the application of $G_y$, but all protons in a row still have same phase. In order to distinguish between elements in a row, one more gradient field is needed along the x-axis — $G_x$. When $G_x$ is applied, the protons in a row will have different Larmor frequencies. This technique is called frequency encoding. In words, after the application of the slice selection gradient field $G_z$, the phase encoding gradient field $G_y$ and the frequency encoding gradient field $G_x$, the spatial information of the slice is encoded into the FID signal, then with a Fourier transform, an MR image can be reconstructed from the FID signal.
1.1.3.2 Imaging sequences

The imaging sequences describe the sequences of the RF pulses (90°, 180°, etc.) and gradients (Gₓ, Gᵧ, and G₂) used for imaging. Many different combinations exist for different purposes of imaging. Here only two major categories are introduced.

1.1.3.2.1 Spin echo

The diagram of a spin echo sequence is illustrated in Figure 1.8. A 90° degree slice selection pulse combined with slice selection gradient G₂ is applied firstly to excite the nuclei at the desired location to tip M down to the x-y plane. Notice a negative gradient in z is applied right after the positive slice selection gradient to compensate the de-phasing of the nuclei in the z direction caused by the positive gradient. A phase encoding gradient Gᵧ is applied immediately after the positive G₂ component and is followed by a 180° pulse. The 180° pulse is used to flip the spins 180° along the x or y axis so to refocus the spins de-phased by the inhomogeneity of the magnetic fields. This 90°-180° pulse combination is called spin echo. After the 180° refocusing pulse, a frequency encoding gradient, or read out gradient, Gₓ, is added when the FID signal is being read out. An additional gradient in the x direction is added at the same time as Gᵧ before the encoding Gₓ to pre-compensate the de-phasing of the nuclei by Gₓ. A positive gradient is used in the read-out direction, instead of a negative one as that in G₂, because later the 180° will invert the direction of the spin’s de-phasing. This sequence is repeated for different phase encoding gradient strengths (Figure 1.8) to scan the whole image.
space. For an image with $N \times N$ pixels, $N$ phase encoding steps are needed. The time between the adjacent two $90^\circ$ pulse is called repetition time (TR) and the time between the $90^\circ$ pulse and the readout gradient is called echo time (TE). Different combinations of the TR and TE are used in MRI to achieve different contrast for imaging. The spin echo imaging sequence can compensate the inhomogeneity of the magnetic field and is the basis of many clinical protocols but the imaging time might be a little longer than the gradient echo sequence, which will be discussed in the following section, because of the added $180^\circ$ pulse.
Figure 1.8 Time course of a spin echo imaging sequence. Horizontal lines next to $G_y$ indicate different phase encoding gradient strengths used for sequential scans.
1.1.3.2.2 Gradient echo

A gradient, instead of a 180° RF pulse in spin echo, can be used to focus the FID signal to be read out. A sequence with such a gradient is called gradient echo sequence. Figure 1.9 shows the diagram of a gradient echo sequence. The slice selection pulse and phase gradient are the same as those in the spin echo sequence. A negative read out gradient is added before the positive gradient to pre-compensate the de-phasing brought by the positive read out gradient. This negative gradient – positive gradient combination will also produce an echo during the signal reading out. The gradient echo require less data acquisition time and can compensate the dephasing effects caused by the read out gradient but cannot compensate for those caused by the inhomogeneous static magnetic field.
1.1.4 Signal to noise ratio

Signal to noise ratio (SNR) is one of the most important characteristics of MRI images. SNR is defined as the ratio of the mean image intensity within a volume of interest (VOI) to the standard deviation of background noise. SNR in MRS/MRI is a
function of static field strength, sample properties, excitation flip angle, receiving coil sensitivity, parameters of the imaging sequence such as spatial resolution, TR, and TE time. To simplify the problem, in this section the $SNR$ is considered only in one pixel and it is assumed that TR is long enough and TE is short enough so that no signal is lost in T1 and T2 relaxation.

### 1.1.4.1 Dependence on the field strength

The signal of NMR/MRI is the voltage or current generated in the receiving coil by the rotating $\mathbf{M}_{xy}$, the x-y component of the net magnetization $\mathbf{M}$, according to Boltzmann statistics, the difference of spin population at different energy levels is proportional to the external magnetic field strength, which means the net magnetization $\mathbf{M}$ is also proportional to the external magnetic field $B_0$. According to Faraday’s law, the voltage or current induced by a changing magnetic field is proportional to the frequency of the magnetic field, the Larmor frequency in the case of NMR/MRI, which is also proportional to the static field strength $B_0$. The principle of reciprocity (Hoult and Richards, 1976; Hoult and Lauterbur, 1979) states that the magnitude of the current induced in the receiving coil by the precessing nuclei is proportional to the magnitude of the $\mathbf{B}_1^-$, the hypothetical field in the receiving coil that rotates in the opposite direction of nuclei precession. The signal in NMR/MRI is the combination of all these terms:

$$M_{xy} = M \sin(\gamma B_1^+ \tau) \quad [1.6]$$
Signal \propto B_0^2 \sin(\gamma B_1^+ \tau) B_1^- \quad \text{[1.7]}

The noise in NMR/MRI comes from the electronic noise induced by the thermal motion of charges in the RF receiving coil and in the sample. The noise produced by a resistor of $R$ is

$$\text{Noise} = \sqrt{4kT\Delta \nu R} \quad \text{[1.8]}$$

where $k$ is the Boltzmann’s constant, $T$ is the absolute temperature of the resistor, and $\Delta \nu$ is the bandwidth. Since the resistance of the coil and the sample is hard to measure and the power dissipated in a resistor is proportional to the resistance when the current is fixed

$$P = I^2 R \propto R \quad \text{[1.9]}$$

$SNR$ can be written as

$$SNR \propto \frac{B_0^2 \sin(\gamma B_1^+ \tau) B_1^-}{\sqrt{4kT_{\text{coil}} \Delta \nu R_{\text{coil}} + 4kT_{\text{sample}} \Delta \nu R_{\text{sample}}}} \propto \frac{B_0^2 \sin(\gamma B_1^+ \tau) B_1^-}{\sqrt{4kT_{\text{coil}} \Delta \nu P_{\text{coil}} + 4kT_{\text{sample}} \Delta \nu P_{\text{sample}}}} \quad \text{[1.10]}$$

Here the coil and the sample are treated as two resistors in series. Eq. 1.10 is the general expression for $SNR$ in MRI. It can be simplified in certain frequency regimes.

1.1.4.2 $SNR$ at low fields

At low fields, where the quasi-static approach is valid and $B_1^+$ and $B_1^-$ distributions are relatively independent of the frequency or $B_0$, in the consideration of
$SNR$ as a function of $B_0$, the terms of $B_1^+$ and $B_1^-$ can be ignored and the signal can be expressed as

$$Signal \propto B_0^2 \quad [1.11]$$

The resistance of the coil is proportional to square root of the frequency because of skin depth effect (Pozar, 1990) and the equivalent resistance for the sample is proportional to the square of the frequency (Hoult and Lauterbur, 1979). The total noise can be expressed as

$$Noise \propto \sqrt{aB_0^2 + bB_0^2} \quad [1.12]$$

where $a$ and $b$ are the co-efficiencies for noise contribution from the coil and from the sample respectively.

$SNR$ at low fields can then be expressed as

$$SNR \propto \frac{B_0^2}{\sqrt{aB_0^2 + bB_0^2}} \quad [1.13]$$

In cases where the field strength is low enough or the sample is small enough that the coil noise dominates, the term for sample noise ($bB_0^2$) becomes negligible and

$$SNR \propto B_0^{\frac{7}{4}} \quad [1.14]$$

When the frequency goes higher or the sample becomes larger and the sample noise dominates, the term for the coil noise can be neglected and

$$SNR \propto B_0 \quad [1.15]$$
1.1.4.3 SNR at high fields

At high fields, where quasi-static assumptions are no longer valid, the distribution of $B_1^+$ and $B_1^-$ needs to be considered in the signal calculation. The sample noise is still dominant but the power dissipated in the sample is not necessarily proportional to the square of the frequency (Keltner et al., 1991; Wen et al., 1997; Collins and Smith, 2001; Liu et al., 2003). The SNR can be written as

$$SNR \propto \frac{B_0^2 \sin(\gamma B_1^+) B_1^-}{\sqrt{P_{\text{sample}}}}$$  \[1.16\]

For all the studies presented in this thesis, sample noise dominance is assumed, which is the case for human body imaging at field strength above 1.5 T.

1.1.5 Hardware

A typical MRI system includes at least a magnet, gradient coils, a transmitter, a receiver, an RF coil and a computer. The patient is put into the magnet. The gradient coils are used to produce the gradient magnetic fields $G_x$, $G_y$, and $G_z$ needed for imaging. The transmitter is used to generate and control the RF pulse and the RF coil is used to produce the $B_1$ field. The MRI signal is also picked up by the RF coil then transferred through the receiver and finally processed and stored in the computer. The computer controls the operation of the whole system (Figure 1.10). Each part is discussed in more detail below.
1.1.5.1 Magnet

The magnet, which is needed to produce a homogeneous static magnetic field $B_0$ within the imaging region, is the most important component in an MRI system. The trend of MRI systems towards higher magnetic fields for higher SNR has brought difficulties such as the inhomogeneity to magnet engineering. The cost also increases with the field strength. Three kinds of magnets are used in MRI systems: a) permanent magnets, b) electromagnets and c) superconducting magnets. The permanent magnets cost less but their field strength is limited (<1 T). Electromagnets are rarely used since the adoption of
the superconducting magnet. The main advantage of superconducting magnets is the higher magnetic field strengths (up to 20 T) and the main disadvantage is the high cost.

### 1.1.5.2 Gradient coils

The gradient coils are used for two purposes in an MRI system: a) to compensate for magnetic field inhomogeneity of the magnet, b) to provide the encoding gradient magnetic fields $G_x$, $G_y$, and $G_z$ that are needed for imaging. They are usually made from copper wires and are driven by their own power supplies and can be switched on and off under computer control. To make an image of high quality, the gradient coils should have high linearity and stability.

### 1.1.5.3 RF coils

There are also two purposes of an RF coil: a) to generate the $B_1$ field, b) to pick up the FID signal. Normally, the coil is tuned to the Larmor frequency and is matched to 50 ohms for efficient power delivery and signal reception by a matching circuit that connects the coil to the transmitter and to the preamplifier. To generate a uniform MRI image, the $B_1$ field needs to be homogeneous and to have a high SNR, the coil needs to have a high sensitivity to MRI signal. It is always difficult to achieve both a high homogeneity and a high sensitivity in an RF coil.

RF coils are one of the most critical parts in an MRI system since they are directly used in exciting the nuclei and receiving the signal. Poorly designed RF coils can
diminish the benefit of high field strength, which usually comes with a high cost. The RF
coil design has always been an interesting field of MRI research because of the
importance, the complexity, the relatively low cost, and the high benefits.

Three kinds of RF coils, surface coils, birdcage coils, and TEM coils, will be
discussed in the following section more specifically. These coils are widely used in MRI
systems and are the topics of the studies that will be presented in the later chapters.

1.1.5.4 Transmitter

The transmitter is used to produce the pulse sequences used in MRI. The most
critical requirement of the transmitter is to control the duration of RF pulse that may be
only several milliseconds long. Another requirement for the transmitter is to provide
proper RF power for different imaging sequences.

1.1.5.5 Receiver

The receiver is used to process the FID signal received by the RF coils, which are
connected to the receiver through preamplifiers. The processes include demodulation,
filtering, and quadrature detection. The processed data is then transformed to the
computer for image reconstruction.
1.1.5.6 Computer

The computer is the central part of the whole MRI system. It controls and synchronizes all the different parts of the system. It also reconstructs and displays the final images.

1.1.6 RF coils: surface coils, birdcage coils, and TEM coils

A surface coil is the simplest RF coil (Ackerman et al., 1980). It is a single loop, usually a circle or a rectangle, embedded with one or multiple capacitors in series to tune the circuit to the Larmor frequency. A matching capacitor is connected to the circuit to match the coil to 50 ohms. A schematic diagram of a surface coil is shown in Figure 1.11 (a). If no wavelength effects are assumed, the current in the surface coil will be a uniform loop current. The field it produces will be in the direction of the coil axis and decreases quickly along that direction (Figure 1.11 (b)). When the coil is used for receiving, this localized RF field prevents the coil detecting noise from the part out of imaging region in the sample thus resulting in high local SNR. Surface coils are used less as transmitting coils because of the limited penetration and the inhomogeneity of the RF field.
Figure 1.11 The circuit of a surface coil that has four fixed tuning capacitors \( C \), an adjustable matching capacitor \( C_m \), and an adjustable tuning capacitor \( C_t \) (a) and the decrease of the magnetic field along the axis of a surface coil with a radius of \( R \) (b).
Introduced by Hayes et al in 1985 (Hayes et al., 1985), birdcage coils have been widely used in NMR/MRI systems because of the highly homogeneous fields they produce (Tropp, 1989; Fitzsimmons et al., 1993; Roffmann et al., 1996; Dardzinski et al., 1998; Matson et al., 1999; Peterson et al., 1999; Akimoto and Candela, 2000; Fujita et al., 2000; Lanz et al., 2001). A typical birdcage coil has two end-rings that are connected by multiple straight elements (rungs, struts, legs) much like a “bird cage”. Capacitors can be placed in the rungs (low-pass), or in the end-rings (high-pass), or in both (band-pass) to tune the coil to the desired frequency. A larger co-axial conductive cylinder, the RF shield, is used to prevent coupling between the RF field within the coil and the environment (gradient coils, mostly). In chapters 2 and 3, the effects of end-ring/shield configuration on $B_1$ homogeneity and SNR in birdcage coils are evaluated. A schematic diagram of a low-pass birdcage coil without the shield and the equivalent circuit is shown in Figure 1.12.
A birdcage coil is a resonant circuit. A birdcage coil with 2N meshes, or legs, has N resonant modes. In mode 1, the currents, which are proportional to the sine of the
azimuthal angles, generate a very homogeneous field within the coil. The magnetic field distribution of mode 1 in the central axial plane of a 16-leg head-size shielded empty birdcage coil driven with one current source is shown in Figure 1.13. Birdcage coils can be used for both transmitting and receiving.

Figure 1.13 The magnetic field distribution of mode 1 in an empty birdcage coil driven with one source at 125 MHz (FDTD simulation).
TEM coils have played an important role in high field NMR/MRI (Vaughan et al., 1994; Baertlein et al., 2000; Vaughan et al., 2002; Ludwig et al., 2004). A TEM coil can be built by replacing each leg of a low-pass birdcage coil with a coaxial transmission line element, which is so constructed that the length of the inner conductors is adjustable and the ends of the inner conductors are connected to the shield with two solid rings. A schematic diagram of a TEM coil is shown in Figure 1.14. Generally, a TEM coil has one more mode than a low-pass birdcage coil and it is also the mode 1 that gives the TEM coil homogeneous field. TEM coils can be tuned to ultra high frequencies (up to 345 MHz for a head size coil) and the self-shielded structure helps to keep RF energy from radiating out. A study of the RF radiation in a TEM coil is presented in chapter 5 of this thesis.
Figure 1.14 A sketch of a TEM coil (a) shown with a cross section (b). The coil is tuned to the desired frequency with the adjustable inner conductor length.
1.1.7 MRI safety

In this section, the biological effects of the three kinds of magnetic fields: static fields, gradient fields, and RF fields in MRI systems will be discussed with the emphasis on RF fields.

1.1.7.1 Static magnetic fields

Most of the human body is diamagnetic, but magnetic susceptibility can vary with concentration of iron, which concentrates more magnetic flux. Magnetic field variation in the local environment might affect the metabolic rate of normal cells. It is generally accepted that the chemical reactions are accelerated in the area with high magnetic susceptibility and are slowed down with low susceptibility (Shung et al., 1992). At high field, alignment of molecules in a diamagnetic anisotropic environment has been demonstrated but no significant physiological impact has been shown.

Electrical charges traveling in a magnetic field whose direction is perpendicular to the direction of the travel experience a Lorentz force that is proportional to the field strength and the velocity of the charges. The blood flow in static magnetic fields will experience such a force although the force does not change the direction or velocity of the blood flow significantly. Moving a conductor in a magnetic field will also produce a current or a voltage in the conductor, so there will be voltage added to the heart cells by the blood flow. A small increase in the amplitude of the T-wave has been observed on ECG as a result, however, no biologic risks are believed to be associated with the change.
1.1.7.2 Gradient magnetic fields

In MRI, the gradient magnetic fields are turned on and off to encode the spatial information into the FID signal. The visual sensation of flashes of light caused by the magnetophosphenes in the eyes induced by extremely low frequency (ELF) fields has been reported (Barnothy, 1963) but not in the pulsed MRI yet. The changing magnetic fields will induce currents in the conductive biological tissues like nerves, skeletal and cardiac muscles. Those currents can override the neuronal control and cause the irregular contraction of the muscles or a sensation of being touched. Heart fibrillation might occur if the currents are high enough to depolarize the myocardial cells. Fortunately, there are no reports of heart malfunction due to MRI although the stimulation of skeletal muscle during echo-planar imaging has been observed.

1.1.7.3 RF fields

The most significant impact of RF fields on the biological tissues is the thermal effects caused by the RF energy absorption. The changing magnetic field produces currents in the conductive tissues and the RF energy dissipates into the tissue as heat. Specific energy absorption rate ($SAR$), which is defined as the RF power absorbed per unit of mass of an object (W/kg), is used to monitor the RF energy dissipated into tissues. There are several $SAR$s need to consider during MRI:

Whole body $SAR$:

$SAR$ averaged over the total mass of the patient’s body and over a specified time
Head SAR:

$SAR$ averaged over the mass of the patient’s head and over a specified time.

Local SAR:

$SAR$ averaged over any 10 grams of tissue of the patient’s body and over a specified time.

The $SAR$ limits set up by International Electrotechnical Commission (IEC, 2002) for the whole body $SAR$, head $SAR$, and local $SAR$ for different body part in normal operation mode are shown in Table 1-2.

Table 1-2 SAR limits in the normal mode of MRI.

