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Abstract

Chapter 1: Endogenous Market Thickness and Honesty: A Quality Trap Model
In this essay I use a dynamic framework to study the problems facing emerging or transition economies in which institutional arrangements (like ISO certification) to credibly signal product quality are absent. I use a two-sided asymmetric information model to analyze this where a seller’s discount factor and a buyer’s valuation for quality are private information. Sellers face a tradeoff between producing high quality which gives low one period profits but increases survival in the market for future periods and low quality which gives higher one period returns but bars the seller from future market activity. Hence, a seller’s choice is based on expected present and future demand. However, demand itself is endogenous and depends on expected quality. Hence, market thickness (buyer-seller ratio), product quality and the distribution of seller types get endogenously determined and multiple steady states may emerge. I find that a sufficient number of sellers need to be patient or profits from investing in high quality must exceed a cutoff value for multiple steady states to exist. With endogenous prices, at least one type of buyer is left with no surplus. Prices also play a role in equilibrium selection. Introducing technology involving ‘learning by doing’ may cause market segregation. Importantly, expectations about market thickness in determining quality matter only if sellers believe that market thickness will be less than one.

Chapter 2: Party Formation and Coalitional Bargaining in a Model of Proportional Representation (with Mandar Oak)
We study a game theoretic model of a parliamentary democracy under proportional representation. In our model, ‘citizen candidates’ form parties prior to the election, voting takes place and then governments are formed. If no single party has an absolute majority, coalition governments may emerge and we study the underlying coalition building process. The type of government that is formed in equilibrium depends on the parties’ seat shares, the size of the rents that ruling parties can extract from holding office and their ideologies. We show that, depending on the relative magnitudes of these factors, a variety of coalition governments may result. Coalitions may be minimal; each member of the coalition is necessary to retain the majority or may contain some redundant parties. Moreover, coalitions may be comprised of parties that are not adjacent in terms of their ideology, that is, there may be a coalition government made up of leftists and rightists from which centrists are excluded. We then look at how the outcomes of the coalition formation game affect the incentives for voter groups to strategically form parties. We compare the results of party formation under proportional representation with that under the plurality rule. We show, in particular, that Duverger’s hypothesis that proportional representation leads to more political competition than does the plurality rule may not hold. Our model explains the diversity of electoral outcomes observed empirically under proportional representation and also contributes to the literature on political competition by integrating models of political entry with models of coalitional bargaining.

Chapter 3: When neighbours report crime: Effects of incentives for crime reporting on crime rates
We examine the incentives for people to report crime in their immediate vicinity using a framework where only immediate neighbours can observe crime, crime affects the immediate neighbours, and reporting crime is a costly activity. As a result, each citizen
would rather have her neighbour do the reporting. Ceteris paribus, an increase in the number of complaints erodes the effectiveness of the police force. Being investigated is also costly to the non-criminals since they have to put up with unpleasant police interrogation. Police choose a level of (costly) effort which determines, for a given number of reports, its success in apprehending criminals and also affects the crime rate. If the cost of interrogation is not too high, multiple equilibria are possible, there are equilibria involving reports only by neighbours of criminals and criminals themselves and there are equilibria in which everyone makes reports regardless of their identity. Moreover, increasing the incentives to report could increase equilibrium crime rates. Hence, from a policy perspective increasing incentives for neighbours to report suspicious activities may prove to be counterproductive.
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1 Endogenous Market Thickness and Honesty: A Quality Trap Model

1.1 Introduction

In this paper we provide an explanation for the wide variation in product quality and demand across economies (or markets) facing similar constraints in that quality of a product is not verifiable before consumption when there are no institutional arrangements (like ISO certification) to provide credible signals. The basic idea is as follows: investment in quality is costly and thus worthwhile only when present and future demand for the product is expected to be high, while demand itself depends on the expected quality. Thus, if investment in high quality goods give low present returns, such investment will be undertaken only if investors (or firms) have optimistic expectations about future market activity. Therefore, in the absence of an ex ante coordinating arrangement, an economy may fall into a low quality, low demand steady state.

This is a scenario typically faced by developing, emerging and transition economies where buyers face uncertainty about the quality of the products they buy in the marketplace and sellers face uncertainty about their present and future prospects in the market. Information in these economies about past market transactions is very limited and exchange in the anonymous market place is potentially risky. In Russia and other transition economies, (see McMillan (1997), for instance, which discusses specific examples) it is reported that people get cheated if they buy from anonymous sellers, which is why very few people do. Since sellers have low expectations about meeting buyers in the market they find it optimal to cheat when they find a buyer, justifying the buyers' fears about transacting in the market. Thus, the economy may be caught in a vicious circle of expectations, which are self fulfilling, even though there may be nothing in the preferences of agents and the technology available to prevent the market from being very active with high quality products being sold. In such a scenario there is scope for intelligent policy intervention, which, by changing expectations, can take the economy from low levels of trade in the marketplace to high levels of self sustaining market activity.

Of course, this scenario is not confined to underdeveloped countries. The internet whose growing volume of trade is now widely acknowledged faces this quality uncertainty problem as well, which in turn affects demand. As a recent article in The New York Times (see References) pointed out ‘fraud had become a problem since the first online auctions...and auction fraud is now the most prevalent computer-related

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1 In other words this is the market for what is commonly called an ‘experience good’.

2 Neuman and Medvinsky (1997) note the dramatic growth of users and organizations reachable on the net in the last couple of years. They still find that actual commerce transacted on the net is low but growing rapidly—we believe the growth is continuing with better security measures for transactions on the net.
crime, according to the Internet Fraud Complaint Center. This affects buyers as well. The report mentions reactions of buyers to this quality uncertainty saying that this affects their future entry in online transactions. Interestingly, in view of the model specification we adopt in this paper, there seems to be some evidence (see Resnick and Zeckhauser (2001)) from eBay that buyers place much less credence on positive messages than on negative ones. Resnick and Zeckhauser’s data suggests that prices and probability of trade do not depend significantly on the number of positive messages received about a seller. However, negative messages seem to matter, and the extreme negative message is a prosecution for fraud. The auction site eBay reports, according to Resnick and Zeckhauser, a small proportion of ‘problem’ trades. Other, less well-known, internet sites might be more prone to such problems. Presumably, the expectation that there will be relatively few problems on eBay is self-fulfilling (at least, this is what this paper will argue).

A few more illustrations about the type of markets which approximate our environment will help us fix ideas. Since we get multiple steady states with some involving high market activity some fairly “thick” markets in developed economies also share features of our model. An example which readily comes to mind is that of phone cards which offer the same number of minutes per dollar but often have differing quality. Again, travel agents offering the same price for air tickets and hotel bookings often end up providing different quality products—some indulge in outright cheating while others provide less facilities than advertised. The same can be said for the quality of service at various hotels as well as the food served in different restaurants whose menucards read alike. This is even more widely seen in the service sector (the large casual labor market for instance) where typically a fair amount of variation is seen in quality, which cannot be inferred from the prices being asked.

1.1.1 Informal description of the model

To analyze this problem we develop a model of two sided asymmetric information to show how expectations about the quality of a product (good or service) induce a certain level of market thickness (by which we mean the relative abundance of buyers to sellers), which in turn induces a certain average level of quality. It is important to note that market thickness defined in this way is used to measure market activity. Thus the absolute size of the market (the total number of buyers and sellers) is not important in our analysis.

More specifically, buyers face uncertainty about the quality of the product they purchase and do not know the time preferences (discount factors) of individual sellers.

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3 Thus demand is endogenous unlike most models dealing with how demand can affect product quality (see Rogerson (1982), for example).

4 This distinction is worth noting as the term ‘market thickness’ has been used differently by different people in the literature. (see McLaren (2000) who, in the introduction, discusses the different uses this term has in the literature.)
Sellers face uncertainty about the valuations that buyers place on the quality of their products. Given the uncertainty that buyers face about quality, they may not find it optimal to purchase the product, while sellers facing this demand uncertainty may not find it worthwhile to invest in production of high quality products. Thus, multiple steady states may emerge - some involving low quality and low market thickness and some involving high quality and high market thickness.

We use a random matching technology to describe the meeting between buyers and sellers such that each seller meets the same expected number of buyers. As the survival of sellers in the market depends on the quality they produce (with high quality sellers surviving longer than low quality ones) we model the population dynamics describing the entry and exit of sellers. (Buyers are, however, short lived which is the same as assuming that they decide whether to purchase from the market each period based on the currently available market statistic without regard to their past experience) Given the population dynamics, the time preferences of sellers and the valuations of buyers, the possible steady state distribution of sellers, the average quality and the market thickness get endogenously determined. To focus our analysis on the characterization of the steady states we do not explicitly model prices to begin with but keep them exogenously fixed. We later relax this assumption and study how different pricing mechanisms affect the steady states and consider the effects on welfare.

1.1.2 Summary of results

We now provide a brief summary of the results we get. An important thing to note is the difference between a static version of our model and the steady state characterization a static version of our model yields the low quality, low market thickness outcome as the unique equilibrium while the dynamic model yields multiple steady states.

In characterizing the steady states, we derive conditions under which multiple steady states emerge. It turns out that the patience of sellers and the profits from investing in high quality must reach a certain cutoff value for multiple steady states to emerge. However, if the relative proportion of high valuation buyers and patient sellers is large compared to low valuation buyers and impatient sellers we get only one steady state apart from the no trade equilibrium. In doing a stability analysis we find that equilibria involving mixed strategies are unstable and those involving pure strategies are stable. We are also able to calculate the ‘push’ that a central authority needs to give to move the economy out of a ‘bad’ steady state.

In comparing fixed prices to (some version of) optimizing pricing behavior we find that most of our qualitative results about the steady states are not affected though for high enough prices intermediate steady states get ruled out which means prices may play a role in equilibrium selection. However, the distributional effects are ambiguous—the higher the proportion of low valuation buyers the lower are prices but
there is a trade-off between quality and consumers surplus—with low prices, expected consumer’s surplus may be higher but average quality is lower.

A couple of important by-products come out from this model. Since the steady state distribution of different sellers gets endogenously determined it points out the importance of time preferences and expectations about market conditions in determining the survival of new firms. A standard explanation in the literature is in terms of different firms facing different random shocks (which determines their exit). This model may perhaps provide a clue towards an alternate explanation. Another important result we get is that market thickness (which is an index of demand) matters only up to a certain level and beyond that quality is driven by cost conditions and preferences only. This is analogous to the market exhibiting Keynesian features (being demand driven) up to a certain point and then exhibiting classical characteristics (where supply conditions drive the economy). This sharp result is partly an artefact of the specific matching technology used but the general intuition is that market thickness is more important when the market is relatively thin. This may partly explain why even flourishing markets are not invasion free from low quality products. Finally, we discuss how this model may shed light on such diverse issues as the problems transition economies are facing to the different quality of health care in various urban centers of a developing nation like India.

1.2 Related literature

A brief survey of earlier related work will help us understand how the present work differs from earlier models of this type. The analysis of quality variation in markets characterized by asymmetric information (leading to market thinness), of course dates back to Akerlof (1970). However, unlike the static lemons model, the distribution of quality and hence the average level of quality gets endogenously determined in our model, which is more representative of certain markets where sellers can typically decide the level of quality they choose to produce. Moreover, the results in Akerlof’s paper are substantively different even if we allow for endogenous quality choice—yielding only one equilibrium. The reason is that the future expected payoffs are what may make costly investment attractive even though one period gains are lower. Hence, a static version of our model yields the low quality low demand equilibrium as the unique outcome.

Thus, in some sense, our model is closely related to the model on endogenous quality choice in the Industrial Organization literature (see Shapiro (1983), for example). These models typically use reputation to sustain equilibria involving investment in quality which show one period losses. On the other hand, we analyze an environment where reputation is hard to develop and there is only some kind of negative reputa-

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6 For variants of this which can yield multiplicity see Mas Colell, Whinston and Green (1998) p 438-444.
tion (or punishment). Hence, in our environment new players are indistinguishable from older ones and the average quality of the market is the only information that potential buyers of the product have. We discuss several markets where this is so.\footnote{The Internet has tried to develop a system of reputation but that is at best partially successful. ebay has a system of buyer feedback. The value of positive feedback left by buyers is of doubtful value, often left by friends or associates of the seller and are thus less reliable than negative feedback which closely corresponds with our model. (see the NYT article earlier cit. and Resnick and Zeckhauser (2001) for details. The latter note that in the data set they look at, existing sellers do not enjoy a boost in price over new entrants.)}

Moreover, demand is not endogenous in these models and thus one gets a unique outcome in these ‘reputation’ models.\footnote{There are endogenous quality choice models in the money literature as well but their purpose is to show how money acts as a uniform quality good and can help reduce the quality uncertainty when goods are exchanged against money as opposed to goods against goods.(See Williamson and Wright (1994), for an example of such work.)} Again, multiple equilibria models are not new in the literature, nor are the multiplier effects of policy intervention or other exogenous perturbations. In that way, this shares some of the features of most multiple equilibria models, in particular, exhibiting strategic complementarities as in the Cooper John model (see Cooper and John, 1988 and Cooper 1999). However, typically these models are static in nature and have an inbuilt increasing returns property either in the matching technology (see for instance Diamond 1982) or in the payoff functions of players. In other words, these pure coordination models represent one shot games where future expectations do not determine current actions.\footnote{For a somewhat different type of model focusing on failure of coordination leading to suboptimal equilibria see Basu (1986). In his model different conjectures lead to different equilibria, showing how crucially expectations matter .}

Such calculations about the future and comparison of current gains against future losses are however typical of a firm’s optimizing behavior-what varies is the value attached by different firms to current as against future payoffs. In this paper, we explicitly model this dynamic optimizing behavior. In its focus on multiplicity of equilibria as an explanation of underdevelopment, this relates to earlier literature, most notably, Murphy, Shleifer and Vishny (MSV,1989) but these papers also do not examine the intertemporal maximization behavior of agents and are more akin to the search models of coordination failure. Since there is no linkage across time it is quite consistent in the MSV model to have an economy coordinating on a high level of economic activity today to coordinate on a low one tomorrow. Thus, these models are not suited to analyzing transition dynamics, learning and the importance of expectations about future market conditions in shaping current behavior. More importantly, these static models cannot be used for studying endogenous quality choice and demand which is determined by the interaction between buyers and sellers. The present model, by contrast, explicitly models this intertemporal process and rationalizes the existence of multiple steady states when firms forego high current profit in order to yield higher future returns and has a structure which can be used to study the importance of out of equilibrium dynamics and forward looking behavior in changing the long run
state of the market or economy. It also models how the distribution of population evolves over time which few of these papers do. (An exception is Banerjee and Newman, 1993 who model how the distribution of population changes. However, their concern is with the interaction between occupational decisions and the distribution of wealth, which is very different from the present work.) This interlinkage across time has been captured in a somewhat different context by Ghosh and Ray (1996) where buyers make repeated entry into the market and hence market history matters but their paper deals with the levels of cooperation which can be sustained when bad past conduct of sellers cannot be punished. Another related work is that of Kranton (1996) which shows how increases in market thickness can cause alternate forms of exchange (like reciprocal exchange) to diminish, while the widespread use of personalized exchange can itself cause markets to remain thin, hence causing such exchanges to persist over time. The questions addressed viz. the interaction between two different institutions for exchange and the model she uses are completely different from this present work.

The paper proceeds as follows. The next section sets up the basic model. In section 4 we do a steady state analysis. Section 5 considers the implications of regulated pricing vs. (some version of) optimized prices. Section 6 discusses illustrations and extensions. Most of the proofs are in Appendix 1. Appendix 2 considers a slight modification of the basic model to show how technologies involving scale economies over time, perhaps due to ‘learning by doing’, helps us in overcoming the uncertainty about quality under certain parameter values.

1.3 The model

This is an infinite horizon model with discrete time. There are two sides of the market, buyers and sellers. Each period the market opens and two types of objects are on sale—a high quality type and a low quality type. Buyers have differing valuations for the high quality good. For simplicity, we consider only two types of buyers, those with a low valuation which we denote by $V_L$ and those with a high valuation denoted by $V_H$. (We assume that low valuation goods are uniformly valued by all buyers and we normalize it to 0.)\footnote{For an interesting study of how history can matter and related dynamics see Adsera and Ray (2000). There are also other dynamic models eg. models of repeated purchases (see Hendel and Lizzeri, 1999)) but their concern is very different from the present work.} Sellers are also of two types, those with a low discount factor $\delta_l$ and those with a high discount factor $\delta_h$.\footnote{The assumption of no gains from trade in the low quality good simplifies the exposition but makes no difference as shown in Appendix 2. Thus this assumption is not restrictive. However, we work out a case in the Appendix to show that it can make a difference when the costs are time dependent perhaps because there is ‘learning by doing’. In that case separating equilibria can emerge.} (This discount factor can be interpreted
as a survival probability.)\textsuperscript{12} Thus, there are four types of agents in the economy.\textsuperscript{13}

Cost, technology and endowments are as follows. High quality goods are produced at a higher cost than low quality goods—we normalize the cost of low quality goods to 0 and the high quality is produced at constant cost per unit denoted by $c$. However, given the asymmetry of information, all goods in the market are sold at a uniform price which we assume to be fixed exogenously at $p$.\textsuperscript{14} Note also that sellers have excess capacity and can produce goods to demand. Given constant unit costs, gains per unit from the high quality good is also a constant every period which we denote by $\pi = p - c$.

There is asymmetric information of the following type - buyers cannot distinguish between the low and high quality good before purchase. \textsuperscript{15} Moreover, they do not know the type of the sellers (i.e. the discount factor of an individual seller) but only the ratio of the two types of sellers in the population. Sellers also cannot distinguish the two types of buyers and know only the ratio of the two types. The past history of market transactions is summarized by two parameters giving the ratio of buyers to sellers (market thickness) and the ratio of high quality goods to low quality goods sold every period. The history of individual transactions that have taken place in the past is not known but these summary statistics are known to all agents. Given these, buyers and sellers have to form expectations about the market thickness and quality at the beginning of each period.

Matching is random in that at any instant in time each seller may meet $q$ buyers (i.e. $q$ is the expected number of buyers per seller every period i.e. it is the market thickness in that period)\textsuperscript{16}

Buyers are expected utility maximizers and base their entry decision on the expected gains from trade. Thus, the buyer’s decision is based on his valuation and his expectation of acquiring the high quality object. He enters if his expected utility from buying the good is greater than the price he pays for it. Sellers are also expected

\textsuperscript{12} By allowing randomizations we get continuity in our endogenous variables (quality and market thickness) which is why we do not believe this framework to be unduly restrictive.

\textsuperscript{13} It is important to mention that though we are dealing with finite numbers, we assume that no agent in the economy believes that his action has any influence on market outcome.

\textsuperscript{14} We endogenize prices in the next section.

\textsuperscript{15} The earlier example of phone cards comes readily to mind as an example of an experience good where one discovers the true value of the card only after one has used it—a high quality one gives the advertised number of minutes and quality sound, a low one would give less minutes than advertised and poor quality of sound.

\textsuperscript{16} The matching technology can be thought of as follows. For the case of less buyers than sellers, assign each buyer to a seller until there are no more buyers. Considering all possible permutations of sellers gives us $n_t/S$ as the matching probability. When there are more buyers than sellers, first assign the same number of buyers to each seller. The remaining buyers are matched as in the case with less buyers than sellers giving us the required result. For a continuum of sellers it is helpful to think of this as some approximation of large discrete numbers. See Binmore and Herrero (1988) who get this result for a continuum. Thus we circumvent the problem that for two continuous interval there is a one to one correspondence between them.
utility maximizers and maximize the present discounted value of lifetime earnings. Each period they can sell a low quality good and face a lower continuation payoff (in the form of being identified by the buyer who presumably spreads this information) from then on or sell a high quality good and face a higher continuation payoff. We assume that the future payoffs after selling the low quality good is 0. This can be thought of as being driven out of the market forever. The seller’s decision thus depends on his forecast about future market conditions (i.e. the future possibilities of being matched with a buyer which is the same as his forecast about future market thickness) and his discount factor. Specifically, we assume that the seller forms point estimates about future market thickness for all future time periods up to infinity. We do not however, specify an explicit mechanism by which his expectations are formed. Thus the sellers’ trade-off is a high one period gain and nothing thereafter to a low one period gain and a probability of future sales in the same market. From now on, selling a low quality object will be referred to as cheating. Buyers’ decisions to purchase from a seller will be referred to as entering the market.

To concentrate on the sellers’ problem we assume the buyers live only one period. Each period there is a potential pool of buyers some of whom choose to enter the market and others choose not to enter. If they enter they may purchase only one unit of the good. Every period there is again a fresh pool who base their decision on the currently available market information (i.e. the summary statistics available of past transactions) about the probability of being cheated.

The population dynamics of the sellers is however more complicated. Every period sellers who are matched and cheat are thrown out (i.e. dishonest sellers who are matched die with probability 1) and a fraction of the unmatched sellers (if \( q \) is less than 1) die. The rate at which they die can be calculated from their survival probabilities. At the same time new sellers of either type enter in an exogenously fixed proportion. Thus, given the fixed ratio at which sellers are replaced, the existing distribution of the two types every period together with the market thickness and the decisions of the two types determine the ratio of the two types (the state variable) in the population next period. In other words, the distribution of population evolves endogenously.

Thus, while for a buyer a strategy is simply an action i.e. a decision whether to enter the market or not enter based on his expectations about the probability of getting a high quality object, for a seller a strategy is more complicated and has to specify his action every period as well as for all periods to come and will essentially be a sequence of actions cheat and not cheat depending on past history and future expectations.

An equilibrium is a strategy profile where every player is playing his best response.

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17 This is merely a convenient assumption. All we need is that those who produce high quality goods face better future payoffs than those who produce low quality goods.

