DYNAMIC VARIATION IN A RESONATING MEMS SHEAR STRESS SENSOR

A Thesis in
Mechanical Engineering
by
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Abstract

Fluid shear stress is an important parameter in the understanding and control of viscous drag, laminar to turbulent transition, flow separation, and turbulent eddies. Laminar flow theory is well defined and understood while turbulent flows are difficult to describe due to their random nature. The measurement of turbulent or fluctuating wall shear stress would contribute to the fundamental understanding of turbulence as well as applications in aerospace, automotive, marine, and biomedical fields. Existing shear stress sensors lack the capability of resolving turbulent flow due to spatial and temporal averaging effects. Micro-electro-mechanical systems (MEMS) can be advantageous for such applications because of their small sizes, tight dimensional tolerances, and enhanced dynamic characteristics. However, concerns regarding bulk micro-machining accuracy and precision, dynamic variation for batch fabrication, and deflection measurement schemes must be resolved before sensor development can continue.

This research utilizes scanning electron microscope linewidth techniques to evaluate the manufacturing accuracy and precision by measuring individual beam widths to develop mean and 95% confidence intervals. The beam dimension measurements exhibited a relative error of 1.75% from design dimensions with a relative uncertainty of 2.83% for the bulk micro-machining process. Dynamic characterization of the out-of-plane resonance is performed using laser doppler vibrometry techniques to determine the resonant frequency variation and investigate laser doppler vibrometry as a deflection measurement scheme. The resonant frequency 95% confidence relative uncertainty is 1.44% averaged from 18 micro-fabricated devices with exactly identical design parameters. Laser doppler vibrometry offers reasonable measurement resolution limited by probe volume and fringe spacing. Range and bandwidth capabilities are limited by the signal processing equipment. However, the technique requires bulky and sensitive components for measurement limiting the application potential. Further investigation and development of a 2-beam in-plane laser doppler vibrometer will better characterize the resolution and system packaging potential for applications.
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List of Symbols

$x, y, z$ cartesian coordinate directions
$u, v, w$ displacement in cartesian coordinate direction
$p$ pressure
$\rho$ density
$g$ acceleration due to gravity
$\mu$ dynamic viscosity
$\nu$ kinematic viscosity
$\tau_{ij}$ fluid shear stress
$\tau_w$ wall shear stress
$\overline{x}$ averaged quantity $x$
$x'$ fluctuating quantity $x$
$U_\infty$ free stream velocity
$\theta$ momentum thickness
$\delta$ displacement thickness
$\delta^*$ dimensionless displacement thickness
$\delta_{ij}$ Kronecker Delta
$\ell_{\text{char}}$ characteristic length scale
$t_{\text{char}}$ characteristic time scale
$b$ beam width
$l$ length
$h$ height
\( n_b \) number of beams in a single serpentine spring structure

\( m_i \) mass of \( i \)

\( M \) applied moment

\( E \) elastic modulus

\( G \) shear modulus

\( \nu \) poisson’s ratio

\( I \) moment of inertia of a plane section

\( \kappa \) radius of curvature

\( \beta \) torsional beam stiffness parameter

\( k_i \) linear spring coefficient of \( i \)

\( c_i \) linear viscous damper coefficient of \( i \)

\( F \) force

\( T \) torque

\( \Theta \) unit angle of twist

\( \theta \) angle of twist

\( \zeta \) damping ratio

\( Q \) damping quality factor

\( \phi \) damping quality factor inverse

\( g_o \) nominal gap

\( \omega \) radian frequency

\( f \) frequency

\( f_r \) resonant frequency

\( f_n \) natural frequency

\( \bar{X} \) X matrix

\( \vec{X} \) X vector

\( D \) flexural rigidity

\( c_{ss} \) speed of sound

\( \gamma \) frequency parameter

\( S_x \) precision index of \( x \)
\( t \) student t distribution
\( S_{ij} \) covariance
\( \theta_i \) absolute sensitivity coefficient
\( U_{X_i} \) uncertainty of \( X_i \)
\( \rho_{ij} \) correlation coefficient
\( f_{rl} \) -3dB lower frequency point
\( f_{rh} \) -3dB higher frequency point
\( \Delta f_r \) resonant bandwidth
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1.1 Introduction

Shear stress is an important parameter in the understanding and control of viscous drag, laminar to turbulent transition, flow separation, and turbulent eddies. Current measurement capabilities allow the laminar or mean shear stress to be measured but are inadequate for turbulent or fluctuating shear stress due to spatial and temporal averaging. Conventional macroscopic sensors lack the ability to measure the turbulent shear stress at the small characteristic length and time scales. Micro-Electro Mechanical Systems (MEMS) offer increased sensitivity and decreased size without compromising bandwidth to measure the turbulent shear stress at length and time scales comparable to those found in turbulent flows. The measurement of shear stress and control of the boundary layer at the solid-fluid interface offers the prospect of reducing skin friction and drag. Accurate measurement and control of the turbulent boundary layer shear stress contributes to the fundamental understanding of turbulent boundary layers as well as applications in aerospace, automotive, marine, and biomedical fields.

In this thesis, we propose a floating element MEMS shear stress sensor with capabilities exceeding that of existing MEMS shear stress sensors. The goal of this sensor is to resolve turbulent shear stress at scales comparable to the characteristic length and time scales of turbulent flows. Figures 1.3-1.5 show the expected characteristic length [1], characteristic frequency [1], and estimated shear stress [2] for typical turbulent boundary layer flows in air and water. The application of these MEMS shear stress sensors will be in arrays containing several to several hundred sensors. However, there are concerns over the static and dynamic calibration, manufacturing accuracy and precision, and displacement measurement scheme of the shear stress sensors. Calibration or characterization of every sensor in large arrays is difficult to nearly impossible. Batch calibration by applying calibration results from a single sensor to the full array offers a potential solution. This assumption requires statistical analysis of a full array to determine the dynamic variation and evaluate assumption validity.
This research measures the out-of-plane fundamental resonant frequency using Laser Doppler Vibrometry (LDV) to characterize the array variation for batch calibration. The in-plane fundamental mode critical for shear stress measurement is expected to exhibit similar variation. An investigation of the bulk micro-machining manufacturing accuracy and precision is performed with direct optical measurement of the sensor beam width. Also, as LDV is a possible optical solution for the dynamic characterization and displacement measurement scheme of the shear stress sensor, this research builds experience and knowledge of techniques and application issues of LDV.

1.2 Fluid Shear Stress Theory

Shear stress is caused by the presence of a velocity gradient in a fluid flow. The velocity gradient arises at a solid-fluid interface due to the no-slip condition applied fluid at the interface. Shear stress will only be addressed in terms of a solid-fluid interface but is also present in fluid-fluid interactions where relative motion occurs such as jet flow. Continuum mechanics apply as typically even the smallest length scales of fluid flows are many times greater than the molecular path length [3, 4]. The theory presented will also apply only to Newtonian fluids. The viscosity of Newtonian fluids is independent of the strain rate in the fluid although dependence on other parameters such as temperature exist. Newtonian fluids are governed by the Navier-Stokes equations [2, 5, 6] written in Equation 1.1.

\[
\rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \rho g_i + \frac{\partial}{\partial x_j} \left[ 2\mu e_{ij} - \frac{2}{3} \mu (\nabla \cdot u) \delta_{ij} \right]
\]

\[
e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

The study of turbulence by statistical theory or numerical simulation is possible but is most frequently looked at in terms of the Reynolds equations. Reynolds performed time averaging on the basic equations of continuity, momentum, and energy with mean \( \bar{u} \) and fluctuating \( u' \) components to define the Reynolds equations. The substitution of Equation 1.2 into Equation 1.1 followed by time averaging leads to the Reynolds Averaged Navier Stokes (RANS) equation in Equation 1.3. The mean momentum equation includes the addition of the turbulent inertia tensor or Reynolds stresses \( \rho \bar{u}_i u'_j \) that complicates analysis as knowledge of the tensor requires physical fluid properties and local flow conditions.

\[
\rho \frac{D\bar{u}_i}{Dt} + \frac{\partial}{\partial x_j} \left( \rho \bar{u}_i u'_j \right) = -\frac{\partial p}{\partial x_i} + \rho g_i + \mu \nabla^2 \bar{u}_i
\]

The wall shear stress \( \tau_w \) can be written as the sum of the mean(laminar) shear stress \( \tau_{mean} \) and fluctuating (turbulent) shear stress \( \tau_{fluc} \). Recognize that \( \tau_{fluc} \) is not an actual viscous stress but rather relates to the turbulent momentum exchange. Manipulation of the RANS equation
for the expression of $\tau_{ij}$ in Equation 1.5 shows the mean shear stress $\tau_{\text{mean}}$ in the first term on the right hand side and the fluctuating shear stress $\tau_{\text{fluc}}$ in the second term on the right hand side.

$$\tau_w = \tau_{\text{mean}} + \tau_{\text{fluc}}$$

$$\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \rho u_i' u_j'$$  (1.5)

The Reynolds stresses $\rho u_i' u_j'$ dominate viscous stresses in turbulent flows. Small length and time scales of the velocity fluctuations make them difficult to measure. Eddy viscosity, Taylor’s vorticity transfer theory, and von Karman’s similarity hypothesis [7] all attempt to address the Reynolds stresses by using empirical constants. Additional modifications have been proposed to these theories but accurate estimation of empirical constants requires high spatial and temporal resolution measurements of shear stress. Accurate shear stress data will allow validation of models which can then be used to predict, estimate, and characterize turbulent flows.

As this research is interested in wall shear stress, the following discussions will briefly introduce both laminar and turbulent boundary layer flow. Both of these topics are detailed and complex with complete books being written on these topics [6]. The laminar and turbulent boundary layers will be discussed in the most canonical case which is the 2-D flat plate shown in Figure 1.1. The fluid enters at the inlet with a uniform velocity profile $U_\infty$. The boundary layer begins at the leading edge of the flat plate and develops as the flow continues down the plate. Outside the boundary layer region the flow maintains a constant velocity $U_\infty$ known as the free stream velocity. The concept of a fully developed boundary layer flow states that variations across (perpendicular to the mean flow velocity) the boundary layer are typically much greater than variations along (parallel to the mean flow velocity) of the boundary layer. Looking at the order-of-magnitude of terms and applying the 2-D assumption, Equations 1.6 are applied to simplify the equations of motion.

$$\frac{\partial()}{\partial x} \approx 0 \left( \frac{\partial}{\partial x} \ll \frac{\partial}{\partial y} \right) \quad \frac{\partial^2()}{\partial x^2} \approx 0 \left( \frac{\partial^2}{\partial x^2} \ll \frac{\partial^2}{\partial y^2} \right) \quad z = 0, \frac{\partial()}{\partial z} = 0$$  (1.6)
1.2.1 Laminar Boundary Layer

Laminar flows are defined by a few characteristics first observed by Reynolds. The fluid follows a well-defined straight path with parallel layers (laminae) moving together. Inherently this means little or no mixing between adjacent layers. Ludwig Prandtl is the first to be credited with laminar boundary layer theory. Prandtl hypothesized that the no-slip boundary condition at the fluid-solid interface must be satisfied. Viscous effects in the free stream can be considered negligible because small velocity gradients exist, however viscous effects in the boundary layer caused by large velocity gradients are significant.

Fully developed and 2-D assumptions are utilized to reduce the full Navier-Stokes equations from Equation 1.1 to a more analytically manageable size. The reduced form of the Navier-Stokes equations for \( x \)-momentum and \( y \)-momentum as well as continuity are shown in Equations 1.7-1.9.

\[
\frac{u}{\partial x} + \frac{v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \tag{1.7}
\]

\[
0 = -\frac{\partial p}{\partial y} \tag{1.8}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1.9}
\]

The most frequently used laminar solutions for a flat plate are the Blasius, Falkner-Skan, and von Karman momentum integral solutions [5, 6, 7]. The Blasius and Falkner-Skan solutions result in similarity solutions based on dimensionless parameters that can be scaled by flow parameters to achieve a solution. After the solution is completed it is possible to calculate the shear stress in the flow. As there are no fluctuating velocity components to laminar flow, the shear stress \( \tau \) is shown in Equation 1.10. Subject to the assumptions of fully developed flow the velocity gradient term parallel to the mean flow velocity is considered negligible. The shear stress \( \tau \) in a laminar boundary layer flow is then approximated by Equation 1.11. More specifically, the wall shear stress \( \tau_w \) can be rewritten as Equation 1.12. Accurate expressions for wall shear stress, skin friction coefficients, and drag coefficients for the similarity solutions based on flow parameters are readily found in most reference books [2, 5, 6, 7].

\[
\tau = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \tag{1.10}
\]

\[
\tau = \mu \frac{\partial u}{\partial y} \tag{1.11}
\]

\[
\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} \tag{1.12}
\]

1.2.2 Turbulent Boundary Layer

Turbulent flows are best defined by the following characteristics [4, 9, 10]:

1. Irregularity - Random fluctuations require a statistical approach as opposed to a determin-
istic approach.

2. Diffusivity - Rapid diffusion and mixing of momentum, heat, and mass transfer.

3. Large Reynolds Number - Inertia effects dominate viscous effects.

4. 3-D Vorticity Fluctuations - Flows have high levels of fluctuating vortices and strong 3-dimensional variations.

5. Dissipative - Flows require a continuous source of energy to make up for deformation work lost to internal energy(heat).

6. Turbulence is dependent on both fluid properties and flow properties.

The governing equations of motion for a turbulent boundary layer on a flat plate are shown in Equations 1.13-1.15 subject to the same fully developed and 2-D assumptions as the laminar case but have been Reynolds averaged [6].

\[
\rho \frac{\partial \bar{u}}{\partial x} + \rho \frac{\partial \bar{v}}{\partial y} = -\frac{\partial \sigma_x}{\partial x} - \frac{\partial}{\partial y} \left( \rho u'v' \right) \quad (1.13)
\]

\[
0 = -\frac{\partial \sigma_y}{\partial y} - \frac{\partial}{\partial x} \rho u'v' \quad (1.14)
\]

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \quad (1.15)
\]

The structure of the fully developed turbulent boundary layer can be described by the Law of the Wall. In this theory the nondimensionalized variables are defined as \(x_+ = \frac{\pi}{u_*} \) and \(y_+ = \frac{y u_*}{\nu} \). The Law of the Wall is described in terms of the dimensionless variables in Figure 1.2 [5]. Note that Kundu and Cohen [5] use the notation of \(U\) as the time averaged mean velocity while this analysis uses \(\bar{u}\). The inner region of the Law of the Wall is composed of three “layers”; they are the viscous(inner) sublayer, the buffer(overlap) layer, and the logarithmic(outter) layer which comprise the “wall region”. The turbulent boundary layer also contains the outer region where transition from the inner region to free stream \(U_\infty\) occurs. In the viscous sublayer(0 < \(y_+ < 5\)) the viscous dissipative effects dominate instead of Reynolds stress terms as expected in turbulent flow. Substantial velocity fluctuations(\(u'\) and \(v'\)) still exist within the viscous sublayer. The buffer layer(5 < \(y_+ < 30\)) is defined by viscous stresses and Reynolds stresses both being important to the dynamics of the layer. The logarithmic layer(30 < \(y_+ < 300\)) is a transition region from the inner region to the outer region. Spalding [11] deduced a single composite formula to describe the inner region as shown in Equation 1.16 where \(\kappa\) and \(B\) are empirical constants.

\[
y_+ = u_+ + e^{-\kappa B} \left[ e^{\kappa u_+} - 1 - \kappa u_+ - \frac{(\kappa u_+)^2}{2} - \frac{(\kappa u_+)^3}{6} \right] \quad (1.16)
\]

The integral momentum relation shown in Equation 1.17 was first derived by von Karman where \(\theta\) is the momentum thickness(Equation 1.18), \(H\) is the momentum shape factor (Equa-
Figure 1.2. Law of the Wall. Shaded region represents typical data. [5]

tion 1.19), and \( \delta^* \) the displacement thickness(Equation 1.20).

\[
\frac{d\theta}{dx} + (2 + H) \frac{\theta}{U_\infty} \frac{dU_\infty}{dx} = \frac{\tau_w}{\rho U_\infty^2} \\
\theta = \int_{0}^{\infty} \frac{\tau}{U_\infty} \left( 1 - \frac{\tau}{U_\infty} \right) dy \\
H = \frac{\delta^*}{\theta} \\
\delta^* = \int_{0}^{\infty} \left( 1 - \frac{\tau}{U_\infty} \right) dy
\]

Solution of the integral momentum relation leads to presentation of the results in the power law shown in Equation 1.21.

\[
\frac{\tau}{U_\infty} = \left( \frac{y}{\delta} \right)^n
\]

The power \( n \) is an empirical relation that can be measured for many different types of flows. High resolution measurement of the shear stress will aid in the determination of empirical constants such as \( n, \kappa, \) and \( B \) for different flows. Knowledge of the shear stress will assist in coefficient and scale determination for statistical distributions used in numerical modeling. Von Karman, Batchelor, and Kolmogorov have all presented statistical techniques to characterize flows that require statistical constants [3, 4, 9].

Characteristic length and time scales are shown in Equations 1.22 and 1.23 [1]. These expressions predict the scale of the smallest structures in space and fluctuations in time for incompressible flows. They allow for estimation of the required resolution and bandwidth of a sensor.
to fully characterize various fluid flows. Estimates for water and air at standard temperature and pressure (STP) and a downstream length of x=1 m for varying velocities are shown in Figures 1.4 and 1.5. The computation of the characteristic length and time require estimates for the mean wall shear stress in the flow. This estimate is made using turbulent boundary layer theory over a flat plate as described in Equation 1.24 [2, 6, 12]. Figure 1.3 plots the estimated shear stress as a function of velocity for water and air. The upper velocity limit for water approaches the maximum velocity capability of research facilities at Penn State while the upper limit for air approximates the limit of the incompressibility assumption at STP.

\[ \ell_{\text{char}} \approx \nu \sqrt{\frac{\rho}{\tau_w}} \]  
(1.22)

\[ t_{\text{char}} = \frac{1}{f_{\text{char}}} \approx \nu \frac{\rho}{\tau_w} \]  
(1.23)

\[ \tau_{w,turb} = \frac{0.0135 \mu^{1/7} \rho^{6/7} U_\infty^{13/7}}{x^{1/7}} \]  
(1.24)

**Estimated Shear Stress for Water and Air @ STP and x=1m**

![Graph showing estimated shear stress for water and air at STP and x=1m vs. free stream velocity U_∞.](image)

**Figure 1.3.** Estimates for the mean shear stress in water and air at STP and x=1m vs. free stream velocity \( U_\infty \).

### 1.3 Micro Electro-Mechanical Systems

Micro Electro-Mechanical Systems (MEMS) are the integration of micro-machined optical, electrical, and mechanical components on a common substrate. A system can be comprised of a single component, all three components, or any combination. These systems are capable of sens-
Figure 1.4. Characteristic length and frequency estimates for water at STP and $x=1\text{m}$ vs. free stream velocity $U_\infty$. Arrows indicate the capabilities of existing shear stress sensors.

Figure 1.5. Characteristic length and frequency estimates for air at STP and $x=1\text{m}$ vs. free stream velocity $U_\infty$. Arrows indicate the capabilities of existing shear stress sensors.

ing, actuating, positioning, regulating, pumping, filtering, or manipulation of the surrounding environment for a specific purpose [13]. They interact with mechanical, electrical, thermal, biological, chemical, optical and magnetic phenomena in nature. The terminology MEMS inferring electrical and mechanical component interactions is often falsely used to describe a device that interacts with any of these phenomena.

The micro-systems are manufactured using integrated circuit(IC) processes for electrical com-
ponents and similar micro-machining processes for mechanical and optical components. Micro-
machining operations include four basic processes; lithography, thin-film deposition, etching, and
metallization [14]. Lithography is the exposure of photoresist or structural materials to elec-
tromagnetic radiation causing the target to become inert or reactive dependent on the specific
process. Thin-film deposition is the application of photoresist, sacrificial, and structural material
to the substrate. Etching is the removal of substrate, sacrificial or structural material through
wet or dry processes. The manufacturing of a MEMS device is a uniquely complex process
determined by device design and characteristics.