<table>
<thead>
<tr>
<th>Averaging time</th>
<th>6 minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Body $SAR$ (W/kg)</td>
<td>Head $SAR$ (W/kg)</td>
</tr>
<tr>
<td>Body region</td>
<td>Whole body</td>
</tr>
<tr>
<td>Normal</td>
<td>2</td>
</tr>
</tbody>
</table>

The local $SAR$ measurement is important because the uneven current distribution in the heterogeneous tissues might cause higher $SAR$ and tissue damage in some “hot spots”
before the whole body SAR reaches the limit. The SAR at one location in tissue can be calculated as

\[
SAR = \frac{\sigma E^2}{2\rho}
\]  

Where \( \sigma \) is the conductivity of the tissue, \( E \) is the magnitude of the electrical field, and \( \rho \) is the mass density of the tissue. This result can be averaged over the volume of interest (head, whole body, or 10 gram region) to compare to IEC limits. Experimental measurement of local SAR is difficult, if possible, because of the unknown \( E \) field distribution inside tissues. Numerical methods with appropriate models have proven useful in local SAR calculations (Grandolfo et al., 1990; Jin and Chen, 1997; Chen et al., 1998; Collins et al., 1998; Gandhi and Chen, 1999; Collins and Smith, 2001; Ho, 2001; Ibrahim et al., 2001; Zhai et al., 2002; Liu et al., 2003; Collins et al., 2004). A study of direct comparison of SAR levels between different body types will be shown in chapter 4.
1.2 Basic Electromagnetic Theory and Resonant Circuits

In this section, the basic electromagnetic theory needed to understand the electromagnetic phenomena in MRI is reviewed. Firstly, the location of the RF fields used in MRI in the electromagnetic radiation spectrum is identified and the concept of frequency, wave number, and wavelength is introduced. Maxwell’s equations, which describe the behavior of electromagnetic waves, are then brought in. A brief review of electromagnetic fields in media will aid in understanding the interaction between the RF fields and the biological tissues. RF power distribution in MRI systems has always been a concern because it is related to both the patient’s safety and the SNR of the system. Section 1.2.4 will provide some background on the electromagnetic power calculation. In section 1.2.5 the principle of reciprocity, which was introduced by Hoult et al into MRI and has been widely used to estimate the SNR, will be discussed. Some concepts about the resonant circuits are introduced in the final section since all RF coils are resonant circuits.
1.2.1 Electromagnetic spectrum

A propagating electromagnetic wave is a function of both space and time,

\[ W(x,t) = W_0 \sin[2\pi(f t - \frac{x}{\lambda})] \]

where \( W \) is the wave function, \( W_0 \) is the amplitude of the wave, \( x \) is the distance the wave has traveled in meter, \( t \) is time in second, \( f \) is the frequency in cycle/second, \( \lambda \) is the wavelength in meter. The frequency and wavelength have the relation,

\[ f = \frac{c}{\lambda} \]

where \( c \) is the speed of the wave propagation. The electromagnetic spectrum shows the entire range of frequencies of the waves (Figure 1.15).

![Electromagnetic spectrum](image)

Figure 1.15 Electromagnetic spectrum.
On the spectrum, the frequency increases, so the wavelength decreases, from the radio frequency wave and microwave at the left to the visible lights in the middle, and to the X-ray and Gamma ray at the right. The energy carried by the wave increases with the frequency. In an MRI system, the frequency of the electromagnetic field needed to excite the nuclei falls within the radio frequency to low microwave frequency range. That is why it is called a radiofrequency field, or RF field. For a 1.5 Tesla MRI system, the frequency is about 64 MHz and for an 8 Tesla MRI system, the strongest system has been used for human imaging, the frequency is about 345 MHz. The typical frequencies of the electromagnetic waves used in daily life are shown in Figure 1.15. The local public radio station WITF operates at 89.5 MHz. A typical cell phone works at 900 MHz. A typical microwave oven works at 2450 MHz and X-ray works at $10^{12}$ MHz. Unlike X-ray or CT, MRI works within a non-ionizing frequency range.

**1.2.2 Maxwell’s equations**

Published by James C. Maxwell in 1873 based on the works of Gauss, Ampere, Faraday, and others, Maxwell’s equations describe the behavior of all electromagnetic waves:

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad [1.20]$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J \quad [1.21]$$

$$\nabla \cdot D = \rho \quad [1.22]$$

$$\nabla \cdot B = 0 \quad [1.23]$$
where 

- \( \mathbf{E} \) is the electric field intensity, in V/m,
- \( \mathbf{B} \) is the magnetic flux density, in Wb/m²,
- \( \mathbf{H} \) is the magnetic field intensity, in A/m,
- \( \mathbf{D} \) is the electric flux density, in Coul/m²,
- \( \mathbf{J} \) is the electric current density, in A/m²,
- \( \rho \) is the electric charge density, in Coul/m³,

The first equation, or Faraday’s law, states that a changing magnetic field produces an electric field. The second equation states that a magnetic field is produced by conductive currents (\( \mathbf{J} \)) or displacement currents (\( \frac{\partial \mathbf{D}}{\partial t} \), the major contribution of Maxwell). The third and fourth equations state the ultimate source of electromagnetic field is electric charges. These four equations are not independent. If it is assumed at some initial time there are no fields anywhere in space, the divergence of both sides of Eq. \( 1.20 \) gives,

\[
\nabla \cdot \nabla \times \mathbf{E} = \nabla \cdot \left( -\frac{\partial \mathbf{B}}{\partial t} \right) \quad [1.24]
\]

since the divergence of the curl of any vector is zero,

\[
0 = \nabla \cdot \left( -\frac{\partial \mathbf{B}}{\partial t} \right) = -\frac{\partial \nabla \cdot \mathbf{B}}{\partial t} \quad [1.25]
\]
\[
\Rightarrow \quad \nabla \cdot \mathbf{B} = 0 \quad [1.26]
\]

Same treatment can deduce Eq. \( 1.22 \) from Eq. \( 1.21 \). So only the first two equations, or Maxwell’s curl equations, need to be considered in the calculation of time-varying electromagnetic fields, which is the case in the FDTD algorithm.
The above equations are for any arbitrary waveforms of electromagnetic field. For a more generally used form, the steady state sinusoidal waves, Maxwell’s equations can be written as

\begin{align*}
\nabla \times E &= -i\omega B \quad [1.27] \\
\nabla \times H &= i\omega D + J \quad [1.28] \\
\n\nabla \cdot D &= \rho \quad [1.29] \\
\n\nabla \cdot B &= 0 \quad [1.30] \\
\end{align*}

where \( i \) is \( \sqrt{-1} \) and \( \omega \) is the angular frequency of the sinusoidal electromagnetic wave.

Sometimes, a hypothetical magnetic current density \( \mathbf{M} \) (\( \text{V/m}^2 \)) is included in the first Maxwell’s equation for a mathematical convenience to represent current loops or some similar magnetic dipole

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} - \mathbf{M} \quad [1.31] \]

For the steady state sinusoidal waves

\[ \nabla \times \mathbf{E} = -i\omega \mathbf{B} - \mathbf{M} \quad [1.32] \]

The Eq. 1.31 will be used to introduce the principle of reciprocity in section 1.2.5. Maxwell’s equations can also be written in integral instead differential form, which will not be discussed here but can be found in almost all electromagnetic textbooks (Lorrain and Corson, 1970; Pozar, 1990; Jackson, 1999).
1.2.3 Electromagnetic fields in media

Linear, isotropic, and nondispersive materials (materials that have field-independent, direction-independent, and frequency-independent electric and magnetic properties) are assumed throughout this work. In the radiofrequency range, biological tissues are generally isotropic. The electric and magnetic properties of biological tissues in MRI are generally frequency-dependent but since only a single frequency (the Larmor frequency) is needed in each calculation, the assumption is still valid. Without an external electric field, the electric dipole moments \( P_e \) in a linear isotropic dielectric material are randomly distributed and the net polarization vector \( P_e \) is zero (Figure 1.16 (a)). When the media is placed in an electric field, the electric dipole moments will align themselves with the external field and produce a polarization vector \( P_e \).

\[
P_e = \varepsilon_0 \chi_e E
\]

[1.33]

where \( \chi_e \) is called the electric susceptibility (Figure 1.16 (b)). The electric field flux density, \( D \), can be written as

\[
D = \varepsilon_0 E + P_e = \varepsilon_0 E + \varepsilon_0 \chi_e E = \varepsilon_0 (1 + \chi_e) E = \varepsilon_0 \varepsilon_r E = \varepsilon E
\]

[1.34]

where \( \varepsilon_r = (1 + \chi_e) \) is the relative electric permittivity and \( \varepsilon \) is the electric permittivity.

A similar treatment can be performed for linear isotropic magnetic materials. The magnetic dipole moments line up with the applied magnetic field \( H \) to produce a magnetic polarization (or magnetization) \( P_m \) that is linearly related to the field \( H \):

\[
P_m = \chi_m H, \text{ where } \chi_m \text{ is the magnetic susceptibility.}
\]

The magnetic field flux density can then be written as
\[ B = \mu_0(H + P_m) = \mu_0(1 + \chi_m)H = \mu H \]  \[1.35\]

where \( \mu = \mu_0(1 + \chi_m) \) is the permeability of the medium. Eq. 1.34 and Eq. 1.35 represent constitutive relations.

The permittivity is a measurement of the polarization of electric dipoles of a medium in an external electric field as the conductivity is a measurement of free charges in the media. Throughout this work, the magnetic permeability is assumed equal to that of free space for all tissues since the effects of \( \mu \) on MRI at radiofrequencies is many orders of magnitude less than those of \( \sigma \) and \( \varepsilon \). To accurately estimate the electromagnetic field in the heterogeneous human body with numerical methods, an anatomically accurate body model with the appropriate electrical properties is needed. Making numerical models and calculating the RF fields will be discussed more in chapter 4.

Figure 1.16 The polarization of electric dipole moments in a media without (a) and with (b) the presence of an external electric field \( E \).
1.2.4 Energy and Poynting theorem

The Poynting theorem dictates energy conservation for electromagnetic fields and sources. In a linear, isotropic, nondispersive volume $V$ that contains electric field $E$ and magnetic field $H$, source current $J_s$, conductivity $\sigma$, letting permittivity $\varepsilon = \varepsilon' - j\varepsilon''$ and permeability $\mu = \mu' - j\mu''$ to be complex to calculate loss, the Poynting theorem can be written as,

$$P_s = P_o + P_h + 2j\omega(W_m - W_e) \quad [1.36]$$

where

$$P_s = -\frac{1}{2} \iiint_E E \cdot J_s^* \, dv \quad [1.37]$$

is the power delivered by the source $J_s$,

$$P_o = \frac{1}{2} \oint (E \times H^*) \cdot ds = \frac{1}{2} \oint S \cdot ds \quad [1.38]$$

is the power flow out of the surface $S$ that encloses the volume $V$; $S = E \times H^*$ is called the Poynting vector, where $*$ indicates the conjugate,

$$P_h = \frac{\sigma}{2} \iiint |E|^2 \, dv + \frac{\omega}{2} \iiint (\varepsilon''|E|^2 + \mu''|H|^2) \, dv \quad [1.39]$$

is the power dissipated as heat in the volume, and

$$W_m = \frac{1}{4} \text{Re} \iiint H \cdot B^* \, dv \quad [1.40]$$

$$W_e = \frac{1}{4} \text{Re} \iiint E \cdot D^* \, dv \quad [1.41]$$
are the time average magnetic energy and electric energy stored in the volume respectively. In words, the power delivered by the source in a volume $V$ is equal to the sum of the power flow out of a surface enclosing the volume, the power dissipated as heat in the volume, and $2\omega$ times the net reactive energy stored in the volume. The real part of $P_s$ and $P_o$ is time-average power.

An MRI system includes the RF sources (coils) and the dielectric materials (biological tissues). For the patient’s safety, the power dissipated in the body as heat has to be monitored closely. The power radiated out of the system, which can be calculated by integrating the Poynting vector over a surface that encloses the RF system, is also of great interests because it is related to the $SNR$ calculation. These topics will be explored more in chapters 4 and 5.

1.2.5 Principle of reciprocity

The principle of reciprocity is used to describe the relationship between the fields produced by two sets of sources. Consider two separate fields $E_1$, $H_1$, and $E_2$, $H_2$ generated by two sets of sources $J_1$, $M_1$ and $J_2$, $M_2$. In MRI, $M_1$ and $M_2$ can represent the spin magnetization $M$. Maxwell’s curl equations for these two sets of sources and fields can be written in phasor notation as

$$\nabla \times E_1 = -j\omega M_1$$  \hspace{1cm} [1.42]

$$\nabla \times H_1 = j\omega E_1 + J_1$$  \hspace{1cm} [1.43]

$$\nabla \times E_2 = -j\omega M_2$$  \hspace{1cm} [1.44]

$$\nabla \times H_2 = j\omega E_2 + J_2$$  \hspace{1cm} [1.45]
Using the vector identity $\nabla \cdot (A \times B) = (\nabla \times A) \cdot B - (\nabla \times B) \cdot A$ it is found

$$
\nabla \cdot (E_1 \times H_2) = (\nabla \times E_1) \cdot H_2 - (\nabla \times H_2) \cdot E_1 \\
= -j \omega \mu H_2 \cdot E_1 - M_2 \cdot H_2 - j \omega \varepsilon E_2 \cdot E_1 - J_2 \cdot E_1 \tag{1.46}
$$

$$
\nabla \cdot (E_2 \times H_1) = (\nabla \times E_2) \cdot H_1 - (\nabla \times H_1) \cdot E_2 \\
= -j \omega \mu H_1 \cdot E_2 - M_1 \cdot H_1 - j \omega \varepsilon E_1 \cdot E_2 - J_1 \cdot E_2 \tag{1.47}
$$

Then subtracting Eq. 1.47 from Eq. 1.46

$$
\nabla \cdot (E_1 \times H_2 - E_2 \times H_1) = -E_1 \cdot J_2 + E_2 \cdot J_1 + H_1 \cdot M_2 - H_2 \cdot M_1 \tag{1.48}
$$

and applying the divergence theorem,

$$
\iiint \nabla \cdot (E_1 \times H_2 - E_2 \times H_1) \, dv = \iiint (E_1 \times H_2 - E_2 \times H_1) \cdot \hat{n} \, ds \tag{1.49}
$$

the general form of the reciprocity theorem can be derived as

$$
\iiint (E_1 \times H_2 - E_2 \times H_1) \cdot \hat{n} \, ds = \iiint (-E_1 \cdot J_2 + E_2 \cdot J_1 + H_1 \cdot M_2 - H_2 \cdot M_1) \, dv \tag{1.50}
$$

when the surface $S$ is a sphere at infinity, the fields can be treated as plane waves that have the relations

$$
E_1 = \frac{\sqrt{\mu}}{\varepsilon} H_1 \times \hat{n} \tag{1.51}
$$

$$
E_2 = \frac{\sqrt{\mu}}{\varepsilon} H_2 \times \hat{n} \tag{1.52}
$$

substituting Eq. 1.51 and Eq. 1.52 into Eq. 1.50, the surface integral at the left side vanishes. The form of reciprocity theorem is then simplified as

$$
\iiint (E_1 \cdot J_2 - H_1 \cdot M_2) \, dv = \iiint (E_2 \cdot J_1 - H_2 \cdot M_1) \, dv \tag{1.53}
$$

In MRI, consider the field produced by a unit current in the receiving RF coil to be field one and the field produced by the magnetization $M$ to be field two so
With the substitution of Eq. 1.54 and \( B_1 = \mu_0 H_1 \), the left side of Eq. 1.53 becomes

\[
I_1 = \oint J_1 \cdot ds = 1 \text{ Ampere}
\]

\[ M_1 = 0 \]
\[ J_2 = 0 \]
\[ M_2 = j \omega \mu_0 M \]

[1.54]

and the right side becomes

\[
\int \int \int - H_1 \cdot M_2 dv = - j \omega \int \int \int B_1 \cdot M dv
\]

[1.55]

where \( V_2 \) is the voltage induced by the magnetization \( M \) in the RF coil. Equating these two sides gives

\[
V_2 = - j \omega \int \int \int B_1 \cdot M dv
\]

[1.57]

This equation is the formula introduced into MRI by Hoult and Richards (Hoult and Richards, 1976). It is accurate when the wavelength is much larger than the dimension of the RF coil-sample system and the phase of the fields in the sample does not change significantly. When the frequency is high enough that the phase difference in the sample cannot be ignored any more, a more precise approach should be taken by considering the effects of the different components of the \( B_1 \) field. Detailed deduction of formula for high fields can be found in (Hoult, 2000). The applications of the formula can be seen in chapters 2 and 4.
1.2.6 Resonant circuits

When the capacitive reactance is equal to the inductive reactance in a circuit at a certain frequency, the circuit is said to be in resonance and the frequency is called the resonant frequency. The resonance can be either series or parallel.

Figure 1.17 (a) shows the diagram of a series resonant circuit with a source $E$, a capacitor $C$, an inductor $L$, and a resistor $R$. The resonant frequency is determined by the following equation

$$f = \frac{1}{2\pi \sqrt{LC}}$$  \[1.58\]

The impedance of the circuit has the minimum value of $R$ at resonance (Figure 1.17 (b)). For a given $V$, the current in the series circuit is the highest at the resonant frequency. The quality factor $Q$ of a resonant circuit is defined as the ratio of the energy stored in the circuit to the energy dissipated in the circuit.
The higher the $Q$, the more efficiently the circuit stores energy. In MRI, since the noise is associated with the energy loss in the coil, high $Q$ resonant circuits are desirable. Notice the voltage across the capacitor in a series circuit is about $Q$ times the applied voltage. This high voltage might cause flashover of the capacitor.

