

The Pennsylvania State University

The Graduate School

Harold and Inge Marcus Department of Industrial and Manufacturing Engineering

**MODELING AND ANALYSIS OF A FOUR STAGE MULTI-PERIOD  
SUPPLY CHAIN**

A Thesis in

Industrial Engineering and Operations Research

by

Uday Nimmala Manikandan

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Submitted in Partial Fulfillment  
of the Requirements  
for the Degree of

Master of Science

December 2008

The thesis of Uday Nimmala Manikandan was reviewed and approved\* by the following:

A. Ravi Ravindran  
Professor of Industrial Engineering and Affiliate Professor of IST School  
Thesis Adviser

Terry P. Harrison  
Professor of Supply Chain and Information Systems  
Smeal College of Business

Richard J. Koubek  
Professor of Industrial Engineering  
Head of the Department of Industrial Engineering and  
Manufacturing Engineering

\*Signatures are on file in the Graduate School

## **ABSTRACT**

Most of the literature in supply chain management either considers only a part of the supply chain or the total supply chain in some simplified manner. This thesis considers most of the strategic and tactical decisions faced by a real world supply chain consisting of all the four stages found in a typical industry, namely, suppliers, manufacturing plants, warehouses and retailers. The modeling decisions will include the supplier selection process integrated with the production amounts, inventory levels, stock-outs, shipment quantities and transportation routes.

The thesis proposes an integrated mathematical programming model for supply chain management. It is a single objective, multi-period, deterministic, centralized supply chain model. The objective of the model is to minimize the total cost involved in running the supply chain. This results in a mixed integer linear program. The model is tested with a set of realistic data and the optimal results are obtained. Risk analysis, such as supplier price increase, cease of a mode of transportation, is performed over the optimized model and the effects of uncertainty are studied.

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## **ACKNOWLEDGEMENTS**

I would like to thank Dr. Ravindran for being my thesis adviser. His guidance, support and patience throughout the research and writing of the thesis were invaluable.

I also want to thank Dr. Harrison for being my thesis reader. His support and guidance were helpful in completing my thesis within the given time limits.

Finally, I would like to thank my family for their continuing support in everything.

## **Chapter 1**

### **INTRODUCTION**

To be the winner in present day's highly competitive market, companies must ensure to satisfy customers demand at the lowest possible costs. This requires efficient supply chain management.

#### **1.1 Supply Chain Management**

Many authors in the literature have defined 'Supply Chain Management' in various interpretations but the basic crux of all the definitions remains the same. SCM according to Lee and Billington (1992) is the co-ordination of the manufacturing, logistics, materials, distribution, and transportation functions within an organization. Leenders and Fearon (1997) defined SCM as "a systems approach to managing the entire flow of information, materials, and services from raw materials suppliers through factories and warehouses to the end customer." Christopher (1998) defined SCM as "Management of upstream and downstream relationships with suppliers and customers to deliver superior customer value at less cost to the supply chain as a whole." Chopra and Meindl (2001) points out that SCM involves the management of flows between and among stages in a supply chain to maximize supply chain profitability. According to Simchi-Levi et al (2002), SCM is "A set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandise is produced and



distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide costs while satisfying service level requirements.” Chen and Paulraj (2004) described that the term ‘SCM’ has been used to explain the planning and control of materials and information flows as well as the logistics activities not only internally within a company, but also externally between companies.

The key elements of Supply Chain and its management from these definitions are therefore the upstream parties, the downstream parties and the integration of all the organizations involved, together with the internal function of an organization itself. The upstream parties, as been described by Handfield and Nichols (1999) & Lambert and Cooper (2000) consists of an organization’s functions, processes and network of suppliers while the downstream function on the other hand concerns the distribution channels, processes and functions where the product passes through to the end customer.

## **1.2 Supply Chain Integration**

With the rapid development of information technology and intense global competition, many manufacturers and service providers are collaborating with their primary suppliers to upgrade the traditional material management functions into part of their corporate strategy. They have adopted an integrated strategic approach to SCM. Although many efforts have failed, it still has become a significant strategic tool for firms striving to achieve competitive advantage. Tan shows (1998a, 2001) that different terminologies have been used to address the various aspects of this new management

philosophy such as supplier integration, partnership, supply base management, supplier alliances, and supply chain synchronization.

Supply chain integration enables members of the supply chain to function as a unified logistics quantity wherein all the supply chain entities and operations including purchasing, manufacturing, and logistics function can be more efficiently and effectively managed.

### **1.3 Supply Chain Modeling**

In the 21st century, there have been many changes in business environment that have contributed to the development of supply chain research and study. Due to the competitive nature of present market, companies have learnt about the importance of the right supply chain structure. Researchers and practitioners have placed increasing attention on the performance, design, and analysis of the supply chain structure as a whole. According to Beamon (1998) this attention is largely a result of the rising costs of manufacturing, the shrinking resources of manufacturing bases, shortened product life cycles, the leveling of the playing field within manufacturing, and the globalization of market economies.

A typical supply chain model usually consists of multiple stages, such as suppliers, manufacturers, distribution centers, warehouses, retailers etc. On a high level, Beamon (1998) says all the processes involved in these stages can be classified into two

broad categories (i) Production Planning and Inventory Control Process and (ii) Distribution and Logistics Process. These processes interact with each other to produce an integrated supply chain.

The single highest cost in a supply chain is inventory which accounts for nearly half of the total logistics costs (Lancioni, 2000). The management of this topic has become a major research area in the field of operations research. Inventory policies affect the cost and responsiveness of the supply chain. Low inventory makes the supply chain more efficient and keeps the inventory costs low, however responsiveness is decreased. Larger amounts of inventory allow firms to satisfy demand and avoid stock outs.

Now considering the broad spectrum of a supply chain, no mathematical model can capture all aspects of the supply chain processes. To compromise the dilemma between model complexity and reality, a model should be designed in such a way that it is reflective of key real world dimensions, yet not too complicated to solve. The model design should at least include decisions, such as supplier selection, assignment of productions to plants, inventory and stock out levels, transportation quantities and routes.

#### **1.4 Thesis Outline**

The supply chain considered in this thesis is a four stage multi-period dynamic system consisting of supplier selection, production, inventory, and transportation

decisions. The four stages considered are *Suppliers*, *Manufacturers*, *Wholesalers* and *Retailers* resulting in a mixed integer linear programming model.

If the decisions in a supply chain are taken by a single decision maker across all the stages as a single collaborative unit, it is a *centralized supply chain*. In a *decentralized supply chain*, each stage is owned by a different company, which acts independently based upon its own priorities and goals. The supply chain considered in this thesis is a centralized supply chain and the only objective function of the model is to minimize the total cost involved across all the stages in the supply chain. The various costs incurred are raw material procurement cost, plant production cost, inventory holding cost at the warehouse and retailers, backorder cost at the retailers, transportation cost for moving the raw materials from suppliers to plants, moving the products from plants to warehouses, and warehouses to retailers. The model assigns the final supplier for each raw material, plant production units (in-plant & finished products), transportation quantities, inventory levels and back-orders taking into account the deterministic demand provided at the retailers for each time period.

The thesis will follow the following format. Chapter 2 will review the literature in the areas of research pertaining to the topic, namely, supplier selection and integrated supply chain modeling. The formulation of the mixed integer linear model to minimize the cost of the supply chain will be discussed in Chapter 3. The assumptions, data, variables and the constraints in all the stages will be described in this chapter. Chapter 4 will contain the implementation of the model for a simulated data set using the

optimization software LINGO followed by Risk Analysis. The effects of supplier price increase and cease of mode of transportation will be studied. Finally, Chapter 5 will include concluding remarks and some recommendations for further research.

## **Chapter 2**

### **LITERATURE REVIEW**

This chapter reviews the relevant work from the literature before we proceed to the problem description and model formulation.

#### **2.1 Supplier Selection**

According to academicians and purchasing managers, there are many criterions that are important in the supplier decision process. Dickson's study (1966) lists 23 such different criteria. These are net price, quality, repair service, delivery, geographic location, financial position, production facilities and capacity, amount of past business, technical capability, management and organization, future purchases, communication systems, operational controls, position in the industry, labor relations record, attitude of each vendor, desire for business, warranties and claim policies, packaging requirements, impression, training aids, compliance with procedures, performance history.

Weber et al (1991) gives a comprehensive review of 74 articles over the past literature since 1966. He lists them all down on the basis of three subentries, Dickinson's vendor selection criteria, Purchasing environment involved and the techniques followed to analyze these criteria. Later he discusses the effect of a new manufacturing trend (JIT) over the vendor selection process. Surprisingly, only ten out of these 74 articles employ

core mathematical techniques. Out of these ten, eight articles employ single objective optimization and two articles discuss multi-objective programming.

A brief review of some of the articles discussing supplier selection as a whole is presented below.

Moore and Fearon (1973) discussed the use of linear programming for supplier selection. Although the article doesn't provide an actual mathematical formulation, a conceptual model is provided stating that price, quality and delivery are important criteria for supplier selection. It also discusses the other applications of computer technology in the purchasing area.

Gaballa (1974) was the first author to apply mathematical programming to supplier selection in a real case. He formulated a mixed integer decision making problem for the Australian Post Office. The objective of this programming model was to minimize the total discounted price of allocated items to the suppliers under capacity and demand constraints.

Anthony and Buffa (1977) formulated a linear programming model to support strategic purchasing scheduling. The objective of the model was to minimize total purchasing and storage costs by considering budget, buyer's demand and supplier storage capacity as constraints. Ordering, transportation and inspection costs were not considered in the model.

Bender et al (1985) describes a mixed integer programming model for supplier selection at IBM to minimize the sum of purchasing, transportation and inventory costs over multiple time periods.

Kingsman (1986) proposed the use of linear programming and dynamic programming to address the issue of minimizing the purchasing cost when commodity prices fluctuate in a stochastic manner over time.

Narasimhan and Stoyhoff (1986) formulated a mixed integer programming model for a large manufacturing firm in the Midwest to determine their suppliers and the order quantities for their multiple production plants.

Turner (1988) presented a mixed integer optimization model used by British coal for their supplier selection. The objective of the model was to minimize the total contract price under demand, supplier capacity, minimum and maximum order quantities and geographic region constraints.

Pan (1989) proposed a linear programming model to select best suppliers under quality, lead time and service constraints. He proposed multiple sourcing to improve the reliability of supply of critical materials.

Buffa and Jackson (1983) presented a multi-criteria linear goal programming model. The model considered the criteria *quality, price, service experience, on-time, late*



*and early deliveries* as supplier attributes and the criteria *material requirement, safety stock* as buyer's specifications.

Sharma et al (1989) presented a non-linear, mixed integer, goal programming model addressing the price, quality and lead time under demand and budget limitations.

Karpak and Kasuganti (1999) used visual interactive goal programming for supplier selection process. The objective of the model is to minimize product acquisition cost and maximize quality and reliability by effectively identifying and allocating order quantities among suppliers.

Seshadri et al. (1991) developed a probabilistic model to represent the connection between multiple sourcing and its consequences, such as number of bids, the seller's profit and the buyer's price.

Petroni and Braglia (2000) use principal component analysis, a multivariate statistical method to develop a supplier selection model for a medium sized bottling machining firm. This method uses information obtained from eigen values to combine different ratio measures defined by every input and output

Ronen and Trietsch (1988) used statistical approach and developed a stochastic stationary inventory model to build a decision support system for purchasing of items for large products.

Soukup (1987) modified the linear weighted point method, the most widely approach for single sourcing in the literature which depends a lot on the human judgment process. The author incorporated the change by using probabilities instead of criterion weights.

Kumar et al (2004) proposed an approach that has the potential to handle realistic situations in a fuzzy environment. They formulated a fuzzy mixed integer goal programming problem considering a triangular membership function. The model has primarily three objectives, minimizing *the net cost*, *the net rejections* and *the net late deliveries* subject to practical constraints such as supplier's capacity, buyer's demand, supplier's quota flexibility, allocation of budget to individual supplier, and purchase value of item etc.

