The Pennsylvania State University

The Graduate School

Department of Economics

ESSAYS IN INTERNATIONAL TRADE
AND INDUSTRIAL ORGANIZATION

A Thesis in
Economics

by
Tatyana Chesnokova

© 2004 Tatyana Chesnokova

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

December 2004
The thesis of Tatyana Chesnokova was reviewed and approved* by the following:

Kala Krishna
Professor of Economics
Thesis Advisor
Chair of Committee

Nezih Guner
Assistant Professor of Economics

Tomas Sjostrom
Professor of Economics

Abdullah Yavas
Professor of Business Administration

Robert C. Marshall
Professor of Economics
Head of the Department of Economics

*Signatures are on file in the Graduate School
ABSTRACT

CHAPTER 1: Immiserizing Deindustrialization: A Dynamic Trade Model with Credit Constraints

In this essay I develop an open economy dynamic model with bequests and credit constraints. The agricultural sector uses only labor, the industrial sector needs an indivisible investment. Under autarky, productive agriculture provides the funds needed for investment in industry and in equilibrium credit constraints are not binding. If agriculture is not sufficiently productive, the price of the industrial good must be high enough to make the industrial sector sustainable.

In an open economy, if the country has the comparative advantage in agriculture, deindustrialization may occur over time. Deindustrialization is welfare reducing when the negative wealth distributional effects swamp the gains from trade.

CHAPTER 2: Return Policies, Market Outcomes, and Consumer Welfare

The effect of return policies on market outcomes is studied in a model where consumers differ in their valuations of time. Product quality is identified with defect rates. Producers first choose quality and then compete in prices. For given defect rates, allowing returns makes products closer substitutes, enhancing competition and reducing prices. Being closer substitutes makes higher qualities less worthwhile, which reduces quality. While the decrease in quality reduces consumer welfare, the decrease in prices raises it. The latter dominates so that aggregate consumer welfare increases with return policy.

CHAPTER 3: Skill Acquisition, Credit Constraints, and Trade

This essay analyzes the role of apprenticeships in overcoming credit constraints that limit the ability of agents to acquire skills. A general equilibrium model is used where the availability of skilled and unskilled labor is endogenously determined. Under the apprenticeship system, trainees can pay for their training by working at below market wages during their apprenticeship. Under the college system, payment must be made up front.

In the static model the response of supply to price depends on the number of skilled agents in the economy. If there are relatively many skilled agents, supply can be
decreasing in price and multiple equilibria may exist. In steady state such non-monotonicity of supply obtains only in the presence of credit constraints. Finally, opening the economy up to trade could reduce welfare if a country imports the good whose relative price has risen due to trade.
# TABLE OF CONTENTS

LIST OF FIGURES ................................................................. vi

ACKNOWLEDGEMENTS ......................................................... vii

CHAPTER 1: Immiserizing Deindustrialization: A Dynamic Trade Model with Credit Constraints ................................................................. 1
  1.1 Introduction ................................................................. 1
  1.2 The Model ................................................................ 7
  1.3 Autarky Equilibrium .................................................... 8
  1.4 The Effects of Trade ...................................................... 16
  1.5 Enriching the Occupational Structure .............................. 24
  1.6 Conclusions ................................................................. 29
  Bibliography ................................................................. 38

CHAPTER 2: Return Policies, Market Outcomes, and Consumer Welfare .... 41
  2.1 Introduction ................................................................. 41
  2.2 The Model ................................................................. 44
  2.3 Equilibrium with No Returns ......................................... 45
  2.4 Equilibrium with Returns ............................................. 47
  2.5 Consumer Welfare ....................................................... 49
  2.6 Extensions and Conclusions ......................................... 51
  Bibliography ................................................................. 53

CHAPTER 3: Skill Acquisition, Credit Constraints, and Trade .................... 55
  3.1 Introduction ................................................................. 55
  3.2 The Model ................................................................. 58
  3.3 Autarky Equilibrium .................................................... 62
  3.4 Effects of Trade .......................................................... 78
  3.5 Conclusions ................................................................. 80
  Bibliography ................................................................. 90

Appendices
  Appendix A: Proofs to Chapter 1 ....................................... 92
  Appendix B: Proofs to Chapter 2 ....................................... 100
  Appendix C: Proofs to Chapter 3 ....................................... 105
LIST OF FIGURES

Figure 1.1. Wealth Dynamics: Productive Agriculture…………………………….. 26
Figure 1.2. Wealth Dynamics: Unproductive Agriculture………………………... 27
Figure 1.3. Market for Industrial Good……………………………………………… 28
Figure 1.4. Price and Wealth Dynamics……………………………………………… 29
Figure 1.5. Welfare in Trade Equilibrium: Productive Agriculture……………… 30
Figure 16. Welfare in Trade Equilibrium: Unproductive Agriculture ……… 31
Figure 1.7. Steady State Supply of Industrial Good: Large Country Case …… 32
Figure 1.8. Returns to Different Occupations……………………………………….. 33
Figure 3.1. Output and Factor Availability………………………………………….. 82
Figure 3.2. Relative Supply: Apprenticeship System……………………………… 83
Figure 3.3. Small Number of Masters……………………………………………….. 84
Figure 3.4. Large Number of Masters……………………………………………….. 85
Figure 3.5. Relative Supply: College System………………………………………… 86
Figure 3.6. Steady State Relative Supply: Apprenticeship System……………… 87
Figure 3.7. Steady State Relative Supply: College System………………………… 88
Figure 3.8. Effects of Trade…………………………………………………………... 89
ACKNOWLEDGEMENTS

I am indebted to my advisor Kala Krishna for her invaluable advice, guidance, and encouragement. I am very grateful to members of my committee Andrew Kleit, Nezih Guner, Tomas Sjostrom, and Abdullah Yavas for helpful discussions and comments.

I also benefited from discussions with Barry Ickes, Robert Marshall, and Cemile Yavas.

Last, but not the least, I thank my husband for his love and support.
To my mother
Chapter 1

Immiserizing Deindustrialization: A Dynamic Trade Model with Credit Constraints

1.1 Introduction

It has been argued that the handloom industry in India collapsed in face of competition from British imports in colonial times. Peter Harnetty (1991, p.505) writes:

“It is clear that in the nineteenth century the [handloom] industry suffered severe decline in terms of its share in the total production and consumption of cloth. This loss of production necessarily involved a fall in income and employment for the handloom weavers, in other words deindustrialization.”

He also states that imperfect credit markets played a substantial role in this process:

“The fall in the market value of handloom goods was one reason for the decline of the industry, but the credit system under which weavers operated was an important factor, too.” (p. 479)

Even if deindustrialization did occur, the standard response of economists would be to interpret deindustrialization as a reallocation of resources across sectors with no aggregate long
term adverse consequences.

This paper argues that such an interpretation can be very misleading when credit markets are imperfect, suggesting that episodes like the deindustrialization of India in colonial times might have been quite damaging. The reason is a crucial discontinuity in welfare which has been overlooked in the literature to date. This discontinuity occurs because trade liberalization can profoundly affect the distribution of wealth, thereby making credit constraints much more or much less binding.

Moreover, it is shown that there is reason to expect a non-monotonic relation between openness to trade and gains from such trade. Relatively productive economies tend to gain from being opened up to trade. Less productive ones, on the other hand, can suffer greater poverty and easily lose from being opened up to trade. This suggests that it may be futile to look for a monotonic relationship between these variables.

That credit constraints are relevant, especially in developing countries, is clear. Tybout (1983), Jaramillo et al. (1996), Gelos and Werner (1999), Bigsten et al. (2000), Banerjee and Duflo (2002), Harrison and McMillan (2003), Love (2003) all show that the evidence is consistent with firms being credit-constrained in a host of developing countries. Moreover, Rajan and Zingales (1998) and Levine et al. (2000) among others show that the development of financial intermediaries or a decrease in credit market imperfections seems to be positively associated with economic growth. This is consistent with less credit-constrained economies tending to gain from trade as suggested in this paper.

Recent empirical work\(^1\) finds a negative correlation between GDP growth and measures of income inequality.\(^2\) An interesting case study is that of South Korea and the Philippines discussed by Benabou (1996). In the early 1960’s, South Korea and the Philippines were similar in many aspects, such as per capita GDP, population, school enrollment, etc. A significant difference was in the distribution of income. The Philippines was much more unequal than South Korea. For instance, the ratio of the income share of the top 20% to the bottom 20% was about twice as large in the Philippines. From 1960 to 1988, Korea experienced growth rate of about 6 percent per year, while for the Philippines it was only 2 percent. Benabou (1996)

\(^1\)See Benabou (1996) for a review of this literature.

\(^2\)Earlier authors had argued that in poor economies, inequality might promote growth by stimulating capital accumulation. See Aghion and Williamson (1998) for more on this.
presents and extends the main theories linking income distribution and growth to explain this puzzle.

This paper provides another explanation for this puzzle. If an economy is sufficiently unequal to start with, then wages are low as labor demand from entrepreneurs cannot keep up with the labor supply from those with too little capital to be anything but workers. In this event, opening up to trade does not allow workers to move into more lucrative occupations and will not lead to an industrial boom.

This paper constructs a dynamic model where credit markets are missing. There is an initial distribution of wealth. Agents live for one period. At the beginning of the period, an agent chooses his occupation: one of an agricultural worker or an industrial producer. In agriculture, only labor is needed and there are constant returns to scale. In industry, an indivisible investment in terms of the agricultural good must be made in order to employ each worker. Only those with inherited wealth sufficient to make this investment have this option. At the end of his life, a worker’s income is divided between his consumption and bequests.

Bequests are modeled as a “warm glow” which enter utility directly. This approach has been commonly used in the development literature as it simplifies the analytics considerably. A Cobb-Douglas utility function is used which results in a constant share of end of period income being left as bequests. However, there is reason to ask whether it is appropriate to use this shortcut.3

There is an alternative way to model the bequest motive, by assuming that agent’s utility depends not on the size of bequest but on his child’s utility, as in Barro (1974). This altruistic specification ensures that there is no discontinuity in utility generated by the lumpiness of investment. However, agents with progenitors who had similar wealth need not be similar in their inherited wealth. Their progenitors could have been indifferent between two choices: high bequests and high consumption, a point made in Mookherjee and Ray (2002, 2003). This specification is intractable, especially outside steady state, which is why we stick to the warm glow approach. We consider Barro’s altruistic specification in Appendix A, restricting attention to steady state outcomes. While Barro’s specification alters the details of the arguments, the

3After all, one might ask, if one knew that a small increase in ones bequest would lead to a large increase in an offspring’s utility, would one not choose to do so? Such a discontinuity in utility, which is generated by the lumpiness of investment, seems disturbing.
basic results are unchanged. For example, it remains true that unanticipated opening to trade can result in ‘involuntary’ deindustrialization.

First, the steady state autarky equilibrium is analyzed. It turns out that the behavior of the economy depends on how productive agriculture is relative to the investment needed in the industrial sector. If agriculture is sufficiently productive, credit market imperfections are not binding in the long run. The economy converges to a unique wealth level and consists of identical non-credit-constrained agents who are indifferent between working in one sector and the other. If agriculture is not productive enough, credit market imperfections are binding in steady state. The group of credit-constrained agents does not shrink. In steady state, there are two distinct classes of agents: ‘poor’ credit-constrained agents working in agriculture and ‘rich’ non-credit-constrained agents working in industry.

Next, an open economy is considered. For a small open economy, trade is shown to raise aggregate welfare if the price of the industrial good exceeds the autarky price so that the industrial good is exported. Basically, this is due to the standard gains from trade: agents working in industry gain as the price of their output rises, while those in agriculture lose and the gains of the former exceed those of the latter. Potentially, trade could have an adverse effect on income distribution. If this made credit constraints more binding, there could be losses from trade. However, when the industrial good is exported, this potentially adverse effect does not occur.

If the autarky price of the industrial good exceeds its world price, the industrial good is imported. The lower price can result in the following situation. Start from the autarky steady state. Since the price of the industrial good falls, the agents who can just afford the investment today can no longer make the bequests needed to ensure their offspring can work in industry. These offspring must now work in agriculture, even if industry is more profitable. However, this is not the end of the story. In the next period, those who were on the margin in second period after opening up to trade are in the same boat! This process results in ‘involuntary’ deindustrialization. This can be immunizerizing for the economy if comparative advantage in agriculture is large, but not too large. In effect, the change in income distribution makes credit constraints more binding and may reduce aggregate welfare. If comparative advantage is small, then the distribution of income is unaffected, while if it is too large, the gains from trade swamp
the income distributional effects.

The large country case is also considered. In particular, the ‘North-South’ trade in which the two countries are distinguished only by differences in credit markets is analyzed. It is shown that differences in credit markets not only create the basis for comparative advantage\(^4\) but may also lead to different short-run and long-run trade effects. In the small country case the world price of the industrial good is exogenous and cannot be affected in the long run. When trading countries are large, the price evolves over time and may take various paths before converging to its steady state value. For example, the following scenario is possible. When a non-credit-constrained country trades with a credit-constrained one, the price of the industrial good falls at first. As a result, deindustrialization occurs in the credit-constrained country and it gets locked into agriculture, so when the price rises later, the country cannot respond.

Finally, the model is augmented to allow a labor market to exist. In the basic model agricultural workers can never move to industry, only industrial workers can move to agriculture. This restricts the ability of price changes to move workers in both directions. Introduction of richer occupational structure eliminates this weakness of the basic model. For relatively high world prices and relatively equal economies, trade is shown to increase the demand for labor and drive wages up. This results in credit-constrained workers becoming unconstrained and being able to invest in more profitable occupations. The level of inequality plays an important role in determining when trade can become a real engine of growth. If economy is too unequal, opening up to trade does not allow occupational mobility and, therefore, does not lead to an industrial boom.

**Related Literature.** The paper is related to two strands of literature: the literature on credit market imperfections and distribution of wealth, and the literature on the effects of trade in the presence of market imperfections. The latter has a venerable tradition, but credit market imperfections and their impact on income distribution is not its prime focus.

There is a thriving field in development which looks at the evolution of wealth and the existence of poverty traps. See, for example, Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997), and Piketty (1997). They employ a nonconvex investment

\(^4\)See Beck (2002) for empirical evidence on a large causal impact of financial development on trade in manufactures.
technology and show how initial conditions determine long-run outcomes. In these papers, the economy displays multiple steady states, where inequality can be high or low. Inequality is persistent: ‘poor’ dynasties are unable to catch up with ‘rich’ ones.\(^5\) However, all of these papers consider a closed economy. Moreover, there is a single final good so that the determination of relative prices of final goods is not an issue. Though this paper is closest to the development literature, it analyzes a new issue: the effects of trade.

Recently, there has been some work in trade in this area. In an influential contribution, Findlay and Kierzkowski (1983) extend the standard Heckscher-Ohlin model by endogenizing the formation of human capital. Much of the work in trade discussed below builds on this paper. They show that trade amplifies initial differences in factor endowments through the Stolper-Samuelson effect: trade raises the reward of the abundant factor in each country. Therefore, trade leads to a decrease in the accumulation of human capital in skill-scarce countries and does the opposite in skill-abundant countries. However, there are no credit constraints.

Cartiglia (1997) incorporates credit constraints into a Findlay-Kierzkowski type model, but uses a static setting. He shows that trade leads to convergence in human capital endowments. A key element in his paper is that skilled labor is used as an input in the formation of skilled labor. Trade liberalization in a skill-scarce country reduces the cost of education and hence weakens credit constraints, resulting in a higher investment in human capital. This effect in fact dominates the Stolper-Samuelson effect of Findlay-Kierzkowski reversing their results.

Ranjan (2001) and Das (2003) are the only papers that use a dynamic framework and allow for credit constraints. Ranjan (2001) looks at the effect of trade liberalization on skill acquisition, the skilled-unskilled wage differential, and the distribution of wealth. He points out a third effect that operates through changes in the distribution of income which influences the accumulation of human capital. However, the aggregate effect of trade is indeterminate in his model. Another prediction is that the degree of credit market imperfections can become a determinant of the pattern of comparative advantage.

Das (2003) looks for the most part at economies with perfect credit markets. Only in a final section does he look at credit market imperfections which are modelled as a fixed differential

---

\(^5\) Another view in the development literature is that economic inequality is inevitable outcome of the market mechanism, even if all agents are identical to begin with and there is no nonconvexity. This point is developed in Ljungqvist (1993), Freeman (1996), Mookherjee and Ray (2002, 2003), and others.
between the borrowing and lending rate. He considers a two-country general equilibrium model
and shows that trade liberalization results in more human capital acquisition in the country
with perfect credit markets. The opposite happens in the country where credit markets are
imperfect. However, both Das (2003) and Ranjan (2001) assume incomplete specialization
under trade so that neither paper actually allows for the possibility of deindustrialization.

1.2 The Model

There is a continuum of agents of unit mass. Each agent lives for one period and has one child.
At the beginning of a period an agent makes occupational and investment choices. At the end
of the period the resulting income is divided between consumption and bequests. The wealth
in the economy evolves through bequests: the initial wealth of an agent in period \( t \), \( w_t \), equals
the bequest from his parent, \( b_{t-1} \). Agents are assumed to be identical with respect to their
abilities and preferences and differ only in their initial wealth endowments. The distribution
of wealth at the beginning of period \( t \) is represented by the distribution function, \( G_t(\cdot) \). Each
agent is endowed with a single unit of labor which he supplies inelastically at no disutility cost.

There are two goods in the economy. The agricultural good is treated as the numeraire. The
price of the industrial good is \( p \). Goods are produced according to the following technologies.
Each agent can use his unit of labor to produce \( n \) units of the agricultural good. Alternatively,
an agent can invest \( k \) units of the numeraire good. Once this investment is made, the agent uses
his unit of labor to produce \( a \) units of the industrial good. Note that this makes the technology
in the industrial sector nonconvex.

There is no credit market, so agents cannot borrow. At the beginning of a period an agent
chooses the sector in which to work. The occupational choice of each agent depends on his
wealth endowment and the returns in each sector. If the agent works in the agricultural sector
his wealth at the end of the period, or his disposable income, is \( Y_t = w_t + n \). If the agent works
in the industrial sector his end of period income is \( Y_t = w_t + ap_t - k \). The disposable income
\( Y_t \) is divided among consumption of the agricultural good, consumption of the industrial good
and bequests. Bequests are made in terms of the numeraire good.

Agents have identical Cobb-Douglas preferences over consumption and bequests. Hence,
optimal consumption and bequests in period $t$ are linear functions of end of period income:

$$c_{At} = \alpha Y_t, \quad p_t c_{It} = \beta Y_t, \quad b_t = \gamma Y_t,$$

where $\alpha + \beta + \gamma = 1$, $c_{At}$ is the consumption of the agricultural good in period $t$, $c_{It}$ is the consumption of the industrial good in period $t$, while $b_t$ is the bequest. The resulting indirect utility is also linear in income:

$$W_t = \frac{A}{p_t} Y_t, \quad \text{where} \quad A = \alpha^\alpha \beta^\beta \gamma^\gamma.$$

1.3 Autarky Equilibrium

1.3.1 Static Equilibrium

Given the wealth distribution at the beginning of the period, the static equilibrium yields the occupational choices of the population and the price, $p_t$, of the industrial good. Agents choose where to work depending on the returns in each sector. A choice of producing the agricultural good yields a payoff of $n$, while the return from working in industry depends on the price of the industrial good, which, in turn, is endogenously determined.

The equilibrium price, $p_t$, and, hence, the return in industry, depends on the distribution of wealth in the economy. This distribution affects both supply of and demand for the industrial good. It determines the aggregate wealth in the economy, which affects demand as well as how many agents work in industry, which, in turn, affects supply. First, we analyze how the occupational choice of each agent depends on the price and then we derive the equilibrium price at the period $t$ for a given wealth distribution $G_t(w)$.

We assume that initially there is a positive proportion of relatively rich agents, i.e., agents with initial wealth more than $k$. This assumption is made to rule out degenerate equilibria where all agents are credit-constrained and the industrial good is not produced.

Since capital markets are missing, all agents with initial wealth $w_t < k$ are unable to borrow to make the investment needed in industry. Hence, all these agents work in agriculture and
have end of period income of \( Y_t = w_t + n \) and indirect utility
\[
W_t(w_t) = \frac{A(w_t + n)}{p^\beta}.
\]

Agents with initial wealth \( w_t \geq k \) have a choice of where to work so that their indirect utility is:
\[
W_t(w_t) = \max \left\{ \frac{A(w_t + n)}{p^\beta}, \frac{A(w_t + ap_t - k)}{p_t^\beta} \right\}. \tag{1.1}
\]

Note that all agents have the same preferences over the sector of work, the only issue is whether they have inherited enough to make working in industry an option. From equation (1.1) it follows that if \( p_t \) is low enough, \( p_t \leq p \equiv \frac{k + n}{a} \), then all agents with initial wealth \( w_t \geq k \) choose to work in the agricultural sector. If \( p_t > p \) then all these agents choose to work in the industrial sector. If \( p_t = p \) then they are indifferent between sectors.

Since both goods are essential for consumers, both goods must be produced in equilibrium ensuring that \( p_t \geq p \).\(^6\) Hence, the supply of the industrial good is horizontal at \( p \) and is vertical at level \( a(1 - G_t(k)) \) above \( p \). If demand intersects supply in the vertical segment, then the equilibrium price is given by the intersection of demand and supply, while if demand intersects supply in the horizontal segment, the equilibrium price is \( p \). We will first derive the price, \( p_t' \), at which demand and this vertical segment of supply intersect. Clearly, the equilibrium price is the maximum of this level and \( p \).

