NUMERICAL STUDY OF FRACTURE APERTURE CHARACTERISTICS AND
THEIR IMPACT ON SINGLE-PHASE FLOW AND CAPILLARY-DOMINATED
DISPLACEMENT

A Dissertation in
Petroleum and Natural Gas Engineering
by
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Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

May 2008
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ABSTRACT

The general problem of immiscible flow in fractured geologic formations has been studied extensively in applications such as hydrology, geo-thermal exploitation, environmental remediation of subsurface contamination, and hydrocarbon recovery. Recovery from oil and gas reservoirs is greatly affected by the presence of natural fractures in the reservoir rock. In naturally fractured reservoirs, the fracture network provides preferential paths for fluids to flow, while the porous rock provides the storage space. Two often encountered difficulties in the study of immiscible flow in fractures are the geometric representation of these fractures, and assigning bulk fracture flow properties representing the complex interactions between the irregular surface of the fractures and the fluids. The objective of this study is to examine both single-phase and two-phase capillary-dominated displacements. A commercially available computational fluid dynamics software was used to investigate single-phase flow. A physically-based percolation model was developed to study capillary-dominated displacement, and validated by using experimental work of Karpyn et al., (2007) as a modeling reference. This study elaborates on the correspondence between structural characteristics of fractures, preferential flow channeling and fracture capillary pressure curves, thus providing a methodology to predict fracture capillary pressure from fracture aperture parameterization.

Knowledge of detailed descriptions of a fracture from Computed Tomography (CT) measurement was used for generating a realistic fracture replica. Single-phase flow simulations were carried out on the CT-scanned fracture by means of Computational Fluid Dynamics (CFD) simulation. The effects of fracture roughness and path tortuosity were explored by visualizations of pressure contours and vector velocity profiles of flows thorough the realistic fracture. The results of CFD simulations were compared against predictions using idealized smooth fractures or parallel plate models. Noticeable discrepancies showed that the cubic law was inadequate in describing flows in fractures because of lack of a valid physical representation of a real fracture. The deviations of
the cubic law predicting flow in a real fracture can be corrected by applying a scaling factor to its hydraulic conductivity.

Although single-phase simulation using CFD provided insightful results, this approach was found inappropriate to describe immiscible displacement at the scale of interest, and especially when capillary forces dominate fluid mobility. A modified invasion percolation (MIP) approach is proposed to model capillary displacement in a rough-walled fracture. A lattice model is generated from the aperture distribution obtained from X-ray CT imaging of a fracture Berea sandstone core. Experimental oil-water saturation maps of primary drainage assist the validation of MIP model. Saturation distribution maps from simulations at various resolutions yield a good agreement with the experimental saturation map. A good match to the experimental observation confirms that MIP model is adequate to describe capillary displacement in rough fractures.

The proposed MIP model was also used to investigate the dependency of fracture capillary behavior on aperture characteristics. A stochastic aperture field generator (COVAR) was used to create a series of artificial fracture models of known geometrical characteristics. Control parameters in these fracture models were mean aperture, standard deviation, and spatial correlation length. The influence of these three parameters on fracture capillary pressure curves was quantified relative to the normalized entry pressure and the irreducible water saturation. Results from this investigation allow us to quantify the effects of fracture morphology on the shape of fracture capillary pressure curve, and to predict the normalized entry pressure and the irreducible water saturation from geostatistical parameters describing the fracture aperture field.
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**NOMENCLATURE**

\( A \) = Isotropic covariance function

\( b \) = Fracture aperture (L)

\( \bar{b} \) = Mean aperture (L)

\( C \) = Curvature number (unitless)

\( C_i \) = Constant that account for fluid properties

\( C_0 \) = Sill value (unitless)

\( g \) = Acceleration of gravity (L/t\(^2\))

\( K_f \) = Hydraulic conductivity (m/L-t\(^2\))

\( K_m \) = Hydraulic conductivity of the matrix rock (m/L-t\(^2\))

\( L \) = Covariance matrix

\( l \) = Lag distance (L)

\( p \) = Pressure (m/L-t\(^2\))

\( P_c \) = Capillary pressure (m/L-t\(^2\))

\( P_c^* \) = Normalized capillary pressure (unitless)

\( P_{c,\text{entry}}^* \) = Normalized entry pressure (unitless)

\( q \) = Flow rate through a fracture (L\(^3\)/t)

\( Q_f \) = Flow through the fracture plane (L\(^3\)/t)

\( Q_m \) = Flow through the matrix rock (L\(^3\)/t)

\( r \) = Core diameter (L)

\( r_i \) = Normal curvature (L)

\( r_i^* \) = Aperture variability of the normal curvature term (\( r_i \))
$r_2$ = In-plane curvature (L)

$\hat{r}_2$ = Dimensionless form of the in-plane curvature term ($r_2$)

$V$ = Variogram (unitless)

$SD^2$ = Variance

$S_{wirr}$ = Irreducible water saturation

$Y$ = Log-normal or normal distribution variable

$\bar{Y}$ = Mean of $Y$

**Greek**

$\alpha$ = Contact angle (degree)

$\varepsilon$ = Vector $\mathcal{N}[0,1]$, normally distributed with mean of zero and SD. of 1

$\beta$ = Local convergence or divergence angle of the fracture wall (degree)

$\gamma$ = Include angle (degree)

$\lambda$ = Spatial correlation length (L)

$\mu$ = Fluid viscosity (m/L-t)

$\rho_w$ = Water density (m/L$^3$)

$\sigma$ = Interfacial tension (m/t$^2$)

$\omega$ = Autocorrelation parameter
ACKNOWLEDGEMENTS

The author would like to acknowledge those people whose support and encouragement were indispensable in the completion of this thesis.

First and foremost I would like to deeply thank my thesis advisor and mentor Dr. Zuleima T. Karpyn for her moral support and continuous encouragement throughout the course of my research. This research would not have been accomplished without her proper guidance and patience.

I gratefully acknowledge the members of my thesis committee Dr. Turgay Ertekin, Dr. Abrahams S. Grader, Dr. Derek Elsworth and Dr. Maria Lopez de Murphy for their intellectual contribution in shaping my academic and professional attitudes. Specially, I would also like to thank the Pennsylvania State Center for Quantitative Imaging for their technical support and access to computers and imaging tools.

My gratitude goes to all my student colleagues who directly and indirectly assisted in this research. My very special cordial thanks to my dad, Chedchai, and my brothers and sister who always give me permanent support.

I am very indebted to my beloved wife, Jureerat, for her moral support and invaluable encouragement throughout the time to complete the degree at the cost of sacrifice of her time and energy. This achievement would not have been possible without her understanding, faith and love.

Finally, I would like to acknowledge the Department of Energy and Mineral Engineering, Daniel J. Allen Memorial Endowment and the Petroleum Research Fund (PRF# 45799-G9) of the American Chemical Society for their financial supports.
Chapter 1

Introduction

Rock fractures provide high permeability pathways for fluid flow in geologic media. There are several circumstances where immiscible fluids flow simultaneously through fractured rocks and porous media, some examples are: oil and water flowing in petroleum reservoirs during water flooding, vapor-liquid transport in geothermal reservoirs, flow of nonaqueous phase liquids in contaminated aquifers, and underground flow in nuclear waste sites. While numerous model formulations of physical phenomena of flows in fractures have been proposed (Pruess and Tsang, 1990; Blunt et al., 1992; Lowry and Miller, 1995; and Hughes and Blunt, 1999), a fundamental understanding of fracture characteristics affecting transport properties of fractures is still lacking. To improve our understanding of key parameters controlling fracture flow properties, additional studies of fluid flow in realistic fracture structures are needed. Previous experimental observations of fluid mapping in a rough-walled fracture (Karpyn et al., 2007) are used as a platform for constructing and validating a percolation model of capillary displacement. Thereafter, flow simulations are performed on several fracture models which are artificially generated by means of a stochastic computer-generated realization program. Then the effects of fracture morphology on fracture flow behaviors are analyzed.

This modeling study presents a valuable tool, not only for predictive purposes, but for systematically organizing knowledge of the relationships between inherent geometric complexities of fractures and fracture flow properties. In particular, the determination of the fracture capillary pressure from an aperture distribution would provide insight into the mechanics of immiscible flow in fractures. As well, it would provide a comprehensive data set for generalizing the dependency of the shape of capillary pressure curves on the geostatistical characteristics of the fracture aperture distribution. The ultimate goal of this numerical work is to develop of a
methodology to predict the fracture capillary pressure from the known aperture distribution, which could lead to a physically supported, pragmatic approach for the representation of transport properties in naturally fractured reservoirs.

1.1 Problem Statement

Many naturally occurring hydrocarbon reservoirs are fractured. Fractures have a significant influence on the transport characteristics of reservoir formations, because they provide preferential flow channels for oil, water, and natural gas; while the rock matrix accounts for the storage capacity of the formation. The internal structures of fractures, as well as the interconnectivity of fracture networks, play a crucial role in the flow characteristics of fracture formations. Several experimental studies suggest that there is a strong dependence between fracture transport properties and the geometric characteristics of fractures (Pyrak-Nolte, et al., 1992; Reitsma and Kueper, 1994; and Keller, et al., 1999). However, mapping fracture networks in their natural environment posts great technological challenges, which consequently add uncertainty on existing fracture flow models. Quantitative characterization of a fracture’s inner structure, formed in a Berea sandstone core has been recently captured using X-ray computed tomography (Karpyn, et al., 2007) as shown in figure 1.1. The structure of this fracture is used in this work to investigate the dependence of fracture transport properties on the structural characterization of a rough fracture. The specific objectives of this work are:

1. To investigate the relationship between structural characteristics of fractures and their transport properties by means of numerical modeling methods.

2. To examine the effect of fracture variations on single-phase flow and investigate potential discrepancies with the parallel-plate model, by using a commercially available computational fluid dynamics software.
3. To develop a physically-based percolation model for describing capillary-dominated displacements of two-phase flow in a rough-walled fracture, and validate the proposed model by comparing to the experimental observations.

4. To identify key parameters controlling fracture aperture characteristics and quantify their relationships with the fracture capillary pressure curve.

5. To provide a methodology to predict fracture capillary pressure from fracture aperture parameterization.

Figure 1.1 Illustration of a three-dimensional map of fracture aperture variation obtained from X-ray CT Scanning (Karpyn et al., 2007)
1.2 Literature Review

1.2.1 Single Phase Flow through a Rough-Walled Fracture

Early models of the study of fluid flow through a fracture were based on the assumption that fracture could be adequately represented as flat parallel plates with a constant aperture equal to the mean aperture of the rough-walled fracture. Starting with the pioneering work by Lomize (1951) and Louis (1969), using parallel glass plates to represent fluid flow through smooth fractures, various approaches have been reported in the literature to study the effect of fracture morphology on transport properties of fractures. The parallel plate model gave origin to the so-called cubic law, which describes flow through a fracture by:

\[ q = C \frac{b^3}{\mu} \frac{dp}{dx} \]  

(1.1)

where \( q \) is the flow rate, \( C \) is a constant that accounts for fluid properties, \( \frac{dp}{dx} \) is the gradient of the hydrostatic pressure, \( \mu \) is the fluid viscosity, and \( b \) is the fracture aperture (Lomize, 1951; Louis, 1969). Equation (1.1) may also be derived from the equation of motion, for fully-developed laminar flow between two stationary planar surfaces. This solution is also called plane Poiseuille flow. The most significant limitation of the cubic law is that it assumes a constant fracture aperture separating the two solid surfaces. In nature, fracture apertures are not constant and show points of contact or asperities that hinder the overall fracture conductivity. Asperities can be seen as flow obstacles that increase fluid channeling and tortuosity of fluid flow paths. Due to the geometrical simplifications imposed on the parallel plate model, predicted fracture flow rates may be overestimated using the cubic law. Nevertheless, the simplicity of this method and the lack of more realistic structural fracture data make the parallel plate model a popular alternative in the study of fluid transport through fractured systems.
Numerous studies have been proposed in the literature concerning the applicability of the cubic law, correction factors, and effective aperture estimations to account for the effect of surface roughness and tortuosity on the description of fracture flow (Iwai, 1976; Witherspoon et al., 1980; Tsang, 1984; Brown, 1987; Pyrak-Nolte et al., 1987; Brown, 1989; Zimmerman and Bodvarsson, 1996; Ge, 1997; and Waite et al., 1998). Another pioneering work in the representation of fractures, and the variation of fracture transmissibility with confining pressure is the bed-of-nails model (Gangi, 1978), where rod-shape asperities determine the degree of separation between the two fracture surfaces under various applied pressures.

A comprehensive review of current methods and state-of-the-knowledge regarding modeling of fluid flow through fractures and fracture characterization is presented in the literature (Berkowitz, 2002; and Neuman, 2005). These confirm the importance of accounting for the highly erratic nature of fractures to properly model flow and transport in fractured rocks. A fracture network model by Gonzalez-Garcia et al. (2000) proposes a three-dimensional reconstruction of intersecting fractures from experimental serial sections to study geometrical characteristics of fractured systems. They used randomly oriented, monodisperse hexagons to represent a fractured block and describe anisotropic flow properties such as permeability and fluid velocity (Gonzalez-Garcia et al., 2000). An integrated approach to describe flow in a rough fracture using experiments, stochastic and numerical simulation is also presented in the literature (Dicman et al., 2004). Results from this approach were in agreement with gravity drainage experiments in a fracture core using X-ray computed tomography (CT) scanning. Experiments using transparent fracture replicas have also assisted in the study of fluid flow through fractures and transport property anisotropy (Auradou et al., 2005). Their numerical model was able to capture permeability anisotropy as a result of the shear displacement of rough fracture walls.

In the first portion of this investigation, we propose a physically-based model using a deterministic computational method based on the solution of Navier-Stokes equations to study single-phase flow through a rough fracture. The hydraulic behavior of a single fracture and how it relates to
fracture morphology and asperities is examined in this study, owing to the available knowledge of
detailed description of a fracture’s structure from X-ray micro-computed tomography (Karpyn et al., 2007). Results from this investigation provide direct quantification of flow and transport in
realistic fracture geometries. In addition, the validity of the cubic law which is extensively used in
describing single phase flow through real fractures can be examined by comparing to numerical
results of flow simulations through a rough-walled fracture.

1.2.2 Immiscible Flow Through a Rough-Walled Fracture

Characterization of fracture morphology and the investigation of fluids transport through fractures
pose difficult challenges to hydrologists, petroleum engineers and geo-environmental engineers.
An underlying question of fluids flowing through fractures is how the complexity of the fracture
geometry affects fluid configurations and the macroscopic transport properties of fracture
systems. Numerous studies show that complexity exists in describing flow through real fractures
such as the heterogeneity in the aperture distribution, and characterization of the tortuosity and
connectivity path of fluid flows (Tsang, 1984; and Brown, 1987; and Keller and Blunt, 1999).
Numerical models representing a realistic fracture with variable aperture also have confirmed the
effect of fracture structure on the fracture transport properties (Pruess and Tsang, 1990; and
Pyrak-Nolte et al., 1992). The knowledge of geometrical description of fractures is crucial and
valuable for constructing a discretized replica of a realistic fracture. Experimental studies using X-
ray computed tomography recently provided the detail geometry of the inner structure of a
fractured rock (Karpyn et al., 2007) so that a fracture replica for simulations can be generated
from the fracture description. In addition, steady state fluid distribution maps at residual saturation
were also obtained from the experimental work and used for comparing with simulation results.
The available fracture description along with geostatistical analysis enables us to characterize
fracture morphology and investigate its effect on the fracture transport properties. Although the
available X-ray CT data allows the reconstruction of conceptualized model of a realistic fracture, an efficient network model cannot be made without a proper model representing necessary flow mechanisms inside fractures. The physical interactions between fluids and irregular solid walls of fractures have to be modeled properly to make reliable predictions of fracture transport properties.

Flow through naturally occurring fractures are frequently found in a narrow range of aperture tens of microns or less; where capillary forces strongly dominate displacement mechanisms. Capillary behavior in fractured matrix rocks was investigated experimentally by Firoozabadi and Hauge (1990), with results that indicated the presence of capillary continuity between the matrix blocks. Their experimental work showed the effect of different contacting media on continuity between two similar rocks, and they found a better continuity was observed for the contacting interfaces with a similar grain size. The prediction on the fracture capillary pressure was studied by several authors (Firoozabadi and Hauge, 1990; Pruess and Tsang, 1990; Kueper and Mc Whorter, 1991; Mc Donald et al., 1991; and Kueper et al., 1992).

Various empirical correlations have been proposed to predict capillary pressure curves of porous media such as Van Genuchten (1980), Su and Brooks (1975), and Brooks and Corey (1964, 1966). All of these correlations express capillary pressure curves as functions of the entry pressure, effective saturation and some geological indices of porous medium properties which can be pore size, pore distribution, porosity, and organic content. However, the accuracy of their prediction depends on specific porous media types, soil texture, soil structure or lithologic types of reservoir formations (Wagner et al., 2001). Reitsma and Kueper (1994) performed an experiment to measure the hysteretic capillary pressure curve for a rough-walled rock fracture under different loads of normal stresses. The measured capillary pressure curves were fitted by the transfer functions of Brooks and Corey (1964) and Van Genuchten (1980), and those curves were used to derive an aperture distribution of the fractured rock. Bertels et al., (2001) determined a fracture aperture distribution by using computed tomography (CT) scanning and measured in situ
saturation along with the capillary pressure and relative permeability for gas/water drainage. They found that the aperture distribution for the basalt sample had an approximately log-normal distribution. The capillary pressure curves did not show monotonic increase as drainage progresses because of the effect of viscous force.

The capillary pressure studies in rough-walled fractures have been investigated by using pore-scale network modeling (Pruess and Tsang, 1990; Blunt et al., 1992; Lowry and Miller, 1995; Karpyn and Piri, 2007; and Piri and Karpyn, 2007). Pore-scale network modeling is a mechanistic approach that incorporates fluid mechanic physics into the conceptualization of porous media geometry. Network models represent the fracture space consisting of apertures as interconnected networks, which give insight of pore scale displacement phenomena (Dullien, 1992). Many researchers have hypothesized that pore-size distributions could be determined directly from capillary pressure curves, and presented relationships between capillary pressure curve and the cumulative distribution function (CDF) of the aperture distribution (Pruess and Tsang, 1990; Reitsuma and Kueper, 1994; and Lowry and Miller, 1995). Most numerical studies using pore-scale modeling are rather schematic model in which aperture distributions were artificially generated. Predictions of fracture capillary pressures from aperture distribution cannot be more realistic unless detail geometry of a fracture void space is available. In this numerical study, computer tomography (CT) data obtained from work of Karpyn et al. (2007) will be used to reproduce a network model of a realistic fracture for capillary pressure prediction.

The invasion percolation approach is one of several modeling techniques used for capillary dominated displacement, in which the algorithm seeks for the least resistant path for invasion by ignoring viscous forces and pressure drop in the fluid flow (Lenormand, 1986). As a result of a neglect of viscosity, the movement of interface curvature is approximately accounted as quasi-static displacement. The local capillary pressures along the invasion front are calculated, and the lowest one will be the most convenient channel for invading fluid to flow. Lenormand and Zarcone (1983) also proposed a series of I-mechanisms which describe capillary displacements in an
etched network. All of those microscopic mechanisms were categorized by calculating the capillary pressure in the functions of meniscus of fluids in a duct and the local pressure of the fluids to decide which mechanisms would be taken place.

Lenormand et al. (1988) took the viscous and capillary effects into their micro pore model, and those two effects were characterized by the capillary number and the ratio of the displacing and displaced viscosities. There are three main basic regimes of displacements which are viscous fingering, capillary fingering and piston-like displacement. Three flow regimes were mapped on to the x-y plane which consists of the logarithm of the capillary number on y-axis and the logarithm of the viscosity ration on x-axis. Hughes and Blunt (1999) implemented I-mechanisms of Leonmand and Zarcone (1983) and the capillary facilitation presented by Glass and Yarrington (1996) on their pore-scale network. A pore-scale network was generated based on an aperture distribution measured from CT-scanner. Fluid distributions of the simulation results yielded a good agreement with experimental pictures. Karpyn and Piri (2007) and Piri and Karpyn (2007) applied displacement mechanisms similar to I-mechanisms on their pore-scale network models and also incorporated the effect of contact angle hysteresis for calculating threshold capillary pressures. The predictions of fluid distributions from their network model perfectly matched with the distribution maps from CT scans. This indicates that the implication of proper deterministic modeling along with the X-ray imaging technique is a rigorous approach to predict fluid distributions in realistic fracture representations.

1.2.3 Invasion Percolation: a Review

Percolation theory is a statistical model used to characterize the randomness in porous media at large scale, which represent the randomness as connectivity of the fracture networks (Stauffer and Aharony, 1992). The possibility of fluid flowing through the connectivity of the fracture system designates the flow pathways of the invasion fluid. The percolation theory can incorporate with physics of fractures and applied on micro network models. Network modeling provides valuable
insight of pore-scale events occurring in the fracture flow (Berkowitz and Ewing, 1998). The early percolation approach for modeling capillary flows in a fracture was started from the standard percolation theory. The standard percolation allows the invading fluid to occupy all aperture sites even if they are not connected to the inlet, so there are no entrapments of defending fluid left at the final stage. Prues and Tsang (1990) generated artificial aperture distribution models and assumed the aperture occupancy dependent only on its local capillary pressure, not its accessibility. The local capillary pressure can be expressed as the radius of curvature normal to the fracture plane, which is well known as Laplace’s equation. The normal curvature is represented in functions of the interfacial tension ($\sigma$), the contact angle ($\alpha$) and the fracture aperture ($b$) as follows:

$$P_c = \frac{2\sigma \cos \alpha}{b}$$

As a result of ignoring the fluid accessibility, the fully saturation of invading fluid and zero entrapment of defending fluid on a capillary pressure curve are observed from the standard percolation model.

