DYNAMIC SUPPLY CHAIN INVENTORY PLANNING MODELS

CONSIDERING SUPPLIER SELECTION AND TRANSPORTATION COSTS

A Dissertation in
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by

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ABSTRACT

In the current competitive global economy, companies are forced to improve their efficiency, and consequently develop appropriate decision support tools that can be used to optimize their entire supply chains. Even though the literature presents extensive mathematical models for analyzing the different processes of a supply chain individually, research covering a system-wide approach is limited and recent. This is the case of inventory planning decisions, which traditionally have been analyzed independently from supplier selection and transportation decisions. In addition, previous research has made assumptions that may not always hold under real world operational settings. This is the case of stationary demand, which is not always true at the tactical and operational levels. Similarly, in distribution the problem is usually simplified by assuming a unique transportation mode, considering neither delivery lead times, nor economies of scale. Therefore, there is an urgent need for integrated models that optimize the entire supply chain, where operations can be accurately represented mathematically.

In the first part of this research, a multi-period single-product inventory lot-sizing model for a serial supply chain problem is developed, where raw materials are purchased from multiple suppliers at the first stage and external demand occurs at the last stage. The demand is known and may change from period to period. The purpose of this model is to determine the optimal inventory policy that coordinates the transfer of materials between consecutive stages of the supply chain from period to period while properly placing purchasing orders to selected suppliers and satisfying customer demand on time. Transportation costs are modeled using an all-units discount structure, where a less-than-truck load (LTL) transportation mode is selected. A general version of the model is formulated as a mixed integer nonlinear program, where transportation costs are either represented by piecewise linear functions or approximated by power or quadratic functions.

In the second part of this research, a mixed integer linear program version of the original single-product model is extended to the multi-product case, where more than one type of raw material may be required for each product. In this scenario, economies of
scale may be obtained by the coordination of replenishment, manufacturing, and transportation orders. Full-truck-load (TL) and LTL transportation modes are both considered in the mathematical formulation. This model provides the appropriate flexibility for practical implementation in real production/distribution systems.

In this research, the convenience of the integrated analysis approach over the sequential method is illustrated. Experimental results obtained in this research show cost savings of up to 6% when the problem is solved using the proposed integrated approach compared to the sequential method, where the production/distribution and supplier selection problems are independently solved in two stages. These experimental results also show that the optimal raw material procurement strategy is affected by production and inventory cost parameters, which may appear to be unrelated to raw material supplier decisions. This finding also ratifies the convenience of using the integrated approach, where the entire supply chain problem is simultaneously analyzed.
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Dedication

This work is dedicated to my loving wife Macarena, and our wonderful children Camila, Benjamin and Cristobal. I am truly thankful for the selflessness and love that you have shown by agreeing to take this journey so far from home. Thanks, Maca, for the support, encouragement, and love that you have provided me throughout all of these years. I will never be able to compensate you for the personal sacrifices that you have made for me and for our children. I can only offer you my unconditional love.

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Chapter 1

Introduction and Overview

1.1 Introduction

In the current global economy, companies have been forced to develop more efficient processes in order to satisfy customer demand at lower costs and with improved service levels. The continuous growth of international markets has significantly impacted supply chain designs and the execution of their associated strategies. Even small companies have customers and suppliers around the world. This situation has been accentuated by increased product customization, resulting in complex distribution networks with thousands of stock keeping units (SKUs) and hundreds of stock locations (Feigin et al., 2003).

During the last two decades, there has been a growing interest in the design, analysis, and performance of supply chains. According to Beamon (1998), this attention is largely due to higher manufacturing costs, scarce resources, faster product life cycles, and the globalization of market economies. Companies now manage their supply chains in highly dynamic environments. Globalization, which expands supply chains across countries and makes boundaries less important, has significantly increased supply chain complexity and flow coordination (Vidal and Goetschalckx, 1997).

In modern supply chains, products can be classified as functional or innovative, resulting in flow dynamics that add additional complexity (Fisher, 1997). Functional products are physical products without any added value that usually have long life cycles and stable demand, but low profit margins. Companies innovate in order to make their products more attractive to customers, which creates competitive advantages. Increased competition shortens the life cycle for innovative products and produces highly dynamic demand patterns. Additionally, customer-focused market orientation trends force supply
chains to satisfy demands for custom products and services, further increasing complexity.

Cook (2006) provides detailed information about logistics costs in the US from 1994 to 2005. In his analysis, inventory, transportation, and logistics administration costs are calculated as a percentage of gross domestic product (GDP). According to this report, the percentage of total logistics costs decreased from the mid-nineties to 2003 due to the strategies that US companies implemented to reduce inventory and transportation costs. However, this trend has reversed since then, showing a sustained increase each year. In 2005, total logistics costs represented 9.5% of GDP, with transportation costs at 6%, inventory costs at 3%, and administrative costs at 0.5%.

Given the growing importance that companies and researchers have placed on supply chain optimization, multiple areas of research have been developed during the last few decades. Extensive literature has been dedicated to inventory, production, transportation and distribution. Most of the work has been focused on analyzing one of these problems or on optimizing individual components of the supply chain. Thus, under current economic conditions characterized by high competitiveness and growing supply chain complexity, integrated analysis of these problems for the entire supply chain has become a high priority.

### 1.2 Supply Chain Structure

According to Min and Zhou (2002), a supply chain is an integrated system that synchronizes interrelated business processes in order to: (1) acquire raw materials and parts; (2) transform those raw materials and parts into finished products; (3) add value to the products; (4) distribute products to retailers and customers; and (5) facilitate information exchanges between the business entities involved in the supply chain. In general terms, the supply chain is integrated by two basic processes: the production planning process and the distribution and logistics process (see Figure 1.1).
According to Chopra and Meindl (2007), a supply chain includes all parties directly or indirectly involved in fulfilling a customer request, including manufacturers, suppliers, transporters, warehouses, retailers, and even customers themselves. Similarly, Bozarth and Handfield (2006) define the supply chain as a network of manufacturers and service providers that work together to convert and move goods from the raw material stage through to the end users.

Supply chains can be classified according to the centralization of their decisions. In centralized supply chains, a single decision maker defines optimal policies for the entire system. Most frequently, centralized supply chains exist in companies that own their distribution networks, including modes of transportation and physical plants. In decentralized supply chains, multiple decision makers make independent decisions in order to optimize their individual objectives. Even though optimal supply chain efficiency is not achieved using this decentralized approach, it is the most common system used in practice (Lee and Whang, 1999).

According to Chopra and Meindl (2007), the success of a supply chain is closely tied to the design and management of supply chain flows. Multiple decisions need to be made simultaneously regarding the flow of information, products, and money. These decisions are classified as strategic, tactical or operational depending on their frequency and duration.
1.3 Supply Chain Management

Multiple definitions are provided in the literature for supply chain management (SCM). Lee and Billington (1992) describe SCM as the coordination of manufacturing, logistics, materials, distribution, and transportation functions within an organization. Thomas and Griffin (1996) define SCM as the management of material and information flows both in and between facilities, vendors, manufacturing and assembly plants, and distribution centers. Chase (1998) states that SCM is an effective and systematic approach used to manage the overall flow of information, materials, and services produced, while customer demand is satisfied. Similarly, Simchi-Levi et al. (2004) state that SCM involves a set of approaches that are used to efficiently integrate all parties involved in a supply chain. Chopra and Meindl (2007) point out that SCM involves managing flows between and among stages in a supply chain in order to maximize its profitability. In all cases, the final objective of SCM is to minimize system-wide costs while maximizing service levels, and to ensure that products are manufactured and distributed in the right quantities to the right locations at the right times.

1.4 Operations Research in Supply Chain Management

Traditional supply chains are managed based on decisions that have been adequate in the past or seem to be intuitively good (Simchi-Levi et al., 2005). However, it has been proven that optimization techniques can significantly improve supply chain efficiency. Geoffrion and Powers (1995) mention that with increased computer capabilities, optimization techniques assume a more relevant role in distribution system design because more realistic models can be created. The extensive literature on quantitative supply chain methods highlights the importance of operations research. This is partially due to problem complexity, where efficient solutions can only be created with the support of appropriated tools. For example, Simchi-Levi et al. (2004) mention that minimizing system-wide costs while maintaining service levels is a challenging goal for
practitioners and researchers. Geunes and Chang (1999) provide a survey of operations research models, with a special focus on supply chain design and decision coordination.

Significant cost savings have been reported by companies applying analytic decision support tools to their operations. Blumenfeld et al. (1987) report cost savings opportunities of $2.9 million per year when decision support tools were applied to the logistics operations at General Motors. Martin et al. (1993) report estimated annual savings of $2.0 million at Libbey-Owens-Ford when a linear programming model was applied to optimize production, inventory, and distribution simultaneously in a multi-plant system.

An efficient approach involves analyzing the entire supply chain, thereby avoiding local sub-optimization produced by the independent analysis of each process. In fact, during the last two decades, companies have recognized that significant cost savings can be achieved by integrating production plans, inventory control, and transportation decisions (Muriel and Simchi-Levi, 2003).

1.5 Inventory Management in Supply Chain

An efficient way to reduce costs and increase working capital is to correctly implement inventory management policies. However, inventories in complex supply chain networks can only be efficiently managed with appropriate analytic decision support tools. Bregman (1990) estimates that a typical firm with annual sales of $500 million could save nearly $100,000 in annual inventory holding costs for every 1% reduction in finished goods inventory. Thus, inventory management decisions are important, given their significant impact on working capital and product availability.

In supply chains, inventory is used to balance the mismatch between supply and demand. It is held in the form of raw materials, work in process, and finished goods. Inventory is a major expense that highly impacts supply chain responsiveness (Chopra and Meindl, 2007). It plays an important role, because the ability to satisfy demand depends on the number of products that are ready and available when demand occurs.
Inventory also has a significant impact on cost reduction, since it can be used to exploit economies of scale in the raw material procurement, production and distribution stages.

Inventory decisions in the distribution network have been analyzed extensively by researchers. Traditionally, the main research focus has been on understanding the structure of optimal decisions (in terms of order quantities or review levels) which meet the objective of minimizing ordering and holding costs when demand patterns are known and stationary (Erengüc et al., 1999). However, those assumptions are not always true, and therefore new alternatives must be considered. Companies traditionally need to manage inventories in multiple stock locations, where transportation decisions become interwoven with inventory control decisions. For inventory control systems with multiple locations, it would be advantageous to simultaneously analyze holding and transportation costs, since in most logistics environments, transportation costs are significantly high.

In complex supply chains where products and components are manufactured in multiple facilities, inventory costs represent a significant proportion of the total network cost (Kaminskya and Kayab, 2008). In these systems, products can be manufactured either using a make-to-order (MTO) strategy or a make-to-stock (MTS) strategy. In facilities managed under an MTO strategy, products are manufactured according to actual demand, while in facilities managed under an MTS strategy, products are manufactured in anticipation of demand and held in inventory. When manufacturing times are short and production networks are relatively uncongested, companies can minimize inventory holding costs by implementing an MTO strategy. However, when the total time required to manufacture a product is greater than the acceptable delivery lead time, companies must plan manufacturing operations in anticipation of specific orders and keep some inventory in their warehouses to be able to meet customer demand in an acceptable amount of time (Kaminskya and Kayab, 2008).

Given the impact of product inventory on supply chain responsiveness and cost efficiency, inventory management plays a significant role in supporting a company’s competitive strategy. Thus, if a company’s competitive strategy requires high levels of responsiveness, then large amounts of inventory can be allocated in proximity to the
customers. However, if a company’s competitive strategy is to be a low cost producer, inventory levels can be reduced through centralized storage locations.

Two main decisions need to be made in inventory models: when to order and how much. In this context, periodic and continuous reviews are the two most common control policies used to minimize costs while satisfying minimum service levels. Under a periodic review policy, a decision maker places periodic orders every \( r \) periods to bring inventories up to a pre-specified level \( S \). For this reason, a periodic review policy is also called a \((r, S)\) policy, where the order quantity is a random variable. On the other hand, under a continuous review policy, a decision maker orders up to a fixed quantity \( Q \) when inventory falls below predefined reorder point \( s \). For this reason, it is also called a \((s, Q)\) policy, where the time between orders is a random variable. The economic order quantity (EOQ) model is the most basic method for determining order quantities in the continuous review system. The EOQ model defines the order quantity \( Q \), minimizing ordering and holding costs. Once \( Q \) is determined, safety stock, reorder points and number of orders per year can also be calculated.

Given the significant importance of inventories in supply chains, companies are constantly reviewing their inventory policies, trying to find the appropriate balance between working capital and supply chain responsiveness. Since supply chain management has become highly complex due to increases in the number of SKUs and competitors, the development of decision support tools in this area is critically important in real world settings.

1.6 Research Objectives and Contributions

The main objective for this research is to develop mathematical programming models that can be used as decision support tools for centralized supply chains. The basic idea is to provide a system-wide analysis for the entire supply chain, considering multiple decisions that traditionally have been analyzed independently. In addition, supplier
selection and transportation costs are incorporated into the supply chain production/distribution planning process.

The objective function for the models minimizes purchasing, production, inventory, and transportation costs, while completely satisfying product demand. The problem is formulated as a multi-period inventory lot-sizing model in a serial supply chain, where raw materials are purchased from multiple suppliers at the first stage and external demand occurs at the last stage. The demand is known and may change from period to period. The stages of this production-distribution serial structure correspond to inventory locations. The first two stages represent storage areas for raw materials and finished products in a manufacturing facility, and the remaining stages symbolize distribution centers or warehouses that take the product closer to customers. Even though the problem is formulated for a serial supply chain network structure, the models can be generalized to more general chains.

The model is initially formulated for a single product problem, where an all-units discount strategy is applied to stimulate demand for larger and more profitable shipments using a less-than-truckload (LTL) transportation mode. Positive lead times are allowed for all stages in the supply chain. The model is later extended to a case involving multiple products and raw materials, where replenishment, production, and transportation operations are jointly considered in order to take advantage of potential economies of scale. Full-truck-load (TL) and LTL transportation modes are considered as alternative shipment options between stages, providing flexibility for practical implementations in real production/distribution systems.

To overcome the disadvantages associated with the increase in problem size, an analytic approach to reduce the size of the respective time-expanded supply chain network is proposed. This method is based on the identification of nodes and arcs in the transshipment network that cannot be used or accessed by any feasible solution due to positive lead times and a finite planning horizon.
1.7 Overview

The remainder of this thesis is organized as follows. Chapter 2 presents a literature review of the main drivers in supply chain optimization, including raw material supplier selection, inventory management, and distribution plans. Initially, a general description of each research area is presented, and then the relevant work for integrated optimization models is described.

Chapter 3 presents a mixed integer nonlinear programming model for the dynamic multi-period inventory problem with supplier selection and transportation costs in a serial supply chain structure. The proposed model analyzes the single-product case, where raw materials are purchased from multiple suppliers. The purpose of this model is to minimize the total variable cost, including purchasing, production, inventory, and transportation costs. Transportation costs are modeled using an all-units discount structure, where the LTL transportation mode is selected. Transportation costs are either represented by piecewise linear functions or approximated by power and quadratic functions. The problem is modeled as a time-expanded network, which is characterized according to the flow feasibility of its nodes and arcs.

Chapter 4 extends the model proposed in Chapter 3 by suggesting a mixed integer linear programming formulation for the multi-product case. The model assumes that multiple raw materials may be used to manufacture each finished product. A joint replenishment cost structure is considered for raw material procurement and production operations. In this model, TL and LTL transportation modes are considered as alternative shipment options between stages.

Chapter 5 presents the concluding remarks obtained from this thesis research and also the new opportunities that have been identified as extension of this work.
Chapter 2

Literature Review

2.1 Introduction

In traditional supply chain management, decisions are made using a sequential approach, where every component of the supply chain is optimized independently. Thus, the optimal solution of one stage is used as input for the next decision stage. However, given that the sequential approach will generally lead to local optimal solutions, further improvement in cost savings can be overlooked. In this chapter, a review of the main drivers of supply chain optimization is provided, including raw material supplier selection, inventory management, and distribution planning. Initially, a general description of research on supplier selection and supply chain inventory models is presented, including models that incorporate transportation considerations. Finally, the relevant work on integrated supply chain optimization models is reviewed, where supplier selection, inventory, and transportation costs are considered simultaneously.

2.2 Supplier Selection Models

In a manufacturing environment, even though raw material orders could be assigned to only one selected supplier, maintaining a multi-supplier structure incents competitiveness while reducing sourcing risks. For example, Geetha and Achary (2000) show that by dividing orders into multiple suppliers, lead time variance can be decreased. Even though it could be argued that administrative costs can be increased under a multi-supplier structure, Bakos and Brynjolfsson (1993) mention that because of information technology advances, administrative costs have been substantially reduced during recent years. Sculli and Shum (1990) explain that a multi-supplier system can be implemented using two alternative mechanisms: 1) placing orders with each supplier according to
some relative frequency; or (2) dividing each order into multiple portions, using all suppliers for each order.

Alternative mechanisms for modeling multi-supplier systems can be found in the literature. Moore and Fearon (1973) suggest a conceptual linear programming (LP) model for supplier selection where price, quality, and delivery are described as important criteria, however no actual model formulation is provided. Gaballa (1974) proposes one of the first mathematical programming formulations for supplier selection, implementing a mixed integer linear programming (MILP) formulation for the Australian post office, where the objective function minimizes the total price of allocated items under capacity and demand constraints. Kingsman (1986) proposes a linear programming and also a dynamic programming (DP) model to minimize purchasing costs under stochastic commodities price conditions. Narasimhan and Stoynoff (1986) formulate a MILP model that allows a large manufacturing firm to determine suppliers and order quantities for multiple production plants. Turner (1988) proposes a model used by a coal company to minimize total contract price subject to demand, supplier capacity, minimum and maximum order quantities, and geographic region constraints. Rosenthal et al. (1995) present a model used to minimize the total purchasing cost over a single period, considering supplier capacity, quality, and demand constraints, including price discounts for product bundles.

At the same time, other scholars address different aspects of emerging supply chain complexities. Pan (1989) presents a LP model to select the best suppliers under quality, demand, and service constraints which shows that multiple sourcing is the best alternative for improving reliability in the supply of critical materials. Chaundry et al. (1993) take price breaks into account when considering the problem of vendor selection under quality, delivery, and capacity constraints.

More recently, researchers have created models that address even deeper levels of complexity. Systems with suppliers that are either fast and expensive or slow and inexpensive have been evaluated to see whether demand information updates improve selection (Yan et al., 2003). The resulting model also applies to multiple-period problems when some demand regularity conditions are satisfied. Fox et al. (2006) consider a
supplier system comprised of one low variable cost (LVC) supplier with a significant fixed cost, and one high variable cost (HVC) supplier with a negligible fixed cost. Defining \((s, S)\) as the base-stock inventory policy, where every time that the inventory position is at or below the reorder point \(s\), an order is placed so that the inventory position is raised up-to-level \(S\). It is shown that depending on the optimal cost function, one of three following policies is advantageous: 1) order exclusively from the LVC supplier using \((s_{LVC}, S_{LVC})\) policy; 2) order exclusively from the HVC supplier using \((s_{HVC}, S_{HVC})\) policy; or 3) use a mixed ordering strategy \((s, S_{HVC}, S_{LVC})\). The mixed strategy is such that, if the inventory position is between \(s\) and \(S_{HVC}\), order up to \(S_{HVC}\) from the HVC supplier. If the inventory position is less than \(s\), order up to \(S_{LVC}\) from the LVC supplier. Finally, if the inventory position is above \(S_{HVC}\), then do not order from either supplier.

Supplier systems have also been represented as queuing networks, where manufacturers maintain constant inventory levels and cost criteria are obtained from a nonlinear function (Arda and Hennet, 2006).

Wadhwa and Ravindran (2007) address the supplier selection problem using a multi-level objective model that considers both qualitative and quantitative factors. In this research, goal programming, weighted objective and compromise programming are analyzed. Ravindran and Wadhwa (2009) present an overview of various multi-criteria techniques in the context of the Department of Defense. These authors suggest a two-step approach for the supplier selection problem. First, a multi-criteria optimization model is used to rank and pre-screen suppliers. Second, alternative multi-criteria methods are used to evaluate the set of suppliers selected from the initial step.

Gencer and Gürpınar (2007) suggest an analytic network process model in an electronic company to evaluate and select the most appropriate supplier with respect to various supplier evaluating criteria, which are classified into three clusters. The interrelationships among the criteria are considered in the selection process.

Federgruen and Yang (2008) analyze the problem where a company needs to satisfy uncertain demand for a given item by procuring supplies from multiple sources. The problem determines the optimal set of suppliers, the aggregated order and its location among suppliers such that total costs are minimized while ensuring that the demand is
met with a given probability. Chou and Chang (2008) apply a fuzzy SMART approach to evaluate the alternative suppliers in an IT hardware manufacturing company. A sensitivity analysis assesses the impact of changes in the risk coefficients in terms of supplier ranking order.

Burke et al. (2010) propose a heuristic procedure for the sourcing problem from multiple suppliers with alternative quantity discounts. In this problem, the total quantity to be procured for a single period is known by the company and communicated to the suppliers. In response, each supplier quotes a price and its capacity limit in terms of the maximum quantity that can be supplied to the buyer. Based on this information, the company decides the quantity allocation between the selected suppliers. The heuristic is tested assuming linear discounts, incremental unit discounts, and all-units discounts. Experimental results show that the heuristic generates near-optimal solutions very quickly.

2.3 Supply Chain Inventory Models

Supply chain inventory systems are commonly implemented by companies when customers are distributed over large geographical areas. In these cases, products pass through multiple stages before they are received by the final customers. Considering the complexity produced by the interactions between different levels, the determination of inventory policies for multi-stage systems is considered to be a difficult problem.

Schwarz (1973) introduces a heuristic solution for a common scenario involving one warehouse and multiple retailers. Such a problem has been found to be very complex, given that order quantities vary even under deterministic demand conditions. Taking this idea further, Williams (1981) presents seven heuristics algorithms for scheduling production and distribution operations in a supply chain network, where the production process is assumed to be an assembly process. These heuristics are used to determine the minimum cost production/distribution schedule that satisfies the final customer demand. Williams (1983) also proposes a dynamic programming algorithm to solve the production
and distribution lot size simultaneously for each node in the supply chain network. The objective function of the algorithm minimizes the average cost per period over an infinite time horizon. Roundy (1985) develops the power-of-two policy, which states that the time between retail orders is a power-of-two multiple of a base period. An optimal solution involving relaxation of the power-of-two policy has also been derived as an alternative method.

Researchers have also studied inventory control, specifically whether it should be centralized or decentralized. Axsäter and Rosling (1993) show that centralized inventory control dominates decentralized control for an N-stage serial supply chain when each location follows a \((s, Q)\) policy, where every time that the inventory position is at or below \(s\), a replenishment order of size \(Q\) is placed. Axsäter and Juntti (1996) show that centralized control dominates when warehouse lead times are long, while decentralized approaches dominate when warehouse lead times are short. The results came from a study comparing worst-case scenarios for an installation versus an echelon stock policy for a two-stage serial supply chain with deterministic demand and simulated stochastic demand. More recently, Abdul-Jalbar et al. (2003) have analyzed the one-warehouse multi-retailer problem by comparing centralized and decentralized structures with deterministic demand. According to this research, under specific conditions of the unit replenishment and holding costs at the warehouse, the centralized policy can provide better solutions.

Multi-criteria optimization models have also been developed to address distribution concerns. Thirumalai (2001) develops such a model for serial cases with three stages that takes both deterministic and stochastic demands into account. Rangarajan and Ravindran (2005) present a base period policy for decentralized supply chains. This policy considers that every retailer orders in integer multiples of some base period, which is arbitrarily set by the warehouse. The problem is solved over a finite planning horizon, using Wagner-Whitin’s (1958) model.

Shang (2008) proposes a two-step heuristic method for finding base order quantities under stochastic demand in a serial inventory system with fixed costs. In the first step, stages are clustered according to cost parameters. In the second step, a single-
stage economic order quantity (EOQ) problem is solved for each cluster. Numerical studies show that the heuristic is near optimal.

Levi et al. (2008) develop an approach to determine inventory ordering policies for a multi-period, capacitated inventory system facing stochastic nonstationary and correlated demand that evolves over time. The proposed approach is computationally efficient and guarantees to produce a policy which total expected cost is no more than twice that of the optimal policy. Among other advantages, the algorithm captures the long-term impact of a decision on system performance in the presence of capacity constraints.

Song et al. (2010) analyze the effect of demand and lead time uncertainties in inventory policies. In particular, this research studies the impact of lead time distribution on the optimal inventory policy, analyzing how it varies as the lead time distribution becomes stochastically smaller or less variable.