Demand is given by:
\[
D_{2t} = \frac{\beta Y_t^A}{p_t}, \tag{1.2}
\]
where \( Y_t^A \), the aggregate disposable income, consists of the aggregate initial wealth and the returns from the production of both goods:
\[
Y_t^A = \int wdG_t(w) + nG_t(k) + (ap_t - k)(1 - G_t(k)).
\]

Recall that \( \int wdG_t(w) \) is the aggregate initial wealth, \( nG_t(k) \) is the aggregate return in the

\(^6\)If the price is below \( p \) then all agents in the economy work in agriculture and the industrial good is not produced. The price \( p_t \) and, as the result, the return to working in industry, will be driven by demand to infinity and agents with initial wealth \( w_t \geq k \) will switch to the industrial sector.
agricultural sector, and \((ap_t - k)(1 - G_t(k))\) is the aggregate return in the industrial sector net of investment. Let \(\theta_t = G_t(k)\) denote the measure of agents with initial wealth \(w_t < k\).

The price \(p'_t\) is equal to:

\[
p'_t = \frac{\beta}{1 - \beta} \int \frac{wdG_t(w) + n\theta_t - k(1 - \theta_t)}{a(1 - \theta_t)}.
\]

The equilibrium price in period \(t\), \(p_t\), is \(\max \{p, p'_t\}\). Since

\[
p'_t - p = \left(\frac{\beta}{1 - \beta}\right) \left[ \int \frac{wdG_t(w) + n}{a(1 - \theta_t)} \right] - \left(\frac{1}{1 - \beta}\right) \left(\frac{n + k}{a}\right)
\]

it follows that the equilibrium price in each period depends on two endogenously determined factors: the aggregate initial wealth, \(\int wdG_t(w)\), and the degree of poverty, \(\theta_t\). The aggregate initial wealth affects the demand side: the greater the aggregate initial wealth, the higher the demand for the industrial good. The degree of poverty, measured as a fraction of population with initial wealth less than investment needed in industry, affects the supply side: a smaller fraction of population working in agriculture leads to a higher supply of the industrial good.

In period \(t\), if either the aggregate initial wealth or the degree of poverty is small enough, then the equilibrium price is equal to its minimal value \(p\). Otherwise, the equilibrium price exceeds \(p\).

Lemma 1 summarizes these results.

**Lemma 1** The equilibrium price in the period \(t\) is

\[
p_t = \begin{cases} p, & \text{if } \beta \int wdG_t(w) + n\theta_t - k(1 - \theta_t) \leq k + (1 - \beta)n \\ \frac{\beta}{1 - \beta} \int \frac{wdG_t(w) + n\theta_t - k(1 - \theta_t)}{a(1 - \theta_t)}, & \text{otherwise.} \end{cases}
\]

1.3.2 Dynamic Equilibrium

Having solved for the occupational structure and the equilibrium price given a wealth distribution, we turn to the behavior of the economy in the long run. It turns out that the behavior
of the economy depends on how productive agriculture is relative to the investment needed in the industrial sector.

**Evolution of bequests.**

Since each agent leaves a share $\gamma$ of his end of period income as bequests, wealth dynamics are described by:

$$w_{t+1} = b_t = \begin{cases} 
\gamma (w_t + n), & \text{if } w_t < k; \\
\gamma (w_t + a p_t - k), & \text{if } w_t \geq k.
\end{cases}$$

(1.3)

This is shown in Figures 1.1 and 1.2 which depict bequests at every inherited wealth level. For credit-constrained agents, who work in agriculture, bequests are independent of price. Therefore, there is a single bequest line $w_{t+1} = \gamma (w_t + n)$ when $w_t < k$. For agents with initial wealth more than $k$ bequests do depend on price, therefore, there is a bequest line for every $p_t$. Recall that $p_t \geq p$. When $p_t = p$ agents earn the same income in the two sectors. As a result, the bequest line at $p_t = p$ coincides with $w_{t+1} = \gamma (w_t + n)$. When $p_t > p$ agents with inherited wealth more than $k$ earn more and hence leave higher bequests than if they had worked in agriculture. As a result, the function relating bequests to inherited wealth is discontinuous at $k$ as depicted.

The intersection of the bequest line $w_{t+1} = \gamma (w_t + n)$ with the 45° line occurs at $w = \frac{\gamma}{1 - \gamma} n$. There are two possibilities: either $w \geq k$ as in Figure 1.1, or $w < k$ as in Figure 1.2. Note that a more productive agriculture shifts the bequest line $w_{t+1} = \gamma (w_t + n)$ upwards. If agriculture is productive enough, i.e., $\gamma n \geq (1 - \gamma) k$, then even the agents with inherited wealth less than $k$ are able to earn enough, over time, to leave bequests more than $k$ and their offspring become non-credit-constrained. Thus, Figure 1.1 corresponds to having a sufficiently productive agriculture.

From Figure 1.1 it is clear that the credit-constrained class (with initial wealth $w_t < k$) must shrink in each period: an agent with inherited wealth $k$ will leave more than $k$ to his offspring who will therefore not be credit-constrained. Moreover, since all credit-constrained agents leave more than the amount they themselves inherited, the credit-constrained class is eliminated in the long run. Hence, credit constraints are not binding in the long run.

Let $\bar{w}(p)$ denote the intersection of the bequest line for the non-credit-constrained agents
at price $p$ with the 45 degree line. Note that in Figure 1.1, for all prices, agents who are not credit-constrained, and who inherit less than $\bar{w}(p)$, will leave more than what they themselves inherited, while agents who are not credit-constrained, and who inherit more than $\bar{w}(p)$, will leave less than what they themselves inherited. As a result, the wealth distribution converges to a single point, $\bar{w}(p)$.

If agriculture is not adequately productive, i.e., $\gamma n < (1 - \gamma)k$, the situation is as depicted in Figure 1.2. It is clear from Figure 1.2 that all agents with wealth less than $k$ leave less than $k$ as bequests. Hence, the credit-constrained group does not shrink over time and credit market imperfections matter for the long-run behavior of the economy.

For agents with initial wealth $w_t \geq k$, bequests depend on the equilibrium price. Let the price $p_t$ be the minimal price at which the offspring of agents with initial wealth $w_t = k$ are not credit-constrained. Solving $\gamma(k + a\hat{p} - k) = k$ gives $\hat{p} = \frac{k}{\gamma a}$.

If $p_t \geq \hat{p}$, then all agents who are not credit-constrained leave more than $k$ as bequests. Therefore, the group of non-credit-constrained agents also does not shrink. If the price is low, $p_t \in [\underline{p}, \hat{p})$, then agents with initial wealth $w_t \in [k, k + a(\hat{p} - p_t))$ leave bequests less than $k$ and agents with initial wealth $w_t \geq k + a(\hat{p} - p_t)$ leave bequests more than $k$. In this event, the group of non-credit-constrained agents shrinks. However, when it shrinks, supply of the good falls. Demand also falls, but, as shown below in Proposition 3, the supply effect dominates. As a result, prices rise. Hence, prices below $\hat{p}$ cannot be sustained in steady state.

**Price and limiting wealth distribution.**

Proposition 2 characterizes the steady state equilibrium when we have a productive agriculture, i.e. $\gamma n \geq (1 - \gamma) k$.

**Proposition 2** If $\gamma n \geq (1 - \gamma) k$, the economy converges to unique autarky wealth level

$$w^A = \frac{\gamma}{1 - \gamma} n \geq k.$$  

The equilibrium autarky price converges to $p^A = \underline{p} = \frac{n + k}{a}$. All agents are indifferent between sectors and have the same level of income and utility.
The aggregate welfare is
\[ W^A = \frac{An}{(1 - \gamma)(p^A)^\beta}. \]

**Proof.** We have already shown that the equilibrium price cannot lie below \( p \) and that the credit-constrained class is eliminated in the long run. All that remains to be shown is that the equilibrium price cannot exceed \( p \).

If the equilibrium price is \( p^A > p \) then in the steady state all agents choose to work in the industrial sector. Since both goods are essential, the agricultural good must be produced in equilibrium, hence, the price \( p^A > p \) cannot be an equilibrium price. Therefore, the equilibrium price is \( p^A = p \). At this price all agents are indifferent between sectors. The wealth dynamics for each agent follows \( w_{t+1} = \gamma(w_t + n) \). Hence the initial wealth and end of period income for each agent converge to
\[ w^A = w = \frac{\gamma}{1 - \gamma}n, \quad Y^A = \frac{w^A}{\gamma}. \]

The aggregate welfare is
\[ W^A = \frac{AY^A}{(p^A)^\beta} = \frac{An}{(1 - \gamma)(p^A)^\beta}. \]

\[ p^A = \frac{\beta}{\alpha} \left[ \frac{n\theta^A - k(1 - \theta^A)}{(1 - \theta^A)a} \right] \geq \hat{p}, \]

If the agricultural sector is productive enough, then credit constraints are not binding in the long run and the credit-constrained class is eliminated. There is perfect equality: all agents are identical in terms of their wealth endowments and, therefore, have the same potential occupational choices. This suggests a reason for greater income equality in early industrialization where productive agriculture was a precondition for takeoff.

What if the agricultural sector is not productive enough?

**Proposition 3** If \( \gamma n < (1 - \gamma)k \), the economy converges to two distinct wealth levels \( \underline{w^A} \) and \( \bar{w^A} \), where
\[ w^A = \frac{\gamma}{1 - \gamma}n < k, \quad \bar{w^A} = \frac{\gamma}{1 - \gamma}(ap^A - k) \geq k. \]

Agents with wealth level \( \underline{w^A} \) work in agriculture and agents with wealth level \( \bar{w^A} \) work in industry. Agents with higher wealth level enjoy higher utility. The equilibrium price converges to
\[ p^A = \frac{\beta}{\alpha} \left[ \frac{n\theta^A - k(1 - \theta^A)}{(1 - \theta^A)a} \right] \geq \hat{p}, \]
where $\theta^A$ is the measure of agents with wealth $w^A$ in steady state, which satisfies

$$\theta^A \geq \hat{\theta} = \frac{(1 - \gamma)(1 - \beta)k}{(1 - \gamma)(1 - \beta)k + \beta \gamma n}.$$

The aggregate welfare is

$$W^A = A(\theta^A - (1 - \theta^A)k)$$

Proof. Recall that $\theta_t = G_t(k)$ denotes the measure of agents in period $t$ with initial wealth $w_t < k$. Consider Figure 1.2. Recall that the group of credit-constrained agents does not shrink over time. Hence, $\theta_t$ does not decrease over time. In particular, the following relationship between $\theta_{t+1}$ and $\theta_t$ holds:

$$\theta_{t+1} = \begin{cases} 
\theta_t, & \text{if } p_t < \hat{p}; \\
= \theta_t, & \text{if } p_t \geq \hat{p}.
\end{cases}$$

Consider Figure 1.3 which depicts demand and supply of the industrial good. Supply of the industrial good, $S_t$, is horizontal at $p$ up to the level $a(1 - \theta)$ and vertical for prices above that. If the equilibrium price is $p < \hat{p}$, then the fraction of the population that is credit-constrained must rise. If it rises by $\Delta$, the supply of the industrial good at a given $p$ shifts in by $a \Delta$ and is depicted by $S'$. Demand also shifts in (depicted by $D'$) as the credit-constrained agents earn less than the non-credit-constrained ones. The shift in demand at a given $p$ is $\frac{\Delta \beta (ap - k - n)}{p}$. But only a part of income is spent on the industrial good, $ap - \beta (ap - k - n) > 0$, and the shift in demand is less than that of supply, so that the price rises to $p'$. This process continues till $p_t$ reaches $\hat{p}$. Note that the fraction of credit-constrained agents, $\theta_t$, also rises till it gets above $\hat{\theta}$, which is determined below.

Thus, the price in the steady state cannot be less than $\hat{p}$ and we can focus on $p \geq \hat{p}$.

It is clear from Figure 1.2 that for any given price $p \geq \hat{p}$ the initial wealth of agents working in agriculture converges to $\underline{w}$ and the initial wealth of agents working in industry converges to $\overline{w}(p) = \frac{\gamma}{1 - \gamma} (ap - k)$. Note that this depends on $p$. Hence we need to see what level of $p$ is consistent, in the steady state, with all the unconstrained agents inheriting wealth $\overline{w}$. Recall that the only effect of a change in $\overline{w}$ is a shift of demand and thereby an increase in price. The measure of agents with $\overline{w}$ in steady state, denoted by $\theta^A$, is determined by the initial distribution of wealth, so that supply is fixed.
Using Lemma 1 to solve for the equilibrium $p$, given $\bar{w}$, gives

$$p(\bar{w}) = \frac{\beta}{1-\beta} \int w dG_t(w) + n\theta^A - k(1-\theta^A)$$

$$= \frac{\beta}{1-\beta} \frac{\theta^A \bar{w} + (1-\theta^A) \bar{w} + n\theta^A - k(1-\theta^A)}{a(1-\theta^A)}.$$

The system

$$\begin{cases}
\bar{w}(p) = \gamma \frac{1-\gamma}{(1-\beta)a \bar{w}} (ap-k) \\
p(\bar{w}) = \frac{\beta}{1-\beta} \frac{\theta^A (w+n+k) - k}{a(1-\theta^A)}
\end{cases} \tag{1.4}$$

is depicted in Figure 1.4. Note that $p(\bar{w})$ line is flatter than $\bar{w}(p)$ line and $p(\bar{w})$ exceeds $\hat{p}$ at $\bar{w} = k$:

$$\text{slope}(p(\bar{w})) = \frac{\beta}{1-\beta} > \frac{1-\gamma}{\gamma a} = \text{slope}(\bar{w}(p)^{-1});$$

$$p(k) = \frac{\beta}{(1-\beta)(1-\gamma a)} \left[ \frac{\theta^A n}{(1-\theta^A)} \right] > \frac{k}{\gamma a} = \hat{p}$$

Hence, the system (1.4) is stable and converges to $(\bar{w}^A, p^A)$.

Solving $p^A \geq \hat{p}$ gives

$$\theta^A \geq \hat{\theta} = \frac{(1-\gamma)(1-\beta)k}{(1-\gamma)(1-\beta)k + \beta\gamma n}.$$  

The aggregate welfare is

$$W^A = \frac{A}{1-\gamma} \frac{\theta^A n + (1-\theta^A)(ap^A - k)}{(p^A)^\beta} = \frac{A(\theta^A n - (1-\theta^A)k)}{\alpha (p^A)^\beta}.$$  

In the case when agriculture is not productive enough, credit constraints are binding in the long run. As a result, the economy in the steady state exhibits inequality and consists of two different classes: credit-constrained agents working in agriculture and non-credit-constrained ones working in industry. Note that the autarky price of the industrial good must be high enough to guarantee the existence of the industrial sector, i.e., price must be above $\hat{p}$. This in turn implies that the proportion of credit-constrained agents is relatively high, i.e. $\theta^A \geq \hat{\theta}$, or, in other words, the supply of the industrial good is sufficiently small.
1.4 The Effects of Trade

In the previous section we have derived steady state equilibrium for a closed economy. Now we want to know how opening up the economy to trade affects occupational choices and wealth distribution and what are the associated welfare effects. First, the effects of trade are analyzed for a small country, and second, large country case is considered.

1.4.1 Small Country Case

This section analyzes effects of trade for the case when the country is small and cannot affect the world price of the industrial good, denoted by $p^W$. If the autarky price of the industrial good is higher (lower) than the world price, the industrial (agricultural) good is imported.

The welfare effects of trade differ greatly depending on whether the credit constraints are binding or not in the autarky steady state. Figures 1.5 and 1.6 help understand these welfare effects.

When agriculture is productive enough, i.e., $\gamma n > (1 - \gamma) k$, all agents have the same wealth level and credit constraints are not binding. Hence, there is nothing to stop agents (who are identical in the autarky steady state) from moving to the more profitable sector when the economy opens up. If the world price lies below $p^A$, then agriculture is more profitable than industry and all agents work in the former and are net buyers of the industrial good. Hence their welfare rises as price falls. If the world price exceeds the autarky price, then the industrial sector is more profitable and, since credit constraints are not binding, all agents will be both willing and able to work in industry. Since all agents are net sellers of the industrial good, welfare rises with its price. Consequently, trade has the same positive effect on everyone and is Pareto superior to autarky. Note that as a result, welfare is at a minimum at the autarky price as depicted in Figure 1.5.

If the agricultural sector is not productive enough, i.e., $\gamma n < (1 - \gamma) k$, then we know from Proposition 3 that the autarky price in steady state exceeds $\hat{p}$. As a result, credit constraints are binding in steady state and there are two groups of agents: credit-constrained ones with wealth endowment $w^A < k$ who work in agriculture, and non-credit-constrained agents with wealth endowment $\bar{w}^A \geq k$ who work in industry.
To understand the welfare effects of trade it is useful to look at the effect trade has on the welfare of the two groups of agents. Consider Figure 1.6, which depicts the welfare under trade for each group as well as aggregate welfare. Agents who are credit-constrained in the autarky steady state always remain credit-constrained and work in agriculture no matter what the world price is. Therefore, they are affected by the world price only via their consumption and their welfare under trade, denoted by $W^T_{cc}$, falls as price rises. Non-credit-constrained agents could, in addition, be affected through the supply side. If the world price exceeds $\hat{p}$, they remain net suppliers of the industrial good and gain from an increase in its price. Hence, their welfare, denoted by $W^T_{ncc}$, rises with price, for $p^W \geq \hat{p}$. However, if price is below $\hat{p}$, but above $p$, non-credit-constrained agents want to produce the industrial good but their bequests are not large enough for their progeny to be able to do so. Of course, if price is below $p$, they choose to work in agriculture. In either event, in steady state, the non-credit-constrained agents become credit-constrained and end up working in agriculture. This is reflected in $W^T_{ncc}$ jumping down to $W^T_{cc}$ when the world price equals $\hat{p}$. Aggregate welfare is just a convex combination of $W^T_{ncc}$ and $W^T_{cc}$ curves and, hence, lies between these two curves for prices above $\hat{p}$ and coincides with $W^T_{cc}$ for prices below $\hat{p}$, as depicted.

We can say one more thing about this aggregate welfare curve. At $p^A$ it must be increasing in price. Aggregate welfare in the trade equilibrium, as a function of the world price, is

$$W^T(p^W) = \begin{cases} W^T_{cc}(p^W), & \text{if } p^W < \hat{p}; \\ \theta A W^T_{cc}(p^W) + (1 - \theta A) W^T_{ncc}(p^W), & \text{if } p^W \geq \hat{p}. \end{cases}$$

Let $x_{cc}$ and $x_{ncc}$ denote the demand for industrial goods from the credit-constrained and unconstrained agents respectively. Due to the homotheticity of preferences, the indirect utility or welfare of an agent is linear in his income and can be written as $\varphi(p)Y$, where $\varphi(p) = \frac{A}{p^\gamma}$. Moreover, recall that disposable income, $Y$, equals earnings as well as inherited wealth, so that in the steady state $Y$ equals earnings scaled up by the factor $\frac{1}{1 - \gamma}$. Using the above and Roy’s
identity we see that

$$\frac{dW}{dp}\bigg|_{p^A} = \theta^A \frac{dW_{cc}}{dp} + (1 - \theta^A) \frac{dW_{ncc}}{dp}$$

$$= \varphi(p) \left[ - \left( \theta^A x_{cc} + (1 - \theta^A) x_{ncc} \right) + (1 - \theta^A) \frac{a}{1 - \gamma} \right]$$

$$= \varphi(p) \left[ -\bar{x} + (1 - \theta^A) a + (1 - \theta^A) \frac{\gamma a}{1 - \gamma} \right]$$

$$= \varphi(p)(1 - \theta^A) \frac{\gamma a}{1 - \gamma} > 0$$

The last equality follows from the observation that $\bar{x} = (\theta^A x_{cc} + (1 - \theta^A) x_{ncc})$ is the aggregate consumption of the industrial good, while $(1 - \theta^A)a$ is the aggregate production. In autarky, these two are equal.

In other words, when the country has a comparative advantage in the industrial good, gains of non-credit-constrained agents exceed losses of credit-constrained ones, and the economy as a whole benefits from trade. However, when the country has a comparative advantage in the agricultural good, there is a net loss in aggregate welfare for world prices close to the autarky level, even though the occupational structure is not affected. The intuition behind these results is as follows. At autarky, the economy is neither a net buyer nor a net seller of the industrial good, so these direct effects vanish. However, an increase in the price of the industrial good raises the bequests of agents working there, which in turn raises steady state level of income of non-credit-constrained agents by $\frac{\gamma a}{1 - \gamma}$. Therefore welfare is not at its minimum at the autarky price, as depicted in Figure 1.6.

More importantly, when the country has a comparative advantage in agriculture and the world price lies below $\hat{p}$, trade affects the occupational structure and deindustrialization results. Aggregate welfare curve jumps at $\hat{p}$, as depicted in Figure 1.6. If the comparative advantage in agriculture is not too large, i.e., the difference between world price of the industrial good and its autarky price is not too large, negative income distributional effects exceed the gains from trade and aggregate welfare falls relative to autarky. In this event, opening the economy up to trade results in immiserizing deindustrialization! If the comparative advantage is too large, gains from trade swamp negative income distributional effects, and the deindustrialization is not welfare reducing. As a result, the country benefits from trade.
Proposition 4 summarizes our results on the steady state equilibrium under trade.

**Proposition 4** If agriculture is productive enough, i.e., \( \gamma n \geq (1 - \gamma) k \), then the opening up to trade results in complete specialization: if the world price of the industrial good is higher (lower) than its autarky price then agricultural (industrial) sector disappears. All agents benefit from trade.