Weber et al (1993) presents a multi-objective approach to systematically analyze the inherent tradeoffs involved in multi-criteria vendor selection problems and in 1998 uses data envelopment analysis to analyze supplier negotiation. Non co-operative supplier negotiation strategies are developed wherein if one supplier is selected then the other is rejected from the solution.

Min (1994) approaches the international supplier selection problem by using multi attribute utility. He used various criteria such as financial terms, quality assurance, perceived risks, service performance, relationships, cultural and communications barriers and trade regulations in his model.

Often complicating the supplier selection decisions are price breaks. These are discounts offered to buyers based on their order quantity size. Many articles, Chaudhry et al. (1991), Guder et al. (1994), Katz et al. (1994), Pirkul and Aras (1985) and Sadrian and Yoon (1994) have considered this difficulty by approaching them as mixed integer linear programming problems by minimizing the purchasing cost even though these problems are actually multi-objective in nature.

According to Wadhwa and Ravindran (2007) supplier selection is a multi-level objective problem that has both qualitative and quantitative factors included in it. The authors solve the multi-criteria problem with the most commonly used techniques, such as goal programming, weighted objective and compromise programming methods. But in all these situations, the authors include only the factors that can be specified in a mathematical fashion. In-order to include the factors that are intangible, software tools such as AHP (Analytical Hierarchy Process) has to be used. Ghodspur (1998) et al approaches this problem using a novel approach of considering both qualitative and quantitative factors by combination of analytical hierarchy process and linear programming. They consider both tangible and intangible factors in choosing the best suppliers and ordering the best quantities such that the total value of purchasing becomes maximum.

Ravindran and Wadhwa (2009), in the forthcoming Handbook of Military Industrial Engineering present an overview of different multiple criteria techniques for the supplier selection process as applicable to the Department of Defense which spends

billions of dollars on procurement. They discuss the existing methods available in the literature and later describe the supplier selection process as a two step technique. The first step involves the use of multi-criteria optimization models for ranking of suppliers and pre-screening of suppliers when there are a large number of suppliers in the supplier base. The second step is the use of various goal programming methods to analyze the short listed suppliers under conflicting objectives. The methods are illustrated with examples.

## **2.2 Integrated Supply Chain Modeling**

According to Min and Zhou (2002), the whole spectrum of supply chain modeling is based on three levels of decisional hierarchy; Competitive strategy, Tactical plans & Operational routines. The classes of supply chain problems coming under the umbrella of competitive strategic analysis are location-allocation decisions, demand planning, distribution planning, strategic alliances, new product development, outsourcing, supplier selection, information technology (IT) selection, pricing, and network restructuring. The tactical problems encountered include inventory control, production/distribution coordination, order/freight consolidation, material handling, equipment selection, and layout design. The problems faced in operations include vehicle routing/scheduling, workforce scheduling, record keeping, and packaging. Most of the models in the supply chain management literature do not show this hierarchy explicitly as there is always some overlapping involved among them.

Erengüç et al (1999) perform a review of many models identifying the relevant decisions that need to be considered in jointly optimizing production/distribution planning in the entire supply chain network. According to the authors the key strategic and tactical decisions that need to be focused are the number of suppliers selected, the criteria used for supplier selection, the number of units supplied under each material category, the various location and allocation decisions in the manufacturing plants and distribution centre network, the amount of inventory stored in the different stages of the network etc.

Min et al (1998) review 33 articles dealing with location-routing problems and classify them on the basis of different categories such as number of stages, deterministic or stochastic nature, number of facilities/vehicles, uncapacitated or capacitated vehicles/facilities, single or multiple objectives, type of data used (hypothetical or real world), static or dynamic, unspecified or soft/hard time windows, specific types of exact or heuristic algorithms used. The authors also suggest appropriately the potential or actual application areas for most of the problems.

Glover et al (1979) developed a single period single objective supply chain model for Agrico Chemical Co. The authors implemented a computer based production, distribution and inventory (PDI) planning system that integrated the three supply chain segments comprised of supply, storage/location, and customer demand planning. This model saved the company \$18 million during the first 3 years of its operation. The

backbone of the PDI system was a network model and diagram that increased the decision maker's insights into supply chain connectivity.

Cohen and Lee (1989) developed a mixed-integer, non-linear, value-added chain network model that coordinated the different processes consisting of supplier shipping of raw materials, centralized production planning, and inter-plant transshipment. The resource deployment decisions in the model incorporated capacity, demand, and production constraints taking into account the exchange rates and transfer prices for the geographically dispersed locations, but failed to capture the various risk factors inherent in a global setting.

Arntzen et al (1995) develop a mixed-integer programming model, called global supply chain model (GSCM). This multi-period, multi-commodity, multistage model takes into account the inter-dependence of production, inventory, bill of materials, shipping, system configuration, offset trade, local content and delivery processes to minimize activity days and total cost. The inclusion of *offset trade* and *local content* is the uniqueness of the paper as it is rarely considered in research. The presented model was evaluated by solving larger problems for Digital Equipment Corporation.

Ashayeri and Rongen (1997) formulated a strategy to determine Distribution centers site locations based on analyses of transportation costs, DC location costs and throughput times. The proposed problem was solved using the multi-criteria method called ELECTRE. Although the proposed model and solution method are simple to use,

they are confined to single-period and un-capacitated problems. The authors showed how the method can be used to locate distribution centers for a European Telecom Company.

Min et al (1999) considers the analytic hierarchy process to solve a multi-objective problem which jointly determines the relocation of manufacturing/distribution facility of a firm named 'Alpha'. The multi-stage supply chain network connects the material flow among suppliers, manufacturers, break-bulk terminals, and customers. In the AHP based model, the decision makers are finally presented with three alternatives for location based on criteria such as site characteristics, cost, traffic access, market opportunity, quality of living and local incentives. The model considers contingency planning based on 'what-if' scenarios associated with supply chain reconfiguration. However, it does not consider multiple periods, capacity constraints and risk factors. The authors extended their work in (2000) by considering multi-periods, capacity and budget constraints by designing a multi-objective, mixed integer programming model that determined the optimal relocation site and phase-out schedule of a combined manufacturing and distribution facility from supply chain perspectives. The model is capable of handling multiple conflicting objectives such as the minimization of cost, traffic accessibility, and local incentives. The results obtained helps the location planner to analyze the various trade offs among the competing objectives.

Another novel methodology was adopted by Melachrinoudis et al (2000) called *physical programming* which in addition to the selection of the new site, determines the optimal schedule to relocate capacity from existing site to the new site and the production

levels at both sites during the transition period. This multi-period problem allows the decision maker to express multi-criteria preferences not in a usual form of weights, but in ranges of different degrees of desirability.

Nozick et al (2001) presents a modeling approach to locate distribution centers by paying careful attention to the inherent trade-offs among facility costs, inventory costs, transportation costs, and customer responsiveness. Though the model deals with multiple objective (service-cost tradeoff) issues, it is confined to a single period and single echelon problem with no capacity constraint. The model is illustrated with an example involving the distribution of finished vehicles by an U.S. automotive manufacturer.

Attai (2003) presents a global supply chain model consisting of four stages located in different countries. The optimal decisions for this supply chain includes facility location, production quantities, shipment amounts, transportation routes, and inventory levels at each stage. At first, a single objective model for maximizing the profit considering the local content rules is applied. Then the model is extended to a multi-objective problem considering the profit, product cycle times and local incentives as three conflicting objectives. The local incentives represent labor quality, tax breaks, loans and customers buying power. The optimal solution for supply chain configuration and sensitivity analysis shows how the different criteria weights affect the strategic planning decisions.



Park (2005) presented a joint approach for production and distributed planning in a multi-plant, multi-retailer, multi-item and multi-period logistic environment and inspected the effectiveness of their integration through a computational study where the only objective is to maximize the total net profit. The author develops mixed-integer models and a heuristic for solving the problem. Computational results on test problems using the proposed heuristic confirm the substantial advantage of the integrated planning approach over the decoupled one. Sensitivity analysis on the input parameters show that the effectiveness of integrating production and distribution functions can be extremely high under the right conditions.

Axsater (2005) considers a two stage distribution inventory system with a central warehouse and multiple retailers. The author provides an approximate technique to find the backorder cost for the warehouse explaining how the warehouse has to optimize its total costs under a complex demand process, i.e. the orders from the retailers.

Haq et al (2006) developed an integrated supplier selection and multi-stage distribution inventory model for an original equipment manufacturing company (tyre industry in Southern India) in a built-to-order supply chain environment. The supplier selection is a qualitative decision making process designed using fuzzy analytical hierarchy process which is integrated with the quantitative mathematical model for the distribution inventory supply chain using a genetic algorithm. The modeling decisions consists of three components (1) Production at each plant for all time periods (2) inventory at plants, distribution centers, wholesalers & retailers for all time periods (3)

Transportation routes between each stage for all time periods. However, the author doesn't consider stock outs in the model. The software used for programming the model using genetic algorithm is C++ language.

Warsing (2008), in the Operations Research and Management Science Handbook, presents the various techniques and mathematical functions that are used to manage inventory, transportation and location decisions in supply chains. The various concepts discussed are inventory modeling, distribution requirement planning, estimating transportation costs, joint transportation and inventory models, centralized and decentralized supply chains, inventory aggregation, and risk pooling. Finally, the various methods to manage dyads individually in supply chain are discussed in order to avoid the bull whip effect which roots up while optimizing the entire supply chain.

### **2.3 Contribution of this thesis**

Based on the above literature review, it is clear that there has been a lot of research done in the areas of supplier selection and supply chain modeling and a few papers combining both, using non traditional methods such as AHP. However, there is no work that integrates both the supplier selection problem along with the inventory, production and transportation problems across the various stages of the supply chain as a single mathematical model. This thesis is a first step in that direction.

## Chapter 3

### PROBLEM DESCRIPTION AND MODEL FORMULATION

This chapter describes the supply chain problem followed by the model formulation. In Chapter 4, we will discuss the model implementation with simulated data and the risk analysis.

#### 3.1 Problem description

The centralized supply chain model consists of four stages; Suppliers, Manufacturing plants, Warehouses and Retailers as illustrated in Fig 3-1.