2.4 Supply Chain Inventory Models with Transportation Costs

Baumol and Vinod (1970) present one of the first models considering multiple supply chain decisions. They propose a theoretic inventory model that integrates transportation cost into inventory decisions. In this model, shipping costs (constant cost/unit), speed (mean lead time) and reliability (variance of lead time) are simultaneously considered. This model has been recognized as one of the significant pioneering efforts to incorporate transportation and inventory costs into model formulations.

Using a more general expression to estimate the variance of the lead time demand, Das (1974) extends the basic theoretic model by independently determining safety stock order quantities. The theoretic model has been further generalized by incorporating rates for less-than-truck load (LTL), full-truck-load (TL), and carload (CL) shipments (Buffa and Reynolds, 1977). Although indifference curves are used to perform a sensitivity analysis, transportation costs are still assumed to be constant per unit shipped. Additional
extensions incorporating backorder costs are presented by Constable and Whybark (1978), where transportation cost is assumed to be linearly-related to shipment volume. The problem is formulated to determine inventory policies that minimize both inventory and transportation costs. It has been found that when compared with the optimal solution obtained from an enumeration process, the solution obtained from a heuristic approach is very similar.

Langley (1980) presents one of the first models incorporating transportation costs by using either estimated or actual freight rates. In this research, different approximation functions are evaluated, including constant, proportional, exponential, inverse, and discrete methods. The problem is solved by enumerating the possible values of $Q$, where transportation costs are incorporated as part of set up costs in the EOQ model. Lee (1986) extends the basic EOQ model to incorporate freight cost as part of the setup cost. The author considers freight rate discounts in order to exploit economies of scale. Three different transportation cost structures are studied in this research, including: (1) all-units discounts, (2) incremental discounts, and (3) stepwise freight costs (proportional to the number of trucks used). Tersine et al. (1989) develop two optimal inventory-transportation decision support algorithms for freight discount. The freight rates are modeled using weight discounts for both the all-weight and incremental freight rate discount structures. Their models calculate the optimal order quantity while minimizing long-term costs. Similarly, Tersine and Barman (1991) incorporate freight discounts and quantity discounts in a deterministic economic order quantity environment. Subsequently, Carter and Ferrin (1996) suggest enumeration techniques to determine the optimal order quantity using actual freight rates.

Expanding the incorporation of transportation costs into supply chain models, Camm et al. (1997) develop an integer programming model for a DC location problem for Proctor & Gamble. The model minimizes the total cost, subject to a maximum number of DCs and customer coverage constraints. In another study, Tyworth and Zeng (1998) perform a sensitivity analysis to estimate the effect of carrier transit time on logistics costs and service for a single stock location following a continuous review under stochastic demand. Lead times are assumed to include transit time (discrete random
variable) and order processing time (fixed variable), and demand is assumed to be a Gamma random variable. Transportation costs are included by fitting the respective freight rate curves where power functions work well for the available data.

Recent models have also taken complex freight costs into account. Swenseth and Godfrey (2002) implement inverse and adjusted inverse freight rate functions in order to determine optimal order quantities. Using a heuristic approach, the model selects between over-declaring a shipment as TL or maintaining a LTL cost structure. Toptal et al. (2003) propose an inventory model that includes fixed transportation costs and a stepwise freight rate proportional to the number of trucks. Two alternative models are developed, one including inbound transportation costs only, and one including both inbound and outbound transportation costs.

Building on the prior research, sophisticated, multi-criteria models integrating transportation and inventory decisions at multiple levels within the supply chain have been developed. Chan et al. (2002) analyze a scenario involving multiple retailers that are served by a warehouse with a single supplier. Linear concave transportation costs are assumed for shipments between the supplier and the warehouse, along with a modified all-units discount structure for shipments between the warehouse and retailers. A zero inventory ordering policy is evaluated for the problem, where a warehouse places orders only if the inventory level is zero. The problem is also formulated as an integer linear programming problem, however only small scale problems can be solved. Consequently, a LP-based heuristic has been developed, which provides good solutions with reasonable computational times for such large-scale problems. A multi-criteria model specifically incorporating actual freight rates through continuous functions has also been proposed for one-warehouse multi-retailer systems (DiFillipo, 2003).

Geunes and Zeng (2003) suggest an integrated analysis of inventory and transportation decisions under a scenario of stochastic demand and shortages. Three different policies are evaluated, including complete expediting, complete backordering and a hybrid policy approach. The problem assumes that the warehouse and customer agree upon a maximum amount of demand in any period, which is defined as maximum current demand shipped (MCDS) every period. Thus, if the customer’s demand exceeds
the inventory available at the warehouse, it expedites the differences between the order quantity and its stock level up to the MCDS. In this scenario, it is guaranteed that the backlogged demand will be delivered in the next period. A heuristic approximation is used to solve the problem.

Recently, Natarajan (2007) has suggested a modified base periodic policy for the one-warehouse, multi-retailer system under decentralized control. In this research, the model is formulated as a multi-criteria problem, which considers transportation costs between echelons. With several efficient solutions generated by the multi-criteria model, the value path method is used to display tradeoffs associated with the efficient solutions to decision makers at each location in the system.

Hu et al. (2008) address the optimal joint control of inventory and transshipment problem under demand and production capacity uncertainty. The problem is analyzed assuming two production locations, where production capacity uncertainty is caused by factors related to downtime, quality problems, and yield, among others. They characterize the optimal production and transshipment policy, and show that uncertainty capacity leads the firm to rotate the inventory that is available for transshipment to the other location.

2.5 Integrated Supplier Selection Models

Most of the supplier selection model formulations described in the literature ignore transportation costs, given that they implicitly assume Freight on Board (FOB) cost structures. In such cases, suppliers are responsible for product shipments and consequently, transportation costs are included in raw material prices. In general, two possible scenarios are assumed: (1) transportation costs are incorporated into the unit price; or (2) transportation costs are incorporated into setup/ordering costs. However, for companies that manage inbound logistics, benefits can be gained by negotiating with freight providers to reduce transportation costs (Carter and Ferrin, 1996). Thus,
integrated supplier selection models that address both inventory and transportation concerns have become strategically important.

Anthony and Buffa (1977) use a LP model to support strategic purchasing decisions. The model minimizes purchasing and storage costs, subject to budget, buyer demand, and supplier storage capacity constraints. Bender et al. (1985) describe a mixed integer optimization model for supplier selection at IBM, where the objective function minimizes the sum of purchasing, transportation and inventory costs over the planning horizon, without exceeding supplier production capacities and various policy constraints. Benton (1991) introduces a nonlinear program and a heuristic approach for supplier selection and lot sizing under conditions with multiple products, resource limitations, and all-units quantity discounts using the EOQ concept. The problem is solved using Lagrangian relaxation, where the objective function minimizes the sum of purchasing, inventory and ordering costs subject to aggregate inventory investment and storage capacity constraints.

Qu et al. (1999) propose a method that allows inventory and transportation decisions to be simultaneously analyzed in a one-warehouse multi-retailer problem. The warehouse is assumed to follow a modified periodic review policy, where fast moving items are ordered every base period, and the remaining items are ordered in multiples of the base period. The proposed approach decomposes the problem into an inventory master problem and a transportation sub-problem, where the transportation problem is analyzed as a traveling sales problem.

The last decade has seen a proliferation of integrated supplier selection models. Ghodsypour and O'Brien (2001) present a mixed integer nonlinear programming (MINLP) model to address supplier selection problems while total logistics costs, including net price, storage, transportation, and ordering costs are minimized. The model assumes that only one order can be assigned to each supplier per order cycle. An uncaptacitated MILP supplier selection model is proposed by Tempelmeier (2002) for a single product with dynamic demand, where suppliers offer all-units and/or incremental quantity discounts which may vary over time. The problem is solved using a heuristic approach. Addressing three-stage problems in the supply chain, Kaminsky and Simchi-
Levi (2003) develop a DP approach to address a problem with first- and third-level manufacturing stages and a second-level transportation stage. Production capacity is considered at both manufacturing stages, and transportation costs are assumed to be in the general concave form. The objective function minimizes production, transportation and inventory holding costs. Basnet and Leung (2005) develop an uncapacitated MILP model that minimizes purchasing, ordering and holding costs subject to demand satisfaction. Other scholars have developed a DP model, where production, inventory, and transportation decisions are integrated based on production capacities and concave cost functions (Van Hoesel et al., 2005). Algorithms run in polynomial time through a planning horizon.

Ding et al. (2005) present a genetic algorithm-based methodology for supplier selection. The proposed method provides possible configurations of the selected suppliers, including transportation modes. Each configuration is then evaluated with respect to the key performance indicators.

Very recently, a goal programming (GP) model has been proposed for multi-period lot sizing problems with supplier selection (Ustun and Dermitas, 2008). GP has also been used to solve problems related to global supplier selection, which has been addressed by a multi-objective decision support system where objective functions are defined depending on the characteristics of the products under analysis (Wadhwa, 2008). Alternative models have also been suggested for tactical, leverage, critical, and strategic products.

Mendoza (2007) proposes effective methodologies for supplier selection and order quantity allocation, where transportation costs are explicitly considered. Addressing both supplier selection and order quantity allocation, Mendoza and Ventura (2008) propose a two-phase method. In the first stage, a list of suppliers is screened and reduced by applying the analytic hierarchy process. In the second stage, a MINLP model is applied to properly allocate order quantities to a selected set of suppliers. The objective function minimizes purchasing, holding and transportation costs, and two continuous functions are used to estimate the actual freight rates. Manikandan (2008) suggests a single objective MILP model for the multi-period supplier selection problem under
dynamic demand. The objective function minimizes the total cost involved in running the supply chain and the model is tested with a set of real data, evaluating the effect of supplier price increases and modes of transportation. Most recently, Mendoza and Ventura (2010) have developed a mathematical model for the N-stage serial system, where supplier selection and inventory control problems are simultaneously considered. A lower bound of the optimal costs is obtained and a 98% effective power-of-two policy is derived.

Che and Wang (2008) propose a genetic algorithm for the supplier selection and quantity allocation problem, where manufactured products may have common and non-common parts. The total utility function for the algorithm includes purchasing, transportation, and assembling costs. The problem selects the most suitable suppliers of parts with the highest quality and minimum time and costs. Tuzkaya and Önüt (2009) analyze an integrated warehousing and transportation network with multiple suppliers, a single warehouse, and multiple manufacturers. They propose a LP model that maximizes profit for both the overall supply chain network and the individual functional units of the supply chain.

Mahnam et al. (2009) develop an inventory model for an assembly supply chain network, where the demand for a single product and the reliability of external suppliers are assumed to be under uncertainty. Sarker and Diponegoro (2009) address an optimal policy for production and procurement in a supply-chain system with multiple non-competing suppliers, a manufacturer, and multiple non-identical buyers. The problem is to determine the production plan, the initial and ending inventories, the cycle beginning and ending times, the number of orders of raw materials in each cycle, and the number of cycles for a finite planning horizon so as to minimize the system cost. In this research, a surrogate network representation is used to find the optimal solution for the problem.

Hajji et al. (2009) address the joint replenishment and production control problem in a two stage supply chain, where supplier and manufacturing stages are unreliable. The problem is defined as an optimal control problem with state constraints and hybrid dynamics in production and replenishment activities. The problem is solved using the Halmilton-Jacobi-Bellman equations of the problem. The robustness of the control policy
generated by the model is tested through a sensibility analysis. Yildiz et al. (2010) propose a MILP model that matches opposite flows of containers from and to the customers and suppliers. The model is implemented in a large automotive supply chain, where the manufacturing company makes all the arrangements for its customers and suppliers based on a centralized optimal solution.

Hajji et al. (2010) consider the supplier selection, replenishment and manufacturing control problem in a dynamic stochastic context. The problem is formulated as an optimal control problem with state constraints and hybrid dynamics. Their experimental results show that it is always profitable to consider multiple suppliers when making replenishment and production decisions. Moreover, it is shown that the availability rates of the suppliers (capability to fulfill an order) and their replenishment lead times are important parameters to consider when choosing the best supplier.

Only a few cases of the supplier selection problem have been investigated where production, inventory, and transportation costs are considered simultaneously. A summary of the most relevant literature related to this research is presented in Table 2.1. Notice that papers are sorted according their similitude with this thesis, with less relevant papers located at the top of the table, and the most closely related papers at the bottom. Even though supplier capabilities and customer demand may change over time, very little research has been dedicated to dynamic models. Indeed, most of the multi-period models assume that model parameters are constant and unchanging over time. However, due to multiple factors associated with market environments, including production and supplier capabilities, model parameters may continuously change between periods. Consequently, further research is still required to develop integrated models for supply chain optimization.
### Table 2.1 Summary of Previous Research on Integrated Models

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Chapter 3

A Dynamic Inventory Model including Supplier Selection and Transportation Costs in a Serial Supply Chain Structure

3.1 Introduction

Nowadays manufacturers and distributors face a myriad of dynamic challenges that require not only exceptional advanced planning, but also a thorough network of communication and coordination mechanisms that allow companies to address changes at a moment's notice. From radical volume changes in customer demand, to variations in prices of raw material and finished products due to currency fluctuations in the global marketplace, to increases in transportation costs due to speculation in the price of crude oil, any number of factors can have a serious effect on corporate revenue projections. Furthermore, when inventory is stuck in various stages of the supply chain, the company may be forced to operate at critical cash flow levels. As such, the ability to intelligently address inbound and outbound issues through effective supply chain inventory and distribution strategies not only keeps the wheels of business turning, but also creates competitive advantages, allowing companies to address supplier concerns and consumer needs in a way that slower, less agile manufacturers are unable to do.

Of the various activities involved in supply chain logistics, purchasing is one of the most strategic because it provides opportunities to reduce costs across the entire supply chain. An essential task within the purchasing process is supplier selection, given that the cost of raw materials and component parts represents the largest percentage of the total product cost in most industries. For instance, in high technology firms, purchased materials and services account for up to 80% of the total product cost (Weber et al., 1991).

According to Chopra and Meindl (2007), inventory is recognized as one of the major drivers of a supply chain. High inventory levels increase the responsiveness of the
supply chain but decrease its cost efficiency because of inventory holding costs. Hence, a relevant problem in supply chain logistics is to determine appropriate inventory levels at the various supply chain stages. Given the prevalence of both supplier selection and inventory management decisions in a supply chain, this chapter addresses both problems simultaneously by studying the production and distribution of a single product or an aggregated unit representing a family of products with similar process plans in a serial supply chain structure. An example of this situation is a manufacturer that purchases raw materials from various preferred suppliers. These raw materials are stored at the manufacturing facility or processed into final products. The products are then either stored at the manufacturer, or transported to a warehouse. At the warehouse stage, products are either stored or transported to a distribution center (DC). In general, a DC may serve products to an entire market area or a set of retailers. Given the possible impact of transportation costs on both supplier selection and inventory replenishment at each stage of the supply chain in modern enterprises, the proposed mixed integer nonlinear programming (MINLP) model considers purchasing, production, inventory, and transportation costs over a planning horizon with time varying demand while considering supplier quality constraints, capacity constraints for both suppliers and the manufacturer, and inventory capacity constraints at all stages.

The scenario described above can be viewed as a generalization of one of the most studied problems in production and inventory planning for a single facility, called the dynamic inventory lot-size problem. This basic problem is to determine the production quantities for each period so that all demands are satisfied on time at minimal production and inventory cost. Wagner and Whitin (1958) use dynamic programming to find an exact solution for the uncapacitated version of the problem in polynomial time. Florian and Klein (1971) show how to handle the case of time-independent production capacity. Additionally, Florian et al. (1980) show that, when the capacity limitation is time-dependent, even for the single item case, the problem is NP-hard. Several authors have considered multi-stage supply chain models under deterministic constant demand. Muckstadt and Roundy (1993) survey and summarize these results. In particular, the sequential structure has been the most common mechanism used to implement
optimization on supply chain logistics. In this case, the optimal results obtained for a particular process are used as input to optimize interrelated processes in the supply chain that are considered independent from a modeling point of view. An integrated optimization approach has shown to produce superior solutions for many of the applications, including problems related to supplier selection and lot sizing production in a supply chain. In fact, substantial research exists on these two topics when they are analyzed independently. However, research covering both topics simultaneously is limited and recent. Basnet and Leung (2005) present one of the first studies where inventory lot sizing is analyzed simultaneously with supplier selection under a dynamic demand environment. They propose an enumerative search algorithm and also a heuristic approach to solve the mixed integer linear programming (MILP) formulation developed for the single stage uncapacitated multiproduct problem. Ustun and Demirtas (2008) propose a mixed integer goal programming model for the single stage lot sizing and supplier selection problem. Additional work has also been performed for the static demand case using some extensions of the economic order quantity (EOQ) model. Mendoza and Ventura (2008) study a serial inventory system with supplier selection and order quantity allocation, where some of the properties of the EOQ model are used to derive a simplified MINLP model for the problem.

This chapter extends previous research developed in inventory lot sizing and supplier selection for a serial supply chain. In particular, the proposed MINLP model solves a dynamic multi-stage single-product capacitated problem. Even though Hoesel et al. (2005) suggest a dynamic programming approach for the multi-stage lot sizing problem in a serial supply chain with production capacities, they neither integrate supplier selection nor consider quality and capacity constraints in their model. Thus, this work intends to cover a novel area of research that has not been studied extensively and, consequently, could allow further research.

The remainder of this chapter is organized as follows. In Section 3.2, the development of the proposed dynamic multi-stage inventory model with supplier selection is presented. Section 3.3 gives a general characterization of the transportation freight rates for the less-than-truck load (LTL) mode. Section 3.4 provides a description
of the two freight rate estimate functions considered in this research. In Section 3.5, a MILP model is formulated for the dynamic inventory lot sizing problem with supplier selection in a serial supply chain structure, where transportation costs are represented by a piecewise linear function. Section 3.6 describes the illustrative example used in this chapter, while Section 3.7 presents the results obtained from the sensitivity analysis performed over some relevant model parameters. Section 3.8 presents a comparison between the integrated and the two-step sequential approaches. Finally, Section 3.9 provides the conclusions of this research and new opportunities that have been identified as extensions of this work.

3.2 The Model

A dynamic lot-sizing model with supplier selection in a serial supply chain structure is presented in this section. Let \( J = \{1,2,\ldots,n_J\} \) be the set of suppliers, \( K = \{1,2,\ldots,n_K\} \) the set of supply chain stages, \( S = \{1,2,\ldots,n_S\} \) the set of customers, and \( T = \{1,2,\ldots,n_T\} \) the set of planning periods. Let \( l_{0j} \) be the number of delivery lead time periods from supplier \( j \in J \) to stage 1, and \( l_k \) the number of delivery lead time periods from stage \( k \in K\setminus\{n_K\} \) to stage \( k + 1 \). Finally, let \( d_s^t \) be the external demand in units of customer \( s \in S \) at time \( t \in T \). Let \( J_t = \{j \in J : t - l_{0j} \geq 0\} \) be the subset of suppliers that can provide raw materials at period \( t \). Note that at time \( t \) only a supplier with a lead time less than \( t \) can supply raw material to stage 1.

The dynamic serial supply chain with multiple suppliers and customers can be modeled as a general transshipment network \( G_D = (N_D, A_D) \), where the set of nodes \( N_D \) and the set of arcs \( A_D \) are defined as follows (see Figure 3.1):

\[
N_D = J \cup K \cup S,
\]

and

\[
A_D = \{(j,1) : j \in J\} \cup \{(k,k+1) : k \in K\setminus\{n_K\}\} \cup \{(n_K,s) : s \in S\}.
\]
As shown in Figure 3.1, stages 1 and 2 represent the raw material and finished product warehouses at the manufacturing facility. \( K_D = \{2,3,4,...,n_K - 1\} \) is the set of intermediate warehousing/distribution stages that hold finished products, including the warehouse at stage 2, and \( n_K \) is the last distribution stage that ships products to customers. Notice that the external demand can be aggregated by consolidating the set of customers into a single demand node \( n_{K+1} \), where \( d^t = \sum_{s \in S} d_s^t \) represents the external demand in units for all customers at time \( t \in T \). In this case, the distribution process from stage \( n_K \) to customers for each period \( t \in T \) can be treated as a separate problem that can be independently solved.

**Figure 3.1.** A dynamic serial supply chain network with multiple suppliers and customers (network \( G_D \))

Considering that the dynamic supply chain represented by network \( G_D \) does not show the effect of changes in demand between consecutive time periods, a (time-expanded) static supply chain network is defined and represented by the general transshipment network \( G_S = (N_S,A_S) \). In this network, the set of nodes \( N_S \) and the set of arcs \( A_S \) are defined as follows:
\[ N_5 = \{(0, j, t) : j \in J, t \in T\} \cup \{(k, t) : k \in K, t \in T\}, \]

and

\[ A_5 = \left\{ \left( (0, j, t), (1, t + l_{0j}) \right) : j \in J, t \in \{1, 2, \ldots, n_T - l_{0j} \} \right\} \cup \left\{ \left( (k, t), (k, t + 1) : k \in K, t \in \{1, 2, \ldots, n_T - l_k \} \right) \right\}. \]

Note that neither set \( S \) nor its aggregated node \( n_K + 1 \) are used in the definition of \( N_5 \). Similarly, neither initial nor ending inventories are included in the definition of \( A_5 \). The raw material flow from supplier \( j \) in period \( t \) is represented as a directed arc connecting nodes \( (0, j, t) \) and \( (1, t + l_{0j}) \). Likewise, the inventory flow between consecutive time periods \( t \) and \( t + 1 \) at stage \( k \) is represented as a directed arc connecting nodes \( (k, t) \) and \( (k, t + 1) \), while the product flow between consecutive stages \( k \) and \( k + 1 \) is represented by a directed arc that connects nodes \( (k, t) \) and \( (k + 1, t + l_k) \). In this scenario, all initial inventories, raw material and finished product pending orders at every stage \( k \) are assumed to be available at the beginning of the first period, and respectively represented by \( i^0_k \), \( q^0_{0j} \), and \( y^0_{k-1} \). In the same way, ending inventories at each stage \( k \) are assumed to be available at the end of the last period and are represented by \( i^{n_T}_k \).

Considering that the MINLP will associate a continuous variable to each arc in \( G_S = (N_S, A_S) \), an efficient approach to reducing the problem size consists of eliminating all arcs and nodes that are not feasible to the problem from the analysis. In this context, an arc or node is considered infeasible if it cannot be used or accessed by any feasible raw material or product flow due to the positive lead times and finite planning horizon. Notice that since only infeasible arcs and nodes are eliminated, the original problem can be formulated in a reduced transshipment network \( G'_S = (N'_S, A'_S) \), where \( N'_S \subseteq N_S \) and \( A'_S \subseteq A_S \). After the reduction, the original and simplified problems will have the same optimal solution, but the simplified problem will have fewer variables and be faster to solve than the original problem.
In the following theorem, a set of feasible time periods $T_k$ is defined for each stage $k \in K$, so that a node $(k, t)$ will be considered feasible if and only if $k \in K$ and $t \in T_k$.

**Theorem 3.1:** For a given stage $k \in K$, let $m_k$ be either zero or the closest preceding stage with positive initial inventory or pending order, i.e., $m_k = \max\{0, k' : y_{k'}^0 > 0, k' \in \{1, \ldots, k\}\}$. Let $m_k^*$ be either $n_k$ or the closest succeeding stage from stage $k$ with positive ending inventory, i.e., $m_k^* = \min\{n_k, k' : i_{k'}^0 > 0, k' \in \{k, \ldots, n_k - 1\}\}$. In addition, let $l_0 = \min\{l_{0j} : j \in J\}$. Finally, let $y_0^0 = \sum_{j \in J} y_{0j}^0$. Then, node $(k, t)$ is a feasible node in the (time-expanded) static network if and only if $t \in T_k$, where

$$T_k = \left\{ t : 1 + \sum_{k' = m_k}^{k-1} l_{k'} \leq t \leq n_T - \sum_{k' = k}^{m_k^*-1} l_{k'}, t \in T \right\}.$$

**Proof:** Considering that the total lead time between stages $m_k$ and $k$ is given by $\sum_{k'=m_k}^{k-1} l_{k'}$, the earliest time $t$ that a flow generated at stage $m_k$ can reach stage $k$ is $1 + \sum_{k'=m_k}^{k-1} l_{k'}$. In addition, given that the lead time between stages $k$ and $m_k^*$ is $\sum_{k'=k}^{m_k^*-1} l_{k'}$, the latest time $t \in T_k$ in which a flow sent from stage $k$ can arrive at stage $m_k^*$ by time $n_T$ is $n_T - \sum_{k'=k}^{m_k^*-1} l_{k'}$. On the other hand, given that node $(k, 1 + \sum_{k'=m_k}^{k-1} l_{k'})$ can always send a product flow to any node $(k, t)$, where $1 + \sum_{k'=m_k}^{k-1} l_{k'} < t < n_T - \sum_{k'=k}^{m_k^*-1} l_{k'}$, then any intermediate node between nodes $(k, 1 + \sum_{k'=m_k}^{k-1} l_{k'})$ and $(k, n_T - \sum_{k'=k}^{m_k^*-1} l_{k'})$ will also be feasible. Therefore, at any stage $k \in K$, a node $(k, t)$ is a feasible node if $t \in T_k$.