18 All this means is that buyers are not basing their decision on entering the market each period based on their past experience but on the current information about market conditions.
to the strategy of others. A steady state is a more restricted class of equilibria where every period the parameters of the model are unchanged—that is every period the same number of buyers enter and the same number of sellers cheat. Births equal deaths and the ratio in which the two types are born is exogenous and given by \( a \) and \( 1 - a \).

In steady state, given this inflow, the distribution of the two types of sellers in the population are unchanged.

More formally, let \( N \) denote the stock of potential buyers and \( n_t \) those who enter the market in period \( t \). Let \( S \) denote the stock of sellers which is a constant.\(^{19}\) Thus \( q_t = n_t/S \) (In steady state the time subscripts can be dropped.)

### Buyers’ objective function

Buyer of type \( i \) solves the following problem \( \max (xV_i - p, 0) \) \( (i = H, L) \) where \( x \) is the probability of meeting an honest seller. Thus, for a buyer, his strategy is simply whether to enter or not enter. We denote his set of strategies (which are the same as his set of available actions) by \( A \).

### Sellers’ objective function

In time period \( t \) seller \( j \) maximizes \( V_j(q_t, q_{t+1}, q_{t+2}, \ldots) = \max (V_C, V_{NC}) \) where the subscripts \( C \) and \( NC \) are used to distinguish the discounted payoffs from deciding to be dishonest in period \( t \) (cheat) and honest in period \( t \) respectively. The \( V \) function looks a little different depending on whether \( q \) is less than or greater than 1.\(^{20}\)

For \( q \) less than 1, \( V_C = pq_t + (1 - q_t)\delta E(V(q')) \) where \( V(q') \) is the continuation payoff if an optimal policy is followed from period \( t + 1 \) \( (q' \) is used as a shorthand for the sequence of future market thickness), \( E \) is used to denote the fact that this is the expected continuation payoff of the sellers based on his (point) estimates about future market thickness. This equation is explained very simply—with probability \( q \) a cheat meets a buyer, gets \( p \) units of money and \( 0 \) thereafter. With a probability \( 1 - q \) he does not meet a buyer and simply goes through the same optimization process again based on the expected value of the current market statistic. This is discounted by seller of type \( j \) \( (j = h, l) \) at the rate \( \delta_j \).

Similarly, \( V_{NC} = q_t(\pi + \delta V(q')) + (1 - q_t)\delta E(V(q')) \).

Thus a seller simply chooses the maximum of these. In the steady state we shall see that he will choose a stationary strategy (or randomize) when called upon to move. The set of possible strategies (which we denote by \( \Omega \)) is of course much larger.

Thus, possible (but not optimal) strategies for a seller are being dishonest in period 1, 2, ..and so on.\(^{20}\)

\(^{19}\)This can be thought of as a limit imposed by the available space in the marketplace—there are only \( S \) slots and we further assume that they are filled every period—hence total births equal total deaths in each period but the distribution of sellers outside the steady state can change. In steady state, the distribution every period is the same but is not equal to the constant proportion of births of the two types every period.

\(^{20}\)Note, that while randomizing, if a player randomizes with probability \( z \) over an action \( A \), we mean that \( z \) fraction of players play \( A \) with probability 1. As we are dealing with large numbers this does not make much difference, though it is, strictly speaking, correct only for a continuum
For \( q > 1 \) \( V_c = pq_t \) and \( V_N = \pi q_t + \delta E(V(q_t)) \)

Thus given \((n, S, A, \Omega, EU, V, \delta, N^L, S^l, x^\infty, q^\infty)\) we have all the components of a Bayesian game \((N^L, S^l)\) and the natural solution concept to employ is a Perfect Bayesian Nash (with out of equilibrium beliefs appropriately defined). However, in the next section we concentrate on a more restricted class of solutions viz. the steady state where the market parameters are unchanging.

### 1.4 Steady State Analysis

Here, we analyze the market when the parameters of the model are unchanging over time. There is a constant ratio of the two types of buyers and a constant ratio of the two types of sellers. Given that, the market thickness \((q = n/S)\) as well as the level of honesty \((x)\) is constant. (We keep \( p \) exogenously fixed in this section for convenience).

Formally a steady state is defined as follows

**Definition 1.1** A steady state is a pair \((n, x)\) for a given price, \( p \), such that

1. Buyers maximize expected utility.
2. Sellers maximize lifetime discounted payoffs.
3. \( 0 \leq x \leq 1, \ 0 \leq n \leq N \).

We now characterize the steady state for the two type case.

Let \( x \) be the probability of meeting an honest seller\(^{21}\) and let \( \gamma \) and \( 1 - \gamma \) be the (steady state) fractions of the two types of sellers \( \delta_h \) and \( \delta_l \) in the market. We use \( s_i \) to denote the probability that type \( i \) seller is honest. \((i = l, h)\) Thus, \( x = \gamma s_h + (1-\gamma)s_l \).

For a buyer, denote by \( b_i \) the probability of entry in the market and by \( n_i \) the number of type \( i \) buyers. \((i = L, H)\). Given our assumption on buyers’ valuations \( (V_L < V_H) \) and sellers’ discount factors, \((\delta_l < \delta_h)\) the admissible randomizations for buyers and sellers can be divided into the following cases:

**Buyer:**
1. \( b_L = b_H = 0 \)
2. \( 0 < b_L < b_H \)
3. \( b_L = 0.4 \) \( 0 < b_L < b_H = 1 \)
4. \( b_L = 1.5 \) \( b_H = 1 \)

**Seller:**
1. \( s_L = s_H = 0 \)
2. \( 0 < s_L < s_H \)
3. \( s_L = 0 \) \( s_H = 1 \)
4. \( s_L = 1.4 \) \( s_H = 1 \)
5. \( s_L = 1.5 \) \( s_H = 1 \)

Steady state equations giving us the fraction of each type of seller is obtained by equating total births and deaths. Births are in an exogenously fixed ratio. In a steady state, the fraction of each type of seller in the market will be unchanged. When there

\(^{21}\) We take that to be equal to the proportion of high quality goods sold which is a fair approximation for a large population. Since we are doing ex ante maximization this seems reasonable.
are fewer buyers than sellers the following two equations provide exact expressions for these quantities:\footnote{Note that the distribution of sellers in the population will not, in general, be equal to the distribution of new born sellers. This is because impatient sellers survive for shorter periods which is why the exogenous ratio of births must have higher proportion of impatient types than in the population.}

\[
S\gamma ((1-q)(1-\delta_h) + q(1-s_h + s_h (1-\delta_h))) = aB 
\]

\( (1) \)

\[
S(1-\gamma)((1-q)(1-\delta_l) + q(1-s_l + s_l (1-\delta_l))) = (1-a)B
\]

\( (2) \)

\[
D \equiv B 
\]

Here, \(a\) denotes the exogenously fixed ratio of type \(\delta_h\) sellers entering the population, \(D\) the number of deaths each period and \(B\) denotes the total number of births in that period. Note that \(D = B\) always holds where \(D\) is obtained from adding the first two equations. This is because we have assumed that the total number of births is always equal to total number of deaths (fixed number of sellers in the market). In addition, in steady state the above two equations hold i.e. in steady state the number of deaths of each type of seller is equal to the number of births of each type. The equation can be explained quite simply—the unmatched fraction of sellers \((1-q)\) die at a rate \(1-\delta\). The fraction of matched sellers is \(q\). They are honest with a probability \(1-s\) in which case they die at their natural rate of \(1-\delta\). The dishonest fraction \((qs)\) die with probability \(1\).\footnote{Of course, there is an integer problem here in that the number of sellers may be fractions. Since we are dealing with large numbers we ignore this. In any case this is not germane to our analysis.}

\(>\)From this we can find out the steady state ratio of sellers for the different types of randomizations. Together with the buyers’ and sellers’ incentive constraints we can get the possible equilibria in our model. The following Lemma gives us the simplified steady state expressions for the buyers’ and sellers’ optimization problem.

**Lemma 1.1** In steady state and for \(q < 1\) the sellers’ decision problem simplifies to the following rule:

Seller \(j\) is honest \(iff\) \(\frac{(p-c)q}{1-\delta_j} \geq \frac{pq}{1-(1-q)\delta_j}\) (is indifferent if this holds with equality).

**Proof.** In steady state \(\gamma_t = \gamma\), \(q_t = q\) and \(x_t = x\). Given this, a seller of type \(j\) chooses to maximize \(V = Max (V_C, V_{NC})\). Putting \(q_t = q\) for all \(t\) we get \(V_C = \frac{pq}{1-(1-q)\delta_j}\) and \(V_{NC} = \frac{(p-c)q}{1-\delta_j}\). Thus, the seller chooses the maximum of the two.

When \(q\) is greater than 1 the steady state equations get simplified. Now there is no uncertainty about meeting a buyer-\(q\) is simply the expected number of buyers every period. Thus, dishonest sellers die with probability 1 and the steady state equations reduce to
\[ S_j(1 - s_h + s_h(1 - \delta_h)) = aD \quad (4) \]
\[ S(1 - \gamma)(1 - s_l + s_l(1 - \delta_l)) = (1 - a)D \quad (5) \]

We can work out the different types of steady states using the admissible values of buyers’ and sellers’ randomizations. Together with the incentive constraints we can find out what steady states can be supported for different parameter values. We look at this in detail in the appendix. Even without the detailed algebra, however, we can understand what is going on quite clearly by drawing graphs of how the thickness of markets change (because of the buyers’ entry decision) with the steady state level of honesty and how the level of honesty changes with the market thickness. The intersection of the two curves gives us the possible steady states.

For certain parametrizations which yield multiplicity, we plot two graphs illustrating this. In figure 1 we plot the level of honesty \((x)\) as a function of market thickness \((q)\). Note that more thickness can never induce less honesty so the curve will never slope down.\(^{24}\) In figure 1 upto \(q_h\) both types prefer to be dishonest so \(x\) is zero. At \(q_h\) the sellers with discount factor \(\delta_h\) are indifferent to being honest and dishonest. So they randomize-as \(s_h\), which is the randomization probability of type \(\delta_h\) being honest rises, so does the steady state level of honesty. Beyond \(q_h\) type \(\delta_h\) strictly prefers to be honest and type \(\delta_l\) still prefers to be honest. However if \(q\) is less than 1 it increases the probability of matching and impatient sellers get knocked out faster which means that the steady state level of the impatient type gets lowered thereby raising the level of honesty. (The mathematics showing the convex shape is worked out in the appendix). At \(q_l\) the \(\delta_l\) type is indifferent and beyond that both types prefer to be honest. Note that this is not the only possible shape of the curve. There are some conditions on the minimum amount of patience and gains from trade for sellers to decide to be honest at some level of market thickness. Moreover, when the market thickness is such that \(q\) is unity or more we will see how these conditions entirely determine sellers behavior (beyond \(q = 1\) market thickness has no effect on the level of honesty). Thus this curve can be thought of as the sellers response curve.

In figure 2 we plot the response of buyers (and hence market thickness) to the level of honesty. The interpretation is similar but there is no convex portion because buyers live only 1 period and thus their steady state is not endogenously determined. We call this the buyers’ response curve.

In figure 3 we plot the two curves together and their intersection gives us the steady states. In general, we get multiple steady states. We now present some propositions about the existence of multiple steady states and bounds on market thickness \((q)\) which induce this. (Formal proofs are in the Appendix.)

**Proposition 1.1** If multiple equilibria exists, \(p\delta_j - c \geq 0\) for at least one \(j\) \((j=l,h)\) and \(N \leq \frac{c(1-\delta)}{(p-c)\delta_j}\). (Necessary condition)

\(^{24}\)The sufficient condition for multiplicity ensures this.
If \( p\delta_j - c \geq 0 \) for all \( j \) and \( N \leq \frac{c(1-\delta_j)}{(p-c)\delta_j} \) for all \( j \) or \( N \geq S \) then multiple equilibria exists. (Sufficient condition)

These are conditions on the minimum level of patience and gains from trade necessary to ensure that at least one type of seller finds it optimal to be honest at some level of market thickness. Moreover the sellers and buyers decision must be mutually consistent in the sense that the level of honesty of type \( j \) seller must induce that market thickness at which they choose to be honest).

**Proposition 1.2** Non degenerate multiple equilibria exist if the necessary and sufficient conditions in Proposition 1.1 are satisfied and in addition \( p/V_H < x^* < p/V_L \) and \( q_h < n_H/S < q_l \) where \( x^* = \text{the maximum value of } x \text{ when } q = q_h, \) where \( q_l = \frac{c(1-\delta_l)}{(p-c)\delta_l} \) and \( q_h = \frac{c(1-\delta_h)}{(p-c)\delta_h} \).

This proposition sets conditions on the proportion of the two types on either side of the market. Intuitively, this means that if there are too many patient sellers or high valuation buyers the only steady state involving positive trade will have the high quality goods on sale with all buyers entering. The intuition is clear from figure 3. As the steady states can be Pareto ranked we can speak of non degenerate states as first, second etc. in ascending order of quality and demand. Given the conditions in this proposition, we can characterize the first non degenerate steady state quite sharply i.e. we can calculate the randomizations by buyers and sellers that support this. Corollary 1 formalizes this.

**Corollary 1** If Proposition 1.2 holds then the first non zero steady state involves randomization by type \( H \) buyers and type \( h \) sellers with the level of honesty at \( x_h \) and the market thickness at \( q_h \). Thus, the corresponding randomizations can be calculated.

Figure 3 makes this quite clear.

**Proposition 1.3** If \( q > 1 \) then the equilibria are entirely dependent on the gains from trade \((p-c)\) and the discount factors \( (\delta) \).

The intuition is that beyond a certain level of market thickness \((q = 1)\) gains from being honest and cheating go up by the same factor so whatever was optimal for a seller at some market thickness \((\text{greater than unity})\) still remains optimal.

\(^{25}x^* \) can be calculated using the steady state equations. In case 2 worked out in Appendix 1 putting \( s_h = 1 \) we can find \( \gamma \) and hence \( x^* \).
1.4.1 A discussion on stability

We give an intuitive analysis of the stability of our equilibria. The difficulty of formalizing the argument arises from the fact that there are three variables which change over time viz. $q, x$ and $\gamma$. Thus it becomes impossible to analyze stability in terms of the figures since the sellers response curves are drawn for steady state values of $\gamma$. We can of course define a law of motion which will map $q_t, x_t, \gamma_t$ into $q_{t+1}, x_{t+1}, \gamma_{t+1}$. However, this will be quite complicated as the law of motion will look different in different segments of the $q, x, \gamma$ plane. We leave such a formal analysis for future research. Here, we provide an informal argument to show how the system behaves when players follow a static expectation rule to decide on their strategies every period. We will analyze the system for local perturbations around the steady state in different regions.

Consider the following rule to analyze stability in this model. If any point $(q, x, \gamma)$ is not a steady state of the model buyers assume that this will be the current value of $x, \gamma$ and optimize accordingly. In similar vein, sellers assume this to be the values of $q, \gamma$ which will prevail forever and reoptimize accordingly. Following this rule it seems that the odd equilibria are stable and the even equilibria are unstable. This is because the odd equilibria involve pure strategies$^{26}$ and the even equilibria involve mixed strategies. Therefore, for local perturbations around a steady state involving mixed strategies a large fraction of agents change their actions (they were indifferent between two pure strategies in equilibrium) causing a big movement away from the steady state while for pure strategies local perturbations (with static expectations) do not change the strategies of agents as the inequalities which decide their optimal action are unaffected.

To see this more clearly it is useful to divide the $(q, x)$ plane into different parts as the state variable (the distribution of $\delta_l$ and $\delta_h$) takes different values in different regions. Thus for any given $q, x$ and a corresponding distribution($\gamma$) of the two types of sellers our adjustment process will give us a new $q, x$ and $\gamma$.

For illustrative purposes, we consider small changes in $q$ and $x$ holding $\gamma$ at the steady state level and then track the system as agents reoptimize using the static expectation rule. Let us denote by $q^*(x)$ and $x^*(q)$ the optimal level of market thickness and honesty as functions of honesty and market thickness respectively. When we have a non steady state value of $x$ buyers adjust their behavior in the sense that next period’s buyers take that to be the expected value of $x$ and behave accordingly. Thus, when $x < x_h$ no buyer enters, so starting from a non steady state pair $(q_t, x_t)$, $q_{t+1} = 0$. Similarly for $x > x_l$ we would have $q_{t+1}$ jumping to $N/S$. For the seller things are a bit more complicated because of their population dynamics. The static adjustment rule is that both types take the expected value of the market statistic $q$ in period $t+1$ to be the same as the observed $q$ in $t$ and optimize accordingly. Thus, for $q < q_h$ both types cheat in period $t + 1$. If $q$ indeed remained

$^{26}$These pure strategies must be strict.
unchanged the population death rate next period would be given by the following expression
\( \frac{(1-\delta_l+q)\gamma_t}{(1-\delta_l+q)(1-\gamma_t)} \). Therefore, in the region where \( q < q_h \) and \( x < x_h \) we would have the ratio of the two types converging to the ratio given by the expression
\( \left( \frac{\gamma}{1-\gamma} \right) = \frac{(1-\delta_l)\alpha}{(1-\delta_l)(1-\alpha)} \). (This is quite intuitive since the initial value of \( x \) was positive which would have implied a higher value of \( \gamma \) for any positive \( q \). This is because at least a proportion of the high type of seller was being honest and hence their death rate was lower than that given by the right hand side of the above expression.

Now with \( x = 0 \) both types are getting knocked out at the rate given by the above expression lowering \( \gamma \) until equality is established).\(^{27}\) Note of course that this only implies local stability of the \((0,0)\) steady state-to see this take \( q = 0 \) and \( x = 1 \) and see that we get cyclical behavior. To see that the second equilibrium is unstable notice that for a value of \( x \) greater than the steady state value \( q \) jumps to \( \frac{2x}{\delta} \) (since at the steady state value they were indifferent any slightly bigger value would cause all of them to strictly prefer to enter the market). Similarly, with \( q \) greater than the equilibrium value all patient sellers switch to honesty, the value of \( \gamma \) goes up and the level of honesty rises. As the \( \delta_l \) sellers die faster \( x \) reaches the limiting value given by the intersection with the buyers response curve and the economy reaches the next steady state. The analysis being similar we do not describe this for all the regions.

1.5 Price Mechanisms

So far prices were exogenous in the model. It was a useful first simplification and it can be justified by thinking of the good in question having a regulated price (we can think of the government, for instance, fixing the price but being unable to control the quality-a typical feature of various developing and transition economies.). We now look at how the model behaves when we explicitly introduce a mechanism for price formation. We briefly consider the implications for this model when prices are posted in the market by sellers and see how it differs depending on whether this is posted before or after matching. We consider alternate specifications to see what sort of equilibrium prices can be supported.

More precisely, suppose the timing of the process is as follows:

Sellers post prices, buyers look at prices and choose the seller with the lower price-randomly if prices are the same. In this case it is not difficult to see that the sellers compete away all their surplus. However, this cannot be an equilibrium since at the level where sellers are robbed of all surplus they would prefer to produce the low quality good. Buyers correctly assess this leading to only low quality goods sold at a price of zero (or no trade) as the only sustainable steady state. This provides one more example of how competitive pricing can be harmful and can justify government intervention in regulating prices even when it cannot regulate quality. The recent debate about the effect of competitive pricing on the quality of LASIK surgery (see,

\(^{27}\)If \( x = 0 \) to begin with the equation is already satisfied.
for instance, The Washington Post, February 22, 2000 for a chatty piece summarizing these fears) seems to be based on this type of argument.

Now think of the same type of story but with different beliefs about the buyers interpreting a lower price as a signal of low quality. In that case, a whole range of prices can be sustained, from the price which satisfies at least one of the sellers incentive constraints to the monopoly price which robs the high type of all the surplus. Whether this entire range can be satisfied of course depend on the parameter values (cost, preferences and valuations) and the analysis would be much the same as with fixed prices. The welfare effects would vary as the different prices leave buyers and sellers with different surplus.

Finally, consider a mechanism a la Diamond (1971).

Buyers are matched with sellers randomly. Sellers announce prices. A buyer has information only about the price announcement of the seller with whom he is matched. A buyer then decides whether to buy or not. If a buyer does not buy, he can be rematched at a small cost $\epsilon$. We assume that in every match each type of buyer will have a cutoff value, if the announced price is above that he will want to be rematched (or leave the market)-otherwise he will buy.

We first provide an informal analysis of what the equilibrium of this game will look like. Intuitively, a few things seem evident—both type of sellers will announce the same price since any lower price will effectively signal his low quality good. This would contradict the fact that we cannot have a separating equilibrium in this environment (see Appendix 2). Moreover, given the rematching cost, any equilibrium which is efficient implies agreement in the first period.\footnote{It might be possible to construct Nash equilibria involving delays but there does not seem reasonable beliefs to support that. In particular, they would not be sequentially rational.} If sellers announce prices such that only the high type of buyer enters, the buyer will be rid of all his surplus. Moreover, in any stationary equilibrium, the price will be such that the low type of buyer will always be kept to his reservation level since at any lower price the seller can always charge a higher price, cause the low type to enter and be better off. This is also intuitive - if he does not enter at that price he gets zero surplus - if he enters the price at which he buys will be such that his expected net utility is zero. Next, we will see that a price of either $p_l$ or $p_h$ will prevail which are the prices which keeps the low and high type buyer with no expected surplus respectively. We can show this by taking any arbitrary lower price and show that the seller who is matched always does better by raising price by $\epsilon$. We formalize this in three propositions one of which basically spells out the incentive constraints which need to be satisfied.

**Proposition 1.4 (Price Proposition1)** For steady states with positive levels of trade to exist the steady state equilibrium price ($p^*$) must be the same for all sellers.