The small dimensions of micro systems offer increased measurement resolution at a reason-
able cost [15, 16]. Figure 1.6 shows the relationship between conventional and micro-fabricated
devices. In the context of shear stress measurement, conventional methods involve techniques

![Figure 1.6. Comparison of sensitivity and bandwidth for conventional and micro-machined systems. (recreated from [17])](image1)

using oil-film interferometry, liquid crystals, and drag balances [18, 19, 20, 21, 22, 23]. These
macroscopic techniques are limited by spatial resolution and small bandwidth making them in-
sufficient for turbulent shear stress measurements [24]. The small physical size of micro-sensors
offers the potential to significantly improve spatial and temporal shear stress resolution [16].
MEMS shear stress sensors can be divided into two categories, in-direct and direct. In-direct
shear stress sensors utilize empirical relations between heat transfer and velocity to calculate the
shear stress at the wall. Direct sensors utilize a floating element in direct contact with the flow,
knowledge of the contact area, stiffness, and deflection allow calculation of the shear stress. The
following sections will perform brief reviews of existing in-direct and direct shear stress sensors.

1.3.1 In-Direct Sensors

In-direct or thermal shear stress sensors operate with the same basic principles as a hot-film
anemometer in free stream flow. The convective heat transfer from the sensing element is related
to the shear stress through empirical relations. Further information regarding the measurement

![Figure 1.7. Direct and in-direct measurement schemes for shear stress measurement.](image2)
technique can be found in [25, 26, 27, 28, 29]. Thermal sensors in literature are generally limited to designs similar to Figure 1.8 where the heating element is mounted on a membrane suspended over a vacuum cavity. The heating element is suspended over the vacuum cavity to limit heat transfer/loss through the substrate. Conventional and MEMS thermal shear stress sensors are agreed to be subject to the following issues [16, 30, 31]:

1. Measurement errors associated with mean temperature drift.

2. Unique calibrations or relationships between heat transfer and wall shear stress are difficult to determine.

3. Heat transfer through the substrate reduces sensitivity and complicates the dynamic response.

4. Heat transfer to the flow can cause perturbations in the flow.

![Figure 1.8. Typical thermal MEMS shear stress sensor.](image)

The first thermal MEMS shear stress sensors were manufactured by Ho and Tai in 1999 [26, 27, 32, 33, 34] using a polysilicon resistor, silicon nitride diaphragm, and silicon substrate. This sensor is the first to utilize the vacuum cavity design to isolate the heating element and reduce conduction through the substrate. The heating element of dimensions 150µm × 3µm has a sensitivity of 1V/Pa and frequency response of 25 kHz but the measurable range is undefined with the peak measurement taken at 2.5 Pa. These thermal sensors are fully integrated into arrays of 25 devices and placed spanwise in a turbulent flow channel. The measurements taken characterized high shear-stress streaks in the flow up to Re=17517. The sensor requires further experimental verification of the dynamic response, noise floor, pressure sensitivity, flow disturbance, and thermal drift before quantitative data can be taken [15, 16].

Sheplak et al. [35, 36] further developed a thin-film platinum sensing element with a silicon-nitride layer stretched over the vacuum cavity. The sensor is calibrated using a laminar flow cell and acoustic plane wave tube [25] with specific attention paid to the noise floor and pressure sensitivity. The sensor is calibrated from 0 to 1.7 Pa statically and from 0.9 mPa to 6.1 mPa dynamically with a resonant frequency of 7 kHz. Sheplak [35] states the sensor has a range from
9μPa to 1.7 Pa with no measurement sensitivity listed and an operating bandwidth < 7kHz. However issues related to ambient temperature sensitivity, variations in dynamic sensitivity at low frequency, noise floor, pressure sensitivity, and flow disturbance due to heating still remain unresolved [35].

Löfdahl et al. [37] manufactured a hot-wire MEMS sensor with 200, 400, and 600 μm wires located at distances above the substrate ranging from 50-250 μm. The device is manufactured on a Silicon-on-Insulator(SOI) wafer by a detailed process explained by Hassl et al [38]. The hot-wire exhibited a measurement range of 50 mPa-2.5 Pa and an operating bandwidth of 5 kHz. Experimental data taken from wind tunnel tests found that predictions and experimental results follow the same trends but do not agree quantitatively [16].

1.3.2 Direct Sensors

Direct MEMS shear stress sensors most often utilize the same floating element concept. A generalized sensor is shown in Figure 1.9 [17]. The floating element is connected to the substrate by beams or tethers that act as springs. As a fluid passes over the floating element a force is exerted on the floating element by the shear stress. With knowledge of the beam stiffness and measurement of the floating element deflection, the force acting on the floating element can be calculated. The force divided by the area of the floating element shown in Equation 1.25 will give the average wall shear stress \( \tau_w \) acting on the floating element. Note that \( \delta \) in Figure 1.9 represents fluid boundary layer thickness while \( \delta \) in Equation 1.25 represents floating element deflection.

\[
\tau_w = \frac{F}{A} = \frac{k\delta}{A}
\]  

(Figure 1.9. Typical floating element MEMS shear stress sensor. [17])

Macroscopic floating element sensors suffer from the following issues [16]:

1. Tradeoff between spatial resolution and force resolution.
5. Cross-axis sensitivity to acceleration and vibration.

6. Sensitivity drifts due to thermal-expansion.

MEMS technology offers solutions to many of these issues. MEMS offer thin, low mass, compliant structures to mitigate the tradeoff between spatial and force resolution. The sensors are manufactured on the substrate and do not require assembly thereby minimizing sensor misalignment, packaging, and installation errors [39]. The gaps necessary for the floating element to deflect are typically less than a few viscous length scales resulting in minimal impact on the flow field [12]. MEMS masses are more than three orders of magnitude smaller than conventional sensors reducing cross-axis sensitivity and vibration effects.

Schmidt et al. [39] developed a floating element sensor with area $500\mu m \times 500\mu m$ using aluminum and polyimide. A capacitive displacement sensing technique measures the differential change in capacitance as the effective area between two plates changes. Figure 1.10 shows the sensor capacitive scheme. The sensor is statically calibrated using a laminar flow cell and results in the measurement range from 0-1 Pa with a $52 \mu V/Pa$ sensitivity. Dynamic calibration is not performed, although analytical estimates predict a resonant frequency of 9.6 kHz. The sensor suffered from moisture variations in the polyimide that caused mechanical sensitivity drift, electromagnetic interference (EMI), and debris accumulation that affected the capacitive sensing scheme [16].

![Figure 1.10. Floating element sensor with capacitive measurement scheme [39].](image1)

![Figure 1.11. Floating element sensor with piezoresistive measurement scheme [40].](image2)

Shajii et al. [40] utilized a design similar to Schmidt, but implemented a piezoresistive measurement scheme with backside electrical contacts and adapted for a perpendicular flow direction. The $120\mu m \times 140\mu m$ sensor is designed for high shear stress applications (1-100kPa), more specifically, industrial polymer screw extrusion. The piezoresistive sensing scheme measure the deflection by assessing the differential change of resistance due to cross sectional area change in the
supporting beams. The sensor is calibrated using a cone and plate viscometer and results show a sensitivity of 13.7 μV/Pa. Although this sensor is not designed for turbulence measurement the backside electrical contacts represent a significant contribution as measurements in turbulent and hydrodynamic flows will require the measurement scheme be electrically isolated from the fluid medium.

Padmanabahn et al. [8, 17, 41] utilized Schmidt’s design with the application of an optical measurement scheme for silicon sensors of area 120μm×120μm and 500μm×500μm. A coherent light source is aimed at the sensor where photo-diode transducers collect the light. The photo-diodes emit a current proportional to the intensity of light on the diode. Measurement of the differential current between two diodes is correlated to the deflection. Figure 1.12 contains a diagram of the measurement scheme. Static calibration is performed via a laminar flow calibration cell with measurement range 0.0014-10 Pa for the 500μm×500μm and 0-1 Pa for the 120μm×120μm sensors. Dynamic calibration is completed up to 10 kHz limited by the acoustic plane wave tube [25]. Resonance estimates are 326 kHz and 100.2 kHz for the 120μm×120μm and 500μm×500μm sensors respectively. The measurement scheme is insensitive to EMI and cross axis-sensitivity to spanwise shear stress, acceleration, and pressure fluctuations [16]. Drawbacks of the sensor are the front side electrical contacts, bulky mounting and electronics of a coherent light source, and mechanical sensitivity to relative motion between the light source and photo-diodes.

Horowitz et al. [42] developed another optical sensor using a geometric Moiré transduction measurement scheme. The sensor is 1280μm×400μm and manufactured using a wafer-bond/thin-back process producing the grating on the backside. A diagram of the sensor is shown in Figure 1.13. The Moiré fringe shift amplifies small displacement by the ratio of the fringe pitch or spacing. Static calibration is performed using a laminar calibration flow cell which results in a range of 0-1.3 Pa and sensitivity 0.26 μm/Pa. Dynamic calibrations is performed in an acoustic plane wave tube and the resonant frequency is 1.7 kHz. This sensing scheme is limited by the imaging system required to analyze the Moiré fringes, large size, and small bandwidth. The technique does offer limited sensitivity to pressure, cross-axis acceleration and vibration, and
also EMI.

Pan and Hyman et al. [43] surface micromachined a passive/active shear stress sensor. The polysilicon/silicon oxide design includes comb finger capacitors for both actuation and sensing. Device floating element areas are 100\(\mu\text{m} \times 100\mu\text{m}\) and 120\(\mu\text{m} \times 120\mu\text{m}\) with sensitivity of 9 Pa/\(\mu\text{m}\) and range 0-5 Pa. No dynamic calibration is performed on the sensor. The devices are subject to EMI effects, not flush mounted by manufacturing definition, and show significant sensitivity to misalignment.

![Figure 1.14. Floating element sensor with passive/active capacitive measurement scheme [43].](image)

### 1.3.3 Sensor to Sensor Variation

The manufacturing of MEMS devices allows for drastic performance and cost transformations by employing batch fabrication and economies of scale successfully exploited by the parent technology in the integrated circuit industry [13]. In the research and development phases of MEMS, small arrays of the MEMS devices are manufactured and characterized. Implementation of the device to commercial or industrial applications would imply the manufacture of arrays containing 10-1000’s of the MEMS. Complete characterization relative to the designed purposes (ex. mechanical, electrical, optical, thermal, acoustic, magnetic, chemical) is nearly impossible for large arrays. Advantages of the batch fabrication processes result in two ideas of importance. First, the production cost of large arrays is marginally greater than the cost of a few devices. Secondly, the precision of the batch fabrication process should allow for calibration of a few devices applied to the entire array. During development and manufacture the primary system parameters should be monitored and evaluated to ensure the batch calibration principle. Due to the small number of devices that have been transitioned to commercial application relatively little work has been performed to characterize the variation of parameters.

Tanner et al. [44] produced a polysilicon electrostatic comb-drive resonator using surface micro-machining processes. Devices with varying beam lengths similar to Figure 1.15 were replicated over the surface of a substrate to investigate the variation of resonant frequency and beam width as a function of location on the substrate. Resonant frequency is measured using a “blur
envelope“ visual technique and beam width by electrical line-width and scanning electron microscope(SEM) line-width techniques. Quantitative results for 18 sensors of each beam length were published for resonant frequency, electrical line-width, and SEM line-width showing variation with location on the substrate wafer. Qualitative conclusions were drawn that a variation existed over the substrate but lacks statistical measures of the manufacturing variation.

Spark et al. [45] investigated the reliability of a resonanting angular rate sensor manufactured from nickel for automotive applications. The resonant sensor is manufactured using techniques similar to LIGA [14] on a CMOS substrate. The reliability interests involved temperature extremes, thermal, fluid immersion, cyclic fatigue, EMI and mechanical shock. While most tests are performed on a single sensor, the cyclic fatigue test is performed on 10 sensors. The resonant frequencies are published in the range of 30-35 kHz with a $6\sigma$ variation listed as ±2 kHz.

Malek et al. [46] performed a direct study of LIGA process variation during manufacture of a microgear. Measurements of critical dimensions were performed at several stages of manufacture using a SEM with resolution specified as 0.04 μm. SEM linewidth measurements are taken after pattern transfer to the x-ray mask blank, deep x-ray lithography, nickel electroplating, and planarization/polishing. Average size deviation with ±3σ(read from plot) for nominal design dimensions of [10, 50, 100] μm are [3.0 ± 1.3, 2.9 ± 2.3, 2.9 ± 2.4] μm. The manufacturing bias independent of feature size is reported as $\approx 2$ μm with expected improved dimensional control as the process is refined.

Rinaldi et al. [47] completed quantitative measurements of simple MEMS cantilever structures manufactured from single crystal silicon using surface micro-machining. The experimental setup closely resembles and is directed towards applications in the characterization of Atomic Force Microscopy(AFM) probe tips. The study investigates cantilevers of nominal dimension $l = [250, 300, 350]$ μm, $w = 35$ μm, and $h = [1, 2]$ μm using LDV to characterize the manufacturing variations and temperature effects. Four cantilevers of $l = [250, 300]$ μm and $h = 2$ μm each were characterized and exhibited an average relative error of 5.4% and 7.1% respectively. Deviations
in resonant frequency are attributed to dimensional variation, test equipment resolution, and non-classical boundary conditions. Post-processing by this research produced average and ±3σ of 15.0 ± 2.9 kHz and 19.9 ± 2.6 kHz for the limited sample size \( N = 4 \).

### 1.4 Scope of Research

The proposed micro fluid shear stress sensor requires investigation of the structural, fluid, and thermal environments as well as product development concerns for eventual application of the sensor. As these sensors can be deployed in arrays from several to hundreds it is prohibitive to calibrate every sensor. Due to the tight manufacturing tolerances, batch fabrication principles are often applied to micro-machined sensors. Also, knowledge of the sensor in-plane stiffness is critical to the floating element principle, however the microscopic scale of deflection makes a direct measurement impractical.

The primary objectives of this research are to validate the batch fabrication principle and indirectly measure sensor out-of-plane stiffness by investigating the sensor structural dynamics. Existing techniques for measuring the in-plane deflection do not offer the resolution and bandwidth required. Evaluating the variational statistics from experiments performs batch fabrication validation and out-of-plane stiffness is measured by relating dynamic parameters to the stiffness with a lumped parameter model. The thesis structure is described in the following paragraphs.

Chapter 2 describes the proposed sensor geometry, manufacture and mechanical analysis. Lumped parameter mass, damping, and stiffness elements are derived for use in single and three degree of freedom model for static and dynamic analysis. Eigenvalue analysis is performed on the lumped parameter dynamic models to solve for resonant frequencies. Modal analysis is performed for a three dimensional solid finite element model to extract the fundamental resonant frequency and visualize mode shapes.

Chapter 3 discusses the experimental setup design and modification as well as data acquisition considerations. A shaker design is presented with lumped parameter and finite element models used to resolve measurement issues caused by the experimental setup. Finally, the principles of laser doppler vibrometry and digital signal processing are introduced.

Chapter 4 contains the data reduction equations, experimental results, and experimental uncertainty analysis. Data reduction equations for the experimental stiffness and half power method damping are presented. Results for beam width measurement, damping, and resonant frequency are used to generate descriptive statistics for each experiment. These statistics are then applied to general uncertainty analysis for mass, damping, experimental, and analytical stiffness to identify sources of variation in the sensor arrays.

Chapter 5 offers a results summary, conclusions, and future work for the shear stress sensor. Results and conclusions for micro-machining accuracy and precision, dynamic characterization and stiffness, sensor array variation for batch fabrication principles, and laser doppler vibrometry are discussed. As a direction for future work, various displacement measurement schemes are discussed as well as packaging considerations and calibration capabilities.
Chapter 2

Shear Stress Sensor Modeling

2.1 Introduction

The mechanical design of a MEMS shear stress sensor that can be scaled to meet the spatial and temporal requirements of turbulent flows is presented in this chapter. A description of the shear stress sensor involving the design, design parameters, considerations, and specific manufacturing process is introduced. Lumped parameter mass, stiffness, and damping elements are derived for use in discrete lumped parameter system models. Single degree of freedom (SDOF) and three degree of freedom (3DOF) lumped parameter system models are developed to predict the fundamental resonant frequency. Finite element analysis of the sensors is completed for the prediction and comparison of resonant frequencies.

2.2 Shear Stress Sensor Description

The shear stress sensor is comprised of two basic elements, a floating plate supported by two identical serpentine beam structures. Figure 2.1 is a diagram of the shear stress sensor where $l$ is the length, $b$ is the beam width, and $h$ is the height of the sensor. SEM pictures of the device are shown in Figure 2.3.

The four parameters $l$, $b$, $h$, and $n_b$ being the number of beams in a single serpentine beam structure fully define the system geometry and modeling parameters. The serpentine beam structure is comprised of $n_b$ basic beam units shown in Figure 2.2. Due to the nature of bulk micro-machining, the parameter $h$ is defined by the height of the working layer in the silicon substrate wafer. Devices of height $[15, 20, 30] \mu$m have been manufactured throughout the research but resonant frequency final results are listed in terms of an $h = 15 \mu$m device height. Devices of various heights have been used for supplementary analysis and experimentation. Parameter $n_b$ is also fixed for this particular research at $n_b = 7$. Changing $n_b$ will increase or decrease the stiffness of the device.
Figure 2.1. Diagram of a single shear stress sensor removed from the substrate.

Figure 2.2. Shear stress sensor basic beam unit.

Figure 2.3. SEM image of a single shear stress sensor.

The device dimensions are selected to keep the predicted out-of-plane resonant frequency within the operating bandwidth of the experimental equipment. The dimensions for three sensors are found in Table 2.1. Beam width and length dimensions are chosen using preferred numbers.[48] The notation used throughout this research for each of the sensors will be 2B50L15H, 4B100L15H and 6B160L15H.

Shear stress sensors are bulk micro-machined from single crystal silicon. The silicon wafer also referred to as the substrate is comprised of the handling layer, silicon oxide sacrificial layer, and device layer shown in Figure 2.4(A). Silicon is an orthotropic material that exhibits different

<table>
<thead>
<tr>
<th></th>
<th>beam width $b$ ($\mu$m)</th>
<th>beam length $l$ ($\mu$m)</th>
<th>beam height $h$ ($\mu$m)</th>
<th>notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor 1</td>
<td>2</td>
<td>50</td>
<td>15</td>
<td>2B50L15H</td>
</tr>
<tr>
<td>Sensor 2</td>
<td>4</td>
<td>100</td>
<td>15</td>
<td>4B100L15H</td>
</tr>
<tr>
<td>Sensor 3</td>
<td>6.3</td>
<td>160</td>
<td>15</td>
<td>6B160L15H</td>
</tr>
</tbody>
</table>

Table 2.1. Shear stress sensor dimensions selected for analysis.
moduli for in-plane and out-of-plane measurements. Simplifications and assumptions are made for this research to reduce the complexity of the orthotropic model. The mechanical properties of silicon used are listed in Table 2.2. Many of the analytical estimations and finite element models used employ an isotropic material model. The appropriate moduli is chosen depending on the expected physical mechanisms. More detailed information regarding silicon properties can be found in [49, 50, 51].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ($\text{kg/m}^3$)</td>
<td>2329</td>
</tr>
<tr>
<td>Elastic modulus (Pa)</td>
<td>$170 \times 10^9$</td>
</tr>
<tr>
<td>$E_{\text{ip}}$</td>
<td>$140 \times 10^9$</td>
</tr>
<tr>
<td>Shear modulus (Pa)</td>
<td>$52 \times 10^9$</td>
</tr>
<tr>
<td>Poisson’s Ratio $\nu$</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 2.2. Single crystal silicon assumed material properties.