On the other hand, since the lead time from stage $m_k$ to stage $k$ is $\sum_{k'=m_k}^{k-1} l_{k'}$, it is straightforward to show that a node $(k, t)$ with $1 \leq t < 1 + \sum_{k'=m_k}^{k-1} l_{k'}$ will not be
feasible. Similarly, since the lead time between stages \( k \) and \( m_k^* \) is \( \sum_{k' = k}^{m_k^* - 1} l_{k'} \), a flow generated in stage \( k \) at time \( t \), \( n_T - \sum_{k' = k}^{m_k^* - 1} l_{k'} < t < n_T \), will always be received at stage \( n_K \) after period \( n_T \).

Based on Theorem 3.1, the following corollary can be stated:

**Corollary 3.1.1**: A raw material order submitted by the manufacturing stage (stage 1) to a given supplier \( j \in J \) at time \( t \) will be feasible if and only if \( t \in T_{0j} \), where

\[
T_{0j} = \{ t : 1 \leq t \leq \max\{t' \in T_1 \} - l_{0j} \}.
\]

**Proof**: Even though a raw material order from supplier \( j \in J \) to the manufacturer stage (stage 1) can always be shipped in period 1, considering that the delivery lead time defined for supplier \( j \) is \( l_{0j} \), the latest time that an order from supplier \( j \) can be shipped is \( \max\{t' \in T_1 \} - l_{0j} \); otherwise, it would be received during an infeasible time period.

Considering the lead times at the different stages of the supply chain network, necessary feasibility conditions regarding the demand at stage \( n_K \) can be established for the dynamic supplier selection and inventory planning problem. These conditions are analyzed in the following lemma and theorem.

**Lemma 3.1**: For a given stage \( k \in K \), if node \( (k, 1) \) is connected to node \( (n_K, t) \) at the demand stage \( n_K \) in period \( t \in T_k \), then every node \( (\hat{k}, 1) \), such that \( k \leq \hat{k} \leq n_K \), is also connected to node \( (n_K, t) \).

**Proof**: Since the total lead time between stages \( \hat{k} \) and \( n_K \) is given by \( \sum_{k' = \hat{k}}^{n_K - 1} l_{k'} \), node \( (\hat{k}, t - \sum_{k' = \hat{k}}^{n_K - 1} l_{k'}) \) is connected to node \( (n_K, t) \). However, since a product flow can be sent from node \( (\hat{k}, 1) \) to node \( (\hat{k}, t - \sum_{k' = \hat{k}}^{n_K - 1} l_{k'}) \), node \( (\hat{k}, 1) \) is always connected to
node \((\bar{k}, t - \sum_{k'=k}^{n_K-1} l_{k'})\). Consequently, through this node, node \((\bar{k}, 1)\) is also connected to the demand node \((n_K, t)\).

**Theorem 3.2:** The supply chain inventory problem with supplier selection can only be feasible if for every demand node \((n_K, t)\), \(1 + \sum_{k'=0}^{n_K-1} l_{k'} \leq t < 1 + \sum_{k'=0}^{n_K-1} l_{k'}\), the following condition holds:

\[
\sum_{k'=k_t}^{n_K} (y_{k'-1}^0 + i_{k'}^0) - \sum_{t'=1}^{t-1} d_t' \geq d_t,
\]

where, \(k_t = \min \{n_k, \ k \in K: \sum_{k'=k}^{n_K-1} l_{k'} < t\}\).

**Proof:** Based on Theorem 3.1, the first feasible node at stage \(n_k\) is given by \((n_k, 1 + \sum_{k'=0}^{k-1} l_{k'})\). In addition, considering that the total lead time from stage 0 to stage \(n_k\) is \(\sum_{k'=0}^{n_K-1} l_{k'}\), the first node at stage \(n_k\) that can be directly supplied with finished products produced by new raw material orders is given by \((n_k, 1 + \sum_{k'=0}^{n_K-1} l_{k'})\). Consequently, any demand node \((n_K, t)\), with \(1 + \sum_{k'=0}^{n_K-1} l_{k'} \leq t < 1 + \sum_{k'=0}^{n_K-1} l_{k'}\), can only be supplied from initial inventory or pending orders. Given that node \((k_t, 1)\) is connected to node \((n_K, t)\), by Lemma 3.1, node \((k_t, 1)\), for \(k_t \leq \bar{k} \leq n_K\), is also connected to node \((n_K, t)\). Thus, the accumulated initial inventory and pending orders accessible to node \((n_K, t)\) can be calculated as \(\sum_{k'=k_t}^{n_K} (y_{k'-1}^0 + i_{k'}^0)\). Moreover, the cumulative demand prior to time \(t\) can be calculated as \(\sum_{t'=1}^{t-1} d_t'\). Therefore, the flow balance at demand stage \(n_K\) in period \(t\) is given by \(\sum_{k'=k_t}^{n_K} (y_{k'-1}^0 + i_{k'}^0) - \sum_{t'=1}^{t-1} d_t'\). Consequently, to achieve feasibility at node \((n_K, t)\), it is necessary to satisfy the condition \(\sum_{k'=k_t}^{n_K} (y_{k'-1}^0 + i_{k'}^0) - \sum_{t'=1}^{t-1} d_t' \geq d_t\). Otherwise, the problem is infeasible.
An illustration of a static supply chain network with three suppliers, five stages, and a planning horizon of six periods is presented on Figure 3.2. Notice that lead times are positive for suppliers 2 and 3, and stages 3 and 4, where orders are delivered one time unit (period) after they are submitted. In this problem, assuming zero initial and ending inventories, and no pending orders, the feasible sets of time periods $T_k$ for each stage $k \in K$ can be determined by applying Theorem 3.1. In this illustration, $T_1 = T_2 = T_3 = \{1, 2, 3, 4\}, T_4 = \{2, 3, 4, 5\}$, and $T_5 = \{3, 4, 5, 6\}$. Similarly, applying Corollary 1.1, sets $T_{0j}, j \in J$, are defined as $T_{01} = \{1, 2, 3, 4\}$ and $T_{02} = T_{03} = \{1, 2, 3\}$. Consequently the reduced network $G_s' = (N_s', A_s')$ can be defined as follows:

$$N_s' = \{(0, j, t): j \in J, t \in T_{0j}\} \cup \{(k, t): k \in K, t \in T_k\},$$

and,

$$A_s' = \left\{((0, j, t), (1, t + l_{0j})) : j \in J, t \in T_{0j} \right\} \cup \left\{((k, t), (k + 1, t + l_k)) : k \in K \setminus nK, t \in T_k \right\}.$$

The lists of remaining parameters, decision variables, and cost function components for the dynamic production planning problem are provided below.

**Remaining Parameters**

- $h_k^t$: unit holding cost at stage $k$ from period $t$ to period $t+1, k \in K, t \in T_k$.
- $b_{0j}^t$: capacity (in units) of supplier $j$ at time $t, j \in J, t \in T_{0j}$.
- $b_1^t$: production capacity (in units) at stage 1 in period $t, t \in T_1$.
- $b_k^t$: distribution capacity at stage $k$ in period $t$ (in units), $k \in K, t \in T_k$.
- $r_k$: inventory capacity (in units) at stage $k, k \in K$.
- $p_{0j}^t$: unit price of raw material for supplier $j$ in period $t, j \in J, t \in T_{0j}$.
- $p_1^t$: unit production cost in period $t, t \in T_1$.
- $f_{0j}^t$: setup cost for an order submitted to supplier $j$ in period $t, j \in J, t \in T_{0j}$.
- $f_1^t$: setup cost for production at stage 1 in period $t, t \in T_1$.
$f_k^t$: setup cost at stage $k$ in period $t$, $k \in K_D, t \in T_k$.

$a_j$: perfect rate of supplier $j$ (probability that a unit is acceptable), $j \in J$.

$a$: minimum acceptable perfect rate.

**Decision Variables**

$q_{0j}^t$: replenishment order quantity (in raw material units) shipped from supplier $j$ to stage 1 in period $t$, $j \in J, t \in T_{0j}$.

$x_1^t$: production lot size (in units of finished product) at the manufacturing stage (stage 1) at the beginning of period $t$, $t \in T_1$.

$y_k^t$: replenishment order quantity (in units of finished product) shipped from stage $k$ to stage $k+1$ in period $t$, $k \in K_D, t \in T_k$.

$i_k^t$: inventory level (in units) held at stage $k$ from period $t$ to period $t + 1$, $k \in K, t \in T_k$.

$w_{0j}^t$: 1 if supplier $j$ receives a replenishment order at time $t$; 0 otherwise; $j \in J, t \in T_{0j}$.

$w_1^t$: 1 if a production order is submitted at time $t$; 0 otherwise; $t \in T_1$.

$w_k^t$: 1 if a replenishment order is shipped from stage $k$ to stage $k + 1$ at time $t$; 0 otherwise; $k \in K_D, t \in T_k$.

**Cost Function Components**

$s_{0j}^t(q_{0j}^t)$: purchasing cost of $q_{0j}^t$ units purchased from supplier $j$ in period $t$, $j \in J, t \in T_{0j}$.

$c_1^t(x_1^t)$: production cost of $x_1^t$ units at stage 1 in period $t \in T_1$. 
Figure 3.2. A (time-expanded) static supply chain network with three suppliers, five stages, and six periods (network $G_S$). The subnetwork circled by the dashed line represents the reduced network $G'_D$ under the assumption of zero initial and ending inventories and no pending orders.
$h_k^t(i^t_k)$: inventory holding cost of $i^t_k$ units stored at stage $k$ from period $t$ to period $t+1$, $k \in K, t \in T_k$.

$u_{ij}^t(q_{0j}^t)$: holding cost for in-transit inventory of $q_{0j}^t$ units shipped from supplier $j$ (stage 0) to stage 1 for periods $t$ to $+l_{0j} - 1$, $j \in J, t \in T_{0j}$. It is calculated as

$$\sum_{t'=t}^{t'+l_{0j}-1} h_{1'}^t(q_{0j}^t).$$

$u_{k+1}^t(y_k^t)$: holding cost for in-transit inventory of $y_k^t$ units shipped from stage $k$ to stage $k+1$ for periods $t$ to $t + l_k - 1$, $k \in K_D, t \in T_k$. It is calculated as

$$\sum_{t'=t}^{t'+l_k-1} h_{k+1}^t(y_k^t).$$

$g_{0j}^t(q_{0j}^t)$: transportation cost of $q_{0j}^t$ units of raw material shipped from supplier $j$ to stage 1 in period $t$, $j \in J, t \in T_{0j}$.

$g_k^t(y_k^t)$: transportation cost of $y_k^t$ units of product shipped from stage $k$ to stage $k+1$ in period $t$, $k \in K_D, t \in T_k$.

**Mixed Integer Nonlinear Programming (MINLP) Model**

Given that purchasing, production, inventory, and transportation cost functions are defined as general expressions, the problem can be formulated as an MINLP, where the objective is to minimize the total cost over $n_T$ planning periods:

(P 3.1)

Minimize $Z = \sum_{j \in J} \sum_{t \in T_{0j}} s_{0j}^t(q_{0j}^t) + \sum_{t \in T_1} c_1^t(x_1^t) + \sum_{k \in K} \sum_{t \in T_k} h_k^t(i_k^t)$

$$+ \sum_{j \in J} \sum_{t \in T_{0j}} u_{ij}^t(q_{0j}^t) + \sum_{k \in K_D} \sum_{t \in T_k} u_{k+1}^t(y_k^t) + \sum_{j \in J} \sum_{t \in T_{0j}} g_{0j}^t(q_{0j}^t)$$

$$+ \sum_{k \in K_D} \sum_{t \in T_k} g_k^t(y_k^t),$$

subject to
In this formulation the total cost is minimized while satisfying all the demand on time. Equation (3.1) represents the total variable cost function, which includes raw material purchasing, production, inventory, and transportation costs for \( n_T \) periods. Notice that no transportation cost is considered between stages 1 and 2, since all these flows occur internally in the manufacturing facility. Set of equations (3.2) guarantees flow balance between raw material and finished product at the manufacturing level. Note that when \( 1 \in T_1 \) then \( \sum_{j \in J} q_{0j}^0 + i_1^0 > 0 \), otherwise \( \sum_{j \in J} q_{0j}^0 + i_1^0 = 0 \). For simplicity of exposition and without loss of generality, a bill of materials (BOM) ratio equal to one is assumed at the manufacturing stage. Consequently one unit of raw material is required for each unit of finished product. Sets of equations (3.3)-(3.5) assure that the balance of finished products between consecutive stages of the supply chain is satisfied, ensuring that customer demand is completely satisfied in each period of the planning horizon. Observe in Equation (3.3), when \( 1 \in T_2 \) then \( x_1^0 + i_2^0 > 0 \), otherwise \( x_1^0 + i_2^0 = 0 \).
Similarly, in Equations (3.4) and (3.5), when \( 1 \in T_k \) then \( y_{k-1}^0 + i_k^0 > 0 \), otherwise \( y_{k-1}^0 + i_k^0 = 0 \). Set of constraints (3.6) guarantees that the minimum acceptable perfect rate for raw materials will be satisfied for each period. Sets of bounds (3.7)-(3.9) represent the capacity constraints for suppliers, manufacturing, and transportation, respectively. Set of constraints (3.10) defines the inventory capacity at each stage in the supply chain. Finally, Sets of constraints (3.11)-(3.14) describe the nature of the variables considered in the model.

Notice that purchasing and production costs usually have a setup and a linear component, which can be respectively expressed as follows:

\[
s_{0j}^t(q_{0j}^t) = f_{0j}^t w_{0j}^t + p_{0j}^t q_{0j}^t, \tag{3.15}
\]
and

\[
c_{1}^t(x_{1}^t) = f_{1}^t w_{1}^t + p_{1}^t x_{1}^t. \tag{3.16}
\]

On the other hand, inventory holding cost functions are frequently assumed to be linear, and can be represented in this manner:

\[
h_{k}^t(i_{k}^t) = h_{k}^t i_{k}^t. \tag{3.17}
\]

Consequently, in-transit holding cost functions can be expressed as follows:

\[
u_{1j}^t(q_{0j}^t) = \sum_{t' = t}^{t+l_{0j}^{-1}} h_{1}^{t'} q_{0j}^{t'}, \tag{3.18}
\]
and

\[
u_{k+1}^t(y_{k}^t) = \sum_{t' = t}^{t+l_{k}^{-1}} h_{k+1}^{t'} y_{k}^{t'}, \tag{3.19}
\]

where \( u_{1j}^t = \sum_{t' = t}^{t+l_{0j}^{-1}} h_{1}^{t'} \) and \( u_{k+1}^t = \sum_{t' = t}^{t+l_{k}^{-1}} h_{k+1}^{t'} \) represent unit holding costs for in-transit inventory from supplier \( j \) to stage 1 and from stage \( k \) to stage \( k+1 \), respectively.
3.3 Analysis of Transportation Freight Rates

Most manufacturing companies rely on third-party providers for the transportation of products through their supply chain (Chan et al., 2002). Assuming trucks as the common means of transportation, freight can be transported using full-truck-load (TL) or less-than-truckload (LTL) options. Even though TL is frequently the less expensive transportation option, when other supply chain costs are simultaneously considered in the analysis, the LTL option can provide more flexibility in the definition of the optimal order size to allow reduction of the overall supply chain cost. This situation is especially true when the shipment sizes are relatively small. According to Swenseth and Godfrey (2002), TL rates are usually expressed on a per-mile basis, while LTL rates are commonly expressed per hundredweight (CWT), defined for a given origin and destination. Justifiably, LTL transportation cost functions are similar to ordering cost functions for inventory replenishment with quantity discounts. The two most commonly used order quantity discount structures are the incremental and all-units discounts. In both cases quantity discounts are obtained as a result of economies of scale. Although both quantity discount structures result in ordering cost functions that are piecewise linear, only incremental quantity discount functions are concave.

Today’s market is characterized by small shipment sizes, which can be modeled using the LTL transportation option. In fact, as a result of increasing product customization, the number of stock-keeping units has dramatically increased during the last two decades, pushing for more frequent shipments. Customer service considerations have also played an important role, promoting decentralized distribution networks where fewer customers are served per DC (Spiegel, 2002). Considering that the all-units discount transportation cost strategy is widely used to stimulate demand for larger and more profitable shipments in the LTL transportation mode, this strategy has been selected to incorporate the transportation costs in Model (P3.1) and can be described as follows:
where \( q \) is the shipment size, \( \beta^0 = 0 < \beta^1 < \beta^2 < \cdots < \beta^n < \beta^{n+1} \) are the breakpoints, \( \alpha_1 \) is the fixed shipment cost for \( 0 < q < \beta^1 \), and \( \alpha_i \) is the unit shipping cost for the order quantity \( q \), when \( \beta^{i-1} < q < \beta^i \), for \( i = 1, \ldots, n \). Notice that \( \alpha_1 > \alpha_2 > \cdots > \alpha \alpha_n > 0 \) and \( \alpha_2 \beta^1 = \alpha_1 \). Thus, the total transportation cost for \( q \) units is calculated as:

\[
g(q) = \min\{\hat{g}(q), \hat{g}(\beta^i)\}, \quad \text{where } \beta^{i-1} \leq q < \beta^i. \tag{3.21}
\]

This function considers the situation where it may be advantageous to declare \( \beta^i \) units, even though only \( q \) units are effectively transported. This concept, commonly known as over-declaring, is frequently used in practice to take advantage of cost reductions obtained by shipping larger sizes. In this case, cost functions are characterized by indifference points, where any number of shipping units beyond these points pays an LTL freight rate corresponding to the next range by over-declaring the shipment size. Thus, shipments are artificially increased to the next higher shipment breakpoint, resulting in a lower total cost (Swenseth and Godfrey, 1996).

As an illustration, Table 3.1 presents the LTL freight rates defined for a specific origin and destination. These rates could represent the transportation cost between a specific supplier and the manufacturing facility. Nominal freight rates represent the rates as stated by the LTL carrier, while actual freight rates represent the transportation costs obtained after the concept of over-declaring has been applied. Notice that even though LTL rates are usually expressed in CWT (Swenseth and Godfrey, 2002), for simplification of this illustration, freight rates are expressed per unit of raw material or finished product. Nominal and actual freight rates are characterized by a minimum and maximum number of units. Subsequent freight ranges are defined by a progressive
reduction of the variable rates. To illustrate the actual freight rates calculation, let us consider the first three nominal rates given by ranges [1, 31], [32, 62], and [63, 124] in units. Using the graphical representation provided in Figure 3.3, it is possible to notice that the total transportation cost given for the range [49, 62] is higher than the total cost obtained when 63 units are declared. Therefore, any shipping size between 49 and 62 units can be over-declared to 63 units to obtain a reduced total transportation cost.

Table 3.1. Example of Nominal and Actual Freight Rates for a Given Origin and Destination

<table>
<thead>
<tr>
<th>Nominal Freight Rate</th>
<th>Actual Freight Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Units</td>
<td>Freight Rate</td>
</tr>
<tr>
<td>1 - 31</td>
<td>519</td>
</tr>
<tr>
<td>32 - 62</td>
<td>$/unit 16.2</td>
</tr>
<tr>
<td>63 - 124</td>
<td>$/unit 12.5</td>
</tr>
<tr>
<td>125 - 312</td>
<td>$/unit 11.3</td>
</tr>
<tr>
<td>313 - 624</td>
<td>$/unit 9.2</td>
</tr>
<tr>
<td>625 - 1,249</td>
<td>$/unit 7.1</td>
</tr>
<tr>
<td>1,250 - 1,874</td>
<td>$/unit 4.8</td>
</tr>
<tr>
<td>1,875 - 2,500</td>
<td>$8,278</td>
</tr>
<tr>
<td>1,711 - 2,500</td>
<td>$</td>
</tr>
</tbody>
</table>

Once all of the indifference points are found, actual freight rates are calculated as they are presented in Table 3.1. These rates alternate between ranges of a constant charge per unit followed by a fixed charge. The fixed charge is the result of over-declaring a LTL shipment to the next LTL transportation range. A graphical representation of actual freight rates is presented in Figure 3.4. Notice that although this function is continuous, it is non-differentiable due to the break and indifference points. Natarajan (2007) states that when actual freight rates are incorporated into analytical models, two problems may arise. First, the determination of the exact rates between every origin and destination can become time consuming and expensive. Second, given that freight rates are function of the volume shipped, transportation cost functions are not differentiable. For similar reasons, several researchers have proposed the use of continuous functions to estimate...
actual freight rates that are also differentiable. In this direction, Section 3.4 presents two continuous functions that can be used to incorporate transportation costs in Model (P3.1).

**Figure 3.3.** Nominal Freight Transportation Cost Function

**Figure 3.4.** Actual Freight Transportation Cost Function
3.4 Continuous Estimates of the Transportation Cost Function

In this section, two alternative approximation functions are described for the LTL transportation mode: a power function and a quadratic function.

**Power Approximation**

Tyworth and Ruiz-Torres (2000) recommend the use of a power approximation function to model the LTL transportation mode. In this case, the unit shipment cost is approximated as a function of the order quantity $q$:

$$F_p(q) = a(q)^b,$$  \hspace{1cm} (3.22)

where $a$ and $b$ correspond to the coefficients of the nonlinear regression function. However, as indicated by Ventura and Mendoza (2008), given that Equation (3.22) can also be represented by Equation (3.23), the coefficients $a$ and $b$ can be easily found by fitting a simple linear regression model.

$$\ln(F_p) = \ln(a) + b \ln(q).$$  \hspace{1cm} (3.23)

Notice that in the all-units discount strategy, the unit transportation cost decreases as the number of units increases. Thus, $F_p(q)$ is a strictly decreasing function for $a > 0$ and $0 < b < 1$. Furthermore, since the transportation cost can be calculated by multiplying the number of transported units by the corresponding unit cost, the resulting transportation cost function will be concave.
**Quadratic Approximation**

As an alternative method, our experimental results have shown that a simple quadratic approximation function can also be used to estimate the freight rates for the LTL transportation mode as a function of the order quantity $q$:

$$F_q(q) = a + b\, q + c\, q^2. \quad (3.24)$$

Using the actual freight rates presented in Table 3.1, the total transportation cost functions generated from the unit cost power and quadratic approximation functions are plotted in Figure 3.5. Both functions are simultaneously plotted to allow a graphical comparison to the actual freight cost. A simple analysis of Figure 3.5 clearly shows that the quadratic function provides a better approximation of the actual freight costs than the traditional power function.

![Graphical Comparison of Power and Quadratic Approximation Functions with the Actual Freight Cost Function](image)

**Figure 3.5.** Graphical Comparison of Power and Quadratic Approximation Functions with the Actual Freight Cost Function
3.5 Alternative Mixed Integer Linear Programming (MILP) Model

Considering the nonlinear characteristics of the power and quadratic approximation functions, modeling transportation costs through any of these two alternatives makes Model (P3.1) nonlinear. In addition, since economies of scale are assumed for transportation costs, both approximation functions are concave. Thus, when the remaining components of the objective function are linear, as defined by Equations (3.15) – (3.19), Model (P3.1) consists of a concave objective function and linear constraints, which is a well known NP-hard problem (Benson, 1995; Pardalos and Schnitger, 1988). In this type of problem, even in semblably simple cases, there can be an exponential number of local minima, making the problem computationally difficult to solve to optimality (Horst et al., 2000). Moreover, given that transportation costs are estimates of actual costs, it cannot be guaranteed that the optimal solution of Model (P3.1) will be the same as the optimal solution for the original problem. Thus, in this section, an MILP model is presented, where purchasing, production, inventory holding, and in-transit inventory costs are represented by linear functions, and transportation costs by piecewise linear functions, as shown in Figure 3.6.