To incorporate the accessibility of fluids, the invasion percolation (IP) is first introduced by Wilkinson and Willemsen (1983). The IP model imposes a fluid continuity algorithm to decide the possibility of filling apertures. The apertures cannot be filled unless there is an escaping route for the displaced fluid. When unoccupied aperture locations are surrounded by occupied ones, there become trapped and will be no longer invaded by displacing fluid. Pyrak-Nolte et al. (1992) used a stratified continuum model to generate fracture void geometries, in which apertures follow a log-normal distribution. The invasion percolation with trapping is used to investigate the effect of trapping, and it confirms there is significant effect of trapping on the transport properties of immiscible fluids. From the work of Pyrak-Nolte et al., a strong relationship between fluid distributions and fracture void geometries was presented.
An intrinsic disadvantage of the invasion percolation approach is that it does not take into account the in-plane radius of the fluid/fluid interface in the calculation capillary pressure, which renders a rough phase structure and overestimated entrapment. From the study of the effect of the in-plane curvature by Glass and Yarrington (1989) and Glass (1993), the experimental observations showed that as the invasion front moves into a new aperture site, the partial fluid on either side of the invasion front also moves in to the aperture. These observations imply that the neighbor apertures play a major role in occupying a filled aperture and forming phase structure. Glass et al. (1998), consequently, incorporated the effect of the in-plane curvature to conform to experimental observations, and named it a modified invasion percolation (MIP) model. MIP model is based on the Young-Laplace equation which expresses the capillary pressure in terms of radii of the normal curvature ($r_1$) and the in-plane curvature ($r_2$) to the fracture plane as following equation:

$$P_c = \sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$$

The MIP approach is implemented in this study for the modeling primary drainage in a fracture whose structure has been mapped from X-ray CT scanning. The proposed method allows the determination of fracture capillary pressure and comparison of invaded fluid structures against experimental observations. The prediction of fluid occupancy is examined qualitatively and quantitatively with the fluid distribution map of primary drainage reconstructed from CT images. A good agreement with the fluid distribution maps from experiment substantiates the accuracy of the proposed model in predicting capillary pressure-saturation curves for other fracture models. Finally, once the proposed model was validated based on the experimental references, the investigation of the relationships between fracture morphology and a fracture capillary pressure can be made.
Chapter 2
Description of Experimental Data

This numerical study is based on the previous experimental work (Karpyn et al., 2007), in which immiscible displacements were conducted on a fractured Berea sandstone core, and imaged using High-resolution X-ray CT scanning. The Berea sandstone core used has approximately 18% of porosity and 200 millidarcy of absolute permeability (Karpyn, 2007). The Berea sandstone core was 25.4 mm in diameter and 101.4 mm long, and was subject to a hydraulic axial compression exerted uniformly on the core until the point of failure. Consequently, a fracture plane was artificially induced along the length of the core sample. The broken two half-pieces of the core sample were placed back together with a slight shift of 1 mm along the longitudinal axis to accentuate aperture variation inside the fracture. Both ends of the cylindrical core were trimmed and ground parallel. Teflon was wrapped around the core sample and prevented a confining fluid entering to the core. Throughout the entire experiment, confining fluid was used to maintain pressure in the core annulus at 0.2 MPa.

A high-resolution computed tomography scanner was used to capture a geometrical description of a fracture inner structure and also fluid distribution maps after flooding the core with fluids. The voxel resolution in the experiment was set at 0.027344 mm x 0.027344 mm x 0.032548 mm, where 0.032548 mm represents the slice thickness. Multiple CT images were captured consecutively through the length of the fractured core to allow the reconstruction of an aperture distribution map as shown in figure 2.1. Apertures in figure 2.1 range from small to large in shades of from blue to red accordingly.
2.1 Experimental Procedure

The immiscible fluids used in the experiment were oil and water. The oil phase was mixed from silicone oil and 30% of n-decane by weight, which yielded approximate viscosity 0.5 cp and a density 0.89 g/cm$^3$ at 25 °C. The water phase was brine with 15% by weight of sodium iodied (NaI) which increased the CT registration and the contrast between the two fluid phases. The viscosity and density of tagged water were 1.2 cp and 1.11 g/cm$^3$, respectively. The measured interfacial tension of the oil-water system was 41.27 mN/m and the contact angle between the two phases was 10° (Karpyn and Piri, 2007; and Piri and Karpyn, 2007).

A sequential schematic of the experimental procedure is illustrated in Figure 2.2 and presented in details in Table 2.1. A dry core was initially scanned to obtain the geometrical description of the fracture. CT data of the fracture description can be used to reconstruct a realistic three dimensional replica of the fracture, and to represent aperture distribution (see figure 2.1) for statistical analysis. The dry core sample was vacuum saturated with tagged water on the second stage. The amount of water required to fully saturate the core sample was measured and used to
determine the effective porosity of the matrix core. Primary drainage was performed on the third stage by injecting oil into the core initially saturated with water, followed by imbibition, simultaneous oil and water injection, and gravitational segregation. Each stage was scanned to capture oil and water distribution in the fracture. Figure 2.2 is a procedure of the experiment, and table 2.1 summarizes the experimental conditions for each stage. Detail information of the experimental procedure is described in Karpyn et al. (2007) and Karpyn (2005).

Figure 2.2 Schematic sequence of experimental stages performed by Karpyn et al. (2007).
Table 2.1 Experimental procedure representing flowing condition and final steady fluid saturation (Karpyn and Piri, 2007; and Piri and Karpyn, 2007).

<table>
<thead>
<tr>
<th>Experimental Stage</th>
<th>Initial Injection Rate (cc/min)</th>
<th>Initial Injection Time (min)</th>
<th>Continued Injection Rate (cc/min)</th>
<th>Continued Injection Time (min)</th>
<th>Final $S_o$</th>
<th>Final $S_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>0.0</td>
<td>1.0</td>
<td>0.0</td>
<td>300</td>
<td>0.65</td>
<td>0.35</td>
</tr>
<tr>
<td>Primary Drainage</td>
<td>0.5</td>
<td>12</td>
<td>0.05</td>
<td>300</td>
<td>0.37</td>
<td>0.63</td>
</tr>
<tr>
<td>Water Imbibition</td>
<td>0.5</td>
<td>12</td>
<td>0.05</td>
<td>300</td>
<td>0.58</td>
<td>0.42</td>
</tr>
<tr>
<td>Secondary Drainage</td>
<td>0.25 each</td>
<td>150</td>
<td>0.1 each</td>
<td>300</td>
<td>0.58</td>
<td>0.42</td>
</tr>
</tbody>
</table>

In this numerical study, the major focus will be on the primary drainage stage. The final saturation map from the experiment will be used as a reference for validating the proposed model. Oil and water are assumed to be absolutely immiscible, and the wettability between fluids and Berea sandstone is expressed as a function of contact angle. The interfacial tension is also an essential parameter to characterize capillary behavior of flow in a fracture. Experimental conditions and the measured fluids and rock properties are accounted in the model development. Further descriptions of the usage of those physical properties for developing the proposed model are explained in the chapter of capillary-dominated displacement in a rough-walled fracture.
2.2 Characterization of Fracture Aperture Field

Fracture morphology can be characterized using geostatistical parameters of its aperture distribution. Frequency distribution or histogram is one of simplest ways to analyze the spatial distribution of apertures. A histogram of aperture distribution is plotted and shown in Figure 2.3. The normal distribution curve of the fracture represents the aperture mean about 0.548 mm and standard deviation 0.25 mm. The peak of histogram shows the aperture values that are most frequently found in the fracture. The biggest aperture on the left tail of the normal curve falls at 2.5 mm, whereas the zero apertures representing contact areas is on the right tail. Contact areas covers 1.56% of the total fracture plane 101.4 x 25.4 mm$^2$, and the calculated fracture volume is $1471.65$ mm$^3$ (Karpyn et al., 2007).

![Fracture aperture histogram](image)

Figure 2.3 Fracture aperture histogram corresponding to figure 1.1 and 2.1, where mean aperture is 0.548 mm.
The variogram is the most commonly used geostatistical technique for describing the spatial relationship of a considered variable. The variation of the aperture in the fracture plane is characterized by a spatial correlation length. If the distance in the fracture plane is smaller than the correlation length the apertures are more likely to be similar, but at distance greater than the correlation length there is little or no correlation between apertures in different locations (Moreno et al., 1988). Variograms are constructed from the aperture distribution in different directions which are isotropic, transverse (y) and longitudinal (x) directions as shown in Figure 2.4. The variograms in the isotropic and the longitudinal (x) direction behave similarly, while the transverse direction grows continuously after passing the short intermediate plateau. The divergent variogram in the transverse (y) direction means the apertures are highly correlated in this direction due to accentuation of a slight shift 1mm of the fracture plane. To determine spatial correlation lengths, variograms have been fitted using the exponential model which gives the best fit with the constructed variograms of the aperture field. The variogram equation of the exponential model can be written as (Kellar and Perez, 2002):

\[ V(L) = C_0 \left[ 1 - \exp \left( -\frac{3l}{\lambda} \right) \right] \]

(2.1)

where \( l \) is the lag distance, \( C_0 \) is the sill value, \( \lambda \) is the spatial correlation length, and \( V \) is the variogram.
Using an exponential curve fitting model, the values of spatial correlation length for each direction are presented in figure 2.4. The spatial correlation length is a geostatistical parameter used to describe the correlation and connectivity between the apertures distributing over the fracture domain. The obtained values of the spatial correlation lengths are 9.9 mm in the isotropic direction and 2.46 mm in the longitudinal direction. The large value of the correlation length in the transverse (y) direction indicates the apertures in this direction are highly correlated and have high connectivity among local apertures. Therefore, fluids prefer to flow in transverse (y) direction in which there are well sorting and good connectivity paths.
Chapter 3

Single-Phase Flow Using CFD Modeling

3.1 Data Conditioning for Flow Simulations

Single phase flow in a rough-walled fracture was investigated herein by implementing Computational Fluid Dynamics (CFD) technique. A three-dimensional replica of a realistic fracture was constructed from X-ray CT data describing the details inner structure of a rock fracture with 27-30 microns (Karpyn et.al, 2007). From information of the CT data, geometric description of a real fracture were all preserved such as the topology of the fracture plane, the interconnectivity of the fracture void space and the contact points of the fracture walls. The 3D fracture geometry was reconstructed from hundred 3116 sections of 2D images of CT data. A visualization software, AMIRA 3.0 (2002), was used to organize those imported CT data to create the 3D fracture as shown in figure 3.1.

Figure 3.1 Volumetric representation of the artificial fracture consisting of tetrahedral grids.
A total of 518,168 small tetrahedral grids constituted a volumetric geometry of a realistic fracture as shown in figure 3.1, where contact areas were depicted in black regions. Surface conditions of the realistic fracture were defined corresponding to physical surfaces of the real fracture in the experiment. Thereafter, 3D post-processing geometry of the realistic fracture was saved in a 3D data exchange file format which supports working on simulator platform. A compatible file format then was delivered to the CFD simulator (Fluent-Inc., 2003), in which single-phase flow simulations were performed. Therefore, a state-of-art of Computational Fluid Dynamics (CFD) technique along with Micro tomography device enabled us to investigate a single-phase flow in a real fracture.

3.2. Rough Fracture Model

Since the matrix rock perpendicular to the fracture plane was assumed to be an impermeable wall, a fluid was allowed to flow in the fracture space only. Flow simulations of water injection took place from the left side of the fracture with a uniform inlet velocity 1 m/s and no back pressure at the right-side outlet boundary. Water used in the simulations was Newtonian and incompressible fluids. That means all of the governing equations used for simulations were collapsed to be the Navier-Stokes equation. The solver was then set up at steady state condition and three-dimensional flow. Finally, governing equations such as mass and momentum equations were numerically solved to obtain the velocity vector and total pressure contour distribution over the fracture domain. The quantification of the effect roughness of fracture surfaces was represented by the amount of pressure drop across the realistic fracture, and the tortuous paths of fluid flows in a real fracture were demonstrated by the vector velocity field in a fracture space. Flow simulations by CFD software, therefore, gave a detailed description of pressure and velocity fields and could be used for deterministic analysis of single-phase flow through a real fracture.
Detailed descriptions of pressure and velocity fields are given in figures 3.2 and 3.3 respectively. Figure 3.2 shows the total pressure contours relative to the fracture aperture distribution. Water follows tortuous paths from inlet (left) to outlet (right) responding to local variations in fracture aperture. The pressure contours are strongly affected by the presence of asperities that act as flow barriers creating additional pressure gradients along the flow path. The pressure profile, therefore, of a real fracture model in figure 3.2 does not decreased uniformly like a pressure profile in a parallel plate model.

Figure 3.2 Total absolute pressure contour of water flowing through the fracture replica.

Figure 3.3 shows a snapshot of the fluid trajectory inside the fracture (stream lines) formed after 0.1 seconds of water injection and the velocity field obtained from the CFD simulation (section A). The main flow channels correspond to fracture regions representing large apertures. On the
stream lines starting from the inlet boundary in figure 3.3, darker gray line depicts a stream line corresponding to a higher velocity magnitude, whereas lighter gray line is a stream line corresponding to a lower velocity magnitude. From the observation of fluid trajectory, stream lines taking tortuous channels around asperities are at high velocity magnitude, whereas stream lines away from asperities are smooth and at low velocity magnitude. Since the fluid flowing through a fracture is conserved in this study, thus the fluid flowing at high velocity into the regions of asperities is observed. The other observations of figure 3.3, when the fluid is flowing with high velocity through the contact areas, are the presence of eddies, vortices and diverging current surrounding the asperities as shown in the enlarged section A illustrating stream lines and velocity distribution.

Figure 3.3 Fracture flow velocity field obtained from CFD simulation. Aperture distribution ranges from 0 (dark blue) to 1.6 mm (red).
These observations therefore suggest that the irregularity of the fracture surfaces and aperture variations causes the tortuosity and vortex of the flow in a rough fracture (Tsang, 1984).

### 3.3. Smooth Fracture Model

In most of applications, single phase flow through fractures is commonly described by the parallel plate model, where the fracture surfaces are assumed smooth and parallel with a constant aperture \( b \) and cross-sectional area \( A = L_y d \) as presented in Figure 3.4. With this idealized geometry, the steady state solution of the Navier-Stokes equations for laminar flow yields the cubic law as expressed in equation 1.1, where the volume flow rate is proportional to the cube of the aperture.

![Figure 3.4 Geometric idealized model of a parallel plate.](image)

However, the parallel plate model can be represented as only a qualitative description of flow through real fractures. The reason is that real fracture surfaces are not smooth parallel plates, but are rough and contact each other at discrete points (Brown and Scholz, 1985a). Many theoretical and experimental works have showed that the effect of roughness of fracture surfaces and tortuosity of flow in a real fracture causes deviation from the cubic law (Iwai, 1976; Witherspoon et al., 1980; and Tsang, 1984). The parallel plate representation of a rough-walled fracture seems
inadequate in describing fluid movement through a fractured medium (Moreno et al., 1988). To demonstrate the lack of realistic presentation of the parallel plate model, an idealized fracture model with smooth wall and constant aperture was generated by a grid generator, GAMBIT, and flow simulations were performed in Fluent.

To examine the differences in fluid flow predictions obtained from our CFD model of a real fracture and a parallel plate model, an idealized fracture model with smooth walls was constructed using a grid generator GAMBIT with dimensions of 100 mm in length and varying sizes of the parallel plate in width. With various parallel plate widths, the study of how the effect of the cubic law’s assumption neglects the velocity in the parallel plate width direction was allowed. There are 20 divisions in the parallel plate width direction and 100 divisions in the fracture length direction as shown in figure 3.5. Grid blocks become thinner as they approach the upper and lower fracture surfaces to aid the delineation of the parabolic velocity profile near the boundary surfaces. The same fluid properties and boundary conditions applied to the real fracture case were also imposed to the smooth fracture model. The Reynolds number was controlled less than 1000 in accordance to the laminar flow condition of the cubic law. The steady, incompressible, 3D equations of momentum and mass conservation were numerically solved. The computational simulations yielded detailed quantitative information of the velocity vector field and the total pressure contour of flow in a smooth fracture model.

Figure 3.5 Idealized fracture model or parallel-plate representation.
The velocity profile was traced until a parabolic shape was steadily formed as shown in Figure 3.6. This means that the pressure gradient and the shear stress in the flow are in balance for a fully developed flow. The average velocity was determined by an area-weighted average after a fully developed flow was achieved, since the fully developed condition obeyed the basic assumption of the cubic law.

![Velocity Vectors Colored By Velocity Magnitude (m/s)](image)

Figure 3.6 Parabolic velocity profile steadily forming after the fully developed length.

To average velocities of the flow through the smooth fracture model, the obtained velocities in the vector field were integrated along the width of a parallel plate model and normalized by cross-sectional area. The flow rate through a fracture was then determined by multiplying the average velocity with cross sectional area perpendicular to the flow direction. The flow rate obtained varies with the cube of aperture or this equation leads to the “cubic law” expressed in equation (1.1). The area-weight average velocity was then compared with the theoretical average velocity derived from the cubic law. The theoretical average velocity or the cubic law velocity was calculated by substituting the gradient pressure along the parallel plate into the cubic equation. The gradient pressure was captured after achieving fully developed region. In figure 3.7, the average velocity obtained from Fluent is compared with the theoretical average velocity. Thus the
comparison of average velocities from Fluent and average velocities from the cubic law allows the validation of the CFD approach for simulating flow in a parallel plate model.

Figure 3.7 Comparison of velocities computed from the parallel-plate grid model of the fracture (CFD) and the analytical cubic law for fracture aperture of 1mm.

The comparison results show the consistency between the average velocity of the flow simulation and the theoretical averaged velocity. The simulation results suggest that the implementation of CFD (Computational Fluid Dynamics) techniques is feasible and applicable to the study of the single phase flow through a parallel plate model. However, the results of aperture size 1 mm in figure 3.7 shows the deviation of the Fluent velocity from the theoretical value, when velocity magnitude exceeds 1.5 m/s. Because of the 3D motion equations used in the simulator, the velocity in parallel plate width direction was taken into account in the vector field. When using a bigger parallel plate width, the greater significance of velocity in the parallel plate width direction was observed. One of basic assumption of the cubic law restricts the parallel plate width at very
small value with respect to the parallel plate length, so the neglect of velocity in the parallel plate width direction can be established. As a result of using the 1 mm plate width which is too big for the imposed conditions in this study, the magnitude of the average velocity computed by the simulator deviates from the 45 degree line, representing no discrepancy between a theoretical velocity value and a Fluent velocity value.

To validate the prediction of flow in a real fracture by using the cubic law, parallel plate meshes were constructed using GAMBIT. The parallel plate geometry contains 1500 hexahedral grids with dimension 100 mm long, 24 mm wide and 0.584 mm thick, which is a mean aperture statistically obtained from the aperture distribution histogram. The boundary conditions and the solver set-up values were exactly defined as same as the real fracture case. Figure 3.8 shows that the pressure drop obtained across the parallel plate model, $3 \times 10^3$ Pascal is noticeably less than the pressure drop in the real fracture, $1.15 \times 10^5$ Pascal. The reason is that the real fracture having rough walls and various aperture sizes violates the geometrical assumption of the parallel plate model. In addition, the discrete points at the contact areas over the fracture increasingly promote the effect of the tortuosity of fluid flow through a real fracture. The lack of roughness and tortuosity effects in the cubic law causes the underestimation of pressure drop in a real fracture as shown in figure 3.8. Figure 3.8 demonstrates the disagreement of pressure drop between the cubic law using the mean aperture of the real fracture as the parallel plate width and the solution of the Navier-Stokes equation of the realistic fracture. This resulting figure shows that the only the constant fracture aperture of the parallel plate model does not completely describe the flow in a real fracture. It is impossible to define an equivalent parallel plate aperture consistent with the observed flow and transport phenomena. The flow through a rock fracture is clearly unlike that though a gap between smooth parallel plates. Real fracture surfaces are rough, implying that the mean aperture used in the cubic law is not the same as the hydraulic aperture representing transport property of a fracture (Brown, 1989).
Figure 3.8 Comparison of pressure decline with distance obtained from a realistic rough fracture model and the parallel-plate representation. Both fractures have the same bulk dimensions and mean aperture (0.584 mm).

Figure 3.8 shows that the parallel plate model, using mean aperture 0.584 mm, does not represent true pressure drop across a fracture. The parallel plate with 0.584 mm width gives the pressure drop per unit length about 31 Pascal/mm, while the pressure drop per unit length of the real fracture is much higher at 1520 Pascal/mm. After using several parallel plate widths, the correct value of equivalent parallel plate width giving the same value of the pressure drop across the real fracture is round to be 0.0964 mm which is called “equivalent hydraulic aperture”.

To incorporate the roughness and tortuosity effect to the parallel plate model, a new effective hydraulic aperture has to be defined with a magnitude smaller than mean aperture to compensate
for the additional pressure drop. The hydraulic conductivity concept is derived from the analogy between the Darcy's law to the cubic law and can be expressed as (Iwai, 1972):

\[ K_j = \frac{b^2}{12\mu} \]  

(3.1)

where \( K_j \) is the hydraulic conductivity, \( b \) is aperture size and \( \mu \) is viscosity. In the other words, the hydraulic conductivity using an arithmetic mean aperture does not properly describe the hydraulic behavior of flow in a real fracture, whereby the reduction of aperture size is a way of forcing the tortuosity and roughness effect into the hydraulic conductivity. In the first part of this investigation, commercial CFD simulator was used to model and analyze single-phase flow through a rough fracture whose inner structure was reconstructed from X-ray computed tomography data. The CFD approach proved to be successful for flow visualization and quantification of pressure and velocity fields during single-phase flow. The results demonstrate the formation of preferential flow channels and their correlation with local structural characteristic of the fracture. When roughness and tortuosity are present in a real fracture, the mismatch of pressure drop predictions from the real fracture and the parallel plate model reached two orders of magnitude. The underestimation of pressure drop at a fixed fracture flow velocity by using the parallel plate model is inherently caused by the assumption of a smooth parabolic profile, excluding the effects of roughness and channel tortuosity. The effect of roughness and tortuosity can be accounted with a scaling factor applied to the hydraulic conductivity term of the cubic law (Witherspoon et al., 1980). Piggot and Elsworth (1993) suggested that the hydraulic conductivity could be approximated as the product of the cube of mean aperture and tortuosity factor.

When two-phase flow becomes drawn to our attention, the computer with high speed processor and very large memory are needed due to the highly complicated surface of a real fracture. The computational cost is very high, and the considered fracture domain has to be small. From the mentioned technical difficulties, CFD approach is not appropriate to capture the problem dealing with the discrete fluid interfaces. Thus the next chapter will introduce a more efficient approach for solving the problem of immiscible flow in a rough-walled fracture.
Chapter 4
Capillary-Dominated Displacement Using Percolation Theory

4.1 Fracture Representation

Due to the complexity of fracture geometry, the fracture model is conceptualized as a two-dimensional lattice with local variation in aperture. The lattice model of this study consists of a number of square lattices which are designated with aperture sizes point by point, and they are all concentric on the same center line as shown in Figure 4.1. A detailed description of aperture size for each grid was obtained from CT data. The dimensions and the number of grid cells of the fracture model correspond to the resolutions of the fracture aperture maps. Table 4.1 shows the details of resolutions in the width and length directions of lattice models according to their statistical information.

Figure 4.1 Conceptualization of the fracture aperture field into a two-dimensional lattice of rectangular elements.
Table 4.1 Lattice model dimensions and statistical parameters for various map resolutions.