If agriculture is not productive enough, i.e., \( \gamma n < (1 - \gamma) k \), and the world price satisfies \( p^W \geq \hat{p} \), then the opening up to trade does not change the occupational structure. When the country has a comparative advantage in the industrial good, gains of unconstrained agents exceed losses of credit-constrained ones and aggregate welfare increases under trade. When the country has a comparative advantage in the agricultural good, there is a net loss if world prices are close to \( p^A \). If \( p^W < \hat{p} \), then the opening up to trade results in deindustrialization; moreover, if \( p^W \in (\underline{p}, \hat{p}) \) this deindustrialization is ‘involuntary’. Deindustrialization is immiserizing if the comparative advantage in agriculture is small.

### 1.4.2 Large Country Case

In this section we examine the ‘North-South’ trade in which the two countries are identical with respect to technologies and distinguished only by differences in credit markets. The South is a developing country with missing credit markets. The North is a developed country with perfect capital markets: agents can costlessly lend and borrow at the same interest rate. We show that not only do differences in credit markets create comparative advantage in the North for the industrial good, but that the short-run and long-run implications of trade can be very different.

**Autarky Equilibrium in the North.**

Since there are perfect credit markets and both goods must be produced, in the steady state equilibrium all agents have the same initial wealth \( w^N \) and are indifferent between occupations. Denote by \( p^N \) the price of industrial good and by \( R^N \) the gross interest rate. Lemma 5 describes the North autarky equilibrium.
Lemma 5 If agriculture is productive enough, i.e., $\gamma_n \geq (1 - \gamma) k$, then there is no borrowing/lending in the steady state equilibrium: all agents have initial wealth more than $k$. The equilibrium price is $p^N = p$.

If agriculture is not productive enough, i.e., $\gamma_n < (1 - \gamma) k$, then in the steady state equilibrium the loan market is active. The interest rate and price are

$$R^N = \max \left\{ 1, \frac{\beta}{\gamma} - \frac{n}{k} \right\}, \quad p^N = \max \left\{ p, \frac{\beta k}{\gamma a} \right\}.$$ 

The formal proof is relegated to Appendix A, but the intuition is as follows.

If the return in agriculture is high enough, then over time the wealth endowment for all agents exceeds the level of investment needed in the industrial sector. Therefore, there is no demand for loans in steady state.

If agriculture is not sufficiently productive, then agents working in industry do not have enough to invest. Hence, they need to borrow from agents working in agriculture. If the demand for the industrial good is quite low, i.e., $\beta$ is relatively small, then the level of production of the industrial good is low as well. Therefore, demand for loans which comes from the producers of the industrial good lies below supply of loans, and, as a result, the equilibrium gross interest rate equals 1. Note that at this interest rate the price at which agents are indifferent between occupations is exactly $p$. If the demand for industrial good is high enough, i.e., $\beta$ is relatively large, then at the interest rate $R^N = 1$ demand for loans exceeds its supply and, as a result, the equilibrium gross interest rate is more than 1, and the price at which agents are indifferent between sectors exceeds $p$.

Trade Equilibrium.

If the agricultural sector is productive enough relative to the investment needed in the industrial sector, i.e., $\gamma_n \geq (1 - \gamma) k$, the steady state equilibrium in the South is the same as that in the North. Therefore, the two countries have identical autarky prices and opening up the economies to trade has no effect.

The imperfections in the South’s credit markets matter only when $\gamma_n < (1 - \gamma) k$. In this case, let $\theta^S$ denote the proportion of agents working in agricultural sector in the South in the
autarky steady state equilibrium. Recall that the South’s autarky price must be high enough: \( p^S \geq \hat{p} \), which in turn implies that the proportion of agents working in agriculture must be also relatively high: \( \theta^S \geq \hat{\theta} \), where \( \hat{\theta} \) denotes the minimal proportion of credit-constrained agents compatible with autarky steady state equilibrium. Note that the price of the industrial good in the South exceeds its price in the North: \( p^N \leq \beta \frac{k}{\gamma a} < \frac{k}{\gamma a} = \hat{p} \leq p^S \). Therefore, better credit markets create comparative advantage in the North for the industrial good. Lemma 6 summarizes this result.\(^7\)

**Lemma 6** Differences in credit markets create comparative advantage in the North for the industrial good.

The trade equilibrium in this case is described in Proposition 7.

**Proposition 7** If agriculture is not productive enough, i.e., \( \gamma_n < (1 - \gamma) k \), the trade equilibrium is as follows.

I. If \( \gamma_n \geq (2\beta - \gamma) k \) then the equilibrium price is \( p^T = p \) and trade results in deindustrialization in the South.

II. If the following conditions are satisfied: \( \beta \geq \frac{1}{2} \) and \( \gamma_n < (2\beta - \gamma) k \), or, \( \beta > \frac{1}{2} \) and \( \gamma_n \leq \frac{(1 - \beta)(1 - \gamma)}{\beta} k \), then the equilibrium price satisfies \( p^T \in (p, \hat{p}) \) and trade results in ‘involuntary’ deindustrialization in the South.

III. If \( \beta > \frac{1}{2} \), \( \gamma_n > \frac{(1 - \beta)(1 - \gamma)}{\beta} k \), and \( \theta^S < 2\hat{\theta} \), then the equilibrium price exceeds \( \hat{p} \) and trade results in ‘involuntary’ deindustrialization in the South.

IV. If \( \beta > \frac{1}{2} \), \( \gamma_n > \frac{(1 - \beta)(1 - \gamma)}{\beta} k \), and \( \theta^S \geq 2\hat{\theta} \), then the equilibrium price exceeds \( \hat{p} \) and the autarky occupational structure in the South does not change.

The formal proof is relegated to Appendix A, but the intuition behind these results is as follows.

Consider Figure 1.7 which depicts the aggregate supply of the industrial good in the steady state. The aggregate supply curve consists of three segments. When the price is \( p \), the industrial good is produced only by the North. At this price the gross interest rate in the North equals

---

\(^7\)Ranjan (2001) and Das (2003) also find that the degree of credit market imperfections can become a determinant of the pattern of comparative advantage.
and all agents are indifferent between sectors. At $p$ the inherited wealth of each agent is less than $k$, hence, some agents work in agriculture and lend the needed funds to industrial workers. Therefore, the maximal supply of the industrial good is less than $a$, which is the amount produced when all agents work in industry. Thus, the aggregate supply curve is horizontal at $p$ as represented by segment $I$. When the price is above $p$ but below $\hat{p}$, the industrial good is again produced only by the North. At this price the gross interest rate exceeds unity, in fact it is such that agents are indifferent between sectors. A higher price of the industrial good not only increases returns in the industrial sector, but also raises the bequests, reducing the amount of loan needed for investment. This in turn increases the proportion of agents working in industry and the supply curve is upward sloping, as represented by segment $II$. For prices above $\hat{p}$ there are two possible cases. If the price falls below $\hat{p}$ along the convergence path to the steady state and deindustrialization occurs in the South, then even though the steady state price is above $\hat{p}$, the deindustrialization in the South is irreversible and the North is the only producer of the industrial good. At prices above $\hat{p}$ the inherited wealth for agents in the North exceeds $k$, therefore, all agents work in industry. Thus, the supply curve is vertical at level $a$, which is represented by segment $III$. In the second case, such deindustrialization does not take place, and the industrial good is produced by both countries. In this event, the aggregate supply curve is vertical at level $(2 - \theta^S)a$ and represented by segment $IV$.

Now we need to relate the type of trade equilibrium, i.e., which segment of the aggregate supply curve intersects demand in the steady state, to the parameters of the model. If the propensity to consume the industrial good is relatively low, i.e., $\beta \leq \frac{1}{2}$, then the aggregate demand for the industrial good is relatively low, and even without specializing in the industrial good, the North can make what is needed by the South. As a result, trade results in a significant fall in the price of industrial good in the South and deindustrialization occurs. In this case, aggregate demand for the industrial good intersects supply at segments $I$ or $II$. As before, welfare effects of opening up to trade for the South depend on whether negative income distributional effects exceed the gains from trade or not.

If $\beta > \frac{1}{2}$, demand from the South cannot be met at North’s autarky price. There are three possible scenarios. The first scenario involves ‘involuntary’ deindustrialization in the South with the steady state price less than $\hat{p}$ and this takes place if the agricultural sector is significantly
unproductive, i.e. \( \gamma_n \leq \frac{(1-\beta)(1-\gamma)}{\beta} k \). In this case, the low return in agriculture results in low income for credit-constrained agents in the South. Even though these agents spend a large share of their income on the industrial good, the quantity demanded is low because their disposable income is low. As a result, at prices above \( \hat{p} \) the aggregate demand is relatively low and the price falls below \( \hat{p} \). In this event, demand for the industrial good intersects supply at segment II.

The second scenario occurs when the agricultural sector is not that unproductive, i.e., \( \gamma_n > \frac{(1-\beta)(1-\gamma)}{\beta} k \), and the supply of the industrial good in the South is relatively low, i.e., \( \theta^S \geq 2\hat{\theta} \). In this case the aggregate demand for industrial good is large enough and is accommodated by the aggregate production in the North and the South at the price above \( \hat{p} \). As a result, in the steady state the South continues to produce the industrial good, and the intersection of demand and supply occurs at segment IV.

The third scenario, which takes place when \( \gamma_n > \frac{(1-\beta)(1-\gamma)}{\beta} k \) and \( \theta^S < 2\hat{\theta} \), is the most interesting. In this event the aggregate demand for the industrial good is large, but the supply in the South is also relatively large. Initially, the price falls below \( \hat{p} \) in the South and deindustrialization results. But with deindustrialization in the South the aggregate supply of the industrial good falls significantly, and, as a result, the price starts to rise until it exceeds \( \hat{p} \). But even though the price in the steady state is higher than \( \hat{p} \), the process of deindustrialization has occurred and now is irreversible. Therefore, in the steady state the industrial good is produced only by the North, and the intersection of demand and supply occurs at segment III in Figure 1.7.

This example points to the differences in short-run and long-run price effects of trade. In the small country case the world price of the industrial good is exogenous and cannot be affected in the long run. When trading countries are large, the price evolves over time, and may take various paths before converging to its steady state value. Moreover, trade can result in ’involuntary’ deindustrialization with the steady state price above \( \hat{p} \), which cannot occur in the small country case.
1.5 Enriching the Occupational Structure

The model is augmented to allow for additional occupations which create an active labor market. In the model outlined in the previous sections a world price below $\hat{p}$ forces industrial producers to move to the agricultural sector. However, agricultural workers are unable to move to industry when the world price of the industrial goods rises! This makes the effect of price changes asymmetric. A richer occupational structure eliminates this weakness of the basic model.

In addition to the two existing occupational choices: agricultural worker and industrial producer (which is now called ‘small-scale entrepreneur’) two new occupational options are introduced. A new technology that allows production of the industrial good by a ‘large-scale entrepreneur’ is posited. An agent can invest $lk$ units of the numeraire good. Once this investment is made, the agent can hire and use his unit of labor to monitor $l \geq 2$ industrial workers, where each worker produces $a$ units of the industrial good. Let $b$ denote the market wage rate. Hence, the payoff from being a large-scale entrepreneur is $(ap - k - b)l$. Therefore, this technology introduces two additional occupations: large-scale entrepreneur and industrial worker (with the return equal to the market wage rate).

Since the objective of this section is to look at the situation when opening up to trade allows credit-constrained agents to move to occupations with higher payoffs, we focus on the case where credit constraints are binding in the autarky steady state: $\gamma n < (1 - \gamma)k$.

1.5.1 Autarky Equilibrium

To derive the autarky equilibrium we analyze how the occupational choice of each agent depends on the price and then derive long-run equilibrium price and the wealth distribution.

Consider Figure 1.8 which depicts return to each occupation as a function of price. As before, all agents with initial wealth $w_t < k$ are credit-constrained but now have two choices: to work in agriculture or to become an industrial worker. What they choose depends on the wage rate. Hence, if the labor market is active, the wage rate must be equal to the return in agricultural sector: $b = n$. Therefore, horizontal line at the level $n$ represents the payoff from being agricultural or industrial worker.

The payoff from working as a small-scale (large-scale) entrepreneur is represented by a
straight line SSE (LSE). Note that SSE is flatter than LSE and intercepts the horizontal axis closer to the origin.\textsuperscript{8}

Agents with initial wealth \( w_t \in [k, lk) \) have two options: either to become small-scale entrepreneurs or to work in agriculture. The intersection of SSE and the horizontal line at the level \( n \) occurs at \( p_L \equiv \frac{k + n}{a} \). Hence, if the price is above \( p_L \) then all these agents choose to become small-scale entrepreneurs.

Agents with initial wealth \( w_t \geq lk \) have all possible options. Similarly, the intersection of LSE and SSE lines occurs at \( p_H \equiv \frac{k + n}{a} \frac{l - 1}{l} \). Therefore, if the price is above \( p_H \) then all agents with initial wealth more than \( lk \) choose to become large-scale entrepreneurs. Note that the labor market is active only when the price is above \( p_H \).

Next, we turn to the analysis of the economy in the long run.

Recall that in previous sections \( \hat{p} \) denotes the minimal price at which the offspring of non-credit-constrained agents are also unconstrained and able to invest. Introduction of two levels of investment results in three price thresholds.

Let \( \hat{p}_L = \frac{k}{\gamma a} \) (which corresponds to \( \hat{p} \) in the model from previous sections) be the the minimal price at which the offspring of small-scale entrepreneurs are able invest \( k \) and become small-scale entrepreneurs. Similarly, the price \( \hat{p}_H \) is the minimal price at which the offspring of large-scale entrepreneurs are not credit-constrained and able to become large-scale entrepreneurs. Solving \( \gamma(lk + (a\hat{p}_H - k - n)l) = lk \) gives \( \hat{p}_H = \frac{k + \gamma n}{\gamma a} > \hat{p}_L \). If the price is above \( \hat{p}_H \), then all large-scale entrepreneurs leave more than \( lk \) as bequests, and, as a result, the group of potential large-scale entrepreneurs does not shrink.

Finally, let the price \( \hat{p}_{LH} \) be the minimal price at which the offspring of small-scale entrepreneurs are able to become large-scale entrepreneurs. Solving \( \frac{\gamma(a\hat{p}_{LH} - k)}{1 - \gamma} = lk \) gives \( \hat{p}_{LH} = \frac{(1 - \gamma)l + \gamma k}{\gamma a} > \hat{p}_H \). If the price is above \( \hat{p}_{LH} \), then all small-scale entrepreneurs leave more than \( lk \) as bequests, therefore, the group of small-scale entrepreneurs shrinks and the group of large-scale ones grow.

\textsuperscript{8}Recall that SSE is given by the equation \((ap - k)\), while for LSE the equation is \((ap - k - b)l\), where \( b = n \). Therefore, SSE is flatter since \( a < al \), and its horizontal intercept is less than that for LSE: \( \frac{k}{a} < \frac{k + n}{a} \). Note that the horizontal intercept for LSE equals \( \hat{p} \).
Since both goods are essential for consumers, in equilibrium the industrial good must be produced. Hence, it is produced either by small-scale entrepreneurs, or by large-scale ones, or by both. We characterize the possible steady state equilibria according to the types of entrepreneurs producing the industrial good.

Type 1. There are only small-scale entrepreneurs in the steady state. Note that in this case the price in the steady state must be in the range \( p^A \in [\hat{p}_L, \hat{p}_H] \). If the price is below \( \hat{p}_L \) then the class of small-scale entrepreneurs shrinks over time. If the price is above \( \hat{p}_H \) then the class of large-scale entrepreneurs emerges.

Type 2. There are small-scale entrepreneurs and large-scale entrepreneurs, hence, the equilibrium price in the steady state satisfies \( p^A \in [\hat{p}_H, \hat{p}_{LH}] \). Similarly, for the prices below \( \hat{p}_H \) the class of large-scale entrepreneurs disappears in the long run and for prices above \( \hat{p}_{LH} \) the class of large-scale entrepreneurs grows over time.

Type 3. There are only large-scale entrepreneurs. In this case the equilibrium price is above \( \hat{p}_{LH} \). Small-scale entrepreneurs become large-scale ones and remain there.

The type of equilibrium that emerges in the long run depends on the relationship between \( p_H \) and \( \hat{p}_L \). Note that inequalities \( p_L < p_H < \hat{p}_H < \hat{p}_{LH} \) and \( p_L < \hat{p}_L \) are always satisfied for \( \gamma n < (1 - \gamma) k \) and \( l \geq 2 \). Moreover, from formulas above it follows that \( p_H \) is increasing with \( n \), while \( \hat{p}_L \) does not depend on \( n \). When agriculture is quite unproductive, i.e., \( \gamma n < \frac{l-1}{l} (1 - \gamma) k \), then \( p_H \) is less than \( \hat{p}_L \). When the return in agriculture increases, \( p_H \) increases as well and for medium levels of productivity in the agricultural sector, i.e., \( \gamma n \in \left[ \frac{l-1}{l} (1 - \gamma) k, (1 - \gamma) k \right] \), \( p_H \) exceeds \( \hat{p}_L \).

Proposition 8 describes steady state equilibrium in autarky.

**Proposition 8** If agriculture is unproductive, i.e., \( \gamma n < \frac{l-1}{l} (1 - \gamma) k \), then there must exist large-scale entrepreneurs in the autarky steady state equilibrium, i.e., depending on the initial wealth distribution the autarky steady state equilibrium is either of Type 2 or of Type 3.

If the agricultural sector is of medium productivity, i.e., \( \gamma n \in \left[ \frac{l-1}{l} (1 - \gamma) k, (1 - \gamma) k \right] \), then, in addition, there may exist small-scale entrepreneurs only.

**Proof.** The logic behind these results is similar to the one behind Proposition 3. First, by the same reasoning as before, the price in the steady state cannot be less than \( \hat{p}_L \). Hence, we
can focus on \( p \geq \hat{p}_L \).

If agriculture is very unproductive, then the following inequality is satisfied: \( \underline{p}_H < \hat{p}_L < \hat{p}_H \). In this case the price in the steady state must be above \( \hat{p}_H \). If \( p < \hat{p}_H \) then the fraction of the population that works as large-scale entrepreneurs\(^9\) must shrink. As a result, supply of the industrial good shifts in. Demand also shifts in as the small-scale entrepreneurs earn less than the large-scale ones, but since only part of their income is spent on the industrial good, the shift in demand is less than that of supply, so that the price rises. This will continue till \( p \) reaches \( \hat{p}_H \). Therefore, in the steady state large-scale entrepreneurs must exist. The equilibrium is of Type 2 or Type 3 depending on how high the equilibrium price is. If the price is above \( \hat{p}_{LH} \) then there are only large-scale entrepreneurs, otherwise small-scale entrepreneurs and large-scale ones coexist in the steady state.

In the case when the agricultural sector is of medium productivity, the following is satisfied: \( \hat{p}_L < \underline{p}_H < \hat{p}_H \). In this case the price in the steady state can be below \( \underline{p}_H \). At this price all agents with wealth more than \( k \) choose to work as small-scale entrepreneurs. Note also that at this price the class of small-scale entrepreneurs does not shrink and does not grow. Hence, the price \( p \in [\hat{p}_L, \underline{p}_H] \) is sustainable for some initial wealth distributions, and in this case the steady state equilibrium is of Type 1. If the price is above \( \hat{p}_H \) but below \( \hat{p}_{LH} \) then the equilibrium is of Type 2, and if it is above \( \hat{p}_{LH} \) then of Type 3.

Unproductive agriculture results in low wage rate, since \( b = n \). Therefore, low labor costs lead to large profits for large-scale entrepreneurs, and they always exist in the long-run equilibrium. If the agricultural sector is of medium productivity, then for some economies the industrial good can be produced only by small-scale entrepreneurs, since high wages and relatively low price make large-scale entrepreneurship unsustainable.

1.5.2 Effects of Trade

Having described the closed economy, we turn to the analysis of the effects of opening up to trade on labor mobility for a small country case. Let \( p^W \) be the world price of the industrial good. Note that the introduction of a labor market does not preclude the possibility of immiserizing deindustrialization. For example, if the world price is below \( \hat{p}_L \) then complete

\(^9\)Note that such class of agents potentially exists since the price is above \( \underline{p}_H \).
deindustrialization occurs independent of type of autarky equilibrium: over time all entrepreneurs leave bequests less than $k$ and their progeny work in the agricultural sector. As before, welfare effects of such deindustrialization depend on whether negative income distributional effects exceed gains from trade or not. Similarly, if the world price is $p^W \in [\hat{p}_L, \hat{p}_H)$, then in the trade equilibrium only small-size entrepreneurs survive: over time large-scale entrepreneurs leave bequests less than $lk$ and their offspring become small-scale entrepreneurs.

In the case when the autarky steady state equilibrium is of Type 2 or Type 3 and the world price is $p^W \in [\hat{p}_H, \hat{p}_{LH})$ the opening up to trade does not change the occupational structure relative to the autarky. The same occupational structure remains for world prices above $\hat{p}_{LH}$ when the autarky equilibrium is of Type 3. Note that in all these cases the opening up to trade does not move workers to occupation with higher payoffs: with trade they earn the same return $n$.

The only case when the opening up to trade results in the change of occupation for workers is as follows. First, in order to increase wages, opening up to trade must allow small-scale entrepreneurs to become large-scale ones, therefore, the autarky equilibrium has to be of Type 1 or 2, so that small-scale entrepreneurs exist in autarky. Second, the world price must be above $\hat{p}_{LH}$ to allow small-scale entrepreneurs to become large-scale ones. Third, the potential labor supply, which consists of credit-constrained agents who are not able to invest into the production of the industrial good, cannot satisfy the demand for labor, which increases with trade.

**Proposition 9** If the initial wealth distribution is such that there exist small-scale entrepreneurs in the autarky equilibrium and the proportion of agents working in the steady state either as industrial or agricultural workers is relatively small, i.e., $\theta < \frac{l}{l+1}$, then if the world price of the industrial good is high enough, i.e., $p^W \geq \hat{p}_{LH}$, opening up to trade leads to perfect equality: all agents in the economy work as small-scale entrepreneurs.