Let $R_{0j} = \{1, 2, ..., n_{R_{0j}}\}$ be the set of freight ranges of the transportation cost function from supplier $j$ to stage 1. Similarly, let $R_k = \{1, 2, ..., n_{R_k}\}$ be the set of freight ranges for the transportation cost between stages $k$ and $k+1$. Notice that in both cases, freight rates alternate between ranges of a fixed charge followed by a constant charge per unit. Thus both sets $R_{0j}$ and $R_k$ can be partitioned in the respective freight range sets of fixed charge ($R^f_{0j}$ and $R^f_k$) and constant charge per unit ($R^c_{0j}$ and $R^c_k$), such that $R_{0j} = R^f_{0j} \cup R^c_{0j}$ and $R_k = R^f_k \cup R^c_k$. Let $q^r$ be a flow variable, which assumes the value of the respective replenishment order quantity $q$, when it falls in transportation range $r$, and is 0 otherwise. Let $\beta^{r-1}$ and $\beta^r$ be the respective lower and upper bounds for range $r$, and $\phi^r$ a binary variable denoting whether ($\phi^r = 1$) or not ($\phi^r = 0$) the transported volume falls in range $r$. Notice that $\beta^0 = 0$ and $\beta^{n_R}$ can be set to the maximum shipping capacity. As an illustration, let us consider the transportation cost function in Figure 3.6,
where $\beta^3 \leq q^4 < \beta^4$. In this case $\varphi^4 = 1$, and $\varphi^r = 0$ for $r = 1, 2, 3$ and 5. Similarly, $q^4 = q$ and $q^r = 0$ for $r = 1, 2, 3$ and 5.

![Figure 3.6. Piecewise Linear Transportation Cost Function](image)

In order to redefine Model (P3.1), let $\beta^r_{0j}$ and $\beta^r_{0j}$ denote the lower and upper limits for range $r$ on the transportation cost function associated with supplier $j$. Similarly, let $\beta^r_{k}$ and $\beta^r_{k}$ denote the respective lower and upper limits for range $r$ on the cost function associated with transportation from stage $k$ to stage $k+1$. Thus, following an approach similar to the one developed by Balarkrishnan and Graves (1989), and Simchi-Levi et al. (2005), Model (P3.1) can be reformulated as an MILP model, according to the following formulation.

**Additional Parameters**

$v^r_{0j}$ : Fixed charge for the actual freight rate for supplier $j$ when order quantity falls in range $r$, $j \in J, r \in R^r_{0j}$. 
\(v_k^r\) : Fixed charge for the actual freight rate from stage \(k\) to stage \(k+1\) if order quantity falls in range \(r\), \(k \in K_D, r \in R_k^\ell\).

\(e_{0j}^r\) : Constant charge per unit for the actual freight rate for supplier \(j\) when order quantity falls in range \(r\), \(j \in J, r \in R_{0j}^c\).

\(e_k^r\) : Constant charge per unit for the actual freight rate from stage \(k\) to stage \(k+1\) if order quantity falls in range \(r\), \(k \in K_D, r \in R_k^c\).

**Additional Variables**

\(\varphi_{0j}^{t,r}\) : 1 if the replenishment order quantity (in units of raw material) sent from supplier \(j\) to stage 1 in period \(t\) falls in transportation range \(r\); 0 otherwise; \(j \in J, t \in T_{0j}, r \in R_{0j}\).

\(\varphi_{k}^{t,r}\) : 1 if the order quantity (in units) sent from stage \(k\) to stage \(k+1\) in period \(t\) falls in transportation range \(r\); 0 otherwise; \(k \in K_D \cup \{n_K\}, t \in T_k, r \in R_k\).

\(q_{0j}^{t,r}\) : defined as \(q_{1j}^t\) when replenishment order quantity (in units of raw material) shipped from supplier \(j\) in period \(t\) falls in transportation range \(r\); 0 otherwise; \(j \in J, t \in T_{0j}, r \in R_{0j}\). Notice that \(q_{0j}^t = \sum_{r \in R_{0j}} q_{0j}^{t,r}\)

\(y_{k}^{t,r}\) : defined as \(y_k^t\) when replenishment order quantity (in units of finished product) shipped from stage \(k\) to stage \(k+1\) in period \(t\) falls in transportation range \(r\); 0 otherwise; \(k \in K_D, t \in T_k, r \in R_k\). Notice that \(y_k^t = \sum_{r \in R_k} y_{k}^{t,r}\).
subject to

Sets of Constraints

\( q_{0j}^{tr} - \beta_{0j}^{r} \varphi_{0j}^{r} \leq 0, \quad j \in J, t \in T_{0j}, r \in R_{0j}, \quad (3.26) \)

\( q_{0j}^{tr} - \beta_{0j}^{r-1} \varphi_{0j}^{r} \geq 0, \quad j \in J, t \in T_{0j}, r \in R_{0j}, \quad (3.27) \)

\( y_{k}^{tr} - \beta_{k}^{r} \varphi_{k}^{r} \leq 0, \quad k \in K_{D}, t \in T_{k}, r \in R_{k}, \quad (3.28) \)

\( y_{k}^{tr} - \beta_{k}^{r-1} \varphi_{k}^{r} \geq 0, \quad k \in K_{D}, t \in T_{k}, r \in R_{k}, \quad (3.29) \)

\[ \sum_{r \in R_{0j}} \varphi_{0j}^{r} \leq 1, \quad j \in J, t \in T_{0j}, \quad (3.30) \]

\[ \sum_{r \in R_{k}} \varphi_{k}^{r} \leq 1, \quad k \in K_{D}, t \in T_{k}, \quad (3.31) \]

\( q_{0j}^{tr} \geq 0, \quad \varphi_{1j}^{r} \in \{0,1\} \quad j \in J, t \in T_{0j}, r \in R_{0j}, \quad (3.32) \)

\( y_{k}^{tr} \geq 0, \quad \varphi_{1k}^{r} \in \{0,1\} \quad k \in K_{D}, t \in T_{k}, r \in R_{k}, \quad (3.33) \)

The objective function in Equation (3.25) represents the total variable cost function which includes purchasing, production, inventory holding, in-transit inventory and transportation costs. Replenishment order quantity variables are disaggregated according to the freight ranges. Transportation cost functions are represented by piecewise linear functions, where freight rates alternate between ranges of constant charge per unit, followed by a fixed charge. Notice that Equations (3.15)-(3.19) are used
to define purchasing, production, inventory holding and in-transit inventory cost functions in the objective function. Sets of constraints (3.2)-(3.14) are defined as in Model (P3.1), except that order quantities are disaggregated: \( q_{0j}^t = \sum_{r \in R_{0j}} q_{ij}^{tr} \) and
\( y_k^t = \sum_{r \in R_k} y_k^{tr} \). Sets of constraints (3.26)-(3.29) guarantee that the order quantities fall in the corresponding freight range. Sets of constraints (3.30) and (3.31) assure that at most one freight range is active for each order quantity. Finally, sets of constraints (3.32) and (3.33) describe the nature of the variables considered in the model.

3.6 Illustrative Example

In this section, a three stage serial supply chain example with three suppliers and six time periods is presented and analyzed. Linear functions are defined for purchasing, production, and holding costs. Three alternative functions are considered to model the LTL transportation costs. In the first case, transportation costs are modeled with a piecewise linear function, which enables an exact representation of the actual costs in the MILP model. In the other two cases, transportation costs are modeled using power and quadratic approximation functions, which are estimates of the actual transportation costs. Thus, Model (P3.2) is used as benchmark to measure the performance of the transportation cost approximations in Model (P3.1).

The illustrative example includes three potential suppliers, one manufacturer and one DC. While raw materials can only be stocked at the manufacturing stage, finished products can be stored at both the manufacturing and distribution stages. Zero delivery lead times are assumed for all stages and suppliers. Consequently, products arrive in the same time period in which they are ordered. No initial and ending inventories are considered for the problem, and neither pending orders. The minimum average acceptable perfect rate for raw materials is assumed to be \( a = 0.95 \). Thus, the mix of raw materials purchased in each period needs to have, on average, a perfect rate greater than or equal to 0.95.
At the supplier level, it is assumed that the three suppliers have been already screened from a larger set of potential suppliers. This selection could have been done using the Analytical Hierarchy Process, as suggested by Mendoza et al. (2008), or following any other suitable method.

Nominal and actual freight rates for suppliers 1, 2 and 3 are provided in Tables 3.1, A.1 and A.2, respectively (see Section 3.3 and Appendix A). Similarly, freight information is also provided for interplant transportation (manufacturing facility to DC) in Table A.3. From the graphical representation of the respective LTL cost functions (see Figure 3.7), it is possible to observe that while supplier 2 has the most expensive transportation cost, interplant shipments happen to use the less expensive route.

Even though it is assumed that all the suppliers may be potentially selected to procure raw materials, they do not necessarily have the same characteristics in terms of ordering costs, perfect rate, and capacity (see Table 3.2). For instance, note that although supplier 1 has the lowest setup and variable costs, considering that its perfect rate is below the minimum acceptable rate, it can only be selected as part of a mix with the other two suppliers. In contrast, even if supplier 2 has lower fixed and variable ordering costs than supplier 3, in an optimal solution this advantage could be offset by higher transportation costs associated with the other two alternatives (see Figure 3.7).

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Capacity (units/period)</th>
<th>Ordering Cost (Fixed $)</th>
<th>Variable ($/unit)</th>
<th>Perfect Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,200</td>
<td>4,000</td>
<td>38</td>
<td>0.93</td>
</tr>
<tr>
<td>2</td>
<td>1,300</td>
<td>4,500</td>
<td>40</td>
<td>0.97</td>
</tr>
<tr>
<td>3</td>
<td>1,100</td>
<td>5,800</td>
<td>42</td>
<td>0.96</td>
</tr>
</tbody>
</table>

The analysis has been performed over a six period planning horizon with time varying customer demand (see Table 3.3). Capacity constraints are only considered at the production stage, where the capacities are assumed to be fixed for the entire planning horizon. Given the anticipated inflation expected in the second half of the planning horizon, production and DC setup costs are increased accordingly. Furthermore, as a
result of seasonality factors, variable production costs are increased for periods 3 and 4. Inventory holding costs are considered constant throughout the entire planning horizon, only depending on the stage and product characteristic (raw material or finished product).

![Figure 3.7](image)

**Figure 3.7.** Actual Transportation Cost Functions for All the Suppliers and Interplant Transportation.

**Table 3.3.** Summary of Parameters for the Illustrative Example

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand (units/period)</th>
<th>Product. Cap. (units/period)</th>
<th>Production Cost ($/order)</th>
<th>Production Cost ($/unit)</th>
<th>DC Cost ($/order)</th>
<th>Holding Costs ($/unit/period)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>900</td>
<td>1,500</td>
<td>3,000</td>
<td>12</td>
<td>2,000</td>
<td>3, 6, 7</td>
</tr>
<tr>
<td>2</td>
<td>950</td>
<td>1,500</td>
<td>3,000</td>
<td>12</td>
<td>2,000</td>
<td>3, 6, 7</td>
</tr>
<tr>
<td>3</td>
<td>1,100</td>
<td>1,500</td>
<td>3,000</td>
<td>16</td>
<td>2,000</td>
<td>3, 6, 7</td>
</tr>
<tr>
<td>4</td>
<td>1,300</td>
<td>1,500</td>
<td>3,900</td>
<td>16</td>
<td>2,600</td>
<td>3, 6, 7</td>
</tr>
<tr>
<td>5</td>
<td>1,500</td>
<td>1,500</td>
<td>5,100</td>
<td>12</td>
<td>3,400</td>
<td>3, 6, 7</td>
</tr>
<tr>
<td>6</td>
<td>1,750</td>
<td>1,500</td>
<td>6,600</td>
<td>12</td>
<td>4,400</td>
<td>3, 6, 7</td>
</tr>
</tbody>
</table>

Based on the information pertaining to actual freight rates for suppliers and interplant transportation, power and quadratic freight approximation functions are fitted using Minitab version 15 (MINITAB, 2009). The results are provided in Table 3.4. Notice that most of the coefficients of determination ($R^2$) are greater than 0.94, with the
only exception of the power estimate function for supplier 2. Considering the values of \(R^2\), the quadratic function always provides the best fit for the actual freight rates.

### Table 3.4  Summary of the Approximation Functions with the Best Fit for Actual Freight Rates

<table>
<thead>
<tr>
<th>Function</th>
<th>Power Estimate function (($))</th>
<th>(R^2)</th>
<th>Quadratic Estimate function (($))</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier 1</td>
<td>(g_1(q) = 112.656 \ (q)^{-0.432307})</td>
<td>0.941</td>
<td>(g_1(q) = 1,027 + 6.371 \ q - 0.001393 \ (q)^2)</td>
<td>0.978</td>
</tr>
<tr>
<td>Supplier 2</td>
<td>(g_2(q) = 78.723 \ (q)^{-0.313660})</td>
<td>0.893</td>
<td>(g_2(q) = 307.8 + 11.72 \ q - 0.002353 \ (q)^2)</td>
<td>0.991</td>
</tr>
<tr>
<td>Supplier 3</td>
<td>(g_3(q) = 123.945 \ (q)^{-0.497339})</td>
<td>0.945</td>
<td>(g_3(q) = 1,096 + 3.971 \ q - 0.000891 \ (q)^2)</td>
<td>0.940</td>
</tr>
<tr>
<td>Interplants</td>
<td>(g_4(q) = 78.813 \ (q)^{-0.497229})</td>
<td>0.945</td>
<td>(g_4(q) = 697.3 + 2.527 \ q - 0.000567 \ (q)^2)</td>
<td>0.940</td>
</tr>
</tbody>
</table>

Similar to the display of transportation cost functions for supplier 1 presented in Figure 3.7, graphical comparisons of power and quadratic estimate functions against the actual freight rates for suppliers 2 and 3 and interplant transportation are provided in Figure B.1 (see Appendix B). Even though both estimate functions provide good approximations of the actual freight rates, the power function appears to be less accurate than the quadratic function. That observation is particularly apparent for shipment sizes above 2,000 units, where the power function overestimates the actual freight rates. In both cases, major estimation errors are produced for shipment sizes between 400 and 2,000 units.

### 3.7  Computational Results

The illustrative example has been implemented on GAMS 21.7 (Rosenthal, 2007) on a Pentium 4 with 3.40 GHz and 1 GB of RAM. The MILP model has been solved using CPLEX 11.0 provided by GAMS, allowing a maximum gap of 0.001% between the LP relaxation and the optimal integer solution. The MINLP models generated by the power and quadratic approximation functions have been solved using the DICOPT
algorithm, also provided by GAMS. The CPU times required to solve the different versions of the problem have not exceeded 5 seconds.

3.7.1 Sensitivity Analysis

Fourteen different cost scenarios have been analyzed in order to capture the effects of holding and production/distribution setup costs on raw material supply decisions. A summary of the results for the different scenarios is shown in Table 3.5, where changes in cost parameters are expressed as percentages of their respective original values. Considering the inverse effect of setup and inventory holding costs on order quantities, and to avoid the neutralization result of these two parameters, setup costs are either maintained or decreased when holding costs are increased. Similarly, when holding costs are decreased, setup costs are maintained or increased.

The illustrative example has been solved using three alternative formulations: two MINLP models, where transportation costs are approximated through power and quadratic transportation cost functions, and a MILP model, where transportation costs are represented by exact piecewise linear functions. After the three alternative model formulations are solved for each instance of the problem, transportation costs are recalculated for the two optimal MINLP solutions using the actual freight rates defined for the problem. The optimal solutions for the MILP model and the two updated solutions for the MINLP models are provided in columns 5-7 of Table 3.5. In addition, transportation cost errors for the power and quadratic functions are calculated as deviation percentages from the actual transportation costs for the 14 cost scenarios. These results, given in the last two columns of the table, indicate that the power estimate function systematically underestimates actual transportation costs, and the quadratic estimate function overestimates actual freight costs. The averages of the absolute transportation cost errors for the power and quadratic estimate functions are 3.08% and 2.22 %, respectively. Consequently, the better fit shown by the quadratic estimate function over the power estimate function (see $R^2$ in Table 3.4) is confirmed.
### Table 3.5 Comparison of Optimal Solutions according to the MILP Model, Power and Quadratic Estimate Functions

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Percentage of Original Cost Value (%)</th>
<th>Total Cost ($)</th>
<th>Gap (%)</th>
<th>Transp. Cost Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Holding</td>
<td>Fixed Prod.</td>
<td>Fixed DC</td>
<td>MILP</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>547,902</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>534,975</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>150</td>
<td>100</td>
<td>558,702</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>100</td>
<td>150</td>
<td>555,102</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>545,775</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>542,175</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>150</td>
<td>150</td>
<td>565,902</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>150</td>
<td>150</td>
<td>552,710</td>
</tr>
<tr>
<td>9</td>
<td>150</td>
<td>100</td>
<td>100</td>
<td>551,683</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>527,583</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>50</td>
<td>100</td>
<td>535,783</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>539,883</td>
</tr>
<tr>
<td>13</td>
<td>50</td>
<td>0</td>
<td>0</td>
<td>496,716</td>
</tr>
<tr>
<td>14</td>
<td>150</td>
<td>0</td>
<td>0</td>
<td>510,683</td>
</tr>
</tbody>
</table>

### Table 3.6 Comparison of Raw Material Order Quantities according to the MILP Model, Power and Quadratic Estimate Functions

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Percentage of Original Cost (%)</th>
<th>MILP</th>
<th>Power</th>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Holding</td>
<td>Fixed Prod.</td>
<td>Fixed DC</td>
<td>Supplier 1</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>2,150</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>3,600</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>150</td>
<td>100</td>
<td>2,150</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>100</td>
<td>150</td>
<td>2,150</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>150</td>
<td>100</td>
<td>3,600</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>3,600</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>150</td>
<td>150</td>
<td>2,150</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>150</td>
<td>150</td>
<td>3,600</td>
</tr>
<tr>
<td>9</td>
<td>150</td>
<td>100</td>
<td>100</td>
<td>6,150</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>6,120</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>50</td>
<td>100</td>
<td>6,120</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>6,120</td>
</tr>
<tr>
<td>13</td>
<td>50</td>
<td>0</td>
<td>0</td>
<td>3,600</td>
</tr>
<tr>
<td>14</td>
<td>150</td>
<td>0</td>
<td>0</td>
<td>6,150</td>
</tr>
</tbody>
</table>

(a) Number of periods where raw material is ordered from one or more suppliers  
(b) Number of periods where raw material is ordered from the specific supplier
Given that the optimal solution of the MILP model is the benchmark solution for the problem, the gap for the solution obtained using the MINLP model with a transportation cost approximation function is equal to the difference between the optimal objective function values of the MINLP and MILP models. The gaps for the power and quadratic estimate functions for the 14 cost scenarios, written as a percentage of the optimal MILP solutions, are given in columns 8 and 9 of Table 3.5. Note that in almost all cases, the gap observed with the quadratic approximation model is substantially lower than the gap obtained with the power approximation model. In fact, the average gap for the power function is more than twice as large as the average gap for the quadratic approximation model (1.18 % vs. 0.55 %). As was expected, neither of the alternatives reached the global optimal solutions due to inaccuracies associated with estimating actual freight costs.

Under the original cost structure (scenario 1, Table 3.6), the optimal solution for the MILP model allocates 42.67% of the total purchased raw material (3,200 of 7,500 units) to supplier 3. This situation can be explained by the relative low transportation cost of this supplier for medium lot size orders (see Figure 3.7), which compensates for its relatively high ordering cost. It is also confirmed that changes on holding and setup costs may affect the optimal sourcing strategy for the problem. In fact, that situation is observed for all three models, where the optimal raw material ordering policy changes based on the inventory holding and setup cost scenario.

Even though the MILP model solution provides the most stable raw material sourcing strategy among the 14 scenarios, the original solution is also affected when holding and set up costs are changed (see Table 3.6). In the optimal solution for the MILP model, when holding costs are reduced (scenarios 2, 5, 6, 8, and 13) the total number of orders is decreased, and consequently the average raw material lot size is increased (see Table 3.6, column b). In this case, the reduction in holding costs triggers a reduction in the number of orders, and as a result it decreases the total raw material set up cost. In contrast, changes in production and distribution set up costs affect the decisions related to order allocation by supplier. In particular, when setup costs are reduced (scenarios 10, 11, 12, and 14) the volume allocated to supplier 3 is increased in the optimal solution for the
MILP model. In addition, raw materials are ordered in every period under consideration (see Table 3.6, column a), enabling the reduction of production and distribution lot sizes, and consequently, the respective holding costs.

Although it may appear that production and distribution lot sizing decisions are unrelated to the raw material ordering policy, the selection of suppliers is affected by changes in production/distribution lot size orders. In fact, given the acceptable perfect rate and low transportation cost of supplier 3, this supplier is preferred under a scenario of small production/distribution lot sizes. On the other hand, due to the relatively high transportation cost of supplier 2, it is not a competitive alternative when lot sizes are reduced. Finally, given that the perfect rate of supplier 1 is lower than the minimum acceptable rate for the problem, this supplier could only be selected in combination with any of the other two suppliers; however, this strategy is competitive under a scenario of reduced production/distribution lot size orders.

The scenario analysis performed in this chapter shows that the optimal procurement strategy may be affected by different conditions of holding and setup costs. Identical solutions are obtained for a group of scenarios sharing similar characteristics (see Table 3.7). In the first group (scenarios 1, 3, 4, and 7), inventory holding costs are maintained and only setup costs are increased. In the second group (scenarios 2, 5, 6, 8, and 13), inventory holding costs are decreased while setup costs are increased, maintained, and decreased. In this group, it is clear that the main driver of the solution is the reduction in inventory holding costs. Finally, in the last group (scenarios 9, 10, 11, 12, and 14) a similar solution is obtained when inventory holding costs are maintained or increased, while setup costs are increased, maintained, or decreased. Note that in this group of scenarios, the effect of inventory holding costs and setup costs are added, given their inverse effect on order quantities.
Table 3.7  Optimal Solutions, by Scenario

<table>
<thead>
<tr>
<th>Solution</th>
<th>Percentage of Original</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Scenario</td>
</tr>
<tr>
<td></td>
<td>Holding 100 100 100</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>100 150 100</td>
</tr>
<tr>
<td>4</td>
<td>100 100 150</td>
</tr>
<tr>
<td>7</td>
<td>100 150 150</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>50 150 100</td>
</tr>
<tr>
<td>6</td>
<td>50 100 150</td>
</tr>
<tr>
<td>8</td>
<td>50 150 150</td>
</tr>
<tr>
<td>13</td>
<td>50 0 0</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>150 100 100</td>
</tr>
<tr>
<td>11</td>
<td>100 50 100</td>
</tr>
<tr>
<td>12</td>
<td>100 100 50</td>
</tr>
<tr>
<td>14</td>
<td>150 0 0</td>
</tr>
</tbody>
</table>

A graphical representation of the three groups of solutions obtained from the scenario analysis is presented in Figure 3.8. In the first group, raw material volume is almost evenly allocated among the three suppliers. In these scenarios, given that the perfect rate of supplier 1 is below the minimum acceptable level (see Table 3.2), it is only possible to include this supplier as part of a mix with the other two alternatives. In the second group, raw material orders are allocated between supplier 1 and 2 with similar raw material allocation percentages. In these scenarios, since the average order lot size is increased, the number of orders is decreased, and consequently only two suppliers are needed. Finally, in the third group, although the three suppliers are considered, supplier 3 captures most of the allocated raw material volume. Considering the low transportation costs of supplier 3 (see Figure 3.7) and its acceptable perfect rates (see Table 3.2), when set up costs are increased and/or holding costs are decreased, it is more convenient to increase the raw material order quantities so that total setup costs can be reduced.
3.7.2 Comparing the Integrated and Sequential Approaches

In supply chain management, raw material procurement and production/distribution decisions can be modeled using either a sequential or an integrated approach. In the sequential approach, the production/distribution problem and the raw material procurement problem are solved separately. In the first stage, assuming infinite raw material supply capacity, the optimal production/distribution strategy is defined so that customer demand is completely satisfied. Thus, the raw material requirements generated by the first stage determine the optimal procurement strategy, where orders are allocated among the available suppliers. In the integrated approach, raw material procurement, production, and distribution decisions are simultaneously made, so that an optimal solution is defined for the entire supply chain problem.
A graphical representation of the integrated supply chain inventory planning problem is provided by the time-expanded transshipment network presented in Figure 3.2. In this network, raw material supplier and production/distribution decisions are simultaneously analyzed. Conversely, in the sequential approach, the production/distribution and supplier selection problems are independently and sequentially solved. In this approach, the production/distribution problem considers stages 2 through 5 in the original time-expanded transshipment network (see Figure 3.2), where flow variable $x^t_1$ is represented by an arc that is only connected to a node in stage 2. Once the optimal production strategy has been defined ($x^t_1$ values), total raw material requirements are calculated based on the respective BOM ratio. Note that in the illustrative example (Section 3.6), the BOM ratio has been assumed to be equal to one, and consequently only one unit of raw material is required to produce one unit of finished product. Similarly, the raw material supply selection problem can be represented by the set of nodes, which represents the raw material suppliers and stage 1 (see Figure 3.2). In this sub-network, the output flows for stage 1 symbolize the raw material requirements for the problem, which are calculated from the optimal production/distribution strategy. Thus, both sub-networks are only connected by a set of linking arcs that connect stages 1 and 2. In the production/distribution problem, flow variable $x^t_1$ represents the optimal production level for period $t$, while in the supplier selection problem, parameter $x^t_1$ represents the raw material requirement in period $t$ needed to satisfy the production plan.