**Proof.** For the same types of sellers and buyers (i.e. discount factor and valuation) suppose $p^*$ was different i.e. the equilibrium price configuration had different
sellers charging different prices. Any price which differs by \( \epsilon \) cannot be an equilibrium since either the seller charging the lower price should increase the price and increase his profit or the seller charging the higher price should decrease his price to increase his expected profit. This is because at price \( p \) if the buyer is buying from a seller so will he at price \( p - \epsilon \). However, at price \( p \), if no one is willing to buy, clearly sellers must lower price for trade to occur. Now consider any set of arbitrary prices differing by more than \( \epsilon \). Consider any but the highest price at which the buyer is willing to trade in this configuration of prices. Call this arbitrarily selected price charged by a seller \( p_a \). He should also be willing to trade at \( p_a + \epsilon \) because of the rematching cost. Thus, for the same types of sellers and buyers (i.e. discount factor and valuation) the argument in Diamond (1971) holds. Suppose, different types of sellers charge different prices. This implies existence of a separating equilibrium which has been shown not to exist. Finally, different buyers cannot be charged different prices as sellers cannot distinguish types, hence price announcements cannot be conditioned on buyer types.

**Proposition 1.5 (Price Proposition 2)** A property of this equilibrium price \( p^* \) is that \( p^* \delta \geq c \) for at least one \( j (j=l,h) \) and \( \max (xV_i - p^*,0) \geq 0 \) for at least one \( i(i = L, H) \) where \( x \) is a steady state level of honesty consistent with \( p^* \). Further, if \( q < 1 \), then \( q \geq \frac{c(1-\delta)}{\delta(p-c)} \) has to hold.

**Proof.** From the necessary condition for multiplicity we know that \( p^* \delta \geq c \) for at least one \( i \), sets bounds below which the equilibrium price cannot fall in a non zero steady state. If this did not hold the seller’s incentive constraint to produce high quality is violated for both types - knowing this buyers will not pay any positive price for the good. The second part of the proposition is the incentive constraint of the buyer to enter which must be satisfied. Finally, the market thickness must satisfy the incentive constraint of the seller.

**Proposition 1.6 (Price Proposition 3)** Let \( p_l \) and \( p_h \) be the maximum prices at which the low type and high type buyer would enter the market (for a given market thickness). The equilibrium price \( p^* = p_l \) or \( p_h \) depending on whether \( \frac{N_s}{S} p_l \geq \frac{n_2}{S} p_h \) or vice versa for all sellers.

**Proof.** Consider some other price \( p' \) different from \( p^* \). Now if \( p' \leq p^* \) a seller should be able to raise it by \( \epsilon/k \) (\( k \geq 1 \)) and it would still be optimal for the buyer to buy (since buying at the next round of matching at \( p' \) is equivalent to buying in the present round at \( p' + \epsilon \)). Again \( p' \geq p^* \) lowers profits in expected terms. The price is \( p_l \) if the gains from selling at \( p_l \) to all buyers exceed that from selling at a higher price \( p_h \) to only the high type of buyer depending on the population distribution of the two types. Since \( \frac{N}{S} p_l \) and \( \frac{n_2}{S} p_h \) gives the expected revenue from the two types. The greater of the two determines the equilibrium price.
As with the case of exogenous pricing, a number of cases arise and many of them are similar. We thus do not do a detailed analysis for all the cases but note that the following possibilities can arise viz. : \( p_l \) is the price and all buyers enter because at that expected level of honesty \( (x) \) all buyers get positive expected utility \( (i.e. xV_i \geq p^* \forall i \) or \( p_h \) is the price and only the high valuation types enter.) Randomizations are of course possible, at \( p_l \) the impatient seller may randomize which may be sustained by a fraction of the low valuation buyers entering.

Some welfare considerations are however worth analyzing in comparing fixed price vs. posted prices. If the low valuation buyer was participating in the market by entering at the fixed price it is likely that he had positive surplus-in the posted price environment he is always held to his reservation level of utility. Thus the low type buyer can never be better off when sellers (optimally) choose prices. The high valued buyer may or may not be better off-if \( p_l \) is the prevailing price it would depend on whether this gives him more surplus than the fixed price situation \( (i.e. \text{ whether } xV_H - p \geq x'V_H - p^* \text{ where } x \text{ and } x' \text{ denote the steady state levels of honesty with fixed and posted prices respectively. Note that there can be no unambiguous answer as there are multiple steady states in both situations). Moreover, the low type creates an externality for the higher type as the higher the proportion of low types in the population of buyers the more likely that \( p_l \) will be charged. What about sellers? It might appear that he would be better off since he is now optimally choosing prices but the analysis is not so simple because we need to specify the steady states across which we are comparing. Thus this is true only when we compare steady states with the same level of market activity. We can also say something about quality. If the optimal \( p \) \( (i.e. \text{ the } p \text{ satisfying the three price proposition}) \text{ satisfies the conditions for both types of sellers to produce the high quality good, consistent with the buyer’s maximization exercise, such a price can effectively serve as a signaling mechanism indicating high quality. Thus, in such a case it might be worthwhile for a central authority with a preference for consumer welfare as well as quality to try fixing the price at a level at which both types of sellers are honest so that the buyer is also left with some surplus.}\n
That, of course, is not the only way to endogenize prices. Another asymmetry comes to mind-if dishonest sellers can be identified it might seem natural that we should also be able to identify honest sellers which means that existing sellers should enjoy a market advantage over new entrants. This is missing in the simple matching technology we have outlined. However, if old sellers charge higher prices, we can find a distribution of prices according to how well established a seller is. An equilibrium configuration would have different subgroups of buyers being catered to by different sellers—thus we would have price variation and market segmentation as we so often do in the real world. That, however, is a topic for a separate line of research which we do not pursue here. In our information structure only dishonesty is identified—the remaining market players are either honest or new entrants who cannot be distinguished. One can think of this as a society where complaints are recorded—a lack of
complaints indicates either good behavior or past inactivity and are indistinguishable for the agents at the beginning of market activity each period. This seems a reasonably good approximation for transition and emerging economies where past history of sellers are hard to come by. There is punishment for bad conduct (perhaps in the form of public announcements. See Greif, Milgrom and Weingast, (1994) for one example of such an authority where complaints could be recorded) but the emerging marketplace has no effective mechanism to record past individual activities. We can also justify this for cases where information is available for pooled samples only, as in the market for milk, where milk from different farmers is mixed and can only be traced back when there are complaints.  

1.6 Extensions and Applications

Here we give some illustrations of markets where our model may provide some insight and discuss some extensions to the basic model.

1.6.1 Illustrations:

1. Transition economies

Russia and several erstwhile communist countries underwent a transition and from centralized planning moved towards a market economy. Hence the potential to switch to high quality was created with a move to a market based structure. However, as noted in McMillan (earlier cit.) ‘New firms find it difficult to sell their products...(due to the) problem of finding potential customers’. Thus, there was inadequate market information and in most of Vietnam private enterprise ‘Most firms ..sold only to local customers’ causing the market to remain thin. To get out of it Mcmillan discusses the role of ‘chambers of commerce, credit bureaus, and trade organizations which seems consistent with the policy recommendations our model has to offer. The fact that this did in fact occur in various economies as a result of conscious policy bears out our argument that there was nothing missing in technology or preferences which was keeping these economies at low levels of trade with dubious quality on sale.

[Note: 29 As noted earlier, in ebay both good and bad comments can be recorded but good comments have less credence as sellers often leave good comments to boost each other’s market. Complaints or bad comments have to be verified.

In traded items across countries some products are not identified by individual seller/supplier. However when found defective the individual seller is tracked down and has to pay a penalty. The same is true for milk from farmers in most states in the US where the milk is pooled before being sold. Tracking farmers would be attempted only if there is a complaint. Moreover, the idea that pooled history is available and determines optimal decision has been explored in a different context by Tirole (1996) which conforms in a milder way to our idea that past indivdual histories are imperfectly observed.

Note, also, that there may be strategic reasons for buyers not to reveal information to others about ‘good’ sellers /products (as in the case of certain software but we do not analyze this here.)]
2. Differing quality of medical facilities in various big cities in India.

Individual buyers of medical services usually try to assess the quality of the service being offered at various centers but are unlikely to have detailed knowledge of individual providers and depend on the reputation of the center or ‘group’ with which the provider is associated. It is part of ‘folk knowledge’ that various cities in India offer medical facilities which show wide variation in quality, especially as perceived by consumers (patients). Some of the ‘Centers of excellence’ enjoy a market share which does not seem to stem from any fundamental difference in primitives i.e. the technology of medical centers and the qualifications of doctors. As perceived quality is hard to measure, there has been no rigorous study of this. However in detailed case studies undertaken by UNDP (see Bandyopadhyay and Gupta 1997 a and b) certain illuminating facts about the organs transplant scenario came out which are worth noting.

There is a wide variation in the number of transplants carried out in East India and South India - comparing big cities in the South, the average number of transplants in 2 major centers in Chennai (a big city in the South) total over 300 and there are quite a few other centers in Chennai where the yearly average is around 100. In contrast, in Calcutta, the major city in the East where such surgery is carried out, the total number in the two major centers in ten years adds to less than four hundred (other centers contribute a negligible amount - transplants are occasional features there.) Thus the ‘market thickness’ is vastly different with Calcutta having, on an average, customers (transplant recipients) of about thirty five in the two major centers as against over three hundred in Chennai. However, the qualifications of doctors are equivalent in both cities and the Hospitals have comparable facilities (both centers in Calcutta satisfy the rigorous specifications required to obtain licence under The Transplantation of Human Organs Rules, 1995). It also appears from the study (earlier cit.) that the quality, as measured by patient satisfaction, is low. From detailed questioning of patients about quality of care, behavior of doctors and nursing staff etc. the authors note widespread satisfaction in patients who have undergone transplants in Madras as opposed to Calcutta. Indeed they conclude ‘The phenomenon of transplant migration has been continuing over the years as more and more patients in need of a kidney move to other parts of India , notably Chennai, Vellore and Bangalore ...hospitals (in Calcutta) have suffered because of charges of negligence ..in health care.’ Thus, the hypothesis of quality being directly related to market thickness seems borne out.

Table 1 shows the differences in market thickness and customer satisfaction in the two cities:

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Location</th>
<th>Average no. of transplants/year (1988-96)</th>
<th>Patient satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apollo</td>
<td>Chennai</td>
<td>100-120</td>
<td>High</td>
</tr>
<tr>
<td>Willingdon</td>
<td>Chennai</td>
<td>100</td>
<td>High</td>
</tr>
<tr>
<td>Belle Vue</td>
<td>Calcutta</td>
<td>20</td>
<td>Low</td>
</tr>
<tr>
<td>Woodlands</td>
<td>Calcutta</td>
<td>15</td>
<td>Low</td>
</tr>
</tbody>
</table>
What is more interesting is that following the passage of The Transplantation of Human Organs Act, 1994, regulating transplants in the country, it has become more difficult to get permission in the South for non-local patients causing a shift towards getting transplanted in their nearest locality. This increased demand has been also matched by increases in the level of quality service provided by the above two centers in Calcutta which has recently seen high turnover coupled with higher patient satisfaction. In fact, this increased demand has caused another center (Wockhardt Medical & Research Centre) to start doing kidney transplants in Calcutta, clearly showing how markets are responding to demand. It is difficult to identify a factor which systematically explains this which is why we think that our multiplicity explanation is convincing.

3 The Service sector: Taxicabs at day and night.

An interesting phenomena that has been observed is the different service provided by taxicabs at day and night at various airports and railway stations. At night, when traffic is thin, cabs tend to provide poor quality, cheating customers by overcharging, taking longer routes and even indulging in outright robbery. At day the service is noticeably better when the flow of passengers is larger. Passengers, in turn, seem to respond to this by relying on taxicabs more heavily during the day and less during the night. This seems to be because the future gains from not cheating is higher at day in the form of getting more passengers than at night. At night time many more tourists make arrangements by asking friends and relatives to meet them or even waiting at the Airport till day before taking a cab. Assuming that the distribution of tourist types and cab drivers are the same in the day and night this seems a good example of demand and quality reinforcing each other. Data on the ratio of tourists who use taxicabs in the day as against those who use it at night should bear this out. Note, that this seems to be a worldwide phenomenon and attempts are made to deal with this by having prepaid cab service available at airports and railway stations where a centralized agency keeps track of passengers assigned to cabs whose license numbers are noted. (This is based on personal experience, conversations with several people and newspaper reports, thus no specific source is cited.)

1.6.2 Extensions and policy issues

We would also like to extend the model to cover cases where reputation matters and try to explain promotion and hiring within firms. Consider different tiers of jobs. Labor starts from the lowest rung (say working at Mc Donald’s at minimum wages), if they shirk they are dismissed by their employer but they can costlessly enter a similar job at the lowest rung. If they perform well, they are rewarded by being assigned jobs in a higher rung next period. Given differing time preferences, there is an equilibrium with labor employed at different rungs. Now consider the effect of improved monitoring. (say different branches of Mc Donald’s can develop a network to identify past cheats.) In general, the equilibrium flow of people moving to different
jobs will now vary. We expect that a society with a similar structure but different information processing environments will have differing levels of welfare—it would be interesting to study the evolution of a society as its information system improves. The ability of agents to signal their types and the costs of signalling could also be important determinants of why such economies could differ.

We think our simple model can be readily modified to capture the idea that there are good and bad pockets in an economy and that the transition from one to the other is often the result of an exogenous change in the environment. The static lemons model is often used to explain the presence of a suboptimal market size when increased trade is beneficial to every agent. This model provides a somewhat different justification for this inoptimality and provides a dynamic framework showing that a central authority facing nearly the same informational constraints can do better by breaking the initial coordination problem. It can successfully explain transitions (why a Mafia ridden state may re-emerge as a tourist attraction spot following stern law enforcement) while the static lemons model would predict that informational constraints will continue to inhibit the market.

A brief policy discussion seems appropriate. This framework suggests that markets with fairly similar fundamentals can converge to quite different steady states. Thus the question arises about what policies can take the economy out of a ‘bad’ steady state? The obvious answer is changing expectations but it is not quite clear what that means in real terms. It might therefore make sense to talk of ‘small’ changes in parameter value, say a one shot marginal increase in law enforcement. A temporary change in a parameter value matters because by perturbing expectations it can take an economy towards a ‘good’ steady state. As a practical matter, setting up quality certification boards would help—if producers have to go through a quality check they will no longer be able to produce low quality goods removing the uncertainty that buyers face. Thus buyers would no longer hesitate to come to the market—effectively once this process starts we would in fact no longer require such boards—we would have self-sustaining system. The market (like a brand name) would be trusted for its quality products and it would continue to live up to that to maintain its future prospects. Thus, in our model quality certification boards only have a temporary role to play. By contrast, the traditional argument that foreign competition would improve quality is not predicted by our model. If anything, matters can become worse—buyers having the option of switching to a known foreign brand would leave the existing market reducing market thickness further and worsen the incentives for sellers to invest in quality. A case in study is Russia post-Perestroika

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30 An interesting possibility is the use of local currency to signal demand to local investors who have to decide whether to invest in a costly technology while facing uncertain demand. This is a possibility suggested by Jayaraman and Oak (2001). However, there are several limitations as pointed out by the authors themselves. In particular, apart from the credibility issue of introducing such currency, for this to work in our set up, consumers must have no uncertainty regarding whether they want a locally produced good.

31 The fact that in a market with imperfect information about quality, late entrants may be
when Russians can travel to Europe and acquire good quality second hand cars—
the Russian car industry continues to produce cars of inferior quality.\textsuperscript{32} Thus, if helping
the domestic industry to grow is an objective, the entry of firms with established
reputation may be harmful. However, liberalization will make the consumer better
off (by lowering prices) at the cost of the domestic producer and welfare analysis
is not possible on an a priori basis. This merely points out that the introduction
of foreign competition is not unambiguously beneficial. The evaluation of welfare
requires a general equilibrium analysis which our model does not permit. A policy
prescription which does come out is that any liberalizing policy which allows access
to cheaper technology for producing high quality goods is unambiguously beneficial
as it increases the incentives for producers to invest in quality.

In conclusion, we would like to list the most promising areas for future research.
An important issue is finding out the conditions for prices to signal quality when low
quality goods also offer some surplus, In Appendix 2 we characterize a separating
equilibrium showing how technologies which involve decreasing costs over time (per-
haps because of ‘learning by doing’) can use prices to effectively separate high good
producers from low good producers, causing market segmentation and resolving the
quality uncertainty. Another area of future research is to study the impact of foreign
competitors in this environment when there is scope for technology diffusion. This
needs to be looked at in greater detail, in particular, studying the impact on easing
borrowing constraints and subsidizing producers who adopt this new technology.
Moreover changing conditions are likely to affect perceptions about the future - thus
it maybe worthwhile looking at how δ evolves over time . Another thing we want to
do is explicitly modeling transitions and learning and looking at the effect of exoge-
nous shocks to the system (like better law enforcement, shift of policy from protecting
domestic markets to liberalizing trade). We hope that this simple framework we have
set up will permit such issues to be explored in the future.

Appendix A: Proofs
We work out the steady states in detail for the case of more sellers than buyers
. For more buyers than sellers the expressions differ but the analysis is simpler—we
briefly discuss that after working through the various cases for $q < 1$.

\textbf{Case 1:} $s_l = s_h = 0$

The steady state ratio $\frac{\gamma}{1 - \gamma} = \frac{(1 - \delta)\alpha}{(1 - \delta_h)(1 - \alpha)}$ when $q < 1$

\textsuperscript{32}The important thing to note is that here, used cars seem to have better reputation than new
cars, perhaps because the counteracting institutions of the type mentioned by Akerlof (earlier cit.)
may have developed. An empirical study by Bond (1982) notes the absence of a market for lemons
in the used trucks industry. Thus, what is important is not the type of product in question but
whether there are credible ways to signal quality.
(otherwise the steady state equations is simply \( \frac{a}{1-a} \). However in equilibrium \( q = 0 \) since \( x = 0 \) and the optimal response is \( b_1 = b_2 = 0 \))

**Case 2:** \( s_l = 0 < s_h < 1 \)

The steady state ratio is found by equating the ratio of deaths of the two types to the exogenously given births (or inflow). Thus we have

\[
\frac{\gamma((1-q)(1-s_h)+q(1-s_l)+s_l(1-\delta_l))}{(1-\gamma)(q+(1-q)(1-\delta_l))} = \frac{a}{1-a}
\]

Now we can solve for \( q \) by looking at the sellers decision problem. For the type \( \delta_h \) to be randomizing it must be that he is indifferent to cheating and being honest. Hence we get

\[
\frac{(p-c)q}{1-\delta_h} = \frac{pq}{1-(1-q)\delta_h},
\]

where the left hand side represents the gains from being honest and the right hand side the gains from cheating. Solving for \( q \) we get

\[
q = \frac{(1-\delta_h)}{(p-c)\delta_h}.
\]

Substituting in the buyers entry problem we can find out the number of buyers who enter and from that \( \gamma \) and hence \( s_h \) can be solved. More precisely since \( S \) (the total stock of sellers) is known from \( q \) we can calculate \( n \) (those buyers who enter in that period). If \( n \) is greater than \( N \) (the population of potential buyers) then there is no solution. If \( n \) is less than the type of \( V_H \) buyers then we calculate the randomization \( \alpha \) so that only \( n \) buyers enter. At this it must be that these buyers are indifferent hence \( xV_H = 1 \) giving us the value of \( x \) and hence \( \gamma \). Thus \( h_2 \) can be calculated. If \( n \) equals the number of type \( V_H \) buyers then \( 1/V_H \leq x < 1/V_L \) and hence admissible ranges of \( \gamma \) and hence \( s_h \) can be found. (This would correspond to a continuum of equilibria). For \( n \) such that type \( V_2 \) always enters and type \( V_L \) is indifferent \( x \) can again be precisely calculated and hence \( \gamma \) and \( s_h \) can be solved. Finally if \( n = N \) then we can again solve for the admissible ranges of steady states and seller randomizations.

**Case 3:** \( s_l = 0 < s_h = 1 \)

The steady state equation is given by

\[
\frac{\gamma(1-\delta_h)}{(1-\gamma)(1-\delta_l)} = \frac{a}{1-a}
\]

and

\[
\frac{c(1-\delta_h)}{(p-c)\delta_h} < q < \frac{c(1-\delta_l)}{(p-c)\delta_l}
\]

must hold. Now for each \( q \) in this range find \( \gamma \) and hence \( x \). This gives us the level of honesty induced by the different values of \( q \). (This gives us the sellers response curve.) Now for each of this \( x \) so found find the level of \( q \) this induces by looking at the buyers maximization problem. This gives the buyers response curve. If the \( x \) induced by a value of \( q \) in turn induces the same \( q \) we have an equilibrium point.

**Case 4:** \( 0 < s_l < s_h = 1 \)

With the impatient sellers randomizing the steady state equation is given by

\[
\frac{\gamma(1-\delta_h)}{(1-\gamma)((1-q)(1-\delta_l)+q(1-s_l)+s_l(1-\delta_l))} = \frac{a}{1-a}.
\]

The value of \( q \) is found by equating the gains from honesty and cheating for the \( \delta_l \) type of seller giving us

\[
q = \frac{c(1-\delta_l)}{(p-c)\delta_l}.
\]

Plugging in the buyers maximization problem we get \( x \) and substituting these values in the steady state equation gives us the equilibrium randomization for the seller.

**Case 5:** \( s_l = s_h = 1 \)

24
With both types being honest the steady state becomes
\[
\frac{\gamma(1-\delta_h)}{(1-\gamma)(1-\delta_l)} = \frac{a}{1-a}
\]
Note this must induce both types of buyers to enter (since \(x = 1, V_i > 1\) for \(i = L, H\) by assumption ) -with only high quality objects on sale all potential buyers must find it optimal to enter(otherwise they would not be potential buyers!).