<table>
<thead>
<tr>
<th>item</th>
<th>Original</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture length (pixels)</td>
<td>3116</td>
<td>779</td>
<td>389</td>
<td>283</td>
<td>222</td>
</tr>
<tr>
<td>Fracture width (pixels)</td>
<td>909</td>
<td>303</td>
<td>113</td>
<td>75</td>
<td>69</td>
</tr>
<tr>
<td>Resolution in length (mm)</td>
<td>0.032548</td>
<td>0.0130192</td>
<td>0.260384</td>
<td>0.358028</td>
<td>0.455672</td>
</tr>
<tr>
<td>Resolution in width (mm)</td>
<td>0.027344</td>
<td>0.082031</td>
<td>0.218750</td>
<td>0.328125</td>
<td>0.355469</td>
</tr>
<tr>
<td>Resolution in aperture (mm)</td>
<td>0.027344</td>
<td>0.027344</td>
<td>0.027344</td>
<td>0.027344</td>
<td>0.027344</td>
</tr>
<tr>
<td>Aperture mean (mm)</td>
<td>0.5838</td>
<td>0.5852</td>
<td>0.5833</td>
<td>0.5817</td>
<td>0.5773</td>
</tr>
<tr>
<td>Aperture Variance (mm²)</td>
<td>0.2500</td>
<td>0.2500</td>
<td>0.2500</td>
<td>0.2500</td>
<td>0.2500</td>
</tr>
</tbody>
</table>

The original high-resolution of aperture distribution map containing 3116x909 pixels was obtained from CT scans and coarsen to generate the other lower-resolution maps for the examination of resolution sensitivity and finding the optimum resolution. To obtain the coarser maps, a mathematical algorithm was applied on the original high-resolution aperture map, which assisted to preserve fracture topology and contact areas. Detailed description of how the coarsening was done can be found at Karpyn and Piri, (2007). Statistical parameters were used to identify potential change in the aperture field due to coarsening. From table 4.1, mean aperture and variance show no significant change in fracture characterization. Each of the four coarse aperture maps consisting of the reduced number of total pixels was used to identify an optimum balance between accurate aperture mapping and reduction of computing time.
4.2 Model Conceptualization

Continuum models extensively used to describe flow in porous media are based on a volume-averaged saturation concept, which is not appropriate to map fluid distribution at the final stage of immiscible flow in a rough-walled fracture. Due to that computational restriction, the modeling approach implemented in this study is an invasion percolation (IP) theory which was originally proposed by Wilkinson and Wilemson (1983). The invasion percolation is a theory focusing on the randomness of the porous media rather than the randomness of the fluid like the most conventional approaches (Berkowitz and Ewing, 1998). The invasion percolation determines the least resistant path for fluid flows and the fluid moving into fractured medium has to be in a continuous phase.

Invasion percolation-based modeling is good for describing capillary-dominated flow of nonwetting fluid in a random, spatially uncorrelated, and two-dimensional fracture system. However, the invasion percolation (IP) approach gives an unrealistic phase structure which is noticeably different from macroscopic smooth structure observed in experiments (Glass and Yarrington, 1989 and Glass, 1993). In order to better represent these smooth fluid structures, Glass et al. (1998) proposed the introduction of an in-plane curvature term to determine local capillary pressure, and his proposed model was called the Modified Invasion Percolation (MIP). In their formulation, the local capillary pressure, required for a nonwetting fluid to invade the aperture field, is given by the Young Laplace equation:

\[
P_c = \sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right)
\]  

(4.1)

where \( r_1 \) and \( r_2 \) are the two principal radii of curvature and \( \sigma \) is the interfacial tension between the fluids. From Figure 4.2, \( r_1 \) represents the principal radii of curvature normal to the fracture plane, and \( r_2 \) is the radii of curvature of invading front.
In figure 4.2, $r_i$ is expressed in terms of the local aperture ($b$) and local convergence or divergence angle of the fracture wall ($\beta$) associated with the contact angle ($\alpha$) of the fluid, and $r_i$ can be written as:

$$ r_i = \frac{b}{2 \cos(\alpha + \beta)} \quad (4.2) $$

Since numerical simulations are solely carried out under drainage process, there is no effect of contact angle hysteresis in oil invasion. Wetting surfaces are represented by $0^\circ < \alpha < 90^\circ$, whereas nonwetting surfaces by $90^\circ < \alpha < 180^\circ$, with the angle measured through the denser phase. The local convergence or divergence angle of the fracture surfaces ($\beta$) spatially varies with the local fracture topology and the direction of interest for determining the local capillary pressure. Through the incorporation of the in-plane curvature, we are able to account not only for local aperture variations, but also for neighboring apertures in the algorithm that determines preferential percolation sites.
$r_2$ is the in-plane curvature of invading front as presented in figure 4.2. From figure 4.3, $r_2$ is expressed as a function of the included angle ($\gamma$) and the spatial correlation length ($\lambda$) of a filled aperture site:

$$r_2 = \left( \frac{\lambda}{2} \right) \tan \left( \frac{\gamma}{2} \right)$$  \hspace{1cm} (4.3)

Figure 4.3 Schematic representation of the calculation of in-plane curvature ($r_2$) as a function of the spatial correlation length ($\lambda$) and the include angle ($\gamma$) (Glass et al., 1998).

The vectors defining $\gamma$ in figure 4.3 are determined as a weighted average of unit vectors extending from the potential invasion site to each neighboring site along the invasion front within a distance $\lambda$. The weighting factor for each unit vector is the reciprocal of the neighbor number raised to a power, and the neighbor number is dependent on the spatial correlation length ($\lambda$) of the aperture field and its resolution.

The included angle between the two vectors ($\gamma$) of the in-plane curvature ($r_2$) is a path dependent angle, which varies with the invasion front, and thus it has to be recalculated as the
invasion advances. Note that from equation 4.3, $r_2$ has become infinite as $\gamma$ approaches $180^\circ$, and $r_2$ gets closed to zero as $\gamma$ approaches $0^\circ$. The local capillary pressure for nonwetting phase invasion is then given by:

$$P_c = \sigma \left( \frac{2 \cos(\alpha + \beta)}{b} + \frac{2}{\lambda \tan(\gamma / 2)} \right)$$  \hspace{1cm} (4.4)$$

The potential aperture to be filled must have a route or connectivity to the outlet for the defending fluid to be able to escape. Since the potential aperture to be filled can be invaded from three connected neighboring apertures, the local capillary pressure has to be determined corresponding to the directions of the neighboring apertures. The lowest capillary pressure from those three connected aperture will be selected as local capillary pressure of the potential aperture to be filled during drainage. After all local capillary pressures for the potential apertures to be filled along the invading front are calculated, the aperture with the minimum local capillary pressure will be occupied first. As invasion advances, the values of local capillaries along the invading front are recalculated and the aperture with lowest local capillary pressure has the highest priority for nonwetting phase to occupy. The occupation process ends when the receding wetting-phase loses connectivity with the fracture outlet, at which point the system is said to reach the irreducible water saturation.

In this numerical study, the investigation of immiscible flow through a fracture mainly focuses on modeling drainage process, where the primary drainage stage of Karpyn et al., (2007) is used as modeling platform. There are a few conditions which have to be considered before applying the invasion percolation theory. One necessary condition for implementing the invasion percolation algorithm is that the flow regime must be under the capillary forces domination with respect to viscous force and gravitational force. The capillary number calculated from experiment conditions described by Karpyn et al. (2007) was approximately $8.7 \times 10^{-4}$, thus falling within the capillary-dominated regime (Hughes et al., 1999).
To apply the invasion percolation to the proposed model, it must be assumed that there is no flow through the matrix rock. The effect of water transferring to matrix rock can be quantified by the ratio of the transferring capability of the matrix rock over that of fracture plane as adapted from Reitsma and Kueper (1994):

\[ Q_m / Q_f = \frac{3K_m \pi r \mu}{(\bar{b})^3 \rho_w g} \]  

(4.5)

where \( K_m \) is the hydraulic conductivity of the matrix, \( r \) is the core diameter of the matrix rock perpendicular to the fracture flow, \( \mu \) is the fluid viscosity, \( \rho_w \) is fluid density which is water in here, and \( \bar{b} \) is the mean aperture. The small value of \( Q_m / Q_f, 2.42 \times 10^{-5} \) indicates that the small amount of water flows through the matrix rock with respect to water flowing through fracture plane. Thus water flow thorough the matrix rock does not play a significant role in oil-water flow through fracture. In this numerical modeling, water movement is assumed to occur only in a fracture void space.

Being oil the nonwettings phase in this system, it prefers to flow through the large channels. Therefore, in a sample with an open fracture, oil invasion occurs through the fracture alone, bypassing the low permeability matrix. This was also confirmed through experimental observation. In addition, observation of gravitational effect from the previous experiment was found negligible. Thus, the immiscible displacement modeled in this work is strongly dominated by capillarity and takes place in the fracture void only. There is no fracture-matrix flow.

Primary drainage is a process at which nonwetting fluid invades into a fractured rock originally saturated with wetting fluid. Because of a strongly water wetted surface of a Berea sandstone used in the experiment (Karpyn et al., 2007), at the beginning by saturating the Berea sample with water, a thin film of water rapidly coated the fracture wall surface. By following oil injection, this water film swelled slowly. Eventually the film was able to fill the centers of local void spaces, starting with the thinnest and progressively filling wider and wider layer as illustrated in Figure 4.4.
This pore scale mechanism is called “snap off” which was originally proposed by Mohanty et al., (1980). From the experimental work of Tokunaga and Wan (1987), they used a probe tensiometer to measure the metric potential of the wetting fluid flowing on a single fracture surface. It was shown that flow at the rate of 2-40 m/d occurred through connected layers of wetting fluid that were between 2 to 70 microns thick. Since film flow observation was not conducted in the previous experiment and the major interest of this study is to focus on the effect of capillary equilibrium, the drainage displacement of this numerical modeling will take account only piston-like displacement, not including film flow mechanism. Figure 4.4 demonstrate a displacement distinction between film flow and piston-like displacement.

Figure 4.4 Immiscible displacement occurs via film flow raising a “snap-off” mechanism (a) and piston-like displacement (b).
The simulation program is written in FORTRAN 90 which requires at least Pentium 4 computer with 500 MB RAM. The source code is in appendix A and can be compiled on UNIX platform for parallel processing. The manual, included in appendix C, presents an example case and program output results. Oil-water saturation distribution of primary drainage conducted by Karpyn et al. (2007) will be used as a reference for validating the proposed percolation model. Flow simulations are performed on fracture models at various resolutions as detailed in table 4.1 for finding out the most effective resolution for saving computational time. Comparison of simulations among different types of percolation algorithms are presented in the following section. It shows how significant roles of the effects of the phase accessibility and the in-plane curvature \( r_2 \) on fluid configuration are.

### 4.3 Implementation of Proposed Percolation Model

In this section, the chronological development of modeling the percolation theory of pore-scale network is presented from the standard percolation, the invasion percolation and the modified invasion percolation. The standard percolation (SP) algorithm is based on a concept that all apertures are able to communicate mutually regardless of accessibility of an invading fluid to the apertures. Since the standard percolation algorithm is not a path dependent invasion, it does not take account the in-plane curvature term \( r_2 \), including to fracture topology which represents in terms of the divergence or divergence angle \( \beta \) of fracture wall of equation (4.2). Thus the capillary pressure can be expressed inversely proportional to an aperture size. At the initial drainage displacement by the standard percolation algorithm, the largest apertures will be occupied first as a preferential flow path. Next pore filling will take place in smaller apertures, and the same algorithm keep searching for occupying smaller apertures until no pores left to be filled. Figure 4.5 demonstrates oil invasion at various saturations on a fracture model at resolution 303x779 pixels of table 4.1, where orange depicts apertures occupied by oil and blue depicts apertures trapped with water as well as contacts areas. All apertures greater than the cut-off
aperture contain oil, while apertures less than the cut-off aperture contain water. Oil saturation is then simply given by the fractional pore volume with apertures greater than the cut-off aperture. The correspondence between the cut-off aperture and the saturation explicitly represents the capillary pressure of the fracture model. This observation confirms an idea of inferring the capillary pressure curve from the aperture distribution. Many researchers presented this correspondence by proposing the cumulative distribution function which correlates the capillary pressure and saturation relations with the aperture distribution (Van Genuchten, 1980; Brooks and Corey, 1964; and Pruess and Tsang, 1990). Although the standard percolation presents the strong relationships between the aperture distribution and the capillary pressure curve, it is still lacking the account for the irreducible water saturation which is often found in a realistic capillary pressure curve. Due to the neglect effect of aperture accessibility of the phase occupancy rule in the standard percolation, the zero irreducible water saturation is observed at the final stage of oil invasion as shown in figure 4.5. Figure 4.5 proves that the aperture occupancy based on the local aperture size criterion is inadequate to capture final map of fluid occupancy, so that the phase accessibility and connectivity should be taken into account by implementing the invasion percolation algorithm to the phase occupancy rule.

Besides the incorporation of inlet accessibility criterion, the invasion percolation algorithm also includes the effects of fracture topology described by the divergence and convergence of the fracture wall ($\beta$) in calculating the local capillary pressure. Figure 4.7 shows the sequence of oil invasion, obtained from the proposed invasion percolation model at different saturations. The inlet accessibility of the oil phase and the continuous escaping route of water phase are preserved throughout the entire displacement. Orange depicts oil phase invasion moving from the left to the right outlet and blue includes both trapped water and the contact areas. The local capillary pressures of the potential apertures to be filled along the invasion front are calculated, and the lowest one will be occupied first. For the next invasion, the local capillary pressures will be recalculated according to new invasion front, and seeking out the apertures requiring the lowest capillary pressure to fill. The same procedure will continue until the wetting phase (in blue) loses
connectivity with the outlet. The final fluid distribution consists of a continuous oil phase which surrounds clusters of trapped water. Figure 4.7 demonstrates a step by step oil advance causing of clusters of trapped water. The algorithms of inlet accessibility and fluid connectivity render the water trapping in the fracture system, which confirms the observation of trapped water found in the previous experiment. Comparison of figure 4.5 and 4.6 shows that phase accessibility and connectivity criterion plays a significant role to map a final fluid distribution of the oil invasion. At the final stage, the invasion percolation algorithm yields the irreducible water saturation, whereas the standard percolation leaves no trapped water in the fracture plane. The standard percolation algorithm is appropriate for system in which aperture are in a state of mutual communication and at equilibrium. However, to satisfy equilibrium, time scales must be long with respect to the communication process over the entire apertures in a fracture (Glass et al., 1998). In practical point of views, communication process may not exist in a real fracture consisted of a large number of apertures. The capillary displacement in primary drainage process is essentially based on a local capillary allowability and the accessibility criteria rather than communication process of the standard percolation concept.

However, the irreducible water saturation 0.567 of the invasion percolation model is still too high with respect to the irreducible water saturation 0.35 from the experiment. This can be explained by the effects of ignoring the in-plane curvature ($r_2$) term of the invasion percolation algorithm. Incorporation of the in-plane curvature ($r_2$) was proposed by Glass et al. (1998) using Young-Laplace equation to calculate a local capillary pressure as given equation (4.4). Oil progression of figure 4.7 shows macroscopic smoother front than that of figure 4.6. Such more realistic invasion front of figure 4.7 indicates the influence of neighboring aperture on the aperture occupancy and the phase structure of oil invasion front. The invasion percolation algorithm incorporating the in-plane curvature for determining a local capillary pressure is called "Modified Invasion Percolation (MIP)" algorithm by which a smooth front of phase structure are satisfied in figure 4.7.
Figure 4.5 Sequence of oil invasion at various saturations by using the standard percolation (SP) algorithm for map 303x779 pixels (see table 4.1).
Figure 4.6 Sequence of oil invasion at various saturations by using the invasion percolation (IP) algorithm for map 303x779 pixels.
Figure 4.7 Sequence of oil invasion at various saturations by using the modified invasion percolation (MIP) for map 303x779 pixels.
The modified invasion percolation (MIP) algorithm incorporates the in-plane curvature \( r_z \) for calculating a local capillary pressure. The MIP approach makes more realistic invasion front than the conventional invasion percolation (IP) approach which expresses the capillary pressure as a function of the normal curvature \( r_1 \) alone. Figure 4.8 presents the fluid distribution map reproduced by the standard percolation (SP), the invasion percolation (IP), and the modified invasion percolation (MIP) algorithms respectively. Despite lack of the effect of phase accessibility, the SP algorithm yields a relatively match to an experimental result at the oil saturation 0.65 where simulation was stopped. Compared to the IP algorithm, the MIP algorithm gives a smother macroscopic structure and less clusters of trapped water. Quantitative comparisons of simulation results and experimental saturation maps show that the percentage match of aperture occupied by oil using the MIP model is in the order of 70%, whereas the IP approach gives 40%. By including the in-plane curvature of the MIP model, it shows better quantitative and quantitative agreement with the experimental observation.

As the invasion of oil progresses, the capillary pressure-saturation history can be captured and used to generate the capillary pressure curve. Figure 4.9 shows the capillary pressure curves constructed from the SP, IP, and MIP algorithms. There is no water trapping from the SP algorithm. The irreducible water saturation \( S_{\text{wir}} \) obtained from the MIP model is 0.394, whereas IP model yields 0.567. The MIP model gives the closer value of \( S_{\text{wir}} \) to the experimental value, 0.35. This shows that the implementation of MIP with the in-plane curvature term leads to final saturation in better agreement with experimental results. The entry pressure of the SP algorithm is less than that of the IP and MIP algorithm due to the neglect of the effect of phase accessibility that adds more resistance to the IP and MIP oil invasion. From comparison of the irreducible water saturation, including the in-plane curvature infers that pore filling does not only come from the adjacent pores, but also from the nearby pores along the invasion front. This confirms the experimental observations conducted by Glass (1993) found that as the invasion front moved into a new aperture location, it also moved partially into apertures on either side, and it caused decreasing of the local curvature.
Primary Drainage from experimental observation at So=0.650

Model 303x779 with SP algorithm at So=0.650

Model 303x779 with IP algorithm at So=0.443

Model 303x779 with MIP algorithm at So=0.606

Figure 4.8 Comparison fluid maps from SP, IP, and MIP against experimental image.
Figure 4.9 Comparison of capillary pressure curves from SP, IP, and MIP algorithms at resolution 303x779 pixels.

The lattice resolution used for each simulation is an important parameter to be considered. Running this model with a high resolution becomes computationally expensive, while a lower resolution map may render significant discrepancy with experimental observations. The optimum resolution will give both accurate and efficient predictions. Figure 4.10 shows simulation results at various resolutions with respect to the actual experimental observations (top image in Figure 4.10). Again, orange depicts a continuous oil phase moving from left to right, and dark blue depicts trapped water. The second image in figure 4.10 was obtained from a simulation using the highest resolution available from CT data. The other three images are simulation results from the coarser models A, B, C, and D respectively which are tabulated in table 4.1. All of the coarser models give similar oil-water distribution maps with respect to the experimental reference, but the coarsest one yields the lowest image quality. Therefore, from the comparison of simulation results, it was decided to select model B as a compromise between computing speed and
structural accuracy. The capillary pressure curves obtained from simulations of the fracture models at different resolutions are shown in figure 4.11. Some sensitivity in irreducible water saturation to model resolution is observed in this figure. The coarser resolution tends to yield a lower irreducible water saturation. Despite the attempt to preserve the geophysical topology of the void space of the fracture during map coarsening, there are aperture structures lost or averaged that create different preferential paths during drainage. And the second reason can be explained by the way of coarsening fracture model is to refine and smooth roughness of a fracture wall by averaging local aperture at each location of coarsening. That allows oil to occupy more in the refined fracture void space. Thus some slight discrepancies in the final oil saturation and the irreducible water saturation ($S_{\text{wir}}$) are observed from the capillary pressure curves of figure 4.11.

An interesting observation from these simulation results is that the final fluid distribution map resembles the original aperture distribution (see figure 2.1). This supports the work of Karpyn et al. (2007) stating that oil prefers to flow through large aperture areas, while water tends to occupy small apertures. This observation confirms many researchers proposing a way to map aperture distribution from capillary pressure curve by using the cumulative distribution function (CDF) such as Van Genuchten (1980), and Brook and Corey (1964, 1966). In other words, the observation indicates the strong correspondence between structural characteristics of fractures and preferential flow channeling of oil invasion through a rough-walled fracture.
Figure 4.10 Comparison of simulated fluid distribution maps, obtained at various pixel resolutions, and the experimental result.
The agreement between of the simulations of fracture models and the experimental observations proves the adequacy of the proposed MIP model to capture fundamental displacement mechanisms taking place during capillary-dominated drainage in a rough-walled fracture. In addition, the obtained capillary pressure curves give irreducible water saturations close to the saturation values obtained in the experiments. Therefore, this MIP approach is used in this study to investigate the relationship between structural characteristics of the fracture aperture field and the associated fracture capillary pressure curves.
Chapter 5

Description of Capillary Pressure Curves from Fracture Morphology

This chapter illustrates the influence of parameters such as mean aperture, standard deviation and spatial correlation length on the magnitude and shape of capillary pressure curves. Those three parameters are key geostatistical variables representing the inner structure and connectivity of fracture aperture distribution. A series of artificial aperture fields are generated by a stochastic computer-generated realizations program, COVAR (El-Kadi and Williams, 2000). The COVAR program requires those three input parameters (mean aperture, standard deviation, and spatial correlation length) to create two-dimensional aperture distribution maps. The artificially created aperture distribution maps are square lattice models that serve as input for the percolation model described in chapter 4. Simulation results yield various fracture capillary pressure curves, visualizations of oil-water distributions, and sequential oil invasion, corresponding to each fracture aperture configuration. This method allows us to investigate the effect of global fracture aperture characteristics such as mean aperture, standard deviation, and spatial correlation length, on the capillary conductivity of fractures.

5.1 Generation of Artificial Aperture Field

To investigate the dependence of fracture capillary pressure on fracture morphology, it is necessary to perform simulations of capillary displacements on different fracture geometries. The stochastic fracture replica with various geostatistical parameters represents the uncertainty and variability of the aperture distribution on the fracture plane. As described in section 4.1, the aperture field is discretized as a grid with variable apertures assigned to each block. Apertures distributing over the fracture plane may be spatially correlated in either anisotropic or isotropic patterns. In various instances, the aperture distributions of fractures are often found to follow log-
normal aperture distributions (Gentier, 1986; and Gale, 1987). For that reason, the entire investigation of this geostatistical analysis focuses on the characterization of log-normal aperture fields. The input for the program is a set of two-dimensional coordinate locations that are used to determine elements of the covariance matrix. The stochastic distribution of aperture is characterized by three main input parameters. Statistical parameters input to the program are mean aperture \((\log \bar{b})\), standard deviation (SD), spatial correlation length \((\lambda)\). COVAR applies a matrix decomposition technique to generate the log-normally distributed values of apertures \((b)\) which are first transformed to the normal distribution \(Y\) by:

\[
Y = \log_{10} b
\]  
(5.1)

The values of \(Y\) are calculated from

\[
Y = L \cdot \varepsilon + \bar{Y}
\]  
(5.2)

where \(\bar{Y}\) is the mean of \(Y\), \(\varepsilon\) is a vector \(N[0,1]\)(i.e., normally distributed with mean of zero and standard deviation of 1), and \(L\) is defined in terms of the covariance matrix

\[
A = LL^T
\]  
(5.3)

and \(T\) stands for transpose.