The diagram of a parallel resonant circuit, or a “tank” circuit, is shown in Figure 1.18 with the impedance-frequency plot. The equations for the resonant frequency and $Q$ of parallel circuits are same as those for series circuits. The impedance of the parallel resonant circuit has the maximum value of $(2\pi fL)^2/R$, or $2\pi fLQ$, at the resonant frequency. Two currents exist in the circuit: A small line current $I_l$ and a circulating current $I_c$ that flows within the parallel $LCR$ portion of the circuit. The circulating current is about $Q$ times the line current. To achieve a high $Q$ parallel circuit, a minimized resistance $R$ and maximized $L/C$ ratio are desired.

$$Q = \frac{2\pi fL}{R}$$

[1.59]

Figure 1.18 The diagram (a) and the impedance change with frequency (b) of a parallel resonant circuit. The impedance of the circuit is defined as $V/I_l$. 
1.3 Finite Difference Time Domain (FDTD) Method for Electromagnetism

The increase of RF frequency with the MRI field strength (up to 8 T for human imaging) makes the quasi-static assumptions used at low fields no longer valid. Full wave Maxwell’s equations need to be solved to accurately model the system. The complex shape of RF coils and samples (head, body, etc.) and the interaction between them make analytic approaches unrealistic. The Finite difference time domain (FDTD) method for electromagnetism (Yee, 1966) is a powerful numerical tool to study the RF system at high fields.

1.3.1 Formula

The FDTD method uses the finite difference approximations to replace the derivatives in space and time in Maxwell’s curl equations so they can be solved directly on a computer in the time domain for arbitrary geometries and material electrical properties. In linear, isotropic, nondispersive materials, Maxwell’s curl equations can be written as

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \\
\frac{\partial \varepsilon \mathbf{E}}{\partial t} = -\sigma \mathbf{E} + \frac{\nabla \times \mathbf{B}}{\mu}
\]

where \( \mathbf{B} \) is magnetic flux density, \( t \) is time, \( \mathbf{E} \) is electrical field intensity, \( \varepsilon \) is electrical permittivity, \( \sigma \) is electrical conductivity, and \( \mu \) is magnetic permeability. In Cartesian coordinates system,
\[
\frac{\partial}{\partial t}(B_i + B_j + B_k) = -\left[\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_z}{\partial z}\right)i + \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_z}{\partial y}\right)j + \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_z}{\partial y}\right)k\right]
\]
\[
\frac{\partial}{\partial t} \varepsilon(E_i + E_j + E_k) = -\sigma(E_i + E_j + E_k) + \frac{1}{\mu} \left[\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)i + \left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}\right)j + \left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial y}\right)k\right]
\]

which can be separated into six equations in the three coordinates:

\[
\frac{\partial B_x}{\partial t} = -\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_z}{\partial z}\right)
\]
\[
\frac{\partial B_y}{\partial t} = -\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_z}{\partial y}\right)
\]
\[
\frac{\partial B_z}{\partial t} = -\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_z}{\partial y}\right)
\]
\[
\frac{\partial \varepsilon_x}{\partial t} = -\sigma \varepsilon_x + \frac{1}{\mu} \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)
\]
\[
\frac{\partial \varepsilon_y}{\partial t} = -\sigma \varepsilon_y + \frac{1}{\mu} \left(\frac{\partial B_x}{\partial y} - \frac{\partial B_x}{\partial y}\right)
\]
\[
\frac{\partial \varepsilon_z}{\partial t} = -\sigma \varepsilon_z + \frac{1}{\mu} \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_y}{\partial x}\right)
\]

Substituting finite difference approximations for the partial derivative with respect to time,

\[
\frac{\partial f}{\partial t} = \frac{f(i,j,k,n+1) - f(i,j,k,n-1)}{2\Delta t}
\]

and for the spatial derivatives,

\[
\frac{\partial f}{\partial x} = \frac{f(i+1,j,k,n) - f(i-1,j,k,n)}{2\Delta x}
\]
\[
\frac{\partial f}{\partial y} = \frac{f(i,j+1,k,n) - f(i,j-1,k,n)}{2\Delta y}
\]
\[
\frac{\partial f}{\partial z} = \frac{f(i,j,k+1,n) - f(i,j,k-1,n)}{2\Delta z}
\]
where \( f \) is one of \( \mathbf{B} \) or \( \mathbf{E} \) field components, \( i, j, \) and \( k \) are indices for positions on the 3D grid in the \( x, y, \) and \( z \) directions, \( n \) is the number of time step, \( \Delta \), is the size of time step, \( \Delta_x, \Delta_y, \) and \( \Delta_z \) are the spatial grid cell size in \( x, y, \) and \( z \) directions respectively, Eq. \( 1.62 \) becomes:

\[
\frac{B_x^{(i,j,k,n+1)} - B_x^{(i,j,k,n-1)}}{2\Delta_t} = \frac{E_y^{(i,j,k,n+1)} - E_y^{(i,j,k,n-1)}}{2\Delta_z} - \frac{E_z^{(i,j,k,n+1)} - E_z^{(i,j,k,n-1)}}{2\Delta_y}
\]

\[
\frac{B_y^{(i,j,k,n+1)} - B_y^{(i,j,k,n-1)}}{2\Delta_t} = \frac{E_z^{(i,j,k,n+1)} - E_z^{(i,j,k,n-1)}}{2\Delta_x} - \frac{E_x^{(i,j,k,n+1)} - E_x^{(i,j,k,n-1)}}{2\Delta_z}
\]

\[
\frac{B_z^{(i,j,k,n+1)} - B_z^{(i,j,k,n-1)}}{2\Delta_t} = \frac{E_x^{(i,j,k,n+1)} - E_x^{(i,j,k,n-1)}}{2\Delta_y} - \frac{E_y^{(i,j,k,n+1)} - E_y^{(i,j,k,n-1)}}{2\Delta_x}
\]

\[
\varepsilon \frac{E_x^{(i,j,k,n+1)} - E_x^{(i,j,k,n-1)}}{2\Delta_t} = -\sigma \varepsilon \frac{E_x^{(i,j,k,n)}}{2\Delta_t}
\]

\[
+ \frac{1}{\mu} \left[ \frac{B_z^{(i,j,k,n+1)} - B_z^{(i,j,k,n-1)}}{2\Delta_z} - \frac{B_y^{(i,j,k,n+1)} - B_y^{(i,j,k,n-1)}}{2\Delta_y} \right]
\]

\[
\varepsilon \frac{E_y^{(i,j,k,n+1)} - E_y^{(i,j,k,n-1)}}{2\Delta_t} = -\sigma \varepsilon \frac{E_y^{(i,j,k,n)}}{2\Delta_t}
\]

\[
+ \frac{1}{\mu} \left[ \frac{B_x^{(i,j,k,n+1)} - B_x^{(i,j,k,n-1)}}{2\Delta_x} - \frac{B_z^{(i+1,j,k,n)} - B_z^{(i,j,k,n)}}{2\Delta_z} \right]
\]

\[
\varepsilon \frac{E_z^{(i,j,k,n+1)} - E_z^{(i,j,k,n-1)}}{2\Delta_t} = -\sigma \varepsilon \frac{E_z^{(i,j,k,n)}}{2\Delta_t}
\]

\[
+ \frac{1}{\mu} \left[ \frac{B_y^{(i+1,j,k,n)} - B_y^{(i,j,k,n)}}{2\Delta_y} - \frac{B_x^{(i,j,k,n+1)} - B_x^{(i,j,k,n-1)}}{2\Delta_x} \right]
\]
These are the basic equations of FDTD methods. The actual algorithm used in electromagnetism computation, Yee algorithm, is more efficient by staggering the field components in space by half cell size and in time by half time step (Yee, 1966):
The positions of \textbf{E} and \textbf{B} components in a Yee cube are shown in Figure 1.19. The \textbf{E} components are located in the middle of the edges and the \textbf{B} components are located at the center of the faces. Every \textbf{E} component is surrounded by four circulating \textbf{B} components and every \textbf{B} component is surrounded by four circulating \textbf{E} components. \textbf{E} components in the whole space at time point \( n \) can be calculated using \textbf{E} components at step \( n-1 \) and \textbf{B} components at \( n-1/2 \). \textbf{B} components at \( n+1/2 \) can then calculated by using \textbf{B} components at \( n-1/2 \) and \textbf{E} components just calculated at \( n \). The \textbf{E} components at \( n+1 \) can then be calculated using \textbf{E} components at \( n \) and \textbf{B} components at \( n+1/2 \). Calculation continues until the final time step is reached. With this leapfrog approach, if all \textbf{E} components and \textbf{B} components at or before time \( t = 0 \) are assumed zero, with appropriately set \textbf{E} components at drive sources at each time step and proper boundary conditions, any field component at any spatial point at any future time point can be solved.
1.3.2 Boundary conditions

The formulation developed in the previous section is suitable for infinite space, however, electromagnetic fields can be solved only in a limited region in space because of the limited storage of a computer. For problems defined in open space, a boundary condition on the outer perimeter of the region is needed to simulate the infinite space.

Radiation boundary conditions (RBCs) or absorbing boundary conditions (ABCs) simulate the infinite space by absorbing all the electromagnetic waves that reach the outer

Figure 1.19 E components (green) and B components (red) in a Yee cell.
boundary of the problem region. In this section, only the Liao boundary condition (Liao et al., 1984), an ABC used in all the calculations in this work, is introduced using notation published previously (Taflove and Hagness, 2000), where the theory for other boundary conditions can also be found.

1.3.2.1 Liao boundary condition

A boundary condition is needed to calculate the field components at the outer boundary $x_{\text{max}}$ at time $t+\Delta t$, $u(x_{\text{max}}, t+\Delta t)$, using the fields that are already available in the computer memory. This cannot be directly calculated with the FDTD formula derived in 1.3.1 because the central difference approximations require information from one half cell beyond $x_{\text{max}}$ which is not available. The Liao boundary condition uses extrapolation in space and time to calculate $u(x_{\text{max}}, t+\Delta t)$.

Consider $L$ known fields $u_l$ within the problem region and are positioned in a straight line perpendicular to the boundary with a uniform spacing $h=\alpha c \Delta t$, where $0\leq \alpha \leq 2$ and $\Delta t$ is the time delay between the fields:
With these $L$ known field values, the field at an arbitrary point $\ell$, $u_\ell = u(x_{\text{max}} - \ell h, t)$, $t-(\ell - 1)\Delta t)$, where $\ell$ is a real number between 1 and $L$, can be estimated by using backward-difference interpolating polynomial originating at $u_1$. In general, a Gregory-Newton backward-difference polynomial originating at 0 for a function $f(x)$ can be written as (Hornbeck, 1975):

$$f(x) = f(0) + x\Delta f_0 + \frac{x(x+1)}{2!}\Delta^2 f_0 + \frac{x(x+1)(x+2)}{3!}\Delta^3 f_0 + ...$$  \[1.68\]

where $\Delta f_0$, $\Delta^2 f_0$, $\Delta^3 f_0$... are the backward differences at 0 defined as follows:

$$\Delta^m f_0 = \sum_{i=0}^{m} (-1)^i C_i^m f(i)$$  \[1.69\]

where $C_i^m$ is the binomial coefficient defined by

$$C_i^m = \frac{m!}{(m-i)i!}$$  \[1.70\]

and $f(i)$ is the known data point at $i$. Letting $\beta = 1-\ell$, then $1-L\leq\beta\leq0$, and
\[ u(\ell) \bigg|_{1 \leq \ell \leq L} = u(\beta) \bigg|_{\beta = 0} + \beta \Delta u \bigg|_{\beta = 0} + \frac{\beta(\beta+1)}{2!} \Delta^2 u \bigg|_{\beta = 0} + \frac{\beta(\beta+1)(\beta+2)}{3!} \Delta^3 u \bigg|_{\beta = 0} + \ldots \]

\[ \approx u \bigg|_{\beta = 0} + \beta \Delta u \bigg|_{\beta = 0} + \frac{\beta(\beta+1)}{2!} \Delta^2 u \bigg|_{\beta = 0} + \frac{\beta(\beta+1)(\beta+2)}{3!} \Delta^3 u \bigg|_{\beta = 0} + \ldots + \frac{\beta(\beta+1)(\beta+2)\ldots(\beta+L-2)}{(L-1)!} \Delta^{L-1} u \bigg|_{\beta = 0} \]

\[ u(\ell) \bigg|_{\tau = 1} = u \bigg|_{\tau = 1} + \beta \Delta u \bigg|_{\tau = 1} + \frac{\beta(\beta+1)}{2!} \Delta^2 u \bigg|_{\tau = 1} + \frac{\beta(\beta+1)(\beta+2)}{3!} \Delta^3 u \bigg|_{\tau = 1} + \ldots + \frac{\beta(\beta+1)(\beta+2)\ldots(\beta+L-2)}{(L-1)!} \Delta^{L-1} u \bigg|_{\tau = 1} \]

Where \( \Delta^m u \bigg|_{\tau = 1} = \sum_{i=1}^{m+1} (-1)^i i^i C_i \Delta_i u(x_{\text{max}} - i h, t - (i-1)\Delta t) \) is the backward difference defined in Eq. 1.69. This is the interpolating polynomial for an arbitrary point within the range \( 1 \leq \ell \leq L \). The Liao boundary condition can be realized by extrapolating this polynomial to \( \ell = 0 \), the desired field \( u_0 \) at the boundary,

\[ u_0 = u(x_{\text{max}} t + \Delta t) = u \bigg|_{\tau = 1} + \beta \Delta u \bigg|_{\tau = 1} + \frac{\beta(\beta+1)}{2!} \Delta^2 u \bigg|_{\tau = 1} + \frac{\beta(\beta+1)(\beta+2)}{3!} \Delta^3 u \bigg|_{\tau = 1} + \ldots + \frac{\beta(\beta+1)(\beta+2)\ldots(\beta+L-2)}{(L-1)!} \Delta^{L-1} u \bigg|_{\tau = 1} \]

Since \( \beta = 1 \), \( \tau = 1 \), the equation can be written as

\[ u_0 = u(x_{\text{max}} t + \Delta t) = u \bigg|_{\tau = 1} + \Delta u \bigg|_{\tau = 1} + \Delta^2 u \bigg|_{\tau = 1} + \Delta^3 u \bigg|_{\tau = 1} + \ldots + \Delta^{L-1} u \bigg|_{\tau = 1} \]

This equation is the Liao ABC of order \( L \) implemented at \( x_{\text{max}} \).

The maximum amplitude error for a plane wave of wavelength \( \lambda \) incident upon the ABC specified by Eq. 1.73 is given by
For a typical setting in the calculations in this work, $c\Delta t = 0.0012\lambda$, $L=2$, the amplitude error is about $5.7 \times 10^{-3} \%$.

In general, $\alpha$ is set between 0.5 and 2 and quadratic interpolation is used to find the known field values for the polynomial. Sometimes $\alpha$ can be chosen such that the field values are explicitly computed in the Yee grid. In all the calculations presented here, $\alpha$ is determined empirically by the software package XFDTD.

### 1.3.3 Stability and accuracy

In the FDTD method, computational stability requires that

$$\frac{1}{\sqrt{\left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}\right)}} > c\Delta t$$

where $c$ is the maximum light velocity in the region concerned. The central-difference approach in space and time used in Yee algorithm has second-order accuracy, or the error of the approximation is proportional to the square of $\Delta t$ or $\Delta x$. Since $\Delta t$ is restricted by the size of Yee cell because of the stability requirement, the accuracy of the calculations can be mainly controlled by the Yee cell size. Normally, the Yee cell dimensions are chosen to be equal or less than 0.1 of the wavelength to achieve desired accuracy.

The FDTD method is straightforward to realize on the computer and the electric properties of media can be easily included into the calculation by assigning proper values
to the conductivity $\sigma$ and permittivity $\varepsilon$ to spatial points. This feature is very useful for
the modeling of human tissues exposed to RF fields in MRI, which is shown in the study
presented in chapters 2 and 4. All the calculations done in this work were set up with the
aid of a commercially available software package XFDTD (Remcom, State College, PA,
USA).
Chapter 2

Effects of End-Ring/Shield Configuration on Homogeneity and Signal to Noise Ratio in a Birdcage-type Coil Loaded with a Human Head: Numerical Calculations
2.1 Abstract

Four different end-ring/shield configurations of a birdcage coil are modeled to examine their effects on RF magnetic ($B_1$) field homogeneity and $SNR$ at 64 MHz and 125 MHz. Configurations include a) a conventional cylindrical shield (conventional), b) a shield with annular extensions to closely shield the end-rings (surrounding shield), c) a shield with annular extensions connected to the rungs (solid connection), and d) a shield with thin wires connected to the rungs (thin wire connection). At both frequencies, the coil with conventional end-ring/shield configuration produces the most homogeneous $B_1$ field when the coil is empty, but produces the least homogeneous $B_1$ field when the coil is loaded with a human head. The surrounding shield configuration results in the most homogeneous $B_1$ and highest $SNR$ in the coil loaded with the human head at both frequencies, followed closely by the solid connection configuration.
2.2 Introduction

In radiofrequency (RF) coil design for MRI systems, signal-to-noise ratio (SNR) and homogeneity of the RF magnetic \( (B_1) \) field, which affects the uniformity of the image, are two of the most important considerations. They are affected by many factors including the current pattern in the coil and the interaction between the fields and the sample. In the absence of wavelength effects, a perfectly homogeneous \( B_1 \) field can be generated within an infinitely long cylinder having surface currents parallel to the cylinder’s axis and proportional to the sine of the azimuthal angles (Figure 2.1). However, the cylinder cannot be infinitely long in practice and there must be some current return path. The currents can return through two end-rings (Figure 2.2), as in a conventional birdcage coil (Hayes et al., 1985); or through the shield (Vaughan et al., 1994; Beck et al., 2000; Ludwig et al., 2002), as in a TEM coil (Vaughan et al., 1994); or through thin wire loops (Wen et al., 1994) (Figure 2.3). The objective of this study is to estimate the effects of these different current patterns on the \( B_1 \) field distribution and SNR in a birdcage coil.
Figure 2.1 Rung currents that produce a homogeneous $B_1$ field in an infinitely long cylinder. Green arrows indicate the direction and the numbers are magnitudes of the currents.
Figure 2.2 The current distribution in a birdcage coil. Green arrows indicate the direction and the numbers are magnitudes of the currents.
Four birdcage coils, identical except for the end-ring/shield configurations, which produce different available current return paths (Figure 2.4), are modeled. The first is a birdcage coil with the conventional cylindrical shield (conventional). The currents in the
rungs can only return through the coil’s two end-rings. The second is a birdcage coil with a shield having conductive annular extensions to closely shield the end-rings (surrounding shield). In this coil, the currents also can only return through the two end-rings, but since the end-rings are closely shielded, there might be significant eddy currents induced in the shield that can affect the RF field. The third coil is a birdcage coil with a shield having conductive annular extensions connected to the rungs (solid connection). The currents can return either through the shield or through the solid annular extensions in a way similar to returning through the end-rings in a conventional birdcage coil. The fourth coil is a birdcage coil with a shield having individual thin wire connections between the shield and the coil’s rungs (thin wire connection). The currents in this coil can only return in a path through the shield. The $B_1$ field distribution and $SNR$ in these four coils are examined at 64 MHz and 125 MHz \( (1.5 \, \text{T and 2.93 T, the systems in our lab}) \) using numerical methods.
2.3 Methods

All coils are identical (27 cm coil diameter, 34 cm shield diameter, and 22 cm length) 16-rung low-pass birdcage coils except for the end-ring/shield configurations.
Each of them was loaded with an anatomically accurate human head model (Collins and Smith, 2001) having 18 tissues with corresponding mass density, water content by percent mass, and electrical permittivity and conductivity at either frequency (Figure 2.5). A part of the shoulder about 4 cm thick was included in the head model to avoid a sharp transition of fields at the end of the neck (Jin et al., 1996; Collins et al., 1998; Kowalski et al., 2000). Sixteen voltage sources, which have the same amplitude and phases equal to the azimuthal angles, were used to perform the function of capacitors in a quadrature coil at ideal mode 1 resonance. This method of simulation has proven accurate up to 128 MHz for a birdcage coil of this size (Alecci et al., 2001). The finite difference time domain (FDTD) (Yee, 1966) method was used to find the steady-state RF electric field (E) and B1 field by solving the full wave Maxwell’s equations. The spatial resolution of the problem region is 5 mm in all three directions. All FDTD calculations were set up and solved with the aid of commercially available software XFDTD (Remcom; State College, PA).