The formal proof is relegated to Appendix A, but the intuition is as follows.

When the world price of the industrial good is high enough, i.e., $p^W \geq \hat{p}_{LH}$, the income of small-scale entrepreneurs increases and over time they are able to leave more than $lk$ to their offspring. Therefore, with trade the class of large-scale entrepreneurs grows and, as a result, the
demand for labor increases. If the proportion of credit-constrained agents is relatively small, then the increased labor demand exceeds labor supply at the current wage equal to \( n \). Wage rate starts to increase, until it rises above \( \frac{1 - \gamma}{\gamma} k \). At this wage industrial workers leave more than \( k \) as bequests, and their offspring are able to become small-scale entrepreneurs, and since the price is high enough, they will eventually be able to invest \( lk \) and become large-scale entrepreneurs. Therefore, the opening up to trade makes industrial workers non-credit-constrained. The wage rate which makes agents indifferent between being worker and large-scale entrepreneur is very high and makes large-scale entrepreneurship unprofitable, and, as a result, all agents become small-scale entrepreneurs.

This case points to the role of inequality in determining when trade can become a real engine of growth. If economy is too unequal, i.e., the proportion of ‘poor’ workers is relatively large, opening up to trade does not allow occupational mobility and, therefore, does not lead to an industrial boom. This may help explain the dissimilarities in the economic performances of Korea and the Philippines.

### 1.6 Conclusions

This paper constructs a simple model where trade liberalization may have adverse wealth distributional effects when credit markets are imperfect. If the world price of the industrial good is below the bequest-sustaining level, opening up to trade results in deindustrialization. Moreover, we show that deindustrialization is welfare reducing if the comparative advantage is small, so that the negative wealth distributional effects swamp the gains from trade.

We conclude with the following observation. The model developed here is a deterministic one, and, therefore, does not allow limiting wealth distributions with continuous support. The main reason for doing this is analytical tractability. If we allow for some kind of uncertainty, then we need to study the interdependent system of stochastic wealth dynamics and endogenous price. In future work we hope to examine this issue.
Figure 1.1. Wealth Dynamics: Productive Agriculture
Figure 1.2. Wealth Dynamics: Unproductive Agriculture
Figure 1.3. Market for Industrial Good
Figure 1.4. Price and Wealth Dynamics
Figure 1.5. Welfare in Trade Equilibrium: Productive Agriculture
Figure 1.6. Welfare in Trade Equilibrium: Unproductive Agriculture
Figure 1.7. Steady State Supply of Industrial Good: Large Country Case
Figure 1.8. Returns to Different Occupations


Chapter 2

Return Policies, Market Outcomes, and Consumer Welfare

2.1 Introduction

Different laws relating to warranties in consumer sales have the same objective—to establish a minimum level of consumer protection from non-conforming goods. In the US the Magnuson-Moss act of 1978 requires that all products with a price over fifteen dollars must have a written warranty. In 1999 European Union approved the Directive 1999/44/EC “on certain aspects of the sale of consumer goods and associated guarantees” to harmonize warranties in the Member States from January 1, 2002 onwards. Prior to the implementation of the Directive, the existing laws concerning consumer protection were somewhat disparate and failed to provide adequate remedies to the consumer against non-conforming goods. The Directive requires that “the seller shall be held liable where the lack of conformity becomes apparent within two years as from delivery of the goods”.¹

This paper examines the role of return policies as warranties in a model of price competition where consumers differ in their valuations of time and products differ in terms of defect rates. We show that return polices tend to reduce quality and prices but to increase aggregate consumer welfare. In other words, the overall effect of return policies on consumers is positive.

Our work is related to two strands of literature: the analysis of warranties and models of price competition with differentiated products. The literature on warranties has dealt mostly with effects of warranties on monopoly pricing. The focus is on the signalling role of warranties which are usually modelled as monetary compensation, i.e., money-back guarantee. Spence (1977), Grossman (1981), and Lutz (1989) have all studied how warranties can serve as a signal of product quality. Shieh (1996) finds that money-back guarantee and price together completely reveal monopolist’s private information about product quality.

A second direction is the use of warranties to sort consumers in an adverse selection framework. Matthews and Moore (1987) consider a monopoly design problem in which each contract specifies quality, price, and warranty. They show that the optimal allocations need not be monotonic: contracts intended for higher types of consumers (who value the good more) have higher prices and yield higher profits to the monopoly, but may not have higher qualities and better warranties. Braverman et al. (1983) analyze the price discrimination issue and demonstrate that a monopoly can bundle a warranty with a quality level to achieve higher profits.

A third direction is the use of warranties to alleviate moral hazard problems. Dybvig and Lutz (1993) analyze the role of warranty choice in the two-sided moral hazard problem. The two-sided moral hazard arises from the assumption that both the producer and the consumer take actions that affect the failure rate of the product: the producer can lower the cost by producing a less durable product and the consumer can increase failures by neglecting to maintain the product. The optimal warranty in the solution trades off these two moral hazard problems.\(^2\)

Finally, warranties modelled as return policies have implication for consumer learning of the experience goods. Che (1996) analyzes a model in which consumers realize idiosyncratic valuations of the good after the purchase. Return policies allow consumers to defer purchasing decisions until they gain some experience with goods. Monopoly seller adopts the return policy if consumers are highly risk-averse or retail costs are high.

All these papers analyze the effects of warranties in monopoly setting. Our paper diverges from the discussed literature and looks at the new issue: the competitive effects of warranties.

This paper is also related to the literature on markets with differentiated products. Early

---

\(^2\)See also Cooper and Ross (1985), Emons (1988), and Mann and Wissink (1989) on warranties under the two-sided moral hazard.
work in this area, see Lancaster (1966), relied on models of horizontal differentiation. In such models firms produce different products, but these cannot be ranked according to their quality because some consumers value one product more than another while others have the opposite ranking. In a series of papers Gabszewicz and Thisse (1979, 1980) and Shaked and Sutton (1982, 1983) developed a model of vertical product differentiation. In this setting all consumers agree on the ranking of products, though they may choose to buy different products since they differ in their marginal valuation of quality and, hence, have different trade-offs with price.

Our approach is similar in that the products with a lower defect rate are of a higher quality than ones with a higher defect rate. Consumers differ in their valuation of time and it affects their trade-off between price and quality. In the standard vertical product differentiation model consumers of type $\theta$ derive utility $\theta q$ from a product of quality $q$ and derive surplus of $\theta q - p(q)$ from its consumption, where $p(q)$ is the price of the product of quality $q$. Hence, $q$ is just some characteristic of the quality that all consumers agree is valuable and $\theta$ captures the differences in how consumers value increments in quality. Our model can be thought of as providing a vertical structure with an alternative explanation. Quality is a lack of defects and differences among consumers relate to their valuation of time.

There exist various ways to protect consumers from defective products: money-back warranties, replacement warranties (or return policies), price reductions, and subsequent repair. In our paper warranties can be interpreted either as return policies or subsequent repairs: consumers are guaranteed an exchange (or a repair) of a defective (non-working) product for a defect-free one. We believe that this particular form of warranty is consistent with the actual consumer protection laws. For example, the Directive 1999/44/EC states that “the consumer shall be entitled to have the goods brought into conformity free of charge by repair or replacement” and if these options fail “the consumer may require an appropriate reduction of the price or have the contract rescinded”.

We analyze how return policies affect the competition between producers compared to the benchmark case where no returns are allowed. In the first stage producers choose defect rates and in the second stage they compete in prices. We show that in the equilibrium with no returns allowed, one producer chooses the lowest possible level of quality while the other chooses a higher one. Returns reduce quality and prices but increase aggregate consumer welfare relative to the
case when returns are not allowed. In effect, returns make consumers more homogeneous, so the demand becomes more responsive to the price changes and, as a result, producers behave more competitively.

2.2 The Model

The good has two characteristics: the value $v$ and the quality, or the probability of the product being defect-free, given by $1 - c$. The lower is $c$ the higher is the quality of the good.

There is a continuum of heterogeneous consumers. If $c = 0$, then all consumers derive the same value $v$ from the consumption of this good. If the good is defective ($c > 0$) the consumer needs to make a new trip to the producer to exchange the good if there are returns, or to buy another unit if there are no returns. Consumers differ in how much they value their time and are distributed uniformly on the unit interval. Their type $t \in [0, 1]$ is interpreted as the disutility of making a trip to the producer to buy or return a good. Each consumer demands at most one unit of the good. All consumers are risk-neutral.

There are two producers in the market. Each producer has zero marginal costs of production. There are positive fixed costs of production given by the common to both producers function $G(c) > 0$, where $c$ is the defect rate. It is assumed that $G(c)$ is a decreasing, convex function: $G'(c) < 0$, $G''(c) > 0$. These fixed costs may be interpreted as an investment cost the producer incurs in order to produce the good of the certain quality. A smaller investment is needed for a less reliable production process.

The game has two stages. At the first stage producers simultaneously choose the characteristics of the good they plan to produce. We assume that the level of value $v > 0$ is exogenously given and is the same for both producers, so they, in fact, choose only the defect rates $(c_1, c_2)$. All the characteristics of the good are observable to the consumers. At the second stage producers simultaneously choose prices $(p_1, p_2)$. Given prices and return policy described below consumers decide from which producer (if any) to buy. The solution concept is subgame perfect equilibrium.

The problem, despite its seeming simplicity, becomes intractable unless limits on consumer heterogeneity are imposed.
Assumption 1. \( c \leq c_{\text{max}} < 1, \quad v > \frac{1}{1 - c_{\text{max}}} \).

This assumption guarantees that both in the case of no returns and the case when returns are allowed the whole market is served and both producers are active\(^3\).

We model return policy in the following way. If the good bought by the consumer turns out to be defective, it can be exchanged for a defect-free one by returning it to the seller free of charge. We assume that after the replacement consumer gets a working good for sure. Thus, the consumer needs to return to the producer at most once and experiences disutility of return also at most once.

The asymmetry between the first visit of the consumer to buy the product and the second visit to return the defective product is consistent with real life observations. Consumers are not able to check the quality of the good the first time because they buy it from the sales counter. When, however, consumers return defective goods they deal with the customer service which are able to check the quality of each product. Return policy modelled this way can also be interpreted as a subsequent repair of a defective product. In this case each producer has access to a zero-cost repair technology. If the good bought by the consumer is defective, then this technology allows producer to repair the good and the consumer is guaranteed a working good after that.

The assumption that a defect-free good is guaranteed on consumer’s second visit to the seller is not crucial and can be relaxed. Results are unchanged if the consumer needs any finite number of returns to get a defect-free good. If, however, the quality is not guaranteed in finite number of returns then the case of returns is essentially the same as the case of no returns: producers choose the same defect rates, realize the same profits, and consumers have the same expected utility.

2.3 Equilibrium with no Returns

This section analyzes the case when there is no return policy for the defective good. We restrict our analysis to a static environment. The producer cannot discriminate among consumers and

\(^3\)These restrictions are similar to the restrictions placed in the vertical differentiation model, see for example Assumptions 1 and 2 in Tirole (1988), p.296.
charges the single price. The returning consumers pay the same price as the consumers who buy the product for the first time. In the case of no returns the consumer pays the price $p$ and incurs disutility $t$ every time she goes to the seller.

Suppose that the consumer follows the strategy of buying a new good every time she gets a defective one. With probability $1 - c$ the good is defect-free, so after incurring the total cost $p + t$ (monetary price plus the cost of time), the consumer gets expected utility of $(1 - c)v - (p + t)$. With probability $c$ the good is defective, so the consumer returns to the store, incurs the cost $p + t$ again, and gets the good that works with probability $1 - c$. With probability $c$ that good is also defective, so the consumer again incurs cost $p + t$. And the process continues in a similar fashion. Hence, the expected utility of consumer $t$ from a good $(v, c)$ is:

$$u = (1 - c)v - (p + t) + c((1 - c)v - (p + t)) + c^2((1 - c)v - (p + t)) + ...$$

$$= (1 - c)(1 + c + c^2 + ...) v - (1 + c + c^2 + ...) (p + t)$$

$$= v - \frac{p + t}{1 - c}.$$ 

If this expected utility is negative, then the consumer decides not to buy the good whether it is her first try or a repeated one. Any other strategy (for example, to buy only fixed number of times) is suboptimal because the consumer faces the same problem every time she decides whether to buy or not.

In the case of no returns the expected full price (total monetary price plus disutility of making trips to the producer) is $\frac{p + t}{1 - c}$. Note the difference between the posted price $p$, the total monetary price $\frac{p}{1 - c}$, and the full price $\frac{p + t}{1 - c}$. With no returns, the full price exceeds the posted one both because the good has to be paid for more than once and because of the time cost. With returns, the total monetary price is equal to the posted one and the full price exceeds the posted one only because of the time cost.

Lemma 10 describes equilibrium in the case when returns are not allowed.

**Lemma 10** When returns are not allowed there exist the unique (up to relabeling producers) equilibrium. One producer chooses $(p_1^{NR}, c_1^{NR})$ and the other producer chooses $(p_2^{NR}, c_2^{NR})$. 

46
Prices and quality levels are:

\[ p_{NR1} = \frac{1}{3} \left( \frac{c_{\text{max}}}{1 - c_{NR}} \right), \quad p_{NR2} = \frac{2}{3} \left( \frac{c_{\text{max}}}{1 - c_{\text{max}}} - c_{NR} \right), \]

\[ c_{NR1} = c_{\text{max}}, \quad c_{NR2} = c_{NR}, \]

where \( c_{NR} \) is determined from \( G'(c_{NR}) = -\frac{4}{9(1 - c_{NR})^2} \).

The producer who chooses \((p_{NR1}^{NR}, c_{NR1})\) serves consumers in \([0, 1/3)\) and the producer who chooses \((p_{NR2}^{NR}, c_{NR2})\) serves consumers in \((1/3, 1]\).

The proof is relegated to the Appendix B.

In equilibrium producers choose distinct defect rates and earn positive profits. The producer who chooses a lower defect rate (higher quality) charges a higher price in equilibrium and earns higher variable profits. These results are similar to Shaked and Sutton (1982).

2.4 Equilibrium with Returns

Under return policy the consumer can exchange the defective good for a defect-free one by returning it to the producer. The consumer pays the price \( p \) only once, when she buys the good.

And the consumer incurs disutility cost \( t \) at most twice: the first time when she buys the good, and the second time with probability \( c \) when she exchanges the defective good for a working one. The consumer realizes value \( v \) with probability \( 1 - c \) in the first period when she buys a working good and with probability \( c \) in the second period when she exchanges the defective good for a working one.

Then the expected utility of consumer \( t \) from a good \((v, c)\) is:

\[ u = (1 - c) v - (p + t) + c (v - t) = v - p - (1 + c)t. \]

If this expected utility is negative the consumer decides not to buy the good.

Lemma 11 describes equilibrium in the case when returns are allowed.

**Lemma 11** When returns are allowed there exist the unique (up to relabeling producers) equilibrium. One producer chooses \((p_{1R}^R, c_{1R}^R)\) and the other producer chooses \((p_{2R}^R, c_{2R}^R)\). Prices and
quality levels are:

\[
p_1^R = \frac{1}{3}(c_{\text{max}} - c^R), \quad p_2^R = \frac{2}{3}(c_{\text{max}} - c^R),
\]

\[
c_1^R = c_{\text{max}}, \quad c_2^R = c^R,
\]

where \(c^R\) is determined from \(G'(c^R) = -\frac{4}{9}\).

The producer who chooses \((p_1^R, c_1^R)\) serves consumers in \([0, 1/3)\) and producer who chooses \((p_2^R, c_2^R)\) serves consumers in \((1/3, 1]\).

The proof is relegated to the Appendix B.

Lemma 1 and Lemma 2 imply that the introduction of return policy leads to a decrease in the quality. The producer with a higher defect rate chooses the same level of defects (the maximal possible) as in the case of no returns, but the producer with a lower defect rate chooses a higher level of defects compared to the benchmark case: \(c_2^R > c_{NR}^R\). Since consumers pay for the product only once the total monetary prices coincide with the posted ones and are equal to \(p_1^R = \frac{1}{3}(c_{\text{max}} - c^R)\) and \(p_2^R = 2p_1^R\). In the case of no returns posted prices are \(p_1^{NR} = \frac{1}{3}(c_{\text{max}} - c_{NR}^R)\) and \(p_2^{NR} = \frac{2}{3}(c_{\text{max}} - c_{NR}^R)\). Since \(c_2^R > c_{NR}^R\) the posted prices, as well as the total monetary prices, decrease with the introduction of return policy.

The intuition behind these results is as follows. For fixed defect rates and fixed prices the introduction of the return policy makes the producer with a lower defect rate worse off and the producer with a higher defect rate better off. This is apparent from noting that with returns, consumers are less concerned about the quality of the good per se, and, as a result, some consumers switch to the producer with a lower price (higher defect rate). It means that the production of a high quality product now yields lower revenue compared to the cost to be paid for this level of quality. Therefore, the producer with a higher price (lower defect rate) has an incentive to move closer to the other producer to recapture the market share lost. Thus, the return policy reduces the quality of products in the market and draws the products closer together in terms of defect rates; the goods are now closer substitutes. It enhances price competition and leads to lower posted prices, as well as total monetary prices, compared to the

\[\text{This follows from convexity of the cost function } G(c).\]
benchmark case of no returns.

This result may seem counterintuitive at the first sight. One might reason that allowing returns would make producers charge the total monetary prices up front, whereas without returns they would not. As a result, one would expect the posted prices with returns to exceed those without returns. This intuition is correct when one deals with a monopoly\(^5\). For oligopoly, however, the opposite is true! It is the strategic effect that drives our result.

Under our specification of the return policy consumers return to the store only once for the defect-free good. Consider the situation when producer exchanges defective good for another good, not necessarily defect-free one. The disutility of return is experienced every time a new trip to the producer is made. Thus, the expected utility of consumer \(t\) from good \((v, c)\) is \(^6\):

\[
u = v - p - \frac{1}{1 - c} t.
\]

One can show that in such case the strategic effect is no longer present and returns do not affect quality. In equilibrium producers choose the same defect rates as in the benchmark case of no returns. Posted prices are just scaled up by the factor \(\frac{1}{1 - c}\). As a result, total monetary prices, as well as full prices, do not change compared to the case when returns are not allowed. Such policy does not affect consumer welfare: each consumer has the same expected utility as in the benchmark case.

The above example demonstrates that for return policies to work it is not sufficient to guarantee that the defective good is replaced. It must be replaced by a defect-free one in a finite number of returns.

### 2.5 Consumer Welfare

Return policy has two different effects on consumers. The decrease in the total monetary prices has a positive effect: consumers now pay less in expected terms. But the decrease in the quality

\(^5\)When the monopolist does not cover the whole market the total monetary price is \(\frac{v}{2}\) both with and without returns, which is the monopoly price of zero-defect good. So, the return policy results in a higher posted price compared to the case of no returns.

\(^6\)Derivation of utility function is similar to the case of no returns; the difference is that now the consumer pays price \(p\) only once.
may have a negative effect; if the decrease is substantial then consumers have to travel more. It turns out that the overall effect of the return policy is positive – aggregate consumer welfare always increases with returns.

Consumers in the interval \([0, \frac{1}{3}]\) always benefit from the return policy. They consume the good of the same quality as in the case of no returns (with defect rate \(c_{\text{max}}\)), but they pay lower total monetary price and experience lower time costs.

The effect on consumers in the interval \((\frac{1}{3}, 1]\) depends on the full price. With the return policy these consumers pay a lower price but consume the good of a lower quality. Utility of the consumer \(t\) in the benchmark case of no returns is \(u^{NR} = v - \frac{p_{2}^{NR} + t}{1 - c^{NR}}\) and in the returns case is \(u^{R} = v - p_{2}^{R} - (1 + c^{R})t\). If \(1 + c^{R} \leq \frac{1}{1 - c^{NR}}\) then all consumers in the interval \((\frac{1}{3}, 1]\) enjoy lower time costs and, as a result, benefit from the returns. If \(1 + c^{R} > \frac{1}{1 - c^{NR}}\) then the most impatient consumers experience lower expected utility in the case of returns, because an increase in the time cost exceeds the benefit resulting from the lower price. The aggregate welfare change for consumers in the interval \((\frac{1}{3}, 1]\) is equal to

\[
\Delta W_{(\frac{1}{3}, 1]} = W_{(\frac{1}{3}, 1]}^{R} - W_{(\frac{1}{3}, 1]}^{NR} =
\]

\[
= \int_{\frac{1}{3}}^{1} (v - p_{2}^{R} - (1 + c^{R})) \, dt - \int_{\frac{1}{3}}^{1} \left( v - \frac{p_{2}^{NR} + t}{1 - c^{NR}} \right) \, dt
\]

\[
= \int_{\frac{1}{3}}^{1} \left( \frac{p_{2}^{NR}}{1 - c^{NR}} - p_{2}^{R} \right) \, dt + \int_{\frac{1}{3}}^{1} \left( \frac{1}{1 - c^{NR}} - (1 + c^{R}) \right) \, dt,
\]

where the first term is the gain in aggregate welfare due to lower total monetary prices and the second term is the loss in aggregate welfare due to increase in travel costs. Straightforward calculations show that

\[
\Delta W_{(\frac{1}{3}, 1]} = \frac{4}{9} \left( \frac{c_{\text{max}}}{1 - c_{\text{max}}} \right)^{2} > 0.
\]

Hence, the positive price effect dominates the negative quality effect. The total effect on aggregate consumer welfare is positive – the expected utility of all consumers in the interval

\[\text{It means that the difference between } c^{NR} \text{ and } c^{R} \text{ is not substantial and consumers travel less with return policy.}\]
increases with returns. Proposition 1 summarizes these findings.