2.3 Shear Stress Sensor Micro-machining

The shear stress sensors with different dimensions were designed in a mask layout editor (L-EDIT from Tanner Research). The design was converted to a positive chromium mask using a laser-writer (Heidelberg DWL66). The mask fabrication process is essentially a lithography process. The starting material was a 5 mm thick glass (transparent material) on which is deposited a thin chromium film (about 200 to 300 nm thick) that acts as the opaque material. A photosensitive polymer called photoresist (AZ1518) was deposited on the chromium. The photoresist was exposed to a 442 nm wavelength laser using the laser writer, which wrote the sensor design (in the software form) onto the photoresist. The mask was developed in MF CD-26 (photo-developer) to obtain photoresist patterns of the sensor on the mask. The chromium was etched in a chrome etchant (1020 Chrome Etch) to transfer the sensor pattern onto the chromium, and the photoresist was removed using Positrip EKC 830. The hard chromium mask can be used multiple times to create photoresist patterns of the shear stress sensors on silicon.

The schematic of the fabrication process is shown in Figure 2.4. The starting material was a silicon-on-insulator (SOI) wafer (A). A SOI wafer consists of three layers, a top device layer made from single crystal silicon on which the sensors are fabricated (20 $\mu$m thick), a middle silicon-dioxide layer that is the sacrificial layer used for making the sensors freestanding (about 2 $\mu$m thick), and a bottom handle silicon layer for handling and processing of the wafer (about 300 to 500 $\mu$m thick). In the first step of the fabrication, a 1.2 $\mu$m layer thick photoresist (Shipley 3012) was spun on the silicon device layer of the SOI wafer (B). The photoresist was exposed to ultraviolet light (UV) through the chromium mask using a mask aligner (Karl Suss MA6), as shown in (B). The wafer was developed (using MF CD-26 developer) to obtain photoresist patterns of the sensor on the device layer of the SOI wafer (C). Deep Reactive Ion Etching (AMS-100 SE I-SPEEDER ICP RIE) or the Bosch Process (DRIE) was used to etch the patterns onto
the silicon device layer with photoresist as the mask (D). This is a dry etching process (SF6 based) and can achieve high aspect ratio features (up to 1:20), otherwise not possible with conventional wet etching. The silicon-dioxide layers acts as the etch stop for the Bosch process. To fabricate the backside etch window, 12 µm thick layer photoresist (SPR 220-7) was patterned on the handle layer of the SOI wafer, using the photolithography process discussed above. The backside pattern of the etch window (on the handle layer) was aligned with the front side pattern of the sensors (on the device layer), as shown in (E). The silicon of the handle layer was etched using the DRIE process to remove silicon from below the sensors (F). The silicon-dioxide sacrificial layer was etched using buffered-oxide etch (BOE 10:10), which is hydro-fluoric acid (HF) etching based process (G). Finally, the photoresist was removed from both the silicon layers using a combination of acetone-isopropyl alcohol rinse and oxygen plasma (M4L) to obtain shear stress sensors fabricated out of single crystal silicon (H).
2.4 Lumped Parameter Models

Lumped parameter system models require the assumption that a distributed continuous system can be defined as discrete lumped elements. This analysis uses lumped mass, linear spring, and linear viscous dampers to describe the distributed system. Mathematical models are built using force equilibrium at degrees of freedom.

2.4.1 Derivations

The analytical lumped parameter models developed for the SDOF and 3DOF models are mass/spring/damper systems. A mass element is a kinetic energy storage element described by $F = ma$ where $F$ is the force acting on the mass, $m$ is the mass of the object, and $a$ is the acceleration of the mass. The linear spring is a potential energy storage element that relates the applied force to the spring displacement by $F = k\Delta x$ where $k$ is the spring constant and $\Delta x$ is the elongation of the spring. A damper is an energy dissipating element that typically is a function of the velocity but could be dependent on acceleration, displacement, or constant given the physical dissipation process occurring. The most frequently used element is a linear viscous damper with the form $F = -cv$ where $c$ is the damping constant and $v$ is the velocity. The derivation of expressions for the constants $m$, $c$, and $k$ for physics relevant to the shear stress sensor are in the following sections.

The nomenclature in Table 2.3 is defined to reduce confusion between the different derived equivalent linear spring stiffnesses.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$bb$</td>
<td>bending stiffness of the serpentine beam</td>
</tr>
<tr>
<td>$be$</td>
<td>bending stiffness of the connection between serpentine beams</td>
</tr>
<tr>
<td>$tb$</td>
<td>torsional stiffness of the serpentine beam</td>
</tr>
<tr>
<td>$te$</td>
<td>torsional stiffness of the connection between serpentine beams</td>
</tr>
<tr>
<td>$si$</td>
<td>equivalent stiffness of some number of beams $i$</td>
</tr>
</tbody>
</table>

Table 2.3. Nomenclature to describe lumped parameter element stiffness.

2.4.1.1 Mass Distribution

In the simplest systems, the mass elements are assumed to be lumped point masses instead of continuous mass distributed through the system. The lumped mass approximation allows a finite number of DOF’s as opposed to an infinite number of DOF’s applied to the distributed mass system. In the interest of mathematical simplicity, it is advantageous to use the least number of DOF’s that will accurately describe the system. Often the distributed mass will have motion different than the assigned DOF’s and requires adjustment for accurate system description. This is accomplished by equating the kinetic energy of the distributed/lumped mass system with a pure lumped mass system.
Figure 2.5. Diagram of the distributed mass system used for KE equivalency.

The shear stress sensor plate will correspond to a DOF and thereby its full mass will contribute to the system, however the serpentine beams attached to the sensor plate and the substrate have some mass $m_b$ but oscillate at an amplitude less than the plate. An equivalent mass for each beam must be calculated to increase the accuracy of the model. The serpentine beam end connections are ignored and the system is shown in Figure 2.5. The velocity profile of the beam is assumed to be a linear profile with zero velocity at the rigid substrate and sensor plate velocity $V_{sp}$ at $x = L$.

$$KE_{lp} = KE_b + KE_{sp}$$

$$\frac{1}{2} m_{lp} V_{sp}^2 = \frac{1}{2} \int_0^L \rho_b A_b V_b^2(x) \, dx + \frac{1}{2} m_b V_{sp}^2$$

Substituting the assumed linear velocity profile from Equation 2.1, integrating, and defining the beam mass as Equation 2.2 leads to the final expression for the equivalent lumped parameter mass $m_{lp}$. The equivalent lumped parameter mass $m_{lp}$ of the distributed system is found in Equation 2.3 where $m_b$ is the beam mass and $m_p$ is the original point mass.

$$V_b(x) = \frac{x}{L} V_{sp} \quad (2.1)$$

$$\frac{1}{2} m_{lp} V_{sp}^2 = \frac{1}{2} \frac{m_b V_{sp}^2}{L^2} \int_0^L x^2 \, dx + \frac{1}{2} m_p V_{sp}^2$$

$$m_b = \rho_b A_b L \quad (2.2)$$

$$m_{lp} = \frac{1}{3} m_b + m_p \quad (2.3)$$

2.4.1.2 Bending Stiffness of Beams

Both the in-plane and out-of-plane analytical models require the derivation of the beam bending stiffness. Bernoulli Beam Theory is used to develop the expression for equivalent stiffness [53, 54, 55]. This theory utilizes compatibility and equilibrium to develop the moment-curvature relationship. The following assumptions are necessary for the derivation of the moment-curvature relationship:

1. Material exhibits linear elastic behavior.
2. The beam contains an axial plane of symmetry.

3. Applied loads lie in the plane of symmetry and are perpendicular to the axis of the beam.

4. The neutral axis of the beam bends but does not stretch.

5. Plane sections of the beam remain plane and perpendicular to the neutral axis.

6. Changes in the cross-sectional dimensions of the beam are negligible.

Compatibility describes the state of strain within the beam and combines this with Hooke’s Law to develop the stress-strain relation. Equilibrium is then instituted by forcing the resultant axial force to vanish, resultant moment in the axial plane must vanish, and the resultant moment about the neutral axis must equal the plane section moment \([54]\). The final form of the moment-curvature relationship is shown in Equation 2.4 where \(\kappa\) is the radius of curvature of the beam, \(M\) is the bending moment on the plane section, \(E\) is the modulus of elasticity of the material, and \(I\) represents the moment of inertia of the plane section about the neutral axis.

\[
\kappa = \frac{M}{EI} \tag{2.4}
\]

Derivation of the equivalent stiffness of a beam in bending requires a relation between the deflection of the beam and the moment rather than the curvature and the moment. Utilizing the small angle approximation of an elastic curve, the deflection is related to the radius of curvature by Equation 2.5.

\[
\kappa = -\frac{d^2v}{dx^2} \tag{2.5}
\]

Equating the differential equation of the elastic curve and moment-curvature relationship results in an expression of the deflection of the beam as a function of the moment \(M\) and the flexural rigidity \(EI\) for a beam of constant cross-sectional area. Solution of Equation 2.6 subject to the appropriate boundary conditions will lead to an expression relating the force and deflection.

\[
EI \frac{d^2v}{dx^2} = -M \tag{2.6}
\]

### 2.4.1.3 Equivalent Stiffness of a Fixed/Free Beam

The fixed/free beam shown in Figure 2.6 is subject to the point load \(F\) applied at the free end of the beam. Appropriate boundary conditions for this case are the kinematic conditions of zero displacement and zero slope at the fixed end of the beam. Solution of Equation 2.6, application of the boundary conditions, and rearranging the solution into the form \(F = k_{eq}v\) the equivalent stiffness of the fixed/free beam is identified in Equation 2.7. This equivalent stiffness is used to model the bending stiffness of the serpentine beam end connection.

\[
k_{be} = \frac{3EI}{l^3} \tag{2.7}
\]
2.4.1.4 Equivalent Stiffness of a Fixed/Fixed Slope Beam

The second type of beam required is fixed at one end and the slope fixed as zero on the other end but the displacement is unconstrained. The additional curvature of the beam required to accommodate this boundary condition will increase the equivalent stiffness of this beam. A diagram of the fixed/fixed slope beam is shown in Figure 2.7 with the appropriate boundary conditions shown to the right. The number of support reactions of this beam exceeds the number of independent equilibrium equations resulting in a statically indeterminate beam. Using the equilibrium equations and the solution of Equation 2.6 subject to all the boundary conditions leads to additional equations necessary for the solution. The equivalent stiffness of a beam subject to the fixed/fixed slope boundary conditions above is written in Equation 2.8 [53]. This is the equivalent stiffness of a serpentine beam in bending.

\[ k_{bh} = \frac{12E_{ip}I}{I^3} \]  

(2.8)

2.4.1.5 Torsional Stiffness of a Beam

The derivation of an elastic rectangular cross-section beam in torsion is a detailed and mathematically intensive process. Barnes [55] derivation involves the use of stress functions, separation of variables, and appropriate boundary conditions to arrive at the solution of a rectangular beam in torsion found in Equation 2.9. Equation 2.9 shows the torque \( T \) as a function of \( \beta \), beam height \( h \), beam width \( b \), shear modulus \( G \), and the unit angle of twist \( \Theta \). The parameter \( \beta \)
(Equation 2.10) is specific to a beam of rectangular cross section [55].

\[
T = \beta \pi b^3 G \Theta 
\]

\[
\beta = \frac{1}{3} \left[ 1 - \frac{192b}{\pi^5 h} \sum_{n=1,3,5...}^{\infty} \frac{\tanh \frac{n\pi h}{2b}}{n^5} \right] 
\]

Figure 2.8. Diagram of a rectangular beam under torsion.

Figure 2.9 shows two adjacent beams of depth \( L \) taken from the serpentine spring before deflection due to torsion occurs. Figure 2.10 shows the same two adjacent beams after the deflection due to torsion. Beam torsion is typically defined by the angle of twist \( \theta \) but the DOF in this analysis is a translational motion. The rotational motion of the beam is coupled to the translational motion using the small angle approximation. The triangle imposed on Figure 2.10 connects the cross-sectional centroids of the fixed beam, undeflected beam position, and deflected beam position. The centroid of the second beam is assumed to displace vertically by some quantity \( v \) relative to the first beam and is located a distance \( 2b \). No bending is assumed in the knuckle connecting the beams forcing the rotation to occur solely in the second beam.

Figure 2.9. Undeformed serpentine beam before a torsion is applied.

Figure 2.10. Rotational deflection of a serpentine beam after a torsion is applied

Assume the force \( F \) is applied as a couple to the end of the beam as shown in Figure 2.10.
(Equation 2.11), the beam twists in a linear fashion (Equation 2.12), and the small angle approximation applies (Equation 2.13).

\[
T = Fh \quad (2.11)
\]

\[
\Theta = \frac{\theta}{L} \quad (2.12)
\]

\[
\theta = \frac{v}{2b} \quad (2.13)
\]

Substituting Equations 2.11-2.13 into Equation 2.9 a force-deflection relationship is derived. The specific equivalent stiffness \( k_{tb} \) of a rectangular beam in torsion is shown in Equation 2.14.

\[
F = \frac{G\beta \theta^2}{2L\nu} \quad (2.14)
\]

An equivalent torsional stiffness for the end connections between serpentine beams must also be derived. Figure 2.11 contains a beam end connection before deflection and Figure 2.12 is the end connection after deflection. The concept is similar to the torsion beam analysis already performed with new definitions for unit angle of twist and the small angle approximation found in Equations 2.15 and 2.16 respectively.

\[
\Theta = \frac{\theta}{2b} \quad (2.15)
\]

\[
\theta = \frac{v}{L} \quad (2.16)
\]

Figure 2.11. Undeformed serpentine beam end connection.  
Figure 2.12. Rotational deflection of a serpentine beam end connection.

After substitution of Equation 2.15 and 2.16 into Equation 2.9 an expression identical to Equation 2.14 is the result. This means that the equivalent stiffness of the serpentine beam end
connections is identical to the torsional stiffness of the beams (Equation 2.14).

\[ k_{tc} = \frac{G\beta b^2}{2l} \]

### 2.4.1.6 Damping

Damping is the method for energy dissipation in lumped parameter systems. Several sources of damping that affect the system are intrinsic mechanical damping, MEMS specific squeeze film damping, and Stokes layer damping. The damping ratio \( \zeta \) of a system represents the ratio of the damping in the system \( c_{sys} \) to the critical damping \( c_c \). The critical damping \( c_c \) is the quantity of damping necessary for the system to return to equilibrium in the shortest time possible. For a system where \( \zeta = 1 \) the system is critically damped, \( \zeta < 1 \) the system is underdamped, and \( \zeta > 1 \) the system is overdamped. Often damping is considered negligible if \( \zeta \ll 1 \). The threshold for this assumption in this analysis is defined as \( \zeta_c < 0.001 \). Another parameter used to characterize damping is the quality factor \( Q \). Quality factor is defined as the ratio of energy stored in a system to the energy dissipated in one cycle. For lightly damped systems with \( \zeta < 0.05 \) the quality factor is related to the damping ratio by Equation 2.17 [56].

\[ Q = \frac{1}{2\zeta} \]  

### (2.17)

### 2.4.1.7 Intrinsic Mechanical Damping

Intrinsic mechanical damping is losses from thermoelastic, surface, and bulk effects of the material. Thermoelastic effects arise from the temperature gradient introduced by compression and expansion of the material under cyclic loading [57]. The thermoelastic loss is a function of material properties as well frequency and temperature. Calculations performed for a silicon cantilever at room temperature in a vacuum determine a loss \( \eta = 2 \times 10^{-5} \) where \( \eta \) is the inverse of quality factor \( Q \). Surface loss effects are difficult to characterize and may be caused by oxidation on the surface, damage to the crystal structure on the surface, or surface roughness. Experiments on the cantilever from above resulted in a loss on the order of \( O \approx 10^{-5} \). Oxidation, damage, and surface roughness are expected to be minimal on a single crystal silicon so surface effects are considered negligible. Bulk dissipation is due to the thermal dissipation of vibrating bonds in the solid. The bulk dissipation of single crystal silicon at room temperature has been measured \( \eta \approx 2 \times 10^{-8} \) making it orders of magnitude smaller than thermoelastic and surface effects [58].

Combining the quality factors and substituting in Equation 2.17 the damping ratio is \( \zeta = 0.000015 \). Mathematically, the estimated damping ratio is well below the defined threshold for \( \zeta_c \) qualifying the assumption that intrinsic mechanical damping is negligible. Experiments to measure the mechanical dissipation of silicon have resulted in extremely high Q values on the order of \( 10^5 \) verifying the assumption to ignore mechanical damping [59, 60, 61, 62].
2.4.1.8 Squeezed Film Damping

Squeeze film damping represents the dissipation of energy caused by the interaction of vibrating MEMS structures with fluids or gases. Most specifically it accounts for the compression/expansion and viscous effects of gases between a fixed and oscillating plate [63]. Figure 2.13 illustrates the basic concept of squeeze film damping.

![Figure 2.13. Illustration of the basic concept of squeeze film damping.](image)

Darling et al. [64] introduced a numerical method for solving the linearized Reynolds equation using a Green’s function method subject to arbitrary acoustic boundary conditions to predict the forces acting on the volume between two plates. Experimental work investigating the quality factors of cantilever beams ranging from 100 to 800 microns in length declared acceptable agreement within and order of magnitude between Darling’s predictions and experimental results [65]. Theoretical analysis using coupled elastic beam theory and the Reynolds equations subject to isothermal and incompressible conditions arrive at similar conclusions [66]. Squeeze film damping is often characterized by the squeeze number $\sigma$ specifically for a square plate in Equation 2.18 where $g_o$ is the nominal gap and $P_o$ is the nominal pressure.

$$\sigma = \frac{12 \mu l^2 \omega}{g_o^2 P_o}$$  \hspace{1cm} (2.18)

The squeeze number $\sigma$ has a $\frac{1}{g_o^2}$ dependence. By design for this research the backside etch of the wafer leaves a nominal gap $g_o$ comparable in dimension to the shear stress sensor length $l$. This large nominal gap results in a small squeeze number leading to the case of an infinite gap. Analysis of a beam resonator by Zhang et al. [66] predicts the quality factor for an infinite gap to be greater than $Q > 1 \times 10^4$.