Figure 2.5 The axial (left), sagittal (middle), and coronal (right) planes in the computer model of the conventional birdcage coil loaded with a human head. The dashed lines indicate the location of the axial plane on which the homogeneity of the field and SNR were calculated.
The magnitude of transverse circularly polarized component of $B_1$ that rotates in the same direction of the nuclear spin precession, $B_1^+$, was calculated on the central axial plane as (Collins and Smith, 2001):

$$B_1^+ = |(\hat{B}_x + i\hat{B}_y) ÷ 2|$$  \[2.1\]

where the circumflex ($^\wedge$) indicates the complex value, $i$ is $\sqrt{-1}$. To calculate $B_1^-$, the magnitude of the other component of $B_1$ that rotates in the opposite direction of $B_1^+$, a second field calculation was performed with the phases of the voltage sources opposite those in the field calculation for $B_1^+$. $B_1^-$ was then calculated as (Collins and Smith, 2001):

$$B_1^- = |(\hat{B}_x - i\hat{B}_y)^* ÷ 2|$$  \[2.2\]

where the asterisk indicates the complex conjugate.

The $SNR$ is calculated on the central axial plane in the coil, as indicated by the dashed lines in Figure 2.5, using the formula (Collins and Smith, 2001):

$$SNR \propto f^2 \frac{\sum_N W_n \sin(V | \hat{B}_{1n}^x | \gamma \tau) | (\hat{B}_{1n}^-)^* |}{\sqrt{P_{abs}}}$$  \[2.3\]

where $f$ is the Larmor frequency, the summation is performed over all $N$ voxels in the plane, $W_n$ is the water content of the $n$th voxel, $V$ is a normalization factor to maximize the amplitude of the total signal contributing to a reconstructed gradient echo image with a 3 ms 90° rectangular RF pulse, $\hat{B}_{1n}^x$ and $\hat{B}_{1n}^-$ are $B_1^+$ and $B_1^-$ of the $n$th voxel, respectively, $\gamma$ is the gyromagnetic ratio of $^1$H, $\tau$ is the duration of the RF pulse, and
\( \sqrt{P_{\text{abs}}} \), the total absorbed power in the entire model, is calculated as (Collins and Smith, 2001):

\[
P_{\text{abs}} = \frac{1}{2} \sum_{n} (\sigma_{xn} E_{xn}^2 + \sigma_{yn} E_{yn}^2 + \sigma_{zn} E_{zn}^2) \Delta_x \Delta_y \Delta_z \tag{2.4}
\]

where the summation is performed over all \( N \) voxels of the entire model. \( \sigma_{xn} \), \( \sigma_{yn} \), \( \sigma_{zn} \) are the conductivity of the material of the \( n \)th voxel, \( E_{xn} \), \( E_{yn} \), and \( E_{zn} \) are the electric field magnitude of the \( n \)th voxel, and \( \Delta_x \), \( \Delta_y \), and \( \Delta_z \) are the dimension of a Yee cell, in x, y, and z direction, respectively. Eq. 2.4 is a numerical formula of the integrity of the power absorbed in an isotropic conductive medium (Pozar, 1990).

Considering the size of the coil-sample system and the field strengths here, sample noise dominance is assumed and the thermal noise and radiation loss of the coil is ignored (Edelstein et al., 1986). This method of calculating SNR has been shown to be in good agreement with experiments in comparison of SNR in the human head at different field strengths (Vaughan et al., 2001).

The homogeneity in the head is defined as the percentage of the area on the central axial plane inside the head that has the magnitude of \( B_1^+ \) within 10% deviation from that at the center of the plane (\( \frac{|B_1^+ (r) - B_1^+ (\text{center})|}{B_1^+ (\text{center})} < 0.1 \)). The homogeneity inside 90% of coil radius on the central axial plane in the empty coil is also calculated to compare with the results of a previous study (Collins et al., 2000).
2.4 Results

The normalized $B_1^+$ distributions on the central axial plane within 90% radius of the unloaded coils for the four end-ring/shield configurations at both 64 MHz and 125 MHz are given in Figure 2.6. The maps and contour plots of the normalized $B_1^+$ on the central axial plane inside the head and the normalized $|B_1|$ on the whole central coronal plane at both frequencies are given in Figure 2.7 and Figure 2.8, respectively. $B_1^+$ at the center of the coil is equal to 1.

![Image of normalized $B_1^+$ distributions](image-url)

Figure 2.6 Normalized $B_1^+$ on the central axial plane in the unloaded coil with the four different end-ring/shield configurations defined in Figure 2.4. $B_1^+$ at the center is normalized to 1.
Figure 2.7 Normalized $B_1^+$ on the central axial plane in the head with the four different end-ring/shield configurations defined in Figure 2.4. $B_1^+$ at the center is normalized to 1.
The normalization factor, $V$, proportional to the input voltage required to produce a maximized amplitude of signal contributing to a reconstructed gradient echo image with a 3 ms $90^\circ$ rectangular RF pulse (Collins and Smith, 2001), is given in Table 2-1 along with absorbed power ($V^2P_{\text{abs}}$), $SNR$, and homogeneity of the unloaded and loaded coil for the four different end-ring/shield configurations at 64 MHz and 125 MHz. All the $SNRs$ are normalized to the $SNR$ of the conventional birdcage at 64 MHz.

Figure 2.8 Normalized $|\mathbf{B}_1|$ on the central coronal plane in the coil with four different end-ring/shield configurations and loaded with the human head model at 64 MHz and 125 MHz. $|\mathbf{B}_1|$ at the center is normalized to 1. The black line inside the coil indicates the position of the head.
Table 2-1 Normalization factor, $V$, total absorbed power ($V^2P_{abs}$), SNR, homogeneity in the empty coil, and homogeneity in the coil loaded with a human head for the four different end-ring/shield configurations at 64 MHz and 125 MHz.

<table>
<thead>
<tr>
<th>$f$</th>
<th>Configuration</th>
<th>$V$</th>
<th>$V^2P_{abs}$ (W)</th>
<th>SNR</th>
<th>Homogeneity$^#$ in empty coil</th>
<th>Homogeneity$^#$ in loaded coil</th>
</tr>
</thead>
<tbody>
<tr>
<td>64 MHz</td>
<td>Conventional</td>
<td>57.05</td>
<td>1.51</td>
<td>1.00</td>
<td>92.48</td>
<td>78.51</td>
</tr>
<tr>
<td></td>
<td>Surrounding Shield</td>
<td>69.30</td>
<td>1.35</td>
<td>1.06</td>
<td>48.04</td>
<td>97.83</td>
</tr>
<tr>
<td>125 MHz</td>
<td>Solid Connection</td>
<td>58.10</td>
<td>1.48</td>
<td>1.01</td>
<td>56.53</td>
<td>96.79</td>
</tr>
<tr>
<td></td>
<td>Thin Wire Connection</td>
<td>84.30</td>
<td>1.56</td>
<td>0.98</td>
<td>44.81</td>
<td>96.79</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>97.50</td>
<td>6.51</td>
<td>1.74</td>
<td>94.47</td>
<td>18.80</td>
</tr>
<tr>
<td></td>
<td>Surrounding Shield</td>
<td>122.90</td>
<td>5.97</td>
<td>1.85</td>
<td>64.54</td>
<td>20.80</td>
</tr>
<tr>
<td></td>
<td>Solid Connection</td>
<td>94.70</td>
<td>6.43</td>
<td>1.78</td>
<td>65.50</td>
<td>20.28</td>
</tr>
<tr>
<td></td>
<td>Thin Wire Connection</td>
<td>156.20</td>
<td>7.37</td>
<td>1.66</td>
<td>51.32</td>
<td>20.02</td>
</tr>
</tbody>
</table>

2.5 Discussion
2.5.1 $B_i^+$ homogeneity

In the unloaded case, the $B_i^+$ distribution in the conventional birdcage coil is visibly more homogeneous than that in the other three coils at both frequencies. This result is consistent with that of the previous study (Collins et al., 2000). When loaded with the head, however, the conventional birdcage coil produces the least homogeneous fields. In the conventional birdcage coil, from the center to the edge of the head, $B_i^+$ drops 30% at 64 MHz and 50% at 125 MHz while in the other three coils, almost the entire area has $B_i^+$ larger than 90% at 64 MHz and 70% at 125 MHz.

The end-ring currents contribute to this change of the homogeneity upon the load in the conventional birdcage coil comparing to the other coils. In the conventional birdcage coil, where the currents in the rungs can only return through the two end-rings, the end-ring currents induce additional transverse field at the coil center (Hayes et al., 1988). This additional transverse field can compensate the low field strength at the center of the unloaded coil. In the loaded coil, however, the $B_i^+$ at the center is higher than that at peripheral areas because of wavelength effects, so the further increase of $B_i^+$ at the center by the end-ring currents actually decreases the homogeneity in the head. In the coil with surrounding shield, although the currents in the rungs also only return through the end-rings, the eddy currents in the shield induced by the end-ring currents are much larger than that in the conventional birdcage coil because of the close shielding, and partially cancel out the effects of the end-ring currents on the $B_i^+$ field homogeneity. In the coil with solid connection, the end-ring currents are limited since much of the current...
in the rungs returns through the shield. In the coil with thin wire connection, there are no end-rings so no end-ring currents can exist at all.

The quantified homogeneity listed in Table 2-1 is consistent with the graphical results. In the unloaded case, the $B_{1+}$ field homogeneity does not change significantly with the increase of the frequency from 64 MHz to 125 MHz because the wavelength (about 4.7 m at 64 MHz and 2.4 m at 125 MHz in free space) is much larger than the coil dimensions at both frequencies. Within the head, the wavelength is much shorter (about 0.49 m at 64 MHz and 0.29 m at 125 MHz, assuming the head is filled with a material having dielectric permittivity equal to the average of that of white matter and gray matter) such that it is comparable to the dimensions of the coil. These wavelength effects account for the significantly degraded $B_{1+}$ field homogeneity in the head at 125 MHz than that at 64 MHz.

2.5.2 SNR

The difference between the highest SNR and the lowest for different end-ring/shield configurations at a given frequency is within 11% of the maximum SNR. At both frequencies, the coil with surrounding shield results in the lowest $V^2P_{abs}$ and the highest SNR. This can be better understood by examining the RF field distribution in the coil-sample system. Figure 2.8 shows the normalized $|B_{1}|$ maps and the contour plots on the center coronal plane of the coils. Compared with the other configurations, the coil with surrounding shield best contains the $B_{1}$ flux within the volume of the coil. This leads to less RF power coupling to the neck and the shoulders, thus better SNR in the central
part of the coil. Among the RF coils currently used in MRI systems, the TEM coil has a similar end-ring/shield configuration to the coil with solid connection, which has the second highest *SNR* of the four coils.

For all four end-ring/shield configurations, a greater-than-quadratic increase of \( V^2 P_{\text{abs}} \) leads to a less-than-linear increase of *SNR* from 64 MHz to 125 MHz because of wavelength effects. This phenomenon was also observed in an experiment with a surface coil on a human subject’s chest (Wen et al., 1997) and in a computer simulation of an unshielded birdcage coil loaded with a human head (Collins and Smith, 2001).

### 2.5.3 Limitations of the methods

The performance of the coils may not be comprehensively represented since homogeneity and *SNR* were only calculated on the central axial plane. The center axial plane was chosen in order to be consistent with previous publications (Collins et al., 1998; Collins et al., 2000; Collins and Smith, 2001) and birdcage coil theory (Hayes et al., 1985), which is based on infinitely long cylinders. In the *SNR* calculation, sample noise dominance was assumed and coil noise and radiation loss were not considered for simplicity. This is consistent with the definition of *ISNR* (Edelstein et al., 1986).

### 2.6 Conclusions

The birdcage coil with a conventional cylindrical shield has the best \( B_1^+ \) homogeneity on the center axial plane in the unloaded coil at 64 MHz and 125 MHz.
When loaded with a human head, however, this configuration has the lowest homogeneity at both frequencies. The coil with surrounding shield has the least energy lost in the sample and the highest SNR at both frequencies. The difference in SNR between different end-ring/shield configurations is within 11%.

# Most of this chapter has been previously published (MRM 51 (1), 217-221, 2004)
Chapter 3

Effects of End-Ring/Shield Configuration on Homogeneity and Signal to Noise Ratio in a Birdcage-type Coil Loaded with a Human Head: Experiments
3.1 Abstract

Four 12-rung linear birdcage-type coils are built to experimentally examine the effects of the end-ring/shield configuration on radiofrequency (RF) magnetic ($B_1$) field homogeneity and SNR at 125 MHz. The coil configurations include a) a conventional cylindrical shield (conventional), b) a shield with annular extensions to closely shield the end-rings (surrounding shield), c) a shield with annular extensions connected to the rungs (solid connection), and d) a shield with thin wires connected to the rungs (thin wire connection). The surrounding shield configuration results in the highest and the thin wire configuration results in the lowest SNR in the central axial slice in a head as predicted by the previous calculations. Although there is no significant difference between the overall SNR of the conventional configuration and the surrounding shield configuration, the surrounding shield configuration has the potential to be tuned to higher frequencies than the conventional configuration. The solid connection configuration has a lower SNR than the conventional configuration and the surrounding shield configuration but a higher SNR than the thin wire connection. The conventional configuration results in the most homogeneous field in the oil phantom as predicted by the previous calculations.

Keywords: end-ring; shield; homogeneity; and SNR
3.2 Introduction

$B_1$ homogeneity and SNR of the coil have always been important issues in RF coil design in MRI. At high fields, where the wavelengths become comparable to the coil and sample dimensions, the RF field distribution depends on the details of the coil configuration. In previous calculations, four different end-ring shield configurations were modeled to examine the effects of different current return paths on $B_1$ homogeneity and SNR at 64 and 125 MHz. Configurations included a) a conventional cylindrical shield (conventional), b) a shield with annular extensions to closely shield the end-rings (surrounding shield), c) a shield with annular extensions connected to the rungs (solid connection), and d) a shield with thin wires connected to the rungs (thin wire connection). It was found that the conventional configuration results in the highest homogeneity in the empty coil while the surrounding shield configuration results in the highest homogeneity and the highest SNR in the coil loaded head at both frequencies although the largest difference of the SNR between different configurations is within 11%. In the calculations, ideally sinusoidal current distribution in the coil rungs was assumed and no matching circuit was needed for the coil. In reality, the matching circuit, which is needed in most cases to maximize the power transmission and signal reception, can disturb the current distribution in the coil and affect the coil performance.

To examine the effects of the end-ring/shield configuration on the $B_1$ homogeneity and SNR experimentally, four head size birdcage-type coils for use at 125 MHz with the same end-ring/shield configurations modeled in the calculations were
constructed. The dimensions of the coils were chosen to be as close to those in the calculations as possible. Images were acquired in an oil phantom and in a human subject’s head for all four configurations. $B_1$ homogeneity in the central axial slice of the oil phantom and the $SNR$ in the central axial, sagittal, and coronal planes were also calculated and compared to the corresponding results in the calculations. RF power absorption was also estimated and compared between different configurations.

3.3 Methods

3.3.1 Coil building

3.3.1.1 Mechanical structure

The mechanical support structures of the four coils are the same. Each coil was built on two coaxial acrylic tubes with the smaller one (inner diameter of 26.05 cm) as the base of the coil and the larger one (outer diameter of 35.56 cm) as the base of the shield. Both tubes have a wall thickness of 0.33 cm and a height of 20.32 cm. A delrin plate with inner diameter of 26.38 cm, outer diameter of 35.23 cm, and thickness of 0.78 cm was placed in the middle between the two tubes to keep the tubes in a coaxial position. Two delrin end plates with inner diameters of 26.05 cm, outer diameters of 35.56 cm, and thickness of 0.78 cm were attached to both ends of the tubes and were held together with the plate in the middle by six supporting delrin rods. The middle plate was also used to support the adjustable tuning and matching capacitors.
3.3.1.2 Electrical circuit

The coils were built as 12-rung linear low-pass birdcage-type with different end-ring/shield configurations (Figure 3.1). All the configurations were built based on the conventional configuration. Initially, a conventional coil with 16 rungs, which is the number of the rungs in the calculations, was built but the capacitor values needed to efficiently tune the coil to 125 MHz were unreasonably small. To build the conventional birdcage coil, twelve 1.29 cm wide and 20.32 cm long strips of copper tape were oriented longitudinally and spaced evenly around the circumference on the inner surface of the smaller tube. Another 1.29 cm wide and 81.83 cm long strip of copper tape was oriented in the circumferential direction and laid down on the top of and connected to the 12 rungs at each end of the tube to form the end-rings of the birdcage coil. The shield is a 20.32 cm long cylinder made by putting strips of copper tape on the outside surface of the larger tube and soldering them together. The surrounding shield configuration has the same rungs and end-rings as the conventional configuration but the shield was extended inwardly to cover the outside surface of the two end plates. The inward part of the shield was 0.78 cm away from the end-rings in the axial direction. The solid configuration has the same structure as the surrounding shield configuration except that it has no end-rings at the end but rungs directly connected to the shield. The thin wire configuration has the same structure as the conventional configuration except that it has no end-rings but rungs directly connected to the shield by extending across the end plates in the radial direction. If the length of the rungs in the conventional configuration and in the surrounding shield configuration is measured as the distance between the middle points of the two end-rings,
it will be about 2.85 cm shorter than that in the solid connection and thin wire connection due to the width of the end-rings and the thickness of the end plates. This is the result of an effort to keep the length of the shield the same for the surrounding shield configuration and the solid connection configuration.