To model the sequential approach, Model (P3.1) is partitioned into two sub-models. In the production/distribution problem, the objective function is defined by four of the original seven terms defined by Equation (3.2). In this objective function, production (second term), holding (third term), in-transit holding (fifth term), and transportation (seventh term) costs are minimized. Most of the constraints are maintained, only excluding Constraints (3.2), (3.6), (3.7), and (3.12), which are used later to solve the supplier selection and replenishment problem. Once the optimal production/distribution strategy has been defined, the required raw material requirements are calculated, so that they can be used as input for the raw material procurement problem. The objective function for the problem minimizes raw material procurement...
(first term), holding (third term), in-transit (fourth term), and transportation (sixth term) costs associated with raw materials. Equation sets (3.2), (3.6), (3.7), and (3.12) are used to constrain the problem, so that raw material balance, minimum acceptable perfect rates, and raw material supplier capacities are satisfied. Note that in Equation (3.2), the original lot size variable $x_1^t$ must be replaced by the raw material requirement level defined for the respective time period, so that the raw material problem can be linked with the production/distribution problem.

The illustrative example described in Section 3.6 has been solved using the sequential and integrated approaches. Required CUP times for both approaches were relatively similar, with 3 seconds for the sequential approach and 4 seconds for the integrated approach. Note that in the sequential approach (see Figure 3.9) production is considered during all time periods, allowing low inventory levels throughout the entire planning horizon. However, when the problem is solved using the integrated approach, the production plan is adjusted so that a higher percentage of the raw material volume is allocated to the less expensive alternatives (suppliers 1 and 2). Notice that given the unacceptable perfect rate of supplier 1, it is only possible to include this supplier as part of a mix with supplier 2. However, when both solutions are compared, almost no improvement is observed with the integrated approach (see Table 3.8). Note that by using the sequential approach, lower inventory and shipping costs are possible; however, the integrated approach generates a solution with lower raw material procurement and fixed costs. Both improvements are almost neutralized, generating similar objective function values for the problem with costs savings of 0.03% of the integrated approach over the sequential approach. One explanation for this lack of improvement could be related to tight production and raw material supplier capacities. In fact, in the integrated approach, all positive production levels are defined at the maximum production capacity, so a reduced number of alternative solutions are possible. A similar situation is also observed with the raw material procurement strategy, where supplier capacities are frequently binding constraints.
Figure 3.9  The Optimal Solution for the Sequential Approach
Figure 3.10  The Optimal Solution for the Integrated Approach
To evaluate the impact of production capacity constraints, both sequential and integrated approaches have been solved under the assumption of unlimited production capacity. Required CUP times for both approaches are relatively similar with 3 seconds for the sequential approach and 4 seconds for the integrated approach. As expected, the production lot sizes of both approaches increases in relation to the original solution for the problem (see Figures 3.11 and 3.12). In this case, the integrated approach generates bigger lot sizes by concentrating the production volume of the last two periods of the sequential approach (periods 5 and 6) into period 5. Even though this strategy increases the inventory levels of the integrated approach, this solution allows additional economies of scale in transportation costs of raw materials and finished products. Note that when production capacity constraints are relaxed, using the integrated approach produces cost savings of around 0.55% compared to the sequential approach (see Table 3.9). The main cost advantages of the integrated approach are observed in transportation operations and fixed production costs, both of which are directly affected by lot size.

### Table 3.8  
Optimal Solutions for the Integrated and Sequential Approaches, under Original Problem Conditions

<table>
<thead>
<tr>
<th>Type</th>
<th>Item</th>
<th>Sequential (a)</th>
<th>Integrated (b)</th>
<th>Savings (a - b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Raw materials</td>
<td>307,800</td>
<td>302,100</td>
<td>5,700</td>
</tr>
<tr>
<td></td>
<td>Shipping</td>
<td>53,778</td>
<td>59,302</td>
<td>(5,524)</td>
</tr>
<tr>
<td></td>
<td>Production</td>
<td>100,600</td>
<td>96,000</td>
<td>4,600</td>
</tr>
<tr>
<td></td>
<td>Holding</td>
<td>7,413</td>
<td>20,100</td>
<td>(12,687)</td>
</tr>
<tr>
<td></td>
<td><strong>Subtotal</strong></td>
<td><strong>469,591</strong></td>
<td><strong>477,502</strong></td>
<td><strong>(7,911)</strong></td>
</tr>
<tr>
<td>Fixed</td>
<td>Raw materials</td>
<td>37,500</td>
<td>34,400</td>
<td>3,100</td>
</tr>
<tr>
<td></td>
<td>Shipping</td>
<td>16,400</td>
<td>14,400</td>
<td>2,000</td>
</tr>
<tr>
<td></td>
<td>Production</td>
<td>24,600</td>
<td>21,600</td>
<td>3,000</td>
</tr>
<tr>
<td></td>
<td><strong>Subtotal</strong></td>
<td><strong>78,500</strong></td>
<td><strong>70,400</strong></td>
<td><strong>8,100</strong></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>548,091</strong></td>
<td><strong>547,902</strong></td>
<td>189</td>
</tr>
</tbody>
</table>
Figure 3.11  The Optimal Solution Generated by the Sequential Approach for a Scenario with Unlimited Production Capacity
Figure 3.12  The Optimal Solution Generated by the Integrated Approach for a Scenario with Unlimited Production Capacity
To properly evaluate the potential impact of the integrated approach on cost reduction, the illustrative example described in Section 3.6 has been revised as follows:

1. Production and inventory capacities are relaxed in all stages, assuming infinite production capacities.
2. Inventory holding costs are assumed to be 50% of the original values.
3. Suppliers 1 and 3 are assumed to have positive lead times, where orders arrive one period after they are placed.
4. The variable raw material price for Supplier 2 is increased from $40/unit to $52/unit, so that this supplier becomes the most expensive alternative.
5. Production cost at period 1 is reduced from $12/unit to $10/unit, so it is advantageous to produce at the beginning of the planning horizon.

The new problem has been solved using the sequential and integrated approaches. The corresponding solutions are provided in Figures 3.13 and 3.14. Note that in the optimal solution for the sequential approach, production operations are only concentrated in periods 1 and 5 (see Figure 3.13), allowing significant reductions in the fixed costs associated with the number of orders placed during the planning horizon. This strategy
also has a positive impact on transportation costs by taking advantage of the all-units discount strategy of these operations. However, when the raw material supplier selection problem is considered, given the positive delivery lead times of suppliers 1 and 3, supplier 2 is the only alternative available to satisfy the raw material requirements in period 1. This example shows the conflict that may be produced when the problem is sequentially solved. Even though the production/distribution strategy appears to be the best alternative, when the raw material supplier problem is considered, it is obvious that this solution is not necessarily the best for the entire supply chain.

The solutions for the sequential and integrated approaches are summarized in Table 3.10. Required CUP times for both approaches were relatively similar, requiring less than 5 seconds in both cases. Note that the sequential approach generates lower fixed costs than the integrated approach, with cost savings of 1.70% over the integrated method. These cost savings are possible given the larger production and distribution lot sizes suggested by the sequential approach. However, when the variable costs are analyzed, the integrated approach generates cost savings of 7.78% over the sequential method. In the integrated analysis, it is acceptable to assume some additional fixed costs, so that a better raw material procurement strategy can be selected.

**Table 3.10** Optimal Solutions Generated by the Integrated and Sequential Approaches for the Revised Scenario

<table>
<thead>
<tr>
<th>Type</th>
<th>Item</th>
<th>Sequential (a)</th>
<th>Integrated (b)</th>
<th>Savings (a- b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw materials</td>
<td>357,500</td>
<td>324,000</td>
<td>33,500</td>
</tr>
<tr>
<td></td>
<td>Shipping</td>
<td>51,745</td>
<td>42,536</td>
<td>9,209</td>
</tr>
<tr>
<td></td>
<td>Production</td>
<td>81,500</td>
<td>88,200</td>
<td>(6,700)</td>
</tr>
<tr>
<td></td>
<td>Holding</td>
<td>33,275</td>
<td>28,975</td>
<td>4,300</td>
</tr>
<tr>
<td></td>
<td><strong>Subtotal</strong></td>
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<td><strong>483,711</strong></td>
<td><strong>40,309</strong></td>
</tr>
<tr>
<td>Fixed</td>
<td>Raw materials</td>
<td>10,300</td>
<td>16,100</td>
<td>(5,800)</td>
</tr>
<tr>
<td></td>
<td>Shipping</td>
<td>7,400</td>
<td>7,400</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Production</td>
<td>8,100</td>
<td>11,100</td>
<td>(3,000)</td>
</tr>
<tr>
<td></td>
<td><strong>Subtotal</strong></td>
<td><strong>25,800</strong></td>
<td><strong>34,600</strong></td>
<td><strong>(8,800)</strong></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>549,820</td>
<td>518,311</td>
<td>31,509</td>
</tr>
</tbody>
</table>
Figure 3.13  The Optimal Solution Generated by the Sequential Approach for the Revised Scenario
Figure 3.14  The Optimal Solution Generated by the Integrated Approach for the Revised Scenario
Overall, the integrated approach generates cost savings of 6.08% over the sequential method, showing the convenience of an integrated approach analysis for the supply chain problem. When the problem is sequentially solved, the optimal solution for the entire problem cannot be guaranteed, and consequently suboptimal solutions may be generated.

### 3.8 Conclusions

In this chapter, alternative mathematical programming formulations have been analyzed for the dynamic inventory planning problem with supplier selection in a serial supply chain. Particular attention is given to analyzing the characteristics of the expanded static transshipment network in order to identify potential problem size reductions. In this process, infeasible solutions are identified, and the associated nodes and arcs are eliminated from the network.

Considering the assumption of the all-units discount transportation cost strategy for the problem, and based on previous research, power and quadratic functions are used to estimate the LTL transportation cost in the proposed MINLP model. Approximations are used because the actual costs are piecewise linear and non-concave. However, given the inaccuracies and increased complexities that these estimate functions introduce into the model, an alternative piecewise linear formulation is developed and incorporated into a MILP model. The experimental results obtained in this chapter show that estimated transportation cost functions may lead to suboptimal solutions, even though the optimal solution for the problem is apparently reached. In particular, power and quadratic functions produce average transportation cost errors of 3.08% and 2.22% in relation to the actual transportation costs for their respective optimal solutions. Similarly, when the optimal solutions for the power and quadratic functions are compared with the optimal solution obtained using the MILP model (piecewise linear function model), average solution gaps of 1.18% and 0.55% are obtained. Consequently, when estimate functions are used to incorporate transportation costs under an all-units discount structure, the
analyst needs to be aware of potential solution gaps that cannot be easily detected without an alternative exact model formulation.

Experimental results show that the integrated approach may generate substantial cost savings over the sequential approach. Even though almost no improvement (0.03%) is observed under the original conditions of the problem, when the production capacities are relaxed, improvements of 0.55% are obtained. This improvement is even larger after a few key parameters are changed. In this scenario, cost savings of 6.08% are obtained as a consequence of implementing the integrated approach. These results show that the integrated approach can produce significant improvements in supply chain efficiency, and consequently it should be preferred.

According to the results obtained in this chapter, inventory holding and production/distribution setup costs may have a significant impact on raw material supplier selection and lot sizing decisions. Even though these parameters may not appear to be directly related to raw material supplier decisions, the analysis of 14 different cost scenarios shows that they play an important role in the optimal sourcing strategy. These results demonstrate the need to develop integrated models in order to correctly minimize the total cost of the entire supply chain network.
Chapter 4

A Dynamic Inventory Model for the Multiproduct Problem including Supplier Selection and Transportation Costs in a Serial Supply Chain Structure

4.1 Introduction

This chapter extends the model presented in Section 3.5 by proposing a mixed integer linear programming (MILP) model for the multiproduct case. In this problem, it is assumed that more than one type of raw material may be required to manufacture a specific finished product. Consequently, the problem is defined in the context of multiple raw materials and multiple finished products. To correctly account for transportation and inventory capacities, the original units of raw materials and finished products are transformed into commensurable equivalent units. In this scenario, significant economies of scale can be obtained by coordinating operations. This coordination can be done by joining replenishment, manufacturing, and transportation orders. In raw material procurement and manufacturing processes, significant setup cost savings can be obtained when multiple raw materials or finished products are combined in the same order. Higher transportation discounts can be obtained in the same way when quantity discounts are offered. Coordinated decision problems occur frequently in supply chain management. Silver (1979) mentions several examples, including problems in scheduling, procurement, and transportation. Stowers and Palekar (1997) provide examples in the contexts of chemical processing and plastics manufacturing. Similarly, Robinson and Lawrence (2004) illustrate such problems in processes related to the industrial manufacturing of lubricants and the coordination of vaccine shipments from a manufacturing facility to a distribution center.

In raw material procurement and manufacturing processes, it is common that setup costs are defined by a major ordering component (independent of the products
included in the order), and several minor ordering components (based on each product included in the order). This problem, known as the joint replenishment problem (JRP), has been intensively studied since it was first described by Starr and Miller (1962), and Shu (1971). The JRP decides the optimal order quantities for a mix of products ordered from the same supplier or manufacturing process, such that ordering and holding costs are minimized. According to Khouja and Goyal (2008), the strategies used to solve the JRP can be classified in two ways: direct grouping strategies and indirect grouping strategies. In a direct grouping strategy, products are partitioned into a predetermined number of sets, where products in the same set are jointly replenished on the same time cycle. In an indirect grouping strategy, replenishments are made at regular time intervals and each product has a replenishment quantity sufficient to last for exactly an integer number of the regular time intervals. Consequently, groups are indirectly formed by products with the same order frequency. According to van Eijs et al. (1992), the indirect grouping strategy outperforms the direct grouping strategy under high major set up costs, since a greater number of products can be jointly replenished.

An extension of the JRP is defined when product demands are deterministic but not uniform over time. The objective is to minimize the total replenishment cost associated with the demand of \( n_p \) products over a planning horizon of \( n_T \) periods. This problem is known as the dynamic joint replenishment problem (DJRP), which has been proven to be NP-hard (Arkin et al., 1989; Joneja, 1990). Different exact algorithms have been proposed in the previous literature to solve this problem, including dynamic programming (Zangwill, 1966; Veinott, 1969; Kao, 1979; Silver, 1979), branch-and-bound (Erenguc, 1988; Federgruen and Tzur, 1994; Kirca, 1995; Robinson and Gao, 1996), branch-and-cut (Raghavan and Rao, 1991; Raghavan, 1993), and Dantzig-Wolfe decomposition (Raghavan and Rao, 1992). However, only computational results for modest size instances have been reported. Boctor et al. (2004) compare six heuristics reported in the literature of DRJP, where the Fogarty-Barringer heuristic (Fogarty and Barringer, 1987) performed the best. Arunachalam and Robynson (2010) propose a six-phase heuristic and a simulated annealing meta-heuristic for the coordinated capacitated lot-size problem. Even though extensive work has been completed for the DRJP, to the
best of our knowledge, no model has been proposed in the context of a serial supply chain. In particular, considering that the DRIP concept is observed in raw material procurement and manufacturing operations, an integrated analysis of the problem can bring substantial cost savings.

In practice, quantity discounts are frequently offered by transportation companies in order to stimulate demand for larger and more profitable quantity orders. In many cases, it may be uneconomical to achieve a discount breakpoint quantity by ordering one single product. Consequently, higher discounts can be achieved by coordinating several products in the same order. In the less-than-truckload (LTL) transportation mode, costs are directly related to the quantity shipped and the distance traveled. LTL cost functions are usually characterized by indifference points, where the number of shipping units beyond these points is charged a higher LTL freight rate corresponding to the next range. These cost functions are invoked when shipment sizes are over-declared. Examples of the LTL transportation mode are third-party carriers such as UPS or FedEx. In general, the LTL transportation mode is preferred for small quantities since it is less expensive than using the full-truck load (TL) mode. In the TL transportation mode, a fixed cost is incurred for each load up to a given capacity. Some examples of TL shipments are overseas containers and trucks designed for specific products. Aucamp (1982) and Lippman (1971) analyze the TL transportation mode, where the cost per load does not change with the number of loads. Conversely, Lee (1986) considers TL shipments for a one-stage model with discounts on the per load costs as the number of truckloads increases.

Although most of the previous research has analyzed TL and LTL transportation modes separately, recent research simultaneously analyzes both transportation modes. Adelwahab and Sargious (1992), and Swenseth and Godfrey (2002) consider TL and LTL as disjoint problems, comparing their inventory policies. Arcelus and Rowcroft (1991) propose models with incremental quantity discounts and three different freight structures. Rieksts and Ventura (2008, 2010) develop inventory policies for the one-stage problem under a constant demand rate. In this research, transportation costs have been incorporated as a combination of the TL and LTL options. These authors mention that
when inventory and setup costs are dominant, the optimal order quantity may be a combination of both transportation options. Thus, if the quantity of partial load is not sufficient to justify another truck, it is optimal to use both modes of freight transportation. Even though the research presented in this chapter has similarities with the research presented by Rieksts and Ventura (2008, 2010), it extends the problem for dynamic demand, explicitly considering raw material supplier selection in the analysis.

In supply chain management, raw material procurement and production/distribution decisions can be modeled by using either a sequential or an integrated approach. In the sequential approach, the production/distribution problem and the raw material procurement process are solved independently. In the first stage, assuming infinite raw material supply capacity, the optimal production/distribution strategy is defined so that customer demand is completely satisfied. Then, based on the raw material requirement generated by the first stage, the optimal raw material procurement strategy is selected in the second stage, where orders are allocated among the available suppliers. In the integrated approach, raw material procurement, production, and distribution decisions are made simultaneously, such that the optimal solution is defined for the entire supply chain problem. Although the integrated optimization approach has been shown to produce superior solutions in the supply chain context (see Chapter 3), the sequential approach has usually been preferred, given the lack of appropriated decision support tools available for analyzing the entire supply chain.

Substantial research exists on raw material procurement and production/distribution when they are analyzed independently. However, research that covers both topics simultaneously is limited and recent. Basnet and Leung (2005) present one of the first studies where inventory lot sizing is analyzed simultaneously with supplier selection under a dynamic demand environment. They propose an enumerative search algorithm and also a heuristic approach to solve the MILP formulation for the single-stage uncapacitated multiproduct problem. Ustun and Demirtas (2008) propose a mixed integer goal programming model for the single-stage lot sizing and supplier selection problem. Additional research has also been conducted for cases of static demand using some extensions of the economic order quantity (EOQ) model. Mendoza and Ventura (2008)
study a serial inventory system with supplier selection and order quantity allocation, where some of the properties of the EOQ model are used to derive a simplified mixed integer nonlinear programming model for the problem.

In this chapter, the DJRP concept is applied for procurement and manufacturing operations. Transportation costs are included as a combination of the TL and LTL transportation modes, taking advantage of the economies of scale provided by the TL mode, and the flexibility of the LTL mode. The problem is solved using an integrated approach, where the entire supply chain is analyzed at once. However, in order to show the advantages of this approach, the problem is also solved using the sequential approach where production/distribution decisions are made independent of the raw material decisions. Thus, this work covers a novel area of research that has not been studied extensively and, consequently, could open new opportunities for improvement.

The remainder of this chapter is organized as follows. Section 4.2 presents a general description of the DJRP, providing the general formulation for the problem. Section 4.3 describes the model formulation for the problem, including the suggested integrated approach and also an alternative formulation for the sequential method. Section 4.4 describes the illustrative example used in this chapter. Section 4.5 presents the computational results obtained from the analysis, including the results for the sequential and integrated approaches. Section 4.6 provides a sensitivity analysis for different levels of ordering and holding costs. Finally Section 4.7 provides the conclusions of this research and new opportunities that have been identified as extensions of this work.

4.2 The Dynamic Joint Replenishment Problem

Even though joint replenishment operations are commonly used in practice, most inventory control theories have focused on situations involving a single product (Goyal and Satir, 1989). In manufacturing, the practice of jointly replenishing multiple products on a single machine has grown with the utilization of flexible machine tools. For
example, Goyal and Satir (1989) mention that when a product is packaged into more than one type of container immediately after production, cost savings are realized when these products are jointly manufactured and then individually packaged. In this case, each product is accountable for a major manufacturing setup cost and an individual packaging setup cost. Chen and Chen (2005) mention that in the automotive industry, it is not uncommon for a supplier to produce several products for a single customer. In these cases, parts manufacturing is conducted in a single manufacturing facility, using the same distribution network and transportation modes for delivery. Robinson and Lawrence (2008) present an application of the DJRP to a chemical company. The firm produces a variety of lubricants that are characterized based on their chemical compositions and package configurations. The manufacturing process considers a major setup cost related to a common operation where raw materials are blended and cooked. After the ingredients are cooled down, they are milled into storage kettles which feed a packaging line. Even though different products can be produced from a single batch of lubricant, minor setup operations are needed to recalibrate the packaging line and change levels every time products are switched. The authors also mention that the delivery of vaccines from manufacturing warehouses to distribution centers is another example of the DJRP. In this case, given quality and security issues, each replenishment shipment from the manufacturing warehouse to a distribution center requires a dedicated refrigerated truck (major setup cost). In addition, each product incurs a minor setup cost related to product labeling, packaging, temperature control, quality inspection, lot-size definition for product tracking and quality control, and forms for the Food and Drug Administration.

In the DJRP, \( n_p \) products must be replenished to satisfy a deterministic dynamic demand over a planning horizon of \( n_T \) time periods. In this context, \( T = \{1, 2, ..., n_T\} \) defines the set of planning periods, and \( P = \{1, 2, ..., n_P\} \) the set of products under consideration. Ordering costs are assumed to be comprised of a major ordering cost and several minor ordering costs. A major ordering cost is incurred each time products are jointly replenished, where the respective cost at period \( t \) is denoted by \( s^t \). On the other hand, a minor ordering cost, denoted by \( s^{p,t} \), is charged each time a product \( p \) is included in an order placed during period \( t \). In this context, \( p^{p,t} \) and \( Q^{p,t} \) represent the price and
order quantity of product $p$ in period $t$. Figure 4.1 provides a graphical representation of the DJRP cost structure for $n_p$ products in period $t$. In this example, the major ordering cost is shared among all products in the order, while each individual product has respective minor and variable ordering costs associated with it.

**Figure 4.1** The Dynamic Joint Replenishment Problem (DJRP) Cost Structure

In the DJRP problem, the demand of product $p$ is assumed to be known for each of the $n_T$ time periods during the planning horizon. Let $d_{p,t}$ represent the demand of product $p$ in period $t$. Inventory holding costs are assumed to be linear, where $h_{p,t}$ denotes the cost per unit of holding product $p$ during inventory in period $t$. The DJRP consists of determining the optimal order quantity $Q_{p,t}$ and inventory level $I_{p,t}$ for each product $p$ and period $t$ during the planning horizon, where $I_{p,0}$ represents the initial inventory of product $p$. Although Boctor et al. (2004) did not consider variable purchasing costs, their DJRP model can be generalized as follows:
In this formulation, purchasing, and inventory holding costs are minimized while satisfying all time demands. Equation (4.1) represents the objective function, which is minimized during the planning horizon. The first term represents the major setup cost which is paid every time an order is placed. The second term corresponds to the minor setup cost which is associated with product \( p \). The third term represents the variable purchasing cost. Notice that if the price is constant (time independent) for each product, the variable purchasing cost will be constant, and consequently can be avoided in the analysis. Finally, the last term in the objective function represents the inventory holding cost, where unitary holding costs are assumed to depend on the time period. Equation set (4.2) guarantees that demand is completely satisfied for each time period. Constraint set (4.3) is used to trigger the binary variable \( \delta_{p,t} \), which is active only if product \( p \) is included in the order placed in period \( t \). In this equation, \( B \) represents a sufficiently large number to be used as the upper bound for the order placed at time \( t \). Notice that this constraint can be tightened by setting \( B \) as a dependent parameter of the product and time period, such that \( B_{p,t} = \sum_{r=t}^{n_t} d_{p,r} \). Constraint set (4.4) is defined to trigger the binary variable \( \delta^t \), which is active during any time period in which an order is placed, independent of the products included in the order. Notice that \( |P| \) represents the
cardinality of the set $P$, which is equivalent to the maximum number of binary variables $\delta^{P,t}$ that can be activated in any time period $t$. Finally, Constraint sets (4.5) and (4.6) describe the nature of the variables considered in the model.