Therefore, from the equation (2) representing the generated process, the mean is given by

\[
E[Y] = LE[\varepsilon] + \bar{Y} = \bar{Y}
\]  
(5.4)

and the covariance can be written as

\[
E[(Y - \bar{Y})(Y - \bar{Y})^T] = LE[\varepsilon\varepsilon^T]L^T = LL^T = A
\]  
(5.5)
in which $E$ stands for the expected value. $Y$ values are characterized by an isotropic covariance function given by

$$A = SD^2 e^{-\omega l}$$

(5.6)

in which $SD^2$ is the variance of $Y$, $l$ is separation lag, and $\omega$ is the autocorrelation parameter in the dimension of inverse length. The exponential form of equation (5.6) shows that apertures will be correlated within a distance of $2/\omega$ and then it might define the correlation length $\lambda$ as $2/\omega$. The expression of equation (5.6) also indicates that the covariance chosen is isotropic.

The geostatistical parameters selected to characterize the aperture distribution herein are mean aperture, standard deviation, and spatial correlation length. The reason for this selection is tied to the role of these structural indicating on phase structure during capillary displacement. Standard deviation is a parameter representing the randomness and variability of aperture distribution, which tends to roughen the invasion front. Spatial correlation length is a parameter commonly used to describe the correlation among apertures, and have a important role to smooth the invasion front. Mean aperture strongly affects the tortuosity and connectivity of flow paths in fractures (Tsang, 1984). Numerical studies by Tsang and Tsang (1990) suggested that these three geostatistical parameters can be used to represent flow patterns through fracture. As above mentioned, these three parameters strongly influence on flow behavior and transport properties of fractures. In this analysis, mean aperture, standard deviation, and spatial correlation length are examined whether there are relationships between these three parameters representing the fracture aperture field and the associated fracture capillary pressure curves. A sensitivity analysis to each of these parameters requires other two to be fixed each time. From the fact that, a single set of these three parameters can create multi-realization fracture models. Thus the sensitivity of realization will be considered on this analysis as well.

Different fracture models with various characteristics and log-normal aperture distributions are artificially generated by controlling those three input parameters. Each artificially generated fracture plane has dimension 25mmX25mm which consists of square lattice 100X100 blocks, and
then the grid resolution is 0.25mm×0.25mm in x and y directions respectively. The following examples demonstrate the fracture-model generations of three cases of when one of the three parameters is analyzed and the other two parameters are fixed, which presents log-normal aperture distributions and visualized created fracture models.

Examples presented in figures 5.1 to 5.6 are some of the patterns used for generating artificial fracture planes for flow simulations that will aid the analysis of the dependency of capillary pressure on fracture morphology. Histograms and log-normal aperture distribution maps given in figures 5.1 and 5.2 present two fracture planes at different mean apertures, 0.18 and 1.18 mm, with constant standard deviation (SD) 0.25, and spatial correlation length (λ) 1.48 mm. Similarly, figures 5.3 and 5.4 illustrate the histograms and aperture distribution maps for fractures different standard deviations, 0.05 and 0.45, at the same mean aperture 0.58 mm. The histogram of the fracture plane with SD= 0.05 shows the sharply peaked amplitude, whereas the histogram with SD= 0.45 gives flatter normal curve. In figures 5.5 and 5.6, fracture planes were generated with different spatial correlation lengths, 1.02 and 9.97 mm, but the same mean aperture 0.58 mm and standard deviation 0.25. From the comparison of aperture distribution maps with spatial correlation length (λ)=1.02 and 9.97 mm, the aperture field with the longer correlation length 9.97 mm yields the larger regions of correlated apertures. The fracture with smaller correlation length (1.02 mm) shows smaller and more scattered correlated regions, indicating less correlation among apertures.
Figure 5.1 Histograms of log-normal aperture distributions at mean aperture 0.18 and 1.18 mm, standard deviation 0.25 and spatial correlation length 1.48 mm.

Figure 5.2 Aperture distribution maps of fractures with mean apertures 0.18 and 1.18 mm, standard deviation 0.25, and spatial correlation length 1.48mm.
Figure 5.3 Histograms of log-normal aperture distributions with standard deviations 0.05 and 0.45, at the same mean aperture 0.58 mm.

Figure 5.4 Aperture distribution maps of fractures with standard deviations 0.05 and 0.45, at the same mean aperture 0.58 mm.
Figure 5.5 Histograms of log-normal aperture distributions with spatial correlation length 1.03 and 9.98 mm at mean aperture 0.58 mm and standard deviation 0.25.

Figure 5.6 Aperture distribution maps of aperture fields with spatial correlation length 1.02 and 9.98 mm, at the same mean aperture 0.58 mm and the standard deviation 0.25.
5.2 Dimensionless Form of Local Capillary Pressure

Fracture planes with mean apertures ranging from 0.38 to 0.58 mm, having the same standard deviation and correlation length were generated using COVAR program. Drainage simulations were then performed on those generated fracture planes. Simulation results yield a family of capillary pressure curves as shown in figure 5.7. It shows that reducing mean aperture causes increase of the entry pressure and the irreducible water. Interestingly, fractures with mean aperture ranging from 0.38 to 0.48 mm give the same values of the irreducible water saturation, but different entry pressures. This observation leads to believe that the fluid invasion may follow a common path, leaving similar clusters of trapped water for certain ranges of mean apertures having the same standard deviation and spatial correlation length.

Figure 5.7 Capillary pressure curves for fractures with mean apertures ranging from 0.38 to 0.58 mm, and standard deviation and spatial correlation length are constant at 0.25 and 1.48 mm, respectively.
Consider the fluid distributions in figure 5.8 at the final stage of fluid saturation maps for the fracture planes with mean aperture 0.38 and 0.48 mm. Orange depicts oil occupied regions and dark blue represents the remaining trapped water and contact areas (zero aperture). In spite of difference in mean apertures, oil invasion in fracture plans (a) and (b) yield identical fluid distribution maps where the irreducible water saturation ($S_{\text{irr}}$) is 0.37.

![Figure 5.8 Final saturation map for fracture planes with mean aperture 0.38 mm (a) and 0.48 mm (b), and standard deviation and spatial correlation length are constant at 0.25 and 1.48 mm, respectively.](image)

Based on the above observation, the expression of capillary pressure in equation (4.4) could be normalized for comparison of fractures in different mean apertures. As a first step toward analysis, and for the sake of simplicity, the topology of the fracture surface expressed in a function of $\beta$ will be ignored. The dimensionless form of equation (4.4) can be written as following equation (Glass, 1998):
\[ P_c^* = \frac{(\cos \alpha)}{r_1^*} + C \frac{1}{r_2^*} \]  

where \[ P_c^* = \frac{\bar{b}}{2\sigma|\cos \alpha|} P_c \quad r_1^* = \frac{b}{\bar{b}} \quad r_2^* = \tan\left(\frac{\gamma}{2}\right) \quad C = \frac{\bar{b}}{\lambda|\cos \alpha|} \]

in which \( \bar{b} \) is the mean aperture, and \( C \) is a dimensionless curvature number that is the weight average of the relative magnitude of the \( r_1^* \) and \( r_2^* \) terms. The notation of \( r_1^* \) throughout this numerical modeling is a positive sign which represents the invasion of the nonwetting phase or primary drainage process are numerically carried out. When the imbibition is of interest, the notation will be negative for \( r_1^* \) term. The measured aperture field is divided by its mean to form a checkboard \( r_1^* \) field with sites at each of the measured locations. \( r_2^* \) is considered as a path-dependent variable associated with \( C \) in a highly nonlinear manner. In the further analysis, the shape of capillary pressure curve will be presented by the two indicators which are the normalized entry pressure (\( P_c^* \) entry) and the irreducible water saturation (\( S_{wirr} \)). To quantify the effects of the fracture morphology on the fracture capillary pressure curve, the normal entry pressure (\( P_c^* \) entry) and the irreducible water saturation (\( S_{wirr} \)) are expressed as functions of mean aperture, standard deviation, and spatial correlation length.
5.3 Effects of Mean Aperture

Studies show that mean aperture has a strong influence on the capillary curve. The magnitude of the mean aperture plays an important role in controlling the effects of tortuosity and connectivity of flow paths in rough fractures (Tsang, 1984). As demonstrated in figure 5.7, it confirms that aperture mean is directly related to the amount of the irreducible water saturation and the magnitude of the entry pressure of a capillary pressure curve commonly used to describe fracture flow behaviors. In cases where the mean aperture is the parameter of interest, the standard deviation and the spatial correlation length of the aperture distribution are fixed with varying mean apertures.

Log-normal aperture distributions of fracture replicas size 25mmX25mm are generated using COVAR. All of the created aperture fields with varying mean apertures from 0.18 to 1.18 mm have the same standard deviation 0.25, correlation length 1.48 mm and similar realization. The simulation results yield the capillary curves as shown in figure 5.9 which shows that the irreducible water saturation decreases as increasing mean aperture, but increasing mean aperture does not alter the normalized entry pressures ($P_{\text{c, entry}}$). The amounts of the irreducible water saturation of the drainage curves are predominantly controlled by the small apertures. At the smaller mean aperture, the effects of tortuosity is greater because there is a large fraction of small apertures found in the aperture field. The smaller the mean aperture of the fracture aperture variation, the more pronounced the water entrapment. Conversely, the effect of connectivity is high when the aperture distribution is skewed to the right of distribution curve or the large aperture. Thus less irreducible water saturation can be found from a fracture model with large mean aperture.
Figure 5.9 Dimensionless capillary pressure curves for fractures with mean apertures ranging from 0.38 to 1.18 mm, and standard deviation and spatial correlation length are constant at 0.25 and 1.48 mm, respectively.

The history of oil invasion and final oil-water distribution are presented in figure 5.10. Invasion of phase structure of the fracture plane with a 0.38 mean aperture is rough, while on the plane with mean aperture 1.18 mm the phase structure progresses with a smoother macroscopic front. This can be explained by the curvature number \( C \) of equation (5.7) which becomes more dominant to the phase structure when increasing mean aperture \( \overline{b} \). As stated by Glass, 2003, the curvature number \( C \) of \( r_2^* \) tends to smooth phase structure, and the aperture variability of \( b \) in the \( r_1^* \) term tends to roughen it. This is the reason why the phase structure of the fracture plane with the mean aperture of 1.18 mm is smoother than that with 0.38 mm, because of the effect of curvature number \( C \) in the fracture case with the larger mean aperture. As a result of smoother progress of oil invasion front, the final sweep area in the 1.18 mm plane is greater than that in the 0.38 mm plane, as illustrated in figure 5.11.
Figure 5.10 Sequential oil invasion in fracture plane with mean aperture =0.38 mm (a) and 1.18 mm (b) at standard deviation=0.25 and spatial correlation length =1.48 mm.

Figure 5.11 Final saturation map for fracture planes with mean aperture 0.38 mm (a) and 1.18 mm (b), and standard deviation and spatial correlation length are constant at 0.25 and 1.48 mm, respectively.
Decreasing the irreducible water saturation by increasing mean aperture can be also observed in other fixed ranges of standard deviations which are SD = 0.05, SD = 0.125, SD = 0.35, and SD = 0.45 as shown in figures 5.12 through 5.15. The normalized entry pressure ($P_c^{*}$ entry) significantly reduced with increasing mean aperture at SD = 0.05 in figure 5.12. At small value of standard deviation, there is little variation in the aperture field, thus fracture plane behaves as a parallel plate rather than a real fracture, in which the cubic law is applicable.

Figure 5.12 Dimensionless capillary pressure curves for fractures with mean apertures ranging from 0.18 to 1.18 mm, and standard deviation and spatial correlation length are constant at 0.05 and 1.60 mm, respectively.
Figure 5.13 Dimensionless capillary pressure curves for fractures with mean apertures ranging from 0.18 to 1.18 mm, and standard deviation and spatial correlation length are constant at 0.125 and 1.60 mm, respectively.
Figure 5.14 Dimensionless capillary pressure curves for fractures with mean apertures ranging from 0.18 to 1.18 mm, and standard deviation and spatial correlation length are constant at 0.35 and 1.31 mm, respectively.
Figure 5.15 Dimensionless capillary pressure curves for fractures with mean apertures ranging from 0.18 to 1.18 mm, and standard deviation and spatial correlation length are constant at 0.45 and 1.08 mm, respectively.
5.4 Effects of Standard Deviation

The variability and randomness of aperture distribution can be described by its standard deviation. From equation (5.7), standard deviation is represented as the variation of the local aperture ($h$) which is the spatial variable in the aperture variability ($r^*$) term. COVAR generates fractures with various standard deviations at mean aperture and small certain range of spatial correlation length. For the reason that the spatial correlation length cannot be fixed for varying standard deviation, this can be explained by the derivation of covariance matrix in COVAR program. The formulation of covariance matrix is based on an isotropic covariance function as expressed in equation 5.9, in which the variance are in an exponential function of the spatial correlation length. This implies that it is not able to vary standard deviation without altering the spatial correlation length.

The simulation results in figure 5.16 show the dimensionless capillary pressure curves in which mean aperture is 0.58 mm for standard deviations ranging from 0.05 to 0.45. The curve of SD=0.05 yields a flat normalized capillary pressure ($P_{C* \text{entry}}$), which indicates a constant pressure drop across a smooth fracture in which there is no effect of tortuosity and roughness. For the same reason, the irreducible water saturation is then zero in the fracture with SD= 0.05 because it is analogous to a smooth fracture with constant aperture. The next step is to investigate the effects of standard deviation on other mean apertures which range from 0.18 to 1.18 mm as shown in figure 5.17. In this analysis, when varying standard deviation, the shape of dimensionless capillary pressure curves are followed by the values of the normalized entry pressure ($P_{C* \text{entry}}$) and the irreducible water saturation ($S_{\text{wir}}$). These two indicators ($P_{C* \text{entry}}$ and $S_{\text{wir}}$) are expressed as functions of standard deviation with family curves of mean apertures as shown in figures 5.17 and 5.18 respectively.
Figure 5.16 Dimensionless capillary pressure curves with standard deviations ranging from 0.05 to 0.45. Mean aperture is 0.58 for all cases while spatial correlation varies from 1.08 to 2.08 mm.

When the standard deviation becomes larger, there is a smoother transition at the level of entry pressure and irreducible water saturation, instead of a sharp stair-step shape. Since the tortuosity and connectivity flow path becomes more dominate on flow through a rough fracture with variable apertures (large standard deviation), these effects significantly influence the shape of the capillary pressure curve in terms of the normalized entry pressure ($P_{c*}$ entry) and the irreducible water saturation ($S_{wir}$). The next step is to investigate the sensitivity of the capillary pressure curve to standard deviation for other mean apertures. The normalized entry pressure ($P_{c*}$ entry) and the irreducible water saturation ($S_{wir}$) are indicators decided to represent the shape of capillary pressure curve, when the standard deviation varies.
Figure 5.17 Dimensionless capillary pressure as a function of standard deviation for fracture planes with mean apertures ranging from 0.18 to 1.18 mm and spatial correlation lengths varying from 1.08 to 2.08 mm.

Figure 5.17 shows the normalized entry pressure ($P_C^{\ast}$ entry) as a function of standard deviation for each mean aperture. For standard deviations ranging from 0.125 to 0.45 in figure 5.17, the normalized entry pressure ($P_C^{\ast}$ entry) tends to decrease as the standard deviation increases for fracture planes having mean aperture ranging from 0.18 to 0.98 mm, whereas the normalized entry pressure ($P_C^{\ast}$ entry) increases with increasing standard deviation for the 1.18 mm plane.
Figure 5.18 shows the irreducible water saturation ($S_{wir}$) as functions of standard deviation and mean aperture. At the narrow range of standard deviation 0.05, fracture with mean aperture ranging from 0.58 to 1.18 mm yields zero irreducible water saturation. This is because fractures with large mean aperture and small standard deviation are similar to a smooth wall fracture. For the fracture with the smallest mean aperture of 0.18 mm, as standard deviation increases the curvature number ($C$) seems to strongly dominate the invasion front and causes the irreducible water saturation to reduce. On the other hand, a fracture with the mean aperture of 1.18 mm becomes greater dominated by the aperture variability ($r_i^*$) as increasing standard deviation. The aperture variability ($r_i^*$) roughens the invasion front and leads to increase the irreducible water saturation. For the fractures with mean apertures from 0.38 to 0.98 mm, the irreducible water
saturation is controlled by the combination of the effects of the aperture variability \( (r_1^*) \) and the curvature number \( (C^*) \). It can be implied that the increase or decrease of the irreducible water saturation is directly controlled by the result of the competition between the aperture variability \( (r_1^*) \) and the curvature number \( (C^*) \).

Figure 5.19 shows phase structures of fractures with mean aperture 0.18 mm by varying standard deviations from 0.05 to 0.45. At the range of standard deviations from 0.125 to 0.45, the irreducible water saturation is reduced due to increasing the influence of the curvature number \( (C^*) \), in which oil can sweep more area. That causes smaller clusters of trapped water remaining in a fracture plane. In the opposite, figure 5.20 shows the increasing effect of the aperture variability \( (r_1^*) \) on the phase structures of fractures for mean aperture 1.18 mm. As increasing standard deviations from 0.05 to 0.45, area sweep of oil invasion is smaller, and invasion front is more complicated because of increasing the effect of the aperture variability \( (r_1^*) \) which tends to roughen the phase structure.
Figure 5.19 Sequence of oil invasion for fracture planes with standard deviations varying from 0.05 to 0.45. Mean aperture is 0.18 mm for all cases while the spatial correlation length varies between 1.1 mm and 2.1 mm.
Figure 5.20 Sequence of oil invasion for fracture planes with standard deviations varying from 0.05 to 0.45. Mean aperture is 1.18 mm for all cases while the spatial correlation length varies between 1.1 mm and 2.1 mm.
5.4 Effects of Spatial Correlation Length

To examine the effect of spatial correlation length on the capillary pressure curve, series of fracture planes are artificially generated using COVAR. The generated fracture models have constant mean aperture and standard deviation with varying spatial correlation lengths ($\lambda$).

Figure 5.21 Normalized variograms for fractures with spatial correlations ranging from 1.03 mm to 9.96 mm, at mean aperture 0.18 mm and standard deviation 0.25.

After artificial aperture fields are stochastically generated by COVAR program, the variograms in the isotropic direction are plotted and fitted by the exponential model to determine the spatial correlation lengths ($\lambda$) which are the input parameters needed for simulations. Figure 5.21 shows the normalized variograms for the fracture planes at mean aperture 0.18 mm and SD=0.25, by varying spatial correlation length from 1.03 to 9.96 mm. Likewise, fractures with other sizes of mean apertures also yields the same family of the isotropic spatial correlation lengths.
Program for calculating variograms and an example case of curve fitting can be found in appendix B and C respectively.

The normalized entry pressure ($P_{c}^{*}$ entry) versus the spatial correlation length is plotted as shown in figure 5.22. As the spatial correlation length ($\lambda$) becomes longer, the normalized entry pressure decreases. However, the irreducible water saturation ($S_{wirr}$) does not decrease with increasing spatial correlation length ($\lambda$) as expected. This is because there are paths of the longer spatial correlation length among apertures. Oil prefers to flow along the paths with high correlation and bypass all the shorter correlated paths. From this observation, it causes oil to bypass and fill up a fracture instead of spanning through small correlation length regions. Therefore, increasing the irreducible water saturation with increasing the spatial correlation length ($\lambda$) is likely observed in figure 5.23.

![Figure 5.22 Entry pressure as a function of the spatial correlation length for fractures with mean apertures ranging from 0.18 to 1.18 mm, at standard deviation 0.25.](image)
Figure 5.23 Irreducible water saturation as a function of the spatial correlation length for fractures with mean apertures ranging from 0.18 to 1.18 mm at standard deviation 0.25.

Figure 5.24 shows the sequence of oil invasion for a fracture plane with the mean aperture 0.18 mm and the standard deviation 0.25, by varying the spatial correlation lengths from 1.01 to 9.97 mm. The irreducible water saturation \( S_{wir} \) tends to increase with increasing the spatial correlation length because of the high correlated paths that allows oil to reach the breakthrough at ease and render more trapped water. Similarly in figure 5.25, fractures with the fixed mean aperture 1.18 mm and the standard deviation by varying spatial correlation lengths also yields more irreducible water saturation due to the increase of the spatial correlation length.
Figure 5.24 The sequence of oil invasion for fracture planes with the spatial correlation lengths varying from 1.01 mm to 9.97 mm at the same mean aperture 0.18 mm and SD = 0.25.
Figure 5.25 the sequence of oil invasion for fracture planes with the spatial correlation lengths varying from 1.01 mm to 9.97 mm at the same mean aperture 1.18 mm and SD = 0.25.
The connectivity in a fracture can also be quantified by considering how much the applied pressure and the areal sweep of oil invasion are required at the breakthrough point. If the breakthrough pressure is low, that is an indication of high connectivity. Figures 5.26 and 5.27 present the breakthrough pressure and oil saturation at breakthrough as a function of the spatial correlation length ($\lambda$). The applied pressure at the breakthrough decreases with increasing spatial correlation length ($\lambda$) as shown in figure 5.26. This implies that a longer spatial correlation length yields the better connectivity. Similarly, increasing the spatial correlation length ($\lambda$) also decreases oil saturation at breakthrough as shown in figure 5.26. Both figures infer that the connectivity among apertures in a fracture is directly related to the spatial correlation length. Longer spatial correlation length indicates the higher correlation and connectivity among apertures.

Figure 5.26 Breakthrough pressure as a function of the spatial correlation length for fractures with mean apertures ranging from 0.18 to 1.18 mm, at standard deviation 0.25.
Figure 5.27 Oil saturation at the breakthrough as a function of the spatial correlation length for fractures with mean apertures ranging from 0.18 to 1.18 mm, at standard deviation 0.25.

Figures 5.28 through 5.33 present the normalized entry pressure ($P_C^{*}$ entry) and the irreducible water saturation ($S_{wirr}$) as functions of spatial correlation length ($\lambda$) and mean aperture with fixed standard deviations ranging from 0.125 to 0.45. Figures 5.22, 5.23 and 5.28 to 5.33 are proposed to quantify the influence of these geostatistical parameters on fracture capillary pressure curves through the normalized entry pressure ($P_C^{*}$ entry) and the irreducible water saturation ($S_{wirr}$). All of these figures also offer a methodology to predict capillary pressures from randomly given geostatistical parameters of fracture models. From interpolations on these figures, the predicted values of the normalized entry pressure ($P_C^{*}$ entry) and the irreducible water saturation ($S_{wirr}$) of a capillary pressure can be obtained. Example cases of predicting fracture capillary pressure curves from random sets of geostatistical parameters can be found in appendix D.
Figure 5.28 Entry pressure as a function of the spatial correlation length for fractures with mean apertures ranging from 0.18 to 1.18 mm, at standard deviation 0.125.