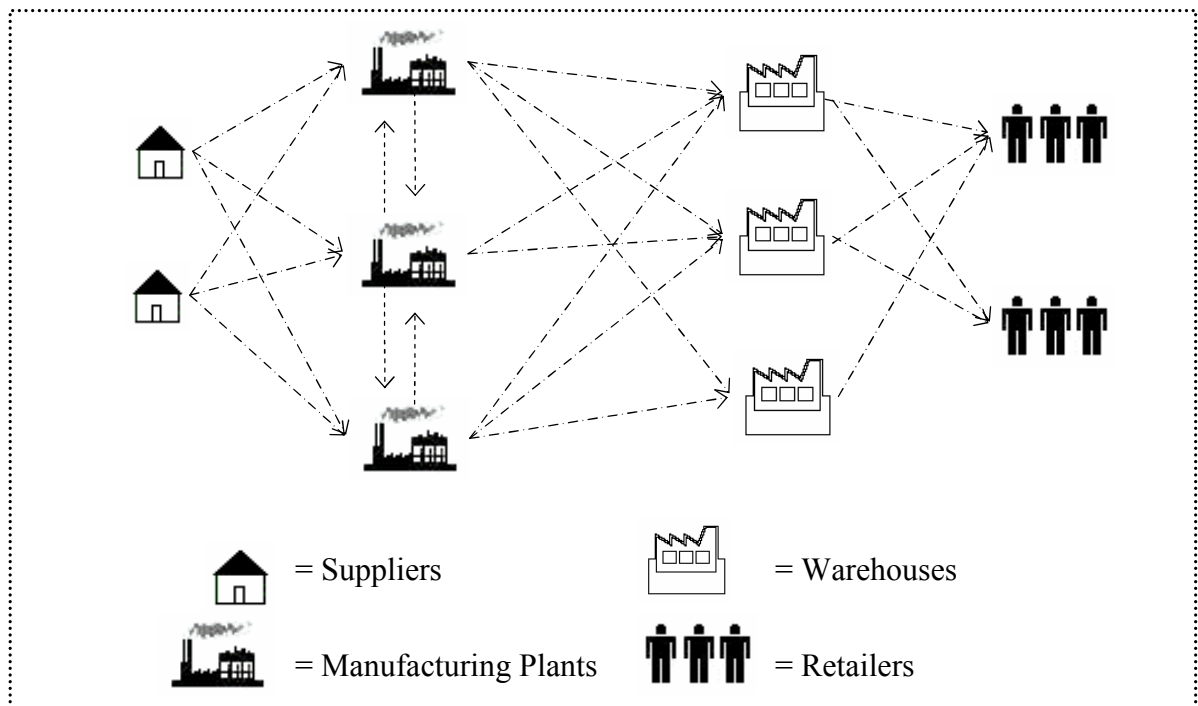


Fig 3-1: Schematic representation of the 4 stages of the supply chain

Suppliers must be selected to minimize the procurement cost of the raw materials purchased. The quality of the item that is being supplied also forms a basic criterion for selection of suppliers. Also, care must be taken that the suppliers have sufficient capacity to supply the items throughout all time periods without any delay or shortage of orders. The raw materials are not only purchased from the suppliers but also certain parts are being manufactured inside the factory along with the finished product. Such parts are termed as *Manufactured In-plant (MIP)* items. Not all the manufacturing sites have the capacity to produce the required number of MIP items at each site. This calls for transportation of MIP items from a manufacturing site to another, based on the requirement of the item at that particular plant. The various costs incurred at the plants are the production costs of the finished product and MIP items and also the inter-plant transportation cost.

The finished products are then transported to the warehouses, which also involves transportation costs. There is only a single mode of transportation between the manufacturing plants and warehouses (air). The products are later shipped to the retailers using three modes of transportation (air, rail and road). Each mode of transportation has a limit to the amount of products that can be shipped using that mode. Costs are also incurred for holding inventory and stock-outs at the retailers. There are capacity restrictions at every stage of the supply chain. The main objective of the problem is to minimize the total cost incurred across all the four stages over multiple time periods.

The optimized model will be analyzed for certain risk conditions, such as, supplier price increase, disruption in the mode of transportation etc. in order to study their impacts.

### **3.2 Model Assumptions**

The following are the underlying assumptions for each stage that needs to be stated before we proceed with the model formulation.

#### **3.2.1 Suppliers:**

- Every supplier can supply more than one raw material for every time period.
- Every chosen supplier has a particular minimum shipment quantity that he supplies and maximum order capacity over which he cannot supply for each type of raw material.
- Only one supplier is selected finally for each type of raw material ordered in every time period.
- Raw materials are shipped from suppliers to manufacturers using 'air' as the mode of transport which has a lead time of 1 time period.

#### **3.2.2 Manufacturers:**

- Manufacturing sites follow JIT production, i.e., there is no provision for holding inventory of raw materials or MIP items or finished products.
- Manufacturing process is instantaneous, i.e., plants can produce and ship immediately when the order arrives without any time delay.
- Backorders are not considered at this stage.
- Each plant has a requirement for the MIP items for every time period.
- Each plant has a capacity limit for producing both MIP items and finished products.

- Inter-transportation of MIP items within manufacturing plant sites is instantaneous.
- A single finished product is produced by combining the different raw materials and MIP items in a particular ratio.

### **3.2.3 Warehouses:**

- Products are shipped from manufacturers to warehouses using dedicated mode of transport (air) with a fixed lead time of one time period.
- In-transit inventory holding cost from the manufacturers to warehouses is neglected, i.e., inventory holding cost comes into play only after the products reach the warehouses.
- Inventory in any period  $t$  represents the inventory level at end of the time period  $t$ .
- Initial inventories are assigned to each warehouse.
- Backorders are not considered at this stage.

### **3.2.4 Retailers:**

- Products are shipped from warehouses to retailers using three different modes of transportation with a fixed lead time of 1 period associated with air, 2 periods for rail, 3 periods for road, and a per unit transportation cost associated with each mode.
- Price breaks for transportation costs are not considered in the model.
- Each mode of transportation has a capacity limit, maximum that can be shipped using that mode.
- In-transit inventory holding cost is again neglected, i.e., the inventory holding cost begins only when the shipment reaches the retailers.

- Inventory in any period  $t$  is the inventory level at end of the time period  $t$ .
- Initial inventories are assigned to each retailer.
- Backorders are allowed at the retailers if they are not able to satisfy customer's demand and a per unit cost is associated with the backorders.
- Backorders at the retailers for the last time period is zero, i.e., all the retailer demands must be satisfied by the end of the planning horizon.

### 3.3 Model Formulation

The index set that will be used in the model formulation are defined below:

$n$  – Set of raw materials

$p$  – Set of MIP items

$h$  – Set of suppliers

$i$  – Set of manufacturing plants

$s$  – Another index defined for manufacturing plants for shipment of MIP items ( $s \subseteq i$ )

$j$  – Set of warehouses

$k$  – Set of retailers

$t$  – Time period index ( $t = 1, 2, 3 \dots T$ )

$m$  – Mode of transport

where  $m=1$  denotes Air

$m=2$  denotes Rail

$m=3$  denotes Road

### 3.3.1 Model Data

#### 3.3.1.1 Supplier Stage

$Qstdset(n)$  – Quality standard set by manufacturer for raw material  $n$

$Qstd(n,h,t)$  – Quality of raw material  $n$  from supplier  $h$  in time period  $t$

$cpu(n,h,t)$  – Cost per unit of raw material  $n$  from supplier  $h$  for time period  $t$

$SCap(n,h)$  – Capacity limit for raw material  $n$  provided by supplier  $h$

$SMin(n,h)$  – Minimum quantity supplied by supplier  $h$  for raw material  $n$

$TSUP(n,h,i)$  – Unit transportation cost of shipping raw material  $n$  from supplier  $h$  to plant  $i$

#### 3.3.1.2 Manufacturer Stage

$PCap(i)$  – Capacity of plant  $i$  to manufacture the finished product

$MIPCap(p,i)$  – Capacity of plant  $i$  to manufacture the  $p^{th}$  MIP item

$Pprod(i)$  – Unit cost of manufacturing the finished product at plant  $i$

$TMIP(p,i,s)$  – Unit transportation cost of shipping the  $p^{th}$  MIP item from plant  $i$  to plant  $s$

$TPROD(i,j)$  – Unit transportation cost for the finished product from plant  $i$  to warehouse  $j$

$CMIP(p,i,t)$  – Unit production cost of the  $p^{th}$  MIP item in plant  $i$  for time period  $t$

$\alpha(n)$  – Number of raw material of type  $n$  required to produce one finished product

$\beta(p)$  – Number of  $p^{th}$  MIP item required to produce one finished product

#### 3.3.1.3 Warehouse Stage

$WCap(j)$  – Capacity limit for warehouse  $j$  for storing finished product.

$WInitialInv(j)$  – Initial Inventory of finished product at warehouse  $j$



$Hw(j)$  – Inventory holding cost per unit of finished product per unit time at warehouse  $j$

$TWAR(j,k,m)$  – Unit transportation cost of the finished product from warehouse  $j$  to retailer  $k$  using shipment mode  $m$

$M1, M2, M3$  – Maximum capacity limit for the number of units transported by each mode  
(Air= $M1$ , Rail= $M2$  and Road= $M3$ )

### 3.3.1.4 Retailer Stage

$R(k,t)$  – Demand for the finished product at retailer  $k$  in time period  $t$

$Hr(k)$  – Inventory holding cost per unit of finished product per unit time at retailer  $k$

$RCap(k)$  – Maximum inventory capacity at retailer  $k$

$RBackcost(k)$  – Unit backorder cost per unit time at retailer  $k$

$RInitialInv(k)$  – Initial Inventory of finished product at retailer  $k$

### 3.3.2 Model Decision Variables

All variables defined are non-negative.

#### 3.3.2.1 Supplier Stage

$S\alpha(n,h,t)$  – Binary variable denoting whether supplier  $h$  is selected in time period  $t$  for raw material  $n$ .

$w(n,h,t)$  – Quantity of raw material  $n$  ordered from selected supplier  $h$  for time period  $t$

$u(n,h,i,t)$  – Quantity of raw material  $n$  transported from supplier  $h$  to plant  $i$  during time period  $t$

### 3.3.2.2 Manufacturer Stage

$v(n,i,t)$  – Quantity of raw material  $n$  reaching plant  $i$  in time period  $t$

$X(i,t)$  – Number of finished products manufactured at plant  $i$  in time period  $t$

$XMIP(p,i,t)$  – Number of  $p^{th}$  MIP item produced in plant  $i$  in time period  $t$

$XMIPS(p,i,s,t)$  – Number of  $p^{th}$  MIP item shipped from plant  $i$  to plant  $s$  in time period  $t$

(Note: When  $i=s$ , it represents the in-plant use of MIP item)

$MIPReq(p,s,t)$  – Requirement of the  $p^{th}$  MIP item in plant  $s$  in each time period  $t$

$y(i,j,t)$  – Number of finished products shipped from plant  $i$  to warehouse  $j$  during time period  $t$

### 3.3.2.3 Warehouse Stage

$\varphi(j,t)$  – Cumulative inventory in warehouse  $j$  at the end of time period  $t$

$z(j,k,t,m)$  – Number of finished products shipped from warehouse  $j$  to retailer  $k$  in time period  $t$  using transport mode  $m$

### 3.3.2.4 Retailer stage

$RBO(k,t)$  – Cumulative backorders at retailer  $k$  at end of time period  $t$

$RInv(k,t)$  – Cumulative inventory at retailer  $k$  at the end of time period  $t$

### 3.3.3 Model Constraints and Objective Function

#### 3.3.3.1 Supplier Selection

According to Wadhwa and Ravindran (2007), Buyer-supplier relationships based solely on price are no longer acceptable for suppliers of critical materials or for organizations that wish to practice the latest innovations in supply chain management. So we consider quality and capacity variables apart from cost variables. The supplier selection model is formulated based on the following preferences of the buyer.

The quality of raw material purchased  $Qstd(n,h,t)$  must meet the specific standard set for that material  $n$  by the manufacturer  $Qstdset(n)$  for every supplier  $h$  in each time period  $t$ :

$$Qstd(n,h,t) \geq Qstdset(n) * S\alpha(n,h,t) \quad \forall n,h,t \quad (3.1)$$

The capacity of the supplier  $h$  for raw material  $n$  must be high enough to meet the quantity of raw material ordered from the supplier in each time period  $t$ :

$$w(n,h,t) \leq SCap(n,h) * S\alpha(n,h,t) \quad \forall n,h,t \quad (3.2)$$

The order quantity of each raw material  $n$  must meet the minimum purchase requirement of supplier  $h$  in every time period  $t$ :

$$w(n,h,t) \geq SMin(n,h) * S\alpha(n,h,t) \quad \forall n,h,t \quad (3.3)$$

For every time period  $t$ , only one supplier is finally selected, but the same supplier can supply more than one raw material:

$$\sum_h S\alpha(n,h,t) = 1 \quad \forall n,t \quad (3.4)$$

$$S\alpha(n,h,t) \in [0,1] \quad \forall n,h,t \quad (3.5)$$

Note: When  $S\alpha(n,h,t) = 0$  then by equation (3.2),  $w(n,h,t)$  is also zero. Thus the final supplier  $h$  selected for every raw material  $n$  in each time period  $t$  corresponds to the set whose value for the variable  $S\alpha(n,h,t)$  is 1. Then, equations (3.1), (3.2) and (3.3) guarantee the quality standards of the buyer and the minimum and maximum order quantity of the supplier.