Figure 3.1 Four 12-rung low-pass linear birdcage-type coils with four different end-ring/shield configurations.
3.3.1.3 Tuning and matching

Three narrow gaps, with a capacitor placed across each of them, were cut into each rung except for the rungs attached with the adjustable matching capacitor (the matching rung) and the adjustable tuning capacitor (the tuning rung, or the rung opposite the matching rung), where the number of gaps is either one or three. This is an effort to choose the capacitor values to minimize the perturbation of the matching circuit. A schematic diagram of matching circuits in the four coils is shown in Figure 3.2. For all configurations except the thin wire one, the central conductor of the RF cable was connected to an adjustable capacitor in series, which was then connected to one side of the fixed capacitor in the middle of the matching rung. The RF cable shield was connected to the coil shield. For the thin wire connection, an inductive drive method was used to decrease the imbalance introduced by the driving mechanism (Bridges, 1988; Hoult and Tomanek, 2002). Instead of being connected to a rung, the central conductor of the RF cable in the thin wire configuration was connected to the coil shield, which was connected to RF shield, to form a driving loop. The transmission return loss curve is plotted in Figure 3.3 for each configuration when loaded with a subject’s head. Resonant mode 1, which is identified as the mode that produces a homogeneous RF field in the coil (measured with a 2.9 cm shielded pickup coil), is tuned to 125.44 MHz and the impedance of the coil loaded with the head was matched to 50 ohms at that frequency. The capacitor values in the matching rung, tuning rung and the other rungs are shown in Table 3-1. Note that the capacitance in the matching rung refers to the value of the fixed disc capacitor in the middle of the rung. An adjustable matching capacitor is also attached
to the matching rung at one point as shown in Figure 3.2 except in the thin wire connection configuration, where the variable matching capacitor was embedded in the separated driving loop.

Figure 3.2 A schematic diagram of the matching circuit for the four configurations.
Figure 3.3 Transmission return loss curves for the four coils. To include all the modes, the curves for the conventional configuration and the surrounding shield configuration span over 270 MHz while those for the solid connection configuration and the thin wire connection configuration span over 60 MHz.
Q values for unloaded coils \( (Q_{\text{unloaded}}) \) and coils loaded with the subject’s head \( (Q_{\text{loaded}}) \) were determined by doubling the Q values at the -3 dB points on the return loss curves on a HP network analyzer 4195A since loading a matched network analyzer halves the Q values of the circuit (Hayes et al., 1988; Chen and Hoult, 1989).

### Table 3-1 Capacitor values in the rungs of the four coils.

<table>
<thead>
<tr>
<th></th>
<th>Matching rung</th>
<th>Tuning rung</th>
<th>Each of other rungs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>2.2 pF</td>
<td>Variable</td>
<td>1.42 pF</td>
</tr>
<tr>
<td>Surrounding</td>
<td>2.7 pF</td>
<td>Variable</td>
<td>2.73 pF</td>
</tr>
<tr>
<td>Solid connection</td>
<td>6.8 pF</td>
<td>Variable</td>
<td>7.33 pF</td>
</tr>
<tr>
<td>Thin wire connection</td>
<td>5.63 pF</td>
<td>Variable</td>
<td>6.21 pF</td>
</tr>
</tbody>
</table>

3.3.2 Imaging parameters

T2-weighted RARE images (Henning et al., 1986) of five axial, sagittal, and coronal slices of a normal Asian male subject (30 yr, 170 cm, 79.4 kg) were acquired on a whole-body MEDSPEC S300 2.94 T research imaging spectrometer (Bruker Instruments, Inc., Karlsruhe, Germany) with TR = 4680 ms, TE = 80.4 ms, matrix size = 256 × 256, NEX = 2, FOV = 25cm, slice thickness = 5 mm, slice interval = 5.5 mm, and RARE factor = 8. SNR was measured with the signal averaged from all brain tissues and divided by the standard deviation of background noise in the image plane and all the SNRs are normalized to the SNR in the conventional configuration. The input RF power is also normalized to that in the conventional coil.

Gradient echo images of a cylindrical vegetable oil phantom (Diameter = 24.7 cm, height = 22.9 cm) were also acquired on the same system to estimate the homogeneity as
in the unloaded coils (Vaughan et al., 2001; Zhang et al., 2003) with TR = 5000 ms, TE = 6 ms, matrix size = 256 × 256, NEX = 1, FOV = 30 cm, slice thickness = 5 mm, and flip angle = 45°. The coils were re-tuned and re-matched for the oil phantom. Here the homogeneity is defined on the central axial plane within 70% of the coil radius as the percentage of the area that has image signal intensity (SI) deviation within 10% of the average image intensity (∣SI(r) - SI_{average}\\/SI_{average}<0.1).

3.4 Results

The images of the axial, sagittal, and coronal slices passing through the center of the coil for the four coils loaded with the subject’s head are shown in Figure 3.4 with the reference images showing the slice positions in the head. The central axial slice for the oil phantom is shown in Figure 3.5 with the dashed line indicating the outer boundary of the area within which the homogeneity is calculated. The homogeneity in the oil phantom, the Q values, the input RF power for the head imaging, and the SNR in the central axial, sagittal, and coronal planes in the head for the four coils are listed in Table 3-2.
Figure 3.4 Image slices of a subject’s head in the four coils. Only the central slices are shown here.
Figure 3.5 The central axial image slice of the oil phantom for the four coils. The circle of the dashed line, which has a diameter about 70% of that of the coil, indicates the outer boundary of the area within which the homogeneity was calculated.
The capacitor values in the conventional configuration and the surrounding shield configuration are smaller than the solid connection and the thin wire connection, indicating larger mutual inductive coupling between rungs in the conventional connection and the surrounding shield configuration than that in the other two configurations (Tropp, 1997). Compared to the conventional configuration, the surrounding shield configuration has less total inductance so could be tuned to a higher frequency. It was also found that during the tuning and matching process, the surrounding shield configuration and the solid connection configuration are less sensitive to the change of outside environment.

Table 3-2: Q values, RF power $P$ in dB, homogeneity in the oil phantom, and SNR in the head for the four coils. The homogeneity is defined on the central axial plane within 70% of the coil radius as the percentage of the area that has image signal intensity deviation within 10% of the average image signal intensity. The RF power is measured for a 3.2 ms 90 degree gauss pulse during head imaging and normalized such that the power used by the conventional coil is equal to 0 dB.

<table>
<thead>
<tr>
<th></th>
<th>Q Unloaded</th>
<th>Q Loaded</th>
<th>$P$ (dB)</th>
<th>Homogeneity in the oil phantom</th>
<th>SNR in the head</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>128</td>
<td>20</td>
<td>0</td>
<td>75.4</td>
<td>1</td>
</tr>
<tr>
<td>Surrounding</td>
<td>162</td>
<td>40</td>
<td>0</td>
<td>39.8</td>
<td>1.04</td>
</tr>
<tr>
<td>Solid connection</td>
<td>222</td>
<td>40</td>
<td>0.9</td>
<td>53.3</td>
<td>0.958</td>
</tr>
<tr>
<td>Thin wire connection</td>
<td>248</td>
<td>48</td>
<td>2.5</td>
<td>36.4</td>
<td>0.761</td>
</tr>
</tbody>
</table>
(position of the RF cable in the magnet, experimenter’s body position, etc.) than the conventional configuration and the thin wire configuration due to the shielding effects of the solid end plates (Figure 3.1).

The wider distribution of modes in the frequency spectrum (Figure 3.3) in the conventional configuration and in the surrounding shield configuration makes the coils more immune to coupling between modes introduced by perturbation of the matching circuit thus having larger tuning range than the solid connection configuration and the thin wire configuration (Tropp, 2001). It is found that even a 3% change of the capacitor in the matching rung in the solid connection (from 6.8 pF to 7 pF) dramatically changes the homogeneity of the image (increased image intensity next to the matching rung and decreased image intensity next to the tuning rung; images not shown here).

3.5.2 B₁ Homogeneity

Considering the fact that the oil phantom occupied about 90% of the coil cross section area, the images in Figure 3.4 show good homogeneity in the oil phantom in all four coils. The higher image intensity at the left and right edges of the image is caused by the high current density in the matching and tuning rungs. The lower image intensity area (dark holes) at the edge is the result of the cancellation of the fields from different rungs. The pattern of the dark holes in the conventional coil is less strong as that in the other three coils. The pattern becomes more obvious and in the solid connection and thin wire connection configurations the pattern is more extended towards the central of the image. The quantitative homogeneity measured from the area within the dashed line (diameter of
18 cm, 70% of the coil diameter) in Figure 3.4 in the conventional coil is about 89.4% higher than that in the surrounding shield configuration, 41.5% higher than that in the solid connection configuration, and 107% higher than that in the thin wire connection configuration. This result verifies the finding in the previous calculations (Liu et al., 2004). Since it is difficult to quantitatively evaluate the $B_1$ homogeneity in the head, only image slices are shown in this work (Figure 3.4). The central axial images show good homogeneity in all four coils except that in the image taken with the solid connection configuration where the left side is slightly brighter than the right side. This inhomogeneity is the perturbation introduced by the matching circuit and may be eliminated by adjusting the capacitance in each rung carefully (like in a TEM coil). Some signal loss is observed on the top of the sagittal slice and coronal slice in the conventional configuration and the surrounding shield configuration but the brain tissue still can be clearly seen. The overall homogeneity in the whole volume of the coil is satisfactory for all four coils.

3.5.3 SNR

The surrounding shield configuration results in the highest SNR in the central axial plane in the head, in agreement with previous numerical calculations (Liu et al., 2004). The SNR in the central axial plane in the surrounding shield configuration is about 4.2% higher than that in the conventional configuration, about 8.8% than that in the solid connection configuration, and about 37% than that in the thin wire connection configuration. The surrounding shield configuration also results in the highest SNR in the
central sagittal plane. The $SNR$ in the central sagittal plane in the surrounding shield configuration is about 2.4% higher than that in the conventional coil, about 21% higher than that in the solid connection configuration, and about 39% higher than that in the thin wire connection. The conventional configuration results in $SNR$ in the central coronal plane about 4.0% higher than the surrounding shield, about 11% higher than the solid connection configuration, and about 32% higher than the thin wire connection configuration. The conventional configuration and the surrounding shield configuration result in $SNR$s in all three planes significantly higher than the solid connection configuration, which results in $SNR$s in all three planes significantly higher than the thin wire connection. There is no significant difference of the overall $SNR$ between the conventional configuration and surrounding shield configuration. The main reason the solid connection configuration results in lower $SNR$ than the conventional configuration and the surrounding shield configuration might be that the solid connection configuration is more sensitive to the perturbation of the field introduced by the matching circuit. As seen in the transmission return loss curves in Figure 3.3, the mode 1 in the conventional configuration and the surrounding shield configuration is separated from the adjacent modes by at least 30 MHz while in the solid connection and the thin wire connection by less than 10 MHz. The asymmetry introduced by the match circuit in the solid connection and the thin wire connection is more likely to increase coupling between modes and degrades both the homogeneity and $SNR$. Another factor affecting the $SNR$ is the length of the rungs, which is about 2.85 cm longer in the solid connection and in the thin wire connection than that in the conventional and in the surrounding shield configuration. So the loading is a little more, which lowers $SNR$, in the solid connection and the thin wire
One possible reason the thin wire connection configuration performs poorer than the calculation predicted is that the decrease of the rung number from 16 to 12, which decreases mutual inductance between rungs, affects the thin wire connection configuration more than other configurations.

The input RF power used by the coils is consistent with the $SNR$ performance of the coils. The surrounding shield configuration uses the same power as the conventional configuration. The solid connection configuration uses 0.9 dB more power and the thin wire connection configuration uses 2.5 dB more power than the conventional configuration. $Q$ values for loaded and unloaded cases are within reasonable range but there is no relation between the ratio of $Q_{\text{unloaded}}$ to $Q_{\text{loaded}}$ and the $SNR$.

3.6 Conclusions

While the surrounding shield configuration results in the highest and the thin wire results in the lowest $SNR$ in the central axial slice in a head as predicted by the previous calculations, there is no significant difference between the overall $SNR$ of the conventional configuration and the surrounding shield configuration. The surrounding shield configuration has the potential to be tuned to higher frequencies than the conventional configuration. The solid connection configuration has a lower $SNR$ than the conventional configuration and the surrounding shield configuration but a higher $SNR$ than the thin wire connection. The conventional configuration results in the most homogeneous field in the oil phantom as predicted by the previous calculation.
Chapter 4

Calculations of $B_1$ Distribution, SAR, and ISNR for a Body-Size Birdcage Coil
Loaded with Different Human Subjects
4.1 Abstract

A numerical model of a female body is developed to study the effects of different body types with different coil orientations and coil drive methods on RF magnetic ($B_1$) field distribution, specific absorption rate (SAR), and intrinsic signal-to-noise ratio (ISNR) for a body-size birdcage coil at 64 MHz and 128 MHz. The coil is loaded with either a larger, more muscular male body model (subject 1) or a newly developed female body model (subject 2), oriented so that rungs are either in plane with the subjects’ arms (rung-on-plane) or not (rung-off-plane), and driven with two (quadrature), four-port, or many (ideal) sources. Loading the coil with subject 1 results in significantly less homogeneous $B_1$ field, higher SAR, and lower ISNR than subject 2 at both frequencies. No significant difference in ISNR between different coil orientations is observed. The conventional quadrature excitation and the four-port excitation result in similar $B_1$ pattern, SAR, and ISNR in all but one case. Results with the ideal excitation are similar to those with conventional quadrature and four-port excitation at 64 MHz and in most cases at 128 MHz but should be used with caution.
4.2 Introduction

Estimating the RF field distribution, RF power deposition and $SAR$, and the $SNR$ in the human body has been an interest in MRI for decades along with the pushing of MRI systems to higher and higher fields. In the early days, experimental measurements have been accompanied by simulations with homogeneous models. Bottomley et al (Bottomley and Andrew, 1978; Bottomley et al., 1985) calculated the RF power deposition and $SAR$ in body-size muscle phantoms exposed to homogeneous fields up to 100 MHz and performed experimental measurements on subjects at 63 MHz. Hoult and Chen et al (Hoult and Lauterbur, 1979; Chen et al., 1986) developed methods to calculate the loss and $SNR$ in human subject. Glover et al (Glover et al., 1985) compared the power deposition and $SNR$ in a lossy body-size phantom and in human bodies for linear and quadrature excitation at 63 MHz. Edelstein et al (Edelstein et al., 1986) introduced the concept of intrinsic $SNR$ into MRI human imaging. Carlson (Carlson, 1989) proposed a general solution for correlation between power deposition and noise in long wavelength and long skin depth approximation. More complex body model were developed in the 90’s. Grandolfo et al (Grandolfo et al., 1990) calculated $SAR$ in a 13-tissue human-torso model up to 64 MHz with impedance method. Wen et al (Wen et al., 1997) measured the power deposition and intrinsic $SNR$ in the heart imaging at 1.5 T, 3 T, and 4 T and Singerman et al (Singerman et al., 1997) simulated the results with multi-layer symmetrical chest model for finite element method. Gandhi et al (Gandhi and Chen, 1999) calculated $SAR$ in a 30-tissue body model, whose anatomy was based on MRI scan
images of a male, from 64 MHz to 350 MHz with finite difference time domain (FDTD) method (Yee, 1966; Kunz and Luebbers, 1993). Collins et al (Collins and Smith, 2001) also developed an anatomically-accurate body model for FDTD by segmenting photo image slices of a male cadaver and presented calculations of $B_1$ field, $SAR$, and $SNR$ matched Wen et al’s experiments well. Interestingly, most subjects, if not all, in the experiments (Bottomley et al., 1985; Glover et al., 1985; Wen et al., 1997) or simulations (Gandhi and Chen, 1999; Collins and Smith, 2001) were males. No direct comparison of $B_1$ field distribution, RF power deposition, or $SNR$ between male body type and female body type has been done yet.