**Proposition 12** *Introduction of return policy increases aggregate consumer welfare.*

This result shows that the introduction of return policy benefits at least some consumers. With return policy some consumers may lose, but the total consumer welfare increases. Hence, laws, targeted to protect consumers in aggregate by imposing obligations on producers for non-conforming products, achieve their goals.

### 2.6 Extensions and Conclusions

This paper examines the role of return policies as warranties in a model of price competition when products differ in terms of defect rates. The introduction of the return policy has two effects. First, it leads to a decrease in the market quality — the producer with the lower defect rate chooses a higher level of defects compared to the benchmark case. Second, the return policy has a competitive effect causing both producers to decrease their prices. The overall effect on the consumers is positive: the aggregate expected utility of consumers increases with the introduction of return policy.

We conclude with the discussion of the extension of the model to the case of endogenous return policy decision. Consider the situation when there is no law that requires return policy for defective goods, but producers are free to introduce return policy on their own. Formally it means that there is a preliminary stage of the game in which producers simultaneously decide whether to adopt return policy or not. To every pair of decisions there corresponds a proper subgame; four in total, because there are two producers and each has two choices available. Every subgame possesses two equilibria. Equilibria when both producers either adopt or do not adopt return policies coincide with those described in Sections 3 and 4. One can show that the two asymmetric subgames (one producer adopts return policy and the other does not) also possess two equilibria each. Those equilibria, however, are not just the permutations of the identities of producers, but have different prices and defect rates. Thus, to find equilibria of extended game it is necessary to compare payoffs for \(4 \times 4 = 16\) situations. However, not all comparisons can be made unless the exact form of cost function \(G(c)\) is specified. Moreover, different specifications may lead to different solutions.
It means that the extended game is likely to possess multiple equilibria. Almost all these equilibria are suboptimal from consumers’ point of view. For example, as shown in Lemma 23 in the Appendix B, the outcome involving at least one of the producers not adopting return policy is an equilibrium. But this equilibrium is welfare dominated for consumers by the situation when both producers allow for returns. It indicates that the introduction of consumer protection laws is indeed desirable; it leads to the increase in aggregate consumer welfare, and cannot be left on the discretion of producers.
Bibliography


Chapter 3

Skill Acquisition, Credit
Constraints, and Trade

(with Kala Krishna)

3.1 Introduction

Different countries at different times have had various forms of transferring skills across generations. A form that has been ubiquitously present is that of apprenticeships. The system has two functions: first to provide hands-on training which is essential for many specific occupations. Such training need not always be called by this name, for example, the internships required of physicians have a significant on the job training component and hence are, at least partly, apprenticeships. The second function is to provide certification of skills.

There is a fairly large literature that models such contracts. It deals with issues such as which labor market imperfections would make firms pay for general training that is transferable across firms, the features of such contracts and their rationale, the inefficiency of training levels provided by firms, as well as the success of such apprenticeship programs in providing a skilled labor force. For example, Chang and Wang (1996), Acemoglu and Pischke (1998), and Malcomson et al. (2003) analyze the effects of asymmetric information between training firms and other potential employers. Acemoglu (1997), Booth and Chatterji (1998) consider the implications of imperfect competition in the skilled labor market. All these papers imply a
worker’s marginal productivity increasing with training by more than the wage the firms pays. This enables the firm to capture some returns to general training and, as a result, the firm finds it profitable to invest in worker training. For a review of this research, see Acemoglu and Pischke (1999) and Smits and Stromback (2001). Such issues are not the subject of this paper. We focus on another, hitherto unstudied aspect of apprenticeships, namely their ability to help circumvent credit constraints.

What is an apprenticeship? Those with the skills to impart (masters) enter into a contract with the unskilled (apprentices) to “teach as best they know” their technical skills. In return, the apprentice undertakes the tasks assigned to him by the master for the (specified) period of his apprenticeship. He is paid below market wages during this period, receiving payment in the form of training instead.\(^1\) Contrast this with the alternative of “going to college” where the training fees have to be paid up front. In the absence of credit markets, which we postulate, only those with the wealth to pay the up front fee could afford the college option. Note however, that even if part of the fee is paid up front, as occurs when the apprentice’s wage is negative, the less well off may be able to afford the apprenticeship route.

We ask, what do apprenticeship forms of training do? Do they make for more equality by loosening credit constraints on the poor? Is there an efficiency effect as well since some of the credit constrained poor may be more talented than some of the rich? How do the college and apprenticeships systems compare in the short run, i.e., in a static setting and in the long run, i.e., in steady state? What does trade do in this setting? Must it always be beneficial or not?

We develop a model where apprenticeships help overcome credit constraints that limit the ability of agents with heterogeneous abilities and wealth to acquire skills. We use a general equilibrium model where the availability of skilled and unskilled labor is endogenously determined. We show that in the static version of our model, under either system, the response of supply to price depends on the number of skilled agents in the economy. If there are relatively few skilled agents, the normal supply response obtains. However, with many skilled agents, supply can be decreasing in price so that multiple equilibria may exist. In steady state, however, such

---

\(^1\)Lane (1996) shows that in late 18th century, apprentices earned 41% of the journeyman (skilled) rate while unskilled workers earned 77%. In some cases, apprentices have even paid for the privilege of learning the trade. In fact, by the 18th century, an up front fee had become the norm. While there was considerable variation in the terms specified between the country and the city as well as across occupations, there were instances of large sums, hundreds of pounds, being paid up front when the trade was particularly well rewarded.
non monotonicity of supply and multiplicity of equilibria obtains only in the presence of credit
constraints. Since credit constraints are stricter in the college system, relative supply of the
skill-intensive good is always higher, at any given price, under the apprenticeship system. There
may or may not be multiple equilibria in steady state: a key determinant is the distribution of
wealth. Finally, we show that opening the economy to trade could easily reduce welfare. Trade
could result in a country importing the good whose relative price has risen due to trade.

Our work is also related to the trade literature on endogenous skill formation and trade.
In an influential contribution Findlay and Kierzkowski (1983) extend the standard Heckscher-
Ohlin model by endogenizing the formation of human capital. They show that trade amplifies
initial differences in factor endowments through the Stolper-Samuelson effect: trade raises the
reward of the abundant factor in each country. Therefore, trade leads to a decrease in the
accumulation of human capital in skill-scarce countries and does the opposite in skill-abundant
countries. However, there are no credit constraints.

Cartiglia (1997) incorporates credit constraints into a Findlay-Kierzkowski type model, but
uses a static setting. He shows that trade leads to convergence in human capital endowments.
A key element in his paper is that skilled labor is used as an input in the formation of skilled
labor. Trade liberalization in a skill-scarce country reduces the cost of education and hence
weakens credit constraints, resulting in a higher investment in human capital. This effect in
fact dominates the Stolper-Samuelson effect of Findlay-Kierzkowski reversing their results.

Ranjan (2001) uses a dynamic framework and looks at the effect of trade liberalization on
skill acquisition, the skilled-unskilled wage differential, and the distribution of wealth. He points
out a third effect that operates through changes in the distribution of income which influences
the accumulation of human capital. However, the aggregate effect of trade is indeterminate in
his model.

All of the above papers do not analyze static and dynamic effects of skill formation. More-
over, they do not model the training sector explicitly and do not compare alternative training
arrangements as we do.

The rest of the paper proceeds as follows. Section 2 lays out the model. Section 3 analyzes
the equilibrium in the single period under each system. Section 4 looks at steady state equilibria
and how they differ from the static equilibria in a closed economy. Section 5 studies how trade
affects outcomes. Section 6 provides concluding remarks and directions for future research.

3.2 The Model

There are two goods, $X$ and $Z$, and one basic factor, unskilled labor, $U$, in the economy. Unskilled labor can be transformed into its skilled counterpart, $S$. However, if $K$ unskilled workers are taken on by a skilled worker then only $G(K)$ units of the skilled worker’s time remains available to him, where $G(K)$ is a decreasing function of $K$. We assume that

$$G(K) = 1 - AK$$

so that $A$ is the time required per trainee.

While good $X$, which will also be termed the agricultural good, uses only unskilled labor, good $Z$, also called the industrial good, uses both skilled and unskilled labor.\(^2\) We normalize units so that the unit labor requirement of unskilled labor in making the agricultural good is unity and take it to be the numeraire. Hence, $p$ denotes the relative price of good $Z$. The production function for the industrial good is

$$Z = S^\alpha U^{1-\alpha}.$$ 

We use an overlapping generations framework. There are $L$ agents born in each period. Each agent lives for two periods and is endowed with one unit of time in each period. An agent is characterized by two parameters: the probability of becoming skilled, or a master, upon undertaking the needed education, $\gamma$, and his initial wealth, $y$. It is assumed that $\gamma$ is distributed uniformly in the unit interval ($\gamma \sim U[0,1]$) and $y$ is distributed according to distribution function $F(\cdot)$ in $[0,y_{\max}]$, where $y_{\max}$ is the maximal wealth level.

In the first period of life an agent makes career choices. He could become an unskilled worker, work in both periods at the unskilled wage, $w$. Alternatively, he could spend part of his time acquiring the skills that give him a chance at becoming a master and allowing him, if

\(^2\)We make this assumption for ease in calculations. However, it will be apparent that all that is needed is that production of $X$ and $Z$ have different relative intensities, so that $Z$ is relatively more skill-intensive than good $X$ at all factor prices.
he so chooses and is successful in his training, to earn a master’s wages in the second period. Agents who try to become a master but fail can work only as unskilled workers in the second period. Masters could also choose to work as unskilled workers were it in their interests to do so.

We study two training systems. In the first, which we call the apprenticeship system, each master hires $K$ apprentices. An apprentice supplies $\beta^A$ hours of his time to the master at a wage $w^A$ and spends $1 - \beta^A$ of his time studying.\(^3\) If a master takes on $K$ apprentices he, thus, obtains $\beta^A K$ units of unskilled labor at cost $w^A \beta^A K$ but has to spend $AK$ hours of his own time in training them.

In the second system, called the college system, unskilled workers pay the master a fee, $w^C$, up-front. The training takes $1 - \beta^C$ units of their time and they work for the remaining time as unskilled workers.\(^4\) Again, depending on his ability, $\gamma$, an unskilled worker may or may not succeed in becoming a master, and failure forces him into the ranks of the unskilled in the second period.

We assume that agents consume only at the end of their lifetimes and have identical Cobb-Douglas preferences. Hence, optimal consumption of each good is a linear function of lifetime income:

$$c_X = \delta Y, \quad c_Z = (1 - \delta) Y,$$

where $c_X$ is the consumption of the agricultural good, $c_Z$ is the consumption of the industrial good, and $Y$ is the lifetime income.

There are no credit markets, so agents cannot borrow. Hence, each agent has to finance any up-front costs only from the initial wealth he was born with.

The key problem we focus on is that since fees must be paid up front, agents with high ability but low initial wealth are barred from becoming skilled under the college system. In the apprenticeship system credit constraints are less binding.\(^5\) In this manner we explore the

---

\(^3\)The master may use all this time himself or sell it on the open market for $w$. We allow $w^A$ to be negative so as to allow for the possibility that being a master is so lucrative that workers are willing to pay for the privilege of being an apprentice.

\(^4\)If the training technologies in the two systems are the same then $\beta^A = \beta^C = \beta$, which will serve as our base case.

\(^5\)It may be that the apprenticeship system is not suited to mass learning, so that its efficiency may be lower than that of the college system if a sufficient mass of skilled agents is present.
implications of apprenticeship as a way of relaxing credit constraints in short-run and long-run settings.

Next we set up the problems under the two systems.

3.2.1 The Apprenticeship System

Since there are constant returns to scale, we can always think of the masters (skilled labor) as running everything and interpret their returns as the earnings of skilled labor. Each master chooses to hire unskilled labor and/or train unskilled workers, who, in return, work part of their time at below market wages and/or pay to be trained. Each apprentice spends $(1 - \beta^A)$ of his time studying and works the rest of the time as an unskilled worker. Unskilled workers are paid wage $w$ and apprentices are paid $w^A$, which may be positive or negative. If it is positive then we say that credit constraints do not operate as anyone who wishes to become an apprentice can do so. If $w^A$ is negative then unskilled workers must pay masters. Only those who have sufficient initial wealth to do so have the option of becoming apprentices and we say that credit constraints operate.

In each period of time there are $M_t$ masters who are the successful trainees from the last period. Each master chooses how many apprentices to take on, $K_t$, and how many unskilled workers to hire, $u_t$, taking into account $w^A_t$, $w_t$, and $p_t$. Profits are

$$
\pi_t^A = p_t (G(K_t))^\alpha (\beta^A K_t + u_t)^{1-\alpha} - u_t - \beta^A w^A_t K_t.
$$

(3.1)

It is convenient to transform the variables from $u_t$ and $K_t$ to $U_t$ and $S_t$ where $u_t + \beta^A K_t = U_t$ and $S_t = 1 - AK_t$. Doing so and substituting in (3.1) yields the following profit-maximization problem

$$
\max_{U_t, S_t} \pi_t^A = p_t S_t^\alpha U_t^{1-\alpha} - U_t - \frac{\beta^A}{A} (1 - w^A_t) S_t + \frac{\beta^A}{A} (1 - w^A_t)
$$

(3.2)

This expression has an intuitive interpretation if one thinks of a master as selling all his time on the market. A unit of master’s time allows him to claim $\frac{\beta^A}{A} (1 - w^A_t)$ from training apprentices. Then he buys back the time, $S_t$, he needs to produce the good. Hence, the earnings

---

6 Increasing returns, when they occur, are external in their nature.
of a master equal the value of output less the cost of hiring all the unskilled labor used, less the opportunity cost of his labor used in production, plus the value of his stock of skilled labor.

The first order conditions with respect to $U_t$ and $S_t$ are:

\[(1 - \alpha) p_t \left( \frac{S_t}{U_t} \right)^\alpha = 1, \quad (3.3)\]
\[\alpha p_t \left( \frac{S_t}{U_t} \right)^{\alpha - 1} = \frac{\beta^A A}{A} \left( 1 - w^A_t \right). \quad (3.4)\]

The marginal value product of an unskilled worker is equated to his wage.\(^7\) Similarly, the value of an additional unit of skilled labor is equated to its opportunity cost.

Using (3.3) and (3.4) we get the demand for unskilled relative to skilled labor for each master to be equal to

\[\frac{U_t}{S_t} = \frac{(1 - \alpha)}{\alpha A} \beta^A (1 - w^A_t) \quad (3.5)\]

Since product is exhausted due to constant returns to scale and perfect competition, the first three terms in (3.2) cancel so that the maximized value of profits equals

\[\pi^A_t = \frac{\beta^A}{A} (1 - w^A_t). \quad (3.6)\]

Note that this is exactly the opportunity cost of the time a master is endowed with. Had he chosen to just train workers, which he could do without facing diminishing returns, and sell the value of the time they offered as payment at market prices this is exactly what he would have obtained.

### 3.2.2 The College System

College students pay the master tuition, $w^c_t$, and spend $(1 - \beta^C)$ hours of their time learning skills. In addition, they can work as unskilled workers $\beta^C$ hours of their time and earn $\beta^C w_t$. A master hires unskilled labor directly and undertakes training of unskilled workers. His profits

---

\(^7\)Note that we do not have to worry about corner solutions since all inputs are essential in production. In addition, note that the unskilled labor hired from the market can be negative: this just means that a master does not use all the apprentice labor he is entitled to, but sells it for $w$ per unit, while paying $w^A$. 

61
are
\[ \pi_t^C = p_t (G(K_t))^\alpha U_t^{1-\alpha} - U_t + w_t^c K_t \]  
(3.7)

Again, it is convenient to substitute for \( G(K_t) = 1 - AK_t = S_t \) in (3.7). Each master solves the following profit-maximization problem:

\[ \max_{S_t, U_t} \pi_t^C = p_t S_t^\alpha U_t^{1-\alpha} - U_t - \frac{w_t^c}{A} S_t + \frac{w_t^c}{A} \]  
(3.8)

The first order conditions are:

\[ (1 - \alpha) p_t \left( \frac{S_t}{U_t} \right)^\alpha = 1 \]  
(3.9)

\[ \alpha Ap_t \left( \frac{S_t}{U_t} \right)^{\alpha - 1} = w_t^c. \]  
(3.10)

Using (3.9) and (3.10) we get the demand for unskilled relative to skilled labor for each master to be

\[ \frac{U_t}{S_t} = \frac{(1 - \alpha)}{\alpha A} w_t^c \]  
(3.11)

The maximized value of each master’s profits is his earnings. Think of the master selling all his skills as a trainer on the market, and buying back his use of \( S_t \) and \( U_t \). Since there are constant returns to scale, product is exhausted, and the first three terms in (3.8) cancel so that the maximized value of profits equals the opportunity cost of the time the master is endowed with

\[ \pi_t^{C*} = \frac{w_t^c}{A}. \]  
(3.12)

### 3.3 Autarky Equilibrium

#### 3.3.1 Static Autarky Equilibrium

In this section we analyze the autarky equilibrium in each period \( t \). First, we describe equilibrium under the apprenticeship system and then under the college system.
The Apprenticeship System

An equilibrium in period $t$ is characterized by a vector of prices $(p_t, w_t^A, w_t)$, where $p_t$ is the price of the industrial good, $w_t^A$ is the wage of the apprentice, and $w_t$ is the wage of unskilled worker. The proportion of agents who become apprentices is denoted by $(1 - \tilde{\gamma}_t^A)$.

Since both goods are essential in the consumption, both goods must be produced in autarky. Therefore, the wage of unskilled workers is equal to the price of the agricultural good, i.e., $w_t = 1$. The equilibrium price, $p_t$, is determined from the condition

$$c \left( \frac{\beta^A}{A} (1 - w_t^A), 1 \right) = p_t$$

where $c(\cdot)$ is the unit cost function. The opportunity cost of a unit of skilled labor is $\frac{\beta^A}{A} (1 - w_t^A)$, while the unskilled wage is unity. This pins down the price for a given $w_t^A$. Using the fact that the production function has the Cobb-Douglas form, we see

$$p_t = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \left( \frac{\beta^A (1-w_t^A)}{A} \right)^{\alpha} = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} (\pi_t^{A*})^{\alpha}$$

(3.13)

Note that as price $p_t$ rises, so does $\pi_t^{A*}$ which, in turn, implies that $w_t^A$ falls. For a high enough price, $w_t^A$ even turns negative so that workers must pay up front to become apprentices. Even under the apprenticeship system credit constraints become binding when $p_t$ is high enough, i.e., when

$$p_t > p_2 = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \left( \frac{\beta^A}{A^*} \right)^{\alpha}.$$

Occupational Choice  A young agent in period $t$, with probability of being talented $\gamma$ and inherited wealth $y$, has two options. The first is to work both periods of his life as unskilled worker. This gives a lifetime income of 2. The second option is to invest in skills hoping to become a master. In this case his first period income equals the apprentice’s wage $\beta^A w_t^A$. In the second period, with probability $\gamma$ he earns the master’s profit, $\pi_{t+1}^{A*}$, and with probability $(1 - \gamma)$ he receives the wage of unskilled worker, $w_t = 1$. The expected lifetime income in this case equals $\beta^A w_t^A + \gamma E_t \pi_{t+1}^{A*} + (1 - \gamma)$. Let $\tilde{\gamma}_t^A$ denote the agent who is indifferent between
these two options. Then $\tilde{\gamma}^A_t$ is determined from

$$2 = \beta^A w^A_t + \gamma E_t \pi^A_{t+1} + (1 - \gamma),$$

or

$$\tilde{\gamma}^A_t = \min \left\{ \frac{1 - \beta^A w^A_t}{E_t \pi^A_{t+1} - 1}, 1 \right\}$$

(3.14)

$= \min \left\{ \frac{1 - \beta^A + A \pi^A_{t+1}}{E_t \pi^A_{t+1} - 1}, 1 \right\}$

where the second equality follows from (3.6).

Agents with a low probability of being talented, $\gamma \in [0, \tilde{\gamma}^A_t]$, work both periods of their life as unskilled workers, while agents with sufficiently high probability, $\gamma \in [\tilde{\gamma}^A_t, 1]$, choose to become apprentices. As expected, higher profits for masters today, i.e., lower wages for apprentices today, raise $\tilde{\gamma}^A_t - fewer$ agents become apprentices. If the expected profits of masters tomorrow rise, i.e., the expected apprentice’s wage tomorrow falls, then $\tilde{\gamma}^A_t$ falls and more agents become apprentices today. Thus

$$\frac{\partial \tilde{\gamma}^A_t}{\partial \pi^A_{t+1}} > 0, \frac{\partial \tilde{\gamma}^A_t}{\partial E_t \pi^A_{t+1}} < 0.$$  

**Equilibrium** Since each agent spends a fixed share of his income on the consumption of each good, the relative demand for the industrial good is equal to

$$RD_t = \frac{Z^D_t}{X^D_t} = \frac{(1 - \delta)}{\delta p_t},$$

(3.15)

where $X^D_t$ and $Z^D_t$ are the aggregate demands for the agricultural and the industrial good respectively.

The derivation of supply of $X$ and $Z$ is slightly more complicated. Take expected profits in the next period, $E_t \pi^A_{t+1}$, and the cutoff level in the previous period, $\tilde{\gamma}^A_{t-1}$, as given. For a particular price, $p_t$, how do we get supply of the industrial and agricultural goods? Note that for each price, we get the return to skilled labor, $\pi^A_t$, and hence the wage for apprentices, from the condition (3.13) that price equals cost for the industrial good. The level of $\pi^A_t$, in turn, determines $\tilde{\gamma}^A_t$ as is apparent from (3.14). The cutoff level, $\tilde{\gamma}^A_t$, and in the credit constrained
case, the level of the apprentice’s wage, $w^A_t$, together determine how many agents are willing and able to become apprentices (the remainder becomes unskilled workers) and hence the time needed to train them! Removing the skilled labor needed for training from the stock of masters gives the supply of skilled labor available for production, $L^S_t$. Adding those who choose to become unskilled workers today to the inherited stock of unskilled workers and unskilled labor supplied by apprentices, gives the supply of unskilled labor available for production, $L^U_t$.