If $g_o << l$ the linear damping coefficient of a square plate can be calculated using Equation 2.19 [63].

$$c_{squeeze} = \frac{96 \mu l^4}{\pi^4 g_o^3}$$  \hspace{1cm} (2.19)

The use of Equation 2.19 is subject to the following assumptions:

1. Nominal gap $g_o$ is much smaller than the lateral dimensions of the plate. ($g_o << L$)
2. Gas motion under the plate can be treated as Stokes flow. (Creeping flow)
3. No pressure gradient exists between the plates. \( \frac{\partial p}{\partial y} = 0 \)

4. Lateral gas motion has a Poiseuille-like velocity profile with no-slip conditions on the plate.

5. Gas obeys the ideal gas law.

6. System is isothermal.

Using \( \zeta_\ell = 0.001 \) the expression for the nominal gap \( g_o \) necessary to consider damping negligible is written in Equation 2.20. This expression is only applicable to a square plate subject to the assumptions listed above.

\[
g_o > 3 \sqrt{\frac{48 \mu d^4}{\zeta \ell \pi^4 m \omega_n}} 
\]  
(2.20)

### 2.4.1.9 Stokes Layer Damping

Stokes layer damping is caused by viscous effects in the boundary layer. However in this case the fluid is considered to be stationary and the sensor plate is in motion creating the boundary layer illustrated in Figure 2.14. The problem of an infinite plate executing oscillatory motion in a parallel direction is known as Stokes second problem [2, 5, 6]. Solution of the governing equation in Equation 2.21 subject to proper boundary conditions leads to the velocity profile in Equation 2.22.

\[
\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} 
\]  
(2.21)

\[
\begin{align*}
    u(0, t) &= U \cos \omega t \\
    u(\infty, t) &= 0 \\
    u(y, t) &= U e^{-y \sqrt{\omega/2\nu}} \cos \left( \omega t - y \sqrt{\frac{\omega}{2\nu}} \right)
\end{align*}
\]  
(2.22)

Differentiation of the velocity profile evaluated at the wall \( y = 0 \) leads to the expression for the force acting on the sensor plate area \( A \).

\[
F(0, t) = \tau_w A = \mu A \left( \frac{\partial u}{\partial y} \right) = -\mu A \sqrt{\omega/2\nu} U \sin(\omega t)
\]
From this a damping coefficient that is a function of $\omega$ for a flat plate with area $A$ can be identified in Equation 2.23.

$$c_{stokes} = \mu A \sqrt{\omega / 2\nu}$$ (2.23)

The expression identifying the limit of $\omega$ for negligible damping is shown in Equation 2.24.

$$\omega < 8\nu \left( \frac{m\omega_0\zeta}{\mu A} \right)^2$$ (2.24)

### 2.4.2 Single Degree of Freedom Lumped Parameter Models

The simplest lumped parameter model possible is the single degree of freedom (SDOF) model. Figure 2.15 shows the most basic SDOF system where all mass, damping, and spring elements have been reduced to a single respective element. The differential equation describing the SDOF system is shown in Equation 2.25.

$$m_{eq} \ddot{x} + c_{eq} \dot{x} + k_{eq} x = 0$$ (2.25)

The exact SDOF model used for the in-plane and out-of-plane analysis is shown in Figure 2.16. The equivalent mass $m_s$ is calculated using the expression in Equation 2.23 for reducing distributed beam mass to a lumped parameter mass. The single equivalent lumped mass for the entire system is expressed as Equation 2.26. This equivalent mass accounts for the total number of basic beam
units $2n_b$ and the sensor plate.

$$m_s = \rho h \left[ \frac{2}{3} n_b (2b^2 + bl) + l^2 \right]$$  \hspace{1cm} (2.26)

The equivalent stiffness $k_{eq}$ for the in-plane and out-of-plane models will be different because different physical deformation mechanisms contribute to the stiffness. Linear springs add in series by Equation 2.27 and in parallel by Equation 2.28.

$$\frac{1}{k_{eq}} = \sum_i \frac{1}{k_i}$$  \hspace{1cm} (2.27)

$$k_{eq} = \sum_i k_i$$  \hspace{1cm} (2.28)

### 2.4.2.1 In-plane Single Degree of Freedom Model

The differential equation describing the in-plane SDOF system is shown in Equation 2.29.

$$m_s \ddot{u} + 2k_{si} u = 0$$  \hspace{1cm} (2.29)

The beams in the in-plane direction only undergo bending deformations. The bending of the serpentine beam as well as the bending of the beam end connections are modeled in series. Equation 2.27 is used to calculate the equivalent stiffness $k_{si}$ of $i$ beams in series as seen in Equation 2.30.

$$k_{si} = \left( \frac{i}{k_{bi}} + \frac{i}{k_{be}} \right)^{-1}$$  \hspace{1cm} (2.30)

Solution of the eigenvalue problem for Equation 2.29 yields the resonant frequency. Table 2.4 contains the in-plane SDOF lumped parameter resonant frequency estimates for $i = n_b = 7$.

<table>
<thead>
<tr>
<th>device dimensions</th>
<th>resonant frequency (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2B50L15H</td>
<td>106.41</td>
</tr>
<tr>
<td>4B100L15H</td>
<td>53.20</td>
</tr>
<tr>
<td>6B160L15H</td>
<td>32.51</td>
</tr>
</tbody>
</table>

Table 2.4. In-plane SDOF lumped parameter resonant frequency predictions.

### 2.4.2.2 Out-of-plane Single Degree of Freedom Model

The differential equation describing the out-of-plane SDOF system is shown in Equation 2.31.

$$m_s \ddot{w} + 2k_{si} w = 0$$  \hspace{1cm} (2.31)

The beams in the out-of-plane direction undergo bending and torsional deformation mechanisms. Once again Equation 2.27 is used to model the beam bending and torsion in series. Equation 2.32 describes the equivalent stiffness of $i$ beams for out-of-plane SDOF system where for this research
\[ i = n_b = 7. \]

\[ k_{si} = \left( \frac{i}{k_{bb}} + \frac{i}{k_{be}} + \frac{i}{k_{tb}} + \frac{i}{k_{te}} \right)^{-1} \quad (2.32) \]

Solution of the eigenvalue problem for Equation 2.31 yields the resonant frequency. Table 2.5 contains the out-of-plane SDOF lumped parameter resonant frequency estimates.

<table>
<thead>
<tr>
<th>device dimensions</th>
<th>resonant frequency (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2B50L15H</td>
<td>163.93</td>
</tr>
<tr>
<td>4B100L15H</td>
<td>98.11</td>
</tr>
<tr>
<td>6B160L15H</td>
<td>56.41</td>
</tr>
</tbody>
</table>

Table 2.5. Out-of-plane SDOF lumped parameter resonant frequency predictions.

### 2.4.3 Three Degree of Freedom Lumped Parameter Models

The three degree of freedom (3DOF) model used for this analysis is shown in Figure 2.17. The 3DOF models are designed to determine the accuracy gained by adding DOF’s and also investigate the resonant frequencies of the 2\(^{nd}\) and 3\(^{rd}\) modes.

![3DOF system representation](image)

Figure 2.17. 3DOF system representation.

The mass of the serpentine springs \(m_b\) and mass of the sensor plate \(m_p\) are modeled by separating the beams from the square sensor plate mass.

\[ m_b = \rho n_b h b (b + l) \]
\[ m_p = \rho h t^2 \]

The stiffness of a single serpentine beam is modeled as two separate linear springs \(k_{si}\) and \(k_{sj}\). \(k_{si}\) is equivalent stiffness of \(i\) beams that are closest to the sensing area mass \(m_p\). \(k_{sj}\) is the stiffness of \(j\) beams further away from \(m_p\) and closer to the fixed substrate. \(i\) and \(j\) must be positive integers subject to the constraint \(i + j = n_b\).

The set of differential equations that describes the system in Figure 2.17 are shown in Equa-
tion 2.33. Here $\vec{u}$ represents a displacement vector of the system’s degrees of freedom.

$$
\begin{bmatrix}
m_b & 0 & 0 \\
0 & m_p & 0 \\
0 & 0 & m_b
\end{bmatrix}
\ddot{\vec{u}} +
\begin{bmatrix}
k_{sj} + k_{si} & -k_{si} & 0 \\
-k_{si} & 2k_{si} & -k_{si} \\
0 & -k_{si} & k_{si} + k_{sj}
\end{bmatrix}
\vec{u} = 0
$$

(2.33)

Solution of the eigenvalue problem leads to three resonant frequencies for each sensor. The three mode shapes of the system defined by the eigenvectors are shown in Figure 2.18. The mode shapes shown to describe the shape but do not portray the absolute or relative displacements of the system. The absolute amplitude of the system is dependent on the initial conditions or forcing function of the system. The relative amplitudes vary for sensors of different dimensions as the mass and stiffness vary. Mode 1 corresponds to the lowest resonant frequency, mode 2 is the second resonance and mode 3 the third resonance.

![Figure 2.18. Mode shapes of the 3DOF system.](image)

### 2.4.3.1 In-plane Three Degree of Freedom Model

The in-plane 3DOF stiffness utilizes the same physical bending mechanisms as the SDOF model. The description of equivalent stiffness $k_{si}$ and $k_{sj}$ for the in-plane 3DOF model are shown below.

$$
k_{si} = \left( \frac{i}{k_{bb} + \frac{i}{k_{be}}} \right)^{-1}
$$

$$
k_{sj} = \left( \frac{j}{k_{bb} + \frac{j}{k_{be}}} \right)^{-1}
$$

The parameters $i$ and $j$ for the in-plane analysis are chosen to be $i = 3$ and $j = 4$ ($i + j = n_b$). The eigensolution of Equation 2.33 leads to the resonant frequencies listed in Table 2.6.
### Table 2.6. In-plane 3DOF lumped parameter resonant frequency predictions.

<table>
<thead>
<tr>
<th>device dimensions</th>
<th>resonant frequencies (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2B50L15H</td>
<td>119.51</td>
</tr>
<tr>
<td></td>
<td>348.39</td>
</tr>
<tr>
<td></td>
<td>384.64</td>
</tr>
<tr>
<td>4B100L15H</td>
<td>59.76</td>
</tr>
<tr>
<td></td>
<td>174.45</td>
</tr>
<tr>
<td></td>
<td>192.32</td>
</tr>
<tr>
<td>6B160L15H</td>
<td>36.46</td>
</tr>
<tr>
<td></td>
<td>107.14</td>
</tr>
<tr>
<td></td>
<td>117.93</td>
</tr>
</tbody>
</table>

### 2.4.3.2 Out-of-plane Three Degree of Freedom Model

The out-of-plane 3DOF stiffness uses identical physical bending and torsion mechanisms as the SDOF model. The description of equivalent stiffness $k_i$ and $k_j$ for the out-of-plane 3DOF model are shown below.

\[
k_{xi} = \left( \frac{i}{k_{bb}} + \frac{i}{k_{bc}} + \frac{i}{k_{tb}} + \frac{i}{k_{tc}} \right)^{-1}
\]

\[
k_{sj} = \left( \frac{j}{k_{bb}} + \frac{j}{k_{bc}} + \frac{j}{k_{tb}} + \frac{j}{k_{tc}} \right)^{-1}
\]

Once again $i = 3$ and $j = 4$ for the analysis. The eigensolution of Equation 2.33 leads to the resonant frequencies listed in Table 2.7.

### Table 2.7. Out-of-plane 3DOF lumped parameter resonant frequency predictions.

<table>
<thead>
<tr>
<th>device dimensions</th>
<th>resonant frequencies (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2B50L15H</td>
<td>163.24</td>
</tr>
<tr>
<td></td>
<td>475.56</td>
</tr>
<tr>
<td></td>
<td>525.39</td>
</tr>
<tr>
<td>4B100L15H</td>
<td>129.11</td>
</tr>
<tr>
<td></td>
<td>376.89</td>
</tr>
<tr>
<td></td>
<td>415.51</td>
</tr>
<tr>
<td>6B160L15H</td>
<td>63.39</td>
</tr>
<tr>
<td></td>
<td>203.92</td>
</tr>
<tr>
<td></td>
<td>224.45</td>
</tr>
</tbody>
</table>

### 2.5 Finite Element Models

Structural Finite Element Analysis (FEA) is an approximate solution obtained by dividing the object continuum into “finite elements”. FEA utilizes the same concept as lumped parameter models to discretize the system into elements but with orders of magnitude more elements and complexity to more accurately represent the continuous distributed system. Numerical methods are used to approximate a solution that is nearly impossible analytically. FEA is still subject to
assumptions that require a knowledge of finite element theory to accurately model systems and interpret results.

ANSYS is the commercial FEA package used for analysis of the shear stress sensor. The finite element chosen for this analysis is a 10-node tetrahedral element. The element exhibits translational degrees of freedom \((u, v, w)\) at each node. For simplicity and more direct comparison to analytical results, a linear isotropic material model with density is used to describe the shear stress sensor.

### 2.5.1 Isotropic vs. Orthotropic Material Models

An orthotropic material exhibits different material properties in orthogonal directions. Silicon is inherently an orthotropic material because of its crystalline structure. The bonds between carbon atoms are highly directional causing anisotropic behavior. FEA models using linear orthotropic and linear isotropic material models were completed for a sensor with dimensions 4B100L20H. The in-plane and out-of-plane resonant frequencies identified for each model are shown in Table 2.8

<table>
<thead>
<tr>
<th>resonance (kHz)</th>
<th>linear isotropic</th>
<th>linear orthotropic</th>
<th>relative error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>in-plane</td>
<td>53.58</td>
<td>52.99</td>
<td>1.11</td>
</tr>
<tr>
<td>out-of-plane</td>
<td>97.24</td>
<td>87.76</td>
<td>10.56</td>
</tr>
</tbody>
</table>

*Table 2.8. Material model comparison for a shear stress sensor of dimension 4B100L20H*

The in-plane resonant frequencies are nearly identical with a relative error of 1.11% well within an engineering approximation range. The error of 10.56% between out-of-plane models is significant but considered acceptable for three reasons. First, the out-of-plane mode is only used to investigate the precision error in the micro-machining process and independent of the resonant frequency accuracy. Secondly, the out-of-plane model is used to estimate the resonance and keep it within the operating bandwidth of the experimental setup. Finally, the shear stress sensor is expected to operate at frequencies below the first resonance making higher out-of-plane resonances unimportant.

### 2.5.2 Geometry and Mesh Generation

The geometry for the FEA models was created within the FEA package. A large block of length, width and height of each sensor dimension was created. The void spaces between the beams were then removed by subtracting volumes resulting in a single continuous volume. The mesh was created using the automatic mesh generator within the FEA package. A 10-node tetrahedral element is simple for the generator to build with few to no errors however typically results in excess nodes. This costs additional CPU time but saves significant personnel time when creating simple models such as this research. Figure 2.20 contains the geometry and mesh of a sensor. The mesh generator allows the user to “smart size” the mesh size by selecting a integer from 1 to 10 with 1 corresponding to a fine mesh and 10 corresponding to a coarse mesh.
The mesh is evaluated for accuracy by investigating the grid convergence of the solution. A grid converged solution will not change significantly as the mesh is further refined. The grid convergence limit is set as < 1% relative difference in resonant frequency for the doubling of nodes. Investigation into grid convergence was completed by checking the out-of-plane natural frequency for “smart size” values of 9, 7, 5, 3, and 1. The natural frequency as a function of “smart size” is shown in Figure 2.19. Sufficient grid convergence occurs at a smart size of 7 relative to 1 where the nodes double from 13195 to 26113 with a difference of 0.78% in the resonant frequency.

![Grid Convergence Investigation](image)

**Figure 2.19.** FEA grid convergence investigation for a 4B100L30H sensor.

### 2.5.3 Boundary Conditions

The modal analysis of the sensors requires boundary conditions to arrest rigid body motion for a solution. Fixed displacement conditions of zero displacement are applied to all nodes in contact with the substrate. These are the areas on the ends of the serpentine beams farthest away from the sensor plate. Figure 2.20 shows the application of the boundary conditions to the sensor model.

### 2.5.4 Finite Element Results

Finite element analysis is performed to extract in-plane longitudinal and out-of-plane resonant frequencies. The in-plane longitudinal resonance relevant to the shear stress measurement is always the first mode because it is designed to have the lowest stiffness relative to the in-plane transverse and out-of-plane modes. The in-plane transverse mode is the second mode and the
out-of-plane mode is the third mode. The in-plane transverse mode is not shown because it is not of interest for this research. Plots showing the deformed shape and colored by contours of the displacement vector magnitude are shown in Figures 2.21- 2.26. The resonances for each sensor are extracted from the FEA analysis and found in Table 2.9.

<table>
<thead>
<tr>
<th>device dimensions</th>
<th>in-plane longitudinal resonant frequency (kHz)</th>
<th>out-of-plane resonant frequency (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2B50L15H</td>
<td>108.45</td>
<td>211.88</td>
</tr>
<tr>
<td>4B100L15H</td>
<td>53.33</td>
<td>90.248</td>
</tr>
<tr>
<td>6B160L15H</td>
<td>32.375</td>
<td>46.55</td>
</tr>
</tbody>
</table>

Table 2.9. FEA resonant frequency predictions for in-plane and out-of-plane modes.

2.6 Discussion and Comparison of Lumped Parameter and Finite Element Models

The predicted resonant frequencies are compiled and shown in Table 2.10. The in-plane SDOF, 3DOF, and FEA models offer excellent agreement for the resonant frequency. The results for the out-of-plane comparison do not match as well as the in-plane comparison but are still acceptable given the modeling assumptions.

The difference between models can be attributed to the boundary conditions, increased DOF’s, and additional assumptions made for the lumped parameter models. Derivation of the lumped parameter equivalent stiffnesses for beams in bending and torsion require that the boundary conditions be applied to each individual basic beam unit. This places increased constraints on the lumped parameter models that may not exist in the physical system. Errors in lumped parameter boundary conditions are compiled as the beams are added in series to evaluate the equivalent stiffness. The FEA model is subject only to the boundary conditions where the sensor contacts the substrate.
The difference is also a consequence of the increased DOF’s of the FEA models. Although the FEA model is a discrete system, the orders of magnitude increase in the model DOF’s allow it to more accurately represent the physical system. The SDOF and 3DOF models utilize very few degrees of freedom to represent a highly complex system.

Lumped parameter models assume that all beams undergo identical deflections as the equiv-
alent stiffness of the lumped parameter system is calculated. The FEA model allows the beams to deflect individually and unequally. The lumped parameter inaccuracy is most evident when investigating the large discrepancy between the results for the out-of-plane 2B50L15H sensor. Figure 2.27 is a close up side view of a single serpentine spring set from a 2B50L20H sensor. Labeling the beam as 1-7 starting at the beam next to the substrate and moving towards the

Table 2.10. Summary of resonant frequency predictions from SDOF, 3DOF, and FEA models.

<table>
<thead>
<tr>
<th></th>
<th>model</th>
<th>2B50L15H</th>
<th>4B100L15H</th>
<th>6B160L15H</th>
</tr>
</thead>
<tbody>
<tr>
<td>in-plane</td>
<td>SDOF (kHz)</td>
<td>106.40</td>
<td>53.20</td>
<td>32.51</td>
</tr>
<tr>
<td></td>
<td>3DOF (kHz)</td>
<td>119.51</td>
<td>59.76</td>
<td>36.46</td>
</tr>
<tr>
<td></td>
<td>FEA (kHz)</td>
<td>108.45</td>
<td>53.33</td>
<td>32.375</td>
</tr>
<tr>
<td>out-of-plane</td>
<td>SDOF (kHz)</td>
<td>163.93</td>
<td>98.11</td>
<td>56.41</td>
</tr>
<tr>
<td></td>
<td>3DOF (kHz)</td>
<td>163.24</td>
<td>129.11</td>
<td>63.39</td>
</tr>
<tr>
<td></td>
<td>FEA (kHz)</td>
<td>211.88</td>
<td>90.25</td>
<td>46.55</td>
</tr>
</tbody>
</table>

Figure 2.27. Isometric view of the 2B50L20H sensor serpentine spring at resonance showing non-equivalent deflections.

sensor plate, beams 1, 2, 6, and 7 are primarily undergoing torsional deflections while beams 3, 4, and 5 undergo minimal torsional deflection. This clearly visualizes how the lumped parameter assumption of equivalent deflection could be incorrect. A study of the aspect ratios and its relation to resonant frequency should be performed to characterize this effect. Re-evaluation of the SDOF lumped parameter analysis with \( n_b = 4 \) results in a resonant frequency of 224.8 kHz much closer to the FEA prediction of 211.8 kHz.
3 Experimental Setup

3.1 Introduction

Design of the experiment and experimental setup are vital to ensure that valid results are obtained. The primary goal of the research is to identify the out-of-plane resonant frequency to evaluate dynamic variation in the sensor array. Various methods of excitation and measurement were investigated and a broadband shaker excitation with laser doppler measurement scheme was chosen. The three primary components of the experimental setup are the shaker, laser vibrometer data acquisition, and signal processing. This chapter will introduce the shaker design and modeling, laser vibrometry data acquisition, and signal processing necessary for the experiment. The complete experimental setup is best described by the block diagram in Figure 3.1.

The shaker input is initiated at the signal generator. This signal passes to the filter/pre-amp where it is filtered to remove unwanted frequency content and amplified. The signal is split and one path goes to the signal analyzer to monitor the input while the other path continues to the power amplifier. The power amplifier increases the voltage of the signal before entering the shaker. In the shaker a piezoelectric crystal vibrates to drive the shear stress sensors according to the input signal. The laser vibrometer measures the velocity component parallel to the beam and converts the optical signal to an electrical signal. The signal analyzer is used to convert the vibrometer output from the time domain to the frequency domain using Fourier Analysis. Power spectra for each individual sensor are calculated and the resonant frequency information extracted via post-processing.