Figure 5.29 Irreducible water saturation as a function of the spatial correlation length for fractures with mean apertures ranging from 0.18 to 1.18 mm, at standard deviation 0.125.
Figure 5.30 Entry pressure as a function of the spatial correlation length for fractures with mean apertures ranging from 0.18 to 1.18 mm, at standard deviation 0.35.

Figure 5.31 Irreducible water as a function of the spatial correlation length for fractures with mean apertures ranging from 0.18 to 1.18 mm, at standard deviation 0.35.
Figure 5.32 Entry pressure as a function of the spatial correlation length for fractures with mean apertures ranging from 0.18 to 1.18 mm, at standard deviation 0.45.

Figure 5.33 Irreducible water as a function of the spatial correlation length for fractures with mean apertures ranging from 0.18 to 1.18 mm, at standard deviation 0.45.
5.6 Sensitivity to Multiple Realizations of the Aperture Field

In this work, the shape of a fracture capillary pressure curve is described by means of two primary indicators, the normalized entry pressure ($P_{C^*\text{ entry}}$) and the irreducible water saturation ($S_{\text{wirr}}$). These two indicators of capillary pressure curves are directly related to the geostatistical parameters of the aperture distribution as shown in section 5.5. Results presented in figures 5.22, 5.23 and 5.28 to 5.33 allow the estimation of the normalized entry pressure ($P_{C^*\text{ entry}}$) and the irreducible water saturation ($S_{\text{wirr}}$) from known geostatistical parameters. However, it is important to recognize that single set of geostatistical variables can correspond to multiple aperture field configurations (realizations) having different capillary pressure characteristics. The sensitivity to multiple realizations of aperture field is examined here for randomly given created aperture fields. For a given set of the geostatistical parameters, the normalized entry pressure ($P_{C^*\text{ entry}}$) and the irreducible water saturation ($S_{\text{wirr}}$) can be predicted from the correspondences shown in the proposed figured. In each case, COVAR generates fractures with two different realizations from the given set of goestatistical parameters, and then simulations are performed on those two different realization fractures. In the following text, three case studies are presented. In each case, two sample fractures are created from the same geostatistical parameter, but at different realizations. Simulation results show the final oil-water saturation, and the sequential oil invasion including the aperture distribution of the two different realization fractures.
Case I. Mean aperture: **0.58** mm, standard Deviation: **0.30**, spatial correlation length **1.48** mm.

Two aperture fields with different realizations are generated using the given set of above geostatistical parameters. Drainage simulations are then performed on the generated aperture fields, and the corresponding capillary pressure curves are resulted in figure 5.34. Thereafter, the given set of the parameters is used to estimate the normalized entry pressure ($P_c^\text{entry}$) and the irreducible water saturation ($S_{wirr}$) using figure 5.17 and 5.18 respectively. The interpolated value of the normalized entry pressure ($P_c^\text{entry}$) obtained from figure 5.17 is 0.974, and the irreducible water saturation ($S_{wirr}$) from figure 5.18 is 0.287. The estimated values of the two capillary curve indicators are in good agreement with those values from the simulation as shown in figure 5.34. Figure 5.35 shows the sequential oil invasion and oil-water saturation from simulations of the two-artificially generated fracture models.

**Figure 5.34** Simulation results of capillary pressure curves for fractures with mean aperture 0.58 mm, standard deviation 0.30, and spatial correlation length 1.48 mm at two different realizations.

**Prediction:** $P_c^\text{entry} = 0.974$

$S_{wirr} = 0.287$

$P_c^\text{entry} = 0.963$

$S_{wirr} = 0.293$
Figure 5.35 Aperture distribution, sequential invasion and oil-water saturation for fracture planes with mean aperture 0.58 mm, standard deviation 0.30, and spatial correlation length 1.45 mm at two different realizations.
**Case II:** Mean aperture: 0.68 mm, standard Deviation: 0.35, spatial correlation length 1.45 mm.

Two-realization fracture models are generated from the same geostatistical characteristics as above. Their geostatistical parameters are used to estimate the normalized entry pressure ($P_{c*}$entry) from figure 5.17 and the irreducible water saturation ($S_{wirr}$) from figure 5.18. Interpolations of the predicted values are calculated between graphs of mean aperture 0.58 and 0.78 mm, which results 0.996 for $P_{c*}$ entry, and 0.263 for $S_{wirr}$. Full reconstruction of the $P_{c*}$ entry using the MIP model is shown in figure 5.36. Estimated $P_{c*}$ entry and the one obtained from drainage simulation show a very close match, but $S_{wirr}$ from figure 5.36 and from interpolation are slightly different, revealing some sensitivity to the actual configuration of the aperture field, or realization. Figure 5.37 depicts the aperture distribution, sequential oil invasion and oil-water saturation from the created fractures with the identical geostatistical parameters for two different realizations.

**Figure 5.36** Simulation result of capillary pressure curves for fractures with mean aperture 0.68 mm, standard deviation 0.35, and spatial correlation length 1.45 mm at two different realizations.
Figure 5.37 Aperture distribution, sequential invasion and oil-water saturation for fracture plane with mean aperture 0.68 mm, standard deviation 0.35, and spatial correlation length 1.45 mm at two different realizations.
Case III. Mean aperture: 0.58 mm, standard Deviation: 0.25, spatial correlation length 5.0 mm.

Similarly, two different realization fractures are generated from the given above geostatistical parameters, in which drainage simulations using MIP model are performed. Capillary pressure curves obtained from the simulations are shown in figure 5.38, and sequential oil invasions and final fluid distribution maps are shown in figure 5.39. The normalized entry pressure ($P_c^*$ entry) and the irreducible water saturation ($S_{wirr}$) are estimated from figures 5.22 and 5.33 respectively. Estimated $P_c^*$ entry are in the range of $P_c^*$ entry from the simulations, but estimated $S_{wirr}$ are slightly off the range of $S_{wirr}$ from the simulations as shown in figure 5.38. Discrepancies between the estimated values and the simulation values present the sensitivity of realizations.

Figure 5.38 Simulation result of capillary pressure curves for fractures with mean aperture 0.58 mm, standard deviation 0.25, and spatial correlation length 5.0 mm at different realizations.
Figure 5.39 Aperture distribution, sequential invasion and oil-water saturation for fracture plane with mean aperture 0.58 mm, standard deviation 0.25, and spatial correlation length 5.0 mm at different realizations.
Results in the sensitivity to realization show fair agreement estimated indicators of capillary pressure curves and the actual curves obtained from drainage simulations using MIP. This supports that fact that the three proposed geostatistical parameters are strongly indications of the shape of a fracture capillary pressure curve, as confirmed many studies (Lowry and Miller 1995; Pruess and Tsang 1990, and Tsang 1984). Comparisons presented in this section also show that realizations may have an important impact in the predicted capillary pressure curves and thus on actual fracture transport characteristics. Extensions that allow analysis of the optimum number of realizations of possible fracture models can be also the further study. From the raised examples of the cases of multi-realization fractures, it presents that this numerical study offers a robust methodology for estimating the fracture capillary pressure from known geostatistical parameters. This could develop to be a pragmatic approach for reservoir simulation.
Chapter 6

Conclusions

This study consists of the investigations of single-phase flow and capillary-dominated displacement in a rough-walled fracture. Single-phase flow was examined using a commercially available computational fluid dynamics software. Simulations were performed on a realistic fracture structure reconstructed from CT-scanned data from a previous experiment. Comparisons of flow behaviors in the realistic fracture and a representative parallel plate models were made. Thereafter, a modified invasion percolation (MIP) model was developed to investigate capillary displacement in a rough-walled fracture, and examine dependency of the fracture capillary pressure curve on fracture morphology. The conclusions of this study can be summarized as follows:

- Discrepancies of pressure drop resulted from CFD simulator and the analytical solution of the cubic law reached two orders of magnitude. The underestimation of pressure drop by the cubic law can be corrected by applying a scaling factor to the hydraulic conductivity term of the cubic law.

- The MIP model using the Young-Laplace equation is valid for modeling primary drainage in a rough-walled fracture under a low capillary number and quasi-static condition. Comparison of results of oil-water saturation distribution from the MIP simulations and the experiment are in a good agreement with 70% match of oil phase location. This indicates that the MIP model provides an accurate representative of the physical mechanisms observed during capillary-dominated drainage in a rough-walled fracture.
During capillary displacement of fluids moving in a rough fracture with variable apertures, phase structure of invasion front was found to be controlled by surface roughness of fracture walls, topography of fracture planes, and the spatial variation of aperture. Those parameters are represented by the aperture variability ($\eta^*$) and the curvature number ($C$) terms in the constitutive relationships describing capillary displacement. The aperture variability ($\eta^*$) tends to roughen the phase structure, whereas the curvature number ($C$) tends to smooth it. A rough or smooth fluid front will develop depending on the relative dominance of the aperture variability ($\eta^*$) or the curvature number ($C$) for a given capillary displacement.

Increasing mean aperture tends to smooth the phase structure due to increase in the curvature number ($C$). With increasing mean aperture, the normalized entry pressure ($P_{c,\text{ entry}}$) also increases. This is because the dimensionless capillary pressure is directly proportional to the mean aperture as expressed in the normalization of Young-Laplace equation. However, entry capillary pressure, expressed in its original form, decreases with increasing mean aperture. Increasing mean aperture reduces the irreducible water saturation ($S_{\text{win}}$) due to the fact that larger mean aperture indicates the presence of large apertures through which oil preferentially flows.

At a narrow range of standard deviation or less variability of local apertures, the aperture field resembles a smooth parallel plate model, in which the cubic law is applicable. A steady entry pressure and zero irreducible water saturation are observed in this case. For a larger standard deviation, the combined effects of the aperture variability ($\eta^*$) and the curvature number ($C$) either increases or decreases the entry pressure and the irreducible water saturation. It depends on which parameters dominate the flow.
• A large spatial correlation length indicates a tendency for high connectivity, correlated paths in the aperture field. At large spatial the correlation length, low normalized entry pressures are observed. However, large spatial correlation lengths do not reduce the irreducible water saturation as expected. On the contrary, an increasing of spatial correlation length causes the irreducible water saturation to increase. A possible explanation for this observation is that oil preferentially flows along highly correlated paths rather than spanning through the ill-sorted paths, thus decreasing sweep efficient drainage and leaving large amount of water behind.

• A single set of geostatistical parameters can represent multiple aperture fields, which impacts the consistency and accuracy of the capillary pressure prediction. These predictions are also tied to the scale at which geostatistical parameters are determined. Further work is required to evaluate the optimum number of realizations which could substantiate the prediction of the fracture capillary pressure and sensibility to scaling effects.
References


APPENDIX A

Source Code of MIP Program

PROGRAM Main
IMPLICIT NONE
INTEGER :: i, j, nx, ny, bM, BT, count, SUMMAP, SUMMAP1, T, PAMN, PAMS, PAME, PAMW, &
           n, mx, my, POT1, start, MIP, fmax
DOUBLE PRECISION :: lx, ly, RES, So, SUMDIS, A, bN, bS, bE, bW, ALPHA, Pc, SIGMA, lamda,&
                    Pmin, Ptrack, Ptrack1, DISN, DISS, DISE, DISW, Soi, Soset, So1, &
                    CONANGLE, Sw
CHARACTER (20) :: FILENAME
PARAMETER (start=0)
PARAMETER (MIP=1)
PARAMETER (Soset=1.0)
DOUBLE PRECISION, ALLOCATABLE :: betaN1(:, :), betaS1(:, :), betaE1(:, :), betaW1(:, :), &
                                   Pji(:, :), OLDpji(:, :), DIS(:, :)
INTEGER, ALLOCATABLE :: POT(:, :), DUMMY(:, :), OLDDUMMY(:, :), PAM(:, :), &
                       OLDMAP(:, :), CONNECT(:, :), OLDCALLNCON(: :), &
                       ORDER(:, :), TRAP(:, :)
INTEGER, ALLOCATABLE :: SUBMAP(:, :), SUBDUMMY(:, :, :)
DOUBLE PRECISION, ALLOCATABLE :: Xi(:, :), Yi(:, :), SUBDIS(:, :, :)

!============================= HEADING ===============================
WRITE(*,*) '                                                                                         ',
WRITE(*,*) '*********************************************************************',
WRITE(*,*) '*                                                                                              *',
WRITE(*,*) '*               MIP Program for Prediction of Pc Curve                *',
WRITE(*,*) '*                                                                                              *',
WRITE(*,*) '*                                            By                                              *',
WRITE(*,*) '*                                                                                              *',
WRITE(*,*) '*                             Tawatchai Petchsingto                             *',
WRITE(*,*) '*                                                                                              *',
WRITE(*,*) '*                    The Pennsylvania State University                   *',
WRITE(*,*) '*                                                                                              *',
WRITE(*,*) '*********************************************************************',

!============= INPUT PARAMETERS OF A LATTICE MODEL ==============
WRITE(*,*) '                                                                                         ',
WRITE(*,*) '                                                                                         ',
WRITE(*,*) '=======  INPUT PARAMETERS OF THE APERTURE DISTRIBUTION  =======',
WRITE(*,*) 'Enter name of input file of a fracture model (e.g., map.txt)',
READ(*,*) FILENAME
WRITE(*,*) 'Enter number of grid blocks of a fracture model in X-direction?',
READ(*,*) nx
WRITE(*,*) 'Enter number of grid blocks of a fracture model in Y-direction?',
READ(*,*) ny
WRITE(*,*) 'Enter resolution of a fracture model in X-direction',
READ(*,*) lx
WRITE(*,*) 'Enter resolution of a fracture model in Y-direction',
READ(*,*) ly
WRITE(*,*) 'Enter resolution of a fracture model in Z-direction',
READ(*,*) RES
WRITE(*,*) 'Enter isotropic spatial correlation length of the aperture filed',
READ(*,*) lamda
mx = NINT(lamda/lx)
my = NINT(lamda/ly)
!============ CHECK SPATIAL CORRELATION LENGTH IS NOT TOO SMALL =============
IF (mx.or.my < 5) THEN
WRITE (*,*) 'Note! the spatial correlation length should be longer than resolution at least 5 times.'
STOP
END IF
IF (mx/=my) THEN
IF (mx>my) THEN
bM = mx
ELSE
bM = my
END IF
END IF
END IF
ELSE
   bM=mx  ! bM is number of blocks used for averaging the BETA angle.
END IF

n=bM  ! n is number of blocks in x direction, where the invasion percolation (IP) will end.

!==================================================  INPUT WETTABILITY OF FLUIDS  ==========================
WRITE(‘*’,*)  
WRITE(‘*’,*)  
WRITE(‘*’,*)  
WRITE(‘*’,*) ‘Enter the contact angle in degree’
READ(*) CONANGLE
WRITE(‘*’,*) ‘Enter the interfacial tension in mdyn/m’
READ(*) SIGMA
ALPHA=CONANGLE/180*22/7

!================================== ALLOCATE ALL VARIABLES ==================================
ALLOCATE  ( betaN1(0:ny+1,0:nx+1),betaS1(0:ny+1,0:nx+1),betaE1(0:ny+1,0:nx+1),
            betaW1(0:ny+1,0:nx+1), Pji(0:ny+1,0:nx+1), OLDPji(0:ny+1,0:nx+1),
            DIS(0:ny+1,0:nx+1) )
ALLOCATE  (   POT(0:ny+1,0:nx+1),  DUMMY(0:ny+1,0:nx+1),  OLDDUMMY(0:ny+1,0:nx+1),
            PAM(0:ny+1,0:nx+1), OLDMAP(0:ny+1,0:nx+1), CONNECT(0:ny+1,0:nx+1),
            OLDCONNECT(0:ny+1,0:nx+1),TRAP(0:ny+1,0:nx+1) )
ALLOCATE  ( SUBMAP(2*my+1,2*mx+1), SUBDUMMY(2*my+1,2*mx+1) )
ALLOCATE  (   XI(2*my+1,2*mx+1),YI(2*my+1,2*mx+1), SUBDIS(2*my+1,2*mx+1) )
26 FORMAT (3000I20)
27 FORMAT ( 3F20.5)
28 FORMAT (3000F20.5)
29 FORMAT ( 2F40.20,I20)

!================================== OPEN INPUT FILE AND OUTPUT FILES ==========================
OPEN (unit=20, file= FILENAME           ,status='unknown')
OPEN (unit=30, file='fluidMAP.txt'          ,status='unknown')
OPEN (unit=40, file='fluidProgress.txt'   ,status='unknown')
OPEN (unit=50, file='CapCurve.txt'        ,status='unknown')

!================================== READING INPUT FILE ========================================
DO j=0,nx+1
   DO i=0,ny+1
      DIS (i,j)=0.0d0
   END DO
END DO
READ(20,*) (DIS(i,j),i=1,nx)
END DO

!= SOMETIMES GENERATED APERTURE FIELD ARE LESS THAN ZERO AT SOME LOCATIONS=
DO j=1,ny
   DO i=1,nx
      IF(DIS(i,j)<0) THEN
         DIS(i,j)=0.0d0
      END IF
   END DO
END DO

SUMDIS=sum(DIS)

!==================================================  CAL BETA FOR NSEW  ==========================
CALL Beta(DIS,nx,ny,lx,ly,bM,RES,betaN1,betaS1,betaE1,betaW1)

!==================================================  PUT ALL INITIAL CONDITIONS  ===========
DO j=0,ny+1
   DO i=0,nx+1
      OLDPji (j,i)=0.0d0
      POT (j,i)=0.0d0
      OLDDUMMY (j,i)=0.0d0
      DUMMY (j,i)=0.0d0
      PAM (j,i)=0.0d0
      OLDMAP (j,i)=0.0d0
      CONNECT (j,i)=0.0d0
      OLDCONNECT(j,i)=0.0d0
      ORDER (j,i)=1.0d0
      Pji (j,i)=1000000.0d0
      TRAP (j,i)=0.0d0
   END DO
END DO
DO j=1,ny  
   IF (DIS(j,i)==0) THEN  
      CONNECT(j,i)=1  
   END IF  
END DO  
END DO  

DO i=1,nx  
   IF (DIS(i,j)==0) THEN  
      CONNECT(j,i)=1  
   END IF  
END DO  
END DO  

!=================================================================
!WHEN START=1, SKIP TO MIP=================================================================

IF (start==1.and.MIP==1) THEN  
   GO TO 100  
END IF  
IF (start==1.and.MIP==0) THEN  
   GO TO 200  
END IF  
Soi=0  

!*************************************************************************************************************!
!***************STEP(3) IP INVADING PROGRESSING IN DISTANCE 2n***********************************************!
!*************************************************************************************************************!

200 IF (start==1.OR.So > Soset) THEN  
   OPEN (unit=70, file='PAM.txt' , status='unknown')  
   OPEN (unit=80, file='DUMMY.txt', status='unknown')  
   OPEN (unit=90, file='CONNECT.txt', status='unknown')  
   OPEN (unit=100, file='Pji.txt', status='unknown')  
   OPEN (unit=110, file='ORDER.txt', status='unknown')  
   OPEN (unit=120, file='TRAP.txt', status='unknown')  
   OPEN (unit=130, file='TandSo.txt', status='unknown')  
   DO j=1, ny  
      READ(70,*) ( PAM(j,i),i=1,nx)  
      READ(80,*) (DUMMY(j,i),i=1,nx)  
      READ(90,*) (CONNECT(j,i),i=1,nx)  
      READ(100,*) (Pji(j,i),i=1,nx)  
      READ(110,*) (ORDER(j,i),i=1,nx)  
      READ(120,*) (TRAP(j,i),i=1,nx)  
   END DO  
   READ(130,*) Ptrack,So,T  
   CLOSE(70)  
   CLOSE(80)  
   CLOSE(90)  
   CLOSE(100)  
   CLOSE(110)  
   CLOSE(120)  
   CLOSE(130)  
END IF  
IF (start==0.and.So < Soset) THEN  
   So =0  
   T =0  
END IF  

COUNT =0  
SUMMAP =10  
SUMMAP1=0  
BT=0  
WRITE(50,*) ' Pc(Pa)  ',' Sw'  
WRITE(50,*) '_________________________________________________'.  
WRITE(50,*) '.  