The objective function minimizes the total purchasing cost given by:

$$\text{Minimize } \sum_{n,h,t} (cpu(n,h,t) * w(n,h,t)) \quad (3.6)$$

### 3.3.3.2 Plant production and inter-plant transportation

#### (i) Transportation of raw materials

The raw materials purchased from the selected suppliers are shipped to all the plants as follows:

$$w(n,h,t) = \sum_i u(n,h,i,t) \quad \forall n,h,t \quad (3.7)$$

$$\sum_h u(n,h,i,t-1) = v(n,i,t) \quad \forall n,i,t \quad t \geq 2 \quad (3.8)$$

The parameter  $(t-1)$  in the variable  $u(n,h,i,t-1)$  indicates that it takes one time period for the raw materials to reach the plants from the suppliers. Raw materials that are purchased by the manufacturer currently would reach the plants only in the next time period. Equation (3.8) is defined only for  $t \geq 2$  and  $v(n,i,t)$  is assumed to be zero for the first time period, i.e.,  $v(n,i,1) = 0$ . The objective function minimizes the total transportation cost of raw materials as follows:

$$\text{Minimize } \sum_{n,h,i,t} (TSUP(n,h,i) * u(n,h,i,t)) \quad (3.9)$$

(ii) Production of MIP items and finished product

The plant's production for both the finished products and MIP items must satisfy their respective capacity limits:

$$X(i,t) \leq PCap(i) \quad \forall i,t \quad (3.10)$$

$$XMIP(p,i,t) \leq MIPCap(p,i) \quad \forall p,i,t \quad (3.11)$$

The raw materials and MIP items are combined together to form the finished product in each plant. For example, two wheels (raw materials) and frame & chain pack (MIP items) are assembled together to form a single bicycle (finished product). Here, we assume that for producing one finished product  $X(i,t)$  in each plant  $i$  for every time period  $t$  we require  $\alpha(n)$  number of raw materials of type  $n$  and  $\beta(p)$  number of  $p^{th}$  MIP items.

$$v(n,i,t) = \alpha(n) * X(i,t) \quad \forall n,i,t \quad (3.12)$$

(Note: By equation (3.12), when  $t=1$ ,  $X(i,1) = 0 \quad \forall i$  as we earlier assumed that  $v(n,i,1)=0$ )

$$MIPReq(p,i,t) = \beta(p) * X(i,t) \quad \forall p,i,t \quad (3.13)$$

The objective function minimizes the total manufacturing cost involved as follows:

$$\text{Minimize } \sum_{i,t} (Pprod(i) * X(i,t)) + \sum_{p,i,t} (CMIP(p,i,t) * XMIP(p,i,t)) \quad (3.14)$$

(iii) Inter-plant transportation of MIP items

Considering the inter-plant transportation within the plants, we have

$$\sum_s XMIPS(p,i,s,t) \leq XMIP(p,i,t) \quad \forall p,i,t \quad (3.15)$$

(Note: When  $s = i$ ,  $XMIPS(p, i, i, t)$  represents of the in-plant use of MIP items)

$$\sum_{\forall i} XMIPS(p, i, s, t) \geq MIPReq(p, s, t) \quad \forall p, s, t \quad (3.16)$$

Equation (3.15) denotes that the number of MIP items shipped from plant  $i$  to all other plants  $s$  to meet their requirement levels plus their own in-plant use must be less than the MIP item production at that plant. Equation (3.16) states that the internal use of MIP items in any plant  $s$  plus the MIP items received from all other plants  $i$  must meet the MIP item requirement level of that plant  $s$  for every MIP item  $p$  in each time period  $t$ .

(Note: The inter-plant transportation of the MIP items is assumed to be instantaneous)

The objective function minimizes the inter-plant transportation cost as follows

$$\text{Minimize } \sum_{\forall p, i, s, t} (TMIP(p, i, s) * XMIPS(p, i, s, t)) \quad (3.17)$$

(Note: When  $s=i$  the unit transportation cost is zero, i.e.,  $TMIP(p, i, i) \equiv 0$  for all  $p$  &  $i$ )

### 3.3.3.3 Warehouse Inventory and Transportation

The finished products assembled in each plant  $i$  are shipped to the different warehouses  $j$  for every time period  $t$ :

$$X(i, t) = \sum_j y(i, j, t) \quad \forall i, t \quad (3.18)$$

(Note: There is no inventory of finished products at the plants)

The inventory stored in the warehouses must be within their capacity limits for each time period  $t$ :

$$\varphi(j,t) \leq WCap(j) \quad \forall j,t \quad (3.19)$$

Considering the products flowing in and out of each warehouse  $j$  and the inventory stored, the flow balance equations can be written as follows,

*Inventory at the end of time period  $t-1$  + Shipment received from manufacturing plants in time period  $t$  = Inventory at the end of time period  $t$  + Shipment sent to retailers in time period  $t$ , i.e.,*

$$\text{For } t \geq 2; \quad \varphi(j,t-1) + \sum_i y(i,j,t-1) = \varphi(j,t) + \sum_{k,m} z(j,k,t,m) \quad \forall j \quad (3.20)$$

$$\text{Note: For } t=1; \quad WInitialInv(j) = \varphi(j,1) + \sum_{k,m} z(j,k,1,m) \quad \forall j \quad (3.21)$$

The parameter  $(t-1)$  of the variable  $y(i,j,t-1)$  in Equation (3.20) indicates that it takes one time period for the finished products to reach the warehouses from the manufacturing plants through air. Equation (3.21) is the same as Equation (3.20) for the first time period  $t=1$  assuming that some initial inventory  $WInitialInv(j)$  is already present in the warehouses at  $t=0$ . The products shipped to the all retailers  $z(j,k,t,m)$  using the various modes of transportation  $m=1$  (Air),  $m=2$  (Rail) and  $m=3$  (Road) must meet their capacity limits  $M1$ ,  $M2$  and  $M3$  respectively.

$$\begin{aligned} z(j,k,t,1) &\leq M1 & \forall j,k,t \\ z(j,k,t,2) &\leq M2 & \forall j,k,t \\ z(j,k,t,3) &\leq M3 & \forall j,k,t \end{aligned} \quad (3.22)$$

The objective function minimizes the total transportation cost and the inventory holding cost for each time period  $t$  as follows:

$$\text{Minimize } \sum_{i,j,t} (TPROD(i,j) * y(i,j,t)) + \sum_{j,t} (Hw(j) * \varphi(j,t)) \quad (3.23)$$

### 3.3.3.4 Retailer Inventory and Transportation

The inventory stored at the retailer must meet the capacity limit for every retailer  $k$  in each time period  $t$ :

$$RInv(k,t) \leq RCap(k) \quad \forall k,t \quad (3.24)$$

Considering the product flowing into the retailers, customer demand, inventory stored and backorders, we write the flow balance equation at the retailer as follows;

*Initial Inventory + Shipment received from warehouse + Backorders at the end of time period  $t =$  Retailer demand + Final inventory + Backorders at the end of time period  $t-1$*

$$\text{For } t \geq 2; RInv(k,t-1) + \sum_{j,m} z(j,k,t-m,m) + RBO(k,t) = R(k,t) + RInv(k,t) + RBO(k,t-1) \quad \forall k \quad (3.25)$$

$$\text{Note: For } t=1; RInitialInv(k) + RBO(k,1) = R(k,1) + RInv(k,1) \quad \forall k \quad (3.26)$$

The parameter  $(t-m)$  of the variable  $z(j,k,t-m,m)$  in equation (3.25) indicates the  $m$  different time periods taken for the products to reach the retailers from the warehouses depending upon the three modes of transportation. The variable is not defined or assumed to be zero when  $t-m$  becomes negative or zero, i.e., for initial cases such as  $t=2, m=3$  and  $t=3, m=3$  respectively. Equation (3.26) is the same as equation (3.25) for the first time period assuming some initial inventory  $RInitialInv(k)$  at every retailer  $k$ .

The backorders at the retailers for the last time period  $T$  is assumed to be zero since all retailer demands have to be met by the end of time period  $T$ .

$$RBO(k,T) = 0 \quad \forall k \quad (3.27)$$



The objective function minimizes the total transportation, inventory holding and backorder costs for each time period  $t$  as follows.

$$\text{Minimize } \sum_{j,k,t,m} (TWAR_{j,k,m} * z(j,k,t,m)) + \sum_{k,t} (Hr(k) * RInv(k,t)) + \sum_{k,t} (RBackcost(k) * RBO(k,t)) \quad (3.28)$$

### 3.4 Mathematical Model

The optimization problem formulated above comprises the objective function which minimizes the total cost involved subject to all the constraints defined earlier;

Minimize Total Cost,  $Z =$

$$\begin{aligned} & \sum_{n,h,t} (cpu(n,h,t) * w(n,h,t)) + \sum_{n,h,i,t} (TSUP(n,h,i) * u(n,h,i,t)) + \sum_{i,t} (Pprod(i) * X(i,t)) \\ & + \sum_{p,i,t} (CMIP(p,i,t) * XMIP(p,i,t)) + \sum_{p,i,s,t} (TMIP(p,i,s) * XMIPS(p,i,s,t)) \\ & + \sum_{i,j,t} (TPROD(i,j) * y(i,j,t)) + \sum_{j,k,t,m} (TWAR(j,k,m) * z(j,k,t,m)) \\ & + \sum_{j,t} (Hw(j) * \varphi(j,t)) + \sum_{k,t} (Hr(k) * RInv(k,t)) + \sum_{k,t} (RBackcost(k) * RBO(k,t)) \end{aligned}$$

subject to the constraints

$$Qstd(n,h,t) \geq Qstdset(n) * S\alpha(n,h,t) \quad \forall n,h,t$$

$$w(n,h,t) \leq SCap(n,h) * S\alpha(n,h,t) \quad \forall n,h,t$$

$$w(n,h,t) \geq SMin(n,h) * S\alpha(n,h,t) \quad \forall n,h,t$$

$$\sum_h S\alpha(n,h,t) = 1 \quad \forall n,t$$

$$S\alpha(n,h,t) \in [0,1] \quad \forall n,h,t$$

$$w(n, h, t) = \sum_i u(n, h, i, t) \quad \forall n, h, t$$

For  $t \geq 2$ ;  $\sum_h u(n, h, i, t-1) = v(n, i, t) \quad \forall n, i, t$

$$X(i, t) \leq PCap(i) \quad \forall i, t$$

$$XMIP(p, i, t) \leq MIPCap(p, i) \quad \forall p, i, t$$

$$v(n, i, t) = \alpha(n) * X(i, t) \quad \forall n, i, t$$

$$MIPReq(p, i, t) = \beta(p) * X(i, t) \quad \forall p, i, t$$

$$\sum_s XMIPS(p, i, s, t) \leq XMIP(p, i, t) \quad \forall p, i, t$$

$$\sum_{\forall i} XMIPS(p, i, s, t) \geq MIPReq(p, s, t) \quad \forall p, s, t$$

$$X(i, t) = \sum_j y(i, j, t) \quad \forall i, t$$

$$\varphi(j, t) \leq WCap(j) \quad \forall j, t$$

For  $t \geq 2$ ;  $\varphi(j, t-1) + \sum_i y(i, j, t-1) = \varphi(j, t) + \sum_{k, m} z(j, k, t, m) \quad \forall j$

For  $t=1$ ;  $WInitialInv(j) = \varphi(j, 1) + \sum_{k, m} z(j, k, 1, m) \quad \forall j$

$$z(j, k, t, 1) \leq M1 \quad \forall j, k, t$$

$$z(j, k, t, 2) \leq M2 \quad \forall j, k, t$$

$$z(j, k, t, 3) \leq M3 \quad \forall j, k, t$$

$$RInv(k, t) \leq RCap(k) \quad \forall k, t$$

For  $t \geq 2$ ;  $RInv(k, t-1) + \sum_{j, m} z(j, k, t-m, m) + RBO(k, t) = R(k, t) + RInv(k, t) + RBO(k, t-1)$

 $\forall k$

$$\begin{aligned} \text{For } t=1; \quad R_{InitialInv}(k) + RBO(k,1) &= R(k,1) + R_{Inv}(k,1) && \forall k \\ RBO(k,T) &= 0 && \forall k \end{aligned}$$

The next chapter illustrates the above problem's implementation and solution for a simulated data set followed by risk analysis.