When \(q > 1\) we know that sellers behavior is independent of \(q\). Thus their choice is dependent only on whether \(p\delta > c\) or the converse. Depending on that each type of sellers behavior is determined. As there is no more uncertainty about matching we do not have the part with \(1 - q\). The analysis is similar except that if \(s = 0\) for any type at \(q \geq 1\) it is always 0.

Proof of proposition 1:

**Proof.** Suppose not, This would imply that \(p\delta < c\). We have to show that this cannot be if the gains from cheating are always lower than the gains from honesty. We can see this from the sellers discounted present value of gains from the two types of behavior

For \(q < 1\) we have \(\frac{pq}{1-\delta} > \frac{pq}{1-(1-q)\delta}\) which gives us \(q > \frac{c(1-\delta)}{(p-c)k\delta}\) (substituting \(p - c\) for \(\pi\)). Since \(q < 1\) implies that \(\frac{c(1-\delta)}{(p-c)k\delta} < 1\). For this to be true we must have must have \(p\delta > c\) contradicting the hypothesis that \(p\delta < c\)

For \(q > 1\) \(\frac{pq}{1-\delta} > pq\) which again gives us \(p\delta > c\).

Sufficiency :Consider the \(q\) for which both types of sellers are honest. (This will always be the case for \(q \geq 1\) since the condition for honest behavior viz. \(p\delta > c\) holds for both \(i\)). Thus, \(x = 1\) which implies both types of buyers enter. Thus apart from the no trade equilibrium the full honesty equilibrium with full entry holds. For this we of course require that the \(q\)at which this occurs is not incompatible with the total number of potential buyers i.e.\(qS \leq n\).

Proof of proposition 2:

**Proof.** Figure 3 makes this quite clear . The first intersection on the positive quadrant is ensured by the given condition. We now need to show at least a second intersection exists. If the configuration is as shown in figure 3 it is obvious. Otherwise by proposition 0 (sufficient condition) we know that at \(q\) max there is an equilibrium with full honesty and full entry.

Proof of corollary 1:

**Proof.** The proof is clear from figure 3. Given condition 2 \(x_2\) must lie to the left of \(x^*\) and \(q_2\) must lie below \(n_2/S\) . Thus, this gives us the first intersection of the 2 curves beyond the origin. The buyers randomization is got by simply equating \(\alpha n_2 = q_2\).\(\alpha\) is the randomization probability of buyers of valuation \(V_2\).For the seller we calculate this from the steady state equation (case 2 ).

Proof of proposition 3:
Proof. When \( n \geq S \) the analysis is much simpler. Now the sellers’ behavior is entirely driven by cost conditions and preferences. That is easy to see. Consider the gains from cheating—that is simply \( pq \) (there is no chance of being unmatched so sellers are driven out from the market after matching and selling the low quality product. From honesty it is simply \( \frac{\pi q}{1-\delta_i} \). Thus, for honesty to be optimal \( pq \geq \frac{\pi q}{1-\delta_i} \), which implies that \( p - p\delta_i \geq \pi \). This can be written as \( p\delta_i \geq c \). This is independent of \( q \). Thus, cost conditions (which determine profit for a given price \( p \)) and preferences (\( \delta \)) entirely determine the behavior of sellers. ■

To show that the steady state level of honesty rises as market thickness increases and the inequalities in case 3 hold.

Proof. The steady state equation is \( \gamma \frac{\pi q}{1-\delta_i} = \frac{\alpha}{1-a} \) which gives us \( \gamma = \frac{\pi q}{1-\delta_i} = b \) \((q + (1-q)(1-\delta_l)) \) where \( b \) is a constant \((= \frac{\alpha}{(1-a)(1-\delta_i)}). \) This simplifies to \( \gamma = 1 - \frac{1}{1+b(1-\delta_l+q\delta_l)} \). Taking derivatives we have \( \frac{d\gamma}{dq} = -\frac{2\delta_l^2}{(1+b(1-\delta_l+q\delta_l))^3} < 0 \) justifying the shape of the curve. ■

Appendix B: Separating equilibria and technology

Here, we show that there is no separating equilibrium (in steady state) given that the low valued good has some value in trade when both sellers face the same technology, which has the same unit cost over time. However, we see that separating equilibria may emerge when cost of technology for individual firms decrease over time.

Let \( V_{H}^h, V_{H}^l, V_{L}^h, V_{L}^l \) denote the value of the high good for the high type of buyer, the value of the low quality good for the high type of buyer, the value of the high good for the low type and the value of the low good for the low type respectively.

One of the conditions for a separating equilibrium is that one type of seller prefers to produce the high type of good and the other the low type and charge separate prices which reveal that.

WLOG let the impatient seller prefer the low quality good and let that be sold at a price \( p_l \) and the high quality at \( p_h \). The market thickness for the two goods are \( q_h \) and \( q_l \). For a separating equilibrium it must be that

\[
\frac{p_l - c_l}{1-\delta_l} q_l \geq \frac{p_h - c_l}{1-\delta_l} q_h
\]

which is independent of the discount factor! Hence no separating equilibrium is possible, because the seller type who gets less can mimic the other.\(^{33}\)

What do we require to have separation?A possibility is where the technology shows scale economies over time (learning by doing perhaps). Let \( k \) denote the scale factor

\(^{33}\)This contrasts with the literature on prices signaling quality. Two papers which show separation and partial separation are Wolinsky (1983) and Bandyopadhyay, Chatterjee and Vasavada(2001). They also have asymmetric information and sellers who can choose the quality to produce which is akin to our environment but their results differ considerably as the tradeoffs are somewhat different. In particular, the sellers have different costs which cause this separation while we allow every seller access to the same technology.
in the sense that \( c_t = k c_{t+1} \).

A separating equilibrium is characterized by a sequence of prices of the two goods, market thickness, costs of production of the two goods. Under the steady state assumption the conditions this needs to satisfy can be summarized by the following equations. (We work this for the case of less buyers than sellers-the case with more buyers as well as where one of the situations involve more buyers than sellers and another the converse can be easily worked out.)

\[
\frac{p_t q_t}{1 - \delta_t} \geq Max(\sum q_h (p_h - k^t c_h) \delta^t, \frac{p_h q_h}{1 - (1 - q_h) \delta^t}) (IC_l)
\]

\[
\sum q_h (p_h - k^t c_h) \delta^t_h \geq Max(\frac{p_h q_h}{1 - (1 - q_h) \delta^t_h}, \frac{p_l q_l}{1 - \delta^t_h}) (IC_h)
\]

\[
V^h_H - p_h \geq V^l_H - p_l (IC_H)
\]

\[
V^l_L - p_l \geq V^h_L - p_h (IC_H)
\]

The first condition \((IC_l)\) says that for a low discount type of seller the gains from producing the low quality good and selling it as one outweighs the one period gain from selling the low quality good as a high quality good (cheating) or from the present discounted value of selling the high quality good. Similarly \((IC_h)\) says the high discount type prefers to sell a high quality product to cheating or selling a low quality product forever. The buyer’s incentive constraints are given by \((IC_H)\) and \((IC_H)\) which says that high valuation buyers prefer to buy the high quality good at the going price to a low quality good while low quality buyers prefer to buy the low quality good. The separating equilibrium is possible because the value of the declining cost of technology has different value to different sellers-the value of future gains from the technology is more to the patient seller. The fact that the low quality good also has surplus implies that for some prices and market thickness impatient sellers prefer selling low quality goods at a lower price to making high one period gains from passing them off as high quality ones. (We assume of course that the individual rationality constraints are satisfied.). Note, of course, that this just shows the existence of such prices—we also need to show under what mechanism these would be equilibrium prices.

This result is robust to prices declining as costs of production in the high technology falls. This is because for the seller who adopts the technology for the first time the costs still remain high as there are no spillovers. With spillovers which make cost decline over time for all producers regardless of whether they adopted this earlier this no longer holds and there may be an optimal time before everyone switches to the technology of the ‘high end’ good. Studying the effects of such spillovers on the evolution of the market is an interesting area for future research.

We note that this does not in any way rule out multiple equilibria - this separating equilibria is one of various possible equilibria.

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References:


2 Party Formation and Coalitional Bargaining in a Model of Proportional Representation

2.1 Introduction

One of the salient features observed in democracies which follow the system of voting by Proportional Representation (PR) is the preponderance of coalition governments. Empirically, the type of such coalition governments seen under PR is fairly diverse, both in terms of size and the ideological closeness of the coalition partners. In particular, minority, minimal winning as well as surplus governments have been seen in various parliamentary democracies under PR. A study of 15 post World War II European democracies by Gallagher, Laver and Mair (1995) finds that about 35% of coalitions are ‘minimal winning’, 36% are minority coalitions (i.e. the coalition members constitute less than half the seats in the legislature) and the rest of the coalitions (29%) are surplus (i.e. they have more than the required number of parties needed to constitute a majority). From the point of view of ‘ideological closeness’ such democracies also show diversity in that coalitions are not necessarily ideologically ‘connected’ and may often leave out the ‘median’ party. However, until recently, coalition government was modeled in the literature as a ‘50% or more coalition’. No minority government was considered. Nor did surplus governments occur in equilibrium.

We construct a model which can explain the broad differences in coalition formation and the diversity in policy outcomes in democracies under the broad rubric of PR documented in the empirical literature. A key feature of our model is the endogenous determination of the number of candidates (or parties), as well as the post election ruling coalition. Our paper contributes to the literature on formal models of parliamentary democracies under PR in two ways. First, it provides an explanation for the various types of coalition governments in terms of the constituent parties’ ideologies and the benefits associated with holding political office. Second, it sheds light on the incentives under PR for various groups to form political parties. This provides a basis for reexamining the Duvergerian hypothesis (see Duverger (1964)) that the number of parties under PR is greater than those under the plurality rule.

A recent paper by Diermeier and Merlo (2000) also allows for the diversity in

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34 Diermeier and Merlo (1999) look at 313 European governments from 11 multiparty democracies out of which only 20 have one party controlling more than half the seats.
35 See also Laver and Schofield (1990) chapters 4 and 5 for details. The distinction they make between the members of government and members who support the government is that the members of the government are directly involved in policy making and enjoy additional benefits of office by having control over various departments of the government.
36 About 25% of coalitions do that. See Laver and Schofield (earlier cit.).
38 It is worth noting that this was formalized much later. See Fey (1997), Feddersen (1992) and Palfrey (1989) for formalizations of the ‘plurality leads to 2 party rule’ part of the law.
outcome in terms of size at the coalition formation stage. However, both their paper and another recent paper by Baron and Diermeier (2001) use ‘efficient’ bargaining in which coalition partners negotiate the efficient policy and redistribute the surplus using unlimited side payments. Baron-Diermeier-Merlo (henceforth BDM) assume that there is perfect transferability of ‘perks of office’ as well as perfect commitment at the coalition formation stage to making such transfers. These assumptions do generate the observed phenomena of diversity in size, but BDM are unable to analyze the ideological diversity of coalitions. Our model can explain both the diversity in size as well as ideological distance by showing how these phenomena are perfectly rationalizable given the trade-offs that parties face between policy compromises and power sharing.

However, our analysis is not confined to the policy making stage alone. Even as we characterize the type of government emerging from a legislature, it is important to study whether such a legislature would emerge from rational agents forming parties, given the payoffs from the ex-post coalition formation game. In particular, the rules governing coalition formation and policy making give rise to certain incentives at the party formation stage for ideological groups to launch parties—i.e. stand for elections. We study how groups will act strategically in deciding whether or not to form parties (at a cost). We find that an equilibrium in which the median group is the only one to form a party always exists. Of course, this equilibrium is not unique. As the equilibrium decision of each group depends on their beliefs about the behavior of other groups we get multiple equilibria. We compare the entry decisions made under PR with those under plurality rule. Our results shed some light on Duverger’s Law (see Duverger (earlier cit.)) which states that while the plurality system leads to a two party regime, PR shows no such tendency. In doing so, we also formalize some of the issues that have led political activists to suggest the superiority of PR over plurality voting.39

A simple illustration will help clarify these issues. There are three potential parties named 1, 2 and 3. Suppose that in every district parties 1 and 2 each enjoy the support of about 40% of voters in each district while about 20% support party 3. Assuming sincere voting, under plurality, either party 1 or party 2 wins in every district. and thus party 3 has no incentive to contest the elections. However, under PR each party will get seats proportional to vote shares and thus 3 will have about 20% of the total seats in the legislature. Hence, under plurality, 3 will have no incentive to stand because it knows it will not get represented. Under PR, party 3 will have an effective role as it may well be asked by either 1 or 2 to be part of a coalition government which means that 3’s supporters get effective representation. However, party 3 may use its power to try to get ‘disproportionate’ gains or switch sides leading to political instability. This is of course a stylized example but it provides an intuitive understanding of some of the issues involved. Thus, this example shows that to get

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39 See the online PR library at http://www.mtholyoke.edu/acad/polit/damy/prlib.htm, Amy (2000) and Barber (2001)
a clear understanding of these issues we need a model which can shed light both on political entry as well as post election coalition formation and policy making. Our paper attempts to do this by building a formal model of PR starting from the party formation stage to government policies undertaken.

There are two crucial features of our treatment of political power. First, we assume that political power is vested in various ministerial offices and the person in charge of a particular ministry is entitled to that power. Without explicitly modeling the issues regarding non-verifiability of such power, we assume that such power cannot be (ex-post) transferred or (ex-ante) credibly promised to the people outside the ministries. Second, bargaining within the parties forming a coalition leads to their agreeing to share the ministries in the proportion of their seat shares. This is an empirical regularity that is consistently observed across time and space in democracies with PR (see Browne and Fendreis (1980) and Laver and Schofield, (1990)).

It should be noted that the idea that politicians value power or rents from office goes back to Downs (1957). The basic idea is that holding office involves receiving certain perks. This may not just be receiving a high salary for being a minister, but includes being able to grant favors to those who patronise the minister. It is thus distinct from the utility that goes with being able to influence policy by being in the government. From now on, we shall use the terms value of power and rents from office interchangeably.

Given the outcome associated with each coalition, several viable coalitions may be possible. Thus, the prerogative to initiate the process of coalition formation becomes crucial. In practice, a party called ‘the formateur’ is conferred such a right. Across the world, there are different rules governing the selection of a formateur. We will consider two protocols that are commonly observed and previously modeled. The first protocol is due to Diermeier and Merlo (1999). According to this protocol, each party in the hung legislature is probabilistically recognized as a formateur with the recognition probability being equal to the fraction of seats won by that party. Diermeier and Merlo argue in favor of empirical support for this protocol. Another protocol is due to Austen-Smith and Banks (1988). According to this protocol, the parties are sequentially recognized as formateurs, starting with the largest party, followed by the second largest party and so on. In each of the protocols, if the attempt at government formation fails, then the status-quo policy is implemented under a caretaker government. We characterize the coalition formation game as described above and then solve backwards for the election and party formation stage. Here is a brief summary of the main features of our model and the important results.

First, at the coalition formation stage we show how, in equilibrium, our model generates the types of coalitions that we see in the real world. Minority, minimal

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40 We describe the bargaining procedure in the next section.
41 For a recent dynamic model which empirically estimates the value of being a politician see Diermeier, Keane and Merlo (2002).
42 This is actually a modified version of Baron and Ferejohn (1989)
winning as well as surplus coalitions occur depending on the identity of the formateur, the relative importance of ideology to the ‘rents of office’, the seat shares of parties and the status quo policy i.e. the policy implemented if a coalition cannot be successfully formed. Interestingly, coalition structure is responsive to the value of power in the following sense. For a certain range of power, the type of coalitions proposed by a formateur both in terms of size and with respect to ideological closeness can keep changing. For example, when the rents are moderate the coalitions formed may not be connected while at higher ranges the coalitions formed in equilibrium may again be connected. However, we find that there exists an upper bound on power beyond which the nature of coalitions formed do not change. Interestingly, beyond that range surplus coalitions do not form in equilibrium. Another important issue which emerges is that once equilibrium considerations are taken into account, what appears as surplus coalitions in the data, are not surplus in the sense that the coalition may not really be carrying extra parties. The reason is as follows—given any formateur, only one of parties A and B may be necessary for the coalition to win the vote of confidence in the house. However, A and B will decide to join a coalition only if both are invited together as their joint presence is necessary to skew the policy sufficiently towards them for it to be more attractive to either than the status quo.

The bargaining protocol also has an important bearing on the outcome. If instead of the one shot random recognition protocol, we follow selection in order, with low power, the unique subgame perfect equilibrium is for the median party to form a minority government. This is independent of the order of recognition and the location of the status quo. The result goes through with endogenous, costly party formation— only one party would form and as cost becomes small, it is formed around the median voter. However, as the gains from office increase both ‘connected’ and ideologically distant coalitions can form which is in keeping with the results of the random recognition protocol.

At the entry (or the party formation) stage the presence of multiple equilibria does not allow us to do a complete characterization of the political process. However, it is interesting to note that the ‘citizen candidate’ framework of candidate entry which we use is robust to the institutional details of coalition formation. In particular, the citizen candidate model looks at equilibrium candidate entry under plurality and depending on costs generate one, or two candidate equilibria. The coalition formation and policy outcome of our model generates similar types of equilibria including equilibria in which the median group is the only candidate as well as equilibria involving spoiler candidates. However, we find that three or more parties can also form unlike plurality rule which lends some support to the theory of greater political competition under PR. We illustrate using some examples why this may not be necessarily true.

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\(^{43}\)See Osborne and Slivinski (1996) and Besley and Coate (1997).

\(^{44}\)The median group is the group containing the median voter. It should also be noted that the median party is the group having the support of the median voter. When the median group contests elections, the two are of course the same.
Hence, our conclusion is that as far as the normative debate on PR vs. plurality is concerned we need to understand institutional details better to make meaningful comparisons.

The rest of the paper is divided as follows. Section 2 presents the related literature in more details. Section 3 presents the formal model, section 4 solves the coalition formation and policy making game and derives the results on the type of coalitions formed including examples in multidimensional policy space. In section 5 we add the party formation stage and prove existence and multiplicity of equilibria. Section 6 does a robustness analysis involving the bargaining procedure and assumptions about voting behavior. Section 7 discusses the empirical implications of our model and concludes. The appendix derives results for a symmetric 3 party case which BDM analyze to facilitate comparison with their model.

### 2.2 Related literature

Our work is related to several strands in the literature, particularly with several papers on coalitional bargaining, party positioning and endogenous party formation. It also draws on the insights provided by classic works on electoral systems, the activist literature on PR and several case studies which throw light on actual coalitional structure.

The study of what type of coalitions will form in equilibrium dates back to Riker (1962). However, the concern in Riker’s work is with the division of a fixed ‘pie’ which members of the winning coalition are entitled to. Hence, he predicts a minimum winning coalition i.e. the minimal winning coalition made of the smallest number of members. Till recently, notwithstanding the empirical evidence on minority and surplus coalitions, most formal work in this area predicted minimal winning coalitions. When ideology is considered there are no longer compelling reasons to predict a minimal winning coalition. Instead, following Axelrod (1970), the natural thing to predict would be a ‘minimum winning connected’ coalition-i.e. a coalition that does not leave out a partner who is in between two coalition partners on the ideological dimension. In practice, even that is not seen and ‘disconnected’ coalitions are not uncommon. Recent work by Diermeier and Merlo (earlier cit.) fills the gap by building a model which is capable of explaining the variation in size of the coalition. However, they assume that parties can commit to a position and make unlimited transfers. This implies that all members apart from the formateur pay to be in the coalition and are left at their reservation level. Moreover, they are unable to analyze how coalition formation depends on party size, as well as the relative importance of power in office to ideology. Our paper attempts to fill this gap, as well as analyze the other extreme when parties cannot commit to policies at the coalition formation stage. Interestingly, the institutional details governing coalition formation imply that even as the value of power gets very high (and hence Riker’s intuition should apply) minimal

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45See Laver and Schofield (earlier cit.) Chapter 5 for details.
coalitions are not the norm—depending on the identity of the formateur minority governments can form. At the opposite end of the spectrum, these institutional procedures imply that even with low gains from office minimal winning connected coalitions may not occur.

There are also a fair number of models of party formation/strategic entry starting from the ‘citizen candidate’ models of endogenous candidate entry to more recent papers by Riviere (2000), Morelli (2001) and Osborne and Tourky (2002) (in the recent papers a distinction is made between a candidate and a party). These papers either assume that the winner is selected by plurality rule or they model the post election policy outcome as a majority rule game, thus missing one of the most important aspects of PR which is coalition formation. There are also papers dealing with strategic entry under more general outcome functions most notably Dutta, Jackson and LeBreton (2000) but the generality of the paper makes it impossible for them to generate any sharp predictions except that under complete information at least one player (candidate) will behave strategically.

An attempt to model the fact that PR promotes diversity has been made by Ortuno Ortin (1997). The paper assumes that the policy is a weighted average of the 2 parties policy with the weights being proportional to seat shares. The main aim is to provide an explanation of why policies pursued might deviate from the median voter’s position. Hamlin and Hjortland (2000) integrate this approach with the citizen candidate literature. However, the assumption of vote weighted average does not capture the institutional details of coalition formation and government policy making which is an important aspect of democracies under PR.

A more complete analysis of the electoral process has been made by Austen Smith and Banks (earlier cit.), Baron (1993) and Baron and Diermeier (earlier cit.). All three papers look at models of parties committing themselves to announcing a policy, given their preferences and voter distributions. However, as pointed out by Alesina (1988) parties committing to positions other than their ideal point, in a one shot game is problematic. Austen Smith and Baron and Diermeier also note that with ‘strategic voting’ (Baron (1993) uses sincere voting) minority parliaments may form in equilibrium and voters do not always vote for the party closest to their ideal point.. The first result is of interest as it provides a justification for dealing with the coalition formation stage under a minority parliament. These papers do not deal with party formation and their parties are only endogenous in the sense that they choose positions.