Here another anatomically accurate body model is developed based on a female body, which is quite different from the male body in size, shape, and composition. The $B_1$ field distribution, $SAR$, and $SNR$ in the female model (subject 2) was directly compared with those in the male model (subject 1) developed by Collins et al (Collins and Smith, 2001) using FDTD method. Both models were loaded into a body size eight-rung high-pass birdcage coil (Hayes et al., 1985) at 64 MHz and 128 MHz. To estimate the effects of coupling between the subject and the coil on the system performance, two coil orientations relative to the subject’s position and three coil drive methods were compared respectively. In the first coil orientation, the subject’s two arms are on the same plane with two coil rungs (rung-on-plane orientation). For the second orientation, the coil is rotated half of the distance between two adjacent rungs from the rung-on-plane orientation so that no rungs are on the same plane with subject’s two arms (rung-off-plane orientation) (Figure 4.1). The three drive methods include the conventional quadrature excitation (Glover et al., 1985), the four-port excitation (Ibrahim et al., 2000;
Ledden et al., 2000), and the ideal excitation (Collins et al., 1998; Collins, 1999), which assumes the current distribution in the coil rungs produces a homogeneous field in an empty coil.
Figure 4.1 Coronal (a), sagittal (b), and axial (c) slices of subject 1 (left) and subject 2 (right) through the center of the coil in the rung-on-plane orientation where two coil rungs and the subject’s two arms are on the same plane and an axial slice (d) of the rung-off-plane orientation where the coil is rotated half of the distance between two adjacent rungs from the rung-on-plane orientation so no rungs are on the same plane with subject’s arms.
4.3 Methods

An eight-rung body size (63 cm diameter, 68 cm shield diameter, and 70 cm length) shielded high-pass birdcage coil loaded with two different subjects was modeled at 64 MHz and 128 MHz using the FDTD method (Figure 4.1). Each rung was modeled as a 10 cm wide copper strip and each end-ring was 0.5 cm thick and 10 cm high. Subject 1 was a male with approximate height of 180.3 cm and mass of 90.3 kg. Subject 2 was a female with approximate height of 165.1 cm and unknown mass. The FDTD model of subject 1 was created by modifying the model used in the previous calculation (Collins and Smith, 2001) — the thickness of skin was decreased to a more realistic degree. The model of subject 2 was newly created by segmenting the images of the female cadaver from the National Libraries of Medicine’s Visible Human Project and then assigning appropriate electric conductivity, electric permittivity, and mass density to each tissue at each frequency (Clauser et al., 1969; Cho et al., 1975; Huang and Wu, 1976; Erdmann and Gos, 1990; Gabriel, 1996; Collins and Smith, 2001). The values of these properties for all the tissue types (33 for the male and 36 for the female) are listed in Table 4-1. The resolution of the FDTD meshes is 5 mm, which has been shown to be adequate for SAR calculations in a head model at 64 MHz (Collins and Smith, 2003), in all three dimensions. The height of the models are the same as the subjects, however, the mass of subject 1 calculated from the model is about 108.9 kg, 20.6% more than the subject’s actual mass. The mass calculated from the model of subject 2 is about 85.5 kg. Although the actual mass of subject 2 is not available, it is expected that the mass calculated from model is also larger than the subject’s actual mass about the same degree
Table 4-1 Conductivity \( \sigma \), relative permittivity \( \varepsilon_r \), and the mass density \( \rho \) assigned to each tissue at different frequencies.

<table>
<thead>
<tr>
<th>Tissue</th>
<th>( \sigma ) (S/m)</th>
<th>( \varepsilon_r )</th>
<th>( \sigma ) (S/m)</th>
<th>( \varepsilon_r )</th>
<th>( \rho ) (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skin</td>
<td>0.431 74.5</td>
<td>0.477 54.2</td>
<td>0.477 54.2</td>
<td>1125</td>
<td></td>
</tr>
<tr>
<td>Muscle</td>
<td>0.757 90.2</td>
<td>0.822 77.2</td>
<td>0.822 77.2</td>
<td>1059</td>
<td></td>
</tr>
<tr>
<td>Fat</td>
<td>0.0294 7.20</td>
<td>0.0337 6.02</td>
<td>0.0337 6.02</td>
<td>1151</td>
<td></td>
</tr>
<tr>
<td>Cortical bone</td>
<td>0.0638 18.6</td>
<td>0.747 16.1</td>
<td>0.747 16.1</td>
<td>1850</td>
<td></td>
</tr>
<tr>
<td>Cancellous bone</td>
<td>0.128 35.7</td>
<td>0.161 27.8</td>
<td>0.161 27.8</td>
<td>1080</td>
<td></td>
</tr>
<tr>
<td>Yellow marrow</td>
<td>0.0177 7.65</td>
<td>0.0227 6.32</td>
<td>0.0227 6.32</td>
<td>943</td>
<td></td>
</tr>
<tr>
<td>Blood</td>
<td>1.57 80.8</td>
<td>1.63 68.5</td>
<td>1.63 68.5</td>
<td>1057</td>
<td></td>
</tr>
<tr>
<td>Grey matter</td>
<td>0.586 110</td>
<td>0.695 78.3</td>
<td>0.695 78.3</td>
<td>1035</td>
<td></td>
</tr>
<tr>
<td>White matter</td>
<td>0.317 73.6</td>
<td>0.387 55.5</td>
<td>0.387 55.5</td>
<td>1027</td>
<td></td>
</tr>
<tr>
<td>Cerebellum</td>
<td>0.653 118</td>
<td>0.755 75.2</td>
<td>0.755 75.2</td>
<td>1035</td>
<td></td>
</tr>
<tr>
<td>CSF</td>
<td>2.27 72.4</td>
<td>2.27 72.4</td>
<td>2.27 72.4</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>Eye sclera</td>
<td>0.881 75.9</td>
<td>0.925 64.3</td>
<td>0.925 64.3</td>
<td>1151</td>
<td></td>
</tr>
<tr>
<td>Eye vitreous humor</td>
<td>1.52 69.7</td>
<td>1.52 69.7</td>
<td>1.52 69.7</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>Eye lens</td>
<td>0.271 50.5</td>
<td>0.29 44.6</td>
<td>0.29 44.6</td>
<td>1151</td>
<td></td>
</tr>
<tr>
<td>Nerve spine</td>
<td>0.342 69.1</td>
<td>0.39 47.9</td>
<td>0.39 47.9</td>
<td>1112</td>
<td></td>
</tr>
<tr>
<td>Cartilage</td>
<td>0.451 65.5</td>
<td>0.505 56.8</td>
<td>0.505 56.8</td>
<td>1171</td>
<td></td>
</tr>
<tr>
<td>Thyroid</td>
<td>0.590 73.8</td>
<td>0.699 67.8</td>
<td>0.699 67.8</td>
<td>1059</td>
<td></td>
</tr>
<tr>
<td>Tongue</td>
<td>0.590 73.8</td>
<td>0.670 66.5</td>
<td>0.670 66.5</td>
<td>1059</td>
<td></td>
</tr>
<tr>
<td>Stomach</td>
<td>0.975 89.6</td>
<td>1.02 79.3</td>
<td>1.02 79.3</td>
<td>1126</td>
<td></td>
</tr>
<tr>
<td>Lung</td>
<td>0.274 38.4</td>
<td>0.301 28.1</td>
<td>0.301 28.1</td>
<td>563</td>
<td></td>
</tr>
<tr>
<td>Aorta</td>
<td>0.494 68.7</td>
<td>0.547 56.6</td>
<td>0.547 56.6</td>
<td>1171</td>
<td></td>
</tr>
<tr>
<td>Heart</td>
<td>0.773 98.7</td>
<td>0.835 75.6</td>
<td>0.835 75.6</td>
<td>1059</td>
<td></td>
</tr>
<tr>
<td>Liver</td>
<td>0.493 82.3</td>
<td>0.551 63.4</td>
<td>0.551 63.4</td>
<td>1158</td>
<td></td>
</tr>
<tr>
<td>Spleen</td>
<td>0.774 113</td>
<td>0.858 80.8</td>
<td>0.858 80.8</td>
<td>1163</td>
<td></td>
</tr>
<tr>
<td>Colon</td>
<td>0.772 95.4</td>
<td>0.833 74.5</td>
<td>0.833 74.5</td>
<td>1059</td>
<td></td>
</tr>
<tr>
<td>Intestine/Bowel content</td>
<td>1.74 128</td>
<td>1.83 95.4</td>
<td>1.83 95.4</td>
<td>1153</td>
<td></td>
</tr>
<tr>
<td>Pancreas</td>
<td>0.552 70.6</td>
<td>0.604 61.8</td>
<td>0.604 61.8</td>
<td>1151</td>
<td></td>
</tr>
<tr>
<td>Gall bladder</td>
<td>0.844 84.1</td>
<td>0.905 72.5</td>
<td>0.905 72.5</td>
<td>928</td>
<td></td>
</tr>
<tr>
<td>Gall bladder bile</td>
<td>1.40 105</td>
<td>1.47 90.5</td>
<td>1.47 90.5</td>
<td>928</td>
<td></td>
</tr>
<tr>
<td>Kidneys/Adrenal gland</td>
<td>0.889 117</td>
<td>0.989 80.2</td>
<td>0.989 80.2</td>
<td>1147</td>
<td></td>
</tr>
<tr>
<td>Bladder/Bladder content</td>
<td>0.269 21.9</td>
<td>0.278 19.6</td>
<td>0.278 19.6</td>
<td>1132</td>
<td></td>
</tr>
<tr>
<td>Testicles</td>
<td>0.975 89.6</td>
<td>1.02 79.3</td>
<td>1.02 79.3</td>
<td>1126</td>
<td></td>
</tr>
<tr>
<td>Prostate</td>
<td>0.552 70.6</td>
<td>0.604 61.8</td>
<td>0.604 61.8</td>
<td>1151</td>
<td></td>
</tr>
<tr>
<td>Mammary tissue</td>
<td>0.0350 7.01</td>
<td>0.0380 6.31</td>
<td>0.0380 6.31</td>
<td>1151</td>
<td></td>
</tr>
<tr>
<td>Uterus/Vagina/Great vestibular gland</td>
<td>0.884 109</td>
<td>0.964 82.7</td>
<td>0.964 82.7</td>
<td>1151</td>
<td></td>
</tr>
<tr>
<td>Ovary</td>
<td>0.690 103</td>
<td>0.8000 75.9</td>
<td>0.8000 75.9</td>
<td>1048</td>
<td></td>
</tr>
<tr>
<td>Tender/Dental filling/Defined</td>
<td>0.552 70.6</td>
<td>0.604 61.8</td>
<td>0.604 61.8</td>
<td>1151</td>
<td></td>
</tr>
</tbody>
</table>
as that for subject 1 since the procedures of making the models are identical. The main cause of the mass difference between the model and the subject is that the volume of cadavers expanded after frozen so the model is bigger, but not taller, than the subject. A subject that has a similar body structure but a bigger size than the original subject is actually modeled here. Other causes include the extra skin tissue added in the model to keep the continuity of skin layer (Collins and Smith, 2001) and the limited resolution of the model. Subject 1 has more muscle tissue (38% of the body mass) than subject 2 (24% of the body mass).

The subjects were loaded in the coil in two different orientations. In the rung-on-plane orientation, the coil had two rungs on the plane where the subject’s two arms were and the rung-off-plane orientation was achieved by rotating the coil half of the distance between two adjacent rungs from the rung-on-plane orientation (Figure 4.1). The arms of the subject are closer to the centers of the two rungs of the coil in the rung-on-plane orientation than in the rung-off-plane orientation. In both orientations, the heart of the subject was on the center axial plane of the coil and the back of the subject (or patient bed) was 10 cm away from the center of the coil.

The coil was driven in conventional quadrature (two ports), in four-port, and ideally. Two equal amplitude voltage sources were placed in the bottom end-ring 90° apart in space and in phase (0° and 90°) for the conventional quadrature drive. For the four-port excitation, four equal amplitude voltages sources with 90° phase shift between each (0°, 90°, 180°, and 270°) were placed in the bottom end-ring. The locations of the voltage sources relative to the subject are different in the rung-off-plane orientation from
those in the rung-on-plane orientation Figure 4.2. Sixteen capacitors in the two end-rings were modeled to tune the coil to the desired resonant frequencies. For the ideal excitation, sixteen voltage sources placed at both end-rings were used to perform the function of the capacitors (Collins, 1999). They had the same amplitude but their phases were equal to the azimuthal angles. A voltage source in the bottom end-ring had a phase 180° shifted from the phase of the voltage source at the same place in the top end-ring. These voltage sources were defined such that they would result in an ideal current distribution if the coil were unloaded. This method proved accurate up to 128 MHz for a head size birdcage coil (Alecci et al., 2001). All FDTD calculations were set up and solved with the aid of commercially available software XFDTD (Remcom; State College, PA). All $B_1$ fields and $E$ fields were scaled as to maximize the amplitude of the total signal contributing to a reconstructed gradient echo image with a 3-ms 90° rectangular RF pulse on the central axial plane (Collins and Smith, 2001).
$B_1^+$, the magnitude of the component of $\mathbf{B}_1$ that rotates in the same direction of the nuclear spin precession, and $B_1^-$, the magnitude of the component of $\mathbf{B}_1$ rotates in the opposite direction of the nuclear spin precession, were calculated from the results of two separate field calculations as (Hoult, 2000):

$$B_1^+ = |(\hat{B}_x + i\hat{B}_y) + 2|$$ \[4.1\]

and

$$B_1^- = |(\hat{B}_x - i\hat{B}_y) * 2|$$ \[4.2\]
where $^\wedge$ indicates the complex value, $i$ is $\sqrt{-1}$, and the asterisk indicates the complex conjugate. The phases of the voltage sources in the field calculation for $B_i^+$ are opposite to those in the field calculation for $B_i^-$. The flip angle of the $n$th Yee cell, $\alpha_n$, on the central axial plane was calculated as (Collins and Smith, 2001):

$$\alpha_n = \gamma B_{1n}^+ \tau$$  \[4.3\]

where $B_{1n}^+$ is $B_1^+$ of the $n$th Yee cell, $\gamma$ is the gyromagnetic ratio of $^1H$ and $\tau$ is the duration of the RF pulse. In this study, $\tau$ is equal to 3 milliseconds. The average flip angle on the whole plane, which would be 90° in a perfectly homogeneous $B_1$ field, was calculated by dividing the summation of the flip angles on the central plane by the total number of Yee cells in that plane.

The ISNR was calculated on the central axial plane of the coil, which is through the heart of the subject, with the formula (Collins and Smith, 2001):

$$ISNR \propto f^2 \frac{\sum W_n (\sin \alpha_n) B_{1n}^-}{N_{plane} \sqrt{|P_{abs}|}}$$  \[4.4\]

where $f$ is the Larmor frequency, $N_{plane}$ is the number of the Yee cells in the plane, $W_n$ is the water content of the $n$th Yee cell, $\alpha_n$ is the flip angle of the $n$th Yee cell, $B_{1n}^-$ is $B_1^-$ of the $n$th Yee cell, and $P_{abs}$, the total absorbed power in the entire model, was calculated as (Collins and Smith, 2001):

$$P_{abs} = \frac{1}{2} N_{body} (\sigma_{xx} E_{xx}^2 + \sigma_{yy} E_{yy}^2 + \sigma_{zz} E_{zz}^2) \Delta x \Delta y \Delta z$$  \[4.5\]
where \( N_{body} \) is the total number of Yee cells in the whole model, \( \sigma \) is the electric conductivity, subscript \( n \) indicates the \( n \)th Yee cell, \( E \) is the magnitude of electric field intensity, \( \Delta_x \), \( \Delta_y \), and \( \Delta_z \) are the dimensions of the Yee cell in three directions. This method of calculating ISNR has been shown to be in good agreement with experiment in comparison of SNR in the human head at different field strengths (Vaughan et al., 2001).

The \( SAR \) in each Yee cell in the body model was calculated as (Collins and Smith, 2001):

\[
SAR = \frac{\sigma_x}{2\rho_x} E_x^2 + \frac{\sigma_y}{2\rho_y} E_y^2 + \frac{\sigma_z}{2\rho_z} E_z^2
\]  \[4.6\]

where \( \rho \) is the mass density of the tissue. The local maximum \( SAR \) in one gram of tissue \((SAR_L)\) (XFDTD, Remcom) and the \( SAR \) averaged over the entire body model \((SAR_W)\) were then calculated. All \( SAR \) values were calculated for a 100% duty cycle.

### 4.4 Results

The distribution of \( B_1^+ \) on the central axial plane in the empty coil and in the coil loaded with subject 1 in the rung-on-plane orientation at 128 MHz with the conventional quadrature excitation, the four-port excitation, and the ideal excitation are shown in Figure 4.3. The \( B_1^+ \) fields are homogeneous in the empty coils. For all three excitations, about 82% of the area of diameter 0.5 m (80% of the coil diameter) has \( |B_1^+(r) - B_1^{+\text{average}}| / B_1^{+\text{average}} \) within 10%. These results indicate the coils are resonant at the right
mode for MRI. Loading the subject, however, distorts the field distribution dramatically. Only about 25% of the area in the body has $|B_1^+(r) - B_1^+_{\text{average}}| / B_1^+_{\text{average}}$ within 10%.

Figure 4.3 The distribution of $B_1^+$ on the central axial plane in the empty coils and in the coils loaded with subject 1 in the rung-on-plane orientation for the conventional quadrature excitation, the four-port excitation, and the ideal excitation at 128 MHz. The $B_1^+$ at the center of the coil is normalized to 1.96 µT, the field strength necessary to produce a 3-ms 90° rectangular RF pulse. $B_1^+$ above 8 µT is expressed as 8 µT.

The average flip angle on the central axial plane is shown in Table 4-2 along with the homogeneity of the flip angle distribution, which is defined as the percentage of the areas that have flip angles within ±30% deviation of the average flip angle on the central axial plane. The larger the number, the more homogeneous the field is. The $P_{\text{abs}}$, $SAR$, $SAR_L$, and relative ISNR on central axial plane are also shown in Table 4-2. ISNR values
are normalized such that the ISNR in subject 1 in the rung-on-plane orientation with ideal excitation at 64 MHz is 1.

Table 4-2 The average flip angle for maximal signal contributing to a reconstructed gradient echo image, homogeneity in the central axial plane, the total absorbed power in the body ($P_{abs}$), whole-body average SAR ($SAR_w$), maximum local SAR in one gram of tissue ($SAR_L$), and ISNR in the central axial plane in different subjects with different orientations of a body-size birdcage coil driven by different methods at 64 MHz and 128 MHz. All results are calculated for a 3-ms 90° rectangular RF pulse with a 100% duty cycle.