4.3 Dynamic Multiproduct Inventory Planning Model in a Serial Supply Chain

A dynamic lot-sizing model with supplier selection in a serial supply chain structure is presented in this section. Let $J = \{1, 2, \ldots, n_J\}$ be the set of suppliers, $K = \{1, 2, \ldots, n_K\}$ be the set of supply chain stages, $S = \{1, 2, \ldots, n_S\}$ be the set of customers, and $M = \{1, 2, \ldots, n_M\}$ be the set of raw materials. In this context, stages 1 and 2 represent the raw material and finished product warehouses at the manufacturing facility, $K_D = \{2, 3, 4, \ldots, n_K - 1\}$ is the set of intermediate warehousing/distribution stages that hold finished products (including the warehouse at stage 2), and $n_K$ is the last distribution stage which ships products to customers (see Figure 3.1). Let $l_{0,j}$ be the number of delivery lead time periods from supplier $j \in J$ to stage 1, and $l_k$ be the number of delivery lead time periods from stage $k \in K_D$ to stage $k + 1$. Finally, let $J_t = \{j \in J : t - l_{0,j} \geq 0\}$ be the subset of suppliers that can provide raw materials at period $t$. Note that at time $t$ only a supplier with a lead time less than $t$ can supply raw material to stage 1.

Similar to the case of the simple product presented in Section 3.2, the dynamic serial supply chain with multiple suppliers and customers can be modeled as a general transshipment network $G_D = (N_D, A_D)$, where the set of nodes $N_D$ and the set of arcs $A_D$ are defined as follows (see Figure 3.1):

\[
N_D = J \cup K \cup S,
\]

and

\[
A_D = \{(j, 1): j \in J\} \cup \{(k, k + 1): k \in K / \{n_K\}\} \cup \{(n_K, s) : s \in S\}.
\]

Although it is assumed that there is only one manufacturing facility, the model can be easily extended to a multi-stage manufacturing supply chain, where multiple
manufacturing processes are sequentially performed. Kaminsky and Simchi-Levi (2003) describe an example from the pharmaceutical industry, where products are manufactured sequentially at several different plants. Notice that the external demand for product \( p \in P \) from customer \( s \in S \) can be aggregated by consolidating the set of customers into a single demand node \( n_{K+1} \), where \( d_{s}^{p,t} = \sum_{s \in S} d_{s}^{p,t} \) represents the external demand in units of product \( p \in P \) for all customers at period \( t \in T \). In this case, the distribution process from stage \( n_{K} \) to customers for each period \( t \in T \) can be treated as a separate problem that can be solved independently.

Considering that the dynamic supply chain represented by network \( G_D \) does not show the effects of changes in demand between consecutive time periods, a (time-expanded) static supply chain network is defined and represented by the general transshipment network \( G_S = (N_S, A_S) \). In this network, the set of nodes \( N_S \) and the set of arcs \( A_S \) are defined as follows:

\[
N_S = \{(0,j,t): j \in J, t \in T\} \cup \{(k,t): k \in K, t \in T\},
\]

and

\[
A_S = \left\{ \left( (0,j,t), (1,t + l_{0,j}) \right): j \in J, t \in \{1,2,\ldots,n_T - l_{0,j}\} \right\} \cup \left\{ \left( (k,t), (k, t + 1): k \in K \setminus nK, t \in 1,2,\ldots,n_T - lk \right) \right\}.
\]

The raw material flow from supplier \( j \) in period \( t \) is represented as a directed arc connecting nodes \( (0,j,t) \) and \( (1, t + l_{0,j}) \), where \( Q_{0,j}^{m,t} \) is the corresponding order lot size of raw material \( m \) ordered at period \( t \) from supplier \( j \). Likewise, the inventory flow between consecutive time periods \( t \) and \( t+1 \) at stage \( k \) is represented as a directed arc connecting nodes \( (k,t) \) and \( (k, t+1) \), where the inventory level of raw material \( m \) at the manufacturing stage (stage 1) and the inventory level of finished product \( p \) at stage \( k \), \( k \in K_D \cup \{n_K\} \), from period \( t \) to period \( t+1 \) are respectively represented by \( m_{1}^{m,t} \) and \( m_{k}^{p,t} \). The product flow between consecutive stages \( k \) and \( k+1 \) is represented by a
directed arc that connects nodes \((k, t)\) and \((k + 1, t + l_k)\), where the manufacturing order lot size at the manufacturing stage and the shipment order lot size for product \(p\) at stage \(k\) from period \(t\) to period \(t + 1\) are respectively represented by \(X_{1}^{p,t}\) and \(Y_{k}^{p,t}\). In this scenario, all initial inventories and pending orders at every stage \(k\) are assumed to be available at the beginning of the first period. Pending orders, initial and ending inventories of raw material \(m\) are respectively represented by \(Q_{0,j}^{m,0}, I_{1}^{m,0}\) and \(I_{1}^{m,n_T}\).

Similarly, pending orders, initial and ending inventories of finished product \(p\) at stage \(k\) are respectively represented by \(Y_{k}^{p,0}, I_{k}^{p,0}\) and \(I_{k}^{p,n_T}\), \(k \in K_D \cup \{n_K\}\). Note that if there is no initial inventory, then \(I_{1}^{m,0}\) and \(I_{k}^{p,0}\) must be set to zero; otherwise they will be left as free variables without associated costs, and consequently the model will set these variables to be equal to the required volume during the planning horizon.

An illustration of a static supply chain network with three suppliers, three stages, and a planning horizon of five periods is presented in Figure 4.2. In this supply chain, two raw materials (1 and 2) are required to manufacture two finished products (1 and 2). Lead times are positive for suppliers 2 and 3, and stage 2, where orders are delivered one time unit (period) after they are submitted. Customer demand for product \(p\) in period \(t\) is defined by \(d_{p,t}\). Notice that in this illustration, given the delivery lead times defined for raw material suppliers, only supplier 1 can satisfy the order quantity \(Q_{0,1}^{m,t}\) in the same period in which it is requested. Similarly, among all the stages, only stage 1 (manufacturing stage) can fulfill orders in the same period in which they are requested. Consequently, due to positive delivery lead times, raw materials and products need to be ordered in advance or maintained in inventory in order to satisfy the demand observed at each time period. In the graphical illustration of the supply chain shown in Figure 4.2, flows are represented as two-dimensional vectors, indicating flows for raw materials (1 and 2) or finished products (1 and 2).
Figure 4.2  A (Time-Expanded) Static Supply Chain Network with Three Suppliers, Three stages, and Five Periods

Assuming that trucks are the primary means of transportation, TL and LTL options are considered as the two transportation modes available for raw materials and finished products. In this case, depending on the supply chain requirements, one or two transportation alternatives could be potentially selected during one or more time periods in the planning horizon. For the TL mode, shipping costs are assumed to be fixed for each route, while the all-units discount cost structure is assumed to apply to the LTL
mode. In particular, considering that the all-units discount transportation cost structure is widely used to stimulate demand for larger and more profitable shipments, it is applicable to the LTL transportation mode. Even though LTL rates are generally stated per hundredweight ($/CWT), in order to simplify our analysis, this chapter assumes that LTL rates are stated by unit ($/unit) per each origin-destination route, which are defined according to the shipment size of raw materials or finished products (see Figure 3.7). On the other hand, TL rates are represented by a fixed cost ($) for each origin-destination. In this context, following an approach similar to the one developed by Balarkrishnan and Graves (1989), and Simchi-Levi et al. (2005), the LTL transportation cost structure is modeled using a piecewise linear cost function.

Similar to Section 3.5, let \( R_{0j} = \{1, 2, ..., n_{R_{0j}}\} \) be the set of freight ranges for the transportation cost function from supplier \( j \) to stage 1. Similarly, let \( R_k = \{1, 2, ..., n_{R_k}\} \) be the set of freight ranges for the transportation cost between stages \( k \) and \( k+1 \). Both sets, \( R_{0j} \) and \( R_k \), can be partitioned into the respective freight range sets of fixed charge \( (R_{0j}^f \text{ and } R_k^f) \) and constant charge per unit \( (R_{0j}^c \text{ and } R_k^c) \), such that \( R_{0j} = R_{0j}^f \cup R_{0j}^c \) and \( R_k = R_k^f \cup R_k^c \). In this context, let \( \beta_{0,j}^{r-1} \) and \( \beta_{0,j}^r \) denote the lower and upper limits for range \( r \) on the transportation cost function associated with supplier \( j \). Similarly, let \( \beta_k^{r-1} \) and \( \beta_k^r \) denote the respective lower and upper limits for range \( r \) on the cost function associated with transportation from stage \( k-1 \) to stage \( k \). Note that rates alternate between ranges of a constant charge per unit followed by a fixed charge, where \( \beta_{0,j}^0 = \beta_k^0 = 0 \) (see Figure 4.3). The fixed charge is the result of over-declaring an LTL shipment to the next LTL transportation range.

Note that in Figure 4.3, shipment size is defined in equivalent units. Given that shipments may include different types of raw materials or finished products, the original units need to be transformed into equivalent units to correctly account for truck capacities and transportation costs for the LTL transportation mode. This transformation is also required in order to properly account for inventory capacities of warehouses and distribution centers, where more than one type of raw material or finished product may be held simultaneously at the same location.
Depending on the application, equivalent units can be defined in terms of weight, volume or even area. In international transportation, a common measure is the twenty-foot equivalent unit (TEU), which corresponds to the volume of a 20-foot long intermodal container. In this context, containers are metal boxes which can be easily transferred between modes of transportation, such as ships, trains, and trucks. According to Swenseth and Godfrey (2002), when trucks are the main transportation means, LTL rates are commonly expressed per hundredweight (CWT), where CWT represents exactly 100 pounds. In military applications, transportation capacities are usually measured in more than one dimension, including volume, area, and weight units. In some retail companies where weight is not relevant, transportation and inventory capacities are usually measured in volume. For example, products can be counted in pallets, where the equivalent factor represents the number of boxes that are contained on each pallet (Cintron, 2010). An example of equivalent unit transformation is presented in Table 4.1. In this illustration, four different finished products are aggregated in a common order, where each product has a different equivalent factor. Thus, if equivalent units represent the volume occupied by each product, product 1 would be the smallest product and...
product 2 would be the largest product included in the order. Consequently, by multiplying the number of units for each product by its respective equivalent factor, all the products are transformed to equivalent commensurable units.

<table>
<thead>
<tr>
<th>Table 4.1</th>
<th>Example of Aggregated Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product</td>
<td>Units</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
</tr>
<tr>
<td>Total</td>
<td>55</td>
</tr>
</tbody>
</table>

The multiproduct dynamic inventory problem with supplier selection in a serial supply chain structure can be formulated as a MILP problem. Note that based on Theorem 3.1 and Corollary 3.1.1 (see Chapter 3), the static (time-expanded) supply chain network can be reduced by defining the feasible nodes in the network. According to this theorem and corollary, nodes \((0, j, t_1)\) and \((k, t_2)\) will be feasible if and only if \(t_1 \in T_{0,j} \) and \(t_2 \in T_k\), where,

\[
T_k = \left\{ t : 1 + \sum_{k' = m_k}^{k-1} l_{k'} \leq t \leq n_T - \sum_{k' = k}^{m_{k} - 1} l_{k'}, t \in T \right\}, \quad k \in K,
\]

and

\[
T_{0,j} = \left\{ t : 1 \leq t \leq \max\{t' \in T_1\} - l_{0,j} \right\}, \quad j \in J,
\]

where

\[
m_k = \begin{cases} 
\max\{k'\}, & \text{if } \sum_{p \in P} (y_{k',-1}^{p,0} + p_{k',0}^0) > 0, k' \in \{2, \ldots, k\}, \\
1, & \text{if } \sum_{m \in M} \left( \sum_{j \in J} Q_{0,j}^{m,0} + l_0^m \right) > 0, \sum_{p \in P} (y_{k',-1}^{p,0} + p_{k',0}^0) = 0, k' \in \{2, \ldots, k\}, \\
0, & \text{otherwise},
\end{cases}
\]
The remaining parameters, variables and formulation for the MILP problem are described as follows:

\[ m_i^* = \begin{cases} 
1, & \sum_{m \in M} I_{1}^{m,n_T} > 0, \\
\min\{k'\}, & \sum_{p \in P} I_{k'}^{p,n_T} > 0, k' \in \{k + 1, \ldots, n_K - 1\}, \sum_{m \in M} I_{1}^{m,n_T} = 0, \\
n_K, & \text{otherwise},
\end{cases} \]

\[ m_k^* = \begin{cases} 
k, & \text{if } k \neq 1, \sum_{p \in P} I_{k}^{p,n_T} > 0, \\
\min\{k'\}, & \text{if } k \neq 1, \sum_{p \in P} I_{k'}^{p,n_T} > 0, k' \in \{k + 1, \ldots, n_K - 1\}, \sum_{p \in P} I_{k}^{p,n_T} = 0, \\
n_K, & \text{otherwise},
\end{cases} \]

and

\[ l_0 = \min\{l_{0j} : j \in J\}. \]

Note that \( m_k \) is the closest preceding stage with a positive initial inventory or a pending order for at least one raw material or finished product. In this case, \( m_k \) will be equal to \( \max\{k'\} \) if there is a stage \( k' \in \{2, \ldots, k\} \) with a positive initial inventory or a pending order for at least one product. Otherwise, \( m_k \) will be defined as 1 if stage 1 has a positive initial inventory or pending order, or 0 if any of the preceding conditions are satisfied. Similarly, \( m_k^* \) represents the closest stage following stage \( k \) with a positive ending inventory for at least one product. Note that two cases are separately defined for \( m_k^* \); in the first case \( m_1^* \) provides the respective expression for \( k = 1 \), and the second expression presents the respective value for \( m_k^* \), when \( k = \{2, \ldots, n_K\} \).

Additional Parameters

\( m^p_m \) : number of units of raw material \( m \) required to produce one unit of finished product \( p \), \( m \in M_p \), \( p \in P \), where \( M_p \) is the set of raw materials used to produce finished product \( p \).
$w_{raw}^m$ : number of equivalent raw material units corresponding to one unit of raw material $m, m \in M$.

$w_f^p$ : number of equivalent finished product units corresponding to one unit of finished product $p, p \in P$.

$r^p$ : production rate (in units/time unit) for product $p, p \in P$.

$u_{0,j}^{m,t}$ : capacity of supplier $j$ for raw material $m$ at time $t$ in units, $j \in J, m \in M, t \in T_{0,j}$.

$b_1^t$ : effective production capacity (in time units) at stage 1 in period $t, t \in T_1$.

$b_{inv,t}^k$ : inventory capacity (in equivalent units) at stage $k$ in period $t, k \in K, t \in T_k$.

Note that $k = 1$ represents the inventory capacity of raw materials, and $k = 2, \ldots, n_K$ indicates the inventory capacity of finished products.

$b_m^p$ : bill of materials ratio. It is defined as the number of units of raw material $m$ required to produce one unit of finished product $p, m \in M, p \in P$.

$a_{0,j}^m$ : perfect rate of supplier $j$ for raw material $m$ (probability that a unit is acceptable), $j \in J, m \in M$.

$a^m$ : minimum acceptable perfect rate for raw material $m, m \in M$.

$h_1^{m,t}$ : unit holding cost for raw material $m$ at the manufacturing stage from period $t$ to period $t+1, m \in M, t \in T_1$.

$h_k^{p,t}$ : unit holding cost for finished product $p$ at stage $k$ from period $t$ to period $t+1, p \in P, k \in K_D \cup \{n_K\}, t \in T_k$.

$u_{1,j}^{m,t}$ : holding cost for in-transit inventory of raw material $m$ shipped from supplier $j$ (stage 0) to stage 1 for periods $t$ to $t + l_{0j} - 1, j \in J, m \in M, t \in T_{0,j}$.

$u_{k+1}^{p,t}$ : holding cost for in-transit inventory of finished product $p$ shipped from stage $k$ to stage $k+1$ for periods $t$ to $t + l_k - 1, k \in K_D, p \in P, t \in T_k$.

$p_{0,j}^{m,t}$ : unit price of raw material $m$ for supplier $j$ in period $t, j \in J, m \in M, t \in T_{0,j}$.

$s_{0,j}^t$ : major ordering cost for each order submitted to supplier $j$ in period $t, j \in J, t \in T_{0,j}$.
minor ordering cost for raw material \( m \) included in the order submitted to supplier \( j \) in period \( t, j \in J, m \in M, t \in T_{0,j} \).

\( p_{1}^{p,t} \) : variable production cost for finished product \( p \) in period \( t, p \in P, t \in T_{1} \).

\( s_{1}^{t} \) : major ordering cost at the manufacturing stage during period \( t, t \in T_{1} \).

\( s_{1}^{p,t} \) : minor ordering cost for each finished product \( p \) manufactured at stage 1 in period \( t, p \in P, t \in T_{k} \).

\( s_{k}^{t} \) : major ordering cost at stage \( k \) during period \( t, k \in K_{p}, t \in T_{k} \).

\( v_{0,j}^{t,r} \) : fixed charge for the actual freight rate from supplier \( j \) to stage 1, when the aggregated order quantity (in equivalent units) in period \( t \) falls in range \( r, j \in J, t \in T_{0,j}, r \in R_{0,j}^{f} \).

\( h_{0,j}^{t} \) : fixed charge for a full truck load from supplier \( j \) to stage 1. It is equivalent to \( v_{0,j}^{t,n_{R_{0,j}}} \), which is the actual fixed charge rate defined for the last range of \( R_{0,j}^{f} \), \( j \in J, t \in T_{0,j} \).

\( e_{0,j}^{t,r} \) : variable unit charge for the actual freight rate from supplier \( j \) to stage 1, when the aggregated order quantity (in equivalent units) in period \( t \) falls in range \( r, j \in J, t \in T_{0,j}, r \in R_{0,j}^{c} \).

\( v_{k}^{t,r} \) : fixed charge for the actual freight rate from stage \( k \) to stage \( k+1 \) when the aggregated order quantity (in equivalent units) in period \( t \) falls in range \( r, k \in K_{p}, t \in T_{k}, r \in R_{k}^{f} \).

\( h_{k}^{t} \) : fixed charge rate for a full truck load from stage \( k \) to stage \( k+1 \). It is equivalent to \( v_{k}^{t,n_{R_{k}}} \), which is the actual fixed charge rate defined for the last range of \( R_{k}^{f} \), \( k \in K_{p}, t \in T_{k} \).

\( e_{k}^{t,r} \) : variable unit charge for the actual freight rate from stage \( k \) to stage \( k+1 \) , when the aggregated order quantity (in equivalent units) in period \( t \) falls in range \( r, k \in K_{p}, t \in T_{k}, r \in R_{k}^{c} \).

\( f_{0,j}^{t} \) : upper bound for transportation range \( r \) (in equivalent units) on the LTL transportation mode from supplier \( j \) to stage 1, \( j \in J, r \in R_{0,j} \).
$\beta^r_k$: upper bound for transportation range $r$ (in equivalent units) on the LTL transportation mode used from stage $k$ to stage $k+1$, $k \in K_D, r \in R_k$.

**Continuous Variables**

$Q_{L0,j}^{t,r}$: portion of the aggregated order quantity (in equivalent units) transported using the LTL mode from supplier $j$ to stage 1 in period $t$. It assumes a positive value when the replenishment order quantity falls in transportation range $r$, $j \in J$, $t \in T_{0,j}, r \in R_{0,j}$.

$Y_{Lk}^{t,r}$: portion of the aggregated order quantity (in equivalent units) transported using the LTL mode from stage $k$ to stage $k+1$ in period $t$. It assumes a positive value when the replenishment order quantity for the LTL mode falls in transportation range $r$, $k \in K_D, t \in T_k, r \in R_k$.

**Integer Variables**

$L_{0,j}^t$: number of full truck loads assigned to transport raw material from supplier $j$ to stage 1 in period $t$, $j \in J$, $t \in T_{0,j}$.

$L_k^t$: number of full truck loads assigned to transport finished product from stage $k$ to stage $k+1$ in period $t$, $k \in K_D, t \in T_k$.

**Binary Variables**

$\delta_{0,j}^t$: 1 if a replenishment order is submitted to supplier $j$ in period $t$; otherwise, 0; $j \in J, t \in T_{0,j}$.

$\delta_{0,j}^{m,t}$: 1 if a replenishment order for raw material $m$ is submitted to supplier $j$ in period $t$; otherwise, 0; $j \in J, m \in M, t \in T_{0,j}$.

$\delta_k^t$: 1 if a replenishment order is placed to stage $k$ in period $t$; otherwise, 0; $k \in k \{n_K\}, t \in T_k$.

$\delta_1^{p,t}$: 1 if a manufacturing order for finished product $p$ is placed to stage 1 in period $t$; otherwise, 0; $p \in P, t \in T_k$. 
\( \varphi_{L0,j}^{t,r} \): 1 if the aggregated LTL replenishment order quantity of raw materials (in equivalent units) sent from supplier \( j \) to stage 1 in period \( t \) falls in transportation range \( r \); otherwise, 0; \( j \in J, t \in T_{0,j}, r \in R_{0,j} \).

\( \varphi_{Lk}^{t,r} \): 1 if the aggregated LTL replenishment order quantity of finished products (in equivalent units) sent from stage \( k \) to stage \( k+1 \) in period \( t \) falls in transportation range \( r \); otherwise, 0; \( k \in K_D, t \in T_k, r \in R_k \).

Assuming free-on-board (FOB) shipping, and following the description presented in Hughes Networks Systems (2010), the buyer (manufacturer or distribution center) pays the freight charges and also owns the goods in-transit. Thus, the in-transit inventory should also be reflected in the total inventory per time unit held by the buyer. Notice that in-transit holding costs are directly related to delivery lead times. Hence, in-transit holding cost functions for raw materials and finished products can be represented by Equations (4.7) and (4.8), respectively:

\[ u_{1,j}^{m,t} \left( Q_{0,j}^{m,t} \right) = \sum_{t'=t}^{t+l_0-1} h_1^{m,t'} Q_{0,j}^{m,t'} = u_{1,j}^{m,t} Q_{0,j}^{m,t}, \]  
(4.7)

and

\[ u_{k+1}^{p,t} \left( y_k^{p,t} \right) = \sum_{t'=t}^{t+l_k-1} h_{k+1}^{p,t'} y_k^{p,t'} = u_{k+1}^{p,t} y_k^{p,t}, \]  
(4.8)

where, \( u_{1,j}^{m,t} = \sum_{t'=t}^{t+l_0-1} h_1^{m,t'} \) and \( u_{k}^{p,t} = \sum_{t'=t}^{t+l_k-1} h_{k+1}^{p,t'} \) represent unit holding costs for in-transit inventory from supplier \( j \) to stage 1 and from stage \( k \) to stage \( k+1 \), respectively.