In summary, we differ from these papers in two major ways. First, we make different assumptions about coalitional bargaining (in particular, by assuming no commitment and non transferable utility). Second, we integrate the ‘institution free’ citizen-candidate approach to politics with the rich institutional details of parliamentary democracy under PR.
2.3 The Model

The form of PR we model here (which is followed in most European countries) is the so called ‘party list’ PR, where the party puts up a list of candidates and voters cast a single vote for a party. We assume sincere voting i.e. we assume that voters cast their vote for the party closest to their ideal point and randomize with equal probability if they are equally close to more than one party. In this paper we abstract away from integer problems and ‘minimum floor rules’. Hence, seat shares are realized exactly in proportion to vote shares for each party. Once seat shares are realized, if any party has more than half the seats that party is called to form the government. Otherwise, parties are selected randomly to form a government with the probability of selection being equal to its relative seat share.

There are $N$ groups of citizens. We denote by $\mathcal{N} = \{1, 2, ..., N\}$ the set of groups of citizens. Let $1 \leq N < \infty$. It is assumed that all the members of a group are identical with regard to preferences on the policy space and $n_i$ is the size of group $i$. The policy space is assumed to be the unit interval. Let $u_i : [0, 1] \to \mathbb{R}$ denote the utility function representing the preferences of a member of group $i \in \mathcal{N}$. We assume that parties and groups have convex distance preferences defined over the policy space given by a utility function $u(|x_i - y|) + Z$ where $x_i$ is agent $i$’s ideal point, $y$ the implemented policy and $u(\cdot)$ satisfies, $u(0) = 0$, $u' < 0$, $u'' < 0$ and $Z$ is a linear term denoting preferences over rents from political office. Thus, parties also care for the benefits associated with office which we shall loosely refer to as power and the total rents available will be denoted by $P$. This can be enjoyed only by parties which serve in the government. There is a cost of contesting an election for a party which we denote by $\delta \geq 0$.

The timeline is as follows:

<table>
<thead>
<tr>
<th>$t=1$</th>
<th>$t=2$</th>
<th>$t=3$</th>
<th>$t=4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>candidate entry</td>
<td>voting</td>
<td>government formation</td>
<td>policy making</td>
</tr>
</tbody>
</table>

In the candidate entry stage groups simultaneously choose whether they will launch a party or not. Let $e^j \in \{0, 1\}$ denote group $j$’s entry decision where $e^j = 1$ if the representative member of the group chooses to launch a party, and $e^j = 0$ otherwise. Let $e = (e^1, e^2, ..., e^N)$ denote the profile of entry decisions. Formally, $C(e) = \{i \in \mathcal{N} : e^i = 1\}$ denotes the set of parties that is generated by the entry profile $e$.

Given an entry profile $e$, and the voting behavior, we can calculate for each $i \in C$, the vote share of $i$. Denote the vote share of entrant $i$ by $s_i$. If the set of parties is $C$, let $V_i(C)$ denote the expected utility of party $i$ in equilibrium in the government.

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46 In practice, there is often a ‘threshold’ number of votes that a party has to win to get elected in the legislature.

47 In principle, we should consider mixed strategies and let $\sigma^j$ denote the mixed strategy of group $j$ with $\sigma$ denoting the profile of mixed strategies. However, it does not add to our model so we do not carry excess notation.
formation stage. The reason we can uniquely map entry decisions into seat shares stems from our assumption about sincere voting. This, together with the assumption that voter preferences are single peaked also enables us to calculate the vote transfers if any one party enters or drops out i.e. the votes gained or lost by a party as a result of entry or exit by another party. The parties which gain or lose as a result of entry or exit by another party will be referred to as neighbors. For any one dimensional policy space this concept can be made more formal as follows.

For each group $i$, denote the set of its non-neighbors, $M(i)$ as follows:

$M(i) = \{ j : \text{there exists a party } k \text{ s.t. } x_i < x_k < x_j \}$

Consequently, denote by $N(i)$ the set of $i$’s neighbors, where $N(i) = \mathbb{N} - \{ i \cup M(i) \}$.

For any set of entrants $C$ and $C'$ such that $i \in C, C'$ and $N(i) \subset C, C'$ we must have $s_i(C) = s_i(C')$, where $s_i(C)$ and $s_i(C')$ denote vote shares of party $i$ when the set of parties is $C$ and $C'$ respectively. This captures the fact that the voters have single peaked preferences.

Let $\{C, \{s_i\}_{i \in C}\}$ denote a legislature composed of parties in the set $C$ where party $i$’s seat share is denoted by $s_i$. Following proportional representation, it is assumed that the parties get seats in the legislature in proportion to their vote shares. Note that we abstract away from minimum vote share requirements and from the integer constraints, thus we denote by $s_i$ both the vote share and the seat share for party $i \in C$. A government is formed out of the parties that constitute the legislature. We assume that the following ‘government formation game’ describes the process of government formation. The government selection process has 3 stages: Formateur selection, Proto Coalition selection and Vote of Confidence. We describe each stage briefly.

1) Formateur selection If there exists a party $i$ with $s_i > \frac{1}{2}$, then party $i$ is asked to be the formateur with probability 1. Otherwise, party $i \in C$ is asked to be the formateur with probability $s_i$. 2) Proto Coalition selection The formateur makes a one shot offer to a subset $D \subseteq C$ to form a government. If the offer is unanimously accepted by the constituents of $D$, then $D$ proceeds to seek a vote of confidence of the legislature. If the offer is not unanimously accepted then a caretaker government is formed. 3) Vote of Confidence The members of the legislature vote for or against $D$ forming the government. If strictly more than 50% of the legislature votes for $D$, then $D$ forms a government. Otherwise a caretaker government is formed.

Following government formation we have the policy making stage. At the policy making stage the parties constituting the government bargain over the policy and share of the spoils of office. If $D (\neq \emptyset)$ is the set of parties in the government and if $\pi_i$ is the relative seat share of $i$ with respect to $D$, then the implemented policy is $x_D = \sum_{i \in D} \pi_i x_i$ and party $i$ receives a share $\pi_i P$ of political power. It there is a caretaker government in office, then a pre-specified policy $x_\emptyset$ is implemented.

48 Note that if a caretaker government takes over none of the parties enjoy any $P$. The caretaker government is not a player in our model and its only role is to implement this prespecified policy $x_\emptyset$ if the government formation game fails.
cases of special interest, when $D = \emptyset$, we are said to have a ‘caretaker’ government; when $D = C$, we are said to have a ‘national’ government; when $|D| = 1$, we have a single party government; and when $|D| > 1$, we have a coalition government in office.

Given that this is a multistage game of perfect information we shall be looking at subgame perfect equilibria. We shall refer to the equilibrium of the whole game as the Political equilibrium and the equilibrium of the continuation game starting with any given legislature as the legislative equilibrium. We postpone a formal definition till we solve the game.

2.4 Solving the Legislative Model

We will solve the game backwards. Thus, we will first solve for the coalition formation and policy making stage for a given legislature. In the next section we shall look at party formation and study the incentives generated by the parliamentary game for party formation. Thus, in this section we start with a given seat share for each party. There are two stages in the legislative game viz. government formation and policy making.

2.4.1 The Government Formation and Policy Making game

We assume that each party in the legislature acts as a cohesive decision making unit which tries to maximize the payoff of its representative member. The bargaining procedure by which this occurs is the following. Once the coalition wins the confidence (investiture) vote, policy making and division of the spoils of office is decided by the members bargaining over the division of weights assigned to their party. The bargaining protocol is a N person Rubinstein alternating offer game (see Rubinstein (1982)) where each member of the coalition gets to make an offer in a prespecified order. Hence, a bigger party gets more turns at proposing. The stationary solution, when the discount factor between periods tend to 1, leads to the outcome being the weighted average of the coalition members.

Let $v_i(D)$ denote the average payoff of a member of party $i$ when $D$ is the ruling coalition. If $i \notin D$, then $v_i(D) = u(|x_i - x_D|)$ and if $i \in D$, then $v_i(D) = u(|x_i - x_D|)$.

See Chen-Ying Huang (2000) who uses the same idea that a bigger coalition gets more turns at proposing.

This follows from the fact that we have concerned ourselves with a division over weights which is equivalent to a pie division problem and the results of the N person game goes through for our environment as well. Together with the assumption that a bigger coalition gets more turns at proposing implies that our result follows directly from Chen-Ying Huang (earlier cit.). To make the status quo non binding, we also need to assume that the break up of a coalition has sufficient cost so that the outside option is sufficiently unattractive to be binding for any member. Hence, any disagreement leads to a reversion to the status quo and coalition members suffer an additional loss in utility from the break up.

Note that the N person problem is reported in several places, see Sutton (1986) for instance, and the discussion in Osborne and Rubinstein (1990).
Let \( s_D \) be the ‘size’ i.e the seat shares of coalition \( D \).

Let \( v_i(\emptyset) \) denote the payoff of a member of party \( i \) when there is a caretaker government. At the vote of confidence stage, the members of party \( i \) will vote for the proposed government \( D \) if \( v_i(D) \geq v_i(\emptyset) \) (we assume that when indifferent, a party member votes for the proposed government). Let \( A(D) \) denote the set of parties that would vote for the proposed government \( D \) and let \( s_{A(D)} \) denote its size. If \( s_{A(D)} > \frac{1}{2} \), then \( D \) forms the government. Let \( W \) denote the set of proto coalitions that will win the vote of confidence. Formally, \( W \equiv \{D \in 2^C \text{ s.t. } s_{A(D)} > \frac{1}{2}\} \). Now we come to the proto coalition selection stage. At this stage the formateur \( k \) must choose the proto coalition. Let \( Y \) denote the set of proto coalitions that are unanimously preferred by its constituents over the status quo. Formally, \( Y \equiv \{D \in 2^C \text{ s.t. } v_i(D) \geq v_i(\phi)\} \).

Thus, every coalition member has a veto power in that it can decide not to be in the coalition. Hence, unanimity is required among the selected members for a coalition to be formed. Let \( D_k \) denote the proto coalition most preferred by a member of party \( k \), i.e. \( D_k = \text{argmax}_{D \in W \cap Y} v_k(D) \). For simplicity, we assume that \( D_k \) is unique for each \( k \) (otherwise choose with equal probability).

Thus, a legislative equilibrium can be formally defined as follows:

**Definition 2.1** A legislative equilibrium is a collection of proto coalition \( D_1, D_2, ..., D_N \) such that \( \forall k \in C, D_k = \text{argmax}_{D \in W \cap Y} v_k(D) \)

Note that existence is not a problem as the sets \( W \) and \( Y \) are well defined. Hence, \( D_k \) is well defined.

### 2.4.2 Results

Before stating our main results on the parliamentary stage it is useful to make precise the types of coalitions we had described in the introduction. If \( s_D < \frac{1}{2} \) we have a minority government, otherwise the government may be minimum winning or surplus. If we find that there is a party \( i \in D \) such that \( s_{D-i} > \frac{1}{2} \), (where \( s_{D-i} \) is the seat share of the coalition \( D \) without the party \( i \)) we would normally say that the government is a surplus government, otherwise it is a minimum winning government. However, \( D - i \) i.e the coalition without party \( i \) may not belong to \( W \cap Y \). In that case, it seems more appropriate to call such governments minimal winning and reserve the term surplus if in addition to \( s_{D-i} > \frac{1}{2} \), \( (D - i) \in W \cap Y \). Hence, in this model the following will be taken to be a surplus coalition.

**Definition 2.2** A coalition \( D \) is said to be surplus if \( s_{D-i} > \frac{1}{2} \) and \( (D - i) \in W \cap Y \).

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51 Note that party \( i \) receives \( \pi_i P \) part of the power. Hence, the per party member share of power is \( \frac{\pi_i P}{s_i} \). Since \( \pi_i = s_i \), we have the per capita share to be \( \frac{1}{s_D} P \).
We will provide an illustration of this while describing one of our results. Regarding ideological closeness, the following definition of connected coalitions will be used.

**Definition 2.3** A coalition is said to be connected if there does not exist a party \(i\) outside the coalition whose ideal point lies inside the line joining the extreme points of the coalition members ideal points.

We now provide the main results of our legislative game regarding coalition size and ideology. As the purpose of the paper is to be able to explain the diversity of coalitions we have a set of results showing what type of conditions lead to what type of government rather than one main result predicting a particular type of government. The first result concerns the size of coalitions that can form in equilibrium.

**Proposition 2.1** An equilibrium government \(D_k\) may be minimal winning, minority, surplus or even caretaker.

**Proof.** We shall show by constructing examples that each of the governments mentioned above can form.

1. **Minimal Winning** Let \(C = \{1, 2, 3\}\) with \(x_1 = 0, x_2 = 0.4, x_3 = 1\) and \(x_\emptyset = 0.7\). Let \(s_1 = s_2 = s_3 = \frac{1}{3}\). Assume that \(P = 0\). Given these parameters, \(W \cap Y = \{\{1, 2\}, \{2\}, \{2, 3\}, \{1, 3\}, \emptyset\}\). Suppose that party 1 is the formateur. If 1 proposes the proto coalition \(\{1, 2\}\), then the implemented policy is 0.2 which is closer to \(x_1\) than the policy of any other government in \(W \cup Y\). Hence \(D_1 = \{1, 2\}\). This is a minimal winning coalition.

2. **Minority** Take the same set up as above except that party 2 is the formateur. Since \(\emptyset \in W \cap Y\), 2 will propose that it should form a minority government.

3. **Caretaker** Same set up as before except that \(x_\emptyset = 0.35\). Let 1 be the formateur. \(W \cap Y = \{\{2\}, \emptyset\}\). Since \(x_\emptyset\) is closer to 1 than \(x_2\), 1 will propose a caretaker government. (Alternatively, 1 will propose a minority government by it and that will get defeated at the investiture stage)

4. **Surplus** This requires a different set up. With three parties surplus government cannot take place. We construct an example using five parties. Let \(C = \{1, 2, 3, 4, 5\}\)

\[
x_1 = 0, s_1 = 0.04, x_2 = 0.1, s_2 = 0.05, x_3 = 0.5, s_3 = 0.4, x_4 = 0.7, s_4 = 0.11, x_5 = 1, s_5 = 0.4, x_\emptyset = 0.7.
\]

Let 3 be the formateur. There is a range of \(P\) for which party 3 chooses a surplus coalition of \(\{2, 3, 4\}\) even though 2 is redundant in the sense that the coalition of \(\{3, 4\}\) can win the confidence vote. Notice that we need to check two things here. First, whether 3 will prefer the \(\{2, 3, 4\}\) surplus coalition to all other coalitions. Second, whether 4 will accept the offer. Calculations show that the surplus coalition will have a policy which is approximately 0.5. It is easy to see that 2 will support the coalition. Therefore, it is sufficient to check this against the minimal winning coalition i.e. the \(\{3, 4\}\) coalition where the policy is 0.54. Thus, we require the following conditions to simultaneously hold.
a) 3 prefers this to the other coalitions in \( W \cap Y \) because \( U_2(0.5 - 0.5) + 0.71P > U(0.5 - 0.54) + 0.78P \), and

b) acceptance of 4 given by \( U_3(0.7 - 0.5) + 0.2P > U_3(0.7 - 0.7) \) i.e. party 4 prefers this to the status quo.

While it is sufficient to construct examples to prove that these types of governments can occur in equilibrium it is worth noting that these are not knife edge results in the sense that changing the numbers slightly do not change the results. Moreover, in the appendix, we analyze a special case which BDM do and show that for that particular scenario, only two types of coalitions can form in equilibrium. Also, certain additional insights follow easily from our coalition formation game. The following propositions formalize them.

**Proposition 2.2** When the median party is the formateur it always forms a single party government.

**Proof.** It is easy to see that this belongs to \( W \) as a majority of members prefer this to the status quo. It is clearly in \( Y \) as the median party prefers this to the status quo. Since the median party is the formateur it maximizes its utility as it implements its ideal point and enjoys the entire \( P \).

**Proposition 2.3** Assume that \( P = 0 \) and that \( x_\emptyset \in [x_{\text{min}}, x_{\text{max}}] \), then there cannot be a national government unless the implemented policy of the national government exactly coincides with \( x_\emptyset \).

**Proof.** Let \( \bar{x} \) denote the seat-share weighted average of all parties’ policy positions. There are two possibilities to consider, 1) \( \bar{x} > x_\emptyset \), and 2) \( \bar{x} < x_\emptyset \). In the first case the party with ideal point \( x_{\text{min}} \) will refuse to be a part of the coalition whereas in the second case the party with ideal point \( x_{\text{max}} \) will refuse to join the government. Hence the national government option, i.e., set \( C \) is not in \( Y \) and therefore not in \( W \cap Y \).

A couple of comments are in order. First, if \( x_\emptyset \notin [x_{\text{min}}, x_{\text{max}}] \), then one can get a national government even if \( P = 0 \). Consider the following example. Let the ideal points of the parties be 0, 0.5 and 0.5 + \( \varepsilon \). Let each party have an equal seat share and let the party with ideal point 0 be the formateur. Assume that \( x_\emptyset = 0.7 \). It is clear that if the party with ideal point 0 invites either of the parties with ideal points 0.5 or 0.5 + \( \varepsilon \) to form a coalition, then they refuse it as the outside option is closer to them. But if the party with ideal point 0 invites both of them together, then the implemented policy is 0.33 + \( \varepsilon/3 \) which both the invitees prefer over 0.7. Hence, the party with ideal point 0 would propose a national government. Second, note that for small amount of power the national government is still the equilibrium. Again, if the outside option were to be too unattractive (e.g. 0.8) then there would be no national government. Thus, we see that a national government can form for moderately unattractive outside option and low power.
This illustrates another point as well. Here, we have an example of a scenario where, by the usual definition, we have a surplus government but if we take equilibrium considerations the government is not surplus as the formateur with ideal point 0 can form a government only if both the parties are invited. Thus, we also provide a new explanation for why we may see coalitions which have additional members who may be left out without the government losing a majority. Also, if $P \neq 0$ then we can have national governments even when the status quo $x_0 \in [x_{\min}, x_{\max}]$. The intuition is similar in that if the formateur is an extreme party other parties agree to join the coalition only if they are invited together in the coalition. As an example let $x_0 = 0.1 = x_2$. Let $x_1 = 0$ and $x_3 = 0.8$. Let 3 have 40% of the seats and 1 and 2 have 30% each. It is easy to see that for moderate $P$ the coalition $\{1,2,3\}$ proposed by 1 belongs to $W \cap Y$ while the coalition $\{1,2\}$ or $\{1,3\}$ does not.

### 2.4.3 Political Power and Coalition structure

Now we turn our attention to another important question. What effect does the change in the amount of political power $P$ have on the outcome of the government formation game? An increase in $P$ makes it more lucrative for all the political parties to be a part of the government. Hence the set of coalitions that are feasible at a lower level of power are feasible at a higher level of power as well. Formally we have,

**Lemma 2.1** \( W \cap Y \) is weakly increasing in $P$, i.e. if $D \in W \cap Y$ for some $P$, then $D \in W \cap Y$ for all $P' > P$.

**Proof.** It suffices to show that both $W$ and $Y$ are non-decreasing in $P$.

First, let us consider $D \in W$. For any party $i$, such that $i \in A(D)$ but $i \notin D$, the payoff from voting for $D$ is unaffected by changes in $P$. However, for any party $i \in D \cap A(D)$, $v_i(D)$ is strictly increasing in $P$. This establishes that the number of parties voting for $D$ is weakly increasing in $P$.

Now consider $Y$. For any $i \in D(\neq \emptyset) \in Y$, $v_i(D)$ is strictly increasing in $P$. Moreover, for any coalition $D$, $v_i(D)$ is strictly increasing in $P$. Hence, the set of proto coalitions that are unanimously preferred by its constituents over the status quo cannot go down. □

The above lemma leads to a useful insight: an increase in political power gives the formateur a greater set of potential proto coalitions. Hence, the payoff of the formateur is higher with higher $P$, even after controlling for the direct effect of a greater $P$.

Given this lemma the following proposition is immediate.

**Proposition 2.4** There exists an upper bound to $P$ beyond which the equilibrium coalition $D_k$ for any formateur $k$ does not change with changes in $P$. Moreover, these ‘limiting coalitions’ cannot be surplus.
Proof. We know that \( v_i(D) \) is strictly increasing in \( P \). Moreover, \( v_i(D) \) is not bounded. Hence, for sufficiently large \( P \) we have \( v_i(D) > v_i(\emptyset) \) for all \( i \in D \). Hence, as \( P \) increases, any \( D \) belongs to \( Y \). We have also proved that \( W \) is non decreasing in \( P \). Given that \( v_i(D) \) is strictly increasing in \( P \) and not bounded this implies that beyond some \( P \) all majoritarian coalitions are in \( W \). Hence, beyond a critical \( P \) the set \( W \) does not change. This means that \( W \cap Y \) does not change. Thus, for every \( k \), \( D_k = \arg\max_{D \in W \cap Y} v_k(D) \) does not change.

It is easy to see that surplus coalitions cannot occur. Consider any surplus coalition and let \( D - i \) denote the coalition without member \( i \). The ideological loss in utility to the formateur \( k \) is \( u_k(|x_i - x_D|) - u(|x_i - x_{D-i}|) \). which is finite. The gain in power is \( \frac{1}{s_{D-i}} P - \frac{1}{s_D} P \). For sufficiently large \( P \) this gain is greater than the ideological loss.

However, at intermediate levels of power coalition structure is responsive to changes in \( P \). The following example illustrates this.