## Chapter 4

### MODEL IMPLEMENTATION AND ANALYSIS

In this chapter, the model formulated earlier is tested for a set of realistic data and the results are discussed followed by risk analysis.

#### 4.1 Given Data

Number of raw materials,  $n = 2$

Number of suppliers,  $h = 3$

Number of plants,  $i = 3$

Number of in-plant items,  $p = 2$

Number of warehouses,  $j = 2$

Number of retailers,  $k = 4$

Number of time periods,  $t = 10$  ( $t = 1, 2, 3, \dots, 10$ )

Number of modes of transport,  $m = 3$

Number of raw materials required to produce one finished product,  $\alpha=1$

Number of MIP items required to produce one finished product,  $\beta=1$

Table 4-1 summarizes the data for the raw materials and suppliers. Values for quality are based on a rating scale (1-Low to 10-High) such that high values represent better quality.

Table 4-2 contains plant data and Table 4-3 includes all the data for the MIP items.

Table 4-1: Raw materials and Suppliers data

	Supplier 1		Supplier 2		Supplier 3	
	Raw Material 1	Raw Material 2	Raw Material 1	Raw Material 2	Raw Material 1	Raw Material 2
Quality Standard set	5	6	5	6	5	6
Quality provided for 10 time periods	8,9,6,2,8, 7,6,8,4,7	8,6,5,9,7, 6,7,6,9,7	5,6,7,8,4, 7,6,5,7,5	8,5,5,8,6, 7,4,8,8,6	6,4,8,5,6, 8,4,7,8,5	5,9,7,6,7, 8,4,7,6,5
Maximum capacity (units)	5500	5200	5400	5000	6750	5750
Minimum order quantity (units)	3800	3750	3500	3250	4500	3900
Cost per unit for 10 time periods (\$)	3,4,1,2,3, 1,5,7,6,8	8,5,3,4,2, 5,4,7,5,6	6,4,6,8,4, 7,5,6,3,9	7,3,4,3,2, 6,4,5,2,8	6,7,8,5,5, 7,6,8,6,4	3,5,4,7,4, 2,5,6,7,8

Table 4-2: Plant data

	Plant 1	Plant 2	Plant 3
Production capacity (number of units)	2000	2500	3500
Production cost for finished products	\$70/unit	\$60/unit	\$65/unit

Table 4-3: MIP items data

	Plant 1		Plant 2		Plant 3	
	MIP item 1	MIP item 2	MIP item 1	MIP item 2	MIP item 1	MIP item 2
Production capacity	2500	2750	3000	3000	2250	3150
Cost per unit for 10 time periods (\$)	12,15,18, 20,16,22, 18,19,15, 12	14,15,16, 12,13,11, 16,18,14, 17	11,20,17, 19,16,15, 13,12,21, 20	15,16,14, 15,16,12, 13,11,14, 13	19,16,18, 17,22,15, 17,19,12, 11	12,16,14, 15,20,23, 24,21,19, 17

For simplicity, we assume that the unit transportation cost for shipping the raw materials from the suppliers to the plants,  $TSUP(n,h,i)$ , is independent of the raw material, supplier, and plant, i.e.,

$$TSUP(n,h,i) = Thi = \$1.5/\text{unit}$$

Similarly, we assume that the unit cost for inter-plant transportation of the  $p^{\text{th}}$  MIP item is independent of the MIP item and plant, i.e.,

$$TMIP(p,i,s) = Tis = \$1.5/\text{unit}$$

Also, we assume that the unit transportation cost of the finished product from plants to warehouses is independent of the plant and warehouse, i.e.,

$$TPROD(i,j) = Tij = \$1.5/\text{unit}$$

Table 4-4: Warehouse data

	Warehouse 1	Warehouse 2
Initial inventory, $WInitialInv(j)$	300	300
Maximum holding capacity, $WCap(j)$	1900	2000
Inventory holding cost per unit time period, $Hw(j)$	\$2.1/unit	\$2.5/unit

Table 4-5: Retailer data

	Retailer 1	Retailer 2	Retailer 3	Retailer 4
Initial inventory, $RInitialInv(k)$	150	150	150	150
Maximum inventory capacity, $RCap(k)$	1100	1200	950	1000
Inventory holding cost per unit time period, $Hr(k)$	\$4.5/unit	\$4.7/unit	\$5.2/unit	\$4.5/unit
Backorder cost per unit time period, $RBackcost(k)$	\$9.5/unit	\$10.2/unit	\$10.5/unit	\$9.5/unit

Table 4-6: Retailer's Demand data

	Customer's demand at the retailers (units)			
	Retailer 1	Retailer 2	Retailer 3	Retailer 4
Time period 1	246	229	398	292
Time period 2	346	231	400	294
Time period 3	124	389	397	229
Time period 4	250	297	245	286
Time period 5	365	406	249	347
Time period 6	269	307	347	268
Time period 7	372	424	251	279
Time period 8	173	153	249	283
Time period 9	235	342	272	285
Time period 10	238	372	382	190

The shipment of finished products from warehouses to retailers is done using three different modes of transportation which have different lead times, unit transportation costs and capacity limits as listed in Table 4-7.

Table 4-7: Warehouse to Retailer Transportation data

	Air	Rail	Road
Lead time of shipment	1 time period	2 time periods	3 time periods
Cost of transportation	\$1.5/unit	\$1/unit	\$0.5/unit
Maximum capacity limit (units)	1000	3000	2000

## 4.2 Results

The optimization software, LINGO is used to generate the mixed integer linear program and solve it. Appendix A is the LINGO code for this problem. Necessary

description for the defined variables, constraints and data are given in the code. The problem generates a total of 1071 variables (60 integer and 1011 continuous) and 1506 constraints. It takes 1791 iterations to find the global solution for this problem using 315 KB of memory space under the Branch and Bound algorithm. The total cost incurred for running the supply chain is \$ 3,573,070. The other results obtained in the solution report are tabulated as follows;

Table 4-8: Supplier selection and Order quantity

	Supplier selected		Quantity ordered	
	Material 1	Material 2	Material 1	Material 2
Time period 1	2	2	3500	3500
Time period 2	2	1	3750	3750
Time period 3	1	3	3900	3900
Time period 4	2	2	3500	3500
Time period 5	1	2	3800	3800
Time period 6	2	2	3500	3500
Time period 7	2	1	3750	3750
Time period 8	2	2	3500	3500
Time period 9	2	2	3500	3500
Time period 10	3	1	4500	3750

It is clear from the above table that the 2<sup>nd</sup> supplier is selected for most of the time periods for both the raw materials. Another interesting observation is that the quantities ordered are very close to the minimum order quantity values for each raw material as seen in Table 4-1.



Table 4-9: Transportation quantity from suppliers to plants

	Raw Material 1			Raw Material 2		
	From Supplier	To Plant	Quantity shipped	From Supplier	To Plant	Quantity shipped
Time Period 1	2	2	2500	2	2	2500
	3	3	1000	2	3	1000
Time Period 2	2	2	2500	1	2	2500
	2	3	1250	1	3	1250
Time Period 3	1	2	2500	3	2	2500
	1	3	1400	3	3	1400
Time Period 4	2	2	2500	2	2	2500
	2	3	1000	2	3	1000
Time Period 5	1	2	2500	2	2	2500
	1	3	1300	2	3	1300
Time Period 6	2	2	2500	2	2	2500
	2	3	1000	2	3	1000
Time Period 7	2	2	2500	1	2	2500
	2	3	1250	1	3	1250
Time Period 8	2	2	2500	2	2	2500
	2	3	1000	2	3	1000
Time Period 9	2	2	2500	2	2	2500
	2	3	1000	2	3	1000
Time Period 10	3	3	4500	1	3	3750

Table 4-10: Finished product and MIP items production in plants

	Plant 1			Plant 2			Plant 3		
	X(1,t)	MIP Item 1	MIP Item 2	X(2,t)	MIP Item 1	MIP Item 2	X(3,t)	MIP Item 1	MIP Item 2
Time period 1	0	0	0	0	0	0	0	0	0
Time period 2	0	2500	0	2500	0	2500	1000	1000	1000
Time period 3	0	0	0	2500	2500	2500	1250	1250	1250
Time period 4	0	0	2750	2500	1650	1150	1400	2250	0
Time period 5	0	500	2750	2500	3000	750	1000	0	0
Time period 6	0	0	1300	2500	2500	2500	1300	1300	0
Time period 7	0	0	500	2500	3000	3000	1000	500	0
Time period 8	0	0	750	2500	3000	3000	1250	750	0
Time period 9	0	1250	500	2500	0	3000	1000	2250	0

	Plant 1			Plant 2			Plant 3		
	X(1,t)	MIP Item 1	MIP Item 2	X(2,t)	MIP Item 1	MIP Item 2	X(3,t)	MIP Item 1	MIP Item 2
Time period 10	0	1250	0	2500	0	3000	1000	2250	500

It can be seen that there is no production of the finished products or MIP items during first period as  $X(i,1)=0$  for all  $i$ (see chapter 3). Plant 1 doesn't produce any finished products at all as it has the highest unit production cost as in Table 4-2 but it still produces MIP items during some time periods which are transported to other plants. Also, plant 2 runs at its full capacity for the finished products during all time periods as it has the lowest production cost.

Table 4-11: Inter-plant transportation of MIP items

	MIP item 1			MIP item 2		
	From Plant	To Plant	Quantity shipped	From Plant	To Plant	Quantity shipped
Time Period 2	1	2	2500	2	2	2500
	3	3	1000	3	3	1000
Time Period 3	2	2	2500	2	2	2500
	3	3	1250	3	3	1250
Time Period 4	2	2	1650	2	2	1150
	3	2	850	1	2	1350
	3	3	1400	1	3	1400
Time Period 5	2	2	2500	1	3	1000
	1	3	500	1	2	1750
	2	3	500	2	2	750
Time Period 6	2	2	2500	1	3	1300
	3	3	1300	2	2	2500
Time Period 7	2	2	2500	1	3	500
	2	3	500	2	3	500
	3	3	500	2	2	2500

	MIP item 1			MIP item 2		
	From Plant	To Plant	Quantity shipped	From Plant	To Plant	Quantity shipped
Time Period 8	2	2	2500	1	3	750
	2	3	500	2	3	500
	3	3	750	2	2	2500
Time Period 9	1	2	1250	2	2	2500
	3	2	1250	1	3	500
	3	3	1000	2	3	500
Time Period 10	1	2	1250	2	2	2500
	3	2	1250	2	3	500
	3	3	1000	3	3	500

Note: When the shipment is between the same plant, i.e., when  $i=s$  (from plant = to plant) it means the internal use of the MIP items. The variables  $v(n,i,t)$  and  $MIPReq(p,i,t)$  are same as  $X(i,t)$  as  $\alpha=1$  and  $\beta=1$ .