3.2 Shear Stress Sensor Actuation

The shaker is a simple design where a piezoelectric crystal sandwiched between two aluminum blocks. Figure 3.2 is an unscaled diagram of the shaker setup. The larger aluminum block is labeled as the “base block”, the piezoelectric crystal composed of lead zirconate titanate is labeled
“PZT”, the smaller aluminum block is labeled “support block”, and the silicon wafer/sensors are labeled as themselves.

Lumped parameter models with 3DOF were derived to investigate the shaker resonances as well as the effect of the shaker on the shear stress sensors. Initial shaker designs exhibited noisy frequency responses in the absence of the support block. Empirical relations and FEA were employed to investigate the mode shapes of the silicon wafer and subsequently make the design change of adding the support block.

Aluminum is assumed to be a linear isotropic material with density $\rho = 2700 \text{ kg/m}^3$, modulus of elasticity $E = 70 \times 10^9 \text{ Pa}$, Poisson’s ratio $\nu = 0.33$, and speed of sound $c_{ss} = 5100 \text{ m/s}$ [55].
The lead zirconate titanate piezoelectric crystal is referred to as “PZT” due to its chemical formula Pb\([\text{Zr}_x\text{Ti}_{1-x}]\)\text{O}_3 where 0<x<1 [67]. A linear orthotropic material model is assigned to the PZT with density \(\rho = 7800\ \text{kg/m}^3\), elastic moduli \(E_{11} = 69 \times 10^9\ \text{Pa}\) and \(E_{33} = 55 \times 10^9\ \text{Pa}\), and Poisson’s ratio \(\nu = 0.31\).

### 3.2.1 Shaker Analytical Models

Lumped parameter models of the shaker were developed to investigate the setup resonances. The lumped parameter system is a semi-definite system meaning that rigid body motion of the system is not constrained. This will result in an eigenvalue of zero from the system analysis signifying the rigid body motion as a resonance. Figure 3.3 is the lumped parameter model diagram and the equations of motion are found in Equation 3.1. Notation used this system is \(m_{bb}\) mass of the base block, \(m_{sb}\) mass of the support block, \(m_{PZT}\) mass of the PZT crystal, \(m_s\) mass of a single sensor, \(c_{sys}\) measured damping coefficient of the shaker setup, \(c_s\) estimated damping coefficient of the shear stress sensor, \(k_{PZT}\) stiffness of the PZT crystal, and \(k_{si}\) stiffness of a serpentine structure with \(i\) beams. The displacement of the shear stress sensors is added as a DOF in this model to investigate the effect of the shaker on the sensors.

![Figure 3.3. Lumped parameter model of the shaker setup.](image)

\[
\begin{bmatrix}
  m_{bb} & 0 & 0 \\
  0 & m_{sb} + m_{PZT} & 0 \\
  0 & 0 & m_s
\end{bmatrix}
\begin{bmatrix}
  \ddot{\mathbf{w}}
\end{bmatrix}
+ \begin{bmatrix}
  c_{sys} & -c_{sys} & 0 \\
  -c_{sys} & c_{sys} + c_s & -c_s \\
  0 & -c_s & c_s
\end{bmatrix}
\begin{bmatrix}
  \dot{\mathbf{w}}
\end{bmatrix}
+ \ldots
\begin{bmatrix}
  k_{PZT} & -k_{PZT} & 0 \\
  -k_{PZT} & k_{PZT} + 2k_{si} & -2k_{si} \\
  0 & -2k_{si} & 2k_{si}
\end{bmatrix}
\begin{bmatrix}
  \mathbf{w}
\end{bmatrix}
= \begin{bmatrix}
  F_{PZT} \\
  F_{PZT}
\end{bmatrix} e^{i\omega t}
\]  

Assume a solution of the form in Equation 3.2.

\[
\mathbf{\ddot{w}}(t) = \mathbf{\ddot{w}}_c e^{i\omega t}
\]  

Substituting the assumed solution into the lumped parameter equations of motion leads to Equation 3.3 where \(\mathbf{M}\) is the mass matrix, \(\mathbf{C}\) is the damping matrix, \(\mathbf{K}\) is the stiffness matrix, and \(\mathbf{F}\) the
force vector, and \( \vec{W} \) the solution amplitude vector.

\[
[-\omega^2 \vec{M} + i\omega \vec{C} + \vec{K}] \vec{W} = \vec{F}
\]  

(3.3)

Complex analysis of Equation 3.3 leads to an solution of \( \vec{W} \) as a function of radian frequency \( \omega \) in Equation 3.4.

\[
\vec{W}(\omega) = \left[-\omega^2 \vec{M} + i\omega \vec{C} + \vec{K}\right]^{-1} \vec{F}
\]  

(3.4)

Solution of Equation 3.4 for the shaker setup with a 4B100L20H sensor attached is plotted in Figure 3.4. As expected two resonances are identified in the plot. The first resonance is the fundamental resonance of the shaker and the second resonance is the expected resonance of the 4B100L20H shear stress sensor. Qualitatively, the sensor resonance will be identified by a significant velocity peak that is not present in a substrate measurement. Experimental measurements of the substrate modes is necessary to distinguish between the substrate and sensor motion. The impedance of the base block is large enough that minimal to negligible motion relative to the support block and shear stress sensors is occurring. Quantitative results in Figure 3.4 represent optimal theoretical predictions therefore experimental amplitudes will be lower.

![Displacement Amplitude](image1)

![Velocity Amplitude](image2)

Figure 3.4. Displacement and velocity predictions for the shaker with a 4B100L20H sensor.

3.2.1.1 Lumped Parameter Masses for the Shaker

The lumped masses of the shaker system are defined as the base block \( m_{bb} \) and support block \( m_{sb} \). The dimensions and mass for each of the aluminum blocks are shown in Table 3.1. The
Table 3.1. Mass of the blocks for the lumped parameter shaker model.

<table>
<thead>
<tr>
<th>block</th>
<th>length (mm)</th>
<th>width (mm)</th>
<th>height (mm)</th>
<th>mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>base block</td>
<td>42.8</td>
<td>28.5</td>
<td>12.8</td>
<td>42.2</td>
</tr>
<tr>
<td>support block</td>
<td>10.0</td>
<td>10.0</td>
<td>4.0</td>
<td>1.1</td>
</tr>
</tbody>
</table>

PZT mass \( m_{PZT} \) is lumped with the support block. It is \( \approx 10\% \) of the support block mass \( m_{sb} \) so the affect is expected to be minimal. The dimensions and mass of the PZT crystal are shown in Table 3.2. The lumped shear stress sensor mass used for the 3DOF shaker analysis is identical to the mass used for the SDOF analysis. Sensor parameters \( h, \ l, \ b \), and \( n_b \) are defined by the device chosen. The SDOF equivalent mass is shown again below for the reader’s convenience.

\[
m_s = \rho h \left[ \frac{2}{3} n_b (2b^2 + bl) + l^2 \right]
\]

### 3.2.1.2 Lumped Parameter Stiffness for the Shaker

The equivalent stiffness of the PZT crystal is written in Equation 3.5 where \( E_{33} \) is the modulus of elasticity, \( A_{PZT} \) is the PZT area, and \( h_{PZT} \) is the height of the PZT. Equation 3.5 is derived from a isotropic member of arbitrary cross-section in tension. The aluminum blocks are assumed to be infinitely stiff.

\[
k_{PZT} = \frac{E_{33} A_{PZT}}{h_{PZT}}
\]

The equivalent shear stress sensor stiffness \( k_{si} \) is equivalent to the expression used in the SDOF analysis subject to the sensor parameters \( h, \ l, \ b \), and \( n_b \) where \( i = n_b \). The out-of-plane SDOF stiffness is shown here:

\[
k_{si} = \left( \frac{i}{k_{th}} + \frac{i}{k_{be}} + \frac{i}{k_{tb}} + \frac{i}{k_{te}} \right)^{-1}
\]

### 3.2.1.3 Lumped Parameter Damping for the Shaker

The lumped parameter damping coefficient for the shaker \( c_{sys} \) is estimated using a SDOF. The assumed system uses the PZT stiffness \( k_{PZT} \) and support block mass \( m_{sb} \) to calculate the damping coefficient. A damping ratio at the limit of the lightly damped system assumption \( \zeta_{sys} = 0.05 \) is applied to SDOF assumed system. The damping coefficient \( c_{sys} \) can be written in the form of...
Equation 3.6.

\[ c_{sys} = 2\zeta_{sys}m_{sys}\omega_{n,sys} \quad (3.6) \]
\[ c_{sys} = 46.4 \text{ kg/s} \]

Minimal damping is assumed for the shear stress sensor with \( \zeta_s = 0.01 \). Mathematically this is performed to limit the sensor resonance amplitude for plotting purposes. Undamped linear systems will exhibit resonance amplitudes of \( \infty \). Similar estimation of the damping coefficient is found in Equation 3.7.

\[ c_s = 2\zeta_s m_s \omega_{n,s} \quad (3.7) \]
\[ c_s = 6.64 \times 10^{-6} \text{ kg/s} \]

3.2.1.4 PZT Force Estimation

The force the PZT exerts on the base block and support block is estimated using the PZT voltage coefficient \( g_{33} = 24.2 \times 10^{-3} \text{ V} \cdot \text{m/N} \), predicted white noise RMS voltage \( V_{rms} = 39.89 \text{ V} \), and PZT height \( h = 2.0 \text{ mm} \).

\[ F_{PZT} = \frac{V_{rms}h}{g_{33}} \]
\[ F_{PZT} = 3.3 \text{ N} \]

3.2.2 Analysis and Design of the Support Block

Initial shaker design attached the silicon substrate directly to the PZT as shown in Figure 3.5. This led to a noisy frequency response caused by bending modes in the silicon substrate. Empirical relations for plate bending modes and finite element models were investigated for the mode shapes and resonant frequencies of the silicon substrate. The dimensions of the original silicon substrate are found in Table 3.3. Redesign of the shaker resulted in addition of the aluminum support block and change in dimension of the silicon substrate. The design of the aluminum support block involved the use of theoretical axial predictions, empirical bending predictions, and finite element analysis.

![Figure 3.5. Shaker with silicon substrate attached to PZT.](image-url)
<table>
<thead>
<tr>
<th>Silicon Substrate</th>
<th>Length (mm)</th>
<th>Width (mm)</th>
<th>Height (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>40</td>
<td>30</td>
<td>0.410</td>
</tr>
</tbody>
</table>

*Table 3.3. Initial silicon substrate dimensions.*

### 3.2.2.1 Numerical Estimation for Plate Vibration

The resonant frequencies of the silicon substrate are estimated using approximated numerical solutions to the transverse displacement of a plate given by Equation 3.8 [68]. Here \( w \) is the displacement, \( D \) is the flexural rigidity defined by Equation 3.9, and \( \rho \) is the density. Equation 3.9 uses \( E \) as the elastic modulus, \( h \) height of the plate, and \( \nu \) poisson’s ratio.

\[
D \nabla^4 w + \rho \frac{\partial^2 w}{\partial t^2} = 0
\]

\[
D = \frac{Eh^3}{12(1-\nu^2)}
\]  

(3.8)  

(3.9)

The use of the flexural rigidity \( D \) assumes the material to be isotropic. Silicon is an orthotropic material so the in-plane elastic modulus \( E_{11} \) is used. The in-plane elastic modulus is used because bending is a function of the stiffness in the direction of the wave propagation. Numerical solution performed by Lemke [69] assumes a solution of the form in Equation 3.10 where \( A_{mn} \) is the amplitude and \( X_m(x) \) and \( Y_n(y) \) are the solutions to beam functions with free-free boundary conditions.

\[
w(x, y) = \sum A_{mn}X_m(x)Y_n(y)
\]

(3.10)

Lemke expanded the solution of Equation 3.8 to six terms and performed the calculation for several different Poisson’s ratios. The amplitude and frequency parameter \( \gamma \) are tabulated so the resonant frequency can be extracted from the frequency parameter. The frequency parameter is defined in Equation 3.11 where \( \omega_n \) is the radian natural frequency, \( a \) is the plate length, \( \rho \) is the density, and \( D \) the flexural rigidity of a plate.

\[
\gamma = \omega_n a^2 \sqrt{\frac{\rho}{D}}
\]

(3.11)

The fundamental resonant frequencies predicted by Lemke for a square silicon plate with dimensions \( l = 35 \text{ mm}, h = 0.410 \text{ mm}, \) and \( \nu = [0.225, 0.343] \) are shown in Table 3.4. The fundamental frequency of the silicon substrate should fall between these values since the Poisson’s ratio of silicon is \( \nu = 0.28 \).

<table>
<thead>
<tr>
<th>Poisson’s ratio (( \nu ))</th>
<th>Frequency Parameter (( \gamma ))</th>
<th>Resonant Frequency (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu = 0.225 )</td>
<td>14.14</td>
<td>1.935</td>
</tr>
<tr>
<td>( \nu = 0.343 )</td>
<td>13.10</td>
<td>1.793</td>
</tr>
</tbody>
</table>

*Table 3.4. Silicon substrate resonant frequency predictions using numerical techniques.*
3.2.2.2 Finite Element Analysis of the Silicon Substrate

Finite element analysis of a plate with dimensions given in Table 3.3 is performed using a linear isotropic material with silicon properties listed in Table 2.2. Once again the orthotropic in-plane elastic modulus is used for the isotropic elastic modulus as the bending modes are a function of the elastic modulus in the direction of the wave propagation. Rigid boundary conditions are applied to the finite element model where the PZT crystal is attached to the silicon substrate. The mesh and boundary conditions of the silicon substrate finite element model are shown in Figure 3.6. The mesh is refined in the two areas where shear stress sensors would be found on the substrate to increase the resolution. The rigid boundary conditions are shown by the apparent lack of nodes at the center of the model where the number of constraints is so large they blend together.

![Mesh and boundary conditions of the silicon substrate.](image)

The solution of the finite element model results in excess of 30 resonant frequencies predicted in a bandwidth of 0-200 kHz. The fundamental resonant frequency is predicted at $f_1 = 5.616$ kHz. Due to the large number of predicted resonances only two figures are shown for illustration purposes. In Figure 3.7, lines showing the location of the sensor arrays are imposed on the contour plot colored by the displacement vector sum. Here the sensors will be relatively unaffected by the plate vibration because the relative amplitude of the plate at these locations is small. Figure 3.8 shows a higher resonance where the plate vibration relative amplitude will have a greater impact on the shear stress sensor vibration.

Comparison of the finite element and numerical approximations shows a large error. The finite element model is most likely more accurate because it utilizes more accurate dimensions as well as boundary conditions. The use of the numerical approximation allows for rapid calculation of the solution order of magnitude and offers insight into the effect of system and material parameters on the resonant frequency.

Evaluation of the large number of resonances in the 0-200 kHz bandwidth suggests the shaker should be modified to reduce the number of resonances in this bandwidth. Several design modifications and materials were investigated to shift the resonances. The modification chosen was the
addition of the aluminum support block to change the flexural rigidity of the silicon substrate by increasing $h$. Figure 3.2.2.2 shows a diagram of the support block where $l_{sb}$ is the length, $h_{sb}$ the height, and $h_s$ is the height of the silicon substrate. The dimensions of the support block are selected as $l_{sb} = 10.0 \text{ mm}$ and $h_{sb} = 4.0 \text{ mm}$. Aluminum is the selected material because of properties, availability, and manufacturing considerations. The dimensions and material are selected so that the fewest number of axial and bending modes are present in the support block to minimize the affect on the shear stress sensors.

3.2.2.3 Theoretical Axial and Empirical Bending Predictions of the Support Block

The differential equation that describes the axial displacement of an 1-D isotropic bar is written in Equation 3.12 with displacement $w$, density $\rho$, and elastic modulus $E$ [70].

$$\frac{\partial^2 w}{\partial z^2} = \frac{\rho}{E} \frac{\partial^2 w}{\partial t^2}$$  \hspace{1cm} (3.12)
Solution of the differential equation and application of fixed-free boundary conditions leads to the expression of the axial resonant frequency in Equation 3.13 where \( n \) is the mode number.

\[
    f_n = \frac{(2n - 1)c_{ss}}{4h_{ab}}
\]  

(3.13)

The axial and bending fundamental resonances of the support block are calculated using Equation 3.13 and Equation 3.11 respectively and displayed in Table 3.5.

<table>
<thead>
<tr>
<th>Resonant Frequency (kHz)</th>
<th>Axial</th>
<th>Bending</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>318.750</td>
<td>130.181</td>
</tr>
</tbody>
</table>

Table 3.5. Axial and bending resonant frequency predictions for the support block.

3.2.2.4 Finite Element Analysis of the Support Block

FEA of the aluminum support block is performed to investigate the predicted resonances. The inertia and stiffness influence of the silicon substrate is considered negligible for this analysis. FEA predicts two resonances at 93.74 kHz and 136.04 kHz over a bandwidth of 0-250 kHz. The presence of these two modes in the experimental range is much more manageable than the 30+ modes of the silicon substrate attached directly to the PZT.

![Figure 3.10. FEA support block resonance at 93.735 kHz.](image)

![Figure 3.11. FEA support block resonance at 136.040 kHz.](image)

3.2.3 Finite Element Analysis of the Shaker Design

A 3-D solid finite element model is generated to represent the system in Figure 3.2. The model uses dimensions of the base block, support block, and PZT described in the previous sections. The silicon substrate attached to the support block is considered negligible as its stiffness and mass contribution are minimal and also the modeling complexity of the interface. The contact area between the PZT and aluminum blocks is treated as a rigid interface.
Modal analysis of the finite element model produced in excess of 20 modes that exist above 25 kHz. Most modes are bending modes of the base block and are expected to have minimal impact on the support block illustrated in Figure 3.12 colored by the relative displacement vector sum. Some modes are expected to contribute significantly to the measured frequency response of the shear stress sensor as shown in Figure 3.13. The large deflection of the support block is expected to affect the shear stress sensor deflection significantly. Figure 3.13 represents the fundamental system resonance and exhibits similar results with 3DOF lumped parameter analysis and experimental data.

![Figure 3.12. FEA shaker resonance with minimal effect.](image)

![Figure 3.13. FEA shaker resonance with maximum effect.](image)

### 3.3 Data Acquisition by Laser Vibrometry

The shear stress sensor velocity is measured through the use of laser doppler vibrometry (LDV). LDV utilizes a coherent light source incident on the surface of interest to measure the shift of the incident wave frequency. The frequency shifting phenomenon caused by relative motion between two objects is known as the doppler effect. The doppler effect has many useful applications beyond vibrometry including temperature measurement, fluid velocity measurement, medical imaging, and radar [71].

#### 3.3.1 Theory of Operation

The laser doppler technique involves the measurement of frequency shifted optical waves proportional to the velocity. The specific laser doppler technique used for this research is the single incident reference beam configuration. Figure 3.14 is a block representation of the vibrometer where O is the object and M is the reference mirror [72]. A source emits a coherent laser beam that is split into an incident beam and a reference beam. The incident beam continues to the object where it is reflected and frequency shifted due to the relative motion. The reference beam is frequency shifted by an acousto-optic modulator to induce directional sensitivity and reflected
from a stationary target. Reflected incident and reference beams are then recombined through a process known as optical heterodyning. Combination of two waves of different frequency leads to a time-varying signal with a frequency of oscillation known as the “beat frequency”. The beat frequency is the difference between the incident and reference beam frequencies. The beat frequency lies in a much more manageable range for signal processing compared to the frequency of light ($\approx 10^{14}$ Hz). A photodetector measures the intensity of the combined signal and converts it to an analog electrical signal. The signal passes to a velocity controller that demodulates the signal and outputs an electrical signal proportional to the velocity. Additional information regarding the theory and application of the laser doppler technique and laser vibrometry can be found in the following resources [71, 72].