**Example 2.1** Let there be three parties with ideal points \( x_1 = 0, x_2 = 0.5, x_3 = 1, s_i = \frac{1}{3} \) for all \( i \) and \( x_\emptyset = 0.4 \). When \( P = 0 \) only caretaker or minority governments form. Let parties have quadratic loss functions viz \( u_i = -(x_i - x)^2 + P \). Beyond \( P = 0.02 \) the \( \{1, 3\} \) coalition forms whenever party 1 or 3 is the proposer. Beyond \( P = 0.025 \) the \( \{1, 2\} \) coalition forms when party 1 is the proposer and the \( \{1, 3\} \) coalition forms when party 3 is the proposer. Beyond \( P = 0.0525 \) ‘connected coalitions’ form each time an extreme party is the formateur.

An important issue to comment on is the relation between connectedness of coalitions and political power. Following Axelrod (earlier cit.) one would predict that at low power parties would form ideologically connected coalitions. However, that is not necessarily the case. We know for instance that minority governments are possible. Even when parties form coalitions the size of parties may cause the coalition to be disconnected even at zero power. The following example illustrates this.

**Example 2.2** There are three parties with ideal points 0, 0.80 and 1. The seat shares are respectively 0.4, 0.4 and 0.2. Suppose \( P=0 \). The status quo is 0.1. Let the party with ideal point 0 be the formateur. It is easy to see that the extreme party with ideal point 1 is chosen and the implemented policy is 0.33 which is better for party 1 than the connected coalition which would lead to 0.5 as the chosen policy.

It is of course true that after controlling for size disconnected coalitions occur only after a certain minimum value of \( P \). Moreover, when \( P \) becomes very large Riker’s prediction does not hold as minority governments can still occur. Riker’s minimal winning prediction only serves as an upper bound to the size of coalitions.
2.5 Party formation

We are now able to define the political equilibrium. We first define the entry stage equilibrium and then the political equilibrium.

**Definition 2.4** Entry-stage Equilibrium: A profile e of entry decisions constitutes an equilibrium if, for all $i \in C$, $V_i(C) - \delta > V_i(C')$, where $V_i(C)$ (respectively $V_i(C')$) is the expected utility of party $i$ from contesting (respectively not contesting) and the set of entrants is denoted by $C$ and $C' = C - i$.

**Definition 2.5** Political Equilibrium: A political equilibrium is a collection \{D^*, e^*\} where $D^* = (D_1, D_2, ..., D_N)$ is a collection of proto coalition decision functions, and $e^*$ is an entry-profile such that,

1. $\forall k \in C, D_k = \arg\max_{D \in W \cap Y} v_k(D)$ and
2. $e^*$ is an equilibrium of the entry game given the proto-coalition decision functions.

2.5.1 Results

Given these definitions we can now easily show existence.

**Proposition 2.5** A Political equilibrium exists.

**Proof.** The number of players ($1 \leq N < \infty$) and the strategy set is finite. Hence, the conditions for existence of a Nash equilibrium holds. In particular, the government formation subgame associated with each formateur also has an equilibrium as $D_k$ is well defined.

Now since the entry decision of each party is dependent on the decisions by other parties it is not very difficult to see that we get multiple equilibria. We shall demonstrate this by giving examples of such multiplicity. However, as our next proposition shows that the median group being the only group to form a party is always an equilibrium.

**Proposition 2.6** There exists a political equilibrium in which the median group stands uncontested and implements its ideal policy in parliament.

**Proof.** If the median group forms a party no group can get more than half the votes by standing on its own.

As the median party retains its absolute majority it still becomes the formateur and implements its ideal point. Thus any group $i$ by launching a party incurs a net cost since its change of utility from standing is $U(x_m) - U(x_m) - \delta = -\delta < 0$. Hence,
no unilateral deviation is profitable. Any group by forming a party only undergoes a cost. In\footnote{This is not robust in the sense that it depends on the simultaneity of the game. Note that this non-robustness is true for the citizen candidate model as well.} 

However, as the next example makes clear there exists parameter values for the status quo for which we have groups which cannot form a government stand so as to increase the chances of the status quo being implemented. These are akin to the ‘spoiler candidates’ in the citizen candidate model. This also allows us to demonstrate multiplicity. We shall have more to say about what the interpretation of the status quo is in the last section.

**Example 2.3** Suppose there are three groups with ideal points $x_i$, $i = 1, 2, 3$. Consider $x_2 < x_\phi < x_3$. There exists configurations in which party 3’s offer will never be accepted nor will it ever be offered by any other party to be part of the coalition for which 3 will still contest. To see this let $x_1 = 0, x_2 = 0.2, x_3 = 1, x_\phi = 0.4, s_i = \frac{1}{3}$ for all i. Let $P=0.1$. It is easy to see that both 1 and 2 prefer the status quo to joining a coalition with 3. However, by withdrawing 3 will give 2 more than half the votes and 2 will implement its ideal point. That gives 3 a utility of -0.64. By forming a party it gets an expected utility of -0.6. Hence, if $\delta < 0.04$, 3 will contest.

It is useful to compare this with plurality voting. In that case the party with ideal point $x_3$ will not contest and the unique equilibrium will be the median group with ideal point $x_2$. This example conforms to the popular notion that PR promotes more political entry while plurality voting leads to a tendency for the median group to get represented. Moreover, by increasing chances of the status quo PR may lead to a failure of successful coalition formation. It is also worth noting that unless groups are symmetrically placed we need voter uncertainty to generate 2 party rule seen in countries using plurality voting.

An interesting point made by Morelli (earlier cit.) is that in a multidistrict model if the population distribution across districts is sufficiently dissimilar Duverger’s predictions are reversed. We demonstrate that even in a one district scenario we can get more parties under plurality voting than under PR.

**Example 2.4** Consider three groups with ideal points 0, $\frac{1}{2}$ and 1. Let the status quo be located at 0.4. Assume that the group sizes are symmetric. Let $P = 0$. Under PR it is easy to see that (given some cost configurations) there is a unique equilibrium in which the median party with ideal point $\frac{1}{2}$ will stand. This is because the group with ideal point 1 can never be successful in forming a government nor does it prefer the status quo to the median party’s ideal point. Hence, it will not form a party. Clearly, all group members with ideal point 1 will then vote for $\frac{1}{2}$. Hence, the group with ideal point 0 will not find it worthwhile to form a party. However, under plurality the two extreme groups standing (and winning with equal probability) is an equilibrium. For higher $P$, three parties contesting (and winning with probability $\frac{1}{3}$ is also an equilibrium.
The intuition for this is that the further extreme party (with ideal point 1) prefers 2’s policy than a coalition of 1 and 2 and hence drops out to give 2 a majority. Given that, 1 also drops out as it has no chance of winning. Thus, while several formalizations of Duverger’s law relied on voters behaving strategically (see references earlier cit.), we have shown in both examples that even with sincere voting the intuition is not very different if there is strategic behavior on the part of parties.

2.5.2 A note on Multidimensional Analysis

In practice, parties care for more than one dimension. This becomes important if there is no unique dimension over which all parties care most. For instance, the Green Party may care more about the environment while a Conservative party may want to promote religious values and a centrist may care more about the economy. Hence, multidimensional analysis is important if we are to have a complete understanding of the political process. We shall now show how the rules governing coalitional policy making that we adopt can be reasonably redefined for multidimensional policy spaces. An interesting by-product of this analysis is that the ‘efficient’ outcome implies ‘log rolling’ among parties—hence, except for very special utility functions parties do not get to implement their ideal point in any dimension for all multiparty coalitions. We need to appropriately define certain terms in multidimensional space.

As there is more than one dimension we need to put relative weights to the different dimensions and specify the particular form the preferences take. In the extreme case where preferences are lexicographic the one dimensional analysis is sufficient for identifying voting behavior in a group. Otherwise we need a multidimensional preference function. For convenience, we will use a quadratic loss function, with different weights being attached to different policies and utility is a weighted sum of the various policies. Thus the utility of group $i$ if the implemented policy vector has elements $(x_1, x_2, ..., x_m)$ is given by $u_i = \sum w^i_j (x^i_j - x_j)^2$. Therefore, given a set of parties, a group votes for the party whose vector of ideal points minimizes its loss. Thus group $i$ chooses $\arg \min_{k \in C} (\sum w^i_j (x^i_j - x^k_j)^2)$. However, the weights being different for different groups bring up fresh problems as this implies that different players will have different perceptions of distance. Hence, party $i$ may believe that party $j$ is closer to it than party $k$ but party $j$ may believe that party $k$ is closer. This problem is of course an important one and Brams et al (2001) analyze this to show how even with single peaked preferences ‘disconnected’ coalitions form in a one dimensional policy space as different players have different perceptions about distance. As their policy space is uni dimensional they require more than 5 players to get ‘disconnected coalitions’—in multidimensional space using their methodology 3 is enough. However, we do not pursue this line of research here but assume a common set of weights which for simplicity we set to be equal for all dimensions. Hence group $i$ has a loss function given by $u_i = \sum (x_i - x)^2$ where $x$ is the implemented policy vector.

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Given this setup, we should be able to do a multidimensional analysis. Note that as in the unidimensional case the trade-off with power still remains and many of our propositions can be generalized. However, except under special circumstances there is no appropriate definition of a ‘median party’. The set of possible coalitions that can form in equilibrium given the random selection rule and the status quo vector can however be analyzed. It is interesting to note that a consequence of parties caring for more than one policy dimension is that the status quo may lie in a position which makes it impossible for any party to form a government. An example will clarify this point.

**Example 2.5** Consider 3 parties 1, 2, 3 with ideal points located on a 2 dimensional policy space with bliss points (0,0), (0,1) and (1,0) Let the status quo be given by \((0,\frac{1}{2})\). Let the party seats shares be 0.1, 0.45 and 0.45 respectively and let \(P=0\). Notice that no two-party coalition is possible. To see this notice that the 1,2 coalition leads to a position \((0,\frac{9}{11})\) which gives 2 lower utility than the status quo. Similarly, the 1,3 coalition gives \((\frac{9}{20},0)\) which is worse for 2. The 1,3 coalition gives \((\frac{9}{20},\frac{9}{20})\) which is worse for 2. Now the grand coalition gives \((\frac{9}{20},\frac{9}{20})\) which is worse that the status quo for all three. Notice that at low cost of entry all three parties entering to make sure that the status quo is maintained is an equilibrium.

At the same time the status quo may be such that more than one party can form a minority government.

## 2.6 Robustness: How critical are the assumptions?

In this section we study the robustness of the equilibrium to the assumptions we made about parliamentary rules (formateur selection procedure/ bargaining outcomes), voting behavior and inability of parties to commit to positions other that their ideal points. We will not deal with the last two issues except make brief remarks about each. However, we shall talk in some detail about two alternate ways to model legislative behavior which are common in the literature.

### 2.6.1 Majority Rule game

Instead of the formateur selection and coalition formation procedure suppose Parliament operated by voting on each issue by majority rule. In a single dimensional policy space this would lead to the Condorcet winner. If a single issue is what matters to the groups we get fairly sharp results for the whole political process. Consider the arbitrary \(N\) groups and assume a unique median exists. In that case we get the following results immediately.

**Proposition 2.7** If after elections the median of the candidates (representing different party positions) is implemented, at most two groups put up candidates in equilibrium.
Proof. First, note that more than two candidates standing cannot be an equilibrium. To see this note that if three (or more) candidates contest in the second stage there will be at least one candidate who will be non-pivotal i.e. whose dropping out will not affect the implemented policy. Hence, it is not optimal for that candidate to contest.

We now show that there can be zero, one or two candidate equilibria.

If cost are very high it is easy to see that no candidate will stand.

To get one candidate equilibria first note that there exists a \( c \) for which if the candidate with ideal point 0 stands, for all \( \epsilon, 1-\epsilon \) will not find it worthwhile to contest and win. This implies that \( u(-1+\epsilon) > P - c \). This implies a continuum of one candidate equilibria where any candidate can stand in equilibrium. However, as \( c \) decreases the range decreases i.e. the marginal candidate who can stand uncontested moves towards the median. Further, as \( c \) decreases we can get two candidate equilibria symmetrically around the median.\(^{53}\)

Notice, that this contrasts with Duverger’s hypothesis that PR leads to a multiparty (more than two) system. Moreover, this range around which symmetric 2 candidate equilibria can occur also keeps shrinking.

Proposition 2.8 As costs go to zero (and the median is unique) the unique equilibrium is for the median citizen to form a party.

Proof. We need to consider only one candidate or two candidate equilibria.

Consider a 1 candidate equilibrium with a group \( x_i \neq x_m \) where \( x_m \) is the ideal point of the median candidate. WLOG, let \( x_i < x_m \). Clearly, any \( j \) such that \( x_i < x_j \leq x_m \) can form a party and get more than half the votes. The net gain to group \( j \) is \( U(x_j) - U(x_i) + P - \delta > 0 \) when \( \delta \to 0 \).

Now consider 2 candidate equilibria. We already know that they must be symmetric around the median. Let us denote the utility to the median group in these symmetric equilibria by \( U(x_s) \). Since, the post election policy gets selected by majority rule, if the median group deviated and formed a party it will get its ideal point in stage 2. Hence, by deviating the median group gets \( U(x_m) - U(x_s) + P - \delta > 0 \) when \( \delta \to 0 \).

We already know that the median group being the only group to form a party is an equilibria. We have shown no other equilibria exists. Hence, as cost go to zero this is the unique equilibrium. \( \blacksquare \)

A comparison with plurality voting is quite interesting. For different cost levels we get one or two candidate equilibria as in the citizen candidate model with plurality voting. As cost decrease our prediction is extremely sharp under PR unlike plurality voting and it predicts a unique outcome. However, this seems hardly representative

\(^{53}\)We have assumed that the group sizes are the same for convenience and they are at the same distance from their neighbors, hence 2 candidate equilibria are possible. Otherwise, as we have shown before we need to introduce voter uncertainty to get 2 candidate equilibria.
of how Parliament works. In particular, even if this were taken to be a way to make decisions we run into problems if the policy space is multidimensional. Different results obtain depending on how voting on different issues take place.

2.6.2 Selection in order

Another rule which is sometimes seen in formateur selection (and mandated by law in Greece) is selection in order analyzed by Austen Smith and Banks (earlier cit.). We now look at ex post coalitions under the ‘Selection in order’ rule (Austen Smith and Banks Protocol). Briefly, this involves a fixed order of asking parties to be the formateur-starting with the largest (in terms of vote shares) and then if the largest fails to form a government the second largest and so on. If all parties fail a national government is formed and the policy implemented is a status quo policy which is implemented by a caretaker government which enjoys no power.

We now present some results which contrast with random selection.

**Proposition 2.9** If power and entry costs are low the unique equilibrium of the political game is for the median group to be the unique party to form.

**Proof.** We first show that if power is ‘low’ in the parliamentary game only the median party will be able to command a majority support. This is because any coalition will have an implemented policy $x_j \neq x_m$ where $x_m$ is the median party’s ideal point. Hence, a majority of members prefer $x_m$ to $x_j$. Hence, the optimal coalition when the median party proposes is for it to propose a coalition consisting only of itself which will be accepted. Therefore, the unique equilibrium of the parliamentary game is for $x_m$ to get implemented.

Clearly, if this is the outcome in the legislature no other group will launch a party in the party formation stage. ■

**Proposition 2.10** If the status quo policy $x_\phi$ is implemented by a caretaker government which enjoys no power, that government will not form in equilibrium.

**Proof.** We are required to show that at least 1 party can form a successful coalition when it is the formateur. Notice that as $x_\phi$ lies between $x_1$ and $x_n$ a coalition of the median party is preferred by a majority to the status quo. So there exists a feasible coalition which dominates the status quo. ■

We notice that this result contrasts with that under proportional selection. In fact, while caretaker governments are not unheard of it is usually the case that even though governments may not form at the first attempt it is almost always the case that some coalition comes to power. The one shot version of the random recognition protocol by cutting off the game in one stage does not allow for any other party to get a chance to propose leading to this ‘extreme’ situation. A further insight that we get is the following.
Corollary 2. An extreme party cannot form a minority government.

Proof. To see this notice that the middle party’s minority government is preferred by a majority of members. Thus there exists at least one coalition which Hence, parties commanding a majority of seats will not accept the proposal of a minority government by an extreme party. ■

A couple of remarks at this point are in order.

Remark 1. Minimal winning, minority and surplus governments are possible in equilibrium. Moreover, the coalitions may be connected or disconnected.

The trade-offs involved are similar to the proportional selection model.

Remark 2. The first party may not be able to form a coalition. Hence, delays may occur in equilibrium. However, it is worth noting that for every equilibrium involving delay there is an equilibrium without delay which leads to the same government.

Notice that the formateur may not be able to form a government including itself and hence the offer it makes to other parties to join a coalition with it will be turned down. However, an equivalent outcome can be achieved by the formateur proposing a coalition excluding itself which lies in $W \cap Y$.

Some points of difference are worth noting.

First, in the random recognition protocol we may get caretaker governments as well as minority governments. Selection in order never leads to a caretaker government in equilibrium. Moreover, only a median party can form a minority government and that too only when power is very low. At the empirical level, while selection in order is not borne out, it is worth investigating if the predictions of the one period random recognition model used in recent papers by Baron and Diermeier (2000) and Diermeier and Merlo (2001) which we have adopted here as well captures important features of the data. Clearly, finite periods of these protocols change the results but it is still not clear what institutional details correspond to this random recognition protocol. Thus, when the largest party is not selected we need to see if this is because a party other than the largest has indicated that it has the support of other parties which would enable it to form a government. Another important thing to look at is how well the ‘random recognition’ model fits the data after accounting for an incumbency bias i.e. where the last party in power is first asked to form the government. The ‘selection in order’ protocol is however something that can be observed and legislated on (as in Greece). However, there is certainly a lot to be said for this ‘random selection’ procedure in terms of capturing the inherent uncertainty that is associated with the political environment in government formation in most countries. Moreover, this random selection model of BDM we have analyzed under modified assumptions lead to fairly interesting results.

Clearly, there are issues which are important in the political process which we have left out. While there is evidence that voters may behave strategically there is
no compelling reason to choose one over the other. As long as representation per se is important there are less reasons for voters to behave strategically under PR than under plurality. There is enough evidence in the recent runoff in the French Presidential elections to believe that voters behaved sincerely. 54 Further work is certainly needed in this area. We have embedded a citizen candidate model in the institutional framework of coalition government formation. Thus, we do not allow parties to credibly commit to positions other than their own. Given that parties may have access to a credible commitment device (often repeated play ensures that, see Alesina (earlier cit.)) it would be interesting to see if this would lead to more divergence or more convergence of party policies. This remains a fascinating area of future research.

2.7 Empirical relevance and concluding remarks

We have presented a model of parliamentary democracy under PR which predicts political coalition formation as a function of party size and the relative importance of power to ideology. Moreover, by endogenizing the political entry stage we have shown how our legislature is consistent with a party formation game under the assumption of sincere voting. Our coalition and policy making stages in particular give rise to certain predictions which contrast with those existing in the literature. In particular, two limiting cases arise, one when parties care only for ideology and another where the rents of office become very large. However, these limiting cases do not give rise to the coalitions predicted by Axelrod (earlier cit.) or Riker (earlier cit.). Part of this comes from an explicit consideration of party size. Thus, when parties care only for ideology they may leave out an ideologically close partner because a large party can tilt the policy too close towards its ideal point because of its increased bargaining strength. Again as power becomes very large Riker’s ‘size principle’ does not apply because the institutional details of what happens if no party can form a protocoalition demand that some policy continues to be in effect under caretaker governments. Hence, if policy matters at all (even if lexicographically) and the inability to form coalitions leads to high but asymmetric costs to different partners, minority coalitions survive. However, Riker’s size principle applies as an upper bound on the size of coalitions that can be formed at high levels of power which is why surplus coalitions cannot occur beyond a certain threshold of power.

We can usefully compare our paper to the papers by Baron-Diermeier-Merlo (BDM) which use efficient bargaining and perfect commitment within the coalition. Given any coalition, our model predicts a policy orientation that is skewed towards larger parties while BDM predicts that party size will be irrelevant. This stems from the bargaining procedure in BDM giving equal weights to all parties. With different weights in proportion to size, our results will match. The more fundamental difference

54 Candidates to the left of Jospin received enough votes which, if combined, could have ensured Chirac’s defeat.
comes from the assumption of no commitment which leads to substantive differences in predictions for coalitions for a given a composition of the legislature. Empirically, we do not believe that perfect commitment or the converse are observed. However, our results are robust to some degree of commitment. We believe that apart from the usefulness of analyzing the polar opposite of BDM no commitment at the coalition formation stage is often a good approximation—this implies that the proposer cannot make a take it or leave it offer such that anything in the status quo’s majority win set would be accepted. This seems consistent with the observed phenomenon of power sharing that is seen in coalitions around the world. As we had pointed out earlier Laver and Schofield (earlier cit.) state that cabinet seats being allotted roughly in proportion to seat shares is one of the most observed empirical regularities of coalition governments. They also provide a discussion for why at the coalition formation stage the manifestos written are not binding and that actual policies are a result of intricate bargaining inside the coalition.\textsuperscript{55} Hence, both as an approximation to what happens in the real world and in terms of predictive power we argue that no commitment is not a bad assumption.

Another issue that we address, but which BDM cannot (because of their assumption of efficient bargaining) is the ideological connectedness of coalitions as functions of power. We are not aware of too many studies which look at this systematically. A paper by Indridason (2001) examines the nature of disconnected coalitions in the context of a few northern European countries and tentatively concludes that this is related to whether the office seeking model is more appropriate or whether the coalition is governed by ideological considerations. Our model makes a set of predictions which are more precise and can provide a useful basis for further case studies along these lines.