Table 4-12: Finished products transportation from plants to warehouses

Time period	Manufacturing plant	Warehouse	Quantity shipped
2	2	2	2500
	3	1	996
	3	2	4
3	2	1	2500
	3	1	154
	3	2	1096
4	2	1	2500
	3	1	105
	3	2	1295
5	2	1	1557
	2	2	943
	3	2	1000
6	2	2	2500
	3	1	1107
	3	2	193
7	2	2	2500
	3	2	1000

Time period	Manufacturing plant	Warehouse	Quantity shipped
8	2	2	2500
	3	2	1250
9	2	2	2500
	3	2	1000
10	2	2	2500
	3	2	1000

We can see that there is zero shipment from plant 1 to any of the warehouses as it doesn't produce any finished products. The shipments during the first time period are zero.

Table 4-13: Warehouse inventory

Time period	Warehouse 1	Warehouse 2
5	1900	1055
6	1900	2000
7	1900	2000

There is no inventory stored during any of the time periods except for periods 5, 6 and 7 for the entire time horizon. Also, the initial inventory stored before the start of the time horizon is shipped immediately during the first time period itself to satisfy retailer demands. From Table 4-13, it is to be noted that the cumulative inventory levels at the end of these three time periods at both the warehouses are their maximum capacity limits except for the 2<sup>nd</sup> warehouse at the end of the 5<sup>th</sup> time period. This is because the inventory holding cost of the 1<sup>st</sup> warehouse is less than that of the 2<sup>nd</sup> warehouse as given in Table 4-4.

Table 4-14: Finished products transportation from warehouses to retailers

Time Period	Air (m=1)			Rail (m=2)			Road (m=3)		
	j	k	z	j	k	z	j	k	z
1	1	3	300	-	-	-	-	-	-
	2	3	300						
3	1	2	996	-	-	-	2	1	47
	2	1	816						
	2	3	690						
	2	4	951						
4	1	2	406	1	2	307	1	1	372
	2	1	365	1	3	347	1	2	424
	2	3	249	1	4	268	1	3	251
	2	4	347	2	1	135	1	4	279
5	2	1	87	-	-	-	1	1	173
							1	3	249
							1	4	283
							2	2	153
6	-	-	-	-	-	-	1	3	272
							1	4	1285
							2	1	656
							2	2	342
7	-	-	-	-	-	-	1	1	917
							1	4	190
							2	2	1572
							2	3	1121
8	-	-	-	-	-	-	1	3	1900
							2	1	2000
							2	2	2000
							2	4	1500
9	-	-	-	-	-	-	2	1	2000
							2	4	1750
10	-	-	-	-	-	-	2	1	2000
							2	3	1500

The index set  $z$  represents the shipment amount transported from warehouse  $j$  to retailer  $k$ . From the table, it can be seen that initially air is used as the mode of transport to meet the retailer's demands quickly as it is the fastest mode of transport. But later as time

progresses, i.e., as production and inventory builds up, road is used as the mode of transport which is the cheapest option even though it is the slowest (see Table 4-7). There are no products transported during the second time period. Another point to be noted is the amount of products shipped using road during the last part of the time horizon which sometimes is almost equivalent to its capacity limit. This indicates the need for products at the retailers during the end of the time horizon to make the cumulative backorders zero as we assumed that  $RBO(k,10)=0$

Table 4-15: Retailer backorders

Time period	Retailer 1	Retailer 2	Retailer 3	Retailer 4
1	96	79	248	142
2	442	310	48	436
3	566	699	445	665

There are backorders only during the first three time periods for the entire time horizon. This indicates that the initial inventory stored at the warehouses and retailers is not able to meet the initial retailer's demand completely. Later as time progresses, the production starts and the shipment flows towards the retailers thus meeting the customer's demands completely. This is also indicative that a product manufactured at the plant at least takes two time periods to reach the retailer which is the sum of the fastest lead times of transportation across the three stages, i.e., plants, warehouses and retailers (considering no production during first time period and air as mode of transport across warehouse to retailer).

Table 4-16: Retailer inventory

Time period	Retailer 1	Retailer 2	Retailer 3	Retailer 4
9	421	-	-	1000
10	1100	1200	739	1000

Inventory is stored only at the end of time periods 9 and 10. This indicates that the plant production is assigned in a manner to meet the customer's demand only and not to store excess inventory. The cumulative inventory at the end of the 9<sup>th</sup> time period is stored in the 1<sup>st</sup> and the 4<sup>th</sup> retailer as the holding cost for these retailers is less when compared to the holding cost for the 3<sup>rd</sup> and 4<sup>th</sup> retailer as in Table 4-5. Also, it is to be noted that the cumulative inventory stored at the 1<sup>st</sup>, 2<sup>nd</sup> and the 4<sup>th</sup> retailer at the end of the time horizon, i.e., at the end of the 10<sup>th</sup> time period are their respective inventory holding capacity limits as in Table 4-5.

### 4.3 Risk Analysis

Two different conditions of risk are simulated to study their effects;

#### 4.3.1 Supplier Price Increase

For the case of illustration, if we assume that the 2<sup>nd</sup> supplier doubles the prices (cost per unit), i.e., an increase of price by 100% (see Appendix B for changed data in LINGO code) then we get a new global optimal solution of \$3,733,080. The total cost increases by 4.4 %.

Table 4-17: Case 1: Supplier selection and Order quantity

	Supplier selected		Quantity ordered	
	Material 1	Material 2	Material 1	Material 2
Time period 1	2	2	3500	3500
Time period 2	<b>1</b>	1	<b>3800</b>	<b>3800</b>
Time period 3	1	3	3900	3900
Time period 4	2	2	3500	3500
Time period 5	1	<b>1</b>	3800	3800
Time period 6	<b>1</b>	<b>3</b>	<b>3900</b>	<b>3900</b>
Time period 7	<b>1</b>	1	<b>3800</b>	<b>3800</b>
Time period 8	2	2	3500	3500
Time period 9	2	2	3500	3500
Time period 10	3	1	4500	3750

The new suppliers and the order quantities for different time periods are tabulated as in Table 4-17. The values that have changed as compared to the values in Table 4-8 are highlighted.

#### 4.3.2 Cease of mode of transportation

Again for the case of illustration, we assume that the 2<sup>nd</sup> mode of transportation between warehouse and retailer (rail) ceases to function, i.e., no units can be shipped using the 2<sup>nd</sup> mode of transport across the last two stages (see Appendix C for changed data in LINGO code). Then we arrive at a new global optimum solution of \$3,575,818. The total cost decreases by just 0.078%.



Table 4-18: Case 2: Finished product transportation from warehouses to retailers

Time Period	Air (m=1)			Road (m=3)		
	From Warehouse	To Retailer	Quantity shipped	From Warehouse	To Retailer	Quantity shipped
1	1	3	300	-	-	-
	2	3	300			
3	1	1	816	1	1	47
	1	2	686			
	1	4	951			
	2	2	310			
	2	3	690			
4	1	1	365	1	1	372
	1	2	406	1	2	424
	1	3	249	1	3	251
	1	4	347	1	4	279
5	1	1	222	1	1	173
				1	2	91
	1	2	307	1	3	249
	1	3	347	2	2	62
	1	4	268	2	4	283
6	-	-	-	1	1	1335
				1	2	342
				1	3	272
				2	4	606
7	-	-	-	1	1	238
				1	2	1572
				1	3	1121
				1	4	869
8	-	-	-	1	3	1900
				2	1	2000
				2	2	2000
				2	4	1500
9	-	-	-	1	3	1250
				2	1	2000
				2	2	500
10	-	-	-	2	1	2000
				2	2	1500

The new shipments quantities obtained using the available two modes of transportation are tabulated in Table 4-18. There is not much of a significant change in the way in which the priority is given to the fastest and slowest modes of transportation as time progresses from the start to the end of the horizon as compared to Table 4-14.

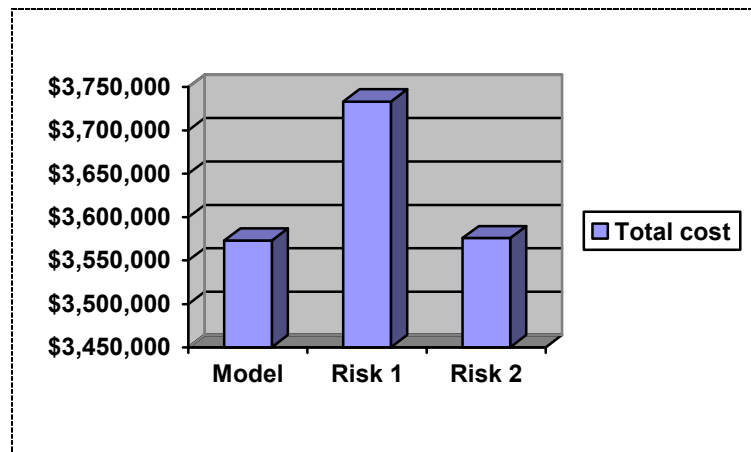


Fig 4-1: Comparison of total cost

The total cost incurred for all the three scenarios discussed so far is compared using a bar chart as in Fig 4-1.

## **Chapter 5**

### **CONCLUSIONS AND FUTURE RESEARCH**

In this chapter we discuss some conclusions and recommendations for further research.

#### **5.1 Conclusions**

The literature pertaining to areas of interest was reviewed. It was concluded that previous work was done in the areas of supplier selection and supply chain modeling but they did not address both the problems collaboratively. The goal of this thesis was to develop a single integrated collaborative model for a four stage supply chain involving most of the strategic and tactical decisions faced by a real world, such as supplier selection, assignment of production quantities, inventory levels, stock-outs and shipment amounts.

The four stages considered were Suppliers, Manufacturers, Wholesalers and Retailers. Supplier selection process involved choosing of suppliers based on quality, capacity and cost. Procurement cost was incurred for all the raw materials ordered by the manufacturing plants. Also, some parts were produced internally by the manufacturing plants known as MIP items. Not all the plants had the capacity to produce these MIP items. This called for inter-plant transportation of the MIP items between the manufacturing plants. Production cost and inter-plant transportation cost was involved at the plant stage. The finished products were shipped to the warehouses using a dedicated

mode of transport. Inventory holding cost was incurred at the warehouse stage. This inventory was shipped to the retailers using three different modes of transportation to satisfy the customer's demand. Costs were also incurred for holding inventory and stock-outs at the retailers. There were capacity restrictions at every stage of the supply chain. The primary objective of the problem was to minimize the total cost incurred across all the four stages over multiple time periods.

The underlying assumptions, variables and constraints were defined as per the problem description. This resulted in a mixed integer linear programming problem. The problem was solved with a set of simulated data using the optimization software, LINGO. The results obtained were tabulated and thus the model implementation was validated accordingly. Finally, the model was analyzed for certain risk conditions, such as, supplier price increase and disruption in a mode of transportation. The effective results were studied and its changes were discussed.

The whole idea behind this research was to provide a universal platform for the decision makers running a firm to take and implement their managerial decisions. This kind of framework would also help them to develop strategies without any lack of communication. The visibility can be achieved by incorporating the framework into the company's infrastructure. The most important significance of this work is that scenarios that can't be visualized clearly due to its practical restrictions can be analyzed by simulating it using this model. These scenarios include changes such as, closing of a particular warehouse, opening of new a retailer outlet, introducing a new product into the product family. The effects of these changes can be studied to decide whether it would benefit or not before implementing it into the corporation's core policy.

## 5.2 Future Research

There is a lot of scope for further research for this work. The supplier selection was accomplished here based only on three important criteria, i.e., quality, capacity and cost. But as such in the real world, supplier selection is a contract negotiation process involving much more complex criteria. Some of these criterions which have a global impact on the supplier selection process are location proximity, geographic conditions, international exchange rates, foreign trade policies etc. Even though most of them are hard to conceptualize in a mathematical format, at least some of them can be taken into account by considering non-traditional decision making methods such as AHP (Analytical hierarchy process).

The single objective model can be extended to a multi-objective problem as most of the supply chain models nowadays not only concentrate on minimizing the total cost involved or maximizing the profit obtained but also emphasize a lot on customer responsiveness. This can be achieved by minimizing the backorders or by reducing the lead time involved in delivering the products. The multi-criteria problem formulated can be solved using goal programming techniques.