3.3.2 System Specifications

The specific system used for this research is a Polytec OFV 2600 velocity controller and Polytec OFV 502 optical interferometer. The optical interferometer generates, splits, captures, recombines, and measures the optical portion of the vibrometer system. The interferometer output signal is passed to the velocity controller where the signal is demodulated, conditioned, and filtered so the electrical signal output is proportional to the velocity. System specifications of important parameters are found in Table 3.6 and Table 3.7 for the velocity controller and sensor head respectively [72].

3.4 Signal Processing

Signal processing is the analysis, interpretation, and manipulation of signals for purposes of storing, extracting, reconstructing, and separating information. The output signal from the laser vibrometer is a continuous, amplitude dependent, time-varying analog signal in the time domain. The signal in this form is difficult to analyze and interpret. The ideal format for analyzing the signal would be a frequency dependent amplitude signal in the frequency domain. The signal
Table 3.6. Selected specifications for the Polytec OFV 2600 velocity controller.

Table 3.7. Selected specifications for the Polytec OFV 502 optical interferometer.

processing performed for this research is Fourier Analysis to extract the frequency spectrum from the continuous time signal.

3.4.1 Discretization and Quantization

The conversion from the analog to the digital domain is the first step necessary for conversion to the frequency domain. A digital signal offers advantages over analog signals because it can be processed using powerful computational techniques. The signal is discretized by sampling the signal at equally spaced time periods $\Delta t_s$. This converts the continuous time signal to a sequence of values of varying amplitude. The sampling period $\Delta t_s$ is related to the sampling frequency $f_s$ by $f_s = \frac{1}{\Delta t_s}$. The discrete sequence is related to the continuous signal by $x_d[n] = x_c(n\Delta t_s)$ where $n$ is an integer value. The expression states that the $n^{th}$ value of the discrete sequence $x_d$ is equivalent to the continuous signal $x_c$ at time $n\Delta t_s$. The total length of the sample in the time domain would be $T = N\Delta t_s$ where $N$ is the sequence length.

The conversion from analog to digital also requires the quantization of continuous signal amplitude. Quantization is the approximation of a continuous range of values by a smaller discrete set. The infinite precision amplitude is truncated at the discretized location $n\Delta t_s$ to a finite precision amplitude. Digital systems are unable to handle infinite precision numbers because of processing and storage contraints. Figure 3.15 represents the discretization and quantization of continuous signal. The sine wave is sampled at discrete locations and then quantized to discrete amplitudes.

3.4.2 Fourier Analysis

The conversion from time domain to frequency domain is completed using Fourier analysis. Fourier analysis states that a continuous time signal can be decomposed into the sum of an infinite number of sinusoidal frequencies by the application of the Fourier transform. The continuous-time Fourier transform is written as a function of the frequency $f$ in Equation 3.14.

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-i2\pi ft} dt$$  \hspace{1cm} (3.14)
Discretization and Quantization of a Continuous Signal

Figure 3.15. Discretization and quantization of a continuous signal.

When the discrete sequence rather than the continuous time signal is used it becomes the Discrete-Time Fourier Transform (DTFT) found in Equation 3.15. This is an approximation of the continuous Fourier transform as the information between each discretization is lost.

\[ X(f) = \sum_{n=-\infty}^{\infty} x[n]e^{-i2\pi fn} \]  

(3.15)

The DTFT must be further modified by using a finite length sequence which becomes the Discrete Fourier Transform (DFT) found in Equation 3.16. This is necessary because it is impossible to compute the sum of an infinite series. The DFT can be thought of as sampling the DTFT at a finite number of frequencies. Here \( N \) is the length of the finite duration sequence.

\[ X[k] = \sum_{n=0}^{N-1} x[n]e^{-i2\pi \frac{k}{N}} \quad \text{for} \quad k = 0, 1, 2 \ldots N - 1 \]  

(3.16)

The DFT is well suited for computational evaluation, especially using the Fast Fourier Transform (FFT). The FFT is a computational algorithm applied to the discrete time signal to extract the discrete frequency content. Further information regarding signal processing can be found in [73, 74, 75, 76, 77].

3.4.3 Digital Signal Processing Considerations

Further considerations must be made for the processing of signals. Aliasing, windowing, and averaging are the three primary considerations. Aliasing is caused by the contamination of the
desired frequency spectrum by unwanted frequencies above the Nyquist frequency. Windowing performs two tasks as to limit the infinite input sequence to finite duration as well as reduce the spectral leakage. Averaging is the combination successive measurements to reduce the precision error inherent in experimental measurements.

The maximum resolvable frequency is defined by the Shannon-Nyquist sampling theorem as \( f_{\text{max}} = \frac{f_s}{2} \). Frequency content above \( f_{\text{max}} \) will appear in the spectrum below \( f_{\text{max}} \) mirrored around \( f_{\text{max}} \). This leads to false frequency content as well as increasing the noise below \( f_{\text{max}} \). This is known as aliasing and arises because the sampling period \( \Delta t_s \) is not infinitely small. Aliasing errors are avoided by the application of an analog low-pass filter with a sharp roll-off at a cut-off frequency of \( f_{\text{cutoff}} = f_{\text{max}} \). This effectively removes the frequency content above \( f_{\text{max}} \) as well as reducing the noise in the measured spectrum [71].

Windowing is the multiplication of a pre-determined sequence with the input sequence to limit the sequence to finite duration and reduce spectral leakage. The calculation of the DFT implies periodicity that the discretized signal \( x[n] \) is a finite length sequence recreated on integer multiples or \( x[n] = x[n + kN] \) where \( k \) is an integer. The window is used to create the finite sequence \( x[1, 2, \ldots, N] \) by multiplication of the window sequence \( w[1, 2, \ldots, N] \) with the infinite sequence \( x[1, 2, \ldots, \infty] \) [75]. The second purpose of windowing is the reduction of spectral leakage. Leakage is caused when the initial and final amplitudes of the periodic finite duration time sequence is not equal \( x[0] \neq x[N] \). The inequality of these values represents a discontinuity that requires infinite frequency content to replicate. The power of the signal is "leaked" out of the desired signal frequency content into the frequency content necessary for the discontinuity. A window will force the tails of the sequence to zero or a value near zero reducing the leakage effect. A side effect of windowing is the reduction of signal power that can be compensated for by evaluating the power contained in the window sequence \( w[n] \) [71].

Error in experimental measurements is caused by the combination of bias error and precision error. Bias error is the fixed, systematic or constant component of the total error. Precision or repeatability error represents the random component of the total error [78]. As subsequent DFT’s are processed the frequency content and amplitude will invariably change with time caused by precision error. DFT’s are averaged to reduce the precision error. Many different averaging techniques are available with the most common being time, RMS time, RMS exponential time, and RMS exponential video.

3.4.4 Signal Analyzer Specifications

The signal analyzer used for this experimentation is a Agilent 89410 Vector Signal Analyzer. The analyzer discretizes, quantizes, and anti-aliasing filters the analog signal to a digital signal. The time domain or frequency domain signal can be visualized or saved from the analyzer. Additional specifications related to the analyzer can be found in the operation manual [79].
Experimental Results and Discussion

4.1 Introduction

A single degree of freedom mathematical model, shown in Equation 4.1, is used to characterize the physical shear stress sensor system. The three experimental parameters required for full characterization of the system are effective mass, damping ratio, and natural frequency. Effective mass \( m_{eff} \) is calculated using nominal design dimensions, damping ratio \( \zeta \) is approximated by the half power method using half power points obtained from LDV measurements, and effective stiffness \( k_{eff} \) is quantified indirectly using \( m_{eff} \) and the experimental resonant frequency \( \omega_n \) measured by LDV.

\[
m_{eff} \ddot{x} + 2\zeta m_{eff} \omega_n \dot{x} + k_{eff} x = 0
\] (4.1)

The uncertainty of \( m_{eff} \), \( \zeta \), and \( k_{eff} \) are evaluated using general uncertainty analysis to investigate the propagation of error through the experiment. Beam width of several random sensors are measured using an SEM line-width technique to determine the mass uncertainty. Damping ratio uncertainty is evaluated by the half power method using the measured frequency spectrum. The uncertainty in \( k_{eff} \), important to the floating element operating principle, is evaluated analytically using the SDOF expression for stiffness and experimentally using the uncertainty of the calculated mass and measured resonant frequency.

4.2 Beam Width Measurements

The beam width of a set of 4B100L20H shear stress sensors are measured using a SEM line-width technique [44]. Five different beams on five different sensors are characterized using images obtained from the SEM. Grayscale images are post-processed using software to identify the beam edge by investigating individual pixel shade. Bias errors are introduced by the grayscale shade interpretation and angle of orientation of the electron beam relative to the surface normal. Bias error affects the mean beam width but will not affect the random error in beam width.
The uncertainty calculated from beam width measurements represents the absolute uncertainty of the manufacturing process. The absolute uncertainty as opposed to relative uncertainty is applied directly to the nominal dimension independent of it’s magnitude. Results from the SEM line-width measurements are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Device</th>
<th>Beam</th>
<th>Beam Width (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4.17</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.11</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.05</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.11</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4.11</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4.07</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.07</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.97</td>
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<tr>
<td></td>
<td>5</td>
<td>4.17</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3.95</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.08</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.08</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4.08</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4.01</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.01</td>
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<td>4.13</td>
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<tr>
<td></td>
<td>4</td>
<td>4.01</td>
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<td></td>
<td>5</td>
<td>4.01</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>4.04</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.09</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.09</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.04</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4.09</td>
</tr>
</tbody>
</table>

**Table 4.1.** SEM line-width beam measurements and statistics for a 4B100L20H sensor.
4.3 Resonant Frequency Measurement

4.3.1 Procedures

Two procedures are used for experimentation pertaining to equipment configuration and data acquisition. A equipment configuration procedure is generalized to allow for variation in configuration dependent on the expected resonant frequency and signal processing requirements. The data acquisition procedure is written for evaluation of a single device and is critical for maintaining the accuracy and repeatability of the data samples.

The following is a generalized procedure for setting up the equipment:

1. IMPORTANT - Set power amplifier gain to zero.

2. Connect equipment using the appropriate input and output connections and power connections.
   - Signal Generator ⇒ Pre-Amp/Filter ⇒ Power-Amplifier ⇒ PZT leads
   - Signal Generator ⇒ Pre-Amp/Filter ⇒ Signal Analyzer
   - Laser Vibrometer ⇒ Signal Analyzer

3. Turn on all equipment and do not acquire data for at least 10 minutes to allow the electronics to reach a steady state operating temperature.

4. Set the waveform, frequency, amplitude, and phase of the input signal in the signal generator.

5. Adjust the filter to the appropriate type, cut-off frequency, and roll-off.

6. Set the gain of the pre-amplifier to desired value.

7. Set necessary signal processing options on the signal analyzer.

8. Align the sensor array parallel/perpendicular to the linear orthogonal traverses to simplify the search process for a device.

9. Mount the laser reference beam module for the laser vibrometer in a vertical position so the mirror is seated on the shoulder and perpendicular to the beam.

10. Turn on the vibrometer laser.

11. Aim vibrometer laser head at target and adjust position to optimize signal return.

12. Increase power amplifier gain to desired level.

13. Begin collecting data.

The following procedure is used to collect data:
1. Visually re-position the laser spot in the area of the shear stress sensor using the traverses on the positioning stage. If looking at the devices there will be apparent increase in laser spot intensity as it moves over a void space or the device cause by diffraction from the micro-machined surfaces. Figure 4.1 contains a diagram representative of the signal level as the laser reflects from a void, serpentine spring, substrate, and device.

2. Move the traverse in a single direction while monitoring the optical signal level to the vibrometer.
   NOTE: A sharp reduction in signal level implies that the void space or a serpentine spring has been located.

3. Move the traverse in the perpendicular direction while monitoring the signal level to the vibrometer.
   NOTE: A sharp reduction in signal level indicates that the void or serpentine spring near a device has been found.

4. Change the signal analyzer to continuous average mode. Using the appropriate traverse, locate the sensor area by recognizing that the signal level at the vibrometer and signal amplitude at the analyzer will reduce significantly when over a void space or serpentine spring.
   NOTE: When located on a device sensing area the the signal level should decrease as the traverse is moved in any direction.

5. Adjust the vertical traverse supporting the laser head to optimize signal return at the vibrometer and signal amplitude from the analyzer. Adjustment of the vertical traverse can shift the spot location.

6. Iterate the previous two steps until the optical signal amplitude in the analyzer has reached an acceptable level.

7. Turn off the continuous average mode and restart the data collection. This will collect the data required to meet the number of averages required and display the output.

8. Save the output to disk or manually record necessary values. Verify that the correct filename is being used.

9. Move to the next shear stress sensor and repeat procedure.

### 4.3.2 Shaker Input

A signal is emitted by the signal generator with the characteristics listed in Table 4.2. The signal is passed to the filter/pre-amplifier with characteristics listed in Table 4.3. After the signal is filter and the gain applied it is split between the power amplifier and signal analyzer. Figure 4.2 is the input signal from the signal analyzer before it is power amplified. The signal analyzer will
Figure 4.1. Expected vibrometer return signal level for locations on the sensor plate, voids, beams, and substrate.

use this signal as the input signal for spectral analysis. The power amplifier increases the voltage to maximize the polarity of the PZT in the shaker and maximize the forcing function.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>waveform</td>
<td>white noise</td>
<td>coupling</td>
<td>AC</td>
</tr>
<tr>
<td>pattern</td>
<td>continuous</td>
<td>low-pass cutoff (kHz)</td>
<td>300</td>
</tr>
<tr>
<td>voltage $V_{pk}$</td>
<td>1</td>
<td>low-pass roll off (dB/oct)</td>
<td>6</td>
</tr>
<tr>
<td>frequency (Hz)</td>
<td>N/A</td>
<td>high-pass cutoff (kHz)</td>
<td>10</td>
</tr>
<tr>
<td>phase (deg)</td>
<td>N/A</td>
<td>high-pass roll off (dB/oct)</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 4.2. Specific signal generator settings

Table 4.3. Specific filter and pre-amp settings

4.3.3 Digital Signal Processing

The shear stress sensor velocity is proportional to the doppler shift of the optical return signal. This optical signal is converted to an electrical signal by the photodetector and passed to the vibrometer controller where it demodulates, filters, and conditions the the analog output signal. The analog electrical signal is passed to the digital signal analyzer where spectral analysis or more specifically a Fast Fourier Transform is performed. Attention should be paid to the selection of appropriate values and options relative to the input, output, and type of processing desired. Important parameters to be considered for spectral analysis are sequence length, frequency bandwidth, anti-aliasing low-pass filter cutoff, bandwidth resolution, window type, amplitude resolution, and averaging. Table 4.4 contains signal analyzer parameters relevant for this research. The sequence length is chosen to provide sufficient frequency resolution while maintaining an acceptable computational time. The hanning window is selected because it minimizes the effects of spectral leakage in an infinite sequence and offers good amplitude and frequency
Figure 4.2. Shaker Input Signal before Power Amplification

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>analyzer</td>
<td>vector</td>
</tr>
<tr>
<td>range (V)</td>
<td>5V</td>
</tr>
<tr>
<td>sequence length</td>
<td>1601</td>
</tr>
<tr>
<td>window type</td>
<td>hanning</td>
</tr>
<tr>
<td>averages</td>
<td>50</td>
</tr>
<tr>
<td>frequency bandwidth (kHz)</td>
<td>20 or 300</td>
</tr>
<tr>
<td>bandwidth resolution (Hz)</td>
<td>18.75 or 287.5</td>
</tr>
</tbody>
</table>

Table 4.4. Specific signal analyzer settings.

resolution.

4.3.4 Resonant Frequency Identification

Spectral analysis of the vibrometer output signal is completed by the signal analyzer. The analyzed spectra data are then saved to file and post-processed via MATLAB. Figure 4.3 is the power spectrum from 0-250 kHz which encompasses the entire operating range of the vibrometer (0-250 kHz).

As frequency \( f \to 0 \) the voltage amplitude of the signal \( V \to \infty \). This is a serious concern as it would imply the system is in rigid body motion. Further investigation identified that the noise floor of the vibrometer when focused at \( \infty \) exhibits similar behavior. The noise floor represents the output of the signal demodulator when the signal return at the photodetector is minimal or a low SNR. The signal demodulator attempts to demodulate the SNR \( \approx 0 \) and returns a voltage amplitude \( V \to \infty \) as \( f \to 0 \).
Figure 4.3. Broadband power spectrum for a 6B160L15H sensor and the substrate.

Qualitatively the measurements taken on shear stress sensors exhibit the same trend as the noise floor while focused at $\infty$, but with a higher amplitude. The SNR off-resonance is too low for correct demodulation of the signal caused by losses in the fiber-optic cable and increased noise levels. The fiber-optic laser vibrometer used for this research is an older system and questions have been raised regarding amplitude accuracy caused by losses in the fiber. Amplitude reduction does not affect this research as long as the reduction is proportional for all frequencies. The shear stress sensor area is on the same order of the laser spot size leading to increased reflection and diffraction from additional surfaces (ex. beams and void edges) in close proximity thereby increasing noise levels. These problematic issues can be resolved by installing new fibers and decreasing the laser spot size.

The substrate exhibits the expected behavior as $f \to 0$ the voltage amplitude $V \to 0$ or the equipment noise floor. This isolates the cause of the increased noise floor for shear stress resonator measurements to the increased noise return from surfaces in close proximity to the shear stress resonator plate. The substrate measurement is performed using the same fiber-optics as the shear stress resonator measurements but is an almost perfectly flat surface under uniform motion. All incident light would be reflected from the uniform surface resulting in low optical noise return.

Narrowband frequency measurements are made near the fundamental resonant frequency to increase the frequency resolution by an order of magnitude. Figure 4.4 contains two sensors and a substrate measurement. Post-processing is performed on the narrowband data to search for the resonant peak and half power (-3dB) points used for damping estimations.