!=================================================================
START FILLING PORES IN DO WHILE LOOP  

DO WHILE (BT==0.and.SUMMAP1/=SUMMAP)  
   START DO WHILE(1)  
   COUNT =COUNT+1  
   IF (count==1) THEN  
      OLDMAP(j,i)=PAM(j,i)  
      OLDCONNECT(j,i)=CONNECT(j,i)  
      Pji(j,i)=1000000.0d0  
      DUMMY(j,i)=0  
   ELSE  
      OLDMAP(j,i)=PAM(j,i)  
      OLDDUMMY(j,i)=DUMMY(j,i)  
      DUMMY(j,i)=0  
      OLDCONNECT(j,i)=CONNECT(j,i)  
   END IF  
   WRITE(50,*) ' Ptrack,So,T  
   WRITE(50,*) '_________________________________________________'.  
   WRITE(50,*) '.  
   WRITE(50,*) 'Pc(Pa)  ',' Sw'  
   WRITE(50,*) '_________________________________________________'.  
   WRITE(50,*) '.  
END DO WHILE(1)  
END DO WHILE (BT==0.and.SUMMAP1/=SUMMAP)  
END IF  
SUMMAP1=SUMMAP  
DO i=1,nx  
   IF (count==1) THEN  
      OLDMAP(j,i)=PAM(j,i)  
      OLDCONNECT(j,i)=CONNECT(j,i)  
      Pji(j,i)=1000000.0d0  
      DUMMY(j,i)=0  
   ELSE  
      OLDMAP(j,i)=PAM(j,i)  
      OLDDUMMY(j,i)=DUMMY(j,i)  
      DUMMY(j,i)=0  
      OLDCONNECT(j,i)=CONNECT(j,i)  
   END IF  
END DO
OLDPji (j,i)=Pji (j,i)
POT (j,i)=0
Pji (j,i)=1000000.0d0
END IF
END DO
END DO
IF (count==1.and.start==0.and.So < Soset) THEN ! LET FIRST ROW BE INVADED EXCEPT ASPERITIES
DO j=1,ny
  IF(DIS(j,1) /=0) THEN
    PAM (j,1)=1
    T = T+1
    ORDER(j,1)=T
    So = DIS(j,1)/SUMDIS+So
    Sw = 1 - So
  END IF
END DO
ENDIF
END IF
DO i=1,nx !==========MARK INVASION FRONT===========
  DO j=1,ny
    IF ((OLDMAP(j,i)==0).and.(DIS(j,i)/=0).and.(OLDCONNECT(j,i)==1).and.(PAM(j,i)==0).and.&
      (PAM(j+1,i)==1).or.(PAM(j-1,i)==1).or.(PAM(j,i+1)==1).or.(PAM(j,i-1)==1)) THEN
      DUMMY(j,i)=1
    END IF
  END DO
END DO
DO i=1,nx !==========CAL LOCAL Pc OF EACH CELL======
  DO j=1,ny
    IF (OLDDUMMY(j,i)==0.and.DUMMY(j,i)==1) THEN
      bN = betaN1(j,i)
      bS = betaS1(j,i)
      bE = betaE1(j,i)
      bW = betaW1(j,i)
      PAMN=PAM(j-1,i)
      PAMS=PAM(j+1,i)
      PAME=PAM(j,i+1)
      PAMW=PAM(j,i-1)
      DISN=DIS(j-1,i)
      DISS=DIS(j+1,i)
      DISE=DIS(j,i+1)
      DISW=DIS(j,i-1)
      A=DIS(j,i)*RES
      CALL IP(DISN,DISS,DISE,DISW, PAMN, PAMS, PAME, PAMW, ALPHA, SIGMA, A, bN, bS, bE, bW,Pc)
      Pji(j,i)=Pc
    END IF
    IF (OLDDUMMY(j,i)==1.and.DUMMY(j,i)==1) THEN
      Pji(j,i)=OLDPji(j,i)
    END IF
  END DO
END DO
Pmin=minval(Pji) !******MINIMUM LOCAL CAPILLARY PRESSURE IS DETERMINED******
IF (count==1.and.start==0.and.So < Soset) THEN
  Ptrack=Pmin
  fmax=1
ENDIF
IF (Ptrack < Pmin.and.count/=1.and.Pmin < 1000000.0d0) THEN
  Ptrack=Pmin
  WRITE(50,27) Ptrack, Sw
ENDIF
DO i=1,nx !======POTENTIAL CELLS ARE OCCUPIED UNTIL REACHING 2n=======
  IF (BT==1) THEN
    EXIT
  END IF
  DO j=1,ny
    IF (Pji(j,i)==Pmin.and.OLDCONNECT(j,i)==1.and.DUMMY(j,i)==1) THEN
      PAM(j,i)=1
      T=T+1
      ORDER (j,i)=T
      So = DIS(j,i)/SUMDIS+So
    END IF
  END DO
END DO
IF (i > fmax. and i < nx) THEN
fmax = i
END IF
IF (PAM(j,2*n) == 1 and MIP == 1) THEN
BT = 1
EXIT
END IF
END IF
END DO
END DO

END IF

END IF

CALL ROUTE(ny, nx, fmax, PAM, OLDCONNECT, DIS, CONNECT, TRAP)
SUMMAP = sum(PAM)

END IF

IF (count == 1) THEN
So1 = So
END IF

Soi = So - So1

WRITE AND OPEN FILE WHEN TIMING IS OVER THE ALLOWABLE MAXIMUM So====
IF (Soi > Soset) THEN
OPEN (unit=70, file='PAM.txt', status='unknown')
OPEN (unit=80, file='DUMMY.txt', status='unknown')
OPEN (unit=90, file='CONNECT.txt', status='unknown')
OPEN (unit=100, file='Pji.txt', status='unknown')
OPEN (unit=110, file='ORDER.txt', status='unknown')
OPEN (unit=120, file='TRAP.txt', status='unknown')
OPEN (unit=130, file='TandSo.txt', status='unknown')
DO j = 1, ny
WRITE(70,26) (PAM(j,i), i=1, nx)
WRITE(80,26) (DUMMY(j,i), i=1, nx)
WRITE(90,26) (CONNECT(j,i), i=1, nx)
WRITE(100,28) (Pji(j,i), i=1, nx)
WRITE(110,26) (ORDER(j,i), i=1, nx)
WRITE(120,26) (TRAP(j,i), i=1, nx)
END DO
WRITE(70,*)
WRITE(80,*)
WRITE(90,*)
WRITE(100,*)
WRITE(110,*)
WRITE(120,*)
WRITE(130,29) Ptrack, So, T
CLOSE(70)
CLOSE(80)
CLOSE(90)
CLOSE(100)
CLOSE(110)
CLOSE(120)
CLOSE(130)
GO TO 200
END IF

WRITE ('(*) 'Pc(Pa)='Ptrack,' Sw=',Sw')
END DO !==END DO WHILE (1)
Ptrack1 = Ptrack
MARK UP FIRST Ptrack COMING OUT FROM IP SECTION
IF (MIP==0) THEN
WRITE('(*) 'IP SUMMAP=', SUMMAP
WRITE('(*) 'Total=', T
DO j = 1, ny
WRITE(30,26) (PAM(j,i), i=1, nx)
WRITE(40,26) (ORDER(j,i), i=1, nx)
END DO
END IF

STEP(4) MIP PROGRESSING UNTIL REACHING Swirr
STEP(4) MIP PROGRESSING UNTIL REACHING Swirr

100 IF (start == 1 OR Soi > Soset) THEN
OPEN (unit=70, file='PAM.txt', status='unknown')
OPEN (unit=80, file='DUMMY.txt', status='unknown')
OPEN (unit=90, file='CONNECT.txt', status='unknown')

OPEN (unit=100, file='Pji.txt' ,status='unknown')
OPEN (unit=110, file='ORDER.txt' ,status='unknown')
OPEN (unit=120, file='TRAP.txt' ,status='unknown')
OPEN (unit=130, file='TandSo.txt' ,status='unknown')
DO j = 1, ny
   READ(70 ,*) ( PAM (j,i),i=1,nx)
   READ(80 ,*) ( DUMMY (j,i),i=1,nx)
   READ(90 ,*) (CONNECT(j,i),i=1,nx)
   READ(100,*) ( Pji    (j,i),i=1,nx)
   READ(110,*) ( ORDER  (j,i),i=1,nx)
   READ(120,*) ( TRAP   (j,i),i=1,nx)
END DO

READ(130,*) Ptrack,So,T
CLOSE(70)
CLOSE(80)
CLOSE(90)
CLOSE(100)
CLOSE(110)
CLOSE(120)
CLOSE(130)
END IF

count  =0
SUMMAP =10
SUMMAP1=0

!================
START FILLING PORES IN DO WHILE LOOP 
===================

DO WHILE (SUMMAP/=SUMMAP1.and.MIP==1) !*********
   SUMMAP1=SUMMAP
   count=count+1
   DO i=1, nx !=============PUT ALL INITIAL CONDITIONS NEEDED=========
      DO j = 1, ny
         IF (count==1.and.start==0) THEN
            OLDDUMMY  (j,i)=0
            OLDMAP    (j,i)=PAM    (j,i)
            OLDCONNECT (j,i)=CONNECT(j,i)
            Pji       (j,i)=1000000.0d0
            DUMMY     (j,i)=0
         ELSE
            OLDMAP    (j,i)=PAM    (j,i)
            OLDDUMMY  (j,i)=DUMMY  (j,i)
            DUMMY     (j,i)=0
            OLDCONNECT (j,i)=CONNECT(j,i)
            OLDPji    (j,i)=Pji     (j,i)
            POT       (j,i)=0
            Pji       (j,i)=1000000.0d0
         END IF
      END DO
   END DO

!==============MARK ALL POTENTIAL CELLS WHICH WILL BE OCCUPIED===========
DO i=1,nx
   CALL POTENT(j,i, ny, nx, my, mx, DUMMY, OLDDUMMY, POT1 )
END IF

!============CHECK AND UPDATE LOCAL Pc ALONG THE INVASION FRONT===========
DO i=1,nx
   IF (OLDMAP(j,i)==0).and.(DIS(j,i)==0).and.(OLDCONNECT(j,i)==1).and.(PAM(j,i)==0).and.&
   ((PAM(j+1,i)==1).or.(PAM(j-1,i)==1).or.(PAM(j,i-1)==1).or.(PAM(j,i+1)==1)) ) THEN
      DUMMY(j,i)=1
   END IF
   IF (POT(j,i)/=0) THEN
      CALL SWEEP(j,i, ny, nx, my, mx, lx,ly, DIS, PAM, DUMMY, &
                  SUBDIS,SUBMAP, SUBDUMMY, Xi, Yi)
      !====CAL LOCAL Pc OF EACH POTENTIAL CELL======
      bN  =betaN1(j,i)
bS = betaS1(j,i)
bE = betaE1(j,i)
bW = betaW1(j,i)
A = DIS(j,i)*RES
CALL CAPILLARY( my, mx, SUBMAP, Xi, Yi, A, bN, bS, bE, bW, RES, &
ALPHA, lamda, SIGMA, Pc)
Pj(j,i) = Pc
END IF
!====UPDATE Pc ABLE TO BE OCCUPIED AND UNCHANGED SURROUNDING CELLS==
!IF(DUMMY(j,i)==1.and.POT(j,i)==0.and.count==1) THEN
Pj(j,i)=OLDPj(j,i)
END IF
END DO
END DO
Pmin=minval(Pj) !=============MINIMUM Pc AT THE INVASION FRONT
!=MARKING AND ORDER SEQUENCE OF OCCUPATION OF POTENTIALS CELLS TO BE FILLED==
DO j=1,nx
IF(Pj(j,i)== Pmin.and.OLDCONNECT(j,i)==1.and.DUMMY(j,i)==1.and.Pmin <1000000.0d0) THEN
PAM (j,i)=1
T=T+1
ORDER (j,i)=T
So=DIS(j,i)/SUMDIS+So
Sw=1-So
IF (i > fmax) THEN
fmax=i
END IF
END IF
END DO
SUMMAP=sum(PAM)
!==========CHECK OIL PATH CONNECTION=============
CALL ROUTE(ny, nx, fmax, PAM, OLDCONNECT, DIS, CONNECT,TRAP)
IF (count==1) THEN
So1=So
END IF
Soi=So-So1
!===================WRITE ON OUT PUT FILE==============
IF (Ptrack < Pmin.and.Pmin < 1000000.0d0)THEN
Ptrack = Pmin
WRITE(50,27) Ptrack, Sw
END IF
WRITE (*,*) 'Pc(Pa)=',Ptrack,'    Sw=',Sw
!==WRITE AND OPEN FILE WHEN TIMING IS OVER THE ALLOWABLE MAXIMUM TIME===
IF (Soi > Soset) THEN
OPEN (unit=70 , file='PAM.txt'           ,status='unknown')
OPEN (unit=80 , file='DUMMY.txt'     ,status='unknown')
OPEN (unit=90 , file='CONNECT.txt' ,status='unknown')
OPEN (unit=100, file='Pji.txt'              ,status='unknown')
OPEN (unit=110, file='ORDER.txt'     ,status='unknown')
OPEN (unit=120, file='TRAP.txt'        ,status='unknown')
OPEN (unit=130, file='TandSo.txt'     ,status='unknown')
DO j = 1 , ny
WRITE(70 ,26) ( PAM  (j,i),i=1,nx)
WRITE(80 ,26) (  DUMMY  (j,i),i=1,nx)
WRITE(90 ,26) (CONNECT(j,i),i=1,nx)
WRITE(100,28) (           Pji  (j,i),i=1,nx)
WRITE(110,26) (  ORDER  (j,i),i=1,nx)
WRITE(120,26) (      TRAP (j,i),i=1,nx)
WRITE(130,29) Ptrack,So,T
CLOSE(70)
CLOSE(80)
CLOSE(90)
CLOSE(100)
CLOSE(110)
CLOSE(120)
CLOSE(130)
END DO
WRITE (130,29) Ptrack,So,T
GO TO 100
END IF
END DO !***************END DO WHILE BIG LOOP (2)**************

!============= WRITE OUTPUT FILE ==============
IF (MIP==1) THEN
   WRITE(*,*) 'SUMMAP=',SUMMAP
   WRITE(*,*) 'Ttotal=',T
   DO j=1, ny
      WRITE(30,26) ( PAM(j,i),i=1,nx)
      WRITE(40,26) (ORDER(j,i),i=1,nx)
   END DO
END IF

END PROGRAM Main
SUBROUTINE Beta(DIS, nx, ny, lx, ly, bM, RES, betaN1, betaS1, betaE1, betaW1)

!Import: DIS, lx, ly, bM, RES, nx, ny
!Export: betaN1, betaS1, betaE1, betaW1
IMPLICIT NONE
INTEGER :: bM, nx, ny
DOUBLE PRECISION, INTENT (IN) :: lx, ly, RES
DOUBLE PRECISION, DIMENSION (0:ny+1, 0:nx+1), INTENT (IN) :: DIS
DOUBLE PRECISION, DIMENSION (0:ny+1, 0:nx+1), INTENT (OUT) :: betaN1, betaS1, betaE1, betaW1

INTEGER :: ii, jj, row, col, k, p, d
DOUBLE PRECISION :: betaN, betaS, betaE, betaW, dN, dN1, LN, LN1, dS, dS1, LS, LS1, dE, dE1, LE, LE1, dW, dW1, LW, LW1, hN, hS, hE, hW
DOUBLE PRECISION, DIMENSION (2*bM+1, 2*bM+1) :: SUBDIS

DO i=1, nx !********
START BIG LOOP********
DO j=1, ny
!==========================SECTION IN bM RADIUS=======================
DO ii=1, 2*bM+1
DO jj=1, 2*bM+1
row=j+jj-bM-1
col=i+ii-bM-1
IF (row <= 0 ) THEN
row=0
END IF
IF (row > ny ) THEN
row=ny+1
END IF
IF (col <= 0 ) THEN
col=0
END IF
IF (col > nx ) THEN
col=nx+1
END IF
SUBDIS (jj,ii)=DIS(row,col)
END DO
END DO
!========================CAL BETA NORTH============================
dN =0
dN1=0
LN =0
LN1=0
k =0
DO jj=bM, 1,-1
p=abs(jj-bM-1)
IF (SUBDIS(jj,bM+1)==0) EXIT
k  = k+1
dN1= (SUBDIS(jj,bM+1))
dN = dN+dN1
LN1= (ly*p)
LN = LN+LN1
END DO
IF (k=0) THEN
END IF

!========================CAL BETA SOUTH============================
dS =0
dS1=0
LS =0
LS1=0
k =0
DO jj=bM+2, 2*bM+1
p=abs(jj-bM-1)
IF (SUBDIS(jj,bM+1)==0) EXIT
k  = k+1
dS1= (SUBDIS(jj,bM+1))
dS = dS+dS1
LS1= (ly*p)
LS = LS+LS1

END DO
END SUBROUTINE
END DO
IF (k/=0) THEN
  dS=dS/k
  LS=LS/k
END IF
!==============================================CAL BETA WEST==============================================
dW =0
dW1=0
LW =0
LW1=0
k  =0
DO ii=bM,1,-1
  p=abs(ii-bM-1)
  IF (SUBDIS(bM+1,ii)==0) EXIT
    k  = k+1
    dW1= (SUBDIS(bM+1,ii))
    dW = dW+dW1
    LW1= (lx*p)
    LW = LW+LW1
END DO
IF (k/=0) THEN
  dW=dW/k
  LW=LW/k
END IF
!==============================================CAL BETA FOR NSEW DIRECTION===============================================
d=DIS(j,i)
hN=0
hS=0
hE=0
hW=0
betaN=0
betaS=0
betaE=0
betaW=0
!==============================================NORTH=======================================================
IF ((j==1).or.(DIS(j-1,i)==0).or.(DIS(j,i)==0)) THEN
  betaN1(j,i)=0
ELSE
  IF (LN/=0) THEN
    hN   = (d-dN)*RES/2
    betaN= atan(hN/LN)
  END IF
  betaN1(j,i)=betaN
END IF
!==============================================SOUTH=====================================================
IF ((j==ny).or.(DIS(j+1,i)==0).or.(DIS(j,i)==0))THEN
  betaS1(j,i)=0
ELSE
  IF (LS/=0) THEN
    hS   = (d-dS)*RES/2
    betaS= atan(hS/LS)
  END IF
  betaS1(j,i)=betaS
END IF
!==============================================EAST========================================================
IF ((i==nx).or.(DIS(j,i+1)==0).or.(DIS(j,i)==0))THEN
  betaE1(j,i)=0
ELSE
  IF (LE/=0) THEN
    hE   = (d-dE)*RES/2
    betaE= atan(hE/LE)
  END IF
  betaE1(j,i)=betaE
END IF
!==============================================WEST========================================================
IF ((i==1).or.(DIS(j,i-1)==0).or.(DIS(j,i)==0))THEN
  betaW1(j,i)=0
ELSE
  IF (LW/=0) THEN
    hW   = (d-dW)*RES/2
  END IF
END IF
betaW = atan(hW/LW)
END IF
betaW1(j,i)=betaW
END IF
END DO
END DO !************END BIG LOOP************

END SUBROUTINE Beta
SUBROUTINE CAPILLARY( my, mx, SUBMAP, Xi, Yi, A, bN, bS, bE, bW, RES, ALPHA, lamda, SIGMA, Pc)
!
!Import: my, mx, SUBMAP, Xi, Yi, A, bN, bS, bE, bW, RES, ALPHA, lamda, SIGMA
!Export: Pc
IMPLICIT NONE
INTEGER , INTENT (IN) :: mx, my
DOUBLE PRECISION, INTENT (IN) :: A, bN, bS, bE, bW, RES, ALPHA, lamda, SIGMA
DOUBLE PRECISION, INTENT(OUT) :: Pc
INTEGER , DIMENSION (2*my+1, 2*mx+1), INTENT (IN) :: SUBMAP
DOUBLE PRECISION, DIMENSION (2*my+1, 2*mx+1), INTENT (IN) :: Xi, Yi
INTEGER :: k, jj, ii, Q14, Q23, Q1, Q2, Q3, Q4, N, Q12, Q34, Q1H, Q2H, Q3H, Q4H
DOUBLE PRECISION :: Xright1, Yright1, Xright, Yright, Xleft1, Yleft1, Xleft, Yleft,&
Xtop1, Ytop1, Xtop, Ytop, Xbot1, Ybot1, Xbot, Ybot,&
Theta1, Theta2, Theta3, Theta4, GammaN, GammaS, GammaE, GammaW,&
Theta1H, Theta2H, Theta3H, Theta4H, TERM2N, TERM2S, TERM2E, TERM2W,&
TERM1N, TERM1S, TERM1E, TERM1W, PI
PARAMETER (PI=22/7)
DOUBLE PRECISION, DIMENSION(4) :: P
!==================SET ARRAY P TO BE ZERO=================
DO k=1,4
  P(k)=1000000.0d0
END DO
!=======================CAL X AND Y FOR LEFT AND RIGHT SIDE==================
!===============RIGHT SIDE==============
Xright1=0
Yright1=0
Xright =0
Yright =0
Q14 =0
N =0
DO jj=1,2*my+1
  DO ii=mx+1,2*mx+1
    Xright1=Xi(jj,ii)+Xright1
    Yright1=Yi(jj,ii)+Yright1
    IF (Xi(jj,ii)/=0.or.Yi(jj,ii)/=0) THEN
      N=N+1
    END IF
  END DO
END DO
IF (N/=0) THEN
  Xright=Xright1/N
  Yright=Yright1/N
  IF (Xright/=0.or.Yright/=0) THEN
    Q14=1
  END IF
END IF
!===============LEFT SIDE==============
Xleft1=0
Yleft1=0
Xleft =0
Yleft =0
Q23 =0
N =0
DO jj=1, 2*my+1
  DO ii=1, mx+1
    Xleft1=Xi(jj,ii)+Xleft1
    Yleft1=Yi(jj,ii)+Yleft1
    IF (Xi(jj,ii)/=0.or.Yi(jj,ii)/=0) THEN
      N=N+1
    END IF
  END DO
END DO
IF (N/=0) THEN
  Xleft=Xleft1/N
  Yleft=Yleft1/N
  IF (Xleft/=0.or.Yleft/=0) THEN
    Q23=1
  END IF
END IF
!=================DESIGNATE THETA==============
Q1=0
Q2=0
Q3=0
Q4=0

!=============RIGHT SIDE=============
Theta1=0
Theta4=0

!Theta1
IF (Xright==0.and.Yright/=0.and.Yright>0.and.Q14==1) THEN
  Theta1=PI/2
  Q1=1
END IF
IF (Yright==0.and.Xright/=0.and.Xright>0.and.Q14==1) THEN
  Theta1=0
  Theta4=0
  Q1=1
  Q4=1
END IF
IF (Xright/=0.and.Yright/=0.and.Xright>0.and.Yright>0.and.Q14==1) THEN
  Theta1=atan(Yright/Xright)
  Q1=1
END IF
!Theta4
IF (Xright/=0.and.Yright/=0.and.Xright>0.and.Yright<0.and.Q14==1) THEN
  Theta4=atan(Yright/Xright)
  Theta4=-Theta4
  Q4=1
END IF
IF (Xright==0.and.Yright/=0.and.Yright<0.and.Q14==1) THEN
  Theta4=PI/2
  Q4=1
END IF

!=============LEFT SIDE==========
Theta2=0
Theta3=0

!Theta2
IF (Xleft==0.and.Yleft/=0.and.Yleft>0.and.Q23==1) THEN
  Theta2=PI/2
  Q2=1
END IF
IF (Yleft==0.and.Xleft/=0.and.Xleft<0.and.Q23==1) THEN
  Theta2=0
  Theta3=0
  Q2=1
  Q3=1
END IF
IF (Xleft/=0.and.Yleft/=0.and.Xleft<0.and.Yleft>0.and.Q23==1) THEN
  Theta2=atan(Yleft/Xleft)
  Theta2=-Theta2
  Q2=1
END IF
!Theta3
IF (Xleft/=0.and.Yleft/=0.and.Xleft<0.and.Yleft<0.and.Q23==1) THEN
  Theta3=atan(Yleft/Xleft)
  Q3=1
END IF
IF (Xleft==0.and.Yleft/=0.and.Yleft<0.and.Q23==1) THEN
  Theta3=PI/2
  Q3=1
END IF

!=============CAL GAMMA NORTH & SOUTH=============
GammaN=0
GammaS=0
IF (Q1==1.and.Q3==1) THEN
  GammaS=PI+Theta1-Theta3
  GammaN=2*PI-GammaS
END IF
IF (Q1==1.and.Q2==1) THEN
  GammaS=PI+Theta1+Theta2
  GammaN=2*PI-GammaS
END IF
IF (Q2==1.and.Q4==1) THEN
  GammaS=PI+Theta2-Theta4
  GammaN=2*PI-GammaS
END IF

IF (Q3==1.and.Q4==1) THEN
  GammaS=PI-Theta3-Theta4
  GammaN=2*PI-GammaS
END IF

!=======CAL TERM2N,TERM2S========
TERM2N=0
TERM2S=0
IF (SUBMAP(my,mx+1)==1.and.Q14==1.and.Q23==1)THEN
  TERM2N=lamda*tan(GammaN/2)
END IF
IF (SUBMAP(my+2,mx+1)==1.and.Q14==1.and.Q23==1)THEN
  TERM2S=lamda*tan(GammaS/2)
END IF