### 4.3.1 The Integrated Model Approach

A MILP model is proposed to solve the dynamic inventory problem with supplier selection. The problem is solved using an integrated approach, where raw material
procurement, manufacturing, and distribution decisions are simultaneously considered. The objective function for the model minimizes the total variable cost over \(n_T\) planning periods, including raw material purchasing, manufacturing, holding, and transportation costs. The formulation of the MILP model is presented below:

\[ \text{(P4.1)} \]

\[
\text{Minimize } Z = \sum_{j \in J} \sum_{t \in T_{0,j}} \left( s_{0,j}^{t} o_{0,j}^{t} + \sum_{m \in M} \left( s_{0,j}^{m,t} o_{0,j}^{m,t} + p_{0,j}^{m,t} c_{0,j}^{m,t} \right) \right) \\
+ \sum_{t \in T_1} \left( s_{1}^{t} o_{1}^{t} + \sum_{p \in P} \left( s_{1}^{p,t} o_{1}^{p,t} + p_{1}^{p,t} x_{1}^{p,t} \right) \right) + \sum_{k \in K_D} \sum_{t \in T_k} s_{k}^{t} b_{k}^{t} \\
+ \sum_{m \in M} \sum_{t \in T_1} h_{1}^{m,t} I_{1}^{m,t} + \sum_{p \in P} \sum_{k \in K_D} \sum_{t \in T_k} h_{k}^{p,t} I_{k}^{p,t} + \sum_{j \in J} \sum_{m \in M} \sum_{t \in T_{0,j}} u_{1,j}^{m,t} q_{0,j}^{m,t} \\
+ \sum_{k \in K_D} \sum_{p \in P} \sum_{t \in T_k} u_{k+1}^{p,t} Y_{k}^{p,t} \\
+ \sum_{j \in J} \sum_{t \in T_{0,j}} \left( \varphi_{0,j}^{t} L_{0,j}^{t} + \sum_{r \in R_{0,j}} \varphi_{L0,j}^{t} Q_{L0,j}^{r} + \sum_{r \in R_{0,j}} \epsilon_{0,j}^{t} Q_{L0,j}^{r} \right) \\
+ \sum_{k \in K_D} \sum_{t \in T_k} \left( \varphi_{k}^{t} L_{k}^{t} + \sum_{r \in R_{k}} \varphi_{Lk}^{t} Q_{Lk}^{r} + \sum_{r \in R_{k}} \epsilon_{k}^{t} Q_{Lk}^{r} \right), \quad (4.9) \]

subject to

\[
\sum_{j \in J_t} Q_{0,j}^{m,t-1} + I_{1}^{m,t-1} = I_{1}^{m,t} + \sum_{p \in P} b_{m}^{p} x_{1}^{p,t}, \quad m \in M, t \in T_1, \quad (4.10) \]

\[
X_{1}^{p,t} + I_{2}^{t-1} = I_{2}^{t} + Y_{2}^{t}, \quad p \in P, t \in T_2, \quad (4.11) \]

\[
Y_{k-1}^{p,t-1} + I_{k}^{t-1} = I_{k}^{t} + Y_{k}^{t}, \quad p \in P, k \in K_D \setminus \{2\}, t \in T_k, \quad (4.12) \]

\[
Y_{nK-1}^{t-1} + I_{nK}^{t-1} = d_{p}^{t} + I_{nK}^{t}, \quad p \in P, t \in T_{nK}, \quad (4.13) \]
\begin{align}
\sum_{j \in J_t} a_{0j}^m Q_{0j}^{m,t-l_{0j}} & \geq a^m \sum_{j \in J_t} Q_{0j}^{m,t-l_{0j}}, & m \in M, t \in T_1, & (4.14) \\
Q_{0j}^{m,t} & \leq b_{0j}^{m,t} \delta_{0j}^{m,t}, & m \in M, j \in J, t \in T_{0j}, & (4.15) \\
\sum_{m \in M} \delta_{0j}^{m,t} & \leq |M| \delta_{0j}^t, & j \in J, t \in T_{0j}, & (4.16) \\
X_1^{p,t} & \leq B \delta_1^{p,t}, & p \in P, t \in T_1, & (4.17) \\
\sum_{p \in P} X_1^{p,t} & \leq b_1^t \delta_1^t, & t \in T_1, & (4.18) \\
\sum_{p \in P} Y_k^{p,t} & \leq B \delta_k^t, & k \in K_D, t \in T_k, & (4.19) \\
\sum_{m \in M} w_{raw}^{m,t} l_1^{m,t} & \leq b_1^{inv,t}, & t \in T_1, & (4.20) \\
\sum_{p \in P} w_f^{p,t} l_k^{p,t} & \leq b_k^{inv,t}, & k \in K_D \cup \{n\}, t \in T_k, & (4.21) \\
\sum_{m \in M} w_{raw}^{m,t} Q_{0j}^{m,t} & = \beta_{0j}^{nR_{0j}} L_0^{t,j} + \sum_{r \in R} Q_{L0,j}^{r,t}, & j \in J, t \in T_{0j}, & (4.22) \\
\sum_{p \in P} w_f^{p,t} Y_k^{p,t} & = \beta_{R_{0j}}^{nR_{0j}} L_k^{t} + \sum_{r \in R} Y_L^{r,t}, & k \in K_D, t \in T_k, & (4.23) \\
Q_{L0,j}^{r,t} & \leq \beta_{0j}^{r} \varphi_{L0,j}^{r,t}, & j \in J, t \in T_{0j}, r \in R_{0j}, & (4.24) \\
Q_{L0,j}^{r,t} & \geq \beta_{0j}^{r-1} \varphi_{L0,j}^{r,t}, & j \in J, t \in T_{0j}, r \in R_{0j}, & (4.25) \\
\sum_{r \in R_{0j}} \varphi_{L0,j}^{r,t} & \leq 1, & j \in J, t \in T_{0j}, & (4.26) \\
y_{Lk}^{r,t} & \leq \beta_{k}^{r} \varphi_{Lk}^{r,t}, & k \in K_D, t \in T_k, r \in R_k, & (4.27) \\
y_{Lk}^{r,t} & \geq \beta_{k}^{r-1} \varphi_{Lk}^{r,t}, & k \in K_D, t \in T_k, r \in R_k, & (4.28) \\
\sum_{r \in R_k} \varphi_{Lk}^{r,t} & \leq 1, & k \in K_D, t \in T_k, & (4.29) \\
Q_{0j}^{m,t} & \geq 0, \delta_{0j}^{m,t} \in \{0,1\}, & j \in J, m \in M, t \in T_{0j}, & (4.30) \\
Q_{L0,j}^{r,t} & \geq 0, \varphi_{L0,j}^{r,t} \in \{0,1\}, & j \in J, m \in M, t \in T_{0j}, r \in R_{0j}, & (4.31) \\
X_1^{p,t} & \geq 0, & p \in P, t \in T_1, & (4.32) \\
l_1^{m,t} & \geq 0, & m \in M, t \in T_1, & (4.33) 
\end{align}
In this formulation, the total cost is minimized while satisfying all time demands. Equation (4.9) represents the total variable cost function which includes purchasing, manufacturing, inventory, and transportation costs for \( n_T \) periods. Notice that no transportation cost is considered between stages 1 and 2, since all flows occur internally in the manufacturing facility. Equation set (4.10) guarantees flow balance between raw materials and finished products at the manufacturing stage. In these equations, in order to correctly calculate the raw material requirement for each unit of finished product, the manufacturing lot size of product \( p \) at the beginning of period \( t \), defined by \( X^p_{1t} \), is multiplied by the respective bill of materials (BOM) ratio \( b^p_m \). Equation set (4.11) defines the product balance at stage 2, where production is directly received from the manufacturing stage. Notice that since stages 1 and 2 are assumed to be in the same location, delivery lead times between these stages are assumed to be zero. Equation set (4.12) guarantees product balance for intermediate stages between stage 2 and the final stage, where customer demand is observed. Equation set (4.13) determines that customer demand must be completely satisfied for each product and time period. Constraint set (4.14) guarantees that the average minimum acceptable perfect rate for each raw material will be satisfied at each period. In this case, given the effect of the delivery lead times, this requirement is imposed at the moment a raw material is received at the manufacturing stage. Constraint set (4.15) guarantees that the supplier capacity for each type of raw material will be satisfied at each period. This constraint set also activates the binary variable \( \gamma^m_{0j} \), associated with each product included in the specific supplier order. Thus, the minor cost associated with each product can be correctly addressed in the objective function. Constraint set (4.16) is defined as a trigger constraint to activate the binary variable \( \delta_{0j}^t \), which identifies the time period \( t \) when an order is allocated to

\[
\begin{align*}
Y^p_{k,t} &\geq 0, & p \in P, k \in K^D, t \in T^k \\
Y^r_{k,r} &\geq 0, & p \in P, k \in K^D, t \in T^k, r \in R^k, \\
L_{0j}^t &\text{ integer, } & j \in J, t \in T_{0j}, \\
L_k^t &\text{ integer, } & k \in K^D, t \in T^k.
\end{align*}
\]
supplier $j$. Considering that the maximum number of raw materials that can be included in the order is equivalent to the total number of raw materials, the cardinality of set $M$ is used as the upper bound for this constraint. Constraint set (4.17) is used to trigger the binary variable $\delta^p_{n,t}$, which is activated every time an order is received by the manufacturing stage to produce product $p$ during period $t$. Constraint set (4.18) defines the maximum production capacity for each time period. Given that a flexible production line is assumed, then manufacturing capacity is defined as joint capacity, where changeover times between products are assumed to be imperceptible.

Assuming unlimited transportation capacity for finished products from the manufacturing stage and DCs, Constraint set (4.19) is only used as a trigger for the variable $\delta^f_k$, where $B$ represents a big number. Sets of inequalities (4.20) and (4.21) correspond to the inventory capacity for raw materials and finished products for different stages in the supply chain. In those cases, inventory capacity is counted in equivalent units of raw material and finished product. Equation set (4.22) calculates the total transportation requirement for each raw material supplier and time period. The right-hand sides of the equations classify the transportation volume in TL and LTL transportation modes. Equation set (4.23) calculates the total transportation requirements for each stage and time period. The right-hand sides of the equations classify the transportation volume for the TL and LTL transportation modes. Constraint sets (4.24)-(4.26) guarantee that the raw material order quantity for the LTL mode will fall in the corresponding freight range. Notice that the transportation cost functions for the LTL transportation mode are represented by piecewise linear functions, where freight rates alternate between ranges of constant charge per unit and fixed charge. Similarly, Constraint sets (4.27) - (4.29) are used to calculate LTL freight rates for transportation of finished products between stages. Finally, Constraint sets (4.30) - (4.37) describe the nature of the variables considered in the model.
4.3.2 The Sequential Model Approach

Even though the integrated approach has been shown to produce superior solutions (see Chapter 3), the sequential approach is the most common method used in practice. When the problem is sequentially solved, raw material supply decisions are made independent of the manufacturing and distribution decisions. In this approach, the production/distribution inventory planning problem is solved first, generating the raw material requirements for each period during the planning horizon. Based on this requirement, a second problem is solved, from which the optimal raw material procurement strategy is defined. Thus, the sequential approach solves the problem in two independent stages, which are only linked by the raw material requirement for each time period (see Figure 4.4).

In the first stage, where the production/distribution inventory planning problem is analyzed, the optimal solution is obtained using Model (P4.2). This MILP model minimizes the production, inventory, and transportation costs of the supply chain problem. The mathematical model is defined as follows:

\[
(P4.2) \quad \text{Minimize } Z = \sum_{t \in T_1} \left( s^t_1 \delta^t_1 + \sum_{p \in P} \left( s^{p,t}_1 \delta^p_t + p^p_1 X^p_1 \right) \right) + \sum_{k \in K_D} \sum_{t \in T_k} s_k^t \delta^t_k \\
+ \sum_{p \in P} \sum_{k \in K_D \cup \{D\}} \sum_{t \in T_k} h^p_t l^p_t + \sum_{k \in K_D} \sum_{p \in P} \sum_{t \in T_k} u^p_{k+1} l^p_k \\
+ \sum_{k \in K_D} \sum_{t \in T_k} \left( \hat{v}^t_k L^t_k + \sum_{r \in R^t_k} v^r_k q^r_k + \sum_{r \in R^t_k} e^r_k Y^r_k \right), \quad (4.38)
\]

subject to Equations (4.11) – (4.13), (4.17) – (4.19), (4.21), (4.23), (4.27) – (4.29), (4.32) – (4.35), and (4.37)
In this formulation, Equation (4.38) represents the total variable cost function which includes production, inventory, and transportation costs for the $n_T$ periods. Note that since no purchasing costs are considered in the analysis, raw material procurement is assumed to be irrelevant to the production and distribution decisions. Remaining constraints are defined as before.
Once the first stage of the sequential problem has been solved, then the optimal raw material procurement strategy is obtained from a second optimization stage. Setting \( q^{m,t} = \sum_{p \in P} b^p_m \xi_{1}^{p,t} \), where \( \xi_{1}^{p,t} \) is the optimal production level for product \( p \) in period \( t \) generated by Model (P4.2), then the MILP model to solve the raw material procurement problem is defined by Model (P4.3) as follows:

\[
\begin{align*}
\text{(P4.3)} \\
\text{Minimize } Z &= \sum_{j \in J} \sum_{t \in T_{0,j}} \left( s_{0,j}^{t} \delta_{0,j}^{t} + \sum_{m \in M} \left( s_{0,j}^{m,t} \delta_{0,j}^{m,t} + p_{0,j}^{m,t} Q_{0,j}^{m,t} \right) \right) + \sum_{m \in M} \sum_{t \in T_1} h_{1}^{m,t} l_{1}^{m,t} \\
+ \sum_{j \in J} \sum_{m \in M} \sum_{t \in T_{0,j}} u_{1,j}^{m,t} Q_{0,j}^{m,t} \\
+ \sum_{j \in J} \sum_{t \in T_{0,j}} \left( \dot{v}_{0,j}^{t} L_{0,j}^{t} + \sum_{r \in R_{0,j}} v_{0,j}^{t,r} \varphi_{L_{0,j}}^{t,r} + \sum_{r \in R_{0,j}} e_{0,j}^{t,r} Q_{L_{0,j}}^{t,r} \right),
\end{align*}
\]

subject to

\[
\sum_{j \in J} Q_{0,j}^{m,t-1} + l_{1}^{m,t-1} = l_{1}^{m,t} + q^{m,t}, \quad m \in M, t \in T_1,
\]

and Equations (4.14) – (4.16), (4.21), (4.23), (4.25) – (4.27), (4.31) – (4.32), and (4.38).

The objective function of this problem is represented by Equation (4.39), which minimizes raw material procurement costs (ordering, variable and transportation costs), and raw material inventory costs. Equation set (4.40) represents raw material balance. Note that this set of equations is equivalent to (4.10), but \( \sum_{p \in P} b^p_m \xi_{1}^{p,t} \) has been replaced by \( q^{m,t} \). Remaining constraints are defined as before. Notice that this problem is solved as the one-stage warehouse problem, where the optimal ordering policy is defined to satisfy a known deterministic demand.
4.4 Illustrative Example

In this section, a three-stage serial supply chain example is presented to illustrate the model. In this supply chain, three raw material suppliers provide the material requirements to the manufacturing stage. The first two stages are assumed to be located at the manufacturing facility, allowing for storage of both raw materials and finished products. The third stage represents a DC, which directly distributes to customers the deterministic demand for two finished products during a five-period planning horizon. A graphical representation of product demand is presented in Figure 4.5. Note that the product demand shows an increase during periods 2 through 4. This characteristic is usually observed in real world operations, where product demand may show seasonality during some time periods of the planning horizon. For example, it is well known that retail companies usually observe some increase in their sales levels during particular days of the month.

![Customer Demand](image)

**Figure 4.5** Product Demand by Period and Product Type

Production and inventory capacities are presented in Table 4.2. Assuming that both finished products are manufactured in a common production line, production capacities are expressed in hours. Note that the production capacities provided in this table have been already reduced by a factor to compensate for the expected overall setup
times for the corresponding time periods. Production rates are assumed to be 1 unit/hr for both finished products. Production capacities are assumed to change through the planning horizon, representing different levels of resource availability. Conversely, inventory capacities are assumed to be constant for the three stages during the entire planning horizon. In this case, equivalent units are expressed in cubic meters (m3), assuming that storage capacity is mainly limited by the space occupied by raw material and finished products.

<table>
<thead>
<tr>
<th>Period</th>
<th>Prod. Cap. (hrs)</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,000</td>
<td>3,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>2</td>
<td>1,200</td>
<td>3,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>3</td>
<td>800</td>
<td>3,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>4</td>
<td>800</td>
<td>3,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>5</td>
<td>2,000</td>
<td>3,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
</tbody>
</table>

Two types of raw materials are considered as part of the manufacturing process. In general, they could represent either particular materials or a set of raw materials with similar characteristics. Raw material requirements are calculated according to the BOM ratio defined for each product (see Table 4.3). In this context, the BOM ratio represents the number of units of raw material needed to produce one unit of a particular finished product. This rate is usually obtained from historical data, where raw material wastes are also included as part of the material requirements to produce a specific finished product.

<table>
<thead>
<tr>
<th>Product $(p)$</th>
<th>Raw Material (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(p)$</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

While raw materials can only be held at the manufacturing stage, finished products can be stored at both the manufacturing and distribution stages. In this context,
different characteristics among types of raw materials and finished products should be
accounted for when defining storage and transportation capacities. Thus, equivalence
factors are needed to convert original units into equivalent units (m$^3$) of raw material and
finished product (see Table 4.4).

**Table 4.4** Equivalence Factors $w^m_{\text{raw}}$ and $w^n_j$ (expressed in m$^3$/unit)

<table>
<thead>
<tr>
<th>Type</th>
<th>Equivalence Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material</td>
</tr>
<tr>
<td>1</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
</tr>
</tbody>
</table>

At the supplier level, it is assumed that the three suppliers have already been
selected from a larger set of potential suppliers. To do this, the Analytical Hierarchy
Process could have been used, as suggested by Mendoza *et al.* (2008), or any other
suitable method. Considering that the minimum acceptable perfect rate for the two raw
materials is assumed to be $a^m = 0.95$, $m \in M$, it must be guaranteed that the quality of
raw materials received during each time period will be greater than or equal to 0.95 at
each time period during the planning horizon.

Even though it is assumed that all suppliers may be potentially selected to provide
the required raw materials, they do not necessarily have the same characteristics in terms
of ordering and transportation costs, perfect rate, and supply capacity. A description of
the three prescreened suppliers is provided in Table 4.5. Following the concept of the
DJRP, raw material ordering costs are defined by a major component which is incurred
anytime an order is placed with a particular supplier, and a minor ordering component
associated with each product included in the order. Notice that even though supplier 1
appears to be the less competitive option (high ordering and variable costs), it has the
largest supply capacity with a zero delivery lead time to the manufacturing stage. Thus,
under raw material deficit, it could be advantageous to assume the respective extra costs
associated with this supplier in order to promptly procure the required raw materials.
Note that all suppliers are assumed to have acceptable perfect rates (greater than or equal
to 0.95). Therefore, raw materials could be purchased from one or several suppliers.
Production operations are assumed to be subject to the DJRP cost structure. A major fixed cost of $5,000 is incurred every time an order is placed, and additional minor fixed costs of $1,500 and $1,250 are incurred respectively when product 1 and product 2 are included in the order. Distribution operations are assumed to be subject only to a major ordering cost of $4,000, which is incurred every time an order is placed. Positive delivery lead times are only considered at stage 2, where orders take one time period to be transported from stage 2 to stage 3.

Inventory holding costs are assumed to be constant during the planning horizon, only increasing as products are moved downstream in the supply chain. The inventory holding cost for raw material 1 is assumed to be higher than that for raw material 2. Similarly, the inventory holding cost for product 2 is assumed to be higher than the corresponding one for product 1 at both stages 2 and 3 (see Table 4.6).

### Table 4.5 Description of Raw Material Suppliers

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Lead Times</th>
<th>Supplier Capacity</th>
<th>Quality Rate</th>
<th>Ordering Cost</th>
<th>Variable Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Material 1</td>
<td>Material 1</td>
<td>Material 2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>9,000</td>
<td>9,000</td>
<td>0.96</td>
<td>9,000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3,500</td>
<td>3,500</td>
<td>0.95</td>
<td>3,500</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4,000</td>
<td>4,000</td>
<td>0.95</td>
<td>4,000</td>
</tr>
</tbody>
</table>

A graphical representation of the transportation cost functions is presented in Figure 4.6. Note that supplier 1 has the most expensive transportation costs, with substantial differences in relation to the other two alternative suppliers. Even though
supplier 2 is slightly more expensive than supplier 3, the differences in their freight rates do not appear to be significant as are the differences with supplier 2. Among all cases, interplant transportation costs are the less expensive route.

![Figure 4.6 Supplier and Interplant LTL Transportation Costs](image)

Initial and ending inventory levels considered for the problem are defined in Table 4.7. In real world environments, companies operate on longer time horizons than the planning horizon considered for a particular analysis as presented in this chapter. Consequently, to guarantee the continuity of the operations, models must satisfy initial and ending conditions for the system, which are defined as a given requirement for the supply chain.

| Table 4.7 Initial and Ending Inventory Levels Required at Each Stage |
|-------------------|-----------------|----------------|----------------|-----------------|-----------------|-----------------|
| Period            | Stage 1 Material 1 | Stage 1 Material 2 | Stage 2 Product 1 | Stage 2 Product 2 | Stage 3 Product 1 | Stage 3 Product 2 |
|                   |                  |                  |                  |                  |                  |                  |
| Initial           | 50               | 70               | 450              | 500              | 900              | 600              |
| Ending            | 500              | 700              | 200              | 150              | 300              | 400              |
4.5 Computational Results

The illustrative example has been implemented on GAMS 21.7 (Rosenthal, 2007) on a Pentium 4 with 3.40 GHz and 1 GB of RAM. The problem has been solved to optimality, using CPLEX 11.0 provided by GAMS. CPU times required to solve different versions of the problem have not exceed 25 seconds.

The problem has been solved using both sequential and integrated approaches, following the models presented in Section 4.3 of this chapter. In the sequential approach, production/distribution decisions are made independent of the raw material procurement decisions, while in the proposed integrated approach, the entire supply chain is analyzed at once.

4.5.1 Solution for the Sequential Model Approach

Based on the information provided in the illustrative example, the problem has been solved using the sequential approach, where Model (P4.2) is used to find the optimal production/distribution strategy (see Section 4.3.2). Then, based on the optimal production levels and considering the respective BOM ratio for each product (see Table 4.3), the required raw materials are calculated for each time period in the planning horizon. Notice that in the optimal solution for Model (P4.2), production operations are only performed in periods 1 and 5 (see Table 4.8). Note also that the production obtained during period 5 is only used to supply the required ending inventories at the manufacturing facility (stage 2).

Once the raw material requirement levels are known, the problem defined by Model (P4.3) is solved in order to determine the optimal procurement strategy (see Section 4.3). The optimal solution for this problem is provided in Table 4.9. Even though supplier 1 has the highest ordering and variable costs (see Table 4.5), the total raw material procured in period 1 is only assigned to this supplier. Considering the positive delivery lead times of suppliers 2 and 3, supplier 1 is the only alternative available to
satisfy the deficit between raw material requirements and initial inventories. Note that the optimal production/distribution strategy would have been infeasible under a scenario of positive delivery lead times for all suppliers, or with a supplier 1 capacity below the raw material requirements for this period. Raw materials procured in periods 4 and 5 are only assigned to supplier 3, which has the lowest transportation and setup costs.

Table 4.8  Optimal Production and Shipping Strategy for the Sequential Approach

<table>
<thead>
<tr>
<th>Period</th>
<th>Production</th>
<th>Shipping</th>
<th>Raw Material Req.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Product 1</td>
<td>Product 2</td>
<td>Product 1</td>
</tr>
<tr>
<td>1</td>
<td>1,250</td>
<td>2,100</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>150</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>1,450</td>
<td>2,250</td>
<td>1,700</td>
</tr>
<tr>
<td>Average</td>
<td>725</td>
<td>1,125</td>
<td>567</td>
</tr>
</tbody>
</table>

Table 4.9  Optimal Raw Material Procurement Strategy for the Sequential Approach

<table>
<thead>
<tr>
<th>Period</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Material 1</td>
<td>Material 2</td>
</tr>
<tr>
<td>1</td>
<td>8,750</td>
<td>8,125</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>8,750</td>
<td>8,125</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Average</td>
<td>8,750</td>
<td>8,125</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Optimal inventory levels for the sequential approach are presented in Table 4.10. Note that at stage 1, only raw material 2 is held in inventory, and in-transit inventories (values between parentheses) are only concentrated during periods 3 and 4, so that production requirements and raw material ending inventories are satisfied in period 5. This situation is explained by the large production lot sizes defined at the beginning of the planning horizon, where most of the procured raw material is immediately used after its arrival. Once the manufacturing process is completed, both finished products are
mainly held in inventory at stage 2, at which point they are gradually delivered to the last stage. In this case, in-transit inventories at stage 3 are essentially produced by the positive delivery lead time between stages 2 and 3.

**Table 4.10**  Optimal Inventory Levels per Stage for the Sequential Approach.

<table>
<thead>
<tr>
<th>Period</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Product 1</td>
</tr>
<tr>
<td>Initial</td>
<td>50</td>
<td>-</td>
<td>70</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>245</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>245</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>245 (+363)</td>
</tr>
<tr>
<td>4</td>
<td>- (+1,350)</td>
<td>608 (+992)</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>500</td>
<td>-</td>
<td>700</td>
</tr>
</tbody>
</table>

*In-transit inventory levels are presented in parentheses

A graphical representation of the sequential approach solution is presented in Figure 4.7. Note that only material 2 is held in inventory at stage 1, while at the other two stages, both finished products are held in inventory. In this solution, stage 2 maintains the inventory levels for the finished product, which is gradually shipped to the last stage, where customer demand is satisfied.
4.5.2 Solution for the Integrated Model Approach

The optimal production and shipping strategy for the integrated approach is presented in Table 4.11. Unlike the solution for the sequential approach, production operations are mainly concentrated in period 2 instead of period 1. Using this strategy,
production operations are allocated during a period with low production costs, but also when it is possible to use the less expensive supplier alternatives (suppliers 2 and 3).

Table 4.11  Optimal Production and Shipping Strategy for the Integrated Approach

<table>
<thead>
<tr>
<th>Period</th>
<th>Production</th>
<th>Shipping</th>
<th>Raw Material Req.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Product 1</td>
<td>Product 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shipping</td>
<td>Product 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>33</td>
<td>450</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>533</td>
</tr>
<tr>
<td>2</td>
<td>1,250</td>
<td>1,549</td>
<td>350</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>667</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>518</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>800</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>150</td>
<td>-</td>
</tr>
</tbody>
</table>

| Total  | 1,450      | 2,250     | 1,700            |
|        | 2,600      | 9,650     | 8,850            |
| Average| 725        | 563       | 425              |
|        | 650        | 2,413     | 2,213            |

The optimal procurement strategy for the integrated approach is presented in Table 4.12. In this solution, raw material orders are mainly allocated to suppliers 2 and 3, assigning to supplier 1 only a small portion of the volume required in period 1. In this solution, the production plan is defined such that raw material procurement is obtained from the least expensive suppliers. Thus, a tradeoff between procurement and production costs is considered.