The complete data tables for all three sensor sizes are shown in Tables 4.5-4.7. These tables include resonant frequency $f_r$, resonant bandwidth $\Delta f_r$, experimental damping ratio $\zeta_{exp}$, and
Figure 4.4. Narrowband power spectra for two 6B160L15H sensors and the substrate experimental stiffness $k_{\exp}$ for each sensor measured. B indicates the specific sensor was broken and G indicates the sensor was glued while attaching the substrate to the support block. Mean, precision index and ±95% confidence intervals are calculated for each parameter. The mass and t-distribution value for each data set are also listed.
Resonant Frequency and Resonant Bandwidth Measurements
2B50L15H

<table>
<thead>
<tr>
<th>Device</th>
<th>Resonant Freq.</th>
<th>Resonant Band.</th>
<th>Zeta</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_r ) (Hz)</td>
<td>( \Delta f_r ) (Hz)</td>
<td>( \zeta )</td>
<td>( k_{exp} ) (N/m)</td>
</tr>
<tr>
<td>A</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>B</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>D</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>E</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>F</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>G</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>I</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>J</td>
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<td>525</td>
<td>0.0020</td>
<td>68.04</td>
</tr>
<tr>
<td>K</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>L</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>M</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>N</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>O</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>P</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Q</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>R</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>S</td>
<td>128475</td>
<td>1113</td>
<td>0.0043</td>
<td>67.96</td>
</tr>
<tr>
<td>T</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>U</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>V</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>W</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>X</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Y</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>MEAN</td>
<td>128513</td>
<td>819</td>
<td>0.0032</td>
</tr>
<tr>
<td></td>
<td>PREC. INDEX</td>
<td>53</td>
<td>416</td>
<td>0.0016</td>
</tr>
<tr>
<td></td>
<td>( \pm 95% ) CONF. INT.</td>
<td>674</td>
<td>5283</td>
<td>0.0206</td>
</tr>
<tr>
<td></td>
<td>ERROR %</td>
<td>0.52</td>
<td>645.05</td>
<td>645.51</td>
</tr>
</tbody>
</table>

\[ t(0.95, \nu) = 12.706 \]

\[
\text{Mass(kg)} = 1.04E-10
\]

G=Glued
B=Broken

**Table 4.5.** Resonant frequency and bandwidth measurements for individual 2B50L15H and mean, precision index, and \( \pm 95\% \) confidence intervals.
### Resonant Frequency and Resonant Bandwidth Measurements

<table>
<thead>
<tr>
<th>Device</th>
<th>Resonant Freq.</th>
<th>Resonant Band.</th>
<th>Zeta</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_r$ (Hz)</td>
<td>$\Delta f_r$ (Hz)</td>
<td>$\zeta_{exp}$</td>
<td>$k_{exp}$ (N/m)</td>
</tr>
<tr>
<td>A</td>
<td>78200</td>
<td>425</td>
<td>0.0027</td>
<td>100.71</td>
</tr>
<tr>
<td>B</td>
<td>78125</td>
<td>487</td>
<td>0.0031</td>
<td>100.52</td>
</tr>
<tr>
<td>C</td>
<td>77400</td>
<td>213</td>
<td>0.0013</td>
<td>98.66</td>
</tr>
<tr>
<td>D</td>
<td>78000</td>
<td>575</td>
<td>0.0042</td>
<td>101.42</td>
</tr>
<tr>
<td>E</td>
<td>78475</td>
<td>663</td>
<td>0.0016</td>
<td>99.11</td>
</tr>
<tr>
<td>F</td>
<td>77575</td>
<td>262</td>
<td>0.0057</td>
<td>100.78</td>
</tr>
<tr>
<td>G</td>
<td>78225</td>
<td>900</td>
<td>0.0056</td>
<td>100.68</td>
</tr>
<tr>
<td>H</td>
<td>79162</td>
<td>256</td>
<td>0.0016</td>
<td>103.21</td>
</tr>
<tr>
<td>I</td>
<td>78252</td>
<td>825</td>
<td>0.0052</td>
<td>100.85</td>
</tr>
<tr>
<td>J</td>
<td>78700</td>
<td>512</td>
<td>0.0032</td>
<td>102.01</td>
</tr>
<tr>
<td>K</td>
<td>78187</td>
<td>888</td>
<td>0.0047</td>
<td>101.97</td>
</tr>
<tr>
<td>L</td>
<td>78687</td>
<td>750</td>
<td>0.0042</td>
<td>101.20</td>
</tr>
<tr>
<td>M</td>
<td>79262</td>
<td>288</td>
<td>0.0018</td>
<td>103.47</td>
</tr>
<tr>
<td>N</td>
<td>78087</td>
<td>1312</td>
<td>0.0084</td>
<td>100.42</td>
</tr>
<tr>
<td>O</td>
<td>78150</td>
<td>1013</td>
<td>0.0064</td>
<td>100.58</td>
</tr>
<tr>
<td>P</td>
<td>78662</td>
<td>950</td>
<td>0.0060</td>
<td>101.91</td>
</tr>
<tr>
<td>Q</td>
<td>78350</td>
<td>1213</td>
<td>0.0077</td>
<td>101.10</td>
</tr>
<tr>
<td>R</td>
<td>79450</td>
<td>313</td>
<td>0.0019</td>
<td>103.96</td>
</tr>
</tbody>
</table>

**Mean**

- Resonant Frequency: 78386 Hz
- Resonant Bandwidth: 658 Hz
- Zeta: 0.0042
- Stiffness: 101.20 N/m

**Precision Index**

- 536
- 341
- 1131
- 719

**Error %**

- 1.44
- 109.17
- 109.42
- 2.89

$t(0.95, \nu) = 2.110$

**Table 4.6.** Resonant frequency and bandwidth measurements for individual 4B100L15H and mean, precision index, and ±95% confidence intervals.
Resonant Frequency and Resonant Bandwidth Measurements
6B160L15H

<table>
<thead>
<tr>
<th>Device</th>
<th>Resonant Freq. $f_r$ (Hz)</th>
<th>Resonant Band. $\Delta f_r$ (Hz)</th>
<th>Zeta $\zeta_{exp}$</th>
<th>Stiffness $k_{exp}$ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>B</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>C</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>D</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>E</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>F</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>G</td>
<td>39912</td>
<td>150</td>
<td>0.0018</td>
<td>66.98</td>
</tr>
<tr>
<td>H</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>I</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>J</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>K</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>L</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>M</td>
<td>39950</td>
<td>200</td>
<td>0.0025</td>
<td>67.11</td>
</tr>
<tr>
<td>N</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>O</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>P</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Q</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>R</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>S</td>
<td>39937</td>
<td>137</td>
<td>0.0017</td>
<td>67.07</td>
</tr>
<tr>
<td>T</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>U</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>V</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>W</td>
<td>39950</td>
<td>187</td>
<td>0.0023</td>
<td>67.11</td>
</tr>
<tr>
<td>X</td>
<td>G</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Y</td>
<td>B</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>MEAN</td>
<td>39937</td>
<td>169</td>
<td>0.0021</td>
<td>67.07</td>
</tr>
<tr>
<td>PREC. INDEX</td>
<td>18</td>
<td>30</td>
<td>0.0004</td>
<td>0.06</td>
</tr>
<tr>
<td>±95% CONF. INT.</td>
<td>57</td>
<td>95</td>
<td>0.0012</td>
<td>0.19</td>
</tr>
<tr>
<td>ERROR %</td>
<td>0.14</td>
<td>56.33</td>
<td>56.23</td>
<td>0.29</td>
</tr>
</tbody>
</table>

$t(0.95, \nu) = 3.182$

$\nu = 1.07 \times 10^{-9}$

G=Glued
B=Broken

Table 4.7. Resonant frequency and bandwidth measurements for individual 6B160L15H and mean, precision index, and ±95% confidence intervals.
4.4 Data Reduction Equations

The complete characterization of the sensor modeled as a SDOF mass/damper/spring system requires the effective mass $m_{eff}$, damping ratio $\zeta$, and effective stiffness $k_{eff}$. In a mathematical SDOF system these are exact values however they are approximated by the calculated and measured values in this research. Knowledge of the sensor length $l$, beam width $b$, sensor height $h$, density $\rho$, resonant frequency $f_r$, and resonant bandwidth $\Delta f_r$ are the parameters required for complete analysis.

\[
m_{\text{eff}} \approx m_{\text{nom}} = \rho h \left[ \frac{2}{3} n_h (2b^2 + bl) + l^2 \right]
\]

\[
\zeta = \frac{c}{4\pi m f_n} \approx \zeta_{\text{exp}} = \frac{1}{2} \frac{\Delta f_r}{f_r}
\]

\[
k_{\text{eff}} = \frac{4\pi^2 m f_n^2}{2} \approx k_{\text{exp}} = 4\pi^2 m_{\text{nom}} f_r^2
\]

The effective mass $m_{\text{eff}}$ is assigned the nominal value calculated using design dimensions in Equation 4.2. This expression is identical to the mass derived for the SDOF lumped parameter analysis. The nominal design value is used as direct measurement of the mass is difficult at this scale. The accuracy and precision of micro-fabrication and the density of single crystal silicon imply that the effective mass is accurately approximated by the nominal mass $m_{\text{eff}} \approx m_{\text{nom}}$. The largest source of error would most likely come from the approximation of a distributed mass system as a lumped mass system.

Mathematically the damping ratio $\zeta$ is defined in terms of the system parameters $c$, $m$, and $f_n$ in Equation 4.3. For lightly damped systems the damping ratio $\zeta_{\text{exp}}$ can be estimated using the half power method. The resonant bandwidth $\Delta f_r$ is the difference of the half power points located -3dB from the peak amplitude at resonance $\Delta f_r = f_{\text{high}}(-3\text{dB}) - f_{\text{low}}(-3\text{dB})$. The half power method is a simple but effective tool for estimating damping as long as the resonances are lightly damped and well separated. Further information regarding the half power method is available in [56, 80, 81, 82].

The effective stiffness $k_{\text{eff}}$ is evaluated using a SDOF relationship utilizing the nominal mass $m_{\text{nom}}$ and experimental measurement of the resonant frequency $f_r$ as shown in Equation 4.4.

4.4.1 Single Shear Stress Sensor Results - Example Calculation

Nominal and experimental values in Table 4.8 are substituted into Equations 4.2-4.4 to evaluate $m_{\text{nom}}$, $\zeta_{\text{exp}}$, and $k_{\text{exp}}$ as an example of the calculations performed for each measured shear stress sensor.

\[
m_{\text{nom}} = 4.172 \times 10^{-10} \text{ kg}
\]

\[
\zeta_{\text{exp}} = 0.00420
\]

\[
k_{\text{exp}} = 134.9 \text{ N/m}
\]
### Table 4.8. Experimental and nominal values for example calculations of a single sensor.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>density $\rho$</td>
<td>$2329 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>length $l$</td>
<td>$100 \mu \text{m}$</td>
</tr>
<tr>
<td>beam width $b$</td>
<td>$4 \mu \text{m}$</td>
</tr>
<tr>
<td>height $h$</td>
<td>$15 \mu \text{m}$</td>
</tr>
<tr>
<td>resonant frequency $f_r$</td>
<td>$78.386 \text{ kHz}$</td>
</tr>
<tr>
<td>half power bandwidth $\Delta f_r$</td>
<td>$0.658 \text{ kHz}$</td>
</tr>
</tbody>
</table>

#### 4.5 Statistics of Experimental Parameters

General uncertainty analysis requires statistics to describe the measured parameter variation for evaluation of the experimental result uncertainty. Mean, precision index, and 95% confidence intervals (CI) are calculated for each measured parameter and applied to standard experimental uncertainty techniques to develop expressions for parameter and experimental result uncertainty.

The standard formulas for mean and precision index are shown in Equations 4.5 and 4.6 where $N$ is the number of samples [74, 78]. The precision index and $t$-distribution of a finite sample population are analogous quantities standard deviation $\sigma$ and $\tau$-distribution for an infinite sample population. Standard deviation and the $\tau$-distribution describe the infinite population only when a gaussian distribution applies to the population. The precision index and $t$-distribution are finite population estimators of the gaussian distributed infinite population standard deviation and $\tau$-distribution.

\[
\overline{X} = \frac{1}{N} \sum_{i=1}^{N} X_i \quad (4.5)
\]
\[
S_X = \left[ \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \overline{X})^2 \right]^{1/2} \quad (4.6)
\]

The $t$-distribution is the same in principle as the $\tau$-distribution however is a function of degrees of freedom $\nu$ as well where $\nu = N - 1$. Equation 4.7 describes the probability CI that a sample $X_i$ lies within $(\overline{X} \pm t(\text{CI}, \nu)S_X)$.

\[
\text{Prob} \left( \overline{X} - t(\text{CI}, \nu)S_X \leq X_i \leq \overline{X} + t(\text{CI}, \nu)S_X \right) = \text{CI} \quad (4.7)
\]

#### 4.5.1 Beam Width Statistics

The beam width analysis is a measured parameter as such only statistics are presented in Table 4.9. This analysis represents the absolute uncertainty $U_{mm}$ of the micro-machining process and should be directly applied as the beam width $U_b$ and length $U_l$ uncertainty. The uncertainty of the device height $U_h$ is provided by the silicon wafer manufacturer. $U_h$ is defined by milling and polishing processes performed to achieve a nearly microscopically flat surface.
Table 4.9. Beam width mean, precision index, and 95% confidence interval for randomly selected 4B100L20H sensors.

<table>
<thead>
<tr>
<th>parameter</th>
<th>4B100L20H</th>
<th>4B100L20H</th>
<th>4B100L20H</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean 5 µm</td>
<td>4.07</td>
<td>4.07</td>
<td>4.07</td>
</tr>
<tr>
<td>precision index</td>
<td>0.0557</td>
<td>0.0557</td>
<td>0.0557</td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>±0.115</td>
<td>±0.115</td>
<td>±0.115</td>
</tr>
</tbody>
</table>

4.5.2 Resonant Frequency and Resonant Bandwidth Statistics

The fundamental resonant frequency and resonant bandwidth are described by mean, precision index and 95% confidence interval based on the sample size collected for each shear stress resonator. Results are listed in Table 4.10. The uncertainty $U_{f_r}$ and $U_{\Delta f_r}$ described by the 95% confidence intervals are used to determine the uncertainty of the damping and stiffness.

Table 4.10. Resonant frequency and resonant bandwidth mean, precision index, and 95% confidence interval for each sensor array.

<table>
<thead>
<tr>
<th>parameter</th>
<th>2B50L15H</th>
<th>4B100L15H</th>
<th>6B160L15H</th>
</tr>
</thead>
<tbody>
<tr>
<td>resonant frequency</td>
<td>128.513</td>
<td>78.386</td>
<td>39.937</td>
</tr>
<tr>
<td>$f_r$ (kHz)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>precision index</td>
<td>0.053</td>
<td>0.536</td>
<td>0.018</td>
</tr>
<tr>
<td>$S_{f_r}$ (kHz)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>±0.674</td>
<td>±1.131</td>
<td>±0.057</td>
</tr>
<tr>
<td>$\pm t(0.95, \nu)S_{f_r}$ (kHz)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>resonant bandwidth</td>
<td>0.819</td>
<td>0.658</td>
<td>0.169</td>
</tr>
<tr>
<td>$\Delta f_r$ (kHz)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>precision index</td>
<td>0.416</td>
<td>0.341</td>
<td>0.030</td>
</tr>
<tr>
<td>$S_{\Delta f_r}$ (kHz)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>±5.283</td>
<td>±0.169</td>
<td>±0.093</td>
</tr>
<tr>
<td>$\pm t(0.95, \nu)S_{\Delta f_r}$ (kHz)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.6 General Uncertainty Analysis

General uncertainty analysis utilizes the statistical uncertainty of independent parameters to describe the dependent parameter uncertainty. This is a measure of how error propagates through the data reduction equations to affect the final result. Detailed uncertainty analysis attempts to identify and consider the propagation of both bias and precision error separately. Assuming that the experimental result $r$ is a function of $m$ parameters $X_i$ and only precision errors exist. The uncertainty of $r$ can be written as Equation 4.8 where $U_{X_i}$ is the parameter uncertainty, $\theta_i$ the
absolute sensitivity coefficient, and \( \rho_{ik} \) the correlation coefficient [78].

\[
\begin{align*}
    r &= r(X_1, X_2, \ldots, X_m) \\
    U_r^2 &= \left[ \sum_{i=1}^{m} \theta_i^2 U_i^2 + \sum_{k=1}^{m} \rho_{ik} \theta_i \theta_k U_i U_k (1 - \delta_{ik}) \right] \\
    \theta_i &= \frac{\partial r}{\partial X_i} \\
    \rho_{ik} &= \frac{S_{ik}}{S_i S_k}
\end{align*}
\]

(4.8) \hspace{1cm} (4.9) \hspace{1cm} (4.10)

The absolute sensitivity coefficient \( \theta_i \) is identified in Equation 4.9. This coefficient describes the sensitivity of the experimental results uncertainty to changes in the uncertainty of a single parameter. The correlation coefficient \( \rho_{ik} \) in Equation 4.10, where \( S_{ik} \) is the covariance, indicates the linear dependence of two variables. Correlation coefficient occurs on the continuous interval \([-1, 1]\) where 0 indicates no dependence, -1 indicates a strong inverse linear relation, and 1 indicates a strong linear relation.

The individual parameter uncertainty \( U_{X_i} \) with the lowest confidence determines the confidence of the experimental result uncertainty \( U_r \). For example, if \( U_{X_i} \) is expressed ≥ 95% confidence for all \( j \) parameters the result \( U_r \) will be expressed with 95% confidence. However if a single parameter is expressed with 90% confidence, the experimental result is expressed with only 90% confidence.

### 4.6.1 Nominal Mass Uncertainty

General uncertainty analysis is performed on the nominal mass equation(Equation 4.2) with the absolute micro-machining uncertainty \( U_{mm} \) expressed at 95% confidence assumed for \( U_b \) and \( U_l \). The relative density uncertainty for single crystal silicon is published by Fujii et al.[50] \( U_d / \rho = 1.1 \times 10^{-7} \) and is converted to an absolute uncertainty for this analysis. The final form of the mass uncertainty equation is shown in Equation 4.11.

\[
\begin{align*}
    \theta_b &= \frac{2}{3} n_b \rho h [4b + l] \\
    \theta_l &= \rho h \left[ \frac{2}{3} n_b b + 2l \right] \\
    U_{mm} &= U_b = U_l = 0.115 \times 10^{-6} \text{ m} \\
    \theta_h &= \rho \left[ \frac{2}{3} n_b (2b^2 + bl) + l^2 \right] \\
    U_h &= 10 \times 10^{-9} \text{ m} \\
    \theta_d &= h \left[ \frac{2}{3} n_b (2b^2 + bl) + l^2 \right] \\
    U_d &= 2.562 \times 10^{-4} \text{ kg/m}^3 \\
    U_m &= \left[ \theta_b^2 U_{mm}^2 + \theta_l^2 U_{mm}^2 + \theta_h^2 U_h^2 + \rho_{bh} \theta_b \theta_l U_m^2 \right]^{1/2} 
\end{align*}
\]

(4.11)
Equation 4.11 is evaluated for the 3 different shear stress sensor dimensions to calculate the 95% confidence interval for each array. Mass, ±95% confidence interval, and relative uncertainty(%) are displayed in Table 4.11. Order of magnitude inspection indicates width and length uncertainty are the primary contributors to the mass uncertainty.

<table>
<thead>
<tr>
<th>parameter</th>
<th>2B50L15H</th>
<th>4B100L15H</th>
<th>6B160L15H</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass $m_{nom}$ (kg)</td>
<td>$1.04 \times 10^{-10}$</td>
<td>$4.17 \times 10^{-10}$</td>
<td>$10.65 \times 10^{-9}$</td>
</tr>
<tr>
<td>95% uncertainty $\pm U_m$ (kg)</td>
<td>$\pm 1.29 \times 10^{-12}$</td>
<td>$\pm 2.59 \times 10^{-12}$</td>
<td>$\pm 4.18 \times 10^{-12}$</td>
</tr>
<tr>
<td>relative uncertainty</td>
<td>1.24%</td>
<td>0.62%</td>
<td>0.39%</td>
</tr>
</tbody>
</table>

Table 4.11. Mass uncertainty for each shear stress sensor array.

4.6.2 Experimental Damping Uncertainty

Damping ratio $\zeta$ is calculated using the simple but effective half power method from Equation 4.3. The mean resonant frequency, mean resonant bandwidth, and 95% confidence for each sized resonator are taken from Table 4.10 and applied to the damping ratio uncertainty equation in Equation 4.12.

$$
\theta_{fr} = \frac{-1}{2} \frac{\Delta f_r}{f_r^2}
$$

$$
U_{fr} = [673, 1131, 57] \text{Hz}
$$

$$
\theta_{\Delta f_r} = \frac{1}{2f_r}
$$

$$
U_{\Delta f_r} = [5283, 719, 95] \text{Hz}
$$

$$
U_\zeta = \left[ \theta_{fr}^2 U_{fr}^2 + \theta_{\Delta f_r}^2 U_{\Delta f_r}^2 + \rho_{fr, \Delta f_r} \theta_{fr} \theta_{\Delta f_r} U_{fr} U_{\Delta f_r} \right]^{1/2}
$$

(4.12)

Results of the damping ratio uncertainty equation are listed in Table 4.12. Initial analysis of the damping ratio uncertainties finds the uncertainties are larger magnitude than the damping ratio itself. While this is alarming there are several explanations. The methods of damping are through mechanical dissipation in the structure, viscous effects, and atmospheric compressibility effects. Theoretical investigation of these effects were all found to be minimal resulting in an extremely lightly damped system so even a small error is on the order of the damping itself. The strength of the half power method lies in its simplicity while providing mediocre accuracy. More accurate higher order methods that utilize complex exponentials or polynomials exist. Alternative excitation methods other than broadband such as burst or step excitations could lead to more accurate estimates also.