<table>
<thead>
<tr>
<th>$f$ (MHz)</th>
<th>Subject</th>
<th>Position</th>
<th>Drive method</th>
<th>Average $\alpha$</th>
<th>Homogeneity</th>
<th>$P_{abs}$ (W)</th>
<th>$SAR_w$ (W/kg)</th>
<th>$SAR_L$ (W/kg)</th>
<th>ISNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>1</td>
<td>on</td>
<td>Quad</td>
<td>87.2°</td>
<td>94.2%</td>
<td>57.27</td>
<td>0.53</td>
<td>14.01</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4-port</td>
<td>86.9°</td>
<td>93.9%</td>
<td>58.65</td>
<td>0.54</td>
<td>14.23</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ideal</td>
<td>86.3°</td>
<td>91.7%</td>
<td>59.04</td>
<td>0.54</td>
<td>14.18</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>off</td>
<td>Quad</td>
<td>88.3°</td>
<td>97.8%</td>
<td>59.28</td>
<td>0.54</td>
<td>15.82</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4-port</td>
<td>88.4°</td>
<td>95.9%</td>
<td>59.78</td>
<td>0.55</td>
<td>14.77</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ideal</td>
<td>88.4°</td>
<td>96.2%</td>
<td>61.39</td>
<td>0.56</td>
<td>15.39</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>on</td>
<td>Quad</td>
<td>88.4°</td>
<td>98.2%</td>
<td>36.47</td>
<td>0.43</td>
<td>8.01</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4-port</td>
<td>88.4°</td>
<td>98.8%</td>
<td>38.26</td>
<td>0.45</td>
<td>9.01</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ideal</td>
<td>88.3°</td>
<td>97.1%</td>
<td>38.74</td>
<td>0.45</td>
<td>9.54</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>off</td>
<td>Quad</td>
<td>88.4°</td>
<td>99.9%</td>
<td>37.10</td>
<td>0.43</td>
<td>7.71</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4-port</td>
<td>88.4°</td>
<td>99.3%</td>
<td>37.43</td>
<td>0.44</td>
<td>7.97</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ideal</td>
<td>88.4°</td>
<td>99.6%</td>
<td>38.56</td>
<td>0.45</td>
<td>8.40</td>
<td>1.12</td>
</tr>
<tr>
<td>128</td>
<td>1</td>
<td>on</td>
<td>Quad</td>
<td>88.6°</td>
<td>66.7%</td>
<td>226.01</td>
<td>2.08</td>
<td>61.14</td>
<td>2.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4-port</td>
<td>84.6°</td>
<td>71.5%</td>
<td>205.24</td>
<td>1.88</td>
<td>57.84</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ideal</td>
<td>96.3°</td>
<td>69.3%</td>
<td>302.89</td>
<td>2.78</td>
<td>90.29</td>
<td>1.72</td>
</tr>
<tr>
<td></td>
<td></td>
<td>off</td>
<td>Quad</td>
<td>83.5°</td>
<td>75.5%</td>
<td>188.88</td>
<td>1.73</td>
<td>45.08</td>
<td>2.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4-port</td>
<td>81.0°</td>
<td>74.7%</td>
<td>179.32</td>
<td>1.65</td>
<td>41.65</td>
<td>2.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ideal</td>
<td>77.7°</td>
<td>71.9%</td>
<td>189.43</td>
<td>1.74</td>
<td>49.72</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>on</td>
<td>Quad</td>
<td>85.8°</td>
<td>82.9%</td>
<td>114.88</td>
<td>1.34</td>
<td>19.53</td>
<td>2.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4-port</td>
<td>85.1°</td>
<td>80.1%</td>
<td>117.15</td>
<td>1.37</td>
<td>21.23</td>
<td>2.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ideal</td>
<td>81.6°</td>
<td>74.7%</td>
<td>119.77</td>
<td>1.40</td>
<td>24.64</td>
<td>2.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>off</td>
<td>Quad</td>
<td>86.9°</td>
<td>85.1%</td>
<td>116.62</td>
<td>1.36</td>
<td>24.91</td>
<td>2.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4-port</td>
<td>86.2°</td>
<td>84.7%</td>
<td>115.85</td>
<td>1.35</td>
<td>22.73</td>
<td>2.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ideal</td>
<td>83.8°</td>
<td>76.4%</td>
<td>123.70</td>
<td>1.45</td>
<td>27.19</td>
<td>2.21</td>
</tr>
</tbody>
</table>
Figure 4.4 contains the plots of the flip angle distribution on the central axial plane for the four-port excitation. Figure 4.5 shows the SAR distribution in the sagittal, coronal, and axial planes through the location where $SAR_L$ occurs with the rung-on-plane orientation and the coil driven with four-port excitation at 128 MHz.

![Figure 4.4](image_url)

**Figure 4.4** The distribution of the flip angle for maximum signal of a reconstructed gradient echo image with a 3-ms rectangular RF pulse on the central axial plane with the four-port excitation.
Figure 4.5 The local SAR distribution in the axial, coronal, and sagittal plane that through the location of the $SAR_L$ at 128 MHz. Both subjects were loaded in the rung-on-plane orientation and the coil was driven with four-port excitation. Linear gray scale is from 0 (black) to 21.23 W/kg (white). Values above 21.23 W/kg are shown with the same (white) color.
4.5 Discussion

4.5.1 Three drive methods

The results for conventional quadrature excitation and the four-port excitation are similar with the four-port excitation results in less (9.6%) $SAR_w$ in one specific case when the coil is loaded with subject 1 in rung-on-plane orientation at 128 MHz. The differences in $B_1^+$ homogeneity, $P_{abs}$, $SAR_w$, and $ISNR$ between these two excitations are all within 5% at 64 MHz and within 10% at 128 MHz. The largest difference of $SAR_L$ is about 11%. Considering the variation of local $SAR$ within the sample, the difference is relatively small.

Unlike the conventional quadrature excitation and the four-port excitation, which are used in experiments (Ledden et al., 2000), the ideal excitation only exists in numerical modeling. With the ideal excitation, the current in each rung of the coil follows the ideal pattern that would produce a perfect homogeneous RF field in an empty birdcage coil and consequently, the interaction between the sample and the coil is ignored. This method requires much less computation time by skipping the fine tuning of the RF coil and has been proven accurate on a head up to 128 MHz where the interaction is not significant (Alecci et al., 2001). In this study, the ideal excitation results in similar $B_1^+$ homogeneity, $P_{abs}$, $SAR_w$, and $SNR$ as the conventional quadrature excitation and the four-port excitation at 64 MHz. The differences in $B_1^+$ homogeneity, $SAR_w$, and $ISNR$ among these three drive methods are all within 7%. For $SAR_L$, the largest difference is about 16%, between the ideal excitation and the quadrature excitation in subject 1 in the
rung-on-plane orientation. At 128 MHz, these three drive methods result in almost identical field patterns in the empty coils (Figure 4.3). When the coil is loaded with a subject, the $B_1^+$ field becomes distorted and the difference of results between ideal excitation and other two excitations increases. The differences of $B_1^+$ homogeneity, $SAR_W$, and $ISNR$, however, are still within 11.2% in all cases except with subject 1 in the rung-on-plane orientation, where the ideal excitation results in $P_{abs}$ and $SAR_W$ about 47.6% larger than the four-port excitation and about 34.0% larger than the conventional quadrature excitation, and an $ISNR$ about 14.0% less than the four-port excitation and about 14.8% than the conventional excitation. These results indicate that with a coil and body models of these sizes, the ideal excitation is still relative accurate in most cases at 128 MHz but should be used with urgent caution. For simplicity, the results of the ideal excitation are not included in the rest of the discussion.

4.5.2 $B_1^+$ distribution

The plots of the flip angle distribution on the central axial plane for the four-port excitation are shown in Figure 4.4. The flip angle distribution plots for the conventional quadrature excitation are similar to those of four-port excitations and are not shown here. In all cases, the distribution of the flip angles, which is proportional to the magnitude of $B_1^+$ field distribution, is more homogeneous at 64 MHz than that at 128 MHz as expected. At 128 MHz, a black band, which represents the area that has smaller flip angles, is observed in the back muscles in the subject along with white spots, which
represent the areas that have larger flip angles, at the edge of two arms (Figure 4.4). The average flip angle ranges more narrowly at 64 MHz (from 86.3° to 88.4°) than at 128 MHz (from 81.0° to 88.6°) with different subjects, different coil orientations, and different drive methods. The homogeneity ranges from 93.9% to 99.9% at 64 MHz and from 66.7% to 85.1% at 128 MHz.

The distribution of the flip angles in subject 1 is always less homogeneous than that in subject 2 because subject 1 has a larger chest size than subject 2. The difference of the homogeneity between subject 1 and subject 2 is less at 64 MHz (2.1%~5.0%) than at 128 MHz (10.7%~20.0%).

Loading the subject in the rung-on-place orientation results in a less homogeneous field distribution than in the rung-off-plane orientation because of the shorter distance and stronger coupling between the arms and the rungs in the rung-on-plane orientation. The difference is from 0.5% to 3.7% at 64 MHz and from 2.6% to 11.7% at 128 MHz.

The $B_1^+$ field distribution produced by the conventional quadrature excitation and the four-port excitation are very similar. Neither one necessarily produces more homogeneous field than the other. In all cases, the difference is within 6.7%.

### 4.5.3 SAR

Averaged among different coil excitation methods and coil orientations, the $SAR_w$ in subject 1 is about 0.54 W/kg at 64 MHz and 1.84 W/kg at 128 MHz. The $SAR_w$ in subject 2 is about 0.44 W/kg at 64 MHz and 1.36 W/kg at 128 MHz, about 18.5% and 26.1% less than that in subject 1 at corresponding frequencies. Considered with the
difference of body mass, flip angle, pulse duration, coil dimension, coil drive method, the results at 64 MHz are in good agreement with experiments (Bottomley et al., 1985). Local SAR levels are the highest in skin and muscle tissues near the periphery of the body in both subjects. At both frequencies, the $SAR_L$ in subject 1 is always located at the skin and muscle tissues near the right clavicle (Figure 4.5). In subject 2, the $SAR_L$ occurs in the triceps of the right arm at 64 MHz and in the triceps of the left arm at 128 MHz (Figure 4.5). Since both subjects’ hands are in contact with the body, these locations might be in a big eddy current loop that connect the arms to the body through hands, which would introduce large SAR (Zhai et al., 2002). Averaged among different coil drive methods and coil orientations, the $SAR_L$ in subject 1 is 14.7 W/kg at 64 MHz and 51.4 W/kg at 128 MHz. The $SAR_L$ in subject 2 is 8.2 W/kg at 64 MHz and 22.1 W/kg at 128 MHz, about 44.2% lower than that in subject 1 at 64 MHz and about 57.0% lower at 128 MHz. This large difference could be explained by the quadratic increase of deposition power with radius from the sample center (Bottomley et al., 1985). Both $SAR_W$ and $SAR_L$ are calculated for a 100% duty cycle here. They can be easily converted to a format that can be directly compared with the IEC regulations (IEC, 2002) for different imaging sequences (Collins et al., 1998).

Loading subject 2 in different orientations results in no significant difference (within 2.2%) of $SAR_W$ at both frequencies. Loading subject 1 in the rung-off-plane orientation results in about 1.9% to 3.8% less $SAR_W$ at 64 MHz and about 12.2% to 16.8% less at 128 MHz than in the rung-on-plane orientation. This result indicates that coil orientation becomes more important when the subject’s body is bigger and the frequency gets higher. Loading the subject into different coil orientations does not affect
the location of $SAR_L$. The difference of the magnitude of $SAR_L$ between these two coil orientations varies from 3.7% to 28% and does not have preference between these two orientations.

No significant difference (<5%) in $SAR_W$ between the conventional quadrature excitation and the four-port drive is observed at 64 MHz. At 128 MHz, the largest difference between $SAR_W$ of the conventional quadrature excitation and the four-port excitation occurs in the subject 1 loaded in the rung-on-plane coil orientation, which has the strongest coupling between coil and subject in all cases. The four-port excitation results in a $SAR_W$ about 9.6% less than that of the quadrature excitation. The difference of $SAR_L$ between these two excitations is below 11.2% at both frequencies.

4.5.4 ISNR

The body coil is used both for transmitting and receiving, so the RF power deposited in the body could also be used to estimate the noise in ISNR calculation (Edelstein et al., 1986; Carlson, 1989). The power absorbed is from 57.3 Watts to 59.8 Watts in subject 1 and from 36.5 Watts to 38.3 Watts in subject 2 with difference coil orientations and drive methods at 64 MHz. These results are also in good agreement with previous experiments (Bottomley et al., 1985) considered with the difference of flip angle, pulse duration, coil dimension, and coil drive method. At 128 MHz, the power absorbed is from 179.3 Watts to 226.0 Watts in subject 1 and from 114.9 Watts to 117.2 Watts in subject 2 with difference coil orientations and drive methods. The absorbed power increases with the frequency as expected theoretically. The slope of the increase,
however, is less than the theoretical value, \( f^2 \). In the case when subject 1 is loaded into rung-on-plane orientation, where the strongest coupling between the coil and the body exists, the power deposition at 128 MHz is about 1.3\% less (for quadrature excitation) and 12.5\% less (for four-port excitation) than the theoretical value. In all other cases, the power deposition is more than 20\% (from 20.3\% to 25\%) less than the theoretical value. This less-than-quadratic increase has been observed in a head model at frequencies above 260 MHz (Collins and Smith, 2001). It occurs here at a frequency as low as 128 MHz because the large size of the body makes the phase variation of \( B_1 \) on any transecting plane significant enough to cause cancellation of the net magnetic flux through the plane and thus less absorbed power according to Farad’s law (Collins and Smith, 2001). This less-than-quadratic increase in \( P_{abs} \) could be used to explain the greater-than-linear increase in ISNR with frequency (from 4.4\% to 7.5 \% more than that predicted linearly) in subject 2. In subject 1, the much less homogeneous flip angle distribution may diminish this effect of less power deposition on ISNR, resulting in an ISNR about 1.4\% to 2.9 \% less than the prediction assuming a linear increase with \( B_0 \).

Loading subject 2 results in higher ISNR than loading subject 1 in all cases. The difference is about from 8.6 \% to 10.7 \% at 64 MHz and from 18.0\% to 18.5\% at 128 MHz with different coil orientations and drive methods. Smaller body size, less muscle tissue, better RF penetration, and more homogeneous \( B_1^+ \) field distribution are among the contributors for the higher ISNR in subject 2 compared to subject 1. Loading the subject in different orientations has no significant effect on the ISNR. The difference is below 3.0\%. The conventional quadrature drive and the four-port drive result in almost identical ISNR (difference < 1.3\% in all cases).
4.6 Conclusions

Loading a larger, more muscular subject results in significantly less homogeneous $B_1^+$ distribution, higher RF power deposition, lower ISNR, higher SAR levels at both 64 MHz and 128 MHz. Only for the larger subject at 128 MHz, rung-off-plane orientation improves RF field homogeneity and reduces SAR level over rung-on-plane orientation with no significant improvement of SNR is observed. The conventional quadrature excitation and the four-port excitation result in similar $B_1^+$ field pattern, SAR levels, and ISNR except for the larger subject at 128 MHz the four-port excitation results less SAR level in the subject than quadrature excitation. Compared to the conventional quadrature excitation and the four-port excitation, the ideal excitation is relative accurate at 64 MHz and in most cases at 128 MHz for these body-size coil simulations but should be used with caution.
Chapter 5

Numerical Evaluation of Radiofrequency Power Radiated in High Field MRI
5.1 Abstract

The power radiated by an 11 cm × 11 cm surface coil placed 1 cm away from a spherical head phantom is numerically evaluated at 125, 175, 300, 345, 400, and 600 MHz. The percentage of input power radiated by the coil increases with the frequency and becomes significant (6.59%) at 300 MHz. The radiated power is also evaluated at 64, 128, and 170 MHz for a head-size shielded birdcage coil and at 170, 300, and 345 MHz for a TEM coil. The percentage of input power radiated becomes significant at 170MHz for the birdcage coil (8.35%) and at 300MHz for the TEM coil (18.4%). All calculations are repeated for a sample that has an electric permittivity different from that of the head phantom to test the effects of sample permittivity on radiation. The results indicate the interaction between the dielectric material in the sample and the RF coil helps to reduce the radiation. The radiation pattern is also calculated for all three coils at certain frequencies and it is found that the main radiation occurs in the directions perpendicular to the coil axis. These results should help the RF coil design at high fields.
5.2 Introduction

In MRI systems, the power radiated by an RF coil is expected to increase with frequency at high fields. Since the RF coil is placed in the gradient coils and magnet and altogether in a RF-screened room, most of the power radiated out of the RF coil is expected to be reflected back by the surface of gradient coils or the metal casing of the magnet and eventually dissipated in the subject (Keltner et al., 1991). This radiated power would increase the specific energy absorption rate (SAR) in the subject when the coil is transmitting and introduce extra coupling to body noise when the coil is receiving. In the early days of MRI, when the field strength was low enough that the wavelength was much larger than the coil-sample dimensions, this part of power loss was ignored (Hoult and Richards, 1976; Hoult and Lauterbur, 1979). Today, with the field strength as high as 8 T for human imaging, the wavelength is approaching the dimension of the coil and the radiated power may no longer be negligible.

Analytic calculations of radiated power have been done for different RF coil-sample systems in MRI. Keltner et al. (Keltner et al., 1991) calculated the radiated power by a surface coil adjacent to a homogeneous sphere with conductivity and permittivity of average brain up to 430 MHz. The radiated power was 15% of the power absorbed in the sphere for a 12-cm-diameter surface coil displaced 2 cm from the sphere at 400 MHz. Harpen (Harpen, 1993) estimated the radiative losses of an unshielded empty head size birdcage coil at 63 MHz and concluded that the radiative losses may account 20% of the total loss. At high fields where the wavelength approaches the dimensions of the RF coil
and with coil structures as complicated as those of shielded birdcage coils and TEM coils, the analytical approach becomes very difficult, if at all possible. With modern computer technology, numerical approaches are more suitable for this kind of problems.

The power radiated by a 11 cm × 11 cm surface coil at 125, 175, 300, 345, 400, and 600 MHz (2.9, 4.0, 7.0, 8.0, 9.4, and 14.0 T); by a head size birdcage coil (Hayes et al., 1985) with a shield at 64, 128, and 170 MHz (1.5, 3.0, and 4.0 T); and by a TEM coil (Vaughan et al., 1994) of the same size at 170, 300, and 345 MHz (4.0, 7.0, and 8.0 T) was calculated by solving full-wave Maxwell’s equations with the finite difference time domain (FDTD) method. To examine the effects of the electric properties of sample on the radiation power, all coils were loaded with two homogeneous head-size spheres having the same conductivity but different electric permittivities.