An extremely important issue is the formateur selection procedure. Diermeier and Merlo (1999) provide the first systematic empirical study but they study only the first stage i.e. they look at whether it is the case that the first party is always asked to form the government against the alternate that this is roughly in proportion to seat shares and find empirical support for the latter. However, some things are worth further investigation—when the party elected to be the formateur is not the largest it is worth looking at whether they have informally waived the right to form the government. Moreover, whether it is the case that the party selected to be the formateur has an informal pre electoral understanding with a set of parties which together can win a majority in parliament. Also, given an incumbency bias in selection we need to check how well the two alternatives perform after controlling for that. Another issue worth pursuing is to see how well proportional selection fits the data after controlling for the number of parties. These issues together with a more specific procedure incorporating institutional details of bargaining among parties inside a coalition will lead the way to a more complete understanding of formateur selection considerations as well as why delays in bargaining over government formation occur.

\textsuperscript{55}See also Laver and Shepsle (1995).
Appendix 2A: 2 dimensional characterization

Here we characterize the equilibrium coalitions under a symmetric 3 party case which would also facilitate comparison with Baron-Diermeier and Merlo (BDM)

Following BDM let the ideal points of party 1,2 and 3 be located on the vertices of an equilateral triangle and the coordinates are respectively \((0,0), (1,0)\) and \((\frac{1}{2}, \frac{\sqrt{3}}{2})\). To make things comparable we assume that \(s_i = \frac{1}{3}\) for all \(i\). We divide the regions by drawing lines from the party’s ideal points which cross at the centroid of the triangle (figure 1).

Diermeier and Merlo (2000) constrain the status quo \(x_\phi\) to lie in the set \((x_1, x_2, x_3)\). We first analyze this case. there are two cases to consider, one where \(x_i (i \in 1,2,3) = x_\phi\) and \(i\) is the formateur and another where \(j \neq i\) is the formateur. In the first case it is easy to see that \(i\) proposes a minority government including only itself and all parties support it as they are indifferent to the government’s policy and the status quo \(x_\phi\) which are the same. In the second case the proposer \(j\) also proposes a minority government and is supported by the party which is different to \(j\)’s policy and the status quo as they are equidistant. Hence, the results in the static version of their paper gives only minority governments under our assumptions.

Now, consider the more general case which Baron and Diermeier (2001) deal with and as a reference point consider one of the six partitions. It is easy to show that only minority or minimal winning governments can form. In each of these six regions (and assuming \(x_\phi \neq x_i\)) if the party closest to the status quo is the proposer a minimal winning coalition forms and the proposer is indifferent to the identity of the other coalition partner. Otherwise, the party closest to the status quo is left out at low power and the other two parties form a coalition. At high enough power any two parties can form a coalition as all parties are willing to be in a coalition rather than out of it. As the formateur is equidistant from the two parties, it is indifferent between the two parties it can invite. It is easy to show that the coalition can never be surplus as the policy from all 3 parties which is at the centroid of the triangle is worse for any 2 parties than a policy which is located at the midpoint of the line joining the 2 parties. Thus, we need more than 3 parties and more asymmetry in positions to get surplus governments or even the type of national governments which are not surplus in the sense we have discussed earlier. For outlying status quo, the same conclusion goes through-if the status quo is mildly outlying a minority or minimal winning coalition forms depending on the identity of the formateur and the distance of the status quo from the ideal points. Clearly, for a status quo which is very far away any formateur gets to form a minority government as that is preferred by all parties to the status quo i.e. \(u_i (|−1|) > u_i (|x_\phi|)\) for all \(i\). Note, this contrasts with Baron and Diermeier’s result as with a very outlying status quo they would have a national government as it would be profitable to extract transfers from all parties.
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3 When neighbours report crime: Effects of

3.1 Introduction

The purpose of this paper is to study the possible effects of giving incentives to people to report crime as well as how unpleasantness of police interrogation affects crime reporting. We study these using a model of crime where neighbours observe and can make reports about criminal activities and examine what effect incentives to reduce costs of reporting have on crime. Thus, crime in our framework is any undesirable activity which has the property that it particularly affects people in the immediate vicinity who are also the people most likely to observe it. In particular, we examine whether lowering the costs of reporting could actually increase criminal activities by thinning the resources of law enforcement over investigating greater number of potential criminal activities, thereby reducing conviction rates and hence expected penalties from committing crime. This has particular relevance in view of recent policy moves which encourage reporting by neighbours.\textsuperscript{56} We argue that apart from the associated welfare cost, this leads to a diversion of resources and may actually encourage criminal activities as the probability of being apprehended goes down with the law enforcement authorities having to divert costly resources to investigate the increased reports of suspicious activities. These counterproductive effects are strongest in the short run when we hold the resources allocated to investigation of criminal activities constant. In the long run, as police budget for the apprehension of criminals increases optimally, some of the resource thinning effect goes down but that may still cause an increase in crime.\textsuperscript{57} In fact, we argue that this call for neighbours to report each other’s suspicious activities will often induce reporting by criminals or people with dubious records to throw suspicion off them and lead to a chain reaction where costly law enforcement resources chase larger and larger groups of suspects, ending up in a lowered crackdown on actual criminal activities.

There is little documented evidence in the crime literature or crime statistics on the consequences on encouraging people to report each other. Widely practised in Soviet Union, East Germany and several erstwhile communist countries the secrecy surrounding their practises do not allow us to get a clear picture of how successful this was in curbing subversive activities. There is a flavour of what such reporting could

\textsuperscript{56} An example of that is the in the guidelines of Operation TIPS (Terrorism Information and Prevention System), promulgated by the US Attorney General Ashcroft, which incidentally never quite got off the ground.

\textsuperscript{57} There are of course associated general equilibrium effects as an increased spending on prevention of potential terrorist activities reduces the resources to spend on other criminal activities thereby encouraging such activities. As noted in a recent article in the New York Times ‘the war on terrorism is also depleting law enforcement resources’ and several states have seen an increase in crime rates. We do not consider these effects in our model as we wish to concentrate on the more striking issue that having to act on an increased number of potential threats following increased vigilance may not, even after a budget increase, decrease such activities. (The NYT June 7, 2003 ‘As Budgets Shrink, Cities See an Impact on Criminal Justice’ by Fox Butterfield.)
lead to during the McCarthy era where people were asked to report on their colleague’s political beliefs. The following incident serves to highlight the chain reactions of inducing people to report. ‘Larry Parks agreed to give evidence to the House of Un-American Activities Committee (HUAC)...insisted that Parks answered all the questions asked. The HUAC had a private session and two days later it was leaked to the newspapers that Parks had named names. Leo Townsend, Isobel Lennart, Roy Huggins, Richard Collins, Lee J. Cobb, Budd Schulberg and Elia Kazan, afraid they would go to prison, were also willing to name people who had been members of left-wing groups.’58 While law enforcement chased these people, it is quite plausible that people genuinely pursuing subversive activities were having a field day. Present day efforts to encourage anonymous ordinary citizens to alert the police to undesirable activities may have similar effects.59

The basic model examines a neighbourhood with a finite number of agents whose potential benefit from committing crime is modeled as a random variable. To make things stark we assume that only immediate neighbours can observe and report on crime. In the base model reporting is costless and no one can be punished for making false reports. The police force (which is not part of the neighbourhood) makes an initial investment in the total amount of budget (or what we shall loosely refer to as the effort expended) anticipating a certain crime level. This investment is costly on the part of the police and the incentive to invest comes from being rewarded for convicting criminals. Given the police budget, the value of the crime opportunity and the anticipated number of reports about criminal activities, people choose whether to commit crime or not and also whether to report on their neighbours. The police investigate all reports and we assume that the conviction probability is decreasing in the number of investigations (the ‘resource thinning effect’). Given that reports are costless, criminals tend to divert attention from themselves by reporting innocent neighbours (we call this the ‘red herring effect’) 60 and this may trigger off a chain reaction where innocent people end up reporting innocent neighbours to reduce their cost of being investigated by the police. Different equilibria emerge depending on how additional reports reduce the probability of conviction, the cost of being investigated

58See, for example the website below from which the lines are taken http://www.spartacus.schoolnet.co.uk/USAmccarthyism.htm
59A recent police investigation at Art Car Museum, an avant-garde gallery in Houston on the basis of a phone call is just one of the several red herrings that law enforcement is chasing in what many have dubbed as the New McCarthyism (see Matthew Rothschild’s articles in the Progressive http://www.progressive.org/0901/roth0102.html for several more illustrations about investigation based on reports and the enormous resources spent on what is dubbed as Ashcroft’s roundup) 60 That people of very dubious background do in fact report crime to divert attention from themselves has enough anecdotal evidence. The following quote from a recent NYT article gives us a flavour of what Ashcroft’s policy can lead to ‘Federal agents, facing intense pressure to avoid another terrorist attack, have acted on information from tipsters with questionable backgrounds and motives, touching off needless scares and upending the lives of innocent suspects’ (NYT, June 19, False Terrorism Tips to F.B.I. Uproot the Lives of Suspects, by Michael Moss)
and the value of having a criminal neighbour removed from the neighbourhood.

The simple model is made more realistic by introducing costs of reporting as well as errors in observation. Cost of reporting leads to a free rider problem, with each neighbour of a criminal preferring the other neighbour to report, thereby getting the benefit of the police investigating the criminal without incurring the costs of reporting. Multiple equilibria emerge, in particular, there is a mixed strategy equilibrium where each agent is indifferent to reporting and not reporting crime.

We next study the effect of providing incentives to report crime. While that leads to a higher number (in expected terms) of criminal activities being reported, the effect on crime rates is unclear. To see why this is so we note that increased reporting leads to two types of effects, it reduces the conviction probability for any given criminal and thereby lowers his threshold for committing crime and it may also induce a criminal to report his innocent neighbour (the ‘red herring effect’) to divert resources away from his crime. Both of these cause an increase in crime. However, increased reporting will cause the police to optimally increase effort which acts in the direction of reducing crime but the net effect may still see an increase in crime. Note this happens even when for a given crime rate, the apprehension probability increases with increased reporting.61 We then look at what happens when crime is observed with error. Two types of errors can occur, with some probability neighbours do not observe crime when crime is committed (the criminal hides his tracks) and with a certain probability neighbours observe what they think are criminal activities when no crime is committed (a false alarm is triggered). We derive several interesting comparative static results on crime and crime reporting in the presence of these errors. The message that we get is that the relation between crime and incentives from crime reporting is fairly complex and incentives for crime reporting may in fact lead to an increase in crime. Hence, the policy of encouraging increased vigilance on neighbours needs reexamination not just from the angle of intruding on the privacy of people but also in terms of its effectiveness in crime prevention.

The rest of the paper is as follows. In section 2 we discuss the related literature. Section 3 outlines the basic model. Section 4 extends the model to examine costly reports, section 5 to errors in detection. and Section 6 discusses various extensions and concludes.

3.2 Related literature

There is a large literature on crime and we will describe only the ones most relevant. There is also a growing body of work on neighbourhood effects. Of course, the neighbourhood effects are of interest beyond the issue of crime and crime policy.

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61 It is worth noting that this issue of providing optimal incentives for increasing the supply of a public good (in this case crime reporting) when there are feedback effects goes beyond the application in this particular model.
The seminal work on crime of course dates back to Becker (1968) who first analyzed crime as an economic decision. Since then there has been several papers extending Becker’s work and developing several aspects of crime and crime fighting policies. In Sah (1991) an explanation is provided as to why similar economies could reach different equilibrium crime rates with a fixed level of resources devoted to law enforcement. With very few criminals accuracy is very high and hence very few people want to be criminals, so we have a low crime equilibrium while with a lot of criminals accuracy is low which is why lots of people find it worthwhile to commit crime and hence we can have a high crime equilibrium. Unlike our model, police effort is not endogenous. Moreover, Sah does not have a neighbourhood structure and Sah’s model cannot be used to analyze the incentives of crime reporting on crime rates. Glaeser et al (1996) examine the issue of neighbourhood effects on crime. They make assumptions on the effect that neighbours have on behaviour, having criminal neighbours induce people to commit crime than having no criminal neighbours. It is interesting that we can get this particular result in our model without having to make assumptions on the way neighbours shape behaviour. The political economy issues in crime have also received some attention recently.

Imrohologru et al. (2000) examine a political economy model of crime where crime as well as police expenditure is chosen endogenously. The purpose of the paper is to look at issues of crime, income redistribution and police expenditures depending on the underlying population characteristics and the apprehension technology. They thereby discuss issues of correlation between crime, redistribution and police expenditures. The issue of errors in detection has been discussed in the context of crime by Benoit and Osborne (1995) as a possible explanation of how severity of punishment is tempered by the possibility of wrongful conviction (a similar idea, though in a different context, is discussed by Lagunoff (2001)). They go on to look at the role of redistribution and policing in crime reduction and look at how the political mechanism decides the shares of expenditure devoted to them. In terms of the broad debate on public vs. private enforcement of law (see the classification and discussion in Polinsky and Shavell (2000) and references therein), our model falls somewhere in between, while enforcement is by a public law enforcement authority, the problems of detection by the police of various criminal activity make them rely on reports by private citizens in selecting the sites to investigate.

We should mention that so far we have not come across any paper in the crime literature, which has this explicit neighbourhood structure nor do we find any work which looks at the issue of offering incentives to report crime on crime levels. However, in terms of modeling, mention may be made of some similarity that this paper has with auditing models (see for example Chatterjee et. al (2002)) and in terms of the two way feedback (i.e. crime affects police effort while police effort affects crime) with Bandyopadhyay (2000) where multiple equilibria is also generated in the context of an asymmetric information model of product quality and the volume of trade.
3.3 A simple model of crime reporting

This is a static model. There are a finite number $N \geq 3$ of people on a circle, agent $i$ receives a crime opportunity $x_i > 0$\(^{62}\), where $x_i$ denotes person $i$'s benefit from crime. Let $F(\cdot)$ denote the distribution of $x$.\(^{63}\) If a crime is committed it can be observed by the two neighbours on either side. There is a police force, which is not part of the population, whose job it is to investigate and apprehend criminals.

Sequence of actions:

Police choose an effort level $e \geq 0$. This is best interpreted as a budget as presumably the police can change effort level conditional on observing crime rate. Agents choose whether to commit crime simultaneously with the choice of $e$ by the police and make reports after having observed their neighbours’ actions. Note if $i$ commits a crime it can be observed only by $i - 1$ and $i + 1$ and not directly by the police. This is essentially a simplification, it embodies the idea that neighbours are better able to keep tabs on suspicious activities than the passing policeman on his or her beat.

Any person, regardless of whether a crime is committed or not can report to the police that a crime has been committed. However, we assume that the police can verify identity of the reporter but the reporter cannot be punished for false allegations.

The police investigate all reports and conditional on crime being committed the apprehension/conviction probability is declining in the number of spots reported i.e. if there are $k$ sites reported the conviction probability is inversely related to $k$ , in particular we shall make a simplifying assumption about the conviction (or apprehension) probability of the criminals and assume that it is given by $\frac{e}{k}$. Further by suitable choice of units $e$ is bounded above by $k$.\(^{64}\) Importantly, we do not assume any errors in conviction so if there are $k$ reports and no crime is committed at a spot, the conviction probability is zero. Moreover, we do not allow the police to condition their investigation on whether two reports or one report has been made about a site.

We make this more formal as follows

At time $t_0$, $x_i$s are realised i.i.d. according to cdf $F(\cdot)$. $x_i \in R_+$ is private information to agent $i$. At $t_1$ each agent chooses $\alpha_i \in \{0, 1\}$ where 1 (0) means commit (don’t commit) crime. Thus, each agent $i$ observes $I_i = (\alpha_{i-1}, \alpha_i, \alpha_{i+1})$. Police does not observe $\alpha_i$’s but choose effort $e$ which is not observed by the agents. At $t_2$ each agent chooses strategy $\beta_i \in \{0, 1\}^2$ where $(0, 0)$ means do not report either neighbour, $(0, 1)$ means report on $i + 1$ but not on $i - 1$ i.e. first entry in the vector

\(^{62}\)This implies of course that everyone is a potential criminal and is not entirely without loss of generality as presumably people could get negative payoffs from committing crime.

\(^{63}\)We also assume that each of the $x$’s are drawn from identical and independent distributions.

\(^{64}\)Note that this is different from having $k$ reports, if two people report the same site the conviction probability does not change.

Also, note that in any particular realization $\frac{e}{k} > 1$ is possible. Thus, the expression is more correctly written as $\min\left(1, \frac{e}{k}\right)$. We could alternately have a more general apprehension technology $g\left(\frac{e}{k}\right)$ constraining $g\left(\frac{e}{k}\right)$ to be between 0 and 1. Here, we will be working with this specific technology $\min\left(1, \frac{e}{k}\right)$. An alternate way to think about $e$ is that it represents the number of spots investigated in which case $e < k$ always holds.
shows report/not report on \(i-1\) and the second entry shows report/not report on \(i+1\). Hence, strategies for agents: \(\{\alpha_i : R_+ \rightarrow \{0,1\}, \beta : \{0,1\}^3 \rightarrow \{0,1\}^2\}\) where \(\alpha_i(\cdot)\) denotes the decision to commit crime as a function of \(x_i\) and \(\beta_i(\cdot)\) denotes crime reporting as a function of information \((I_i)\). The strategy for the police is simply a choice of \(\epsilon\).\(^{65}\)

After investigation payoffs are obtained as follows:

Let \(\beta_i^j\) denote \(i\)'s reporting strategy with respect to neighbor \(j\). Hence, \(\beta_i^{i+1} = 1\) means \(i+1\) reports \(i\) to the police.

Now suppose that \(i\) commit crime, then his expected net benefit is given by

\[
x_i = E_k[\max\{\beta_{i-1}^i, \beta_{i+1}^i\}(\frac{c}{k}\theta + \gamma(\frac{c}{k}))]
\]

A non-criminal’s payoff is

\[
-E_k[\max\{\beta_{i-1}^i, \beta_{i+1}^i\} \gamma(\frac{c}{k})]
\]

if there is no criminal neighbour,

\[
-E_k \left[ \max\{\beta_{i-1}^i, \beta_{i+1}^i\} \gamma(\frac{c}{k}) + 1 - \max\{\beta_{i-1}^i, \beta_{i-2}^i\}(1 - w)\frac{c}{k} \right]
\]

if there is one criminal neighbour and

\[
-E_k(\max\{\beta_{i-1}^i, \beta_{i+1}^i\} \gamma(\frac{c}{k}) + 2 - \max\{\beta_{i-1}^i, \beta_{i-2}^i\}(1 - w)\frac{c}{k} - \max\{\beta_{i+1}^i, \beta_{i+2}^i\}(1 - w)\frac{c}{k})
\]

if there are two criminal neighbours. Here \(-w\) is the payoff if a criminal is removed to the criminal’s neighbour. The intuition for this is that the criminal neighbour is replaced by the population average. In that case, a good approximation is to write the payoff in terms of the crime rate, so if \(u\) is the crime rate, the probability of a criminal neighbour replaced by another criminal is \(u\) and hence \(w = u\). It must be kept in mind that this is a static model so we do not really look at the dynamics of replacement. We merely assume that once a criminal is removed with a probability equal to \(u\) we get criminal behaviour again which is implicitly captured in the payoffs.

Police’s net payoff is

\[
E_k(\pi(u(\frac{c}{k})) - c(e))N
\]

where \(\pi\) is the reward per criminal convicted, \(c(e)\) is the cost of effort and is convex in \(e\). Essentially, for the police both crime rate and the number of reports are given (i.e. has expectations over the crime rate and reports) and he chooses the optimal \(e\). It is worth pointing out that this specification is not without loss of generality as

\(^{65}\)Note, this rules out agents being able to punish other agents for reporting them. In a repeated game framework, this would could lead to interesting equilibria where no one makes reports because the reporter is punished. Indeed, this may capture salient features of some crime prone neighbourhoods. However, we do not pursue this issue further in the paper.
presumably the police could get positive payoffs from having lower crime rates per se. From now on we shall drop the expectation term when writing the payoffs for notational simplicity.

**Equilibrium:**

We solve for the Nash equilibria of this game.

Hence, police maximizes \( \pi(u(e_k)N - c(e)) \), yielding \( (N\pi u)\frac{1}{k} - c'(e) = 0 \) which is non linear in \( e \).

This yields \( e^* = c^{-1}(\frac{N\pi u}{k}) \) which is as expected increasing in the number of criminals but decreasing in the number of reports.

While in its most general form we cannot get a closed form solution for this problem, several interesting properties about the equilibria of this model can be pinpointed. First, we note that there cannot exist a no crime equilibrium. To see this, note that if no one commits crime, rewards are zero, hence optimal effort is zero, however, at zero effort, everyone will commit crime. Also, there is a ‘contagion equilibrium’ in the sense that in equilibrium all agents make reports irrespective of whether they have committed crime. This follows from the fact that if everyone is reporting everyone else it makes sense for no agent to deviate as that would bring no benefit but increase his personal cost of being investigated by the police.

The following claims formalize this:

**Lemma 3.1** There cannot exist an equilibrium in which no one commits crime.

**Proof.** To see this note that if \( u \) (crime rate) is 0, then the reward is 0. Given \( c(e) > 0 \) for all \( e \), the optimal \( e = 0 \) However, at \( e = 0 \), \( u = 1 \). Hence, \( u = 0 \) cannot be an equilibrium. ■

**Proposition 3.1** There exists an equilibrium in which all agents report a crime, irrespective of whether a crime has been committed. \((p=1)\).

**Proof.** When all \( N \) report, a uniltateral deviation still causes all \( N \) spots to be reported. Hence, payoffs of both criminals and non criminals do not change. Thus, no profitable deviation exists. ■

We now look at the conditions under which this is unique. For this, we need two more lemmas.

**Lemma 3.2** In any symmetric equilibrium, a criminal always reports both neighbours and in turn is always reported on by both neighbours.