4.6.3 Analytical and Experimental Stiffness Uncertainty

The experimental stiffness is evaluated by applying a SDOF model to the shear stress sensor that relates natural frequency, mass, and stiffness. The mass is calculated using nominal values
but with experimental uncertainties. For a system that is lightly damped the natural frequency is considered equivalent to the fundamental resonant frequency. With knowledge of these two parameters the stiffness can be estimated using Equation 4.4 and compared to the analytically derived lumped parameter stiffness from Equation 2.32 written again in a reduced form in Equation 4.13. Uncertainty analysis of the analytic stiffness is performed as an estimator and comparison for the experimentally measured stiffness.

\[
k_{\text{exp}} = k_{\text{exp}}(m, f_r)
k_{\text{exp}} = 4\pi^2 m_{\text{nom}} f_r^2
\]

\[
\theta_{m_{\text{nom}}} = 4\pi^2 f_r^2
\]

\[
\theta_{f_r} = 8\pi^2 m_{\text{nom}} f_r
\]

\[
k_{\text{lp}} = k_{\text{lp}}(b, t, h)
\]

\[
k_{\text{lp}} = \frac{2E}{n_b} \left( \frac{-\beta b^2 h^3}{\beta b^2 + 4\beta b^4 + 8(1 + \nu) b h^3} \right)
\] (4.13)

The uncertainty of \( k_{\text{lp}} \) is simplified by assuming that the modulus of elasticity \( E \) and poisson’s ratio \( \nu \) of silicon are exact quantities. This analysis is primarily concerned with the variation of parameters due to the micro-machining process not from material property variation. Also the resonator arrays are manufactured from a single ingot of homogeneous single crystal silicon, making the variation of material properties minimal on the order of the relative uncertainty of the density \( (O \sim 10^{-7}) \). \( E \) and \( \nu \) are most likely biased estimates of the true material properties but the precision uncertainty analysis is invariant to bias errors. The uncertainties of the parameters are listed followed by the uncertainty analysis equations in Equations 4.14 and 4.15 for \( U_{k_{\text{exp}}} \) and \( U_{k_{\text{lp}}} \), respectively.

\[
U_m = [1.04, 4.17, 10.65] \times 10^{-12} \text{ kg}
\]

\[
U_{f_r} = [673, 1131, 57] \text{ Hz}
\]

\[
U_{m_{\text{mm}}} = U_b = U_l = 1.15 \times 10^{-7} \text{ m}
\]

\[
U_h = 10 \times 10^{-9} \text{ m}
\]

\[
U_{k_{\text{exp}}} = \left[ \frac{\theta_m^2 U_m^2 + \theta_{f_r}^2 U_{f_r}^2}{2} \right]^{1/2}
\] (4.14)

\[
U_{k_{\text{lp}}} = \left[ \frac{\theta_{m_{\text{mm}}}^2 U_{m_{\text{mm}}}^2 + \theta_{h}^2 U_h^2}{2} \right]^{1/2}
\] (4.15)
Uncertainty analysis results for the analytical and experimental stiffness are found in Table 4.13. The analytical relative uncertainty $U_{k_{lp}}$ exhibits a inverse relationship to sensor dimensions. With the fixed absolute micro-machining uncertainty, it is expected that the stiffness uncertainty will decrease with the decreasing relative dimensional error. Numerical inspection of the analytical stiffness absolute sensitivity coefficients $\theta_i$ also shows that the primary contribution to the analytical stiffness uncertainty $U_{k_{lp}}$ is from the beam width $b$. Once again the relative error of beam width $U_{mm}/b$ compared to height $U_h/h$ or beam length $U_{mm}/l$ is much greater so the greatest contribution is expected from beam width.

Experimental stiffness relative uncertainty $U_{k_{exp}}/k_{exp}$ does not exhibit the same expected inverse relationship as the analytical stiffness relative uncertainty. It is difficult to draw conclusions by comparing the experimental stiffness due to the small population size of the 2B50L15H ($n = 2$) and 6B160L15H ($n = 4$) data sets. These data sets offer insight into the expected mean stiffness but provide little confidence to evaluate the variation or uncertainty. Additional experimentation to increase the population size would increase the confidence of conclusions about uncertainty.

<table>
<thead>
<tr>
<th>parameter</th>
<th>2B50L15H</th>
<th>4B100L15H</th>
<th>6B160L15H</th>
</tr>
</thead>
<tbody>
<tr>
<td>experimental stiffness</td>
<td>68.0</td>
<td>101.2</td>
<td>67.1</td>
</tr>
<tr>
<td>$k_{exp}$ (N/m)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95% uncertainty</td>
<td>0.71</td>
<td>2.92</td>
<td>0.19</td>
</tr>
<tr>
<td>$\pm U_{k_{exp}}$ (N/m)</td>
<td>1.05</td>
<td>2.89</td>
<td>0.29</td>
</tr>
<tr>
<td>relative uncertainty %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>analytical stiffness</td>
<td>110.6</td>
<td>158.5</td>
<td>133.7</td>
</tr>
<tr>
<td>$k_{lp}$ (N/m)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95% uncertainty</td>
<td>12.32</td>
<td>7.88</td>
<td>3.50</td>
</tr>
<tr>
<td>$\pm U_{k_{lp}}$ (N/m)</td>
<td>11.13</td>
<td>4.97</td>
<td>2.61</td>
</tr>
<tr>
<td>relative uncertainty %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>relative error %</td>
<td>(k_{lp} - k_{exp})/k_{exp}</td>
<td>62.7</td>
<td>56.0</td>
</tr>
</tbody>
</table>

Table 4.13. Experimental and analytical stiffness uncertainty analysis for each sensor array.

Comparison of the experimental stiffness data and predicted analytical stiffness shows a large relative error. Analytical models in this research consistently over predict the sensor stiffness. These simple discrete lumped models are developed to identify behavior trends and draw qualitative conclusions about the complex continuous 3-D system. Inaccuracy of the SDOF lumped parameter stiffness is explained through material models, modeling assumptions, and boundary conditions.

Analytical estimates utilize isotropic material models while silicon is an orthotropic material. The elastic modulus of silicon is constant in-plane and the bending and torsional stiffness are functions of the in-plane modulus so orthotropic effect are expected to be minimal. Finite element models predict $\sim 10\%$ reduction in natural frequency when a orthotropic material model is used instead of isotropic.

Several modeling assumptions are made while reducing the complex 3-D continuous system to a lumped parameter SDOF discrete system. The small angle approximation is used while deriving the linear stiffness of individual elements. Equivalent deflection of each element is assumed as
the stiffness is combined using series and parallel approximations. The reduction to a single degree of freedom forces complex interaction between bending elements, torsion elements, and deflections across the beams to be modeled from a SDOF. More accurate stiffness estimates would be expected as the DOF’s of the model are increased and the lumped parameter configuration is optimized to more accurately represent the physical system.

The application of boundary approximations is always a source of error in models. Assumptions regarding the deflection and slope of the beam-boundary interaction are made, specifically the boundaries are assumed perfectly rigid. These assumptions while accurate are not exact, small deviations from the boundary conditions will affect the stiffness of the connecting element. The combination of the stiffness in series compounds the boundary assumption inaccuracies meaning a small deviation at the first beam will propagate into a large deviation at the last beam.
Chapter 5

Conclusions and Future Work

The research goals of this project were met by the measurement of the variation for batch fabrication, resonant frequency measurement for indirect evaluation of stiffness, and experience using a laser doppler vibrometer system. Each of these results are discussed in the context of the eventual application to a shear stress sensor. Conclusions and important details of other parts of this research are discussed in the general conclusions section. A short description of future work, evaluation of displacement sensing schemes, and water tunnel calibration is included at the end of the chapter.

5.1 Micro-Machining Accuracy and Precision

Characterization of the bulk micro-machining accuracy and precision is completed using a SEM linewidth technique and image analysis software. The width of 25 different beams on 5 randomly selected sensors were measured and exhibited mean and a 95% confidence interval of 4.07 ± 0.115 µm and relative uncertainty of 2.83%. The mean beam width $\bar{b} = 4.07$ µm represents a 1.75% deviation from the nominal design value of $b = 4$ µm. The precision measurement of ±0.115 µm is absolute and should be applied to any dimension. The relative uncertainty will increase for smaller dimensions and decrease for larger dimensions. The bulk micro-machining process maintains acceptable mean and precision errors relative to the current nominal design dimensions. Refinement of the specific micro-machining process and increased resolution of the lithography mask could further decrease the bias and precision errors in bulk micro-machining.

5.2 Resonator Array Variation for Batch Calibration

The out-of-plane fundamental resonant frequency was measured using a laser doppler vibrometry technique and broadband shaker excitation. Statistical analysis of data for each resonator size displayed a mean and ±95% confidence interval [128.5 ± 0.67, 78.4 ± 1.13, 39.9 ± 0.17] kHz with
relative error [0.52%, 1.44%, 0.14%]. This indicates small precision error for resonant frequency measurements validating the batch calibration principle for these resonators.

Frequency response and amplitude measurement of the resonators for full dynamic characterization requires resolution of the laser vibrometer signal to noise issue. This is achieved by decreasing the laser spot size and increasing the resonator amplitude off resonance. Reduction of the spot size will decrease the noisy optical signal return from reflective surfaces in close proximity to the resonator area. The laser spot size is most easily reduced by switching to a newer integrated LDV and microscope system but could be reduced by developing a more accurate experimental design and procedure for the current fiber-optic system. The off resonance vibration amplitude can be increased by increasing the gain of the power amplifier and substitution of a more reactive PZT crystal variety thereby increasing displacement amplitude. Qualitatively these same conclusions can be applied to the in-plane dynamic characterization. A in-plane LDV system will require a measurement probe area less than the area of the shear stress sensor to maximize the signal to noise ratio and also amplitude resolution greater than the magnitude of the resonator.

Effective stiffness is a system parameter more relevant to the eventual application of shear stress measurement than resonant frequency. Evaluation of out-of-plane stiffness uncertainty should provide a qualitative description of expectations for in-plane stiffness uncertainty. Out-of-plane sensor stiffness is evaluated using the mathematical SDOF undamped system description $k = 4\pi^2 m f_n^2$. The SDOF model assumption is qualified by the strong isolated peak in Figure 4.3 indicating a lightly damped system with a single resonance. The mean resonator stiffness for each nominal size is [68.0, 101.2, 67.1] N/m. Uncertainty analysis performed to predict the propagation of measurement uncertainty leads to 95% confidence intervals of [± 0.71, ± 2.92, ± 0.19] N/m and relative uncertainty [1.05%, 2.89%, 0.29%]. Limited population size of experimental data sets preclude experimental conclusions relating uncertainty and size; however, analytical results applied qualitatively to in-plane stiffness indicate that relative uncertainty is inversely related to sensor dimensions. Inspection of the analytical absolute sensitivity coefficients show that micro-machined sensor dimensions represent the most significant contribution to uncertainty. Accuracy and precision of bulk micro-machining will be vital as the sensor sizes are reduced to dimensions on the order of turbulent length and time scales. Experimentally the resonant frequency measurement propagates the largest uncertainty to the sensor stiffness. Development of static methods to directly evaluate the sensor stiffness would mitigate this resonant frequency error and increase the precision of stiffness measurement.

### 5.3 Shear Stress Sensor Uncertainty Analysis

The principle of measurement for floating element shear stress sensor utilizes knowledge of the sensor contact area, sensor stiffness, and measured sensor deflection to evaluate the shear stress acting on the MEMS sensor area. The formula for shear stress is shown in Equation 5.1 with stiffness $k \sim 10^4$, deflection $\delta \sim 10^{-8}$, and length $l \sim 10^{-4}$. Order of magnitude uncertainty
analysis is performed and the absolute sensitivity coefficients $\theta_i$ are shown with the expected order of magnitude.

$$\tau_w = \frac{k\delta}{l^2}$$

$$\theta_k = \frac{\delta}{l^2} \sim 10^6$$

$$\theta_\delta = \frac{k}{l^2} \sim 10^3$$

$$\theta_i = \frac{-k\delta}{l^4} \sim 10^8$$

$$U_{\tau_w}^2 = (\theta_k U_k)^2 + (\theta_\delta U_\delta)^2 + (\theta_i U_{mm})^2$$

The absolute sensitivity coefficients indicate that the shear stress will be very sensitive to uncertainty in the deflection and area measurements. Minimizing these uncertainties requires precise deflection measurement schemes $U_\delta$ and bulk micro-machining $U_{mm}$ to limit the measured wall shear stress uncertainty $U_{\tau_w}$.

### 5.4 Evaluation of Laser Doppler Vibrometry

Laser doppler vibrometry is an effective non-intrusive technique to accurately measure velocity. The current fiber optic LDV offers spot size $\approx 50 - 100 \, \mu m$, bandwidth of $250 \, kHz$, and resolution of $0.5 \, \mu m$ for out-of-plane measurements. The user interface, fiber-optics, and controller allows for simple measurement and easy alignment in a moderately portable package. While the fiber-optics offer some advantages, the disadvantages are increased laser spot size due to the variable focal length lens. MEMS specific microscope systems offer spot sizes $\geq 1 \, \mu m$, bandwidth of $20 \, MHz$, and resolution of $1 \, pm$ for out-of-plane measurements. These systems offer increased performance in a more complex, expensive, and bulkier package. Single reference beam systems such as these only offer measurement in the direction parallel to the beam.

In-plane dynamic characterization will require the measurement of velocity in the plane perpendicular to the laser beam. The two options that exist for in-plane measurements are stroboscopic video microscopy(SVM) and techniques similar to laser doppler velocimetry used for fluid flows. Stroboscopic video microscopy analyzes video images of strobbed optical pulses shifted in phase and synchronized with the setup excitation to infer the displacement and velocity. This system is expensive, complex and requires the use of a microscope for measurement. The other option is development of techniques similar to laser doppler velocimetry in fluid flows. Velocimetry measures the doppler burst from a particle moving through a fringe pattern created by the interference of two coherent light sources. This technique would require significant adaptation for application to vibrometry. Important considerations in the application of a dual beam system are measurement probe volume, reflected light collection, and signal processing.
5.5 General Conclusions

Initial experiments were plagued by small resonator amplitudes on the same order as the substrate. The causes were determined to be large amplitude substrate deflection shapes and squeeze-film damping of the resonator. Addition of the support block to the experimental setup increased the substrate stiffness and mass. The result was the shifting of substrate modes to higher frequencies and lower amplitude vibrations. MEMS researchers often ignore or consider the substrate effects negligible because of the macroscopic nature of the substrate or limit the operating bandwidth of the MEMS device to regions far from the analytically predicted resonance. As the performance expectations of MEMS are increased it will require the broadband experimental dynamic characterization of the device as well as the influence of the substrate.

Squeeze film damping from the viscous and compressibility effects in the gap between the resonator and substrate significantly damped resonance amplitude. Substantial development was put into the backside-etch process by members of Dr. Aman Haque’s research group to increase the gap size. The reduction in damping from the back-side etch process increased the amplitude at resonance. MEMS are typically thought of planar systems because of the fabrication processes. The backside-etch process adds a degree of freedom to the deflection possibilities and hence the application possibilities of MEMS.

5.6 Future Work

Future work necessary for the development of the MEMS shear stress sensor is the displacement measurement scheme, packaging and waterproofing of the MEMS, and experimental calibration of the arrays in a wind or water tunnel.

5.6.1 Development of Sensing Method

A displacement sensing method is necessary for the calculation of wall shear stress. The scheme requires high resolution and large bandwidth to accurately resolve the shear stress at length and time scales of turbulent flow. Application of a shear stress sensor in research and industry environments have two different specifications. A shear stress sensor for research facilities would require high resolution, large measurement range, and large bandwidth for a variety of flows and fluids. The measurement scheme equipment size would be less important and support is available to resolve technical implementation and operation issues. A sensor in industry applications could be tailored to meet resolution, range, and bandwidth requirements of the specific flow and fluid. Robust streamlined "plug and play" integration would be ideal to minimize process downtime and technical support of the shear stress sensor. Optical, piezoresistive, and capacitive schemes will be presented in the context of implementation, performance, and technical concerns.
5.6.1.1 Optical Techniques

An optical scheme for the in-plane displacement is 2-beam laser doppler vibrometry. This technology is undeveloped for vibrometry applications but maintains analogy to the well developed laser doppler velocimetry technique. Stroboscopic Video Microscopy is not a solution because it uses synchronous strobing and excitation of the sensor.

Implementation of 2-beam LDV is a non-intrusive scheme that requires visual access to the device. Current optical methods require mounting of a coherent light source on the front side with photodetector mounted on the back-side. Two-beam LDV could be performed on the front side or back-side of the sensor but back-side would reduce optical signal disturbance as it passes through the flow field. The technique is very sensitive to the surroundings requiring a robust setup to minimize these effects.

Theoretically the performance of the technique is limited by the probe volume and fringe spacing of the coherent beams and signal processing capacity. 2-beam LDV offers reasonable resolution, good range, and large bandwidth for measurement. Development of the process is necessary to better characterize the resolution capabilities of laser doppler vibrometry.

Technical concerns for this technique are due to the complexity of the optical systems and signal processing required. The optical systems must constantly be aligned and calibrated to ensure valid data collection.

5.6.1.2 Resistive Techniques

Resistive or piezo-resistive techniques measure the differential change of electrical resistance through a structure under deflection. The displacement is inferred from strain relations to the differential resistance change identical to a strain gauge concept. Implementation of a resistive technique requires isolation of electrical components from the flow field. This raises issues relative to packaging and isolation from the fluid for the shear stress sensor.

The performance prospects are good for resolution, range, and bandwidth. Bandwidth is limited by the sensor dynamic response as the resistive properties change under large strains. Range would be proportional to the strain limit of the material. Resolution is related to the strain rate and electro-mechanical properties of the material. Materials with properties more desirable than single crystal silicon could be exploited through deposition and metallization processes.

Sensor operations are sensitive to temperature drifts because the resistive properties of materials are a function of temperature. The cross-axis sensitivity to deflection would need to be determined. Also the power dissipated by heat through the resistance must be considered. Heating affects in the flow similar to hot-film anemometry are possible. The use of a resistive scheme greatly increases the micro-machining complexity requiring significant time for development.

5.6.1.3 Capacitive Techniques

Capactive techniques measure the differential change of capacitance as the structure deflects. The differential capacitance is related to the effective area of two parallel plates. Relative motion
between the plates will cause the effective area to change and a change in capacitance. The presence of electrical components requires more complex packaging for electrical leads and also isolation from the fluid.

The performance of capacitive techniques are good for resolution and range but poor bandwidth. The resolution is limited by the electronics ability to measure the differential change in capacitance. Range is limited by the maximum sensor deflection. Current applications of capacitive sensing schemes have are limited to low frequency and would require further development.

Technical concerns involve the fouling of the capacitor and cross-axis sensitivity. The presence of particulate matter between the capacitor plates would cause error in differential capacitance measurement. Also the cross axis-sensitivity to deflection would need to be characterized and minimized for sensor application. The use of a capacitive scheme requires additional complex steps to be added to the micro-machining process increasing the difficulty of manufacturing a sensor.

5.6.2 Packaging and Waterproofing

MEMS devices require post-processing after their manufacture to be prepared for the application. Packaging of the device can be just as difficult as the manufacturing. Device packaging is the issues of integrating micro components with the macro world. Electrical leads must be transitioned from the MEMS to a macroscopic size for data collection and analysis. The device must be mounted on the solid/fluid interface without affecting the properties of the sensor. Hydro and liquid applications of the shear stress sensor will require the isolation of electrical components from the fluid. This could involve the deposition of a highly compliant material in the gaps. Packaging and waterproofing raise many questions that will need to be answered before calibration and measurements can be made.

5.6.3 Experimental Calibration in a Water Tunnel

The Applied Research Laboratory at Penn State maintain facilities that could be used for calibration of the shear stress sensor. The 12-inch water tunnel can operate at velocities $\approx 25 \text{ m/s}$ and capable of producing mean shear stress up to $\approx 200 \text{ Pa}$. 
Bibliography