The transportation between the initial stages, i.e., from suppliers to plants and plants to warehouses is assumed to follow a single mode of shipment. This assumption can be relaxed to include all three modes of shipments. Also, the transportation costs for all modes are assumed to follow a linear cost structure. This can be made more realistic by including price breaks or quantity discount models, i.e., by considering a stepwise function instead of a linear function. These kind of slightly complex freight rate functions

gives an opportunity to conceptualize real world situations such as, FTL (Full Truck Load) and LTL(less than truck load) etc.

The manufacturing process and also the inter-plant transportation of the MIP items is assumed to be instantaneous here which is unlike as in real world scenarios. There has to be some lead time associated with them. Although we assume JIT production to be followed at the manufacturing plants, there has to be some provision for holding inventory at the manufacturing plants too.

The cost structure in the plants considers only the production costs based on the number of units manufactured but there are also some other fixed and variable costs associated with plants in real world situations such as, machinery set-up cost, maintenance and repair costs, plant labor cost and other miscellaneous costs. Similarly, there are various costs associated with warehouses too. These can be incorporated into the model to an extent by considering the concept of owned and leased warehouses.

The supply chain modeled here has the provision for multiple raw materials and MIP products but there is only a single finished product flowing in the supply chain after the plant stage. But generally in practical conditions, there are many finished products manufactured by a single firm.

The demand at the retailers is assumed as deterministic in nature but generally even after extensive market research and forecasting, customer's demand usually ends up being stochastic in nature.

The model was validated using simulated data and the results were analyzed based on the same. But if available, it would be always interesting to study the effects of

actual real world data on the model to help the decision makers to incorporate managerial strategies into their company's core policies.

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## APPENDIX A

### LINGO CODE

```

model:
sets:
material/1..2/:Qstdset;
supplier/1..3/;
plant/1..3/:PCap,Pprod;
InplantItem/1..2/;
warehouse/1..2/:WCap, Hw, WInitialInv;
retailer/1..4/:RCap, Hr, RInitialInv, RBackcost;
period/1..10/;
transport/1..3/:TWAR;

masu(material, supplier):SCap, SMin;
Inpl(InplantItem, plant):MIPCap;
plper(plant,period):X;
waper(warehouse,period):phi;
reper(retailer,period):R,RInv,RBO;

plwaper(plant,warehouse,period):y;
maplpe(material, plant, period):v;
Inplpe(InplantItem, plant, period):CMIP, XMIP,MIPReq;
masupe(material, supplier, period):Qstd, cpu, w, Salpha;

masupape(material, supplier, plant, period):u;
Inplplpe(InplantItem, plant, plant, period):XMIPS;
warepertr(warehouse,retailer,period,transport):z;

endsets

!Formulation;

!Total cost objective function;

[totalcost]min=@sum(masupe(n,h,t):(cpu(n,h,t)*w(n,h,t)))+@sum(period(t)
:@sum(plant(i):X(i,t)*Pprod(i))
+ @sum(Inplplpe(p,i,s,t)|i#NE#s:(Tis*XMIPS(p,i,s,t))) +
@sum(Inplpe(p,i,t):(CMIP(p,i,t)*XMIP(p,i,t))
+ @sum(masupape(n,h,i,t):Thi*u(n,h,i,t)) +
@sum(plwaper(i,j,t):Tij*y(i,j,t))
+ @sum(waper(j,t):phi(j,t)*Hw(j)) +
@sum(reper(k,t):RInv(k,t)*Hr(k)) +
@sum(warepertr(j,k,t,m):z(j,k,t,m)*TWAR(m))
+ @sum(reper(k,t):RBO(k,t)*RBackcost(k));

!Supplier Selection;

@for(masupe(n,h,t):Qstd(n,h,t)>Qstdset(n)*Salpha(n,h,t));
@for(masupe(n,h,t):w(n,h,t)<SCap(n,h)*Salpha(n,h,t));
@for(masupe(n,h,t):w(n,h,t)>SMin(n,h)*Salpha(n,h,t));

```

```

@for(masupe(n,h,t):@BIN(Salpha(n,h,t)));
@for(period(t):@for(material(n):@sum(supplier(h):Salpha(n,h,t)=1)));

!Supplier Plant Transition phase;

@for(masupe(n,h,t):w(n,h,t)=@sum(plant(i):u(n,h,i,t)));
!For Time period 1;
@for(material(n):@for(plant(i):v(n,i,1)=0));
!For Time period 2 or more;
@for(maplpe(n,i,t)|t#GE#2:v(n,i,t)=@sum(supplier(h):u(n,h,i,t-1)));

!Plant equations;

@for(plper(i,t):X(i,t)<PCap(i));
@for(Inplpe(p,i,t):XMIP(p,i,t)<MIPCap(p,i));
@for(maplpe(n,i,t):X(i,t)=alpha*v(n,i,t));
@for(Inplpe(p,i,t):X(i,t)=beta*MIPReq(p,i,t));

@for(Inplpe(p,i,t):@sum(plant(s):XMIPS(p,i,s,t)<XMIP(p,i,t));
@for(Inplpe(p,s,t):@sum(plant(i):XMIPS(p,i,s,t)>MIPReq(p,s,t));

!Plant Warehouse Transition phase;

@for(plper(i,t):X(i,t)=@sum(warehouse(j):y(i,j,t)));

!Warehouse equations;

@for(waper(j,t):phi(j,t) < WCap(j));
!Period 1;
@for(warehouse(j):phi(j,1)=WInitialInv(j)-
@sum(retailer(k):@sum(transport(m):z(j,k,1,m))));
!Period 2 or more;
@for(waper(j,t)|t#GE#2:phi(j,t)=phi(j,t-1)+@sum(plant(i):y(i,j,t-1))-
@sum(retailer(k):@sum(transport(m):z(j,k,t,m))));
@for(warepertr(j,k,t,m):Z(j,k,t,1)<M1);
@for(warepertr(j,k,t,m):Z(j,k,t,2)<M2);
@for(warepertr(j,k,t,m):Z(j,k,t,3)<M3);

!Retailer equations;

!Period 1;
@for(retailer(k):R(k,1)+RInv(k,1)-RBO(k,1)=RInitialInv(k));
!Period 2;
@for(retailer(k):@sum(warehouse(j):z(j,k,1,1))=R(k,2)+RInv(k,2)-
RInv(k,1)+RBO(k,1)-RBO(k,2));
!Period 3;
@for(retailer(k):@sum(warehouse(j):z(j,k,2,1)+z(j,k,1,2))=R(k,3)+
RInv(k,3)-RInv(k,2)+RBO(k,2)-RBO(k,3));
!Period greater than 3;
@for(reper(k,t)|t#GT#3:@sum(warehouse(j):@sum(transport(m):z(j,k,t-
m,m)))=R(k,t)+RInv(k,t)-RInv(k,t-1)+RBO(k,t-1)-RBO(k,t));
@for(reper(k,t):RInv(k,t)<RCap(k));
@for(retailer(k):RBO(k,10)=0);

```

```
!Data set;
```

```
data:
```

```
Qstdset= 5,6;
Qstd=8,9,6,2,8,7,6,8,4,7,
5,6,7,8,4,7,6,5,7,5,
6,4,8,5,6,8,4,7,8,5,
8,6,5,9,7,6,7,6,9,7,
8,5,5,8,6,7,4,8,8,6,
5,9,7,6,7,8,4,7,6,5;
SCap= 5500,5400,6750,5200,5000,5750;
SMin= 3800,3500,4500,3750,3250,3900;
cpu=3,4,1,2,3,1,5,7,6,8,
6,4,6,8,4,7,5,6,3,9,
6,7,8,5,5,7,6,8,6,4,
8,5,3,4,2,5,4,7,5,6,
7,3,4,3,2,6,4,5,2,8,
3,5,4,7,4,2,5,6,7,8;
PCap=2000,2500,3500;
Pprod=70,60,65;
MIPCap=2500,3000,2250,2750,3000,3150;
CMIP=12,15,18,20,16,22,18,19,15,12,
11,20,17,19,16,15,13,12,21,20,
19,16,18,17,22,15,17,19,12,11,
14,15,16,12,13,11,16,18,14,17,
15,16,14,15,16,12,13,11,14,13,
12,16,14,15,20,23,24,21,19,17;
alpha=1;
beta=1;
Tis=1.5;
Thi=1.5;
Tij=1.5;
WCap=1900,2000;
RCap=1100,1200,950,1000;
RInitialInv = 150,150,150,150;
WInitialInv = 300,300;
Hw=2.1,2.5;
Hr=4.5,4.7,5.2,4.5;
RBackcost=9.5,10.2,10.5,9.5;
R=246,346,124,250,365,269,372,173,235,238,
229,231,389,297,406,307,424,153,342,372,
398,400,397,245,249,347,251,249,272,382,
292,294,229,286,347,268,279,283,285,190;
TWAR = 1.5,1,0.5;
M1=1000;
M2=3000;
M3=2000;
```

```
enddata
```

```
end
```

## APPENDIX B

### CASE 1: LINGO CODE

```

model:
sets:
material/1..2/:Qstdset;
supplier/1..3/;
plant/1..3/:PCap,Pprod;
InplantItem/1..2/;
warehouse/1..2/:WCap, Hw, WInitialInv;
retailer/1..4/:RCap, Hr, RInitialInv, RBackcost;
period/1..10/;
transport/1..3/:TWAR;

masu(material, supplier):SCap, SMin;
Inpl(InplantItem, plant):MIPCap;
plper(plant,period):X;
waper(warehouse,period):phi;
reper(retailer,period):R,RInv,RBO;

plwaper(plant,warehouse,period):y;
maplpe(material, plant, period):v;
Inplpe(InplantItem, plant, period):CMIP, XMIP,MIPReq;
masupe(material, supplier, period):Qstd, cpu, w, Salpha;

masupape(material, supplier, plant, period):u;
Inplplpe(InplantItem, plant, plant, period):XMIPS;
warepertr(warehouse,retailer,period,transport):z;

endsets

!Formulation;

!Total cost objective function;

[totalcost]min=@sum(masupe(n,h,t):(cpu(n,h,t)*w(n,h,t)))+@sum(period(t)
:@sum(plant(i):X(i,t)*Pprod(i))
+ @sum(Inplplpe(p,i,s,t)|i#NE#s:(Tis*XMIPS(p,i,s,t))) +
@sum(Inplpe(p,i,t):(CMIP(p,i,t)*XMIP(p,i,t))
+ @sum(masupape(n,h,i,t):Thi*u(n,h,i,t)) +
@sum(plwaper(i,j,t):Tij*y(i,j,t))
+ @sum(waper(j,t):phi(j,t)*Hw(j)) +
@sum(reper(k,t):RInv(k,t)*Hr(k)) +
@sum(warepertr(j,k,t,m):z(j,k,t,m)*TWAR(m))
+ @sum(reper(k,t):RBO(k,t)*RBackcost(k));

!Supplier Selection;

@for(masupe(n,h,t):Qstd(n,h,t)>Qstdset(n)*Salpha(n,h,t));
@for(masupe(n,h,t):w(n,h,t)<SCap(n,h)*Salpha(n,h,t));
@for(masupe(n,h,t):w(n,h,t)>SMin(n,h)*Salpha(n,h,t));