**Proof.** If a criminal is reported on and there are \( k-1 \) reported spots without the criminal reporting, his marginal gain from reporting is \( \gamma(\frac{e}{k-1}) - \gamma(\frac{e}{k}) + \frac{e}{k-1} - \frac{e}{k} > 0 \). For each neighbour, marginal gain from reporting is \( (1-u)(\frac{e}{k-2} - \frac{e}{k-1}) > 0 \) if the criminal does not report and \( (1-u)(\frac{e}{k-1} - \frac{e}{k}) + \gamma(\frac{e}{k-1}) - \gamma(\frac{e}{k}) > 0 \). Either way the neighbours report, in which case the best response is for the criminal to report. ■
Lemma 3.3  If a non-criminal agent surrounded by non-criminal neighbours is reported on, he always reports both neighbours. In other words, suppose agent $i$, $i+1$, and $i-1$ are not criminals and $i-1$ reports $i$, $i$ will report both $i-1$ and $i+1$.

Proof. In this case the only effect present is the change in the cost of being investigated. For any number (say $k$) reports, agent $i$’s incremental gain from reporting one neighbour is $\gamma\left(\frac{e}{k}\right) - \gamma\left(\frac{e}{k+1}\right) > 0$ and from reporting two neighbours is $\gamma\left(\frac{e}{k}\right) - \gamma\left(\frac{e}{k+2}\right) > \gamma\left(\frac{e}{k}\right) - \gamma\left(\frac{e}{k+1}\right)$. Hence, both neighbours are reported. ■

Proposition 3.2  If the marginal gains from being investigated less thoroughly always outweigh the marginal losses from a lower probability of removing a criminal neighbour i.e. $\gamma\left(\frac{e}{k}\right) - \gamma\left(\frac{e}{k-1}\right) > (\frac{e}{k} - \frac{e}{k-1})$, for all $k$, the equilibrium in proposition 1 is unique.

Proof. Suppose there is at least one criminal located in say spot $i$ in society. By Lemma 1 we know that this must be true. In that case, by lemma 2, the criminal $i$ will report his neighbours $i+1$, $i-1$. Now, both $i+1$, $i-1$ will report their neighbours $i+2$, $i-2$ to reduce $\gamma$ since from the condition given in the claim, $\gamma\left(\frac{e}{k}\right) - \gamma\left(\frac{e}{k-1}\right) > (\frac{e}{k} - \frac{e}{k-1})$ for all $i$. Now, if $i+2$ and $i-2$ have innocent neighbours they must report both neighbours by Lemma 3. If on the other hand they have a criminal neighbour, the condition of the proposition implies that they will report. Thus, we see that ultimately everyone ends up reporting their neighbour in equilibrium. Hence $k = N$ in equilibrium. ■

To see what the equilibrium crime levels look like we construct a simple example.

Example 3.1  Let the random variable $x$ be uniform on $(0,1)$. For simplicity, let $\gamma = 0$. Thus, for a penalty $\theta$, effort level $e$ and $k$ reports, $x^* = \theta \frac{e}{k}$. Thus, the expected crime rate $u$ is $1 - \theta \frac{e}{k}$. Let the police’s objective function be given by $E_k(\pi N \frac{e}{k} u) - e^2$. Plugging the expression for $u$ in the police’s objective function we get $e^* = \frac{2Nu}{k}$. Now, let us consider what happens when all agents report. In that case, $e^* = \pi u$. However, the expected value of $u$ is $1 - \frac{\theta e}{N}$. Substituting the value of $u$ in $e^* = \pi u$, we get $e^* = \pi(1 - \frac{\theta e}{N})$. Rearranging, we solve for the optimal $e$ to get $e^* = \frac{N\pi}{N+\pi\theta}$. Correspondingly, $u = \frac{N}{N+\pi\theta}$. As $N$ gets large this (crime rate) goes to unity. For other equilibria i.e. equilibria where only criminals report and are reported on, such closed from solutions are more difficult to get. The reason is that even for a given realizations of $\theta$’s the reporting rate for any particular reporting strategy will be influenced by the way the $\theta$’s are located. For example if there are five agents and two criminals, it matters if the two are neighbours or they are surrounded by innocent people. In general, we can say that if $n$ agents are criminals, the minimum number of reports are $\min (n+2, N)$ and the maximum number of reports are $\min (3n, N)$.

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66 This, in any case does not affect the equilibrium decision in the scenario where all people make reports.
Notice that all these examples presuppose a positive level of $e$, with myopic beliefs about the crime rate, it is possible to construct an equilibrium in which everyone commits crime, no one reports crime and the police do not expend any effort. The intuition for this is that if everyone commits crime, there is no benefit to replacing a criminal neighbour, assuming that people hold expectations about the crime rate remaining the same.

**Proposition 3.3** Suppose $w=u$, there exists an equilibrium such that $u=1$, $k=0$, $e=0$

**Proof.** Notice that with $u = 1$, gains from reporting is 0. Hence, starting from $k = 0$, no unilateral deviation is profitable, as no one reports, optimal $e = 0$.

It is worth remarking that such an equilibrium would not be robust if, for instance we allowed some random checks by the police. In this model police investigate crime only if reports are made. In a sense the reports play a procedural role here. However, even in situations in which crime is discovered by neighbours and investigated only if there is a report, one cannot realistically rule out random checks. If we use noisy signals so that everybody reports in a high crime noisy equilibria, we could have self fulfilling equilibria as well (report out of fear of the neighbour’s activities as well as get a red herring effect).

Note that noise can play a fairly substantive role and different results can be obtained depending on how we model noise.

Consider, for example, the scenario where only a fraction $\mu$ of crimes are noticed by neighbours. In that case, if a criminal reports his payoff (again noting that this is in expected terms) is $\frac{e}{k}$ (assuming $k - 1$ reports) and if he does not report, it is $\mu \frac{e}{k}$. Clearly, for large $k$ remaining silent is better if neighbours do not report when no crime is observed. One can argue that this will itself induce people to report, but with positive cost of reporting the story is not so compelling. In fact, this may hold even with no cost of reporting but a cost of investigation. Another important issue is what happens when a crime is not reported but the police investigate a report next to the location of crime. It is plausible to assume that the crime gets discovered with some probability. Alternately, we could allow the police to draw inferences about the person reporting (whether he is a criminal or non criminal). Either way, silent equilibria are possible.

In the next few sections we shall examine the following issues more closely. First, we consider the public good problem i.e. when reporting is costly and hence each

\[67 \text{To see this note that with random checks, there is an expected penalty } \theta \frac{e}{s}, \text{ where } s \text{ is the number of spots checked. Clearly, there are values of } \theta \text{ for which people with low realizations will not commit crime. In turn, with } u < 1, (1 - u) > 0, \text{ making it profitable for someone to report. Whether this causes a complete unraveling of course depends on whether the conditions in proposition 2 are satisfied.}

In fact for any mixed strategy by the police and all agents but } i \text{, it is not a best response for agent } i \text{ not to report. Similarly, with all agents (other than the police) mixing over their reporting strategy, } e = 0 \text{ is not, in general, not optimal. Hence, this equilibrium would not be trembling hand perfect.}
neighbour of a criminal would like to free ride on the other neighbour to make a report. Second, in this scenario we consider more and more realistic cases, where crime is observed with error and look at how they affect reporting, police effort and hence crime levels. Finally, in the conclusion, a few more extensions are also considered. For instance, should all reports be investigated. While a democratic state may require that the law enforcement authorities should take note of all citizen’s complaints, this may not be the optimal policy to follow if one would like to minimize crime. Again, we examine if a fixed cost in the police budget i.e. a certain fixed investment may deter crime which may cause the variable cost to be low. Lastly, we consider how with heterogeneity in tastes for minority races, as well as migration possibilities, segregated neighbourhoods may endogenously emerge.

3.4 The model with costly reporting

We now look at what happens with costly reporting. Of particular interest is whether there is a certain amount of ‘free riding’, if \( i \) commits a crime, \( i - 1 \) would prefer \( i + 1 \), i.e. the other neighbour to report and save on the cost of reporting. This could give rise to multiple equilibria including those involving mixed strategies. We first look at the mixed strategy equilibria. We denote by \( a \) the probability of reporting a criminal neighbour by a non criminal and by \( a' \) the probability of reporting a criminal neighbour by a criminal. Note that the reporting probabilities for the criminal and the non criminal on their criminal neighbour(s) will be different. This is because the criminal, by not reporting, faces a different payoff from the non criminal (in terms of being reported and apprehended).

To calculate the mixed strategy, we equate the payoffs from reporting and not reporting to get

\[
-c + \left( \frac{e}{k} \right)(-u) + (1 - \frac{e}{k})(-1) = a((\frac{e}{k})(-u) + (1 - \frac{e}{k})(-1)) + (1 - a)(-1)
\]

Solving for \( a \), gives \( a^* = 1 - \frac{ek}{(1-u)e} \), which is as expected i.e. increasing in police effort, decreasing in \( c \) and \( k \) (more reports exacerbate the underreporting for an individual as the additional gains are less). It is also decreasing in \( u \), as the higher the crime rate the lower are the gains from having a population average as opposed to a criminal neighbour. For this we of course need, \( \frac{ek}{(1-u)e} < 1 \). There are also coordination equilibria where one neighbour reports and the other does not report. The person who reports gets a payoff of \( -c + \left( \frac{e}{k} \right)(-u) + (1 - \frac{e}{k})(-1) \) and the one who does not report gets \( (\frac{e}{k})(-u) + (1 - \frac{e}{k})(-1) \). For these to be equilibria we need \( -c + \left( \frac{e}{k} \right)(-u) + (1 - \frac{e}{k})(-1) \geq -1 \) which on simplification yields \( -c + (1 - u)e \frac{e}{k} \geq 0 \) or \( (1 - u)e \frac{e}{k} \geq c \) which is quite intuitive, it says that the expected benefit from replacing a criminal (difference in payoffs from replacing a criminal with a random sample drawn from the population multiplied by the arrest probability) must be greater than the cost of reporting.

We also note that, in general, a criminal will have a higher probability of reporting a criminal neighbour. The following lemma proves it.
Lemma 3.4 In any mixed strategy equilibrium with costly reports, a criminal’s probability of reporting \((a')\) is greater than a non criminal’s probability of reporting \((a)\).

Proof. Notice, a criminal’s (incremental) payoff from reporting a criminal neighbour is 
\[-c + \left(\frac{c}{k}\right)(-u) + (1 - \frac{c}{k})(-1) + (\frac{c}{k - 1} - \frac{c}{k})\theta.\]
Equating this with the payoff from not reporting we get 
\[-c + \left(\frac{c}{k}\right)(-u) + (1 - \frac{c}{k})(-1) + (\frac{c}{k - 1} - \frac{c}{k})\theta = a'((\frac{c}{k})(-u) + (1 - \frac{c}{k})(-1) + (\frac{c}{k - 1} - \frac{c}{k})\theta) + (1 - a')(1).\]
Solving for \(a'\) we get 
\[a' = 1 - \frac{ck}{(1-u)(k+1)} + a'' > a'.\]

Given these conditions, we can characterize candidate equilibria in this environment. The equilibria involve the following actions by agents-non criminals are not reported on, criminals are either reported on by a neighbour on one side (co ordination equilibria) or the criminals’ neighbours randomize on reporting neighbours. In the latter case, i.e. in the mixed strategy equilibrium, criminals report other criminals with a higher probability.

Proposition 3.4 If \((1 - u)\frac{c}{k} \geq c\), \(\frac{ck}{(1-u)c} < 1\) and \(\frac{\theta e}{k(k+1)}(1 - (1-a')^2) < c\), there are multiple equilibria, one of which involves mixed strategies by neighbours of criminals, in all such equilibria a criminal never reports an innocent neighbour.

Proof. Notice that \((1 - u)\frac{c}{k} \geq c\) ensures that it is always worthwhile for a neighbour of a criminal to report a criminal if the criminal’s other neighbour has not reported. Hence, there are co ordination equilibria. Moreover, whenever this condition is satisfied and \(\frac{ck}{(1-u)c} < 1\), there are mixed strategy equilibria which generate the same payoffs to both neighbours and is equal to the reporting neighbour’s payoff in the co ordination equilibria.\(^{68}\) Lemma 4 ensures that the criminal’s randomizing strategy is also satisfied in the mixed strategy equilibrium (i.e. \(a' > 0\)) whenever \(\frac{ck}{(1-u)c} < 1\). Finally, \(\frac{\theta e}{k(k+1)}(1 - (1-a')^2) < c\) guarantees that a criminal never reports an innocent neighbour i.e. the red herring effect does not outweigh the costs of reporting.

As before, silent equilibrium can be sustained with costly reporting. However, this should be more robust.

Proposition 3.5 There exists an equilibrium with costly reporting such that everyone commits crime and no one reports with the optimal effort by the police being zero. Moreover, this is robust to small perturbations in the sense that with a small probability the police may randomly investigate a site.

Proof. The proof for the first part is as in Proposition 3. The robustness is shown as follows. Let the police expend effort \(\varepsilon\) and further assume that each of the

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\(^{68}\)Notice that because of the public good nature of the crime, the payoffs in the mixed strategy to each player are identical with what each player gets if he reports crime in the two co ordination equilibria.

67
$N$ spots are searched with a probability $\frac{1}{N}$. In that case, probability of conviction is $\varepsilon^N$ and the expected disutility is $-\theta \varepsilon^N$. In expected terms the crime rate $u < 1$ in that case. Moreover, assume that if anyone reports the police do not search randomly but investigate the report. However, $(1 - u)\varepsilon < c$ if $\varepsilon$ is close to 0. Hence, with costly reports the marginal benefit is too small and no one reports.

Now, we need to see what happens if costs fall for both effort level of the police and the level of crime.

To do this let us look at the cutoff (in a mixed strategy equilibrium):

$$x_i^* = (1 - (1 - a)^2)\theta e_k - \gamma(\varepsilon)$$

Thus, police solves $\pi(1 - u')\varepsilon - C(e)$, where $u'$ and $k'$ denote the new beliefs that the police have about the crime rate and reports.

The question is what is the effect on $u$, the crime rate in the economy when $a$ goes up. (because of decreasing cost of reporting). Notice that while a closed form solution is not possible, we see that $x_i^*$ can both increase as well as decrease with an increase in $a$ and $a'$. It will increase (therefore expected crime will go down) because each crime has a higher probability of being reported but as $e$ is decreasing in the number of reports, the net effect is ambiguous.

### 3.5 Noisy observation with and without costly reports

Criminal acts are rarely observed with precision. In particular, it is reasonable to assume that a neighbour observes a crime only with a probability $\mu$. It may be natural to expect that this would lead to a change in some of the results. For instance, if neighbours of criminals do not report, the criminal may prefer to keep silent in the hope that they will not be reported and that the crime will be undiscovered. With costless reports it turns out that this does not matter. In fact, both criminals and non criminals prefer to report. However, with costly reporting there is a difference. We calculate both the cases.

The probability that a crime is committed when no one observes it is given by

$$\frac{u(1 - \mu)}{(1 - u) + u(1 - \mu)} = \frac{u(1 - \mu)}{1 - u\mu}.$$ Note, it assumes that of all crimes committed a fraction $\mu$ is not observed (the criminal hides his tracks). If, instead we assumed that each neighbour has a probability $\mu$ of not observing the crime, the probability that crime is unobserved by anyone is \(\left\{\frac{u(1 - \mu)}{1 - u\mu}\right\}^2\). If reports are not costly, the unique equilibrium is for all agents to report. Clearly there is nothing to lose by reporting, the gain for a non criminal is \(\frac{u(1 - \mu)(1 - w)}{e} > 0\). With costly reports we are back to the scenario in the earlier section except that not reporting a neighbour when there is no crime is potentially costly. Equating marginal cost to marginal benefit we have $\frac{u(1 - \mu)(1 - w)}{e} > c$ for any neighbour to want to report. Note, we should have coordination equilibria as before as well as mixed strategy equilibria. There would be equilibria where the criminal does not report as well as one where he does report. Notice that the cutoff cost for reporting when there is no crime is positively related

\(^{69}\)In that sense, the equilibrium in the basic model where all persons report is more robust.
to the crime rate $u$ and negatively related to the accuracy with which a crime is observed. This makes intuitive sense, if there is more crime then the conditional probability of having a criminal is more for a given level of detection probability, hence the expected cost of not reporting is higher. Similarly, if criminal activities are observed with a great deal of accuracy, then for a given level of crime the conditional probability of having a criminal neighbour when no criminal activity is observed goes down and hence the expected benefit from making a report.

However, this is not the only type of imperfection in observation possible. A possible source of error is getting a signal about crime when in fact no crime has been committed. Let us denote by $\beta$ the probability that a neighbour observes crime when no crime has been committed. Hence, we can now compute the following probabilities: 1. probability of a crime being committed when there is no signal and 2. probability of no crime being committed when there is a signal. Hence, conditional on receiving a signal the probability of crime is $\frac{(1-u)\beta}{\beta(1-u)+\mu}$ (1-$u$). Now, the question is what is the effect of this kind of error on crime reporting and ultimately on crime rates. Clearly, what such errors do is reduce the gains from crime reporting. Hence, increases in false alarms reduce crime reporting incentives while increases in detecting crime (accuracy) would ceteris paribus cause people to report crime. What is interesting is that expectations about crime level increase reporting probabilities, however that itself (ceteris paribus) increases crime. The effect on police effort is more complex. If there are a large number of mistakes in reporting, the incentive of the police to invest decreases, hence increases in reporting as a result of noise may indeed cause increased crime. The following proposition formalizes this.

**Proposition 3.6** For any given distribution of crime opportunities, and cost of reporting, an increase in $\beta$ causes a lower level of optimal police effort, however the effect on expected crime is ambiguous.

**Proof.** With the possibility of a false alarm, the expected gain from reporting is $\frac{(1-u)\beta}{\beta(1-u)+\mu} (1-u)\mu$ which is decreasing in $\beta$. Hence, either no one will report as $\beta$ increases (this happen if $\frac{(1-u)\beta}{\beta(1-u)+\mu} (1-u)\mu < c$ as $\beta$ increases) or in a mixed strategy equilibrium the probability of reporting will decrease or in a pure strategy equilibrium, number of reports will stay unchanged. However, police effort will go down as the police now equate the marginal gains from apprehension which are now lower to marginal cost. Thus, police equate $\mu u (\frac{\beta}{\beta(1-u)+\mu} (1-u)\beta\mu)$ with $c(e)$. Since, $\frac{(1-u)\beta}{\beta(1-u)+\mu} < 1$, clearly optimal effort will be lower. Note, that the effect on crime is

70 Taking derivatives with respect to $u$ and $\mu$ yields $\frac{1-\mu}{1-\mu u} > 0$, and $\frac{\mu(1-u)}{(1-\mu u)^2} < 0$ respectively.

71 Polinsky and Shavell (1999) note that both type 1 and type 2 error have the effect of increasing crime. Thus, they conclude that only an increase in penalty will lead to the same level of deterrence. In our model, type 1 error encourages reporting which, ceteris paribus increases crime. At the same time it may reduce the police’s optimal $e$ as the gains from investigation go down. This may further increase crime. Type 2 errors however discourage reporting. Hence, we may have equilibria in which no one reports crime.
unclear as with a lower level of reporting, the apprehension probability (and hence, the expected disutility from crime) may go up. Notice, that in a pure strategy equilibrium, if the number of reports stay the same, crime will go up as less effort is expended over the same number of reports.

3.6 Extensions and conclusion

We have developed a simple model of a neighbourhood where neighbours can observe each others activities and tried to analyze the role of increased vigilance by neighbours on each others activities. In general, we see that increased vigilance may not reduce criminal activities. Thus, being your neighbours gate keeper may actually increase the likelihood of your neighbour misbehaving. Hence, policies that reduce the costs associated with reporting crime need to be carefully evaluated as it may simultaneously also end up encouraging crime.

Our model can be usefully extended in several directions. One extension which is immediate is to have only the neighbours observe criminal activities but to allow the criminal activities to impact the entire neighbourhood. This will make it even more relevant to analyzing issues of terrorist activities which typically affect a larger section of the population than just the terrorists’ neighbours. In terms of the results in this model it gives another channel for the so called contagion effect to stop—any agent who is innocent and reported on may not want to report an innocent neighbour as it may decrease the apprehension probability of criminals who are not his immediate neighbours but whose actions affect his payoffs. Hence, Lemma 3 need not hold i.e. a pool of innocent people do not necessarily report each other if they have been reported on. Moreover, this would make the actual distribution of the criminals in the neighbourhood less relevant as the total number of criminals is what matters in determining the payoffs.

An important issue we have ignored here is the optimality of the investigating mechanism. While many democracies require that the complaints of all citizens be investigated, clearly this may not be the optimal crime reducing policy. Thus investigating only a certain number of reported crimes may actually increase apprehension rates and hence reduce crime than if all reports were investigated. It is worth looking at whether one should investigate a spot only if both neighbours report and investigate the reporter if only one report is made about a spot. This may have the effect of reducing the ‘free rider’ problem while also preventing the ‘red herring’ effect as a criminal knows that if he is alone in reporting a neighbour he will end up being investigated by the police. Notice, the no report equilibria where everyone commits crime becomes more robust in such a scenario as one person deviating (by reporting) does not induce police effort.

Another promising area deals with endogenizing the choice of neighbourhood by agents in a multiperiod framework. It would be useful to see if this can lead to initial shocks (realization of crime opportunities) determining the long run distribution of
crime in two otherwise identical neighbourhoods. It is also worth looking at how the particular realizations of crime opportunities within a neighbourhood in terms of location matter in determining the crime level. For instance, if the high realizations of crime opportunities occur in locations close to each other rather than in very dispersed locations would the crime rate change. Moreover, would rational discrimination in effort expended by the police in different neighbourhoods on the basis of past crime rates actually cause the crime rates in each neighbourhood to perpetuate? These dynamic issues remain fascinating areas for future research.
References:


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Appendix 3A: Important Illustrations

Seller’s curve

Fig 1     \( q_h \)    \( q_l \)    \( q \)
Fig 2

Fig 3

Equilibrium
Triangle

\[ \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) \]
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