This, in effect, gives the size of the Rybczynski box depicted in Figure 3.1, where the supply of skilled labor is on horizontal axis and the supply of unskilled labor is on vertical axis. Note that in Figure 3.1, $L^S_i$ and $L^U_i$ denote the skilled and unskilled labor used in the production of good $i \in \{Z, X\}$. Now we can use the Rybczynski box to get the supply of $Z$ and $X$.

From (3.5) we know the relative demand for skilled and unskilled labor in $Z$, i.e., we know the input mix used in production of $Z$:

$$\frac{U_t}{S_t} = \frac{1 - \alpha}{\alpha} \pi^{A*}_t$$

Therefore, the ratio of unskilled and skilled labor used in the production of the industrial good must lie on the ray $O_zP$, which has slope equal to $\frac{1 - \alpha}{\alpha} \pi^{A*}_t$, and whose length is proportional to the supply of $Z$. Since only unskilled labor is needed in the production of agricultural good, the division of labor between the two goods is given by the intersection of $O_zP$ and $O_xA$, which is the point $P$. Hence, the supply of good $X$ is given by $O_xP$.

More formally, using (3.3) we get the supply of good $Z$ to be equal to

$$Z^S_t = M_t (S_t)\alpha (U_t)^{1-\alpha}$$

$$= M_t S_t \left( \frac{U_t}{S_t} \right)^{1-\alpha}$$

$$= L^S_Z \left( \frac{1 - \alpha}{\alpha} \pi^{A*}_t \right)^{\alpha},$$

where $M_t$ is the number of masters in period $t$ and $L^S_Z = M_t S_t$. Using this we get that the supply of good $Z$ equals $O_zA$ multiplied by $\left( \frac{1 - \alpha}{\alpha} \pi^{A*}_t \right)^{\alpha}$. Then, the relative supply equals

$$\frac{O_zA}{O_xP} \left( \frac{1 - \alpha}{\alpha} \pi^{A*}_t \right)^{\alpha}.$$
Hence, every price \( p_t \) corresponds to a point on the relative supply curve as depicted in Figure 3.2—moving \( p_t \) traces out the relative supply curve.

In Figure 3.2, price \( p_1 \) corresponds to \( \pi_t^{A^*} = 1 \). When price is below \( p_1 \), the return to skilled labor is less than 1 (the wage of unskilled worker), as a result the option of working as unskilled worker is more profitable than the option of being a master. For all prices below \( p_1 \) the supply of the industrial good, and hence the relative supply, is zero. At price \( p_1 \) skilled workers are indifferent between two options so that relative supply is horizontal. Price \( p_2 \) corresponds to \( \pi_t^{A^*} = \frac{\beta^A}{A} \) and, as we can see from (3.6), to \( w_t^A = 0 \). For all prices below \( p_2 \) the apprentice’s wage is positive, so credit constraints do not operate and the relative supply is denoted by \( RS_{t}^{ncc} \). For prices above \( p_2 \) the apprentice’s wage is negative, i.e., workers pay to become apprentices. In this event, agents are subject to credit constraints. Relative supply in this region is denoted by \( RS_{t}^{cc} \). When price exceeds \( p_3 \) the apprentice’s wage is so low that the option of investing in skills is dominated and there are no apprentices (\( \bar{\gamma}_t^A = 1 \)). Finally, at \( p_4 \) all unskilled labor is used in the production of the industrial good, the supply of the agricultural good is zero and the relative supply goes to infinity.

Next, we turn to the shape of the relative supply curve and the nature of static equilibrium. An equilibrium in which the apprentice’s wage is positive is a non-credit-constrained (NCC) equilibrium. An equilibrium in which the apprentice’s wage is negative is a credit-constrained (CC) equilibrium.

**Proposition 13** Under the apprenticeship system, if the number of masters in period \( t \) is small enough, i.e., \( M_t \leq \tilde{M}^A = \frac{2A}{(1 + A - \beta^A)} \), then relative supply is increasing in price. The static equilibrium could be credit-constrained or not. If there are enough masters, i.e., \( M_t > \tilde{M}^A \), then relative supply need not be increasing in price. Multiple equilibria are possible, but there is at most one non-credit-constrained equilibrium.

We relegate the formal proof to the Appendix C and focus on the intuition behind this result here. Suppose that the price increases. This results in a higher return to skilled labor and in a lower apprentice’s wage.\(^8\) As a result, the input mix in production moves away from skilled labor. In a static setting, the fall in the apprentice’s wage makes investing in skills less

\(^8\)If the there are credit constraints, this translates into the fee paid by an apprentice going up.
profitable and $\tilde{\gamma}_t^A$ increases. Hence, the supply of unskilled labor in period $t$ rises. As fewer agents wish to be apprentices, masters spend less time training them and the supply of skilled labor available for production increases. Note that both skilled and unskilled labor available for production rise so that their relative availability may rise or fall.

The effects on relative supply can be decomposed into two parts. First, the part due to change in the availability of skilled relative to unskilled labor for production purposes. An increase in the relative skilled labor availability raises $Z/X$, the relative supply of the skilled labor intensive good, a la Rybczynski. A decrease in the relative skilled labor availability does the opposite. Second, the part due to factor price changes and hence input mix changes. An increase in $p$ moves the input mix towards unskilled labor, the ray in Figures 3.3 and 3.4 moves from $P''$ to $P'$. For given factor supplies, this raises the relative supply of the skill-intensive good. This is the basis of the usual positive supply response in general equilibrium.

In Figure 3.3 both effects raise relative supply of $Z$. When there are few masters, $M_t > \tilde{M}^A$, then the supply of skilled labor is relatively small to begin with. As a result, any given increase in $\tilde{\gamma}_t^A$ release what amounts to a large percentage increase in the supply of skilled labor so that skilled labor becomes relatively more abundant, and relative supply of $Z$ rises.

If there are many masters, $M_t > \tilde{M}^A$, as in Figure 3.4, then an increase in price results in an increase in the relative availability of unskilled labor. With many masters, the supply of skilled labor is relatively large. Any change in $\tilde{\gamma}_t^A$ translates into a small percentage increase in the supply of skilled labor and, as a result, skilled labor becomes relatively less abundant and the relative supply of $Z$ may fall! In this case the two effects work in opposite directions and relative supply may be downward sloping.

It is shown in the Appendix C that when $M_t \leq \tilde{M}^A$, relative demand curve can intersect the relative supply curve at most once: either in its non-credit-constrained part, or in its credit-constrained part. If $M_t > \tilde{M}^A$ there may be multiple equilibria with at most one non-credit-constrained one. Multiple equilibria in this static set-up arise from the interaction of credit constraints and prices. When price is low, so is the return to skilled labor. In this case, the apprentice’s wage is high, there are no credit constraints and a large fraction of the

---

9The reason is that independent of the number of masters, relative supply is monotonic in the absence of credit constraints. Hence, it may intersect relative demand at most once.
population becomes apprentices. Since $M_t$ is large, despite this, there is a lot of skilled labor available for production and output is high. Since price is low, demand is high and this can be an equilibrium. On the other hand, if price is high, so is the return to skilled labor and for this, apprentice’s wages are negative. Credit constraints operate and many agents cannot become apprentices. While this does free up some skilled labor for production, masters are abundant and there is an ample supply of unskilled workers. Hence the relative supply of skilled workers is low, as is the relative supply of $Z$.

Note that not all static equilibria are consistent with steady state: for example, if the intersection occurred at prices above $p_3$, $\tilde{\gamma}_t^A = 1$ and nobody would invest in skills. If there are no masters in period $t + 1$, then the return to skilled labor would be infinite which is not consistent with expectations or possible in steady state.

**Comparative Statics** What is the effect of a decrease in expected profits in period $t + 1$ on period $t$’s equilibrium price? From (3.14) we can see that a decrease in the expected profit from becoming a master in the next period increases $\tilde{\gamma}_t^A$ – fewer agents choose to become apprentices. This fall in the number of agents wishing to be apprentices results, as argued above, in an increase in the supply of skilled and unskilled labor. However, in contrast to the effects of a price change, there is no change in the returns to skilled or unskilled labor and hence no effect on the input mix. The effect on relative supply depends only on whether the supply of unskilled labor increases relatively more or less than the supply of skilled labor.

In Figure 3.3 the supply of skilled labor increases relatively more than the supply of skilled labor since $M_t < \tilde{M}^A$. As we can see at this given price the relative supply goes up:

$$\frac{A'P'}{Ox'P'} > \frac{AP}{OP}.$$  

So, the relative supply curve shifts out, resulting in a lower equilibrium price. From (3.13) we can see that as a result the return to skilled labor falls and apprentice’s wage rises.

In Figure 3.4 the opposite happens – a decrease in master’s profit expected in next period results in a higher percentage increase in the supply of unskilled labor relative to that of skilled
labor. As a result the relative supply decreases:

\[
\frac{A'P''}{O'P''} < \frac{AP}{O'P'}
\]

As the relative supply curve shifts in, the equilibrium price in period \( t \) rises, resulting in higher return to skilled labor and lower apprentice’s wage.

Lemma 14 summarizes these results.

**Lemma 14** If there are few masters in period \( t \), i.e., \( M_t \leq \bar{M}^A \), then a decrease in expected earnings of a master in the subsequent period results in a lower equilibrium price in the current period, as well as a lower return to skilled labor and a higher apprentice’s wage. If \( M_t > \bar{M}^A \), then the opposite occurs.\(^{10}\)

**College System**

An equilibrium in period \( t \) is characterized by a vector of prices \((p_t, w^C_t, w_t)\), where \( p_t \) is the price of the industrial good, \( w^C_t \) is the tuition students pay to masters, and \( w_t \) is the wage of unskilled worker. As shown below, agents with abilities above the cutoff level, \( \tilde{\gamma}^C_t \), choose to get trained, so that \((1 - \tilde{\gamma}^C_t)\) is the proportion of agents who become college students.

As in the previous section, the wage of unskilled workers is equal to the price of the agricultural good, i.e., \( w_t = 1 \).

**Occupational Choice** Let \( \tilde{\gamma}^C_t \) denote the ability of the agent who is indifferent between the two options: to work both periods as unskilled worker or to invest in skills hoping to become a master in the second period. The first option gives a total lifetime income of 2. If agent chooses the second option, then his first period income equals the wage of unskilled worker multiplied by \( \beta^C \) units of time less the tuition. In the second period, with probability \( \gamma \) he earns the master’s profit, \( \pi^C_{t+1} \), and with probability \((1 - \gamma)\) he receives the wage of unskilled worker, \( w_t = 1 \). The expected lifetime income in this case equals \( \beta^C - w^C_t + \gamma E_t \pi^C_{t+1} + (1 - \gamma) \). Then \( \tilde{\gamma}^C_t \) is determined from

\[
2 = \beta^C - w^C_t + \gamma E_t \pi^C_{t+1} + (1 - \gamma),
\]

\(^{10}\)It will become obvious that this lemma also holds for the college system.
or

\[
\tilde{\gamma}_t^C = \min \left\{ \frac{1 - \beta^C + w_t^C}{E_t \pi_{t+1}^C - 1}, 1 \right\} \tag{3.17}
\]

\[
= \min \left\{ \frac{1 - \beta^C + A \pi_t^C}{E_t \pi_{t+1}^C - 1}, 1 \right\}
\]

where the second equality follows from (3.12).

Comparing (3.14) and (3.17) we can see that in both apprenticeship and college systems the proportion of agents deciding to invest in skills is given by the same function. As in the apprenticeship system, higher profit for masters today is associated with a lower payoff for trainees, whether through a reduction in the apprentice’s wage or an increase in the cost of college tuition, which raises \( \tilde{\gamma}_t^C \) – fewer agents become college students. Similarly, if the expected profits of masters tomorrow rise, then \( \tilde{\gamma}_t^C \) falls and more agents become college students today. Thus,

\[
\frac{\partial \tilde{\gamma}_t^C}{\partial \pi_{t+1}^C} > 0, \quad \frac{\partial \tilde{\gamma}_t^C}{\partial E_t \pi_{t+1}^C} < 0.
\]

**Equilibrium** As in the previous section, the relative demand is given by (3.15). We can use the approach used in the analysis of the apprenticeship system to derive the relative supply here as well. It is easy to check that when there are few masters, the relative supply under the college system is upward-sloping, but if the number of masters is large then the relative supply can be downward-sloping. Moreover, note that the comparative statics results of Lemma 14 have their analogue for the college system.

**Proposition 15** Under the college system, if \( M_t \leq \tilde{M}^C = \frac{2A}{1 + A - \beta^C} \) then relative supply is increasing in price and there is a unique equilibrium. If \( M_t > \tilde{M}^C \), then relative supply need not be increasing in price and there may be multiple equilibria.

Suppose that the training technologies are the same in the two systems: \( \beta^A = \beta^C = \beta \).

What can we say about the relative position of relative supply curves under the college system and under the apprenticeship system? As before, from the condition that the price of the good

\[11\text{Note that in this case } M^A = M^C = \tilde{M}.\]
\( Z \) equals cost, we get the master’ profit corresponding to any price:

\[
p_t = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \left( \pi^{C*}_t \right)^\alpha
\]

Then, from (3.17) we get \( \tilde{\gamma}_t^C \). These \( \tilde{\gamma}_t^C \) and \( \pi^{C*}_t \) determine the available supply of skilled and unskilled labor in a way that is very similar to the apprenticeship system when agents are subject to credit constraints. For any price there is a common level of earnings for skilled labor, \( \pi^*_t \), under both systems, and corresponding to this, an education cost – the tuition fee, \( w^C_t \), in the college system or the implicit price of \( \beta(1 - w^A_t) \) under the apprenticeship system. All of the education cost is paid up front in the college system, while only \( \max\{0, -\beta w^A_t\} \) is paid up front in the apprenticeship system, so that more is always paid up front under the college system. Hence, at any given \( p_t \) (or, at any \( \pi^*_t \) corresponding to this price) a larger proportion of agents are credit-constrained under the college system.

At any given price \( p_t \), both systems have the same cutoff level for ability. However, since credit constraints are stricter in the college system, fewer agents actually become trainees, the rest of those who want to do so cannot and remain unskilled so that the supply of unskilled workers is larger at any given \( p \) under the college system. The smaller number of trainees under the college system requires less skilled labor to train them. As a result, the skilled labor available for production is also larger under the college system. What can we say about relative availability? As before, when there are few masters the relative availability of skilled labor is higher under the college system, while when there are many masters the relative availability of skilled labor is lower under the college system. Hence, the effect on relative supply follows from the relative factor availability under the two systems.

If there are few masters, \( M \leq \tilde{M} \), then at any given \( p \), the relative supply of the skill-intensive good is more than that under the apprenticeship system. If there are many masters, \( M > \tilde{M} \), it is less than that in the apprenticeship system. This is depicted in Figure 3.5. In Figure 3.5 (a) the number of masters is small, and, as a result, the relative supply curves for both systems are upward-sloping. At any price below \( p_3 \) (so that the proportion of agents who invest in skills is positive) the relative supply curve under the college system is to the right of the relative supply curve under the apprenticeship system. When price is above \( p_3 \) there are no
agents who invest in skills and the two curves coincide. Therefore, there is a unique equilibrium in each system, and the equilibrium price under the college system is lower than the equilibrium price under the apprenticeship system. Since the equilibrium price is lower under the college system, we cannot conclude that the tuition in college is higher than the full fee under the apprenticeship system.

In Figure 3.5 (b) the number of masters is large. The relative supply curve under the college system is to the left of the relative supply curve under the apprenticeship system. And, as a result, the equilibrium price in the college system is higher. Moreover, as the price is higher and so is the full fee and hence the up-front education fee under the college system.

Proposition 16 summarizes these results.

**Proposition 16** If the training technologies are the same in the two systems then at any given \( p \), relative supply of the skill-intensive good is higher under the college system than that under the apprenticeship system as long as \( M \leq \tilde{M} \). As a result, the equilibrium price is lower than the equilibrium price under the apprenticeship system. If \( M > \tilde{M} \), relative supply of the skill-intensive good is lower under the college system than that under the apprenticeship system. Consequently, the equilibrium price under the college system is higher than the equilibrium price under the apprenticeship system.

3.3.2 Autarky Steady State Equilibrium

In this section we solve for autarky steady state equilibrium. First, we describe steady state equilibrium under the apprenticeship system, and then under the college system.

**Apprenticeship System**

Steady state equilibrium is characterized by a vector of prices \((p, w^A, w)\), where \( p \) is the price of the industrial good, \( w^A \) is the wage of the apprentice, and \( w \) is the wage of unskilled worker. The proportion of agents who become apprentices is denoted by \( 1 - \gamma^A \).

As in the previous section the wage of unskilled workers is equal to the price of the agricultural good, i.e., \( w = 1 \).
**Occupational Choice**  Let $\tilde{\gamma}^A$ denote the ability of the agent who is indifferent between the two options: to work both periods as unskilled worker or to invest in skills hoping to become a master in the second period. The first option gives a total lifetime income of 2. If agent chooses the second option, then his first period income equals the apprentice’s wage rate $\beta^A w^A$. In the second period, with probability $\gamma$ he earns the master’s profit, $\pi^A*$, with probability $(1 - \gamma)$ he receives the wage of unskilled worker, $w = 1$. The lifetime income in this case equals $\beta^A w^A + \gamma \pi^A* + (1 - \gamma)$. Then $\tilde{\gamma}^A$ is determined from

$$2 = \beta^A w^A + \gamma \pi^A* + (1 - \gamma)$$

Using $\pi^A* = \frac{\beta^A}{A}(1 - w^A)$ we have

$$\tilde{\gamma}^A = \min \left\{ \frac{(1 - \beta^A) + A \pi^A*}{\pi^A* - 1}, 1 \right\} = \begin{cases} 1, & \text{if } \pi^A* \leq \frac{2 - \beta^A}{1 - A}, \\ \frac{(1 - \beta^A) + A \pi^A*}{\pi^A* - 1}, & \text{if } \pi^A* > \frac{2 - \beta^A}{1 - A}, \end{cases} \quad (3.18)$$

Note that a higher return to skilled labor means a lower apprentice’s wage $w^A$, hence there is a trade-off between a higher earnings tomorrow and a lower wage today. But since a fall in $w^A$ raises the earnings of a master in the next period by more than it reduces the wage of an apprentice in the current period, a higher $\pi^A*$ leads to a lower $\tilde{\gamma}^A$: more agents become apprentices. Thus,

$$\frac{\partial \tilde{\gamma}^A}{\partial \pi^A*} < 0.$$

Note that the relationship between $\tilde{\gamma}^A$ and the return to skilled labor has the opposite sign from that in the static set-up. In steady state the return to the option of investing in skills is increasing in $\pi^A*$ as explained above. In the static set-up, the master’s earnings expected in the next period are fixed and agents take into account only the apprentice’s wage in the current period, and as a higher $\pi^A*$ means a lower apprentice’s wage, the return to the option of investing in skills is decreasing in $\pi^A*$.

Note also that for low levels of profit ($\pi^A* \leq \frac{2 - \beta^A}{1 - A}$) the option of investing in skills is dominated and all agents choose to work as unskilled labor. What happens if the return to skilled labor becomes large? Using $\pi^A* = \frac{\beta^A}{A}(1 - w^A)$ we can rewrite the return to the option
of investing in skills as $\beta^A + (1 - \gamma) + \pi^{A*} (\gamma - A)$. Then, for $\gamma < A$ this return is clearly less than the lifetime income of unskilled worker. Hence, all agents with $\gamma < A$ choose not to invest in skills. As profit goes to infinity ($\pi^{A*} \to \infty$) the proportion $A$ of agents work as unskilled labor and $(1 - A)$ become apprentices, i.e., $\tilde{\gamma}^A \to A$. Therefore, in steady state the proportion of agents who become apprentices never exceeds $1 - A$.

**Equilibrium** As in the previous section, the relative demand equals

$$RD = \frac{Z^d}{X^d} = \frac{(1 - \delta)}{\delta p}$$

We can use the approach used in the static set-up to derive the relative supply here as well. Take any price $p$. From the condition that price equals cost for the industrial good we get the return to skilled labor, $\pi^{A*}$:

$$p = \alpha^{-\alpha} (1 - \alpha)^{- (1 - \alpha)} (\pi^{A*})^{\alpha}$$

Then, the level $\pi^{A*}$ uniquely determines $\tilde{\gamma}^A$ as is apparent from (3.18). The cutoff level $\tilde{\gamma}^A$ and, in the credit constrained case, the level of the apprentice’s wage, together determine the available supply of skilled and unskilled labor. Note the following difference between steady state and static supply of skilled and unskilled labor. In static set-up the number of masters as well as the amount of unskilled labor from the previous period is exogenous, while in steady state these are not fixed but endogenous and determined by $\tilde{\gamma}^A$.

Next, we can use the Rybczynski box to derive the supply of good $X$ and good $Z$. As in the previous section every $p$ gives $\pi^{A*}$, $\gamma^A$, $X^S$ and $Z^S$. In other words, every $p$ corresponds to a point on the relative supply curve as depicted in Figure 3.6.

In Figure 3.6 $\pi_1$ denotes the return to skilled labor above which $\gamma^A < 1$. Price $p_1$ corresponds to this $\pi_1$. For all prices below $p_1$ nobody invests in skills, so production of the industrial good is zero. Hence, relative supply is zero as well. Price $p_2$ corresponds to $\pi^{A*} = \frac{\beta^A}{A}$ and, as in the static set-up, to $w^A = 0$. For all prices below $p_2$ the apprentice’s wage is positive, so that credit constraints are not binding and relative supply is denoted by $RS^{ncce}$. For prices above $p_2$ the apprentice’s wage is negative, and agents are subject to credit constraints. Relative supply
in this region is denoted by $RS^{cc}$. Finally, at $p_3$ all unskilled labor is used in the production of the industrial good, the supply of the agricultural good is zero and relative supply goes to infinity.