Table 4.12  Optimal Raw Material Procurement Strategy for the Integrated Approach

<table>
<thead>
<tr>
<th>Period</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Material 1</td>
<td>Material 2</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>-</td>
<td>3,146</td>
<td>3,427</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

| Total  | 50         | 3,146      | 3,427      | 6,904     | 6,053     | 10,100     | 9,480     |
| Average| 50         | 3,146      | 3,427      | 3,452     | 2,018     | 5,050      | 3,160     |

Optimal inventory levels for the integrated approach are presented in Table 4.13. At stage 1, raw materials are held in inventory only during time period 4 in order to satisfy the production requirements and ending inventory levels. In-transit inventories of raw materials are observed at periods 1, 3, and 4, which are explained by the positive
delivery lead times of suppliers 2 and 3. At stage 2, inventory is held at periods 2 and 3. Observe that in this case, unlike the sequential approach, the initial production is delayed from period 1 to period 2, allowing the use of less expensive suppliers.

Table 4.13 Optimal Inventory Levels per Stage for the Integrated Approach

<table>
<thead>
<tr>
<th>Period</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Product 1</td>
</tr>
<tr>
<td>Initial</td>
<td>50</td>
<td>70</td>
<td>450</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>+7,146</td>
<td>3 (+6,844)</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>900</td>
</tr>
<tr>
<td>3</td>
<td>- (+2,904)</td>
<td>- (+1,048)</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>1,350</td>
<td>12 (+1,588)</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>500</td>
<td>700</td>
<td>200</td>
</tr>
</tbody>
</table>

* In-transit inventory levels are presented in parentheses

A graphical representation of the optimal solution for the integrated approach is presented in Figure 4.8. In this figure, it can be seen that raw materials are only held in inventory during the last periods of the planning horizon in order to satisfy ending inventory requirements. Finished products are held in inventory at stages 2 and 3, with the highest inventory levels at stage 3.
Figure 4.8  Optimal Solution for the Integrated Approach
4.5.3 Comparing the Sequential and Integrated Approaches

Required CUP times for these two approaches are slightly different. While the two subproblems for the sequential approach have a solution time of about 8 seconds, the optimal solution for the integrated approach has a solution time of 13 seconds.

The optimal procurement strategy is significantly impacted by the optimization approach selected. The percentage of volume allocated to each supplier for each type of raw material is graphically represented in Figure 4.9. Graphs (a) and (b) show the supplier percentage allocation for the sequential approach, while graphs (c) and (d) present the supplier percentage allocations for the integrated approach. In the sequential approach solution, the raw material requirement levels defined by the production/distribution problem are mainly concentrated at the beginning of the planning horizon. In turn, when the raw material supplier selection problem is considered, this approach is shown to be inefficient, given its high raw material procurement costs. In the sequential approach, around 87% of raw material 1 and 86% of raw material 2 are allocated to supplier 1. In this solution, given the reduced production costs at the beginning of the planning horizon (see Table 4.6), the optimal solution for the production/distribution problem assigns around 86% and 93% of the total production of products 1 and 2 to the first time period (see Table 4.8). To satisfy the respective raw material requirements, given the positive delivery lead times of suppliers 2 and 3, supplier 1 is the only feasible alternative for the problem. Consequently, the optimal solution for the production/distribution problem forces the selection of the most expensive raw material supplier among the available alternatives. Thus, when the problem is sequentially solved, it is not possible to realize that this decision negatively impacts the procurement strategy. Conversely, when the integrated approach is implemented, the optimal solution considers a production plan where supplier 1 is almost eliminated from the procurement strategy. In this solution, supplier 2 captures around 68% and 64% of the total volume procured of raw materials 1 and 2, while the remaining raw material requirement is allocated to supplier 3.
Figure 4.9 Optimal Allocation of Raw Materials to Suppliers for the Integrated and Sequential Approaches

A graphical representation of the final inventory levels for each time period (expressed in equivalent units) is presented in Figure 4.10. In this figure, it can be seen that the sequential approach tends to maintain higher and more stable inventory levels than the integrated approach. In defining an optimal production/distribution strategy, the sequential approach concentrates high production levels at the beginning of the planning horizon, assuming a positive impact on production and distribution costs. However, this strategy negatively impacts total inventory levels across the entire supply chain. In fact, in the sequential approach, positive inventory levels are observed during all time periods and stages, with the only exception of period 4 at stage 2. At stage 1, the sequential approach solution generates inventory levels of raw material 2 almost constantly during
periods 1 through 3, with a clear increase after period 4. However, in the integrated approach solution, only positive inventory levels are observed at the beginning and end of the planning horizon. A similar situation is also observed at stage 2, where the integrated approach outperforms the sequential method with lower inventory levels. Finally, in stage 3, inventory levels proposed by both solutions are relatively similar with only some differences during period 2, where the sequential approach has higher inventory levels. Thus, both approaches tend to generate similar inventory levels as products are moved closer to end customers. On average, the sequential approach generates higher inventory levels than the integrated approach. This situation is explained by the larger production lot sizes of this approach relative to the integrated method.

In general, the integrated approach should produce a superior solution than the sequential approach. For the problem under analysis, cost savings of 5.73% are obtained when the integrated approach is used (see Table 4.14). Even though the sequential approach tries to take advantage of the economies of scale produced by larger production and distribution lot sizes, the integrated approach generates a superior solution by considering the effects of raw material procurement decisions. In fact, when fixed costs are analyzed, the sequential approach generates the most competitive alternative, producing savings of 1.83% over the integrated method. However, when variable costs are analyzed, the integrated approach compensates for the extra fixed costs by generating cost savings of 7.56% in variable costs (see Table 4.14). Consequently, the integrated approach is able to account for all of the factors that finally produce a superior solution for the problem.
Figure 4.10  Optimal Inventory Levels of Raw Materials and Finished Products for Each Stage and Optimization Approach (Sequential and Integrated)
Table 4.14  Optimal Solutions for the Integrated and Sequential Approaches

<table>
<thead>
<tr>
<th>Type</th>
<th>Item</th>
<th>Sequential (a)</th>
<th>Integrated (b)</th>
<th>Saving (a-b) $</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Raw material</td>
<td>389,493</td>
<td>206,112</td>
<td>183,380</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shipping</td>
<td>123,075</td>
<td>133,673</td>
<td>(10,598)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Production</td>
<td>1,113,000</td>
<td>1,166,890</td>
<td>(53,890)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Holding</td>
<td>124,148</td>
<td>112,686</td>
<td>11,462</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Subtotal</strong></td>
<td><strong>1,749,716</strong></td>
<td><strong>1,619,362</strong></td>
<td><strong>130,354</strong></td>
<td><strong>7.56%</strong></td>
</tr>
<tr>
<td>Fixed</td>
<td>Raw material</td>
<td>45,500</td>
<td>60,500</td>
<td>(15,000)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shipping</td>
<td>12,000</td>
<td>16,000</td>
<td>(4,000)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Production</td>
<td>15,500</td>
<td>28,000</td>
<td>(12,500)</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Subtotal</strong></td>
<td><strong>73,000</strong></td>
<td><strong>104,500</strong></td>
<td><strong>(31,500)</strong></td>
<td><strong>-1.83%</strong></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1,822,716</td>
<td>1,723,862</td>
<td>98,854</td>
<td>5.73%</td>
</tr>
</tbody>
</table>

4.6  Sensitivity Analysis

In order to evaluate the effects of ordering and holding costs on raw material supply decisions, 18 different scenarios have been evaluated for the problem under analysis. A summary of the results is shown in Table 4.15, where changes in cost parameters are expressed as percentages of their respective original values. In this case, holding and ordering costs are maintained (100%), decreased (0 and 50%), or increased (150%). While changes in holding and major ordering costs affect all stages in the supply chain, variations in minor ordering costs only affect the raw material and manufacturing stages.

Considering the inverse effect of ordering and inventory holding costs on order quantities, it is important to avoid the neutralization effects of these two parameters. Thus, when holding costs are increased, ordering costs are either maintained or decreased. Similarly, when holding costs are decreased, ordering costs are either maintained or increased. Note that in Table 4.15, the percentages of raw material allocated to each supplier are rounded to the closest second decimal, so that small quantities can be identified.
The original optimal solution for the problem (scenario 1) allocates most of the volume to suppliers 2 and 3 to satisfy the raw material requirements during the planning horizon, leaving a limited volume for supplier 1 (0.50%). In this solution, 68.36% of raw material 1 and 63.85% of raw material 2 are allocated to supplier 3, and 31.15% of raw material 1 and 36.15% of raw material 2 are allocated to supplier 2. In this scenario, supplier 1 is almost not considered, given its disadvantages in transportation and raw material purchasing costs. An identical solution is obtained for scenarios 2 to 5 and a similar solution is obtained for scenarios 6 and 7. In these cases, individual changes in ordering or inventory holding costs do not produce any significant change in the optimal solution for the problem.

When major and minor ordering costs are individually decreased (scenarios 8 to 12), significant changes are observed in the total volume allocated for raw material 2, but not for raw material 1. The volume of raw material 2 allocated to supplier 2 is increased from 36.15% to 53.08%, while the volume allocated to supplier 3 is decreased from 63.85% to 46.92%. In these scenarios, when ordering costs are reduced, it becomes attractive to order more frequently from supplier 2, which originally had high ordering costs and low variable raw material costs (see Table 4.5).

When major and minor ordering costs are simultaneously decreased (scenario 13), differences in the raw material procurement strategy are observed for both raw materials. The total allocated volume of raw material 1 to supplier 2 is increased from 31.15% to 46.53%, and the allocated volume of raw material 2 to the same supplier is increased from 36.15% to 58.72%. The explanation for this decision is explained by similar low variable purchasing costs, which become attractive when high ordering costs are decreased.

When major ordering costs are increased and holding costs are decreased (scenario 14), supplier 1 is also considered as an option. Even though supplier 1 is the most expensive alternative among the three suppliers, this supplier is also considered in the optimal procurement strategy. It is acceptable to assume the respective extra raw material procurement costs in order to take advantage of larger production lot sizes at the beginning of the planning horizon. Thus, by reducing holding costs, it is advantageous to
increase production lot sizes, and consequently the number of production orders, so it is possible to compensate for the negative effect of the higher ordering costs.

Finally, when one or both ordering costs are omitted, raw materials are procured only from suppliers 2 and 3, not considering supplier 1 as part of the optimal raw material procurement strategy (scenarios 15 to 18). In this case, the total volume allocated to supplier 2 is almost identical to the original conditions (scenario 1), whereas the total volume allocated to supplier 3 is reduced to almost half of the original order quantity.

In general, when ordering costs are reduced, the volume allocated to supplier 2 is increased, reducing the production and distribution lot sizes, and consequently the respective holding costs. Even though it may appear that production and distribution lot sizing decisions are unrelated to the raw material ordering policy, the selection of suppliers is affected by changes in production/distribution lot size orders. Thus, it is clear that both decisions are closely related, and consequently should not be analyzed separately.
Table 4.15  Sensitivity Analysis of the Optimal Solution, Under Different Scenarios of Ordering and Holding Costs

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Setup Cost (%)</th>
<th>Holding Time (seconds)</th>
<th>Total Cost ($)</th>
<th>Raw Material 1</th>
<th>Raw Material 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Major (%)</td>
<td>Minor (%)</td>
<td>Variable</td>
<td>Fixed</td>
<td>Total</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>100</td>
<td>22</td>
<td>424,665</td>
<td>1,299,196</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>100</td>
<td>24</td>
<td>392,225</td>
<td>1,275,293</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>150</td>
<td>17</td>
<td>424,665</td>
<td>1,322,696</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>100</td>
<td>10</td>
<td>457,105</td>
<td>1,323,099</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
<td>100</td>
<td>16</td>
<td>434,665</td>
<td>1,317,946</td>
</tr>
<tr>
<td>6</td>
<td>150</td>
<td>150</td>
<td>50</td>
<td>394,543</td>
<td>1,323,273</td>
</tr>
<tr>
<td>7</td>
<td>150</td>
<td>150</td>
<td>10</td>
<td>426,983</td>
<td>1,347,388</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>100</td>
<td>17</td>
<td>401,185</td>
<td>1,264,758</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
<td>100</td>
<td>9</td>
<td>411,185</td>
<td>1,283,758</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>100</td>
<td>10</td>
<td>443,625</td>
<td>1,307,625</td>
</tr>
<tr>
<td>11</td>
<td>50</td>
<td>50</td>
<td>10</td>
<td>411,185</td>
<td>1,258,758</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>50</td>
<td>16</td>
<td>421,185</td>
<td>1,277,758</td>
</tr>
<tr>
<td>13</td>
<td>50</td>
<td>50</td>
<td>150</td>
<td>442,059</td>
<td>1,283,860</td>
</tr>
<tr>
<td>14</td>
<td>150</td>
<td>100</td>
<td>28</td>
<td>420,608</td>
<td>1,270,952</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
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<tr>
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<td>9</td>
<td>418,418</td>
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</table>
4.7 Conclusions

This chapter has introduced a new MILP formulation for the multiproduct dynamic inventory problem with supplier selection in a serial supply chain structure. Although the supply chain problem is sequentially analyzed in most real world supply chain settings, the present chapter suggests an integrated approach, where raw material procurement, production, and distribution decisions are simultaneously analyzed. Mathematical models have been presented and analyzed for both optimization approaches, describing the advantages and disadvantages for each case. The problem has been analyzed in the context of the DJRP, where major and minor ordering costs are incurred every time an order is placed at the raw material procurement, and manufacturing stages. This approach explicitly includes the economies of scale associated with larger order lot sizes, and also the logistic and economic advantages obtained by coordinating product replenishment orders.

This chapter has simultaneously incorporated TL and LTL transportation modes as alternative shipment options between stages. This assumption provides sufficient flexibility to the model for evaluating the tradeoff between large order lot sizes and their respective inventory costs. Two types of raw materials and two finished products with a deterministic dynamic demand over a planning horizon of five time periods have been considered as part of the illustrative example. Experimental results show the advantage of the integrated approach, with cost savings of up to 5.73% over the solution generated by the sequential method. Although the sequential approach reduces ordering costs by increasing production and distribution lot sizes, significant costs in raw material purchasing are incurred because raw material procurement decisions are not considered. Thus, by considering the entire supply chain, it is possible to evaluate the tradeoff between ordering and variable costs, selecting the best strategy for all stages. It is possible to notice that potentially infeasible solutions could be generated when the
problem is sequentially solved, proposing large lot production sizes that may not be able to be satisfied with the available raw material supplier capacity.

According to the results described in this chapter, inventory holding and production/distribution ordering costs may have a significant impact on raw material supplier selection and lot sizing decisions. The analysis of 18 different cost scenarios shows that these parameters play an important role in the optimal sourcing strategy. In general, it is observed that the volume allocated by a supplier can significantly change when some of these parameters are modified. This is the case for supplier 1, which allocated volume increases from 0.50% to 12.38% for material 1, and from 0.00% to 19.78% for raw material 2 when major ordering costs are increased and holding costs are decreased. These results demonstrate the need to develop integrated models in order to correctly analyze the entire supply chain.

Further research could consider the effect of variability in some key parameters on the solution generated by the proposed integrated model. Among other factors, demand variability and lead times may generate significant impacts on the optimal solution for the problem. Additional model extensions could also be considered for the non-serial distribution network problem. Although most of the networks can be simplified to a serial supply chain by identifying the bottleneck path, in some cases it could be necessary to consider more general chain structures. In those cases, additional work may be required to adapt the proposed integrated model.
Chapter 5

Conclusions and Future Research

5.1 Conclusions

The models proposed in this research extend the dynamic version of Wagner and Within’s (1958) economic lot-size inventory problem for centralized supply chains. However, the interrelated effects of raw material supplier selection and transportation decisions on the entire supply chain are also recognized in this thesis. In particular, the inherent characteristics of different suppliers in terms of procurement costs, raw material quality, and supply capacity are taken into account. Considering the potential economies of scale that can be obtained by an integrated analysis of transportation operations, the effects of these decisions in the supply chain production/distribution planning problem are explicitly included in the models proposed in this research.

In this research, the strategic importance of including supplier selection and transportation costs in supply chain production/distribution planning is emphasized. Most previous research studies have used sequential analysis, where the optimal solution of one stage (e.g., production and distribution decisions) is used as input for the next stage in the supply chain (e.g., raw material supplier selection). In this approach, based on the observed product demand, the optimal production/distribution strategy is defined first, and then the respective raw material requirements that satisfy the strategy are used as input for the supplier selection problem. Even though this approach simplifies the analysis, when all the supply chain decisions are made simultaneously, superior solutions can be obtained.

The problem has been initially analyzed for a single product case, where the objective function minimizes total variable cost, including purchasing, production, inventory, and transportation costs. The supplier selection problem has been analyzed assuming that only one raw material is needed to produce one type of finished product.
Transportation costs have been incorporated assuming an all-units discount structure, where a less-than-truck load (LTL) transportation mode is selected. The problem has been modeled using both nonlinear and linear programming. In the first case, two continuous and concave approximations of the transportation cost function have been implemented in a mixed integer nonlinear programming model, while in the second case, a piecewise linear function has been used to represent transportation costs in a mixed integer linear programming (MILP) model.

Previous research has frequently assumed that the raw material supplier problem can be modeled as a single raw material problem even though this case is rarely observed in production processes. In particular, this simplification may be considered true when raw materials can be aggregated into a single group, or when there is a particular raw material that plays a critical role in the production process. However, when raw materials are not homogenous, or when suppliers provide different mixes of the required raw materials, this assumption cannot be held. For this reason, the MILP model for a single product has also been extended to cases where multiple raw materials are required in a manufacturing process. Using the concept of bill of materials, the respective requirement levels for each raw material are calculated based on the amount of finished product manufactured.

The incorporation of the multiproduct concept provides substantial benefits to the analysis of the problem considered in this thesis. Even though previous research has frequently assumed that the multiproduct problem can be separated (and consequently independently solved), when joint production and storage capacity restrictions are present or when potential economies of scale for transportation operations can be obtained, the problem cannot be partitioned. In addition, the model proposed in this research assumes the presence of joint replenishment costs at the raw material and production stages. In this case, it is assumed that a major ordering cost is incurred each time one or more products are jointly replenished, while a minor ordering cost is charged each time a product is included in the combined order. In this scenario, significant economies of scale can be obtained by coordinating operations. Similarly, when quantity discounts are offered in transportation operations by coordinating several materials or products in the
same order, higher transportation discounts can be obtained. In particular, two frequent transportation cost structures have been explicitly incorporated into the model, including full-truck-load (TL) and LTL options.

Even though real operations frequently consider LTL and TL transportation modes simultaneously, related literature assumes that the problem can be modeled using only one of these two alternatives. However, when inventory and ordering costs are dominant, the optimal order quantity may be a combination of TL and LTL. Thus, if the quantity of a partial load is not sufficient to justify another truck load using TL, it is optimal to use both modes of freight transportation.

To overcome the inherent increase in the size of the optimization model for the integrated approach and make the proposed methodology more scalable, an analytic approach for reducing the problem complexity by eliminating suboptimal solutions from the analysis has been proposed. Considering that the model assigns a continuous variable to each arc of the time-expanded network, all the arcs and nodes that cannot be used or accessed by feasible raw material or product flows due to positive lead times and a finite planning horizon are eliminated from the model.

According to the results described in Chapters 3 and 4, inventory holding and production/distribution ordering costs may have a significant impact on raw material supplier selection and lot sizing decisions. Even though these parameters may not appear to be directly related to raw material supplier decisions, the analysis of 14 different cost scenarios for the single product case and 18 different cost scenarios for the multiproduct case show that in both situations, these parameters play an important role in the optimal sourcing strategy. These results demonstrate the need to develop integrated models in order to fully minimize the total cost of the entire supply chain network.

Experiment results also show cost savings of up to 6% when the problem is solved with the proposed integrated approach over the sequential method, where the production/distribution and supplier selection problems are independently solved in two consecutive stages. It appears that the advantages of the integrated approach are accentuated in situations involving high production capacities, with limited raw material supplier alternatives. This situation can be exacerbated when the suppliers have positive
lead times. In this case, the production/distribution problem can concentrate a significant proportion of the production in a few periods without considering the effect on raw material procurement. Thus, it is possible that the best raw material suppliers may be unavailable to satisfy the raw material requirements generated by the optimal production/distribution strategy. Consequently, in this scenario, it may be necessary to allocate a significant proportion of the raw material requirements to suppliers that would normally be avoided in the integrated approach.

5.2 Future Research

Although this research has assumed that customer demand and lead times are deterministic and known in advance, uncertainty is frequently present in real operational environments. One example is when new products are launched in the market, with no available historical information. In those cases, the accuracy of demand forecasts can be highly variable, and consequently, significant uncertainty can be related to the expected product demand. Other examples can be observed in maritime operations, where transportation lead times can change significantly based on the weather and seasonal conditions. In all of these cases, significant opportunities for improvement could be realized by incorporating stochastic features into the proposed models.

When product demand is only known for a few periods, a rolling horizon strategy could be implemented. In this approach, the model is run for several time periods, however only a few of the initial time periods are operationally implemented. Thus, the model is successively run as new information becomes available. Using this approach, multiple analyses could be performed, including decisions regarding the optimal length of the planning horizon, number of implementation periods, and others. In particular, it would be interesting to evaluate the effect of period length on the quality of the optimal solution for the problem. It is clear that by decreasing the period length, it may be possible to obtain better accuracy with the model; however, the problem size could increase substantially, along with the associated computational times. Therefore, a
tradeoff analysis could be performed to decide the optimal period length for each specific problem. A similar situation can also be observed when establishing the number of time periods in the planning horizon. As may be expected, increasing the number of periods allows for better visibility of future events that can be captured in the analysis, ensuring that the optimal solution for the initial periods will also be appropriate for the entire planning horizon. However, in the context of the rolling horizon strategy, by incorporating additional time periods, it cannot be guaranteed that the optimal solution obtained for the implemented periods will necessarily change and consequently it cannot be guaranteed that the additional complexity added to the problem could be necessary. Given that the rolling horizon strategy is frequently used in real operational environments, a careful analysis of this idea could substantially benefit future implementations of the models proposed in this research.

In many cases, raw material purchasing prices may be affected by quantity discount structures. As such, the implementation of a concept similar to the one applied to transportation costs could be useful in order to represent a more realistic condition for the problem. In addition, considering that the problem is defined in the context of a serial supply chain structure, a natural extension of this research would be to extend the models to more general supply chains with multiple facilities in each stage.
### Appendix A

#### Nominal and Actual Freight Rates

**Table A.1.** Nominal and Actual Freight Rates for Supplier 2

<table>
<thead>
<tr>
<th>Number Units</th>
<th>Nominal Freight Rate</th>
<th>Actual Freight Rate</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>1 - 31</td>
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<td>617</td>
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<td>32 - 62</td>
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<td>63 - 124</td>
<td>16.1</td>
<td>53 - 62 $1,016</td>
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<td>125 - 312</td>
<td>14.0</td>
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<tr>
<td>313 - 624</td>
<td>12.3</td>
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<td>625 - 1,249</td>
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<tr>
<td>1,875 - 2,500</td>
<td>14,520</td>
<td>313 - 487 $12.3</td>
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**Note:**
- Nominal Freight Rate
- Actual Freight Rate

**Table A.2.** Nominal and Actual Freight Rates for Supplier 3

<table>
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<th>Number Units</th>
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<td>63 - 124</td>
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<td>125 - 312</td>
<td>8.7</td>
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<td></td>
<td>1,711 - 2,500 $5,533</td>
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**Note:**
- Nominal Freight Rate
- Actual Freight Rate

**Table A.3.** Nominal and Actual Freight Rates for Interplant Transportation

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</thead>
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<td>4.4</td>
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<td>1,712 - 2,500 $3,521</td>
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</tbody>
</table>

**Note:**
- Nominal Freight Rate
- Actual Freight Rate
Appendix B
Comparison between Estimate and Actual Freight Cost Function

Figure B.1  Graphical Comparison Estimate Function and actual Freight Cost Function for: (a) Supplier 2, (b) Supplier 3 and (c) Interplant Transportation (Manufacturing Facility to DC).
Bibliography


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Victor Alejandro Valdebenito Candia

Victor Valdebenito studied at the Universidad de Concepcion (Chile), where he received a B.S. degree in Forest Resources in 1992. In 2005, he was awarded with a scholarship from Fulbright, and with a scholarship from Conicyt-Chile in 2007. In 2010, he received a Master of Science degree in Forest Resources and Operations research, and a Doctor of Philosophy degree in Industrial Engineering and Operations Research from The Pennsylvania State University. His research interests include supply chain logistics, operations management, and transportation.