!====================
CAL X AND Y FOR TOP AND BOTTOM SIDE====================

!=============TOP SIDE==========
Xtop1=0
Ytop1=0
Xtop =0
Ytop =0
Q12=0
N=0
DO jj=1,my+1
  DO ii=1,2*mx+1
    Xtop1=Xi(jj,ii)+Xtop1
    Ytop1=Yi(jj,ii)+Ytop1
    IF (Xi(jj,ii)/=0.or.Yi(jj,ii)/=0) THEN
      N=N+1
    END IF
  END DO
END DO
END IF
IF (N/=0) THEN
  Xtop=Xtop1/N
  Ytop=Ytop1/N
END IF
IF (Xtop/=0.or.Ytop/=0) THEN
  Q12=1
END IF

!=============BOTTOM SIDE==========
Xbot1=0
Ybot1=0
Xbot =0
Ybot =0
Q34=0
N=0
DO jj=my+1, 2*my+1
  DO ii=1,2*mx+1
    Xbot1=Xi(jj,ii)+Xbot1
    Ybot1=Yi(jj,ii)+Ybot1
    IF (Xi(jj,ii)/=0.or.Yi(jj,ii)/=0) THEN
      N=N+1
    END IF
  END DO
END DO
END IF
IF (N/=0) THEN
  Xbot=Xbot1/N
  Ybot=Ybot1/N
END IF
IF (Xbot/=0.or.Ybot/=0) THEN
  Q34=1
END IF

!=============TOP SIDE============
Q1H=0
Q2H=0
Q3H=0
Q4H=0
!Theta1
Theta1H=0
Theta2H=0
IF (Ytop==0.and.Xtop/=0.and.Xtop>0.and.Q12==1) THEN
  Theta1H=0
  Q1H=1
END IF
IF (Xtop==0.and.Ytop/=0.and.Ytop>0.and.Q12==1) THEN
  Theta1H=Pi/2
  Theta2H=Pi/2
  Q1H=1
  Q2H=1
END IF
IF (Xtop/=0.and.Ytop/=0.and.Xtop>0.and.Ytop>0.and.Q12==1) THEN
  Theta1H=atan(Ytop/Xtop)
  Q1H=1
END IF
!Theta2
IF (Xtop/=0.and.Ytop/=0.and.Xtop<0.and.Ytop>0.and.Q12==1) THEN
  Theta2H=atan(Ytop/Xtop)
  Theta2H=-Theta2H
  Q2H=1
END IF
IF (Ytop==0.and.Xtop/=0.and.Xtop<0.and.Q12==1) THEN
  Theta2H=0
  Q2H=1
END IF
!=============BOTTOM SIDE==========
!Theta3
Theta3H=0
Theta4H=0
IF (Ybot==0.and.Xbot/=0.and.Xbot<0.and.Q34==1) THEN
  Theta3H=0
  Q3H=1
END IF
IF (Xbot==0.and.Ybot/=0.and.Ybot<0.and.Q34==1) THEN
  Theta3H=Pi/2
  Theta4H=Pi/2
  Q3H=1
  Q4H=1
END IF
IF (Ybot/=0.and.Xbot/=0.and.Xbot<0.and.Ybot<0.and.Q34==1) THEN
  Theta3H=atan(Ybot/Xbot)
  Q3H=1
END IF
!Theta4
IF (Xbot/=0.and.Ybot/=0.and.Xbot>0.and.Ybot<0.and.Q34==1) THEN
  Theta4H=atan(Ybot/Xbot)
  Theta4H=-Theta4H
  Q4H=1
END IF
IF (Ybot==0.and.Xbot/=0.and.Xbot>0.and.Q34==1) THEN
  Theta4H=0
  Q4H=1
END IF
!=============CAL GAMMA EAST & WEST==========
GammaW=0
GammaE=0
IF (Q2H==1.and.Q3H==1) THEN
  GammaE=2*Pi-Theta2H-Theta3H
  GammaW=2*Pi-GammaE
END IF
IF (Q1H==1.and.Q4H==1) THEN
  GammaE=Theta1H+Theta4H
  GammaW=2*Pi-GammaE
END IF
IF (Q2H==1.and.Q4H==1) THEN
  GammaE=Pi+Theta4H-Theta2H
  GammaW=2*Pi-GammaE
END IF
IF (Q1H==1.and.Q3H==1) THEN
    GammaE=PI+Theta1H-Theta3H
    GammaW=2*PI-GammaE
END IF

!==========CAL TERM2E,TERM2W=======
TERM2E=0
TERM2W=0
IF (SUBMAP(my+1,mx)==1.and.Q12==1.and.Q34==1) THEN
    TERM2W=lamda*tan(GammaW/2)
END IF
IF (SUBMAP(my+1,mx+2)==1.and.Q12==1.and.Q34==1) THEN
    TERM2E=lamda*tan(GammaE/2)
END IF

!=============CAL APERTURE TERM OR BETA=============
TERM1N=0
TERM1S=0
TERM1E=0
TERM1W=0
IF (SUBMAP(my,mx+1)==1) THEN
    TERM1N=(2*cos(ALPHA+bN))/A
END IF
IF (SUBMAP(my+2,mx+1)==1) THEN
    TERM1S=(2*cos(ALPHA+bS))/A
END IF
IF (SUBMAP(my+1,mx+2)==1) THEN
    TERM1E=(2*cos(ALPHA+bE))/A
END IF
IF (SUBMAP(my+1,mx)==1) THEN
    TERM1W=(2*cos(ALPHA+bW))/A
END IF

!=============CAL LOCAL CAPILLARY PRESSURE=============
IF (SUBMAP(my,mx+1)==1) THEN
    P(1)=SIGMA*(TERM1N+2/TERM2N)
END IF
IF (SUBMAP(my+2,mx+1)==1) THEN
    P(2)=SIGMA*(TERM1S+2/TERM2S)
END IF
IF (SUBMAP(my+1,mx+2)==1) THEN
    P(3)=SIGMA*(TERM1E+2/TERM2E)
END IF
IF (SUBMAP(my+1,mx)==1) THEN
    P(4)=SIGMA*(TERM1W+2/TERM2W)
END IF
Pc=minval(P)
END SUBROUTINE CAPILLARY
SUBROUTINE IP(DISN,DISS,DISE,DISW, PAMN, PAMS, PAME, PAMW, ALPHA, SIGMA, A, bN, bS, bE, bW, Pc)

! Import: DISN, DISS, DISE, DISW, PAMN, PAMS, PAME, PAMW, ALPHA, SIGMA, A, bN, bS, bE, bW
! Export: Pc

IMPLICIT NONE

INTEGER, INTENT(IN) :: PAMN, PAMS, PAME, PAMW
DOUBLE PRECISION, INTENT(IN) :: ALPHA, SIGMA, A, bN, bS, bE, bW, DISN, DISS, DISE, DISW
DOUBLE PRECISION, INTENT(OUT) :: Pc

INTEGER :: k
DOUBLE PRECISION, DIMENSION(4) :: P

! ================== SET ARRAY P TO BE ZERO ==================
DO k=1,4
  P(k)=1000000.0d0
END DO

! =============== CAL Pc CONSIDERING ONLY APERTURE SIZE ==========
IF (DISN/=0.and.PAMN==1) THEN
  P(1)=SIGMA*(2*cos(ALPHA+bN))/A
END IF
IF (DISS/=0.and.PAMS==1) THEN
  P(2)=SIGMA*(2*cos(ALPHA+bS))/A
END IF
IF (DISE/=0.and.PAME==1) THEN
  P(3)=SIGMA*(2*cos(ALPHA+bE))/A
END IF
IF (DISW/=0.and.PAMW==1) THEN
  P(4)=SIGMA*(2*cos(ALPHA+bW))/A
END IF

Pc=minval(P)

END SUBROUTINE IP
SUBROUTINE POTENT(j, i, ny, nx, my, mx, DUMMY, OLDDUMMY, POT1)

!Import: j, i, ny, nx, my, mx, DUMMY, OLDDUMMY
!Export: POT1
IMPLICIT NONE
INTEGER :: j, i, nx, ny, mx, my
INTEGER :: POT1
INTEGER :: DUMMY, OLDDUMMY
INTEGER :: my, mx
DOUBLE PRECISION :: CHECK, SUBOLDDUMMY, SUBDUMMY

!=============SET INITIAL CONDITION============
POT1 = 0

!========== SWEEPING IN RADIUS LAMDA===========
DO ii = 1, 2*mx+1
DO jj = 1, 2*my+1
row = j+jj-my-1
col = i+ii-mx-1
IF (row <= 0) THEN
row = 0
END IF
IF (row > ny) THEN
row = ny + 1
END IF
IF (col <= 0) THEN
col = 0
END IF
IF (col > nx) THEN
col = nx + 1
END IF
SUBOLDDUMMY(jj,ii) = OLDDUMMY(row, col)
SUBDUMMY(jj,ii) = DUMMY(row, col)
CHECK(jj,ii) = 0
END DO
END DO

!===========CHECK THERE ARE ANY CHANGES==========
DO ii = 1, 2*mx+1
DO jj = 1, 2*my+1
IF (SUBDUMMY(jj,ii) /= SUBOLDDUMMY(jj,ii)) THEN
CHECK(jj,ii) = 1
END IF
END DO
END DO

POT1 = SUM(CHECK)
IF (POT1 /= 0) THEN
POT1 = 1
END IF

END SUBROUTINE POTENT
SUBROUTINE ROUTE(ny, nx,fmax, PAM, OLDCONNECT, DIS, CONNECT,TRAP)

!Import: ny, nx,fmax, PAM, OLDCONNECT, DIS
!Export: CONNECT,TRAP
IMPLICIT NONE
INTEGER          , INTENT (IN) :: nx,ny,fmax
INTEGER          ,DIMENSION (0:ny+1,0:nx+1), INTENT (IN) :: PAM, OLDCONNECT
DOUBLE PRECISION,DIMENSION (0:ny+1,0:nx+1), INTENT (IN) :: DIS
INTEGER          ,DIMENSION (0:ny+1,0:nx+1), INTENT(IN OUT) :: CONNECT, TRAP
INTEGER          :: i,j,SUMCONNECT, SUMCONNECT1,SUMCONNECT_BIG,&
                     SUMCONNECT1_BIG,countroute,fmin,SUMTRAP

!===========CLEAN UP CONNECT TO BE ZERO FROM i=fmax to fmin
IF (fmax < 5 )THEN
DO i=1,fmax
  DO j=0,ny+1
    CONNECT(j,i)=0
  END DO
  fmin = i
END DO
END IF
IF (fmax > 5) THEN
DO i=fmin,fmax
  DO j=0,ny+1
    CONNECT(j,i)=0
  END DO
END DO
SUMCONNECT_BIG =10
SUMCONNECT1_BIG=0
countroute=0

!==================FINDING CONNECTED ROUTE===================
DO WHILE (SUMCONNECT_BIG/=SUMCONNECT1_BIG)
  countroute=countroute+1
  SUMCONNECT1_BIG=SUMCONNECT_BIG
  SUMCONNECT_BIG=SUMCONNECT
  countroute=0
  DO WHILE (SUMCONNECT/=SUMCONNECT1)
    DO j=1,ny
      IF(i==nx.and.PAM(j,i)==0.and.DIS(j,i)/=0) THEN
        CONNECT(j,i)=1
      ELSE
        IF (PAM(j,i)==0.and.DIS(j,i)/=0.and.CONNECT(j,i)==0.and.&
            (CONNECT(j-1,i)==1.or.CONNECT(j+1,i)==1.or.CONNECT(j,i-1)==1.or.&
            CONNECT(j,i+1)==1).and.TRAP(j,i)==0) THEN
          CONNECT(j,i)=1
        IF (PAM(j-1,i)==0.and.DIS(j-1,i)/=0.and.CONNECT(j-1,i)==0.and.TRAP(j-1,i)==0) THEN  !NORTH
          CONNECT(j-1,i)=1
        IF (PAM(j+1,i)==0.and.DIS(j+1,i)/=0.and.CONNECT(j+1,i)==0.and.TRAP(j+1,i)==0) THEN  !SOUTH
          CONNECT(j+1,i)=1
        IF (PAM(j,i-1)==0.and.DIS(j,i-1)/=0.and.CONNECT(j,i-1)==0.and.TRAP(j,i-1)==0) THEN  !WEST
          CONNECT(j,i-1)=1
        IF (PAM(j,i+1)==0.and.DIS(j,i+1)/=0.and.CONNECT(j,i+1)==0.and.TRAP(j,i+1)==0) THEN  !EAST
          CONNECT(j,i+1)=1
      END IF
    END IF
  END DO
  SUMCONNECT=0
  DO j=ny,1,-1
    IF(i==nx.and.PAM(j,i)==0.and.DIS(j,i)/=0) THEN
      CONNECT(j,i)=1
    ELSE
      END IF
  END DO
END WHILE (1)
END DO
END IF

SUMCONNECT_BIG=SUMCONNECT1_BIG
!====================SWEEP OVER FROM RIGHT TO LEFT================
DO i=fmax,fmin-1
  SUMCONNECT_BIG =10
  SUMCONNECT1_BIG =0
  DO WHILE (SUMCONNECT_BIG/=SUMCONNECT1_BIG)
    DO j=1,ny
      IF(i==nx.and.PAM(j,i)==0.and.DIS(j,i)/=0) THEN
        CONNECT(j,i)=1
      ELSE
        IF ((PAM(j-1,i)==0.and.DIS(j-1,i)/=0.and.CONNECT(j-1,i)==0).and.&
            (CONNECT(j-1,i+1)==1.or.CONNECT(j-1,i-1)==1.or.CONNECT(j-1,i)==1.or.&
            CONNECT(j-1,i+1)==1).and.TRAP(j-1,i)==0) THEN
          CONNECT(j-1,i)=1
        IF (PAM(j-1,i)==0.and.DIS(j-1,i)/=0.and.CONNECT(j-1,i)==0.and.TRAP(j-1,i)==0) THEN  !NORTH
          CONNECT(j-1,i)=1
        IF (PAM(j+1,i)==0.and.DIS(j+1,i)/=0.and.CONNECT(j+1,i)==0.and.TRAP(j+1,i)==0) THEN  !SOUTH
          CONNECT(j+1,i)=1
        IF (PAM(j-1,i)==0.and.DIS(j-1,i)/=0.and.CONNECT(j-1,i)==0.and.TRAP(j-1,i)==0) THEN  !WEST
          CONNECT(j-1,i)=1
        IF (PAM(j-1,i)==0.and.DIS(j-1,i)/=0.and.CONNECT(j-1,i)==0.and.TRAP(j-1,i)==0) THEN  !EAST
          CONNECT(j-1,i)=1
      END IF
    END IF
  END DO
  SUMCONNECT_BIG =SUMCONNECT
  SUMCONNECT1_BIG=0
  SUMCONNECT_BIG =SUMCONNECT
END DO

SUMCONNECT=0
DO j=ny,1,-1
  IF(i==nx.and.PAM(j,i)==0.and.DIS(j,i)/=0) THEN
    CONNECT(j,i)=1
  ELSE
    END IF
END DO
IF (PAM(j,i)==0.and.DIS(j,i)!=0.and.CONNECT(j,i)==0.and. &
(CONNECT(j-1,i)==1.or.CONNECT(j+1,i)==1.or.CONNECT(j-1,i)==1.or. &
CONNECT(j+1,i)==1)).and.TRAP(j,i)==0) THEN
    CONNECT(j,i)=1
ENDIF
IF (PAM(j-1,i)==0.and.DIS(j-1,i)!=0.and.CONNECT(j-1,i)==0.and.TRAP(j-1,i)==0) THEN  !NORTH
    CONNECT(j-1,i)=1
ENDIF
IF (PAM(j+1,i)==0.and.DIS(j+1,i)!=0.and.CONNECT(j+1,i)==0.and.TRAP(j+1,i)==0) THEN  !SOUTH
    CONNECT(j+1,i)=1
ENDIF
END IF
SUMCONNECT=SUMCONNECT+PAM(j,i)
END DO
END DO
END DO WHILE (1)
END DO !END DO WHILE (1)

Inicializar =fmin.fmax
SUMCONNECT =10
SUMCONNECT1=0
DO WHILE (SUMCONNECT/=SUMCONNECT1) !START DO WHILE (2)
    SUMCONNECT1=SUMCONNECT
    DO j=ny,1,-1 !SWEEP FROM TOP TO BOTTOM
        IF(i==nx.and.PAM(j,i)==0.and.DIS(j,i)!=0) THEN
            CONNECT(j,i)=1
        ELSE
            IF (PAM(j,i)==0.and.DIS(j,i)!=0.and.CONNECT(j,i)==0.and. &
                (CONNECT(j-1,i)==1.or.CONNECT(j+1,i)==1.or.CONNECT(j,i-1)==1.or. &
                  CONNECT(j,i+1)==1)).and.TRAP(j,i)==0) THEN
                CONNECT(j,i)=1
            END IF
            IF (PAM(j-1,i)==0.and.DIS(j-1,i)!=0.and.CONNECT(j-1,i)==0.and.TRAP(j-1,i)==0) THEN  !NORTH
                CONNECT(j-1,i)=1
            END IF
            IF (PAM(j+1,i)==0.and.DIS(j+1,i)!=0.and.CONNECT(j+1,i)==0.and.TRAP(j+1,i)==0) THEN  !SOUTH
                CONNECT(j+1,i)=1
            END IF
            IF (PAM(j,i-1)==0.and.DIS(j,i-1)!=0.and.CONNECT(j,i-1)==0.and.TRAP(j,i-1)==0) THEN  !WEST
                CONNECT(j,i-1)=1
            END IF
            IF (PAM(j,i+1)==0.and.DIS(j,i+1)!=0.and.CONNECT(j,i+1)==0.and.TRAP(j,i+1)==0) THEN  !EAST
                CONNECT(j,i+1)=1
            END IF
        END IF
    END DO
    SUMCONNECT=0
    DO j=1,ny !SWEEP FROM BOTTOM TO TOP
        IF(i==nx.and.PAM(j,i)==0.and.DIS(j,i)!=0) THEN
            CONNECT(j,i)=1
        ELSE
            IF (PAM(j,i)==0.and.DIS(j,i)!=0.and.CONNECT(j,i)==0.and. &
                (CONNECT(j-1,i)==1.or.CONNECT(j+1,i)==1.or.CONNECT(j,i-1)==1.or. &
                  CONNECT(j,i+1)==1)).and.TRAP(j,i)==0) THEN
                CONNECT(j,i)=1
            END IF
            IF (PAM(j-1,i)==0.and.DIS(j-1,i)!=0.and.CONNECT(j-1,i)==0.and.TRAP(j-1,i)==0) THEN  !NORTH
                CONNECT(j-1,i)=1
            END IF
            IF (PAM(j+1,i)==0.and.DIS(j+1,i)!=0.and.CONNECT(j+1,i)==0.and.TRAP(j+1,i)==0) THEN  !SOUTH
                CONNECT(j+1,i)=1
            END IF
            IF (PAM(j,i-1)==0.and.DIS(j,i-1)!=0.and.CONNECT(j,i-1)==0.and.TRAP(j,i-1)==0) THEN  !WEST
                CONNECT(j,i-1)=1
            END IF
            IF (PAM(j,i+1)==0.and.DIS(j,i+1)!=0.and.CONNECT(j,i+1)==0.and.TRAP(j,i+1)==0) THEN  !EAST
                CONNECT(j,i+1)=1
            END IF
        END IF
    END DO
END DO !SWEEP FROM BOTTOM TO TOP

ELSE
    IF (PAM(j,i)==0.and.DIS(j,i)!=0.and.CONNECT(j,i)==0.and. &
        (CONNECT(j-1,i)==1.or.CONNECT(j+1,i)==1.or.CONNECT(j,i-1)==1.or. &
          CONNECT(j,i+1)==1)).and.TRAP(j,i)==0) THEN
        CONNECT(j,i)=1
    END IF
    IF (PAM(j-1,i)==0.and.DIS(j-1,i)!=0.and.CONNECT(j-1,i)==0.and.TRAP(j-1,i)==0) THEN  !NORTH
        CONNECT(j-1,i)=1
    END IF
    IF (PAM(j+1,i)==0.and.DIS(j+1,i)!=0.and.CONNECT(j+1,i)==0.and.TRAP(j+1,i)==0) THEN  !SOUTH
        CONNECT(j+1,i)=1
    END IF
    IF (PAM(j,i-1)==0.and.DIS(j,i-1)!=0.and.CONNECT(j,i-1)==0.and.TRAP(j,i-1)==0) THEN  !WEST
        CONNECT(j,i-1)=1
    END IF
    IF (PAM(j,i+1)==0.and.DIS(j,i+1)!=0.and.CONNECT(j,i+1)==0.and.TRAP(j,i+1)==0) THEN  !EAST
        CONNECT(j,i+1)=1
    END IF
END IF

END IF
IF (PAM(j,i+1)==0.and.DIS(j,i+1)==0.and.CONNECT(j,i+1)==0.and.TRAP(j,i+1)==0) THEN !EAST
  CONNECT(j,i+1)=1
END IF
END IF
END IF

SUMCONNECT=SUMCONNECT+PAM(j,i)
END DO
END DO !END DO WHILE (2)

END DO
SUMCONNECT_BIG = sum(CONNECT)
END DO
!======================CHECK ENTRAPMENT===========================!
DO =fmin,fmax
  DO j=1, ny
    IF (TRAP(j,i)==0.and.CONNECT(j,i)==0) THEN
      TRAP (j,i)=1
    END IF
  END DO
END DO

!==================CHECK A STARTING COLUMN OF SWEEPING==================
IF (fmax > 5.and.fmax < nx-5) THEN
  SUMTRAP=0
  DO j=1,ny
    SUMTRAP=TRAP(j,fmin)+SUMTRAP
  END DO
  IF (SUMTRAP==ny) THEN
    fmin=fmin+1
    WRITE (*,*) 'fmin=',fmin,'fmax=',fmax
    PAUSE
  END IF
END IF

END SUBROUTINE ROUTE
SUBROUTINE SWEEP(j, i, ny, nx, my, mx, lx, ly, DIS, PAM, DUMMY, SUBDIS, SUBMAP, SUBDUMMY, Xi, Yi)

!Import: j, i, ny, nx, my, mx, lx, ly, DIS, PAM, DUMMY
!Export: SUBDIS, SUBMAP, SUBDUMMY, Xi, Yi

IMPLICIT NONE

INTEGER         , INTENT (IN)  :: j, i, nx, ny, mx, my
DOUBLE PRECISION, INTENT (IN)  :: lx, ly

INTEGER         ,DIMENSION (0:ny+1,0:nx+1), INTENT (IN) :: PAM, DUMMY
DOUBLE PRECISION,DIMENSION (0:ny+1,0:nx+1),INTENT (IN) :: DIS

INTEGER         ,DIMENSION (2*my+1, 2*mx+1), INTENT (OUT):: SUBMAP, SUBDUMMY
DOUBLE PRECISION,DIMENSION (2*my+1, 2*mx+1), INTENT (OUT): Xi, Yi, SUBDIS

INTEGER          :: jj, ii, row, col
DOUBLE PRECISION :: px, py

DO ii=1,2*mx+1
  DO jj=1,2*my+1
    Xi (jj,ii)=0 !====SET ZERO EVERY CHECKING
    Yi (jj,ii)=0
  END DO
END DO