Next, we turn to the shape of the relative supply curve and the nature of steady state equilibrium. First, we prove that when credit constraints are not operating the relative supply curve is upward-sloping. Suppose that price rises. Then the return to skilled labor rises as well, and this results in lower $\tilde{\gamma}^A$: more agents choose to invest in skills at a higher price. Since more agents become apprentices, the supply of unskilled labor falls. What can we say about the supply of skilled labor? Subtracting the skilled labor needed for training from the stock of masters gives the supply of skilled labor available for production:

$$L^S = M - AK = \frac{1}{2} \left(1 - (\tilde{\gamma}^A)^2\right) - A \left(1 - \tilde{\gamma}^A\right),$$

where $M = \frac{1}{2} \left(1 - (\tilde{\gamma}^A)^2\right)$ is the number of masters and $K = 1 - \tilde{\gamma}^A$ is the number of apprentices. As price rises, the number of apprentices increases, as does the number of masters. The total effect on the supply of skilled labor is positive — $L^S$ increases with the price since:

$$\frac{dL^S}{dp} = \frac{d\tilde{\gamma}^A}{dp} (A - \tilde{\gamma}^A) > 0$$

Hence, a higher price leads to a higher supply of skilled labor available for production of good $Z$ and a lower aggregate supply of unskilled labor.

What about the output of the skill-intensive good? The supply of the skill-intensive good is

$$Z^S = (L^S)^\alpha (L_z^U)^{1-\alpha} = L^S \left(\frac{1-\alpha}{\alpha} \pi^{A*}\right)^\alpha$$

from (3.16).

Then, as $L^S$ and $\pi^{A*}$ both increase, we can conclude that the supply of good $Z$ increases as well. Since the amount of unskilled labor used in the production of skill-intensive good
equals $L_U^z = \left( \frac{1 - \alpha}{\alpha} \pi^{A^z} \right) L^S$, it is clear that $L_U^z$ goes up. As the total available unskilled labor decreases and the amount of unskilled labor used in the production of good $Z$ increases at this higher price, the amount of unskilled labor available for production of good $X$ decreases, so that supply of the agricultural good falls. Thus, we can conclude that when credit constraints are not operating, relative supply is increasing in price.

When agents are subject to credit constraints, the effects of an increase in price on the supply of unskilled and skilled labor are ambiguous and depend on the distribution of wealth. If a higher price results in unskilled labor becoming relatively more abundant, then the relative supply of the skill-intensive good may fall. If, for example, there are many agents who become credit-constrained at this higher price, then a large proportion of agents who invested in skills at lower price cannot afford to do so. Hence, the supply of unskilled labor rises, and the supply of skilled labor available for production of good $Z$ falls. As a result, the relative supply of good $Z$ may be lower at this higher price. Thus, when credit constraints operate, the shape of the relative supply curve depends on the distribution of wealth and can be either upward-sloping or downward-sloping.

As relative supply need not be monotonic multiple steady state equilibria may arise. When price is low, so is the return to skilled labor. In this case, the apprentice’s wage is positive, credit constraints are not binding and a large fraction of the population become apprentices. There is a lot of skilled labor available for production and output is high. At this low price, demand is high and this can be an equilibrium. On the other hand, if price is high, so is the return to skilled labor and, as a result, apprentice’s wage is negative. Credit constraints operate and many agents cannot become apprentices. This results in a relatively small supply of skilled labor and an ample supply of unskilled workers. Hence the relative supply of skilled workers is low, as is the relative supply of the skill-intensive good.

Proposition 17 summarizes these results.

**Proposition 17** Under the apprenticeship system, if $p \leq p_2$ then credit constraints are not binding and steady state relative supply is increasing in price. If $p > p_2$ then credit constraints are binding and steady state relative supply need not be increasing in price. Multiple steady state equilibria may exist.
College System

Steady state equilibrium is characterized by a vector of prices \((p, w^C, w)\), where \(p\) is the price of the industrial good, \(w^C\) is the tuition students pay to masters, and \(w\) is the wage of unskilled worker. The proportion of agents who become college students is denoted by \(1 - \tilde{\gamma}^C\). As before, the wage of unskilled workers is equal to the price of the agricultural good.

Occupational Choice  Let \(\tilde{\gamma}^C\) denote the ability of the agent who is indifferent between the two options: to work both periods of his life as unskilled worker or to go to college. The first option gives a total lifetime income of 2. If agent chooses the second option, then his lifetime income equals \(-w^C + \beta^C + \gamma \pi^{C^*} + (1 - \gamma)\). Then \(\tilde{\gamma}^C\) is determined from

\[
2 = -w^C + \beta^C + \gamma \pi^{C^*} + (1 - \gamma)
\]

Using \(\pi^{C^*} = \frac{w^C}{A}\) we have

\[
\tilde{\gamma}^C = \min \left\{ \frac{1 - \beta^C + w^C}{\pi^{C^*} - 1}, 1 \right\} = \begin{cases} 
1, & \text{if } \pi^{C^*} \leq \frac{2 - \beta^C}{1 - A} \\
\frac{(1 - \beta^C) + A \pi^{C^*}}{\pi^{C^*} - 1}, & \text{if } \pi^{C^*} > \frac{2 - \beta^C}{1 - A}
\end{cases}
\]  

(3.19)

Comparing (3.18) and (3.19) we can see that in both apprenticeship and college systems the proportion of agents deciding to invest in skills in steady state is given by the same function. As in the apprenticeship system, higher return to skilled labor decreases \(\tilde{\gamma}^C\) – more agents become college students.

Equilibrium  We can use the approach used in the analysis of steady state equilibrium under the apprenticeship system to derive the relative supply here as well. Then, since under the college system credit constraints operate at all prices, we can conclude that the shape of the relative supply curve depends on the distribution of wealth and can be either upward-sloping or downward-sloping. There may or may not be multiple equilibria in steady state: a key determinant again is the distribution of wealth.
Proposition 18. Under the college system credit constraints are always binding. Hence, steady state relative supply need not be increasing in price. Multiple steady state equilibria may exist.

Suppose that the two training technologies are the same: $\beta^A = \beta^C = \beta$. What can we say about the relative position of steady state relative supply curves under the college system and under the apprenticeship system? At any given price, both systems have the same cutoff level of ability. However, since credit constraints are stricter in the college system, fewer agents actually become trainees, the rest of those who want to do so cannot and remain unskilled so that the supply of unskilled workers is larger at any given price under the college system. The smaller number of agents acquiring skills under the college system results in a lower supply of skilled labor available for production. As a result, at any price the supply of good $Z$ relative to good $X$ is lower under the college system. Figure 3.7 depicts relative supply curves in both system, where relative supply under the college system is denoted by $RS^C$, relative supply under the apprenticeship system when credit constraints are not binding is denoted by $RS^{Acc}$ and when credit constraints are binding by $RS^{Acc}$. As depicted, relative supply curve under the college system is to the left of the relative supply curve under the apprenticeship system. And, as a result, the equilibrium price under the college system is higher. Moreover, as the price is higher, so is the full fee and hence the up-front education fee under the college system.

Proposition 19 summarizes these results.

Proposition 19. If the two training technologies are the same, then at any given $p$, steady state relative supply is lower under the college system than that under the apprenticeship system. As a result, the equilibrium price is higher than the equilibrium price under the apprenticeship system.

3.4 Effects of Trade

Having described the closed economy, we turn to the analysis of the effects of opening the economy up to trade. We analyze welfare effects of trade for the case when the country is small and cannot affect the world price of the industrial good, denoted by $p^W$.

Suppose that the country opens up to trade in period $t$. From the condition that the price equals unit cost for the industrial good, the return to skilled labor is determined by the world
price $p^W$. Let $\pi_{t+1}^{T_*}$ denote the return to skilled labor in period $t + 1$ under trade. Then for all periods after opening up to trade, profits are constant at level $\pi^{T_*}$, where

$$\pi^{T_*} = \alpha^\alpha (1 - \alpha)^{(1-\alpha)} \left( p^W \right)^{-1}. $$

Similarly, the cut-off level, which depends on master’s earnings in the current period and in the next period, is also fixed after first period at:

$$\tilde{\gamma}^T = \frac{(1 - \beta) + A\pi^{T_*}}{\pi^{T_*} - 1}. $$

In period $t$ the number of masters is inherited from the previous period. Hence, the output levels are determined by the number of masters as well as by the world price. In period $t + 1$, the number of masters equals $\frac{1}{2} \left( 1 - (\tilde{\gamma}^T)^2 \right)$ and depends only on $p^W$, so that the output levels in period $t + 1$ are determined only by the world price. Thus, we can conclude that starting from period $t + 1$ the relative price of the industrial good, the return to skilled labor, the education fee, and the cut-off level are fixed, so that the economy is in steady state equilibrium. Then, for any given world price the relative supply is given by the corresponding point on the steady state relative supply curve constructed in Section 4. Similarly, the point on the relative demand curve corresponding to the world price gives the relative demand for the industrial good in trade steady state equilibrium.

From Section 4 we know that when credit constraints operate, steady state relative supply need not be increasing in price and multiple equilibria may exist under either system. As we show below, non-monotonicity of relative supply may result in trade equilibria where the country ends up importing the industrial good at world prices higher than its autarky price and, as a result, loses from opening up to trade.

Consider the situation depicted in Figure 3.8. The relative supply is non-monotonic and there are two stable steady state autarky equilibria: $E_1^A$ and $E_2^A$. At equilibrium $E_1^A$ the price, $p_1^A$, is low and relative supply of the industrial good is high, while at $E_2^A$ the price, $p_2^A$, is high and supply is low. Suppose that initially the economy is in autarky equilibrium $E_1^A$. What happens when the country opens up to trade and faces the world price of the industrial good which is higher than its autarky price? As the price rises, the earnings of a master rise as well,
so that the option of investing in skills becomes more attractive. As more agents decide to invest in skills, the up-front education fee goes up. Credit constraints become tighter and more agents are unable to afford education. Consequently, the supply of the skilled labor decreases, resulting in lower output of the industrial good as well as lower relative supply, denoted by $RS_T$. As the price goes up, the relative demand for the industrial good, denoted by $RD_T$, falls. If a significant proportion of agents becomes credit constrained at this higher price, then the decrease in relative supply is considerable and exceeds the decrease in relative demand: opening up to trade allows only few rich agents to invest in skills and, as a result, the supply of the skill-intensive good falls dramatically and cannot satisfy domestic demand at this price. Thus, the country imports the industrial good even though the world price is higher than the autarky price! Since the country loses its comparative advantage in the industrial good and has to import this good at a higher price, the aggregate welfare can be lower with trade.

**Proposition 20** Opening up an economy to trade need not be welfare-improving. Aggregate welfare may fall with trade if a country imports the industrial good at world prices higher than autarky price.

### 3.5 Conclusions

In this paper we develop a model where apprenticeships help overcome credit constraints that limit the ability of agents with heterogeneous abilities and wealth to acquire skills. We show that in the static version of our model, under either system, the response of supply to price depends on the number of skilled agents in the economy. If there are relatively few skilled agents, the normal supply response obtains. However, with many skilled agents, supply can be decreasing in price so that multiple equilibria may exist. In steady state, however, such nonmonotonicity of supply and multiplicity of equilibrium obtains only in the presence of credit constraints. Since credit constraints are stricter in the college system, relative supply of the skill intensive good is always higher, at any given price, under the apprenticeship system. There may or may not be multiple equilibria in steady state: a key determinant is the distribution of wealth. Finally, we show that opening the economy to trade could easily reduce welfare. In fact, trade could result in a country importing the good whose relative price has risen due to
trade.
Figure 3.1. Output and Factor Availability
Figure 3.2. Relative Supply: Apprenticeship System
Figure 3.3. Small Number of Masters
Figure 3.4. Large Number of Masters
Figure 3.5. Relative Supply: College System
Figure 3.6. Steady State Relative Supply: Apprenticeship System
Figure 3.7. Steady State Relative Supply: College System
Figure 3.8. Effects of Trade
Bibliography


Appendix A

Proofs to Chapter 1

Proof of Lemma 5. Since there are perfect credit markets and both good must be produced, in the steady state equilibrium all agents must have the same wealth endowment \( w^N = \frac{\gamma n}{(1 - \gamma R)} \) and must be indifferent between occupations. Hence, the following condition holds:

\[
ap - Rk = n
\]  

(A.1)

Agents borrow and lend non-zero amounts only if \( w^N < k \). In the case of \( \gamma n \geq (1 - \gamma) k \), the initial wealth \( w^N \geq k \) for all \( R \geq 1 \), therefore, there is no borrowing/lending in this case.

Consider now the case when \( \gamma n < (1 - \gamma) k \). Note that \( p > \bar{p} \) if \( R > 1 \). Denote by \( \delta \) the proportion of agents working in industry. The supply of loans comes from agents working in agriculture and the demand for loans is from agents working in industry:

\[
S^L = (1 - \delta) w^N, \quad D^L = \delta (k - w^N)
\]  

(A.2)

Then equilibrium in the loans market is given by

\[
R = 1, \text{ if } S^L_{|R=1} > D^L_{|R=1}
\]  

and

\[
R : S^L = D^L, \text{ otherwise}
\]  

(A.3)
From (A.1) the price of industrial good equals

\[ p = n + \frac{Rk}{a} \]  

(A.4)

The supply and the demand for industrial good are

\[ S^2 = \begin{cases} \delta a, & \text{if } p > \frac{p}{p} \\ \in [0, \delta a], & \text{if } p = \frac{p}{p} \end{cases}, \quad D^2 = \frac{\beta Y}{p} = \frac{\beta}{p} \frac{n}{1 - \gamma R} \]

Then equilibrium in the market for industrial good is given by intersection of supply and demand:

\[ S^2 = D^2 \]  

(A.5)

The solution to (A.3), (A.4), and (A.5) gives the equilibrium interest rate and price of industrial good:

\[ R^N = \frac{\beta}{\gamma} - \frac{n}{k} > 1, \quad p^N = \frac{\beta k}{\gamma a} > p \text{ if } n < \frac{\beta - \gamma k}{\gamma}, \]

\[ R^N = 1, \quad p^N = p, \text{ otherwise} \]

\[ \text{Proof of Proposition 7.} \quad \text{Note that the price in the steady state trade equilibrium must satisfy } p^T \geq \hat{p} \text{ since at prices below } \hat{p} \text{ all agents in both countries choose to work in agriculture and industrial good is not produced. Hence, there are two possible cases for equilibrium price: either } p^T \in [p, \hat{p}) \text{, or } p^T \geq \hat{p}. \]

\[ \text{Suppose that } p^T \in [p, \hat{p}). \text{ In this case trade results in deindustrialization in the South: the South produces only agricultural good and imports industrial good. Since } p^T < \hat{p}, \text{ some proportion of agents in the North must work in agricultural sector in order to lend } (k - w^N) \text{ to agents working in industrial sector, as } w^N = \frac{\gamma}{1 - \gamma}(ap - k) < k. \text{ Interest rate is linked to the price by (A.1). Denote by } \delta \text{ the proportion of agents working in industry in the North. Using} \]
(A.1) and (A.2) the supply of industrial good is

\[
S = \delta a = \begin{cases} 
\frac{\gamma n}{k - \gamma (ap - n)} a, & \text{if } p^T > p; \\
0, & \text{if } p^T = p.
\end{cases}
\]

Excess demand in the South is

\[
ED^S = \frac{\beta}{1 - \gamma} \frac{n}{p}, \tag{A.6}
\]

and excess supply in the North is

\[
ES^N = \delta a - \frac{\beta}{1 - \gamma} \frac{n}{p} \left\{ \begin{array}{ll}
\frac{n}{k - \gamma (ap - n)} \left( \frac{\gamma a - \beta k}{p} \right), & \text{if } p^T > p \\
0, & \text{if } p^T = p
\end{array} \right. \tag{A.7}
\]

(A.6) and (A.7) intersect at the price

\[
p^T = \begin{cases} 
\frac{(2 - \gamma) k + \gamma \beta n}{\gamma (1 + \beta - \gamma)} > p, & \text{if } n < \frac{2\beta - \gamma}{\gamma} k \\
p, & \text{otherwise.}
\end{cases}
\]

Since we are in case \( \gamma n < (1 - \gamma) \), the inequality \( n < \frac{2\beta - \gamma}{\gamma} k \) holds for all \( \beta > \frac{1}{2} \). The price satisfies condition \( p^T < \hat{p} \) if \( \gamma n < \frac{1 - \beta}{\beta} (1 - \gamma) k \).

Suppose now that \( p^T \geq \hat{p} \). There are two possible scenarios: the occupational structure in the South does not change with trade, or there is deindustrialization. Consider the first scenario first. At this price the interest rate in the North is \( R^N = 1 \) and all agents work in the industrial sector. Excess demand in the South is

\[
ED^S = \frac{\beta}{1 - \gamma} \frac{\theta^S n + (1 - \theta^S) (ap - k)}{p} - \frac{(1 - \theta^S)}{p} \tag{A.8}
\]

\[
= \frac{\beta}{1 - \gamma} \frac{\theta^S n - (1 - \theta^S) k}{p} - \frac{\alpha}{1 - \gamma} (1 - \theta^S) a.
\]

\(^1\)This inequality is always satisfied if \( \beta \leq \frac{1}{2} \).
and excess supply in the North is

\[ ES^N = a - \frac{\beta}{1-\gamma} \frac{ap - k}{p} = \frac{\alpha}{1-\gamma}a + \frac{\beta k}{(1-\gamma)p} \]  

(A.9)

(A.8) and (A.9) intersect at the price

\[ p^T = \frac{\beta \theta^S n - (2 - \theta^s k)}{\alpha (2 - \theta^S)} \]

Simple calculations show that \( p^T \) is below \( \hat{p} \) for all \( \theta^S \in [\hat{\theta}, 1] \) if \( \beta \leq \frac{1}{2} \). In the case when \( \beta > \frac{1}{2} \) the price is below \( \hat{p} \) if \( \gamma n < \frac{1-\beta}{\beta} (1 - \gamma) k \). If the last inequality is not satisfied, then \( p^T \geq \hat{p} \) for \( \theta^S \geq 2 \hat{\theta} \).

The second scenario occurs when trade equilibrium involves deindustrialization in the South and \( p^T \geq \hat{p} \). This situation happens for \( \theta^S \in [\hat{\theta}, 2 \hat{\theta}] \) when \( \beta > \frac{1}{2} \) and \( \gamma n \geq \frac{1-\beta}{\beta} (1 - \gamma) k \).

**Proof of Proposition 9.** The industrial workers remain credit-constrained in the steady state if the wage rate is low enough: 
\[ \frac{\gamma b}{1-\gamma} < k, \text{ or } b < \frac{1-\gamma}{\gamma} k. \]
If the wage is above \( \frac{1-\gamma}{\gamma} k \) then the industrial workers are able to become small-scale entrepreneurs, and since the world price is high enough, \( p^W \geq \hat{p}_{LH} \), they will eventually be able to invest \( lk \) and become large-scale entrepreneurs.

The demand for labor exists when the return from working as a large-scale entrepreneur exceeds that of a small-scale one, i.e.,

\[ (ap^W - k - b) l > ap^W - k. \]

Thus, the maximum wage when the demand for labor is positive equals \( \frac{l-1}{l} (ap^W - k) \). Note that this wage exceeds \( \frac{1-\gamma}{\gamma} k \). At this wage agents prefer to become entrepreneurs rather than workers:

\[ (ap^W - k - b) l > b \]

Therefore, if the wage is above \( \frac{1-\gamma}{\gamma} k \) and below \( \frac{l-1}{l} (ap^W - k) \) then all agents prefer to be large-scale entrepreneurs, and the supply of labor is zero. Thus, the wage in this interval cannot be an equilibrium wage.
If the wage rate is less than $\frac{1-\gamma}{\gamma}k$, then small-scale entrepreneurs choose to become large-scale ones. However, the supply of labor is not sufficiently large to satisfy the increased demand for labor: $\theta < \frac{l}{1+l}$. Thus, the wage $b < \frac{1-\gamma}{\gamma}k$ cannot be an equilibrium wage either.

This implies that there is no wage at which labor demand and supply intersect, hence, the labor market is not active. Therefore, in the state trade equilibrium all agents work as small-scale entrepreneurs.

**Altruistic Preferences.** Consider the model described in Section 2. Suppose now that dynasties are linked by fully altruistic preferences as in Barro (1974). Then generation $t$ payoff is given by

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_{A,\tau}, c_{I,\tau}),$$

where $u(c_{A,\tau}, c_{I,\tau})$ is one-period utility function, which depends on $c_{A,\tau}$ (consumption of the agricultural good) and $c_{I,\tau}$ (consumption of the industrial good), $\beta < 1$ is discount rate.

The objective of this analysis is to show that the main result of the paper, namely, that opening up to trade may result in involuntary deindustrialization, is robust to the specification of preferences. In order to show that we need to identify the range of world prices that result in involuntary deindustrialization for altruistic preferences.

We restrict our analysis to the particular Cobb-Douglas specification of the one-period utility function:

$$u(c_A, c_I) = c_A^\alpha c_I^{1-\alpha}$$

This results in indirect utility being linear in income spent on consumption and precludes the transfer of wealth across generations for consumption purposes. The only motive for bequests is to provide future generations with investment opportunities.