5.3 Method

5.3.1 Coil-sample models

A single-channel surface coil was modeled as an 11 cm × 11 cm square copper loop with four capacitors evenly distributed in the loop to tune the coil to 125, 175, 300, 345, 400, and 600 MHz (Figure 5.1). The coil was placed 1 cm away from a homogeneous spherical phantom (18 cm diameter) that contains one of two samples: sample one having relative permittivity ($\varepsilon_r$) of average brain (average of values for gray matter and white matter) and sample two having $\varepsilon_r$ of free space. Both samples have electrical conductivity ($\sigma$) of average brain. Calculations were performed in a 2-meter cubic space. The power
radiated out of a box S (50 cm × 50 cm × 78 cm) that enclosed the coil and the sample was calculated. A 5 mm spatial resolution was used in all three dimensions throughout the problem space.

Figure 5.1 The model of the surface coil with a spherical phantom in the whole problem region. Power radiated out of box S is calculated. The origin is set at the center of the phantom and the axis of the coil is in the x direction.

A shielded head-size (27cm diameter, 34cm shield diameter, and 22cm length) eight-rung high-pass birdcage coil at 64, 128, and 170 MHz and an eight-rung TEM coil
with the same dimensions at 170, 300, and 345 MHz were modeled (Figure 5.2). Sixteen lumped-element capacitors were placed in the end-rings of the birdcage coil to tune the coil to each resonant frequency. In the TEM coil, the dielectric material and the length of the inner conductors of the rungs were adjusted to tune the coil to the desired resonant frequencies. Both coils were driven in quadrature with two voltage sources that have same amplitude but 90° shifted phases. Both coils were loaded with the phantom used in the surface coil calculation. Calculations were performed in a 5-meter cubic space. The power radiated out of a box (2.5 m × 2.5 m × 2.5 m) that enclosed the coil and the sample was calculated. A higher spatial resolution sub-grid (5 mm in all three dimensions) was used to model the coil and the sample, and a lower resolution main grid (25 mm in all three dimensions) was used to model a large amount of free space surrounding the coil.
In each calculation, the origin was set at the center of the phantom. Following MRI conventions, the axis of the surface coil was set in the x direction and the axis of the birdcage coil and the TEM coil were set in the z direction. The elevation angle $\theta$ and azimuthal angle $\phi$ used in the radiation gain calculation and radiation pattern plot were also defined in Figure 5.1 for the surface coil and in Figure 5.2 for the volume coils. A second-order Liao absorbing boundary condition was used in all calculations and near-zone fields were monitored to conform the steady state was reached at each frequency.

Figure 5.2 The model of the volume coil-sample system in the whole problem region (a) and the close look of the spherical phantom in the birdcage (b) and the TEM coil (c) with half of the coil removed to show the details of coil structure. The capacitors in the end-rings of the birdcage coil are not shown here. Power radiated out of box S is calculated. The origin is set at the center of the phantom and the coil axis is in the z direction.
The large free space surrounding the coil-sample system is not used to calculate the far zone fields directly (the space is still not far enough for that approach) but to obtain more accurate near zone fields for the near-to-far-fields transformation (Kunz and Luebbers, 1993), which is used to calculated the far zone radiation pattern.

5.3.2 Formulas

After the steady state RF fields were found, the time-average radiation power was calculated by integrating the real part of Poynting vector over the surface of the box that enclosed the coil and the sample (Pozar, 1990):

\[ P_r = \frac{1}{2} \oint_S \text{Re}[E \times H^*] d\hat{s} \]  \[5.1\]

where \( P_r \) indicates the radiated power, \( S \) is the surface of the closed box, Re is to take the real part of a complex value, \( E \) is the complex electric field intensity, \( H \) is the complex magnetic field intensity, and * indicates conjugate. The power dissipated in the sample was calculated as (Collins and Smith, 2001):

\[ P_{abs} = \frac{1}{2} \sum_N (\sigma_{zn} E_{zn}^2 + \sigma_{yn} E_{yn}^2 + \sigma_{xn} E_{xn}^2) \Delta x \Delta y \Delta z \]  \[5.2\]

where the summation is performed over all \( N \) voxels of the whole sample, the \( \sigma_{xn}, \sigma_{yn}, \) and \( \sigma_{zn} \) are the conductivities of the \( n \)th voxel in three dimensions, and \( E_{xn}, E_{yn}, \) and \( E_{zn} \) are the electric field magnitude in the \( n \)th voxel in three dimensions. The percentage of the input power lost to radiation, \( e \), was calculated by taking the ratio of the radiation power to the input power, \( P_{in} \), which is the sum of the radiation power and the absorbed power:
The gain relative to a lossless isotropic antenna is given by (Kunz and Luebbers, 1993):

\[ e = \frac{P_r}{P_{in}} = \frac{P_r}{P_r + P_{abs}} \]  \[5.3\]

where \( \theta \) and \( \phi \) are the elevation and azimuthal angle as defined in Figure 5.1 and Figure 5.2 for the surface coil and volume coils, respectively. \( E_\theta \) and \( E_\phi \) are the \( E \) fields in the \( \theta \) and \( \phi \) directions at far zone transformed from the near zone fields. \( \eta_0 \) is the impedance of free space.

\[ \text{Gain}(\theta, \phi) = \frac{(|E_\theta|^2 + |E_\phi|^2)}{P_{in} / 4\pi \eta_0} \]  \[5.4\]

5.3.3 Verification of the methods

All the modeling was carried out with a commercially available software package XFDTD (www.remcom.com), which has been widely used in MRI (Alecci et al., 2001; Collins and Smith, 2001; Ho, 2001; Vaughan et al., 2001; Zhai et al., 2002) and other fields. Examples and verification of radiation power calculations for antennas with XFDTD are available (Kunz and Luebbers, 1993).

5.4 Results

The percentage of input power radiated by the surface coil loaded with sample 1 and sample 2 at 125, 175, 300, 345, 400, and 600 MHz is shown in Table 5-1 and plotted in Figure 5.3 with a solid line for sample 1 and a dashed line for sample 2. The input power radiated by the surface coil loaded with sample 1, which has electrical
conductivity and permittivity of average brain, increases with frequency from 2.93% at 125 MHz to 26.0% at 600 MHz. The power radiated by the surface coil loaded sample 2, which has electrical conductivity of average brain and permittivity of free space, increases from 4.24% at 125 MHz to 42.8% at 600 MHz.

Table 5-1 The percentage of the input power radiated, \( e \), calculated by taking the ratio of power radiated over the sum of power radiated and the power dissipated in the sample, for a surface coil loaded with different samples at different frequencies \( f \).

<table>
<thead>
<tr>
<th>( f ) (MHz)</th>
<th>Sample</th>
<th>( \sigma ) (S/m)</th>
<th>( \varepsilon_r )</th>
<th>( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>1</td>
<td>0.541</td>
<td>66.92</td>
<td>2.93%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.541</td>
<td>1</td>
<td>4.24%</td>
</tr>
<tr>
<td>175</td>
<td>1</td>
<td>0.585</td>
<td>58.95</td>
<td>3.68%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.585</td>
<td>1</td>
<td>7.17%</td>
</tr>
<tr>
<td>300</td>
<td>1</td>
<td>0.657</td>
<td>50.56</td>
<td>6.59%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.657</td>
<td>1</td>
<td>16.4%</td>
</tr>
<tr>
<td>345</td>
<td>1</td>
<td>0.675</td>
<td>49.00</td>
<td>8.65%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.675</td>
<td>1</td>
<td>20.2%</td>
</tr>
<tr>
<td>400</td>
<td>1</td>
<td>0.695</td>
<td>47.68</td>
<td>11.0%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.695</td>
<td>1</td>
<td>25.1%</td>
</tr>
<tr>
<td>600</td>
<td>1</td>
<td>0.764</td>
<td>45.02</td>
<td>26.0%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.764</td>
<td>1</td>
<td>42.8%</td>
</tr>
</tbody>
</table>
The percentage of input power radiated by the birdcage coil at 64, 128, and 170 MHz and by the TEM coil at 170, 300, and 345 MHz loaded with sample 1 and sample 2 is shown in Table 5-2 and plotted in Figure 5.4 with a solid line for sample 1 and dashed line for sample 2. When the coil is loaded with sample 1, the percentage of the input power radiated increases with the frequency from 3.78% at 64 MHz to 8.35% at 170 MHz for the birdcage coil and from 2.77% at 170 MHz to 27.3% at 345 MHz for the TEM coil. When the coil is loaded with sample 2, the percentage of the input power radiated increases from 4.70% at 64 MHz to 17.39% at 170 MHz for the birdcage coil and from 6.15% at 170 MHz to 29.91% at 345 MHz for the TEM coil.

Figure 5.3 The percentage of input power radiated by a surface coil loaded with sample 1 (blue) and sample 2 (red).
Snapshots of the E field magnitude at an arbitrary time step after the fields reach steady state and the radiation pattern in the central x-z plane for the surface coil loaded with both samples at 600 MHz are shown in Figure 5.5. Those for the birdcage coil at 170 MHz and the TEM coil at 345 MHz loaded with sample 1 are shown in Figure 5.6. The E field is normalized such that the maximum E field magnitude is equal to 1. The radiation pattern is the plot of gain (dBi) versus θ, which is defined in Figure 5.1 for the surface coil and in Figure 5.2 for the volume coils. Field distributions and radiation patterns for the volume coils loaded with sample 2 are similar.

Table 5-2 The percentage of the input power radiated by a high-pass birdcage coil and by a TEM coil loaded with different samples at different frequencies f.

<table>
<thead>
<tr>
<th>Coil</th>
<th>f (MHz)</th>
<th>Sample</th>
<th>σ(S/m)</th>
<th>ε_r</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Birdcage</td>
<td>64</td>
<td>1</td>
<td>0.451</td>
<td>91.93</td>
<td>3.78%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.451</td>
<td>1</td>
<td>4.70%</td>
</tr>
<tr>
<td></td>
<td>128</td>
<td>1</td>
<td>0.545</td>
<td>66.27</td>
<td>6.79%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.545</td>
<td>1</td>
<td>12.1%</td>
</tr>
<tr>
<td></td>
<td>170</td>
<td>1</td>
<td>0.582</td>
<td>59.5</td>
<td>8.35%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.582</td>
<td>1</td>
<td>17.1%</td>
</tr>
<tr>
<td></td>
<td>170</td>
<td>1</td>
<td>0.582</td>
<td>59.5</td>
<td>2.77%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.582</td>
<td>1</td>
<td>6.15%</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>1</td>
<td>0.657</td>
<td>50.56</td>
<td>18.4%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.657</td>
<td>1</td>
<td>21.0%</td>
</tr>
<tr>
<td></td>
<td>345</td>
<td>1</td>
<td>0.674</td>
<td>49</td>
<td>27.1%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.674</td>
<td>1</td>
<td>29.9%</td>
</tr>
</tbody>
</table>
Figure 5.4 The percentage of input power radiated by a head-size birdcage coil (square) and TEM coil (triangle) loaded with two different samples versus frequency. Blue lines represent sample 1 and red lines represent sample 2.
Figure 5.5 Snapshots of $E$ field magnitude and radiation pattern in the central x-z plane of the surface coil loaded with sample 1 (a, c) and with sample 2 (b, d) at 600 MHz. The field is normalized such that the maximum $E$ field magnitude is equal to 1. The white circle in (a, b) indicates the location of the sample. The radiation pattern is the plot of gain (dBi) versus $\theta$, which is defined in Figure 5.1. The dashed line indicates the direction of maximum gain.
Figure 5.6Snapshots of \( \mathbf{E} \) field magnitude and radiation pattern in the central x-z plane for the birdcage coil at 170 MHz (a, c) and for the TEM coil at 345 MHz (b, d). Both coils are loaded with sample 1. The field is normalized such that the maximum \( \mathbf{E} \) field magnitude is equal to 1. The radiation pattern is the plot of gain (dBi) versus \( \theta \), which is defined in Figure 5.2. The dashed line indicates the direction of maximum gain.
5.5 Discussion

Previous computer modeling and experimental studies at 7.0 T demonstrated that sample 1 yielded a similar RF field distribution in the human brain at 300 MHz (Yang et al., 2002). Thus, its perturbation to the RF fields should approximate that of the human head. Sample 2 has the conductivity same as sample 1 but the permittivity of free space. It does not simulate a real situation but to conceptually test the effect of the permittivity on the radiation. The power radiated in the surface-coil-sample-2 system is always larger than that in the surface-coil-sample-1 system. This is because the larger permittivity in sample 1 facilitates RF wave penetration and propagation in the sample (Yang et al., 2002) so a smaller portion of the input power is radiated. This effect can be seen graphically in the E field distribution inside and outside the two samples (Figure 5.5 (a), (b)). More field penetration is seen inside the sample in the coil-sample-1 system while stronger field is seen outside the sample in the coil-sample-2 system. The radiation patterns for both coil-sample systems are shown in Figure 5.5 (c) and (d). The two systems have similar radiation pattern with the surface-coil-sample-2 system having larger radiation gain. In the surface-coil-sample-1 system, the dielectric permittivity helps to cut the radiation down to almost half (51%) of that in the surface-coil-sample-2 system averaged over all six frequencies. This dramatic decrease of radiated power with the permittivity is also observed in the birdcage coil and in the TEM coil. The radiation in birdcage-coil-sample-1 system is only about 61% of that in the birdcage-coil-sample-2 system and the radiation in the TEM-coil-sample-1 system is about 75% of that in the TEM-coil-sample-2 system, averaged over corresponding frequencies. Perhaps dielectric
materials can be used not only to manipulate the near fields (Foo et al., 1992; Alsop et al., 1998; Zhang et al., 2001; Yang et al., 2002) but also to reduce far zone radiation.

The power radiated by a surface coil placed 1 cm away from a sphere of 18 cm diameter of sample 1, which should simulate the head, remains less than 5% at frequencies up to 300 MHz. At 400 MHz, the percentage of the input power radiated is about 11%, which agrees well with the analytic result published by Keltner et al (1991), which can be calculated to be 13%. The problem of radiation becomes even more severe at 600 MHz where about 26% of the input power is radiated. As shown in Figure 5.5 (c), the direction of maximum radiation gain is in an angle about 21° from the y-z plane. The supposed direction of the maximum gain of a surface coil with axis in x direction and without the presence of the sample would be 0° from the y-z plane.

The input power radiated by a birdcage coil loaded with sample 1 is about 8.35% at 170 MHz but for the TEM coil it is only 2.77%. Better RF shielding of the TEM coil might contribute to the lower amount of radiation. The direction of the maximum radiation gain for a birdcage coil is in the transverse plane. An elongated RF shield or an inwardly extended shield (Collins et al., 1998; Liu et al., 2004) that closely shields the two eng-rings in the birdcage coil might help to reduce the radiation. The radiation in the TEM coil becomes significant at frequencies above 175MHz. The radiation pattern for a TEM coil is similar to that of a birdcage coil with the maximum radiation gain is in the transverse plane. The asymmetry in the radiation patterns is caused by the non-ideally-symmetric current distribution in the coils.

As shown in our models, the radiation of an RF system really depends on the details of the system. The results in this study may only apply to the coil with similar
dimensions, geometry, and samples. Sample loss dominance is assumed and the resistive loss of the coils is ignored in the calculations.

5.6 Conclusions

The radiation becomes significant at a frequency as low as 170 MHz for a shielded birdcage coil loaded with a head phantom. The power radiated by a surface coil or by a TEM coil loaded with the same phantom also becomes significant at 300 MHz. The interaction between the RF coil and the dielectric material in the sample helps to reduce the radiation. The main radiation is in the plane perpendicular to the coil axis. While the models used in this study are typical settings in MRI practice, results should be used with caution since the radiation depends on the details of the system.
Summary

The FDTD method has been proven a reasonably accurate and valuable tool in the RF field calculation in high field MRI. Calculations of $B_1$ field, $SNR$, $SAR$, and radiation loss with different coils loaded with different samples agree with the experiments conducted in this study or published previously and provide useful information like local $SAR$ distribution that could not be obtained easily with experiments or analytical approaches. It is shown that end-ring/shield configuration affects the $B_1$ field distribution and $SNR$ in a birdcage coil. With a newly created anatomically accurate female body model, the calculation of $B_1$ field distribution, $SAR$, and $SNR$ are compared between different body types. It is found that loading a larger, more muscular subject into the RF coil results in significantly less homogeneous $B_1^+$ distribution, lower $SNR$, higher $SAR$ levels. The RF radiation loss in a surface coil, in a head size birdcage coil, and in a head size TEM coil loaded with different phantoms at a frequency range from 64 MHz to 600 MHz is also evaluated numerically and results show that the radiation becomes significant at high fields and the interaction between the RF field and the dielectric material in the sample helps to reduce the radiation. These results provide useful information for RF design and MRI safety guideline at high fields. More models of different body types, those of children for example, and better models of RF coils (higher resolution, modeling of loss in the coil, modeling of matching circuit, etc.) should be pursued in the future to improve the accuracy of the calculations. Other future projects should include a systematic study of radiation loss and its quantitative relation with $SNR$. 
at high fields; evaluation of power distribution (power dissipated in the sample, in the coil, power radiated) in MRI systems; and modeling parallel imaging system, which has shown promising improvement on $B_1$ homogeneity among other benefits.
Bibliography


Beck BL, Brooker HR and Blackband SJ (2000). The coaxial reentrant cavity (ReCav) coil for high frequency large volume MRI/S. *Proc. The eighth annual meeting ISMRM, p.1388*, Denver, Colorado, USA.


Gabriel C (1996). Compilation of the dielectric properties of body tissues at RF and microwave frequencies. Number


VITA

Wanzhan Liu
Wanzhan.liu@gmail.com

Education

Sept. 1996-Sept. 1999: M.S. in Biomedical Engineering; Graduate School of University of Science and Technology of China, Beijing, China;

Sept. 1991-July 1995: B.S. in Electrical Engineering; Huazhong University of Science and Technology, Wuhan, Hubei, China;

Selected Publications and presentations


Liu, W., C. M. Collins and M. B. Smith (2002). “Effects of end-ring configuration on homogeneity and signal to noise ratio in a birdcage coil loaded with the human head.” Proc. The tenth annual meeting ISMRM, p.774, Honolulu, Hawaii, USA.


Liu, W., Q. X. Yang, C. M. Collins and M. B. Smith (2002). “Numerical evaluation of power radiated and dissipated by a loaded surface coil at high field.” Proc. The tenth annual meeting ISMRM, p.915, Honolulu, Hawaii, USA.