!========== SWEEPING IN RADIUS LAMDA==========
DO ii=1,2*mx+1
  DO jj=1,2*my+1
    row=j+jj-my-1
    col=i+ii-mx-1
    IF (row <=  0    ) THEN
      row = 0
    END IF
    IF (row >  ny    ) THEN
      row = ny + 1
    END IF
    IF (col <=  0    ) THEN
      col = 0
    END IF
    IF (col >  nx    ) THEN
      col = nx + 1
    END IF
    SUBDIS  (jj,ii)=DIS  (row,col)
    SUBMAP  (jj,ii)=PAM  (row,col)
    SUBDUMMY(jj,ii)=DUMMY(row,col)
    !================CAL Xi Yi OF EACH POTENTIAL CELL================
    px=abs(ii-mx-1)
    py=abs(jj-my-1)
    IF (SUBDUMMY(jj,ii)==1.and.px/=0) THEN
      Xi(jj,ii)=(px*lx/SQRT((px*lx)**2+(py*ly)**2))**(1/px)
    END IF
    IF (SUBDUMMY(jj,ii)==1.and.py/=0) THEN
      Yi(jj,ii)=(py*ly/SQRT((px*lx)**2+(py*ly)**2))**(1/py)
    END IF
  END DO
END DO

!===============DESIGNATE Xi Yi FOR EACH QUADRANT===============
DO jj=my+1, 2*my+1
  Do ii=1, 2*mx+1
    Yi(jj,ii)=-Yi(jj,ii)
  END DO
END DO
DO jj=1, 2*my+1
  Do ii=1, mx+1
    Xi(jj,ii)=-Xi(jj,ii)
  END DO
END DO

END SUBROUTINE SWEEP
APPENDIX B

Source Code of Variogram Program

PROGRAM Main
IMPLICIT NONE
INTEGER :: j, i, jj, ii, n, num, nx, ny, block, mx, my
DOUBLE PRECISION :: lx, ly, RES, ResA, tor, DIF, lag, lagP, lagN, r, VAR, bigLx, bigLy, bigL, nmax
CHARACTER (20) :: FILENAME
INTEGER, ALLOCATABLE :: IND (:,:)
DOUBLE PRECISION, ALLOCATABLE :: DIS (:,:)

!=============================HEADING====================================
WRITE(*,*) '                                                                     '
WRITE(*,*) '*********************************************************************'
WRITE(*,*) '*                                                                                   *'
WRITE(*,*) '*                        Calculation for Variogram                              *'
WRITE(*,*) '*                                                                                              *'
WRITE(*,*) '*                                             By                                             *'
WRITE(*,*) '*                                                                                              *'
WRITE(*,*) '*                              Tawatchai Petchsingto                            *'
WRITE(*,*) '*                                                                                              *'
WRITE(*,*) '*                       The Pennsylvania State University                *'
WRITE(*,*) '*                                                                                              *'
WRITE(*,*) '*********************************************************************'

!==================INPUT PARAMETERS OF A LATTICE MODEL ======================

WRITE(*,*)'                                                                     '
WRITE(*,*)'                                                                     '
WRITE(*,*)'===========  INPUT PARAMETERS OF THE APERTURE DISTRIBUTION  ========='
WRITE(*,*)'                                                                     '
WRITE(*,*)'Enter name of input file of a fracture model (e.g., map.txt)'
READ (*,*) FILENAME
WRITE(*,*) 'Enter number of grid blocks of a fracture model in X-direction?'
READ (*,*) nx
WRITE(*,*) 'Enter number of grid blocks of a fracture model in Y-direction?'
READ (*,*) ny
WRITE(*,*) 'Enter resolution of a fracture model in X-direction'
READ (*,*) lx
WRITE(*,*) 'Enter resolution of a fracture model in Y-direction'
READ (*,*) ly
WRITE(*,*) 'Enter resolution of a fracture model in Z-direction'
READ (*,*) ResA
ALLOCATE (IND(0:ny+1,0:nx+1))
ALLOCATE (DIS(0:ny+1,0:nx+1))

!=================================INPUT PARAMETERS OF A LATTICE MODEL ===========
WRITE(*,*) 'Enter number of grid blocks of a fracture model in Z-direction'
READ (*,*) ResA
ALLOCATE (IND(0:ny+1,0:nx+1))
ALLOCATE (DIS(0:ny+1,0:nx+1))

!=================================INPUT PARAMETERS OF A LATTICE MODEL ===========

IF (lx/=ly) THEN
  IF(lx>ly) THEN
    RES=ly
  ELSE
    RES=lx
  END IF
ELSE
  tor=ABS(lx-ly)
END IF

END IF
bigLx=nx*lx
bigLy=ny*ly

!=================================SELECT THE SHORTEST LENTHT FROM BOTH SIDES OF A FRACTURE PLANE
IF (bigLx/=bigLy) THEN
  IF(bigLx>bigLy) THEN
    bigL=bigLx
  ELSE
    bigL=bigLy
  END IF
ELSE
  bigL=bigLx
END IF
bigL=bigLx
END IF

nmax=NINT(bigL/RES/2)!==========MAXIMUM NUMBER OF BLOCKS FOR LOOP
26 FORMAT (5F20.5)
OPEN (unit=20, file= FILENAME ,status='unknown')
OPEN (unit=30, file='VARr.txt',status='unknown')
!===================HEADING ON OUTPUT FILE======================
WRITE(30,*) '              lag      ','           VAR'
WRITE(30,*) '_________________________________________________'
WRITE(30,*) '                                                 '
DO j=1,ny
READ (20,*) (DIS(j,i),i=1,nx)
END DO
WRITE (30,26) 0, 0
DO n=1,nmax !=======START THE OUTEST LOOP=====
!(1) ASSIGN INDEX OF EACH LAG
DO i=1, nx
    DO j=1, ny
        IF (DIS(j,i)/=0) THEN
            IND(j,i)=1
        ELSE
            IND(j,i)=0
        END IF
    END DO
END DO
!(2) ASSIGN PARAMETER FOR EACH LAG DISTANCE
lag  =RES*n
lagP =lag+tor
lagN =lag-tor
DIF  =0
VAR =0
num =0
r   =0
!(3) CAL. VAR NUMBER
DO i=1,nx
    DO j=1, ny
        IF (IND(j,i)/=0) THEN
            IND(j,i)=0
            DO ii=1,nx
                DO jj=1, ny
                    IF (IND(jj,ii)/=0) THEN
                        mx=abs(i-ii)
                        my=abs(j-jj)
                        r=SQRT((lx*mx)**2+(ly*my)**2)
                    END IF
                END DO
            END DO
        END IF
    END DO
END DO
!(4) CAL VARIOGRAM VS. LAG
IF (num/=0) THEN
    VAR = DIF/(2*num)
END IF
WRITE   (*,*) 'lag=',lag,'VARr=',VAR
WRITE (30,26)  lag, VAR
END DO !============= END THE OUTEST LOOP========

END PROGRAM Main
STEP 1  A fracture model size 2.5mm x 2.5 mm is generated using COVAR program, in which aperture field is normal distribution with mean aperture 0.548 mm, standard deviation 0.25. The fracture model is discretized into 100x100 girds. Its aperture distribution is depicted as following.

![Aperture distribution generated from COVAR program with the input parameters which are mean aperture 0.548 mm, standard deviation 0.25, and spatial correlation length 2.445 mm.](image)

Figure C.1 Aperture distribution generated from COVAR program with the input parameters which are mean aperture 0.548 mm, standard deviation 0.25, and spatial correlation length 2.445 mm.

STEP 2  Determine the isotropic correlation length of the fracture by using the program calculating semivariogram as following example.

=============INPUT PARAMETERS OF THE APERTURE DISTRIBUTION=============  
Enter name of input file of a fracture model (e.g.,map.txt).  
d548SD025.txt  
Enter number of grid blocks of a fracture model in X-direction.  
100  
Enter number of grid blocks of a fracture model in Y-direction.
Enter resolution of a fracture model in X-direction.
0.25
Enter resolution of a fracture model in Y-direction.
0.25
Enter resolution of a fracture model in Z-direction.
1.0

RESULT:

<table>
<thead>
<tr>
<th>lag</th>
<th>VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.25000</td>
<td>0.01733</td>
</tr>
<tr>
<td>0.50000</td>
<td>0.02998</td>
</tr>
<tr>
<td>0.75000</td>
<td>0.03940</td>
</tr>
<tr>
<td>1.00000</td>
<td>0.04629</td>
</tr>
<tr>
<td>1.25000</td>
<td>0.05047</td>
</tr>
<tr>
<td>1.50000</td>
<td>0.05436</td>
</tr>
<tr>
<td>1.75000</td>
<td>0.05681</td>
</tr>
<tr>
<td>2.00000</td>
<td>0.05833</td>
</tr>
<tr>
<td>2.25000</td>
<td>0.05949</td>
</tr>
<tr>
<td>2.50000</td>
<td>0.06079</td>
</tr>
<tr>
<td>2.75000</td>
<td>0.06138</td>
</tr>
<tr>
<td>3.00000</td>
<td>0.06257</td>
</tr>
<tr>
<td>3.25000</td>
<td>0.06253</td>
</tr>
<tr>
<td>3.50000</td>
<td>0.06377</td>
</tr>
<tr>
<td>3.75000</td>
<td>0.06389</td>
</tr>
<tr>
<td>4.00000</td>
<td>0.06412</td>
</tr>
<tr>
<td>4.25000</td>
<td>0.06454</td>
</tr>
<tr>
<td>4.50000</td>
<td>0.06469</td>
</tr>
<tr>
<td>4.75000</td>
<td>0.06516</td>
</tr>
<tr>
<td>5.00000</td>
<td>0.06658</td>
</tr>
<tr>
<td>5.25000</td>
<td>0.06461</td>
</tr>
<tr>
<td>5.50000</td>
<td>0.06442</td>
</tr>
<tr>
<td>5.75000</td>
<td>0.06455</td>
</tr>
<tr>
<td>6.00000</td>
<td>0.06472</td>
</tr>
<tr>
<td>6.25000</td>
<td>0.06520</td>
</tr>
<tr>
<td>6.50000</td>
<td>0.06446</td>
</tr>
<tr>
<td>6.75000</td>
<td>0.06335</td>
</tr>
<tr>
<td>7.00000</td>
<td>0.06260</td>
</tr>
<tr>
<td>7.25000</td>
<td>0.06425</td>
</tr>
<tr>
<td>7.50000</td>
<td>0.06398</td>
</tr>
<tr>
<td>7.75000</td>
<td>0.06268</td>
</tr>
<tr>
<td>8.00000</td>
<td>0.06284</td>
</tr>
<tr>
<td>8.25000</td>
<td>0.06344</td>
</tr>
<tr>
<td>8.50000</td>
<td>0.06360</td>
</tr>
<tr>
<td>Distance (mm)</td>
<td>Variogram Value</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>8.75000</td>
<td>0.06526</td>
</tr>
<tr>
<td>9.00000</td>
<td>0.06515</td>
</tr>
<tr>
<td>9.25000</td>
<td>0.06327</td>
</tr>
<tr>
<td>9.50000</td>
<td>0.06376</td>
</tr>
<tr>
<td>9.75000</td>
<td>0.06377</td>
</tr>
<tr>
<td>10.00000</td>
<td>0.06540</td>
</tr>
<tr>
<td>10.25000</td>
<td>0.06440</td>
</tr>
<tr>
<td>10.50000</td>
<td>0.06313</td>
</tr>
<tr>
<td>10.75000</td>
<td>0.06330</td>
</tr>
<tr>
<td>11.00000</td>
<td>0.06370</td>
</tr>
<tr>
<td>11.25000</td>
<td>0.06616</td>
</tr>
<tr>
<td>11.50000</td>
<td>0.06471</td>
</tr>
<tr>
<td>11.75000</td>
<td>0.06483</td>
</tr>
<tr>
<td>12.00000</td>
<td>0.06527</td>
</tr>
<tr>
<td>12.25000</td>
<td>0.06569</td>
</tr>
<tr>
<td>12.50000</td>
<td>0.06584</td>
</tr>
</tbody>
</table>

**STEP 3** Construct variogram from above data for fitting curve and determining the isotropic spatial correlation length as following graph.

![Variogram](image)

Figure C.2 Variogram constructed from the output file of the Variogram program for determining the spatial correlation length.
From fitting curve, it yields the value of spatial correlation length 2.445 mm which is an input parameter for MIP program.

**STEP 4** Deliver the text file of the generated fracture to MIP program for running simulation and the results of MIP simulation are the capillary pressure curve, the final fluid saturation map and the sequential oil invasion progress.

============== INPUT PARAMETERS OF THE APERTURE DISTRIBUTION ===============
Enter name of input file of a fracture model (e.g., map.txt).
d548SD025.txt
Enter number of grid blocks of a fracture model in X-direction.
100
Enter number of grid blocks of a fracture model in Y-direction.
100
Enter resolution of a fracture model in X-direction.
0.25
Enter resolution of a fracture model in Y-direction.
0.25
Enter resolution of a fracture model in Z-direction.
1.0
Enter isotropic spatial correlation length of the aperture filed.
2.445

======================== INPUT FLUID PROPERTIES =========================
Enter the contact angle in degree.
10
Enter the interfacial tension in mdyn/m.
41.27
RESULTS:

(1) The output file of the capillary pressure curve names CapCurve.txt which presents this following data.

<table>
<thead>
<tr>
<th>Pc(Pa)</th>
<th>Sw</th>
</tr>
</thead>
<tbody>
<tr>
<td>51.03</td>
<td>0.98994</td>
</tr>
<tr>
<td>121.43</td>
<td>0.95533</td>
</tr>
<tr>
<td>124.69</td>
<td>0.95432</td>
</tr>
<tr>
<td>129.38</td>
<td>0.95300</td>
</tr>
<tr>
<td>133.89</td>
<td>0.95153</td>
</tr>
<tr>
<td>135.20</td>
<td>0.95039</td>
</tr>
<tr>
<td>135.64</td>
<td>0.94938</td>
</tr>
<tr>
<td>137.68</td>
<td>0.92209</td>
</tr>
<tr>
<td>138.48</td>
<td>0.92071</td>
</tr>
<tr>
<td>140.12</td>
<td>0.91968</td>
</tr>
<tr>
<td>141.71</td>
<td>0.91863</td>
</tr>
<tr>
<td>141.73</td>
<td>0.91662</td>
</tr>
<tr>
<td>142.63</td>
<td>0.91549</td>
</tr>
<tr>
<td>144.56</td>
<td>0.91413</td>
</tr>
<tr>
<td>146.79</td>
<td>0.91306</td>
</tr>
<tr>
<td>148.13</td>
<td>0.91204</td>
</tr>
<tr>
<td>151.78</td>
<td>0.91098</td>
</tr>
<tr>
<td>152.15</td>
<td>0.67831</td>
</tr>
<tr>
<td>152.44</td>
<td>0.67688</td>
</tr>
<tr>
<td>153.27</td>
<td>0.67572</td>
</tr>
<tr>
<td>154.18</td>
<td>0.67326</td>
</tr>
<tr>
<td>156.53</td>
<td>0.67225</td>
</tr>
<tr>
<td>157.52</td>
<td>0.67101</td>
</tr>
<tr>
<td>159.10</td>
<td>0.66978</td>
</tr>
<tr>
<td>159.27</td>
<td>0.61666</td>
</tr>
<tr>
<td>159.99</td>
<td>0.61007</td>
</tr>
<tr>
<td>160.72</td>
<td>0.60900</td>
</tr>
<tr>
<td>160.87</td>
<td>0.60234</td>
</tr>
<tr>
<td>163.26</td>
<td>0.60107</td>
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<tr>
<td>163.89</td>
<td>0.59983</td>
</tr>
<tr>
<td>164.47</td>
<td>0.59853</td>
</tr>
<tr>
<td>165.45</td>
<td>0.59581</td>
</tr>
<tr>
<td>168.09</td>
<td>0.59237</td>
</tr>
<tr>
<td>170.45</td>
<td>0.59128</td>
</tr>
<tr>
<td>171.35</td>
<td>0.58777</td>
</tr>
<tr>
<td>174.60</td>
<td>0.58672</td>
</tr>
<tr>
<td>176.39</td>
<td>0.58332</td>
</tr>
<tr>
<td>177.82</td>
<td>0.58215</td>
</tr>
<tr>
<td>178.64</td>
<td>0.55616</td>
</tr>
<tr>
<td>183.34</td>
<td>0.55511</td>
</tr>
<tr>
<td>184.87</td>
<td>0.55410</td>
</tr>
<tr>
<td>188.61</td>
<td>0.55297</td>
</tr>
<tr>
<td>190.09</td>
<td>0.50447</td>
</tr>
<tr>
<td>192.71</td>
<td>0.50346</td>
</tr>
<tr>
<td>196.39</td>
<td>0.50236</td>
</tr>
<tr>
<td>197.54</td>
<td>0.50129</td>
</tr>
</tbody>
</table>
The capillary pressure curve from output file, CapCurve.txt, can construct as following capillary pressure curve.

Figure C.3 Capillary pressure curve reconstructed from CapCurve.txt, the output data of the MIP program.

(2) The second output file of final oil-water saturation, fluidmap.txt, can be visualized as figure below:
Figure C.4 Final map of oil-water saturation reconstructed from fluidmap.txt, the output data file of the MIP program.

where orange depicts oil phase and dark blue regions mean trapped water.

(3) The third file name, fluidProgress.txt, represent the sequence of oil invasion.

Figure C.5 Sequential oil invasion reconstructed from fluidProgress.txt, the output data file of the MIP program.
APPENDIX D

Prediction of Capillary Pressure Indicators

For the first two cases, two sets of geostatistical parameters are randomly selected for testing the proposed prediction charts. Based on those sampled parameter, the normalized entry pressures ($P_{c*}$ entry) and the irreducible water saturations ($S_{wirr}$) are interpolated from the prediction charts. The next step is to generate fracture model from the given geostatistical parameters. Simulations are performed on the generated fracture models, and yield the normalized entry pressures ($P_{c*}$entry) and the irreducible water saturations ($S_{wirr}$). These two representations of capillary pressure from simulations are compared to those from the interpolations. In the third case, a set of geostatistical parameter are identical to the geostatistical parameters of the fracture used in the previous experiment (Karpyn et al., 2007). Simulation is performed on the COVAR-generated fracture, and obtained $P_{c*}$ entry and $S_{wirr}$ are compared to that of from Model 113x389.

Case Studies:

Case I. Mean aperture: 0.5 mm, standard deviation: 0.125, spatial correlation length 1.5 mm.

The normalized entry pressure ($P_{c*}$entry) and the irreducible water saturation ($S_{wirr}$) are estimated from interpolation of figure 5.28 and 5.29. Figures D.1 and D.2 demonstrate the interpolations to estimate the value of $P_{c*}$ entry and $S_{wirr}$ from figure 5.28 and 5.29 respectively. Thereafter, a log-normal fracture model is artificially generated using COVAR program with the input parameters which are mean aperture 0.5 mm, standard deviation 0.125, and spatial correlation length 1.5 mm. Then the variogram of the aperture distribution is constructed and fitted by the exponential model which yields isotropic correlation length 5.136 mm. The simulation result yields the capillary pressure curve as shown in figure D.3. Those two indicators, which are $P_{c*}$entry and $S_{wirr}$, are representation of the capillary pressure curve of the generated fracture model.
Figure D.1 Demonstrative interpolation on the prediction chart of standard deviation 0.125 (figure 5.28) for determining $P_{c*entry}$ in CASE I.

Figure D.2 Demonstrative interpolation on the prediction chart of standard deviation 0.125 (figure 5.29) for determining $S_{win}$ in CASE I.
Figure D.3 Capillary pressure curve resulted from simulation for an artificially generated fracture with mean aperture 0.5 mm, standard deviation 0.125, and spatial correlation length 5.136 mm.

The normalized entry pressure ($P_c^{\text{entry}}$) and the irreducible water saturation ($S_{\text{wirr}}$) from the simulation are in a good agreement with those from interpolations of figures 5.28 and 5.29.
CASE II. Mean aperture: 0.65 mm, standard deviation: 0.35, spatial correlation length: 4.167 mm.

Similarly, an artificial fracture model has been created of 25 x 25 mm in which the fracture plane is discretized into 100x100 grids. The log-normal aperture distribution has a mean aperture of 0.65 mm, standard deviation 0.35 and fitted isotropic correlation length 4.167 mm. The capillary pressure curve from the simulation is shown in figure D.6. The irreducible water saturation (S_{wirr}) of figure D.6 is in a good agreement with the S_{wirr} obtained from interpolation of figure 5.31 as shown in figure D.5. The normalized entry pressure (P_{c*entry}) from the simulation of figure D.6 is relatively different from the interpolation of figure 5.30 as shown in figure D.4. The discrepancies of P_{c*} entry values might come from the effect of aperture configurations.

Figure D.4 Demonstrative interpolation on the generalization chart of standard deviation 0.35 (figure 5.30) for determining P_{c*} entry in CASE II.
Figure D.5 Demonstrative interpolation on the prediction chart of standard deviation 0.35 (figure 5.31) for determining $S_{\text{swirr}}$ in CASE II.

Figure D.6 Capillary pressure curve resulted from simulation for an artificially generated fracture with mean aperture 0.65 mm, standard deviation 0.35, and spatial correlation length 4.167 mm.
CASE III. Mean aperture: 0.548 mm, standard deviation: 0.35, spatial correlation length: 2.4 mm.

A fracture model with normal aperture distribution has been generated using COVAR program, where there is mean aperture of 0.548 mm, standard deviation 0.25 mm and isotropic correlation length 2.4 mm. These parameters are exactly the same as the parameters of the fracture from the previous experiment. However, fractures are in different dimensions. The dimension of the generated fracture model is 25x25 mm, whereas the dimension of the actual fracture is 25.4x101.2 mm. Resolution of the actual fracture model chosen is 113x389 grids, and the artificial fracture model is 100x100 grids. MIP simulations are performed on both fracture models and the results of capillary pressures are shown in figure D.7.

![Figure D.7](image.png)

Figure D.7 Comparison capillary pressure curves between the artificial fracture and the actual model with identical geostatistical parameter which are the same of mean aperture 0.548 mm, standard deviation 0.25 mm and spatial correlation length 2.4 mm.
The discrepancies are found in the values of the irreducible water saturation \( (S_{\text{win}}) \). This is because of the different aperture configuration of the artificial fracture and Model 113x389, which create the different flow paths in the fracture models. Many studies show that the characteristics of aperture configurations play an important role to define the tortuous paths and connectivity of the invasion fluid as well as the fluid occupancies (Tsang, 1984; and Pruess and Tsang, 1990). Thus discrepancies of the irreducible water saturation can be observed. However, the normalized entry pressure \( (P_{c* \text{ entry}}) \) from the artificial model gives a good agreement with that from the actual model.
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