```



```

@for(masupe(n,h,t):@BIN(Salpha(n,h,t)));
@for(period(t):@for(material(n):@sum(supplier(h):Salpha(n,h,t)=1)));

!Supplier Plant Transition phase;

@for(masupe(n,h,t):w(n,h,t)=@sum(plant(i):u(n,h,i,t)));
!For Time period 1;
@for(material(n):@for(plant(i):v(n,i,1)=0));
!For Time period 2 or more;
@for(maplpe(n,i,t)|t#GE#2:v(n,i,t)=@sum(supplier(h):u(n,h,i,t-1)));

!Plant equations;

@for(plper(i,t):X(i,t)<PCap(i));
@for(Inplpe(p,i,t):XMIP(p,i,t)<MIPCap(p,i));
@for(maplpe(n,i,t):X(i,t)=alpha*v(n,i,t));
@for(Inplpe(p,i,t):X(i,t)=beta*MIPReq(p,i,t));

@for(Inplpe(p,i,t):@sum(plant(s):XMIPS(p,i,s,t))<XMIP(p,i,t));
@for(Inplpe(p,s,t):@sum(plant(i):XMIPS(p,i,s,t))>MIPReq(p,s,t));

!Plant Warehouse Transition phase;

@for(plper(i,t):X(i,t)=@sum(warehouse(j):y(i,j,t)));

!Warehouse equations;

@for(waper(j,t):phi(j,t) < WCap(j));
!Period 1;
@for(warehouse(j):phi(j,1)=WInitialInv(j)-
@sum(retailer(k):@sum(transport(m):z(j,k,1,m))));
!Period 2 or more;
@for(waper(j,t)|t#GE#2:phi(j,t)=phi(j,t-1)+@sum(plant(i):y(i,j,t-1))-
@sum(retailer(k):@sum(transport(m):z(j,k,t,m))));
@for(warepertr(j,k,t,m):Z(j,k,t,1)<M1);
@for(warepertr(j,k,t,m):Z(j,k,t,2)<M2);
@for(warepertr(j,k,t,m):Z(j,k,t,3)<M3);

!Retailer equations;

!Period 1;
@for(retailer(k):R(k,1)+RInv(k,1)-RBO(k,1)=RInitialInv(k));
!Period 2;
@for(retailer(k):@sum(warehouse(j):z(j,k,1,1))=R(k,2)+RInv(k,2)-
RInv(k,1)+RBO(k,1)-RBO(k,2));
!Period 3;
@for(retailer(k):@sum(warehouse(j):z(j,k,2,1)+z(j,k,1,2))=R(k,3)+
RInv(k,3)-RInv(k,2)+RBO(k,2)-RBO(k,3));
!Period greater than 3;
@for(reper(k,t)|t#GT#3:@sum(warehouse(j):@sum(transport(m):z(j,k,t-
m,m)))=R(k,t)+RInv(k,t)-RInv(k,t-1)+RBO(k,t-1)-RBO(k,t));
@for(reper(k,t):RInv(k,t)<RCap(k));
@for(retailer(k):RBO(k,10)=0);

```

```
!Data set;
```

```
data:
```

```
Qstdset= 5,6;
Qstd=8,9,6,2,8,7,6,8,4,7,
5,6,7,8,4,7,6,5,7,5,
6,4,8,5,6,8,4,7,8,5,
8,6,5,9,7,6,7,6,9,7,
8,5,5,8,6,7,4,8,8,6,
5,9,7,6,7,8,4,7,6,5;
SCap= 5500,5400,6750,5200,5000,5750;
SMin= 3800,3500,4500,3750,3250,3900;
cpu=3,4,1,2,3,1,5,7,6,8,
12,8,12,16,8,14,10,12,6,18,
6,7,8,5,5,7,6,8,6,4,
8,5,3,4,2,5,4,7,5,6,
14,6,8,6,4,12,8,10,4,16,
3,5,4,7,4,2,5,6,7,8;
PCap=2000,2500,3500;
Pprod=70,60,65;
MIPCap=2500,3000,2250,2750,3000,3150;
CMIP=12,15,18,20,16,22,18,19,15,12,
11,20,17,19,16,15,13,12,21,20,
19,16,18,17,22,15,17,19,12,11,
14,15,16,12,13,11,16,18,14,17,
15,16,14,15,16,12,13,11,14,13,
12,16,14,15,20,23,24,21,19,17;
alpha=1;
beta=1;
Tis=1.5;
Thi=1.5;
Tij=1.5;
WCap=1900,2000;
RCap=1100,1200,950,1000;
RInitialInv = 150,150,150,150;
WInitialInv = 300,300;
Hw=2.1,2.5;
Hr=4.5,4.7,5.2,4.5;
RBackcost=9.5,10.2,10.5,9.5;
R=246,346,124,250,365,269,372,173,235,238,
229,231,389,297,406,307,424,153,342,372,
398,400,397,245,249,347,251,249,272,382,
292,294,229,286,347,268,279,283,285,190;
TWAR = 1.5,1,0.5;
M1=1000;
M2=3000;
M3=2000;
```

```
enddata
```

```
end
```

## APPENDIX C

### CASE 2: LINGO CODE

```

model:
sets:
material/1..2/:Qstdset;
supplier/1..3/;
plant/1..3/:PCap,Pprod;
InplantItem/1..2/;
warehouse/1..2/:WCap, Hw, WInitialInv;
retailer/1..4/:RCap, Hr, RInitialInv, RBackcost;
period/1..10/;
transport/1..3/:TWAR;

masu(material, supplier):SCap, SMin;
Inpl(InplantItem, plant):MIPCap;
plper(plant,period):X;
waper(warehouse,period):phi;
reper(retailer,period):R,RInv,RBO;

plwaper(plant,warehouse,period):y;
maplpe(material, plant, period):v;
Inplpe(InplantItem, plant, period):CMIP, XMIP,MIPReq;
masupe(material, supplier, period):Qstd, cpu, w, Salpha;

masupape(material, supplier, plant, period):u;
Inplplpe(InplantItem, plant, plant, period):XMIPS;
warepertr(warehouse,retailer,period,transport):z;

endsets

!Formulation;

!Total cost objective function;

[totalcost]min=@sum(masupe(n,h,t):(cpu(n,h,t)*w(n,h,t)))+@sum(period(t)
:@sum(plant(i):X(i,t)*Pprod(i))
+ @sum(Inplplpe(p,i,s,t)|i#NE#s:(Tis*XMIPS(p,i,s,t))) +
@sum(Inplpe(p,i,t):(CMIP(p,i,t)*XMIP(p,i,t)))
+ @sum(masupape(n,h,i,t):Thi*u(n,h,i,t)) +
@sum(plwaper(i,j,t):Tij*y(i,j,t))
+ @sum(waper(j,t):phi(j,t)*Hw(j)) +
@sum(reper(k,t):RInv(k,t)*Hr(k)) +
@sum(warepertr(j,k,t,m):z(j,k,t,m)*TWAR(m))
+ @sum(reper(k,t):RBO(k,t)*RBackcost(k));

!Supplier Selection;

@for(masupe(n,h,t):Qstd(n,h,t)>Qstdset(n)*Salpha(n,h,t));
@for(masupe(n,h,t):w(n,h,t)<SCap(n,h)*Salpha(n,h,t));
@for(masupe(n,h,t):w(n,h,t)>SMin(n,h)*Salpha(n,h,t));

```

```

@for(masupe(n,h,t):@BIN(Salpha(n,h,t)));
@for(period(t):@for(material(n):@sum(supplier(h):Salpha(n,h,t)=1)));

!Supplier Plant Transition phase;

@for(masupe(n,h,t):w(n,h,t)=@sum(plant(i):u(n,h,i,t)));
!For Time period 1;
@for(material(n):@for(plant(i):v(n,i,1)=0));
!For Time period 2 or more;
@for(maplpe(n,i,t)|t#GE#2:v(n,i,t)=@sum(supplier(h):u(n,h,i,t-1)));

!Plant equations;

@for(plper(i,t):X(i,t)<PCap(i));
@for(Inplpe(p,i,t):XMIP(p,i,t)<MIPCap(p,i));
@for(maplpe(n,i,t):X(i,t)=alpha*v(n,i,t));
@for(Inplpe(p,i,t):X(i,t)=beta*MIPReq(p,i,t));

@for(Inplpe(p,i,t):@sum(plant(s):XMIPS(p,i,s,t))<XMIP(p,i,t));
@for(Inplpe(p,s,t):@sum(plant(i):XMIPS(p,i,s,t))>MIPReq(p,s,t));

!Plant Warehouse Transition phase;

@for(plper(i,t):X(i,t)=@sum(warehouse(j):y(i,j,t)));

!Warehouse equations;

@for(waper(j,t):phi(j,t) < WCap(j));
!Period 1;
@for(warehouse(j):phi(j,1)=WInitialInv(j)-
@sum(retailer(k):@sum(transport(m):z(j,k,1,m))));
!Period 2 or more;
@for(waper(j,t)|t#GE#2:phi(j,t)=phi(j,t-1)+@sum(plant(i):y(i,j,t-1))-
@sum(retailer(k):@sum(transport(m):z(j,k,t,m))));
@for(warepertr(j,k,t,m):Z(j,k,t,1)<M1);
@for(warepertr(j,k,t,m):Z(j,k,t,2)<M2);
@for(warepertr(j,k,t,m):Z(j,k,t,3)<M3);

!Retailer equations;

!Period 1;
@for(retailer(k):R(k,1)+RInv(k,1)-RBO(k,1)=RInitialInv(k));
!Period 2;
@for(retailer(k):@sum(warehouse(j):z(j,k,1,1))=R(k,2)+RInv(k,2)-
RInv(k,1)+RBO(k,1)-RBO(k,2));
!Period 3;
@for(retailer(k):@sum(warehouse(j):z(j,k,2,1)+z(j,k,1,2))=R(k,3)+
RInv(k,3)-RInv(k,2)+RBO(k,2)-RBO(k,3));
!Period greater than 3;
@for(reper(k,t)|t#GT#3:@sum(warehouse(j):@sum(transport(m):z(j,k,t-
m,m)))=R(k,t)+RInv(k,t)-RInv(k,t-1)+RBO(k,t-1)-RBO(k,t));
@for(reper(k,t):RInv(k,t)<RCap(k));
@for(retailer(k):RBO(k,10)=0);

```

```
!Data set;
```

```
data:
```

```

Qstdset= 5,6;
Qstd=8,9,6,2,8,7,6,8,4,7,
5,6,7,8,4,7,6,5,7,5,
6,4,8,5,6,8,4,7,8,5,
8,6,5,9,7,6,7,6,9,7,
8,5,5,8,6,7,4,8,8,6,
5,9,7,6,7,8,4,7,6,5;
SCap= 5500,5400,6750,5200,5000,5750;
SMin= 3800,3500,4500,3750,3250,3900;
cpu=3,4,1,2,3,1,5,7,6,8,
6,4,6,8,4,7,5,6,3,9
6,7,8,5,5,7,6,8,6,4,
8,5,3,4,2,5,4,7,5,6,
7,3,4,3,2,6,4,5,2,8,
3,5,4,7,4,2,5,6,7,8;
PCap=2000,2500,3500;
Pprod=70,60,65;
MIPCap=2500,3000,2250,2750,3000,3150;
CMIP=12,15,18,20,16,22,18,19,15,12,
11,20,17,19,16,15,13,12,21,20,
19,16,18,17,22,15,17,19,12,11,
14,15,16,12,13,11,16,18,14,17,
15,16,14,15,16,12,13,11,14,13,
12,16,14,15,20,23,24,21,19,17;
alpha=1;
beta=1;
Tis=1.5;
Thi=1.5;
Tij=1.5;
WCap=1900,2000;
RCap=1100,1200,950,1000;
RInitialInv = 150,150,150,150;
WInitialInv = 300,300;
Hw=2.1,2.5;
Hr=4.5,4.7,5.2,4.5;
RBackcost=9.5,10.2,10.5,9.5;
R=246,346,124,250,365,269,372,173,235,238,
229,231,389,297,406,307,424,153,342,372,
398,400,397,245,249,347,251,249,272,382,
292,294,229,286,347,268,279,283,285,190;
TWAR = 1.5,1,0.5;
M1=1000;
M2=0;
M3=2000;

enddata
end

```