The agent’s problem can be rewritten in the following form:

$$V_t(w_t, p_t) = \max_{i_t \in \{a, m\}, w_{t+1}} \left\{ \frac{A}{p_t} (w_t - w_{t+1} + R_{i_t}) + \beta V_{t+1}(w_{t+1}, p_{t+1}) \right\},$$

96
where $i_t \in \{\text{agriculture, industry}\}$ is the occupational choice, $R_{it}$ is the earned income:

$$R_{it} = \begin{cases} n, & \text{if } i_t = \text{agriculture} \\ a p_t - k, & \text{if } i_t = \text{industry} \end{cases}$$

and $A = \alpha^\alpha (1 - \alpha)^{1-\alpha}$.

This specification of preferences makes analytics intractable, especially outside steady state. Thus, we restrict our analysis only to steady state outcomes.

**Autarky.** We construct steady state autarky equilibrium for the following mutually exhaustive cases: $n \geq k$ and $k - in \leq n < k - (i - 1) n$, $i = 1, 2, \ldots$, in turns.

Consider the case $n \geq k$. In such a case an agent working in agriculture is able to leave $k$ as a bequest to his offspring. Therefore, at most one generation is needed to switch from agriculture to industry.

**Lemma 21** If $n \geq k$ there are two groups of agents in the steady state autarky equilibrium: constrained agents with initial wealth $w_t = 0$, who work in agriculture and leave bequests of 0, and unconstrained agents with initial wealth $\pi_t = k$, who work in industry and leave bequests of $k$. The price is

$$p^A = \frac{k + \beta n}{\beta a}.$$

**Proof.** Denote by $V_1(0)$ the indirect utility of an agent with initial wealth $w_t = 0$, who works in agriculture and leaves bequest $w_{t+1} = 0$. Similarly, $V_2(k)$ is the indirect utility of an agent with initial wealth $w_t = k$, who works in industry and leaves bequest $w_{t+1} = k$. Price $p$ is compatible with steady state if and only if

$$V_1(0) = \frac{An}{p^{1-\alpha}} + \beta V_1(0) \geq \frac{A(n - k)}{p^{1-\alpha}} + \beta V_2(k); \quad (A.10)$$

$$V_2(k) = \frac{A(ap - k)}{p^{1-\alpha}} + \beta V_2(k) \geq \frac{A(ap)}{p^{1-\alpha}} + \beta V_1(0) \quad (A.11)$$

Inequality (A.10) represents the non-deviation condition for agents with initial wealth $w_t = 0$. The left hand side is the payoff from consuming all income and leaving bequest of $w_{t+1} = 0$, so that the next generation continues to work in agriculture. The right hand side is the payoff from
sacrificing consumption by \( k \), so that the next generation is able to switch to more profitable occupation: working in the industrial sector.

Inequality (A.11) represents the non-deviation condition for agents with initial wealth \( w_t = k \). The left hand side is the payoff from leaving bequest of \( w_{t+1} = k \), so that the next generation continue to work in industry. The right hand side is the payoff from increasing consumption by \( k \), so that the next generation do not have sufficient initial wealth and have to switch to less profitable occupation: working in agriculture.

Inequalities (A.10) and (A.11) are equivalent to:

\[
\frac{Ak}{p^{1-a}} = \beta (V_2(k) - V_1(0)) = \frac{\beta}{1 - \beta} \frac{A(ap - k - n)}{p^{1-a}}
\]

or,

\[
p = \frac{k + \beta n}{\beta a}
\]

Lemma 22 If \( n \in \left[ \frac{k}{i + 1}, \frac{k}{i} \right] \) there are two groups of agents in the steady state autarky equilibrium: constrained agents with initial wealth \( w = 0 \), who work in agriculture and leave bequests of 0, and unconstrained agents with initial wealth \( \overline{w} = k \), who work in industry and leave bequests of \( k \). The price satisfies

\[
p^A \in \left[ \frac{k + \beta n}{\beta a}, \frac{(1 - \beta + \beta^{i+1}) k + (\beta + \beta i - i) n}{\beta^{i+1}a} \right]
\]

Proof. Price \( p \) is compatible with steady state if and only if

\[
V_1(0) = \frac{An}{p^{1-a}} + \beta V_1(0) \geq \frac{A(n - (k - in))}{p^{1-a}} + \beta^{i+1}V_2(k); \quad (A.12)
\]

\[
V_2(k) = \frac{A(ap - k)}{p^{1-a}} + \beta V_2(k) \geq \frac{A(ap)}{p^{1-a}} + \beta V_1(0) \quad (A.13)
\]

Inequality (A.12) is similar to (A.10) The left hand side is the payoff from consuming all income and leaving bequest of \( w_{t+1} = 0 \). The right hand side is the payoff from sacrificing consumption
of \((k - in)\), so that the \((i + 1)\)th generation is able to switch to more profitable occupation: working in industry. Inequality (A.13) represents the non-deviation condition for agents with initial wealth \(w_t = k\) and is the same as (A.11) because industrial workers are always able to switch to agriculture in the next generation.

From (A.12) and (A.13) it follows that the price must satisfy

\[
\frac{k + \beta n}{\beta a} \leq p^A \leq \frac{(1 - \beta + \beta^{i+1}) k + (\beta + \beta i - i) n}{\beta^{i+1} a}
\]

### Small Open Economy

If the world price is \(p^W < \frac{k + \beta n}{\beta a}\) then conditions (A.11) and (A.13) are violated. Therefore, agents with initial wealth \(w = k\) choose bequests of \(w_{t+1} = 0\) and deindustrialization results. If the world price satisfies \(p^W \in \left[ \frac{k + \beta n}{\beta a} \right] \) then deindustrialization is ‘involuntary’ in the sense that even though the price is high enough, that working in the industrial sector yields a higher one-period income than working in the agricultural sector, it is not high enough to provide incentives to sacrifice today’s consumption for future investment.
Appendix B

Proofs to Chapter 2

Proof of Lemma 10. Suppose that producer 1 chooses $c_1$, producer 2 chooses $c_2$ and $c_1 > c_2$.

We first solve the price stage. Consumer $\tilde{t}$ is indifferent between products if

$$v - \left( \frac{p_1 + \tilde{t}}{1 - c_1} \right) = v - \left( \frac{p_2 + \tilde{t}}{1 - c_2} \right).$$

Then the marginal consumer is

$$\tilde{t} = \frac{p_2(1 - c_1)}{c_1 - c_2} - \frac{p_1(1 - c_2)}{c_1 - c_2}.$$

Consumers in $[0, \tilde{t})$ prefer product with defect rate $c_1$ and price $p_1$, and consumers in $(\tilde{t}, 1]$ prefer product with defect rate $c_2$ and price $p_2$.

Producer 1 faces the demand

$$D_1(p_1, p_2, c_1, c_2) = \left[ \frac{p_2(1 - c_1)}{c_1 - c_2} - \frac{p_1(1 - c_2)}{c_1 - c_2} \right] \frac{1}{1 - c_1}$$

where the term in brackets is the share of the market served by producer 1 and the second term accounts for the repeated purchases due to defects.

Producer 1 solves the following problem:

$$\max_{p_1} p_1 D_1(p_1, p_2, c_1, c_2).$$
From the first order conditions the best response of producer 1 is

\[ p_1 = \frac{p_2}{2} \left( \frac{1-c_1}{1-c_2} \right). \]

Producer 2 faces the demand

\[ D_2(p_1, p_2, c_1, c_2) = \left[ 1 - \left( \frac{p_2(1-c_1)}{c_1-c_2} - \frac{p_1(1-c_2)}{c_1-c_2} \right) \right] \frac{1}{1-c_2}. \]

Similarly, producer 2 solves:

\[
\begin{align*}
\text{Maximize} & \quad p_2 D_2(p_1, p_2, c_1, c_2) \\
\text{to get his best response to be} & \quad p_2 = \left( \frac{c_1 - c_2}{2 \cdot 1 - c_1} \right) + \frac{p_1}{2} \left( \frac{1 - c_2}{1 - c_1} \right). \\
\end{align*}
\]

The intersection of best responses gives the equilibrium prices and quantities:

\[
\begin{align*}
p_1 &= \frac{1}{3} \left[ \frac{c_1 - c_2}{1 - c_2} \right], \quad p_2 = \frac{2}{3} \left[ \frac{c_1 - c_2}{1 - c_1} \right], \\
D_1(c_1) &= \frac{1}{3} \left[ \frac{1}{1 - c_1} \right], \quad D_2(c_2) = \frac{2}{3} \left[ \frac{1}{1 - c_2} \right].
\end{align*}
\]

Next, we turn to the solution for the first stage: choosing the level of defects. Producer 1 chooses the level of defects to maximize his profits:

\[
\begin{align*}
\text{Maximize} & \quad \pi_1 = p_1 D_1(c_1) - G(c_1) = \frac{1}{9} \frac{c_1 - c_2}{(1 - c_1)(1 - c_2)} - G(c_1).
\end{align*}
\]

The profit function is increasing in \( c_1 \):

\[
\frac{\partial \pi_1}{\partial c_1} = \frac{1}{9(1-c_1)^2} - G'(c_1) > 0.
\]

As a result, producer 1’s choice is independent of the choice of producer 2 and \( c_1 = c_{\text{max}} \).

Similarly, producer 2 chooses \( c_2 \) to solve

\[
\begin{align*}
\text{Maximize} & \quad \pi_2 = p_2 D_2(c_2) - G(c_2) = \frac{4}{9} \frac{c_1 - c_2}{(1 - c_1)(1 - c_2)} - G(c_2).
\end{align*}
\]
The first order condition for producer 2 is independent of $c_1$:

$$G'(c_2) = -\frac{4}{9(1 - c_2)^2}.$$ 

**Proof of Lemma 11.** We first solve the price stage. Consumer $\tilde{t}$ is indifferent between products if

$$v - p_1 - (1 + c_1)\tilde{t} = v - p_2 - (1 + c_2)\tilde{t}$$

Then the marginal consumer is

$$\tilde{t} = \frac{p_2 - p_1}{c_1 - c_2}$$

Consumers in $[0, \tilde{t})$ prefer product with defect rate $c_1$ and price $p_1$, and consumers in $(\tilde{t}, 1]$ prefer product with defect rate $c_2$ and price $p_2$.

Producer 1 faces the demand

$$D_1(p_1, p_2, c_1, c_2) = \frac{p_2 - p_1}{c_1 - c_2}$$

and solves the following problem:

$$\begin{align*}
Maximize & \quad p_1 D_1(p_1, p_2, c_1, c_2) \\
\end{align*}$$

From the first order conditions the best response of producer 1 is

$$p_1 = \frac{p_2}{2}.$$ 

Producer 2 faces the demand

$$D_2(p_1, p_2, c_1, c_2) = 1 - \frac{p_2 - p_1}{c_1 - c_2}$$

and solves:

$$\begin{align*}
Maximize & \quad p_2 D_2(p_1, p_2, c_1, c_2) \\
\end{align*}$$
to get his best response to be 

\[ p_2 = \frac{c_1 - c_2}{2} + \frac{p_1}{2}. \]

The intersection of best responses gives the equilibrium prices and quantities:

\[ p_1 = \frac{1}{3}(c_1 - c_2), \quad p_2 = \frac{2}{3}(c_1 - c_2), \quad D_1(c_1) = \frac{1}{3}, \quad D_2(c_2) = \frac{2}{3} \]

Next we turn to the solution for the first stage: choosing the level of defects. Producer 1 chooses the level of defects to maximize his profits:

\[ \text{Maximize } \pi_1 = \frac{1}{9}(c_1 - c_2) - G(c_1) \]

The profit function is increasing in \( c_1 \):
\[
\frac{\partial \pi_1}{\partial c_1} = \frac{1}{9} - G'(c_1) > 0.
\]
As a result, producer 1’s choice is independent of the choice of producer 2 and \( c_1 = c_{\text{max}} \).

Similarly, producer 2 chooses \( c_2 \) to solve

\[ \text{Maximize } \pi_2 = \frac{4}{9}(c_1 - c_2) - G(c_2) \]

The first order condition for producer 2 is \( G'(c_2) = -\frac{4}{9} \). \( \blacksquare \)

**Lemma 23** At least one of the producers choosing not to adopt return policy constitutes an equilibrium of the extended game.

**Proof of Lemma 23.** The equilibria in a two-stage asymmetric subgame (only one producer adopts return policy) are:

In equilibrium 1 producer with no return policy chooses \((p_1, c_1)\) and producer with return policy chooses \((p_2^*, c_2^*)\), where:

\[ p_1 = \frac{1}{3} \left( c_{\text{max}} - c^R (1 - c_{\text{max}}) \right), \quad p_2 = \frac{2}{3} \left( c_{\text{max}} - c^R (1 - c_{\text{max}}) \right), \]

\[ c_1 = c_{\text{max}}, \quad c_2 = c^R, \]

103
In equilibrium 2 producer with no return policy chooses \((p_1^*, c_1^*)\) and the producer with return policy chooses \((p_2^*, c_2^*)\), where:

\[
p_1 = \frac{2}{3} \left( c_{\text{max}} - c^{NR} \right), \quad p_2 = \frac{1}{3} \left( c_{\text{max}} - c^{NR} \right) - \frac{c^{NR}}{1 - c^{NR}}, \\
c_1 = c^{NR}, \quad c_2 = c_{\text{max}}.
\]

In each equilibrium producer with defect rate \(c_{\text{max}}\) serves consumers in \([0, 1/3)\) and the other serves consumers in \((1/3, 1]\).

Equilibrium 2 exists if condition \(c_{\text{max}} > \frac{c^{NR}}{1 - c^{NR}}\) is satisfied.

Fix the equilibria in all subgames, in particular, assume that producer 1 (row player) always chooses the defect rate \(c_{\text{max}}\). Then, the game is:

<table>
<thead>
<tr>
<th></th>
<th>No returns</th>
<th>Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>No returns</td>
<td>((c_{\text{max}}, c^{NR}))</td>
<td>((c_{\text{max}}, c^{R}))</td>
</tr>
<tr>
<td>Returns</td>
<td>((c_{\text{max}}, c^{NR}))</td>
<td>((c_{\text{max}}, c^{R}))</td>
</tr>
</tbody>
</table>

Denote by \(\pi_{ij}^1\) the profit of producer 1, where \(i \in \{\text{No returns}, \text{Returns}\}\) is the choice of producer 1 and \(j \in \{\text{No returns}, \text{Returns}\}\) is the choice of producer 2. Then the following inequalities hold:

\[
\pi_{1}^{NR,NR} > \pi_{1}^{R,NR} \quad \text{and} \quad \pi_{1}^{NR,NR} > \pi_{1}^{R,NR},
\]

where

\[
\pi_{1}^{NR,NR} = \frac{1}{9} \left( \frac{c_{\text{max}} - c^{NR} (1 - c_{\text{max}})}{1 - c^{NR}} \right) - G(c_{\text{max}}), \\
\pi_{1}^{R,NR} = \frac{1}{9} \left( \frac{c_{\text{max}} (1 - c^{NR}) - c^{NR}}{1 - c^{NR}} \right) - G(c_{\text{max}}), \\
\pi_{1}^{NR,R} = \frac{1}{9} \left( \frac{c_{\text{max}} - c^{R} (1 - c_{\text{max}})}{1 - c_{\text{max}}} \right) - G(c_{\text{max}}), \\
\pi_{1}^{R,R} = \frac{1}{9} (c_{\text{max}} - c^{R}) - G(c_{\text{max}}).
\]

It implies that ‘No returns’ is the dominant strategy for producer 1. Thus, there exists an equilibrium in the extended game in which at least one producer chooses not to adopt return policy. ■
Appendix C

Proofs to Chapter 3

Proof of Proposition 13. For a given price $p_t$ the return to skilled labor is determined from the condition that the price is equal to cost (3.13). Then, for given expected profits in the next period we get $\bar{\gamma}_t^A$ corresponding to this $\pi_t^{A*}$ from (3.14). For given $\bar{\gamma}_t^{A-1}$ the available supplies of skilled and unskilled labor when there are no credit constraints equal

$$L_{t,ncc}^S = M_t - A (1 - \bar{\gamma}_t^A) \quad \text{(C.1)}$$

$$L_{t,ncc}^U = \beta^A (1 - \bar{\gamma}_t^A) + \bar{\gamma}_t^A + 1 - M_t \quad \text{(C.2)}$$

When agents are subject to credit constraints ($w_t^A < 0$) the supply of unskilled labor is

$$L_{t,cc}^S = M_t - A (1 - \bar{\gamma}_t^A) (1 - F(-w_t^A)) \quad \text{(C.3)}$$

$$= L_{t,ncc}^S + A (1 - \bar{\gamma}_t^A) F(-w_t^A)$$

and the supply of skilled labor is

$$L_{t,cc}^U = \bar{\gamma}_t^A + F(-w_t^A) (1 - \bar{\gamma}_t^A) + \beta^A (1 - F(-w_t^A)) (1 - \bar{\gamma}_t^A) + 1 - M_t \quad \text{(C.4)}$$

$$= (\beta^A (1 - \bar{\gamma}_t^A) + \bar{\gamma}_t^A) (1 - F(-w_t^A)) + F(-w_t^A) + 1 - M_t$$

$$= L_{ncc}^U + (1 - \beta^A) (1 - \bar{\gamma}_t^A) F(-w_t^A)$$

The supply of the agricultural good is equal to the total unskilled labor available less the
total unskilled labor used by the masters. Thus

\[ X_t^s = L_t^U - M_t U_t \]

The aggregate supply of the industrial good is equal to

\[ Z_t^S = M_t (S_t)^\alpha (U_t)^{1-\alpha} \]

Using (3.3) it follows that

\[ Z_t^S = M_t U_t \left( \frac{S_t}{U_t} \right)^\alpha = L_t^U \frac{1}{(1 - \alpha) p_t} \]

Then, relative supply equals

\[ RS_t = \frac{Z_t^S}{X_t^s} = \frac{1}{(1 - \alpha) p_t} \frac{L_t^U}{L_t^U X_t^1} \]

Differentiating this expression with respect to \( p_t \) we get

\[ \frac{dRS_t}{dp_t} = \frac{1}{(1 - \alpha) p_t} \left( \left( \frac{L_t^U}{L_t^U X_t^1} \right)' - \left( \frac{L_t^U}{L_t^U X_t^1} \right) p_t \right) \]

So, we have that if

\[ \frac{d (L_t^U)}{dp_t} / L_t^U - \frac{d (L_t^U)}{dp_t} / L_t^U < \frac{1}{p_t} \]

then the relative supply curve is upward-sloping. This condition is always satisfied if

\[ \frac{d (L_t^S)}{dp_t} / L_t^S > \frac{d (L_t^U)}{dp_t} / L_t^U \]

Using (C.1) – (C.4) we get

\[ \frac{d (L_t^S)}{dp_t} L_t^U - \frac{d (L_t^U)}{dp_t} L_t^S = \left( \frac{dL_t^S}{d\gamma_t^A} L_t^U - \frac{dL_t^U}{d\gamma_t^A} L_t^S \right) d\gamma_t^A \frac{d (\pi_t^{A^*})}{dp_t} - \frac{d (\pi_t^{A^*})}{dp_t} \]

\[ = \left( 2A - M_t (1 + A - \beta^A) \right) \frac{d\gamma_t^A}{d (\pi_t^{A^*})} \frac{d (\pi_t^{A^*})}{dp_t} \]

Therefore, if the number of masters is less than \( \bar{M}^A = \frac{2A}{(1 + A - \beta^A)} \), then the percentage
increases in the supply of skilled labor is more than that in the supply of unskilled labor and, as a result, the relative supply curve is upward-sloping.

We can rewrite the relative supply in the following way:

\[ RS_t = \frac{1}{p_t} R(p_t) \]

where

\[
R(p_t) = \frac{1}{(1 - \alpha)} \left( \frac{L^U}{L^X} \right)
\]

\[
= \frac{1}{(1 - \alpha)} \left( \frac{1 - \alpha \pi^A L^S}{\alpha \pi^A L^S} \right)
\]

Using (C.1) and (C.2) it is straightforward to show that in the region when the apprentice's wage is positive, \( R(p_t) \) is increasing in \( p_t \). Since \( \frac{RD_t}{p_t} = \frac{(1 - \delta)}{\delta} \) is a constant and \( \frac{RS_t}{p_t} = R(p_t) \) is an increasing function, there is at most one \( p_t \) at which \( \frac{RD_t}{p_t} \) and \( \frac{RS_t}{p_t} \) intersect. Therefore, there exists at most one equilibrium of NCC-type.
Tatyana Chesnokova

Vita

Department of Economics
608 Kern Building
The Pennsylvania State University
University Park, PA 16802

Phone: (814) 237-8447 (Home)
E-mail: tat159@psu.edu
Website: http://www.econ.psu.edu/~tat159

Citizenship: • Russian Federation

Education: • Ph.D., Economics, The Pennsylvania State University, expected December 2004
• M.A., Economics, New Economic School, Moscow, 1999
• Diploma, Applied Mathematics, Moscow State University, 1997

Ph.D. Thesis: • “Essays in International Trade and Industrial Organization”

Thesis Advisor: Professor Kala Krishna

Fields: • Primary: International Economics, Industrial Organization
• Secondary: Microeconomic Theory

Papers: • “Immiserizing Deindustrialization: A Dynamic Trade Model with Credit Constraints”, 2003
• “Return Policies, Market Outcomes and Consumer Welfare”, 2003

Teaching Experience: • Graduate Instructor: Intermediate Micro (2 semesters)
• Teaching Assistant: Mathematical Economics (graduate), Advanced International Trade (4 semesters)

Research Experience: • Research Assistant to Professor Kala Krishna on project on indivisible consumer goods and complementarities, Summer 2002

Presentations & Professional Activities:
• Mid-West International Economics Meeting, Indiana University, Bloomington, Fall 2003
• European Association for Research in Industrial Economics (EARIE), Madrid, Spain, 2002